Inflation Expectations, Interest Rates, and Bond Returns*

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VERY PRELIMINARY, PLEASE DO NOT CIRCULATE

Abstract

Using consensus forecasts by major financial institutions during 1989-2022, we study the link between inflation expectations and interest rates in 18 advanced economies. Consensus forecasts exhibit horizon-increasing overreaction that links to excess interest rate volatility: higher inflation forecasts predict higher future bond returns, especially at longer maturities. To rationalize horizon-increasing overreaction, we present a new model where expectations are formed by retrieving past inflation episodes from a dynamically evolving database, based on their similarity to the current inflation state. The model performs well qualitatively and quantitatively, capturing cross-country variability in overreaction and return predictability based on countries' inflation histories.

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1 Introduction

Inflation expectations play a key role in the macroeconomy. A main reason for this is the Fisher Equation, which states that the nominal interest rate i is equal to the equilibrium ex-ante real rate r plus the expected rate of inflation π^e:

$$i = r + \pi^e. \tag{1}$$

Under flexible prices, higher expected inflation increases the nominal interest rate one-to-one, the so called "Fisher effect", leaving the real rate and thus economic activity unaffected. With rigid prices, and depending on central bank policy, higher expected inflation can increase or decrease the real rate, reducing or boosting real activity (see Galí (2015)).

Traditional analyses of the Fisher equation assume rational expectations, contrary to much survey evidence (D'Acunto et al. (2023)). Non-rational inflation expectations can however exert important effects. Ex-ante, they can distort the equilibrium real rate, affecting consumption and investment. Ex-post, they cause a systematic redistribution between borrowers and lenders that, under financial or other frictions, can affect real activity.

Do non-rational inflation expectations affect interest rates? And if so, how? We study these questions theoretically and empirically for 18 advanced economies over 1989-2022. We gather consensus forecasts for inflation at short, medium and long horizons by major financial institutions from Consensus Economics (CE). We combine this data with nominal interest rates at short, medium and long maturities, and with realized inflation in the same countries. CE has three advantages: i) it surveys major investors, whose forecasts better proxy for bond market expectations compared to other surveys, ii) its cross-country dimension helps detect broad regularities, and crucially iii) it allows to study variation in belief biases across forecast horizons and countries, throwing new light on the mechanism.

In Section 2 we first ask: during high inflation states, do investors over or under-estimate future inflation? We detect horizon-increasing overreaction: a high current forecast predicts sharp over-estimation of inflation at the two, three, five, and 6-10 years horizons, but it predicts if anything under-estimation at shorter horizons. Previous studies show that short-horizons US consensus forecasts underreact (Coibion and Gorodnichenko (2012)). At longer horizons, strong overreaction emerges. We next connect expectations to interest rates: consistent with the Fisher equation, higher average expected inflation over a given horizon correlates with a higher nominal interest rate on bonds maturing at the same horizon.

Does belief overreaction affect interest rates? In Section 3 we use the expected average inflation at horizon h to predict: i) the future real yield to maturity h and ii) the excess return on maturity h bonds. Under the expectation-hypothesis and rationality, predictability should be zero. We instead find strong predictability. Consistent with overreaction: higher inflation

forecasts predict higher real yields to maturity and bond price appreciation, especially for long term bonds. These results are robust to controlling for proxies of ex-ante real rates and for the level and slope of the yield curve, the leading proxies for time varying risk premia. During high inflation states investors appear to demand an excessive compensation for inflation and vice-versa during low states, causing interest rates to overreact, especially at long maturities.

These results challenge leading models of belief formation. Consensus overreaction is at odds with information frictions (Sims (2003), Woodford (2003), Mankiw and Reis (2002), Gabaix (2014)), its horizon increasing pattern is at odds with standard formulations of diagnostic expectations (Bordalo et al. (2018), Bianchi et al. (2024b)). Adaptive expectations (Cagan (1956)) are backward looking and make no predictions across horizons. To account for the data, Section 4 presents a model in which expectations are formed by retrieving past episodes from a database, influenced by numerical and semantic similarity to the current inflation state, which acts as a memory cue. Numerically, seeing 3% inflation makes it easier to recall past 3% episodes. Semantically, seeing 3% inflation makes it easier to recall what happened after 3% inflation episodes. Recollections and then used to imagine the future. The memory database accumulates dynamically. We characterize the long run converge of beliefs.

In this model the database anchors beliefs toward frequently experienced values, promoting underreaction to the changing inflation state. Numerical similarity anchors beliefs to current inflation, promoting overreaction. Semantic similarity instead produces a form of statistical conditioning, promoting forward looking behavior. The equilibrium forecast combines these forces based on the similarity weights and features of the DGP. For a stationary DGP, numerical similarity causes long run beliefs to exhibit horizon increasing overreaction. Under an AR(1) DGP, beliefs converge to an AR(1) whose persistence is excessive (overreaction) at long horizons and it can be insufficient (underreaction), at short horizons. Belief biases depend on the memory database: overreaction is stronger when inflation is less persistent and more volatile.

The model has two degrees of freedom, the numerical and semantic similarity weights. The time varying inflation cue and the database are instead observable. In Section 5 we estimate similarity weights by matching model implied expectations to CE forecasts in the pooled dataset. The model captures the untargeted predictability of forecast errors and returns of Sections 2 and 3. We test the memory mechanism by evaluating the ability of the model to capture cross country heterogeneity in belief biases, which is significant in the data. Model-implied beliefs capture sizable cross-country variation in over-reaction, which is stronger in countries with less persistent and volatile inflation. The model also reproduces cross country variation in return predictability.

Much macro finance work studies interest rates and their link to inflation expectations. Starting with Fama (1975) nominal rates are known to predict future inflation, a finding interpreted through the lens of rational expectations (Mishkin (1992), Evans and Lewis (1995),

Kozicki and Tinsley (2001), Gürkaynak et al. (2010). Other papers document excess volatility of US interest rates (Shiller (1979), Mankiw and Summers (1984), Giglio and Kelly (2018)), and predictability of US excess bond returns based on the yield curve (Campbell and Shiller (1991), Fama and Bliss (1987), Cochrane and Piazzesi (2005)). Cieslak (2018) shows that US bond returns can also be predicted by survey forecasts of the federal funds rate. These findings are interpreted as capturing risk premia variation (Wachter (2006), Bansal and Shaliastovich (2013),Vayanos and Vila (2021)), again maintaining rationality. We directly test for rationality using measured forecasts, which predicts returns even conditionally to yield curve proxies. Measured on-rational beliefs help reconcile bond return anomalies across major economies.

We complement growing work on the macroeconomic impact of non-rational beliefs. Bordalo et al. (2024b) and Bordalo et al. (2021) study beliefs about TFP. Bianchi et al. (2024b) and L'Huillier et al. (2024) show in representative agent models that overreacting inflation expectations distort the ex-ante real interest rates, affecting savings and investment. We document overreaction in inflation expectations and interest rates, and highlight a key implication: it causes systematic and quantitatively meaningful redistribution between borrowers and savers. In heterogeneous agent models, this mechanism can affect resource allocation even under flexible prices. The idea that borrowers suffer during periods of deflation dates back to Fisher (1933) and has been documented by Doepke and Schneider (2006).

Foundations of non-rational beliefs have explored several approaches, including diagnostic expectations (Bordalo et al. (2018), Maxted (2024)), level-k (García-Schmidt and Woodford (2019), Farhi and Werning (2019), Iovino and Sergeyev (2023)), adaptive expectations (Nagel (2024)), natural expectations (Fuster et al. (2012)), or incorrect models (Gabaix (2019), Angeletos et al. (2021), Molavi et al. (2024)). We build on the memory model in Bordalo et al. (2023) but allow for a dynamically evolving database, following Bonaglia and Gennaioli (2025), which studies a more general setting. Belief distortions are an emergent property of the interaction between time varying cues and a gradually accumulating database, producing different biases depending on measurable factors such as inflation histories and cues. Relative to time-decay experience effects (Malmendier and Nagel (2016)) or constant-gain learning (Evans and Honkapohja (2009)), similarity allows beliefs to be state *an* history dependent. Investors will overreact to an inflation surge in a country which experienced hyperinflation but underreact in a country without such experience, in line with Gennaioli et al. (2024). Our model founds overreaction to news in diagnostic expectations, and bridges it with models of slow Bayesian learning (Farmer et al. (2024)). In the latter, agents weigh different models using a misspecified prior. Mnemonic learning is model-free, so the misspecified prior emerges endogenously due to selective retrieval based on current cues and past history.

2 Inflation Expectations and Interest Rates

2.1 Data Sources and Sample

We measure investor expectations using Consensus Economics (CE), which surveys macroeconomic forecasts for Western and Eastern European, the Asian-Pacific, and Latin American countries from major financial institutions worldwide. Short-term (same- and next- year) forecasts are surveyed monthly at the individual forecaster level. Long term forecasts are surveyed quarterly after 2014 (January, April, July and October), only twice a year before. For long term forecasts CE only reports the consensus (arithmetic average forecast), minimum and maximum forecasts, and standard deviation.

Our key measure is the consensus forecast for the percent change in the CPI h calendar years ahead, where $h = 0, 1, 2, 3, 4, 5, 6-10$. $h = 0$ is same-year and $h = 6-10$ averages CPI changes in the 6-10 years window. We obtain realized yearly CPI inflation, also in percentage points, from the IMF's Data Mapper and the World Development Indicators (WB). We average the two measures when they are available. We collect bond prices for maturities form 1 to 10 years from Refinitiv. The 4-year maturity has large gaps, so we discard it. We also use forecasts about next year's 3-month and 10 years interest rates from CE. Tables from 7 and 13 in Appendix A describe our data.

We focus on the IMF group of "advanced economies," which have comparable levels of development and do not feature hyperinflations over the period. To ensure a meaningful time series, at each horizon h we focus on countries that have bond yields, inflation forecasts, 3-month and 10 years interest rate forecasts from at least 1999Q4. Our final sample comprises 18 countries and ranges from 1989Q4 to 2022Q1. Table 14 reports the countries at each maturity.[1] Figures 4 to 6 in Appendix A plot in sample realized inflation for the 18 countries.

2.2 Inflation Expectations

To understand bond markets, a first-order question is whether during a "high inflation state" investors over- or under-estimate future inflation, which determines whether interest rates compensate excessively or insufficiently for inflation. We thus predict the consensus forecast error for country c at horizons $t + h$ using two proxies for the country's inflation state at t. The first is the current quarter inflation forecast for h calendar years ahead, $\mathbb{F}_t(\pi_{c,t+hY})$. High expected inflation means a high inflation state. We pool countries, estimating:

[1] Among the IMF's "major advanced economies" the UK does not met this criteria because its CE forecasts are only available from 2004. Our results are robust to including the UK and also all other 18 countries at all maturities (including those for which they have data gaps). Appendix Tables 28 to 32 report this robustness for Tables 1 to 5 in the paper.

$$\pi_{c,t+hY} - \mathbb{F}_t\left(\pi_{c,t+hY}\right) = \alpha_h + \delta_h \cdot \mathbb{F}_t\left(\pi_{c,t+hY}\right) + \varepsilon_{c,t+hY}. \tag{2}$$

When $\delta_h < 0$ forecasters overestimate inflation in a high inflation state and viceversa in a low state, capturing overreaction. When $\delta_h > 0$ they underreact.

Our second proxy is a country's average inflation in the prior two calendar years $\overline{\pi}_{c,t} = (\pi_{c,t-1} + \pi_{c,t-2})/2$. Now t is a year, we do not observe quarterly inflation, and the forecast error is computed by averaging forecasts within a year. Again, a negative predictive coefficient says that the consensus is too pessimistic during a high inflation state, pointing to overreaction.

Table 1 reports estimates for Equation (2) in Panel A and for the realized inflation proxy in Panel B. We denote the forecast error at horizon h by $FE_{c,h} \equiv \pi_{c,t+hY} - \mathbb{F}_t\left(\pi_{c,t+hY}\right)$. We include country dummies to absorb cross-sectional differences in inflation regimes, and control for heteroskedasticity and spatio-temporal correlations using Driscoll-Kraay standard errors.

Table 1: Predicting the consensus forecast error using the current inflation state

	Panel A					
	(1)	(2)	(3)	(4)	(5)	(6)
	$FE_{c,0}$	$FE_{c,1}$	$FE_{c,2}$	$FE_{c,3}$	$FE_{c,5}$	$FE_{c,6-10}$
$\mathbb{F}_{\mathrm{t}}\left(\pi_{\mathrm{t+hY}}\right)$	0.037	-0.356**	-1.004***	-1.114***	-1.655***	-1.207***
	(0.045)	(0.140)	(0.300)	(0.274)	(0.273)	(0.108)
Country fixed effects	✓	✓	✓	✓	✓	✓
N	1029	1029	1171	1033	1093	656
Adj. R^2	0.020	0.062	0.157	0.176	0.252	0.553
	Panel B					
	(1)	(2)	(3)	(4)	(5)	(6)
	$FE_{c,0}$	$FE_{c,1}$	$FE_{c,2}$	$FE_{c,3}$	$FE_{c,5}$	$FE_{c,6-10}$
$\overline{\pi}_{\mathrm{c,t}}$	-0.041	-0.218*	-0.295**	-0.339**	-0.626***	-0.490***
	(0.039)	(0.119)	(0.141)	(0.127)	(0.189)	(0.074)
Country fixed effects	✓	✓	✓	✓	✓	✓
N	420	420	462	416	462	331
Adj. R^2	-0.002	0.048	0.057	0.088	0.229	0.426

Note. We predict inflation consensus forecast error from two proxies for the current inflation state. Each column reports the country pooled regression at horizon $h = 0, ..., 6-10$: the dependent variable is the inflation consensus forecast error $\pi_{c,t+hY} - \mathbb{F}_t\left(\pi_{c,t+hY}\right)$ over horizon h and country c. The two proxies for the current inflation state are: (i) the consensus forecast for country c and horizon h $\mathbb{F}_t\left(\pi_{c,t+hY}\right)$ (Panel A), and (ii) the average realized inflation of country c over the past two years, $\overline{\pi}_{c,t} = (\pi_{c,t-1} + \pi_{c,t-2})/2$ (Panel B). We control for country fixed effects. The sample ranges from 1989Q4 to 2022Q1, Table 14 reports the countries available at each maturity. Standard errors in parenthesis are Driscoll-Kraay. *** **, and * indicate significance at the 1, 5, and 10 percent levels.

In Panel A, δ_h is positive at the shortest horizon, indicative of underreaction, but not significantly so. At longer horizons, $h \geq 1$, overreaction emerges: high expected inflation predicts

inflation overestimation. The magnitude of predictability increases with h: overreaction is stronger at longer horizons. These patterns are confirmed if we estimate Equation (2) at the yearly frequency and if we look at future forecast revisions: high expected inflation predicts downward expectation revisions, especially at long horizons, see Tables 19 and 26 in the Appendix. Horizon increasing overreaction is confirmed in Panel B: starting from $h = 1$ high recent inflation predicts overestimation of future inflation, more so at longer horizons.

Magnitudes are large. In Panel A, a one standard deviation higher forecast predicts a 0.217 standard deviations lower error at the one year horizon. The effect increases with h, reaching a 0.826 standard deviations decrease at the 6-10 years horizon. In Panel B a one standard deviation higher recent inflation reduces the error by 0.205 standard deviations at the one year horizon and by 0.667 standard deviations at the 6-10 year horizon.

Several papers show systematic errors in inflation forecasts. A key debate concerns whether beliefs over or under-react and what is the underlying mechanism. Four quarters ahead professional forecasts of US inflation exhibit consensus under-reaction to *recent* news, which is explained with information frictions (Coibion and Gorodnichenko, 2015). At the individual level though, the same forecasters over-react, departing from rational updating. Bordalo et al. (2020) argue that this force may cause consensus overreaction at long horizons, as in the case of stock analysts' long term earnings growth forecasts (Bordalo et al., 2024c).

Consistent with these results, short term consensus forecasts (i.e. same year forecasts) tend if anything to under-react, while overreaction prevails at longer horizons. There are two possible mechanisms. First, forecasters may overreact to recent "high inflation" news. Second, holding news constant, forecasters may be persistently pessimistic due to high past inflation. To separate these forces we predict errors using a proxy for recent news, the four-quarter consensus revision $\mathbb{F}_t(\pi_{c,t+hY}) - \mathbb{F}_{t-4}(\pi_{c,t+hY})$, and a proxy for the past state, the consensus four-quarters ago $\mathbb{F}_{t-4}(\pi_{c,t+hY})$. Following Bordalo et al. (2024c) we estimate:

$$\pi_{c,t+h} - \mathbb{F}_t(\pi_{c,t+h}) = \alpha_h + \delta_{1,h} \cdot [\mathbb{F}_t(\pi_{c,t+h}) - \mathbb{F}_{t-4}(\pi_{c,t+h})] + \delta_{2,h} \cdot \mathbb{F}_{t-4}(\pi_{c,t+h}) + \varepsilon_{c,t+h}. \quad (3)$$

A positive coefficient on the forecast revision, $\delta_{1,h} > 0$, captures under-reaction to recent news, a negative coefficient $\delta_{1,h} < 0$ captures overreaction. A negative coefficient on the lagged forecast, $\delta_{2,h} < 0$, captures persistent overreaction: forecasters are too pessimistic/optimistic if last year's inflation state was high/low.

Table 2 reports the estimates, again including country dummies and using Driscoll-Kraay standard errors. At horizons up to one year, $\delta_{1,h}$ is positive but not statistically different from zero, pointing to some short term underreaction to news. At longer horizons, $h \geq 2$, we instead see significant overreaction to news, $\delta_{1,h} < 0$. Coefficient $\delta_{2,h}$, shows overreaction to past inflation states: it is negative and significant at all horizons. Consensus overreaction to news *and* to the past inflation state are stronger at long horizons. Starting from $h \geq 2$ the

two coefficients become similar, $\delta_{1,h} = \delta_{2,h}$ cannot be rejected at conventional levels for $h \geq 2$. Equation (2), which imposes $\delta_{1,h} = \delta_{2,h}$, captures the key features in the data.[2]

Table 2: Predicting consensus error from recent news and past inflation states

	(1)	(2)	(3)	(4)	(5)	(6)
	$FE_{c,0}$	$FE_{c,1}$	$FE_{c,2}$	$FE_{c,3}$	$FE_{c,5}$	$FE_{c,6-10}$
$\mathbb{F}_t(\pi_{c,t+hY}) - \mathbb{F}_{t-4}(\pi_{c,t+hY})$	0.160	0.215	-0.847*	-1.092***	-1.297***	-1.123***
	(0.098)	(0.240)	(0.448)	(0.411)	(0.257)	(0.110)
$\mathbb{F}_{t-4}(\pi_{c,t+hY})$	-0.105**	-0.625**	-1.232***	-1.318***	-1.765***	-0.988***
	(0.045)	(0.246)	(0.433)	(0.399)	(0.380)	(0.107)
Country fixed effects	✓	✓	✓	✓	✓	✓
N	973	970	1104	969	1008	618
Adj. R^2	0.088	0.103	0.137	0.151	0.183	0.398

Note. We predict the inflation consensus forecast error from consensus revision (recent news) and past inflation state. Each column reports the country pooled regression at horizon $h = 0, ..., 6-10$: the dependent variable is the inflation consensus forecast error $\pi_{c,t+hY} - \mathbb{F}_t\left(\pi_{c,t+hY}\right)$ over horizon h and country c. The consensus revision is $\mathbb{F}_t\left(\pi_{c,t+hY}\right) - \mathbb{F}_{t-4}\left(\pi_{c,t+hY}\right)$, while past inflation state is $\mathbb{F}_{t-4}\left(\pi_{c,t+hY}\right)$. We control for country fixed effects. The sample ranges from 1989Q4 to 2022Q1, Table 14 reports the countries available at each maturity. Standard errors in parenthesis are Driscoll-Kraay. *** **, and * indicate significance at the 1, 5, and 10 percent levels.

Effects are large. A one standard deviation increase in the consensus revision is associated with a 0.163 standard deviations decrease in the forecast error at the 2 year horizons, and with a 0.403 decrease at the 6-10 year horizon.[3] A one standard deviation increase in lagged forecasts leads to a 0.381 standard deviations decrease in the forecast error at one year, a 0.428 decrease at 2-year, and a 0.728 decrease at the 6-10 year horizons.

Overall, consensus inflation expectations sharply overreact, more so at longer horizons. As we show in Section 4, these patterns discriminate among leading theories of expectations and motivate our memory based approach. Before doing so, we study whether measured expectations matter for interest rates.

2.3 Inflation Expectations and Nominal Interest Rates

As a first step, we assess the contemporaneous link between expected inflation and nominal interest rates. Let $r^{(h)}$ be the ex-ante real rate on a zero-coupon bond with maturity h. By the

[2]If we estimate Equation (3) for the US only, the lagged forecast and the revision become more negative at longer horizons but only the former is a significant predictor. We performed the same test using SPF forecasts for the CPI (Appendix Table 16), which cover 1981Q3-2024Q2. Here we observe forecast revisions at most four-quarters out. At this short horizon we confirm no predictability from the revision and statistically significant negative predictability from lagged forecasts. SPF also contains forecasts for average inflation over the next 4 to 9 but here the forecast revision is not available.

[3]Overreaction to news cannot be due to measurement error in individual analyst forecasts because such errors is averaged out in the consensus. Predictability is also negative - and typically larger - for the lagged forecast, again inconsistent with measurement error bias.

Fisher equation the bond's nominal rate $i_t^{(h)}$ is equal to

$$i_t^{(h)} = r^{(h)} + \mathbb{E}_t\left(\overline{\pi_{t+1,h}}\right), \tag{4}$$

where $\overline{\pi_{t+1,h}} = (\pi_{t+1Y} + ... + \pi_{t+hY})/h$ is average inflation between t and $t + hY$, and $\mathbb{E}_t$ is the market expectation at t. Motivated by this equation, we construct expected average inflation in the next h calendar years, our proxy for $\mathbb{E}_t(\overline{\pi_{t+1,h}})$, and estimate:

$$i_{c,t}^{(h)} = \alpha_h + \lambda_h \cdot \mathbb{F}_t\left(\overline{\pi_{c,t+1,h}}\right) + \varepsilon_{c,t}^{(h)}. \tag{5}$$

We measure the nominal rate $i_{c,t}^{(h)}$ at maturity h and time t in country c using the zero-coupon price $P_{c,t}^{(h)}$ and the fact $\ln P_{c,t}^{(h)} = -h \cdot i_{c,t}^{(h)}$. Prices are available monthly, so we compute quarterly rates as monthly averages. We do not expect a unit coefficient in Equation (5) because: i) our proxy for investor expectations may be imperfect, and ii) the ex-ante real rate may be time varying, and in particular it may co-move with expected inflation, which as we argued can occur in leading models (e.g. featuring price rigidity).

Table 3 Panel A estimates Equation (5), ranging from the the one year, Column (1), to the 6-10 years maturity, Column (5). To capture movements in ex-ante real rates due to expectations about central bank policy or risk premia, Panel B controls for CE forecasts of nominal rates next year, available for the 3 months and 10 year maturities. To roughly match maturities, Panel B controls for the expected 3 months rate for $h = 1, 2, 3$, Columns (1)-(3), and the 10 year rate for $h = 5, 6 - 10$, Columns (4) and (5). We do not jointly include the two expected rates due to collinearity.[4] We include country dummies and use Driscoll-Kraay standard errors.

In Panel A higher expected inflation for the next h years is associated with higher nominal rates at the same maturity. The R^2 is sizable, ranging from 31% to 54%. It is lower at the 6-10 years maturity, probably due to limited data.[5] The estimated coefficient is indistinguishable from 1 at $h = 1$, and from 2 at longer horizons. Quantitatively, a one standard deviation higher inflation forecast corresponds to a 0.457 standard deviations higher nominal rate at 1 year, which then increases to about 0.7 standard deviations at maturities $h \geq 2$.

When we introduce expected nominal rates in Panel B, we see that they are tightly anchored to current rates (the coefficient on expected rates is very close to 1).[6] Expected inflation however continues to positively correlate with current rates. The coefficient on expected inflation is

[4]The VIFs for the regressions of Panel B, including both three months and ten years yields' forecasts are larger than the recommended threshold of 10 for horizons 1 to 5 (for one or both controls). The VIFs for inflation forecasts, instead, are always below 10, ranging from 3.18 to 5.77.

[5]The R^2 with country fixed effects alone is, in the order of maturities: 0.270, 0.188, 0.167, 0.212, 0.217. The gain in R^2 obtained by adding CE forecasts is sizable, especially at longer horizons.

[6]Controlling for expected nominal rates sharply increases the regression R^2 because these expectations are highly correlated with contemporaneous rates (ranging from 0.967 to 0.980). Expected rates instead exhibit a more moderate correlation with inflation expectations (ranging from 0.606 to 0.730).

Table 3: Nominal yields and consensus inflation forecast

	Panel A				
	(1)	(2)	(3)	(4)	(5)
	$i_{c,t}^{(1)}$	$i_{c,t}^{(2)}$	$i_{c,t}^{(3)}$	$i_{c,t}^{(5)}$	$i_{c,t}^{(6-10)}$
$\mathbb{F}_t\left(\overline{\pi_{c,t+1,h}}\right)$	1.090***	2.144***	2.324***	2.658***	2.545***
	(0.173)	(0.303)	(0.269)	(0.282)	(0.543)
Country fixed effects	✓	✓	✓	✓	✓
N	972	1163	1079	1296	550
Adj. R^2	0.339	0.488	0.540	0.527	0.314
	Panel B				
	(1)	(2)	(3)	(4)	(5)
	$i_{c,t}^{(1)}$	$i_{c,t}^{(2)}$	$i_{c,t}^{(3)}$	$i_{c,t}^{(5)}$	$i_{c,t}^{(6-10)}$
$\mathbb{F}_t\left(\overline{\pi_{c,t+1,h}}\right)$	0.039	0.067**	0.243***	0.369***	0.213**
	(0.026)	(0.034)	(0.074)	(0.089)	(0.103)
$\mathbb{F}_t[i_{c,t+1}^{(h)}]$	0.982***	1.029***	0.949***	1.011***	0.938***
	(0.018)	(0.016)	(0.023)	(0.033)	(0.029)
Country fixed effects	✓	✓	✓	✓	✓
N	972	1146	1060	1124	550
Adj. R^2	0.962	0.973	0.938	0.965	0.948

Note. We study the association between nominal yields to maturity h and the consensus forecast for the average inflation over h years (Panel A). Additionally, we control for the consensus forecast of interest rates (Panel B). Each column reports the country pooled regression at horizon $h = 1, ..., 6-10$. The dependent variable is the nominal yield to maturity h. The consensus forecast for the average inflation over h years for country c, $\mathbb{F}_t\left(\overline{\pi_{c,t+1,h}}\right)$, is computed as: $\frac{1}{h}\sum_{h'\geq 1}\mathbb{F}_t\left(\pi_{c,t+h'Y}\right)$. To roughly match short and long maturities, Panel B controls for the expected 3 months rate for $h = 1, 2, 3$, Columns (1)-(3), and for the expected 10 year rate for $h = 5, 6-10$, Columns (4) and (5). We control for country fixed effects. The sample ranges from 1989Q4 to 2022Q1, Table 14 reports the countries available at each maturity. Standard errors in parenthesis are Driscoll-Kraay. *** **, and * indicate significance at the 1, 5, and 10 percent levels.

lower, in part because expected rates also co-move with expected inflation, in part because they may capture changes in real rates. Quantitatively, a one-standard deviation increase in expected inflation results in a 0.016 standard deviations increase in the nominal rate at 1 year and in larger increases at higher maturities up to 0.058 standard deviations at 6-10 years.

Overall, CE forecasts reliably proxy for investor expectations and, consistent with theory, positively co-move with interest rates at different horizons. Does overreaction in expectations create excess interest rates volatility? We assess this question next.

3 Inflation Expectations and Return Predictability

We derive two return predictability tests linking overreaction to excess interest rate volatility. For simplicity we derive them under a constant ex-ante real rate $r^{(h)}$, but we relax this assumption in our regressions.

Real yield to maturity. This is the difference between the bond's nominal rate at issuance $i_{c,t}^{(h)}$ and average realized inflation over the bond's lifetime. By Equation (4), the real yield on a bond issued at t in country c with maturity h is equal to

$$\widetilde{r}_{c,t,t+h}^{(h)} = r^{(h)} - \left[\overline{\pi_{t+1,h}} - \mathbb{E}_t\left(\overline{\pi_{t+1,h}}\right)\right], \tag{6}$$

If inflation is lower than expected, $[\overline{\pi_{t+1,h}} - \mathbb{E}(\overline{\pi_{t+1,h}})] < 0$, the real yield to maturity is above the ex-ante real rate $\widetilde{r}_{c,t,t+h}^{(h)} > r^{(h)}$, redistributing resources from borrowers to lenders. If inflation is higher than expected, the opposite occurs. Under rational expectations these effects are unsystematic because forecast errors are on average zero and unpredictable. With non-rational expectations inflation surprises are systematic, creating predictable movements in the real yield to maturity. If investors share the overreacting beliefs of CE forecasters, inflation is systematically lower than expected after high inflation states and vice-versa after low states, giving our first predictability test.

Prediction 1. *If investors overreact, more strongly so at longer maturities, the regression*:

$$\widetilde{r}_{c,t,t+h}^{(h)} = \theta_{0,h}^{r} + \theta_{h}^{r} \cdot \mathbb{F}(\overline{\pi_{c,t+1,h}}) + \epsilon_{c,t,t+h}^{(h)} \tag{7}$$

should yield positive coefficients $\theta_h^r > 0$ with magnitude increasing in h.

Due to maturity increasing overreaction, in high inflation states nominal rates are too high, and so realized real rates are too high, especially at longer maturities. Key to predictability here is the presence of systematic inflation forecast errors.

