The Exchange Rate as an Industrial Policy*

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Abstract

We study the role of exchange rates in industrial policy. We construct an open-economy macroeconomic framework with Marshallian production externalities and imperfect capital mobility, and provide conditions under which foreign exchange interventions that keep the currency undervalued can improve welfare. A quantitative analysis applied to China's growth take-off shows that the observed foreign exchange interventions significantly increased output growth. Our analysis shows that in economies featuring a dynamic path of externalities, the optimal exchange rate industrial policy can lead to sizable welfare gains, especially when combined with time-invariant conventional industrial policies.

Keywords: Exchange rates, industrial policy, imperfect financial markets, growth takeoff, capital controls

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1. Introduction

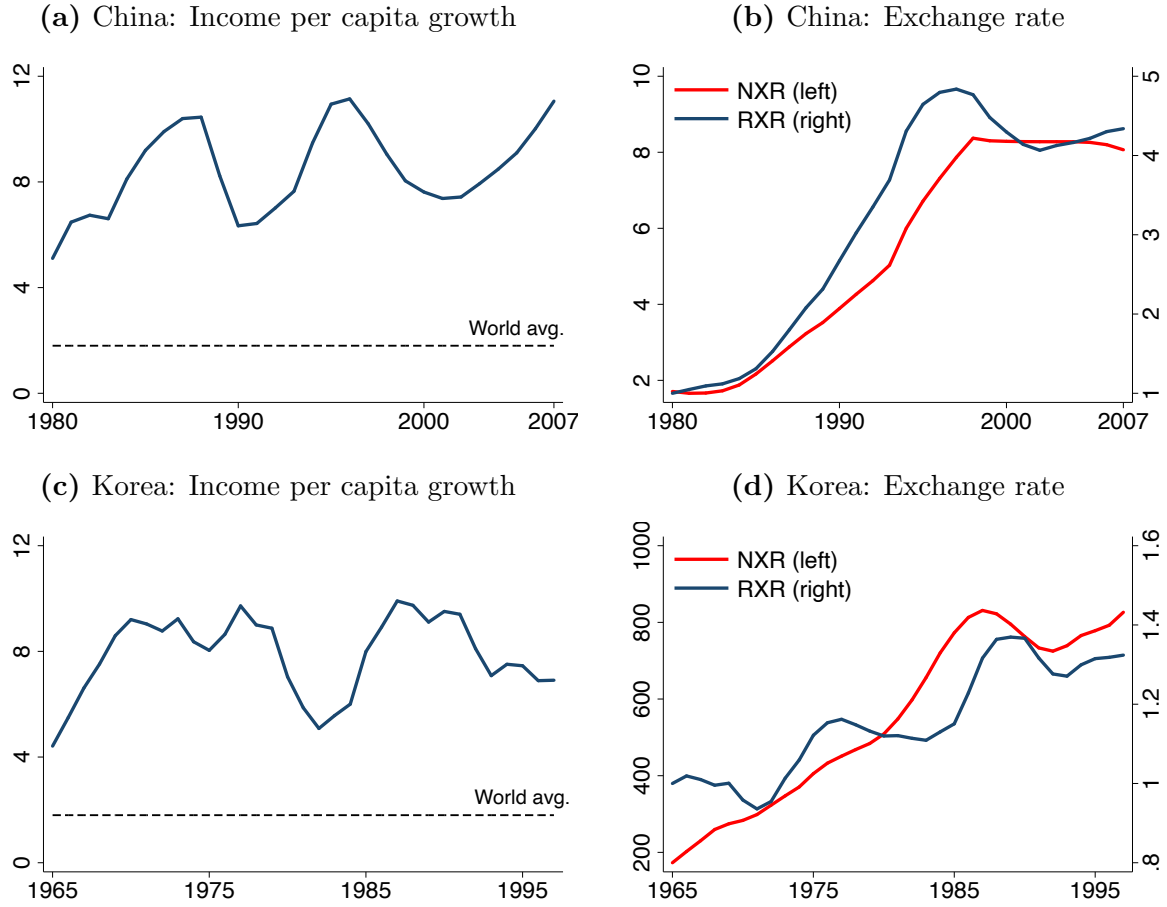
A long-standing view in policy circles is that the exchange rate can be used as a tool to foster development. The broad idea is that maintaining a depreciated exchange rate can stimulate strategic sectors by enhancing their competitiveness and speed up economic growth. This narrative is based on salient examples of prolonged growth in emerging-market economies, including South Korea from the 1960s to the 1990s and China from the 1980s to the 2010s. These economies experienced decades of high per capita output growth, averaging more than three times the global rate, a process that was also accompanied by significant depreciations of both nominal and real exchange rates (see Figure 1).

In this paper, we study how the exchange rate can be used as an industrial policy from a theoretical and quantitative perspective. To do so, we develop an open-economy macroeconomic framework with production externalities and imperfect capital mobility. We first establish conditions related to the dynamic path of externalities under which foreign exchange interventions that keep the currency undervalued can be desirable and effective. We then conduct a quantitative analysis of China's growth take-off and show that the observed foreign exchange interventions significantly increased output growth. Additionally, we show that the optimal "exchange rate industrial policy" can generate sizable welfare gains, particularly when combined with time-invariant conventional industrial policies.

The paper begins by constructing a theoretical framework to study the role of exchange rates as an industrial policy. The model focuses on Marshallian production externalities—often regarded as the textbook case for industrial policy (e.g., Harrison and Rodríguez-Clare, 2010)—that exhibit a dynamic pattern, with stronger spillovers in economies that are further from the technological frontier (e.g., Redding, 1999). We introduce these externalities into a canonical open-economy framework with tradable and nontradable goods and imperfect international financial markets, which allow the government to influence the path of the real exchange rate through foreign exchange interventions (e.g., Gabaix and Maggiori, 2015).

In this framework, the first-best allocation can be attained with time-varying, sector-specific labor subsidies. In the absence of these tools, there can be scope for exchange rate

Figure 1: The Macroeconomic Effects of Exchange Rate Industrial Policy



*N*otes: Panels (a) and (c) show the 5-year moving average of the annual per capita GDP growth rate. Panels (b) and (d) show the 5-year moving average of the nominal exchange rate per USD and the multilateral real exchange rate (expressed as domestic currency per unit of a basket of foreign currencies). Data sources: BIS, OECD, World Bank.

industrial policies, based on foreign exchange interventions, as second-best policies. We show that the desirability of these policies critically depends on the dynamic patterns of externalities. When externalities are stronger in the earlier stages of development, economies that are converging to the technological frontier can benefit from foreign exchange interventions aimed at keeping the currency undervalued in the early stages of the transition, increasing labor supply, and directing resources to the tradable sector. The intuition for this result follows from second-best policy theory (e.g., Bhagwati, 1969): By introducing a second-order distortion in the domestic Euler equation, the government can achieve a first-order

welfare gain from internalizing the production externalities. In contrast, in economies that are not converging—either because they are stagnating or because they are at the technological frontier—there is little scope for these policies. Although these economies may feature externalities, foreign exchange interventions are not the appropriate tool to address them if externalities do not exhibit a dynamic pattern.

We then conduct a quantitative analysis of our model applied to China's take-off to assess the quantitative relevance of exchange rate industrial policy. For this, we build on the methodology proposed by Bartelme, Costinot, Donaldson and Rodriguez-Clare (2019) to estimate sector-specific production externalities using cross-country sectoral trade data and an instrumental variables approach. We expand on this work by estimating how these externalities depend on a country's distance to the technological frontier and find that they are stronger the farther a country is from the frontier, as considered in our theoretical framework. We then calibrate the model to match salient features of the Chinese economy, incorporating the observed path of reserve accumulation.

Our quantitative analysis leads to two main results. First, the output effect of exchange rate industrial policy can be quantitatively large. Using our calibrated model, we estimate that the observed reserve accumulation in China between 1980 and 2008 contributed to an annual growth rate that was 0.4 percentage points higher. During the period after 2000, when reserve accumulation was particularly large, the effect on annual growth was 1.60 percentage points. Second, the optimal design of these policies can also generate considerable welfare gains. In isolation, the optimal exchange rate industrial policy captures 17% of the welfare gains from the first-best allocation. Additionally, we show that these policies complement time-invariant traditional policy instruments. For instance, the welfare gains from time-invariant labor subsidies amount to 66% of the first-best gains in isolation and increase to 90% when combined with the optimal exchange rate industrial policy.

Finally, we use our framework to study what determines the effectiveness of exchange rate industrial policy. Our analysis highlights two key dimensions. First, the importance of international capital mobility. On the one hand, regarding financial flows, a necessary condition for this policy to be effective is imperfect capital mobility, so that foreign exchange

interventions can influence the exchange rate and the macroeconomy. In fact, the allocations under the optimal exchange rate industrial policy can also be implemented through a capital control tax, a result that can be recast as "capital controls as industrial policy." On the other hand, in terms of physical capital accumulation, a necessary condition is opening the economy to foreign investors. Otherwise, the increase in domestic rates of return can hinder capital accumulation and, consequently, backfire by reducing economic growth. Second, our analysis highlights the importance of labor markets and the sectoral composition of dynamic externalities. These policies are most effective in environments with highly elastic labor supply, where changes in the exchange rate generate large reallocations of labor and production. They are also most desirable when dynamic externalities are present in sectors that can more easily attract additional labor as they become more competitive.

We conclude our analysis of the effectiveness of the exchange rate as an industrial policy by using our framework to interpret historical experiences. The Asian growth miracles are often cited as emblematic examples of export-led growth under undervalued currencies. Through the lens of our model, these economies exhibit the central ingredients required for effective exchange rate industrial policies: a process of convergence to the technological frontier, initially underdeveloped financial markets, and "demographic dividends" that imply a highly elastic labor supply to the tradable sector. The salient characteristics of the Asian examples contrast with those of Latin American experiences, which are often referenced as failures of these types of policies. Most Latin American economies did not undergo convergence processes and featured tradable production tilted toward commodity sectors, which are less likely to display sizable externalities. They also faced larger costs for sectoral labor reallocation and had a relatively more open capital account.

Related Literature. Our paper is related to several strands of the literature. First, it builds on the new generation of macroeconomic models of the exchange rate and imperfect financial markets, as surveyed by Maggiori (2022). These models have been used to study exchange rate dynamics, their connection with the macroeconomy, and the effectiveness of foreign exchange interventions (see, for example, Gabaix and Maggiori, 2015; Fanelli and

Straub, 2021; Itskhoki and Mukhin, 2021a,b). We contribute to this literature by analyzing how international capital market imperfections play a central role in the desirability of exchange rate industrial policy.

Second, the paper contributes to the literature that studies the role of exchange rates in economic development (see, for example, Hirschman, 1958; Rodrik, 1986; Krugman, 1987; Baldwin and Krugman, 1989; Rodrik, 2008). This literature has developed models showing how maintaining an undervalued exchange rate and managing capital inflows can be desirable in the presence of various types of production externalities in the tradable sector (see, for example, Michaud and Rothert, 2014; Korinek and Serven, 2016; Guzman, Ocampo and Stiglitz, 2018; Itskhoki and Moll, 2019; Benigno, Fornaro and Wolf, 2022; Bergin, Choi and Pyun, 2024). We contribute to this literature by conducting a quantitative investigation of the effects of exchange rate industrial policies on growth, based on empirical estimates of Marshallian externalities.

Our paper is also related to the vast literature on industrial policy (see Harrison and Rodríguez-Clare, 2010; Juhasz, Lane and Rodrik, 2024, for a survey of recent advances in this literature).[1] We build on this literature by modeling and estimating production externalities. Our contribution is to show that conventional tools for industrial policy (e.g., import tariffs, taxes or subsidies to sectoral production, and direct financial interventions) can be complemented with exchange rate policy: while the former have been shown to affect allocation in the long run (Choi and Levchenko, 2021), the latter exploits the time-varying dimension of these policies.

Finally, our paper is also related to the literature on how capital flows to fast-growing developing economies (see, for example, Lucas, 1990; Alfaro, Kalemli-Ozcan and Volosovych, 2008; Aguiar and Amador, 2011; Gourinchas and Jeanne, 2013, among others). Closest to our work, a growing literature studies China's integration into international capital markets.

[1] Notable contributions in the area of international trade include Redding (1999); Melitz (2005); Bartelme *et al.* (2019); Lashkaripour and Lugovskyy (2023). Other applications have been studied in the context of firm heterogeneity (Gaubert, Itskhoki and Vogler, 2021; Choi, Levchenko, Ruzic and Shim, 2024), network economies (Liu, 2019), technology diffusion (Buera and Trachter, 2024; Bai, Jin, Lu and Wang, 2024), financial frictions (Itskhoki and Moll, 2019; Farhi and Tirole, 2021), and geoeconomics (Clayton, Maggiori and Schreger, 2023, 2024).

This literature has highlighted the central role of exchange rate policy and capital controls in the process of international integration (see, for example, Song, Storesletten and Zilibotti, 2011; Jeanne, 2013; Song, Storesletten and Zilibotti, 2014; Farhi and Maggiori, 2019; Bahaj and Reis, 2020; Clayton, Dos Santos, Maggiori and Schreger, 2022). We contribute to this by showing that growth processes without capital inflows can result from the use of exchange rates and capital controls as industrial policies to redirect resources to strategic sectors.

The rest of the paper is organized as follows. Section 2 presents the baseline theoretical framework and characterizes the optimal exchange rate industrial policy. Section 3 conducts the quantitative analysis and applies it to China's takeoff. Section 4 examines the determinants of the effectiveness and desirability of this policy, focusing on the role of international capital mobility, labor market characteristics, production technologies, and sectoral heterogeneity, and discusses historical experiences through the lens of our model. Finally, Section 5 concludes.

2. Theoretical Framework

We consider a canonical small-open-economy model with tradable and nontradable goods. There are three types of agents in the domestic economy: households, firms, and the government. In this setting, we incorporate dynamic production externalities and segmented asset markets. The rest of the world trades tradable goods and an external asset with the domestic economy. We study the optimal exchange rate industrial policy when the economy undergoes a growth process and externalities dissipate as it converges to the technological frontier.

2.1. Environment

Households. The environment is deterministic, and time is infinite, discrete, and denoted by $t = 0, 1, ...$ The representative household has preferences over an infinite stream of

consumption, C_t, and labor, L_t,

$$\sum_{t=0}^{\infty} \beta^t \left[\frac{{C_t}^{1-\sigma}}{1-\sigma} - \phi \frac{L_t^{1+\nu}}{1+\nu} \right]. \tag{1}$$

where $1/\sigma$ is the intertemporal elasticity of substitution, and $1/\nu$ is the Frisch elasticity of labor supply. The consumption good is a composite aggregator of tradable C_{Tt} and nontradable C_{Nt} consumption

$$C_t = \left[\omega^{\frac{1}{\eta}} (C_{Tt})^{1-\frac{1}{\eta}} + (1-\omega)^{\frac{1}{\eta}} (C_{Nt})^{1-\frac{1}{\eta}} \right]^{\frac{\eta}{\eta-1}}, \tag{2}$$

where $\omega \in (0,1)$ is the weight on the tradable good and $\eta > 0$ is the elasticity of substitution between tradable and nontradable consumption. Similarly, labor is a composite aggregate of labor supply in the tradable sector L_{Tt} and nontradable sector L_{Nt}

$$L_t = \left[L_{Tt}^{1+\frac{1}{\zeta}} + L_{Nt}^{1+\frac{1}{\zeta}} \right]^{\frac{\zeta}{\zeta+1}}, \tag{3}$$

where $\zeta > 0$ is the elasticity of substitution between labor supply in different sectors. Households receive their income from labor and profits from domestic firms and can save or borrow using a domestic currency bond. Their budget constraint expressed in domestic currency is given by

$$P_{Tt} C_{Tt} + P_{Nt} C_{Nt} + B_{t+1} = W_{Tt} L_{Tt} + W_{Nt} L_{Nt} + \Pi_t + T_t + R_t B_t, \tag{4}$$

where P_{Tt}, P_{Nt} are the prices of tradables and nontradables; B_{t+1} are the bonds purchased in t that mature in $t+1$; R_t is the domestic currency interest rate; W_{Tt}, W_{Nt} are the nominal wages in the tradable and nontradable sectors; Π_t denotes profits from firms in both sectors; and T_t denotes transfers from the government.

The household's problem is to choose allocations $\{C_t, C_{Tt}, C_{Nt}, L_t, L_{Tt}, L_{Nt}, B_{t+1}\}_{t=0}^{\infty}$ that maximize utility (1), subject to the aggregation technologies (2)-(3), the sequence of budget constraints (4), given a sequence of prices, profits, and transfers, and an initial level

of bonds B_0.

Throughout this section, we make the following parametric assumptions to provide analytical results. These assumptions are relaxed in the quantitative analysis in the following sections.

ASSUMPTION 1. *Suppose* $\sigma = \eta = 1$, $\zeta = \frac{1}{\nu}$, *and* $\nu \to 0$.

The first condition corresponds to the preference parameterization of Cole and Obstfeld (1991). The second and third conditions correspond to a linear disutility of labor, as in Rogerson (1988), and imply that sectoral wages are equal, $W_{Tt} = W_{Nt} \equiv W_t$. Given these assumptions, the first-order conditions that characterize the solution to the household's problem are:

$$\left(\frac{1-\omega}{C_{Nt}}\right) = p_t \left(\frac{\omega}{C_{Tt}}\right), \tag{5}$$

$$\left(\frac{\omega}{C_{Tt}}\right) \frac{W_t}{P_{Tt}} = \phi, \tag{6}$$

$$\left(\frac{\omega}{C_{Tt}}\right) = \beta R_{t+1} \frac{P_{Tt}}{P_{Tt+1}} \left(\frac{\omega}{C_{Tt+1}}\right), \tag{7}$$

where $p_t \equiv P_{Nt}/P_{Tt}$ is the relative price of nontradable goods. The first equation relates the marginal utility of consuming tradables and nontradables to their relative price. The second equation equates the marginal disutility of supplying labor with the product of the real wage in tradable goods and the marginal utility of consuming tradables. The last equation is the Euler equation, where the relevant interest rate is the real interest rate of the bond in local currency.

Firms. There is a representative firm in each sector. Firms in each sector have access to a production technology that uses labor l_{it} as an input:

$$y_{it} = A_t L_{it}^{\gamma_{it}} l_{it}^{\alpha}, \tag{8}$$

for $i = T, N$. Firms' productivity, $Z_{it} \equiv A_t L_{it}^{\gamma_{it}}$, is the product of an exogenous and an endogenous component. The exogenous component, A_t, evolves according to

$$\log(A_t) = \rho \log\left(\varphi \overline{A}\right) + (1-\rho)\log(A_{t-1}), \tag{9}$$

for $t \geq 1$, where $\overline{A}$ is the technological frontier; $\varphi \in (0,1)$ is the fraction of the technological frontier to which the country converges in the steady state; $A_0 \leq \varphi \overline{A}$ is the initial productivity; and $\rho \in (0,1)$ governs the speed of convergence. The endogenous component, $L_{it}^{\gamma_{it}}$, captures the Marshallian production externalities by which aggregate sectoral labor L_{it} increases the productivity of firms operating in that sector (see Harrison and Rodríguez-Clare, 2010, for a detailed discussion of this type of externality and their foundations). In equilibrium, $L_{it} = l_{it}$, given the representative firm assumption. We assume that the externalities are sector-dependent and a function of the distance to the technological frontier, i.e., $\gamma_{it} = \Gamma_i(\overline{A}/A_t)$, and make the following assumption regarding the relative strength and dynamics of these sectoral externalities.

Assumption 2. *Suppose that Γ_i is weakly increasing in $\overline{A}/A_t$, and $\Gamma_i(\overline{A}/A_t) \in [0, 1-\alpha]$, with $\alpha \in (0,1)$. Additionally, suppose that $\Gamma_N(\overline{A}/A_t) = 0$.*

The first condition assumes that externalities are stronger the further the economy is from the technological frontier. This captures the idea that externalities are larger in the initial growth phase of a sector, when the role of learning and knowledge acquisition is more relevant (see, for example, Redding, 1999; Melitz, 2005; Itskhoki and Moll, 2019, for studies on industrial policies in economies with dynamic externalities that dissipate as sectors grow). We provide empirical evidence on this assumption in Section 3.1. Note that we do not impose any assumption on the level of externalities once the economy reaches its steady-state productivity $\varphi\overline{A}$; it could be that economies at the steady state feature a permanently positive externality. The second condition, an upper bound on the level of externalities, ensures concavity of the planner's problem.[2]

[2]Under Assumption 1, which features linear disutility of labor, concavity of the planner's problem requires decreasing returns to scale in the aggregate. In the quantitative analysis, which features an upward-sloping

The last condition assumes that externalities are only present in the tradable sector and is common in the literature on industrial policy in open economies (see, for example, Krugman, 1987). The rationale for this assumption is that learning-by-doing and knowledge spillovers are more likely to be present in exporting sectors such as manufacturing and less so in the nontradable sectors of developing economies, which, prior to growth takeoff, are more concentrated in local agricultural sectors. In Appendix A.6, we relax this assumption and characterize the optimal exchange rate industrial policy when the economy features externalities in the nontradable sector that could be stronger or weaker than those in the tradable sector. In addition, we study the effects of externalities in the nontradable sector in our quantitative analysis in Section 3.

Firms choose labor to maximize their profits, $\Pi_{it} = P_{it} A_t L_{it}^{\gamma_{it}} l_{it}^{\alpha} - W_t l_{it}$, which gives rise to the following aggregate labor demand:

$$\alpha A_t L_{it}^{\alpha+\gamma_{it}-1} = W_t / P_{it}. \tag{10}$$

Government. The government manages a portfolio of bonds in local and foreign currency and transfers its proceeds to households as lump-sum payments. Its budget constraint is given by

$$F_{t+1} + \mathcal{E}_t F_{t+1}^* + T_t = R_t F_t + \mathcal{E}_t R^* F_t^*, \tag{11}$$

where F_{t+1} and F_{t+1}^* are the local and foreign currency bonds purchased in period t, respectively; R^* is the foreign currency interest rate; and $\mathcal{E}_t$ is the nominal exchange rate expressed as domestic currency per unit of foreign currency.

Rest of the world. The rest of the world exchanges tradable goods and foreign currency bonds with the government of the small open economy and provides a perfectly elastic supply of funds at the interest rate R^*. Financial markets are segmented, and the rest of the world

labor supply, we can relax these conditions and still obtain a concave planner's problem with $\gamma_{Tt} > 0$ and $\alpha = 1$.

cannot trade domestic currency bonds. Finally, we assume that the law of one price holds for tradable goods and normalize the foreign currency price of tradables so that $P_{Tt} = \mathcal{E}_t$.

Competitive equilibrium. We now define a competitive equilibrium for given government policies.

***Definition* 1** (Competitive equilibrium)**.** *Given initial asset positions F_0, F_0^*, a competitive equilibrium is a sequence of private allocations $\{C_t, C_{Tt}, C_{Nt}, L_t, B_{t+1}, L_{Tt}, L_{Nt}\}_{t=0}^{\infty}$, prices $\{P_{Tt}, P_{Nt}, W_t, \mathcal{E}_t, R_t\}_{t=0}^{\infty}$, and government policies $\left\{F_{t+1}, F_{t+1}^*, T_t\right\}_{t=0}^{\infty}$ such that:*

1. *Allocations solve the households' and firms' problem, given prices;*
2. *Government policies satisfy the government budget constraint;*
3. *Markets clear:*

$$C_{Nt} = A_t L_{Nt}^{\alpha}, \tag{12}$$

$$F_{t+1} + B_{t+1} = 0. \tag{13}$$

Equations (12) and (13) are the market-clearing conditions for nontradable goods and local currency bonds. The equilibrium allocations of labor and foreign currency bonds are demand-determined because their supply is perfectly elastic.

We now derive the equations that characterize the competitive equilibrium allocations. These will serve as implementability conditions for the optimal policy problem. By combining (5), (6), and (10), we obtain

$$\left(\frac{1-\omega}{\omega}\frac{C_{Tt}}{C_{Nt}}\right) = \frac{L_{Tt}^{\alpha+\gamma_{Tt}-1}}{L_{Nt}^{\alpha-1}}, \tag{14}$$

$$\frac{\phi}{(\omega/C_{Tt})} = \alpha A_t L_{Tt}^{\alpha+\gamma_{Tt}-1}. \tag{15}$$

The first equation equates the marginal rate of substitution between tradable and nontradable goods to their private marginal rate of transformation. The second equation equates

the marginal rate of substitution between tradables and labor with the private marginal product of labor. Finally, competitive equilibrium allocations are also characterized by the market-clearing condition for nontradables (12) and the balance of payments condition (or the market-clearing condition for tradable goods),

$$C_{Tt} - A_t L_{Tt}^{\alpha+\gamma_{Tt}} = R^* F_t^* - F_{t+1}^*, \tag{16}$$

which states that net imports must be financed with external debt. Note that the household's Euler equation is not an implementability condition but is used to pin down the local currency interest rate R_t.

2.2. First-best allocation

We begin by characterizing the first-best allocation, which serves as a useful benchmark.

***Definition* 2** (First best)**.** *A first-best allocation is the allocation* $\tilde{\mathbf{x}}_t \equiv \left\{\tilde{C}_{Tt}, \tilde{C}_{Nt}, \tilde{L}_{Tt}, \tilde{L}_{Nt}, F_{t+1}^*, A_t\right\}$ *that maximizes utility* (1)*, subject to the aggregation technologies* (2)-(3)*, the balance of payments condition* (16)*, and market-clearing conditions for nontradable goods* (12)*.*

The first-order conditions that characterize the first-best allocation are

$$\left(\frac{1-\omega}{\omega}\frac{\tilde{C}_{Tt}}{\tilde{C}_{Nt}}\right) = \frac{(\alpha+\gamma_{Tt})}{\alpha}\frac{\tilde{L}_{Tt}^{\alpha+\gamma_{Tt}-1}}{\tilde{L}_{Nt}^{\alpha-1}}, \tag{17}$$

$$\frac{\phi}{\left(\omega/\tilde{C}_{Tt}\right)} = (\alpha+\gamma_{Tt}) A_t \tilde{L}_{Tt}^{\alpha+\gamma_{Tt}-1}, \tag{18}$$

$$\left(\frac{\omega}{\tilde{C}_{Tt}}\right) = \beta R^* \left(\frac{\omega}{\tilde{C}_{Tt+1}}\right). \tag{19}$$

The first equation equates the marginal rate of substitution between tradable and nontradable goods to their social marginal rate of transformation. The second equation equates the marginal rate of substitution between tradables and labor with the social marginal product of labor. The last equation is the Euler equation, which equates the intertemporal marginal rate of substitution to the foreign currency interest rate.

The social marginal rate of transformation and the social marginal product of labor are higher than their private counterparts due to production externalities in the tradable sector. These differences introduce wedges in the intratemporal allocation of labor and consumption in the competitive equilibrium, relative to the first-best allocation, that cannot be eliminated by foreign exchange (FX) intervention. The following proposition formalizes this result.

***Proposition* 1** (Impossibility result)**.** *The first-best allocation cannot be attained through FX intervention.*

We include all proofs in Appendix A. FX intervention affects the intertemporal margin of consumption by influencing the path of the exchange rate and the rate of return on domestic savings. This policy cannot attain the first-best allocation because the wedges introduced by the production externality distort the intratemporal allocation of consumption and labor. On the other hand, as the next proposition states, fiscal policy can attain the first-best allocation through time- and sector-specific labor subsidies.

***Proposition* 2.** *The first-best allocation can be attained with FX intervention and the following time-varying labor subsidies to the tradable sector:*

$$\tau_{Tt}^{L} = \frac{\gamma_{Tt}}{\alpha + \gamma_{Tt}}.$$

This is a familiar result from the macro-public finance literature. Labor subsidies undo the wedges between the social and private marginal rates of transformation, and FX intervention ensures that the returns on saving in local and foreign currency are equal. While this is the most desirable policy from a social perspective, it may be difficult to implement from a political economy standpoint. In particular, since externalities dissipate during the convergence process, these optimal subsidies are time-varying, with stronger subsidies in the earlier stages of the transition. Additionally, sector-specific subsidies are likely to face practical restrictions under WTO regulations (see, for example, Rodrik *et al.*, 2009).

2.3. Optimal exchange rate industrial policy

The exchange rate industrial policy is a government policy that maximizes the lifetime utility of households subject to the implementability conditions that characterize a competitive equilibrium. This constitutes a second-best policy. We formally define this problem below.

***Definition* 3.** *An optimal exchange rate industrial policy is a government policy that solves the following problem:*

$$\max_{\{C_{it},L_{it},F^*_{t+1}\}_{t\geq 0}^{i=T,N}} \sum_{t=0}^{\infty} \beta^t \left[\log C_t - \phi \left(L_{Tt} + L_{Nt}\right)\right] \quad \textit{subject to} \tag{P1}$$

$$\left(\frac{1-\omega}{\omega}\frac{C_{Tt}}{C_{Nt}}\right) = \frac{L_{Tt}^{\alpha+\gamma_{Tt}-1}}{L_{Nt}^{\alpha-1}},$$

$$\frac{\phi}{(\omega/C_{Tt})} = \alpha A_t L_{Tt}^{\alpha+\gamma_{Tt}-1},$$

$$C_{Tt} - A_t L_{Tt}^{\alpha+\gamma_{Tt}} = R^* F_t^* - F_{t+1}^*,$$

the consumption aggregator definition (2)*, and the market-clearing condition for nontradable goods* (12)*.*

This problem is characterized by the following modified Euler equation

$$\left(\frac{\omega}{C_{Tt}}\right) = \beta R^* \frac{\theta(\mathbf{x}_{t+1}, \gamma_{Tt+1})}{\theta(\mathbf{x}_t, \gamma_{Tt})} \left(\frac{\omega}{C_{Tt+1}}\right), \tag{20}$$

where $\theta(\mathbf{x}_t, \gamma_{Tt})$ is a function that depends on the allocations of the economy, $\mathbf{x}_t \equiv \left\{C_{Tt}, C_{Nt}, L_{Tt}, L_{Nt}, F^*_{t+1}, A_t\right\}$, and the strength of the externality at a given time period t. We provide an expression for this function in Appendix A.3.

We contrast the allocations under the optimal exchange rate industrial policy with a benchmark allocation that corresponds to a competitive equilibrium in which the government is a "passive" agent, in the sense that it intermediates capital flows as if households have direct access to saving and borrowing at the foreign currency interest rate but does not take into account the effect of its FX interventions on production externalities. We formalize this benchmark notion as follows.

***Definition* 4.** *A laissez-faire competitive equilibrium is a competitive equilibrium with an associated government policy in which uncovered interest rate parity (UIP) holds, i.e.,* $R_{t+1} = R^* \frac{\mathcal{E}_{t+1}}{\mathcal{E}_t}$.

Quadratic-linear approximation to the policy problem. Under Assumption 1, the equilibrium nontradable allocations are independent of intertemporal considerations and, therefore, also independent of policy, and are given by

$$C_{Nt} = A_t \left[\frac{\alpha(1-\omega)}{\phi}\right]^{\alpha}, \tag{21}$$

$$L_{Nt} = \frac{\alpha(1-\omega)}{\phi}. \tag{22}$$

Using this result, in Appendix A.4, we show that the optimal policy problem can be approximated by a quadratic-linear problem in terms of log deviations of tradable consumption and labor from their first-best allocations.[3] In particular, we can approximate the welfare loss from the first-best allocation to second order as

$$-\frac{1}{2}\sum_{t=0}^{\infty}\beta^t\left[\omega z_t^2 + \left[\omega(\alpha+\gamma)^2 + \omega(\alpha+\gamma)\right]x_t^2\right], \tag{23}$$

where $z_t = \log(C_{Tt}) - \log(\tilde{C}_{Tt})$ and $x_t = \log(L_{Tt}) - \log(\tilde{L}_{Tt})$ are the log deviations from the first-best tradable consumption and labor, respectively, and γ is a weighted function of the path of γ_{Tt}, with the expression provided in Appendix A.4. Furthermore, we can also approximate the implementability conditions to first order as

$$z_t = \psi_t + (\alpha+\gamma-1)x_t, \tag{24}$$

$$\sum_{t=0}^{\infty}\left(\frac{1}{R^*}\right)^t\left(z_t - (\alpha+\gamma)x_t\right) = 0, \tag{25}$$

[3] See Benigno and Woodford (2003) and Itskhoki and Mukhin (2023) for a description of this approximation approach to a general class of policy problems.

where $\psi_t \equiv \log \alpha - \log(\alpha + \gamma_{Tt}) \leq 0$. The first equation equates the marginal rate of substitution between tradables and labor with the private marginal product of labor. The second equation represents the intertemporal budget constraint of the economy. With this simplified problem, we can characterize the optimal policy.

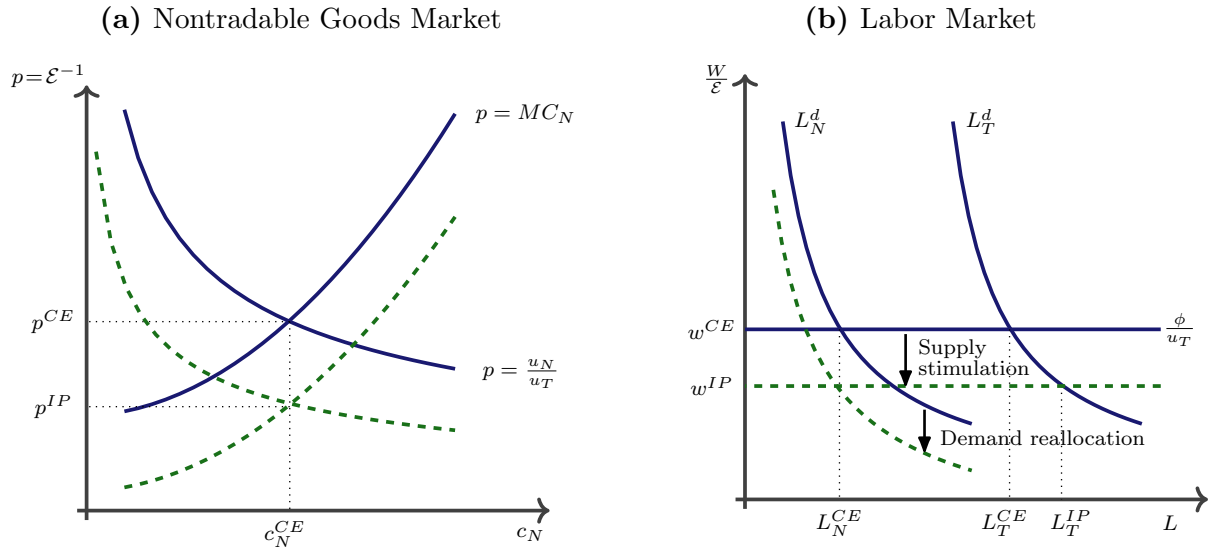
***Lemma* 1.** *The allocations that solve the policy problem* (P1) *can be approximated by those that maximize* (23) *subject to* (24) *and* (25).

