An Analytic Theory of Frictional Firm Dynamics*

Vasco M. Carvalho
University of Cambridge and CEPR

Hanbaek Lee
University of Cambridge

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Abstract

We develop a heterogeneous-firm general equilibrium model where frictional firm-level capital dynamics are characterized analytically under a parsimonious set of assumptions. In equilibrium, missing firm-level investments due to endogenous inaction lead to the misallocation of aggregate investment and output, all of which we characterize in closed form. After aggregating firm-level allocations, we show - analytically - that the aggregate investment and output functions are highly nonlinear in the aggregate TFP, displaying heightened volatility and sharp asymmetry over the business cycle, as observed in the data. This nonlinearity stems from the interaction between the endogenous quantity, quality, and sorting margins in the missing investments of inactive firms over the business cycle.

Keywords: Firm dynamics, investment, adjustment costs, misallocation, nonlinear aggregate fluctuations.

JEL Classification: D21, E22, E32

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1 Introduction

How do firm-level frictions shape firm dynamics and macroeconomic aggregates? Despite abundant empirical evidence on the presence of adjustment costs and the resulting inaction and lumpiness that characterizes firm-level investment, there is a lingering debate on the macroeconomic implications of these firm-level frictions. Arguably, one of the main reasons for this inconclusiveness is that models featuring such frictions are often highly complex and, therefore, computational. Given the dearth of analytical insights and benchmarks, debates then ensue on alternative computational strategies, narrow modeling, parameterization choices, degrees of freedom relating to which moments to match, or how to measure such moments in data in the first place.

Against this background, this paper develops an analytically tractable general equilibrium model featuring heterogeneous firms facing non-convex capital adjustment costs. We characterize, analytically, the role of firm-level investment frictions on both micro behavior – i.e. equilibrium firm dynamics and micro-level inaction – and macro outcomes – i.e., both aggregate misallocation at steady state and general equilibrium aggregate dynamics in non-stationary frictional economies. In particular, we: (i) analytically characterize micro-level inaction bands, (ii) derive sufficient conditions establishing when micro-level frictions have no bearing on aggregate dynamics, (iii) provide closed-form solutions for the behavior of equilibrium aggregate investment and output in this class of economies, as nonlinear functions of aggregate productivity and structural parameters alone and, as a result, (iv) show that equilibrium aggregates in frictional economies are more volatile and endogenously negatively skewed, for a given symmetric aggregate productivity process. Finally, (v) we quantitatively show the significant economic impact of the nonlinearities implied by frictional aggregate dynamics.

To model the aggregate investment and output in a frictional economy, we develop a highly tractable general equilibrium model where heterogeneous, profit-maximizing firms combine labor and capital in order to produce consumption goods. While firms face frictionless labor markets, the investment technology features both fixed and quadratic adjustment costs, the former giving rise to periods of firm-level inaction in investment. As is customary in these environments,

firms face both idiosyncratic and aggregate-level, persistent productivity shocks.[1] A representative household makes both consumption and labor supply decisions in spot markets - taking as given the respective prices - and additionally makes consumption-saving decisions, where the sole inter-temporal asset is given by the market portfolio on the underlying firms. Finally, prices are determined in general equilibrium. Relative to the literature, our key modeling assumption is that firms' production technologies are such that the resulting profit functions are homogeneous of degree one with respect to firm-level capital stock, with our leading example being that of capital-augmenting constant returns to scale Cobb-Douglas technology. Analogously to the methodological contribution of Moll (2014) in the context of financial frictions, this assumption is key in delivering tractability: in our setting, firm-level investment becomes purely forward-looking and independent of past history, thus simplifying considerably the state-space of the model and implying that equilibrium aggregation is feasible given knowledge of the productivity of inactive firms in any given period.

Our first theoretical contribution is to provide an analytical solution for firm-level investment under adjustment costs. As is typical in this class of models, the introduction of a fixed cost of adjustment entails an inaction band where firms choose not to invest. We solve analytically for this equilibrium Ss inaction band, whose length depends only on the size of fixed costs, a parameter controlling the importance of convex costs of adjustment and the aggregate productivity state. Intuitively, an increase in either investment adjustment friction parameter lengthens the inaction band. Stochastic changes in aggregate productivity both lengthen the band counter-cyclically – so that frictions have more bite along the extensive margin of inactive firms in recessions – and alter its location with respect to firm-level productivity states – so that, in recessions, the typical 'missing investment firm' is of higher productivity.

Our second theoretical contribution is to characterize aggregate equilibrium investment and output in this frictional general equilibrium setting. We first derive the analytical expressions for equilibrium aggregate investment and output, showing how they can be written in deviations from the frictionless benchmark

[1] We additionally allow for an ansatz firm-level Poisson shock that prevents a firm from undertaking any investment in a given period. This shock does not play a key role in any of our results, but we include it for general purposes.

due to aggregate misallocation stemming from non-convex adjustment costs. Moreover, we analytically decompose the gap between the equilibrium allocation and the benchmark into quantity, quality, and sorting margins in closed forms, all of which are economically interpretable and easily quantifiable. The quantity channel equates to extensive margin fluctuations in the number of inactive firms; the quality channel refers to the fluctuations in the average productivity of inactive firms; the sorting channel stands for fluctuations in the correlation between the foregone investment size and the future idiosyncratic productivity.

Based on our analytical expressions, we first give sufficient conditions for the irrelevance of capital adjustment frictions on the levels and dynamics of aggregate investment. Specifically, we show that whenever the firm-level productivity distribution is symmetric around the midpoint of the Ss inaction band, the presence of firm-level inaction is irrelevant for aggregate investment. Intuitively, within the inaction band, two types of firm-level 'missing investments' co-exist: the missing investment of inactive firms that, absent frictions, would like to add to their capital stock and the missing disinvestment of inactive firms that would optimally reduce their capital stock. Our result shows that whenever this symmetry condition holds, the two effects cancel out; more generally, we show that the closer we are to this symmetry condition, the more muted the effects of frictions on general equilibrium aggregates.[2]

Importantly, we additionally show that macroeconomic irrelevance is generically impossible. This is because, as we show, firm-level inaction is at most neutral in its impact on either aggregate investment or aggregate output, but not both. In other words, even when the more traditional investment irrelevance condition holds, firm-level inaction remains relevant for aggregate output. This is because of persistent firm-level productivity, which renders foregone output growth due to the missing investment greater than the one due to the missing disinvestment.[3]

Our third and main result provides a general equilibrium representation result for aggregate output and investment dynamics for our frictional firm dynamics

[2]This result generalizes the main result in Elsby and Michaels (2019), which in turn is foreshadowed in Caplin and Spulber (1987). The former paper shows that symmetry leads to an 'approximate neutrality' (through Taylor expansions) in partial equilibrium, while our result is derived from the exact analytical solution in general equilibrium. This neutrality condition from the symmetry is independent from the smoothing force of the general equilibrium effect, thus distinct from the result of Khan and Thomas (2008).

[3]According to our analytical decomposition, this effect is due to the sorting channel.

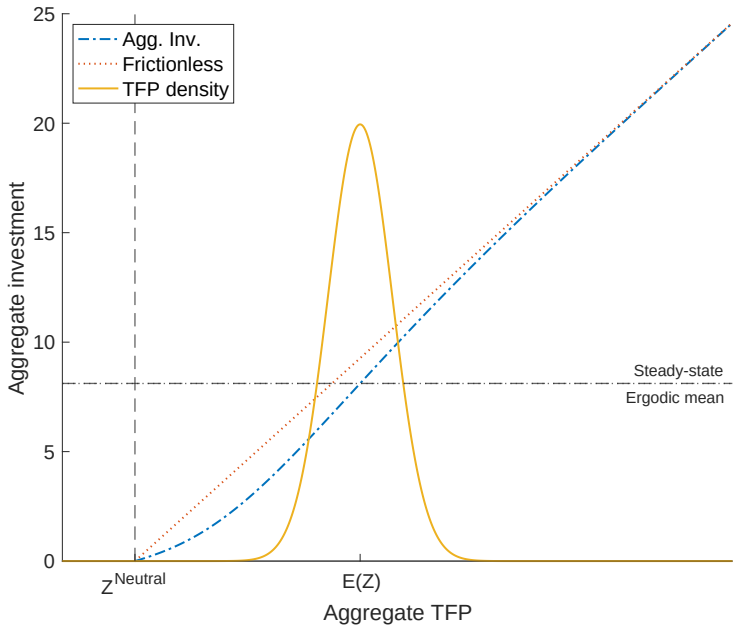
setup under the additional assumption that the representative household's period return function is linear (and separable) in consumption. We then revisit the impact of this assumption in a numerical exercise with concave utility. Under this assumption, and building on our previous set of results, we show that equilibrium aggregate investment is given by the following expression:

$$I(Z) \approx aZ + b - c\phi'_{SN}\left(\frac{d}{Z+e} + f\right)$$

where a, b, c, d, e, f are constants (analytically) pinned down by combinations of structural parameters of the model, Z is the realization of the aggregate productivity shock, and ϕ'_{SN} is the derivative of the standard normal productivity distribution evaluated at the point given by the expression in parenthesis. In the main text, we additionally derive an analogous expression for aggregate output growth. To interpret further the result above, we first note that $aZ + b$ is the general equilibrium solution for aggregate investment under no fixed costs of adjustment (in which case, c is zero). Intuitively, in the absence of non-convex costs of adjustment, the model recovers the equilibrium solution of the representative firm RBC model under risk neutrality, where aggregate investment scales linearly with aggregate productivity. Second, in the aggregate, frictional firm dynamics resolve in an additional aggregate nonlinear term, whose value depends on model parameters and, again, on the realization of the aggregate productivity shock; this is because the function ϕ'_{SN} is nonlinear over its domain. Thus, the aggregation result above delivers a simple yet nonlinear representation of aggregate dynamics in general equilibrium where, despite the complexity of the environment featuring heterogeneous firms and non-convex adjustment costs, aggregate productivity emerges as a sufficient statistic to understand aggregate investment (and output growth, as we also show). To summarize this discussion and understand its implications, Figure 1 plots the aggregate investment function above.

Figure 1 overlays three objects under a standard parameterization of the model's structural parameters. First, it plots the probability density function of the aggregate productivity process with mean $E(Z)$. Second, it plots the frictionless aggregate investment solution as a function of Z realizations, with constant slope b, again an analytic function of structural parameters of the model. Third, it plots the full nonlinear aggregate investment function above, derived under frictional

Figure 1: Aggregate investment function



firm dynamics in general equilibrium.

We see that the two solutions coincide for (very) large realizations of aggregate productivity. Mathematically, this is because the derivative of the standard normal function (ϕ'_{SN}) asymptotes to zero for large values of its argument. Economically, these are cases where the aggregate productivity realization (and hence returns to investment) is so high that fixed adjustment costs do not have a bearing in firms' investment decisions. We additionally see that the two functions intersect for a given low threshold realization of aggregate productivity at $Z^{Neutral}$. This is the aggregate productivity value below which aggregate investment under the frictionless case is negative.[4]

More importantly, in the relevant part of the domain – where aggregate productivity realizations have non-negligible probability – we see the impact of the nonlinearity in aggregate investment function under firm-level non-convex adjustment costs. In particular, what emerges from this encoding of misallocation over the business cycle is an asymmetry of aggregate investment response across

[4]While here we do not plot the behaviour of aggregate investment below this point (because these only occur with vanishing probability) our full solution in the main text extends to this part of the domain as well.

low and high realizations of aggregate productivity, whereby investment declines by more in recessions than rises in expansions. This is to be contrasted to the frictionless case where aggregate investment responds linearly (and thus symmetrically) to aggregate productivity. An important immediate consequence of this nonlinear aggregate behaviour in the frictional economy is that frictional aggregate dynamics are (i) more volatile and (ii) endogenously negatively skewed despite the assumed symmetry of the aggregate productivity process. Further (iii) it is immediate that the ergodic average investment is lower than the steady-state level which, as we argue further, magnifies the welfare costs of business cycles.

Our fourth and last contribution is quantitative. This serves both to quantify the economic significance of the nonlinearity driven by the frictional firm dynamics and to check whether results are quantitatively robust to the inclusion of the concave utility (and its general equilibrium effect through inter-temporal smoothing). We exploit a calibration strategy that is based on analytical expressions for key target moments, which helps sharpen the quantification. According to the simulation based on the fully analytical model with calibrated parameters, firm-level frictional dynamics substantially amplify the volatility and generate significant negative skewness in aggregate investment, output, and consumption. For example, the volatility and skewness of aggregate investment are amplified by 40% and 39%, respectively, via endogenous fluctuations of the inaction band. Due to this endogenous nonlinearity, the model economy's feature business cycle asymmetry, with recessions particularly worse than they would otherwise be, featuring significant drops in investment, output, and consumption.

We then compare the results from the fully analytical model with the quantitative results under log utility, where additional general equilibrium effects potentially can dampen nonlinearity (Khan and Thomas, 2008). The latter is solved with a global nonlinear solution method in the sequence space developed in Lee (2025). The general equilibrium effect now significantly flattens the aggregate investment function: the slope of the linearly approximated frictional investment function decreases by 20% after the inclusion of the log utility. However, the nonlinearity in aggregate dynamics remains economically meaningful: results regarding significantly greater volatility and negative skewness in the frictional case, when compared to the frictionless benchmark, are essentially unaffected.

Related Literature Our paper is primarily related to the literature studying micro level lumpiness and its aggregate implications. Early work in this literature established the theoretical conditions under which the impact of the firm-level Ss cycle on aggregate dynamics is neutralized. One of the seminal results is the neutrality studied by Caplin and Spulber (1987). The paper theoretically shows that the initial uniform distribution of firms over the Ss cycle leads to aggregate neutrality of firm-level inaction. Caballero and Engel (1993) extends the neutrality condition to an environment with firm-level heterogeneity and strategic complementarity. The recent work by Elsby and Michaels (2019) generalizes the sufficient condition using the approximated aggregate investment function. Specifically, when the idiosyncratic productivity distribution becomes symmetric over the inaction band, the missing investment and the disinvestment stemming from the inaction balance out, leading to the neutrality.

Our paper contributes to the literature by sharpening the neutrality condition through exact analytical investment functions at the firm level without approximation. Moreover, we theoretically show that the neutrality cannot hold for both investment and output simultaneously due to the positive correlation between the investment size and the future productivity under the persistent idiosyncratic productivity process.

