Constrained School Choice and the Demand for Effective Schools *

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Abstract

We study how a constraint in the length of school application lists affects parents' application behavior, equilibrium allocations, and what we can learn from observed application lists. We exploit a policy change in the Barbados education market for secondary school admissions that imposed a binding constraint on the length of application lists. We show that parents react by omitting the most selective schools from their applications. Although this behavior change has a minor effect on the equilibrium allocation, it affects our inferences regarding what parents value when choosing schools. Assuming application lists reveal truthful preferences, we infer that parents do not value school effectiveness after controlling for peer quality. However, by estimating preferences using a method robust to strategic behavior, we infer that parents value effective schools even after controlling for peer quality. The difference in results is due to parents omitting selective and effective schools from their application lists when constrained in their applications' length.

Keywords: School choice, Secondary education, Education policy.

JEL codes: I21, I24, I28, J24.

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1 Introduction

Centralized education markets that allocate students to schools are expanding worldwide (Neilson, 2024). Many of these markets rely on the Student Proposing Deferred Acceptance mechanism, which incentivizes truthful revelation of preferences when applicants are unconstrained in the number of schools they can list. However, most centralized education markets constrain the length of application lists, which may lead to parents not revealing their true preferences when applying to schools.

Previous literature studies the theoretical effects of imposing a binding constraint on the length of application lists (Haeringer and Klijn, 2009). An important finding is that parents no longer reveal truthful preferences in their applications when constrained. There is also laboratory evidence in line with the theoretical predictions (Calsamiglia et al., 2010). However, less is known regarding the effects of constraining the length of application lists in an actual market. Furthermore, it may also affect what we can infer from observed application lists regarding parental valuation of school effectiveness.

We take advantage of a policy change in the centralized secondary admission market in Barbados that imposed a binding constraint on the number of schools parents can list. The policy change allows us to study the effect of the constraint on application lists and placement outcomes. Furthermore, we merge these data with information on students' post-secondary outcomes, which enables us to examine parental preferences for peer quality and school effectiveness.

First, we use event studies to evaluate how the constraint in the length of application lists affects the content of lists and whether the policy change affects the equilibrium allocation. After the policy change, we show that parents change their application lists in patterns consistent with skipping the impossible. That is, they exclude from their application lists the traditionally most selective schools in the market. However, the change in application lists does not affect the equilibrium allocation. This result is consistent with parents making pay-off irrelevant mistakes. To the extent that out-of-reach schools are, in fact, out of reach, excluding these schools from the application lists does not affect the equilibrium.

Second, we estimate parental preferences for schools following two approaches. The first

one assumes application lists to represent truthful preferences. The second one assumes the matching outcome to be a stable equilibrium (Fack et al., 2019; Artemov et al., 2023). We combine information on choices and students' educational outcomes to decompose valuations for schools into preferences for peer composition, school effectiveness, and match quality Abdulkadiroğlu et al. (2020). Under the truth-telling assumption, parents do not value school effectiveness once we control for peer quality. Under the stable equilibrium assumption, parents value school effectiveness and peer quality.

The truth-telling assumption implies that parents rank listed schools truthfully and prefer listed schools to excluded ones. However, as our event study evidence shows, parents skip the impossible in the application lists under a binding constraint in the number of schools they can list. By doing so, they may skip selective and effective schools from their applications, leading a researcher to conclude that they do not value school effectiveness. Estimating preferences under the stability of the matching equilibrium is robust to this type of deviation from truth-telling, allowing us to get closer to parents' school valuations.

Assumptions matter when studying preferences for school effectiveness when school choice is constrained, even if the constraint does not affect the equilibrium allocation. This is particularly important when the most selective schools are also among the most effective, and parents exclude them from their choice sets to avoid being unmatched. Our results show that even in a context where parents value school effectiveness, implementation choices in the system's design can limit the role of parental choice in fostering school competition on school effectiveness.

Our paper contributes to two strands of the literature. First, we contribute to the literature by studying the effects of imposing a constraint on the number of schools students can apply to. Haeringer and Klijn (2009) theoretical results show that constraining parents' application lists forces them to strategize. Calsamiglia et al. (2010) design a laboratory experiment and find that the constraint reduces truth-telling. To our knowledge, we are the first to study the effects of imposing a constraint in a real market.

Second, we contribute to the literature studying parental valuation for school effectiveness. Most of the literature assumes that parents reveal their true preferences in their application lists and find mixed results regarding parental valuation for school effectiveness

after controlling for peer quality (Abdulkadiroğlu et al., 2020; Ainsworth et al., 2023; Campos and Kearns, 2024). Beuermann et al. (2023) considers the possibility of non-truthful behavior and uses a modified version of the estimator when assuming truth-telling to recover parental preferences. We instead follow (Fack et al., 2019) and use the market stability to recover parental valuations in a context where we know parents are not being truthful.

The remainder of the paper proceeds as follows. Section 2 describes the institutional background. Section 3 details the administrative data we use for the analysis. Section 4 contains event study evidence regarding the effects of the policy change. Section 5 contains the analysis of parental valuation for school effectiveness in a constrained environment. Section 6 concludes.

