*Green*back?
Sustainable Investment and the International Transmission of Monetary Policy

Marco Garofalo*

This draft: March 20, 2025

Abstract

In this paper I assemble a novel granular dataset to study the international transmission of US monetary policy to green and brown firms. I document two new facts: foreign brown companies experience a starker reduction in (i) their stock prices and (ii) their investment than their green counterparties in response to Fed monetary tightening. To rationalise these findings, I build a novel two-country model with two sectors (green and brown) and international intermediaries subject to financial frictions. Following a domestic monetary policy hike, investors reallocate their portfolios away from foreign assets into higher-yielding domestic assets; however, this capital outflow is significantly attenuated for foreign green positions, when lenders exhibit a bias for sustainable investing. I take the model to the data: my empirical analyses of investor-level cross-border flows from US mutual funds supports this attenuation mechanism. A quantitative simulation exercise suggests that this "green bias" might have contributed to the observed cross-country resilience to the latest US tightening cycle, and that greener economies stand to lose a valuable buffer from the current reconsideration of sustainable investing in the US.

JEL Codes: E52, F34, F41, F42, G12, G14, G15, G30
Key Words: Climate, monetary policy, international spillovers, firm-level data, cross-border lending, international fund flows.

*University of Oxford & Bank of England & CfM, email: marco.garofalo@economics.ox.ac.uk.
I am grateful to Ozge Akinci, Martin Bodenstein, Davide Brignone, Matteo Cacciatore, Ambrogio Cesa-Bianchi, Francesca Diluiso, Aydan Dogan, Sarah Duffy, Sergio de Ferra, Andrea Ferrero, Chris Hyland, Alba Patozi, Federica Romei, Sanjay Singh and Francesco Zanetti for their valuable comments and suggestions. We would also like to thank for helpful discussions the participants to the Macro-group seminars at University of Oxford. All errors and omissions remain my own.
The views expressed in this paper are those of the author and not necessarily those of the Bank of England and its committees.

1 Introduction

In recent years a burgeoning literature has emerged, exploring the interplay between issues related to climate change and monetary policy (Abiry et al., 2022; Ferrari & Nispi Landi, 2020; Ferrari Minesso & Pagliari, 2023; Giovanardi et al., 2023; Macaire & Naef, 2021; Mongelli et al., 2022; Muller, 2021). Whether central banks are assigned an explicit climate mandate, their policies can nonetheless affect green and brown firms unevenly, and thus impact countries' efforts towards greening their economies. For instance, Patozi (2023), Benchora et al. (2023) and Döttling and Lam (2023) present evidence that green US firms are less sensitive to Fed's monetary policy than brown firms. In Europe, Bauer et al. (2024) similarly find that brown firms are more responsive to ECB policies, while Fornari and Groß (2024) find the opposite, i.e. a relatively higher sensitivity for green firms. Papoutsi et al. (2022) find that the ECB's corporate bond holdings are skewed more heavily towards high-emission European sectors compared to a market portfolio of sectoral capital stocks. While this literature has so far concentrated on *domestic* monetary policy within the US and the Euro Area, little is known about the relationship between firms' greenness and *international* monetary spillovers. Do brown and green firms respond heterogeneously to foreign monetary policy? And, if so, what drives these differences and do they matter? As the global climate transition is widely under way, my paper sheds light on these pressing questions, contributing to the ongoing debate in this literature.

In this paper I construct a novel granular dataset to study the heterogeneous spillovers on foreign brown and green firms of US monetary policy interventions. I document two new facts. First, non-US brown businesses see larger declines in their stock prices than green listed companies do following a US monetary tightening. Second, foreign brown firms also decrease their investment by more than green ones do in response to the same shock. These results are in line with findings on domestic US firms in most of the closed-economy literature (Benchora et al., 2023; Döttling & Lam, 2023; Havrylchyk & Pourabbasvafa, 2025; Patozi, 2023). In order to gain insights on what might be driving my findings, I then build a two-country model where each open economy has two sectors, i.e. green and brown. Crucially, in order for my model to match those new facts, my framework also includes international intermediaries that are subject to financial frictions, constraining their ability to accumulate both green and brown assets, but to a different extent. Specifically, a bias towards green assets leads to a brown premium in steady state, and an attenuation of portfolio reallocations away from foreign green assets in favour of higher-yielding domestic assets following a domestic monetary policy tightening. To validate this key mechanism of the model, I conducted empirical analyses using investor-level cross-border flows from US mutual funds, which support an attenuation effect on cross-border outflows from a green bias. Finally, I use my model to run a quantitative simulation exercise, whose results suggest that this "green bias" might have contributed to the observed cross-country resilience to the latest US tightening cycle, and that greener economies stand to lose a valuable buffer from the current reconsideration of sustainable investing in the US. This highlights the crucial role of sustainable investing in shaping international capital flows and mitigating the impacts of monetary policy changes.

As described above, the existing literature does not appear to have reached a consensus yet on what the relative sensitivity of brown and green firms to central banks' interventions in

a domestic setting is, and thus it is unable to provide a widely agreed prior hypothesis when considering the international transmission of monetary policy. On the one hand, several studies have uncovered large heterogeneity in the sign and magnitude of firm-level responses to *domestic* policy driven by businesses' features such as size (Gertler & Gilchrist, 1994), age (Cloyne et al., 2019), liquidity holdings (Jeenas, 2018), default risk (Ottonello & Winberry, 2020) and excess bond premia (Ferreira et al., 2023), as well as to *foreign* shocks whose effects might be modulated through corporate involvement in international trade and financing (Adams & Verdelhan, 2022; De Leo et al., 2023; di Giovanni & Rogers, 2022; Di Giovanni & Hale, 2022; Gopinath & Itskhoki, 2022; Hegarty et al., 2023; Kalemli-Ozcan, 2019; Miranda-Agrippino & Nenova, 2022; Miranda-Agrippino & Rey, 2020). If any of these corporate features were clearly correlated with green or brown status, I could use the existing body of literature to formulate an expected relationship between international spillovers and firms' greenness. However, my novel dataset of non-US listed firms constructed through several sources and spanning 38 countries (including 21 advanced and 17 emerging economies) over the selected sample period from 2010 to 2019, does not suggest substantial heterogeneity along these dimensions: firms directly and indirectly producing fewer GHG emissions as a share of firms' sales — a common definition of corporate "greenness" in the literature — are no more likely than brown ones to be significantly younger, larger, with more liquidity, riskier, selling or investing more, or relying relatively more on US Dollar funding.

Thus, without an established result from the literature to guide my analysis, I must begin with documenting novel facts on the international transmission of monetary policy spillovers. I investigate heterogeneous responsiveness between brown and green firms selecting two firm-level outcomes commonly considered in the literature on the transmission of monetary policy: stock prices and investment.

Using a procedure commonly employed in the monetary policy literature (Anderson & Cesa-Bianchi, 2024), I combine daily stock returns at Fed's policy announcement days with high-frequency policy surprises from Jarociński and Karadi (2020). Specifically, using these series as proxies for monetary policy shocks, I run panel local projections (Jordà, 2005) for the cumulative changes in equity prices and find a pronounced heterogeneity in the responses, with brown firms exhibiting greater sensitivity to these shocks compared to their green counterparts. This disparity persists when I consider quarterly investment as firm-level real outcome variable, underscoring that the significance of carbon intensity as a critical factor in determining firms' reaction to monetary policy changes is not limited to asset prices. My results control for known determinants of firm-level heterogeneous response to monetary policy, such as firm size, age, profits, leverage and default risk, which help isolate the effect of carbon intensity. Importantly, these findings hold even when controlling for the possibility that brown firms import disproportionally more production inputs in US Dollar, and in turn debt denomination (Gopinath & Itskhoki, 2022; Mukhin, 2022). To address further these potential sources of confounding factors I include firm and country-sector-time fixed effects in all specifications.

As my second main contribution, I rationalise these novel findings developing a novel two-country model featuring two distinct sectors – green and brown – alongside international intermediaries, and exhibiting two forms of financial frictions. First, green and brown firms in

each economy need to issue bonds to finance part of their capital expenditure. Second, financial intermediaries fund themselves through domestic deposits in order to invest into domestic central bank's reserves, domestic and foreign corporate bonds. Intermediaries are limited in their ability to hold assets by a leverage constraint linked to their net worth. Crucially, they exhibit a bias towards green holdings, no matter their source. As a result, in steady state the model yields excess returns of brown over green bonds, matching what commonly found in the literature to be the "carbon premium".

These parsimonious yet flexible set-up is enough to deliver my new documented facts. Following a domestic monetary policy hike, investors reallocate their portfolios away from foreign assets in favour of higher-yielding domestic alternatives. However, returns on green foreign assets decrease by less than brown returns in a counterfactual where investors have no bias towards green assets. Similarly, comparing the model results with and without this bias for sustainable investment for green and brown foreign investment, my framework is able to deliver the observed heterogeneous response from my second fact.

Key to matching these new facts is the interaction between the bias towards (international) green holdings and the portfolio reallocation mechanism triggered by a domestic monetary tightening. As the latter pulls more inflows into the domestic economy, the outflows away from foreign green assets are attenuated with respect to foreign brown assets as a result of investors' bias for sustainable investment.

Furthermore, my chosen option for modelling the bias towards sustainable investment captures in a simple and parsimonious way several micro-foundations suggested by the literature. For instance, households might enjoy non-pecuniary utility from green investing; investors could or might be forced by regulation to consider green businesses less exposed to transition risk (Bauer et al., 2024; Benchora et al., 2023; Pástor et al., 2022; Patozi, 2023). Ultimately, what is crucial is that any of these sources of heterogeneity between green and brown investment leads to a green bias towards green assets, in turn generating an attenuation effect of outflows from foreign economies caused by monetary policy tightening in the domestic economy.

Given the centrality of the portfolio reallocation mechanism I introduced in my model to match my new facts, I take my framework to the data by testing this crucial attenuation effect for international outflows following a US monetary policy shock. I use granular data from US mutual funds cross-border flows and holdings to estimate the heterogeneous response of capital flows to US monetary policy shocks, focusing on funds with varying degrees of sustainability preferences. Specifically, I follow the related closed-economy literature (Patozi, 2023) and compare the responses of capital flows for funds with and without an ESG mandate. The results, controlled for various fund characteristics and fixed effects, reveal that ESG-focused funds adjust their cross-border positions by less in response to US monetary policy. This resilience to international monetary spillovers is particularly evident in equity and index funds, where ESG mandates significantly mitigate the impact of Fed shocks on capital flows. These findings underscore the role of sustainable investment preferences in attenuating the effects of monetary policy on international capital flows.

Finally, I investigate the role played by the attenuation mechanism of the international monetary transmission I have uncovered during the latest Fed tightening cycle. Specifically, I

simulate my model to yield a similar global outlook to what observed for the US and the rest of the world post Covid pandemic. This period has been particularly characterised by surging inflation in most economies, including the US, where the Fed has raised interest rates in order to quell price pressures. As a result of tighter financial conditions in the US, foreign economies where expected to experience capital outflows from international investors searching for higher US yields.

While other factors, such as local central banks also raising rates, might have contributed to the overall unexpected resilience, my simulation shows that the green attenuation channel played a sizeable role. Indeed, I compare to my baseline simulation one where I assume no green bias in international financing. Without green bias international capital outflows would have been half as much as the model's predicted change. The more economies have "gone green", the larger the shielding effect that green bias might have granted them. Importantly, this means that greener economies stand to lose a valuable buffer from the current reconsideration of sustainable investing in the US. This simulation exercise highlights the crucial role of sustainable investing in shaping international capital flows and mitigating the impacts of monetary policy changes.

Going forward, there are several avenues for future research. For instance, my analysis purposely employs a parsimonious approach to modelling bias towards green investment, encompassing different potential micro-foundations. However, the source of this bias might be crucial post-climate transition: if preference-driven, the attenuation effect brought about by this bias may persist; but if based on transition risk, it may fade. Through these mechanisms, the anti-climate stance of the second Trump administration could also influence the detected attenuation effect of international US monetary spillovers. Regarding policy implications, brown firms might be facing a "double whammy" of higher interest rates and disproportionately drying international funding. For a policymaker this is, of course, a first-order issue given that brown firms are precisely the ones that need more support to catch up and accelerate their climate transition.

Related literature With this work I build on and contribute to two main strands of the literature.

First, as already mentioned above, a burgeoning literature is emerging on the relationship between monetary policy and climate change (Abiry et al., 2022; Ferrari & Nispi Landi, 2020; Ferrari Minesso & Pagliari, 2023; Giovanardi et al., 2023; Macaire & Naef, 2021; Mongelli et al., 2022; Muller, 2021; Papoutsi et al., 2022), with a particular focus on the heterogenous effects of monetary policy on green and brown firms (Bauer et al., 2022; Benchora et al., 2023; Döttling & Ratnovski, 2021; Fornari & Groß, 2024; Havrylchyk & Pourabbasvafa, 2025; Patozi, 2023). My paper extends existing studies to the international dimension, considering how green and brown firms might react heterogeneously to cross-border spillovers from US monetary policy. It concludes that the differential responsiveness between green and brown firms, so far documented by several studies only within closed-economy setting, also carries out when considering the international transmission of monetary policy through cross-border financial channels.

Second, my work is closely related to the strand of the literature in international macroeconomics and finance which investigates international spillovers of monetary policy. The seminal

work by Rey (2015) found that economies which open to international capital flows are inevitably exposed to foreign developments, even if they let their currencies' exchange rate float. Fed's monetary contractions are particularly powerful given the centrality of the US in the international monetary and financial system: they induce comovements in the international financial variables, a truly Global Financial Cycle, in turn leading to retrenchments of international credit flows (Miranda-Agrippino & Nenova, 2022; Miranda-Agrippino & Rey, 2020). Against this backdrop, a literature has developed examining the heterogenous effects of international spillovers of foreign shock and, in particular, US monetary policy on firms, highlighting transmission mechanisms through international trade and financial linkages (di Giovanni & Rogers, 2022; Gopinath & Itskhoki, 2022; Hegarty et al., 2023; Kalemli-Ozcan, 2019). Within this tradition, my work is the first, to my knowledge, to uncover how green bias in international investment provides economies with some extent of shielding against adverse foreign shocks causing capital outflows.

The remainder of this paper is organised as follows. Section 2 sets the scene drawing prior expectations from the relevant literature, describes the datasets I employ in my analysis, and documents two new facts for non-US brown and green companies. In Section 3 I layout my open-economy model to reconcile these facts and draw implications, which I then proceed to test through my empirical analysis of mutual funds flows and holdings in Section 4. In Section 5, I describe my simulation exercise. Finally, I conclude in Section 6 suggesting further avenues for future research. I relegate all additional details to the Appendix.

2 New facts on the international transmission of US monetary policy

The current literature lacks a clear consensus on the relative sensitivity of brown and green firms to central bank interventions within the *domestic* economy. This gap makes it challenging to formulate a definitive hypothesis regarding the *international* transmission of monetary policy. Previous studies have identified significant variability in how firms respond to domestic policy and foreign shocks, influenced by factors such as firm size (Gertler & Gilchrist, 1994), age (Cloyne et al., 2019), liquidity holdings (Jeenas, 2018), default risk (Ottonello & Winberry, 2020), and excess bond premia (Ferreira et al., 2023). These responses can also be shaped by a firm's involvement in international trade and finance (Adams & Verdelhan, 2022; De Leo et al., 2023; di Giovanni & Rogers, 2022; Di Giovanni & Hale, 2022; Gopinath & Itskhoki, 2022; Hegarty et al., 2023; Kalemli-Ozcan, 2019; Miranda-Agrippino & Nenova, 2022; Miranda-Agrippino & Rey, 2020).

If there was a clear correlation between these firm-level characteristics and their green or brown status, the existing literature could provide me with a basis for expecting a specific relationship between international spillovers and a firm's greenness.

In the remaining of this section I thus first detail the granular datasets I employ in my analysis. My constructed firm-level dataset, which includes non-US listed firms across 38 countries (21 advanced and 17 emerging economies) from 2010 to 2019, does not seem to indicate substantial differences along these dimensions. Given the absence of established findings to guide

the analysis, I will then proceed with documenting two novel facts for brown and green firms outside the US, which will motivate my quantitative framework and empirical investigation (Section 3 and 4, respectively).

2.1 Data

I use two main types of data to put together novel facts on the international transmission of monetary policy. The first is announcement dates and proxy measures of monetary policy shocks for the Fed.[1] I consider shocks series calculated following state-of-the-art high-frequency identification. The second is firm-level data on real and nominal variables, such as GHG emissions and investment, as well as stock prices.

2.1.1 Monetary policy surprises

In my analysis I employ shocks series from monetary policy surprises for the Fed from Jarociński and Karadi (2020). The authors use high-frequency changes in short-term interest rate derivatives prices around FOMC policy announcements to isolate unexpected shocks to monetary policy. They then use information from equity prices to purge these shocks from the effects of information about the economic outlook that is released alongside the policy announcements (Miranda-Agrippino & Ricco, 2021).

A few technical adjustments are worth mentioning here. First, I normalise these series so that they can be interpreted in terms of basis points changes of the selected rate of reference for my analysis: these are the daily yields for the one-year constant-maturity Treasury for the US (Figure 1).[2] Finally, for my analysis at lower frequencies, I aggregate up the daily shocks into quarterly series using a simple sum.

2.1.2 Firm-level information

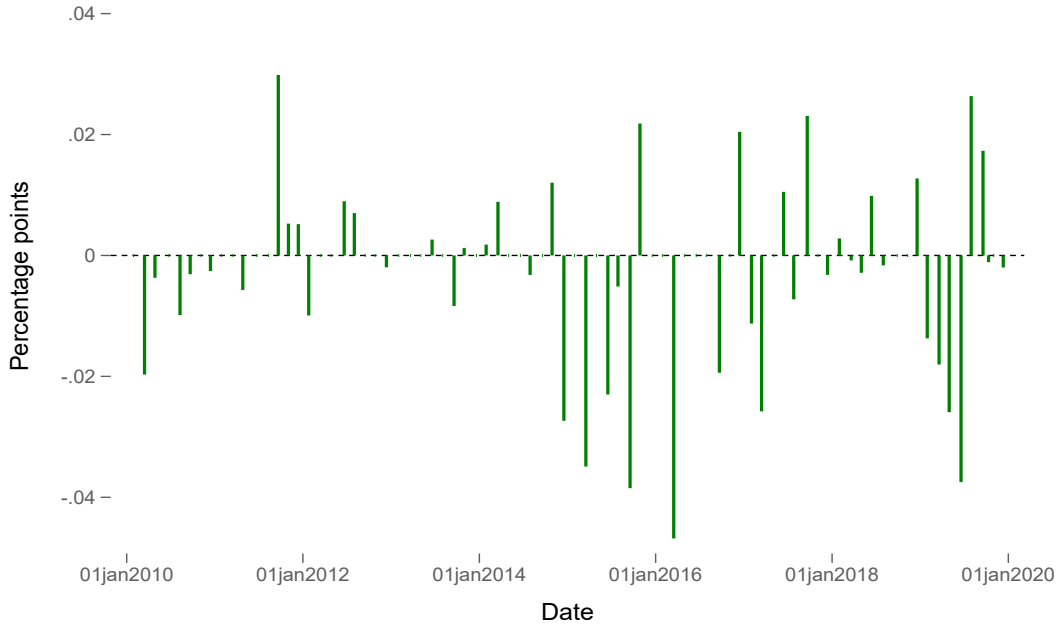
I construct a novel dataset linking together information on firm-level emissions, stock markets and balance sheets. I consider only observations dated from 2010 and before 2020, in order to ensure that my results are not driven by the Global Financial Crisis or the Covid pandemic. As highlighted by Döttling and Lam (2023), this is also a period of relatively greater climate change concerns and better emissions data coverage, making it an ideal time span for my analysis. I retain publicly listed companies. I exclude all those that are not operating, with country of incorporation or address being the USA, or a tax haven according to Hines (2010), whose country of the headquarters location and country of incorporation are not the same, and for which the currency of document is the domestic currency of the country in which the firm is located and incorporated (Adams & Verdelhan, 2022).

In addition, I carry out a series of adjustments common in the corporate finance literature, in particular studies using firm-level information to investigate issues in international macroeco-

[1] In Appendix **??** I carry out the same analyses also for the ECB using similarly constructed Euro-Area monetary policy shocks from Jarociński and Karadi (2020).

[2] I use the German one-year government bond yield for the EA. As noted by Jarociński and Karadi (2020) and Gertler and Karadi (2015), using rates with longer maturity than the central bank policy rates has the advantage of incorporating the impact of unconventional monetary policy tools, thus ensuring those rates remain valid measures of policy stance also during the ZLB period.

Figure 1: Monetary policy surprises on Fed announcement dates



Notes: Monetary policy shocks for the period 2010 to 2019, normalised by the market yield on US Treasury securities at 1-year constant maturity. The shock series is based on the "Poor Man's" high frequency monetary policy surprises robust to information effects from Jarociński and Karadi (2020).

nomics and finance (Adams & Verdelhan, 2022; Hegarty et al., 2023). I exclude firms in utilities, finance and insurance, real estate and public sectors[3], which are likely to be too idiosyncratic to be treated in this general framework of analysis.

Emissions I source corporate carbon emissions data from Trucost. Trucost's Environment dataset provides annual global greenhouse gas (GHG) emissions data for around 15,000 of the world's largest listed companies, which account for 95% of global market capitalization.[4]

I drop observations for countries or sectors that have one or no firm in the whole sample, and that have only one firm per year. The broad country and sector coverage of my final sample is reported in table , respectively.

This dataset includes not only Scope 1, which I use in my analysis in line with most of the literature (Döttling & Lam, 2023), but also 2 and 3 indicators. Measures of carbon emissions for Scope 1 include direct emissions from sources that are owned or controlled by the business itself, e.g. from using fuel for machines and vehicles. On the other hand, a company might be brown or green based on indirect emissions through its operations. Indeed, Scope 2 measures capture indirect emissions, e.g. from consumption of purchased electricity or heating, while Scope 3 from indirect activities attributable to suppliers.[5]

[3]For primary NAICS 2-digit sector codes: 22, 52, 53 and 92, respectively. For more details on NAICS classification, see here. Within the remaining sample, I also excludes firms assigned according to Trucost's simple industry classification to Banks, Capital Markets, Consumer Finance, Electric Utilities, Financial Services, Gas Utilities, Independent Power and Renewable Electricity Producers, Insurance, Multi-Utilities, Office REITs, Real Estate Management and Development, Water Utilities.

[4]See Appendix A.1.1 for more details on how Trucost puts together the data.

[5]Another reason why it is helpful to look at measures other than Scope 1 is to control for non-financial channels

Other firm-level indicators are available from Trucost, for example detailed information specific to Fossil Fuels and Energy Data. However, given their more limited data coverage, I focus my analysis solely on Scope 1 and 2 carbon emissions. For these and in line with the literature, I produce results identifying brown and green firms by both their level (units) and intensity (units over revenues) of carbon emissions.

Finally, as the dataset is reported by fiscal year, I assign each observation to the relevant calendar year in order to reflect the appropriate time period when matched with the datasets. I do this by assigning an observation to the previous year when the reported period-end date is earlier than July. For firm-year observations that have more than one entry after this adjustment, I keep the latest one.

Stock prices From Compustat North America and Global Security Daily dataset I collect information on common stocks. For a company with multiple issues of stock, I keep only current primary issue to represent market data for the company as a whole. I filter out stale price data, and stocks where the first two digits of their ISIN code do not match the ISO code of the firm's country. Finally, I use price-close to compute the daily return as the change in that price on date from the previous day. Returns are winsorised at 1% level.

Balance sheet After selecting through the procedure described above a panel of companies from the Trucost database, I source corresponding firm-level quarterly information on a series of variables of interest from Compustat North America and Global.

Importantly, the capital stock of a firm is not readily available in Compustat. It must be constructed. I follow the vast literature on this (Clementi & Palazzo, 2019; Salinger & Summers, 1983), and use the perpetual inventory method (PIM) to build the capital stock of each firm in my sample from the book value of tangible capital.[6]

Finally, all variables turned into real values using country-specific GDP deflators from OECD.[7]

Liabilities I source firm-level outstanding liabilities by currency and debt instrument from S&P Capital IQ (Adams & Verdelhan, 2022; Rodnyansky et al., 2022). The dataset includes active and inactive public and private ultimate-parent companies filing financial statements. As the emissions data, this information are available mostly at annual rather than quarterly frequency. Liabilities are grouped into Bonds and Notes, Term Loans, Capital Leases and Other Borrowing. When computing currency shares of debt, Capital IQ first converts liabilities into US Dollars.

In order to draw a better picture of the sample captured by my analysis, Table 1 summarises

behind the heterogeneity in spillovers across central banks, as details below in Section 2.3.

[6]See Appendix A.1 for more details. Recent studies have started to include also intangible capital in firm-level analyses of the capital stock (Peters & Taylor, 2017), and its response to monetary policy (Döttling & Ratnovski, 2021). Following most of the literature on the heterogeneous response of monetary policy across firms, in particular Ottonello and Winberry (2020), I consider solely tangible capital given the difficulties with dealing with intangible assets data at firm level.

[7]See OECD Quarterly GDP and components - expenditure approach - volume and price indices here for more info.

the main moments of the relevant variables constructed through matching all these different sources using Capital IQ's unique firm-level identifiers.[8] A few important findings emerge.

First, there is large variation across the sample for several variables. For example, the mean age of a firm is relatively mature, around 60 years, but the distribution is strongly skewed to the right with some firms even older than 100 years. Similarly, the average firm appears to hold a sizable share of its balance sheet into liquid assets (about 50% of total assets in current assets, and about 15% in cash and short-term investments), and have only limited leverage (around 25% of total assets). However, there is a long tail of firms exhibiting larger values for liquidity measures and leverage.

[8]Specifically, I mostly use *gvkey* from Compustat.

Table 1: Summary statistics of main firm-level covariates

(a) All firms	Mean	Median	S.D.	95th P.	Max	N
Emissions	1279608.03	35727.43	6174109.59	5645486.74	2.11e+08	55083
Emissions/revenues	267.28	24.15	1086.23	1225.12	52940.28	55083
dlnk	0.01	-0.00	0.06	0.10	0.52	90809
Real sales growth	0.02	0.00	0.32	0.39	10.33	87560
Age	58.01	40.00	56.18	155.00	653.00	54028
Real total assets	6.84	6.79	1.79	9.92	12.84	90809
Current assets ratio	0.46	0.45	0.21	0.81	1.00	90809
Leverage	0.25	0.24	0.18	0.54	6.98	90809
d2d	7.55	6.54	5.24	16.80	81.58	63994
Che assets ratio	0.13	0.10	0.13	0.37	0.98	90740
USD share	0.11	0.00	0.28	0.96	1.00	70876
EUR share	0.02	0.00	0.13	0.01	1.00	70876

(b) Brown firms	Mean	Median	S.D.	95th P.	Max	N
Emissions	2129226.89	89658.62	8141867.71	11036957.15	2.11e+08	28360
Emissions/revenues	466.89	51.00	1475.62	2231.19	52940.28	28360
dlnk	0.01	-0.00	0.06	0.10	0.52	28360
Real sales growth	0.01	0.00	0.28	0.33	10.33	27748
Age	59.11	40.00	57.34	155.00	528.00	27732
Real total assets	7.63	7.60	1.65	10.43	12.84	28360
Current assets ratio	0.41	0.39	0.20	0.76	1.00	28360
Leverage	0.27	0.25	0.16	0.54	2.43	28360
d2d	7.48	6.53	5.09	16.63	55.06	22671
Che assets ratio	0.12	0.09	0.11	0.33	0.96	28337
USD share	0.14	0.00	0.30	0.97	1.00	23116
EUR share	0.03	0.00	0.15	0.14	1.00	23116

(c) Green firms	Mean	Median	S.D.	95th P.	Max	N
Emissions	377943.14	16100.02	2577948.12	766034.73	59777007.94	26723
Emissions/revenues	55.45	15.27	184.70	200.20	3029.87	26723
dlnk	0.01	-0.00	0.06	0.11	0.52	62449
Real sales growth	0.02	0.01	0.34	0.42	9.83	59812
Age	56.86	39.00	54.90	155.00	653.00	26296
Real total assets	6.49	6.45	1.73	9.49	12.40	62449
Current assets ratio	0.48	0.48	0.21	0.82	1.00	62449
Leverage	0.25	0.23	0.19	0.54	6.98	62449
d2d	7.60	6.55	5.32	16.86	81.58	41323
Che assets ratio	0.14	0.10	0.13	0.39	0.98	62403
USD share	0.09	0.00	0.26	0.94	1.00	47760
EUR share	0.02	0.00	0.12	0.00	1.00	47760

Source: CIQ, Trucost and author's calculations. Notes: Summary statistics of panel of firms from 2010 to 2019. Emission measures refer to Scope 1. Firms are brown (green) if their emission intensity is larger (smaller) than country-sector median across all years. Listed variables from top to bottom: emission levels (tCO2e); carbon intensity (tCO2e/$M); log first-difference change in the (real) firm-level capital stock (decimals); log first-difference change in (real) firm-level sales (decimals); age of the firm (years since foundation); log of real total assets; current assets (share of total assets); debt over assets (share of total assets); distance to default (units of firm's value volatility); cash and short-term investments (share of total assets); USD liabilities (share of total liabilities). For all details on these variables and the cleaning procedures applied to construct my final sample, please see Appendix A.1.

Second, these descriptive statistics from my collected dataset do not seem to indicate substantial differences between green and brown businesses along these common dimensions of firm-level heterogeneous response to monetary policy. I define firms as brown (green) if their emission intensity, i.e. GHG emissions relative to sales, is larger (smaller) than country-sector median across all years. Non-US brown and green firms do not appear to differ consistently in terms of age, size, liquidity, risk, sales, investment, or reliance on US Dollar or Euro financing. As shown in Table A.3 in the Appendix, these findings carry on also when considering Scope 2 rather than Scope 1 emission measures.

As none of the determinants of heterogeneous response to domestic monetary policy and its cross-border spillovers identified by the literature appear to correlate with a firm's "green" status, I must proceed with documenting novel facts using my dataset. Specifically, in the rest of this section I first set out the general empirical specifications. I then use these to examine the differential responses of brown and green firms by analysing two key firm-level outcomes: stock prices and investment.

2.2 Empirical specifications

In order to produce the new facts on the different response of green and brown firms to international monetary spillovers reported in Section 2.3, I employ two main empirical specifications. First, I estimate the heterogeneous effect on stock prices using event-study panel regressions considering the days of FOMC policy announcements. I then bring the analysis to lower frequency, and estimate quarterly panel regressions for the firm-level investment response to the Fed interventions.

Specifically, for Fact 1 I adopt the high-frequency approach estimating the following local projection regression (Jordà, 2005):

$$d \ln p_{i,\tau+h} = \beta^h \cdot \varepsilon_\tau^M \times \ln b_{i,t-1} + \gamma^h \cdot X_{i,t-1} + \alpha^h + \delta_f^h + \delta_{j,c,s,\tau}^h + u_{i,\tau+h} \tag{1}$$

where the coefficient of interest β^h measures how the response (cumulative logarithmic difference) to a monetary policy shock ε_τ^M, as measured by the surprise series described in section 2.1.1, of the closing quotes for company i's (ordinary) stock price on the day τ of the FOMC announcements and subsequent h days, with respect to the day before, $d \ln p_{i,\tau+h}$, changes across the distribution of firm-level emissions according to selected measure $b_{i,t-1}$ in the previous year $t-1$ with respect to day τ. If β^h is significantly different from zero, this would be evidence of heterogeneous responses between brown and green firms. I include as controls X, all corresponding to the previous year or quarter with respect to τ according to their available time frequency, the term $\ln b_{i,t-1}$, firms' size (total assets), profits, book leverage, market-to-book ratio, cash, short-term debt, retained earnings, dividends, distance-to-default, sales growth, momentum, current assets as a share of total, age, share of USD and EUR liabilities. Because heterogeneous responses might stem from states other than the one I consider, i.e. green status, I include all interactions of those controls with the monetary policy shock to avoid potential omitted variable bias (Cloyne et al., 2023). I also add a constant α, firm fixed effects δ_f^h, and $\delta_{j,c,s,t}$ currency-country-sector-time fixed effects, where s refers to 2-digit NAICS industry

categories. Finally, I also include a dummy for the stock exchange. This set of restrictive fixed effects aims at controlling for time-varying and invariant global, country, currency and sector-related features, e.g. the impact of movements in exchange rates or industry-specific production systems disproportionally requiring certain inputs. Finally, I cluster standard errors at firm-time level to account for correlation within firms and time.

As already pointed out by many studies (Anderson & Cesa-Bianchi, 2024; Patozi, 2023), beyond allowing to uncover short-term dynamic effects that might be obscured when examining data at lower frequencies, this high-frequency empirical approach offers two main advantages over other empirical strategies to identify heterogenous responses. First, it mitigates the risk of aggregation bias that can arise when interest rate surprises need to be combined for empirical analysis at lower frequencies, such as quarterly or annually (Ramey, 2016). Second, although the surprise measures are designed to be exogenous and may not require controlling for potential confounding factors, analysing daily fluctuations through this method reduces the likelihood of reverse causality, where market movements might influence monetary policy decisions.

Given the unavailability of higher frequency data, to study the heterogenous response to a real firm-level outcome such as investment for Fact 2, I adjust my local projection set-up as follows:

$$d \ln k_{i,t+h} = \beta^h \ln b_{i,t-1} \varepsilon_t^M + \gamma^h X_{i,t-1} + \alpha^h + \delta_f^h + \delta_{c,s,t}^h + u_{i,t+h} \tag{2}$$

where the coefficient of interest β^h now measures how the response of real capital accumulation from quarter $t-1$ to t and following periods, $d \ln k_{i,t+h}$, to a monetary policy shock in quarter t changes across the distribution of firm-level emissions according to selected measure $b_{i,t-1}$ in the previous year $t-1$ with respect to day τ. Again, if β^h is significantly different from zero, this would be evidence of heterogeneous responses between brown and green firms. Given the lower frequency of this set-up, I include a modified list of controls X, all corresponding to the previous year or quarter with respect to t according to their available time frequency: the term $\ln b_{i,t-1}$, firms' size (total assets), profits, book leverage, market-to-book ratio, cash, short-term debt, retained earnings, dividends, distance-to-default, sales growth, current assets as a share of total, age, share of USD and EUR liabilities; and their interactions with the shock. I again add a constant α, firm fixed effects δ_f^h, and $\delta_{c,s,t}$ country-sector-time fixed effects, where s refers to 2-digit NAICS industry categories. I include a dummy indicating the fiscal quarter following Ottonello and Winberry (2020), but I do not include a currency fixed effect as I have deflated capital by country-specific deflators in order to obtain a real measure. Clustering of standard errors is at firm-time level to account for correlation within firms and time.

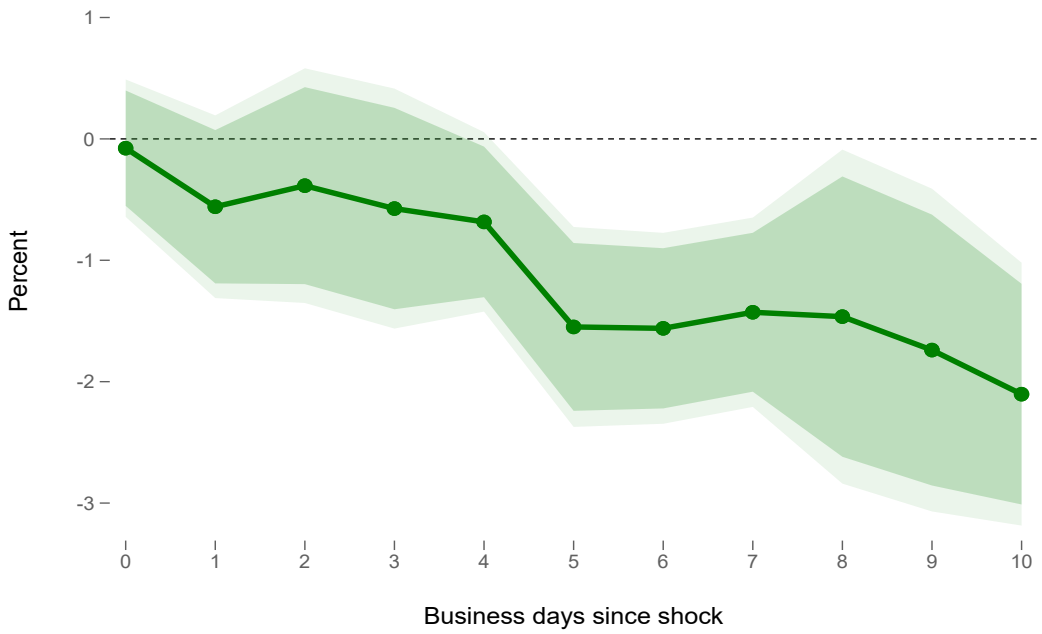
Finally, for both specifications I also report in Appendix A.2 the average (mostly negative) effect of the monetary policy shock only to give a benchmark to its interaction terms with the emission measures above, which are the main focus of my analysis. In order to do this I have to drop the time fixed effect from those specification, as the monetary policy surprises series vary only over time.

2.3 New facts on climate and international monetary spillovers

Fact 1. *The stock returns of foreign brown firms decline by more than their green counterparties in response to US monetary policy.*

As my baseline, I plug in equation (1) the Scope 1 emission intensity measure from Table 1, and estimate the coefficients on its interaction term with monetary policy shocks. Figure 2 plots the results. It shows how brown firms, i.e. exhibiting higher emission intensity, experience a more adverse impact from a 25 basis point tightening of the Fed monetary stance compared to foreign green firms. This goes to aggravate the decline experienced by the average foreign firms as a result of the same shock (Figure A.2).

