Incomplete Information, Incomplete Markets and the Monetary Transmission Mechanism*

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Abstract

Incomplete information and incomplete markets are commonly believed to help in accounting for the demand transmission mechanism. We introduce the two main classes of beliefs formation processes, sticky information and signal extractions with entropy minimization (encompassing rational inattention or robust control), in macro models with uninsurable risk, borrowing constraints and nominal rigidities, to examine the monetary transmission mechanism. Both beliefs formation processes generate high average MPC, but also induce an excessively amplified consumption and investment response, which we directly compare to data evidence. Under sticky information the amplification is due to agents that anticipate future impediments in adjustment, even more so when they face liquidity constraints. Under signal extraction it is due to the state-contingent nature of the SDF. General equilibrium effects also contribute to the amplification. Persistence inducing device, such as habit preferences and interest rate smoothing, maintain promises in accounting for dampened dynamics. The second may come from central banks' incidental model's mis-specification and learning, or purposeful desire to influence the term structure and long term rates.

Keywords: *Beliefs Formation Processes, Incomplete Markets, Monetary Transmission, Micro Elasticities, Macro Elasticities.*

JEL classification: *J22, J23, J31, J62, E21, D31.*

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1. Introduction

Reconciling the micro and macro responses of consumption and other demand components has been a main challenge since at least the work of Kuznets (1946). A joint inclusion of liquidity constraints and myopia has often been proposed as a resolution to observed inconsistencies (see for instance Flavin (1981) or Flavin (1983)). Both are very realistic and have proven helpful to explain many empirical observations. In this paper we ask whether augmenting macro models with both, incomplete markets and modern formalizations of beliefs formation processes, can reconcile the MPC observed in the data and the macro responses as we know them from Christiano et al. (1999) and, if discrepancies arise, which other simple macro frictions provide a way forward. Our focus is on the monetary transmission mechanism, hence the response to changes in interest rates.

We introduce one of the two main classes of beliefs formation processes, sticky information (Mankiw and Reis (2002)) and signal extraction with entropy minimization (this encompasses various alternatives such as Sims (2003), Hansen and Sargent (2001) or Gabaix (2019))).[1] in the incomplete market macro model, with uninsurable risk, borrowing constraints and nominal rigidities.[2] The latter are needed to study the monetary transmission mechanism. Note that beliefs affect the stochastic discount factors, which enter future firms' profits, hence they affect the transmission of shocks more broadly. We study those models performance in reconciling the micro elasticities, such as the MPC, with the monetary transmission mechanism as we know it from Christiano et al. (1999). We compare them to various benchmarks: the representative agent model, the macro model with incomplete markets, also augmented with simple discount factor shocks.

Let us first motivate why those beliefs formation processes may help to dampen the consumption response. In the sticky information model a fraction of households does not update beliefs, hence part of the consumption demand is sticky. In the signal extraction problem households choose beliefs optimally by balancing utility maximization with minimization of

1 This class of models encompasses also the Caplin and Leahy (2019) framework of wishful thinking or the Brunnermeier and Parker (2005) of optimism. This is so when the sign of the penalty on entropy induces optimal beliefs that assign higher weights to the good states. Even in this case the essence of our results, which depends on the state contingent nature of the SDF remains.

2 Our baseline model features one asset, since we wish to isolate and put under microscope the interaction between beliefs and incomplete markets. But we also show results with the two assets model.

the entropy costs: a positive penalty on the information cost delivers beliefs skewed toward worst case scenarios.[3] This may enhance precautionary behaviour. As we explain below several effects are operative, both in the partial and the general equilibrium responses, so whether those models can reconcile the micro and macro responses ultimately requires a quantitative assessment, with plausible frictions and parametrization.

We find that: both models with beliefs formation deliver high average MPC,[4] but also an excessively amplified response of aggregate consumption and investment, which we compare directly to the data evidence a' la Christiano et al. (2005). The incomplete market model alone suffers of the same catch-22 condundrum, and this is known as we explain in our literature review below. This, as we also clarify in the draft, is largely due to the amplification from the general equilibrium effects. The model with sticky information delivers high MPC and a response of aggregate consumption that is slightly below the one of the incomplete market model, but still much higher than in the data, and does not exhibit hump shaped responses. Adding also discount factor shocks may help to dampen the dynamic, but does not resolve the puzzle. The model with signal extraction delivers the highest MPC, but also the most amplified aggregate consumption response. Note that in all cases an implausibly high degree of price rigidity (in the order of a non-adjustment Calvo probability of 0.99) brings the response on impact close to the data, albeit not hump-shaped: but this avenue was already considered, and excluded for DSGE models (see Christiano et al. (2005)). Alternatively, augmenting the models also with persistent discount factor shocks, akin to habit preferences, or interest rate smoothing dampens the consumption response, albeit with no humps. Note that the second may be considered a form of incomplete information on the side of central bankers, eventually leading to model mis-specification, or their purposeful intention to affect long term rates.

We discuss, with analytical derivations, an quantify, with numerical decompositions, the channels behind those results and how they balance out. We summarize them briefly here. In the sticky information framework a fraction of agents cannot update expectations, and

3 This is the case in the robust control a' la Hansen (1985) or Sims (2006).

4 The estimate that those models typically target are from Parker et al. (2013) or Fagereng et al. (2021), which range form 0.3 to 0.5. Note recent estimates, obtained under alternative experimental scenarios are smaller and in the order of 0.15 to 0.20 (see Orchard et al. (2024), Boehm et al. (2023) or Colarieti et al. (2024).

hence adjust consumption. This tames the response of aggregate consumption. The agents that adjust do so in response to all future shocks, as they anticipate that with positive probability they won't be able to adjust consumption in the future. In absence of liquidity constraints, the response to future interest rate shocks would increase precautionary savings and dampen the consumption response. But in presence of borrowing constraints, the result is the opposite: the MPC is very large and so it the consumption response. In the signal extraction model with penalty on information costs optimal beliefs are pessimistic and this may enhance precautionary savings. Those same optimal beliefs, however, are a function of future value functions, hence of all future shocks, and as such they render the stochastic discount factor (SDF since now on) state-contingent. This makes all consumption adjustment much more volatile. Besides the channels affecting the individual policy functions, both models feature general equilibrium effects, which are a typical by product of incomplete market models and which generally amplify the consumption response. Indeed, consistently with McKay et al. (2016), the absence of such general equilibrium effects would dampen the consumption response, even to the forward guidance policy. Our simulation results show that the balance of those effects in both models still leaves the consumption, and also the investment, responses too amplified relatively to the data. A decomposition of the impulse response functions into direct, or partial equilibrium effects, and indirect, or general equilibrium effects, show that the latter are mostly responsible for the amplification.

Our results also indicate that curbing the general equilibrium effects is possibly the most promising avenue for a reconciliation of the micro and macro responses. Those, which are typical of models with uninsurable risk, arise since the solution involves an iteration between individual policy functions and the subsequent changes in the interest rates that clear the market, based on the aggregation of the policy functions. One possibility to curb those effects is the k-level thinking proposed by Farhi and Werning (2019): this mechanism stops the iterations between the adjustment in the policy functions and the market clearing at a finite order, effectively dampening the general equilibrium amplification. We explore other simpler and empirically established mechanisms, which act as persistence inducing devices. Those include persistent stochastic discount factor shocks, which are akin to habit preferences, and

interest rate smoothing.[5] Past works have shown that interest-rate smoothing in particular is either an incidental manifestation of model mis-specification of a central bankers that learns from past observations (Rudebusch (2002) or Del Negro and Schorfheide (2009)), or a purposeful choice, due to the monetary policies such as forward-guidance aiming at affecting the term structure and long-term yields (see Coibion and Gorodnichenko (2012)). Those policies may include forward guidance. We show that interest rate smoothing is the most promising as it curbs the iterative adjustment on the interest rate and brings the consumption response closer to the data, although without delivering an hump-shaped response.

Related Literature. Reconciling the consumption response to anticipated and unanticipated shocks, as well as the micro and macro response, is a long standing issue. Early attempts for micro-founded macro models also relied on the idea of exploring the joint role of incomplete markets and myopia (see Flavin (1981) or Flavin (1983)).

DSGE models too have dealt with the challenges of reproducing persistent and hump-shaped response of demand components, particularly consumption. Christiano et al. (1999) and Christiano et al. (2005) jointly had already highlighted that macro models, with plausible degree of price rigidities, fail to replicate the persistent response of macro variables to monetary policy. They showed that a combination of habit in consumption,[6] frictions on investment and wage rigidity helps.[7] DSGE models however did not delve into the reconciliation of micro and macro responses, as they did not feature heterogeneity. Models with incomplete markets emerged to account for the high marginal propensity to consume and they successfully did so by including hand-to-mouth agents. An extensive literature emerged and extended the incomplete markets models to include nominal rigidities and other rigidities to study the monetary transmission mechanism (see Kaplan et al. (2018), McKay et al. (2016) or Auclert and Rognlie (2018) among many others others). A distinctive feature of those models is the possibility to decompose the monetary transmission mechanism into direct and indirect, or general equilibrium effects: changes in the consumption-saving margins, triggered by

5 The appeal of interest rate smoothing over the years was also due to its direct observation in the data. See Barro (1988).

6 An important contribution by Campbell and Cochrane (1999) had established that consumption habit also resolve asset price puzzles.

7 Habit curbs the consumption response on impact. Frictions on investment, and wage rigidity, curb the response of future marginal costs, hence inflation, nominal rates and future consumption. The latter in turn affects the current response of consumption in forward looking models.

monetary policy shocks, lead to second order changes in interest rates through asset market adjustment. Several contribution highlighted the role of those channels for the amplification in the response of the macro variables.

To reconcile the response of the aggregate variables to the hump-shaped ones observed in the data, even in response to forward-guidance policies, some authors have proposed to extend the incomplete market models further with deviations from rational expectations. Farhi and Werning (2019) introduce k-level thinking to explain the forward guidance puzzle. Prior to that McKay et al. (2016) argued that the forward guidance puzzle could be solved in a model with incomplete markets: crucially their model however neutralized the distributional effects of the policy, which is one of the elements that induce large average MPC and, hence, large responses in aggregate consumption. Furthermore, Auclert et al. (2020) show that a combination of various frictions, including sticky information to macro shocks, can bring an estimated macro model of incomplete market close to the data. Our paper also explores the combined role of incomplete markets and various types of belief formation processes and argues that a solid way to reconcile micro and macro responses is to curb the general equilibrium effects, which can be done also with k-level thinking or other macro persistence-inducing devices.

Our work is also related to the large literature that studies belief formation processes and its role for the macroeconomy. We focus on two main classes of belief formation processes. On the one side, there is the sticky information or epidemiological approach whose main contributions include Mankiw and Reis (2002) and Carroll (2003). In those models agents are subject to a fixed poisson shock that prevents them from adjusting their expectations. The second class of imperfect information models features a signal extraction problem, where agents face a cost of gathering information. Those set-ups all features a two stage problem in which the agent first chooses the optimal belief, by trading off precision in the decision problem against the cost of gathering information, proxied by an entropy constraint. In Hansen and Sargent (2001) the entropy measure is given by the Kullbach-Leibler, in Sims (2006) it is given by the Shannon measure, in Gabaix (2019) the optimal beliefs are chosen in a linear quadratic framework. In all those cases the resulting optimal beliefs tend to

assign larger weights to bad states, presumably enhancing cautionary behaviour. In Caplin and Leahy (2019) the entropy constraint is introduced so that the optimal beliefs result in higher weights to the good states.[8] In the second stage, households choose consumption, by weighting utilities with their optimal subjective beliefs. Both those types of beliefs formation processes are realistic and grounded in experimental evidence on micro decisions. Our paper examines whether alone they can reconcile the micro and macro responses, and consider which macro feature may complement their role.

In studying both these classes of belief formation processes in a general equilibrium model with incomplete markets, we are able to dissect and quantitatively account for their role in individual policy functions and for their interaction with general equilibrium effects.

2. The Macro Model with Uninsurable Risk and Belief Formations

We present first the baseline dynamic general equilibrium model with incomplete markets and nominal rigidities and assess its monetary transmission mechanism against the data. In this context we also consider the role of discount factor shocks. We then extend it to include sticky information or signal extractions and re-assess the monetary transmission mechanism.