2 Institutional background

Barbados possesses a centralized school choice system that governs the admission process to public secondary schools. At the end of primary school (after grade 6), students register to take the Barbados Secondary School Entrance Examination (BSSEE) and provide a list of ranked secondary school choices to the Ministry of Education.[1] The BSSEE is comprised of three subjects that all students take: mathematics, English language, and an essay. The total BSSEE score is the sum of the scores on the individual sections and ranges from 0 to 200. Students are ranked by their BSSEE score and gender. Individual school capacity by gender is pre-determined. With the exceptions of the two single-sex schools (one for girls and another for boys), all other coed schools have half of their sits pre-determined for girls and the other half for boys. The algorithm assigns the highest ranked student to her first choice. It then moves on to the second and treats her similarly. At some point, the procedure will reach a student whose first choice is full. At that point, it tries to assign the student to her second choice. If full, to the third choice and so on. Only once this student has been assigned to a school does the algorithm move onto the next person.[2]

[1]The list of ranked schools is submitted before taking the BSSEE and this list can not be modified afterwards.

[2]BSSEE results provided to the students include their score and the allocated secondary school. However, school admissions cutoff for each school are not revealed.

Between 1987 and 1995, students could rank up to fifteen schools (out of the 22 schools operating in the country) without any geographical restrictions. Starting in 1996, some restrictions were implemented. First, students were further constrained to list up to nine school choices. Second, the first two school choices of the ranked list had no geographical restrictions. However, schools listed between the third and ninth position were restricted to the geographical zone where the student resided. For this purpose, the island was divided into three geographical zones. Third, each secondary school was required to take at least 30 percent of its students from within its zone. Finally, both of the single-sex schools were not subject to neither the zone related restrictions for choices nor student intake.

Secondary school begins in first form (the equivalent of 7th grade) and ends at fifth form (the equivalent of 11th grade) when students take the Caribbean Secondary Education Certification (CSEC) examinations. These are the Caribbean equivalent of the British Ordinary levels (O-levels) examinations and are externally graded by the Caribbean Examinations Council (CXC). The CSEC examinations are given in 33 subjects. To be eligible for university admission, one must pass five or more subjects including English and mathematics. In addition, entry level positions in the public sector require at least five CSEC subject passes. Therefore, the CSEC is a high-stakes examination.

3 Data

We observe the full population of students who applied to a public secondary school in Barbados between 1987 and 2011.[3] We obtained the official individual-level administrative BSSEE data for each of these years. These data include date of birth, gender, primary school attended, parish of residence, total score on the BSSEE exam, the ranked list of secondary schools the student wished to attend, and the administrative assignment by the Ministry of Education.

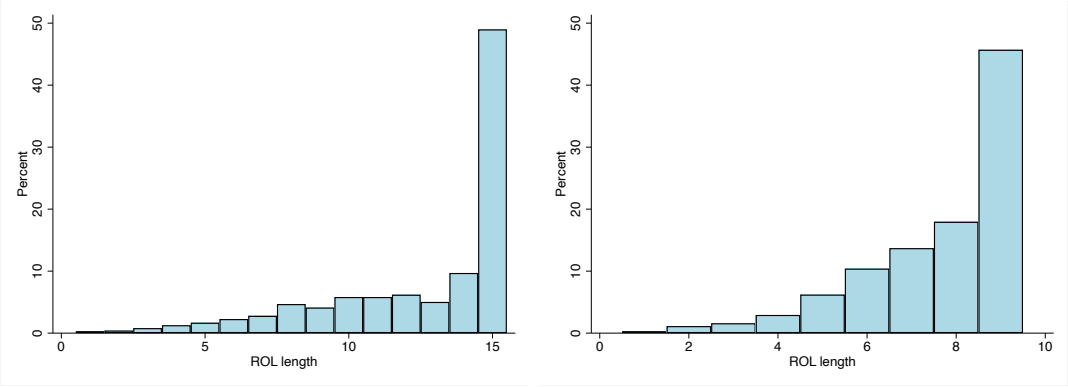
To track student performance in secondary school we collected individual-level data on the CSEC examinations (taken five years after secondary school entry, typically at age 16). The CSEC data are available for all years between 1993 and 2016. These data include the

[3]Around 91 percent of secondary students in Barbados are enrolled in the public education system.

scores for each subject examination taken, and the secondary school attended. The CSEC data were linked to the 1987 through 2011 BSSEE cohorts by full name (first, middle, and last), gender, and date of birth.[4]

Figure 1 shows how the policy change moves the market from an environment with a binding constraint at fifteen to a an environment with a binding constraint at nine.

Figure 1: ROL length



(a) ROL length 1995 (b) ROL length 1997

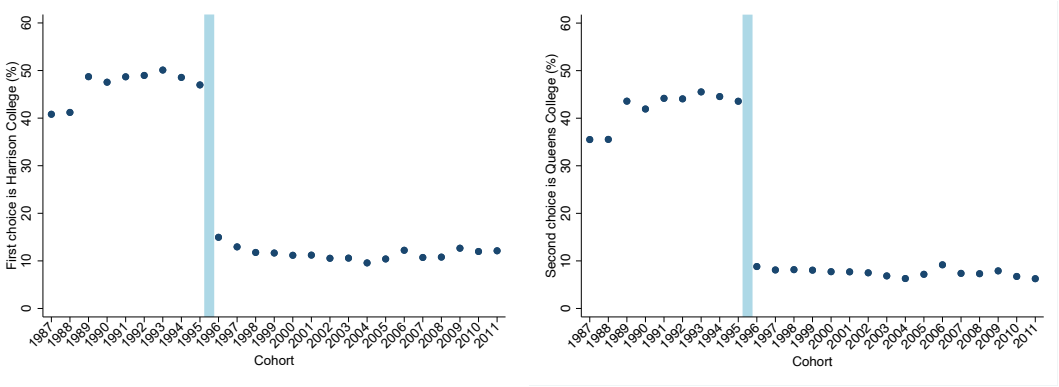
NOTE: This figure shows the distribution of application lengths before (panel a) and after (panel b) the policy change.

