Market Entry and Plant Location in Multi-Product Firms

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Abstract

Multinational firms offer diverse products across countries due to variation in consumer tastes and production costs. Cannibalization forces further influence product variety. Using car-industry data, we estimate a quantitative model in which manufacturers decide where to produce and sell their products, featuring complementarities and substitutabilities across choices due to cannibalization, returns to scale and transport cost-savings. We provide a new algorithm to solve this class of combinatorial discrete choice problems. We examine the impact of demand and supply factors on product variety and study the effect of country-specific subsidies on global production, market offerings, and prices. Counterfactuals highlight the relevance of interdependencies and demonstrate how consumption and production subsidies affect foreign outcomes through firms' production relocation.

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1 Introduction

Multinational, multiproduct firms play an outsized role in shaping the global economy and account for a large share of global manufacturing output and employment. Although the literature on international trade often treats these two facets—their multiproduct nature and multinational presence—in isolation, the choices of the group of markets to serve, the set of products to sell and to export in each of those markets, and the location of production of these products are inherently interdependent.

In this paper, we study how policies influence the global production decisions of multiproduct firms, highlighting the interactions between country-specific demand, supply, and trade policies and shocks. As firms optimize their production and market entry decisions jointly, policy changes in one country can trigger non-trivial adjustments in product availability, sourcing, pricing, and trade patterns across the global economy.

To address these questions, we integrate a canonical export platform model, akin to Tintelnot (2017), with a standard multiproduct firm framework inspired by Hottman et al. (2016). In our model, a firm with a given product portfolio makes two key sets of discrete decisions: (1) selecting the markets in which to sell each product and (2) choosing the locations where production takes place. On the demand side, firms face a nested constant elasticity of substitution (CES) utility structure, which allows the elasticity of demand between a firm's own products to differ from the elasticity of substitution between varieties supplied by different firms. On the production side, firms can operate as multinationals, producing in multiple countries and potentially serving multiple markets from a single production location, though they must incur trade costs. Each decision entails a corresponding fixed cost that firms must account for.

In this setup, firms face a number of tradeoffs. First, the model features cannibalization across the products of a firm in a given market, i.e. when deciding to introduce a product in a given market a firm internalizes that a part of the market share of that product is going to come at the expense of its other products. Second, the model features cannibalization across production plants within the same firm: an additional plant diminishes the profitability of the firm's other plants, as is typical of export platform models. At the same time, there are increasing returns at the production plant level that introduce complementarities between producing and selling the same product in different markets. These decisions richly interact with trade costs, as there is a proximity-concentration tradeoff by which product entry and plant location decisions are co-determined; the benefits of serving a market depend on where the product is produced, and the value of a plant on the markets it serves. Due to the existence of fixed costs for both sets of discrete decisions, the problem inevitably delves into a combinatorial discrete choice problem.

The solution to this problem requires addressing the computational challenge of solving a model in which firms make discrete, interdependent choices, with some decisions exhibiting pairwise positive and others negative interdependencies. To circumvent these difficulties, we develop a novel algorithm for high-dimensional single-agent discrete choice problems that operates on the space of all sublattices of the original decision lattice. By defining a set-increasing mapping on these

sublattices, we leverage Tarski's fixed point theorem to obtain bounds on the optimal solution even when the objective function is neither globally supermodular nor submodular. The key conditions required to apply the algorithm are that the sign of the interdependencies between any two decisions is known and remains invariant to other remaining choices.

The mapping works by operating on the Boolean lattice of decision vectors and its complete sublattices. For each sublattice and each decision, we define a "best-case" scenario by choosing the supremum value for coordinates with positive interdependencies and the infimum value for those with negative interdependencies. Conversely, the "worst-case" scenario uses the infimum for positive interdependencies and the supremum for negative ones. Decisions with negative marginal benefits—even in their best case—are fixed at 0, while those with nonnegative marginal benefits—even in their worst case—are fixed at 1. Iterating this set-increasing mapping yields convergence, by Tarski's fixed point theorem, to bounds on the optimal decision vector.

We examine these margins of participation in the international economy in the global car industry. This industry provides an ideal setting for studying these questions, as it comprises multinational, multiproduct firms producing tradable goods. To this end, we leverage a rich global dataset that links quantities from the production location of individual car models to their country of sale. We complement this dataset with data on car prices and characteristics at the car model level for a subset of countries that account for more than 80% of global car sales.

The global car industry exemplifies the defining features of multinational, multiproduct firms, with automakers managing extensive product portfolios and strategically deciding over production and sales across markets. Brands offer an average of 3.56 models per country and segment, with some featuring as many as 23, and the typical model is sold in 12 markets, sometimes reaching up to 75. Production is similarly dispersed, with models manufactured in an average of 1.5 locations and roughly 44.5% of production exported when serving multiple markets. These patterns underscore the industry's rich structure, where firms must navigate trade-offs between market access, and production, fixed and trade costs—factors that our dataset allows us to study in detail.

To estimate the parameters of the demand equation, as well as the determinants of trade and marginal production costs, we use our merged dataset on prices and quantities. To estimate fixed costs, we introduce a moment inequality approach, constructing moments that are monotonic in firms' discrete choices. When we evaluate computationally feasible upper and lower bounds for the firms' optimal choices on the moment function we obtain valid moment inequalities due to this monotonicity. By using these bounds for estimation instead of the model-implied optimal choices, we avoid a common challenge in estimating combinatorial discrete choice problems—namely, the need to arbitrarily assign values to decisions where the bounds have not converged.

With our estimated model, we examine how production subsidies, consumption subsidies, and tariffs favoring electric vehicles could shape the global automobile industry. For each policy, we assess the impact of the United States enacting each policy at different values. The goal of our analysis is to shed light on the effectiveness of these policies in achieving their intended goals, rather than to judge whether there are normative reasons to implement these policies.

Our findings indicate that demand-side EV subsidies in the U.S. favor production in Europe and encourage EV market entry, while marginal cost subsidies shift production toward the U.S. Both of these policies reduce the share of non-electric alternative being offered, not only in the U.S., but also in other markets. Tariffs, in contrast, have limited effects on global production structures due to the high fixed costs associated with plant relocation, these policy tools induce production relocation of some products – those that are very appealing to US customers, but lead to reductions in overall variety offered for consumers. Additionally, U.S. policies generate spillover effects on other countries by influencing firms' production location decisions.

Our paper closely relates to different branches of the literature. First, it relates to papers that quantitatively study multinational production such as Ramondo and Rodríguez-Clare (2013), Tintelnot (2017), Arkolakis et al. (2018), Arkolakis et al. (2023), Castro-Vincenzi (2024) and Head et al. (2025)). In particular, our paper is related to those that allow for export platforms, whereby firms can operate plants that serve multiple markets, and increasing returns at the plant level. Second, this paper connects with the literature that studies multiproduct firms in international markets, such as Eckel and Neary (2010), Dhingra (2013), Mayer et al. (2014), Hottman et al. (2016), Head and Mayer (2019) and Arkolakis et al. (2021). Our paper merges these two literatures by studying the problem of jointly deciding in which locations to produce and in which markets to sell its goods for multinational, multiproduct firms in a quantitative framework.[1]

Third, our paper relates to the methodological literature concerned with solving models with multiple discrete choices. The methods developed in this literature have exclusively focused on problems in which the sign of the interdependencies is common across all the potential choices of the economic agent of interest, by devising algorithms that leverage the super- or sub-modularity of the agents' objective function (e.g. Antràs et al. 2017, Arkolakis et al. 2023 or Alfaro-Ureña et al. 2024). Our contribution to the literature is to provide a solution method for solving single-agent static problems featuring simultaneous positive and negative interdependencies. Our algorithm relates to methods developed to solve entry games, including Jia (2008) and Sabal (2025), which also require restrictions on the sign of the interdependence across firms' discrete choices.

Fourth, this paper contributes to the empirical literature on moment inequalities, as surveyed in Pakes et al. (2015) and Kline et al. (2021). Our approach differs from the literature in that we nest our solution approach in the estimation procedure instead of exploiting deviations from observed behavior to bound the parameters of the model. Similarly to standard procedures like simulated GMM or MLE, we apply a solution method at multiple realizations of latent variables and parameter vectors and compare simulated and observed moments to estimate the parameters of the model. However, our computationally feasible solution approach, in general, only provides bounds on the solution. Our contribution is to use such bounds to build moment inequalities that are valid whenever the moments are monotone in firms' discrete choices.

Closest to our paper is Head et al. (2025), who also study multinational production deci-

[1]It is worth noting that two papers, Bernard et al. (2018) and Helpman and Niswonger (2023), theoretically explore the multiproduct and multinational margins for large firms.

sions in the automotive sector, focusing on multi-stage global supply chains in the production of battery-electric vehicles (BEVs). Their setting similarly features complementarities across stages of the supply chain and substitutabilities within each stage, generating a combinatorial problem akin to ours. Methodologically, they introduce a novel approach by recasting the problem as a mixed-integer linear programming (MILP) model, allowing them to leverage fast commercial solvers specifically designed for this class of problems. However, their method cannot be directly applied to our context, as the interactions among firms' product-entry decisions—specifically, cannibalization effects—non-linearly affect the variable profits of each product. They use their model to explore how policies such as subsidies, tariffs, and local production requirements can have complex and sometimes unintended impacts on firms' production-location decisions and overall battery production and EV adoption.

Finally, the paper belongs to the literature that studies multinational production in the automotive industry and the analysis of government policy effects in the sector. It builds on work examining multinational production decisions in the automotive industry (e.g., Coşar et al. 2016, Head and Mayer 2019, Castro-Vincenzi 2024, Head et al. (2025))). While these prior studies analyze bilateral frictions, home market advantages, or production location choices, they do not fully integrate firms' endogenous decisions on both production and product entry. This paper also extends research on government policies in the automobile market (e.g., Goldberg 1995, Barwick et al. 2024, Allcott et al. 2024, Sabal 2025, Head et al. (2025)) by studying their effect on the global production decisions of these multiproduct firms, although abstracting from strategic interactions across firms.

The rest of the paper is structured as follows. In Section 2, we present a quantitative model of multinational, multiproduct firms. Section 3 describes the solution algorithm. Section 4 introduces the data and explains the estimation strategy. In Section 5, we present the model estimates, and we discuss counterfactual results in Section 6. Section 7 concludes.

2 Model

In this section, we develop a quantitative model of plant location decisions for multiproduct firms. Brand $b \in \mathcal{B}$ competing in segment s with a given set of products Ω^{bs} decides in which subset of potential countries $\mathcal{N} = \{1, 2, ..., N\}$ to produce and sell each of its products. Each product $m \in \Omega^{bs}$ is characterized by a set of demand and marginal cost shifters that potentially vary across destination and origin countries. We assume that brands are atomistic and do not internalize the effect of their decisions on segment-country expenditure.

We model brands' portfolio and market entry choices in two stages. In Stage 1, each brand b competing in a segment s draws for each product $m \in \Omega^{bs}$ a country in which it can produce at zero fixed cost, a set of plant location fixed cost shocks $\{\nu^p_{mo}\}_{m\in\Omega^{b,s},o}$, and set of market entry fixed cost shocks $\{\nu^e_{mn}\}_{m\in\Omega^{b,s},n}$.[2] Upon observing the realizations of these unobservables (to the econo-

[2]We denote collections of variables $\{X_m\}_{m\in\Omega^{b,s}}$ or $\{Y_n\}_{n\in\mathcal{N}}$ by $\{X_m\}_m$ and $\{Y_n\}_n$, respectively, for ease of

metrician), brand b simultaneously chooses in which countries to produce and in which countries to sell each of its products. In Stage 2, brand b chooses prices for each product it chose to sell in each country.

2.1 Demand, Marginal Costs, and Prices

In each country $n \in \mathcal{N}$, the representative consumer has nested CES preferences. Consumers in country n spend an exogenous amount E_{sn} in segment s, which we define by body type categories (e.g., SUV).[3] The consumption index of segment s in country n is given by,

$$C_{sn} = \Big(\int_{b \in \Omega_{sn}} (\psi_{bsn} C_{bsn})^{\frac{\sigma^B - 1}{\sigma^B}} db \Big)^{\frac{\sigma^B}{\sigma^B - 1}}, \tag{1}$$

where ψ_{bsn} is an exogenous brand-segment-country demand shifter and σ^B denotes the elasticity of substitution across brands within a segment. We parameterize ψ_{bsn} as,

$$\log(\psi_{bsn}) = \phi_b + \beta_1 \mathbb{1}\{h(b) = n\} + \beta_2 \log dist_{nh(b)} + u_{bsn},$$

where $h(b)$ denotes the headquarters country of brand b and u_{bsn} is an unobserved (to the econometrician) demand shock.

Brand b is unable to affect segment-country expenditure because it is atomistic. However, brand b internalizes the effect of its plant location and market entry decisions on its consumption index C_{bsn}, which is given by,

$$C_{bsn} = \Big(\sum_{m=1}^{M_{bs}} I_{mn} (\psi_{mn} C_{mn})^{\frac{\sigma^M - 1}{\sigma^M}} \Big)^{\frac{\sigma^M}{\sigma^M - 1}}, \tag{2}$$

where ψ_{mn} is an exogenous product-country demand shifter and σ^M denotes the elasticity of substitution across models within a brand-segment. If the brand offers product m in country n, $I_{mn} = 1$; otherwise, $I_{mn} = 0$. We parametrize demand shifter ψ_{mn} as,

$$\log(\psi_{mn}) = \phi_m + \beta_3 \log(HP_m) \log(GDPpc_n) + \beta_4 \log(Weight_m) \log(GDPpc_n) + u_{mn}, \tag{3}$$

where $GDPpc_n$ denotes the GDP per capita of country n, HP_m denotes the horsepower of product m and u_{mn} denotes an unobserved (to the econometrician) demand shock.

The revenue share of brand b within segment s in country n is given by,

$$S_{bsn} = \frac{(P_{bsn}/\psi_{bsn})^{1-\sigma^B}}{\sum_{b' \in \mathcal{B}} (P_{b'sn}/\psi_{b'sn})^{1-\sigma^B}}, \tag{4}$$

notation.

[3] A possible micro-foundation is a Cobb-Douglas nest over automobiles and an outside good, followed by a Cobb-Douglas nest over a discrete set of possible segments (body types).

where the brand-segment-country exact price index is given by,

$$P_{bsn} = \Big(\sum_{m=1}^{M_{bs}} I_{mn} \Big(\frac{P_{mn}}{\psi_{mn}} \Big)^{1-\sigma^M} \Big)^{\frac{1}{1-\sigma^M}}. \tag{5}$$

The price index P_{bsn} is a sufficient statistic capturing all endogenous factors that affect revenues for brand b in segment-country sn.

The brand chooses which of its M_{bs} products to introduce in each country. The demand equation for product m in country n is,

$$C_{mn} = A_{sn}(\psi_{bsn})^{\sigma^B-1}(\psi_{mn})^{\sigma^M-1}(P_{bsn})^{\sigma^M-\sigma^B}(P_{mn})^{-\sigma^M}, \tag{6}$$

where A_{sn} is an exogenous segment-country demand shifter and the price of product m in country n is denoted by P_{mn}. Thus, the revenue share of product m *within* brand-segment bs in country n is,

$$S_{mn} = \frac{(P_{mn}/\psi_{mn})^{1-\sigma^M}}{\sum_{k=1}^{M_{bs}}(P_{kn}/\psi_{kn})^{1-\sigma^M}}. \tag{7}$$

Products within a brand are more substitutable than across brands, causing $\sigma^M > \sigma^B$. Holding fixed prices at finite values, it follows that if brand b offers product m in country n, this leads to positive demand for product m in country n, as per equation (6), but also to a decrease in the the brand-segment price index, as observed in equation (5). The decline in the brand-segment price index reflects the greater availability of products within a brand, which causes a decline in the demand for other products offered by the same brand in country n. We refer to this phenomenon as *cannibalization*.

We assume that marginal production costs of product m in origin country o are given by,

$$c_{mo} = \kappa_b\kappa_o\kappa_s dist_{oh(b)}^{\kappa_5} \exp(\kappa_6 \mathbb{1}\{o = h(b)\}) HP_m^{\kappa_7} Weight_m^{\kappa_8}, \tag{8}$$

where $dist_{oh(b)}$ denotes the distance between the origin country and the headquarters country of brand b and HP_m denotes the horsepower of product m. The marginal cost of supplying a unit of product m in country n when produced in country o depends additionally on iceberg trade costs,

$$\tau_{on} = \tau_n tariff_{on}^{\kappa_1} \exp(\kappa_2 dist_{on} + \kappa_3 dist_{on}^2 + \kappa_4 \mathbb{1}\{n = o\}). \tag{9}$$

At Stage 2, firms choose prices for products in each country given product offerings $\boldsymbol{I}_n^{bs} := (I_{1n}, I_{2n}, ..., I_{M_{bs}n})$ and production locations $\boldsymbol{D}^{bs} := [D_{mo}]_{m\in\Omega_{bs},o}$. The optimal pricing decision is given by a constant markup rule,

$$P_{mn}(\boldsymbol{D}_m) = \frac{\sigma^B}{\sigma^B - 1} \times \min_{o:D_{mo}=1} \{\tau_{on} c_{mo}\}, \tag{10}$$

where $\boldsymbol{D}_m = \{D_{mo}\}_{o\in\mathcal{N}}$ and $D_{mo} = 1$ if product m is produced in o (and is zero otherwise). A tractable feature of this model is that the price of each product is independent of the set of products that the brand sells in country n. The production location matrix induces a minimal marginal cost for each product m in each destination country n. At the pricing stage, firms perfectly know marginal production and sourcing costs and choose to source from a unique production location.

2.2 Variable Profits

Given production location and market entry decisions, as well as optimal prices, the variable profits for product m in country n are given by,

$$\pi_{mn}(\boldsymbol{I}_n^{bs}, \boldsymbol{D}^{bs}) = A_{bsn}\big(P_{bsn}(\boldsymbol{I}_n^{bs}, \boldsymbol{D}^{bs})\big)^{\sigma^M-\sigma^B}(P_{mn}(\boldsymbol{D}_m)/\psi_{mn})^{1-\sigma^M}, \tag{11}$$

where

$$P_{bsn}(\boldsymbol{I}_n^{bs}, \boldsymbol{D}^{bs}) = \Big(\sum_{m=1}^{M_{bs}} I_{mn}\Big(\frac{P_{mn}(\boldsymbol{D}_m)}{\psi_{mn}}\Big)^{1-\sigma^M}\Big)^{\frac{1}{1-\sigma^M}}. \tag{12}$$

The exogenous demand shifter $A_{bsn} := A_{sn}(\psi_{bsn})^{\sigma^B-1}$ summarizes the exogenous market conditions for brand b in segment s and country n.

2.3 Plant Location and Market Entry

Brand b in segment s simultaneously chooses its production location and market entry decisions for all products Ω^{bs} so as to maximize total profits. We assume the fixed cost of offering product m in market n is denoted by,

$$F_{mn}^e = \gamma_0 + \gamma_1 dist_{nh(b)} + \nu_{mn}^e \tag{13}$$

and the fixed cost of producing product m in origin o is,

$$F_{mo}^p = (1 - d_{mo}^p)(\gamma_2 + \nu_{mo}^p), \tag{14}$$

where d_{mo}^p follows a multinomial distribution with N possible realizations (as many as the number of origin countries), one trial, and success probability $p_{mo} = \frac{e^{\gamma_3 \mathbf{1}\{o=h(b)\}}}{\sum_{k=1}^N e^{\gamma_3 \mathbf{1}\{k=h(b)\}}}$.[4] Fixed cost unobservables (to the econometrician) are iid and normally distributed, i.e., $\nu_{mn}^e \sim_{iid} \mathbb{N}(0, \sigma_e)$ and $\nu_{mo}^p \sim_{iid} \mathbb{N}(0, \sigma_p)$. Our specifications allow for both production plant fixed costs and market entry fixed costs that vary at the product-country level. In the data, we only observe products that are produced in at least one country. By assuming that each product can be produced in some country at zero fixed cost, we directly match this feature of the data.[5]

[4] By definition of the multinomial distribution, a restriction is that $\sum_{o\in\mathcal{N}} p_{mo} = 1$.

[5] Absent this assumption, the model with normal fixed costs only matches this feature of the data in certain regions of the support of the fixed cost distribution, generating a selection problem.

Given fixed costs and variable profits, brand b in segment s solves:

$$\max_{\boldsymbol{I}^{bs},\boldsymbol{D}^{bs}} \Pi_{bs}(\boldsymbol{I}^{bs},\boldsymbol{D}^{bs}) = \sum_{n=1}^{N}\sum_{m=1}^{M_{bs}}\{I_{mn}(\pi_{mn}(\boldsymbol{I}_n^{bs},\boldsymbol{D}^{bs}) - F_{mn}^{e}) - D_{mn}F_{mn}^{p}\}. \tag{15}$$

Brand b solves for the $M_{bs} \times N$ matrices $\boldsymbol{I}^{bs}$ and $\boldsymbol{D}^{bs}$ to maximize its profits, internalizing the interdependence across each of the multiple binary choices it faces. A feature of this problem is that brand b must choose among many binary choices, some of which are pairwise complements and some pairwise substitutes.[6] For example, in this model, producing and selling a given product are complementary decisions. Indeed, producing product m can only reduce trade costs for such a product in any country, which increases the returns from offering the product. However, selling product m in country n decreases the returns from offering product $m' \neq m$ in country n, due to cannibalization. Therefore, such binary choices are substitutes.

In what follows, we leverage such properties of the model to tractably study this problem. A brute force solution approach is clearly infeasible in light of large number of potential combinations that must be evaluated to find the solution.[7]

3 Solution Method

In this section, we provide a method for solving combinatorial discrete choice problems with both complementary and substitutable discrete choices. We first provide an approach that bounds the solution of the problem. Then, we show that we can implement the branching algorithm from Arkolakis et al. (2023) to obtain a full solution of the problem, starting from the bounds.

3.1 First-Step Algorithm

Brands solve a problem of the form,

$$C^* = \text{argmax}_{C\in\mathcal{B}^J}\pi(C) \tag{16}$$

where $\mathcal{B}^J$ is the set of all 2^J $J-$dimensional binary vectors, i.e., the J-dimensional Boolean set. To formalize the notion of interdependence across binary choices, we start by defining the marginal value of any discrete choice i under C.

Definition 1 (Marginal Value) *The marginal value of discrete choice i under C is given by,*

$$\sigma_i(C) := \pi(C^{i\to 1}) - \pi(C^{i\to 0}) \tag{17}$$

where $C^{i\to 1}$ is the vector C with the i^{th} coordinate set to 1 (analogously for $C^{i\to 0}$).

[6] We discuss formally what we mean by "complements" and "substitutes" in Section 3 and prove properties of the model in Appendix A.

[7] With two products and 15 countries, a brute-force solution approach requires evaluating the profit function $2^{2\times 2\times 15} \approx 10^{18}$ times, which is computationally infeasible.

Thus, $\sigma_i(C)$ is the change in $\pi(C)$ when changing the i^{th} coordinate from zero to one holding all other coordinates constant. We also define the cross-partial of i and j under C.

Definition 2 (Cross-Partial) *For any two choices $i, j \in \{1, 2, ..., J\}$, we define the cross-partial of i and j under C as*

$$\sigma_{ij}(C) := \sigma_j(\sigma_i(C)). \tag{18}$$

Thus, $\sigma_{ij}(C)$ is the difference in marginal value of coordinate i when j is set to 1 relative to when j is set to 0.

A key requirement for our algorithm is that for all pairs of discrete choices i and j, the *sign* of $\sigma_{ij}(C)$ is known and independent of C. We state this assumption formally below.

Assumption 1 *For all $C \in \mathcal{B}^J$, and any $i, j \in \{1, 2, ..., J\}$, the sign of $\sigma_{ij}(C)$ is known and independent of C.*

Throughout this paper, we refer to two binary choices i and j as being *complements* when $\sigma_{ij} > 0$ and as *substitutes* when $\sigma_{ij} < 0$.

To develop our algorithm, we exploit Tarski's fixed-point theorem (Tarski 1955), stated below.

Theorem 1 (Tarski 1955) *Let $(L, \leq)$ be a complete lattice. Suppose $F : L \to L$ is monotone increasing; i.e., for all $x, y \in L$, $x \leq y \implies F(x) \leq F(y)$. The set of fixed points of F is then a complete lattice with respect to $\leq$.*

Denote as $S(\mathcal{B}^J)$ the set of complete sublattices of $\mathcal{B}^J$.[8] If $J = 2$, then $\mathcal{B}^J = \{(0,0), (1,0), (0,1), (1,1)\}$ and

$$\begin{aligned} S(\mathcal{B}^J) = \big\{&\emptyset, \{(0,0)\}, \{(1,0)\}, \{(0,1)\}, \{(1,1)\}, \{(0,0),(1,0)\}, \{(0,0),(0,1)\}\} \\ &\{(0,0),(1,1)\}, \{(1,0),(1,1)\}, \{(0,1),(1,1)\}, \{(0,0),(1,0),(1,1)\}, \ldots, \\ &\{(0,0),(1,0),(0,1),(1,1)\}\big\}. \end{aligned}$$

The set $S(\mathcal{B}^J)$ together with the partial order $\subseteq$ yields lattice $(S(\mathcal{B}^J), \subseteq)$ with supremum $\mathcal{B}^J$ and infimum $\emptyset$. As a non-empty and finite lattice, it is a complete lattice. Thus, Tarski (1955) applies to any $F : S(\mathcal{B}^J) \to S(\mathcal{B}^J)$ that is monotone increasing.

