Knowledge Network and Asset Pricing *

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Abstract

We develop a model with multiple sectors connected through both knowledge and physical input-output networks, in which network structures determine economic dynamics and constitute systematic risk in asset pricing. The model predicts that the sparsity of knowledge network increases aggregate consumption and leads to positive risk premium. Our model implications receive empirical support as follows: (i) consumption growth increases with changes in knowledge network sparsity; (ii) firms with higher exposures to knowledge network sparsity carry higher expected stock returns; and (iii) knowledge network sparsity exists in the stochastic discount factor and helps price the cross section of stock returns.

Keywords: Patents, intellectual property, input-output network, asset pricing, cross-section of stock returns
JEL Codes: G10, L10, O30

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1 Introduction

In modern economy, the specialization in production and heterogeneity in human and natural resources lead to large-scale, complex supply chains and cooperation network among firms (Hidalgo and Hausmann, 2009). With the advancement of technologies, firms nowadays are relying on more variety of intangible and tangible inputs than ever. From the aggregate perspective, the whole economy can be regarded as a network that encompasses various supply chains that may influence or intervene each other. The structure and disruptions within such a network may have a profound effect on all sectors of an economy (Acemoglu et al., 2012; Carvalho, 2014; Acemoglu and Tahbaz-Salehi, 2024).[1] Thus, production network plays an important role in economic fluctuations, and its structural changes may lead to systematic risk in financial markets.

Herskovic (2018) develops a multi-sector network model that aims to capture the input-output dynamics of different sectors in a production economy. He presents a competitive equilibrium and derives a closed-form solution in which consumption growth is a linear function of network sparsity and network concentration. In particular, consumption growth increases with changes in network sparsity and decreases with changes in network concentration; thus, changes in network sparsity are positively priced and changes in network concentration are negatively priced. These model predictions are supported by his empirical analyses based the input-output matrix.

However, existing input-output matrix data mainly cover tangible capital/inputs and miss the role of knowledge/intellectual property (IP) in production network (Robbins, Streitwieser, Jolliff, and Robbins, 2010). Such a miss likely biases the estimation of network-related systematic risk and weakens the explanatory power of network models for economic dynamics and financial markets. The IP is taking the lion's share in today's knowledge-based economy. At the aggregate level, IP-intensive industries contributed to nearly 41% of U.S. GDP and accounted for 33% of all U.S. employment in 2019.[2] At the firm-level, the market positions and competitive advantages of top US companies (such as Apple, Microsoft, Alphabet, Amazon, Nvidia, and Meta) are built on IP rather than real manufacturing. Also, Belo et al. (2022) show that the contribution of knowledge capital from innovation activities to a firm's market value increased from 24.9% in the 1970s to 44.8% in the 2010s, whereas the contribution of physical capital has declined from 30.9% to 20.7% in the same period. At the product level, although a smartphone consists of hundreds of chips

[1]For instance, the shortage of integrated circuit (IC) chips due to the outbreak of COVID-19 not only caused the delays in consumer electronics and automobile manufacturing but also pushed up inflation. https://www.forbes.com/sites/forbestechcouncil/2022/05/19/impacts-of-the-global-chip-shortage-and-how-to-prepare-as-the-backlog-stabilizes/?sh=4a66a18824d7

[2]These statistics are based on an official report "Intellectual property and the U.S. economy: Third edition" published by the USPTO: https://www.uspto.gov/ip-policy/economic-research/intellectual-property-and-us-economy

and elements, it also involves more than 250,000 active patents.[3]

Figure 1 highlights the differences between the knowledge network and the physical network within the two-digit North American Industry Classification System (NAICS) sectors. Panel (a) illustrates the network based on the U.S. input-output table for 2019, provided by the Bureau of Economic Analysis (BEA), while Panel (b) depicts an intersectoral network derived from the U.S. patent reference networks in 2019 (our measure for knowledge input-output network following Ziedonis (2004)). It's noteworthy that, even at two-digit sector level, firms frequently utilize knowledge from other sectors, and different sectors may play varied roles in two networks.[4] For instance, sectors like durable and nondurable manufacturing and finance are key in providing both physical and knowledge inputs. In contrast, real estate and enterprises' management play a significant role in physical input but not knowledge input, whereas sectors like healthcare and retail trade are crucial for knowledge provision but are not physical input suppliers.

In this paper, we construct a multi-sector model that accounts for both knowledge and physical inputs in the production network, present the equilibrium conditions, and discuss the roles of three network features (knowledge network sparsity, physical network sparsity, and network concentration) in consumption growth. Specifically, network concentration depends on the sectoral output shares, while knowledge (physical) network sparsity is a characteristic of the knowledge (physical) input-output network.[5] In this economy, a sector's production employs both knowledge and physical inputs produced by itself and other sectors, and at the same time, it produces both knowledge and physical outputs.

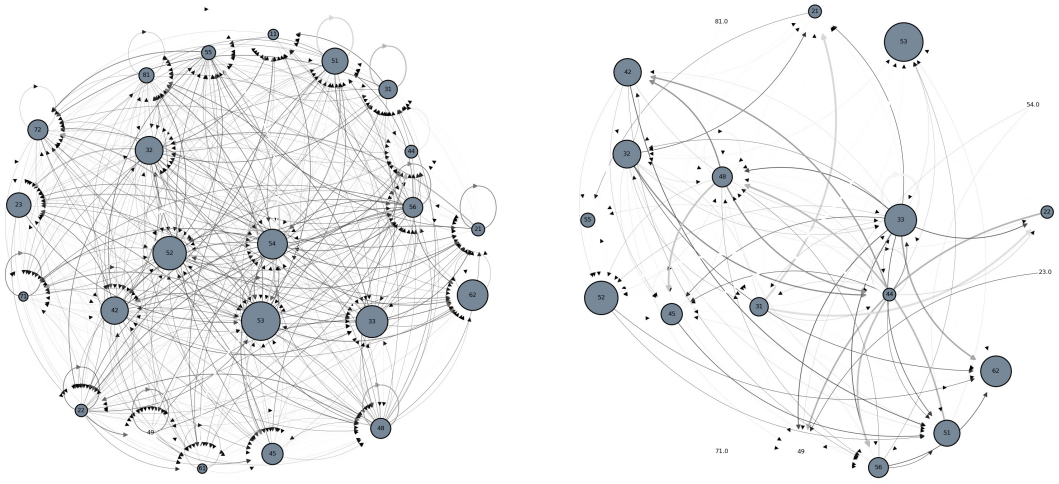
Our model deviates from Acemoglu et al. (2012) and Herskovic (2018), both of which only consider physical network. Adding knowledge network allows us to explicitly study the independent roles of knowledge production and the change of knowledge input-output network structure in generating aggregate fluctuations. As a result, our framework is able to generate additional asset pricing implications from the knowledge network. Moreover, some sectors are important in production whereas others are central in the technological front. For example, the energy sector is essential to the physical input-output network, but plays little role in the knowledge input-output

[3] https://www.bbc.com/news/business-15343549

[4] As per Acemoglu et al. (2012), we assess the outdegree of knowledge or physical input provision for each sector, which is calculated as the total of either knowledge or physical input shares across their customer sectors. It serves as an indicator of a sector's significance as a provider of physical or knowledge input. The Spearman ranking correlation between the outdegrees of the two networks is 0.212.

[5] The evolution of knowledge network can be characterized as a reduced-form combination of three sources of innovation. The first is 'creative destruction', where firms innovate and replace existing one made by other firms (Stokey, 1988; Segerstrom et al., 1990; Grossman and Helpman, 1991; Aghion and Howitt, 1992; Klette and Kortum, 2004). The second is that the incumbent firms improve their own technology without replacing products made by any other firms (Krusell, 1998; Lucas and Moll, 2014). Last, firms may introduce new ideas and push the technology frontier forward, as in Romer (1990), Acemoglu (2003), and Jones (2016).

Figure 1. Physical and knowledge network



(a) Physical network (b) Knowledge network

Note: Panel (a) displays an intersectoral network corresponding to the U.S. input–output table in 2019, provided by the Bureau of Economic Analysis (BEA). Panel (b) shows an intersectoral network corresponding to the U.S. patent reference networks in 2019, using the patent references to track knowledge inputs as in Ziedonis (2004). Each node (circle) represents a different sector at the two-digit North American Industry Classification System (NAICS) code level, with the size of nodes representing output shares. Node labels specify the two-digit NAICS sector. An arrow from sector i to j means that sector j provides physical inputs (or patents) to i; the intensity of the arrow (transparency) indicates the input share from sector i relatively to the total inputs (or patents) used by sector j.

network.

Our model suggests that consumption growth increases with both knowledge network sparsity and physical network sparsity. This is because that an exogenous increase in knowledge (physical) network sparsity reflects that an economy's input structure becomes more specialized, which brings in efficiency gain and more outputs. Consequently, firms' production costs reduce, leading to higher corporate profits, and higher aggregate consumption. Thus, the risks associated with knowledge and physical network sparsity are positively priced.

On the other hand, concentration reflects the degree to which economy's outputs are dominated by a few large sectors. In our model, the consumption share of each sector is fixed by households' preference. Thus, a positive shock to concentration has to come from the increased (decreased) usage of these large (small) sectors' products as inputs to other sectors. Given that the output elasticity of each input is less than one, total outputs drop and lead to a negative price of risk for the concentration factor.[6]

[6] As the increase in outputs due to using more inputs from the large sectors cannot compensate the loss in outputs

One major challenge for our empirical testing is the lack of comprehensive, publicly available data to measure a sector's knowledge output being used as an input in another sector's production. To overcome this challenge, we propose to use the frequencies of cross-citations and the economic values of patents from Kogan et al. (2017) to approximate the transfers of unobservable "licensing fees" across sectors.[7] We then use Compustat customer segment data to measure the physical input-output matrix.

We construct empirical proxies of three network risks, which are the changes in the levels of knowledge network sparsity, physical network sparsity, and network concentration for the sample period 1980-2019. The three network factors we construct carry a modest level of autocorrelation. Consistent with our model implication, consumption growth is significantly and positively related to knowledge network sparsity and physical network sparsity, and is significantly and negatively related to network concentration. This finding offers empirical evidence for our three network risks being systematic in financial markets.

We then implement asset pricing tests to substantiate the roles of network risks in expected stock returns. We find a monotonically increasing pattern in average returns and alphas in tercile portfolios sorted by stocks' prior exposures to knowledge or physical network sparsity. More importantly, the average monthly returns and alphas on the high-minus-low portfolios based on knowledge (physical) network sparsity exposure are in the range between 0.26% and 0.35% (0.28% and 0.41%) with statistical significance. On the other hand, we find a monotonically decreasing pattern in average monthly returns and alphas in tercile portfolios sorted by stocks' prior exposures to network concentration. In addition, the average returns and alphas on the high-minus-low portfolios are in the range between -0.40% and -0.46% with statistical significance.

Two empirical findings are worth further discussion. First, the economic magnitude of the high-minus-low portfolio alphas is fairly close to that of its average returns. Thus, our network risks have low correlation with conventional risk factors. Second, our estimation of risk exposures includes all three network factors together, which ensures that the exposures to three network factors are diagonal to each other. Overall, our sorted portfolio analyses confirms the unique role of network-related risk, especially that of knowledge network.

due to using fewer inputs from the small sectors (because the output elasticity of each input is less than one). Our finding is consistent with that of Richmond (2019), who shows that countries that are important in international trade network have lower interest rates and lower currency risk premia compared to countries with limited engagement in global trade.

[7] It is common in the literature to use patent references to track knowledge inputs (Ziedonis, 2004). The use of reference data is appropriate due to (i) the territorial principle of U.S. patent laws; (ii) the necessity of an active patenting strategy in today's fierce patent competition (Kortum and Lerner, 1998; Hall and Ziedonis, 2001; Hall, 2004); and (iii) the laws and incentive designs that ensure the relevance and comprehensiveness of patent references that have legal implications (Trajtenberg, 1990; Lanjouw and Schankerman, 2001; Von Graevenitz et al., 2011, 2013; Hsu et al., 2022).

To further connect our model to data, we derive the model's implication on the cross-section of expected stock returns using a simplified, two-period model. In this model, we are able to approximate stock values and expected stock returns in the form of exposures to knowledge and physical network sparsity as well as network concentration. We further show that these model-implied betas can explain empirical betas, which validates the cross-sectional implication of our model.

To substantiate our proposition that network risks serve as systematic risk and are priced in equity markets, we estimate a linear stochastic discount factor (SDF) that includes all three network factors using the GMM test of Cochrane (1996, 2001).[8] Our primary test asset set includes the Fama-French 30 industry portfolios, 25 size-B/M portfolios, 25 size-investment portfolios, 25 size-profitability portfolios, and 25 size-momentum portfolios – all are fairly standard stock portfolios in the literature.[9] We find that the loading on the knowledge network sparsity factor is significantly positive. More importantly, the knowledge sparsity factor has distinct explanatory power for stock returns and reduces pricing errors when it is added to the SDF. Overall, our GMM results confirm our model implications on systematic network risk and highlight the unique and important role of knowledge network.

This study adds to the recent development in production network research. This field primarily explores how a micro shock is propagated through production network and shape aggregate outcomes and firm distribution.[10] Our contribution to this body of work is twofold. First, we shift the focus from the typical analysis of production networks' influence on the real economy, such as growth volatility and innovation distribution, to asset pricing. Second, we integrate a knowledge network with the physical network, which enables the interplay between these two input sources to shape economic dynamics. In particular, we embed knowledge/IP in a production model based on multi-sector intermediary inputs, as in Cai and Li (2019) and Liu and Ma (2021).[11] Thus, we present a more general, simplified model to characterize the knowledge network's progression

[8]This method enables us to interpret the effect of knowledge network risk in expected stock returns in a standard, straightforward way, such as Lewellen et al. (2010), Papanikolaou (2011), Kogan and Papanikolaou (2014), Belo et al. (2017), and Lin et al. (2020).

[9]To ensure that our results are not sensitive to specific test asset set, we consider two alternative test asset sets: first, we replace Fama-French 30 industry portfolios with SIC two-digit portfolios. Second, we consider a smaller test asset set that includes Fama-French 10 industry portfolios and 4 sets of 2-by-3 double sorted portfolios (6 size-B/M portfolios, 6 size-investment portfolios, 6 size-profitability portfolios, and 6 size-momentum portfolios).

[10]See, for example, Jovanovic (1987), Gabaix (2011), Foerster et al. (2011), Acemoglu et al. (2012), Carvalho and Gabaix (2013), Di Giovanni et al. (2014), Johnson (2014), Barrot and Sauvagnat (2016), Baqaee and Farhi (2019), Huo et al. (2019), Bigio and La'o (2020), Herskovic et al. (2020), Carvalho et al. (2021), Foerster et al. (2022), and among others.

[11]Cai and Li (2019) build upon the Schumpeterian growth model as in Klette and Kortum (2004) and extend to a multi-sectoral framework. The extension allows to characterize the variation in R&D intensity across different sectors and its implication on economic growth. Liu and Ma (2021) incorporate a multi-sector setting into the endogenous growth model such as Romer (1990) and quantify the welfare gain/cost of R&D misallocation.

and its subsequent impact on asset pricing.

This study also adds to the literature on the role of technological innovation in financial markets in the following two ways. First, our model for a knowledge input-output network differs from prior models that treat technologies as a single component in aggregate production or a firm-level characteristic.[12] Second, we propose empirical measures for knowledge network to analyze the dynamics of different sectors' IP inflows and outflows, and connect those to risk premia in expected stock returns.

Moreover, we contribute to the asset pricing literature on supply chains and networks. Prior studies in this area examine how the cross-section of stock returns is related to industry concentration (Hou and Robinson, 2006), economic connections among firms (Cohen and Frazzini, 2008), network centrality (Ahern, 2013), and distance to end consumers (Gofman et al., 2020). However, prior studies in this literature focus on physical production and sales, and leave the role of IP in supply chains largely unexplored.

The remainder of the paper is organized as follows. Section 2 sets up our model. Section 3 discusses the closed-form solution for our model and its implication on asset pricing. Section 4 reports the data sources and describes how we construct network risk factors. Section 5 presents the empirical results for the roles of our proposed network risk factors in asset pricing. Section 6 explores the model's implication on the cross-section of expected stock returns. Section 7 concludes.

2 Modeling Networks of Knowledge and Physical Goods

In this section, we set up a parsimonious multisector model with both knowledge and physical input-output networks, present the equilibrium conditions, and discuss the role of the two networks. Firms in different sectors produce physical and knowledge inputs, trade them with each other, and thus form two networks to interconnect sectors. The representative household consumes and trades firms' stocks.

[12]The role of an aggregate technology component in asset pricing has been discussed in Greenwood et al. (1997), Hobijn and Jovanovic (2001), Pástor and Veronesi (2009), Hsu (2009), Garleanu et al. (2012), Papanikolaou (2011), Kogan and Papanikolaou (2014), and Kogan et al. (2020). Other studies focus on the role of firm-level innovation investment in asset pricing, such as Lin (2012), Cohen et al. (2013), Hirshleifer et al. (2013), Kung and Schmid (2015), Hirshleifer et al. (2018), and Bena and Garlappi (2020). Moreover, technological spillovers also explain stock returns (Lee et al., 2019; Lin et al., 2020). Given the important role of technological innovation and IP in investment, our paper is also extends the investment- and production-based asset pricing literature based on Cochrane (1991, 1996), Jermann (1998, 2010), Belo (2010), Papanikolaou (2011), Belo and Yu (2013), Belo et al. (2014a,b), and Kuehn and Schmid (2014).

