Optimal Income Taxation in an Integrated Economy *†

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Abstract

Using the AMNE database, we first show that personal income taxation affects the inward and outward activity of multinationals, which in turn change occupation choices, matching quality of workers and managers, and distributions of earnings. We then propose a framework to explore the optimal income taxation in an integrated economy where occupation choices and multinational activities jointly determine the sorting, matching, and income distributions. We find that a closed economy framework overlooks the gains from FDI inflows and thus over-predicts the optimal income tax level. Meanwhile, a framework without endogenous matching underestimates the loss from the complementarity effect, leading to under-predictions of the optimal income tax rate.

Keywords: territorial taxation, income inequality, optimal income taxation, offshoring

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1 Introduction

In a globalized economy, technological and institutional advancements have increasingly blurred the borders between national labor markets, facilitating the formation of cross-country teams and labor movement across borders. These changes have reshaped how workers and managers are matched: high-skill workers can generate higher value by partnering with more productive international managers, while high-skill managers can leverage their expertise at lower costs by employing cheaper foreign labor. However, theoretical frameworks on optimal income taxation with endogenous wages – where individuals' wages are determined by their own abilities and their co-workers' abilities – often overlook cross-border dynamics, as they typically assume a fixed distribution of labor skills and stable labor supply within a closed economy. These ignoring the cross-border dynamics may restrict our understanding of how personal income taxation affects the formation of international teams and income distribution, potentially resulting in misguided policy recommendations on optimal income taxation. Therefore, in this paper, we incorporate cross-border dynamics and heterogeneous agents into the analysis to examine how personal income tax schedules impact individual occupational choices and the cross-border activities of multinational firms, thereby influencing labor matching quality and income distribution in an integrated economy.

We model an economy where production requires both physical inputs and knowledge. In this economy, a continuum of agents with heterogeneous skills makes occupational choices between becoming workers or managers and sorts into teams competitively. Building on Garicano (2000) and Garicano and Rossi-Hansberg (2004, 2006, 2012), we model firms as knowledge-based hierarchies where less skilled agents (workers) focus on production tasks, while more skilled agents (managers) specialize in knowledge-intensive tasks. More capable managers enhance the productivity of all the workers in their teams, as their greater knowledge allows them to solve a wider range of problems encountered in production. Similarly, more skilled workers enable managers to oversee larger teams, as these workers solve more problems independently, requiring less assistance from their managers. This creates a complementarity between the abilities of managers and workers, which determines the composition of production teams in general equilibrium. As we will explore later, this endogenous

matching of workers and managers in production units consistently exhibits positive sorting, where better workers are always matched with better managers. This dynamic, driven by comparative advantage, also shapes agents' occupational choices: more capable agents prefer to establish their own firms and manage their own teams rather than work under other managers.

To explore the impact of personal income tax rates (relative to business profit taxes) on the formation of international teams and income distribution, we consider a one-sector, two-country model with a territorial tax system where workers are immobile, but managers can move across borders. Under current international tax frameworks, some countries apply a worldwide tax system, taxing the global earnings of resident corporations, while others use a territorial tax system, exempting foreign-earned income from domestic taxation. Most countries, including all G7 and the majority of OECD nations, now follow a territorial approach, where foreign-earned profits are only taxed in the country where the business is conducted. In this paper, we focus on a territorial tax system, meaning managers pay corporate income taxes exclusively in the countries where they conduct production, regardless of their nationality. Tax revenues, collected from both immobile domestic workers and managers operating domestically, are rebated to domestic citizens through lump-sum transfers.

In our model, the two countries differ only in their labor skill distributions: the North has a skill distribution with a relatively high mean, while the South has a relatively low mean. Since managers are mobile and the North has more skilled agents, globalization leads to the formation of cross-country teams, where northern managers oversee southern workers. This leads to an increase in the number of workers in the South and more managers in the North, bringing an inflow of multinational firms into the South, compared to a closed economy.

We initially study the case where personal income tax rates and corporate income tax rates are symmetric across the two countries. Then, we compare this to an "asymmetric" tax scenario, where the tax on managers' rents remains the same, but the tax on workers' wages differs between the countries.

We first show that since agents make occupational choices based on after-tax earnings, a unilateral decrease in the southern personal income tax rate results in more workers and fewer

managers in the South. This finding is consistent with earlier research on optimal income taxation in models with endogenous wages and occupational choices (Scheuer, 2014; Moresi, 1997; Stiglitz, 1982; Naito, 1999). Simultaneously, in response to the lower personal income tax rate in the South, the North adjusts by supplying fewer workers but more managers in the new equilibrium. This shift occurs because some highly skilled workers in the South, who were formerly managers, now choose to work as employees. Moreover, as the skill level of workers increases, the marginal value of managers' skills also rises, making it more profitable for some northern agents to switch from workers to managers.

As a result, our theory predicts that when the South lowers its personal income tax rate, it attracts an inflow of northern managers. This prediction aligns with the empirical findings in Section 3, where, using the OECD Analytical Activity of Multinational Enterprises (AMES) database, we reveal two key facts. First, personal income tax rates (relative to corporate tax rates) have been decreasing over time across OECD countries. Second, countries with lower personal income tax rates tend to have higher multinational outputs. These findings underscore the importance of incorporating the mechanism by which domestic personal income tax schedules influence multinationals' sourcing decisions in the study of optimal income taxation.

Next, we examine how personal income tax reductions affect individual earnings levels and structures in both countries. A unilateral tax cut reduces the take-home earnings of relatively unskilled workers in both countries.[1] This occurs mainly because the marginal return to skill of workers is driven by the quality of the match. With the North supplying more managers, some previously skilled northern workers now become managers of the least-skilled workers, which lowers the marginal productivity and thus pre-tax wages for these unskilled workers globally. Meanwhile, in the South, the total amount of lump-sum transfers, which contributes to take-home earnings, decreases after the tax cut, further lowering income for unskilled workers. As a result, low-productivity workers experience worse match, leading to a rise in within-worker wage inequality in both countries.

Besides, the effect of income taxation on between-occupation inequality follows directly

[1]In this paper, we define "after-tax" earnings as nominal earnings minus taxes, while "take-home" earnings mean after-earnings plus the lump-sum transfer.

from the changes in occupational choices. Lower personal income tax rates in the South encourage more workers and fewer managers. As some former managers transition to workers, between-occupation inequality in the South decreases. In contrast, as the North supplies more managers and fewer workers, between-occupation inequality in the North increases. Furthermore, research using individual-level data indicates that within-occupation wage variation accounts for a significant portion of overall earnings inequality.[2] Our model supports this finding, highlighting the important role of within-occupation wage variation: although between-occupation inequality may respond differently to a unilateral decrease in personal income tax, the impact of within-occupation inequality is more pronounced. Hence, a unilateral reduction in personal income tax leads to an increase in within-occupation inequality, which in turn raises total wage inequality in both countries.

What is the optimal southern personal income tax policy that maximizes aggregate consumption in the South? Following Antras et al. (2006), we assume managers consume only in their own country, so a country's aggregate consumption is defined by the take-home earnings of its citizens. Since utility is linear in the consumption of homogenous outputs, welfare is as well. In the South, the equilibrium effect on aggregate consumption can be decomposed into two parts. First, a lower personal income tax attracts more FDI inflows, increasing total tax revenue and thus aggregate consumption. Second, a unilateral tax cut reduces the take-home earnings of relatively unskilled workers, as they experience worse matches. We show that, when the personal income tax is above a certain threshold, the first offshoring effect dominates the second complementarity effect, making aggregate consumption in the South concave with respect to the tax rate and identifying a unique optimal tax rate that maximizes southern aggregate consumption.

This paper highlights two key points: first, personal income taxation influences multinationals' sourcing decisions; second, there is endogenous matching between workers and managers. We conclude by examining two alternative frameworks to assess the importance of these two adjustment margins. First, we consider a "closed-economy" scenario with no

[2]For instance, Akerman et al. (2013) find that within-occupation and within-industry wage variation accounted for 59 percent of the variance in log wages in Sweden in 2001 and 66 percent of the change in inequality from 2001 to 2007. Helpman et al. (2016) report that within-group variation was responsible for the majority of the overall change in Brazilian wage inequality during the trade liberalization period that began in 1991.

cross-border activities. In this autarky setting, we show that maximum southern aggregate consumption is achieved when workers and managers are subject to the same income tax rate, aligning with the general view that tax disparities between income types introduce distortions that should be avoided.[3] However, this closed-economy framework overlooks the benefits of FDI inflows, thereby overestimating the optimal income tax level. Next, we consider a "fixed-matching" framework in which personal income tax is independent of the quality of matching between workers and managers. Although this framework captures gains from FDI inflows, it ignores the market failure caused by mismatches between agents. Therefore, the fixed-matching framework underestimates the losses from the complementarity effect, leading to under-predictions of optimal income tax rates.

2 Related Literature

This paper contributes to the literature on optimal income taxation in models with endogenous wages and occupational choice, building on foundational works by Feldstein (1973), Allen (1982), and Stiglitz (1982). In Stiglitz's (1982) two-type model, a higher marginal tax rate on low types improves social welfare by reducing wage inequality. Specifically, a higher tax rate on low types decreases labor supply among low types, which increases their wages while, due to production complementarity, reducing the wages of high types. Sachs, Tsyvinski, and Werquin (2020) extend Stiglitz's model by introducing a continuum of income types, while Rothschild and Scheuer (2013) further depart from Stiglitz's framework by examining optimal taxation in settings with multidimensional skill heterogeneity, where individuals have different skills across occupations. Although these more complex models similarly suggest that higher marginal tax rates at the bottom can raise wages in low-wage occupations, they show a diminished role for endogenous wages in offsetting redistribution compared to Stiglitz's simpler framework. However, this strand of literature generally overlooks cross-border dynamics, as they assume a fixed distribution of labor skills and stable labor supply in a closed economy. In contrast, our paper highlights that ignoring the impact

[3]For example, Mirrlees, Crawford, and Freedman (2010) argue that the tax system should aim at neutrality and align tax rates for the employed and self-employed.

of personal income taxation on multinational firms' sourcing decisions fails to account for the benefits of lower personal taxation on FDI inflows. This omission leads to an overestimation of the optimal income tax level for low-wage occupations.

This paper also contributes to the literature of knowledge-based hierarchies, where firm productivity depends on the organization of production. Most mainstream economic models assume a fixed organization of production, employing production functions, such as the constant-elasticity-of-substitution production function, that depend on total quantities of a pre-determined and inflexible set of inputs. In contrast, in Garicano (2000)'s model and in several follow-up papers, managers decide whom to hire and how many workers of each type to employ, aiming to minimize production costs. This approach highlights how firm productivity arises from interactions among team members and the organizational structure of these interactions. Caliendo and Rossi-Hansberg (2012) and Antras et al. (2006) extend these frameworks to explore their applications in international trade and the global organization of production. Specifically, Caliendo and Rossi-Hansberg (2012) analyze how the gains from trade liberalization are affected by the changes in organization that they trigger, while Antras et al. (2006) show that how matching individuals in international production teams affects wages across countries with varying skill distributions in ways that are consistent with the empirical evidence. Building on this literature, our work incorporates organizational decision-making to examine optimal taxation in an open economy. We analyze how changes in personal income taxation affect the inward and outward activities of multinationals, which influence the organization of work – such as worker-manager matching quality and firm size – and, in turn, shape the structure of earnings and the patterns of consumption in the global economy.

This paper also contributes to the literature on how tax policies influence multinationals' sourcing decisions. While most existing studies focus on the effects of corporate tax policies, such as Barrios et al. (2012), Huizinga and Voget (2009), Huizinga et al. (2014), and Wang (2020), which show that higher corporate tax rates in host countries reduce cross-border economic activities and multinational engagement, our study shifts the focus to personal income taxation. Following the approaches of Huizinga and Voget (2009) and Wang (2020),

we first use a trade gravity model to estimate the effect of personal income tax rates on bilateral multinational activities. We then develop a theoretical framework where heterogeneous individuals can migrate, switch occupations, and form international teams, demonstrating how personal income tax policies influence the formation of international teams and generate significant cross-country effects on social welfare.

3 Empirical Evidence

Prior to conducting comparative static analyses and counterfactual simulations, we initially provide empirical evidence illustrating that variations in personal income tax rates across different nations significantly impact multinational activities.

3.1 Data

Our empirical analysis requires data on three key aspects: (1) personal income tax rates across countries, (2) multinational output, and (3) bilateral trade gravity measures along with other macroeconomic data.

Personal income tax rate data is sourced from the OECD tax database. The primary variable of interest is the average personal income tax rate (tax wedge), defined as the ratio between taxes paid by a single worker at various income multiples (67%, 100%, 133%, 167% of average earnings) and the corresponding total labor cost for the employer. For robustness, we also include the average personal income tax rate (all-in) as an additional check. Unlike the tax wedge, the all-in tax rate is calculated as the ratio of taxes paid to earnings, rather than total labor costs. Furthermore, we incorporate statutory corporate income tax rates for each country as control variables in our reduced-form regressions.

The data on multinational output across countries is sourced from the OECD Analytical Activity of Multinational Enterprises (AMNE) database, which provides valuable insights into the role of multinational enterprises in the global economy. This dataset covers 34 industries classified under ISIC Rev. 4 and 59 economies, spanning the period from 2005 to 2016. The dataset we use is structured as a matrix of world gross output by host country,

industry, and country of ownership, reflecting the output of foreign-owned firms from each parent country operating in host countries across different industries.

Bilateral trade data is derived from the CEPII Gravity Data. This dataset provides extensive information on trade frictions between pairs of countries, as well as several macroeconomic indicators for each country. Additional macroeconomic data is sourced from the Penn World Table.

After merging the various data sources, we obtain a comprehensive dataset covering 37 countries and 34 industries from 2005 to 2016. Tables A1 and A2 in the appendix present the countries and industries included in our analysis.

3.2 Specification

Inspired by Huizinga and Voget (2009) and Wang (2020), we estimate the following reduced-form gravity equation:

$$logX_{ildt}^{MP} = \beta_1 log(1 - \kappa_{dt}) + grav_{idt}\beta_2 + FE_i + FE_d + FE_l + T_t + u_{ildt}$$

where X_{ildt}^{MP} represents the bilateral multinational output produced by firms in industry l, owned by parent country i, in host country d at year t. κ_{dt} denotes the personal income tax rate in host country d at year t, as discussed in the previous subsection. The trade gravity variables between host country d and source country i include factors such as contiguity, common language, common religion, common currency, and the log of weighted distance.

We also include fixed effects for the host country (FE_d), source country (FE_i), industry (FE_l), and time (T_t) to account for unobserved heterogeneity. The regression is run on both the full sample and a partial sample excluding domestic production (i.e., observations where the host country and parent country are the same).

We expect a positive value for β_1, consistent with the intuition that higher personal income tax rates hinder international teams and multinational production.

3.3 Empirical Results

The regression results presented in Table 1 provide strong support for the foundational assumptions of our model. The first four columns show the regression results for the full sample, while the last four columns focus on the partial sample, excluding domestic production observations. For each regression, we use the average personal income tax rate (tax wedge) at 67%, 100%, 133%, and 167% of the average wage in each country to regress multinational output. The coefficient β_1 is significantly positive across all regressions, indicating that higher personal income tax rates hinder multinational activities. Furthermore, the results suggest that multinational output is more sensitive to changes in the personal income tax rate for higher-income groups compared to lower-income groups. **The two facts above aligns with the implications of our model, which posits that changes in personal income tax rates influence labor's career choices and FDI decisions, particularly for high-skilled labor.**

In Table A3, we utilize the average all-in personal income tax rate as an alternative measure of personal income taxation and re-estimate the reduced-form regressions to conduct robustness checks. The results are largely consistent with those presented in Table 1, thereby reinforcing the robustness and reliability of our empirical findings.

