Private Capital Interim Performance and Its Predictability

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March 17, 2025

Abstract

This paper studies the predictability of interim performance measures that are widely used in private capital (PC) markets, focusing on buyout (BO), venture capital (VC), and private debt (PD) funds. Amid a plethora of existing performance measures in the literature, I provide a unified framework within which most of the measures can be understood in a consistent manner, by introducing a generalized concept of TVPI (generalized TVPI; gTVPI). Then, I decompose the interim performance measures by taking log linearization, leading to two different decomposition approaches—the ROI approach and NPV approach. Each decomposition leads to a variance decomposition approach à la Campbell and Shiller (1988), imposing valuable structure on the predictability of the measures and its economic implications. In the empirical investigation of the ROI and NPV approaches, I find that the ROI approach provides cross-sectional performance predictability for certain fund types (particularly for VC funds) but virtually no time-series predictability, when future performance measures are regressed on current values. The NPV approach improves cross-sectional performance predictability for some fund types (particularly for BO funds) and leads to time-series performance predictability for BO funds and PD funds when distributed capital is assumed to be reinvested in private markets. In both approaches, gTVPI-like measures (total return measures) turn out to be the most useful in predicting future performance.

JEL: G00, G12, G24.

Keywords: Private capital, PC performance measure, Private equity, Private debt, Buyout funds, Venture capital.

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1 Introduction

The private capital (PC) market has been experiencing rapid growth in the past decade, increasing in the aggregate assets under management (AUM) from $3.9 trillion in 2013 to $12.3 trillion in 2023 globally ($2.2 trillion to $7.5 trillion in North America), according to Preqin.[1] The substantial growth of the sector is often attributed to PC funds' historical outperformance compared with public market benchmarks, as documented in some of the literature.[2] With an expectation of contiuning outperformance in the future, more investors—including global institutional investors—are projected to plow capital into this market.[3] Therefore, correct assessment of PC fund performance would be crucial to their investment decisions.

Since PC funds have a quite long life, and their ultimate performance is fully realized at liquidation, investors in a PC fund—or limited partners (LPs)—need to rely heavily on interim performance measures for a substantial portion of the fund's life. For a typical PC fund, it takes at least ten years from its inception through liquidation. While interim performance tends to converge to the final performance as the fund matures (e.g., Hochberg, Ljungqvist, and Vissing-Jørgensen (2013) for interim IRR), for a notable portion of the fund's life, interim performance measures provide a noisy signal on its final performance. Because investors in PC funds generally need to make real-time (additional) investment decisions, LPs have to rely on interim performance measures for those decisions—waiting until the full

[1]Future of Alternatives 2029, September 2024, Preqin. Private capital markets refer to debt and equity investments in private companies or assets, such as private credit/debt (PD), private equity (PE) and real assets such as infrastructure and real estate.

[2]The outperformance of PE funds, including venture captial (VC) funds, is documented in studies such as (Ljungqvist and Richardson (2003), Robinson and Sensoy (2010), and Harris, Jenkinson, and Kaplan (2014), among others. For PD funds, studies including Munday, Hu, True, and Zhang (2018), Böni and Manigart (2022), and Suhonen (2024) document their outperformance, although Erel, Flanagan, and Weisbach (2024) find no outperformance.

[3]For example, Norges Bank's letter in response to an inquiry from the Ministry of Finance of Norway states that "analyses of historical returns indicate that investments in private equity could give higher returns after costs than listed equities in the long term." See the link for further details.

liquidation of the funds to obtain the ultimate performance would rarely be an option.

However, there are only a limited number of studies (e.g., Jung-Hee Noh and Tae (2023) for interim IRR) that focus on the predictability of interim performance measures, and this paper attempts to fill the gap by studying the PC fund performance predictability of interim measures within a unified framework.[4] While there can be many other reasons, the lack of such studies may be partly attributed to 1) a plethora of performance measures, making it difficult to pick and justify one of the measures when their predictability differs, and 2) a lack of a theoretical framework and, as a result, a lack of economic interpretations of their predictability.

One may find, both in the literature or in practice, that there are (perhaps too) many different performance measures. Examples include, but not exhaustively, the total value to paid-in capital (TVPI), internal rate of return (IRR), modified IRR (MIRR), public market equivalent (PME; Long and Nickels (1996) and Kaplan and Schoar (2005)), generalized PME (GPME; Korteweg and Nagel (2016)), and direct alpha (Gredil, Griffiths, and Stucke (2023)). However, there has been "no consensus about a single best measure or approach to evaluate" PC fund performance (Korteweg and Sorensen (2023)). As a result, it is a common practice to use multiple measures in combination.[5] Therefore, to the extent that multiple performance measures are needed to gain a holistic perspective on PC fund performance, understanding those measures in a single (theoretical) framework would be crucial.

[4]The literature on interim performance measures tends to focus on the accuracy of net asset values (NAVs), which are self-reported by general partners (GPs), rather than the interim measures' usefulness in predicting future performance. Those studies—including Jenkinson, Sousa, and Stucke (2013), Barber and Yasuda (2017), Chakraborty and Ewens (2018), Brown, Gredil, and Kaplan (2019), and Hüther (2023))—find that GPs have an incentive to and do inflate NAVs around fundraising events. Yet, "in nonfundraising periods, the evidence suggests that GPs tend to be conservative in their reporting of NAVs" (Kaplan and Sensoy (2015)). While the accuracy of NAVs affects the informativeness of interim performance measures, even if NAVs are assumed to be perfectly accurate, interim performance measures' predictive power for future performance is not guaranteed and, thus, is a separate question.

[5]Unlike the academic literature emphasizing a discounted cash flow (DCF) valuation approach, in practice both the GP and LP appear to rely on money multiples (TVPI) and IRR when assessing investment and fund performance (Gompers, Kaplan, and Mukharlyamov (2016) and Da Rin and Phalippou (2014), among others).

In the first part of the paper, I provide a unified framework, within which most of the performance measures may be integrated. More specifically, I introduce a concept of the generalized TVPI (gTVPI) that can nest performance measures designed to estimate, roughly speaking, total returns over the life of a fund—such as TVPI, PME, and GPME. It is shown that most of the other measures, which are intended to gauge the rate of returns, have a quite straightforward relation to the gTVPI, either approximately or exactly.[6] Those rate of return measures—such as IRR, direct alpha, and MIRR—can be nested into the generalized internal rate of return (gIRR) or the generalized modified internal rate of return (gMIRR). An additional advantage of the unified framework is that the cost of capital and the return on distributed capital do not need to be equal, which allows a larger set of performance measures than typical Stochastic Discount Factor (SDF) valuation approaches do.

The proposed framework gives straightforward economic interpretations of the "generalized" performance measures: the gTVPI can be regarded as the total return on investment (ROI) in excess of the cost of capital, and the gIRR/gMIRR can be thought of as the average excess ROI over the effective duration/physical duration of the fund. The total ROI is an attainable return based on a tangible trading strategy, and, therefore, the average excess ROI can be considered as a version of the average "alpha," and the total excess ROI can be thought of as the total alpha. Given these interpretations, the predictability of interim performance measures means the predictability of the total or average alpha, depending on the (type of the) measure.

The "alpha" interpretation of the excess ROI naturally leads to the simplest decomposition of interim performance measures that involves a similar interpretation for the predictability of those measures—namely the "ROI" approach. With this approach, the current (log) value of the gTVPI can be decomposed into its future (log) value and the (log) inter-

[6]While the suggested framework itself does not pin down which measure to use for performance assessment, it helps to classify those performance measures based on the economic interpretation of their predictability in later part of the paper.

mediate excess ROI, defined by the ratio of the future excess ROI and the current excess ROI. Hence, the current gTVPI—the current total "alpha"—predicts the future gTVPI—the future total "alpha"—and/or the intermediate excess ROI—the future "alpha". However, the intermediate excess ROI may not be attainable from a tangible trading strategy, making the ROI approach somewhat vague in its interpretation.

Next, as an alternative approach, I propose another decomposition of interim performance measures, namely the NPV approach. With this approach, the current net gTVPI (i.e., $gTVPI_t - 1$) is regarded as the current net present value (NPV) of the entire portfolio—residual assets plus reinvested distributions minus capital calls—scaled by the current value of the capital calls. The current net gTVPI can be decomposed into the (appropriately adjusted) future net gTVPI and the sum of the (appropriately adjusted) intermediate excess return (between now and the future). Economically, this decomposition can be interpreted as the current (scaled) NPV predicting the future (discounted and scaled) NPV and/or the (discounted and scaled) intermediate excess gain. Interpreting the (discounted and scaled) NPV as total "alpha," again, the current total "alpha" predicts the future total "alpha" and/or the future "alpha."

These two decompositions using log linearization are similar to that of Campbell and Shiller (1988). As Campbell-Shiller decomposition does, the variance decomposition based on the two approaches imposes valuable structure on the predictability of interim performance measures and its economic implications: the current value of performance measures can be decomposed into its (appropriately adjusted) future value and the intermediate excess returns. Therefore, the predictability of interim performance measures for their future values is closely tied to the predictability of those measures for the intermediate excess returns. From the modern portfolio theory perspective, the intermediate excess returns (between now and the future) are particularly of interest to investors, and, thus, so are the predictability of interim performance measures.

As a benchmark, perfect predictability of measures based on gTVPI (total return measures) implies (excess) returns between now and the future is not predicted by those measures. On the other hand, perfect predictability of rate of return measures implies that (excess) returns between now and the future stay the same as the current rate of return. Therefore, in term of the "alpha" language, total "alpha" does not predict the future "alpha" if gTVPI-like measures are perfectly predictable, and the average "alpha" perfectly predicts the future "alpha" if gIRR/gMIRR-like measures are perfectly predictable. From investors' point of view, the predictability of gIRR/gMIRR-like measures would be particularly appealing, as the (average) past performance predicts the same degree of performance from now.

Lastly, I perform empirical investigation for both ROI and NPV approaches, using the buyout (BO), venture capital (VC), and private debt (PD) fund data from Preqin. The ROI approach yields cross-sectional performance predictability for certain fund types—particularly for VC funds—and certain specifications. However, cross-sectional performance predictability is largely limited to gTVPI (total return) measures. For time-series performance predictability, the ROI approach provides virtually no predictability for most performance measures. The missing time-series predictability might be particularly disappointing to investors as the current *within-fund* outperformance (and also underperformance) fully reverts back in the future.

For the NPV approach, cross-sectional performance predictability improves substantially for certain fund types—particularly for BO funds. The NPV approach yields time-series performance predictability for BO funds—and for PD funds when alternative risky debt market benchmarks are used—when distributed capital is reinvested in private markets. Overall, the NPV approach provides useful time-series predictability, when reinvesting distributions in private markets is allowed, and improved cross-sectional predictability for BO and PD funds. Note that the NPV approach also provides clearer economic interpretations than the ROI approach because the NPV approach involves actual returns. In both approaches, gTVPI

measures (total return measures) turn out to be the most useful measures for predicting future performance—the average "alpha" persists through the future only for limited cases, making the gIRR/gMIRR-like measures less useful.

This paper has three major contributions: this study 1) provides a unified framework within which most existing measures (or their approximations) of PC performance can be understood in a consistent manner, by introducing generalized concepts of those measures, 2) propose two different decomposition approaches—the ROI and NPV approach—yielding a variance decomposition approach à la Campbell and Shiller (1988) with a straightforward economic interpretation, 3) empirically examine the two approaches in cross-sectional and time-series regressions.

2 Conceptual Framework

In this section, I introduce a generalized version of "Total Value to Paid-In" capital (TVPI), named "generalized TVPI" or gTVPI, and show that up to a first-order approximation, most of the extant measures of private capital fund performance can be linked to gTVPI in a straightforward manner. Note that the purpose of introducing gTVPI is not to add another measure to the already long list of measures of private capital fund performance. Rather, the newly introduced concept helps to understand different measures of private capital fund performance in a unified framework.

2.1 Generalized TVPI

Consider a fund that liquidates at time T—the liquidity timing T is referred to as the maturity of the fund for simplicity. Define a cumulative (return-adjusted) capital call $\mathcal{C}_T$ at

maturity and $\mathcal{C}_t$ at any period $0 < t < T$:

$$\mathcal{C}_T \equiv \sum_{\tau=0}^{T-1} R^C_{\tau,T} C_\tau \quad , \quad \mathcal{C}_t \equiv \sum_{\tau=0}^{t-1} R^C_{\tau,t} C_\tau \quad , \quad R^C_{\tau,t} \equiv \prod_{\tau'=\tau+1}^{t} R^C_{\tau'} \, , \tag{1}$$

where R^C_{t+1} is the cost of capital between t and $t+1$. Similarly, define a cumulative (return-adjusted) distribution $\mathcal{D}_T$ at maturity and $\mathcal{D}_t$ at any period $0 < t < T$:

$$\mathcal{D}_T \equiv \sum_{\tau=1}^{T} R^D_{\tau,T} D_\tau \quad , \quad \mathcal{D}_t \equiv \sum_{\tau=1}^{t} R^D_{\tau,t} D_\tau \quad , \quad R^D_{\tau,t} \equiv \prod_{\tau'=\tau+1}^{t} R^D_{\tau'} \, , \tag{2}$$

where R^D_{t+1} is the return between t and $t+1$ on distributed capital, i.e., the cumulative distributed capital $\mathcal{D}_t$ at time t is invested in a portfolio that yields the return of R^D_{t+1} between t and $t+1$. Note that there is no distribution at period 0 and no capital call at period T.

A cumulative capital call $\mathcal{C}_t$ in Equation (1) can be rewritten as

$$\mathcal{C}_t = R^C_t \sum_{\tau=0}^{t-1} R^C_{\tau,t-1} C_\tau = R^C_t \left(\sum_{\tau=0}^{t-2} R^C_{\tau,t-1} C_\tau + C_{t-1} \right) = R^C_t \left(\mathcal{C}_{t-1} + C_{t-1} \right) \, , \tag{3}$$

since $R^C_{t,t} = 1$ for $\forall t$. Similarly, a cumulative distribution $\mathcal{D}_t$ in Equation (2) can be rewritten as

$$\mathcal{D}_t = R^D_t \sum_{\tau=1}^{t-1} R^D_{\tau,t-1} D_\tau + D_t = R^D_t \mathcal{D}_{t-1} + D_t \, , \tag{4}$$

since $R^D_{t,t} = 1$ for $\forall t$. Note that $\mathcal{C}_0 = \mathcal{D}_0 = 0$. Denote the value of the fund's asset at time t (before capital calls in the same period) by V_t. Then the generalized TVPI (gTVPI) at time

t is defined as

$$gTVPI_t \equiv \frac{V_t + \mathcal{D}_t}{\mathcal{C}_t} \ . \tag{5}$$

Note that both V_t and $\mathcal{C}_t$ do not include capital calls at time t. Commonly used measures of PC (largely PE) performance, particularly performance ratios such as TVPI and Kaplan-Schoar PME (KS PME) can easily be nested into gTVPI. To see this, when both R_t^C and R_t^D are set to 1 for all t—no (net) cost of capital and no (net) return on distributions—the gTVPI is equivalent to the classic TVPI. If both R_t^C and R_t^D are set to R_t^m, the (public) market return from $t-1$ to t, gTVPI at maturity T becomes the KS PME:

$$gTVPI_T = \frac{\mathcal{D}_T}{\mathcal{C}_T} = \frac{\sum_{\tau=1}^{T} R_{\tau,T}^m D_\tau}{\sum_{\tau=0}^{T-1} R_{\tau,T}^m C_\tau} = (KS\ PME\ )_T \ ,$$

since $V_T = 0$. While Kaplan and Schoar (2005) by and large considers liquidated funds, studies often extend the KS PME to funds where cash flows are not fully realized (e.g., Brown et al. (2019)) by including interim valuation of V_t. Then, gTVPI at time t becomes the interim KS PME when $R_t^C = R_t^D = R_t^m$:

$$gTVPI_t = \frac{\mathcal{D}_t}{\mathcal{C}_t} = \frac{V_t + \sum_{\tau=1}^{t} R_{\tau,t}^m D_\tau}{\sum_{\tau=0}^{t-1} R_{\tau,t}^m C_\tau} = (KS\ PME\ )_t \ .$$

While the gTVPI framework nests most of existing "total return/multiple" measures of PC (or PE) performance—e.g., generalized PME (Korteweg and Nagel (2016))—this approach differs from typical Stochastic Discount Factor (SDF) valuation approaches in that returns on distributions (R_{t+1}^D) do not need to be equal to the cost of capital (R_{t+1}^C). As discussed in Brown et al. (2019), changes in total return/multiple measures in excess of the benchmark (the cost of capital in the gTVPI framework) tend to moderate over a fund's life as the fund distributes its assets back to LP. However, if the distributed capital was invested

in assets that are similar to the residual assets in the fund's current portfolio, such "artificial" moderation in excess performance would disappear—as Weighted PME in Brown et al. (2019) attempts to address. Therefore, the additional possibility of $R^C_{t+1} \neq R^D_{t+1}$ included in the gTVPI framework may provide a useful tool to better understand fund performance by considering investment opportunities available to fund investors (LPs), outside of the benchmark generating the cost of capital.

2.2 Rate of Return: Relation to gTVPI

Another PC performance measure that is widely used both in practice and in the literature is the internal rate of return (IRR). The IRR is designed to capture an annual compounding return over the entire life of a PC fund, with an implicit assumption that the single return discounts both capital calls and distributed capital. Note that mathematically the IRR may not exist or may have multiple values.

In Appendix Section A.1, I derive an approximate relation between the IRR and the TVPI:

$$irr_T \equiv log(1 + IRR_T) \approx \frac{tvpi_T}{Dur_T} \quad , \quad Dur_T \equiv \frac{\sum_{\tau=1}^{T} \tau D_\tau}{\sum_{\tau=1}^{T} D_\tau} - \frac{\sum_{\tau=0}^{T-1} \tau C_\tau}{\sum_{\tau=0}^{T-1} C_\tau} \, , \tag{6}$$

where $tvpi_T \equiv \log(TVPI_T)$, and Dur_T is the Macaulay duration difference between distributions and capital calls (with no discount). Note that in this approximation, the duration difference Dur_T is roughly the "effective duration" (e.g., Gredil et al. (2023), among others) or "duration of performance" (e.g., Kocis, Bachman, Long, and Nickels (2009), among others) that are often used by practitioners:

$$Dur_T \approx (\textit{Effective Duration}) \equiv \frac{tvpi_T}{\log(1 + IRR_T)} \, . \tag{7}$$

Before liquidation, the interim IRR (IRR_t for $t < T$) is calculated under an assumption that assets held in the fund are liquidated at the NAV in the corresponding period. The interim IRR can be approximated as

$$irr_t \equiv log(1 + IRR_t) \approx \frac{tvpi_t}{Dur_t} \quad , \quad Dur_t \equiv \frac{\sum_{\tau=1}^{t} \tau D_\tau + tV_t}{\sum_{\tau=1}^{t} D_\tau + V_t} - \frac{\sum_{\tau=0}^{t-1} \tau C_\tau}{\sum_{\tau=0}^{t-1} C_\tau} \, . \tag{8}$$

which is also derived in Appendix Section A.1. Note that for $t = T$, the interim IRR becomes the final IRR defined in Equation (6) as $V_T = 0$ at liquidation.