2.1. The Baseline Model with Uninsurable Risk

In the baseline model agents face uninsurable risk and borrowing constraints and invest in one asset. An intermediary channels households' savings to firms. Firms in the production sector are monopolistic competitive, use labour and capital to produce and face adjustment costs on prices and investment. We maintain this set of macro frictions to make our model comparable to modern macro medium scale frameworks. The fiscal authority maintains a balanced budget with lump sum taxes and the monetary policy is set based on a Taylor rule.

We purposefully focus on the one asset model as we want to cleanly highlight the marginal

8 Another model of optimistic beliefs is Brunnermeier and Parker (2005).

role of deviations from rational expectations. In Appendix A.1 we also derive and describe the monetary transmission mechanism in variants with two assets and wage rigidity.

One reason to entertain deviations from rational expectations is that they curb the response to future expectations. For this reason in the baseline version of the model we also consider the role of discount factor shocks.

Households Decisions. At the beginning of each period t, households experience an idiosyncratic productivity shock e_t, which follows an n_e-state Markov process with transition matrix $P(e_{t+1}|e_t)$ and a stationary distribution $f(e)$. Since now on, the state space is $s_t = [e_t, \epsilon_t]$, where ϵ_t is the monetary policy shock and where the idiosyncratic and the monetary shocks are independent from each other. We can also define at each period t the history of shocks as $s^t = [s_0, ..., s_t] \in S^t$. Given the initial state (a_0, s_0, ϵ_0), each agent chooses a sequence of state contingent plans for consumption, labour supply and assets, (c_t, n_t, a_t), where, since now on, the sub-index t indicates that the policy function is conditional to the history of shocks up to period t. Moreover since now on we define the expectation operator E_t, where the sub-index t indicates that the expectation is taken with respect to the unconditional joint probability of all the states in the state space, idiosyncratic and aggregate ones, that we define as $\pi(s_t)$ and as $\pi(s^t)$ when taken as conditional to the whole history. Real labor income ξ_t, at time t, is a function of labor hours supplied n_t, and real wages w_t: $\xi_t = e_t w_t n_t$. To focus on the consumption-saving decision, and its implication under incomplete markets, in the baseline we set labour hours as chosen homogenously by the household head and being equal across members experiencing different realization of income. But we also present the impulse responses to monetary shocks to various variants of the baseline incomplete market models, which include the one with heterogenous hours.

Each household, in period t, chooses consumption, c_t, savings in liquid, a_t, assets to maximize the following value function:

$$\begin{aligned} V_t(s_t, a_{t-1}) &= \max_{c_t, a_t} u(c_t, n_t) + \beta E_t V_{t+1}(s_{t+1}, a_t) \\ \text{s.t.} \quad & c_t + a_t = \xi_t + (1 + r_t^a) a_{t-1}; \qquad a_t \geq 0 \end{aligned} \tag{1}$$

where $V_t(s_t, a_{t-1})$ is the value function in time t. The first-order condition with respect to assets is:

$$u_c(c_t, n_t) = \mu_t + \beta E_t \partial_b V_{t+1}(s_{t+1}, a_t) \tag{2}$$

where m_t is the Lagrange multiplier on the borrowing limit, $u_c(c_t, n_t)$ is the marginal utility of consumption. The envelope condition implies: $\partial_a V_t(s_t, a_{t-1}) = u_c(c_t)(1 + r_t^a)$.

Labour Supply. In the baseline model variant labor hours per capita are chosen by the family as a whole and are determined through the following intra-temporal first-order condition:

$$\varphi n_t^{\rho} = \int u_c(s_t, a_{t-1}) \frac{\partial \xi_t}{\partial n_t} dD(e_t, a_{t-1}) \tag{3}$$

where $dD(e_t, a_{t-1})$ is the density of the share of households in each triplet of the endogenous state space $[e_t, a_t]$. Equation (3) states that the marginal utility of labour income equalizes the aggregate labour dis-utility. The solution to the optimization problem is a collection of type-specific policy functions, $c_t(s_t, a_{t-1})$, $a_t(s_t, a_{t-1})$, that depend on the path $\{r_k^a, w_k\}_{k>t}$, taken as given by households.

Household Aggregates, Fixed Point Equilibrium and Arbitrage. A key tenet of heterogenous agent models with incomplete markets is a fixed point equilibrium, in which each individual policy function depends upon market prices, and the latter solve market clearing conditions, that involve aggregation of the individual policy functions. In the baseline framework consumption-saving decisions are heterogenous across households, hence the procedure involves a fixed point iteration with the asset market clearing condition. The aggregate policy functions for households' savings and consumption, a_t, in every period $i \geq t$ are:$\mathcal{A}_t(r_i^a, \tau_i) = \Big(\int a(s_t, a_{t-1}) dD(e_t, a_{t-1}, b_{t-1}) \Big), \mathcal{C}_t(r_i^a, \tau_i) = \Big(\int c(s_t, a_{t-1}) dD(e_t, a_{t-1}) \Big)$. Per capita labour hours are homogenous across agents experiencing different income realization, hence aggregate effective labor supply is given by the per capita hours weighted by the shares of agents in each income realization:$N_t^{Supply} = n_t \int e_t dD(e_t, a_{t-1})$.

Asset Market Clearing and Financial Intermediary. Under monopolistic competition

firms earn non-zero profits, hence households saving are channeled through a competitive intermediary into claims on firms' future dividends. The asset market clearing condition then reads as follows: $\mathcal{A}_t = v_t$. The zero profit condition of the financial intermediary implies:$(1 + r_t^a)\mathcal{A}_{t-1} = \frac{d_t+v_t}{v_{t-1}} v_{t-1}$, where d_t are firms' dividends and v_t is the value of all firms in the economy. The no arbitrage condition implies: $E_t(1 + r_{t+1}) = \frac{E_t(d_{t+1}+v_{t+1})}{v_t} = \frac{1+i_t}{E_t(1+\pi_{t+1})}$, with i_t being the central bank's rate.

2.2. Production Sector

Firms produce varieties, y_t, over which they hold monopoly power. A representative competitive firm aggregates varieties through a CES aggregate consumption bundle. The optimal demand for each variety is then given by $p_{i,t} = \left(\frac{Y_t}{y_t}\right)^{\frac{1}{\eta}} p_t$, where p_t is the aggregate price index and η is the elasticity of substitution across varieties. Firms choose capital and labour input, investment and prices to solve the following optimization problem:

$$
\begin{aligned}
J_i\left(k_{t-1}\right) = \max_{p_{i,t},k_t,I_t,l_t} \quad & \left\{ \frac{p_{i,t}}{p_t} y_t - w_t l_t - I_t - \frac{1}{2\varkappa\varepsilon_I}\left(\frac{k_t - k_{t-1}}{k_{t-1}}\right)^2 k_{t-1} - \frac{\eta}{2\kappa}\ln(1+\pi_t)^2 Y_t + \right. \\
& \left. + \frac{J_{i,t+1}(k_t)}{1+r_{t+1}} \right\} \\
\text{s.t.} \quad & k_t = (1-\delta)k_{t-1} + I_t - k_t^{adj}; \;\; p_{i,t} = \left(\frac{Y_t}{y_t}\right)^{\frac{1}{\eta}} p_t; \;\; y_t = z_t k_{t-1}^{\nu} l_t^{1-\nu} \quad (4)
\end{aligned}
$$

The first order condition for labour and capital demand lead to:

$$
l_{o,t} = \left(\frac{mc_t(1-\nu)\alpha_o}{w_t^o}\right)^{\frac{1}{1-\sigma}} \left(z_t k_{t-1}^{\nu}\right)^{\frac{\sigma}{(1-\nu)(1-\sigma)}} y_t^{\frac{\sigma-1+\nu}{(1-\nu)(\sigma-1)}}, \tag{5}
$$

$$
(1 + r_{t+1})q_t = \nu z_{t+1}\left(\frac{L_{t+1}}{k_t}\right)^{1-\nu} mc_{t+1} - \left[\frac{k_{t+1}}{k_t} - (1-\delta) + \frac{1}{2\varkappa\varepsilon_I}\left(\frac{k_{t+1}-k_t}{k_t}\right)^2\right] + \frac{k_{t+1}}{k_t} q_{t+1} \tag{6}
$$

where we defined q_t as the Lagrange multiplier on the capital accumulation and, hence, as the shadow price of capital, and its expression is obtained from the first order condition

on investment: $q_t = 1 + \frac{1}{\varkappa \varepsilon_I}\left(\frac{k_t - k_{t-1}}{k_{t-1}}\right)$, and where mc_{t+1} is the Lagrange multiplier on the production constraint and represents the real marginal cost. The first order condition with respect to prices leads to the Phillips curve:

$$\log(1+\pi_t) = \kappa(mc_t - \frac{1}{\mu_p}) + \frac{Y_{t+1}}{Y_t}\log(1+\pi_{t+1})\Psi_{t,t+1} \tag{7}$$

where π_t is inflation, mc_t are marginal costs, $\mu_p = \frac{\eta}{\eta - 1}$ and $\Psi_{t,t+1}$ is the stochastic discount factor and is equal to $\frac{1}{1+r_{t+1}}$.

Monetary and Fiscal Policy Stance. Monetary policy follows a simple Taylor-type rule $i_t = r^* + \phi_\pi \pi_t$. The fiscal authority keeps a balanced budget, so that $r_t B^g + G = T_t$, where T_t is the tax revenue, G stands for government expenditures and B^g is the constant level of government debt. There is a flat tax rate, τ_t, on households' income, which is: $\xi_t = (1 - \tau_t) e_t w_t n_t$. Tax revenue, aggregated across households, are $tau_t(e_t w_t n_t)$. Like before the fiscal authority adjust taxes to balance the budget.

Definition. Competitive Equilibrium. *A Competitive Equilibrium of the economy satisfies the following definition: the sequence $[c_t, a_t]_{t=0}^{\infty}$ solves households' consumption-saving and labour hours decision in Equation (1), given the distribution of idiosyncratic shocks, $P(e_{t+1}|e_t)$ and the sequence of prices r_t^a, r_t. Labour hours are set homogenously according to 3 and are aggregated according to shares of households in each wealth percentile. Aggregate asset holdings and consumption of the households are equal to the product of the individual optimal functions and the distribution of households across assets. Firms choose labor demand, l_t, and capital inputs, k_t to solve discounted profit optimization, given in Equation (4). Market clearing for financial assets, and the aggregate resource constraint, are satisfied. Monetary policy determines the short term interest rate according to the Taylor rule and fiscal policy sets a balanced budget, with lump sum taxes.*

Numerical Solution and Calibration. The stochastic steady state is solved combining the endogenous grid method (see Carroll (2006)) and a multivariate Newton method. The dynamic response to monetary shock is computed using recent approximated methods based on numerical derivatives in sequence space: see Boppart et al. (2020) or Auclert et al. (2021).[9].

9 The underlying rationale of the method lies in finding a solution in sequence space, in which all operations

Specifically the responses of policy functions to sequences of shocks are given by numerical derivatives in sequence space. Hence they provide the linearized solution in response to the path of shocks. The equilibrium distribution of agents across wealth percentiles is then obtained by verifying that the aggregated policy functions solve a set of macro targets, typically given by the full general equilibrium conditions. The parameters used in the calibration are largely taken from the literature. In Appendix B we describe their values and the reason for their choices, both for the baseline model and the variants we examine.

MPCs in the Baseline Model. Before examining the transmission of the monetary shock it is instructive to examine how the baseline model performs in terms of matching the marginal propensity to consume in face of an unexpected shock. Empirical estimates vary significantly. The estimate that those models typically target are from Parker et al. (2013) or Fagereng et al. (2021). The first estimates the MPC in response to the economic stimulus of 2008, using additional questions from the Consumer Expenditure Survey and exploiting the randomized nature of the policy. The second uses sizable lottery prizes in Norwegian administrative panel data to explore how transitory income shocks are spent and saved over time. Jointly those studies give numbers for the average MPC ranging form 0.3 to 0.5. It shall be noted that those estimates are currently being revised downward under alternative experimental scenarios: latest estimates, coming from Orchard et al. (2024), Boehm et al. (2023) or Colarieti et al. (2024), report estimates in the order of 0.15 to 0.20.

