Credit Shocks and House Price Volatility

Juhana Siljander*
Imperial College London†

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Abstract

This paper develops a dynamic general equilibrium housing model to explain how credit supply shocks drive housing boom-bust cycles with excess house price volatility. The model introduces a novel mechanism linking investors' capital allocation to real estate dynamics, emphasizing credit-constrained firms rather than household borrowing. It embeds an endogenous credit cycle, generating a bust without requiring an explicit shock reversal. The model also captures key housing market dynamics, including house price momentum, large consumption elasticities from housing wealth, and declining interest rates during the boom. Quantitatively, it explains much of the early 2000s U.S. house price surge, attributing it to credit relaxation that also increased long-run aggregate productivity by 9–13%. The large productivity effects suggest that the economic costs of macroprudential policies restricting credit access may be significantly higher than previously understood.

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†Imperial College Business School Department of Finance. Contact: j.siljander@imperial.ac.uk

1 Introduction

Real estate bubbles and crashes often follow financial liberalization, as seen in Finland, Norway, and Sweden (late 1980s–early 1990s), "the Asian financial crisis" (mid-1990s), "the Japanese asset price bubble" (1986–1991), and the U.S. and European housing booms of the early 2000s. Kindleberger and Aliber (2011) write that while "*most expansions of money and credit do not lead to mania … every mania has been associated with the expansion of credit.*"

However, the role of credit supply shocks in generating sizable housing booms remains debated. Some models link credit relaxation to housing booms (Favilukis et al., 2017; Justiniano et al., 2019; Greenwald, 2018; Greenwald and Guren, 2021), while others argue that unrealistic assumptions—such as ignoring rental markets—invalidate this link, and (potentially irrational) beliefs drive the booms (Iacoviello and Neri, 2010; Kiyotaki et al., 2011; Justiniano et al., 2015; Kaplan et al., 2020).[1] A key puzzle remains: Why do house prices fluctuate so much in the data when models struggle to replicate such volatility? Piazzesi and Schneider (2016) call this the "house price volatility puzzle."

I propose a new theoretical mechanism where credit shocks impact house prices through credit-constrained entrepreneurs rather than household demand. This connects to empirical research on financial frictions and capital allocation (Buera et al., 2011; Chaney et al., 2012; Adelino et al., 2015, 2016; Schmalz et al., 2017; Bahaj et al., 2020; Gupta et al., 2022). Two key observations underpin the mechanism.

First, like bonds, investment properties have homogeneous expected returns across potential owners of the particular properties. In contrast, the productivity of business capital may vary substantially depending on its owner-operator (Fagereng et al., 2020).[2] When productive entrepreneurs expand, less productive investors accommodate this capital demand by shifting their portfolios toward housing and other financial assets. The price impacts depend on marginal investors' portfolio incentives and the capital and housing supply elasticities between these investors, as the credit-driven capital demand shock propagates to the housing market via this *portfolio*

[1] U.S. household debt approximately doubled during 2000 – 2007, largely fueled by an increase in private-label securitization of mortgage backed securities (MBS), which contributed 46% of the MBS market in dollar volume in 2004, up from 22% in 2003 (Levitin and Wachter, 2011). Loan-to-value (LTV) ratios for new mortgage originations also rose from approximately 85 % in 2000 to 95% in 2005 (Duca, Muellbauer and Murphy, 2011).

[2] Fagereng et al. (2020, Table 7) report that returns to net worth among business owners are highly persistent, and they exhibit substantial heterogeneity with a three-fold difference between the median and the 90th percentile.

rebalancing channel.

Second, rental prices depend on asset productivity through the *general equilibrium demand channel*. In a heterogeneous-agent setting, credit constraints affect capital allocation and productivity (Buera et al., 2011; Buera and Moll, 2015; Buera and Nicolini, 2020). Higher aggregate income boosts housing demand, raising rents and house prices. Thus, credit markets influence house prices, as credit relaxation reallocates factor inputs to productive but credit-constrained entrepreneurs.

Figure 1: House price amplification for a credit shock

The figure demonstrates the propagation of the credit shock from the capital market to the housing market: (a) savers value capital at price $p_k = \underline{a}$ and provide perfectly elastic capital supply to entrepreneurs, prohibiting capital price effects; (b) house prices increase with output, which affects rents via the general equilibrium demand channel.

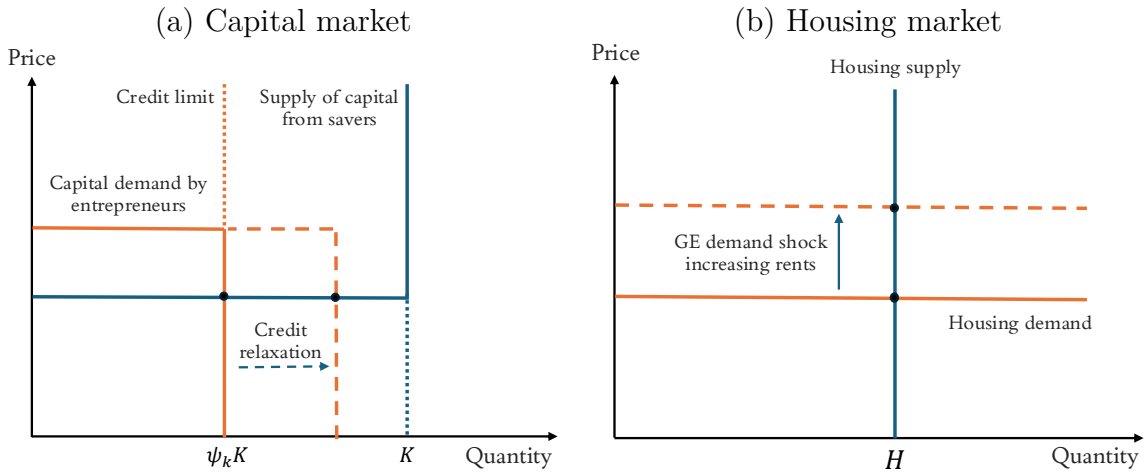


Figure 1 illustrates this mechanism in a model with two types of agents: productive *entrepreneurs* and less productive *savers*. Entrepreneurs facing binding credit constraints have a higher valuation for business capital than savers. Credit relaxation allows them to acquire capital. If capital supply is elastic, capital prices remain stable, but house prices rise due to increased output and demand-driven rent growth.

More broadly, capital supply elasticity between investors is key to understanding house price effects. Prior research has focused on "tenure supply elasticity"—how rental investors supply housing to homebuyers (Greenwald and Guren, 2021). My framework highlights capital supply elasticity from savers to entrepreneurs as an equally relevant factor.

If this supply is inelastic, capital prices react to credit shocks, triggering a financial accelerator that further amplifies the demand channel. I show that the capital supply elasticity is a sufficient statistic for quantifying the impact of the financial accelerator.

Combining the forces implies that the general equilibrium spillover to house prices may be significant enough to outweigh direct capital price effects.

I formalize these ideas in a dynamic heterogeneous-agent model. I solve the model using global solution methods (Brunnermeier and Sannikov, 2016) and consider a credit relaxation that lifts the productive agents completely off their credit constraints (cf. occasionally binding constraints). This causes a sustained boom, where house prices keep increasing disproportionately, justifying even lower rental yields and, thus, higher price-to-rent ratios, while maintaining the marginal investors' no-arbitrage condition between different assets. In particular, the model produces house price momentum akin to Guren (2018), consistent with strong autocorrelation in house prices documented by Case and Shiller (1988).

Calibrating housing supply elasticity with rent and investment data and using observed pre-crisis loan-to-value changes, my model predicts a 34.4% house price increase—closely matching the 40.0% rise in 2003–2007. Moreover, the model replicates the low housing Sharpe ratios observed in data, addressing the "house price volatility puzzle."

Despite the boom, it is short-lived. Entrepreneurs' higher consumption propensity erodes their wealth over time, eventually leading to a reversal. Once credit constraints rebind, entrepreneurs liquidate assets, interest rates drop discontinuously, and output falls due to worsening capital allocation. These "flight-to-safety" dynamics mirror the U.S. financial crisis. Guerrieri and Lorenzoni (2017) show that these dynamics are challenging to produce in standard asset pricing models, and they solve the problem by introducing an exogenous credit tightening, which introduces an additional wedge to the Euler equation that drives asset pricing. My model endogenizes the bust as a natural outcome of credit expansion without an explicit credit shock, consistent with the empirical evidence by Mian and Sufi (2014), who argue that no such shock occurred.

Importantly, my mechanism holds even with frictionless rental markets and endogenous housing supply. Moreover, unlike much of the prior work emphasizing household credit constraints, in my model, housing collateral constraints do not drive asset prices—consistent with Kaplan, Mitman and Violante (2020), who find that constrained households are too few to generate a demand boom. Instead, in my model, the key constraint affects entrepreneurs' business capital.

The model also explains two additional empirical findings. First, it generates large consumption elasticities from housing wealth, similar to Berger, Guerrieri, Lorenzoni and Vavra (2017), but now with endogenous—rather than exogenous—house prices.

Prior macro models have struggled to produce realistic house price booms from calibrated shocks, and the secular increase in house prices has often been explained by increasing housing preferences or better amenities. Such channels would, however, shut down the substitution effect crucial for driving the large consumption elasticities in Berger et al. (2017). My model contributes to the literature by producing the consumption effects in a model with an endogenous housing boom.

Second, my model accounts for the falling interest rates observed during the early 2000s. Initially, consistent with the data, credit relaxation raises interest rates, but over time, agents with low consumption propensities and high savings rates accumulate wealth. This lowers aggregate consumption propensity and drives down interest rates, similar to Mian, Straub and Sufi (2021).

Finally, while the model generates endogenous boom-bust cycles, credit relaxation improves capital allocation, leading to a 9–13% increase in aggregate productivity in the long run. This suggests that macroprudential policies limiting credit access may come with substantial economic costs.

My paper is related to the empirical (Mian and Sufi, 2009; Levitin and Wachter, 2011; Mian et al., 2020) and quantitative theory literature (Kiyotaki et al., 2011; Justiniano et al., 2015, 2019; Favilukis et al., 2017; Greenwald, 2018; Greenwald and Guren, 2021) on how credit conditions affect house prices. I propose a new price amplification mechanism, where a credit shock propagates through the financial markets to generate a housing boom. Concerning such mechanisms, the Kiyotaki and Moore (1997) financial accelerator has been previously considered in models with housing or land by Iacoviello (2005) and Liu, Wang and Zha (2013). I augment this financial accelerator literature by documenting a novel amplification mechanism building on a specific interaction between the capital and housing markets; notably, my mechanism amplifies house prices even without the financial accelerator considered in this literature.[3]

The mechanism builds on two key observations: (i) financial constraints affect capital allocation and productivity, an idea discussed earlier in the theoretical (Buera et al., 2011; Moll, 2014; Buera and Moll, 2015; Buera and Nicolini, 2020) and empirical literature (Gomme et al., 2011; Buera et al., 2011; Chaney et al., 2012; Adelino et al., 2015, 2016; Schmalz et al., 2017; Bahaj et al., 2020; Doerr, 2020, 2021), and (ii) shocks to the constraint propagate through financial markets as investors rebalance their portfolios. The latter relates to literature considering investors' and households'

[3]In Appendix B.2.4, I use a simplified version of the model to illustrate a novel strategic interaction between the housing and capital markets, embedded in my model mechanism.

portfolio choices during housing boom-bust episodes (Pennacchi and Rastad, 2011; Martinez-Toledano, 2020; DeFusco et al., 2022; Gargano and Giacoletti, 2022).

The paper is organized as follows. Section 2 presents my modeling framework. Section 3 presents a theoretical analysis of the main features of my model. Section 4 considers model dynamics in case of an occasionally binding credit constraint, incl. an additional discussion of the empirical relevance of the model mechanisms in section 4.4. Section 5 produces the quantitative analysis with an estimate of the housing supply elasticity used in model calibration. Section 6 concludes.

2 Modeling framework

2.1 Demographics

Time is continuous and horizon is infinite. At time $t = 0$, a mass one of agents is born. The agents are indexed by $j \in [0, 1]$ and they have heterogeneous time discount rates $\rho^j > 0$. At each instant, each agent dies with Poisson probability ξ. On the other hand, whenever an agent of type j dies, another agent of type j is born, thus keeping the type distribution fixed.

Agents derive utility from a utility flow $u(c_t^j, s_t^j)$, where c_t^j and s_t^j, respectively, denote non-durable consumption and consumption of housing services of an agent j at time t. I assume the agents are perfectly altruistic towards future generations, in the sense that they maximize life-time utility ignornig the possibility of dying.[4]

2.2 Production technology

Each agent can purchase productive capital on the capital market, and has access to a linear production technology $a^j k_t$, which is used to produce non-durable consumption goods, which is the numéraire in the economy and are traded freely in the goods market. All capital is homogeneous and the total stock of productive capital is denoted by K_t. Without loss of generality, I assume the indexation of agents to satisfy $a^j \geq a^i$ for all $j \geq i$.

[4]This is a technical assumption pertinent for producing economically sensible results. I interpret the birth and death as firm entry and exit, respectively. If the agents optimized their behavior as if the firm exit corresponds to a literal death, that would generate economically insensible results with counterfactually high consumption propensities, given the relatively low life expectancy of a typical start-up. An alternative modeling approach would be to consider switching types, which would likely produce similar results.

2.3 Capital investments

Each agent can choose to invest part of their output, implying a law of motion for capital as

$$\frac{dk_t^j}{k_t^j} = \Phi_k(i_{kt}^j) - \delta_k\, dt, \quad \Phi_k(i) \equiv \frac{1}{\kappa_k}\log(1+\kappa_k i),$$

where i_{kt}^j is the investment rate, δ_k the depreciation rate of capital, and $\Phi_k(\cdot)$ is the adjustment cost function with elasticity parameter κ_k. Capital investments $i_{kt}k_t^j$ are paid for by the production of non-durable consumption goods.

With this specification the return on holding capital is given by

$$r_{kt}^j \equiv \max_{i_{kt}^j} \frac{a^j - i_{kt}^j}{p_{kt}} + \Phi_k(i_{kt}^j) - \delta_k + \mu_{kt}^p,$$

where p_{kt} is the price of productive capital and $\mu_{kt}^p\, dt \equiv \frac{dp_{kt}}{p_{kt}}$ is the rate of capital price appreciation. By taking the first order condition w.r.t. i_{kt}^j, we obtain

$$p_{kt} = 1 + \kappa_k i_{kt}. \tag{2.1}$$

2.4 Housing production

Housing services s_t are produced one-to-one using housing capital h_t and are traded freely on a rental market at a rental price r_{st}. Housing capital is denoted by H_t and it is all homogeneous.

Each house owner can choose to redevelop their property to increase the size of the house. The property is unusable during the redevelopment, yielding no rental income. In that case, the housing stock evolves as[5]

$$\frac{dh_t^j}{h_t^j} = \Phi_h(i_{ht}^j) - \delta_h\, dt, \quad \Phi_h(i) = \frac{1}{\kappa_h}\log(1+\kappa_h i),$$

where δ_h is the depreciation rate of housing, $i_{ht}^j \in [0,1]$ is the investment rate, as a percentage of the housing stock h_t, and $\Phi_h(\cdot)$ is the adjustment cost function, with housing supply elasticity parameter κ_k.

[5] An alternative approach to model housing production is to consider a construction sector, which uses land permits to expand housing supply (Kaplan et al., 2020). These two modeling alternatives correspond to two different views on, how housing supply expands. My approach corresponds to housing expansion in a city, where land is scarce, and housing supply is increased by redeveloping existing lots, thus increasing the density of housing. The alternative is to consider geographic expansion of city borders, where the supply of land permits is a key determinant of the housing supply elasticity.

Moreover, with this specification, the return on housing is given by

$$r_{ht} \equiv \max_{i^j_{ht}} \frac{[1 - i^j_{ht}]r_{st}}{p_{ht}} + \Phi_h(i^j_{ht}) - \delta_h + \mu^p_{ht},$$

where p_{ht} is the house price and $\mu^p_{ht}\, dt \equiv \frac{dp_{ht}}{p_{ht}}$ is the rate of house price appreciation.

By taking the first order condition on i^j_{ht}, it follows that the supply schedule for housing investments takes the form[6]

$$\frac{p_{ht}}{r_{st}} = 1 + \kappa_k i_{ht}. \tag{2.2}$$

Note that by equation (2.2), the response of housing supply to one percentage increase in housing price-to-rent ratio is governed by the parameter κ_h as

$$\frac{d\Phi_h(i_h)}{\frac{d(p_h/r_s)}{p_h/r_s}} = \Phi'_h(i_h)\frac{di_h}{\frac{d(p_h/r_s)}{p_h/r_s}} = \frac{1}{\kappa_h}.$$

2.5 Liquid savings

Household can invest in a one-period risk-free bond b_t, which is in zero net supply, and pays interest r_{ft}. However, they face a collateral constraint given by

$$-b^j_t \leq (1 - \phi_k)p_{kt}k^j_t + (1 - \phi_h)p_{ht}h^j_t, \quad \phi_k, \phi_h \in [0, 1]. \tag{2.3}$$

Note that all debt is short-term; each agent owns a firm with the linear productive capability. Thus, I concentrate on business loans, which typically have variable interest rates and many such loans have covenants allowing the lender to force loan pre-payment if a collateral condition is violated.[7] Since the model has perfect foresight, the distinction between short- and long-term debt is not crucial.

2.6 Portfolio choice

The agents can freely trade housing, productive capital as well as the risk-free bond b_t subject to short-selling constraints $k^j_t \geq 0$ and $h^j_t \geq 0$ as well as the collateral

[6] Note that, unlike price-to-rent ratio, house price is not stationary along the balanced growth path of the model.

[7] For instance, Glennon and Nigro (2005) document that in their sample of the Small Business Administration (SBA) 7(a) loan guarantee program 84% of loans had variable interest rate. Moreover, Chodorow-Reich and Falato (2017) report that 37% of loans in the Shared National Credit Program data were terminated or reduced before maturity during the crisis years of 2008 – 2009, approximately matching the fraction of loans with a covenant violation.

constraint (2.3). I denote an agent's net worth by $n_t^j = p_{kt}k_t^j + p_{ht}h_t^j + b_t^j$. For simplicity I will drop the superscripts j, when not necessary to emphasize the heterogeneity between agents.

Denote the portfolio shares of capital and housing, respectively, by

$$\theta_{kt} = \frac{p_{kt}k_t}{n_t} \quad \text{and} \quad \theta_{ht} = \frac{p_{ht}h_t}{n_t}.$$

With this notation I rewrite the collateral constraint (2.3) as

$$\phi_k\theta_{kt} + \phi_h\theta_{ht} \leq 1. \tag{2.4}$$

Using this notation, an agent's wealth evolves as

$$\frac{dn_t^j}{n_t^j} = -\frac{c_t^j}{n_t^j} - \frac{r_{st}s_t^j}{n_t^j} + r_{ft} + \theta_{kt}^j \underbrace{\left[r_{kt}^j - r_{ft}\right]}_{\substack{\text{Excess return} \\ \text{on capital}}} + \theta_{ht}^j \underbrace{\left[r_{ht} - r_{ft}\right]}_{\substack{\text{Excess return} \\ \text{on housing}}} dt. \tag{2.5}$$

The first two terms on the wealth evolution describe consumption of non-durable consumption goods and housing services, respectively. On the other hand, the agent's portfolio return is given by the risk-free rate and the excess returns on capital and housing investments. Note that the returns on capital are heterogeneous across agents.

Upon death, I assume an agent leaves "accidental bequest," i.e., all their wealth is equally redistributed to all newborns.

2.7 Equilibrium

I look for a recursive competitive equilibrium driven by an aggregate state vector $\mathbf{S}$. Observe that given the aggregate state, an agent's opportunity set is fully determined by her net worth n, which is the agent's state. Given capital and house prices $p_{kt} = p_k(\mathbf{S}_t), p_{ht} = p_h(\mathbf{S}_t)$, risk-free interest rate $r_{ft} = r_f(\mathbf{S}_t)$, rental price $r_{st} = r_s(\mathbf{S}_t)$ and a law of motion $d\mathbf{S}_t$ for the aggregate state $\mathbf{S}_t$, agents' policies for non-durable consumption c_t, housing consumption s_t as well as portfolio shares on capital θ_{kt} and housing θ_{ht} solve the agent's problem, maximizing their lifetime utility:

$$V(n;\mathbf{S}) = \max_{\{c_t,s_t,\theta_{kt},\theta_{ht}\}_{t\geq\tau}} \mathbb{E}_\tau\left[\int_\tau^\infty e^{-\rho t}u(c_t,s_t)\,dt\right]$$

where the maximization is subject to the wealth evolution (2.5), the LTV constraint (2.4) and short-selling constraints $\theta_{kt} \geq 0$ and $\theta_{ht} \geq 0$, given initial states $n_\tau = n$ and

$\mathbf{S}_\tau = \mathbf{S}$. Note that in this generality, the aggregate state $\mathbf{S}$ consists of the whole wealth distribution of agents.