The generalized IRR (gIRR) can be defined as an annual compounding return on a hypothetical stream of cash flows where each cash flow is replaced by its future value at liquidation—for capital calls C_t by $R^C_{t,T} C_t$, and for distributed capital D_t by $R^D_{t,T} D_t$. Similar to the relation between the IRR and TVPI, in Appendix Section A.1, an approximate relation between the gIRR and gTVPI is derived:

$$girr_t \approx \frac{gtvpi_t}{gDur_t} \quad , \quad gDur_t \equiv \frac{\sum_{\tau=1}^{t} \tau R^D_{\tau,t} D_\tau + tV_t}{\sum_{\tau=1}^{t} R^D_{\tau,t} D_\tau + V_t} - \frac{\sum_{\tau=0}^{t-1} \tau R^C_{\tau,t} C_\tau}{\sum_{\tau=0}^{t-1} R^C_{\tau,t} C_\tau} \, , \tag{9}$$

where $gtvpi_t \equiv \log(gTVPI_t)$. Note that when $R^C_t = R^D_t = 1$—no cost of capital and no return on distributed capital—gIRR becomes IRR. When $R^C_t = R^D_t = R^m_t$, gIRR becomes the direct alpha in Gredil et al. (2023) and $girr$ (approximately) becomes the alpha in Phalippou and Gottschalg (2008).

A variant of the IRR that is advocated by some academics and practitioners (e.g., Phalippou (2008)) is the modified IRR (MIRR). The MIRR naturally fits into the gTVPI framework in this paper as the measure nests the concepts of cost of capital and return on distribution. The MIRR is defined by $(R - 1)$ where R satisfies

$$R^T \sum_{\tau=0}^{T-1} \frac{C_\tau}{R^C_{0,\tau}} = \sum_{\tau=1}^{T} R^D_{\tau,T} D_\tau \, . \tag{10}$$

I define the generalized MIRR (gMIRR) at liquidation as follows:

$$gMIRR_T \equiv \frac{(1 + MIRR)}{\left(R^C_{0,T}\right)^{1/T}} \ . \tag{11}$$

The gMIRR can be thought of as the average rate of return earned on distributed capital in excess of the cost of capital over the entire life of the fund. As derived in Appendix Section A.2, the gMIRR and gTVPI has the following relation:

$$gmirr_T = \frac{gtvpi_T}{T} \quad , \quad r^C_{0,T} \equiv \log(R^C_{0,T}) \ . \tag{12}$$

The interim version of the gMIRR has a similar relation to the interim gTVPI, as also derived in Appendix Section A.2:

$$gmirr_t = \frac{gtvpi_t}{t} \quad , \quad r^C_{0,t} \equiv \log(R^C_{0,t}) \ . \tag{13}$$

Note that when $R^C_t = R^D_t = 1$, the $gmirr_t$ reads

$$gmirr_t = \frac{tvpi_t}{t} \ . \tag{14}$$

In Larocque, Shive, and Sustersic Stevens (2022), this version of the gMIRR is defined as the "multiple-implied return" and used to compute the "timing gap."

2.3 Economic Interpretation

$gTVPI_t$ can be regarded as a measure of the total (gross) excess return—in excess of the cost of capital—up to time t on the (discounted) value of the called capital at time 0. To

see this, consider

$$gTVPI_t \equiv \frac{V_t + \mathcal{D}_t}{\mathcal{C}_t} = \frac{1}{R^C_{0,t}} \left(\frac{V_t + \mathcal{D}_t}{\mathcal{C}_t / R^C_{0,t}} \right) \equiv \frac{ROI_{0,t}}{R^C_{0,t}} \,, \tag{15}$$

where $R^C_{0,t}$ is the cost of capital between time 0 and t. The numerator of the first expression is the total value at time t: $V_t + \mathcal{D}_t$ is the combined value of the residual portfolio (V_t) and the (reinvestment) portfolio of the distributed capital ($\mathcal{D}_t$). The denominator $\mathcal{C}_t$ of the first expression is the total cost (at time t) of the capital called up to time t, and the denominator $\mathcal{C}_t / R^C_{0,t}$ in the parenthesized factor of the second expression is the (discounted) value of the cumulative capital at time 0. Therefore, the parenthesized factor—denote it by $ROI_{0,t}$—can be thought of as the total return on the (discounted) invested/called capital (i.e., return on investment/ROI) between time 0 and t, and $gTVPI_t$ can be interpreted as the total ROI in excess of the cost of capital.[7]

Conceptually, the total return $ROI_{0,t}$ can be attained by the following trading strategy: (a) at time 0, invest capital $\mathcal{C}_t / R^C_{0,t}$ in the benchmark which generates $R^C_{0,t}$ between time 0 and t, (b) every time there is a capital call (possibly including time 0), meet the capital call by selling the benchmark assets for the needed amount, (c) every time there is a capital distribution, invest the distributed capital (through a separate account) in assets that generate $R^D_{0,t}$ between time 0 and t, and (d) at time t, all the capital invested in the benchmark assets are liquidated, the value of the residual portfolio in the fund is V_t, and the value of the reinvested distributions is $\mathcal{D}_t$.[8] This trading strategy does not involve liquidating (part of) the fund or the reinvested distribution portfolio and, hence, is still valid when the fund or the reinvested distributions are difficult to be liquidated.

[7] While denoted as ROI, the parenthesized factor is in fact equivalent to the total MIRR. To avoid confusion with generalized MIRR, and since ROI and MIRR are conceptually quite similar, the ROI denotation is used throughout the paper.

[8] It is assumed that the benchmark assets are liquid and can be sold at no cost.

Now taking a log on both sides of Equation (15) yields

$$gtvpi_t = \log\left(gTVPI_t\right) = -\log\left(R^C_{0,t}\right) + \log\left(ROI_{0,t}\right) = roi_{0,t} - r^C_{0,t}\ , \tag{16}$$

where $roi_{0,t} \equiv \log\left(ROI_{0,t}\right)$ and $r^C_{0,t} \equiv \log\left(R^C_{0,t}\right)$. Thus, the log of $gTVPI_t$ is the log difference between $ROI_{0,t}$ and the cost of capital between time 0 and t.

This relation in combination with the approximate relation between $gIRR_t$ and $gTVPI_t$ in Equation (9) gives

$$girr_t \approx \frac{gtvpi_t}{gDur_t} = \frac{roi_{0,t} - r^C_{0,t}}{gDur_t}\ , \tag{17}$$

Therefore, the log of $gIRR_t$ can be regarded as the average log rate of return on invested/called capital in excess of the average (log) cost of capital over the (generalized) "duration" of the fund.

Similarly, Equation (13) together with Equation (16) yields

$$gmirr_t = \frac{gtvpi_t}{t} = \frac{roi_{0,t} - r^C_{0,t}}{t}\ . \tag{18}$$

As a result, the log of $gMIRR_t$ can be interpreted as the average log rate of return on invested/called capital in excess of the average (log) cost of capital over the interim "physical" life of the fund.

Lastly, since $gDur_t \leq t$, the $gmirr_t$ and $girr_t$ have the following relation:

$$gmirr_t \lessapprox girr_t\ . \tag{19}$$

Therefore, $gmirr_t$ can be thought of as a more conservative measure of the (log) rate of return compared to the $girr_t$.

3 Log Linearization: ROI approach

In this section, I apply a log linearization approach—based on the ROI interpretation of the measures—to decompose the generalized performance measures discussed in Section 2. This log linearization yields a decomposition that leads to the simplest form of predictive regressions of generalized performance measures.

3.1 Log Linear Decomposition

Before starting, define the intermediate ROI between time t and $t+1$ as typically the (holding period) return is defined:

$$ROI_{t+1} \equiv \left(\frac{ROI_{0,t+1}}{ROI_{0,t}} \right) \quad , \quad ROI_{0,t} = \prod_{s=1}^{t} ROI_s \ , \tag{20}$$

where $ROI_{0,0} = 1$. However, note that ROI_{t+1} is not an actual return between time t and $t+1$ except for certain special cases.[9] Therefore, it is generally not feasible to attain the return of ROI_{t+1} between t and $t+1$ from the portfolio yielding $ROI_{0,t}$ at time t.

Now starting from the relation in Equation (16), the difference between $gtvpi_{t+1}$ and $gtvpi_t$ reads

$$gtvpi_{t+1} - gtvpi_t = (roi_{0,t+1} - roi_{0,t}) - \left(r^C_{0,t+1} - r^C_{0,t}\right) = roi_{t+1} - r^C_{t+1} \ , \tag{21}$$

where $roi_{t+1} \equiv \log\left(ROI_{t+1}\right)$. Iterating Equation (21) gives

$$gtvpi_t = gtvpi_{t+n} - \sum_{k=1}^{n} \left(roi_{t+k} - r^C_{t+k}\right) \ , \tag{22}$$

where n can be any integer between 1 and $T-t$.

[9] Detailed discussions on this point are provided in Section A.4.

The log linear decomposition for the gTVPI can easily be extended to measures of the rate of return: gIRR and gMIRR. For gIRR, using Equation (9), one can rewrite Equation (22) as

$$\frac{gDur_t}{gDur_{t+n}}\, girr_t \approx girr_{t+n} - \frac{1}{gDur_{t+n}} \sum_{k=1}^{n} \left(roi_{t+k} - r^C_{t+k} \right) \ , \tag{23}$$

Rearranging this equation gives

$$girr_t \approx girr_{t+n} - \frac{\Delta_n gDur_{t+n}}{gDur_{t+n}} \left(\frac{1}{\Delta_n gDur_{t+n}} \sum_{k=1}^{n} \left(roi_{t+k} - r^C_{t+k} \right) - girr_t \right) \ , \tag{24}$$

where

$$\Delta_n gDur_{t+n} \equiv gDur_{t+n} - gDur_t \ .$$

The first term on the right hand side of Equation (24) reflects the future value of $girr_t$, and the second term—those in the parenthesis—reflects the average log excess ROI, computed over the change of the (generalized) duration between t and $t+n$, in excess of $girr_t$. The factor in the parenthesis is multiplied by the share of the change of the duration over the entire duration, as the average (log) excess ROI between t and $t+n$ only contributes to the change of generalized IRR in proportion to the share.

Turning to the gMIRR, using Equation (13) one can rewrite Equation (22) as

$$\frac{t}{t+n} gmirr_t = gmirr_{t+n} - \frac{1}{t+n} \sum_{k=1}^{n} \left(roi_{t+k} - r^C_{t+k} \right) \ ,$$

Rearranging this equation yields

$$gmirr_t = gmirr_{t+n} - \frac{n}{t+n} \left(\frac{1}{n} \sum_{k=1}^{n} \left(roi_{t+k} - r^C_{t+k} \right) - gmirr_t \right) \ . \tag{25}$$

The first term reflects the future value of $gmirr_t$, and the second term—those in the parenthesis—reflects the average log excess ROI between t and $t+n$ in excess of $gmirr_t$. The factor in the parenthesis is multiplied by the share of the physical time passed between t and $t+n$ over the entire physical life of the fund, as the average (log) excess ROI between t and $t+n$ only contributes to the change of generalized MIRR in proportion to the share.

The first terms in the RHS of Equation (22), (24), and (25), respectively, are the future value of each performance measure. Therefore, the decomposition in Equation (22), (24), and (25) provides a useful framework to interpret regressions of future (log) performance measures on their respective current (log) values. Such regressions are the simplest way to examine the predictability of interim performance measures (e.g., Jung-Hee Noh and Tae (2023)).

3.2 Economic Interpretation

The log linearization in Equation (22) yields a decomposition of $gtvpi_t$ that is analogous to that of the price-dividend ratio in Campbell and Shiller (1988):

$$p_t - d_t \approx \rho^n \left(p_{t+n} - d_{t+n}\right) - \sum_{k=1}^{n} \rho^{k-1} \left(r_{t+k} - \Delta d_{t+k}\right) + (\text{Constant}) \,, \tag{26}$$

where $p_t - d_t$ is the log dividend-price ratio at time t, r_{t+1} is the log stock return between time t and $t+1$, Δd_{t+1} is the log dividend growth between time t and $t+1$, and $\rho \equiv \frac{\exp(\overline{p-d})}{\exp(\overline{p-d})+1}$ where $\overline{p-d}$ is the long-term mean of $p_t - d_t$. Note that the existence of the long-run mean $\overline{p-d}$ relies on an assumption that the log dividend-price ratio is stationary.[10]

[10]While there are apparent similarities between the decomposition in Equation (22) and the Campbell-Shiller decomposition in Equation (26), there are some notable differences. First, the coefficient on the future value (of $gtvpi_t$) in Equation (22) is equal to 1, but ρ^n of the Campbell-Shiller decomposition in Equation (26) is less than 1. Second, unlike the Campbell-Shiller decomposition, which can extend to an infinite n as stock has no maturity, PC funds have finite life, and, as a result, n can only extend to the maturity $T-t$. Third and relatedly, the decomposition in Equation (22) does not require the stationarity of $gtvpi_t$. Lastly, the decomposition in Equation (22) is an exact relation while the Campbell-Shiller decomposition is an approximate relation.

As the Campbell-Shiller decomposition is typically interpreted, the decomposition in Equation (22) can be interpreted as follows: the current value of $gtvpi_t$ either predicts its future value $gtvpi_{t+n}$ or the log excess ROI $\sum_{k=1}^{n}\left(roi_{t+k}-r_{t+k}^{C}\right)$ between now and the future, or both. Similarly, the relation in Equation (24) and (25) can be interpreted as follows: the current value of $girr_t$ (or $gmirr_t$) either predicts its future value $girr_{t+n}$ (or $gmirr_{t+n}$) or the average log excess ROI in excess of $girr_t$ (or $gmirr_t$), or both. Thus, the log linearization imposes economically meaningful structures on the predictability of interim performance measures, in that it links predictability on the future measure and that on the (intermediate) excess return.

As a benchmark, consider the case of the full predictability of $gtvpi_t$: $gtvpi_t$ is an unbiased ex-ante measure of $gtvpi_{t+n}$, i.e., $gtvpi_t = E_t\left[gtvpi_{t+n}\right]$. In this case, the log excess ROI between t and $t+n$ is zero in expectation, i.e., the (intermediate) ROI is equal to the cost of capital. However, the full predictability of the rate of return measures ($girr_t = E_t\left[girr_{t+n}\right]$ or $gmirr_t = E_t\left[gmirr_{t+n}\right]$) has a different interpretation: the average log excess ROI is equal to the current value of the rate of return measures ($girr_t$ or $gmirr_t$) ex ante. In other words, while the full predictability of $gtvpi_t$ implies zero future excess ROI in expectation, the full predictability of $girr_t$ or $gmirr_t$ indicates future excess ROI being at the same rate as the current $girr_t$ or $gmirr_t$ ex ante. Therefore, the predictability of the total return measure ($gtvpi_t$) and that of the rate of return measures ($girr_t$ or $gmirr_t$) differ starkly by their implications for future excess returns, measured by the excess ROI.

Since gTVPI can be interpreted as the total excess ROI, conceptually it can be thought of as a version of "total alpha," which is the (appropriate) sum of past alpha—where the excess ROI can be considered as alpha. Similarly, gIRR/gMIRR can be thought of as the "average alpha" over the effective duration/physical duration. Hence, the full predictability of gTVPI can be regarded as total alpha forecasting zero alpha between now and the future—the current total alpha has no forecasting power for future alpha. On the other hand, the full

predictability of gIRR/gMIRR can be thought of as the average alpha forecasting the same alpha between now and the future—the current average alpha (over the past) predicting perfectly persistent alpha.

While the log linearization in Equation (22), (24), and (25) leads to a decomposition that can be related to the simplest form of predictive regressions, its economic interpretations may not be very straightforward. The essence of the decomposition is that the current performance measures can be decomposed into their future value and the intermediate (adjusted) returns. However, as discussed in Section A.4, the (intermediate) ROI is not an actual return but rather a conceptual and hypothetical return. Therefore, the return portion of the decomposition does not have a clear economic counterpart in reality, which makes its economic interpretation vague.

3.3 Variance Decomposition

As the Campbell-Shiller log linearization can be translated into a variance decomposition, the log linearization in Equation (22), (24), and (25), respectively, leads to

$$1 = \beta^n_{gtvpi} - \beta^n_{roi^{ex}} \quad , \quad 1 \approx \beta^n_{girr} - \beta^n_{\overline{roi}^{ex}-girr} \quad , \quad 1 = \beta^n_{gmirr} - \beta^n_{\overline{roi}^{ex}-gmirr} \, , \tag{27}$$

where

$$\beta^n_{gtvpi} \equiv \frac{Cov(gtvpi_t, gtvpi_{t+n})}{Var(gtvpi_t)} \quad , \quad \beta^n_{girr} \equiv \frac{Cov(girr_t, girr_{t+n})}{Var(girr_t)} \quad , \quad \beta^n_{gmirr} \equiv \frac{Cov(gmirr_t, gmirr_{t+n})}{Var(gmirr_t)} \, ,$$

and

$$\begin{aligned}
\beta^{n}_{roi^{ex}} &\equiv \frac{Cov\left(gtvpi_t, \sum_{k=1}^{n}\left(roi_{t+k} - r^{C}_{t+k}\right)\right)}{Var(gtvpi_t)} \ , \\
\beta^{n}_{\overline{roi}^{ex}-girr} &\equiv \frac{Cov\left(girr_t, \frac{\Delta_n gDur_{t+n}}{gDur_{t+n}}\left(\frac{1}{\Delta_n gDur_{t+n}}\sum_{k=1}^{n}\left(roi_{t+k} - r^{C}_{t+k}\right) - girr_t\right)\right)}{Var(girr_t)} \ , \\
\beta^{n}_{\overline{roi}^{ex}-gmirr} &\equiv \frac{Cov\left(gmirr_t, \frac{n}{t+n}\left(\frac{1}{n}\sum_{k=1}^{n}\left(roi_{t+k} - r^{C}_{t+k}\right) - gmirr_t\right)\right)}{Var(gmirr_t)} \ .
\end{aligned}$$

Note that since Equation (22), (24), and (25) hold ex post, the variance decomposition in Equation (27) holds both conditionally and unconditionally.

The equivalence between volatility tests and return forecasting regressions is well established in the literature (e.g., Cochrane (1991)). Similarly, the variance decomposition in Equation (27) is equivalent to regressions of each of the two right hand side terms on the left hand side term in Equation (22), (24), and (25), respectively. The interpretation of the variance decomposition is also similar to that of the Campbell-Shiller decomposition: the variance of $gtvpi_t$ is either explained by its covariance with the future value or its covariance with the future excess ROI (relative to the cost of capital), or both. The variance of $girr_t/gmirr_t$ is either explained by its covariance with the future value or its covariance with the average future excess ROI (relative to the cost of capital) in excess of the current $girr_t/gmirr_t$, or both.

