Finance and Productivity Growth: The Role of Intangibles*

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March 20, 2025

Abstract

This paper investigates the impact of credit frictions on the contribution of intangible capital to productivity growth, using a structural model fitted to UK firm data and matching stylized facts on the relationship between intangibles, debt and TFP at the firm level. The model incorporates investment decisions in tangible and intangible assets, alongside debt issuance, with financial frictions modelled as a debt premium dependent on earnings-based leverage and intangible intensity. Employing a novel neural network estimation method for value function parameters, we find a significant decline in intangible capital's contribution to TFP growth post-2010, from approximately 8% to 3%. This decline is attributed to increased leverage constraining intangible investment. However, we also find evidence of a post-GFC increase in intangible-specific credit frictions, resulting in a 15% TFP growth reduction by 2018.

JEL Classification: D22, D24, D25, O30, O47, C45.
Keywords: Intangible Capital, Credit Constraints, Total Factor Productivity, Neural Networks.

*We would like to thank Saleem Bahaj, Maarten De Ridder, Jesus Fernández-Villaverde, Andreas Joseph, Sebnem Kalemli-Ozcan, Marco Pagano, Neeltje van Horen and three anonymous referees, as well as participants in the 2021 China Meeting of the Econometric Society, the 2021 and 2024 EEA-ESEM, the 2024 Meeting of the Royal Economic Society, the 2024 IARIW Meeting, and seminar participants at the Athens University of Economics and Business for their useful discussions and comments. Any views expressed here are solely those of the authors and do not represent those of the Bank of England or any of its committees. We remain solely responsible for the content of this paper and any errors or omissions that could remain.

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1 Introduction

Intangibles, including R&D, have become an increasingly important driver of innovation and economic growth in many economies around the world (see, e.g., Haskel and Westlake, 2017; Brown et al., 2009; De Ridder, 2018; Moran and Queralto, 2018; Anzoategui et al., 2019). According to the *World Intangible Investment Highlights*, intangible investments grew at three times the rate of tangible investments over the period 2008-2023.[1] However, intangibles are harder to finance than tangible investments for two reasons. First, intangible investments are subject to more severe information asymmetries, leading to investors assessing them as riskier. One of the reasons for severe information asymmetries is the high information complexity of intangible assets which makes them hard to evaluate (see, e.g., Gu and Wang, 2005). Second, intangibles suffer from low redeployability and uncertain liquidation values which limit their use as loan collateral (Williamson, 1988; Shleifer and Vishny, 1992; Falato et al., 2022).

Since credit frictions have been shown to affect the real economy through their impact on investment (see, e.g., Huber, 2018; Fakos et al., 2022), as well as innovation and productivity growth (see, e.g., Levine and Warusawitharana, 2021), the question arises to what extent financial frictions affecting intangible investment have contributed to weak Total Factor Productivity (TFP) growth since the global financial crisis (GFC). While intangible investment and financial frictions have separately been identified as important drivers of productivity growth, comprehensive evidence investigating how financial frictions affect the contribution of intangibles to productivity growth is still sparse.

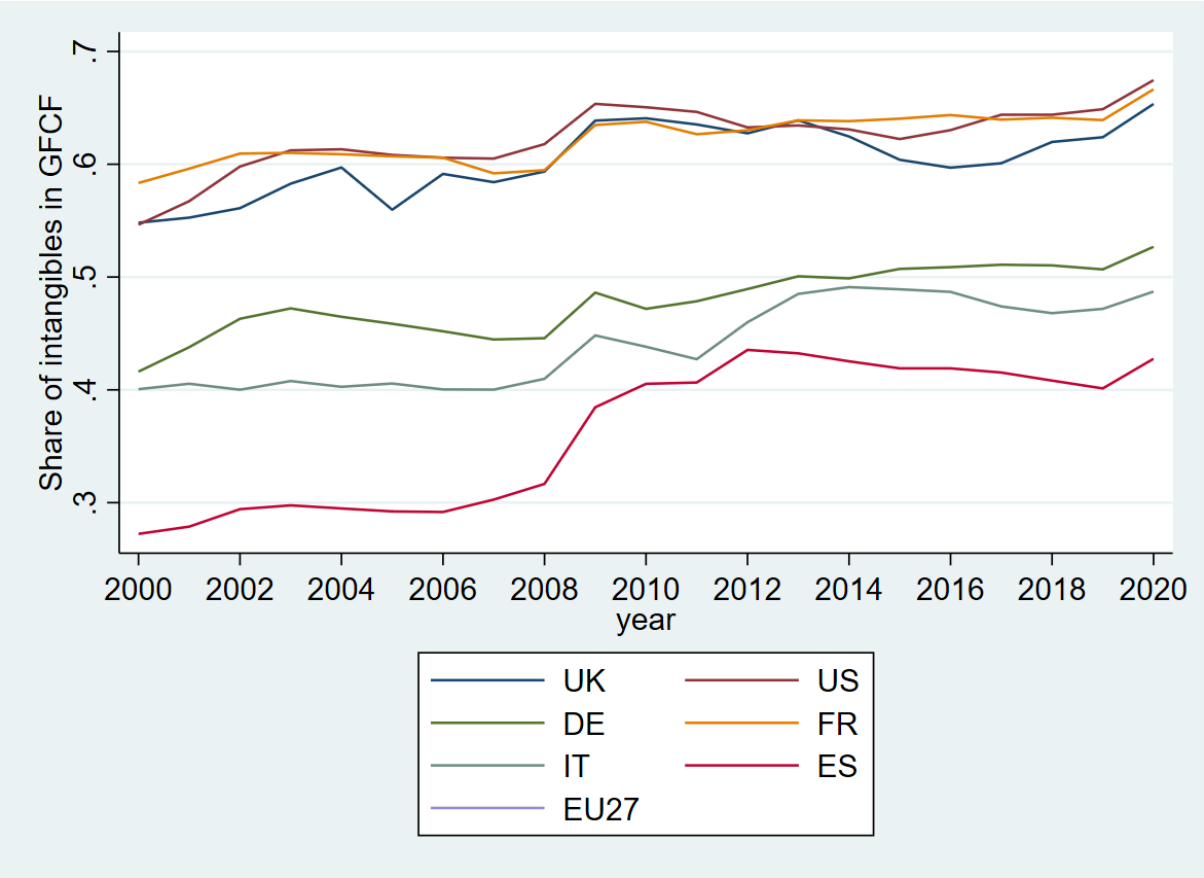
In this paper, we explore the contribution of intangibles to productivity growth in the UK, where financing frictions related to intangibles are likely to be particularly relevant. The UK's share of intangible investment in total investment is similar to that of the US, and higher than in most EU-27 economies (see Figure 1). In addition, the UK has experienced a significant TFP growth slowdown since the GFC (see Figure 2). In 2022, TFP was approximately 17% below trend, had it continued to grow at the average annual growth rate experienced over 1970-2007.

First, we document stylized facts about the relationship between intangibles and firm-level TFP using a panel of listed UK firms. We find a significantly positive association between intangibles (both capital and investment) and firm-level TFP. We also explore how intangibles mediate the relationship between firm-level TFP and leverage. Aggregate UK data suggests a strong positive correlation between corporate debt and investment, while the correlation between debt and productivity is more tenuous. However, at the firm level, the literature suggests that high corporate debt leads to lower investment, especially in times of crisis, with negative subsequent effects on productivity. Typically, leverage is assumed to be "good" in normal times, as it allows firms to invest in their productive capacity. Debt then becomes "bad" in a downturn owing to debt overhang reasons. Rather than focusing on *state*-dependency (i.e., effects of debt depending

[1] See https://www.wipo.int/pressroom/en/articles/2024/article_0008.html

on the state of the business cycle), we study *type*-dependency (i.e., effects of debt depending on what type of investments the firm undertakes). Interestingly, we find that high levels of debt are not necessarily bad for firm-level TFP, if the debt is accompanied by high levels of productive (intangible) investment, especially R&D. We adopt measures to mitigate endogeneity issues, such as using the unanticipated TFP component as our dependent variable. That said, our empirical results should be mainly interpreted as correlations that inform our structural model.

Figure 1: **Share of intangibles in total market economy investment**



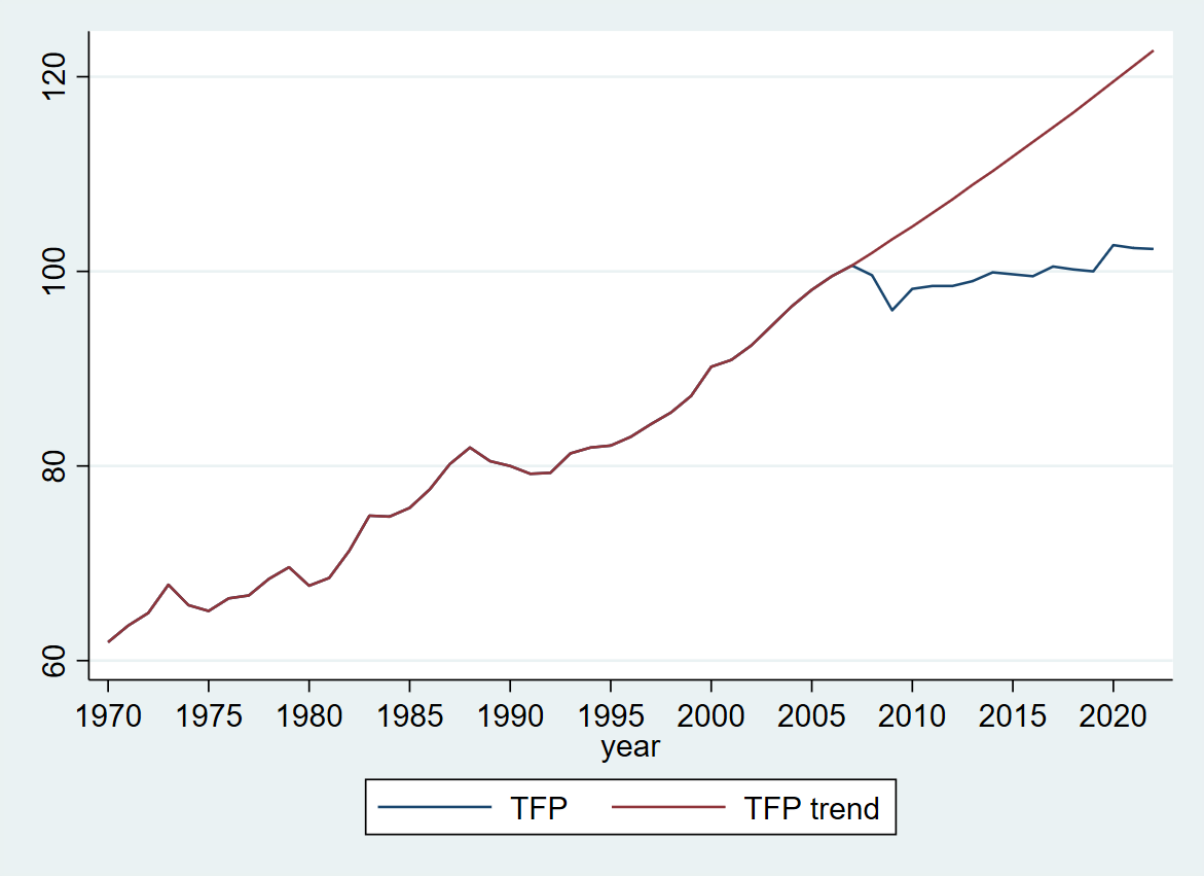
Notes: Market economy includes all NACE Rev.2 industries, excluding L, O, P, Q, T and U. Share defined as Total intangible GFCF over (Total intangible GFCF + Total tangible GFCF). GFCF measured at current prices, in millions of national currencies. Source: EUKLEMS and ITANProd.

Second, we develop a structural model which is consistent with the stylized facts on the relationship between tangibles, intangibles and firm-level TFP, as well as the significant interaction between intangibles and debt. This model is set up to replicate the salient features of the data. It builds on the traditional neoclassical model of Eberly et al. (2008), Warusawitharana (2015) and Levine and Warusawitharana (2021) (see also Moll (2014) and Midrigan and Xu (2014) for similar approaches). Importantly, we augment the model with an option for external debt financing. Debt enters the model both as a control and a state variable, which aids in a fuller characterization of the solution in a more realistic setting and allows us to replicate important stylized facts. Financial frictions enter our model through a premium on the cost of debt that depends on the firm's earnings-based leverage ratio, i.e. the ratio of debt to the firm's earnings.

This is essentially an earnings-based constraint (EBC), which links firms' current earnings and their access to debt (see, e.g., Drechsel (2023) for the macroeconomic implications of EBCs). In addition, the cost of debt increases with the share of intangible capital in the firm's overall capital stock, consistent with the fact that intangible capital is harder to liquidate in the event of default, making the firm riskier for lenders. These two financial frictions are controlled by parameters that can be estimated separately.

Methodologically, we propose a new way to estimate value function parameters using neural network (NN) methods. Our method is similar to the methods used in Maliar et al. (2021) and Fernández-Villaverde et al. (2023), who introduce the use of deep learning for approximating value functions. They show that NN methods are more efficient at approximating and optimizing unknown functions when the state space is large. We contribute to the literature by proposing a new way to estimate value function *parameters* based on the observed states and choice variables in our data set. Our approach is a moments-matching approach, where we make the parameters of the value function a part of the state space.

Figure 2: **UK market sector TFP, 1970 to 2022 (2019 = 100)**



Notes: Multi Factor Productivity (MFP) (2019 = 100), total market sector (SIC 2007 A to U). The TFP trend assumes an annual growth rate of 1.3% from 2008 onwards (average growth rate for 1970-2007). Source: ONS.

Finally, we use our model to quantify the effect of intangible investment, and the financial

frictions affecting it, on aggregate TFP growth in the UK. Our analysis of intangible capital's contribution to TFP growth reveals a notable shift around 2010. Prior to this period, intangible capital growth accounted for approximately 8% of TFP gains on average. However, this contribution subsequently declined to roughly 3% between 2010 and 2018. This deterioration can partly be attributed to a marked increase in earnings-based leverage that constrained intangible investment. However, a comparative analysis of the pre- and post-GFC periods also highlights a significant worsening of credit frictions specifically related to intangible assets, ultimately leading to an estimated 15% reduction in TFP growth by 2018.

The rest of the paper is organized as follows. Section 2 briefly surveys the related literature and highlights our contributions. Section 3 presents some stylized facts on the relationship between intangibles and firm-level TFP, as well as the mediating role of intangibles in the relationship between debt and TFP. Section 4 presents a structural model that is consistent with these stylized facts and lays out how we estimate its parameters using neural network methods. Section 5 presents the counterfactual exercises to shed light on the relevance of intangibles for TFP growth and quantify the effect of financial frictions. Section 6 concludes.