2.1 Production of physical goods

Suppose there exist n sectors in our economy, each of which has a continuum of homogeneous firms on the interval $[0, 1]$. We assume that each firm in sector i (for convenience, we use the term 'firm i' to refer to a firm in sector i) uses a knowledge input collection x_{it} and a physical input bundle m_{it} to produce physical goods y_{it} with the following production technology:

$$y_{it} = z_{it} x_{it}^{\phi} m_{it}^{\psi}, \quad where \quad \phi + \psi < 1, \tag{1}$$

where z_{it} is the sectoral productivity,[13] and ψ and ϕ govern the knowledge and physical input share, respectively. Following Herskovic (2018), we assume $\phi + \psi < 1$. It is straightforward to extend the model by incorporating other primary factors, such as labor, land, and capital. The current specification can be interpreted as a case where these inputs are also used with a share of $1 - \phi - \psi$, but with a fixed supply. Moreover, the model can be extended to treat these inputs as *endogenous* variables. For instance, Section A of the Internet Appendix presents qualitatively similar asset pricing implications when we incorporate labor into the production technology and add leisure to the representative household's utility.

It is well documented in the literature that endogenous innovation and knowledge diffusion interact within a representative industry, determining the shape of the productivity distribution and generating aggregate growth, for example, see Jovanovic and Rob (1989), Acemoglu et al. (2007), Luttmer (2012), Lucas and Moll (2014), Perla and Tonetti (2014), Buera and Oberfield (2020), Benhabib et al. (2021), König et al. (2022), and among others. As in Liu and Ma (2021) and Cai and Tian (2023), we extend the intra-industry interaction to a multisectoral setting and allow firms to apply their own knowledge and adopt some from firms in other sectors. Specifically, suppose that firm i adopts knowledge inputs x_{jit} from firm j, for $j \in \{1, 2, \ldots, n\}$, and the knowledge input collection to produce is formed as

$$x_{it} = \prod_{j=1}^{n} x_{jit}^{\theta_{jit}}, \tag{2}$$

where θ_{jit} governs the cost share of knowledge inputs adopted from the firm j with $\sum_{j=1}^{n} \theta_{jit} = 1$. Our setting here is also consistent with Akcigit et al. (2018), which find that most patents result from collaboration, and interacting with better inventors is highly associated with higher subsequent productivity.

Also, the physical input bundle m_{it} is composite of physical inputs m_{jit} purchased from firm j

[13] z_{it} captures components in total factor productivity other than technology advancement (Kung and Schmid, 2015; Kogan et al., 2017), such as frictions to resource reallocation (Restuccia and Rogerson, 2008; Hsieh and Klenow, 2009), markups (De Loecker et al., 2020; Corhay et al., 2020; Edmond et al., 2023), financial frictions (Bigio and La'o, 2020), and among others. For a thorough review, see the survey by Kogan and Papanikolaou (2019).

for $j \in \{1, 2, \ldots, n\}$ as

$$m_{it} = \prod_{j=1}^{n} m_{jit}^{\omega_{jit}}, \tag{3}$$

where ω_{jit} governs the share of physical input expenditure from firm j with $\sum_{j=1}^{n} \omega_{jit} = 1$. Note that we define the knowledge and physical input collections as in Equation (2) or (3) following standard practice in the network literature (see, e.g., Gabaix, 2011; Acemoglu, Carvalho, Ozdaglar, and Tahbaz-Salehi, 2012; Foerster, Sarte, and Watson, 2011, among many others). This specification allows sector productivity to be interpreted as the residual output unexplained by inputs, for which we can easily find its empirical counterpart—total factor productivity (TFP).

Suppose that the price of physical goods in sector i is p_{it}, and the price of knowledge inputs in sector i is q_{it}. Because firms' optimization problem is time-separable, thus it is equivalent to solving the problem period by period. In this case, given the unit cost of the knowledge input collection, denoted as $\bar{q}_{it}$, and the unit cost of the physical input bundle, denoted as $\bar{p}_{it}$, firm i chooses the knowledge input collection x_{it} and the physical input bundle m_{it} to maximize its dividends as

$$d_{it} = \max_{x_{it}, m_{it}} p_{it} z_{it} x_{it}^{\phi} m_{it}^{\psi} - \bar{q}_{it} x_{it} - \bar{p}_{jt} m_{it}. \tag{4}$$

The first-order conditions are given by

$$(x_{it}) \qquad \phi p_{it} z_{it} x_{it}^{\phi-1} m_{it}^{\psi} = \bar{q}_{it}; \tag{5}$$

$$(m_{it}) \qquad \psi p_{it} z_{it} x_{it}^{\phi} m_{it}^{\psi-1} = \bar{p}_{it}. \tag{6}$$

Thus, the physical outputs by firm i is given by

$$y_{it} = \left(z_{it} p_{it}^{\phi+\psi} \left(\frac{\phi}{\bar{q}_{it}} \right)^{\phi} \left(\frac{\psi}{\bar{p}_{it}} \right)^{\phi} \right)^{\frac{1}{1-\phi-\psi}}. \tag{7}$$

Moreover, taking the knowledge input prices $\{q_{jt}\}$ as given, we have the unit cost of the knowledge input collection for firm i as the minimal cost of the knowledge inputs to produce one unit of the knowledge input collection as

$$\bar{q}_{it} = \min_{\{x_{jit}\}} \sum_{j} q_{jt} x_{jit}, \quad s.t. \quad 1 = \prod_{j=1}^{n} x_{jit}^{\theta_{jit}}.$$

Solving the problem, we have the unit cost of the knowledge input collection for firm i as

$$\bar{q}_{it} = \prod_{j} \left(\frac{q_{jt}}{\theta_{jit}} \right)^{\theta_{jit}}. \tag{8}$$

Since the composition of the knowledge input collection is constant return to scale, we have the knowledge inputs used by firm i from firm j to make x_{it} units of the knowledge input collection as, for $\forall\, i \in \{1, 2, \dots, n\}$,

$$x_{jit} = \theta_{jit} \frac{\bar{q}_{it} x_{it}}{q_{jt}}. \tag{9}$$

Similarly, given the product prices $\{p_{jt}\}$, we have the unit cost of the physical input bundle as

$$\bar{p}_{it} = \prod_j \left(\frac{p_{jt}}{\omega_{jit}} \right)^{\omega_{jit}}, \tag{10}$$

and the physical inputs are needed by firm i from firm j as, for $\forall\, j \in \{1, 2, \dots, n\}$,

$$m_{jit} = \omega_{jit} \frac{\bar{p}_{it} m_{it}}{p_{jt}}. \tag{11}$$

Combining Equation (5) with (9), we have the firm i's expenditure on the knowledge inputs from firm j is a fraction ($\phi\theta_{jit}$) of the sales as

$$q_{jt} x_{jit} = \phi \theta_{jit} p_{it} y_{it}, \tag{12}$$

and the expenditure on the physical inputs from firm j is given by

$$p_{jt} m_{jit} = \psi \omega_{jit} p_{it} y_{it}. \tag{13}$$

Discussions on the prices of physical and knowledge inputs. The input share (θ_{ji} or ω_{ji}) affects the price index of input collections, as shown in Equations (8) and (10), respectively, not only as a weighting factor (the exponent on price p_j or q_j) but also as a share shifter (θ_{jit} or ω_{jit} in the denominator). As we will show later, some combinations of these share shifters determine how sparsity factors from knowledge or physical input networks affect consumption. Alternatively, one can normalize the input collection using the input shares (θ_{ji} or ω_{ji}) by setting $x_{it} = \prod_{j=1}^{n} \left(\frac{x_{jit}}{\theta_{jit}} \right)^{\theta_{jit}}$ or $m_{it} = \prod_{j=1}^{n} \left(\frac{m_{jit}}{\omega_{jit}} \right)^{\omega_{jit}}$. In this approach, the input shares serve only as a weighting factor in the price index of input collections, with the share shifter effectively removed, i.e., $\bar{q}_{it} = \prod_j q_{jt}^{\theta_{jit}}$ or $\bar{p}_{it} = \prod_j p_{jt}^{\omega_{jit}}$. As a result, we cannot directly derive the sparsity factors from either the physical or knowledge network. In this case, sectoral productivity can no longer be interpreted simply as a residual output, as it must account for the share shifter. When constructing sectoral productivity later in the empirical part, we need to use the TFP, adjusting for the share shifters from both physical and knowledge input networks. This adjustment implicitly reintroduces the two sparsity factors from these networks and yields the same three aggregate

factors and their relationship with consumption, as shown in Equation (34). Therefore, for our purpose of constructing risk factors empirically, this alternative modeling choice is equivalent to our current one.

2.2 Production of knowledge

We assume full depreciation of knowledge input in our main model, and discuss its accumulation later in this section. We adopt this simplification for closed-form solutions, but argue that this assumption is reasonably acceptable when the IP market is efficient (i.e., all knowledge input can be licensed) and/or given the fast obsolescence of new technologies. Firm i's R&D department transforms h_{it} units of physical goods into k_{it} units of knowledge with the following technology:

$$k_{it} = a_{it} h_{it}, \tag{14}$$

where a_{it} is sector i's knowledge productivity, measuring the efficiency of transformation. Here, in the same spirit of Benhabib et al. (2021) and Lucas and Moll (2014), we assume that a_{it} are exogenously given. Due to the linear technology, we have

$$q_{it} = \frac{p_{it}}{a_{it}}, \tag{15}$$

where p_{it} and q_{it} are, respectively, the prices of sector i's physical goods and knowledge inputs. Equation (15) implies the knowledge price is lower if it is more efficient to transform, i.e., a higher a_{it}.

Discussions on the flow or stock of knowledge. Knowledge is modeled here as a flow rather than a stock variable because, in our empirical analysis, the market value of knowledge is proxied by patent licensing fees. We adopt this simplified framework of knowledge production for a multisector model with minimal yet essential components ensures tractability while allowing us to derive meaningful and testable predictions that directly connect to empirical data. This approach also enhances the clarity of the underlying economic intuition, which is essential for both empirical validation and theoretical insight. However, our model can be extended to incorporate knowledge *accumulation*. In Section B of the Internet Appendix, we build on the framework of Liu and Ma (2021) and extend the model to allow sectoral knowledge productivity (a_{jt}) to grow *endogenously*. This extension aligns with endogenous/Schumpeterian growth theory, as in Romer (1990), Aghion and Howitt (2009), and Acemoglu (2009), which typically examine knowledge spillovers and sectoral interdependencies in long-run economic outcomes, though these aspects are beyond the scope of this paper. Most importantly, Section B of the In-

ternet Appendix shows that under reasonable assumptions, the predictions regarding knowledge network factors and their roles as systematic risks remain unchanged.

2.3 Evolution of knowledge networks

Sectors are interconnected through a knowledge input-output network, in addition to the physical input-output network used in Acemoglu et al. (2012) and Herskovic (2018). In particular, we characterize these two networks with the following two $n \times n$ matrices:

$$\Theta_t = \begin{bmatrix} \theta_{11t} & \dots & \theta_{1nt} \\ \vdots & \ddots & \vdots \\ \theta_{n1t} & \dots & \theta_{nnt} \end{bmatrix} \quad and \quad \Omega_t = \begin{bmatrix} \omega_{11t} & \dots & \omega_{1nt} \\ \vdots & \ddots & \vdots \\ \omega_{n1t} & \dots & \omega_{nnt} \end{bmatrix}. \tag{16}$$

Here, Θ_t is the matrix for knowledge network, and each element of Θ_t, i.e., θ_{jit}, is the expenditure share of knowledge inputs adopted by firm i from firm j as in Equation (2). Ω_t is the matrix for physical input network, and each element of Ω_t, i.e., ω_{jit}, is the expenditure share of physical inputs purchased by firm i from firm j as in Equation (3).

The evolution of matrix Θ_t can be treated as a reduced-form combination of three sources of innovation. Consider a simple case where only firm i innovates. First, firm i's knowledge may replace existing one made by other firms. This process is commonly known as 'creative destruction', which goes back to Schumpeter (1939), followed by Stokey (1988), Segerstrom et al. (1990), Grossman and Helpman (1991), Aghion and Howitt (1992), Klette and Kortum (2004), and among others.[14] In our setting, the creative destruction process can be captured by shifting some knowledge share from another firm(s) to firm i. In other words, $\Delta\theta_{iit} = -\Delta\theta_{jit} > 0$, for some $j \neq i$. Second, as argued by Krusell (1998) and Lucas and Moll (2014), the innovation may improve firm i's technology without displacing other firms. In this case, the share of knowledge used by firm i its own, i.e., θ_{iit}, increases whereas all other shares universally decline as $\sum_j \theta_{jit} = 1$. Third, firm i can introduce new ideas and push the technology frontier forward, as Romer (1990) pioneered the work, followed by some recent contributions by Acemoglu (2003) and Jones (2016). Even though we do not explicitly model innovation on this extensive margin, it is equivalent to a rise in θ_{iit}. Garcia-Macia et al. (2019) use the U.S. plant-level data and find that the second channel contributes the most to TFP growth. However, our paper aims to quantify the overall implication of the evolution of such a knowledge network on asset pricing rather than investigating how and why the network evolves. Thus, we take as given the evolution of the knowledge network as the

[14] For a thorough review on the effects of 'creative destruction' on macroeconomic activity, see the survey by Aghion et al. (2014).

combined effects of three channels.

2.4 Representative household

Suppose there exists a representative household with a CRRA Bernoulli preference as

$$U(C_t) = \frac{C_t^{1-\eta} - 1}{1-\eta}, \tag{17}$$

where η measures risk aversion and C_t is a consumption bundle. Given the price index and the dividends, shareholding, and value of firms, the household chooses the consumption bundle and the shareholding at $t+1$ to maximize their utility, subject to the budget constraint, as

$$\max_{C_t, \{s_{i,t+1}\}} \sum_{\tau=0}^{\infty} \beta^\tau \mathbf{E}_t \left[U(C_{t+\tau})\right] \quad s.t. \quad P_t C_t + \sum_{i=1}^{n} s_{i,t+1}(v_{it} - d_{it}) = \sum_{i=1}^{n} s_{it} v_{it}, \tag{18}$$

where β is the discount factor, $\{d_{it}\}$ is defined in Equation (4), and $\mathbf{E}_t$ is the conditional expectation operator at time t. Solving the household's problem, the cum-dividend value of firm i, denoted by v_{it}, can be expressed recursively by

$$v_{it} = d_{it} + \mathbf{E}_t \left[\mathbf{M}_{t,t+1} v_{i,t+1}\right], \tag{19}$$

where $\mathbf{M}_{t,t+1} = \mathbf{E}_t \left[\beta \left(\frac{C_{t+1}}{C_t}\right)^{-\eta} \frac{P_t}{P_{t+1}}\right]$ is the pricing kernel that prices all assets in the economy. Moreover, the consumption bundle is a composite of goods from all sectors in a Cobb-Douglas form as

$$C_t = \prod_{i=1}^{n} c_{it}^{\zeta_i}, \tag{20}$$

where c_{it} is the quantity of consumed goods provided by firm i, and ζ_i governs the expenditure share with $\sum_i \zeta_i = 1$.[15] Also, the price index of the bundle is given by

$$P_t = \prod_{i=1}^{n} \left(\frac{p_{it}}{\zeta_i}\right)^{\zeta_i}. \tag{21}$$

It is straightforward to show that the expenditure of consumption on firm i's goods is given by

$$p_{it} c_{it} = \zeta_i P_t C_t. \tag{22}$$

[15] Notably, the expenditure share ζ_i is assumed to be time invariant.

2.5 Market clearing condition

We have three market clearing conditions. First, physical goods can be used as either consumption or inputs to create knowledge or physical inputs. Thus, all good markets clear as, for any $i \in \{1, 2, \dots, n\}$

$$y_{it} = c_{it} + h_{it} + \sum_{j=1}^{n} m_{ijt}, \tag{23}$$

where y_{it} is the supply of good i, c_{it} is consumed by household, h_{it} is input demanded by its R&D department, and m_{jit} is physical inputs used by firm j. Second, all knowledge markets clear as, for any $i \in \{1, 2, \dots, n\}$

$$k_{it} = \sum_{j=1}^{n} x_{ijt} \tag{24}$$

where k_{it} is the knowledge supply, and x_{ijt} is the knowledge adopted by firm j. Third, all asset markets clear as, for any $i \in \{1, 2, \dots, n\}$

$$s_{it} = 1. \tag{25}$$

This implies that the representative household owns all firms in equilibrium.

3 Equilibrium Analysis

In this section, we define the competitive equilibrium, and discuss the close-form solution and its implication on asset pricing.