Regarding the non-neutrality, Caballero and Engel (1991) analyzes the off-the-steady-state deterministic dynamics of the aggregate allocation, which can be explained by the dynamics of the summary indices in analytical forms. According to the result, the firm-level heterogeneity in the Ss cycle crucially impacts the convergence speed to the steady state during the post-shock period. The same authors extend the result (Caballero and Engel, 1999), arguing that firm-level endogenous inaction leads to the nonlinear aggregate dynamics based on the generalized Ss framework, which incorporates a probabilistic Ss cycle. Cooper et al. (1999) shows that the number of firms adjusting capital crucially drives the aggregate investment, and the cross-sectional distribution of the investment vintage affects the shock sensitivity of the aggregate investment.

Building upon these findings, Baley and Blanco (2021) develops a sufficient-statistic approach to explain the convergence speed in the post-shock periods, where the sufficient statistics are two steady-state cross-sectional moments: the

dispersion of capital-to-productivity ratio and the covariance of the ratio and the time from the last adjustment. From the steady-state moments identified from the plant-level data, the authors show the relevance of the micro-level Ss cycle in the aggregate dynamics. Relatedly, Alvarez and Lippi (2022) develops a novel analytical framework that can track the whole profile of the impulse response of any cross-sectional moments to an aggregate shock and applies the method to the model of Caballero and Engel (1999) analyzing the relevance of the firm-level Ss cycle on the aggregate dynamics.

Our framework characterizes the role of firm-level frictions on the aggregate dynamics of investment and output through analytical aggregation. Furthermore, the distance between the frictional aggregate allocations and the frictionless benchmark is decomposed into economically interpretable channels, which helps understand how firm-level frictions affect aggregate allocation. Another advantage of our approach is that the dynamics of aggregate allocations are globally and nonlinearly characterized in general equilibrium.

Thomas (2002) puts forward the role of general equilibrium as an important reason for the aggregate neutrality result. The paper shows that the impact of the firm-level heterogeneity on the aggregate fluctuations is approximately neutralized once the general equilibrium effect is considered. In a similar context, Khan and Thomas (2003) focuses on the nonlinearity in the aggregate dynamics driven by the firm-level Ss cycle in the partial equilibrium and shows that the nonlinearity disappears in the general equilibrium. The same authors show that the approximate neutrality of the firm-level Ss cycle in the general equilibrium remains unaffected, even after the empirically supported firm-level heterogeneity is considered in the model in Khan and Thomas (2008). House (2014) shows that this neutrality is driven by the near-infinite elasticity of firm-level (long-lived) capital adjustment timing to the general equilibrium effect.

However, neutrality via general equilibrium effects is still a topic of debate in the literature. Gourio and Kashyap (2007) points out that the neutrality result is possibly a numerical outcome; a different parametric or structural setup leads to non-neutral aggregate outcomes stemming from firm-level lumpy adjustment. Bachmann et al. (2013) shows that the observed pro-cyclical shock sensitivity and the nonlinear dynamics of the aggregate investment are driven by the firm-level lumpy investments under the assumption of the necessity for the maintenance

investment for operation. Winberry (2021) shows that once the empirically supported interest rate dynamics are considered, the general equilibrium effect does not fully neutralize the impact of the firm-level Ss cycle, resulting in the procyclical aggregate investment sensitivity, as observed in the data. In the paper, an important part of the model is incorporating the observed investment sensitivity to the general equilibrium effect (the interest rate). This is done by matching the semi-elasticity of investment inferred indirectly by the investment sensitivity to the tax credit studied in Zwick and Mahon (2017). Koby and Wolf (2020) shows that the convex adjustment cost can discipline firm-level investment sensitivity at the empirically plausible level.

Our paper contributes to this debate by theoretically characterizing the sufficient condition for the investment neutrality and showing the impossibility of joint neutrality in investment and output in general equilibrium. Moreover, we show that the general equilibrium effect significantly smooths both frictional and frictionless investment volatility, but at the same time, the gap between the two stemming from the firm-level Ss cycle remains significantly large.

On the empirical side, Caballero et al. (1995) empirically analyzes how the frictional investment at the firm level shapes the aggregate investment dynamics in comparison with the frictionless benchmark based on the plant-level capital adjustment data. According to the result, the frictional investment leads to the extra skewness and the kurtosis of the aggregate investment. According to Doms and Dunne (1998), this empirical pattern is due to the firm-level spiky investments, and the fluctuations in the number of firms making large-scale investments are the key drivers of ups and downs in aggregate investments.

These empirical findings are highly consistent with our analytical investment at the firm and macro levels. In particular, the fluctuations in the number (mass) of firms actively investing are the dominant channel for the ups and downs in the investment gap, substantially contributing to the nonlinearity of the aggregate investment dynamics: the aggregate investment features significantly higher skewness and kurtosis than the frictionless benchmark.

Lastly, our model's assumptions and underlying economics are closely related to Moll (2014). In the paper, heterogeneous entrepreneurs save, consume, and operate their own businesses using the constant returns-to-scale Cobb-Douglas technology. Due to the constant returns to scale assumption, the financial con-

straint always binds, so the business return becomes linear in the current wealth. Then, the decision on the saving rate (saving over the wealth) is independent of the current wealth under the CRRA utility. This independence is the key ingredient for the model's analytical tractability.

Similarly, our model's firm produces using constant returns to scale production technology and invests for future operations. Due to the convex adjustment cost that is not dependent upon the capital stock, both intensive and extensive margin investment decisions become isolated from the current capital stock of a firm (Theorem 1). As a result, the inaction band and the active firms' investment size are all characterized in the analytical form.[5]

Roadmap The rest of this paper proceeds as follows. Section 2 explains our baseline model. Section 3 analytically characterizes the firm-level equilibrium allocations. Section 4 analytically aggregates the firm-level allocations and analyzes the aggregate-level allocations. Section 5 explains the parametrization and quantitative exercises. Section 6 concludes.

2 Model

Time is discrete and lasts forever. A unit measure of ex-ante homogeneous atomless firms is considered. A persistent firm-level TFP process makes firms ex-post heterogeneous. Each firm owns the capital stock, making an investment decision one period ahead of the utilization. At the beginning of period t, a firm i is given the capital stock k_{it} and the productivity level z_{it}. Therefore the individual state variables s_{it} are as follows:

$$s_{it} = \{k_{it}, z_{it}\}.$$

In each period, the rational household and firms understand the aggregate

[5]More generally, our paper is also related to the literature that studies the misallocation at the micro level and its macroeconomic implications (Restuccia and Rogerson, 2008; Hsieh and Klenow, 2009; Asker et al., 2014; David and Venkateswaran, 2019) We provide the analytical characterization of the misallocation and its fluctuations over the business cycle. Therefore, the role of firm-level parameters such as idiosyncratic productivity volatility on the misallocation can be analytically investigated. In our framework, misallocation is the key source of the nonlinearity in the business cycle.

productivity level Z_t and the distribution Φ_t of the individual states s_{it}. Therefore, the vector of the aggregate state variables S_t are as follows:

$$S_t = \{Z_t, \Phi_t\}.$$

2.1 Technology

Assumption 1 (Production technology)**.**
Firm i operates using a production function F that leads to a homogeneous-of-degree-one operating profit Π with respect to capital stock k_{it}:

$$\lambda\Pi(z_{it}Z_tk_{it}; S_t) = \Pi(\lambda z_{it}Z_tk_{it}; S_t), \text{ for } \forall\lambda > 0.$$
$$s.t.\ \Pi(z_{it}Z_tk_{it}; S_t) = \max_{n_{it}} F(z_{it}Z_tk_{it}, n_{it}; S_t) - w_tn_{it}$$

where z_{it} is capital-augmenting idiosyncratic productivity; Z_t is capital-augmenting aggregate productivity; k_{it} is capital stock; n_{it} is labor demand; S_t is the aggregate state; w_t is wage.

Then, from the homogeneity property, the following equation holds:

$$\Pi(z_{it}Z_tk_{it}; S_t) = \Pi(1; S_t)z_{it}Z_tk_{it}.$$

For example, given a capital stock k_{it} and the productivity z_{it}, consider a firm maximizes the operating profit using the CRS Cobb-Douglas production function where the aggregate and idiosyncratic productivities are capital-augmenting:[6]

$$\begin{aligned}\Pi_{it} :&= \max_{n_{it}} (Z_tz_{it})^\alpha k_{it}^\alpha n_{it}^{1-\alpha} - w_tn_{it} \\ &= \alpha\left(\frac{1-\alpha}{w_t}\right)^{\frac{1-\alpha}{\alpha}} Z_tz_{it}k_{it},\end{aligned}$$

where w_t is wage.

The firm-level productivity z_{it} and aggregate productivity Z_t follow AR(1)

[6] Moll (2014) also assumes the capital-augmenting productivity process.

processes:

$$z_{it+1} = \rho z_{it} + \epsilon_{it+1}, \quad \epsilon_{it+1} \sim_{iid} G^z, \quad \text{s.t. } \mathbb{E}\epsilon_{it+1} = \bar{\epsilon} > 0 \text{ and } Var(\epsilon_{it+1}) = \sigma^2_{\epsilon}.$$
$$Z_{t+1} = \rho_{agg} Z_t + \epsilon_{t+1}, \quad \epsilon_{t+1} \sim_{iid} G^z_{agg}, \quad \text{s.t. } \mathbb{E}\epsilon_{t+1} = \bar{\epsilon}_{agg} > 0 \text{ and } Var(\epsilon_{it+1}) = \sigma^2_{agg}.$$

The theoretical model implications stay unaffected under different productivity specifications. The particular specification above is assumed for the analytical tractability, which is one of the most common forms in the literature.

2.2 Firm-level investment and the real and financial frictions

A firm's capital stock evolves in the following law of motion:

$$k_{it+1} = (1 - \delta)k_{it} + I_{it}$$

where I_{it} is the endogenously determined firm-level investment. The optimal investment decision is subject to the following frictions: 1) time to build. 2) convex adjustment cost, 3) fixed adjustment cost, and 4) financial constraint. The details of the assumptions are elaborated as follows:

Assumption 2 (Frictional capital adjustment)**.**
Firm i's non-zero capital adjustment is subject to a unit period of time to build and entails convex adjustment cost C_{it} and fixed adjustment cost $\xi > 0$, where Ψ_{it} takes the following form:

$$\Psi_{it} := \Psi(I_{it}) = \frac{\mu}{2} I^2_{it}$$

Lastly, with a probability of $\varphi > 0$, an i.i.d Poisson financial friction shock arrives, which makes the firm not able to adjust a capital stock exogenously.

Any non-zero investment takes a fixed adjustment cost $\xi > 0$. Therefore, net depreciation occurs in the capital stock when a firm does not invest. The firm's capital stock increases only if the firm makes positive investment and pays a convex and fixed adjustment cost. Therefore, a firm-level investment follows an (S, s) cycle in the equilibrium. Each firm faces a financial constraint in its

investment decision at a Poisson rate $\varphi > 0$.[7] We assume financially constrained firms cannot make an adjustment in the capital.[8]

2.3 Recursive formulation of household's problem

In this section, we present the recursive form of the representative household's problem. For the brevity of the notation, we drop the time subscript and denote the future allocation with an apostrophe. In the beginning of a period, the representative household is given with an equity portfolio a. The household has information on the current aggregate states $S = \{Z, \Phi\}$, where Z is the contemporaneous aggregate productivity; Φ is the contemporaneous distribution of individual states s. Households rationally expect how the endogenous aggregate state evolves in the next period: $\Phi' = M_H(S)$. Based on the rational expectation, the household decides consumption, saving, and labor supply. The recursive problem of the household is as follows:

$$
\begin{aligned}
V(a;S) = \max_{c,a',l_H} \quad & u(c,l_H) + \beta \mathbb{E} V(a';S') \\
\text{s.t. } & c + \int \Gamma_{Z,Z'} q(S,S') a'(k,z) d\Phi' = w(S) l_H + \int a(k,z) d\Phi \\
& M_H(S) = \Phi', \quad S = \{\Phi, Z\}
\end{aligned}
$$

where V is the value function of the household; c is consumption; a' is a future equity portfolio; l_H is labor supply; $w(S)$ is wage; $\Gamma_{Z,Z'}$ is the exogenous aggregate state transition probability from A to A'; $q(S,S')$ is the stochastic discount factor. The market portfolio is a financial vehicle that aggregates the all the existing firms.

2.4 Recursive formulation of firms' problem

In this section, we present the recursive form of the firm-level problem. For the brevity of the notation, we drop the subscripts and denote future allocations with the apostrophe.

[7] The exogenous *i.i.d.* Poisson shock is similar to the Calvo price shock in the New Keynesian models.

[8] This assumption can be further generalized combined with a constraint, but we have not included in the baseline model for the sake of the tractability.

At the beginning of each period, a firm is given individual state variables $s = \{k, z\}$ and informed of the current aggregate states $S = \{Z, \Phi\}$, where Z is the contemporaneous aggregate productivity; Φ is the contemporaneous distribution of individual firms. Firms rationally expect how the aggregate states evolve in the next period: $S' = G(S)$. The recursive form of the firm-level problem is as follows:

$$
\begin{aligned}
J(k, z; S) &= \Pi(zZk; S) + (1 - \varphi) \max\{R^*(k, z; S) - \xi, R^c(k, z; S)\} + \varphi R^c(k, z; S) \\
R^*(k, z; S) &= \max_I -I - \Psi(I) + \mathbb{E}_{z,S} q(S, S') J(k(1 - \delta) + I, z'; S') \\
R^c(k, z; S) &= \frac{1}{R(S)} \mathbb{E}_{z,S} J(k(1 - \delta), z'; S')
\end{aligned}
$$

$$
\begin{aligned}
\text{(Operating profit)} \quad & \Pi(zZk; S) := \max_n F(zZk, n; S) - wn \\
\text{(Idiosyncratic productivity)} \quad & z' = G_z(z) \text{ (AR(1) process)} \\
\text{(Stochastic discount factor)} \quad & q(S, S') = \beta u_c(c', l'_H) / u_c(c, l_H) \\
\text{(Aggregate states)} \quad & S = \{Z, \Phi\} \\
\text{(Aggregate law of motion)} \quad & \Phi' = M(S)
\end{aligned}
$$

where J is the value function of the firm; R^* and R^c are the interim value functions for capital adjusters and non-adjusters, respectively.

2.5 Recursive competitive equilibrium

In this section, we define the recursive competitive equilibrium in the economy. $(g_c, g_a, g_{l_H}, g_I, g_n, V, J, R^*, R^c, q, w, M, M_H)$ is a recursive competitive equilibrium if the following conditions are satisfied.