Figure 2: Response of non-US stock prices to US monetary policy: Scope 1 emission intensity



Notes: Dynamics of the interaction coefficient between emission intensity and monetary shocks over time. Shock normalised to be 25 basis points on the market yield on US Treasury Securities at 1-year constant maturity. Connected line reports the estimated coefficient β^h from equation (1). Two areas go from darker to lighter shades corresponding to 90% and 95% confidence intervals. Standard errors clustered at firm and event-day level.

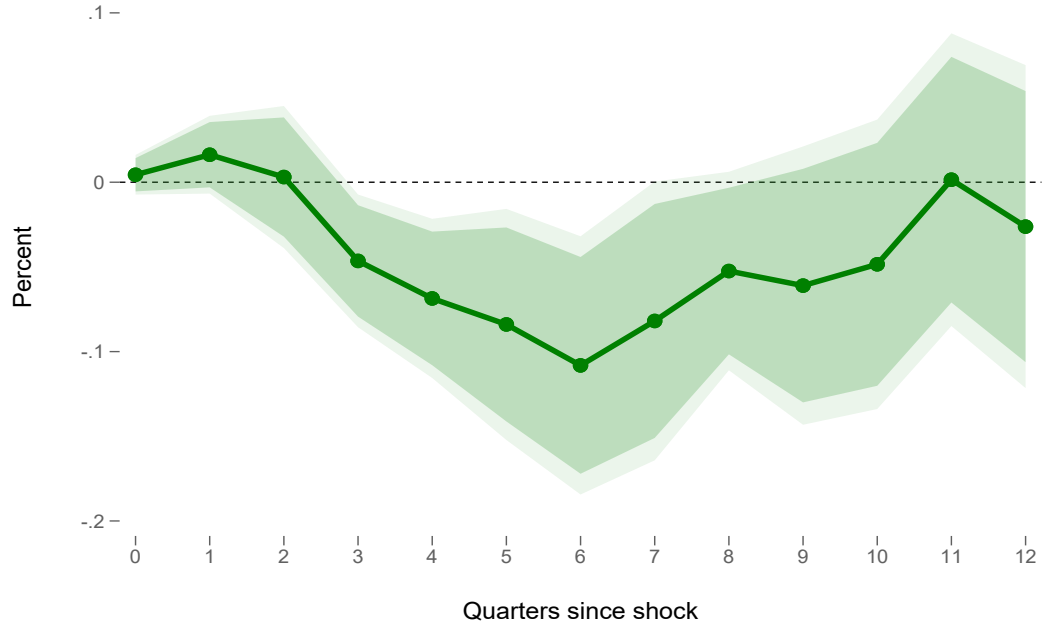
In the Appendix A.3, I show how these findings are broadly robust to a series of cross-checks: (i) using the "median rotation" shock series as shown in Figure A.1; (ii) considering Scope 2 rather than 1 emissions; (iii) using a non-linear emission measure to sort firms into brown and green as done in Table 1, where I define firms as brown (green) if their Scope 1 emission intensity is larger (smaller) than country-sector median across all years; (iv) excluding emissions which have been estimated by Trucost in favour of keeping only those reported by firms themselves; (v) considering only Advanced or Emerging Market Economies.

Fact 2. *The capital accumulation of foreign brown firms declines by more than their green counterparties in response to US monetary policy hikes.*

Figure 3 shows how a 25 basis point hike of the US policy rate disproportionately drags

on brown firms' investment compared to foreign green firms. Again, this worsens the decrease experienced by the average foreign firms as a result of the same shock (Figure A.10).

Figure 3: Response of non-US stock prices to US monetary policy: brown vs. green firms



Notes: Dynamics of the interaction coefficient between brown dummy based on emission intensity and monetary shocks over time. Shock normalised to be 25 basis points on the market yield on US Treasury Securities at 1-year constant maturity. Connected line reports the estimated coefficient β^h from equation (2). Two areas go from darker to lighter shades corresponding to 90% and 95% confidence intervals. Standard errors clustered at firm and event-day level.

In the Appendix A.4, I show how these findings are again broadly robust to the same battery of robustness tests as carried out for Fact 1.

Armed with these new facts, I will now show how a two-country two-sector New Keynesian model can rationalise what I detected in the data through a set-up including financial frictions which interact with a bias of investors towards green assets.

3 A two-country two-sector model of monetary spillovers with bias for green investment

In this section I set out the model I built to rationalise the empirical findings from the previous sections. I keep my framework purposely parsimonious, and restrict my attention to those modelling features that I identified to be essential for the characterisation of the main channel reconciling my results: a bias for sustainable investment in cross-border flows. First, in Section 3.1 I briefly describe the main features and innovative components of my framework, relegating a detailed exposition of the model and its equations to Appendix A.7. Finally, I use my model to draw implications in Section 3.2, which I then proceed to test in Section 4. Finally, I use my model to run a simulation exercise mimicking the international outlook over the recent past to quantify the attenuation effect on international monetary spillovers of the green bias.

3.1 The model at a glance

My model builds on the medium-scale open-macro DSGE New Keynesian framework with financial frictions, price and wage rigidities proposed by Gertler and Karadi (2011, 2013), and in the recent work by Sims and Wu (2021a, 2021b).

Following those contributions, my setup allows for the joint interaction of households, labour unions, several types of production firms, financial intermediaries, a fiscal authority, and a central bank. Importantly, this economy exhibits two financial frictions crucial to match the new facts from section 2. First, as common in the literature, I assume that wholesale firms must issue perpetual bonds (Woodford, 2001) to finance capital acquisition and obtain funds from intermediaries (Gertler & Karadi, 2011). This thus creates scope for external financing through bonds issuance. Second, intermediaries finance themselves through net worth and taking deposits (short-term debt), which they invest into central banks' reserves and, crucially, corporate bonds from wholesale firms. I also introduce a form of market segmentation preventing households from holding long-term bonds directly.[9] Because of a costly enforcement problem, a non-absconding constraint requires the value of continuing as an intermediary to be higher than the fraction of assets that can be retained in case of bankruptcy (Gertler & Karadi, 2011; Sims & Wu, 2021b). This constraint binds at all times, and leads to the emergence of excess returns on bonds in steady state.

On the other hand, my model exhibits some important innovations on the considered benchmarks, expanding on the framework by Wu et al. (2024).

First, each economy is made up of *two* rather than just one sector: brown and green.[10] The two sectors are exactly the same in terms of their composition into firms and labour unions, and a final consumption entity allows the households to consume both green and brown goods, whether they are produced domestically or abroad. Notice I define *green* in a general sense. It could refer to firms producing fewer emissions, or adapting faster to the green transition. Ultimately, it does not matter for my quantitative results, as I do not include a climate block in the model allowing for a adverse feedback through emissions. This is because I want to focus on the effect on US monetary policy transmission of the channel from a bais for sustainable investment only. Thus, in order to zero-in on that, I abstract from other climate-related considerations on the supply and demand effects of emissions, as well as explicitly modelling the firm- and aggregate-level risk from the green transition and/or capital scraping or higher incidence of natural disasters. These considerations are, of course, important in the academic and policy debate on the economic effects of climate change, but they are not directly germane to my specific analysis.

Second, I introduce in each economy a layer of financial intermediaries holding corporate bonds from both domestic and foreign brown and green firms, and exhibiting a tighter leverage constraint (Gertler & Karadi, 2011) on brown bonds. This specific type of financial friction leads

[9] Other studies, such as Gertler and Karadi (2013), allow for households' direct securities and bonds holdings. Limited participation in asset markets by households and thus incomplete arbitrage are then achieved through the assumption of transactions costs. For simplicity and to keep the model parsimonious, I do not allow households to hold directly bonds.

[10] Garofalo and Poyntner (2025) adjusts a similar framework to study the international spillovers of central bank's corporate bond purchases. Specifically, in that setting the two sectors include firms whose bonds are eligible and ineligible for purchases by their domestic central bank.

to a "green" bias in intermediaries' portfolio allocation. This is a parsimonious modelling choice to capture in an easy way several potential micro-foundation for this bias towards sustainable investment. The bias could stem from the fact that brown firms might be riskier because of the green transition (Döttling & Lam, 2023) or green assets might provide special utility to climate-minded households and investors (Patozi, 2023). Same goes for using a mean-variance model as proposed by Benchora et al. (2023) for the US economy. No matter the underlying micro-foundation, ultimately brown firms will face higher excess returns on their bonds, which then would make them more sensitive to monetary policy, in general, as shown by the literature cited above.

As wholesale firms and intermediaries exhibit the main novel features of my framework, I focus on them in what follows, referring the reader to the Appendix A.7 for details on all the relevant equations for this and the other parts of my model.

3.1.1 Wholesale firms

I include four types of production firms in each sector of the two economies. A competitive final good producer turns retail output into a final good. A continuum of monopolistically competitive and price sticky retail firms sales wholesale output to the final good firm. And a representative investment good firm makes new physical capital from unused output through a convex adjustment cost technology. Finally, the sector's representative wholesale firm inputs in its Cobb-Douglas technology capital purchased from the investment good producer and labour from unions, which are subject to wage stickiness.

Crucially, each sector s's wholesale firm exhibits the following "loan in advance constraint" (Carlstrom et al., 2017; Gertler & Karadi, 2011):

$$\psi_s P^k_{s,t} I_{s,t+1} \leq Q_{s,t} F_{m,s,t} \quad \text{for} \quad s = 1, 2 \tag{3}$$

Equation **??** requires that, in order to finance a constant fraction, $\psi_s \in [0, 1]$, of its capital acquisition $I_{s,t+1}$ at price $P^k_{s,t}$, each sector's wholesaler must issue perpetual bonds $F_{m,s,t}$ at market price $Q_{s,t}$ with decaying coupon payments, whose decaying parameter is $\kappa \in [0, 1]$. $F_{m,s,t}$ is the total coupon liability due in period t, i.e. $F_{m,s,t-1} = CF_{m,s,t-1} + \kappa CF_{m,s,t-2} + \kappa^2 CF_{m,s,t-3} + ...$ The latter, through forward iteration, implies that one needs only keep track of the period t and $t-1$ total coupon liabilities, $CF_{m,s,t} = F_{m,s,t} - \kappa F_{m,s,t-1}$. This constraint creates scope for external financing and distorts the standard asset pricing decisions.

3.1.2 Financial intermediaries

I model financial intermediaries by taking the framework proposed by Wu et al. (2024) to a two-sector set-up. Specifically, households own and hold deposits at intermediaries. Each period and within each sector s a fraction $1 - \sigma_s$ with $\sigma_s \in [0, 1]$ stochastically exit and return their net worth $N_{s,t}$ to their household owner.[11] Once exited, they are replaced by an equal number of new intermediaries — these start up with X_s real funds from their household owner. As

[11] Notice I have dropped the single intermediary index i here to simplify the notation, as all intermediaries behave in the same way and are subject to the same constraints within each sector.

long as intermediaries keep operating, they finance their activities through their net worth $N_{s,t}$ and deposits $D_{s,t}$. These activities of each intermediary consist in investing into central bank reserves $RE_{s,t}$ and corporate bonds from the domestic and foreign wholesale firms across sectors $F_{s,t}$, in order to maximise its expected terminal net worth where discounting by the stochastic discount factor of the household $\Lambda_{t,t+1}$:

$$V_{s,t} = \max(1-\sigma)\mathbb{E}_t \sum_{j=1}^{\infty} \sigma^{j-1} \Lambda_{t,t+j} n_{s,t+j} \quad \text{for} \quad s = 1,2 \tag{4}$$

where $n_{s,t} = N_{s,t}/P_t$ is real net worth, with P_t the price level of final output.

Again, for the sake of clarity, I focus here on the different financial constraints of our set-up with respect to the benchmarks in the literature, and relegate all the equations and details to the Appendix A.7.

Crucially, intermediaries in Home and Foreign economies are equally constrained by a non-absconding requirement (Gertler & Karadi, 2011), which prevents them from declaring bankruptcy and retain the entirety of their balance sheet. They can only divert away from households a stochastic and exogenous fraction θ_t of their assets $\Delta Q_t^g b_{H,t}^{g,FI} + Q_t^b b_{H,t}^{b,FI}$ and $\left[\Delta Q_t^{*g} b_{F,t}^{g,FI} + Q_t^{*b} b_{F,t}^{b,FI}\right] \mathcal{Q}_t$, where $\mathcal{Q}_t$ is the real exchange rate, b_H and b_F are the real (divided by price level P_t) domestic private bond holdings of domestic and foreign bonds respectively, and $0 \leq \Delta \leq 1$ is the steady state difference in recoverability between the two types of bonds, *green* (g) and *brown* (b).[12] As a result of this, for households to be still willing to lend funds, intermediaries must guarantee (also in steady state) excess returns of long term private green and brown bonds over the deposit rate, with the ones on the former being lower due to the higher recoverability through Δ. Crucially, this set-up allows foreign and domestic green bonds to have a smaller excess return over deposits than their brown counterparties. This means that the return on assets from brown firms is higher than on green, as commonly found in the literature on climate finance and dubbed "carbon premium" (Bauer et al., 2022, 2024; Bolton & Kacperczyk, 2021; Pástor et al., 2021).[13] The constraints for the two types of intermediaries are thus:

$$V_t \geq \theta_t \left\{ \Delta Q_t^g b_{H,t}^{g,FI} + Q_t^b b_{H,t}^{b,FI} + \left[\Delta Q_t^{*g} b_{F,t}^{g,FI} + Q_t^{*b} b_{F,t}^{b,FI}\right] \mathcal{Q}_t \right\} \tag{5}$$

3.2 Quantitative results

I describe in detail the calibration of the model in terms of its parameters and targeted moments in Appendix **??**. In this section I instead focus on how the results from my parsimonious model rationalise the facts from section 2.

[12] As already mentioned above, this adjustment captures in a stylised way the bias against brown bonds in the intermediaries' private brown loan portfolio. In models with financial frictions (Gertler & Karadi, 2013; Sims & Wu, 2021b), this recoverability parameter commonly reflects the fact that some private loan portfolio is arguably more exposed to bank malfeasance than others. This is the case because depositors monitor the performance of the former with more difficulty than the latter.

[13] Strictly speaking the carbon premium is usually defined as the fact that brown stocks have higher expected returns compared to green stocks. The bonds in my set-up can also be thought of as shares, given that the constraint (3) links them directly to capital accumulation, and thus the net worth of a wholesale firm.

Specifically, I solve[14] the model to produce impulse responses to an exogenous monetary policy shock by the Home central bank. My objective is to compare those to my empirical findings, and assess whether my relatively simple set-up does a good job at matching those observations.

3.2.1 The international spillovers of US monetary policy on foreign brown and green firms

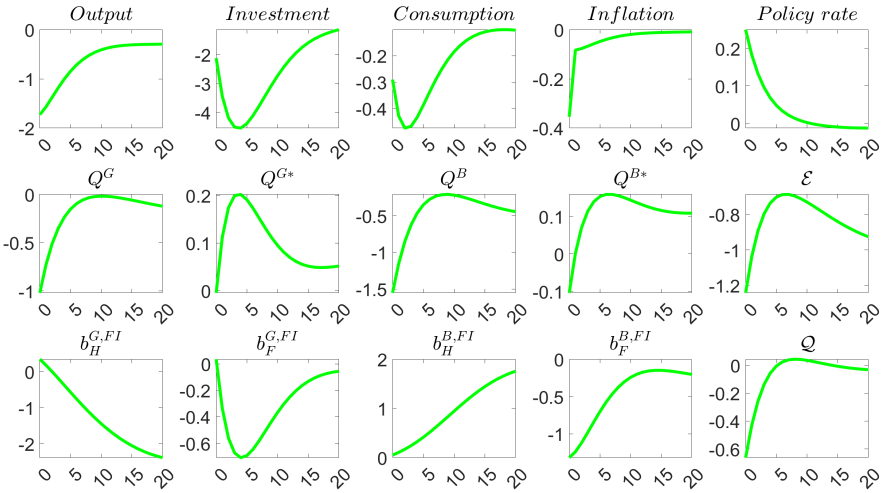
In order to quantify the international spillovers of Home central bank's monetary policy, I use my model to produce impulse responses to an exogenous monetary policy shock which increases the domestic policy rate by 25 basis points, and quickly goes back to its steady-state value within two years.

Looking at the Home economy (Figure 4), as expected, higher interest rates generate a downturn, with output dropping along with consumption and investment, and prices declining. The effect of Home monetary policy spills over onto the Foreign economy, where the picture for the economic outlook is flipped: demand and inflation increase as a result of the depreciation of the Foreign currency boosting the trade balance. This expenditure switching channel dominates over the depressing impact of lower demand from Home mainly because my baseline model assumes Producer Currency Pricing: having a different regime such as Local (LCP) or Dominant Currency Pricing (DCP) would weaken the expenditure switching channel, and potentially lead the impact of lower demand from Home to dominate (Gopinath & Itskhoki, 2022). This would not however change my main results matching the new facts from section 2 (TBU - link to Appendix with results for DCP). In response to the stronger economic outlook, the Foreign central bank raises its own interest rates, but to a lesser extent than the Home central bank[15], the nominal (and real) exchange rate appreciate, as the Home currency strengthens against the Foreign currency.

[14] I solve the model with Dynare via a linear approximation about the non-stochastic steady state.

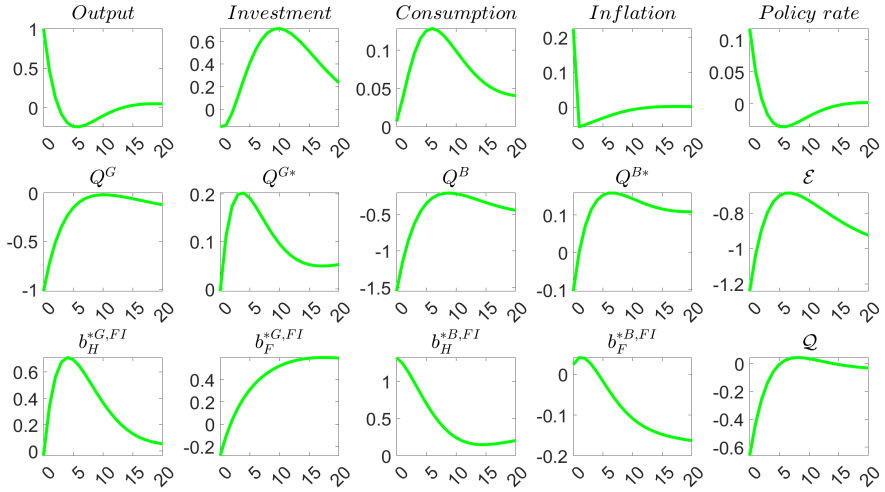
[15] Notice my analysis is completely positive. I let the Taylor rule endogenously drive the policy decisions in Foreign. This means that the path for interest rates in Foreign does not necessarily represent the *optimal* response to the tightening in Home.

Figure 4: IRFs for Home macroeconomic variables from Home monetary policy shock



Notes: I calibrate the Home monetary policy shock to yield a 25 basis point increase in the domestic policy rate. All variables in percentage change from steady-state, expect for inflation and policy rate which are in percentage points.

Figure 5: IRFs for Foreign macroeconomic variables from Home monetary policy shock



Notes: I calibrate the Home monetary policy shock to yield a 25 basis point increase in the domestic policy rate. All variables in percentage change from steady-state, expect for inflation and policy rate which are in percentage points.

Importantly, comparing capital accumulation of green and brown firms in the Foreign economy shows Home monetary interventions might have unequal cross-border effects (Figure 6). The increase in the yield of domestic assets following the policy hike leads intermediaries to reallocate their portfolios away from foreign and more towards domestic bonds. The increased (decreased) domestic holdings of domestic (foreign) bonds by domestic intermediaries make the yield curve faced by domestic (foreign) firms when financing less (more) steep, and thus stimulate their capital accumulation. This occurs through the reduction (rise) in their *real long term yield*, i.e. the overall spread capturing the difference between the net long term private yield and the net deposit rate, plus the real net deposit rate, i.e. the difference between the net deposit rate and expected inflation. As in Sims and Wu (2021b), this variable is the relevant metric for assessing monetary policy transmission to the firms in the model.[16]

3.2.2 Attenuation effect and testable implications

I now turn to how the model can rationalise this attenuation effect through the lens of its predicted dynamics for cross-border flows. This will allow me to draw testable predictions that I will then take to the data in section 4.

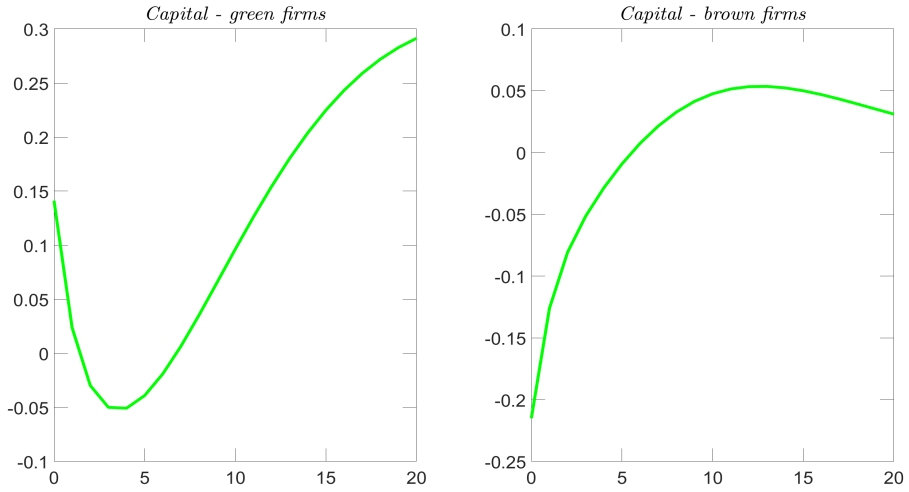
Specifically, Figure 4 shows that, as expected and in line with the rich literature mentioned above, the increase in interest rates in Home makes investing domestically relatively more attractive than buying Foreign bonds with lower returns. This drives down Home holdings of Foreign bonds. At the same time, higher domestic rates act as a pulling force on investment from Foreign investors, leading to an outflow of capital trying to take advantage of the higher returns from holding Home bonds (Figure 5).

Crucially though, the portfolio reallocation by domestic intermediaries is less severe for green than brown foreign bonds, that is the financial spillovers are significantly attenuated. As shown in Figure 4, $b_F^{G,FI}$ declines by only half as much as its brown counterpart $b_B^{G,FI}$. In the model this is the result of the green bias of investors, which leads them to decrease their green holdings by less than their brown ones, as the relatively returns of Home and Foreign bonds diverge following the monetary policy tightening. This translates in a smaller adjustment for Home investors in terms of Foreign bonds position in their portfolios.

From my discussion above, it is then straightforward to see how this dampening mechanism feeds back into a smaller decline in activity and investment, rationalising the differential responses depicted for capital accumulation in Figure 6, and equivalently for investment in 7.

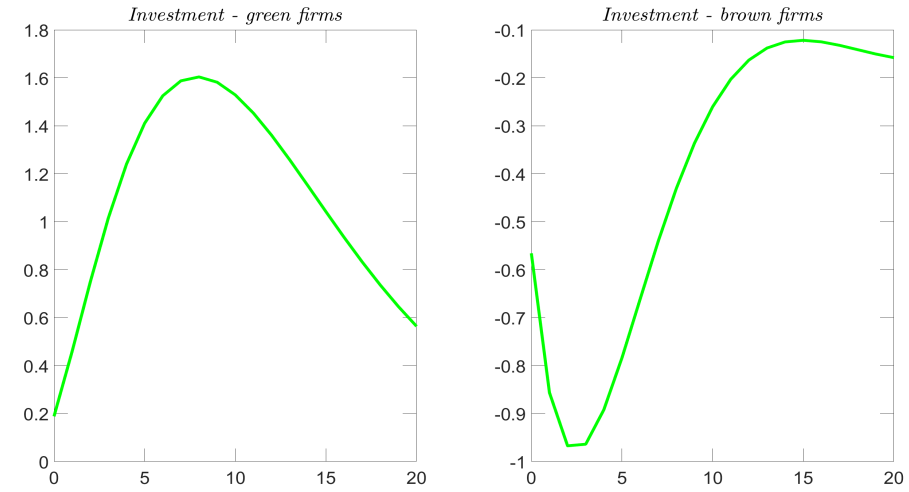
[16] Sims and Wu (2021b) highlight that having long term yields as the central driver in the economy is in line with (Wu & Zhang, 2016, 2019) and the other studies in the literature using the shadow rate of Wu and Xia (2016) as a summary statistic for the effects of unconventional policies.

Figure 6: IRFs for Foreign investment: green vs. brown



Notes: I calibrate the Home monetary policy shock to yield a 25 basis point increase in the domestic policy rate. All variables in percentage change from steady-state, expect for inflation and policy rate which are in percentage points.

Figure 7: IRFs for Foreign investment: green vs. brown



Notes: I calibrate the Home monetary policy shock to yield a 25 basis point increase in the domestic policy rate. All variables in percentage change from steady-state, expect for inflation and policy rate which are in percentage points.

The model predicts that, at the heart of the attenuated outlook for US monetary spillovers onto non-US firms documented in section 2 and matched by my quantitative framework, lies a differential resilience of green investment opportunities which benefits from a green bias. Given the centrality of this dynamic for my framework, in the following section of the paper I bring my model to the data by testing this key channel and prediction of my model.

4 Testing model mechanisms

In this section I take the testable implications predicted by my quantitative framework in Section 3 to the data. In particular, I look at whether the reaction to a US monetary policy shock of the funding behaviour of US mutual funds differs depending on the climate-mindedness of these financial intermediaries.

4.1 Mutual funds

In this section I report empirical evidence from monthly capital flows of US mutual funds. I first describe the data and the empirical specification I employ to identify the effect of preferences for green investment which exploits variation across mutual funds mandates within management company and fund's type and strategy. Finally, I present evidence of the attenuation channel predicted by my model to be at play in the dynamics of cross-border investment by mutual funds.

4.2 Data

For this analysis I use the CRSP mutual fund database, which is free from survivorship bias, and includes information on a fund's monthly returns and total net assets, along with data on portfolio holdings, expense ratios, and associated fees, as well as other fund's characteristics, such as its investment objective, fund family, type (e.g., equity, bond, hybrid), and founding date.[17] Given the focus of my analysis, I keep only funds that specialise in international investment.

I follow van der Beck (2023) to identify mutual funds with a potential "green bias". I label a fund as an "ESG fund" if it exhibits at least one sustainability keyword in its name. [18]

Finally, following Patozi (2023) I compute capital flows for mutual fund i in month t as the net growth in i's total net assets $A_{i,t}$ adjusted for price changes through i's monthly return $r_{m,t}$ between $t-1$ and t:

$$Flows_{i,t} = \frac{A_{i,t} - A_{i,t-1}(1 - r_{i,t})}{A_{i,t-1}} \tag{6}$$

As fund's returns within the month are accounted for, this measures accurately captures the net investments flowing in or out of the fund in that month.

[17] I use this dataset because it covers also non-US assets, while other sources such as the data from 13F holdings for institutional investors considers only US equities.

[18] The considered keywords are: Environment, social, governance, green, sustainable, responsible, SRI, ESG, climate, clean, carbon, impact, fair, gender, solar, renewable, ethical, thematic, and conscious.

4.3 Empirical specification

To test the prediction from my model, I run the following panel regression:

$$Flows_{i,t+1} = \beta \cdot \varepsilon_t^M \times ESG_{i,t-1} + \gamma \cdot X_{i,t-1} + \alpha + \delta_{fund} + \delta_{mgmt,t} + \delta_{lipper,t} + u_{i,t} \quad (7)$$

where ε_t^M is the monetary policy shock aggregated to a monthly frequency. Following Patozi (2023) the set of control variables $X_{i,t-1}$ includes the lagged term $ESG_{i,t-1}$, fund return, fund return 12-month-rolling volatility, fund size, fund age, fund's expense ratio, and an interaction of the monetary policy shock with the fund's return volatility. $\delta_{c,t}$ represents management firm - time fixed effects (e.g., Vanguard, Blackrock, etc.), and $\delta_{s,t}$ controls for how funds of different types and with different investment strategies evolve over time. Fund types and investment strategies are defined according to the Lipper classification system. This controls for the fund type (e.g., ETF, Index, or Mutual Fund) as well as the fund's investment objective (e.g., US Mid Cap Blend, Global Large Cap Blend, etc.). Finally, I saturate the regression and control for time-invariant fund-specific features adding fund fixed effects.

Crucially, using this set-up means that the identification strategy here relies on within-fund variation: by definition of the ESG dummy, this occurs because the fund might change its name to include some of the listed keywords, and thus start to exhibit a "green bias".

4.4 Results

To begin with I drop the time fixed effects from my baseline regression, so to be able to estimate the average effect across mutual fund's flows, as done in section 2. A monetary policy tightening appears to lower capital flows in line with what found by Patozi (2023) considering a sample not focused on international investors (Table 2). The estimated coefficients are mostly negative, but not always significant probably as a result of the smaller sample of mutual funds considered.

Table 2: Mutual Fund International Flows - Average response to US monetary policy

	Equity			**Index (equity)**			**Bond**		
	(1) All	(2) Inst.	(3) Retail	(4) All	(5) Inst.	(6) Retail	(7) All	(8) Inst.	(9) Retail
Fed shock	-0.20	-0.08	-0.48**	-0.58	-0.30	-2.31**	0.04	0.66	-0.30
	(0.16)	(0.20)	(0.24)	(0.43)	(0.44)	(1.13)	(0.35)	(0.50)	(0.42)
Observations	118104	64652	53220	16782	13084	3679	16704	8490	8214
R^2	0.092	0.079	0.116	0.099	0.098	0.139	0.091	0.087	0.090
Fund FE	✓	✓	✓	✓	✓	✓	✓	✓	✓
Mgmt Firm FE	✓	✓	✓	✓	✓	✓	✓	✓	✓
Lipper Class. FE	✓	✓	✓	✓	✓	✓	✓	✓	✓
Fund Controls	✓	✓	✓	✓	✓	✓	✓	✓	✓

Notes: This table reports estimates of equation (7) where time fixed effects have been dropped in order to allow for the estimation of the coefficient on the monetary policy term, as well as its interaction with ESG mandate. Standard errors clustered at the firm level in parentheses with ***, ** and * denoting coefficients significantly different from zero at the 1, 5 and 10% levels, respectively. For more details on the monetary policy shocks used, see the notes to Figure 1.

Moving to my baseline specification, Table 3 reports the estimated coefficients on the interaction term between the monetary policy shock and the indicator of ESG mandate proxying

Table 3: Mutual Fund International Flows - Interaction ESG mandate & US monetary policy

	Equity			Index (equity)			Bond		
	(1) All	(2) Inst.	(3) Retail	(4) All	(5) Inst.	(6) Retail	(7) All	(8) Inst.	(9) Retail
ESG Mandate × Fed shock	0.31***	0.22	0.36**	0.31	0.28	0.00	-0.07	0.00	-0.29
	(0.10)	(0.16)	(0.17)	(0.28)	(0.30)	(.)	(0.39)	(.)	(0.41)
Observations	113867	58933	48947	15255	11564	2539	15632	6823	7463
R^2	0.237	0.247	0.349	0.337	0.323	0.554	0.322	0.347	0.496
Fund FE	✓	✓	✓	✓	✓	✓	✓	✓	✓
Mgmt Firm-time FE	✓	✓	✓	✓	✓	✓	✓	✓	✓
Lipper-time FE	✓	✓	✓	✓	✓	✓	✓	✓	✓
Fund Controls	✓	✓	✓	✓	✓	✓	✓	✓	✓

Notes: This table reports estimates of equation 7. Standard errors clustered at the firm level in parentheses with ***, ** and * denoting coefficients significantly different from zero at the 1, 5 and 10% levels, respectively. For more details on the monetary policy shocks used, see the notes to Figure 1.

for green bias. Three main points emerge. First, in line with the attenuation mechanism in my model most of the coefficients are estimated to be positive. Second, estimates are statistically significant when considering all equity funds, where retail funds seem to be driving the results. Third, the reaction to monetary policy is not significantly different from the average for bonds institutional investors — although the sample reduces substantially for those.

After having validated the main mechanisms of my model, I move to the next section where I use my framework to quantify how much the presence of this attenuation chennel from "green bias" might have contributed to the observed cross-country resilience of the latest US tightening cycle.

5 Simulation

In this section I use my quantitative framework to investigate the role played by the attenuation mechanism of the international monetary transmission I have uncovered during the latest Fed tightening cycle. Specifically, I simulate my model to yield a similar global outlook to what observed for the US and the rest of the world post Covid pandemic. This period has been particularly characterised by surging inflation in most economies, including the US, where the Fed has raised interest rates in order to quell price pressures. As a result of tighter financial conditions in the US, foreign economies where expected to experience capital outflows from international investors searching for higher US yields.

5.0.1 Calibration and scenarios

My exercise consists of two steps.

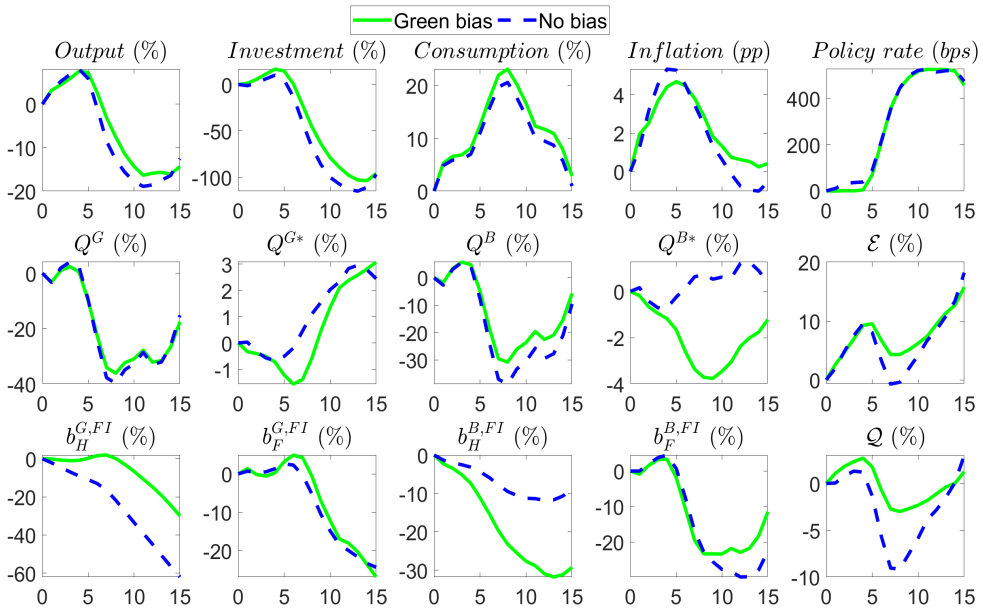
First, I simulate my model in order to match the shape of the increase in US Personal Consumption Expenditures Price Index inflation and the Federal Funds Effective Rate post pandemic. The former climbed from 2% in 2021 Q1 to peak around 7% in 2022 Q4, and then gradually revert back towards 2% by 2024. Following this surge in prices, the Fed raised the rates leading its target Federal Funds Effective Rate to increase from around 0% to about 5% by the end of 2023. Following the latest assessments in the literature (Giannone & Primiceri, 2024),

which suggests that US price pressures were driven mainly by demand forces and unexpectedly accommodative monetary policy, I let my baseline model choose a combination of demand and monetary policy shocks in order to match the observed patterns in the data described above.

Second, I then run another simulation feeding to the model the shocks identified in my first step, but this time assuming no green bias by setting Δ close to one.

The results of this exercise are reported in Figure 8, where the green lines refer to the post-pandemic changes for my baseline model from the first step, while the blue lines the counterfactual without green bias.

Figure 8: Simulation results for Home macroeconomic variables - TBU



Notes: Baseline with green bias calibrated to match the shape of the increase in US Personal Consumption Expenditures Price Index inflation and the Federal Funds Effective Rate post pandemic. All variables in deviation with respect to values at beginning of post-Covid period, i.e. 2021 Q1. Units specified in subplots' labels.

While other factors, such as local central banks also raising rates, might have contributed to the overall unexpected resilience across countries to the latest US tightening cycle, my simulation shows that the green attenuation channel might have played a sizeable role. Indeed, comparing the green and blue lines for $b_F^{G,FI}$ suggests that green bias might have led to a peak effect of around 5%, as opposed to only half of that without any bias towards sustainable investing.

The more economies have "gone green", the larger the shielding effect that green bias might have granted them. This simulation exercise highlights the crucial role of sustainable investing in shaping international capital flows and mitigating the impacts of monetary policy changes.

6 Conclusion

In this paper I examine how US monetary policy impacts green and brown firms internationally. My analysis reveals two key insights: foreign brown companies face a more significant decline in both their stock prices and investment compared to their green counterparts when the Fed tightens its monetary policy. To explain these findings, I develop an novel two-country model

that includes green and brown sectors, along with international intermediaries subject to financial constraints. After a domestic monetary policy tightening, investors shift their portfolios from foreign assets to higher-yielding domestic ones. However, this capital outflow is notably reduced for foreign green positions when lenders prioritize sustainable investing. I then validate the model using data on cross-border investor flows and holdings from US mutual funds. A quantitative simulation indicates that embracing green practices may have substantially contributed to the observed resilience across countries to the recent US tightening cycle.