Henceforth, all results refer to the solution of the approximate quadratic-linear problem. This problem highlights the trade-offs faced by the planner. Ideally, the planner would like to set tradable consumption and labor to their first-best levels in every period. However, private choices impose a static relationship between them. Therefore, the planner can only choose the path of net savings in the economy, which in turn determines the path of labor and consumption given private choices.

Prior to characterizing the optimal policy, it is useful to explain the macroeconomic effects of increasing aggregate savings in the initial period (see Figure 2). Higher savings reduce current tradable consumption, which affects current allocations through two channels. First, lower tradable consumption generates a reallocation of labor demand. Depressed aggregate demand reduces the demand for both tradable and nontradable goods, depreciates the exchange rate, and lowers labor demand in the nontradable sector. Second, lower tradable consumption stimulates aggregate labor supply. Under the parameterization in Assumption 1, the positive supply-side effect and the negative demand-side reallocation effect in the nontradable labor market cancel out, implying the same level of nontradable production as in the laissez-faire equilibrium. In the tradable sector, both effects contribute to higher labor.

The case of converging economies. We now study the optimal exchange rate industrial policy (XR-IP) in converging economies. We begin by characterizing the optimal policy in economies undergoing transitional dynamics, converging to steady-state productivity levels closer to the technological frontier. These economies exhibit a path of production externalities in the tradable sector that are stronger in the early phase of the growth process.

Figure 2: The Macroeconomic Effects of Exchange Rate Industrial Policy in Initial Periods



Notes: This figure shows the allocations of the laissez-faire competitive equilibrium (blue line) and the optimal exchange rate industrial policy (green dotted line) in the nontradable goods market and the labor market at $t = 0$.

In the case of converging economies, the optimal policy features high saving rates in the initial periods, when tradable production externalities are stronger, to induce greater labor in the tradable sector. The following proposition characterizes the optimal exchange rate industrial policy in comparison to the laissez-faire competitive equilibrium (CE).

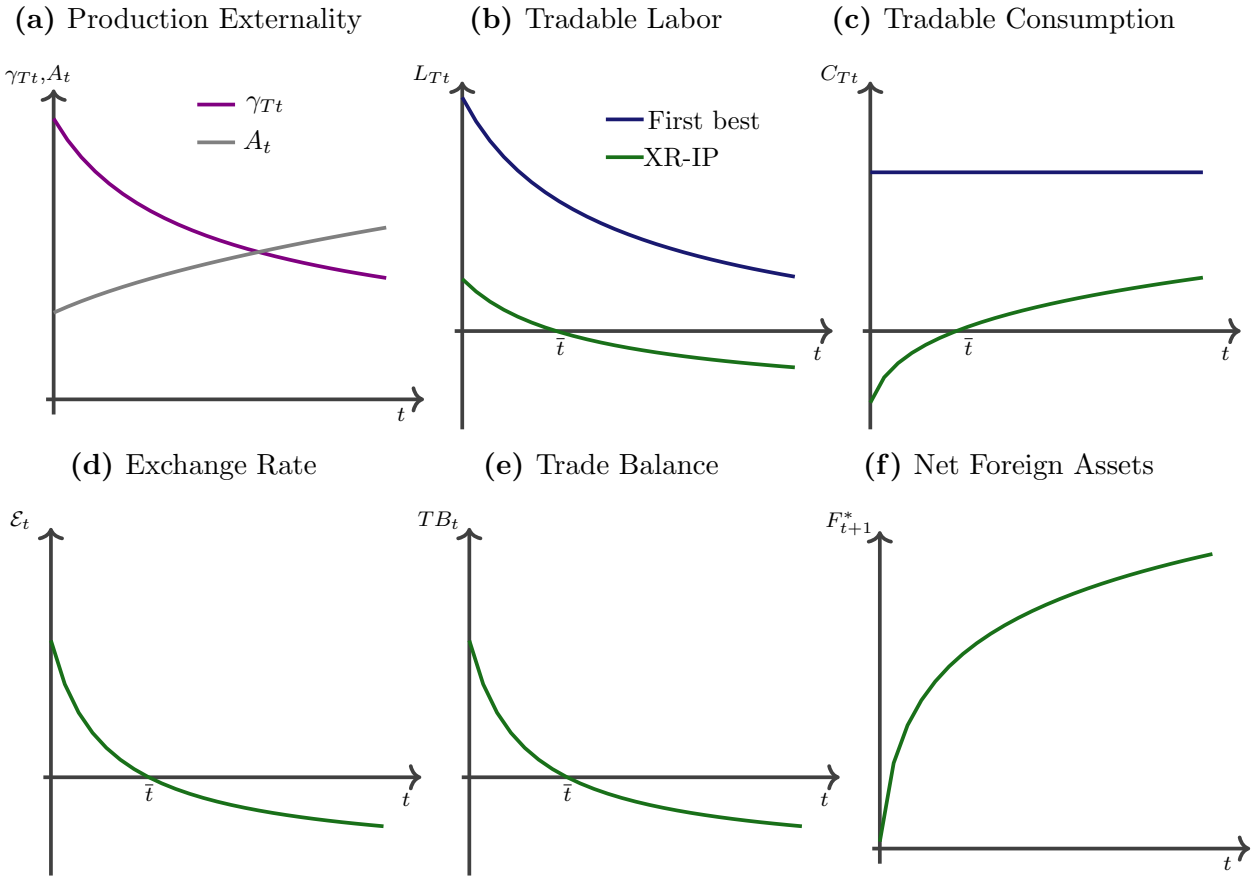
***Proposition* 3** (Exchange rate industrial policy in converging economies)**.** *Suppose that the economy starts below its steady-state level of productivity (i.e., $A_0 < \varphi\overline{A}$), which implies a decreasing path of externalities in tradable production (i.e., γ_{Tt} decreasing in t). Assuming $\beta R^* = 1$, the optimal exchange rate industrial policy in these economies implies $\exists\ \bar{t} > 0$ such that:*

$$\mathcal{E}_t^{IP} > \mathcal{E}_t^{CE}, \quad L_{Tt}^{IP} > L_{Tt}^{CE}, \quad C_{Tt}^{IP} < C_{Tt}^{CE} \qquad \text{if } t < \bar{t},$$

with opposite inequalities if $t > \bar{t}$. This is achieved through trade balance surpluses, $TB_t^{IP} > TB_t^{CE}$ if $t < \bar{t}$, and $F_{t+1}^{*IP} > F_{t+1}^{*CE}$ for all t. Furthermore, $\tilde{L}_{Tt} > L_{Tt}^{IP}$ for all t.

Figure 3 shows the dynamic path of variables for the optimal policy relative to the

Figure 3: Exchange Rate Industrial Policy Dynamics



Notes: This figure shows the dynamics of the allocations under the first-best (blue line) and the optimal exchange rate industrial policy (green line) in deviations from those of the laissez-faire competitive equilibrium.

laissez-faire competitive equilibrium. In the initial periods, when the externality is stronger, the economy exhibits lower tradable consumption, higher tradable labor, and a depreciated exchange rate relative to the laissez-faire competitive equilibrium exchange rate. The depreciated exchange rate is achieved through currency market interventions and the accumulation of international reserves. By generating a trade surplus and accumulating reserves, the economy establishes a net creditor position, which leads to a trade deficit, higher tradable consumption, lower tradable production, and a more appreciated exchange rate in future periods when the production externality dissipates.

Figure 3 also shows that tradable labor under the optimal policy is always below its first-best level. This is because achieving the same labor allocation as in the first-best

scenario would require a large distortion in the intertemporal consumption margin, making this allocation suboptimal.

The case of non-converging economies. We now characterize the optimal exchange rate industrial policy in economies that are either not converging to the technological frontier or are already at the technological frontier. In this case, since externalities do not exhibit a dynamic pattern, there is no role for this type of policy. Any exchange rate depreciation induced by the government in early periods would be accompanied by an appreciation in later periods when the trade balance is reversed. The following proposition formalizes this result.

***Proposition* 4** (Exchange rate industrial policy in non-converging economies)**.** *Consider an economy that is either not converging to the technological frontier or is already at the technological frontier (i.e., $A_0 = \varphi\overline{A}$), so that externalities are constant. Assuming $\beta R^* = 1$, the allocations under the optimal exchange rate industrial policy coincide with those in the laissez-faire competitive equilibrium.*

This result implies that when externalities are constant, FX intervention is not the appropriate tool to address them. Therefore, in our framework, a necessary condition for exchange rate industrial policies to be welfare-enhancing, relative to the laissez-faire competitive equilibrium, is that the economy is converging to the technological frontier so that externalities exhibit a dynamic pattern.

It is worth emphasizing that this no-intervention result holds for general preferences and does not rely on the parametric assumptions of Assumption 1.[4] However, this no-intervention result applies only when considering interventions that satisfy the country's intertemporal budget constraint (25), i.e., we rule out interventions that induce permanent current account surpluses (by ruling out free disposal technologies in agents' problems). In environments where externalities are sufficiently large, there may be scope for considering these other types of policies. We leave the study of such policies for future research.

[4]This result assumes $\beta R^* = 1$. If we relax this assumption and use the parameter values from Section 3 for the Chinese economy, we obtain a modest optimal FX intervention, an order of magnitude lower than that under the observed convergence process.

3. Quantitative Analysis

This section conducts a quantitative analysis of exchange rate industrial policy by applying the theoretical framework to China's growth takeoff. Section 3.1 provides new empirical evidence on the dynamic path of Marshallian externalities. Section 3.2 discusses model parameterization based on this empirical evidence and model validation. Finally, Section 3.3 examines the effects of the observed policy and the optimal exchange rate industrial policy.

3.1. Estimating dynamic Marshallian externalities

The most novel aspect of our model parameterization relates to production externalities. We assume the functional form $\Gamma_i(\mathrm{df}_t) = \gamma_i^0 + \gamma_i^1 \mathrm{df}_t$, where the distance to the technological frontier is given by $\mathrm{df}_t = \frac{\bar{A}}{A_t} - 1$. This specification implies that γ_i^0 governs the production externalities of sector i at the technological frontier, while γ_i^1 determines how externalities vary with the distance to the technological frontier.

To estimate these parameters, we build on the methodology proposed by Bartelme *et al.* (2019) (henceforth, BCDR), who estimate production spillovers in tradable sectors. In particular, we estimate sectoral production functions $Y_{cit} = A_{cit} L_{cit}^{1+\Gamma_i(\mathrm{df}_{ct})}$, where Y_{cit} is the output of country c and sector i in period t. This equation is a multi-country version of (30), which assumes constant returns to scale at the firm level ($\alpha = 1$). The only departure from the BCDR method is that we allow the production externality to be a function of the country's distance to the frontier, df_{ct}. Following BCDR, in Appendix B.1, we show that if one assumes a CES demand structure and uses agents' optimality conditions, the production spillover parameters γ_i^0, γ_i^1 can be obtained by estimating the empirical model:

$$x_{cit} = a_c + a_i + a_t + \gamma_i^0 \log L_{cit} + \gamma_i^1 \mathrm{df}_{ct} \log L_{cit} + \varepsilon_{cit}, \tag{26}$$

where x_{cit} is the average log expenditure on goods from country c in sector i and period t (adjusted by trade elasticity)[5]; L_{cit} is a measure of employment in sector i in country c in

[5]Formally, $x_{cit} \equiv \left(\frac{1}{J}\sum_d \log X_{cd,i,t}\right)/\theta_i$, where $X_{cd,i,t}$ is the expenditure on goods from country c in sector i and destination d in period t; J is the total number of destinations; and θ_i is the trade elasticity in sector

period t; df_{ct} is the distance to the technological frontier of country c in period t; a_c, a_i, and a_t are country, sector, and time fixed effects that absorb productivity shifters as well as other country, sectoral, and time-varying characteristics, respectively; and ε_{cit} is a random error term. Note that, unlike our baseline model, the framework of BCDR assumes multiple tradable sectors and imperfect substitutability of home varieties relative to those of the rest of the world. In Section 4.4, we show that we obtain similar quantitative results to those from our baseline model when we consider extensions with multiple tradable sectors and a differentiated home tradable good facing a downward-sloping demand from the rest of the world.

The estimation of (26) uses bilateral trade flows from sectoral input-output data and population data across countries, along with labor productivity to proxy the distance to the frontier for each country. Following BCDR, we estimate (26) using a demand-side instrumental variable strategy, where we instrument country-sector employment using the country's population and sectoral expenditure shares. The logic behind this instrument is that, within each sector, employment should be higher in countries that are larger and/or have a stronger preference for goods from that sector. Appendix B.1 provides further details on the data, estimation sample, and instrumental variable strategy.

Appendix Table B.2 reports the results from estimating (26) for each manufacturing sector included in the sample. For all sectors, we obtain positive estimates of γ_i^0 and γ_i^1, reflecting production externalities at the technological frontier and showing that these externalities increase with the distance to the technological frontier, as assumed in our theoretical framework.[6] Given that our baseline model features a single tradable sector, we set the externality parameters to the sales-weighted average across tradable sectors. This implies an average externality at the technological frontier of $\gamma^0 = 0.14$ and a loading on the distance to the technological frontier of $\gamma^1 = 0.01$. In Section 4.4, we consider a multi-sector extension of our model in which we use all sectoral externality estimates $\{\gamma_i^0, \gamma_i^1\}$.

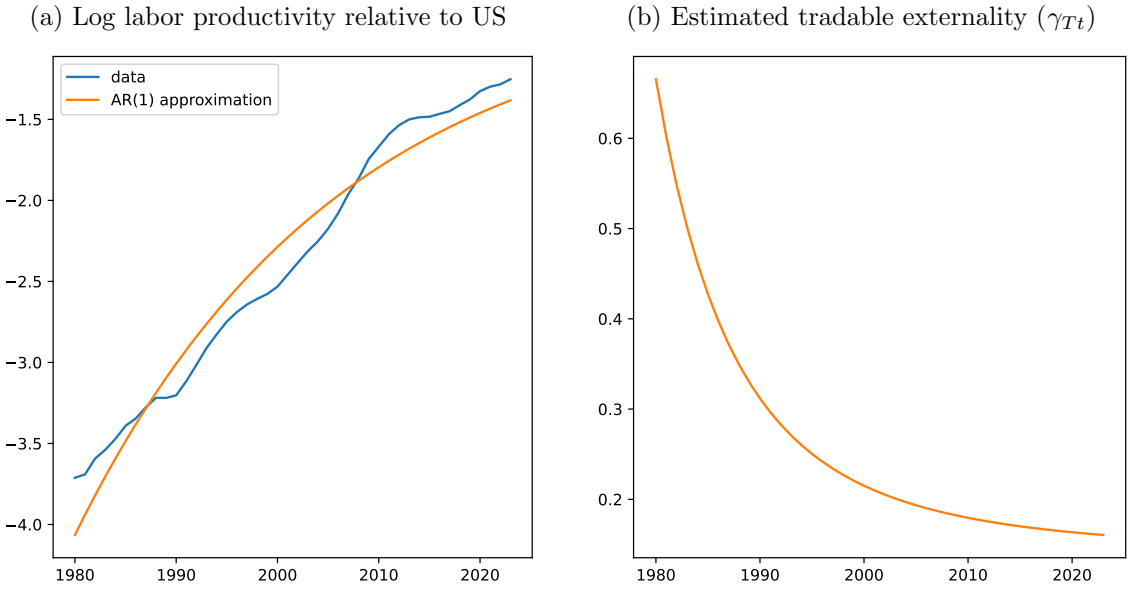
To apply these estimates to our parameterized model, we estimate the process (9) using

i.

[6]We estimate significant variation in externalities across sectors, with most commodity-related sectors (including oil, metals, minerals, and wood) displaying lower externalities than the average.

observed data on labor productivity since 1980, as depicted in Panel (a) of Figure 4. We restrict φ such that China's productivity level cannot exceed half that of the U.S. (in line with the long-term growth estimates of Fernández-Villaverde, Ohanian and Yao, 2023; OECD, 2023), and normalize $\overline{A} = 1$.[7] The estimated parameters (A_0, ρ, φ) are reported in Table 1. Panel (b) shows the path of production externalities obtained by combining the estimated productivity process with the estimates of production externalities. The estimated tradable externality declines from $\gamma_{T0} = 0.67$ in 1980—when U.S. productivity was more than 40 times that of China—to $\gamma_{T30} = 0.18$ in 2010.

Figure 4: Estimated Externality for China



Notes: This figure shows the approximation of the exogenous productivity process for China and the implied production externality path. Panel (a) presents the data and the approximation of labor productivity. Panel (b) displays the estimated path for the production externality, using the implied distance to the frontier and the average of the estimates of γ_i^0 and γ_i^1 from Table B.2. Further details are provided in Appendix B.1.

3.2. Calibration

Standard parameters. We use the general preferences in (1)-(3) for the quantitative analysis. Table 1 reports the values used in our baseline model parameterization. Our model

[7] We obtain similar quantitative results when we assume full convergence to the technological frontier (see Appendix Table B.3).

features a subset of parameters that are standard in open-economy macroeconomic models. We set the coefficient of relative risk aversion to $\sigma = 1$; the elasticity of substitution between tradable and nontradable goods to $\eta = 0.8$; the weight of tradables in consumption, $\omega = 0.35$, to match China's average manufacturing share in output. We set the Frisch elasticity of labor supply to $\frac{1}{\nu} = 1$, and $\zeta = 1/\nu$ which gives separable labor supply preferences across sectors. Consistent with our externality estimation procedure we assume constant returns to scale at the firm level in production, $\alpha = 1$. We conduct an annual calibration with the gross international interest rate $R^* = 1.05$. We set the initial period of our exercise to 1980, when China's growth started to take off and set the initial level of foreign currency bonds $F_0^* = 0.04$ to match China's international-reserves-to-GDP in 1980. We calibrate households' subjective discount factor to $\beta = 0.98$ so that the laissez-faire competitive equilibrium features no foreign currency bond accumulation from 1980–2008, as a benchmark for our exchange rate industrial policy analysis. For all the quantitative exercises replicating the growth take-off period, we feed in the estimated productivity and externality processes given in Figure 4.

Validation. We validate the calibrated model by comparing it with data moments for China during the growth takeoff, as shown in Table 2. To do this, we feed in the path of foreign-currency bond holdings by the government during the transition to match the observed level of reserves (see Figure B.1). The model aligns well with the observed growth in output and consumption, as well as the dramatic increase in net exports, and it predicts a currency depreciation, although smaller than in the data.

3.3. The Effects of Exchange Rate Industrial Policy

Observed policy. In this section, we assess the macroeconomic effects of the observed exchange rate industrial policy, measured by the path of reserve accumulation. Appendix Figure B.1 shows China's reserve accumulation during the 1980–2008 period, which accelerated particularly after 2000. Table 3 reports the macroeconomic effects of this observed reserve accumulation policy for both the full period and the 2000—2008 subperiod. We evaluate these effects by comparing the model predictions under the observed policy to a

Table 1: Parameter values

Parameter		Value
Preferences		
Coefficient of relative risk aversion	σ	1
Discount factor	β	0.98
Frisch elasticity of labor supply	$1/\nu$	1
Tradable-nontradable elasticity	η	0.8
Disutility of labor	ϕ	0.02
Weight on tradables in CES	ω	0.35
Technology		
Returns to scale	α	1
Steady-state productivity relative to the technological frontier	φ	0.48
Initial productivity	A_0	0.02
Speed of convergence	ρ	0.04
Externality at the technological frontier	γ^0	0.14
Externality distance-to-frontier coefficient	γ^1	0.01
International asset markets		
Gross interest rate	R^*	1.05
Initial foreign currency bond position	F_0^*	0.04

Notes: This table presents the baseline parameter values used in applying our model to China's takeoff.

Table 2: Data and Model predictions under the Observed Reserves Policy

Average annual growth, %	Data	Model
Output	9.0	7.8
Consumption	7.5	7.1
Reserves	15.4	15.4
Net exports	16.0	18.6
Real exchange rate	4.9	2.4

Notes: This table shows the average growth for China from 1980–2008. The data for output, consumption, and net exports are per capita in constant local currency units. Consumption refers to household consumption expenditure, and reserves are per capita in constant USD. The model column represents the path of foreign-currency bond holdings that match the observed reserve accumulation policy and the estimated productivity and externality processes. Data sources: China National Bureau of Statistics, OECD, World Bank.

counterfactual scenario with no reserve accumulation.

We estimate a large impact of reserve accumulation on output, with an additional 1.6 percentage points of annual growth during the 2000—2008 period and 0.4 percentage points during the full period. This policy had a particularly strong effect on the tradable sector, where the impact was four times larger than in the aggregate economy and contributed to currency depreciation. The counterpart of this policy is a decline in consumption, which grew less than in the counterfactual economy with no reserve accumulation. Appendix Table B.3 shows that these conclusions are robust to alternative parameterizations of preferences and technology.

Table 3: Effects of Observed Reserves Policy Relative to No Reserves Accumulation

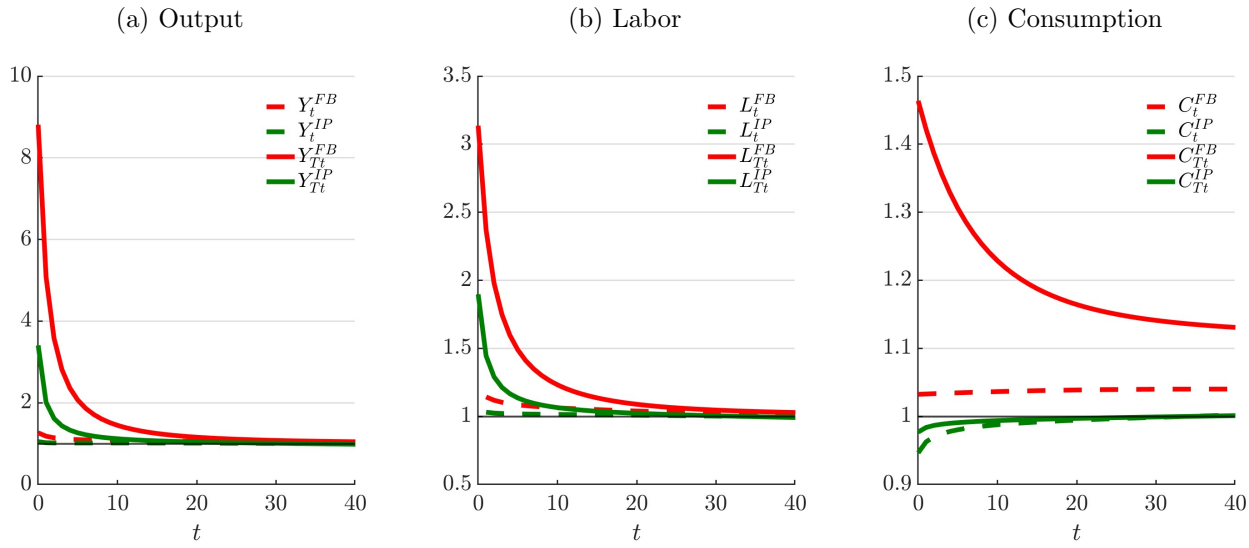
Average annual growth, p.p.		1980-2008	2000-2008
a. Output	Aggregate	0.4	1.6
	Tradable	1.6	6.1
b. Labor	Aggregate	0.3	1.2
	Tradable	0.8	3.3
c. Consumption	Aggregate	-0.2	-0.8
	Tradable	-0.1	-0.5
d. External sector	Reserves	15.4	31.1
	Net exports	26.2	60.8
	Real exchange rate	0.5	1.8

Notes: This table shows the difference in the average annual growth rate during China's growth takeoff for the observed reserve accumulation policy relative to constant reserves (no accumulation), measured in percentage points. Output, consumption, and net exports are measured in units of domestic consumption, while reserves are measured in tradables.

Optimal policy. In this section, we analyze the effects of the optimal exchange rate industrial policy. Figure 5 depicts the paths of output, labor, and consumption under the optimal exchange rate industrial policy relative to the laissez-faire competitive equilibrium in the first four decades of the transitional dynamics and compares them to the first-best

allocation. Under the optimal exchange rate industrial policy, aggregate output is 2.1 percent higher on average during the first decade of the policy, and labor is 2.3 percent higher. The expansionary effects on output and labor are smaller than in the first-best allocation due to the intertemporal cost associated with the exchange rate policy, which is reflected in a consumption path that is 2.5 percent lower relative to the competitive equilibrium in the first decade.

Figure 5: Allocations in the Transitional Dynamics: First-Best and Optimal Exchange Rate Industrial Policy Relative to Competitive Equilibrium

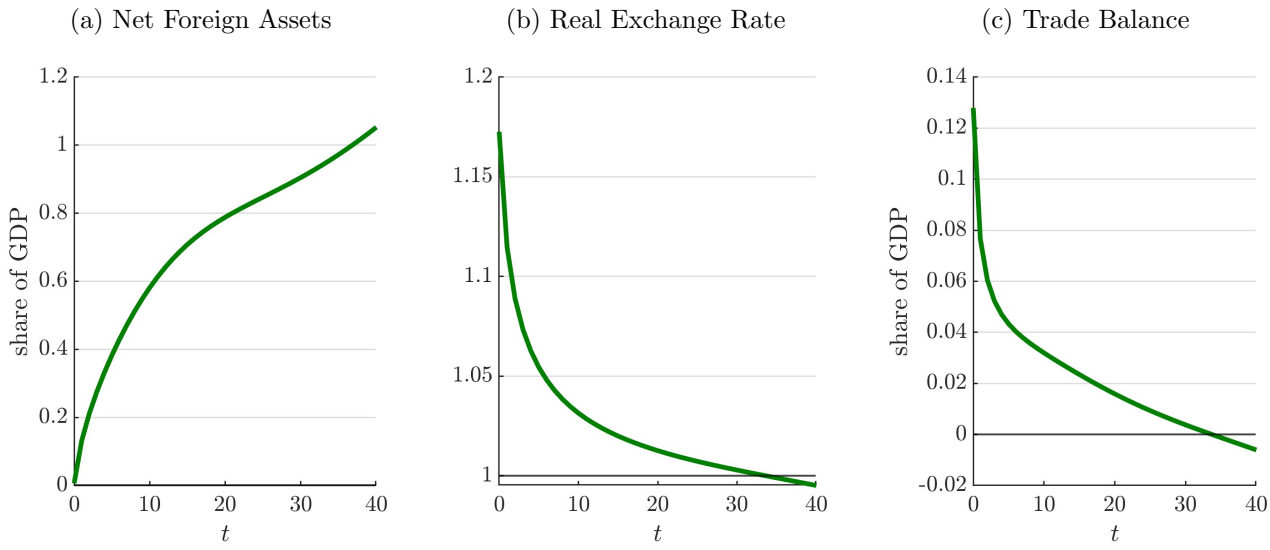


Notes: This figure shows the allocations for output, labor, and consumption under the first-best and optimal exchange rate industrial policy, expressed as deviations relative to the laissez-faire competitive equilibrium. First-best allocations are denoted by the superscript "FB," while optimal exchange rate industrial policy allocations are denoted by the superscript "IP." The horizontal axis measures the number of years since the initial period, 1980. Panel (a) shows output, Panel (b) shows labor, and Panel (c) shows consumption, both in aggregate and for the tradable sector.

Panel (a) of Figure 6 shows that the allocation under the exchange rate industrial policy is achieved through substantial reserve accumulation, reaching 55 percent of GDP in the first decade and later decelerating over the following decades. This policy keeps the currency undervalued by an average of 10.4 percent during the first decade and leads to "export-led growth," as reflected in a larger trade balance relative to the competitive equilibrium (see Panels (b) and (c)). Note that the timing of the optimal policy stands in contrast to the observed path of reserves that accelerated two decades after the beginning of the growth

takeoff.

Figure 6: Foreign Exchange Interventions Under the Optimal Exchange Rate Industrial Policy



Notes: The variables in Panels (a)–(c) represent the optimal exchange rate industrial policy, expressed as deviations relative to the laissez-faire competitive equilibrium. The horizontal axis measures the number of years since the initial period, 1980. Panels (a) and (c) show differences as a share of GDP, while Panel (b) presents the ratio. Figure B.2 provides these values for the first-best allocation.

In terms of welfare, the optimal exchange rate industrial policy increases welfare by 0.07% of permanent consumption, which is equivalent to one-fifth of the welfare gains from the first-best allocation (see Table 4).

We also analyze the effects of other commonly implemented industrial policies and their interaction exchange rate industrial policy. These include a constant subsidy to all labor and a subsidy to labor in the tradable sector.[8] We begin by analyzing the welfare effects of time-invariant implementation of these policies in isolation and combined with exchange rate industrial policy, reported in Table 4. The main takeaway from this analysis is that exchange rate industrial policy complements these time-invariant policies. While time-invariant policies generate large welfare gains by exploiting the average level of production externalities,

[8] Itskhoki and Moll (2019) provide a historical account of these types of policies in East Asian growth miracles and show their desirability when externalities arise from financial frictions. These policies include general labor market tools that affect real wages, such as restrictions on unionization and upper bounds on wage growth.

exchange rate industrial policy leverages their dynamic patterns. A combination of a time-invariant subsidy to tradable labor and exchange rate industrial policy achieves 80% of the first-best welfare gains.

Table 4: Welfare of Exchange Rate Industrial Policy and "Classic" Industrial Policies

Policy		Δ Welfare
a. XR-IP		0.07
b. First best		0.41
c. Other policy mixes	Labor subsidy	0.09
	Tradable labor subsidy	0.27
	Labor subsidy + XR-IP	0.15
	Tradable labor subsidy + XR-IP	0.35

Notes: Welfare is expressed in consumption-equivalent terms as the percentage increase in per-period consumption required to equate welfare with the laissez-faire competitive equilibrium in each case. The labor subsidy refers to the optimal constant uniform labor subsidy of 5.8% applied to both sectors. The tradable labor subsidy refers to the optimal constant labor subsidy of 17.1% applied only to the tradable sector.

Appendix Table B.4 studies simple time-varying implementation of labor subsidies through step functions, which introduce a temporary subsidy that is later removed or reduced. These policies achieve welfare gains similar to those attained by a policy mix of time-invariant policies and exchange rate industrial policies.

4. On the Efficiency of Exchange Rate Industrial Policy

In this section, we examine the factors that determine the effectiveness of using the exchange rate as an industrial policy from both a theoretical and quantitative perspective. We analyze the role of international capital mobility, labor market characteristics, production technologies and sectoral heterogeneity, and henceforth focus on converging economies.

4.1. International capital mobility

Model with global financial intermediaries. Consider an extension of the baseline model in which households can trade in international capital markets that operate imperfectly, as in Gabaix and Maggiori (2015). Suppose there is a unit measure of foreign financial intermediaries that engage in carry trade by buying and selling bonds in different currencies. Their aggregate balance sheet is given by $Q^*_{t+1} = -Q_{t+1}/\mathcal{E}_t$, where Q^*_{t+1} and Q_{t+1} are bonds purchased in period t in foreign and local currency, respectively. Their demand for local currency assets is given by

$$Q_{t+1} = \frac{1}{\Gamma_I}\left[\mathcal{E}_t - \frac{R^*}{R_{t+1}}\mathcal{E}_{t+1}\right], \tag{27}$$

where $\Gamma_I \geq 0$ is a measure of intermediaries' risk-bearing capacity.[9] When $\Gamma_I = 0$, there is free capital mobility, and the equilibrium features the UIP condition. In this case of a completely open capital account, FX interventions become ineffective because households undo them by trading with the rest of the world in a frictionless manner. At the other extreme, when $\Gamma_I \to \infty$, no intermediation is possible, and the model collapses to the baseline model. The market-clearing condition for domestic currency bonds is given by

$$F_{t+1} + B_{t+1} + Q_{t+1} = 0,$$

and the balance of payments condition is

$$C_{Tt} - A_t L_{Tt}^{\alpha+\gamma_{Tt}} = R^* F^*_t - F^*_{t+1} + \frac{Q_{t+1}}{\mathcal{E}_t} - R_t \frac{Q_t}{\mathcal{E}_t}.$$

In this setup, we show the following theoretical result. All the theoretical results in this section are derived under the parametric assumptions of Section 2.

***Proposition* 5** (Exchange rate industrial policy with international capital mobility)**.** *Consider the economy with international intermediaries in the initial period. Suppose that the*

[9] This demand arises from an optimization problem of intermediaries that maximize next period's profits subject to an incentive compatibility constraint. See Gabaix and Maggiori (2015) for further details.

economy starts below its steady-state level of productivity and converges to it in the next period (i.e., $A_0 < \varphi \overline{A}$ and $\rho = 1$ with $\gamma_{T0} > \gamma_{Tt} = 0$ for $t \geq 1$). The optimal exchange rate industrial policy ("IP-B") implies:

$$\mathcal{E}_0^{IP} > \mathcal{E}_0^{IP-B} > \mathcal{E}_0^{CE}, \quad L_{T0}^{IP} > L_{T0}^{IP-B} > L_{T0}^{CE}, \quad C_{T0}^{IP} < C_{T0}^{IP-B} < C_{T0}^{CE}, \quad CA_0^{IP} > CA_0^{IP-B} > CA_0^{CE}.$$

In an economy with intermediaries, the social planner faces an additional cost from distorting intertemporal consumption choices. This distortion creates a wedge between the returns in local and foreign currency, which intermediaries exploit through carry trades to extract positive profits from the economy. The planner's optimal response to this environment is to reduce the strength of exchange rate interventions.

We also conduct a quantitative analysis of this model extension. Appendix C.1 provides further details on the model calibration, which is disciplined with data on private capital flows to China during the growth takeoff. Table 5 shows that the observed reserve accumulation during the 2000—08 period increased the output growth rate by 1.5 p.p., which is similar to the estimated effect in the model without foreign intermediaries.

Capital controls as an industrial policy. In this section, we show that capital controls can be used in a time-varying manner to replicate the allocations of exchange rate industrial policy. This occurs because both capital controls and FX interventions affect the economy by creating a distortion in the household's Euler equation.

Consider a simplified version of the model in which households trade only foreign currency bonds with the rest of the world. Suppose further that the government has access to a capital control policy in the form of a time-specific tax on households' savings and borrowing, τ_t^B. The following result establishes an equivalence between the allocations attained under the optimal exchange rate industrial policy in the baseline model and those in this economy with optimal capital controls.