To construct a set-increasing mapping in $S(\mathcal{B}^J)$, define, for any $\boldsymbol{C} \in S(\mathcal{B}^J)$,

$$\overline{\Omega}(\boldsymbol{C}) = \{i \in \{1, 2, ..., J\} : \sigma_i(\pi(\sup_i(\boldsymbol{C}))) < 0\} \tag{19}$$

$$\underline{\Omega}(\boldsymbol{C}) = \{i \in \{1, 2, ..., J\} : \sigma_i(\pi(\inf_i(\boldsymbol{C}))) \geq 0\} \tag{20}$$

[8] A lattice is a partially ordered set $(L, \leq)$ such that for any $\{a, b\} \subseteq L$, the infimum and the supremum are both in L. A complete lattice is a lattice $(L, \leq)$ such that any subset $S \subseteq L$ has a supremum and infimum in L. Note that any finite lattice is complete.

where, for each $i \in \{1, 2, ..., J\}$ and $j \in \{1, 2, ..., J\}$,

$$[\sup_i(\boldsymbol{C})]_j := \mathbb{1}[\sigma_{ij} \geq 0][\sup(\boldsymbol{C})]_j + \mathbb{1}[\sigma_{ij} < 0][\inf(\boldsymbol{C})]_j \tag{21}$$

$$[\inf_i(\boldsymbol{C})]_j := \mathbb{1}[\sigma_{ij} \geq 0][\inf(\boldsymbol{C})]_j + \mathbb{1}[\sigma_{ij} < 0][\sup(\boldsymbol{C})]_j. \tag{22}$$

We define mapping $F : S(\mathcal{B}^J) \to S(\mathcal{B}^J)$ as,

$$F(\boldsymbol{C}) = \{C \in \boldsymbol{C} \colon C_i = 0 \text{ and } C_{i'} = 1 \, \forall i \in \overline{\Omega}(\boldsymbol{C}) \text{ and } \forall i' \in \underline{\Omega}(\boldsymbol{C})\}. \tag{23}$$

Intuitively, by Assumption 1, it is always possible to define a coordinate-specific supremum and infimum. Since the sign of all pairwise interdependencies are known and independent of third choices, one can always define the vectors $\sup_i(\boldsymbol{C})$ and $\inf_i(\boldsymbol{C})$ that maximize and minimize the marginal value of i, respectively. If at the "worst-case scenario" for i, $\inf_i(\boldsymbol{C})$, the marginal value of i is positive, then $i \in \underline{\Omega}(\boldsymbol{C})$ and $F(\boldsymbol{C})$ returns only vectors C with $C_i = 1$. If at "best-case scenario" for i, $\sup_i(\boldsymbol{C})$, the marginal value of i is negative, then $i \in \underline{\Omega}(\boldsymbol{C})$ and $F(\boldsymbol{C})$ returns only vectors C with $C_i = 0$.

We prove that the mapping F is set-increasing.

Theorem 2 (Set-Increasing F) *The mapping $F : S(\mathcal{B}^J) \to S(\mathcal{B}^J)$ is set-increasing. That is, $\boldsymbol{C} \subseteq \boldsymbol{C}'$ implies that $F(\boldsymbol{C}) \subseteq F(\boldsymbol{C}')$.*

Proof. See Appendix B.1. ■

Corollary 1 (Set-Increasing $\overline{F}$ and $\underline{F}$) *Mappings $\overline{F}$ and $\underline{F}$ are set-increasing, where*

$$\overline{F}(\boldsymbol{C}) = \{C \in \boldsymbol{C} \colon C_i = 0 \ \forall i \in \overline{\Omega}(\boldsymbol{C})\}, \tag{24}$$

$$\underline{F}(\boldsymbol{C}) = \{C \in \boldsymbol{C} \colon C_i = 1 \ \forall i \in \underline{\Omega}(\boldsymbol{C})\}. \tag{25}$$

Proof. Follows immediately from the proof of Theorem 2. ■

Theorem 2 results from the monotonicity of the infimum and supremum together with Assumption 1. Note that for $\boldsymbol{C} \subseteq \boldsymbol{C'}$, it must be that for all i, $\sigma_i(\pi(\sup_i(\boldsymbol{C}))) \leq \sigma_i(\pi(\sup_i(\boldsymbol{C'})))$ and $\sigma_i(\pi(\inf_i(\boldsymbol{C}))) \geq \sigma_i(\pi(\inf_i(\boldsymbol{C'})))$. Thus, $\overline{\Omega}(\boldsymbol{C'}) \subseteq \overline{\Omega}(\boldsymbol{C})$ and $\underline{\Omega}(\boldsymbol{C'}) \subseteq \underline{\Omega}(\boldsymbol{C})$, implying $F(\boldsymbol{C}) \subseteq F(\boldsymbol{C'})$. Intuitively, starting at worse "worst-case scenarios" and better "best-case scenarios" solves fewer coordinates, leading F to rule out fewer vectors.

We describe the first-step procedure in Algorithm 3 for future reference.

Algorithm 3 (First-Step Algorithm) *We implement the algorithm as follows:*

1. *Start with $\boldsymbol{C}_0 = \mathcal{B}^J$.*
2. *Apply F to obtain $\boldsymbol{C}_1 = F(\boldsymbol{C}_0)$; note that $\boldsymbol{C}_1 \in S(\mathcal{B}^J)$.*
3. *Obtain $\boldsymbol{C}_k = F(\boldsymbol{C}_{k-1})$ for $k = 1, 2, ...$ until convergence.*

Proposition 1 states that Algorithm 3 converges monotonically to a set $\boldsymbol{C}^f$ such that $C^* \in \boldsymbol{C}^f$.

Proposition 1 *Algorithm 3 has the following properties:*

1. $\boldsymbol{C}_k = F(\boldsymbol{C}_{k-1}) \subseteq \boldsymbol{C}_{k-1}$ *for all* $k \in \mathbb{N}$.
2. *It converges to a fixed point* $\boldsymbol{C}^f$ *in a finite number of iterations* K *i.e.,* $F(\boldsymbol{C}_T) = \boldsymbol{C}^f$ *for all* $T \geq K$.
3. *The solution to the problem* C^* *is an element of* $\boldsymbol{C}^f$.

Proof. Due to Tarski (1955), we can start at $\boldsymbol{C}_0 = \mathcal{B}^J$ and iteratively apply F until convergence to fixed point $\boldsymbol{C}^f$, which is the supremum of the set of fixed points. Convergence occurs monotonically because F is set-increasing, so that $\boldsymbol{C}_1 = F(\boldsymbol{C}_0) \subseteq \boldsymbol{C}_0$ implies that $F(\boldsymbol{C}_1) \subseteq F(\boldsymbol{C}_0)$. The algorithm convergences in a finite number of iterations, because the number of iterations is bounded above by J, the number of binary decisions that are to be solved. $\{C^*\}$, the unique (almost surely) solution to the brand's problem is also a fixed point, since $C_i^* = 1$ if and only if the marginal value of i is positive i.e., $\sigma_i(C^*) \geq 0$. Thus, $C^* \in \boldsymbol{C}^f$. ■

We have proven that we can apply an iterative solution procedure that bounds the argmax of any combinatorial discrete choice problem that features simultaneous complementarities and substitutabilties, satisfying Assumption 1.

Finally, we note that a consequence of Corollary 1 and Proposition 1 is trivially that an algorithms that mimic Algorithm 3 but that apply mappings $\overline{F}$ or $\underline{F}$ instead of F also converge to sets $\overline{\boldsymbol{C}}^f$ and $\underline{\boldsymbol{C}}^f$ with the properties that $\boldsymbol{C}^f \subseteq \overline{\boldsymbol{C}}^f$ and $\boldsymbol{C}^f \subseteq \underline{\boldsymbol{C}}^f$.

3.1.1 Interdependencies in the Multinational Brand's Problem

The objective function of each brand b features an interdependence structure across discrete choices that features both complementary and substitutable discrete choices. Under $\sigma^M > \sigma^B$, the sign of the interdependence across any pair of binary choices is independent of third choices, and satifies Assumption 1. That is, any pair of binary choices are always either complements of substitutes regardless of the value of $(\boldsymbol{I}^{bs}, \boldsymbol{D}^{bs})$.

1. D_{mo} and I_{mn} are complementary across markets for any m and any $o, n \in \mathcal{N}$ due to trade costs and increasing returns at the production plant level. Indeed, producing product m in more markets can only decrease the marginal cost of selling product m in any given country, which increases product m sales. Selling product m in a country increases the revenues for the product and therefore the returns from producing it.

2. D_{mo} and I_{sn} for $m \neq s$, and for any $o, n \in \mathcal{N}$, are substitutes. This is due to cannibalization. Indeed, producing model m in any country o can only decrease the price of model m in country n. This has the effect of attracting sales towards model m and away from other models $s \neq m$ sold in the same country.

3. I_{mn} and I_{sn} for $m \neq s$ are substitutes due to cannibalization within markets.

4. I_{mn} and I_{st} are independent for $n \neq t$ and for all $m, s \in \Omega^{bs}$ because demand across markets is independent.

5. D_{mn} and D_{mt} are substitutes for $n \neq t$. The marginal gains (from reduced trade costs) of producing a product in a given plant can only decrease as the product is produced in more plants, since fewer countries will find it optimal to source from the given plant.

6. D_{mn} and D_{so} are substitutes for $m \neq s$. This substitutability is potentially strict only in markets where m and s are both offered. The intuition in these cases is that the weak decline in the price of some model m caused by an additional production plant leads to a lower increase in sales whenever product s is also produced at lower prices.

We exploit this property of the brand's payoff function to apply Algorithm 3. We use this algorithm both at the estimation stage and for performing counterfactual exercises.

3.1.2 Simple Example

We provide a simple example to illustrate how Algorithm 3 works. Consider a case with three choices such that $\sigma_{12} > 0$, $\sigma_{13} < 0$, and $\sigma_{23} < 0$. A payoff function that yields this interdependence structure is,

$$\pi(C) = \sum_{i=1}^{3} C_i[\pi_i(C) - F_i],$$

where

$$\pi_i(C) = A_i + \sum_{j \neq i} \gamma_{ij} C_j,$$

and $\gamma_{12} = 1/4$, $\gamma_{13} = -2$, $\gamma_{23} = -1$, $(A_1, A_2, A_3) = (1/2, 1, 4)$, and $F_i = 1$. Then,

$$\boldsymbol{C}_0 = \{(0,0,0), (1,0,0), (0,1,0), (0,0,1), (1,1,0), (1,0,1), (0,1,1), (1,1,1)\}.$$

As $\sigma_{12} > 0$, $\sigma_{13} < 0$, $\sup(\boldsymbol{C}_0) = (1,1,1)$, and $\inf(\boldsymbol{C}_0) = (0,0,0)$, it holds that

$$\sup_1(\boldsymbol{C}_0) = [\cdot, [\sup(\boldsymbol{C}_0)]_2, [\inf(\boldsymbol{C}_0)]_3] = [\cdot, 1, 0].$$

Similarly,

$$\sup_2(\boldsymbol{C}_0) = [1, \cdot, 0] \qquad \sup_3(\boldsymbol{C}_0) = [0, 0, \cdot]$$

and

$$\inf_1(\boldsymbol{C}_0) = [\cdot, 0, 1] \qquad \inf_2(\boldsymbol{C}_0) = [0, \cdot, 1] \qquad \inf_3(\boldsymbol{C}_0) = [1, 1, \cdot].$$

Under this payoff function,

$$\{\sigma_1(\pi(\sup_1(\boldsymbol{C}_0))) = -1/4 < 0, \sigma_2(\pi(\sup_2(\boldsymbol{C}_0))) = 1/4 \geq 0, \sigma_3(\pi(\sup_3(\boldsymbol{C}_0))) = 13/4 \geq 0\}$$
$$\{\sigma_1(\pi(\inf_1(\boldsymbol{C}_0))) < 0, \sigma_2(\pi(\inf_2(\boldsymbol{C}_0))) = -3 < 0, \sigma_3(\pi(\inf_3(\boldsymbol{C}_0))) \geq 0\}$$

then

$$\overline{\Omega}(\boldsymbol{C}_0) = 1 \qquad \text{and} \qquad \underline{\Omega}(\boldsymbol{C}_0) = 3.$$

Then, applying mapping F, we obtain,

$$F(\boldsymbol{C}_0) = \{C \in \boldsymbol{C}_0 : C_1 = 1, C_3 = 0\} = \{(0,1,1), (0,0,1)\} = \boldsymbol{C}_1.$$

The algorithm proceeds by applying F to $\boldsymbol{C}_1$. In effect, the algorithm has already solved for decisions 1 and 3. In the next iteration, the algorithm checks,

$$\sigma_2(\pi(\sup_2(\boldsymbol{C}_1))) = A_2 - F_2 + \gamma_{23} = 1 - 1 - 1 = -1 < 0,$$

and so $F(\boldsymbol{C}_1) = \{(0,0,1)\}$.

3.2 Applying Algorithm 3 to the Value Function

While Algorithm 3 is guaranteed to converge to a set containing the solution, the set may still be large when directly applied to an arbitrary payoff function π. Equivalently, the solution for only a relatively small number of discrete choices might be found at convergence. In our model of multiproduct, multinational production, there is an economic reason why the algorithm may fail to provide informative bounds on the solution when applied directly to Π_{bs}. For simplicity, consider a single-product firm deciding whether to set a production plant for product m in country o, D_{mo}. Algorithm 3 solves for a 1 if the marginal value of setting $D_{mo} = 1$ relative to $D_{mo} = 0$ at the decision-specific infimum is positive. In other words, if in the most unfavorable conditions for D_{mo} given the interdependence structure, the marginal gains from setting this choice to 1 are positive. At the initialization of the algorithm, the most unfavorable conditions for producing m in country o consist of having plants in all countries $k \neq o$ open for product m, and selling product m nowhere. Crucially, *conditional* on selling product m nowhere, the marginal gains from producing product m in country o will be zero minus the fixed cost of production and, therefore, unlikely to be positive.[9]

A way around this practical problem is to apply Algorithm 3 not to Π_{bs}, but to $V_{bs}(\boldsymbol{D}^{bs}) =$

[9]Under our assumption of normal fixed costs, it is possible that the fixed cost of production is negative but with a small probability under a large mean and relatively small variance.

$\max_{\boldsymbol{I}^{bs}} \Pi_{bs}(\boldsymbol{I}^{bs}, \boldsymbol{D}^{bs})$. To ensure that the algorithm can be applied, what is required is to show that the discrete choices in $\boldsymbol{D}^{bs}$ satisfy Assumption 1 under objective function V_{bs}. To see why this would help, note that in the previous example, we are now not comparing the gains from setting $D_{mo} = 1$ relative to $D_{mo} = 0$ conditional on selling m nowhere, but rather solving for product m offerings decisions under $D_{mo} = 1$ and separately under $D_{mo} = 0$ (conditional on all other production locations for product m being open). Such an evaluation is more likely to yield positive profits if the optimal offering choices conditional on $D_{mo} = 1$ yield higher profits than the optimal offering choices under $D_{mo} = 0$.

We show that for single-product brands, mapping $V_{bs}(\boldsymbol{D}^{bs})$ satisfies Assumption 1.

Proposition 2 *If $M_{bs} = 1$, the value function $V_{bs}(\boldsymbol{D}^{bs}) = \max_{\boldsymbol{I}^{bs}} \Pi_{bs}(\boldsymbol{I}^{bs}, \boldsymbol{D}^{bs})$ is such that D_{1o} and $D_{1o'}$ are substitutes for $o \neq o'$.*

Proof. See Appendix A.2.1. ■

When brands are multiproduct, we develop an algorithm based on the value function:

$$V_{b,s,m}(\boldsymbol{D}_m; \boldsymbol{I}_{-m}, \boldsymbol{D}_{-m}) := \max_{\boldsymbol{I}_m} \sum_{n=1}^{N} I_{mn} \Delta\pi_n^m(\boldsymbol{D}_m; \boldsymbol{I}_{-m}, \boldsymbol{D}_{-m}) - \sum_{n=1}^{N} D_{mn} F_{mn}^p, \tag{26}$$

where $\Delta\pi_n^m$ is the marginal value of selling product m in country n. The value function in equation (26) essentially conditions on all production location choices for all products and sales choices for products other than m, and maximizes the additional profits that the brand can earn by optimally choosing its sales decisions for model m across countries. In Appendix A.2.3, we show that conditional on the optimal solution $(\boldsymbol{I}^*_{-m}, \boldsymbol{D}^*_{-m})$ for all decisions not concerning product m, the program in equation (26) returns $\boldsymbol{I}^*_m$.

Of course, the objective is to find the optimal solution, so we cannot condition on $(\boldsymbol{I}^*_{-m}, \boldsymbol{D}^*_{-m})$. We prove that starting from any known bounds of the solution, we can iteratively apply Algorithm 3 product by product to obtain bounds of the solution. To apply Algorithm 3 under $V_{b,s,m}$, we prove in Appendix A.2 that mapping $V_{b,s,m}$ exhibits (i) decreasing differences in $\boldsymbol{D}_m$, and (ii) decreasing differences in $(\boldsymbol{I}_{-m}, \boldsymbol{D}_{-m})$. We describe the algorithm below.

Algorithm 4 (Production Location Solver)

1. *Start at the known bounds of the solution. At the first iteration, initialize with $(\overline{\boldsymbol{I}}^b, \overline{\boldsymbol{D}}^b) = (\mathbf{1}, \mathbf{1})$ and $(\underline{\boldsymbol{I}}^b, \underline{\boldsymbol{D}}^b) = (\mathbf{0}, \mathbf{0})$.*
2. *Start at $m = 1$.*
3. *Apply Algorithm 3 to $V_{b,s,1}$ until convergence. Construct the decision-specific infima and suprema using knowledge of substitutability across all discrete choices.*
4. *Repeat Step 3 for all products $m \in \{2, ..., M_{bs}\}$.*
5. *Repeat Steps 2-3 until convergence.*

Intuitively, the algorithm starts at any known bounds of the solution and exploits substitutability across all discrete choices according to the $V_{b,s,m}$ mapping. Consider the problem of solving for $D_{mo} = 1$. Due to the substitutabilty properties proven in Appendix A.2, the algorithm evaluates the marginal change in $V_{b,s,m}$ by setting $D_{mo'} = 1$ for all $o' \neq o$, and $D_{m'n} = I_{m'n} = 1$ for all $m' \neq m$ and for all $n \in \mathcal{N}$. If this marginal change is *positive*, the algorithm says that $\underline{D}_{mo} = 1 = D^*_{mo}$. Consider the problem of solving for $D_{mo} = 0$. In this case, the algorithm evaluates the marginal change in $V_{b,s,m}$ by assuming that $D_{mo'} = 0$ for all $o' \neq o$, and $D_{m'n} = I_{m'n} = 0$ for all $m' \neq m$ and for all $n \in \mathcal{N}$. If this marginal change is *negative*, the algorithm says that $\overline{D}_{mo} = 0 = D^*_{mo}$.

Proposition 3 *Algorithm 4 converges to bounds on the solution $\boldsymbol{D}^{bs,*}$ of brand b in segment s.*

Proof. See Appendix B.2. ■

Algorithm 4 consists of applying Algorithm 3 iteratively to different products under the $V_{b,s,m}$ function. It is, therefore, a composition of set-increasing mappings. A composition of increasing functions is itself increasing. As such, we can apply the Algorithms 3 and 4 iteratively and in any order provided we start at any known bounds of the solution. In practice, we start with Algorithm 4 to solve for production location decisions, and then apply Algorithm 3 to Π_{bs} to solve for both sales and production location decisions. We describe the implementation in detail in Appendix C.

3.3 Second-Step Algorithm

Algorithms 3 and 4 converge to sets of possible vectors (or matrices) containing the solution. To obtain a full solution, we show that we can implement a branching algorithm akin to Arkolakis et al. (2023) to obtain a full solution to the brand's problem. The branching algorithm consists of the following:

1. Choose any decision I_{mn} or D_{mo} such that:

$$[\underline{I}_{mn} = 0 \text{ and } \overline{I}_{mn} = 1] \text{ or } [\underline{D}_{mo} = 0 \text{ and } \overline{D}_{mo} = 1]$$

2. Apply the first-step algorithm conditional on 1 and conditional on 0.

3. If a full solution is obtained both conditional on 1 and conditional on 0, choose the one that yields the highest profits.

4. Otherwise, in each branch, choose another decision to set to both 1 and 0 and solve conditional on each of these cases.

5. Repeat until convergence.

Proposition 4 *The branching algorithm, when applied after our algorithms 3 or 4, yields the global solution of the brand's problem.*

Proof. See Appendix B.3. ■

Intuitively, the first-step algorithms can be seen as solving for a subset of the discrete decision of the brand e.g., if $\underline{D}_{mo} = 1$, this is equivalent to stating that $D^*_{mo} = 1$; if $\overline{D}_{mo} = 0$, the algorithm states that $D^*_{mo} = 0$. For those binary choices that remain unsolved, the branching algorithm picks one, conditions on values of 0 or 1 and then solves conditional on each of these values by applying in each "branch" the first-step procedure. This process is repeated in each of the branches until solutions are found in each of the branches. Given a finite number of possible solutions, these are compared by brute-force to find the maximum. As discussed in Arkolakis et al. (2023), a limitation of this procedure is that there is no guarantee that it will converge quickly to the solution. Indeed, in some cases, we find that the branching algorithm performs poorly and does not yield a solution in a reasonable amount of time. In practice, we implement the branching procedure but limit the number of possible branches – if that limit is surpassed, we simply use the bounds obtained from the first-step procedure.

4 Estimation

We estimate the model in two steps. In the first step, we estimate demand and marginal cost parameters, including the demand elasticities for each nest and product-, brand- and segment-specific demand shifters, and trade and marginal production costs. In the second stage, we estimate the determinants of the distributions of fixed costs associated with introducing a product in a country and producing a product in a particular location. We develop a moment inequality approach based on our solution method and moments that are monotone in brands' binary entry decisions.

4.1 Data

Our analysis uses three types of data: data on global car sales and production, data on car prices and product characteristics for a subset of countries, and data on bilateral measures of trade costs, namely distances and tariffs, and country characteristics.

Our global car sales and production data corresponds to 2019 and is compiled by IHS Markit using new car registrations of individual car products identifying the plant of assembly and the country of sale. The dataset contains information for 1,130 car products, 124 car brands, 56 countries of sale, and 30 production countries. A product is defined as a parent-brand-body type-nameplate.[10] We allow for 7 market segments defined by body type categories, namely, convertible, coupe, hatchback, MPV, SUV, Sedan and Wagon. In total, our dataset contains information on production and sales for 330 brand-segments. The average brand-segment has 3.5 car models in its portfolio, has production facilities in 2.4 countries and sells in 18.9 markets. The average model is produced in 1.5 country and sold in 12.1 markets.

We leverage data on car prices and characteristics at the product level for Australia, Brazil, China, Germany, France, India, Italy, Japan, Mexico, Spain, the United Kingdom and the United

[10] For example, a Toyota Corolla Sedan.

Table 1: Summary Statistics

Number of:	Mean (unweigh.)	Mean (weight.)	p25	p50	p75	p90	Max
Models (per brand-segment)	3.5	10.2	1	2	5	8	23
Sales countries (per model)	12.1	30.7	1	2	17	43	75
Production countries (per model)	1.5	3.1	1	1	1	3	12
Models sold (per country)	41.4	137.9	11	31	65	102	387
Sales countries (per brand-segment)	18.9	49.8	1	5	40	62	77
Production countries (per brand-segment)	2.4	8.1	1	1	2	5	19
Share exported (by model-prod. loc.)	38.7%	12.9%	0%	6.4%	94%	100%	100%
Share produced in HQs (by brand-segment)	80.2%	44.1%	63.2%	100%	100%	100%	100%
Share sold in HQs (by brand-segment)	61.2%	30.9%	19.9%	78.7%	100%	100%	100%

States for 2019. This dataset contains information about the list price and a host of characteristics of the car product, among which we use horsepower, car weight, and car length. For these car products and countries, we use the global car production dataset to identify their assembly location.

We measure trade costs using bilateral applied tariffs for each pair of countries and for cars with different engine types, sourced from ITC (2022). Additionally, we utilize standard bilateral geographical distance measures from CEPII (Mayer and Zignago 2011). Finally, we gather data on country characteristics, including GDP, population, average family size, and urbanization rates, from the World Bank, to project demand shifters on country-level characteristics.

4.2 Demand and Marginal Costs

First, we describe how we estimate trade costs and marginal production costs. From equations (8)-(10), we write estimating equation,

$$\begin{aligned}\log P_{mn} = {} & \alpha_n + \kappa_1 \log \text{tariff}_{on} + \kappa_2 \text{dist}_{on} + \kappa_3 \text{dist}_{on}^2 + \kappa_4 \mathbb{1}\{o = n\} \\ & + \alpha_b + \alpha_o + \alpha_s + \kappa_5 \log dist_{oh(b)} + \kappa_6 \mathbb{1}\{o = h(b)\} + \kappa_7 \log HP_m \\ & + \kappa_8 \log Weight_m + \varepsilon_{mn}. \end{aligned} \tag{27}$$

Relative to equations (8)-(10), equation (27) also includes an error term ε_{mn}. We include $\alpha_n = \log \tau_n$ to control for destination country-specific unobserved factors and policies, such as VAT or sales taxes, that determine observed final consumer prices (MSRP). We interpret ε_{mn} as measurement error and rule out the possibility that it is observed at the time of making offering, production, or sourcing decisions, which would generate a selection problem. Under this assumption, equation (27) can be estimated using OLS.

In this regression, we are identifying trade costs from differences in prices of specific car models

that are produced in the same location, and are sold in different countries.[11] These regression estimates allows to construct estimated trade costs as

$$\hat{\tau}_{on} = \hat{\tau}_n tariff_{on}^{\hat{\kappa}_1} \exp(\hat{\kappa}_2 dist_{on} + \hat{\kappa}_3 dist_{on}^2 + \hat{\kappa}_4 \mathbb{1}\{n = o\}). \tag{28}$$

The remaining parameters and fixed effects provide estimates of marginal production costs,

$$\log \hat{c}_{mo} = \hat{\alpha}_b + \hat{\alpha}_o + \hat{\alpha}_s + \hat{\kappa}_5 \log dist_{oh(b)} + \hat{\kappa}_6 \mathbb{1}\{o = h(b)\} + \hat{\kappa}_7 \log HP_m + \hat{\kappa}_8 \log Weight_m. \tag{29}$$

To estimate the demand shifters and elasticities, we leverage that our dataset contains both information on quantities and prices to construct revenue shares, which permits exploiting equations (4) and (7) for estimation. Based on such equations, we follow a two-step estimation procedure.

