Labor Market Sorting and Public Pensions in Developing Countries*

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Abstract

We build an equilibrium life-cycle labor search model in which firms post contracts specifying job formality and wages. We calibrate the model to match key aspects of Brazilian labor market — worker transitions across the formal and informal sectors over the life cycle and sorting between firm size, formality, and worker skills in the cross section. We consider two sets of pension reforms — pension reforms with an alternative share rule of pension contribution between workers and firms and pension reforms with alternative pension contribution financing. We find that fixing total pension tax rate but increasing firm's share increases labor market power of more productive firms, leading to less vacancy posting, higher non-employment rate, and lower labor share. Reforms weakening the linkage between formality and public pension contributions lead to an increase in aggregate vacancies and formality share, which stimulate the fluidity of the labor market and benefit low-skilled workers more. The results highlight the importance of the behavior response of firms in policy reforms in developing countries.

Keywords: Labor Market Power, Sorting, Formal and Informal Sectors, Public Pension.

JEL Classification Numbers: D15, H55, O17.

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1 Introduction

Public pension programs are designed to provide income to retirees as labor earnings fade. While details of these systems vary across countries, the public pension systems are typically financed through payroll taxes on both workers and firms and provide older workers with a pension during their retirement years. In order for the public insurance system to operate functionally, the government should be able to accurately monitor incomes of workers, which postulates that the workers being formally registered to the government with an employment position in a firm. This formality-based public pension system, which typically works well in developed countries, presents two challenges for governments in developing countries.

On the one hand, a large population of unregistered workers — "informal workers" — is not covered by the public system (Meghir, Narita, and Robin, 2015; Ulyssea, 2018; Haanwinckel and Soares, 2021; Dix-Carneiro, Goldberg, Meghir, and Ulyssea, 2024). Unlike in developed countries, where workers' lifetime earnings — consequently the pension benefits workers received — are often an accurate representation of the labor market history of the worker, incomes of workers in developing countries are not accurately observed by the government due to the presence of large informal labor markets.

Furthermore, as is common in most developing countries, the workers who are covered by the public pension system are more likely to be more educated, work in larger firms, and earn higher wages, while the low–skilled, presumably those who demand more insurance from government redistribution, are missed out from the coverage of public pension system. Achieving better redistribution by expanding coverage to these informal workers can lead to large welfare gains. For example, recent papers, such as (Egger, Haushofer, Miguel, Niehaus, and Walker, 2022; Gerard, Naritomi, and Silva, 2021; Muralidharan, Niehaus, and Sukhtankar, 2023) examine the effects of cash transfers and highlight the role of expanding formal employment with a stimulation of aggregate demand.

On the other hand, firms in developing countries can typically exert a stronger market power over the labor market (Felix, 2022; Amodio and De Roux, 2024; Armangué-Jubert, Guner, and Ruggieri, 2024; Lobel, 2024). The ability of firms to act strategically with the presence of labor market power casts doubts on predictions of policy experiments made under the assumption of perfect labor market competition. For example, when the labor market is perfectly competitive, workers are compensated to the point that the total of wage and pension contribution is equal to the amount of marginal product, which is invariant to policy. Therefore, it does not matter whether the pension contribution tax is paid by the worker or the firm. However, when labor market competition is imperfect and the public pension contributions are partially made by the firm, a policy reform can alter the relative position of employers in the labor market, affecting their decision on the amount

of vacancies to post and incentive to hire formally. Therefore, policy reforms will impact both the coverage of the pension system and the size of pensions received by workers in the economy, leading to heterogeneous effects on workers and firms of different characteristics.

In this paper, we study the case of the Brazilian economy in which over 30 percent of overall employment and up to 50 percent of employment among low-education workers are informal. As Brazil operates a pay-as-you-go (PAYG) Social Security system in which taxes levied on earnings and payroll within the formal economy finance retirement pensions, this high informal labor share leads to low coverage of the public pension program and low pension income of workers who are likely to spend significant portions of their working lives in the informal sector. We build an equilibrium life-cycle labor search model in which specifically models the firm choice of hiring formal and informal workers with labor market power to obtain a thorough understanding of policy impacts through firms' strategic interactions and distributional effects.

Using data from a nationally representative household survey data *Pesquisa Nacional por Amostra de Domicilios Continua* (continuous PNAD, PNADC), we document three main aspects about the Brazilian labor market. First, the Brazilian labor market is fluid. There are frequent transitions between formal and informal jobs for workers over their life cycle, which cannot be fully explained by the argument that the informal sector serves as a subsistence sector to insure against the risk of unemployment in the formal sector. Furthermore, job transitions across sectors have strong implications for future job search prospects and earnings growth. To be more specific, job switches within the same sector occur at a higher rate than those between sectors, whereas a job switch leads to large wage changes only when it is associated with a change of job formality status. Second, government regulations affect firm operation and hiring incentives, which is evident from the fact that the wage distribution of formal workers is compressed by the federal minimum wage in Brazil, with a significant fraction of workers—amounting to 20% of formal workers—bunching at exactly the minimum wage. As noted in Pries and Rogerson (2005), minimum wages can prevent firms from partly transferring the payroll tax that is levied on themselves to workers via lower wages. The large amount of marginal jobs with a binding minimum wage constraint suggests that policies can potentially have an impact on the labor market through firms' monetary budget incentives. Third, using the firm size variable from the PNADC, we document that larger firms significantly hire a larger share of formal workers. The educational attainments and wages of workers are also higher in larger firms. The strong sorting pattern between firm size, formality, and worker skills suggests that policies affecting incentive of a single firm may eventually lead to complicated aggregate impacts through firms' strategic behavioral responses.

Motivated by the unique data features and policy background of the Brazilian labor market, we build an equilibrium life-cycle labor search model in which firms post contracts specifying job formality and wage on the job. The contract-posting framework captures firm's joint choice of

formality and wages, which gives rise to the role of minimum wage in the presence of a wage distribution. In the model, all individuals can accumulate private savings to insure themselves against labor market risks and finance retirement. However, only those employed formally can accumulate public pension wealth. Individuals search for a job both on- and off-the-job in a random search fashion, and their human capital evolves contingent on employment status. We allow formal and informal jobs to differ in several important dimensions. First, informal jobs can evade corporate income taxes, public pension payroll taxes, and the minimum wage that formal jobs are subject to. Instead, informal jobs are faced with the risk of being randomly screened, upon which they hand in all firm profits as penalty. Second, human capital growth and job security may intrinsically be different between formal and informal jobs. Third, motivated by the fact that high-skilled workers sort into formal jobs, we allow search efficiency to differ contingent on individuals' skills, employment status, and the targeted sector. Firms post vacancies on formal and informal job markets and offer contracts specifying wage piece rate conditional on formality and firm productivity. Individuals fully internalize the prospects of job transitions and income flows and decide whether to accept a job offer. We close the model with a convex vacancy posting cost function which pins down equilibrium firm size distribution.

We calibrate the model to match two salient aspects of the Brazilian economy, namely, life cycle patterns of employment share of formal and informal jobs and transition rates between sectors, as well as the sorting between firm size, formality, and worker skills in the cross section. We find that formal jobs are associated with higher job security and increased human capital growth. The job search efficiency is higher within the sector than across the sector. High-skilled individuals are more efficient in the search for formal jobs. The optimal share of formal vacancies is 0 for lowest productive firms because they run into net losses if paying above the minimum wage. The optimal share of formal vacancies keeps increasing as firm productivity increases given that the penalty of being informal in convex in output. The optimal piece rate increases in firm productivity in most of the region, in the same way as in Burdett and Mortensen (1998). Not surprisingly, due to compensating differentials, the piece rate is lower in a formal job than an informal job posted by the same firm productivity type. We further decompose the value of vacancy into three margins: probability of forming a match, human capital of the worker, and duration of the job and find that all three are important in explaining the difference of vacancy yields across firm productivity types.

Armed with a well-fitted and validated model, we simulate various reforms to the Brazilian public pension system. In the first exercise, we maintain the formality-based public pension system but vary the split of pension contribution between worker and firm. Although the total contribution as a fraction of wage income is kept 28% as under the current policy, the policy leads to non-trivial change in labor market position across firms. To be more specific, when the firm's part of contribution is higher, firms with lower productivity are less able to post above the statutory

minimum wage, strengthening the market power of more productive firms. We find that a higher contribution share for firms reduces the amount of vacancies and labor compensation share as the labor market power of more productive firms increases. Furthermore, the non-employment rate increases and firms increase usage of capital relative to labor. While increasing firms' contribution share leads to change in formal employment share for small and large firms in opposite directions, the overall effect is dominated by large firms that is increasing in the share of firm contribution.

In our second policy experiment, we deviate from the formality-based pension contribution rule and consider three alternative contribution rules that weaken the linkage between formal employment and pension contribution. Under the first policy scheme, all individuals make a fixed amount of contribution. Under the second, unemployed individuals make a fixed amount of contribution and employed individuals make additionally a constant fraction of their wage incomes in spite of the formality of their jobs. The last policy reform scheme involves a linear consumption tax which is also universal to everyone. We find that albeit the specific way of implementation, all three proposed policy reforms stimulate labor market fluidity, increase aggregate output and workers' compensation share. Due to the relaxation of governmental regulation in terms of the minimum wage and required payroll contribution, firms, especially mid-productivity firms, expand vacancy size and raise the piece rate, which creates fiercer competition As the equilibrium formal employment share increases, firm also find more profitable to increase the share of formal vacancies as within-sector poaching is more efficient. In the new steady state equilibrium, unemployment rate decreases and formal employment share increases. We also find that the policy reforms benefit low-skilled workers more with a more prominent increase in their formal employment share. The policy reforms reduces dramatically inequality of public pension wealth with only a minor increase in skill wage premium. We additionally find that partial equilibrium that do not take into account the behavioral response of firms generate misleading predictions about both the aggregate and distributional impacts. Our results highlight the role of firms' behavioral responses in understanding results of policy reforms.

Related literature Our paper is primarily related to literature studying Social Security reforms in developing countries with the presence of large informality, such as Jung and Tran (2012), Joubert (2015), Esteban-Pretel and Kitao (2021), McKiernan (2021, 2022), and Kudrna, Piggott, and Poonpolkul (2022), or more generally, labor market policies in developing economies, such as Albrecht, Navarro, and Vroman (2009), Bosch and Esteban-Pretel (2012), Rud and Trapeznikova (2021), Donovan, Lu, and Schoellman (2023), and Engbom, Malmberg, Porzio, Rossi, and Schoellman (2024). Compared with the previous literature, we focus on the labor market implications of the formality-based public pension system by explicitly modeling firms' informality choice in the presence of worker and firm heterogeneity and search frictions, and show that firms behavioral

responses are the key to understanding the policy effects.

Second, our paper contributes to the large literature on labor market sorting and market power, such as Abowd, Kramarz, and Margolis (1999), Card, Heining, and Kline (2013), Lise, Meghir, and Robin (2016), and Bonhomme, Lamadon, and Manresa (2019). In particular, we document a strong sorting pattern between productivity and formality in Brazil and highlight its implications for public pension reforms. A burgeoning literature has documented that labor market power of firms plays an important role in understanding the trends in the labor market and the welfare effects of policy reforms (Berger, Herkenhoff, and Mongey, 2022; Yeh, Macaluso, and Hershbein, 2022; Lamadon, Mogstad, and Setzler, 2022; Card, 2022; Bagga, 2023; Bachmann, Bayer, Stüber, and Wellschmied, 2024). Furthermore, recent studies have found that firms in developing countries have a stronger market power than firms in developing countries and that the market power of firms is the key to understanding policy effects (Lobel, 2024; Baumgartner, Corbi, and Narita, 2022).

Finally, our paper is also related to the large literature studying the source of formality in developing countries. Satchi and Temple (2009), Meghir, Narita, and Robin (2015) and Narita (2020) focus on search frictions on the labor market. D'Erasmo and Boedo (2012), Franjo, Pouokam, and Turino (2022), and Erosa, Fuster, and Martinez (2023) highlight the role of financial friction. Bobba, Flabbi, Levy, and Tejada (2021) and Bobba, Flabbi, and Levy (2022) study the role of human capital accumulation. Haanwinckel (2020) and Haanwinckel and Soares (2021) focus on the role of skill distribution and its implications for wage dispersion. Ulyssea (2010, 2018, 2020) study the impacts of government regulation of the informal sector. Bosch, Goñi-Pacchioni, and Maloney (2012), McCaig and Pavcnik (2018), Dix-Carneiro, Goldberg, Meghir, and Ulyssea (2021), and Felix (2022) highlight the role of trade shocks. Motivated by unique data features and the institutional background of Brazil, we build a unified framework incorporating differences between formal and informal jobs in terms of government regulation, labor market, and financial market frictions, and let the data speak to the relative importance of each factor on aggregates as well as heterogeneous firms.

Layouts The remainder of the paper proceeds as follows. Section 2 discusses the policies present in the Brazilian economy. Section 3 shows the empirical facts we establish about the Brazilian labor market which suggest mechanisms how policy reforms can potentially have an impact on the economy. Section 4 details the quantitative model used and Section 5 discusses the calibration of the model parameters. Section 6 details the policy experiment performed.

2 Policy Background

As in many countries, access to government assistance programs is associated with formal work history in Brazil. Specifically, formal sector workers have access to the Social Security system, must earn at least the minimum wage, and receive unemployment and job displacement insurance in the event of job loss.[1]

2.1 Social Security

Brazil operates under a pay-as-you-go Social Security System for private sector, formal workers (*Regime Geral de Previdência Social (RGPS)*).[2] Workers contribute between 7.5 and 14 percent of earnings up to a monthly earnings cap.[3] Employers pay a flat rate of 20 percent (22.5 percent if the firm operates in the financial sector). Workers are eligible for retirement at age 65 (age 62 for women) with 20 years of contributions to the system (15 years for women). The benefit amount is equal to 60 percent of the average formal earnings of the worker with an additional 2 percent per year of contributions over the 20 required years.[4]

There are also two programs available to elderly workers who do not qualify for the pension program. First, a non-contributory pension program (*Benefício de Prestação Continuada da Assistência Social (BCP-LOAS*) is available to those over 65 years of age with a household income per capita less than one-quarter of the minimum wage. The benefit provided under this system is equivalent to the minimum wage. Second, rural workers over the age of 60 (men) and 55 (women) who have worked for at least 15 years are eligible for an additional program (*Previdência Rural*). This benefit is also equal to the minimum wage.

The PBM Reform in Brazil It is worth noting that there is a large reduction in payroll taxes for selected sectors in Brazil since 2011 — the *Plano Brasil Maior (PBM)* industries policies. The policy reform replaces the 28% pension payroll tax with a 2% firm profit tax. Despite limited coverage of reform (1.7% of firms and 6.2% of employment nationwide), recent papers shed light on the role of labor market power. Baumgartner, Corbi, and Narita (2022) show that sectors that experience a greater reduction in payroll tax rates saw a larger increase in employment (40% firm

[1] Brazil runs a universal health care system (*Sistema Único de Saúde (SUS)*) available to all workers, regardless of sex, race, occupation, and other characteristics (https://www.gov.br/saude/pt-br/sus).

[2] The government runs a separate pension scheme for public sector employees (*Regime Próprios de Previdência Social (RPPS)*)

[3] Self-employed workers contribute 20 percent of monthly covered declared earnings. This contribution is lower for self-employed individuals who choose to forego the option of a pension through years of contributions. Under these circumstances, the worker contributed 11 percent for larger businesses and 5 percent for small businesses.

[4] The Brazilian pension system underwent a reform in 2019 which eliminated a path to benefit through years of contributions and made small changes to the benefit function and worker contribution rates. In this work, we focus on the post-reform system.

entry, 60% expansion of existing firms). Furthermore, these effects are more pronounced in local labor markets that were previously less concentrated. Using the same quasi-natural experiment, Lobel (2024) documents that reducing the payroll tax increases the firm's labor market power and replaces capital with labor. Furthermore, Lobel (2024) shows that high-skilled workers benefit more from this reform.

2.2 Minimum Wage and Job Displacement Insurance

The national minimum wage in Brazil is set as a floor on the nominal monthly income. This minimum wage is applied to full-time formal sector workers with a smaller (proportional) amount available to part-time workers. The value of this minimum wage is equal to roughly 640 Brazilian Reals throughout the years 2012-2018.[5]

Formal workers also have access to two main sources of income in the event of job displacement (Gerard, Naritomi, and Silva, 2021): unemployment insurance and a lump-sum severance payment. Formal private sector workers with at least 6 months of tenure and who are involuntarily laid off have access to unemployment insurance benefits (*Seguro Desemprego*). Benefits received are 80 percent earnings in the three months prior to layoff and range from 100-187 percent of the minimum wage. Benefit duration ranges from 3-5 months and are received as long as the worker is not re-employed. The unemployment insurance program is financed by a 0.65% tax on firm sales.

In addition to this program, which pays in the event of continued unemployment, two programs provide lump-sum payments in the event of dismissal. First, a severance savings account program (*Fundo de Garantia de Tempo de Serviçio (FGTS)*) requires employers to pay 8 percent of a workers wages into an account. Workers gain access to the balance of the account if they are involuntarily laid off. Second, employers must pay severance pay equal to a months wage and a fine of 40 percent of the balance of the worker's FGTS account.

3 Data

Data Source and Sample Our main data source is the PNADC, a nationally representative Brazilian household survey which contains information about social-demographics, education, employment and incomes.[6] Approximately 211,000 households are interviewed each quarter and

[5]For more details about the minimum wage in Brazil, see Engbom and Moser (2022).