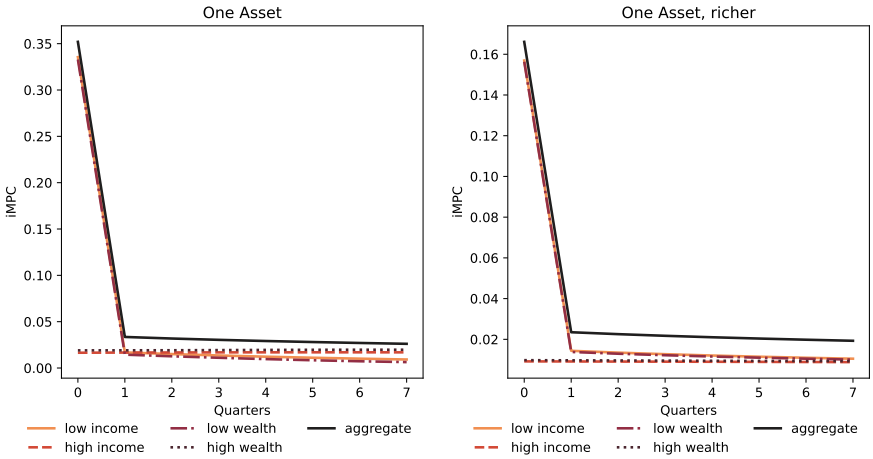
Figure 1 shows the marginal propensity to consumers generated by the baseline model, also for a higher calibration of the idiosyncratic shock volatility. The MPC are shown for high and low wealth and income individuals. As one may expect the MPC are higher for low income and low wealth households, as they are more likely to become borrowed constraints. The highest MPC is for low income households in the baseline model calibration and peaks 0.35. The aggregate MPC is around the estimates that are used as targets from the literature (see Parker et al. (2013) or Fagereng et al. (2021)). The MPC becomes much smaller if we increase the variance of uninsurable risk as this strengthens the incentives for precautionary

like summation and multiplication are defined for sequences, see, for example, Dunford and Schwartz (1958)). Any variable in the model is therefore an infinite sequence.

saving, hence dampening the partial equilibrium consumption response (see bottom panel of figure 1.

Note that we purposefully focus on the one asset model, as we aim at cleanly disentangling the role of belief formation. It is however well known that the two-asset variants of models with incomplete markets (Kaplan et al. (2018) or Bayer et al. (2019)) has had more success in matching the average MPC, due to the presence of wealthy agents that become borrowing constrained. In appendix B.2 indeed we show that this is the case. However, those models deliver both a higher MPC, but also a even more amplified response of aggregate consumption. Hence the portfolio choice does not contribute to the micro-macro response reconciliation.

Figure 1: Marginal propensities to consumer from unexpected income shocks in the one asset model. Yellow, red and burgundy lines, both solid and dashed show the MPCs of high and low wealth and income households. The black line shows the aggregate MPC.



(a) MPC **(b) MPC, higher risk**

2.3. The Monetary Transmission Mechanism in the Baseline and in Christiano et al. (1999)

Before extending the model with deviations from rational expectations, it is instructive to examine the monetary transmission mechanism and its comparison to the data counterparts. For the latter we take the impulse responses reported in the VAR with Choleski decomposition a' la Christiano et al. (1999). Other empirical specifications, such as local projections or alternative identification schemes, may be possible. However there is large agreement that the patterns identified in Christiano et al. (1999) are generally valid.

Figure 2: Monetary Policy Shock in a One Asset Model. The figure shows impulse-responses to a monetary policy shock in different versions of the one-asset model. Variables are in percentage deviations from the steady state. All model IRFs are in response to a monetary policy shock with 0.6 persistence and a value of the shock normalized to match the empirical trajectory of the Central Bank rate. Empirical impulse-responses from Christiano et al. (2010) are shown in violet. Shaded area stands for the 95% confidence interval.

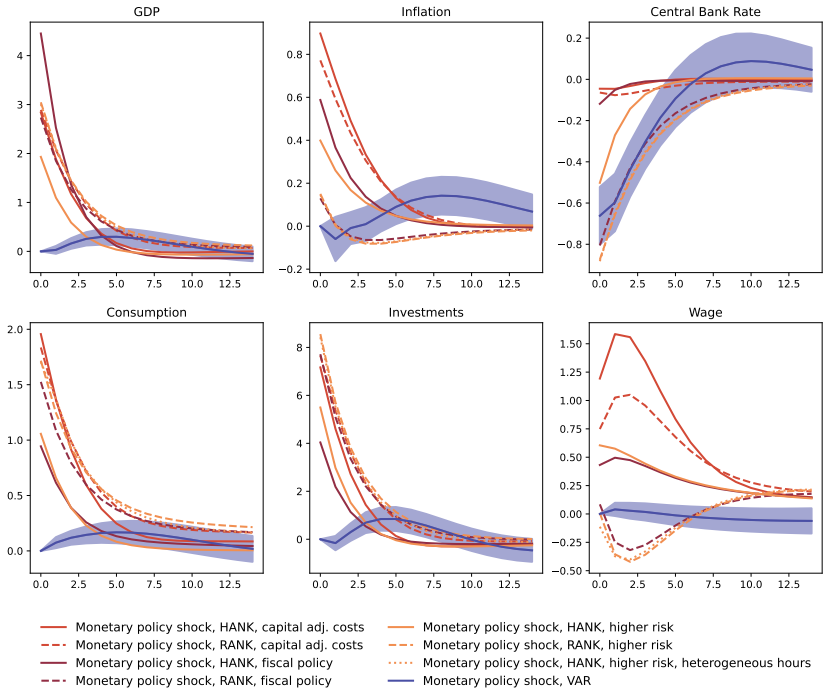
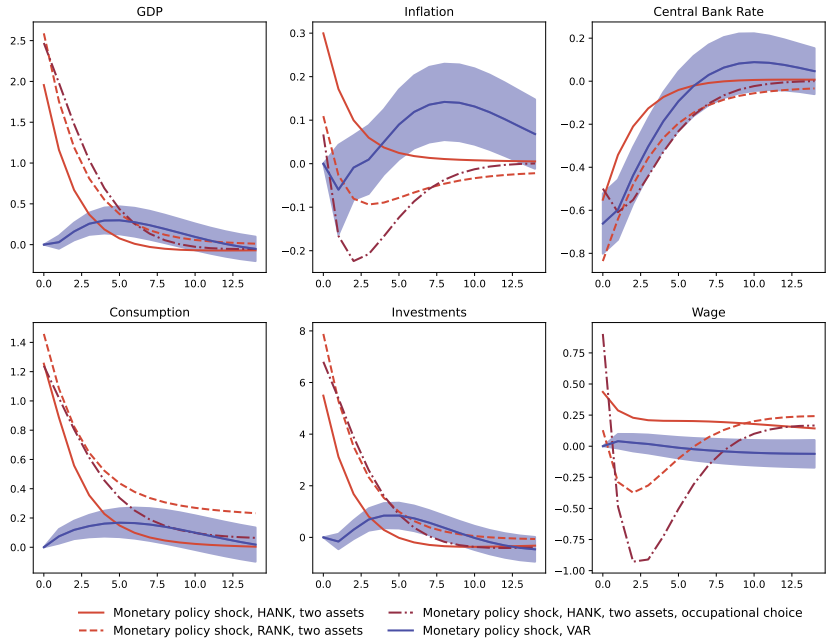


Figure 2 shows impulse responses to 25 basis point shock to the interest rate. In each panel we compare the impulse responses to a monetary shock in the model with incomplete markets to the empirical ones and the ones implied by the equivalent representative agent model. To provide a thorough assessment of the incomplete market models, we also show the monetary transmission mechanisms in several variants: with higher volatility of idiosyncratic risk, active fiscal policy and heterogenous labours hours.

The main result from this comparison is that the response of consumption, and investment, in the models is much higher than in the data and surely not hump shaped. The response of investment is about three times that of consumption, thereby in line with the business cycle relative fluctuations one would expect. But as the response of consumption is so amplified relatively to the data, so is the one of investment. As known, in absence of habit preferences, the consumption responses in the representative agent model is also higher than the empirical one.

Can Other Frictions Rescue the Empirical Validity. For completeness we examine

Figure 3: Monetary Policy Shock in Two Assets Model. The figure shows impulse-responses to a monetary policy shock in a two-assets model. Variables are shown in percentage deviations from the steady state. Model IRFs are in response to a monetary policy shock (easing) with 0.6 persistence. Empirical impulse-responses from Christiano et al. (2010) are shown in violet. Shaded area stands for the 95% confidence interval.

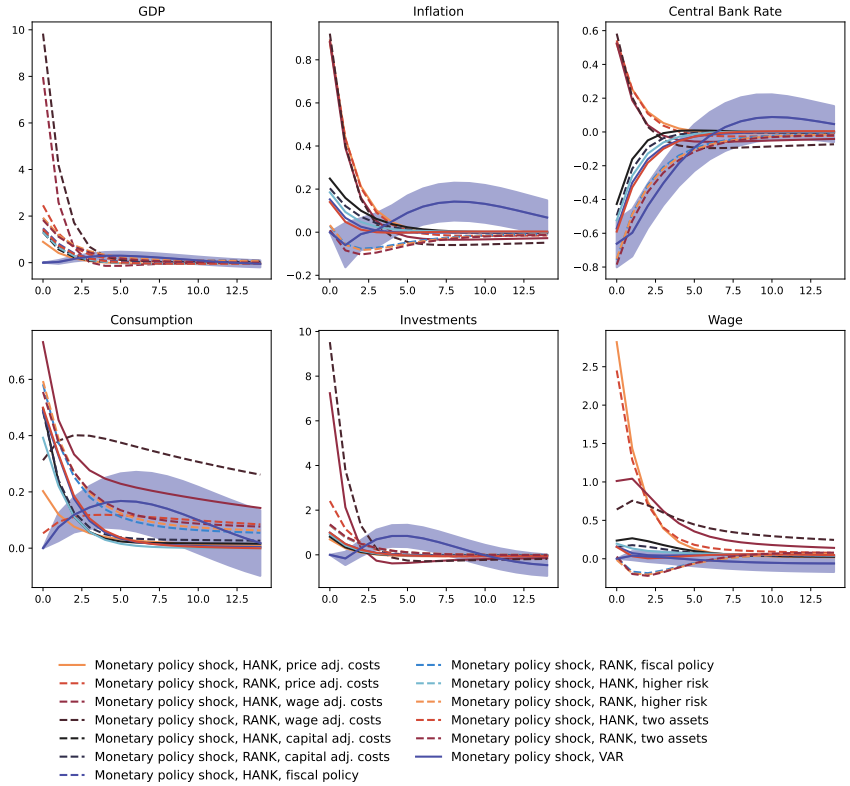


the monetary transmission mechanism also in more complex variants of the incomplete market model. Figure 3 shows the impulse responses in the model with two assets, a liquid and non-liquid. Figure 4 considers a large set of model variants, including the one with wage rigidity. The derivations for the model with two assets and with nominal wage rigidity are presented in Appendix A.1, their calibration is in Appendix B. The model variants presented in 4 include also the case with an extremely high adjustment cost on price, shown with the yellow and the dashed red lines.[10] In all cases, with the exception of the implausibly high price rigidity, the consumption response is much larger than in the data, and in no case it is hump shaped.

Can the Incomplete Market Model be Extended to Include Habit Preferences? In DSGE models hump shaped response of consumption can be obtained with habit preferences, which imply that current consumption depends on past values. This avenue however is not valuable and feasible in models with incomplete markets. Introducing external habits would reduce the MPC much below the one observed in the data. Introducing internal

10 Quantitatively the calibration of the adjustment cost delivers a Phillips curve, whose slope corresponds in the Calvo model to a probability of non-adjusting prices of 0.99.