Definition 2.1 (Recursive competitive equilibrium)**.** A recursive competitive equilibrium consists of

(i) value functions $V^j(n^j; \mathbf{S})$;

(ii) policy functions $c^j, s^j, \theta_h^j, \theta_k^j$ depending on n^j and $\mathbf{S}$;

(iii) prices $p_h(\mathbf{S}), p_k(\mathbf{S}), r_s(\mathbf{S}), r_f(\mathbf{S})$, and

(iv) aggregate state evolution $d\mathbf{S} = \boldsymbol{\mu}^S dt$

such that

1. $V^j(n^j; \mathbf{S})$ and the above associated policy functions solve the individual Hamilton-Jacobi-Bellman equations (cf. equation (A.1) in the Appendix A) given prices and state evolution.

2. Markets clear

   - Goods market: $\int_o^1 c_t^j \, dj = \int_o^1 (a^j - i_{kt}^j) k_t^j \, dj$;
   - Rental market: $\int_o^1 s_t^j \, dj = \int_o^1 (1 - i_{ht}^j) h_t^j \, dj$;
   - Housing market: $\int_o^1 h_t^j \, dj = H_t$;
   - Capital market: $\int_o^1 k_t^j \, dj = K_t$;
   - Bond market: $\int_o^1 b_t^j \, dj = 0$.

3. The aggregate state evolution $d\mathbf{S}$ is consistent with agents' equilibrium choices, given policy functions and market clearing conditions.

In what follows, I will denote the aggregate consumption and economy-wide total wealth, respectively, by

$$C_t \equiv \int_o^1 c_t^j \, dj \quad \text{and} \quad N_t \equiv \int_o^1 n_t^j \, dj.$$

3 Model: theoretical analysis

3.1 Pricing decompositions

Let us denote the relative valuation of the capital stock, the relative wealth of agent j and the capital allocation, respectively, by

$$\nu_t \equiv \frac{p_{kt}K_t}{p_{ht}H_t + p_{kt}K_t} = \frac{p_{kt}K_t}{N_t}, \quad \eta^j \equiv \frac{n_t^j}{N_t}, \quad \text{and} \quad \psi_{kt}^j \equiv \frac{k_t^j}{K_t}.$$

I begin with pricing decompositions for capital and house prices. These follow directly from goods market clearing without assuming optimizing agents.

While their proofs are trivial, the key insight is that the relative wealth and capital distributions $(\eta^j, \psi_{kt}^j)_{j\in[0,1]}$ and relative capital valuation ν_t remain stationary on the balanced growth path, despite k_t^j, p_{ht}, H_t, K_t, and N_t being non-stationary. This makes the formulas highly useful for deriving intuition in my subsequent analyses. For simplicity, I present results under inelastic capital and housing supply $\kappa_k = \kappa_h = \infty$ and refer to the Supplemental Appendix for the general case.

Proposition 3.1. *Suppose $\kappa_k = \kappa_h = \infty$. Then the prices of productive capital and housing can be expressed, respectively, as*

$$p_{kt} = \frac{\int_o^1 a^j \psi_{kt}^j \, dj}{\int_o^1 \eta^j \frac{c_t^j}{n_t^j} \, dj} \nu_t \quad \textit{and} \quad p_{ht} = \frac{\int_o^1 a^j \psi_{kt}^j \, dj}{\int_o^1 \eta^j \frac{c_t^j}{n_t^j} \, dj} \frac{K}{H}(1-\nu_t). \tag{3.1}$$

In order to highlight the generality of this result, I present the proof here instead of postponing it to the Appendix.

Proof. By the goods market clearing we have

$$\frac{C_t}{N_t} = \frac{1}{N_t}\int_o^1 a^j k_t^j \, dj = \frac{p_{kt}K}{p_{kt}N_t}\int_o^1 a^j \frac{k_t^j}{K} \, dj = \frac{\nu_t}{p_{kt}}\int_o^1 a^j \psi_{kt}^j \, dj.$$

Therefore

$$p_{kt} = \frac{\int_o^1 a^j \psi_{kt}^j \, dj}{C_t/N_t}\nu_t.$$

The results follow from observing that by the definition of ν_t

$$p_{ht} = p_{kt}\frac{K}{H}\frac{1-\nu_t}{\nu_t}, \quad \text{and} \quad \frac{C_t}{N_t} = \frac{\int_o^1 c_t^j \, dj}{N_t} = \int_o^1 \frac{n_t^j}{N_t}\frac{c_t^j}{n_t^j} \, dj = \int_o^1 \eta^j \frac{c_t^j}{n_t^j} \, dj.$$

□

In the following related proposition, I characterize the housing rental rate under an additional assumption of unit elasticity of substitution between housing and non-durable consumption.

Proposition 3.2. *Suppose* $\kappa_k = \kappa_h = \infty$[8] *and that the utility function has the form* $u(c, s) = u(c^\alpha s^{1-\alpha})$ *for a constant* $\alpha \in (0, 1)$*. Then the housing rental rate is given by*

$$r_{st} = \underbrace{\int_o^1 a^j \psi_{kt}^j \, dj}_{\substack{\text{Average productivity} \\ \text{of capital } (\equiv A_t)}} \times \underbrace{\frac{K}{H}}_{\substack{\text{Supply of capital relative to} \\ \text{supply of housing}}} \times \underbrace{\frac{1-\alpha}{\alpha}}_{\substack{\text{Preferences for housing} \\ \text{vs. non-durable consumption}}} . \tag{3.2}$$

The supply of capital K and average productivity $A_t \equiv \int_0^1 a^j \psi_{kt}^j \, dj$, determined by capital allocation ψ_{kt}^j, shape the economy's productive capacity and aggregate income. In equilibrium, this drives total consumption demand, including housing services. The term $(1-\alpha)/\alpha$ governs, how the output is divided between housing and nondurable consumption, defining aggregate housing demand. This contrasts with housing supply, H, in the denominator of the second component. Thus, the rental rate is simply the ratio of housing demand to supply, measured in the economy's numéraire. Moreover, we obtain the following immediate corollary of the above results.

Corollary 3.3. *Suppose* $\kappa_k = \kappa_h = \infty$ *and that the utility function has the form* $u(c, s) = u(c^\alpha s^{1-\alpha})$ *for a constant* $\alpha \in (0, 1)$*. Then the housing price-to-rent ratio is given by*

$$\frac{p_{ht}}{r_{st}} = \underbrace{\frac{\alpha}{1-\alpha}}_{\substack{\text{Preferences for housing} \\ \text{vs. non-durable consumption}}} \times \underbrace{\frac{1-\nu_t}{\int_o^1 \eta^j \frac{c_t^j}{n_t^j} \, dj}}_{\text{Discounted portfolio incentives}} .$$

This corollary states that when capital and housing supply are inelastic, the price-to-rent ratio depends only on the relevant discount factor—closely linked to the interest rate (as shown in Proposition 3.6)—and portfolio incentives captured in ν_t, which I will discuss in detail in Section 4.2.

Notably, mortgage or interest rates do not appear in the rental rate (3.2) or price-to-rent ratio. Here it is due to the absence of investments ($\kappa_h = \kappa_k = \infty$). However, this observation, nevertheless, highlights that interest rates affect rents only through aggregate consumption, in contrast to explanations where mortgage costs influence

[8] I relax this assumption in the Supplemental Appendix.

rents more directly through the investor problem. This raises the question of what frictions absent in the current model (e.g., market power) justify a direct impact of mortgage rates on rents beyond the general equilibrium effects.

Regarding Proposition 3.1, the house price formula differs from the rental rate r_{st} in two ways. First, the numerator in (3.2), $(1-\alpha)/\alpha$, is replaced by $1-\nu$. With unit elastic substitution in a representative agent steady-state, $\nu_t = \alpha$, showing that ν_t is largely preference-driven. However, in a heterogeneous agent setup, prices also reflect portfolio incentives. I will explore ν_t further in section 4.

Second, the denominator in (3.2) is replaced by the aggregate consumption propensity:

$$\frac{C}{N} = \int_o^1 \eta^j \frac{c_t^j}{n_t^j}\, dj. \tag{3.3}$$

This measure, one minus the savings rate, links higher savings to greater asset demand and higher prices. I will now examine this in more detail.

3.2 Consumption propensity and the interest rate

For the consumption analysis, I assume logarithmic utility similarly to Greenwald (2018).[9] Apart from analytic convenience, the logarithmic utility specification has two main reasons. First, Piazzesi, Schneider and Tuzel (2007), Davis and Ortalo-Magné (2011) and Aguiar and Hurst (2013) document that the elasticity of substitution between housing and non-durable consumption is close to 1. This argues for a utility specification of the form $u(c^\alpha s^{1-\alpha})$. Moreover, the model has no uncertainty and thus the logarithmic form corresponds to setting intertemporal elasticity of substitution to 1, which does not seem far-fetched relative to the reality. Finally, logarithmic utility is convenient also because then the agents are PIH[10] consumers. We have the following Lemma.

Lemma 3.4. *Suppose an agent j has logarithmic utility*

$$u(c,s) \equiv \alpha \log c + (1-\alpha)\log s, \quad \alpha \in (0,1). \tag{3.4}$$

Then

$$\frac{c_t^j}{n_t^j} = \alpha\rho^j \quad \textit{and} \quad \frac{r_{st}s_t^j}{n_t^j} = (1-\alpha)\rho^j \quad \textit{for all } t.$$

[9] Greenwald (2018) considers logarithmic utility over non-durable consumption and housing services, but contrary to my model, where output is produced by capital only, his utility specification includes a third term for disutility of working, as he considers a model with labor-only production.

[10] Permenant income hypothesis.

This result has several important consequences. First, similarly to Berger, Guerrieri, Lorenzoni and Vavra (2017), it follows that consumption response to a house price shock is given by

$$\frac{\partial c_t}{\partial p_{ht}/p_{ht}} = \frac{c_t}{n_t} \times p_{ht} h_t, \tag{3.5}$$

when keeping all other prices and portfolios constant, but allowing net worth to evolve with the change in house prices: the consumption response for a house price shock by PIH agents is purely due to a wealth effect. In particular, given large increases in house prices, the model produces large consumption responses as documented empirically by Mian, Rao and Sufi (2013).

Moreover, as discussed in the following Proposition, for highly levered agents, the consumption responses are relatively higher; this corresponds to empirical findings by Mian and Sufi (2014), who document strong and significant effect of leverage on marginal propensity to consume out of housing wealth.

Proposition 3.5. *Suppose an agent j has logarithmic utility of the form (3.4) and let*

$$\eta^j_{t,APC} \equiv \frac{\eta^j_t \frac{c^j_t}{n^j_t}}{\int_o^1 \eta^j \frac{c^j_t}{n^j_t}\, dj} \tag{3.6}$$

denote the average-consumption-propensity (APC) weighted wealth share. Keeping all other prices and the agent's portfolio constant, but allowing consumption to react to a change in net worth, the consumption elasticities for capital price shocks are then given by the corresponding portfolio shares as

$$\frac{\partial c^j_t/c^j_t}{\partial p_{ht}/p_{ht}} = \theta^j_{ht} \quad \text{and} \quad \frac{\partial c^j_t/c^j_t}{\partial p_{kt}/p_{kt}} = \theta^j_{kt}. \tag{3.7}$$

Moreover, the aggregate consumption response is given by

$$\frac{\partial C_t/C_t}{\partial p_t/p_t} = \int_o^1 \eta^j_{t,APC} \frac{\partial c^j_t/c^j_t}{\partial p_t/p_t}\, dj,$$

where $p_t \in \{p_{kt}, p_{ht}\}$.

Finally, in case no agent is credit-constrained, we have the following characterization of the interest rate.

Proposition 3.6. *Suppose $\kappa_k = \kappa_h = \infty$, all agents have log utility of the form (3.4)*

and that no agent is credit constrained. Then the risk-free rate is given by

$$r_{ft} = g_t + \int_o^1 \rho^j \eta^j_{t,APC}\, dj, \tag{3.8}$$

where $\eta^j_{t,APC}$ *are as in* (3.6) *and* $g_t \equiv \dot{A}_t/A_t$ *is the aggregate productivity growth in the economy.*

The risk-free interest rate is a weighted average of agents' time discount rates. Consumption propensities c^j_t/n^j_t are typically smooth due to consumption smoothing incentives and under logarithmic utility (3.4), in particular, they are constant: $\frac{c^j_t}{n^j_t} = \alpha\rho^j$.

Therefore, the interest rate is quantitatively driven by two key forces: (i) economic growth g_t and (ii) shifts in the wealth distribution η^j_t affecting weights $\eta^j_{t,APC}$ via (3.6) in (3.8). Along transition paths, agents with low consumption propensities tend to accumulate wealth, reducing the second term in (3.8). If this effect outweighs growth g_t, interest rates decline. This aligns with Mian et al. (2021), where wealth distribution shifts drive interest rates. However, I derive this result in a standard heterogeneous agent model without relying on non-homothetic preferences. I will revisit this issue in my quantitative analysis in Section 5.

4 Model dynamics with an occassionally binding credit constraint

The key economic assumption in the modeling framework of section 2 is the heterogeneity in capital returns, whereas return on housing are homogeneous.[11] A natural implication of this assumption is that agents with high capital return invest heavily on capital, eventually yielding them credit-constrained, if they are not able to fully finance their productive endeavors. However, for this to be the case, they must have fully leveraged the other potential margins of adjustment. In particular, we have the following result.

[11] This relates to empirical literature documenting substantial heterogeneity in returns to wealth across business owners (Fagereng et al., 2020). Moreover, it seems intuitive that the rental yield on any particular housing property cannot substantially vary across agents, given that the set of prospective renters for that particular property does not usually depend on the owner of the property. In contrast, business capital—such as commercial real estate or land—can be used in very different ways, depending on the comparative advantage of the owner.

Proposition 4.1 (Irrelevance of housing collateral)**.** *A financially constrained agent does not own housing, i.e., $h_t^j = 0$ necessarily. Consequently, relaxation of the housing collateral constraint ϕ_h does not affect house prices.*

This result is striking yet natural, given the absence of frictions that could overturn it. While it could be relaxed by extending the model,[12] it directly addresses the debate on credit conditions and house prices. Most studies focus on housing collateral, where relaxing constraints boosts demand. However, Kaplan et al. (2020) argue that financially constrained households are too few to drive a housing boom. Proposition 4.1 replicates this in an extreme setup where no constrained agents own housing. Here, collateral constraints in housing investments are irrelevant, as affected agents are not financially constrained. This highlights my model's focus on firm credit, differing from the literature on household credit. I explore this distinction's empirical relevance in Section 4.4.

4.1 House price amplification and a financial accelerator

So far, my analysis has used a general heterogeneous-agent framework with minimal assumptions.[13] To further describe the equilibrium and house price amplification, I now assume two agent types: productive *entrepreneurs* and less productive *savers*. While my insights extend to a broader heterogeneous-agent model, this two-agent setup enables a precise analysis of model dynamics, visualized in a two-dimensional phase diagram. I now state the assumption.

Assumption 4.2. The economy consists of entrepreneurs with mass λ, indexed by $j \in [0, \lambda]$, and savers of mass $1 - \lambda$ indexed by $j \in (\lambda, 1]$. The entrepreneurs have productivity $a^j = a$ and they discount future at rate $\rho^j = \rho$ for all $j \in [0, \lambda]$. Savers have productivity $a^j = \underline{a} < a$ and they discount future at rate $\tilde{\rho}^j = \underline{\rho} < \rho$ for all $j \in (\lambda, 1]$.

[12] For example, One could introduce a behavioral friction requiring agents to own housing for consumption. Since the model already drives a disproportionate house price increase after a credit shock, this friction would further amplify the effect, benefiting entrepreneurs through rising collateral values (Kiyotaki and Moore, 1997; Doerr, 2020). Theorem B.4 in the Supplemental Appendix B.2.4 formalizes this in a static setup, showing that when entrepreneurs hold housing in their initial portfolio, amplification strengthens, potentially leading to multiple equilibria due to new strategic interactions between capital and housing markets.

[13] The main assumption is heterogeneous capital returns and homogeneous housing returns.

I denote, respectively, by

$$\psi_{kt} \equiv \frac{\int_o^\lambda k_t^j \, dj}{K_t} \quad \text{and} \quad \eta_t \equiv \frac{\int_o^\lambda n_t^j \, dj}{N_t}$$

the entrepreneur share of total capital and wealth. It is not difficult to see that this economy with two types of agents aggregates to an economy with two representative agents, one for the entrepreneurs and another one for the savers.[14] To further simplify the notation, I will use non-underlined and underlined objects to denote the entrepreneur saver quantities, respectively, as

$$n_t \equiv \int_0^\lambda n_t^j \, dj \quad \text{and} \quad \underline{n}_t \equiv \int_\lambda^1 n_t^j \, dj,$$

and similarly for $k_t, \underline{k}_t, h_t, \underline{h}_t, b_t, \underline{b}_t, c_t, \underline{c}_t$ and $s_t, \underline{s}_t$. These denote the endogenous model objects for the two representative agents.

Before analyzing the full model dynamics, let's build intuition by examining how an unanticipated credit relaxation affects the saver's portfolio when the entrepreneur is credit-constrained. By Proposition 4.1, the saver, as the marginal agent, determines asset prices. The credit relaxation allows constrained entrepreneurs to buy capital, increasing aggregate productivity $A(\psi_k)$ and raising the rental rate (3.2). In a static model, where capital prices remain unchanged, this leads to a positive house price shock as depicted in Figure 1:

$$\frac{\underline{a}}{p_k} = \frac{r_s(\psi_k) \uparrow}{p_h \uparrow}.$$

Savers, facing a demand shock from entrepreneurs, supply capital elastically and shift portfolios toward housing, driving a disproportionate house price boom even without a financial accelerator.

In a dynamic model, the initial demand shock also affects capital prices, introducing two new forces. First, the no-arbitrage condition includes price appreciation; if house prices are expected to rise faster than capital prices, another amplification effect occurs:

$$\frac{\underline{a}}{p_{kt} \uparrow} + \frac{dp_{kt}}{p_{kt}} = \frac{r_{st}(\psi_{kt}) \uparrow}{p_{ht} \uparrow\uparrow\uparrow} + \underbrace{\frac{dp_{ht}}{p_{ht}}}_{\text{Assume } \frac{dpht}{p_{ht}} > \frac{dp_{kt}}{p_{kt}}}. \tag{4.1}$$

Second, as savers now account for price appreciation, their capital supply may no

[14] Note that with linear ak production technology an entrepreneur may be credit-constrained if and only if all entrepreneurs are credit-constrained, consistent with the entrepreneurs aggregating to a representative agent.

longer be fully elastic, causing capital prices to respond to the credit shock. This could dampen the effect relative to the static model, but also introduce a financial accelerator, amplifying house price effects through the rental rate $r_{st}(\psi_{kt})$. The following result formalizes this.

Theorem 4.3 (Sufficient statistic)**.** *Let $\psi_{ko}, \phi_{ko}, p_{ko}$ and $\psi_{k1}, \phi_{k1}, p_{k1}$ denote the capital allocation, credit constraint parameter, and price of capital before and (immediately) after an unanticipated credit shock. If the entrepreneur is credit-constrained before and after the shock, then the new endogenous capital allocation ψ_{k1} after the impact satisfies*

$$\frac{\psi_{k1}}{\psi_{ko}} = \frac{\phi_{ko}}{\phi_{k1}} + \underbrace{\frac{1-\phi_{ko}}{\phi_{k1}}\left(1 - \frac{p_{ko}}{p_{k1}}\right)}_{\textit{Financial accelerator}} \tag{4.2}$$

In particular, denoting the elasticities of ψ_k and p_k with respect to ϕ_k by $\varepsilon^{\phi}_{\psi}$ and $\varepsilon^{\phi}_{p_k}$, taking the limit $\phi_{k1} \to \phi_{ko}$ yields

$$\varepsilon^{\phi}_{\psi} = -1 + \frac{1-\phi_k}{\phi_k}\varepsilon^{\phi}_{p_k}. \tag{4.3}$$

By Proposition 4.1, credit-constrained entrepreneurs do not own housing, so their collateral constraint is:

$$-b \leq (1-\phi_k)p_{kt}k_t = (1-\phi_k)p_{kt}\psi_{kt}K_t. \tag{4.4}$$

If the credit relaxation does not affect prices ($p_{ko} = p_{k1}$), the change in ϕ_k fully determines capital reallocation (4.2). However, if prices rise, the collateral constraint (4.4) further loosens, enabling more capital purchases, following the Kiyotaki and Moore (1997) financial accelerator mechanism. The key question is how much additional capital the entrepreneur can acquire due to the accelerator effect, driven by the strategic interaction of p_k and ψ_k. Proposition 4.3—specifically equation (4.3)—shows that the impact of this mechanism depends entirely on the elasticity of capital price to the credit shock.