[6]Micro data is available for download from https://www.ibge.gov.br/estatisticas/sociais/trabalho/9171-pesquisa-nacional-por-amostra-de-domicilios-continua-mensal.html. The PNADC also asks several questions about the characteristics of the firms in which the individual is employed, such as the size of the firm categorized into several bins. We use the cleaning procedure developed by Data Zoom (available at: https://www.econ.puc-rio.br/datazoom/english/pnadc.html)

interviewed for five consecutive quarters. By appealing to the panel feature of the data, we can construct the annual change of a variable by comparing its values in wave 1 and wave 5.

We use a sample of civilian, private sector, non-farm male workers between the ages of 20 and 64 from quarter 1 of 2012 to quarter 3 of 2019. The final sample includes 3,609,117 observations for 1,220,490 individuals. In some statistics, we exploit the fact that this dataset is a short panel, which allows us to track individuals over the span of a year. When we restrict our attention to only those workers who we observe in waves 1 and 5, we are left with a sample of 278,477 individuals who are in the labor force (employed or unemployed) in both waves and 246,914 who are employed in both waves.[7] Using the survey question on years of schooling, we further categorize workers into three educational groups: less than high school, high school, and college.[8] Among the final sample, 20% have a college degree.

Definition of Formality We restrict our attention to self-employed or remunerated employees and define those who have a contract in their current work or are currently contributing to a Social Security program as formal workers and those who do not as informal.[9] This definition is slightly different from that used in other works. Notably, Meghir, Narita, and Robin (2015) and Engbom and Moser (2022) define informality as all workers who are self-employed or those in remunerated employment without an official work permit. As our focus is the Social Security system, we choose to include those workers who are contributing to a pension program as formal.

As a preview of the empirical facts, we document several salient features of the Brazilian labor market that have strong implications for policy evaluation. First, we find evidence that the Brazilian labor market is frictional, with workers gradually climbing up the job ladder for formal jobs over the life cycle. Furthermore, workers' formality status is the key predictor of wage growth and future job transition. Second, in the cross section, there is a strong sorting pattern between worker skills, firm size, and formality. Last, government regulation plays an important role in determining the aggregate formality, as evident by a clear wage compression of wages around the Federal minimum wage for formal workers.

[7] 243,191 individuals are observed in all waves; 202,264 are employed (either formally or informally) in all waves

[8] Workers who have completed 11 or fewer years of education are classified as "less than high school", those with 12-15 years of schooling are classified as "high school", and those who have completed 16 or more years of education are classified as "college".

[9] We use the primary job in order to classify individuals. Less than 2 percent of year-quarter observations in our sample report having a second or third job.

3.1 Employment by Formality over the Life Cycle

A key feature of the Brazilian labor market is the large informal sector. During the period that we study, informality represents roughly 30 percent of the labor force and approximately one-third of employment. Figure 1 shows the share of the workforce employed formally, employed informally and unemployed at various ages.

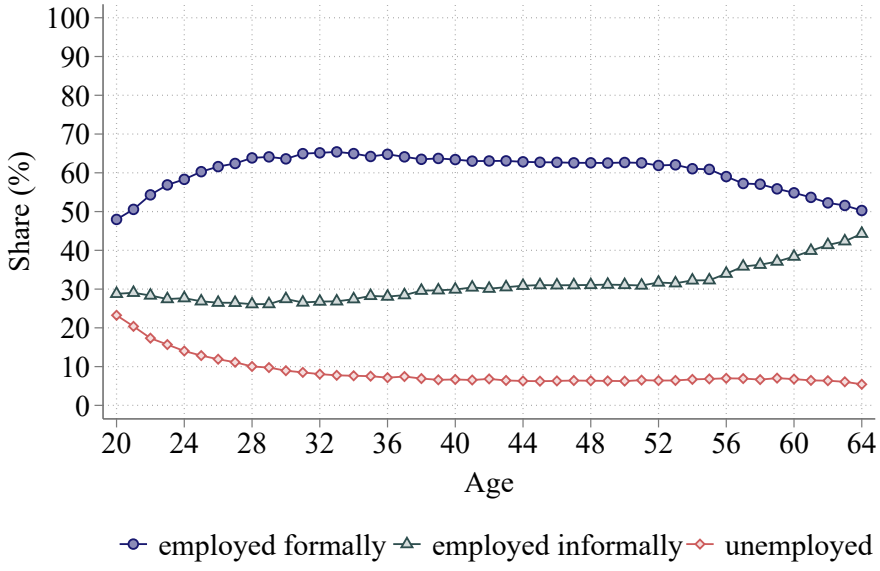


Figure 1: Employment Shares by Age

Figure 1 clearly shows life cycle patterns of formality that can be rationalized by a frictional labor market. The share of workers who are unemployed falls from a high of over 20 percent for those who are aged 20 to a low of under 10 percent for those who are nearing retirement. Workers gradually climb the job ladder to move to formal jobs on the early career path. The share of formal workers stabilizes around 65% between the ages 30 and 50 and gradually decreases afterward. Overall, the share of the workforce that is employed formally exhibits an inverse U shape. However, the share of working informally increases in higher age groups–up to over 40 percent for workers who are nearly retired.[10]

Further dividing workers based on their educational groups, we find that while the level of formality differs across educational groups, the overall life cycle patterns look alike. To be more specific, Figure 2 shows that formality levels for college educated workers reach to over 80 percent of the labor force for those around the age of 40, while this share is roughly 50 percent for those

[10]Note that these shares do not include those workers who are not in the labor force. This increase in informality at older ages is largely driven by a labor force composition effect due to the retirement of formal sector workers. As shown in Figure A.3 in Appendix A, when a non-employment state is included, the share of informal employment is roughly flat over the life cycle, while formal employment decreases beginning around age 55.

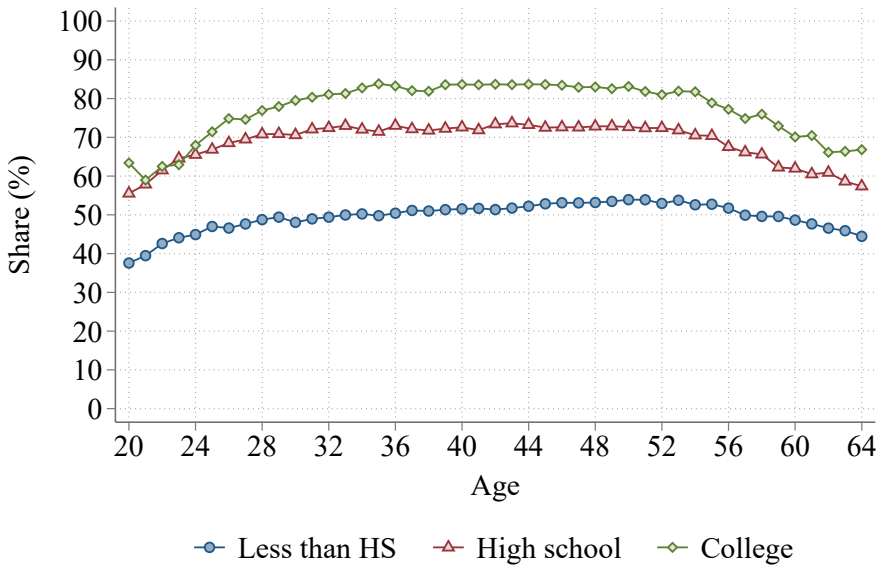


Figure 2: Formality Shares by Age and Education

with less than a high school degree.

3.2 Income Dynamics and Job Transition by Formality

We present the probability of an annual transition between labor market states in Table 1. As Table 1 shows, the Brazilian labor market is fluid: workers who are formally employed when we observe them in the first wave exhibit a 85 percent probability of being formally employed when we observe them a year later; those who are informally employed when we observe them in wave 1 have a 76 percent probability of being informally employed in wave 5.[11] Unemployment is less persistent with 63 percent of those who are unemployed transitioning out one year later. Approximately equal portions transition from the unemployment to the formal sector as from the unemployment to the informal sector. These features remain across the education and age groups (shown in Tables A.2 and A.3 in Appendix A).

Besides future job transition prospects, formality is also an important predictor of workers' wage growth. As well documented in the literature, the job switch is a major source of wage growth. (See, for example, Burdett and Mortensen (1998), Postel-Vinay and Robin (2002), and Cahuc, Postel-Vinay, and Robin (2006).) However, in the context of the Brazilian labor market, the

[11] When computing these transitions, we look only at employment statuses in Waves 1 and 5 rather than incorporating information on transitions in waves 2,3, and 4. Among those who have formal employment in Waves 1 and 5, nearly 90 percent are formally employed throughout the 5 observed quarters, while among those informally employed at the endpoints, 70 percent are informally employed in Waves 2-4. Among those who transition, over 80 percent do not transition through the third state.

Table 1: Annual Employment Transition Rates

	Unemp.	Stayer	Switcher	
Employed formally	0.042	0.851	0.107	Formal 0.074 Informal 0.033
Employed informally	0.067	0.758	0.176	Formal 0.072 Informal 0.104
Unemployed	0.373	Job finding 0.627		Formal 0.327 Informal 0.299

Notes: Individuals who report changing sectors without changing jobs are assumed to stay in the sector from period t_0. Workers are assumed to have switched jobs if their tenure in period t_1 is less than 12 months.

majority of wage changes are associated with a change of worker's formality status. As shown by Figure 3, while the within-sector job switch is only associated with annual wage growth less than 2 percent, the across-sector job switch is associated with much larger changes in wages — moving from an informal job to a formal one leads to over 8 percent wage increase for all education groups, amounting to 13 percent for the college-educated, and moving from a formal job to an informal one leads to over 12 percent wage decrease. Therefore, to better understand the welfare effects of policy reforms, we need to incorporate the frictional features of the labor market, as well as its impact on wage progression, into the framework.

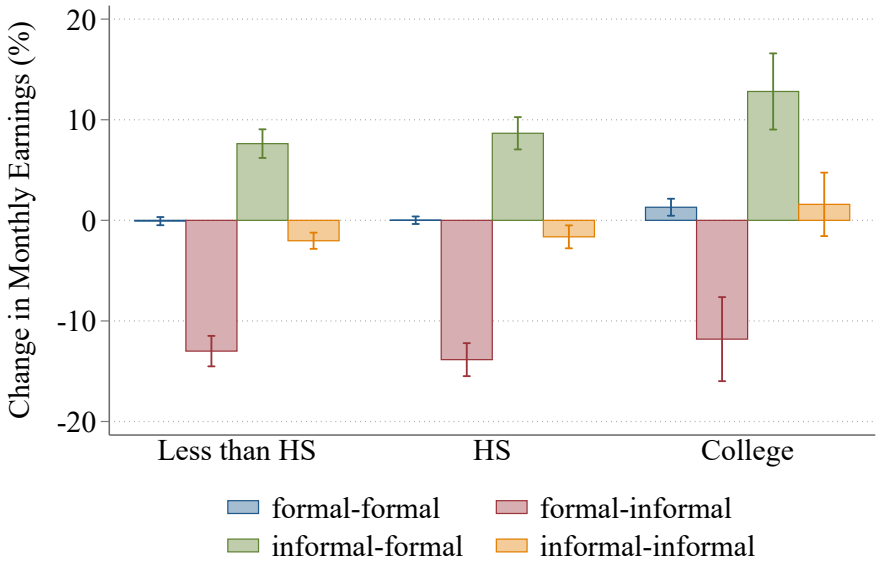


Figure 3: Earnings Growth by Formality Status

Table 1 and Figure 3 clearly demonstrate that formality is the key dimension to understand the

future job transition and wage growth for workers on the Brazilian labor market. We conclude this subsection by showing the wage growth by educational group in Figure 4. As Figure 4 shows, there is a large wage growth gap between those who have a college degree and those who do not, and this gap closes after age 40. In Table A.1 in Appendix A, we provide further more detailed statistics on the large wage premium across educational groups.

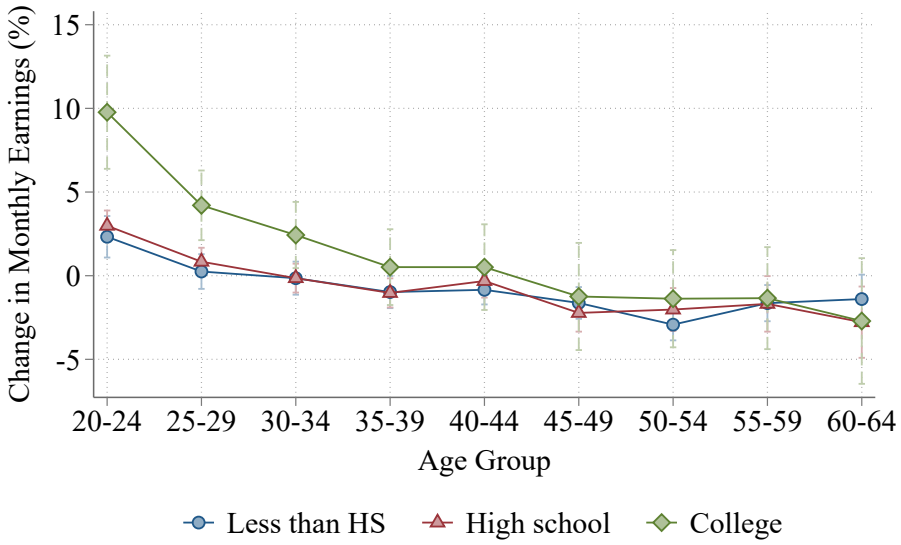


Figure 4: Earnings Growth by Education and Age

3.3 Minimum Wage and Wage Distribution

In Figure 5, we present the wage distributions[12] in log multiples of the minimum wage by the formality status of the workers. As Figure 5(a) clearly shows, little mass of workers have reported income below the federal minimum wage. Consistent with Engbom and Moser (2022), we also observe that the presence of the minimum wage compresses the income distribution of formal workers, with a clear clustering of around 13% formal workers who receive exactly the minimum wage. In comparison, the informal income distribution has a larger mass below the minimum wage, as shown by Figure 5(b). The average income for formal workers is 0.46 log points higher than that of informal workers.

Figure 5 demonstrates that formal regulation, in particular the minimum wage, apparently affects firm formality choice through financial incentives. The significant fraction of formal jobs

[12]To collect information on income survey asks: "Qual era o rendimento bruto mensal que...recebia normalmente nesse trabalho?" (What is the gross monthly income normally received from this job?). Therefore, we interpret this income as pre-tax earnings. The value reported is either monetary or the value of goods received as payment.

operating around the minimum wage suggests that the Social Security policy reform, which is tightly linked to the worker's formality status, may have an impact on the labor market.

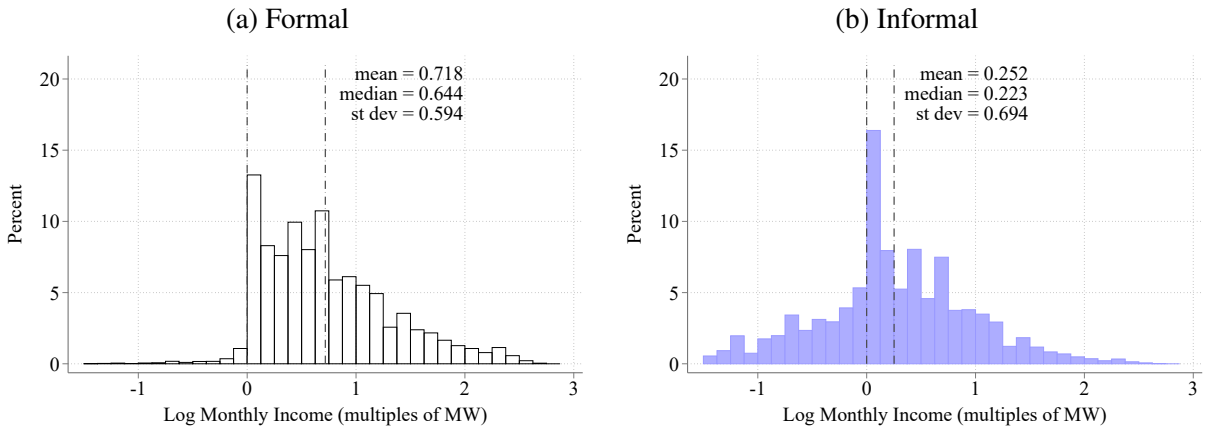


Figure 5: Distribution of Log Monthly Incomes by Formality

3.4 Sorting between Firm Size, Worker Skill, and Formality

Finally, we document how formality vary by firm characteristics. As is common for household survey data, the PNADC does not link a worker to the exact firm where they are employed. Alternatively, we make use of the PNADC firm size variable and present summary statistics of workers who are employed in firms within the same firm size bin.[13]

We consider five firm size bins, namely 1 to 4 employees, 5 to 9 employees, 10 to 19 employees, 20 to 49 employees, and over 50 employees. Taking into account the large number of informal firms, the Brazilian economy is populated by small firms, with only 0.8% of the firms having more than 50 employees. As shown by the purple bars in Figure 6, the majority of employment is concentrated among the smallest and largest firms in the economy, firms of 1 to 4 employees and firms of more than 50 employees each account for more than 30% of the total employment, while the three middle size bins each account for less than 10% of the total employment. Furthermore, the white bars Figure 6 demonstrate that the share of formal employment is strictly increasing in the size of firm—while only 30% of workers employed among the smallest firms are formal, more than 90% of workers among firms of more than 50 employees are formal.