4 Event studies

This section explores changes in student application lists induced by the policy change. We first focus on the first and second choices. For these choices, students could still list any school in the market after the policy change. We then focus on overall changes in the application lists (i.e., extensive margin) measured by the inclusion of any selective school in them. Lastly, we explore changes in equilibrium sorting.

[4]We matched 90 percent of individuals observed in the CSEC administrative records to the BSSEE records. The 10 percent rate of unmatched individuals closely mimics the 9 percent enrollment rate in private secondary schools who would not have taken the BSSEE.

Figure 2: Most popular first and second choices



(a) Harrison College (b) Queens College

NOTE: This figure shows the percentage of parents that list Harrison College as their first choice (panel a) and the percentage of parents that list Queens College as their second choice (panel b). The x-axis indicates the application cohort.

4.1 Applications lists

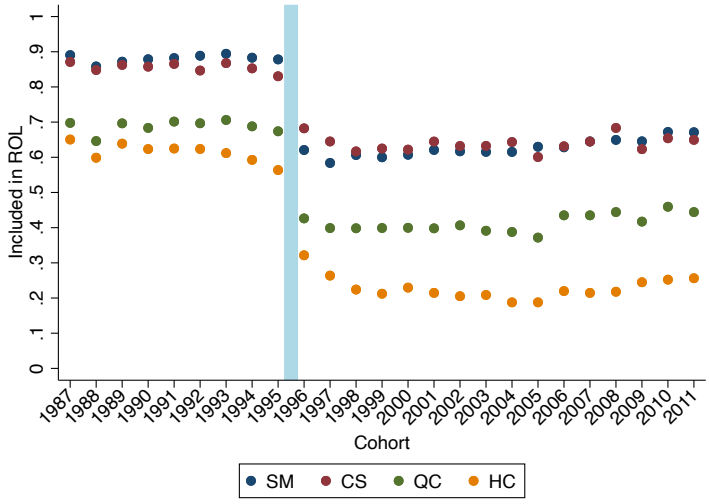
The most popular first choice school is Harrison College. Before the policy change an average of 47% of students listed this school as their first choice. The most popular second choice school is Queens College. Before the policy change, an average of 42% of students listed this school as their second choice. Consequently, these two schools had the highest admission cut-offs.

Figure 2 panel (a) shows that after the policy change, there is a dramatic decrease of 25 percentage points in the share of students that list Harrison College as their first choice. Figure 2 panel (b) shows a similar pattern for Queens College. The decrease in the share of students that list Queens College as their second choice is 30 percentage points.

The previous figures show drastic changes in application behavior but allow for the possibility that parents could still be applying to selective schools in a different order. To show evidence of parents completely omitting certain schools from their application lists, we group the most popular schools in the market and study whether parents react by not including these schools in their applications.

The four most popular schools in the market are Harrison College (HC), Queens College (QC), Combemere School (CS), and Saint Michaels (SM). For ease of exposition, we label

Figure 3: Big 4 included in ROL



NOTE: This figure shows the percentage of parents that list any of the Big 4 schools in their application lists. The x-axis indicates the application cohort.

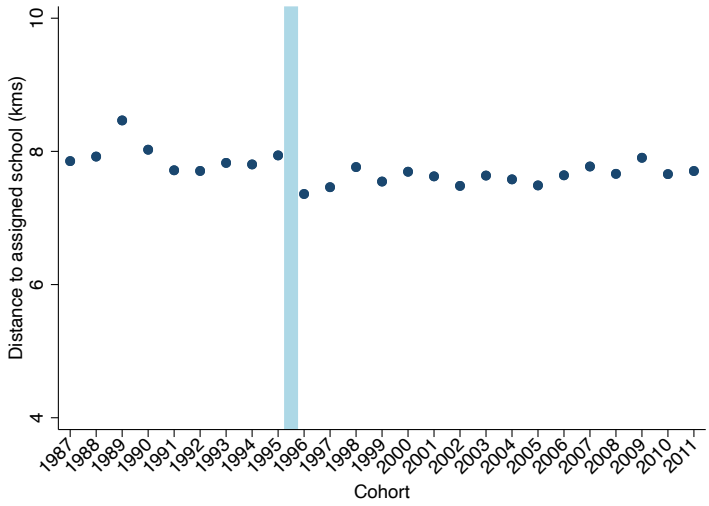
these schools as the Big Four. Beyond the policy change affecting the schools students list as their first or second choices (i.e., intensive margin), it also affects whether students include Big Four schools in their applications (i.e., extensive margin). Figure 3 shows that the policy change reduced the percentage of students that listed any Big Four schools in their application lists. The decrease goes from 20 to 30 percentage points for Combemere School and Queens College, respectively.