First, we estimate σ^M and ψ_{mn} using equation (7) and exploiting variation in product-country revenue shares within brands, segments, and destination countries. Combining equation (3) for ψ_{mn} with equation (7), we obtain estimating equation,

$$\begin{aligned} \log S_{mn} = {} & (1-\sigma^M) \log P_{mn} + \alpha_{bsn} \\ & + \tilde{\phi}_m + \tilde{\beta}_3 \log(HP_m) \log(GDPpc_n) + \tilde{\beta}_4 \log(Weight_m) \log(GDPpc_n) + \tilde{u}_{mn}, \end{aligned} \tag{30}$$

where $\tilde{\theta} := (\sigma^M - 1)\theta$. We estimate this regression by instrumenting the price of each product, $\log(P_{mn})$, with $\log \text{dist}_{on}$ and $\log \text{tariff}_{on}$ to deal with potential endogeneity issues.[12]

To estimate σ^B and ψ_{bsn}, we regress the log of S_{bsn} (given in equation (4)) on the log prices of that brand-segment and characteristics of the brand. We construct these prices by using the regression equation (30) and noticing that the structure of our model implies that $\log(P_{bsn}) \equiv \frac{\alpha_{bsn}}{(\sigma^M - 1)}$. Thus, we estimate via OLS,

$$\log(S_{bsn}) = (1-\sigma^B) \log(P_{bsn}) + \phi_{sn} + \check{\phi}_b !\text{`} \check{\beta}_1 \mathbb{1}\{h(b) = n\} + \check{\beta}_2 \log dist_{nh(b)} + \check{u}_{bsn}, \tag{31}$$

where $\check{\theta} := (\sigma^B - 1)\theta$ and $\phi_{sn} = (\sigma^B - 1) \log(P_{sn})$.

Finally, we recover segment-country demand shifter A_{sn} by exploiting that equation (6) implies,

$$P_{mn} C_{mn} = A_{sn} (P_{bsn})^{\sigma^M - \sigma^B} (\psi_{bsn})^{\sigma^B - 1} (\psi_{mn})^{\sigma^M - 1} (P_{mn})^{1-\sigma^M}. \tag{32}$$

The main challenge for the estimation of this equation is that we do not observe prices in all destination countries, n, but just for a subset. To solve this shortcoming of the data, we leverage our previous estimates for the demand shifters ψ_{mn} and ψ_{bsn}, and our estimates of marginal costs, to construct a predicted price $\hat{P}_{mn}$ for product m in destination country n being supplied from the relevant production location o, and we construct the related brand-segment-market level price

[11] See, e.g., Anderson and van Wincoop (2003), Atkin and Donaldson (2015) and Donaldson (2018).

[12] Strictly speaking, due to the CES assumption, equilibrium prices do not depend on unobserved demand factors, eliminating the need for an instrument.

index P_{bsn} using its formula (given in equation (5)). Then, we construct the destination-segment-shifter as follows, where we assume that we observe revenues with a measurement error that varies at the product-destination level,

$$\hat{P}_{mn}C_{mn} \equiv \frac{P_{mn}C_{mn}}{(P_{bsn})^{\sigma^M-\sigma^B}(\psi_{bsn})^{\sigma^B-1}(\psi_{mn})^{\sigma^M-1}(P_{mn})^{1-\sigma^M}} = A_{sn}e^{v_{mn}}, \tag{33}$$

so that we can estimate the A_{sn} as a segment-country fixed effect from the regression,

$$\log \hat{P}_{mn}C_{mn} = \log A_{sn} + v_{mn}. \tag{34}$$

This concludes our estimation strategy for the demand and marginal cost parameters of the model.

4.3 Fixed Costs: Moment Inequality Estimation

Given the demand and marginal cost parameters, we use moment inequalities to estimate fixed-cost parameters. Given the large number of discrete choices, implementing (simulated) GMM or MLE is generally infeasible, as they require computing the full solution of the brands' problem under multiple parameter vectors and draws of unobserved shocks.[13] To construct moment inequalities, we combine Algorithm 3 and 4 to obtain computationally feasible upper and lower bounds on brands' optimal choices. We combine these bounds on the firms' optimum with moment functions that are weakly increasing in the choices, thereby formulating valid moment inequalities.[14]

Formally, conditional on the demand and marginal cost estimates, a realization of brand-segment fixed cost shocks $\boldsymbol{\nu}_{bs} \equiv \{\nu^e_{mn}, \nu^p_{mn}\}_{m\in\Omega^{b,s},n}$ and a realization of the free plant for each product, our first-step solution procedure yields lower bounds $\{\underline{I}_{mn}(\boldsymbol{\nu}_{bs}), \underline{D}_{mn}(\boldsymbol{\nu}_{bs})\}_{m,n}$ and upper bounds $\{\overline{I}_{mn}(\boldsymbol{\nu}_{bs}), \overline{D}_{mn}(\boldsymbol{\nu}_{bs})\}_{m,n}$. Consider monotone moment functions $g(I, D; \boldsymbol{\gamma})$. We construct moment inequalities of the form,

$$\frac{1}{M}\sum_{m=1}^{M} \mathbb{E}_\nu[g_m(\underline{\boldsymbol{I}}_m(\boldsymbol{\nu}_{bs}), \underline{\boldsymbol{D}}_m(\boldsymbol{\nu}_{bs}); \boldsymbol{\gamma})] \leq \frac{1}{M}\sum_{m=1}^{M} g_m(\boldsymbol{I}_m, \boldsymbol{D}_m; \boldsymbol{\gamma}) \leq \frac{1}{M}\sum_{m=1}^{M} \mathbb{E}_\nu[g_m(\overline{\boldsymbol{I}}_m(\boldsymbol{\nu}_{bs}), \overline{\boldsymbol{D}}_m(\boldsymbol{\nu}_{bs}); \boldsymbol{\gamma})],$$

where $\mathbb{E}_\nu$ denotes that the expectation is with respect to random draws from the distribution of fixed cost shocks and the multinomial distribution determining the realization of the free plant for each product, and M denotes the total number of products in our sample. To implement these moment inequalities, we simulate 10 draws from the distribution of fixed cost unobservables, apply our solution method for each of the draws under each parameter vector $\boldsymbol{\gamma}$, and implement the procedure from Andrews and Soares (2010) to determine whether such a parameter vector $\boldsymbol{\gamma}$ is included in the 95% confidence set.

The procedure in Andrews and Soares (2010) is applicable to a set of unconditional moment

[13] As discussed in Section 3, while our two-step solution procedure is guaranteed to find the solution, only the first step of the algorithm converges rapidly. The branching algorithm has the disadvantage of not being guaranteed to converge quickly to the solution.

[14] We do not need moments to be increasing, we need them to be monotonic.

inequalities. We employ the following monotone moment functions to derive bounds on $\boldsymbol{\gamma}$.

1. Share of destination countries in which product m is offered:

$$g_m^1(\boldsymbol{I}_m, \boldsymbol{D}_m) = \frac{1}{N-1} \sum_{n=1}^{N} I_{mn}.$$

   This moment intuitively bounds parameter γ_0 (the mean fixed cost of offering product m in country n). If the simulated lower bound of this moment is larger than the observed value at $\boldsymbol{\gamma}$, then γ_0 is too small; if the simulated upper bound of this moment is smaller than the observed value, then γ_0 is too large.

2. Sold in destination in profit bin k:

$$g_m^{2,k}(\boldsymbol{I}_m, \boldsymbol{D}_m) = \mathbb{1}\Big\{ \sum_{n=1}^{N} I_{mn} \mathbb{1}\big\{n \in B^k\big\} > 0 \Big\} \text{ for } k \in \{1,2,3,4,5\},$$

   where B^k are quintile categories of profitability index,

$$A_{sn} \psi_{bsn}^{\sigma^B - 1} (\psi_{mn} / \overline{\phi}_m)^{\sigma^M - 1},$$

   and $\overline{\phi}_m$ is the average marginal cost of product m across all possible origins. The model predicts that more profitable products are more likely to be offered. However, the extent to which this is true also depends on the variance of the fixed cost distribution. If the variance of unobserved factors affecting fixed costs (σ_e) is very large, product offerings should appear less related to demand and marginal cost fundamentals. Therefore, the moments $g_m^{2,k}$ (and the bounds thereof) provide information about σ_e.

3. Percentiles p of sales destinations interacted with distance to HQ:

$$g_m^{3,p}(\boldsymbol{I}_m, \boldsymbol{D}_m) = Q_p\big(\big\{I_{mn} dist_{nh(b)} \mathbb{1}\{h(b) \neq n\}\big\}_{n=1}^{N}\big) \text{ for } p \in \{25, 50, 75\}.$$

   This moment provides information on γ_1, the parameter determining the relationship between market entry fixed costs and distance to the brand's headquarters country. If the true value of γ_1 were large, relatively fewer instances of market entry would be observed in distant countries; if it were small, relatively more entry would be observed even in distant countries.

4. Share of production locations entered (excluding HQ country):

$$g_m^4(\boldsymbol{I}_m, \boldsymbol{D}_m) = \frac{1}{N} \sum_{o=1}^{N} D_{mo} \mathbb{1}\big\{h(b) \neq o\big\}.$$

   This moment provides information on γ_2, the mean fixed cost of setting up a (non-free) production plant for product m. At low production plant fixed costs, it is more likely that

the brand chooses to set up a plant for a product. The opposite is true at high fixed costs.

5. Produced in location in profit bin k (exc. HQ) and produced in at least 2 locations:

$$g_m^{5,k}(\boldsymbol{I}_m, \boldsymbol{D}_m) = \mathbb{1}\Big\{\sum_{o=1}^{L} D_{mo}\mathbb{1}\big\{o \in W^k\big\}\mathbb{1}\big\{h(b) \neq o\big\} > 0\Big\}\mathbb{1}\Big\{\sum_{o=1}^{L} D_{mo} > 1\Big\},$$

for $k \in \{1, 2, 3, 4, 5\}$, where origin profit bins are defined using quintile categories of,

$$\phi_o^{1-\sigma^M} \sum_{n=1}^{N} \tau_{on}^{1-\sigma^M} A_{sn} \psi_{bsn}^{\sigma^B - 1} (\psi_{mn})^{\sigma^M - 1}, \tag{35}$$

which mimics the market access measure from Redding and Venables (2004). The model predicts that production locations that are "nearby" profitable destination countries for a given product m are more attractive (given fixed costs) due to lower trade costs. The model rationalizes deviations from such behavior through lower production plant fixed costs. Thus, a less pronounced relationship between market access and production location choices reflects a higher variability in production location fixed costs. This set of moments intuitively provides bounds on σ_p.

6. Produced in the HQ country:

$$g_m^6(\boldsymbol{I}_m, \boldsymbol{D}_m) = \mathbb{1}\big\{D_{mh(b)} = 1\big\}.$$

The probability that a product is produced in the brand's headquarters country intuitively provides information on γ_3, the parameter determining the probability that the free plant for product m is in the brand's headquarters country.

While our approach has the advantage of being computationally tractable, the informativeness of the procedure is an empirical question.

5 Estimation Results

In this section, we describe our parameter estimates.

5.1 Trade and Production Cost Estimates

We estimate trade and marginal cost parameters using 13,275 list prices for the products in the 12 countries for which we have data. We report the estimates in Table 2. The estimated elasticity of trade costs with respect to tariffs, κ_1, is 0.772^a. We find that trade costs increase concavely in geographic distance, with $\hat{\kappa}_2 > 0$ and statistically significant, but $\hat{\kappa}_3 < 0$ and statistically significant. We also find that conditional on geographic distance and tariffs, producing in the same

Table 2: Demand and Marginal Cost Parameter Estimates

	Parameter estimate	**Standard error**
Demand (Inner Nest) ($N = 8085$)		
σ^M	6.55	1.09
(log) Horsepower × (log) GDPpc ($\tilde{\beta}_3$)	0.134	0.058
(log) Weight × (log) GDPpc ($\tilde{\beta}_4$)	0.183	0.130
Demand (Upper Nest) ($N = 3420$)		
σ^B	2.97	0.130
Home Bias ($\check{\beta}_1$)	0.836	0.151
(log) Distance to Brand HQ ($\check{\beta}_2$)	-0.293	0.050
Marginal Costs (log) ($N = 7684$)		
Trade Costs		
(log) Tariff (κ_1)	0.772	0.063
Distance to Production Country (κ_2)	0.202	0.026
Squared Distance to Production Country (κ_3)	-0.061	0.017
$\mathbb{1}\{o = n\}$ (κ_4)	-0.004	0.008
Production Costs		
(log) Distance to Brand HQ (κ_5)	0.034	0.008
$\mathbb{1}\{o = h(b)\}$ (κ_6)	0.007	0.031
(log) Horsepower (κ_7)	0.650	0.049
(log) Weight (κ_8)	0.857	0.075

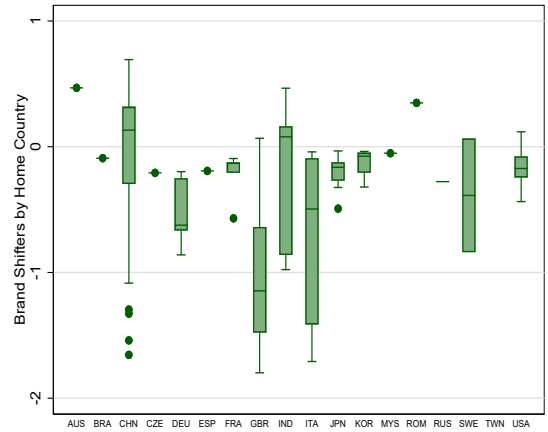
Notes: The inner nest demand estimation is conducted via IV-2SLS, instrumenting prices with tariffs and distance between the origin and destination country. Product and brand-segment-destination country fixed effects are included in the specification. Robust standard errors are reported. The upper nest demand estimation is conducted via OLS. We include segment-destination country fixed effects and report robust standard errors. The marginal cost specification is estimated via OLS, includes destination country fixed effects, and robust standard errors are reported.

country as the destination country reduces transport costs, though in a statistically insignificant manner (i.e., $\hat{\kappa}_4 \approx 0$).

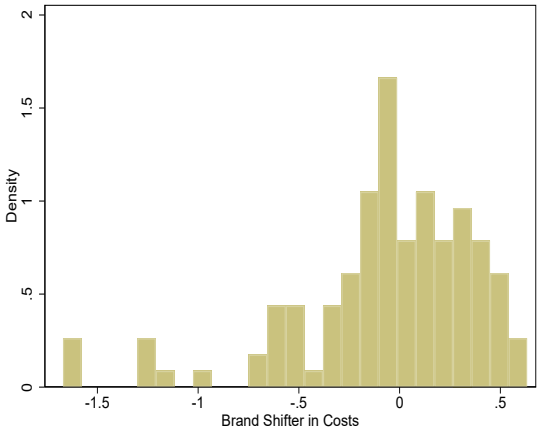
We estimate that the elasticity of marginal production costs with respect to the distance between the location of production and the headquarters is 0.034^a. Moreover, we estimate statistically-significant elasticities of marginal costs to horsepower (κ_7) and weight (κ_8) equal to 0.650 and 0.857, respectively.

In Figure 1, we present the estimates of the fixed effects for brands, production locations, headquarters locations, and bodytype. As shown in Figures 1a and 1b, there is substantial heterogeneity in brand productivity, that can be partially explained by the headquarters location of those brands, with China and Western Europe accounting for the majority of low-cost brands. Figure 1c highlights that production location differ quite substantially in the origin-specific component of marginal costs. Countries like Canada and those in Western Europe have much larger shifters than countries such as Russia and those in Asia, and Latin America. Figure 1d show that sports cars are substantially costlier to produce.

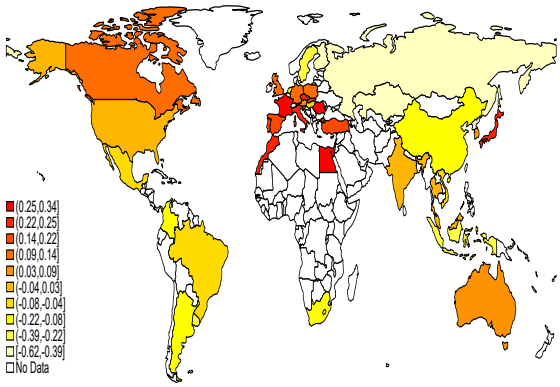
Figure 1: Estimates of Marginal Cost Function



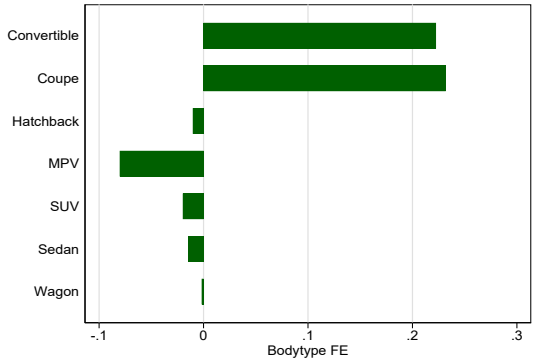
(a) Brand Shifters by Home Country



(b) Brand FE Histogram



(c) Origin Shifter



(d) Bodytype Shifter

Notes: In this figure, we graphically represent the estimates for the different components in Equation 29. In Panel (a), we present averages of the α_b by the home market of the brands. In Panel (b), we present a histogram for the estimated α_b. In Panel (c), we present the values of the origin fixed effects, α_o. In Panel (d), we present the values of the segment shifters, α_s.

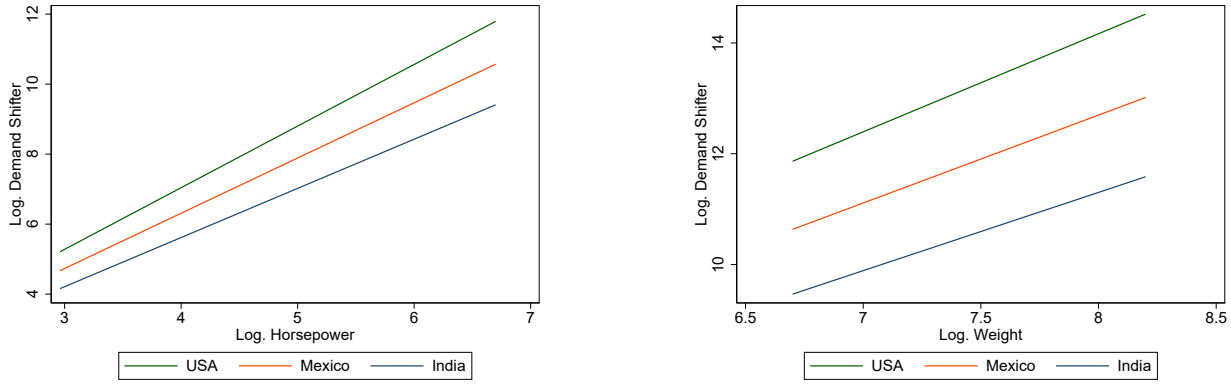
5.2 Demand Estimates

We report demand estimates in Table 2. The estimated elasticity across models within a brand-market-segment, σ^M, is of 6.55^a, and the demand elasticity for brands within market-segments, σ^B, is estimated to be equal to 2.97^a. Our estimates imply that $\hat{\sigma}^M > \hat{\sigma}^B$ (statistically significantly), which has two implications. First, there is sizable cannibalization, which implies that when a brand decides to introduce a new product in a country it significantly reduces the sales for its other products sold in the same country. The second implication is that the price-marginal cost markup, which is determined by σ^B, is around 1.51.[15]

Regarding the product-destination demand shifters, we find that, conditional on product fixed

[15]Pursuing the same strategy of estimating the elasticities sequentially, we can estimate an elasticity across segments, σ^S. We estimate a value of 2.81 with a robust standard error of 0.23. We do not use this strategy in our estimation procedure as it fails to provide demand shifters for each possible market due to data limitations.

Figure 2: Estimates of Car Model Characteristics on Demand Shifters

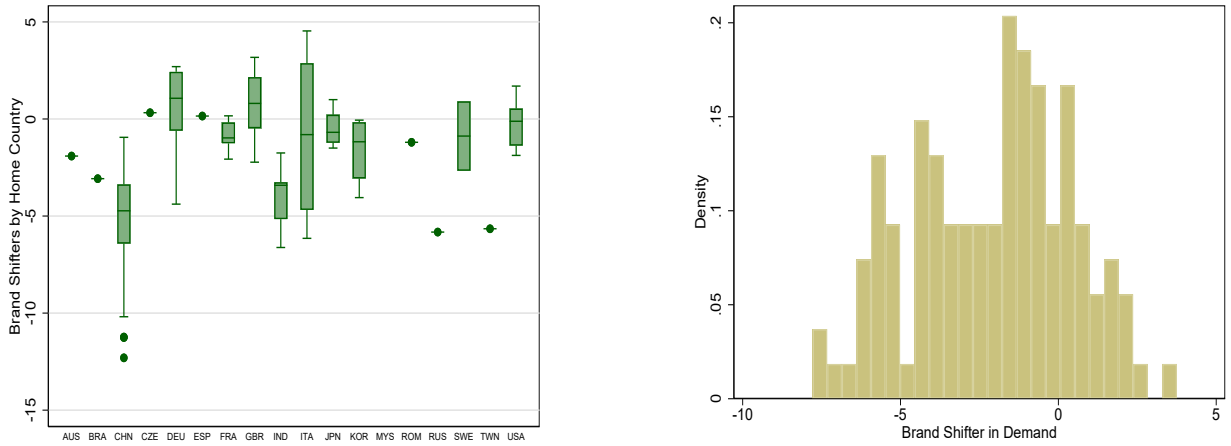


(a) Preference for Horsepower

(b) Preference for Weight

Notes: In this figure, we present the estimates of the heterogeneous effects of horsepower and weight on the demand shifters from Equation 30 conditioning in three values of GDP per capita, for USA, Mexico and India. In Panel A, we show the plots for horsepower, and in Panel B, for Weight.

Figure 3: Estimates of Brand Fixed Effects in Demand



(a) Brand FE by Home Country

(b) Brand FE Histogram

Notes: In this figure, we present the values of the demand brand shifters, $\check{\phi}_b$, introduced in Equation 31. In Panel A, we show the values of these demand shifters by the home country of the brands. In Panel B, we present a histogram for $\check{\phi}_b$.

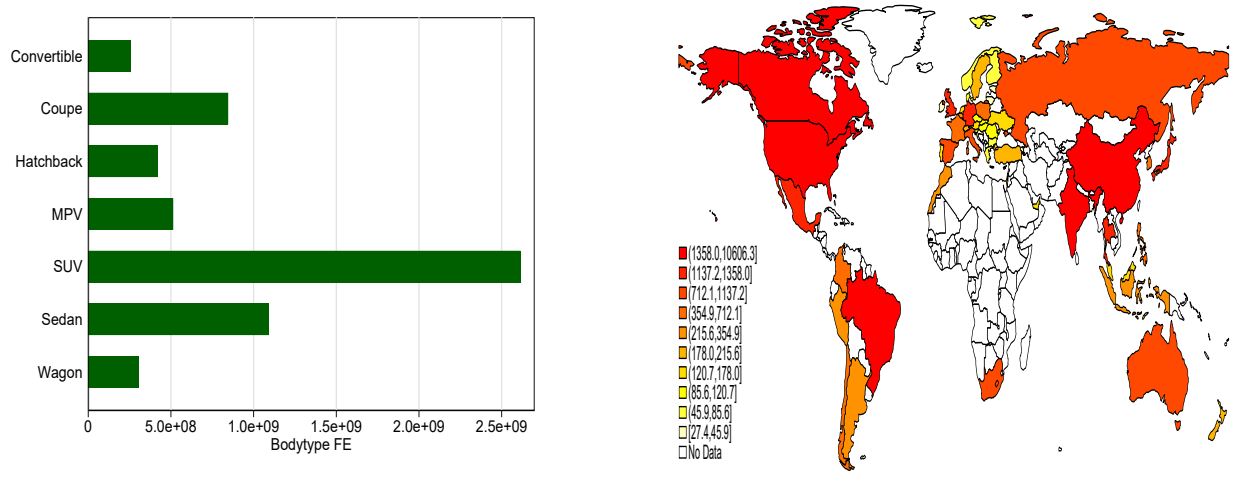
effects, products with both more horsepower and more weight, are preferred in countries with higher GDP per capita, as displayed in Figure 2.

At the upper nest, we estimate a home bias effect in demand of 1.42 (r.s.e. equal to 0.098) which is in line with Coşar et al. (2016). In this nest, we also estimate brand fixed effects that capture vertical quality differences across brands. In Figure 3, we plot these brand fixed effects. Comparing these plots with those in Figure 1a, we find that a positive relationship between cost and quality. Expensive brands from countries like the United States, Great Britain, Italy or Germany seem to have very large quality shifters, whereas brands from China and India are perceived as relatively

lower quality.

Averages across destination markets and across segments of the estimates for the market-segment shifters, A_{sn}, are shown in Figure 4. On average, SUVs are the segment with the largest demand shifters, whereas station wagons are the bodytype with the smallest shifters. These demand shifters are largest in larger and/or richer countries, such as the United States, China, India, Great Britain, or Canada.

Figure 4: Estimates of Market-Segment Shifters



(a) Average A_{sn} by Segment

(b) Average A_{sn} by Destination Market

Notes: In this figure, we graphically represent the estimates for the estimates of A_{sn} from Equation 34. In Panel (a), we present averages of the A_{sn} by segment. In Panel (b), we present averages of the A_{sn} by destination market.

5.3 Fixed Cost Estimates

We implement our moment inequality procedure, discussed in section 4.3, using a grid-search procedure in combination with Andrews and Soares (2010). Table 3 reports the limits of the 95% confidence sets for each of the estimated parameters. We find that the constant of the per-product market entry fixed costs lies between -3 and 1 million USD, while the semi-elasticity with respect to distance to the brand's headquarters country is between 1.5 and 3.0. Thus, a 1% increase in distance to the brand's headquarters country leads to an increase in the product-market entry fixed cost of $15,000 - 30,000$ USD. Moreover, we find that the standard deviation of the market entry fixed cost distribution is between 4 and 6 million USD.