The share of formal employment by firm size presented in Figure 6 reflects both the share of formally registered businesses, the *extensive margin*, and the share of formal workers within the

[13] Admittedly, our approach abstracts from the variations of firms within the firm size bin. To further examine firm differences along other dimensions, one needs to use more detailed matched employer-employee data as in Haanwinckel and Soares (2021) and Engbom and Moser (2022).

firm, the *intensive margin*. A major limitation of using household survey data is that we do not directly observe the formality status of the firm, i.e., the extensive margin. However, as argued by Ulyssea (2018), the intensive margin explains a large share of total informal employment, and more than 40% informal workers are employed by formal firms. Furthermore, as will become clear in the following sections, our approach overcomes this problem by calibrating a structural model, which has a clear notion of intensive and extensive margins, only by matching the model counterparts of formality share by firm size with the data documented in Figure 6.



Figure 6: Employment, Formality Shares by Firm Size Category

This increase in formality share in firm size can be partially explained by the higher level of education among workers, as shown in the purple bars in Figure 7. Although the share of employees with at least a high school degree increases from around 40 percent among employees at firms of 1-4 employees to nearly 65 percent among employees at firms of over 50 employees. Furthermore, as is clear by the blue line in Figure 7, monthly income is also increasing in the size of firms——the earnings gap between the largest and smallest firms is over 30 percent. Applying the framework of Abowd, Kramarz, and Margolis (1999) that decomposes the wage into fixed effects of the firm and the worker to Brazilian data, Alvarez, Benguria, Engbom, and Moser (2018) and Engbom and Moser (2022) document that the firm component explains a large proportion of wage inequality and its dynamics over time. Consistent with Engbom et al. (2022), we also find that informal jobs are associated with greater volatility of income compared to formal ones.

Taking together Figure 6 and Figure 7, we easily come to a striking conclusion that those workers who are employed formally are more likely to have a higher education degree and earn higher wages, while those who earn less, presumably having a larger demand for social insurance,

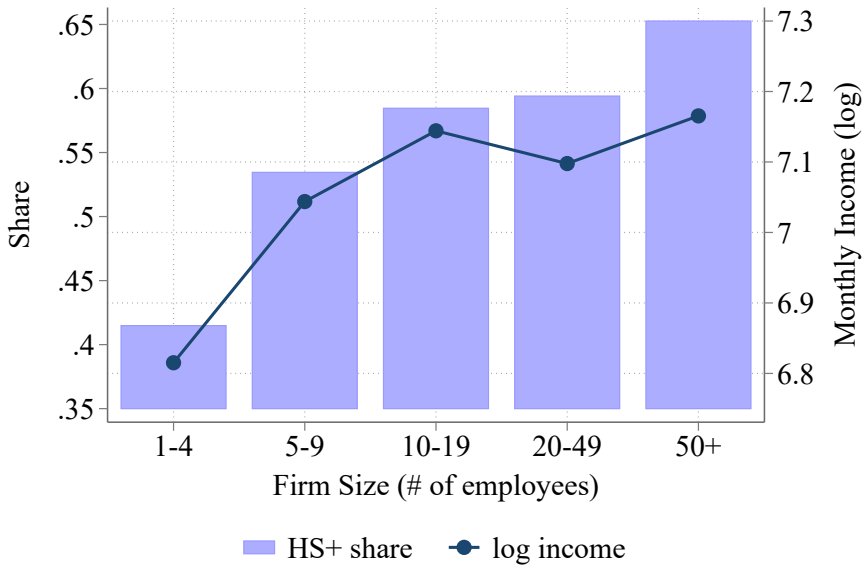


Figure 7: Education Share and Income by Firm Size Category

are more likely to be employed informally and excluded from the coverage of social insurance programs. As formality and recruitment decisions are jointly made by firms, the strong sorting pattern between firms, workers, and formality suggests that pension reforms should take firms' behavioral responses into account.

4 Model

The data section clearly demonstrates formality as a key dimension in understanding the Brazilian labor market, which is determined endogenously through the interaction between workers, firms, and government policies. In this section, we present a life cycle labor search model to match these salient features of the Brazilian labor market.

Our model builds on the seminal work of Burdett and Mortensen (1998), whose framework is further extended by Aizawa and Fang (2020) to allow multiple components in a firm contract and by Meghir, Narita, and Robin (2015) and Engbom and Moser (2022) to allow multiple sectors. Our framework is geared towards large scale policy experiments at both household and firm levels by introducing two key features to the model: first, we model formality as an equilibrium outcome of firms' contract posting problem subject to workers' consent; second, we introduce rich life cycle features and frictional labor market which accounts for households' incentive to balance value of formal and informal offers in terms of sector-contingent employment and wage growth prospects. The integration of rich two-sided heterogeneity endogenously determines wage growth and job

switch over the life cycle and sorting between workers and firms in the cross section.

4.1 Setting

Time is discrete. We present a version of the model at the stationary equilibrium and will drop the time subscript of the model. The economy is populated by an infinitely lived government, infinitely lived firms that differ in productivity z, and J overlapping generations of individuals. Throughout the model, the notation of *firm* should be thought of as a multi-worker firm which consists of multiple worker-job matches and *job* is a position to which a single worker is matched.[14] Individuals are risk averse and maximize lifetime utility. Firms are risk neutral and maximize the discounted present value of profit flows. Individuals and firms have the same discounting factor β. The cohort size of population grows at a rate of g_n. Age is indexed by $j = \{1, 2, ..., J\}$ and age-dependent conditional survival probability is denoted as θ_j. Upon entering the model economy at age $j = 1$, each individual is endowed with a permanent skill level $s = \{1, ..., S\}$. Individuals can work up to age $J_R - 1$. After J_R, all individuals retire.

Jobs within the same firm use the same firm productivity and employ workers at different productivity levels. All jobs are full-time.[15] In the model, formality is specified at the match level, i.e., a firm may employ both formal and informal employment by posting vacancies in markets for formal and informal contracts. Firms differ in the intensive margin of formality—the share of formal workers in total employment, which is affected by firm's vacancy postings. The extensive margin of informality is manifested in the share of firms where all employment are informal.

Firms post vacancies on markets for formal jobs ($\psi = 1$) and informal jobs ($\psi = 2$). We abstract from realistic characters of firms such as industries and focus on the heterogeneity of firms in their productivity z. We will use formality and sectors interchangeably. Given productivity z, firms choose the total number of vacancies subject to a convex vacancy cost function. In addition to the formality of the job, firms also choose a piece rate ϕ on the output of a match that workers will receive as their wages. The combination of formality ψ and piece rate ϕ is referred to as a contract. As in models featuring imperfect labor market competition following the work of Burdett and Mortensen (1998), firms in the model have monoposony power by competing over contracts in anticipation of other firms' optimal behavior. A firm may set piece rates differently for formal and informal vacancies, but will set the same piece rate for all vacancies conditional on formality as an equilibrium outcome. We use the vector of states $\mathbf{y} \equiv (z, \psi, \phi)$ to denote the job-related states for an individual. For unemployed and retired individuals, z, ψ, ϕ are assumed to be 0.

Individuals differ in age j, individual states $\mathbf{x} \equiv (a, b, s, h)$ and job states $\mathbf{y}$. a is private savings. b is average lifetime earnings on formal jobs which evolves according to the worker's

[14] To simplify the model, we assume labor is divisible and the number of workers in a firm take continuous values.
[15] According to PNADC, only 6.6 % of prime-age, employed men are working less than 30 hours weekly.

employment history and serves as the basis to calculate social security benefits after retirement. h is the general human capital of the worker, which accumulates on the job and depreciates during unemployment. The distribution of individuals is denoted as $\mu(j, \mathbf{x}, \mathbf{y})$.

At the beginning of a period, workers employed in the previous period are hit by an exogenous shock to be separated from the current job with probability λ_d. For simplicity, we assume that the newly-separated will rest from search in the current period. The rest of the workers, including both unemployed and non-separated, search for a job. The job search is random. Individuals' search efficiency vary by age, skill, employment status, and their current and targeted sector. To distinguish from the distribution by employment status, we denote the distribution of searchers by $\mu_+(j, \mathbf{x}, \mathbf{y})$. For simplicity, we assume that individuals can receive at most one job offer in every period. After receiving a job offer, individuals decide whether they want to move to the new job based on a comparison between the utility of staying as the current status and the utility of the new job. Once the matching decisions are made, the operative matches continue to the production stage. After production, workers and firms split the output according to the piece rate specified in the contract and pay taxes according to government obligations at the worker and firm levels. Individuals decide on consumption and savings for the next period, after which time proceeds to the next period.

4.2 Search and Matching

Vacancies are segmented by sectors ψ. The aggregate vacancy in the market ψ is denoted by V_ψ. Workers have a probability of meeting vacancies in both markets. The search efficiency of the worker is affected by the worker's age, skill level, current employment status ψ_{-1}, and the target sector ψ, and we denote it by $\alpha(j, s, \psi_{-1}, \psi)$.

The total measure of effective searchers on market $\psi \in \{1, 2\}$

$$U_\psi = \sum_{j=1,\ldots,J_R-1} \sum_{s=\{1,\ldots,S\}} \sum_{\psi_{-1}=\{0,1,2\}} \alpha(j, s, \psi_{-1}, \psi) e(j, s, \psi_{-1}).$$

Aggregate matching function for both sectors are given by

$$M_\psi = \mathcal{M}(U_\psi, V_\psi).$$

The probability for an individual in sector ψ_{-1} to find a job in sector ψ is given by

$$\lambda_e(j, s, \psi_{-1}, \psi) = \alpha(j, s, \psi_{-1}, \psi) \frac{M_\psi}{U_\psi}, \tag{1}$$

and the probability for a vacancy in sector ψ to contact a searcher is given by

$$q_\psi = \frac{M_\psi}{V_\psi}. \tag{2}$$

For simplicity, we assume that a worker can only receive at most one job offer every period. Upon meeting a job vacancy, an individual decides whether to take the job offer based on the comparison between the value of staying with the current employment status and the value of working for the new employer. As in Aizawa and Fang (2020), we smooth the switching probability by introducing a preference shock for making a move to the poaching firm ε. The variance of the preference shocks is assumed to be σ_e for job-to-job transitions and σ_n for unemployment-to-employment transitions. The preference shock captures non-pecuniary factors such as job flexibility and demand shock for liquidity moving to informal jobs, which rationalizes potential wage cuts after voluntary job switches. Once matched, a worker-job match starts to produce. An unmatched job vacancy expires with value 0.

4.3 Government

Regulation by formality On the employer side, formal jobs have to pay above the minimum wage $w_{\min}$ and are subject to corporate income tax at a proportional rate τ_{ci} on profits after paying wages and the firm part of payroll taxes. The workers employed formally pay a progressive wage income tax on the wage incomes (τ_l). Under the current rule, Social Security payroll taxes are $\tau_{sl} = 28\%$ of wage income in total, with $\tau_{sl}^f = 20\%$ of wages contributed by the employer and the remaining $\tau_{sl}^w = 8\%$ contributed by the worker.

Informal jobs can avoid all formal regulations. However, they are randomly screened by the government authorities whose role is to enforce tax and labor laws. Once caught, the firm turns in all profits in the current period to the government after paying wages. The history of being caught does not affect firms' operation in the future, and the caught match can continue to the next period with probability of being caught again. The probability of being caught for an informal job is convex in the output y and is capped by a maximum probability $\eta_{\max}$:

$$\eta(y) = \min\{\eta_0 y^{\eta_1+1}, \eta_{\max}\}, \quad \eta_0, \eta_1 > 0. \tag{3}$$

The convex screening technology $\eta(.)$ ensures that more productive jobs are more likely to operate formally, allowing the model to capture the empirical fact that more productive firms have a larger share of formal workers.

Social Security Social security benefits are affected by both the number of contribution years and the total contribution.[16] To reduce the computational burden, we use only a single state—the Social Security contributions b—to approximate the accumulated social security wealth. b increases by $1/35$ of worker's wage income in every period of formal employment and is capped at a maximum contribution $b_{\max}$.[17] Retired individuals collect social security benefits $w_r(b)$ based on their contribution level b. Non-employed individuals before age 65 are eligible for basic unemployment income w_n. To simplify the problem, we assume that w_n is independent of the previous employment histories of the individuals.

Fiscal budget Motivated by the fact that the old-age assistance program is largely financed by general fiscal revenue instead of the Social Security funds. We do not have a separate Social Security budget. Rather, the revenue and expenditure of the Social Security program come together with other fiscal programs.

The government makes an exogenous expenditure on general public goods provision which is valued at G and issues one-period risk-free debt D', which pays the interest rate of r. Fiscal revenues are raised from taxes on formal jobs, penalties on informal jobs, and universal taxes on capital income and consumption. Consumption is taxed at a proportional rate τ_c. The government budget constraint is satisfied in every period:

$$\begin{aligned}&\sum_{\chi}\eta(y(\chi))\mathbb{1}_{[\psi=1]}\mu(\chi)+\sum_{\chi}[\tau_{sl}w(\chi)+\tau_l(w(\chi))+\tau_{ci}y(\chi)]\mathbb{1}_{[\psi=2]}\mu(\chi)\\&+\sum_{\chi}[ra(\chi)\tau_a+\tau_c c(\chi)]\mu(\chi)+D'=G+\sum_{\chi}w_r(\chi)\mu(\chi)+(1+r)D\end{aligned}\tag{4}$$

As will be clear in the following subsection, the public good provision enters the utility function of households. When the Social Security runs a larger deficit, the cost will pass through to the planner in the form that household derive less utility from general public good provision.

4.4 Firms

Firms post vacancies at the beginning of each period. Firms choose the number of vacancies to post in each sector and the contract (ϕ, ψ) associated with vacancies on each market. After the matching process, the newly formed matches together with matches continued from the previous period then proceed to the production stage. A firm's profit is simply the sum of profits in all worker-job matches.

[16] McKiernan (2022) provides a detailed description of the social security accumulation and benefit rules.
[17] See Appendix A for more details about the approximation method.

Production The effective human capital for a worker with general human capital h at skill level s is $x = x_s h$. The firm can choose for each of the matches from traditional or modern production technology. We denote the total surplus of a match by $\zeta(z, x) = \max\{y_0, y_1\}$.

The traditional production technology is labor-only,

$$y_0 = zx.$$

The modern production technology additionally uses capital k,

$$y_1 = zk^{\alpha}x^{1-\alpha}.$$

The maximum capital a firm can borrow is $k \leq \mu_{\psi} y_0$, where μ measures the degree of financial development, which is allowed to differ between sectors. $\mu = \infty$ refers to a perfect financial market.

The choice of production technology by firms can thus be characterized by a cutoff rule. Firms with a lower productivity than the cutoff $\bar{z}$ choose traditional technology and firms with a higher productivity than $\bar{z}$ choose modern technology. By introducing the choice of technology, we obtain the equilibrium outcome that more productive firms are more capital intensive, as is consistent with Franjo, Pouokam, and Turino (2022) and Erosa, Fuster, and Martinez (2023).[18]

Profits of a job Workers employed informally receive a ϕ fraction of the total output as their wages. Informal workers do not pay labor income taxes. The informal job gets all remaining profits $(1 - \phi)\zeta$ if it is not caught and gets 0 otherwise. The expected net profit flow for an informal job is thus given as

$$\pi(x, z, \phi, \psi = 1) = (1 - \eta(\zeta))(1 - \phi)\zeta(z, x). \tag{5}$$

To account for the bunching of wages around the minimum wage as in the data, we assume that formal jobs always pay at least the minimum wage. The wage of a formal worker is thus given as $w = \max\{w_{\min}, \phi\zeta\}$. Formal firms pay Social Security contributions $\tau_{sl}^{f} w$ on the behalf of the worker and pay corporate income taxes on gross profits $(\zeta - w - \tau_{sl}^{f} w)$ at a flat rate τ_{ci}. Wage income taxes are paid by workers thus do not enter firms budget. A formal job's net profit given piece ϕ is

$$\pi(x, z, \phi, \psi = 2) = (1 - \tau_{ci})(\zeta - (1 + \tau_{sl}^{f}) \max\{w_{\min}, \phi\zeta\}). \tag{6}$$

[18] Itskhoki and Moll (2019) argues that suppressing factor prices at early stage of development can accelerate development through faster wealth accumulation of entrepreneurs which further relax their borrowing constraints at later stages.

As can be seen from equations (5) and (6), profit flow for an informal job is always non-negative, while profit flow for a formal job can possibly be negative due to the binding minimum wage constraint. We assume that those matches yielding negative profit flows can never be formed even if profits can possibly turn positive in the future with human capital growth of the worker.

The expected revenue yield per vacancy given $(j, \mathbf{x}, \mathbf{y})$—the total discounted present value of profit flows conditional on the match is not separated—is thus given by

$$\Pi(j, \mathbf{x}, \mathbf{y}) = \sum_{t=j}^{J_R-1} \beta^{t-j} \int (1 - \tilde{\lambda}_t(\mathbf{x_t}, \mathbf{y})) \mathbb{E}\pi(t, \mathbf{x_t}, \mathbf{y}) d\mathcal{G}(\mathbf{x_t}|\mathbf{x}), \tag{7}$$

where $\mathcal{G}(\mathbf{x_t}|\mathbf{x})$ is the expected distribution of worker states conditional on the worker being continuously employed by the same firm $\mathbf{y}$ until horizon t and $\tilde{\lambda}_t(\mathbf{x_t}, \mathbf{y})$ is the cumulative separation rate given states $(\mathbf{x_t}, \mathbf{y})$, which takes into account of risks of exogenous and endogenous separation and mortality.