***Proposition* 6** (Capital controls as an industrial policy)**.** *Consider a model variant in which households can save or borrow in foreign currency, and the government can impose a capital*

Table 5: Effectiveness of Observed Reserves Policy 2000-2008

	Output		Labor		Consumption		Ext. sector	
Average annual growth, p.p.	Agg.	Trad	Agg.	Trad	Agg.	Trad	NX	RXR
a. Baseline	1.6	6.1	1.2	3.3	-0.8	-0.5	60.8	1.8
b. Int. capital mobility	1.5	4.0	1.0	2.1	-0.5	-0.3	11.0	1.2
c. Labor mkt characteristics								
High Frisch elasticity	1.8	6.0	1.3	3.6	-0.7	-0.4	60.2	1.4
Low Frisch elasticity	1.5	6.2	1.1	2.9	-1.1	-0.6	61.6	2.3
d. Production technology								
Foreign capital	1.2	2.5	0.8	1.4	-0.6	-0.5	57.5	0.4
Domestic capital	-1.6	-1.6	-1.1	-1.1	0.5	0.4	74.2	-0.3
Imported inputs	1.5	5.0	1.1	2.8	-0.8	-0.5	59.9	1.4
e. Sectoral structure								
Multiple sectors	1.6	7.0	3.1	3.7	-0.8	-0.4	60.7	2.2
Nontradable externality	1.6	5.9	1.2	3.2	-0.9	-0.5	60.0	1.8
Home-foreign goods	1.1	8.6	1.6	5.4	-2.8	-1.4	69.5	9.2

Notes: This table shows the difference in the average annual growth rate for China's observed reserve accumulation policy relative to constant reserves (no accumulation), measured in percentage points for the period 2000–2008 across each model variant. Output, consumption, and net exports are measured in units of domestic consumption. Output refers to domestic value added, while reserves are measured in tradables. In Panel (c), high (low) Frisch labor supply elasticity corresponds to $\frac{1}{\nu} = \frac{4}{3}$ ($\frac{1}{\nu} = \frac{3}{4}$).

control. The allocations under the optimal exchange rate industrial policy can be attained by imposing the following time-varying capital control:

$$\tau_t^B = \frac{\theta(\mathbf{x}_{t+1}, \gamma_{Tt+1})}{\theta(\mathbf{x}_t, \gamma_{Tt})} - 1.$$

This equivalence result arises because, in both economies, the government can control the intertemporal allocation of consumption through FX intervention in the baseline model and capital controls in this model. Therefore, the allocations under both optimal policies coincide. This result echoes Farhi, Gopinath and Itskhoki (2014), who show that exchange

rate devaluations can be replicated with a combination of fiscal tools.

We also analyze quantitatively the optimal path of capital controls in our baseline model calibration. Appendix C1 presents the sequence of taxes on savings that replicate the allocation of the optimal exchange rate industrial policy during the transitional dynamics.

Finally, an alternative policy approach is to regulate the capital account by restricting private capital flows in a time-invariant manner and complementing this policy with exchange rate industrial policy through foreign exchange interventions. In the context of the model extension with financial intermediaries, this can be achieved by taxing the profits from carry trade. As shown in Gabaix and Maggiori (2015), a government tax on intermediaries' profits, τ^{π}, is equivalent to considering an alternative economy with lower risk-bearing capacity from intermediaries: $\Gamma_I^{eff} = \frac{\Gamma_I}{(1-\tau^{\pi})}$. This result implies that the government can effectively eliminate the cost of foreign exchange intervention by taxing the profits of intermediaries engaged in carry trade. In this sense, this time-invariant capital flow policy complements the use of exchange rate industrial policy. This result helps explain the joint use of capital account restrictions and exchange rate interventions by China during its growth process (Clayton *et al.*, 2022).

4.2. Labor market characteristics

This section examines the relevance of labor mobility for the desirability and effectiveness of exchange rate industrial policy. Consider now a generalization of the parametric assumptions of Section 2, where preferences are given by

$$\sum_{t=0}^{\infty} \beta^t \left[\frac{{C_t}^{1-\sigma}}{1-\sigma} - \phi \frac{L_{Tt}^{1+\nu}}{1+\nu} - \phi \frac{L_{Nt}^{1+\nu}}{1+\nu} \right], \tag{28}$$

where ν^{-1} is the Frisch elasticity of sector-specific labor supply.[10] In this setup, we establish the following result.

Proposition 7 (Labor supply elasticity)**.** *Consider a version of the model with preferences*

[10]This corresponds to the case where $\zeta = \nu^{-1}$. See Berger, Herkenhoff and Mongey (2022) for an example of similar preferences in the context of firm-specific labor supply.

given by (28). *The optimal exchange rate industrial policy implies the following initial allocations:*

$$\frac{\mathcal{E}_0^{IP}}{\mathcal{E}_0^{CE}} > 1, \quad \frac{L_{T0}^{IP}}{L_{T0}^{CE}} > 1, \quad \frac{C_{T0}^{CE}}{C_{T0}^{IP}} > 1, \quad \frac{CA_0^{IP}}{CA_0^{CE}} > 1, \quad \frac{F_1^{*IP}}{F_1^{*CE}} > 1,$$

and all these ratios increase with the elasticity of labor supply, ν^{-1}.

This proposition states two key results. First, the same policy prescriptions hold in this more general setup. Second, the strength of the optimal intervention and its effects on exchange rate depreciation in the initial periods are larger when labor supply is more elastic. When labor supply is highly elastic, the government can more effectively induce higher employment in the tradable sector and exploit the production externality during the early stages of convergence, when it is strongest.

Table 5 shows the quantitative relevance of labor supply elasticity. The effect of the observed reserve accumulation on output growth is 1.8 percentage points in a model parameterization with a Frisch elasticity of 1.25, compared to 1.5 percentage points when the Frisch elasticity is 0.75.

The case of fixed labor supply. In Appendix C.3.2, we also study the case of fixed aggregate labor supply and show that the optimal exchange rate industrial policy shares the same characteristics as the baseline model. In this model, the optimal policy does not feature a supply stimulation channel and affects allocations only through the reallocation of sectoral labor demand. This case emphasizes that the main results of the paper do not rely on the presence of wealth effects in labor supply.

4.3. Production technology

Economies with capital. Consider an economy where capital is an input for production. In this variant of the model, firms produce according to the following technology:

$$y_{it} = A_t \left(L_{it}^{1-\theta_i} K_{it}^{\theta_i}\right)^{\gamma_{it}} \left(l_{it}^{1-\theta_i} k_{it}^{\theta_i}\right)^{\alpha}, \tag{29}$$

where k_{it} is the capital employed by a firm in sector i at time t, K_{it} is the aggregate level of capital, and θ_i is the capital share, which we allow to be sector-specific. We show below that whether the main features of the optimal policy are preserved in this economy depends on whether capital is accumulated by foreign or domestic investors.

We first focus on the model where foreign investors accumulate capital. In this version of the model, firms can rent capital at the rental rate $r_{kt} = R^* + \delta - 1$, expressed in tradable goods. Households and the government face the same problems as in the baseline model. Appendix C.4 outlines the implementability conditions of this model. We can analytically characterize the optimal policy for the case in which $\theta_N = 0$, which allows for separability between the tradable and nontradable blocks of the model; we relax this assumption in the quantitative analysis. The following proposition characterizes the optimal policy for converging economies in this case.

***Proposition* 8** (Economy with foreign capital)**.** *In the economy where foreign investors accumulate capital, if $\theta_N = 0$, the optimal exchange rate industrial policy exhibits the same dynamic properties as in the baseline model. Initially, it features a depreciated exchange rate that gradually appreciates over time relative to its laissez-faire value.*

The optimal policy is qualitatively similar to that in the baseline model. An initially depreciated exchange rate makes labor cheaper for tradable firms, which increases their demand for labor and capital, bringing allocations closer to those in the first-best.

We also analyze the quantitative relevance of this model extension with foreign capital beyond the analytical case in Proposition 8. Appendix C.4 describes the calibration of this model. Our estimates indicate that observed reserve accumulation stimulated output growth by 1.2 percentage points, a magnitude similar to that in our baseline model without capital (see Table 5).

Next, we consider the model in which capital is accumulated domestically. All other features of the model remain the same as in the previous version. In this model variant, domestic agents can accumulate capital, and the supply of capital is determined by an equal return condition between the return on capital and the return on local currency bonds, $r_{kt} = R_t + \delta - 1$. This introduces an additional cost to exchange rate industrial policy.

When the government initially accumulates reserves and issues local currency bonds, the exchange rate depreciates, and the rate of return on local currency bonds increases. Because agents now face a higher interest rate, the capital supply decreases so that it yields a higher return in equilibrium. This additional cost is traded off against the benefits of exchange rate industrial policy.

We also analyze the quantitative relevance of the model extension with domestic capital formation. Appendix C.4 describes the calibration of this model. In this case, we estimate that the observed policy decreases output growth by 1.6 percentage points (see Table 5), indicating that the output costs of postponing domestic investment by increasing domestic rates of return are quantitatively large. This analysis highlights that an essential ingredient of exchange rate industrial policy is allowing capital to be accumulated by foreign investors.

Economies with imported inputs. Consider now an economy where production relies on imported inputs. In this variant, firms produce according to the following technology:

$$y_{it} = A_t \left(L_{it}^{1-\xi_i} M_{it}^{\xi_i} \right)^{\gamma_{it}} \left(l_{it}^{1-\xi_i} m_{it}^{\xi_i} \right)^{\alpha}, \tag{30}$$

where m_{it} (M_{it}) represents the individual (aggregate) imported inputs used in sector i at time t, and ξ_i denotes the imported input share. The price of imported inputs in foreign currency is given by P_{Mt}^*. The following proposition shows that this model is equivalent to the model with foreign capital.

***Proposition* 9.** *The model with imported inputs has an isomorphic setup to the one with foreign capital, where k_{it} is replaced by m_{it}, r_{kt} is replaced by P_{Mt}^*, and θ_i is replaced by ξ_i.*

A corollary of this proposition is that the same qualitative features of the optimal exchange rate industrial policy are preserved in the empirically relevant case of a model with imported inputs in the tradable sector, where $\xi_T > \xi_N = 0$.

We then quantitatively analyze the case of imported inputs using a calibration that includes imported inputs in both the tradable and nontradable sectors. Appendix C.5 provides details on the calibration of this model extension. Table 5 shows that the observed policy

stimulated output growth by 1.5 percentage points in the model with imported inputs, a slightly lower effect than that observed in the baseline model.

4.4. Sectoral structure

Multiple tradable sectors. As discussed in Section 3.1, tradable sectors can differ substantially in terms of their Marshallian externalities. In this section, we characterize the optimal exchange rate industrial policy in the presence of sectoral heterogeneity. For simplicity, we consider a variant of the model with two tradable sectors, $j = 1, 2$. To allow for heterogeneity in labor supply elasticities across sectors, we assume that preferences are given by

$$\sum_{t=0}^{\infty} \beta^t \left[\frac{{C_t}^{1-\sigma}}{1-\sigma} - \phi \frac{L_{T1t}^{1+\nu_1}}{1+\nu_1} - \phi \frac{L_{T2t}^{1+\nu_2}}{1+\nu_2} - \phi \frac{L_{Nt}^{1+\nu_N}}{1+\nu_N} \right], \tag{31}$$

where the consumption good is given by the composite of tradable and nontradable aggregators (2), and tradable consumption is an aggregator of tradable-sector varieties,

$$C_{Tt} = C_{T1t}^{1/2} C_{T2t}^{1/2}.$$

In this environment, we establish the following sufficient statistic result. Define $\epsilon_t = \log(\mathcal{E}_t) - \log(\tilde{\mathcal{E}}_t)$ as the log deviation from the first-best exchange rate, and $\psi_{jt} = \log \alpha - \log(\alpha + \gamma_{Tjt}) \leq 0$.

***Proposition* 10** (Multiple sectors)**.** *In both the single-tradable-sector model and the multiple-tradable-sector model, the optimal exchange rate industrial policy follows the same law of motion:*

$$(1+D)\epsilon_t + D\psi_t = (1+D)\epsilon_{t+1} + D\psi_{t+1},$$

where $D > 0$ and ψ_t are model-specific. In the single-tradable-sector model,

$$D = \frac{(\alpha+\gamma)^2 + (1+\nu)(\alpha+\gamma)}{(\alpha+\gamma-1-\nu)^2}, \quad \psi_t = \log\alpha - \log(\alpha+\gamma_{Tt}).$$

In the multiple-tradable-sector model,

$$D = \frac{1}{2}\left[D_1 + D_2\right], \quad \psi_t = \frac{D_1}{D_1+D_2}\psi_{1t} + \frac{D_2}{D_1+D_2}\psi_{2t},$$

where the sector weights are given by

$$D_j = \frac{(\alpha+\gamma)^2 + (1+\nu_j)(\alpha+\gamma)}{(\alpha+\gamma-1-\nu_j)^2}, \quad \textit{for } j = 1, 2.$$

This proposition shows that the optimal exchange rate industrial policy in the model with multiple tradable sectors behaves similarly to the baseline model. It approximately follows the path of a weighted average of the production externalities of both tradable sectors. In addition, the optimal policy places greater weight on sectors with more elastic labor supply, as stronger externalities in these sectors can be exploited more effectively by the policy.[11]

We also analyze the case of multiple sectors quantitatively. For this, we use our sectoral externalities estimates from Section 3 and data on sectoral sizes of China to calibrate the model. Appendix C.6 provides further details on the model calibration. Table 5 summarizes the quantitative results of this model extension, which show that the estimated output effects of the observed reserve accumulation policy are similar to those in the baseline model with a single tradable sector.

Externalities in the non-tradable sector. Consider the baseline model extended to include externalities in the non-tradable sector, $\gamma_{NTt} > 0$, in addition to those in the tradable sector. In this case, the optimal policy path depends on assumptions about preferences and labor supply. Under the baseline parameterization of Assumption 1, the optimal policy coincides with that in the economy without non-tradable externalities. In particular, the

[11] See Palazzo (2024) for evidence on heterogeneous responses of sectors to currency undervaluation.

optimal policy in that case is independent of the path of non-tradable externalities. This occurs because the negative effect of the policy on the demand for non-tradables exactly offsets its positive effect on labor supply, implying a non-tradable equilibrium allocation that is independent of intertemporal considerations. Outside the baseline parameterization, the optimal policy will depend on the path of externalities in the tradable and non-tradable sectors. If the latter have stronger dynamic effects than the former, the equilibrium policy may imply opposite dynamics, i.e., currency appreciation in the initial stages.

To assess the quantitative relevance of non-tradable externalities, we consider a case in which the non-tradable sector follows the same path of externalities estimated for the tradable sector. Table 5 shows that the policy's effects on output are similar to those in our baseline model, indicating that the theoretical parametric assumptions used in the theoretical characterization are close to a quantitatively realistic calibration.

Home-foreign goods. We also consider an extension with home and foreign goods, where the rest of the world features a downward-sloping demand for the home good. This extension incorporates the assumption of differentiated home goods used in Bartelme *et al.* (2019) to estimate sector externalities. Appendix C.6 provides details on the model calibration. Table 5 shows that the estimated quantitative effects of the observed policy are similar to those in the baseline model with an undifferentiated tradable good.

4.5. A discussion of historical experiences

Our framework can be used to interpret historical experiences with exchange rate interventions aimed at accelerating economic development. Emblematic cases of these policies include the Asian growth miracles and, more recently, the Chinese growth process (see, for example, Page, 1994; Song *et al.*, 2014). Through the lens of our model, these economies arguably met the necessary condition for the desirability of such policies—namely, undergoing a convergence process. In addition, they exhibited two characteristics that, in our model, make these policies more effective and desirable. First, the policies were conducted in environments with capital account interventions and initially underdeveloped financial markets,

which made FX interventions more effective in influencing exchange rates. Second, these economies had a high degree of labor mobility across sectors. In the salient case of China, there was significant labor migration from the local rural sector to the urban manufacturing sector (see Cai, 2016, for a discussion of the demographic dividend in China).

The characteristics of the Asian examples stand in contrast to those of Latin American experiences, which are often referenced as failures of these types of policies. Most Latin American economies did not experience convergence processes and were specialized in commodity sectors that, as discussed in Section 3.1, are less clearly associated with production externalities. They also faced greater costs in sectoral labor reallocation and had a more open capital account. Through the lens of our model, this configuration renders exchange rate industrial policy less effective and desirable. Moreover, in Latin America, these policies were often implemented through monetary policy aimed at targeting an undervalued real exchange rate (see, for example, Calvo, Reinhart and Vegh, 1995). While such policies can address temporary exchange rate misalignments, they may also generate higher inflation and instability if used to target the exchange rate in the medium run (see, for example, Uribe, 2003). In contrast, our framework highlights sterilized foreign exchange interventions as a more appropriate policy tool for targeting the real exchange rate in the medium run.

5. Conclusions

In this paper, we studied how exchange rate policies can be used to accelerate economic development. Our quantitative framework indicates that the effects of these policies can be substantial and highlights two key conditions for their desirability. First, production externalities must be dynamic, which can occur when economies undergo convergence processes. Second, international capital mobility must be imperfect, either due to underdeveloped financial markets or policies regulating the capital account. Our analysis also emphasizes that these policies may be ineffective or undesirable in economies that fail to meet either of these conditions. These context-based policy prescriptions align with the discussion of historical experiences.

Although our analysis has focused on policies from the perspective of individual economies, our framework can be extended to study interactions in the global economy. An interesting application in this regard is the idea of "currency wars," which gained prominence during China's growth takeoff. Our framework could be used to examine the extent to which these global dynamics arise as a result of multiple economies attempting to exploit the dynamic patterns of production externalities. We leave the study of these global interactions for future research.

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A. Baseline theoretical model appendix

A.1. Proof of Proposition 1: Impossibility result

We show the proposition for more general preferences $\sum_{t=0}^{\infty}\beta^t\left[\frac{{C_t}^{1-\sigma}}{1-\sigma}-\phi\frac{L_t^{1+\nu}}{1+\nu}\right]$, where $\zeta=\frac{1}{\nu}$.

Given the initial foreign currency asset position F_0^*, the conditions that characterize the competitive equilibrium allocation $\{C_{Tt},C_{Nt},L_{Tt},L_{Nt},F_{t+1}^*\}_{t=0}^{\infty}$ are

$$\left(\frac{(1-\omega)}{\omega}\frac{C_{Tt}}{C_{Nt}}\right)^{\frac{1}{\eta}}=\frac{L_{Tt}^{\alpha+\gamma_{Tt}-1-\nu}}{L_{Nt}^{\alpha-1-\nu}}, \tag{A.1}$$

$$\frac{\phi L_{Tt}^{\nu}}{(\omega/C_{Tt})^{\frac{1}{\eta}}\,C_t^{\frac{1}{\eta}-\sigma}}=\alpha A_t L_{Tt}^{\alpha+\gamma_{Tt}-1}, \tag{A.2}$$

$$\left(\frac{\omega}{C_{Tt}}\right)^{\frac{1}{\eta}}C_t^{\frac{1}{\eta}-\sigma}=\beta R_{t+1}\frac{L_{Nt}^{\alpha-1-\nu}/L_{Tt}^{\alpha+\gamma_{Tt}-1-\nu}}{L_{Nt+1}^{\alpha-1-\nu}/L_{Tt+1}^{\alpha+\gamma_{Tt+1}-1-\nu}}\left(\frac{\omega}{C_{Tt+1}}\right)^{\frac{1}{\eta}}C_{t+1}^{\frac{1}{\eta}-\sigma}, \tag{A.3}$$

$$C_{Nt}=A_tL_{Nt}^{\alpha},$$

$$C_{Tt}-A_tL_{Tt}^{\alpha+\gamma_{Tt}}=R^*F_t^*-F_{t+1}^*,$$

where in equation (A.3) we substitute $P_{Tt}=\frac{L_{Nt}^{\alpha-1-\nu}}{L_{Tt}^{\alpha+\gamma_{Tt}-1-\nu}}$ into (5) after normalizing $P_{Nt}\equiv 1$ without loss of generality, and combine firms' labor demand from (10).

The conditions that characterize the first-best allocation $\{\tilde{C}_{Tt},\tilde{C}_{Nt},\tilde{L}_{Tt},\tilde{L}_{Nt},\tilde{F}_{t+1}^*\}_{t=0}^{\infty}$ are

$$\left(\frac{(1-\omega)}{\omega}\frac{\tilde{C}_{Tt}}{\tilde{C}_{Nt}}\right)^{\frac{1}{\eta}}=\left(\frac{\alpha+\gamma_{Tt}}{\alpha}\right)\frac{\tilde{L}_{Tt}^{\alpha+\gamma_{Tt}-1-\nu}}{\tilde{L}_{Nt}^{\alpha-1-\nu}}, \tag{A.4}$$

$$\frac{\phi\tilde{L}_{Tt}^{\nu}}{\left(\omega/\tilde{C}_{Tt}\right)^{\frac{1}{\eta}}\tilde{C}_t^{\frac{1}{\eta}-\sigma}}=(\alpha+\gamma_{Tt})A_t\tilde{L}_{Tt}^{\alpha+\gamma_{Tt}-1}, \tag{A.5}$$

$$\left(\frac{\omega}{\tilde{C}_{Tt}}\right)^{\frac{1}{\eta}}\tilde{C}_t^{\frac{1}{\eta}-\sigma}=\beta R^*\left(\frac{\omega}{\tilde{C}_{Tt+1}}\right)^{\frac{1}{\eta}}\tilde{C}_{t+1}^{\frac{1}{\eta}-\sigma}, \tag{A.6}$$

$$\tilde{C}_{Nt}=A_t\tilde{L}_{Nt}^{\alpha},$$

$$\tilde{C}_{Tt}-A_t\tilde{L}_{Tt}^{\alpha+\gamma_{Tt}}=R^*\tilde{F}_t^*-\tilde{F}_{t+1}^*.$$

Observe that satisfying the first-best intertemporal optimality condition (A.6) in the

competitive equilibrium (A.3) requires government foreign exchange intervention $\{F^*_{t+1}\}_{t=0}^{\infty}$ such that $R_{t+1} = R^* \frac{L_{Nt+1}^{\alpha-1-\nu}/L_{Tt+1}^{\alpha+\gamma_{Tt+1}-1-\nu}}{L_{Nt}^{\alpha-1-\nu}/L_{Tt}^{\alpha+\gamma_{Tt}-1-\nu}}$ for all t. Further, equations (A.4)–(A.5) for the first best and (A.1)–(A.2) in the competitive equilibrium only coincide if $\gamma_{Tt} = 0$ for all t. Therefore, in the presence of production externalities, $\gamma_{Tt} > 0$ for some t, then the first-best allocation cannot be achieved in the competitive equilibrium for any $\{F^*_t\}_{t=0}^{\infty}$.

A.2. Proof of Proposition 2: Time-sector specific subsidies

We show the proposition for more general preferences $\sum_{t=0}^{\infty}\beta^t\left[\frac{C_t^{1-\sigma}}{1-\sigma} - \phi\frac{L_t^{1+\nu}}{1+\nu}\right]$, where $\zeta = \frac{1}{\nu}$.

With tradable and nontradable sector-specific labor subsidies τ^L_{it}, the firms' problem for each sector $i \in \{T, N\}$ is

$$\max_{l_{it}} \quad \pi_{it} = P_{it}A_t l_{it}^{\alpha} L_{it}^{\gamma_{it}} - (1-\tau^L_{it})W_{it}l_{it}.$$

Firms profit maximization for labor demand in each sector l_T and l_N give

$$\alpha A_t L_{Tt}^{\alpha+\gamma_{Tt}-1} = (1-\tau^L_{Tt})\frac{W_{Tt}}{P_{Tt}},$$
$$\alpha A_t L_{Nt}^{\alpha-1} = (1-\tau^L_{Nt})\frac{W_{Nt}}{P_{Nt}}.$$

The government budget constraint is

$$F_{t+1} + \mathcal{E}_t F^*_{t+1} + T_t + \tau^L_{Tt}W_{Tt}L_{Tt} + \tau^L_{Nt}W_{Nt}L_{Nt} = R_t F_t + \mathcal{E}_t R^* F^*_t,$$

which, combined with the household budget constraint and firms' profits, gives the balance of payments condition (16).

Given τ_{Tt}^L, τ_{Nt}^L, the conditions that characterize the competitive equilibrium are

$$\begin{aligned}
\left(\frac{(1-\omega)}{\omega}\frac{C_{Tt}}{C_{Nt}}\right)^{\frac{1}{\eta}} &= \frac{(1-\tau_{Nt}^L)}{(1-\tau_{Tt}^L)}\frac{L_{Tt}^{\alpha+\gamma_{Tt}-1-\nu}}{L_{Nt}^{\alpha-1-\nu}},\\
\frac{\phi L_{Tt}^{\nu}}{(\omega/C_{Tt})^{\frac{1}{\eta}}\, C_t^{\frac{1}{\eta}-\sigma}} &= \frac{1}{(1-\tau_{Tt}^L)}\alpha A_t L_{Tt}^{\alpha+\gamma_{Tt}-1},\\
\left(\frac{\omega}{C_{Tt}}\right)^{\frac{1}{\eta}} C_t^{\frac{1}{\eta}-\sigma} &= \beta R_{t+1}\frac{L_{Nt}^{\alpha-1-\nu}/L_{Tt}^{\alpha+\gamma_{Tt}-1-\nu}}{L_{Nt+1}^{\alpha-1-\nu}/L_{Tt+1}^{\alpha+\gamma_{Tt+1}-1-\nu}}\left(\frac{\omega}{C_{Tt+1}}\right)^{\frac{1}{\eta}} C_{t+1}^{\frac{1}{\eta}-\sigma},\\
C_{Nt} &= A_t L_{Nt}^{\alpha},\\
C_{Tt} - A_t L_{Tt}^{\alpha+\gamma_{Tt}} &= R^* F_t^* - F_{t+1}^*.
\end{aligned} \tag{A.7}$$

Setting

$$\begin{aligned}
\tau_{Tt}^L &= \frac{\gamma_{Tt}}{\alpha+\gamma_{Tt}},\\
\tau_{Nt}^L &= 0,
\end{aligned}$$

gives identical conditions to the first best (A.4)–(A.5) for the competitive equilibrium. Government policies $\left\{F_{t+1}, F_{t+1}^*, T_t\right\}_{t=0}^{\infty}$ can then be used to equate $R_{t+1} = R^*\frac{L_{Nt+1}^{\alpha-1-\nu}/L_{Tt+1}^{\alpha+\gamma_{Tt+1}-1-\nu}}{L_{Nt}^{\alpha-1-\nu}/L_{Tt}^{\alpha+\gamma_{Tt}-1-\nu}}$ for all t, so (A.7) in the competitive equilibrium is equivalent to first-best condition (A.6).

A.3. Optimal exchange rate industrial policy: General problem

We show the proposition for more general preferences $\sum_{t=0}^{\infty}\beta^t\left[\frac{C_t{}^{1-\sigma}}{1-\sigma} - \phi\frac{L_t^{1+\nu}}{1+\nu}\right]$, where $\zeta = \frac{1}{\nu}$.

In this section we solve the optimal exchange rate industrial policy problem (P1). After substituting the labor aggregator and nontradable goods market-clearing condition, the Lagrangian is

$$\mathcal{L} = \sum_{t=0}^{\infty} \beta^t \Bigg\{ \frac{C_t^{1-\sigma}}{1-\sigma} - \phi \frac{L_{Tt}^{1+\nu}}{1+\nu} - \phi \frac{\left((C_{Nt}/A_t)^{\frac{1}{\alpha}} \right)^{1+\nu}}{1+\nu}$$
$$+ \zeta_t \left[\frac{L_{Tt}^{\alpha+\gamma_{Tt}-1-\nu}}{(C_{Nt}/A_t)^{\frac{\alpha-1-\nu}{\alpha}}} - \left(\frac{(1-\omega)}{\omega} \frac{C_{Tt}}{C_{Nt}} \right)^{\frac{1}{\eta}} \right]$$
$$+ \mu_t \left[\alpha A_t L_{Tt}^{\alpha+\gamma_{Tt}-1-\nu} - \frac{\phi}{\omega^{\frac{1}{\eta}}} C_{Tt}^{\frac{1}{\eta}} \frac{1}{C_t^{\frac{1}{\eta}-\sigma}} \right]$$
$$+ \lambda_t \left[R^* F_t^* - F_{t+1}^* - C_{Tt} + A_t L_{Tt}^{\alpha+\gamma_{Tt}} \right] \Bigg\}.$$

The first-order conditions for $C_{Tt}, C_{Nt}, L_{Tt}, F_{t+1}^*$ are

$$\left(\frac{\omega}{C_{Tt}} \right)^{\frac{1}{\eta}} C_t^{\frac{1}{\eta}-\sigma} - \zeta_t \frac{1}{\eta} \left(\frac{1-\omega}{\omega} \frac{1}{C_{Nt}} \right)^{\frac{1}{\eta}} C_{Tt}^{\frac{1}{\eta}-1} - \mu_t \left[\frac{1}{\eta} \frac{\phi}{\omega^{\frac{1}{\eta}}} C_{Tt}^{\frac{1}{\eta}-1} \frac{1}{C_t^{\frac{1}{\eta}-\sigma}} - \left(\frac{1}{\eta} - \sigma \right) \phi \frac{1}{C_t^{-\sigma+1}} \right] = \lambda_t,$$

$$\left(\frac{1-\omega}{C_{Nt}} \right)^{\frac{1}{\eta}} C_t^{\frac{1}{\eta}-\sigma} - \frac{\phi}{\alpha} C_{Nt}^{\frac{1+\nu-\alpha}{\alpha}} A_t^{\frac{-1-\nu}{\alpha}} + \zeta_t \left[\frac{\nu+1-\alpha}{\alpha} C_{Nt}^{\frac{\nu+1-2\alpha}{\alpha}} \frac{L_{Tt}^{\alpha+\gamma_{Tt}-1-\nu}}{(1/A_t)^{\frac{\alpha-1-\nu}{\alpha}}} + \frac{1}{\eta} \left(\frac{(1-\omega)}{\omega} C_{Tt} \right)^{\frac{1}{\eta}} \frac{1}{C_{Nt}^{\frac{1}{\eta}+1}} \right]$$
$$+ \mu_t \left(\frac{1}{\eta} - \sigma \right) \frac{\phi}{\omega^{\frac{1}{\eta}}} C_{Tt}^{\frac{1}{\eta}} \frac{1}{C_t^{-\sigma+1}} \left(\frac{1-\omega}{C_{Nt}} \right)^{\frac{1}{\eta}} = 0,$$

$$\phi L_{Tt}^{\nu} - \zeta_t \frac{(\alpha + \gamma_{Tt} - 1 - \nu) L_{Tt}^{\alpha+\gamma_{Tt}-\nu-2}}{(C_{Nt}/A_t)^{\frac{\alpha-1-\nu}{\alpha}}} - \mu_t (\alpha + \gamma_{Tt} - 1 - \nu) \alpha A_t L_{Tt}^{\alpha+\gamma_{Tt}-\nu-2} = \lambda_t (\alpha + \gamma_{Tt}) A_t L_{Tt}^{\alpha+\gamma_{Tt}-1},$$

$$\lambda_t = \beta R^* \lambda_{t+1}.$$

Combining these expressions gives

$$\left(\frac{\omega}{C_{Tt}} \right)^{\frac{1}{\eta}} C_t^{\frac{1}{\eta}-\sigma} = \frac{\theta(\mathbf{x}_{t+1}, \gamma_{Tt+1})}{\theta(\mathbf{x}_t, \gamma_{Tt})} \beta R^* \left(\frac{\omega}{C_{Tt+1}} \right)^{\frac{1}{\eta}} C_{t+1}^{\frac{1}{\eta}-\sigma},$$

where

$$\theta(\mathbf{x}_t, \gamma_{Tt}) \equiv \frac{1}{T_t} + \frac{1}{T_t(\omega/C_{Tt})^{\frac{1}{\eta}}} \frac{1}{C_t^{\frac{1}{\eta}-\sigma}} \left\{ \frac{\phi}{(\alpha+\gamma_{Tt}-1-\nu)\alpha A_t} L_{Tt}^{2\nu-\alpha-\gamma_{Tt}+1} M_t \right.$$
$$\left. + \frac{S_t}{Q_t} \left[\left(\frac{1-\omega}{C_{Nt}} \right)^{\frac{1}{\eta}} C_t^{\frac{1}{\eta}-\sigma} - \frac{\phi}{\alpha} C_{Nt}^{\frac{1+\nu-\alpha}{\alpha}} A_t^{\frac{-1-\nu}{\alpha}} + \frac{\phi}{(\alpha+\gamma_{Tt}-1-\nu)\alpha A_t} L_{Tt}^{2\nu-\alpha-\gamma_{Tt}+1} N_t \right] \right\},$$

$$M_t \equiv \frac{1}{\eta} \frac{\phi}{\omega^{\frac{1}{\eta}}} C_{Tt}^{\frac{1}{\eta}-1} \frac{1}{C_t^{\frac{1}{\eta}-\sigma}} - \left(\frac{1}{\eta} - \sigma \right) \phi \frac{1}{C_t^{-\sigma+1}},$$

$$N_t \equiv \left(\frac{1}{\eta} - \sigma \right) \frac{\phi}{\omega^{\frac{1}{\eta}}} C_{Tt}^{\frac{1}{\eta}} \frac{1}{C_t^{-\sigma+1}} \left(\frac{1-\omega}{C_{Nt}} \right)^{\frac{1}{\eta}},$$

$$Q_t \equiv \left[\frac{1}{\alpha} \frac{A_t^{\frac{\nu+1-2\alpha}{\alpha}}}{C_{Nt}^{\frac{\alpha-1-\nu}{\alpha}}} N_t - \frac{\nu+1-\alpha}{\alpha} C_{Nt}^{\frac{\nu+1-2\alpha}{\alpha}} \frac{L_{Tt}^{\alpha+\gamma_{Tt}-1-\nu}}{(1/A_t)^{\frac{\alpha-1-\nu}{\alpha}}} - \frac{1}{\eta} \left(\frac{(1-\omega)}{\omega} C_{Tt} \right)^{\frac{1}{\eta}} \frac{1}{C_{Nt}^{\frac{1}{\eta}+1}} \right],$$

$$S_t \equiv \left[\frac{1}{\alpha} \frac{A_t^{\frac{\nu+1-2\alpha}{\alpha}}}{C_{Nt}^{\frac{\alpha-1-\nu}{\alpha}}} M_t - \frac{1}{\eta} \left(\frac{1-\omega}{\omega} \frac{1}{C_{Nt}} \right)^{\frac{1}{\eta}} C_{Tt}^{\frac{1}{\eta}-1} \right],$$

$$T_t \equiv \left[1 - \frac{(\alpha+\gamma_{Tt})}{(\alpha+\gamma_{Tt}-1-\nu)\alpha} L_{Tt}^{1+\nu} M_t + \frac{(\alpha+\gamma_{Tt})}{(\alpha+\gamma_{Tt}-1-\nu)\alpha} L_{Tt}^{1+\nu} N_t \right].$$

A.4. Proof of Lemma 1: Log-quadratic approximation

We first show that under Assumption 1, the optimal exchange rate industrial policy (XR-IP) problem is independent of the nontradables block $\{C_{Nt}, L_{Nt}\}_{t=0}^{\infty}$. To see this for the more general case that allows for externalities in both the tradable and nontradable goods sectors, we have the two constraints

$$\left(\frac{\omega}{C_{Tt}} \right) = \phi \frac{1}{\alpha A_t L_{Tt}^{\alpha+\gamma_{Tt}-1}},$$
$$\left(\frac{\omega}{C_{Tt}} \right) = \frac{\left(\frac{C_{Nt}}{A_t} \right)^{\frac{\alpha+\gamma_{Nt}-1}{\alpha+\gamma_{Nt}}}}{L_{Tt}^{\alpha+\gamma_{Tt}-1}} \left(\frac{1-\omega}{C_{Nt}} \right).$$

Combining these two equations gives

$$\frac{\phi}{\alpha A_t} = \left(\frac{C_{Nt}}{A_t}\right)^{\frac{\alpha+\gamma_{Nt}-1}{\alpha+\gamma_{Nt}}} \left(\frac{1-\omega}{C_{Nt}}\right),$$
$$\Rightarrow C_{Nt} = A_t \left[\frac{\alpha(1-\omega)}{\phi}\right]^{\alpha+\gamma_{Nt}}, \tag{A.8}$$

and the nontradable goods market-clearing condition $C_{Nt} = A_t L_{Nt}^{\alpha+\gamma_{Nt}}$ determines L_{Nt}, which shows that the nontradables block $\{C_{Nt}, L_{Nt}\}_{t=0}^{\infty}$ is exogenous for this analytical case.