Excess holding period return. This is the one period return from buying and selling a bond with maturity $h > 1$ in excess of the short rate. Suppose the investor buys a bond with unit face value at t for price $\ln P_t^{(h)} = -h \cdot i_t^{(h)}$. Selling the bond after one period gives return $y_{t,t+1}^{(h)} = P_t^{(h)}/P_{t+1}^{(h-1)} - 1$, which approximates to $y_{t,t+1}^{(h)} \approx i_t^{(h)} + (h-1)\cdot(i_t^{(h)} - i_{t+1}^{(h-1)})$. By Equation (4) the excess return relative to the short rate $i_t^{(1)}$ is:

$$rx_{t,t+1}^{(h)} = rx^{(h)} - (h-1)\left[\mathbb{E}_{t+1}\left(\overline{\pi_{t+2,h}}\right) - \mathbb{E}_t\left(\overline{\pi_{t+2,h}}\right)\right]. \tag{8}$$

where $rx^{(h)} \equiv -(h-1)r^{(h-1)} + hr^{(h)} - r^{(1)}$ is a constant. When at $t+1$ investors become

more pessimistic about inflation, $\mathbb{E}_{t+1}(\overline{\pi_{t+2,h}}) - \mathbb{E}_t(\overline{\pi_{t+2,h}}) > 0$, the $t+1$ nominal rate increases, reducing the price at which the bond is sold. As a result, the excess return is low. When investors become more optimistic, the reverse occurs. Under rational expectations forecast revisions are unpredictable, and so are excess returns. With non-rational expectations, forecast revisions are systematic, creating predictable excess return movements. If investors share the overreacting beliefs of CE forecasters, revisions are systematically negative after high inflation states and vice-versa after low states, giving our second test.

Prediction 2. *If investors overreact, more strongly so at longer maturities, the regression:*

$$rx^{(h)}_{c,t,t+1} = \theta^{ex}_{0,h} + \theta^{ex}_{h} \cdot \mathbb{F}(\overline{\pi_{c,t+1,h}}) + \epsilon^{(h)}_{c,t,t+h} \tag{9}$$

should yield positive coefficients $\theta^{ex}_h > 0$ with magnitude increasing in h.

Due to maturity increasing overreaction, in high inflation states nominal rates are too high and hence bond prices are too low. Thus, bonds appreciate in the future, especially at longer maturities. Key to predictability here is systematic belief revision.

It is known that US interest rates are "too volatile" (Shiller (1979), Mankiw and Summers (1984)) and excess bond returns are predictable (Duffee, 2013).[7] Standard explanations maintain rational expectations, assuming that the ex-ante real rate/risk premium varies over time. Because risk premia are unobserved, we cannot rule out that predictability may be driven by non-rational expectations about inflation and other outcomes. We assess the explanatory power of non-rational beliefs using survey inflation expectations.

3.1 Predictable Real Yields to Maturity

To test Prediction 1 we estimate regression (7). We compute $\widetilde{r}^{(h)}_{c,t,t+h}$ as the difference between the nominal rate of a maturity h bond purchased at t in country c and average realized inflation in the country between t and $t+h$. To allow for time varying ex-ante real rates, we also estimate a version of (7) controlling for the difference between the current nominal rate and expected inflation, $i^{(h)}_{c,t} - \mathbb{F}_t(\overline{\pi_{c,t+1,h}})$. To validate this proxy we compare it to ex-ante real rates inferred from inflation-adjusted bonds (King and Low, 2014), which are traded and highly liquid in the US and UK. The correlation between our proxy and this measure for these countries ranges from 70% to 90% (Appendix Table 39), indicating that our proxy is valid.

Table 4 reports the estimates. Columns (1)-(5) run the basic specification, Columns (6)-(10) control for the ex-ante real rate proxy. We include country dummies and use Driscoll-Kraay standard errors. Consistent with Prediction 1, a higher inflation forecast predicts higher real

[7] In Fama and Bliss (1987) a higher spread between the h year forward and sport rates predicts higher excess returns on the h maturity bond. Cochrane and Piazzesi (2005) build a related predictor based on forward rates.

yield to maturity. This is true also conditional on the ex-ante real return. This test does not mechanically follow from error predictability because the ex-ante real rate may covary with inflation expectations. Important, and also in line with Prediction 1, predictability is stronger for higher h, consistent with stronger overreaction at longer maturities. Results are similar if we estimate a yearly regression (see Table 27).[8]

The effects are quantitatively large: a one standard deviation increase in the inflation forecast results in a 0.163 standard deviations decrease in the ex-post real return, with the effect reaching the maximum of 0.688 at $h = 5$. As we control for ex-ante real rates, the effect decreases but remains sizable: at the one-year horizon, column (6), a one standard deviation rise in the inflation forecast leads to a 0.127 standard deviations increase in the ex-post real return. We later quantify this systematic redistribution between lenders and borrowers.

3.2 Predictable Excess Returns

To test Prediction 2 we compute the "one year holding period excess bond return" between t and $t+1$ in country c for a bond with maturity h as $rx^{(h)}_{c,t,t+1} = i^{(h)}_{c,t} + (h-1) \cdot \left(i^{(h)}_{c,t} - i^{(h-1)}_{c,t+1}\right) - i^{(1)}_{c,t}$. We average monthly yields over the quarter and compute quarterly excess returns (results are robust to using an annual frequency). To smooth short-term bond price volatility, we average excess returns over two years:

$$\overline{rx}^{(h)}_{c,t,t+1} = \frac{rx^{(h)}_{c,t,t+1} + rx^{(h)}_{c,t+1,t+2}}{2}. \tag{10}$$

Table 5 estimates Equation (9) including country dummies and using Driscoll-Kraay standard errors. Panel A reports the basic specification. Panel B includes the level and slope of the yield curve at t as controls. Level is the one year nominal rate, slope is the difference between the ten and one year rates. They are leading predictors of excess returns, viewed as proxies of time-varying real rates (Fama (1984)). Panel B thus tests whether CE forecasts contain genuine information about market expectations, predicting bond returns over and beyond current rates. This is a very stringent test: level and slope are directly affected by market expectations, while CE inflation forecasts are only a noisy proxy for them.

Consistent with Prediction 2, higher inflation expectations predict higher future excess returns. Predictability is stronger for higher maturity bonds, consistent with stronger overreaction at the horizons relevant for these maturities. Predictability is robust to controlling for level and slope, and quantitatively large. Focusing on Panel B, a one standard deviation higher inflation

[8] We have substantially fewer observations for the 6-10 years maturity because we can only include observations up to 2012. To build the realized real rate for this maturity we need realized inflation data for the subsequent 10 years, and our inflation data only extends up to 2022. Additionally, we have fewer data points for long-term maturity nominal yields, further contributing to the lower number of observations.

Table 4: Predicting ex-post real rates using consensus inflation forecast

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
	$\tilde{r}^{(1)}_{c,t,t+1}$	$\tilde{r}^{(2)}_{c,t,t+2}$	$\tilde{r}^{(3)}_{c,t,t+3}$	$\tilde{r}^{(5)}_{c,t,t+5}$	$\tilde{r}^{(6-10)}_{c,t+6,t+10}$	$\tilde{r}^{(1)}_{c,t,t+1}$	$\tilde{r}^{(2)}_{c,t,t+2}$	$\tilde{r}^{(3)}_{c,t,t+3}$	$\tilde{r}^{(5)}_{c,t,t+5}$	$\tilde{r}^{(6-10)}_{c,t+6,t+10}$
$\mathbb{F}_t\left(\overline{\pi_{c,t+1,h}}\right)$	0.483	1.798***	2.003***	2.302***	-0.894	0.375***	0.470***	0.579***	0.796***	0.613***
	(0.299)	(0.392)	(0.329)	(0.264)	(0.563)	(0.131)	(0.143)	(0.145)	(0.125)	(0.208)
$i^{(h)}_{c,t} - \mathbb{F}_t\left(\overline{\pi_{c,t+1,h}}\right)$						1.202***	1.161***	1.098***	1.031***	1.054***
						(0.125)	(0.105)	(0.076)	(0.059)	(0.090)
Country fixed effects	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
N	972	1163	1019	1080	230	972	1163	1019	1080	230
Adj. R^2	0.058	0.235	0.326	0.384	0.885	0.682	0.736	0.802	0.880	0.885

Note. We study the association between ex-post real rates to maturity h and the consensus forecast for the average inflation over h years (Columns (1) to (5)). Additionally, we control for ex-ante real rates (Columns (6) to (10)). Each column reports the country pooled regression at horizon $h = 1, ..., 6-10$. Real ex-post rates $\widetilde{r}^{(h)}_{c,t,t+h}$ over horizon h and country c are defined as $i^{(h)}_{c,t} - \left(\overline{\pi_{c,t+1,h}}\right)$. The consensus forecast for the average inflation over h years for country c, $\mathbb{F}_t\left(\overline{\pi_{c,t+1,h}}\right)$, is computed as: $\frac{1}{h}\sum_{h' \geq 1} \mathbb{F}_t\left(\pi_{c,t+h'Y}\right)$. Ex-ante required real rates are defined as $i^{(h)}_{c,t} - \mathbb{F}_t\left(\overline{\pi_{c,t+1,h}}\right)$. We control for country fixed effects. The sample ranges from 1989Q4 to 2022Q1, Table 14 reports the countries available at each maturity. Standard errors in parenthesis are Driscoll-Kraay. *** **, and * indicate significance at the 1, 5, and 10 percent levels.

Table 5: Predicting realized excess returns using consensus inflation forecasts

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	$\overline{rx}^{(2)}_{c,t,t+1}$	$\overline{rx}^{(3)}_{c,t,t+1}$	$\overline{rx}^{(5)}_{c,t,t+1}$	$\overline{rx}^{(6-10)}_{c,t,t+1}$	$\overline{rx}^{(2)}_{c,t,t+1}$	$\overline{rx}^{(3)}_{c,t,t+1}$	$\overline{rx}^{(5)}_{c,t,t+1}$	$\overline{rx}^{(6-10)}_{c,t,t+1}$
$\mathbb{F}_t\left(\overline{\pi_{c,t+1,h}}\right)$	0.307***	0.429**	3.059***	5.402**	0.097	0.409***	0.926**	3.291***
	(0.103)	(0.165)	(0.916)	(2.311)	(0.072)	(0.149)	(0.407)	(1.158)
$i^{(1)}_{c,t}$					0.270***	0.137	0.867***	0.750**
					(0.036)	(0.089)	(0.137)	(0.326)
$i^{(10)}_{c,t} - i^{(1)}_{c,t}$					0.381***	0.349**	1.458***	2.439***
					(0.081)	(0.148)	(0.234)	(0.520)
Country fixed effects	✓	✓	✓	✓	✓	✓	✓	✓
N	944	873	712	466	941	871	712	466
Adj. R^2	0.138	0.267	0.211	0.121	0.417	0.300	0.516	0.409

Note. We predict one-year holding period realized bond excess returns with maturity h from consensus forecast of the average inflation over h years (Columns (1) to (4)). Additionally, we control for level and slope of nominal yields (Columns (5) to (8)). Each column reports the country pooled regression at horizon $h = 2, 3, 5, 6-10$: the dependent variable is the smoothed one-year holding period excess returns over horizon h and country c, defined as $\overline{rx}^{(h)}_{c,t,t+1} = \dfrac{rx^{(h)}_{c,t,t+1} + rx^{(h)}_{c,t+1,t+2}}{2}$. The raw one-year holding period excess returns $rx^{(h)}_{c,t,t+1}$ is defined as $i^{(h)}_{c,t} + (h-1)\cdot\left(i^{(h)}_{c,t} - i^{(h-1)}_{c,t+1}\right) - i^{(1)}_{c,t}$. The level and slope for country c and maturity h are defined respectively as $i^{(1)}_{c,t}$ and $i^{(10)}_{c,t} - {}^{(1)}_{c,t}$.We control for country fixed effects. The sample ranges from 1989Q4 to 2022Q1, Table 14 reports the countries available at each maturity. Standard errors in parenthesis are Driscoll-Kraay. *** **, and * indicate significance at the 1, 5, and 10 percent levels.

forecast predicts a 0.134 standard deviations higher return at $h = 3$, and a 0.405 standard deviations higher return at $h = 6 - 10$. The results are robust to smoothing returns over three years, see Appendix Table 21. [9]

3.3 Economic Implications

Doepke and Schneider (2006) show that US unexpected inflation causes significant redistribution toward debtors (younger and middle-class households with fixed mortgage debt or the government), and hurts bondholders (wealthy or older households). Under overreaction, such redistribution is systematic after low inflation states, when nominal rates are too low, and takes the reverse direction after high inflation states, when nominal rates are too high. To quantify these effects, consider a household or firm borrowing $L^{(h^*)}$ under a simple loan with h^* years maturity. The loan specifies nominal yearly repayments that entail a constant expected real repayment $P^{(h^*)}$ ((expressed in dollars at the issuance date). Ex-ante, then, the real repayment is set to fulfill:

$$L^{(h^*)} = \sum_{h=1}^{h^*} \prod_{s=1}^{h} \frac{P^{(h^*)}}{1 + r_s}, \tag{11}$$

where r_s is the ex-ante real rate, which is assumed to be time invariant. As inflation deviates from its expected level, the borrower's real payments differ from those initially agreed upon when the contract was signed. Discounted at the fixed ex-ante real rate, the present value of these payments is

$$\hat{L}^{(h^*)} = \sum_{h=1}^{h^*} \prod_{s=1}^{h} \frac{P^{(h^*)}}{(1 + r_s)(1 + \pi_s - \pi_s^e)}. \tag{12}$$

where π_s^e is ex ante expected inflation s years ahead and π_s is actual realized inflation. When forecast errors are predictable, the real value of the borrower's payments may systematically differ from the ex-ante value of the loan $L^{(h^*)}$.

We consider the effect of an increase in inflation expectations π_s^e starting from a perfect foresight scenario, and in particular focus on a parallel shift where π_s^e changes by π^e for all s. Given over-reaction, this change in beliefs is associated with systematic overestimation of future inflation, leading to larger real payments. Table 6 estimates the consequences of such change using our estimates of forecast error predictability from previous sections; the details are in Appendix H.

The first row of Table 6 reports the semi-elasticity of $\hat{L}^{(h^*)}$ to the parallel shift in inflation expectations. Higher inflation expectations substiatially increase the cost of longer maturity

[9] Appendix Table 22 adds the expected 10-year rate as a control. Now, the level of interest rates loses its predictive power, while inflation expectations retain theirs. We can also directly link excess-returns to the unfolding of forecast errors by predicting excess-returns using the forecast errors fitted with the current forecast, again controlling for level and slope. Appendix Tables 20 and 46 report this exercise.

Table 6: A simple quantification of the redistribution from borrowers to savers

	Loan Maturity				
	1	2	3	5	10
$\Delta \log \hat{L}^{(h^*)}$	0.36	0.86	1.39	2.77	6.04
$L^{(h^*)}$	$\$24k$	$\$48k$	$\$72k$	$\$118k$	$\$234k$
$\hat{L}^{(h^*)} - L^{(h^*)}$	\$85	\$410	\$992	$\$3,267$	$\$14,119$

Note. The upper section of the table reports the semi-elasticity of the loan value to a parallel shift in inflation expectations. The lower section provides a simple example of loan structured to maintain a fixed real payment of $\$24,000$ across all maturities before the change in expectations. The table reports the initial loan value and its absolute change following a 1 percentage point increase in inflation expectations across all horizons. The analysis is based on the estimates from Panel A of Table 1.

loans, reaching a value of approximately 6 for the 10-year loan. This means that a 1 percentage point rise in inflation expectations for the next ten years increases the loan cost by about 6%.

The next two rows compute the absolute changes in loan cost following a 1 percentage point increase in inflation expectations, assuming that the loan was initially structured to maintain a constant annual real payment of $\$24,000$. For a 10-year loan, the real burden of payments increases by about $\$14,000$.

Bianchi et al. (2024b) and L'Huillier et al. (2024) integrate diagnostic expectations into standard NK models and demonstrate that overreaction to inflation can distort ex-ante rates, shaping consumption and investment. Here we document overreaction in inflation expectations and nominal interest rates. The systematic redistribution that we quantify can play an important role when departing from the representative agent assumption. Fisher (1933) famously argued that *debt deflation* - the increase in the debt burden caused by deflation - was a key factor in the severity of the Great Depression. Growing work on heterogeneous-agent NK models shows that redistribution between borrowers and savers plays a prominent role in the transmission of monetary policy due to the different propensities to consume or invest of these agents (Auclert (2019); Sterk and Tenreyro (2018)). Non-rational inflation expectations can thus influence resource allocation by creating systematic redistribution.

Redistribution can also concern banks and firms and have important effects in the presence of borrowing constraints. Gomes et al. (2016) for instance shows that lower-than-expected inflation increases the real burden of nominal debt, raising default risk and reducing output. In Brunnermeier and Sannikov (2016), lower inflation increases the debt burden of banks, which engage in maturity transformation by issuing nominal securities (inside money) while investing in illiquid long-term assets. In the context of sovereign debt, Broner et al. (2013) show that emerging economies prefer short-term borrowing because it is cheaper than long-term debt, particularly during crises when long-term borrowing costs increase. Given that many emerging

economies have a history of high inflation, our findings could help explain why short-term debt remains cheap compared to long term debt.

4 Selective Memory and Inflation Expectations

Horizon increasing overreaction challenges leading approaches to expectations. Models of information frictions or inattention (Sims, 2003; Woodford, 2003), by underreaction of US short-term forecasts (Coibion and Gorodnichenko, 2015), predict: i) consensus underreaction to recent news, $\delta_{1,h} > 0$, but ii) error unpredictability from the past state $\delta_{2,h} = 0$. These models cannot account for the sharp consensus overreaction in the data.

Diagnostic expectations (DE), (Bordalo et al. (2018), Bordalo et al. (2020), Bianchi et al. (2024a)) relax rational updating, motivated by overreaction of individual analyst forecasts (Bordalo et al. (2020)). They can produce $\delta_{1,h} < 0$. In formulations with a long look-back period (Bianchi et al. (2024b), Bordalo et al. (2024c)), they also predict overreaction to past states, $\delta_{2,h} < 0$. These formulations of DE do not however produce horizon-dependence, and in particular cannot yield short-term underreaction.[10]

Non-rational adaptive rules (Nerlove (1958), Cagan (1956)), model inflation expectations as a distributed lag of past inflation, with coefficients that decay in temporal distance. A similar property holds for constant gain learning or experience effects models (Marcet and Nicolini (2003), Malmendier and Nagel (2016)). These models were also motivated by underreaction of inflation expectations to news. They can give overreaction to recent news if the weight on recent inflation is high, but then struggle to produce overreaction to past states because high sensitivity to news calls for a low weight on past states.[11]. In addition, adaptive models are not forward-looking (Lucas Jr, 1972), so they do not make predictions on the term structure of forecasts, and hence on horizon increasing overreaction.[12]

To capture horizon increasing overreaction we offer a new model built on regularities in human recall. Memory can naturally generate state-dependent overreaction to salient cues and persistent weighting of remote states populating the memory database. One key result here is that belief distortions endogenously vary across countries characterized by different inflation histories. We will test this prediction in the data.

[10] A combination of information frictions and DE can give consensus underreaction and individual level overreaction (Bordalo et al. (2020)). This may allow to rationalize $\delta_{1,h} > 0$ and $\delta_{2,h} < 0$. Underreaction is however observed in the forecasts of some analysts, which cannot be produced by DE.

[11] The adaptive one-step ahead inflation forecast is $\pi^e_{t+1} = p \cdot [\pi_t - \pi^e_{t-1}] + (1-p) \cdot \pi^e_{t-1}$. If inflation is AR(1) with persistence ρ the forecast error has coefficient $(\rho - \alpha)$ on the inflation news and coefficient $(\rho + \alpha - 1)$ on the lagged forecast. Higher α reduces the former coefficient and increases the second. This model does not allow to specify a process for forecast revisions without specifying additional parameters

[12] Appendix Figures 13 and 15 shows that indeed CE forecasts exhibit forward-looking updating, with gradual yet insufficient mean reversion of longer horizon forecasts, consistent with a combination of forward looking logic and over-reaction to recent news.

4.1 The Model

We model an agent (a forecaster) who has lived through or studied a country's inflation history, which forms her memory database. When making a forecast at horizon h, the current state cues her to retrieve similar inflation episodes which are then used to imagine the future. The agent may also use statistical inputs as in (Bordalo et al. (2024a)) but for simplicity we abstract from this. What is key is that salient recollections act as belief anchors, perhaps because she does not fully trust her model.

Vast memory work documents thee strong regularities in recall (Kahana, 2012): frequency, similarity to cues, and interference.[13] Frequency means that if an inflation level, say 2%, has occurred more often in a country's history, the agent is more likely to recall it. Similarity means that the inflation state cues episodes sharing some of its contextual or semantic features. Contextually, if current inflation is 7%, the agent is more likely to recall numerically similar episodes. Semantic similarity captures "cause-effect" logic; The current 7% state prompts recall of what happened $after$ similar past episodes: if 7% was historically followed by 5%, the agent recalls 5%. Interference captures competition of events in recall: retrieved events block others which are forgotten. Thus, changing inflation states and databases bring some episodes to mind throwing others into oblivion, causing state dependence.[14]

Memory is random, so even identical agents may recall different episodes and make different forecasts. Because however they share the same cue and database, their consensus forecast exhibits systematic movements. We formalize these ideas building on Bonaglia and Gennaioli (2025), who generalize the memory model in Bordalo et al. (2023) and study its dynamics.

The Database. Upon entering $t = 0, 1, ...$ it is the set of dated past inflation realizations $\Pi_t \equiv \{\pi_k\}_{0 \leq k \leq t-1}$. Memory traces could be vectors containing also the monetary policy regime, unemployment, etc. and cues could depend on these features. Bonaglia and Gennaioli (2025) allow for this possibility.

Recall and Beliefs. When thinking at t about inflation at $t + h$, the agent draws a sample from her database, where each draw retrieves π_k with probability $r_{t,h}(\pi_k)$. Her mnemonic

[13] In standard experiments, subjects recall words from a previously studied list. Frequency means that "apple" is more likely to be recalled if it is repeated in the list. Similarity means, semantically, that cuing the word "wheel" fosters recall of "car" because wheels are part of cars. It means, contextually, that cuing "wheel" fosters recall of "eel", for they share spelling and phonetic features. Interference means that words compete in recall, so frequent and similar items cause forgetting of others.

[14] This structure yields well known contiguity and recency effects (Kahana, 2012). By semantic similarity, the current inflation state prompts recall of its past successors (contiguity). By numerical similarity, the current state cues recall of similar inflation levels, many of which are recent due to inflation persistence. We abstract from primacy effects, which are studied by Gennaioli et al. (2024) for households in the NYFed and Michigan surveys. Here we do not observe the inflation experience first seen by forecasters.

expectation is the average recollection π_k. The consensus of many identical agents is equal to:

$$\mathbb{E}_t^m(\pi_{t+h}) = \sum_{\pi_k \in \Pi_t} \pi_k \cdot r_{t,h}(\pi_k). \tag{13}$$

The database Π_t encodes the frequency of different past values. The current state π_t and forecast horizon h shape similarity, affecting the retrieval function $r_{t,h}(\pi_k)$ as we now formalize.

Cues, Similarity, Retrieval. A cue is a salient signal q_t igniting associative (i.e. similarity-based) retrieval. In our setting, the cue is the current state, $q_t = \pi_t$. Following multidimensional scaling and recognition (Nosofsky, 1992; Torgerson, 1958), similarity between the cue π_t and an episode π_k exponentially drops in the distance between their contextual and semantic features. Context is captured by the numerical distance between the cue and the episode $|\pi_t - \pi_k|$. Semantic distance is occurs between the cue and the h step predecessor of the k-episode, $|\pi_t - \pi_{k-h}|$. Feature-level distances enter an overall discrepancy $d_h(\pi_k, q_t)$. The probability of retrieving π_k when forecasting at t inflation for $t+h$ is equal to:

$$r_{t,h}(\pi_k) = \frac{\exp\left[-d_h(\pi_k, \pi_t)\right]}{\sum_{\pi_s \in \Pi_t} \exp\left[-d_h(\pi_s, \pi_t)\right]}. \tag{14}$$

The numerator captures associativity: it is easier to retrieve inflation states that are similar to the current one, semantically and numerically. It also captures frequency: repeating the same state (e.g. 5%) at different times k increases its recall. The denominator captures interference: a highly frequent or similar episode π_s hinders recall of π_k.

Expectations. Equations (14) and (13) characterize beliefs at every horizon h as a function of: i) the database Π_t, ii) numerical and semantic distances $|\pi_t - \pi_k|$ and $|\pi_t - \pi_{k-h}|$ with the cue π_t. The database, the similarity features and the cue are observable from a country's inflation history. Knowledge of the similarity function pins down mnemonic beliefs. We later estimate a similarity function and simulate the model. Before doing so, we study the long run converge and properties of mnemonic beliefs under a known DGP.

4.2 Long Run Beliefs

Based on Bonaglia and Gennaioli (2025), we characterize the beliefs of an agent who lives through a long history produced by an inflation DGP. To get tractability, we second-order approximate the similarity function:

$$d_h(\pi_k, q_t) = \alpha \cdot (\pi_{k-h} - \pi_t)^2 + \beta \cdot (\pi_k - \pi_t)^2, \tag{15}$$

where $\alpha \geq 0$ and $\beta \geq 0$ capture the respective weight of semantic and context features.

We first study a general Gaussian stationary case and then focus on an AR(1) process. To grasp the idea, consider an agent entering time s with database Π_s and observing cue $q_s = \pi$. Given a Borel-set $A \subset \mathbb{R}$ having positive measure under the DGP, the subjective probability of A is the share of past episodes in the database that fulfill $\pi_k \in A$ and that are successfully retrieved according to (14). That is, $P(A|\Pi_s, h, \pi) = \sum_{\pi_k \in A} r_{s,h}(\pi_k)$. Such probability can be zero when the database is sparse (s is small) but it eventually becomes positive as the agent experiences some $\pi_k \in A$.

Bonaglia and Gennaioli (2025) prove that the sequence $(P(A|\Pi_s, h, \pi))_s$ of h-periods ahead beliefs about a generic set A conditional on cue π converges in probability to the probability of the event under a Gaussian kernel $\Lambda_\infty(\cdot|h, \pi_t)$ that depends on the cue π. We study the average forecast when such kernel is produced under similarity (15). For a stationary Gaussian DGP we say that the agent "overreacts" to the current state π_t if the mnemonic expectations is more sensitive to it than the rational expectation.

Proposition 1 *Let $(\pi_t)_t$ be zero-mean, stationary and Gaussian. If $\beta > 0$, then there exists an horizon $\tilde{h}$ such that, for all forecasts at horizons $h \geq \tilde{h}$ there is overreaction. If $\beta = 0$ and $\alpha \to \infty$ the mnemonic expectation is rational at all horizons.*

Numerical similarity produces horizon increasing overreaction. Upon seeing 3% inflation, this realization becomes more salient when thinking about all horizons. Under a stationary process, this produces overreaction for sufficiently long forecast horizons. Semantic similarity produces forward looking behavior, and in particular rational one-step conditioning.

Important, mnemonic expectations will also depend on the database, which captures features of the DGP. To characterize this dependence, suppose that inflation follows an AR(1) process

$$\pi_k = \rho \pi_{k-1} + \sigma \cdot \epsilon_k, \tag{16}$$

with persistence $\rho \in (0, 1)$ and volatility $\sigma > 0$.

Proposition 2 *Let $\sigma_\pi^2 \equiv \sigma^2 / (1 - \rho^2)$. Expected inflation at t for $t + h$ is equal to:*

$$\mathbb{E}_t^m (\pi_{t+h}) = \frac{\alpha \rho^h + \beta + 2\alpha\beta\sigma_\pi^2 \left(1 - \rho^{2h}\right)}{\alpha + \beta + 2\alpha\beta\sigma_\pi^2 (1 - \rho^{2h}) + 1/2\sigma_\pi^2} \cdot \pi_t. \tag{17}$$

The forecast at horizon h is proportional to current inflation π_t. Mnemonic learning recovers the AR(1) structure of the DGP, with distorted persistence. In Gabaix (2019) and Angeletos et al. (2021)) agents exaggerate the true persistence of an AR(1) DGP. DE produce the same bias under a long look-back period ($J \to \infty$ in Bianchi et al. (2024b)). Excess persistence is a special case of Equation (17), which however also produces underreaction and horizon-dependent

distortions. The emergence of different biases depends on four measurable parameters: the DGP's true persistence ρ, its volatility σ_π^2 (both reflect the long run database), and the similarity weights α and β. Consider three extreme cases. If $\alpha \to \infty$ and $\beta \to 0$, recall is fully shaped by semantic similarity. In this case, as shown for a general DGP, the forecast is rational:

$$\lim_{\alpha\to\infty,\beta\to 0} \mathbb{E}_t^m(\pi_{t+h}) = \rho^h \cdot \pi_t. \tag{18}$$

If $\alpha \to 0$ and $\beta \to \infty$, recall is fully shaped by contextual similarity. In this case, the agent expects the present π_t to persist into the future:

$$\lim_{\alpha\to 0,\beta\to\infty} \mathbb{E}_t^m(\pi_{t+h}) = \pi_t, \tag{19}$$

Recall of inflation rates similar to π_t causes "static expectations." Given mean reversion, $\rho < 1$, this is a force toward overreaction. Finally, if $\alpha, \beta \to 0$ retrieval is only based on frequency. In this case, the forecast is anchored to the long run mean $\pi_\infty = 0$.

$$\lim_{\alpha,\beta\to 0} \mathbb{E}_t^m(\pi_{t+h}) = 0. \tag{20}$$

Here the agent expects immediate reversal of changes in π_t. Given persistence, $\rho > 0$, this is a force toward underreaction.

Expectations in (17) combine these three forces based on the similarity weights and parameters of the DGP. The role of similarity weights is immediate: higher semantic similarity α renders beliefs "more rational", higher contextual similarity β causes stronger overreaction, and weaker similarity - lower α and β - enhance the role of frequency, boosting underreaction.

Consider next the database, i.e. the DGP's parameters. Volatile inflation, high σ, boosts the weight on static expectations: $\lim_{\sigma\to\infty} \mathbb{E}_t^m(\pi_{t+h}) = \pi_t$. Upon seeing higher π_t the agent recalls many memories of higher past inflation, promoting overreaction. Stable inflation, low σ, boosts the weight on long run expectations: $\lim_{\sigma\to 0} \mathbb{E}_t^m(\pi_{t+h}) = 0$. Now, under any cue, the agent recalls many episodes near $\pi_\infty = 0$, promoting underreaction to changes in π_t.