1. g_c, g_{lH}, V and g_a, solves the household's problem.
2. g_I, g_{n_d}, J, R^*, and R^c solve a firm's problem.
3. Market Clearing:

$$
\begin{aligned}
\text{(Labor Market)} \quad & g_{lH}(\Phi; S) = \int g_{n_d}(k, z; S) d\Phi \\
\text{(Product Market)} \quad & g_c(\Phi; S) = \int \Big(\Pi(1; S) zZk - g_I(k, z; S) - \Psi(g_I(k, z; S)) \Big) d\Phi
\end{aligned}
$$

4. Consistency Condition:[9]

$$M_H(S) = M(S) = \Phi', \text{ where for } \forall K' \subseteq \mathbb{K} \text{ and } z' \in \mathbb{Z},$$
$$\Phi'(K', z') = \int \Gamma_{z,z'} \left(\mathbb{I}\{(g_I(k, z; S) + (1-\delta)k) \in K'\} \right) d\Phi$$

3 Analytics of frictional firm dynamics

3.1 The unconstrained optimal investment

We define the firm-level unconstrained optimal investment $I^*(s; S)$ as follows:

$$I^*(s; S) := \arg\max_I -I - \frac{\mu}{2} I^2 + \mathbb{E}_{z,S} q(S, S') J(k(1-\delta) + I, z'; S')$$

which is the optimal investment level when a firm adjust its capital stock.

Theorem 1 (Forward-looking affine investment)**.**
Unconstrained firm-level optimal investment is:

$$I^*(z; S) = \frac{A(z; S) - 1}{\mu}$$

where $A(z; S)$ the sum of the discounted future marginal profit and can be expressed in the following affine form in z:

$$A(z; S) = \mathcal{G}(S) z + \mathcal{H}(S)$$

for some $\mathcal{G}, \mathcal{H} : \mathbb{S} :\to \mathbb{R}$ where $\mathbb{S}$ is measurable w.r.t. filtration of the current period.

Proof. See Appendix. ■

The intuition behind the theoretical result is that if a unit of capital stock is added, the capital contributes to the production in the entire future period in a linearly additive way, independent of the future capital adjustment or non-adjustment. In an economy with decreasing returns to scale production function, the future marginal benefit of a unit capital investment depends on the future

[9] $\mathbb{K}$ and $\mathbb{Z}$ are the supports of the marginal distributions of capital and productivity induced from Φ.

capital stock, as the scale matters. Then, optimal investment needs to consider different benefits contingent on future capital adjustments, which often disturbs the analytical characterization.

In our model, investment decisions are determined by the sum of discounted expected future marginal benefits, making firm-level investment independent of existing capital stock. This formulation renders investment purely a function of individual productivity shocks and aggregate economic conditions. This approach differs fundamentally from traditional models where firms must consider their accumulated capital when making investment decisions. Such traditional frameworks implicitly make firms partially backward-looking since capital stock reflects past shock realizations. Our model, by contrast, establishes a purely forward-looking investment mechanism focused on anticipated economic benefits rather than historical idiosyncratic shocks. Nevertheless, investment still exhibits persistence due to the inherent persistence in the shock process itself.

Another distinctive feature of our firm dynamics model is the absence of a satiation point in capital accumulation. Many conventional firm dynamic models characterize investment incentives as the gap between current and optimal future capital stock, which creates an implicit satiation point where investment naturally decelerates. Our model, however, posits that firms maintain infinite growth appetite regardless of accumulated capital—they remain "infinitely greedy." In this framework, without intensive margin adjustment frictions such as convex adjustment costs, firm size would become theoretically undefined. The introduction of convex adjustment costs establishes equilibrium by identifying the investment level where marginal benefits and costs intersect, thereby anchoring firm size appropriately. Despite this theoretical possibility for unbounded growth, firm size remains bounded in practice due to the stationary, mean-reverting nature of the idiosyncratic productivity process.

The optimal investment strictly increases in the idiosyncratic productivity z and strictly decreases in the convex adjustment cost parameter μ. $\mathcal{G}$ is the slope of the future marginal benefit that interacts with contemporaneous idiosyncratic productivity z. $\mathcal{H}$ is the intercept of the marginal benefit independent of z. $\mathcal{G}$ and $\mathcal{H}$ fluctuate over the business cycle depending on the aggregate state realization S. In Section 4, we provide the fully analytical form of $\mathcal{G}$ and $\mathcal{H}$ under a set of mild assumptions.

3.2 The constrained optimal investment

In this section, we analytically characterize the firm-level investment decision in the extensive margin: whether to invest or not. The optimal investment size decision is independent of the existing capital stock as shown from Theorem 1. Similarly, the extensive margin decision of investing also becomes independent of the firm's capital stock, which means the idiosyncratic productivity is the only state that matters for the extensive margin decision. The following theorem characterizes the threshold rule of a firm's investment with respect to the productivity level.

Theorem 2 (The inaction region of productivity)**.**
Under Assumption 1 and 2, a firm does not invest when the productivity z lies in the inaction interval $\Omega = [z_L, z_H]$ specified as below:

$$\Omega(S) := [z_L(S), z_H(S)] = \left[\frac{1 - \sqrt{2\mu\xi} - \mathcal{H}(S)}{\mathcal{G}(S)}, \frac{1 + \sqrt{2\mu\xi} - \mathcal{H}(S)}{\mathcal{G}(S)} \right]$$

Proof. See Appendix. ■

It is worth noting that the inaction band is not firm-specific and varies over the business cycle. Then, the width of the stochastic inaction band is as follows:

$$|z_H(S) - z_L(S)| = \frac{2\sqrt{2\mu\xi}}{\mathcal{G}(S)}$$

Corollary 1 (The role of frictional parameters on the inaction)**.**
An increase in the adjustment friction parameters μ and ξ widens the inaction band.

Proof. See Appendix. ■

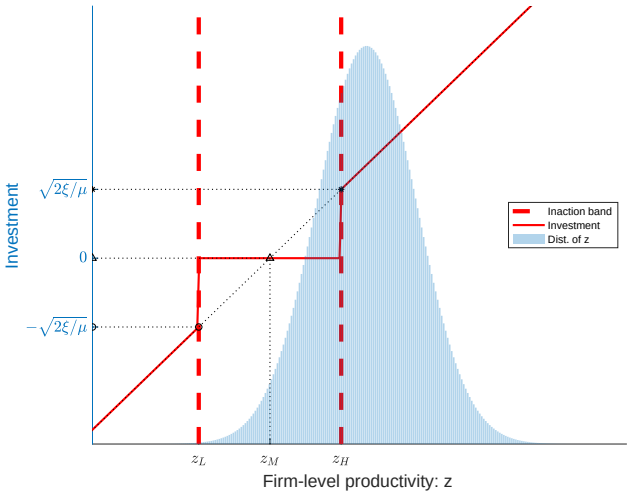
Our analytical framework sharply distinguishes the role of two adjustment cost parameters in shaping firm-level investment dynamics: the convex adjustment cost parameter μ affects the optimal investment level and the inaction band, while the fixed adjustment cost ξ affects the firm-level investment only through the extensive margin (inaction).

Figure 2 illustrates the optimal investment and the inaction band. In the figure, the horizontal axis is the idiosyncratic productivity, and the vertical axis is

the level of investment. The bell-shaped curve illustrates the probability density function of idiosyncratic productivity. The thick line is the constrained-optimal investment curve as a function of idiosyncratic productivity. In the figure, the investment curve becomes flat for the productivity levels belonging to the inaction band, as the investment becomes zero due the inaction.

In this economy, the distribution of the idiosyncratic productivity stays unchanged over the business cycle. However, the unconstrained optimal investment function $I^*(z;S)$ and the inaction band shifts over the business cycle, resulting in the aggregate investment fluctuations.

Figure 2: The optimal firm-level investment as a function of productivity



In the aggregate investment fluctuations, one of the key variables is the missing investments belonging to the inaction band. There are two relevant margins in the missing investments: the measure of the missing firms (quantity) and the productivity (quality). The length of inaction band and the mass of the firms belonging to the band will be the core determinants of the quantity margin. The key allocation for the quality margin is the midpoint of the inaction band, which can be analytically characterized:

Corollary 2 (The characteristics of the inaction band)**.**

The midpoint of the inaction band z_M satisfies

$$z_M(S) = \frac{1 - \mathcal{H}(S)}{\mathcal{G}(S)},$$

and the unconstrained optimal investment at the midpoint is zero: $I^*(z_M; S) = 0$.

Proof. See Appendix. ■

The inaction midpoint z_M fluctuates over the business cycle, determining the quality of the missing investments. The quantity and the quality margins are the key forces in shaping the aggregate investment dynamics and the nonlinearity.

Thus, an alternative expression of the inaction boundaries is as follows:

$$z_H(S) = z_M(S) + \frac{\sqrt{2\mu\xi}}{\mathcal{G}(S)}$$
$$z_L(S) = z_M(S) - \frac{\sqrt{2\mu\xi}}{\mathcal{G}(S)}$$

On top of the real frictions analyzed so far, a financial friction shock (with probability φ) constrains the investment opportunity in the extensive margin. Therefore, the constrained optimal investment $\widehat{I}(s; S)$ is as follows:

$$\widehat{I}(z; S) = \begin{cases} \frac{\mathcal{G}(S)z + \mathcal{H}(S) - 1}{\mu} & \text{if } z \notin \Omega(S), \text{ with probability } 1 - \varphi \\ 0 & \text{otherwise} \end{cases}$$

We close the firm-level analysis by establishing a theoretical linkage between the firm-level investments and the macroeconomic dynamics through the inaction band dynamics. The analytical characterizations indicate that the macroeconomic conditions affect the firm-level incentive to invest through $\mathcal{G}$ and $\mathcal{H}$. In particular, under the fixed adjustment cost, $\mathcal{G}$ and $\mathcal{H}$ shape the dynamics of the inaction band, which bring in two distinct effects: the quantity and quality effects. The former is through the (de-)synchronization of investment timings across the firms, and the latter is through the selection effect driven by the endogenous choice of inactions.

Proposition 1 (Synchronization: quantity margin)**.**
If a shift in the aggregate TFP shock leads to $\Delta\mathcal{G}(S) > 0$, then the inaction band shrinks.

Proof. It is immediate from $\frac{\partial |z_H - z_L|}{\partial \mathcal{G}(S)} < 0$. ■

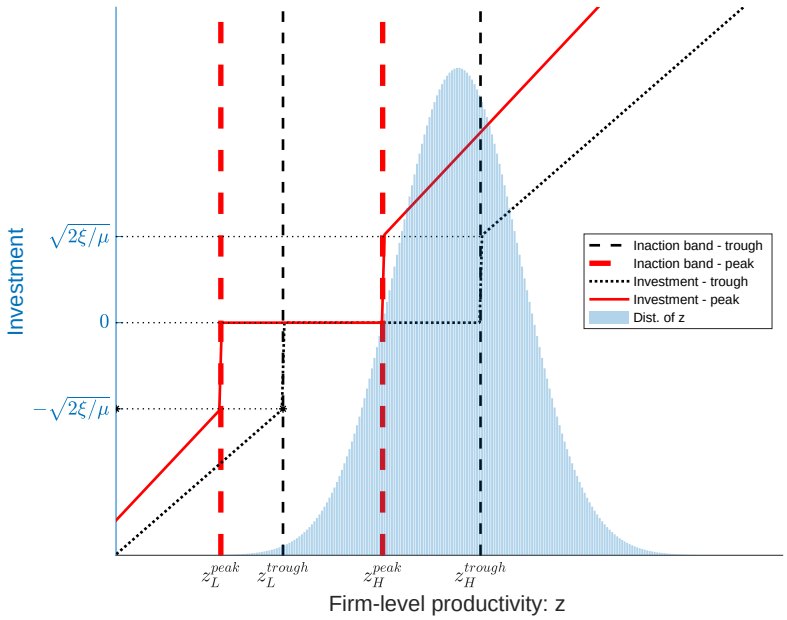
Proposition 1 states the inaction band shrinks when a macroeconomic environment increases $\mathcal{G}$, the inaction band shrinks. Therefore, a greater portion of firms invest together, which is the synchronization effect. Conversely, if $\mathcal{G}$ decreases, de-synchronization happens.

Proposition 2 (Aggregate-level selection effect: quality margin)**.**
If a shift in the aggregate TFP shock leads to $\Delta\mathcal{G}(S) > 0$ *and* $\Delta\mathcal{H}(S) > 0$*, then the inaction midpoint decreases.*

Proof. See Appendix. ■

Proposition 2 states that when a macroeconomic environment increases both $\mathcal{G}(S)$ and $\mathcal{H}(S)$, the average quality of inactive firms decreases. Therefore, relatively poor-quality firms jump into investment, which is a negative selection effect. Conversely, if both $\mathcal{G}$ and $\mathcal{H}$ decrease, a positive selection happens. Figure 3 illustrates the constrained optimal investment at the firm level for peak and

Figure 3: Shift of inaction bands and the firm-level investment policy



trough periods.[10] These two channels crucially determine the aggregate misallocation level driven by the firm-level inactions. Later, under a fully analytical setup, we show that that $d\mathcal{G}(S)/dZ > 0$ and $d\mathcal{H}(S)/dZ > 0$ (Proposition 10).

4 Analytical aggregation and the macroeconomy

4.1 Analytical aggregation: investment

We first define the frictionless aggregate investment assuming zero fixed adjustment cost as a benchmark.[11] Among the frictions, we focus on the role of fixed adjustment cost, as it is the crucial friction that qualitatively distinguishes the capital dynamics in the heterogeneous firm models from the representative counterpart. We denote Φ as the stationary distribution of the idiosyncratic productivity. The frictionless aggregate investment $I^{NoFixed}$ is defined as follows:

$$\begin{aligned} I^{NoFixed}(S) &:= (1-\varphi)\int I^*(z;S)d\Phi(z) = (1-\varphi)\frac{\mathcal{G}(S)\mathbb{E}z + \mathcal{H}(S) - 1}{\mu} \\ &= \frac{1-\varphi}{\mu}\left(\mathcal{G}(S)\left(\frac{\bar{\epsilon}}{1-\rho}\right)\right) + \frac{1-\varphi}{\mu}\left(\mathcal{H}(S) - 1\right) \\ &= \frac{1-\varphi}{\mu}\mathcal{G}(S)\left(\frac{\bar{\epsilon}}{1-\rho} - z_M(S)\right) \end{aligned} \tag{1}$$

It is worth noting that in Equation (1), the integration is taken over the entire support of the firm-level productivity z without the inaction band considered.

To characterize the frictional aggregate investment, we define moments of the idiosyncratic productivity distribution inside the inaction bands.

Definition 1 (Moments over the inaction band)**.**
$M_n(S)$ is defined as the n^{th} (non-central) moment of idiosyncratic productivity z over the inaction band $\Omega(S)$.

$$M_n(S) := \int_{z\in\Omega(S)} z^n d\Phi_z$$

[10] The functions are based on the calibrated parameters in Section 5.

[11] For this definition, the financial shock and the convex adjustment cost are still considered. Without the convex adjustment cost, the investment size cannot be well defined in this model environment.