Table 1: The Impact of Personal Income Tax Rate on Multinational Output

VARIABLES	Log of Multinational Output (millions of US dollars)							
	All Sample				Host Country is not Source Country			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Log of (1 - tax wedge 67)	0.961***				0.661*			
	(0.341)				(0.338)			
Log of (1 - tax wedge 100)		2.155***				1.819***		
		(0.339)				(0.335)		
Log of (1 - tax wedge 133)			1.545***				1.236***	
			(0.277)				(0.274)	
Log of (1 - tax wedge 167)				1.511***				1.299***
				(0.251)				(0.247)
Log of (1 - statutory corporate income tax rate)	0.114	-0.071	0.053	0.017	0.198	0.006	0.124	0.075
	(0.279)	(0.275)	(0.273)	(0.274)	(0.277)	(0.273)	(0.271)	(0.272)
Control Variables	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Host and Source Country Fixed Effect	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Industry Fixed Effect	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Year Effect	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Observations	827,676	827,676	827,676	827,676	812,580	812,580	812,580	812,580
R^2	0.413	0.413	0.413	0.413	0.387	0.387	0.387	0.387

Notes: Columns (1)–(4) present the regression results for the full sample, whereas columns (5)–(8) report the results for the partial sample, which excludes observations related to domestic production. The key explanatory variable, the tax wedge, is measured as the average personal income tax rate (tax wedge). It is defined as the ratio of taxes paid by a single worker at different income levels (67%, 100%, 133%, and 167% of average earnings) to the corresponding total labor cost for the employer in each host country in a given year. The set of control variables includes the logarithm of GDP for both the source and host countries, indicators for whether the host and source countries share a border, whether they have a common official or primary language, whether they share a language spoken by at least 9% of the population, whether they share a common religion, whether they use a common currency, and the logarithm of the weighted distance (in kilometers) between the host and source countries. Additionally, we incorporate host country fixed effects, source country fixed effects, industry fixed effects, and year effects to account for unobserved heterogeneity. Robust standard errors are reported in parentheses. *** p<0.01, ** p<0.05, * p<0.1.

4 Model

4.1 Basic Environment

Endowment: Consider a world economy with countries indexed by $i \in N$, one factor of production (labor), and one homogeneous output. In particular, a country i is endowed with a continuum of agents with ability z derived from a cumulative distribution function $G_i(z)$ with density $g_i(z)$ and support in $[0, \bar{z}_i]$. Agents make an occupation choice between managers and workers: workers are immobile, but managers are mobile across the border.

Preference and Market Structure: Assume that utility is linear with respect to the consumption of homogeneous outputs, and trade costs are zero, so that agents are income maximizers. Both the goods and the labor markets are perfectly competitive in these equilibria.

Technology: The organization of the structure follows Garicano (2000), where each production team consists of one manager and a few production workers. Each agent has one unit of time and a skill level z. Workers have to solve a problem to carry out production. If a worker knows the solution, she solves it and produces one unit of output. If she does not know the solution, she can ask her manager. If the manager knows the solution, she then produces one unit of output; otherwise, no production occurs. It takes a manager $0 < h < 1$ units of time to communicate about this with the worker, whether the manager knows the solution or not. Thus, the skill level of an agent determines the set of problems she can solve. An agent with skill z can solve all problems that require knowledge between 0 and z. Here, we normalize the set of problems so that the skill level z also represents the proportion of problems an agent can solve.

Proposition 1. *Each manager hires workers of homogeneous ability.*

Given Lemma 1, a manager in a team with n workers of homogeneous ability z_p faces the following time constraint:

$$h(1 - z_p)n = 1,$$

and so can deal with $n(z_p) = 1/[h(1-z_p)]$ workers. Hence, the total output in a production team formed by a manager with skill level z_m and workers with skill z_p is therefore given by $z_m n(z_p)$. Let $w_i(\cdot)$ be an equilibrium wage function in country i. Given wages, managers choose the ability z_p of their workers to maximize rents,

$$r_i(z_m) = \max_{z_p} \big(z_m - w_i(z_p)\big) n(z_p), \tag{1}$$

subject to

$$h(1-z_p)n(z_p) = 1.$$

The first-order condition of this problem is given by

$$w_i'(z_p) = \frac{z_m - w_i(z_p)}{1-z_p}. \tag{2}$$

Taxation: In the current system of international taxation, some countries tax the worldwide earnings of resident corporations (a worldwide tax system), while others exempt foreign-earned income of multinational businesses from their domestic tax base (a territorial tax system). Many countries have transitioned from a worldwide to a territorial tax system, with the most recent example being the United States through the 2017 Tax Cuts and Jobs Act (Langenmayr and Liu, 2023). In fact, most countries, including all G7 countries and the majority of OECD countries, now apply a territorial approach to taxing their multinationals, ensuring that foreign-earned profits are only subject to taxation in the foreign country in which they were earned.

Hence, this paper considers a territorial tax system, wherein managers pay corporate income taxes exclusively to the countries where they conduct production, regardless of these managers' nationalities. Formally, let t_i^M and t_i^W denote the corporate income tax rate and personal income tax rate collected by country i on managers' rents and workers' wages, respectively. Besides, we assume that tax revenues are rebated to domestic *citizens* through lump-sum transfers T_i.[4]

[4]That is, in this paper, a person, who is born in country i and works in country j, pays the taxes to the government

Agents choose whether to become managers or workers, and if they become managers, which country to establish a production team to maximize their utility, that is, their *take-home earnings*. In other words, agents with ability z and nationality i solve the following maximization problem:

$$\max\{\max_j(1-t_j^M)r_j(z)+T_i,(1-t_i^W)w_i(z)+T_i\},$$

or equivalently, agents maximize their *after-tax earnings*:

$$\max\{\max_j(1-t_j^M)r_j(z),(1-t_i^W)w_i(z)\}.$$

Proposition 2. *The equilibrium wage and matching quality depend on workers' nationalities, but both are independent of managers' nationalities.*

Proposition 2 implies that no matter an agent works for a domestic or a foreign firm, the skill level of her matched managers is the same. Since the marginal return to skill of workers is driven by the quality of matched managers, a worker receives the same amount of wage payment regardless of her manager's nationality. Hence, we denote $m_i(z_p)$ as the skill level of the manager of a worker with ability z_p in country i, and denote $w_i(z_p)$ as an equilibrium before-tax wage function in country i.

As we prove in Proposition 3 that an equilibrium allocation of this economy has to satisfy positive sorting, and thus, m is invertible. Meanwhile, from Proposition 3, we know that in each country, there exists an equilibrium occupational choice threshold such that agents in country i with skill in $[0,z_i^*]$ become workers and agents with skill in $[z_i^*,\bar{z}_i]$ become managers. Agents with knowledge z_i^* are indifferent. Hence, the labor market clearing condition yields

$$\sum_{i=1}^{N}\int_0^{z_p}g_i(z)dz=\sum_{i=1}^{N}\int_{m_i(0)}^{m_i(z_p)}n(m_i^{-1}(z))g_i(z)dz,\ \text{for all } z_p\leq\max_i\{z_i^*\},\tag{3}$$

where $m_i^{-1}(z)$ is the ability of the workers hired by a manager of ability z in country i. The left-hand side of this equation is the total supply of workers between 0 and z_p in the

j and receives lump-sum transfer from the government i.

whole economy. The right-hand side is the total demand for workers by managers between $m_i(0)$ and $m_i(z_p)$ in all countries. Since the set of workers and the set of managers are connected, we have $m_i(0) = z_i^*$. Meanwhile, since the equilibrium exhibits positive sorting between workers and managers, it implies that $m_i(z_i^*) = \bar{z}_i$ where $\bar{z}_i$ represents the ability of the most productive agent in country i. Labor market clearing is guaranteed when the supply and demand of production workers equalize for every skill level of workers with ability $z_p \leq \max_i\{z_i^*\}$.

Furthermore, the balanced budget condition of the government implies that

$$T_i = t_i^W \underbrace{\int_0^{z_i^*} w_i(z)g_i(z)dz}_{\text{domestic workers' wages}} + t_i^M \Big[\underbrace{\int_0^{z_i^*} m_i(z)g_i(z)dz}_{\text{domestic workers' outputs}} - \underbrace{\int_0^{z_i^*} w_i(z)g_i(z)dz}_{\text{domestic workers' wages}} \Big], \tag{4}$$

where T_i is the total lump-sum transfer to the citizens of country i. The government collects taxes from workers and managers. Equation (1) implies that managers' rents are equal to the difference between domestic workers' physical output and labor costs.

A competitive equilibrium in the economy is defined by a wage function w_i, a rent function r_i, a matching function m_i, and occupational choice thresholds z_i^*, such that managers maximize rents (Equation (1) is satisfied and $\lim_{\epsilon \to 0} \frac{\partial \Pi(\bar{z}, z^* + \epsilon)}{\partial \epsilon} < 0$),[5] agents maximize utility ($(1 - t_i^W)w_i(z_i^*) = (1 - t_i^M)r_i(z_i^*)$), labor markets clear (Equation (3) is satisfied together with $m_i(0) = z_i^*$ and $m_i(z_i^*) = \bar{z}_i$), and the government's budget is balanced (Equation (4) is satisfied). The following proposition shows that such an equilibrium of the economy exists as long as h is smaller than a threshold h^*.[6]

Proposition 3. *There exists a threshold $h^* > 0$ such that if $h \in [0, h^*]$, there exists a unique competitive equilibrium of this economy. In equilibrium, the equilibrium exhibits positive sorting. The earnings function is strictly convex. The set of managers and the set of workers are connected, and the equilibrium allocation is efficient.*

[5] The second condition is needed to guarantee that managers at $\bar{z}_i$ do not profit from hiring agents with abilities slightly above z_i^*.

[6] We follow Antras, Garicano, and Rossi-Hansberg (2006) that we do not allow agents to be self-employed, so we need to restrict h for an equilibrium to exist. If we were to allow agents to work on their own, we could guarantee existence for all $0 < h < 1$.

4.2 Equilibrium in the World Economy

Consider a world formed by two independent countries, wherein agents can form teams with agents in their own countries or with agents in the other country. Assume that the first one, which we call the "North" (or "N"), has a uniform distribution of skills in the population, $G_N(z) = z$ for $z \in [0, 1]$ with density $g_N(z) = 1$. That is, the best agents in the North can solve all the problems that arise in production. The second economy, which we call "the South" (or "S"), has a uniform distribution of skills $G_S(z) = z/\alpha$ for $z \in [0, \alpha]$ with density $g_S(z) = 1/\alpha$, and we require $\alpha < 1$. The best Southern agents can thus solve only a fraction α of the problems during production. The North, therefore, is relatively better endowed with skilled agents, but both countries are identical in all other respects, including population size. Besides, we assume that the cost of communicating the solution to a problem, h, is the same whether communication happens between agents in the same country or in different countries.

Let M_S and M_N be the sets of managers who establish firms in the South and the North. Since the sets of managers are connected intervals, we could write $M_S = [m_S(0), m_S(z_S^*)]$ and $M_N = [m_N(0), m_N(z_N^*)]$.

Proposition 4. *If there exists an overlapping of the skill distribution of managers who establish firms in the South and the North, denoted as $M_S \cap M_N = [a, b]$ where $a < b$, then the following conditions hold:*

1. $a = \min\{z_S^*, z_N^*\}$ *and* $b = \min\{m_S(z_S^*), m_N(z_N^*)\}$.

2. *For any* $c \in [a, b]$,

$$m_S^{-1}(c) = m_N^{-1}(c),$$

$$w_S(m_S^{-1}(c)) = w_N(m_N^{-1}(c)).$$

3. $t_N^M = t_S^M$.

The second condition of Proposition 4 suggests that for any managers who are indifferent between producing in the South and forming teams in the North, they are matched with

the same type of workers and pay the same amount of pre-tax wages to local workers in two countries. Hence, we omit the subscripts of wage and matching functions and instead use $w(.)$ and $m(.)$ from now on. The third condition of Proposition 4 implies that $t_N^M = t_S^M$ (that is, the event that corporate income taxes are equal across borders) is a necessary condition for the existence of overlapping skill sets of managers from two countries.

4.3 Symmetric Tax Equilibrium

We start with a symmetric equilibrium as a benchmark, wherein two governments impose symmetric tax rates $t^W(\equiv t_S^W = t_N^W)$ on workers' wages and $t^M(\equiv t_S^M = t_N^M)$ on managers' rents. Depending on the value of the communication cost h and skill overlap measure α, there exist two types of equilibria in the economy.[7]

The first one is an equilibrium in which all agents in the South are workers, while the second type of equilibrium is the one where some of the agents in the South are managers. Please refer to Section III.A. of Antràs, Garicano, and Rossi-Hansberg (2016) for the discussion on the first type of equilibria. In this paper, we focus on the second type of equilibria where managers come from both countries to capture the reality.

First, deriving the labor market clearing condition (3) with respect to z_p, we obtain that

$$m'(z) = h(1-z)\frac{g(z)}{g(m(z))}, \tag{5}$$

where the distribution of skills in the world population is given by the sum of the distribution of skills in the South and in the North, as

$$g(z) = \begin{cases} (1+\alpha)/\alpha & 0 < z < \alpha \\ 1 & \alpha < z < 1. \end{cases} \tag{6}$$

Denote z_α as the worker type that works for the best agent in the South such that

[7] In general, as long as tax regimes are common across countries, the set of parameter values that determines the boundary between these two equilibria is a nonlinear function of h and α, as plotted in Figure 1. Intuitively, as the communication cost h increases, managers can have smaller teams, so in equilibrium, there are more managers and fewer workers. Thus, some skilled agents in the South are more likely to become managers to meet the demand. Meanwhile, as α increases, it is profitable for some high-ability agents to become managers in the South. In all, the possibility of the existence of the second type of equilibrium increases as α or h goes up.

$m(z_\alpha) = \alpha$. In order to guarantee the least amount of offshoring, assume that international teams are formed only if managers strictly prefer to hire foreign workers than domestic ones. That is, southern agents with skill lower than or equal to z_α work for managers in their own country, and workers with skill greater than z_α work with northern managers. Then, Equations (5) and (6) imply that

$$m'(z) = \begin{cases} h(1-z) & 0 < z < z_\alpha \\ \dfrac{1+\alpha}{\alpha} h(1-z) & z_\alpha < z < z^*. \end{cases}$$

Together with the boundary conditions: $m(0) = z^*$ and $m(z^*) = 1$ and the continuity of the matching function at z_α,[8] we use these differential equations to solve for the matching function

$$m(z) = \begin{cases} z^* + hz(1 - z/2) & 0 < z < z_\alpha \\ z^* - \dfrac{hz_\alpha}{\alpha}(1 - z_\alpha/2) + \dfrac{1+\alpha}{\alpha} hz(1 - z/2) & z_\alpha < z < z^*. \end{cases}$$

and an occupational choice threshold[9]

$$z^* = \frac{1 + h - \sqrt{1 + h^2 + ((1-\alpha)/(1+\alpha))2h}}{h}.$$

Using the definition of z_α, we also get that

$$z_\alpha = 1 - \sqrt{1 - 2(\frac{\alpha - z^*}{h})}.$$

Equation (2), together with $(1 - t^M)r(z^*) = (1 - t^W)w(z^*)$ and the continuity of wages at z_α, implies that the equilibrium wage function is given by

$$w(z) = \begin{cases} \dfrac{h}{2} z^2 - \sigma_1(1-z) + z^* & 0 < z < z_\alpha \\ \dfrac{1+\alpha}{\alpha} \dfrac{h}{2} z^2 - \sigma_2(1-z) + z^* - \dfrac{hz_\alpha}{\alpha}(1 - z_\alpha/2) & z_\alpha < z < z^*. \end{cases}$$

[8] We have to guarantee that the matching function is continuous for all $z < z^*$, in particular at z_α, in order for the wage function to be differentiable.