Higher persistence ρ has conflicting effects. On the one hand, it increases the volatility of inflation, fostering overreaction as discussed above. On the other hand, higher ρ increases the volatility of rational expectations. The latter effect dampens overreaction, which in fact disappears as $\rho \to 1$, when expectations become rational $\lim_{\rho\to 1} \mathbb{E}_t^m(\pi_{t+h}) = \pi_t$. An important implication follows: overreaction is stronger when inflation is less persistent.

These comparative statics allow mnemonic expectations to creates variation in belief distortions across different inflation histories.

4.3 Predictions of Selective Memory

Consider first the predictability of mnemonic forecast errors from the current mnemonic revision and lagged mnemonic forecast, the counterpart of Equation (3).[15]

Proposition 3 *Under Equation (17) the forecast revision at t and the forecast at $t-1$ predict the forecast error at $t+h$ with coefficients:*

$$\delta_{1,h} = \delta_{2,h} = \delta_h \equiv \rho^h \cdot \frac{\alpha + \beta + 2\alpha\beta\sigma_\pi^2(1-\rho^{2h}) + 1/2\sigma_\pi^2}{\alpha\rho^h + \beta + 2\alpha\beta\sigma_\pi^2(1-\rho^{2h})} - 1, \tag{21}$$

If $\beta > 0$ mnemonic expectations have two properties:

i) They overreact at long horizons: $\lim_{h\to\infty} \delta_h = -1$. They also underreact at short horizons, $h = 1$, if and only if:

$$\rho\,(1+\rho) > 2\beta\sigma^2\left(1 + 2\alpha\sigma^2\right). \tag{22}$$

ii) they overreact more at longer horizons, δ_h decreases in h.

Consistent with Table 2, at long horizons mnemonic expectations produce overreaction, $\delta_{1,h} < 0$ and persistence, $\delta_{2,h} < 0$ of similar magnitude. This occurs as long as contextual similarity plays a role, $\beta > 0$. At short horizons underreaction may instead prevail. This occurs provided inflation is very persistent and little volatile (ρ is high and σ is low) or similarity is weak (α and β are low). Crucially, mnemonic expectations rationalize stronger overreaction at longer horizons. If beliefs at $h = 1$ underreact, there is a threshold $\tilde{h} > 1$ so that beliefs underreact if $h < \tilde{h}$ and overreact otherwise.[16]

By producing horizon-increasing overreaction, mnemonic expectations also produces return predictability. We also show this property in the AR(1) long run case.

Proposition 4 *If $\beta > 0$, the mnemonic expectation of average inflation $\mathbb{E}_t^m(\overline{\pi_{t+1,h}})$ predicts the ex-post real yield to maturity h with coefficient:*

$$\theta_h^r = -\sum_{j=1}^{h} \delta_j \cdot w_j; \tag{23}$$

where $(w_j)_{1\le j\le h}$ are positive weights with $\sum_{j=1}^h w_j = 1$. If $\alpha = 0$, $\theta_h^r = 1 - \frac{\rho(1+2\beta-\rho^2)(1-\rho^h)}{2h\beta(1-\rho)}$, which is increasing in h and positive when h is high enough.

[15] This regression is not identical to Equation (17). In the latter inflation is observed annually, forecasts and their revisions quarterly or semi-annually. Our regression results are however consistent if we only use annual variation in forecasts as shown in Appendix Table 15.

[16] The intuition for the tradeoff between under and over-reaction goes as follows. At short horizons, low h, the "static" forecast $\mathbb{E}_t^m(\pi_{t+h}) = \pi_t$ is close to rational. Formally, (18) is close to (19). Thus, short term distortions take the form of database-driven anchoring to $\pi_\infty = 0$, creating under-reaction. At long horizons, by contrast, beliefs $\mathbb{E}_t^m(\pi_{t+h}) = \pi_\infty$ are close to rational. Formally, (18) is close to (20). Here distortions come from similarity-driven anchoring to π_t, creating over-reaction.

The same expectation also predicts the excess return on a maturity h bond with coefficient:

$$\theta_h^{rx} = h \cdot [\theta_h^r \cdot (1-\rho) + \delta_1 \cdot w_1 - \rho \cdot \delta_h \cdot w_h]; \tag{24}$$

When $\alpha = 0$, $\theta_h^{rx} = (h-1)(1-\rho)$. Thus, excess return predictability coefficients are positive and increase with the horizon h.

By fostering overreaction, contextual similarity $\beta > 0$ implies that higher current inflation π_t causes an excessive upward adjustment of the nominal interest rate at t. Subsequently, lower than expected inflation causes abnormally high real yields to maturity, producing positive and maturity increasing coefficients θ_h^r.

For holding period excess bond returns, the picture is more complicated. Long term bonds overreact more on impact, so they may revert less during the holding period. In Equation (24), excess return predictability is not fully aligned to real yields predictability, and it may be even negative. The model does not predict a one-to-one correspondence between them. [17] Both forms of predictability are stronger at longer horizons, where mnemonic beliefs overreact more. Predictability coefficients depends on features of the DGP, giving a broad implication: predictability should dpennd on inflation histories. We test this property in our data.

5 Model Quantification

We estimate similarity parameters α and β by fitting model-implied expectations to CE forecasts in the pooled dataset (so α and β are constant across c and t). We then assess the model's ability to produce realistic predictability of forecast errors (Section 5.2) and bond returns (Section 5.3), which are untargeted in our estimation. We then test whether memory can reproduce cross-country variation in predictability of errors and returns based on different inflation histories, which shape the database.

5.1 The Database and Similarity Parameters

In each country c we construct the memory database at time t using the country's past annual inflation, $\Pi_{c,t} = \{\pi_{c,t}\}_{k=t_{c,0},\ldots t-1}$, where $t_{c,0}$ is the first inflation level observed for c (going from as early as 1960 to 1996, see Table 8). Given (α, β), the we compute retrieval $r_{t,h}(\pi_{c,k})$ for $\pi_{c,k}$ in Equation (14) using a similarity function that is negative exponential in distance:

$$d_h(\pi_{c,k}, \pi_{c,t}) = \alpha \cdot |\pi_{c,k-h} - \pi_{c,t}| + \beta \cdot |\pi_{c,k} - \pi_{c,t}| \quad \text{for each } \pi_{c,k} \in \Pi_{c,t}, \tag{25}$$

[17] At short horizons these effects can go in the opposite direction if belief underreaction prevails, but this is never the case in our CE

We compute the mnemonic expectation using (13), which we denote as $\mathbb{E}_t^m(\pi_{c,t+h}|\alpha,\beta)$. Consistent with CE, for $h = 6-10$ we average beliefs over the entire horizon. We construct a grid of over 10,000 values of $(\alpha,\beta) \in [0,1]^2$ with a step size of 0.01. Each parameter vector yields a panel of model-implied forecasts. We select the vector (α,β) that minimizes the average distance between CE and model-implied forecasts:

$$\min_{\alpha,\beta} \sum_{c,t,h\in\{1,2,3,5,6-10\}} |\mathbb{F}_t(\pi_{c,t+h}) - \mathbb{E}_t^m(\pi_{c,t+h}|\alpha,\beta)|,$$

where $\mathbb{F}_t(\pi_{c,t+h})$ is the CE forecast. The simulation and estimation algorithm is described in Appendix K. The parameter point estimate and 99% confidence interval[18] are given by:

$$\widehat{\alpha} = 0.11 \pm 0.031 \quad \text{and} \quad \widehat{\beta} = 0.14 \pm 0.002.$$

CE forecasts are best described by a model in which contextual similarity matters, $\widehat{\beta} > 0$, yielding overreaction, and semantic similarity receives a similar weight, $\widehat{\alpha} = 0.11 \approx \widehat{\beta}$, creating some forward looking behavior. The fairly low magnitude of both parameters suggests that also frequency plays an important role, creating a tendency to underreact. The forces shaping mnemonic expectations all play a significant role in the data.

5.2 Explanatory Power for Inflation Expectations

We assess the model's ability to produce the forecast error predictability coefficients δ_h in Table (1), Panel A. For each country c and time t we compute the forecast error at horizon h by subtracting from future realized inflation $\pi_{c,t+h}$ the model-implied expectation in country c and time t for horizon $t+h$. We predict this error with the same model-implied expectation. Panel A in Figure 1 reports in red the estimate of δ_h using CE forecasts and in blue the estimate from model-implied forecasts.[19]. The model does a good job at reproducing horizon increasing overreaction. Model-implied coefficients are statistically indistinguishable from actual ones.

Table (1) focused on pooled estimates, extracting general patterns across countries. We now consider cross country variation in belief biases. Estimating Equation (2) separately for each country reveals significant heterogeneity in forecast error predictability coefficients $\delta_{c,h}$, visible from the boxplots of Appendix Figure 12, Panel A. At the shortest (same year) horizon, the median coefficient is about 0.06 and the inter-quartile range is [0.01, 0.06]. At the longest

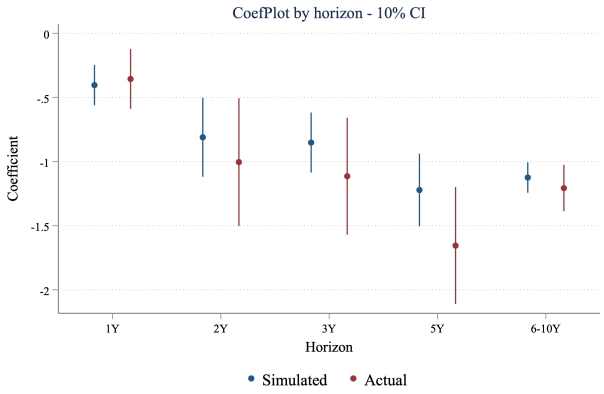
[18]We performed block-bootstrap (Künsch, 1989) for each country's time series of realized inflation, obtaining 1000 bootstrapped samples. For each sample we estimate the parameters α and β according to the algorithm described in Appendix K. Confidence intervals are symmetric percentile bootstrap confidence intervals, which have the advantage of being robust under model misspecification (Kolesár, 2025).

[19]Because the database and cues vary annually, the model-based regression is annual. We thus benchmark it to the annual CE estimate from Appendix Table 15

(6-10) horizon the median coefficient is about -1 and the inter-quartile range is [-1.3, -0.7]. Maturity increasing overreaction holds, but with sharply different strengths between the top and bottom quartile. This variation is not noise, the F-test strongly rejects coefficient equality, at conventional significance levels at all maturities.

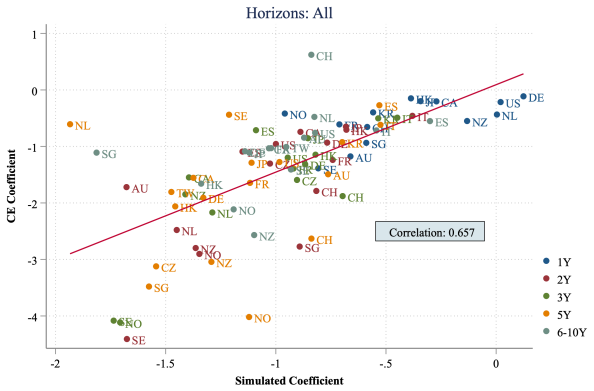
Does this variation stems from country histories? To answer this question, we estimate Equation (2) country by country using model implied expectations (both for constructing forecast errors and as a predictor). Figure (1) Panel B plots on the y-axis the estimates $\delta_{c,h}$ obtained using CE forecasts, on the x-axis those obtained using the model.

Figure 1: Performance for Forecast Error Predictability



(a) Panel A: actual and simulated δ_h

Note. The figure displays the point estimates and their 90% confidence intervals (vertical bars) for each variable across horizons. Each horizon is a different regression. Country fixed effects are included. Sample varies with the horizons according to Table 14. Standard errors are Driscoll-Kraay.



(b) Panel B: actual and simulated $\delta_{h,c}$

Note. The figure displays the country-horizon specific coefficients from the simulated (y-axis) against the CE (x-axis) datasets. The line indicates the fit between the two. Pooled (across countries and horizons) correlation is reported in the lower-right corner of the graph. The identifier for each data point corresponds to the ISO 3166-2 country abbreviation.

The model captures significant variation in the extent of error predictability across countries and horizons, exhibiting a 65% correlation with the coefficient estimates from CE.[20] To focus on cross country differences, we regress the error predictability coefficient $\delta_{c,h}$ estimated using CE data on h-dummies. The regression yields an R^2 of 0.2, which is entirely due to common horizon increasing overreaction across countries. Adding the predictability coefficient $\delta_{c,h}$ estimated from model-implied forecasts yields an R^2 of 0.47 (see Appendix Table 34). The explanatory power more than doubles due to the ability of memory to capture cross country variation in overreaction, shaped by different country-specific histories.

In our long run AR(1) analysis we tied forecast biases to inflation persistence and volatility. AR(1) is not an accurate model for our countries, due to changing inflation regimes from high

[20]The model also does a good job at matching the separate coefficients δ_{1h} and δ_{2h} for the revision and lagged forecast. The pooled results are shown in the Appendix Figure 9, and the regression is in Table 18.

and unstable inflation in the 70s-80s to stability in our sample period. The properties of our long run model fit more the latter period. They predict more overreaction if the cue π_t is less objectively predictive of higher future inflation and if inflation shocks are more volatile. It also predicts strong overreaction if accumulating "high inflation" episodes in the database are not much followed by high future inflation.

To capture biases tied to cues, we compute persistence $\rho_{h,c}$ of inflation in country c at horizon h and the standard deviation of inflation shocks $\sigma_{h,c}$ in the same country at horizon h by regressing inflation at $t+h$ on inflation at t. We expect more overreaction when $\rho_{h,c}$ is low and $\sigma_{h,c}$ is high. To capture biases due to the database, we regress inflation at $t+h$ on the average in-sample inflation $\overline{\pi}_t$ experienced up to t. Low predictive coefficient $\overline{\rho}_{h,c}$ should lead to more overreaction due to the database. To test these mechanisms, we pool countries and horizons and regress the estimated predictability coefficients $\delta_{h,c}$ on these proxies. We obtain:

$$\delta_{h,c} = \underset{(0.241)}{-0.803^{***}} - \underset{(0.142)}{0.237^{*}} \cdot \sigma_{h,c} + \underset{(0.402)}{1.770^{***}} \cdot \rho_{h,c} + \underset{(0.092)}{0.463^{***}} \cdot \overline{\rho}_{h,c} + \varepsilon_{h,c}. \tag{26}$$

Countries in which cues or the database are more predictive of future inflation exhibit less overreaction, countries with more volatile inflation shocks exhibit more overreaction, consistent with the model. The R^2 of the regression is 0.41, and does not change if we add forecast horizon dummies to the regression. Similar results obtain if we regress model implies predictability coefficients on the same proxies, except that now the R^2 increases to 0.6. This analysis confirms the ability of the model to capture variation across countries and horizons based on the histories shaping cues and the database.

One intriguing question, then, is whether country level differences in overreaction translate into country level differences in bond return predictability. This is what we tackle next.

5.3 Explanatory Power for Bond Returns

To assess whether memory can produce return predictability similar to that of Section 3, we take $\mathbb{E}_t^m(\pi_{c,t+h}|\alpha,\beta)$ and construct the model-implied forecast for average inflation between t and $t+h$ held at t for country c. We use this measure to predict, for each country, time, and maturity, the realized bond return. We consider both the real yield to maturity, which points to predictable forecast errors, and the excess return, which points to forecast revisions. We first consider pooled estimates and then move to cross country variation.

Consider the real yield to maturity first. Figure 2 reports in Panel A the predictability coefficient θ_h^r in Equation (7) obtained using CE forecasts (red) and the one obtained using model-based forecasts (blue).[21] In the pooled estimates the memory model captures the positive

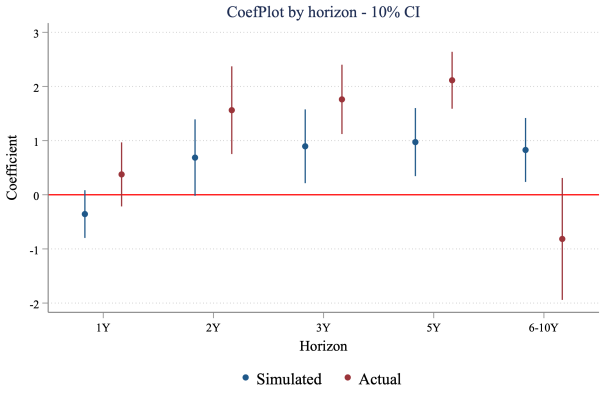
[21]Estimates using CE forecasts are from Columns (1) to (5) of Appendix Table 27, while estimates using model-implied forecasts are from Appendix Table 17. Both use annual data.

and maturity increasing predictability of the real yield, with magnitudes that, except for the 6-10 years horizon, are comparable to those estimated in the data.

Consider cross country variation next. Also here variability if significant, see the boxplots of θ_h^r in Appendix Figure 12, Panel B. The median predictability of the real yield at a one year maturity is 0.6, the inter-quartile range is $[0.007, 0.8]$. At the 6-10 years maturities the median is 0.9 and the inter-quartile range is $[0.3, 3.2]$. Positive and maturity increasing real yield predictability is present across countries. Inter-quartile ranges however confirm significant variability in its strength, which here ties to variation in the degree market efficiency.

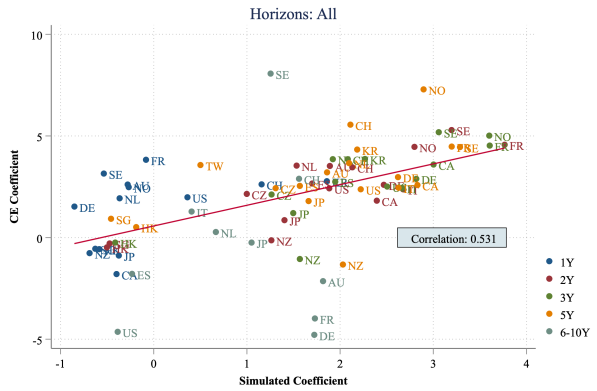
To assess the role of inflation histories, Panel B reports the predictability coefficients obtained using CE forecasts (y-axis) and the model-implied forecasts (x-axis) as predictors. The correlation between predictability coefficients is 53%, confirming the model's explanatory power for cross sectional differences. Adding the model-implies coefficient to maturity dummies improves the R^2 from 23% to 41% (see Appendix Table 35), confirming the explanatory power of inflation histories for excess interest rate volatility.

Figure 2: Performance for Real Yield to Maturity



(a) Panel A: Coefplot by horizon of real interest rates' regression

Note. The figure displays the point estimates and their 90% confidence intervals (vertical bars) for each variable across horizons. Each horizon is a different regression. Country fixed effects are included. Sample varies with the horizons according to Table 14 and regressions are performed on the simulated dataset. Standard errors are Driscoll-Kraay.



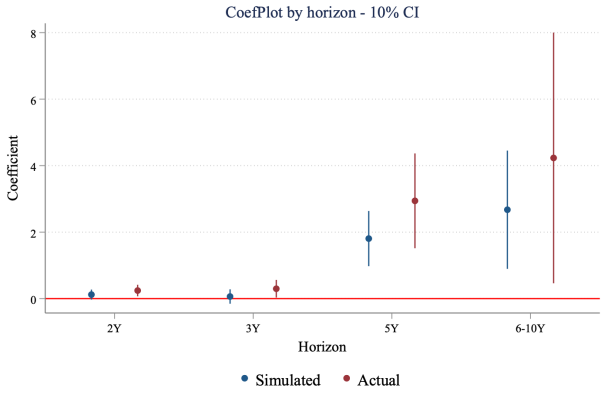
(b) Panel B: Scatterplot pooling countries and maturities of real interest rates' regression

Note. The figure reports the coefficients from CE dataset (y-axis) at annual frequency against simulated dataset (x-axis). Coefficients are pooled across countries and maturities. The line indicates the fit between the two datasets' coefficients. Correlation is reported in the lower-right corner of the graph. The identifier for each data point corresponds to the ISO 3166-2 country abbreviation.

We conclude by predicting the excess return in country c between t and $t+1$ on a maturity h bond (again at the annual frequency) on the model-implied average inflation expected between t and $t+h$ for the same country, following Equation (9). Excess returns are smoothed over two years, as in Section 3. The estimates in Panel A are from Appendix Table 23 and 43, respectively. Figure 3 reports in Panel A the pooled regression coefficient θ_h^{ex} of the model-

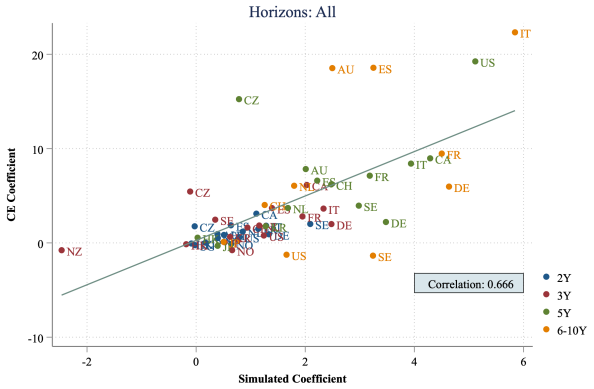
implied (blue) and CE-estimated (red) predictability coefficients. Model-implied excess return predictability tracks fairly well the positive and maturity increasing pattern obtained when using CE forecasts, again with similar coefficient magnitudes.

Figure 3: Performance for Excess Returns



(a) Panel A: Coefplot by horizon of bond excess returns' regression

Note. The figure displays the point estimates and their 90% confidence intervals (vertical bars) for each variable across horizons. Each horizon is a different regression. Country fixed effects are included. Sample varies with the horizons according to Table 14 and regressions are performed on the simulated dataset. Standard errors are Driscoll-Kraay.



(b) Panel B: Scatterplot pooling countries and maturities of bond excess returns' regression

Note. The figure reports the coefficients from CE dataset (y-axis) at annual frequency against simulated dataset (x-axis). Coefficients are pooled across countries and maturities. The line indicates the fit between the two datasets' coefficients. Correlation is reported in the lower-right corner of the graph. The identifier for each data point corresponds to the ISO 3166-2 country abbreviation.

We observe significant cross country variation also in excess return predictability, visible in the boxplots of estimates for θ_h^{ex}, reported in Appendix Figure 12, Panel B. At the $h = 2$ horizon the median coefficient is 0.3, the interquartile range is $[-0.1, 0.6]$. At the 6-10 years maturity they are 3.5 and $[2.4, 10]$, respectively. Again, this confirms that maturity increasing overreaction in interest rates to the inflation state is a robust feature of the data, but also the significant cross country variability in its strength.

To assess the explanatory power of memory for this variation, Panel B reports the country by country model-implied (x-axis) versus CE-estimated (y-axis) predictability coefficient θ_h^{ex}. Again, model performance is good. The correlation between the two coefficients is 67%. Adding the mnemonic coefficient to maturity dummies improves the R^2 for CE-predictability coefficients from 29% to 47% (see Appendix Table 37).[22]

[22]In the previous exercises we compared the ability of model implied and CE forecasts to predict the same actual bond returns. We can also assess the extent to which memory alone can produce a joint distribution of current expectations and future returns that is similar to the one observed in the data. To do so, we impute a constant ex-ante required return for each country c and maturity h, and then obtain "artificial" nominal interest rates by adding model-implied expectations to it, following the Fisher Equation. In Appendix G we implement this strategy and find that the results are similar to those found above: model-implied return predictability

Overall, mnemonic inflation expectations can account for the average horizon increasing overreaction in inflation forecasts and for maturity increasing predictability of bond returns in a sample of 18 advanced economies as well as for observed cross country variation in the strength of these effects. Time varying cues and accumulating past experiences can cause different belief biases in different countries, causing changing inflation states to exert different redistributive effects between lenders and borrowers.

6 Conclusion

[INCOMPLETE] Using a survey on inflation expectations by large financial institutions across 18 advanced economies we show that consensus inflation forecasts exhibit horizon increasing overreaction. This phenomenon links to maturity-increasing excess volatility of interest rates detected through systematic predictability of bond returns based on expectations data. Existing models of non-rational beliefs have a hard time accounting for these patterns. We show that the they can be explained, qualitatively and quantitatively, by a memory based model of belief formation. The model offers new predictive power on cross-country variation in belief biases, a feature of the data that has so far received little attention.

Memory offers a foundation for a rich set of biases, most notably the forward looking overreaction of diagnostic expectations and the opposite phenomenon of underreaction, giving testable conditions tied to historical features of the DGP when either should prevail. Mnemonic learning is gradual, forward-looking, but does not converge to the truth because it exhibits persistent instability due to salient cues.

The role of histories in shaping biases in inflation expectations can offer new insights on optimal momentary policy. A Central bank's influence on inflation plays a dual role in our model: it shapes both the gradually accumulating database and the cues igniting state-dependent retrieval. This dual role may create policy dilemmas, for instance between stable inflation and state-dependent expectations management, which may be interesting to study in future work.

has 43% and 44% correlations with the real yield and excess return predictability in the actual (and CE) data, respectively.

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A Data Description

Table 7: Description of Data Sources

Variable	Data Source	Series Identifier	Sample Period	Coverage Notes
Bond Yields	LSEG	RF *COUNTRY* GVT BMK BID YLD 1Y - RED. YIELD	1962m2–2022m10	Refer to Tables 11 to 13 for coverage and countries.
Bond Yields	LSEG	RF *COUNTRY* GVT BMK BID YLD 2Y - RED. YIELD	1976m2–2022m10	Refer to Tables 11 to 13 for coverage and countries.
Bond Yields	LSEG	RF *COUNTRY* GVT BMK BID YLD 3Y - RED. YIELD	1962m2–2022m10	Refer to Tables 11 to 13 for coverage and countries.
Bond Yields	LSEG	RF *COUNTRY* GVT BMK BID YLD 4Y - RED. YIELD	1988m11–2022m10	Refer to Tables 11 to 13 for coverage and countries.
Bond Yields	LSEG	RF *COUNTRY* GVT BMK BID YLD 5Y - RED. YIELD	1962m2–2022m10	Refer to Tables 11 to 13 for coverage and countries.
Bond Yields	LSEG	RF *COUNTRY* GVT BMK BID YLD 6Y - RED. YIELD	1995m1–2022m10	Refer to Tables 11 to 13 for coverage and countries.
Bond Yields	LSEG	RF *COUNTRY* GVT BMK BID YLD 7Y - RED. YIELD	1986m1–2022m10	Refer to Tables 11 to 13 for coverage and countries.
Bond Yields	LSEG	RF *COUNTRY* GVT BMK BID YLD 8Y - RED. YIELD	1994m8–2022m10	Refer to Tables 11 to 13 for coverage and countries.
Bond Yields	LSEG	RF *COUNTRY* GVT BMK BID YLD 9Y - RED. YIELD	1992m5–2022m10	Refer to Tables 11 to 13 for coverage and countries.
Bond Yields	LSEG	RF *COUNTRY* GVT BMK BID YLD 10Y - RED. YIELD	1962m2–2022m10	Refer to Tables 11 to 13 for coverage and countries.

Table 8: Description of Data Sources

Variable	Data Source	Series Identifier	Sample Period	Coverage Notes
Realized CPI inflation	International Monetary Fund Data Mapper	PCPIPCH	1980–2023	CZ from 1996. AU and NZ up to 2022. NL missing in 1980 and 1986.
Realized CPI inflation	World Bank's World Development Indicators	FP.CPI.TOTL.ZG	1960–2023	CZ from 1992, HK from 1982, SG from 1961. TW is not covered.
Realized CPI inflation	OECD	Consumer price indices (CPIs, HICPs), COICOP 1999	2024Q1	HK, JP, SG, and TW are not covered.
Realized CPI inflation	Statistics Bureau of Japan	2020-Base Consumer Price Index	2024M1–2024M3	JP only.
Realized CPI inflation	Census and Statistics Department	Table 510-60001	2024M1–2024M3	HK only.
Realized CPI inflation	Department of Statistics, Directorate General of Budget, Accounting & Statistics	Price Indices	2024M1–2024M3	TW only.
Realized CPI inflation	Singapore Department of Statistics	Consumer Price Index	2024Q1	SG only.
Realized CPI inflation	Bureau of Labor Statistics	CUSR0000SA0	1947M1–2024M7	US only.
Realized GDP Deflator Inflation	Bureau of Economic Analysis	Implicit Price Deflators for Gross Domestic Product	1947Q1-2024Q2	US only.

Note. the two-letter identifier in the coverage notes corresponds to the ISO 3166-2 country abbreviation.

Table 9: Description of Data Sources

Variable	Data Source	Series Identifier	Sample Period	Coverage Notes
Forecasted Bond Yields	Consensus Economics	3-month interest rate (3-month period)	1989m10–2022m1	NL, ES, SE, and CH from 1989m11. AU and NZ from 1990m1. SG from 1990m11. HK from 1990m12. KR and TW from 1994m12. CZ from 1998m5. NO from 1998m6. NZ missing from 1991m4 to 1991m12 and from 1993m1 to 1994m11.
Forecasted Bond Yields	Consensus Economics	3-month interest rate (12-month period)	1989m10–2022m1	NL, ES, SE, and CH from 1989m11. AU and NZ from 1990m1. SG from 1990m11. HK from 1990m12. KR and TW from 1994m12. CZ from 1998m5. NO from 1998m6. NZ missing from 1991m4 to 1991m12 and from 1993m1 to 1994m11.
Forecasted Bond Yields	Consensus Economics	Prime Lending Rate (3-month period)	1989m10–2022m1	AU from 1990m11. NZ, SG, and KR from 1994m12. NL, ES, and SE from 1995m1. NO and CH from 1998m6. CZ and TW from 2006m3. HK not covered.
Forecasted Bond Yields	Consensus Economics	Prime Lending Rate (12-month period)	1989m10–2022m1	AU from 1990m11. NZ, SG, and KR from 1994m12. NL, ES, and SE from 1995m1. NO and CH from 1998m6. CZ and TW from 2006m3. HK not covered.

Note. the two-letter identifier in the coverage notes corresponds to the ISO 3166-2 country abbreviation.