3.1 Competitive equilibrium

We now define a competitive equilibrium as follows:

Definition 1 (Competitive equilibrium). *Given the sequence of sectoral productivity $\{z_{it}\}$ and knowledge transformation efficiency $\{a_{it}\}$, a competitive equilibrium is a sequence of good prices $\{p_{it}\}$, knowledge prices $\{q_{it}\}$, good supply $\{y_{it}\}$, consumption of goods $\{c_{it}\}$, inputs for knowledge $\{h_{it}\}$, physical inputs $\{m_{jit}\}$, knowledge supply $\{k_{it}\}$, knowledge demand by producers $\{x_{jit}\}$, firms' value $\{v_{it}\}$, firms' dividends $\{d_{it}\}$, share held by the household, $\{s_{it}\}$, consumption bundle C_t, and price index P_t such that, at any time t: for any $i \in \{1, 2, \dots, n\}$,*

1. *given $\{p_{it}\}$ and $\{q_{it}\}$, firm i chooses knowledge inputs $\{x_{jit}\}$ and physical inputs $\{m_{jit}\}$ to form the knowledge input collection x_{it} and the physical input bundle m_{it} and then maximizes dividends as in Equation (4);*

2. *given p_{it} and q_{it}, the R&D department of firms i transforms inputs into knowledge with the efficiency a_{it};*

3. *given $\{v_{it}\}$, $\{d_{it}\}$, and P_t, the representative household maximizes her utility as in Equation (18);*

4. *the good prices $\{p_{it}\}$ clear the good markets; the knowledge prices $\{q_{it}\}$ clears the knowledge markets; the stock-holdings $\{s_{it}\}$ clear the assets markets; and the price index P_t is normalized to 1.*

3.2 Solution

Now, we briefly discuss the solution of our model in the competitive equilibrium. First, using Equations (12), (13), (15), (22), and (24), sectoral output revenue can be expressed as

$$p_{it}y_{it} = \zeta_i C_t + \phi \sum_{j=1}^{n} \theta_{ijt} p_{jt} y_{jt} + \psi \sum_{j=1}^{n} \omega_{ijt} p_{jt} y_{jt}. \tag{26}$$

We then define the vector that stacks endogenous variables across sectors together as

$$\mathbf{x}_t = [\mathbf{x}_{1t}, \mathbf{x}_{2t}, \dots, \mathbf{x}_{nt}]^T, \quad for \;\; \mathbf{x} \in \{p,\; y,\; z,\; a,\; \zeta\}.$$

Thus, stacking Equation (26) across sectors, the vector of sectoral sales is given by

$$p_t \circ y_t = (\mathbf{I}_n - \phi\Theta_t - \psi\Omega_t)^{-1}\zeta C_t, \tag{27}$$

where $\circ$ is the Hadamard product, $\mathbf{I}_n$ is the $n \times n$ identity matrix, and Θ_t and Ω_t are network matrices defined in Equation (16). $(\mathbf{I}_n - \phi\Theta_t - \psi\Omega_t)^{-1}$ is the generalized version of the Leontief inverse matrix. This matrix can be rewritten as

$$(\mathbf{I}_n - \phi\Theta_t - \psi\Omega_t)^{-1} = \underbrace{\mathbf{I}_n}_{\text{direct effects}} + \underbrace{\phi\Theta_t + \psi\Omega_t + (\phi\Theta_t + \psi\Omega_t)^2 + (\phi\Theta_t + \psi\Omega_t)^3 + \dots}_{\text{indirect effects}}. \tag{28}$$

Clearly, not only does it capture the direct effects of change in consumption on sectoral output, but indirectly domino effects through either physical input-output network, knowledge networks, or interaction of the two. Then, we define a vector δ_t as

$$\delta_t = [\delta_{1t}, \delta_{2t}, \dots, \delta_{nt}]^T = (1 - \phi - \psi)(\mathbf{I}_n - \phi\Theta_t - \psi\Omega_t)^{-1}\zeta. \tag{29}$$

The ith element of the vector δ_t, i.e., δ_{it}, characterizes the relationship between the consumption bundle and sectoral sales. In particular, δ_{it} summarizes how any sectoral productivity shocks, improvement in knowledge transformation efficiency, or changes in either knowledge or physical input-output network directly or indirectly affect the consumption bundle and further the pricing kernel. Moreover, using Equations (5) and (6), we have

$$d_{it} = (1 - \phi - \psi) p_{it} y_{it}, \tag{30}$$

and the budget constraint by the representative household implies, we have, in equilibrium,

$$C_t = \sum_{i=1}^{n} d_{it} = (1 - \phi - \psi) \sum_{i=1}^{n} p_{it} y_{it}, \tag{31}$$

where the second equation is due to Equation (30). Combining Equation (27) with (31), we have the ith element of the vector δ_t is equal to the sales share, also known as Domar weights, as

$$\delta_{it} = \frac{p_{it} y_{it}}{\sum_{i=1}^{n} p_{it} y_{it}}. \tag{32}$$

This implies that the overall effects on the production side on the consumption growth are proportional to the sales share. A relatively large sector has more significant impacts on the consumption bundle.

Next, we want to account for the interactions among sectoral production and drive some aggregate factors that affect the pricing kernel. In doing so, we first combine Equation (7) with (8) and (10), and solve the production as a function of knowledge and good prices, respectively, as

$$y_{it} = \left(z_{it} p_{it}^{\phi+\psi} \left(\phi \prod_{j=1}^{n} \left(\frac{\theta_{jit}}{q_{jt}} \right)^{\theta_{jit}} \right)^{\phi} \left(\psi \prod_{j=1}^{n} \left(\frac{\omega_{jit}}{p_{jt}} \right)^{\omega_{jit}} \right)^{\psi} \right)^{\frac{1}{1-\phi-\psi}}. \tag{33}$$

Then, we take the logarithm on both sides of Equation (33), substitute $p_{it} y_{it}$ with Equation (27), and utilize the fact that the price index P_t is normalized to one.[16] The logarithm of the consumption bundle can be rewritten as

$$\begin{aligned} \log C_t \quad = \quad & constant - \sum_i \delta_{it} \log \delta_{it} + \frac{1}{1-\phi-\psi} \left(\phi \sum_i \delta_{it} \sum_j \theta_{jit} \log \theta_{jit} \right. \\ & \left. + \psi \sum_i \delta_{it} \sum_j \omega_{jit} \log \omega_{jit} + \sum_i \delta_{it} \log z_{it} + \phi \sum_i \delta_{it} \sum_j \theta_{jit} \log a_{jt} \right) \end{aligned} \tag{34}$$

[16] Please refer to Section C in the Internet Appendix for details.

Here, the logarithm of aggregate consumption evolves as the input-output linkage, knowledge network, sectoral productivity, and the knowledge transformation efficiency change. Proposition 1 describes the closed-form solution to consumption growth.

Proposition 1. *The aggregate consumption growth in equilibrium is given by*

$$\Delta \log C_{t+1} = -\Delta \mathcal{N}^{c}_{t+1} + \Delta \mathcal{N}^{sx}_{t+1} + \Delta \mathcal{N}^{sm}_{t+1} + \Delta \mathcal{N}^{e}_{t+1} \tag{35}$$

where Δ is the first-difference operator, and $\mathcal{N}^{c}$ is defined as the sectoral concentration, $\mathcal{N}^{s,x}$ and $\mathcal{N}^{s,m}$ are, respectively, the sparsities of the knowledge and physical networks, and $\mathcal{N}^{e}$ is the weighted average sectoral productivity, defined as, respectively,

$$\mathcal{N}^{c}_{t} = \sum_{i} \delta_{it} \log \delta_{it} \tag{36}$$

$$\mathcal{N}^{sx}_{t} = \frac{\phi}{1-\phi-\psi} \sum_{i} \delta_{it} \sum_{j} \theta_{jit} \log \theta_{jit} \tag{37}$$

$$\mathcal{N}^{sm}_{t} = \frac{\psi}{1-\phi-\psi} \sum_{i} \delta_{it} \sum_{j} \omega_{jit} \log \omega_{jit} \tag{38}$$

$$\mathcal{N}^{e}_{t} = \frac{1}{1-\phi-\psi} \sum_{i} \delta_{it} \left(\log z_{it} + \phi \sum_{j} \theta_{jit} \log a_{jt} \right). \tag{39}$$

Proof: please refer to Section C of the Internet Appendix.

3.3 Discussion and economic intuition

Proposition 1 implies that an exogenous increase in sectoral concentration leads to lower consumption while an exogenous increase in the sparsity of either knowledge or physical network leads to higher consumption. In this subsection, we discuss the economic intuition behind these implications.

Sparsity Before we discuss the economic intuition behind the positive relation between consumption and network sparsity, it is helpful to define the knowledge network sparsity of sector i as

$$\mathcal{N}^{sx}_{it} = \sum_{j=1}^{n} \theta_{jit} \log \theta_{jit}$$

and its physical network sparsity as

$$\mathcal{N}_{it}^{sm} = \sum_{j=1}^{n} \omega_{jit} \log \omega_{jit} .$$

Knowledge (physical) network sparsity measures the specialization of the knowledge (physical) inputs used in the production. If the production of sector i uses no specialized inputs, i.e., θ_{jit}'s (ω_{jit}'s) are evenly distributed across inputs, knowledge (physical) network sparsity is low. On the contrary, if the production of sector i uses specialized inputs, i.e., some sectors provides most of knowledge (physical) inputs, knowledge (physical) network sparsity is high. The overall knowledge (physical) network sparsity of the economy depends on the average knowledge (physical) network sparsity of the sectors, weighted by each sector's output share. Thus, the overall knowledge (physical) network sparsity is higher if sectors uses more specialized inputs on average, and if the sectors with higher output shares use more specialized inputs than those with lower output shares.

To illustrate the intuition behind why an increase in knowledge networksparsity leads to higher consumption, consider two knowledge input providers, j and l, for firms in sector i (we use firm i and l here for simplicity). Assume that provider j supplies a larger fraction of inputs than provider l, i.e., $\theta_{ji} > \theta_{li}$. Thus, the marginal rate of technical substitution between input x_{ji} and x_{li} is given by $\frac{\theta_{li} x_{ji}}{\theta_{ji} x_{li}}$. This indicates that for one unit less of input l, firm i needs to increase its usage of input j, by $\theta_{li}/\theta_{ji} x_{ji}$ units, to maintain the same level of output. Now, consider a case where, all else being equal, firm i shifts a small fraction of inputs from l to j, thereby increasing j's input share. Mathematically, for a small $\epsilon > 0$, we have $\theta'_{ji} = \theta_{ji} + \epsilon$ and $\theta'_{li} = \theta_{li} - \epsilon$. This change makes firm i's input usage more sparse. Consequently, for one unit less of input l, firm i now needs to increase its usage of input j by $\theta'_{li}/\theta'_{ji} x_{ji}$ units to maintain the same output. Since $\theta'_{li}/\theta'_{ji} < \theta_{li}/\theta_{ji}$, firm i incurs lower input costs to produce the same output, resulting in higher profits and thus more dividend distributions. As a result, consumption increases.[17]

Concentration Concentration measures the cross sector dispersion in terms of *output* share δ, while in contrast sparsity measures the dispersion in terms of *input* shares. An exogenous increase in network concentration indicates that the dispersion in output share becomes larger. That is, large sectors in an economy in terms of output share becomes larger while small sectors becomes smaller on average. Imagine a simple case where with $\delta_i > \delta_j$, sector i's output share goes up, sector j's goes down, and everything else stays the same. Clearly, network concentration of the economy goes up. The product of a sector is used either as consumption goods or as inputs

[17] Analogously, the same intuition can be applied to the response of consumption to a higher physical sparsity.

of other sectors (including itself). Given that household's expenditure share on each sector's product is fixed at ζ_i as in Equation (20), an increase in a sector i's output share has to come from the increase in the demand of its product as inputs. Equations (2) and (3) indicate that the sectoral production function is decreasing return to scale with respect to individual input, which implies that the marginal productivity of an input decreases with the quantity of that input, and more importantly, the speed of decrease increases with the quantity.[18] Consequently, all else equal, the gain from increasing the input with larger quantity (input from sector i in our example) cannot compensate the loss from decreasing the input with smaller quantity (input from sector j), resulting in net losses in output. Therefore, increase in network concentration leads to lower output and thus lower consumption, implying a negative price of risk for the concentration factor.

To put it simply, a positive shock to knowledge (physical) network sparsity increases the use of the more productivity inputs while a positive shock to network concentration does the opposite. Consequently, knowledge (physical) network sparsity increases while network concentration decreases the consumption and their prices of risk have opposite signs.

4 Data and Factor Construction

We collect US public firms' stock data from the Center for Research in Security Prices (CRSP), and their accounting data are from the Compustat fundamental dataset. Our sample includes NYSE/NASDAQ/AMEX common stocks. We consider stocks with exchange code being 1, 2, or 3, and share code being 10, 11, or 12. To avoid the concern of micro stocks, we only include stocks with a price above \$1 at portfolio formation. We use SIC two-digit industries as our sector unit.[19]

4.1 Back-to-envelope estimation

In order to empirically test our model implication, we need to construct empirical proxies for the levels of knowledge network sparsity ($\mathcal{N}_t^{sx}$), physical network sparsity ($\mathcal{N}_t^{sm}$), and network concentration ($\mathcal{N}_t^{c}$) in Proposition 1. We then use the changes in their levels (i.e., first-order

[18] Since at the equilibrium, the marginal productivity of each input must be the same. The differential impacts of the changes in input shares on output has to come from the second-order effect.

[19] We use Compustat historical SIC industry code first. If it is missing, we then use the available SIC industry code from CRSP.

differences) are our empirical proxies for three network risk factors:

$$\Delta\mathcal{N}_t^{sx} = \frac{\phi}{1-\phi-\psi}\left[\sum_i \delta_{it}\sum_j \theta_{jit}\log\theta_{jit} - \sum_i \delta_{it-1}\sum_j \theta_{jit-1}\log\theta_{jit-1}\right] \tag{40}$$

$$\Delta\mathcal{N}_t^{sm} = \frac{\psi}{1-\phi-\psi}\left[\sum_i \delta_{it}\sum_j \omega_{jit}\log\omega_{jit} - \sum_i \delta_{it-1}\sum_j \omega_{jit-1}\log\omega_{jit-1}\right] \tag{41}$$

$$\Delta\mathcal{N}_t^{c} = \sum_i \delta_{it}\log\delta_{it} - \sum_i \delta_{it-1}\log\delta_{it-1} \tag{42}$$

The key parameters for equations above follow the first-order conditions as in Equations (5) and (6) as

$$\phi_{it} = \frac{\sum_j q_{jt}x_{jit}}{p_{it}y_{it}} = \frac{sector\ i's\ expenditure\ on\ all\ patents\ in\ t}{sector\ i's\ total\ revenue\ in\ t}, \tag{43}$$

$$\theta_{jit} = \frac{q_{jt}x_{jit}}{\sum_l q_{lt}x_{lit}} = \frac{sector\ i's\ expenditure\ on\ sector\ j's\ patents\ in\ t}{sector\ i's\ expenditure\ on\ all\ patents\ in\ t}, \tag{44}$$

$$\psi_{it} = \frac{\sum_j p_{jt}m_{jit}}{p_{it}y_{it}} = \frac{sector\ i's\ expenditure\ on\ all\ physical\ inputs\ in\ t}{sector\ i's\ total\ revenue\ in\ t}, \tag{45}$$

$$\omega_{jit} = \frac{p_{jt}m_{jit}}{\sum_l p_{lt}m_{lit}} = \frac{sector\ i's\ expenditure\ on\ sector\ j's\ physical\ inputs\ in\ t}{sector\ i's\ expenditure\ on\ all\ physical\ inputs\ in\ t}. \tag{46}$$

4.2 Data for knowledge network

To measure knowledge network sparsity and its changes, we use the cross-references in the patent data. Patent data are arguably the best data source that allow researchers to have paper trails of knowledge transfers, which are analogous to the input-output table based on physical capital transfers, for the following three reasons. First, the territorial principle of U.S. patent laws requires inventors all around the world to file and register their patents in the U.S. to protect their intellectual property. Second, the establishment of the Court of Appeals for the Federal Circuit (a court that specializes in patent lawsuits) in 1982 and a few highly publicized patent lawsuits in the mid-1980s motivated firms to actively patent their intellectual property (Kortum and Lerner, 1998; Hall and Ziedonis, 2001; Hall, 2004). Finally, U.S. patent laws require patent applicants and examiners to list all prior patents that are closely related to claims of a patent application in the latter's reference based on "duty of candor and good faith". In other words, the reference list of a granted patent needs to be comprehensive in principle. Also, the procedures of patent examination ensure that the finalized reference list contains all relevant and only those relevant patents cited (Trajtenberg, 1990; Ziedonis, 2004). Thus, it is reasonable for us to assume that patent data contain most technologies and that the associated reference data track the flow of knowledge and

IP across sectors.

We collect patent and reference data from the Patentsview Database, which covers all patents granted by the USPTO. To measure a sector's use of knowledge flows from another sector (i.e., the former's expenditure on the latter's patents), we collect the economic value of each patent ("patent value") being assigned to every public firm from the patent value database of Kogan et al. (2017).[20]

Our key idea is to decompose a patent's value by its frequency of being cited by subsequent patents from different sectors: if a patent of a firm in sector A is worth \$1 million and it has been cited by 2 subsequent patents of firms in sector B and 8 subsequent patents of firms in sector C before its expiration date, the patent creates knowledge input of \$ 0.2 million (\$ 0.8 million) to sector B (sector C). In other words, sectors B and C will spend \$ 0.2 million and \$ 0.8 million on sector A's knowledge, respectively. A patent can also be cited by its creator and other firms in the same sector as its creator – in this case, the citation will be attributed as the knowledge used by the creator's own sector. This approach may be considered as following the concept of "licensing fees" – when a patent is referred by another subsequent patent, the former is utilized by the latter and can thus charge a licensing fee from the latter.[21] This analogy, albeit not perfectly realistic, offers us an intuitive way to capture the knowledge network across sectors.

We acknowledge that our approach has the following two limitations. First, we can only consider public firms' patents and citations from public firms' patents because we only have these firms' patent values (based on stock market reactions) and SIC codes for us to assign sectors and to calculate the weight factor. Second, it neglects the economic value a firm uses its patents in its own productions and products. Nevertheless, to our best knowledge, patent references offer us the best way to track knowledge flows across sectors.