4.2 Model dynamics

I now proceed to consider model dynamics in a closed-form (η_t, ν_t) -phase diagram, relying on the following Theorem.

Theorem 4.4. *Suppose Assumption 4.2 holds. The evolution of the wealth distribution, characterized by η_t, and relative prices, characterized by ν_t, are given by*

$$\frac{\dot{\eta}_t}{\eta_t(1-\eta_t)} = \underbrace{\frac{\underline{c}_t}{\underline{n}_t} - \frac{c_t}{n_t} + r_{st}\left(\frac{\underline{s}_t}{\underline{n}_t} - \frac{s_t}{n_t}\right)}_{\textit{Consumption difference}} + \theta_{kt} \underbrace{[r_{kt} - \underline{r}_{kt}]}_{\textit{Return difference}} + \underbrace{\frac{\xi}{1-\eta_t}\left(\frac{\lambda}{\eta_t} - 1\right)}_{\textit{Demographic evolution}} \tag{4.5}$$

$$\frac{\dot{\nu}_t}{\nu_t(1-\nu_t)} = \underbrace{\frac{(1-i_{ht})r_{st}}{p_{ht}} - \frac{a^j - i_{kt}}{p_{kt}}}_{\substack{\textit{Yield difference} \\ \textit{(net of investments)}}}, \tag{4.6}$$

where j is the identity of the marginal (representative) investor: $a^j = a$ for $\eta_t > \phi_k \nu_t$ and $a^j = \underline{a}$ for $\eta_t \leq \phi_k \nu_t$.

Wealth accumulation between the saver and entrepreneur depends on their consumption rates, return differences, and demographics. The accidental bequest setup pushes wealth distribution toward equal per capita levels; this overlapping generations framework ensures an interior steady-state.

The relative prices of capital and housing, represented by ν_t, evolve based on yield differences for the marginal agent investing in both assets. By the no-arbitrage condition, total returns must be equal, so capital gains differences are determined by yield differences. The identity of the marginal agent depends on whether the collateral constraint binds. In (η_t, ν_t) space, the line $\eta_t = \phi_k \nu_t$ separates regions where the constraint is binding or not.

Finally, note that it is not difficult to show that in equilibrium[15]

$$\psi_{kt} = \min\left\{\frac{\eta_t}{\phi_k \nu_t}, 1\right\}, \quad \theta_{kt} = \frac{\psi_{kt}}{\eta_t}\nu_t, \quad \text{and} \quad r_{kt} - \underline{r}_{kt} = \frac{a - \underline{a}}{p_{kt}}.$$

Combining these observations with Propositions 3.1 and 3.2, the investment FOCs (2.1) and (2.2), and Lemma 3.4, Theorem 4.4 provides a fully specified closed-form characterization of the model dynamics via a two-dimensional (η_t, ν_t)-phase diagram associated with the non-linear dynamic system described by equations (4.5) and (4.6).

[15]The first and second equations are direct consequences of the asset market clearing condition and the collateral constraint. The third one follows easily from agents' capital market optimization. For additional details, see Appendix A and the Supplemental Appendix.

4.3 Phase diagram and a collateral credit relaxation

Now, consider these dynamics in a phase diagram illustrating equilibrium in steady-state and along the transition. Figure 2 shows the (η, ν) phase diagram: the orange line in panel (a) represents the model's saddle path. The solid blue line marks the credit constraint, dividing the plane into constrained (left) and unconstrained (right) regions. Crossing this line shifts the marginal agent from saver to entrepreneur, causing a non-differentiability due to the drop in productivity from a (entrepreneur) to $\underline{a}$ (saver).[16]

Panel (b) examines an unanticipated credit shock rotating the blue line. Similar to Kiyotaki and Moore (1997), the steady-state lies in the constrained region. After the shock, if large enough, entrepreneurs become unconstrained, and the economy shifts to the right region. The house price relative to capital (ν_t) jumps on impact and continues rising, fueling a house price boom until the economy returns to the constrained region. This reversal is due the entrepreneurs' higher consumption propensities, which eventually trigger capital liquidation, deteriorating capital allocation, and partial price reversal.

Figure 3 translates the phase diagram dynamics from Figure 2 into house price effects along the saddle path. The binding credit constraint creates a non-differentiability point, mirroring Figure 2. At the impact of an unanticipated credit shock, the economy adjusts to a new saddle path with an initial price jump, followed by a sustained house price boom. This aligns with empirical literature on house price momentum, previously explored by Guren (2018). My model offers a frictionless explanation for this "house price momentum puzzle" (Piazzesi and Schneider, 2016) without relying on nominal rigidities.

Meanwhile, entrepreneurs' high consumption propensities reduce their wealth. In this two-agent setup, Theorem 3.6 implies:

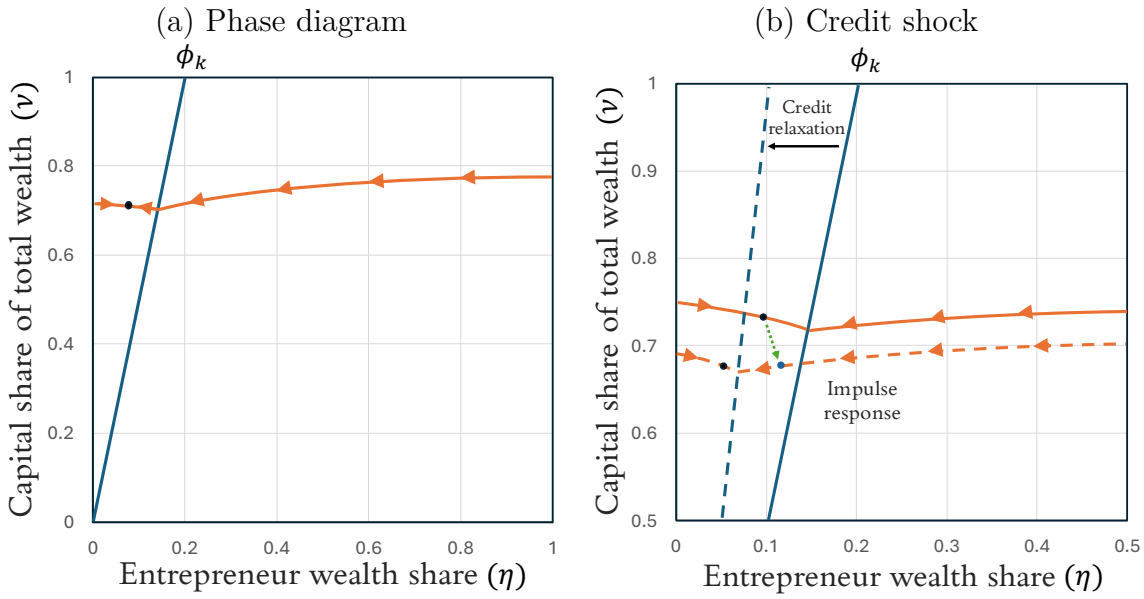
$$r_{ft} = \rho\eta_{t,APC} + \underline{\rho}(1 - \eta_{t,APC}),$$

where $g_t = 0$ when agents are unconstrained. Since $\rho > \underline{\rho}$ and η_t declines, interest rates decreasing during the boom, replicating pre-2008 dynamics and offering a new explanation for this phenomenon previously studied by Mian et al. (2021).

[16] Recall that with linear ak production technology an entrepreneur may be credit-constrained if and only if all entrepreneurs are credit-constrained, consistent with the entrepreneurs aggregating to a representative agent.

Figure 2: Phase diagram

The figure demonstrates the model dynamics and the impulse response to an unanticipated, exogenous credit shock. Panel (a) shows the saddle path in a (η, ν) phase diagram for $\phi_k = 0.20$. The line $\eta = \phi_k \nu$ separates the region, where the credit constraint is binding (left) from the non-binding region (right). Panel (b) considers an unanticipated, exogenous credit relaxation $\phi_k = 0.20 \to 0.10$ and depicts the impulse response, as the economy readjusts onto the new saddle path. The credit shock is represented by a rotation of the $\eta = \phi_k \nu$ line.



Moreover, by the no-arbitrage condition:

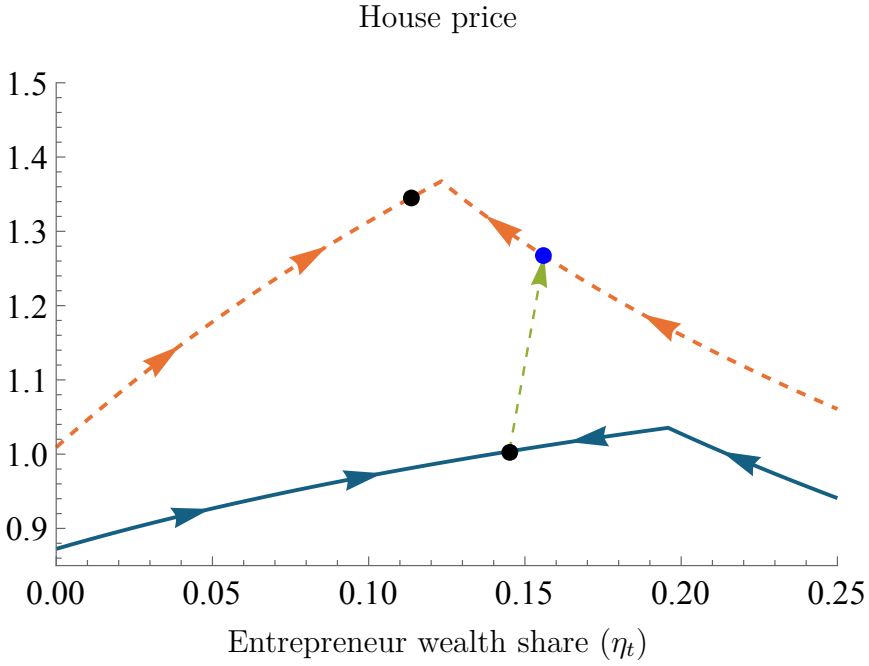
$$r_{ft} = \frac{r_{st}}{p_{ht}} + g_{ht} + \mu^p_{ht}.$$

Since the house price appreciation rate μ^p_{ht} drops when the credit constraint begins to bind, interest rates drop discontinuously. Simultaneously, entrepreneurs begin to liquidate their capital, reducing productive capacity. The resulting drop in interest rates and output resembles "credit crunch" and "flight-to-safety" dynamics considered previously by Guerrieri and Lorenzoni (2017).

Standard asset pricing models struggle with simultaneous output and interest rate declines, since such dynamics would typically violate the Euler equation. Guerrieri and Lorenzoni (2017) address this via exogenous credit tightening, introducing a pricing multiplier into the Euler equation. My model generates similar dynamics endogenously, without an explicit credit tightening, consistent with empirical evidence by Mian and Sufi (2014), who argue that no such credit shock occurred at the time of the bust.

Figure 3: House price dynamics with an impulse response

The picture depicts equilibrium price schedules as a function of the entrepreneur wealth share across the state space for $\eta \in [0, 0.25]$. For $\phi_k = 0.27$ and $\phi_k = 0.18$ the transitions are depicted in solid blue and dashed orange lines, respectively, arrows describing the direction of the dynamics. The economy is initialized at $\eta = 0.145$ and the impulse response at the impact of the shock is shown (dashed green), given an unanticipated exogenous shift in the value of ϕ_k from $\phi_k = 0.27$ to $\phi_k = 0.18$. The new steady state on the orange dashed line is depicted as a black point at $\eta_{ss} = 0.11$.



4.4 Model features and empirical relevance

Several of the model features can be mapped to empirical evidence in the existing applied micro literature.

4.4.1 Financial constraints and aggregate productivity

A large body of empirical research has shown that collateral constraints affect investments and business activity at both micro and macro levels (Chaney et al., 2012; Liu et al., 2013; Adelino et al., 2015, 2016; Schmalz et al., 2017; Bahaj et al., 2020; Doerr, 2021). Building on this empirical motivation, I embed my model with a mechanism that guarantees that credit conditions affect aggregate output, aligning with prior theory literature on the topic (Buera and Moll, 2015; Buera and Nicolini, 2020; Moll, 2014). While I motivate this via the entrepreneurial finance setting, the model only requires that financial frictions affect aggregate output, an assumption deeply rooted in the existing literature (Buera and Shin, 2013; Midrigan and Xu, 2014; Whited

and Zhao, 2021). In particular, my model makes no predictions on entry or exit of entrepreneurs.[17] For a more thorough survey of the role of borrowing constraints for aggregate output, I refer to Hopenhayn (2014).

Beyond this general motivation, an immediate corollary of Proposition 4.1 predicts that firms holding real estate (savers) are less productive than those without (entrepreneurs), consistent with the empirical findings by Doerr (2020).[18]

Regarding aggregate productivity, Gomme, Ravikumar and Rupert (2011) document that U.S. business capital returns rose from 3.8% to 11.8% during the 2000–2006 housing boom, reaching their highest level since 1975. Even excluding capital gains, returns increased throughout the early 2000s. Meanwhile, U.S. GDP grew at an average of 3.9% annually from 2003 to 2006, peaking at 7.0% in Q3/2003. These statistics align with my model prediction that a credit supply shock, driven by private-label mortgage-backed securities, led to a productivity-enhancing capital reallocation (Levitin and Wachter, 2011; Buera et al., 2011; Buera and Shin, 2013).

Finally, I note that the model takes no stance on the exact source of the increase in capital productivity. For instance, if a low-productivity entrepreneur is willing to increase their working hours to expand their business, this could increase the aggregate productivity of capital while decreasing labor productivity. Similarly, the model is agnostic about the beneficiaries of the boom—whether entrepreneurs (higher profits), employees (higher wages), or consumers (lower prices). Consequently, it makes no predictions on labor productivity,[19] income distribution,[20] or sectoral effects (Mian,

[17] Decker et al. (2014) show that entrepreneurship and business dynamism have declined in the U.S. over recent decades. This suggests capital reallocation occurs through existing firms rather than new firm entry, aligning with my model in section 2. However, the model could be adapted to incorporate business creation.

[18] Doerr (2020) argues that rising house prices lower aggregate productivity by benefiting less productive firms, which typically have high real estate holdings. My mechanism agrees that these firms are less productive but adds that higher collateral values help smaller entrepreneurs grow, potentially boosting productivity and employment (Adelino et al., 2015). While Doerr (2020) focuses on publicly listed firms with significant real estate assets, my model is more relevant in the context of private start-ups with minimal real estate holdings (Haltiwanger, 2015; Doerr, 2021).

[19] The cyclicality of labor productivity is debated and varies by period (Fernald and Wang, 2016). A common view is that—due to decreasing returns—it should be countercyclical since adding workers during a boom lowers average productivity (Biddle, 2014). Firms also retain only their most productive employees in recessions, while hiring less productive workers in booms. Given these complexities, changes in average labor productivity during a boom are not a reliable test of my mechanism.

[20] Mian and Sufi (2009) find a negative correlation between income and credit growth across U.S. zip codes, suggesting credit expansion in the early 2000s was driven by a credit supply shock rather than income growth (Levitin and Wachter, 2011). My mechanism agrees on the importance of credit supply, but also allows for rising aggregate productivity, even if income declines locally. If productivity gains benefit firms' customers or employees outside the zip code, income growth may not appear where credit expands. I investigates this question in more detail in the Supplemental

Sufi and Verner, 2020, cf. section 4.4.3 below).

4.4.2 Portfolio rebalancing

A key mechanism in my model is that credit relaxation leads marginal asset price setters (savers) to shift their portfolios toward housing. Macro housing literature emphasizes that investor responses to demand shocks—the "tenure supply elasticity"—determine house price effects (Kaplan et al., 2020; Greenwald and Guren, 2021). In Kaplan et al. (2020), investors sell homes to new buyers, increasing homeownership with minimal price effects. In contrast, my model focuses on capital supply: savers elastically provide capital to entrepreneurs and shift their portfolios toward housing.

While the empirical evidence on the topic is still limited, if anything the evidence seems to point to the housing market attracting more investor activity during the boom, consistent with my theory of portfolio rebalancing: (i) "speculation" by individual investors increased (DeFusco, Nathanson and Zwick, 2022), (ii) institutional investors, such as pension funds, doubled real estate holdings from 3.1% in 2000 to 6.5% in 2008 (Pennacchi and Rastad, 2011), and (iii) individual households increased housing portfolio shares (Martinez-Toledano, 2020; Gargano and Giacoletti, 2022).

4.4.3 Productivity or demand?

The model presents an interesting identification challenge: housing rents and prices rise due to demand, driven by higher productivity. This suggests a strong demand-driven effect on house prices, aligning with empirical research on housing booms. However, the ultimate cause of this demand is increased aggregate productivity, which also has strong empirical backing, as documented above. I discuss this empirically in more detail in the Supplemental Appendix, but note here that this highlights a key debate.

Mian, Sufi and Verner (2020) study whether credit supply shocks operate by increasing the economy's "productive capacity" or via "household demand". They leverage Bahadir and Gumus (2016), who show demand shocks are inflationary while productivity shocks are not. Therefore, demand shocks should differentially affect the non-tradeable sector, which Mian et al. (2020) confirm using natural experiments in the U.S.A. in the 1980s.

Appendix

However, ruling out productivity effects by merely considering differences in tradable vs. non-tradeable economic activity seems difficult, since collaterialized lending can very plausibly be concentrated on the non-tradeable sector. Another strand of the literature has documented that financial frictions may significantly impact productivity (Buera, Kaboski and Shin, 2011; Whited and Zhao, 2021). If credit expansions only increase demand, why does macroeconomic research attribute substantial productivity effects to financial misallocation?

5 Quantitative analysis

5.1 Model calibration

There are four crucial sets of parameters affecting my mechanism: (i) κ_h determines the housing supply elasticity and directly affects house price responses for economic shocks; (ii) the parameters $a, \underline{a}$ and $\rho, \underline{\rho}$ determine the extent of price amplification within the constrained region; (iii) the evolution of K_t relative to H_t affects house prices and is governed by the investment technology parameters $\kappa_k, \delta_k, \delta_h$, and κ_h; and (iv) the extent of the credit liberalization, which is the main experiment I consider, and is governed by the pre- and post-shock levels of ϕ_k. The parameter values I consider are shown in Table 1.

I use the linear relation in (2.2) to estimate κ_h. I construct annual price-to-rent ratios for 2015 – 2024 using county-level rental and house price data from Zillow. Housing investment rate is measured at county-level as the change in the number of housing units, obtained from the U.S. census. I do not have data on repairs and maintenance, and I will exclude this component of housing investments and correspondingly set $\delta_h = 0$. I report the regression results in Table 2. For the baseline calibration, I use the OLS estimate reported in column (3) of the table. The reciprocal of the estimate is used to calibrate $\kappa_h = 1/0.000669 = 1495$. In the Supplemental Appendix, I show that the main quantitative results are robust for an alternative calibration, relying on an IV strategy following Chaney, Sraer and Thesmar (2012), reported in column (4) of table 2.

Finally, to maintain symmetry between the housing and capital supply, I calibrate $\kappa_k = \kappa_h = 1495$ and $\delta_k = 0$. This guarantees that some artificial difference in supply elasticities is not driving my results. Moreover, commercial real estate is a plausible source of business capital for many businesses and it seems reasonable to assume its supply elasticity follows closely that of residential construction.

Table 1: Model calibration

Parameter	Name	Value	Source/Target
		Experiment	
Capital LTV constraint	ϕ_k	$0.27 \rightarrow 0.18$	Shift in LTV ratios
		Parameter values	
Housing preferences	$1-\alpha$	0.20	Housing exp. share
saver time discount	$\underline{\rho}$	0.005	High saving propensity
Entrepreneur time discount	ρ	0.02	Low saving propensity
Entrepreneur productivity	a	1.84	Fagereng et al. (2020)
saver productivity	$\underline{a}$	1.00	Normalization
Housing adjustment cost	κ_h	1495	See Table 2
Capital adjustment cost	κ_k	1495	See text
Housing depreciation rate	δ_h	0.00	See text
Capital depreciation rate	δ_k	0.00	See text
Death rate	ξ	0.20	5-year average start-up survival
Entrepreneur share	λ	0.11	See text
Initial capital stock	K_o	1.35	Price normalization
Initial housing stock	H_o	1.00	Normalization

Note that the housing supply here is less elastic than in Kaplan et al. (2020), who use the 30-year elasticity reported by Saiz (2010). Saiz (2010) studies, how geography affects the long-run housing supply. My calibration is better interpreted in the context of short-run investment responses, abstracting away from potential long-run geographical impediments to housing construction. In particular, Cunningham and Orlando (2024, Table 1) report that the average time-to-completion for a U.S. multi-family home was 27 months during 2008 – 2022. The short-run housing supply is, therefore, plausibly quite inelastic.