Overall, our event studies show evidence of parents reacting to the policy change by skipping the impossible in their application lists. However, a change in applications does not necessarily imply a change in assignments. In the next subsection, we explore changes in equilibrium assignments.

4.2 Equilibrium

Application changes affect the equilibrium allocation depending on whether these changes are payoff-relevant. For instance, the previous section showed that parents reacted by skipping the impossible schools in their application lists. This reaction can only affect the equilibrium allocation when the omitted schools are ex-post feasible. When the impossible schools were, in fact, impossible, the changes in application lists may not affect the equilibrium allocation

Figure 4: Distance



NOTE: This figure shows the average distance to the assigned school. We measure distance in kilometers between primary and secondary school locations. The x-axis indicates the application cohort.

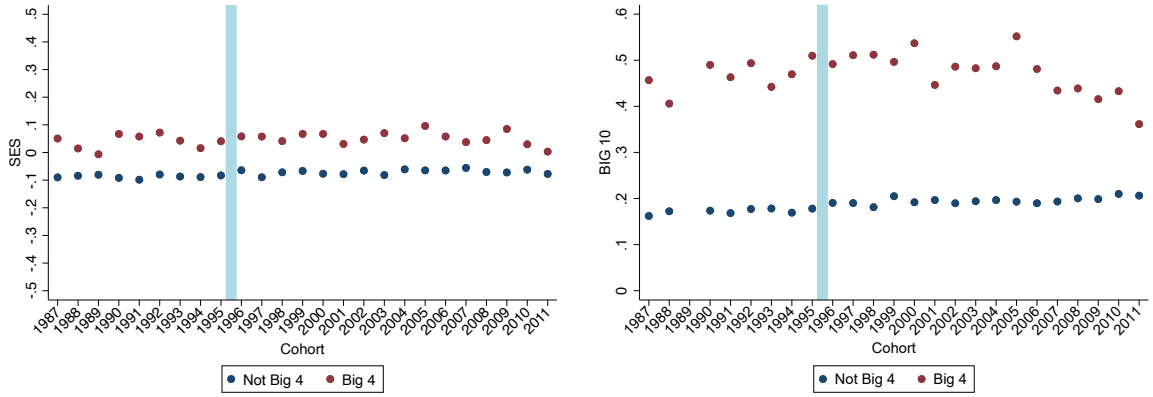
(i.e., payoff irrelevant).

We first consider changes in distance to the assigned school. We take distance to a school as a measure of welfare to the extent that it is one of the most important determinants of parental preferences. Figure 4 shows that the average distance to the assigned schools has not changed much after the policy change.

We next evaluate whether the policy change affected the socioeconomic composition of students assigned to Big Four schools. This could occur if the application changes we observed affected different subgroups of students, such as low-SES students who may no longer apply to Big Four schools. We can only measure socioeconomic status at the primary school level using the primary school locations and constructing an SES index by the smallest census regions.

Figure 5 panel (a) shows that students assigned to Big 4 schools have higher SES than students at other schools, but this has not changed over time or because of the policy change of 1996. It is important to highlight that being able to measure SES at the primary school level may partially explain this result. Figure 5 panel (b) shows that every year almost half the assigned students to Big 4 schools come from the same 10 primary schools in the market. Therefore, since this pattern has not changed over time or because of the policy change, we

Figure 5: SES and school of origin



(a) SES index

(b) Big 10 elementary

NOTE: Panel (a) shows the average SES index of students assigned to Big 4 and non-Big 4 schools. The SES index measures socieconomic status at the primary school level. Panel (b) shows the share of students from Big 10 primary schools assigned to Big 4 and non-Big 4 schools. We define Big 10 primary schools as the set of schools that most commonly get students assigned to Big 4 schools. The x-axis indicates the application cohort.

are also unlikely to detect changes in SES composition.

The lack of changes in equilibrium sorting shows two issues. First, the policy did not create more inequalities. Second, it shows that even though it may seem that parents are free to choose, they are, in fact, severely constrained by the combination of the one-shot exam priority and the binding constraint on application list length.

5 Preferences for school effectiveness

When parents omit the most selective schools from their application lists, it affects what we can learn from them about their preferences. For example, consider a case where the most selective schools are the most effective, and parents omit these schools from their application lists when constrained. From the observed application lists, we may infer that parents do not value school effectiveness when they do not list effective schools because they are unfeasible. Therefore, in such a context, parental applications may not incentivize school competition on effectiveness, even though parents do care about effective schools.

In this section, we study parental preferences for school effectiveness in an environment

where we know parents have omitted selective schools from their applications. We estimate school effectiveness first and later link these estimates to parental preferences.

5.1 School effectiveness

To measure school effectiveness, we estimate a value-added specification with match effects. Consider the following potential outcomes:

$$Y_{ij} = \alpha_j + X_i'\beta_j + \epsilon_{ij},$$

where Y_{ij} measures a post-secondary educational outcome. X_i is a set of observable variables that include the admission exam score and gender. We demean the observable covariates such that we can interpret α_j as the effect for the average student. β_j measures match effects between observable variables and school j. ϵ_{ij} captures unobservables.