4 Log Linearization: NPV approach

In this section, I apply a different log linearization approach from that in Section 3 to decompose generalized performance measures. This log linearization yields a decomposition regarding the predictability of the future discounted NPV (or its rate) based on the current NPV (or its rate).

4.1 Log Linear Decomposition

The intermediate gain between t and $t+1$ in excess of the cost of capital can be written as:

$$(V_{t+1} + \mathcal{D}_{t+1} - \mathcal{C}_{t+1}) - R^C_{t+1}(V_t + \mathcal{D}_t - \mathcal{C}_t) = (V_t + \mathcal{D}_t + C_t)\left(R^W_{t+1} - R^C_{t+1}\right) , \tag{28}$$

where R^W_{t+1} is defined as

$$R^W_{t+1} \equiv \frac{V_{t+1} + \mathcal{D}_{t+1}}{V_t + \mathcal{D}_t + C_t} , \tag{29}$$

which is the holding period return on a combined portfolio of (a) the residual assets in the fund (V_t), (b) the reinvested distribution portfolio ($\mathcal{D}_t$), and (c) a capital call at time t (C_t). The intermediate gain in Equation (28) can be thought of as ex-post value generated from the combined portfolio between t and $t+1$.

Then, the total excess gain from t to $t+1$, discounted by the cost R^C_{t+1} of capital calls to time t and scaled by the value $\mathcal{C}_t$ of cumulative capital calls, reads as follows:

$$\frac{(V_{t+1} + \mathcal{D}_{t+1} - \mathcal{C}_{t+1})}{R^C_{t+1}\mathcal{C}_t} - \frac{V_t + \mathcal{D}_t - \mathcal{C}_t}{\mathcal{C}_t} = \frac{V_t + \mathcal{D}_t + C_t}{\mathcal{C}_t}\frac{R^W_{t+1} - R^C_{t+1}}{R^C_{t+1}} , \tag{30}$$

Rewriting this equation in terms of gTVPI yields:

$$\frac{\mathcal{C}_{t+1}}{R^C_{t+1}\mathcal{C}_t}(gTVPI_{t+1} - 1) - (gTVPI_t - 1) = \left(gTVPI_t + \frac{C_t}{\mathcal{C}_t}\right)\frac{R^W_{t+1} - R^C_{t+1}}{R^C_{t+1}} . \tag{31}$$

Iterating this relation leads to

$$gTVPI_t - 1 = \frac{1}{\prod_{k=1}^{n} R^C_{t+k}} \frac{\mathcal{C}_{t+n}}{\mathcal{C}_t} \left(gTVPI_{t+n} - 1\right) - \sum_{k=1}^{n} \left(gTVPI_{t+k-1} + \frac{C_{t+k-1}}{\mathcal{C}_{t+k-1}}\right) \frac{\mathcal{C}_{t+k-1}}{\mathcal{C}_t} \frac{R^W_{t+k} - R^C_{t+k}}{\prod_{j=1}^{k} R^C_{t+j}} \ . \tag{32}$$

Approximating this relation using $\log(x) \approx \frac{2(x-1)}{x+1}$ gives

$$\frac{gTVPI_t+1}{2} gtvpi_t \approx \frac{1}{\prod_{k=1}^{n} R^C_{t+k}} \frac{\mathcal{C}_{t+n}}{\mathcal{C}_t} \left(\frac{gTVPI_{t+n}+1}{2} gtvpi_{t+n}\right) - \sum_{k=1}^{n} \left(gTVPI_{t+k-1} + \frac{C_{t+k-1}}{\mathcal{C}_{t+k-1}}\right) \frac{\mathcal{C}_{t+k-1}}{\mathcal{C}_t} \frac{R^W_{t+k} - R^C_{t+k}}{\prod_{j=1}^{k} R^C_{t+j}} \ . \tag{33}$$

Now from this relation and the relation $girr_t \approx \frac{gtvpi_t}{gDur_t}$, for gIRR, one can derive

$$\frac{gTVPI_t+1}{2} girr_t \approx \frac{1}{\prod_{k=1}^{n} R^C_{t+k}} \frac{\mathcal{C}_{t+n}}{\mathcal{C}_t} \frac{gTVPI_{t+n}+1}{2} girr_{t+n} - \frac{\Delta_n gDur_{t+n}}{gDur_{t+n}} \times \left(\frac{1}{\Delta_n gDur_{t+n}} \sum_{k=1}^{n} \left(gTVPI_{t+k-1} + \frac{C_{t+k-1}}{\mathcal{C}_{t+k-1}}\right) \frac{\mathcal{C}_{t+k-1}}{\mathcal{C}_t} \frac{R^W_{t+k} - R^C_{t+k}}{\prod_{j=1}^{k} R^C_{t+j}} - \frac{gTVPI_t+1}{2} girr_t\right) \ . \tag{34}$$

Based on Equation (33) and the relation $gmirr_t = \frac{gtvpi_t}{t}$, for gMIRR, the following relation can be derived:

$$\frac{gTVPI_t+1}{2} gmirr_t \approx \frac{1}{\prod_{k=1}^{n} R^C_{t+k}} \frac{\mathcal{C}_{t+n}}{\mathcal{C}_t} \frac{gTVPI_{t+n}+1}{2} gmirr_{t+n} - \frac{n}{t+n} \times \left(\frac{1}{n} \sum_{k=1}^{n} \left(gTVPI_{t+k-1} + \frac{C_{t+k-1}}{\mathcal{C}_{t+k-1}}\right) \frac{\mathcal{C}_{t+k-1}}{\mathcal{C}_t} \frac{R^W_{t+k} - R^C_{t+k}}{\prod_{j=1}^{k} R^C_{t+j}} - \frac{gTVPI_t+1}{2} gmirr_t\right) \ . \tag{35}$$

4.2 Economic Interpretation

To gain economic insights into the decomposition in Equation (33), (34), and (35), I start with the interpretation of Equation (30). In Equation (30), the first term on the left hand side is the NPV at time $t+1$ discounted to t, scaled by the cumulative capital $\mathcal{C}_t$ at time

t. The second term is the NPV at time t, again scaled by $\mathcal{C}_t$. Thus, the terms on the left hand side represent the scaled (discounted) NPV at time t. The change of the (appropriately scaled and discounted) NPV between t and $t+1$ is equivalent to the (appropriately scaled and discounted) intermediate gain between t and $t+1$, which is the term on the right hand side:

$$\frac{NPV_{t+1}}{R^C_{t+1}\mathcal{C}_t} - \frac{NPV_t}{\mathcal{C}_t} = \frac{(\text{Total Excess Gain})_{t,t+1}}{R^C_{t+1}\mathcal{C}_t} \ ,$$

Equation (33) is an approximation of Equation (32), which is directly derived from Equation (30).

Therefore, the decomposition in Equation (33) can be interpreted as follows: the current (scaled) NPV (i.e., $gTVPI_t - 1$) either predicts the future discounted (and scaled) NPV or the discounted (and scaled) total excess gain between now and the future, or both. This decomposition provides a meaningful structure on the predictability of the future (scaled and discounted) NPV. In corporate finance theory, NPV is considered as (arguably) the best metric for assessing the value of a project. Accordingly, in an ideal world, investors are supposed to care about the (appropriately adjusted) future NPV of a fund when making investment decisions. Scaling the NPV by the current value of cumulative capital calls, the net gTVPI (i.e., $gTVPI-1$) can be interpreted as the NPV per the value of invested dollars, i.e., NPVI.

Equation (34) and (35) are derived from (33), where the interpretation is given in the previous paragraph. The factor $\frac{gTVPI+1}{2}$ in the term on the left hand side of Equation (34) and (35) is for adjusting $\log(gTVPI)$ to the level of the net gTVPI (i.e., $gTVPI - 1$). If gTVPI is close to 1, the adjusting factor is also close to 1. The adjusted $girr_t$ and $gmirr_t$

can be written as

$$\frac{gTVPI_t+1}{2} girr_t \approx \frac{gTVPI_t-1}{gDur_t} = \frac{1}{\mathcal{C}_t}\frac{NPV_t}{gDur_t} \quad , \quad \frac{gTVPI_t+1}{2} gmirr_t \approx \frac{gTVPI_t-1}{t} = \frac{1}{\mathcal{C}_t}\frac{NPV_t}{t} \ ,$$

indicating that the adjusted $girr_t$ (or adjusted $gmirr_t$) can be interpreted as the (scaled) rate of generated value (NPV) over the effective duration (or physical duration for the adjusted $gmirr_t$) of the fund.

The decomposition in Equation (34) and (35) can be interpreted in a similar way as that in Equation (24) and (25) is interpreted. The current value of the adjusted $girr_t$ (or adjusted $gmirr_t$) either predicts its future (discounted and scaled) value or the average total excess gain in excess of the adjusted $girr_t$ (or adjusted $gmirr_t$), or both. In other words, the current (scaled) rate of NPV generation either predicts its future (discounted and scaled) rate or its rate of (discounted and scaled) NPV generation between now and the future in excess of the current rate, or both.

Therefore, the predictability of the net gTVPI in Equation (33) and the predictability of the adjusted $girr/gmirr$ in Equation (34) and (35) have quite different interpretations—as the predictability of $gtvpi$ in Equation (22) differs from the predictability of $girr/gmirr$ in Equation (24) and (25) in Section 3.2. As a benchmark, the full predictability of the (adjusted) net gTVPI implies that the current NPV of the fund is an unbiased predictor of the future discounted NPV:

$$\frac{gTVPI_t+1}{2} girr_t \approx gTVPI_t - 1 = \frac{NPV_t}{\mathcal{C}_t} = \frac{1}{\mathcal{C}_t} E_t\left[\frac{\mathcal{C}_{t+n}(gTVPI_{t+n}-1)}{R^C_{t,t+n}}\right] = \frac{1}{\mathcal{C}_t} E_t\left[\frac{NPV_{t+n}}{R^C_{t,t+n}}\right] \ .$$

From Equation (33), the full predictability of the (adjusted) net gTVPI implies that the

(appropriately adjusted) intermediate gain between now and the future is zero in expectation:

$$E_t\left[\sum_{k=1}^{n}\left(gTVPI_{t+k-1}+\frac{C_{t+k-1}}{\mathcal{C}_{t+k-1}}\right)\frac{\mathcal{C}_{t+k-1}}{\mathcal{C}_t}\frac{R^W_{t+k}-R^C_{t+k}}{\prod_{j=1}^{k}R^C_{t+j}}\right]=0\ .$$

By contrast, the full predictability of the adjusted *girr* (or adjusted *gmirr*) implies that the rate of (discounted) NPV generation stays the same at the current rate. To see this, the full predictability of the adjusted *girr* gives

$$\frac{gTVPI_t+1}{2}girr_t\approx\frac{1}{\mathcal{C}_t}\frac{NPV_t}{gDur_t}=\frac{1}{\mathcal{C}_t}E_t\left[\frac{1}{R^C_{t,t+n}}\frac{NPV_{t+n}}{gDur_{t+n}}\right]\approx E_t\left[\frac{1}{\prod_{k=1}^{n}R^C_{t+k}}\frac{\mathcal{C}_{t+n}}{\mathcal{C}_t}\frac{gTVPI_{t+n}+1}{2}girr_{t+n}\right]\ ,$$

and the full predictability of the adjusted *gmirr* yields

$$\frac{gTVPI_t+1}{2}gmirr_t\approx\frac{1}{\mathcal{C}_t}\frac{NPV_t}{t}=\frac{1}{\mathcal{C}_t}E_t\left[\frac{1}{R^C_{t,t+n}}\frac{NPV_{t+n}}{t+n}\right]\approx E_t\left[\frac{1}{\prod_{k=1}^{n}R^C_{t+k}}\frac{\mathcal{C}_{t+n}}{\mathcal{C}_t}\frac{gTVPI_{t+n}+1}{2}gmirr_{t+n}\right]\ .$$

From Equation (34) and Equation (35), the full predictability of the adjusted *girr* (or adjusted *gmirr*) indicates that the rate of the (appropriately adjusted) intermediate gain between now and the future is the same as the current rate—adjusted *girr* (or adjusted *gmirr*) at time t.

Since net gTVPI can be interpreted as NPV per invested dollar (NPVI), conceptually it can be thought of as a version of "total alpha," which is the (appropriate) sum of past alpha—where the (appropriately discounted and scaled) intermediate gain can be considered as alpha. Similarly, adjusted gIRR/gMIRR can be thought of as the "average alpha" over the effective duration/physical duration. Hence, the full predictability of net gTVPI can be regarded as total alpha forecasting zero alpha between now and the future—the current total alpha has no forecasting power for future alpha. On the other hand, the full predictability of adjusted gIRR/gMIRR can be thought of as the average alpha forecasting the same alpha between now and the future—the current average alpha (over the past) predicting perfectly

persistent alpha.

Lastly, the return portion of the decomposition in Equation (33), (34), and (35) involves actual returns, and the remaining part of the decomposition is the discounted future value of respective performance measures. Therefore, the decomposition of the NPV approach has clear economic counterparts in reality, making its interpretation more than simply conceptual.

4.3 Variance Decomposition

As the Campbell-Shiller log linearization can be translated into a variance decomposition, the log linearization in Equation (33), (34), and (35), respectively, leads to

$$1 \approx \beta^n_{\widetilde{gtvpi}} - \beta^n_{G^{ex}} \quad , \quad 1 \approx \beta^n_{\widetilde{girr}} - \beta^n_{\widetilde{G}^{ex}-\widetilde{girr}} \quad , \quad 1 \approx \beta^n_{\widetilde{gmirr}} - \beta^n_{\widetilde{G}^{ex}-\widetilde{gmirr}} \ , \tag{36}$$

where

$$\begin{aligned}
\beta^n_{\widetilde{gtvpi}} &\equiv \frac{Cov\left(\frac{gTVPI_t+1}{2} gtvpi_t, \frac{1}{\prod_{k=1}^n R^C_{t+k}} \frac{\mathcal{C}_{t+n}}{\mathcal{C}_t} \frac{gTVPI_{t+n}+1}{2} gtvpi_{t+n}\right)}{Var(gTVPI_t - 1)} \ , \\
\beta^n_{\widetilde{girr}} &\equiv \frac{Cov\left(\frac{gTVPI_t+1}{2} girr_t, \frac{1}{\prod_{k=1}^n R^C_{t+k}} \frac{\mathcal{C}_{t+n}}{\mathcal{C}_t} \frac{gTVPI_{t+n}+1}{2} girr_{t+n}\right)}{Var\left(\frac{gTVPI_t+1}{2} girr_t\right)} \ , \\
\beta^n_{\widetilde{gmirr}} &\equiv \frac{Cov\left(\frac{gTVPI_t+1}{2} gmirr_t, \frac{1}{\prod_{k=1}^n R^C_{t+k}} \frac{\mathcal{C}_{t+n}}{\mathcal{C}_t} \frac{gTVPI_{t+n}+1}{2} gmirr_{t+n}\right)}{Var\left(\frac{gTVPI_t+1}{2} gmirr_t\right)} \ ,
\end{aligned}$$

and

$$\begin{aligned}
\beta^n_{G^{ex}} &\equiv \frac{Cov\left(\frac{gTVPI_t+1}{2} gtvpi_t, \sum_{k=1}^n \left(gTVPI_{t+k-1} + \frac{C_{t+k-1}}{\mathcal{C}_{t+k-1}}\right) \frac{\mathcal{C}_{t+k-1}}{\mathcal{C}_t} \frac{R^W_{t+k} - R^C_{t+k}}{\prod_{j=1}^k R^C_{t+j}}\right)}{Var\left(\frac{gTVPI_t+1}{2} gtvpi_t\right)} \ , \\
\beta^n_{\widetilde{G}^{ex}-\widetilde{girr}} &\equiv \frac{Cov\left(\frac{gTVPI_t+1}{2} girr_t, \frac{\Delta_n gDur_{t+n}}{gDur_{t+n}} \left(\frac{1}{\Delta_n gDur_{t+n}} \sum_{k=1}^n \left(gTVPI_{t+k-1} + \frac{C_{t+k-1}}{\mathcal{C}_{t+k-1}}\right) \frac{\mathcal{C}_{t+k-1}}{\mathcal{C}_t} \frac{R^W_{t+k} - R^C_{t+k}}{\prod_{j=1}^k R^C_{t+j}} - \frac{gTVPI_t+1}{2} girr_t\right)\right)}{Var\left(\frac{gTVPI_t+1}{2} girr_t\right)} \ , \\
\beta^n_{\widetilde{G}^{ex}-\widetilde{gmirr}} &\equiv \frac{Cov\left(\frac{gTVPI_t+1}{2} gmirr_t, \frac{n}{t+n} \left(\frac{1}{n} \sum_{k=1}^n \left(gTVPI_{t+k-1} + \frac{C_{t+k-1}}{\mathcal{C}_{t+k-1}}\right) \frac{\mathcal{C}_{t+k-1}}{\mathcal{C}_t} \frac{R^W_{t+k} - R^C_{t+k}}{\prod_{j=1}^k R^C_{t+j}} - \frac{gTVPI_t+1}{2} gmirr_t\right)\right)}{Var\left(\frac{gTVPI_t+1}{2} gmirr_t\right)} \ .
\end{aligned}$$

Again, note that since Equation (33), (34), and (35) hold ex post, the variance decomposition in Equation (36) holds both conditionally and unconditionally.

The interpretation of the variance decomposition is similar to that in Section 3.3: the variance of the net gTVPI is either explained by its covariance with the future discounted value or its covariance with the sum of the future adjusted total excess gain, or both. The variance of the adjusted $girr_t/gmirr_t$ is either explained by its covariance with the future discounted value or its covariance with the future rate of (discounted and scaled) NPV generation in excess of the current $girr_t/gmirr_t$, or both.

Comparing the variance decomposition in Equation (36) with that in Equation (27), there are a few differences. First, the NPV approach in Equation (36) involves an adjusting factor of $\frac{gTVPI+1}{2}$. Second, for future values of (adjusted) measures, there is an additional adjusting factor of

$$\frac{1}{\prod_{k=1}^{n} R_{t+k}^{C}} \frac{\mathcal{C}_{t+n}}{\mathcal{C}_t} \ .$$

This adjusting factor takes into consideration of capital calls between time t and $t+n$. Note that this adjusting factor is 1 when there are no capital calls between t and $t+n$, in which case the ROI over the period is exactly the same as the (holding period) return on the combined portfolio. Except for this special case, the adjusting factor is not 1, and since the adjusting factor is directly affected by the benchmark return, the choice of the benchmark is likely to affect the regression coefficients in Equation (36) more than those in Equation (27). Third and lastly, the second beta coefficients in each relation in Equation (36) involve actual (holding period) returns R_t^W rather than (generally) hypothetical ROIs for those in Equation (27).