Similar to the frictionless benchmark, the aggregate investment is defined as follows:

$$\begin{aligned} I(S) &:= \int \widehat{I}(z;S) d\Phi(z) \\ &= I^{NoFixed}(S) - \Delta_I \end{aligned} \tag{2}$$

where $\Delta_I := \frac{1-\varphi}{\mu} \left(\mathcal{G}(S) M_1(S) + (\mathcal{H}(S) - 1) M_0(S) \right)$ represents the missing aggregate investment (misallocation) due to the presence of the inaction band. We interchangeably call this term investment gap. Δ_I can be dribbled further to a more intuitive form.

$$\begin{aligned} \Delta_I &= \frac{1-\varphi}{\mu} \left(\mathcal{G}(S) M_1(S) + (\mathcal{H}(S) - 1) M_0(S) \right) \\ &= \frac{1-\varphi}{\mu} \mathcal{G}(S) M_0(S) \left(\frac{M_1(S)}{M_0(S)} - \underbrace{z_M(S)}_{\text{Inaction midpoint}} \right) \end{aligned} \tag{3}$$

From Equation (3), we deduce the following equivalence:

$$\Delta_I = 0 \iff \underbrace{\frac{M_1(S)}{M_0(S)}}_{\text{Average productivity among inaction}} - \underbrace{z_M(S)}_{\text{Inaction midpoint}} = 0, \tag{4}$$

when $M_0 > 0$. If $M_0 = 0$ is true, the investment gap trivially becomes zero. M_1/M_0 is the ratio between the first and the zero moments, which can be interpreted as the average productivity of the inaction band. Line (4) shows that the investment gap is zero if the average productivity among the inactive firms is the same as the inaction midpoint. That is, the extensive margin in the firm-level investment is perfectly washed out after aggregation if Φ distribution is symmetric around the midpoint on the inaction band.

Proposition 3 (The sufficient condition for the investment neutrality)**.**
If the stationary distribution Φ is symmetric around z_M on the inaction band $\Omega(S)$, the presence of firm-level lumpy investment is irrelevant to the aggregate investment:

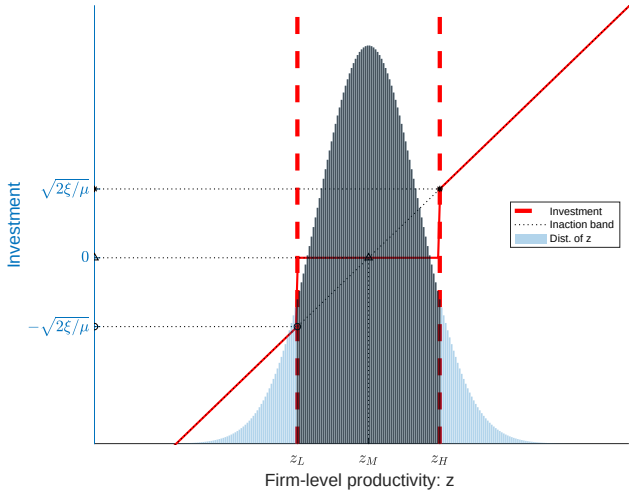
$$I(S) = I^{NoFixed}(S)$$

Proof. See Appendix. ■

The intuition behind the result is in the balance between the positive and the negative investments when the idiosyncratic productivity distribution is symmetric around the inaction mid point. It is worth noting that the unconstrained optimal investment is zero at the inaction midpoint according to Corollary 2. Then, the total missing investments due to the inaction becomes zero ($\Delta_I = 0$) by the exact cancelling out between the positive and negative investments, leading to the neutrality. Here the exact cancel out of the optimal investment is due to the firm-level unconstrained optimal investment I^* being linear in z as shown in Theorem 1.

Figure 4 illustrates an example of the neutrality of the firm-level lumpiness. The horizontal axis is idiosyncratic productivity and the square bracket indicates a hypothetical inaction band. Then, the sum of unconditional optimal investments inside the inaction band is zero, satisfying the aggregate neutrality.

Figure 4: An example of the irrelevance case



The aggregate neutrality of the firm-level lumpy investment has been an important issue in the firm dynamics and the business cycle literature. Proposition 3 analytically characterize the sufficient condition when the lumpy investment becomes irrelevant in the macroeconomy. The result is similar to the main result of Elsby and Michaels (2019). Their paper shows that the symmetry leads to

the neutrality based on the analytical approximations, while our result is derived from the exact analytical solution. This neutrality condition from the symmetry is independent from the smoothing force of the general equilibrium effect, thus distinguished from the result of Khan and Thomas (2008).

In the following proposition, we provide the analytical characterization of the investment gap:

Proposition 4 (The analytical investment gap)**.**
The investment gap Δ_I satisfies the following relationship:

$$\Delta_I = I^{NoFixed}\left(\Phi^{SN}(z_H^*(S)) - \Phi^{SN}(z_L^*(S)) + \frac{\phi^{SN}(z_H^*(S)) - \phi^{SN}(z_L^*(S))}{z_M^*(S)}\right)$$

where $z_j^* := \frac{z_j - \frac{\bar{\epsilon}}{1-\rho}}{\sqrt{\frac{\sigma_\epsilon^2}{1-\rho^2}}}$, $j \in \{H, M, L\}$ is the normalized stochastic variable. ϕ^{SN} and Φ^{SN} are the probability density function and cumulative distribution function of the standard normal distribution.

Proof. See Appendix. ■

The analytical form is closely related to the inverse Mills ratio, both conceptually and technically. The inverse Mills ratio captures the conditional probability of a random variable, possibly under the selection. Similarly, the investment gap is the conditional sum of the investment over the inaction band, where the inaction is a type of selection. Moreover, due to the assumption of the idiosyncratic productivity following the normal distribution, the analytical form of the investment gap closely mimics the inverse Mills ratio, except for the normalization. Using the analytical form, we can characterize the aggregate investment in the following form:

$$\begin{aligned} I(S) &= I^{NoFixed}(S) - \Delta_I \\ &= I^{NoFixed}\left(1 - \underbrace{(\Phi^{SN}(z_H^*(S)) - \Phi^{SN}(z_L^*(S)))}_{\Delta_I\text{: Quantity channel}} - \underbrace{\frac{\phi^{SN}(z_H^*(S)) - \phi^{SN}(z_L^*(S))}{z_M^*(S)}}_{\Delta_I\text{: Quality channel}}\right) \end{aligned}$$

In the analytical form of the investment gap, two distinct channels are additively separated: the quantity and the quality margins. The quantity margin refers to $\Phi^{SN}(z_H^*(S)) - \Phi^{SN}(z_L^*(S))$ which captures the measure of the inactive firms. The quality margin refers to the remaining term, which is a function of the inverse normalized inaction midpoint. When the inaction midpoint lies below the average productivity, z_M^* is negative, and $\phi^{SN}(z_H^*) > \phi^{SN}(z_L^*)$, which makes Δ_I increases in z_M^*. Later, we show that the empirically supported variations of the inaction band are when z_M lies below the average productivity.

Then, we determine the sign of the investment gap in relation to the frictionless aggregate investment.

Proposition 5 (The sign of the investment gap)**.**
The investment gap Δ_I and the frictionless investment $I^{NoFixed}$ satisfy the following relationships:

$$\begin{aligned} &i) \quad \Delta_I / I^{NoFixed} > 0, \\ &ii) \quad |I^{NoFixed}| > |\Delta_I|. \end{aligned}$$

Proof. See Appendix. ■

Proposition 5 implies that as long as the frictionless benchmark is positive, the aggregate investment gap is positive, and the magnitude is smaller than the frictionless benchmark. If frictionless benchmark is negative, the aggregate investment gap is negative, while the magnitude is still smaller than the frictionless benchmark. Therefore, we the following statement holds:

$$\begin{aligned} I^{NoFixed}(S) > 0 \implies I^{NoFixed}(S) > I(S) \quad &\text{[Underinvestment]} \\ I^{NoFixed}(S) < 0 \implies I^{NoFixed}(S) < I(S) \quad &\text{[Overinvestment]} \end{aligned}$$

When the frictionless aggregate investment is positive, the investment gap leads to the underinvestment, and vice versa for the positive frictionless aggregate investment. Likewise, the signs of the aggregate allocations tightly co-move as summarized in the following corollary:

Corollary 3 (The equivalence relations I)**.**

The following equivalence conditions hold:

$$
\begin{aligned}
&(i) \quad z_M^*(S) = 0 \iff I^{NoFixed}(S) = 0 \iff \Delta_I(S) = 0 \iff I(S) = 0 \\
&(ii) \quad z_M^*(S) < 0 \iff I^{NoFixed}(S) > 0 \iff \Delta_I(S) > 0 \iff I(S) > 0 \\
&(iii) \quad z_M^*(S) > 0 \iff I^{NoFixed}(S) < 0 \iff \Delta_I(S) < 0 \iff I(S) < 0
\end{aligned}
$$

Proof. The equivalences are immediate from the definition of $I^{NoFixed}$ and Proposition 5. ■

As we will show later, the empirically supported case is (ii) where the inaction midpoint is left to the average idiosyncratic productivity, and the misallocation is positive leading to the underinvestment in equilibrium compared to the frictionless benchmark.

In the analytical characterization of the investment gap, the two key channels are quantity and quality channels. In the following approximation, we show that the dynamics of the inaction midpoint can summarize the dynamics of the combined effects from the two channels. The approximated form is as follows:[12]

$$
\begin{aligned}
\Delta_I &\approx \Delta_I^{Approx} \\
&:= I^{NoFixed}\left(\frac{2\sqrt{2\mu\xi}}{\mathcal{G}(S)\sqrt{\frac{\sigma_\epsilon^2}{1-\rho^2}}}(1-\zeta_2)\phi^{SN}(\zeta_1 z_M^*)\right) \\
&= \frac{(1-\varphi)2\sqrt{2\mu\xi}(1-\zeta_2)}{\mu}\phi^{SN}(\zeta_1 z_M^*)(-z_M^*) \\
&= \frac{(1-\varphi)2\sqrt{2\mu\xi}(1-\zeta_2)}{\zeta_1\mu}(\phi^{SN})'(\zeta_1 z_M^*)
\end{aligned}
$$

where ϕ^{SN} is the probability density function of the standard normal distribution; $\zeta_1, \zeta_2 \in \mathbb{R}$ are the coefficients of approximation that minimize the approximation error for the quantity and the quality channels. In the quantitative exercise, we verify the accuracy of the approximation using the calibrated parameters. Using the approximated investment gap, we can also approximate the aggregate invest-

[12] The details of the approximation are explained in Appendix A.

ment in the following form:

$$
\begin{aligned}
I(S) &\approx I^{Approx} \\
&:= I^{NoFixed} - \frac{(1-\varphi)2\sqrt{2\mu\xi}(1-\zeta_2)}{\zeta_1\mu}(\phi^{SN})'(\zeta_1 z_M^*)
\end{aligned}
$$

Our theoretical results throughout the paper do not rely on the approximated form above, as the equilibrium allocations are already analytically characterized. However, the approximated form has its value in that it provides a simple and intuitive form to understand the dynamics, which is summarized by the inaction midpoint variation.

4.2 Analytical aggregation: output

In this section, we analytically aggregate the aggregate output. Similar to the aggregate investment, the output can be characterized by the combination of a frictionless output benchmark $Y^{NoFixed}$ and the output gap Δ_Y that is driven by the last period's missing investments of the inactive firms. We denote the last period's variables by the subscript of -1.

$$
\begin{aligned}
Y(S) &= \frac{\Pi(1;S)}{\alpha} Z \int z k d\Phi^{joint}(k,z) \\
&= \frac{\Pi(1;S)}{\alpha} Z \int (\rho z_{-1} + \epsilon)((1-\delta)k_{-1} + \widehat{I}(z_{-1};S_{-1}))d\Phi^{joint} \\
&= Y^{NoFixed}(S) - \frac{\Pi(1;S)}{\alpha} Z \int_{z_{-1}\in\Omega(S_{-1})} (\rho z_{-1} + \epsilon) I^*(z_{-1};S_{-1}) d\Phi \\
&= Y^{NoFixed}(S) - \Delta_Y
\end{aligned}
$$

where Φ^{joint} is the joint distribution of the firm-level capital and productivity; $\Delta_Y := \frac{\Pi(1;S)}{\alpha} \int_{z_{-1}\in\Omega(S_{-1})} (\rho z_{-1} + \epsilon) I^*(z_{-1};S_{-1}) d\Phi$ is the output gap driven by the last period's inactive firms. We also define the frictionless benchmark of the output growth $dY^{NoFixed}$ that would be achieved by frictionless investment:

$$
\begin{aligned}
dY^{NoFixed} &:= \frac{\Pi(1;S)}{\alpha} Z(1-\varphi) \int (\rho z_{-1} + \epsilon)\, I^*(z_{-1};S_{-1}) d\Phi \\
&= \frac{\Pi(1;S)}{\alpha} Z(1-\varphi) \int (\rho z_{-1} + \epsilon)\, \frac{A(z_{-1};S_{-1}) - 1}{\mu} d\Phi
\end{aligned}
$$

The frictional counterpart is the output growth dY that is driven by the firm-level frictional investments:

$$\begin{aligned} dY &:= \frac{\Pi(1;S)}{\alpha} Z \int_{z_{-1}\notin\Omega(S_{-1})} (\rho z_{-1}+\epsilon)\, I^*(z_{-1};S_{-1}) d\Phi \\ &= \frac{\Pi(1;S)}{\alpha} Z \int_{z_{-1}\notin\Omega(S_{-1})} (\rho z_{-1}+\epsilon)\, \frac{A(z_{-1};S_{-1})-1}{\mu} d\Phi \end{aligned}$$

Thus, the following relationship holds:

$$dY = dY^{NoFixed} - \Delta_Y.$$

By combining the analytical form of the firm-level investment and the moments in the inaction domain, we can decompose the output gap into two parts: the level and the sorting channels.

$$\begin{aligned} \Delta_Y &:= \frac{\Pi(1;S)}{\alpha} Z(1-\varphi) \int_{z_{-1}\in\Omega(S_{-1})} (\rho z_{-1}+\epsilon)\, \frac{A(z_{-1};S_{-1})-1}{\mu} d\Phi \\ &= \frac{\Pi(1;S)}{\alpha} Z\bar{z}\Delta_I(S_{-1}) + \frac{\Pi(1;S)}{\alpha} Z\rho\Delta_{II}(S_{-1}) \\ &= \frac{\Pi(1;S)}{\alpha} Z \left(\underbrace{\bar{z}\Delta_I(S_{-1})}_{\text{level channel}} + \underbrace{\rho\Delta_{II}(S_{-1})}_{\text{sorting channel}} \right) \end{aligned} \tag{5}$$

where $\bar{z} = \frac{\bar{\epsilon}}{1-\rho}$ is the average idiosyncratic productivity; the sorting channel Δ_{II} is defined as

$$\begin{aligned} \Delta_{II}(S) :&= \frac{1-\varphi}{\mu} \mathcal{G}(S) \left(M_2(S) - (z_M(S)+\bar{z}) M_1(S) + z_M(S)\bar{z} M_0(S) \right). \\ &= \frac{1-\varphi}{\mu} \mathcal{G}(S) M_1(S) \left(\frac{M_2(S)}{M_1(S)} - z_M(S) \right) - \bar{z}\Delta_I(S) \end{aligned}$$

where $\bar{z} = \frac{\bar{\epsilon}}{1-\rho}$ is the average idiosyncratic productivity. The level channel indicates the missing output due to the past period's missing investments. This channel assumes the average productivity for the missing capital stock for the calculation of the output.