Assuming selection on observables, we estimate the parameters under the following empirical specification:

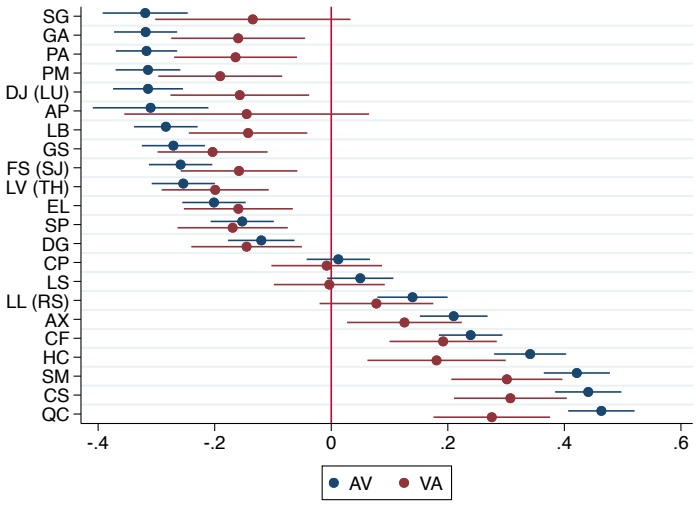
$$Y_i = \sum_{j=1}^{J} D_{ij}\alpha_j + \sum_{j=1}^{J} \beta_j D_{ij} \times X_i + \epsilon_i,$$

where D_{ij} is a dummy variable indicating the assignment of student i to school j.

As the selection on observables is a strong assumption, we validate our value-added specification by exploiting admission discontinuities to higher-ranked schools in parental application lists.

Figure 6 shows estimated $\hat{\alpha}$ parameters using CSEC performance as the outcome variable. We show the estimated parameters and the school outcome averages for comparison purposes. Panel (a) shows estimates using data before the policy change, while Panel (b) uses data afterward. Results do not depend on the policy environment. The main result of these figures is that for the most selective schools, even though school averages overestimate school effectiveness, they do correlate positively with school effectiveness. In other words, the most selective schools are also the most effective.

Figure 6: CSEC approved 5 subjects



NOTE: This figure shows estimates of school effectiveness and average outcomes. The red dots plot $\hat{\alpha}_j$ estimates without any control variable or match effects. The blue dots plot $\hat{\alpha}_j$ estimates when controlling for the one-shot exam score, gender, and school zone and including match effects. Horizontal bars indicate 95% confidence intervals.

5.2 Preferences

In a constrained environment, parents no longer have incentives to reveal their preferences when choosing their application lists. Thus, to recover student preferences, we use the method proposed by (Fack et al., 2019), which assumes that the equilibrium is stable and only relies on school assignments. Under the stability assumption, each parent has a personalized choice set. The choice set depends on the admission score and the ex-post school cut-offs. Each parent's feasible choice set is the set of schools with admission cut-offs lower than their children's admission exam score. Stability implies that parents obtain their preferred school within their feasible choice set. Parental choice problem is:

$$\arg\max_{j \in \Omega_i} \quad U_{ij},$$

Where U_{ij} is the utility the obtain when assigned to school j and Ω_i indicates their choice set. We denote k_j school j admission cut-off. Then the ex-post feasible choice set of parent i is:

$$\Omega_i = \{j : s_i \geq k_j\}.$$

To allow for heterogeneous preferences we define define cells $c(i)$ that group students by gender. We define the utility agent i obtains from school j as follows:

$$U_{ij} = V_{ij} + \epsilon_{ij} = \delta_{c(i)j} + \lambda_{c(i)} d_{ij} + \epsilon_{ij},$$

where $\delta_{c(i)j}$ captures average tastes for school j within cell $c(i)$, d_{ij} measures the distance from student i to school j such that $\lambda_{c(i)}$ measures distaste for distance within cell $c(i)$, and ϵ_{ij} capture all remaining unobserved tastes. Assuming that the distribution of ϵ_{ij} is type I extreme value we choice probabilities are as follow:

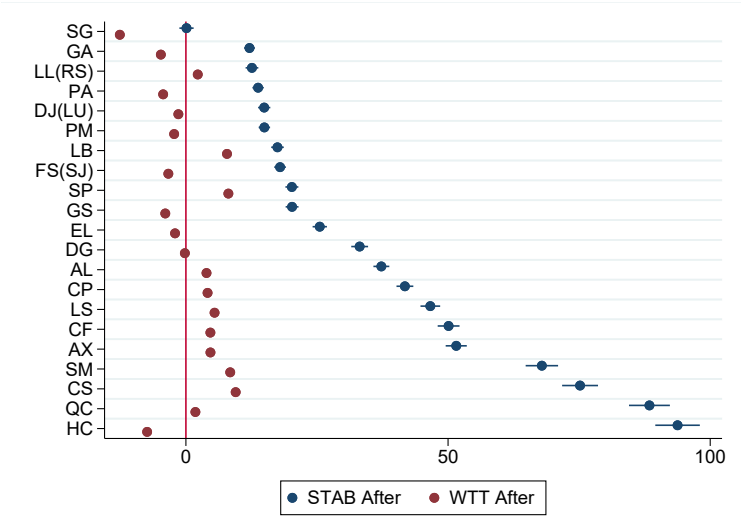
$$Pr(D_i = j) = \frac{\exp V_{ij}}{\sum_{k \in \Omega_i} \exp V_{ik}}$$

Estimation of the model parameter is akin to the estimation of a discrete choice model with heterogeneous choice sets. We estimate the model parameters via maximum likelihood.