Figure 4: Monetary Policy Shock in Variants of the One Asset Model. The figure shows impulse-responses to a monetary policy shock in different versions of the one-asset model: with wage rigidity, active fiscal policy, higher idiosyncratic risk. Variables are in percentage deviations from the steady state. All model IRFs are in response to a monetary policy shock with 0.6 persistence and a value of the shock normalized to match the empirical trajectory of the Central Bank rate. Empirical impulse-responses from Christiano et al. (2010) are shown in violet. Shaded area stands for the 95% confidence interval.



habits [11] implies that a solution of the fixed point equilibrium involves both current and past consumption, a procedure that can hardly achieve convergence.

Un-bundling the Mechanism. Direct and Indirect Channels and the Role of the Wealth Distribution. The response of the macro variables in incomplete market models is due to the direct impact of the monetary shocks on the policy functions and a general equilibrium effect, by which changes in the policy functions trigger further changes in interest rates through adjustment in the asset markets. Understanding the relative relevance of the two provides insights on how to bring the model dynamic close to the data. Figure 5 shows a decomposition of the impulse responses into the direct and the general equilibrium effects. Figure Figure 6 provides a closely related decomposition into changes in the policy functions and changes in the distribution. Appendix B.1 presents formal derivations that clarify the link between the two decompositions. The decomposition is presented again for various model variants, with different volatilities of idiosyncratic risk, active fiscal policy, heterogenous hours, nominal wage rigidities and the two asset variants. The general equilibrium component accounts, in all cases, for the bulk of the transmission channels. Indeed any amplification of the aggregate variables is induced by this channel.

2.4. Extending the Model with Sticky Information

We now extend the baseline model to include sticky information along the lines of Carroll et al. (2020), Mankiw and Reis (2002) or Auclert et al. (2020). Households in the model are randomly hit by a shock that follows a binomial or Poisson structure, so that with probability θ they adjust their expectations and with probability $(1-\theta)$ they don't. Given the history of events, $s^t = [s_0, ..., s_t]$, which includes both the realization of idiosyncratic and aggregate

11 Several authors show that adding peer effects, hence consumption dependence from the average, helps to resolve several puzzles from asset prices (see Campbell and Cochrane (1999), Abel (1990), Constantinides (1990), Tallarini and Zhang (2005) among many others.

Figure 5: Direct and Indirect Effects. All figures show the decomposition into a direct and an indirect effect. Direct effect incorporates the response to (liquid) interest rate. Indirect responses include the responses to wage, labor hours, taxes and illiquid interest rate depending on the model variant.

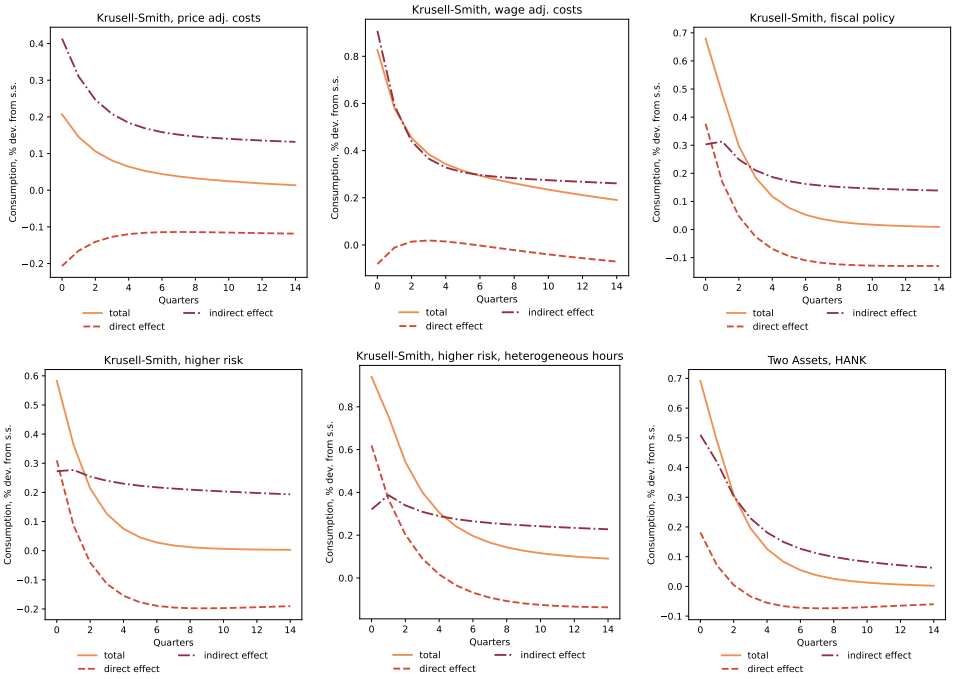
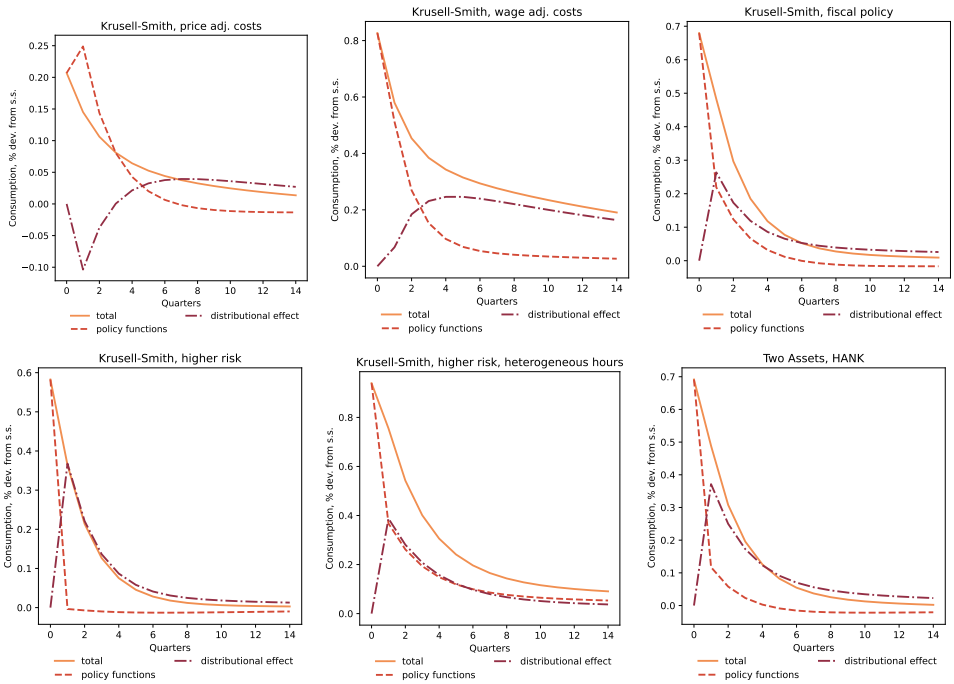


Figure 6: Policy Functions and Distributional Effect. All figures show the decomposition into a change induced by the change in policy functions and a change induced by a change in the distribution. All impulse-responses are in response to the same shock as in Figure 5, Figure 2 and Figure 3. Distributional effect is calculated by setting to 0 the first row of the $\mathcal{F}$ matrix in the fake news algorithm.



shocks, up to period t, the household that can update at time t chooses a plan for current and future consumption so as to solve:

$$V_t(s_t, a_{t-1}) = \max_{c_{t+i}, a_{t+i}} [\sum_{i=0}^{\infty} \beta^i (1-\theta)^i u(c_{t+i,i}, n_{t+i,i}) + \beta\theta \sum_{i=0}^{\infty} \beta^i (1-\theta)^i E_t[V_{t+1}(s_{t+1+i}, a_{t+i})]$$
$$\text{s.t.} \quad c_{t+i} + a_{t+i} = \xi_{t+i} + (1 + r^a_{t+i}) a_{t+i-1} \qquad a_t \geq 0 \tag{8}$$

Taking first order conditions, between any period $t+1$ and $t+i+k$, with respect to consumption and assets leads to:

$$\beta^i (1-\theta)^i E_t[u_c(c_{t+i,i}, n_{t+i,i}) - \mu_{t+i}] = \beta\theta \sum_{k=i}^{\infty} \beta^k (1-\theta)^k E_t[\partial_a V_{t+1}(s_{t+1}, a_t) R^a_{t+i,t+k+1}] \quad i = 0, 1, 2... \tag{9}$$

$$\partial_a V(s_{t+1}, a_t) = \beta\theta \sum_{k=i}^{\infty} \beta^k (1-\theta)^k E_t[V_{t+1+k}(s_{t+1+k}, a_{t+k}) R^a_{t,t+k+1}] \tag{10}$$

where μ_t is the Lagrange multiplier on the borrowing constraint, $R^a_{t+i,t+k+1}$ and $R^a_{t+i,t+k+1}$ are the compound returns between any period $t+1$ and $t = k+1$ and all others variables are defined as in 9.

In sum, the joint distribution of the updating event enters the Euler equation, rescaling the future marginal utility. Intuitively this implies that the infrequent update may potentially dampen the transmission of shocks, through its impact on future expectations. How powerful is this mechanism and whether it can generate a hump shape dynamic is an quantitative question to be addressed in the full structural model.

Lemma 1. *If there are no liquidity constraints a shock that increases the interest induces the household to increase precautionary saving. If there are liquidity constraints a future increase in the interest rate also increases the tightness of the borrowing limit. This in turn reduces future marginal utility and increases current consumption by more.*

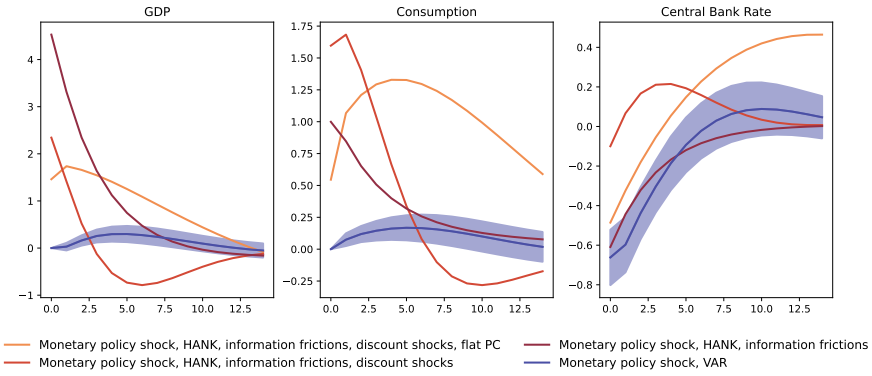
Equation 9 is behind Lemma 1. For part one of the lemma we set the Lagrange multiplier, μ, equal to zero. In this case higher expected interest rates reduce spending today and encourage savings. If the agent is liquidity constraint today the MPC rises. If the agent expects to be liquidity constrained in the future, hence unable to spend savings and with a probability (1θ) unable to adjust the plan, its MPC rises by more. Lemma 1 clarifies

that, in presence of liquidity constraints, multiple effects operate on the policy function at household level. Some go in the direction of increasing precautionary savings and some in the direction of increasing the current MPC. The response of the aggregate consumption function will weight also the consumption of the agents that do not update, and this may tame the response. Eventually how those effects balance out is a quantitative question to be addressed in the simulated model.

The Transmission Mechanism with Sticky Information. Figure 7 presents with the burgundy line the results for the one asset model with sticky information. In the sticky information model the impulse response of consumption is somewhat more muted than in the baseline, but the difference is not large and we detect no hump shaped response. Infrequent expectation updates adds three main effects to the model. The first affects the individual consumption function of the agents that update their expectations: as they are aware that in the future their update is infrequent, they adjust consumption today much more. This is even more so if those agents also happen to be borrowing constrained. Hence this effects amplifies the consumption response. Second, the aggregate consumption response includes the policy functions of the households that do not update. This tames the aggregate consumption response. Third, there are general equilibrium effects, stemming from second order changes in interest rate that follow the asset market adjustment. As explained earlier, those tend to amplify the aggregate consumption response. The overall response depends on the balance of those three effects.