We now discuss more details of our construction of the input-output data for knowledge network. For patent X granted in sector j filed in date τ, we assume that it generates knowledge

[20] We thank them for making the data publicly available at https://github.com/KPSS2017/Technological-Innovation-Resource-Allocation-and-Growth-Extended-Data. We use the nominal patent value in the data set. Under the efficient market hypothesis and the rational expectation hypothesis, a firm's market reaction to news should reflect the fair economic value associated with it. In particular, a patent's value is measured as a firm's market value change upon the the patent's grant announcement, which has been adjusted for return volatility, day-of-week fixed effect, and firm-year fixed effects. We drop patents assigned to multiple CRSP permnos.

[21] It is worth noting that there is no comprehensive licensing fee data (or even anything with acceptable coverage) for public firms to our knowledge. Thus, our use of references is an approximate for licensing fees based on the following logic: since references are closely related to claims (which define the protection scope of a patent), it is common for researchers often use reference list to construct measures for potential patent licensing, legal disputes, and infringement likelihoods (Trajtenberg, 1990; Lanjouw and Schankerman, 2001; Von Graevenitz et al., 2011, 2013; Hsu et al., 2022). We do not intend to claim that all patents in the reference list deserve licensing fees; nevertheless, under the law of large numbers, we can use the cross-referencing records to track licensing royalties across sectors.

inflow to other patents that cite it until its expiration (i.e., 20 years from its filing date). Such knowledge flow can be quantified as a charging a licensing fee per forward cite, which is its patent value divided by the number of forward citations from other patents until its expiration. Although we do not know all forward citations before a patent's expiration, we use a linearly extrapolation approach to calculate its forward citations in year t *before* its expiration date to prevent forward-looking bias.[22] Then, sector i's expenditure on sector j's patents in year t is summed licensing fees that all patents in sector j can charge all patents in sector i granted in year t. Also, sector i's expenditure on all patents in year t is the sum across all sectors.

4.3 Data for physical network and other parameters

Following Herskovic (2018), we estimate physical input-output data based on Compustat customer segment data.[23] WRDS provides a supply chain linking data of this customer-supplier relation based on Compustat segment data. The data includes the identity of both customers and suppliers as well as sales from each supplier to its customers. We only include unique customer-supplier pairs in each datadate.[24] The details are provided in Section D of the Internet Appendix.

In each year, following our discussion in Section 4.1, sector i's expenditure on all inputs is the sum of all sales from its suppliers, which include physical input from the Compustat customer segment data. In addition, sector i's expenditure on sector j's physical inputs is the sum of all sales from its suppliers in sector j.

[22]For example, if a patent was filled on January 1, 1991, its expiration date will be January 1,2011. Its number of forward citations in 1992 is based on the number of citations it received from patents filed up to 1992 and linearly extrapolate to January 1, 2011. Its number of forward citations in 2002 is based on the number of citations it received from patents filed up to 2002 and linearly extrapolate to January 1, 2011. Its number of forward citations in 2012 is based on the number of citations it received from patents filed up to January 1, 2011. The extrapolation could potentially make the number of forward citations as a fraction number instead of an integer. We use the largest integer below this fraction in this case.

[23]Regulation SFAS No. 131 requires firms to disclose the identity of any customer representing more than 10% of the total reported sales.

[24]The segment data is collected at the supplier level with datadate being suppliers' datadate. To identify the datadate for the customer, we keep the closest customer datadate to each supplier datadate among the three consecutive years of previous, contemporaneous, and next year centered around the year of supplier datadate. To avoid using future information, we set the year to be the later of the customer and supplier datadate for each customer-supplier pair. This treatment can potentially create an issue that one customer might have two different datadates for the same year. In this case, it would have different sales and potentially be in two different sectors if the company changes industry code from one year to another year. To avoid this complication, we only keep customers with the same industry code in a year (less than 0.5% observations are dropped) and take an average of the sales. To further exclude potential data errors in this data, we eliminate observations where total customer sales are more than total sales of the supplier reported in Compustat fundamental data. Further, to avoid the case where the number of companies is too few in a sector especially in earlier years, we require more than 5 companies each year for a customer sector when computing network factors.

4.4 Summary statistics

Other parameters for Section 4.1 are calculated as follows: δ_{it} is sector i's total revenue divided by the total revenue of all sectors. For ϕ (ψ), we use the average of all ϕ_{it} (ψ_{it}) using an expanding window until year t.

The above procedure leads to the level of the network concentration ($\mathcal{N}_t^c$), the knowledge network sparsity ($\mathcal{N}_t^{sx}$), and the physical network sparsity ($\mathcal{N}_t^{sm}$), in each year from 1979 to 2019. We present the summary statistics of them in Panel A of Table 1. Perhaps not surprising, all three levels reveal strong autocorrelation. We then take changes of them to form our three factors ($\Delta\mathcal{N}_t^{sx}$, $\Delta\mathcal{N}_t^{sm}$, and $\Delta\mathcal{N}_t^{c}$). As shown in Panel B of Table 1, these factors fluctuate around zero with modest autocorrelation, which are consistent with our intention to measure the innovation in our three network components. Their time series are presented in Figure 1.

4.5 Other data

We have collected other data from various sources: we obtain macro data from Fed St Louis including consumption, GDP, CPI, unemployment rate and total factor productivity (TFP). Consumption growth is the growth of seasonally adjusted real personal consumption in nondurable goods and services per capita. GDP growth is the growth of seasonally adjusted GDP per capita. Inflation is the growth of seasonally adjusted CPI, and unemployment growth is the growth of seasonally adjusted unemployment rate. In addition, we download asset pricing factors and portfolio returns from Fama and French data library.

We also consider residual TFP (RTFP) as the residual from regressing TFP growth on three factors following Herskovic (2018). Specifically, we obtain TFP data from Fed St Louis. We first compute TFP growth as year-over-year growth rate of TFP, then regress this TFP growth on three network factors, and take the residuals as RTFP. This factor helps us capture other productivity-related innovation that are diagonal to network factors.

5 Empirical Analyses

5.1 Network factors and consumption

The key feature of our model is that we derive the consumption growth to be a linear function of innovations in knowledge network sparsity, physical network sparsity, and network concentration as shown in Equation (35) of Proposition 1. These network factors lead to systematic risk in

asset pricing because consumption growth increases with both knowledge and physical network sparsity, and decreases with network concentration. Thus, our knowledge network sparsity factor $\Delta\mathcal{N}_t^{sx}$ and physical network sparsity factor $\Delta\mathcal{N}_t^{sm}$ are positively associated with consumption growth and are negatively related to marginal utility, which results in a positive price of risk. In addition, our concentration factor, $\Delta\mathcal{N}_t^{c}$, is negatively associated with consumption growth and positively related to marginal utility, which results in a negative price of risk.

To empirically test Proposition 1 and to justify how network factors reflect systematic risks, we estimate the following regression that is an empirical analogy of Equation (35):

$$\Delta \log C_t = b_0 + b_1 \Delta\mathcal{N}_t^{sx} + b_2 \Delta\mathcal{N}_t^{sm} + b_3 \Delta\mathcal{N}_t^{c} + \gamma' \mathbf{X}_t + \eta \Delta \log C_{t-1} + \epsilon_t, \tag{47}$$

where $\Delta \log C_t$ denotes consumption growth in year t. $\mathbf{X}_t$ denotes a set of macro control variables including GDP growth, inflation, and unemployment growth. All these variables have been defined in Section 4.5. The sample is annual from 1980 to 2019.

Table 2 shows that the knowledge and physical network sparsity factors have positive explanatory power for consumption growth, and that the network concentration factor has negative explanatory power for consumption growth – all consistent with our model predictions. Such a relation is robust to various model specifications when we include different sets of control variables. For instance, even when we include lagged consumption growth and several macro controls in Column (3), we still find significantly positive coefficients on $\Delta\mathcal{N}_t^{sx}$ and $\Delta\mathcal{N}_t^{sm}$ and a significantly negative coefficient on $\Delta\mathcal{N}_t^{c}$.

5.2 Network risk exposures and stock portfolios

To examine our model predictions on asset pricing, we measure firms' exposure to network risks using beta-sorted portfolios because our network factors are not return-based. The idea is that firms with higher exposures to knowledge or physical network sparsity factors are expected to provide higher expected stock returns; in addition, firms with higher exposures to the network concentration factor are expected to provide lower expected stock returns.

We first measure each stock's exposures (i.e., betas) using trailing-window regressions following the asset pricing literature. (Later in Section 6.2, we will validate our empirical betas by connecting it to model-implied betas based on a simplified model with a closed-form solution for betas.) In particular, at the end of June in each year t, a stock i's betas to each factor is estimated from regressing its annual excess returns on all factors using the following trailing window

regression over the past five years: for $\forall\, n \in \{1, \ldots, 5\}$,

$$r_{i,t-n} = \alpha + \beta_{\Delta\mathcal{N}^{sx},t}\Delta\mathcal{N}^{sx}_{t-n} + \beta_{\Delta\mathcal{N}^{sm},t}\Delta\mathcal{N}^{sm}_{t-n} + \beta_{\Delta\mathcal{N}^{c},t}\Delta\mathcal{N}^{c}_{t-n} + \beta_{RTFP,t}\mathbf{RTFP}_{t-n} + \epsilon_{it}, \tag{48}$$

where $r_{i,t-n}$ denotes stock i's annual excess returns in the past five years and α denotes the intercept term. The estimation of Equation (48) leads to $\beta_{\Delta\mathcal{N}^{sx},t}$ as stock i's exposure to the knowledge network sparsity risk, $\beta_{\Delta\mathcal{N}^{sm},t}$ as its exposure to the physical network sparsity risk, $\beta_{\Delta\mathcal{N}^{c},t}$ as its exposure to network concentration risk, and $\beta_{RTFP,t}$ as its exposure to TFP risk that captures productivity growth. We include all three network factors in our estimation of risk exposures to ensure that the estimated exposures to three network factors are diagonal to each other.

Next, at the end of June of year t, we sort all stocks into tercile groups based on their exposures to each of the three network factors.[25] We then construct a value-weighted portfolio in each group and track its return in each month from July of year t to June of year $t+1$. Finally, we take a long position in the high exposure portfolio and a short position in the low exposure portfolio. This high-minus-low (H-L) portfolio serves as a mimicking portfolio for each network risk, and its monthly returns approximate the risk premium of corresponding network risk.

We present the averages and t-statistics of the monthly returns on aforementioned portfolios in the row with the label Ret^e in Table 3. Panel A.1 is based on knowledge network sparsity beta ($\beta_{\Delta\mathcal{N}^{sx}}$), Panel A.2 is based on physical network sparsity beta ($\beta_{\Delta\mathcal{N}^{sm}}$), and Panel A.3 is based on concentration beta ($\beta_{\Delta\mathcal{N}^{c}}$). As shown in Panel A.1, the average monthly returns on the portfolios on high, middle, and low exposure to knowledge network sparsity are 0.53%, 0.69%, and 0.84%, respectively. The average monthly return on the H-L portfolio is as high as 0.31%, which is statistically significant with a t-statistic of 2.48.

In Panel A.2 for physical network sparsity, the average monthly returns on the high, middle, and low portfolios are 0.55%, 0.65%, and 0.90%, respectively. In addition, the average monthly return on the H-L portfolio is 0.35% with a t-statistic of 2.33. In Panel A.3 for network concentration, the high, middle, and low portfolios' monthly excess returns are 0.88%, 0.68%, and 0.48%, respectively. More importantly, the average monthly return on the H-L portfolio is -0.40% with a t-statistic of -2.64. The monotonic patterns in portfolio returns offer empirical support to our model implementation on positively priced sparsity risk and negatively priced concentration risk.

However, average returns and their spreads on the aforementioned portfolios have not been adjusted for common systematic risk. We thus regress the monthly excess return on each portfolio

[25] Following the asset pricing literature, we use a firm's prior exposures to risk factors as the best predictors for its risk exposures at the end of June in each year t. To ensure the segment data is publicly available in our asset pricing tests, we follow Fama and French (1992) timing to align year t factors with asset returns from July of year $t+1$ to June of year $t+2$.

on several common risk factors including the market factor (CAPM), Fama-French three factors (FF3) that further include the size factor and the value factor, Fama-French four factors (FF4) that further add the momentum factor, and Fama-French five factors (FF5) that add the investment and profitability factors to FF3. The alpha (in percentage) and corresponding t-statistic of each portfolio is presented in Table 3.

We again find monotonic patterns in alphas across high, middle, and low portfolios in each panel of Table 3 Panel A. In particular, the H-L portfolio's alphas of CAPM, FF3, FF4, and FF5 models are 0.32%, 0.35%, 0.26%, and 0.30% (with t-statistics of 2.48, 2.63, 2.08, and 1.98), respectively, in Panel A.1. A similar or even stronger result is also found in Panels A.2 and A.3. We also note that the economic magnitude of the alphas is fairly close to that of average returns, suggesting that network risk premiums have limited covariation with conventional risk factors. This finding confirms the unique role of network-related risk, especially knowledge network sparsity, in equity markets that cannot be explained by common systematic risk. Thus, our network model with both knowledge and physical capital delivers a new risk factor from the perspective of intellectual property.

To further visualize the return spreads of these beta sorted portfolios, Figure 2 plots the high-minus-low portfolio spreads sorted based on the knowledge sparsity beta, physical network sparsity beta, and the network concentration beta.

In addition to stock returns and alphas, we also examine firm characteristics of aforementioned beta-sorted portfolios. For each portfolio, we calculate the annual average of one of the following firm characteristics: market beta β_{CAPM}, market equity, book to market ratio, asset growth, operating profitability, sales-to-PP&E (Property, Plant, and Equipment) ratio, and R&D capital. We then calculate the time series average of each portfolio and compare the difference between the high and low portfolios. As shown in Table 3 Panel B, the high- and low-beta portfolios have fairly similar firm characteristics except for sales-to-PP&E ratio – we find that firms with high knowledge network sparsity beta have significantly higher sales-to-PP&E ratios.

Table 3 Panel B suggests the following: first, firms with high and low network betas are indifferent in major characteristics; thus, the return spreads presented in Panel A are not driven by those firm characteristics. Second, firms with high knowledge network sparsity betas are substantial in terms of market size: their average size of $3,159 million is commensurate to the average size of $3,029 million in the portfolio of low knowledge network sparsity betas. Finally, the higher sales-to-PP&E ratio of firms with high knowledge network sparsity betas is intuitive: when firms' revenues are much higher than their physical capital (measured by PP&E), they are likely to rely more on knowledge and IP capital and are thus more subject to systematic risk related to knowledge network sparsity.

5.3 Pricing of knowledge network risk

To further substantiate our proposition that knowledge network factors serve as new systematic risk, we estimate the following linear stochastic discount factor (SDF) that corresponds to Proposition 1:

$$m_t = 1 - b_{mkt} MKT_t - b_{sx} \Delta\mathcal{N}_t^{sx} - b_{sm} \Delta\mathcal{N}_t^{sm} - b_c \Delta\mathcal{N}_t^{c} - b_{RTFP} RTFP_t, \tag{49}$$

where b_{mkt}, b_{sx}, b_{sm}, b_c, and b_{RTFP} are the loadings of MKT_t (the market factor), $\Delta\mathcal{N}_t^{sx}$, $\Delta\mathcal{N}_t^{sm}$, $\Delta\mathcal{N}_t^{c}$, and $RTFP_t$, respectively. Equation (49) can be tested using the standard procedure as in Cochrane (1996) and Cochrane (2001), and is an asset pricing test of Equation (35) as SDF is a function of consumption growth in almost all utility functions in the asset pricing literature.[26] The significance of b_{sx}, b_{sm}, and b_c (the loadings on the three network factors factors) will inform us whether the three network factors indeed exist in the SDF. As suggested in our model, we expect positive b_{sm} and b_{sx} and negative b_c because the two network sparsity factors are positively priced and the network concentration factor is negatively priced. In particular, the loading on b_{sx} suggests if the knowledge network sparsity factor helps price stock portfolios in addition to the other two network factors.

To test Equation (49), we use a broad test asset set that includes the Fama-French 30 industry portfolios, 25 size-B/M portfolios, 25 size-investment portfolios, 25 size-profitability portfolios, and 25 size-momentum portfolios – all are fairly standard stock portfolios in the literature. We consider a broader set of test assets to relax the strong factor structure of FF25 portfolios mentioned by Daniel and Titman (2006) and Lewellen et al. (2010).

In Table 4, we present the estimated loadings of all factors following the GMM procedure of Cochrane (2001). We first notice that the loadings on $\Delta\mathcal{N}^{sx}$ are significantly positive and confirm that knowledge network sparsity risk is positively priced. Those on $\Delta\mathcal{N}^{sm}$ are significantly positive in all model specifications except model (4), supporting that physical network sparsity risk is positively priced. Finally, those on $\Delta\mathcal{N}^{c}$ are all significantly negative, indicating negative priced network concentration risk is negatively pried. All of our findings are consistent with our model predictions and Table 3.