Contracts For $\psi \in \{1, 2\}$, the expected vacancy yields are given as

$$\tilde{\Pi}(z, \psi) = \max_{\phi} \{ \int \mathbb{E}[\Pi(j, \mathbf{x}, \mathbf{y} = (z, \phi, \psi)) \mathbb{1}_A(j, \mathbf{x}, \mathbf{y}', \mathbf{y})] d\mu_+(j, \mathbf{x}, \mathbf{y}') \}, \tag{8}$$

where $\mathbb{1}_A(j, \mathbf{x}, \mathbf{y}', \mathbf{y})$ denotes that an individual with states $j, \mathbf{x}, \mathbf{y}'$ accepts an job offer $\mathbf{y}$. We allow piece rates $\{\phi_1, \phi_2\}$ to be contingent on formality of the job. When choosing ϕ, firms internalize the fact that a higher ϕ generates a lower revenue margin per vacancy but increases the probability of forming a match, as is standard in models of wage competition such as Burdett and Mortensen (1998).

Given the vacancy yield by formality, firms choose ψ to maximize expected yield for each vacancy:

$$\Pi(z|\epsilon_f) = \max_{\psi \in \{1,2\}} \{\Pi(z, \psi = 1), \Pi(z, \psi = 2) - \sigma_f \epsilon_f\}.$$

where ϵ_f is an unobserved management shock that is i.i.d. across all vacancies.[19] By assuming that ϵ_f follows type I extreme value distribution with standard deviation σ_f, we obtain a close-form

[19]The management shock captures all unobserved non-market shocks that affect firm's decision of posting formal vacancies, such as social norm within the local market.

expression for the probability of posting a formal vacancy $\Delta(z)$:

$$\Delta(z) = \begin{cases} \frac{\exp(\frac{\Pi(z,2)}{\sigma_f})}{\exp(\frac{\Pi(z,2)}{\sigma_f})+\exp(\frac{\Pi(z,1)}{\sigma_f})} & \text{if } \Pi(z,2) > 0 \\ 0 & \text{if } \Pi(z,2) \leq 0. \end{cases} \tag{9}$$

At the firm level, $\Delta(z)$ is also the share of formal vacancies. Apparently, $\Delta(z)$ increases as the value of a formal vacancy relative to an informal vacancy increases. The size of the shock σ_f determines the elasticity of the formal vacancy share to policy changes.

Vacancy posting and firms size The expected yield is $q_1\Pi(z,1)$ for an informal vacancy and $q_2\Pi(z,2)$ for a formal vacancy. Firms are subject to a convex vacancy posting cost $\mathcal{C}(v)$, and firms choose v to maximize the expected vacancy yield of profits net of vacancy costs. Using the first-order condition, the vacancy per firm v^* equalizes the marginal cost and expected marginal yield of vacancy posting:

$$\mathcal{C}'(v^*) = (1-\Delta)q_1\tilde{\Pi}(z,1) + \Delta q_2\tilde{\Pi}(z,2). \tag{10}$$

A firm then posts Δv^* formal vacancies and $(1-\Delta)v^*$ informal vacancies. To fix the idea, we follow Kaas and Kircher (2015) and Lise and Robin (2017) and assume that the vacancy cost function is characterized by a level parameter γ_0 and a curvature parameter γ_1:

$$\mathcal{C}(v) = \gamma_0 v^{\gamma_1+1}, \quad \gamma_0, \gamma_1 > 0. \tag{11}$$

To pin down the firm size in equilibrium, we assume that firms with the same productivity have the same employment size and the same employment composition within the firm. The measure of firms by productivity is simply the total number of vacancies by firm productivity over the amount of vacancies per firm. The size of a firm is given as the population of workers employed in firms of the same productivity divided by the measure of these firms.[20]

4.5 Individuals

The flow utility for an individual of skill level s in period j is given as

$$u_j(c,l) + v(G) + \gamma_{s,\psi},$$

where c is private consumption and l is leisure. $v(G)$ is the utility derived from general public good provision. Workers of skill level s differ in the non-pecuniary utility $\gamma_{s,\psi}$ from working in

[20] See Appendix B for the analytical derivations.

sector ψ, which gives us an additional degree of freedom to match sorting by skills across sectors. Individuals also derive warm-glow utility $\mathcal{B}(.)$ from leaving bequests to future generations.

General human capital h lies in equi-spaced grids $\mathcal{H}$, with grid step size Δh. All individuals enter the economy with the lowest level of human capital. Worker human capital accumulates on the job and depreciates during unemployment. In particular, for an unemployed individual with human capital level h, human capital evolves following

$$h' = \begin{cases} h - \Delta h & \text{with prob. } \rho_n(h) \\ h & \text{with prob. } 1 - \rho_n(h). \end{cases}$$

We allow human capital evolution to differ by sector ψ to account for the difference in human capital growth in formal and informal jobs, as highlighted by Bobba et al. (2021):

$$h' = \begin{cases} h + \Delta h & \text{with prob. } \rho_\psi(h) \\ h & \text{with prob. } 1 - \rho_\psi(h). \end{cases}$$

In our setup, firms make strictly positive profits net vacancy costs. We assume that aggregate bequests and aggregate firm profits are reimbursed to households in a uniform lump sum fashion, and the lump sum transfer is denoted as w_{ls}.

Recursive life cycle decisions We denote value for retired workers as V^R, for employed and unemployed workers as V^E and V^N respectively. Individual's problem can be formulated recursively and solved by backward induction.

Between the period $j = J_R, ..., J$, all individuals are retired. Individuals receive social security benefits $w_r(b)$ and make simple consumption-savings decisions. The problem for retired individuals are given as :

$$\begin{aligned} V_j^R(\mathbf{x}) = & \max_{c,a'} \{u_j(c,0) + \beta\{(1-\chi_j)\mathcal{B}(a') + \chi_j V_{j+1}^R(\mathbf{x})\}\} \\ s.t. \quad & a' = (1+r)a - (1+\tau_c)c + w_r(b) + w_{ls} \\ & a' \geq 0 \end{aligned} \tag{P1}$$

Between period $j = 1, .., J_R - 2$, the employed individual's problem can be formulated as:

$$
\begin{aligned}
V_j^E(\mathbf{x}, \mathbf{y}) = \max_{c,a'} \{ & u_j(c, 1) + \gamma_{s,\psi} + \beta \{ (1 - \chi_j) \mathcal{B}(a') + \chi_j [\lambda_d \mathbb{E}(V_{j+1}^N(\mathbf{x}')) \\
& + \int \int \lambda_e(\mathbf{x}', \mathbf{y}') \max \{ \mathbb{E}(V_{j+1}^E(\mathbf{x}', \mathbf{y})), \mathbb{E}(V_{j+1}^E(\mathbf{x}', \mathbf{y}')), \mathbb{E}(V_{j+1}^N(\mathbf{x}')) + \sigma_w \epsilon \} dF(\mathbf{y}') d\Phi(\epsilon_w) \\
& + (1 - \lambda_d - \lambda_e) \int \mathbb{E}(\max \{ V_{j+1}^E(\mathbf{x}', \mathbf{y}), \mathbb{E}(V_{j+1}^N(\mathbf{x}')) + \sigma_n \epsilon \}) d\Phi(\epsilon)] \} \} \\
s.t. \quad a' = & \ (1 + r)a - (1 + \tau_c)c + w(\mathbf{x}, \mathbf{y}) - \mathbb{1}_{\psi=2} T_l(w) + w_{ls} \\
b' = & \begin{cases} b + \frac{1}{35} \min \{ w(\mathbf{x}, \mathbf{y}), b_{\max} \} & \text{if } \psi = 2 \\ b & \text{otherwise} \end{cases} \\
h' = & \begin{cases} h + \Delta h & \text{with prob. } \rho_\psi(h) \\ h & \text{with prob. } 1 - \rho_\psi(h) \end{cases} \\
a' \geq & \ 0
\end{aligned}
\tag{P2}
$$

Between period $j = 1, .., J_R - 2$, unemployed individual's problem is similarly given as

$$
\begin{aligned}
V_j^N(\mathbf{x}) = \max_{c,a'} \{ & u_j(c, 0) + \beta \{ (1 - \chi_j) \mathcal{B}(a') + \chi_j [(1 - \lambda_n) \mathbb{E}(V_{j+1}^N(\mathbf{x}')) \\
& + \int \int \lambda_n(\mathbf{x}', \mathbf{y}) \mathbb{E}(\max \{ V_{j+1}^E(\mathbf{x}', \mathbf{y}), V_{j+1}^N(\mathbf{x}') + \sigma_n \epsilon \}) dF(\mathbf{y}) d\Phi(\epsilon)] \} \} \\
s.t. \quad a' = & (1 + r)a - (1 + \tau_c)c + w_n + w_{ls} \\
b' = & \ b \\
h' = & \begin{cases} h - \Delta h & \text{with prob. } \rho_n(h) \\ h & \text{with prob. } 1 - \rho_n(h) \end{cases} \\
a' \geq & \ 0
\end{aligned}
\tag{P3}
$$

The problems in the final working period $J_R - 1$ are similar to problems (P2) and (P3). The only difference is that the expected continuation values in the next period are replaced by the retirement value in the period J_R.

4.6 The Coexistence of Formal and Informal Job in the Model

In our model, the formal and informal sectors coexist. In Table 2, we list the advantages and disadvantages that an informal job brings to different agents.

Table 2: Pros and Cons of Informality in the Model

Pros	Cons
Workers	
• alternative to unemployment • avoid income-based taxation • preference for liquidity/job flexibility	• restricted access to government programs (Social Security, unemployment insurance, etc.) • lower wages/no minimum wage guarantee • less job stability • potential consequences if caught
Firms	
• avoid contributions • non-binding minimum wage	• lower human capital workers • potential consequences if caught
Government	
• job creation for low-skilled	• decreased income based tax revenue

4.7 Definition of a Competitive Equilibrium

A stationary competitive equilibrium consists of government policies $\{G, w_{min}, w_n, \tau_l(.), \tau_{sl}, \tau_{ci}, \eta(.)\}$, a lump-sum transfer w_{ls}, interest rate r, a distribution of individuals $\mu(j, \mathbf{x})$, labor market contact rates q_ψ for firms and $\lambda(.)$ for individuals, value functions of individuals $\{V^N, V^E, V^R\}$, policy functions of individuals $\{c, a'\}$, value functions of firms $\Pi(z, \psi)$, and policy functions of firms $\{\Delta, v_1, v_2, \phi_1, \phi_2\}$ such that:

1. Given contact offer rates and distribution, individuals maximize with optimal choice of consumption, savings and offer acceptance as in problems (P1), (P2), and (P3);

2. Given policies, prices, and distribution of individuals, firms maximize with optimal choice of vacancy numbers on formal and informal markets and piece rates conditional on formality as given by (8), (9), and (10);

3. Labor market contact rates and job offer distributions are induced by firms' optimal contract offering and vacancy posting and distribution of searchers as given by equations (1) and (2);

4. Given capital rental rate r, each match chooses production technology and capital demand subject to sector-specific borrowing limit;

5. Individual distribution induced by offer distribution and individuals' optimal choices;

6. Capital market clears;

7. The fiscal budget (4) is balanced;

8. The lump-sum transfer w_{ls} is the sum of aggregate firm profits and aggregate bequests;

9. The distribution of individuals across states is stationary as the flow balance equations of individuals hold.

The equilibrium concept is standard and fully detailed in the Appendix.

5 Calibration

We use data between 2012 and 2019 and treat the data as an approximation of a steady state equilibrium. We classify individuals' permanent skill type by their education. We consider college graduates as high-skilled, which accounts for around 20% of the population and all the rest as low-skilled. We calibrate the model to match the aggregate and distributional moments of the Brazilian economy. After presenting the functional specifications of the rest of model elements to complete the model, we layout our calibration strategy which is further divided into two subsections for two block of parameters. In the first, we present the pre-determined parameters that are directly taken from the data or calibrated to target standard moments as in the literature. Table 3 summarizes those pre-determined parameters. In the second, we detail how we infer the key parameters related to informality from moments specific to our context, as summarized in Table 4. After presenting the parameters, we show the implications of the model in terms of contract posting and vacancy yield by firm productivity.

5.1 Functional Specification

Preferences Individuals period utility is CRRA in consumption and leisure

$$u_j(c,l) = \frac{(c^{\rho_j} l^{1-\rho_j})^{1-\gamma}}{1-\gamma},$$

where γ is a parameter for risk aversion, and ρ_j is the age-dependent utility weight on consumption. Leisure is set to be 1 for unemployed and retired individuals and $\frac{2}{3}$ for employed individuals.

Following Heathcote, Storesletten, and Violante (2017), we assume that the utility for the general public good is $v(G) = \psi_g log(G)$ and set the parameter $\psi_g = 0.233$.

Following De Nardi (2004) and Lockwood (2018), we use the following bequest utility function:

$$\mathcal{V}(b) = (\frac{\phi_b}{1-\phi_b})^{\tilde{\rho}} \frac{(\frac{\phi_b}{1-\phi_b} c_b + b)^{1-\tilde{\rho}}}{1-\rho}.$$

$\tilde{\rho} = 1-\gamma(\rho-1)$ captures the weight on consumption, which is consistent with the flow utility $u_j(.)$ for $j = J_R$. The parameter $\phi_b \in [0, 1)$ is the marginal propensity to bequeath and the parameter $c_b > 0$ is the threshold consumption level below which people do not leave bequests.

Search and matching Both sectors have a common aggregate matching function

$$\mathcal{M}(U, V) = \min\{\alpha_0 U^{1-\alpha_1} V^{\alpha_1}, U, V\}.$$

Search efficiency contains an age component α_j, a skill component α_s, and it is also affected by individual's current employment status ψ_{-1} and the targeted sector ψ:

$$\log \alpha(j, s, \psi_{-1}, \psi) = \alpha_j + \alpha_w \mathbb{1}_{[\psi_{-1}=\psi]} + \alpha_b \mathbb{1}_{[\psi_{-1}\neq\psi]} + \alpha_s \mathbb{1}_{[\psi=2]}. \tag{12}$$

Albeit simple, the function (12) is flexible enough to capture the variations in the transition rates by worker characteristic observed in the data. To begin with, we allow the search efficiency to have an age component α_j, which can potentially rationalize the increasing pattern of informality at older ages. We also allow search efficiency to be different by sector—search efficiency of on-the-job search relative to search off unemployment is α_w if conducted within-sector and α_b if conducted between-sector. To account for the potential difference by skill type in the ability to search for formal jobs, we additionally introduce a skill component α_s when the target sector $\psi = 2$.

Human capital We set $s_{col} = 0.6$ which is the initial human capital difference between college and non-college to match the average income ratio between two educational groups 2.5. In the model, the skill premium also reflects the fact that college graduates sort into more productive firms and formal jobs, where they also enjoy faster human capital growth.

As a natural normalization, the lower bound of the human capital grid, which is human capital for the entrants to the labor market, is set to $h_{\min} = 1$. We set the upper bound to be $h_{\max} = 4$ to match the variance of log wages in the cross section. The human capital upgrading shock is

modeled as

$$\rho_\psi(h) = \rho_{\psi,0}(h^{-\rho_{\psi,1}} - {h_{\max}}^{-\rho_{\psi,1}}), \quad \rho_{\psi,0}, \rho_{\psi,1} > 0,$$

which decreases in the level of human capital stock h and converges to 0 when $h = h_{\max}$. Similarly, the human capital downgrading shock is modeled as

$$\rho_n(h) = \rho_{n,0}(h - h_{\min})^{\rho_{n,1}}, \quad \rho_{n,0}, \rho_{n,1} > 0,$$

which takes the value 0 when $h = h_{\min}$ and increases in the level of human capital stock h.

5.2 Pre-determined Parameters

Environment Brazil has been going through a decline in population growth, from 1.7% in the early 1990s to 0.8% in 2020. We set $g_n = 1.0\%$, which is the average during the periods studied. Mortality rates are taken from McKiernan (2022), as shown Figure 8(a). Monetary terms, such as nominal wages and profits, are normalized by the federal minimum wage, which is monthly 640 Brazilian Reals at the price level of year 2000.

Preferences We set the discount factor parameter $\beta = 0.98$ to target an aggregate capital-to-output ratio of 2.7. In our model, the formal sector is more capital-intensive. To obtain aggregate capital and aggregate output, we take the sum of the formal and informal sectors. We parameterize the utility weight on consumption by age ρ_j as a cubic polynomial in age for $j \leq J_R - 1$ to match the employment rate by age and $\rho_j = \rho_{J_R-1}$ for all $j > J_R$, as shown in Figure 8(b). We set the marginal propensity $\psi_b = 0.7$ and the threshold $c_b = 0.5$ to target the fraction of households that leave bequests $= 0.8$.