The XR-IP problem is then to solve for the tradables block C_{Tt}, L_{Tt}, and F_{t+1}^*

$$\max_{\{C_{Tt}, L_{Tt}, F_{t+1}^*\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \left[\omega \log C_{Tt} - \phi L_{Tt}\right] + constant$$
$$s.t. \quad \left(\frac{\omega}{C_{Tt}}\right) = \phi \frac{1}{\alpha A_t L_{Tt}^{\alpha+\gamma_{Tt}-1}},$$
$$C_{Tt} - A_t L_{Tt}^{\alpha+\gamma_{Tt}} = R^* F_t^* - F_{t+1}^*,$$
$$F_0^* \text{ given.}$$

To derive the approximation of this XR-IP problem, we first define the reference balanced trade (BT) allocation $\{\overline{C}_T, \overline{L}_T\}$ by

$$\overline{C}_T = \overline{Y}_T = \overline{A}\ \overline{L}_T^{\alpha+\gamma},$$
$$\frac{\phi}{\left(\omega/\overline{C}_T\right)} = (\alpha+\gamma)\overline{A}\ \overline{L}_T^{\alpha+\gamma-1},$$

where γ is defined below. Therefore, in the BT allocation

$$\overline{L}_T = \frac{\omega(\alpha+\gamma)}{\phi}.$$

To approximate the welfare function, we take a second-order approximation of the welfare function around the BT allocation

$$\mathbb{W}_0 = \sum_{t=0}^{\infty} \beta^t \left[\omega \log C_{Tt} - \phi L_{Tt}\right].$$

A second-order Taylor expansion for the tradable consumption term around the BT is

$$\omega \log C_{Tt} = \omega c_{Tt} + \omega \log \overline{C}_T,$$

where $c_{Tt} \equiv \log C_{Tt} - \log \overline{C}_T$, and similarly for l_{Tt}, y_{Tt}. A second-order Taylor expansion for the tradable labor term around the BT is

$$-\phi L_{Tt} = -\phi \overline{L}_T e^{l_{Tt}} = \ -\phi \overline{L}_T - \phi \overline{L}_T l_{Tt} - \frac{1}{2}\phi \overline{L}_T l_{Tt}^2$$

Therefore, welfare in terms of deviations, ignoring terms independent of c_{Tt} and l_{Tt}, is

$$\mathbb{W}_0 = \sum_{t=0}^{\infty} \beta^t \left[\omega c_{Tt} - \phi \overline{L}_T (l_{Tt} - \frac{1}{2} l_{Tt}^2) \right],$$

and similarly for the first best (FB) approximated around the BT. We now approximate the resource constraint relative to the BT. A first-order approximation of the LHS is

$$c_{Tt} - a_t - (\alpha + \gamma) l_{Tt} - (\gamma_{Tt} - \gamma) \log \overline{L}_T = R^* f_t^* - f_{t+1}^*,$$

where $a_t \equiv \log A_t - \log \overline{A}$, $f_t^* \equiv \frac{F_t^*}{\overline{Y}_T}$. For welfare, a second-order approximation of the LHS around the BT is

$$\begin{aligned} c_{Tt} + \frac{1}{2} c_{Tt}^2 - a_t - \frac{1}{2} a_t^2 - (\alpha + \gamma) l_{Tt} - \frac{1}{2}(\alpha + \gamma)^2 l_{Tt}^2 \\ -(\gamma_{Tt} - \gamma) \log \overline{L}_T - \frac{1}{2}(\gamma_{Tt} - \gamma)^2 (\log \overline{L}_T)^2 = R^* f_t^* - f_{t+1}^*. \end{aligned}$$

Iterating this forward and using the transversality condition $\lim_{s \to \infty} \beta^s f_{t+s}^* = 0$,

$$\begin{aligned} &\sum_{t=0}^{\infty} \beta^t c_{Tt} \\ &= \ -\sum_{t=0}^{\infty} \beta^t \left[\frac{1}{2} c_{Tt}^2 - a_t - \frac{1}{2} a_t^2 - (\alpha + \gamma) l_{Tt} - \frac{1}{2}(\alpha + \gamma)^2 l_{Tt}^2 - (\gamma_{Tt} - \gamma) \log \overline{L}_T - \frac{1}{2}(\gamma_{Tt} - \gamma)^2 (\log \overline{L}_T)^2 \right] \\ &+ \frac{1}{\beta} f_0^*, \end{aligned}$$

and similarly for the FB allocation relative to the BT. Taking the difference in welfare and substituting using the iterated resource constraint

$$\begin{aligned}\mathbb{W}_0 - \tilde{\mathbb{W}}_0 = & -\sum_{t=0}^{\infty}\beta^t\omega\left[\frac{1}{2}c_{Tt}^2 - (\alpha+\gamma)l_{Tt} - \frac{1}{2}(\alpha+\gamma)^2 l_{Tt}^2\right] \\ & + \sum_{t=0}^{\infty}\beta^t\omega\left[\frac{1}{2}\tilde{c}_{Tt}^2 - (\alpha+\gamma)\tilde{l}_{Tt} - \frac{1}{2}(\alpha+\gamma)^2\tilde{l}_{Tt}^2\right] \\ & - \sum_{t=0}^{\infty}\beta^t\left[\phi\overline{L}_T(l_{Tt} - \frac{1}{2}l_{Tt}^2)\right] + \sum_{t=0}^{\infty}\beta^t\left[\phi\overline{L}_T(\tilde{l}_{Tt} - \frac{1}{2}\tilde{l}_{Tt}^2)\right] \\ = & -\frac{1}{2}\sum_{t=0}^{\infty}\beta^t\left[\omega z_t^2 + \left[\omega(\alpha+\gamma)^2 + \omega(\alpha+\gamma)\right]x_t^2\right], \end{aligned} \tag{A.9}$$

denoting deviations from the FB $z_t \equiv \log C_{Tt} - \log \tilde{C}_{Tt}$, $x_t \equiv \log L_{Tt} - \log \tilde{L}_{Tt}$, and using that $\phi\overline{L}_T = \omega(\alpha+\gamma)$, and as we now show the interaction terms are zero to second order.

Combining the second-order approximations of the resource constraint

$$\begin{aligned} c_{Tt} - \tilde{c}_{Tt} + \frac{1}{2}c_{Tt}^2 - \frac{1}{2}\tilde{c}_{Tt}^2 - (\alpha+\gamma)(l_{Tt} - \tilde{l}_{Tt}) - (\alpha+\gamma)^2(l_{Tt}^2 - \tilde{l}_{Tt}^2) &= R^*\check{f}_t^* - \check{f}_{t+1}^* \\ \tilde{c}_{Tt}(c_{Tt} - \tilde{c}_{Tt}) - \tilde{c}_{Tt}(\alpha+\gamma)(l_{Tt} - \tilde{l}_{Tt}) + h.o.t. &= \tilde{c}_{Tt}(R^*\check{f}_t^* - \check{f}_{t+1}^*), \end{aligned}$$

where $\check{f}_t^* \equiv f_t^* - \tilde{f}_t^*$. Substituting for $\tilde{c}_{Tt}$ gives

$$\begin{aligned} & \tilde{c}_{Tt}(c_{Tt} - \tilde{c}_{Tt}) - a_t(\alpha+\gamma)(l_{Tt} - \tilde{l}_{Tt}) - \tilde{l}_{Tt}(\alpha+\gamma)^2(l_{Tt} - \tilde{l}_{Tt}) \\ -(\gamma_{Tt} - \gamma)\log\overline{L}_T(\alpha+\gamma)(l_{Tt} - \tilde{l}_{Tt}) - (R^*\tilde{f}_t^* - \tilde{f}_{t+1}^*)(\alpha+\gamma)(l_{Tt} - \tilde{l}_{Tt}) + h.o.t. &= \tilde{c}_{Tt}(R^*\check{f}_t^* - \check{f}_{t+1}^*) \end{aligned}$$

$$\begin{aligned} & \sum_{t=0}^{\infty}\beta^t\Big[\tilde{c}_{Tt}(c_{Tt} - \tilde{c}_{Tt}) - a_t(\alpha+\gamma)(l_{Tt} - \tilde{l}_{Tt}) - \tilde{l}_{Tt}(\alpha+\gamma)^2(l_{Tt} - \tilde{l}_{Tt}) \\ -(\gamma_{Tt} - \gamma)\log\overline{L}_T(\alpha+\gamma)(l_{Tt} - \tilde{l}_{Tt}) - (R^*\tilde{f}_t^* - \tilde{f}_{t+1}^*)(\alpha+\gamma)(l_{Tt} - \tilde{l}_{Tt})\Big] + h.o.t. &= \sum_{t=0}^{\infty}\beta^t\Big[\tilde{c}_{Tt}(R^*\check{f}_t^* - \check{f}_{t+1}^*)\Big] \end{aligned}$$

Therefore, let γ such that

$$\sum_{t=0}^{\infty}\beta^t\Big[[\tilde{l}_{Tt}(\alpha+\gamma) + s_t - [\alpha+\gamma+1]l_{Tt}](l_{Tt} - \tilde{l}_{Tt})\Big] = 0$$

where $s_t = a_t + (\gamma_{Tt} - \gamma)\log\overline{L}_T + R^*\tilde{f}_t^* - \tilde{f}_{t+1}^*$.

The interaction terms simplify to

$$\sum_{t=0}^{\infty}\beta^t\left[\omega\tilde{c}_{Tt}(c_{Tt}-\tilde{c}_{Tt})-\omega(\alpha+\gamma)^2 l_{Tt}(l_{Tt}-\tilde{l}_{Tt})-\omega(\alpha+\gamma)l_{Tt}(l_{Tt}-\tilde{l}_{Tt})\right]+h.o.t.$$

$$\begin{aligned}&=\sum_{t=0}^{\infty}\beta^t\left[\omega\tilde{c}_{Tt}(R^*\check{f}_t^*-\check{f}_{t+1}^*)\right]\\&=\sum_{t=0}^{\infty}\beta^t\left[\omega\check{f}_{t+1}^*(\tilde{c}_{Tt+1}-\tilde{c}_{Tt})\right]\\&=0,\end{aligned}$$

given by the FB optimality condition $\tilde{c}_{Tt}=\tilde{c}_{Tt+1}$ and $\check{f}_0^*=0$.

Next, solve the constraints in terms of z_t, x_t. The loglinear resource constraint gives

$$z_t-(\alpha+\gamma)x_t=R^*\check{f}_t^*-\check{f}_{t+1}^*. \tag{A.10}$$

The loglinear MRS = MPL constraint for the XR-IP is

$$\log\phi-\log\omega+\log C_{Tt}=\log\alpha+\log A_t+(\alpha+\gamma-1)\log L_{Tt}+(\gamma_{Tt}-\gamma)\log L_{Tt},$$

and for the FB is

$$\log\phi-\log\omega+\log\tilde{C}_{Tt}=\log(\alpha+\gamma_{Tt})+\log A_t+(\alpha+\gamma-1)\log\tilde{L}_{Tt}+(\gamma_{Tt}-\gamma)\log\tilde{L}_{Tt}.$$

Combining the XR-IP and FB gives

$$\begin{aligned}z_t&=\psi_t+(\alpha+\gamma-1)x_t+(\gamma_{Tt}-\gamma)(\log L_{Tt}-\log\tilde{L}_{Tt})\\&=\psi_t+(\alpha+\gamma-1)x_t,\end{aligned} \tag{A.11}$$

where $\psi_t\equiv\log\alpha-\log(\alpha+\gamma_{Tt})\leq 0$ and the second line uses a first-order approximation around the BT, which gives $(\gamma_{Tt}-\gamma)(\log L_{Tt}-\log\tilde{L}_{Tt})=0$.

Combining (A.9), (A.11), and (A.10), the approximate XR-IP problem is

$$\begin{aligned}\max_{\{z_t,x_t,\check{f}^*_{t+1}\}_{t=0}^{\infty}} \quad & -\frac{1}{2}\sum_{t=0}^{\infty}\beta^t\left[\omega z_t^2+\left[\omega(\alpha+\gamma)^2+\omega(\alpha+\gamma)\right]x_t^2\right] \\ s.t. \quad & z_t=\psi_t+(\alpha+\gamma-1)x_t, \\ & z_t-(\alpha+\gamma)x_t=R^*\check{f}^*_t-\check{f}^*_{t+1}, \\ & \check{f}^*_0=0.\end{aligned} \tag{A.12}$$

$\beta R^*=1$ and iterating (A.12) using the transversality condition $\lim_{s\to\infty}\beta^s f^*_{t+s}=0$ gives

$$\sum_{t=0}^{\infty}\beta^t\left(z_t-(\alpha+\gamma)x_t\right)=0,$$

which shows the Lemma.

A.5. Proof of Proposition 3: Optimal XR-IP in converging economies

We first solve the optimal XR-IP problem in this case, then characterize the solution relative to the laissez-faire competitive equilibrium (LF-CE).

Combining the constraints, we can solve the XR-IP problem for x_t

$$\begin{aligned}\max_{\{x_t\}_{t=0}^{\infty}} \quad & -\frac{1}{2}\sum_{t=0}^{\infty}\beta^t\left[\omega(\psi_t+(\alpha+\gamma-1)x_t)^2+\left[\omega(\alpha+\gamma)^2+\omega(\alpha+\gamma)\right]x_t^2\right] \\ s.t. \quad & \sum_{t=0}^{\infty}\beta^t\left[\psi_t-x_t\right]=0.\end{aligned}$$

Let λ be the multiplier on the lifetime resource constraint. The first-order condition for x_t is

$$-\beta^t\left[\omega(\psi_t+(\alpha+\gamma-1)x_t)(\alpha+\gamma-1)+\left[\omega(\alpha+\gamma)^2+\omega(\alpha+\gamma)\right]x_t\right]=\beta^t\lambda.$$

We get a loglinear Euler equation to characterize the XR-IP solution

$$\psi_t+\left[(\alpha+\gamma-1)+c\right]x_t^{IP}=\psi_{t+1}+\left[(\alpha+\gamma-1)+c\right]x_{t+1}^{IP},$$

where $c = \frac{(\alpha+\gamma)^2+(\alpha+\gamma)}{(\alpha+\gamma-1)} < 0$. Therefore, for all $t \geq 1$

$$\begin{aligned}\psi_0 + (\alpha+\gamma-1+c)x_0^{IP} &= \psi_t + (\alpha+\gamma-1+c)x_t^{IP} \\ \Rightarrow x_t^{IP} &= \frac{(\psi_0-\psi_t)}{(\alpha+\gamma-1+c)} + x_0^{IP}.\end{aligned}$$

To show that $x_t^{IP} < 0$, i.e., $\tilde{L}_{Tt} > L_{Tt}^{IP}$ for all t, from the lifetime resource constraint

$$\begin{aligned}\frac{x_0^{IP}}{1-\beta} + \sum_{t=1}^{\infty}\beta^t\frac{(\psi_0-\psi_t)}{(\alpha+\gamma-1+c)} &= \sum_{t=0}^{\infty}\beta^t\psi_t \\ x_0^{IP} = (1-\beta)\psi_0 - \frac{\beta\psi_0}{(\alpha+\gamma-1+c)} + (1-\beta)\frac{(\alpha+\gamma+c)}{(\alpha+\gamma-1+c)}\sum_{t=1}^{\infty}\beta^t\psi_t &< 0,\end{aligned}$$

since $(\alpha+\gamma+c) < 0$, given $\gamma_{Tt} > \gamma_{Tt+1} > 0$ for all t then $\psi_t < \psi_{t+1} < 0$, $\sum_{t=1}^{\infty}\beta^t\psi_t < 0$. Therefore

$$x_t^{IP} = \frac{(1-\beta)\psi_0(\alpha+\gamma+c)}{(\alpha+\gamma-1+c)} - \frac{\psi_t}{(\alpha+\gamma-1+c)} + (1-\beta)\frac{(\alpha+\gamma+c)}{(\alpha+\gamma-1+c)}\sum_{t=1}^{\infty}\beta^t\psi_t < 0.$$

The LF-CE is characterized by

$$\begin{aligned}z_t &= z_{t+1}, \\ z_t &= \psi_t + (\alpha+\gamma-1)x_t, \\ z_t - (\alpha+\gamma)x_t &= R^*\check{f}_t^* - \check{f}_{t+1}^*, \\ \check{f}_0^* &= 0.\end{aligned}$$

Combining the first two equations

$$\psi_t + (\alpha+\gamma-1)x_t^{CE} = \psi_{t+1} + (\alpha+\gamma-1)x_{t+1}^{CE},$$

so the CE allocation is given by the XR-IP with setting $c = 0$. Note that

$$x_t^{CE} = \frac{(\psi_0-\psi_t)}{(\alpha+\gamma-1)} + x_0^{CE}.$$

Therefore, from the lifetime resource constraint

$$x_0^{CE} = (1-\beta)\psi_0 - \frac{\beta\psi_0}{(\alpha+\gamma-1)} + (1-\beta)\frac{(\alpha+\gamma)}{(\alpha+\gamma-1)}\sum_{t=1}^{\infty}\beta^t\psi_t,$$

and

$$\begin{aligned} x_0^{IP} - x_0^{CE} &= \underbrace{\frac{\beta\psi_0}{(\alpha+\gamma-1+c)(\alpha+\gamma-1)}c}_{>0} + \underbrace{\frac{(1-\beta)\sum_{t=1}^{\infty}\beta^t\psi_t}{(\alpha+\gamma-1+c)(\alpha+\gamma-1)}[-c]}_{<0} \\ &> \frac{\beta\psi_0}{(\alpha+\gamma-1+c)(\alpha+\gamma-1)}c + \frac{(1-\beta)\sum_{t=1}^{\infty}\beta^t\psi_0}{(\alpha+\gamma-1+c)(\alpha+\gamma-1)}[-c] \\ &= \frac{\beta\psi_0}{(\alpha+\gamma-1+c)(\alpha+\gamma-1)}c + \frac{\beta\psi_0}{(\alpha+\gamma-1+c)(\alpha+\gamma-1)}[-c] \\ &= 0. \end{aligned}$$

From the lifetime resource constraint

$$\begin{aligned} \sum_{t=0}^{\infty}\beta^t(x_t^{IP} - x_t^{CE}) &= 0 \\ \underbrace{x_0^{IP} - x_0^{CE}}_{>0} &= -\sum_{t=1}^{\infty}\beta^t(x_t^{IP} - x_t^{CE}), \end{aligned}$$

so for at least one $t \geq 1$, $(x_t^{IP} - x_t^{CE}) < 0$. Note, for any $t \geq 1$

$$x_t^{IP} - x_t^{CE} = \frac{-(\psi_0 - \psi_t)}{(\alpha+\gamma-1+c)(\alpha+\gamma-1)}c + x_0^{IP} - x_0^{CE}.$$

Therefore

$$x_{t+1}^{IP} - x_{t+1}^{CE} - (x_t^{IP} - x_t^{CE}) = \frac{\psi_{t+1} - \psi_t}{(\alpha+\gamma-1+c)(\alpha+\gamma-1)}c < 0,$$

since $\psi_{t+1} - \psi_t > 0$ and $c < 0$, so $(x_t^{IP} - x_t^{CE})$ is strictly decreasing in t. Then, together with $(x_0^{IP} - x_0^{CE}) > 0$ and $(x_t^{IP} - x_t^{CE}) < 0$ for some t it must be that $\exists\ \bar{t} > 0$ such that $(x_t^{IP} - x_t^{CE}) > 0$ (i.e. $L_{Tt}^{IP} > L_{Tt}^{CE}$) for $t < \bar{t}$ and $(x_t^{IP} - x_t^{CE}) < 0$ (i.e. $L_{Tt}^{IP} < L_{Tt}^{CE}$) for $t > \bar{t}$.

For both the XR-IP and LF-CE

$$C_{Tt} = \frac{\omega\alpha A_t}{\phi}\frac{1}{L_{Tt}^{1-\alpha-\gamma_{Tt}}},$$
$$\mathcal{E}_t = \left(\frac{C_{Nt}}{A_t}\right)^{\frac{\alpha-1}{\alpha}} L_{Tt}^{1-\alpha-\gamma_{Tt}},$$

where C_{Nt} coincides for the XR-IP and LF-CE. Therefore, for $t < \bar{t}$ since $L_{Tt}^{IP} > L_{Tt}^{CE}$

$$C_{Tt}^{IP} < C_{Tt}^{CE},$$
$$\mathcal{E}_t^{IP} > \mathcal{E}_t^{CE}.$$

By definition of the trade balance

$$TB_t = A_t L_{Tt}^{\alpha+\gamma_{Tt}} - C_{Tt},$$
$$\Rightarrow TB_t^{IP} > TB_t^{CE},$$

for $t < \bar{t}$.

It is straight forward that similarly for $t > \bar{t}$ when $L_{Tt}^{IP} < L_{Tt}^{CE}$ that $C_{Tt}^{IP} > C_{Tt}^{CE}$, $\mathcal{E}_t^{IP} < \mathcal{E}_t^{CE}$, and $TB_t^{IP} < TB_t^{CE}$.

To examine the path of assets F_{t+1}^*, for both the XR-IP and CE

$$\psi_t - x_t = R^* \check{f}_t^* - \check{f}_{t+1}^*,$$
$$\check{f}_0^* = 0.$$

Therefore, at any t

$$\sum_{s=0}^{t} \beta^s [\psi_s - x_s] + \beta^t \check{f}_{t+1}^* = 0$$
$$\beta^t \check{f}_{t+1}^* = \sum_{s=0}^{t} \beta^s [x_s - \psi_s].$$

Substituting in for the solution for x_t gives

$$\begin{aligned}
x_t - \psi_t &= \frac{(1-\beta)\psi_0(\alpha+\gamma+c)}{(\alpha+\gamma-1+c)} - \frac{(\alpha+\gamma+c)\psi_t}{(\alpha+\gamma-1+c)} + (1-\beta)\frac{(\alpha+\gamma+c)}{(\alpha+\gamma-1+c)}\sum_{t=1}^{\infty}\beta^t\psi_t \\
&< \frac{(1-\beta)\psi_0(\alpha+\gamma+c)}{(\alpha+\gamma-1+c)} - \frac{(\alpha+\gamma+c)\psi_t}{(\alpha+\gamma-1+c)} + \beta\frac{(\alpha+\gamma+c)}{(\alpha+\gamma-1+c)}\psi_0 \\
&< \frac{(1-\beta)\psi_0(\alpha+\gamma+c)}{(\alpha+\gamma-1+c)} - \frac{(\alpha+\gamma+c)\psi_0}{(\alpha+\gamma-1+c)} + \beta\frac{(\alpha+\gamma+c)}{(\alpha+\gamma-1+c)}\psi_0 \\
&= 0.
\end{aligned}$$

Therefore

$$\beta^t \check{f}^*_{t+1} = \sum_{s=0}^{t}\beta^s[x_s - \psi_s] < 0,$$

which shows that $\tilde{F}^*_{t+1} > F^{*IP}_{t+1}$ for all t.

Combining the expression above for $\check{f}^*_{t+1}$ for the XR-IP and CE

$$\beta^t[(\check{f}^*_{t+1})^{IP} - (\check{f}^*_{t+1})^{CE}] = \sum_{s=0}^{t}\beta^s[x^{IP}_s - x^{CE}_s].$$

We know that

$$\begin{aligned}
\sum_{s=0}^{\infty}\beta^s[x^{IP}_s - x^{CE}_s] &= 0 \\
\sum_{s=0}^{t}\beta^s[x^{IP}_s - x^{CE}_s] + \sum_{s=t+1}^{\infty}\beta^s[x^{IP}_s - x^{CE}_s] &= 0,
\end{aligned}$$

and that $(x^{IP}_0 - x^{CE}_0) > 0$, $(x^{IP}_s - x^{CE}_s)$ is strictly decreasing in s and $(x^{IP}_t - x^{CE}_t) < 0$ for all $t > \bar{t}$. Therefore

$$\begin{aligned}
\beta^t[(\check{f}^*_{t+1})^{IP} - (\check{f}^*_{t+1})^{CE}] = -\sum_{s=t+1}^{\infty}\beta^s[x^{IP}_s - x^{CE}_s] > 0 \\
\Rightarrow F^{*IP}_{t+1} > F^{*CE}_{t+1} \text{ for all } t.
\end{aligned}$$

A.6. Proof of Proposition 4: Optimal XR-IP in non-converging economies

If the economy is not converging or is at the technological frontier, then $\gamma_{Tt} = \gamma_T$ for all $t \geq 0$. In either case, the solution to the XR-IP problem shown in Appendix A.5 is

$$\psi + [(\alpha + \gamma - 1) + c]\, x_t^{IP} = \psi + [(\alpha + \gamma - 1) + c]\, x_{t+1}^{IP}$$
$$x_t^{IP} = x_{t+1}^{IP},$$

and for the LF-CE is

$$\psi + (\alpha + \gamma - 1) x_t^{CE} = \psi + (\alpha + \gamma - 1) x_{t+1}^{IP}$$
$$x_t^{CE} = x_{t+1}^{CE}.$$

The other conditions are identical, so the allocations for the optimal XR-IP and LF-CE must coincide.

To see that this no-intervention result holds more generally observe that the XR-IP and LF-CE conditions coincide apart from the XR-IP modified Euler equation

$$\left(\frac{\omega}{C_{Tt}}\right)^{\frac{1}{\eta}} C_t^{\frac{1}{\eta}-\sigma} = \beta R^* \frac{\theta(\mathbf{x}_{t+1}, \gamma_{Tt+1})}{\theta(\mathbf{x}_t, \gamma_{Tt})} \left(\frac{\omega}{C_{Tt+1}}\right)^{\frac{1}{\eta}} C_{t+1}^{\frac{1}{\eta}-\sigma},$$

where $\theta(\mathbf{x}_t, \gamma_{Tt})$ and $\mathbf{x}_t \equiv \left\{C_{Tt}, C_{Nt}, L_{Tt}, L_{Nt}, F_{t+1}^*, A_t\right\}$. If $\beta R^* = 1$, $A_t = A$, and $\gamma_{Tt} = \gamma_T$ for all $t \geq 0$ then we can conjecture and confirm that the solution coincides with the LF-CE Euler equation

$$\left(\frac{\omega}{C_{Tt}}\right)^{\frac{1}{\eta}} C_t^{\frac{1}{\eta}-\sigma} = \left(\frac{\omega}{C_{Tt+1}}\right)^{\frac{1}{\eta}} C_{t+1}^{\frac{1}{\eta}-\sigma},$$

B. Quantitative model appendix

B.1. Additional details in the estimation of production externalities

Methodology. This section extends the methodology proposed by Bartelme *et al.* (2019) (BCDR) to a setting with multiple periods and a time-varying externality. As in BCDR we allow for arbitrary sectoral employment subsidies and bilateral trade tariffs.

Firms in country c tradable sector i with destination j produce using a constant-returns-to-scale production function at the firm level ($\alpha = 1$)

$$y_{cj,i,t} = A_{cj,i,t} l_{cj,i,t} L_{cit}^{\Gamma_{ict}}$$

with a sector i production externality Γ_{ict}, where $L_{cit} = \sum_j l_{cj,i,t}$. The optimality condition for labor $l_{cj,i,t}$, given producer price $P_{cj,i,t}$, wage W_{ct} and employment subsidy s_{cit} is

$$A_{cj,i,t} L_{cit}^{\Gamma_{ict}} = \frac{(1 - s_{cit}) W_{ct}}{P_{cj,i,t}}.$$

Taking logs and combining with the same expression for reference sector i_0 in country c and taking the average across destination countries j

$$\begin{aligned}\frac{1}{J}\sum_j \log P_{cj,i,t} - \frac{1}{J}\sum_j \log P_{cj,i_0,t} = & -\Gamma_{ict} \log L_{cit} + \Gamma_{i_0ct} \log L_{ci_0t} + \log(1 - s_{cit}) - \log(1 - s_{ci_0t}) \\ & - \frac{1}{J}\sum_j \log A_{cj,i,t} + \frac{1}{J}\sum_j \log A_{cj,i_0,t}. \end{aligned} \tag{B.1}$$

Households in country j have CES preferences over consumption of goods from sectors i across source countries c

$$u_j(c_{cj,i,t}) = \left(\sum_i \beta_{jit}^{\frac{1}{\iota}} C_{jit}^{1-\frac{1}{\iota}} \right)^{\frac{\iota}{\iota - 1}},$$

$$C_{jit} = \left(\sum_c c_{cj,i,t}^{1-\frac{1}{\theta_i}} \right)^{\frac{\theta_i}{\theta_i - 1}},$$

where β_{jit} is a preference shifter for country j and sector i normalized so that $\sum_i \beta_{jit} = 1$, ι

is the elasticity of substitution between goods from different sectors, and θ_i is the elasticity of substitution between goods from different origins. If the household faces consumer prices $p_{cj,i,t}$, the first-order condition w.r.t. $c_{cj,i,t}$ is

$$p_{cj,i,t} = \left(\sum_i \beta_{jit}^{\frac{1}{\iota}} C_{jit}^{1-\frac{1}{\iota}} \right)^{\frac{1}{\iota-1}} \beta_{jit}^{\frac{1}{\iota}} C_{jit}^{-\frac{1}{\iota}} \left(\sum_c c_{cj,i,t}^{1-\frac{1}{\theta_i}} \right)^{\frac{1}{\theta_i-1}} c_{cj,i,t}^{-\frac{1}{\theta_i}}.$$

Taking the ratio of this with the same expression for reference source country $c = 0$

$$\frac{p_{cj,i,t}}{p_{0j,i,t}} = \frac{c_{cj,i,t}^{-\frac{1}{\theta_i}}}{c_{0j,i,t}^{-\frac{1}{\theta_i}}}$$
$$\frac{X_{cj,i,t}}{X_{0j,i,t}} = \frac{c_{cj,i,t}^{1-\frac{1}{\theta_i}}}{c_{0j,i,t}^{1-\frac{1}{\theta_i}}},$$

where $X_{cj,i,t} \equiv p_{cj,i,t} c_{cj,i,t}$ is expenditure by households in j on sector i in source country c, then

$$\frac{X_{cj,i,t}}{X_{0j,i,t}} = \left[\frac{p_{cj,i,t}}{p_{0j,i,t}} \right]^{1-\theta_i},$$

and taking the average across countries j gives

$$\frac{1}{1-\theta_i} \frac{1}{J} \sum_j \log X_{cj,i,t} - \frac{1}{1-\theta_i} \frac{1}{J} \sum_j \log X_{c_0 j,i,t} = -\frac{1}{J} \sum_j \log p_{cj,i,t} + \frac{1}{J} \sum_j \log p_{c_0 j,i,t}. \tag{B.2}$$

Bartelme *et al.* (2019) allow for import tariffs $t^m_{cj,i,t}$ and export taxes $t^x_{cj,i,t}$ so that consumer $p_{cj,i,t}$ and producer $P_{cj,i,t}$ prices are related by

$$p_{cj,i,t} = \frac{(1 + t^m_{cj,i,t})}{(1 - t^x_{cj,i,t})} P_{cj,i,t}.$$

Taking logs and the average across countries j gives

$$\frac{1}{J} \sum_j \log p_{cj,i,t} = \frac{1}{J} \sum_j \log P_{cj,i,t} + \frac{1}{J} \sum_j \log(1 + t^m_{cj,i,t}) - \frac{1}{J} \sum_j \log(1 - t^x_{cj,i,t}). \tag{B.3}$$

Combining (B.3) with the household equation (B.2) and firms equation (B.1)

$$x_{cit} = a_c + a_i + a_t + \gamma_i^0 \log L_{cit} + \gamma_i^1 \mathrm{df}_{ct} \log L_{cit} + \varepsilon_{cit}, \tag{B.4}$$

where

$$\begin{aligned}
x_{cit} &\equiv \frac{1}{1-\theta_i}\frac{1}{J}\sum_j \log X_{cj,i,t}, \\
\varepsilon_{cit} &\equiv \frac{1}{J}\sum_j \log \alpha_{cj,i,t} - \mathbb{E}\left[\frac{1}{J}\sum_j \log \alpha_{cj,i,t} \mid c\right] - \mathbb{E}\left[\frac{1}{J}\sum_j \log \alpha_{cj,i,t} \mid i\right] - \mathbb{E}\left[\frac{1}{J}\sum_j \log \alpha_{cj,i,t} \mid t\right] \\
&\quad + \mathbb{E}\left[\frac{1}{J}\sum_j \log \alpha_{cj,i,t}\right], \\
\alpha_{cj,i,t} &\equiv A_{cj,i,t}(1 - t^x_{cj,i,t})/[(1 - s_{cit})(1 + t^m_{cj,i,t})], \\
\Gamma_{ict} &= \gamma_i^0 + \gamma_i^1 \mathrm{df}_{ct},
\end{aligned}$$

a_c, a_i, a_t are country, sector and time fixed effects, and the shocks ε_{cit} are constructed so that

$$\begin{aligned}
\mathbb{E}[\varepsilon_{cit}|c] &= 0 \text{ for all } c, \\
\mathbb{E}[\varepsilon_{cit}|i] &= 0 \text{ for all } i, \\
\mathbb{E}[\varepsilon_{cit}|t] &= 0 \text{ for all } t.
\end{aligned}$$

The second and first stages for the IV estimator in dummy variable notation are

$$\begin{aligned}
x_{cit} = &\sum_{n\in\mathcal{C}} a_n \times \mathbb{1}_{n=c} + \sum_{n\in\mathcal{I}} a_n \times \mathbb{1}_{n=i} + \sum_{n\in\mathcal{T}} a_n \times \mathbb{1}_{n=t} \\
&+ \sum_{n\in\mathcal{I}} \gamma_n^0 \times (\mathbb{1}_{n=i} \times \log L_{cnt}) + \sum_{n\in\mathcal{I}} \gamma_n^1 \times \mathrm{df}_{ct}(\mathbb{1}_{n=i} \times \log L_{cnt}) + \varepsilon_{cit},
\end{aligned} \tag{B.5}$$

$$\begin{aligned}
(\mathbb{1}_{n=i} \times \log L_{cnt}) = &\sum_{m\in\mathcal{C}} \widetilde{a}_{mn} \times \mathbb{1}_{m=c} + \sum_{m\in\mathcal{I}} \widetilde{a}_{mn} \times \mathbb{1}_{m=i} + \sum_{m\in\mathcal{T}} \widetilde{a}_{mn} \times \mathbb{1}_{m=t} \\
&+ \sum_{m\in\mathcal{I}} \widetilde{\gamma}_{mn} \times (\mathbb{1}_{m=i} \times \log \widehat{L}_{cmt}) + \widetilde{\varepsilon}_{cnt}, \quad \text{for all } n \in \mathcal{I},
\end{aligned} \tag{B.6}$$

where $\mathcal{C}$, $\mathcal{I}$ and $\mathcal{T}$ denote the set of countries, sectors and years, respectively. L_{cit} is sector

size at time t, equal to the sales share of country c population $\widehat{L}_{ct}$, constructed as $L_{cit} = (S_{cit}/S_{ct})\widehat{L}_{ct}$, where $S_{cit} = \sum_j X_{cj,i,t}$, $S_{ct} = \sum_{j,i} X_{cj,i,t}$, and $X_{cj,i,t}$ are bilateral trade flows from country c to j in current USD.