Can Shocks to the Stochastic Discount Factor Rescue the Impulse Responses of the Model? Since the pioneering work by Flavin (1981) and Flavin (1983) the goal of aligning the consumption dynamic in the model and the data has called attention onto the role of discount factor. A high MPC arises when liquidity constraints impart a heightened consumption response in booms. A dampened response of the aggregate consumption function may be obtained by taming the response to future expectations. We therefore now extend the model with sticky information to include also an AR(1) shock to the discount factor. Results are shown with the orange line in figure 7, where the yellow line captures the model variant with discount factor shocks. As reference curves we include the empirical responses

Figure 7: Monetary Policy Shock in a One Asset Model with Discount Factor Shocks. The figure shows impulse-responses to a monetary policy shock in different versions of the one-asset model augmented with a discount factor shock. Variables are shown in percentage deviations from the steady state. All model IRFs are in response to a monetary policy shock with 0.6 persistence and a value of the shock normalized to match the empirical trajectory of the Central Bank rate. Empirical impulse-responses from Christiano et al. (2010) are shown in violet. Shaded area stands for the 95% confidence interval.



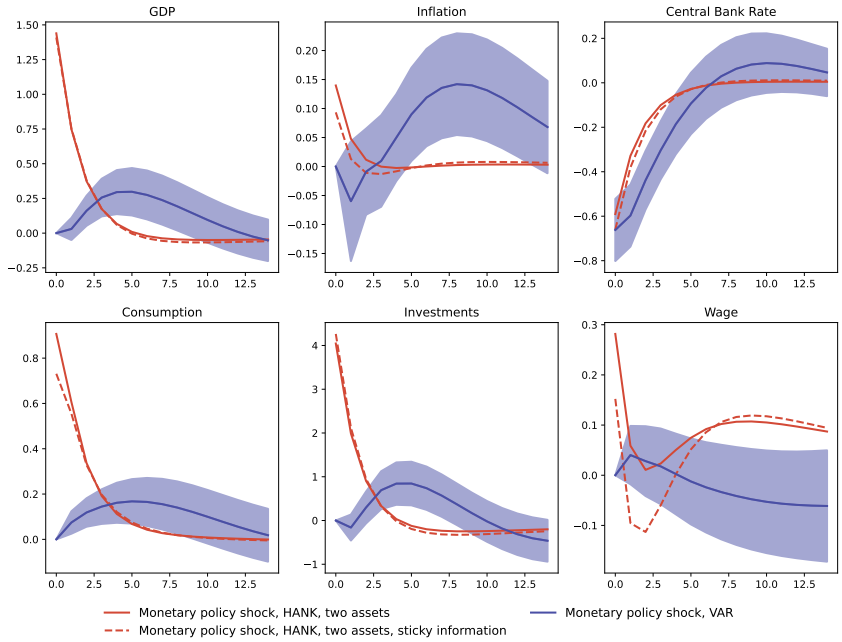
alongside the one asset model and its variant with an almost flat Phillips curve, or else with a very high degree of price stickiness. As already known implausibly high degree of price stickiness can dampen the consumption response and bring it closer to the data. Under the mode with sticky information and a discount factor shock the impulse response becomes hump shaped, albeit it remains significantly above the empirical response.

How does sticky information performs in the two asset model. The incomplete market model with two assets has been shown to perform very well in matching high MPC, primarily for the possibility that a fraction of high wealthy individual can become liquidity constraints. We therefore now include sticky information also in the two asset model, with non-liquid assets featuring liquidation costs. The baseline derivations of the two asset model, as well as additional details on the calibration, are presented in Appendix A.1. Impulse responses are shown in figure 8. The model with sticky information, the red dashed line, is compared a usual with the baseline incomplete market model and the empirical impulse responses. Once again, in all cases the response of aggregate consumption lacks persistence and is higher than in the data. Similar conclusions hold for investment, GDP and inflation.

2.5. Incomplete Market Model Augmented with Signal Extraction

We now examine the performance of the incomplete market model augmented with a signal extraction problem. The households' optimization now features two-stages. In the first

Figure 8: Monetary Policy Shock in Two Assets Model with Sticky Information. The figure shows impulse-responses to a monetary policy shock the model with sticky information. Variables are shown in percentage deviations from the steady state. Model IRFs are in response to a monetary policy shock (easing) with 0.5 persistence. Empirical impulse-responses from Christiano et al. (2010) are shown in violet. Shaded area stands for the 95% confidence interval.



stage households choose the optimal beliefs by trading off higher precision in the optimized value function against a higher cost of information, proxied by a penalty parameter on an entropy constraint. The second stage consists in the consumption-saving decision based on the value functions weighted by the optimal beliefs derived in the first stage. Broadly speaking, signal extraction modifies the households' stochastic discount factor through the weighting with the optimal beliefs. Under specifications with positive penalty parameters (Hansen and Sargent (2001) or Sims (2006)) optimal beliefs are skewed toward worst case scenarios. As this enhances precautionary behaviour, those beliefs may contribute to dampen the consumption response. Next, we lay down the model's derivations and discuss all the channels that characterize it.

The Optimization Problem. Time is discrete. As before there is a continuum of households indexed by $i \in [0;1]$ that maximize the discounted utility $\sum_{t=0}^{\infty} u(c_t, n_t)$. The utility function is increasing, concave, differentiable and displays decreasing absolute risk aversion, a class which includes constant relative risk aversion. Each period, each household i receives a productivity shock e_t and, as before, the state space is $s_t = [e_t, \epsilon_t]$, where ϵ_t is the

monetary policy shock. Given the initial state $(a_0, b_0, s_0, \epsilon_0)$, each agent chooses a sequence of state contingent plans a_t, b_t, unconditional subjective beliefs $\pi(s^t)$, where $s^t = [s_0, ..., s_t] \in S^t$ is the history of idiosyncratic and aggregate shock realizations through period t. Furthermore, we indicate future subjective beliefs, conditional to the history up to t, as $\pi(s_{t+1} \mid s^t)$. Their maximization problem is:

$$V(s_0, a_0) = min_{\pi(s^t)} max_{(c_t, a_t)} \sum_{t=0}^{\infty} \beta^t \sum_{s^t \in S^t} \pi(s^t) u(c_t, n_t) - \frac{1-\beta}{\vartheta} \sum_{t=1}^{\infty} \beta^t \sum_{s^t \in S^t} \pi(s^t) ln \frac{\pi(s^t)}{\overline{\pi}(s^t)} \tag{11}$$

$$\text{s.t.} \quad c_t + a_t = \xi_t + (1 + r_t^a) a_{t-1}; \qquad a_t \geq 0; \qquad \sum_{s^t \in S^t} \pi(s^t) = 1$$

where the agent faces an entropy costs, $\sum_{s^t \in S^t} \pi(s^t) ln \frac{\pi(s^t)}{\overline{\pi}(s^t)}$, given by the *Kullbach-Leibler* measure, and where the Lagrange multiplier ϑ is a penalty parameter on the information cost. We allow agents to choose beliefs at all horizons. We discount the distortion of future beliefs at the same rate as future utility. The factor $1 - \beta$ compensates for the fact that distorting beliefs $\pi(s^t)$ also distort $\pi(s^t)$ for all $k > t$, so that $\pi(s^t)$ enters the sequence problem with a weight $\frac{\beta^t}{1-\beta}$. Appendix A.4 shows that the value function is recursive. If so, we can rewrite the maximization problem as follows:

$$V(s_t, a_{t-1}) = min_{\pi(s_{t+1} \mid s^t)} max_{(c_t, a_{t-1})} u(c_t, n_t) + \beta \sum_{s_{t+1} \in S_{t+1}} \pi(s_{t+1} \mid s^t) V(s_{t+1}, a_t) \tag{12}$$

$$- \frac{\beta}{\vartheta} \sum_{s_{t+1} \in S_{t+1}} \pi(s_{t+1} \mid s^t) ln \frac{\pi(s_{t+1} \mid s^t)}{\overline{\pi}(s_{t+1} \mid s^t)}$$

$$\text{s.t.} \quad c_t + a_t = \xi_t + (1 + r_t^a) a_{t-1}; \qquad a_t \geq 0; \qquad \sum_{s^t \in S^t} \pi(s_{t+1} \mid s^t) = 1$$

The maximization problem can be broken in tow stages.

First Stage. In the first stage the household chooses $\pi(s_{t+1})$ to solve the minimization problem, which leads to the following optimal beliefs:

$$\pi(s_{t+1} \mid s^t) = \frac{\overline{\pi}(s_{t+1} \mid s^t) exp(\vartheta V(e_{t+1}, a_t))}{\sum_{s_{t+1} \in S_{t+1}} \overline{\pi}(s_{t+1} \mid s^t) exp(\vartheta V(s_{t+1}, a_t))} \tag{13}$$

Derivations of 13 are in Appendix A.

Second Stage. In the second stage we substitute the optimal belief into the recursive value functions. This delivers:

$$V(s_t, a_t) = u(c_t, n_t) + \frac{1}{\vartheta} ln E_t exp(\vartheta V(s_{t+1}, a_t)) \tag{14}$$

The above delivers the Euler:

$$u_c(c_t, n_t) - \mu_t = \beta E_t[\frac{\overline{\pi}(s_{t+1}) exp(\vartheta V(s_{t+1}, a_t))}{\sum_{s_{t+1} \in S_{t+1}} \overline{\pi}(s_{t+1}) exp(\vartheta V(s_{t+1}, a_t))} u_c(c_t, n_t)(1 + r^a_{t+1})] \tag{15}$$

Tom simplify matters labour hours are chosen homogenously by the household head. Since the optimal beliefs weight the value function, they enter all households decisions.

Channels Affecting Individual Policy Functions. Before discussing the numerical results it is instructive to examine the channels that potentially affecting consumption-saving decisions in this model. As before different channels emerge.

From equation 15, and for given objective probabilities, the stochastic discount factor of this model reads as follows:

$$m_{t+1} = \frac{exp(\vartheta V(s_{t+1}, a_t))}{\sum_{s_{t+1} \in S_{t+1}} exp(\vartheta V(s_{t+1}, a_t))} \tag{16}$$

The conditional deviation m_{t+1} affects how agents assign different subjective probabilities, relatively to the objective ones, to future events. In particular, if for some states $m_{t+1} > 1$ agents are assigning higher subjective probabilities to those states, while if $m_{t+1} < 1$ the opposite holds. Given this, the ambiguity parameter θ captures the degree to which agents are uncertain about the probability measure. Note that in past literature this parameter was generally considered fixed and positive (see Hansen and Sargent (2001) or Sims (2006)), but variants of it have been considered (Caplin and Leahy (2019) consider a negative parameter, Bassanin et al. (2021) model a time-varying parameter that matches survey evidence). In what follows we consider the baseline case of a positive penalty on information costs.

Lemma 2. *If $\theta < 0$, $m_{t+1} < 1$ for future good states and $m_{t+1} > 1$ for future bad states. Hence, beliefs endogenously emerge as left-skewed and agents act with pessimism.*

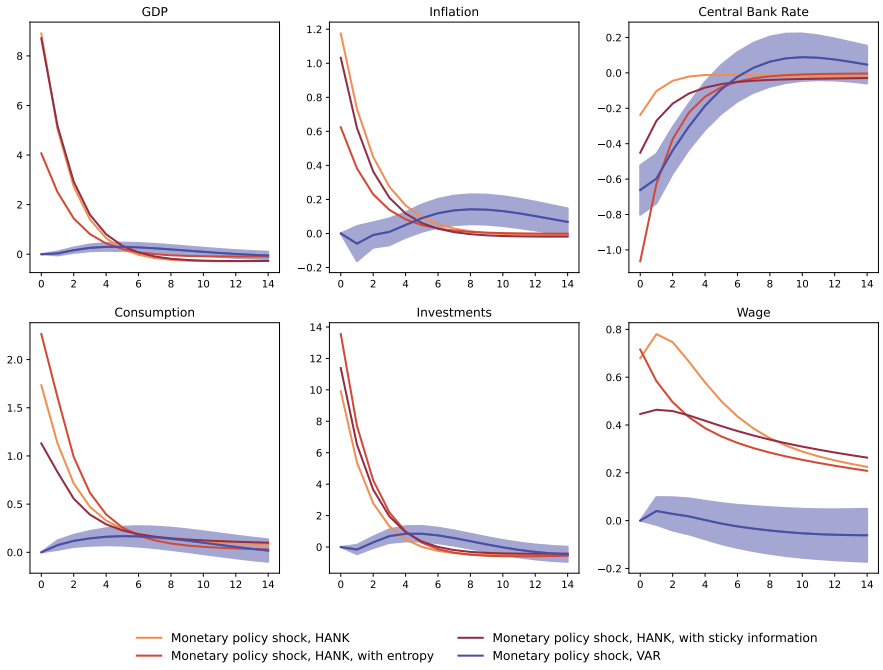
Proof. *When $\theta < 0$ for future high utility states ($V(s_{t+1}, a_t) > \mathbb{E}_t\{V(s_{t+1}, a_t\})$ the condition $\exp\{\vartheta V(s_{t+1}, a_t)\} > \mathbb{E}_t\{\exp\{\vartheta V(s_{t+1}, a_t)\}\}$ holds, namely the risk-adjusted value function for future good states is smaller than the average one. This implies that $m_{t+1} > 1$ for bad states, agents overweight the worst case scenarios, hence they are pessimist.*

The above lemma has two implications. In absence of liquidity constraints, left-skewed beliefs may enhance precautionary saving. However, two additional effects deliver higher MPC and higher aggregate consumption response. First, the stochastic discount factor is state contingent, hence any change in consumption is larger than in the baseline economy. Second, once we re-introduce liquidity constraints, left skewed beliefs also increase the expectations of being liquidity constrained in the future, and hence they increase MPC today.