Going forward, my work has left few avenues unexplored. For instance, my framework purposely employs a parsimonious approach to modelling bias towards green investment, which encompasses different potential micro-foundations. However, the source of the green bias might be relevant for the period following the climate transition: if the bias comes from genuine preferences, the attenuation effects I document might persist; should it stem from transition risk, it would naturally be expected to dissipate, and the heterogenous effect on international monetary spillovers with it. Relatedly, depending on what the ultimately source of the green bias is, the anti-climate stance of the new US administration under Trump's second term might affect the attenuation mechanism of international monetary spillovers in the future. Another example relates to the policy implications highlighted by my paper for the transition period: brown firms appear to be subject to a "double whammy", as they are hit by both higher interest rates and see their international funding dry up disproportionately quicker. For a policymaker this is, of course, a first-order issue given that brown firms are precisely the ones that need more support to catch up and accelerate climate transition. I leave these interesting avenues of further investigation to future research.

References

Abiry, R., Ferdinandusse, M., Ludwig, A., & Nerlich, C. (2022). *Climate change mitigation: How effective is green quantitative easing?* Publications Office.

Adams, P., & Verdelhan, A. (2022). Exchange Rate Risk in Public Firms.

Anderson, G., & Cesa-Bianchi, A. (2024). Crossing the Credit Channel: Credit Spreads and Firm Heterogeneity. *American Economic Journal: Macroeconomics*, *16*(3), 417–446.

Bauer, M. D., Huber, D., Rudebusch, G. D., & Wilms, O. (2022). Where is the carbon premium? Global performance of green and brown stocks. *Journal of Climate Finance*, *1*, 100006.

Bauer, M. D., Offner, E. A., & Rudebusch, G. D. (2024). Green Stocks and Monetary Policy Shocks: Evidence from Europe. *Working Paper*.

Benchora, I., Leroy, A., & Raffestin, L. (2023, March). Is Monetary Policy Transmission Green?

Bolton, P., & Kacperczyk, M. (2021). Do investors care about carbon risk? *Journal of Financial Economics*, *142*(2), 517–549.

Carlstrom, C. T., Fuerst, T. S., & Paustian, M. (2017). Targeting Long Rates in a Model with Segmented Markets. *American Economic Journal: Macroeconomics*, *9*(1), 205–242.

Clementi, G. L., & Palazzo, B. (2019). Investment and the Cross-Section of Equity Returns. *The Journal of Finance*, *74*(1), 281–321.

Cloyne, J., Ferreira, C., Froemel, M., & Surico, P. (2019). Monetary policy, corporate finance and investment. *Banco de España Working Paper Series*, 78.

Cloyne, J., Jordà, Ò., & Taylor, A. M. (2023, February). State-Dependent Local Projections: Understanding Impulse Response Heterogeneity.

De Leo, P., Gopinath, G., & Kalemli-Ozcan, S. (2023). Monetary Policy Cyclicality in Emerging Economies.

di Giovanni, J., & Rogers, J. H. (2022). The Impact of U.S. Monetary Policy on Foreign Firms. *SSRN Electronic Journal*.

Di Giovanni, J., & Hale, G. (2022). Stock Market Spillovers via the Global Production Network: Transmission of U.S. Monetary Policy. *The Journal of Finance*, *77*(6), 3373–3421.

Döttling, R., & Lam, A. (2023, September). Does Monetary Policy Shape the Path to Carbon Neutrality?

Döttling, R., & Ratnovski, L. (2021, August). *Monetary Policy and Intangible Investment* (SSRN Scholarly Paper No. 3612304). Social Science Research Network. Rochester, NY.

Ferrari, A., & Nispi Landi, V. (2020). *Whatever it takes to save the planet?: Central banks and unconventional green policy*. Publications Office.

Ferrari Minesso, M., & Pagliari, M. S. (2023). No country is an island. International cooperation and climate change. *Journal of International Economics*, *145*, 103816.

Ferreira, T. R. T., Ostry, D. A., & Rogers, J. (2023). Firm Financial Conditions and the Transmission of Monetary Policy. *Finance and Economics Discussion Series*, (2023-037), 1–108.

Fornari, F., & Groß, J. (2024, December). Green and Glowing or Brown in Disguise? How do monetary policy shocks shape the cross section of equity returns?

Garofalo, M., & Poyntner, P. (2025). The international spillovers of corporate bonds purchases. *Working Paper*.

Gertler, M., & Gilchrist, S. (1994). Monetary Policy, Business Cycles, and the Behavior of Small Manufacturing Firms*. *The Quarterly Journal of Economics*, *109*(2), 309–340.

Gertler, M., & Karadi, P. (2011). A model of unconventional monetary policy. *Journal of Monetary Economics*, *58*(1), 17–34.

Gertler, M., & Karadi, P. (2013). QE 1 vs. 2 vs. 3. . . : A Framework for Analyzing Large-Scale Asset Purchases as a Monetary Policy Tool. *International Journal of Central Banking*, *9*, 49.

Gertler, M., & Karadi, P. (2015). Monetary Policy Surprises, Credit Costs, and Economic Activity. *American Economic Journal: Macroeconomics*, *7*(1), 44–76.

Giannone, D., & Primiceri, G. (2024). The Drivers of Post-Pandemic Inflation. *NBER Working Paper*.

Giovanardi, F., Kaldorf, M., Radke, L., & Wicknig, F. (2023). The preferential treatment of green bonds. *Review of Economic Dynamics*, *51*, 657–676.

Gopinath, G., & Itskhoki, O. (2022, January). Dominant Currency Paradigm: A review. In G. Gopinath, E. Helpman, & K. Rogoff (Eds.), *Handbook of International Economics* (pp. 45–90, Vol. 6). Elsevier.

Havrylchyk, O., & Pourabbasvafa, P. (2025, January). Firms' Carbon Emissions and Monetary Policy.

Hegarty, C., Moretti, M., Ottonello, P., & Perez, D. J. (2023). Global Borrowing Costs and Firms' Risk in Open Economies.

Hines, J. R. (2010). Treasure Islands. *Journal of Economic Perspectives*, *24*(4), 103–126.

International Monetary Fund. (2022). *Fiscal Monitor, October 2022: Helping People Bounce Back*.

Jarociński, M., & Karadi, P. (2020). Deconstructing Monetary Policy Surprises— The Role of Information Shocks. *American Economic Journal: Macroeconomics*, *12*(2), 1–43.

Jeenas, P. (2018). Monetary Policy Shocks, Financial Structure, and Firm Activity: A Panel Approach. *SSRN Electronic Journal*.

Jordà, Ò. (2005). Estimation and Inference of Impulse Responses by Local Projections. *American Economic Review*, *95*(1), 161–182.

Kalemli-Ozcan, S. (2019, September). *U.S. Monetary Policy and International Risk Spillovers* (tech. rep. No. w26297). National Bureau of Economic Research. Cambridge, MA.

Macaire, C., & Naef, A. (2021). Greening Monetary Policy: Evidence from the People's Bank of China.

Miranda-Agrippino, S., & Nenova, T. (2022). A tale of two global monetary policies. *Journal of International Economics*, *136*, 103606.

Miranda-Agrippino, S., & Rey, H. (2020). U.S. Monetary Policy and the Global Financial Cycle. *The Review of Economic Studies*, *87*(6), 2754–2776.

Miranda-Agrippino, S., & Ricco, G. (2021). The Transmission of Monetary Policy Shocks. *American Economic Journal: Macroeconomics*, *13*(3), 74–107.

Mongelli, F. P., Pointner, W., & Van Den End, J. W. (2022). The Effects of Climate Change on the Natural Rate of Interest: A Critical Survey. *SSRN Electronic Journal*.

Mukhin, D. (2022). An Equilibrium Model of the International Price System. *American Economic Review*, *112*(2), 650–688.

Muller, N. Z. (2021, November). On the Green Interest Rate.

Ottonello, P., & Winberry, T. (2020). Financial Heterogeneity and the Investment Channel of Monetary Policy. *Econometrica*, *88*(6), 2473–2502.

Papoutsi, M., Piazzesi, M., & Schneider, M. (2022). How unconventional is green monetary policy?, 51.

Pástor, Ľ., Stambaugh, R. F., & Taylor, L. A. (2021). Sustainable investing in equilibrium. *Journal of Financial Economics*, *142*(2), 550–571.

Pástor, Ľ., Stambaugh, R. F., & Taylor, L. A. (2022). Dissecting green returns. *Journal of Financial Economics*, *146*(2), 403–424.

Patozi, A. (2023). Green Transmission: Monetary Policy in the Age of ESG. *SSRN Electronic Journal*.

Peters, R. H., & Taylor, L. A. (2017). Intangible capital and the investment-q relation. *Journal of Financial Economics*, *123*(2), 251–272.

Ramey, V. (2016). Macroeconomic Shocks and Their Propagation. In *Handbook of Macroeconomics* (pp. 71–162, Vol. 2). Elsevier.

Rey, H. (2015, May). *Dilemma not Trilemma: The Global Financial Cycle and Monetary Policy Independence* (tech. rep. No. w21162). National Bureau of Economic Research. Cambridge, MA.

Rodnyansky, A., Timmer, Y., & Yago, N. (2022). Intervening against the Fed. *Working Paper*.

Salinger, M., & Summers, L. H. (1983). Tax Reform and Corporate Investment: A Microeconometric Simulation Study. In *Behavioral Simulation Methods in Tax Policy Analysis* (pp. 247–288). University of Chicago Press.

Schmitt-Grohé, S., & Uribe, M. (2003). Closing small open economy models. *Journal of International Economics*, *61*(1), 163–185.

Sims, E., & Wu, J. C. (2021a). Wall Street QE vs. Main Street Lending, 32.

Sims, E., & Wu, J. C. (2021b). Evaluating Central Banks' tool kit: Past, present, and future. *Journal of Monetary Economics*, *118*, 135–160.

van der Beck, P. (2023). Flow-Driven ESG Returns. *Swiss Finance Institute Research Paper No. 21-71*.

Woodford, M. (2001). Fiscal Requirements for Price Stability [Publisher: [Wiley, Ohio State University Press]]. *Journal of Money, Credit and Banking*, *33*(3), 669–728.

Wu, J. C., & Xia, F. D. (2016). Measuring the Macroeconomic Impact of Monetary Policy at the Zero Lower Bound. *Journal of Money, Credit and Banking*, *48*(2-3), 253–291.

Wu, J. C., Xie, Y., & Zhang, J. (2024). The Role of International Financial Integration in Monetary Policy Transmission. *IMF Economic Review*.

Wu, J. C., & Zhang, J. (2016, November). A Shadow Rate New Keynesian Model.

Wu, J. C., & Zhang, J. (2019). Global effective lower bound and unconventional monetary policy. *Journal of International Economics*, *118*, 200–216.

Online Appendix for:

*Green*back? Sustainable Investment and the International Transmission of Monetary Policy

by Marco Garofalo

A.1 Data

A.1.1 Data sources

Trucost's corporate carbon emissions I source firm-level information on carbon emissions from Trucost Environmental suite.

Trucost follows four steps to build its data. First, it links company business segments to activities in its model. Next, it creates an estimated profile for each firm using an environmentally extended input/output (EEIO) model. Then, it gathers public data like regulatory filings, sustainability reports, and third-party sources, correcting any detected errors and inconsistencies. Finally, it works with companies to verify that the data is accurate across several dimensions.

Compustat I source balance-sheet data from Compustat (North America abd Global) Daily Updates - Fundamentals Quarterly via Wharton Research Data Services (WRDS). The time range is from 1961-03 (earliest available) to 2022-08; the data are quarterly and uniquely identified by a firm ID (GVKEY) and year-quarter combination. I screen the panel considering only consolidated accounts, industrial format, standard data format, active and inactive — so there is no "survivor bias" — domestic (US) firms, and calendar (so actual date rather than corresponding fiscal) quarterly entries.

As specified in Section 2.1 in the main text, I trim my Compustat sample in line with related literature. Specifically, I drop financial (SIC 6000-6799) and utilities firms (SIC 4900-4999).

National Accounts I calculate depreciation rates following Ottonello and Winberry (2020) from BEA Interactive Data Tables - National Data - Fixed Assets Accounts Tables.[19] From "Section 3 - Private Fixed Assets by Industry", the Fixed Assets Tables provide data on the stock, investment, and depreciation of (i) equipment, (ii) structures, and (iii) intellectual property products. These data are disaggregated by NAICS industries. From the stock (in billions of dollars) and the depreciation (in billions of dollars), one can calculate the rate as the ratio of the latter over the former. These series, which are industry specific, are at annual frequency, starting from 1947 and ending in 2021 (the latest year available). To go from annual to quarterly frequency, one can assume the same depreciation figures within each year, and use the relationship linking the quarterly (δ_{ind}) to the annual ($\delta_{ind,ann}$) depreciation rate:

$$\delta_{ind} = 1 - (1 - \delta_{ind,ann})^{(1/4)} \tag{8}$$

[19] `https://apps.bea.gov/iTable`.

which just comes from

$$1 - \delta_{ind,ann} = (1 - \delta_{ind})^4 \tag{9}$$

i.e. annual *un-depreciation* equals the product of quarterly *un-depreciations*.

A.1.2 Data cleaning

Following the literature, in particular Ottonello and Winberry (2020), I apply a series of cleaning steps to my dataset.

With the aim to exclude extreme observations or unreasonable values from the final dataset, I make sure that the firm-quarter observations in my sample do not satisfy any of the following conditions:

- Negative capital or assets.
- Acquisitions larger than 5% of assets. Incidentally, this requires that my sample starts in 1983q3, when the data on acquisitions becomes available.
- Investment rate is in the top and bottom 1% of the distribution.
- Net current assets as a share of total assets higher than 10 or below -10.
- Leverage higher than 10 or negative.
- Negative sales or liquidity.

Finally, I winsorise observations of leverage and distance to default at the top and bottom 0.5% of the distribution.

A.1.3 Data coverage

Table A.1: Country coverage of final sample

	2010	2011	2012	2013	2014	2015	2016	2017	2018	2019
AUS	111	110	119	125	132	131	129	138	149	229
AUT	15	16	16	17	16	17	17	17	17	17
BEL	23	23	21	24	25	26	26	26	26	28
BRA	53	56	59	61	62	62	63	66	66	66
CAN	147	152	156	158	164	170	172	179	181	179
CHL	11	12	12	12	12	12	12	12	12	12
CHN	57	48	44	52	63	71	769	856	875	921
COL	3	2	2	3	3	4	4	4	4	4
DEU	105	110	110	109	114	120	121	124	126	137
DNK	27	28	30	24	24	25	22	24	26	28
EGY	7	8	7	10	10	11	13	14	14	15
ESP	33	34	33	32	35	37	37	39	40	39
FIN	29	29	31	33	32	34	36	38	41	43
FRA	110	113	115	120	127	134	133	134	136	140
GBR	132	135	135	132	136	139	140	135	139	181
GRC	12	12	12	13	12	13	13	12	12	13
HUN	3	2	2	2	2	2	4	4	3	2
IDN	31	34	27	26	28	30	29	34	34	36
IND	28	134	140	147	164	172	163	163	181	187
ISR	22	26	24	24	25	25	25	25	28	28
ITA	24	26	26	28	31	34	38	39	41	43
JPN	88	88	86	89	94	97	99	102	93	75
KOR	400	341	346	81	63	56	53	48	48	75
MEX	27	28	27	32	32	31	32	35	34	35
MYS	84	90	98	104	108	112	108	110	114	118
NLD	28	28	25	27	31	28	33	33	34	39
NOR	24	22	24	21	22	23	22	25	25	27
NZL	16	19	20	22	24	23	22	27	28	31
PAK	16	17	18	16	16	16	16	18	18	22
PHL	10	13	16	19	22	22	21	22	22	25
POL	21	23	25	25	26	27	28	30	31	33
PRT	6	6	6	7	7	7	7	7	7	6
RUS	7	10	11	11	12	16	22	21	21	23
SWE	68	73	77	76	79	83	87	109	113	141
THA	67	65	65	72	78	81	87	90	92	92
TUR	26	27	32	34	35	36	37	38	39	41
TWN	33	30	38	51	371	433	444	431	439	642
ZAF	50	51	54	51	50	54	59	60	63	68
AEs	1453	1421	1450	1213	1564	1655	1676	1712	1749	2141
G20	1396	1463	1487	1256	1307	1353	2059	2172	2226	2436
G20 - EMEs	279	388	394	414	446	472	1174	1273	1313	1377
G20 - EMEs ex. China	222	340	350	362	383	401	405	417	438	456
EMEs	501	620	639	677	723	759	1467	1577	1623	1700
Oil Producers	178	184	191	190	198	209	216	225	227	229
BRICS	195	299	308	322	351	375	1076	1166	1206	1265

Source: Trucost and author's calculations. Notes: Number of firms. Country groupings follow IMF classification (International Monetary Fund, 2022). For all details on my final sample, please see Appendix A.1.

Table A.2: Sector coverage of final sample

	2010	2011	2012	2013	2014	2015	2016	2017	2018	2019
Accommodation and Food Services	40	38	39	38	45	47	55	52	54	57
Administrative and Support and Waste Management and Remediation Services	33	34	38	41	45	45	55	58	62	74
Agriculture, Forestry, Fishing and Hunting	23	24	24	25	27	25	33	34	34	34
Arts, Entertainment, and Recreation	13	17	19	18	19	19	21	22	22	25
Construction	111	114	117	109	113	120	136	141	142	149
Educational Services	7	7	7	7	8	11	12	14	15	15
Health Care and Social Assistance	20	22	23	25	26	33	42	45	49	54
Information	155	157	154	151	162	172	204	216	231	282
Manufacturing	1007	1066	1100	904	1189	1260	1753	1824	1861	2160
Mining, Quarrying, and Oil and Gas Extraction	128	137	142	140	147	147	170	180	185	201
Professional, Scientific, and Technical Services	110	119	130	126	144	155	213	227	240	266
Retail Trade	123	121	113	120	138	148	175	197	195	225
Transportation and Warehousing	87	91	92	95	113	120	140	141	147	149
Wholesale Trade	97	94	91	91	111	112	134	138	135	150

Source: Trucost and author's calculations. Notes: Number of firms. For all details on my final sample, please see Appendix A.1.

Table A.3: Summary statistics of main firm-level covariates - Scope 2

(a) All firms	Mean	Median	S.D.	95th P.	Max	N
Emissions	267913.01	33832.27	1610759.21	1200824.00	1.56e+08	55083
Emissions/revenues	65.67	25.29	355.75	235.41	41724.72	55083
dlnk	0.01	-0.00	0.06	0.10	0.52	90809
Real sales growth	0.02	0.00	0.32	0.39	10.33	87560
Age	58.01	40.00	56.18	155.00	653.00	54028
Real total assets	6.84	6.79	1.79	9.92	12.84	90809
Current assets ratio	0.46	0.45	0.21	0.81	1.00	90809
Leverage	0.25	0.24	0.18	0.54	6.98	90809
d2d	7.55	6.54	5.24	16.80	81.58	63994
Che assets ratio	0.13	0.10	0.13	0.37	0.98	90740
USD share	0.11	0.00	0.28	0.96	1.00	70876
EUR share	0.02	0.00	0.13	0.01	1.00	70876

(b) Brown firms	Mean	Median	S.D.	95th P.	Max	N
Emissions	415499.29	62903.20	2213610.43	1915923.46	1.56e+08	28035
Emissions/revenues	105.17	44.98	493.85	328.82	41724.72	28035
dlnk	0.01	-0.00	0.06	0.09	0.52	28035
Real sales growth	0.01	0.00	0.27	0.32	10.33	27485
Age	58.01	39.00	53.44	155.00	409.00	27389
Real total assets	7.63	7.58	1.59	10.33	12.61	28035
Current assets ratio	0.40	0.39	0.19	0.74	1.00	28035
Leverage	0.27	0.26	0.16	0.55	2.67	28035
d2d	7.56	6.66	5.12	16.22	81.58	22419
Che assets ratio	0.12	0.09	0.11	0.32	0.96	28017
USD share	0.14	0.00	0.31	0.98	1.00	22911
EUR share	0.03	0.00	0.13	0.06	1.00	22911

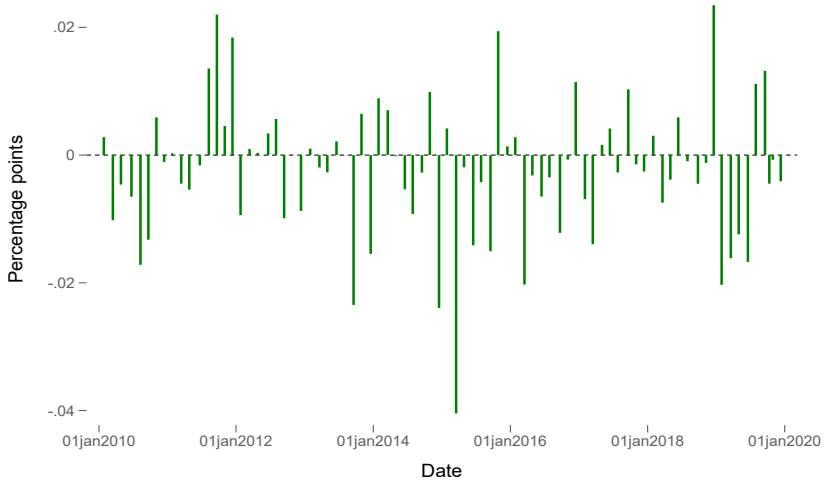
(c) Green firms	Mean	Median	S.D.	95th P.	Max	N
Emissions	114941.21	18186.50	398752.20	482766.00	9618669.97	27048
Emissions/revenues	24.73	15.26	40.69	65.35	1197.18	27048
dlnk	0.01	-0.00	0.06	0.11	0.52	62774
Real sales growth	0.02	0.01	0.35	0.42	9.83	60075
Age	58.02	40.00	58.86	154.00	653.00	26639
Real total assets	6.49	6.45	1.76	9.58	12.84	62774
Current assets ratio	0.48	0.48	0.21	0.82	1.00	62774
Leverage	0.25	0.23	0.19	0.53	6.98	62774
d2d	7.55	6.47	5.31	17.11	69.86	41575
Che assets ratio	0.14	0.10	0.13	0.39	0.98	62723
USD share	0.09	0.00	0.26	0.93	1.00	47965
EUR share	0.02	0.00	0.13	0.00	1.00	47965

Source: CIQ, Trucost and author's calculations. Notes: Summary statistics of panel of firms from 2010 to 2019. Emission measures refer to Scope 2. Firms are brown (green) if their emission intensity is larger (smaller) than country-sector median across all years. Listed variables from top to bottom: emission levels (tCO2e); carbon intensity (tCO2e/$M); log first-difference change in the (real) firm-level capital stock (decimals); log first-difference change in (real) firm-level sales (decimals); age of the firm (years since foundation); log of real total assets; current assets (share of total assets); debt over assets (share of total assets); distance to default (units of firm's value volatility); cash and short-term investments (share of total assets); USD liabilities (share of total liabilities). For all details on these variables and the cleaning procedures applied to construct my final sample, please see Appendix A.1.

A.2 Further robustness

A.2.1 Fed analysis: monetary surprises

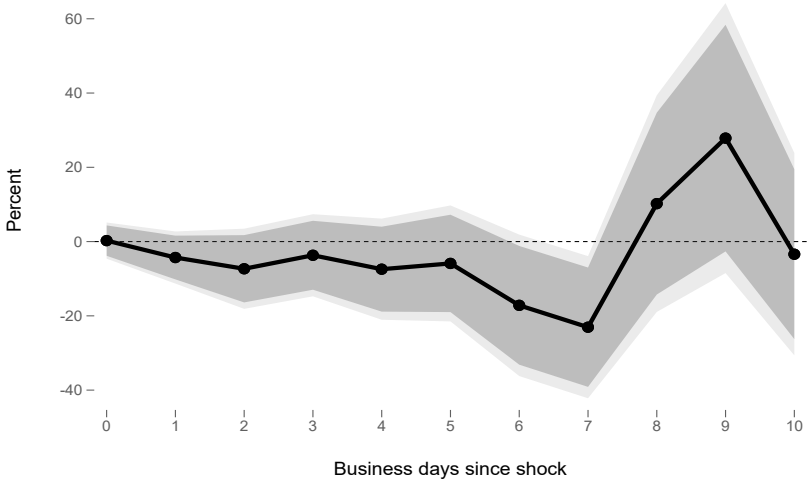
Figure A.1: Monetary policy surprises on Fed announcement dates



Notes: Monetary policy shocks for the period 2010 to 2019, normalised by the market yield on US Treasury securities at 1-year constant maturity. The shock series is based on the "Median Rotation" high frequency monetary policy surprises robust to information effects from Jarociński and Karadi (2020).

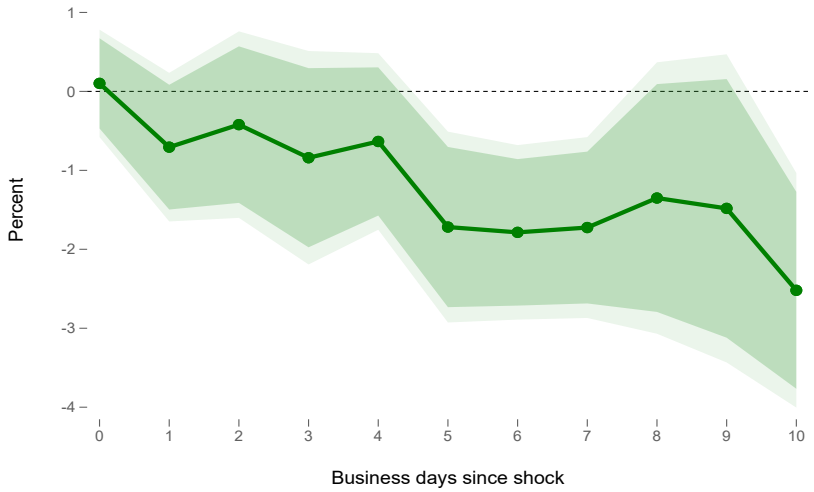
A.3 Fed analysis: stock prices

Figure A.2: Response of non-US stock prices to US monetary policy: average



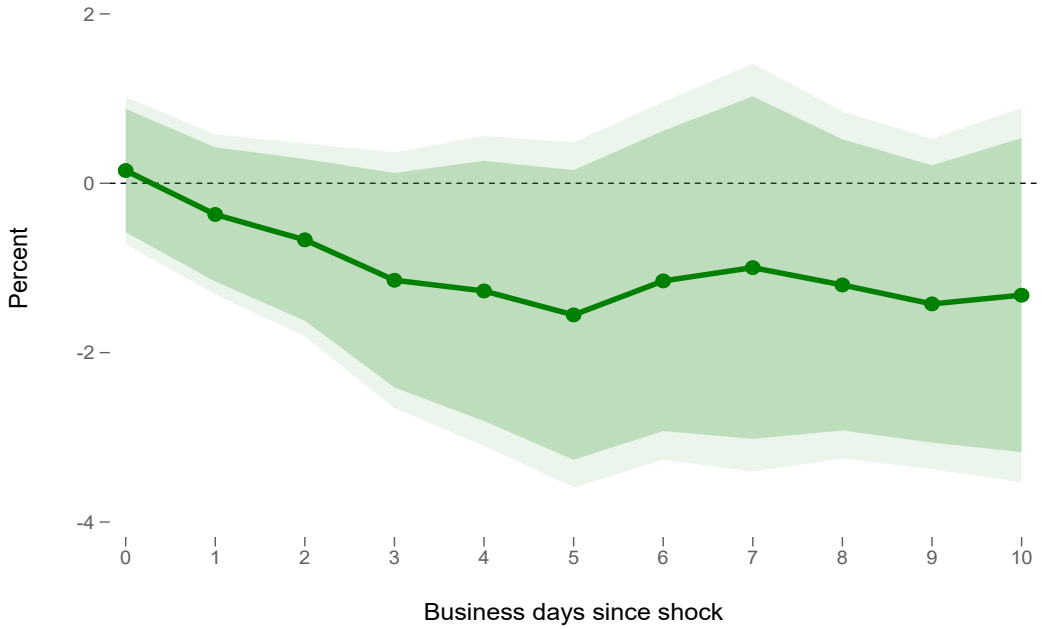
Notes: Dynamics of the coefficient on monetary shocks over time. Shock normalised to be 25 basis points on the market yield on US Treasury Securities at 1-year constant maturity. Connected line reports the estimated coefficient β^h from equation (1). Two areas go from darker to lighter shades corresponding to 90% and 95% confidence intervals. Standard errors clustered at firm and event-day level.

Figure A.3: Response of non-US stock prices to US monetary policy ("Median Rotation" shocks): Scope 1 emission intensity



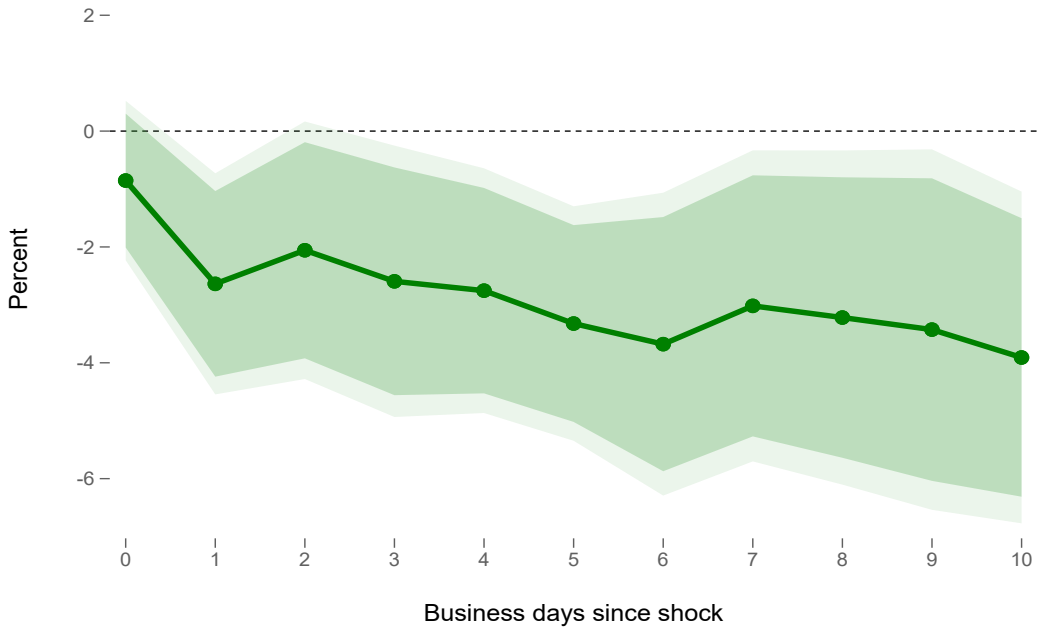
Notes: Dynamics of the interaction coefficient between emission intensity and monetary shocks over time. Shock normalised to be 25 basis points on the market yield on US Treasury Securities at 1-year constant maturity. Connected line reports the estimated coefficient β^h from equation (1). Two areas go from darker to lighter shades corresponding to 90% and 95% confidence intervals. Standard errors clustered at firm and event-day level.

Figure A.4: Response of non-US stock prices to US monetary policy: Scope 2 emission intensity



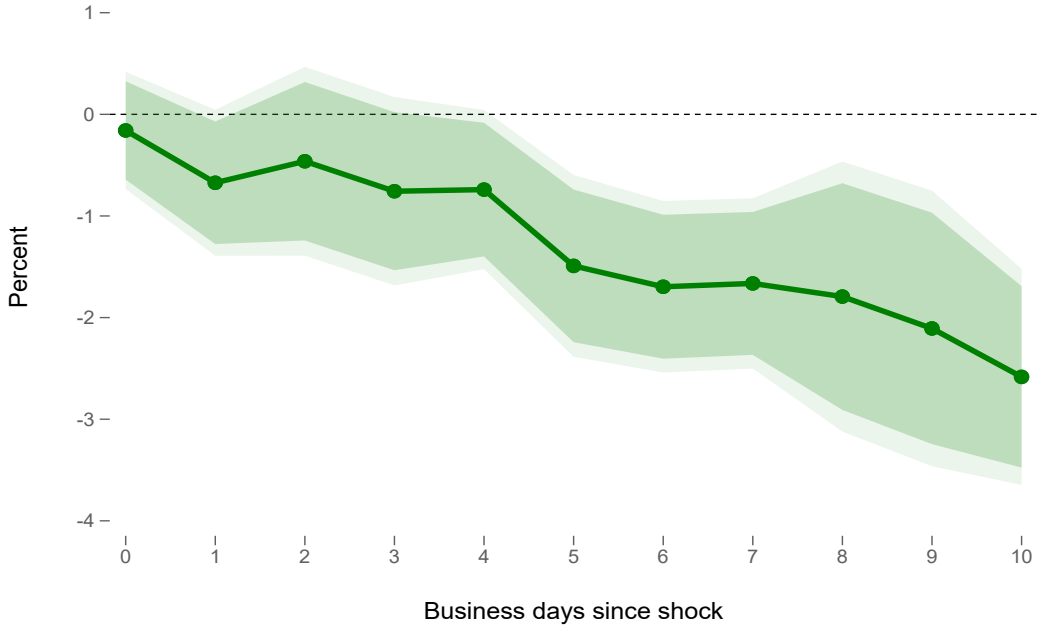
Notes: Dynamics of the interaction coefficient between emission intensity and monetary shocks over time. Shock normalised to be 25 basis points on the market yield on US Treasury Securities at 1-year constant maturity. Connected line reports the estimated coefficient β^h from equation (1). Two areas go from darker to lighter shades corresponding to 90% and 95% confidence intervals. Standard errors clustered at firm and event-day level.

Figure A.5: Response of non-US stock prices to US monetary policy: brown vs. green firms (Scope 1)



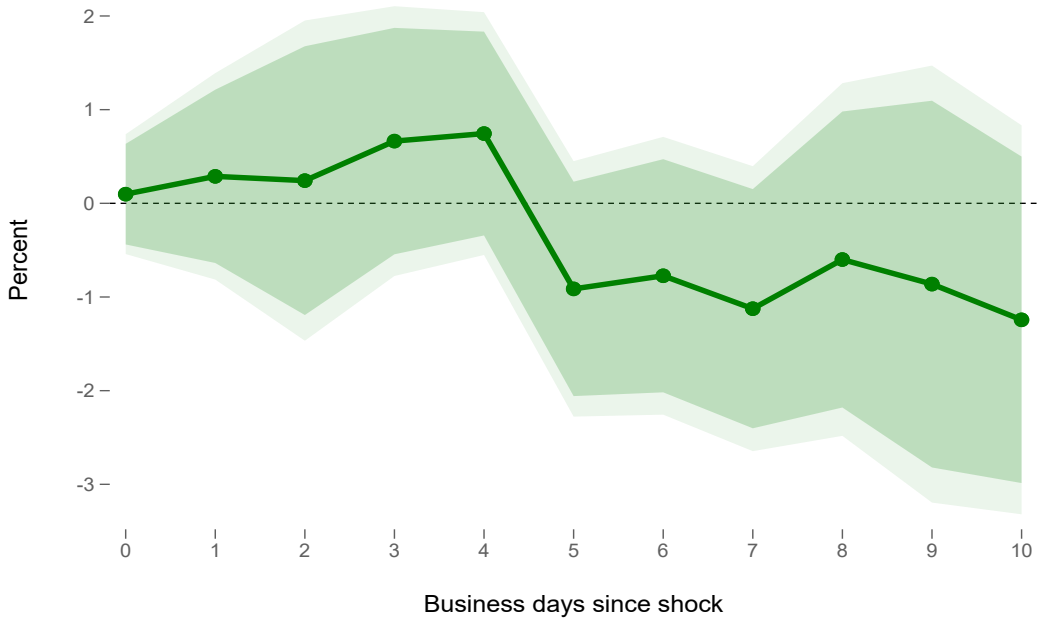
Notes: Dynamics of the interaction coefficient between brown dummy based on emission intensity and monetary shocks over time. Shock normalised to be 25 basis points on the market yield on US Treasury Securities at 1-year constant maturity. Connected line reports the estimated coefficient β^h from equation (1). Two areas go from darker to lighter shades corresponding to 90% and 95% confidence intervals. Standard errors clustered at firm and event-day level.

Figure A.6: Response of non-US stock prices to US monetary policy: Scope 1 absolute emissions



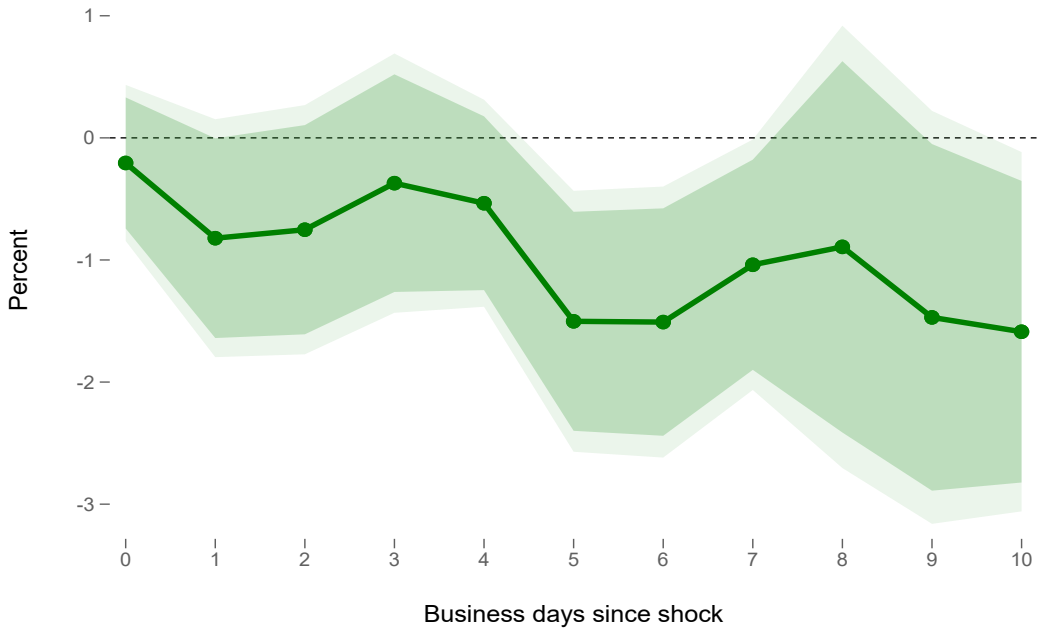
Notes: Dynamics of the interaction coefficient between absolute emissions and monetary shocks over time. Shock normalised to be 25 basis points on the market yield on US Treasury Securities at 1-year constant maturity. Connected line reports the estimated coefficient β^h from equation (1). Two areas go from darker to lighter shades corresponding to 90% and 95% confidence intervals. Standard errors clustered at firm and event-day level.