Table 10: Description of Data Sources

Variable	Data Source	Series Identifier	Sample Period	Coverage Notes
Forecasted CPI Inflation	Consensus Economics	Consumer Prices	1989Q4–2022Q1	AU from 1991q2. HK, NL, NZ, SG, KR, ES, SE, and TW from 1995q2. NO and CH from 1998q4. Semiannual data, quarterly after 2014q2
Forecasted CPI Inflation	Survey of Professional Forecasters	CPIh, $h = 1, \ldots 6$	1981Q3–2024Q2	US only. $h = 1$ denotes the "forecast" for the previous quarter, $h = 2$ denotes the forecast for the current quarter, $h = 3, ..6$ denote forecasts for the four quarters succeeding the current. Missing in 1969Q1-1969Q3 and 1970Q1 for $h = 6$.
Forecasted GDP Deflator	Survey of Professional Forecasters	PGDPh, $h = 1, \ldots 6$	1968Q4–2024Q2	US only. $h = 1$ denotes the "forecast" for the previous quarter, $h = 2$ denotes the forecast for the current quarter, $h = 3, ..6$ denote forecasts for the four quarters succeeding the current. Missing in 1969Q1-1969Q3 and 1970Q1 for $h = 6$.

Note. the two-letter identifier in the coverage notes corresponds to the ISO 3166-2 country abbreviation.

Table 11: Detailed Coverage of LSEG Data for Bond Yields

Horizon	Australia	Canada	Czech Rep
1Y	1995m1–2022m10	1994m4–2022m10	1995m1–2022m10
2Y	1976m2–2022m10	1986m1–2022m10	1999m4–2022m10
3Y	2005m4–2022m10	1994m4–2022m10	1998m3–2022m10
4Y	2005m4–2022m10	1994m4–2022m10	1999m9–2022m10
5Y	1976m2–2022m10	1986m5–2022m10	1998m4–2022m10
6Y	2000m2–2022m10	not covered	2004m10–2022m10
7Y	2005m4–2022m10	1994m4–2022m10	2000m4–2022m10
8Y	1994m9–2022m10	not covered	2003m7–2022m10
9Y	2005m4–2022m10	not covered	2004m11–2022m10
10Y	1976m2–2022m10	1986m1–2022m10	2000m5–2022m10
Horizon	France	Germany	Hong Kong
1Y	1995m1–2022m10	1994m3–2022m10	1991m7–2022m10
2Y	1989m2– 2022m10	1980m2–2022m10	1991m12–2022m10
3Y	1986m10–2022m10	1987m11–2022m10	1993m11–2022m10
4Y	2000m2–2022m10	2001m7–2022m10	2009m6–2022m10
5Y	1989m2–2022m10	1986m2–2022m10	1994m10–2022m10
6Y	1997m1–2022m10	2001m7–2022m10	not covered
7Y	1998m2–2022m10	1988m11–2022m10	1995m12–2022m10
8Y	1998m10–2022m10	2001m7–2022m10	not covered
9Y	2000m2–2022m10	2001m7–2022m10	not covered
10Y	1986m2–2022m10	1960m1–2022m10	1996m11–2022m10

Note. country names correspond to those used by LSEG.

Table 12: Detailed Coverage of LSEG Data for Bond Yields

Horizon	Italy	Japan	Netherlands
1Y	2009m8–2022m10	1995m1–2022m10	1995m1–2022m10
2Y	1989m7–2022m10	1979m4–2022m10	1993m2–2022m10
3Y	1998m9–2022m10	1986m2–2022m10	1994m9–2022m10
4Y	1988m11–2022m10	2000m10–2022m10	2001m12–2022m10
5Y	1988m12–2022m10	1986m1–2022m10	1994m2–2022m10
6Y	1995m4–2022m10	2000m11–2022m10	2001m12–2022m10
7Y	1990m9–2022m10	1995m3–2022m10	1986m1–2022m10
8Y	2001m7–2022m10	2000m11–2022m10	2001m12–2022m10
9Y	2001m7–2022m10	2000m11–2022m10	2001m12–2022m10
10Y	1991m3–2022m10	1966m10–2022m10	1987m2–2022m10
Horizon	New Zealand	Norway	Singapore
1Y	1992m7–2022m10	1995m3–2022m10	1997m5–2022m10
2Y	1985m4–2022m10	1994m3–2022m10	1998m2–2022m10
3Y	1992m7–2022m10	1996m4–2022m10	not covered
4Y	not covered	not covered	not covered
5Y	1985m4–2022m10	1992m11–2022m10	1990m7–2022m10
6Y	not covered	1995m7–2022m10	not covered
7Y	not covered	not covered	1998m2–2022m10
8Y	1994m8–2022m10	not covered	not covered
9Y	not covered	not covered	not covered
10Y	1985m4–2022m10	1987m2–2022m10	1998m7–2022m10

Note. country names correspond to those used by LSEG.

Table 13: Detailed Coverage of LSEG Data for Bond Yields

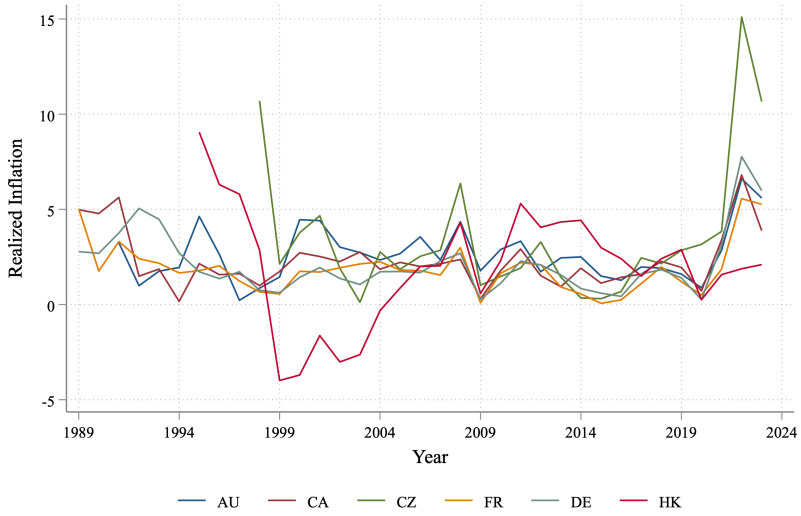
Horizon	S. Korea	Spain	Sweden
1Y	1997m12–2022m10	2010m11–2022m10	1984m1–2022m10
2Y	2002m3–2022m10	1998m7–2022m10	1986m5–2022m10
3Y	1997m12–2022m10	1991m2–2022m10	1988m2–2022m10
4Y	2002m3–2022m10	1992m5–2022m10	1993m11–2022m10
5Y	1997m12–2022m10	1988m6–2022m10	1985m2–2022m10
6Y	not covered	2001m12–2022m10	2005m10–2022m10
7Y	not covered	1991m8–2022m10	1987m2–2022m10
8Y	not covered	2001m12–2022m10	1998m5–2022m10
9Y	not covered	1992m5–2022m10	1993m2–2022m10
10Y	2000m11–2022m10	1991m6–2022m10	1987m2–2022m10
Horizon	Switzerland	Taiwan	US
1Y	1996m2–2022m10	not covered	1962m2–2022m10
2Y	1995m2–2022m10	2002m6–2022m10	1976m6–2022m10
3Y	1996m2–2022m10	not covered	1962m2–2022m10
4Y	1995m1–2022m10	not covered	2000m11–2022m10
5Y	1994m2–2022m10	1999m5–2022m10	1962m2–2022m10
6Y	1995m1–2022m10	not covered	2000m11–2022m10
7Y	1995m1–2022m10	not covered	2009m6–2022m10
8Y	1995m1–2022m10	not covered	2000m11–2022m10
9Y	1995m1–2022m10	not covered	2000m11–2022m10
10Y	1994m2 -2022m10	1999m5–2022m10	1962m2–2022m10

Note. country names correspond to those used by LSEG.

Table 14: Composition of samples across different maturities

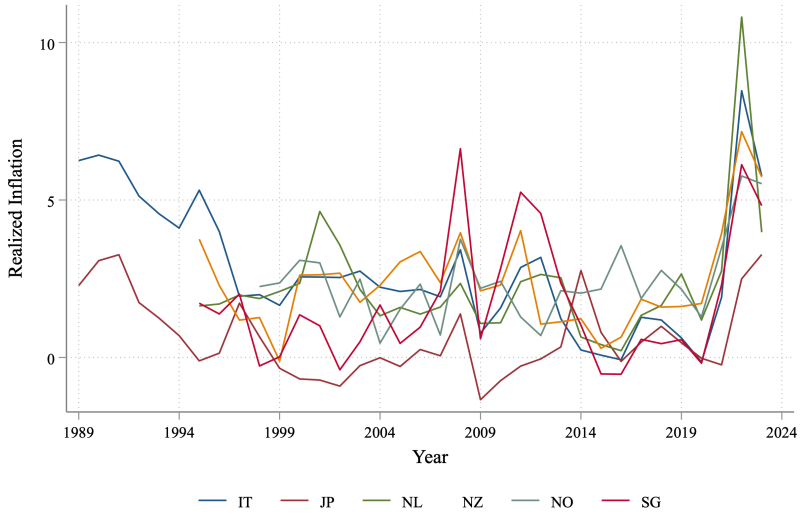
Country	**Samples**					
	Cumulative	By Maturity				
		H:0Y and H:1Y	H:2Y	H:3Y	H:5Y	H:(6-10)Y
Australia	✓	✓	✓		✓	✓
Canada	✓	✓	✓	✓	✓	✓
Czech Republic	✓		✓	✓	✓	
France	✓	✓	✓	✓	✓	✓
Germany	✓	✓	✓	✓	✓	✓
Hong Kong	✓	✓	✓	✓	✓	✓
Italy	✓		✓	✓	✓	✓
Japan	✓	✓	✓	✓	✓	✓
Netherlands	✓	✓	✓	✓	✓	✓
New Zealand	✓	✓	✓	✓	✓	✓
Norway	✓	✓	✓	✓	✓	✓
Singapore	✓	✓	✓		✓	✓
South Korea	✓	✓		✓	✓	
Spain	✓		✓	✓	✓	✓
Sweden	✓	✓	✓	✓	✓	✓
Switzerland	✓	✓	✓	✓	✓	✓
Taiwan	✓				✓	✓
United States	✓	✓	✓	✓	✓	✓

Figure 4: Realized Inflation over the sample



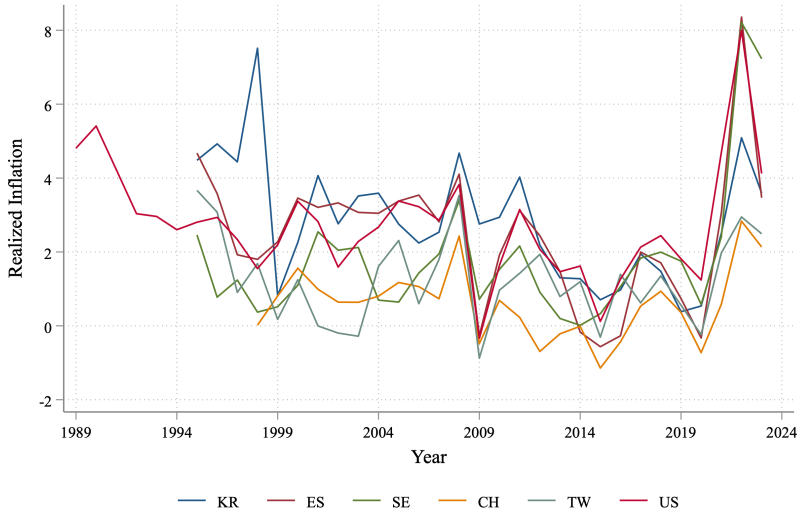
Note. the figure displays the time series for annual realized inflation for 6 out of 18 countries in our sample for which inflation forecasts are available. Sample varies according to Table 8. The identifier for each series corresponds to the ISO 3166-2 country abbreviation.

Figure 5: Realized Inflation over the sample



Note. the figure displays the time series for annual realized inflation for 6 out of 18 countries in our sample for which inflation forecasts are available. Sample varies according to Table 8. The identifier for each series corresponds to the ISO 3166-2 country abbreviation.

Figure 6: Realized Inflation over the sample



Note. the figure displays the time series for annual realized inflation for 6 out of 18 countries in our sample for which inflation forecasts are available. Sample varies according to Table 8. The identifier for each series corresponds to the ISO 3166-2 country abbreviation.

B Additional Tables and Figures

Table 15: Forecast errors on inflation forecast revision and lagged inflation forecast - Annual Frequency

	(1)	(2)	(3)	(4)	(5)	(6)
	$FE_{c,0}$	$FE_{c,1}$	$FE_{c,2}$	$FE_{c,3}$	$FE_{c,5}$	$FE_{c,6-10}$
$\mathbb{F}_t(\pi_{t+hY}) - \mathbb{F}_{t-4}(\pi_{t+hY})$	0.324	0.481**	-0.434	-0.635	-1.265***	-1.115***
	(0.199)	(0.232)	(0.356)	(0.378)	(0.313)	(0.137)
$\mathbb{F}_{t-4}(\pi_{t+hY})$	-0.091	-0.426*	-0.923**	-1.051**	-1.588***	-1.015***
	(0.055)	(0.235)	(0.421)	(0.405)	(0.359)	(0.120)
Country fixed effects	✓	✓	✓	✓	✓	✓
N	406	405	445	400	437	314
Adj. R^2	0.137	0.092	0.091	0.114	0.165	0.389

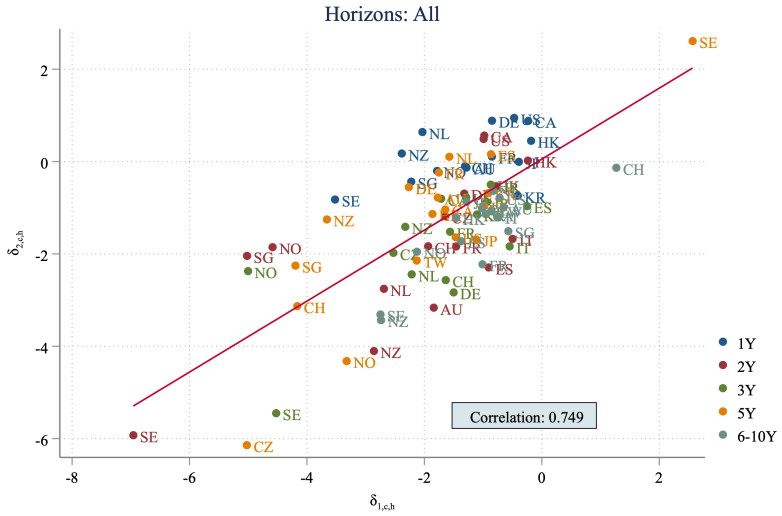
Note. We predict the inflation consensus forecast error from consensus revision (recent news) and past inflation state. Each column reports the country pooled regression at horizon $h = 0, ..., 6 - 10$: the dependent variable is the inflation consensus forecast error $\pi_{c,t+hY} - \mathbb{F}_t\left(\pi_{c,t+hY}\right)$ over horizon h and country c. The consensus revision is $\mathbb{F}_t\left(\pi_{c,t+hY}\right) - \mathbb{F}_{t-4}\left(\pi_{c,t+hY}\right)$, while past inflation state is $\mathbb{F}_{t-4}\left(\pi_{c,t+hY}\right)$. We control for country fixed effects. The sample ranges from 1989 to 2022, Table 14 reports the countries available at each maturity. Standard errors in parenthesis are Driscoll-Kraay. *** **, and * indicate significance at the 1, 5, and 10 percent levels.

Table 16: Inflation forecast error on inflation forecast revision from inflation forecast revision and lagged inflation forecast

	(1)
	$\pi_{t+4} - \mathbb{F}_t^{SPF}(\pi_{t,t+4})$
$\mathbb{F}_t^{SPF}(\pi_{t+4}) - \mathbb{F}_{t-1}^{SPF}(\pi_{t+3})$	0.017
	(0.407)
$\mathbb{F}_{t-1}^{SPF}(\pi_{t+3})$	-0.463***
	(0.103)
α	1.304***
	(0.322)
N	168
Adj. R^2	0.111

Notes. We predict the inflation consensus forecast error from consensus revision (recent news) and past inflation state. Column (1) reports the regression for the US at horizon $h = 4$, where h here denote quarters: the dependent variable is the CPI inflation SPF forecast error $\pi_{t+4} - \mathbb{F}_t\left(\pi_{t+4}\right)$. The consensus revision is $\mathbb{F}_t^{SPF}\left(\pi_{t+4}\right) - \mathbb{F}_{t-1}^{SPF}\left(\pi_{t+3}\right)$, while past inflation state is $\mathbb{F}_{t-1}^{SPF}\left(\pi_{t+3}\right)$. The sample ranges from 1981Q4 to 2023Q3, coverage varies according to Tables 8 and 10. Standard errors in parenthesis. *** **, and * indicate significance at the 1, 5, and 10 percent levels.

Figure 7: Scatterplot of simulated and actual inflation forecast errors' regressions - Table 2's results



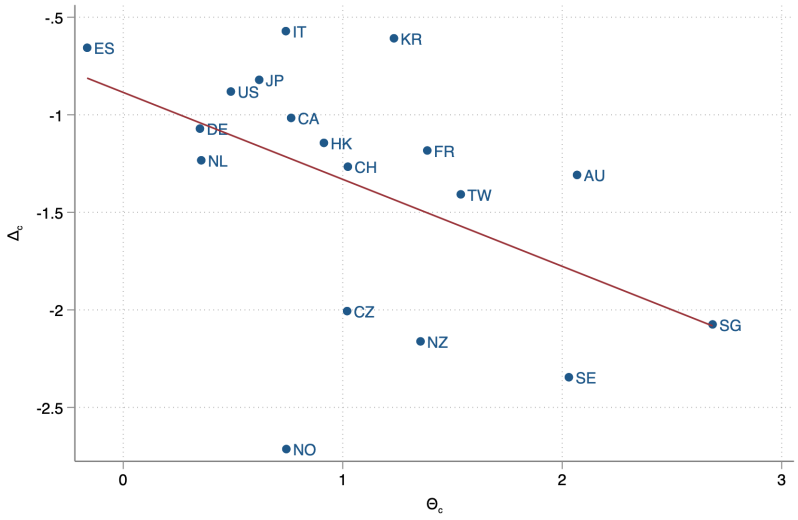
Notes. The figure reports the coefficients from 3, plotting $\delta_{2,c,h}$ (y-axis) on $\delta_{1,c,h}$ (x-axis) for each country and horizon h=0,1,2,3,5,6-10. For each horizon, the sample varies according to Table 14. The scatterplot visualizes the correlation between volatility and persistence across countries and horizons. The line indicates the fit between the two coefficients. Correlation is reported in the lower-right corner of the graph. The identifier for each data point corresponds to the ISO 3166-2 country abbreviation.

Table 17: Predicting ex-post real rates using model-implied inflation forecasts

	(1)	(2)	(3)	(4)	(5)
	$\tilde{r}^{(1)}_{c,t}$	$\tilde{r}^{(2)}_{c,t}$	$\tilde{r}^{(3)}_{c,t}$	$\tilde{r}^{(5)}_{c,t}$	$\tilde{r}^{(6-10)}_{c,t}$
$\mathbb{F}_t(\overline{\pi_{c,t+1,h}})$	-0.058	0.681	0.771**	0.922**	0.609***
	(0.269)	(0.410)	(0.359)	(0.343)	(0.193)
Country fixed effects	✓	✓	✓	✓	✓
N	376	443	395	439	109
Adj. R^2	0.041	0.140	0.184	0.241	0.343

Note. We study the association between ex-post real rates to maturity h and the consensus forecast for the average inflation over h years Columns (1) to (5). Each column reports the country pooled regression at horizon $h = 1, ..., 6-10$. Real ex-post rates $\tilde{r}^{(h)}_{c,t,t+h}$ over horizon h and country c are defined as $i^{(h)}_{c,t} - \left(\overline{\pi_{c,t+1,h}}\right)$. The consensus forecast for the average inflation over h years for country c, $\mathbb{F}_t\left(\overline{\pi_{c,t+1,h}}\right)$, is computed as: $\frac{1}{h}\sum_{h'\geq 1}\mathbb{F}_t\left(\pi_{c,t+h'Y}\right)$. We control for country fixed effects. The sample ranges from 1989 to 2022, Table 14 reports the countries available at each maturity. Standard errors in parenthesis are Driscoll-Kraay. *** **, and * indicate significance at the 1, 5, and 10 percent levels.

Figure 8: Country-Level estimations of Δ_c and Θ_c



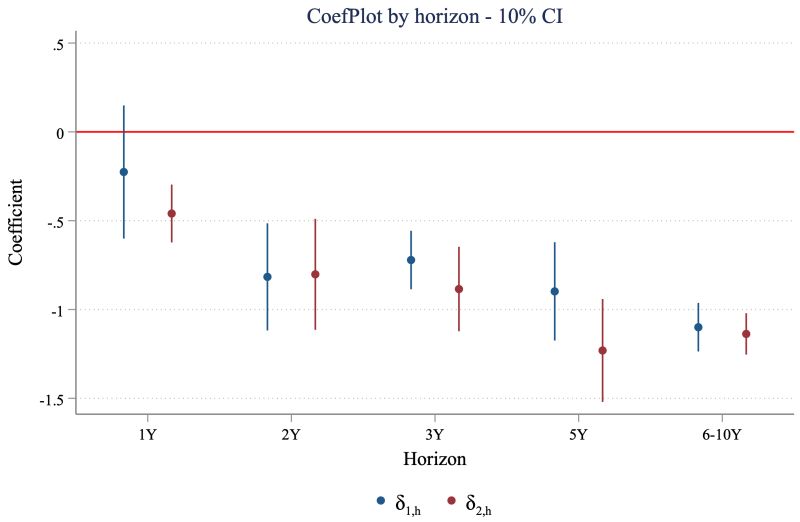
Notes. Starting from Equation 2, we compute for each country $\Delta_c = \frac{\sum_{s=1}^{6-10} \delta_{c,s}}{n_c}$, where n_c represents the number of terms in the numerator, varying across countries due to differences in data availability (horizon 4 is excluded from this calculation). A similar procedure is applied to calculate the average coefficients of inflation expectations shown in Panel B of Table 4 (defined as Θ_c). The scatterplot reports the country-level estimations of both Δ_c and Θ_c. The red line refers to the correlation between the two average coefficients across countries. The identifier for each data point corresponds to the ISO 3166-2 country abbreviation.

Table 18: Forecast errors on inflation forecast revision and lagged inflation forecast - Simulated Dataset

	(1)	(2)	(3)	(4)	(5)
	$FE^m_{c,1}$	$FE^m_{c,2}$	$FE^m_{c,3}$	$FE^m_{c,5}$	$FE^m_{c,6-10}$
$\mathbb{F}^m_t(\pi_{t+hY}) - \mathbb{F}^m_{t-4}(\pi_{t+hY})$	-0.226	-0.816***	-0.721***	-0.898***	-1.100***
	(0.221)	(0.178)	(0.097)	(0.163)	(0.080)
$\mathbb{F}^m_{t-4}(\pi_{t+hY})$	-0.460***	-0.802***	-0.885***	-1.231***	-1.137***
	(0.096)	(0.184)	(0.140)	(0.170)	(0.068)
Country fixed effects	✓	✓	✓	✓	✓
N	406	446	401	438	321
Adj. R^2	0.088	0.213	0.270	0.350	0.768

Note. We predict the inflation consensus forecast error from consensus revision (recent news) and past inflation state. Each column reports the country pooled regression at horizon $h = 1, ..., 6-10$: the dependent variable is the inflation consensus forecast error $\pi_{c,t+hY} - \mathbb{F}^m_t(\pi_{c,t+hY})$ over horizon h and country c. The consensus revision is $\mathbb{F}^m_t(\pi_{c,t+hY}) - \mathbb{F}^m_{t-4}(\pi_{c,t+hY})$, while past inflation state is $\mathbb{F}^m_{t-4}(\pi_{c,t+hY})$. We control for country fixed effects. The sample ranges from 1989 to 2022, Table 14 reports the countries available at each maturity. Standard errors in parenthesis are Driscoll-Kraay. *** **, and * indicate significance at the 1, 5, and 10 percent levels.

Figure 9: Coefplot by horizon of Table 18's results



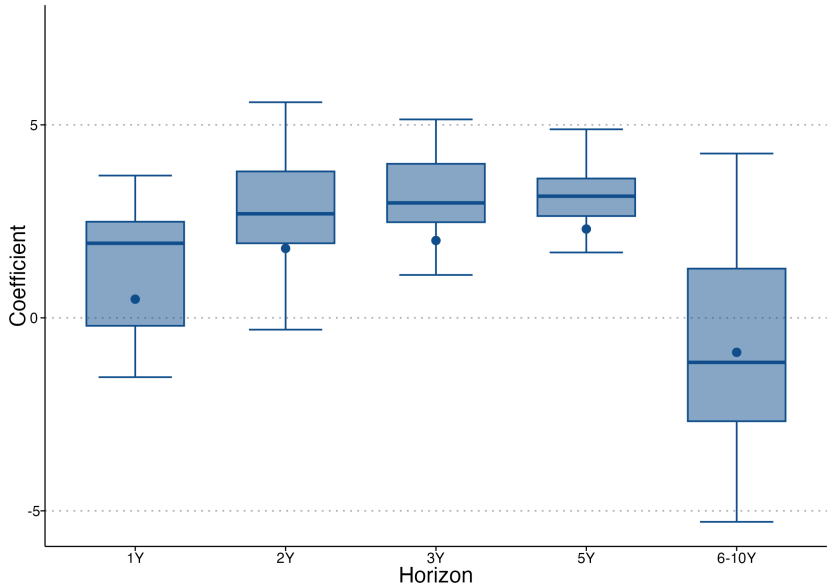
Note. the figure displays the point estimates and their 90% confidence intervals (vertical bars) for each variable across horizons. Each horizon is a different regression. Country fixed effects are included. Sample varies with the horizons according to Table 14. and regressions are performed on the simulated dataset. Standard errors are Driscoll-Kraay.

Table 19: Predicting the consensus forecast error using current inflation state - Annual Frequency

	(1)	(2)	(3)	(4)	(5)
	$FE_{c,1}$	$FE_{c,2}$	$FE_{c,3}$	$FE_{c,5}$	$FE_{c,6-10}$
$\mathbb{F}_t\left(\pi_{t+hY}\right)$	-0.271*	-0.763**	-0.937***	-1.549***	-1.211***
	(0.151)	(0.286)	(0.275)	(0.297)	(0.123)
Country fixed effects	✓	✓	✓	✓	✓
N	420	462	416	462	331
Adj. R^2	0.036	0.117	0.154	0.268	0.545

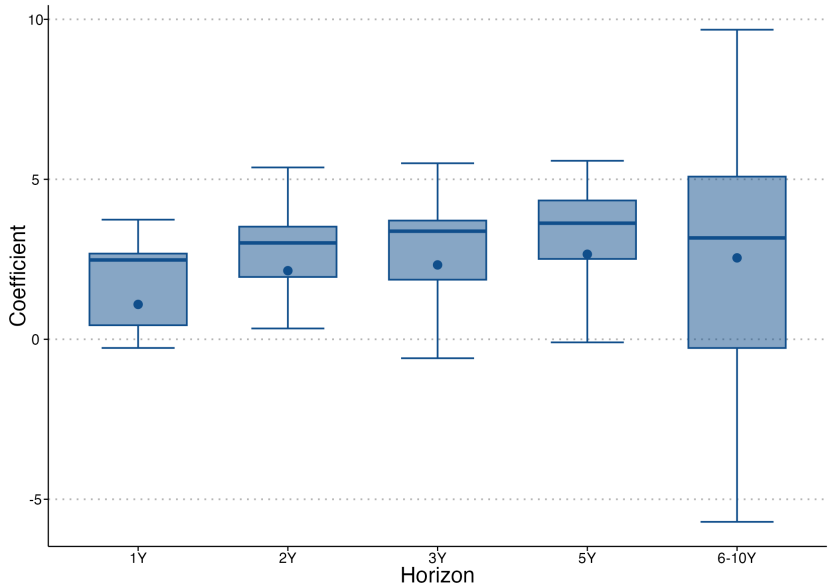
Note. We predict inflation consensus forecast error from the current inflation state. Each column reports the country pooled regression at horizon $h = 1, ..., 6-10$: the dependent variable is the inflation consensus forecast error $\pi_{c,t+hY} - \mathbb{F}_t\left(\pi_{c,t+hY}\right)$ over horizon h and country c. The consensus forecast for country c and horizon h is $\mathbb{F}_t\left(\pi_{c,t+hY}\right)$. We control for country fixed effects. The sample ranges from 1989 to 2022, Table 14 reports the countries available at each maturity. Standard errors in parenthesis are Driscoll-Kraay. *** **, and * indicate significance at the 1, 5, and 10 percent levels. Sample varies with the horizons according to Table 14.

Figure 10: Boxplots of Table 4 Panel A's results



Notes. The figure displays the boxplots and the pooled estimates from Table 4's Panel A for each horizon h=1,2,3,5,6-10. For each horizon, the sample varies according to Table 14. The boxplot illustrates the distribution of the country-coefficient. The box represents the interquartile (IQR) range, with the median indicated by the line inside the box. The whiskers extend to 1.5 times the IQR above the upper quartile and below the lower quartile. Points beyond the whiskers are considered outliers and are removed from the boxplots. The plot displays also the pooled estimates (dots) for each horizon from Table 4's Panel A.

Figure 11: Boxplots of Table 3 Panel A's results



Notes. The figure displays the boxplots and the pooled estimates from Table **??** for each horizon h=1,2,3,5,6-10. For each horizon, the sample varies according to Table 14. The boxplot illustrates the distribution of the country-coefficient. The box represents the interquartile (IQR) range, with the median indicated by the line inside the box. The whiskers extend to 1.5 times the IQR above the upper quartile and below the lower quartile. Points beyond the whiskers are considered outliers and are removed from the boxplots. The plot displays also the pooled estimates (dots) for each horizon from Table 3's Panel A.