Besides the channels that affect individual policy functions, this model too features the general equilibrium effects characterizing incomplete markets models, which generally tend to amplify the response of consumption.

The Monetary Transmission Mechanism with Signal Extraction. Figure 9 shows the impulse response functions to a 25 basis point monetary shock in the baseline one asset model with uninsurable risk to the one augmented with sticky information and with signal extraction (red line for the latter). Figure 12 shows a similar comparison, but excluding the empirical counterpart. The first, burgundy line, is the one with sticky information. The second is an incomplete market model, where the monetary policy rules embeds a simple interest rate smoothing. Adding sticky information dampens the response of consumption relatively to the one of the baseline incomplete market model. Signal extraction instead amplifies it. The response of investment is not much different across the three models and it is excessively amplified in all. The same is true for the response of GDP, wages and inflation.

Figure 9: Monetary Policy Shock in a one Asset Model with Signal Extraction. The figure shows impulse-responses to a monetary policy shock in different versions of the one-asset model augmented with optimal beliefs from a signal extraction problem. Variables are shown in percentage deviations from the steady state. All model IRFs are in response to a monetary policy shock with 0.6 persistence and a value of the shock normalized to match the empirical trajectory of the Central Bank rate. Empirical impulse-responses from Christiano et al. (2010) are shown in violet. Shaded area stands for the 95% confidence interval.

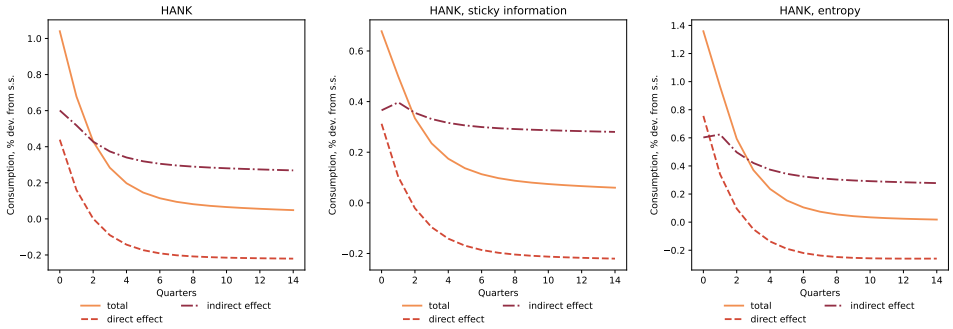


2.6. The Role of the General Equilibrium Effects

In all those models the response of consumption also depends on the extent of the general equilibrium effects, namely the fluctuations induced by the changes in prices need to cleat the markets. We noted and discussed their role in the baseline incomplete market model. Those effects represent the key distinctive feature that characterizes incomplete market models compared to the representative agent model and they are responsible for about 2/3 of the transmission of monetary policy shocks. Therefore, they are largely responsible for the amplified response of aggregate consumption and other demand components.

We now examine the decomposition between the direct and the indirect channels, described before, in the models with sticky information and signal extraction. We also compare them to the same decomposition in the baseline one asset model. The main results are very similar across the three models: the indirect or general equilibrium effects are the main drivers of the dynamic in the impulse response function. This is even more so in the model with sticky in formation.

Figure 10: Direct and Indirect Effects across Models. All figures show the decomposition into a direct and an indirect effect in three models, the baseline one asset model with uninsurable risk, the one augmented with sticky information and the one augmented with signal extraction. Direct effect incorporates the response to (liquid) interest rate. Indirect responses include the responses to wage, labor hours, taxes and illiquid interest rate depending on the model variant.



2.7. Persistence Inducing Devices

We had previously shown that adding persistent AR(1) discount factor shocks to the one asset model with stick information seem to hold promise in terms of generating a hump shaped response of consumption, albeit not dampened enough as to reach the data equivalent. The stochastic discount factor shock is methodologically akin to habit preferences, which as discussed earlier are not easily applicable in incomplete market models.

We now consider a second persistence inducing devise, interest-rate smoothing. Note that potentially there maybe several macro frictions that curb the transmission mechanism. We are proposing some that have two features. The first is to be simple and induce consistent dynamic across models' variants. The second is their possibility of curbing either the stochastic discount factor or the general equilibrium effects.

We have argued that a large part of the amplification stems from consumption response to higher order changes in interest rates due to market clearing adjustment. It seems therefore reasonable to include a smoothing devise precisely on the interest rate adjustment. Traditionally interest rate smoothing has been very popular among central bankers since it is clearly detectable in the data. Furthermore, it has often been associated to learning or model mis-specification by the central banker (see Rudebusch (2002) or Del Negro and Schorfheide (2009)): it therefore seems to align with the assumptions of a model featuring information frictions. An alternative interpretation of the interest rate smoothing is that of an intentional choice, implemented when the monetary authority aims at control or affect

the term structure and long term rates. Those policies do include the forward guidance. The yellow line in figure 9 plots the impulse responses of the incomplete market model with interest rate smoothing. The consumption response is now much closer to the data equivalent, though still not hump shaped.

3. Conclusions

An expanding literature is developing macro models with heterogeneity which can match the distributions and the micro elasticities, whose empirical estimates are now more widely available. So far most of the models focused on the role of incomplete markets, but traditionally deviations from rational expectations were considered to have an important role in explaining the dynamic of the consumption-saving margin. We solve three incomplete market models, two of which features modern beliefs formation processed and assess their ability to reconcile micro and macro consumption, or other demand components, elasticities.

Our main results show that both incomplete markets and deviations from rational expectations tend to increase MPC or micro elasticities, albeit for different reasons. Beliefs formation processes operate mostly through modifications of the individual SDFs. However, all of them also imply an excessively amplified response of aggregate consumption. We detect the main reason for this amplification in the peculiar channel that characterizes macro models with uninsurable risk, namely the general equilibrium effects activated by the changes in the asset distribution. Those effects become even larger and more prominent with endogenous subjective beliefs. We suggest that the resolution of those puzzles most likely lie in a combination of incomplete markets that help to match the micro elasticities and simple learning processes on the policy actions.

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A. Analytical Derivations

A.1. Incomplete Market Models Variants

In this section we describe the model blocks for some variants of the baseline incomplete market model. We describe first the variant in which we extend the household portfolio choice to include two assets, one liquid and one non-liquid, with an adjustment costs for shifting out of the non-liquid asset. The second variant we present include wage rigidity in the production.

Two-Assets Case. In this case, given the initial state $(a_0, b_0, s_0, \epsilon_0)$, each agent chooses a sequence of state contingent plans, (c_t, n_t, a_t, b_t), where a_t is the non-liquid asset and b_t is the liquid one to maximize the following value function:

$$\begin{aligned} V_t(s_t, a_{t-1}, b_{t-1}) &= \max_{c_t, a_t, b_t} u(c_t, n_t) + \beta E_e V_{t+1}(s_{t+1}, a_t, b_t,) \\ \text{s.t.} \quad c_t + a_t + b_t &= \xi_t + (1 + r_t^a)a_{t-1} + (1 + r_t^b)b_{t-1} - \Phi(a_t, a_{t-1}); \qquad a_t \geq 0, \quad b_t \geq \underline{b} \end{aligned} \tag{A.1}$$

Following an established tradition (see Baumol (1952) and Tobin (1956)) we include a portfolio adjustment cost. We adopt the formulation proposed in Straub (2019), which encompasses both a fixed and a convex component, and reads as follows: $\Phi(a_t, a_{t-1}) = \frac{\chi_1}{\chi_2}\left|\frac{a_t - (1-r_t^a)a_{t-1}}{(1+r_t^a)a_{t-1}+\chi_0}\right|^{\chi_2}[(1 + r_t^a)a_{t-1} + \chi_0]$, with $\chi_0 > 0$, $\chi_1 > 0$ and $\chi_2 > 1$. Note that $\Phi(a_t, a_{t-1})$ is bounded, differentiable,[12] and convex in both arguments. The first-order conditions with respect to liquid and non-liquid assets are:$u_c(c_t, n_t)[1+\Phi_{a_t}(a_t, a_{t-1})] = \mu_t^1 + \beta E_t \partial_a V_{t+1}(s_{t+1}, a_t, b_t)$ and $u_c(c_t, n_t) = \mu_t^2 + \beta E_t \partial_b V_{t+1}(s_{t+1}, a_t, b_t)$, where $u_c(c_t, n_t)$ is the marginal utility of consumption. The envelope condition implies: $\partial_a V_t(s_t, a_{t-1}, b_{t-1}) = u_c(c_t)(1 + r_t^a - \Phi_{a_{t-1}}(a_t, a_{t-1}))$ and $\partial_b V_t(s_t, a_{t-1}, b_{t-1}) = u_c(c_t)(1 + r_t^b)$. The first order condition for the labour supply remains the same, except that now the marginal utilities are also function of the both assets. The aggregate policy functions for households' savings and consumption, a_t, in every period $i \geq t$ are: $\mathcal{A}_t(r_i^a, \tau_i) = \left(\int a(s_t, a_{t-1}) dD(e_t, a_{t-1}, b_{t-1})\right)$, $\mathcal{B}_t(r_i^b, \tau_i) = \left(\int b(s_t, a_{t-1}) dD(e_t, a_{t-1}, b_{t-1})\right)$, $\mathcal{C}_t(r_i^a, \tau_i) = \left(\int c(s_t, a_{t-1}) dD(e_t, a_{t-1})\right)$. At last, arbitrage in this case is extended to include the equalization of the expected returns on liquid and non-liquid assets.