Figure A.7: Response of non-US stock prices to US monetary policy: Scope 1 emission intensity (reporting only)



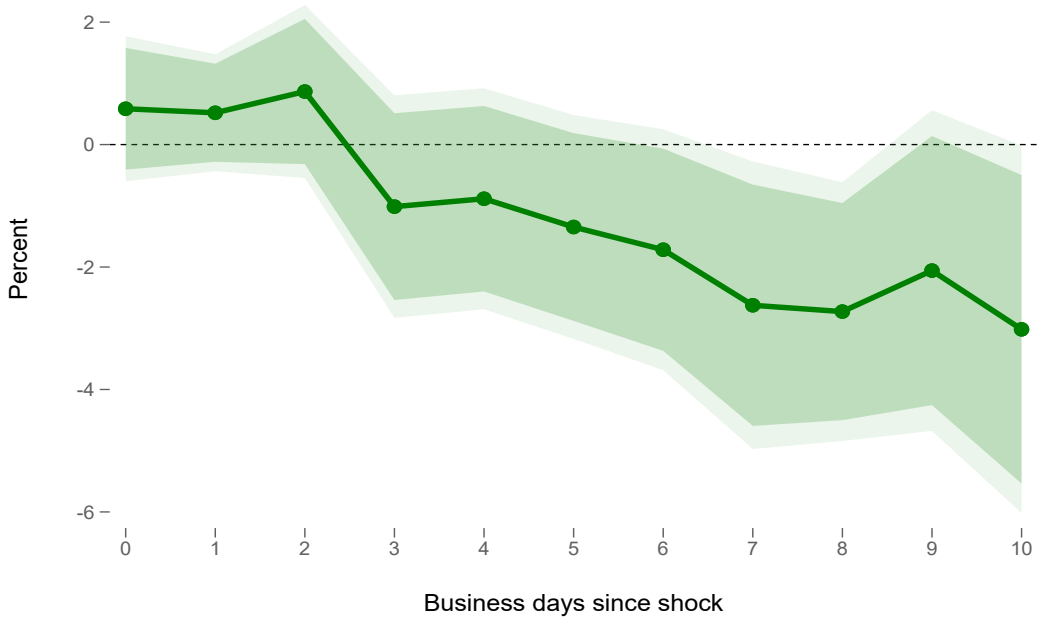
Notes: Dynamics of the interaction coefficient between emission intensity and monetary shocks over time. Shock normalised to be 25 basis points on the market yield on US Treasury Securities at 1-year constant maturity. Connected line reports the estimated coefficient β^h from equation (1). Two areas go from darker to lighter shades corresponding to 90% and 95% confidence intervals. Standard errors clustered at firm and event-day level.

Figure A.8: Response of non-US stock prices to US monetary policy: Scope 1 emission intensity (AEs)



Notes: Dynamics of the interaction coefficient between emission intensity and monetary shocks over time. Shock normalised to be 25 basis points on the market yield on US Treasury Securities at 1-year constant maturity. Connected line reports the estimated coefficient β^h from equation (1). Two areas go from darker to lighter shades corresponding to 90% and 95% confidence intervals. Standard errors clustered at firm and event-day level.

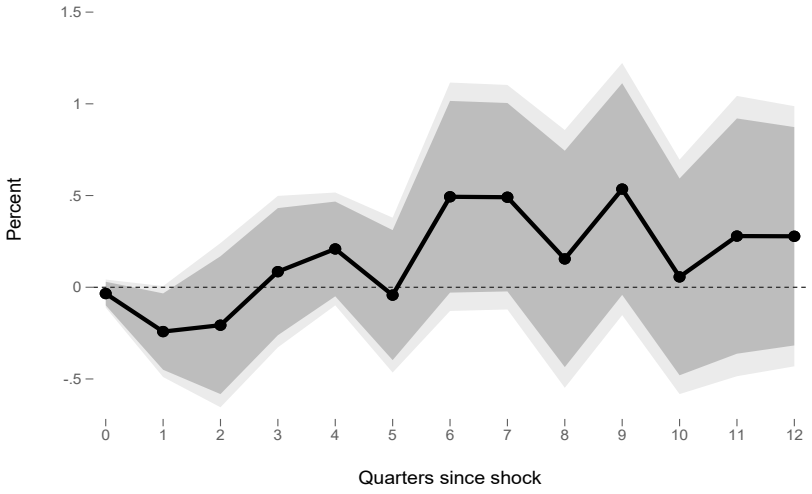
Figure A.9: Response of non-US stock prices to US monetary policy: Scope 1 emission intensity (EMEs)



Notes: Dynamics of the interaction coefficient between emission intensity and monetary shocks over time. Shock normalised to be 25 basis points on the market yield on US Treasury Securities at 1-year constant maturity. Connected line reports the estimated coefficient β^h from equation (1). Two areas go from darker to lighter shades corresponding to 90% and 95% confidence intervals. Standard errors clustered at firm and event-day level.

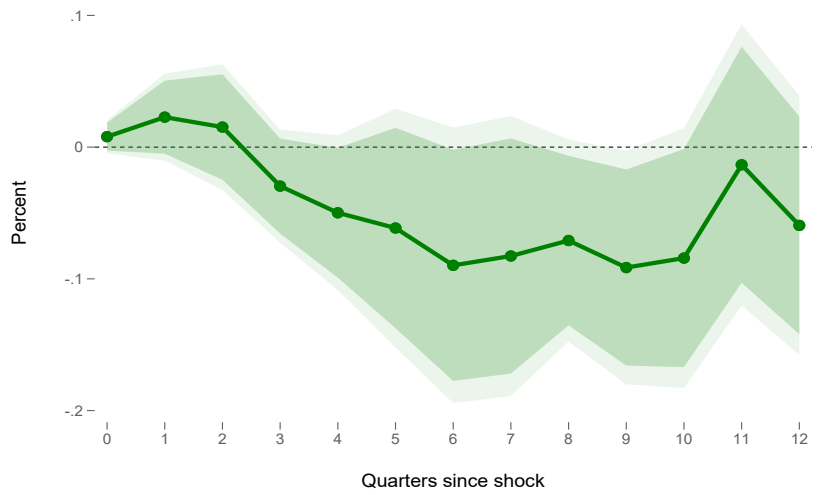
A.4 Fed analysis: investment

Figure A.10: Response of non-US firm-level investment to US monetary policy: average



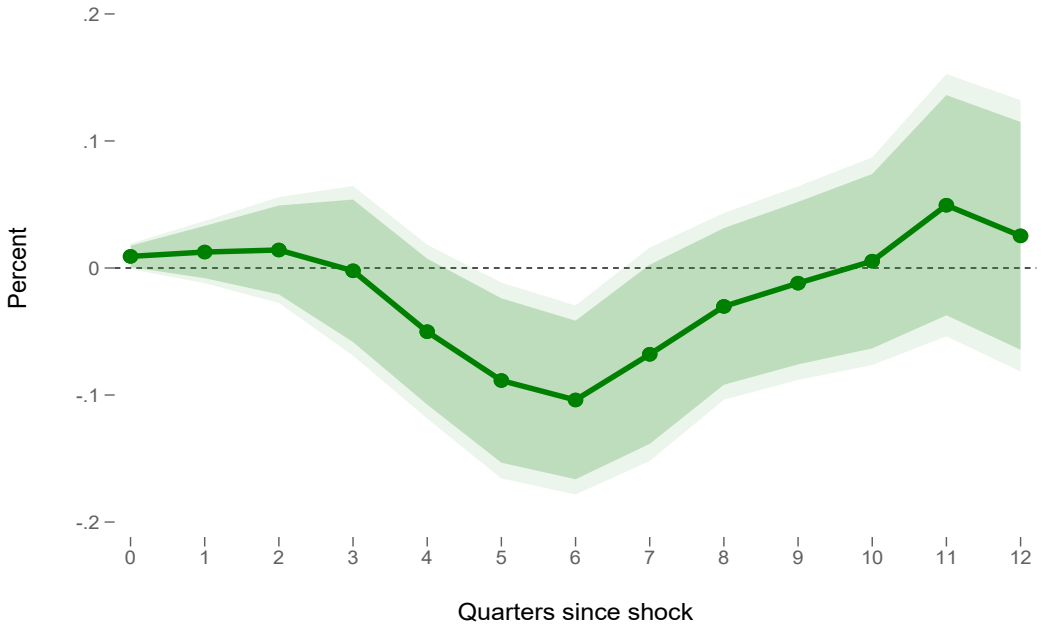
Notes: Dynamics of the coefficient on monetary shocks over time. Shock normalised to be 25 basis points on the market yield on US Treasury Securities at 1-year constant maturity. Connected line reports the estimated coefficient β^h from equation (2). Two areas go from darker to lighter shades corresponding to 90% and 95% confidence intervals. Standard errors clustered at firm and event-day level.

Figure A.11: Response of non-US firm-level investment to US monetary policy ("Median Rotation" shocks): Scope 1 emission intensity



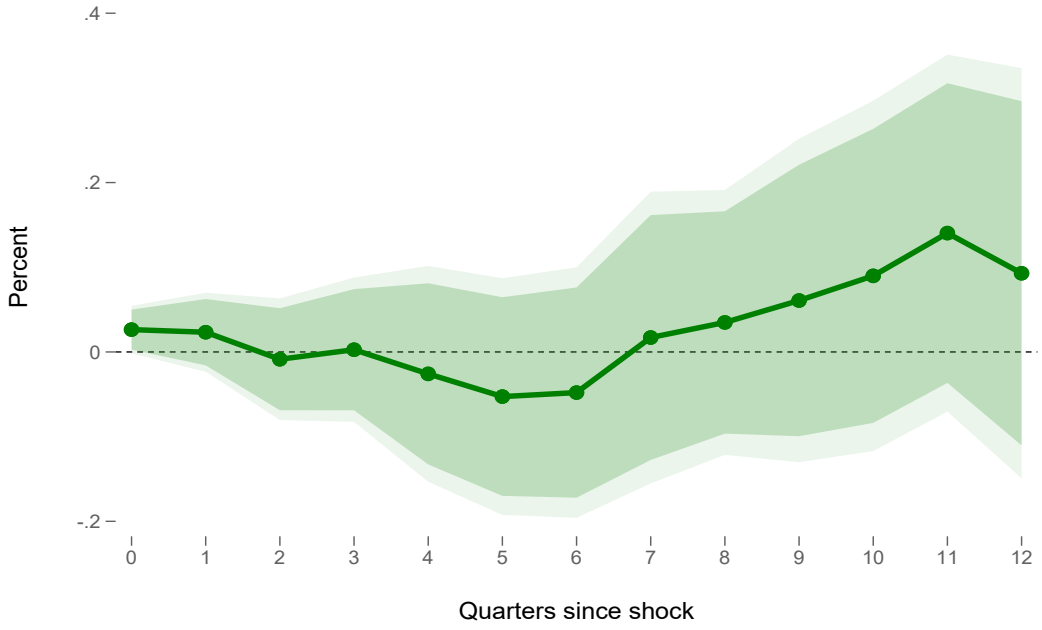
Notes: Dynamics of the interaction coefficient between emission intensity and monetary shocks over time. Shock normalised to be 25 basis points on the market yield on US Treasury Securities at 1-year constant maturity. Connected line reports the estimated coefficient β^h from equation (2). Two areas go from darker to lighter shades corresponding to 90% and 95% confidence intervals. Standard errors clustered at firm and event-day level.

Figure A.12: Response of non-US firm-level investment to US monetary policy: Scope 2 emission intensity



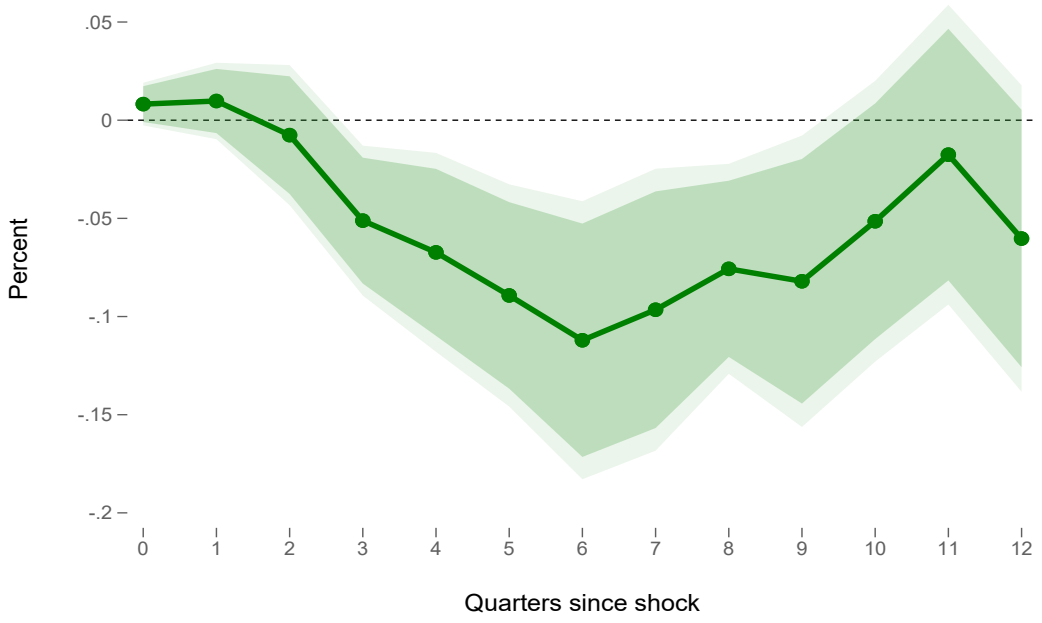
Notes: Dynamics of the interaction coefficient between emission intensity and monetary shocks over time. Shock normalised to be 25 basis points on the market yield on US Treasury Securities at 1-year constant maturity. Connected line reports the estimated coefficient β^h from equation (2). Two areas go from darker to lighter shades corresponding to 90% and 95% confidence intervals. Standard errors clustered at firm and event-day level.

Figure A.13: Response of non-US firm-level investment to US monetary policy: brown vs. green firms (Scope 1)



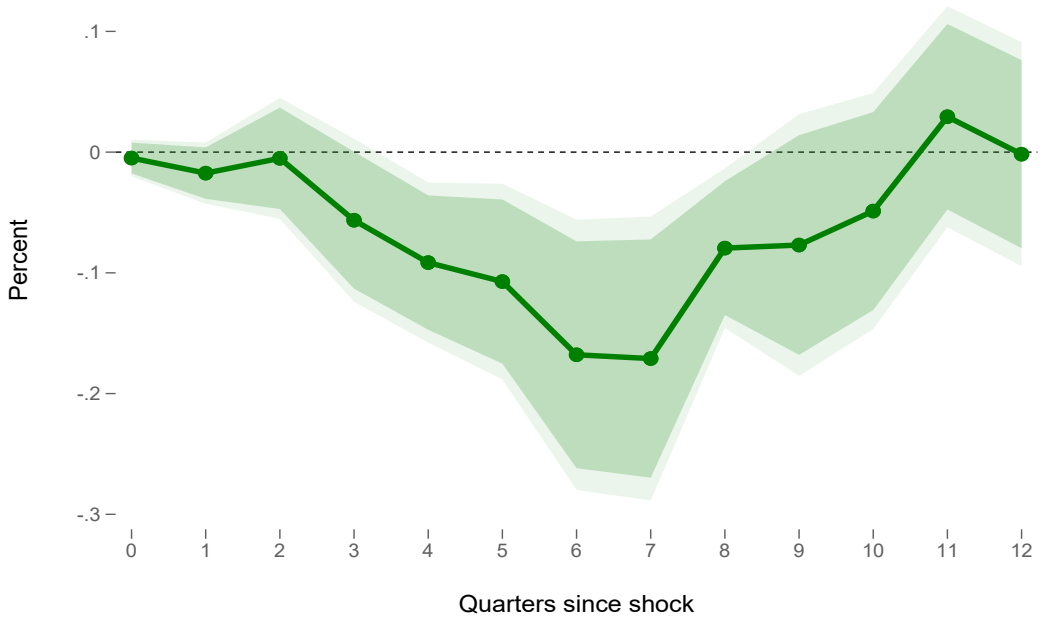
Notes: Dynamics of the interaction coefficient between brown dummy based on emission intensity and monetary shocks over time. Shock normalised to be 25 basis points on the market yield on US Treasury Securities at 1-year constant maturity. Connected line reports the estimated coefficient β^h from equation (2). Two areas go from darker to lighter shades corresponding to 90% and 95% confidence intervals. Standard errors clustered at firm and event-day level.

Figure A.14: Response of non-US firm-level investment to US monetary policy: Scope 1 absolute emissions



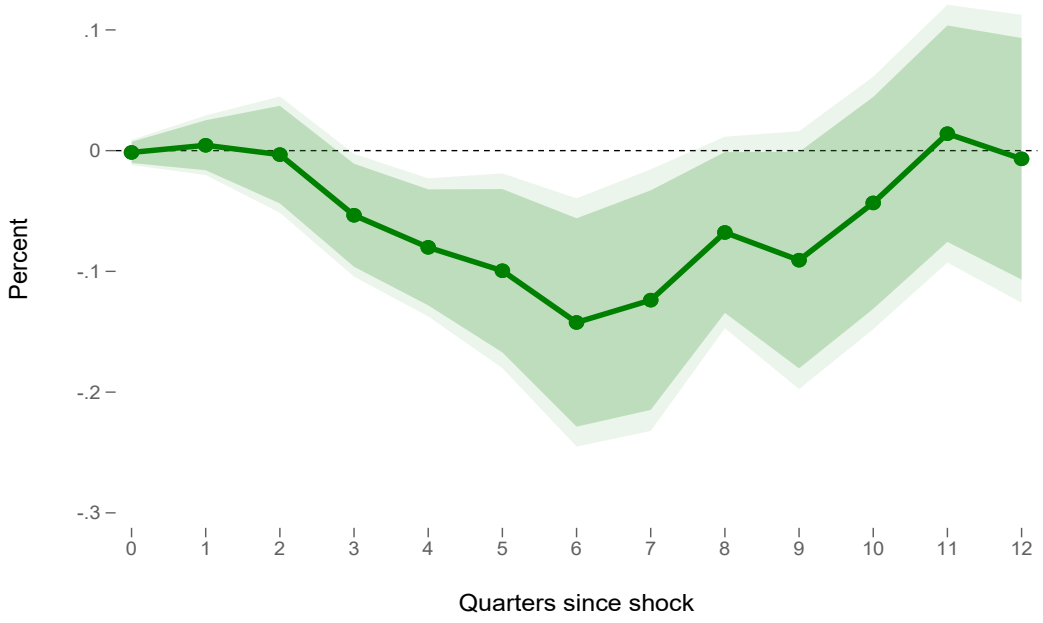
Notes: Dynamics of the interaction coefficient between absolute emissions and monetary shocks over time. Shock normalised to be 25 basis points on the market yield on US Treasury Securities at 1-year constant maturity. Connected line reports the estimated coefficient β^h from equation (2). Two areas go from darker to lighter shades corresponding to 90% and 95% confidence intervals. Standard errors clustered at firm and event-day level.

Figure A.15: Response of non-US firm-level investment to US monetary policy: Scope 1 emission intensity (reporting only)



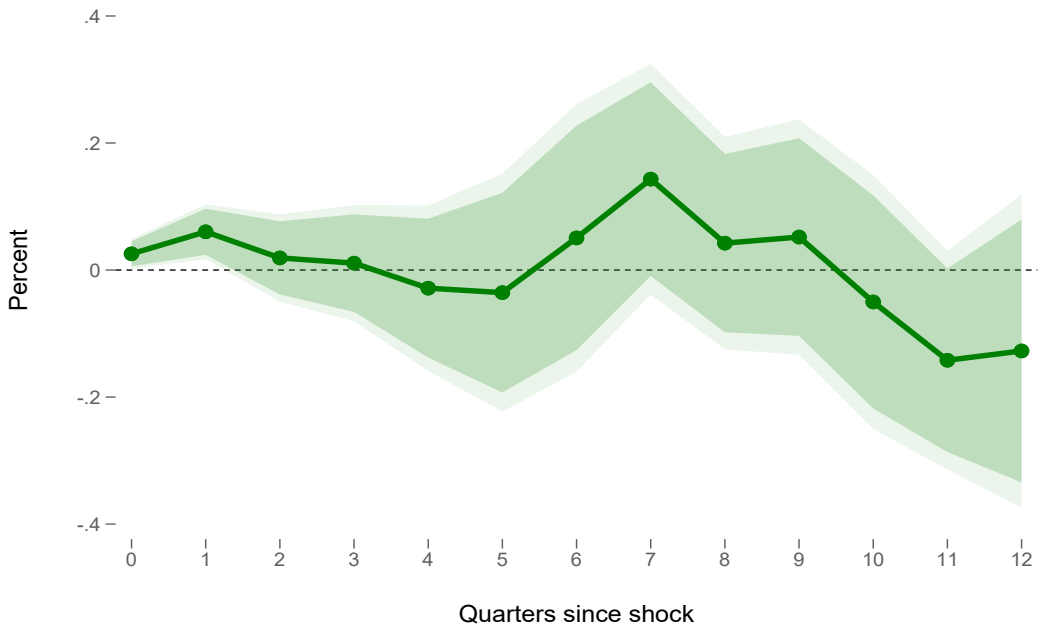
Notes: Dynamics of the interaction coefficient between emission intensity and monetary shocks over time. Shock normalised to be 25 basis points on the market yield on US Treasury Securities at 1-year constant maturity. Connected line reports the estimated coefficient β^h from equation (2). Two areas go from darker to lighter shades corresponding to 90% and 95% confidence intervals. Standard errors clustered at firm and event-day level.

Figure A.16: Response of non-US firm-level investment to US monetary policy: Scope 1 emission intensity (AEs)



Notes: Dynamics of the interaction coefficient between emission intensity and monetary shocks over time. Shock normalised to be 25 basis points on the market yield on US Treasury Securities at 1-year constant maturity. Connected line reports the estimated coefficient β^h from equation (2). Two areas go from darker to lighter shades corresponding to 90% and 95% confidence intervals. Standard errors clustered at firm and event-day level.

Figure A.17: Response of non-US firm-level investment to US monetary policy: Scope 1 emission intensity (EMEs)

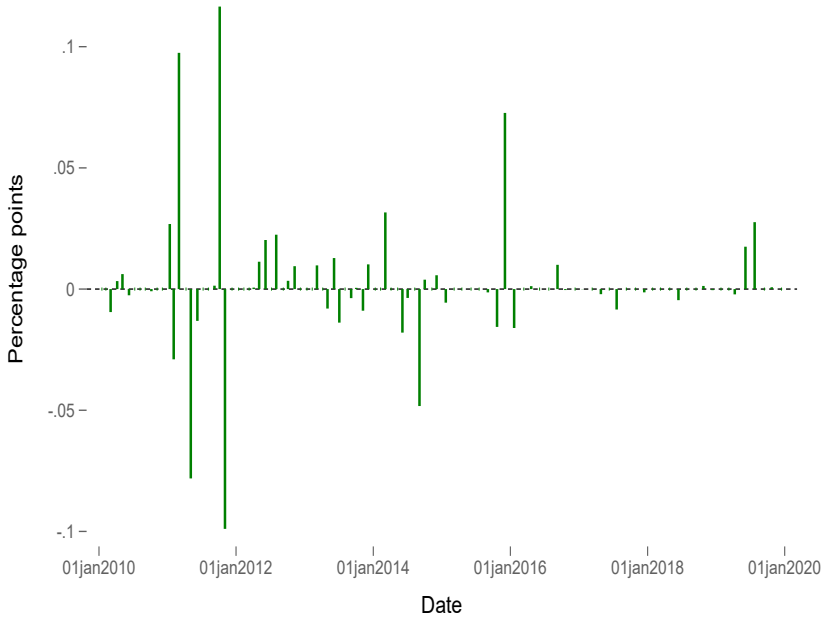


Notes: Dynamics of the interaction coefficient between emission intensity and monetary shocks over time. Shock normalised to be 25 basis points on the market yield on US Treasury Securities at 1-year constant maturity. Connected line reports the estimated coefficient β^h from equation (2). Two areas go from darker to lighter shades corresponding to 90% and 95% confidence intervals. Standard errors clustered at firm and event-day level.

A.4.1 ECB analysis: monetary surprises

For this analysis, given the existence of closed-economy literature for the EA (Bauer et al., 2024; Fornari & Groß, 2024; Havrylchyk & Pourabbasvafa, 2025) and as I am interested in international spillovers, I further exclude from my sample all firms with country of incorporation or address being a member of the Euro Area.[20]

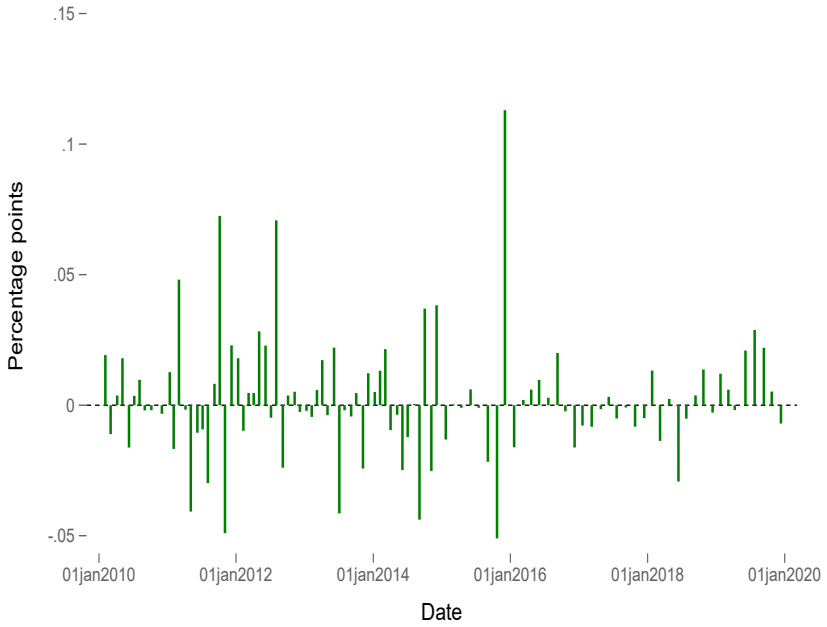
Figure A.18: Monetary policy surprises on ECB announcement dates



Notes: Monetary policy shocks for the period 2010 to 2019, normalised by the market yield on the German one-year government bond. The shock series is based on the "Poor Man's" high frequency monetary policy surprises robust to information effects from Jarociński and Karadi (2020).

[20] See here. These are Austria, Belgium, Cyprus, Estonia, Finland, France, Germany, Greece, Ireland, Italy, Latvia, Lithuania, Luxembourg, Malta, Netherlands, Portugal, Slovakia, Slovenia, Spain.

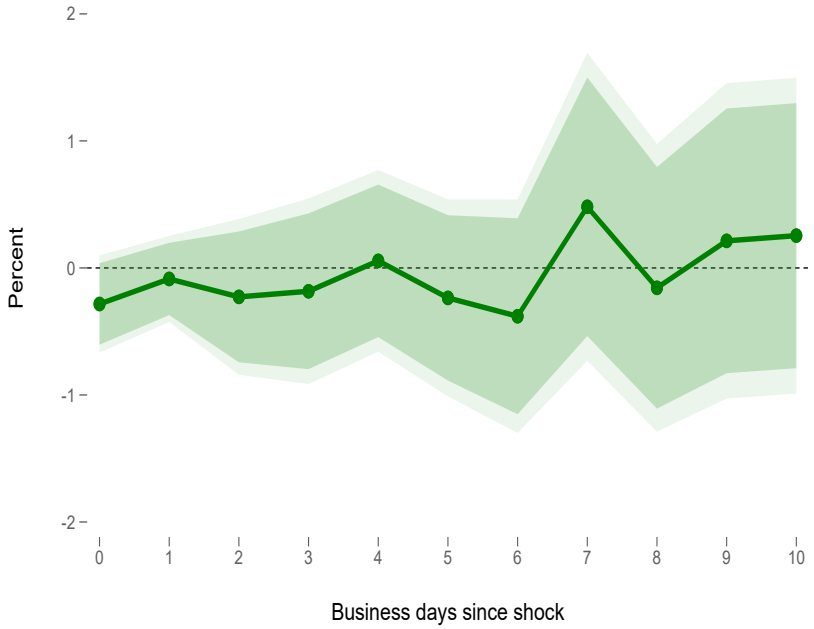
Figure A.19: Monetary policy surprises on ECB announcement dates



Notes: Monetary policy shocks for the period 2010 to 2019, normalised by the market yield on the German one-year government bond. The shock series is based on the "Median Rotation" high frequency monetary policy surprises robust to information effects from Jarociński and Karadi (2020).

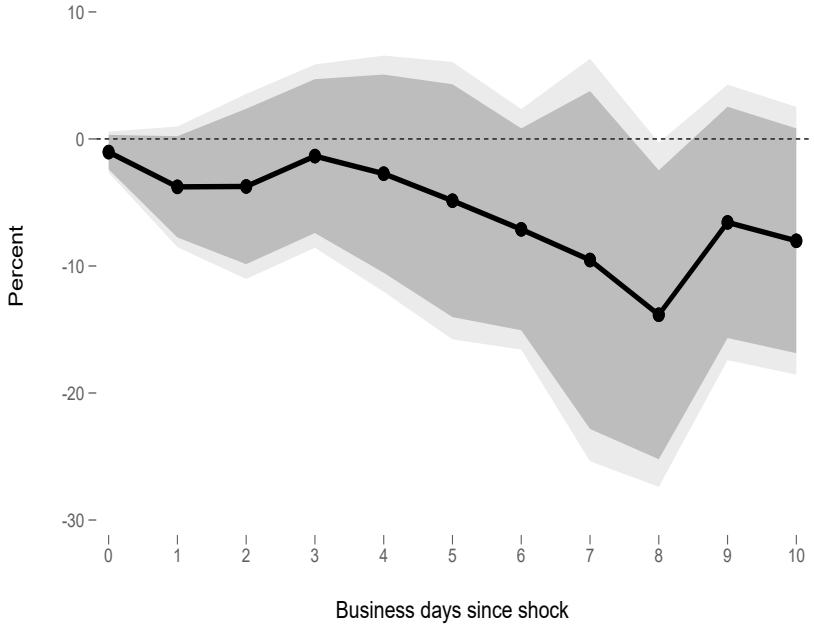
A.5 ECB analysis: stock prices

Figure A.20: Response of non-US non-EA stock prices to ECB monetary policy: Scope 1 emission intensity



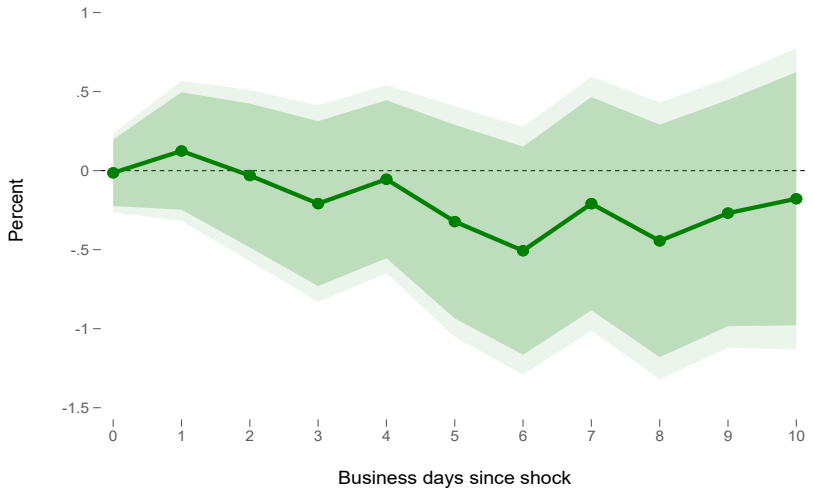
Notes: Dynamics of the interaction coefficient between emission intensity and monetary shocks over time. Shock normalised to be 25 basis points on the German one-year government bond. Connected line reports the estimated coefficient β^h from equation (1). Two areas go from darker to lighter shades corresponding to 90% and 95% confidence intervals. Standard errors clustered at firm and event-day level.

Figure A.21: Response of non-US non-EA stock prices to ECB monetary policy: average



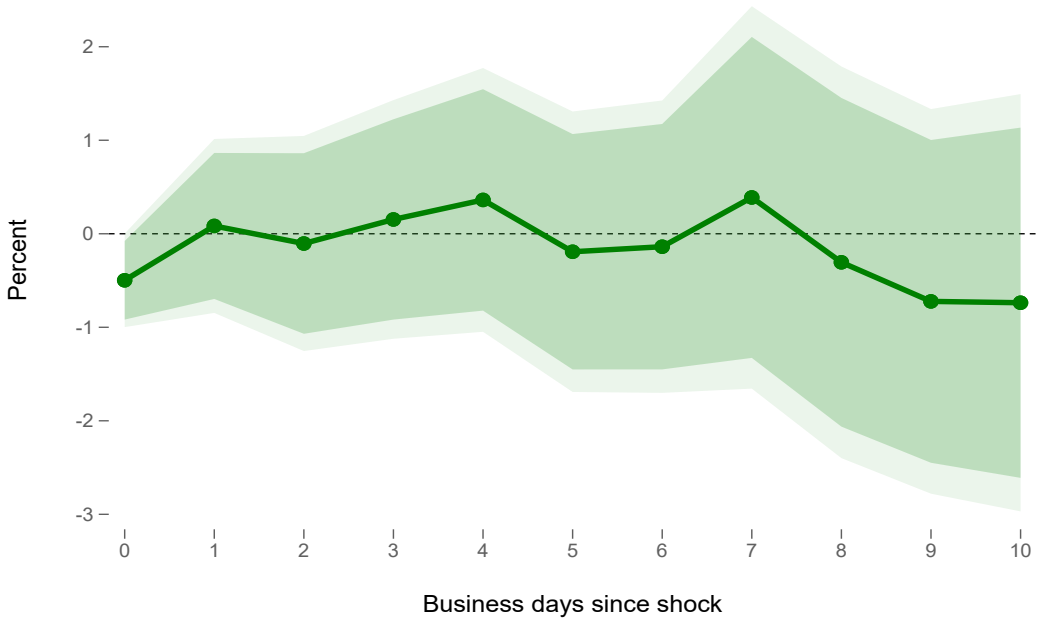
Notes: Dynamics of the coefficient on monetary shocks over time. Shock normalised to be 25 basis points on the German one-year government bond. Connected line reports the estimated coefficient β^h from equation (1). Two areas go from darker to lighter shades corresponding to 90% and 95% confidence intervals. Standard errors clustered at firm and event-day level.

Figure A.22: Response of non-US non-EA stock prices to ECB monetary policy ("Median Rotation" shocks): Scope 1 emission intensity



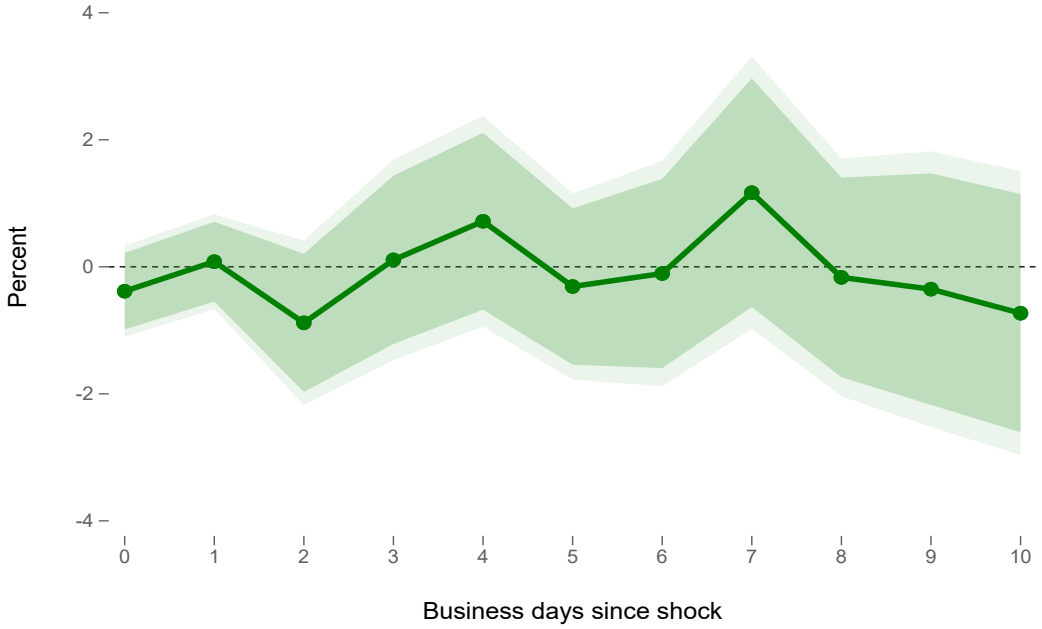
Notes: Dynamics of the interaction coefficient between emission intensity and monetary shocks over time. Shock normalised to be 25 basis points on the German one-year government bond. Connected line reports the estimated coefficient β^h from equation (1). Two areas go from darker to lighter shades corresponding to 90% and 95% confidence intervals. Standard errors clustered at firm and event-day level.