We also estimate preferences under weak truth-telling for comparison purposes. Weak truth-telling assumes ROLs represent true preferences between the ranked schools. We describe the estimation in Appenddix.

Figure 6 shows estimated tastes for each school in the market measured in willingness to travel. $\frac{\delta_{c(i)j}}{\lambda_{c(i)}}$. The blue dots represent our estimates under the stability of the market assumption and the red dots represent the estimates assuming weak truth telling. We highlight two things from this figure. First, the estimates quite different overall. Second, the stability based estimates show a higher tastes for the most oversubscribed schools, while the weak truth telling based estimates do not show that pattern. In fact, according to the WTT estimates parents do not value particularly high the Big Four schools, whereas the STAB estimates shows that these are the schools parents value the most.

Figure 7: Average tastes in willingness-to-travel



NOTE: This figure shows estimates of mean parental valuation for schools in willingness-to-travel. The red dots show estimates assuming truth-telling. The blue dots show estimates under the stability of the market assumption.

5.3 Preferences for effectiveness

In this section, we combine what we learned in the previous sections to investigate whether parents value school effectiveness after controlling for peer quality and match effects. We use our value-added estimates to decompose school-level outcomes into three components: peer quality (Q_j), school effectiveness (ATE_j), and average match effects for students in a given cell (M_{jc}). We then regress our cell-specific average tastes against these components. We have two sets of average taste estimates, one under weak truth-telling and the other under stability.

$$\hat{\delta}_{jct} = \kappa_{ct} + \rho_1 Q^*_{jt} + \rho_2 ATE^*_{jt} + \rho_3 M^*_{jct} + \xi_{cjt}$$

Table 1 shows our estimation results. We are in an environment where parents have already reacted by skipping the impossible from their application lists. The first column shows that in this environment if we assume parents reveal their truthful preferences in their application lists, we learn that they do not value school effectiveness when controlling for peer quality and match effects. On the other hand, when we recover parental preferences using a method robust to non-truthful behavior, we learn that parents value both peer quality and

school effectiveness.

Table 1: Preferences determinants: 1997-2002

	WTT	Stability
Peer quality	0.635***	0.784***
	(0.054)	(0.032)
ATE	-0.021	0.185***
	(0.053)	(0.031)
Match effect	0.064	0.000
	(0.048)	(0.028)
Observations	264	264

NOTE: This table shows the results of regressing average taste parameters against our estimates of peer quality, school effectiveness, and match effects. All variables are scaled in standard deviation units.

We take this as evidence that even if parents value school effectiveness, non-truthful reporting of preferences may lead to demand pressure for schools to compete on peer quality and not effectiveness. Notice that, ultimately, parental reported preferences, not their true preferences, influence the equilibrium allocation and induce incentives for schools to compete.

6 Conclusions

Most centralized education markets around the world impose a constraint on the number of applications students can list. We show that this practice leads to many parents not applying to the most selective schools in the market. However, this behavior does not necessarily affect the equilibrium allocation (i.e., pay-off irrelevant). When schools rank students using a one-shot exam, the omitted selective schools are likely to be ex-post infeasible for many students, rendering their omission irrelevant. We take this as evidence that a combination of restricted length in application lists and admission priorities that rely on a one-shot exam may effectively limit parental choice.

We further show that in an environment with limited parental choice, even if parents

do value school effectiveness, application lists may not reflect so. Thus, the policies behind that environment may limit the competitive effect of school choice. Concerning educational policies, our results show that another way to improve centralized markets would be to ensure that their design implementation choices enhance choice rather than reduce it