We find large production plant fixed costs. Producing a product in an additional country (above and beyond the "free" location) costs brands 1.1-2.6 billion USD. The parameter γ_3, which determines how likely it is that the "free plant" is located at the brand's headquarters country, is between 3.8 and 4.3. These parameter values imply a 61-72% probability that the "free plant" for a product is located in the same country as the brand's headquarters. Finally, we find substantial unobserved heterogeneity in the production plant fixed cost distribution, with a standard deviation

Table 3: Fixed Cost Estimates

	95% Confidence Set Limits
Market Entry Fixed Cost	
γ_0 (Constant)	[-3.0, 1.0]
γ_1 (Log Distance to HQ)	[1.5, 3.0]
σ_e (Std Shock)	[4.0, 6.0]
Production Plant Fixed Costs	
γ_2 (Constant)	[1100.0, 2600.0]
γ_3 (Free HQ plant probability)	[3.8, 4.3]
σ_p (Std Shock)	[400.0, 1000.0]
Observations	1130

Notes: Confidence sets computed using Andrews and Soares (2010) and a grid search. Units are in millions of USD. We cluster the variance-covariance matrix at the brand-segment level.

of 400-1000 million USD.

6 The Effect of US EV Policies on Global Automobile Production

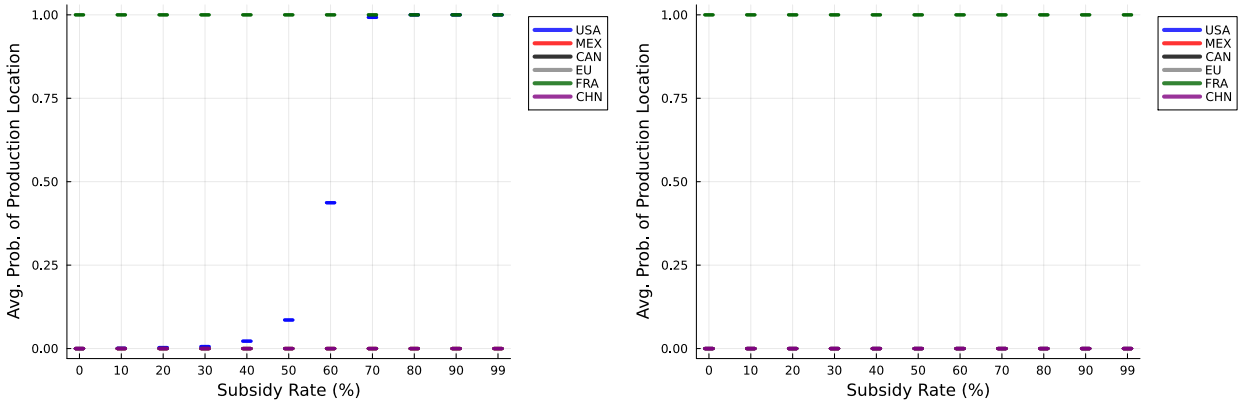
Accounting for 3% of global GDP, the global automobile industry has historically been subject to important policy packages and government intervention. In recent years, the automobile industry has again captured policymakers' attention, with environmental policies seeking to incentivize the adoption of environmentally-friendly vehicles, and trade policies aiming to promote and protect domestic manufacturers. The goal of our analysis is to shed light on the effectiveness of these policies in achieving their intended goals, rather than to judge whether there are normative reasons to implement these policies. For instance, a current topic in policy discussions is the extent to which tariffs on manufactured goods produced abroad can induce firms to re-locate their production in the country imposing the tariff. Through the lens of our quantitative model, the answer to this question will depend crucially on the interaction between demand-side and supply-side factors that enter firms' plant location decision. If demand for such goods is large in the United States and the fixed cost of setting up a plant is relatively small, our model predicts that firms will profitably choose to re-locate production to the United States. If production fixed costs are very large and the product is not particularly demanded in the US market, this is less likely. A crucial factor affecting how profitable a product is in a country is cannibalization. Tariffs on foreignly-produced electric vehicles may fail to induce brands to re-locate the United States if such brands abstain from selling electric vehicles that cannibalize sales of their non-electric varieties.

We use our quantitative model to evaluate the effect of multiple policies enacted in the United States on global production in the car industry. First, we "zoom in" and focus on the decision of Peugeot competing in the wagon segment. This exercise allows us showcase our methodology and the economics behind brands' production location choices. Then, we provide the effects of several policies on overall automobile market outcomes worldwide.

6.1 Case Study: Peugeot in the Market for Wagons

We study the market entry and plant location decisions of Peugeot within the wagon segment. In this segment, we observe Peugeot offering two products: Peugeot 308 (non-electric) and Peugeot 508 (electric).[16] In the data, we observe that Peugeot produces both of these products only in France, and that neither product is offered in the United States. First, we draw 96,000 fixed cost shock and free plant realizations for Peugeot. We study two cases. In both cases, we match the empirical observation that both products are produced only in France, and select draws such that the non-electric product is offered in the United States. However, in one case, we select draws such that the electric product is not offered in the United States.[17] In another case, we condition on the electric product being offered there. Under these two sets of draws, we study how Peugeot responds to various policies favoring the purchase or production of electric products in America.

Figure 5: Probability of Production Location - Marginal Cost Subsidy



(a) Electric (b) Non-Electric

Notes: These plots shows the probability that the electric and non-electric products in the Peugeot wagon segment are produced in different countries/regions at different US marginal cost subsidy rates. The mean is computed across fixed cost draws that condition on production of both products in France, the offering of the non-electric product in the United States, and the non-offering of the electric product in the United States. We also condition on the market offering fixed cost being positive for the electric product in the United States.

6.1.1 Electric Product Not Initially Offered in the United States

First, we study the effects of US policies on Peugeot's plant location and market entry decisions, conditional on not initially selling the product in the United States (as in the data). Figures 5-6 show the effects of different marginal cost subsidies on electric vehicles in the United States on outcomes of interest worldwide. Figure 5 shows that a subsidy rate of 60% is required so that Peugeot produces the electric vehicle with 50% probability in the United States. At subsidy rates higher than 70% the probability approaches 1. Figure 6 shows that the the probability that the

[16] We define a product as electric whenever the majority of its sales in our price data (where we observe fuel type) correspond to the electric version within our product definition.

[17] We implement this exercise at $\gamma_0 = 1$, $\gamma_1 = 1.5$, $\sigma_e = 4$, $\gamma_2 = 1100$, $\gamma_3 = 4$, $\sigma_p = 400$.

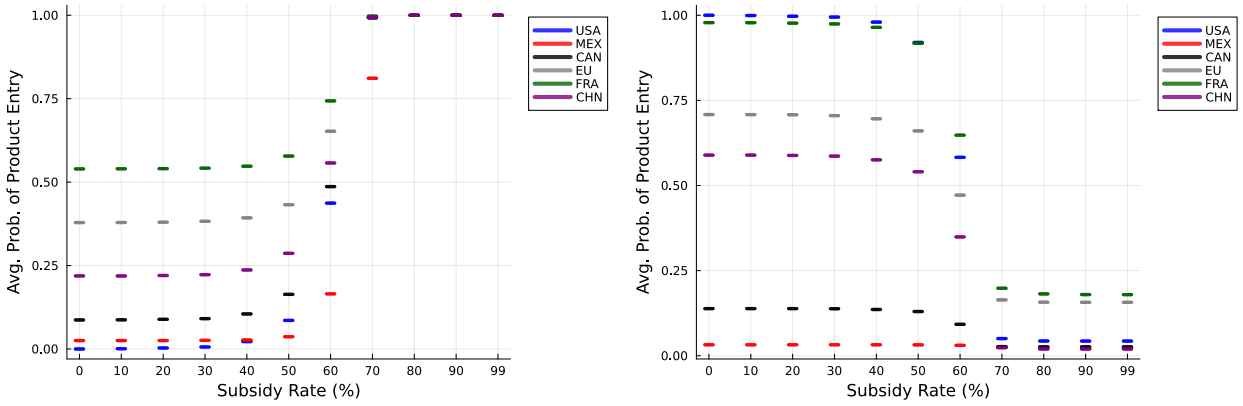
electric product is offered mirrors the probability that Peugeot produces the product in the United States. Two interesting findings emerge, which illustrate the interdependencies in the framework. As the probability of production in the United States rises, the probability that the electric product is offered in markets other than the United States increases, with important increases in Mexico, Canada, Europe, and China. Interestingly, the entry of the electric product globally comes at the expense of a reduced probability that the non-electric product is offered in the United States, France, and China. This highlights the importance of cannibalization in causing changes in market outcomes.

Figures 7-8 show the effects of a US consumer subsidy on electric vehicles on Peugeot's product offerings and production location choices. The probability that Peugeot offers the electric product rises quickly with the consumer subsidy rate, and a 40% subsidy rate leads to a probability close to 1 that the product is offered in the United States. While offering the product in the United States becomes more likely with a high consumer subsidy, this is not the case with production - a consumer subsidy of over 90% is required for production in the United States to be higher than 50%. However, the probability of production in a non-France European country rises with the consumer subsidy rate. As the American demand for this product rises, Peugeot has an incentive to set the production plant in countries that can cheaply serve the US market.

6.1.2 Electric Product Initially Offered in the United States

Figure 9 shows that, conditional on initially selling the electric product in the United States, the marginal cost subsidy needs to be at least 60% for Peugeot to find it worthwhile to set the

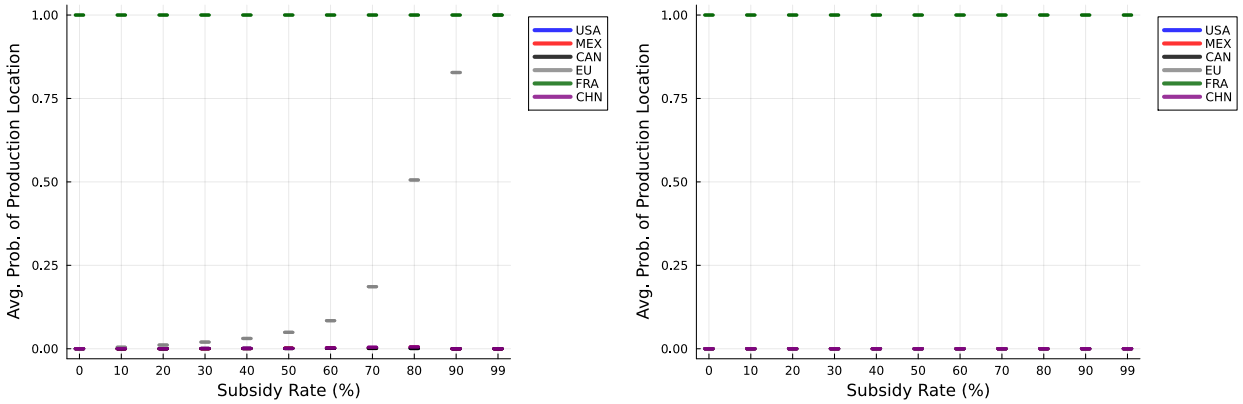
Figure 6: Probability of Product Offering - Marginal Cost Subsidy



(a) Electric (b) Non-Electric

Notes: These plots shows the probability that the electric and non-electric products in the Peugeot wagon segment are sold in different countries/regions at different US marginal cost subsidy rates. The mean is computed across fixed cost draws that condition on production of both products in France, the offering of the non-electric product in the United States, and the non-offering of the electric product in the United States. We also condition on the market offering fixed cost being positive for the electric product in the United States.

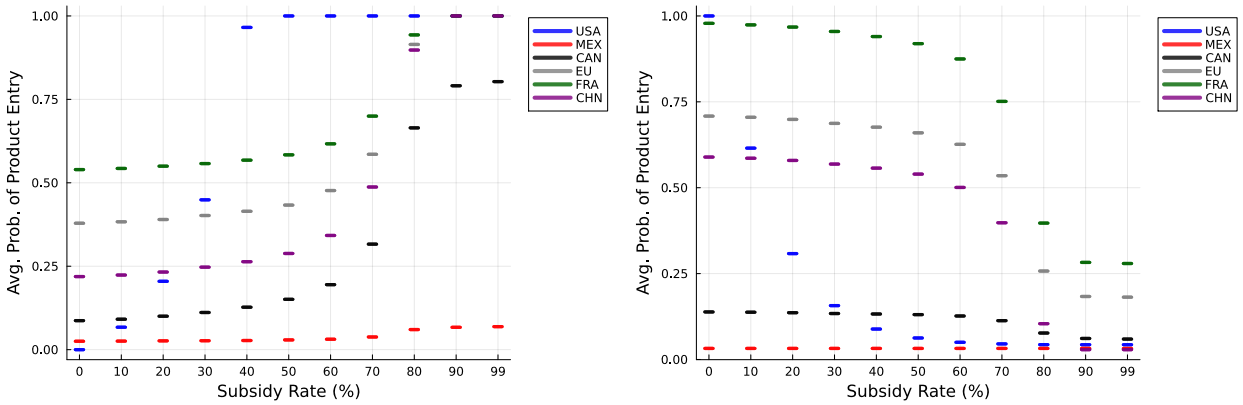
Figure 7: Probability of Production Location - Consumer Subsidy



(a) Electric (b) Non-Electric

Notes: These plots shows the probability that the electric and non-electric products in the Peugeot wagon segment are produced in different countries/regions at different US consumer subsidy rates. The mean is computed across fixed cost draws that condition on production of both products in France, the offering of the non-electric product in the United States, and the non-offering of the electric product in the United States. We also condition on the market offering fixed cost being positive for the electric product in the United States.

Figure 8: Probability of Product Offering - Consumer Subsidy



(a) Electric (b) Non-Electric

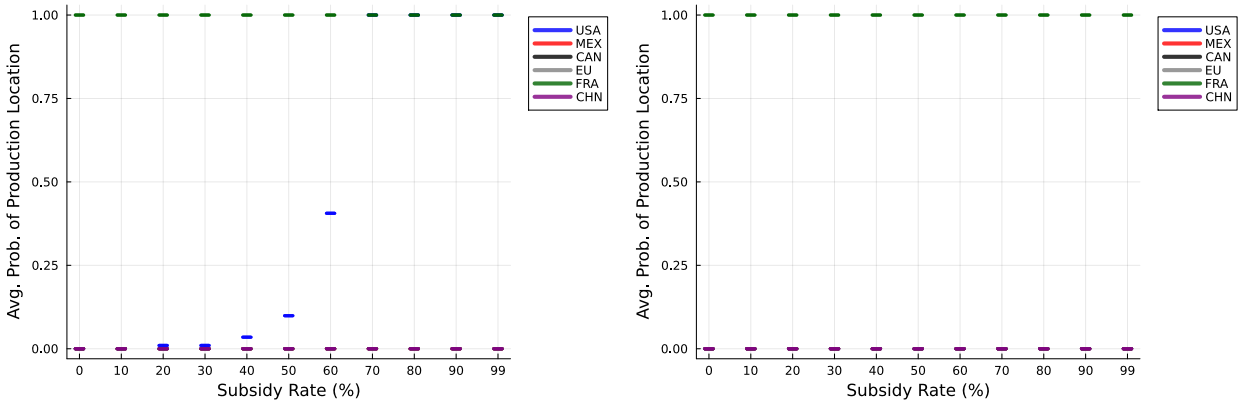
Notes: These plots shows the probability that the electric and non-electric products in the Peugeot wagon segment are sold in different countries/regions at different US consumer subsidy rates. The mean is computed across fixed cost draws that condition on production of both products in France, the offering of the non-electric product in the United States, and the non-offering of the electric product in the United States. We also condition on the market offering fixed cost being positive for the electric product in the United States.

plant in the United States with probability higher than 40%. Figure 9 looks similar to Figure 5.

The marginal cost subsidy in the United States affects other countries only if it induces the opening of a plant. As a result, Figure 10 mirrors Figure 6 – a low cost plant in the United States induces the entry of the electric product in other countries, and simultaneously induces the exit of the non-electric product.

Even if the electric product were initially offered in the United States, the model predicts that

Figure 9: Probability of Production Location - Marginal Cost Subsidy



(a) Electric

(b) Non-Electric

Notes: These plots shows the probability that the electric and non-electric products in the Peugeot wagon segment are produced in different countries/regions at different US marginal cost subsidy rates. The mean is computed across fixed cost draws that condition on production of both products in France, and the offering of the non-electric and electric products in the United States. We also condition on the market offering fixed cost being positive for the electric product in the United States.

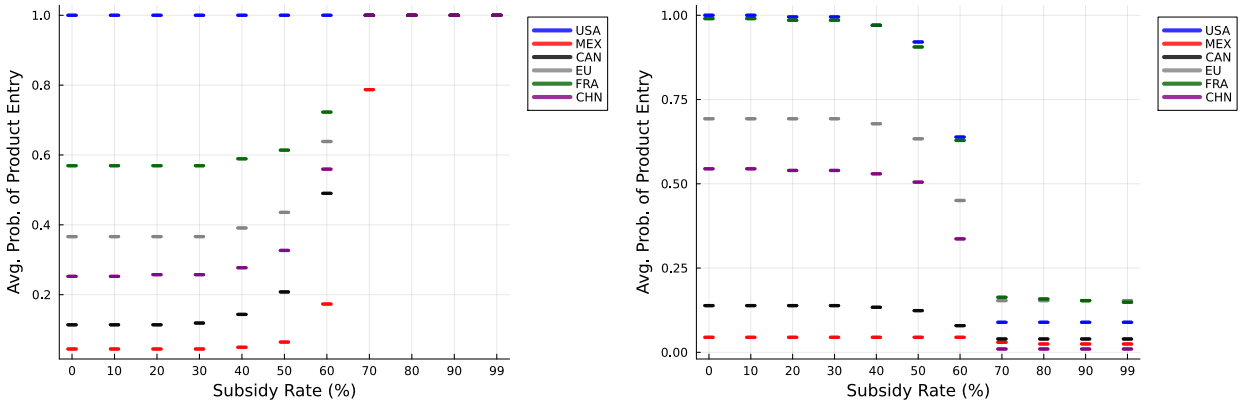
that a higher consumer subsidy will mostly induce the entry of plants in relatively lower cost countries for that product, which are typically not the United States.[18]

6.2 Aggregate Effects of US Policies Favoring EVs

We study the aggregate effects of policies implemented in the United States on the global structure of automobile production. We evaluate the effects of various levels of subsidies and tariffs on aggregate outcomes, both on the extensive and the intensive margin. To solve for the optimal production and offering choices of firms under different policies, we implement our solution method at 128 draws of fixed costs and under each of the points in the confidence set of fixed cost parameters $\boldsymbol{\gamma}$. Let $h_{bs}(\boldsymbol{\gamma}, \tau, r)$ denote an outcome for brand b in segment s under parameter vector $\boldsymbol{\gamma}$, policy τ, and realization of fixed cost shocks r. Our solution method yields bounds on production location and product entry decisions $(\boldsymbol{I}^{bs}, \boldsymbol{D}^{bs})$, and as such, bounds on outcomes h that are weakly monotone in such choices. To obtain bounds on aggregates, we first average across the 128 realizations of shocks to obtain averaged lower and upper bounds, $\underline{h}^a_{bs}(\boldsymbol{\gamma}, \tau)$ and $\overline{h}^a_{bs}(\boldsymbol{\gamma}, \tau)$, respectively. Then, we sum across all brands and all segments to obtain aggregate bounds $\underline{H}^a(\boldsymbol{\gamma}, \tau)$ and $\overline{H}^a(\boldsymbol{\gamma}, \tau)$. Finally, we obtain bounds on the counterfactual *levels* by minimizing and maximizing $\underline{H}^a(\boldsymbol{\gamma}, \tau)$ and $\overline{H}^a(\boldsymbol{\gamma}, \tau)$ over $\boldsymbol{\gamma}$ in the confidence set CS, respectively. This procedure yields bounds on levels $\underline{H}(\tau) = \min_{\boldsymbol{\gamma} \in CS} \underline{H}^a(\boldsymbol{\gamma}, \tau)$ and $\overline{H}(\tau) = \max_{\boldsymbol{\gamma} \in CS} \underline{H}^a(\boldsymbol{\gamma}, \tau)$.

[18] There is a small probability at which production occurs in the United States rather than in Europe when the subsidy is 70-80%. This happens at draws for which the fixed cost of production in the United States is much lower than the fixed cost of production in Europe. When this is the case, even if Europe has lower marginal costs, Peugeot would rather open the plant in the United States to save on fixed costs. However, once the demand-side subsidies increase in the United States, it becomes relatively more important to save on marginal costs rather than fixed costs.

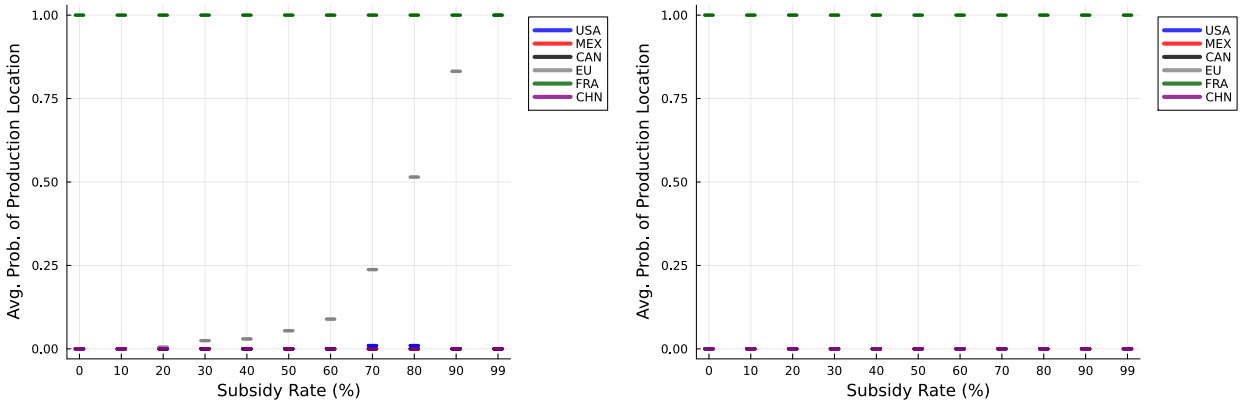
Figure 10: Probability of Product Offering - Marginal Cost Subsidy



(a) Electric (b) Non-Electric

Notes: These plots shows the probability that the electric and non-electric products in the Peugeot wagon segment are sold in different countries/regions at different US marginal cost subsidy rates. The mean is computed across fixed cost draws that condition on production of both products in France, and the offering of the non-electric and electric products in the United States. We also condition on the market offering fixed cost being positive for the electric product in the United States.

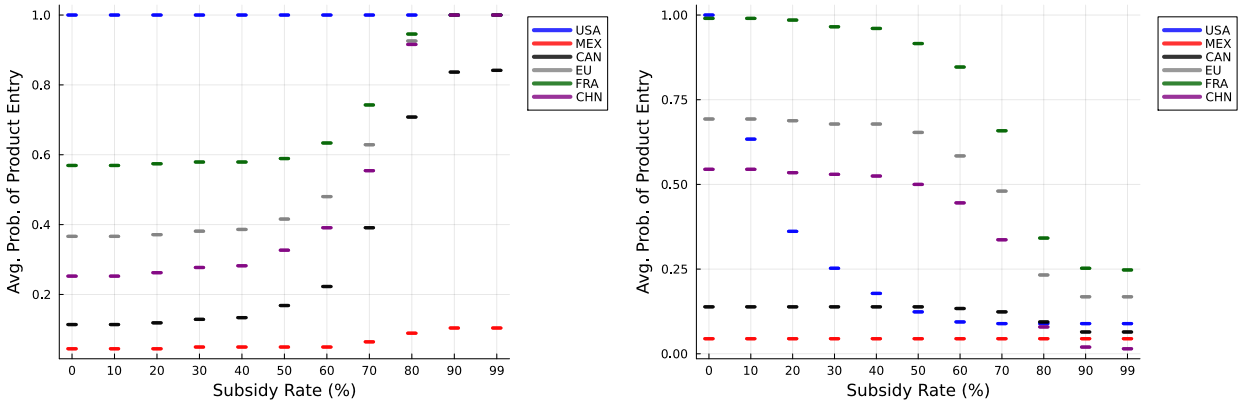
Figure 11: Probability of Production Location - Consumer Subsidy



(a) Electric (b) Non-Electric

Notes: These plots shows the probability that the electric and non-electric products in the Peugeot wagon segment are produced in different countries/regions at different US consumer subsidy rates. The mean is computed across fixed cost draws that condition on production of both products in France, and the offering of the non-electric and electric products in the United States. We also condition on the market offering fixed cost being positive for the electric product in the United States.

Figure 12: Probability of Product Offering - Consumer Subsidy



(a) Electric (b) Non-Electric

Notes: These plots shows the probability that the electric and non-electric products in the Peugeot wagon segment are sold in different countries/regions at different US consumer subsidy rates. The mean is computed across fixed cost draws that condition on production of both products in France, and the offering of the non-electric and electric products in the United States. We also condition on the market offering fixed cost being positive for the electric product in the United States.