As a measure of model fit, we report the annualized mean absolute pricing errors (MAE) in percentage, the JT-statistic (JT-Test), and the p-value associated with the over-identification test. The p-values of JT-statistics in model (1) to (4) are all close to zero, suggesting significant pricing errors (measured by the sum of squared pricing errors). On the other hand, the p-value of J-

[26] The GMM test of Cochrane (1996) and Cochrane (2001) is widely used in the literature (Lewellen et al., 2010; Papanikolaou, 2011; Kogan and Papanikolaou, 2014; Belo et al., 2017; Lin et al., 2020).

statistics in model (5), our full model, is 0.83, indicating insignificant pricing errors. This results confirm that our full SDF model that encompasses the market factor, three network factors, and the residual TFP can price our test assets reasonably well.

Finally, in the bottom row, we report the p-values for the difference of the J-statistics from comparing model (1) with model (2), comparing model (3) with model (4), and comparing model (5) with model (4). In the first two comparisons (1 vs. 2 and 3 vs. 4), p-values are 0.00 and suggest that the existence of three network factors indeed improves the SDF model fitting for our test assets. More importantly, in the last comparison, the fact that model (5) outperforms model (4) with a significant difference in the J-statistics (p-value = 0.00) again substantiates the unique role of knowledge network risk – its explanatory power for stock returns is diagonal to that of the other two network factors.

To ensure that our results are not sensitive to specific test asset set, we also replace Fama-French 30 industry portfolios with SIC two-digit portfolios and find consistent results in Table A1 in Section E of the Internet Appendix. In addition, we also consider a smaller test asset set that includes Fama-French 10 industry portfolios and 4 sets of 2-by-3 double sorted portfolios (6 size-B/M portfolios, 6 size-investment portfolios, 6 size-profitability portfolios, and 6 size-momentum portfolios) and find fairly consistent results in Table A2 in Section E of the Internet Appendix.

Overall, our test results suggest that knowledge network risk is indeed priced in stock markets in addition to physical network risk; more importantly, accounting for knowledge network risk significantly reduces the pricing errors. Our empirical evidence thus confirm our model implications on systematic risk related to knowledge transfers in supply chains.

6 Cross-sectional Implications

In this section, we attempt to develop the cross-sectional implications of our model using a two-period cash flow structure, which enables us to derive an analytical solution for sectoral exposures to network risk factors (i.e., model-implied betas). Using a three-sector model simulation, we show that the sectoral exposures derived from the simplified two-period cash flow model are consistent with those obtained from the infinite-horizon model. Finally, we apply the exposures from the simplified model to guide the construction of empirical counterparts, finding them in line with the empirical exposures estimated in Section 5.2.

6.1 Model-implied relation

To illustrate the intuition, we assume that the model only has two periods t and $t+1$, implying that $v_{i,t+1} = d_{i,t+1}$, for $\forall i$, in order to obtain analytical solution on the sectoral exposure to each risk factors. Thus, the gross return of the stocks for firms in sector i is given as[27]

$$R_{i,t+1} = \frac{v_{i,t+1}}{v_{it} - d_{it}} = \frac{d_{it}}{v_{it} - d_{it}} \frac{d_{i,t+1}}{d_{it}} \tag{50}$$

Thus, the net return is given by

$$r_{i,t+1} = \log \frac{d_{it}}{v_{it} - d_{it}} + \Delta \log d_{i,t+1} \tag{51}$$

Also, denote the log pricing kernel as $\mathbf{m}_{t,t+1} = \log \mathbf{M}_{t,t+1} = \log \beta - \eta \Delta \log C_{t+1}$. We can show that the expected excess return on sector i is given by

$$\begin{aligned} xr_{i,t+1} &\equiv \log \mathbb{E}_t[e^{r_{i,t+1}}] - r_{ft} \\ &\simeq \beta_i^c \mathbf{var}_t\left(\Delta \mathcal{N}_{i,t+1}^c\right) + \beta_i^{sm} \mathbf{var}_t\left(\Delta \mathcal{N}_{i,t+1}^{sm}\right) + \beta_i^{sx} \mathbf{var}_t\left(\Delta \mathcal{N}_{i,t+1}^{sx}\right) + \beta_i^e \mathbf{var}_t\left(\Delta \mathcal{N}_{i,t+1}^e\right) \end{aligned} \tag{52}$$

where β_i^f measures the sector i's exposure to the aggregate factor as

$$\beta_i^c = -\frac{\mathbf{cov}_t\left(\Delta \delta_{i,t+1}, \Delta \mathcal{N}_{i,t+1}^c\right)}{\delta_{it} \mathbf{var}_t\left(\Delta \mathcal{N}_{i,t+1}^c\right)}, \quad and \quad \beta_i^f = \frac{\mathbf{cov}_t\left(\Delta \delta_{i,t+1}, \Delta \mathcal{N}_{i,t+1}^f\right)}{\delta_{it} \mathbf{var}_t\left(\Delta \mathcal{N}_{i,t+1}^f\right)}, \quad for \ \ \forall\, f \in \{sm, sx, e\}. \tag{53}$$

If we only consider the first-order of indirect effects in Leontief inverse matrix in Equation (28), then we have, for $i \in \{1, 2, \ldots, n\}$,

$$\Delta \delta_{i,t+1} = (1 - \phi - \psi) \sum_j \left(\phi \zeta_j \Delta \theta_{ij,t+1} + \psi \zeta_j \Delta \omega_{ij,t+1}\right)$$

Notably, $\{\theta_{ijt}\}$ and $\{\omega_{ijt}\}$ are independent both within their group and across groups. Thus, using the Delta method, we obtain the covariances between the change in the output share of

[27] In Section F of the Internet Appendix, we provide detailed analytic derivations.

industry i and the change of each aggregate factor as

$$\mathbf{cov}_t\left(\Delta\delta_{i,t+1}, \Delta\mathcal{N}^c_{i,t+1}\right) = (1-\phi-\psi)^2\left(1+\log\delta_{it}\right)\sum_j \phi^2\zeta_j^2\mathbf{var}\left(\Delta\theta_{ij,t+1}\right) + \psi^2\zeta_j^2\mathbf{var}\left(\Delta\omega_{ij,t+1}\right), \quad (54)$$

$$\begin{aligned}\mathbf{cov}_t\left(\Delta\delta_{i,t+1}, \Delta\mathcal{N}^{sm}_{i,t+1}\right) &= (1-\phi-\psi)\phi^2\psi\sum_j \zeta_j^2\omega_{jit}\log\omega_{jit}\mathbf{var}\left(\Delta\theta_{ij,t+1}\right)\\ &+\psi^2\sum_j \zeta_j\left((1-\phi-\psi)\psi\zeta_j\omega_{jit}\log\omega_{jit} + \left(1+\log\omega_{ijt}\right)\delta_{jt}\right)\mathbf{var}\left(\Delta\omega_{ij,t+1}\right), \quad (55)\end{aligned}$$

$$\begin{aligned}\mathbf{cov}_t\left(\Delta\delta_{i,t+1}, \Delta\mathcal{N}^{sx}_{i,t+1}\right) &= (1-\phi-\psi)\phi\psi^2\sum_j \zeta_j^2\theta_{jit}\log\theta_{jit}\mathbf{var}\left(\Delta\omega_{ij,t+1}\right)\\ &+\phi^2\sum_j \zeta_j\left((1-\phi-\psi)\phi\zeta_j\theta_{jit}\log\theta_{jit} + \left(1+\log\theta_{ijt}\right)\delta_{jt}\right)\mathbf{var}\left(\Delta\theta_{ij,t+1}\right). \quad (56)\end{aligned}$$

Note that the above sectoral beta to each risk factor is computed based on the nearest dividend strip of the sector only for tractability. Presumably, the betas should depend on all dividend strips in the future, although arguably, the nearest strip has the largest pricing impact due to discounting. We then test this approximation through simulations (see Section F in the Internet Appendix for details). First, we calibrate our model to the US economy, considering two sectors: manufacturing and services sectors. We simulate the economy and compute the sectoral stock returns from the full-fledged model, i.e., $r_{it} = d_{it}/(v_{it} - d_{it})$, where v_{it} is solved in Equation (19), and our approximation of firms' exposure to aggregate risk as shown in Equation (53). We first regress the sectoral stock returns on aggregate factors. To examine whether the betas from our two-period approximation align with those derived from the full model, we perform a regression of sectoral stock returns on aggregate factors, controlling for industry fixed effects. The top panel of Table 5 presents the point estimates.

Next, we approximate the β coefficients in Equation (53) by applying a two-period cash-flow assumption. This approach uses the covariance between changes in industry i's output share and changes in aggregate factors, as detailed in Equations (54) through (56). The bottom panel of Table 5 presents the approximated β coefficients and their standard errors from the simulation. We find that the approximated exposures derived from the two-period cash flow is consistent with the point estimates from the full model in terms of the signs and the relative magnitudes of the sectoral β's.

6.2 Relation between model-implied and empirical betas

To connect our empirical evidence to our model and to validate the model-implied betas in Equation (53), we empirically examine the relation between them and empirical betas in Section 5.2

by estimating the following panel regression:

$$\beta_{i,t}^{data} = \alpha + b_{\Delta\mathcal{N}^{sx}}\beta_{\Delta\mathcal{N}^{sx},t}^{model} + b_{\Delta\mathcal{N}^{sm}}\beta_{\Delta\mathcal{N}^{sm},t}^{model} + b_{\Delta\mathcal{N}^{c}}\beta_{\Delta\mathcal{N}^{c},t}^{model} + \gamma\mathbf{X}_{it} + \eta_i + \zeta_t + \epsilon_{it}, \tag{57}$$

in which $\beta_{i,t}^{data}$ denotes one of the estimated $\beta_{\Delta\mathcal{N}^{sx},t}$, $\Delta\beta_{\mathcal{N}^{sm},t}$, and $\beta_{\Delta\mathcal{N}^{c},t}$. Also, $\beta_{\mathcal{N}^{sx},t}^{model}$, $\beta_{\mathcal{N}^{sm},t}^{model}$, and $\beta_{\mathcal{N}^{c},t}^{model}$ are estimated following Equation (53) for each industry every year. We assign model-implied betas for each industry to its constitute firms, and then estimate panel regressions for each one of the three empirical betas in Table 3 with and without other control variables. Table 6 reports the results.

Table 6 reports the results for knowledge network sparsity beta in Panel A, physical network sparsity beta in Panel B, and network concentration beta in Panel C.[28] Column (1) shows that the model-implied betas are all positively and significantly related to the corresponding empirical betas after controlling for firm and year fixed effects. The pattern remains after we further add various controls such as book to market, market equity, profitability, asset growth and past returns in Columns (2)-(4). In particular, we include log of book to market ratio ($\log BM$), log of market equity ($\log ME$), operating profitability (OP) defined as revenues minus cost of goods sold, interest expense, and selling, general, and administrative expenses divided by book equity, asset growth (AG) defined as the growth rate of total assets, momentum (MOM) defined as the cumulative stock returns over the past 12 months skipping the most recent month, short-run reversal (SR) defined as stock return in the last month, and long-run reversal (LR) defined as the stock returns over the past 36 months skipping the most recent 12 months. Therefore, this table suggests that the model-implied betas derived in Section 6.1 are consistent with empirical betas.

7 Conclusion

While the cross-sector transfer and production of intellectual property constitute the foundation of today's knowledge-based economy, how they influence economic dynamics and financial markets remains unknown thus far. This paper aims to fill this gap in the literature from both theoretical and empirical perspectives. We first develop a multi-sector network model, in which sectors are connected through not only a physical input-output network, but also a knowledge input-output network. Three characteristics of the economy's network structure are critical to aggregate risks: knowledge network sparsity, physical network sparsity, and network concentration, which command positive, positive, and negative risk premiums, respectively. The knowledge network risk we develop is a novel and timely addition to the literature that focuses on

[28] The sample is annual from 1984 to 2019. The t-statistics in the parentheses are based on standard errors clustered by firm. All variables are winsorized at the 1st and 99th percentiles.

physical input-output network (Acemoglu et al., 2012; Herskovic, 2018).

Empirical tests are designed and implemented to examine our model predictions. We resort to patent citation data to construct empirical measures for knowledge input-output flows across sectors. We show that consumption growth is positively related to the changes in knowledge and physical network sparsity (as our knowledge and physical network sparsity factors) and is negatively related to the changes in network concentration (as our concentration factor). We then implement a sorted portfolio approach in which we measure each stock's ex ante exposure to each network risk factor and find a monotonic pattern in average returns and alphas of sorted portfolios in each network factor. Our portfolio analyses not only confirm the positive (negative) risk premium on knowledge and physical network sparsity (network concentration) implied by our model but also highlight network risk being distinct from conventional systematic risk. We also implement an asset pricing test to show that all three network factors exist in a linear SDF model and help explain the cross-section of stock returns. Finally, we further derive the cross-sectional implications of our model, which delivers model-implied betas that are strongly correlated with our empirical betas. Overall, our empirical evidence highlights the important role of knowledge network in today's complex supply chains from the perspective of equity markets.

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Table 1 Summary statistics

This table reports the summary statistics (mean, standard deviation, and the first-order autocorrelation coefficients AC(1)) of knowledge sparsity, physical network sparsity and concentration in level in Panel A and factor (i.e., change in level) in Panel B. The sample is annual from 1979 to 2019 in Panel A and 1980 to 2019 in Panel B.

	Knowledge sparsity	Physical sparsity	Concentration
	Panel A: Level		
Mean	-0.0123	-0.0378	-2.6415
Std.	0.0076	0.0027	0.2128
AC(1)	0.90	0.73	0.84
	Panel B: Shock		
Mean	-0.0007	-0.0001	-0.0233
Std.	0.0008	0.0019	0.0940
AC(1)	0.05	0.18	-0.11

Table 2 Contemporaneous Relation between factors and Consumption

This table reports the coefficients from the contemporaneous regression of annual consumption growth on knowledge sparsity, physical network sparsity and concentration factors, macro controls, and lag consumption growth. Consumption growth is defined as growth of real personal consumption in nondurable goods and services per capita. Macro controls include GDP growth (Growth_{GDP}) defined as growth of GDP per capita, inflation defined as growth of CPI, and unemployment growth ($\text{Growth}_{Unemployment}$) defined as growth of unemployment rate. All macro data are seasonally adjusted. We report t-statistics based on standard errors estimation using the Newey-West procedure adjusted for six lags and adjusted R-square (Adj. R^2) in percentage. The sample is annual from 1980 to 2019.

	(1)	(2)	(3)
$\Delta\mathcal{N}^{sx}$	2.38	2.52	2.13
	(2.82)	(2.84)	(3.24)
$\Delta\mathcal{N}^{sm}$	0.82	0.82	0.65
	(2.61)	(2.62)	(2.61)
$\Delta\mathcal{N}^{c}$	-0.03	-0.03	-0.02
	(-2.50)	(-2.50)	(-2.00)
$\Delta \log GDP_t$	0.48	0.50	0.46
	(10.50)	(7.08)	(9.12)
$\Delta \log CPI_t$	-0.36	-0.38	-0.36
	(-10.00)	(-7.80)	(-8.20)
$\Delta \mathbf{U}_t$		0.00	0.01
		(0.48)	(0.86)
$\Delta \log C_{t-1}$			0.27
			(4.07)
b_0	0.01	0.01	0.00
	(2.32)	(2.13)	(1.61)
Adj. R^2	70.32	69.53	75.48

Table 3 Beta-sorted portfolios

This table reports returns (Panel A) and firm characteristics (Panel B) of one-way value-weighted tercile beta-sorted portfolios. For each stock, at the end of June in each year t, beta is estimated from the following annual rolling window regression over the past five years: $r_t = \alpha + \beta_{\Delta\mathcal{N}^{sx}}\Delta\mathcal{N}_t^{sx} + \beta_{\Delta\mathcal{N}^{sm}}\Delta\mathcal{N}_t^{sm} + \beta_{\Delta\mathcal{N}^{c}}\Delta\mathcal{N}_t^{c} + \beta_{RTFP}RTFP_t$. Each month from July of year t to June of year $t+1$, we sort all stocks in the sample into three value-weighted portfolios based on knowledge sparsity beta $\beta_{\Delta\mathcal{N}^{sx}}$ in Panels A.1 and B.1, physical network sparsity beta $\beta_{\Delta\mathcal{N}^{sm}}$ in Panels A.2 and B.2, or concentration beta $\beta_{\Delta\mathcal{N}^{c}}$ in Panels A.3 and B.3. Panel A report the average excess returns and alphas (in percentages) from CAPM, Fama-French three-factor (FF3), four-factor (FF4), and five-factor (FF5) models of the bottom, the middle, and the top tercile portfolios as well as the spread between the top and the bottom tercile portfolios. Panel B reports the time series average of cross sectional mean firm characteristics at the end of June each year including: β_{CAPM} estimated from CAPM using monthly returns over the past 60 months with a minimum of 36 months, market equity (ME, in millions of dollars), book to market ratio (BM) defined as book equity in the previous year divided by market equity in last December, asset growth (AG) defined as the growth rate of total assets in the previous year, operating profitability (OP) defined as revenues minus cost of goods sold, interest expense, and selling, general, and administrative expenses divided by book equity in the previous year, Sale/Ppegt defined as sale divided by Property, Plant, and Equipment in the previous year, and RDME defined by five-year cumulative R&D expenses with a 20% annual depreciation in the previous year divided by market equity in last December. Variables in Panel B are winsorized at the 1st and 99th percentiles. The sample includes NYSE/NASDAQ/AMEX common stocks (exchange code is 1, 2, or 3, and share code is 10, 11, or 12) with a price above $1 at portfolio formation. We report t-statistics based on standard errors estimation using the Newey-West procedure adjusted for three lags. The sample is monthly from July 1986 to June 2022 in Panel A and 1986 to 2022 in Panel B.