The logic of the instrument $\widehat{L}_{cit}$ is that large countries, or countries with stronger tastes for particular sectoral consumption, are expected to be relatively productive (and, therefore, have relatively lower prices) in sectors with relatively larger production externalities.

Data and sample. Our data and sample of countries and sectors follows Bartelme *et al.* (2019) (BCDR). We use data from the OECD's Inter-Country Input-Output tables, which provide bilateral trade among the 61 major economies in Table B.1, including both advanced and developing economies. These data report all bilateral flows, including domestic sales, in each sector we use to construct aggregate measures of expenditure and sales by country and sector as described in Section 3.2. For population data, we use Penn World Tables version 9.0. The 15 2-digit manufacturing sectors used in BCDR are listed in Table B.2.

The measure of population $\widehat{L}_{ct}$ is "POP" in the Penn World Tables. $\widehat{L}_{cit}$ is the demand-predicted sector size IV, constructed as $\widehat{L}_{cit} = \widehat{\beta}_{cit} \times \widehat{L}_{ct}$. From the nested CES preferences the demand shifter $\widehat{\beta}_{cit}$ is given by

$$\widehat{\beta}_{cit} = \frac{S_{cit}/(P_{cit})^{1-\iota}}{\sum_l S_{clt}/(P_{clt})^{1-\iota}},$$

where $P_{cit} \equiv \left(\sum_j p_{jc,i,t}^{1-\theta_i}\right)^{1/(1-\theta_i)}$ is sector i's price index in country c. Bartelme *et al.* (2019) estimate $\log P_{cit} = \frac{1}{C}\sum_j \log(X_{jc,i,t}/X_{cit})/(\theta_i - 1)$, $\iota = 1.28$, and use estimates for θ_i from the literature. For relative productivity $\frac{\overline{A}}{A_t}$ we use GDP per capita relative to the U.S. measured in PPP from the World Bank. We estimate (B.5) and (B.6) pooling across all available cross-section years as Bartelme *et al.* (2019), $\mathcal{T} = \{1995, 2000, 2005, 2010\}$.

Results. Table B.2 shows results from the estimation of γ_i^0 and γ_i^1. The final column shows the value of the externality by sector $\Gamma_{i,China,2022} = \gamma_i^0 + \gamma_i^1 \mathrm{df}_{China,2022}$, where $\mathrm{df}_{China,2022} = 2.43$.

Table B.1: Countries in empirical analysis to estimate production externalities

Advanced	Emerging Markets
Australia	Argentina
Austria	Brazil
Belgium	Brunei Darussalam
Canada	Bulgaria
Denmark	Cambodia
Finland	Chile
France	China
Germany	Hong Kong
Greece	Colombia
Iceland	Costa Rica
Ireland	Croatia
Israel	Cyprus
Italy	Czech Republic
Japan	Estonia
Korea	Hungary
Luxembourg	India
Netherlands	Indonesia
New Zealand	Latvia
Norway	Lithuania
Portugal	Malaysia
Singapore	Malta
Spain	Mexico
Sweden	Philippines
Switzerland	Poland
United Kingdom	Romania
United States	Russian Federation
	Saudi Arabia
	Slovakia
	Slovenia
	South Africa
	Taiwan
	Thailand
	Tunisia
	Turkey
	Vietnam

Notes: Sample of countries used in the empirical analysis to estimate production externalities, as in Bartelme *et al.* (2019).

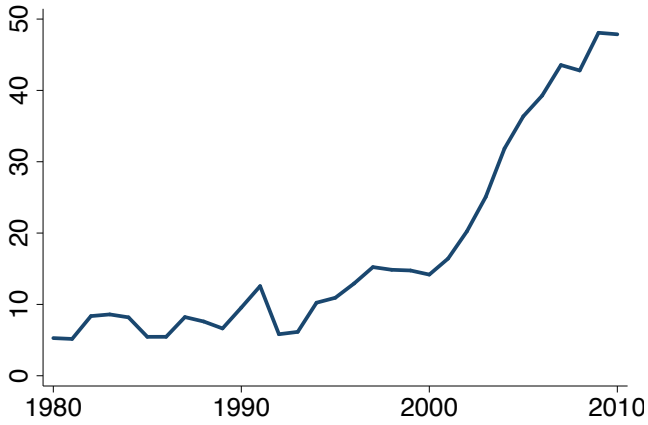
B.2. Additional results

Table B.2: Sector-level Results from the Estimation of Production Externalities

Sector (ISIC code)	γ_i^0	γ_i^1	Γ_i China 2022
Rubber and Plastics (22)	0.361	0.046	0.473
	(0.082)	(0.025)	
Chemicals (20)	0.254	0.003	0.261
	(0.027)	(0.005)	
Computers and Electronics (26)	0.236	0.006	0.252
	(0.025)	(0.007)	
Other Transport Equipment (30)	0.181	0.008	0.199
	(0.023)	(0.004)	
Motor Vehicles (29)	0.192	0.000	0.191
	(0.016)	(0.007)	
Food, Beverages and Tobacco (10-12)	0.180	0.004	0.187
	(0.022)	(0.004)	
Mineral Products (23)	0.133	0.005	0.146
	(0.015)	(0.003)	
Wood Products (16)	0.109	0.012	0.138
	(0.016)	(0.007)	
Paper Products (17-18)	0.118	0.006	0.133
	(0.015)	(0.004)	
Textiles (13-15)	0.078	0.011	0.104
	(0.013)	(0.003)	
Fabricated Metals (25)	0.067	0.014	0.100
	(0.014)	(0.005)	
Basic Metals (24)	0.061	0.009	0.084
	(0.014)	(0.004)	
Machinery and Equipment (28)	0.033	0.015	0.070
	(0.015)	(0.007)	
Electrical Machinery (27)	0.023	0.014	0.056
	(0.015)	(0.006)	
Petroleum Products (19)	0.031	0.009	0.054
	(0.082)	(0.005)	
Average ($N = 912$)	**0.137**	**0.011**	**0.163**

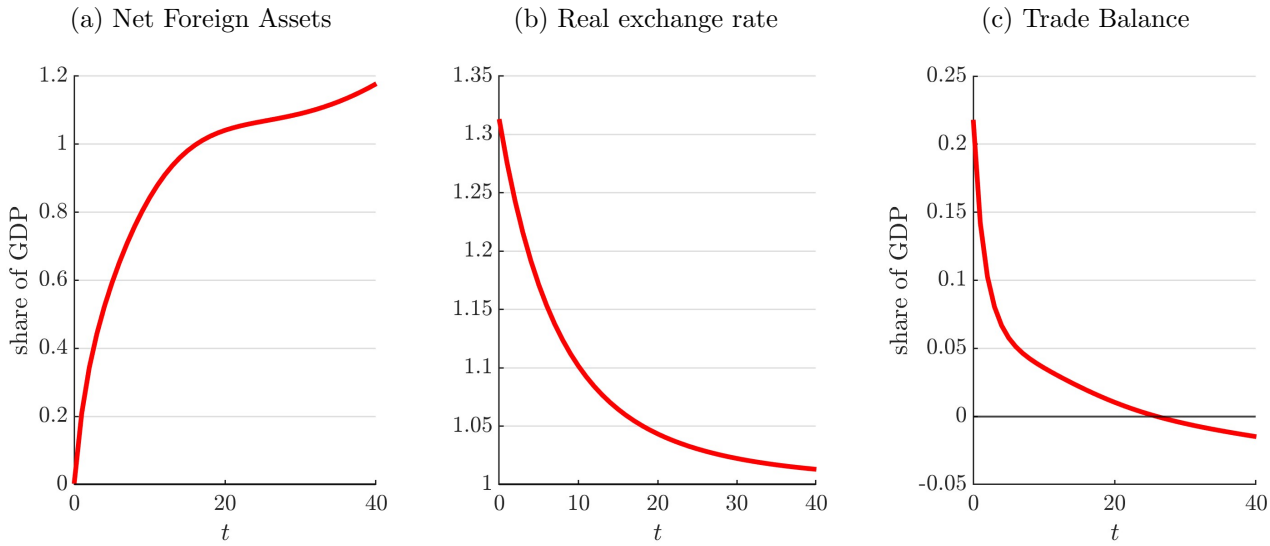
Notes: This table shows the results from estimating (26) using the instrumental variable approach in Bartelme *et al.* (2019). The final column shows the value of the externality by sector using $\text{df}_{China,2022}$. The average is weighted using sector sales shares.

Figure B.1: China: Reserves % of GDP



Notes: Reserves of foreign exchange and gold. Data source: World Bank.

Figure B.2: Foreign Exchange Interventions Under the First Best



Notes: The variables in Panels (a)-(c) are for the first best, expressed in deviations relative to the laissez-faire competitive equilibrium. The horizontal axis measures years since the initial period 1980. Panels (a) and (c) are the difference as a share of GDP, and Panel (b) is the ratio.

C. Model extensions appendix

C.1. Model with global financial intermediaries

C.1.1. Proof of Proposition 5: Economy with financial intermediaries

We first derive the XR-IP problem with international capital mobility. Given that intermediaries operate only in the initial period by choosing Q_1, $Q_0 = 0$, and $Q_t = 0$ for all $t \geq 2$,

Table B.3: Effectiveness of Observed Reserves Policy 2000-2008: Robustness

	Output		Labor		Consumption		Ext. sector	
Average annual growth, p.p.	Agg.	Trad	Agg.	Trad	Agg.	Trad	NX	RXR
a. Baseline	1.6	6.1	1.2	3.3	-0.8	-0.5	60.8	1.8
b. Alternative parameters								
Cole-Obstfeld	1.6	5.4	1.2	3.0	-0.9	-0.9	60.4	1.6
Coeff. RRA $\sigma = 2$	2.3	7.2	1.7	4.0	-0.6	-0.2	61.9	2.1
Dec. returns $\alpha = 0.75$	1.5	6.3	1.3	3.5	-1.2	-0.6	61.9	2.6
Full convergence $\varphi = 1$	1.5	5.6	1.1	3.0	-0.8	-0.5	60.6	1.7
Fast convergence $\rho = 0.1$	1.9	7.0	1.4	3.8	-1.0	-0.6	61.1	2.2

Notes: This table shows the difference in average annual growth rate for China's observed reserves accumulation policy relative to constant reserves (no accumulation) in percentage points for 2000-2008 for each model variant. Output, consumption and net exports measured in units of domestic consumption. Output is domestic value added. Reserves measured in tradables.

Table B.4: Welfare of "Step Function" Labor Subsidies

Temporary policy	Δ Welfare
Labor subsidy	0.11
Tradable labor subsidy	0.36
Labor subsidy + XR-IP	0.15
Tradable labor subsidy + XR-IP	0.39

Notes: Welfare expressed in consumption equivalent terms as the percentage increase in per-period consumption from the laissez-faire competitive equilibrium to equate welfare in each case. These "step function" labor subsidies revert to their steady state optimal level. The temporary labor subsidy is the optimal initial 10-year labor subsidy for both sectors of 12.1%, and 4.3% thereafter. The temporary tradable labor subsidy is the optimal initial 10-year tradable-sector only labor subsidy of 30.1%, and 12.5% thereafter.

and $\gamma_{T0} > \gamma_{Tt} = 0$ for all $t \geq 1$, the balance of payments condition is

$$C_{T0} - A_0 L_{T0}^{\alpha+\gamma_{T0}} = R^* F_0^* - F_1^* + \frac{Q_1}{\mathcal{E}_0}$$

$$C_{T1} - A_1 L_{T1}^{\alpha} = R^* F_1^* - F_2^* - R_t \frac{Q_1}{\mathcal{E}_1}$$

$$C_{Tt} - A_t L_{Tt}^{\alpha} = R^* F_t^* - F_{t+1}^* \qquad \text{for } t \geq 2.$$

Combining and iterating these forward and using the transversality condition gives the intertemporal resource constraint

$$\sum_{t=0}^{\infty} \frac{1}{(R^*)^t} \left(A_t L_{Tt}^{\alpha+\gamma_{Tt}} - C_{Tt}\right) + Q_1 \left(\frac{1}{\mathcal{E}_0} - \frac{1}{\mathcal{E}_1}\right) = -R^* F_0^*.$$

Substituting the optimality condition for the intermediaries $Q_1 = \frac{1}{\Gamma_I}\left[\mathcal{E}_0 - \frac{R^*}{R_1}\mathcal{E}_1\right]$ gives

$$\underbrace{\sum_{t=0}^{\infty} \frac{\left(A_t L_{Tt}^{\alpha+\gamma_{Tt}} - C_{Tt}\right)}{(R^*)^t}}_{\text{NPV net exports}} + \underbrace{\frac{1}{\Gamma_I}\left(1 - \frac{R^*}{R_1}\frac{\mathcal{E}_1}{\mathcal{E}_0}\right)\left(1 - \frac{R_1}{R^*}\frac{\mathcal{E}_0}{\mathcal{E}_1}\right)}_{\leq 0} = -R^* F_0^*. \tag{C1}$$

From the HH Euler equation,

$$\frac{C_{Tt+1}}{C_{Tt}} = \beta R_{t+1} \frac{\mathcal{E}_t}{\mathcal{E}_{t+1}}.$$

Substituting the constraint for C_{Tt} gives

$$\frac{A_{t+1} L_{Tt+1}^{\alpha+\gamma_{Tt+1}-1}}{A_t L_{Tt}^{\alpha+\gamma_{Tt}-1}} = \beta R_{t+1} \frac{\mathcal{E}_t}{\mathcal{E}_{t+1}} = \frac{R_{t+1}}{R^*} \frac{\mathcal{E}_t}{\mathcal{E}_{t+1}}.$$

Substituting the constraint for C_{Tt}, the intertemporal resource constraint from (C1) is

$$\sum_{t=0}^{\infty} \frac{1}{(R^*)^t} \left[\frac{\omega\alpha}{\phi} A_t L_{Tt}^{\alpha+\gamma_{Tt}-1} - A_t L_{Tt}^{\alpha+\gamma_{Tt}}\right] - \frac{1}{\Gamma_I}\left(1 - \frac{A_0 L_{T0}^{\alpha+\gamma_{T0}-1}}{A_1 L_{T1}^{\alpha-1}}\right)\left(1 - \frac{A_1 L_{T1}^{\alpha-1}}{A_0 L_{T0}^{\alpha+\gamma_{T0}-1}}\right) = R^* F_0^*.$$

The XR-IP problem is then to solve for the tradables block C_{Tt}, L_{Tt}, and F_{t+1}^*

$$\max_{\{C_{Tt}, L_{Tt}, F_{t+1}^*\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \left[\omega(\alpha + \gamma_{Tt} - 1)\log L_{Tt} - \phi L_{Tt}\right] + constant$$

$$s.t. \quad \sum_{t=0}^{\infty} \frac{1}{(R^*)^t} \left[\frac{\omega\alpha}{\phi} A_t L_{Tt}^{\alpha+\gamma_{Tt}-1} - A_t L_{Tt}^{\alpha+\gamma_{Tt}}\right] - \frac{1}{\Gamma_I}\left(1 - \frac{A_0 L_{T0}^{\alpha+\gamma_{T0}-1}}{A_1 L_{T1}^{\alpha-1}}\right)\left(1 - \frac{A_1 L_{T1}^{\alpha-1}}{A_0 L_{T0}^{\alpha+\gamma_{T0}-1}}\right) = R^* F_0^*,$$

$$F_0^* \text{ given.}$$

Let λ be the multiplier on the constraint. The first-order conditions w.r.t L_{T0}, L_{T1}, and

L_{Tt} for $t \geq 2$ are

$$\omega(\alpha+\gamma_{T0}-1)\frac{1}{L_{T0}} - \phi - \lambda\left[\frac{\omega\alpha}{\phi}(\alpha+\gamma_{T0}-1)A_0L_{T0}^{\alpha+\gamma_{T0}-2} - (\alpha+\gamma_{T0})A_0L_{T0}^{\alpha+\gamma_{T0}-1}\right]$$
$$-\lambda\left[\frac{1}{\Gamma_I}\left[(\alpha+\gamma_{T0}-1)\frac{A_0L_{T0}^{\alpha+\gamma_{T0}-2}}{A_1L_{T1}^{\alpha-1}} - (\alpha+\gamma_{T0}-1)\frac{A_1L_{T1}^{\alpha-1}}{A_0L_{T0}^{\alpha+\gamma_{T0}}}\right]\right] = 0$$
$$\beta\left[\omega(\alpha-1)\frac{1}{L_{T1}} - \phi\right] - \lambda\frac{1}{R^*}\left[\frac{\omega\alpha}{\phi}(\alpha-1)A_1L_{T1}^{\alpha-2} - \alpha A_1L_{T1}^{\alpha-1}\right]$$
$$-\lambda\left[\frac{1}{\Gamma_I}\left[(1-\alpha)\frac{A_0L_{T0}^{\alpha+\gamma_{T0}-1}}{A_1L_{T1}^{\alpha}} - (1-\alpha)\frac{A_1L_{T1}^{\alpha-2}}{A_0L_{T0}^{\alpha+\gamma_{T0}-1}}\right]\right] = 0$$
$$\beta^t\left[\omega(\alpha-1)\frac{1}{L_{Tt}} - \phi\right] - \lambda\frac{1}{(R^*)^t}\left[\frac{\omega\alpha}{\phi}(\alpha-1)A_tL_{Tt}^{\alpha-2} - \alpha A_tL_{Tt}^{\alpha-1}\right] = 0 \text{ for } t \geq 2.$$

Given that A_t is constant for $t \geq 2$, the final equation gives the usual XR-IP intertemporal optimality condition for $t \geq 2$

$$L_{T2} = L_{Tt+1}.$$

Combining the first-order conditions gives

$$\begin{aligned}\frac{1}{A_0L_{T0}^{\alpha+\gamma_{T0}-1}}\theta_0(L_{T0},L_{T1},\Gamma_I) &= \frac{1}{A_1L_{T1}^{\alpha-1}}\theta_1(L_{T0},L_{T1},\Gamma_I) \\ &= \frac{1}{A_1L_{Tt}^{\alpha-1}} \text{ for } t \geq 2\end{aligned} \tag{C2}$$

where

$$\theta_0(L_{T0},L_{T1},\Gamma_I) \equiv \frac{\frac{\omega\alpha}{\phi}(\alpha+\gamma_{T0}-1)\frac{1}{L_{T0}} - \alpha}{\frac{\omega\alpha}{\phi}(\alpha+\gamma_{T0}-1)\frac{1}{L_{T0}} - (\alpha+\gamma_{T0}) + \frac{1}{\Gamma_I}(\alpha+\gamma_{T0}-1)\left[\frac{A_0L_{T0}^{-1}}{A_1L_{T1}^{\alpha-1}} - \frac{A_1L_{T1}^{\alpha-1}}{A_0L_{T0}^{2(\alpha+\gamma_{T0})-1}}\right]}$$
$$\theta_1(L_{T0},L_{T1},\Gamma_I) \equiv \frac{\frac{\omega\alpha}{\phi}(\alpha-1)\frac{1}{L_{T1}} - \alpha}{\frac{\omega\alpha}{\phi}(\alpha-1)\frac{1}{L_{T1}} - \alpha + R^*\frac{1}{\Gamma_I}(1-\alpha)\left[\frac{A_0L_{T0}^{\alpha+\gamma_{T0}-1}}{A_1L_{T1}^{2\alpha-1}} - \frac{A_1L_{T1}^{-1}}{A_0L_{T0}^{\alpha+\gamma_{T0}-1}}\right]}.$$

With no financial intermediaries, $\Gamma_I \to \infty$, then back to the baseline XR-IP model and $\theta_0(L_{T0})$ and $\theta_1 = 1$.

The solution to the model for L_{T0} and L_{T1} is characterized by equation (C2) above and

from the balance of payments after substituting the optimality conditions

$$G\left(L_{T0},\theta_0,\theta_1\right)-\underbrace{H(L_{T0},L_{T1})}_{\leq 0}-\frac{1}{\beta}F_0^*=0,$$

where

$$\begin{aligned}
G\left(L_{T0},\theta_0,\theta_1\right) \equiv & \left(1+\beta\frac{\theta_1}{\theta_0}+\frac{\beta^2}{1-\beta}\frac{1}{\theta_0}\right)\frac{\omega\alpha}{\phi}L_{T0}^{\alpha+\gamma_{T0}-1} \\
& -L_{T0}^{\alpha+\gamma_{T0}}-\left(\beta\left[\frac{\theta_1}{\theta_0}\right]^{\frac{\alpha}{\alpha-1}}+\frac{\beta^2}{1-\beta}\left[\frac{1}{\theta_0}\right]^{\frac{\alpha}{\alpha-1}}\right)\left(L_{T0}^{\alpha+\gamma_{T0}-1}\right)^{\frac{\alpha}{\alpha-1}}, \\
H(L_{T0},L_{T1}) \equiv & \frac{1}{\Gamma_I}\left(1-\frac{A_0L_{T0}^{\alpha+\gamma_{T0}-1}}{A_1L_{T1}^{\alpha-1}}\right)\left(1-\frac{A_1L_{T1}^{\alpha-1}}{A_0L_{T0}^{\alpha+\gamma_{T0}-1}}\right)\leq 0.
\end{aligned}$$

Summarize the XR-IP solution for the economy with intermediaries as

$$G\left(L_{T0}^{IP-B},\theta_0(L_{T0},L_{T1},\Gamma_I),\theta_1(L_{T0},L_{T1},\Gamma_I)\right)-\underbrace{H(L_{T0}^{IP-B},L_{T1}^{IP-B})}_{\leq 0}-\frac{1}{\beta}F_0^*=0,$$

where below we prove that $\theta_1(L_{T0},L_{T1},\Gamma_I)\in(0,1)$ and $\theta_0(L_{T0},L_{T1},\Gamma_I)\in(0,1)$.

The solution for the baseline XR-IP model with no intermediaries is

$$G\left(L_{T0}^{IP},\theta_0(L_{T0},L_{T1},\infty),1\right)-0-\frac{1}{\beta}F_0^*=0,$$

where $\theta_0(L_{T0},L_{T1},\infty)\in(0,1)$.

The solution for the LF-CE is

$$G\left(L_{T0}^{CE},1,1\right)-0-\frac{1}{\beta}F_0^*=0.$$

We now compare the LF-CE with the XR-IP with intermediaries' ("IP-B") allocations.

The LF-CE allocation without intermediaries is

$$A_0L_{T0}^{\alpha+\gamma_{T0}-1}=A_1L_{T1}^{\alpha-1},$$

where L_{T0} satisfies the intertemporal resource constraint. In the economy with intermedi-

aries, this leads to $H(L_{T0}^{CE}, L_{T1}^{CE}) = 0$, so it is feasible.

Now consider a small increase in L_{T0}, $dL_{T0} > 0$, from the LF-CE allocation in the economy with intermediaries. We show that this, combined with a decrease in L_{T1}, $dL_{T1} < 0$ increases household utility and is feasible. The change in welfare from each is

$$\begin{aligned}\frac{\partial W}{\partial L_{T0}} dL_{T0} &= \left[\omega(\alpha + \gamma_{T0} - 1)\frac{1}{A_0 L_{T0}} - \phi\right] dL_{T0} \\ \frac{\partial W}{\partial L_{T1}} dL_{T1} &= \beta\left[\omega(\alpha - 1)\frac{1}{A_1 L_{T1}} - \phi\right] dL_{T1}.\end{aligned}$$

Consider the welfare neutral change around the LF-CE allocation

$$\begin{aligned} dW = \frac{\partial W}{\partial L_{T0}} dL_{T0} + \frac{\partial W}{\partial L_{T1}} dL_{T1} = 0 \\ \left[\omega(\alpha + \gamma_{T0} - 1)\frac{1}{A_0 L_{T0}} - \phi\right] dL_{T0} + \beta\left[\omega(\alpha - 1)\frac{1}{A_1 L_{T1}} - \phi\right] dL_{T1} = 0.\end{aligned}$$

The resource constraint is

$$RC = R^* F_0^* - \sum_{t=0}^{\infty} \frac{1}{(R^*)^t}\left[\frac{\omega\alpha}{\phi} A_t L_{Tt}^{\alpha+\gamma_{Tt}-1} - A_t L_{Tt}^{\alpha+\gamma_{Tt}}\right] + \frac{1}{\Gamma_I}\left(2 - \frac{A_0 L_{T0}^{\alpha+\gamma_{T0}-1}}{A_1 L_{T1}^{\alpha-1}} - \frac{A_1 L_{T1}^{\alpha-1}}{A_0 L_{T0}^{\alpha+\gamma_{T0}-1}}\right).$$

The change in the resource constraint from the change in L_{T0} and L_{T1} is

$$\begin{aligned}\Delta RC = &- \left[\frac{\omega\alpha}{\phi}(\alpha + \gamma_{T0} - 1) A_0 L_{T0}^{\alpha+\gamma_{T0}-2} - (\alpha + \gamma_{T0}) A_0 L_{T0}^{\alpha+\gamma_{T0}-1}\right] dL_{T0} \\ &- \beta\left[\frac{\omega\alpha}{\phi}(\alpha - 1) A_1 L_{T1}^{\alpha-2} - \alpha A_1 L_{T1}^{\alpha-1}\right] dL_{T1} \\ &\underbrace{- \frac{1}{\Gamma_I}\frac{1}{L_{T0}}\left[(\alpha + \gamma_{T0} - 1)\frac{A_0 L_{T0}^{\alpha+\gamma_{T0}-1}}{A_1 L_{T1}^{\alpha-1}} - (\alpha + \gamma_{T0} - 1)\frac{A_1 L_{T1}^{\alpha-1}}{A_0 L_{T0}^{\alpha+\gamma_{T0}-1}}\right] dL_{T0}}_{=0} \\ &\underbrace{- \frac{1}{\Gamma_I}\frac{1}{L_{T1}}\left[(1 - \alpha)\frac{A_0 L_{T0}^{\alpha+\gamma_{T0}-1}}{A_1 L_{T1}^{\alpha-1}} - (1 - \alpha)\frac{A_1 L_{T1}^{\alpha-1}}{A_0 L_{T0}^{\alpha+\gamma_{T0}-1}}\right] dL_{T1}}_{=0} \\ = &\ \gamma_{T0} A_0 L_{T0}^{\alpha+\gamma_{T0}-1} dL_{T0} > 0\end{aligned}$$

So this welfare-neutral change enables leftover resources to increase tradable consumption and raise overall welfare. Therefore, we can raise welfare in the economy with interme-

diaries relative to the LF-CE by increasing L_{T0} and decreasing L_{T1}. We can do so similarly by decreasing any L_{Tt} for $t \geq 2$. If we do the opposite change and decrease L_{T0} and increase L_{T1}, the signs are reversed and this will leave utility constant but reduce resources. A perturbation of L_{T1} and L_{Tt} for any $t \geq 2$ leads to no change in welfare or resources.

Therefore, this shows that locally around the LF-CE allocation

$$\begin{aligned} L_{T0}^{IP-B} &> L_{T0}^{CE}, \\ C_{T0}^{IP-B} &< C_{T0}^{CE}, \\ \mathcal{E}_0^{IP-B} &> \mathcal{E}_0^{CE}. \end{aligned}$$

We now compare the baseline XR-IP with the XR-IP with intermediaries' allocations. The XR-IP allocation in the economy with no intermediaries is

$$A_0 L_{T0}^{\alpha+\gamma_{T0}-1} = \frac{1}{\theta_0} A_1 L_{T1}^{\alpha-1},$$

where

$$\theta_0(L_{T0}, L_{T1}, \Gamma_I = \infty) \equiv \frac{\frac{\omega\alpha}{\phi}(\alpha+\gamma_{T0}-1)\frac{1}{L_{T0}} - \alpha}{\frac{\omega\alpha}{\phi}(\alpha+\gamma_{T0}-1)\frac{1}{L_{T0}} - (\alpha+\gamma_{T0}) + \frac{1}{\Gamma}(\alpha+\gamma_{T0}-1)\left[\frac{A_0 L_{T0}^{-1}}{A_1 L_{T1}^{\alpha-1}} - \frac{A_1 L_{T1}^{\alpha-1}}{A_0 L_{T0}^{2(\alpha+\gamma_{T0})-1}}\right]}$$

and $\Gamma_I \to \infty$ implies $\theta_0(L_{T0}) \in (0,1)$ and L_{T0} satisfies the intertemporal resource constraint.

In the economy with intermediaries, $H(L_{T0}^{IP}, L_{T1}^{IP}) < 0$ enters the resource constraint, so the baseline XR-IP allocation without intermediaries is not feasible here. We therefore consider a small change from the XR-IP allocation with intermediaries in the baseline economy without intermediaries. The term $H(L_{T0}, L_{T1}) = 0$, so there are additional resources leftover and the allocation with intermediaries cannot be optimal in this case.

Again consider the welfare-neutral change of increasing L_{T0}, $dL_{T0} > 0$ and decreasing L_{T1}, $dL_{T1} < 0$.

$$\begin{aligned} dW = \frac{\partial W}{\partial L_{T0}} dL_{T0} + \frac{\partial W}{\partial L_{T1}} dL_{T1} = 0 \\ \left[\omega(\alpha+\gamma_{T0}-1)\frac{1}{A_0 L_{T0}} - \phi\right] dL_{T0} + \beta\left[\omega(\alpha-1)\frac{1}{A_1 L_{T1}} - \phi\right] dL_{T1} = 0. \end{aligned}$$

The change in the resource constraint in the economy without intermediaries is

$$\begin{aligned}\Delta RC = & -\left[\frac{\omega\alpha}{\phi}(\alpha+\gamma_{T0}-1)A_0 L_{T0}^{\alpha+\gamma_{T0}-2} - (\alpha+\gamma_{T0})A_0 L_{T0}^{\alpha+\gamma_{T0}-1}\right] dL_{T0} \\ & -\beta\left[\frac{\omega\alpha}{\phi}(\alpha-1)A_1 L_{T1}^{\alpha-2} - \alpha A_1 L_{T1}^{\alpha-1}\right] dL_{T1} \\ & = \gamma_{T0} A_0 L_{T0}^{\alpha+\gamma_{T0}-1} dL_{T0} > 0,\end{aligned}$$

after following the same steps as above, and $\Delta RC > 0$ since $dL_{T0} > 0$.

Therefore, this welfare-neutral reallocation further increases available resources for tradable consumption to raise utility. This must be preferred to the welfare-neutral change of decreasing L_{T0} and increasing L_{T1}, which strictly reduces the resources available for consumption. A similar change in both L_{T1} and L_{Tt} for any $t \geq 2$ yields no change in utility or the resource constraint. This must also be strictly preferred to increasing utility from only changing either L_{T0} or L_{T1}, since changing both allows for additional resources.

Therefore, this shows that locally around the IP-B allocation

$$\begin{aligned}L_{T0}^{IP} &> L_{T0}^{IP-B} \\ C_{T0}^{IP} &< C_{T0}^{IP-B} \\ \mathcal{E}_0^{IP} &> \mathcal{E}_0^{IP-B}.\end{aligned}$$

The results for CA_0 directly follow, which shows the Proposition.

C.1.2. Quantitative analysis

We solve the quantitative model in Section 3 with international capital mobility. We set $\Gamma_I = 0.1$, which implies average private net foreign assets in the model that align with those in the data during the growth takeoff period (see Table C1). This value of Γ_I is also used in Gabaix and Maggiori (2015). All other externally set parameters are as the same as in the baseline model, and the calibrated ones follow the same calibration strategy.

Table C1: Data and Model predictions under the Observed Reserves Policy: International Capital Mobility

Average, % of GDP	Data	Model
Private NFA	-10.9	-13.2

Notes: This table shows the average for private net foreign assets for China from 1981-2008 in the data (which are excluding gold), defined as net foreign assets less reserves, and the model counterpart with international capital mobility is $-Q_t$. The earliest data available are for 1981 and in the model we assume intermediaries hold zero assets in the initial period 1980. Data source: Milesi-Ferretti (2024).

C.2. Model with capital controls

We first describe the model variant in which households can save or borrow in foreign currency and the government can impose a capital control tax.

Households. Households can save or borrow in foreign currency at R^* and the government imposes a time-varying capital control tax τ_t^B. The household budget constraint expressed in domestic currency is given by

$$P_{Tt}C_{Tt} + P_{Nt}C_{Nt} + \frac{1}{(1+\tau_t^B)}\mathcal{E}_t B_{t+1}^* = W_{Tt}L_{Tt} + W_{Nt}L_{Nt} + \Pi_t + T_t + \mathcal{E}_t R^* B_t^*, \qquad \text{(C3)}$$

where B_{t+1}^* are the foreign currency bonds purchased in t that mature in $t+1$ and R^* is the foreign currency interest rate. The other elements of the household problem are as in the baseline model. We show the proposition for more general preferences $\sum_{t=0}^{\infty}\beta^t\left[\frac{{C_t}^{1-\sigma}}{1-\sigma} - \phi\frac{L_t^{1+\nu}}{1+\nu}\right]$, where $\zeta = \frac{1}{\nu}$.

The household's problem is to choose allocations $\{C_t, C_{Tt}, C_{Nt}, L_t, L_{Tt}, L_{Nt}, B_{t+1}^*\}_{t=0}^{\infty}$ that maximize utility, subject to the aggregation technologies (2)-(3); the sequence of budget constraints (C3), given a sequence of prices, profits, and transfers; and an initial level of bonds

B_0^*. The first-order conditions that characterize the solution to the household's problem are

$$\begin{aligned}
\left(\frac{1-\omega}{C_{Nt}}\right)^{\frac{1}{\eta}} &= p_t \left(\frac{\omega}{C_{Tt}}\right)^{\frac{1}{\eta}}, \\
\left(\frac{\omega}{C_{Tt}}\right)^{\frac{1}{\eta}} C_t^{\frac{1}{\eta}-\sigma} \frac{W_{Tt}}{P_{Tt}} &= \phi L_{Tt}^{\nu}, \\
\left(\frac{1-\omega}{C_{Nt}}\right)^{\frac{1}{\eta}} C_t^{\frac{1}{\eta}-\sigma} \frac{W_{Nt}}{P_{Nt}} &= \phi L_{Nt}^{\nu}, \\
\left(\frac{\omega}{C_{Tt}}\right)^{\frac{1}{\eta}} C_t^{\frac{1}{\eta}-\sigma} &= \beta R^*(1+\tau_t^B) \frac{P_{Tt}}{P_{Tt+1}} \frac{\mathcal{E}_{t+1}}{\mathcal{E}_t} \left(\frac{\omega}{C_{Tt+1}}\right)^{\frac{1}{\eta}} C_{t+1}^{\frac{1}{\eta}-\sigma}.
\end{aligned}$$

Firms. As in Section 2.