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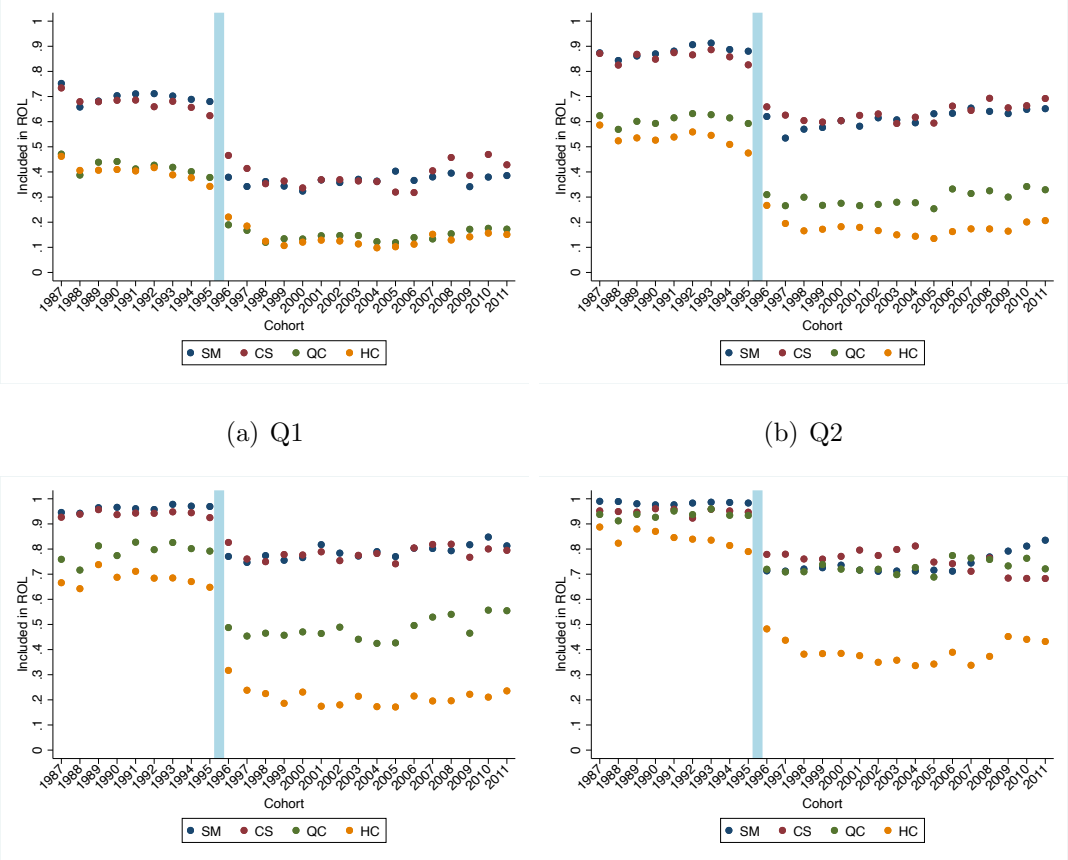
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A Appendices

A.1 Big 4 by one-shot exam quartiles

Figure A.1: Big 4 included in ROL by admission score



(a) Q1

(b) Q2

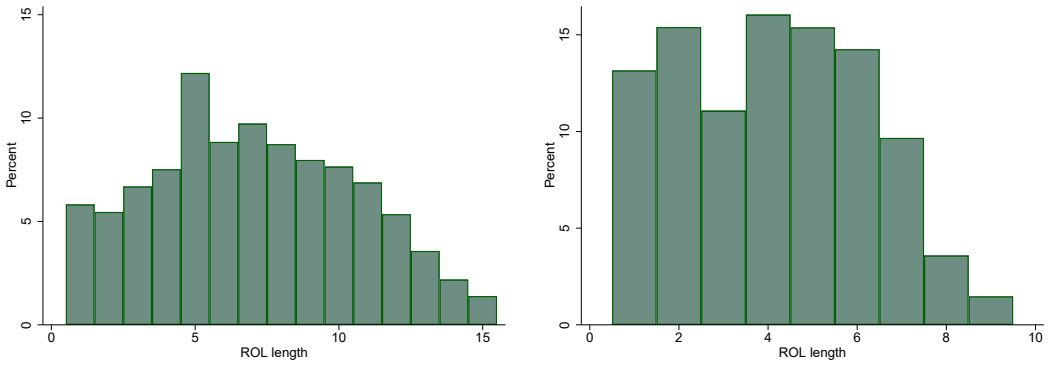
(c) Q3

(d) Q4

Note: This figure shows...

A.2 ROL position

Figure A.2: Position in ROL matched

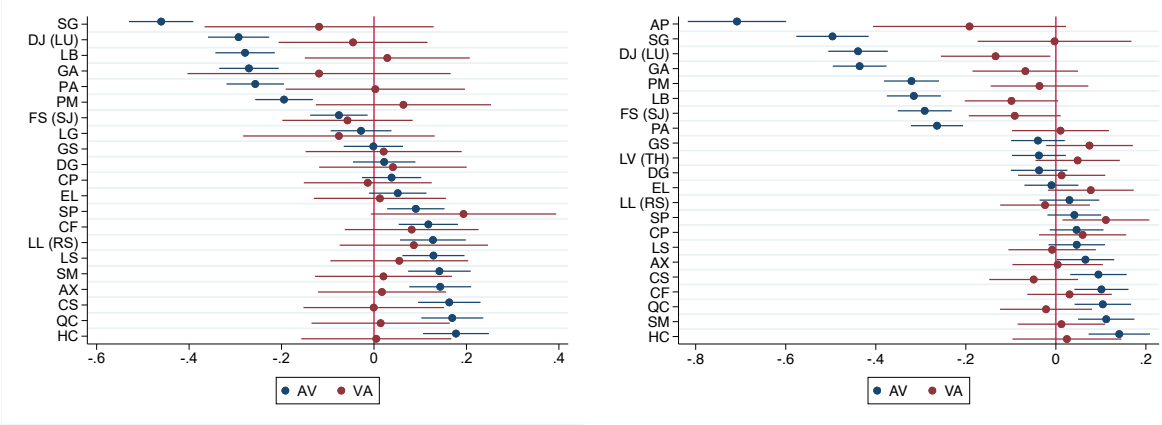


(a) Pooled 1993-1995 (b) Pooled 1996-1998

NOTE: This figure shows...