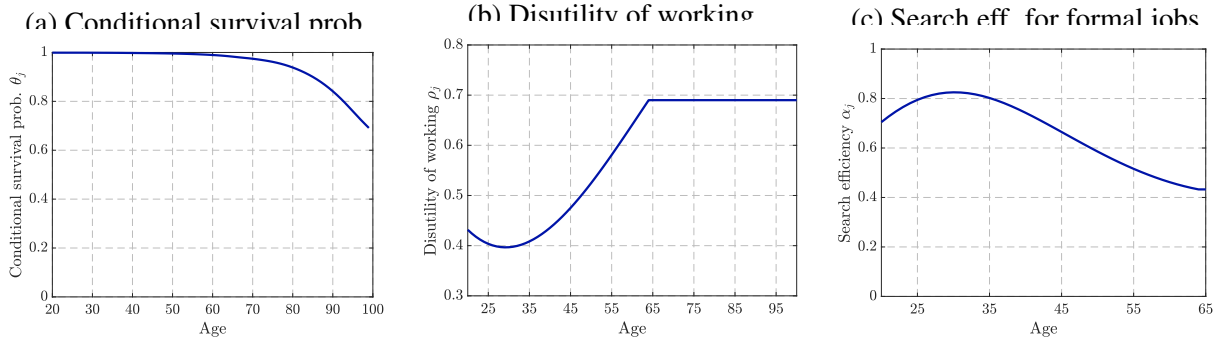


Figure 8: Age-dependent Parameters

Table 3: Pre-determined Parameters

Parameter		Value	Parameter		Value
Panel A. Environment					
J	Max age: 100	80	J_R	Retirement age: 60	40
β	Discounting factor	0.98	γ	Risk aversion	1.50
c_b	Bequest, threshold	0.5	ϕ_b	Bequest, marginal propensity	0.70
$\{\rho_j\}_{j=1,...,J}$	Disutility of working		$\{\theta_j\}_{j=1,...,J}$	Conditional survival prob.	
g_n	Population growth rate	0.01	ψ_g	Taste for public good	0.23
Panel B. Firms and labor market					
α	Production tech., capital share	0.30	δ	Depreciation rate of capital	0.06
λ_0	Sep. rate: informal to unemp.	0.067	λ_1	Sep. rate: formal to unemp.	0.042
α_1	Matching func: weight on vac.	0.50	μ_2	Collateral param.: formal	∞
Panel C. Government policies					
	Formal			All	
$w_{\min}$	Minimum wage	1.0	τ_c	Consumption tax rate	0.20
w_n	Unemployment benefits	0.01	τ_k	Capital income tax	0
τ_{sl}^f	SS firm contribution rate	0.20			
τ_{sl}^f	SS worker contribution rate	0.08			
τ_{ci}	Corporate income tax rate	0.23			
$\tau_l(.)$	Wage income tax schedule				
$w_r(.)$	Social security benefit rule				

Firms and labor market Following Erosa, Fuster, and Martinez (2023), we set the capital share of the modern production function $\alpha = 0.30$ and the capital depreciation rate $\delta = 0.06$. We directly take the separation rate to non-employment by sector from the data. Without further information, we set the weight on the vacancy of the aggregate matching function $\alpha_1 = 0.5$ and calibrate the constant parameter α_0 to match the transition rate. We further assume that there is a perfect capital market for the formal sector, i.e., $\eta_2 = \infty$, and calibrate the collateral parameter for the informal sector η_1 to match the capital-to-output ratio in the informal sector in the data.

Government policies We take the government policies for regulating the formal sector directly from the data. To start with, we set the minimum wage $w_{min} = 1$ as a normalization of the monetary unit in the model. Unemployment insurance is a small component of social insurance in

Brazil. The UI pays around 1.3 minimum monthly income, which lasts for 3 to 5 months. Despite the small size and short duration, the UI also has very small coverage, and we set w_n to be a small number.[21]

Following Meghir, Narita, and Robin (2015), we set the Social Security contribution to be 28% of pre-tax wages. We approximate pension benefits as a piecewise linear function of average lifetime income b as follows:

$$w_r(b) = \begin{cases} 0 & \text{for } b \in [0, 0.8405) \\ 2.0447b & \text{for } b \in [0.8405, 1.4340) \\ 0.9568(b - 1.4340) + 1.2136 & \text{for } b \in [1.4340, 6.0411) \\ 0.5639(b - 6.0411) + 5.6215 & \text{for } b \in [6.0411, 7.4575) \\ 6.4202 & \text{for } b \in [7.4575, +\infty) \end{cases}$$

To obtain the approximation, we first simulate the employment and wage profiles of workers using a statistical model, calculate the pension income using the formula from McKiernan (2022) with the simulated data, and then approximate the function using the mean benefits in each average lifetime income percentile bin.

Wage income tax is progressive and the marginal tax rates $\tau_l(.)$ are taken from McKiernan (2022):

$$\tau_l(w) = \begin{cases} 0 & \text{for } w \in [0, 2.0320) \\ 0.075 & \text{for } w \in [2.0320, 3.0167) \\ 0.15 & \text{for } w \in [3.0167, 4.0033) \\ 0.225 & \text{for } w \in [4.0033, 4.9783) \\ 0.275 & \text{for } w \in [4.9783, +\infty) \end{cases}$$

The government imposes a 23 percent corporate income tax rate on profits of the formal sector. Besides the tax regulations on the formal sector, a 20 percent consumption tax rate is additionally levied on consumption of all households, independent of the employment status of the household.

5.3 Internally Calibrated Parameters

In this subsection, we provide more information on how we identify the key parameters associated with the choice of formality in the model. As is common for this type of large-scale quantitative model, there is no one-to-one mapping between a parameter and a moment, as most

[21] The severance pay is also small and is dependent on duration of employment. For more details, see https://www.oecd.org/employment/emp/Brazil.pdf.

of the moments are affected by multiple parameters. However, each moment may be more informative about certain aspects of the model. Table 4 summarizes these correspondences, which are further detailed in the following.

Table 4: Internally Calibrated Parameters

Parameter	Value	Description	Target moment	Data	Model
Preference shock variances					
σ_e	1.0	Pref. shock var. J2J	Δw formal to informal	-0.10	-0.12
σ_n	0.1	Pref. shock var. non-emp.	NE job finding	0.63	0.58
σ_f	1.5	Formal vac. posting shock var.	Agg. formal emp. share	0.65	0.65
Firm productivity and production					
$z_{\min}$	0.10	Min firm productivity	Mean log wage: size 1-5	6.94	6.74
$z_{\max}$	1.60	Max firm productivity	Mean log wage: size 50+	7.38	7.59
μ_1	2.5	Collateral param.: informal	K to Y ratio: informal	1.04	1.07
Vacancy posting					
γ_0	6.0	Vacancy cost: constant	Firm share: size 1-5	0.91	0.92
γ_1	2.2	Vacancy cost: curvature	Firm share: size 50+	0.01	0.01
Screening technology					
η_0	0.05	Screen prob.: constant	Formal share: size 1-5	0.31	0.30
η_1	3.0	Screen prob.: curvature	Formal share: size 50+	0.96	0.90
Job search efficiency					
α_w	0.86	Within-sector	Within-sec. trans. rate	0.08	0.07
α_b	0.45	Between-sector	Between-sec. trans. rate	0.04	0.03
α_{s1}	0.82	Target formal: low-skilled	Trans. to formal, low-skilled	0.10	0.07
α_{s2}	1.0	Target formal: high-skilled	Normalization		

Variances of preference shocks In the model, we allow for a few preference shocks, which on one hand smooths the problem, on the other hand, it also allows for the role of unobserved factors that are not present in the model. The size of the preference shock variance determines the elasticity of workers' behavioral response and thus informs the impact of policy experiments in the following sections.

The preference shock for job-to-job transition captures aspects of job transitions other than monetary returns such as differences of non-pecuniary utilities across formal and informal jobs. We calibrate the shock variance parameter σ_e to match the wage cut when agents are moving

from formal to informal jobs. In the data, workers transiting from a formal to an informal job experience a 10 percent wage cut. When the shock variance is smaller, it is less likely that workers sacrifice monetary returns from working on a formal job for non-pecuniary utility on an informal job, leading to a smaller mean wage cut in equilibrium.

The preference shock for accepting a job offer for an unemployed worker captures non-pecuniary utility of working. We calibrate the variance parameter σ_n to match the aggregate job finding rate for unemployed workers.[22] Similarly, we calibrate the parameter for vacancy posting preference shock σ_f to match the employment share of formal workers in equilibrium.

Firm productivity and production The spread of firm productivity determines wage dispersion across firms in equilibrium. After normalizing the monetary terms by the minimum wage, we calibrate the bounds of the firm productivity grids to match the mean log wage for workers employed in the bottom and the top firm size bin. Our model predicts a slightly larger firm size wage premium than that in the data. The spread of firm productivity determines the fraction of workers bound by minimum wage in the model through firms' formality choice and vacancy creation. In Table 5, we present the model fit of the distribution of workers bound by the minimum wage by their characteristics. To be more specific, informal workers and low-skilled workers are much more likely to receive wages below the minimum wage. Over the life cycle, the fraction falling below the minimum wage is u-shaped, as is consistent with the data. In the model, informal jobs are more likely to be associated with lower firm productivity and lower capital intensity, we set η_1 to match the capital to output ratio in the informal sector.

Vacancy posting As can be seen from equation (11), a lower curvature parameter γ_1 implies that more productive firms can increase vacancies with less steeply increasing costs, leading to a larger firm size in equilibrium. We calibrate $\gamma_0 = 6.0$ and $\gamma_1 = 2.2$ to target the share of firms in the bottom and the top firm size bins. Table 6 further report the model fit of the size and employment distributions of firms by firm size bin.

Screening of informal sector In a similar fashion, the curvature parameter for screening of the informal sector η_1 determines how fast the incentive to avoid the informal regulation increases as the firm productivity increases. We thus calibrate the screening probability function to match the employment share of formal employees in the bottom and the top firm size bins.

[22] An extreme case is that when the variance parameter σ_n goes to infinity, the job acceptance rate approaches 50% as the acceptance decision is completely independent of the monetary returns.

Table 5: Model Fit of Fraction of Workers Bound by Minimum Wage

	Data			Model		
	$\leq w_{min}$	$< w_{min}$	$= w_{min}$	$\leq w_{min}$	$< w_{min}$	$= w_{min}$
Overall	27.1	11.8	15.3	18.9	9.6	9.3
By formality						
formal	16.5	2.2	14.3	8.6	0	8.6
informal	50.3	32.9	17.5	41.9	31.1	10.8
By education						
High school and below	30.8	13.6	17.2	20.7	11.0	9.7
College graduate	7.9	2.8	5.1	9.8	2.8	7.0
By age						
$25-34$	27.4	10.5	16.9	25.9	14.0	11.9
$35-44$	25.5	10.9	14.6	10.6	3.5	7.1
$45-54$	26.8	12.7	14.1	8.0	2.0	6.0
$55-64$	31.5	17.5	14.0	11.2	3.7	7.5

Notes: "$< w_{min}$" refers to fraction workers who make below 90% of minimum wage; "$= w_{min}$" refers to fraction of workers who make between 90% and 110% of minimum wage.

Table 6: Model Fit of Firm Size Distribution

	Firm number counts		Employment	
	Data	Model	Data	Model
Size of employees				
≤ 5	0.9083	0.9147	0.4079	0.2088
$6-10$	0.0440	0.0311	0.1178	0.0790
$11-20$	0.0251	0.0249	0.0823	0.1311
$21-50$	0.0145	0.0181	0.0533	0.2134
$50+$	0.0083	0.0111	0.3388	0.3676

Notes: Data of firm number count distribution are based on statistics conditional on formality of firms in Ulyssea (2018); data of employment share are by authors' own calculation using Continuous PNAD.

Job search efficiency We calibrate the parameters for the function (12) to match the job transition rates by previous employment status, age, and skill type. In particular, the age component α_j is approximated with a cubic polynomial in age to match the job-finding rates of unemployed

individuals by age, with calibrated values plotted in Figure 8(c). We recover that the job search efficiency is higher for job transition within the same sector and for the high-skilled in searching for formal jobs. Table 7 reports model fit in terms of the share of employment and the transition rates by formality and characteristics of workers.

Table 7: Model Fit of Employment and Transition Rates by Worker Skill and Age

	Overall	By Skill		By Age			
		Low	High	[25,35)	[35,45)	[45,55)	[55,65)
Panel A. Data							
Employment share							
Informal	0.272	0.350	0.150	0.272	0.311	0.336	0.398
Formal	0.653	0.572	0.789	0.625	0.613	0.603	0.555
Unemployment	0.075	0.078	0.066	0.100	0.076	0.061	0.047
Transition rates							
Informal → informal	0.104	0.150	0.095	0.157	0.109	0.080	0.006
Informal → formal	0.072	0.055	0.056	0.125	0.076	0.048	0.025
Formal → informal	0.033	0.030	0.013	0.049	0.035	0.023	0.023
Formal → formal	0.074	0.120	0.080	0.123	0.077	0.048	0.033
Unemp. → informal	0.299	0.356	0.143	0.250	0.321	0.360	0.362
Unemp. → formal	0.327	0.204	0.303	0.340	0.353	0.283	0.250
Panel B. Model							
Employment share							
Informal	0.254	0.301	0.168	0.374	0.207	0.184	0.248
Formal	0.670	0.603	0.747	0.540	0.721	0.745	0.678
Unemployment	0.076	0.096	0.086	0.086	0.071	0.071	0.074
Transition rates							
Informal → informal	0.108	0.115	0.110	0.119	0.092	0.092	0.117
Informal → formal	0.068	0.063	0.100	0.079	0.076	0.059	0.045
Formal → informal	0.011	0.012	0.010	0.005	0.006	0.008	0.023
Formal → formal	0.055	0.051	0.073	0.069	0.061	0.048	0.046
Unemp. → informal	0.285	0.336	0.238	0.313	0.253	0.260	0.307
Unemp. → formal	0.217	0.166	0.270	0.209	0.255	0.223	0.184

Other internally calibrated parameters We calibrate parameters for human capital dynamics (ρ) to match wage growth of continuous employment in the same sector and wage growth off unemployment. For the sake of space, we delegate the additional model fit to Appendix B.

5.4 Solution to the Firm's Problem

Figure 9(a) depicts the value of a vacancy by formality conditional on meeting a worker, and Figure 9(b) depicts the share of formal vacancies following equation (9). When firm productivity is low, the value of a formal vacancy is 0 due to the binding minimum wage constraint, while the value of an informal vacancy is positive. It thus follows that low-productivity firms only post informal vacancies. The value of both types of vacancies increases monotonically, but the slope for the value of a formal vacancy is steeper. For mid-productivity firms, the yields of posting a formal and an informal vacancy are close, firms will post around half of their vacancies in the formal sector. The share of formal vacancies keeps increasing as the firm's productivity increases.

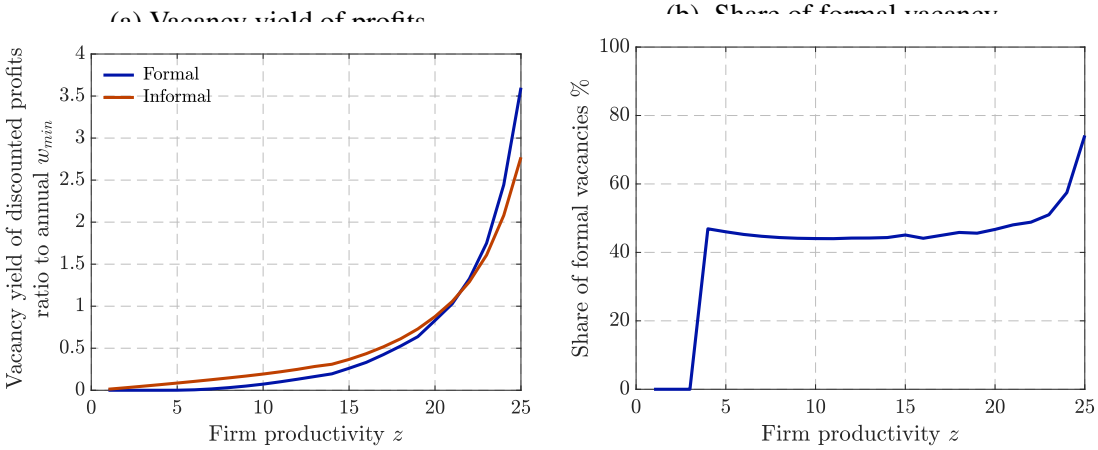


Figure 9: Solution to the Vacancy Posting Problem

Several factors are contributing to this pattern. First, the probability of being screened of an informal job is convex in output, while taxes on a formal job are linear, thus the increase in value is lower on an informal job. Second, a formal job intrinsically has a lower separation rate into non-employment and provides higher human capital growth, as depicted in Figure B.1. The last reason is less obvious but closely related to the overall sorting pattern. In equilibrium, high-productivity workers are more likely to work formally, which means that the vacancy yield of formal vacancies is higher because they can more easily poach workers from jobs operating in the same sector.

We further decompose the value of the vacancy into three margins: probability of forming a match, human capital of the worker, and duration of the job and plot the log difference to the lowest firm productivity type in Figure 10. As can be seen from Figure 10(a), vacancies posted by the

highest firm productivity type are two times more likely to form a match, and the newly-matched workers' human capital are on average 1.5 times more productive. Figure 10(b) further shows that the total discounted present value of dynamic profit flows is 4.6 times higher for the highest type, which can be further decomposed to the profit flow being 3.2 times higher per period and 1.4 times longer duration.