[9] In Appendix B.7, we show that in order for the world to be in the second type of equilibria where managers come from both countries, i.e. $z^* < \alpha$, we need to guarantee that $h < \frac{2(\alpha-1)}{\alpha^2-\alpha-2}$.

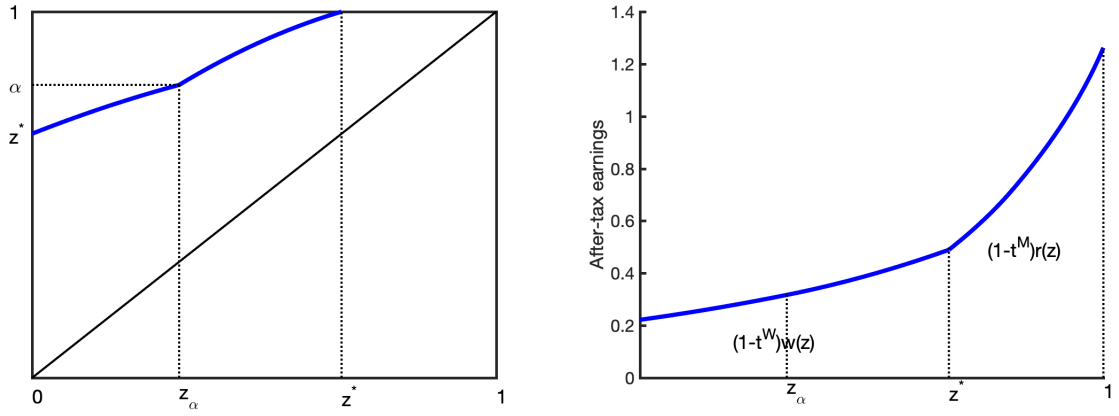
where

$$\sigma_1 = \frac{h[\frac{1+\alpha}{\alpha}\frac{h}{2}(z^*)^2 + \frac{h}{\alpha}z_\alpha(1-z^*) + z^* - \frac{hz_\alpha}{\alpha}(1-z_\alpha/2)]}{(1-t^M)/(1-t^W) + h - hz^*},$$

and

$$\sigma_2 = \sigma_1 - \frac{h}{\alpha}z_\alpha.$$

Figure 1 summarizes what we have discussed about a symmetric tax equilibrium. First, as shown in Figure 1(a), all agents with skill between 0 and z^* become workers, and agents with skill between z^* and 1 are managers. The equilibrium exhibits positive sorting such that more skilled managers lead teams with more productive workers. Moreover, agents with skill in $[0, z_\alpha]$ work for managers with skill in $[z^*, \alpha]$, while agents with skill in $[z_\alpha, z^*]$ work for northern managers with skill in $[\alpha, 1]$. Second, as indicated in Figure 1(b), the after-tax wage function of workers and the after-tax rent function of managers are strictly increasing, continuous, and differentiable everywhere except for z^* where the marginal return to skill of managers is larger than that of workers.



(a) Matching

(b) Wages and rents

Figure 1: Symmetric tax equilibrium

4.4 Asymmetric Tax Equilibrium

In this subsection, we examine an asymmetric tax equilibrium, where the taxes on managers' rents remain consistent across regions ($t^M \equiv t_S^M = t_N^M$), while the taxes on workers' wages differ ($t_S^W \neq t_N^W$).

Proposition 5. *If there exists an overlapping of the skill distribution of managers who establish firms in the South and the North, denoted as* $M_S \cap M_N = [a, b]$ *with* $a < b$*, and if the taxes on Southern workers' wages differ from those on Northern workers' wages, that is,* $t_S^W \neq t_N^W$*, the following conditions hold:*

1. *If* $t_S^W < t_N^W$*, then* $z_N^* < z_S^* \leq \alpha < 1$.

2. *If* $t_S^W > t_N^W$*, then* $z_S^* < z_N^* \leq \alpha < 1$.

Proposition 5 indicates that the thresholds for occupational choices differ when personal income taxes vary between the two countries. Specifically, lower personal income taxes result in higher occupational ability thresholds. Agents make occupational choices based on their after-tax earnings, making them more likely to become workers when the tax on local workers is lower. In this paper, we focus on the scenario where the tax on Southern workers is lower. However, the same logic applies if the tax on Northern workers is lower.

We define z_α as the skill level at which workers are employed by the best Southern managers, such that $m(z_\alpha) = \alpha$, and we let z_S represent the skill level of workers employed by the worst Southern managers, such as $m(z_S) = z_S^*$. It's important to note that positive sorting implies that $z_S < z_\alpha$. Furthermore, from Proposition (5), we know that $z_N^* < z_S^*$ when the South imposes lower personal income taxes. However, the numerical relationship between z_α and z_N^* depends on parameter values. Specifically, there are three possible numerical relationships: (i) $0 < z_S < z_\alpha < z_N^* < z_S^*$; (ii) $0 < z_S < z_N^* < z_\alpha < z_S^*$; and (iii) $0 < z_N^* < z_S < z_\alpha < z_S^*$. To avoid duplications, we provide the details of deriving the equilibrium matching and wage functions for the first case of $0 < z_S < z_\alpha < z_N^* < z_S^*$ in the main manuscript, while the other two cases are described in Appendix C. It is important to note that, during the estimation process, for any arbitrary combination of parameter values $\{\alpha, h, t_i^W, t_i^M\}$, one has to consider all three cases before determining the unique equilibrium allocation.

First, we derive the labor market clearing (3) with respect to z_p, resulting in the following

equations:

$$m'(z) = \begin{cases} \dfrac{1+\alpha}{\alpha} h(1-z) & 0 < z < z_S \\ h(1-z) & z_S < z < z_\alpha \\ \dfrac{1+\alpha}{\alpha} h(1-z) & z_\alpha < z < z_N^* \\ \dfrac{1}{\alpha} h(1-z) & z_N^* < z < z_S^*. \end{cases}$$

These differential equations, along with the boundary conditions: $m(0) = z_N^*$ and $m(z_S^*) = 1$, and the definitions of z_S and z_α, allow us to determine the equilibrium matching function:

$$m(z) = \begin{cases} \dfrac{1+\alpha}{\alpha} hz(1-z/2) + z_N^* & 0 < z < z_S \\ hz(1-z/2) + \dfrac{1}{\alpha} hz_S(1-z_S/2) + z_N^* & z_S < z < z_\alpha \\ \dfrac{1+\alpha}{\alpha} hz(1-z/2) - \dfrac{1}{\alpha} hz_\alpha(1-z_\alpha/2) + \dfrac{1}{\alpha} hz_S(1-z_S/2) + z_N^* & z_\alpha < z < z_N^* \\ \dfrac{1}{\alpha} hz(1-z/2) + 1 - \dfrac{1}{\alpha} hz_S^*(1-z_S^*/2) & z_N^* < z < z_S^*. \end{cases}$$

Besides, the continuity of the matching function at z_S, z_α and z_N^* implies that $\{z_S, z_\alpha, z_N^*, z_S^*\}$ satisfy the following conditions:

$$\frac{1+\alpha}{\alpha} hz_S(1-z_S/2) + z_N^* = z_S^*, \tag{7a}$$

$$hz_\alpha(1-z_\alpha/2) + \frac{1}{\alpha} hz_S(1-z_S/2) + z_N^* = \alpha, \tag{7b}$$

$$hz_N^*(1-z_N^*/2) - \frac{1}{\alpha} hz_\alpha(1-z_\alpha/2) + \frac{1}{\alpha} hz_S(1-z_S/2) + z_N^* = 1 - \frac{1}{\alpha} hz_S^*(1-z_S^*/2). \tag{7c}$$

Meanwhile, the rent-maximization problem of managers (2) yields

$$w(z) = \begin{cases} \dfrac{1+\alpha}{2\alpha} hz^2 - \sigma_1(1-z) + z_N^* & 0 < z < z_S \\ \dfrac{1}{2} hz^2 - \sigma_2(1-z) + \dfrac{1}{\alpha} hz_S(1-z_S/2) + z_N^* & z_S < z < z_\alpha \\ \dfrac{1+\alpha}{2\alpha} hz^2 - \sigma_3(1-z) - \dfrac{1}{\alpha} hz_\alpha(1-z_\alpha/2) + \dfrac{1}{\alpha} hz_S(1-z_S/2) + z_N^* & z_\alpha < z < z_N^* \\ \dfrac{1}{2\alpha} hz^2 - \sigma_4(1-z) + 1 - \dfrac{1}{\alpha} hz_S^*(1-z_S^*/2) & z_N^* < z < z_S^*. \end{cases}$$

The continuity of the wage function at z_S, z_α, and z_N^* is guaranteed when $\{\sigma_1, \sigma_2, \sigma_3, \sigma_4\}$ satisfy the following conditions:

$$\sigma_2 = \sigma_1 + \frac{1}{\alpha} h z_S, \tag{7d}$$

$$\sigma_3 = \sigma_2 - \frac{1}{\alpha} h z_\alpha, \tag{7e}$$

$$\sigma_4 = \sigma_3 + h z_N^*. \tag{7f}$$

Additionally, agents at z_N^* are indifferent between workers and managers, i.e. $(1 - t_N^W) w(z_N^*) = (1 - t^M) r(z_N^*)$, which yields:

$$(1 - t_N^W) \left[\frac{1}{2\alpha} h (z_N^*)^2 - \sigma_4 (1 - z_N^*) + 1 - \frac{1}{\alpha} h z_S^* (1 - z_S^*/2) \right] = (1 - t^M) \frac{\sigma_1}{h}. \tag{7g}$$

Likewise, agents at z_S^* are indifferent between occupations, i.e. $(1-t_S^W) w(z_S^*) = (1-t^M) r(z_S^*)$, which is equivalent to:

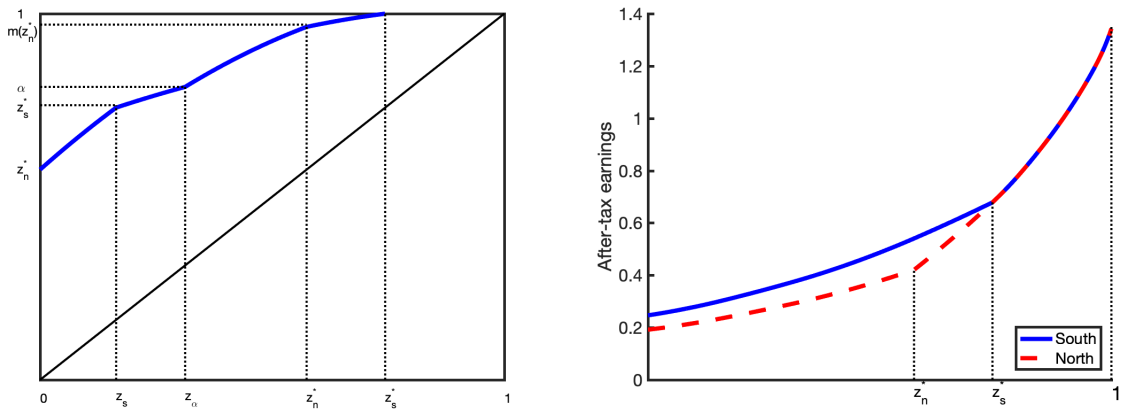
$$(1-t_S^W) \left[\frac{1}{2\alpha} h (z_S^*)^2 - \sigma_4 (1 - z_S^*) + 1 - \frac{1}{\alpha} h z_S^* (1 - z_S^*/2) \right] = (1-t^M) \frac{z_S^* - \left[\frac{1+\alpha}{2\alpha} h z_S^2 - \sigma_1 (1 - z_S) + z_N^* \right]}{h(1 - z_S)}. \tag{7h}$$

Hence, conditions (7a)-(7h) jointly determine the values of $\{z_S, z_\alpha, z_N^*, z_S^*, \sigma_1, \sigma_2, \sigma_3, \sigma_4\}$.

Finally, let Figure 2 summarize what we have demonstrated for an asymmetric tax equilibrium. First, agents with skill in $[0, z_S]$ work for managers with skill in $[z_N^*, z_S^*]$. Agents with skill in $[z_S, z_\alpha]$ work for managers with skill in $[z_S^*, \alpha]$. Agents with skill between z_α and z_S^* work for northern managers with skill in $[\alpha, 1]$. Besides, the after-tax earnings function is strictly increasing, continuous, and differentiable with respect to skill in both countries, and the marginal return to skill of managers is larger than that of workers.

5 Effects of Unilateral Personal Income Tax Deductions

In this section, we examine the impact of changes in personal income taxation on the composition of international teams, occupational choices, and the distributions of earnings. To accomplish this, we compare the symmetric tax equilibrium presented in Section 4.3, where the tax rate on workers is uniform across two countries, with the asymmetric tax equilib-



(a) Matching

(b) Wages and rents

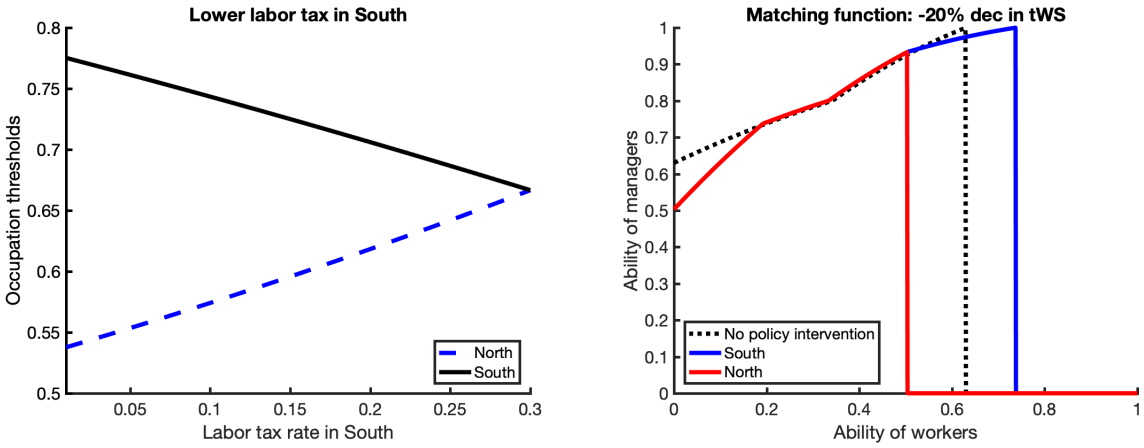
Figure 2: Asymmetric tax equilibrium

rium described in Section 4.4, where the tax rate on southern workers is lower than that on northern workers.

5.1 Matching, Occupational Choices, and Firm Characteristics

Before we delve into the effects of changes in taxation on the distributions of earnings, it will be useful to examine their impacts on individual occupational choices and the matches that form among groups of workers and managers. Figure 3(a) illustrates how personal income tax policies in one country affect individual occupational choices in both countries. Specifically, it shows that when the tax on southern workers becomes lower, the occupational ability threshold that separates the ability of the agents who choose to be workers or managers decreases in the South. This implies that lower personal income tax rates induce more workers and fewer managers in the South. Intuitively, workers choose occupations based on take-home occupational income. [10] Meanwhile, lower personal income tax in the South increases the occupational threshold in the North, indicating that the North supplies fewer workers but more managers in the new equilibrium. This is because of two reasons. First, in order to ensure that the labor market clears for every skill level of workers, the North has to supply more managers with the South now supplying more workers. Second, since the

[10] In fact, researchers in public and labor economics have found that personal tax rates may affect workers' occupational choices (Gentry and Hubbard, 2004; Bohacek and Zubricky, 2012; Powell and Shan, 2012; Rothschild and Scheuer, 2013; Cubas and Silos, 2020; Silos and Cubas, 2017).



(a) Occupational Choice (b) Matching

Figure 3: Effects of a 20 percent decrease in the southern personal income tax rate on the occupation thresholds and the matching quality of workers in two countries, while keeping the corporate income tax rate in both countries and the personal income tax rate in the North fixed. The occupation thresholds separate the ability of the agents who choose to be workers or managers.