5 Empirical Results

This section empirically investigates the predictability of the (generalized) performance measures discussed in Section 3 and 4. In particular, I consider four different cases: 1) $R_t^C = R_t^D = 1$, 2) $R_t^C = R_t^D = R_t^m$, where R_t^m is the market benchmark return, 3) $R_t^C = R_t^m$ and $R_t^D = R_t^R$, where R_t^R is the return on residual fund assets (and capital calls, if any), and 4) $R_t^C = R_t^m$ and $R_t^D = R_t^I$, where R_t^I is the private market index return for the corresponding asset class.[11]

The first two cases are the most widely used performance measures in practice: the gTVPI in the first case corresponds to typical money multiples (TVPI), and that in the second case corresponds to KS PME by Kaplan and Schoar (2005). Similarly, the gIRR in the first case corresponds to the typical IRR, and that in the second case (approximately) corresponds to direct alpha. Since versions of those measures are often used in practice, an empirical examination of the first two cases would shed light on the predictability of commonly used performance measures.

The two remaining cases assume that distributed capital can be invested in private capital markets. The third case, where $R_t^D = R_t^R$, assumes that distributed capital can be reinvested in the fund itself, similar to a typical assumption for the computation of liquid asset returns (e.g., stock returns). If private capital fund performance relative to the benchmark persists, as Brown et al. (2019) points out, investing distributed capital into the benchmark would lead to moderation of excess performance (of the combined portfolio of residual fund assets and reinvested distributions) during the later phase of the fund's life. The $R_t^D = R_t^R$ assumption can be considered as a version of Weighted PME in Brown et al. (2019). However, there could be a criticism that reinvesting distributed capital into the fund is generally infeasible.

[11] For the market benchmark return R_t^m, I take the S&P 500 Index return as typically done in the literature. For the private market index, Preqin Buyout Index is taken for BO funds, Preqin Venture Capital Index for VC funds, and Preqin Private Debt Index for PD funds.

To mitigate such a criticism, in the last case, I consider a case where $R_t^D = R_t^I$, where R_t^I is a private market index for the corresponding asset class. [12]

Note that in the empirical exercises, this paper takes the net asset value (NAV) reported by funds as an empirical proxy of V_t. Previous studies—including Jenkinson et al. (2013), Barber and Yasuda (2017), Chakraborty and Ewens (2018), Brown et al. (2019), and Hüther (2023))—suggest that GPs have an incentive to inflate NAVs, particularly around fundraising events. Despite the possibility of NAV manipulation, there is no good alternative to NAV that can replace V_t at the fund level, and, as a result, NAV is taken as the empirical counterpart of V_t.

5.1 Data and Summary Statistics

For empirical exercises, this paper mostly relies on the Private Capital and Hedge Fund Data by Preqin, which is a commercial data provider, via S&P Xpressfeed. Note that Preqin reports net-of-fees performance—including the TVPI and IRR—of each fund but not gross-of-fees performance. Therefore, it might be helpful for readers to keep in mind that empirical tests in this study entail net-of-fees performance only. While Preqin reports cash flows between the GP and LPs, its coverage appears quite poor compared with the performance coverage.[13] As a result, I mainly use the performance table for inferring cash flow information.[14]

Commercial databases on private capital are often criticized about possible biases associated with sample selection (and relatedly, survivorship) stemming from their data collection and reporting methods. Yet, Harris et al. (2014) shows that performance in Preqin is "qual-

[12] It is also not clear if investing in the private market "index" is entirely feasible, but presumably it would be more feasible than investing in the fund itself.

[13] Korteweg and Sorensen (2017) states that the Preqin data "do not contain cash flows between the GP and LPs." Based on the statement at the time, the cash flow table seems relatively new in Preqin, which could explain the limited coverage.

[14] A detailed description of cash flow inference from the performance table is provided in Appendix Section B.2.

itatively similar to that in Burgiss," which is derived entirely from limited partners (LPs) and, hence, is much less subject to selection biases. Thus, such selection biases are unlikely to be systematic in Preqin, at least to the extent that those biases are readily identifiable. Still, empirical results in this paper using Preqin need to be interpreted with this caveat in mind.

The sample is limited to funds that focus on North America and are denominated in United States dollars. Their performance data spans from March 1983 through September 2023, but the coverage looks somewhat limited in the 80s and 90s. I also exclude single deal funds, which only take a very small portion of the entire funds. The sample is further limited to funds that are either closed or liquidated, mainly to exclude funds that are still in their fundraising stage. The main sample consists of 1,763 BO funds, 2,285 VC funds, and 1,117 PD funds, respectively.

Table 1 displays the summary statics of the sample, without applying any additional cleaning steps. Since I mainly work with the performance table, fund characteristics are attached to each performance record. As a result, funds that report longer performance—either due to better reporting or longer lifespan—are more represented in the table.

For BO funds, the average (median) size is roughly \$1,200 (\$520) million, showing a notable skewness towards large funds. The median vintage year is 2003 and the vast majority of the sample is performance records between 1992 and 2015. The average (median) years from vintage—where the reference date is the beginning of the vintage year—is 12 (11) years, displaying the long-term nature of investments in PC funds. The average (median) percent of called capital to committed capital is 86% (96%), showing that typical funds tend to call a substantial portion of the committed capital at the early stage of their life. Then, the summary statistics on the commonly used money multiples—the distributed to paid-in capital (DPI), the residual value to paid-in capital (RVPI), and the TVPI—are reported. The average (median) DPI is 1.19 (1.08), indicating that on average the funds succeed to

return at least the par value to the investors while performance is reported. The average (median) RVPI is 0.45 (0.3), implying that the average fund that report performance has a nonnegligible share of assets compared to the paid-in capital. The average (median) TVPI is 1.68 (1.52), and the average (median) IRR is 13.7% (13%).

For VC funds, the average (median) size is roughly $230 ($154) million, substantially smaller than BO funds. The average (median) years from vintage is 15 (14) years, somewhat longer than the sample BO funds. Regarding performance, the average (median) is 1.29 (0.83) for the DPI, 0.39 (0.1) for the RVPI, 1.73 (1.31) for the TVPI, and 12.9% (7.2%) for the IRR. Overall, based on the TVPI and IRR, performance measures for VC funds are more skewed than those for BO funds.

For PD funds, the average (median) size is roughly $850 ($460) million, a bit smaller than BO funds but notably larger than VC funds. The average (median) years from vintage is 11 (9) years, somewhat shorter than the sample BO funds, likely reflecting the recent rapid growth of the private debt sector (and the PD fund sample including disproportionately more funds that are recently launched). For performance, the average (median) DPI is 0.96 (1.01), the average (median) RVPI is 0.42 (0.25), the average (median) TVPI is 1.41 (1.32), and the average (median) IRR is 10.8% (10.1%). The TVPI and IRR for PD funds show that their performance skewness is not as severe as that for VC funds and comparable to that for BO funds.

5.2 ROI approach

Since the variance decomposition in Equation (27) is based on an accounting identity, those relations should hold both in cross section and time series. Yet, as is typical in the asset management industry, the number of funds (roughly on the order of thousands) is notably larger than the number of time periods (roughly 50 quarters), the vast majority of performance variations come from cross-sectional variations. In addition, time-series regression

estimates are subject to well-known biases (e.g., Kendall (1954) and Nickell (1981)). I start with cross-sectional regressions and turn to time-series regressions, addressing biases utilizing a half-panel jackknife fixed-effects estimator proposed by Chudik, Pesaran, and Yang (2018).

5.2.1 Cross Section

When running a cross-sectional regression, one needs to determine a peer group within which a fund is compared with others. By adding a time-fixed effect, a peer group is set to be the entire set of funds that exist at the time period. Yet, funds that have started in different years are likely to experience different performance due to market conditions. For example, funds of 2005 vintage would exhibit different performance from performance of 2009 vintage funds. Therefore, funds in the same vintage year are taken as a peer group for each fund, incorporated by adding a time×vintage fixed effect in the following set of regressions:[15][16]

$$\begin{aligned} gtvpi_{i,t+n} &= \beta^n_{gtvpi} gtvpi_{i,t} + \epsilon^{gtvpi}_{i,t+n} \,, \\ girr_{i,t+n} &= \beta^n_{girr} girr_{i,t} + \epsilon^{girr}_{i,t+n} \,, \\ gmirr_{i,t+n} &= \beta^n_{gmirr} gmirr_{i,t} + \epsilon^{gmirr}_{i,t+n} \,. \end{aligned} \tag{37}$$

These regression coefficients are empirical estimates of the first beta of the three relations in Equation (27). Black solid lines in Figure 1 display these beta estimates for $n = 1, ..., 8$ for BO funds, VC funds, and PD funds, respectively. The gray shaded (fully filled) area represents the 95% confidence interval for the beta estimates.[17] By adding a time×vintage fixed effect, dispersion of fund performance measures with respect to the average measure amongst

[15] Barber and Yasuda (2017) takes a similar approach by assessing a fund's performance relative to its vintage year cohort funds.

[16] The computation of *girr* is adjusted to ensure that the second relation in the variance decomposition of Equation (27) holds exactly—details are provided in Section B.3.

[17] Standard errors are two-way clustered by fund and time.

funds with the same vintage year at the same period determines the beta coefficients—cross-sectional regression with a peer group of the same vintage year.

Similarly, running the following set of regressions—with a time×vintage fixed effect—empirically estimates the second beta of the three relations in Equation (27):

$$\begin{aligned}
\sum_{k=1}^{n}\left(roi_{t+k}-r^{C}_{t+k}\right) &= \beta^{n}_{roi^{ex}}gtvpi_{i,t}+\epsilon^{gtvpi}_{i,t+n}\,,\\
\frac{\Delta_n gDur_{t+n}}{gDur_{t+n}}\left(\frac{1}{\Delta_n gDur_{t+n}}\sum_{k=1}^{n}\left(roi_{t+k}-r^{C}_{t+k}\right)-girr_t\right) &= \beta^{n}_{\overline{roi}^{ex}-girr}girr_{i,t}+\epsilon^{girr}_{i,t+n}\,, \qquad (38)\\
\frac{n}{t+n}\left(\frac{1}{n}\sum_{k=1}^{n}\left(roi_{t+k}-r^{C}_{t+k}\right)-gmirr_t\right) &= \beta^{n}_{\overline{roi}^{ex}-gmirr}gmirr_{i,t}+\epsilon^{gmirr}_{i,t+n}\,.
\end{aligned}$$

Red long dashed lines in Figure 1 display these beta estimates for $n = 1, ..., 8$ for BO funds, VC funds, and PD funds, respectively. The pink shaded (with shaded line) area represents the 95% confidence interval for the beta estimates. Note that the difference between the black and red lines is 1 by an accounting identity.

A common observation across different measures (with different assumptions on the cost of capital and the return on distributed capital) and across different fund types is that the predictive beta—the first beta of the three relations in Equation (27)—is never statistically greater than 1. In fact, except for gTVPI, the predictive beta on gIRR and gMIRR are all statistically smaller than 1. Based on the variance decomposition in Equation (27), these findings imply that (a) high *gtvpi* compared with the fund's peer group does not predict higher intermediate excess ROI, and (b) high *girr* (or *gmirr*) compared with the fund's peer group predicts a lower rate of intermediate excess ROI relative to the current *girr* (or *gmirr*). Therefore, in cross section, higher performance measures do not predict higher performance between today and the future, measured by either excess ROI, or the rate of excess ROI relative to the current rate of return measure—interpreted as outperforming the current performance.

Based on the "total alpha" or "average alpha" interpretation of current performance measures—with alpha being the excess ROI—these findings imply that past outperformance of a fund relative to its peer group does not predict intermediate outperformance, again relative to its peer group. Higher total alpha does not predict higher future alpha, and higher average alpha does not predict its perfect persistence in cross section. In particular, for $girr/gmirr$ all the predictive betas are below 1, even for small n, indicating that there is no short-term (perfect) persistence of cross-sectional outperformance. While the "average alpha" tends to revert to its cross-sectional average, generally it does not fully revert back, displaying a certain degree of persistence.

Another finding across different fund types is that the predictability of the future $gtvpi$ based on its current value is notably better than the predictability of $girr$ and $gmirr$, based on their respective current value. To the extent that future performance measures are of interest to fund investors (LPs), current $gtvpi$—relative to the fund's peer group—would be the most useful measure for predicting its future value.

Lastly, particularly for $gtvpi$, there are some differences amongst fund types. For BO funds, β^n_{gtvpi} drops to statistically below 1 as n increases, as can be seen from Figure 1a. Yet, the same beta coefficient is always statistically indifferent from 1 for VC funds for all specifications as shown in Figure 1b. For PD funds (Figure 1c), β^n_{gtvpi} is statistically indifferent from 1 for the KS PME setup ($R^C_t = R^D_t = R^m_t$), and close to 1 for the money multiple setup ($R^C_t = R^D_t = 1$), but for the rest two setups (private market reinvestment), the beta coefficient drops to notably below 1 as n increases.

5.2.2 Time Series

When running a time series regression of a variable's future value on the current value, it is well known that the regression coefficient is subject to a bias (e.g., Kendall (1954) and Marriott and Pope (1954), and more recently Stambaugh (1999)). While there are structural

approaches that are developed to address such a bias (e.g., Amihud and Hurvich (2004) and Boudoukh, Israel, and Richardson (2022)), in the panel setting of this paper, it would be difficult to directly apply those approaches.

Instead, I utilize a reduced-form approach of a half-panel jackknife method by Chudik et al. (2018). A time series regression in my panel setting is achieved by adding a fund-fixed effect to the regressions in Equation (37) and (38). All variations come from within-fund variations, which are variations over time. With a fund-fixed effect, the regression coefficients are subject to biases well-documented by Nickell (1981). The half-panel jackknife fixed-effects estimator proposed by Chudik et al. (2018) corrects for such biases. Another advantage of their approach is that the asymptotic distribution of the regression coefficient is given in a closed form.

Similar to the cross-sectional regressions, black solid lines in Figure 2 display beta estimates of Equation (37) for $n = 1, ..., 8$ for BO funds, VC funds, and PD funds, respectively. Virtually all predictive betas become zero or close to zero as n increases, except for the $R_t^C = R_t^m$ and $R_t^D = R_t^R$ setup for VC funds (only for the gTVPI). Therefore, betas for ROIs in Equation (38) largely become -1 as n increases: higher performance measures *within fund* reverse over time, both for the excess ROI or the rate of the excess ROI relative to the current rate. These results are consistent with typical findings of Campbell and Shiller (1988) decomposition, where predictability mostly comes from returns, although the ROI in this approach is not an actual return.

Based on the "total alpha" or "average alpha" interpretation of current performance measures—with alpha being the excess ROI—these findings imply that past outperformance of a fund does not persist at all. Both higher total alpha and higher average alpha—as well as lower total alpha and lower average alpha—dissipate over time and fully revert back in the long run. Thus, in the long run, the *within-fund* predictive power of interim performance measures for their future values is virtually zero, from the ROI perspective. However, in the

short run, the predictive betas seem to dissipate the most slowly for *gtvpi*. Hence, for the purpose of short-term (within-fund) performance prediction, *gtvpi* would still be the most useful measure.

5.3 NPV approach

While the ROI approach leads to the simplest form of predictive regressions, since the ROI is generally not an actual return, the interpretation of the variance decomposition in (27) is economically less meaningful compared with the NPV approach.

Although the variance decomposition in (36) gives more complicated relations due to discounting and scaling, the economic interpretation is rather clearer: (a) the current (scaled) NPV either predicts future (discounted and scaled) NPV or the sum of (appropriately adjusted) excess return between now and the future, or both, and (b) the current rate of (scaled) NPV generation either predicts the future (discounted and scaled) rate of NPV generation or the rate of excess gain generation relative to the current rate between now and the future, or both.

In this subsection, I empirically examine this approach and study if the NPV approach provides additional advantages over the ROI approach besides a better economic interpretation.

5.3.1 Cross Section

Similar to the ROI approach, a time×vintage fixed effect is added to the following set of regressions, corresponding to the first beta of the three relations in Equation (36):

$$\begin{aligned}
\tfrac{1}{\prod_{k=1}^{n} R_{t+k}^{C}} \tfrac{\mathcal{C}_{i,t+n}}{\mathcal{C}_{i,t}} \left(gTVPI_{i,t+n} - 1\right) &= \beta^{n}_{\widetilde{gtvpi}} \left(gTVPI_{i,t} - 1\right) + \epsilon^{\widetilde{gtvpi}}_{i,t+n} \,, \\
\tfrac{1}{\prod_{k=1}^{n} R_{t+k}^{C}} \tfrac{\mathcal{C}_{i,t+n}}{\mathcal{C}_{i,t}} \left(\tfrac{gTVPI_{i,t+n}-1}{gDur_{i,t+n}}\right) &= \beta^{n}_{\widetilde{girr}} \left(\tfrac{gTVPI_{i,t}-1}{gDur_{i,t}}\right) + \epsilon^{\widetilde{girr}}_{i,t+n} \,, \\
\tfrac{1}{\prod_{k=1}^{n} R_{t+k}^{C}} \tfrac{\mathcal{C}_{i,t+n}}{\mathcal{C}_{i,t}} \left(\tfrac{gTVPI_{i,t+n}-1}{t+n}\right) &= \beta^{n}_{\widetilde{girr}} \left(\tfrac{gTVPI_{i,t}-1}{t}\right) + \epsilon^{\widetilde{gmirr}}_{i,t+n} \,.
\end{aligned} \tag{39}$$

Note that instead of taking an approximation of $gTVPI - 1 \approx \frac{gTVPI+1}{2} gtvpi$, the raw form involving the net gTVPI is taken to guarantee that all three relations in Equation (36) hold exactly, rather than approximately.[18]

For the second beta of the three relations in Equation (36):

$$\begin{aligned}
&\sum_{k=1}^{n} \left(gTVPI_{i,t+k-1} + \tfrac{C_{i,t+k-1}}{\mathcal{C}_{i,t+k-1}}\right) \tfrac{\mathcal{C}_{i,t+k-1}}{\mathcal{C}_{i,t}} \tfrac{R^{W}_{i,t+k} - R^{C}_{t+k}}{\prod_{j=1}^{k} R^{C}_{t+j}} = \beta^{n}_{G^{ex}} \left(gTVPI_{i,t} - 1\right) + \epsilon^{\widetilde{gtvpi}}_{i,t+n} \,, \\
&\tfrac{\Delta_n gDur_{i,t+n}}{gDur_{i,t+n}} \left(\tfrac{1}{\Delta_n gDur_{i,t+n}} \sum_{k=1}^{n} \left(gTVPI_{i,t+k-1} + \tfrac{C_{i,t+k-1}}{\mathcal{C}_{i,t+k-1}}\right) \tfrac{\mathcal{C}_{i,t+k-1}}{\mathcal{C}_{i,t}} \tfrac{R^{W}_{i,t+k} - R^{C}_{t+k}}{\prod_{j=1}^{k} R^{C}_{t+j}} - \tfrac{gTVPI_{i,t}-1}{gDur_{i,t}} \right) \\
&= \beta^{n}_{\widetilde{G^{ex}-girr}} \left(\tfrac{gTVPI_{i,t}-1}{gDur_{i,t}}\right) + \epsilon^{\widetilde{girr}}_{i,t+n} \,, \\
&\tfrac{n}{t+n} \left(\tfrac{1}{n} \sum_{k=1}^{n} \left(gTVPI_{i,t+k-1} + \tfrac{C_{i,t+k-1}}{\mathcal{C}_{i,t+k-1}}\right) \tfrac{\mathcal{C}_{i,t+k-1}}{\mathcal{C}_{i,t}} \tfrac{R^{W}_{i,t+k} - R^{C}_{t+k}}{\prod_{j=1}^{k} R^{C}_{t+j}} - \tfrac{gTVPI_{i,t}-1}{t} \right) \\
&= \beta^{n}_{\widetilde{G^{ex}-gmirr}} \left(\tfrac{gTVPI_{i,t}-1}{t}\right) + \epsilon^{\widetilde{gmirr}}_{i,t+n} \,.
\end{aligned} \tag{40}$$

Adding a time×vintage fixed effect achieves cross-sectional regressions, in comparison with funds with the same vintage year at each point of time.