The savers' productivity parameter is normalized to $\underline{a} = 1.0$. This is without loss of generality and corresponds to fixing the units of measurement for productive capital. I interpret the savers as bondholders, corresponding to pension and insurance funds as well as high net-worth individuals in the data.

I use business owners' average (3.41%) and top quartile (6.28%) returns on wealth from Fagereng et al. (2020, Table 7) to calibrate the production heterogeneity between savers and entrepreneurs, respectively. This yields $a = 6.28/3.41 = 1.84$. This represents a modest degree of heterogeneity in the agents' productivity, reflecting

Table 2: Calibration of housing supply elasticity

The table reports the regression of Investment rate on Price-to-rent ratio, used in the calibration of housing supply elasticity. The baseline calibration is based on the OLS estimate in column (3). Additionally, results are shown for an IV strategy, instrumenting price-to-rent ratio with the interaction of the Saiz (2010) housing supply elasticity with the 30-year mortgage rate. The robustness for this calibration and details on the exclusion restriction are discussed in detail in the Supplemental Appendix.

	Dependent variable:			
	Price-to-rent ratio	Investment rate		
	First stage	*Reduced form*	*OLS*	*IV*
	(1)	(2)	(3)	(4)
Housing supply elasticity	0.1329***	0.0003***		
× 30-year mortgage	(0.0114)	(2.96×10^{-5})		
Price-to-rent ratio			0.0007***	0.0018***
			(0.0001)	(0.0003)
MSA Fixed-effects	Yes	Yes	Yes	Yes
Observations	4,066	9,919	4,771	3,949
R^2	0.73	0.48	0.60	0.55
Within R^2	0.026	0.013	0.060	

Clustered (MSA) standard-errors in parentheses
*Signif. Codes: ***: 0.01, **: 0.05, *: 0.1*

differences in the agents' ability for value-adding production with the assets (e.g. land or commercial real estate) they own. The heterogeneity in productivity of capital and homogeneity in rental yields are key assumptions of my model. This relates to the intuition that while land or commercial real estate can be used for several different purposes yielding very different returns depending on the abilities of the business owner, the rental yield on an investment property depends on the market demand for that particular property and is less affected by who happens to own it.

The time discount rates $\rho = 0.02$ and $\underline{\rho} = 0.005$ are chosen to produce modest differences in marginal consumption propensities. Misra and Surico (2014) document that approximately half of the population have marginal consumption propensities

close to zero, whereas another 20% consume over 50% out of unanticipated transitory income. My calibration produces conservative heterogeneity in consumption propensities relative to observed extremes in the data, and yields pre-shock risk-free interest rate of 0.5%.

Before the crisis most business loans were originated at LTVs close to 0.70 (Titman, Tompaidis and Tsyplakov, 2005; Benmelech, Garmaise and Moskowitz, 2005). I calibrate the LTV shock using the maximal commercial mortgage LTVs of approximately 0.73 in 2000 and 0.82 in 2004, reported by An and Sanders (2010). These correspond to $\phi_k = 0.27$ and $\phi_k = 0.18$ before and after the credit shock, respectively. Note that mortgages constitute to a substantial part of small business loans, and calibrating the LTV shock according to commercial mortgages, therefore, seems appropriate.[21]

Finally, the preference parameter α is matched to the expenditure share of housing services: housing and utilities expenditure has been relatively stable at approximately 20% of total GDP at least for the past 40 years.[22] The initial housing stock is normalized to $H_o = 1$, and I set $K_o = 1.35$ to target the initial house price of $p_{ho} = 1.0$. Finally, I initialize the economy at $\eta_o = 0.145$ to match a 5-year boom duration.[23]

Finally, I set the death rate at $\xi = 20\%$ to match the average 33% 5-year survival rate of new businesses (Fairlie et al., 2023). The entrepreneur share $\lambda = 11\%$ mostly affects the location of the steady state in the η_t axis in Figure 3; I set it at 11% approximately matching the share of entrepreneurs in the U.S. economy and to help matching the proper duration of the boom.

5.2 Results: impulse responses for an unanticipated LTV relaxation

As shown in Figures 4 and 5, the credit relaxation leads to 34.4% increase in house prices, while output increases by 13.8% during the same period. This explains the

[21]For instance, in Q1/2002, the total value of U.S. proprietorship and partnership mortgages was $1.46 trillions, constituting 76% of their total debt holdings, whereas mortgages added up to 35 % of all debt for U.S. small businesses of less than 500 employees, which constitute to 99% of all U.S. firms and majority of total U.S. employment.

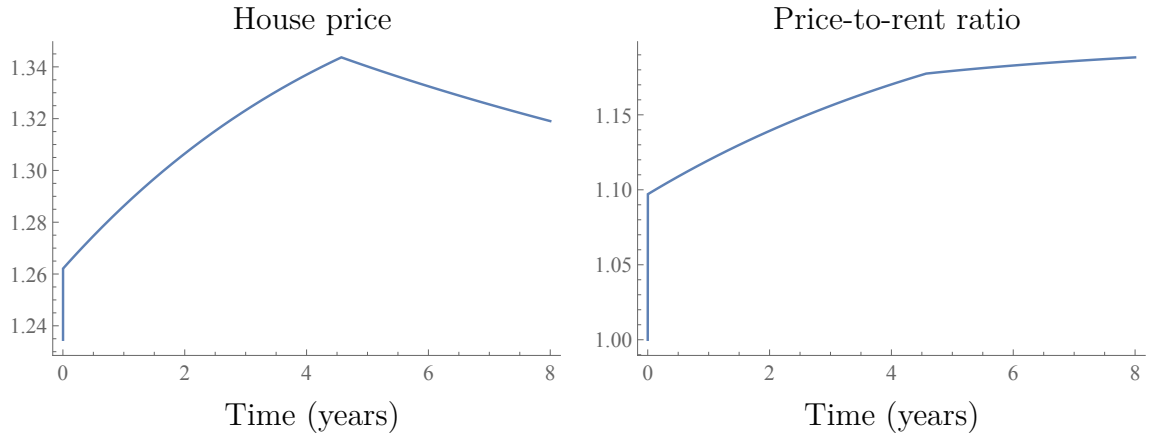
[22]Source: Personal Consumption Expenditures on Housing and utilities [DHUTRC1Q027SBEA] divided by total Personal Consumption Expenditures [PCEC], Federal Reserve Back of St. Louis.

[23]The quantitative size of the house price boom is robust for initializing the economy at the pre-shock steady state, but endowing the entrepreneurs with slightly higher wealth share helps in producing slightly longer boom period.

40.0% house price increase between Q1/2003 and Q1/2007, during the "house price bubble"; Levitin and Wachter (2011) document that the "bubble" really formed only after 2003, as the preceding house price growth in the early 2000s was supported by increasing rents, and the price-to-rent ratios began to increase only later in 2003 – 2004.

Figure 4: Impulse responses for house price and price-to-rent ratio.

The shock lifts the entrepreneurs off their credit constraint; a non-differentiable change in the impulse responses occurs later, when the entrepreneurs hit the constraint again.



At impact of the credit shock house prices are increased by 26.2% together with a 9.7% change in the price-to-rent ratio. The price-to-rent ratio keeps increasing, having increased 17.8% by the time of the bust.

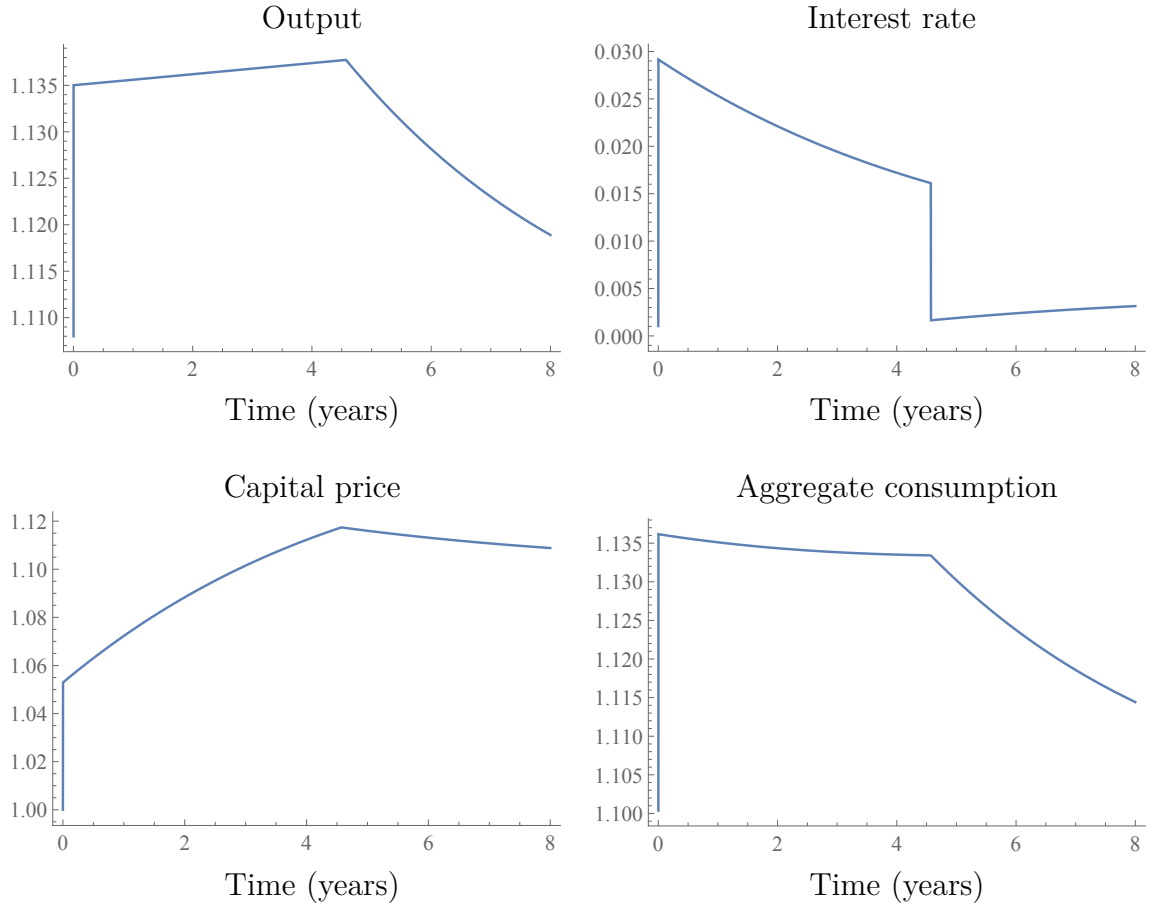
Entrepreneurs are lift off their credit constraints and they use the additional collateral capacity to purchase more productive capital and to finance higher consumption during a sustained boom lasting for 4.6 years, after which they again hit the constraint and prices begin to decrease.

Figure 5 shows that the price increase in productive capital is substantially smaller compared to house prices. On the one hand, the savers are willing to sell their capital to the entrepreneurs due to their differences in productivity; for housing investments savers remain marginal, and the demand for housing versus capital relative to the supply of each, therefore remains high: savers rebalance their portfolio towards housing, creating a housing boom. On the other hand, the entrepreneurs know that they will not hold the capital forever, but only for a few years during the boom, after which they prefer to liquidate it for financing higher future consumption. After the boom, capital is bought back by the savers, who are not willing to pay a high price for it due to their lower productivity. This affects the entrepreneurs' valuation for capital

them being forward-looking.

Figure 5: Output, interest rate, capital price and aggregate consumption.

The credit relaxation creates a strong boom in the affected sectors. Once the entrepreneurs hit the credit constraint, after being lift off the constraint at the impact of the shock, a decrease in the growth rate of real output is accompanied with a discontinuous downward jump in the real estate rate, resembling a credit crunch. Aggregate consumption increases together with the increased output at the impact of the shock, but the effect is dampened in the long run.



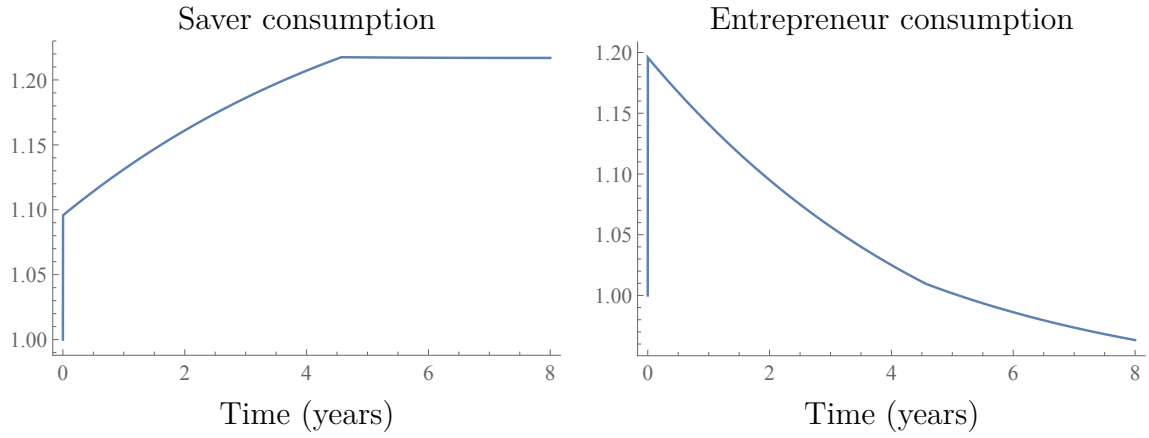
Once this liquidation begins, output decreases. House price growth flattens as savers' demand for housing is lowered, since now they are also marginal for capital investments. As the growth rates for capital and house prices decrease, the returns on these assets decrease, as well. Risk-free interest rate drops in order to balance the returns across different assets.

Note that the interest rate evolution follows the observed dynamics: during the PLS market expansion in 2003 – 2004, the long-running decrease in interest rates stopped, as the 30-year mortgage rate increased from 5.2% in June 2003 to 6.4% by August of the same year. Afterwards, the rates remained mostly flat until the crisis, when they dropped again from 6.4% in October 2008 to 4.8% in April 2009.

In conclusion, the bust in the model is created endogenously four years after the original credit supply shock.

Figure 6: Saver and entrepreneur consumption

Impulse responses for aggregate, saver and entrepreneur consumption as well as for entrepreneur net worth. The credit relaxation creates a consumption boom, especially for the home owners (savers), who enjoy from a large house price appreciation.



The credit shock also produces a consumption boom, most notably for the home-owner savers, who enjoy from their increased housing wealth; see Figures 5 and 6. This corresponds to the evidence of Mian, Rao and Sufi (2013), who document large consumption responses out of housing wealth, and the theoretical analysis of Berger et al. (2017), which I augment in Proposition 3.5. Savers' consumption increases 21.8% during the boom, constituting to a large consumption elasticity close to 1 out of increasing housing wealth. This additional consumption is partially produced by the additional production due to improved capital allocation, as entrepreneurs hold more of the productive capital stock after the shock. The entrepreneurs experience similar 20% consumption boom at the impact of shock, driven by the increase of the price of capital, which increases their net worth. However, this is reversed in the long-run, since due to their high consumption propensity, they consume their newly acquired wealth, eventually reversing any gains they initially experienced.

5.3 Sharpe ratios

An important aspect of the house price volatility puzzle is the low Sharpe ratios observed for housing investments in the aggregate data. As is typical for asset pricing models, also in my model, the optimal portfolio allocation equalizes Sharpe ratios for different assets for the marginal investor. However, not all investors are marginal in

all assets, which implies that Sharpe ratios do not need to coincide, when measured from aggregate price indices.

To make this argument rigorous, I introduce AR(1) aggregate shocks into the LTV credit constraint:

$$\phi_{k,t+1} = \bar{\phi}_k + \rho_k(\phi_{kt} - \bar{\phi}_k) + \epsilon^{\phi}_{k,t+1}.$$

I calibrate $\bar{\phi}_k = 0.27$, assume no investments ($\kappa_h = \kappa_k = \infty$), linearize the model around its steady state as implemented by Dynare (Adjemian et al., 2024), and use monthly data on price-to-rent ratios from Zillow during 2015 – 2024 to estimate a sequence of shocks using the Kalman smoother. I simulate the linearized model with the estimated shocks and consider the excess returns and standard deviations of returns for the business capital and housing, respectively.

I note that the dynamic house price amplification mechanism in equation (4.1) now actually dampens the relative house price effects since $\frac{\partial p_{ht}}{p_{ht}} < \frac{\partial p_{kt}}{p_{kt}}$ around the steady-state. This highlights the importance of considering the global non-linear dynamics discussed in section 5.2 to properly evaluate the effects of large credit shocks. This notwithstanding, the entrepreneurs' excess returns on capital still generates larger Sharpe ratios for capital investments than for housing in the aggregate data.

We observe that the monthly return volatility for capital and housing investments co-incide up to the fifth decimal at 0.00563. The excess returns, however, differ. As expected, the excess return on housing is an exact zero: $r_h - r_f = 0$, but for capital this is not true in aggregate due to the excess return earned by the entrepreneurs: $\psi_k r_k + (1-\psi_k)\underline{r}_k - r_f = 0.00019$ (monthly). Since the model does not have aggregate risk, we should not expect large excess returns, but this exercise documents that qualitatively the model can produce low observed Sharpe ratios for housing (technically 0) and larger aggregate Sharpe ratios for other investments.

6 Conclusion

The Great Financial Crisis was the deepest U.S. recession since the Great Depression in the 1930s. Nevertheless, the economics community lacks consensus on the drivers of the housing "bubble" preceding and contributing to the crisis. Understanding these drivers is crucial for proper design of economic polic. Mian and Sufi (2010) write:

> "*From a policy perspective, it is important to understand whether a rise in leverage is driven by demand-side productivity shocks or supply-side finan-*

cial factors. A supply-driven rise in leverage may not be innocuous. For example, if leverage growth is driven by risk-shifting supply-side incentives such as regulatory arbitrage or expectations of government bailouts, then there may be a role for intervention to realign incentives."

This paper proposes a third explanation for the crisis, with substantial policy implications. I build a model, where an increase in credit access affects capital allocation, leading to increased aggregate productivity, which drives up house prices. While these dynamics are initially caused by a credit supply shock, similarly to the Mian-Sufi narrative, the house price effects are greatly influenced by the increase in aggregate productivity. The mechanism relies on a new amplification channel emphasizing the role of the portfolio incentives of the marginal agents in a heterogeneous-agent set-up.

Much of the macroprudential policy following the crisis, such as the Dodd-Frank Act (2010) in the U.S.A., aimed to limit household leverage that shoot off due to the increase in credit. In my model calibration the credit relaxation preceding the crisis increases long-run aggregate productivity by 13% in the height of the boom, and 9% in the long-run (steady state comparative statics). While higher leverage ratios may make the economy more vulnerable to economic shocks, and also in my model, the dynamics lead to an eventual bust endogenously, these high productivity effects suggest that the economic costs of macroprudential policy limiting leverage ratios may be substantially higher than previously understood (Richter et al., 2019).

A Proofs of the theoretical results

In this section, I present the proofs of the main results in the paper. Additional details and generalizations (e.g. $\kappa_h, \kappa_k < \infty$) may be found in the Supplemental Appendix.

Proof of Proposition 3.2. The Hamilton-Jacobi-Bellman equation associated with the problem of an agent j is given by

$$\begin{aligned}
&\rho^j V(n;\mathbf{S}) \\
&= \max_{c,s,\theta_k,\theta_h} u(c,s) + \nabla_{\mathbf{S}} V \cdot d\mathbf{S} \\
&\quad + \frac{\partial V}{\partial n} n \left[-\frac{c}{n} - \frac{r_s s}{n} + r_f + \theta_k \left(r_k^j - r_f \right) + \theta_h \left(r_h - r_f \right) \right] \\
&\text{s.t.} \quad \phi_k \theta_k + \phi_h \theta_h \leq 1, \quad \theta_k \geq 0, \quad \theta_h \geq 0,
\end{aligned} \tag{A.1}$$

where $\nabla_{\mathbf{S}} V$ denotes the gradient of V with respect to the aggregate state variable $\mathbf{S}$.