Government. The government budget balances each period with revenue from the capital control tax τ_t^B distributed lump-sum to the household

$$-\frac{\tau_t^B}{(1+\tau_t^B)} \mathcal{E}_t B_{t+1}^* = T_t.$$

Rest of the world. The domestic economy consumes C_{Tt} and produces $A_t L_{Tt}^{\alpha+\gamma_{Tt}}$ of the tradable good, and saves B_{t+1}^* abroad at the real interest rate R^*. The value in domestic currency must be equal, giving the balance of payments

$$C_{Tt} - A_t L_{Tt}^{\alpha+\gamma_{Tt}} = R^* B_t^* - B_{t+1}^*.$$

As in the baseline model, we assume the law of one price holds for tradable goods and normalize the foreign currency price of tradables, so that $P_{Tt} = \mathcal{E}_t$.

C.2.1. Proof of Proposition 6: Capital controls as industrial policy

We now show the proposition for the model.

The competitive equilibrium allocation $\{C_{Tt}, C_{Nt}, L_{Tt}, L_{Nt}, B_{t+1}^*\}_{t=0}^{\infty}$ is characterized by

combining households' and firms' optimality conditions and market clearing to give

$$\left(\frac{1-\omega}{\omega}\frac{C_{Tt}}{C_{Nt}}\right)^{\frac{1}{\eta}} = \frac{L_{Tt}^{\alpha+\gamma_{Tt}-1-\nu}}{L_{Nt}^{\alpha-1-\nu}}, \tag{C4}$$

$$\frac{\phi L_{Tt}^{\nu}}{(\omega/C_{Tt})^{\frac{1}{\eta}} C_t^{\frac{1}{\eta}-\sigma}} = \alpha A_t L_{Tt}^{\alpha+\gamma_{Tt}-1}, \tag{C5}$$

$$\left(\frac{\omega}{C_{Tt}}\right)^{\frac{1}{\eta}} C_t^{\frac{1}{\eta}-\sigma} = \beta R^*(1+\tau_t^B)\left(\frac{\omega}{C_{Tt+1}}\right)^{\frac{1}{\eta}} C_{t+1}^{\frac{1}{\eta}-\sigma}, \tag{C6}$$

$$C_{Nt} = A_t L_{Nt}^{\alpha}, \tag{C7}$$

$$C_{Tt} - A_t L_{Tt}^{\alpha+\gamma_{Tt}} = R^* B_t^* - B_{t+1}^*. \tag{C8}$$

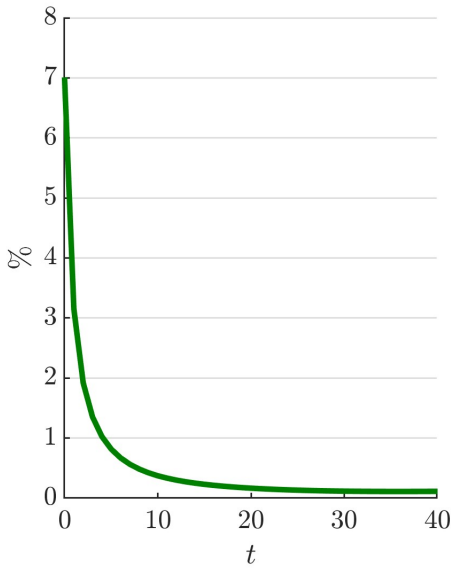
By setting the sequence of capital control taxes

$$\tau_t^B = \frac{\theta(\mathbf{x}_{t+1}, \gamma_{Tt+1})}{\theta(\mathbf{x}_t, \gamma_{Tt})} - 1,$$

where $\mathbf{x}_t \equiv \{C_t^{IP}, C_{Tt}^{IP}, C_{Nt}^{IP}, L_t^{IP}, L_{Tt}^{IP}, L_{Nt}^{IP}, A_t\}$ is the optimal exchange rate industrial policy allocation, the competitive equilibrium conditions (C4)–(C8) with the capital control policy $\{\tau_t^B\}_{t=0}^{\infty}$ are equivalent to the XR-IP and, therefore, attain the some allocation.

C.2.2. Quantitative analysis

Figure C1: Transitional Dynamics: Capital Controls as Industrial Policy



Notes: This figure shows the sequence of capital controls (savings subsidies) that can be used instead of foreign exchange interventions to replicate the allocation of the optimal exchange rate industrial policy during the transitional dynamics in the quantitative model. For additional details of the economy in which the government can use subsidies on savings instead of foreign exchange interventions, see Section 4.1. For additional details on the model parameterization, see Section 3.2.

C.3. Labor market characteristics

C.3.1. Proof of Proposition 7: Labor supply

The first-order conditions that characterize the solution to the household's problem are

$$\begin{aligned}
\left(\frac{1-\omega}{C_{Nt}}\right)^{\frac{1}{\eta}} &= p_t \left(\frac{\omega}{C_{Tt}}\right)^{\frac{1}{\eta}}, \\
\left(\frac{\omega}{C_{Tt}}\right)^{\frac{1}{\eta}} C_t^{\frac{1}{\eta}-\sigma} \frac{W_{Tt}}{P_{Tt}} &= \phi L_{Tt}^{\nu}, \\
\left(\frac{1-\omega}{C_{Nt}}\right)^{\frac{1}{\eta}} C_t^{\frac{1}{\eta}-\sigma} \frac{W_{Nt}}{P_{Nt}} &= \phi L_{Nt}^{\nu}, \\
\left(\frac{\omega}{C_{Tt}}\right)^{\frac{1}{\eta}} C_t^{\frac{1}{\eta}-\sigma} &= \beta R_{t+1} \frac{P_{Tt}}{P_{Tt+1}} \left(\frac{\omega}{C_{Tt+1}}\right)^{\frac{1}{\eta}} C_{t+1}^{\frac{1}{\eta}-\sigma}.
\end{aligned}$$

Substituting aggregate labor demand from the firms' problem for $i = T, N$

$$\alpha A_t L_{it}^{\alpha+\gamma_{it}-1} = W_{it}/P_{it},$$

for the competitive equilibrium then

$$\frac{\phi L_{Tt}^{\nu}}{(\omega/C_{Tt})^{\frac{1}{\eta}} C_t^{\frac{1}{\eta}-\sigma}} = \alpha A_t L_{Tt}^{\alpha+\gamma_{Tt}-1},$$
$$\frac{\phi L_{Nt}^{\nu}}{((1-\omega)/C_{Nt})^{\frac{1}{\eta}} C_t^{\frac{1}{\eta}-\sigma}} = \alpha A_t L_{Nt}^{\alpha-1}.$$

Substituting nontradable market clearing $C_{Nt} = A_t L_{Nt}^{\alpha}$ gives

$$\frac{\phi}{(\omega/C_{Tt})^{\frac{1}{\eta}} C_t^{\frac{1}{\eta}-\sigma}} = \alpha A_t L_{Tt}^{\alpha+\gamma_{Tt}-1-\nu},$$
$$\frac{\phi}{(1-\omega)^{\frac{1}{\eta}} C_t^{\frac{1}{\eta}-\sigma}} = \alpha A_t^{1-\frac{1}{\eta}} L_{Nt}^{\alpha-1-\frac{\alpha}{\eta}-\nu}.$$

These will serve as implementability conditions for the XR-IP problem.

For Cole and Obstfeld (1991) preferences ($\sigma = \eta = 1$), the second equation is given by

$$\frac{\phi}{(1-\omega)^{\frac{1}{\eta}}} = \alpha L_{Nt}^{-1-\nu}$$
$$L_{Nt} = \left[\frac{\alpha (1-\omega)^{\frac{1}{\eta}}}{\phi}\right]^{\frac{1}{1+\nu}},$$

so the nontradable block $\{C_{Nt}, L_{Nt}\}$ is also exogenous in the economy with elastic labor supply. The tradables constraint is given by

$$\frac{\phi}{(\omega/C_{Tt})} = \alpha A_t L_{Tt}^{\alpha+\gamma_{Tt}-1-\nu},$$

and similarly for the FB allocation.

We now derive the approximate problem as in Lemma 1, where the BT allocation is

$$\overline{C}_T = \overline{A}\ \overline{L}_T^{\alpha+\gamma},$$
$$\frac{\phi}{\left(\omega/\overline{C}_T\right)} = (\alpha+\gamma)\overline{A}\ \overline{L}_T^{\alpha+\gamma-1-\nu}.$$

Therefore, in the BT allocation

$$\overline{L}_T = \left[\frac{\omega(\alpha+\gamma)}{\phi}\right]^{\frac{1}{1+\nu}}.$$

The first-order loglinear approximation of the MRS = MRT constraint is

$$z_t = \psi_t + (\alpha+\gamma-1-\nu)x_t.$$

The welfare function is

$$\mathbb{W}_0 = \sum_{t=0}^{\infty}\beta^t\left[\omega\log C_{Tt} - \phi\frac{L_{Tt}^{1+\nu}}{1+\nu}\right],$$

where the other terms are exogenous. A second-order Taylor expansion for the tradable labor term around the BT is

$$-\phi\frac{L_{Tt}^{1+\nu}}{1+\nu} = -\phi\frac{\overline{L}_T^{1+\nu}}{1+\nu}e^{(1+\nu)l_{Tt}} = -\phi\frac{\overline{L}_T^{1+\nu}}{1+\nu} - \phi\overline{L}_T^{1+\nu}l_{Tt} - \frac{1}{2}\phi(1+\nu)\overline{L}_T^{1+\nu}l_{Tt}^2.$$

Therefore, welfare in terms of deviations and ignoring terms independent of c_{Tt}, l_{Tt} is

$$\mathbb{W}_0 = \sum_{t=0}^{\infty}\beta^t\left[\omega c_{Tt} - \phi\overline{L}_T^{1+\nu}(l_{Tt} - \frac{1}{2}(1+\nu)l_{Tt}^2)\right],$$

and similarly for the FB $\tilde{\mathbb{W}}_0$.

Taking a second-order approximation of the resource constraint gives

$$\begin{aligned}\mathbb{W}_0 - \tilde{\mathbb{W}}_0 = & -\sum_{t=0}^{\infty}\beta^t\omega\left[\frac{1}{2}c_{Tt}^2 - (\alpha+\gamma)l_{Tt} - \frac{1}{2}(\alpha+\gamma)^2 l_{Tt}^2\right] \\ & +\sum_{t=0}^{\infty}\beta^t\omega\left[\frac{1}{2}\tilde{c}_{Tt}^2 - (\alpha+\gamma)\tilde{l}_{Tt} - \frac{1}{2}(\alpha+\gamma)^2\tilde{l}_{Tt}^2\right] \\ & -\sum_{t=0}^{\infty}\beta^t\left[\phi\overline{L}_T^{1+\nu}(l_{Tt} - \frac{1}{2}(1+\nu)l_{Tt}^2)\right] + \sum_{t=0}^{\infty}\beta^t\left[\phi\overline{L}_T^{1+\nu}(\tilde{l}_{Tt} - \frac{1}{2}(1+\nu)\tilde{l}_{Tt}^2)\right] \\ = & -\frac{1}{2}\sum_{t=0}^{\infty}\beta^t\left[\omega z_t^2 + \left[\omega(\alpha+\gamma)^2 + (1+\nu)\omega(\alpha+\gamma)\right]x_t^2\right],\end{aligned}$$

following the same steps as Lemma 1 and using that $\phi\overline{L}_T^{1+\nu} = \omega(\alpha+\gamma)$.

Therefore, the approximate XR-IP problem relative to the FB is

$$\begin{aligned}\max_{\{z_t,x_t,\check{f}_{t+1}^*\}_{t=0}^{\infty}} \quad & -\frac{1}{2}\sum_{t=0}^{\infty}\beta^t\left[\omega z_t^2 + \left[\omega(\alpha+\gamma)^2 + (1+\nu)\omega(\alpha+\gamma)\right]x_t^2\right] \\ s.t. \quad & z_t = \psi_t + (\alpha+\gamma-1-\nu)x_t, \\ & z_t - (\alpha+\gamma)x_t = R^*\check{f}_t^* - \check{f}_{t+1}^*, \\ & \check{f}_0^* = 0.\end{aligned}$$

Combining the constraints and iterating gives

$$\sum_{t=0}^{\infty}\beta^t\left[\psi_t - (1+\nu)x_t\right] = 0,$$

imposing the transversality condition for net foreign assets $\lim_{s\to\infty}\beta^s f_{t+s}^* = 0$.

We can solve the XR-IP problem for x_t

$$\begin{aligned}\max_{\{x_t\}_{t=0}^{\infty}} \quad & -\frac{1}{2}\sum_{t=0}^{\infty}\beta^t\left[\omega(\psi_t + (\alpha+\gamma-1-\nu)x_t)^2 + \left[\omega(\alpha+\gamma)^2 + (1+\nu)\omega(\alpha+\gamma)\right]x_t^2\right] \\ s.t. \quad & \sum_{t=0}^{\infty}\beta^t\left[\psi_t - (1+\nu)x_t\right] = 0.\end{aligned}$$

Let λ be the multiplier on the lifetime resource constraint. The first-order condition for

x_t is

$$-\beta^t \left[\omega(\psi_t + (\alpha+\gamma-1-\nu)x_t)(\alpha+\gamma-1-\nu) + \left[\omega(\alpha+\gamma)^2 + (1+\nu)\omega(\alpha+\gamma)\right] x_t\right] = \beta^t(1+\nu)\lambda.$$

We get a loglinear Euler equation to characterize the XR-IP solution

$$\psi_t + \left[(\alpha+\gamma-1-\nu)+c\right] x_t^{IP} = \psi_{t+1} + \left[(\alpha+\gamma-1-\nu)+c\right] x_{t+1}^{IP},$$

where $c = \frac{(\alpha+\gamma)^2+(1+\nu)(\alpha+\gamma)}{(\alpha+\gamma-1-\nu)} < 0$. Therefore,

$$x_t^{IP} = \frac{(\psi_0 - \psi_t)}{(\alpha+\gamma-1-\nu+c)} + x_0^{IP}.$$

The LF-CE is characterized by

$$\psi_t + (\alpha+\gamma-1-\nu)x_t^{CE} = \psi_{t+1} + (\alpha+\gamma-1-\nu)x_{t+1}^{CE},$$

and

$$x_t^{CE} = \frac{(\psi_0 - \psi_t)}{(\alpha+\gamma-1-\nu)} + x_0^{CE}.$$

Substituting into the lifetime resource constraint

$$x_0^{IP} = \frac{(1-\beta)}{(1+\nu)}\psi_0 - \frac{\beta\psi_0}{(\alpha+\gamma-1-\nu+c)} + \frac{(1-\beta)}{(1+\nu)}\frac{(\alpha+\gamma+c)}{(\alpha+\gamma-1-\nu+c)}\sum_{t=1}^{\infty}\beta^t\psi_t.$$

Similarly for the LF-CE gives

$$x_0^{CE} = \frac{(1-\beta)}{(1+\nu)}\psi_0 - \frac{\beta\psi_0}{(\alpha+\gamma-1-\nu)} + \frac{(1-\beta)}{(1+\nu)}\frac{(\alpha+\gamma)}{(\alpha+\gamma-1-\nu)}\sum_{t=1}^{\infty}\beta^t\psi_t.$$

Therefore

$$\begin{aligned}
x_0^{IP} - x_0^{CE} &= \frac{\beta\psi_0}{(\alpha+\gamma-1-\nu+c)(\alpha+\gamma-1-\nu)}c \\
&\quad + \frac{(1-\beta)\sum_{t=1}^{\infty}\beta^t\psi_t}{(1+\nu)(\alpha+\gamma-1-\nu+c)(\alpha+\gamma-1-\nu)}\left[c(\alpha+\gamma-1-\nu)-(\alpha+\gamma)c\right] \\
&= \underbrace{\frac{\beta}{(\alpha+\gamma-1-\nu+c)(\alpha+\gamma-1-\nu)}c}_{<0}\ \underbrace{\left[\psi_0 - \frac{(1-\beta)}{\beta}\sum_{t=1}^{\infty}\beta^t\psi_t\right]}_{<0} \\
&> \frac{\beta}{(\alpha+\gamma-1-\nu+c)(\alpha+\gamma-1-\nu)}c\left[\psi_0 - \frac{(1-\beta)}{\beta}\frac{\beta}{1-\beta}\psi_0\right] \\
&= 0.
\end{aligned}$$

To determine the sign of $\frac{\partial(x_0^{IP}-x_0^{CE})}{\partial\nu}$, observe that

$$\frac{\partial(x_0^{IP}-x_0^{CE})}{\partial\nu} = \underbrace{\frac{\partial Q}{\partial\nu}}_{>0}\underbrace{\left[\psi_0 - \frac{(1-\beta)}{\beta}\sum_{t=1}^{\infty}\beta^t\psi_t\right]}_{<0} < 0,$$

$$\text{where } Q \equiv \frac{\beta}{(\alpha+\gamma-1-\nu+c)(\alpha+\gamma-1-\nu)}c.$$

To see that $\frac{\partial Q}{\partial\nu} > 0$, note that $\frac{\partial c}{\partial\nu} = \frac{2(\alpha+\gamma)^2}{(\alpha+\gamma-1-\nu)^2} > 0$. Then

$$\frac{\partial Q}{\partial\nu} = \underbrace{\frac{\beta}{\left[(\alpha+\gamma-1-\nu+c)(\alpha+\gamma-1-\nu)\right]^2}}_{>0} \times \underbrace{\left[2(\alpha+\gamma)^2 + c(\alpha+\gamma-1-\nu) + c(\alpha+\gamma-1-\nu+c)\right]}_{>0}.$$

This implies $(x_0^{IP} - x_0^{CE}) > 0$, i.e., the approximation of $\frac{L_{T0}^{IP}}{L_{T0}^{CE}} > 1$, is decreasing in ν. The Frisch elasticity of labor supply is ν^{-1}, so if the labor supply becomes more elastic ν decreases, then $\frac{L_{T0}^{IP}}{L_{T0}^{CE}}$ increases. For both the XR-IP and LF-CE

$$C_{Tt} = \frac{\omega\alpha A_t}{\phi}\frac{1}{L_{Tt}^{1-\alpha-\gamma_{Tt}}},$$
$$\mathcal{E}_t = \frac{\omega}{1-\omega}\frac{C_{Nt}}{C_{Tt}},$$

where C_{Nt} coincides for the XR-IP and LF-CE. The results for L_{T0} then imply that

$$\begin{aligned}\frac{\mathcal{E}_0^{IP}}{\mathcal{E}_0^{CE}} &> 1,\\ \frac{C_{T0}^{CE}}{C_{T0}^{IP}} &> 1,\\ \frac{CA_0^{IP}}{CA_0^{CE}} &> 1,\\ \frac{F_1^{*IP}}{F_1^{*CE}} &> 1,\end{aligned}$$

are decreasing in ν, which shows the Proposition.

C.3.2. Fixed labor supply

We first characterize the competitive equilibrium and solve the optimal exchange rate industrial policy problem with fixed labor supply, then characterize the solution relative to the laissez-faire competitive equilibrium.

With fixed labor supply of 1 unit by the households, the labor market-clearing condition is $L_{Tt} + L_{Nt} = 1$.

Competitive equilibrium. From the households' first-order conditions for C_{Tt} and C_{Nt}, combined with the firms' optimal labor demand, and nontradable goods market-clearing

$$\begin{aligned}\left(\frac{(1-\omega)}{\omega}\frac{C_{Tt}}{C_{Nt}}\right)^{\frac{1}{\eta}} &= \frac{L_{Tt}^{\alpha+\gamma_{Tt}-1}}{L_{Nt}^{\alpha-1}}\\ \left(\frac{(1-\omega)}{\omega}C_{Tt}\right)^{\frac{1}{\eta}} &= \frac{A_t^{\frac{1}{\eta}}L_{Tt}^{\alpha+\gamma_{Tt}-1}}{(1-L_{Tt})^{\alpha-1-\frac{\alpha}{\eta}}},\end{aligned}$$

which characterizes the competitive equilibrium allocation and is the implementability condition for the optimal exchange rate industrial policy problem. The remaining conditions for the laissez-faire competitive equilibrium are as in the baseline model.

Exchange rate industrial policy. The optimal exchange rate industrial policy problem with fixed labor supply is

$$\begin{aligned}
&\max_{\{C_{it},L_{it},F^*_{t+1}\}^{i=T,N}_{t\geq 0}} \sum_{t=0}^{\infty} \beta^t \frac{C_t^{1-\sigma}}{1-\sigma} \quad \text{subject to} \\
&\left(\frac{1-\omega}{\omega}\frac{C_{Tt}}{C_{Nt}}\right)^{\frac{1}{\eta}} = \frac{L_{Tt}^{\alpha+\gamma_{Tt}-1}}{L_{Nt}^{\alpha-1}}, \\
&L_{Tt} + L_{Nt} = 1, \\
&C_{Tt} - A_t L_{Tt}^{\alpha+\gamma_{Tt}} = R^* F_t^* - F_{t+1}^*,
\end{aligned}$$

the consumption aggregator definition (2), and the market-clearing conditions for nontradable goods (12).

Analytical case. Suppose that the economy starts below the steady-state level of productivity and converges to it in the next period. After substituting for nontradable consumption and labor, the XR-IP problem is given by

$$\begin{aligned}
\max_{\{C_{Tt},L_{Tt},F^*_{t+1}\}_{t=0}^{\infty}} \quad & \sum_{t=0}^{\infty} \beta^t [\omega \log C_{Tt} + (1-\omega)\alpha \log(1-L_{Tt})] \\
s.t. \quad & \frac{\omega}{C_{Tt}} = \frac{(1-L_{Tt})^{-1}}{A_t L_{Tt}^{\alpha+\gamma_{Tt}-1}}(1-\omega), \qquad \text{(C9)} \\
& C_{Tt} - A_t L_{Tt}^{\alpha+\gamma_{Tt}} = R^* F_t^* - F_{t+1}^*, \\
& F_0^* \text{ given.}
\end{aligned}$$

Substituting out C_{Tt} gives

$$\begin{aligned}
\max_{\{L_{Tt},F^*_{t+1}\}_{t=0}^{\infty}} \quad & \sum_{t=0}^{\infty} \beta^t [\omega(\alpha+\gamma_{Tt}-1)\log L_{Tt} + [\omega + (1-\omega)\alpha]\log(1-L_{Tt})] + constant \\
s.t. \quad & \frac{\omega}{(1-\omega)} A_t L_{Tt}^{\alpha+\gamma_{Tt}-1} - \frac{1}{(1-\omega)} A_t L_{Tt}^{\alpha+\gamma_{Tt}} = R^* F_t^* - F_{t+1}^*, \\
& F_0^* \text{ given.}
\end{aligned}$$

The first-order conditions are

$$\begin{aligned}
&\omega(\alpha+\gamma_{Tt}-1)\frac{1}{L_{Tt}}-[\omega+(1-\omega)\alpha]\frac{1}{(1-L_{Tt})}\\
&=\lambda_t\left[\frac{\omega}{(1-\omega)}(\alpha+\gamma_{Tt}-1)A_tL_{Tt}^{\alpha+\gamma_{Tt}-2}-\frac{1}{(1-\omega)}(\alpha+\gamma_{Tt})A_tL_{Tt}^{\alpha+\gamma_{Tt}-1}\right],\\
\lambda_t&=\beta R^*\lambda_{t+1}.
\end{aligned}$$

From the first first-order condition

$$\frac{(1-L_{Tt})^{-1}}{A_tL_{Tt}^{\alpha+\gamma_{Tt}-1}}(1-\omega)\left[\frac{\omega(\alpha+\gamma_{Tt}-1)\frac{1}{L_{Tt}}-(\alpha+\omega\gamma_{Tt})}{\omega(\alpha+\gamma_{Tt}-1)\frac{1}{L_{Tt}}-(\alpha+\gamma_{Tt})}\right]=\lambda_t.$$

Substituting λ_t from the second first-order condition gives the modified XR-IP Euler equation in this case

$$\begin{aligned}
\left(\frac{\omega}{C_{Tt}}\right)&=\beta R^*\frac{\theta(L_{Tt+1},\gamma_{Tt+1})}{\theta(L_{Tt},\gamma_{Tt})}\left(\frac{\omega}{C_{Tt+1}}\right),\\
\theta(L_{Tt},\gamma_{Tt})&\equiv\frac{\omega(\alpha+\gamma_{Tt}-1)\frac{1}{L_{Tt}}-(\alpha+\omega\gamma_{Tt})}{\omega(\alpha+\gamma_{Tt}-1)\frac{1}{L_{Tt}}-(\alpha+\gamma_{Tt})}\in(0,1].
\end{aligned}\tag{C10}$$

We now characterize the optimal XR-IP solution.

For $t\geq 1$ with $\gamma_{Tt}=0$, from (C10) $\theta_t=\theta_{t+1}=1$, therefore

$$\begin{aligned}
L_{T1}&=\left[\frac{A_1}{A_{t+1}}\right]^{\frac{1}{1-\alpha}}L_{Tt+1},\\
C_{T1}&=C_{Tt+1}.
\end{aligned}\tag{C11}$$

For $t=0$

$$\begin{aligned}
\frac{(1-L_{T0})^{-1}}{A_0L_{T0}^{\alpha+\gamma_{T0}-1}}\theta_0&=\frac{(1-L_{T1})^{-1}}{A_1L_{T1}^{\alpha-1}},\\
\Rightarrow(1-L_{T0})A_0L_{T0}^{\alpha+\gamma_{T0}-1}&=(1-L_{T1})A_1L_{T1}^{\alpha-1}\theta_0.
\end{aligned}\tag{C12}$$

The sequence of foreign currency bonds $\{F_{t+1}^*\}_{t=0}^{\infty}$ given F_0^* is determined by the balance

of payments

$$C_{T0} - A_0 L_{T0}^{\alpha+\gamma_{T0}} = R^* F_0^* - F_1^*,$$
$$C_{Tt} - A_t L_{Tt}^{\alpha} = R^* F_t^* - F_{t+1}^* \text{ for } t \geq 1.$$

Substituting (C11) and C_{T0}, C_{T1} from (C9), and iterating the second balance of payments equation forward

$$\frac{\beta}{(1-\beta)} \frac{\omega}{(1-\omega)} A_1 (1 - L_{T1}) L_{T1}^{\alpha-1} - A_1 \widetilde{A}_1 L_{T1}^{\alpha} = F_1^*,$$

and substituting for F_1^* into the first balance of payments equation

$$\frac{\omega}{(1-\omega)} A_0 (1 - L_{T0}) L_{T0}^{\alpha+\gamma_{T0}-1} - A_0 L_{T0}^{\alpha+\gamma_{T0}} + \frac{\beta}{(1-\beta)} \frac{\omega}{(1-\omega)} A_1 (1 - L_{T1}) L_{T1}^{\alpha-1} - A_1 \widetilde{A}_1 L_{T1}^{\alpha} - \frac{1}{\beta} F_0^* = 0.$$

Substituting the optimality condition (C12)

$$H(L_{T0}, \theta_0(L_{T0})) = L_{T1}, \tag{C13}$$
$$H(L_{T0}, \theta_0(L_{T0})) \equiv \frac{1}{(A_1 \widetilde{A}_1)^{\frac{1}{\alpha}}} \left\{ \frac{\omega}{(1-\omega)} A_0 (1 - L_{T0}) L_{T0}^{\alpha+\gamma_{T0}-1} - A_0 L_{T0}^{\alpha+\gamma_{T0}} \right.$$
$$\left. + \frac{\beta}{(1-\beta)} \frac{\omega}{(1-\omega)} \frac{1}{\theta_0} A_0 (1 - L_{T0}) L_{T0}^{\alpha+\gamma_{T0}-1} - \frac{1}{\beta} F_0^* \right\}^{\frac{1}{\alpha}}.$$

We solve for L_{T0} in the XR-IP by plugging L_{T1} from (C13) into (C12)

$$A_0 L_{T0}^{\alpha+\gamma_{T0}-1} - A_0 L_{T0}^{\alpha+\gamma_{T0}} - (1 - H(L_{T0}, \theta_0)) A_1 (H(L_{T0}, \theta_0))^{\alpha-1} \theta_0 = 0. \tag{C14}$$

Following the same steps to solve for L_{T0} in the laissez-faire competitive equilibrium

$$A_0 L_{T0}^{\alpha+\gamma_{T0}-1} - A_0 L_{T0}^{\alpha+\gamma_{T0}} - (1 - H(L_{T0}, 1)) A_1 (H(L_{T0}, 1))^{\alpha-1} = 0. \tag{C15}$$

We can sign the following

$$\frac{\partial H}{\partial L_{T0}} = \frac{1}{A_1\widetilde{A}_1}\frac{1}{\alpha}H(L_{T0},\theta_0)^{1-\alpha}\Bigg\{\frac{\omega}{1-\omega}\left(-(1-\alpha-\gamma_{T0})A_0L_{T0}^{\alpha+\gamma_{T0}-2}-(\alpha+\gamma_{T0})A_0L_{T0}^{\alpha+\gamma_{T0}-1}\right)$$
$$-(\alpha+\gamma_{T0})A_0L_{T0}^{\alpha+\gamma_{T0}-1}+\frac{\beta}{(1-\beta)}\frac{\omega}{(1-\omega)}\frac{1}{\theta_0}A_0\left[-(1-\alpha-\gamma_{T0})L_{T0}^{\alpha+\gamma_{T0}-2}-(\alpha+\gamma_{T0})L_{T0}^{\alpha+\gamma_{T0}-1}\right]\Bigg\}$$
$$<0,$$
$$\frac{\partial H}{\partial \theta_0} = -\frac{1}{A_1\widetilde{A}_1}\frac{1}{\alpha}H(L_{T0},\theta_0)^{1-\alpha}\frac{\beta}{(1-\beta)}\frac{\omega}{(1-\omega)}\frac{1}{\theta_0^2}(1-L_{T0})L_{T0}^{\alpha+\gamma_{T0}-1}<0.$$

Treating L_{T0} as a function of θ_0 and differentiating (C14) with respect to θ_0 gives

$$-\left[(1-\alpha-\gamma_{T0})A_0L_{T0}^{\alpha+\gamma_{T0}-2}-(\alpha+\gamma_{T0})A_0L_{T0}^{\alpha+\gamma_{T0}-1}\right]\frac{\partial L_{T0}}{\partial\theta_0}$$
$$+A_1(H(L_{T0},\theta_0))^{\alpha-1}\theta_0\left[\frac{\partial H}{\partial L_{T0}}\frac{\partial L_{T0}}{\partial\theta_0}+\frac{\partial H}{\partial\theta_0}\right]$$
$$+(1-\alpha)(1-H(L_{T0},\theta_0))A_1\theta_0(H(L_{T0},\theta_0))^{\alpha-1}\left[\frac{\partial H}{\partial L_{T0}}\frac{\partial L_{T0}}{\partial\theta_0}+\frac{\partial H}{\partial\theta_0}\right]$$
$$-(1-H(L_{T0},\theta_0))A_1(H(L_{T0},\theta_0))^{\alpha-1}=0.$$

Since $\frac{\partial H}{\partial\theta_0}<0$

$$A_1(H(L_{T0},\theta_0))^{\alpha-1}\theta_0\left[\frac{\partial H}{\partial\theta_0}\right]$$
$$+(1-\alpha)(1-H(L_{T0},\theta_0))A_1\theta_0(H(L_{T0},\theta_0))^{\alpha-1}\left[\frac{\partial H}{\partial\theta_0}\right]$$
$$-(1-H(L_{T0},\theta_0))A_1(H(L_{T0},\theta_0))^{\alpha-1}<0.$$

Therefore, the remaining terms must satisfy

$$\frac{\partial L_{T0}}{\partial\theta_0}\Bigg\{-\left[(1-\alpha-\gamma_{T0})A_0L_{T0}^{\alpha+\gamma_{T0}-2}-(\alpha+\gamma_{T0})A_0L_{T0}^{\alpha+\gamma_{T0}-1}\right]$$
$$+A_1(H(L_{T0},\theta_0))^{\alpha-1}\theta_0\left[\frac{\partial H}{\partial L_{T0}}\right]$$
$$+(1-\alpha)(1-H(L_{T0},\theta_0))A_1\theta_0(H(L_{T0},\theta_0))^{\alpha-1}\left[\frac{\partial H}{\partial L_{T0}}\right]\Bigg\}>0.$$

The terms in braces are negative, since $\frac{\partial H}{\partial L_{T0}} < 0$, which means that it must be that

$$\frac{\partial L_{T0}}{\partial \theta_0} < 0.$$

This shows for the XR-IP where $\theta_0 < 1$, then compared with the CE solution to (C15),

$$L_{T0}^{IP} > L_{T0}^{CE}.$$

For both the XR-IP and CE

$$C_{T0} = \frac{\omega}{1-\omega}(1 - L_{T0})A_0 L_{T0}^{\alpha+\gamma_{T0}-1} \Rightarrow \frac{\partial C_{T0}}{\partial L_{T0}} < 0,$$
$$\mathcal{E}_0 = \frac{(1 - L_{T0})^{\alpha-1}}{L_{T0}^{\alpha+\gamma_{T0}-1}} \Rightarrow \frac{\partial \mathcal{E}_0}{\partial L_{T0}} > 0,$$

therefore, since $L_{T0}^{IP} > L_{T0}^{CE}$, $C_{T0}^{IP} < C_{T0}^{CE}$, $\mathcal{E}_0^{IP} > \mathcal{E}_0^{CE}$.

By definition of the current account balance

$$CA_0 = L_{T0}^{\alpha+\gamma_{T0}} - C_{T0} + (R^* - 1)F_0^* \Rightarrow CA_0^{IP} > CA_0^{CE}.$$

Given F_0^*, for both the XR-IP and CE,

$$F_1^* = R^* F_0^* + A_0 L_{T0}^{\alpha+\gamma_{T0}} - C_{T0} \Rightarrow F_1^{*IP} > F_1^{*CE}.$$

This shows the same result as Proposition 3 in the initial period for the model with fixed labor supply.