2 Related literature

This paper is related to four strands of the literature. The first strand demonstrates that intangibles are an important driver of innovation and economic growth. Anzoategui et al. (2019) apply an endogenous growth model to US data and find that the long-term productivity slowdown is mainly due to lower R&D investment and spillovers. Moran and Queralto (2018) estimate a strong and persistent effect of R&D shocks on TFP with US aggregate data. And OECD (2016) finds a causal link between R&D spending and productivity in a large panel of advanced-economy firms. These studies speak to the fact that an important channel through which financial frictions affect productivity growth is their impact on innovation (see, e.g., Aghion et al., 2010; Levine and Warusawitharana, 2021)

The second strand of literature relates to the funding of intangible assets, namely the specific financial frictions affecting intangible investment. While tangible assets, such as property, plant, and equipment, can be used as collateral to obtain loans, theory predicts that lenders are more reluctant to lend against intangibles because they are more likely to be firm specific or have more uncertain liquidation values (Williamson, 1988; Shleifer and Vishny, 1992). Empirical evidence broadly supports that theory. Intangible intensive firms are more credit constrained and invest mostly from internal funds (see, e.g., Almeida and Campello, 2007; Brown et al., 2009, 2012; Chen, 2014; Haskel and Westlake, 2017; Lei et al., 2018; Demmou et al., 2019; Demmou et al., 2020; Dell'Ariccia et al., 2021; Falato et al., 2022; Lee and Paluszynski, 2022; Bøler et al., 2023).[2]. Falato et al. (2022) argue that the shift toward intangible capital reduces firms' debt

[2]Kermani and Ma (2023) distinguish themselves from the consensus in the literature and provide evidence that the increasing prevalence of intangible assets has not significantly reduced the liquidation

capacity and leads them to hoard more cash – explaining much of the secular upward trend in US corporate cash holdings. Lee and Paluszynski (2022) document that intangible-intensive firms disproportionately borrow from non-bank lenders and argue that higher collateral requirements drive them away from bank borrowing.

The third related strand of literature looks at the macroeconomic implications of intangibles through the lens of financing frictions. De Ridder (2018) shows that tighter credit conditions during the financial crisis had a negative effect on R&D and intangible investment and TFP growth after the crisis.[3] Lopez and Olivella (2018) show that intangible capital amplifies the effects of financial shocks on the real economy by tilting investment towards tangibles. Caggese and Pérez-Orive (2022) show that the increase in the intensity of intangible capital in production has decreased the sensitivity of investment to lower interest rates. This is because intangible-intensive firms rely relatively more on corporate savings, the accumulation of which is slowed down by low interest rates. Bøler et al. (2023) make use of a natural experiment to estimate the magnitude of collateral constraints and to quantify the aggregate impact of these constraints on productivity. They find quantitatively large gains in output per worker in sectors dominated by constrained and intangible-intensive firms. Finally, Döttling and Ratnovski (2023) show that intangible investment responds less to monetary policy compared to tangible investment, particularly so among financially constrained firms, indicating that corporate intangible capital weakens the credit channel of monetary policy transmission.

Our paper is also indirectly related to the literature studying the link between debt and productivity. Duval et al. (2020) study the effects of various financial vulnerabilities on firm-level TFP in advanced economies, finding that firms with weak balance sheets prior to the financial crisis performed worse in terms of TFP since the crisis. Following a more structural approach, Gopinath et al. (2017) develop a model that helps to explain how financial frictions can lead to capital misallocation and lower aggregate productivity. Franklin et al. (2015) find that contractions in credit supply after the financial crisis led to lower productivity in a sample of UK firms. Doerr et al. (2018) report negative effects of credit shocks on investment and productivity of Italian firms. Huber (2018) finds evidence for negative aggregate demand effects of lower credit on innovation and productivity for German firms, irrespective of their direct exposure to the credit shocks. In the context of the UK, Bahaj et al. (2017) show evidence of a U-shaped relationship between firms' indebtedness and levels of productivity. Coricelli et al. (2012) find an inverted U-shaped relationship between leverage and TFP growth in a sample of Central and Eastern European countries that experienced strong credit growth - providing evidence that "excessive" leverage can be unproductive. Generally, these papers do not explore how the type of investment a firm undertakes affects the relationship between debt and TFP or TFP growth.

values of non-financial firms in the US.

[3]By contrast, De Ridder (2022) develops a general-equilibrium model, and also finds empirical evidence, for a negative long-term effect of intangible investment on TFP, as large firms carrying out intangible investment gain market power, which deters entry of new firms and creative destruction.

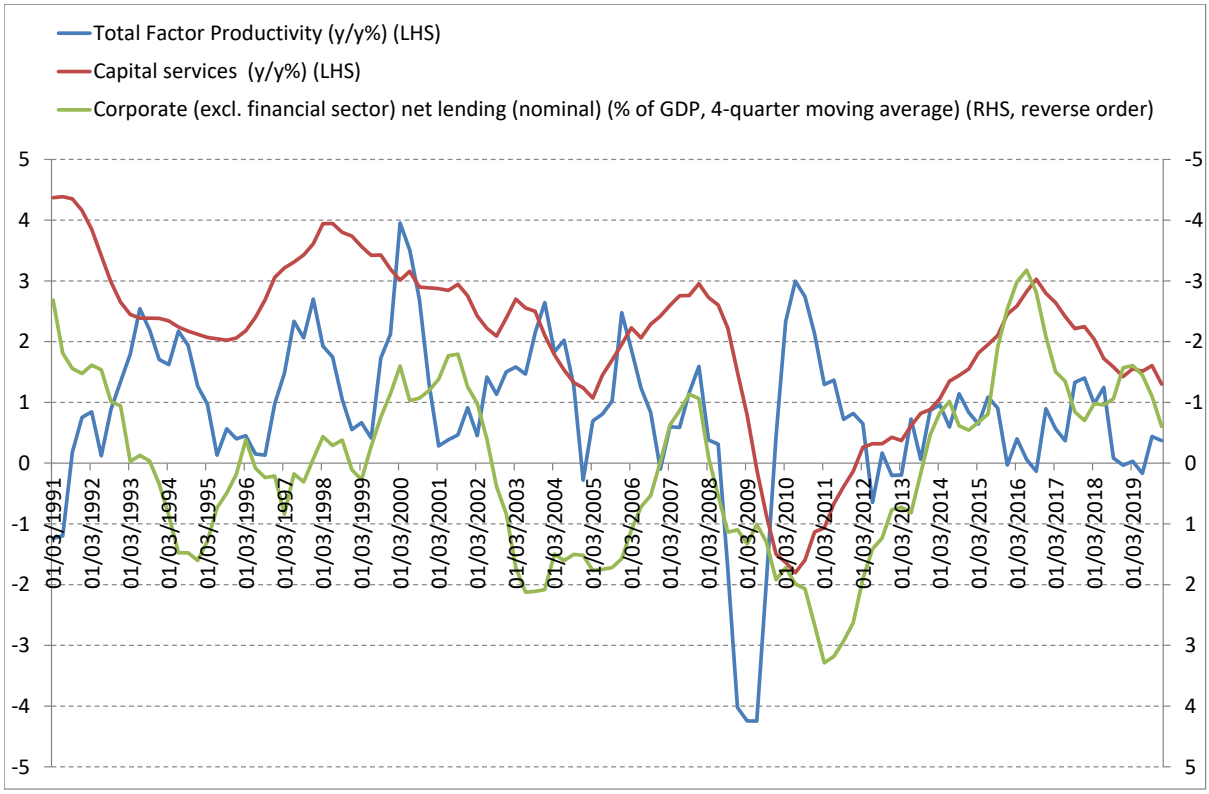
We contribute to the various strands of literature above by exploring how financial frictions that specifically affect intangibles influence the latter's contribution to productivity growth. We achieve this by developing a stylized structural model where firms optimally choose to invest in two types of assets (tangible and intangibles assets) and whether to issue new debt in order to do so. The model is itself motivated by two stylized facts derived from a sample of large UK firms. In the model, debt financing costs increase in the firm's leverage and the share of intangible capital in the overall capital stock. Higher intangible intensity is assumed to magnify the impact of leverage on external financing costs because of the specific frictions affecting intangibles, including information asymmetry and low deployability as collateral. Our contribution also lies in the methodological domain, where we rely on neural network methods to estimate value function parameters consistent with the data. To the best of our knowledge, we are the first to propose such an approach. Finally, the estimated model is used to quantify the contribution of intangibles, and the financial frictions affecting them, to TFP growth.

3 Stylized facts

3.1 Empirical model

The relationship between investment, debt, and TFP is not straightforward. Figure 3 shows aggregate time series on investment (measured by the growth in capital services), TFP growth and the level of corporate sector debt as a percentage of GDP in the UK. As the chart suggests, there is a relatively strong positive correlation between investment and debt in the past few decades (the contemporaneous correlation coefficient is 0.6) - firms typically use debt to invest. However, the link between investment and TFP growth, and debt and TFP growth, is much more tenuous (the correlation coefficient between both investment and TFP growth, and debt and TFP growth, is 0.1). There is a myriad of factors that affect TFP, and there are time lags in their effects. In addition, not all investment is driven by technological progress that translates into higher TFP. We aim to shed light on these relationships by going beyond the aggregate numbers, and analyzing firm-level heterogeneity.

Figure 3: **Aggregate TFP, capital services and corporate debt**



Notes: TFP is calculated from a basic Cobb-Douglas production function, with a labor share of 2/3.
Sources: ONS and own calculations.

We use the natural logarithm of firm-level TFP as the left-hand side variable.[4] We use various measures of tangibles and intangibles as our main explanatory variables (see Section 3.2), namely the firm's tangible capital stock and investment and its intangible capital stock and investment. The purpose of making this distinction is to study whether different types of investment have heterogeneous effects on TFP, and explore whether they affect the relationship between debt and TFP. Therefore, we also include a measure of leverage and its interaction with tangibles and intangibles. We also control for various firm-level characteristics (age, size, cash holdings and profits[5]) that can be expected to have an effect on TFP. All our baseline regressions include firm and time fixed effects. For our baseline analysis, we use simple OLS panel regressions with lagged explanatory variables[6] of the following form:

[4]Note that the structural model of Section 4 delivers predictions on the *level* of firm-level TFP, so our empirical analysis is formulated accordingly.

[5]We include profits as a control variable to distinguish between profitable and productive firms. Although the two concepts can be closely related empirically, in our sample the correlation coefficient between the two variables is below 0.1.

[6]These estimations are subject to obvious endogeneity issues, in particular related to simultaneity bias. Our baseline method mitigates endogeneity somewhat by including lagged, rather than contemporaneous, values of all the explanatory variables. It does not, however, eradicate it (see, for example, Leszczensky and Wolbring, 2022). Therefore, we also estimated a system-GMM version of the model (Arellano and Bover, 1995; Blundell and Bond, 1998). We do not report the results of the system-GMM for brevity, but they are qualitatively in line with our baseline results.

$$z_{it} = \alpha + \beta_1 D_{i,t-1} + \beta_2 R_{i,t-1} + \beta_3 D_{i,t-1} R_{i,t-1} + c_i + f_t + X_{i,t-1} + e_{it} \quad (1)$$

where z_{it} is the natural logarithm of TFP of firm i at time t, $D_{i,t-1}$ is the debt ratio (leverage) of firm i at time $t-1$, $R_{i,t-1}$ is one of four measures of tangible or intangible intensity, $X_{i,t-1}$ is a vector of firm characteristics (sector, age, size, profits, cash) and e_{it} is an i.i.d. error term. To facilitate the interpretation of the interaction between leverage and tangible and intangible intensity, we use binary measures of the latter (see Section 3.2). The way we use interaction terms between continuous and dummy variables is common in the recent micro-level panel data literature (see, e.g., Buera and Karmakar, 2021; Chodorow-Reich, 2014; Giroud and Mueller, 2017; and Joseph et al., 2020).[7]. We use industry-median corrected values for the debt ratio[8]. The terms c_i and f_t are firm and time fixed effects, respectively.

3.2 Data

The firm-level data comes from Refinitiv Worldscope, which is a proprietary dataset that includes financial account information on large (mainly listed) UK firms. Even though the number of firms in our sample is a small proportion of all firms in the UK economy, the tail of large firms can have significant aggregate-level productivity effects, as shown in Dacic and Melolinna (2022). Our sample spans the period 1990-2018. Table 1 provides definitions for all the variables used in the analysis, as well as their Worldscope identifiers. The key items for our analysis are the dependent variable, productivity (measured by firm-level TFP), and the main explanatory variables, i.e. leverage, and measures of tangible and intangible intensity. The exact definitions of the firm-level financial account data series and the aggregate-level deflators are provided in Appendix A.1.

Table 1: **Variable definitions**

Variable	Definition	Worldscope code
TFP	Ackerberg-Caves-Frazer (2015) measure of TFP (log level)	n/a
Intangible intensity - stock	Intangible capital stock over total assets	n/a
Intangible intensity - flow	Year-on-year change in intangible capital stock over total assets	n/a
Leverage ratio	Total debt over total assets (corrected for industry medians)	totdebt/totass
Tangible intensity - stock	Fixed tangible assets over total assets	ppegross/totass
Tangible intensity - flow	Capital expenditure over total assets	(ppegross(t)-ppegross(t-1))/totass
Age	Years since incorporation	age
Size	Real total assets (log level, deflated by aggregate GDP deflator)	totass
Profitability	Profits (EBITDA) over total assets	ebitda/totass
Cash ratio	Cash and short-term investment over total assets	csti/totass
Interest expense ratio	Interest expense over total debt	intex/totdebt

[7]This approach is also consistent with the way we derive data moments for the structural model - see Section 4 for details.

[8]This approach is similar to that of Levine and Warusawitharana (2021) and provides a way of controlling for time-varying industry-specific effects. The results below are also robust to not using the median correction.

There are different options for measuring firm-level TFP, none of which is without challenges. TFP is not easy to calculate from firm-level data because only proxies for capital, labour and other inputs are typically available in the data, and there is no firm-level data on prices.[9] We use an established TFP-estimation method based on a production function approach suggested by Ackerberg et al. (2015) (building on the method originally introduced by Olley and Pakes, 1996).[10] We use both tangible (i_{it}) and intangible (ii_{it}) investment as instruments to yield the following proxy for TFP (ω_{it}):

$$\omega_{it} = h(i_{it}, ii_{it}, \mathbf{x}_{it}) \tag{2}$$

where $\mathbf{x}_{it}$ is the state variable (capital stock) and $h(.)$ is a monotonous, increasing function of unknown form.[11] From the point of view of the production function estimation, TFP is assumed to be an exogenous Markov process, while the TFP process in our empirical specifications can potentially depend on variables other than the ones included in the production function. The inclusion of both tangible and intangible investment in Equation (2) helps to mitigate this unavoidable conceptual discrepancy.

Appendix A.1 defines the variables used in the production function estimation. The estimation is carried out at the 1-digit SIC sector level. We do not report the detailed estimation results for the sake of brevity, but it is worth mentioning that the weighted (by number of firms per sector) average coefficients for labour and capital inputs are 0.74 and 0.17, respectively. These coefficients are statistically significant at the 1% level for nearly all sectors, and the assumption of constant returns to scale is rejected for most sectors. We use these average coefficients to calibrate our structural model (see Section 4).

We measure a firm's tangible and intangible intensity as follows:

- Intangible intensity is defined as intangible assets over total assets (stock measure) or intangible investment (change in intangible assets) over total assets (flow measure). Our methodology for measuring intangible assets is explained in Appendix A.2.

[9] Strictly speaking, the measure we are using is therefore revenue TFP (TFPR). In the absence of firm-level price data, quantity-based TFP (TFPQ) cannot be estimated. This is an important caveat to keep in mind when interpreting our results, and implies that the regression coefficients are downward biased, and the TFPR measure may include a demand shock component (see Barterlsman and Wolf, 2018). Nevertheless, using real values (deflated with relevant aggregate GDP and investment deflators) for the revenue TFP measure is much closer conceptually to technological progress than revenue labour productivity. We call it TFP for simplicity in the analysis that follows.

[10] Another popular control function approach was introduced by Wooldridge (2009). We have replicated our main analysis with this TFP measure. The results are very similar to our baseline results reported below.