A drawback of performing the counterfactual exercises in levels is that the parameter vector $\boldsymbol{\gamma}$ minimizing (maximizing) a given counterfactual outcome could vary across different policy levels, even though there is only one true parameter that generates the data. An alternative approach that addresses this concern is to report counterfactual changes. In Appendix D, we report aggregate changes in counterfactual outcomes by computing:

$$\begin{aligned}\Delta\overline{H}(\boldsymbol{\gamma},\tau) := \max\{&\overline{H}^a(\boldsymbol{\gamma},\tau) - \underline{H}^a(\boldsymbol{\gamma},\tau_0), \overline{H}^a(\boldsymbol{\gamma},\tau) - \overline{H}^a(\boldsymbol{\gamma},\tau_0),\\ &\underline{H}^a(\boldsymbol{\gamma},\tau) - \overline{H}^a(\boldsymbol{\gamma},\tau_0), \underline{H}^a(\boldsymbol{\gamma},\tau) - \underline{H}^a(\boldsymbol{\gamma},\tau_0)\}\\ \Delta\underline{H}(\boldsymbol{\gamma},\tau) := \min\{&\overline{H}^a(\boldsymbol{\gamma},\tau) - \underline{H}^a(\boldsymbol{\gamma},\tau_0), \overline{H}^a(\boldsymbol{\gamma},\tau) - \overline{H}^a(\boldsymbol{\gamma},\tau_0),\\ &\underline{H}^a(\boldsymbol{\gamma},\tau) - \overline{H}^a(\boldsymbol{\gamma},\tau_0), \underline{H}^a(\boldsymbol{\gamma},\tau) - \underline{H}^a(\boldsymbol{\gamma},\tau_0)\},\end{aligned}$$

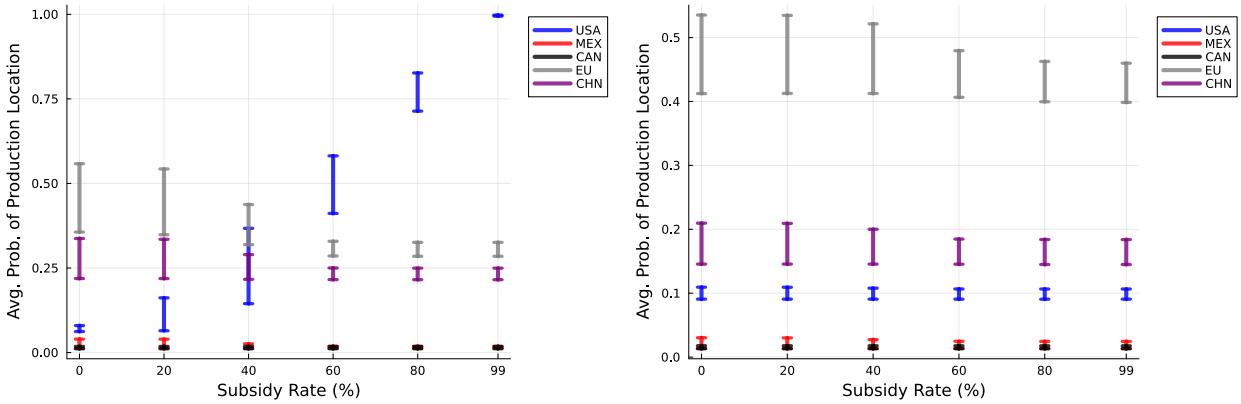
where τ_0 denotes the case without policies in the United States (or anywhere). Then, we report $\Delta\overline{H}(\tau) = \max_{\boldsymbol{\gamma}\in CS}\Delta\overline{H}(\boldsymbol{\gamma},\tau)$ and $\Delta\underline{H}(\tau) = \min_{\boldsymbol{\gamma}\in CS}\Delta\underline{H}(\boldsymbol{\gamma},\tau)$.

Using this approach, we study the effects of consumer and producer subsidies, and tariffs implemented in the United States on global automobile production. The reported bounds reflect both the fact that it is computationally feasible in some cases to find the full solution (the branching algorithm applied after the first-step algorithm does not converge after a maximal number of iterations), and statistical uncertainty regarding the value of $\boldsymbol{\gamma}$.

6.2.1 Marginal Cost Subsidy on Electric Vehicles in the United States

Figures 13-16 show the effects of different levels of US marginal cost subsidies on global production, product offerings, and profits. The first important observation is that as the marginal

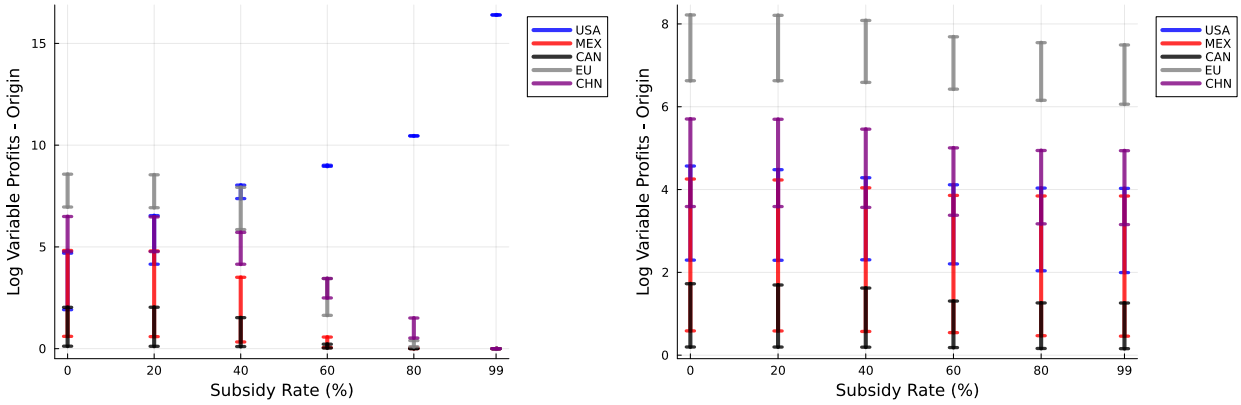
Figure 13: Probability of Production Location - Marginal Cost Subsidy



(a) Electric (b) Non-Electric

Notes: These plots show the lower and upper bound levels $\underline{H}(\tau)$ and $\overline{H}(\tau)$ of probability that an electric (panel (a)) and non-electric (panel (b)) product is produced in several countries/regions under different US marginal cost subsidy rates. The probability is computed by calculating the average number of 1's per electric (non-electric) product. The bounds reflect both uncertainty arising from unsolved binary decisions due to computational infeasibility, as well as from statistical uncertainty regarding the fixed cost parameter vector.

Figure 14: Log Variable Profits (Origin) - Marginal Cost Subsidy

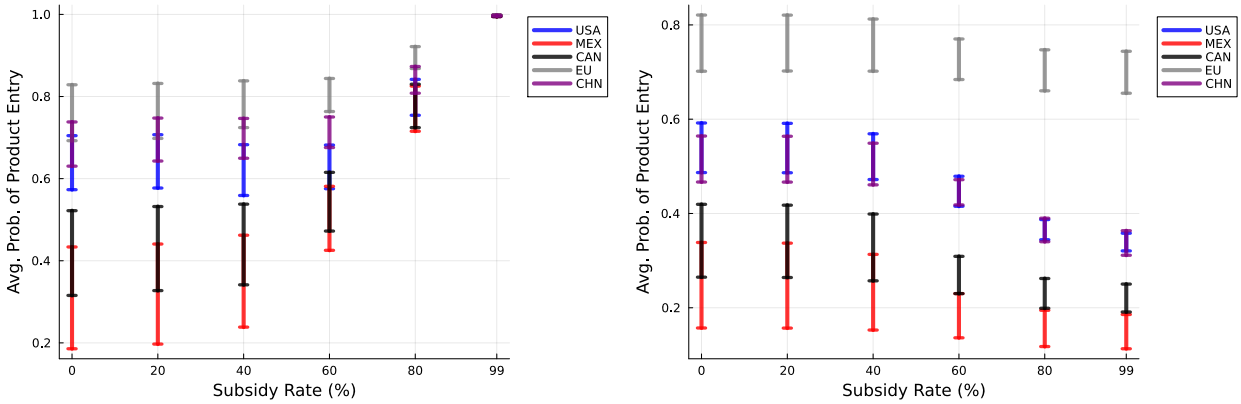


(a) Electric (b) Non-Electric

Notes: These plots show the lower and upper bound levels $\underline{H}(\tau)$ and $\overline{H}(\tau)$, respectively, of the (log) of total variable profits across electric (panel (a)) and non-electric (panel (b)) products at the origin-level for several production countries/regions under different US marginal cost subsidy rates on electric products. The bounds reflect both uncertainty arising from unsolved binary decisions due to computational infeasibility, as well as from statistical uncertainty regarding the fixed cost parameter vector.

cost subsidy increases, there is an increase in the lower and upper bound of the share of electric products that are produced in the United States. However, even at a 20% marginal cost subsidy, there is at most a 10 percentage point increase in the share of electric products produced in the United States. A subsidy rate higher than 60% would be required for the United States to ensure that half of the world's electric products are produced stateside. At high marginal cost subsidy rates, certain products are no longer produced in China or in Europe, though there is limited scope

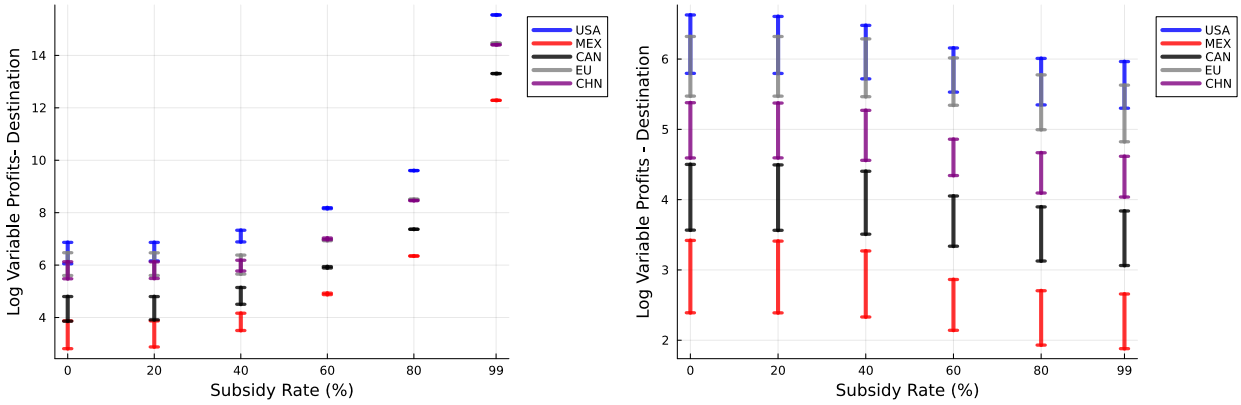
Figure 15: Probability of Product Offering - Marginal Cost Subsidy



(a) Electric (b) Non-Electric

Notes: These plots show the lower and upper bound levels $\underline{H}(\tau)$ and $\overline{H}(\tau)$ of probability that an electric (panel (a)) and non-electric (panel (b)) product is offered in several countries/regions under different US marginal cost subsidy rates on electric products. The probability is computed by calculating the average number of 1's per electric (non-electric) product. The bounds reflect both uncertainty arising from unsolved binary decisions due to computational infeasibility, as well as from statistical uncertainty regarding the fixed cost parameter vector.

Figure 16: Log Variable Profits (Destination) - Marginal Cost Subsidy



(a) Electric (b) Non-Electric

Notes: These plots show the lower and upper bound levels $\underline{H}(\tau)$ and $\overline{H}(\tau)$, respectively, of the (log) of total variable profits across electric (panel (a)) and non-electric (panel (b)) products at the origin-level for several production countries/regions under different US marginal cost subsidy rates on electric products. The bounds reflect both uncertainty arising from unsolved binary decisions due to computational infeasibility, as well as from statistical uncertainty regarding the fixed cost parameter vector.

for production plant closure, with at most a 25 percentage point decrease in the share of electric products produced in Europe, even at the most extreme US subsidy rates. This is partly due to the fact that European and Chinese brands draw "free" plants in their home jurisdictions. Figure 14 shows that variable profits from Chinese and European EV plants decline sharply at a 60% US marginal cost subsidy, though a subsidy larger than 80% would be required to completely push European and Chinese EV makers out of business. Panel (b) of Figure 13 shows that marginal

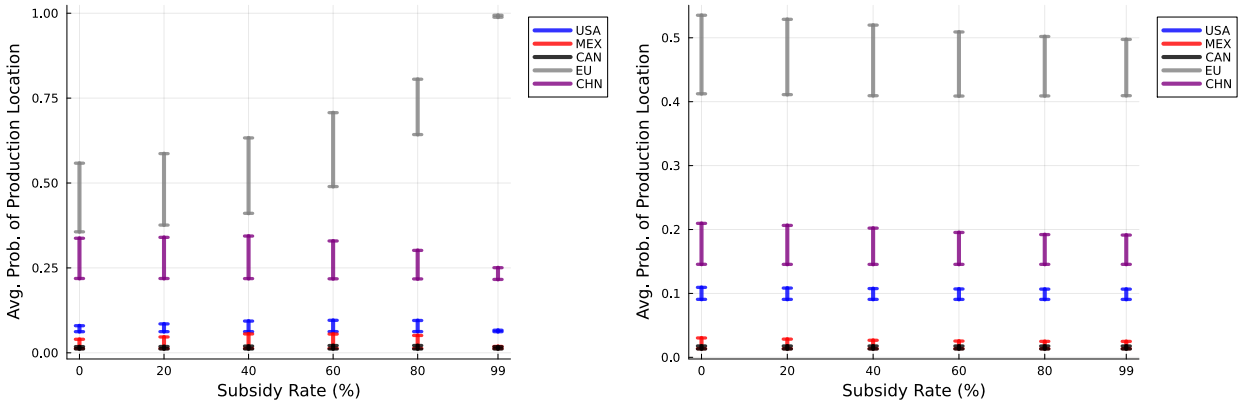
cost subsidies favoring electric products in the United States have limited effects on the production location choices for non-electric products.

Figure 15 shows that large enough US marginal cost subsidies on electric products lead to increased offering probabilities of electric products in other jurisdictions. At subsidy rates 40% or higher, there is a significantly positive increase in the lower and upper bound share of electric products offered in Mexico or Canada. Higher subsidy rates in the United States (greater than 60%) are required for the lower and upper bounds to increase for Europe and China. At such subsidy rates, there is also significant exit of non-electric products in the United States and in China, and both the upper and lower bounds of the offering rates decrease in all jurisdictions. Figure 20 shows the analogous effects on the intensive margin. As marginal cost subsidies rise, US-produced electric vehicles become more appealing to consumers abroad, causing an increase in the EV variable profits globally, particularly at marginal cost subsidy rates 40% or higher. Panel (b) of Figure 20 shows that at such subsidy rates, there is a corresponding decline in global non-electric earnings, as consumers substitute away from non-subsidized products that are now relatively more costly to produce.

6.2.2 Consumer Subsidies on Electric Vehicles in the United States

Figure 17 shows how consumer subsidies on electric products in the United States affects brands' production location choices. Panel (a) shows that larger consumer subsidies cause brands to open additional electric-product plants in Europe. This is due to European countries having a comparative advantage producing electric products to serve the US market. In the limit when the US

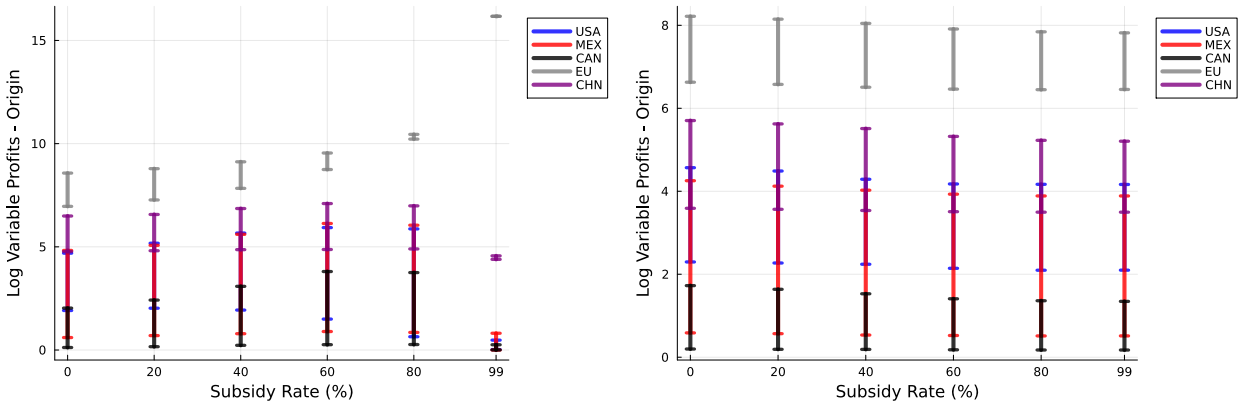
Figure 17: Probability of Production Location - Consumer Subsidy



(a) Electric (b) Non-Electric

Notes: These plots show the lower and upper bound levels $\underline{H}(\tau)$ and $\overline{H}(\tau)$ of probability that an electric (panel (a)) and non-electric (panel (b)) product is produced in several countries/regions under different US consumer subsidy rates on electric products. The probability is computed by calculating the average number of 1's per electric (non-electric) product. The bounds reflect both uncertainty arising from unsolved binary decisions due to computational infeasibility, as well as from statistical uncertainty regarding the fixed cost parameter vector.

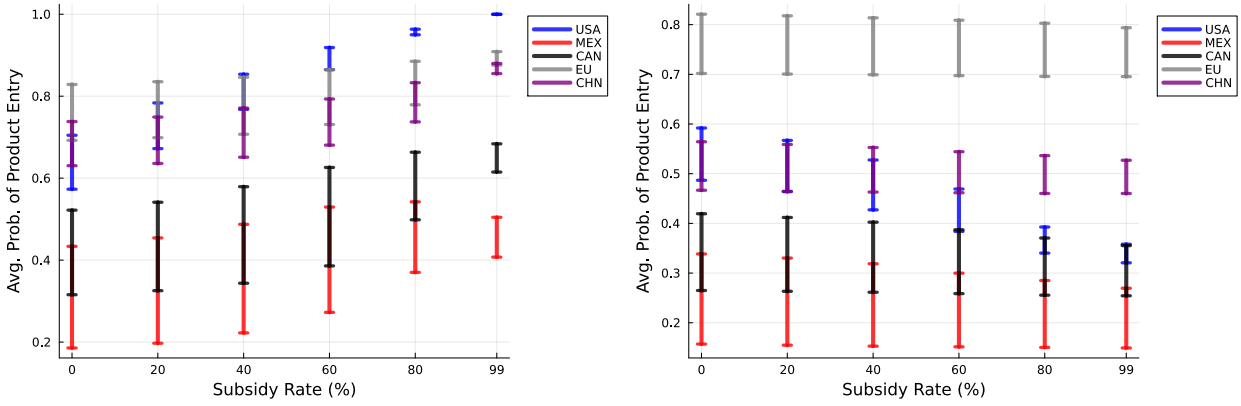
Figure 18: Log Variable Profits (Origin) - Consumer Subsidy



(a) Electric (b) Non-Electric

Notes: These plots show the lower and upper bound levels $\underline{H}(\tau)$ and $\overline{H}(\tau)$, respectively, of the (log) of total variable profits across electric (panel (a)) and non-electric (panel (b)) products at the origin-level for several production countries/regions under different US consumer subsidy rates on electric products. The bounds reflect both uncertainty arising from unsolved binary decisions due to computational infeasibility, as well as from statistical uncertainty regarding the fixed cost parameter vector.

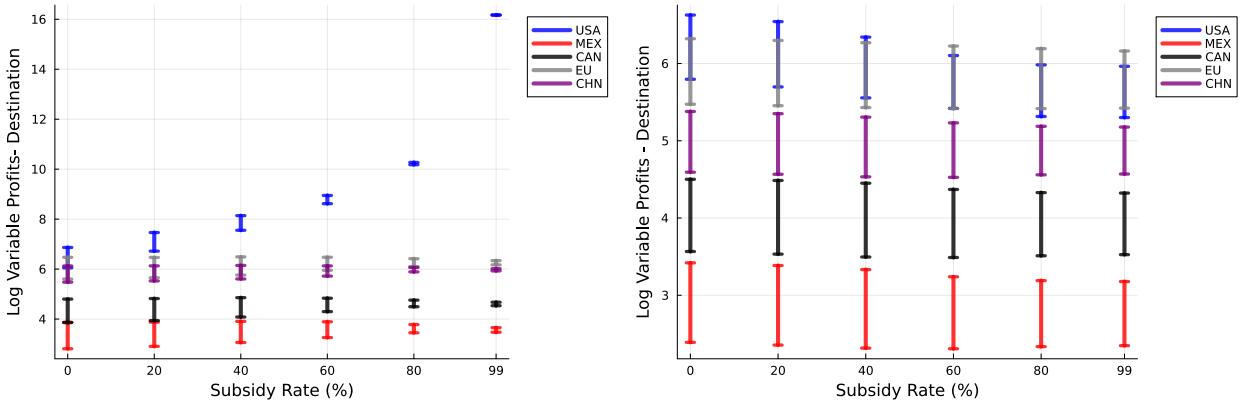
Figure 19: Probability of Product Offering - Consumer Subsidy



(a) Electric (b) Non-Electric

Notes: These plots show the lower and upper bound levels $\underline{H}(\tau)$ and $\overline{H}(\tau)$ of probability that an electric (panel (a)) and non-electric (panel (b)) product is offered in several countries/regions under different US consumer subsidy rates on electric products. The probability is computed by calculating the average number of 1's per electric (non-electric) product. The bounds reflect both uncertainty arising from unsolved binary decisions due to computational infeasibility, as well as from statistical uncertainty regarding the fixed cost parameter vector.

Figure 20: Log Variable Profits (Destination) - Consumer Subsidy



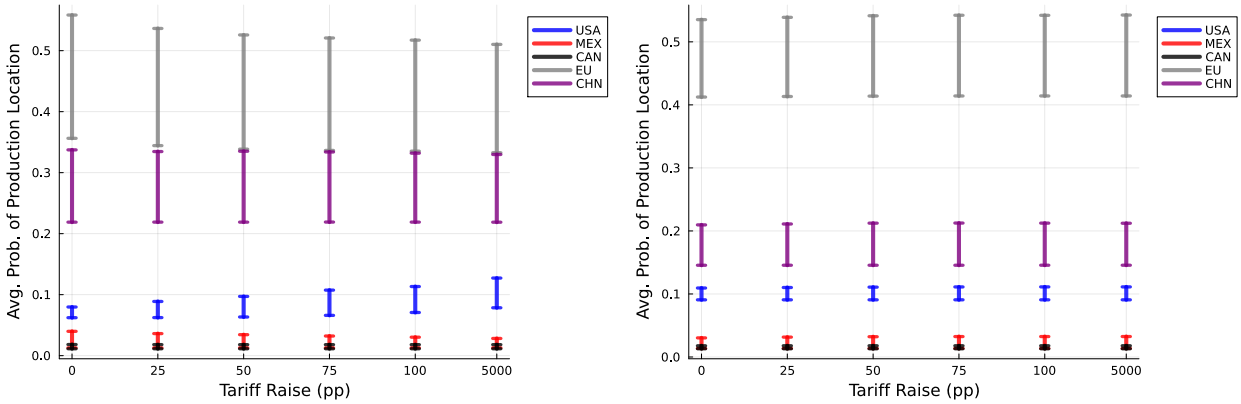
(a) Electric (b) Non-Electric

Notes: These plots show the lower and upper bound levels $\underline{H}(\tau)$ and $\overline{H}(\tau)$, respectively, of the (log) of total variable profits across electric (panel (a)) and non-electric (panel (b)) products at the origin-level for several production countries/regions under different US consumer subsidy rates on electric products. The bounds reflect both uncertainty arising from unsolved binary decisions due to computational infeasibility, as well as from statistical uncertainty regarding the fixed cost parameter vector.

government subsidizes most of the price of electric products, we find that all potential electric products are produced in Europe. Meanwhile, Panel (b) shows that production of non-electric products decreases, particularly in Europe and in China, especially when consumer subsidy rates are higher than 40%. This shows that cannibalization forces matter and can drive the exit of substitutable non-subsidized products. Figure 18 confirms that profits at the origin-level rise substantially in Europe, and in the limit mostly European and Chinese plants produce electric products, with some production taking place in Mexico. Perhaps surprisingly, we do not find that demand-side interventions alone can attract production of electric products to the United States.

The opening up of new plants in Europe as a result of US demand-side interventions has implications for global consumers. Figure 19 shows that such European plants export such additional products to destination markets other than the United States. Canadian, Mexican, Chinese, and European consumers have access to additional electric products as a consequence of US policy. Panel (b) of Figure 19 shows that due to cannibalization forces, as demand-side subsidies increase, some non-electric products exit the US market, but also other markets, as a result of the closure of production production plants for such products. Figure 20 shows that destination-level variable profits for electric vehicles rise primarily in the United States. Thus, while additional electric products are offered in the other countries in response to the subsidies, such products are not so appealing to non-American consumers who do not receive a government incentive. Panel (b) also shows that while cannibalization forces lead to the exit of non-electric products, there are more limited non-electric variable profits; the non-electric products that exit are the least appealing to consumers and the least profitable to firms.

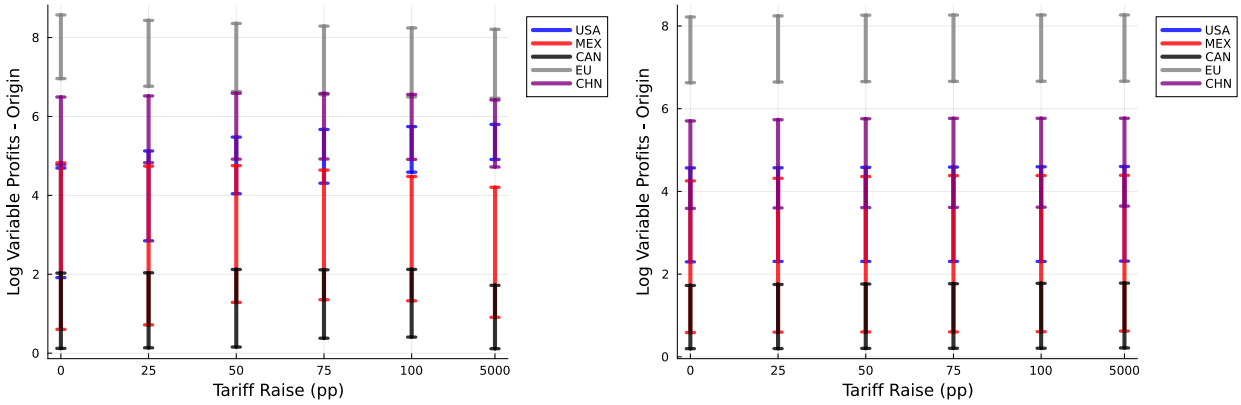
Figure 21: Probability of Production Location - Tariffs on ROW



(a) Electric (b) Non-Electric

Notes: These plots show the lower and upper bound levels $\underline{H}(\tau)$ and $\overline{H}(\tau)$ of probability that an electric (panel (a)) and non-electric (panel (b)) product is produced in several countries/regions under different US tariff raises on electric products. The probability is computed by calculating the average number of 1's per electric (non-electric) product. The bounds reflect both uncertainty arising from unsolved binary decisions due to computational infeasibility, as well as from statistical uncertainty regarding the fixed cost parameter vector.