Denote the multiplier on the collateral constraint by λ^j_{LTV} and the multipliers on the nonnegativity constraints by λ^j_k and λ^j_h, respectively. I suppress the aggregate state variable from the value function and denote $V'(n) := \frac{\partial V}{\partial n}$, The FOCs are

$$r^j_k = r_f + \frac{\lambda^j_{LTV}\phi_k - \lambda^j_k}{V'(n^j)n^j}, \tag{A.2}$$

$$r_h = r_f + \frac{\lambda^j_{LTV}\phi_h - \lambda^j_h}{V'(n^j)n^j}, \tag{A.3}$$

$$u_c = V'(n^j), \tag{A.4}$$

$$u_s = V'(n^j)r_s. \tag{A.5}$$

It immediately follows that $u_s/u_c = r_s$. For the utility specification $u(c^\alpha s^{1-\alpha})$ this gives

$$\frac{1-\alpha}{\alpha}c = r_s s \implies r_s = \int_o^1 a^j k^j \, dj \frac{K}{H}\frac{1-\alpha}{\alpha}, \tag{A.6}$$

where I used the goods market clearing $C = \int_o^1 a^j k^j \, dj$ for $\kappa_h = \kappa_k = \infty$. □

Proof of Lemma 3.4. Standard homotheticity arguments imply, that with logarithmic utility, an agent's value function is of the form[24]

$$V^j(n; \boldsymbol{S}) = \frac{1}{\rho^j}\log\left[\omega(\boldsymbol{S})n\right].$$

The results follows from (A.4) and (A.5). □

Proof of Proposition 3.5. An agent's wealth is given by $n^j = p_h h^j + p_k k^j + b^j$. By Lemma 3.4, we have $c^j = \alpha n^j$. Therefore,

$$\frac{\partial c}{\partial p_h} = \alpha\frac{\partial}{\partial p_h}n = \alpha h = \alpha\frac{p_h h}{p_h} = \theta_h\frac{\alpha n}{p_h} = \theta_h\frac{c}{p_h},$$

and similarly for $\frac{\partial c}{\partial p_k}$. Note also that $\alpha = \frac{c}{n}$ and, thus, the above also proves equation (3.5).

[24] For more details, I refer to the Supplemental Appendix.

Finally,

$$\frac{\partial C_t/C_t}{\partial p_t/p_t} = \frac{1}{C_t}\int_o^1 \frac{\partial c_t^j}{\partial p_t/p_t}\,dj = \int_o^1 \frac{c_t^j}{C_t}\frac{\partial c_t^j/c_t^j}{\partial p_t/p_t}\,dj$$
$$= \int_o^1 \frac{n_t^j}{C_t}\frac{c_t^j}{n_t^j}\frac{\partial c_t^j/c_t^j}{\partial p_t/p_t}\,dj = \int_o^1 \frac{\eta_t^j \frac{c_t^j}{n_t^j}}{C_t/N_t}\frac{\partial c_t^j/c_t^j}{\partial p_t/p_t}\,dj,$$

which finishes the proof by (3.3). □

Proof of Proposition 3.6. First of all, note that the total wealth in the economy is given by

$$N_t = p_{kt}K + p_{ht}H = \frac{\int_o^1 a^j k_t^j\,dj}{C/N} \equiv \frac{A_t K}{C_t/N_t},$$

where I have denoted by $A_t \equiv \int_o^1 a^j k_t^j/K\,dj$ the total factor productivity (TFP) of the economy. This follows from the pricing decompositions (3.1). Consequently

$$\dot{N}_t = -\frac{A_t K}{(C_t/N_t)^2}\frac{\partial}{\partial t}\left(\frac{C_t}{N_t}\right) + N_t\frac{\dot{A}_t}{A_t}.$$

Now by Lemma 3.4 we obtain

$$\frac{\partial}{\partial t}\left(\frac{C_t}{N_t}\right) = \frac{\partial}{\partial t}\int_o^1 \eta_t^j\frac{c_t^j}{n_t^j}\,dj = \alpha\int_o^1 \frac{\partial \eta_t^j}{\partial t}\rho^j\,dj$$

where by the evolution of dn_t^j for non-constrained agents and Lemma 3.4 we have

$$\frac{\partial \eta_t^j}{\partial t} = \frac{\partial}{\partial t}\frac{n_t^j}{N_t} = \frac{\dot{n}_t^j}{N_t} - \frac{n_t^j}{N_t}\frac{\dot{N}_t}{N_t} = \frac{(-\rho^j + r_{ft})n_t^j}{N_t} - \eta_t^j\frac{\dot{N}_t}{N_t} = -\rho^j\eta_t^j + \left(r_{ft} - \frac{\dot{N}_t}{N_t}\right)\eta_t^j.$$

Therefore, using the fact that $N_t = A_t K/(C_t/N_t)$ we obtain

$$\dot{N}_t = -\frac{A_t K}{(C_t/N_t)^2}\left[\left(r_{ft} - \frac{\dot{N}_t}{N_t}\right)\int_o^1 \eta_t^j\cdot(\alpha\rho^j)\,dj - \alpha\int \eta_t^j\left(\rho^j\right)^2 dj\right] + N_t\frac{\dot{A}_t}{A_t}$$
$$= -\frac{N_t}{C_t/N_t}\left(r_{ft} - \frac{\dot{N}_t}{N_t}\right)\frac{C_t}{N_t} - \alpha N_t\int \frac{\eta_t^j}{C_t/N_t}\left(\rho^j\right)^2 dj + N_t\frac{\dot{A}_t}{A_t}.$$

Simplifying and reorganizing this equation implies

$$r_{ft} = g_t + \alpha\int \frac{\eta_t^j}{C_t/N_t}\left(\rho^j\right)^2 dj = g_t + \int_o^1 \rho^j\eta_{t,APC}^j\,dj,$$

as required. □

Proof of Proposition 4.1. Since bonds are in zero net supply, there must be an agent, who is not credit-constrained: $\lambda^j_{LTV} = 0$ for some j. Therefore, by (A.3), $r_h \leq r_f$ with equality if and only if $\lambda^j_h = 0$. This implies that if $\lambda^j_{LTV} > 0$, necessarily $\lambda^j_h > 0$ as well. □

Proof of Theorem 4.3. By Proposition 4.1, the entrepreneur does not own any housing. Therefore, their wealth immediately after the impact is given by

$$\begin{aligned} n_1 &= p_{k1}\psi_{ko}K + b_o = \psi_{ko}p_{k1}K - (1-\phi_{ko})\psi_{ko}p_{ko}K \\ &= \phi_{ko}\psi_{ko}p_{k1}K - (1-\phi_{ko})(\psi_{ko}p_{ko}K - \psi_{ko}p_{k1}K), \end{aligned}$$

where I have suppressed the time index on the total capital stock K_t, since the capital stock is pre-determined at the time of the impact.

On the other hand, after the capital reallocation following the credit shock, we must have

$$n_1 = p_{k1}\psi_{k1}K + b_1 = p_{k1}\psi_{k1}K - (1-\phi_1)p_{k1}\psi_{k1}K = \phi_{k1}p_{k1}\psi_{k1}K.$$

Combining the above equations yields

$$1 = \frac{\phi_{ko}\psi_{ko}}{\phi_{k1}\psi_{k1}} - \frac{1-\phi_{ko}}{\phi_{k1}}\left(\frac{p_{ko}\psi_{ko}}{p_{k1}\psi_{k1}} - \frac{\psi_{ko}}{\psi_{k1}}\right).$$

We obtain

$$\frac{\psi_{k1}}{\psi_{ko}} = \frac{\phi_{ko}}{\phi_{k1}} + \frac{1-\phi_{ko}}{\phi_{k1}}\left(1 - \frac{p_{ko}}{p_{k1}}\right)$$

Moreover, we can rewrite this as

$$\frac{\psi_{k1}-\psi_{ko}}{\psi_{ko}} = -\frac{\phi_{ko}}{\phi_{k1}}\frac{\phi_{k1}-\phi_{ko}}{\phi_{ko}} + \frac{1-\phi_{ko}}{\phi_{k1}}\frac{p_{k1}-p_{ko}}{p_{k1}}.$$

Dividing by $\frac{\phi_{k1}-\phi_{ko}}{\phi_{ko}}$ and taking the limit $\phi_{k1} \to \phi_{ko}$ yields

$$\frac{d\log\psi_k}{d\log\phi_k} = -1 + \frac{1-\phi_k}{\phi_k}\frac{d\log p_k}{d\log\phi_k},$$

which finishes the proof. □

Proof of Theorem 4.4. Recall the definition of $\nu_{kt} = p_{kt}K_t/(p_{kt}K_t + p_{ht}H_t)$. Differen-

tiating ν with respect to time yields

$$\begin{aligned}
\dot{\nu}_t &= \frac{\dot{p}_{kt}}{p_{kt}}\nu_t + \frac{\dot{K}_t}{K_t}\nu_t - \left(\dot{p}_{kt}K_t + p_{kt}\dot{K}_t + \dot{p}_{ht}H_t + p_{ht}\dot{H}_t\right)\frac{p_{kt}K_t}{(p_{kt}K_t + p_{ht}H_t)^2} \\
&= (\mu^p_{kt} + g_{kt})\nu_t - \left[(\mu^p_{kt} + g_{kt})p_{kt}K_t + (\mu^p_{ht} + g_{ht})p_{ht}H_t\right]\frac{p_{kt}K_t}{(p_{kt}K_t + p_{ht}H_t)^2} \\
&= (\mu^p_{kt} + g_{kt})\nu_t - (\mu^p_{kt} + g_{kt})\nu_t^2 - (\mu^p_{ht} + g_{ht})\nu_t(1-\nu_t) \\
&= (\mu^p_{kt} + g_{kt} - \mu^p_{ht} - g_{ht})\nu_t - (\mu^p_{kt} + g_{kt} - \mu^p_{ht} - g_{ht})\nu_t^2 \\
&= (\mu^p_{kt} + g_{kt} - \mu^p_{ht} - g_{ht})\nu_t(1-\nu_t).
\end{aligned}$$

Denoting by a^j the capital productivity of the marginal investor, we obtain by equations (A.2) and (A.3) that

$$\dot{\nu}_t = \nu_t(1-\nu_t)\left[\frac{(1-i_{ht})r_{st}}{p_{ht}} - \frac{a^j - i_{kt}}{p_{kt}}\right].$$

Moreover,

$$\begin{aligned}
dn_t &= -c_t - r_{ht}s_t + r_{ft}n_t + \theta_{kt}n_t[r_{kt} - r_{ft}] + \theta_{ht}n_t[r_{ht} - r_{ft}] - \xi n_t + \xi\lambda N_t \\
&= -c_t - r_{ht}s_t + r_{ft}n_t + \frac{\lambda_{LTV,t}}{V'(n_t)n_t}n_t - \xi n_t + \xi\lambda N_t
\end{aligned}$$

and similarly for $d\underline{n}_t$. Here we used the fact that for any agent investing in housing $r_{ht} = r_{ft}$, and if $k_t > 0$ and

$$r_{kt} - r_{ft} = \frac{\lambda_{LTV,t}\phi_k}{V'(n_t)n_t} > 0,$$

then the collateral constraint is binding and $\theta_{kt} = 1/\phi_k$.

It follows that

$$\frac{d(n_t + \underline{n}_t)}{n_t + \underline{n}_t} = \eta_t\frac{dn_t}{n_t} + (1-\eta_t)\frac{d\underline{n}_t}{\underline{n}_t}\,dt = \eta_t\left[r_{ft} - \frac{c_t}{n_t} - \frac{r_{ht}s_t}{n_t} + \frac{\lambda_{LTV,t}}{V'(n_t)n_t}\right] + (1-\eta_t)\left[r_{ft} - \frac{\underline{c}_t}{\underline{n}_t} - \frac{r_{ht}\underline{s}_t}{\underline{n}_t}\right].$$

We obtain

$$\begin{aligned}
\frac{d\eta_t}{\eta_t} &= \frac{d\left(\frac{n_t}{n_t + \underline{n}_t}\right)}{\frac{n_t}{n_t + \underline{n}_t}} = r_{ft} - \frac{c_t}{n_t} - \frac{r_{ht}s_t}{n_t} + \frac{\lambda_{LTV,t}}{V'(n_t)n_t} + \xi\left(\frac{\lambda}{\eta_t} - 1\right) - \frac{d(n_t + \underline{n}_t)}{n_t + \underline{n}_t} \\
&= (1-\eta_t)\left[\frac{\underline{c}_t}{\underline{n}_t} - \frac{c_t}{n_t} + \frac{\lambda_{LTV,t}}{V'(n_t)n_t}\right] + (1-\eta_t)\left[\frac{r_{ht}\underline{s}_t}{\underline{n}_t} - \frac{r_{ht}s_t}{n_t}\right] + \xi\left(\frac{\lambda}{\eta_t} - 1\right),
\end{aligned}$$

where

$$\frac{\lambda_{LTV,t}}{V'(n_t)n_t} = \frac{1}{\phi_k}[r_{kt} - \underline{r}_{kt}] = \theta_{kt}[r_{kt} - \underline{r}_{kt}].$$

This finishes the proof.

□

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Entrepreneurs, Collateral Channel, and the House Price Volatility Puzzle

Supplemental Appendix

Juhana Siljander

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B Theoretical appendix with additional robustness results

B.1 Model extension: $\kappa_h, \kappa_k < \infty$

The following lemma generalizes the main results of the paper in to the case $\kappa_h, \kappa_k < \infty$. For simplicity, I write these results under Assumption 4.2, although the generalization to the more general case is trivial (just replace $A(\psi_k)$ by $\int_o^1 a^j \psi_k \, dj$ everywhere).

Lemma B.1. *Propositions 3.1 and 3.2 generalize for $\kappa_h, \kappa_k < \infty$ as follows:*

$$
\begin{aligned}
p_k &= \frac{1+\kappa_k A(\psi_k)}{\nu + \kappa_k C/N}\nu, & p_h &= \frac{1+\kappa_k A(\psi_k)}{\nu + \kappa_k C/N}\frac{K}{H}(1-\nu) \\
r_s &= \frac{1+\kappa_k A(\psi_k)}{1+\kappa_h}\frac{1-\nu+\kappa_h \frac{r_s S}{N}}{\nu + \kappa_k C/N}\frac{K}{H}, & \frac{p_h}{r_s} &= \frac{1+\kappa_h}{1-\nu+\kappa_h \frac{r_s S}{N}}(1-\nu), \\
i_k &= \frac{A(\psi_k)\nu - C/N}{\nu + \kappa_k C/N}, & i_h &= \frac{1-\nu-\frac{r_s S}{N}}{1-\nu+\kappa_h \frac{r_s S}{N}}.
\end{aligned}
\tag{B.1}
$$

Proof. By the goods market clearing we have

$$
C = A(\psi_k)K - i_k K = \frac{A(\psi_k)}{p_k} p_k K - \frac{p_k - 1}{\kappa_k} K = \frac{\kappa_k A(\psi_k)}{\kappa_k p_k} p_k K + \frac{p_k K}{\kappa_k p_k} - \frac{p_k K}{\kappa_k}.
$$

This implies

$$
C + \frac{p_k K}{\kappa_k} = \frac{1+\kappa_k A(\psi_k)}{\kappa_k p_k} p_k K
$$

Dividing by N an noting that $\nu \equiv p_k K/N$, we obtain

$$
\frac{C}{N} + \frac{\nu}{\kappa_k} = \frac{1+\kappa_k A(\psi_k)}{\kappa_k p_k}\nu,
$$

which yields

$$
p_k = \frac{1+\kappa_k A(\psi_k)}{\nu + \kappa_k C/N}\nu \quad \text{and} \quad p_h = \frac{1+\kappa_k A(\psi_k)}{\nu + \kappa_k C/N}\frac{K}{H}(1-\nu), \tag{B.2}
$$

where I used the fact that $\frac{p_k K}{p_h H} = \nu/(1-\nu)$

It also follows that the total wealth is given by

$$
N = p_k K + p_h H = \frac{1+\kappa_k A(\psi_k)}{\nu + \kappa_k C/N} K.
$$

On the other hand, by the rental market clearing we obtain

$$\begin{aligned} S = (1-i_h)H &= H - \frac{1}{\kappa_h}\left(\frac{p_h}{r_s} - 1\right) H \\ &= \left(1 + \frac{1}{\kappa_h}\right) H - \frac{1}{\kappa_h}\frac{p_h H}{r_s}, \end{aligned}$$

which yields

$$r_s S + \frac{1}{\kappa_h} p_h H = \left(1 + \frac{1}{\kappa_h}\right) r_s H = \left(1 + \frac{1}{\kappa_h}\right) \frac{p_h H}{p_h / r_s}.$$

By dividing by N and by noting that $p_h H/N = 1 - \nu$, we may rewrite this as

$$\frac{p_h}{r_s} = \left(1 + \frac{1}{\kappa_h}\right) \frac{1-\nu}{(1-\nu)/\kappa_h + r_s S/N} = \frac{1+\kappa_h}{1 - \nu + \kappa_h \frac{r_s S}{N}} (1-\nu).$$

We may plug in these formulas for p_k and p_h into equation (**??**) to solve for i_h and i_k as

$$i_k = \frac{1}{\kappa_k} \frac{\nu + \kappa_k A(\psi_k)\nu - (\nu + \kappa_k C/N)}{\nu + \kappa_k C/N} = \frac{A(\psi_k)\nu - C/N}{\nu + \kappa_k C/N}$$

and

$$\begin{aligned} i_h = \frac{1}{\kappa_h}\left(\frac{p_h}{r_s} - 1\right) &= \frac{1}{\kappa_h}\left(\frac{1+\kappa_h}{1 - \nu + \kappa_h \frac{r_s S}{N}}(1-\nu) - 1\right) \\ &= \frac{1}{\kappa_h} \frac{(1+\kappa_h)(1-\nu) - (1 - \nu + \kappa_h \frac{r_s S}{N})}{1 - \nu + \kappa_h \frac{r_s S}{N}} \\ &= \frac{1 - \nu - \frac{r_s S}{N}}{1 - \nu + \kappa_h \frac{r_s S}{N}}. \end{aligned}$$

Finally, by (A.6) we also have

$$\begin{aligned} r_s = \frac{A(\psi_k) - i_k}{1 - i_h} \frac{K}{H} \frac{1-\alpha}{\alpha} &= \frac{\frac{\left[1+\kappa_k A(\psi_k)\right] C/N}{\nu + \kappa_k C/N}}{\frac{[1+\kappa_h]\frac{r_s S}{N}}{1 - \nu + \kappa_h \frac{r_s S}{N}}} \frac{K}{H} \frac{1-\alpha}{\alpha} \\ &= \frac{\left[1 + \kappa_k A(\psi_k)\right] C/N}{[1+\kappa_h]\frac{r_s S}{N}} \frac{1 - \nu + \kappa_h \frac{r_s S}{N}}{\nu + \kappa_k C/N} \frac{K}{H} \frac{1-\alpha}{\alpha} \\ &= \frac{1 + \kappa_k A(\psi_k)}{1 + \kappa_h} \frac{1 - \nu + \kappa_h \frac{r_s S}{N}}{\nu + \kappa_k C/N} \frac{K}{H}. \end{aligned}$$

□

B.2 Model extension: house price amplification with non-linear production

In this section I present a simplified one-period static model, which can be used to further illustrate some of the model mechanics with non-linear production function $y = ak^{\gamma}, \gamma < 1$. In this model I also show analytically that a sufficiently large credit relaxation will necessarily imply a disproportionate increase in house prices, even if capital supply from savers to entrepreneurs is not fully elastic. Under some additional conditions, the model also has strong enough amplification to produce multiple equilibria; the related discussion also demonstrates robustness to a model version, where entrepreneurs might initially own housing (cf. the discussion on Proposition 4.1).

B.2.1 The model environment

The one-period economy has a unit mass of two types of agents: entrepreneurs and savers. All agents are price-takers acting in a competitive economy. In the morning of the period, the agents wake up with some pre-existing portfolio holdings of productive capital k, housing h, and bonds b. Capital and housing are in fixed supply K and H, respectively. Bonds are in zero net supply $b + \underline{b} = 0$, where b denotes the combined holdings of the entrepreneurs and $\underline{b}$ the combined holdings of the savers.

For simplicity, I assume that the initial portfolios are identical within a type. I look for a symmetric equilibrium, where agents of the same type make identical choices, and the economy aggregates to two representative agents, one for each type. In what follows, I consider the problem of these representative agents. I denote the saver quantities with underlined objects when emphasizing their different behavior from the entrepreneurs.