12 Except at $a_t = (1 - r_t^a)a_{t-1}$.

Definition. Competitive Equilibrium for a Two Asset Economy with Incomplete Markets. *A Competitive Equilibrium of the economy satisfies the following definition: the sequence* $[c_t, a_t, b_t]_{t=0}^{\infty}$ *solves households' consumption-saving and labour hours decision in Equation (A.1), given the distribution of idiosyncratic shocks,* $P(e_{t+1}|e_t)$ *and the sequence of prices* r_t^a, r_t^b, r_t*. Labour hours are set homogenously according to 3 and it is aggregated according to the shares of households in each wealth percentile. Aggregate asset holdings and consumption of the households are equal to the product of the individual optimal functions and the distribution of households across occupations and assets. Firms choose labor demand for each occupation,* l_t*, and capital inputs,* k_t *to solve discounted profit optimization, given in Equation (4). Market clearing, for financial assets and labour markets for each occupation, and the aggregate resource constraints are satisfied. Monetary policy determines the short term interest rate according to the Taylor rule and fiscal policy sets a balanced budget.*

Nominal Wage Rigidity with Homogenous Labour Hours. We introduce wage rigidity both, in the model with homogenous and heterogenous labour hours. The comparison provides further insights on the interplay between nominal rigidities and adjustment in labour supply. When wages are rigid, and individuals all work the same labour hours, the likelihood of unvoluntary labour supply is higher than when they can adjust labour hours in response to their income shocks. Consider first the case in which labour hours are set homogenously across individuals, according to Equation (3), we assume that unions face adjustment costs when setting wages (see Galí (2015) or Auclert et al. (2023)). Labour services are provided to intermediate good firms at nominal wages, w_t, by competitive labour packers, who combine specialized services $n_{k,t}$, through a CES technology that reads as follows:$n_t = \left(\int_k n_{k,t}^{\frac{(\epsilon_w - 1)}{\epsilon_w}} dk \right)^{\frac{\epsilon_w}{\epsilon_w - 1}}$. Each of the labour services is offered by a union, indexed by k. Given the above aggregator, the packers choose the optimal demand for each labour service k, which reads as follows: $n_{k,t} = (\frac{w_{k,t}}{w_t})^{-\epsilon_w} n_t$, where $w_{k,t}$ is the nominal wage set by union k. Union k combines efficiency units of labour supplied by households into union-specific labour services, $n_{k,t} = \int_i e_{i,t} n_{i,k,t} dD_i$, and sets nominal wages, $w_{k,t}$, facing quadratic adjustment costs: $\frac{\epsilon_w}{2\kappa_w} log(\frac{w_{k,t}}{w_{k,t-1}\Pi_w})^2$, where i for brevity stands for (e_t, a_{t-1}). Note that, under the homogenous labour hours case, the individual labour services are uniformly supplied by each individual

i, hence $n_{i,k,t} = n_{k,t}$, or else they are chosen by a household head equally for all. Given the demand for its labour services, and taking households' consumption saving decisions as given, union k chooses the nominal wage $w_{k,t}$ and the labour services, $n_{k,t}$, that maximize the average utility of households members:

$$\sum_{s=0}^{\infty} \beta^s \{ \int_i \{ \frac{c_{i,t+s}^{1-\sigma} - 1}{1-\sigma} - \int_k \varphi \frac{n_{i,k,t+s}^{1+\rho}}{1+\rho} dk \} dD_i - \frac{\epsilon_w}{2\kappa_w} log(\frac{w_{k,t}}{w_{k,t-1}\Pi_w})^2 \} \tag{A.2}$$

subject to the demand function, $n_{k,t} = (\frac{w_{k,t}}{w_t})^{-\epsilon_w} n_t$. After imposing symmetry across unions, and after defining the real wage as, $w_t^r = \frac{w_t}{p_t}$, the union's first order condition with respect to wages reads as follows:

$$0 = -\frac{\epsilon_w}{\kappa_w} \frac{log(\frac{\Pi_{w,t}}{\Pi_w})}{w_t^r} + \beta \frac{\epsilon_w}{\kappa_w} \frac{log(\frac{\Pi_{w,t+1}}{\Pi_w})}{w_t^r} + \int_i e_{i,t} n_t c_{i,t}^{-\sigma} dD_i + \lambda_t [-\epsilon_w (w_t^r)^{-1} n_t] \tag{A.3}$$

and the one for labour demand is:$0 = \int_i e_{i,t} w_t^r c_{i,t}^{-\sigma} dD_i - \varphi n_t^{\rho} - \lambda_t$, where $\Pi_{w,t} = \frac{w_t}{w_{t-1}}$ is the wage inflation and $\lambda_{k,t} = \lambda_t$ is the multiplier associated with the labour demand constraint. Merging the two conditions delivers the following wage Phillips curve:

$$log\left(\frac{\Pi_{w,t}}{\Pi_w}\right) = \kappa_w \left(\varphi n_t^{\rho} - \frac{\epsilon_w - 1}{\epsilon_w} w_t^r \int_i e_{i,t} c_{i,t}^{-\sigma} dD_i \right) n_t + \beta log\left(\frac{\Pi_{w,t+1}}{\Pi_w}\right) \tag{A.4}$$

where $\mu_w = \frac{\epsilon_w}{\epsilon_w - 1}$ is the wage mark-up. The definition of the competitive equilibrium for this economy reads as follows:

Competitive Equilibrium, Incomplete Market Model with Wage Ridigity. *A Competitive Equilibrium of the economy satisfies the following definition: the sequence* $[c_t, a_t]_{t=0}^{\infty}$ *solves households' consumption-saving in Equation (1), given the distribution of idiosyncratic shocks,* $P(e_{t+1}|e_t)$ *and the sequence of prices* r_t^a, r_t*. Wages and labour hours are set by labour unions respectively based on A.3 and* **??**. *Aggregate asset holdings and consumption of the households are equal to the product of the individual optimal functions and the distribution of households across occupations and assets. Firms choose labor demand,* l_t*, and capital inputs,* k_t *to solve discounted profit optimization, given in Equation (4). Market clearing, for financial assets and labour markets for each occupation, and the aggregate*

resource constraints are satisfied. Monetary policy determines the short term interest rate according to the Taylor rule and fiscal policy sets a balanced budget.

A.2. Derivations of Equation 13

The households chooses $\pi(s_{t+1} \mid s^t)$ to solve the following optimization problem:

$$V(s_t, a_{t-1}) = min_{\pi(s_{t+1}|s^t)} max_{(a_{t-1})} u(c_t, n_t) + \beta \sum_{s_{t+1} \in S_{t+1}} \pi(s_{t+1} \mid s^t) V(s_{t+1}, a_t) - \tag{A.5}$$
$$- \frac{\beta}{\vartheta} \sum_{s_{t+1} \in S_{t+1}} \pi(s_{t+1} \mid s^t) ln \frac{\pi(s_{t+1} \mid s^t)}{\overline{\pi}(s_{t+1} \mid s^t)}; \qquad s.to \sum_{s^t \in S^t} \pi(s_{t+1} \mid s^t) = 1$$

where we assume to have substituted the budget constraint in the utility function. The first order conditions with respect to $\pi(s_{t+1} \mid s^t)$ in state s_{t+1} leads to:

$$V(s_{t+1}, a_t, b_t) - \frac{1}{\vartheta} ln(\frac{\pi(s_{t+1} \mid s^t)}{\overline{\pi}(s_{t+1} \mid s^t)}) - \frac{1}{\vartheta} \pi(s_{t+1} \mid s^t) \frac{\overline{\pi}(s_{t+1} \mid s^t)}{\pi(s_{t+1} \mid s^t)} \frac{1}{\overline{\pi}(s_{t+1} \mid s^t)} - \lambda_{t+1} = 0 \tag{A.6}$$

Rearranging leads to:

$$V(s_{t+1}, a_t, b_t) - \frac{1}{\vartheta} ln(\frac{\pi(s_{t+1} \mid s^t)}{\overline{\pi}(s_{t+1} \mid s^t)}) - \frac{1}{\vartheta} - \lambda_{t+1} = 0 \tag{A.7}$$

$$ln(\frac{\pi(s_{t+1} \mid s^t)}{\overline{\pi}(s_{t+1} \mid s^t)}) = \vartheta V(s_{t+1}, a_t, b_t) - 1 - \vartheta \lambda_{t+1} \tag{A.8}$$

$$\pi(s_{t+1} \mid s^t) = \overline{\pi}(s_{t+1} \mid s^t) exp[\vartheta V(s_{t+1}, a_t, b_t) - 1 - \vartheta \lambda_{t+1}] \tag{A.9}$$

The first order conditions with respect to state s'_{t+1} leads to:

$$\pi(s'_{t+1} \mid s^t) = \overline{\pi}(s'_{t+1} \mid s^t) exp[\vartheta V(s'_{t+1}, a_t, b_t) - 1 - \vartheta \lambda_{t+1}] \tag{A.10}$$

We can now substitute both of them into the $\sum_{s^t \in S^t} \pi(s_{t+1} \mid s^t) = 1$ leads to:

$$1 = \sum_{s^t \in S^t} \pi(s_{t+1} \mid s^t) = \overline{\pi}(s_{t+1} \mid s^t) exp[\vartheta V(s_{t+1}, a_t, b_t) - 1 - \vartheta \lambda_{t+1}] + \tag{A.11}$$
$$+ \sum_{s'_{t+1} \in S_{t+1}/s_{t+1}} \overline{\pi}(s'_{t+1} \mid s^t) exp[\vartheta V(s'_{t+1}, a_t, b_t) - 1 - \vartheta \lambda_{t+1}]$$

which implies:

$$exp[-1 - \vartheta \lambda_{t+1}] = [\sum_{s_{t+1} \in S_{t+1}} exp[\vartheta V(s'_{t+1}, a_t, b_t)]]^{-1} \tag{A.12}$$

Substituting A.12 into A.9 leads to:

$$\pi(s_{t+1} \mid s^t) = \frac{\overline{\pi}(s_{t+1} \mid s^t) exp(\vartheta V(e_{t+1}, a_t))}{\sum_{s_{t+1} \in S_{t+1}} \overline{\pi}(s_{t+1} \mid s^t) exp(\vartheta V(s_{t+1}, a_t))} \tag{A.13}$$

A.3. Derivations of Equation 14

Substituting A.13 into the value function leads to:

$$V(s_t, a_{t-1}) = max_{(a_{t-1})} u(c_t, n_t) + \tag{A.14}$$
$$+ \beta \sum_{s_{t+1} \in S_{t+1}} \frac{\overline{\pi}(s_{t+1} \mid s^t) exp(\vartheta V(e_{t+1}, a_t))}{\sum_{s_{t+1} \in S_{t+1}} \overline{\pi}(s_{t+1} \mid s^t) exp(\vartheta V(s_{t+1}, a_t))} V(s_{t+1}, a_t) -$$
$$- \frac{\beta}{\vartheta} \sum_{s_{t+1} \in S_{t+1}} \frac{\overline{\pi}(s_{t+1} \mid s^t) exp(\vartheta V(e_{t+1}, a_t))}{\sum_{s_{t+1} \in S_{t+1}} \overline{\pi}(s_{t+1} \mid s^t) exp(\vartheta V(s_{t+1}, a_t))} ln\left(\frac{\frac{\overline{\pi}(s_{t+1}|s^t) exp(\vartheta V(e_{t+1}, a_t, b_t))}{\sum_{s_{t+1} \in S_{t+1}} \overline{\pi}(s_{t+1}|s^t) exp(\vartheta V(s_{t+1}, a_t))}}{\overline{\pi}(s_{t+1} \mid s^t)}\right)$$

Rearranging leads to:

$$V(s_t, a_t) = u(c_t, n_t) + \frac{1}{\vartheta} ln \tilde{E}_t exp(\vartheta V(s_{t+1}, a_t)) \tag{A.15}$$

where the $\tilde{E}_t$ is the expectation operator with respect to the optimal subjective beliefs.