South supplies some highly skilled workers who were formerly managers, some highly skilled northern workers are matched with lower-skilled managers in the new integrated equilibrium. Given that the marginal value of workers' skill increases with the skill of the managers with whom they are matched, these highly skilled northern workers find it more profitable to switch from workers to managers.

Figure 3(b) presents the matching functions before and after a decrease in the southern workers' tax rate. The first key change is in the opportunities of the agents with $z_N^* < z < z^*$ in the North. Because the occupational ability threshold in the North decreases from z^* to z_N^*, some previously highly skilled workers now become managers of low-quality workers. Since the equilibrium exhibits positive sorting between workers and managers, a decline in z_N^* implies that matches of workers with sufficiently low skill necessarily become worse. The second key change results from the agents with $z^* < z < z_S^*$ in the South. Because of an increase in the southern occupational threshold, some previously low-quality managers now become the more productive workers in the economy. As a result, some highly skilled workers' matches suffer from the increasing competition of better southern workers who were previously managers. Hence, both low- and high-skilled workers experience worse matches

after a unilateral decrease in southern workers' personal income tax rates. [11]

There are two opposing forces for middle-skilled workers who remain workers in both countries. On one hand, they now have less competition, since some of their highly skilled competitors from the North, particularly the ones who were previously matched with the best managers, have become managers. On the other hand, they now face increasing competition, because some of the southern managers who were previously managing low-quality workers have become better workers. As a result, the joint effect on these workers' matches is ambiguous.

Proposition 6. *After a unilateral decrease in personal income taxation, low- and high-skilled workers are matched with worse managers, while low- and high-skilled managers are matched with better workers in both countries.*

Proposition 1 implies that a unilateral decrease in personal income taxation has a negative impact on the marginal return to the skill of low- and high-skilled workers (which is driven by the quality of the match). Such a policy intervention may not necessarily improve these workers' take-home salary. Besides, since the size of a firm is an increasing function of workers' quality, the second part of Proposition 1 implies that some surviving businesses will become larger, since some managers have improved their match. This leads to the following corollary.

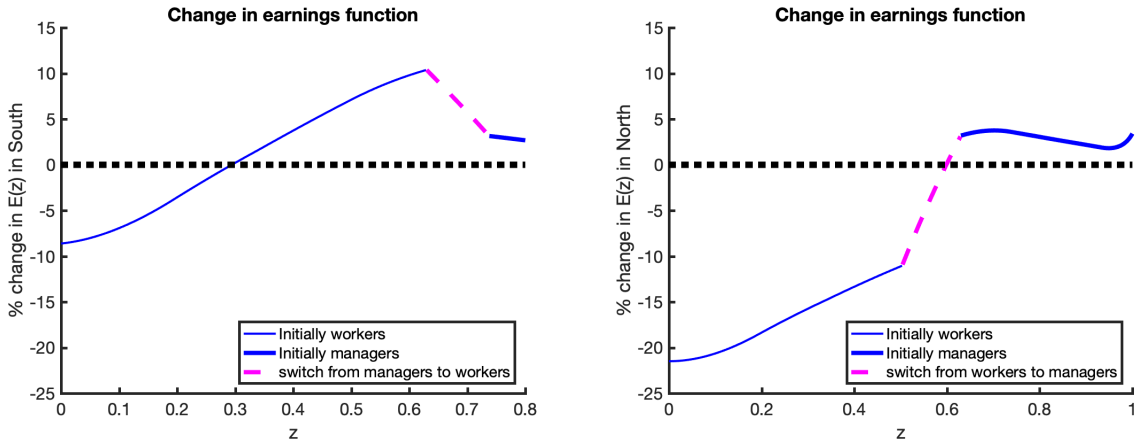
Corollary 1. *A unilateral decrease in personal income taxation increases the size of the least- and most-productive firms.*

5.2 Wages, Rents, and Income Inequality

The previous subsection focused on the implications of our theory for quantities. Corresponding to those quantities are equilibrium conditions for workers' wages and managerial earnings, as we have described in the previous section.

Figure 4(a) presents the impact of personal income tax deductions on take-home earn-

[11] In the Appendix, Figure A2 presents that a unilateral decrease in the *northern* workers' personal income tax implies worse matches of low- and high-ability workers in the integrated economy for the same reasons as above.



(a) South (b) North

Figure 4: Effects of a 20 percent decrease in the southern personal income tax rate on the changes in take-home wages of workers and take-home rents of managers in two countries.

ings in the South.[12] The equilibrium effect on low-skill workers' take-home wages can be decomposed into two components. First, the marginal return to skill of workers is driven by the quality of the match, which decreases for all low-skill workers. Second, as expected, the lump-sum transfer is higher than the tax collected from low-income workers. Hence, when the southern government collects a lower personal income tax rate, the amount of lump-sum transfer minus income taxes decreases for low-skill workers. As a result, a unilateral personal income tax deduction leads to lower take-home earnings for relatively unskilled southern workers. Besides, Figure 4(a) shows that the change in the take-home income function is increasing in workers' ability. This is mainly driven by the fact that total tax payments are proportional to wages, and thus high-pay workers benefit more from the personal income taxation deduction. Hence, as workers become more productive, their take-home income increases more significantly.

Figure 4(b) exhibits a relatively different pattern for wage levels in the North. It suggests that a unilateral decline in southern workers' personal income tax rates decreases the after-tax wages of northern workers at all skill levels. This is the result of two effects: first, the marginal return to skill of workers decreases for low-skill workers because they are matched with worse managers. Second, as we will later formalize in Section 6.1, the gross national

[12] In this paper, we define "after-tax" earnings as nominal earnings minus taxes, while "take-home" earnings mean after-tax earnings plus the lump-sum transfer.

product in the North becomes smaller when the South uses lower personal taxation to attract FDI inflow. This implies that the lump-sum transfer to the northern agents, which is proportional to the value of gross national product, will shrink. Hence, the wage level decreases for northern workers at all skill levels, especially the low-skill workers.[13]

Our model also allows us to derive some conclusions on managers' after-tax earnings. In the North, there are three forces. First, from Summary 1, we know that all northern workers' wages decrease, and so the managers' net profits increase. Second, from Proposition 1, we know that low- and high-skilled managers are matched with better workers, while middle-quality managers do not necessarily improve their match. This implies that the marginal return to skill of low and high-ability managers will increase even more. Third, when the South attracts more FDI inflow, the lump-sum transfer in the North becomes smaller. As a result, as shown in Figure 4(b), the tax cuts in the South *unambiguously* increase after-tax earnings of northern managers at all skill levels, especially the low- and high-skilled managers.[14]

The picture is more ambiguous for southern managers. On one hand, only some relatively unskilled workers' wages decrease, while others' wages increase; on the other hand, only the low- and high-skilled managers have better matches. As a result, since the worst managers of the economy come from the North (as suggested by $z_S^* > z_N^*$), even the least productive southern managers may not necessarily work with sufficiently low-skill workers whose wages become lower or have better matches. Hence, in Figure A4(a) in the appendix, we illustrate that, conditional on the values of parameters α and h, a unilateral tax cut in the South *sometimes* increases and sometimes decreases take-home earnings of southern managers. We summarize these findings below.

Summary 1. *A unilateral decrease in personal income taxation has the following effects on*

[13]Note that the North keeps the same personal income taxation policy. Hence, the tax benefit is not present for northern workers.

[14]We provide graphical illustrations of the changes in take-home rent in the North in Figure A3(d). Specifically, the dotted line 1 represents the take-home rent function without any policy interventions. The dashed line 2 is the after-tax rent function (with the same amount of lump-sum transfer as in line 1) after a unilateral personal tax deduction in South. The solid line 3 characterizes the take-home rent with (the adjusted lump-sum transfer) after the policy intervention. Figure A3(d) shows that the first two forces increase the rent function from line 1 to line 2, while the third force decreases the rent function from line 2 to line 3. In fact, we verify in Figure A4(b) that, for different combinations of α and h, the take-home rent of northern managers increases at all skill levels.

take-home earnings:

(i) It decreases the take-home wages of low-skill workers, but it increases the marginal return to skill of workers at all skill levels in both countries.

(ii) It unambiguously decreases workers' wages and increases managers' rents at all skill levels in the country that fails to cut taxes.

Research using individual-level data finds that within-occupation wage variation contributes at least as much as between-occupation variation to the overall level of earnings inequality in the United States, Germany, Sweden, and Brazil.[15] Hence, we use the Theil index of income inequality as a measure to explore how personal income taxation affects the distribution of earnings within and between occupations.[16]

Figure 5 plots the Theil index of inequality against the southern personal income tax, suggesting that the within-occupation component accounts for a substantial share of overall inequality. Besides, Figure 5 exhibits a negative correlation between within-occupation inequality and income tax. In fact, from Summary 1, we know that a fall in income taxation decreases low-skill workers' wages and increases the marginal return to skill of workers at all skill levels in both countries. As a result, within-worker wage inequality increases in response to a unilateral decline in personal income taxation.

The analysis of the impact of income taxation on between-occupation inequality follows quite directly from the changes in occupational choice thresholds. Lower southern personal income tax rates induce more workers and fewer managers in the South. Hence, as some

[15] See, e.g., Card, Heining, and Kline (2013) for Germany, Akerman et al. (2013) for Sweden, Helpman et al. (2016) for Brazil, Mouw and Kalleberg (2010) for the United States, and others.

[16] For each occupation $k = 1, ..., K$, the overall Theil index is

$$T = \frac{1}{N} \sum_k \sum_i \frac{y_{ik}}{\bar{y}} \ln \frac{y_{ik}}{\bar{y}}$$

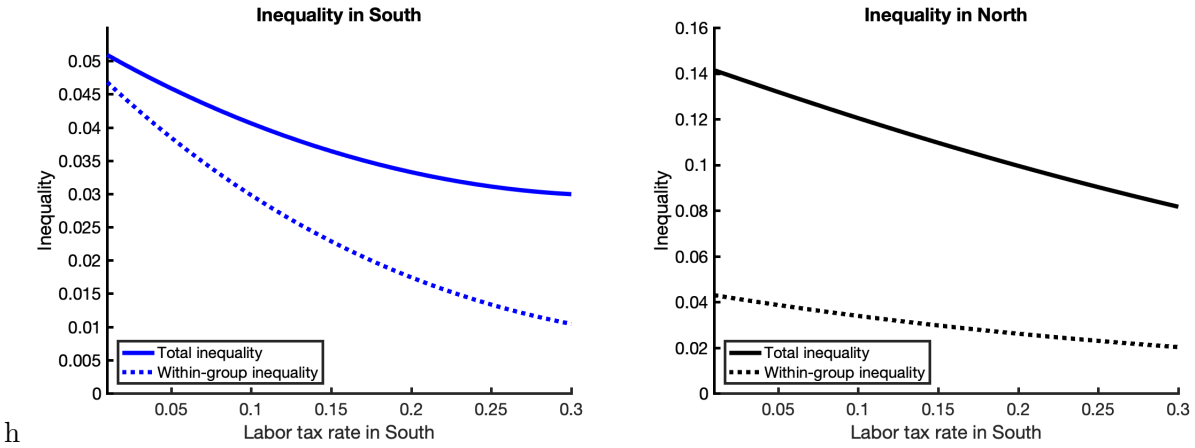
where y_{ik} is the income of worker i in occupation k, $N = \sum_k N_k$ and $\bar{y}$ is the mean income. We compute the "within occupation component" as

$$T_{within} = \sum_k s_k T_k$$

where $s_k = \frac{N_k \bar{y}_k}{N \bar{y}}$ is the income share of occupation k, and $T_k = \frac{1}{N_k} \sum_{i=1}^{N_k} (\frac{y_i}{\bar{y}_k} \ln \frac{y_i}{\bar{y}_k})$ is the Theil index of inequality for occupation k. Besides, the "between occupation component" as is

$$T_{between} = \sum_k s_k \ln \frac{\bar{y}_k}{\bar{y}},$$

so that $T = T_{within} + T_{between}$.



h

(a) a (b) h

Figure 5: Effects of a decrease in the southern personal income tax rate on the within- and between-occupation inequality in two countries.

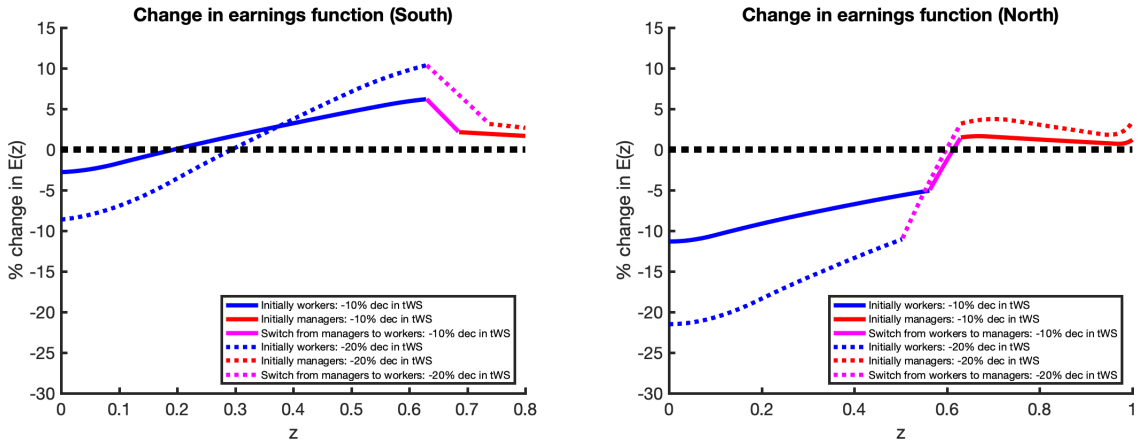
formerly managers now become workers, between-occupation inequality becomes smaller. Meanwhile, lower southern personal income tax decreases the occupational threshold in the North, indicating that some previously high-skilled workers now are managers, and thus, between-occupation inequality becomes larger. We summarize these results below:

Summary 2. *A unilateral decrease in southern personal income taxation has the following effects on income inequality:*

(i) It increases within-worker wage inequality in both countries.

(ii) It decreases between-occupation income inequality in the South, but increases between-occupation income inequality in the North.

Finally, we turn to compare the changes in take-home earnings when the southern income tax rate is decreased by 10% and 20% respectively. Figure 6 indicates that when the southern workers' income tax decreases from 10% to 20%, the wages of low-skill workers in both countries decrease even more for two reasons. First, lower personal income tax in the South induces more managers in the North, thus more previously high-skilled workers now become managers of low-quality workers. Since there exists positive sorting between workers and managers, the least productive workers experience worse matches. Second, the amount of lump-sum transfers decreases in both countries. In the South, as the South collects a lower personal income tax, the amount of lump-sum transfer minus income taxes decreases for low-



(a) South (b) North

Figure 6: Effects of a decrease in the southern personal income tax rate on the take-home wages of workers and on the take-home rents of managers in two countries, depending on different southern personal income tax levels.

skilled southern workers. In the North, as the multinationals move their production sites from the North to the South, the gross national product and thus the lump-sum transfer to the northern citizens become smaller. As a result, a further decline in southern personal income tax has a more significant negative impact on the take-home wages of low-skill workers in both countries.

It follows that the same logic that the northern managers benefit more from lower wages, and some low- and high-skilled managers are matched with even better workers in response to a further decline in the southern workers' income tax. Hence, as shown in Figure 6(b), managers in the North become better off. Meanwhile, if a unilateral decline in the southern workers' personal income tax has a positive effect on southern managers' take-home earnings, a further decline in personal income tax increases the southern managers' rents even more; see Figure 6(a). Otherwise, the southern managers' rents decrease more significantly, as shown in Figure A5(a) in the appendix.