Figure A.23: Response of non-US non-EA stock prices to ECB monetary policy: Scope 2 emission intensity



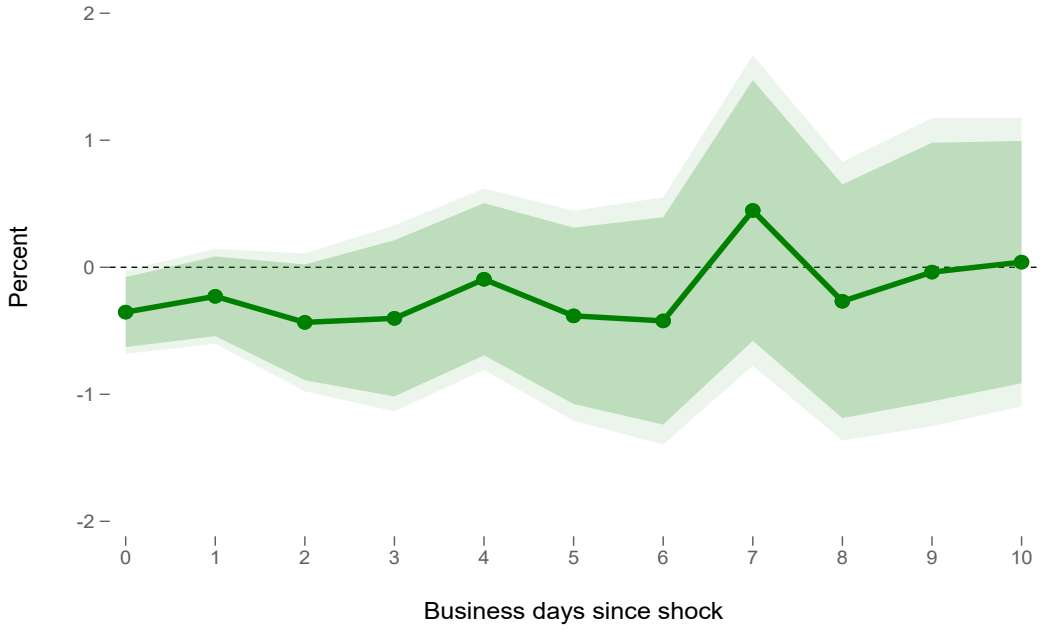
Notes: Dynamics of the interaction coefficient between emission intensity and monetary shocks over time. Shock normalised to be 25 basis points on the German one-year government bond. Connected line reports the estimated coefficient β^h from equation (1). Two areas go from darker to lighter shades corresponding to 90% and 95% confidence intervals. Standard errors clustered at firm and event-day level.

Figure A.24: Response of non-US non-EA stock prices to ECB monetary policy: brown vs. green firms (Scope 1)



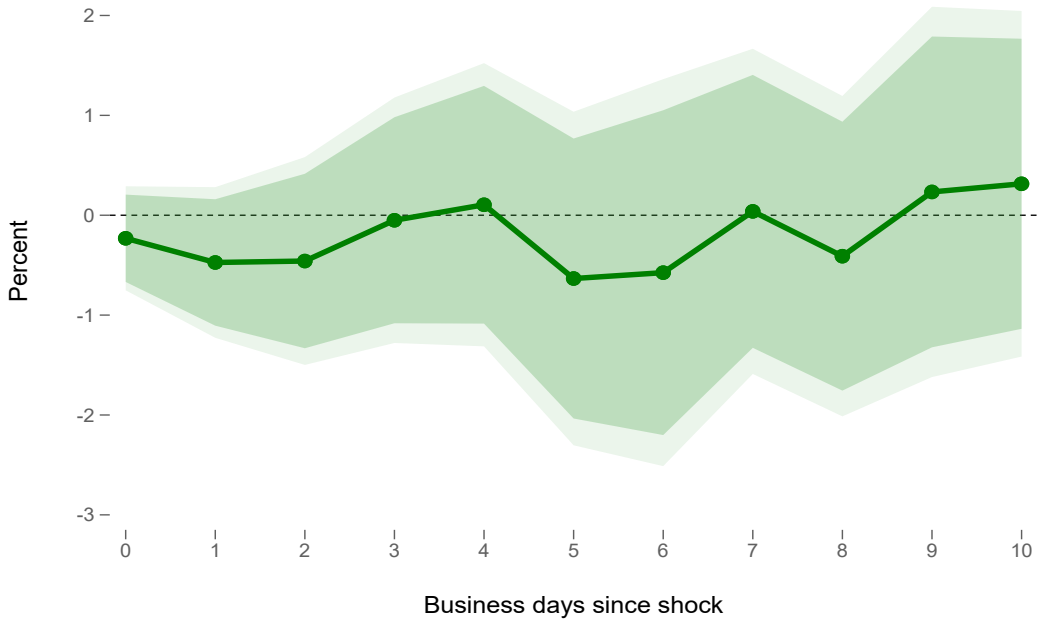
Notes: Dynamics of the interaction coefficient between brown dummy based on emission intensity and monetary shocks over time. Shock normalised to be 25 basis points on the German one-year government bond. Connected line reports the estimated coefficient β^h from equation (1). Two areas go from darker to lighter shades corresponding to 90% and 95% confidence intervals. Standard errors clustered at firm and event-day level.

Figure A.25: Response of non-US non-EA stock prices to ECB monetary policy: Scope 1 absolute emissions



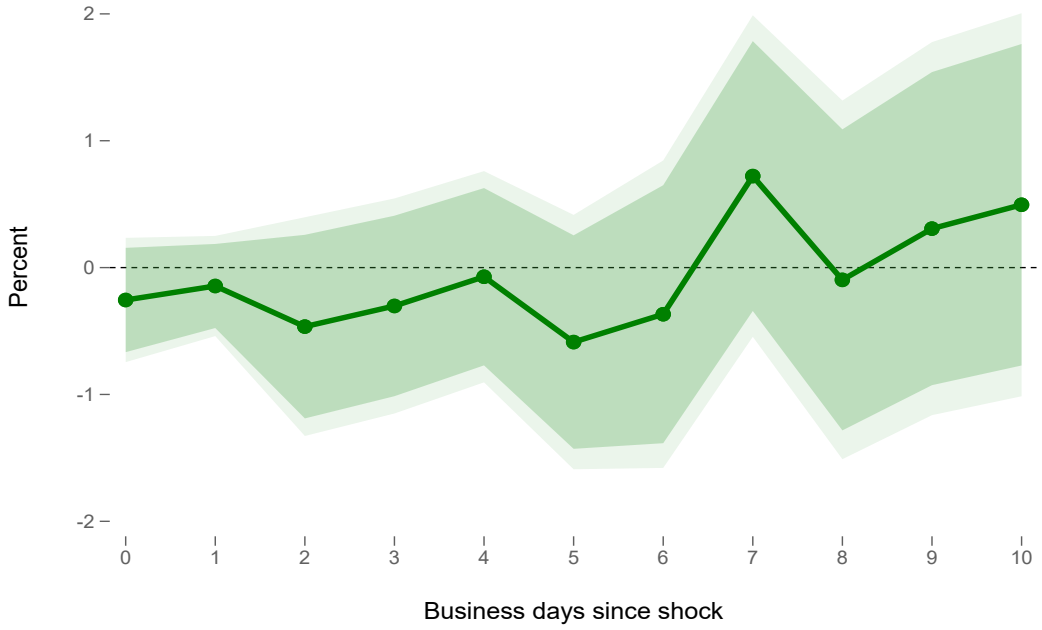
Notes: Dynamics of the interaction coefficient between absolute emissions and monetary shocks over time. Shock normalised to be 25 basis points on the German one-year government bond. Connected line reports the estimated coefficient β^h from equation (1). Two areas go from darker to lighter shades corresponding to 90% and 95% confidence intervals. Standard errors clustered at firm and event-day level.

Figure A.26: Response of non-US non-EA stock prices to ECB monetary policy: Scope 1 emission intensity (reporting only)



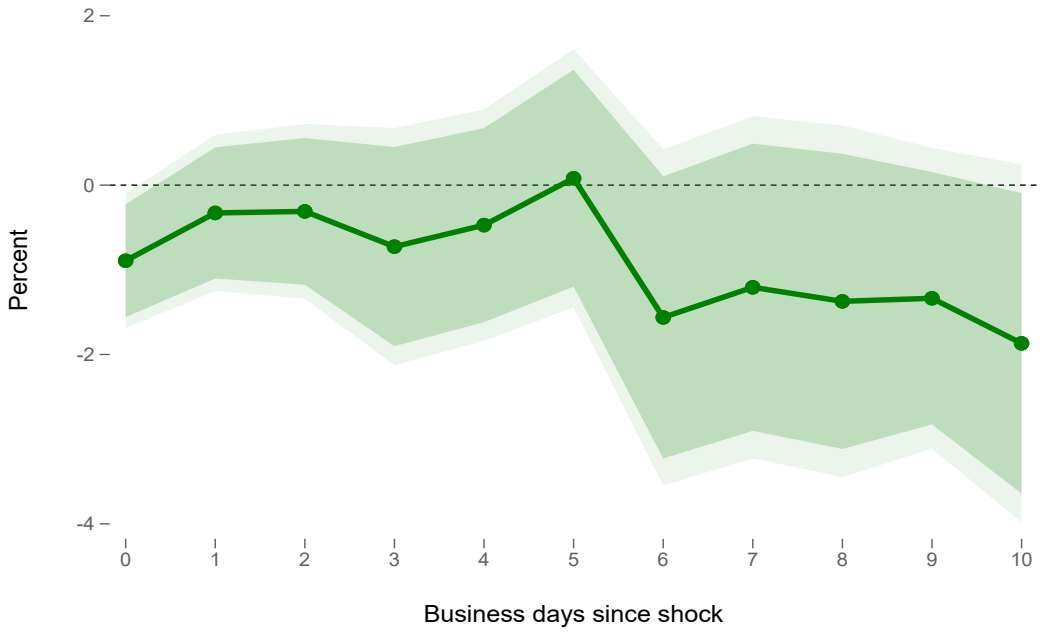
Notes: Dynamics of the interaction coefficient between emission intensity and monetary shocks over time. Shock normalised to be 25 basis points on the German one-year government bond. Connected line reports the estimated coefficient β^h from equation (1). Two areas go from darker to lighter shades corresponding to 90% and 95% confidence intervals. Standard errors clustered at firm and event-day level.

Figure A.27: Response of non-US non-EA stock prices to ECB monetary policy: Scope 1 emission intensity (AEs)



Notes: Dynamics of the interaction coefficient between emission intensity and monetary shocks over time. Shock normalised to be 25 basis points on the German one-year government bond. Connected line reports the estimated coefficient β^h from equation (1). Two areas go from darker to lighter shades corresponding to 90% and 95% confidence intervals. Standard errors clustered at firm and event-day level.

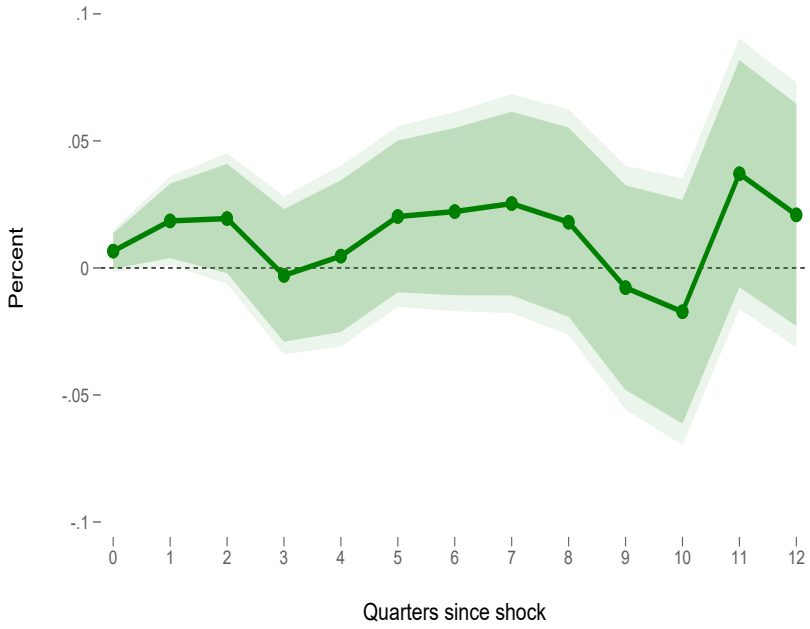
Figure A.28: Response of non-US non-EA stock prices to ECB monetary policy: Scope 1 emission intensity (EMEs)



Notes: Dynamics of the interaction coefficient between emission intensity and monetary shocks over time. Shock normalised to be 25 basis points on the German one-year government bond. Connected line reports the estimated coefficient β^h from equation (1). Two areas go from darker to lighter shades corresponding to 90% and 95% confidence intervals. Standard errors clustered at firm and event-day level.

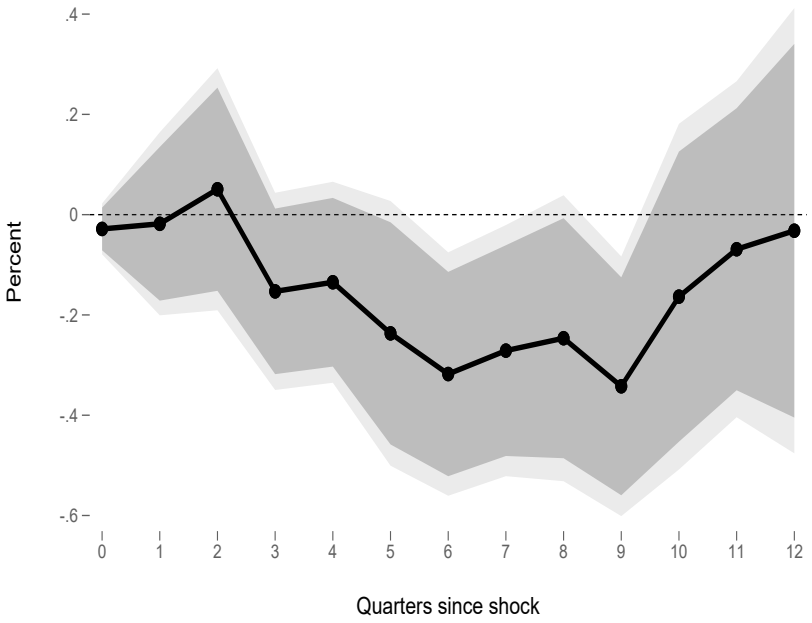
A.6 ECB analysis: investment

Figure A.29: Response of non-US non-EA firm-level investment to ECB monetary policy: Scope 1 emission intensity



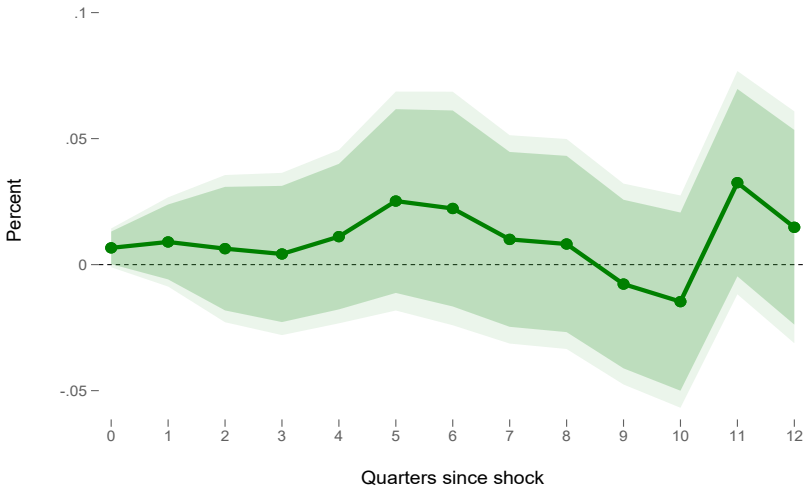
Notes: Dynamics of the interaction coefficient between emission intensity and monetary shocks over time. Shock normalised to be 25 basis points on the German one-year government bond. Connected line reports the estimated coefficient β^h from equation (2). Two areas go from darker to lighter shades corresponding to 90% and 95% confidence intervals. Standard errors clustered at firm and event-day level.

Figure A.30: Response of non-US non-EA firm-level investment to ECB monetary policy: average



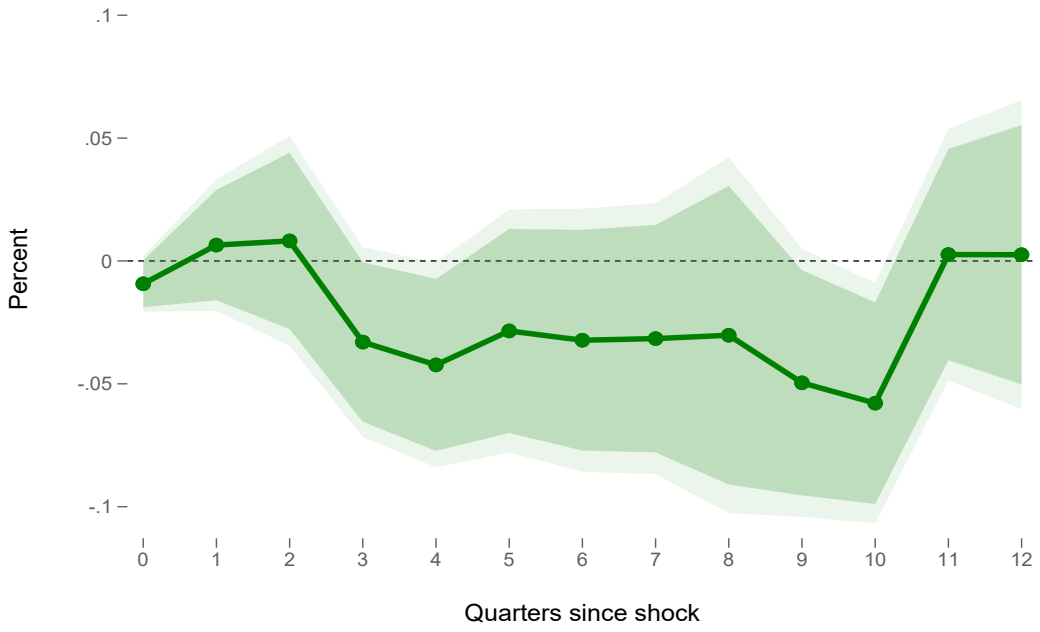
Notes: Dynamics of the coefficient on monetary shocks over time. Shock normalised to be 25 basis points on the German one-year government bond. Connected line reports the estimated coefficient β^h from equation (2). Two areas go from darker to lighter shades corresponding to 90% and 95% confidence intervals. Standard errors clustered at firm and event-day level.

Figure A.31: Response of non-US non-EA firm-level investment to ECB monetary policy ("Median Rotation" shocks): Scope 1 emission intensity



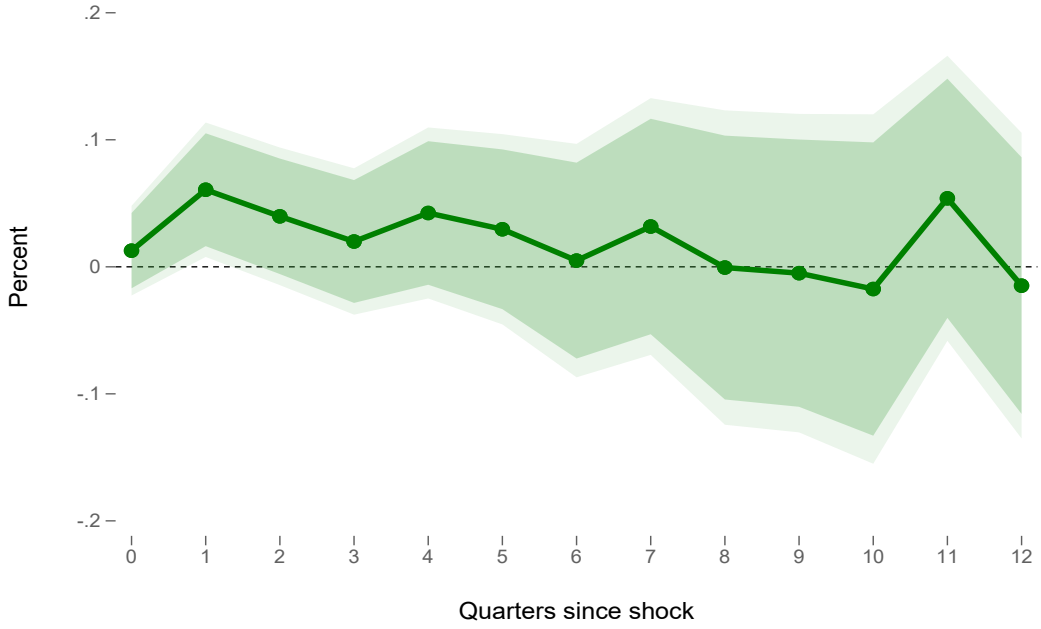
Notes: Dynamics of the interaction coefficient between emission intensity and monetary shocks over time. Shock normalised to be 25 basis points on the German one-year government bond. Connected line reports the estimated coefficient β^h from equation (2). Two areas go from darker to lighter shades corresponding to 90% and 95% confidence intervals. Standard errors clustered at firm and event-day level.

Figure A.32: Response of non-US non-EA firm-level investment to ECB monetary policy: Scope 2 emission intensity



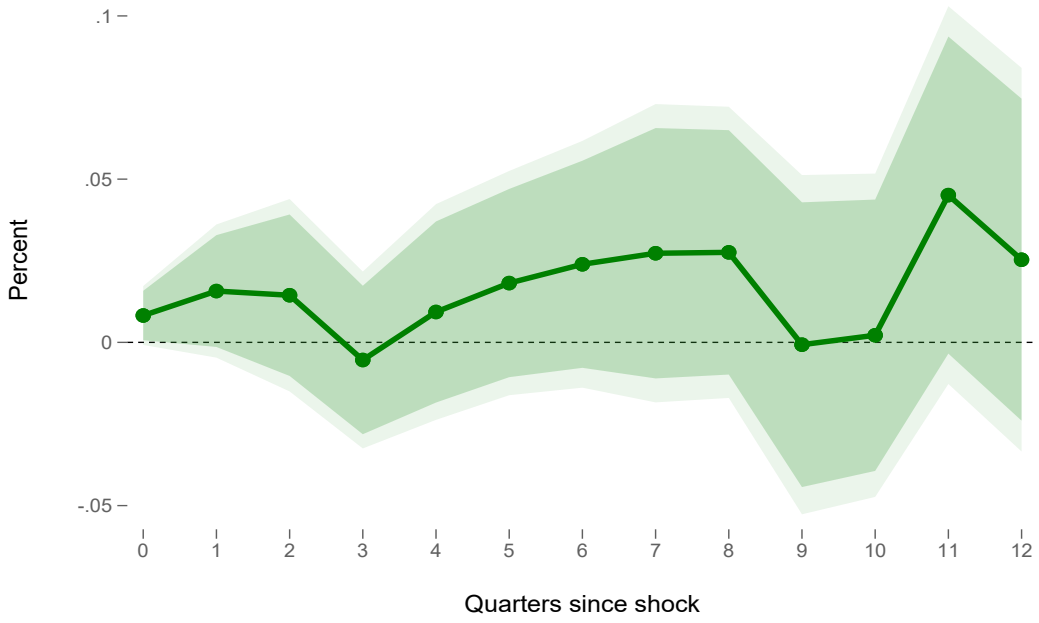
Notes: Dynamics of the interaction coefficient between emission intensity and monetary shocks over time. Shock normalised to be 25 basis points on the German one-year government bond. Connected line reports the estimated coefficient β^h from equation (2). Two areas go from darker to lighter shades corresponding to 90% and 95% confidence intervals. Standard errors clustered at firm and event-day level.

Figure A.33: Response of non-US non-EA firm-level investment to ECB monetary policy: brown vs. green firms (Scope 1)



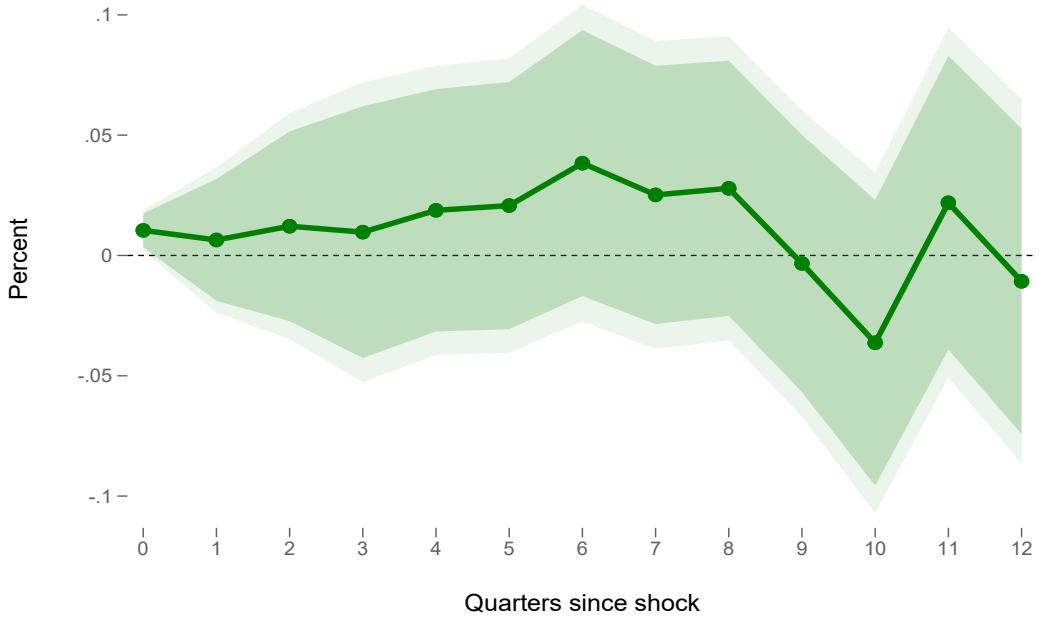
Notes: Dynamics of the interaction coefficient between brown dummy based on emission intensity and monetary shocks over time. Shock normalised to be 25 basis points on the German one-year government bond. Connected line reports the estimated coefficient β^h from equation (2). Two areas go from darker to lighter shades corresponding to 90% and 95% confidence intervals. Standard errors clustered at firm and event-day level.

Figure A.34: Response of non-US non-EA firm-level investment to ECB monetary policy: Scope 1 absolute emissions



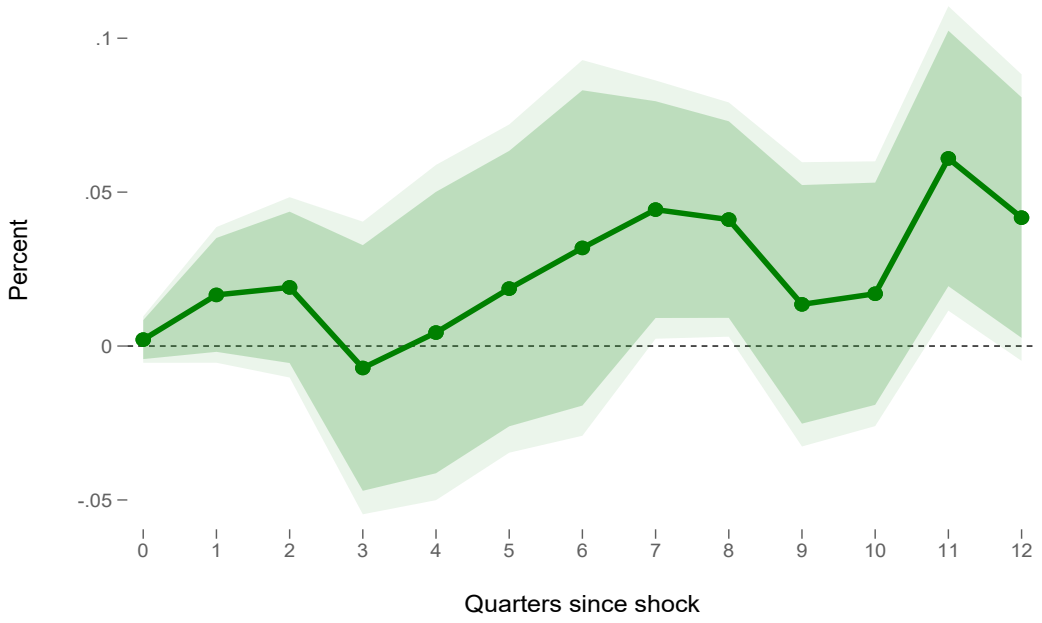
Notes: Dynamics of the interaction coefficient between absolute emissions and monetary shocks over time. Shock normalised to be 25 basis points on the German one-year government bond. Connected line reports the estimated coefficient β^h from equation (2). Two areas go from darker to lighter shades corresponding to 90% and 95% confidence intervals. Standard errors clustered at firm and event-day level.

Figure A.35: Response of non-US non-EA firm-level investment to ECB monetary policy: Scope 1 emission intensity (reporting only)



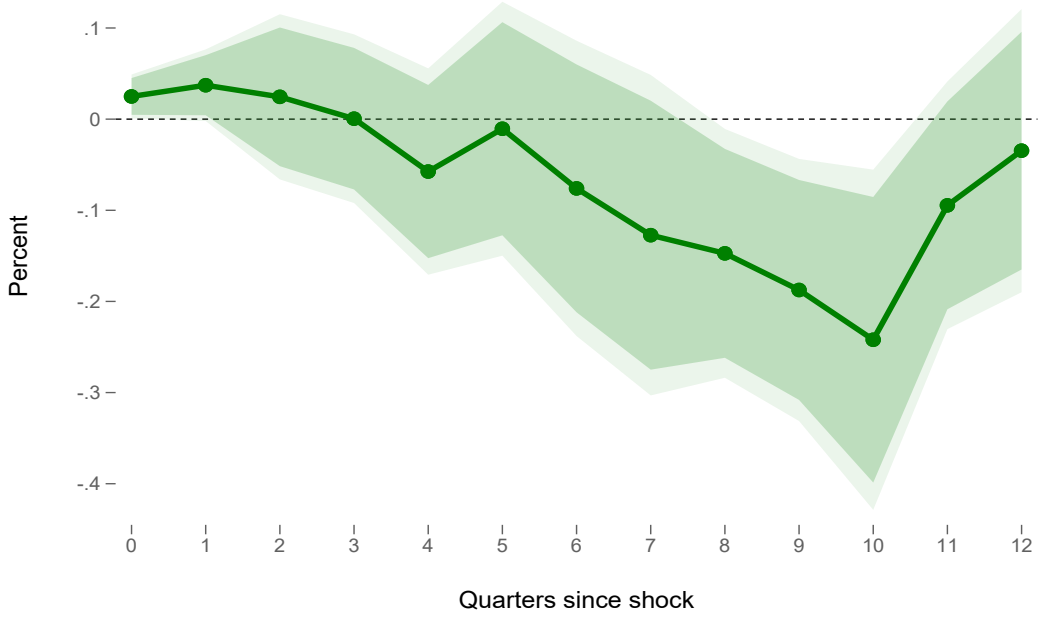
Notes: Dynamics of the interaction coefficient between emission intensity and monetary shocks over time. Shock normalised to be 25 basis points on the German one-year government bond. Connected line reports the estimated coefficient β^h from equation (2). Two areas go from darker to lighter shades corresponding to 90% and 95% confidence intervals. Standard errors clustered at firm and event-day level.

Figure A.36: Response of non-US non-EA firm-level investment to ECB monetary policy: Scope 1 emission intensity (AEs)



Notes: Dynamics of the interaction coefficient between emission intensity and monetary shocks over time. Shock normalised to be 25 basis points on the German one-year government bond. Connected line reports the estimated coefficient β^h from equation (2). Two areas go from darker to lighter shades corresponding to 90% and 95% confidence intervals. Standard errors clustered at firm and event-day level.

Figure A.37: Response of non-US non-EA firm-level investment to ECB monetary policy: Scope 1 emission intensity (EMEs)



Notes: Dynamics of the interaction coefficient between emission intensity and monetary shocks over time. Shock normalised to be 25 basis points on the German one-year government bond. Connected line reports the estimated coefficient β^h from equation (2). Two areas go from darker to lighter shades corresponding to 90% and 95% confidence intervals. Standard errors clustered at firm and event-day level.

A.7 Details on the model

In what follows, I detail the parts of the two-country model used in my analysis. Throughout the exposition recall that: (i) starred variables correspond to the Foreign economy; (ii) the two countries are symmetric bar for their size and parameter values.

Crucially, there are two types of wholesale firms in each country, brown and green. I define *green* in a general sense. They could finance activities of firms producing fewer emissions, or be aimed at the green transition. Ultimately, it does not matter as I do not include a climate block in the model for a feedback through emissions. This is because I want to focus on the effect on US monetary policy transmission of the channel from sustainable preferences only. Thus, in order to zero-in on that, I abstract from other climate-related considerations on the supply and demand effects of emissions, as well as the firm- and aggregate-level risk from the green transition and/or capital scraping or higher incidence of natural disasters. These considerations are, of course, important in the academic and policy debate on the economic effects of climate change, but they are not directly germane to my specific analysis.

A.7.1 Households - Intratemporal decision

Home households consume bundle

$$C_t = \Bigg\{ [\nu\Upsilon]^{\frac{1}{\vartheta}} (C^b_{H,t})^{\frac{\vartheta-1}{\vartheta}} + [(1-\nu)\Upsilon]^{\frac{1}{\vartheta}} (C^b_{F,t})^{\frac{\vartheta-1}{\vartheta}} + [\nu(1-\Upsilon)]^{\frac{1}{\vartheta}} (C^g_{H,t})^{\frac{\vartheta-1}{\vartheta}} + [(1-\nu)(1-\Upsilon)]^{\frac{1}{\vartheta}} (C^g_{F,t})^{\frac{\vartheta-1}{\vartheta}} \Bigg\}^{\frac{\vartheta}{\vartheta-1}} \tag{10}$$

while Foreign households

$$C^*_t = \Bigg\{ [\nu^*\Upsilon^*]^{\frac{1}{\vartheta}} (C^{*b}_{H,t})^{\frac{\vartheta-1}{\vartheta}} + [(1-\nu^*)\Upsilon^*]^{\frac{1}{\vartheta}} (C^{*b}_{F,t})^{\frac{\vartheta-1}{\vartheta}} + [\nu^*(1-\Upsilon^*)]^{\frac{1}{\vartheta}} (C^{*g}_{H,t})^{\frac{\vartheta-1}{\vartheta}} + [(1-\nu^*)(1-\Upsilon^*)]^{\frac{1}{\vartheta}} (C^{*g}_{F,t})^{\frac{\vartheta-1}{\vartheta}} \Bigg\}^{\frac{\vartheta}{\vartheta-1}} \tag{11}$$

where $C^b_{H,t}$ ($C^{*b}_{H,t}$) and $C^b_{F,t}$ ($C^{*b}_{F,t}$) represent Home (Foreign) consumption of Home and Foreign brown final goods respectively, while the corresponding green variables are $C^g_{H,t}$ ($C^{*g}_{H,t}$) and $C^g_{F,t}$ ($C^{*g}_{F,t}$). The weight of the Home (Foreign) good in the Home (Foreign) bundle, ν (ν^*), captures households' preferences for the domestic good, and it might differ between Home and Foreign economies. Specifically, $\nu = 1-(1-\gamma)v$ and $\nu^* = \gamma v$, where $\gamma \in [0,1]$ and $(1-\gamma)$ measures the (relative) population size of the Home (Foreign) country, while $v \in [0,1]$ and $(1-v)$ measures the degree of trade openness. Notice that the framework nests a closed-economy set-up retrieved when $v = 0$; when $v = 1$ there is no home bias, while for $0 < v < 1$ the bias is towards home goods. Υ (Υ^*) is the share of brown goods in Home (Foreign) consumption. Finally, ϑ is the elasticity of substitution between Home and Foreign goods.

For both Home and Foreign households the intratemporal problem requires to maximise nominal consumption net of the costs of the Home and Foreign goods bundled together, that is optimally picking the allocation of consumption expenditure between domestic and foreign goods:

$$\max_{\{C^b_{H,t},C^b_{F,t},C^g_{H,t},C^g_{F,t}\}} P_tC_t - P^b_{H,t}C^b_{H,t} - P^b_{F,t}C^b_{F,t} - P^g_{H,t}C^g_{H,t} - P^g_{F,t}C^g_{F,t}$$

$$\max_{\{C^{*b}_{H,t},C^{*b}_{F,t},C^{*g}_{H,t},C^{*g}_{F,t}\}} P^*_tC^*_t - P^{*b}_{H,t}C^{*b}_{H,t} - P^{*b}_{F,t}C^{*b}_{F,t} - P^{*g}_{H,t}C^{*g}_{H,t} - P^{*g}_{F,t}C^{*g}_{F,t}$$

where $P^b_{H,t}$ ($P^{*b}_{H,t}$) is the price of the Home brown goods in Home (Foreign), while $P^b_{F,t}$ ($P^{*b}_{F,t}$) is the price of the Foreign brown goods in Home (Foreign), all in the corresponding domestic currency and at time t. Similarly, the prices for Home and Foreign green goods in Home (Foreign) are $P^g_{H,t}$ ($P^{*b}_{H,t}$) and $P^b_{F,t}$ ($P^{*b}_{F,t}$). respectively. P_t (P^*_t) is the CPI index of the domestic (foreign) economy, to be defined shortly below.