The first and second beta estimates (black solid lines and red long dashed lines, respectively) are shown in Figure 3, respectively for BO, VC, and PD funds. Shaded areas represent

[18] Details on these adjustments are provided in Section B.3.

the 95% confidence interval (gray fully filled areas for black lines and pink areas with shaded lines for red lines).

While there appear some (nuanced) improvements in predictive power for VC and PD funds, the starkest difference compared with the ROI approach comes from BO funds (Figure 3a). Basically, the predictive betas for BO funds are either equal to 1 or statistically greater than 1 for the net gTVPI. In comparison with the ROI approach where the predictive betas for BO funds are all statistically below 1 (Figure 1a), this result is qualitatively different. In particular, for the third and fourth specifications where distributed capital is invested in private markets, $\beta^n_{\widetilde{gtvpi}} > 1$, indicating that higher net gTVPI compared with the fund's peer group predicts a higher excess return between today and the future, again compared with the fund's peer group. Furthermore, for these two specifications for BO funds, $\beta^n_{\widetilde{girr}}$ is either statistically indifferent from 1 or slightly below 1, implying that higher (adjusted) $girr$, compared with the fund's peer group, well predicts its (appropriately adjusted) future value.

Based on the "total alpha" interpretation of net gTVPI—with alpha being NPV generation per invested dollar—$\beta^n_{\widetilde{gtvpi}} > 1$ for BO funds implies that past higher total alpha relative to the fund's peer group predicts higher future alpha, again relative to its peer group, when investment opportunities in private markets are available to LPs of BO funds. This result indicates that "artificial" moderation in excess performance due to capital distributions to LPs could be of notable magnitude, at least for BO funds.

Similarly, based on the "average alpha" interpretation of adjusted $girr$, $\beta^n_{\widetilde{girr}}$ being close to 1 for BO funds implies that past higher average alpha relative to the fund's peer group predicts similar alpha in the future, again relative to its peer group. Therefore, in terms of average alpha, past outperformance persists almost perfectly in cross section, at least for BO funds.

5.3.2 Time Series

For time-series regressions, a fund fixed effect is added to regressions in Equation (39) and (40). Those regression coefficients are displayed in Figure 4, respectively for BO, VC, and PD funds.

Again, the starkest difference compared with the ROI approach comes from BO funds (Figure 4a). For the ROI approach, regardless of fund type, most predictive betas are close to zero as n increases. However, for BO funds when distributed capital is reinvested in private markets, $\beta^n_{\widetilde{gtvpi}}$ is statistically indifferent from 1 for $n = 8$. Thus, higher net gTVPI *within fund* is (roughly) an unbiased estimate of the future (appropriately adjusted) net gTVPI. However, the confidence interval is quite wide, which is not too surprising as returns are typically volatile compared to its mean.

Based on the "total alpha" interpretation of net gTVPI—with alpha being NPV generation per invested dollar—these results for BO funds implies that past higher total alpha does not predict higher future alpha (nor lower future alpha), when investment opportunities in private markets are available to LPs of BO funds. Since past total alpha does not predict future alpha, (appropriately adjusted) future total alpha is ex ante the same as the past total alpha.

Unlike BO funds, VC funds' time-series predictability does not improve much by taking the NPV approach (Figure 4b). Yet, for PD funds (Figure 4c), in the $R^C_t = R^m_t$ and $R^D_t = R^R_t$ setting, $\beta^n_{\widetilde{gtvpi}}$ is statistically indifferent from 1 for $n = 8$. Therefore, the NPV approach provides a certain degree of time-series predictability for BO and PD funds, in that (appropriately adjusted) future net gTVPI can be predicted by current net gTVPI, when distributed capital is reinvested in private markets, unlike the ROI approach displaying virtually no time-series predictability.

5.3.3 Alternative Benchmark

While this paper uses S&P 500 Index returns as the market benchmark for the baseline setup, some may argue that different market benchmarks might be more appropriate for private funds. For example, companies that are held by BO funds tend to be smaller firms than large corporations that drive most of the S&P 500 Index return. For VC funds, portfolio companies would be even smaller firms as those funds invest in firms at their very early stage. For PD funds, since most of their assets are risky debts, a benchmark for risk debt market would be a better proxy for the market benchmark.

To alleviate such concerns, I run both the cross-sectional and time-series regressions for the NPV approach taking alternative benchmarks for the market benchmark R_t^m. For BO and VC funds, I take small cap stock indexes—Russell 2000 Index and S&P SmallCap 600 Index, and for PD funds, I take risky debt market indexes—ICE BofA US High Yield Index and Morningstar/LSTA US Leveraged Loan Index.

These results are shown in Figure 5 (cross section) and Figure 6 (time series). While VO funds do not exhibit much qualitative changes taking the alternative benchmarks, predictive betas for BO funds and PD funds—particularly $\beta^n_{\widetilde{gtvpi}}$—increase for the alternative benchmarks. In particular, $\beta_{\widetilde{gtvpi}}$ for PD funds increase meaningfully. As a result, both for cross section and time series, $\beta_{\widetilde{gtvpi}}$ is statistically indifferent from or greater than 1 for PD funds when distributed capital is invested in private markets. Therefore, taking the alternative risky debt market benchmarks improves the predictability of net gTVPI for PD funds.

As can be seen from Figure A1, taking the alternative benchmarks for the ROI approach does not change any cross-sectional results in a qualitative way. For time-series results, as shown in Figure A2, taking the alternatives for the ROI approach tend to worsen the predictability of performance measures—most notably for PD funds—although not to a substantial extent. Therefore, improvements in predictability by taking alternative benchmarks largely apply only to the NPV approach.

6 Conclusion

This paper studies the predictability of interim PC performance measures. Amid a plethora of existing PC (largely PE) performance measures in the literature, I provide a unified framework within which most of the measures can be understood in a consistent manner, by introducing a generalized concept of TVPI (gTVPI). Next, I decompose the performance measures by taking a log linearization, leading to two different decomposition approaches—the ROI and NPV approach. Those approaches yield a variance decomposition à la Campbell and Shiller (1988), imposing valuable structure on the predictability of the measures and its economic implications. In the empirical examination of the two approaches, the NPV approach seems more useful in that it provides time-series performance predictability, which does not exist in the ROI approach, under certain conditions. In both approaches, total return measures (gTVPI measures) turn out to be the most useful to predict future performance.

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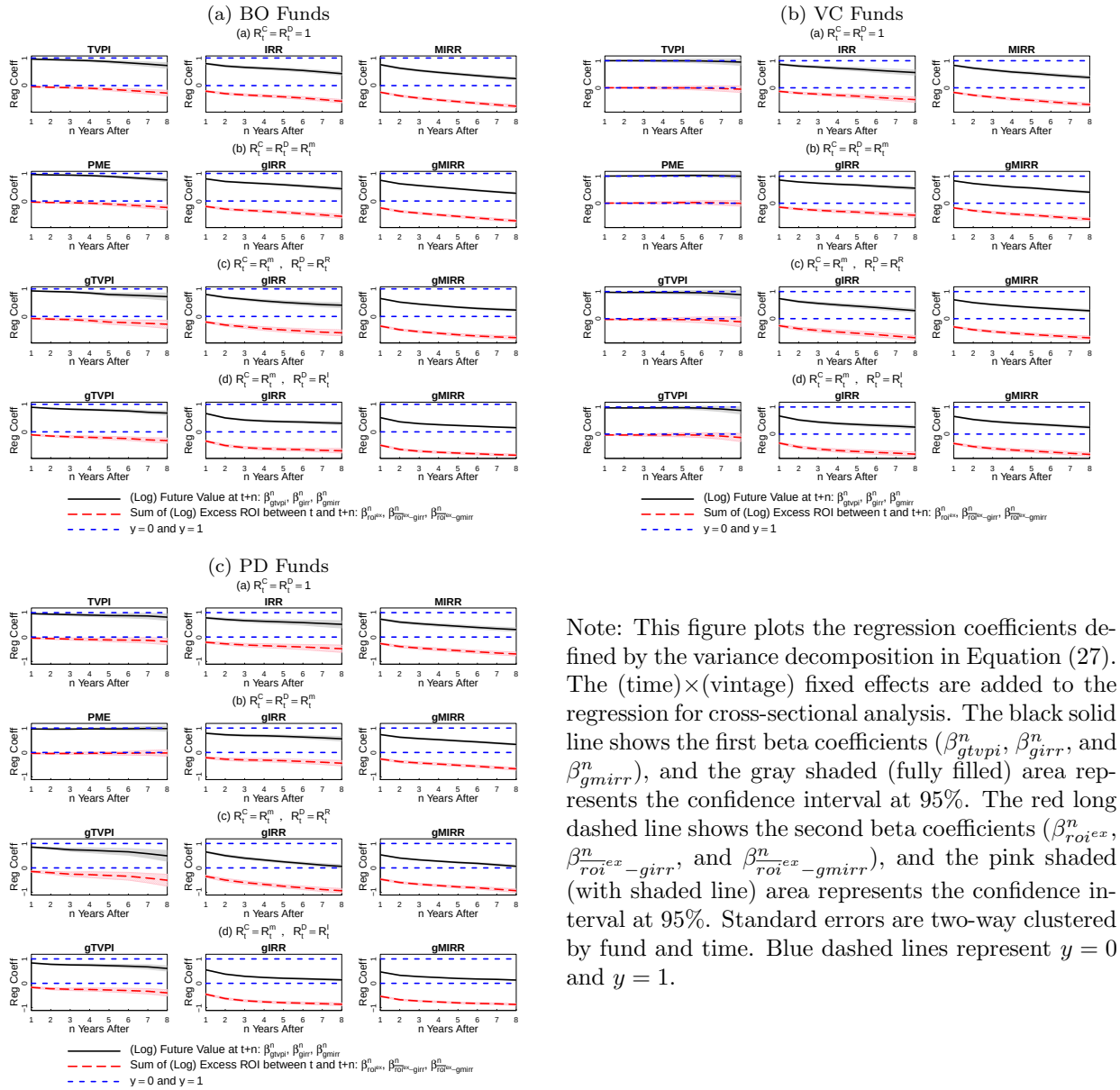
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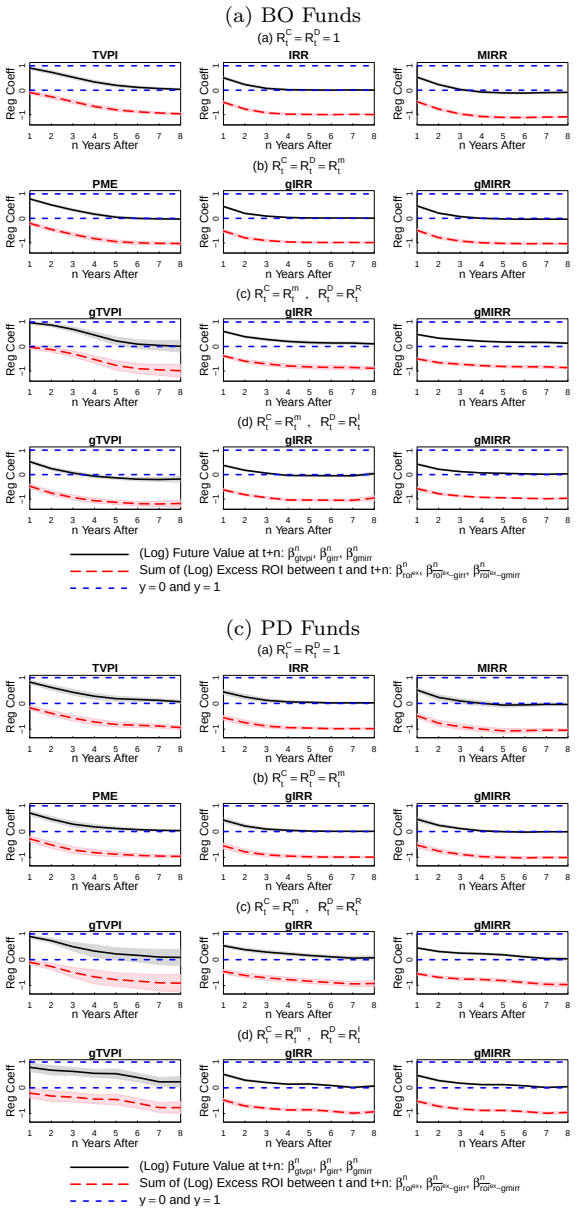
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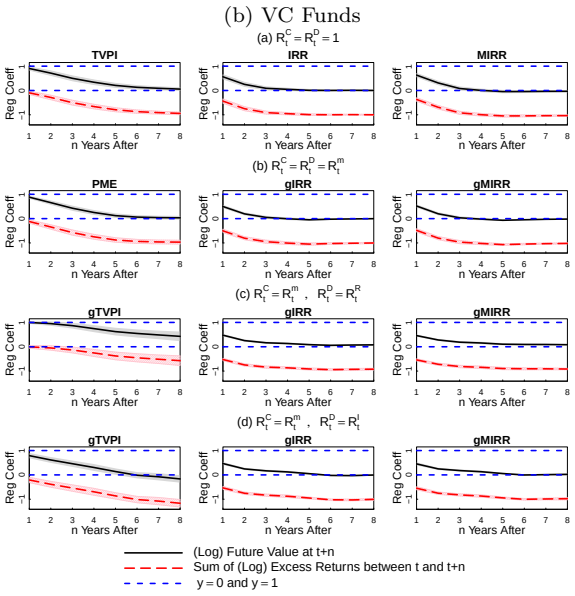
Figure 1: (ROI approach) Cross-Sectional Regression Coefficients of the Variance Decomposition in Equation (27)



Note: This figure plots the regression coefficients defined by the variance decomposition in Equation (27). The (time)×(vintage) fixed effects are added to the regression for cross-sectional analysis. The black solid line shows the first beta coefficients (β^n_{gtvpi}, β^n_{girr}, and β^n_{gmirr}), and the gray shaded (fully filled) area represents the confidence interval at 95%. The red long dashed line shows the second beta coefficients ($\beta^n_{roi^{ex}}$, $\beta^n_{roi^{ex}-girr}$, and $\beta^n_{roi^{ex}-gmirr}$), and the pink shaded (with shaded line) area represents the confidence interval at 95%. Standard errors are two-way clustered by fund and time. Blue dashed lines represent $y = 0$ and $y = 1$.

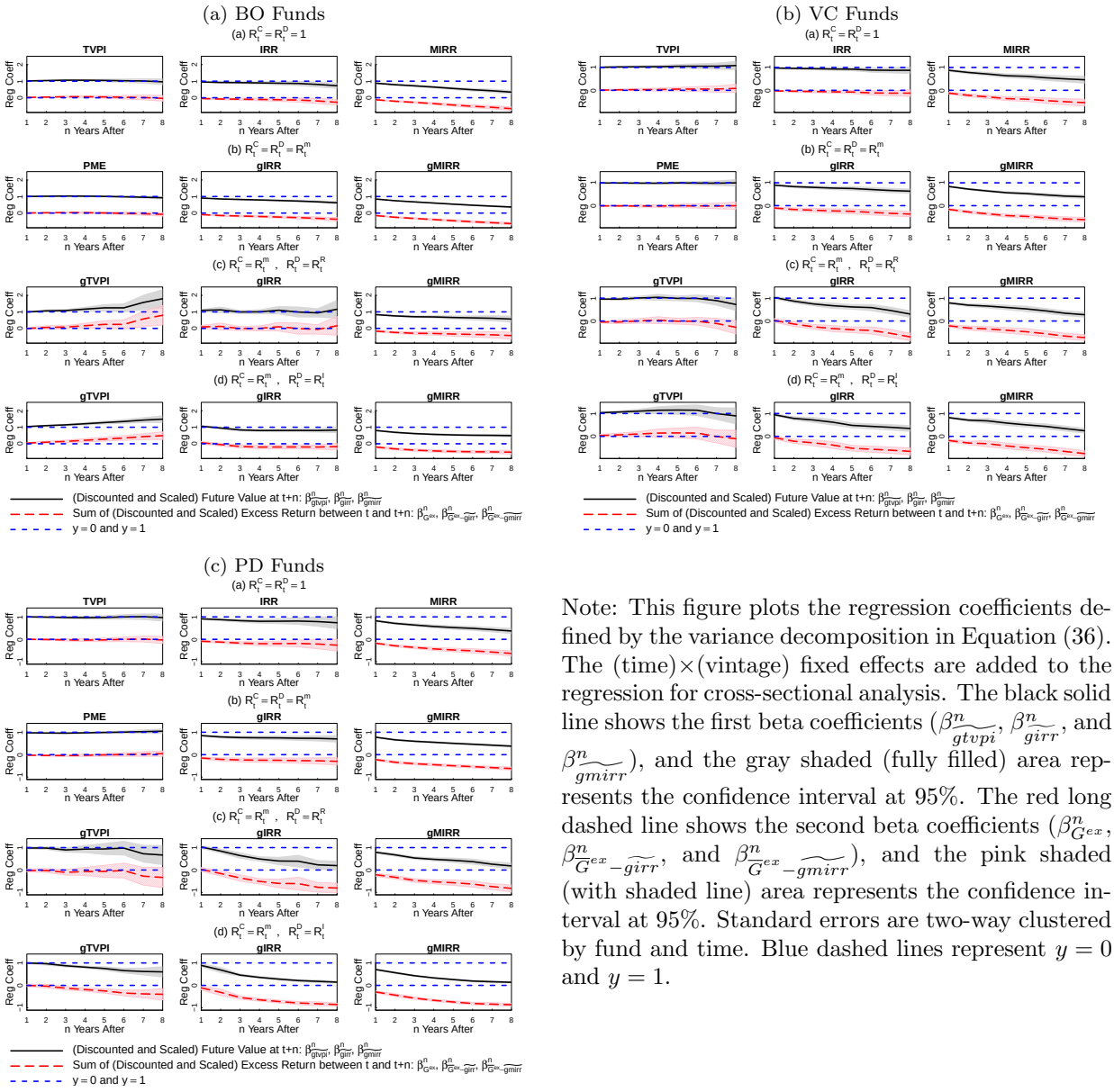
Figure 2: (ROI approach) Time-Series Regression Coefficients of the Variance Decomposition in Equation (27)