[11] The role of $h(.)$ in the solution methods, and the Stata prodest package we use is described in more detail in Mollisi and Rovigatti (2017).

- Tangible intensity is defined as tangible fixed assets over total assets (stock measure) or tangible investment (change in tangible fixed assets) over total assets (flow measure).

Table 2 shows the main features of selected key variables used in the analysis. TFP is available for 27,712 observations, which will mainly dictate the sample size for the regressions below. Overall, the dataset is large enough for meaningful analysis to be carried out with the panel data methods we use. Standard deviations are high and the differences between the largest and smallest values are large, as is to be expected in firm-level data. The mean values of the variables look generally sensible. Average TFP growth is around 1.3% per annum, broadly in line with estimates of aggregate TFP growth in the UK during the same period. We also find that there is a positive contemporaneous correlation between TFP and intangible intensity (for both stocks and flows), and a negative relationship between TFP and tangible intensity (for both stocks and flows) (not shown in the Table).

Table 2: **Selected key variables - descriptive statistics**

	Observations	Mean	Std. Dev.	Min	Max
TFP	27,712	1.057	0.538	-0.769	4.093
TFP growth	26,008	0.013	0.233	-1.585	1.942
Intangible intensity - stock	27,712	0.504	0.562	0.000	4.618
Intangible intensity - flow	26,677	0.042	0.219	-2.599	2.208
Leverage	27,712	0.201	0.192	0	1.365
Tangible intensity - stock	27,599	0.510	0.359	0.004	1.830
Tangible intensity - flow	26,097	0.003	0.108	-0.687	0.642
Age	27,712	33.983	32.741	0	164
Size	27,712	4.660	2.088	-3.413	12.370
Profitability	27,566	0.061	0.243	-2.635	0.516
Cash ratio	27,705	0.134	0.163	0	0.906

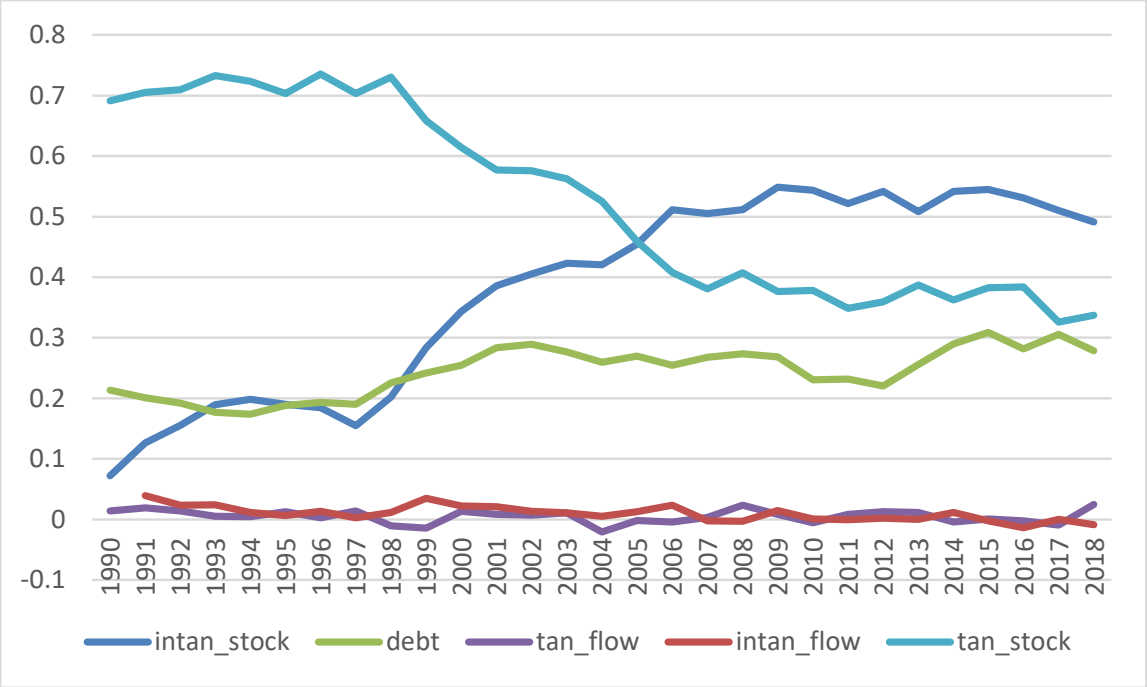
Notes: All variables are winsorised at the 1st and 99th percentiles. For definitions, see Appendix A.1.

Figure 4 shows the weighted medians of selected key variables over time. While this hides firm-level heterogeneity, it is a useful sense-check on the data. Some interesting facts emerge. First, the median leverage ratio dipped slightly immediately after the GFC, but has since increased to elevated levels. Second, median tangible intensity as measured by the stock variable has trended down strongly, possibly due to reduced investment and increased divestment and scrappage of physical assets. Third, median intangible intensity as measured by the stock variable increased strongly in the late 2000's until the financial crisis, but has been relatively flat since.[12] Fourth, both intangible and tangible intensity as measured by the flow variable have both been declining

[12]The recent weakness in intangible investment growth and its structural macroeconomic consequences has been analyzed by Bailey et al. (2022).

over time.

Figure 4: **Selected data series - medians over time**



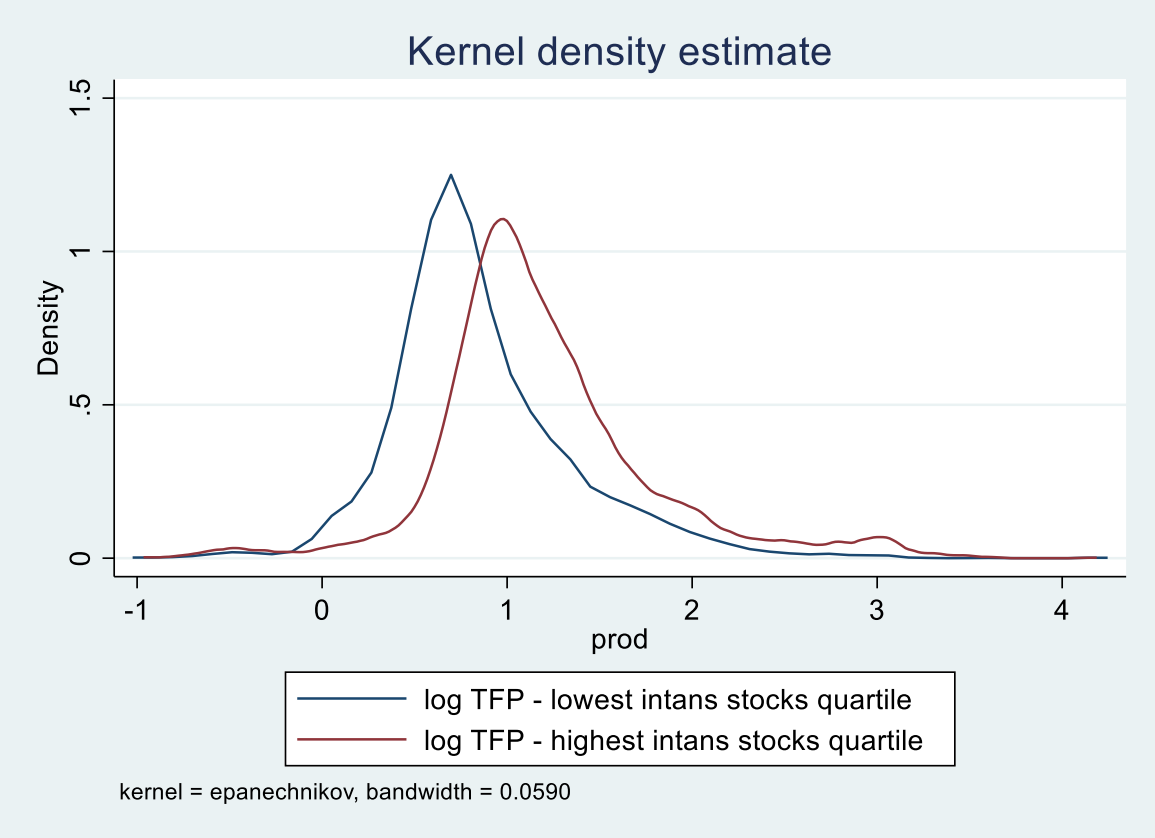
Notes: Employment-weighted medians. Sources: Worldscope and own calculations.

The characteristics of intangible-intensive firms are of special interest to our analysis. The correlations in our sample suggest that intangible stocks (and flows) are higher in firms that are less indebted, younger, smaller, more cash rich[13] and less profitable than those with smaller intangible stocks (and flows). Figure 5 shows that the TFP distribution of firms with high intangible intensity lies to the right of that of firms with low intangible intensity. This clearly points to a positive contemporaneous and unconditional relationship between the level of TFP and intangible intensity. In terms of industry decomposition, high intangible-intensity firms are heavily concentrated in the manufacturing and ICT sectors.[14]

[13]This is a common finding in the literature (see, e.g., Dao and Maggi, 2018). Typically, firms with high intangible spending tend to have more cash, as it is harder to use external financing for intangible investment projects due to their less collateralisable nature.

[14]We have also compared our data with data from the ONS Innovation Survey (2017) (https://www.gov.uk/government/statistics/uk-innovation-survey-2017-main-report), which suggests that internal R&D spending covers over half of all innovation-related expenditure in the surveyed UK firms. This underlines the importance of taking into account accumulated R&D spending in our measure of intangible assets, as it is a crucial component of innovation and potentially TFP growth. Furthermore, consistent with our industry decomposition, according to the survey, innovation activities are especially important in high-tech manufacturing and knowledge-intensive services.

Figure 5: **TFP distribution, by intangible intensity**



Notes: The blue line is the kernel density estimate of the distribution of log(TFP) for the first quartile of intangible intensity (stock definition). The red line is the kernel density estimate of the distribution of log(TFP) for the fourth quartile of intangible intensity (stock definition). Sources: Worldscope and own calculations.

In terms of correlations between the sample investment variables and the corresponding aggregate macroeconomic variables (to the extent these can be measured), the correlation between the ratio of R&D spending to total sales and the relevant ONS aggregate series (R&D spending to GDP) is positive at around 0.37. The ratio of R&D spending to total sales is higher in our sample than in the economy as a whole. This is not surprising, as our sample includes most of the largest UK firms, which carry out the bulk of R&D spending. A similar comparison between the sum of capital expenditure spending divided by total sales with the corresponding aggregate ONS series shows a relatively strong positive correlation of around 0.51, emphasizing the importance of large firms for driving business investment dynamics in the UK.

3.3 Main Results

Table 3 presents the results of our baseline panel regressions in Equation (1). The correlation between (lagged) intangible intensity and the level of TFP is strongly positive for both the stock and flow measures (columns (1)-(2)). By contrast, the correlation between TFP and tangible intensity (both stocks and flows) is strongly negative (columns (3)-(4)). The relationship between

the lagged leverage ratio and TFP is positive, but insignificant in all the regressions. In the last row of Table 3, we report the p-value of the full effect of leverage on TFP, taking into account its direct effect and its interaction with the firm's investment profile. The total effect is significantly positive when taking into account the mediating role of high intangible intensity, whereas it is insignificant in the case of high tangible intensity. In other words, leverage has a separate positive effect on TFP for those firms that are in the highest quartile of the ratio of intangibles (stocks and flows) to total assets, as reflected in the positive interaction term. The coefficients on the other control variables are mostly significant and intuitive. On average, TFP is higher for older, larger, and more profitable firms, while liquid assets are not a significant driver of TFP.

Table 4 summarizes the economic size of the relationships of interest, namely the correlation between TFP and tangible or intangible intensity and the total effect of debt after taking into account its interaction with the latter. There is a large and significant effect of 9.7% on TFP for companies that are in the highest quartile of the intangible stock ratio.[15] This effect is lower, but significantly positive, for firms that are in the highest quartile of the intangible flow ratio. By contrast, the effects are significantly negative for firms with high tangible intensity (measured by both stocks and flows). While this may appear counter-intuitive, it is possible that tangible investment has positive longer-term effects on firms' revenues, or on a measure of labor productivity (sales/number of employees), even if the TFP effects are negative or neutral. Overall, these results support the existing evidence on the positive effect of intangibles on TFP. It is worth noting that the strongest positive TFP effects accrue from the intangible stocks rather than the flows. This is consistent with the effects of intangible investment being subject to significant time lags, typically several years, documented in the innovation literature (see, for example, Hall et al., 2010).

The results on the total effect of debt indicate that leverage has a differential impact on TFP depending on the type of investment the firm undertakes. A 10pp increase in the leverage ratio predicts a 1.1% increase in TFP if the firm is in the highest quartile of the intangible stock ratio. The effect is 0.9% when the firm is in the highest quartile of the intangible flow ratio. By contrast, high tangible intensity does not interact significantly with debt and the total effect of debt remains insignificant for firms with high tangible intensity (as measured by both stocks and flows). These results suggest that a combination of high debt and high "productive" investment (as proxied by intangible investment) can be associated with high levels of TFP, even when the direct correlation between debt and TFP is more ambiguous.

[15] As noted above, due to the nature of our TFP measure, the regression coefficients may underestimate the true size of the effects, and hence the estimates in Table 4 should be seen as conservative.

Table 3: **OLS panel regression results**

ln(TFP)	(1)	(2)	(3)	(4)
Leverage (t-1)	0.00820	0.0310	0.0446	0.0431
	(0.0338)	(0.0359)	(0.0410)	(0.0383)
Intangible intensity - stock (t-1)	0.0972***			
	(0.0129)			
Intangible intensity - flow (t-1)		0.0240***		
		(0.00672)		
Tangible intensity - stock (t-1)			-0.0249**	
			(0.0117)	
Tangible intensity - flow (t-1)				-0.0238***
				(0.00504)
Leverage*RHS variable (t-1)	0.105*	0.0556	0.0119	0.0188
	(0.0594)	(0.0391)	(0.0500)	(0.0313)
Size (t-1)	0.0280***	0.0174**	0.0156**	0.0139*
	(0.00780)	(0.00797)	(0.00792)	(0.00804)
Age (t-1)	0.00604***	0.00733***	0.00676***	0.00721***
	(0.000939)	(0.00100)	(0.000962)	(0.00100)
Profitability (t-1)	0.118***	0.115***	0.102***	0.0939***
	(0.0218)	(0.0222)	(0.0223)	(0.0223)
Cash ratio (t-1)	0.0163	-0.0127	-0.0317	-0.0343
	(0.0390)	(0.0403)	(0.0402)	(0.0410)
Constant	0.660***	0.687***	0.732***	0.721***
	(0.0399)	(0.0401)	(0.0387)	(0.0397)
Observations	24,302	23,293	24,209	22,827
R-squared	0.793	0.792	0.792	0.798
Joint debt effect (p-value)	0.049	0.066	0.179	0.115

Notes: Robust standard errors in parentheses. *** p<0.01, ** p<0.05, * p<0.1. All regressions include time and firm fixed effects. RHS variable (t-1) stands for Intangible intensity - stock (t-1) in column (1), Intangible intensity - flow (t-1) in column (2), Tangible intensity - stock (t-1) in column (3), and Tangible intensity - flow (t-1) in column (4).