During the morning, the agents freely trade on the assets. Capital and housing prices are endogenous and denoted by p_k and p_h, respectively. Since there is no consumption during the morning, when asset trades take place, there is no real interest rate. I normalize the nominal interest rate to zero, implying that the bond is a zero-return savings technology from the morning until the afternoon. The trading is subject to short-selling constraints $k, h \geq 0$ and to a collateral constraint

$$-b \leq (1 - \phi_k)p_k k + (1 - \phi_h)p_h h. \tag{B.3}$$

In the afternoon, production occurs, possible debt positions are cleared, and consumption takes place in that order. All agents derive utility from non-durable con-

sumption c and from housing services s with utility $u(c^{\alpha}s^{1-\alpha})$, where $u(\cdot)$ satisfies Inada conditions. The consumption good c, which is the numéraire in the economy, is produced with $ak^{\gamma}, \gamma \in (0,1]$, technology, where the saver's and the entrepreneur's productivity parameters are given, respectively, by $\underline{a}$ and a. I assume that the entrepreneur is more productive at any given level of capital holdings: $a > \underline{a}$. Housing services are produced one-to-one by houses h and are traded in the afternoon at a frictionless rental market with an endogenous rental price r_s.

For an initial portfolio (k_{io}, h_{io}, b_{io}), an agent i solves

$$\begin{aligned} \max_{c_i, s_i, k_i, h_i, b_i} \quad & u(c_i^{\alpha} s_i^{1-\alpha}) \\ \text{s.t.} \quad & c_i + r_s s_i \leq a_i k_i^{\gamma} + r_s h_i + b_i. \\ & p_k k_i + p_h h_i + b_i = p_k k_{io} + p_h h_{io} + b_{io}, \end{aligned} \tag{B.4}$$

subject to the collateral constraint (B.3) and the short-selling constraints on h and k.

B.2.2 Equilibrium prices

I denote the (representative) entrepreneur's ownership share of capital by $\psi_k \equiv k/K$, the average productivity of capital by

$$A(\psi_k) \equiv \frac{ak^{\gamma} + \underline{a}\,\underline{k}^{\gamma}}{K} = \left[a\psi_k^{\gamma} + \underline{a}(1-\psi_k)^{\gamma}\right] K^{\gamma-1},$$

the total wealth in the economy by $N \equiv p_k K + p_h H$, aggregate consumption by $C \equiv c + \underline{c}$, and the value of total capital stock relative to the total wealth in the economy by $\nu \equiv p_k K/(p_k K + p_h H)$. The following result characterizes the endogenous prices of the model in terms of ψ_k, ν, and the aggregate consumption propensity C/N and replicates Theorem 3.1 in section 3.

Proposition B.2. *The rental rate r_s, house price p_h, and price of capital p_k can be characterized via the following formulas:*

$$r_s = A(\psi_k)\frac{K}{H}\frac{1-\alpha}{\alpha} \quad \textit{and} \quad p_h = A(\psi_k)\frac{K}{H}\frac{(1-\nu)}{C/N},$$

as well as

$$p_k = A(\psi_k)\frac{\nu}{C/N}.$$

Proof. The proof is identical to that of Proposition 3.1, and is omitted here. □

B.2.3 House price amplification

The key equation determining the extent of house price amplification is that of the saver's no-arbitrage condition, which with $\gamma < 1$ takes the form

$$\frac{\gamma \underline{a}\, \underline{k}^{\gamma-1}}{p_k} = \frac{r_s}{p_h}. \tag{B.5}$$

The numerator on the left-hand side of the equation includes the marginal productivity of saver's capital, which in case of a linear technology was directly given by the saver's productivity parameter $\underline{a}$, but more generally takes the form $\gamma \underline{a}\, \underline{k}^{\gamma-1} = \gamma \underline{a}\, (1-\psi_k)^{\gamma-1} K^{\gamma-1}$. A credit relaxation affecting capital allocation also affects this marginal productivity, with two main implications for asset prices.

First, part of the credit shock is now absorbed by capital price p_k, as the supply of capital from savers to entrepreneurs is no longer fully elastic. Increasing entrepreneur capital share ψ_k also increases the saver's marginal capital productivity. If this direct effect on capital productivity is more significant than the indirect, general equilibrium effect on the rental rate r_s, house prices might not respond as strongly as the price of capital.

Second, since the price of capital is now responding to a credit shock, this will introduce a financial accelerator à la Kiyotaki and Moore (1997) into the model. Credit relaxation increases the price of capital, further relaxing the credit constraint and generating a strategic complementarity between asset prices and credit access. The exact magnitudes of these forces depend on the model parameters. However, independent of these parameters, it is necessarily true that for sufficiently constrained agents (i.e., sufficiently low entrepreneur capital share ψ_k), the price of housing increases disproportionately for a small credit shock:

Theorem B.3. *Let*

$$g(\psi_k) \equiv \frac{p_h}{p_k}.$$

Then $g'(0) > 0$.

Proof. By (B.5), we can write the ratio of house and capital prices as a function of ψ_k as[1]

$$g(\psi_k) = \frac{p_h}{p_k} = \frac{r_s}{\gamma \underline{a}\, \underline{k}^{\gamma-1}} = \frac{A(\psi_k)}{\gamma \underline{a} (1-\psi_k)^{\gamma-1} K^{\gamma-1}} \frac{K}{H} \frac{1-\alpha}{\alpha}. \tag{B.6}$$

[1]Note that if the entrepreneur is not credit-constrained, then combining the no-arbitrage conditions for both agents yields

$$\frac{a}{\underline{a}} = \left(\frac{\psi_k}{1-\psi_k}\right)^{1-\gamma},$$

The result follows by differentiation. □

Note that in particular, the price ratio depends on the credit constraint parameter ϕ_k only through its effect on the capital allocation ψ_k. As long as the entrepreneur cannot afford to buy too much of the capital, i.e. ψ_k is close to zero, a (small) relaxation of the constraint allowing an increase in ψ_k will disproportionately increase the price of housing (i.e., $g' > 0$).

B.2.4 Strategic complementarity and multiple equilibria

While the idea behind the financial accelerator discussed in the previous section is well-known (Kiyotaki and Moore, 1997), when combined with the housing market, the interaction between the housing and capital markets provides a new and additional source of amplification. This force is due to the disproportionate increase in house prices discussed in section 4.1 and it is strong enough to generate multiple equilibria in this static model.[2]

Theorem B.4. *Suppose $\gamma < 1$. Then there exists an entrepreneur initial portfolio (k_o, h_o, b_o) with $k_o = 0$ such that the model admits two possible equilibria (after trading).*

Intuitively, when a productive entrepreneur initially only holds housing, the equilibrium house price amplification mechanism discussed in section 4.1 producing a disproportionate house price boom now disproportionately benefits the entrepreneur. This amplifies the financial accelerator mechanism enough to potentially produce multiple equilibria.

Note that the intuition behind the proof of this result is also simple. Assuming an entrepreneur portfolio with $k_o = 0$, their wealth is given by

$$n = p_h h_o + b_o = A(\psi_k) K \frac{h_o}{H} \frac{1-\alpha}{\alpha} + b_o, \tag{B.7}$$

which depends on the endogenous capital allocation ψ_k. On the other hand, we know

which pins down the capital allocation ψ_k, in which case the state of the economy is characterized by the allocation of housing $\psi_h = h/H$. Therefore, we can consider ψ_k as a state variable describing the state of the economy only in the case of credit-constrained entrepreneurs (when $\psi_h = 0$ necessarily). In particular, the fact that the right hand side of (B.6) equals zero at $\psi_k = 1$ for $\gamma < 1$ has no economic meaning, as $\psi_k = 1$ can never represent an equilibrium of the model for $\gamma < 1$.

[2]Theorem B.4 provides a sufficient, but not a necessary, condition for the existence of multiple equilibria.

that, if the entrepreneur is credit-constrained after trading, their wealth must equal

$$\begin{aligned} n = p_k k + b &= p_k k + (1-\phi_k) p_k k = \phi_k p_k k \\ &= \phi_k \gamma \underline{a}\, \underline{k}^{\gamma-1} k \\ &= \phi_k \gamma \underline{a} (1-\psi_k)^{\gamma-1} \psi_k K^{\gamma}. \end{aligned} \tag{B.8}$$

The question is, under which conditions does this nonlinear system of equations, (B.7) and (B.8), have two solutions. Note that this is a system of two equations on two unknowns n and ψ_k.

To illustrate the mechanism, I first calibrate the model parameters to produce a quantitative example. I begin by normalizing $\underline{a} = 1, K = 1$ and $H = 1$. For calibrating the heterogeneity in productivity, I rely on Fagereng, Guiso, Malacrino and Pistaferri (2020, Table 7), who report that among business owners, the return to net worth is 6.28% in the top quartile and 10.09% in the top decile of the return distribution. I use the ratio between these two to calibrate $a = 1.61$. Furthermore, I set $\gamma = 0.5$ and $\alpha = 0.8$, the latter of which reflects an approximate 20% consumption share of housing net of utilities (Davis and Ortalo-Magné, 2011).[3] I also assume the entrepreneurs initially own $h_o = 15\%$ of the housing stock with $k_o = 0$. Following the proof of Theorem B.4, I also set

$$b_o = \underline{a} K^{\gamma} \frac{h_o}{H} \frac{1-\alpha}{\alpha} - 0.005,$$

to guarantee that the entrepreneurs would be underwater, if the capital allocation was the worst possible with $\psi_k = 0$. Finally, I set the collateral constraint parameter at $\phi_{ko} = 0.15$ and $\phi_{k1} = 0.01$, corresponding to an 85% loan-to-value ratio, relaxed to 1% by the credit shock (Greenwald, 2018).

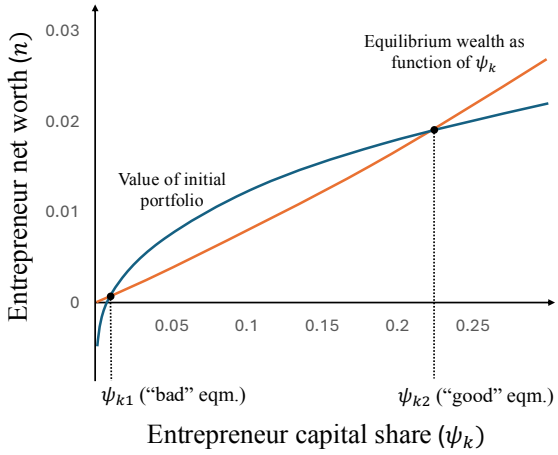
In panel (a) of Figure 1, I depict the entrepreneurs' net worth n as a function of the capital allocation ψ_k, showing the relations specified by both (B.7) and (B.8). These equations have two solutions corresponding to two different equilibria: $\psi_{k1} = 0.0096$ ("bad" equilibrium) and $\psi_{k2} = 0.225$ ("good" equilibrium). In the good equilibrium, the levered entrepreneurs trade off their 15% housing share to 22.5% ownership share on the productive capital. However, if everyone believes that the output and prices should be lower, the economic fundamentals also support another equilibrium with much worse capital allocation, with the entrepreneurs owning only 0.96% of the capital

[3] Davis and Ortalo-Magné (2011) report 24% consumption share for housing. I follow Greenwald (2018) in assuming a 4% cost for utilities to come up with a 20% housing consumption share, net of utilities.

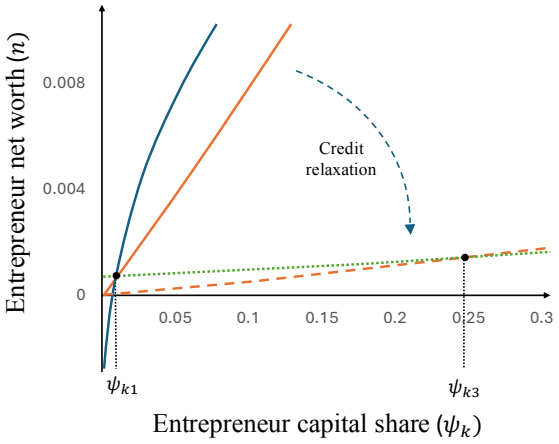
Figure 1: Multiplicity of equilibria and the financial accelerator

The figure illustrates the strategic complementarity between house prices and the capital allocation for entrepreneurs initialized with housing wealth as well as the related financial accelerator mechanism. The orange line in panel (a) depicts the equilibrium relation between capital allocation and entrepreneur wealth for credit-constrained entrepreneurs. The curved blue line, on the other hand, shows how the value of the entrepreneur's initial portfolio depends on the equilibrium capital allocation. The model equilibria are characterized by the intersections of these two lines. Panel (b) of the figure (with rescaled y-axis) considers a credit relaxation, depicted as a rotation of the orange line. The dotted green line describes how the entrepreneur's wealth depends on the capital allocation, assuming an initial portfolio associated with the "bad" equilibrium. The new equilibrium is found at the intersection of the dashed orange and dotted green lines, where the entrepreneur wealth relation and the equilibrium condition coincide.

(a) Multiplicity of equilibria



(b) Financial accelerator



stock.

This multiplicity relies on the strategic complementarity between the entrepreneurs' capital holdings and house prices. Higher house prices allow the entrepreneurs, whose initial holdings consist of housing, to take more debt to purchase more of the business capital. Higher entrepreneur business capital holdings reflect better capital allocation, which increases house prices as detailed in section 4.1. Such an increase in house prices further relaxes the entrepreneurs' credit constraints generating a strong feedback mechanism, yielding multiple equilibria. These two solutions (the "good" and "bad" equilibria) are shown in panel (a) of Figure 1

Consider now an economy starting from the bad equilibrium and facing an exogenous, unanticipated credit relaxation, reducing the collateral credit parameter ϕ_k from $\phi_{k1} = 0.15$ to $\phi_{k2} = 0.01$, corresponding to 99% maximal loan-to-value ratio. The economy must readjust to satisfy the new equilibrium conditions associated with this new parameter setup. I depict this adjustment in panel (b) of Figure 1.

Note that $n = \psi_k = 0$ is always a solution for equation (B.8). In particular,

a relaxation of the credit constraint, represented by a reduction in the value of ϕ_k, can, therefore, be visually described as a clock-wise rotation of the graph describing the relation between n and ψ_k specified by equation (B.8), and shown as the dashed orange line in panel (b) of Figure 1.

Given the agents' portfolio holdings in the initial bad equilibrium, the adjustment affecting prices will also affect the wealth of all agents. The dotted green line shows how the value of the entrepreneur's portfolio depends on the capital allocation. The new equilibrium after the credit shock is characterized by the capital allocation, for which the value of the entrepreneur portfolio coincides with the equilibrium condition described by equation (B.8). The new equilibrium is, therefore, depicted at the intersection of the dotted green and dashed orange lines in panel (b) of Figure 1.

Under the new parameter values, the bad equilibrium is no longer viable, and the credit relaxation leads to a large reallocation of capital with entrepreneurs owning $\psi_{k3} = 24.8\%$ of the capital stock after the credit relaxation. The improvement in capital allocation leads to a 44.8% increase in house prices.

B.2.5 Proof of Theorem B.4

Proof. We consider an entrepreneur with an initial portfolio consisting only of housing h_o and bonds b_o. Since the saver also invests in the bond with unit return, it must be the case that $p_h = r_s$. Therefore, we may write the the entrepreneur net worth, before trading, as

$$n_{\text{pre}} = n_{\text{pre}}(\psi_k) = p_h h_o + b_o = A(\psi_k) K \frac{h_o}{H} \frac{1-\alpha}{\alpha} + b_o.$$

On the other hand, after trading, for a credit-constrained entrepreneur we must have

$$n_{\text{post}} = n_{\text{post}}(\psi_k) = \phi_k p_k k = \phi_k \gamma \underline{a}\, \underline{k}^{\gamma-1} k = \phi_k \gamma \underline{a} (1-\psi_k)^{\gamma-1} \psi_k K^{\gamma}.$$

By setting $n = n_{\text{pre}} = n_{\text{post}}$, we may use these two equations to solve for the equilibrium capital allocation ψ_k after trading. Due to the nonlinearity of this system of equations, in general, this may have multiple solutions. I proceed to argue for a particular example of such a multiplicity, confirming that both solutions correspond to an economically meaningful equilibrium.

Let us first note that

$$n_{\text{post}}(0) = 0 \quad \text{and} \quad n'_{\text{post}}(0) = \phi_k \gamma \underline{a} K^{\gamma}.$$

Similarly, we have

$$n_{\text{pre}}(0) = \underline{a}K^{\gamma}\frac{h_o}{H}\frac{1-\alpha}{\alpha} + b_o \quad \text{and} \quad n'_{\text{pre}}(0) = +\infty.$$

Therefore, for any $\varepsilon > 0$ small enough, by choosing

$$b_o = -\underline{a}K^{\gamma}\frac{h_o}{H}\frac{1-\alpha}{\alpha} - \varepsilon, \tag{B.9}$$

we can guarantee one solution ψ_{k1} arbitrarily close to 0, with $n_{\text{pre}}(0) < n_{\text{post}}(0) = 0$ and $n_{\text{pre}}(\psi_k) > n_{\text{post}}(\psi_k)$ for every $\psi_k \in (\psi_{k1}, \psi_{k1} + \delta)$ and for some $\delta > 0$ chosen small enough.

To guarantee the existence of a second solution, by the continuity of n_{pre} and n_{post}, we only need to establish that $n_{\text{pre}}(\psi_k^*) < n_{\text{post}}(\psi_k^*)$, where ψ^* is defined via

$$\frac{a}{\underline{a}} = \left(\frac{\psi_k^*}{1-\psi_k^*}\right)^{1-\gamma}. \tag{B.10}$$

Indeed, if the entrepreneur is not credit-constrained, then the portfolio problems of both agents dictate that

$$\frac{\gamma a k^{\gamma-1}}{p_k} = 1 = \frac{\gamma \underline{a}\,\underline{k}^{\gamma-1}}{p_k},$$

which implies (B.10) for $\psi_k = \psi_k^*$. Therefore, if in equilibrium $\psi_k < \psi_k^*$, the entrepreneur must be credit-constrained. This implies that, if $n_{\text{pre}}(\psi^*) < n_{\text{post}}(\psi^*)$ and $n_{\text{pre}}(\psi_k) > n_{\text{post}}(\psi_k)$ for some $\psi_k > \psi_{k1}$, there must exist another root ψ_{k2} for $n = n_{\text{pre}} = n_{\text{post}}$ on (ψ_{k1}, ψ_k^*); since by the definition of ψ_k^*, the entrepreneur is credit-constrained with the capital allocation characterized by ψ_{k2}, this produces another equilibrium of the model.

Therefore, it remains to establish a sufficient condition for $n_{\text{pre}}(\psi_k^*) < n_{\text{post}}(\psi_k^*)$. However, since $n_{\text{post}}(\psi_k^*)$ does not depend on h_o and every term of $n_{\text{pre}}(\psi_k^*)$ does, we can always guarantee this by choosing h_o small enough.

To recap the argument, we first choose ψ_k^* to satisfy (B.10). We then choose h_o small enough so that

$$A(\psi_k^*)K\frac{h_o}{H}\frac{1-\alpha}{\alpha} - \underline{a}K^{\gamma}\frac{h_o}{H}\frac{1-\alpha}{\alpha} < \phi_k\gamma\underline{a}(1-\psi_k^*)^{\gamma-1}\psi_k^*K^{\gamma}.$$

For b_o chosen as in (B.9), this guarantees $n_{\text{pre}}(\psi_k^*) < n_{\text{post}}(\psi_k^*)$ and $n_{\text{pre}}(0) < n_{\text{post}}(0)$,

for every $\varepsilon > 0$. Finally, we choose $\varepsilon > 0$ small enough to guarantee that there exists $\psi_{k1} \in (0, \psi_k^*)$ such that $n_{\text{pre}}(\psi_k) > n_{\text{post}}(\psi_k)$ for every $\psi_k \in (\psi_{k1}, \psi_{k1} + \delta)$, for some $\delta > 0$ chosen small enough. This guarantees the existence of two equilibria, in both of which the entrepreneur is credit-constrained. □

B.3 Quantitative robustness: housing supply elasticity

By equation (2.1), we have

$$i_h = -\frac{1}{\kappa_h} + \frac{1}{\kappa_h}\frac{p_h}{r_s}.$$

In particular, the model dictates how housing investment i_h respond to the price-to-rent ratio p_h/r_s. When price-to-rent ratio is high, building new housing units is expected to be profitable, and housing developers thus construct more units to meet the demand.