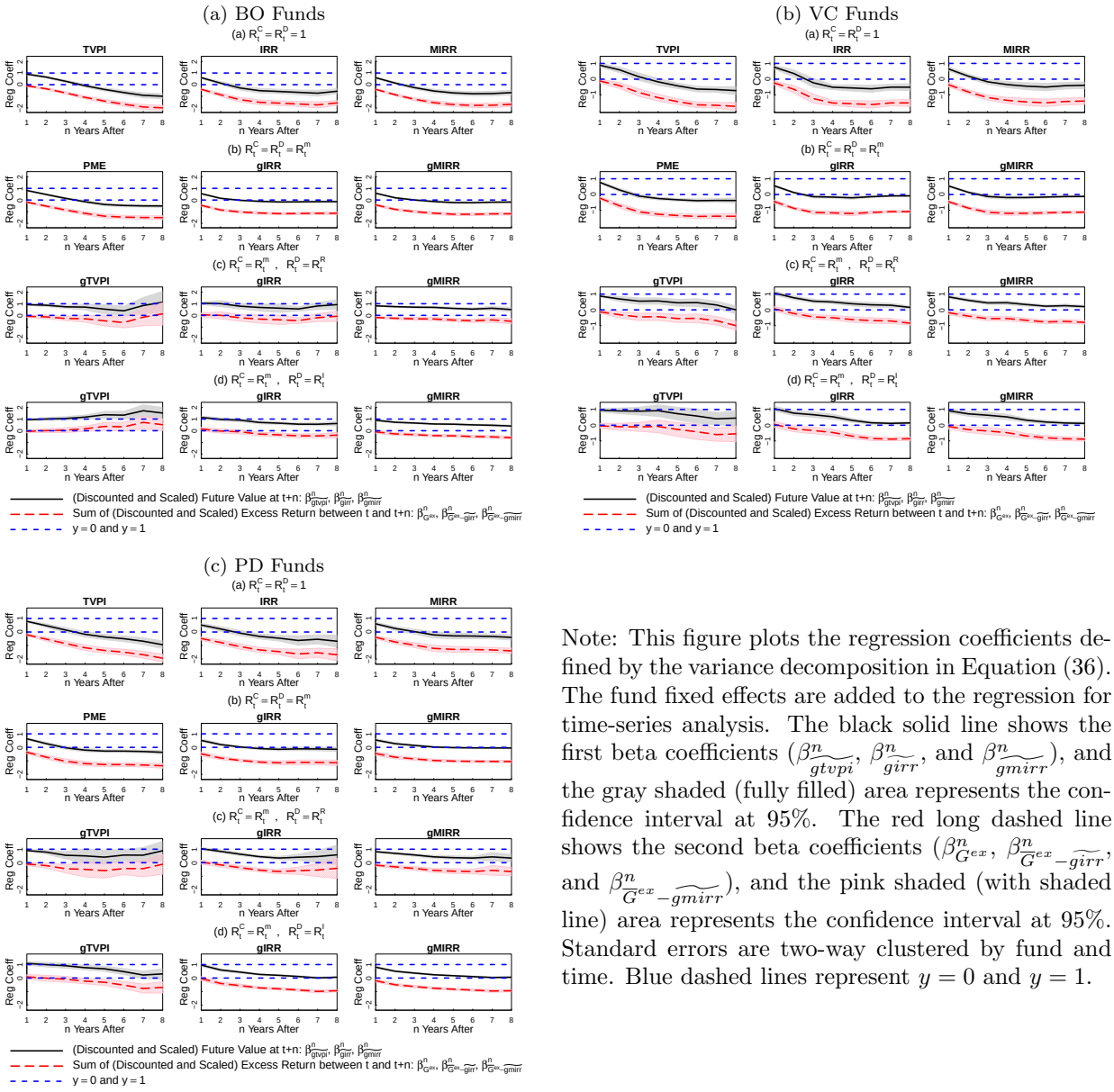
Note: This figure plots the regression coefficients defined by the variance decomposition in Equation (27). The fund fixed effects are added to the regression for time-series analysis. The black solid line shows the first beta coefficients (β^n_{gtvpi}, β^n_{girr}, and β^n_{gmirr}), and the gray shaded (fully filled) area represents the confidence interval at 95%. The red long dashed line shows the second beta coefficients ($\beta^n_{roi^{ex}}$, $\beta^n_{\overline{roi}^{ex}-girr}$, and $\beta^n_{\overline{roi}^{ex}-gmirr}$), and the pink shaded (with shaded line) area represents the confidence interval at 95%. Standard errors are two-way clustered by fund and time. Blue dashed lines represent $y = 0$ and $y = 1$.

Figure 3: (NPV approach) Cross-Sectional Regression Coefficients of the Variance Decomposition in Equation (36)



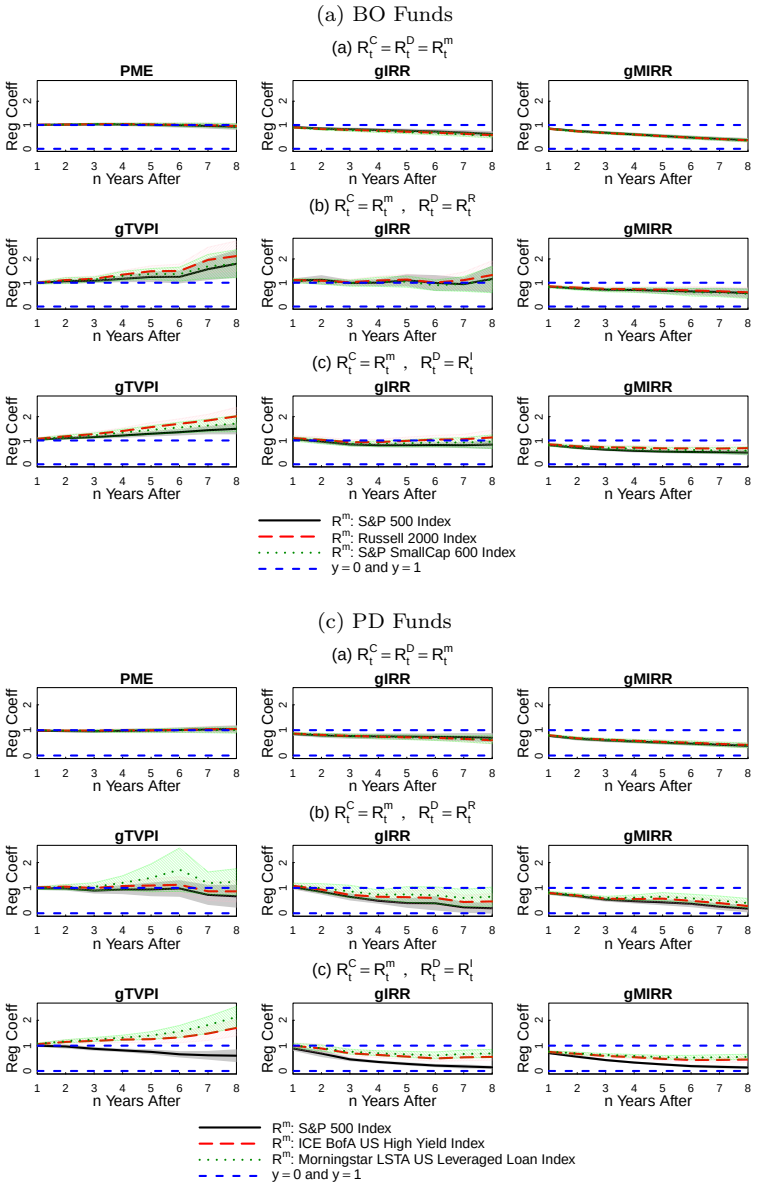
Note: This figure plots the regression coefficients defined by the variance decomposition in Equation (36). The (time)×(vintage) fixed effects are added to the regression for cross-sectional analysis. The black solid line shows the first beta coefficients ($\beta^n_{\widetilde{gtvpi}}$, $\beta^n_{\widetilde{girr}}$, and $\beta^n_{\widetilde{gmirr}}$), and the gray shaded (fully filled) area represents the confidence interval at 95%. The red long dashed line shows the second beta coefficients ($\beta^n_{G^{ex}}$, $\beta^n_{G^{ex}-\widetilde{girr}}$, and $\beta^n_{G^{ex}-\widetilde{gmirr}}$), and the pink shaded (with shaded line) area represents the confidence interval at 95%. Standard errors are two-way clustered by fund and time. Blue dashed lines represent $y = 0$ and $y = 1$.

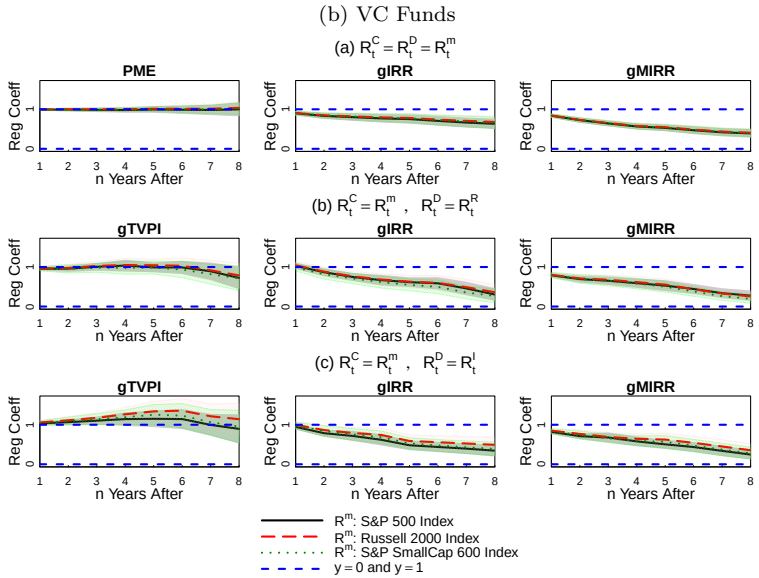
Figure 4: (NPV approach) Time-Series Regression Coefficients of the Variance Decomposition in Equation (36)



Note: This figure plots the regression coefficients defined by the variance decomposition in Equation (36). The fund fixed effects are added to the regression for time-series analysis. The black solid line shows the first beta coefficients ($\beta^n_{\widetilde{gtvpi}}$, $\beta^n_{\widetilde{girr}}$, and $\beta^n_{\widetilde{gmirr}}$), and the gray shaded (fully filled) area represents the confidence interval at 95%. The red long dashed line shows the second beta coefficients ($\beta^n_{G^{ex}}$, $\beta^n_{G^{ex}-\widetilde{girr}}$, and $\beta^n_{G^{ex}-\widetilde{gmirr}}$), and the pink shaded (with shaded line) area represents the confidence interval at 95%. Standard errors are two-way clustered by fund and time. Blue dashed lines represent $y = 0$ and $y = 1$.

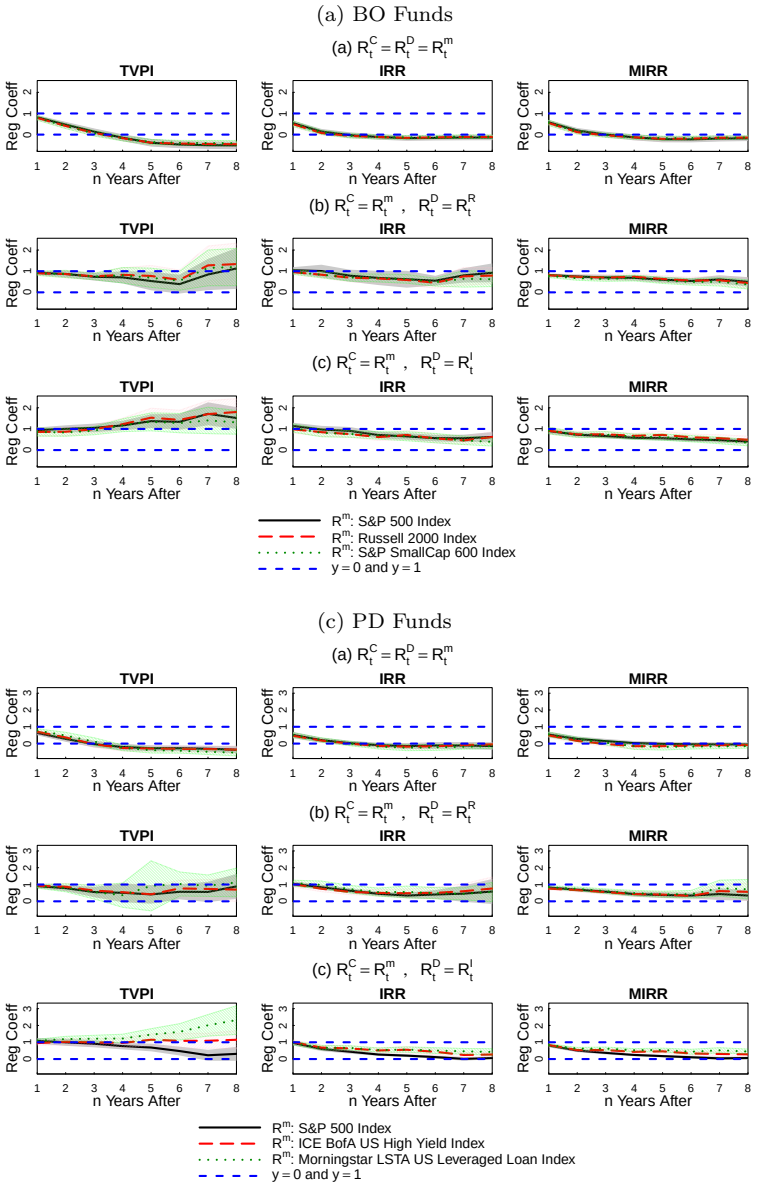
Figure 5: (NPV approach) Cross-Sectional Regression Coefficients of the Variance Decomposition in Equation (36); Alternative Benchmark

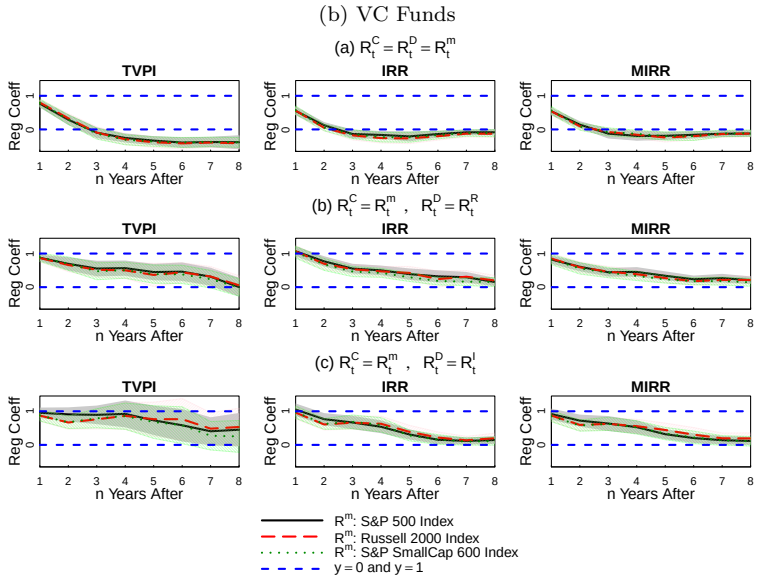




Note: This figure plots the first regression coefficient of the three relations in the variance decomposition in Equation (36). The (time)×(vintage) fixed effects are added to the regression for cross-sectional analysis. Each line except blue dashed lines shows the first beta coefficients ($\beta^n_{\widetilde{gtvpi}}$, $\beta^n_{\widetilde{girr}}$, and $\beta^n_{\widetilde{gmirr}}$) with a different benchmark (benchmark described in the legend). The shaded area represents the confidence interval at 95% of the line of its corresponding color. Standard errors are two-way clustered by fund and time. Blue dashed lines represent $y = 0$ and $y = 1$.

Figure 6: (NPV approach) Time-Series Regression Coefficients of the Variance Decomposition in Equation (36); Alternative Benchmark





Note: This figure plots the first regression coefficient of the three relations in the variance decomposition in Equation (36). The fund fixed effects are added to the regression for time-series analysis. Each line except blue dashed lines shows the first beta coefficients ($\beta^n_{\widetilde{gtvpi}}$, $\beta^n_{\widetilde{girr}}$, and $\beta^n_{\widetilde{gmirr}}$) with a different benchmark (benchmark described in the legend). The shaded area represents the confidence interval at 95% of the line of its corresponding color. Standard errors are two-way clustered by fund and time. Blue dashed lines represent $y = 0$ and $y = 1$.

Tables

Table 1: Summary Statistics

(a) BO Funds

	Truncated Mean	Truncated S.D.	10th Pctl	25th Pctl	Median	75th Pctl	90th Pctl	N Obs
Fund Size ($ Million)	1179.02	1766.62	103.4	240	521	1287.1	3400	66813
Vintage Year	—	—	1992	1997	2003	2008	2015	66813
Years from Vintage	11.89	8.02	2.25	5	10.5	17.5	24	66813
Called Capital (%)	85.94	26.55	37	78.76	96.18	100	108.91	66160
DPI	1.19	1.04	0	0.27	1.08	1.84	2.6	65794
RVPI	0.45	0.47	0	0	0.3	0.9	1.13	65486
TVPI	1.68	0.8	0.87	1.1	1.52	2.06	2.75	65263
IRR (%)	13.68	19.35	-7.9	4.6	13	23	35.2	64178

(b) VC Funds

	Truncated Mean	Truncated S.D.	10th Pctl	25th Pctl	Median	75th Pctl	90th Pctl	N Obs
Fund Size ($ Million)	234.98	250.56	30	63	154	300	565.7	74302
Vintage Year	—	—	1987	1993	2000	2005	2012	82124
Years from Vintage	14.8	9.41	3	6.5	13.75	22	28.75	82124
Called Capital (%)	89.72	21.44	52.93	91.5	100	100	100	81215
DPI	1.29	1.57	0	0.18	0.83	1.78	3.28	80305
RVPI	0.39	0.51	0	0	0.1	0.78	1.12	80150
TVPI	1.73	1.47	0.53	0.88	1.31	2.09	3.59	79368
IRR (%)	12.87	32.79	-15.88	-3.95	7.2	20.35	46.1	75605

(c) PD Funds

	Truncated Mean	Truncated S.D.	10th Pctl	25th Pctl	Median	75th Pctl	90th Pctl	N Obs
Fund Size ($ Million)	848.49	1076.69	93	181	460	1030	2125	29215
Vintage Year	—	—	1993	1999	2006	2012	2017	30729
Years from Vintage	10.61	7.36	2.25	4.5	9	15.75	22.25	30729
Called Capital (%)	87.87	25.67	44.87	80.6	97.67	100	106.22	30067
DPI	0.96	0.71	0.02	0.31	1.01	1.45	1.88	29537
RVPI	0.42	0.44	0	0	0.25	0.87	1.05	29678
TVPI	1.41	0.45	0.98	1.12	1.32	1.58	1.98	29262
IRR (%)	10.75	10.71	-1.17	6	10.1	14.8	22.88	29074

Note: For truncated mean and standard deviation, top and bottom 1% are trimmed to remove outliers.