The sorting channel captures missing output due to the time-varying cross-sectional correlation between the capital stock and the ex-post productivity real-

ization. The decision of inaction is made a period ahead of the capital utilization for the production. Therefore, there is a ex-post inefficiency that a firm's action or inaction turns out to be over-investment or under-investment after a future productivity realization. If many of potentially good firms chooses inaction, the under-investment is made among ex-post good firms, adding extra positive output gap as the misallocation. In this case, the positive sorting between the capital stock and the productivity is weaker than the frictionless benchmark, so the sorting channel increases Δ_Y.

In contrast, if many of potentially good firms aggressively chooses to invest, the over-investment is realized among good firms, adding extra negative output gap as the misallocation. In this case, the positive sorting between the capital stock and the productivity is stronger than the frictionless benchmark, so the sorting channel decreases Δ_Y.[13] Over the business cycle both cases are realized depending on the level of the aggregate TFP.

The output gap in conjunction with investment gap shape the aggregate frictional dynamics that possibly deviate from the frictionless benchmarks. An important unsettled debate regarding these gaps is whether such gaps are qualitative nature of two economies. Does a micro-level friction theoretically generates macro-level fluctuations that are not replicated by the frictionless benchmark at any combination of parameters?

Regarding this fundamental question, Proposition 6 answers that the gap is the qualitative nature of arising from the frictions, which does not disappear regardless of a quantitative setup.

Proposition 6 (Impossibility of the macroeconomic neutrality)**.**
Either $\Delta_Y(S) \neq 0$ *or* $\Delta_I(S) \neq 0$ *holds. That is, the presence of firm-level inaction necessarily makes the aggregate frictional dynamics from the frictionless benchmark either through the output gap or investment gap.*

Proof. See Appendix. ■

The theoretical result holds because if the investment gap is zero, the output gap through the sorting channel is inevitably non-zero: it becomes positive as

[13] It is worth noting that there are no externality or frictional components in the model that disturb the first welfare theorem. The comparison in this paper is made against the frictionless benchmark.

Corollary 4 states; if the output gap is zero, the inaction midpoint cannot be located at the mean of the idiosyncratic productivity distribution, meaning a non-zero investment gap by Proposition 3.

Corollary 4 (Output non-neutrality under the investment neutrality)**.**
If the stationary distribution Φ_z is symmetric on the inaction region $[z_L, z_H]$, the presence of firm-level lumpy investment makes the aggregate output deviate from the frictionless benchmark:

$$\Delta_I(S) = 0 \textit{ and } \Delta_{II}(S) > 0 \implies \Delta_Y(S) > 0$$

Proof. See Appendix. ■

From the technical point of view, in the analytical characterization of the two aggregate gaps, the key components are the two ratios of the moments in the inaction band: M_2/M_1 and M_1/M_0. These two ratios are equated at z_M if and only if both of the two aggregate gaps are zero.

$$\Delta_Y(S) = 0 = \Delta_I(S) \iff \underbrace{\frac{M_2(S)}{M_1(S)} = z_M = \frac{M_1(S)}{M_0(S)}}_{\text{Impossible equality}}$$

However, those two ratios cannot be equated due to the Jensen's inequality: $M_2 M_0 > M_1^2$.

We derive the analytical forms of the sorting channel in Proposition 7 and the aggregate output gap in Corollary 5. Like the aggregate investment gap, the both of the analytical forms are a product of frictionless aggregate investment $I^{NoFixed}$ and an amplifying or dampening component that varies over the business cycle.

Proposition 7 (The analytical sorting channel)**.**
The sorting channel Δ_{II} takes the following closed form:

$$\Delta_{II} = I^{NoFixed} \left(\left(-\frac{\sqrt{\frac{\sigma_\epsilon^2}{1-\rho^2}}}{z_M^*} \right) \left(\Phi^{SN}(z_H^*) - \Phi^{SN}(z_L^*) \right) + (z_H - z_L) \frac{\phi^{SN}(z_H^*) + \phi^{SN}(z_L^*)}{2 z_M^*} \right)$$

where $z_j^* := \frac{z_j - \frac{\bar{\epsilon}}{1-\rho}}{\sqrt{\frac{\sigma_\epsilon^2}{1-\rho^2}}}$, $j \in \{H, M, L\}$ *is the normalized stochastic variable.* ϕ^{SN} *and* Φ^{SN} *are the probability density function and cumulative distribution function of the standard normal distribution.*

Proof. See Appendix. ■

Corollary 5 (The analytical output gap)**.**
The output gap Δ_Y *has the following analytical characterization:*

$$\Delta_Y = \frac{\Pi(1;S)}{\alpha} Z I^{NoFixed} \left(\left(\frac{\bar{\epsilon}}{1-\rho} - \frac{\rho\sqrt{\frac{\sigma_\epsilon^2}{1-\rho^2}}}{z_M^*} \right) \left(\Phi^{SN}(z_H^*) - \Phi^{SN}(z_L^*) \right) + \rho(z_H - z_L) \frac{\phi^{SN}(z_H^*) + \phi^{SN}(z_L^*)}{2z_M^*} + \frac{\bar{\epsilon}}{1-\rho} \frac{\phi^{SN}(z_H^*) - \phi^{SN}(z_L^*)}{z_M^*} \right)$$

where $z_j^* := \frac{z_j - \frac{\bar{\epsilon}}{1-\rho}}{\sqrt{\frac{\sigma_\epsilon^2}{1-\rho^2}}}$, $j \in \{H, M, L\}$ *is the normalized stochastic variable.* ϕ^{SN} *and* Φ^{SN} *are the probability density function and cumulative distribution function of the standard normal distribution.*

Proof. The analytical form is immediate by combining Equation (5), Proposition 4, and Proposition 7. ■

Proposition 8 shows that the output gap is positive when the frictionless aggregate investment is positive. From

Proposition 8 (The sign of the output gap)**.**
If the frictionless investment $I^{NoFixed}$ *is positive, the output gap* Δ_Y *is also positive:*

$$I^{NoFixed} > 0 \implies \Delta_Y > 0.$$

Proof. See Appendix. ■

Similar to the approximation to the frictional aggregate investment, the output

gap from the sorting channel can be approximated as follows:

$$\begin{aligned}\Delta_{II} &\approx \Delta_{II}^{Approx} \\ &= I^{NoFixed}(-\zeta_3)\phi^{SN}(\zeta_1 z_M^*)(z_H - z_L) \\ &= \left(\frac{-\zeta_3}{1-\zeta_2}\right)\sqrt{\frac{\sigma_\epsilon^2}{1-\rho^2}}\Delta_I^{Approx}\end{aligned}$$

where ϕ^{SN} is the probability density function of the standard normal distribution; $\zeta_3 \in \mathbb{R}$ is the coefficient of approximation, which is obtained from the minimization problem of the approximation error for the sorting channel. It is worth noting that the approximation is accurate only if $\Delta_I > 0$. If $\Delta_I = 0$, Corollary 4 predicts that $\Delta_{II} > 0$, which immediately shows the inaccurate approximation.

Then, we deduce that the output gap is approximated as follows:

$$\begin{aligned}\Delta_Y &\approx \Delta_Y^{Approx} \\ &= \frac{\Pi(1;S)}{\alpha} Z\left(\rho\Delta_{II}^{Approx}(S_{-1}) + \frac{\overline{\epsilon}}{1-\rho}\Delta_I^{Approx}(S_{-1})\right) \\ &= \frac{\Pi(1;S)}{\alpha} Z\left(\rho\left(\frac{-\zeta_3}{1-\zeta_2}\right)\sqrt{\frac{\sigma_\epsilon^2}{1-\rho^2}} + \frac{\overline{\epsilon}}{1-\rho}\right)\Delta_I^{Approx}(S_{-1}) \qquad (6)\end{aligned}$$

According to the approximation, the output and investment gaps positively comove over the business cycle. Moreover, the sufficient statistics to explain these approximated dynamics are simply the inaction midpoint z_M and the exogenous aggregate TFP Z.

On top of the aggregate investment and output, other aggregate allocations are obtained as follows:

$$\begin{aligned}C(S) &= Y(S) - I(S) - \int\left(\frac{\mu}{2}I(k,z)^2 + M_0\right)d\Phi^{joint}(k,z) \\ K'(S) &= K(1-\delta) + I(S)\end{aligned}$$

where Φ^{joint} is the joint distribution of the firm-level capital stock and the productivity.

4.3 The stationary equilibrium

In this section, we provide the analytical characterization of the allocations in the stationary equilibrium and investigate their nature. We assume the aggregate TFP stays at $Z = \overline{Z}$ in the stationary equilibrium. In the stationary recursive competitive equilibrium, $\mathcal{G}(S)$ and $\mathcal{H}(S)$ are specified as follows:

$$\mathcal{G}^{ss} = \Pi^{ss}(1)\left[\frac{\rho\beta\overline{Z}}{1-\rho\beta(1-\delta)}\right]$$
$$\mathcal{H}^{ss} = \Pi^{ss}(1)\left[\frac{\beta\overline{\epsilon}\overline{Z}}{(1-\rho\beta(1-\delta))(1-\beta(1-\delta))}\right]$$

Then, using $\mathcal{G}^{ss}$ and $\mathcal{H}^{ss}$ the inaction band $\Omega = [z_L^{ss}, z_H^{ss}]$ and the inaction midpoint are specified as follows:

$$z_H^{ss} = \frac{(1+\sqrt{2\mu\xi})(1-\rho\beta(1-\delta))}{\Pi^{ss}(1)\rho\beta\overline{Z}} - \frac{\overline{\epsilon}}{\rho(1-\beta(1-\delta))}$$
$$z_L^{ss} = \frac{(1-\sqrt{2\mu\xi})(1-\rho\beta(1-\delta))}{\Pi^{ss}(1)\rho\beta\overline{Z}} - \frac{\overline{\epsilon}}{\rho(1-\beta(1-\delta))}$$
$$z_M^{ss} = \frac{1-\rho\beta(1-\delta)}{\Pi^{ss}(1)\rho\beta\overline{Z}} - \frac{\overline{\epsilon}}{\rho(1-\beta(1-\delta))}.$$

where $\Pi^{ss}(1)$ is the marginal profit of an effective capital stock in the stationary equilibrium. It is worth noting that the entire inaction band shifts to the left if the marginal profit or the aggregate TFP level increases.

The idiosyncratic productivity's volatility σ does not affect the inaction band. However, the parameter affects the equilibrium allocations through the measure of firms belonging to the inaction band.

The misallocation terms are characterized as follows:

$$\Delta_I = \frac{(1-\varphi)\Pi^{ss}(1)}{\mu}\left[\frac{\rho\beta\overline{Z}(1-\delta)M_0^{ss}}{1-\rho\beta(1-\delta)}\right]\left(\frac{M_1^{ss}}{M_0^{ss}} - \left(\frac{1-\rho\beta(1-\delta)}{\Pi^{ss}(1)\rho\beta\overline{Z}} - \frac{\overline{\epsilon}}{\rho(1-\beta(1-\delta))}\right)\right)$$

$$\Delta_{II} = \frac{(1-\varphi)\Pi^{ss}(1)}{\mu}\left[\frac{\rho\beta\overline{Z}(1-\delta)M_1^{ss}}{1-\rho\beta(1-\delta)}\right]\left(\frac{M_2^{ss}}{M_1^{ss}} - \left(\frac{1-\rho\beta(1-\delta)}{\Pi^{ss}(1)\rho\beta\overline{Z}} - \frac{\overline{\epsilon}}{\rho(1-\beta(1-\delta))}\right)\right) - \frac{\overline{\epsilon}}{1-\rho}\Delta_I$$

$$\Delta_Y = \frac{\Pi^{ss}(1)}{\alpha}\overline{Z}\left(\frac{\overline{\epsilon}}{1-\rho}\Delta_I + \rho\Delta_{II}\right)$$

The convex adjustment cost parameter impacts the misallocation terms through two channels. One is the intensive margin: the size of an optimal investment decreases in the convex adjustment cost. Thus, the misallocation also decreases in the cost parameter. The second is the extensive margin: the inaction band increases in the convex adjustment cost parameter. Then, the mass of firms belonging to the inaction band increases in the convex adjustment cost parameter, leading to a greater misallocation. Due to these two counter-veiling channels, the net impact of the convex adjustment cost parameter on the misallocation is unclear.

We analyze the local property of the aggregate investment around the stationary equilibrium through the impulse responses to an unexpected instantaneous shock to the future marginal benefit of investment $A(z;S)$.

Proposition 9 (Asymmetric impulse response of investment)**.**
Consider an instantaneous arbitrary shock $\epsilon > 0$ to $A(z;S)$. Denote the response of aggregate investment as $I(z;S,\epsilon)$. If $z_M < \bar{z}$, then the following inequality holds:

$$|I(z;S,+\epsilon) - I(z;S,0)| < |I(z;S,-\epsilon) - I(z;S,0)|$$

That is, the instantaneous response of investment is asymmetric and more sensitive to a negative shock.

Proof. See Appendix. ■

Proposition 9 states that when an instantaneous shock hits the stationary equilibrium, despite the same magnitude, the consequence of the economy can be different depending on the sign of the shock. In the shock experiments used in

Proposition 9, due to the symmetric nature of the shock, the response of the inaction band and the optimal investment are symmetric as well between the positive and the negative shocks. However, the changes in the measure of firms belonging to the inaction band are asymmetric due to the inaction band's skewed location. This quantity effect drives the theoretical result.

However, in the responses of the frictionless benchmark investment, such quantity effect is missing due to the absence of the inaction band. Therefore, the symmetry holds between the positive and negative shock, as stated in Corollary 6.