6 Planners' Problems

In this section, we analyze the effects of changes in personal income taxation policy on aggregate production, tax revenue, aggregate consumption, and the determination of an

optimal tax policy. Furthermore, we investigate the implications of changes in skill overlap (α) and communication costs (h) on the optimal tax policy. We conclude by introducing two alternative frameworks to assess the importance of two margins of adjustment: one where personal income taxation has no impact on the inward and outward activity of multinationals, and another where it has no effect on the matching quality between workers and managers.

6.1 Production, Tax Revenue, and Consumption

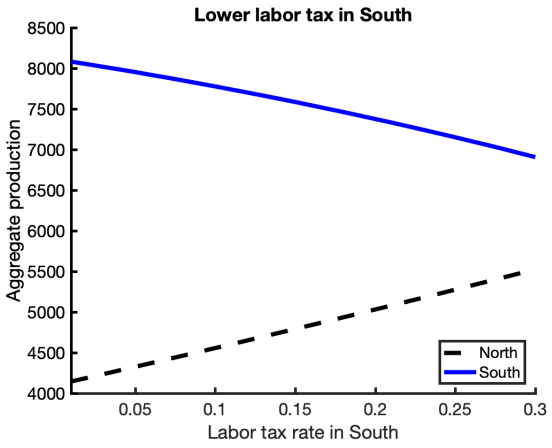
Note that a worker of ability z works for a manager with ability $m(z)$. This means that the total output produced by this worker is given by $m(z)$. Therefore, the total physical output of a country i is given by

$$Y_i = \int_0^{z_i^*} m(z)g(z)dz.$$

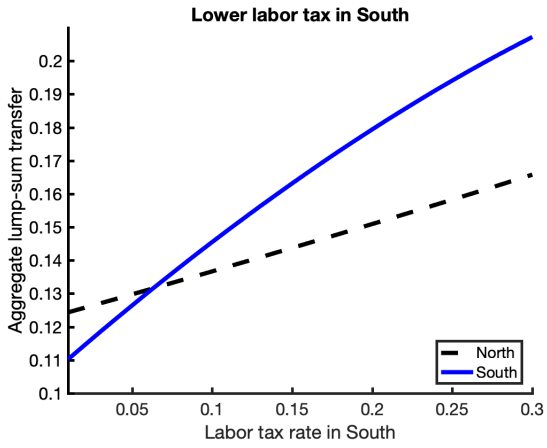
Hence, when the tax on southern workers becomes lower, the South supplies more workers while the North supplies more managers. Since the managers are mobile and workers are immobile, some Northern managers move to the South to conduct production. As a result, as illustrated in Figure 7(a), lower personal income tax in the South leads to an increase in the total physical output in the South but a decrease in physical output in the North.

In Figure 7(b), we are interested in studying the effect of personal income tax policy on the total tax revenue, which is rebated to domestic residents through lump-sum transfers, in two countries. Recall that this paper focuses on a territorial tax system where managers pay corporate income tax to the country where they conduct production regardless of managers' nationality. Hence, when the South collects a lower personal income tax, more northern managers move to the South, and thus the total tax revenue decreases in the North. Besides, the effect of tax policy on southern tax revenue is the result of two effects: on one hand, lower personal income tax in South attracts more FDI inflow and thus the southern corporate income tax revenue increases. On the other hand, lower personal income tax implies lower tax collected from southern workers. Figure 7(b) suggests that the second effect is always larger than the first effect because the South features net exports of "workers."

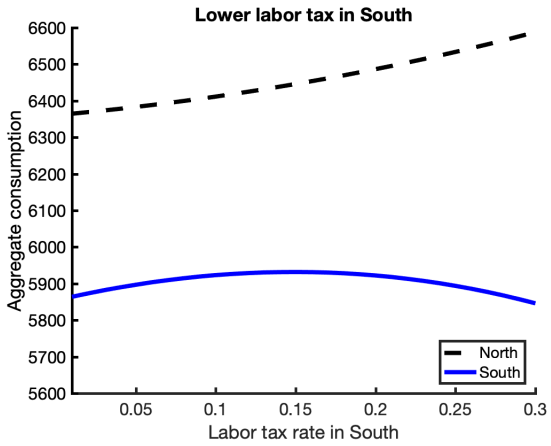
In Figure 7(c), we study the effect on aggregate consumption. It follows the assumption in Antras, Garicano, and Rossi-Hansberg (2006) that managers consume in their own country,



(a) production



(b) lump-sum transfer



(c) consumption/welfare

Figure 7: Effects of a decrease in the southern personal income tax rate on the aggregate production, total tax revenue, and aggregate consumption in two countries.

and thus managers' after-tax rents have to be repatriated. That is, we define a country's aggregate consumption as the amount of take-home earnings of its citizens. Since utility is linear with respect to the consumption of homogeneous outputs, so does welfare. In the North, northern managers have to pay corporate income tax to the southern government if they work with workers abroad. Hence, when the South collects a lower personal income tax, the total tax revenue that is rebated to the northern citizens as a lump-sum transfer becomes lower, and so does the aggregate consumption in the North.

In the South, the equilibrium effect on the aggregate consumption can again be decomposed in two. First, lower personal income tax attracts more FDI inflow, so the total tax revenue and thus the aggregate consumption increases in the South. Second, from Proposition 1 we know that a unilateral decrease in personal income taxation has a negative impact on the marginal return to skill of low-skilled workers because these workers now work with worse managers. In Figure 7(c), we show that the first offshoring effect dominates the second complementarity effect with the personal income tax above a certain threshold, and so the aggregate consumption in the South is concave in the southern personal income tax.

Summary 3. *A unilateral decrease in southern personal income taxation has the following effects on physical production, tax revenue, and consumption:*

(i) It increases physical production in the South and decreases physical production in the North;

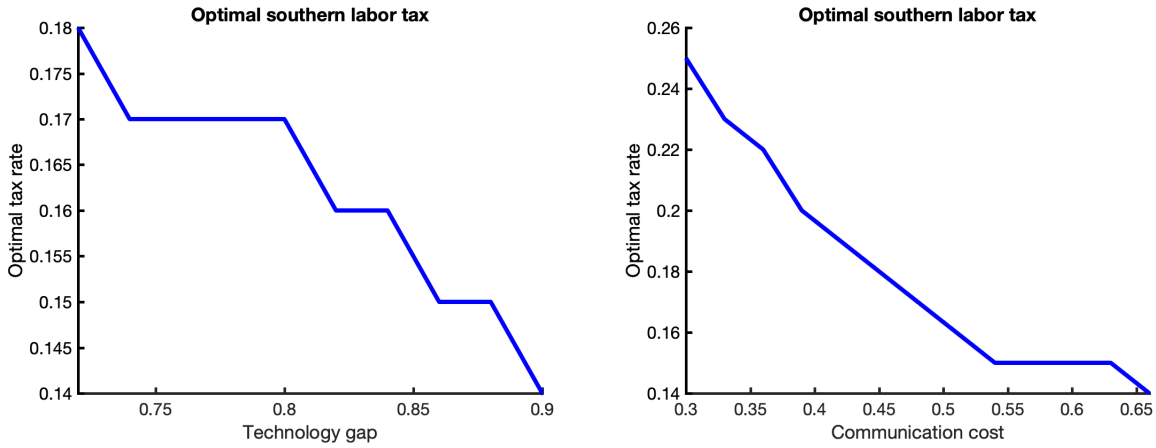
(ii) It increases tax revenue (and thus lump-sum transfer) in the South and decreases tax revenue (and thus lump-sum transfer) in the North;

(iii) It increases aggregate consumption (and thus welfare) in the South with the southern personal income tax above a threshold but decreases it for the rest;

(iv) It decreases aggregate consumption (and thus welfare) in the North.

6.2 Optimal Policy

To explore the optimal southern personal income taxation policy where the maximum amount of southern aggregate consumption is achieved, we first note that the optimal southern income tax rate, at which the offshoring effect and complementarity effect are identical,



(a) a (b) h

Figure 8: The optimal southern personal income tax rate depending on different parameters.

is a function of the parameters α and h. First, the complementarity effect decreases with α and increases with h. An increase in α means that agents in the South become relatively more skilled. This implies that low-skill workers are matched with better managers, and thus the complementarity effect becomes smaller. A fall in h indicates that managers have access to a communication technology that allows them to deal with a larger team. It follows that the worst workers are matched with much better managers, and thus the complementarity effect is smaller. Second, the offshoring effect decreases with both α and h. As α increases, southern agents become more skilled, and thus the number of workers in the South shrinks. As a result, the North exports fewer knowledge services, and the offshoring effect becomes weaker. Meanwhile, a fall in h leads to an increase in the size of all firms in the economy. So, as the northern managers are expanding their businesses and hiring more workers, the offshoring effect becomes stronger. The overall effect of α and h on the optimal tax policy results from the combination of these two effects. In Figure 8, we show that the optimal southern income tax decreases with α and h, respectively.

Summary 4. *An increase in the skill overlap (α) has the following effects on the optimal tax policy: (i) It improves the match for low-skill workers, so the loss from the complementarity effect decreases; (ii) It decreases the number of workers in the South, so the gains from the offshoring effect decrease; (iii) It decreases the optimal personal income tax rate in the South.*

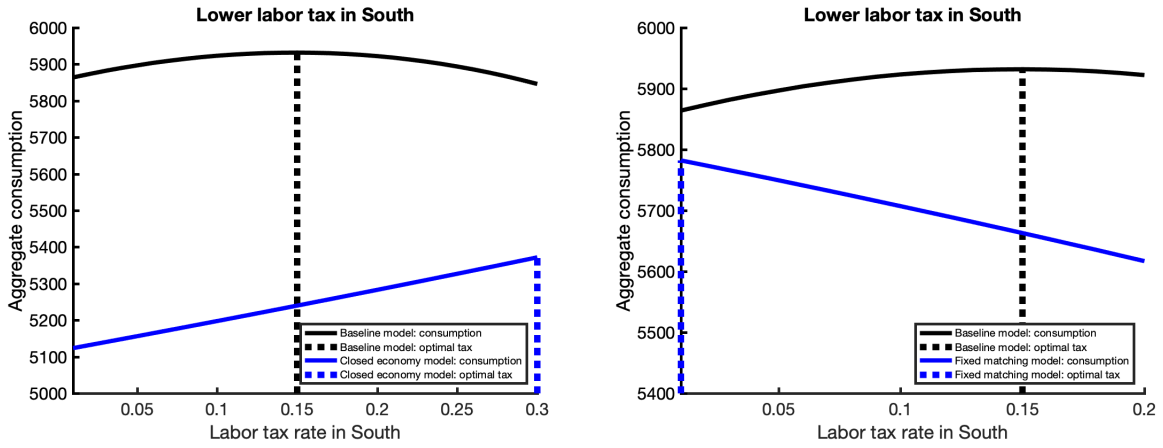
A decrease in communication costs (h) has the following effects on the optimal tax policy: (i) It improves the match for low-skill workers, so the loss from the complementarity effect decreases; (ii) It increases the size of all firms, so the gains from the offshoring effect increase; (iii) It increases the optimal personal income tax rate in the South.

6.3 Alternative Frameworks

Our study draws attention to two important facts: first, personal income taxation affects the inward and outward activity of multinational firms; second, there exists endogenous matching between heterogeneous workers and heterogeneous managers. We conclude by considering two alternative frameworks to assess the importance of these two margins of adjustment.

First, we consider a "closed-economy" scenario where personal income taxation does NOT affect the inflow and outflow of multinational activities. Specifically, we allow personal taxation to affect occupational choices but assume both managers and workers are immobile. Then, under the assumption that the corporate income tax rate equals 30% in the South, we plot the aggregate consumption (and thus welfare) against the southern personal income tax rate in Figure 9(a). Figure 9(a) suggests that the maximum amount of southern aggregate consumption is achieved when the personal tax rate is equal to the corporate income tax. Intuitively, since managers no longer work abroad, the gains from the offshoring effect are not present in a closed-economy framework. Meanwhile, from Proposition 1, we know there exists a mismatch between workers and managers when the government collects different personal and corporate income taxes. Hence, in a closed-economy model where multinationals are not present, a market is efficient when the government collects the same amount of income tax from workers and managers. In other words, a closed-economy framework overlooks the gains from FDI inflows and thus over-predicts the optimal income tax level.

Second, we consider a "fixed-matching" scenario where personal income taxation does NOT affect the matching quality between workers and managers. To do so, we first recover the measures of workers and managers (and thus the matching function between workers and managers) when personal and corporate income tax rates are equal to 30% in both countries. Then, taking the matching function as exogenously given, we predict the optimal tax policy



(a) consumption (b) production

Figure 9: The optimal southern personal income tax rate depending on different frameworks. Personal income tax does not affect the location choices of multinationals in the closed-economy framework, while it does not affect the matching quality of workers and managers in the fixed-matching framework.

in the South when the southern personal income tax rate is decreased from 30% to 10%. Figure 9(b) indicates that a lower southern personal income tax results in higher aggregate consumption (and thus higher welfare). Intuitively, a lower personal income tax attracts more FDI inflow in an integrated economy. Hence, the gains from the offshoring effect become significant in response to a unilateral decline in domestic personal income taxation. Meanwhile, under the assumption that the matching between workers and managers is fixed, a unilateral decrease in personal income taxation no longer causes market failure due to mismatching between agents. As a result, a fixed-matching scenario underestimates the loss from the complementarity effect, leading to under-predictions of the optimal income tax rate.

7 Conclusion

This paper examines the complex interplay between personal income taxation, cross-border team formation, and income distribution in a globalized economy. By incorporating endogenous matching between heterogeneous agents and cross-border dynamics, we provide a framework to analyze how tax policies influence occupational choices, the quality of labor matching, and multinational firms' sourcing decisions.

Our findings highlight two significant mechanisms: (1) the impact of personal income taxation on the allocation of skilled managers across borders, which shapes the formation of international teams, and (2) the endogenous matching between workers and managers, which directly affects wage inequality. A unilateral reduction in personal income tax rates in a developing region attracts foreign managers, boosting FDI inflows and tax revenues but also increasing within-occupation wage inequality and reducing take-home earnings for less-skilled workers.

From a policy perspective, we demonstrate that a concave relationship exists between the personal income tax rate and level of social welfare within a country. This suggests the existence of an optimal tax rate that balances the benefits of FDI inflows with the adverse effects on worker matching and income distribution. Furthermore, by comparing alternative scenarios, we show the importance of accounting for cross-border dynamics and endogenous matching when designing tax policies. Ignoring these factors either overestimates or underestimates the optimal tax rate, potentially leading to suboptimal outcomes.

In conclusion, our work underscores the necessity of integrating globalized labor market dynamics into the study of optimal income taxation. Future work could extend this analysis by exploring the implications of alternative tax systems, such as progressive taxation or global minimum tax agreements, to better address the challenges and opportunities presented by international labor mobility and multinational firm activities.