Figure 22: Log Variable Profits (Origin) - Tariffs on ROW



(a) Electric (b) Non-Electric

Notes: These plots show the lower and upper bound levels $\underline{H}(\tau)$ and $\overline{H}(\tau)$, respectively, of the (log) of total variable profits across electric (panel (a)) and non-electric (panel (b)) products at the destination level for several production countries/regions under different US tariff raises on electric products. The bounds reflect both uncertainty arising from unsolved binary decisions due to computational infeasibility, as well as from statistical uncertainty regarding the fixed cost parameter vector.

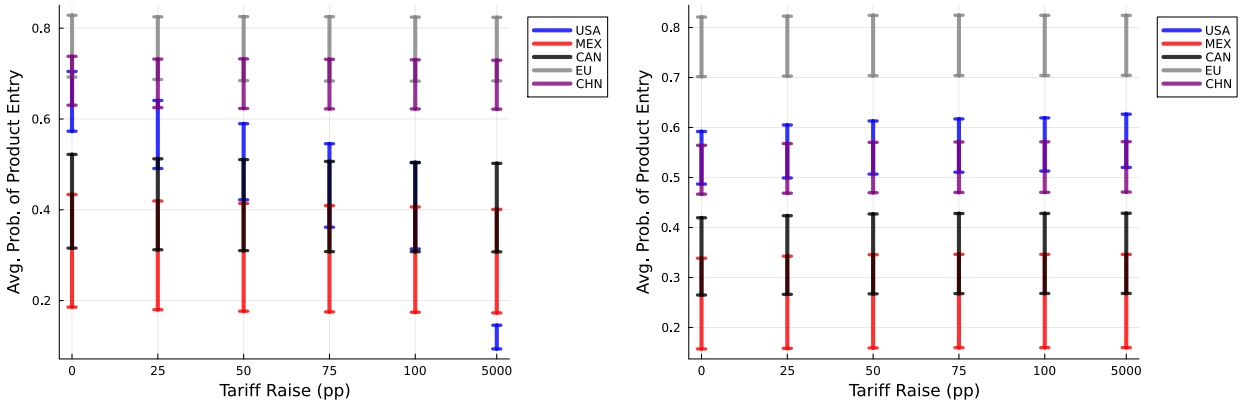
6.2.3 Tariffs on Electric Vehicles Not Produced in the United States

Figure 21 shows the effect of increasing tariffs on electric vehicles not produced in the United States on brands' production location choices. We find that as tariffs increase, brands shift the production of some of their electric products to the United States. At 0 additional tariffs, 7-9% of electric products are produced in the United States, while at a 5000 percentage point increase

in tariffs, the bounds shift to 9-12%. Figure 22 shows that profits originating from electric vehicle plants in the United States increase from 1.9-4.7 to 4.8-5.8 log points, which implies that some of the most profitable products for the US car market move to the United States in response to a large tariff. Thus, tariffs can attract the production of electric products that are highly popular in the United States. For these products, brands find it worth it to incur the large production fixed costs in response to tariffs, and to save on the additional unit costs that they would have to pay otherwise. Figures 21-22 show that the bounds do not change significantly for non-electric products, suggesting limited effects on the global production of non-electric vehicles.

Figure 23 shows sizable effects of the tariffs on product availability in the United States. As the tariffs increase, the share of electric products offered in the United States decreases, from 58-78% at zero additional tariffs to 2-12% at a 5000 percentage point increase. Panel (b) shows that the exit of some of the electric products induces the entry of some non-electric products in the United States. Figure 24 show that higher tariffs on electric products lead to smaller electric-vehicle earnings in the United States, and modest increases in the bounds on non-electric variable profits. Thus, both on the extensive and the intensive margin, tariffs on electric products lead to lower sales of electric products in the United States.

Figure 23: Probability of Product Offering - Tariffs on ROW



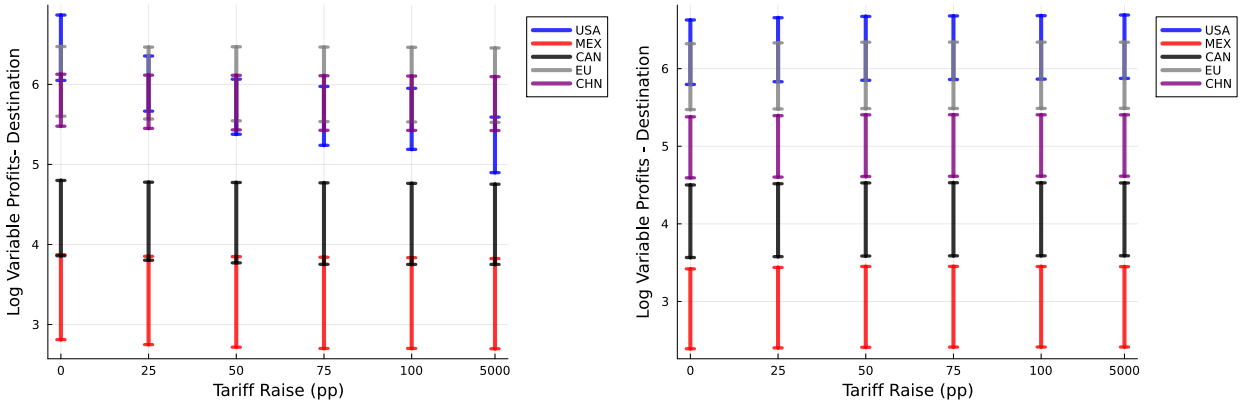
(a) Electric (b) Non-Electric

Notes: These plots show the lower and upper bound levels $\underline{H}(\tau)$ and $\overline{H}(\tau)$ of probability that an electric (panel (a)) and non-electric (panel (b)) product is offered in several countries/regions under different US tariff raises on electric products. The probability is computed by calculating the average number of 1's per electric (non-electric) product. The bounds reflect both uncertainty arising from unsolved binary decisions due to computational infeasibility, as well as from statistical uncertainty regarding the fixed cost parameter vector.

7 Conclusion

Our analysis highlights how multinational, multiproduct firms make interconnected production and market entry decisions in response to policy changes. By modeling these choices within a unified framework, we show that subsidies and tariffs have distinct effects on global production. Demand-

Figure 24: Log Variable Profits (Destination) - Tariffs on ROW



(a) Electric (b) Non-Electric

Notes: These plots show the lower and upper bound levels $\underline{H}(\tau)$ and $\overline{H}(\tau)$, respectively, of the (log) of total variable profits across electric (panel (a)) and non-electric (panel (b)) products at the destination level for several production countries/regions under different US tariff raises on electric products. The bounds reflect both uncertainty arising from unsolved binary decisions due to computational infeasibility, as well as from statistical uncertainty regarding the fixed cost parameter vector.

side EV subsidies in the U.S. favor European production and EV market entry, while marginal cost subsidies shift production toward the U.S., both reducing the availability of non-electric alternatives worldwide. Tariffs, in contrast, have limited effects on global production structures due to high fixed costs, inducing selective production relocation but reducing product variety.

More broadly, our framework illustrates how national policies can reshape firms' global strategies, creating spillover effects across markets. The presence of fixed costs and economies of scale generates a proximity-concentration tradeoff, meaning that decisions in one country influence firms' entire global operations. These findings underscore the importance of considering cross-market effects when analyzing trade and industrial policies in industries dominated by multinational firms.

References

Alonso Alfaro-Ureña, Juanma Castro-Vincenzi, Sebastian Fanelli, and Eduardo Morales. Firm dynamics with interdependent markets. *mimeo*, January 2024.

Hunt Allcott, Reigner Kane, Max Maydanchik, Joseph Shapiro, and Felix Tintelnot. The effects of "buy american": Electric vehicles and the inflation reduction act. Mimeograph, October 2024.

James E. Anderson and Eric van Wincoop. Gravity with Gravitas: A Solution to the Border Puzzle. *American Economic Review*, 93(1):170–192, 2003.

Donald W.K. Andrews and Gustavo Soares. Inference for Parameters Defined by Moment Inequalities Using Generalized Moment Selection. *Econometrica*, 78:119–157, 2010.

Pol Antràs, Teresa Fort, and Felix Tintelnot. The margins of global sourcing: Theory and evidence from u.s. firms. *American Economic Review*, 107(9):2514–2564, 2017.

Costas Arkolakis, Natalia Ramondo, Andrés Rodríguez-Clare, and Stephen Yeaple. Innovation and production in the global economy. *American Economic Review*, 108(8):2128–73, August 2018. doi: 10.1257/aer.20141743. URL https://www.aeaweb.org/articles?id=10.1257/aer.20141743.

Costas Arkolakis, Sharat Ganapati, and Marc-Andreas Muendler. The extensive margin of exporting products: A firm-level analysis. *American Economic Journal: Macroeconomics*, 13(4): 182–245, October 2021. doi: 10.1257/mac.20150370. URL https://www.aeaweb.org/articles?id=10.1257/mac.20150370.

Costas Arkolakis, Fabian Eckert, and Rowan Shi. Combinatorial discrete choice: A quantitative model of multinational location decisions*. *mimeo*, November 2023.

David Atkin and Dave Donaldson. "Who's Getting Globalized? The Size and Implications of Intra-national Trade Costs". *Mimeo*, 2015.

Panle Jia Barwick, Hyuk-Soo Kwon, and Shanjun Li. Attribute-based subsidies and market power: an application to electric vehicles. Working Paper 32264, National Bureau of Economic Research, March 2024. URL http://www.nber.org/papers/w32264.

Andrew B. Bernard, J. Bradford Jensen, Stephen J. Redding, and Peter K. Schott. Global firms. *Journal of Economic Literature*, 56(2):565–619, June 2018. doi: 10.1257/jel.20160792. URL https://www.aeaweb.org/articles?id=10.1257/jel.20160792.

Juanma Castro-Vincenzi. Climate hazards and resilience in the global car industry. *mimeo*, February 2024.

Kerem A. Coşar, Paul L. E. Grieco, Shengyu Li, and Felix Tintelnot. What drives home market advantage? *Journal of International Economics*, 110:135–150, 2016.

Swati Dhingra. Trading Away Wide Brands for Cheap Brands. *American Economic Review*, 103(6):2554–2584, October 2013. URL https://ideas.repec.org/a/aea/aecrev/v103y2013i6p2554-84.html.

Dave Donaldson. Railroads of the raj: Estimating the impact of transportation infrastructure. *American Economic Review*, 108(4-5):899–934, April 2018. doi: 10.1257/aer.20101199. URL https://www.aeaweb.org/articles?id=10.1257/aer.20101199.

Carsten Eckel and J. Peter Neary. Multi-Product Firms and Flexible Manufacturing in the Global Economy. *The Review of Economic Studies*, 77(1):188–217, 01 2010. ISSN 0034-6527. doi: 10.1111/j.1467-937X.2009.00573.x. URL https://doi.org/10.1111/j.1467-937X.2009.00573.x.

Pinelopi Koujianou Goldberg. Product differentiation and oligopoly in international markets: The case of the u.s. automobile industry. *Econometrica*, 63(4):891–951, 1995. ISSN 00129682, 14680262. URL http://www.jstor.org/stable/2171803.

Keith Head and Thierry Mayer. Brands in motion: How frictions shape multinational production. *American Economic Review*, 109(9):3073–3124, September 2019. doi: 10.1257/aer.20161345. URL https://www.aeaweb.org/articles?id=10.1257/aer.20161345.

Keith Head, Thierry Mayer, Marc Melitz, and Chenying Yang. Industrial policies for multi-stage production: The battle for battery-powered vehicles. Manuscript, 2025. URL https://github.com/ckhead/HMMY_BEV.

Elhanan Helpman and Benjamin Niswonger. Dynamics of Large Multinationals. *Journal of the European Economic Association*, 21(5):1994–2042, 01 2023. ISSN 1542-4766. doi: 10.1093/jeea/jvad004. URL https://doi.org/10.1093/jeea/jvad004.

Colin J. Hottman, Stephen J. Redding, and David E. Weinstein. Quantifying the Sources of Firm Heterogeneity. *The Quarterly Journal of Economics*, 131(3):1291–1364, 2016. URL https://ideas.repec.org/a/oup/qjecon/v131y2016i3p1291-1364..html.

ITC. Market access map, 2022. URL Accessedfromwww.macmap.org.

Panle Jia. What Happens When Wal-Mart Comes to Town: An Empirical Analysis of the Discount Retailing Industry. *Econometrica*, 76(6):1263–1316, 2008.

Brendan Kline, Ariel Pakes, and Elie Tamer. Chapter 5 - moment inequalities and partial identification in industrial organization. In Kate Ho, Ali Hortaçsu, and Alessandro Lizzeri, editors, *Handbook of Industrial Organization, Volume 4*, volume 4 of *Handbook of Industrial Organization*, pages 345–431. Elsevier, 2021. doi: https://doi.org/10.1016/bs.hesind.2021.11.005. URL https://www.sciencedirect.com/science/article/pii/S1573448X21000054.

Thierry Mayer and Soledad Zignago. Notes on Cepii's Distances Measures: the geodist database. *CEPII Working Paper*, 2011-25, 2011.

Thierry Mayer, Marc J. Melitz, and Gianmarco I. P. Ottaviano. Market size, competition, and the product mix of exporters. *American Economic Review*, 104(2):495–536, February 2014. doi: 10.1257/aer.104.2.495. URL https://www.aeaweb.org/articles?id=10.1257/aer.104.2.495.

Ariel Pakes, Jack R. Porter, Katherine Ho, and Joy Ishii. Moment Inequalities and their Application. *Econometrica*, 83(1):315–334, 2015.

Natalia Ramondo and Andrés Rodríguez-Clare. Trade, multinational production, and the gains from openness. *Journal of Political Economy*, 121(2):273–322, 2013. doi: 10.1086/670136. URL https://doi.org/10.1086/670136.

Stephen Redding and Anthony J. Venables. Economic geography and international inequality. *Journal of International Economics*, 62(1):53–82, 2004. ISSN 0022-1996. doi: https://doi.org/

10.1016/j.jinteco.2003.07.001. URL https://www.sciencedirect.com/science/article/pii/S0022199603000965.

Alejandro Sabal. Product entry in the global automobile industry. *mimeo*, 2025.

Alfred Tarski. A lattice-theoretical fixpoint theorem and its applications. *Pacific Journal of Mathematics*, 5(2):285 – 309, 1955.

Felix Tintelnot. Global production with export platforms. *The Quarterly Journal of Economics*, 132(1):157–209, 2017.

Appendices

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A Model Properties

In this section, we derive the key model properties that guarantee that Assumption 1 is satisfied under different specifications of the payoff function. We begin by proving the interdependence structure across the discrete choices that enter Π_{bs}. We then prove the interdependence properties of value functions V_{bs} and $V_{b,s,m}$.

A.1 Interdependence Properties under Π_{bs}

In this subsection, we prove the properties of Π_{bs} stated in section 3.1.1. Total profits for brand in segment s can be written as,

$$\Pi_{bs}(\boldsymbol{I}^{bs}, \boldsymbol{D}^{bs}) = \sum_{n=1}^{N} \sum_{m=1}^{M_{bs}} \{I_{mn}(\pi_{mn}(\boldsymbol{I}_n^{bs}, \boldsymbol{D}^{bs}) - F_{mn}^{e}) - D_{mn}F_{mn}^{p}\} \tag{A1}$$

$$= \sum_{n=1}^{N} A_{bsn} P_{bsn}(\boldsymbol{I}_n^{bs}, \boldsymbol{D}^{bs})^{1-\sigma^B} - \sum_{m,n} (I_{mn}F_{mn}^{e} + D_{mn}F_{mn}^{p}), \tag{A2}$$

where

$$P_{bsn}(\boldsymbol{I}_n^{bs}, \boldsymbol{D}^{bs}) = \Big(\sum_{m=1}^{M_{bs}} I_{mn} \big(P_{mn}(\boldsymbol{D}_m)/\psi_{mn}\big)^{1-\sigma^M} \Big)^{\frac{1}{1-\sigma^M}}. \tag{A3}$$

A.1.1 Property 1: D_{mo} and I_{mn} are complements for any m and any $o, n \in \mathcal{N}$

To prove this property, consider the marginal value of I_{mn},

$$\sigma_{I_{mn}}(\boldsymbol{I}^{bs}, \boldsymbol{D}^{bs}) = A_{bsn}[P_{bsn}(\boldsymbol{I}_n^{b,m\to 1}, \boldsymbol{D}^{bs})^{1-\sigma^B} - P_{bsn}(\boldsymbol{I}_n^{b,m\to 0}, \boldsymbol{D}^{bs})^{1-\sigma^B}] - F_{mn}^{e}. \tag{A4}$$

To prove Property 1, we need to show that $\sigma_{I_{mn}}(\boldsymbol{I}^{bs}, \boldsymbol{D}^{bs})$ is increasing in $\boldsymbol{D}_m$. Consider $\hat{\boldsymbol{D}}_m \geq \boldsymbol{D}_m$. From the perspective of country n sales, there are two possibilities. More production location decisions for product m could either: (i) leave optimal sourcing decisions unchanged, and therefore $P_{mn} = \hat{P}_{mn}$, or (ii) decrease the cost of supplying m in n, causing $\hat{P}_{mn} < P_{mn}$. Thus, the difference-in-differences yields,

$$\sigma_{I_{mn}}(\boldsymbol{I}^{bs}, \hat{\boldsymbol{D}}^{b}) - \sigma_{I_{mn}}(\boldsymbol{I}^{bs}, \boldsymbol{D}^{bs}) = A_{bsn}[P_{bsn}(\boldsymbol{I}_n^{b,m\to 1}, \hat{\boldsymbol{D}}^{b})^{1-\sigma^B} - P_{bsn}(\boldsymbol{I}_n^{b,m\to 0}, \hat{\boldsymbol{D}}^{b})^{1-\sigma^B} \tag{A5}$$

$$- P_{bsn}(\boldsymbol{I}_n^{b,m\to 1}, \boldsymbol{D}^{bs})^{1-\sigma^B} + P_{bsn}(\boldsymbol{I}_n^{b,m\to 0}, \boldsymbol{D}^{bs})^{1-\sigma^B}] \tag{A6}$$

$$= A_{bsn}[P_{bsn}(\boldsymbol{I}_n^{b,m\to 1}, \hat{\boldsymbol{D}}^{b})^{1-\sigma^B} - P_{bsn}(\boldsymbol{I}_n^{b,m\to 1}, \boldsymbol{D}^{bs})^{1-\sigma^B}] \tag{A7}$$

where the second equality follows from the fact that whenever $I_{mn} = 0$, the brand-segment price index is equal under $\hat{\boldsymbol{D}}^{b}$ and under $\boldsymbol{D}^{bs}$. Now, under case (i) with $P_{mn} = \hat{P}_{mn}$, such a difference is clearly zero, while under case (ii) the difference is clearly positive. Thus,

$$\sigma_{I_{mn}}(\boldsymbol{I}^{bs}, \hat{\boldsymbol{D}}^{b}) - \sigma_{I_{mn}}(\boldsymbol{I}^{bs}, \boldsymbol{D}^{bs}) \geq 0,$$

which proves complementarity.

We can also prove complementarity in the other direction. Consider the marginal value of D_{mo},

$$\sigma_{D_{mo}}(\boldsymbol{I}^{bs}, \boldsymbol{D}^{bs}) = \sum_n A_{bsn}[P_{bsn}(\boldsymbol{I}^{bs}_n, \boldsymbol{D}^{b,mo\to 1})^{1-\sigma^B} - P_{bsn}(\boldsymbol{I}^{bs}_n, \boldsymbol{D}^{b,mo\to 0})^{1-\sigma^B}] - F^p_{mo}. \tag{A8}$$

Consider the differences-in-differences,

$$\sigma_{D_{mo}}(\boldsymbol{I}^{b,mk\to 1}, \boldsymbol{D}^{bs}) - \sigma_{D_{mo}}(\boldsymbol{I}^{b,mk\to 0}, \boldsymbol{D}^{bs}). \tag{A9}$$

Clearly, for countries $n \neq k$, the terms in the differences-in-differences cancel out, since the marginal contribution of D_{mo} to country n profits is independent of the value of I_{mk}. Therefore,

$$\sigma_{D_{mo}}(\boldsymbol{I}^{b,mk\to 1}, \boldsymbol{D}^{bs}) - \sigma_{D_{mo}}(\boldsymbol{I}^{b,mk\to 0}, \boldsymbol{D}^{bs}) \tag{A10}$$

$$= A_{bsk}[P_{bsk}(\boldsymbol{I}^{b,m\to 1}_k, \boldsymbol{D}^{b,mo\to 1})^{1-\sigma^B} - P_{bsk}(\boldsymbol{I}^{b,m\to 1}_k, \boldsymbol{D}^{b,mo\to 0})^{1-\sigma^B}] \geq 0, \tag{A11}$$

where the equality follows from the fact that the terms with no sales in country n of model m cancel out and the inequality follows from potentially lower prices from $D_{mo} = 1$ and $\sigma^B > 1$.

A.1.2 Property 2: D_{mo} and I_{kn} are substitutes for $m \neq k$ and for any $o, n \in \mathcal{N}$

To prove Property 2, consider the marginal value of I_{sn},

$$\sigma_{I_{kn}}(\boldsymbol{I}^{bs}, \boldsymbol{D}^{bs}) = A_{bsn}[P_{bsn}(\boldsymbol{I}^{b,k\to 1}_n, \boldsymbol{D}^{bs})^{1-\sigma^B} - P_{bsn}(\boldsymbol{I}^{b,k\to 0}_n, \boldsymbol{D}^{bs})^{1-\sigma^B}] - F^e_{kn}. \tag{A12}$$

To prove Property 2, we need to show that $\sigma_{I_{kn}}(\boldsymbol{I}^{bs}, \boldsymbol{D}^{bs})$ is decreasing in $\boldsymbol{D}_m$. Consider $\hat{\boldsymbol{D}}_m \geq \boldsymbol{D}_m$. Clearly, if $I_{mn} = 0$, changes in production location choices for product m will not affect the brand-segment price index. Thus, consider the case in which $I_{mn} = 1$. In this case, more production locations for model m can only lead to declines in the price of model m in country n. Differentiating with respect to P_{mn}, we obtain,

$$\frac{\partial \sigma_{I_{kn}}(\boldsymbol{I}^{bs}, \boldsymbol{D}^{bs})}{\partial P_{mn}} = \frac{A_{bsn}}{\psi_{mn}} \left(\frac{P_{mn}}{\psi_{mn}}\right)^{-\sigma^M} (1-\sigma^B)[P_{bsn}(\boldsymbol{I}^{b,k\to 1}_n, \boldsymbol{D}^{bs})^{\sigma^M - \sigma^B} - P_{bsn}(\boldsymbol{I}^{b,k\to 0}_n, \boldsymbol{D}^{bs})^{\sigma^M - \sigma^B}] \tag{A13}$$

$$\geq 0, \tag{A14}$$

where the inequality follows from $\sigma^B > 1$ and the fact that the price index is decreasing in varieties offered and there is cannibalization, $\sigma^M > \sigma^B$. Therefore, increases in prices of model m in country n increase the marginal value of offering product k in country n. Or, increases in production locations for model m, which weakly decrease prices for product m in country n, decrease the marginal value of offering product k country n. It follows that Property 2 holds.

A.1.3 Property 3: I_{kn} and I_{mn} are substitutes in country n for $k \neq m$

The proof for this property mimics the proof for Property 2. Indeed, we proved that $\sigma_{I_{kn}}(\boldsymbol{I}^{bs}, \boldsymbol{D}^{bs})$ is increasing in P_{mn}. Offering product m in country n is equivalent to decreasing its price from ∞ to a

finite value. Therefore, $\sigma_{I_{kn}}(\boldsymbol{I}^{bs}, \boldsymbol{D}^{bs})$ is also decreasing in I_{mn}.

A.1.4 Property 4: I_{mn} and I_{kt} are independent for $n \neq t$ for all m, s

This property follows by inspection of equation (A4) defining $\sigma_{I_{mn}}$, which clearly does not depend on the sales decisions in markets other than n.

A.1.5 Property 5: D_{mo} and D_{mt} are substitutes for $o \neq t$

To prove Property 5, consider the marginal value of producing model m in country o,

$$\sigma_{D_{mo}}(\boldsymbol{I}^{bs}, \boldsymbol{D}^{bs}) = \sum_n A_{bsn}[P_{bsn}(\boldsymbol{I}_n^{bs}, \boldsymbol{D}^{b,mo\to 1})^{1-\sigma^B} - P_{bsn}(\boldsymbol{I}_n^{bs}, \boldsymbol{D}^{b,mo\to 0})^{1-\sigma^B}] - F_{mo}^p. \tag{A15}$$

We introduce notation and write,

$$\sigma_{D_{mo}}(\boldsymbol{I}^{bs}, \boldsymbol{D}^{bs}) = \sum_n \Delta_n^o(\boldsymbol{D}_m) - F_{mo}^p. \tag{A16}$$

Note that $\Delta_n^o(\boldsymbol{D}_m) \geq 0$ for all n and all $\boldsymbol{D}_m$. We abuse notation and write Δ_n^o as a function of only $\boldsymbol{D}_m$ even though it also depends on production location of other products and of country-n offerings. We prove Property 5 by showing that $\Delta_n^o(\overline{\boldsymbol{D}}_m) \leq \Delta_n^o(\underline{\boldsymbol{D}}_m)$ for all n. This shows that the marginal value of producing m in o is smaller when m is produced in more locations. Consider two cases:

1. $\Delta_n^o(\overline{\boldsymbol{D}}_m) = 0$. In this case, producing product m in country o causes no change in variable profits in country n because m is not sourced to n from o. Thus, necessarily $0 = \Delta_n^o(\overline{\boldsymbol{D}}_m) \leq \Delta_n^o(\underline{\boldsymbol{D}}_m)$.