Using the definition of consumption bundles above (10) and (11), the FOCs for $C^b_{H,t}$, $C^b_{F,t}$, $C^g_{H,t}$, $C^g_{F,t}$, $C^{*b}_{H,t}$, $C^{*b}_{F,t}$, $C^{*g}_{H,t}$ and $C^{*g}_{F,t}$ yield respectively:

$$C^b_{H,t} = \left(\frac{P^b_{H,t}}{P_t}\right)^{-\vartheta} \nu\Upsilon C_t \tag{12}$$

$$C^b_{F,t} = \left(\frac{P^b_{F,t}}{P_t}\right)^{-\vartheta} (1-\nu)\Upsilon C_t \tag{13}$$

$$C^g_{H,t} = \left(\frac{P^g_{H,t}}{P_t}\right)^{-\vartheta} \nu(1-\Upsilon) C_t \tag{14}$$

$$C^g_{F,t} = \left(\frac{P^g_{F,t}}{P_t}\right)^{-\vartheta} (1-\nu)(1-\Upsilon) C_t \tag{15}$$

$$C^{*b}_{H,t} = \left(\frac{P^{*b}_{H,t}}{P^*_t}\right)^{-\vartheta} \nu^*\Upsilon^* C^*_t \tag{16}$$

$$C^{*b}_{F,t} = \left(\frac{P^{*b}_{F,t}}{P^*_t}\right)^{-\vartheta} (1-\nu^*)\Upsilon^* C^*_t \tag{17}$$

$$C^{*g}_{H,t} = \left(\frac{P^{*g}_{H,t}}{P^*_t}\right)^{-\vartheta} \nu^*(1-\Upsilon^*) C^*_t \tag{18}$$

$$C^{*g}_{F,t} = \left(\frac{P^{*g}_{F,t}}{P^*_t}\right)^{-\vartheta} (1-\nu^*)(1-\Upsilon^*) C^*_t \tag{19}$$

$$\tag{20}$$

One can plug these demand equations into (10) and (11) to obtain the CPI index P_t and P^*_t for Home and Foreign, respectively:

$$P_t = \left[\nu\Upsilon(P^b_{H,t})^{1-\vartheta} + (1-\nu)\Upsilon(P^b_{F,t})^{1-\vartheta} + \nu(1-\Upsilon)(P^g_{H,t})^{1-\vartheta} + (1-\nu)(1-\Upsilon)(P^g_{F,t})^{1-\vartheta}\right]^{\frac{1}{1-\vartheta}} \tag{21}$$

$$P^*_t = \left[\nu^*\Upsilon^*(P^{*b}_{H,t})^{1-\vartheta} + (1-\nu^*)\Upsilon^*(P^{*b}_{F,t})^{1-\vartheta} + \nu^*(1-\Upsilon^*)(P^{*g}_{H,t})^{1-\vartheta} + (1-\nu^*)(1-\Upsilon^*)(P^{*g}_{F,t})^{1-\vartheta}\right]^{\frac{1}{1-\vartheta}} \tag{22}$$

At this point, I assume that the law of one price (LOOP) holds, that is the prices of Home and Foreign differentiated goods in the Home economy are equal to prices in Foreign converted in domestic currency: for brown (green) prices $P^b_{H,t}(h) = \mathcal{E}_t P^{*b}_{H,t}(h)$ ($P^g_{H,t}(h) = \mathcal{E}_t P^{*g}_{H,t}(h)$) and $P^b_{F,t}(f) = \mathcal{E}_t P^{*b}_{F,t}(f)$ ($P^g_{F,t}(f) = \mathcal{E}_t P^{*g}_{F,t}(f)$), with $\mathcal{E}_t$ being the nominal exchange rate, i.e. the price of one unit of Foreign currency in terms of Home currency. This also implies:

$$P^b_{H,t} = \mathcal{E}_t P^{*b}_{H,t} \tag{23}$$
$$P^g_{H,t} = \mathcal{E}_t P^{*g}_{H,t} \tag{24}$$
$$P^b_{F,t} = \mathcal{E}_t P^{*b}_{F,t} \tag{25}$$
$$P^g_{F,t} = \mathcal{E}_t P^{*g}_{F,t} \tag{26}$$

When $v < 1$, that is when there is not full openness and thus home bias exists, purchasing power parity does not hold: $P_t \neq \mathcal{E}_t P^*_t$. This means that the real exchange rate is different from one, and defined as

$$\mathcal{Q}_t \equiv \mathcal{E}_t \frac{P^*_t}{P_t} \tag{27}$$

Thus, an increase in $\mathcal{E}_t$ indicates a nominal depreciation of the Home currency, and an increase in $\mathcal{Q}_t$ a real depreciation.

Because the CPI indexes are non-stationary, I must parse them out from the expressions for the real exchange rate:

$$\mathcal{E}_t = \mathcal{Q}_t \frac{P^*_t}{P_t} \;\Rightarrow\; \frac{\mathcal{Q}_t}{\mathcal{Q}_{t-1}} = \frac{\mathcal{E}_t}{\mathcal{E}_{t-1}} \frac{P^*_t}{P^*_{t-1}} \frac{P_{t-1}}{P_t} \;\Rightarrow\; \frac{\mathcal{Q}_t}{\mathcal{Q}_{t-1}} = \Delta\mathcal{E}_t \frac{\Pi^*_t}{\Pi_t}$$

where

$$\Pi_t \equiv P_t / P_{t-1} \tag{28}$$
$$\Pi^*_t \equiv P^*_t / P^*_{t-1} \tag{29}$$

are (gross) Home and Foreign CPI inflation respectively, and $\Delta\mathcal{E}_t \equiv \mathcal{E}_t/\mathcal{E}_{t-1}$ is the gross rate of nominal de/appreciation of the domestic currency. Furthermore, (gross) Home and Foreign PPI inflation are respectively:

$$\Pi^{b}_{H,t} \equiv P^{b}_{H,t}/P^{b}_{H,t-1} \tag{30}$$
$$\Pi^{g}_{H,t} \equiv P^{g}_{H,t}/P^{g}_{H,t-1} \tag{31}$$
$$\Pi^{*b}_{F,t} \equiv P^{*b}_{F,t}/P^{*b}_{F,t-1} \tag{32}$$
$$\Pi^{*g}_{F,t} \equiv P^{*g}_{F,t}/P^{*g}_{F,t-1} \tag{33}$$

Finally, terms of trade are defined as the ratio between the price of imports and the price of exports (and using LOOP):

$$\mathcal{T}^{bb}_{t} \equiv \frac{P^{b}_{F,t}}{P^{b}_{H,t}} = \frac{P^{*b}_{F,t}}{P^{*b}_{H,t}} \tag{34}$$
$$\mathcal{T}^{gg}_{t} \equiv \frac{P^{g}_{F,t}}{P^{g}_{H,t}} = \frac{P^{*g}_{F,t}}{P^{*g}_{H,t}} \tag{35}$$
$$\mathcal{T}^{bg}_{t} \equiv \frac{P^{b}_{F,t}}{P^{g}_{H,t}} = \frac{P^{*b}_{F,t}}{P^{*g}_{H,t}} \tag{36}$$
$$\mathcal{T}^{gb}_{t} \equiv \frac{P^{g}_{F,t}}{P^{b}_{H,t}} = \frac{P^{*g}_{F,t}}{P^{*b}_{H,t}} \tag{37}$$

A.7.2 Households - Intertemporal decision

Households in each economy solve a utility maximisation problem to allocate consumption over time. They have the same preferences over consumption and labour supply:

$$\mathbb{E}_t \sum_{j=0}^{\infty} \beta^j \left[u(C_t) - g(L_t) \right]$$

where

$$u(C_t) = \frac{(C_{t+j} - bC_{t+j-1})^{1-\zeta} - 1}{1-\zeta}$$
$$g(L_t) = \chi \frac{L^{1+\eta}_{t+j}}{1+\eta}$$

with $\beta \in (0,1)$ being the households' subjective discount factor, $b < 1$ captures the degree of internal habit formation, $\zeta > 0$ is the inverse of elasticity of intertemporal substitution, $\eta \geq 0$ the inverse of the Frisch elasticity of labour supply, and χ the scaling parameter on the disutility of labour.

Home households

Home households solves the maximisation problem:

$$\max_{\{C_t, L^g_t, L^b_t, D_t\}_{t=0}^{\infty}} \mathbb{E}_t \sum_{j=0}^{\infty} \beta^j \left[\frac{(C_{t+j} - bC_{t+j-1})^{1-\zeta} - 1}{1-\zeta} - \chi^g \frac{(L^g_{t+j})^{1+\eta}}{1+\eta} - \chi^b \frac{(L^b_{t+j})^{1+\eta}}{1+\eta} \right]$$

subject to the budget constraint in nominal terms

$$P_t C_t + D_t \leq MRS_t^g L_t^g + MRS_t^b L_t^b + R_{t-1}^d D_{t-1} + DIV_t - P_t X - P_t T_t \tag{38}$$

where C_t is consumption, L_t^g (L_t^b) labour supply paid at nominal wage MRS_t^g (MRS_t^b) in domestic currency by the labour union working with the green (brown) wholesaler, D_t holdings of one-period deposits denominated in domestic currency and yielding nominal interest rate from t to $t+1$, DIV_t nominal dividends from the domestic intermediary and firms, X is the real (start-up) transfer to the financial intermediary, and finally T_t is a real lump-sum tax paid to the domestic government.[21]

I rewrite the budget constraint deflating it by P_t

$$C_t + d_t \leq mrs_t L_t + \frac{R_{t-1}^d}{\Pi_t} d_{t-1} + div_t - X - T_t$$

where I have defined $d_t \equiv D_t/P_t$, $div_t \equiv DIV_t/P_t$, $mrs_t \equiv MRS_t/P_t$, I used the definition of inflation π_t to get rid of the (non-stationary) price levels.

Then the Lagrangian for Home households' maximisation problem is

$$\mathcal{L}_t = \mathbb{E}_t \sum_{j=0}^{\infty} \beta^j \Bigg\{ \left(\frac{(C_{t+j} - bC_{t+j-1})^{1-\zeta} - 1}{1-\zeta} - \chi^g \frac{(L_{t+j}^g)^{1+\eta}}{1+\eta} - \chi^b \frac{(L_{t+j}^b)^{1+\eta}}{1+\eta} \right) - \mu_t \left[C_t + d_t - mrs_t^g L_t^g - mrs_t^b L_t^b - \frac{R_{t-1}^d}{\Pi_t} d_{t-1} - div_t + X_t + T_t \right] \Bigg\}$$

where μ_t is the lagrangian multiplier on the households' budget constraint and the FOCs $\forall t = 0, ..., \infty$ for consumption, labour supply and deposits are:

$$\frac{\partial \mathcal{L}_t}{\partial C_t} : \quad \mu_t = (C_t - bC_{t-1})^{-\zeta} - b\beta \mathbb{E}_t (C_{t+1} - bC_t)^{-\zeta} \tag{39}$$

$$\frac{\partial \mathcal{L}_t}{\partial L_t^g} : \quad mrs_t^g \mu_t = \chi^g (L_t^g)^{\eta} \tag{40}$$

$$\frac{\partial \mathcal{L}_t}{\partial L_t^b} : \quad mrs_t^b \mu_t = \chi^b (L_t^b)^{\eta} \tag{41}$$

$$\frac{\partial \mathcal{L}_t}{\partial d_t} : \quad 1 = R_t^d \mathbb{E}_t \left[\Lambda_{t,t+1} \Pi_{t+1}^{-1} \right] \tag{42}$$

where

$$\Lambda_{t,t+1} = \beta \frac{\mu_{t+1}}{\mu_t} \tag{43}$$

is households' real stochastic discount factor.

Finally, as it will be clear from the derivations below, the expression for households' dividends received from retailers, wholesalers, capital producers, labour unions and financial inter-

[21] Notice how every element of the budget constraint needs to be in domestic price/currency. For example, consumption C_t is in units of CPI bundle, hence I multiply it by P_t.

mediaries is:

$$div_t \equiv \frac{DIV_t}{P_t} = \frac{P^b_{H,t}}{P_t} Y^b_t + \frac{P^g_{H,t}}{P_t} Y^g_t - mrs^g_t L^g_t - mrs^b_t L^b_t - I_t + (1-\sigma)\lfloor_t \\ + Q^g_t (b^g_t - \kappa b^g_{t-1}\Pi_t^{-1}) - b^g_{t-1}\Pi_t^{-1} + Q^b_t (b^b_t - \kappa b^b_{t-1}\Pi_t^{-1}) - b^b_{t-1}\Pi_t^{-1} \tag{44}$$

Foreign households

Foreign households solve the same maximisation problem as Home households:

$$\max_{\{C^*_t, L^{*g}_t, L^{*b}_t, D^*_t\}_{t=0}^{\infty}} \mathbb{E}_t \sum_{j=0}^{\infty} \beta^j \left[\frac{(C^*_{t+j} - bC^*_{t+j-1})^{1-\zeta} - 1}{1-\zeta} - \chi^g \frac{(L^{*g}_{t+j})^{1+\eta}}{1+\eta} - \chi^b \frac{(L^{*b}_{t+j})^{1+\eta}}{1+\eta} \right]$$

subject to the budget constraint in nominal terms

$$P^*_t C^*_t + D^*_t \leq MRS^{*g}_t L^{*g}_t + MRS^{*b}_t L^{*b}_t + R^{*d}_{t-1} D^*_{t-1} + DIV^*_t - P^*_t X^* - P^*_t T^*_t \tag{45}$$

Furthermore, similarly to what done for Home household, I rewrite the budget constraint deflating it by P^*_t:

$$C^*_t + d^*_t \leq mrs^{*g}_t L^{*g}_t + mrs^{*b}_t L^{*b}_t + \frac{R^{*d}_{t-1}}{\Pi^*_t} d^*_{t-1} + div^*_t - X^* - T^*_t$$

Then the Lagrangian for Foreign households' maximisation problem is

$$\mathcal{L}^*_t = \mathbb{E}_t \sum_{j=0}^{\infty} \beta^j \Bigg\{ \left(\frac{(C^*_{t+j} - bC^*_{t+j-1})^{1-\zeta} - 1}{1-\zeta} - \chi^g \frac{(L^{*g}_{t+j})^{1+\eta}}{1+\eta} - \chi^b \frac{(L^{*b}_{t+j})^{1+\eta}}{1+\eta} \right) \\ - \mu^*_t \left[C^*_t + d^*_t - mrs^{*g}_t L^{*g}_t - mrs^{*b}_t L^{*b}_t - \frac{R^{*d}_{t-1}}{\Pi^*_t} d^*_{t-1} - div^*_t + X^*_t + T^*_t \right] \Bigg\}$$

and the FOCs $\forall t = 0, ..., \infty$ are:

$$\frac{\partial \mathcal{L}^*_t}{\partial C^*_t}: \quad \mu^*_t = (C^*_t - bC^*_{t-1})^{-\zeta} - b\beta \mathbb{E}_t (C^*_{t+1} - bC^*_t)^{-\zeta} \tag{46}$$

$$\frac{\partial \mathcal{L}^*_t}{\partial L^{*g}_t}: \quad mrs^{*g}_t \mu^*_t = \chi^g (L^{*g}_t)^{\eta} \tag{47}$$

$$\frac{\partial \mathcal{L}^*_t}{\partial L^{*b}_t}: \quad mrs^{*b}_t \mu^*_t = \chi^b (L^{*b}_t)^{\eta} \tag{48}$$

$$\frac{\partial \mathcal{L}^*_t}{\partial d^*_t}: \quad 1 = R^{*d}_t \mathbb{E}_t \left[\Lambda^*_{t,t+1} (\Pi^*_{t+1})^{-1} \right] \tag{49}$$

where

$$\Lambda^*_{t,t+1} = \beta \frac{\mu^*_{t+1}}{\mu^*_t} \tag{50}$$

Also, dividends are:

$$div^*_t \equiv \frac{DIV^*_t}{P^*_t} = \frac{P^{*b}_{F,t}}{P^*_t} Y^{*b}_t + \frac{P^{*g}_{F,t}}{P^*_t} Y^{*g}_t - mrs^{*g}_t L^{*g}_t - mrs^{*b}_t L^{*b}_t - I^*_t + (1-\sigma)\lfloor^*_t + Q^{*g}_t (b^{*g}_t - \kappa b^{*g}_{t-1}(\Pi^*_t)^{-1}) - b^{*g}_{t-1}(\Pi^*_t)^{-1} + Q^{*b}_t (b^{*b}_t - \kappa b^{*b}_{t-1}(\Pi^*_t)^{-1}) - b^{*b}_{t-1}(\Pi^*_t)^{-1} \tag{51}$$

A.7.3 Firms - Final-good producers

At the consumer-facing end of the supply chain for the Home economy lies a green and a brown representative firms producing final-good output Y^g_t and Y^b_t respectively through the CES technologies:

$$Y^g_t = \left(\int_0^{\gamma(1-\phi_g)} Y_t(h)^{\frac{e_p-1}{e_p}} dh \right)^{\frac{e_p}{e_p-1}} \tag{52}$$

$$Y^b_t = \left(\int_{\gamma(1-\phi_g)}^{\gamma} Y^b_t(h)^{\frac{e_p-1}{e_p}} dh \right)^{\frac{e_p}{e_p-1}} \tag{53}$$

where e_p is the labour elasticity of substitution, $Y^g_t(h)$ ($Y^b_t(h)$) are domestic green (brown) intermediate goods, whose price is $P^g_{H,t}(h)$ ($P^b_{H,t}(h)$). Differentiated output is provided by retail firms indexed by $h \in [0, \gamma)$. Of these, a fraction $1 - \phi_g$, where $0 \leq \phi_g \leq 1$, provides differentiated output using intermediate goods from green wholesalers, while a fraction ϕ_g sales goods produced with inputs from brown intermediate-good producers.

Focusing on the green final-good producer, her problem involves maximising the present discounted value of real profits using the household's stochastic discount factor:

$$\max_{Y^g_t, \{Y^g_t(h)\}_{h\in[0,\gamma(1-\phi_g))}} \mathbb{E}_t \sum_{j=0}^{\infty} \Lambda_{t,t+j} \left[\frac{P^g_{H,t+j}}{P_{t+j}} Y^g_{t+j} - \int_0^{\gamma(1-\phi_g)} \frac{P^g_{H,t+j}(h)}{P_{t+j}} Y^g_{t+j}(h) dh \right]$$

subject to the technology (52).

Simplifying the problem to a choice of inputs $Y^g_t(h)$ only, by plugging this constraint into the objective function, the FOC for a generic $Y^g_t(h)$ is (notice I simplified out the common deflator P_t)

$$P^g_{H,t}\left(\int_0^{\gamma(1-\phi_g)} Y^g_t(h)^{\frac{e_p-1}{e_p}}dh\right)^{\frac{1}{e_p-1}} Y^g_t(h)^{-\frac{1}{e_p}} - P^g_{H,t}(h) = 0$$

$$\Rightarrow P^g_{H,t}(Y^g_t)^{\frac{1}{e_p}} Y^g_t(h)^{-\frac{1}{e_p}} = P^g_{H,t}(h)$$

$$\Rightarrow Y^g_t(h) = \left(\frac{P^g_{H,t}(h)}{P^g_{H,t}}\right)^{-e_p} Y^g_t \quad (54)$$

where the last equation is the downward-sloping demand function for the differentiated good h faced by green retailers.

The aggregate green price index P^g_t satisfies:

$$(P^g_{H,t})^{1-e_p} = \frac{1}{\gamma(1-\phi_g)}\int_0^{\gamma(1-\phi_g)} (P^g_{H,t}(h))^{1-e_p}dh \quad (55)$$

Similar equations apply for brown retailers

$$Y^b_t(h) = \left(\frac{P^b_{H,t}(h)}{P^b_{H,t}}\right)^{-e_p} Y^b_t \quad (56)$$

$$(P^b_{H,t})^{1-e_p} = \frac{1}{\gamma\phi_g}\int_{\gamma(1-\phi_g)}^{\gamma} (P^b_{H,t}(h))^{1-e_p}dh \quad (57)$$

and in the Foreign economy:

$$Y^{*g}_t(f) = \left(\frac{P^{*g}_{F,t}(f)}{P^{*g}_{F,t}}\right)^{-e_p} Y^{*g}_t \quad (58)$$

$$(P^{*g}_{F,t})^{1-e_p} = \frac{1}{(1-\gamma)(1-\phi_g)}\int_{\gamma}^{1-(1-\gamma)\phi_g} (P^{*g}_{F,t}(h))^{1-e_p}dh \quad (59)$$

$$Y^{*b}_t(f) = \left(\frac{P^{*b}_{F,t}(f)}{P^{*b}_{F,t}}\right)^{-e_p} Y^{*b}_t \quad (60)$$

$$(P^b_{F,t})^{1-e_p} = \frac{1}{(1-\gamma)\phi_g}\int_{1-(1-\gamma)\phi_g}^{1} (P^b_{F,t}(h))^{1-e_p}dh \quad (61)$$

A.7.4 Firms - Retail producers

Here I first focus on the green retailers, as the derivation for the brown ones is similar.

There is a continuum of Home green retailers indexed by $h \in [0, \gamma(1-\phi_g))$, each operating with a simple "repackaging" technology represented by the production function:

$$Y^g_t(h) = Y^g_{w,t}(h),$$

where $Y^g_{w,t}(h)$ is the intermediate good produced by green wholesalers.

With this technology, the nominal profits the retail firm tries to maximise are:

$$DIV^g_{R,t}(h) = (P^g_{H,t}(h) - P^g_{w,t})Y^g_t(h)$$

While intermediate bundles (and goods) markets are perfectly competitive, the retail market is not: firms operate under monopolistic competition, that is they are able to set the price of their retail goods given the demand for those from the final good firms, i.e. $Y^g_t(h) = \left(\frac{P^g_{H,t}(h)}{P^g_{H,t}}\right)^{-e_p} Y^g_t$. This allows firms to select prices that could be different from the flexible prices / competitive equilibrium outcome. This decision is the result of the firm's maximisation problem below and is affected by nominal rigidities in the form of Calvo pricing, that is the firm can adjust its prices in each period with probability $1 - \phi_p$, where $0 \leq \phi_p \leq 1$.

Thus, each retailer able to re-optimise wants to maximise the present discounted value of its (real) profits (using the discount factor of the household Λ):

$$\max_{P^g_{H,t}(h)} \mathbb{E}_t \sum_{j=0}^{\infty} \phi_p^j \Lambda_{t,t+1} P^{-1}_{t+j} \left[(P^g_{H,t}(h))^{1-e_p} (P^g_{H,t+j})^{e_p} Y^g_{t+j} - P^g_{w,t+j} (P^g_{H,t}(h))^{-e_p} (P^g_{H,t+j})^{e_p} Y^g_{t+j} \right]$$

where I have substituted in the demand function for $Y^g_t(h)$, so that the problem involves picking only the price.

The FOC is:

$$\frac{\partial \mathcal{L}_{R,t}}{\partial P^g_{H,t}(h)} : \quad (e_p - 1)(P^g_{H,t}(h))^{-e_p} \mathbb{E}_t \sum_{j=0}^{\infty} \phi_p^j \Lambda_{t,t+j} P^{-1}_{t+j} (P^g_{H,t+j})^{e_p} Y^g_{t+j} =$$
$$e_p P^g_{H,t}(h)^{-e_p-1} \mathbb{E}_t \sum_{j=0}^{\infty} \phi_p^j \Lambda_{t,t+j} P^{-1}_{t+j} P^g_{H,w,t+j} (P^g_{H,t+j})^{e_p} Y^g_{t+j}$$

Define the auxiliary variables:

$$X^g_{1,t} \equiv \sum_{j=0}^{\infty} \phi_p^j \Lambda_{t,t+j} P^{-1}_{t+j} P^g_{w,t+j} (P^g_{H,t+j})^{e_p} Y^g_{t+j}$$
$$X^g_{2,t} \equiv \sum_{j=0}^{\infty} \phi_p^j \Lambda_{t,t+j} P^{-1}_{t+j} (P^g_{H,t+j})^{e_p} Y^g_{t+j}$$

Equivalently, in the more convenient recursive form (grouping $p^g_{w,t} Y^g_t$ and Y^g_t respectively):

$$X^g_{1,t} \equiv p^g_{w,t} (P^g_{H,t})^{e_p} Y^g_t + \phi_p \mathbb{E}_t \Lambda_{t,t+1} (P^g_{H,t+1})^{e_p} X^g_{1,t+1}$$
$$X^g_{2,t} \equiv P^{-1}_t (P^g_{H,t})^{e_p} Y^g_{H,t} + \phi_p \mathbb{E}_t \Lambda_{t,t+1} P^{-1}_{t+1} (P^g_{H,t+1})^{e_p} X^g_{2,t+1}$$

where $p^g_{w,t} \equiv \frac{P^g_{w,t}}{P_t}$.

I now divide the first variable by $(P^g_{H,t})^{e_p}$ and the second by $P^{-1}_t (P^g_{H,t})^{e_p}$ to define $x^g_{1,t}$ and $x^g_{2,t}$ (also $x^g_{1,t} = X^g_{1,t}/(P^g_{H,t})^{e_p} \Rightarrow x^g_{1,t+1} = X^g_{1,t+1}/(P^g_{H,t+1})^{e_p}$ and $x^g_{2,t} = X^g_{2,t}/(P^{-1}_t (P^g_{H,t})^{e_p}) \Rightarrow$

$x^g_{2,t+1} = X^g_{2,t+1}/(P^{-1}_{t+1}(P^g_{H,t+1})^{e_p}))$:

$$x^g_{1,t} \equiv p^g_{w,t} Y^g_t + \phi_p \mathbb{E}_t \Lambda_{t,t+1} (\Pi^g_{H,t+1})^{e_p} x^g_{1,t+1} \tag{62}$$

$$x^g_{2,t} \equiv Y^g_t + \phi_p \mathbb{E}_t \Lambda_{t,t+1} \Pi^{-1}_{t+1} (\Pi^g_{H,t+1})^{e_p} x^g_{2,t+1} \tag{63}$$

Consider a symmetric equilibrium where firms pick the same price, then $P^g_{H,t}(h) = P^{g,o}_{H,t}$ and the FOC simplifies to:

$$p^{g,o}_{H,t} = \frac{e_p}{e_p - 1} \frac{x^g_{1,t}}{x^g_{2,t}} \tag{64}$$

where $p^{g,o}_{H,t} \equiv \frac{P^{g,o}_{H,t}}{P_t}$.

At this point I can integrate the demand function for retailers:

$$\int_0^{\gamma(1-\phi_g)} Y^g_t(h) dh = \int_0^{\gamma(1-\phi_g)} \left(\frac{P^g_{H,t}(h)}{P^g_{H,t}} \right)^{-e_p} Y^g_t dh$$

Because each retailer produces the same amount, i.e. an equal share of the total output of the domestic country as shown by the demand function for each $Y^g_t(h)$ (and thus use equal amounts of $Y^g_{w,t}(h)$ as inputs), and given technology $Y^g_t(h) = Y^g_{w,t}$, the left-hand side of the equation above sums up to a fraction of the total output for intermediate output $Y^g_{w,t}$: $\int_0^{\gamma(1-\phi_g)} Y^g_t(h) dh = \int_0^{\gamma(1-\phi_g)} Y^g_{w,t}(h) dh = \gamma(1-\phi_g) Y^g_{w,t}$.

$$\gamma(1-\phi_g) Y^g_{w,t} = \int_0^{\gamma(1-\phi_g)} \left(\frac{P^g_{H,t}(h)}{P^g_{H,t}} \right)^{-e_p} Y^g_t dh \quad \Rightarrow \quad Y^g_{w,t} = v^{g,p}_t Y^g_t \tag{65}$$

where $v^{g,p}_t = \frac{1}{\gamma(1-\phi_g)} \int_0^{\gamma(1-\phi_g)} \left(\frac{P^g_{H,t}(h)}{P^g_{H,t}} \right)^{-e_p} dh$ is a measure of price dispersion.

One can use the Calvo assumption to split $v^{g,p}_t$ noting that $(1-\phi_p)$ of retailers can update setting the same price $P^{g,o}_{H,t}$, and the rest can only retain the price from the previous period:

$$v^{g,p}_t = \frac{1}{\gamma(1-\phi_g)} \int_0^{\gamma(1-\phi_g)(1-\phi_p)} \left(\frac{P^{g,o}_{H,t}}{P^g_{H,t}} \right)^{-e_p} dh + \frac{1}{\gamma(1-\phi_g)} \int_{\gamma(1-\phi_g)(1-\phi_p)}^{\gamma(1-\phi_g)} \left(\frac{P^g_{H,t-1}(h)}{P^g_{H,t}} \right)^{-e_p} dh \tag{66}$$

which becomes by multiplying and dividing by $P^{-e_p}_t$ the first term and by $(P^g_{H,t-1})^{-e_p}$ the second term on the right-hand side respectively, and rearranging terms out of the integrals:

$$v^{g,p}_t = (1-\phi_p)(p^{g,o}_{H,t})^{-e_p} P^{-e_p}_t (P^g_{H,t})^{e_p} + \frac{1}{\gamma(1-\phi_g)} (P^g_{H,t})^{e_p} (P^g_{H,t-1})^{-e_p} \int_{\gamma(1-\phi_g)(1-\phi_p)}^{\gamma(1-\phi_g)} \left(\frac{P^g_{H,t-1}(h)}{P^g_{H,t-1}} \right)^{-e_p} dh$$

For the definition of gross inflation and the price dispersion term, the expression simplifies to:

$$v_t^{g,p} = (1-\phi_p)(p_{H,t}^{g,o})^{-e_p} P_t^{-e_p} (P_{H,t}^g)^{e_p} + \frac{\gamma(1-\phi_g)\phi_p}{\gamma(1-\phi_g)} (\Pi_{H,t}^g)^{e_p} v_{t-1}^{g,p}$$

$$\Rightarrow \quad v_t^{g,p} = (1-\phi_p)(p_{H,t}^{g,o} P_t / P_{H,t}^g)^{-e_p} + \phi_p (\Pi_{H,t}^g)^{e_p} v_{t-1}^{g,p} \tag{67}$$

For the same Calvo price properties, the price index (55) can be rewritten as:

$$(P_{H,t}^g)^{1-e_p} = (1-\phi_p)(P_{H,t}^{g,o})^{1-e_p} + \frac{1}{\gamma(1-\phi_g)} \int_{\gamma(1-\phi_g)(1-\phi_p)}^{\gamma(1-\phi_g)} (P_{H,t-1}^g(h))^{1-e_p} dh$$

$$\Rightarrow \quad (P_{H,t}^g)^{1-e_p} = (1-\phi_p)(P_{H,t}^{g,o})^{1-e_p} + \phi_p (P_{H,t-1}^g)^{1-e_p}$$

$$\Rightarrow \quad 1 = (1-\phi_p)(p_{H,t}^{g,o} P_t / P_{H,t}^g)^{1-e_p} + \phi_p (\Pi_{H,t}^g)^{e_p - 1} \tag{68}$$

As for final-good firms, the same equations apply for brown retailers

$$p_{H,t}^{b,o} = \frac{e_p}{e_p - 1} \frac{x_{1,t}^b}{x_{2,t}^b} \tag{69}$$

$$x_{1,t}^b \equiv p_{w,t}^b Y_t^{*b} + \phi_p \mathbb{E}_t \Lambda_{t,t+1} (\Pi_{H,t+1}^b)^{e_p} x_{1,t+1}^b \tag{70}$$

$$x_{2,t}^b \equiv Y_t^b + \phi_p \mathbb{E}_t \Lambda_{t,t+1} (\Pi_{t+1})^{-1} (\Pi_{H,t+1}^b)^{e_p} x_{2,t+1}^b \tag{71}$$

$$Y_t^b = v_t^{b,p} Y_t^b \tag{72}$$

$$v_t^{b,p} = (1-\phi_p)(p_{H,t}^{b,o} P_t / P_{H,t}^b)^{-e_p} + \phi_p (\Pi_{H,t})^{e_p} v_{t-1}^{b,p} \tag{73}$$

$$1 = (1-\phi_p)(p_{H,t}^{b,o} P_t / P_{H,t}^b)^{1-e_p} + \phi_p (\Pi_{H,t})^{e_p - 1} \tag{74}$$

and the Foreign economy. In particular:

$$p_{F,t}^{*g,o} = \frac{e_p}{e_p - 1} \frac{x_{1,t}^{*g}}{x_{2,t}^{*g}} \tag{75}$$

$$x_{1,t}^{*g} \equiv p_{w,t}^{*g} Y_t^{*g} + \phi_p \mathbb{E}_t \Lambda_{t,t+1}^* (\Pi_{F,t+1}^{*g})^{e_p} x_{1,t+1}^{*g} \tag{76}$$

$$x_{2,t}^{*g} \equiv Y_t^{*g} + \phi_p \mathbb{E}_t \Lambda_{t,t+1}^* (\Pi_{t+1}^*)^{-1} (\Pi_{F,t+1}^{*g})^{e_p} x_{2,t+1}^{*g} \tag{77}$$

$$Y_t^{*g} = v_t^{*g,p} Y_t^{*g} \tag{78}$$

$$v_t^{*g,p} = (1-\phi_p)(p_t^{*g,o} P_t^* / P_{F,t}^{*g})^{-e_p} + \phi_p (\Pi_{F,t}^*)^{e_p} v_{t-1}^{g,p} \tag{79}$$

$$1 = (1-\phi_p)(p_{F,t}^{*g,o} P_t^* / P_{F,t}^{*g})^{1-e_p} + \phi_p (\Pi_{F,t-1}^*)^{e_p - 1} \tag{80}$$

$$p_{F,t}^{*b,o} = \frac{e_p}{e_p - 1} \frac{x_{1,t}^{*b}}{x_{2,t}^{*b}} \tag{81}$$

$$x_{1,t}^{*b} \equiv p_{w,t}^{*b} Y_t^{*b} + \phi_p \mathbb{E}_t \Lambda_{t,t+1}^{*} (\Pi_{F,t+1}^{*b})^{e_p} x_{1,t+1}^{*b} \tag{82}$$

$$x_{2,t}^{*b} \equiv Y_t^{*b} + \phi_p \mathbb{E}_t \Lambda_{t,t+1}^{*} (\Pi_{t+1}^{*})^{-1} (\Pi_{F,t+1}^{*b})^{e_p} x_{2,t+1}^{*b} \tag{83}$$

$$Y_t^{*b} = v_t^{*b,p} Y_t^{*b} \tag{84}$$

$$v_t^{*b,p} = (1 - \phi_p)(p_t^{*b,o} P_t^* / P_{F,t}^{*b})^{-e_p} + \phi_p (\Pi_{F,t}^{*})^{e_p} v_{t-1}^{b,p} \tag{85}$$

$$1 = (1 - \phi_p)(p_t^{*b,o} P_t^* / P_{F,t}^{*b})^{1-e_p} + \phi_p (\Pi_{F,t-1}^{*})^{e_p - 1} \tag{86}$$

A.7.5 Firms - Green and brown capital producer

In each country a representative green (brown) capital producer takes as input unconsumed final green (brown) output I_t^g (I_t^b), where

$$I_t = I_t^g + I_t^b \tag{87}$$

and similarly in Foreign

$$I_t^* = I_t^{*g} + I_t^{*b}, \tag{88}$$

and uses the following technology to create new physical green (brown) capital:

$$\hat{I}_t^g = \left[1 - S\left(\frac{I_t^g}{I_{t-1}^g}\right)\right] I_t^g \tag{89}$$

$$\hat{I}_t^b = \left[1 - S\left(\frac{I_t^b}{I_{t-1}^b}\right)\right] I_t^b \tag{90}$$

with convex function of adjustment cost $S(I_t/I_{t-1}) = \frac{\kappa}{2}(I_t/I_{t-1} - 1)^2$.
Similarly for Foreign:

$$\hat{I}_t^{*g} = \left[1 - S\left(\frac{I_t^{*g}}{I_{t-1}^{*g}}\right)\right] I_t^{*g} \tag{91}$$

$$\hat{I}_t^{*b} = \left[1 - S\left(\frac{I_t^{*b}}{I_{t-1}^{*b}}\right)\right] I_t^{*b} \tag{92}$$

Each capital producer maximises the present discounted value of its dividends

$$DIV_{k,t} = P_t^k \hat{I}_t - P_t I_t = P_t^k \left[1 - S\left(\frac{I_t}{I_{t-1}}\right)\right] I_t - P_t I_t$$

or in real terms

$$div_{k,t} = p_t^k \left[1 - S\left(\frac{I_t}{I_{t-1}}\right)\right] I_t - I_t$$

using the household's stochastic discount factor:

$$\max_{I_t} \mathbb{E}_t \sum_{j=0}^{\infty} \Lambda_{t,t+j} \left\{ p_{t+j}^k \left[1 - S\left(\frac{I_{t+j}}{I_{t+j-1}} \right) \right] I_{t+j} - I_{t+j} \right\}$$