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A Additional tables and figures

Table A1: 37 countries in the data

ISO-3 Code	Country	ISO-3 Code	Country	ISO-3 Code	Country
AUS	Australia	HUN	Hungary	NZL	New Zealand
AUT	Austria	ISL	Iceland	NOR	Norway
BEL	Belgium	IRL	Ireland	POL	Poland
CAN	Canada	ISR	Israel 1	PRT	Portugal
CHL	Chile	ITA	Italy	SVK	Slovak Republic
COL	Colombia	JPN	Japan	SVN	Slovenia
CZE	Czech Republic	KOR	Korea	ESP	Spain
DNK	Denmark	LVA	Latvia	SWE	Sweden
EST	Estonia	LTU	Lithuania	CHE	Switzerland
FIN	Finland	LUX	Luxembourg	TUR	Turkey
FRA	France	MEX	Mexico	GBR	United Kingdom
DEU	Germany	NLD	Netherlands	USA	United States
GRC	Greece				

Table A2: 34 industries in the data

Industry Code	Industry	ISIC Rev.4
A	Agriculture, forestry and fishing	01, 02, 03
B	Mining and extraction of energy producing products	05, 06, 07, 08, 09
C10T12	Food products, beverages and tobacco	10, 11, 12
C13T15	Textiles, wearing apparel, leather and related products	13, 14, 15
C16	Wood and products of wood and cork	16
C17T18	Paper products and printing	17, 18
C19	Coke and refined petroleum products	19
C20T21	Chemicals and pharmaceutical products	20, 21
C22	Rubber and plastic products	22
C23	Other non-metallic mineral products	23
C24	Basic metals	24
C25	Fabricated metal products	25
C26	Computer, electronic and optical products	26
C27	Electrical equipment	27
C28	Machinery and equipment, nec	28
C29	Motor vehicles, trailers and semi-trailers	29
C30	Other transport equipment	30
C31T33	Other manufacturing; repair and installation of machinery and equipment	31, 32, 33
DTE	Electricity, gas, water supply, sewerage, waste and remediation services	35,36, 37, 38, 39
F	Construction	41, 42, 43
G	Wholesale and retail trade; repair of motor vehicles	45, 46, 47
H	Transportation and storage	49, 50, 51, 52, 53
I	Accommodation and food services	55, 56
J58T60	Publishing, audiovisual and broadcasting activities	58, 59, 60
J61	Telecommunications	61
J62T63	IT and other information services	62, 63
K	Financial and insurance activities	64, 65, 66
L	Real estate activities	68
MTN	Other business sector services	69, 70-75, 77, 78-82
O	Public admin. and defence; compulsory social security	84
P	Education	85
Q	Human health and social work	86, 87, 88
RTS	Arts, entertainment, recreation and other service activities	90, 91, 92, 93,94,95, 96
T	Private households with employed persons	97, 98

Table A3: Robustness Checks on the Impact of Personal Income Tax Rate on Multinational Output

VARIABLES	Log of Multinational Output (millions of US dollars)							
	All Sample				Originality is not Destination			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Log of (1 - all-in tax 67)	0.085				-0.185			
	(0.379)				(0.374)			
Log of (1 - all-in tax 100)		1.998***				1.614***		
		(0.418)				(0.413)		
Log of (1 - all-in tax 133)			1.301***				0.966***	
			(0.320)				(0.317)	
Log of (1 - all-in tax 167)				1.504***				1.208***
				(0.293)				(0.289)
Log of (1-Statutory Corporate Income Tax Rate)	0.330	-0.0150	0.139	0.0834	0.401	0.0665	0.204	0.147
	(0.279)	(0.278)	(0.273)	(0.274)	(0.277)	(0.275)	(0.271)	(0.272)
Control Variables	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Host and Source Country Fixed Effect	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Industry Fixed Effect	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Year Effect	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Observations	827,676	827,676	827,676	827,676	812,580	812,580	812,580	812,580
R^2	0.413	0.413	0.413	0.413	0.387	0.387	0.387	0.387

Robust standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

Notes: Columns (1)–(4) present the regression results for the full sample, whereas columns (5)–(8) report the results for the partial sample, which excludes observations related to domestic production. The key explanatory variable, all-in tax, is measured as the average personal income tax rate (all-in). It is defined as the ratio of taxes paid by a single worker at different income levels (67%, 100%, 133%, and 167% of average earnings) to the corresponding earnings for the employer in each host country in a given year. The set of control variables includes the logarithm of GDP for both the source and host countries, indicators for whether the host and source countries share a border, whether they have a common official or primary language, whether they share a language spoken by at least 9% of the population, whether they share a common religion, whether they use a common currency, and the logarithm of the weighted distance (in kilometers) between the host and source countries. Additionally, we incorporate host country fixed effects, source country fixed effects, industry fixed effects, and year effects to account for unobserved heterogeneity. Robust standard errors are reported in parentheses. *** p<0.01, ** p<0.05, * p<0.1.

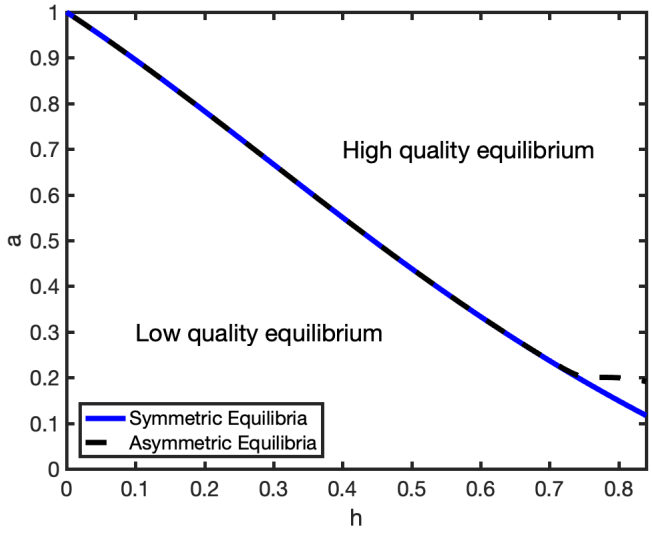


Figure A1: Types of Equilibria

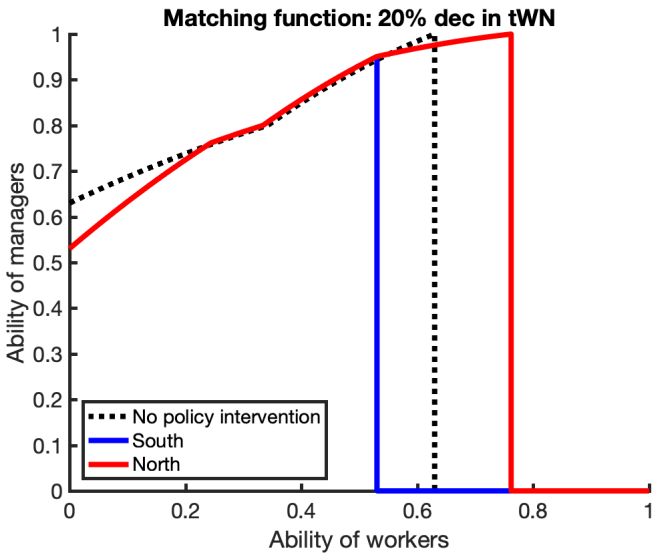


Figure A2: Effects of a 20 percent decrease in the *northern* personal income tax rate on the matching quality of workers in two countries.

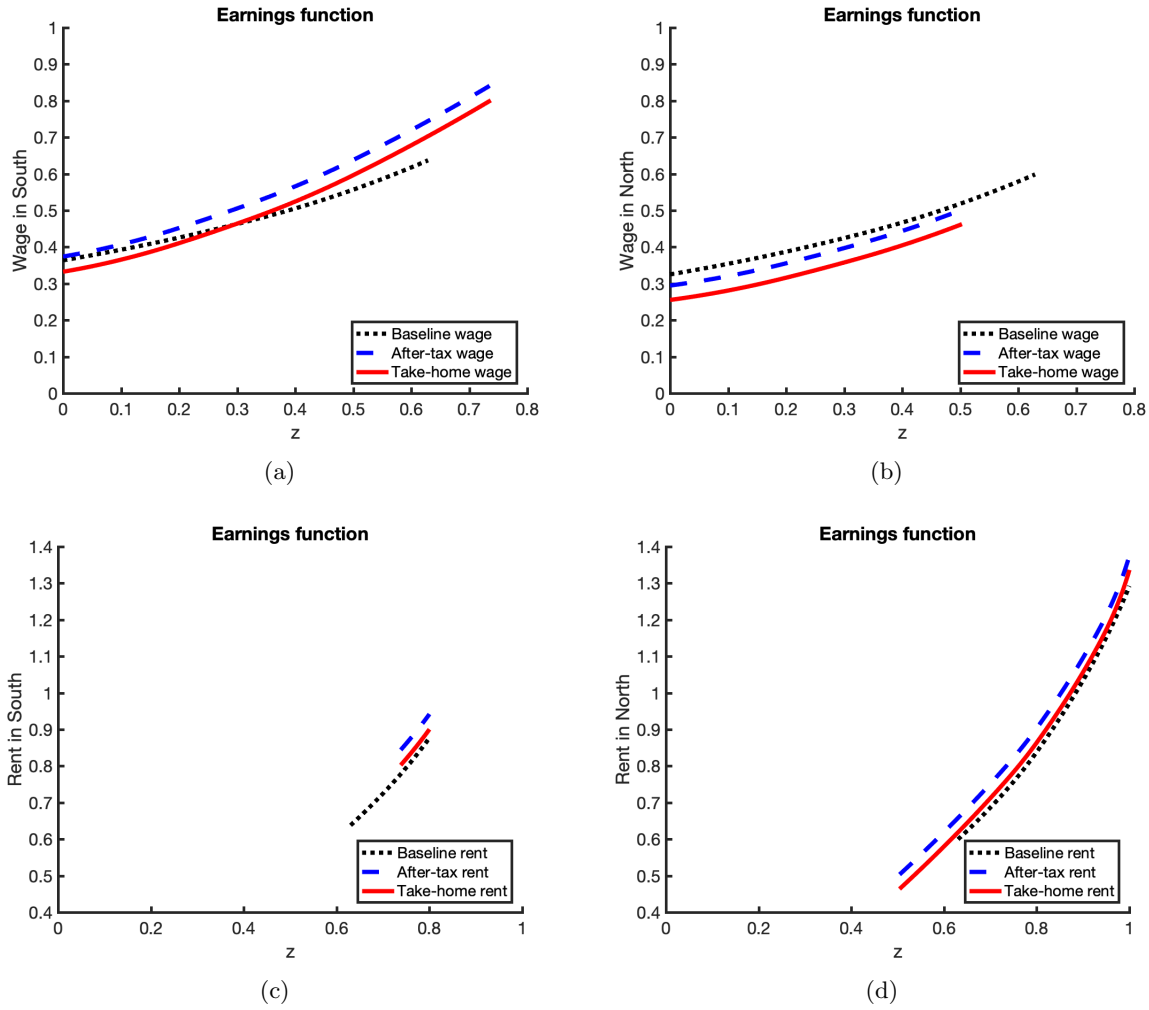
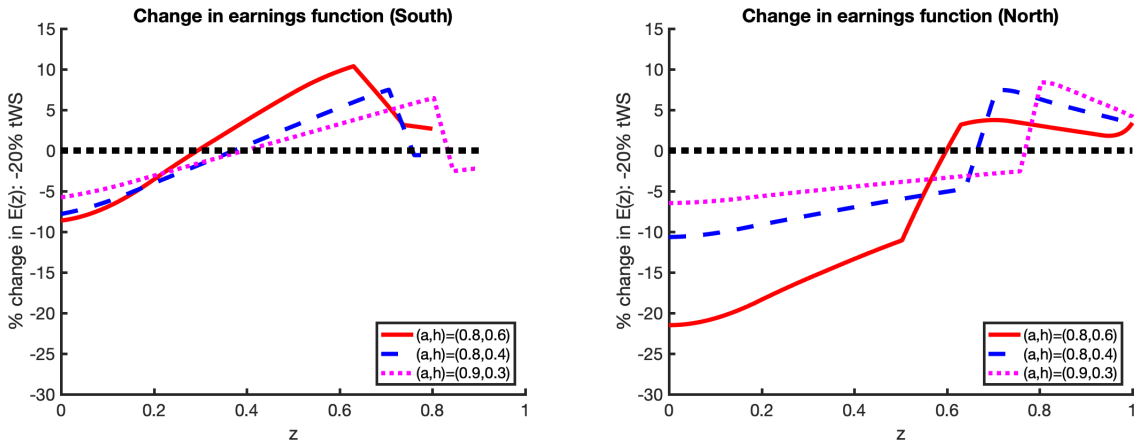
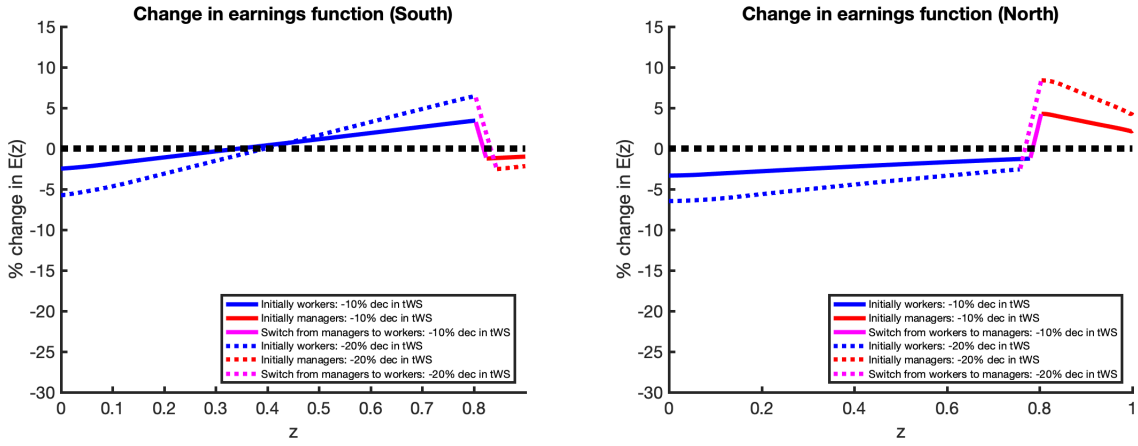


Figure A3: Effects of a 20 percent decrease in the southern personal income tax rate on the take-home wages of workers and on the take-home rents of managers in two countries.



(a) South (b) North

Figure A4: Effects of a 20 percent decrease in the southern personal income tax rate on the take-home wages of workers and on the take-home rents of managers in two countries, depending on different combinations of parameters.



(a) South (b) North

Figure A5: Effects of a decrease in the southern personal income tax rate on the take-home wages of workers and on the take-home rents of managers in two countries, depending on different southern personal income tax levels.

B Proof

B.1 Proof of Proposition 1

To demonstrate that each manager hires only one worker type, we will first establish that a manager cannot be matched with an interval of workers.

Lemma 1. *A particular manager cannot be matched with an interval of workers.*

Proof. Suppose by contradiction that the matching function is such that for some interval with positive Lebesgue measure F, $M'(z_p) = 0, z_p \in F$. Then, from the labor market clearing condition,

$$\int_F g(z)dz = 0.$$

However, this results in a contradiction, and the contradiction arises from the constraints imposed by measurability. □

Next, we turn to the problem of a manager who hires the set of workers $A = \{z_1, ..., z_m\}$ as follows

$$r(z_m) = \max_{n,z} \sum_{j=1}^{m} n_j(z_m - w(z_j)), \tag{7}$$

$$s.t.\ 1 = h\sum_{j}^{m}(1 - z_j)n_j.$$

The first order conditions must be hold for all j:

$$\frac{z_m - w(z_j)}{h(1 - z_j)} = \lambda,$$

$$w'(z_j) = \lambda h.$$

Lemma 2. *A particular worker type may not be matched with several different managers.*

Proof. Suppose by contradiction that a particular worker type z_p is matched with two manager types, z_m and z'_m, with $z_m < z'_m$. Then, we have

$$\frac{\partial w(z_p)}{\partial z_p} = \frac{z_m - w(z_p)}{1 - z_p},$$

and

$$\frac{\partial w(z_p)}{\partial z_p} = \frac{z'_m - w(z_p)}{1 - z_p},$$

However, this leads to a contradiction, as

$$\frac{z_m - w(z_p)}{1 - z_p} < \frac{z'_m - w(z_p)}{1 - z_p}.$$

Consequently, we conclude that a particular worker type is not matched with different managers. □

Therefore, the matching function $m(z)$ is a single-value function, resulting in each manager z_m being matched with a specific worker type z_j.