Appendix A Derivations and Proofs

A.1 Approximate Relation between gIRR and gTVPI

I first relate IRR to TVPI and generalize the relation. The IRR is defined as $R-1$ where R satisfies the following relation:

$$\sum_{\tau=0}^{T-1} \frac{C_\tau}{R^\tau} = \sum_{\tau=1}^{T} \frac{D_\tau}{R^\tau} \quad \Longleftrightarrow \quad \sum_{\tau=0}^{T-1} R^{T-\tau} C_\tau = \sum_{\tau=1}^{T} R^{T-\tau} D_\tau \quad , \quad \forall R > 0 \, , \tag{A.1}$$

where there are no capital calls at liquidation and no distribution at the inception of the fund.

For nonnegative series $\{C_t\}$ and $\{D_t\}$ where neither is zero for all t, one can take a log of both sides of Equation (A.1):

$$\log\left(\sum_{\tau=0}^{T-1} R^{T-\tau} C_\tau\right) = \log\left(\sum_{\tau=1}^{T} R^{T-\tau} D_\tau\right) \, .$$

A Taylor expansion of each side of the equation up to the first order at $r \equiv \log R = 0$ reads

$$\log\left(\sum_{\tau=0}^{T-1} C_\tau\right) + \frac{\sum_{\tau=0}^{T-1}(T-\tau)C_\tau}{\sum_{\tau=0}^{T-1} C_\tau} r \approx \log\left(\sum_{\tau=1}^{T} D_\tau\right) + \frac{\sum_{\tau=1}^{T}(T-\tau)D_\tau}{\sum_{\tau=1}^{T} D_\tau} r \, .$$

Rearranging terms to solve for r gives

$$r \approx \frac{\log\left(\sum_{\tau=1}^{T} D_\tau\right) - \log\left(\sum_{\tau=0}^{T-1} C_\tau\right)}{\frac{\sum_{\tau=1}^{T} \tau D_\tau}{\sum_{\tau=1}^{T} D_\tau} - \frac{\sum_{\tau=0}^{T-1} \tau C_\tau}{\sum_{\tau=0}^{T-1} C_\tau}} = \frac{tvpi_T}{(Duration\ of\ \{D_t\}) - (Duration\ of\ \{C_t\})} \, , \tag{A.2}$$

where $tvpi_T \equiv \log(TVPI_T)$, and "duration" means Macaulay duration for each cash flow stream with no discount.

For notational clarity, I denote $r = \log(1 + IRR)$ by irr, mainly because r is mostly used for log returns throughout this paper.[19]

Before a fund is fully liquidated, the interim version of the IRR is defined by $R - 1$ that solves

$$\sum_{\tau=0}^{t-1} \frac{C_\tau}{R^\tau} = \sum_{\tau=1}^{t} \frac{D_\tau}{R^\tau} + V_t \quad \Longleftrightarrow \quad \sum_{\tau=0}^{t-1} R^{t-\tau} C_\tau = \sum_{\tau=1}^{t} R^{t-\tau} D_\tau + R^t V_t \,. \tag{A.3}$$

Then, $irr_t \equiv \log(1 + IRR_t)$ where IRR_t is the interim IRR can be approximated as

$$irr_t \approx \frac{\log\left(\sum_{\tau=1}^{t} D_\tau + V_t\right) - \log\left(\sum_{\tau=0}^{t-1} C_\tau\right)}{\frac{\sum_{\tau=1}^{t} \tau D_\tau + t V_t}{\sum_{\tau=1}^{t} D_\tau + V_t} - \frac{\sum_{\tau=0}^{t-1} \tau C_\tau}{\sum_{\tau=0}^{t-1} C_\tau}} = \frac{tvpi_t}{Dur_t} \,, \tag{A.4}$$

where Dur_t is the duration difference between distributions and capital calls assuming that the residual value V_t of the fund is realized at time t.

Similar to the IRR, define the generalized IRR (gIRR) where $R = 1 + gIRR > 0$ satisfies

$$\sum_{\tau=0}^{T-1} \frac{R^C_{\tau,T} C_\tau}{R^\tau} = \sum_{\tau=1}^{T} \frac{R^D_{\tau,T} D_\tau}{R^\tau} \quad \Longleftrightarrow \quad \sum_{\tau=0}^{T-1} R^{T-\tau} R^C_{\tau,T} C_\tau = \sum_{\tau=1}^{T} R^{T-\tau} R^D_{\tau,T} D_\tau \,, \tag{A.5}$$

A Taylor expansion of each side of the equation up to the first order at $r = \log R = 0$ yields the following approximation:

$$girr_T = r \approx \frac{gtvpi_T}{gDur_T} \quad , \quad gDur_T \equiv \frac{\sum_{\tau=1}^{T} \tau R^D_{\tau,T} D_\tau}{\sum_{\tau=1}^{T} R^D_{\tau,T} D_\tau} - \frac{\sum_{\tau=0}^{T-1} \tau R^C_{\tau,T} C_\tau}{\sum_{\tau=0}^{T-1} R^C_{\tau,T} C_\tau} \,, \tag{A.6}$$

where $gtvpi_T \equiv \log(gTVPI_T)$. Similar to the IRR, the interim version of the gIRR at time

[19] This notation usage is few exceptions since the IRR stands for the internal "rate of return" instead of the return as opposed to other returns considered in this paper; variables with lowercase letters indicate a log of the variables. Yet, $irr = \log(1 + IRR)$ is approximately a rate of return analogous to other log returns.

t can be approximated as

$$girr_t \approx \frac{gtvpi_t}{gDur_t} \quad , \quad gDur_t \equiv \frac{\sum_{\tau=1}^{t} \tau R_{\tau,t}^{D} D_\tau + tV_t}{\sum_{\tau=1}^{t} R_{\tau,t}^{D} D_\tau + V_t} - \frac{\sum_{\tau=0}^{t-1} \tau R_{\tau,t}^{C} C_\tau}{\sum_{\tau=0}^{t-1} R_{\tau,t}^{C} C_\tau} \ , \tag{A.7}$$

where $gtvpi_t \equiv \log(gTVPI_t)$.

A.2 Relation between gMIRR and gTVPI

Starting from Equation (10), I first assume constant cost of capital and constant return on distribution, which are assumptions often made in practice:

$$R^T \sum_{\tau=0}^{T-1} \frac{C_\tau}{(R^C)^\tau} = \sum_{\tau=1}^{T} \left(R^D\right)^{T-\tau} D_\tau \ . \tag{A.8}$$

Rewriting this equation gives

$$\left(\frac{R}{R^C}\right)^T \sum_{\tau=0}^{T-1} \left(R^C\right)^{T-\tau} C_\tau = \sum_{\tau=1}^{T} \left(R^D\right)^{T-\tau} D_\tau \ . \tag{A.9}$$

Taking a log on both sides of the equation yields

$$T(r - r^C) = \log\left(\frac{\sum_{\tau=1}^{T} \left(R^D\right)^{T-\tau} D_\tau}{\sum_{\tau=0}^{T-1} \left(R^C\right)^{T-\tau} C_\tau}\right) = gtvpi_T \quad \Longleftrightarrow \quad r = r^C + \frac{gtvpi_T}{T} \ ,$$

where I denote $r - r^C$ by $gmirr \equiv \log(gMIRR)$. This relation is exact and not an approximation, unlike the relation between gTVPI and gIRR. Without the assumption of constant cost of capital and constant return on distribution, the $gmirr$ can be rewritten as

$$gmirr_T \equiv \log(1 + MIRR_T) - \frac{1}{T} r_{0,T}^{C} = \frac{gtvpi_T}{T} \quad , \quad r_{0,T}^{C} \equiv \log(R_{0,T}^{C}) \ . \tag{A.10}$$

An intermediate version of the gMIRR can be defined in a similar way to that of the

gIRR:

$$gmirr_t \equiv \log(1 + MIRR_t) - \frac{1}{t} r^C_{0,t} = \frac{gtvpi_t}{t} \ , \tag{A.11}$$

where the interim MIRR is defined by $R - 1$ that solves

$$R^t \sum_{\tau=0}^{t-1} \frac{C_\tau}{R^C_{0,\tau}} = \sum_{\tau=1}^{t} R^D_{\tau,t} D_\tau + V_t \ . \tag{A.12}$$

A.3 Half-panel Jackknife Method and Asymptotic Distribution

For the fund fixed-effect approach of the time-series analysis, the standard OLS regression estimator suffers from small-T bias as shown in Nickell (1981). To address such bias, this paper employs the half-panel jackknife method proposed by Chudik et al. (2018). I start with a basic regression setup with fund-fixed effects:

$$y_{i,t} = \mu_i + \beta x_{i,t} + \epsilon_{i,t} \ ,$$

where i is an index for funds, and t is an index for time. Note that this is a narrower setup than that in Chudik et al. (2018) as they also include time-fixed effects. The FE estimator of β reads

$$\hat{\beta} = \hat{Q}^{-1}\hat{q} \quad , \quad \hat{Q} \equiv \frac{1}{\sum_{i=1}^N T_i} \sum_{i=1}^{N} \sum_{t=1}^{T_i} \left(x^*_{i,t}\right)^2 \quad , \quad \hat{q} \equiv \frac{1}{\sum_{i=1}^N T_i} \sum_{i=1}^{N} \sum_{t=1}^{T_i} x^*_{i,t} y_{i,t} \ , \tag{A.13}$$

where $x^*_{i,t} = x_{i,t} - \bar{x}_i$ and $\bar{x}_i = \frac{1}{T_i} \sum_{t=1}^{T_i} x_{i,t}$.

In their proposed half-panel jackknife method, for each fund i, observations are split into the first $T_i/2$ and second $T_i/2$.[20] Denote the first half sample by a and the second half sample

[20] When T_i is not an even number, the baseline approach Chudik et al. (2018) take is to delete the first observations so that the sample consists of an even number of observations for each fund.

by b. For each half-sample, the FE estimator $\hat{\beta}_a$ and $\hat{\beta}_b$ are given by

$$\hat{\beta}_a = \hat{Q}_a^{-1}\hat{q}_a \quad , \quad \hat{Q}_a \equiv \frac{2}{\sum_{i=1}^N T_i}\sum_{i=1}^N\sum_{t=1}^{T_i/2}\left(x_{i,t}^{a,*}\right)^2 \quad , \quad \hat{q}_a \equiv \frac{2}{\sum_{i=1}^N T_i}\sum_{i=1}^N\sum_{t=1}^{T_i/2} x_{i,t}^{a,*}y_{i,t} \ ,$$
$$\hat{\beta}_b = \hat{Q}_b^{-1}\hat{q}_b \quad , \quad \hat{Q}_b \equiv \frac{2}{\sum_{i=1}^N T_i}\sum_{i=1}^N\sum_{t=T_i/2+1}^{T_i}\left(x_{i,t}^{b,*}\right)^2 \quad , \quad \hat{q}_b \equiv \frac{2}{\sum_{i=1}^N T_i}\sum_{i=1}^N\sum_{t=T_i/2+1}^{T_i} x_{i,t}^{b,*}y_{i,t} \ ,$$

where $x_{i,t}^{a,*} = x_{i,t} - \bar{x}_i^a$, $\bar{x}_i^a = \frac{2}{T_i}\sum_{t=1}^{T_i/2} x_{i,t}$, $x_{i,t}^{b,*} = x_{i,t} - \bar{x}_i^b$, and $\bar{x}_i^b = \frac{2}{T_i}\sum_{t=T_i/2+1}^{T_i} x_{i,t}$. Then, the half-panel jackknife FE estimator of β, denoted by $\tilde{\beta}$, is given by

$$\tilde{\beta} = 2\hat{\beta} - \frac{1}{2}\left(\hat{\beta}_a + \hat{\beta}_b\right) \ .$$

Asymptotically $\tilde{\beta}$ is equivalent to

$$\sqrt{\sum_{i=1}^N T_i}\left(\tilde{\beta} - \beta\right) \overset{a}{\sim} Q^{-1}\frac{1}{\sqrt{\sum_{i=1}^N T_i}}\sum_{i=1}^N\sum_{t=1}^{T_i} d_{i,t}^*\epsilon_{i,t} \ ,$$

where

$$d_{i,t}^* = 1_{\{t\leq T_i/2\}}d_{i,t}^{a,*} + 1_{\{t>T_i/2\}}d_{i,t}^{b,*} \ , \quad d_{i,t}^{a,*} = x_{i,t} - (2\bar{x}_i - \bar{x}_i^a) \ , \quad d_{i,t}^{b,*} = x_{i,t} - (2\bar{x}_i - \bar{x}_i^b) \ ,$$

where $1_{\{.\}}$ is an indicator function. Therefore, the asymptotic distribution of $\tilde{\beta}$ is given by

$$\sqrt{\sum_{i=1}^N T_i}\left(\tilde{\beta} - \beta\right) \xrightarrow{d} N(0, Q^{-1}RQ^{-1}) \ ,$$

where

$$R = \lim_{N,T_i\to\infty}\frac{1}{\sum_{i=1}^N T_i}E\left[\left(\sum_{i=1}^N\sum_{t=1}^{T_i} d_{i,t}^*\epsilon_{i,t}\right)^2\right] \ . \tag{A.14}$$

A consistent estimator of the asymptotic variance of $\tilde{\beta}$ is given by

$$\widehat{AsyVar}\left(\sqrt{\sum_{i=1}^{N} T_i}\tilde{\beta}\right) = \hat{Q}^{-1}\hat{R}\hat{Q}^{-1} ,$$

where $\hat{Q}$ is given by Equation (A.13). An estimation of $\hat{R}$, which is the estimator of R defined in Equation (A.14), depends on assumptions on $\epsilon_{i,t}$. For example, if one attempts to estimate clustered standard errors with clusters being funds (N clusters), $\hat{R}$—without any small sample correction—is given by

$$\hat{R} = \frac{1}{\sum_{i=1}^{N} T_i} \sum_{i=1}^{N} \left(\sum_{t=1}^{T_i} d_{i,t}^{*} \hat{\epsilon}_{i,t} \right)^2 ,$$

where

$$\hat{\epsilon}_{i,t} = y_{i,t} - \bar{y}_i - \tilde{\beta}\left(x_{i,t} - \bar{x}_i\right) \quad , \quad \bar{y}_i = \frac{1}{T_i}\sum_{t=1}^{T_i} y_{i,t}.$$

A.4 Discussions on ROI

By its definition, ROI from time 0 to $t+1$ can be written as

$$ROI_{0,t+1} = \frac{V_{t+1} + \mathcal{D}_{t+1}}{\mathcal{C}_{t+1}/R_{0,t+1}^{C}}, \tag{A.15}$$

Based on the trading strategy discussed in Section 2.3, the strategy yields the following return between time 0 and t:

$$\widetilde{ROI}_{0,t}^{t+1} = \frac{V_t + \mathcal{D}_t + C_t}{\mathcal{C}_{t+1}/R_{0,t+1}^{C}} \neq \frac{V_t + \mathcal{D}_t}{\mathcal{C}_t/R_{0,t}^{C}} = ROI_{0,t}$$

The difference between the two reads

$$\begin{aligned}
\widetilde{ROI}_{0,t}^{t+1} - ROI_{0,t} &= \frac{V_t + \mathcal{D}_t + C_t}{\mathcal{C}_{t+1}/R_{0,t+1}^C} - \frac{V_t + \mathcal{D}_t}{\mathcal{C}_t/R_{0,t}^C} = \frac{C_t}{\mathcal{C}_{t+1}/R_{0,t+1}^C} + \frac{V_t + \mathcal{D}_t}{\mathcal{C}_{t+1}/R_{0,t+1}^C} - \frac{V_t + \mathcal{D}_t}{\mathcal{C}_t/R_{0,t}^C} \\
&= \frac{C_t}{\mathcal{C}_{t+1}/R_{0,t+1}^C} + \frac{V_t + \mathcal{D}_t}{\mathcal{C}_t/R_{0,t}^C}\left(1 - \frac{R_{t+1}^C C_t}{\mathcal{C}_{t+1}}\right) - \frac{V_t + \mathcal{D}_t}{\mathcal{C}_t/R_{0,t}^C} \\
&= \frac{C_t}{\mathcal{C}_{t+1}/R_{0,t+1}^C} - \frac{V_t + \mathcal{D}_t}{\mathcal{C}_t/R_{0,t}^C}\frac{R_{t+1}^C C_t}{\mathcal{C}_{t+1}} = \frac{C_t}{\mathcal{C}_{t+1}/R_{0,t+1}^C} - \frac{V_t + \mathcal{D}_t}{\mathcal{C}_t}\frac{C_t}{\mathcal{C}_{t+1}/R_{0,t+1}^C} \\
&= -\frac{V_t + \mathcal{D}_t - \mathcal{C}_t}{\mathcal{C}_t}\frac{C_t}{\mathcal{C}_{t+1}/R_{0,t+1}^C} = -\left(gTVPI_t - 1\right)\frac{C_t}{\mathcal{C}_{t+1}/R_{0,t+1}^C} \, .
\end{aligned}$$

Thus, unless (i) there is no capital call at time t, or (ii) gTVPI is exactly 1, $\widetilde{ROI}_{0,t}^{t+1}$ is different from $ROI_{0,t}$. This implies that the ROI between t and $t+1$ defined by Eqution (20) differs from the actual return between t and $t+1$ for the trading strategy achieving $ROI_{0,t+1}$:

$$ROI_{t+1} = \frac{ROI_{0,t+1}}{ROI_{0,t}} = \frac{R_{t+1}^C \mathcal{C}_t}{\mathcal{C}_{t+1}}\frac{V_{t+1} + \mathcal{D}_{t+1}}{V_t + \mathcal{D}_t} \neq \frac{ROI_{0,t+1}}{\widetilde{ROI}_{0,t}^{t+1}} = \frac{V_{t+1} + \mathcal{D}_{t+1}}{V_t + \mathcal{D}_t + C_t} = R_{t+1}^W \, ,$$

where the last equality comes from Equation (29). Therefore, ROI_{t+1} may not be an achievable return between t and $t+1$—the sum of (excess) log ROI in each period presented in Equation (22), (24), and (25) is not "actual" returns. Rather, ROI_{t+1} is a conceptual return derived from the ratio between two "total returns" ($ROI_{0,t+1}$ and $ROI_{0,t}$).