A.3 CSEC take-up

Figure A.3: CSEC take-up

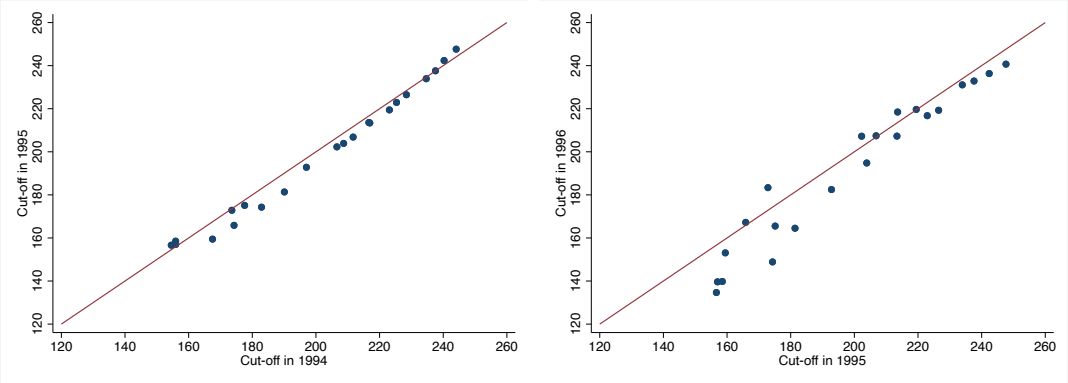


(a) Before 1996 (b) After 1996

NOTE: This figure shows...

A.4 Cut-offs and peers

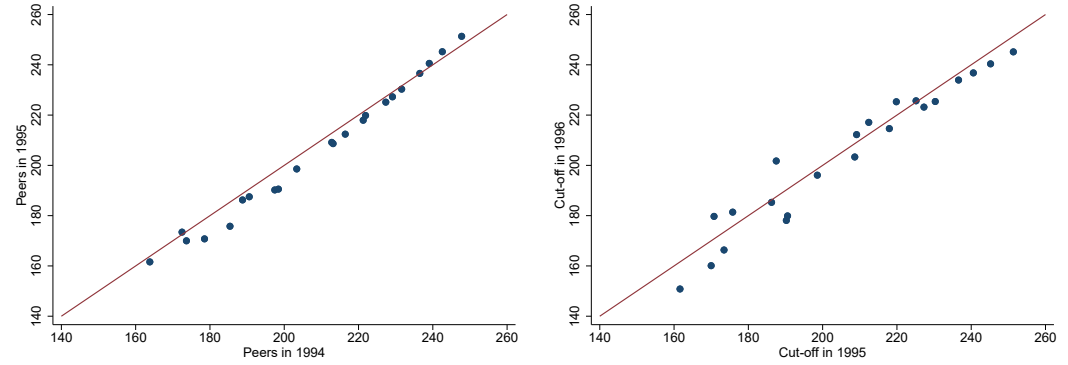
Figure A.4: Cut-offs



(a) Cut-offs 1994-1995 (b) Cut-offs 1995-1996

NOTE: This figure shows...

Figure A.5: Peers



(a) Peers 1994-1995 (b) Peers 1995-1996

NOTE: This figure shows...

A.5 Validation

$$Y_i = \kappa_0 + \phi \hat{Y}_i + \sum_{l=1}^{L} \kappa_l p_{il} + e_i$$

$$\hat{Y}_i = \pi_0 + \sum_{l=1}^{L} [\pi_l D_{il} + \omega_l p_{il}] + v_i$$

A.6 Weak truth telling

$$P[y_i^R = (j_1, ..., j_R) \mid x_i; \beta] = \frac{exp(x'_{ij_1}\beta)}{\sum_{l\in\mathcal{J}} exp(x'_{il}\beta)} \times ... \times \frac{exp(x'_{ij_R}\beta)}{\sum_{l\in\mathcal{J}\setminus\{j_1...j_{R-1}\}} exp(x'_{il}\beta)}$$

A.7 Decomposition

Realized outcome:

$$Y_i = \sum_j \mathbb{1}[D_i = j] Y_{ij}$$

Population moment can be decomposed:

$$\begin{aligned} A_i =& (1/J)\sum_j Y_{ij} \\ =& (1/J)\sum_j \alpha_j + X_i'\beta + (1/J)\sum_j \epsilon_{ij} \\ =& \bar{\alpha} + X_i'\beta + \bar{\epsilon}_i \end{aligned}$$

$$\begin{aligned} Y_{ij} =& (\bar{\alpha} + X_i'\beta + \bar{\epsilon}_i) + \alpha_j + X_i'\beta + \epsilon_{ij} - (\bar{\alpha} + X_i'\beta + \bar{\epsilon}_i) \\ =& (\bar{\alpha} + X_i'\beta + \bar{\epsilon}_i) + (\alpha_j - \bar{\alpha}) + (\epsilon_{ij} - \bar{\epsilon}_i) \\ =& A_i + ATE_j + M_{ij} \end{aligned}$$

$$\begin{aligned} E[Y_i \mid D_i = j] =& E[A_i \mid D_i = j] + E[ATE_j \mid D_i = j] + E[M_{ij} \mid D_i = j] \\ =& Q_j + ATE_j + E[M_{ij} \mid D_i = j] \end{aligned}$$