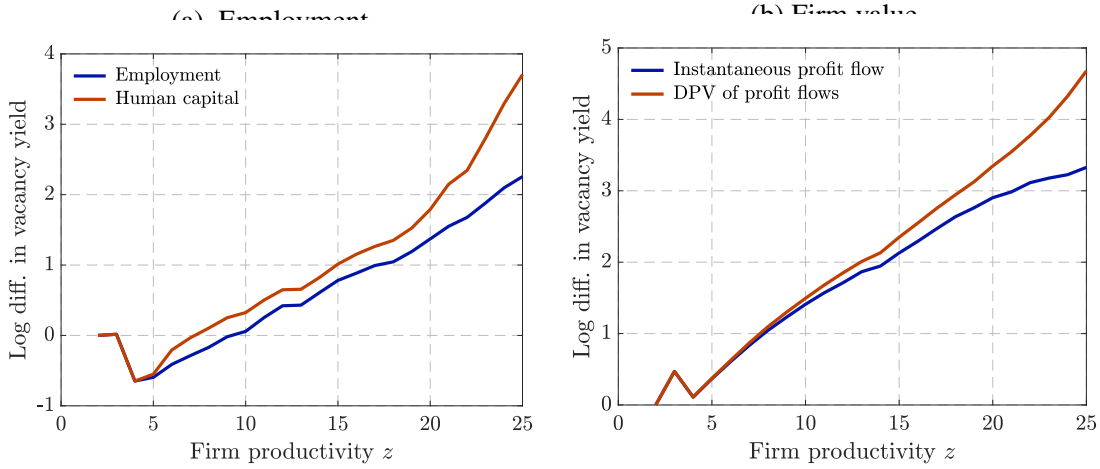


Figure 10: Vacancy Yield by Firm Productivity

In Figure 11, we plot the optimal piece rate set by the firms. In a simpler setting with only wage competition for homogeneous workers, Burdett and Mortensen (1998) prove that the wage is increasing in firm productivity because a more productive firm can always offer a slightly higher wage than its less productive competitors. In our setup, firms compete over piece rates rather than wages, and wage piece rates are set conditional on formality.

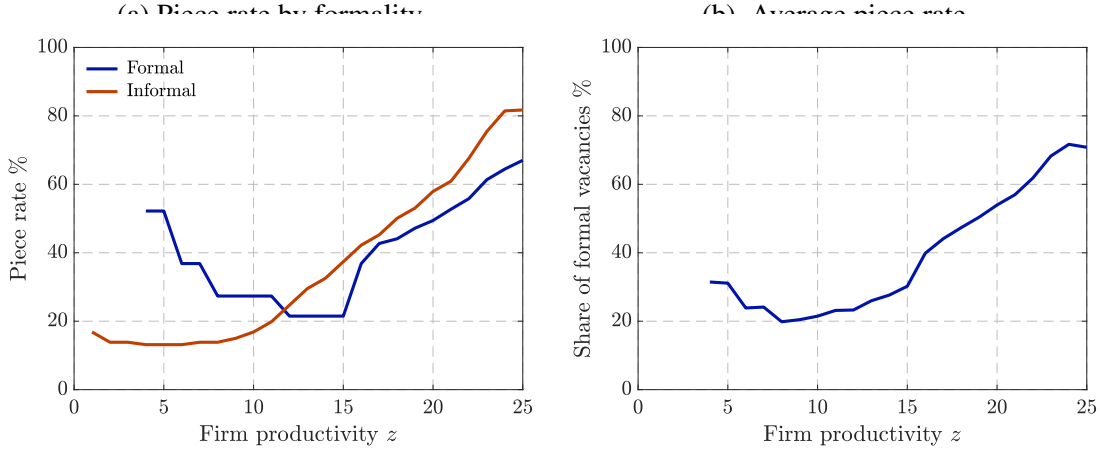


Figure 11: Firms' Choice of Wage Piece Rate

As can be seen in Figure 11(a), consistent with the argument in Burdett and Mortensen (1998),

the piece rate is increasing in firm productivity for informal vacancies as well as formal vacancies posted by mid- to high-productivity firms. However, the piece rate decreases in productivity for low-productivity formal vacancies. With the presence of minimum wage, a firm always needs to pay the minimum wage once they meet a low productivity worker even if they have posted a low piece rate. On the other hand, a lower piece rate may discourage high-skilled workers from accepting the job offer. To increase the probability of forming a match, less productive firms may promise a higher piece rate. Figure 11(b) plots the average piece rate by firm productivity.

6 Policy Experiment

Under the current policy, the total Social Security tax for formal employees, which is 28.5% of the wage income, is jointly contributed by the firm and the worker. In this section, we consider several alternative policy reforms to illustrate how the presence of informality and the labor market power of firms may affect the effects of policy reforms.

6.1 Alternative Contribution Share

To start with, we keep the total payroll tax rate at 28. 5%, but change the share rule between the worker and the firm. It is worth noting that the share rule varies across countries in the real world. For example, the payroll tax is divided equally between the firm and the worker in the US but is contributed entirely by the firm in Australia.

In Figure 12, we plot the firms' policy function of formal vacancy share and wage piece rate under three firm contribution rate levels, 0%, 14%, and 28%, each corresponding to the case that firms pay none of payroll tax, half of the payroll taxes, or all of the payroll taxes, respectively.

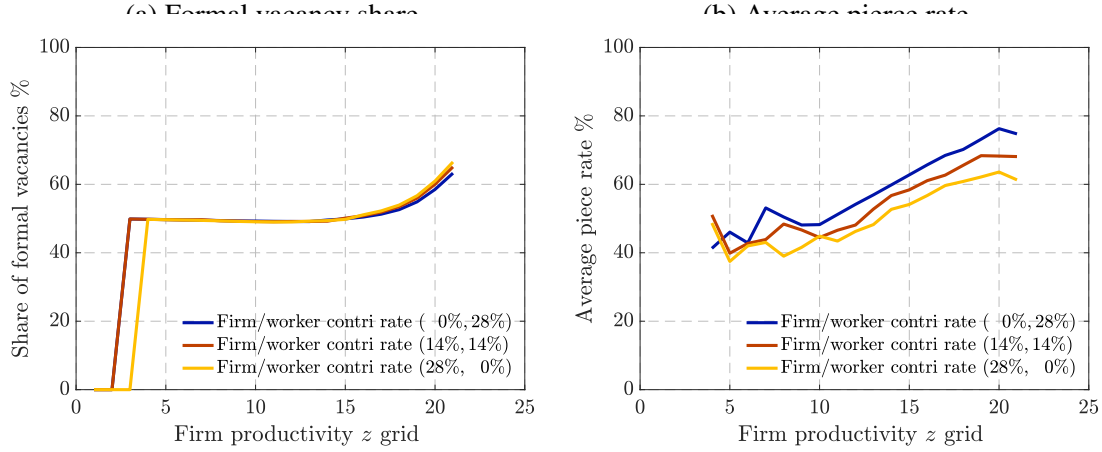


Figure 12: Policy Impacts on Firm Vacancy Posting Policy

As Figure 12(a) shows, when the contribution rate of the firm increases, the expected profit of posting a formal vacancy turns from positive to negative for low-productivity firms. Therefore, the productivity cutoff that firms post formal vacancy shifts to the right. In addition, more productive firms increase the share of formal vacancies because they face less competition in the formal sector. Faced with less competition for hiring formal workers, the more productive firms enjoy greater labor market power and decrease their wage rate, as presented by Figure 12(b).

Figure 13 further reports the impacts of changing the share rule on the aggregate moments. When more productive firms gain greater market power, they tend to reduce the number of vacancies, cut the employment size, but maintain a higher skill composition. Therefore, as Figure 13(a) shows, the aggregate number of vacancies decreases and capital usage increases. Furthermore, Figure 13(b) indicates that a greater labor market power for larger firms, which allows them to reduce labor compensation, leads to a decrease in aggregate labor share.

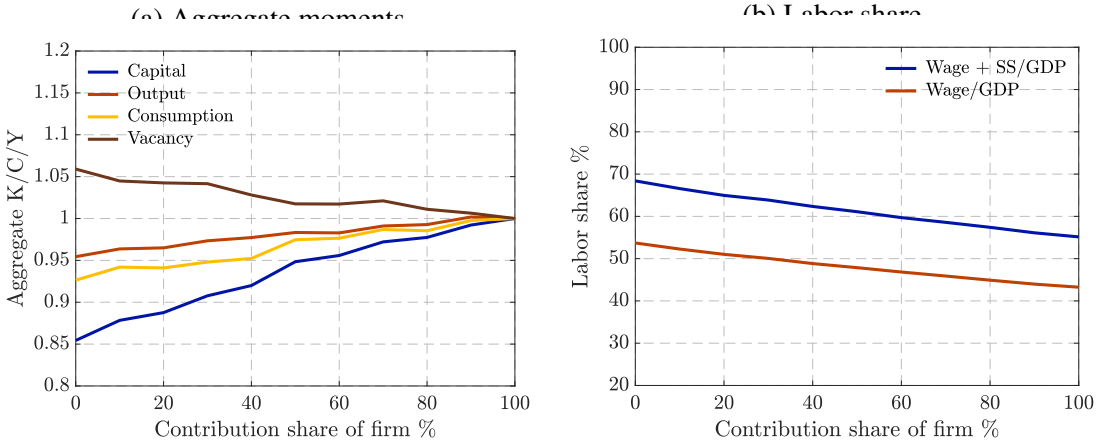


Figure 13: Policy Impacts on Aggregate Variables and Labor Share

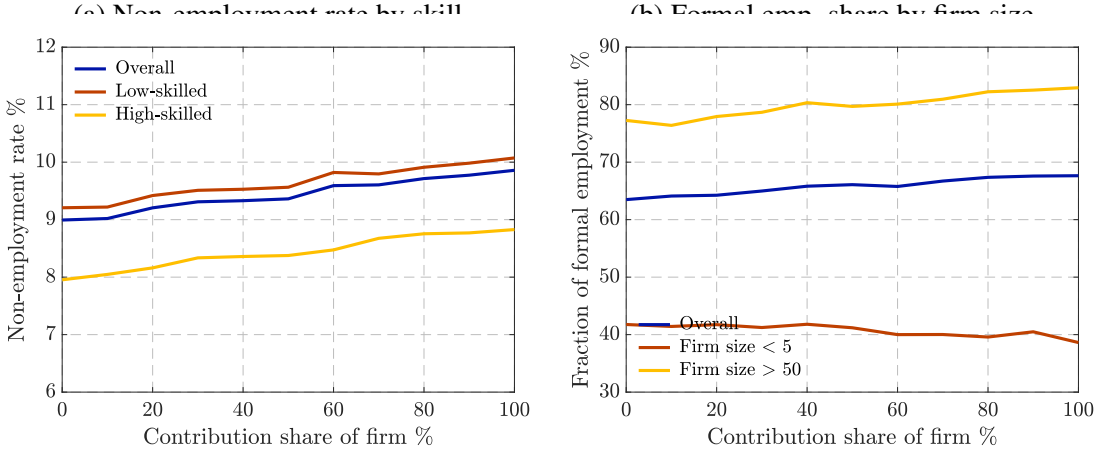


Figure 14: Policy Impacts on Aggregate Employment

Not surprisingly, as firms gain greater market power and reduce the number of vacancies, the share of non-employment increases with the contribution share of the firm, as shown in Figure 14(a). The impact on formal employment share is more difficult. On the one hand, as the firm contribution share increases, low-productivity firms post less formal vacancies because they are less able to afford the payroll tax on top of the minimum wage. On the other hand, more productive firms cut the size of hiring, but increase the formal vacancy share. The overall impact is dominated by the more productive firms. As Figure 14(b) indicates, the overall formality increases in the share of payroll contribution by firm.

6.2 Alternative Social Security Financing

Under the current policy, only workers employed formally can accumulate social security contribution while individuals non-employed or employed informally, who are more likely to be less-skilled and asset-poor are excluded from the system. In this section, we consider several alternative policy schemes that weaken the formality-social security linkage and evaluate their impacts on aggregate and distributional outcomes. An option is to make social security compulsory for all individuals. Another option is to levy an additional consumption tax as in Esteban-Pretel and Kitao (2021) or more broadly the VAT tax Piggott and Whalley (2001). We thus consider the three possible reform schemes as follows:

- All individuals, including individuals employed formally and informally and non-employed individuals, contribute a fixed amount of 30% of current minimum wage;
- Non-employed individuals contribute a fixed amount of 10% of minimum wage. All employed individuals, including employed formally and informally, contribute an amount of 10% minimum wage plus 15% of their wages;
- All individuals pay an additional 20% consumption tax as social security contributions.

All of the above schemes apply to individuals between the ages of 20 and 65. The social security budget of the government is balanced in each case. Across all policy counterfactuals, we fix all policy regulations as in the benchmark, including the minimum wage. In terms of the Social Security, we change only the way how Social Security contributions are collected but fix the Social Security benefit rule the same as the benchmark. We further assume that social security contributions are paid by workers. To help us better understand the role of firm's responses, we consider both a partial equilibrium environment where job arrival rates, job contract distributions, as well as interest rates and lump-sum transfers are fixed to the benchmark values and a general equilibrium where all variables are allowed to adjust. In implementing the reforms, we assume that individuals

have full compliance with the policy. In cases of compulsory savings where the household asset is sufficiently low, we assume that the government will fully subsidize households to meet the contribution rule.

Impact on firms' decisions Figure 15 presents the policy impacts on firm's policy functions in terms of the share of formal vacancies and the average piece rate in both the benchmark and the first counterfactual. Figure 16 presents the difference in the firms' choices between these two cases.

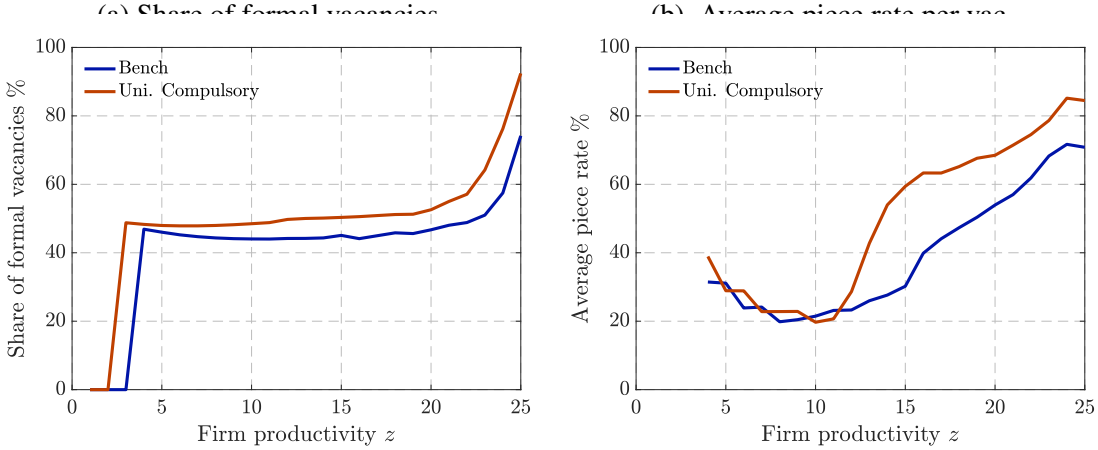


Figure 15: Policy Impacts on Firm's Decisions

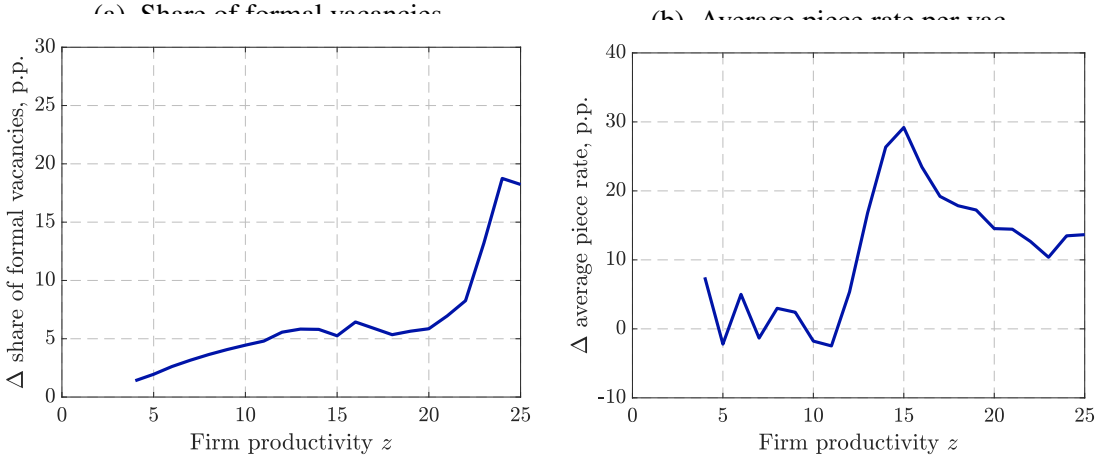


Figure 16: Policy Impacts on Firm's Decisions

As will be shown in the next section, the policy reform is associated with an increase in aggregate formality share. Since within-sector poaching is more efficient, firms will respond by increasing the share of formal vacancies. In addition, more productive firms hire more formal

workers and rely more on poaching formal workers, they thus have a larger increase in the share of formal vacancies, as shown in Figure 16(a). Firms' piece rate policy function displays a different pattern. Mid-productivity firms are more likely to be bound by the minimum wage constraint and reimburse all their profit margins to workers by a 30% increase in offered wages after the policy change, as shown in Figure 16(b).