Table 4: **Economic effects**

What is the effect on TFP at time t of...	
... the firm being in Q4 of:	
variable (t-1)	**effect**
Intangible intensity - stock	9.7%***
Intangible intensity - flow	2.4%***
Tangible intensity - stock	-2.5%**
Tangible intensity - flow	-2.4%***
...a 10pp increase in leverage and the firm being in Q4 of:	
variable (t-1)	**effect**
Intangible intensity - stock	1.1%**
Intangible intensity - flow	0.9%*
Tangible intensity - stock	0.6%
Tangible intensity - flow	0.6%

Notes: *** p<0.01, ** p<0.05, * p<0.1.

To shed further light on the role of intangibles, we examine the TFP effect of different intangible components, namely balance sheet intangibles, R&D spending, SG&A spending and goodwill (see Appendix C Table 1 for details). The results suggest that the positive TFP effect of intangibles is largely driven by R&D and organizational capital (SG&A) (columns (2)-(3) and (6)-(7)). By contrast, balance sheet measures of intangibles are not significant drivers of TFP (columns (1) and (5)), although investments in goodwill are weakly significant (column (8)). Moreover, the positive interaction between debt and high intangible intensity is purely driven by R&D investment (columns (2) and (7)). Hence, it is ultimately high R&D intensity that drives the positive total effect of debt on TFP.

3.4 Selected Robustness Checks

The main issue with our baseline results is potential reverse causality[16]. Specifically, a firm can invest in its inputs (including intangibles) in anticipation of an improvement in its TFP. While this issue is mitigated by the use of lagged control variables, it cannot be excluded entirely. Therefore, we follow Levine and Warusawitharana (2021) and split TFP into an anticipated and unanticipated component, and then use the unanticipated TFP component as our dependent variable in Equation (1). The effect of intangibles remains significantly positive (see Appendix C Table 2, columns (1)-(2)), implying that the effects identified earlier run from intangible intensity to TFP, rather than the other way round. The joint debt effect also remains significant for the stock (at the 10% level) and flow (at the 5% level) measures of intangible intensity. Hence,

[16]See Levine and Warusawitharana (2021) for a discussion of this topic.

reverse causality does not appear to be driving our results.

We also run several robustness checks regarding the assumptions made in the construction of intangible assets in Appendix A.2, specifically on the depreciation rates and the share of SG&A spending that is assumed to be intangible investment. Following Peters and Taylor (2017), we allow the R&D depreciation rate to vary between 10% and 25%, and the SG&A depreciation rate between 10% and 30%. We also vary the share of SG&A that is assumed to be intangible investment between 10% and 50%. The results are in Appendix C Table 2, columns (3)-(10). The coefficients on high intangible intensity remain highly significantly positive and the joint effect of debt remains significantly positive for intangible intensive firms in most cases.

4 Structural model

We develop a structural model that is consistent with the findings on the relationship between debt, investment in tangible and intangible capital and TFP in Section 3. The model builds on a standard firm profit-maximisation problem using dynamic optimization, as in Warusawitharana (2015) and Levine and Warusawitharana (2021). In this partial equilibrium neoclassical model, the firm maximizes future profits. Its overall capital stock consists of tangible and intangible assets. While both tangible and intangible investment increase the firm's overall capital stock, intangible investment also potentially increases TFP. In other words, the firm's intangible capital stock has a direct positive effect on how efficiently the firm uses its overall capital stock. This approach is similar to a large number of notable contributions in the innovation literature (see, e.g., Klette and Kortum, 2004, and Warusawitharana, 2015), and is consistent with our stylized facts. Firms also face convex capital adjustment costs related to both types of investment.[17]

We innovate by augmenting the model with an option for external debt financing, which can be used to finance either type of investment and carries an interest that needs to be paid on the accumulated debt in future periods. The novelty of our approach is that the firm takes into account how its debt financing and investment decisions influence its leverage and the cost of borrowing. Our approach is arguably more realistic, as it explicitly links investment and financing decisions taken by firms to their state of indebtedness, and it also allows us to model the interactions between debt, different types of investment and TFP directly. Importantly, the model is consistent with the following stylized facts: (1) higher intangible intensity is associated with higher TFP, while higher tangible intensity is not; and (2) firms that take on higher debt and invest more in tangible capital have lower TFP, while firms that take on more debt and invest to a greater extent in intangible capital have higher TFP.

The setup is as follows. The firm maximizes its profits by choosing investment into tangible and intangible capital, both of which are inputs in the production function. To the extent that investment cannot be financed by current dividends, it will be financed with debt. New debt

[17] Convex adjustment costs are a standard assumption in these types of models. See Eberly et al. (2008), who provide evidence for this with firm-level data.

financing increases the stock of external debt. Financial frictions enter our model through a premium on the cost of debt that depends on the firm's earnings-based leverage ratio, i.e. the ratio of debt to the firm's earnings. This is essentially an earnings-based constraint, which links firms' current earnings and their access to debt (see, e.g., Lian and Ma, 2021; Drechsel, 2023). In addition, the cost of debt increases with the share of intangible capital in the firm's overall capital stock, consistent with the fact that a higher intangible intensity reduces a firm's capacity to provide collateralisable assets (see, e.g. Falato et al., 2022). Firms take these frictions into account when choosing their optimal investment expenditures.

The firm's firm's value is determined by its effort to maximize the present discounted value of future expected dividends,

$$V = \max_{i,r} E_0 \int_0^\infty e^{\rho t} \pi(z, k, s, b, i, r) dt, \tag{3}$$

subject to the following laws of motion for productivity z, the tangible capital stock k, the intangible capital stock s, and the stock of debt b:

$$\dot{z} = \mu_z - \rho_z(z - \mu_z) + \sigma_z dW \ , \tag{4}$$

$$\dot{k} = \delta_k k + i \ , \tag{5}$$

$$\dot{s} = \delta_s s + r \ , \tag{6}$$

$$\dot{b} = \text{if } (\pi < 0)\pi - bx, \text{ else } -bx \ . \tag{7}$$

The parameter ρ captures the discount factor. This value function can be rewritten as the sum of current dividends and the change in the value function with respect to the state variables, as implied by Ito's lemma:

$$\begin{aligned} &\rho V(z, k, s, b) = \pi(z, k, s, b, i, r) \\ &+ \frac{\partial V}{\partial k}(-\delta_k k + i) + \frac{\partial V}{\partial s}(-\delta_s s + r) + \frac{\partial V}{\partial b}(-xb + f) + \frac{\partial V}{\partial z}(\mu_z - \rho_z(z - \mu_z)) + \frac{\sigma_z^2}{2}\frac{\partial^2 V}{\partial^2 z} \ . \end{aligned} \tag{8}$$

Note that $\frac{\partial V}{\partial t} = 0$ holds as the optimal solution to the value function does not change with respect to time and hence this term is omitted. Exogenous total factor productivity (z) follows an Ornstein-Uhlenbeck process with mean μ_z and persistence parameter ρ_z. Tangible and intangible capital are accumulated when the firm invests in the relevant capital stock and depreciate at rates δ_k and δ_s, respectively. Debt is accumulated whenever current dividends (or net earnings, see Equation (9)) are negative and the principal is paid back at rate x. In other words, a fraction x of the stock of debt b has to be paid back in every period. Debt accumulation is equal to $f = -\pi$ when $\pi < 0$ and $f = 0$ otherwise. As it is more practical to solve, we solve for the Hamilton-Jacobi-Bellman Equation (8). The firm maximizes this value function for the space spanned by the four state variables: exogenous TFP, i.e. TFP not explained by intangible capital

(z), tangible capital (k), intangible capital (s), and the stock of debt (b) by optimally choosing investment in tangible capital (i) and intangible capital (r).

Current dividends at any point in time, $\pi(z, k, s, b, i, r)$, are a function of these four state variables, as well as the investment decisions. We assume that $f \geq 0$, i.e. firms cannot pay back their debt principal faster than at the rate x when they get a positive productivity realisation and dividends are positive. This assumption means that equity holders extract any dividends that are not optimally invested back into the firm. In other words, we assume that retaining earnings within the firm and paying off any debt beyond the required rate (x) is assumed to not be optimal for the firm's shareholders. The current dividend of the firm is the sum of income from production ($y(z, k, s)$), minus the cost of tangible and intangible investment, minus the cost of debt and the repayment of the principal. It is defined as follows:

$$\pi(z,k,s,b,i,r) = y(z,k,s) - i - r - \lambda_k \frac{k}{2}\left(\frac{i}{k} - \delta_k\right)^2 - \lambda_s \frac{s}{2}\left(\frac{r}{s} - \delta_s\right)^2 - b\left(x + B(z,k,s,b,\psi_1,\psi_2)\right) \tag{9}$$

The functional form of the convex capital adjustment costs is chosen to reflect the typical setup described in Hayashi (1982). The parameters λ_k and λ_s determine the size of capital adjustment costs for tangible and intangible capital. Current dividends are further decreased by the cost of debt (see Equation 12 below), which is a function of the firm's leverage and the share of intangible capital in the capital stock.

Our main functional choice for the firm's output is (assuming labour input=1 for simplicity):[18]

$$y(z, s, k) = (z + s^{\gamma_1})(s + k)^{\alpha} \ . \tag{10}$$

Equation (10) is our preferred functional form as it is the equivalent of the production function used to estimate TFP as per Equation (2). When $\gamma_1 > 0$, intangible capital increases TFP directly by construction. One disadvantage of this specification, however, is that it requires $\gamma_1 > 0$ for investment in tangible and intangible capital to be positive when $\delta_k > \delta_s$. In order to verify that this condition holds in the data (and is not arbitrarily imposed by the model), we therefore estimate the model first assuming the following Cobb-Douglas production function:

$$y(z, s, k) = z s^{\gamma_2} k^{\alpha} \ . \tag{11}$$

[18]This simplifying assumption can be relaxed without affecting the main analytical results to derive a profit-maximization condition with regard to wages and labour input. Also note that since we are not focusing on labour markets or the household sector in a general equilibrium framework, there is no condition linking labour productivity to the marginal product or wages in the model. However, for our purposes, this partial equilibrium model is sufficient.

When $\gamma_2 > \alpha$, intangible capital increases TFP, with the latter defined as $zs^{\gamma_2-\alpha}$. Importantly, this production function is equally well defined when $\gamma_2 < \alpha$, i.e. it allows for intangible capital to decrease TFP. Hence, we first use this unrestricted Cobb-Douglas specification to verify that $\gamma_2 > \alpha$, in other words that intangible capital indeed contributes positively to TFP. Once this is verified, we use the output specification in Equation (10) to fit the model to the data. This ensures that we fit the values of the parameters (including γ_1) in a way that best explains the data. For Equation (10), we further require $\alpha, \gamma_1 \in (0,1)$ and $\gamma_1 < 1 - \alpha$ so that the firm faces diminishing returns to scale in both capital stocks. For Equation (11) we require $\alpha + \gamma_2 \in (0,1)$ and $\alpha, \gamma_2 \in (0,1)$.

Finally, we turn to the relationship between leverage and the cost of debt. Equation (12) captures the idea that leverage increases borrowing costs:

$$B(z,k,s,b,\psi_1,\psi_2) = \xi_0 + \psi_1 \frac{b}{y}\left(1 + \frac{\psi_2}{\psi_1}\frac{s}{s+k}\right) . \tag{12}$$

This function is essentially a penalty function as described in Den Haan and De Wind (2012). It allows us to model the cost of debt as a function of earnings-based leverage ($\frac{b}{y}$) and the share of intangible capital in the capital stock ($\frac{s}{s+k}$). The impact of leverage on the cost of debt is controlled by the parameter ψ_1. Intuitively, the function captures the idea that debt servicing costs typically increase as a firm becomes increasingly leveraged (and risky).[19] In addition, a higher intangible capital share increases the cost of debt when $\psi_2 > 0$, consistent with the fact that intangible capital is harder to liquidate in the event of default, making the firm riskier for lenders.

We now solve for the firm's optimal investment using the Hamilton-Jacobi-Bellman Equation (8) and the laws of motion. The first-order conditions for optimal investment are dependent on whether the firm's profit after the investment choice is positive or negative. If it is positive, the debt state does not change (debt does not increase) and the marginal cost of increased debt does not enter the investment decision. If the firm needs to take on new debt to invest, and the cost of increasing debt given the current state is high, then the firm will invest less (see Appendix B.1 for detailed derivations of the first-order conditions).

i: $[1 + \lambda_k(\frac{i}{k} - \delta_k)](1 - \frac{\delta V}{\delta b}) = \frac{\delta V}{\delta k}$ when $\pi < 0$ and $1 + \lambda_k(\frac{i}{k} - \delta_k) = \frac{\delta V}{\delta k}$ when $\pi \geq 0$

r: $[1 + \lambda_s(\frac{r}{s} - \delta_s)](1 - \frac{\delta V}{\delta b}) = \frac{\delta V}{\delta s}$ when $\pi < 0$ and $1 + \lambda_s(\frac{r}{s} - \delta_s) = \frac{\delta V}{\delta s}$ when $\pi \geq 0$

The steady states for k and s when $b = 0$ can be obtained by solving the frictionless value function. They are given by Equations (13) and (14) in the case of production function (10), and Equations (15) and (16) in the Cobb-Douglas case:

[19] This concept is similar to putting penalty functions on consumers (see Algan et al., 2014) instead of hard debt constraints to avoid consumer Ponzi schemes.

$$s = k\frac{\gamma}{\alpha\frac{\delta_s-\delta_k}{\delta_s+\delta_k}+\gamma} = \psi k \ , \tag{13}$$

$$k = \left(\frac{\alpha\psi^{\gamma}}{(\rho+\delta_k)(1+\psi)^{1-\alpha}}\right)^{\frac{1}{1-(\alpha+\gamma)}} \ , \tag{14}$$

$$zs^{\gamma}k^{\alpha-1} = \frac{\rho+\delta_k}{\alpha\zeta} \tag{15}$$

$$s^{\gamma-1}k^{\alpha} = \frac{\rho+\delta_s}{\gamma\zeta} \tag{16}$$

When we allow for capital adjustment costs and the negative effect of debt and leverage on period profits specified in Equation (12), the value function has to be solved numerically.

4.1 Solving and estimating the value function

We solve the value function for the firm's optimal policies given the states. We approximate the value function of the firm, given these states, with a neural network. We show in Appendix B.2 that our solution algorithm produces a solution to a simplified version of our problem with two states that is similarly accurate as the implicit method solution outlined in Achdou et al. (2022). While our solution method takes longer to solve the problem in this simple case, it is easier to implement in a high-dimensional setting with a large number of state variables because of code parsimony.

Our method is similar to the methods used in Maliar et al. (2021) and Fernández-Villaverde et al. (2023), who introduce the use of deep learning for approximating value functions. They show that neural network methods are more efficient at approximating and optimizing unknown functions when the state space is large. In other words, neural networks have the ability to break the "curse of dimensionality". They can approximate a multi-dimensional unknown function, such as the value function presented above. We contribute to the literature by proposing a new way to estimate value function *parameters* based on the observed state and control variables in the data. Our approach is a moments-matching approach, where we make the parameters of the value function a part of the state space. We write the states of the problem as $X_1 = [z, k, s, b]$ and the parameter space as $\Psi = [\rho, \alpha, \gamma, \delta_k, \delta_s, \lambda_k, \lambda_s, \psi_1, \psi_2]$.