C.4. Models with capital formation

C.4.1. Proof of Proposition 8: Economy with foreign capital

In this economy, firms in sector $i = T, N$ choose labor and capital to maximize their profits $\Pi_{it} = P_{it}A_t \left(L_{it}^{1-\theta_i} K_{it}^{\theta_i}\right)^{\gamma_{it}} \left(l_{it}^{1-\theta_i} k_{it}^{\theta_i}\right)^{\alpha} - W_{it} l_{it} - \mathcal{E}_t r_{kt} k_{it}$, which gives rise to the following

aggregate labor and capital demand

$$\alpha(1-\theta_i)A_t L_{it}^{(1-\theta_i)(\alpha+\gamma_{it})-1} K_{it}^{\theta_i(\alpha+\gamma_{it})} = W_{it}/P_{it},$$
$$\alpha\theta_i A_t L_{it}^{(1-\theta_i)(\alpha+\gamma_{it})} K_{it}^{\theta_i(\alpha+\gamma_{it})-1} = \mathcal{E}_t r_{kt}/P_{it}.$$

The rest of the world exchanges tradable goods, capital with the firms, and foreign currency bonds with the government of the small open economy, and provides a perfectly elastic supply of funds at the interest rate R^*. This implies that the local rental rate of capital satisfies $r_{kt} = (R^* + \delta - 1)$.

Combining factor demands with the household optimality conditions gives the implementability conditions for the optimal exchange rate industrial policy problem

$$\left(\frac{1-\omega}{\omega}\frac{C_{Tt}}{C_{Nt}}\right)^{\frac{1}{\eta}} = \frac{(1-\theta_T)L_{Tt}^{(1-\theta_T)(\alpha+\gamma_{Tt})-1}K_{Tt}^{\theta_T(\alpha+\gamma_{Tt})}}{(1-\theta_N)L_{Nt}^{(1-\theta_N)\alpha-1}K_{Nt}^{\theta_N\alpha}},$$
$$\frac{\phi}{(\omega/C_{Tt})^{\frac{1}{\eta}}C_t^{\frac{1}{\eta}-\sigma}} = \alpha(1-\theta_T)A_t L_{Tt}^{(1-\theta_T)(\alpha+\gamma_{Tt})-1}K_{Tt}^{\theta_T(\alpha+\gamma_{Tt})},$$
$$\alpha\theta_T A_t L_{Tt}^{(1-\theta_T)(\alpha+\gamma_{Tt})}K_{Tt}^{\theta_T(\alpha+\gamma_{Tt})-1} = R^*+\delta-1,$$
$$\alpha\theta_N A_t L_{Nt}^{(1-\theta_N)\alpha}K_{Nt}^{\theta_N\alpha-1} = \frac{R^*+\delta-1}{\left(\frac{1-\omega}{\omega}\frac{C_{Tt}}{C_{Nt}}\right)^{\frac{1}{\eta}}},$$
$$C_{Tt} - A_t\left(L_{Tt}^{1-\theta_T}K_{Tt}^{\theta_T}\right)^{\alpha+\gamma_{Tt}} + (R^*+\delta-1)(K_{Tt}+K_{Nt}) = R^*F_t^* - F_{t+1}^*,$$
$$C_{Nt} = A_t\left(L_{Nt}^{1-\theta_N}K_{Nt}^{\theta_N}\right)^{\alpha}.$$

For $\theta_N = 0$, combining these equations and nontradable market clearing, the nontradable allocations are independent of intertemporal considerations and given by (21)-(22).

Normalizing $P_{Tt}^* = 1$, and substituting the condition for r_{kt} gives the XR-IP problem to

solve for the tradables block C_{Tt}, L_{Tt}, K_{Tt}, and F^*_{t+1}

$$\begin{aligned}
\max_{\{C_{Tt},L_{Tt},K_{Tt},F^*_{t+1}\}_{t=0}^{\infty}} \quad & \sum_{t=0}^{\infty}\beta^t\left[\omega\log C_{Tt}-\phi L_{Tt}\right]+constant \\
s.t. \quad \left(\frac{\omega}{C_{Tt}}\right) &= \phi\frac{1}{\alpha(1-\theta_T)A_tL_{Tt}^{(1-\theta_T)(\alpha+\gamma_{Tt})-1}K_{Tt}^{\theta_T(\alpha+\gamma_{Tt})}}, \\
\alpha\theta_TA_tL_{Tt}^{(1-\theta_T)(\alpha+\gamma_{Tt})}K_{Tt}^{\theta_T(\alpha+\gamma_{Tt})-1} &= R^*+\delta-1, \\
C_{Tt}-A_t\left(L_{Tt}^{1-\theta_T}K_{Tt}^{\theta_T}\right)^{\alpha+\gamma_{Tt}}+(R^*+\delta-1)K_{Tt} &= R^*F^*_t-F^*_{t+1}, \\
F^*_0 \text{ given.}&
\end{aligned}$$

We show this can be approximated by a quadratic-linear problem similar to Lemma 1. The first best (FB) problem is

$$\begin{aligned}
\max_{\{C_{Tt},C_{Nt},L_{Tt},L_{Nt},K_{Tt},F^*_{t+1}\}_{t=0}^{\infty}} \quad & \sum_{t=0}^{\infty}\beta^t\left[\omega\log C_{Tt}+(1-\omega)\log C_{Nt}-\phi L_{Tt}-\phi L_{Nt}\right] \\
s.t. \quad C_{Tt}-A_t\left(L_{Tt}^{1-\theta_T}K_{Tt}^{\theta_T}\right)^{\alpha+\gamma_{Tt}}+(R^*+\delta-1)K_{Tt} &= R^*F^*_t-F^*_{t+1}, \\
C_{Nt} &= A_tL_{Nt}^{\alpha}, \\
F^*_0 \text{ given.}&
\end{aligned}$$

The FB tradable MRS = social MRT condition is

$$\left(\frac{\omega}{C_{Tt}}\right) = \phi\frac{1}{(\alpha+\gamma_{Tt})(1-\theta_T)A_tL_{Tt}^{(1-\theta_T)(\alpha+\gamma_{Tt})-1}K_{Tt}^{\theta_T(\alpha+\gamma_{Tt})}},$$

and condition for capital is

$$(\alpha+\gamma_{Tt})\theta_TA_tL_{Tt}^{(1-\theta_T)(\alpha+\gamma_{Tt})}K_{Tt}^{\theta_T(\alpha+\gamma_{Tt})-1}=R^*+\delta-1.$$

The reference balanced trade (BT) allocation $\{\overline{C}_T, \overline{L}_T, \overline{K}_T\}$ is given by

$$\begin{aligned}
\overline{C}_T &= \overline{A}\ \left(\overline{L}_T^{1-\theta_T}\overline{K}_T^{\theta_T}\right)^{\alpha+\gamma} - (R^* + \delta - 1)\overline{K}_T \equiv \overline{Y}_T \\
\frac{\phi}{(\omega/\overline{C}_T)} &= (\alpha+\gamma)(1-\theta_T)\overline{A}\ \overline{L}_T^{(1-\theta_T)(\alpha+\gamma)-1}\overline{K}_T^{\theta_T(\alpha+\gamma)}, \\
(\alpha+\gamma)\theta_T\overline{A}\ \overline{L}_T^{(1-\theta_T)(\alpha+\gamma)}\overline{K}_T^{\theta_T(\alpha+\gamma)-1} &= (R^* + \delta - 1)
\end{aligned}$$

where γ is defined below. Therefore, in the BT allocation

$$\overline{L}_T = \frac{\omega(\alpha+\gamma)(1-\theta_T)}{\phi(1-(\alpha+\gamma)\theta_T)}.$$

We take a second-order approximation of the welfare function around the BT allocation as for the baseline model.

A first-order approximation of the resource constraint relative to the BT gives

$$\begin{aligned}
c_{Tt} - \frac{1}{(1-(\alpha+\gamma)\theta_T)}\Big[a_t + (1-\theta_T)(\alpha+\gamma)l_{Tt} + (1-\theta_T)(\gamma_{Tt}-\gamma)\log\overline{L}_T + \theta_T(\gamma_{Tt}-\gamma)\log\overline{K}_T\Big] \\
= R^* f_t^* - f_{t+1}^*.
\end{aligned}$$

For welfare, a second-order approximation of the LHS, iterating the resource constraint, and using the transversality condition gives

$$\begin{aligned}
\sum_{t=0}^{\infty}\beta^t c_{Tt} = &-\sum_{t=0}^{\infty}\beta^t\Bigg[\frac{1}{2}c_{Tt}^2 - \frac{1}{(1-(\alpha+\gamma)\theta_T)}\Big[a_t + \frac{1}{2}a_t^2 + (1-\theta_T)(\alpha+\gamma)l_{Tt} + \frac{1}{2}(1-\theta_T)^2(\alpha+\gamma)^2 l_{Tt}^2 \\
&+ \frac{1}{2}[\theta_T^2(\alpha+\gamma)^2 - (\alpha+\gamma)\theta_T]k_{Tt}^2 + (1-\theta_T)(\gamma_{Tt}-\gamma)\log\overline{L}_T + \frac{1}{2}(1-\theta_T)^2(\gamma_{Tt}-\gamma)^2(\log\overline{L}_T)^2 \\
&+ \theta_T(\gamma_{Tt}-\gamma)\log\overline{K}_T + \frac{1}{2}\theta_T^2(\gamma_{Tt}-\gamma)^2(\log\overline{K}_T)^2\Big]\Bigg] + \frac{1}{\beta}f_0^*,
\end{aligned}$$

and similarly for the FB allocation relative to the BT. Taking the difference in welfare and substituting the iterated resource constraint gives

$$\mathbb{W}_0 - \tilde{\mathbb{W}}_0 = -\frac{1}{2}\sum_{t=0}^{\infty}\beta^t\left[\omega z_t^2 + \omega\frac{(1-\theta_T)^2(\alpha+\gamma)^2 + (1-\theta_T)(\alpha+\gamma)}{(1-(\alpha+\gamma)\theta_T)}x_t^2 + \omega\theta_T(\alpha+\gamma)u_t^2\right],$$

denoting deviations from the FB $z_t \equiv \log C_{Tt} - \log \tilde{C}_{Tt}$, $x_t \equiv \log L_{Tt} - \log \tilde{L}_{Tt}$, $u_t \equiv \log K_{Tt} - \log \tilde{K}_{Tt}$. This uses $\overline{L}_T$, and the interaction terms are zero to second order similar to the baseline model. γ is given by

$$\sum_{t=0}^{\infty} \beta^t \Big[[\tilde{l}_{Tt} \frac{(1-\theta_T)^2(\alpha+\gamma)^2}{(1-(\alpha+\gamma)\theta_T)^2} + s_t - \frac{(1-\theta_T)^2(\alpha+\gamma)^2 + (1-\theta_T)(\alpha+\gamma)}{(1-(\alpha+\gamma)\theta_T)} l_{Tt}](l_{Tt} - \tilde{l}_{Tt}) \\ -\omega\theta_T(\alpha+\gamma)\tilde{k}_{Tt}(k_{Tt} - \tilde{k}_{Tt}) \Big] = 0$$

where $s_t = \frac{(1-\theta_T)(\alpha+\gamma)}{(1-(\alpha+\gamma)\theta_T)^2} \Big[a_t + (1-\theta_T)(\gamma_{Tt} - \gamma) \log \overline{L}_T + \theta_T(\gamma_{Tt} - \gamma) \log \overline{K}_T + R^* \tilde{f}_t^* - \tilde{f}_{t+1}^* \Big]$.

Solving the constraints in terms of z_t, x_t, u_t gives the approximate XR-IP problem

$$\max_{\{z_t, x_t, u_t, \check{f}_{t+1}^*\}_{t=0}^{\infty}} \quad -\frac{1}{2} \sum_{t=0}^{\infty} \beta^t \left[\omega z_t^2 + \omega \frac{(1-\theta_T)^2(\alpha+\gamma)^2 + (1-\theta_T)(\alpha+\gamma)}{(1-(\alpha+\gamma)\theta_T)} x_t^2 + \omega\theta_T(\alpha+\gamma) u_t^2 \right]$$

$$s.t. \quad z_t = \psi_t + [(1-\theta_T)(\alpha+\gamma) - 1] x_t + \theta_T(\alpha+\gamma) u_t,$$

$$\psi_t + (1-\theta_T)(\alpha+\gamma) x_t + [\theta_T(\alpha+\gamma) - 1] u_t = 0,$$

$$z_t - \frac{(1-\theta_T)(\alpha+\gamma)}{(1-(\alpha+\gamma)\theta_T)} x_t = R^* \check{f}_t^* - \check{f}_{t+1}^*,$$

$$\check{f}_0^* = 0.$$

Combining the first and second constraints gives

$$z_t = \frac{1}{1-\theta_T(\alpha+\gamma)} \psi_t + \frac{\alpha+\gamma-1}{1-\theta_T(\alpha+\gamma)} x_t.$$

Substituting for z_t, $\beta R^* = 1$ into the third constraint, iterating and using the transversality condition gives

$$\sum_{t=0}^{\infty} \beta^t \left[\psi_t - x_t \right] = 0.$$

The XR-IP problem simplifies to

$$\max_{\{x_t\}_{t=0}^{\infty}} \quad -\frac{1}{2}\sum_{t=0}^{\infty}\beta^t\left[\frac{(\psi_t+(\alpha+\gamma-1)x_t)^2}{1-\theta_T(\alpha+\gamma)}+[(1-\theta_T)^2(\alpha+\gamma)^2+(1-\theta_T)(\alpha+\gamma)]x_t^2\right.$$
$$\left.+\frac{\theta_T(\alpha+\gamma)}{1-\theta_T(\alpha+\gamma)}\left(\psi_t+(1-\theta_T)(\alpha+\gamma)x_t\right)^2\right]$$
$$s.t. \quad \sum_{t=0}^{\infty}\beta^t\left[\psi_t-x_t\right]=0.$$

This gives a loglinear Euler equation to characterize the XR-IP solution

$$\psi_t+H^{IP}x_t^{IP}=\psi_{t+1}+H^{IP}x_{t+1}^{IP},$$

where

$$H^{IP}\equiv\frac{(\alpha+\gamma-1)^2+(1-\theta_T)^2(\alpha+\gamma)^2+(1-\theta_T)(\alpha+\gamma)-\theta_T(1-\theta_T)(\alpha+\gamma)^2}{\alpha+\gamma-1+\theta_T(1-\theta_T)(\alpha+\gamma)^2}.$$

We show $H^{IP}-(\alpha+\gamma-1)<0$, which will be used below.

$$H^{IP}-(\alpha+\gamma-1)=\frac{(1-\theta_T)^2(\alpha+\gamma)^2+(1-\theta_T)(\alpha+\gamma)-\theta_T(1-\theta_T)(\alpha+\gamma)^3}{\alpha+\gamma-1+\theta_T(1-\theta_T)(\alpha+\gamma)^2},$$

where the numerator is positive since

$$(1-\theta_T)^2(\alpha+\gamma)^2+(1-\theta_T)(\alpha+\gamma)-\theta_T(1-\theta_T)(\alpha+\gamma)^3$$
$$>(1-\theta_T)^2(\alpha+\gamma)^2+\theta(1-\theta_T)(\alpha+\gamma)^3-\theta_T(1-\theta_T)(\alpha+\gamma)^3>0,$$

and the denominator is negative since

$$\alpha+\gamma-1+\theta_T(1-\theta_T)(\alpha+\gamma)^2<\alpha+\gamma-1+(\alpha+\gamma)^2<\alpha+\gamma-1+\alpha+\gamma<0.$$

Therefore, $H^{IP}<0$.

To solve for x_0^{IP}, use the loglinear Euler equation and lifetime resource constraint

$$x_0^{IP} = (1-\beta)\psi_0 - \frac{\beta\psi_0}{H^{IP}} + (1-\beta)\frac{H^{IP}+1}{H^{IP}}\sum_{t=1}^{\infty}\beta^t\psi_t.$$

The LF-CE is characterized by

$$\begin{aligned} z_t &= z_{t+1}, \\ z_t &= \psi_t + [(1-\theta_T)(\alpha+\gamma)-1]x_t + \theta_T(\alpha+\gamma)u_t, \\ \psi_t + (1-\theta_T)(\alpha+\gamma)x_t + [\theta_T(\alpha+\gamma)-1]u_t &= 0, \\ z_t - \frac{(1-\theta_T)(\alpha+\gamma)}{(1-(\alpha+\gamma)\theta_T)}x_t &= R^*\check{f}_t^* - \check{f}_{t+1}^*, \\ \check{f}_0^* &= 0. \end{aligned}$$

Combining these conditions gives

$$\psi_t + H^{CE}x_t^{CE} = \psi_{t+1} + H^{CE}x_{t+1}^{CE},$$

where $H^{CE} = (\alpha+\gamma-1) < 0$. Therefore, from the lifetime resource constraint and using $H^{IP} < H^{CE} < 0$ then

$$\begin{aligned} x_0^{IP} - x_0^{CE} &= -\beta\psi_0\left[\frac{1}{H^{IP}} - \frac{1}{H^{CE}}\right] + (1-\beta)\sum_{t=1}^{\infty}\beta^t\psi_t\left[\frac{H^{IP}+1}{H^{IP}} - \frac{H^{CE}+1}{H^{CE}}\right] \\ &> -\beta\psi_0\left[\frac{1}{H^{IP}} - \frac{1}{H^{CE}}\right] + (1-\beta)\sum_{t=1}^{\infty}\beta^t\psi_0\left[\frac{1}{H^{IP}} - \frac{1}{H^{CE}}\right] \\ &= 0. \end{aligned}$$

As in the proof of Proposition 3, from the lifetime resource constraint

$$\underbrace{x_0^{IP} - x_0^{CE}}_{>0} = -\sum_{t=1}^{\infty}\beta^t(x_t^{IP} - x_t^{CE}),$$

so for at least one $t \geq 1$, $(x_t^{IP} - x_t^{CE}) < 0$. Note, for any $t \geq 1$

$$x_t^{IP} - x_t^{CE} = (\psi_0 - \psi_t)\left[\frac{1}{H^{IP}} - \frac{1}{H^{CE}}\right] + x_0^{IP} - x_0^{CE},$$

therefore

$$x_{t+1}^{IP} - x_{t+1}^{CE} - (x_t^{IP} - x_t^{CE}) = -(\psi_{t+1} - \psi_t)\left[\frac{1}{H^{IP}} - \frac{1}{H^{CE}}\right] < 0,$$

since $\psi_{t+1} - \psi_t > 0$ and $\frac{1}{H^{IP}} - \frac{1}{H^{CE}} > 0$, so $(x_t^{IP} - x_t^{CE})$ is strictly decreasing in t. Then, together with $(x_0^{IP} - x_0^{CE}) > 0$ and $(x_t^{IP} - x_t^{CE}) < 0$ for some t it must be that $\exists\ \bar{t} > 0$ such that $(x_t^{IP} - x_t^{CE}) > 0$ (i.e. $L_{Tt}^{IP} > L_{Tt}^{CE}$) for $t < \bar{t}$ and $(x_t^{IP} - x_t^{CE}) < 0$ (i.e. $L_{Tt}^{IP} < L_{Tt}^{CE}$) for $t > \bar{t}$.

To show the results for the remaining variables

$$z_t^{IP} - z_t^{CE} = \underbrace{\frac{\alpha + \gamma - 1}{1 - \theta_T(\alpha + \gamma)}}_{<0}(x_t^{IP} - x_t^{CE}),$$
$$u_t^{IP} - u_t^{CE} = \underbrace{\frac{(1 - \theta_T)(\alpha + \gamma)}{1 - \theta_T(\alpha + \gamma)}}_{>0}(x_t^{IP} - x_t^{CE}),$$

therefore $C_{Tt}^{IP} < C_{Tt}^{CE}$ and $K_{Tt}^{IP} > K_{Tt}^{CE}$ for $t < \bar{t}$ since $L_{Tt}^{IP} > L_{Tt}^{CE}$. For the XR-IP and LF-CE

$$\mathcal{E}_t = \left(\frac{\omega}{1 - \omega}\frac{C_{Nt}}{C_{Tt}}\right),$$

where C_{Nt} coincides for the XR-IP and LF-CE. Therefore, for $t < \bar{t}$ $\mathcal{E}_t^{IP} > \mathcal{E}_t^{CE}$.

By definition of the trade balance

$$\begin{aligned} TB_t &= A_t\left(L_{Tt}^{1-\theta_T} K_{Tt}^{\theta_T}\right)^{\alpha+\gamma_{Tt}} - (R^* + \delta - 1)K_{Tt} - C_{Tt} \\ &= (1 - \alpha)A_t\left(L_{Tt}^{1-\theta_T} K_{Tt}^{\theta_T}\right)^{\alpha+\gamma_{Tt}} - C_{Tt}, \\ \Rightarrow TB_t^{IP} &> TB_t^{CE}, \end{aligned}$$

for $t < \bar{t}$.

It is straight forward that similarly for $t > \bar{t}$ when $L_{Tt}^{IP} < L_{Tt}^{CE}$ that $K_{Tt}^{IP} < K_{Tt}^{CE}$, $C_{Tt}^{IP} > C_{Tt}^{CE}$, $\mathcal{E}_t^{IP} < \mathcal{E}_t^{CE}$, and $TB_t^{IP} < TB_t^{CE}$.

The result $F_{t+1}^{*IP} > F_{t+1}^{*CE}$ for all t follows directly the steps as for Proposition 3.

C.4.2. Economy with domestic capital

In this economy, the household budget constraint expressed in domestic currency is given by

$$P_{Tt}C_{Tt} + P_{Nt}C_{Nt} + B_{t+1} + Q_tK_{t+1} = W_tL_t + \Pi_t + T_t + R_tB_t + r_{kt}K_t + Q_t(1-\delta)K_t,$$

where Q_t is the price of capital. Households purchase K_{t+1} at period t, and in period $t+1$ rent capital to firms and re-sell undepreciated capital at price Q_{t+1}.

The household Euler equations are

$$\left(\frac{\omega}{C_{Tt}}\right)^{\frac{1}{\eta}} C_t^{\frac{1}{\eta}-\sigma} = \beta R_{t+1}\frac{P_{Tt}}{P_{Tt+1}}\left(\frac{\omega}{C_{Tt+1}}\right)^{\frac{1}{\eta}} C_{t+1}^{\frac{1}{\eta}-\sigma},$$
$$\left(\frac{\omega}{C_{Tt}}\right)^{\frac{1}{\eta}} C_t^{\frac{1}{\eta}-\sigma} = \beta\frac{P_{Tt}}{P_{Tt+1}}\left(\frac{r_{kt+1}+Q_{t+1}(1-\delta)}{Q_t}\right)\left(\frac{\omega}{C_{Tt+1}}\right)^{\frac{1}{\eta}} C_{t+1}^{\frac{1}{\eta}-\sigma}.$$

Firms in sector $i = T, N$ choose labor and capital to maximize their profits $\Pi_{it} = P_{it}A_t\left(L_{it}^{1-\theta_i}K_{it}^{\theta_i}\right)^{\gamma_{it}}\left(l_{it}^{1-\theta_i}k_{it}^{\theta_i}\right)^{\alpha} - W_{it}l_{it} - r_{kt}k_{it}$, which gives rise to the following aggregate labor and capital demand

$$\alpha(1-\theta_i)A_tL_{it}^{(1-\theta_i)(\alpha+\gamma_{it})-1}K_{it}^{\theta_i(\alpha+\gamma_{it})} = W_{it}/P_{it},$$
$$\alpha\theta_iA_tL_{it}^{(1-\theta_i)(\alpha+\gamma_{it})}K_{it}^{\theta_i(\alpha+\gamma_{it})-1} = r_{kt}/P_{it}.$$

Capital accumulation is subject to adjustment costs

$$K_{t+1} = \Phi\left(\frac{I_t}{K_t}\right)K_t + (1-\delta)K_t,$$

where I_t is aggregate investment in which units of tradable goods are used to produce new capital and $\Phi\left(\frac{I_t}{K_t}\right) = \frac{\delta^{1/\phi_k}}{1-1/\phi_k}\left(\frac{I_t}{K_t}\right)^{1-1/\phi_k} - \frac{\delta}{\phi_k-1}$. Capital producers choose I_t to maximize

$Q_t\Phi\left(\frac{I_t}{K_t}\right)K_t - P_{Tt}I_t$, which implies

$$\frac{Q_t}{P_{Tt}} = \frac{1}{\Phi'\left(\frac{I_t}{K_t}\right)} = \left(\frac{I_t/K_t}{\delta}\right)^{1/\phi_k}.$$

In the laissez-faire competitive equilibrium, the return on saving in bonds and capital for the household must be equal which gives

$$R^* = \frac{\frac{r_{kt+1}}{P_{Tt+1}} + \frac{Q_{t+1}}{P_{Tt+1}}(1-\delta)}{\frac{Q_t}{P_{Tt}}}. \tag{C16}$$

The balance of payments is

$$C_{Tt} - A_t\left(L_{Tt}^{1-\theta_T}K_{Tt}^{\theta_T}\right)^{\alpha+\gamma_{Tt}} + I_t = R^*F_t^* - F_{t+1}^*.$$

C.4.3. Quantitative analysis

We solve the quantitative model for the economies with capital for the parameterization in Section 3.2, setting the capital share $\theta_T = \theta_N = 1/3$ and annual depreciation rate $\delta = 0.08$. For the economy with domestic capital we set the capital adjustment costs parameter $\phi_k = 4$ following Ottonello and Winberry (2020). When solving for the observed reserves policy, we calibrate the initial capital stock K_0 to target the average capital stock to GDP ratio for 2000-2008 for China of 2.6 (source: IMF). In the model, this moment is 3.0. All other externally set parameters are as the same as in the baseline model, and the calibrated ones follow the same calibration strategy.

C.5. Model with imported inputs

C.5.1. Proof of Proposition 9: Economy with imported inputs

In this economy, firms in sector $i = T, N$ choose labor and imported inputs to maximize their profits $\Pi_{it} = P_{it}A_t\left(L_{it}^{1-\xi_i}M_{it}^{\xi_i}\right)^{\gamma_{it}}\left(l_{it}^{1-\xi_i}m_{it}^{\xi_i}\right)^{\alpha} - W_{it}l_{it} - \mathcal{E}_tP_{Mt}^*m_{it}$, which gives rise to

the following aggregate labor and capital demand

$$\alpha(1-\xi_i)A_t L_{it}^{(1-\xi_i)(\alpha+\gamma_{it})-1} M_{it}^{\xi_i(\alpha+\gamma_{it})} = W_t/P_{it},$$
$$\alpha\xi_i A_t L_{it}^{(1-\xi_i)(\alpha+\gamma_{it})} M_{it}^{\xi_i(\alpha+\gamma_{it})-1} = \mathcal{E}_t P_{Mt}^*/P_{it}.$$

The rest of the world exchanges tradable goods, inputs with the firms at foreign currency price P_{Mt}^*, and foreign currency bonds with the government of the small open economy, and provides a perfectly elastic supply of funds at the interest rate R^*.

Combining factor demands with the household optimality conditions gives the implementability conditions for the optimal exchange rate industrial policy problem

$$\left(\frac{1-\omega}{\omega}\frac{C_{Tt}}{C_{Nt}}\right)^{\frac{1}{\eta}} = \frac{(1-\xi_T)L_{Tt}^{(1-\xi_T)(\alpha+\gamma_{Tt})-1} M_{Tt}^{\xi_T(\alpha+\gamma_{Tt})}}{(1-\xi_N)L_{Nt}^{(1-\xi_N)\alpha-1} M_{Nt}^{\xi_N\alpha}}, \tag{C17}$$

$$\frac{\phi}{(\omega/C_{Tt})^{\frac{1}{\eta}} C_t^{\frac{1}{\eta}-\sigma}} = \alpha(1-\xi_T)A_t L_{Tt}^{(1-\xi_T)(\alpha+\gamma_{Tt})-1} M_{Tt}^{\xi_T(\alpha+\gamma_{Tt})}, \tag{C18}$$

$$\alpha\xi_T A_t L_{Tt}^{(1-\xi_T)(\alpha+\gamma_{Tt})} M_{Tt}^{\xi_T(\alpha+\gamma_{Tt})-1} = P_{Mt}^*, \tag{C19}$$

$$\alpha\xi_N A_t L_{Nt}^{(1-\xi_N)\alpha} M_{Nt}^{\xi_N\alpha-1} = \frac{P_{Mt}^*}{\left(\frac{1-\omega}{\omega}\frac{C_{Tt}}{C_{Nt}}\right)^{\frac{1}{\eta}}}, \tag{C20}$$

$$C_{Tt} - A_t\left(L_{Tt}^{1-\xi_T} M_{Tt}^{\xi_T}\right)^{\alpha+\gamma_{Tt}} + P_{Mt}^*(M_{Tt}+M_{Nt}) = R^* F_t^* - F_{t+1}^*, \tag{C21}$$

$$C_{Nt} = A_t\left(L_{Nt}^{1-\xi_N} M_{Nt}^{\xi_N}\right)^{\alpha}. \tag{C22}$$

For $M_{it} = K_{it}$, $P_{Mt}^* = r_{kt}$, $\xi_i = \theta_i$, equations (C17)-(C22) are equivalent to the economy with foreign capital in Appendix C.4.1. Therefore, for $m_{it} = k_{it}$ these economies are isomorphic.

A corollary is for $\xi_N = 0$ we have the same result as Proposition 8 for the economy with imported inputs.

C.5.2. Quantitative analysis

We solve the quantitative model for the economy with imported inputs for the parameterization in Section 3.2. We normalize $P^*_{Mt} = 1$. When solving for the observed reserves policy, we calibrate $\xi_T = \xi_N = 0.06$ to match China's imports of intermediate goods as a share of GDP from 2000-2008 of 6.4% (source: World Bank). In the model, this moment is 6.4%. All other externally set parameters are as the same as in the baseline model, and the calibrated ones follow the same calibration strategy.

C.6. Multiple tradable sectors

C.6.1. Proof of Proposition 10: Economy with multiple tradable sectors

In this economy, the household budget constraint expressed in domestic currency is given by

$$P_{T1t}C_{T1t} + P_{T2t}C_{T2t} + P_{Nt}C_{Nt} + B_{t+1} = W_{T1t}L_{T1t} + W_{T2t}L_{T2t} + W_{Nt}L_{Nt} + \Pi_t + T_t + R_tB_t.$$

Define the aggregate price of tradables $P_{Tt} = P_{T1t}^{1/2}P_{T2t}^{1/2}$, and similarly for the foreign currency price P^*_{Tt}. Assume the law of one price holds for each variety of tradable good $P_{T1t} = \mathcal{E}_t P^*_{T1t}$ and $P_{T2t} = \mathcal{E}_t P^*_{T2t}$.

The first-order conditions that characterize the solution to the household's problem are

$$\begin{aligned}
\left(\frac{\omega}{2C_{T1t}}\right) &= \frac{P_{T1t}}{P_{T2t}}\left(\frac{\omega}{2C_{T2t}}\right) \\
\left(\frac{1-\omega}{C_{Nt}}\right)^{\frac{1}{\eta}} &= \frac{P_{Nt}}{P_{T1t}}\left(\frac{\omega}{C_{Tt}}\right)^{\frac{1}{\eta}}\left(\frac{C_{Tt}}{2C_{T1t}}\right), \\
\left(\frac{\omega}{C_{Tt}}\right)^{\frac{1}{\eta}} C_t^{\frac{1}{\eta}-\sigma}\left(\frac{C_{Tt}}{2C_{T1t}}\right)\frac{W_{T1t}}{P_{T1t}} &= \phi L_{T1t}^{\nu_1}, \\
\left(\frac{\omega}{C_{Tt}}\right)^{\frac{1}{\eta}} C_t^{\frac{1}{\eta}-\sigma}\left(\frac{C_{Tt}}{2C_{T2t}}\right)\frac{W_{T2t}}{P_{T2t}} &= \phi L_{T2t}^{\nu_2}, \\
\left(\frac{1-\omega}{C_{Nt}}\right)^{\frac{1}{\eta}} C_t^{\frac{1}{\eta}-\sigma}\frac{W_{Nt}}{P_{Nt}} &= \phi L_{Nt}^{\nu_N}, \\
\left(\frac{\omega}{C_{Tt}}\right)^{\frac{1}{\eta}} C_t^{\frac{1}{\eta}-\sigma}\left(\frac{C_{Tt}}{2C_{T1t}}\right) &= \beta R_{t+1}\frac{P_{T1t}}{P_{T1t+1}}\left(\frac{\omega}{C_{Tt+1}}\right)^{\frac{1}{\eta}} C_{t+1}^{\frac{1}{\eta}-\sigma}\left(\frac{C_{Tt+1}}{2C_{T1t+1}}\right).
\end{aligned}$$

Firms in tradable sectors $1, 2$ and the nontradable sector choose labor to maximize their

profits, which gives rise to the following aggregate labor demand

$$\begin{aligned}\alpha A_t L_{T1t}^{\alpha+\gamma_{T1t}-1} &= W_{T1t}/P_{T1t}\\ \alpha A_t L_{T2t}^{\alpha+\gamma_{T2t}-1} &= W_{T2t}/P_{T2t}\\ \alpha A_t L_{Nt}^{\alpha-1} &= W_{Nt}/P_{Nt}.\end{aligned}$$

Normalize $P^*_{T1t} = 1$, which gives $P_{T1t} = \mathcal{E}_t$. Let $p^*_t \equiv \frac{P^*_{T2t}}{P^*_{T1t}} = \frac{P_{T2t}}{P_{T1t}}$ and $p_{1t} \equiv \frac{P_{Nt}}{P_{T1t}}$. Normalize $P_{Nt} \equiv 1$, then $p_{1t} = \mathcal{E}_t^{-1}$. For the competitive equilibrium (CE) for Cole and Obstfeld (1991) preferences ($\sigma = \eta = 1$), the nontradable block is exogenous, and with $\beta R^* = 1$, the laissez-faire CE for $\{C_{T1t}, C_{T2t}, L_{T1t}, L_{T2t}, F^*_{t+1}\}_{t\geq 0}$ is given by

$$\left(\frac{C_{T1t}}{C_{T2t}}\right) = p^*_t, \tag{C23}$$

$$\frac{\phi}{(\omega/(2C_{T1t}))} = \alpha A_t L_{T1t}^{\alpha+\gamma_{T1t}-1-\nu_1}, \tag{C24}$$

$$\frac{\phi}{(\omega/(2C_{T2t}))} = \alpha A_t L_{T2t}^{\alpha+\gamma_{T2t}-1-\nu_2}, \tag{C25}$$

$$C_{T1t} + p^*_t C_{T2t} = A_t L_{T1t}^{\alpha+\gamma_{T1t}} + p^*_t A_t L_{T2t}^{\alpha+\gamma_{T2t}} + R^* F^*_t - F^*_{t+1}, \tag{C26}$$

$$\left(\frac{\omega}{2C_{T1t}}\right) = \left(\frac{\omega}{2C_{T1t+1}}\right).$$

Equations (C23)-(C26) serve as implementability conditions for the XR-IP problem

$$\max_{\{C_{T1t},C_{T2t},L_{T1t},L_{T2t},F^*_{t+1}\}_{t\geq 0}} \sum_{t=0}^{\infty} \beta^t \left[\frac{\omega}{2}\left(\log C_{T1t} + \log C_{T2t}\right) - \phi\frac{L_{T1t}^{1+\nu_1}}{1+\nu_1} - \phi\frac{L_{T2t}^{1+\nu_2}}{1+\nu_2}\right] \quad \text{subject to}$$

$$\begin{aligned}\left(\frac{C_{T1t}}{C_{T2t}}\right) &= p^*_t,\\ \frac{\phi}{(\omega/(2C_{T1t}))} &= \alpha A_t L_{T1t}^{\alpha+\gamma_{T1t}-1-\nu_1},\\ \frac{\phi}{(\omega/(2C_{T2t}))} &= \alpha A_t L_{T2t}^{\alpha+\gamma_{T2t}-1-\nu_2},\\ C_{T1t} + p^*_t C_{T2t} &= A_t L_{T1t}^{\alpha+\gamma_{T1t}} + p^*_t A_t L_{T2t}^{\alpha+\gamma_{T2t}} + R^* F^*_t - F^*_{t+1},\\ F^*_0 &\text{ given.}\end{aligned}$$

We now derive the approximate quadratic-linear problem as in Lemma 1.