Then, the FOCs for green and brown Home producers are:

$$1 = p_t^{g,k} \left[1 - S\left(\frac{I_t^g}{I_{t-1}^g} \right) - S'\left(\frac{I_t^g}{I_{t-1}^g} \right) \frac{I_t^g}{I_{t-1}^g} \right] + \mathbb{E}_t \Lambda_{t,t+1} p_{t+1}^{g,k} S'\left(\frac{I_{t+1}^g}{I_t^g} \right) \left(\frac{I_{t+1}^g}{I_t^g} \right)^2 \tag{93}$$

$$1 = p_t^{b,k} \left[1 - S\left(\frac{I_t^b}{I_{t-1}^b} \right) - S'\left(\frac{I_t^b}{I_{t-1}^b} \right) \frac{I_t^b}{I_{t-1}^b} \right] + \mathbb{E}_t \Lambda_{t,t+1} p_{t+1}^{b,k} S'\left(\frac{I_{t+1}^b}{I_t^b} \right) \left(\frac{I_{t+1}^b}{I_t^b} \right)^2 \tag{94}$$

Similarly for Foreign:

$$1 = p_t^{*g,k} \left[1 - S\left(\frac{I_t^{*g}}{I_{t-1}^{*g}} \right) - S'\left(\frac{I_t^{*g}}{I_{t-1}^{*g}} \right) \frac{I_t^{*g}}{I_{t-1}^{*g}} \right] + \mathbb{E}_t \Lambda_{t,t+1}^* p_{t+1}^{*g,k} S'\left(\frac{I_{t+1}^{*g}}{I_t^{*g}} \right) \left(\frac{I_{t+1}^{*g}}{I_t^{*g}} \right)^2 \tag{95}$$

$$1 = p_t^{*b,k} \left[1 - S\left(\frac{I_t^{*b}}{I_{t-1}^{*b}} \right) - S'\left(\frac{I_t^{*b}}{I_{t-1}^{*b}} \right) \frac{I_t^{*b}}{I_{t-1}^{*b}} \right] + \mathbb{E}_t \Lambda_{t,t+1}^* p_{t+1}^{*b,k} S'\left(\frac{I_{t+1}^{*b}}{I_t^{*b}} \right) \left(\frac{I_{t+1}^{*b}}{I_t^{*b}} \right)^2 \tag{96}$$

A.7.6 Firms - Wholesalers

In each country there are two types of wholesale firms: green g and brown b. Each wholesaler produces intermediate output $Y_{w,t}$ using a Cobb-Douglas technology with productivity A_t and combining capital K_t purchased from the investment-good producer and labour $L_{d,t}$ from unions:

$$Y_{w,t}^g = A_t^g (u_t^g K_t^g)^\alpha (L_{d,t}^g)^{1-\alpha} \tag{97}$$

$$Y_{w,t}^b = A_t^b (u_t^b K_t^b)^\alpha (L_{d,t}^b)^{1-\alpha} \tag{98}$$

where $0 < \alpha < 1$ is the capital share in production. Similarly in Foreign:

$$Y_{w,t}^{*g} = A_t^{*g} (u_t^{*g} K_t^{*g})^\alpha (L_{d,t}^{*g})^{1-\alpha} \tag{99}$$

$$Y_{w,t}^{*b} = A_t^{*b} (u_t^{*b} K_t^{*b})^\alpha (L_{d,t}^{*b})^{1-\alpha} \tag{100}$$

Notice u_t reflects the degree of capital utilization, which shows up both in production and in capital accumulation:

$$K_{t+1}^g = \hat{I}_t^g + (1 - \delta(u_t^g)) K_t^g, \tag{101}$$

$$K_{t+1}^b = \hat{I}_t^b + (1 - \delta(u_t^b)) K_t^b, \tag{102}$$

with $\hat{I}_t$ as new physical capital purchased from the capital producer and $\delta(u_t)$ as a functional

form linking capital utilisation to its depreciation. In Foreign similarly:

$$K^{*g}_{t+1} = \hat{I}^{*g}_t + (1 - \delta(u^{*g}_t))K^{*g}_t, \tag{103}$$

$$K^{*b}_{t+1} = \hat{I}^{*b}_t + (1 - \delta(u^{*b}_t))K^{*b}_t, \tag{104}$$

Each wholesale firm exhibits the following "loan in advance constraint" (Carlstrom et al., 2017; Gertler & Karadi, 2011; Sims & Wu, 2021b):

$$\psi P^{g,k}_t \hat{I}^g_t \leq Q^g_t (B^g_t - \kappa B^g_{t-1}) \tag{105}$$

$$\psi P^{b,k}_t \hat{I}^b_t \leq Q^b_t (B^b_t - \kappa B^b_{t-1}) \tag{106}$$

Dividing by P_t and rearranging (by multiplying and dividing by P_{t-1}) so to feature inflation terms:

$$\psi p^{g,k}_t \hat{I}^g_t \leq Q^g_t \left(\frac{B^g_t}{P_t} - \kappa \frac{B^g_{t-1}}{P_{t-1}} \Pi^{-1}_t \right) \tag{107}$$

$$\psi p^{b,k}_t \hat{I}^b_t \leq Q^b_t \left(\frac{B^b_t}{P_t} - \kappa \frac{B^b_{t-1}}{P_{t-1}} \Pi^{-1}_t \right) \tag{108}$$

where $p^{g,k}_t \equiv P^{g,k}_t / P_t$ and $p^{b,k}_t \equiv P^{b,k}_t / P_t$. In Foreign:

$$\psi p^{*g,k}_t \hat{I}^{*g}_t \leq Q^{*g}_t \left(\frac{B^{*g}_t}{P^*_t} - \kappa \frac{B^{*g}_{t-1}}{P^*_{t-1}} (\Pi^*_t)^{-1} \right) \tag{109}$$

$$\psi p^{*b,k}_t \hat{I}^{*b}_t \leq Q^{*b}_t \left(\frac{B^{*b}_t}{P^*_t} - \kappa \frac{B^{*b}_{t-1}}{P^*_{t-1}} (\Pi^*_t)^{-1} \right) \tag{110}$$

Equation (107) and (108) requires that, in order to finance a constant fraction, $\psi \in (0, 1]$, of its investment $\hat{I}_t$ at price P^k_t, each wholesaler must issue bonds as perpetuities $B_t - \kappa B_{t-1}$ at market price Q_t with decaying coupon payments, whose decaying parameter is $\kappa \in (0, 1)$ (Woodford, 2001). This means that one unit of bonds today yields a coupon payment of one unit of the domestic currency next period, but this payment decays at rate κ. Then B_{t-1} is the total coupon liability due in period t, i.e. $B_{t-1} = CB_{t-1} + \kappa CB_{t-2} + \kappa^2 CB_{t-3} + ...$ with CB as the new nominal issuance in each period. The latter, through forward iteration, implies that one needs only keep track of the period t and $t-1$ total coupon liabilities, $CB_t = B_t - \kappa B_{t-1}$.

Similarly, because of the assumed perpetual bond structure, the price in t of bonds issued in $t - j$ is $\kappa^j Q_t$ for $j \geq 0$, and thus knowing the price for the current issue allows to know the entire sequence of prices for past issues too. This keeps the model tractable, as for example the value of the (outstanding) bond holdings in period t can be expressed as $Q_t B_t = Q_t CB_t + \kappa Q_t CB_{t-1} + \kappa^2 Q_t CB_{t-2} + ...$ If in period t one sells $Q_{t-1} B_{t-1}$ worth of bonds from $t-1$, the realised return would be the sum of the total coupon liability due in t and the value of the bond portfolio from $t-1$ in t (thus at price κQ_t) divided by the value of that same portfolio in $t-1$:

$R_t = \frac{B_{t-1}+\kappa Q_t B_{t-1}}{Q_{t-1}B_{t-1}} \Rightarrow R_t = \frac{1+\kappa Q_t}{Q_{t-1}}$. Thus we have:

$$R_t^g = \frac{1+\kappa Q_t^g}{Q_{t-1}^g} \tag{111}$$

$$R_t^b = \frac{1+\kappa Q_t^b}{Q_{t-1}^b} \tag{112}$$

and in Foreign:

$$R_t^{*g} = \frac{1+\kappa Q_t^{*g}}{Q_{t-1}^{*g}} \tag{113}$$

$$R_t^{*b} = \frac{1+\kappa Q_t^{*b}}{Q_{t-1}^{*b}} \tag{114}$$

Crucially, constraints (107) and (108) create scope for external financing, distort the standard asset pricing decisions, and in turn represent the main mechanism through which corporate bonds purchases transmit to the real economy.

Each wholesaler provides the bundling firm with intermediate goods within a competitive market, and maximise their present discounted value (using households' stochastic discount factor) of dividends

$$DIV_{w,t}^g = P_{w,t}^g A_t^g (u_t^g K_t^g)^\alpha L_{d,t}^{g\,1-\alpha} - W_t^g L_{d,t}^g - P_t^{g,k} \hat{I}_t^g - B_{t-1}^g + Q_t^g (B_t^g - \kappa B_{t-1}^g)$$

$$DIV_{w,t}^b = P_{w,t}^b A_t^b (u_t^b K_t^b)^\alpha L_{d,t}^{b\,1-\alpha} - W_t^b L_{d,t}^b - P_t^{b,k} \hat{I}_t^b - B_{t-1}^b + Q_t^b (B_t^b - \kappa B_{t-1}^b)$$

In real terms (dividing by P_t and rearranging by multiplying and dividing by P_{t-1}) so to feature inflation terms):

$$div_{w,t}^g = p_{w,t}^g A_t^g (u_t^g K_t^g)^\alpha (L_{d,t}^g)^{1-\alpha} - w_t^g L_{d,t}^g - p_t^{g,k} \hat{I}_t^g - \frac{B_{t-1}^g}{P_{t-1}} \Pi_t^{-1} + Q_t^g \left(\frac{B_t^g}{P_t} - \kappa \frac{B_{t-1}^g}{P_{t-1}} \Pi_t^{-1} \right)$$

$$div_{w,t}^b = p_{w,t}^b A_t^g (u_t^b K_t^b)^\alpha (L_{d,t}^b)^{1-\alpha} - w_t^b L_{d,t}^b - p_t^{b,k} \hat{I}_t^b - \frac{B_{t-1}^b}{P_{t-1}} \Pi_t^{-1} + Q_t^b \left(\frac{B_t^b}{P_t} - \kappa \frac{B_{t-1}^b}{P_{t-1}} \Pi_t^{-1} \right)$$

with real wages $w_t^g \equiv W_t^g/P_t$ and $w_t^b \equiv W_t^b/P_t$, and output prices $p_{w,t}^g \equiv P_{w,t}^g/P_t$ and $p_{w,t}^b \equiv P_{w,t}^b/P_t$.

Notice how I have already substituted in the production technology (97) and (98), and that wholesalers take into account their costs for labour, past financing and capital accumulation, net of the part that is paid with external financing (leading to future liabilities to be paid and reducing dividends in the following periods).

The maximisation for the green wholesaler is carried out considering the process for capital accumulation (101) and loan-in-advance constraint (107), and thus the lagrangian for the problem is:

$$\mathcal{L}^g_{w,t} = \mathbb{E}_t \sum_{j=0}^{\infty} \Lambda_{t,t+j} \Bigg\{ p^g_{w,t+j} A^g_{t+j} (u^g_{t+j} K^g_{t+j})^{\alpha} (L^g_{d,t+j})^{1-\alpha} - w^g_{t+j} L^g_{d,t+j} - p^{g,k}_{t+j} \hat{I}^g_{t+j} - \frac{B^g_{t+j-1}}{P_{t+j-1}} \Pi^{-1}_{t+j}$$
$$+ Q^g_t \left(\frac{B^g_{t+j}}{P_{t+j}} - \kappa \frac{B^g_{t+j-1}}{P_{t+j-1}} \Pi^{-1}_{t+j} \right) + \nu^g_{1,t+j} \left[\hat{I}^g_{t+j} + (1 - \delta(u^g_{t+j})) K^g_{t+j} - K^g_{t+j+1} \right]$$
$$+ \nu^g_{2,t+j} \left[Q^g_{t+j} \left(\frac{B^g_{t+j}}{P_{t+j}} - \kappa \frac{B^g_{t+j-1}}{P_{t+j-1}} \Pi^{-1}_{t+j} \right) - \psi p^{g,k}_{t+j} \hat{I}^g_{t+j} \right] \Bigg\}$$

where $\nu^g_{1,t}$ and $\nu^g_{2,t}$ are the lagrangian multipliers on constraints (101) and (107), respectively. The FOCs $\forall t = 0, ..., \infty$ for consumption, labour supply and deposits are:

$$\frac{\partial \mathcal{L}^g_{w,t}}{\partial L^g_{d,t}} : \quad p^g_{w,t} (1-\alpha) A^g_t (u^g_t K^g_t)^{\alpha} (L^g_{d,t})^{-\alpha} - w^g_t = 0$$
$$\frac{\partial \mathcal{L}^g_{w,t}}{\partial \hat{I}^g_t} : \quad -p^{g,k}_t + \nu^g_{1,t} - \psi p^{g,k}_t \nu^g_{2,t} = 0$$
$$\frac{\partial \mathcal{L}^g_{w,t}}{\partial u^g_t} : \quad \alpha p^g_{w,t} A^g_t (u^g_t K^g_t)^{\alpha-1} K^g_t (L^g_{d,t})^{1-\alpha} - \nu^g_{1,t} \delta'(u^g_t) K^g_t = 0$$
$$\frac{\partial \mathcal{L}^g_{w,t}}{\partial K^g_{t+1}} : \quad \mathbb{E}_t \Lambda_{t,t+1} \left[\alpha p^g_{w,t+1} A^g_{t+1} (u^g_{t+1} K^g_{t+1})^{\alpha-1} u_{t+1} (L^g_{d,t+1})^{1-\alpha} + \nu^g_{1,t+1} (1 - \delta(u^g_{t+1})) \right] - \nu^g_{1,t} = 0$$
$$\frac{\partial \mathcal{L}^g_{w,t}}{\partial B_t} : \quad (1 + \nu^g_{2,t}) \frac{Q^g_t}{P_t} - \mathbb{E}_t \Lambda_{t,t+1} \left[\frac{1}{P_t} \Pi^{-1}_{t+1} + \kappa \frac{Q^g_{t+1}}{P_t} \Pi^{-1}_{t+1} + \nu^g_{2,t+1} \kappa \frac{Q^g_{t+1}}{P_t} \Pi^{-1}_{t+1} \right] = 0$$

Defining $M^g_{1,t} = 1 + \psi \nu^g_{2,t}$ and $M^g_{2,t} = 1 + \nu^g_{2,t}$ so that:

$$\frac{M^g_{1,t} - 1}{M^g_{2,t} - 1} = \psi, \tag{115}$$

the FOCs simplify to:

$$\frac{\partial \mathcal{L}^g_{w,t}}{\partial L^g_{d,t}}: \quad w^g_t = p^g_{w,t}(1-\alpha)A^g_t(u^g_t K^g_t)^\alpha (L^g_{d,t})^{-\alpha} \tag{116}$$

$$\frac{\partial \mathcal{L}^g_{w,t}}{\partial \hat{I}^g_t}: \quad \nu^g_{1,t} = p^{g,k}_t M^g_{1,t} \tag{117}$$

$$\frac{\partial \mathcal{L}^g_{w,t}}{\partial u^g_t}: \quad p^k_t M^g_{1,t}\delta'(u^g_t) = \alpha p^g_{w,t} A^g_t (u^g_t K^g_t)^{\alpha-1}(L^g_{d,t})^{1-\alpha} \tag{118}$$

$$\frac{\partial \mathcal{L}^g_{w,t}}{\partial K^g_{t+1}}: \quad p^{g,k}_t M^g_{1,t} = \mathbb{E}_t \Lambda_{t,t+1}\left[\alpha p^g_{w,t+1} A^g_{t+1}(u^g_{t+1})^\alpha (K^g_{t+1})^{\alpha-1}(L^g_{d,t+1})^{1-\alpha} + (1-\delta(u^g_{t+1}))p^{g,k}_{t+1} M^g_{1,t+1}\right] \tag{119}$$

$$\frac{\partial \mathcal{L}^g_{w,t}}{\partial B_t}: \quad Q^g_t M^g_{2,t} = \mathbb{E}_t \Lambda_{t,t+1}\Pi^{-1}_{t+1}\left[1+\kappa Q^g_{t+1} M^g_{2,t+1}\right] \tag{120}$$

For the brown wholesaler we have:

$$w^b_t = p^b_{w,t}(1-\alpha)A^b_t(u^b_t K^b_t)^\alpha (L^b_{d,t})^{-\alpha} \tag{121}$$

$$\nu^b_{1,t} = p^{b,k}_t M^b_{1,t} \tag{122}$$

$$p^k_t M^b_{1,t}\delta'(u^b_t) = \alpha p^b_{w,t} A^b_t (u^b_t K^b_t)^{\alpha-1}(L^b_{d,t})^{1-\alpha} \tag{123}$$

$$p^k_t M^b_{1,t} = \mathbb{E}_t \Lambda_{t,t+1}\left[\alpha p^b_{w,t+1} A^b_{t+1}(u^b_{t+1})^\alpha (K^b_{t+1})^{\alpha-1}(L^b_{d,t+1})^{1-\alpha} + (1-\delta(u^b_{t+1}))p^{b,k}_{t+1} M^b_{1,t+1}\right] \tag{124}$$

$$Q^b_t M^b_{2,t} = \mathbb{E}_t \Lambda_{t,t+1}\Pi^{-1}_{t+1}\left[1+\kappa Q^b_{t+1} M^b_{2,t+1}\right] \tag{125}$$

$$\frac{M^b_{1,t}-1}{M^b_{2,t}-1} = \psi \tag{126}$$

Similarly in Foreign:

$$w^{*g}_t = p^{*g}_{w,t}(1-\alpha)A^{*g}_t(u^{*g}_t K^{*g}_t)^\alpha (L^{*g}_{d,t})^{-\alpha} \tag{127}$$

$$\nu^{*g}_{1,t} = p^{*g,k}_t M^{*g}_{1,t} \tag{128}$$

$$p^{*k}_t M^{*g}_{1,t}\delta'(u^{*g}_t) = \alpha p^{*g}_{w,t} A^{*g}_t (u^{*g}_t K^{*g}_t)^{\alpha-1}(L^{*g}_{d,t})^{1-\alpha} \tag{129}$$

$$p^{*k}_t M^{*g}_{1,t} = \mathbb{E}_t \Lambda^*_{t,t+1}\left[\alpha p^{*g}_{w,t+1} A^{*g}_{t+1}(u^{*g}_{t+1})^\alpha (K^{*g}_{t+1})^{\alpha-1}(L^{*g}_{d,t+1})^{1-\alpha} + (1-\delta(u^{*g}_{t+1}))p^{*g,k}_{t+1} M^{*g}_{1,t+1}\right] \tag{130}$$

$$Q^{*g}_t M^{*g}_{2,t} = \mathbb{E}_t \Lambda^*_{t,t+1}(\Pi^*_{t+1})^{-1}\left[1+\kappa Q^{*g}_{t+1} M^{*g}_{2,t+1}\right] \tag{131}$$

$$\frac{M^{*g}_{1,t}-1}{M^{*g}_{2,t}-1} = \psi \tag{132}$$

$$w_t^{*b} = p_{w,t}^{*b}(1-\alpha)A_t^{*b}(u_t^{*b}K_t^{*b})^{\alpha}(L_{d,t}^{*b})^{-\alpha} \tag{133}$$

$$\nu_{1,t}^{*b} = p_t^{*b,k}M_{1,t}^{*b} \tag{134}$$

$$p_t^{*k}M_{1,t}^{*b}\delta'(u_t^{*b}) = \alpha p_{w,t}^{*b}A_t^{*b}(u_t^{*b}K_t^{*b})^{\alpha-1}(L_{d,t}^{*b})^{1-\alpha} \tag{135}$$

$$p_t^{*k}M_{1,t}^{*b} = \mathbb{E}_t\Lambda_{t,t+1}^{*}\left[\alpha p_{w,t+1}^{*b}A_{t+1}^{*b}(u_{t+1}^{*b})^{\alpha}(K_{t+1}^{*b})^{\alpha-1}(L_{d,t+1}^{*b})^{1-\alpha} + (1-\delta(u_{t+1}^{*b}))p_{t+1}^{*b,k}M_{1,t+1}^{*b}\right] \tag{136}$$

$$Q_t^{*b}M_{2,t}^{*b} = \mathbb{E}_t\Lambda_{t,t+1}^{*}(\Pi_{t+1}^{*})^{-1}\left[1+\kappa Q_{t+1}^{*b}M_{2,t+1}^{*b}\right] \tag{137}$$

$$\frac{M_{1,t}^{*b}-1}{M_{2,t}^{*b}-1} = \psi \tag{138}$$

A.7.7 Financial intermediaries

As international financial markets are integrated, in each country a representative intermediary can hold both Home and Foreign private bonds issued by (the two types of) wholesale firms. I model financial intermediaries by taking the framework proposed by Wu et al. (2024) to a two-firm set-up within each country.

Each period financial intermediaries pay a fraction $1-\sigma$ (with $\sigma \in [0,1]$) of their beginning balance (to be defined shortly) to and receive X real funds from their household owners. As long as intermediaries keep operating, they finance their activities through their net worth N_t and deposits D_t, exclusively from domestic households. These activities of each intermediary consist in investing into domestic central bank reserves RE_t and corporate bonds from the two types of wholesale firms from Home and Foreign, i.e. $B_{H,t}^{g,FI}$, $B_{H,t}^{b,FI}$, $B_{F,t}^{g,FI}$ and $B_{F,t}^{b,FI}$. The resulting balance sheet condition is:

$$Q_t^g B_{H,t}^{g,FI} + Q_t^b B_{H,t}^{b,FI} + Q_t^{*g} B_{F,t}^{g,FI}\mathcal{E}_t + Q_t^{*b} B_{F,t}^{b,FI}\mathcal{E}_t + RE_t = D_t + N_t \tag{139}$$

where Q_t^g, Q_t^b, Q_t^{*g} and Q_t^{*b} are the prices of the (holdings of the) bonds. Foreign bonds are denominated in the currency of the country of issuance, and thus their value in the balance sheet condition is converted into domestic currency using the nominal exchange rate $\mathcal{E}_t$.

In real terms:

$$Q_t^g b_{H,t}^{g,FI} + Q_t^b b_{H,t}^{b,FI} + Q_t^{*g} b_{F,t}^{g,FI}\mathcal{E}_t + Q_t^{*b} b_{F,t}^{b,FI}\mathcal{E}_t + re_t = d_t + n_t \tag{140}$$

The beginning balance in each period t is thus:

$$\mathcal{B}_t = (R_t^g - R_{t-1}^d)Q_{t-1}^g B_{H,t-1}^{g,FI} + (R_t^b - R_{t-1}^d)Q_{t-1}^b B_{H,t-1}^{b,FI} \tag{141}$$

$$+ \left(R_t^{*g}\frac{\mathcal{E}_t}{\mathcal{E}_{t-1}}\left[1-\omega/2(b_{F,t-1}^{g,FI}/b_F^{g,FI}-1)^2\right] - R_{t-1}^d\right)Q_{t-1}^{*g}B_{F,t-1}^{g,FI}\mathcal{E}_{t-1} \tag{142}$$

$$+ \left(R_t^{*b}\frac{\mathcal{E}_t}{\mathcal{E}_{t-1}}\left[1-\omega/2(b_{F,t-1}^{b,FI}/b_F^{b,FI}-1)^2\right] - R_{t-1}^d\right)Q_{t-1}^{*b}B_{F,t-1}^{b,FI}\mathcal{E}_{t-1} \tag{143}$$

$$+ (R_{t-1}^{re} - R_{t-1}^d)RE_{t-1} + R_{t-1}^d N_{t-1} \tag{144}$$

where returns are defined in (111), (112), (113) and (114), while R^{re}_{t-1} is the nominal interest rate on reserves of the domestic central bank. The balance sums up the excess returns over the cost of deposits of the domestic green and brown bonds, as well as the foreign ones. Importantly, I assume quadratic transaction costs for (real) Foreign bonds, whose generated revenues are paid to the government. I define $b^{g,FI}_{F,t} \equiv B^{g,FI}_{F,t}/P^*_t$ and $b^{b,FI}_{F,t} \equiv B^{b,FI}_{F,t}/P^*_t$ with $b^{g,FI}_F$ and $b^{b,FI}_F$ their steady-state values. Also notice the smaller ω, the higher the degree of financial openness of a country. This additional friction is a common way in the literature to achieve the existence of a determinate steady state and a stationary solution in open-economy models with domestic holdings of Foreign bonds whose returns are exogenously determined abroad (Schmitt-Grohé & Uribe, 2003). The last term in the balance sheet condition measures savings from using own net worth as opposed to deposits as source of financing.

Intermediaries have thus net worth

$$N_t = \sigma \mathcal{B}_t + P_t X \tag{145}$$

and want to maximise the present discounted value of their future dividends discounting by the stochastic discount factor of the household $\Lambda_{t,t+1}$:

$$V_t = \max (1-\sigma) \mathbb{E}_t \sum_{j=1}^{\infty} \Lambda_{t,t+j} \lfloor_{t+j} \tag{146}$$

where $\lfloor_t \equiv \mathcal{B}_t/P_t$ and $(1-\sigma)\lfloor_t$ are real initial balance and dividend payment, respectively. Notice the real-term expression for (145) is:

$$n_t = \sigma \lfloor_t + X \tag{147}$$

and thus

$$n_t = \sigma \Pi_t^{-1} \Bigg\{ (R^g_t - R^d_{t-1}) Q^g_{t-1} b^{g,FI}_{H,t-1} + (R^b_t - R^d_{t-1}) Q^b_{t-1} b^{b,FI}_{H,t-1} \tag{148}$$

$$+ \left(R^{*g}_t \frac{\mathcal{E}_t}{\mathcal{E}_{t-1}} \left[1 - \omega/2 (b^{g,FI}_{F,t-1}/b^{g,FI}_F - 1)^2 \right] - R^d_{t-1} \right) Q^{*g}_{t-1} b^{g,FI}_{F,t-1} \mathcal{Q}_{t-1} \tag{149}$$

$$+ \left(R^{*b}_t \frac{\mathcal{E}_t}{\mathcal{E}_{t-1}} \left[1 - \omega/2 (b^{b,FI}_{F,t-1}/b^{b,FI}_F - 1)^2 \right] - R^d_{t-1} \right) Q^{*b}_{t-1} b^{b,FI}_{F,t-1} \mathcal{Q}_{t-1} \tag{150}$$

$$+ (R^{re}_{t-1} - R^d_{t-1}) re_{t-1} + R^d_{t-1} n_{t-1} \Bigg\} + X \tag{151}$$

This problem can be rewritten in recursive form:

$$V_t = \max \left[(1-\sigma) \mathbb{E}_t \Lambda_{t,t+1} \lfloor_{t+1} + \mathbb{E}_t \Lambda_{t,t+1} V_{t+1} \right] \tag{152}$$

and substituting in the expression for $\lfloor_{t+1}$ (notice $\lfloor_{t+1} = \mathcal{B}_{t+1}/P_{t+1}$ and thus inflation terms appear in the expression below for the elements within $\mathcal{B}_{t+1}$ that are lagged - through multiplying and dividing by P_t):

$$V_t = \max \Bigg\{ (1-\sigma)\mathbb{E}_t \Lambda_{t,t+1} \Pi_{t+1}^{-1} \Bigg\{ (R_{t+1}^g - R_t^d) Q_t^g b_{H,t}^{g,FI} + (R_{t+1}^b - R_t^d) Q_t^b b_{H,t}^{b,FI}$$
$$+ \left(R_{t+1}^{*g} \frac{\mathcal{E}_{t+1}}{\mathcal{E}_t} \left[1 - \omega/2 (b_{F,t}^{g,FI} / b_F^{g,FI} - 1)^2 \right] - R_t^d \right) Q_t^{*g} b_{F,t}^{g,FI} \mathcal{Q}_t$$
$$+ \left(R_{t+1}^{*b} \frac{\mathcal{E}_{t+1}}{\mathcal{E}_t} \left[1 - \omega/2 (b_{F,t}^{b,FI} / b_F^{b,FI} - 1)^2 \right] - R_t^d \right) Q_t^{*b} b_{F,t}^{b,FI} \mathcal{Q}_t$$
$$+ (R_t^{re} - R_t^d) re_t + R_t^d n_t \Bigg\} + \mathbb{E}_t \Lambda_{t,t+1} V_{t+1} \Bigg\} \tag{153}$$

where $b_{H,t}^{g,FI} = B_{H,t}^{g,FI}/P_t$, $b_{H,t}^{b,FI} = B_{H,t}^{b,FI}/P_t$, $n_t = N_t/P_t$, $d_t = D_t/P_t$ and $re_t = RE_t/P_t$.

Intermediaries in both countries are constrained by a non-absconding requirement (Gertler & Karadi, 2011), which prevents them from declaring bankruptcy and retain the entirety of their balance sheet. While reserves at the domestic central bank are fully recoverable by depositors, intermediaries can only divert away from households a stochastic and exogenous fraction θ_t of their real holdings of green and brown domestic (foreign) bonds $\Delta Q_t^g b_{H,t}^{g,FI} + Q_t^b b_{H,t}^{b,FI} + \left[\Delta Q_t^{*g} b_{F,t}^{g,FI} + Q_t^{*b} b_{F,t}^{b,FI} \right] \mathcal{Q}_t$, where $0 \leq \Delta \leq 1$ is the steady state difference in recoverability between those bonds. As a result of this, for households to be still willing to lend funds, intermediaries must guarantee (also in steady-state) excess returns of long term private bonds over the deposit rate, with the spread between green and brown bonds calibrated through Δ.

The non-absconding constraint for intermediaries is thus:

$$V_t \geq \theta_t \Bigg\{ \Delta Q_t^g b_{H,t}^{g,FI} + Q_t^b b_{H,t}^{b,FI} + \left[\Delta Q_t^{*g} b_{F,t}^{g,FI} + Q_t^{*b} b_{F,t}^{b,FI} \right] \mathcal{Q}_t \Bigg\} \tag{154}$$

The maximisation problem in (153) subject to (154) has Lagrangian

$$\mathcal{L}_{Int,t} = \max (1+\lambda_t) \mathbb{E}_t \Bigg\{ (1-\sigma) \Lambda_{t,t+1} \Pi_{t+1}^{-1} \Bigg\{ (R_{t+1}^g - R_t^d) Q_t^g b_{H,t}^{g,FI} + (R_{t+1}^b - R_t^d) Q_t^b b_{H,t}^{b,FI}$$
$$+ \left(R_{t+1}^{*g} \frac{\mathcal{E}_{t+1}}{\mathcal{E}_t} \left[1 - \omega/2 (b_{F,t}^{g,FI} / b_F^{g,FI} - 1)^2 \right] - R_t^d \right) Q_t^{*g} b_{F,t}^{g,FI} \mathcal{Q}_t$$
$$+ \left(R_{t+1}^{*b} \frac{\mathcal{E}_{t+1}}{\mathcal{E}_t} \left[1 - \omega/2 (b_{F,t}^{b,FI} / b_F^{b,FI} - 1)^2 \right] - R_t^d \right) Q_t^{*b} b_{F,t}^{b,FI} \mathcal{Q}_t$$
$$+ (R_t^{re} - R_t^d) re_t + R_t^d n_t \Bigg\} + \Lambda_{t,t+1} V_{t+1} \Bigg\}$$
$$- \lambda_t \theta_t \Bigg\{ \Delta Q_t^g b_{H,t}^{g,FI} + Q_t^b b_{H,t}^{b,FI} + \left[\Delta Q_t^{*g} b_{F,t}^{g,FI} + Q_t^{*b} b_{F,t}^{b,FI} \right] \mathcal{Q}_t \Bigg\}$$

where λ_t is the Lagrangian multiplier on constraint (154).

The FOCs are:

$$\frac{\partial \mathcal{L}_{Int,t}}{\partial b^g_{H,t}}: \quad (1+\lambda_t)\mathbb{E}_t\left\{(1-\sigma)\Lambda_{t,t+1}\Pi^{-1}_{t+1}\left\{(R^g_{t+1}-R^d_t)Q^g_t\right\}+\Lambda_{t,t+1}\frac{\partial V_{t+1}}{\partial \lfloor_{t+1}}\right\}-\lambda_t\theta_t\Delta Q^g_t=0$$

$$\frac{\partial \mathcal{L}_{Int,t}}{\partial b^b_{H,t}}: \quad \nu^g_{1,t}=p^k_t M^g_{1,t}$$

$$\frac{\partial \mathcal{L}_{Int,t}}{\partial b^{*g}_{F,t}}: \quad w^g_t=p^g_{w,t}(1-\alpha)A^g_t(u^g_t K^g_t)^\alpha(L^g_{d,t})^{-\alpha}$$

$$\frac{\partial \mathcal{L}_{Int,t}}{\partial b^{*b}_{F,t}}: \quad \nu^g_{1,t}=p^k_t M^g_{1,t}$$

$$\frac{\partial \mathcal{L}_{Int,t}}{\partial re_t}: \quad \nu^g_{1,t}=p^k_t M^g_{1,t}$$

Defining the auxiliary variable $\Omega_{t+1}\equiv 1-\sigma+\frac{\partial V_{t+1}}{\partial \lfloor_{t+1}}$, the FOCs simplify to:

$$\frac{\lambda_t}{1+\lambda_t}\theta_t\Delta=\mathbb{E}_t\Lambda_{t,t+1}\Omega_{t+1}\Pi^{-1}_{t+1}(R^g_{t+1}-R^d_t) \tag{155}$$

$$\frac{\lambda_t}{1+\lambda_t}\theta_t=\mathbb{E}_t\Lambda_{t,t+1}\Omega_{t+1}\Pi^{-1}_{t+1}(R^b_{t+1}-R^d_t) \tag{156}$$

$$\frac{\lambda_t}{1+\lambda_t}\theta_t\Delta=\mathbb{E}_t\Lambda_{t,t+1}\Omega_{t+1}\Pi^{-1}_{t+1}\left\{R^{*g}_t\frac{\mathcal{E}_{t+1}}{\mathcal{E}_t}\left[1-\frac{\omega}{2}(b^{g,FI}_{F,t}/b^{g,FI}_F-1)(3b^{g,FI}_{F,t}/b^{g,FI}_F-1)\right]-R^d_t\right\} \tag{157}$$

$$\frac{\lambda_t}{1+\lambda_t}\theta_t=\mathbb{E}_t\Lambda_{t,t+1}\Omega_{t+1}\Pi^{-1}_{t+1}\left\{R^{*b}_t\frac{\mathcal{E}_{t+1}}{\mathcal{E}_t}\left[1-\frac{\omega}{2}(b^{b,FI}_{F,t}/b^{b,FI}_F-1)(3b^{b,FI}_{F,t}/b^{b,FI}_F-1)\right]-R^d_t\right\} \tag{158}$$

$$0=\mathbb{E}_t\Lambda_{t,t+1}\Omega_{t+1}\Pi^{-1}_{t+1}(R^{re}_t-R^d_t) \tag{159}$$

When the constraints binds and thus $\lambda_t>0$, the FOCs show that the model generates excess returns of investing in bonds while financing through deposits. These returns get closer across types of bonds as Δ approaches one, which increases the required returns of green bonds. As in Wu et al. (2024), I assume that (154) always binds, so that the slope of the yield curve is never zero and always positive, as usually the case in reality.

Defining

$$\phi\equiv V/(\sigma\theta\lfloor_t) \tag{160}$$

and by definition of Ω_{t+1}, I get the expression:

$$\Omega_t=1-\sigma+\sigma\theta\phi_t \tag{161}$$

By definition of ϕ the binding constraint (154) implies:

$$\phi = \frac{\Delta Q_t^g b_{H,t}^{g,FI} + Q_t^b b_{H,t}^{b,FI} + \left[\Delta Q_t^{*g} b_{F,t}^{g,FI} + Q_t^{*b} b_{F,t}^{b,FI}\right] \mathcal{Q}_t}{\sigma \lfloor_t} \tag{162}$$

Combining this with the FOCs and the beginning balance (144):

$$\mathbb{E}_t \left[\Lambda_{t,t+1} \Omega_{t+1} \lfloor_{t+1}\right] = \sigma \frac{\lambda_t}{1+\lambda_t} \theta \phi_t \lfloor_t + \mathbb{E}_t \left[\Lambda_{t,t+1} \Omega_{t+1} \Pi_{t+1}^{-1}\right] R_t^d n_t + \Phi_t \tag{163}$$

with

$$\Phi_t = \omega \mathbb{E}_t \Lambda_{t,t+1} \Omega_{t+1} \Pi_{t+1}^{-1} R_{t+1}^{*g} \frac{\mathcal{E}_{t+1}}{\mathcal{E}_t} \left(\frac{b_{F,t}^{g,FI}}{b_F^{g,FI}} - 1 \right) \frac{b_{F,t}^{g,FI}}{b_F^{g,FI}} Q_t^{*g} b_{F,t}^{g,FI} \mathcal{Q}_t \tag{164}$$

$$+ \omega \mathbb{E}_t \Lambda_{t,t+1} \Omega_{t+1} \Pi_{t+1}^{-1} R_{t+1}^{*b} \frac{\mathcal{E}_{t+1}}{\mathcal{E}_t} \left(\frac{b_{F,t}^{b,FI}}{b_F^{b,FI}} - 1 \right) \frac{b_{F,t}^{b,FI}}{b_F^{b,FI}} Q_t^{*b} b_{F,t}^{b,FI} \mathcal{Q}_t \tag{165}$$

Rewriting (152) through (160), (161) and (163), and dividing by $\lfloor_t$, I get the expression for:

$$\phi_t = \frac{1+\lambda_t}{\sigma\theta} \left\{ \mathbb{E}_t \Lambda_{t,t+1} \Omega_{t+1} \Pi_{t+1}^{-1} R_t^d \frac{n_t}{\lfloor_t} + \frac{\Phi_t}{\lfloor_t} \right\} \tag{166}$$

The financial intermediary pays domestic households real dividends $div_t^{FI} = (1-\sigma)\lfloor_t$.