	Low	t-Stat.	Middle	t-Stat.	High	t-Stat.	H-L	t-Stat.
Panel A: Returns of beta-sorted portfolios								
Panel A.1: Knowledge-sparsity-beta-sorted portfolios								
Ret^e	0.53	(2.11)	0.69	(3.37)	0.84	(3.48)	0.31	(2.48)
α_{CAPM}	-0.18	(-2.07)	0.09	(1.55)	0.14	(2.15)	0.32	(2.48)
α_{FF3}	-0.19	(-2.15)	0.03	(0.85)	0.16	(2.38)	0.35	(2.63)
α_{FF4}	-0.12	(-1.42)	0.04	(0.95)	0.14	(2.19)	0.26	(2.08)
α_{FF5}	-0.13	(-1.40)	-0.07	(-1.72)	0.17	(2.12)	0.30	(1.98)
Panel A.2: Physical-sparsity-beta-sorted portfolios								
Ret^e	0.55	(2.08)	0.65	(3.12)	0.90	(3.89)	0.35	(2.33)
α_{CAPM}	-0.18	(-1.83)	0.04	(0.66)	0.23	(2.69)	0.41	(2.60)
α_{FF3}	-0.18	(-2.04)	-0.01	(-0.33)	0.23	(2.84)	0.41	(2.89)
α_{FF4}	-0.12	(-1.31)	-0.01	(-0.26)	0.22	(2.62)	0.33	(2.34)
α_{FF5}	-0.13	(-1.37)	-0.09	(-2.11)	0.15	(1.90)	0.28	(1.92)
Panel A.3: Concentration-beta-sorted portfolios								
Ret^e	0.88	(3.50)	0.68	(3.29)	0.48	(1.89)	-0.40	(-2.64)
α_{CAPM}	0.20	(2.02)	0.08	(1.28)	-0.26	(-2.95)	-0.46	(-2.73)
α_{FF3}	0.18	(1.92)	0.03	(0.59)	-0.25	(-3.04)	-0.43	(-2.74)
α_{FF4}	0.22	(2.20)	0.01	(0.27)	-0.19	(-2.27)	-0.41	(-2.48)
α_{FF5}	0.21	(2.18)	-0.07	(-1.45)	-0.22	(-2.62)	-0.43	(-2.67)

Table 3 (cont.) Beta-sorted portfolios

	Low	*t*-Stat.	Middle	*t*-Stat.	High	*t*-Stat.	H-L	*t*-Stat.
Panel B: Characteristics of beta-sorted portfolios								
Panel B.1: Knowledge-sparsity-beta-sorted portfolios								
β_{CAPM}	1.17	(20.49)	0.94	(17.18)	1.11	(27.91)	-0.06	(-1.57)
ME	3028.89	(3.67)	4496.76	(4.18)	3159.83	(4.07)	130.94	(0.40)
BM	0.79	(21.82)	0.85	(22.33)	0.79	(21.64)	0.00	(0.08)
AG	0.13	(9.00)	0.10	(12.60)	0.14	(11.51)	0.01	(0.74)
OP	0.15	(18.16)	0.20	(38.09)	0.15	(12.93)	0.00	(0.25)
Sale/Ppegt	4.17	(21.86)	4.05	(24.43)	4.52	(37.19)	0.35	(1.87)
RDME	0.10	(18.08)	0.07	(17.64)	0.10	(12.04)	0.00	(-0.23)
Panel B.2: Physical-sparsity-beta-sorted portfolios								
β_{CAPM}	1.15	(21.57)	0.95	(18.52)	1.12	(16.25)	-0.04	(-0.43)
ME	3022.16	(4.13)	4601.44	(3.99)	3061.85	(3.84)	39.69	(0.11)
BM	0.78	(23.12)	0.85	(21.84)	0.81	(20.83)	0.03	(1.15)
AG	0.14	(11.27)	0.10	(11.71)	0.13	(10.49)	-0.01	(-0.74)
OP	0.14	(10.16)	0.20	(39.33)	0.16	(16.94)	0.02	(1.52)
Sale/Ppegt	4.37	(28.11)	4.03	(23.10)	4.34	(32.88)	-0.03	(-0.23)
RDME	0.10	(11.86)	0.07	(17.93)	0.09	(17.98)	-0.01	(-1.40)
Panel B.3: Concentration-beta-sorted portfolios								
β_{CAPM}	1.12	(19.36)	0.94	(17.40)	1.16	(20.95)	0.04	(0.55)
ME	2660.09	(4.81)	4562.42	(4.04)	3462.94	(3.36)	802.85	(1.26)
BM	0.81	(18.61)	0.84	(20.93)	0.78	(24.50)	-0.03	(-0.81)
AG	0.13	(12.77)	0.09	(12.33)	0.14	(8.90)	0.01	(0.71)
OP	0.16	(16.84)	0.20	(33.50)	0.14	(11.57)	-0.01	(-0.98)
Sale/Ppegt	4.31	(18.93)	4.04	(25.54)	4.39	(44.22)	0.08	(0.36)
RDME	0.09	(18.48)	0.07	(18.35)	0.10	(10.61)	0.01	(0.90)

Table 4 Estimating the market price of knowledge and physical network risk

This table presents GMM estimates of the parameters of a five-factor linear SDF model, with knowledge sparsity factor ($\Delta\mathcal{N}^{sx}$), physical network sparsity factor ($\Delta\mathcal{N}^{sm}$), concentration factor ($\Delta\mathcal{N}^{c}$), residual TFP factor (**RTFP**), and the market factor (**MKT**). RTFP factor is the residual from regressing TFP growth on $\Delta\mathcal{N}^{sx}$, $\Delta\mathcal{N}^{sm}$, and $\Delta\mathcal{N}^{c}$. We use the Fama-French 30 industry portfolios and 4 sets of 5-by-5 double sorted portfolios (25 size-B/M portfolios, 25 size-INV portfolios, 25 size-OP portfolios, and 25 size-MOM portfolios) as test assets. We do the normalization such that $E[m] = 1$ (see, for example, Cochrane, 2001), and standardize the factors to a unit standard deviation. We report t-statistics based on standard errors estimation using the Newey-West procedure adjusted for three lags. As a measure of fit, we report the annualized mean absolute pricing errors (MAE) in percentage, the JT-statistics, the p-value associated with the over-identification test, and the p-value for the difference of the JT-statistics when comparing model (1) with model (2), model (3) with model (5), and model (4) with model (5). The sample is monthly from July 1981 to June 2021.

	(1)	(2)	(3)	(4)	(5)
MKT	0.42	0.23	0.11	0.15	0.16
	(9.50)	(5.59)	(3.03)	(3.40)	(3.45)
RTFP			0.51	0.20	0.13
			(5.42)	(1.79)	(1.09)
$\Delta\mathcal{N}^{sx}$		0.34			0.31
		(3.76)			(2.86)
$\Delta\mathcal{N}^{sm}$		0.22		0.13	0.20
		(2.62)		(1.44)	(2.08)
$\Delta\mathcal{N}^{c}$		-0.87		-0.77	-0.63
		(-10.45)		(-7.50)	(-4.92)
MAE (%)	13.92	2.40	2.94	4.19	4.07
JT-statistics	868.07	228.80	347.64	176.07	109.62
p-value	0.00	0.00	0.00	0.00	0.83
JT-Diff test		1 vs. 2		3 vs. 4	4 vs. 5
p-value(JT-Diff)		0.00		0.00	0.00

Table 5 Comparison of sectoral exposures to aggregate factors

This table compares betas implied by the full-fledged model and approximated betas under two-period cash flow assumption. Simulation is based on the calibration to the US data with two sectors (see Appendix F for details). The model has been simulated for five hundred periods. The top panel shows the point estimates of a panel regression of sectoral stock returns on aggregate risk factors. Stock return for sector i is computed as $r_{it} = d_{it}/(v_{it} - d_{it})$, where v_{it} is solved in Equation (19). The bottom panel displays the average and standard deviation of approximated betas under two-period cash flow assumption. Sectoral exposures to the aggregate factors are calculated based on Equation (53). The t-statistics in the parentheses are based on standard errors clustered by sector.

	Manufacturing	Services
Regression from the full-fledge model		
$\mathbf{D}_i \times \mathbf{var}\left(\Delta\mathcal{N}^{c}_{t+1}\right)$	-33.37***	-26.69***
	(0.000)	(0.000)
$\mathbf{D}_i \times \mathbf{var}\left(\Delta\mathcal{N}^{sx}_{t+1}\right)$	-34.01***	-42.16***
	(0.000)	(0.000)
$\mathbf{D}_i \times \mathbf{var}\left(\Delta\mathcal{N}^{sm}_{t+1}\right)$	43.24***	47.01***
	(0.000)	(0.000)
Obs	800	
adjusted R^2	0.001	
Approximated β using two-period cash flows		
β^{c}_{i}	-6.41	-2.37
	(0.97)	(2.08)
β^{sx}_{i}	-0.38	-0.80
	(0.06)	(0.10)
β^{sm}_{i}	2.34	5.02
	(0.67)	(1.14)

Table 6 Relationship between model-implied betas and empirical betas

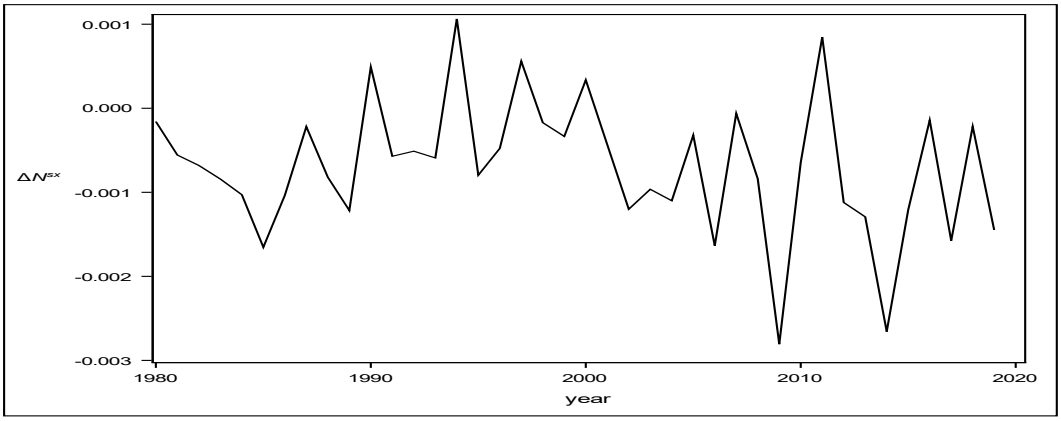
This table runs a panel regression of each one of the three empirical betas (knowledge sparsity beta in Panel A, physical network sparsity beta in Panel B and Concentration beta in Panel C) in Table 3 on the three model-implied betas ($\beta^{s,x}$, $\beta^{s,m}$, and β^{c}) in Equation (53) in column (1). All point estimates for $\beta^{s,x}$ are scaled by 1,000 for interpretational convenience. We add firm-level controls in columns (2)-(4) including log of book to market ratio, log of market equity, operating profitability (OP) defined as revenues minus cost of goods sold, interest expense, and selling, general, and administrative expenses divided by book equity, asset growth (AG) defined as the growth rate of total assets, momentum (MOM) defined as the cumulative stock returns over the past 12 months skipping the most recent month, short-run reversal (SR) defined as stock return in the last month, and long-run reversal (LR) defined as the stock returns over the past 36 months skipping the most recent 12 months. We assign model-implied betas for each industry to its constitute firms. We control for firm and year fixed effects. The t-statistics in the parentheses are based on standard errors clustered by firm. Variables are winsorized at the 1st and 99th percentiles. The sample is annual from 1984 to 2019.

	Panel A: Knowledge sparsity beta ($\beta_{N^{s,x}}$)				Panel B: Physical sparsity beta ($\beta_{N^{s,m}}$)				Panel C: Concentration beta ($\beta_{\mathcal{N}^c}$)			
	(1)	(2)	(3)	(4)	(1)	(2)	(3)	(4)	(1)	(2)	(3)	(4)
$\beta^{s,x}$	2.07	1.73	1.81	1.68	-4.91	-4.31	-4.59	-4.67	-0.15	-0.13	-0.14	-0.14
	(2.99)	(2.48)	(2.57)	(2.39)	(-6.86)	(-6.04)	(-6.38)	(-6.48)	(-6.02)	(-5.29)	(-5.68)	(-5.75)
$\beta^{s,m}$	0.82	1.65	1.82	1.94	8.12	6.29	6.40	6.45	0.09	0.04	0.04	0.04
	(0.59)	(1.14)	(1.24)	(1.32)	(5.84)	(4.46)	(4.48)	(4.51)	(1.93)	(0.81)	(0.90)	(0.93)
β^{c}	-58.86	-40.62	-38.51	-36.54	177.08	122.14	124.24	125.58	3.01	1.45	1.52	1.55
	(-2.97)	(-1.92)	(-1.79)	(-1.71)	(8.67)	(5.87)	(5.89)	(5.93)	(4.40)	(2.06)	(2.13)	(2.17)
$\log BM$		91.53	98.31	26.11		56.41	-28.84	-85.37		3.84	-0.39	-1.67
		(2.59)	(2.65)	(0.67)		(1.79)	(-0.85)	(-2.37)		(3.52)	(-0.33)	(-1.36)
$\log ME$		-157.95	-107.03	-74.16		60.79	65.20	82.40		2.28	4.10	4.52
		(-5.69)	(-3.83)	(-2.64)		(2.36)	(2.44)	(3.06)		(2.56)	(4.47)	(4.90)
OP			-163.70	-113.22			9.01	44.60			-3.18	-2.32
			(-2.55)	(-1.76)			(0.15)	(0.75)			(-1.59)	(-1.16)
AG			-314.55	-222.68			40.28	84.09			-8.12	-6.89
			(-5.30)	(-3.74)			(0.82)	(1.68)			(-4.79)	(-4.00)
MOM			-10.97	-124.28			-248.66	-304.98			-13.16	-14.71
			(-0.22)	(-2.41)			(-7.41)	(-8.51)			(-11.33)	(-11.87)
SR				273.70				-395.08				-3.41
				(1.97)				(-3.62)				(-0.89)
LR				-187.25				-90.58				-2.51
				(-8.39)				(-6.30)				(-4.64)

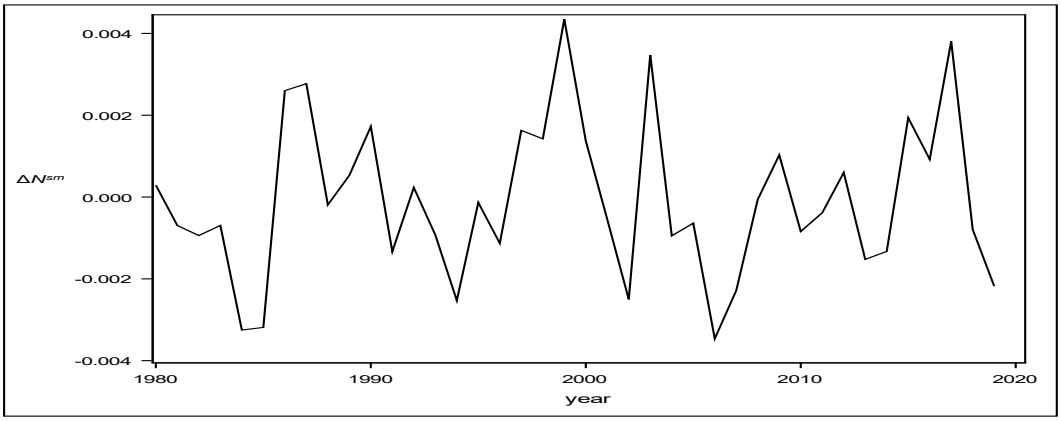
Figure 1. Times series of concentration and sparsity factors

This figure plots the time series of the knowledge sparsity factor, physical network sparsity factor, and concentration factor. The sample is annual from 1980 to 2019.