Usually, we would estimate a subset of parameters $\hat{\Psi} \in \Psi$ of this value function given data moments Y. We would estimate $\hat{\Psi}$ by solving the value function for a guess of the parameter values to be estimated, compute the likelihood and update our guess. We would then solve the value function for each new guess until a point is found where the likelihood given the parameters is maximized.[20] Instead, given the comparative ease with which a neural network can be used to solve the value function for multiple states, we propose the following alternative estimation. Take states $X_1 = [z, k, s, b]$ and add the parameters $\Psi_X = [\gamma, \lambda_k, \lambda_s, \psi_1, \psi_2]$ as additional states to solve the value function for. Other, calibrated parameters are summarized as $\bar{\Psi} = [\alpha, \delta_k, \delta_s]$. We can then estimate the parameters Ψ_X governing the value function given data Y with Algorithm 1 (below). The advantage of this approach is that we are able to estimate parameters that would be difficult to calibrate based on empirical analysis.

[20]For comparison, more details on this conventional algorithm are set out in in Appendix B.2.

The reason this approach works within a reasonable amount of time is that the neural network will approximate the parameter space, making the computation for matching moments a task of minimizing the distance between the neural network function and the observable states and policies by varying the estimated parameters, and thereby maximizing the likelihood of the model producing the observed data. This is similar to Kase et al. (2022), though in our approach we are approximating the model, rather than the likelihood function, as a function of to-be-estimated parameters.

Algorithm 1 Estimating the value function by solving for to-be-estimated parameters

Solve $V([X\Psi_X]|\bar{\Psi})$ and produce estimated moments $\hat{Y}(\Psi_X)$.

Alter Ψ_X such that $\min ||\hat{Y}(\Psi_X) - Y||$, e.g. $\min_{\Psi_X}(\hat{Y}(\Psi_X) - Y)W^{-1}(\hat{Y}(\Psi_X) - Y)'$.

The final parameter estimates $\hat{\Psi}_X$ are where $||\hat{Y}(\hat{\Psi}_X) - Y||$ is minimised.

The advantage of our approach is that once the neural network is trained, evaluating the distance of our model to the empirically observed moments, it is computationally cheap. We can therefore use standard methods to find a global minimum within the approximated parameter space. A further advantage of our method is its coding speed and the higher control provided for avoiding local minima, failures to converge, or impossible parameter shifts in the approximated parameter space.

One can describe our approach as matching moments. Concretely, we are matching the investment policies, debt financing choice, and productivity explained by intangible capital to different quartiles of the state space found in the data. We do this by choosing the value function among the set of value functions solved in the parameter space that best fits the outcomes in the data for investment and financing behaviour as well as productivity explained by intangible capital. While there has lately been a lot of progress in the literature in using neural networks as universal approximators in economic and econometric problems, this is, to our knowledge, the first time the parameters determining a value function have been estimated in this way.

4.2 Estimation results

We estimate the parameters λ_k, λ_s, ψ_1, ψ_2 and γ with our method and solve the value function for the parameter space. The results of our estimations are shown in Table 5. Our estimates suggest that $\gamma_1 > 0$ in the baseline case and $\gamma_2 > \alpha$ in the Cobb-Douglas case. The elasticity is of similar magnitude in both cases. Table 3 in the Appendix shows the calibrated parameter values in our model. These are a combination of convenient choices (adjustment costs and unexplained TFP), values typically used in the literature (discount factor) and values based on UK data (e.g., exponent on capital and depreciation rates).

The fit of our estimation is shown in Appendix Figure 3. We match the moments by varying the parameters to-be-estimated to find a minimum loss in the parameter space, where we have

Table 5: **Estimated model parameters**

Estimated Parameters	ψ_1	ψ_2	λ_k	λ_s	γ
Fit baseline	0.0010	0.0012	6.2108	12.82	0.147
Fit C-D	0.0003	0.0146	0.85	9.50	0.171

Notes: Results are reported for both the baseline and the Cobb-Douglas (C-D) output function. Model parameters were fit using a loss function weighted by the inverse standard deviations of the data moments. In the unweighted estimation, each data moment was given equal weight in the loss function.

solved the value function. The top panels show the fit of physical and intangible investment, conditional on states K, S, B, respectively. The conditional data moment is represented by the solid blue line, while the standard deviation interval is shown with a blue dashed line. The red line shows the predictions for investment conditional on the states from the fitted model. The fitted model matches investment patterns for both tangible (top left-hand panel) and intangible capital (top right-hand panel) as the size of debt, tangible and intangible capital vary. The bottom left panel shows the equivalent conditional moments and model predictions for debt financing. Except for very small firms, the model matches debt financing well. The bottom right panel in Figure 3 shows estimated firm productivity. We see in the data that estimated productivity increases conditional on intangible capital and decreases mildly conditional on physical capital - matching the stylized facts.

For robustness, we also show the estimated fit for the model where the production function has the more common Cobb-Douglas specification in Appendix Figure 4. The model also matches investment, debt financing, and productivity patterns well, doing slightly better on matching debt financing, but slightly worse on matching the physical capital investment decisions conditional on the state variables.

4.3 Predictions on investment, debt, and TFP

Our model is consistent with the patterns of TFP, investment and debt financing observed in the data. The first chart in Figure 5 shows the relationship between tangible and intangible investment (horizontal axes) and the TFP state explained by intangible capital (vertical axis). The second and third charts show the relationship between the two types of investment and debt financing. Taken together, the plots show that: (1) higher intangible intensity is associated with higher TFP, while higher tangible intensity is not; and (2) firms that take on higher debt and invest more in tangible capital have lower TFP, while firms that take on more debt and invest to a greater extent in intangible capital have higher TFP. Investment in tangible capital that is accompanied by high debt may decrease future TFP-enhancing intangible investment. This is because most of the firm's current income is spent on debt payments and the firm has to turn to external financing to invest further. In this case, the optimality condition for investment in intangible capital ($r = [\frac{\frac{\delta V}{\delta s}}{1-\frac{\delta V}{\delta b}} \frac{1}{\lambda_s} + \delta_s]s$) means that the marginal cost of additional external debt will reduce the amount of intangible investment. These patterns are in line with our stylized

facts.

5 Counterfactual exercises

5.1 Contribution of intangibles to aggregate productivity growth

To assess the contribution of intangible capital growth to aggregate productivity growth, we conduct a counterfactual exercise utilizing the estimated model parameters. Specifically, we employ the estimated parameter vector $\hat{\Psi}_x = \{\psi_1, \psi_2, \lambda_k, \lambda_s, \gamma\}$ to simulate firm investment decisions. This exercise enables us to quantify the extent to which intangible capital growth explains aggregate productivity growth, and importantly how this has changed over time.

The procedure involves the following steps:

1. State-space discretization: For each year in the data, we discretize the state space by computing terciles for physical capital (k), intangible capital (s), and debt (b). This discretization yields 27 distinct state-space combinations.

2. State-space averaging and normalization: We compute the average values of k, s, and b for each of the 27 state-space combinations within each year. At this stage, we ensure that the grid space of the observed data is consistent with the model's state-space representation.

3. Optimal investment policy: Using the solved optimal investment policies derived from the firms' optimized value functions $V(z, k, s, b)$, we determine the implied intangible investment for each firm, conditional on the estimated parameters $\hat{\Psi}_x$ and the firm's location within the discretized state space. Omitting the subscript j for clarity, optimal investment is:

$$r = \frac{s}{\lambda_s} \left[\frac{\frac{\partial V(z,k,s,b,\hat{\Psi}_x)}{\partial s}}{1 - \frac{\partial V(z,k,s,b,\hat{\Psi}_x)}{\partial b} \mathbb{1}_{\pi<0}} \right] - 1 + s\delta_s \tag{17}$$

4. Productivity calculation: The implied TFP growth generated by the predicted investment for each discretized state space (r_j^{γ}) is then aggregated into explained productivity growth as a weighted average, where the weights are determined by the mean capital stock of that state space.

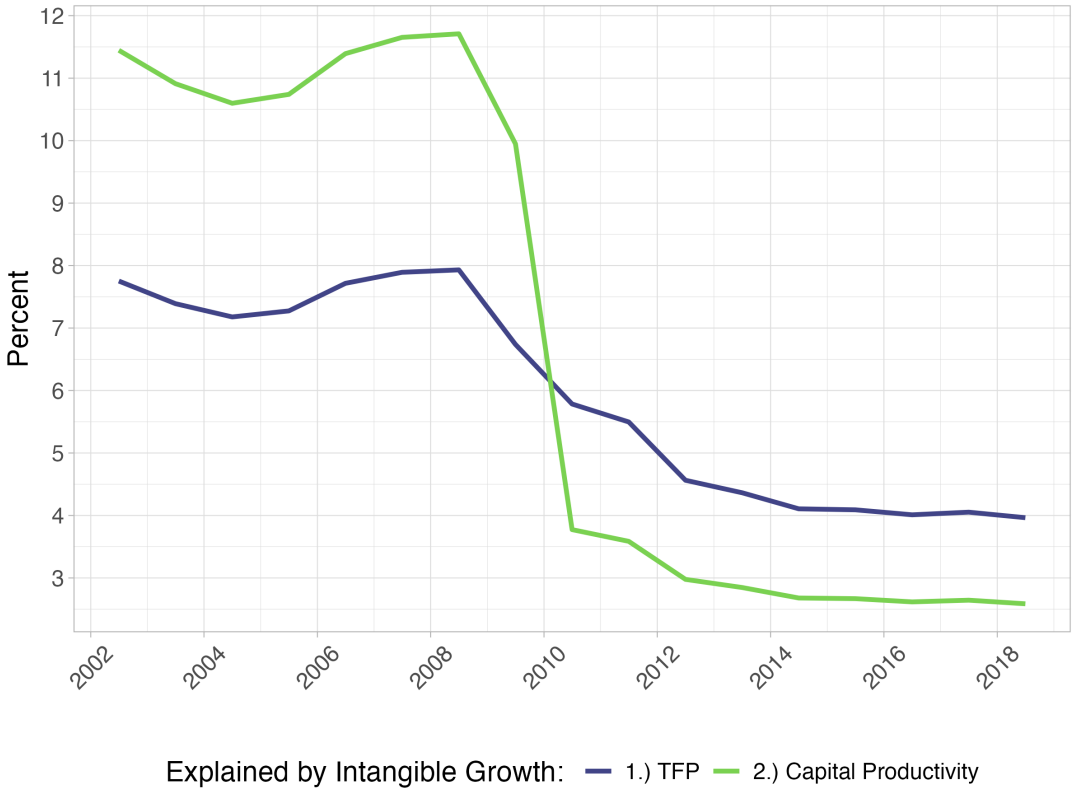
$$\text{Explained TFP growth}_t = \sum_{j=1}^{J} r_j^{\gamma} \frac{s_j + k_j}{\sum_{j=1}^{J}(s_j + k_j)} \tag{18}$$

5. Benchmark comparison: By comparing the aggregate productivity growth implied by the model with the actual growth observed in the data, we quantify the extent to which intangible capital growth explains aggregate productivity growth. We perform this exercise for two distinct measures of productivity growth produced by the ONS, namely TFP

growth and capital productivity growth, where capital productivity is defined as output per unit of capital (i.e. GVA divided by capital services).

The fraction of aggregate productivity growth explained by intangible investment according to the model is shown in Figure 6. Our results indicate that on average intangible capital growth accounts for approximately 8% of TFP growth and 11% of capital productivity growth prior to 2010. Subsequently, between 2010 and 2018, these contributions decline to roughly 3% and 4%, respectively. In the absence of a deterioration in financial frictions (measured by ψ_1 and ψ_2), the decline in intangible investment (and the associated TFP growth foregone) is explained by a sharp increase in earnings-based leverage, $\frac{b}{y}$ (see Figure 7).

Figure 6: **TFP and capital productivity growth explained by predicted optimal intangible investment**



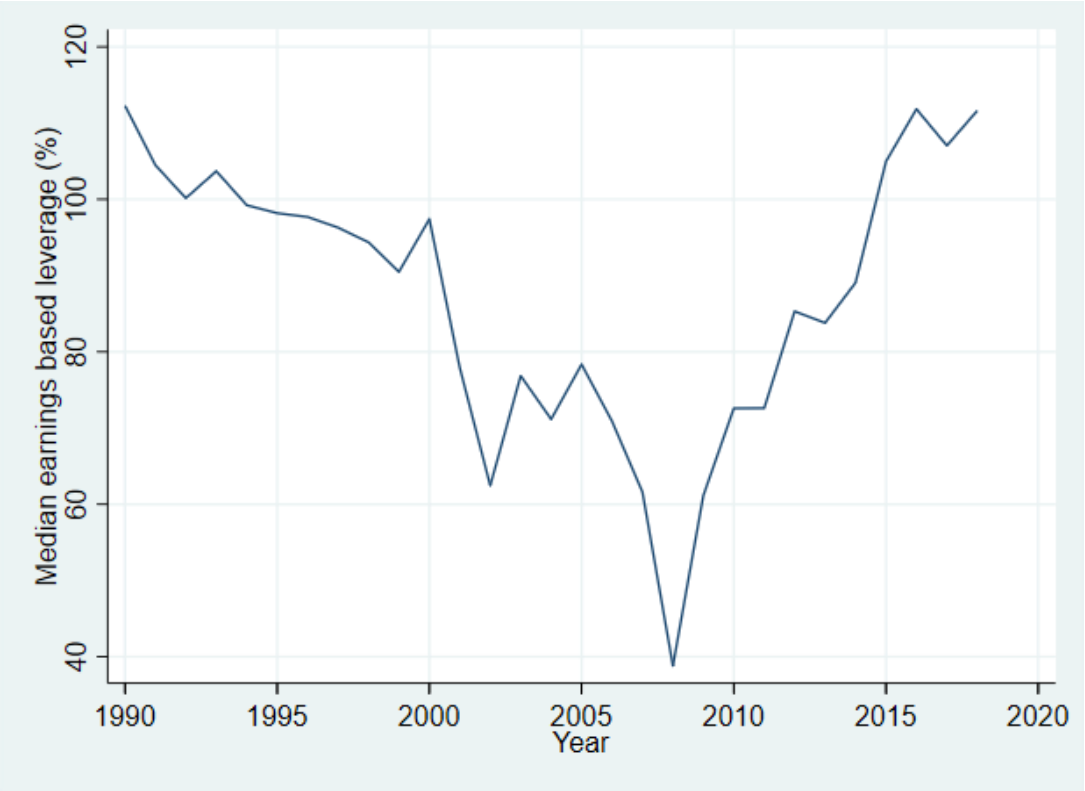
Source: ONS and authors' calculations.

For robustness, we also check that the intangible investment activity directly observed in the data (as opposed to that predicted by the model) delivers similar results on the contribution of intangible investment to productivity growth (see Appendix Figure 6).

5.2 Credit frictions and post-GFC intangible investment

This section examines the potential impact of heightened credit frictions, as captured by the parameters ψ_1 and ψ_2, on intangible investment and TFP growth following the Global Financial

Figure 7: **Median earnings-based leverage (in %)**



Source: Authors' calculations.

Crisis (GFC). ψ_1 measures the strength of the earnings-based constraint, whereas ψ_2 measures the strength of the friction related to the non-collateralisable nature of intangibles. Specifically, we investigate whether these frictions have intensified and quantify their contribution to the observed decline in intangible investment and TFP growth.

Our methodology proceeds as follows:

1. Baseline parameterization: We maintain the parameters λ_k, λ_s, and γ, estimated using the full-sample state space (z, k, s, b), to serve as a baseline.

2. Period-specific credit friction estimation: We re-estimate the parameters ψ_1 and ψ_2, representing credit frictions, to accurately reflect investment and financing behavior in the pre-2008 and post-2008 periods, respectively, for each state-space location.