Corollary 6 (Symmetric impulse response of frictionless investment)**.**
Consider an instantaneous arbitrary shock $\epsilon > 0$ to $A(z;S)$. Denote the response of frictionless investment as $I^{NoFixed}(z;S,\epsilon)$. Then, the following equality holds:

$$|I^{NoFixed}(z;S,+\epsilon) - I^{NoFixed}(z;S,0)| = |I^{NoFixed}(z;S,-\epsilon) - I^{NoFixed}(z;S,0)|$$

That is, the instantaneous response of frictionless investment is symmetric.

Proof. See Appendix. ■

4.4 A general equilibrium model with the full analytics

In this section, we analyze the general equilibrium of an economy where the allocations are fully characterized in analytic forms. In particular, we consider the household's GHH preference with the risk aversion parameter $\sigma \to 0$:

$$\lim_{\sigma \to 0} \frac{(c - \eta l_H)^{1-\sigma}}{1-\sigma} \tag{7}$$

We define the marginal profit of an effective capital stock $\Pi(1) := \alpha \left(\frac{1-\alpha}{\eta}\right)^{\frac{1-\alpha}{\alpha}}$. Then, the sum of the present discounted marginal benefit of the investment has

the following explicit form:

$$
\begin{aligned}
A(z;S) &= \mathcal{G}(S)z + \mathcal{H}(S), \\
\mathcal{G}(S) = \mathcal{G}(Z) &= \frac{\alpha\left(\frac{1-\alpha}{\eta}\right)^{\frac{1-\alpha}{\alpha}}\beta\rho\left[\rho_{agg}Z + \frac{\bar{\epsilon}_{agg}}{1-\rho\beta(1-\delta)}\right]}{1-\rho_{agg}\rho\beta(1-\delta)} \\
\mathcal{H}(S) = \mathcal{H}(Z) &= \frac{\alpha\left(\frac{1-\alpha}{\eta}\right)^{\frac{1-\alpha}{\alpha}}\beta\left[\rho_{agg}Z\bar{\epsilon} + \frac{\bar{\epsilon}_{agg}\bar{\epsilon}(1-\rho\rho_{agg}\beta^2(1-\delta)^2)}{(1-\beta(1-\delta))(1-\rho\beta(1-\delta))}\right]}{(1-\rho_{agg}\rho\beta(1-\delta))(1-\rho_{agg}\beta(1-\delta))}
\end{aligned}
$$

While the persistence of the aggregate TFP process ρ_{agg} affects the future marginal benefit of the investment through both $\mathcal{G}$ and $\mathcal{H}$, the volatility parameter σ_{agg} does not affect the marginal benefit.

Proposition 10 (Marginal benefit monotonicity in aggregate productivity)**.** *In the analytical equilibrium, the following properties hold:*

$$
\frac{\partial\mathcal{G}(Z)}{\partial Z} > 0, \quad \frac{\partial\mathcal{H}(Z)}{\partial Z} > 0
$$

Proof. The proof is immediate from the analytical form of $\mathcal{G}$ and $\mathcal{H}$. ■

It is worth noting that $\mathcal{G}$ and $\mathcal{H}$ are affine functions of the aggregate TFP Z. Therefore, the frictionless aggregate investment $I^{NoFixed}$ is also an affine function of Z:

$$
\begin{aligned}
I^{NoFixed} &= \frac{1-\varphi}{\mu}\left(\mathcal{G}(S)\left(\frac{\bar{\epsilon}}{1-\rho}\right)\right) + \frac{1-\varphi}{\mu}\left(\mathcal{H}(S)-1\right) \\
&= aZ + b,
\end{aligned}
$$

for some $a, b \in \mathbb{R}$. The analytical form of a and b are available in Appendix.

Given $\mathcal{G}$ and $\mathcal{H}$, the stochastic inaction band and the midpoint are analytically characterizable:

$$
z_M(Z) = \frac{1-\mathcal{H}(Z)}{\mathcal{G}(Z)}, \quad z_H(Z) = z_M(Z) + \frac{\sqrt{2\mu\xi}}{G(Z)}, \quad z_L(Z) = z_M(Z) - \frac{\sqrt{2\mu\xi}}{G(Z)}.
$$

Then, based on the inaction bands and the midpoint, the aggregate investment

$I(Z)$ analytically characterizable by

$$I(Z) = (aZ + b)\left(1 - \underbrace{\Phi^{SN}(z_H^*(Z)) + \Phi^{SN}(z_L^*(Z))}_{\Delta_I\text{: Quantity channel}} - \underbrace{\frac{\phi^{SN}(z_H^*(Z)) - \phi^{SN}(z_L^*(Z))}{z_M^*(Z)}}_{\Delta_I\text{: Quality channel}}\right).$$

Figure 1 plots the aggregate investment (dash-dotted line) and the frictionless benchmark (dotted line) as functions of the aggregate TFP in the horizontal axis. The solid line plots the probability density of the aggregate TFP. $Z^{Neutral}$ on the horizontal axis indicates the TFP level where the investment gap becomes zero. At this TFP level, the inaction band lies at the center of the productivity distribution, which satisfies the sufficient condition for the neutrality specified in Proposition 3. The neutrality point has the analytical form:

Proposition 11 (The neutrality point)**.**
In the analytical equilibrium, there exists a level of TFP, where the investment gap is zero:

$$Z^{Neutral} = \frac{1}{\rho_{agg}\overline{\epsilon}}\left(\frac{(1-\rho)(1-\rho_{agg}\beta(1-\delta))}{\alpha\left(\frac{1-\alpha}{\eta}\right)^{\frac{1-\alpha}{\alpha}}\beta} - \frac{\overline{\epsilon}_{agg}\overline{\epsilon}}{1-\beta(1-\delta)}\right)$$

Proof. See Appendix. ■

The following corollary summarizes the signs of aggregate investment, the frictionless benchmark, and the investment gap. The key determinant of the signs is the location of the inaction midpoint relative to the average idiosyncratic productivity.

Corollary 7 (The equivalence relations II)**.**
The following equivalences hold:

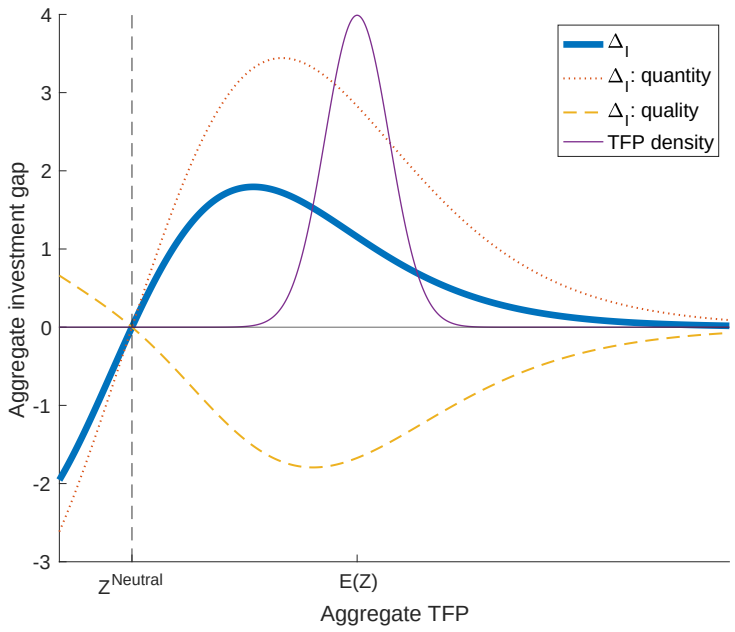
$$\begin{aligned}
(i) \quad & z_M^*(Z) = 0 \iff Z = Z^{Neutral} \\
(ii) \quad & z_M^*(Z) < 0 \iff Z > Z^{Neutral} \\
(iii) \quad & z_M^*(Z) > 0 \iff Z < Z^{Neutral}
\end{aligned}$$

Proof. The proof is immediate from Corollary 3 and Proposition 11. ■

For example, if the inaction midpoint is below the average idiosyncratic productivity, the frictionless firm-level investment is on average positive. This is because the optimal investment at the midpoint is zero, as shown in Corollary 2 and because the optimal investment is the affine function of idiosyncratic productivity. The other equivalences are directly from Corollary 3.

In Figure 1, the investment gap Δ_I is the vertical distance between the aggregate investment and the frictionless benchmark, which varies over the aggregate TFP Z.[14] As the aggregate TFP decreases from a high level to the moderately low level in the positive domain, the investment gap increases. Then, if the aggregate TFP level goes further below a certain level, the investment gap starts to decrease. The following proposition states the existence of such point where the investment gap is maximized.

Figure 5: Decomposition of the investment gap: quantity vs. quality



[14]The plot is based on the calibrated parameters, so the location and the dispersion of the aggregate TFP distribution is specifically tailored to match the empirically observed aggregate dynamics. The focus of the current section is on the theoretical property of the investment function that is invariant over different parameter levels. The parameter calibration is discussed in Section 5

Proposition 12 (Maximum investment gap)**.**
There exists $Z^{Max} \in \mathbb{R}$ *such that*

$$\Delta_I(Z^{Max}) > \Delta_I(Z) \text{ for } \forall Z \in \mathbb{R}$$

Proof. See Appendix. ■

Due to the presence of the maximum point, an increase in the aggregate TFP volatility does not always lead to an increase in the average misallocation over the business cycle. In Appendix, we show that the average misallocation is an inverted U-shape over the aggregate TFP volatility, using the comparative static analysis.

The aggregate investment is a nonlinear function of the aggregate TFP. This nonlinearity generates a gap in the aggregate investment level between the stationary equilibrium and the ergodic mean:

$$\underbrace{I\left(\int Z d\Phi^Z\right)}_{\text{stationary equilibrium}} \neq \underbrace{\int I(Z) d\Phi^Z}_{\text{ergodic mean}}$$

The ranking between the two is determined depending on the location $\mathbb{E}Z$ and the dispersion σ_{agg} of the aggregate TFP distribution. In other words, the fluctuations in the aggregate TFP could possibly bring in a cost or benefit to an economy through the nonlinearity of aggregate investment. According to the calibrated parameters, the ergodic mean is slightly lower than the stationary equilibrium level, which we interpret as a cost of the business cycle. Taking a step further, we theoretically show that the ergodic average is lower than the stationary equilibrium level, under an empirically plausible set of parametric assumptions.

Proposition 13 (The cost of business cycle)**.**
If $I^{NoFixed}(Z) > 0$ *and* $\mathbb{P}(Z < Z^{Max})$ *is sufficiently small,*

$$\mathbb{E}I(Z) < I(\mathbb{E}Z)$$

Proof. See Appendix. ■

Moreover, under the same set of assumptions, we show that the inaction-

driven frictional aggregate investment feature a greater volatility and a more negative skewness than the frictionless benchmark.

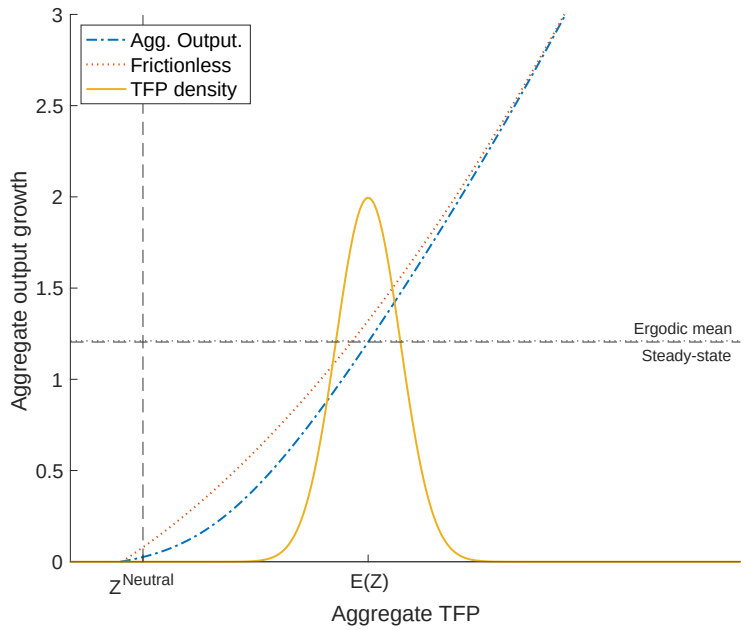
Proposition 14 (Skewed business cycles)**.**
If $I^{NoFixed}(Z) > 0$ *and* $\mathbb{P}(Z < Z^{Max})$ *is sufficiently small,*

(i) $s.d.(I) > s.d.(I^{NoFixed})$

(ii) $skewness(I) < skewness(I^{NoFixed})$

Proof. See Appendix. ■

Figure 6: Aggregate output function



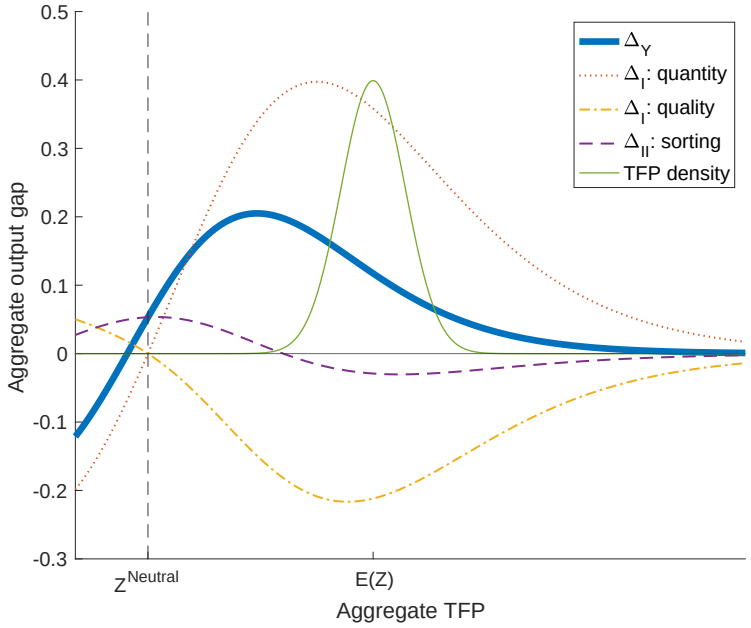
The nonlinear dynamics of the investment gap is the outcome of the two distinct channels: the quantity and quality channels. Figure 5 plots the investment gap (thick solid line), the quantity channel (dotted line) and the quality channel (dashed line). The sign of quantity and the quality channel depends on the level of the aggregate TFP Z and the location of the inaction midpoint z_M compared to the average idiosyncratic productivity level. According to the calibrated parameter level, the quantity channel dominates the quality channel, and the dominance decays as the aggregate TFP increases beyond a certain high level. According to

Doms and Dunne (1998) and Gourio and Kashyap (2007), the fluctuations in the aggregate investment is largely explained by the fluctuations in the number of firms making large-scale investments. The quantity channel's dominance in our model is consistent with the empirical evidence. The nonlinearity in the two channels are due to the nonlinear dynamics of the inaction bands and the curvature of the normal probability density function.