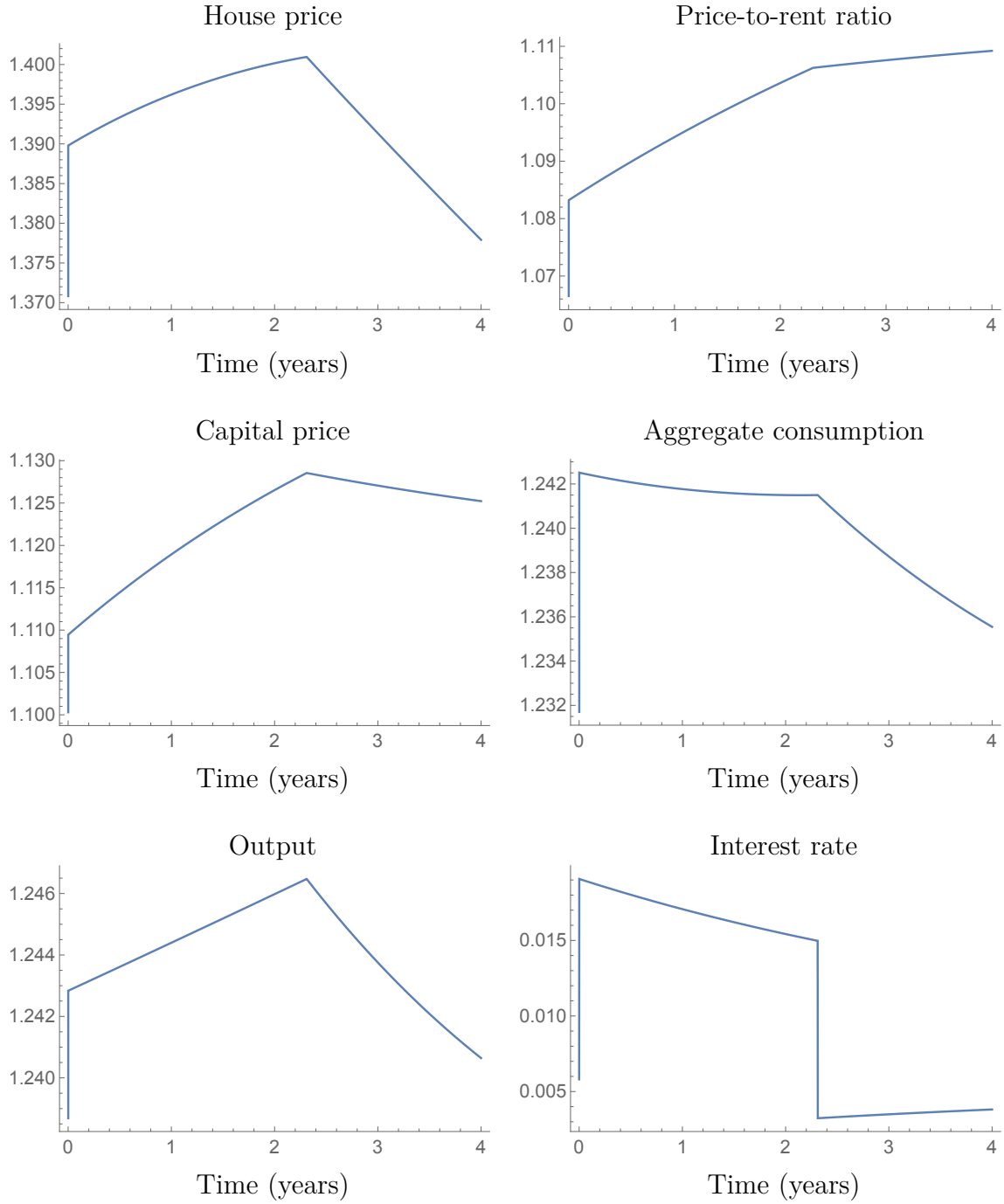
We estimate this regression using US annual data on county-level housing units from the Census and price and rent indicies from Zillow. To consider the robustness of my baseline results for alternative calibrations of κ_h, I consider an instrument, following the identification argument of Chaney, Sraer and Thesmar (2012) and interact the Saiz (2010) land topography supply elasticity measure with 30-year mortgage rates. While this instrument has been widely used in the literature for house prices, the argument here is more subtle, since my outcome variable is housing investment rate, which we for sure should expect to correlate with the supply elasticity also via other channels than the price-to-rent ratio, thus potentially violating the exclusion restriction.

To alleviate this concern, I include MSA fixed-effects into the regression. Since the supply elasticity is measured at the MSA-level, these fixed-effects capture all regional variation in supply elasticities due to the MSA-level geographical differences as measured by Saiz (2010). My identification argument therefore relies on the interaction of these supply elasticities with the time-varying credit conditions. These credit shocks provide plausibly exogenous variation in demand conditions, therefore, producing demand-driven price shocks, isolated from regional supply-conditions potentially also affecting investments.

Intuitively, the two key assumption behind this strategy are that (i) the long-run housing supply elasticity affects, how house prices react to credit-driven demand shocks, since forward-looking housing investors take these supply conditions into account in their pricing decisions, and (ii) the housing developers competitively react to the price information.

Figure 2: Robustness check for lower housing supply elasticity

The figure reports impulse responses for unanticipated credit relaxation $\phi_k = 0.27 \to 0.18$ but with $\kappa_k = \kappa_h = 555$. Additionally, we initialize the model at the steady-state unlike in our baseline calibration. This slightly lowers the duration of the boom. House price, price-to-rent ratio, capital price, aggregate consumption, and output are here normalized to 1, before the shock.



Note that the second assumption would be violated, for example, if the housing developer had market power, allowing their investment decisions to affect house prices. To the extent that this IV strategy might not be fully convincing, I note that the IV estimate for $1/\kappa_h$ is higher than the OLS estimate, thus making it more difficult for the model to generate large price effects compared to a moment-matching exercise merely using the OLS-estimate. The quantitative results are reported in Figure 2, when using the value $\kappa_k = \kappa_h = 555 = 1/0.00180$ associated with column (4) of Table 2.

C Empirical Appendix

C.1 Data

My empirical analyses combine two main data sets: (i) Community Reinvestment Act business loan data and (ii) County Business Patterns data on business activity. In addition, I use county-level Zillow house price indices to measure house price growth rates during 2000 – 2022.

C.1.1 Community Reinvestment Act business loan data

Community Reinvestment Act was enacted in 1977 to improve credit access for businesses in low- and moderate-income (LMI) neighborhoods. The law aimed to remove "redlining" practices that discriminated against loan applications from LMI neighborhoods. According to the law, banking institutions are regularly evaluated on how well they meet their communities' requirements and are given a CRA rating (Outstanding, Satisfactory, Needs to Improve, Substantial Noncompliance). These ratings are used by federal banking agencies, for example, when approving new branches or bank mergers.[4]

The Act operates mainly at the level of U.S. census tracts. The median household income in each tract is compared to the median in the corresponding Metropolitan Statistical Area (MSA) or Metropolitan Division (MD). Tracts with a median household income below 50% of the MSA/MD median are considered low-income, and tracts with household income between 50% and 80% of the MSA/MD median are considered moderate-income tracts.

[4]The relevant federal banking agencies are the Board of Governors of the Federal Reserve System, the Office of the Comptroller of the Currency (OCC), and the Federal Deposit Insurance Corporation.

Each banking institution is assigned a CRA Assessment Area to evaluate its performance based on its presence in the neighborhood (e.g., number and location of branches). The Act provides five different sets of evaluation criteria, e.g., depending on the bank's size, describing the requirements for its presence and operations in the LMI neighborhoods within its assessment area. Based on the value of their assets, banks may be considered Small Banks or Intermediate Small Banks (ISB), both of which have different criteria from larger institutions.[5]

As part of the evaluation process, each institution not considered a Small Bank or an ISB must disclose its lending activity annually. These disclosure data are shared publicly at a bank-county level each year since 1996 by the Federal Financial Institutions Examination Council (FFIEC). The data includes the number and total value of small business loans originated by each bank by loan size at origination ($\leq$ \$100,000, between \$100,000 and \$250,000, and between \$250,000 and \$1,000,000), as well as aggregate counts and values for all business loans originated for firms with annual revenue less than \$1,000,000. These bank-county-level business loan data constitute the primary data for my analyses.

C.1.2 County Business Patterns

I merge the CRA loan data at the county-year level with County Business Patterns (CBP) data on employment and establishment counts published by the U.S. Census Bureau. The CBP data includes employment by county, industry and establishment size (measured in number of employees) between 1987 and 2022 as of March of the reported year. In all the analyses, I use the 2-digit National American Industry Classification System (NAICS) level and exclude the residential construction (NAICS 1997 category 2332; NAICS 2002- category 2361) and real estate (NAICS category 53) sectors to ensure that the specifics of the housing boom-bust cycles do not drive my results.

[5]The Small Bank asset threshold was set at \$250 million in 1995, increased to one billion in 2005, and has then been adjusted annually, and is currently (as of January 1, 2024) set at \$1.564 billion. The ISB threshold was initially set at \$250 million in 2005, has been adjusted annually since, and is now (as of January 1, 2024) set at \$391 million.

Table 1: Small business loan originations and house prices

The table reports the results for an annual county-level regression of growth in small business loan originations ($\leq$ \$250,000 at origination) on house price growth rates during 2000 – 2022. The loan origination counts are from Community Reinvestment Act data and house prices from the Zillow house price indices at the county level. House price growth is instrumented by interacting the Saiz (2010) housing supply elasticity measure with the 30-year mortgage rate. Standard errors are clustered at the state level, and the regressions are weighted by county-level (log) population in 2010. Controls include the county-level share of the white population, homeownership rate, share of the population in the labor force, unemployment rate, and share of the population with a college degree, all measured in 2010. The loan growth data is restricted only to include loan growth rates between the 10th and 90th percentiles to remove outliers and infinite growth rates (associated with zero originations).

	Dependent variable:					
	House price growth	Growth in loan originations				
	First stage	*Reduced form*	*OLS*		*Instrumental variable (2SLS)*	
	(1)	(2)	(3)	(4)	(5)	(6)
Housing supply elasticity	-0.0005***	-0.0010***				
× 30-year mortgage rate	(0.0001)	(0.0002)				
House price growth			0.119***	0.116***	1.91***	1.74***
			(0.012)	(0.012)	(0.412)	(0.404)
State FE	Yes	Yes	Yes	Yes	Yes	Yes
Year FE	Yes	Yes	Yes	Yes	Yes	Yes
Controls	No	No	No	Yes	No	Yes
Observations	13,431	13,305	38,454	38,454	13,431	13,431
R^2	0.564	0.572	0.359	0.360		

Clustered (state) standard-errors in parentheses
*Signif. Codes: ***: 0.01, **: 0.05, *: 0.1*

In some small counties, employment data for all industries is not disclosed due to privacy reasons, and only a size class is provided. In such cases, I use the midpoint of the category, except for "over 100,000 employees," where 200,000 employees are used as a proxy for the employment count. The data also includes the full county-level employment and establishment counts. However, in my regression specifications, I only use industry-level data, which allows for removing the residential construction and real estate sectors.

Throughout the paper, I restrict both the CRA loan growth rates and the CBP employment and establishment growth rates between the 10th and 90th percentiles to remove infinite values and other outliers.[6]

C.2 Stylized facts

Adelino et al. (2015) argue that increases in real estate prices affect employment growth by facilitating collateralized external financing and creating or expanding small business activity via the collateral channel. They consider the differential effect of house prices on the expansion of employment between establishments of different sizes: assuming small business financing is more likely to rely on real estate collateral, house prices should have a differential impact on small business expansion compared to large corporations. In order to reason for a causal effect, they instrument house prices by the housing supply elasticity, measured by land topography differences by Saiz (2010). In addition to the effect of house prices on employment, Adelino et al. (2015) document the intermediary step, by showing the effect of house prices on home equity refinancing and on home equity line of credit loans. For business loans they consider commercial and industrial loans reported in bank Call Reports. However, for these they do not find a statistically significant effect. While arguing for the same collateral effect of Adelino et al. (2015), I augment their analysis by considering small business loan originations in the CRA business loan data. I show that unlike in bank Call Reports, for the CRA business loan originations, we find similar effects than Adelino et al. (2015) find for the home equity loans.

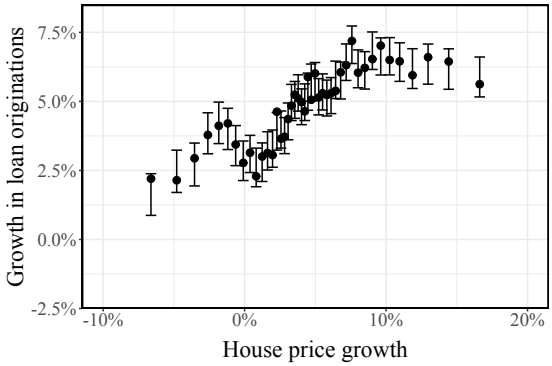
Figure 3: Collateral channel, business loans, and business activity

Panel A plots Cattaneo et al. (2024) binscatter between house prices and business loan originations ($\leq$ \$250,000 at origination). The left-hand side graph uses county-level data, whereas the right-hand side figure uses bank-county-level data during 2000 – 2022. House price data is from Zillow, and business loan originations are from Community Reinvestment Act data provided by the U.S. Federal Financial Institutions Examination Council. Panel B shows similar binscatters documenting the relation between small business loan originations and county-level annual growth rates in the number of establishments and employment in County Business Patterns data. The establishment and employment counts exclude residential construction and real estate sectors.

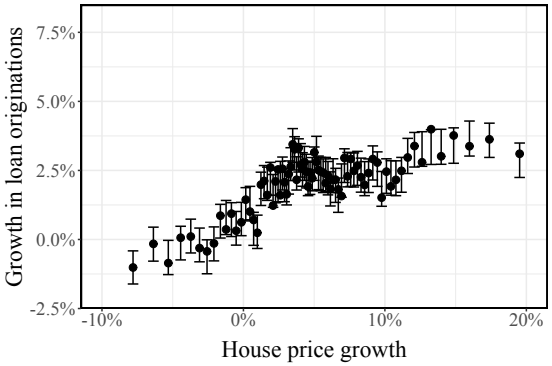
Panel A

House prices and small business loan origination

(i) County panel



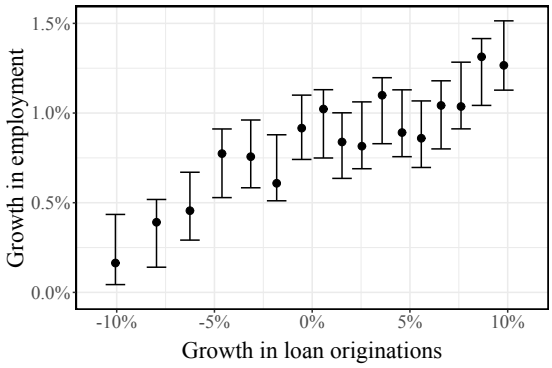
(ii) Bank-County panel



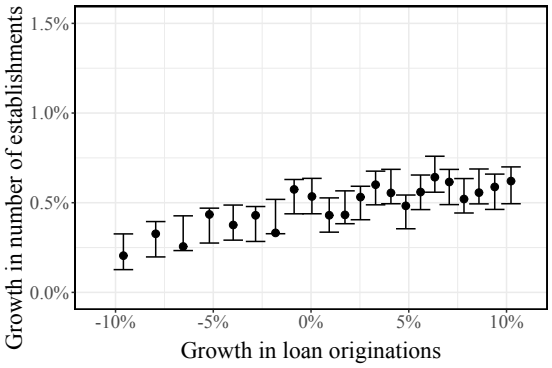
Panel B

Small business loans origination and business activity

(i) Employment growth



(ii) Growth in number of establishments



C.2.1 House prices and small business loans

In Panel A of Figure 3, I plot the Cattaneo et al. (2024) binscatter between growth rates in small business loan originations ($\leq$ \$250,000 at origination) and house prices. On the left, I depict this across U.S. counties using county-level panels from 2000 to 2022. On the right, I leverage the bank-level data and depict the binscatter using the bank-county level panel data from 2000 to 2022. The figure demonstrates a strong correlation between these variables, which is consistent with the findings of Adelino et al. (2015). I validate this relationship using two different regression specifications. First, I consider a county-year-level regression reported in Table 1. I instrument the house price growth rates with the IV strategy of Chaney et al. (2012), interacting the MSA-level Saiz (2010) land topography housing supply elasticity measure with the 30-year mortgage rates. The first stage of the IV specification is given by:[7]

$$\text{House price growth}_{it} = \alpha_j + \lambda_t + \rho\, \text{Elasticity}_k \times \text{Mortgage rate}_t + \gamma' X_i + \varepsilon_{it}, \quad \text{(C.1)}$$

where α_j are state fixed effects, λ_t are year fixed effects, housing supply elasticity is measured at the MSA-level (k), and X_i is a vector of controls, including the county-level share of the white population, homeownership rate, share of the population in the labor force, unemployment rate, and share of the population with a college degree, all measured in 2010. The regression is weighted by each county's 2010 (log) population, and standard errors are clustered at the state level.

I specify the second state of the IV regression as follows:

$$\text{Loan origination growth}_{it} = \theta_j + \delta_t + \beta\, \text{House price growth}_{it} + \nu' X_i + \varepsilon_{it}, \quad \text{(C.2)}$$

where θ_j are state and δ_t year fixed effects. In both regressions, I restrict the sample between the 10th and 90th percentiles of loan growth rates to remove infinite (or otherwise large) growth rates stemming from small counties with zero loan originations in any particular year.

The OLS regressions re-establish the strong correlation in Panel A of Figure 3, whereas the IV setup suggests a large elasticity of 1.91. Notably, this estimated elasticity has the same order of magnitude as reported by Adelino et al. (2015) for home equity refinancing (1.60) and for cash-out HELOC loans (1.72).

[6] Especially in the bank-level data, there are many bank-county pairs with zero observations in some particular years. These lead to infinite growth rates, which are removed by this sample restriction, which is applied throughout the paper.

[7] All growth rates are measured as log differences.

Table 2: Small business loan originations and house prices at the bank level

The table reports the results for an annual bank-county-level regression of growth in small business loan originations ($\leq$ \$250,000 at origination) on house price growth rates during 2000 – 2022. The loan origination counts are from Community Reinvestment Act data and house prices from the Zillow house price indices at the county level. House price growth is instrumented by interacting the Saiz (2010) housing supply elasticity measure with the 30-year mortgage rate. Standard errors are clustered at the bank level, and the regressions are weighted by bank-level (log) small business loan origination counts. Controls include the county-level share of the white population, homeownership rate, share of the population in the labor force, unemployment rate, and share of the population with a college degree, all measured in 2010. The loan growth data is restricted only to include loan growth rates between the 10th and 90th percentiles to remove outliers and infinite growth rates (associated with zero originations).

	Dependent Variable: Growth in loan originations			
	OLS		*IV (2SLS)*	
	(1)	(2)	(3)	(4)
House price growth	0.082***	0.082***	1.00**	1.03***
	(0.023)	(0.023)	(0.394)	(0.349)
Bank FE	Yes	Yes	Yes	Yes
State FE	Yes	Yes	Yes	Yes
Year FE	Yes	Yes	Yes	Yes
Controls	No	Yes	No	Yes
Observations	1,002,521	1,002,521	491,509	491,509
R^2	0.088	0.088		

Clustered (bank) standard-errors in parentheses
*Signif. Codes: ***: 0.01, **: 0.05, *: 0.1*

To provide further confidence on the large estimated elasticity from house price growth rates to business loan originations, I augment this analysis by considering the CRA bank-level panel data on small business loan originations, with the second stage regression specified as

$$\text{Loan origination growth}_{it} = \eta_i + \theta_j + \delta_t + \beta\,\text{House price growth}_{lt} + \nu' X_l + \varepsilon_{it}, \quad \text{(C.3)}$$

where η_i are bank, θ_j state and δ_t year fixed effects, house price growth rates are measured at the county level (l), and X_l is the vector of county-level controls, as before. This specification estimates the within-bank elasticity, which excludes the effects on the extensive margin when new banks enter and exit the market during the business cycle. Therefore, we should expect the estimated elasticity to be lower than

in the county-level analysis, as (C.3) reflects only the intensive margin effects. On the other hand, including bank fixed effects in the regression should provide further confidence in the strong connection between collateral valuations and business loan originations.[8]

I report the results of this regression in Table 2; column (3) documents the elasticity of a 1.1 (log) point increase in the growth of within-bank loan originations for a one (log) point increase in the house price growth rate. Column (4) of the table demonstrates the robustness of the result for including a set of county-level controls. The drop in the elasticity from 1.9 to 1.1 between the county- and bank-level analyses suggests a substantial role for the extensive margin as new banks enter and exit the small business loan ($\leq$ \$250,000 at origination) market during the business cycle.

C.2.2 Small business loans and business activity

The second key component of the collateral channel concerns the effect of business loans on business activity, which I measure by employment and number of establishments at the county-year level. Panel B of Figure 3 demonstrates a strong correlation between business loan origination and both measures of business activity.