Lemma 3. *The matching function $m(z)$ is continuous.*

Proof. The solution to a manager's problem (7) exists for all z_m since it is the maximization of a continuous function over a compact set. Thus, the maximizers z_j and n_j also vary continuously with z_m. □

Lemma 4. *Each manager hires only one worker type.*

Proof. Suppose by contradiction that a manager hires two worker types, z_1 and z_2. The first order condition of the manager's problem yields

$$\frac{w(z_1) - w(z_2)}{z_1 - z_2} = \lambda h.$$

Hence, the manager is indifferent in hiring any worker $z \in [z_1, z_2]$ if wages are such that

$$\tilde{w}(z) = w(z_1) + \lambda h(z - z_1).$$

Since the manager is hiring both z_1 and z_2, we know that

$$w(z_2) = \tilde{w}(z_2) = w(z_1) + \lambda h(z - z_1).$$

For the wage function to be consistent, it must be greater than $\tilde{w}(z)$ for all z in the

interval $[z_1, z_2]$. If it were smaller, the manager would prefer to hire any worker within that interval. If it were equal, the manager would hire all of them. Therefore,

$$w(z) > \tilde{w}(z) \text{ for all } z \in [z_1, z_2].$$

Since $w(z_2) = \tilde{w}(z_2)$, the slope of the wage function must be higher than λh for some interval H and lower for some other interval L. Now consider a manager z_m^H hiring workers from interval H, and let the slope of the wage function at the ability of the workers z_m^H hires, denoted as z_p^H, be given by $s > \lambda h$. Then, z_m^H is indifferent in hiring workers with different abilities if wages are such that

$$\tilde{w}^H(z) = w(z_p^H) + s(z - z_p^H),$$

which implies that

$$\tilde{w}^H(z_2) > w(z_2) = \tilde{w}(z_2)$$

since $w(z_p^H) > \tilde{w}(z_p^H)$. Hence z_m^H would have an incentive to hire the workers of type z_2, and offer them a wage of $w(z_2) + \epsilon$ for ϵ small enough. This contradicts the assertion that z_m hires workers of type z_2. Thus, we conclude that managers hire only one type of worker. □

B.2 Proof of Proposition 2

Proof. We begin by assuming perfect mobility of workers in their respective countries, which implies that for all $i \in \{S, N\}$ and $z \in [a, b]$ with $a \leq b$, we have:

$$(1 - t_i^W) w_i^S(z) = (1 - t_i^W) w_i^N(z),$$

or

$$w_i^S(z) = w_i^N(z).$$

This indicates that workers' wages are independent of the nationality of their matched managers. If not, we assume that $w_S^S(z_p) > w_N^S(z_p)$ for some arbitrary $z_p \in [a, b]$, it follows that

$m_N^S(z_p)$ is not a real-valued function on the interval $[a, b]$, as no workers are willing to work for North managers with ability $m_N^S(z_p)$.

We next to show that

$$m_i^S(z) = m_i^N(z) \text{ for all } i \in \{S, N\} \text{ and } z \in [a, b], \text{ where } a \leq b.$$

Again, by contradiction, let's assume that there exists a $z_p \in (a, b)$ such that $m_N^N(z_p) \neq m_N^S(z_p)$, and both $m_N^N(z_p)$ and $m_N^S(z_p)$ are real-valued on the interval of $[a, b]$, implying that managers from both the South and the North establish firms in the North in the equilibrium.

The manager's rent-maximization problem yields:

$$w_N^{N'}(z_p) = \frac{m_N^N(z_p) - w_N^N(z_p)}{1 - z_p},$$
$$w_N^{S'}(z_p) = \frac{m_N^S(z_p) - w_N^S(z_p)}{1 - z_p}.$$

Utilizing the definition of differentiability, we can derive:

$$\begin{aligned} w_N^{N'}(z_p) &= \lim_{z \to z_p} \frac{w_N^N(z_p) - w_N^N(z)}{z_p - z} \\ &= \lim_{z \to z_p} \frac{w_N^S(z_p) - w_N^S(z)}{z_p - z} \\ &= w_N^{S'}(z_p). \end{aligned}$$

where the second equality comes from the perfect labor mobility assumption.

Consequently, we have:

$$\begin{aligned} & w_N^{N'}(z_p) = w_N^{S'}(z_p) \\ \Leftrightarrow & \frac{m_N^N(z_p) - w_N^N(z_p)}{h(1 - z_p)} = \frac{m_N^S(z_p) - w_N^S(z_p)}{h(1 - z_p)} \\ \Leftrightarrow & m_N^N(z_p) = m_N^S(z_p) \end{aligned}$$

Symmetrically, we can conclude that

$$m_S^N(z_p) = m_S^S(z_p).$$

□

B.3 Proof of Proposition 3

Proof. We first demonstrate that if an equilibrium exists, it exhibits positive sorting.

Lemma 5. *The equilibrium exhibits positive sorting.*

Proof. We denote $\Pi(z_m, z_p)$ as the rent earned by a manager with ability z_m who hires workers with ability z_p. Since managers optimally choose the ability of their workers, it follows that

$$\frac{\partial \Pi(z_m, z_p)}{\partial z_p} = 0.$$

Differentiating this expression yields:

$$\frac{\partial z_m}{\partial z_p} = -\frac{\partial^2 \Pi(z_m, z_p)/\partial z_p^2}{\partial^2 \Pi(z_m, z_p)/\partial z_p \partial z_m}.$$

The numerator is negative because managers maximize their rents in equilibrium. To show that the denominator is positive, notice that

$$\frac{\partial^2 \Pi(z_m, z_p)}{\partial z_p \partial z_m} = \frac{\partial (1/h(1-z_p))}{\partial z_p} = \frac{1}{h(1-z_p)^2} > 0.$$

Hence, we conclude that

$$\frac{\partial z_m}{\partial z_p} > 0.$$

This argument holds for all workers, leading us to the conclusion that in an equilibrium allocation, $m'(z) > 0$ for all workers with ability z. □

Lemma 6. *The earnings function is strictly convex.*

Proof. To show that the earnings functions is strictly convex, first notice that the wage function is convex:

$$w''(z_p) = h\frac{g(z_p)}{g(m(z_p))} > 0.$$

Meanwhile, the rent function is strictly convex:

$$r''(z_m) = \frac{(m^{-1})'(z_m)}{h(1-m^{-1}(z_m))^2} > 0,$$

where the inequality results from positive sorting. □

Let $W(w)$ and $M(w)$ be the equilibrium set of agents who become workers and managers, respectively. We next show that the set of managers and the set of workers are connected intervals in equilibrium.

Lemma 7. *$M(w)$ and $W(w)$ are connected intervals.*

Proof. First, suppose by contradiction that $M(w) = [a, b] \cup [c, d]$ with $b < c$. Positive sorting implies that $m_S^{-1}(b) = m_S^{-1}(c) \equiv z_1$. Managers' rents are given by

$$r(b) = \frac{b - w(z_1)}{h(1-z_1)},$$
$$r(c) = \frac{c - w(z_1)}{h(1-z_1)}.$$

Denote $e = \frac{1}{2}b + \frac{1}{2}c$, and it follows that

$$r(e) = \frac{e - w(z_1)}{h(1-z_1)} = \frac{1}{2}r(b) + \frac{1}{2}r(c). \tag{8}$$

Meanwhile, from Lemma (6) we know that the rent function is strictly convex:

$$r(\frac{1}{2}b + \frac{1}{2}c) < \frac{1}{2}r(b) + \frac{1}{2}r(c).$$

This contradicts condition (8). Hence, we conclude that the set of managers is connected.

Next, we want to show that $W(w)$ is connected. Suppose that is it not. In particular, suppose that $W(w) = [a, b] \cup [c, d]$ with $b < c$. Positive sorting implies that $m(b) = m(c) \equiv z_2$

with $b \neq c$. This contradicts Proposition 1, which states that each manager hires only one worker type. Hence, $W(w)$ is a connected interval of the form $[0, z^*]$. □

Therefore, there exists a unique allocation that (i) satisfies two differential equations (2) and (5), (ii) satisfies two boundary conditions: $m(0) = z^*$, and $m(z^*) = \bar{z}$, and (iii) yields a continuous and strictly convex earnings function. The final step is to prove that such an allocation is in fact an equilibrium. For this we need to prove that there exists an $h^* > 0$ such that a manager with ability $\bar{z}$ has no incentive to hire a worker with ability $z^* + \epsilon$, that is, $\lim_{\epsilon \to 0} \frac{\partial \Pi(\bar{z}, z^* + \epsilon)}{\partial \epsilon} < 0$.

Notice that the profits of managers with ability $\bar{z}$ are given by

$$\Pi(\bar{z}, z^* + \epsilon) = \frac{\bar{z} - w(z^* + \epsilon)}{h(1 - (z^* + \epsilon))}$$

so,

$$\lim_{\epsilon \to 0} \frac{\partial \Pi(\bar{z}, z^* + \epsilon)}{\partial \epsilon} = \frac{-r'(z^*)}{h(1 - z^*)} + \frac{\bar{z} - r(z^*)}{h(1 - z^*)^2}.$$

Since $r'(z^*) = \frac{1}{h}$ and $r(z^*) = \frac{z^* - w(0)}{h}$, then $\lim_{\epsilon \to 0} \frac{\partial \Pi(\bar{z}, z^* + \epsilon)}{\partial \epsilon} < 0$ is equivalent to

$$h\bar{z} + w(0) < 1.$$

Hence, since $w(0) < 1$ and $\bar{z} < 1$, this implies that there exists an $h^* > 0$ such that for all $h < h^*$, we have $\lim_{\epsilon \to 0} \frac{\partial \Pi(\bar{z}, z^* + \epsilon)}{\partial \epsilon} < 0$. Therefore, for all $h < h^*$, there exists a unique competitive equilibrium in this economy. Given that markets are complete and competitive, this implies that the equilibrium allocation in the economy is Pareto optimal. □

B.4 Proof of Proposition 4

Proof of Proposition 4(1). $M_S \cap M_N = [a, b]$ implies that for any manager with ability $c \in [a, b]$, they receive the same after-tax payoff when building teams in both countries. Mathematically, this can be expressed as:

$$(1 - t_S^M) r_S(c) = (1 - t_N^M) r_N(c)$$

Since $r_S(.)$ and $r_N(.)$ are continuous functions, both functions are differentiable at interior points. We can rewrite this relationship as:

$$r'_S(c) = \frac{r_S(c) - r_S(c-\epsilon)}{\epsilon} = \frac{1-t_N^M}{1-t_S^M}\frac{r_N(c) - r_N(c-\epsilon)}{\epsilon} = \frac{1-t_N^M}{1-t_S^M}r'_N(c)$$

Using the envelope condition, the marginal return to skill for managers is given by

$$r'_i(z) = \frac{1}{h(1-m_i^{-1}(z))}$$

Hence, we can establish the equation:

$$\frac{1-t_N^M}{1-t_S^M} = \frac{r'_S(c)}{r'_N(c)} = \frac{1-m_N^{-1}(c)}{1-m_S^{-1}(c)}$$

Given that t_S^M, t_N^M, m_N^{-1}, and m_S^{-1} are all within the range (0, 1), we can deduce:

$$\frac{t_N^M}{t_S^M} = \frac{m_N^{-1}(c)}{m_S^{-1}(c)}$$

Let us denote the constant as $M \equiv \frac{t_N^M}{t_S^M}$. In equilibrium, $m_i^{-1}(z)$ can be expressed in a form of $m_i^{-1}(z) = \sqrt{h^2 + 2hk(c-z)}$ where k and c are some constants derived from the matching function $m(.)$ and are independent of z. Thus, we can rewrite the equation as

$$M \equiv \frac{t_N^M}{t_S^M} = \frac{m_N^{-1}(c)}{m_S^{-1}(c)} = \frac{\sqrt{h^2 + 2hk_N(c_N - z)}}{\sqrt{h^2 + 2hk_S(c_S - z)}}, \text{ for } z \in (a, b)$$

Solving this equation yields $k_N = k_S$, $c_N = c_S$, and $M \equiv \frac{t_N^M}{t_S^M} = 1$, which implies, $t_N^M = t_S^M$. □

Proof of Proposition 4(2). Since $t_N^M = t_S^M$ and $(1-t_S^M)r_S(c) = (1-t_N^M)r_N(c)$, we can conclude that

$$r_S(c) = r_N(c), \text{ for any } c \in (a, b)$$

Besides, since $r_i(c) = \frac{c - w_i(m_i^{-1}(c))}{h(1-m_i^{-1}(c))}$ and $m_N^{-1}(c) = m_S^{-1}(c)$, we can deduce

$$w_S(m_S^{-1}(c)) = w_N(m_N^{-1}(c))$$

□

Proof of Proposition 4(3). Let us denote $\underline{z} \equiv \min\{z_S^*, z_N^*\}$. Considering the positive sorting and labor market clearing condition, we can infer that $m_S(0) = \underline{z}$ or/and $m_N(0) = \underline{z}$. Without loss of generality, let's assume that $m_S(0) = \underline{z}$.

Since $M_S \cap M_N = [a, b]$ and both M_S and M_N are connected intervals as per Proposition 3, we have $m_S(0) = a$ and/or $m_N(0) = a$. If $m_S(0) = a$, then $m_S^{-1}(a) = m_N^{-1}(a) = 0$, with the first equality derived from Proposition 1. Similarly, if $m_N(0) = a$, then $m_N^{-1}(a) = m_S^{-1}(a) = 0$, where $a = \underline{z} = \min\{z_S^*, z_N^*\}$.

Likewise, we have $m_S(z_S^*) = b$ or/and $m_N(z_N^*) = b$. If $m_S(z_S^*) = b$, then $m_S^{-1}(b) = m_N^{-1}(b) = z_S^*$, which is feasible only if $b \in M_N$ or $m_N(z_S^*) \leq m_N(z_N^*)$. Using the same logic, we have $m_S^{-1}(b) = m_N^{-1}(b) = z_N^*$ if $m_S(z_N^*) \leq m_S(z_S^*)$. In all, we have $b = \min\{m_S(z_S^*), m_N(z_N^*)\}$.

□

B.5 Proof of Proposition 5

Proof. First, we want to show that $z_S^* < z_N^*$. Agents with z_S^* are indifferent between being workers or managers in the South. Thus, we have

$$(1 - t^M)r(z_S^*) = (1 - t_S^W)w(z_S^*) < (1 - t_N^W)w(z_S^*),$$

where the inequality is due to the condition $t_S^W > t_N^W$. Hence, agents with z_S^* strictly prefer becoming workers in the North. Equivalently, this implies $z_S^* < z_N^*$.

Next, we need to to show that $z_N^* \leq \alpha$. Suppose, by contradiction, that $\alpha < z_N^*$. This would imply a gap in the set $M_S \cup M_N$, where $[\alpha, z_N^*]$ is not a subset of $M_S \cup M_N$. However, according to the positive sorting and labor market clearing condition, there exists a z_p such that $m^{-1}(\alpha) = z_p$ and $m^{-1}(z_N^*) = z_p$. Thus, there exists an equilibrium where workers with

ability z_p in a particular country i can be assigned to managers with skill levels either α or z_N^* and receive the same wages. This leads to the following equation:

$$w_i'(z_p) = \frac{\alpha - w_i(z_p)}{1 - z_p} = \frac{z_N^* - w_i(z_p)}{1 - z_p}.$$

Solving this equation reveals that $\alpha = z_N^*$. This contradicts our initial assumption that $\alpha < z_N^*$. Therefore, we conclude that $z_S^* < z_N^* \leq \alpha$.

Using the same logic, we can also demonstrate that if $t_S^W < t_N^W$, then $z_N^* < z_S^* \leq \alpha$. □

B.6 Proof of Proposition 6

Proof. We denote the matching function as $m(z)$ and $\hat{m}(z)$ before and after a unilateral decrease in personal income taxation in the South. As a result, the mass of southern workers will increase, denoted by $z_S^* < \hat{z}_S^*$, while the mass of northern workers will decrease, that is, $z_N^* > \hat{z}_N^*$.