Table 20: Cumulative forecast errors on cumulative current inflation forecasts, yield curve's level and yield curve's slope - 1st Stage

	(1)	(2)	(3)	(4)
	$h=2$	$h=3$	$h=5$	$h=6-10$
	$\overline{FE}_{c,2}$	$\overline{FE}_{c,2}$	$\overline{FE}_{c,2}$	$\overline{FE}_{c,2}$
$\mathbb{F}_t\left(\overline{\pi_{c,t+1,2}}\right)$	-0.632***	-0.542***	-0.844***	-1.359***
	(0.194)	(0.190)	(0.236)	(0.440)
$i_{c,t}^{(1)}$	-0.136	-0.133	-0.008	0.021
	(0.084)	(0.081)	(0.084)	(0.089)
$i_{c,t}^{(10)} - i_t^{(1)}$	-0.422*	-0.386*	-0.423	-0.593
	(0.223)	(0.222)	(0.284)	(0.365)
Country fixed effects	✓	✓	✓	✓
N	941	871	713	467
Adj. R^2	0.221	0.210	0.172	0.254

Note. We predict the consensus forecast for the average inflation over h years. Each column reports the country pooled regression at horizon. Horizons on top of each column indicate the horizon of the excess return for which the regression is instrumenting. The dependent variable is the inflation consensus cumulative forecast error $\overline{\pi_{t+1,2}} - \mathbb{F}_t\left(\overline{\pi_{t+1,2}}\right)$ for country c at horizon 2. The consensus forecast for the average inflation over 2 years for country c, $\mathbb{F}_t\left(\overline{\pi_{c,t+1,2}}\right)$, is computed as: $\frac{1}{2}\sum_{h'\geq 1}\mathbb{F}_t\left(\pi_{c,t+h'}\right)$. The level and slope for country c and maturity h are defined respectively as $i_{c,t}^{(1)}$ and $i_{c,t}^{(10)} - {}_{c,t}^{(1)}$. The sample ranges from 1989Q1 to 2020Q4, Table 14 reports the countries available at each maturity. Standard errors in parenthesis are Driscoll-Kraay. *** **, and * indicate significance at the 1, 5, and 10 percent levels.

Table 21: Predicting realized excess returns (smoothed over three years) using consensus inflation forecasts

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	$\overline{rx}^{(2)}_{c,t+1,t}$	$\overline{rx}^{(3)}_{c,t+1,t}$	$\overline{rx}^{(5)}_{c,t+1,t}$	$\overline{rx}^{(6-10)}_{c,t+1,t}$	$\overline{rx}^{(2)}_{c,t+1,t}$	$\overline{rx}^{(3)}_{c,t+1,t}$	$\overline{rx}^{(5)}_{c,t+1,t}$	$\overline{rx}^{(6-10)}_{c,t+1,t}$
$\mathbb{F}_t\left(\overline{\pi_{c,t+1,h}}\right)$	0.193**	0.268**	2.192***	3.401**	0.026	0.273**	0.557*	2.214**
	(0.077)	(0.119)	(0.604)	(1.399)	(0.079)	(0.129)	(0.313)	(0.851)
$i^{(1)}_{c,t}$					0.261***	0.133*	0.813***	0.785***
					(0.028)	(0.072)	(0.109)	(0.269)
$i^{(10)}_{c,t} - i^{(1)}_{c,t}$					0.304***	0.198	1.233***	2.089***
					(0.091)	(0.142)	(0.294)	(0.447)
Country fixed effects	✓	✓	✓	✓	✓	✓	✓	✓
N	880	813	656	426	877	811	656	426
Adj. R^2	0.154	0.314	0.247	0.161	0.446	0.338	0.581	0.511

Note. We predict one-year holding period realized bond excess returns with maturity h from consensus forecast of the average inflation over h years (Columns (1) to (4)). Additionally, we control for level and slope of nominal yields (Columns (5) to (8)). Each column reports the country pooled regression at horizon $h = 2, 3, 5, 6-10$: the dependent variable is the smoothed one-year holding period excess returns over horizon h and country c, defined as $\overline{rx}^{(h)}_{c,t,t+1} = \frac{rx^{(h)}_{c,t,t+1} + rx^{(h)}_{c,t+1,t+2} + rx^{(h)}_{c,t+2,t+3}}{3}$. The raw one-year holding period excess returns $rx^{(h)}_{c,t,t+1}$ is defined as $i^{(h)}_{c,t} + (h-1)\cdot\left(i^{(h)}_{c,t} - i^{(h-1)}_{c,t+1}\right) - i^{(1)}_{c,t}$. The level and slope for country c and maturity h are defined respectively as $i^{(1)}_{c,t}$ and $i^{(10)}_{c,t} - i^{(1)}_{c,t}$. We control for country fixed effects. The sample ranges from 1989Q4 to 2019Q4, Table 14 reports the countries available at each maturity. Standard errors in parenthesis are Driscoll-Kraay. *** **, and * indicate significance at the 1, 5, and 10 percent levels.

Table 22: Predicting realized excess returns (smoothed over two years) using consensus inflation forecasts and interest rates forecasts

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	$\overline{rx}^{(2)}_{c,t,t+1}$	$\overline{rx}^{(3)}_{c,t,t+1}$	$\overline{rx}^{(5)}_{c,t,t+1}$	$\overline{rx}^{(6-10)}_{c,t,t+1}$	$\overline{rx}^{(2)}_{c,t,t+1}$	$\overline{rx}^{(3)}_{c,t,t+1}$	$\overline{rx}^{(5)}_{c,t,t+1}$	$\overline{rx}^{(6-10)}_{c,t,t+1}$
$\mathbb{F}_t\left(\overline{\pi_{c,t+1,h}}\right)$	0.307***	0.429**	3.059***	5.402**	0.215**	0.987***	1.758***	3.134***
	(0.103)	(0.165)	(0.916)	(2.311)	(0.085)	(0.229)	(0.527)	(1.134)
$i^{(1)}_{c,t}$					-0.040	-0.149	-0.296	-1.197***
					(0.083)	(0.153)	(0.213)	(0.399)
$\mathbb{F}_t[i^{(10)}_{c,t+1}]$					0.302***	0.203	1.252***	2.079***
					(0.079)	(0.128)	(0.249)	(0.453)
	✓	✓	✓	✓	✓	✓	✓	
N	944	873	712	466	845	789	651	466
Adj. R^2	0.138	0.267	0.211	0.121	0.379	0.313	0.519	0.388

Note. We predict one-year holding period realized bond excess returns with maturity h from consensus forecast of the average inflation over h years (Columns (1) to (4)). Additionally, we control for level of nominal yields and long-term nominal yields forecasts (Columns (5) to (8)). Each column reports the country pooled regression at horizon $h = 2, 3, 5, 6-10$: the dependent variable is the smoothed one-year holding period excess returns over horizon h and country c, defined as $\overline{rx}^{(h)}_{c,t,t+1} = \dfrac{rx^{(h)}_{c,t,t+1} + rx^{(h)}_{c,t+1,t+2}}{2}$. The raw one-year holding period excess returns $rx^{(h)}_{c,t,t+1}$ is defined as $i^{(h)}_{c,t} + (h-1)\cdot\left(i^{(h)}_{c,t} - i^{(h-1)}_{c,t+1}\right) - i^{(1)}_{c,t}$. The level and slope for country c and maturity h are defined respectively as $i^{(1)}_{c,t}$ and $i^{(10)}_{c,t} - {}^{(1)}_{c,t}$. We control for country fixed effects. The sample ranges from 1989Q4 to 2020Q4, Table 14 reports the countries available at each maturity. Standard errors in parenthesis are Driscoll-Kraay. *** **, and * indicate significance at the 1, 5, and 10 percent levels.

Table 23: Bond excess returns (smoothed over two years) on model-implied current inflation forecasts

	(1)	(2)	(3)	(4)
	$\overline{rx}^{(2)}_{c,t,t+1}$	$\overline{rx}^{(3)}_{c,t,t+1}$	$\overline{rx}^{(5)}_{c,t,t+1}$	$\overline{rx}^{(6-10)}_{c,t,t+1}$
$\mathbb{F}^m_t\left(\overline{\pi_{c,t+1,h}}\right)$	0.095	0.017	1.806***	2.674**
	(0.088)	(0.123)	(0.487)	(1.036)
Country fixed effects	✓	✓	✓	✓
N	372	343	269	172
Adj. R^2	0.070	0.141	0.229	0.176

Note. We predict one-year holding period realized bond excess returns with maturity h from model-implied forecast of the average inflation over h years. Each column reports the country pooled regression at horizon $h = 2, 3, 5, 6-10$: the dependent variable is the smoothed one-year holding period excess returns over horizon h and country c, defined as $\overline{rx}^{(h)}_{c,t,t+1} = \frac{rx^{(h)}_{c,t,t+1} + rx^{(h)}_{c,t+1,t+2}}{2}$. The raw one-year holding period excess returns $rx^{(h)}_{c,t,t+1}$ is defined as $i^{(h)}_{c,t} + (h-1)\cdot\left(i^{(h)}_{c,t} - i^{(h-1)}_{c,t+1}\right) - i^{(1)}_{c,t}$. We control for country fixed effects. The sample ranges from 1989 to 2020, Table 14 reports the countries available at each maturity. Standard errors in parenthesis are Driscoll-Kraay. *** **, and * indicate significance at the 1, 5, and 10 percent levels.

Table 24: Bond excess returns (smoothed over three years) on current forecasts for average expected inflation over the period $t, t+h$ and interest rate forecasts

	(1)	(2)	(3)	(4)
	$\overline{rx}^{(2)}_{c,t,t+1}$	$\overline{rx}^{(3)}_{c,t,t+1}$	$\overline{rx}^{(5)}_{c,t,t+1}$	$\overline{rx}^{(6-10)}_{c,t,t+1}$
$\mathbb{F}_t\left(\overline{\pi_{c,t+1,h}}\right)$	0.095	0.777***	1.527***	2.190**
	(0.071)	(0.201)	(0.421)	(0.821)
$\mathbb{F}_t[i^{(3m)}_{c,t+1}]$	0.660***	-0.916***	0.465	-0.849**
	(0.143)	(0.264)	(0.291)	(0.322)
$\mathbb{F}_t[i^{(10)}_{c,t+1}]$	-1.040***	-0.221	-1.575**	-1.634
	(0.204)	(0.385)	(0.687)	(1.022)
$i^{(1)}_{c,t}$	0.581***	1.111***	1.798***	3.100***
	(0.145)	(0.270)	(0.412)	(0.571)
$i^{(10)}_{c,t} - i^{(1)}_{c,t}$	1.240***	0.574	2.744***	3.999***
	(0.162)	(0.360)	(0.553)	(0.735)
Country fixed effects	✓	✓	✓	✓
N	785	733	599	426
Adj. R^2	0.546	0.402	0.628	0.564

Note. We predict one-year holding period realized bond excess returns with maturity h from consensus forecast of the average inflation over h years. Additionally, we control for the consensus forecast of the 3-month and 10-year interest rates, and for the level and slope of nominal yields. Each column reports the country pooled regression at horizon $h = 2, 3, 5, 6-10$: the dependent variable is the smoothed one-year holding period excess returns over horizon h and country c, defined as $\overline{rx}^{(h)}_{c,t,t+1} = \dfrac{rx^{(h)}_{c,t,t+1} + rx^{(h)}_{c,t+1,t+2} + rx^{(h)}_{c,t+2,t+3}}{3}$. The raw one-year holding period excess returns $rx^{(h)}_{c,t,t+1}$ is defined as $i^{(h)}_{c,t} + (h-1)\cdot\left(i^{(h)}_{c,t} - i^{(h-1)}_{c,t+1}\right) - i^{(1)}_{c,t}$. The consensus forecast of the 3-month and 10-year interest rates are $\mathbb{F}_t\left[i^{(3m)}_{c,t+1}\right]$ and $\mathbb{F}_t\left[i^{(10)}_{c,t+1}\right]$, respectively. The level and slope for country c and maturity h are defined respectively as $i^{(1)}_{c,t}$ and $i^{(10)}_{c,t} - {}^{(1)}_{c,t}$. We control for country fixed effects. The sample ranges from 1989Q4 to 2019Q4, Table 14 reports the countries available at each maturity. Standard errors in parenthesis are Driscoll-Kraay. *** **, and * indicate significance at the 1, 5, and 10 percent levels.

Table 25: Bond excess returns on current forecasts for average expected inflation over the period $t, t+h$, interest rate forecasts, and required real rates

	(1)	(2)	(3)	(4)
	$\overline{rx}^{(2)}_{c,t,t+1}$	$\overline{rx}^{(3)}_{c,t,t+1}$	$\overline{rx}^{(5)}_{c,t,t+1}$	$\overline{rx}^{(6-10)}_{c,t,t+1}$
$\mathbb{F}_{\mathrm{t}}\left(\overline{\pi_{\mathrm{c,t+1,h}}}\right)$	0.281***	0.352***	1.964***	4.282***
	(0.071)	(0.123)	(0.547)	(1.586)
$i^{(h)}_{\mathrm{c,t}} - \mathbb{F}_{\mathrm{t}}\left(\overline{\pi_{\mathrm{c,t+1,h}}}\right)$	0.291***	0.319***	0.937***	1.091***
	(0.035)	(0.086)	(0.131)	(0.374)
Country fixed effects	✓	✓	✓	✓
N	845	789	651	466
Adj. R^2	0.399	0.390	0.513	0.340

Note. We predict one-year holding period realized bond excess returns with maturity h from consensus forecast of the average inflation over h years. Additionally, we control for ex-ante real rates. Each column reports the country pooled regression at horizon $h = 2, 3, 5, 6-10$: the dependent variable is the smoothed one-year holding period excess returns over horizon h and country c, defined as $\overline{rx}^{(h)}_{c,t,t+1} = \dfrac{rx^{(h)}_{c,t,t+1} + rx^{(h)}_{c,t+1,t+2}}{2}$. The raw one-year holding period excess returns $rx^{(h)}_{c,t,t+1}$ is defined as $i^{(h)}_{c,t} + (h-1)\cdot\left(i^{(h)}_{c,t} - i^{(h-1)}_{c,t+1}\right) - i^{(1)}_{c,t}$. Ex-ante required real rates are defined as $i^{(h)}_{\mathrm{c,t}} - \mathbb{F}_{\mathrm{t}}\left(\overline{\pi_{\mathrm{c,t+1,h}}}\right)$. We control for country fixed effects. The sample ranges from 1989Q4 to 2020Q4, Table 14 reports the countries available at each maturity. Standard errors in parenthesis are Driscoll-Kraay. *** **, and * indicate significance at the 1, 5, and 10 percent levels.

Table 26: Predictability regressions by horizon

	(1)	(2)	(3)	(4)
	$FR_{c,2}$	$FR_{c,3}$	$FR_{c,5}$	$FR_{c,6-10}$
$\mathbb{F}_{\mathrm{t}}\left(\pi_{\mathrm{c,t+hY}}\right)$	-0.236**	-0.355***	-0.819***	-0.974***
	(0.090)	(0.097)	(0.131)	(0.150)
Country fixed effects	✓	✓	✓	✓
N	1107	973	1021	847
Adj. R^2	0.145	0.213	0.462	0.536

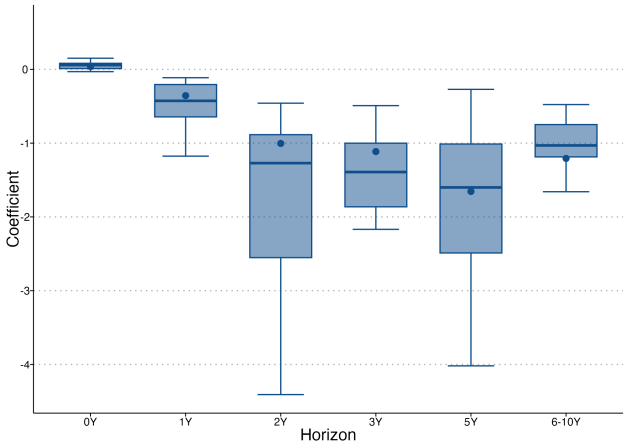
Note. We predict inflation consensus forecast revision from the current inflation state. Each column reports the country pooled regression at horizon $h = 2, 3, 5, 6-10$: the dependent variable is the inflation consensus forecast revision $\mathbb{F}_{t+h-1}\left(\pi_{c,t+hY}\right) - \mathbb{F}_t\left(\pi_{c,t+hY}\right)$ over horizon h and country c. The current inflation state is the consensus forecast for country c and horizon h $\mathbb{F}_t\left(\pi_{c,t+hY}\right)$. We control for country fixed effects. The sample ranges from 1989 to 2021, Table 14 reports the countries available at each maturity. Standard errors in parenthesis are Driscoll-Kraay. *** **, and * indicate significance at the 1, 5, and 10 percent levels.

Table 27: Ex-post real returns on current average forecast for inflation over the period $t+1, t+h$, controlling for required real rates - Annual frequency

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
	$\tilde{r}^{(1)}_{c,t,t+1}$	$\tilde{r}^{(2)}_{c,t,t+2}$	$\tilde{r}^{(3)}_{c,t,t+3}$	$\tilde{r}^{(5)}_{c,t,t+5}$	$\tilde{r}^{(6-10)}_{c,t+6,t+10}$	$\tilde{r}^{(1)}_{c,t,t+1}$	$\tilde{r}^{(2)}_{c,t,t+2}$	$\tilde{r}^{(3)}_{c,t,t+3}$	$\tilde{r}^{(5)}_{c,t,t+5}$	$\tilde{r}^{(6-10)}_{c,t,t+6,t+10}$
$\mathbb{F}_t\left(\overline{\pi_{c,t+1,h}}\right)$	0.376	1.562***	1.762***	2.115***	-0.816	0.262*	0.331**	0.446***	0.749***	0.556**
	(0.349)	(0.478)	(0.378)	(0.309)	(0.642)	(0.136)	(0.134)	(0.142)	(0.147)	(0.224)
$i^{(h)}_{c,t} - \mathbb{F}_t\left(\overline{\pi_{c,t+1,h}}\right)$						1.183***	1.146***	1.081***	1.010***	1.047***
						(0.135)	(0.119)	(0.081)	(0.066)	(0.113)
Country fixed effects	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
N	390	458	409	456	119	390	458	409	456	119
Adj. R^2	0.046	0.220	0.293	0.384	0.865	0.706	0.773	0.831	0.883	0.865

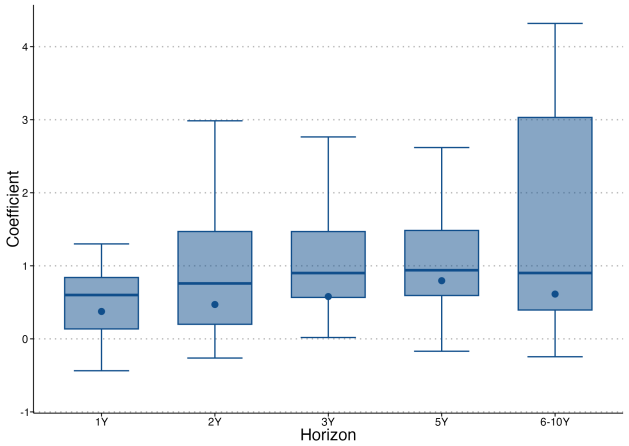
Notes. We study the association between ex-post real rates to maturity h and the consensus forecast for the average inflation over h years (Columns (1) to (5)). Additionally, we control for ex-ante real rates (Columns (6) to (10)). Each column reports the country pooled regression at horizon $h = 1, ..., 6-10$. Real ex-post rates $\tilde{r}^{(h)}_{c,t,t+h}$ over horizon h and country c are defined as $i^{(h)}_{c,t} - \left(\overline{\pi_{c,t+1,h}}\right)$. The consensus forecast for the average inflation over h years for country c, $\mathbb{F}_t\left(\overline{\pi_{c,t+1,h}}\right)$, is computed as: $\frac{1}{h}\sum_{h'\geq 1}\mathbb{F}_t\left(\pi_{c,t+h'Y}\right)$. Ex-ante required real rates are defined as $i^{(h)}_{c,t} - \mathbb{F}_t\left(\overline{\pi_{c,t+1,h}}\right)$. We control for country fixed effects. The sample ranges from 1989 to 2022, Table 14 reports the countries available at each maturity. Standard errors in parenthesis are Driscoll-Kraay. *** **, and * indicate significance at the 1, 5, and 10 percent levels.

Figure 12: Combined figure with two subfigures side-by-side on the first row and one centered subfigure on the second row.



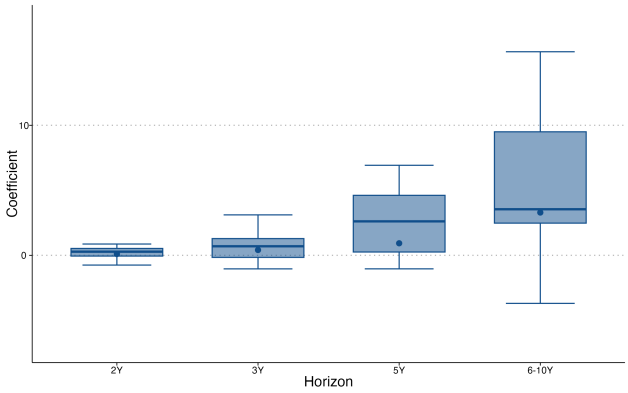
(a) Panel A: Boxplot of Table 1 Panel A's results.

Note. The figure displays the boxplots and the pooled estimates from Table 1's Panel A, for each horizon $h = 0, 1, 2, 3, 5, 6\text{-}10$. For each horizon, the sample varies according to Table 14. The boxplot illustrates the distribution of the country-coefficients. The box represents the interquartile (IQR) range, with the median indicated by the line inside the box. The whiskers extend to 1.5 times the IQR above the upper quartile and below the lower quartile. Points beyond the whiskers are considered outliers and are removed from the boxplots. The plot also displays the pooled estimates (dots) for each horizon from Table 1's Panel A.



(b) Panel B: Boxplots of Table 4 Panel B's results.

Note. The figure displays the boxplots and the pooled estimates from Table 4's Panel B for the coefficients on inflation forecasts for each horizon $h = 1, 2, 3, 5, 6\text{-}10$. For each horizon, the sample varies according to Table 14. The boxplot illustrates the distribution of the country-coefficient. The box represents the interquartile (IQR) range, with the median indicated by the line inside the box. The whiskers extend to 1.5 times the IQR above the upper quartile and below the lower quartile. Points beyond the whiskers are considered outliers and are removed from the boxplots. The plot also displays the pooled estimates (dots) of the inflation forecast's coefficient for each horizon from Table 4's Panel B.



(c) Panel C: Boxplots of Table 5 Panel B's results.

Note. The figure displays the boxplots and the pooled estimates from Table 5's Panel B for each horizon $h = 1, 2, 3, 5, 6\text{-}10$. For each horizon, the sample varies according to Table 14. The boxplot illustrates the distribution of the country-coefficient. The box represents the interquartile (IQR) range, with the median indicated by the line inside the box. The whiskers extend to 1.5 times the IQR above the upper quartile and below the lower quartile. Points beyond the whiskers are considered outliers and are removed from the boxplots. The plot also displays the pooled estimates (dots) of the coefficients for each horizon from Table 5's Panel B.

C Term Structure

$$\pi_{t+hY} = \alpha + \beta_1 \pi_t + \varepsilon_{t+hY} \tag{27}$$

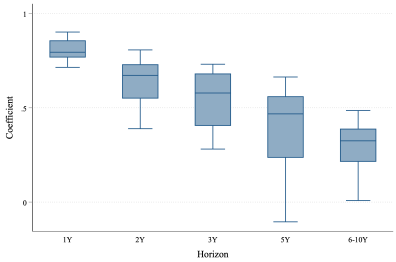
$$\mathbb{E}_t \left[\pi_{t+hY}\right] = \alpha + \beta_2 \mathbb{E}_t \left[\pi_{t+1Y}\right] + \varepsilon_{t+hY} \tag{28}$$

$$\pi_{t+hY} = \alpha + \beta_3 \mathbb{E}_t \left[\pi_{t+1Y}\right] + \varepsilon_{t+hY} \tag{29}$$

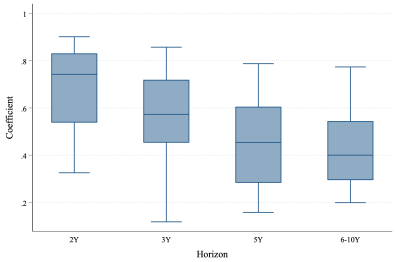
$$\pi_{t+hY} = \alpha + \beta_4 \mathbb{E}_{t-1Y} \left[\pi_{t+1Y}\right] + \varepsilon_{t+hY} \tag{30}$$

$$\mathbb{E}_t \left[\pi_{t+hY}\right] = \alpha + \beta_5 \mathbb{E}_{t-1Y} \left[\pi_{t+1Y}\right] + \varepsilon_{t+hY} \tag{31}$$

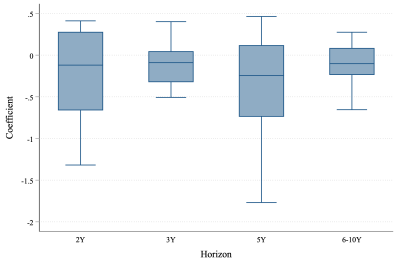
Figure 13: Boxplot of the β coefficient by horizon of the country-by-country regressions



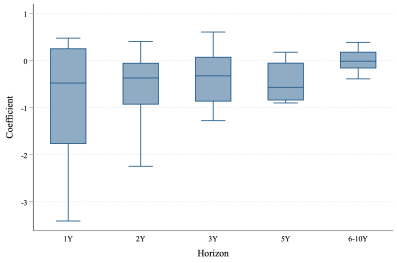
Equation (27)



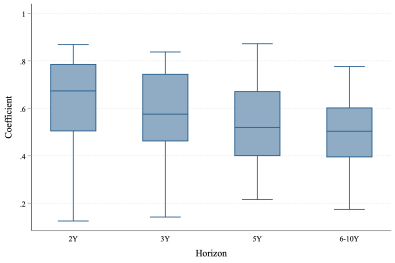
Equation (28)



Equation (29)

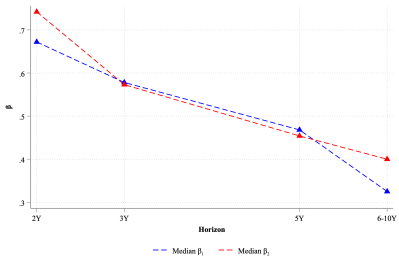


Equation (30)

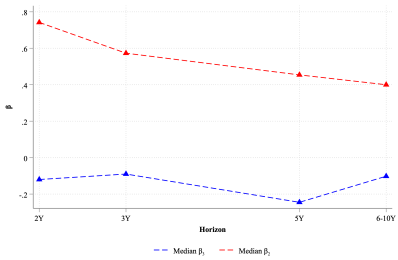


Equation (31)

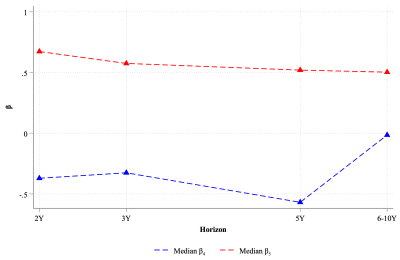
Figure 15: Plot of the median β coefficient by horizon of the country-by-country regressions



(a) Equation (27) vs Equation (28)



(b) Equation (28) vs Equation (29)



(c) Equation (30) vs Equation (31)

D Robustness

Table 28: Inflation forecast errors on current inflation forecasts and past realized inflation. Full cumulative, including UK

	Panel A					
	(1)	(2)	(3)	(4)	(5)	(6)
	$FE_{\mathrm{c,0}}$	$FE_{\mathrm{c,1}}$	$FE_{\mathrm{c,2}}$	$FE_{\mathrm{c,3}}$	$FE_{\mathrm{c,5}}$	$FE_{\mathrm{c,6-10}}$
$\mathbb{F}_{\mathrm{t}}\left(\pi_{\mathrm{c,t+hY}}\right)$	0.039	-0.406**	-0.984***	-1.201***	-1.671***	-1.146***
	(0.054)	(0.155)	(0.286)	(0.279)	(0.277)	(0.110)
Country fixed effects	✓	✓	✓	✓	✓	✓
N	1366	1366	1361	1285	1131	745
Adj. R^2	0.018	0.066	0.160	0.187	0.249	0.483
	Panel B					
	(1)	(2)	(3)	(4)	(5)	(6)
	$FE_{\mathrm{c,0}}$	$FE_{\mathrm{c,1}}$	$FE_{\mathrm{c,2}}$	$FE_{\mathrm{c,3}}$	$FE_{\mathrm{c,5}}$	$FE_{\mathrm{c,6-10}}$
$\overline{\pi}_{\mathrm{c,t}}$	-0.065	-0.265*	-0.298**	-0.340**	-0.638***	-0.428***
	(0.043)	(0.130)	(0.141)	(0.125)	(0.193)	(0.069)
Country fixed effects	✓	✓	✓	✓	✓	✓
N	554	554	534	515	477	376
Adj. R^2	0.002	0.057	0.065	0.086	0.232	0.348

Note. We predict inflation consensus forecast error from two proxies for the current inflation state. Each column reports the country pooled regression at horizon $h = 0, ..., 6 - 10$: the dependent variable is the inflation consensus forecast error $\pi_{c,t+hY} - \mathbb{F}_t\left(\pi_{c,t+hY}\right)$ over horizon h and country c. The two proxies for the current inflation state are: (i) the consensus forecast for country c and horizon h $\mathbb{F}_t\left(\pi_{c,t+hY}\right)$ (Panel A), and (ii) the average realized inflation of country c over the past two years, $\overline{\pi}_{c,t} = (\pi_{c,t-1} + \pi_{c,t-2})/2$ (Panel B). We control for country fixed effects. The sample ranges from 1989Q4 to 2022Q1. For every horizon, we include all of the countries listed in Table 14, as well as United Kingdom. Standard errors in parenthesis are Driscoll-Kraay. *** **, and * indicate significance at the 1, 5, and 10 percent levels.

Table 29: Inflation forecast error predictability from inflation forecast revision and lagged inflation forecast. Full cumulative, including UK

	(1)	(2)	(3)	(4)	(5)	(6)
	$FE_{c,0}$	$FE_{c,1}$	$FE_{c,2}$	$FE_{c,3}$	$FE_{c,5}$	$FE_{c,6-10}$
$\mathbb{F}_t(\pi_{c,t+hY}) - \mathbb{F}_{t-4}(\pi_{c,t+hY})$	0.193	0.299	-0.840*	-1.235***	-1.299***	-1.155***
	(0.122)	(0.317)	(0.425)	(0.438)	(0.272)	(0.125)
$\mathbb{F}_{t-4}(\pi_{c,t+hY})$	-0.108***	-0.612**	-1.181***	-1.427***	-1.805***	-0.952***
	(0.040)	(0.246)	(0.393)	(0.408)	(0.392)	(0.131)
Country fixed effects	✓	✓	✓	✓	✓	✓
N	1290	1286	1282	1205	1042	701
Adj. R^2	0.108	0.108	0.141	0.163	0.185	0.339

Note. We predict the inflation consensus forecast error from consensus revision (recent news) and past inflation state. Each column reports the country pooled regression at horizon $h = 0, ..., 6 - 10$: the dependent variable is the inflation consensus forecast error $\pi_{c,t+hY} - \mathbb{F}_t\left(\pi_{c,t+hY}\right)$ over horizon h and country c. The consensus revision is $\mathbb{F}_t\left(\pi_{c,t+hY}\right) - \mathbb{F}_{t-4}\left(\pi_{c,t+hY}\right)$, while past inflation state is $\mathbb{F}_{t-4}\left(\pi_{c,t+hY}\right)$. We control for country fixed effects. The sample ranges from 1989Q4 to 2022Q1. For every horizon, we include all of the countries listed in Table 14, as well as United Kingdom. Standard errors in parenthesis are Driscoll-Kraay. *** **, and * indicate significance at the 1, 5, and 10 percent levels.