3. Optimal investment: Using the period-specific estimates of ψ_1 and ψ_2, alongside the baseline parameters and the mean capital and debt states for each year within the discretized state space, we compute the optimal intangible investment and the associated productivity growth. Intangible investment for the pre-GFC (post-GFC) period is calculated using the corresponding pre-GFC (post-GFC) estimates of ψ_1 and ψ_2. The methodology is otherwise the same as in the preceding section.

4. Pre- and post-GFC comparison: We isolate the effect of the deterioration in credit frictions

by comparing the model's predictions on aggregate TFP growth driven by intangible investment based on the post-GFC estimates of credit frictions with the predictions based on the pre-GFC estimates, holding the other parameters of the model constant.

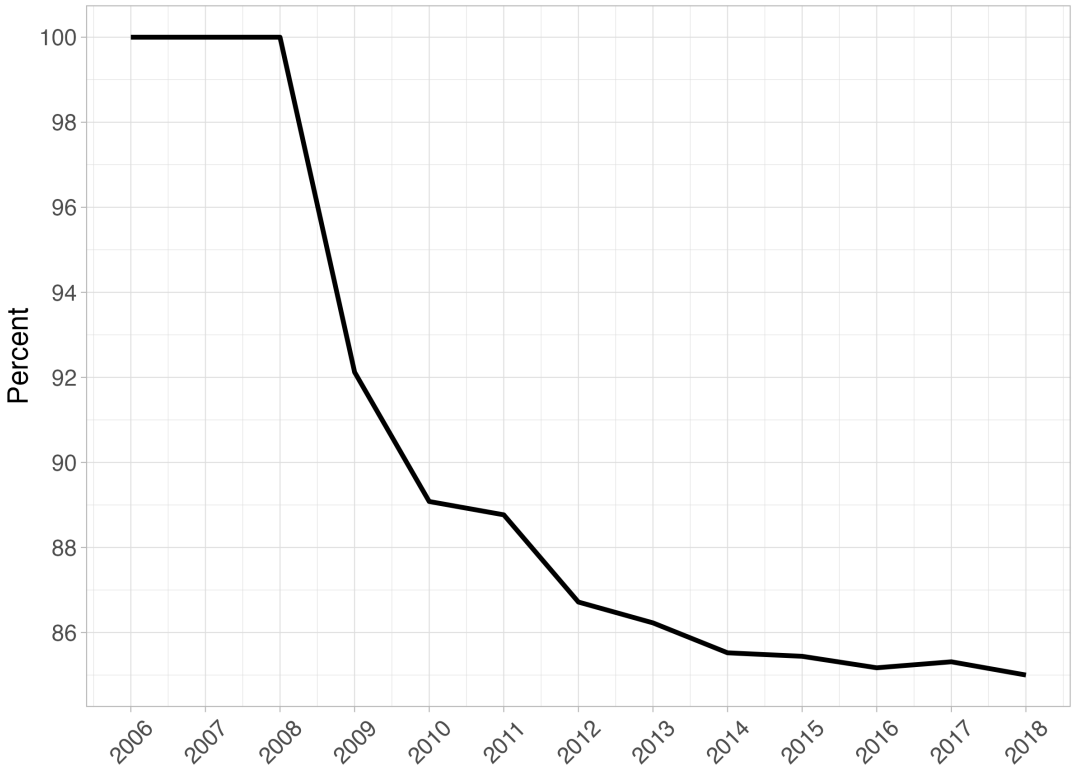
This counterfactual approach allows us to determine the extent to which the observed changes in intangible investment and TFP growth can be attributed to the evolution of credit frictions since the GFC. Table 6 shows the estimates of credit frictions. It suggests that credit frictions, as measured by the parameters, increased significantly after the GFC. While ψ_1 nearly doubled after the GFC, the most dramatic change is seen in ψ_2, with a post-GFC/pre-GFC ratio of 315. These results suggest a significant tightening of credit constraints for intangible intensive firms.

Table 6: **Pre- and post-GFC estimates of credit frictions**

	ψ_1	ψ_2
Pre-GFC	1.11E-05	3.68E-05
Post-GFC	1.96E-05	0.0116
Post-GFC/Pre-GFC ratio	1.8	315

The results on TFP growth are in Figure 8. Figure 8 shows the ratio (in percent) between the annual TFP growth that is generated by intangible investment as predicted by the model, assuming that the parameters ψ_1 and ψ_2 are equal to their estimated values based on pre-GFC data and post-GFC data. For example, a ratio of about 89% in 2011 means that the deterioration in credit frictions resulted in an 11% loss in aggregate TFP growth generated by intangible investment. Overall, Figure 8 shows that the tightening of credit frictions (mainly psi_2) between the pre- and post-GFC periods entails a growing loss of TFP growth, peaking at around 15% in 2018. For robustness, we also check that the intangible investment activity directly observed in the data (as opposed to that predicted by the model) delivers similar results on the TFP growth losses due to worsening credit frictions (see Appendix Figure 7).

Figure 8: **Impact of worsening credit frictions on aggregate TFP growth generated by intangible investment**



Source: ONS and authors' calculations.

6 Conclusions

The GFC has shown that financial frictions can leave long-lasting scars on the real economy through their impact on investment, as well as innovation and productivity growth. In a world where intangibles have become an increasingly important driver of innovation and economic growth, this begs the question of to what extent financial frictions affecting intangible investment have contributed to weak TFP growth since the GFC. Comprehensive evidence on this question is still sparse.

We contribute by developing a structural model that enables us to quantify the effect of intangible investment, and the financial frictions affecting it, on aggregate TFP growth. We apply the model to the UK, an intangible-intensive economy that has witnessed weak TFP growth since the GFC. In our model, firms optimally choose to invest in two types of assets (tangibles and intangibles) and whether to issue new debt in order to do so. The model is consistent with stylized facts derived from a sample of large UK firms, namely a positive association between intangibles and firm-level TFP and a positive interaction between leverage and intangible intensity. Financial frictions enter our model through a premium on the cost of debt that depends on the firm's earnings-based leverage ratio and the share of intangible capital in

the firm's overall capital stock - reflecting that intangibles are harder to liquidate in the event of default. We contribute methodologically by proposing a new way to estimate value function parameters using neural network (NN) methods.

Our analysis of intangible capital's contribution to TFP growth reveals a notable shift around 2010. Prior to this period, intangible capital growth accounted for approximately 8% of TFP gains on average. However, this contribution subsequently declined to roughly 3% between 2010 and 2018. This deterioration can partly be attributed to a marked increase in earnings-based leverage that constrained intangible investment. However, a comparative analysis of the pre- and post-GFC periods also highlights a significant worsening of credit frictions specifically related to intangible assets, ultimately leading to an estimated 15% reduction in TFP growth by 2018. The economic magnitude of the gains in TFP growth from removing the intangible-specific credit friction is substantial. This suggests that policies aimed at alleviating credit constraints for intangible-intensive firms are an important means to promote growth.

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Appendix

A Data

A.1 Firm-level data

The exact definitions of the variables used in the analysis are as follows:

TFP: Measure of firm-level real (revenue) TFP (in log), estimated using the Ackerberg et al. (2015) methodology. The main challenge in estimating firm-level production functions is the endogeneity between inputs (capital, labour and intermediate inputs) and output, causing OLS estimates of the production function coefficients to be biased. As is now standard in the literature, we resort to control-function (CF) approaches to estimate TFP. In particular, we use two common approaches: Olley-Pakes (OP) with the Ackerberg-Caves-Frazer correction and Wooldridge. For brevity, we only report results for the first approach.

Intangible intensity - stock: Stock-based measure of intangible intensity, defined as the stock of intangible assets (defined in Appendix A.2) divided by total assets. In our regressions, we use a dummy variable equal to 1 for the highest quartile of this ratio and 0 otherwise.

Intangible intensity - flow: Flow-based measure of intangible intensity, defined as investment in intangible assets (year-on-year change in the stock of intangible assets) divided by total assets. In our regressions, we use a dummy variable equal to 1 for the highest quartile of this ratio and 0 otherwise.

Leverage: Total debt divided by total assets (corrected for industry medians).

Tangible intensity - stock: Stock-based measure of tangible intensity, defined as the stock of tangible assets (total fixed assets) divided by total assets. In our regressions, we use a dummy variable equal to 1 for the highest quartile of this ratio and 0 otherwise.

Tangible intensity - flow: Flow-based measure of tangible intensity, defined as investment in tangible assets (year-on-year change in the stock of tangible assets) divided by total assets. In our regressions, we use a dummy variable equal to 1 for the highest quartile of this ratio and 0 otherwise.

Age: Number of years since the firm's incorporation.

Size: Real total assets (log level, deflated by GDP deflator).

Profitability: Profits (EBITDA) divided by total assets.

Cash ratio: Cash and short-term investments divided by total assets.

Interest expense ratio: Interest expenses divided by total debt (used as an instrument in system GMM regressions only).

To mitigate the effects of outliers, all variables are winsorised at the 1st and 99th percentiles, as is conventional in the literature.

The following variables are used as inputs into the TFP production function estimations:

Output: Log level of revenue (Worldscope: netsales), deflated by GDP deflator.

Capital input: Log level of the sum of net property, plant and equipment (Worldscope: netppe) and the stock of intangible assets (defined in Appendix A.2), deflated by GFCF deflator.

Labour input: Log level of cost of goods sold (Worldscope: cogs), deflated by GDP deflator.

Tangible investment: Log level of capital expenditure (Worldscope: capex), deflated by GFCF deflator.

Intangible investment: Log level of year-on-year change in gross intangible assets (as defined in Appendix A.2), deflated by GFCF deflator.

In addition to the firm-level data, we use aggregate GDP and GFCF deflators, published by the Office for National Statistics, to deflate some of the variables (inputs for the TFP production function estimation and total assets). We do not report any stylized facts on these series for brevity, but from 1990 to 2018, the average annual growth rate of the GDP deflator is around 2.4% and the growth rate of the GFCF deflator is around 2%. We also use aggregate data on real GDP, total employment hours and capital services to calculate aggregate TFP (where we also assume the labour share to be 2/3 and capital share 1/3, as is standard in the literature.). All the aggregate data is available on the website of the Office for National Statistics (www.ons.gov.uk).

A.2 Definition of intangible capital

There is no uniformly agreed measure of intangible assets in the literature. For firm-level data, a particular challenge is the fact that some intangible investments (like R&D spending) are not capitalized into the balance sheet, as they are recorded as expenses in the P&L account. However, a recent paper by Peters and Taylor (2017) introduces a methodology for accumulating expenses on intangibles into stocks. We follow their methodology, as laid out below.

The intangible capital stock (K_{it}^{int}) is defined as:

$$K_{it}^{int} = INT_{it} + A_{it} + B_{it} \tag{19}$$

where INT_{it} is intangible assets on the balance sheet for firm i at time t, A_{it} is accumulated R&D (RD) spending (defined below) and B_{it} is accumulated sales, general and administrative (SGA) spending (defined below). Accumulated R&D spending is defined as follows:

$$A_{it} = (1 - d_{rd})A_{i,t-1} + RD_{it} \tag{20}$$

where d_{rd} is the R&D depreciation rate (assumed to be 15% economy-wide, following previous literature). Accumulated SG&A spending is defined as follows:

$$B_{it} = (1 - d_{sga})B_{i,t-1} + 0.3 * SGA_{it} \tag{21}$$

where d_{sga} is the SG&A depreciation rate (assumed to be 20% economy-wide, following previous

literature), and 30% of SG&A is assumed to be related to intangible investment.[21]

A choice needs to be made on how to determine the starting R&D and SG&A stocks (A_{i0} and B_{i0}), as the firms are not usually observed from the year they were formed. We assume these to be zero[22]. There is also a small number of cases, where a firm exits the dataset and then re-enters n number of years later. For the accumulation of R&D (and analogously for SG&A), we use the following proxy formula to calculate the accumulated stocks in the year of re-entry:

$$A_{it} = (1 - d_{rd})^n A_{i,t-n} + (n-1)\left(\frac{(1-d_{rd})^n RD_{i,t-n} + RD_{it}}{2}\right) \tag{22}$$

Note that our measure of intangible assets excludes goodwill, but the results are broadly similar when goodwill is included.

B Details of the Structural Model and Algorithm

B.1 First-order conditions

The first-order conditions can be derived by taking the first derivative of equation (3) with respect to tangible investment i and intangible investment r:

$$\rho V(k,s,b) = \pi(k,s,b,i,r) + \frac{\partial V}{\partial k}(-\delta_k k + i) + \frac{\partial V}{\partial s}(-\delta_s s + r) + \frac{\partial V}{\partial b}(-xb + f)\ . \tag{23}$$

The first-order condition for tangible capital i is:

$$\frac{\partial V(k,s,b)}{\partial i} = 0 = -1 - \lambda_k\left(\frac{i}{k} - \delta_k\right) = \frac{\partial V}{\partial k}$$

when $\pi \geq 0$ and:

$$\frac{\partial V(k,s,b)}{\partial i} = 0 = -1 - \lambda_k\left(\frac{i}{k} - \delta_k\right) = \frac{\partial V}{\partial k} + \left[1 + \lambda_k\left(\frac{i}{k} - \delta_k\right)\right]\frac{\partial V}{\partial b}$$

when $\pi < 0$.

The first-order condition for intangible capital r is:

$$\frac{\partial V(k,s,b)}{\partial r} = 0 = -1 - \lambda_s\left(\frac{r}{s} - \delta_s\right) = \frac{\partial V}{\partial s}$$

when $\pi \geq 0$ and

[21] We also examine the robustness of our results to alternative assumptions on the depreciation rates as well as the intangible share of SG&A.

[22] Peters and Taylor (2017) make a similar assumption, but also apply a more complicated method for accumulating stocks from non-zero initial stocks. They find that their results look very similar with either method, and given the lack of data on the UK, we do not pursue this avenue.

$$\frac{\partial V(k,s,b)}{\partial r} = 0 = -1 - \lambda_k \left(\frac{r}{s} - \delta_s\right) = \frac{\partial V}{\partial s} + \left[1 + \lambda_s \left(\frac{r}{s} - \delta_s\right)\right] \frac{\partial V}{\partial b}$$

when $\pi < 0$.

We assume that the firm either invests optimally when choosing to take on external debt financing or when not, but cannot take on debt for only one type of investment at any point. This simplifies the decision-making problem of the firm, while coming at little cost to the model structure due to the continuous time framework where re-optimisation of external financing is continuously possible. It can be interpreting as reflecting an inability of the firms management to direct funds away from areas that need them when they choose to turn to external borrowing. Hence the firm first decides optimal investment in either case and then decides optimally whether or not to turn to external financing. The firm has to rely on external financing for all investment that cannot be financed from current income. Thus for investment which exceeds income from current dividends the firm has to take the consequences of an increased debt level into account.

The optimal investment policy is then straightforward to solve for. When optimal total investment exceeds current firm income their optimal level is: $i : 1 + \lambda_k(\frac{i}{k} - \delta_k) = \frac{\partial V}{\partial k}$ and $r : 1 + \lambda_k(\frac{r}{s} - \delta_s) = \frac{\partial V}{\partial s}$. Note that only the marginal benefit of getting more capital of either type enters, which is set equal to the marginal capital adjustment cost. In contrast, when $i + r \leq \pi$ then the optimal investment choice is $[1 + \lambda_k(\frac{i}{k} - \delta_k)](1 - \frac{\partial V}{\partial b}) = \frac{\partial V}{\partial k}$ and $[1 + \lambda_s(\frac{r}{s} - \delta_s)](1 - \frac{\partial V}{\partial b}) = \frac{\partial V}{\partial s}$. Note that in this case the cost of financing enters additionally. As $\frac{\partial V}{\partial b} < 0$ this will mean that optimal investment is lower than in the first case as the firm optimally accounts for the cost of an additional unit of debt when choosing investment.