The solution to the FB problem

$$\max_{\{C_{T1t},C_{T2t},L_{T1t},L_{T2t},F^*_{t+1}\}_{t\geq 0}} \sum_{t=0}^{\infty} \beta^t \left[\frac{\omega}{2} \left(\log C_{T1t} + \log C_{T2t} \right) + (1-\omega) \log C_{Nt} \right.$$
$$\left. - \phi \frac{L_{T1t}^{1+\nu_1}}{1+\nu_1} - \phi \frac{L_{T2t}^{1+\nu_2}}{1+\nu_2} - \phi \frac{L_{Nt}^{1+\nu_N}}{1+\nu_N} \right] \quad \text{subject to}$$
$$C_{T1t} + p_t^* C_{T2t} = A_t L_{T1t}^{\alpha+\gamma_{T1t}} + p_t^* A_t L_{T2t}^{\alpha+\gamma_{T2t}} + R^* F_t^* - F_{t+1}^*,$$
$$C_{Nt} = A_t L_{Nt}^{\alpha},$$
$$F_0^* \text{ given},$$

gives the optimality condition in the FB

$$\left(\frac{\tilde{C}_{T1t}}{\tilde{C}_{T2t}} \right) = p_t^*.$$

The reference BT allocation is given by

$$\overline{C}_{T1} = \overline{A}\ \overline{L}_{T1}^{\alpha+\gamma_1}$$
$$\frac{\phi}{\left(\omega/(2\overline{C}_{T1})\right)} = (\alpha+\gamma_1)\overline{A}\ \overline{L}_{T1}^{\alpha+\gamma_1-1-\nu_1},$$

and similarly for $T2$. Therefore, in the BT allocation

$$\overline{L}_{T1}^{1+\nu_1} = \frac{\omega(\alpha+\gamma_1)}{2\phi},$$

and similarly for $T2$.

The loglinear constraint on consumption across tradable sectors for the XR-IP is

$$z_{1t} = z_{2t},$$

where $z_{jt} \equiv \log C_{Tjt} - \log \tilde{C}_{Tjt}$. The first-order loglinear approximation of the MRS = MRT constraint in each sector is

$$z_{1t} = \psi_{1t} + (\alpha + \gamma_1 - 1 - \nu_1) x_{1t},$$

and similarly for $T2$, where $x_{jt} \equiv \log L_{Tjt} - \log \tilde{L}_{Tjt}$, $\psi_{jt} \equiv \log \alpha - \log(\alpha + \gamma_{Tjt}) \leq 0$.

The loglinear balance of payments constraint is given by

$$z_{1t} + z_{2t} - (\alpha + \gamma_1)x_{1t} - (\alpha + \gamma_2)x_{2t} = R^* \check{f}_t^* - \check{f}_{t+1}^*.$$

A second-order approximation of the welfare function around the BT allocation gives

$$\begin{aligned}\mathbb{W}_0 - \tilde{\mathbb{W}}_0 = & -\frac{1}{2}\sum_{t=0}^{\infty}\beta^t\omega\Bigg\{\frac{1}{2}z_{1t}^2 + \frac{1}{2}z_{2t}^2 + \frac{1}{2}\left[(\alpha+\gamma_1)^2 + (1+\nu_1)(\alpha+\gamma_1)\right]x_{1t}^2 \\ & + \frac{1}{2}\left[(\alpha+\gamma_2)^2 + (1+\nu_2)(\alpha+\gamma_2)\right]x_{2t}^2\Bigg\},\end{aligned}$$

following the same steps as Lemma 1 and using $\overline{L}_{Tj}$. Therefore, the approximate XR-IP problem relative to the FB is

$$\begin{aligned}\max_{\{z_{1t},z_{2t},x_{1t},x_{2t},\check{f}_{t+1}^*\}_{t=0}^{\infty}} \quad & -\frac{1}{2}\sum_{t=0}^{\infty}\beta^t\omega\Bigg\{\frac{1}{2}z_{1t}^2 + \frac{1}{2}z_{2t}^2 + \frac{1}{2}\left[(\alpha+\gamma_1)^2 + (1+\nu_1)(\alpha+\gamma_1)\right]x_{1t}^2 \\ & + \frac{1}{2}\left[(\alpha+\gamma_2)^2 + (1+\nu_2)(\alpha+\gamma_2)\right]x_{2t}^2\Bigg\} \\ s.t. \quad & z_{1t} = z_{2t}, \\ & z_{1t} = \psi_{1t} + (\alpha + \gamma_1 - 1 - \nu_1)x_{1t}, \\ & z_{2t} = \psi_{2t} + (\alpha + \gamma_2 - 1 - \nu_2)x_{2t}, \\ & z_{1t} + z_{2t} - (\alpha + \gamma_1)x_{1t} - (\alpha + \gamma_2)x_{2t} = R^* \check{f}_t^* - \check{f}_{t+1}^*, \\ & \check{f}_0^* = 0.\end{aligned}$$

Combining the constraints we get the condition for x_{1t} and x_{2t}

$$\begin{aligned}&\psi_{1t} + (\alpha + \gamma_1 - 1 - \nu_1)x_{1t} = \psi_{2t} + (\alpha + \gamma_2 - 1 - \nu_2)x_{2t} \\ &x_{2t} = U_t + Vx_{1t},\ U_t \equiv \frac{\psi_{1t} - \psi_{2t}}{(\alpha + \gamma_2 - 1 - \nu_2)},\ V \equiv \frac{(\alpha + \gamma_1 - 1 - \nu_1)}{(\alpha + \gamma_2 - 1 - \nu_2)} > 0.\end{aligned}$$

Combining the constraints and iterating the resource constraint gives

$$\sum_{t=0}^{\infty} \beta^t \left[\psi_{1t} - (1+\nu_1)x_{1t} + \psi_{2t} - (1+\nu_2)x_{2t}\right] = 0.$$

We can solve the XR-IP problem for x_{1t}, x_{2t}

$$\begin{aligned}\max_{\{x_{1t},x_{2t}\}_{t=0}^{\infty}} \quad & -\frac{1}{2}\sum_{t=0}^{\infty}\beta^t\omega\Bigg\{(\psi_{1t} + (\alpha+\gamma_1-1-\nu_1)x_{1t})^2 + \frac{1}{2}\left[(\alpha+\gamma_1)^2 + (1+\nu_1)(\alpha+\gamma_1)\right]x_{1t}^2 \\ & \qquad + \frac{1}{2}\left[(\alpha+\gamma_2)^2 + (1+\nu_2)(\alpha+\gamma_2)\right]x_{2t}^2\Bigg\} \\ s.t. \quad & x_{2t} = U_t + Vx_{1t} \\ & \sum_{t=0}^{\infty}\beta^t\left[\psi_{1t} - (1+\nu_1)x_{1t} + \psi_{2t} - (1+\nu_2)x_{2t}\right] = 0.\end{aligned}$$

We get a loglinear Euler equation for x_{1t} to characterize the XR-IP solution

$$\begin{aligned}&\psi_{1t}(\alpha+\gamma_1-1-\nu_1) + \frac{1}{2}\left[(\alpha+\gamma_2)^2 + (1+\nu_2)(\alpha+\gamma_2)\right]U_tV \\ &+ \left\{(\alpha+\gamma_1-1-\nu_1)^2 + \frac{1}{2}\left[(\alpha+\gamma_1)^2 + (1+\nu_1)(\alpha+\gamma_1)\right] + \frac{1}{2}\left[(\alpha+\gamma_2)^2 + (1+\nu_2)(\alpha+\gamma_2)\right]V^2\right\}x_{1t} \\ &= \psi_{1t+1}(\alpha+\gamma_1-1-\nu_1) + \frac{1}{2}\left[(\alpha+\gamma_2)^2 + (1+\nu_2)(\alpha+\gamma_2)\right]U_{t+1}V \\ &+ \left\{(\alpha+\gamma_1-1-\nu_1)^2 + \frac{1}{2}\left[(\alpha+\gamma_1)^2 + (1+\nu_1)(\alpha+\gamma_1)\right] + \frac{1}{2}\left[(\alpha+\gamma_2)^2 + (1+\nu_2)(\alpha+\gamma_2)\right]V^2\right\}x_{1t+1}.\end{aligned}$$

Substituting $z_{1t} = \psi_{1t} + (\alpha+\gamma_1-1-\nu_1)x_{1t} = z_{2t}$, the Euler equation simplifies to

$$\begin{aligned}&z_{1t} + \left[\frac{1}{2}D_1 + \frac{1}{2}D_2\right]z_{1t} - \left[\frac{1}{2}D_1\psi_{1t} + \frac{1}{2}D_2\psi_{2t}\right] \\ &= z_{1t+1} + \left[\frac{1}{2}D_1 + \frac{1}{2}D_2\right]z_{1t+1} - \left[\frac{1}{2}D_1\psi_{1t+1} + \frac{1}{2}D_2\psi_{2t+1}\right],\end{aligned} \tag{C27}$$

where $D_j = \frac{(\alpha+\gamma_j)^2+(1+\nu_j)(\alpha+\gamma_j)}{(\alpha+\gamma_j-1-\nu_j)^2} > 0$.

The single-tradable-sector model Euler equation for $z_t = \psi_t + (\alpha+\gamma-1-\nu)x_t$ is

$$z_t + Dz_t - D\psi_t = z_{t+1} + Dz_{t+1} - D\psi_{t+1}, \tag{C28}$$

where $D = \frac{(\alpha+\gamma)^2+(1+\nu)(\alpha+\gamma)}{(\alpha+\gamma-1-\nu)^2} > 0$.

We can map the multiple-tradable-sector model to the form

$$z_t + D^M z_t - D^M \psi_t^M = z_{t+1} + D^M z_{t+1} - D^M \psi_{t+1}^M,$$

by setting $D^M = \frac{1}{2}[D_1 + D_2]$. Then to map

$$\begin{aligned} D^M \psi_t^M &= \frac{1}{2} D_1 \psi_{1t} + \frac{1}{2} D_2 \psi_{2t} \\ \psi_t^M &= \frac{1}{D^M}\left[\frac{1}{2} D_1 \psi_{1t} + \frac{1}{2} D_2 \psi_{2t}\right] \\ &= \frac{D_1}{D_1 + D_2}\psi_{1t} + \frac{D_2}{D_1 + D_2}\psi_{2t}. \end{aligned}$$

Next, to show the connection between z_{1t} and the log deviation of the first-best exchange rate $\epsilon_t \equiv \log(\mathcal{E}_t) - \log(\tilde{\mathcal{E}}_t)$, observe the optimality condition

$$\left(\frac{2(1-\omega)}{\omega}\frac{C_{T1t}}{C_{Nt}}\right) = p_{1t} = \mathcal{E}_t^{-1}.$$

Taking logs and combining with the same expression for the FB exchange rate $\tilde{\mathcal{E}}_t$ gives

$$z_{1t} = -\epsilon_t,$$

and similarly for the single-tradable-sector model $z_t = -\epsilon_t$. Substituting these into (C27) and (C28) shows the proposition.

Finally, we can also show that D_j is increasing in sector j labor elasticity ν_j^{-1} and externality γ_j

$$\frac{\partial D_j}{\partial \nu_j} = \frac{(\alpha+\gamma_j)(\alpha+\gamma_j-1-\nu_j)[3(\alpha+\gamma_j)+1+\nu_j]}{(\alpha+\gamma_j-1-\nu_j)^4} < 0,$$

and

$$\begin{aligned} \frac{\partial D_j}{\partial \gamma_j} &= \frac{(\alpha+\gamma_j-1-\nu_j)\left[[2(\alpha+\gamma_j)+(1+\nu_j)](\alpha+\gamma_j-1-\nu_j) - 2[(\alpha+\gamma_j)^2+(1+\nu_j)(\alpha+\gamma_j)]\right]}{(\alpha+\gamma_j-1-\nu_j)^4} \\ &> 0. \end{aligned}$$

C.6.2. Quantitative analysis

Model. We consider a general version with $i = 1, ..., I$ tradable sectors and assume the same labor supply elasticity across sectors. Preferences are given by

$$\sum_{t=0}^{\infty} \beta^t \left[\frac{{C_t}^{1-\sigma}}{1-\sigma} - \sum_{i=1}^{I} \phi \frac{L_{Tit}^{1+\nu}}{1+\nu} - \phi \frac{L_{Nt}^{1+\nu}}{1+\nu} \right],$$

where the budget constraint in domestic currency is given by

$$\sum_{i=1}^{I} P_{Tit} C_{Tit} + P_{Nt} C_{Nt} + B_{t+1} = \sum_{i=1}^{I} W_{Tit} L_{Tit} + W_{Nt} L_{Nt} + \Pi_t + T_t + R_t B_t,$$

where tradable consumption is a CES aggregator over tradable sectors

$$C_{Tt} = \left[\sum_{i=1}^{I} s_i^{\frac{1}{\iota}} (C_{Tit})^{1-\frac{1}{\iota}} \right]^{\frac{\iota}{\iota-1}},$$

with weights $\sum_{i=1}^{I} s_i = 1$ and elasticity of substitution across sectors ι.

Define the aggregate price of tradables $P_{Tt} = \left(\sum_{i=1}^{I} s_i P_{Tit}^{1-\iota} \right)^{\frac{1}{1-\iota}}$, and similarly for the foreign currency price P_{Tt}^*. Assume the law of one price holds for each variety i of tradable good $P_{Tit} = \mathcal{E}_t P_{Tit}^*$.

Tradable sector $i = 1, ..., I$ firms choose labor to maximize their profits, and produce with the following constant-returns-to-scale at the firm level ($\alpha = 1$) production technology

$$y_{Tit} = A_t a_i L_{Tit}^{\gamma_{Tit}} l_{Tit},$$

where A_t is the exogenous aggregate productivity component, a_i is a constant tradable sector i productivity component, $L_{Tit}^{\gamma_{Tit}}$ is the endogenous tradable sector i productivity component. Nontradable firms production $y_{Nt} = A_t l_{Nt}$ is as in the baseline model. This gives rise to the following aggregate labor demand

$$\begin{aligned} A_t a_i L_{Tit}^{\gamma_{Tit}} &= W_{Tit}/P_{Tit} \text{ for all } i = 1, ..., I, \\ A_t &= W_{Nt}/P_{Nt}. \end{aligned}$$

Normalize $P^*_{T1t} = P_{Nt} = 1$. Let $p^*_{it} \equiv \frac{P^*_{Tit}}{P^*_{T1t}} = \frac{P_{Tit}}{P_{T1t}}$ for $i = 2, ..., I$ and $p_{1t} \equiv \frac{P_{Nt}}{P_{T1t}}$, then $p_{1t} = \mathcal{E}_t^{-1}$. Therefore, $p^*_{it} = p_{1t} P_{Tit}$.

For the laissez-faire competitive equilibrium the $2I+3$ variables $\{C_{Tit}, L_{Tit}\}_{i=1}^I, C_{Nt}, L_{Nt}, F^*_{t+1}$ are characterized by the following $2I + 3$ equations

$$\begin{aligned}
\left(\frac{s_i C_{T1t}}{s_1 C_{Tit}}\right)^{\frac{1}{\rho}} &= p^*_{it} \text{ for all } i = 2, ..., I, \\
\frac{\phi L^{\nu}_{Tit}}{(\omega/C_{Tt})^{\frac{1}{\eta}} C_t^{\frac{1}{\eta}-\sigma} (s_i C_{Tt}/C_{Tit})^{\frac{1}{\rho}}} &= A_t a_i L^{\gamma_{Tit}}_{Tit} \text{ for all } i = 1, ..., I, \\
\frac{\phi L^{\nu}_{Nt}}{((1-\omega)/C_{Nt})^{\frac{1}{\eta}} C_t^{\frac{1}{\eta}-\sigma}} &= A_t, \\
C_{Nt} &= A_t L_{Nt}, \\
C_{T1t} + \sum_{i=2}^{I} p^*_{it} C_{Tit} &= A_t L^{1+\gamma_{T1t}}_{T1t} + \sum_{i=2}^{I} p^*_{it} A_t a_i L^{1+\gamma_{Tit}}_{Tit} + R^* F^*_t - F^*_{t+1}, \\
\left(\frac{\omega}{C_{Tt}}\right)^{\frac{1}{\eta}} C_t^{\frac{1}{\eta}-\sigma} \left(\frac{s_1 C_{Tt}}{C_{T1t}}\right)^{\frac{1}{\rho}} &= \beta R^* \left(\frac{\omega}{C_{Tt+1}}\right)^{\frac{1}{\eta}} C_{t+1}^{\frac{1}{\eta}-\sigma} \left(\frac{s_1 C_{Tt+1}}{C_{T1t+1}}\right)^{\frac{1}{\rho}}.
\end{aligned}$$

The nominal exchange rate is then given by

$$\left(\frac{1-\omega}{\omega}\frac{C_{Tt}}{C_{Nt}}\right)^{\frac{1}{\eta}} \left(\frac{C_{T1t}}{s_1 C_{Tt}}\right)^{\frac{1}{\rho}} = \mathcal{E}_t^{-1}.$$

Identification. The section details the identification of a_i, p^*_{it}, s_i, and γ_{Tit} for $i = 1, ..., I$.

We assume constant relative foreign currency prices across sectors $p^*_{it} = 1$ for $i = 1, ..., I$ with variation in relative tradable-sector productivity a_i, which is isomorphic to both a_i and p^*_i varying across sectors. We calibrate a_i to match average production shares $\overline{y}_{Ti}$ for tradable sectors for China from the OECD's Inter-Country Input-Output tables, i.e.

$$\overline{y}_{Tit} \equiv \frac{P_{Tit} Y_{Tit}}{\sum_{i=1}^{I} P_{Tit} Y_{Tit}}.$$

Substituting the production function and using the law of one price

$$\overline{y}_{Tit} = \frac{\mathcal{E}_t P^*_{Tit} A_t a_i L^{1+\gamma_{Tit}}_{Tit}}{\sum_{i=1}^{I} \mathcal{E}_t P^*_{Tit} A_t a_i L^{1+\gamma_{Tit}}_{Tit}} = \frac{a_i L^{1+\gamma_{Tit}}_{Tit}}{\sum_{i=1}^{I} a_i L^{1+\gamma_{Tit}}_{Tit}}.$$

We calibrate tradable sector preference weights s_i to match average expenditure shares $\overline{x}_{Ti}$ for tradable sectors for China from the OECD's Inter-Country Input-Output tables, i.e.

$$\overline{x}_{Tit} \equiv \frac{P_{Tit}C_{Tit}}{\sum_{i=1}^{I} P_{Tit}C_{Tit}}.$$

From the household first-order condition for tradable sector i we have

$$s_i = s_1 \left(p_{it}^*\right)^{\rho-1} \left(\frac{P_{Tit}C_{Tit}}{P_{T1t}C_{T1t}}\right)$$
$$P_{T1t}C_{T1t} \left(p_{it}^*\right)^{1-\rho} \frac{s_i}{s_1} = P_{Tit}C_{Tit}.$$

Substituting $P_{Tit}C_{Tit}$ into $\overline{x}_{Tit}$ gives

$$\overline{x}_{Tit} = \frac{\frac{s_i}{s_1}\left(p_{it}^*\right)^{1-\rho}}{\sum_{i=1}^{I} \frac{s_i}{s_1}\left(p_{it}^*\right)^{1-\rho}}.$$

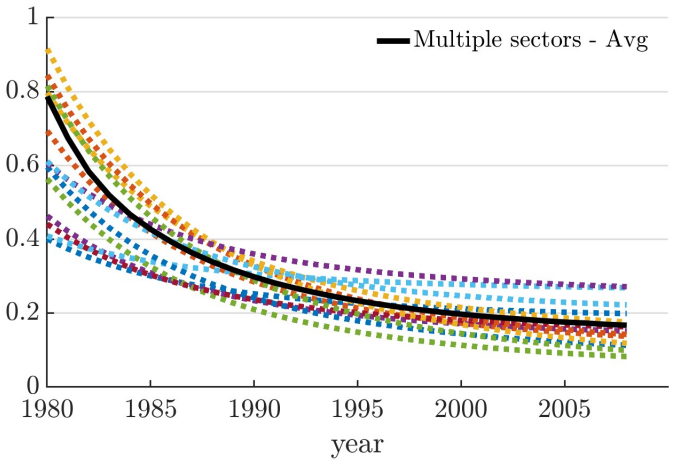
Given $p_{it}^* = 1$ we set $s_i = \overline{x}_{Ti}$, the average expenditure share for all $i = 1, ..., I$.

For γ_{Tit}, we use the same approximation of the China productivity growth process to compute the sector-specific γ_{Tit} for tradable sector i using the sectoral γ_0^i, γ_1^i estimates from Section 3.1 given in Appendix Table B.2. We drop the sectors with highest and lowest externality estimates: Plastic, Rubber ($\gamma_{Ti0} = 3.00$, Y_{Ti} share 0.02); and Motor Vehicles ($\gamma_{Ti0} = 0.18$, Y_{Ti} share 0.12). Figure C2 shows the estimated γ_{Tit} for the 13 sectors.

We solve the quantitative model for the parameterization in Section 3.2, and set the elasticity of substitution between tradables $\iota = 1.28$ consistent with Bartelme *et al.* (2019). All other externally set parameters are as the same as in the baseline model, and the calibrated ones follow the same calibration strategy.

Table C2 shows the calibrated model matches well the sector shares for China during the takeoff period.

Figure C2: Mutiple Sectors: Externality by Sector



Notes: This figure shows the estimated γ_{Tit} for the tradable sectors in the model with multiple sectors.

Table C2: Multiple sectors calibration

Sector i	Externality in 2000	Y_{Ti} share		C_{Ti} share	
	γ_{Ti2000}	Data	Model	Data	Model
Food	0.21	0.21	0.21	0.31	0.31
Textiles	0.17	0.13	0.13	0.09	0.09
Wood	0.22	0.00	0.00	0.00	0.00
Paper	0.17	0.01	0.01	0.01	0.01
Petrol Prod.	0.11	0.01	0.01	0.02	0.02
Chemicals	0.28	0.03	0.03	0.01	0.01
Minerals Prod.	0.18	0.02	0.02	0.01	0.01
Basic Metals	0.14	0.03	0.03	0.01	0.01
Fabr. Metals	0.19	0.03	0.03	0.01	0.01
Computers	0.17	0.19	0.19	0.12	0.12
Electric Mach.	0.29	0.13	0.13	0.13	0.13
Machinery	0.15	0.18	0.18	0.24	0.24
Other Transp.	0.25	0.04	0.04	0.04	0.04

Notes: Y_{Ti} are average production shares and C_{Ti} are average expenditure shares for tradable sector i. Data are the average for China in 1995, 2000, 2005 and 2010. Model shares are the average for 1980-2008 for the observed reserve accumulation. Data source: OECD.

C.7. Model with nontradable externalities

Consider a generalization of the baseline model in which $\gamma_{Nt} > 0$ for $t \geq 0$. In principle, the presence of time- and sector-specific production externalities can imply different paths of exchange rate policies depending on the relative strength of externalities. However, the following characterizes the optimal policy under Assumption 1 for an arbitrary path of externalities in the nontradable sector, $\gamma_{Nt} > 0$.

Appendix A.5 showed that under Assumption 1 the optimal exchange rate industrial policy depends only on the tradable block of the model for any arbitrary path of γ_{Nt}.

Nontradable production and consumption are independent of the optimal policy, which can be seen from equation (A.8) and nontradable goods market clearing.

In this case, the modified Euler equation for the optimal exchange rate industrial policy takes the form

$$\left(\frac{\omega}{C_{Tt}}\right) = \beta R^* \frac{\theta(L_{Tt+1}, \gamma_{Tt+1})}{\theta(L_{Tt}, \gamma_{Tt})} \left(\frac{\omega}{C_{Tt+1}}\right),$$

which does not depend on γ_{Nt}.

Therefore, the optimal policy directly follows Proposition 3.

C.8. Model with home and foreign goods

Households. We extend the model with elastic labor supply so that the tradable good is a CES aggregate of home and foreign tradable goods

$$C_{Tt} = \left[(1-\omega_F)^{\frac{1}{\eta_T}} (C_{Ht})^{1-\frac{1}{\eta_T}} + \omega_F^{\frac{1}{\eta_T}} (C_{Ft})^{1-\frac{1}{\eta_T}}\right]^{\frac{\eta_T}{\eta_T - 1}},$$

where $\omega_F \in (0,1)$ is the weight on foreign tradable goods and $\eta_T > 0$ is the elasticity of substitution between home and foreign tradable goods. The home good C_{Ht} is a CES aggregate of varieties $j \in [0,1]$ given by $C_{Ht} = \left(\int c_{Hjt}^{\frac{\theta-1}{\theta}} dj\right)^{\frac{\theta}{\theta-1}}$. The household budget constraint expressed in domestic currency is given by

$$\int P_{Hjt} c_{Hjt} dj + P_{Ft} C_{Ft} + P_{Nt} C_{Nt} + B_{t+1} = W_{Tt} L_{Tt} + W_{Nt} L_{Nt} + \Pi_t + T_t + R_t B_t,$$

where P_{Hjt}, P_{Ft} are the prices of home varieties and foreign tradable goods.

The optimal consumption allocation gives

$$c_{Hjt} = \left(\frac{P_{Hjt}}{P_{Ht}}\right)^{-\theta} C_{Ht},$$

where $P_{Ht} \equiv \left(\int P_{Hjt}^{1-\theta} dj\right)^{\frac{1}{1-\theta}}$, and between home and foreign tradable goods

$$\begin{aligned} C_{Ht} &= (1-\omega_F)\left(\frac{P_{Ht}}{P_{Tt}}\right)^{-\eta_T} C_{Tt}, \\ C_{Ft} &= \omega_F \left(\frac{P_{Ft}}{P_{Tt}}\right)^{-\eta_T} C_{Tt}, \end{aligned} \tag{C29}$$

where $P_{Tt} \equiv [(1-\omega_F)P_{Ht}^{1-\eta_T} + \omega_F P_{Ft}^{1-\eta_T}]^{\frac{1}{1-\eta_T}}$. The household optimality conditions are

$$\begin{aligned} \left(\frac{1-\omega_F}{C_{Ht}}\right)^{\frac{1}{\eta_T}} &= \frac{P_{Ht}}{P_{Ft}}\left(\frac{\omega_F}{C_{Ft}}\right)^{\frac{1}{\eta_T}}, \\ \left(\frac{1-\omega}{C_{Nt}}\right)^{\frac{1}{\eta}} &= \frac{P_{Nt}}{P_{Ht}}\left(\frac{\omega}{C_{Tt}}\right)^{\frac{1}{\eta}}\left(\frac{(1-\omega_F)C_{Tt}}{C_{Ht}}\right)^{\frac{1}{\eta_T}}, \\ \left(\frac{\omega}{C_{Tt}}\right)^{\frac{1}{\eta}} C_t^{\frac{1}{\eta}-\sigma}\left(\frac{(1-\omega_F)C_{Tt}}{C_{Ht}}\right)^{\frac{1}{\eta_T}} \frac{W_{Tt}}{P_{Ht}} &= \phi L_{Tt}^{\nu} \\ \left(\frac{1-\omega}{C_{Nt}}\right)^{\frac{1}{\eta}} C_t^{\frac{1}{\eta}-\sigma} \frac{W_{Nt}}{P_{Nt}} &= \phi L_{Nt}^{\nu} \\ \left(\frac{\omega}{C_{Tt}}\right)^{\frac{1}{\eta}} C_t^{\frac{1}{\eta}-\sigma}\left(\frac{(1-\omega_F)C_{Tt}}{C_{Ht}}\right)^{\frac{1}{\eta_T}} &= \beta R_{t+1}\frac{P_{Ht}}{P_{Ht+1}}\left(\frac{\omega}{C_{Tt+1}}\right)^{\frac{1}{\eta}} C_{t+1}^{\frac{1}{\eta}-\sigma}\left(\frac{(1-\omega_F)C_{Tt+1}}{C_{Ht+1}}\right)^{\frac{1}{\eta_T}}. \end{aligned}$$

Firms. Domestic tradable sector firms produce varieties of home goods and are owned by domestic households. A continuum of tradable sector firms j hire labor from the household l_{Tjt} to produce variety j of the home good, with a production technology $y_{Hjt} = A_t L_{Tt}^{\gamma_{Tt}} l_{Tjt}^{\alpha}$. Prices are perfectly flexible so firms can freely adjust their price each period. Each firm j faces domestic demand from the household problem

$$c_{Hjt} = \left(\frac{P_{Hjt}}{P_{Ht}}\right)^{-\theta} C_{Ht},$$

and similarly for foreign demand

$$c^*_{Hjt} = \left(\frac{P^*_{Hjt}}{P^*_{Ht}}\right)^{-\theta} C^*_{Ht}.$$

Similar to (C29) foreign demand for home goods is given by

$$C^*_{Ht} = \omega_F \left(\frac{P^*_{Ht}}{P^*_{Tt}}\right)^{-\eta_T} C^*_{Tt},$$

where C^*_{Tt} is aggregate foreign tradable consumption and $P^*_{Tt} \equiv [(1-\omega_F)(P^*_{Ft})^{1-\eta_T} + \omega_F (P^*_{Ht})^{1-\eta_T}]^{\frac{1}{1-\eta_T}}$.

The firm takes the aggregate domestic price P_{Ht} and demand C_{Ht}, and aggregate foreign currency price P^*_{Ht} and demand C^*_t as given. As in the baseline model, we assume the law of one price for home varieties and foreign tradable goods, $P_{Hjt} = \mathcal{E}_t P^*_{Hjt}$ and $P_{Ft} = \mathcal{E}_t P^*_{Ft}$.

Firm j profits are $\Pi_{jt} = P_{Hjt} c_{Hjt} + \mathcal{E}_t P^*_{Hjt} c^*_{Hjt} - W_{Tt} l_{Tjt}$ and is subject to the constraint that $y_{Hjt} \geq c_{Hjt} + c^*_{Hjt}$. Given that this constraint will hold with equality and the law of one price, the firm j price-setting problem for $p \equiv P_{Hjt}$ after substituting the demand functions and dividing by P_{Ht} is given by

$$\max_p \ \Pi_t(p) = \left(\frac{p}{P_{Ht}} - w_t\right) p^{-\theta} \left[\left(\frac{1}{P_{Ht}}\right)^{-\theta} C_{Ht} + \omega_F \left(\frac{1}{\mathcal{E}_t P^*_{Ht}}\right)^{-\theta} \left(\frac{P^*_{Ht}}{P^*_{Tt}}\right)^{-\eta_T} C^*_{Tt}\right]$$

where w_t is the effective real wage $w_t = \frac{W_{Tt}}{P_{Ht} A_t L_{Tt}^{\gamma_{Tt}} l_{Tt}^{\alpha-1}}$. This problem for each firm j is identical. The first-order condition for the optimal price gives

$$p = \frac{\theta}{\theta - 1} w_t P_{Ht}$$

Equilibrium. For our parameterization $\alpha = 1$, in equilibrium since each firm j is identical, $p = P_{Ht}$, therefore

$$w_t = \frac{\theta - 1}{\theta} \Leftrightarrow \frac{W_{Tt}}{P_{Ht} A_t L_{Tt}^{\gamma_{Tt}}} = \frac{\theta - 1}{\theta}.$$

The tradable labor market clearing condition is

$$\int l_{Tjt} dj = L_{Tt},$$

and in a symmetric equilibrium with unit mass of firms j, $l_{Tjt} = L_{Tt}$.

The home tradable good market clearing condition is

$$\begin{aligned} C_{Ht} + C^*_{Ht} &= A_t L_{Tt}^{1+\gamma_{Tt}} \\ C_{Ht} + \omega_F \left(\frac{P^*_{Ht}}{P^*_{Tt}}\right)^{-\eta_T} C^*_{Tt} &= A_t L_{Tt}^{1+\gamma_{Tt}}, \end{aligned}$$

and the nontradable good market clearing condition is $C_{Nt} = A_t L_{Nt}$.

We normalize the domestic price of the home tradable good $P_{Ht} \equiv 1$ and foreign price of the foreign tradable good $P^*_{Ft} \equiv 1$, so $1/\mathcal{E}_t = P^*_{Ht}$, $\mathcal{E}_t = P_{Ft}$. By definition of P_{Tt}, P^*_{Tt}

$$\frac{P^*_{Ht}}{P^*_{Tt}} = P^*_{Ht}[(1-\omega_F) + \omega_F (P^*_{Ht})^{1-\eta_T}]^{-\frac{1}{1-\eta_T}} = [(1-\omega_F)\mathcal{E}_t^{1-\eta_T} + \omega_F]^{-\frac{1}{1-\eta_T}}.$$

The balance of payments is given by

$$\begin{aligned} P_{Ft} C_{Ft} - \mathcal{E}_t P^*_{Ht} C^*_{Ht} &= \mathcal{E}_t R^* F^*_t - \mathcal{E}_t F^*_{t+1} \\ C_{Ft} - \frac{1}{\mathcal{E}_t} \omega_F [(1-\omega_F)\mathcal{E}_t^{1-\eta_T} + \omega_F]^{\frac{\eta_T}{1-\eta_T}} C^*_{Tt} &= R^* F^*_t - F^*_{t+1} \end{aligned}$$

The household optimality condition between home and foreign tradable goods gives

$$\mathcal{E}_t = \left(\frac{\omega_F}{C_{Ft}}\right)^{\frac{1}{\eta_T}} \left(\frac{1-\omega_F}{C_{Ht}}\right)^{-\frac{1}{\eta_T}}. \tag{C30}$$

C.8.1. Quantitative analysis

We solve the quantitative model for the parameterization in Section 3.2, and set the elasticity of substitution between home and foreign tradable goods $\eta_T = 1.5$ (Feenstra, Luck, Obstfeld and Russ, 2018), and the elasticity of substitution between home varieties $\theta = 6$ (Gali and Monacelli, 2005), which implies a markup of 20% .

We feed in a path of world demand for tradable goods C^*_{Tt} to match the growth in

world (excluding China) real imports relative to China's population since 1980 (measured in constant USD, source: World Bank).

We calibrate the share of foreign goods in the tradable consumption basket $\omega_F = 0.45$ to match China's average manufacturing share in output as in the baseline model. All other externally set parameters are as the same as in the baseline model, and the calibrated ones follow the same calibration strategy.