Table 30: Nominal yields on current average forecast for average inflation over the period $t+1, t+h$ and forecast of nominal yields for $t+1$. Full cumulative, including UK

	Panel A				
	(1)	(2)	(3)	(4)	(5)
	$i_{c,t}^{(1)}$	$i_{c,t}^{(2)}$	$i_{c,t}^{(3)}$	$i_{c,t}^{(5)}$	$i_{c,t}^{(6-10)}$
$\mathbb{F}_t\left(\overline{\pi_{c,t+1,h}}\right)$	1.074***	2.109***	2.318***	2.637***	2.512***
	(0.161)	(0.295)	(0.268)	(0.284)	(0.518)
Country fixed effects	✓	✓	✓	✓	✓
N	1146	1326	1180	1347	652
Adj. R^2	0.357	0.488	0.526	0.519	0.291
	Panel B				
	(6)	(7)	(8)	(9)	(10)
	$i_{c,t}^{(1)}$	$i_{c,t}^{(2)}$	$i_{c,t}^{(3)}$	$i_{c,t}^{(5)}$	$i_{c,t}^{(6-10)}$
$\mathbb{F}_t\left(\overline{\pi_{c,t+1,h}}\right)$	0.049*	0.058*	0.234***	0.338***	0.134
	(0.026)	(0.031)	(0.071)	(0.094)	(0.104)
$\mathbb{F}_t[i_{c,t+1}^{(h)}]$	0.986***	1.028***	0.949***	1.017***	0.953***
	(0.017)	(0.015)	(0.021)	(0.034)	(0.031)
Country fixed effects	✓	✓	✓	✓	✓
N	1146	1309	1161	1175	646
Adj. R^2	0.957	0.973	0.939	0.963	0.947

Note. We study the association between nominal yields to maturity h and the consensus forecast for the average inflation over h years (Panel A). Additionally, we control for the consensus forecast of interest rates (Panel B). Each column reports the country pooled regression at horizon $h = 1, ..., 6-10$. The dependent variable is the nominal yield to maturity h. The consensus forecast for the average inflation over h years for country c, $\mathbb{F}_t\left(\overline{\pi_{c,t+1,h}}\right)$, is computed as: $\frac{1}{h}\sum_{h' \geq 1} \mathbb{F}_t\left(\pi_{c,t+h'Y}\right)$. To roughly match short and long maturities, Panel B controls for the expected 3 months rate for $h = 1, 2, 3$, Columns (1)-(3), and for the expected 10 year rate for $h = 5, 6-10$, Columns (4) and (5). We control for country fixed effects. The sample ranges from 1989Q4 to 2022Q1. For every horizon, we include all of the countries listed in Table 14, as well as United Kingdom. Standard errors in parenthesis are Driscoll-Kraay. *** **, and * indicate significance at the 1, 5, and 10 percent levels.

Table 31: Ex-post real returns on current average forecast for inflation over the period $t+1, t+h$, controlling for required real rates. Full cumulative, including UK

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
	$\tilde{r}^{(1)}_{c,t,t+1}$	$\tilde{r}^{(2)}_{c,t,t+2}$	$\tilde{r}^{(3)}_{c,t,t+3}$	$\tilde{r}^{(5)}_{c,t,t+5}$	$\tilde{r}^{(6-10)}_{c,t+6,t+10}$	$\tilde{r}^{(1)}_{c,t,t+1}$	$\tilde{r}^{(2)}_{c,t,t+2}$	$\tilde{r}^{(3)}_{c,t,t+3}$	$\tilde{r}^{(5)}_{c,t,t+5}$	$\tilde{r}^{(6-10)}_{c,t+6,t+10}$
$\mathbb{F}_t\left(\overline{\pi_{c,t+1,h}}\right)$	0.397	1.779***	2.025***	2.284***	-0.739*	0.308**	0.502***	0.621***	0.809***	1.037***
	(0.290)	(0.387)	(0.342)	(0.265)	(0.412)	(0.142)	(0.150)	(0.150)	(0.124)	(0.312)
$i^{(h)}_{c,t} - \mathbb{F}_t\left(\overline{\pi_{c,t+1,h}}\right)$						1.200***	1.151***	1.087***	1.026***	1.105***
						(0.136)	(0.108)	(0.079)	(0.060)	(0.103)
Country fixed effects	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
N	1146	1325	1111	1118	268	1146	1325	1111	1118	268
Adj. R^2	0.098	0.238	0.328	0.383	0.878	0.615	0.726	0.799	0.880	0.878

Note. We study the association between ex-post real rates to maturity h and the consensus forecast for the average inflation over h years (Columns (1) to (5)). Additionally, we control for ex-ante real rates (Columns (6) to (10)). Each column reports the country pooled regression at horizon $h = 1, ..., 6-10$. Real ex-post rates $\tilde{r}^{(h)}_{c,t,t+h}$ over horizon h and country c are defined as $i^{(h)}_{c,t} - \left(\overline{\pi_{c,t+1,h}}\right)$. The consensus forecast for the average inflation over h years for country c, $\mathbb{F}_t\left(\overline{\pi_{c,t+1,h}}\right)$, is computed as: $\frac{1}{h}\sum_{h'\geq 1}\mathbb{F}_t\left(\pi_{c,t+h'Y}\right)$. Ex-ante required real rates are defined as $i^{(h)}_{c,t} - \mathbb{F}_t\left(\overline{\pi_{c,t+1,h}}\right)$. We control for country fixed effects. The sample ranges from 1989Q4 to 2022Q1. For every horizon, we include all of the countries listed in Table 14, as well as United Kingdom. Standard errors in parenthesis are Driscoll-Kraay. *** **, and * indicate significance at the 1, 5, and 10 percent levels.

Table 32: Bond excess returns on current forecasts for average expected inflation over the period $t, t+h$ and interest rate forecasts. Full cumulative, including UK

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	$\overline{rx}^{(2)}_{c,t,t+1}$	$\overline{rx}^{(3)}_{c,t,t+1}$	$\overline{rx}^{(5)}_{c,t,t+1}$	$\overline{rx}^{(6-10)}_{c,t,t+1}$	$\overline{rx}^{(2)}_{c,t,t+1}$	$\overline{rx}^{(3)}_{c,t,t+1}$	$\overline{rx}^{(5)}_{c,t,t+1}$	$\overline{rx}^{(6-10)}_{c,t,t+1}$
$\mathbb{F}_t\left(\overline{\pi_{c,t+1,h}}\right)$	0.307***	0.452**	3.049***	6.089***	0.072	0.392**	0.934**	2.829***
	(0.104)	(0.181)	(0.929)	(2.113)	(0.067)	(0.154)	(0.426)	(1.026)
$i^{(1)}_{c,t}$					0.270***	0.163*	0.867***	0.864**
					(0.040)	(0.095)	(0.158)	(0.355)
$i^{(10)}_{c,t} - i^{(1)}_{c,t}$					0.371***	0.364**	1.508***	2.667***
					(0.079)	(0.150)	(0.231)	(0.515)
Country fixed effects	✓	✓	✓	✓	✓	✓	✓	✓
N	1041	964	758	549	1038	962	758	549
Adj. R^2	0.136	0.260	0.201	0.117	0.411	0.300	0.511	0.415

Note. We predict one-year holding period realized bond excess returns with maturity h from consensus forecast of the average inflation over h years (Columns (1) to (4)). Additionally, we control for level and slope of nominal yields (Columns (5) to (8)). Each column reports the country pooled regression at horizon $h = 2, 3, 5, 6-10$: the dependent variable is the smoothed one-year holding period excess returns over horizon h and country c, defined as $\overline{rx}^{(h)}_{c,t,t+1} = \dfrac{rx^{(h)}_{c,t,t+1} + rx^{(h)}_{c,t+1,t+2}}{2}$. The raw one-year holding period excess returns $rx^{(h)}_{c,t,t+1}$ is defined as $i^{(h)}_{c,t} + (h-1)\cdot\left(i^{(h)}_{c,t} - i^{(h-1)}_{c,t+1}\right) - i^{(1)}_{c,t}$. The level and slope for country c and maturity h are defined respectively as $i^{(1)}_{c,t}$ and $i^{(10)}_{c,t} - {}^{(1)}_{c,t}$.We control for country fixed effects. The sample ranges from 1989Q4 to 2022Q1. For every horizon, we include all of the countries listed in Table 14, as well as United Kingdom. Standard errors in parenthesis are Driscoll-Kraay. *** **, and * indicate significance at the 1, 5, and 10 percent levels.

E Coefficient Predictability

In what follows, we discuss two versions of a test aimed at assessing how much of the variation in the coefficients δ_h estimated from Specification (2) can be explained by our model. To do so, Table 33 reports, in the first column, the regression of the predictability coefficient, pooled across countries and horizons, on horizon-specific dummy variables. The second column includes also the model-implied coefficients, resulting in a marked increase in R^2.

Table 33: Inflation forecast error predictability coefficients from Specification (2) on horizon dummies and simulated coefficients - without country fixed effects

	(1)	(2)
	δ_h	δ_h
δ_h^m		1.479***
		(0.246)
α_h	-0.532**	0.087
	(0.251)	(0.231)
Horizon fixed effects	✓	✓
N	79	79
R^2	0.211	0.473

Notes. We study the relation between the actual and model-implied coefficients estimated for Specification (2). In Column (1) reports the country and horizon pooled regression on horizon dummies, while in Column (2) we control also for the model-implied cooefficient. Table 14 reports the countries available at each horizon $h = 1, \ldots, 6-10$. *** **, and * indicate significance at the 1, 5, and 10 percent levels.

Table 34 presents a version of the same test conducted including country fixed effects.

We performed an analogous test for the specification of Figure 2,[23] i.e.,

$$\widetilde{r}^{(h)}_{c,t,t+h} = \alpha_h + \delta_h \cdot \mathbb{F}_{\mathrm{t}} \left(\overline{\pi_{\mathrm{c,t+1,h}}} \right) + \varepsilon_{c,t,t+h} \tag{32}$$

Finally, we conducted the same test for the specification of Figure 3,[24] i.e.,

$$\overline{rx}^{(h)}_{c,t,t+1} = \alpha_h + \delta_h \cdot \mathbb{F}_{\mathrm{t}} \left(\overline{\pi_{\mathrm{c,t+1,h}}} \right) + \varepsilon_{c,t,t+h} \tag{33}$$

[23] This is the data equivalent of Equation (7)

[24] This is the data equivalent of Equation (9)

Table 34: Inflation forecast error predictability coefficients from Specification (2) on horizon dummies and simulated coefficients - including country fixed effects

	(1)	(2)
	δ_{h}	δ_{h}
δ_h^m		0.756**
		(0.305)
α_h	-0.509	-0.146
	(0.414)	(0.422)
Horizon fixed effects	✓	✓
Country fixed effects	✓	✓
N	79	79
R^2	0.612	0.650

Notes. We study the relation between the actual and model-implied coefficients estimated for Specification (2). In Column (1) reports the country and horizon pooled regression on country and horizon dummies, while in Column (2) we control also for the model-implied cooefficient. Table 14 reports the countries available at each horizon $h = 1, \ldots, 6-10$. *** **, and * indicate significance at the 1, 5, and 10 percent levels.

Table 35: Inflation forecast error predictability coefficients from Specification (32) on horizon dummies and simulated coefficients - without country fixed effects

	(1)	(2)
	δ_{h}	δ_{h}
δ_h^m		1.091***
		(0.239)
α_h	1.310**	1.440***
	(0.596)	(0.525)
Horizon fixed effects	✓	✓
N	73	73
R^2	0.229	0.412

Notes. We study the relation between the actual and model-implied coefficients estimated for Specification (32). In Column (1) reports the country and horizon pooled regression on horizon dummies, while in Column (2) we control also for the model-implied cooefficient. Table 14 reports the countries available at each horizon $h = 1, \ldots, 6-10$. *** **, and * indicate significance at the 1, 5, and 10 percent levels.

Table 36: Inflation forecast error predictability coefficients from Specification (32) on horizon dummies and simulated coefficients - including country fixed effects

	(1)	(2)
	δ_{h}	δ_{h}
δ_h^m		0.232
		(0.336)
α_h	1.659*	1.647*
	(0.858)	(0.862)
Horizon fixed effects	✓	✓
Country fixed effects	✓	✓
N	73	73
R^2	0.722	0.725

Notes. We study the relation between the actual and model-implied coefficients estimated for Specification (32). In Column (1) reports the country and horizon pooled regression on country and horizon dummies, while in Column (2) we control also for the model-implied cooefficient. Table 14 reports the countries available at each horizon $h = 1, \ldots, 6-10$. *** **, and * indicate significance at the 1, 5, and 10 percent levels.

Table 37: Inflation forecast error predictability coefficients from Specification (33) on horizon dummies and simulated coefficients - without country fixed effects

	(1)	(2)
	δ_{h}	δ_{h}
δ_h^m		1.899***
		(0.456)
α_h	1.040	-0.290
	(1.188)	(1.083)
Horizon fixed effects	✓	✓
N	55	55
R^2	0.291	0.473

Notes. We study the relation between the actual and model-implied coefficients estimated for Specification (33). In Column (1) reports the country and horizon pooled regression on horizon dummies, while in Column (2) we control also for the model-implied cooefficient. Table 14 reports the countries available at each horizon $h = 1, \ldots, 6-10$. *** **, and * indicate significance at the 1, 5, and 10 percent levels.

Table 38: Inflation forecast error predictability coefficients from Specification (33) on horizon dummies and simulated coefficients - including country fixed effects

	(1)	(2)
	δ_{h}	δ_{h}
δ_h^m		3.144***
		(0.660)
α_h	4.962*	3.399
	(2.779)	(2.208)
Horizon fixed effects	✓	✓
Country fixed effects	✓	✓
N	55	55
R^2	0.568	0.741

Notes. We study the relation between the actual and model-implied coefficients estimated for Specification (33). In Column (1) reports the country and horizon pooled regression on country and horizon dummies, while in Column (2) we control also for the model-implied cooefficient. Table 14 reports the countries available at each horizon $h = 1, \ldots, 6 - 10$. *** **, and * indicate significance at the 1, 5, and 10 percent levels.

F Comparison of Ex-Ante Real Rates

To assess the validity of the proxy we use for the ex-ante real rate, we compare it with two market-derived rates of the inflation-adjusted sovereign bonds issued by the UK and the US. Table 39 displays the correlation of these market measures with the proxy we have computed with CE data. In what follows, we provide more details on the UK and US rates.

UK. For UK data, we use the "real spot rates" published by the Bank of England. These are the rates for a "index-linked zero coupon bond" which has "its value linked to movements in a suitable price index to prevent inflation eroding its purchasing power."[25] We use the series at daily frequency, from January 1999 to June 2024, and then aggregate it at quarterly frequency taking the simple average in order to make it comparable with the data we use in the rest of the analysis. To be consistent with CE data, we take the average of the rates for the horizons 6- to 10-year ahead.

US. For US data, we use "TIPS yields" by the Federal Reserve Board following the methodology of Gürkaynak et al. (2010).[26] Among available series, we used that of of zero-coupon inflation protected treasuries. We use the series at daily frequency, from January 1999 to July 2024, and then aggregate it at quarterly frequency taking the simple average in order to make it comparable with the data we use in the rest of the analysis. To be consistent with CE data, we take the average of the rates for the horizons 6- to 10-year ahead.

Table 39: Correlation between CE ex-ante real yields and actual (FED or BoE) real yields.

			United Kingdom		
	$h=2$	$h=3$	$h=4$	$h=5$	$h=6-10$
ρ	NA	.9183***	.9317***	.9365***	.9522***
N	NA	48	51	51	51
			United States		
	$h=2$	$h=3$	$h=4$	$h=5$	$h=6-10$
ρ	.8723***	.9078***	.708***	.9331***	.8899***
N	62	62	59	62	42

Note. Sample sizes and correlation coefficients vary with the horizons. ***, **, and * indicate significance at the 1, 5, and 10 percent levels respectively.

[25] See `https://www.bankofengland.co.uk/statistics/yield-curves/terminology-and-concepts`.

[26] See `https://www.federalreserve.gov/data/tips-yield-curve-and-inflation-compensation.htm`.

G "Artificial" Nominal Interest Rates

We define first the following measures, computed using model-implied forecast data:

- $r_{c,t}^{(h),m} = i_{c,t}^{(h)} - \mathbb{F}_{\mathrm{t}}^{m}\left(\overline{\pi_{\mathrm{c,t+1,h}}}\right)$, for $h = 1, 2, 3, 5, 6-10$, ex-ante real rate, model implied;
- $r_{c}^{(h),m} = \frac{1}{T_c}\sum_{t=T_{\mathrm{start},c}}^{T_{\mathrm{end},c}}\left(r_{c,t}^{(h),m}\right)$, for $h = 1, 2, 3, 5, 6-10$, average ex-ante real rate over the period in which CE forecast data is available, model implied;
- $i_{c,t}^{*(h),m} = r_{c}^{(h),m} + \mathbb{F}_{t}^{m}\left(\overline{\pi_{c,t+1,h}}\right)$, for $h = 1, 2, 3, 5, 6-10$, ex-ante nominal rate, model implied;
- $rx_{\mathrm{c,t,t+1}}^{(h),m} = -(h-1)\cdot i_{c,t+1}^{*(h-1),m} + h\cdot i_{c,t}^{*(h),m} - i_{c,t}^{*(1),m}$, for $h = 1, 2, 3, 5, 6-10$, one year excess return, model implied;[27]
- $\overline{rx}_{\mathrm{c,t,t+1}}^{(h),m} = \frac{1}{2}\left(rx_{\mathrm{c,t,t+1}}^{(h),m} + rx_{\mathrm{c,t+1,t+2}}^{(h),m}\right)$, for $h = 1, 2, 3, 5, 6-10$, smoothed (over two years) excess returns, model implied;

G.1 Expected Real Rates

Table 40 reports the horizon-specific results of the following specification:

$$r_{c,t}^{(h)} = \alpha_h + \delta_h \cdot \mathbb{F}_{\mathrm{t}}\left(\overline{\pi_{\mathrm{c,t+1,h}}}\right) + \varepsilon_{c,t+h}, \tag{34}$$

Table 41 reports the horizon-specific results of the following specification:

$$r_{c,t}^{(h),m} = \alpha_h + \delta_h \cdot \mathbb{F}_{\mathrm{t}}^{m}\left(\overline{\pi_{\mathrm{c,t+1,h}}}\right) + \varepsilon_{c,t+h}. \tag{35}$$

G.2 Excess returns

Table 43 reports the horizon-specific results of the following specification:

$$\overline{rx}_{\mathrm{c,t,t+1}}^{(h)} = \alpha_h + \delta_h \cdot \mathbb{F}_{\mathrm{t}}\left(\overline{\pi_{\mathrm{c,t+1,h}}}\right) + \varepsilon_{c,t+h}. \tag{36}$$

Table 44 reports the horizon-specific results of the following specification:

$$\overline{rx}_{\mathrm{c,t,t+1}}^{(h),m} = \alpha_h + \delta_h \cdot \mathbb{F}_{\mathrm{t}}^{m}\left(\overline{\pi_{\mathrm{c,t+1,h}}}\right) + \varepsilon_{c,t+h}. \tag{37}$$

[27] Note that the formula for the one year excess return for the for (6-10) horizon is different from the one used in the CE dataset. Indeed, for (6-10), the one year excess return is simply computed according to the formula described above, and not as the average of the excess returns for the horizons from 6 to 10. This is because, by construction, the variables used are already computed using averages over those horizons.

Table 40: Ex-ante real returns on current average (CE) forecast for inflation over the period $t+1, t+h$ - Annual frequency

	(1)	(2)	(3)	(4)	(5)
	$r^{(1)}_{\mathrm{c,t,t+1}}$	$r^{(2)}_{\mathrm{c,t,t+2}}$	$r^{(3)}_{\mathrm{c,t,t+3}}$	$r^{(5)}_{\mathrm{c,t,t+5}}$	$r^{(6-10)}_{\mathrm{c,t,t+6,t+10}}$
$\mathbb{F}_{\mathrm{t}}\left(\overline{\pi_{\mathrm{c,t+1,h}}}\right)$	0.096	1.083**	1.254***	1.532***	1.315
	(0.213)	(0.419)	(0.372)	(0.369)	(0.819)
Country fixed effects	✓	✓	✓	✓	✓
N	390	474	439	528	209
Adj. R^2	0.119	0.253	0.303	0.324	0.098

Note. We study the association between ex-ante real rates to maturity h and the consensus forecast for the average inflation over h years. Each column reports the country pooled regression at horizon $h = 1, ..., 6-10$. Ex-ante rates $r^{(h)}_{c,t,t+h}$ over horizon h and country c are defined as $i^{(h)}_{\mathrm{c,t}} - \mathbb{F}_{\mathrm{t}}\left(\overline{\pi_{\mathrm{c,t+1,h}}}\right)$. The consensus forecast for the average inflation over h years for country c, $\mathbb{F}_{\mathrm{t}}\left(\overline{\pi_{\mathrm{c,t+1,h}}}\right)$, is computed as: $\frac{1}{h}\sum_{h'\geq 1}\mathbb{F}_{\mathrm{t}}\left(\pi_{c,t+h'Y}\right)$. We control for country fixed effects. The sample ranges from 1989 to 2022, Table 14 reports the countries available at each maturity. Standard errors in parenthesis are Driscoll-Kraay. *** **, and * indicate significance at the 1, 5, and 10 percent levels.

Table 41: Model-implied ex-ante real returns on current average (model-implied) forecast for inflation over the period $t+1, t+h$

	(1)	(2)	(3)	(4)	(5)
	$r^{(1),m}_{\mathrm{c,t,t+1}}$	$r^{(2),m}_{\mathrm{c,t,t+2}}$	$r^{(3),m}_{\mathrm{c,t,t+3}}$	$r^{(5),m}_{\mathrm{c,t,t+5}}$	$r^{(6-10),m}_{\mathrm{c,t,t+6,t+10}}$
$\mathbb{F}^m_{\mathrm{t}}\left(\overline{\pi_{\mathrm{c,t+1,h}}}\right)$	-0.742***	-0.380	-0.282	-0.256	-0.729*
	(0.225)	(0.384)	(0.372)	(0.397)	(0.358)
Country fixed effects	✓	✓	✓	✓	✓
N	390	474	439	528	209
Adj. R^2	0.231	0.101	0.090	0.059	0.210

Note. We study the association between model-implied ex-ante real rates to maturity h and the model-implied forecast for the average inflation over h years. Each column reports the country pooled regression at horizon $h = 1, ..., 6-10$. Ex-ante rates ${r^m_{c,t,t+h}}^{(h)}$ over horizon h and country c are defined as $i^{(h)}_{\mathrm{c,t}} - \mathbb{F}^m_{\mathrm{t}}\left(\overline{\pi_{\mathrm{c,t+1,h}}}\right)$. The consensus forecast for the average inflation over h years for country c, $\mathbb{F}^m_{\mathrm{t}}\left(\overline{\pi_{\mathrm{c,t+1,h}}}\right)$, is computed as: $\frac{1}{h}\sum_{h'\geq 1}\mathbb{F}^m_{\mathrm{t}}\left(\pi_{c,t+h'Y}\right)$. We control for country fixed effects. The sample ranges from 1989 to 2022, Table 14 reports the countries available at each maturity. Standard errors in parenthesis are Driscoll-Kraay. *** **, and * indicate significance at the 1, 5, and 10 percent levels.

Table 42: Correlation between Figure 16's estimated (at annual frequency) and model-implied coefficients

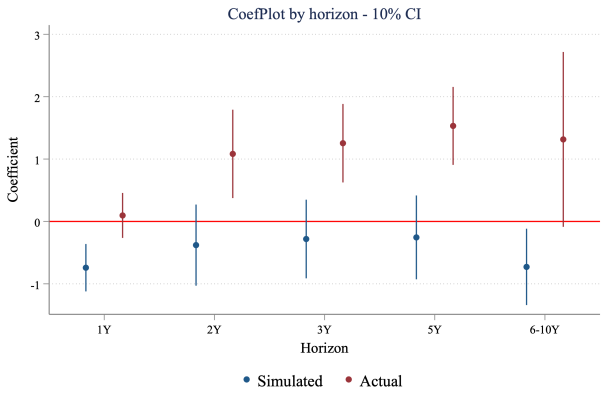
	$h=1$	$h=2$	$h=3$	$h=5$	$h=6-10$
ρ	.6257**	.4127	.4707*	.3937	.4998
N	14	16	15	18	10

(a) Correlation between estimated (at annual frequency) and simulated coefficients across horizons.

	$h=1$	$h=2$	$h=3$	$h=\{5, 6-10\}$
ρ	.6257**	.4127	.4707**	.396**
N	14	16	15	28

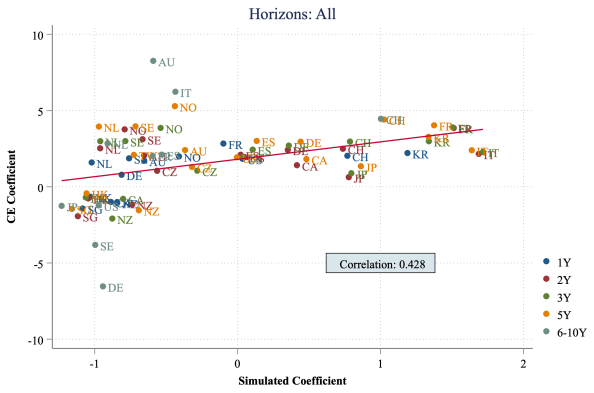
(b) Correlation between estimated (at annual frequency) and simulated coefficients across horizons, joining 5 and 6-10 horizons.

Figure 16: Coefplot and Scatterplot of model-implied and CE ex-ante real returns' regression



(a) Panel A: Coefplot by horizon of model-implied model-implied ex-ante real returns' regression

Note. the figure displays the point estimates and their 90% confidence intervals (vertical bars) for each variable across horizons. Each horizon is a different regression. Country fixed effects are included. Sample varies with the horizons according to Table 13 and regressions are performed on the simulated dataset. Standard errors are Driscoll-Kraay.



(b) Panel B: Scatterplot pooling countries and maturities of model-implied ex-ante real returns' regression

Note. the figure reports the coefficients from CE dataset (y-axis) at annual frequency against simulated dataset (x-axis). Coefficients are pooled across countries and maturities. The line indicates the fit between the two datasets' coefficients. Correlation is reported in the lower-right corner of the graph. The identifier for each data point corresponds to the ISO 3166-2 country abbreviation.

Table 43: Bond excess returns (smoothed over two years) on current average (CE) forecasts for inflation over the period $t+1, t+h$ - Annual frequency

	(1)	(2)	(3)	(4)
	$\overline{rx}^{(2)}_{c,t,t+1}$	$\overline{rx}^{(3)}_{c,t,t+1}$	$\overline{rx}^{(5)}_{c,t,t+1}$	$\overline{rx}^{(6-10)}_{c,t,t+1}$
$\mathbb{F}_t\left(\overline{\pi_{c,t+1,h}}\right)$	0.244**	0.298*	2.941***	4.232*
	(0.102)	(0.157)	(0.836)	(2.195)
Country fixed effects	✓	✓	✓	✓
N	371	342	269	172
Adj. R^2	0.088	0.152	0.173	0.093

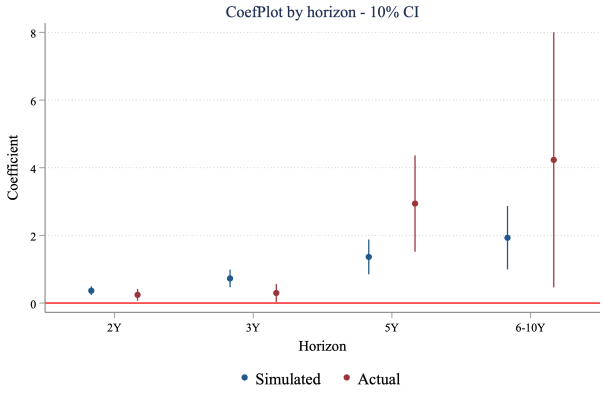
Note. We predict one-year holding period realized bond excess returns with maturity h from consensus forecast of the average inflation over h years. Each column reports the country pooled regression at horizon $h = 2, 3, 5, 6-10$: the dependent variable is the smoothed one-year holding period excess returns over horizon h and country c, defined as $\overline{rx}^{(h)}_{c,t,t+1} = \dfrac{rx^{(h)}_{c,t,t+1} + rx^{(h)}_{c,t+1,t+2}}{2}$. The raw one-year holding period excess returns $rx^{(h)}_{c,t,t+1}$ is defined as $i^{(h)}_{c,t} + (h-1)\cdot\left(i^{(h)}_{c,t} - i^{(h-1)}_{c,t+1}\right) - i^{(1)}_{c,t}$. We control for country fixed effects. The sample ranges from 1989 to 2020, Table 14 reports the countries available at each maturity. Standard errors in parenthesis are Driscoll-Kraay. *** **, and * indicate significance at the 1, 5, and 10 percent levels.