Panel A: knowledge sparsity factor



Panel B: Physical sparsity factor



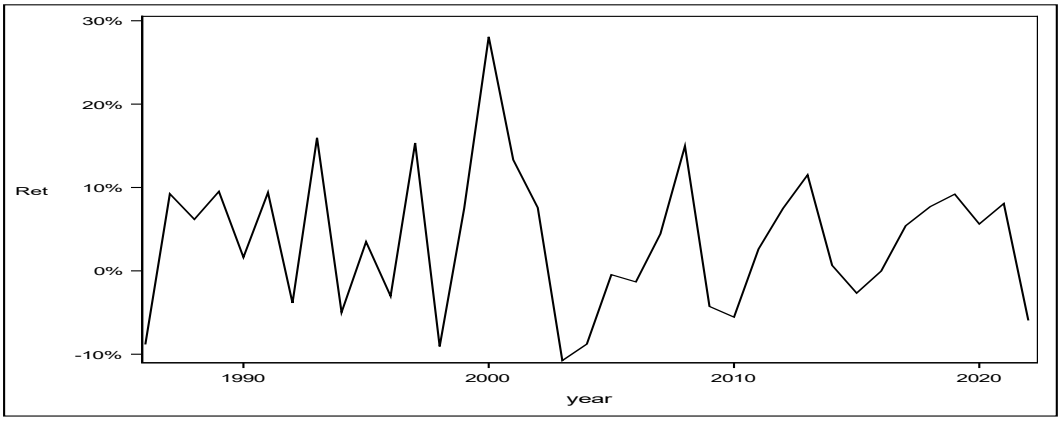
Panel C: Concentration factor



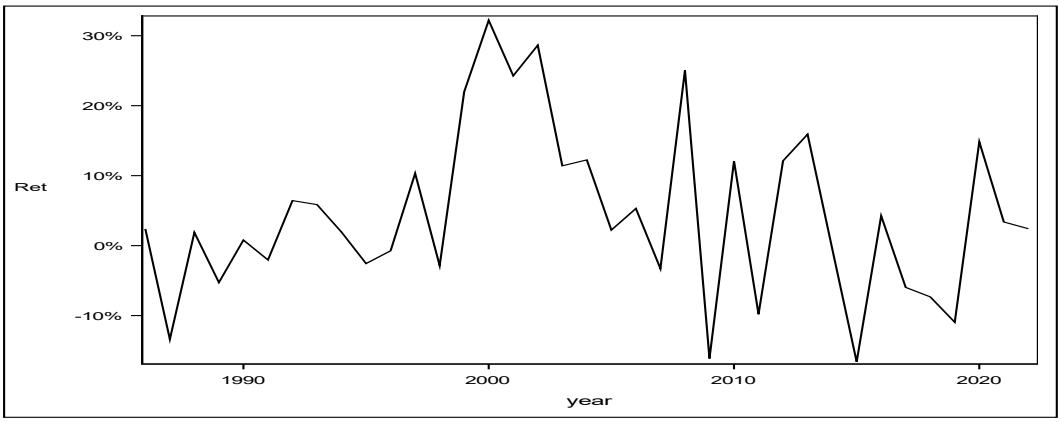
Figure 2. High-minus-low portfolio returns of concentration and sparsity factor betas

This figure plots the value-weighted returns on the high-minus-low portfolios sorted based on the knowledge sparsity factor beta, physical network sparsity factor beta, and concentration factor beta (as formed in Table 3) on a per annum basis from July 1986 to June of 2022. There are only six months in 1986 and 2022.

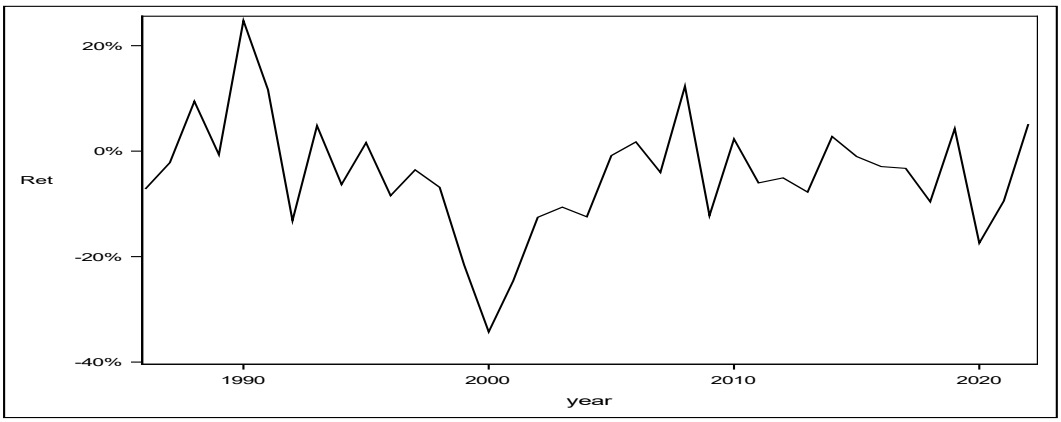
Panel A: knowledge-sparsity-beta-sorted portfolios



Panel B: Physical-sparsity-beta-sorted portfolios



Panel C: Concentration-beta-sorted portfolios



Internet Appendix

A Model with labor

Now, we consider the case in which firms utilize labor as an additional input alongside physical and knowledge inputs. Accordingly, we modify the production technology of firm i in Equation (1) to:

$$y_{it} = z_{it} l_{it}^{\alpha} x_{it}^{\phi} m_{it}^{\psi}, \quad where \quad \alpha + \phi + \psi < 1, \tag{58}$$

where l_{it} represents the labor employed by firm i, and α determines the labor share in production. Given the wage rate (W_t), the firm maximizes its profits, leading to the first-order condition for labor, in addition to Equations (5) and (6):

$$\alpha p_{it} l_{it}^{\alpha-1} x_{it}^{\phi} m_{it}^{\psi} = W_t. \tag{59}$$

Now, the dividends by firm i can be expressed as

$$d_{it} = (1 - \alpha - \phi - \psi) p_{it} y_{it} \tag{60}$$

and the output of firm i can by expressed as

$$y_{it} = \left(z_{it} p_{it}^{\alpha+\phi+\psi} \left(\frac{\alpha}{W_t} \right)^{\alpha} \left(\frac{\phi}{\bar{q}_{it}} \right)^{\phi} \left(\frac{\psi}{\bar{p}_{it}} \right)^{\phi} \right)^{\frac{1}{1-\phi-\psi}}. \tag{61}$$

On the supply side, the representative household provides labor and earns a wage. Accordingly, her preferences are given by:

$$U(C_t, L_t) = \frac{C_t^{1-\eta} - 1}{1 - \eta} - \gamma \frac{L_t^{1+\epsilon}}{1 + \epsilon}, \tag{62}$$

where γ captures the relative importance of leisure compared to consumption, and ϵ represents the inverse of the Frisch elasticity of labor supply. Given the price index, dividends, shareholdings, and firm values, the household optimizes utility by choosing the consumption bundle and shareholdings at $t + 1$, subject to the budget constraint, as:

$$\max_{C_t, L_t, \{s_{i,t+1}\}} \sum_{\tau=0}^{\infty} \beta^{\tau} \mathbf{E}_t \left[U(C_{t+\tau}, L_{t+\tau}) \right] \quad s.t. \quad C_t + \sum_{i=1}^{n} s_{i,t+1}(v_{it} - d_{it}) = W_t L_t + \sum_{i=1}^{n} s_{it} v_{it}. \tag{63}$$

The first-order condition establishes the relationship between the consumption bundle and labor supply:

$$W_t = \gamma L_t^{\epsilon} C_t^{\eta}. \tag{64}$$

Finally, the market-clearing condition for the labor market is given by:

$$L_t = \sum_i l_{it}. \tag{65}$$

Combining Equation (59) with (65), we have

$$W_t L_t = \alpha \sum_i p_{it} y_{it} \tag{66}$$

Using Equations (60) and (66), the household's budget constraint in equilibrium becomes:

$$C_t = \sum_i d_{it} + W_t L_t = (1 - \phi - \psi) \sum_i p_{it} y_{it} \tag{67}$$

Thus, combinding Equation (64) and (66) with (67), we have

$$W_t = \tilde{\gamma} C_t^{\frac{\epsilon+\eta}{1+\eta}}, \tag{68}$$

where $\tilde{\gamma} = \left(\gamma \left(\frac{\alpha}{1-\phi-\psi}\right)^{\epsilon}\right)^{\frac{1}{1+\epsilon}}$. Since Equation (26) still holds, the vector δ_t, defined in Equation (29), continues to be equal to the Domar weights, as given in Equation (32). Taking the logarithm on both sides of Equation (61), and substituting $p_{it} y_{it}$ with Equation (27), W_t with Equation (68), $\bar{p}_{it}$ with Equation (10), and $\bar{q}_{it}$ with Equation (8) and (15), we can rewrite the logarithm of the consumption bundle as:

$$\begin{aligned} \log C_t \quad = \quad & \frac{1}{1 + \frac{\alpha(\epsilon+\eta)}{(1-\alpha-\phi-\psi)(1+\eta)}} \Bigg(cons - \sum_i \delta_{it} \log \delta_{it} + \frac{1}{1-\alpha-\phi-\psi} \Bigg(\phi \sum_i \delta_{it} \sum_j \theta_{jit} \log \theta_{jit} \\ & + \psi \sum_i \delta_{it} \sum_j \omega_{jit} \log \omega_{jit} + \sum_i \delta_{it} \log z_{it} + \phi \sum_i \delta_{it} \sum_j \theta_{jit} \log a_{jt} \Bigg) \Bigg) \end{aligned} \tag{69}$$

where $cons = \frac{(1-\alpha-\phi-\psi)\log(1-\phi-\psi) + \alpha \log \alpha + \phi \log \phi + \psi \log \psi - \alpha \log \tilde{\gamma}}{1-\alpha-\phi-\psi} + (1-\phi-\psi)\zeta' \log \zeta$. Notably, the introduction of labor affects the consumption bundle in Equation (34) in two key ways. First, with labor as an additional input, the effects of knowledge and physical input networks on firms' outputs must be adjusted to account for the labor share. Second, as wages influence household income, labor introduces a general equilibrium (GE) effect on the consumption bundle. This effect is captured by the first term on the right-hand side of Equation (69). It is straightforward to show that incor-

porating labor into the technology leads to qualitatively similar implications for asset pricing as in Proposition 1.

B Endogenous growth model

In this section, we extend our benchmark model and allow sectoral knowledge productivity to grow endogenously. We will show later that our main result still holds under this new setting.

Here, we set up the production of goods and knowledge and the preference of the representative household the same as in Section 2. Following Liu and Ma (2021), we assume that our economy now evolves in continuous time instead, and the law of motion for the sector i's knowledge productivity is given by

$$\mathbf{d}\log a_{it} = \chi\left(\log e_{it} + \sum_j \gamma_{ji}\log a_{jt} - \log a_{it}\right)\mathbf{d}t \tag{70}$$

where e_{it} is the inputs used for innovations that push forward the sector i's technological frontier, γ_{ji} governs the share of frontier knowledge that sector i adopts from sector j's knowledge in the innovation process, and χ is a parameter to measure the elasticity of the sector i's technological growth to the innovation inputs. Here, we treat innovation and application separately. The process of the knowledge productivity summarizes the innovation process, whereas the interconnection of the knowledge inputs characterises how sectors applies their own as well as others sectors' knowledge. Therefore, the current-value Hamiltonian of a social planner's problem can be written as

$$\begin{aligned}\mathcal{H} \;=\; & \frac{C_t^{1-\eta}}{1-\eta} + \sum_i \mu_{it}\left(z_{it}x_{it}^{\phi}m_{it}^{\psi} - c_{it} - \sum_j m_{ijt} - h_{it} - e_{it}\right) + \lambda_t\left(\prod_i c_{it}^{\zeta_i} - C_t\right) \\ & + \sum_i \tau_{it}\left(\prod_j x_{jit}^{\theta_{jit}} - x_{it}\right) + \sum_i \rho_{it}\left(a_{it}h_{it} - \sum_j x_{ijt}\right) \\ & + \sum_i \sigma_{it}\left(\prod_j m_{jit}^{\theta_{jit}} - m_{it}\right) + \chi\sum_i \nu_{it}\left(\log e_{it} + \sum_j \gamma_{ji}\log a_{jt} - \log a_{it}\right)\end{aligned} \tag{71}$$

where $\{\mu_{it}\}$, λ_t, $\{\tau_{it}\}$, $\{\rho_{it}\}$, $\{\sigma_{it}\}$, and $\{\nu_{it}\}$ are Lagrangian multipliers for constraints. By the maxi-

mum principle, we have

$$\mathcal{H}_{C_t} = 0 \quad \Rightarrow \quad C_t^{-\eta} = \lambda_t \tag{72}$$

$$\mathcal{H}_{c_{it}} = 0 \quad \Rightarrow \quad \zeta_i \lambda_t C_t = \mu_{it} c_{it} \tag{73}$$

$$\mathcal{H}_{x_{it}} = 0 \quad \Rightarrow \quad \phi \mu_{it} z_{it} x_{it}^{\phi} m_{it}^{\psi} = \tau_{it} x_{it} \tag{74}$$

$$\mathcal{H}_{x_{ijt}} = 0 \quad \Rightarrow \quad \rho_{it} x_{ijt} = \tau_{jt} \theta_{ijt} x_{jt} \tag{75}$$

$$\mathcal{H}_{h_{it}} = 0 \quad \Rightarrow \quad \mu_{it} = \rho_{it} a_{it} \tag{76}$$

$$\mathcal{H}_{m_{it}} = 0 \quad \Rightarrow \quad \psi \mu_{it} z_{it} x_{it}^{\phi} m_{it}^{\psi} = \sigma_{it} m_{it} \tag{77}$$

$$\mathcal{H}_{m_{ijt}} = 0 \quad \Rightarrow \quad \mu_{it} m_{ijt} = \sigma_{jt} \omega_{ijt} m_{jt} \tag{78}$$

$$\mathcal{H}_{e_{it}} = 0 \quad \Rightarrow \quad \mu_{it} e_{it} = \chi \nu_{it} \tag{79}$$

$$\mathcal{H}_{\log a_{it}} = 0 \quad \Rightarrow \quad \rho_{it} a_{it} h_{it} + \chi \left(\sum_j \gamma_{ij} \nu_{jt} - \nu_{it} \right) = \beta \nu_{it} - \mathbf{d}\nu_{it}/\mathbf{d}t \tag{80}$$

We first show that the transversality condition, i.e., $\lim_{t\to\infty} e^{-\beta t}\mathcal{H} = 0$, implies $\mathbf{d}\nu_{it}/\mathbf{d}t = 0$ for all i. Next, we normalize all prices by the shadow price of the aggregate consumption λ_t and quantities used for innovation ($\{e_{it}\}$) and application ($\{h_{it}\}$) by the aggregate consumption C_t. Denote $\tilde{\mu}_{it} = \mu_{it}/\lambda_t$, $\tilde{\tau}_{it} = \tau_{it}/\lambda_t$, $\tilde{\rho}_{it} = \rho_{it}/\lambda_t$, and $\tilde{\sigma}_{it} = \sigma_{it}/\lambda_t$. Then, we define that $\bar{h}_{it} = \tilde{\rho}_{it} a_{it} h_{it}/C_t$ and $\bar{\nu}_{it} = \nu_{it}/\lambda_t/C_t$. For now, we want to show that if the vector $\bar{h}_t$, defined as $\bar{h}_t = \left[\bar{h}_{1t}, \ldots, \bar{h}_{nt}\right]'$, is time-invariant, then the vector $\bar{\nu}_t$, defined as $\bar{\nu}_t = [\bar{\nu}_{1t}, \ldots, \bar{\nu}_{nt}]'$ is time-invariant as well. Using Equation (80), we have

$$\mathbf{d}\bar{\nu}_{it}/\mathbf{d}t = -\bar{h}_t + ((\beta + \chi)\mathbf{I} - \chi\Gamma)\,\bar{\nu}_t \tag{81}$$

where $\mathbf{I}$ is a n-by-n identity matrix, and Γ is a matrix of γ_{ij} on the (i, j) cell. If the vector $\bar{h}_t$ is time-invariant, solving the problem, we have

$$\bar{\nu}_t = \mathbf{e}^{((\beta+\chi)\mathbf{I}-\chi\Gamma)t}\bar{\nu}_0 - \left(\mathbf{e}^{((\beta+\chi)\mathbf{I}-\chi\Gamma)t} - \mathbf{I}\right)((\beta + \chi)\mathbf{I} - \chi\Gamma)^{-1}\,\bar{h}_0, \tag{82}$$

where $\bar{h}_t = \bar{h}_0$ for all t. Our assumption requires that $\tilde{\mu}_{it} h_{it}$ is proportional to the aggregate consumption and the proportion is time-invariant and summed up equal to one. By transversality condition, i.e., $\lim_{t\to\infty} e^{-\beta t}\bar{\nu}_t = 0$, we have

$$\bar{\nu}_t = ((\beta + \chi)\mathbf{I} - \chi\Gamma)^{-1}\,\bar{h}_0. \tag{83}$$

Combining Equation (79) with (83), we have $\tilde{\mu}_{it} e_{it}$ is proportional to the aggregate consumption as well. Moreover, we have $\chi \sum_i \bar{\nu}_{it} = 1$.