2. $\Delta_n^o(\overline{\boldsymbol{D}}_m) > 0$. If this is the case, then necessarily country o is the minimum cost location to source product m to country n, under $\overline{\boldsymbol{D}}_m$. The fact that $\underline{\boldsymbol{D}_m} \leq \overline{\boldsymbol{D}}_m$ implies that $\Delta_n^o(\underline{\boldsymbol{D}}_m) \geq 0$ – if o is the lowest cost production location under a superset of possible sourcing locations, it must also be the lowest cost location under a subset. However, the change in cost (and therefore in price) must be weakly smaller under $\overline{\boldsymbol{D}}_m$ than under $\underline{\boldsymbol{D}}_m$, which implies that $\Delta_n^o(\overline{\boldsymbol{D}}_m) \leq \Delta_n^o(\underline{\boldsymbol{D}}_m)$.

We have therefore proven Property 5.

A.1.6 Property 6: D_{mo} and D_{kt} are substitutes for $m \neq k$

To prove this property, it suffices to prove that $\frac{\partial}{\partial P_{kn}}\Delta_n^o(\boldsymbol{D}_m) \geq 0$. This follows from the fact that, as seen in equation (A15), the marginal value of producing m in country o consists of the change in variable profits across all countries arising from this decision minus the production fixed cost. If some other product k is produced is now produced in some country t, this could potentially reduce the price of product k in any country, affecting the marginal contribution of D_{mo} through the brand-segment price index.

We have,

$$\frac{\partial}{\partial P_{kn}}\Delta_n^o(\boldsymbol{D}_m) = \frac{A_{bsn}}{\psi_{kn}}\left(\frac{P_{kn}}{\psi_{kn}}\right)^{-\sigma^M}(1-\sigma^B)[P_{bsn}(\boldsymbol{I}_n^{bs},\boldsymbol{D}^{b,mo\to 1})^{\sigma^M-\sigma^B} - P_{bsn}(\boldsymbol{I}_n^{bs},\boldsymbol{D}^{b,mo\to 0})^{\sigma^M-\sigma^B}] \quad \text{(A17)}$$

$$\geq 0, \quad \text{(A18)}$$

since $\sigma^B > 1$, $\sigma^M > \sigma^B$ and the brand-segment price index is weakly decreasing in the set of production locations. We have therefore proven Property 6.

A.2 Interdependence Properties under $V_{b,s,m}$

We now prove the interdependence across production locations under the mapping,

$$V_{b,s,m}(\boldsymbol{D}_m;\boldsymbol{I}_{-m},\boldsymbol{D}_{-m}) := \max_{\boldsymbol{I}_m}\sum_{n=1}^{N} I_{mn}\Delta\pi_n^m(\boldsymbol{D}_m;\boldsymbol{I}_{-m},\boldsymbol{D}_{-m}) - \sum_{n=1}^{N} D_{mn}F_{mn}^p. \quad \text{(A19)}$$

We first consider the single-product firm case and prove Proposition 2. We then prove general properties that hold in the general case with multiproduct firms.

A.2.1 Proof of Proposition 2

Consider the mapping,

$$V_{b,s}(\boldsymbol{D}_m) := \max_{\boldsymbol{I}_m}\sum_{n=1}^{N} I_{mn}\Delta\pi_n^m(\boldsymbol{D}_m) - \sum_{n=1}^{N} D_{mn}F_{mn}^p. \quad \text{(A20)}$$

The aim is to prove that D_{mo} and $D_{mo'}$ are substitutes under $V_{b,s}$. It suffices to prove that, holding fixed all production location decisions except for two, we have that,

$$V_{b,s}((1,1),\boldsymbol{D}_m) - V_{b,s}((0,1),\boldsymbol{D}_m) \leq V_{b,s}((1,0),\boldsymbol{D}_m) - V_{b,s}((0,0),\boldsymbol{D}_m). \quad \text{(A21)}$$

While b is single-product in segment s, we label the product as m (this helps with notation when we extend the result).

Introducing notation, let $I_{mn}^{(i,j)}$ denote the optimal product entry decision rule under the production location decision implied by $(i,j) \in \{(1,1),(1,0),(0,1),(0,0)\}$. We abuse notation and write $((i,j),\boldsymbol{D}_m)$ as the argument of the function, which should be interpreted as all production locations other than (i,j) are fixed at the values corresponding to $\boldsymbol{D}_m$.

The first step of the proof is to realize that,

$$I_{mn}^{(1,1)} = I_{mn}^{(1,0)} + I_{mn}^{(0,1)} - I_{mn}^{(1,0)}I_{mn}^{(0,1)}. \quad \text{(A22)}$$

Equation (A22) says that if product m is sold in country n under production vector $(1,1)$, it must be sold either under production vector $(1,0)$ or under production vector $(0,1)$ (or both). This property arises due

to single-origin sourcing – sourcing from two locations cannot be beneficial. In addition, we must have that,

$$I_{mn}^{(1,1)} \geq \max\{I_{mn}^{(1,0)}, I_{mn}^{(0,1)}\}, \tag{A23}$$

and

$$I_{mn}^{(0,0)} \leq \min\{I_{mn}^{(1,0)}, I_{mn}^{(0,1)}\} \tag{A24}$$

Inequality (A23) says that if m is sold in n under either $(1,0)$ or $(0,1)$, then it is necessarily sold under $(1,1)$. Inequality (A24) says that if m is sold in n under $(0,0)$, it must be sold either under $(1,0)$ or $(0,1)$. Combining inequalities (A22)-(A24), we obtain,

$$I_{mn}^{(1,1)} - I_{mn}^{(0,1)} = I_{mn}^{(1,0)} - I_{mn}^{(1,0)} I_{mn}^{(0,1)} \leq I_{mn}^{(1,0)} - I_{mn}^{(0,0)}. \tag{A25}$$

We are now ready to prove the proposition. Expanding inequality (A21), the goal is to prove that

$$\begin{aligned} &\sum_n \left(I_{mn}^{(1,1)} - I_{mn}^{(0,1)}\right) \Delta\pi_n^m(1,1) + \sum_n I_{mn}^{(0,1)} \left[\Delta\pi_n^m(1,1) - \Delta\pi_n^m(1,1)\right] \\ &\qquad\qquad \leq \\ &\sum_n \left(I_{mn}^{(1,0)} - I_{mn}^{(0,0)}\right) \Delta\pi_n^m(1,0) + \sum_n I_{mn}^{(0,0)} \left[\Delta\pi_n^m(1,0) - \Delta\pi_n^m(0,0)\right] \end{aligned}$$

or equivalently,

$$\begin{aligned} &\sum_n \left(I_{mn}^{(1,1)} - I_{mn}^{(0,1)}\right) \Delta\pi_n^m(1,1) + \sum_n (I_{mn}^{(0,1)} - I_{mn}^{(0,0)}) \left[\Delta\pi_n^m(1,1) - \Delta\pi_n^m(0,1)\right] \\ &\qquad\qquad + \sum_n I_{mn}^{(0,0)} [\Delta\pi_n^m(1,1) - \Delta\pi_n^m(0,1)] \\ &\qquad\qquad \leq \\ &\sum_n \left(I_{mn}^{(1,0)} - I_{mn}^{(0,0)}\right) \Delta\pi_n^m(1,0) + \sum_n I_{mn}^{(0,0)} \left[\Delta\pi_n^m(1,0) - \Delta\pi_n^m(0,0)\right]. \end{aligned} \tag{A26}$$

Due to Property 5 of the model (proven in subsection A.1.5),

$$\Delta\pi_n^m(1,1) - \Delta\pi_n^m(0,1) \leq \Delta\pi_n^m(1,0) - \Delta\pi_n^m(0,0), \tag{A27}$$

which follows from the fact that production locations are substitutes. Combining inequalities (A26)-(A27), it follows that a sufficient condition to prove is,

$$\sum_n (I_{mn}^{(1,1)} - I_{mn}^{(0,0)}) \Delta\pi_n^m(1,1) \leq \sum_n (I_{mn}^{(0,1)} - I_{mn}^{(0,0)}) \Delta\pi_n^m(0,1) + \sum_n (I_{mn}^{(1,0)} - I_{mn}^{(0,0)}) \Delta\pi_n^m(1,0). \tag{A28}$$

Note that the LHS of inequality (A28) across the variable profits in all countries for which the brand enters only when the product is produced in either of the two countries. The RHS sums profits over all destination countries for which sourcing occurs from the second country and over all countries for which

sourcing occurs from the first country. It must be that the LHS is equal to the RHS, which concludes the proof.

A.2.2 Corollary of Proposition 2: $V_{b,s,m}$ exhibits decreasing differences in $\boldsymbol{D}_m$

A corollary of Proposition 2 is that the function $V_{b,s,m}$, defined in equation (A19), exhibits decreasing differences in $\boldsymbol{D}_m$. The proof follows immediately from the arguments in section A.2.1. Note that all of the arguments apply even in the case with multiproduct firms, given that they do not depend on the values of $(\boldsymbol{I}_{-m}, \boldsymbol{D}_{-m})$.

A.2.3 Argmax of $V_{b,s,m}(\boldsymbol{D}_m; \boldsymbol{I}^*_{-m}, \boldsymbol{D}^*_{-m})$ is $\boldsymbol{D}^*_m$

In this subsection, we prove the following proposition.

Proposition 5 *The optimal production location decisions for product m, $\boldsymbol{D}^*_m$, that solve the program $\max_{\boldsymbol{I}^{bs}, \boldsymbol{D}^{bs}} \Pi_{bs}(\boldsymbol{I}^{bs}, \boldsymbol{D}^{bs})$ are also the argmax of $V_{b,s,m}(\boldsymbol{D}_m; \boldsymbol{I}^*_{-m}, \boldsymbol{D}^*_{-m})$ is $\boldsymbol{D}^*_m$.*

Proof. The problem $\max_{\boldsymbol{I}^{bs}, \boldsymbol{D}^{bs}} \Pi_{bs}(\boldsymbol{I}^{bs}, \boldsymbol{D}^{bs})$ has a unique solution almost surely. Note that this problem can trivially be written as,

$$\max_{\boldsymbol{D}_m} \max_{\boldsymbol{I}_m} \max_{\boldsymbol{I}_{-m}, \boldsymbol{D}_{-m}} \Pi_{bs}(\boldsymbol{I}^{bs}, \boldsymbol{D}^{bs}) = \sum_{n=1}^{N} \sum_{m=1}^{M_{bs}} \{I_{mn}(\pi_{mn}(\boldsymbol{I}^{bs}_n, \boldsymbol{D}^{bs}) - F^e_{mn}) - D_{mn} F^p_{mn}\}. \tag{A29}$$

Let $(\boldsymbol{I}_{-m}(\boldsymbol{I}_m, \boldsymbol{D}_m), \boldsymbol{D}_{-m}(\boldsymbol{I}_m, \boldsymbol{D}_m))$ denote the argmax of the innermost maximization problem. We can then write the problem as,

$$\max_{\boldsymbol{D}_m} \max_{\boldsymbol{I}_m} \sum_{n=1}^{N} \{I_{mn}(\Delta\pi^m_n(\boldsymbol{D}_m; \boldsymbol{I}_{-m}(\boldsymbol{I}_m, \boldsymbol{D}_m), \boldsymbol{D}_{-m}(\boldsymbol{I}_m, \boldsymbol{D}_m))) - D_{mn} F^p_{mn}\}. \tag{A30}$$

This step follows because conditional on production location choices, market entry decisions are independent across countries. As such, given any $\boldsymbol{D}_m$, the inner loop of problem (A30) simply chooses optimal entry decisions in each country independently, accounting for how such entry decisions affect overall profits. Now, program (A30) has the form of $V_{b,s,m}$, which proves Proposition 5. ■

A.2.4 $V_{b,s,m}$ exhibits decreasing differences in $\boldsymbol{D}_{-m}$ and $\boldsymbol{I}_{-m}$

To prove this result, consider the difference

$$\Delta V^o_{b,s,m}(\boldsymbol{D}_m; \boldsymbol{I}_{-m}, \boldsymbol{D}_{-m}) = V_{b,s,m}(\boldsymbol{D}^{o\to 1}_m; \boldsymbol{I}_{-m}, \boldsymbol{D}_{-m}) - V_{b,s,m}(\boldsymbol{D}^{o\to 0}_m; \boldsymbol{I}_{-m}, \boldsymbol{D}_{-m}) - F^p_{mo}.$$

The goal is to prove that $\Delta V^o_{b,s,m}(\boldsymbol{D}_m; \boldsymbol{I}_{-m}, \boldsymbol{D}_{-m})$ is decreasing in $\boldsymbol{I}_{-m}$ and $\boldsymbol{D}_{-m}$. We are interested in establishing that $\Delta V^o_{b,s,m}(\boldsymbol{D}_m; \underline{\boldsymbol{I}}_{-m}, \boldsymbol{D}_{-m}) - \Delta V^o_{b,s,m}(\boldsymbol{D}_m; \overline{\boldsymbol{I}}_{-m}, \boldsymbol{D}_{-m}) \geq 0$ and $\Delta V^o_{b,s,m}(\boldsymbol{D}_m; \boldsymbol{I}_{-m}, \underline{\boldsymbol{D}}_{-m}) - \Delta V^o_{b,s,m}(\boldsymbol{D}_m; \boldsymbol{I}_{-m}, \overline{\boldsymbol{D}}_{-m}) \geq 0$ for $\overline{\boldsymbol{D}}_{-m} \geq \underline{\boldsymbol{D}}_{-m}$ and $\overline{\boldsymbol{I}}_{-m} \geq \underline{\boldsymbol{I}}_{-m}$. Using $\underline{V}_{b,s,m}$ to denote dependence on $\underline{\boldsymbol{I}}_{-m}$ or $\underline{\boldsymbol{D}}_{-m}$, and $\overline{V}_{b,s,m}$ to denote dependence on $\overline{\boldsymbol{I}}_{-m}$ or $\overline{\boldsymbol{D}}_{-m}$, the difference-in-differences can be

expanded as,

$$\begin{aligned}\Delta\underline{V}^{o}_{b,s,m}(\boldsymbol{D}_m) - \Delta\overline{V}^{o}_{b,s,m}(\boldsymbol{D}_m) = &\sum_n \underline{I}^1_{mn}[\Delta\underline{\pi}^m_n(\boldsymbol{D}^{o\to1}_m) - \Delta\overline{\pi}^m_n(\boldsymbol{D}^{o\to1}_m)] \\ &+ \sum_n (\underline{I}^1_{mn} - \overline{I}^1_{mn})\Delta\overline{\pi}^m_n(\boldsymbol{D}^{o\to1}_m) \\ &- \sum_n \underline{I}^0_{mn}[\Delta\underline{\pi}^m_n(\boldsymbol{D}^{o\to0}_m) - \Delta\overline{\pi}^m_n(\boldsymbol{D}^{o\to0}_m)] \\ &- \sum_n (\underline{I}^0_{mn} - \overline{I}^0_{mn})\Delta\overline{\pi}^m_n(\boldsymbol{D}^{o\to0}_m),\end{aligned} \tag{A31}$$

where under and over-lines are used to denote dependence on $\underline{\boldsymbol{D}}_{-m}(\underline{\boldsymbol{I}}_{-m})$ and $\overline{\boldsymbol{D}}_{-m}(\overline{\boldsymbol{I}}_{-m})$, respectively and superscripts on I_{mn} denote the optimal product-market entry decision under $D_{mo} = j$. Partition the set of countries as follows:

$$\begin{aligned}G_m &= \{n : \underline{I}^1_{mn} - \overline{I}^1_{mn} > \underline{I}^0_{mn} - \overline{I}^0_{mn}\} \\ E_m &= \{n : \underline{I}^1_{mn} - \overline{I}^1_{mn} = \underline{I}^0_{mn} - \overline{I}^0_{mn}\} \\ L_m &= \{n : \underline{I}^1_{mn} - \overline{I}^1_{mn} < \underline{I}^0_{mn} - \overline{I}^0_{mn}\}.\end{aligned}$$

We can partition equation (A31) and write,

$$\Delta\underline{V}^{o}_{b,s,m}(\boldsymbol{D}_m) - \Delta\overline{V}^{o}_{b,s,m}(\boldsymbol{D}_m) = \Sigma_{G_m} + \Sigma_{E_m} + \Sigma_{L_m} \tag{A32}$$

where Σ_{G_m} denotes the components of the sum in equation (A31) that sum over G_m, Σ_{E_m} denotes the components that sum over E_m, and Σ_{L_m} denotes the components that sum over L_m. It suffices to show that each of these components is weakly positive. Starting with G_m, note that if $n \in G_m$, it must be that $\underline{I}^1_{mn} = 1$ and $\overline{I}^1_{mn} = \underline{I}^0_{mn} = \overline{I}^0_{mn} = 0$, which implies,

$$\Sigma_{G_m} = \sum_{n\in G_m} \Delta\underline{\pi}^m_n(\boldsymbol{D}^{o\to1}_m) \geq 0, \tag{A33}$$

where the inequality follows from $n \in G_m \implies I^1_{mn} = 1$. Focusing now on Σ_{E_m}, note that for all $n \in E_m$, either $\underline{I}^1_{mn} = \overline{I}^1_{mn} = \underline{I}^0_{mn} = \overline{I}^0_{mn} = 0$, in which case the difference-in-differences is $\Sigma_{E_m} = 0$, or $\underline{I}^1_{mn} = \overline{I}^1_{mn} = \underline{I}^0_{mn} = \overline{I}^0_{mn} = 1$, in which case,

$$\Sigma_{E_m} = \sum_{n\in E_m : \underline{I}^1_{mn}=\overline{I}^1_{mn}=\underline{I}^0_{mn}=\overline{I}^0_{mn}=1} [(\Delta\underline{\pi}^m_n(\boldsymbol{D}^{o\to1}_m) - \Delta\overline{\pi}^m_n(\boldsymbol{D}^{o\to1}_m)) - (\Delta\underline{\pi}^m_n(\boldsymbol{D}^{o\to0}_m) - \Delta\overline{\pi}^m_n(\boldsymbol{D}^{o\to0}_m))] \geq 0, \tag{A34}$$

where the inequality follows because the marginal value of producing m at country o is decreasing in the set of production and sales locations for products $-m$, as shown in Properties 2 and 6. Finally, consider the summation across $n \in L_m$. Note that if $n \in L_m$, then necessarily $\underline{I}^0_{mn} = \underline{I}^1_{mn} = \overline{I}^1_{mn} = 1$ and $\overline{I}^0_{mn} = 0$.

As such,

$$\Sigma_{L_m} = \sum_{n \in L_m} \{[(\Delta \underline{\pi}_n^m(\boldsymbol{D}_m^{o\to 1}) - \Delta \overline{\pi}_n^m(\boldsymbol{D}_m^{o\to 1})) - (\Delta \underline{\pi}_n^m(\boldsymbol{D}_m^{o\to 0}) - \Delta \overline{\pi}_n^m(\boldsymbol{D}_m^{o\to 0}))] - \Delta \overline{\pi}_n^m(\boldsymbol{D}_m^{o\to 0})\} \geq 0,$$

where the inequality follows from Properties 2 and 6 and from the fact that for $n \in L_m$, $\overline{I}_{mn}^0 = 0 \implies \Delta \overline{\pi}_n^m(\boldsymbol{D}_m^{o\to 0}) < 0$.

We have therefore proven that $V_{b,s,m}$ exhibits decreasing differences in $\boldsymbol{D}_{-m}$ and $\boldsymbol{I}_{-m}$.

B Method Proofs

B.1 Proof of Theorem 2

For $\boldsymbol{C} \subseteq \boldsymbol{C'}$, it must be that for all i, $\sigma_i(\pi(\sup_i(\boldsymbol{C}))) \leq \sigma_i(\pi(\sup_i(\boldsymbol{C'})))$ and $\sigma_i(\pi(\inf_i(\boldsymbol{C}))) \geq \sigma_i(\pi(\inf_i(\boldsymbol{C'})))$. Thus, $\overline{\Omega}(\boldsymbol{C'}) \subseteq \overline{\Omega}(\boldsymbol{C})$ and $\underline{\Omega}(\boldsymbol{C'}) \subseteq \underline{\Omega}(\boldsymbol{C})$, which proves that $F(\boldsymbol{C}) \subseteq F(\boldsymbol{C'})$.

B.2 Proof of Proposition 3

Given the proof for Proposition 1, the proof of Proposition 3 requires three steps. First, note that conditional on m and on $(\boldsymbol{I}_{-m}, \boldsymbol{D}_{-m})$, we can apply Algorithm 3 using the mapping defined in Corollary 1 which uses the decision-specific infima constructed using knowledge of substitutability across D_{mo} and $D_{mo'}$, as established in Corollary A.2.2. By Theorem 2, this mapping, which we denote by $\underline{F}_m(\boldsymbol{I}_{-m}, \boldsymbol{D}_{-m})$ is set-increasing and therefore converges to a fixed point. Second, we show that $\underline{F}_m(\overline{\boldsymbol{I}}_{-m}, \overline{\boldsymbol{D}}_{-m})$ converges to a set containing a lower bound of $\boldsymbol{D}_m^*$ provided $(\overline{\boldsymbol{I}}_{-m}, \overline{\boldsymbol{D}}_{-m})$ are upper bounds of the solution $(\boldsymbol{I}_{-m}^*, \boldsymbol{D}_{-m}^*)$. This follows because by definition of mapping $\underline{F}_m(\overline{\boldsymbol{I}}_{-m}, \overline{\boldsymbol{D}}_{-m})$, the infimum $\overline{\boldsymbol{D}}_m$ of the fixed point lattice must satisfy

$$\begin{aligned}
\underline{D}_{mo} = 1 &\iff V_{b,s,m}(\underline{\boldsymbol{D}}_m^{o\to 1}; \underline{\boldsymbol{I}}_{-m}, \underline{\boldsymbol{D}}_{-m}) - V_{b,s,m}(\underline{\boldsymbol{D}}_m^{o\to 0}; \underline{\boldsymbol{I}}_{-m}, \underline{\boldsymbol{D}}_{-m}) \geq 0 \\
&\implies V_{b,s,m}(\boldsymbol{D}_m^{*,o\to 1}; \boldsymbol{I}_{-m}^*, \boldsymbol{D}_{-m}^*) - V_{b,s,m}(\boldsymbol{D}_m^{*,o\to 0}; \boldsymbol{I}_{-m}^*, \boldsymbol{D}_{-m}^*) \geq 0 \qquad \text{(A35)} \\
&\implies D_{mo}^* = 1, \qquad \text{(A36)}
\end{aligned}$$

where implication (A35) arises from the decreasing differences results proved in sections A.2.2 and A.2.4 and implication (A36) stems from the argmax result proven in section A.2.3. We can similarly prove that mapping $\overline{F}_m(\overline{\boldsymbol{I}}_{-m}, \overline{\boldsymbol{D}}_{-m})$ (also defined in Corollary 1) converges to a set containing an upper bound of $\boldsymbol{D}_m^*$ provided $(\overline{\boldsymbol{I}}_{-m}, \overline{\boldsymbol{D}}_{-m})$ are upper bounds of the solution. This follows because by definition of mapping $\overline{F}_m(\overline{\boldsymbol{I}}_{-m}, \overline{\boldsymbol{D}}_{-m})$, the supremum $\overline{\boldsymbol{D}}_m$ of the fixed point lattice must satisfy

$$\begin{aligned}
\overline{D}_{mo} = 0 &\iff V_{b,s,m}(\overline{\boldsymbol{D}}_m^{o\to 1}; \overline{\boldsymbol{I}}_{-m}, \overline{\boldsymbol{D}}_{-m}) - V_{b,s,m}(\overline{\boldsymbol{D}}_m^{o\to 0}; \overline{\boldsymbol{I}}_{-m}, \overline{\boldsymbol{D}}_{-m}) < 0 \\
&\implies V_{b,s,m}(\boldsymbol{D}_m^{*,o\to 1}; \boldsymbol{I}_{-m}^*, \boldsymbol{D}_{-m}^*) - V_{b,s,m}(\boldsymbol{D}_m^{*,o\to 0}; \boldsymbol{I}_{-m}^*, \boldsymbol{D}_{-m}^*) < 0 \qquad \text{(A37)} \\
&\implies D_{mo}^* = 0, \qquad \text{(A38)}
\end{aligned}$$

where (A37)-(A38) again follow from sections A.2.2-A.2.4. The final step is to prove that composing the mappings across products m, as described in Algorithm 4, still yields bounds of the solution. This follows from the fact that if F and G are both set-increasing mappings defined on sets of complete sublattices, then the composition $F \circ G$ is also set-increasing (because the composition of two increasing functions is increasing). Moreover, if G converges to a complete lattice containing the solution, then necessarily F converges to a complete lattice containing the solution. We can apply this reasoning to compose such set-increasing mappings *in any order* to obtain bounds of the solution.[19]

B.3 Proof of Proposition 4

We proceed by induction on the dimensionality of the problem and under payoff function π. If the dimensionality of the problem is $k = 1$, branching trivially returns the solution, as it simply compares the payoff under $C_1 = 0$ and under $C_1 = 1$. Suppose that branching yields the global solution C^* under dimensionality of the problem K. That is, $B(\boldsymbol{C}^K) = C^*$, where $\boldsymbol{C}^K$ is defined to be any lattice with K decisions yet unsolved, and B denotes the branching mapping.

We prove that $B(\boldsymbol{C}^{K+1}) = C^*$. In this case, branching chooses some coordinate i to fix and applies the first-step mapping to lattices $\boldsymbol{C}_{i=1}^{K+1}$ and $\boldsymbol{C}_{i=0}^{K+1}$, which denote the subsets of $\boldsymbol{C}^K$ that includes only the complete sublattices with elements $C_i = 1$ or $C_i = 0$, respectively. The first-step mapping is set-increasing, so by construction $F(\boldsymbol{C}_{i=1}^{K+1}) \subseteq \boldsymbol{C}_{i=1}^{K+1}$ and $F(\boldsymbol{C}_{i=0}^{K+1}) \subseteq \boldsymbol{C}_{i=0}^{K+1}$. By the inductive hypothesis, $B(F(\boldsymbol{C}_{i=0}^{K+1})) = C_{i=0}^*$ and $B(F(\boldsymbol{C}_{i=1}^{K+1})) = C_{i=1}^*$, where $C_{i=1}^*$ is the argmax conditional on $i = 1$ and $C_{i=0}^*$ is the argmax conditional on $i = 1$. The global maximum must have either $C_i^* = 1$ or $C_i^* = 0$. It follows that $B(\boldsymbol{C}^{K+1}) = C^*$.