B.2 Comparison of the Neural Network to the implicit Method solution

We compare our solution to the implicit method for solving a continuous time value function outlines in the Appendix of Achdou et al. (2022). For this we simplify the problem to the standard problem of a firm investing under uncertainty with a productivity state z and a capital state k. The firm optimally chooses investment i,

$$V = \max_{i,} E_0 \int_0^\infty e^{\rho t} \pi(z,k) dt, \tag{24}$$

subject to the laws of motion $\dot{z} = \mu_z - \rho_z(z - \mu_z) + \sigma_z dW$ and $\dot{k} = \delta_k k + i$.

The optimal investment policy for both solutions is plotted in Figure 1. We assume that $\mu_z = 1$ in steady state. The solutions are almost undistinguishable. However, one can compare the detailed Hamilton-Jacobi-Bellman error. Here the Neural network does comparable for the steady state and significantly better for out of steady state approximations as shown in Figure 2.

B.3 More details on the algorithm

As mentioned in the main text, the usual way to estimate a subset of parameters $\hat{\Psi} \in \Psi$ of a value function given data Y is as described in Algorithm 2. One estimates $\hat{\Psi}$ by solving the value function for a guess for the parameter values to be estimated, computes the likelihood and updates our guess. We solve the value function for each new guess until we believe to have found a point where the likelihood given the parameters is maximised.

Figure 1: **Optimal investment policy for either approximation of the value function**

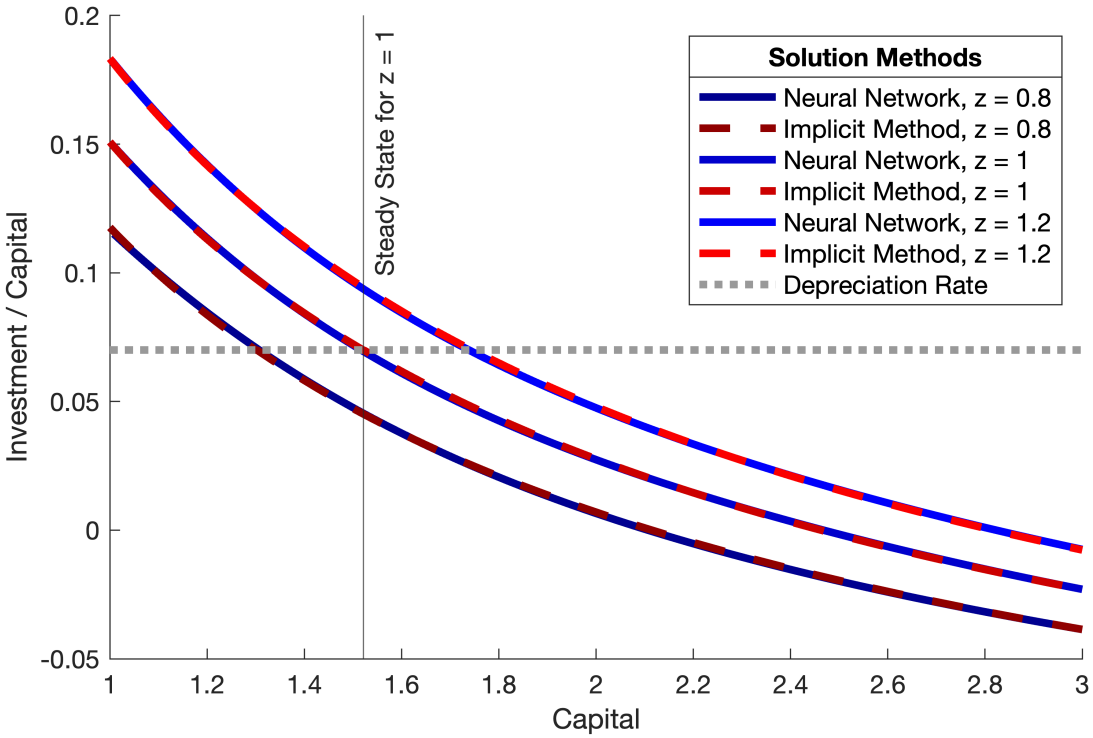
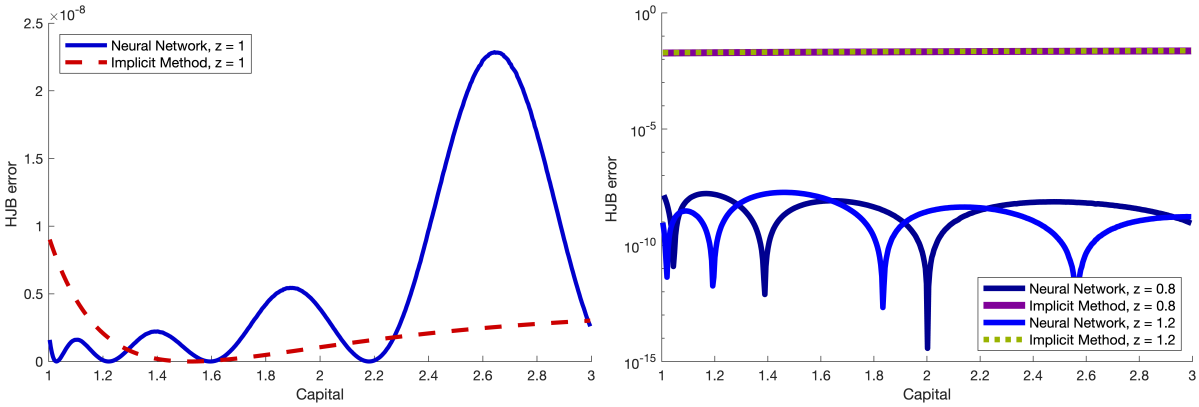


Figure 2: **Comparison of the Hamilton-Jacobi-Bellman error when the function is solved with a Neural Network or the implicit method**



Compared with this solution method, Algorithm 1 may be computationally more intensive, but it is both easier to code and easier to avoid local minima within the approximated range of the value function. With our approach, once the neural network is trained, evaluating the distance of our model to the empirically observed moments is computationally cheap. We can use standard methods to find a global minimum within the approximated parameter space. To avoid local minima we can use stochastic starting points. We can also avoid failures of the algorithm to converge, and avoid impossible parameters in the approximated parameter space.

Algorithm 2 Estimating Value function standard

Solve $V(X|\Psi_1)$ producing output $\hat{Y}$ ($\hat{Y}$ can be conditional on X)
while N less than Maxit **do**
  Check weighted distance $||\hat{Y} - Y||$
  Update Ψ given distance e.g. Newton
  Solve $V(X_1|\Psi_{N+1})$ producing output $\hat{Y}$
  if $|\Psi_{N+1} - \Psi_N| < \epsilon$ **then** Break the Loop
  end if
end while Then $\Psi_N = \hat{\Psi}$

C Extra tables and figures

Table 1: **OLS panel regression results - Intangible components**

ln(TFP)	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Leverage (t-1)	0.0238	0.00694	0.0447	0.0375	0.0439	0.0267	0.0356	0.0618*
	(0.0324)	(0.0305)	(0.0352)	(0.0340)	(0.0347)	(0.0318)	(0.0356)	(0.0374)
Intangible intensity - stock (balance sheet) (t-1)	-0.00326							
	(0.0119)							
R&D intensity - stock (t-1)		0.0521***						
		(0.0172)						
SG&A intensity - stock (t-1)			0.0879***					
			(0.0142)					
GW intensity - stock (t-1)				0.0227				
				(0.0147)				
Intangible intensity - flow (balance sheet) (t-1)					-0.00601			
					(0.00626)			
R&D intensity - flow (t-1)						0.0162*		
						(0.00844)		
SG&A intensity - flow (t-1)							0.0462***	
							(0.00773)	
GW intensity - flow (t-1)								0.0131*
								(0.00672)
Leverage*RHS variable (t-1)	0.113	0.149**	-0.00165	0.145	0.0418	0.111**	0.0388	0.0519
	(0.0694)	(0.0748)	(0.0579)	(0.0902)	(0.0436)	(0.0561)	(0.0393)	(0.0519)
Size (t-1)	0.0163**	0.0183**	0.0307***	0.0151*	0.0156*	0.0156**	0.0198**	0.0176**
	(0.00783)	(0.00772)	(0.00832)	(0.00817)	(0.00795)	(0.00791)	(0.00800)	(0.00851)
Age (t-1)	0.00656***	0.00648***	0.00569***	0.00648***	0.00732***	0.00730***	0.00732***	0.00681***
	(0.000955)	(0.000948)	(0.000939)	(0.00106)	(0.000999)	(0.000999)	(0.001000)	(0.00116)
Profitability (t-1)	0.104***	0.105***	0.115***	0.108***	0.105***	0.108***	0.126***	0.106***
	(0.0221)	(0.0221)	(0.0220)	(0.0224)	(0.0222)	(0.0223)	(0.0221)	(0.0227)
Cash ratio (t-1)	-0.0254	-0.0237	-0.0116	-0.00954	-0.0254	-0.0217	-0.00821	-0.00949
	(0.0390)	(0.0388)	(0.0387)	(0.0402)	(0.0404)	(0.0398)	(0.0402)	(0.0429)
Constant	0.730***	0.710***	0.666***	0.734***	0.706***	0.700***	0.668***	0.713***
	(0.0383)	(0.0389)	(0.0411)	(0.0464)	(0.0390)	(0.0393)	(0.0407)	(0.0483)
Observations	24,302	24,302	24,302	21,877	23,293	23,293	23,293	20,417
R-squared	0.791	0.792	0.793	0.798	0.792	0.792	0.792	0.803
Joint debt effect (p-value)	0.063	0.039	0.436	0.052	0.125	0.038	0.119	0.086

Notes: Robust standard errors in parentheses. *** p<0.01, ** p<0.05, * p<0.1. All regressions include time and firm fixed effects. All intangible components are ratios to total assets. Intangible components are: balance sheet intangibles, research and development (R&D), selling, general and admin expenses (SG&A), and goodwill (GW).

Table 2: **Robustness checks**

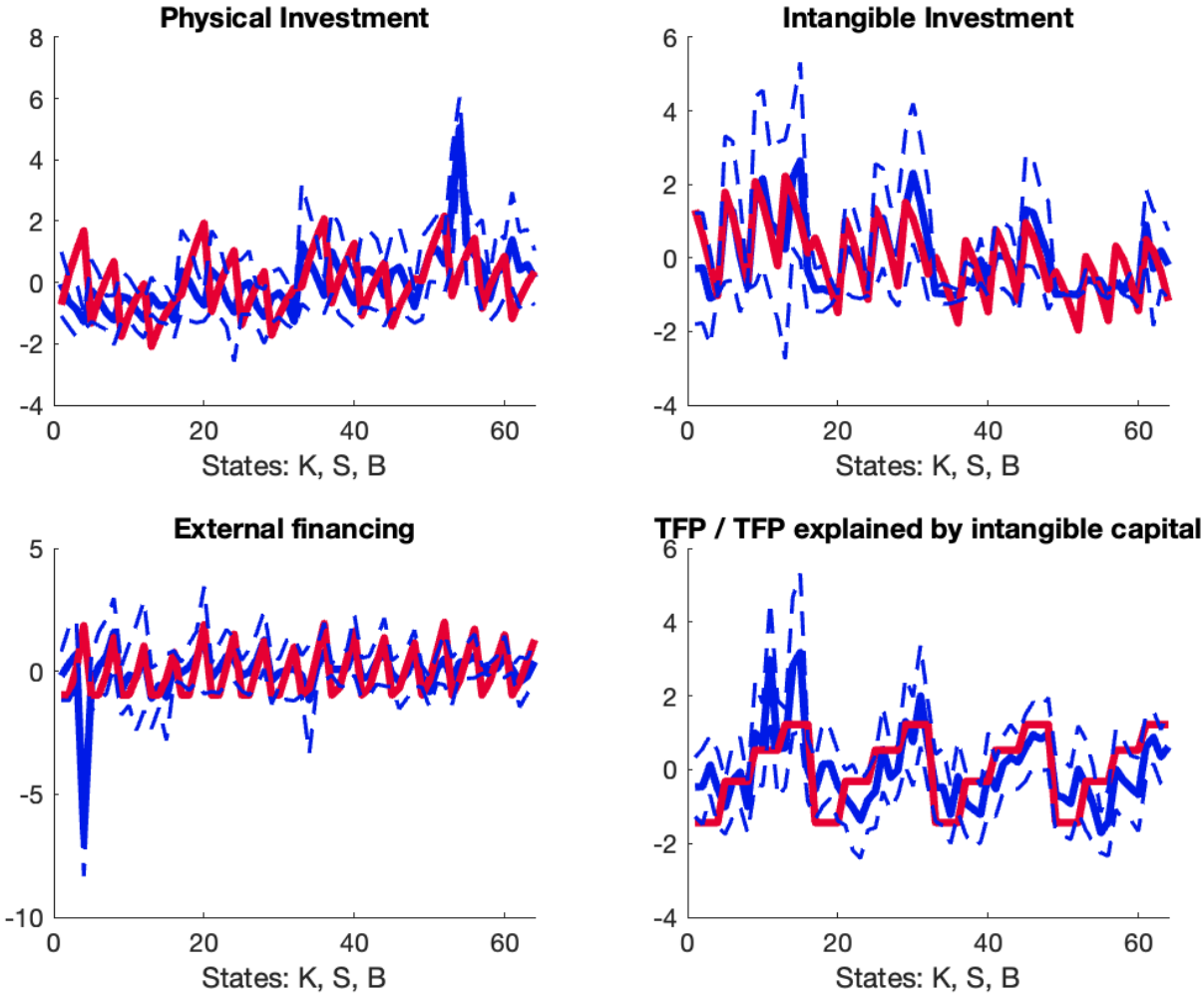
	(1) TFP_η	(2) TFP_η	(3) TFP low SG&A	(4) TFP low SG&A	(5) TFP low depr	(6) TFP low depr	(7) TFP high SG&A	(8) TFP high SG&A	(9) TFP high depr	(10) TFP high depr
Leverage (t-1)	0.0431	0.0617*	0.00938	0.0266	0.0152	0.0251	0.0228	0.0306	0.0324	0.0373
	(0.0287)	(0.0324)	(0.033)	(0.0351)	(0.0326)	(0.0356)	(0.0342)	(0.0359)	(0.0341)	(0.0343)
Intangible intensity - stock (t-1)	0.0972***		0.0513***		0.0956***		0.106***		0.0887***	
	(0.0096)		(0.0132)		(0.013)		(0.0139)		(0.0127)	
Intangible intensity - flow (t-1)		0.0116**		0.0144***		0.0267***		0.0295***		0.0222***
		(0.00556)		(0.00652)		(0.00683)		(0.0072)		(0.00722)
Leverage*RHS variable (t-1)	0.0556	0.0387	0.109*	0.0740*	0.0754	0.0587	0.055	0.049	0.105*	0.0493
	(0.0533)	(0.0329)	(0.0596)	(0.0395)	(0.0599)	(0.0395)	(0.0596)	(0.0403)	(0.0588)	(0.0415)
Observations	24,307	23,298	24,302	23,293	24,302	23,293	24,302	23,293	24,302	23,293
R-squared	0.565	0.563	0.799	0.799	0.805	0.804	0.803	0.802	0.797	0.796
Joint debt effect (p-value)	0.06	0.012	0.045	0.037	0.123	0.074	0.171	0.092	0.048	0.085

Notes: Robust standard errors in parentheses. *** p<0.01, ** p<0.05, * p<0.1. Regression specifications for controls (not reported) and fixed effects are similar to baseline OLS models (Table 3). Variable TFP_η is the unanticipated TFP component, as defined by Levine and Warusawitharana (2021). Columns with "low depr" contain results with a depreciation rate of 10% for R&D and SG&A. Columns with "high depr" contain results with depreciation rates of 25% and 30% for R&D and SG&A, respectively. Columns with "low SG&A" contain results with 10% of SG&A spending assumed to be intangible investment, and columns with "high SG&A" use a share of 50%.