Table 44: Model-implied bond excess returns (smoothed over two years) on current average (model-implied) forecast for inflation over the period $t+1, t+h$

	(1)	(2)	(3)	(4)
	$\overline{rx}^{(2),m}_{c,t,t+1}$	$\overline{rx}^{(3),m}_{c,t,t+1}$	$\overline{rx}^{(5),m}_{c,t,t+1}$	$\overline{rx}^{(6),m}_{c,t,t+1}$
$\mathbb{F}^m_t\left(\overline{\pi_{c,t+1,h}}\right)$	0.366***	0.728***	1.365***	1.930***
	(0.076)	(0.153)	(0.303)	(0.552)
Country fixed effects	✓	✓	✓	✓
N	446	416	397	291
Adj. R^2	0.890	0.599	0.626	0.653

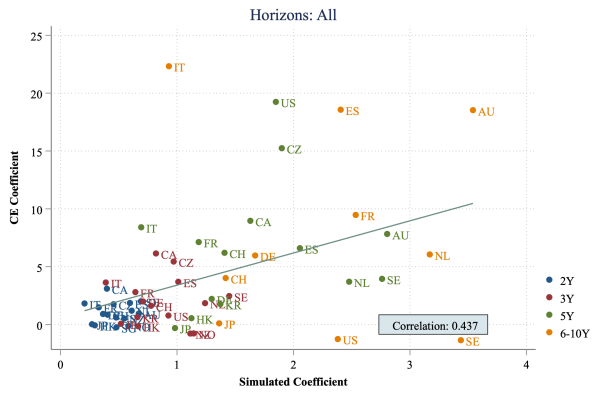
Note. We predict one-year holding period realized model-implied bond excess returns with maturity h from consensus forecast of the model-implied average inflation over h years. Each column reports the country pooled regression at horizon $h = 2, 3, 5, 6-10$: the dependent variable is the smoothed one-year holding period excess returns over horizon h and country c, defined as $\overline{rx}^m_{c,t,t+1}{}^{(h)} = \frac{rx^m_{c,t,t+1}{}^{(h)} + rx^m_{c,t+1,t+2}{}^{(h)}}{2}$. The raw one-year holding period excess returns $rx^m_{c,t,t+1}{}^{(h)}$ is defined as $i^{*(h),m}_{c,t} + (h-1)\cdot\left(i^{*(h),m}_{c,t} - i^{*(h-1),m}_{c,t+1}\right) - i^{*(1),m}_{c,t}$. We control for country fixed effects. The sample ranges from 1989 to 2020, Table 14 reports the countries available at each maturity. Standard errors in parenthesis are Driscoll-Kraay. *** **, and * indicate significance at the 1, 5, and 10 percent levels.

Figure 17: Coefplot and Scatterplot of model-implied and CE excess returns' regression



(a) Panel A: Coefplot by horizon of model-implied excess returns' regression

Note. the figure displays the point estimates and their 90% confidence intervals (vertical bars) for each variable across horizons. Each horizon is a different regression. Country fixed effects are included. Sample varies with the horizons according to Table 13 and regressions are performed on the simulated dataset. Standard errors are Driscoll-Kraay.



(b) Panel B: Scatterplot pooling countries and maturities of model-implied excess returns' regression

Note. the figure reports the coefficients from CE dataset (y-axis) at annual frequency against simulated dataset (x-axis). Coefficients are pooled across countries and maturities. The line indicates the fit between the two datasets' coefficients. Correlation is reported in the lower-right corner of the graph. The identifier for each data point corresponds to the ISO 3166-2 country abbreviation.

Table 45: Correlation between Figure 17's estimated (at annual frequency) and model-implied coefficients

	$h=2$	$h=3$	$h=5$	$h=6-10$
ρ	.062	-.0522	.204	-.082
N	16	15	14	10

(a) Correlation between estimated (at annual frequency) and simulated coefficients across horizons.

	$h=2$	$h=3$	$h=\{5,6-10\}$
ρ	.062	-.0522	.0742
N	16	15	24

(b) Correlation between estimated (at annual frequency) and simulated coefficients across horizons, joining 5 and 6-10 horizons.

H Economic Implications

We are interested in finding the change in $\hat{L}^{(h^*)}$ for a given change in expected inflation π_j^e due to predictable forecast errors. Formally, if we let $Q^{(h^*)} \equiv \hat{L}^{(h^*)}/P^{(h^*)}$, then

$$\frac{\partial \hat{L}^{(h^*)}}{\partial \pi_j^e} = P^{(h^*)} \frac{\partial Q^{(h^*)}}{\partial \pi_j^e}$$

or, using the definition of $Q^{(h^*)}$,

$$\frac{1}{\hat{L}^{(h^*)}} \cdot \frac{\partial \hat{L}^{(h^*)}}{\partial \pi_j^e} = \frac{1}{Q^{(h^*)}} \cdot \frac{\partial Q^{(h^*)}}{\partial \pi_j^e}.$$

Note that the left-hand side captures the semi-elasticity of $\hat{L}^{(h^*)}$ to a change in expectations: $\partial \log \hat{L}^{(h^*)}/\partial \pi_j^e$.

More specifically, we will consider a parallel change of inflation expectations at all horizons, that is we assume $\pi_s^e = \tilde{\pi}_s^e + \pi^e$ and compute the derivative with respect to π^e. We have

$$\begin{aligned}
\frac{1}{Q^{(h^*)}} \cdot \frac{\partial Q^{(h^*)}}{\partial \pi^e} =& \frac{1}{Q^{(h^*)}} \cdot \frac{\partial}{\partial \pi^e} \sum_{h=1}^{h^*} \prod_{s=1}^{h} \frac{1}{1 + i_{s-1} - \pi_s} \\
=& -\frac{1}{Q^{(h^*)}} \left[\frac{1}{(1 + i_0 - \pi_1)^2} \cdot \frac{\partial (i_0 - \pi_1)}{\partial \pi^e} + ... \right. \\
& \left. + \prod_{s=1}^{h^*} \frac{1}{1 + i_{s-1} - \pi_s} \sum_{j=1}^{h^*} \frac{1}{1 + i_{j-1} - \pi_j} \cdot \frac{\partial (i_{j-1} - \pi_j)}{\partial \pi^e} \right].
\end{aligned}$$

We can further simplify the expression by approximating one-period inflation with average inflation over the loan duration:

$$i_{s-1} - \pi_s \approx \frac{1}{h} \sum_{j=1}^{h} (i_{s-1} - \pi_s) \equiv i^{(h)} - \overline{\pi}_{0,h}, \, \forall s,$$

where $\overline{\pi}_{0,h}$ denotes average inflation between 0 and h. We then have

$$\frac{1}{Q^{(h^*)}} \cdot \frac{\partial Q^{(h^*)}}{\partial \pi^e} = -\frac{1}{Q^{(h^*)}} \sum_{h=1}^{h^*} \frac{1}{(1 + i^{(h)} - \overline{\pi}_{0,h})^{h+1}} \sum_{j=1}^{h} \frac{\partial (i_{j-1} - \pi_j)}{\partial \pi^e}.$$

Finally, using the decomposition (**??**), we can rewrite the right-hand side into two components:

$$A^{(h^*)} \equiv -\frac{1}{Q^{(h^*)}} \sum_{h=1}^{h^*} \frac{1}{(1 + i^{(h)} - \overline{\pi}_{0,h})^{h+1}} \cdot \frac{\partial}{\partial \pi^e} \sum_{j=1}^{h} (i_{j-1} - \pi_j^e)$$

and

$$B^{(h^*)} \equiv -\frac{1}{Q^{(h^*)}} \sum_{h=1}^{h^*} \frac{1}{(1+i^{(h)}-\overline{\pi}_{0,h})^{h+1}} \cdot \frac{\partial}{\partial \pi^e} \sum_{j=1}^{h} (\pi_j^e - \pi_j).$$

We focus only on the impact of inflation expectations on forecast errors, i.e., part $B^{(h^*)}$. The sum $\sum_{j=1}^{h}(\pi_j^e - \pi_j)$ is the (negative of the) cumulative forecast error over horizon h, thus, $\frac{\partial}{\partial \pi^e}\sum_{j=1}^{h}(\pi_j^e - \pi_j)$ captures the change in the cumulative forecast error following from a parallel change in expectations over the loan duration. Equation (2) estimates the sensitivity of the h-period-ahead forecast error to a change in the average inflation expectation. Using the notation of this section, we can rewrite Equation (2) as

$$\pi_h - \pi_h^e = \alpha_h + \delta_h \pi_h^e + \varepsilon^{(h)}.$$

Using also our assumption that $\pi_j^e = \tilde{\pi}_j^e + \pi^e$ for all h, it follows that

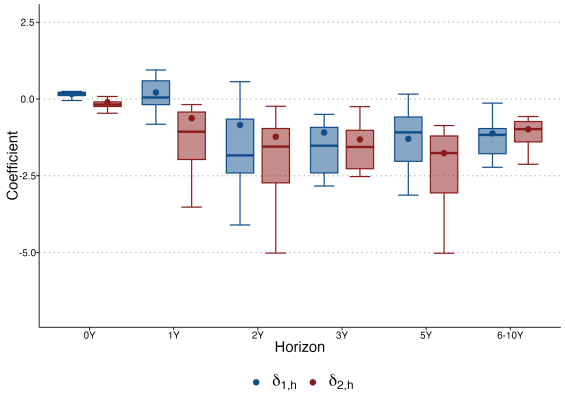
$$B^{(h^*)} = \frac{1}{Q^{(h^*)}} \sum_{h=1}^{h^*} \frac{\sum_{j=1}^{h} \delta_j}{(1+i^{(h)}-\overline{\pi}_{0,h})^{h+1}}.$$

I Temporary Appendix

I.1 Content removed from Section 2

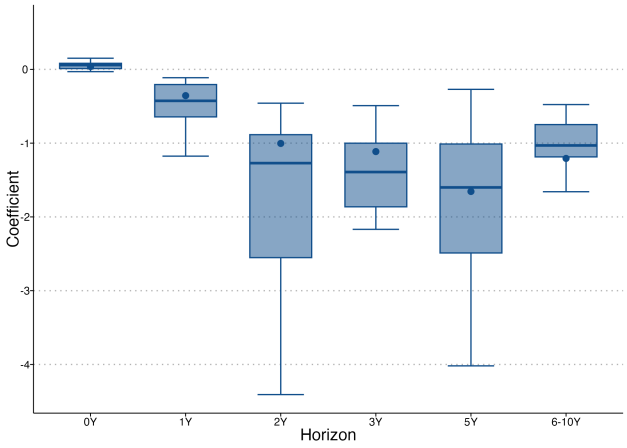
Do these patterns hold, to varying degrees, across countries? Figure 18 reports, in Panel A, the boxplot of country-specific estimates $\delta_{1,c,h}$ (red) and $\delta_{2,c,h}$ (blue) at different horizons h. Each box reports the interquartile range of estimates. The line in the box is the median estimate. Consistent with Table 1, in most countries there is overreaction to news and persistence, and these distortions tend to increase with the forecast horizon. The strength of these distortions, though, exhibits significant cross-country variation. Such variation is systematic: the cross-country correlation between overreaction and excess persistence is 75% (see Figure 7 in Appendix). Consistent with Table 1, there is a tight connection between $\delta_{1,c,h}$ and $\delta_{2,c,h}$ which suggests that in some countries expectations tend to exhibit both more overreaction and persistence than in others. Thus, upward inflation signals trigger a stronger excess upward movement in beliefs in some countries compared to others.

Figure 18: Boxplots of inflation forecast errors' regressions



(a) Panel A: Boxplot of Table 2's results

Notes. The figure displays the boxplots and the pooled estimates from (3) for each each of the two coefficients $\delta_{1,h}$, $\delta_{2,h}$ and for each horizon h=0,1,2,3,5,6-10. For each horizon, the sample varies according to Table 14. The boxplot illustrates the distribution of the country-coefficients from specification 3. The box represents the interquartile (IQR) range, with the median indicated by the line inside the box. The whiskers extend to 1.5 times the IQR above the upper quartile and below the lower quartile. Points beyond the whiskers are considered outliers and are removed from the boxplots. The plot displays also the pooled estimates (dots) for each horizon from Table 2.



(b) Panel B: Boxplot of the δ country-level coefficient from Equation (2).

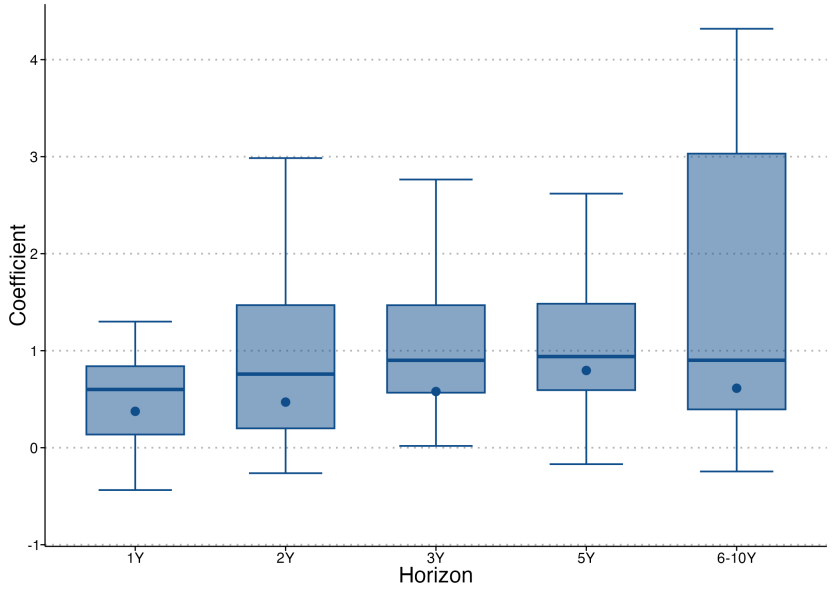
Note. The figure displays the boxplots and the pooled estimates from (2) of δ_h, for each horizon h=0,1,2,3,5,6-10. For each horizon, the sample varies according to Table 14. The boxplot illustrates the distribution of the country-coefficients from specification 2. The box represents the interquartile (IQR) range, with the median indicated by the line inside the box. The whiskers extend to 1.5 times the IQR above the upper quartile and below the lower quartile. Points beyond the whiskers are considered outliers and are removed from the boxplots. The plot displays also the pooled estimates (dots) for each horizon from Table 1'S Panel A.

Figure 18, Panel B reports the country-by-country estimation of Equation (2), with the

pooled estimate (the dot i the box) and the median estimate (the line). The pooled estimate is above the median at shorter horizons, whereas it drops below this value for the 5-year and (6-10)-year horizon (for the latter it drops even below the lower quartile). The pooled estimation table is reported in the appendix (Table 1's Panel A). The estimation confirms that δ_h is generally negative and more so at longer forecast horizons. This pattern is present in most countries, but stronger in some countries compared to others.

I.2 Content removed from Section 3

Figure 19: Boxplots of Table 4 Panel B's results



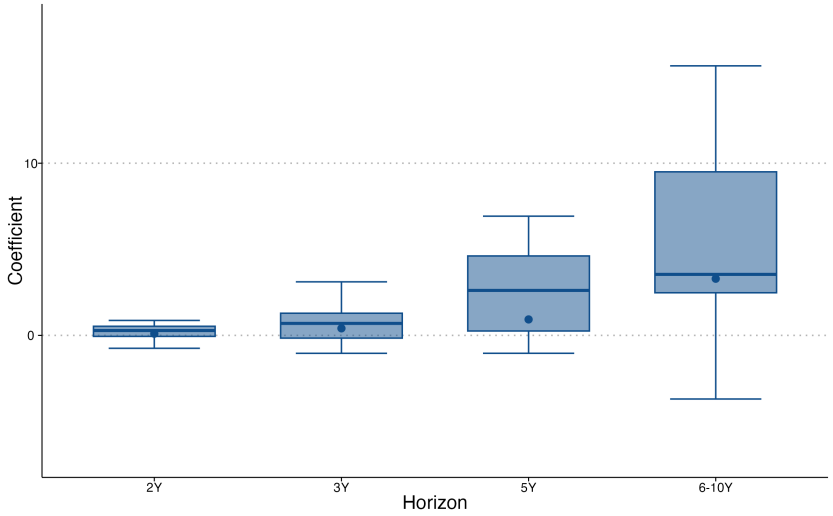
Note. The figure displays the boxplots and the pooled estimates from Table 4's Panel B for the coefficients on inflation forecasts for each horizon h=1,2,3,5,6-10. For each horizon, the sample varies according to Table 14. The boxplot illustrates the distribution of the country-coefficient. The box represents the interquartile (IQR) range, with the median indicated by the line inside the box. The whiskers extend to 1.5 times the IQR above the upper quartile and below the lower quartile. Points beyond the whiskers are considered outliers and are removed from the boxplots. The plot displays also the pooled estimates (dots) of the inflation forecast's coefficient for each horizon from Table 4's Panel B.

Figure 19 reports the boxplot for the country-by-country specification of Panel B (the boxplot for Panel A is similar and reported in Figure 10). The graph confirms that the same predictability patterns occur in most countries, but also shows that their strength varies considerably in the cross-section. This suggests that the redistributive forces from over-reacting expectations may be more severe in some countries than in others. Across the 18 countries in our sample, there is a negative correlation (equal to -0.492) between the average predictability coefficient of inflation forecast errors in a given country and the average predictability coefficient for the ex-post real rate estimated in the same country.[28] Countries in which inflation

[28]Figure 8 in the Appendix illustrates the country-level data points for these estimations, with the correlation

expectations over-react more are associated with stronger predictability of ex-post real yields to maturity. Section 5, which contains a quantitative evaluation of our memory model, will link the evidence on forecast errors and return predictability to a country's inflation history.

Figure 20: Boxplots of Table 5 Panel B's results



Note. The figure displays the boxplots and the pooled estimates from Table 5's Panel B for each horizon h=1,2,3,5,6-10. For each horizon, the sample varies according to Table 14. The boxplot illustrates the distribution of the country-coefficient. The box represents the interquartile (IQR) range, with the median indicated by the line inside the box. The whiskers extend to 1.5 times the IQR above the upper quartile and below the lower quartile. Points beyond the whiskers are considered outliers and are removed from the boxplots. The plot displays also the pooled estimates (dots) of the coefficients for each horizon from Table 5's Panel B.

Figure 20 reports the boxplot of the coefficients estimated country-by-country (together with the pooled estimates). The figure shows that excess return predictability occurs in most countries, with a strength that exhibits sizable cross-country variation.[29] In Section 5, we will connect this variation to countries' inflation histories through the lens of our memory model.

We conclude this section by linking return predictability to belief biases using a two-stage approach. In the first stage, we predict the inflation forecast error using the current forecast for average inflation at the relevant horizon, controlling for level and slope (Appendix Table 20 reports the first stage). In the second stage, we regress excess returns on the predicted forecast error from the first stage, again controlling for level and slope:[30,31]

$$\overline{rx}^{(h)}_{t,t+1Y} = \alpha_h + \theta_h \widehat{FE_INF}_{t,t+2Y} + \beta_l i^{(1)}_t + \beta_s (i^{(10)}_t - i^{(1)}_t) + \varepsilon^{(h)}_{t+1Y}$$

line shown in red.

[29]XXX Unlike in the case of the ex-post yield, our model does not imply a monotonic connection between the excess return predictability coefficient $\theta^{rx}_{c,h}$ in country c and the extent of predictability of inflation forecast errors in the same country $\delta_{c,h}$.

[30]In the first-stage regression, the dependent variable is the forecast error two years ahead because this is the horizon over which we compute bond returns. We ensure that the sample varies across horizons to match the one used in the second stage, XXX as detailed in Table 14.

[31]We do not perform this test for the ex-post real yield to maturity because the second stage regression would tautologically assigns coefficient -1 to the forecast error +1 to the ex-ante real rate.

Table 46: Bond excess returns on predicted forecast errors yield curve's slope and level - 2nd Stage

	(1)	(2)	(3)	(4)
	$\overline{rx}^{(2)}_{c,t+1,t}$	$\overline{rx}^{(3)}_{c,t+1,t}$	$\overline{rx}^{(5)}_{c,t+1,t}$	$\overline{rx}^{(6-10)}_{c,t+1,t}$
$\widehat{FE_INF}_{\mathrm{c,t,t+2Y}}$	-0.153	-0.600**	-0.803**	-1.592**
	(0.114)	(0.244)	(0.384)	(0.654)
$i^{(1)}_{\mathrm{c,t}}$	0.249***	0.064	0.852***	0.693**
	(0.037)	(0.099)	(0.133)	(0.308)
$i^{(10)}_{\mathrm{c,t}} - i^{(1)}_{\mathrm{c,t}}$	0.316***	0.124	1.141***	1.585***
	(0.099)	(0.154)	(0.211)	(0.382)
Country fixed effects	✓	✓	✓	✓
N	941	871	713	467
Adj. R^2	0.417	0.299	0.517	0.423

Note. We predict one-year holding period realized bond excess returns with maturity h from predicted cumulative forecast error from the 1st stage regression, Table 20. Additionally, we control for level and slope of nominal yields. Each column reports the country pooled regression at horizon $h = 2, 3, 5, 6-10$: the dependent variable is the smoothed one-year holding period excess returns over horizon h and country c, defined as $\overline{rx}^{(h)}_{c,t,t+1} = \frac{rx^{(h)}_{c,t,t+1} + rx^{(h)}_{c,t+1,t+2}}{2}$. The raw one-year holding period excess returns $rx^{(h)}_{c,t,t+1}$ is defined as $i^{(h)}_{c,t} + (h-1) \cdot \left(i^{(h)}_{c,t} - i^{(h-1)}_{c,t+1}\right) - i^{(1)}_{c,t}$. The level and slope for country c and maturity h are defined respectively as $i^{(1)}_{c,t}$ and $i^{(10)}_{c,t} - {}^{(1)}_{c,t}$. We control for country fixed effects. The sample ranges from 1989Q4 to 2020Q4, Table 14 reports the countries available at each maturity. Standard errors in parenthesis are Driscoll-Kraay. *** **, and * indicate significance at the 1, 5, and 10 percent levels.

Second stage estimates are reported in Table 46. Intuitively, a systematic positive inflation surprise (i.e., a higher than expected realized inflation) should be associated with lower excess returns. The predictable forecast has the expected negative sign, confirming that over-reacting inflation expectations act as a source of excess bond return predictability.

I.3 Content removed from Section 4

I.4 Content removed from Section 5

I.5 Content removed from Section 6

J Proofs

Formal Preliminaries We consider a generic horizon h. Under the assumptions of stationary autoregressive DGP and with similarity specification described in (15), we sum up the model in three essential ingredients:

- A probability space $(\mathbb{R}^\infty, \mathcal{B}(\mathbb{R}^\infty), \mathcal{P})$ where $\mathcal{P}$ the (shift invariant) Kolmogorov extension law induced by the stationary AR(1) stochastic process $(X_t)_t$ whose time t realization is denoted by π_t and $\mathcal{B}(\mathbb{R}^\infty)$ is the cylinder sigma algebra. We indicate by $\omega := (\pi_1, \pi_2, ...) \in \mathbb{R}^\infty$ any sequence of real numbers.
- A sequence of (random) mnemnonic expectations $(\mathbb{E}^m_t)_{t\in\mathbb{N}}$ such that

$$\mathbb{E}^m_t : \omega \mapsto \frac{\sum_{k=h}^t \pi_k \cdot e^{-\alpha(\pi_t - \pi_{k-h})^2 - \beta(\pi_t - \pi_k)^2}}{\sum_{k=h}^t e^{-\alpha(\pi_t - \pi_{k-h})^2 - \beta(\pi_t - \pi_k)^2}}$$

  which is Borel measurable by standard arguments.

Proposition 1 Proof. Fix $x \in \mathbb{R}$. For all $t \in \mathbb{N}$, consider the x-parameterized (random) mnemnonic expectation

$$\mathbb{E}^{m,x}_t : \omega \mapsto \frac{\sum_{k=h}^t \pi_k \cdot e^{-\alpha(x - \pi_{k-h})^2 - \beta(x - \pi_k)^2}}{\sum_{k=h}^t e^{-\alpha(x - \pi_{k-h})^2 - \beta(x - \pi_k)^2}}$$

We first show that for each $\mathbb{E}^{m,x}_t$ converges almost surely to some $\Lambda(x)$ and we algebraically characterize the limit $\Lambda(x)$. Then we we will prove that $\Lambda(X_t)$ is the p-lim of $(\mathbb{E}^m_t)_{t\in\mathbb{N}}$, namely

$$\lim_{t\to\infty} \mathcal{P}(\{|\mathbb{E}^m_t - \Lambda(X_t)| < \epsilon\}) = 1$$

for all $\epsilon > 0$.

P-lim characterization: Consider the numerator of $\mathbb{E}^{m,x}_t(\omega)$. We know that the map

$$S_x \circ T \circ ... \circ T : \omega \mapsto \pi_t \cdot e^{-\alpha(x - \pi_{t-h})^2 - \beta(x - \pi_t)^2}$$

when the shift operator T is applied t times is bounded and $\mathbb{R}^2$-Borel measurable for each t (hence integrable with respect to any probability measure). Hence the Pointwise Bierkoff Ergodic Theorem applies with respect to the shift operator which is a well-known measure preserving transformation of $\mathcal{P}$. We conclude that,

$$\lim_{t\to\infty} \frac{\sum_{k=h}^t \pi_k \cdot e^{-\alpha(x - \pi_{k-h})^2 - \beta(x - \pi_k)^2}}{t} = \int \tilde{\pi} \cdot e^{-\alpha(x - \tilde{\pi}_{-h})^2 - \beta(x - \tilde{\pi})^2} d\tau(\tilde{\pi}, \tilde{\pi}_{-h}) \tag{38}$$

almost surely, where τ is the unique stationary probability measure for the process (X_{t-h}, X_t) with realizations (π_{t-h}, π_t). An analogous reasoning applies to the denominator. Hence we conclude that

$$\lim_{t\to\infty} \frac{\sum_{k=h}^{t} e^{-\alpha(x-\pi_{k-h})^2-\beta(x-\pi_k)^2}}{t} = \int e^{-\alpha(x-\tilde{\pi}_{-h})^2-\beta(x-\tilde{\pi})^2} d\tau(\tilde{\pi}, \tilde{\pi}_{-h})$$

almost surely. By properties of the exponential function, we know that the denominator is strictly positive almost surely. Consequently we infer that

$$\lim_{t\to\infty} \mathbb{E}_t^{m,x} = \Lambda(x) := \frac{\int \tilde{\pi} \cdot e^{-\alpha(x-\tilde{\pi}_{-h})^2-\beta(x-\tilde{\pi})^2} d\tau(\tilde{\pi}, \tilde{\pi}_{-h})}{\int e^{-\alpha(x-\tilde{\pi}_{-h})^2-\beta(x-\tilde{\pi})^2} d\tau(\tilde{\pi}, \tilde{\pi}_{-h})}$$

almost surely. We said that τ is the stationary probability measure for the stochastic process (X_{t-h}, X_t). Denote by Y the bivariate normal having marginals X_{-h}, X distributed identically to the limit distribution of the one dimensional original stochastic process, namely according to the normal $\mathcal{N}(0, \frac{\sigma^2}{1-\rho^2} = \sigma_\pi^2)$. In order to determine the stationary distribution of the two dimensional stochastic process (X_{t-h}, X_t) we need to compute the covariance matrix of Y. Variances coincide with σ_π^2, while the two covariances are defined by the following expression:

$$\begin{aligned} COV[X_{-h}, X] = COV[X, X_{-h}] &= \mathbb{E}[X_{-h}, X] - \mathbb{E}[X_{-h}]\mathbb{E}[X] \\ &= \mathbb{E}[X_{-h}(\rho^h X_{-h} + \tilde{\epsilon})] \\ &= \rho^h \mathbb{E}[(X)^2] = \frac{\rho^h \cdot \sigma^2}{1-\rho^2} \end{aligned}$$

By applying the Radon-Nikodym Theorem and reformulating the RHS of (38) using quadratic forms we get

$$\Lambda(x) = \frac{\int_{\mathbb{R}^2} \pi \cdot \exp\left(-0.5(\pi_{-h}, \pi)^\top \Sigma_1^{-1}(\pi_{-h}, \pi) - 0.5(\pi_{-h} - x, \pi - x)^\top \Sigma_2^{-1}(\pi_{-h} - x, \pi - x)\right) d(\pi_{-h}, \pi)}{\int_{\mathbb{R}^2} \exp\left(-0.5(\pi_{-h}, \pi)^\top \Sigma_1^{-1}(\pi_{-h}, \pi) - 0.5(\pi_{-h} - x, \pi - x)^\top \Sigma_2^{-1}(\pi_{-h} - x, \pi - x)\right) d(\pi_{-h}, \pi)}$$

Where one can note that the denominator is just the normalizing constant. By the characterization of the product of two bivariate Gaussian densities, we know it is normal $\mathcal{N}^*(\mathbf{m}, \mathbf{\Sigma_3})$ such that:

$$\begin{aligned} \mathbf{\Sigma_3} &= \left(\begin{bmatrix} \frac{\sigma^2}{1-\rho^2} & \frac{\sigma^2\cdot\rho}{1-\rho^2} \\ \frac{\sigma^2\cdot\rho}{1-\rho^2} & \frac{\sigma^2}{1-\rho^2} \end{bmatrix}^{-1} + \begin{bmatrix} 2\alpha & 0 \\ 0 & 2\beta \end{bmatrix} \right)^{-1} \\ \mathbf{m} &= \mathbf{\Sigma_3} \begin{bmatrix} 2\alpha & 0 \\ 0 & 2\beta \end{bmatrix} \begin{pmatrix} x \\ x \end{pmatrix}. \end{aligned}$$

By performing the algebra, one gets that

$$\mathbf{m}(2) = \Lambda(x) = \frac{\alpha\rho^h + \beta + 2\alpha\beta\sigma_\pi^2(1-\rho^{2h})}{\alpha + \beta + 2\alpha\beta\sigma_\pi^2(1-\rho^{2h}) + \frac{1}{2\sigma_\pi^2}} \cdot x.$$

Convergence in Probability: We want to show that

$$\lim_{t\to\infty} \mathcal{P}(\{\ \mathbb{E}_t^m - \Lambda(X_t)| \geq \epsilon\}) = 0$$

for all $\epsilon > 0$. Note that, for all $t \in \mathbb{N}$ and for all $B > 0$,

$$\mathcal{P}(\{\ |\mathbb{E}_t^m - \Lambda(X_t)| \geq \epsilon\}) \leq \mathcal{P}\left(\{X_t \in [-B,B]^c\}\right) + \mathcal{P}(\{\ |\mathbb{E}_t^m - \Lambda(X_t)| \cdot 1_{[X_t \in [-B,B]]} \geq \epsilon\}\Big). \quad (39)$$