A.4. Recursivity of the Value Function with Optimal Subjective Beliefs

The general optimization problem reads as follows:

$$V(s_0, a_0,) = min_{\pi(s^t)} max_{(c_t, a_t)} \sum_{t=0}^{\infty} \beta^t \sum_{s^t \in S^t} \pi(s^t) u(c_t, n_t) - \tag{A.16}$$
$$- \frac{1-\beta}{\vartheta} \sum_{t=1}^{\infty} \beta^t \sum_{s^t \in S^t} \pi(s^t) ln(\frac{\pi(s^t)}{\overline{\pi}(s^t)})$$
$$\text{s.t.} \quad c_t + a_t = \xi_t + (1 + r_t^a) a_{t-1}; \qquad a_t \geq 0; \qquad \sum_{s^t \in S^t} \pi(s^t) = 1$$

Once the budget constraint is substituted in the utility function the objective function reads as follows:

$$V(s_0, a_0) = min_{\pi(s^t)} max_{(a_t)} \sum_{t=0}^{\infty} \beta^t \sum_{s^t \in S^t} \pi(s^t) u(a_t, s_t) - \tag{A.17}$$
$$- \frac{1-\beta}{\vartheta} \sum_{t=1}^{\infty} \beta^t \sum_{s^t \in S^t} \pi(s^t) ln(\frac{\pi(s^t)}{\overline{\pi}(s^t)})$$

Note that we can write the cost on entropy as a function of a specific realization at time $t = 1$:

$$\sum_{t=1}^{\infty} \beta^t \sum_{s^t \in S^t} \pi(s^t) ln \frac{\pi(s^t)}{\overline{\pi}(s^t)} = \tag{A.18}$$
$$= \sum_{y_1 \in Y} [\sum_{s^t \in S^t, s.t. s_1 \in S^t} \pi(s_1) \pi(s^t \mid s_1) ln(\frac{\pi(s_1)\pi(s^t \mid s_1)}{\overline{\pi}(s_1)\overline{\pi}(s^t \mid s_1)})] =$$
$$= \sum_{s_1 \in S} [\pi(s_1) ln(\frac{\pi(s_1)}{\overline{\pi}(s_1)}) + \sum_{s^t \in S, s.t. s_1 \in s^t} \pi(s_1) \pi(s^t \mid s_1) ln(\frac{\pi(s^t \mid s_1)}{\overline{\pi}(s^t \mid s_1)})]$$

this implies that:

$$\frac{1-\beta}{\vartheta}\sum_{t=1}^{\infty}\beta^t\sum_{s^t\in S^t}\pi(s^t)ln\frac{\pi(s^t)}{\overline{\pi}(s^t)}= \tag{A.19}$$
$$=\frac{1-\beta}{\vartheta}\sum_{t=1}^{\infty}\sum_{s_1\in S}[\pi(s_1)ln(\frac{\pi(s_1)}{\overline{\pi}(s_1)})+\sum_{s^t\in S,s.t.s_1\in s^t}\pi(s_1)\pi(s^t\mid s_1)ln(\frac{\pi(s^t\mid s_1)}{\overline{\pi}(s^t\mid s_1)})]=$$
$$=\frac{\beta}{\vartheta}\sum_{s_1\in S}\pi(s_1)ln(\frac{\pi(s_1)}{\overline{\pi}(s_1)})+\sum_{s_1\in S}\beta\pi(s_1)\frac{1-\beta}{\vartheta}\sum_{t=2}^{\infty}\beta^{t-1}\sum_{s^t\in S,s.t.s_1\in s^t}\pi(s_1)\pi(s^t\mid s_1)ln(\frac{\pi(s^t\mid s_1}{\overline{\pi}(s^t\mid s_1)})]$$

We can now decompose the utility as follows:

$$\sum_{t=0}^{\infty}\sum_{s\in S}\pi(s^t)u(a_{t-1},s_t)=u(s_1,a_0)+ \tag{A.20}$$
$$+\beta\sum_{t=1}^{\infty}\beta^{t-1}\sum_{s_1\in S}\sum_{s^t\in Y^t}\pi(s_1)\pi(s^t\mid s_1)u(s_t,a_t)=$$
$$=u(s_1,a_0)+\beta\sum_{s_1\in S}\pi(s_1)\sum_{t=1}^{\infty}\beta^{t-1}\sum_{s^t\in Y^t}\pi(s^t\mid s_1)u(s_t,a_{t-1})$$

the merging things together delivers the following:

$$V(s_0,a_0)=max_{(a_t))}\sum_{t=0}^{\infty}\beta^t\sum_{s^t\in S^t}\pi(s^t)u(c_t,n_t)- \tag{A.21}$$
$$-\frac{1-\beta}{\vartheta}\sum_{t=1}^{\infty}\beta^t\sum_{s^t\in S^t}\pi(s^t)ln(\frac{\pi(s^t)}{\overline{\pi}(s^t)})=$$
$$=max_{(a_t)}[u(s_1,a_0)+\frac{\beta}{\vartheta}\sum_{s_1\in S}\pi(s_1)ln(\frac{\pi(s_1)}{\overline{\pi}(s_1)})]+$$
$$+\beta\sum_{s_1\in S}\pi(s_1)[\sum_{t=1}^{\infty}\beta^{t-1}\sum_{s^t\in Y^t}\pi(s^t\mid s_1)u(s_t,a_{t-1})+$$
$$+\frac{1-\beta}{\vartheta}\sum_{t=2}^{\infty}\beta^{t-1}\sum_{s^t\in S,s.t.s_1\in s^t}\pi(s_1)\pi(s^t\mid s_1)ln(\frac{\pi(s^t\mid s_1)}{\overline{\pi}(s^t\mid s_1)})]=$$
$$=max_{(a_t)}[u(s_1,a_0)-\frac{\beta}{\vartheta}\pi(s_1)ln(\frac{\pi(s_1)}{\overline{\pi}(s_1)})+\beta\sum_{s_1\in S}\pi(s_1)V(s_1,a_0)]$$

B. Models Calibration

In this section we describe the calibration choice and summarize the parameters used to calibrate either the one or the two asset version of the model. Other parameters, such as adjustment costs on prices, wages and capital, when present in the model are common across the two main model variants with one and two assets. We describe the US calibration in details below. Calibration to other countries follows the same principles.

Baseline Calibration for One Asset Model. Time in the model is in quarters. Most parameters are set following the literature on models with heterogeneous agents and uninsurable risk, whereby values are set based on targets for asset distributions. All of the parameters related to the preferences and production sector are set following Auclert et al. (2021). The steady-state interest rate on government bonds is calibrated to 0.0125, close to the 5-year average of the Effective Federal Funds Rate. The time discount factor, β, is then obtained from the asset market clearing condition. The earning shock, e, follows an autoregressive process with its autocorrelation equal to 0.966 and the standard deviation equal to 0.92. The steady-state markup is set to 1.015.[13] We nevertheless verify that the dynamic of variables remains the same under different values of the mark-ups. The steady-state level of government spending is set to 0.2 of GDP, a widely used "great ratio" from the literature. The value of taxes is then obtained from the fiscal policy balanced budget and is equal to 0.4. Parameters characterizing the production sector are set as follows. The capital depreciation rate is set to 0.02. The capital share ν is calibrated to 0.40. The baseline σ is set to 0.2.

Additional Parameters for the Two Asset Economy. The portfolio adjustment cost parameter, χ_1, is set to 6.19 to match the quarterly interest rate of 0.0125 and the debt-to-output ratio of 1.04. Following Auclert et al. (2021) the parameters χ_0 and χ_2 are set respectively to 0.25 and 2. The steady-state markup is set to 1.015 in order to match the steady-state sum of liquid and non-liquid assets.

13 This is the value chosen in models with two assets in order to match the steady-state sum of liquid and non-liquid assets, which in our model is 16.26. To maintain consistency between the one-asset model and the one with assets, which we present next, we keep this value here as well.

B.1. The Dynamic of the Wealth Distribution

Here, we describe formally how, in the context of the numerical solution that we employ (see Auclert et al. (2021)), the dynamic of the wealth distribution affects the dynamic of the macro variables. Let us first introduce some notation: the state space of the economy is composed of the vector (e, k_-) which includes uninsurable income shocks and capital; in the baseline Krusell et al. (2010) economy the vector of aggregate variables $\mathbf{Y}_t = (K_t, C_t)$ includes capital and consumption; finally $\mathbf{D}_t$ is the fraction of agents at time t at each point on this state grid. The solution to the optimal consumption-saving decision for each household delivers the optimal value function and the policy functions for saving-investment that is used to obtain the transition matrix, Λ'_t, across the grid points over time, that is from (e, k_-) to (e', k). To obtain the response of the aggregate variables in the economy the sequence space Jacobian relies on computing a first order Taylor approximation or else the Jacobian matrix: $\mathcal{J}_{t,s} = \frac{d\mathcal{Y}_{t,s}}{dx} = \frac{\mathcal{Y}_t^s - \mathcal{Y}_{ss}}{dx}$ where dx represent a small shock. In this context: $\mathcal{Y}_t^s = (\mathbf{y}_t^s)'\mathbf{D}_t^s$, defines the output, or vector of aggregate variables, at time t in response to the shock at time s, and the distribution evolves as as follow: $\mathbf{D}_{t+1}^s = (\Lambda_t^s)'\mathbf{D}_t^s$; $\mathbf{y}_t^s$ is the vector of individual policy functions, y. Since y and Λ are both differentiable, for small shocks dx differentiating the aggregate variables delivers:

$$d\mathcal{Y}_t^s = \mathbf{y}'_{ss} d\mathbf{D}_t^{ss} + (d\mathbf{y}_t^s)'\mathbf{D}_{ss} \tag{B.1}$$

Equation B.1 shows that the change in the aggregate consumption/assets is driven by the change in policy functions multiplied by the steady state distribution and the change in the distribution times the steady state policy functions. The fake news algorithm calculates those two objects through the matrix $\mathcal{F}_{t,s}dx \equiv d\mathcal{Y}_t^s - d\mathcal{Y}_{t-1}^{s-1}$. Subtracting $d\mathcal{Y}_t^s$ from $d\mathcal{Y}_{t-1}^{s-1}$ and using the fact that by recursivity of the policy function $d\mathbf{y}_t^s = d\mathbf{y}_{t-1}^{s-1}$ one obtains:

$$\mathcal{F}_{t,s}dx = \mathbf{y}'_{ss}(d\mathbf{D}_t^s - d\mathbf{D}_{t-1}^{s-1}) \tag{B.2}$$

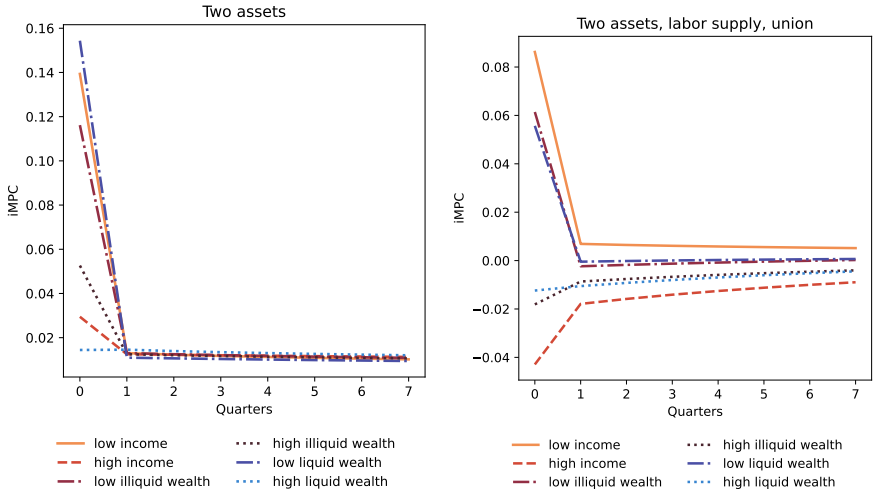
Lemma 2 in Auclert et al. (2021) shows that $\mathbf{y}'_{ss}(d\mathbf{D}_t^s - d\mathbf{D}_{t-1}^{s-1}) = (\Lambda'_{ss})^{t-1} d\mathbf{D}_1^s$. So, $\mathcal{F}_{t,s}$ can be interpreted as the impulse response to a "date-s fake news shock". Numerically it

represents a matrix whose first rows is given by the response of the aggregate policy function at any period t and the remaining rows the responses in the distributions weighted by the expectation operator of the future time path of the aggregate variables for the households on each of the grid points. Our decomposition is made based on those two groups of rows.

B.2. MPC in Two Assets Model

Figure 11 shows the marginal propensities to consume across wealth and income percentiles for two model variants: one with labour supply and one without. The MPC are larger in the second case, as labour supply provides an additional insurance device. The MPCs are larger for wealthy households, as those are more exposed to fluctuations in the value of non-liquid assets and have likelihood of becoming borrowing constrained.

Figure 11: MPC from unexpected income shocks. Partial equilibrium marginal propensities to consume from the baseline model under the baseline calibration for the income process and under a calibration with higher volatility. Left panel shows the case with endogenous labour supply and right panel the case with no labour supply.



(a) MPC **(b) MPC higher risk**

C. Other Impulse Response Functions

Figure 12 shows the impulse response to a monetary shock comparing the one asset model with uninsurable risk to the equivalent augmented with sticky information a' la (**?**) and to the one augmented with a signal extraction choice.

Figure 12: Monetary Policy Shock, comparing One Asset Model, with the one augmented with sticky information and the one augmented with Signal Extraction. The figure shows impulse-responses to a monetary policy shock in different versions of the one-asset model augmented with optimal beliefs from a signal extraction problem. Variables are shown in percentage deviations from the steady state. All model IRFs are in response to a monetary policy shock with 0.6 persistence and a value of the shock normalized to match the empirical trajectory of the Central Bank rate. Shaded area stands for the 95% confidence interval.