In section C.2.1, I argued that house prices have a robust aggregate effect on business loan originations, consistent with existing micro-level evidence on collateral valuations' role in entrepreneurial financing and investments (Schmalz et al., 2017; Chaney et al., 2012; Bahaj et al., 2020). In this case, the validity of the housing supply elasticity instrument for house price growth rates - argued, e.g. by Mian and Sufi (2011) - should suggest that the supply elasticity measure can also instrument business loan originations. Thus, I consider the correlations of panel B of Figure 3 in regression setups using the housing supply elasticity as an instrument. In my analyses, I exclude real estate and residential construction sectors to ensure that the specifics of the housing boom-bust cycles during my study period do not drive my results and to alleviate some of the concerns raised by Kerr et al. (2022).

[8] To implement this regression, I restrict the data by only including observations, where the loan growth rate is between the 10th and 90th percentiles.

Table 3: Business loan originations and employment

The table reports the results for an annual county-level regression of growth in employment on growth in small business loan originations ($\leq$ \$250,000 at origination) during 2000 – 2022. The loan origination counts are from Community Reinvestment Act data. Employment information collected from County Business Patterns excludes construction and real estate sectors. I use the Zillow house price indices at the county level to control house prices. Growth in loan originations is instrumented by interacting the Saiz (2010) housing supply elasticity measure with the 30-year mortgage rate. Standard errors are clustered at the state level. The regressions are weighted by county-level (log) population in 2010. The loan and employment growth rates are restricted between the 10th and 90th percentiles.

	Dependent variable: Employment growth						
	Reduced form	*OLS*			*Instrumental variable (2SLS)*		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Housing supply elasticity	-0.0005***						
× 30-year mortgage rate	(9.02×10^{-5})						
Growth in loan originations		0.016***	0.015***	0.013***	0.480***	0.472***	0.379***
		(0.003)	(0.003)	(0.003)	(0.099)	(0.107)	(0.136)
House price growth			0.073***	0.069***		0.016	0.028
			(0.008)	(0.008)		(0.022)	(0.023)
State FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Year FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Controls	No	No	No	Yes	No	No	Yes
Observations	9,244	23,673	23,673	23,673	9,244	9,244	9,244
R^2	0.156	0.077	0.085	0.101			

Clustered (state) standard-errors in parentheses
*Signif. Codes: ***: 0.01, **: 0.05, *: 0.1*

Table 4: Business loan originations and number of establishments

The table reports the results for an annual county-level regression of growth in number of establishments on growth in small business loan originations ($\leq$ \$250,000 at origination) during 2000 – 2022. The loan origination counts are from Community Reinvestment Act data. Information on the number of establishments collected from County Business Patterns excludes construction and real estate sectors. I use the Zillow house price indices at the county level for house prices. Growth in loan originations is instrumented by interacting the Saiz (2010) housing supply elasticity measure with the 30-year mortgage rate. Standard errors are clustered at the state level. The regressions are weighted by county-level (log) population in 2010. The loan and establishment growth rates are restricted between the 10th and 90th percentiles.

	Dependent variable: Growth in number of establishments						
	Reduced form	*OLS*			*Instrumental variable (2SLS)*		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Housing supply elasticity × 30-year mortgage rate	-0.0003*** (7.84×10^{-5})						
Growth in loan originations		0.014*** (0.001)	0.013*** (0.001)	0.012*** (0.001)	0.375*** (0.093)	0.380*** (0.103)	0.276** (0.123)
House price growth			0.050*** (0.004)	0.048*** (0.004)		-0.008 (0.023)	0.009 (0.023)
State FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Year FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Controls	No	No	No	Yes	No	No	Yes
Observations	12,903	35,385	35,385	35,385	12,903	12,903	12,903
R^2	0.280	0.166	0.178	0.207			

Clustered (state) standard-errors in parentheses
*Signif. Codes: ***: 0.01, **: 0.05, *: 0.1*

Loan originations and employment. I regress the annual growth rate in employment on the growth in small business loan originations ($\leq$ \$250,000 at origination) across U.S. counties during 2000 – 2022:

$$\text{Employment Growth}_{it} = \theta_j + \delta_t + \beta\,\text{Loan growth}_{it} + \nu' X_{it} + \varepsilon_{it},$$

where δ_j are state and θ_t year fixed effects, and the vector of controls X_{it} is as in section C.2.1, augmented with annual house price growth rates in some regression specifications reported in Table 3.

The first stage regression follows the structure of (C.1) and is given by[9]

$$\text{Loan growth}_{it} = \alpha_j + \lambda_t + \rho\,\text{Elasticity}_k \times \text{Mortgage rate}_t + \gamma' X_i + \varepsilon_{it}. \tag{C.4}$$

The results for this first stage are reported in column (2) of Table 1, excluding controls. We observe a statistically significant negative coefficient, documenting a negative association between housing supply elasticity and loan growth rates. This is consistent with the expectation of inelastic areas enjoying higher house prices and (collateralized) loan growth rates.

This IV regression of employment growth on loan growth rates documents an elasticity between 0.379 and 0.480, depending on the exact regression specification. In the OLS regressions of columns (3) and (4) of Table 3, house price growth rates are highly statistically significant when introduced as controls. Nevertheless, the coefficient on loan originations remains stable. Most notably, however, when instrumenting the loan origination growth rates with housing supply elasticity, the residual correlation of house prices for employment growth drops to zero. This provides further evidence for the results of Adelino et al. (2015), arguing that the collateral channel provides an essential source of external financing for small businesses, with aggregate implications for employment growth rates.

Note that the instrumented elasticity in columns (5) to (7) are substantially higher than the OLS estimates in columns (2) to (4) of Table 3. Similarly to the analyses in section 5, this is likely due to two reasons. First, for privacy reasons, the County Business Patterns data only provides "employment flags" showing the size class of the county-industry pair for such pairs, where there are not enough firms to maintain the anonymity of the data. In my analysis, which uses industry-level data to exclude the real estate and residential construction sectors, I encode these employment flags

[9] I restrict the loan and employment data between the 10th and 90th percentiles.

using the midpoint of the specified employment class. This produces measurement error generating attenuation bias.

Second, the OLS estimate likely reflects the fact that the Saiz IV captures the local average treatment effect (LATE) among the complier counties that respond to the housing supply elasticity "shock" (i.e. to the exogenous variation in supply elasticity). For example, through the lens of my theory, the house price increase is due to improved capital allocation, when entrepreneurs use the new collateral capacity associated with the credit relaxation (Duca, Muellbauer and Murphy, 2011; Levitin and Wachter, 2011) to purchase business capital. However, there may be natural cross-regional differences in business dynamism; high-growth areas, e.g., in San Francisco or New York, may have more entrepreneurial activity and, therefore, have a differential ability to benefit from such a credit shock compared to other low-supply elasticity areas such as Fort Lauderdale or Los Angeles. In such a case, the IV specification only captures the local average treatment effect among those complier counties that, in reality, have enough entrepreneurs who can benefit from the credit relaxation. In particular, this implies that we should interpret these estimates with cautiousness, as the large local average treatment effect does not necessarily imply a large average treatment effect.

Loan originations and business creation. In addition to employment, we can consider the effect of loan originations on business creation measured by the annual growth rate in number of establishments across U.S. counties:[10]

$$\text{Growth in \# of establishments}_{it} = \theta_j + \delta_t + \beta\,\text{Loan growth}_{it} + \nu' X_{it} + \varepsilon_{it},$$

where θ_j and δ_t are again state and year fixed effects, respectively, and X_{it} is a vector of controls as in (C.2.1). The first stage for the IV regression is still given by (C.4).

I report the results in Table 4. The elasticity for the growth rate in the number of establishments with respect to the growth in small business loan originations is estimated between 0.276 and 0.375, depending on the exact regression specification (with or without controls). In broad terms, this is similar to the one for employment.

Notably, after instrumenting the loan growth rates with the housing supply elasticity, the residual effect of house prices on business creation is zero, again similar to the employment effects reported in Table 3. Finally, I note that the same caveats related to the substantial difference between the OLS and IV estimates, as discussed in relation to the employment effects, also apply here (cf. measurement error and

[10] Again, I restrict the loan data and the data on establishment growth rates between the 10th and 90th percentiles.

attenuation bias, and LATE estimates).

C.3 Empirical tests

This paper proposes a new mechanism for explaining how credit relaxations may disproportionately affect house prices. Relaxing a collateral credit constraint leads to increased aggregate productivity, which generates a positive house price effect via the general equilibrium demand channel. Therefore, a key testable prediction of the model is whether increased economic activity indeed generates a causal impact on house prices and, in particular, how strong that channel is quantitatively.

Existing literature suggests substantial income elasticity of housing demand (Davis and Ortalo-Magné, 2011). Therefore, we should expect economic growth to generate a large house price effect, given a fixed housing supply in the short run. In order to further quantify the combined effect of a collateral credit relaxation on economic growth and the effect of growth on house prices, I use County Business Patterns data on employment together with Zillow house price data, to estimate this impact. I exclude the real estate and residential construction sectors from the analysis to make sure none of the results are driven by the specifics of the housing boom-bust cycles during my study period of 2000 – 2022 (Kerr, Kerr and Nanda, 2022). I also restrict the employment and establishment growth rates between 10th and 90th percentiles to remove outliers and observations with infinite values. For a more thorough description of the data, I refer to Appendix C.

My analysis consists of two components. First, in Appendix C.2, I reproduce the results of Adelino, Schoar and Severino (2015) to produce an estimate of how house prices affect economic activity, measured by employment. Second, in section C.4, I use the Bartik instrument to estimate the elasticity of how an increase in employment affects house prices. I conclude in section C.5 by combining these estimates to evaluate the quantitative significance of the feedback mechanism. In all these empirical analyses, I interpret the business capital of my model to correspond to land in the data and use house prices as a proxy for land valuations.

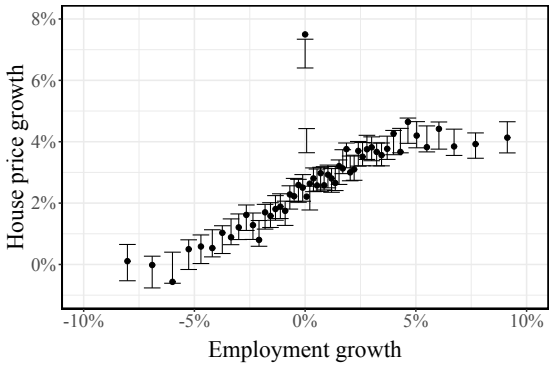
I note that the empirical contribution of this analysis relative to the existing literature is modest and this analysis is only provided to produce additional back-of-the-envelope evidence on the quantitative significance of my mechanism.[11]

[11]My modest empirical contribution here is two-fold: (i) while Adelino et al. (2015) document the effect of house prices on home equity refinancing and HELOC loans, they do not find effects on commercial and industrial lending in bank Call Reports data. I use the Community Reinvestment Act business loan data from 2000 – 2022 to augment their analysis and document similar elastic-

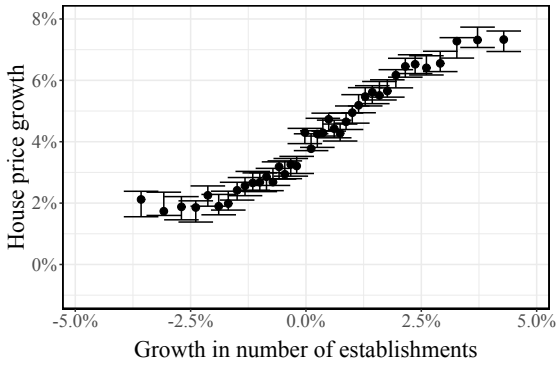
Figure 4: Business activity and house prices

The figure plots the Cattaneo et al. (2024) binscatter between annual growth in business activity (employment, number of establishments) and house price growth across U.S. counties during 2000 – 2022. House price data is from Zillow. Employment and establishments counts are from County Business Patterns data and exclude residential construction and real estate sectors.

(i) Employment growth



(ii) Growth in number of establishments



C.4 Business activity and house prices in the data

In Figure 4, I plot the Cattaneo et al. (2024) binscatters between two measures of business activity, annual county-level employment growth and growth in number of establishments, respectively, and house price growth rates. Both measures of business activity demonstrate a strong positive correlation with house price growth rates during 2000 – 2022.

I further test this relationship in a county-level annual panel regression of the Zillow house price index on employment growth, instrumented with the Bartik (1991) instrument, across U.S. counties during 2000 – 2022:

$$\text{House price growth}_{it} = \theta_i + \delta_t + \beta\, \text{Employment growth}_{it} + \varepsilon_{it},$$

where θ_i are county and δ_t year fixed effects. I weigh the regression with the county-level (log) population in 2010.

For the IV regression, I define the Bartik instrument in a standard manner, interacting the aggregate industry growth rates with county-level industry composition,

ities for business loan originations than Adelino et al. (2015) find for home equity loans in their analysis; (ii) I combine these effects of house price growth rates on employment with the reverse effect of employment on house prices using the Bartik instrument. This combination allows for producing a quantitative evaluation of the combined feedback mechanism. This augments earlier work by Liebersohn (2017), who considers the effect of "Bartik shocks" on house prices.

both measured annually:

$$\text{Bartik}_{it} \equiv \sum_{l=1}^{N} w_{il,t-1} g_{lt},$$

where industries (l) are 2-digit NAICS industries (excluding real estate and residential construction), g_{lt} is the nationwide employment growth rate in industry l between years $t-1$ and t, and $w_{il,t-1}$ is the employment share of industry l in county i in year $t-1$. The first stage regression now is given by

$$\text{Employment growth}_{it} = \alpha_i + \lambda_t + \rho \, \text{Bartik}_{it} + \varepsilon_{it},$$

where α_i and λ_t are county and year fixed effects, respectively.

Table 5: Employment growth and house prices

This table documents the results for a county-level regression of annual house price growth rates on county-level employment growth during 2000 – 2022. The growth rate in employment is instrumented with the Bartik instrument. The regressions are weighted by county-level (log) population in 2010. The employment growth rate is restricted between 10th and 90th percentiles.

	Dependent Variable:			
	Employm. growth	House price growth		
	First stage	*Reduced form*	*OLS*	*IV (2SLS)*
	(1)	(2)	(3)	(4)
Employm. Bartik	0.567***	0.344***		
	(0.075)	(0.074)		
Employm. growth			0.127***	0.606***
			(0.010)	(0.100)
County FE	Yes	Yes	Yes	Yes
Year FE	Yes	Yes	Yes	Yes
Observations	26,978	26,978	26,980	26,978
R^2	0.243	0.505	0.508	

Clustered (county) standard-errors in parentheses
*Signif. Codes: ***: 0.01, **: 0.05, *: 0.1*

I report the results in Table 5 and, respectively, for the analogous regression using establishment growth rates in Table 6. The instrumented elasticities are statistically highly significant and economically substantial, 0.606 for employment and 0.438 for establishment growth rates, respectively, for both measures of business activity.

Table 6: Establishment growth and house prices

This table documents the results for a county-level regression of annual house price growth rates on growth in the number of establishments during 2000 – 2022. The growth rate in the number of establishments is instrumented with the Bartik instrument. The regressions are weighted by county-level (log) population in 2010. The establishment growth rate is restricted between 10th and 90th percentiles.

	Dependent Variable:			
	Estab. growth	House price growth		
	First stage	*Reduced form*	*OLS*	*IV (2SLS)*
	(1)	(2)	(3)	(4)
Estab. growth Bartik	0.547***	0.240**		
	(0.047)	(0.118)		
Estab. growth			0.283***	0.438**
			(0.017)	(0.212)
County FE	Yes	Yes	Yes	Yes
Year FE	Yes	Yes	Yes	Yes
Observations	39,556	39,556	39,556	39,556
R^2	0.321	0.555	0.560	

Clustered (county) standard-errors in parentheses
*Signif. Codes: ***: 0.01, **: 0.05, *: 0.1*

The instrumented elasticity for employment is substantially higher than the OLS estimate. This is due to two reasons. First, for privacy reasons, the County Business Patterns data only provides "employment flags" showing the size class of the county-industry pair for such pairs, where there are not enough firms to maintain the anonymity of the data. In my analysis, which uses industry-level data to exclude the real estate and residential construction sectors, I encode these employment flags using the midpoint of the specified employment class. For details, I refer to section C.1.2, but note here that this does bring measurement error into the analysis, generating attenuation bias in the OLS estimate.

Second, the OLS estimate likely reflects the fact that the Bartik IV captures the local average treatment effect (LATE) among the complier counties that respond to the Bartik shock. For example, if the aggregate industry growth rate is driven by a demand shock in a particular 4-digit industry, it is plausible that other 4-digit industries within the same 2-digit industry would not respond to such a shock. Therefore, the industry exposure to the particular 2-digit industry is an imperfect measure of the county's exposure to such a shock, and not all counties with a high industry share in that 2-digit industry would respond to the shock similarly. The IV specification, therefore, only captures the local average treatment effect among those counties that, in reality, have true exposure to the particular 4-digit industry experiencing the shock.

Consequently, one should be cautious about how to interpret the results of this IV specification. In the following subsection, I use these estimates to produce a quantitative evaluation of the potential significance of my model mechanism. It should, however, be noted that this analysis is merely suggestive, and further empirical research testing my model mechanism is highly justified. In particular, the underlying assumption in using the LATE estimates in this analysis is that the economic shocks driving the house price fluctuations are broad-based enough to generate aggregate responses throughout the economy instead of merely affecting a small subset of complier industries.

C.5 Aggregation

The empirical analyses in sections C.2 and 5 aim to gauge whether the quantitatively substantial house price effects observed in section **??** are broadly consistent with the data. To make progress on this question, I provide a back-of-the-envelope calculation building on the reduced form estimates in Tables 5, 1, and 3.

I consider a collateral credit relaxation to reflect the relaxation of credit conditions in the early 2000s preceding the financial crisis. Indeed, Duca et al. (2011) document that the loan-to-value (LTV) ratios for mortgage originations increased from approximately 85% in 2000 to 95% in 2005. These LTV ratios, however, incorporate considerable heterogeneity. Levitin and Wachter (2011) document that a substantial fraction of mortgages originated at LTV ratios exceeding 100%, whereas An and Sanders (2010) show that for commercial mortgages, in particular, LTV ratios increased from the average of 73% pre-crisis to 82% during the peak of the boom. In particular, the exact calibration of the credit shock is complex without detailed microdata on the LTV ratios for the particular class of entrepreneurs who are the most relevant for my mechanism. Therefore, and for comparability with the existing literature, I follow Greenwald (2018) and consider a credit relaxation, where the LTV constraint is relaxed from 85% to 99%, i.e., a 16.5% increase in real estate collateral when assuming that loans are originated at the credit constraint (or at the relevant pricing discontinuity).

For the sake of argument, I assume that the existing market equilibrium is not supported after such an exogenous credit relaxation; I look for a new equilibrium where house prices have increased by x %. Given the shock, the total housing collateral has then increased by a factor of $1.165 \cdot (1 + x/100)$.

In Appendix C.2.1—cf. equation (C.2)—I consider the effect of house price shocks on business loan originations. I interpret the results of this analysis as describing how real estate collateral affects loan originations. I estimate an instrumented elasticity of 1.91, implying that the credit shock should (in this back-of-the-envelope calculation) imply a $1.91 \cdot 100 \cdot [1.165 \cdot (1 + x/100) - 1]$ percent increase in business loan originations. By column (5) of Table 3 this translates to $0.480 \cdot 1.91 \cdot 100 \cdot [1.165 \cdot (1 + x/100) - 1]$ increase in employment, which by column (4) of Table 5 implies $0.606 \cdot 0.480 \cdot 1.91 \cdot 100 \cdot [1.165 \cdot (1 + x/100) - 1]$ increase in house prices. Assuming that land, the price of which is proxied by house prices, is the relevant collateral for the entrepreneur, this must be consistent with the original assumption of a x % increase in house prices:

$$0.606 \cdot 0.480 \cdot 1.91 \cdot 100 \cdot [1.165 \cdot (1 + x/100) - 1] = x.$$

This linear equation is solved by $x = 25.9\%$, explaining 64.8% of the total 40.0% real house price increase preceding the financial crisis during Q1/2003 – Q1/2007.

While this back-of-the-envelope analysis is somewhat sensitive to the elasticities used in the calculation and is not robust for the Lucas critique, it demonstrates that

the estimated elasticities relevant to my house price mechanism are large enough to plausibly generate substantial house price effects.[12] This augments the quantitative evidence produced in section 5. These combined quantitative and reduced form empirical evidence suggest that the mechanism proposed in this paper may be useful for understanding how credit conditions affect house prices.

[12]For example, using the elasticities from the regressions with a full set of control variables (1.74 and 0.379 respectively for loan originations and employment growth), the house price increase associated with the above credit relaxation would amount to 12.3%, i.e., 30.8% of the total 40.0% real house price increase. Note that while estimating the price effects using nominal house price data, when estimated in log differences, price inflation is captured by the regression fixed effects, and the estimated elasticities govern real price effects.