Then, we want to show that the solution to the social planner's problem is the same as in our

benchmark model, as long as $\bar{h}_t$ stays constant over the time. Equation (73) implies that

$$\prod_i \left(\frac{\tilde{\mu}_{it}}{\zeta_i}\right)^{\zeta_i} = 1 \tag{84}$$

Combining Equation (74) with (77), we have

$$y_{it} = \left(z_{it}\tilde{\mu}_{it}^{\phi+\psi}\left(\frac{\phi}{\tilde{\tau}_{it}}\right)^{\phi}\left(\frac{\psi}{\tilde{\sigma}_{it}}\right)^{\phi}\right)^{\frac{1}{1-\phi-\psi}} \tag{85}$$

and Equation (75) and (76) imply

$$\tilde{\tau}_{it} = \prod_j \left(\frac{\tilde{\mu}_{jt}}{a_{jt}\theta_{jit}}\right)^{\theta_{jit}} \tag{86}$$

and Equation (78) implies

$$\tilde{\sigma}_{it} = \prod_j \left(\frac{\tilde{\mu}_{jt}}{\omega_{jit}}\right)^{\omega_{jit}} \tag{87}$$

Combining Equation (73), (74), (75), (77), and (78) into the resource constraint, we have

$$\tilde{\mu}_t \tilde{y}_t = \left(\mathbf{I} - \phi\Theta - \psi\Omega\right)^{-1}\left(\zeta + \chi\bar{v}\right)C_t, \tag{88}$$

where Θ and Ω are defined in Equation (16). Now, define $\delta = \frac{1}{2}(1-\phi-\psi)\left(\mathbf{I} - \phi\Theta - \psi\Omega\right)^{-1}\left(\zeta + \chi\bar{v}\right)$. Moreover, the ith element of the vector δ can be rewritten as the Domar weight of sector i as

$$\delta_{it} = \frac{(1-\phi-\psi)\tilde{\mu}_{it}y_{it}}{C_t} = \frac{\tilde{\mu}_{it}y_{it}}{\sum_j \tilde{\mu}_{jt}y_{jt}} \tag{89}$$

where the aggregate consumption can be rewritten as

$$C_t = \frac{1}{2}\sum_i(\zeta_i + \chi\bar{v}_i)C_t = \sum_i \tilde{\mu}_{it}y_{it} - \sum_i \tilde{\mu}_{it}\sum_j m_{ijt} - \sum_i \frac{\tilde{\mu}_{it}}{a_{it}}\sum_j x_{ijt} = (1-\phi-\psi)\sum_i \tilde{\mu}_{it}y_{it}.$$

Thus, Equation (88) implies

$$\begin{aligned}\log\delta_{it} + \log C_t \quad = \quad & \log(1-\phi-\psi) + \frac{1}{1-\phi-\psi}\Bigg(\log z_{it} + \phi\sum_j \theta_{jit}\log a_{jt} + \log\tilde{\mu}_{it} + \phi\log\phi + \psi\log\psi \\ & + \phi\sum_j \theta_{jit}\log\theta_{jit} - \phi\sum_j \theta_{jit}\log\tilde{\mu}_{jt} + \psi\sum_j \omega_{jit}\log\omega_{jit} - \psi\sum_j \omega_{jit}\log\tilde{\mu}_{jt}\Bigg)\end{aligned}$$

We stack the equations above across sectors as

$$\begin{aligned}\log \delta_t + \mathbf{1}\log C_t &= constant + \frac{1}{1-\phi-\psi}\left(\log z_t + \phi\Theta'\log a_t + \right.\\ &\left. \phi\mathbf{diag}\left(\Theta'\log\Theta\right) + \psi\mathbf{diag}\left(\Omega'\log\Omega\right) + \left(\mathbf{I} - \phi\Theta' - \psi\Omega'\right)\log\tilde{\mu}_{it}\right)\end{aligned}$$

Combining it with Equation (84), we have

$$\begin{aligned}\log C_t &= constant - \sum_i \delta_{it}\log\delta_{it} + \frac{1}{1-\phi-\psi}\left(\phi\sum_i \delta_{it}\sum_j \theta_{jit}\log\theta_{jit} + \right.\\ &\left. +\psi\sum_i \delta_{it}\sum_j \omega_{jit}\log\omega_{jit} + \sum_i \delta_{it}\log z_{it} + \phi\sum_i \delta_{it}\sum_j \theta_{jit}\log a_{jt}\right) \qquad (90)\end{aligned}$$

Clearly, the aggregate consumption in Equation (90) is the proportional to the one in Equation (34), and moreover the proportion is fixed over time.

Last, we want to establish the balanced growth path. In this case, we need to focus on the case where $\eta = 1$, i.e., $\lambda_t C_t = 1$. The low of motion for the knowledge productivity in Equation (70) becomes

$$\mathbf{d}\log a_t/\mathbf{d}t = \chi\left(\log\chi + \log\bar{\nu} + (\Gamma - \mathbf{I})\log a_t\right).$$

Then we take the derivative on both sides with respect to t as

$$(\Gamma - \mathbf{I})\,\mathbf{d}\log a_t/\mathbf{d}t = 0. \qquad (91)$$

As Γ is a random matrix, thus we have $\mathbf{d}\log a_t/\mathbf{d}t$ to be en eigenvector of Γ, and moreover $\mathbf{d}\log a_t/\mathbf{d}t$ is fixed over time. Thus, there exists a vector a, where $a' = a'\Gamma$ and $\sum_i a_i = 1$, such that the economy grows at the rate of g where $g = \chi\log\chi + a'\log\bar{\nu}$.

C Solution to the model

Given Equation (15), we take the logarithm on both sides of Equation (33) as

$$\log p_{it}y_{it} = \frac{1}{1-\phi-\psi}\left(\log z_{it} + \log p_{it} + \phi\left(\log\phi + \sum_j \theta_{jit}\left(\log\theta_{jit} + \log a_{jt} - \log p_{jt}\right)\right) + \psi\left(\log\psi + \sum_j \omega_{jit}\left(\log\omega_{jit} - \log p_{jt}\right)\right)\right) \qquad (92)$$

Stacking Equation (92) together, we have

$$\begin{aligned}\log p_t \circ y_t &= \frac{1}{1-\phi-\psi}\left(\log z_t + \phi\Theta'\log a_t + \left(\phi\log\phi + \psi\log\psi\right)\mathbf{1}_n\right.\\ &\left. + \left(\mathbf{I}_n - \phi\Theta' - \psi\Omega'\right)\log p_t + \phi\mathbf{diag}\left(\Theta'\log\Theta\right) + \psi\mathbf{diag}\left(\Omega'\log\Omega\right)\right) \qquad (93)\end{aligned}$$

where $\circ$ is the Hadamard product, $\mathbf{1}_n$ is a vector of ones, $\mathbf{I}_n$ is a n-dimension identify matrix, $\mathbf{diag}(M)$ is a vector of diagonal elements of the matrix D, and Θ and Ω are defined in Equation (16). Using Equation (27) and (29), we have

$$\log p_t \circ y_t = \log \delta_t + \log C_t \mathbf{1}_n - \log(1-\phi-\psi)\mathbf{1}_n \tag{94}$$

Combining Equation (93) with (94) and multiplying both sides by

$$\delta' = (1-\phi-\psi)\zeta' \left(\mathbf{I}_n - \phi\Theta' - \psi\Omega'\right)^{-1},$$

we have

$$\begin{aligned}\delta'(\log \delta_t + \log C_t \mathbf{1}_n) \quad = \quad & \frac{\delta'}{1-\phi-\psi}\Big(\log z_t + \phi\Theta' \log a_t + (\phi\log\phi + \psi\log\psi + \log(1-\phi-\psi))\,\mathbf{1}_n \\ & + \phi\mathbf{diag}\left(\Theta'\log\Theta\right) + \psi\mathbf{diag}\left(\Omega'\log\Omega\right)\Big) + \zeta'\log\zeta.\end{aligned}$$

Since $\delta'\mathbf{1}_n = 1$, then Equation (33) is established with $constant = \zeta'\log\zeta + \frac{\phi\log\phi+\psi\log\psi+\log(1-\phi-\psi)}{1-\phi-\psi}$.

D Details about Compustat Segment Data

Following Herskovic (2018), we estimate physical network factors based on the information from Compustat customer segment data. Regulation SFAS No. 131 requires firms to disclose the identity of any customer representing more than 10% of the total reported sales. WRDS provides a supply chain linking data of this customer-supplier relation based on Compustat segment data. The data includes the identity of both customers and suppliers as well as sales from each supplier to its customers. We only include unique customer-supplier pair in each data.

The segment data is collected at the supplier level with datadate being suppliers' datadate. To identify the datadate for the customer, we keep the closest customer datadate to each supplier datadate among the three consecutive years of previous, contemporaneous, and next year centered around the year of supplier datadate. To avoid using future information, we set the year to be the later of the customer and supplier datadate for each customer-supplier pair. This treatment can potentially create an issue that one customer might have two different datadates for the same year. In this case, it would have different sales and potentially be in two different sectors if the company changes industry code from one year to another year. To avoid this complication, we only keep customers with the same industry code in a year (less than 0.5% observations are dropped) and take an average of the sales. Further, to avoid the case where the number of com-

panies is too few in a sector especially in early years, we require more than 5 companies each year for a customer sector when computing network factors.

To further exclude potential data errors in the segment data, we eliminate observations where total customer sales are more than total sales of the supplier reported in Compustat fundamental data. Sector is defined by SIC-2 digit industry code. We use Compustat historical SIC industry code first. If it is missing, we then use the available SIC industry code from CRSP.

E Additional Robustness Checks

Table A1 Estimating the market price of risk, alternative testing assets

This table presents GMM estimates of the parameters of a five-factor linear SDF model, with knowledge sparsity factor ($\Delta\mathcal{N}^{sx}$), physical network sparsity factor ($\Delta\mathcal{N}^{sm}$), concentration factor ($\Delta\mathcal{N}^{c}$), residual TFP factor (**RTFP**), and the market factor (**MKT**). RTFP factor is the residual from regressing TFP growth on $\Delta\mathcal{N}^{sx}$, $\Delta\mathcal{N}^{sm}$, and $\Delta\mathcal{N}^{c}$. We use the SIC 2-digit industry portfolios and 4 sets of 5-by-5 double sorted portfolios (25 size-B/M portfolios, 25 size-INV portfolios, 25 size-OP portfolios, and 25 size-MOM portfolios) as test assets. We only include SIC 2-digit industry portfolios with more than 20 stocks in every month of the sample. We do the normalization such that $E[m] = 1$ (see, for example, Cochrane, 2001), and standardize the factors to a unit standard deviation. We report t-statistics based on standard errors estimation using the Newey-West procedure adjusted for three lags. As a measure of fit, we report the annualized mean absolute pricing errors (MAE) in percentage, the JT-statistics, the p-value associated with the over-identification test, and the p-value for the difference of the JT-statistics when comparing model (1) with model (2), model (3) with model (5), and model (4) with model (5). The sample is monthly from July 1981 to June 2021.

	(1)	(2)	(3)	(4)	(5)
MKT	0.33	0.26	0.14	0.20	0.21
	(8.54)	(5.60)	(3.87)	(4.73)	(4.61)
RTFP			0.49	0.17	0.12
			(4.81)	(1.52)	(0.90)
$\Delta\mathcal{N}^{sx}$		0.24			0.23
		(2.25)			(1.96)
$\Delta\mathcal{N}^{sm}$		0.37		0.27	0.31
		(4.15)		(3.08)	(3.22)
$\Delta\mathcal{N}^{c}$		-0.96		-0.71	-0.72
		(-9.70)		(-6.58)	(-5.52)
MAE (%)	9.29	2.04	2.24	2.17	2.46
JT-statistics	878.99	245.49	301.68	142.89	109.81
p-value	0.00	0.00	0.00	0.08	0.74
JT-Diff test		1 vs. 2		3 vs. 4	4 vs. 5
p-value(JT-Diff)		0.00		0.00	0.00

Table A2 Estimating the market price of risk, alternative testing assets

This table presents GMM estimates of the parameters of a five-factor linear SDF model, with knowledge sparsity factor ($\Delta\mathcal{N}^{sx}$), physical network sparsity factor ($\Delta\mathcal{N}^{sm}$), concentration factor ($\Delta\mathcal{N}^{c}$), residual TFP factor (**RTFP**), and the market factor (**MKT**). RTFP factor is the residual from regressing TFP growth on $\Delta\mathcal{N}^{sx}$, $\Delta\mathcal{N}^{sm}$, and $\Delta\mathcal{N}^{c}$. We use the Fama-French 10 industry portfolios and 4 sets of 2-by-3 double sorted portfolios (6 size-B/M portfolios, 6 size-INV portfolios, 6 size-OP portfolios, and 6 size-MOM portfolios) as test assets. We do the normalization such that $E[m] = 1$ (see, for example, Cochrane, 2001), and standardize the factors to a unit standard deviation. We report t-statistics based on standard errors estimation using the Newey-West procedure adjusted for three lags. As a measure of fit, we report the annualized mean absolute pricing errors (MAE) in percentage, the JT-statistics, the p-value associated with the over-identification test, and the p-value for the difference of the JT-statistics when comparing model (1) with model (2), model (3) with model (5), and model (4) with model (5). The sample is monthly from July 1981 to June 2021.

	(1)	(2)	(3)	(4)	(5)
MKT	0.34	0.27	0.12	0.18	0.18
	(6.76)	(3.90)	(2.16)	(3.04)	(2.73)
RTFP			1.11	0.29	0.03
			(3.92)	(1.08)	(0.10)
$\Delta\mathcal{N}^{sx}$		0.39			0.56
		(1.76)			(2.46)
$\Delta\mathcal{N}^{sm}$		0.51		0.50	0.41
		(2.54)		(2.55)	(1.90)
$\Delta\mathcal{N}^{c}$		-0.88		-0.53	-0.53
		(-4.54)		(-2.44)	(-2.42)
MAE (%)	9.01	1.83	1.83	2.19	3.42
JT-statistics	104.21	35.64	53.28	32.04	20.25
p-value	0.00	0.22	0.01	0.37	0.89
JT-Diff test		1 vs. 2		3 vs. 4	4 vs. 5
p-value(JT-Diff)		0.00		0.00	0.00

F Simulation for cross-sectional tests

In this section, we quantitatively assess our approximation (betas) for firms' exposures to aggregate risk factors. We begin by calibrating our model using US data across two sectors: (1) manufacturing, and (2) services. Next, we simulate the model to calculate industrial stock returns as implied by Equation (19), along with the corresponding exposures to aggregate risks as in Equation (53). We then test their correlation using Equation (48). Overall, our findings show that the point estimates are in a line with our model's predictions.

First, we derive the expression for excessive returns in Equation (52) as

$$\begin{aligned} xr_{i,t+1} &\equiv \log \mathbb{E}_t[e^{r_{i,t+1}}] - r_{ft} \\ &= \eta\Big(\mathbf{cov}_t\left(\Delta \log \delta_{i,t+1}, \Delta \log C_{t+1}\right) + \mathbf{var}_t\left(\Delta \log C_{t+1}\right)\Big), \\ &\simeq -\mathbf{cov}_t\left(\Delta \log \delta_{i,t+1}, \Delta \mathcal{N}^c_{i,t+1}\right) + \mathbf{cov}_t\left(\Delta \log \delta_{i,t+1}, \Delta \mathcal{N}^{sm}_{i,t+1}\right) \\ &\quad + \mathbf{cov}_t\left(\Delta \log \delta_{i,t+1}, \Delta \mathcal{N}^{sx}_{i,t+1}\right) + \mathbf{cov}_t\left(\Delta \log \delta_{i,t+1}, \Delta \mathcal{N}^{e}_{i,t+1}\right) \\ &= -\beta^c_i \mathbf{var}_t\left(\Delta \mathcal{N}^c_{i,t+1}\right) + \beta^{sm}_i \mathbf{var}_t\left(\Delta \mathcal{N}^{sm}_{i,t+1}\right) + \beta^{sx}_i \mathbf{var}_t\left(\Delta \mathcal{N}^{sx}_{i,t+1}\right) + \beta^{e}_i \mathbf{var}_t\left(\Delta \mathcal{N}^{e}_{i,t+1}\right) \end{aligned}$$

where for the second equation, we use Equation (27), (29), and (30) to obtain $d_{it} = \delta_{it} C_t$; the third equation is due to Equation (35).

Next, we simulate the model and explore the correlation of sectoral exposures between two-period cash flow model and the full-fledge model. In doing so, we set the discount factor to 0.96 and the risk-aversion parameter to 3. We apply a similar strategy as in Section 4.4 to calibrate the rest of parameters. For ϕ (ψ), we use the average of all ϕ_{it} (ψ_{it}). Here, we set $\phi = 0.17$ and $\psi = 0.45$. Then, we take the average of each sectors' knowledge (physical) input expenditure share, i.e., θ_{ijt} (ω_{ijt}), over time as the knowledge (physical) input expenditure share in the steady state. Here, we have

$$\Theta = \begin{bmatrix} 0.82 & 0.49 \\ 0.18 & 0.51 \end{bmatrix}, \quad \textit{and} \quad \Omega = \begin{bmatrix} 0.76 & 0.49 \\ 0.24 & 0.51 \end{bmatrix}.$$

In simulation, we need to alter the value of these input shares so that the standard deviation for these input shares are calculated as

$$\mathbf{sd}(\Delta\Theta) = \begin{bmatrix} 0.08 & 0.11 \\ 0.08 & 0.11 \end{bmatrix}, \quad \textit{and} \quad \mathbf{sd}(\Delta\Omega) = \begin{bmatrix} 0.42 & 0.46 \\ 0.42 & 0.46 \end{bmatrix}.$$

Similarity, we take the average of the output shares, denoted δ_{it}, over time as the output shares in steady state. Using Equation (29), we calculate the model-consistent consumption share, ζ_i. Here, we have

$$\delta = \begin{bmatrix} 0.55 & 0.45 \end{bmatrix}^T, \quad \textit{and} \quad \zeta = \begin{bmatrix} 0.48 & 0.52 \end{bmatrix}^T.$$

After simulating the model for five hundred periods, we compute the stock return for sector i as $r_{it} = d_{it}/(v_{it} - d_{it})$, where v_{it} is solved in Equation (19). We then compute firms' exposures to the aggregate factors as defined in Equation (53). Notably, our analysis emphasizes the evolution of two networks rather than aggregate shocks, keeping sectoral productivities constant throughout the simulations. To examine whether the betas from our two-period approximation align with

those derived from the full model, we perform a regression of sectoral stock returns on three aggregate factors, controlling for industry fixed effects. The top panel of Table 5 presents the point estimates.

Next, we approximate the β coefficients in Equation (53) by applying a two-period cash-flow assumption. This approach uses the covariance between changes in industry i's output share and changes in aggregate factors, as detailed in Equations (54) through (56). The bottom panel of Table 5 presents the approximated β coefficients and their standard errors from the simulation. It is notable that the coefficient of the concentration factor for each sector should approximate $-\beta_i^c$, as indicated in Equation (52). We find that the approximated exposures derived from the two-period cash flow is consistent with the point estimates from the full model.