Figure 6 plots the output counterpart of Figure 1, which plots the analytical output growth function dY (dash dotted line), frictionless benchmark $dY^{NoFixed}$ (dotted line), and the TFP density function. Both the output growth function and frictionless growth benchmark are highly nonlinear in the aggregate TFP, while their monotonic patterns are similar to the aggregate investment counterpart. The output gap is the vertical difference between the two growth functions. In the figure, consistent with Corollary 4, the output gap is positive even when the aggregate TFP is at the neutrality level $Z = Z^{Neutral}$.

In Figure 7, we decompose the aggregate output gap into three components: quantity, quality and sorting channels. The quantity and quality channels are scaled version of the ones from the investment gap. The sorting channel flips the sign from positive to negative as the aggregate TFP increases beyond a threshold.

Figure 7: Decomposition of the output gap



Lastly, we propose the fully analytical approximation to the aggregate investment and output growth function. The functions are in a relatively simple form even though it preserves the nonlinear effect from the aggregate TFP fluctuations.

Proposition 15 (Analytical approximation to the aggregate investment and output growth)**.** *In the analytical equilibrium, we have the following analytical approximation to the aggregate investment and the aggregate output growth:*

$$I(Z) \approx \underbrace{aZ+b}_{=I^{NoFixed}} \underbrace{- c(\phi^{SN})'\left(\frac{d}{Z+e}+f\right)}_{=\Delta_I^{Approx}}$$

$$dY(Z) \approx \underbrace{gZ^2+hZ}_{=dY^{NoFixed}} \underbrace{- mZ\overbrace{c(\phi^{SN})'\left(\frac{d}{Z+e}+f\right)}^{=\Delta_I^{Approx}}}_{=\Delta_Y^{Approx}}$$

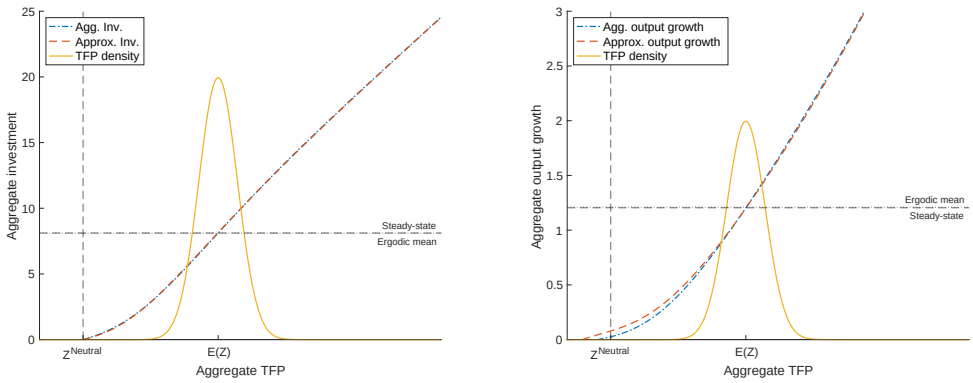
for $a, b, c, d, e, f, g, h, m \in \mathbb{R}$.

Proof. See Appendix. ■

In the analytical approximation, the key nonlinear component is the Gaussian derivative $(\phi^{SN})'$. To check the accuracy of the approximation, based on the calibrated parameters in Section 5, we plot both the original function and the approximation for the aggregate investment (panel (a)) and output growth (panel (b)) in Figure 8. For both panels, the dash-dotted line is for the original analytical function and the dashed line is the analytical approximation. For both investment and output growth, the approximation is highly accurate, as can be seen from the two curves being tightly aligned. For output growth, the approximation error becomes larger when the aggregate TFP is close to the $Z^{neutral}$. This is due to the fact that the original function's output gap is positive at $Z = Z^{neutral}$, while the approximation leads to zero output gap at the same point. Under the calibrated parameters, the average approximation error of the aggregate investment function is 0.47% of the steady-state investment level, and the average approximation error of the aggregate output growth function is 0.83% of the steady-state output

growth level.[15]

Figure 8: Approximated aggregate investment and output growth



(a) Investment (b) Output growth

5 Quantitative analysis

This section analyzes the business cycle using our analytical model with calibrated parameters. There are two main goals in this analysis. The first is to quantify the business cycle implication of the firm-level frictions by utilizing the full potential of the model, the usage of which is not limited to theoretical analysis. The model includes important frictional components in a standard way, which are well-known as crucial for the firm dynamics and the business cycle. Our analytical framework enables an accurate quantitative analysis of the role of these frictions over the business cycle without complicated computation and computational errors.

Second, only through quantitative analysis can we verify that the investment gap and the output gap significantly matter for the business cycle. Despite the theoretical non-neutrality (Proposition 6), if the gaps only lead to quantitatively negligible differences, they are quantitatively only near-neutrality. In particular, when a concave utility is considered, an inter-temporal smoothing force for an aggregate investment coming from the general equilibrium effect mutes the non-linearity of the investment and output growth gaps (Khan and Thomas, 2008). In

[15]The average approximation error is obtained by averaging the TFP-specific approximation error using the aggregate TFP distribution.

this section, we quantify such GE effect and quantitatively establish the role of the nonlinear misallocation over the business cycle.

5.1 Calibration

We calibrate three parameters of the fully analytical model, which are convex adjustment cost parameter μ, fixed adjustment cost parameter ξ, and the aggregate TFP shock volatility σ_Z. For the rest of the parameters, We follow standard levels in the firm dynamics literature. The unit period is a quarter. The calibration is based on two steps: 1) calibration of the frictional parameters based on the stationary equilibrium and 2) calibration of the TFP volatility parameter based on the stochastic equilibrium.

The analytical equilibrium allocations, such as the inaction band, show how the micro-level frictional parameters affect the equilibrium. On top of the equilibrium allocations, some key moments are also characterized in a closed form. We suggest a calibration strategy for the two frictional parameters based on the analytically tractable moments. The first moment is the inaction portion:

$$\Phi(z_H) - \Phi(z_L)$$

Next is the dispersion to average ratio of firm-level investments:

$$\frac{s.d.(\widehat{I})}{mean(\widehat{I})} = I + \left(\left(\frac{H-1}{\mu} \right)^2 - 2I \left(\frac{H-1}{\mu} \right) \right) \frac{M_0}{I} + 2 \left(\frac{H-1}{\mu} - I \right) \left(\frac{G}{\mu} \right) \frac{M_1}{I} + \left(\frac{G}{\mu} \right)^2 \frac{M_2}{I}$$

For the stochastic equilibrium, the log output volatility $std(log(Y))$ is computed using a Monte Carlo simulation without discretizing the TFP support. This is feasible because the investment, output, and consumption are accurately characterized for any level of aggregate TFP Z under the analytical framework. The volatility is obtained after applying the HP filter for both model and the data. Table 1 reports the level of the calibrated and fixed parameters.

Table 2 reports the target moments in the data and the model-implied moments in the upper part of the table. The last column specifies the source of the moments. The bottom part of the table reports the selected equilibrium moments regarding the misallocation. In the stationary equilibrium with the calibrated pa-

Table 1: Calibrated and fixed parameters

Parameters	Description	Value
Stationary eq. parameters		
α	Capital share	0.3300
δ	Depreciation rate	0.0250
ρ	Firm-level productivity shock persistence	0.9300
σ	Firm-level productivity shock volatility	0.1100
μ	Convex adjustment cost*	0.0185
ξ	Fixed adjustment cost*	0.5330
$\overline{\epsilon}$	Firm-level productivity shock average	0.0800
η	Labor disutility	1.9800
φ	Financial shock	0.1000
β	Household's discount factor	0.9900
Aggregate shock parameters		
ρ_Z	Aggregate productivity shock persistence	0.9700
σ_Z	Aggregate productivity shock volatility*	0.0105

Notes: The parameters with asterisk (*) are calibrated.

rameters, the investment gap and the output gap are around 14% and 10% of each equilibrium allocation, respectively.

Table 2: Targeted moments and selected equilibrium moments

	Model	Data	Reference
Target moments			
Inaction portion (%)	30.50	30.20	Zwick and Mahon (2017)
$sd(I^*)/mean(I^*)$	6.49	6.50	Compustat data
$std(log(Y))$	1.47	1.44	NIPA data (HP-filtered)
Untargeted moments			
Investment semi-elasticity	8.50	7.20	Zwick and Mahon (2017)
Selected equilibrium moments			
Δ_I/I (%)	14.24		
Δ_Y/dY (%)	9.72		

According to the target moments, around 30% of firms are inactive. The calibrated parameters indicate that the inaction region is located left to the mean idiosyncratic productivity. This implies that the bottom 30% of firms do not invest due to their low productivity.[16]

5.2 Business cycle analysis

Using the fully analytical model with the calibrated parameters, we simulate the model for 5,000 periods to analyze the model-implied business cycle. Table 3 reports the business cycle statistics implied by the fully analytical model. The the statistics for investment, output, and consumptions are compared with the corresponding frictionless benchmark, and the statistics for the mass of the inactive firms $P(\Omega)$, investment gap, sorting channel, and the output gap are reported. All the statistics are computed after taking log for each variable. It is worth noting that volatility of aggregate allocations are significantly greater in the frictional case compared to the frictionless benchmark. The investment volatility is 40% greater; the output volatility is 26% greater; and the consumption volatility is 27% greater in the frictional case than in the frictionless case. This is consistent with the theoretical prediction illustrated in Figure 1. Moreover, the frictional dynamics lead to significantly more negatively skewed investment, output, and consumption. This is because during the downturn, the investment gap and the output gap increases due to the nonlinearity: The mass of inactive firms and all the misallocation terms are negatively skewed and counter-cyclical. As a result, the investment negative skewness is 39% greater; the output and the consumption negative skewnesses are more than twice greater in the frictional case than in the frictionless case.

The greater volatility and the negative skewness of the frictional investment compared to the frictionless benchmark is well captured in Figure 9. Panel (a) plots the simulated investment dynamics of the frictional case (solid line) and the frictionless case (dashed line) in % deviation from the average. The frictional investment is significantly more volatile than the frictionless benchmark. Panel (b) plots the same time series in the absolute value. Therefore, the distance be-

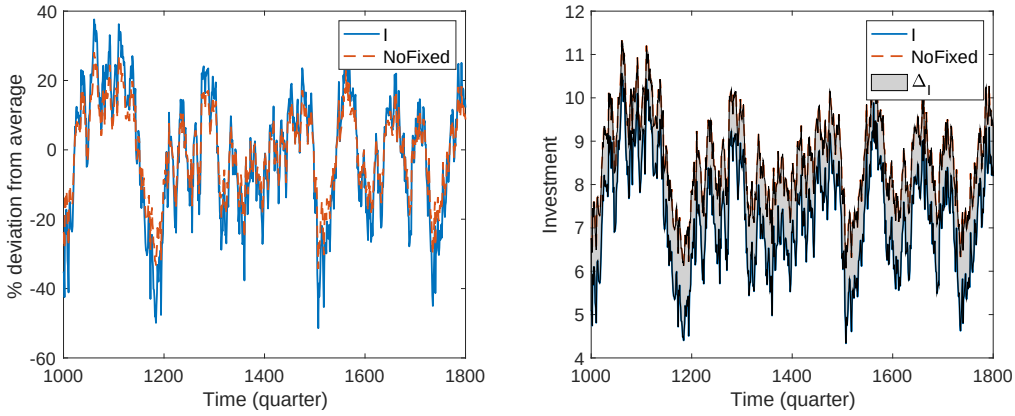
[16]There is a negligible portion of firms making negative investment, which is also consistent with the data.

Table 3: Business cycle statistics

	I		Y		C		$\mathbb{P}(\Omega)$	Δ_I	Δ_{II}	Δ_Y
Fixed cost	Yes	No	Yes	No	Yes	No	Yes	Yes	Yes	Yes
Volatility	0.196	0.140	0.152	0.121	0.155	0.122	0.281	0.222	0.302	0.254
Skewness	-0.576	-0.414	-0.129	-0.062	-0.134	-0.052	-0.262	-0.520	-0.386	-0.469
Kurtosis	3.526	3.257	2.449	2.456	2.413	2.384	2.884	3.060	2.945	3.019
Corr. with Y	0.789	0.824	1.000	1.000	0.974	0.982	-0.829	-0.824	-0.827	-0.825
Auto-corr.	0.966	0.967	0.997	0.996	0.999	0.999	0.968	0.967	0.967	0.967

tween the two series indicate the investment gap Δ_I. As can be seen from the figure, the investment gap increases when the frictionless investment level is low, dragging the frictional investment further down. In this economy, recessions are particularly bad due to the nonlinearly dropping aggregate investment.

Figure 9: Investment dynamics: frictional vs frictionless

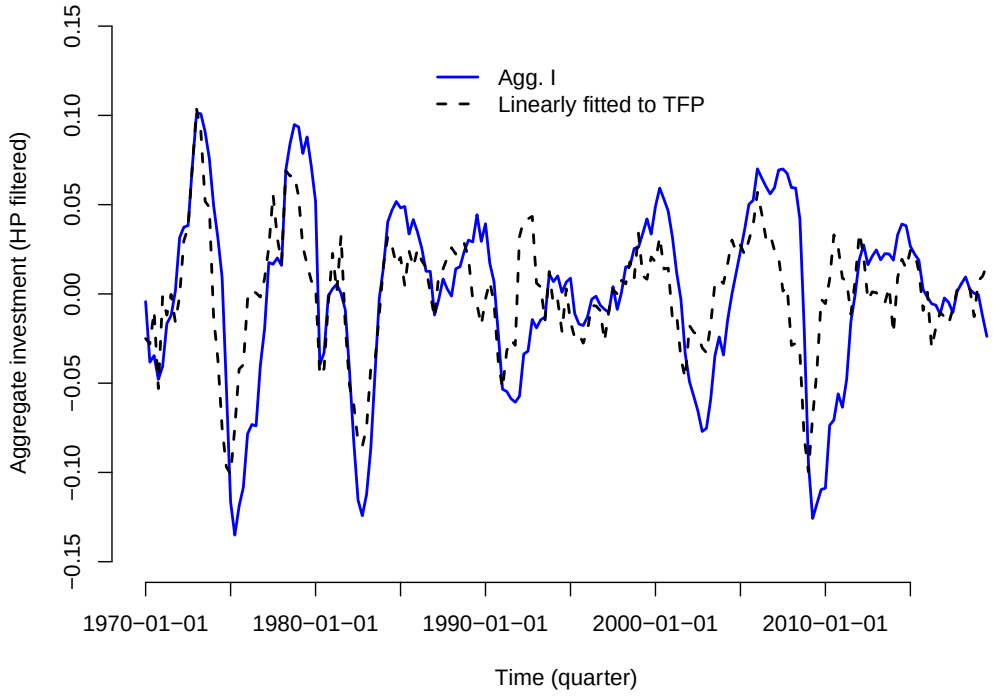


(a) Normalized investment

(b) Unnormalized investment

This nonlinear investment dynamics is well-supported by the data. Figure 10 plots the time series of the fixed investment (solid line) of the U.S. from National Income and Product Accounts (NIPA) together with the investment series that is linearly fitted to the TFP series from Fernald (2014) (dashed line). Both series are HP filtered. The investment linearly fitted to the TFP series is the data counterpart of the frictionless benchmark in the analytical framework. As shown from the figure, the distance between the two series tend to increase significantly during the downturn, consistent with the theory prediction.