Similarly for the foreign economy we have:

$$\frac{\lambda_t^*}{1+\lambda_t^*} \theta_t^* \Delta = \mathbb{E}_t \Lambda_{t,t+1}^* \Omega_{t+1}^* (\Pi_{t+1}^*)^{-1} (R_{t+1}^{*g} - R_t^{*d}) \tag{167}$$

$$\frac{\lambda_t^*}{1+\lambda_t^*} \theta_t^* = \mathbb{E}_t \Lambda_{t,t+1}^* \Omega_{t+1}^* (\Pi_{t+1}^*)^{-1} (R_{t+1}^{*b} - R_t^{*d}) \tag{168}$$

$$\frac{\lambda_t^*}{1+\lambda_t^*} \theta_t^* \Delta = \mathbb{E}_t \Lambda_{t,t+1}^* \Omega_{t+1}^* (\Pi_{t+1}^*)^{-1} \left\{ R_t^g \frac{\mathcal{E}_t}{\mathcal{E}_{t+1}} \left[1 - \frac{\omega}{2} (b_{H,t}^{*g,FI} / b_H^{*g,FI} - 1)(3 b_{H,t}^{*g,FI} / b_H^{*g,FI} - 1) \right] - R_t^{*d} \right\} \tag{169}$$

$$\frac{\lambda_t^*}{1+\lambda_t^*} \theta_t^* = \mathbb{E}_t \Lambda_{t,t+1}^* \Omega_{t+1}^* (\Pi_{t+1}^*)^{-1} \left\{ R_t^b \frac{\mathcal{E}_t}{\mathcal{E}_{t+1}} \left[1 - \frac{\omega}{2} (b_{H,t}^{*b,FI} / b_H^{*b,FI} - 1)(3 b_{H,t}^{*b,FI} / b_H^{*b,FI} - 1) \right] - R_t^{*d} \right\} \tag{170}$$

$$0 = \mathbb{E}_t \Lambda_{t,t+1}^* \Omega_{t+1}^* (\Pi_{t+1}^*)^{-1} (R_t^{*re} - R_t^{*d}) \tag{171}$$

$$Q_t^{*g} b_{F,t}^{*g,FI} + Q_t^{*b} b_{F,t}^{*b,FI} + Q_t^g b_{H,t}^{*g,FI} \mathcal{E}_t^{-1} + Q_t^b b_{H,t}^{*b,FI} \mathcal{E}_t^{-1} + re_t^* = d_t^* + n_t^* \tag{172}$$

$$n_t^* = \sigma \lfloor_t^* + X^* \tag{173}$$

$$n_t^* = \sigma(\Pi_t^*)^{-1}\Bigg\{(R_t^{*g} - R_{t-1}^{*d})Q_{t-1}^{*g}b_{F,t-1}^{*g,FI} + (R_t^{*b} - R_{t-1}^{*d})Q_{t-1}^{*b}b_{F,t-1}^{*b,FI} \tag{174}$$

$$+ \left(R_t^g \frac{\mathcal{E}_{t-1}}{\mathcal{E}_t}\left[1 - \omega/2(b_{H,t-1}^{*g,FI}/b_H^{*g,FI} - 1)^2\right] - R_{t-1}^{*d}\right) Q_{t-1}^g b_{H,t-1}^{*,FI}/\mathcal{Q}_{t-1} \tag{175}$$

$$+ \left(R_t^b \frac{\mathcal{E}_{t-1}}{\mathcal{E}_t}\left[1 - \omega/2(b_{H,t-1}^{*b,FI}/b_H^{*b,FI} - 1)^2\right] - R_{t-1}^{*d}\right) Q_{t-1}^b b_{H,t-1}^{*b,FI}/\mathcal{Q}_{t-1} \tag{176}$$

$$+ (R_{t-1}^{*re} - R_{t-1}^{*d})re_{t-1}^* + R_{t-1}^{*d}n_{t-1}^*\Bigg\} + X^* \tag{177}$$

$$\Omega_t^* = 1 - \sigma + \sigma\theta^*\phi_t^* \tag{178}$$

$$\phi^* = \frac{\Delta Q_t^{*g}b_{F,t}^{*g,FI} + Q_t^{*b}b_{F,t}^{*b,FI} + \left[\Delta Q_t^g b_{H,t}^{*g,FI} + Q_t^b b_{H,t}^{*b,FI}\right]/\mathcal{Q}_t}{\sigma \lfloor_t^*} \tag{179}$$

$$\Phi_t^* = \omega\mathbb{E}_t\Lambda_{t,t+1}^*\Omega_{t+1}^*(\Pi_{t+1}^*)^{-1}R_{t+1}^g\frac{\mathcal{E}_t}{\mathcal{E}_{t+1}}\left(\frac{b_{H,t}^{*g,FI}}{b_H^{*g,FI}} - 1\right)\frac{b_{H,t}^{*g,FI}}{b_H^{*g,FI}}Q_t^g b_{H,t}^{*g,FI}/\mathcal{Q}_t \tag{180}$$

$$+ \omega\mathbb{E}_t\Lambda_{t,t+1}^*\Omega_{t+1}^*(\Pi_{t+1}^*)^{-1}R_{t+1}^b\frac{\mathcal{E}_t}{\mathcal{E}_{t+1}}\left(\frac{b_{H,t}^{*b,FI}}{b_H^{*b,FI}} - 1\right)\frac{b_{H,t}^{*b,FI}}{b_H^{*b,FI}}Q_t^b b_{H,t}^{*b,FI}/\mathcal{Q}_t \tag{181}$$

$$\phi_t^* = \frac{1 + \lambda_t^*}{\sigma\theta^*}\left\{\mathbb{E}_t\Lambda_{t,t+1}^*\Omega_{t+1}^*(\Pi_{t+1}^*)^{-1}R_t^{*d}\frac{n_t^*}{\lfloor_t^*} + \frac{\Phi_t^*}{\lfloor_t^*}\right\} \tag{182}$$

A.7.8 Labour market

The labour market in each country exhibits a similar system as the one of final-good firms and retailers, but separately for agents facing labour demand from green and brown wholesalers.

There are two labour packers which combines differentiated labour from the labour unions into final labour used in production through the CES technology:

$$L_{d,t}^g = \left(\int_0^{\gamma(1-\phi_g)} L_{d,t}^g(l)^{\frac{e_w-1}{e_w}}dl\right)^{\frac{e_w}{e_w-1}} \tag{183}$$

$$L_{d,t}^b = \left(\int_{\gamma(1-\phi_g)}^{\gamma} L_{d,t}^b(l)^{\frac{e_w-1}{e_w}}dl\right)^{\frac{e_w}{e_w-1}} \tag{184}$$

where e_w is the labour elasticity of substitution, $L_{d,t}^g(l)$ ($L_{d,t}^b(l)$) is differentiated labour from suppliers of green (brown) packers whose price is nominal wage $W_t^g(l)$ ($W_t^b(l)$). Differentiated labour is provided by labour unions indexed by $l \in [0, \gamma)$. Of these, a fraction $1 - \phi_g$, where $0 \leq \phi_g \leq 1$, provides differentiated labour which is combined to satisfy labour demand of green wholesalers. Conversely, a fraction ϕ_g would go to brown packers and, in turn, brown intermediate-good producers.

The maximisation problem for the labour packers is:

$$\max_{L^g_{d,t},\left\{L_{d,t}(l)\right\}_{l\in[0,\gamma(1-\phi_g))}} \mathbb{E}_t \sum_{j=0}^{\infty} \Lambda_{t,t+j} \left[\frac{W^g_{t+j}}{P_{t+j}} L^g_{d,t+j} - \int_0^{\gamma(1-\phi_g)} \frac{W^g_{t+j}(l)}{P_{t+j}} L^g_{d,t+j}(l) dl \right]$$

subject to technology (183), and

$$\max_{L^b_{d,t},\left\{L_{d,t}(l)\right\}_{l\in[\gamma(1-\phi_g),\gamma)}} \mathbb{E}_t \sum_{j=0}^{\infty} \Lambda_{t,t+j} \left[\frac{W^b_{t+j}}{P_{t+j}} L^b_{d,t+j} - \int_{\gamma(1-\phi_g)}^{\gamma} \frac{W^b_{t+j}(l)}{P_{t+j}} L^b_{d,t+j}(l) dl \right]$$

subject to technology (184).

Simplifying the problem to a choice of inputs $L_{d,t}(l)$ only, by plugging the technology functions into the objective function, the FOC for a generic $L_{d,t}(l)$ is (notice I simplified out the common deflator P_t)

$$W^g_t \left(\int_0^{\gamma(1-\phi_g)} L^g_{d,t}(l)^{\frac{e_w-1}{e_w}} dl \right)^{\frac{1}{e_w-1}} L^g_{d,t}(l)^{-\frac{1}{e_w}} - W^g_t(l) = 0$$

$$\Rightarrow W^g_t (L^g_{d,t})^{\frac{1}{e_w}} L^g_{d,t}(l)^{-\frac{1}{e_w}} = W^g_t(l)$$

and thus

$$L^g_{d,t}(l) = \left(\frac{W^g_t(l)}{W^g_t} \right)^{-e_w} L^g_{d,t} \tag{185}$$

where the last equation is the downward-sloping demand function for the differentiated labour faced by unions. Similarly for brown wholesalers' demand:

$$L^b_{d,t}(l) = \left(\frac{W^b_t(l)}{W^b_t} \right)^{-e_w} L^b_{d,t} \tag{186}$$

The aggregate wage levels W^g_t and W^b_t satisfy:

$$(W^g_t)^{1-e_w} = \frac{1}{\gamma(1-\phi_g)} \int_0^{\gamma(1-\phi_g)} W^g_t(l)^{1-e_w} dl \tag{187}$$

$$(W^b_t)^{1-e_w} = \frac{1}{\gamma\phi_g} \int_{\gamma(1-\phi_g)}^{\gamma} W^b_t(l)^{1-e_w} dl \tag{188}$$

Unions buy labour $L^g_t(l)$ and $L^b_t(l)$ from households at price MRS^g_t and MRS^b_t respectively. They operate with a simple "repackaging" technology represented by the production function $L^g_{d,t}(l) = L^g_t(l)$ and $L^b_{d,t}(l) = L^b_t(l)$. With this technology and the demand functions above, the nominal profits the union tries to maximise are:

$$DIV^g_{L,t}(l) = [W^g_t(l) - MRS^g_t](W^g_t(l))^{-e_w}(W^g_t)^{e_w} L^g_{d,t}$$

$$DIV^b_{L,t}(l) = [W^b_t(l) - MRS^b_t](W^b_t(l))^{-e_w}(W^b_t)^{e_w}L^b_{d,t}$$

In real terms (dividing by P_t and rearranging so to feature real wages w^g_t and w^b_t):

$$div^g_{L,t}(l) = (W^g_t(l))^{1-e_w}P_t^{e_w-1}(w^g_t)^{e_w}L^g_{d,t} - mrs^g_t(W^g_t(l))^{-e_w}P_t^{e_w}(w^g_t)^{e_w}L^g_{d,t}$$

$$div^b_{L,t}(l) = (W^b_t(l))^{1-e_w}P_t^{e_w-1}(w^b_t)^{e_w}L^b_{d,t} - mrs^b_t(W^b_t(l))^{-e_w}P_t^{e_w}(w^b_t)^{e_w}L^b_{d,t}$$

In this market there are Calvo-type wage rigidities, that is the union can adjust its wages in each period with probability $1-\phi_w$, where $0 \leq \phi_w \leq 1$.

Thus, each union able to re-optimise wants to maximise the present discounted value of its (real) profits using the discount factor of the household. I develop here the case for the green labour union, but the derivation for the others is similar.

$$\max_{W^g_t(l)} \mathbb{E}_t \sum_{j=0}^{\infty} \phi^j_w \Lambda_{t,t+j} \left[(W^g_t(l))^{1-e_w}P_{t+j}^{e_w-1}(w^g_{t+j})^{e_w}L^g_{d,t+j} - mrs^g_{t+j}(W^g_t(l))^{-e_w}P^{e_w}_{t+j}(w^g_{t+j})^{e_w}L^g_{d,t+j} \right]$$

The FOC is:

$$\frac{\partial \mathcal{L}^g_{L,t}}{\partial W^g_t(l)}: \quad (e_w-1)W^g_t(l)^{-e_w}\mathbb{E}_t \sum_{j=0}^{\infty} \phi^j_w \Lambda_{t,t+j} P_{t+j}^{e_w-1}(w^g_{t+j})^{e_w}L^g_{d,t+j} = e_w(W^g_t(l))^{-e_w-1}\mathbb{E}_t \sum_{j=0}^{\infty} \phi^j_w \Lambda_{t,t+j} mrs^g_{t+j} P^{e_w}_{t+j}(w^g_{t+j})^{e_w}L^g_{d,t+j}$$

Define the auxiliary variables:

$$F^g_{1,t} \equiv \sum_{j=0}^{\infty} \phi^j_w \Lambda_{t,t+j} mrs^g_{t+j} P^{e_w}_{t+j}(w^g_{t+j})^{e_w}L^g_{d,t+j}$$
$$F^g_{2,t} \equiv \sum_{j=0}^{\infty} \phi^j_w \Lambda_{t,t+j} P_{t+j}^{e_w-1}(w^g_{t+j})^{e_w}L^g_{d,t+j}$$

Equivalently, in the more convenient recursive form:

$$F^g_{1,t} \equiv mrs^g_t P^{e_w}_t (w^g_t)^{e_w}L^g_{d,t} + \phi_w \mathbb{E}_t \Lambda_{t,t+1} F^g_{1,t+1}$$
$$F^g_{2,t} \equiv P_t^{e_w-1}(w^g_t)^{e_w}L^g_{d,t} + \phi_w \mathbb{E}_t \Lambda_{t,t+1} F^g_{2,t+1}$$

I now divide the first variable by $P^{e_w}_t$ and the second by $P_t^{e_w-1}$ to define $f^g_{1,t}$ and $f^g_{2,t}$ (also $f^g_{1,t} = F^g_{1,t}/P^{e_w}_t \Rightarrow f^g_{1,t+1} = F^g_{1,t+1}/P^{e_w}_{t+1}$ and $f^g_{2,t} = F^g_{2,t}/P_t^{e_w-1} \Rightarrow f^g_{2,t+1} = F^g_{2,t+1}/P_{t+1}^{e_w-1}$):

$$f_{1,t}^g \equiv mrs_t^g (w_t^g)^{e_w} L_{d,t}^g + \phi_w \mathbb{E}_t \Lambda_{t,t+1} \Pi_{t+1}^{e_w} f_{1,t+1}^g \tag{189}$$

$$f_{2,t}^g \equiv (w_t^g)^{e_w} L_{d,t}^g + \phi_w \mathbb{E}_t \Lambda_{t,t+1} \Pi_{t+1}^{e_w - 1} f_{2,t+1}^g \tag{190}$$

Consider a symmetric equilibrium where unions pick the same wage, then $W_t^g(l) = W_t^{g,o}$ and the FOC simplifies to:

$$w_t^{g,o} = \frac{e_w}{e_w - 1} \frac{f_{1,t}^g}{f_{2,t}^g} \tag{191}$$

where $w_t^{g,o} \equiv \frac{W_t^{g,o}}{P_t}$.

At this point I can integrate the demand function (185):

$$\int_0^{\gamma(1-\phi_g)} L_{d,t}^g(l) dl = \int_0^{\gamma(1-\phi_g)} \left(\frac{W_t^g(l)}{W_t^g} \right)^{-e_w} L_{d,t}^g dl$$

Because each union provides the same amount, i.e. an equal share of the total $L_{d,t}^g$ as shown by the demand function (and thus use equal amounts of $L_t^g(l)$ as inputs), and given unions' technology $L_{d,t}^g(l) = L_t^g(l)$, the left-hand side of the equation above sums up to a fraction of the total supply L_t: $\int_0^{\gamma(1-\phi_g)} L_{d,t}^g(l) dl = \int_0^{\gamma(1-\phi_g)} L_t^g(l) dl = \gamma(1-\phi_g) L_t^g$.

$$\gamma(1-\phi_g) L_t^g = \int_0^{\gamma(1-\phi_g)} \left(\frac{W_t^g(l)}{W_t^g} \right)^{-e_w} L_{d,t}^g dl \quad \Rightarrow \quad L_t^g = v_t^g L_{d,t}^g \tag{192}$$

where $v_t^g = \frac{1}{\gamma(1-\phi_g)} \int_0^{\gamma(1-\phi_g)} \left(\frac{W_t^g(l)}{W_t^g} \right)^{-e_w} dl$ is a measure of wage dispersion.

One can use the Calvo assumption to split v_t^g noting that $(1-\phi_g)$ of unions can update setting the same wage $w_t^{g,o}$, and the rest can only retain the wage from the previous period:

$$v_t^g = \frac{1}{\gamma(1-\phi_g)} \int_0^{\gamma(1-\phi_g)(1-\phi_w)} \left(\frac{W_t^{g,o}}{W_t^g} \right)^{-e_w} dl + \frac{1}{\gamma(1-\phi_g)} \int_{\gamma(1-\phi_g)(1-\phi_w)}^{\gamma(1-\phi_g)} \left(\frac{W_{t-1}^g(l)}{W_t^g} \right)^{-e_w} dl$$

which becomes by multiplying and dividing by $P_t^{-e_w}$ the first term and by $(W_{t-1}^g)^{-e_w}$ the second term on the right-hand side respectively, and rearranging terms out of the integrals:

$$v_t^g = (1-\phi_w) \left(\frac{w_t^{g,o}}{w_t^g} \right)^{-e_w} + \frac{1}{\gamma(1-\phi_g)} (W_t^g)^{e_w} (W_{t-1}^g)^{-e_w} \int_{\gamma(1-\phi_g)(1-\phi_w)}^{\gamma(1-\phi_g)} \left(\frac{W_{t-1}^g(l)}{W_{t-1}^g} \right)^{-e_w} dl$$

For the definition of gross inflation and the wage dispersion term, the expression simplifies to:

$$v_t^g = (1-\phi_w) \left(\frac{w_t^{g,o}}{w_t^g} \right)^{-e_w} + \phi_w \Pi_t^{e_w} \left(\frac{w_t^g}{w_t^g} \right)^{-e_w} v_{t-1}^g \tag{193}$$

For the same Calvo price properties, the wage level (187) can be rewritten as:

$$(W_t^g)^{1-e_w} = (1-\phi_w)(W_t^{g,o})^{1-e_w} + \frac{1}{\gamma(1-\phi_g)}\int_{\gamma(1-\phi_g)(1-\phi_w)}^{\gamma(1-\phi_g)} W_{t-1}^g(l)^{1-e_w}dl$$

$$\Rightarrow \quad (W_t^g)^{1-e_w} = (1-\phi_w)(W_t^{g,o})^{1-e_w} + \phi_w(W_{t-1}^g)^{1-e_w}$$

$$\Rightarrow \quad (w_t^g)^{1-e_w} = (1-\phi_w)(w_t^{g,o})^{1-e_w} + \phi_w\Pi_t^{e_w-1}(w_{t-1}^g)^{1-e_w} \tag{194}$$

Similar equations apply for unions linked to the brown packer:

$$w_t^{b,o} = \frac{e_w}{e_w-1}\frac{f_{1,t}^b}{f_{2,t}^b} \tag{195}$$

$$f_{1,t}^b = mrs_t(w_t^b)^{e_w}L_{d,t}^b + \phi_w\mathbb{E}_t\Lambda_{t,t+1}\Pi_{t+1}^{e_w}f_{1,t+1}^b \tag{196}$$

$$f_{2,t}^b = (w_t^b)^{e_w}L_{d,t}^b + \phi_w\mathbb{E}_t\Lambda_{t,t+1}\Pi_{t+1}^{e_w-1}f_{2,t+1}^b \tag{197}$$

$$L_t^b = v_t^b L_{d,t}^b \tag{198}$$

$$v_t^b = (1-\phi_w)\left(\frac{w_t^{b,o}}{w_t^b}\right)^{-e_w} + \phi_w\Pi_t^{e_w}\left(\frac{w_t^b}{w_t^b}\right)^{-e_w}v_{t-1}^b \tag{199}$$

$$(w_t^b)^{1-e_w} = (1-\phi_w)(w_t^{b,o})^{1-e_w} + \phi_w\Pi_t^{e_w-1}(w_{t-1}^b)^{1-e_w} \tag{200}$$

And similar equations apply in Foreign:

$$w_t^{*g,o} = \frac{e_w}{e_w-1}\frac{f_{1,t}^{*g}}{f_{2,t}^{*g}} \tag{201}$$

$$f_{1,t}^{*g} = mrs_t^*(w_t^{*g})^{e_w}L_{d,t}^{*g} + \phi_w\mathbb{E}_t\Lambda_{t,t+1}^*\Pi_{t+1}^{*e_w}f_{1,t+1}^{*g} \tag{202}$$

$$f_{2,t}^{*g} = (w_t^{*g})^{e_w}L_{d,t}^{*g} + \phi_w\mathbb{E}_t\Lambda_{t,t+1}^*(\Pi^*)_{t+1}^{e_w-1}f_{2,t+1}^{*g} \tag{203}$$

$$L_t^{*g} = v_t^{*g}L_{d,t}^{*g} \tag{204}$$

$$v_t^{*g} = (1-\phi_w)\left(\frac{w_t^{*g,o}}{w_t^{*g}}\right)^{-e_w} + \phi_w\Pi_t^{*e_w}\left(\frac{w_t^{*g}}{w_t^{*g}}\right)^{-e_w}v_{t-1}^{*g} \tag{205}$$

$$(w_t^{*g})^{1-e_w} = (1-\phi_w)(w_t^{*g,o})^{1-e_w} + \phi_w(\Pi_t^*)^{e_w-1}(w_{t-1}^{*g})^{1-e_w} \tag{206}$$

$$w_t^{*b,o} = \frac{e_w}{e_w-1}\frac{f_{1,t}^{*b}}{f_{2,t}^{*b}} \tag{207}$$

$$f_{1,t}^{*b} = mrs_t^*(w_t^{*b})^{e_w}L_{d,t}^{*b} + \phi_w\mathbb{E}_t\Lambda_{t,t+1}^*\Pi_{t+1}^{*e_w}f_{1,t+1}^{*b} \tag{208}$$

$$f_{2,t}^{*b} = (w_t^{*b})^{e_w}L_{d,t}^{*b} + \phi_w\mathbb{E}_t\Lambda_{t,t+1}^*(\Pi^*)_{t+1}^{e_w-1}f_{2,t+1}^{*b} \tag{209}$$

$$L_t^{*b} = v_t^{*b}L_{d,t}^{*b} \tag{210}$$

$$v_t^{*b} = (1-\phi_w)\left(\frac{w_t^{*b,o}}{w_t^{*b}}\right)^{-e_w} + \phi_w\Pi_t^{*e_w}\left(\frac{w_t^{*b}}{w_t^{*b}}\right)^{-e_w}v_{t-1}^{*b} \tag{211}$$

$$(w_t^{*b})^{1-e_w} = (1-\phi_w)(w_t^{*b,o})^{1-e_w} + \phi_w(\Pi_t^*)^{e_w-1}(w_{t-1}^{*b})^{1-e_w} \tag{212}$$

A.7.9 Monetary policy

Home and Foreign central banks follow a Taylor rule in setting the domestic nominal (policy) interest rate R_t^{re}, targeting domestic PPI inflation and output fluctuations:

$$\left(\frac{R_t^{re}}{R^{re}}\right) = \left(\frac{R_{t-1}^{re}}{R^{re}}\right)^{\rho_r} \left[\left(\frac{\Pi_{H,t}}{\Pi_H}\right)^{\phi_\pi} \left(\frac{Y_t}{Y_{t-1}}\right)^{\phi_y}\right]^{1-\rho_r} \exp\left(s_r \varepsilon_t^r\right) \tag{213}$$

$$\left(\frac{R_t^{*re}}{R^{*re}}\right) = \left(\frac{R_{t-1}^{*re}}{R^{*re}}\right)^{\rho_r} \left[\left(\frac{\Pi_{H,t}^{*}}{\Pi_H^{*}}\right)^{\phi_\pi} \left(\frac{Y_t^{*}}{Y_{t-1}^{*}}\right)^{\phi_y}\right]^{1-\rho_r} \exp\left(s_{*r} \varepsilon_t^{*r}\right) \tag{214}$$

with monetary policy shocks $\varepsilon_t^r \sim N(0, s_r^2)$ and $\varepsilon_t^{*r} \sim N(0, s_{*r}^2)$. The variables without subscript are the corresponding steady-state targets followed by the central banks. The parameter $0 < \rho_r < 1$ measures the degree of inertia of the nominal interest rate adjustment, while $\phi_\pi > 1$ and $\phi_y > 0$ modulates the reaction of the policy rate to domestic inflation and output, respectively.

The central bank in each country issues reserves to finance its purchases of corporate bonds, the form of unconventional monetary policy included in this setting

$$Q_t^g B_t^{g,CB} + Q_t^b B_t^{b,CB} = RE_t \tag{215}$$

$$Q_t^{*g} B_t^{*g,CB} + Q_t^{*b} B_t^{*b,CB} = RE_t^{*} \tag{216}$$

is the central bank's balance-sheet condition, while the exogenous rules governing changes in domestic (real) bond holdings are

$$b_t^{g,CB} - b^{g,CB} = \rho_g (b_t^{g,CB} - b^{g,CB}) + s_g \varepsilon_t^{g,CB} \tag{217}$$

$$b_t^{b,CB} - b^{b,CB} = \rho_b (b_t^{b,CB} - b^{b,CB}) + s_b \varepsilon_t^{b,CB} \tag{218}$$

$$b_t^{*g,CB} - b^{*g,CB} = \rho_g (b_t^{*g,CB} - b^{*g,CB}) + s_{*g} \varepsilon_t^{*g,CB} \tag{219}$$

$$b_t^{*b,CB} - b^{*b,CB} = \rho_b (b_t^{*b,CB} - b^{*b,CB}) + s_{*b} \varepsilon_t^{*b,CB} \tag{220}$$

where I define $b_t^{g,CB} \equiv B_t^{g,CB}/P_t$, $b_t^{b,CB} \equiv B_t^{b,CB}/P_t$, $b_t^{*g,CB} \equiv B_t^{*g,CB}/P_t^*$ and $b_t^{*b,CB} \equiv B_t^{*b,CB}/P_t^*$. Innovations $\varepsilon_t^{g,CB} \sim N(0, s_g^2)$, $\varepsilon_t^{*g,CB} \sim N(0, s_{*g}^2)$, $\varepsilon_t^{b,CB} \sim N(0, s_b^2)$, $\varepsilon_t^{*b,CB} \sim N(0, s_{*b}^2)$ are policy shocks. The variables without subscript are again the corresponding steady-state values, and the parameters $0 < \rho_g, \rho_b < 1$ are autoregressive coefficients.

The profit made by the central bank on its balance sheet, which is then transferred to the government, is

$$T_t^{CB} = (1 + \kappa Q_t^g)\Pi_t^{-1} b_{t-1}^{g,CB} + (1 + \kappa Q_t^b)\Pi_t^{-1} b_{t-1}^{b,CB} - R_{t-1}^{re}\Pi_t^{-1} re_{t-1} \tag{221}$$

For Foreign a similar expression holds:

$$T_t^{*CB} = (1+\kappa Q_t^{*g})(\Pi_t^*)^{-1} b_{t-1}^{*g,CB} + (1+\kappa Q_t^{*b})(\Pi_t^*)^{-1} b_{t-1}^{*b,CB} - R_{t-1}^{*re}(\Pi_t^*)^{-1} re_{t-1}^* \tag{222}$$

For simplicity, there is no government spending nor debt, which means that the government's budget constraint simplifies to:

$$0 = P_t T_t + P_t T_t^{CB} \tag{223}$$
$$+ R_t^{*g} \frac{\mathcal{E}_t}{\mathcal{E}_{t-1}} \omega/2 (b_{F,t-1}^{g,FI}/b_F^{g,FI} - 1)^2 Q_{t-1}^{*g} B_{F,t-1}^{g,FI} \mathcal{E}_{t-1} \tag{224}$$
$$+ R_t^{*b} \frac{\mathcal{E}_t}{\mathcal{E}_{t-1}} \omega/2 (b_{F,t-1}^{b,FI}/b_F^{b,FI} - 1)^2 Q_{t-1}^{*b} B_{F,t-1}^{b,FI} \mathcal{E}_{t-1} \tag{225}$$

which means that the profit remittance from central bank's asset holdings and from the transaction cost on foreign assets are paid to domestic households in the form of subsidies, i.e. negative taxes.

Similarly for Foreign:

$$0 = P_t^* T_t^* + P_t^* T_t^{*CB} \tag{226}$$
$$+ R_t^{g} \frac{\mathcal{E}_{t-1}}{\mathcal{E}_t} \omega/2 (b_{H,t-1}^{*g,FI}/b_H^{*g,FI} - 1)^2 Q_{t-1}^{g} B_{H,t-1}^{*g,FI}/\mathcal{E}_{t-1} \tag{227}$$
$$+ R_t^{b} \frac{\mathcal{E}_{t-1}}{\mathcal{E}_t} \omega/2 (b_{H,t-1}^{*b,FI}/b_H^{*b,FI} - 1)^2 Q_{t-1}^{b} B_{H,t-1}^{*b,FI}/\mathcal{E}_{t-1} \tag{228}$$

A.7.10 Market clearing

In equilibrium the resources constraint must be satisfied and markets must clear.

As far as domestic goods are concerned, it must hold that the sum of domestic and foreign consumption of home goods together with domestic investment equals domestic output. Thus, the aggregate resource constraint for the each economy is:

$$Y_t = C_{H,t} + C_{H,t}^* + I_t \tag{229}$$
$$Y_t^* = C_{F,t}^* + C_{F,t} + I_t^* \tag{230}$$

Market clearing for bonds:

$$b_t^g = b_{H,t}^{g,FI} + \frac{1-\gamma}{\gamma} b_{H,t}^{*g,FI} + b_t^{g,GB} \tag{231}$$

$$b_t^b = b_{H,t}^{b,FI} + \frac{1-\gamma}{\gamma} b_{H,t}^{*b,FI} + b_t^{b,GB} \tag{232}$$

$$b_t^{*g} = b_{F,t}^{*g,FI} + \frac{\gamma}{1-\gamma} b_{F,t}^{g,FI} + b_t^{*g,GB} \tag{233}$$

$$b_t^{*b} = b_{F,t}^{*b,FI} + \frac{\gamma}{1-\gamma} b_{F,t}^{b,FI} + b_t^{*b,GB} \tag{234}$$

A.7.11 Exogenous processes

The model exhibits three types of exogenous shocks within each economy: the conventional monetary policy shock, the green and brown asset purchase shocks, and a productivity shock. The latter features in the AR(1) process for productivity of green and brown wholesalers A_t and A_t^* in logarithmic form:

$$\ln A_t^g = \rho_A \ln A_{t-1}^g + s_A \varepsilon_t^{A,g} \tag{235}$$

$$\ln A_t^b = \rho_A \ln A_{t-1}^b + s_A \varepsilon_t^{A,b} \tag{236}$$

$$\ln A_t^* = \rho_A \ln A_{t-1}^* + s_{*A} \varepsilon_t^{*A,g} \tag{237}$$

$$\ln A_t^* = \rho_A \ln A_{t-1}^* + s_{*A} \varepsilon_t^{*A,b} \tag{238}$$

A.7.12 Equilibrium

I collect here the equilibrium conditions of the model.

Financial intermediaries

Home

(111), (112), (155), (156), (157), (158), (159), (140), (147), (151), (161), (162), (165), (166).

Foreign

(113), (114), (167), (168), (169), (170), (171), (172), (173), (177), (178), (179), (181), (182).

Households: Intratemporal decision

Home

All: (21).
Green: (14), (15).
Brown: (12), (13).

Foreign

All: (22).
Green: (18), (19).
Brown: (16), (17).

Households: Intertemporal decision

Home

(39), (40), (41), (42), (43), (38), (44).

Foreign

(46), (47), (48), (49), (50).

Production - Wholesalers

Home

Green: (97), (101), (107), (116), (118), (119), (120), (115).
Brown: (98), (102), (108), (121), (123), (124), (125), (126).

Foreign

Green: (99), (103), (109), (127), (129), (130), (131), (132).
Brown: (100), (102), (108), (121), (123), (124), (125), (126).

Production - Retailers

Home

Green: (64), (62), (63), (65), (67), (68).
Brown: (69), (70), (71), (72), (73), (74).

Foreign

Green: (75), (76), (77), (78), (79), (80).
Brown: (81), (82), (83), (84), (85), (86).

Production - Capital producer

Home

Green: (89), (93).
Brown: (90), (94).
Green + Brown: (87).

Foreign

Green: (91), (95).
Brown: (92), (96).
Green + Brown: (88).

Labour market

Home

Green: (191), (189), (190), (192), (193), (194).
Brown: (195), (196), (197), (198), (199), (200).

Foreign

Green: (201), (202), (203), (204), (205), (206).
Brown: (207), (208), (209), (210), (211), (212).

Government and central bank

Home

(225), (215), (221), (213).

Foreign

(228), (216), (222), (214).

Market clearing

Home

(229), (231), (232).

Foreign

(230), (233), (234).

Whole economy

Home

(28), (31), (30).

Foreign

(29), (33), (32).

Cross economies

(23), (25), (24), (26), (27), (34), (35), (36), (37).

Exogenous processes

Home

(217), (218), (235), (236).

Foreign

(219), (220), (237), (238).

There are 160 equations for 160 endogenous variables

$$\{R_t^g, Q_t^g, R_t^b, Q_t^b, R_t^{re}, R_t^d, \Pi_t, \Lambda_{t,t+1}, \Omega_t, \lambda_t, \phi_t, \mu_t, C_t, L_t, mrs_t, w_t^{g,o}, f_{1,t}^g, f_{2,t}^g, w_t^{b,o}, f_{1,t}^b, f_{2,t}^b, w_t^g, w_t^b, L_{d,t}^g, L_{d,t}^b,$$
$$p_{H,t}^o, x_{1,t}, x_{2,t}, \tilde{p}_{m,t}, p_{w,t}^g, p_{w,t}^b, Y_t, Y_{m,t}, Y_{w,t}^g, Y_{w,t}^b, u_t^g, K_t^g, p_t^{g,k}, M_{1,t}^g, M_{2,t}^g, I_t^g, \hat{I}_t^g, u_t^b, K_t^b, p_t^{b,k}, M_{1,t}^b, M_{2,t}^b, I_t^b, \hat{I}_t^b,$$
$$I_t, b_t^g, b_t^b, T_t, T_t^{CB}, b_t^{g,CB}, b_t^{b,CB}, re_t, v_t^p, v_t^{g,w}, v_t^{b,w}, d_t, n_t, A_t^g, A_t^b, C_{H,t}, C_{F,t}, b_{H,t}^{g,FI}, b_{H,t}^{b,FI}, b_{F,t}^{g,FI}, b_{F,t}^{b,FI},$$
$$p_{H,t}, p_{F,t}, \Pi_{H,t}, P_t, \Phi_t, \lfloor_t,$$
$$R_t^{*g}, Q_t^{*g}, R_t^{*b}, Q_t^{*b}, R_t^{*re}, R_t^{*d}, \Pi_t^*, \Lambda_{t,t+1}^*, \Omega_t^*, \lambda_t^*, \phi_t^*, \mu_t^*, C_t^*, L_t^*, mrs_t^*, w_t^{*g,o}, f_{1,t}^{*g}, f_{2,t}^{*g}, w_t^{*b,o}, f_{1,t}^{*b}, f_{2,t}^{*b}, w_t^{*g}, w_t^{*b},$$
$$L_{d,t}^{*g}, L_{d,t}^{*b}, p_{H,t}^{*o}, x_{1,t}^*, x_{2,t}^*, \tilde{p}_{m,t}^*, p_{w,t}^{*g}, p_{w,t}^{*b}, Y_t^*, Y_{m,t}^*, Y_{w,t}^{*g}, Y_{w,t}^{*b}, u_t^{*g}, K_t^{*g}, p_t^{*g,k}, M_{1,t}^{*g}, M_{2,t}^{*g}, I_t^{*g}, \hat{I}_t^{*g}, u_t^{*b}, K_t^{*b}, p_t^{*b,k},$$
$$M_{1,t}^{*b}, M_{2,t}^{*b}, I_t^{*b}, \hat{I}_t^{*b}, I_t^*, b_t^{*g}, b_t^{*b}, T_t^*, T_t^{*CB}, b_t^{*g,CB}, b_t^{*b,CB}, re_t^*, v_t^{*p}, v_t^{*g,w}, v_t^{*b,w}, d_t^*, n_t^*, A_t^{*g}, A_t^{*b}, C_{H,t}^*, C_{F,t}^*,$$
$$b_{H,t}^{*g,FI}, b_{H,t}^{*b,FI}, b_{F,t}^{*g,FI}, b_{F,t}^{*b,FI}, p_{H,t}^*, p_{F,t}^*, \Pi_{H,t}^*, P_t^*, \Phi_t^*, \lfloor_t^*,$$
$$\mathcal{T}_t, \mathcal{E}_t, \mathcal{Q}_t, div_t, \}$$

and 8 exogenous shock $\{\varepsilon_t^A; \varepsilon_t^r; \varepsilon_t^{g,CB}; \varepsilon_t^{b,CB}; \varepsilon_t^{*A}; \varepsilon_t^{*r}; \varepsilon_t^{*g,CB}; \varepsilon_t^{*b,CB}\}$.