Overall impacts Panel A of Table 8 presents the impact of policy on aggregate moments. Column 2 presents the moments under the benchmark. For the first counterfactual where all individuals contribute a constant of 30% minimum wage every period, we present both the PE and GE outcomes, as shown in columns 3 and 4. In this counterfactual, the Social Security coverage increases to 100% by construction. When we fix the job offer distribution and job arrival rates, the formal employment share decreases from 61.9% in the benchmark to 60.5%. This is because under the benchmark, conditional on the same productivity type, informal jobs offer higher wages as compensating differentials, as shown in Figure 11(a). However, once we allow firms' decisions to respond to policies, the employment share of formal jobs increases to 74.9%. This comes together with an increase in the aggregate number of vacancies by 12% and the share of formal vacancies from 52.5% under the benchmark to 57.8% under the counterfactual.

The increase in vacancy posting reflects an increase in firm competition. As firms increase vacancies, the unemployment rate decreases from 7.2% under the benchmark to 5.7% under the counterfactual. We further find that GDP increases by 3.5%, and the compensation share, which is evaluated as social security contributions and wages paid by firms to the GDP ratio, increases from 54.3% to 56.7%.

Panel B of Table 8 presents the policy impacts by skill group. The PE simulation is again misleading in predicting the formal employment share. We find that the formality share increases more for low-skilled workers. The skill wage premium increases mildly from 65.6% to 68.7%. However, the inequality in social security wealth decreases to 0.

In columns 5 and 6, we present policy impacts of the other two alternative social security schemes. Compared with the first policy counterfactual, the contribution amount differs by heterogeneous households. Albeit the specific social security collection rule, we find that breaking the formality-social security contribution linkage leads to a large increase in formal vacancy, thus formal employment share, and a reduction in social security wealth inequality.

Table 8: Impacts of Social Security Policy Reforms

	Benchmark	Compulsory		Compulsory	Cons. Tax
	$0.28w$ formal	$0.3w_{min}$		$0.1w_{min} + 0.15w$	$0.2c$
		PE	GE	GE	GE
Panel A: Aggregate impacts					
SS coverage	0.7454	1	1	0.9655	0.8805
Aggregate vacancy	1	1	1.1188	1.1435	1.2003
Formal vac. share	0.5257	0.5257	0.5776	0.5650	0.5756
Formal emp. share	0.6194	0.6052	0.7493	0.7309	0.7398
Non-emp. share	0.0719	0.0617	0.0567	0.0570	0.0580
Compensation/GDP	0.5434	0.5674	0.5661	0.5599	0.5571
Output	1	1.0049	1.0348	1.0018	1.0396
Panel B: Impacts by skill					
Formality share					
Low-skilled	0.4472	0.4413	0.6639	0.6336	0.6430
High-skilled	0.7923	0.7691	0.8344	0.8282	0.8360
SS coverage					
Low-skilled	0.5392	1	1	0.9458	0.8196
High-skilled	0.9525	1	1	0.9852	0.9409
Wage ratio	1.6559	1.6573	1.6872	1.7012	1.6983
SS income ratio	3.3811	1	1	1.3722	1.5765

Notes: Aggregate vacancy and output are normalized by values in the benchmark economy.

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Appendix

A Data

In Section 3, we present various stylized facts related to how employment and wages vary by education, formality status, and firm size. Here, we discuss various details related to construction and robustness to these facts. Our robustness exercises focus on how employment shares, education levels, and formality vary over time and how these facts change (or do not change) if female labor force participants are included in the estimation sample.

When we analyze time series variation, we will combine the PNADC data used in the main empirical results (covering years 2012-2019) with its predecessor , *Pesquisa Nacional por Amostra de Domicilios* (PNAD) that covers years 2001-2009 and 2011-2015. Similarly to PNADC, PNAD is a nationally representative Brazilian household survey that contains information on labor supply and income. Microdata is available for download from the Institute Brasileiro de Geografia e Estatistica.[23] We use the cleaning procedure developed by Data Zoom (available at: https://www.econ.puc-rio.br/datazoom/english/pnadMicro.html) We follow a similar sample selection criteria and select a sample of civilian, private sector, non-farm male workers between the ages of 20 and 64. We are left with a dataset of 1,144,598 observations, an average of 81,757 observations per year. Unlike PNADC, PNAD is an annual cross-sectional dataset. Therefore, we cannot construct some of the facts (i.e. employment transition rates and earnings growth statistics) that require a panel dimension available in PNADC.

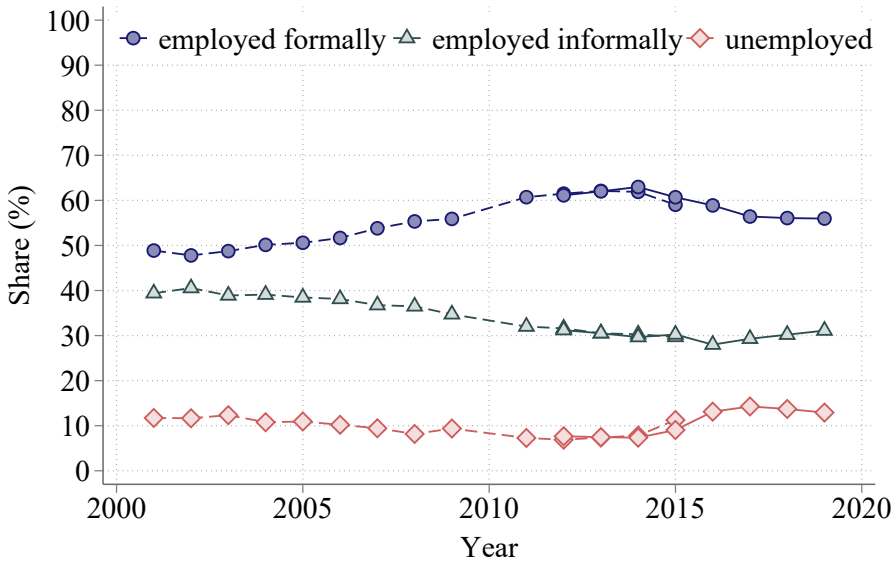
In the main results, we use a sample excluding women as we want to focus on a group of employees who are more attached to the labor market. However, here we also consider robustness for each of these facts using a sample that includes women. The remaining sample selection criteria stay the same; in these exercises, we use a sample of civilian, private sector, non-farm workers between the ages of 20 and 64. The final sample includes 7,983,774 observations for 2,610,375 individuals. When we restrict attention to only those workers who we observe in both waves 1 and 5, we are left with a sample of 468,297 individuals who are in the labor force (employed or unemployed) in both waves and 406,468 individuals who are employed in both waves.

A.1 Employment and Formality Shares

Figure 1 shows how participants in the labor force are distributed in formal employment, informal employment, and unemployment at various ages; Figure 2 shows how the share of the labor force employed formally varies by education and age. We focus specifically on the primary

[23] https://www.ibge.gov.br/en/statistics/social/labor/20620-summary-of-indicators-pnad2.html?=&t=downloads.

Figure A.1: Employment Shares by Year



Source: PNAD 2001-2009, 2011-2015 and Continuous PNAD 2012-2019, men ages 20-64, authors' calculations. *Notes*: figure shows the share of the labor force in a given year who is employed formally, employed informally and unemployed. Workers are employed formally if they either have a contract in their current work or are contributing to a pension system. Individuals are unemployed if he is not working but has looked for work in the last 30 days.

employment of individuals as under 5 percent of the sample

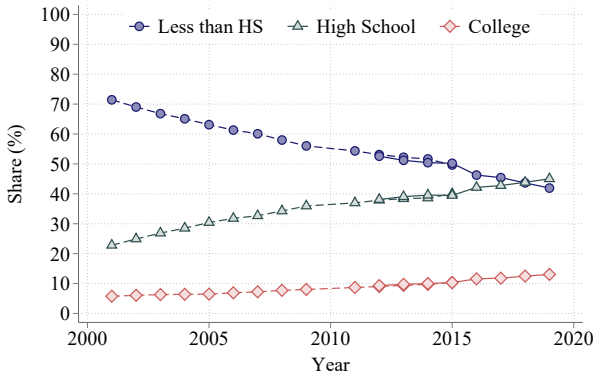
Time Series

Figure A.1 shows how the employment shares shown in Figure 1 have changed over time. There has been a roughly 10 percentage point decrease in the share of the labor force working informally. This is mainly offset by an increase in the share of the workforce that is working formally.

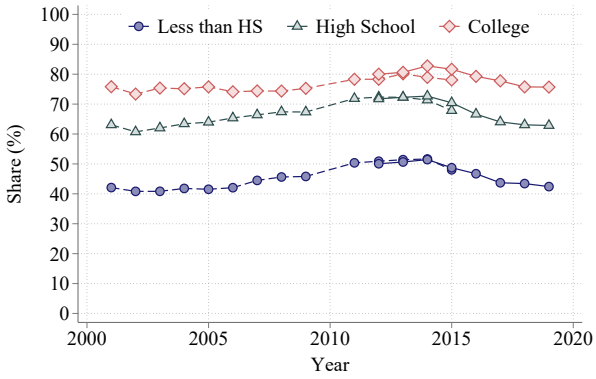
Figure A.2 shows how the share of the population and the formality of each educational group evolve over the same period of time. There has been a large decline in the percentage of the population that has less than a high school degree (shown in Figure A.2(a)); this share has dropped from around 70 percent in 2001 to around 40 percent in 2019. Over this time, the share with only a high school degree has increased from around 20 percent to over 40 percent. The share with a college degree has also increased from only 5 percent of the population to over 10 percent. However, the share of formal employment by education (shown in Figure A.2(b)), however, has remained fairly constant.

Figure A.2: Education and Formality by Year

(a) Education Shares



(b) Formality by Education



Source: PNAD 2001-2009, 2011-2015 and Continuous PNAD 2012-2019, men ages 20-64, authors' calculations.

Including Non-Employment

By focusing on labor force participants, we eliminate the impact of extensive margin movements, such as retirements, on employment shares. Figure A.3 extends the shares shown in Figure 1 to include those who are retired or are not in the labor force. We note three main results. First, while the share of people not in the labor force is very low early in the life cycle, this share increases beginning around age 50. The share of those not employed reaches nearly 50 percent by age 65. Second, much of this increase in non-employment seems to be driven by declines in formal employment. The formal employment share declines from 60 percent to around 30 percent between age 50 and 65. Finally, once we account for non-employment, the informal labor share is roughly flat over the life cycle. This indicates that the increase in the share of informal labor in Figure 1 is related to the change in the composition of the labor force due to the retirements of the formal sector.

Relationship to Minimum Wage

One concern related to formality and public finance is how the minimum wage interacts with the formal labor decision. To attempt to address this issue (although the minimum wage is not the direct topic of this work), Figure A.4 shows the evolution of formality, education, and the minimum wage over time. Notably, we see that both the formality share and the share with a high school degree or more have both increased over the sample period. The increase in the share with a higher level of education has increased more than the formal employment share. At the same time, the minimum wage has increased drastically during this period. This indicates that formal employment is determined by more than a policy decision, such as increasing the minimum wage.

Figure A.3: Labor Force Shares (Including Non-Employment) by Age



A.2 Wage Distribution by Formality

In the main text, wage distributions are shown as log-multiples of the minimum wage for both the formal and informal sectors (Figure 5). Table A.1 shows additional details on the distribution of log-incomes (mean and variance) by sector, education, and age.

A.3 Employment Transition Rates

By Age, Education

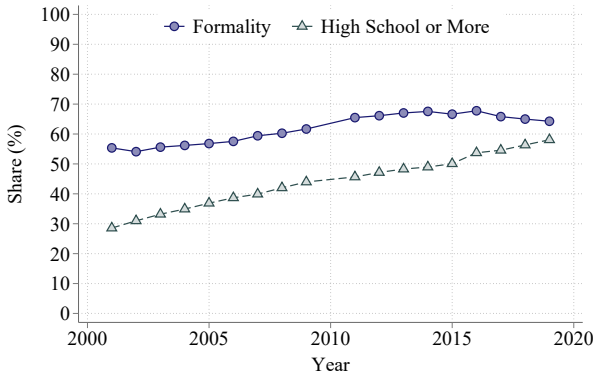
Tables A.2 and A.3 demonstrate how the annual transition rates in Table 1 vary by age and education, respectively.

When analyzing how transition rates vary by age (Table A.2, we observe some facts. First, younger workers are more likely to transition into unemployment (both formality and informality) and less likely to transition out of unemployment than older workers. Prime-age (ages 30-44) workers are the least likely to transition to unemployment and most likely to move out of unemployment. Second, the youngest workers are most likely to transition from informality to formality, while the oldest workers are most likely to stay in their current position–whether it be a formal or an informal position. It is likely that patterns of informal self-employment at older ages is a motivator for older workers remaining in informal employment.

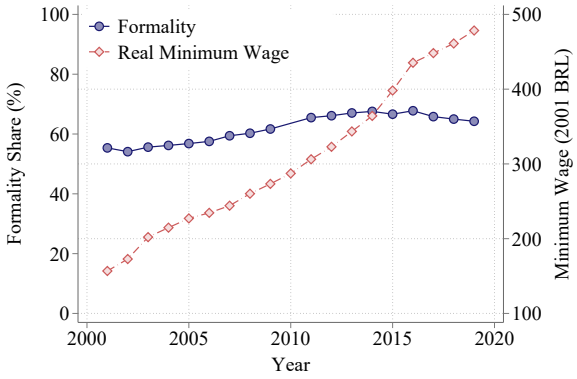
When studying how transition rates vary by education (Table A.3) we observe that while those

Figure A.4: Formality, Education, and Minimum Wage by Year

(a) Formality and Education



(b) Formality and Minimum Wage



Source: PNAD 2001-2009, 2011-2015 and Continuous PNAD 2012-2019, men ages 20-64, authors' calculations.
Notes:

without a high school degree and those with only a high school degree look fairly similar, those workers with a college degree appear quite different. College-educated workers are less likely to become unemployed (from either a formal or an informal job). These workers are also more likely to remain in formal jobs and less likely to transition from a formal job to an informal job. Finally, conditional on being unemployed, workers of lower education levels experiment higher job finding rates, with many transitions out of unemployment being to informal jobs.

A.4 Earnings and Wage Growths

Wage Growth by Education, Formality Status

The results in Figures 3 and 4 are constructed using changes in monthly earnings; Figures A.5 and A.6, rather, are constructed using hourly wages. This is done to remove the impact that changes in hours worked over time on the growth statistics. These results are very similar, with only small differences in the magnitude of growth; this is consistent with the observation that hours do not vary much from year to year. Specifically, most Brazilian workers who are employed in both years work full-time hours in each year. Figure A.5 shows that when we consider wage growth rather than earnings growth, we observe larger growth in earnings for those workers.

Sample Using only Job Switchers

The results in Figures 3 and 4 in the main text are calculated using all workers, those who switch jobs and those who stay in the same position. Figures A.7 and A.8 perform the same earnings growth calculations using a sample of only workers who switch jobs. The main results

Table A.1: Summary Statistics of Log Incomes

	Formal		Informal	
	mean	*var.*	*mean*	*var.*
Less than high school				
25-34	6.824	0.163	6.344	0.552
35-44	6.906	0.224	6.438	0.605
45-54	6.948	0.273	6.420	0.656
55-64	6.932	0.330	6.348	0.703
Average	6.859	0.234	6.326	0.614
High school graduate				
25-34	6.984	0.225	6.531	0.570
35-44	7.121	0.296	6.670	0.642
45-54	7.210	0.367	6.761	0.660
55-64	7.265	0.440	6.779	0.732
Average	7.005	0.278	6.541	0.612
College graduate				
25-34	7.541	0.404	7.084	0.668
35-44	7.746	0.436	7.301	0.740
45-54	7.830	0.459	7.336	0.780
55-64	7.872	0.499	7.389	0.836
Average	7.643	0.454	7.177	0.750

hold, although the smaller sample size leads to a decrease in precision in the estimates.