First, we want to show that $m(z) - \hat{m}(z) > 0$ for some certain low-skilled workers. Let z_S and $\hat{z}_S$ be the worker type that works for the worst southern managers with ability z_S^* and $\hat{z}_S^*$. Then, within the interval $0 < z < \min\{z_S, \hat{z}_S\}$, we have $m(z) - \hat{m}(z) = z_N^* - \hat{z}_N^* > 0$ where the inequality follows from the fact that $z_N^* > \hat{z}_N^*$. Consequently, for some low skill workers with ability $0 < z < \min\{z_S, \hat{z}_S\}$, their matches with managers worsen after the policy change.

Next, we want to show that $m(z) - \hat{m}(z) > 0$ for some certain highly skill workers. In the interval $\max\{z_N^*, \hat{z}_N^*\} < z < \min\{z_S^*, \hat{z}_S^*\}$, or equivalently, $z_N^* < z < z_S^*$, we can rewrite it as

$$m(z) - \hat{m}(z) = -\frac{1}{\alpha} h z_S^* (1 - \frac{z_S^*}{2}) + \frac{1}{\alpha} h \hat{z}_S^* (1 - \frac{\hat{z}_S^*}{2}) > 0,$$

where the inequality follows from the condition $z_S^* < \hat{z}_S^*$ and the fact that $z(1 - \frac{z}{2})$ is a nondecreasing function of z for any $z \in [0, 1]$. Hence, some certain highly skilled workers with ability in the range $z_N^* < z < z_S^*$ now find themselves matched with less competent managers.

Finally, since the matching functions are monotonic and invertible, at the same point

where workers are matched with worse managers, managers are paired with more capable workers. □

B.7 Proof of Footnote 9

Proof. In order for the world to be in an equilibrium where managers come form both countries, we need to guarantee that $z^* < \alpha$, or equivalently,

$$\begin{aligned}
&\frac{1+h-\sqrt{1+h^2+2h(\frac{1-\alpha}{1+\alpha})}}{h} < \alpha \\
&\Leftrightarrow 1+h-\alpha h < \sqrt{1+h^2+2h(\frac{1-\alpha}{1+\alpha})} \\
&\Leftrightarrow 1+h^2+\alpha^2h^2+2h-2\alpha h-2\alpha h^2 < 1+h^2+2h(\frac{1-\alpha}{1+\alpha}) \\
&\Leftrightarrow \alpha^2 h+2-2\alpha-2\alpha h < 2(\frac{1-\alpha}{1+\alpha}) \\
&\Leftrightarrow (\alpha^2-2\alpha)h < 2(\frac{1-\alpha}{1+\alpha})-2+2\alpha \\
&\Leftrightarrow h < \frac{2(\alpha-1)}{\alpha^2-\alpha-2}.
\end{aligned}$$

□

C Empirics

In our main manuscript, we previously derived equilibrium matching and wage functions for the case where $0 < z_S < z_\alpha < z_N^* < z_S^* < \alpha$. Now, we will describe the equilibrium allocations for the other two cases.

Case 1: $0 < z_N^* < z_S < z_\alpha < z_S^* < \alpha$

Let z_α be such that $m(z_\alpha) = \alpha$, and let z_N be such that $m(z_S) = z_S^*$. In this scenario, the equilibrium is characterized by schedules of wages $\boldsymbol{w} \equiv w(z)$, schedules of matches $\boldsymbol{m} \equiv m(z)$, schedules of rents $\boldsymbol{r} \equiv r(z)$, and occupation cutoffs $\{z_N^*, z_S^*\}$. The equilibrium conditions are as follows:

1. matching function

$$m(z) = \begin{cases} \frac{1+\alpha}{\alpha} hz(1-\frac{z}{2}) + z_N^* & \text{if } z \in [0, z_N^*] \\ \frac{1}{\alpha} hz(1-\frac{z}{2}) + hz_N^*(1-\frac{z_N^*}{2}) + z_N^* & \text{if } z \in (z_N^*, z_S] \\ \frac{1}{1+\alpha} hz(1-\frac{z}{2}) + \frac{1}{\alpha(1+\alpha)} hz_S(1-\frac{z_S}{2}) + hz_N^*(1-\frac{z_N^*}{2}) + z_N^* & \text{if } z \in (z_S, z_\alpha) \\ \frac{1}{\alpha} hz(1-\frac{z}{2}) + 1 - \frac{1}{\alpha} hz_S^*(1-\frac{z_S^*}{2}) & \text{if } z \in (z_\alpha, z_S^*] \end{cases}$$

where

(i)

$$\frac{1}{\alpha(1+\alpha)} hz_S(1-\frac{z_S}{2}) + hz_N^*(1-\frac{z_N^*}{2}) + z_N^* = \frac{1}{\alpha(1+\alpha)} hz_\alpha(1-\frac{z_\alpha}{2}) + 1 - \frac{1}{\alpha} hz_S^*(1-\frac{z_S^*}{2})$$

(ii)

$$\frac{1}{\alpha} hz_S(1-\frac{z_S}{2}) + hz_N^*(1-\frac{z_N^*}{2}) + z_N^* = z_S^*$$

(iii)

$$\frac{1}{\alpha} hz_\alpha(1-\frac{z_\alpha}{2}) + 1 - \frac{1}{\alpha} hz_S^*(1-\frac{z_S^*}{2}) = \alpha$$

2. wage function

$$w(z) = \begin{cases} \frac{1}{2}\frac{1+\alpha}{\alpha} hz^2 - A_1(1-z) + z_N^* & \text{if } z \in [0, z_N^*] \\ \frac{1}{2}\frac{1}{\alpha} hz^2 - A_2(1-z) + hz_N^*(1-\frac{z_N^*}{2}) + z_N^* & \text{if } z \in (z_N^*, z_S] \\ \frac{1}{2}\frac{1}{1+\alpha} hz^2 - A_3(1-z) + \frac{1}{\alpha(1+\alpha)} hz_S(1-\frac{z_S}{2}) + hz_N^*(1-\frac{z_N^*}{2}) + z_N^* & \text{if } z \in (z_S, z_\alpha) \\ \frac{1}{2}\frac{1}{\alpha} hz^2 - A_4(1-z) + 1 - \frac{1}{\alpha} hz_S^*(1-\frac{z_S^*}{2}) & \text{if } z \in (z_\alpha, z_S^*] \end{cases}$$

where

(iv)

$$A_2 = A_1 + hz_N^*$$

(v)

$$A_3 = A_2 + \frac{1}{\alpha(1+\alpha)} h z_S$$

(vi)

$$A_4 = A_3 - \frac{1}{\alpha(1+\alpha)} h z_\alpha$$

(vii)

$$\begin{aligned}\frac{1}{h(1-z_S)}[z_S^* - (\frac{1}{2}\frac{1}{\alpha}hz_S^2 - A_2(1-z_S) + hz_N^*(1-\frac{z_N^*}{2}) + z_N^*)] \\ = \frac{1}{t'}[\frac{1}{2}\frac{1}{\alpha}h(z_S^*)^2 - A_4(1-z_S^*) + 1 - \frac{1}{\alpha}hz_S^*(1-\frac{z_S^*}{2})]\end{aligned}$$

(viii)

$$\frac{A_1}{h} = \frac{1}{t}[\frac{1}{2}\frac{1+\alpha}{\alpha}h(z_N^*)^2 - A_1(1-z_N^*) + z_N^*]$$

3. rent function

$$r(z) = \begin{cases} \frac{1}{\phi_1(z)}[z - \frac{1+\alpha}{2\alpha}\frac{(h-\phi_1(z))^2}{h} + A_1\frac{\phi_1(z)}{h} - C_1] & \text{if } z \in [z_N^*, m(z_N^*)] \\ \frac{1}{\phi_2(z)}[z - \frac{1}{2\alpha}\frac{(h-\phi_2(z))^2}{h} + A_2\frac{\phi_2(z)}{h} - C_2] & \text{if } z \in (m(z_N^*), z_S^*] \\ \frac{1}{\phi_3(z)}[z - \frac{1}{2}\frac{1}{1+\alpha}\frac{(h-\phi_3(z))^2}{h} + A_3\frac{\phi_3(z)}{h} - C_3] & \text{if } z \in (z_S^*, \alpha] \\ \frac{1}{\phi_4(z)}[z - \frac{1}{2\alpha}\frac{(h-\phi_4(z))^2}{h} + A_4\frac{\phi_4(z)}{h} - C_4] & \text{if } z \in (\alpha, 1] \end{cases}$$

where

$$\begin{aligned}\phi_1(z) &= \sqrt{h^2 + \frac{2h\alpha}{1+\alpha}(C_1 - z)} \\ \phi_2(z) &= \sqrt{h^2 + 2h\alpha(C_2 - z)} \\ \phi_3(z) &= \sqrt{h^2 + 2h(1+\alpha)(C_3 - z)} \\ \phi_4(z) &= \sqrt{h^2 + 2h\alpha(C_4 - z)}\end{aligned}$$

and

$$
\begin{aligned}
C_1 &= z_N^* \\
C_2 &= hz_N^*(1-\frac{z_N^*}{2}) + z_N^* \\
C_3 &= \frac{1}{\alpha(1+\alpha)} hz_S(1-\frac{z_S}{2}) + hz_N^*(1-\frac{z_N^*}{2}) + z_N^* \\
C_4 &= 1 - \frac{1}{\alpha} hz_S^*(1-\frac{z_S^*}{2})
\end{aligned}
$$

Please note that conditions (i) to (viii) apply to determine the values of $\{z_N^*, z_S^*, z_S, z_\alpha, A_1, A_2, A_3, A_4\}$.

Case 2: $0 < z_S < z_N^* < z_\alpha < z_S^* < \alpha$

In this case, we have the same definitions for z_α and z_S, and the equilibrium can be characterized in a similar manner. The equilibrium conditions are identical to those in Case 1, with the exception that the matching and earnings functions are adjusted as follows:

1. matching function

$$
m(z) = \begin{cases}
\frac{1+\alpha}{\alpha} hz(1-\frac{z}{2}) + z_N^* & \text{if } z \in [0, z_S] \\
hz(1-\frac{z}{2}) + \frac{1}{\alpha} hz_S(1-\frac{z_S}{2}) + z_N^* & \text{if } z \in (z_S, z_N^*] \\
\frac{1}{1+\alpha} hz(1-\frac{z}{2}) + \frac{\alpha}{1+\alpha} hz_N^*(1-\frac{z_N^*}{2}) + \frac{1}{\alpha} hz_S(1-\frac{z_S}{2}) + z_N^* & \text{if } z \in (z_N^*, z_\alpha) \\
\frac{1}{\alpha} hz(1-\frac{z}{2}) + 1 - \frac{1}{\alpha} hz_S^*(1-\frac{z_S^*}{2}) & \text{if } z \in (z_\alpha, z_S^*]
\end{cases}
$$

where

(i)

$$
\frac{\alpha}{1+\alpha} hz_N^*(1-\frac{z_N^*}{2}) + \frac{1}{\alpha} hz_S(1-\frac{z_S}{2}) + z_N^* = \frac{1}{\alpha(1+\alpha)} hz_\alpha(1-\frac{z_\alpha}{2}) + 1 - \frac{1}{\alpha} hz_S^*(1-\frac{z_S^*}{2})
$$

(ii)

$$
\frac{1+\alpha}{\alpha} hz_S(1-\frac{z_S}{2}) + z_N^* = z_S^*
$$

(iii)

$$\frac{1}{\alpha}hz_\alpha(1-\frac{z_\alpha}{2})+1-\frac{1}{\alpha}hz_S^*(1-\frac{z_S^*}{2})=\alpha$$

2. wage function

$$w(z)=\begin{cases}\frac{1}{2}\frac{1+\alpha}{\alpha}hz^2-A_1(1-z)+z_N^* & \text{if } z\in[0,z_S]\\ \frac{1}{2}hz^2-A_2(1-z)+\frac{1}{\alpha}hz_S(1-\frac{z_S}{2})+z_N^* & \text{if } z\in(z_S,z_N^*]\\ \frac{1}{2}\frac{1}{1+\alpha}hz^2-A_3(1-z)+\frac{\alpha}{1+\alpha}hz_N^*(1-\frac{z_N^*}{2})+\frac{1}{\alpha}hz_S(1-\frac{z_S}{2})+z_N^* & \text{if } z\in(z_N^*,z_\alpha)\\ \frac{1}{2}\frac{1}{\alpha}hz^2-A_4(1-z)+1-\frac{1}{\alpha}hz_S^*(1-\frac{z_S^*}{2}) & \text{if } z\in(z_\alpha,z_S^*]\end{cases}$$

where

(iv)

$$A_2=A_1+\frac{1}{\alpha}hz_S$$

(v)

$$A_3=A_2+\frac{\alpha}{1+\alpha}hz_N^*$$

(vi)

$$A_4=A_3-\frac{1}{\alpha(1+\alpha)}hz_\alpha$$

(vii)

$$\begin{aligned}&\frac{1}{h(1-z_S)}[z_S^*-(\frac{1}{2}\frac{1+\alpha}{\alpha}hz_S^2-A_1(1-z_S)+z_N^*)]\\ &=\frac{1}{t'}[\frac{1}{2}\frac{1}{\alpha}h(z_S^*)^2-A_4(1-z_S^*)+1-\frac{1}{\alpha}hz_S^*(1-\frac{z_S^*}{2})]\end{aligned}$$

(viii)

$$\frac{A_1}{h}=\frac{1}{t}[\frac{1}{2}h(z_N^*)^2-A_2(1-z_N^*)+\frac{1}{\alpha}hz_S(1-\frac{z_S}{2})+z_N^*]$$

3. rent function

$$r(z) = \begin{cases} \frac{1}{\phi_1(z)}[z - \frac{1+\alpha}{2\alpha}\frac{(h-\phi_1(z))^2}{h} + A_1\frac{\phi_1(z)}{h} - C_1] & \text{if } z \in [z_N^*, z_S^*)] \\ \frac{1}{\phi_2(z)}[z - \frac{1}{2}\frac{(h-\phi_2(z))^2}{h} + A_2\frac{\phi_2(z)}{h} - C_2] & \text{if } z \in (z_S^*, m(z_N^*)] \\ \frac{1}{\phi_3(z)}[z - \frac{1}{2}\frac{1}{1+\alpha}\frac{(h-\phi_3(z))^2}{h} + A_3\frac{\phi_3(z)}{h} - C_3] & \text{if } z \in (m(z_N^*), \alpha] \\ \frac{1}{\phi_4(z)}[z - \frac{1}{2\alpha}\frac{(h-\phi_4(z))^2}{h} + A_4\frac{\phi_4(z)}{h} - C_4] & \text{if } z \in (\alpha, 1] \end{cases}$$

where

$$\begin{aligned} \phi_1(z) &= \sqrt{h^2 + \frac{2h\alpha}{1+\alpha}(C_1 - z)} \\ \phi_2(z) &= \sqrt{h^2 + 2h(C_2 - z)} \\ \phi_3(z) &= \sqrt{h^2 + 2h(1+\alpha)(C_3 - z)} \\ \phi_4(z) &= \sqrt{h^2 + 2h\alpha(C_4 - z)} \end{aligned}$$

and

$$\begin{aligned} C_1 &= z_N^* \\ C_2 &= \frac{1}{\alpha}hz_S(1 - \frac{z_S}{2}) + z_N^* \\ C_3 &= \frac{\alpha}{1+\alpha}hz_N^*(1 - \frac{z_N^*}{2}) + \frac{1}{\alpha}hz_S(1 - \frac{z_S}{2}) + z_N^* \\ C_4 &= 1 - \frac{1}{\alpha}hz_S^*(1 - \frac{z_S^*}{2}) \end{aligned}$$

The conditions (i) to (viii) still apply to determine the values of $\{z_N^*, z_S^*, z_S, z_\alpha, A_1, A_2, A_3, A_4\}$.