Fix $\delta > 0$ and pick $B > 0$ large enough so that, if $\mathbf{X}$ is distributed according to the stationary distribution, $\mathcal{P}\left(\{\mathbf{X} \in [-B,B]^c\}\right) < \delta$. Let us now focus on $\mathcal{P}(\{\ |\mathbb{E}_t^m - \Lambda(X_t)| \cdot 1_{[X_t \in [-B,B]]} \geq \epsilon\})$. By Markov Inequality,

$$\mathcal{P}(\{\ |\mathbb{E}_t^m - \Lambda(X_t)| \cdot 1_{[X_t \in [-B,B]]} \geq \epsilon\}) \leq \frac{\int_{\mathbb{R}^{t-1}\times[-B,B]} |\mathbb{E}_t^m - \Lambda(X_t)| d\mathcal{P}}{\epsilon}. \quad (40)$$

By Construction of $\mathcal{P}$ and by properties of the Gaussian stationary AR(1) processes, for all $t \in \mathbb{N}$

$$\begin{aligned}\int_{\mathbb{R}^{t-1}\times[-B,B]} |\mathbb{E}_t^m - \Lambda(X_t)| d\mathcal{P} &= \int_{\mathbb{R}^{t-1}\times[-B,B]} |\mathbb{E}_t^m - \Lambda(x_t)| f(\mathbf{x}_{t-1}) \cdot f(x_t|\mathbf{x_{t-1}}) d\mathbf{x}_t \\ &\leq \int_{\mathbb{R}^{t-1}\times[-B,B]} |\mathbb{E}_t^m - \Lambda(x_t)| \frac{f(\mathbf{x}_{t-1})}{\sqrt{2\pi\sigma^2}} d\mathbf{x}_t\end{aligned}$$

where $\frac{1}{\sqrt{2\pi\sigma^2}} \geq f(x_t|\mathbf{x_{t-1}})$ everywhere, justifying the above inequality. By applying Fubini-Tonelli's theorem we get

$$\int_{\mathbb{R}^{t-1}\times[-B,B]} \frac{|\mathbb{E}_t^m - \Lambda(x_t)| f(\mathbf{x}_{t-1})}{\sqrt{2\pi\sigma^2}} d\mathbf{x}_t = \int_{-B}^{B} \left(\int \frac{|\mathbb{E}_t^{m,y} - \Lambda(y)| f(\mathbf{x}_{t-1})}{\sqrt{2\pi\sigma^2}} d\mathbf{x}_{t-1} \right) dy, \quad (41)$$

and for notational simplicity consider

$$\Phi_t(y) := \int \frac{|\mathbb{E}_t^{m,y} - \Lambda(y)| f(\mathbf{x}_{t-1})}{\sqrt{2\pi\sigma^2}} d\mathbf{x}_{t-1}$$

hence, $\Phi_t(y) = \dfrac{||\mathbb{E}_t^{m,y} - \Lambda(y)||_{L^1(\mathcal{P})}}{\sqrt{2\pi\sigma^2}}$. By Bierkoff Mean Ergodic Theorem we know that [32] for all $y \in \mathbb{R}$

$$\lim_{t\to\infty} ||\mathbb{E}_t^{m,y} - \Lambda(y)||_{L^1(\mathcal{P})} = 0, \tag{42}$$

yielding that $\Phi_t \to 0$ pointwise. We are almost ready to apply Lebesgue Dominated Convergence theorem to conclude the argument. First, by the triangle inequality, for all $t \in \mathbb{N}$, for all $x \in [-B, B]$,

$$\int_{\mathbb{R}^{t-1}} |\mathbb{E}_t^{m,x}| + |\Lambda(x)| d\mathcal{P} \geq \Phi_t(x). \tag{43}$$

Now, an inspection of $|\mathbb{E}_t^{m,B}|$ yields that for all $x \in [-B, B]$,

$$\int_{\mathbb{R}^{t-1}} |\mathbb{E}_t^{m,B}| \; d\mathcal{P} \geq \int_{\mathbb{R}^{t-1}} |\mathbb{E}_t^{m,x}| \; d\mathcal{P},$$

whence, by (43),

$$\sup_{t\in\mathbb{N}} \int |\mathbb{E}_t^{m,B}| + |\Lambda(B)| d\mathcal{P} \geq \int_{\mathbb{R}^{t-1}} |\mathbb{E}_t^{m,x}| + |\Lambda(x)| d\mathcal{P} \geq \Phi_t(x)$$

for all $t \in \mathbb{N}$ and for all $x \in [-B, B]$. Finally, by (42) and the fact that any convergent sequence is bounded, $\sup_{t\in\mathbb{N}} \int_{\mathbb{R}^{t-1}} |\mathbb{E}_t^{m,B}| d\mathcal{P} < \infty$, whence,

$$\int_{-B}^{B} \left(\sup_{t\in\mathbb{N}} \int_{\mathbb{R}^{t-1}} |\mathbb{E}_t^{m,B}| \; d\mathcal{P} \right) dy < \infty,$$

proving that $(\Phi_t)_{t\in\mathbb{N}}$ is dominated by the integrable (constant) function. Because of this fact, by (40), (41) and (42), we apply the Lebesgue Dominated Convergence Theorem and we infer that

$$0 \leq \lim_{t\to\infty} \mathcal{P}(\{ \; |\mathbb{E}_t^m - \Lambda(X_t)| \cdot 1_{[X_t \in [-B,B]]} \geq \epsilon\}) \leq \lim_{t\to\infty} \frac{\int_{-B}^{B} \Phi_t(y) dy}{\epsilon} = 0. \tag{44}$$

By (39) and the fact that $(X_t)_{t\in\mathbb{N}}$ is a Gaussian stationary AR(1) process, we infer that

$$\lim_{t\to\infty} \mathcal{P}(\{ \; |\mathbb{E}_t^m - \Lambda(X_t)| \geq \epsilon\}) \leq \lim_{t\to\infty} \mathcal{P}(\{X_t \in [-B, B]^c\}) = \mathcal{P}(\{\mathbf{X} \in [-B, B]^c\}) < \delta,$$

but since $\delta > 0$ is arbitrary, the proof follows. ■

The next two corollaries show two key properties of mnemnonic expectations when they are generated under suitable stable DGP.

[32] Note that the Mean and the Pointwise Ergodic theorems are different theorems. The Pointwise ergodic theorem here is simply used to ensure that that the Pointwise and mean limit coincides.

Corollary 1 *Suppose $(X_t)_t$ is any stationary ergodic gaussian process. Then,*

$$\beta = 0 \implies \lim_{\alpha \to \infty} \Lambda(\cdot|h, X_t) = \mathbb{E}[X_{t+h}|X_t].$$

Proof. Thanks to the assumptions of stationarity, ergodicity and Gaussianity of the DGP, the steps applied in the first part of the main theorem apply verbatim. Hence we obtain that DM's mnemonic expectations converges in probability to stochastic process of mnemonic expectation with gaussian kernel deduced by convoluting bivariate Gaussian densities. In particular, we know it is normal $\mathcal{N}^*(\mathbf{m}, \mathbf{\Sigma_3})$ such that:

$$\mathbf{\Sigma_3} = \left(\begin{bmatrix} \text{Var}(X_t) & \text{Cov}(X_{t+h}, X_t) \\ \text{Cov}(X_{t+h}, X_t) & \text{Var}(X_t) \end{bmatrix}^{-1} + \begin{bmatrix} 2\alpha & 0 \\ 0 & 2\beta \end{bmatrix} \right)^{-1}$$
$$\mathbf{m} = \mathbf{\Sigma_3} \begin{bmatrix} 2\alpha & 0 \\ 0 & 2\beta \end{bmatrix} \begin{pmatrix} X_t \\ X_t \end{pmatrix}.$$

By performing the algebra under the assumption that $\beta = 0$, one gets that

$$\mathbf{m}(2) = \frac{2\alpha \text{Cov}(X_{t+h}, X_t) \left([\text{Var}(X_t)]^2 - [\text{Cov}(X_{t+h}, X_t)]^2 \right)}{[\text{Var}(X_t)]^2 + 2\alpha \cdot \left(\text{Var}(X_t)^2 - (\text{Cov}(X_{t+h}, X_t))^2 \right) \cdot \text{Var}(X_t) - (\text{Cov}(X_{t+h}, X_t))^2} X_t,$$

By sending $\alpha \to \infty$ one gets

$$\mathbf{m}(2) = \Lambda_\infty(\cdot|h, X_t) = \frac{\text{Cov}(X_{t+h}, X_t)}{\text{Var}(X_t)} X_t = \mathbb{E}[X_{t+h}|X_t]$$

where the last equality is an implication of the well-known characterization of conditional expectations for stationary gaussian processes. ■

By iterating the same steps above, we get that the mnemnonic belief under cue X_t is defined by

$$X_t \frac{\text{Cov}(X_{t+h}, X_t) \cdot 2\alpha \left([\text{Var}(X_t)]^2 - \text{Cov}(X_{t+h}, X_t)^2 \right)}{\left(\text{Var}(X_t) + 2\alpha \left([\text{Var}(X_t)]^2 - \text{Cov}(X_{t+h}, X_t)^2 \right) \right) \text{Var}(X_{t+h}) - \text{Cov}(X_{t+h}, X_t)^2}$$
$$+ X_t \frac{2\beta \left([\text{Var}(X_t)]^2 - \text{Cov}(X_{t+h}, X_t)^2 \right) \left(2\alpha \left([\text{Var}(X_t)]^2 - \text{Cov}(X_{t+h}, X_t)^2 \right) + \text{Var}(X_t) \right)}{\left(\text{Var}(X_t) + 2\alpha \left([\text{Var}(X_t)]^2 - \text{Cov}(X_{t+h}, X_t)^2 \right) \right) \text{Var}(X_{t+h}) - \text{Cov}(X_{t+h}, X_t)^2}.$$

In order to deduce overreaction for long enough horizons, let us focus on the coefficient that multiplies X_t. Letting $h \to \infty$ and relying on the hypotheses, we get that the first addendum

goes to zero, while coefficient of the second one converges to

$$\frac{2\beta[\text{Var}(X_t)]^2\left[2\alpha[\text{Var}(X_t)]^2+\text{Var}(X_t)\right]}{[\text{Var}(X_t)]^2+2\alpha[\text{Var}(X_t)]^3}>0=\lim_{h\to\infty}\frac{\text{Cov}(X_{t+h},X_t)}{\text{Var}(X_t)},$$

proving the desired claim.

■

Natural Expectations (Level) The steps above can be repeated by simply using the stationary distribution for AR(2)

$$y_{t+1}=\rho_1 y_t+\rho_2 y_{t-1}+\epsilon_t$$

Instead of the one of an AR(1). Note that the stationary variance-covariance matrix of (Y_t,Y_{t+1}) is

$$\begin{bmatrix}\dfrac{\sigma^2}{1-\rho_1^2\left(\dfrac{1+\rho_2}{1-\rho_2}\right)-\rho_2^2} & \dfrac{\rho_1}{1-\rho_2}\dfrac{\sigma^2}{1-\rho_1^2\left(\dfrac{1+\rho_2}{1-\rho_2}\right)-\rho_2^2}\\ \dfrac{\rho_1}{1-\rho_2}\dfrac{\sigma^2}{1-\rho_1^2\left(\dfrac{1+\rho_2}{1-\rho_2}\right)-\rho_2^2} & \dfrac{\sigma^2}{1-\rho_1^2\left(\dfrac{1+\rho_2}{1-\rho_2}\right)-\rho_2^2}\end{bmatrix}$$

Hence we get as limit mnemonic conditional expectation:

$$E(x)=\frac{2ab\alpha x\left(-b^2+1\right)+2a\beta x\left(-b^2+1\right)\left(2a\alpha\left(-b^2+1\right)+1\right)}{\left(1+2a\alpha\left(-b^2+1\right)\right)\left(1+2a\beta\left(-b^2+1\right)\right)-b^2}$$

Where the stationary variance is

$$a=\frac{\sigma^2}{1-\rho_1^2\left(\dfrac{1+\rho_2}{1-\rho_2}\right)-\rho_2^2}$$

and the stationary correlation is

$$b=\frac{\rho_1}{1-\rho_2}.$$

Note that if the Humean component explodes while $\beta=0$,

$$\lim_{\alpha\to\infty}\frac{2ab\alpha x\left(-b^2+1\right)+2a\beta x\left(-b^2+1\right)\left(2a\alpha\left(-b^2+1\right)+1\right)}{\left(1+2a\alpha\left(-b^2+1\right)\right)\left(1+2a\beta\left(-b^2+1\right)\right)-b^2}=bx=\frac{\rho_1}{1-\rho_2}x.$$

Note that this value coincides with the stationary correlation value of the AR(2) DGP.

As for the AR(1) case, the Humean component of memory allows the DM to "mnemonically learn" the one period corellation structure of the DGP. Still, if under the AR(1)-Markovian structure mnemonic learning was sufficiently sophisticated learning heuristic to achieve the rational expectation benchmark, with a slightly more involved DGP, Humean Rationality leads to a forecasting behavior which is biased. Indeed note that the bias B is such that

$$B(x_t, x_y) = \left(\frac{\rho_1}{1-\rho_2} - \rho_1\right) x_t - \rho_2 x_y = \left(\frac{\rho_1 \rho_2}{1-\rho_2}\right) x_t - \rho_2 x_y.$$

Proof of Proposition 3. We first derive the expressions for the coefficients in the theoretical counterpart of the OLS regression (3), i.e.,

$$\pi_{t+h} - \mathbb{E}_t^m[\pi_{t+h}] = \alpha_h + \delta_{1,h}\left(\mathbb{E}_t^m[\pi_{t+h}] - \mathbb{E}_{t-1}^m[\pi_{t+h}]\right) + \delta_{2,h}\mathbb{E}_{t-1}^m[\pi_{t+h}] + \varepsilon_{t+h}. \tag{45}$$

From (17), $e_h \equiv \mathbb{E}_t^m[\pi_{t+h}]/\pi_t$ is independent of π_t. We can then define the forecast error $FE_{t,t+h} \equiv \pi_{t+h} - \mathbb{E}_t^m[\pi_{t+h}] = \pi_{t+h} - e_h\pi_t$ and the forecast revision $FR_{t,t+h} \equiv \mathbb{E}_t^m[\pi_{t+h}] - \mathbb{E}_{t-1}^m[\pi_{t+h}] = e_h\pi_t - e_{h+1}\pi_{t-1}$. To obtain the OLS coefficients, we use (45) and the assumption of orthogonality of ε_{t+h}, to obtain

$$\begin{aligned}
Cov(FE_{t,t+h}, FR_{t,t+h}) &= \delta_{1,h}Var(FR_{t,t+h}) + \delta_{2,h}Cov(FR_{t,t+h}, \mathbb{E}_{t-1}^m[\pi_{t+h}]),\\
Cov(FE_{t,t+h}, \mathbb{E}_{t-1}^m[\pi_{t+h}]) &= \delta_{1,h}Cov(FR_{t,t+h}, \mathbb{E}_{t-1}^m[\pi_{t+h}]) + \delta_{2,h}Var(\mathbb{E}_{t-1}^m[\pi_{t+h}]).
\end{aligned}$$

Solving for $\delta_{1,h}$ and $\delta_{2,h}$ yields

$$\delta_{1,h} = \frac{Cov(FE_{t,t+h}, FR_{t,t+h})Var(\mathbb{E}_{t-1}^m[\pi_{t+h}]) - Cov(FE_{t,t+h}, \mathbb{E}_{t-1}^m[\pi_{t+h}])Cov(\mathbb{E}_{t-1}^m[\pi_{t+h}], FR_{t,t+h})}{Var(FR_{t,t+h})Var(\mathbb{E}_{t-1}^m[\pi_{t+h}]) - Cov(FR_{t,t+h}, \mathbb{E}_{t-1}^m[\pi_{t+h}])^2},$$

$$\delta_{2,h} = \frac{Cov(FE_{t,t+h}, \mathbb{E}_{t-1}^m[\pi_{t+h}])Var(FR_{t,t+h}) - Cov(FE_{t,t+h}, FR_{t,t+h})Cov(FR_{t,t+h}, \mathbb{E}_{t-1}^m[\pi_{t+h}])}{Var(FR_{t,t+h})Var(\mathbb{E}_{t-1}^m[\pi_{t+h}]) - Cov(FR_{t,t+h}, \mathbb{E}_{t-1}^m[\pi_{t+h}])^2}.$$

We compute each of the theoretical moments of interests:

$$\begin{aligned}
Var(FR_{t,t+h}) &= (e_h^2 + e_{h+1}^2 - 2\rho e_h e_{h+1})Var(\pi_t),\\
Var(\mathbb{E}_{t-1}^m[\pi_{t+h}]) &= e_{h+1}^2 Var(\pi_{t-1}),\\
Cov(FR_{t,t+h}, \mathbb{E}_{t-1}^m[\pi_{t+h}]) &= (\rho e_h e_{h+1} - e_{h+1}^2)Var(\pi_{t-1}),\\
Cov(FE_{t,t+h}, \mathbb{E}_{t-1}^m[\pi_{t+h}]) &= (\rho^{h+1}e_{h+1} - \rho e_h e_{h+1})Var(\pi_{t-1}),\\
Cov(FE_{t,t+h}, FR_{t,t+h}) &= (\rho^h e_h - \rho^{h+1}e_{h+1} - e_h^2 + \rho e_h e_{h+1})Var(\pi_t).
\end{aligned}$$

Substituting the above moments into the expression for $\delta_{1,h}$ yields

$$\begin{aligned}\delta_{1,h} &= \frac{(\rho^h e_h - \rho^{h+1}e_{h+1} - e_h^2 + \rho e_h e_{h+1})e_{h+1}^2 - (\rho^{h+1}e_{h+1} - \rho e_h e_{h+1})(\rho e_h e_{h+1} - e_{h+1}^2)}{(e_h^2 + e_{h+1}^2 - 2\rho e_h e_{h+1})e_{h+1}^2 - (\rho e_h e_{h+1} - e_{h+1}^2)^2} \\ &= \frac{\rho^h(1-\rho^2)e_h e_{h+1}^2 - (1-\rho^2)e_h^2 e_{h+1}^2}{(1-\rho^2)e_h^2 e_{h+1}^2} \\ &= \frac{\rho^h}{e_h} - 1.\end{aligned}$$

Using the definition of e_h, together with (17), gives $\delta_{1,h} = \delta_h$, where δ_h is given by (21). Analogous steps prove that $\delta_{2,h} = \delta_h$.

Part (i). Differentiating (21) with repsect to h,

$$\begin{aligned}\frac{\partial \delta_h}{\partial h} =& \frac{\rho^h \log \rho}{2\sigma_\pi^2[\alpha\rho^h + \beta + 2\alpha\beta\sigma_\pi^2(1-\rho^{2h})]^2} \times \\ & \big([2\sigma_\pi^2(\alpha+\beta) + 4\alpha\beta(\sigma_\pi^2)^2(1-3\rho^{2h}) + 1][\alpha\rho^h + \beta + 2\alpha\beta\sigma_\pi^2(1-\rho^{2h})] \\ & -[2\sigma_\pi^2(\alpha+\beta) + 4\alpha\beta(\sigma_\pi^2)^2(1-\rho^{2h}) + 1][\alpha\rho^h - 4\alpha\beta\sigma_\pi^2\rho^{2h}]\big),\end{aligned}$$

which is negative if and only if

$$\begin{aligned}&\big([2\sigma_\pi^2(\alpha+\beta) + 4\alpha\beta(\sigma_\pi^2)^2(1-3\rho^{2h}) + 1][\alpha\rho^h + \beta + 2\alpha\beta\sigma_\pi^2(1-\rho^{2h})] \\ &-[2\sigma_\pi^2(\alpha+\beta) + 4\alpha\beta(\sigma_\pi^2)^2(1-\rho^{2h}) + 1][\alpha\rho^h - 4\alpha\beta\sigma_\pi^2\rho^{2h}]\big) > 0\end{aligned}$$

or, after straightforward algebra,

$$\beta + 2\beta^2\sigma_\pi^2 + 2\alpha\beta\sigma_\pi^2[2 + 4\beta\sigma_\pi^2(1-\rho^{2h}) + \rho^{2h}] + 4\alpha^2\beta(\sigma_\pi^2)^2[2\beta\sigma_\pi^2(1-\rho^{4h})^2 + 1 + \rho^{2h} - 2\rho^{3h}] > 0.$$

The first square bracket is clearly positive. The second square bracket is also positive since $\rho^{2h} > \rho^{3h}$ for $0 < \rho < 1$ and $h \geq 1$. As a result, δ_h is decreasing in h. Finally, from equation (21), it is immediate to verify that $\delta_h \to -1$.

Part (ii). From (21) with $h = 1$,

$$\begin{aligned}\delta_1 &= \frac{\alpha\rho + \beta\rho + 2\alpha\beta\rho\sigma_\pi^2(1-\rho^2) + \rho/2\sigma_\pi^2}{\alpha\rho + \beta + 2\alpha\beta\sigma_\pi^2(1-\rho^2)} - 1 \\ &= \frac{-2\beta(1-\rho) - 4\alpha\beta(1-\rho)\sigma^2 + (1-\rho^2)\rho/\sigma^2}{2\alpha\rho + 2\beta + 4\alpha\beta\sigma^2} \\ &= (1-\rho)\frac{\rho/\sigma^2 + \rho^2/\sigma^2 - 2\beta - 4\alpha\beta\sigma^2}{2\alpha\rho + 2\beta + 4\alpha\beta\sigma^2}.\end{aligned}$$

The latter is negative if and only if

$$\rho + \rho^2 - 2\beta\sigma^2(1 + 2\alpha\sigma^2) < 0,$$

which is condition (22). ■

Proof of Proposition 4. We derive an expression for the coefficient θ_h^r of the OLS regression (**??**). From the definition of real rate (**??**), we have

$$\widetilde{r}_t^{(h)} = r^{(h)} - \frac{1}{h}\sum_{j=1}^{h}\pi_{t+j} + \frac{1}{h}\sum_{j=1}^{h}\mathbb{E}_t^m[\pi_{t+j}]$$

or, using the process for inflation (16),

$$\widetilde{r}_t^{(h)} = r^{(h)} - \frac{1}{h}\sum_{j=1}^{h}(\rho^j\pi_t - \mathbb{E}_t^m[\pi_{t+j}]) + \sigma\frac{1}{h}\sum_{j=1}^{h}\epsilon_{t+j}.$$

From (17), we let $e_h \equiv \mathbb{E}_t^m[\pi_{t+h}]/\pi_t$, which is independent of π_t. As a result,

$$\widetilde{r}_t^{(h)} = r^{(h)} - \pi_t\frac{1}{h}\sum_{j=1}^{h}(\rho^j - e_j) + \sigma\frac{1}{h}\sum_{j=1}^{h}\epsilon_{t+j}.$$

Similarly,

$$\mathbb{E}_t^m\left(\overline{\pi_{t+1,h}}\right) = \frac{1}{h}\sum_{j=1}^{h}\mathbb{E}_t^m\pi_{t+j} = \pi_t\frac{1}{h}\sum_{j=1}^{h}e_j.$$

The OLS coefficient θ_h^r satisfies

$$\begin{aligned}\theta_h^r &= \frac{Cov\left(\widetilde{r}_t^{(h)}, \mathbb{E}_t^m\left(\overline{\pi_{t+1,h}}\right)\right)}{Var\left(\mathbb{E}_t^m\left(\overline{\pi_{t+1,h}}\right)\right)} \\ &= -\frac{\left(\frac{1}{h}\sum_{j=1}^{h}(\rho^j - e_j)\right)\left(\frac{1}{h}\sum_{j=1}^{h}e_j\right)}{\left(\frac{1}{h}\sum_{j=1}^{h}e_j\right)^2} \\ &= -\frac{\sum_{j=1}^{h}e_j\delta_j}{\sum_{j=1}^{h}e_j},\end{aligned}$$

where the last line uses the fact that $\delta_h = \rho^j/e_j - 1$ from Proposition 3. Equation (23) follows from letting

$$w_j \equiv \frac{e_j}{\sum_{j=1}^{h}e_j}. \tag{46}$$

Next, we derive an expression for the coefficient θ_h^{rx} of the OLS regression (**??**). Above we showed that $\mathbb{E}_t^m\left(\overline{\pi_{t+1,h}}\right) = \left(\pi_t \sum_{j=1}^h e_j\right)/h$. By the same logic,

$$\mathbb{E}_{t+1}^m\left(\overline{\pi_{t+2,h}}\right) = \pi_{t+1}\frac{1}{h-1}\sum_{j=1}^{h-1} e_j$$

and

$$\mathbb{E}_t^m\left(\overline{\pi_{t+2,h}}\right) = \pi_t\frac{1}{h-1}\sum_{j=2}^{h} e_j.$$

Substituting the latter into (8) gives

$$\begin{aligned} rx_{t,t+1}^{(h)} &= -(h-1)r^{(h-1)} + hr^{(h)} - r^{(1)} - \pi_{t+1}\sum_{j=1}^{h-1} e_j + \pi_t \sum_{j=2}^{h} e_j \\ &= -(h-1)r^{(h-1)} + hr^{(h)} - r^{(1)} - \pi_t\sum_{j=1}^{h-1}\rho e_j + \pi_t\sum_{j=2}^{h} e_j - \sigma\epsilon_{t+1}\sum_{j=1}^{h-1} e_j. \end{aligned}$$

Therefore,

$$\begin{aligned} \theta_h^{rx} &= \frac{Cov\left(rx_{t,t+1}^{(h)}, \mathbb{E}_t^m\left(\overline{\pi_{t+1,h}}\right)\right)}{Var\left(\mathbb{E}_t^m\left(\overline{\pi_{t+1,h}}\right)\right)} \\ &= -\frac{\left(\sum_{j=1}^{h-1}\rho e_j - \sum_{j=2}^{h} e_j\right)\left(\frac{1}{h}\sum_{j=1}^{h} e_j\right)}{\left(\frac{1}{h}\sum_{j=1}^{h} e_j\right)^2} \\ &= -\frac{\sum_{j=1}^{h-1}\rho e_j - \sum_{j=2}^{h} e_j}{\frac{1}{h}\sum_{j=1}^{h} e_j}. \end{aligned} \tag{47}$$

Using the fact that $\delta_j = \rho^j/e_j - 1$ yields

$$\begin{aligned} \sum_{j=1}^{h-1}\rho e_j - \sum_{j=2}^{h} e_j &= \sum_{j=1}^{h-1}(\rho^{j+1} - \rho e_j\delta_j) - \sum_{j=2}^{h}(\rho^j - e_j\delta_j) \\ &= \sum_{j=2}^{h} e_j\delta_j - \rho\sum_{j=2}^{h} e_{j-1}\delta_{j-1}. \end{aligned}$$

Substituting the latter into (47) and rearranging gives

$$\theta_h^{rx} = h\Big(-\frac{\sum_{j=2}^{h} e_j\delta_j}{\sum_{j=1}^{h} e_j} + \rho\frac{\sum_{j=2}^{h} e_{j-1}\delta_{j-1}}{\sum_{j=1}^{h} e_j}\Big).$$

From (23) and (46), the first term in the bracket equals $\theta_h^{rx}+\delta_1 w_1$, while the second term equals $-\rho(\theta_h^{rx}+\delta_h w_h)$. Therefore,

$$\begin{aligned}\theta_h^{rx} &= h\Big(-\frac{\sum_{j=2}^{h} e_j\delta_j}{\sum_{j=1}^{h} e_j}+\rho\frac{\sum_{j=2}^{h} e_{j-1}\delta_{j-1}}{\sum_{j=1}^{h} e_j}\Big)\\ &= h\Big((1-\rho)\theta_h^{rx}+\delta_1 w_1-\rho\delta_h w_h)\Big),\end{aligned}$$

which is (24).

Evaluating (17) at $\beta = 0$ yields

$$e_h = \frac{\alpha\rho^h}{\alpha + 1/2\sigma_\pi^2},$$

hence, $\rho e_h = e_{h+1}$, which implies $\theta_h^{rx} = 0$ from (47).

Evaluating (17) at $\alpha = 0$ yields

$$e_h = \frac{\beta}{\beta + 1/2\sigma_\pi^2},$$

hence, $e_h = e_{h+1}$, which implies $\theta_h^{rx} = (1-\rho)(h-1)$ from (47). ■

K Simulation Algorithm

1. **Parameters' Inizialitation:**

   - Define forecast horizons $h = 1, 2, \ldots, 10$ years.
   - Construct a grid of parameter values for $(\alpha, \beta) \in [0, 1]^2$ with a step size of 0.01, yielding over 10,000 possible parameter pairs.

2. **Data Loading:** For each country c, load annual realized inflation data, $\pi_{c,t}$, covering the period from t_0 to T as specified in Appendix Table 8, which details the starting and ending dates for each country based on the available data sources.

3. **Forecasts' Simulations:** For each parameter combination (α, β), for each country c, for each horizon h (from 1 to 10), and for each period t:

   - Calculate the model-implied expectation $E_t^m(\pi_{t+h,c} \mid \alpha, \beta)$ as the weighted forecast:

$$E_t^m(\pi_{t+h,c} \mid \alpha, \beta) = \frac{\sum_{k=t_0}^{t} \pi_k^c \cdot e^{-\alpha(\pi_{k-h}^c-\pi_t^c)^2-\beta(\pi_k^c-\pi_t^c)^2}}{\sum_{k=t_0}^{t} e^{-\alpha(\pi_{k-h}^c-\pi_t^c)^2-\beta(\pi_k^c-\pi_t^c)^2}}$$

- For the 6-10 year horizon, calculate the average model-implied expectation:

$$E_t^m(\pi_{t+6,\ldots,t+10,c} \mid \alpha, \beta) = \frac{1}{5} \sum_{h=6}^{10} E_t^m(\pi_{t+h,c} \mid \alpha, \beta)$$

4. **Parameters' Estimation:**

- For each (α, β), compute the average absolute distance between observed and model-implied forecasts, where $F_t(\pi_{t+h,c})$ represents the observed forecast from the CE dataset averaged annually. Select the optimal parameter pair $(\hat{\alpha}, \hat{\beta})$ that minimizes this distance acccordingly:

$$\min_{\alpha,\beta} \sum_{t,c,h \in \{1,2,3,5,6-10\}} |F_t(\pi_{t+h,c}) - E_t^m(\pi_{t+h,c} \mid \alpha, \beta)|$$