Appendix B Details on Empirical Analyses

B.1 Inferring Cash Flow Information from Performance Table

Inferring cash flow information from the performance table involves two variables in the performance table: (a) the called percent and (b) the fund's distributed to paid-in (DPI) ratio. The called percent variable is how much capital has been called as a share of the

amount of committed capital. The DPI ratio of a fund is given by the ratio of the sum of distributed capital to the amount of called capital. Using these two variables, one can infer capital calls C_t and distributions D_t scaled by the amount of committed capital.

Denote the amount of committed capital to a fund by Γ. The called percent and DPI can then be written as

$$\left(\text{Called Percent}\right)_t = \frac{\sum_{s=0}^{t} C_s}{\Gamma} \quad , \quad \left(\text{DPI}\right)_t = \frac{\sum_{s=0}^{t} D_s}{\sum_{s=0}^{t} C_s} \ .$$

As a result, $\sum_{s=0}^{t} D_s/\Gamma$ is given by

$$\frac{\sum_{s=0}^{t} D_s}{\Gamma} = \frac{\sum_{s=0}^{t} C_s}{\Gamma} \times \frac{\sum_{s=0}^{t} D_s}{\sum_{s=0}^{t} C_s} = \left(\text{Called Percent}\right)_t \times \left(\text{DPI}\right)_t \ .$$

Therefore, conceptually C_t/Γ and D_t/Γ can be written as

$$\begin{aligned}
\frac{\tilde{C}_t}{\Gamma} &= \left(\text{Called Percent}\right)_t - \left(\text{Called Percent}\right)_{t-1} \ , \\
\frac{\tilde{D}_t}{\Gamma} &= \left(\text{Called Percent}\right)_t \times \left(\text{DPI}\right)_t - \left(\text{Called Percent}\right)_{t-1} \times \left(\text{DPI}\right)_{t-1} \ ,
\end{aligned}$$

where the tilde indicates conceptual equality. Note that for the first observation for each fund, the lag variables are considered as zero.

In practice, $\tilde{C}_t/\Gamma$ and $\tilde{D}_t/\Gamma$ are occasionally negative, and negative cash flows can generate issues such as negative gross returns—leading to undefined log returns. To avoid such issues, C_t/Γ and D_t/Γ are derived as follows:

$$\frac{C_t}{\Gamma} = \max\left\{\frac{\tilde{C}_t}{\Gamma}, 0\right\} + \max\left\{-\frac{\tilde{D}_t}{\Gamma}, 0\right\} \quad , \quad \frac{D_t}{\Gamma} = \max\left\{\frac{\tilde{D}_t}{\Gamma}, 0\right\} + \max\left\{-\frac{\tilde{C}_t}{\Gamma}, 0\right\} \ .$$

Hence, negative capital calls are interpreted as positive distributions, and negative distributions are interpreted as positive capital calls.

Lastly, while not cash flows, the NAV of a fund—the empirical counterpart of V_t—scaled by Γ is given by

$$\frac{V_t}{\Gamma} = (\text{Called Percent})_t \times (\text{RVPI})_t = \frac{\sum_{s=0}^{t} C_s}{\Gamma} \times \frac{V_t}{\sum_{s=0}^{t} C_s} \, .$$

Therefore, since all the performance-related variables (e.g., gTVPI and ROI) are ratios among the above inferred variables, one can readily compute those performance-related metrics from the performance table.

B.2 Performance Data Cleaning Procedure

In addition to the basic data cleaning procedure described in Section 5.1, this subsection provides details on further data cleaning procedure for the performance table. Observations with non-positive called percent, missing called percent, missing DPI, and missing RVPI are eliminated. Observations with negative TVPI, which is the sum of DPI and RVPI, are also deleted.

After getting rid of those observations, except for the case of $R_t^C = R_t^D = 1$, funds of which the first observation appears later than by the end of the year after the vintage year are excluded. The reason for this step is that the computation of $\mathcal{C}_t$ and $\mathcal{D}_t$ involves compounding cash flows with volatile returns, and missing a non-negligible period of cash flows (although their sum is recorded) could distort $\mathcal{C}_t$ and $\mathcal{D}_t$ meaningfully—particularly more so for $\mathcal{C}_t$ as distributions tend to occur at later stages of a fund's life. Note that the case of $R_t^C = R_t^D = 1$ is not subject to this issue.

Then, observations that have not passed certain minimum called percent (30%) are excluded.[21] The rationale for this filter is that funds might not have an incentive to/be able to value their assets correctly when only a small portion of their planned investments are

[21] There are cases where once a fund passes the minimum called percent but its called percent goes back to below the minimum later on. These are very rare occasions and are not excluded.

made. Also, observations with called percent greater than 1,000% are deleted as they are likely to be (fat-finger) errors. Next, observations that are past the point where $(\text{RVPI})_t = 0$ are excluded—$(\text{RVPI})_t = 0$ indicates that there is no residual assets in the fund, and all its distributions are made. These observations are dropped because performance measures after all the distributions are made do not depend on the fund any longer.

Lastly, to remove outliers, for each performance measure, the top and bottom 1% are excluded. Naturally, observations with a missing or invalid (corresponding) performance measure are eliminated. Then, for performance (predictive) regressions, the sample is limited to observations that do not miss any quarterly observation between the current period (time t) and the prediction period (time $t + n$).

B.3 Additional Steps for Variance Decomposition

There are some steps that are taken to ensure the exact equality for the variance decomposition in Section 3 and 4—Equation (27) and (36). Otherwise, regressions for both the future performance measure and the intermediate return need to be tracked, which is done only for the first few sets of the results for demonstrative purpose.[22] To eliminate such redundancy, the empirical part of this paper takes an approach that guarantees the equality of every relation in Equation (27) and (36).

For the variance decomposition of the ROI approach in Equation (27), the log gIRR is computed according to Equation (9), not from its definition. This ensures that the second (approximate) relation in the equation holds exactly. An exception is the $R_t^C = R_t^D = 1$ case: the net IRR is given in the raw data, and the effective duration is computed as

$$Dur_t \approx \frac{tvpi_t}{irr_t} \,, \tag{B.1}$$

[22]The exact and approximate regressions exhibit only minor quantitative differences and rarely any qualitative differences.

where the "generalized" denotation ("g") is omitted to emphasize $R_t^C = R_t^D = 1$, where gTVPI is simply TVPI, and gIRR is simply IRR. This definition of the effective duration guarantees that Equation (24) holds exactly rather than approximately.

For the variance decomposition of the NPV approach in Equation (36), different adjustments are made to ensure that all the relations in Equation (36) are held exactly. First, since Equation (32) is an exact relation and $gTVPI_t - 1 \approx \frac{gTVPI_t+1}{2} gtvpi_t$, replacing $\frac{gTVPI_t+1}{2} gtvpi_t$ with $gTVPI_t - 1$ when computing betas in the first relation of Equation (36) guarantees that the relation holds exactly.

For the second relation, dividing Equation (32) by $gDur_{t+n}$ and rearranging terms give

$$\frac{gTVPI_t-1}{gDur_t} = \frac{1}{\prod_{k=1}^{n} R_{t+k}^C} \frac{\mathcal{C}_{t+n}}{\mathcal{C}_t} \frac{gTVPI_{t+n}-1}{gDur_{t+n}} - \frac{\Delta_n gDur_{t+n}}{gDur_{t+n}} \times \left(\frac{1}{\Delta_n gDur_{t+n}} \sum_{k=1}^{n} \left(gTVPI_{t+k-1} + \frac{C_{t+k-1}}{\mathcal{C}_{t+k-1}} \right) \frac{\mathcal{C}_{t+k-1}}{\mathcal{C}_t} \frac{R_{t+k}^W - R_{t+k}^C}{\prod_{j=1}^{k} R_{t+j}^C} - \frac{gTVPI_t-1}{gDur_t} \right), \tag{B.2}$$

where both the LHS and RHS, respectively, are approximations of those in Equation (34) since

$$\frac{gTVPI_t-1}{gDur_t} \approx \frac{gTVPI_t+1}{2} \frac{gtvpi_t}{gDur_t} \approx \frac{gTVPI_t+1}{2} girr_t \, .$$

Thus, replacing $\frac{gTVPI_t+1}{2} girr_t$ with $\frac{gTVPI_t-1}{gDur_t}$ when computing betas in the second relation of Equation (36) guarantees that the relation holds exactly. Note that for the $R_t^C = R_t^D = 1$ case, the effective duration is computed as Equation (B.1), which makes the second approximate equality of the above equation holds as an exact equality.

For the third relation of Equation (36), one can derive a similar relation to Equation

(B.2) from Equation (32):

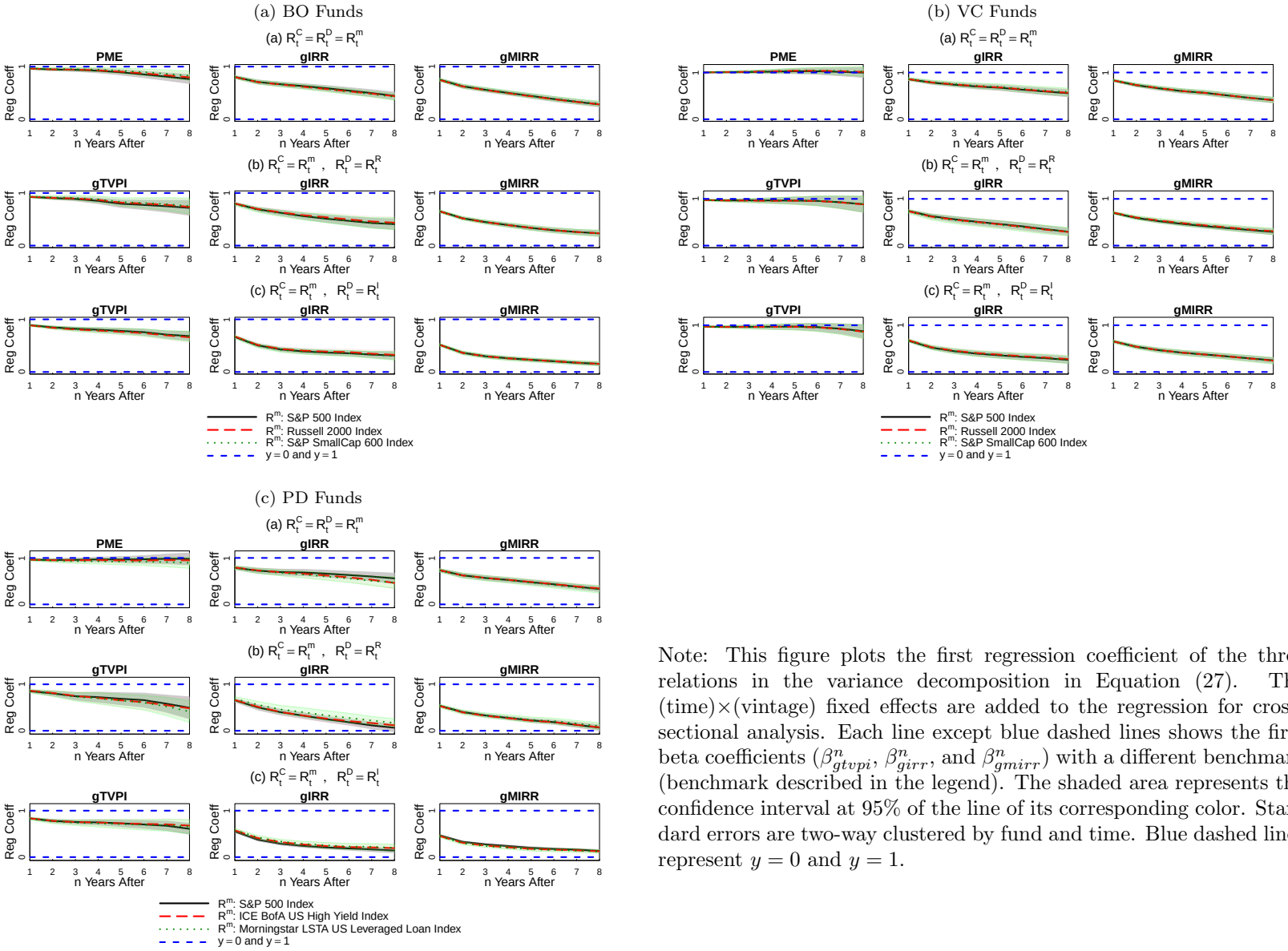
$$\frac{gTVPI_t-1}{t} = \frac{1}{\prod_{k=1}^{n} R_{t+k}^{C}} \frac{\mathcal{C}_{t+n}}{\mathcal{C}_t} \frac{gTVPI_{t+n}-1}{t+n} - \frac{n}{t+n} \times \left(\frac{1}{n} \sum_{k=1}^{n} \left(gTVPI_{t+k-1} + \frac{C_{t+k-1}}{\mathcal{C}_{t+k-1}} \right) \frac{\mathcal{C}_{t+k-1}}{\mathcal{C}_t} \frac{R_{t+k}^{W} - R_{t+k}^{C}}{\prod_{j=1}^{k} R_{t+j}^{C}} - \frac{gTVPI_t-1}{t} \right) ,$$

where both the LHS and RHS, respectively, are approximations of those in Equation (35) since

$$\frac{gTVPI_t-1}{t} \approx \frac{gTVPI_t+1}{2} \frac{gtvpi_t}{t} = \frac{gTVPI_t+1}{2} gmirr_t \, .$$

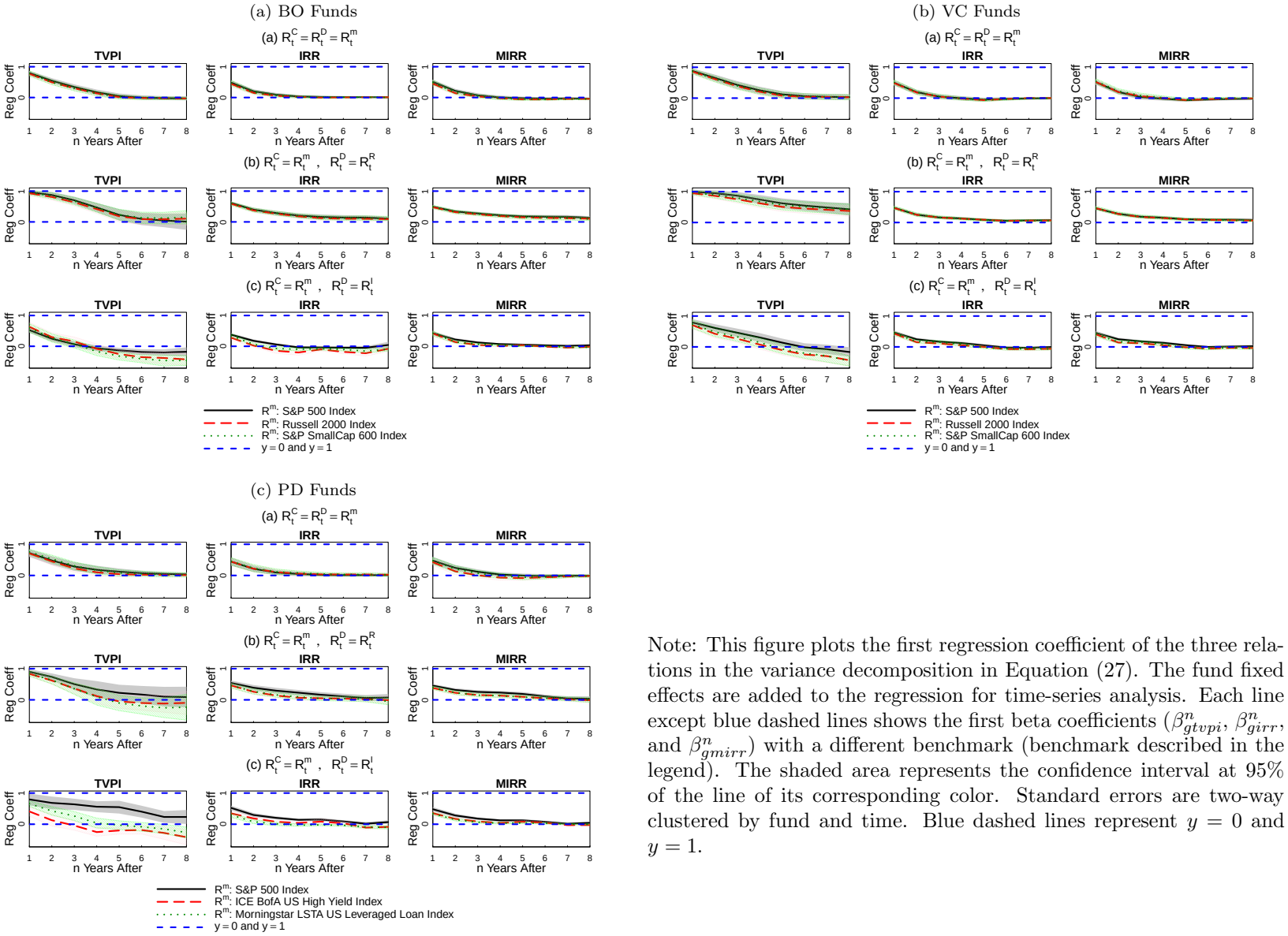
Therefore, replacing $\frac{gTVPI_t+1}{2} gmirr_t$ with $\frac{gTVPI_t-1}{t}$ when computing betas in the third relation of Equation (36) guarantees that the relation holds exactly.

Figure A1: (ROI approach) Cross-Sectional Regression Coefficients of the Variance Decomposition in Equation (27); Alternative Benchmark



Note: This figure plots the first regression coefficient of the three relations in the variance decomposition in Equation (27). The (time)×(vintage) fixed effects are added to the regression for cross-sectional analysis. Each line except blue dashed lines shows the first beta coefficients (β^n_{gtvpi}, β^n_{girr}, and β^n_{gmirr}) with a different benchmark (benchmark described in the legend). The shaded area represents the confidence interval at 95% of the line of its corresponding color. Standard errors are two-way clustered by fund and time. Blue dashed lines represent $y = 0$ and $y = 1$.

Figure A2: (ROI approach) Time-Series Regression Coefficients of the Variance Decomposition in Equation (27); Alternative Benchmark



Note: This figure plots the first regression coefficient of the three relations in the variance decomposition in Equation (27). The fund fixed effects are added to the regression for time-series analysis. Each line except blue dashed lines shows the first beta coefficients (β^n_{gtvpi}, β^n_{girr}, and β^n_{gmirr}) with a different benchmark (benchmark described in the legend). The shaded area represents the confidence interval at 95% of the line of its corresponding color. Standard errors are two-way clustered by fund and time. Blue dashed lines represent $y = 0$ and $y = 1$.