Figure 10: Aggregate investment in the U.S. data



5.3 The general equilibrium effect of log utility

The fully analytical model assumes linear utility, which abstracts the inter-temporal smoothing force from the concave utility's general equilibrium effect. Khan and Thomas (2008) has shown that the general equilibrium effect can possibly dampen all the meaningful nonlinearity arising from the firm-level inactions. According to our theoretical prediction, the nonlinearity cannot be perfectly washed out for both investment and output, establishing the theoretical relevance of the firm-level inaction (Proposition 6). Nevertheless, the quantitative relevance of such nonlinearity has yet to be investigated. In this section, we quantitatively analyze the economic relevance of the frictional dynamics under the household's log utility.

The household's consumption decision affects each firm's inter-temporal decision through the marginal benefit channel: $\mathcal{G}$ and $\mathcal{H}$. Conversely, each firm's investment decision affects the consumption through the dividend stream. More-

over, given the marginal benefit, the aggregate investment is a nonlinear function of Z. Therefore, the problem is a highly complex nonlinear fixed point problem, which is computationally demanding. For this problem, I apply the global nonlinear solution method in the sequence space developed in Lee (2025), which efficiently computes the dynamically consistent solution using the theoretical property of the simulated path's ergodicity. The detailed explanation of the method application is available in Appendix.

Figure 11 plots the aggregate investment for the log utility case (dash dotted line) and the linear utility case (thick solid line) and the frictionless benchmark for the log utility case (dotted line) and the linear utility case (dashed line), together with the aggregate TFP density function (thin solid line).[17] The general equilibrium effect of the log utility significantly flattens the investment functions. When each investment functions are fitted to a linear line, the slope of the investment function decreases by 20% after the inclusion of the log utility, and that of the frictionless investment decreases by 30%. However, the investment gap for the log utility still significantly increases when the aggregate TFP decrease over the meaning domain, which implies the nonlinearity effect coming out of the firm-level inactions.

Table 4 reports the business cycle statistics under the log utility. As in Table 3, the firm-level friction significantly amplifies the volatility of the aggregate investment, output, and consumption. Also, the negative skewness becomes starker in these aggregate allocations after the inclusion of the firm-level fixed adjustment cost. These results show that the general equilibrium effect of log utility does not fully neutralize the nonlinear effect of the firm-level endogenous inaction.

5.4 Impulse response analysis

In this section, we analyze the impulse response function of aggregate investment and output under the perfect foresight assumption.[18] We consider positive and negative TFP shocks with magnitudes of one and three standard deviations

[17] An aggregate TFP realization beyond three standard deviations requires substantially long periods of simulation, which we have not included in this computation. This is the reason why the figure includes a narrower range of the aggregate TFP domain than the fully analytical case (Figure 1).

[18] As the analytical solution is known, computation of the stochastic response path is also feasible.

Figure 11: The general equilibrium effect of log utility

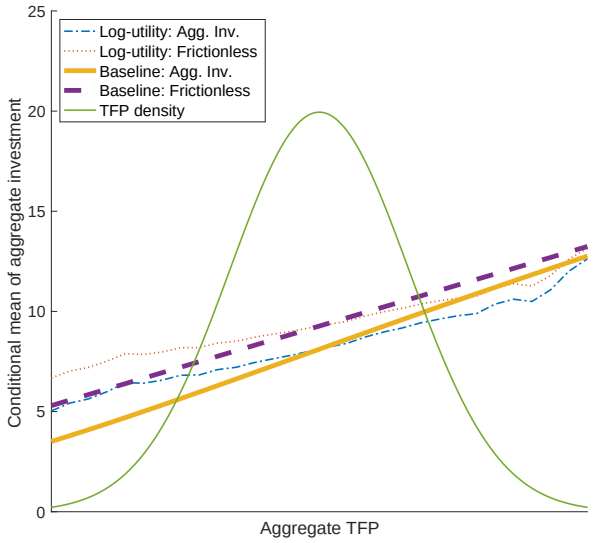
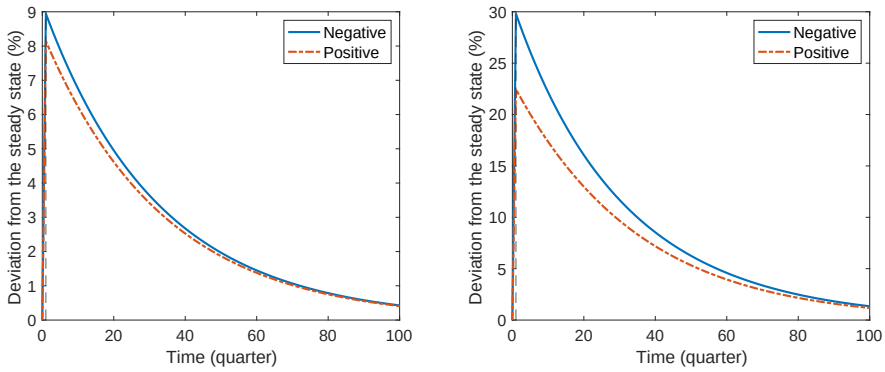


Table 4: Business cycle statistics: log utility case

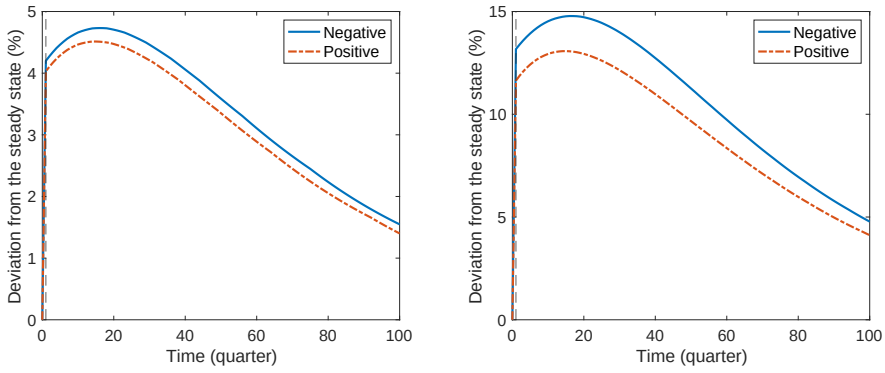
	I		Y		C		$\mathbb{P}(\Omega)$	Δ_I	Δ_{II}	Δ_Y
Fixed cost	Yes	No	Yes	No	Yes	No	Yes	Yes	Yes	Yes
Volatility	0.216	0.159	0.208	0.160	0.205	0.159	0.245	0.301	0.404	0.373
Skewness	-0.321	-0.107	-0.080	0.093	-0.070	0.100	-0.775	-2.327	-2.044	-2.315
Kurtosis	5.001	4.996	3.654	3.582	3.546	3.435	4.542	13.792	11.936	14.074
Corr. with Y	0.979	0.989	1.000	1.000	0.998	0.999	-0.986	-0.918	-0.937	-0.928
Auto-corr.	0.995	0.995	0.999	0.998	0.999	0.998	0.996	0.994	0.995	0.994

for rich comparison. Figure 12 plots the impulse responses. The solid line is for a negative TFP shock and the dash-dotted line is for a positive TFP shock.

There are two main takeaways in this figure. The first is the asymmetry in the impulse responses between the positive and the negative shocks. All the allocations display significantly greater sensitivity to the negative shock than to the positive shock. The second is the nonlinearity in the magnitude of the response. Although the shock magnitudes in the right-hand-side panels are three times greater than the ones on the left-hand side, the magnitudes of the responses on the right-hand side are beyond three times those on the left-hand side. Moreover, the asymmetry between the positive and the negative shocks is greater as the magnitude of the shock increases.



(a) I: 1-$s.d.$ shock (b) I: 3-$s.d.$ shock



(c) Y: 1-$s.d.$ shock (d) Y: 3-$s.d.$ shock

Figure 12: Asymmetric (absolute) impulse responses to an aggregate TFP shock

6 Concluding remarks

This paper develops a heterogeneous-firm general equilibrium model where frictional firm-level capital dynamics are characterized analytically. Under a set of parsimonious assumptions, we show that the firm-level frictional investment, including inactions, becomes tractable due to its history-independence. At the aggregate level, the shift in the inaction band generates substantial nonlinearity in the aggregate investment and output, all of which we characterize in closed form. Moreover, we provide a sufficient condition under which the impact of the firm-level inaction on the aggregate investment is neutralized and the impossibility of macroeconomic neutrality due to the output misallocation channel. The nonlinearity of the frictional aggregate dynamics leads to substantially greater volatility and negative skewness in the aggregate investment, output, and consumption.

References

Alvarez, F., H. L. Bihan, and F. Lippi (2016): "The Real Effects of Monetary Shocks in Sticky Price Models: A Sufficient Statistic Approach," *The American Economic Review*, 106, 2817–2851.

Alvarez, F. and F. Lippi (2022): "The Analytic Theory of a Monetary Shock," *Econometrica*, 90, 1655–1680.

Asker, J., A. Collard-Wexler, and J. De Loecker (2014): "Dynamic Inputs and Resource (Mis)Allocation," *Journal of Political Economy*, 122, 1013–1063.

Bachmann, R. and C. Bayer (2014): "Investment Dispersion and the Business Cycle," *American Economic Review*, 104, 1392–1416.

Bachmann, R., R. J. Caballero, and E. M. R. A. Engel (2013): "Aggregate Implications of Lumpy Investment: New Evidence and a DSGE Model," *American Economic Journal: Macroeconomics*, 5, 29–67.

Baley, I. and A. Blanco (2021): "Aggregate Dynamics in Lumpy Economies," *Econometrica*, 89, 1235–1264.

Bertola, G. and R. Caballero (1990): "Kinked Adjustment Costs and Aggregate Dynamics," in *NBER Macroeconomics Annual 1990, Volume 5*, National Bureau of Economic Research, Inc, 237–296.

Bertola, G. and R. J. Caballero (1994): "Irreversibility and Aggregate Investment," *The Review of Economic Studies*, 61, 223–246.

Caballero, R. J. and E. M. R. A. Engel (1991): "Dynamic (S, s) Economies," *Econometrica*, 59, 1659–1686.

——— (1993): "Heterogeneity and Output Fluctuations in a Dynamic Menu-Cost Economy," *The Review of Economic Studies*, 60, 95–119.

——— (1999): "Explaining Investment Dynamics in U.S. Manufacturing: A Generalized (S, s) Approach," *Econometrica*, 67, 783–826.

Caballero, R. J., E. M. R. A. Engel, J. C. Haltiwanger, M. Woodford, and R. E. Hall (1995): "Plant-Level Adjustment and Aggregate Investment Dynamics," *Brookings Papers on Economic Activity*, 1995, 1.

Caplin, A. S. and D. F. Spulber (1987): "Menu Costs and the Neutrality of Money," *The Quarterly Journal of Economics*, 102, 703–726.

Cooper, R., J. Haltiwanger, and L. Power (1999): "Machine Replacement and the Business Cycle: Lumps and Bumps," *American Economic Review*, 89, 32.

Cooper, R. W. and J. C. Haltiwanger (2006): "On the Nature of Capital Adjustment Costs," *The Review of Economic Studies*, 73, 611–633.

David, J. M. and V. Venkateswaran (2019): "The Sources of Capital Misallocation," *American Economic Review*, 109, 2531–67.

Doms, M. E. and T. Dunne (1998): "Capital Adjustment Patterns in Manufacturing Plants," *Review of Economic Dynamics*, 1, 409–429.

Elsby, M. W. and R. Michaels (2019): "Fixed adjustment costs and aggregate fluctuations," *Journal of Monetary Economics*, 101, 128–147.

Fernald, J. (2014): "A Quarterly, Utilization-Adjusted Series on Total Factor Productivity," *Federal Reserve Bank of San Francisco, Working Paper Series*, 01–28.

Gourio, F. and A. K. Kashyap (2007): "Investment spikes: New facts and a general equilibrium exploration," *Journal of Monetary Economics*, 54, 1–22.

House, C. L. (2014): "Fixed costs and long-lived investments," *Journal of Monetary Economics*, 68, 86–100.

Hsieh, C.-T. and P. J. Klenow (2009): "Misallocation and Manufacturing TFP in China and India," *The Quarterly Journal of Economics*, 124, 1403–1448.

Khan, A. and J. K. Thomas (2003): "Nonconvex factor adjustments in equilibrium business cycle models: do nonlinearities matter?" *Journal of Monetary Economics*, 50, 331–360.

——— (2008): "Idiosyncratic Shocks and the Role of Nonconvexities in Plant and Aggregate Investment Dynamics," *Econometrica*, 76, 395–436.

Koby, Y. and C. K. Wolf (2020): "Aggregation in Heterogeneous-Firm Models: Theory and Measurement," *Working Paper*, 66.

Kydland, F. E. and E. C. Prescott (1982): "Time to Build and Aggregate Fluctuations," *Econometrica*, 50, 1345–1370.

Lee, H. (2025): "Dynamically Consistent Global Nonlinear Solutions in the Sequence Space: Theory and Applications," *Working paper*, 42.

Moll, B. (2014): "Productivity Losses from Financial Frictions: Can Self-Financing Undo Capital Misallocation?" *American Economic Review*, 104, 3186–3221.

Restuccia, D. and R. Rogerson (2008): "Policy distortions and aggregate productivity with heterogeneous establishments," *Review of Economic Dynamics*, 11, 707–720.

——— (2013): "Misallocation and productivity," *Review of Economic Dynamics*, 16, 1–10.

Thomas, J. (2002): "Is Lumpy Investment Relevant for the Business Cycle?" *Journal of Political Economy*, 110, 508–534.

Winberry, T. (2021): "Lumpy Investment, Business Cycles, and Stimulus Policy," *American Economic Review*, 111, 364–396.

Zwick, E. and J. Mahon (2017): "Tax Policy and Heterogeneous Investment Behavior," *American Economic Review*, 107, 217–248.