A.5 Firm Size Statistics

Figures A.9 and A.10 show how the log of real monthly earnings and the log of real hourly wage varies by both firm size category and education level. We observe similar patterns across both these measures. Notably, we see higher earnings and wages not only among those with higher

Table A.2: Annual Employment Transition Rates by Age

Ages 20-29				
	Unemployed	Stayer	Switcher	
Employed Formally	0.067	0.761	0.172	Formal 0.123 Informal 0.049
Employed Informally	0.106	0.611	0.282	Formal 0.125 Informal 0.157
Unemployed	0.410	Job finding 0.590		Formal 0.340 Informal 0.250
Ages 30-44				
	Unemployed	Stayer	Switcher	
Employed Formally	0.038	0.850	0.112	Formal 0.077 Informal 0.035
Employed Informally	0.062	0.753	0.185	Formal 0.076 Informal 0.109
Unemployed	0.326	Job finding 0.674		Formal 0.353 Informal 0.321
Ages 45-54				
	Unemployed	Stayer	Switcher	
Employed Formally	0.031	0.898	0.071	Formal 0.048 Informal 0.023
Employed Informally	0.051	0.820	0.128	Formal 0.048 Informal 0.080
Unemployed	0.357	Job finding 0.643		Formal 0.283 Informal 0.360
Ages 55-64				
	Unemployed	Stayer	Switcher	
Employed Formally	0.032	0.918	0.056	Formal 0.033 Informal 0.023
Employed Informally	0.047	0.870	0.083	Formal 0.025 Informal 0.058
Unemployed	0.388	Job finding 0.612		Formal 0.250 Informal 0.362

Notes: Individuals who report changing sectors without changing jobs are assumed to stay in the sector from period t_0. Workers are assumed to have switched jobs if their tenure in period t_1 is less than 12 months. In the initial period, t_0, 24.4% are ages 20-29 (15.0% employed formally, 6.2% employed informally, 3.2% unemployed); 42.8% are ages 30-44 (28.8% employed formally, 11.3% employed informally, 2.7% unemployed); 21.3% are ages 45-54 (14.2% employed formally, 6.0% employed informally, 1.1% unemployed); 11.6% are ages 55-64 (7.2% employed formally, 3.8% employed informally, 0.6% unemployed)

Table A.3: Annual Employment Transition Rates by Education

Less than high school

	Unemployed	Stayer	Switcher	
Employed Formally	0.031	0.818	0.151	Formal 0.115 Informal 0.036
Employed Informally	0.068	0.730	0.202	Formal 0.042 Informal 0.160
Unemployed	0.440	Job finding 0.560		Formal 0.204 Informal 0.356

High school graduate

	Unemployed	Stayer	Switcher	
Employed Formally	0.033	0.817	0.150	Formal 0.125 Informal 0.025
Employed Informally	0.074	0.715	0.210	Formal 0.069 Informal 0.141
Unemployed	0.513	Job finding 0.487		Formal 0.287 Informal 0.200

College graduate

	Unemployed	Stayer	Switcher	
Employed Formally	0.021	0.885	0.093	Formal 0.080 Informal 0.013
Employed Informally	0.054	0.795	0.151	Formal 0.056 Informal 0.095
Unemployed	0.554	Job finding 0.446		Formal 0.303 Informal 0.143

Notes: Individuals who report changing sectors without changing jobs are assumed to stay in the sector from period t_0. Workers are assumed to have switched jobs if their tenure in period t_1 is less than 12 months. In the initial period, t_0, 37.6% are less than high school (19.8% employed formally, 15.0% employed informally, 2.9% unemployed); 47.1% are high school graduates (33.0% employed formally, 10.4% employed informally, 3.7% unemployed); 15.4% are college graduates (12.2% employed formally, 2.3% employed informally, 0.8% unemployed)

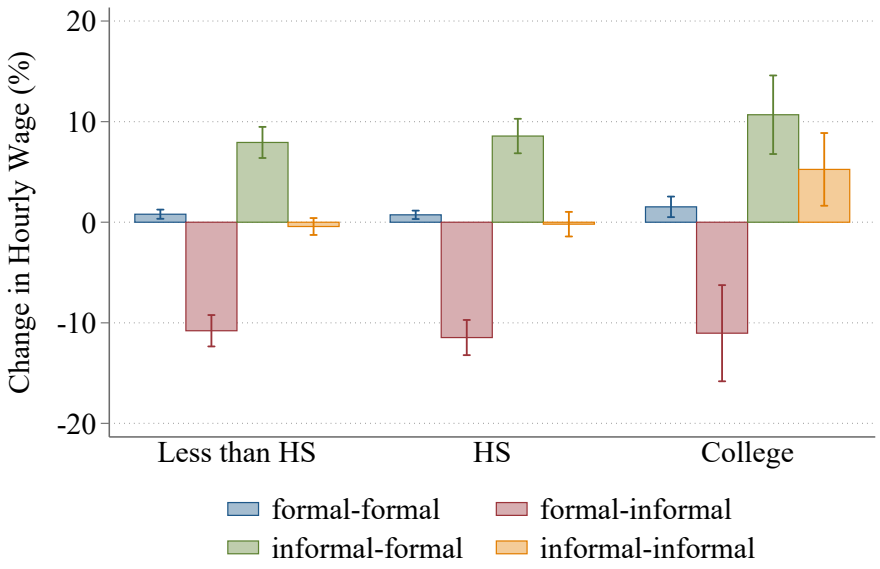


Figure A.5: Wage Growth by Formality Status

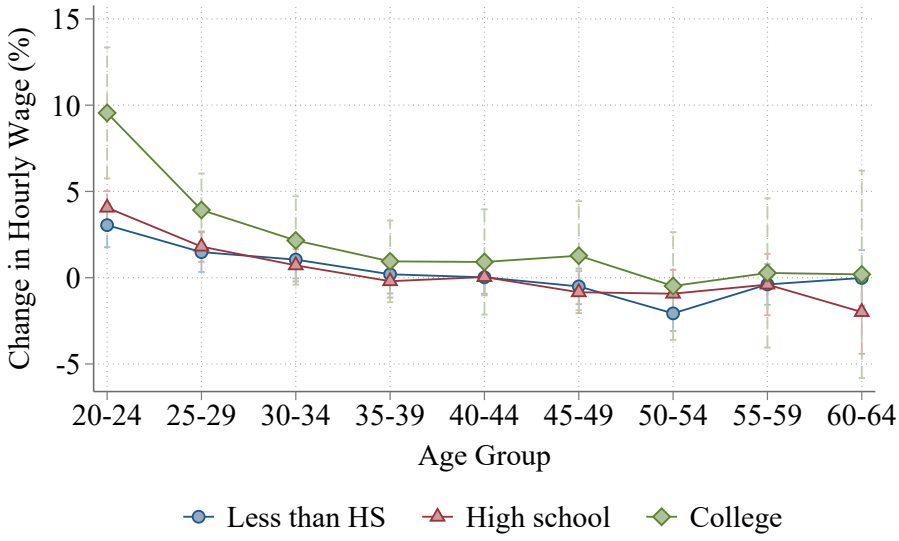


Figure A.6: Wage Growth by Education and Age

education (as shown in A.1) but also by firm size. Larger firm–who also tend to employee more workers formally–also pay higher wages.

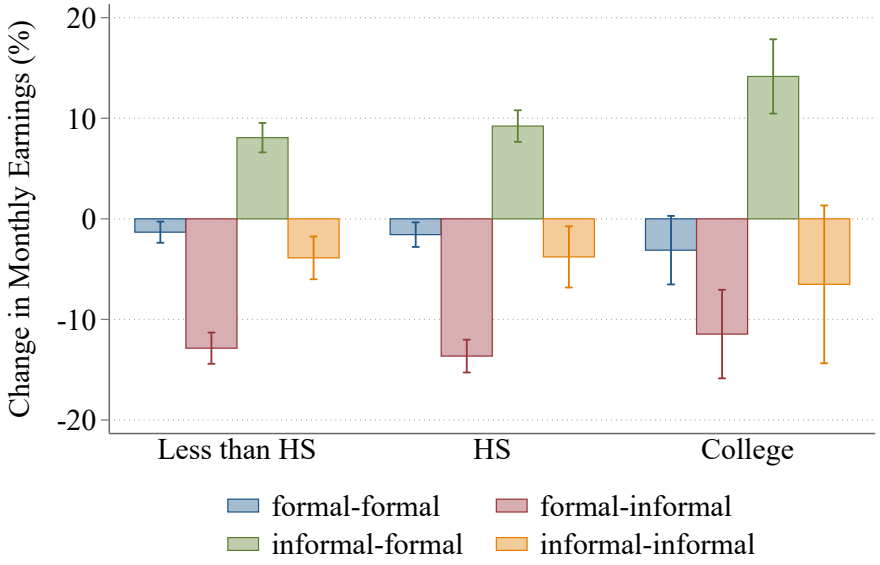


Figure A.7: Earnings Growth by Formality Status (Only Job Switchers)

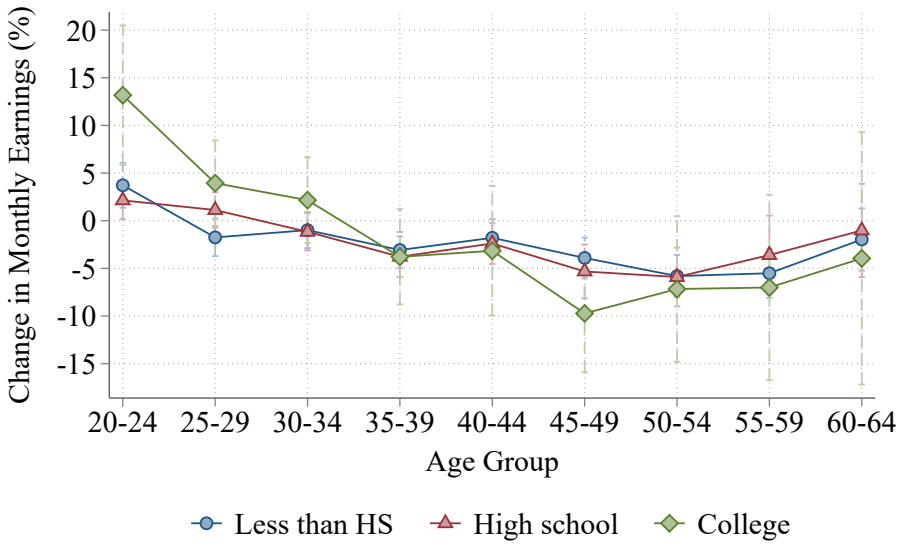


Figure A.8: Earnings Growth by Education and Age (Only Job Switchers)

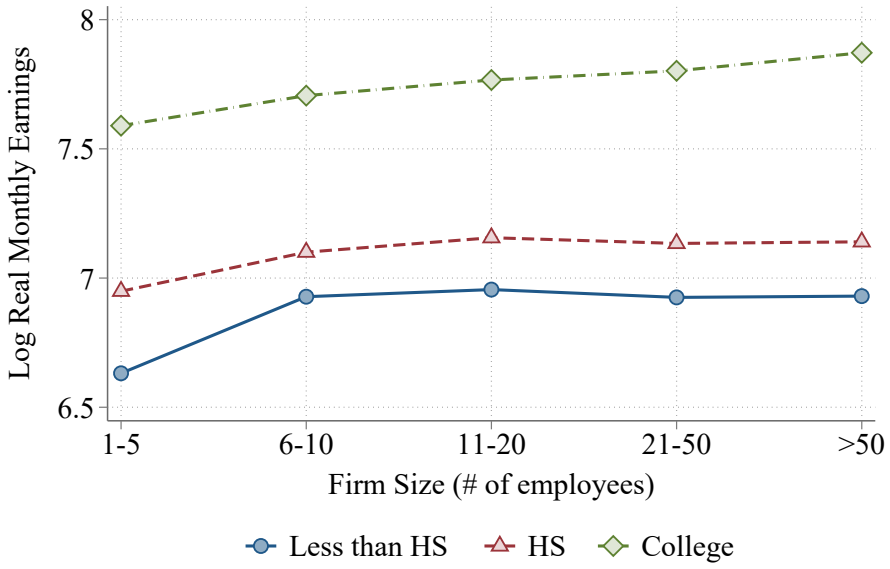


Figure A.9: Monthly Earnings by Education and Firm Size Category

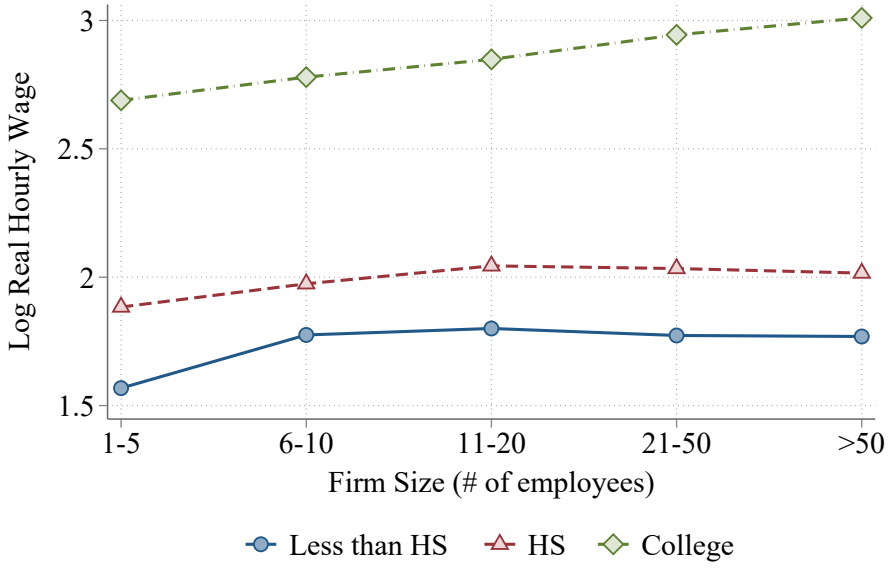


Figure A.10: Hourly Wage by Education and Firm Size Category

B Model

B.1 Firm Size Distribution in the Model

To pin down the firm size distribution in the model, we assume that firms of the same productivity z have the same firm size. We use capital letters to denote variables aggregated over firms of the same productivity – amount of firms $\Gamma(z)$, total vacancies $V(z)$ and total employment $E(z)$, and lower case letters to denote the average of firms – vacancies per firm $v(z)$, average employment per firm $e(z)$, average separation rate $\delta(z)$ and average employment vacancy yield $\xi(z)$. In equilibrium, the number of new hiring net the number of job separations equals total employment growth, which is at the same rate of population growth on the balanced growth path, i.e.,

$$\Gamma(z)v(z)q\xi(z) - E(z)\delta(z) = E(z)g_n.$$

After obtaining $q, \delta(z), v(z), \xi(z)$ from the main iterations/simulations, we can then solve for the average firm size as well as the number of firms

$$e(z) = \frac{E(z)}{\Gamma(z)} = \frac{v(z)q\xi(z)}{\delta(z)}, \tag{B.1}$$

$$\Gamma(z) = \frac{E(z)\delta(z)}{v(z)q\xi(z)}. \tag{B.2}$$

B.2 Solution Algorithm

The model does not admit an analytical solution; thus we numerically solve the stationary equilibrium.

1. Make initial guesses on
   - Distribution of individuals $e(\mathbf{x}, \mathbf{y}, j)$
   - Firms' vacancies $v(z, \psi)$
   - Firms' piece rate function $\phi(z, \psi)$
2. Use the aggregate matching function to obtain labor market contact rates for firms and workers as well as the job offer distribution $G(z, \psi)$
3. Given prices, government policies, labor market contact rates and offer distribution:
   - Solve individuals' life cycle problem by backward induction
   - Simulate 20,000 individuals' life cycle

- Update distribution of individuals

4. Given policies and updated individual distributions, solve firm's problem
   - Given individuals' policy functions, obtain DPV of profits conditional on formality and piece rate $\phi(j, \mathbf{x}, \mathbf{y})$ using equation (7)
   - Solve for ϕ that maximize expected profit yield per vacancy conditional on formality $\tilde{\Pi}(z, \psi)$ using equation (8), and obtain the choice probability $\Delta(z)$
   - Solve for optimal vacancy numbers using F.O.C. as in equation (9)
5. Iterate until all above objects converge
6. Solve for firm size distribution using equations (B.1) and (B.2)

B.3 Benchmark Model

Calibration Table B.1 summarizes the rest of the model parameters that are calibrated internally. We calibrate human capital growth rates (ρ) to match wage growth of continuous employment in the same sector. We plot corresponding human capital growth functions in Figure B.1.

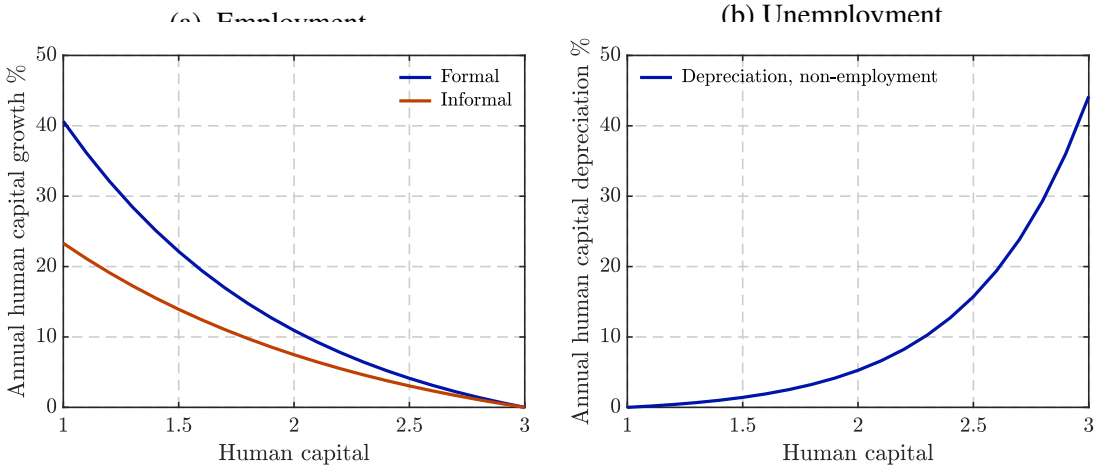


Figure B.1: Human Capital Evolution by Employment Status

Table B.1: Additional Calibrated Parameters

Parameter	Value	Description
Preferences		
$\gamma_{l,1}$	0	Disutility of low-skilled working in informal sector
$\gamma_{h,1}$	-0.24	Disutility of high-skilled working in informal sector
Matching function		
α_0	0.25	Matching function: constant
Human capital evolution		
s_{col}	0.6	Initial human capital premium of college
$h_{\min}$	1	Lower bound of human capital grids
$h_{\max}$	3	Upper bound of human capital grids
$\rho_{n,0}$	0.50	Unemployment: HC depreciation, constant
$\rho_{n,1}$	2.00	Unemployment: HC depreciation, slope
$\rho_{1,0}$	0.25	Informal jobs: HC growth, constant
$\rho_{1,1}$	0.75	Informal jobs: HC growth, slope
$\rho_{2,0}$	0.45	Formal jobs: HC growth, constant
$\rho_{2,1}$	1.00	Formal jobs: HC growth, slope