Table 3: **Parameter list and calibration**

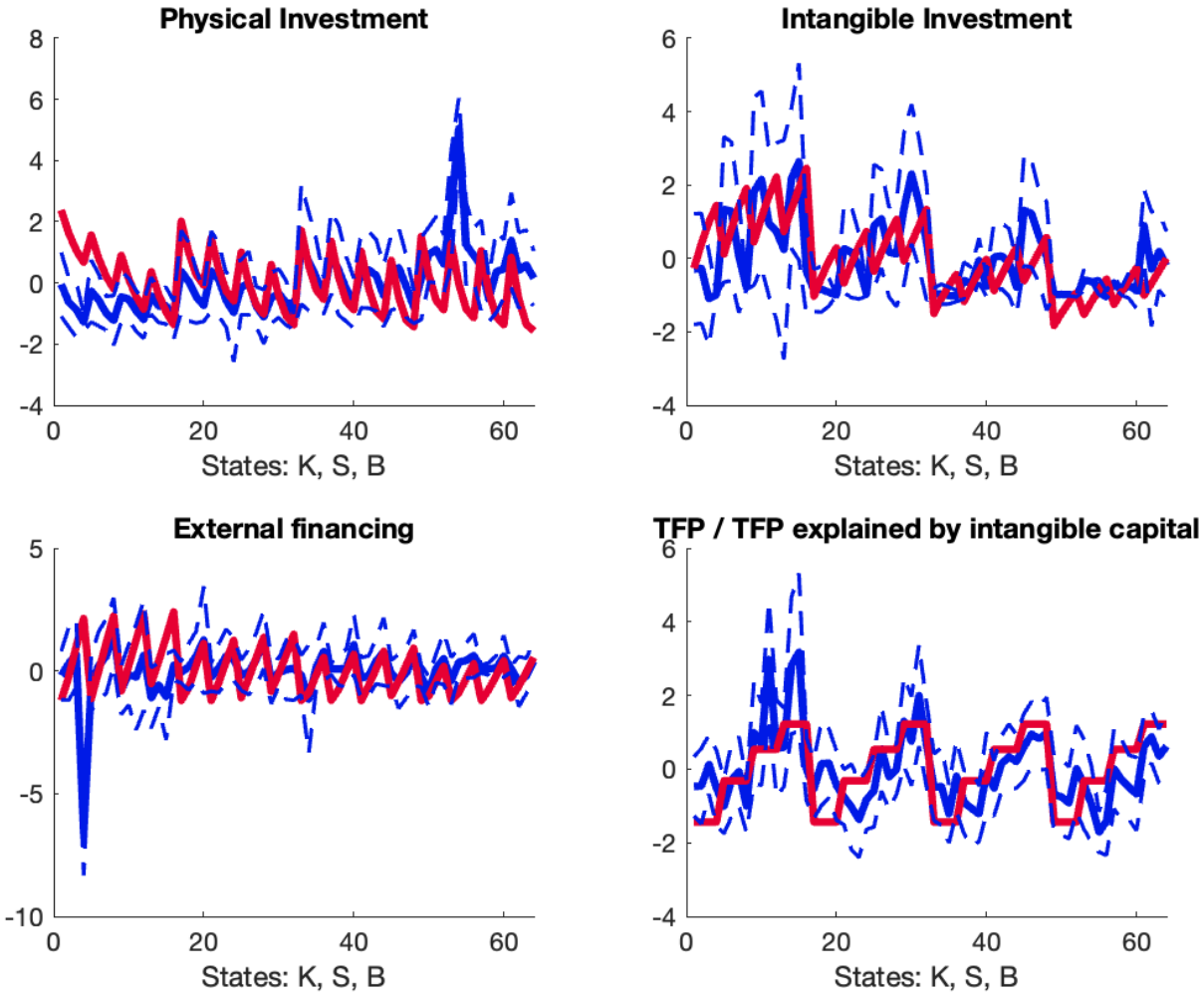
Parameter	Value	Description
ρ	0.05	Annual discount factor of 0.9512.
δ_k	0.07	Depreciation rate of physical capital
δ_s	0.2	Depreciation rate of intangible capital
α	0.17	Exponent on combined (Baseline) or physical (C-D) capital
x	0.1	Principal pay back rate of loans
ζ	1.83 (Baseline) 1.3 (C-D)	Otherwise unexplained TFP (for steady state normalisation)
	Estimated Parameters	
γ_1	Estimated	Exponent on intangible capital
ψ_1	Estimated	Cost of leverage
ψ_2	Estimated	Cost of intangible intensity
λ_k	Estimated	Adjustment cost of physical capital
λ_s	Estimated	Adjustment cost of intangible capital

Figure 3: **Matched moments from the model to the data for the baseline model**



Note: Estimated baseline model results from the data are shown in red for each of physical Capital (K), intangible capital (S), and debt (B) combination. The blue line shows the model moments conditional on the states. The dashed lines show the standard deviation of the moments conditional on the states. The red lines show the predicted model outcomes of the fitted model. The data and model results are normalised. All state combinations are increasing from left to right.

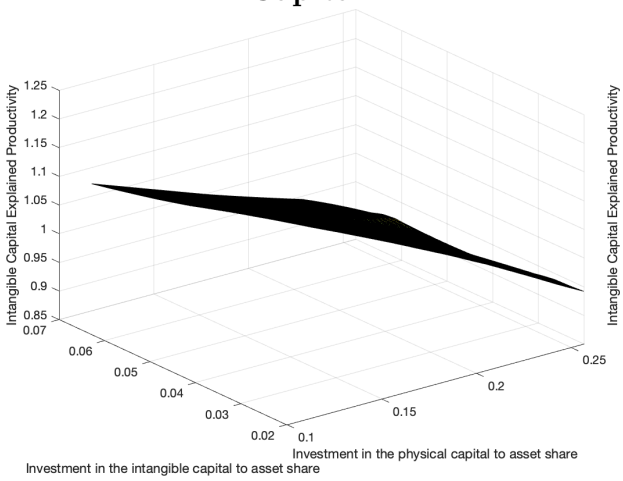
Figure 4: **Matched moments from the model to the data for the Cobb-Douglas model**



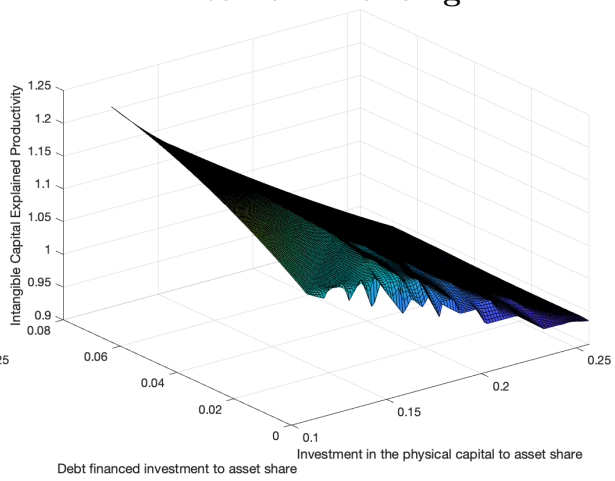
Note: Estimated Cobb-Douglas model results from the dtata are shown in red for each of physical Capital (K), intangible capital (S), and debt (B) combination. The blue line shows the model moments conditional on the states. The dashed lines show the standard deviation of the moments conditional on the states. The red lines show the predicted model outcomes of the fitted model. The data and model results are normalised. All state combinations are increasing from left to right.

Figure 5: **Estimated model results for productivity explained by intangible capital based on investment activity**

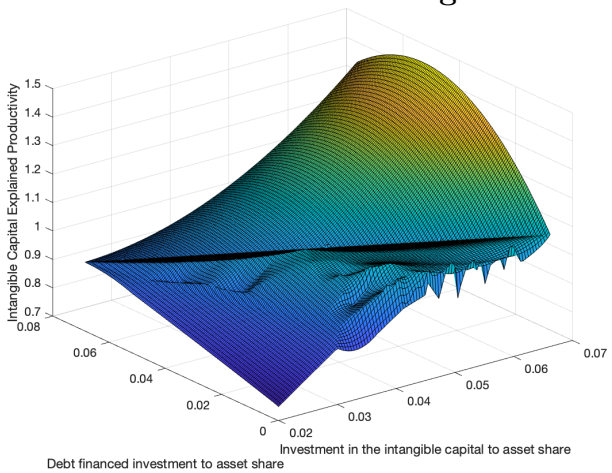
Investment in Physical & Intangible Capital



Investment in Physical Capital & External Financing

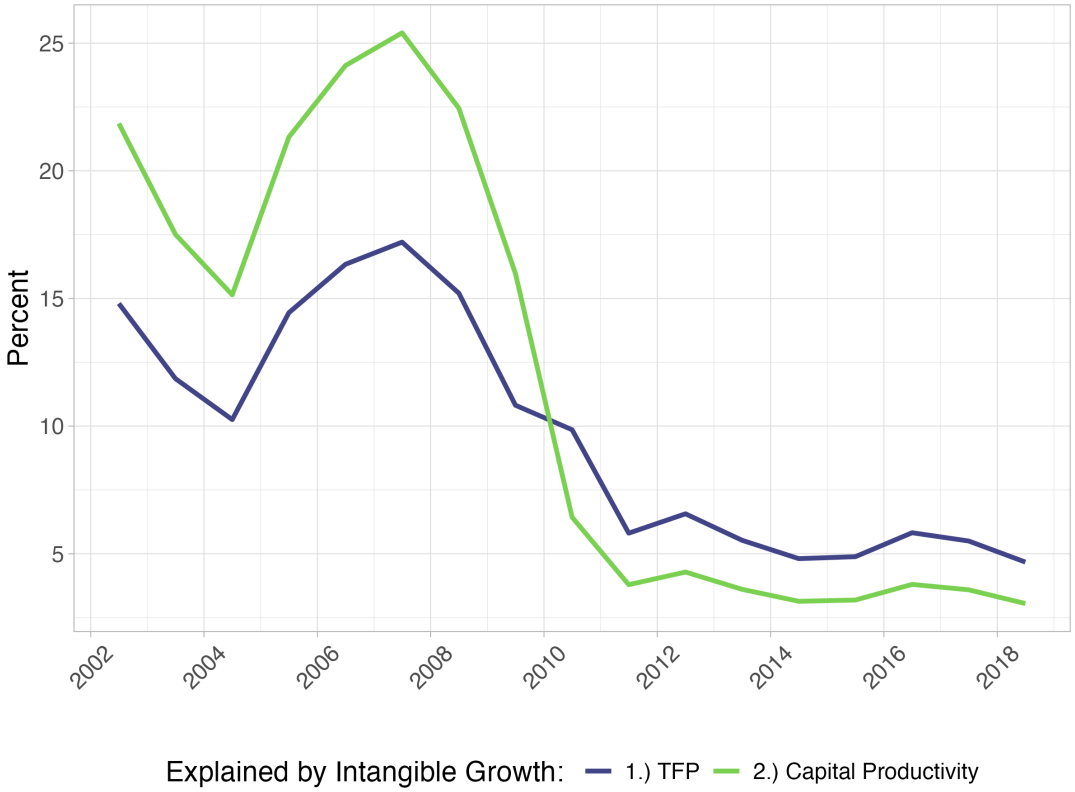


Investment in Intangible Capital & External Financing



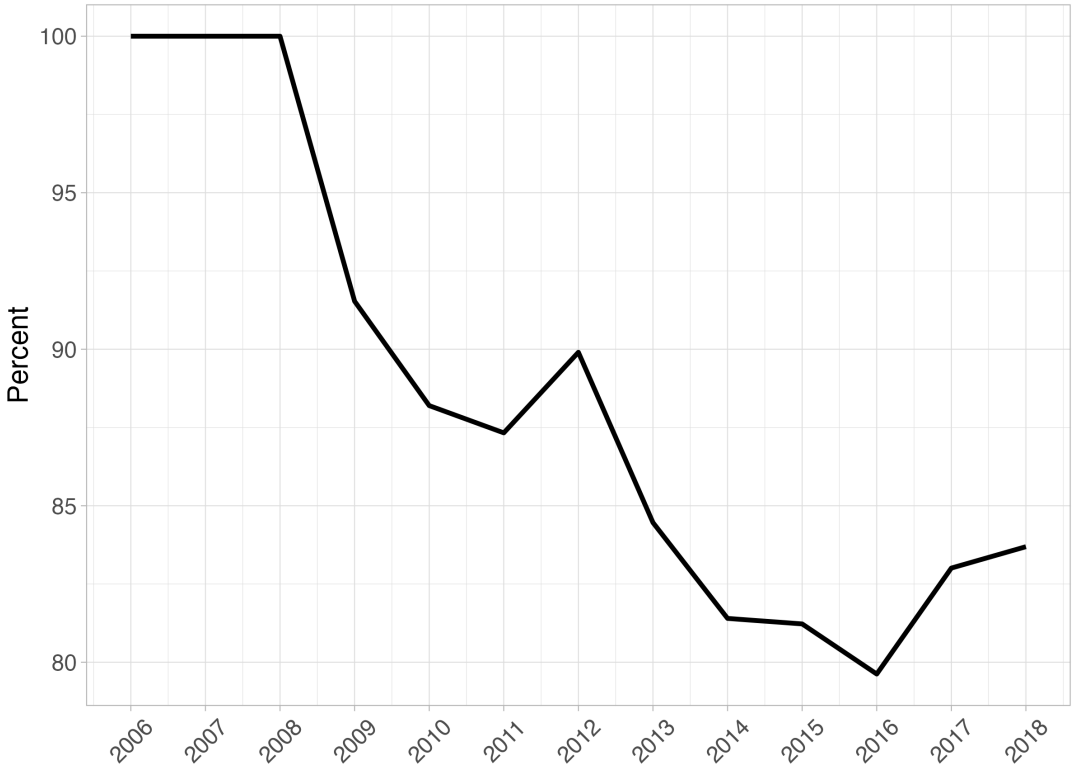
Note: The model is parameterised according to the calibration and estimation. The results are then interpolated based on the model predictions, collapsing the model predictions for investment activity given the three model state variables and the predicted investment outcomes.

Figure 6: **TFP and capital productivity growth explained by observed intangible investment**



Source: ONS and authors' calculations. This chart uses the intangible investment activity directly observed in the data (as opposed to that predicted by the model) to calculate productivity growth.

Figure 7: **Impact of worsening credit frictions on aggregate TFP growth generated by intangible investment**



Source: Authors' calculations. This chart uses the intangible investment activity directly observed in the data (as opposed to that predicted by the model) to calculate TFP growth.