C Practical Implementation of Algorithms 3 and 4

Table 4 provides a pseudocode description of our first-step algorithm implementation, which combines Algorithms 3 and 4. The code is a straightforward for-loop over products (indexed by m) and potential production locations (indexed by l) for brand b in segment s. We describe each of the steps of the code in words:

- Step 1: In this step, we initialize a while loop by setting an exit condition equal to `true`. We also start at known bounds of the solution. The brand has $M_{bs} \times (N + L)$ binary decisions to make, where N denotes the potential destination countries and L the potential production countries. The matrix `fixed` is an $M_{bs} \times (N + L)$ matrix with values 0.5 at each coordinate, denoting that no binary decision has been solved for at the initialization step.

- Step 2: The second step stores in `output_prev` the matrix `output` computed in the previous iteration.

- Step 3: For-loop over products $m \in \{1, 2, ..., M_{bs}\}$ and potential production locations $l \in \{1, 2, ..., L\}$. If the matrix `output_prev` is 0.5 at coordinate $(m, N + l)$ (at production location l), the algorithm first computes the upper bound using the `MVmax_sup` function, which computes the largest possible

[19] In Appendix C, we discuss our practical implementation.

Table 4: Pseudocode description of the `CMMS_prod` function.

Step	Pseudocode Description
1	**Initialize:** Set `exit_condition = true`. Copy `fixed` matrix of binary decisions to `output`.
2	**While** `exit_condition is true`, **do**: Store previous output: `output_prev = output`.
3	**For each product** m: **For each potential production location** l: **If** `output_prev[m, N+l] == 0.5`, then: Set `index = (m, N+l)`. Upper bound: `upper = MVmax_sup(output, index, profit_func)` **If** `upper < 0.0`, then set `output[index] = 0.0`. **Else:** Lower bound: `lower = MVmax_inf(output, index, profit_func)` **If** `lower` $\geq$ `0.0`, then: Set `output[index] = 1.0`
4	**Update offering decisions for product** m**:** Call `CMMS_sales(output, profit_func, m)` to adjust sales.
5	**Check convergence:** **If** `output_prev == output`, set `exit_condition = false`.
6	**Return:** Output the final production allocation: `return output`.

marginal value of (m, l) under function $V_{b,s,m}$ (see equation (26)) – that is, setting $(\boldsymbol{I}_{-m}, \boldsymbol{D}_{-m})$ at lower bounds, as per the substitutability. If such an upper bound is strictly negative, we update the solution matrix `output` and set coordinate $(m, N + l)$ to 0. Otherwise, we compute the lower bound by applying `MVmax_inf`, which computes the smallest possible marginal value of (m, l) under function $V_{b,s,m}$ – that is, setting $(\boldsymbol{I}_{-m}, \boldsymbol{D}_{-m})$ at upper bounds, as per the substitutability. If such a lower bound is weakly positive, we update the solution matrix `output` and set coordinate $(m, N+l)$ to 1.

- Step 4: Given the solved 0's or 1's for the production locations of product m, we apply function `CMMS_sales` to product m offering decisions. The function `CMMS_sales` effectively applies Algorithm 3 to all offering choices of product m. We provide the pseudocode description of `CMMS_sales` in Table 5.

- Step 5: After looping over all products, the algorithm checks whether binary decisions were solved in Steps 3 and 4. If none were solved, the algorithm sets `exit_condition = false` and exits the `while` loop. Otherwise, the algorithm repeats Steps 3-4.

- If `exit_condition = false`, the algorithm returns `output`, which is a matrix containing 0's, 1's, and 0.5's.

Our implementation of the algorithm composes Algorithms 3 and 4, which we proved were both set-increasing. Thus, starting at known bounds of the solution, such a composition converges to bounds of the solution of the brand's problem.

Table 5: Pseudocode description of the `CMMS_sales` function.

Step	Pseudocode Description
1	**Initialize:** Copy `fixed` matrix of binary decisions to `output`.
2	**Iterate over sales locations:** **For each sales destination** $n = 1$ to N: **If** `output[m, n] == 0.5`, then: Set `index = (m, n)`
3	**Compute bounds and update** `output`**:** Upper bound: `upper = MVsup(output, index, profit_func)` **If** `upper < 0.0`, then set `output[index] = 0.0`. **Else:** Lower bound: `lower = MVinf(output, index, profit_func)` **If** `lower` $\geq$ `0.0`, then set `output[index] = 1.0`
5	**Return:** Output the updated sales allocation: `return output`.

Formally, in our practical implementation, we apply set-increasing mapping $F^{bs} = (F^D_{M_{bs}} \circ F^I_{M_{bs}}) ... \circ (F^D_m \circ F^I_m) \circ ... \circ (F^D_2 \circ F^I_2) \circ (F^D_1 \circ F^I_1)$, which is a composition of set-increasing mappings, where, for $\boldsymbol{C} \in S(\mathcal{B}^{M_{bs} \times (N+L)})$,

$$F^I_m(\boldsymbol{C}) = \{C \in \boldsymbol{C} : C_i = 0 \text{ and } C_{i'} = 1 \ \forall i \in \overline{\Omega}^I_m(\boldsymbol{C}) \text{ and } \forall i' \in \underline{\Omega}^I_m(\boldsymbol{C})\}$$
$$F^D_m(\boldsymbol{C}) = \{C \in \boldsymbol{C} : C_i = 0 \text{ and } C_{i'} = 1 \ \forall i \in \overline{\Omega}^D_m(\boldsymbol{C}) \text{ and } \forall i' \in \underline{\Omega}^D_m(\boldsymbol{C})\},$$

and

$$\overline{\Omega}^I_m(\boldsymbol{C}) = \{n \in \{1, 2, ..., N\} : \sigma_{(m,n)}(\Pi_{bs}(\sup_{(m,n)}(\boldsymbol{C})) < 0\}$$
$$\underline{\Omega}^I_m(\boldsymbol{C}) = \{n \in \{1, 2, ..., N\} : \sigma_{(m,n)}(\Pi_{bs}(\inf_{(m,n)}(\boldsymbol{C})) \geq 0\}$$
$$\overline{\Omega}^D_m(\boldsymbol{C}) = \{l \in \{1, 2, ..., L\} : \sigma_{(m,N+l)}(V_{b,s,m}(\sup_{(m,N+l)}(\boldsymbol{C})) < 0\}$$
$$\overline{\Omega}^D_m(\boldsymbol{C}) = \{l \in \{1, 2, ..., L\} : \sigma_{(m,N+l)}(V_{b,s,m}(\inf_{(m,N+l)}(\boldsymbol{C})) \geq 0\}.$$

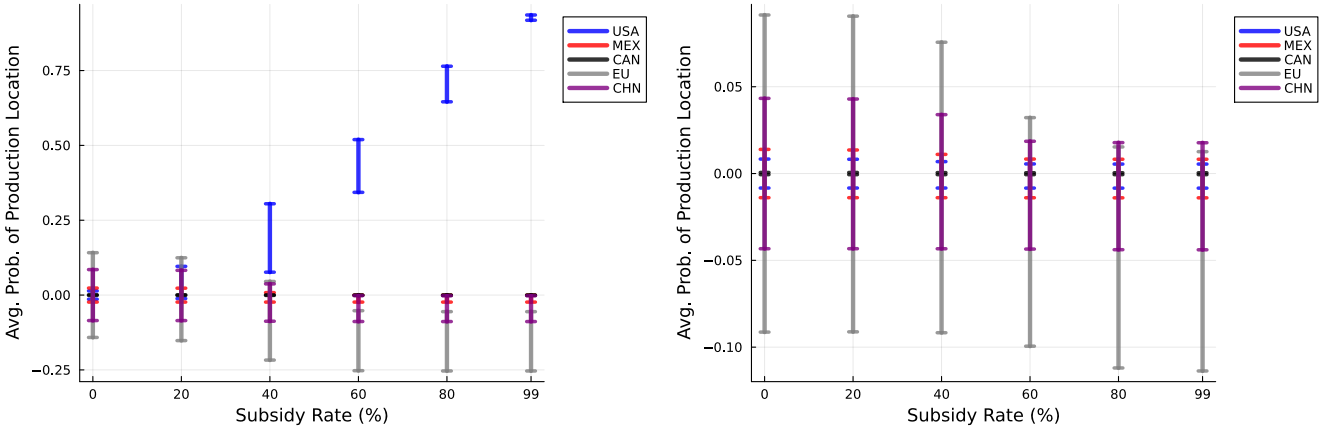
Due to Propositions 1-3, the algorithm we implement in practice is a composition of set-increasing mappings, which is itself set-increasing, and therefore converges to bounds of the solution of the problem under the interdependence properties proven under Π_{bs} and $V_{b,s,m}$.

D Aggregate Counterfactual Exercises in Changes

In this section, we report aggregate changes in counterfactual outcomes.

D.1 Marginal Cost Subsidy on Electric Vehicles in the United States

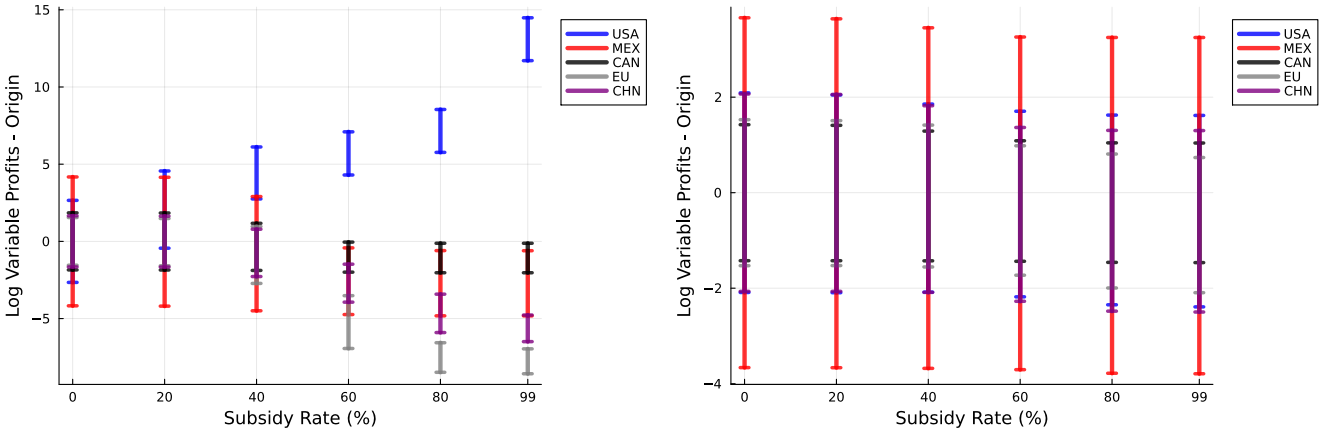
Figure 25: Change in the Probability of Production Location - Marginal Cost Subsidy



(a) Electric (b) Non-Electric

Notes: These plots show the lower and upper bound changes $\Delta\underline{H}(\tau)$ and $\Delta\overline{H}(\tau)$ in the probability that an electric (panel (a)) and non-electric (panel (b)) product is produced in several countries/regions under different US marginal cost subsidy rates. The probability is computed by calculating the average number of 1's per electric (non-electric) product. The bounds reflect both uncertainty arising from unsolved binary decisions due to computational infeasibility, as well as from statistical uncertainty regarding the fixed cost parameter vector.

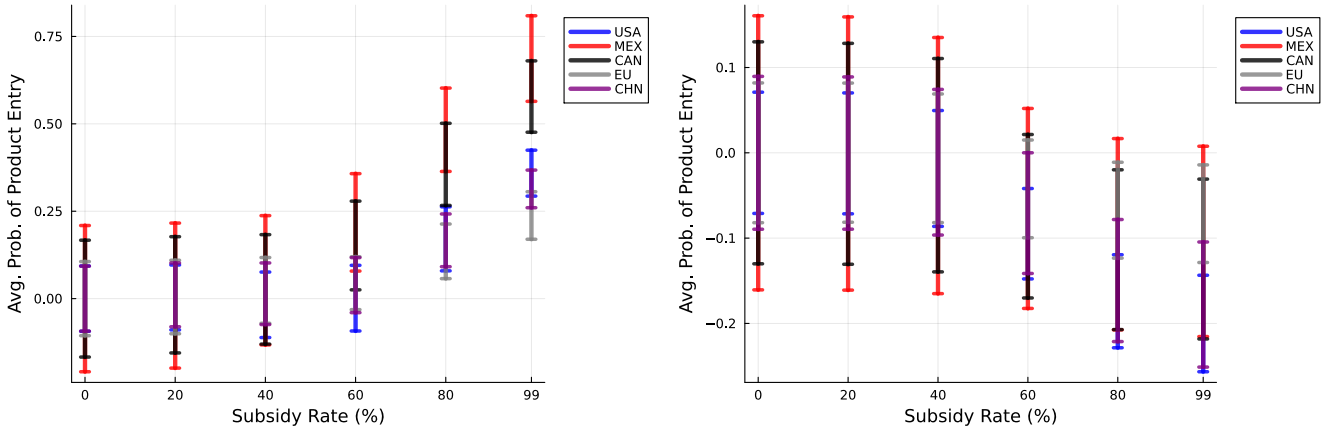
Figure 26: Change in Log Variable Profits (Origin) - Marginal Cost Subsidy



(a) Electric (b) Non-Electric

Notes: These plots show the lower and upper bound changes $\Delta\underline{H}(\tau)$ and $\Delta\overline{H}(\tau)$, respectively, in the the (log) of total variable profits across electric (panel (a)) and non-electric (panel (b)) products at the origin-level for several production countries/regions under different US marginal cost subsidy rates on electric products. The bounds reflect both uncertainty arising from unsolved binary decisions due to computational infeasibility, as well as from statistical uncertainty regarding the fixed cost parameter vector.

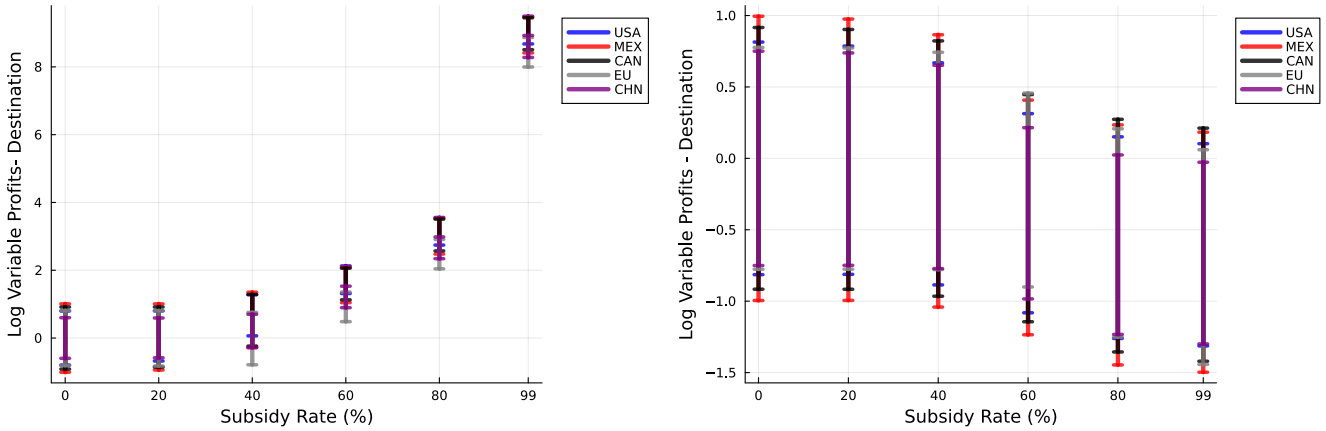
Figure 27: Change in the Probability of Product Offering - Marginal Cost Subsidy



(a) Electric (b) Non-Electric

Notes: These plots show the lower and upper bound changes $\Delta\underline{H}(\tau)$ and $\Delta\overline{H}(\tau)$ in the probability that an electric (panel (a)) and non-electric (panel (b)) product is offered in several countries/regions under different US marginal cost subsidy rates on electric products. The probability is computed by calculating the average number of 1's per electric (non-electric) product. The bounds reflect both uncertainty arising from unsolved binary decisions due to computational infeasibility, as well as from statistical uncertainty regarding the fixed cost parameter vector.

Figure 28: Change in Log Variable Profits (Destination) - Marginal Cost Subsidy

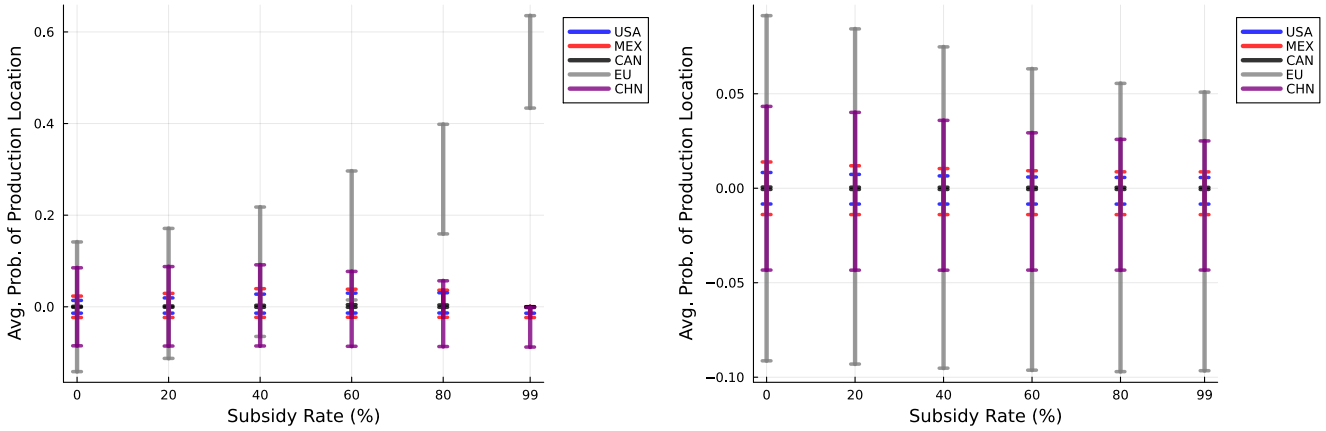


(a) Electric (b) Non-Electric

Notes: These plots show the lower and upper bound changes $\Delta\underline{H}(\tau)$ and $\Delta\overline{H}(\tau)$, respectively, in the (log) of total variable profits across electric (panel (a)) and non-electric (panel (b)) products at the origin-level for several production countries/regions under different US marginal cost subsidy rates on electric products. The bounds reflect both uncertainty arising from unsolved binary decisions due to computational infeasibility, as well as from statistical uncertainty regarding the fixed cost parameter vector.

D.2 Consumer Subsidies on Electric Vehicles in the United States

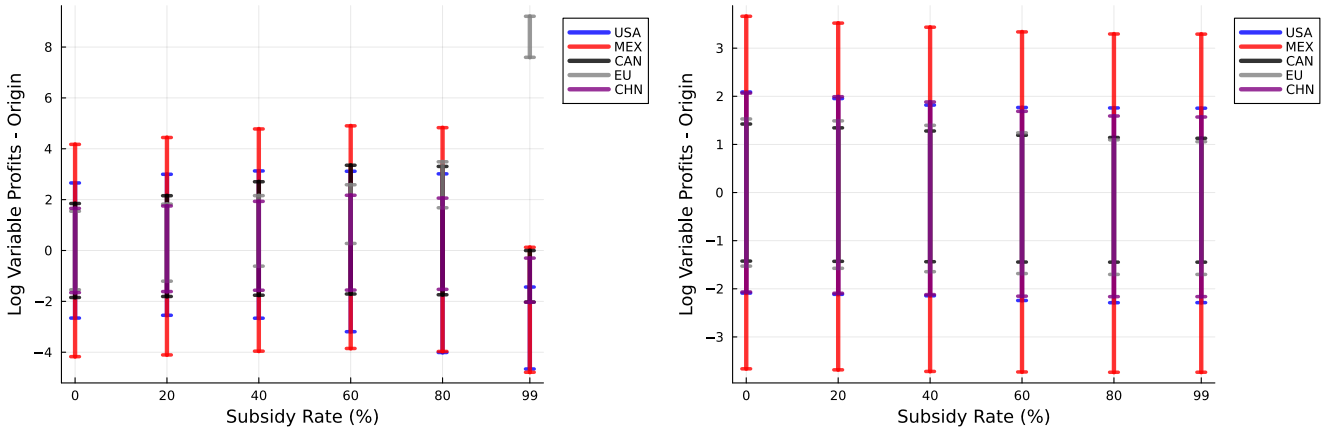
Figure 29: Change in Probability of Production Location - Consumer Subsidy



(a) Electric (b) Non-Electric

Notes: These plots show the lower and upper bound changes $\underline{H}(\tau)$ and $\overline{H}(\tau)$ in the probability that an electric (panel (a)) and non-electric (panel (b)) product is produced in several countries/regions under different US consumer subsidy rates on electric products. The probability is computed by calculating the average number of 1's per electric (non-electric) product. The bounds reflect both uncertainty arising from unsolved binary decisions due to computational infeasibility, as well as from statistical uncertainty regarding the fixed cost parameter vector.

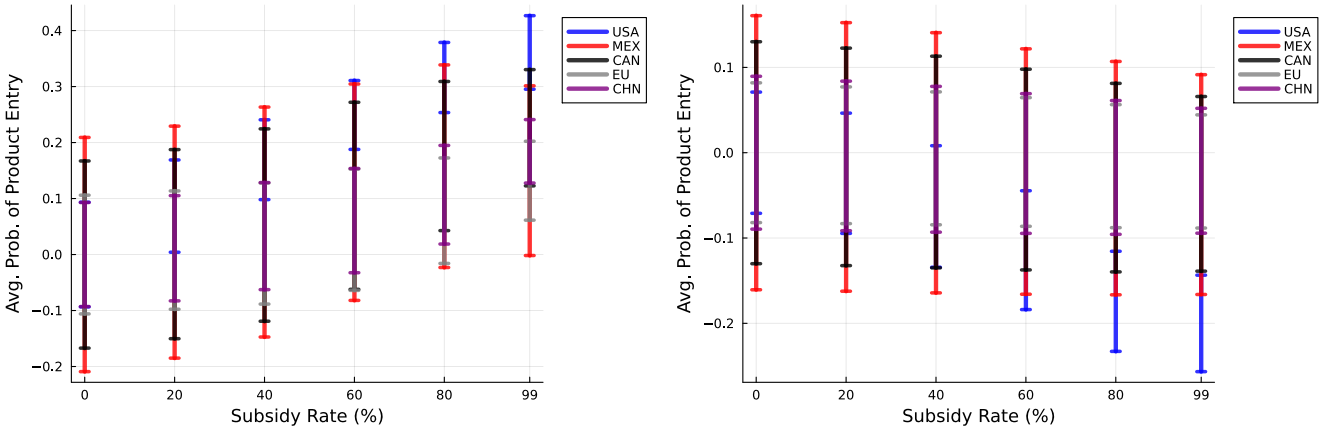
Figure 30: Change in Log Variable Profits (Origin) - Consumer Subsidy



(a) Electric (b) Non-Electric

Notes: These plots show the lower and upper bound changes $\Delta\underline{H}(\tau)$ and $\Delta\overline{H}(\tau)$, respectively, in the (log) of total variable profits across electric (panel (a)) and non-electric (panel (b)) products at the origin-level for several production countries/regions under different US consumer subsidy rates on electric products. The bounds reflect both uncertainty arising from unsolved binary decisions due to computational infeasibility, as well as from statistical uncertainty regarding the fixed cost parameter vector.

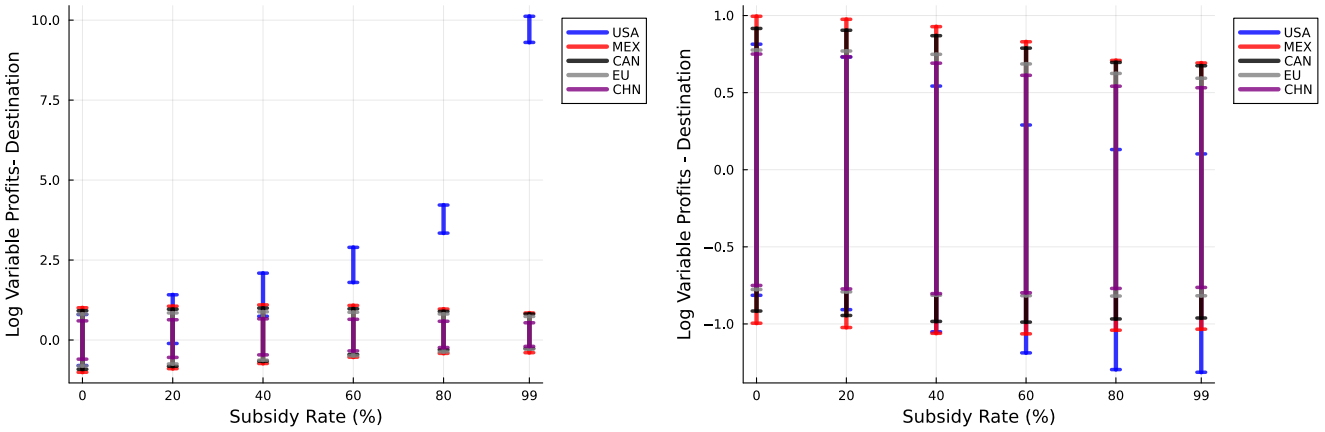
Figure 31: Change in Probability of Product Offering - Consumer Subsidy



(a) Electric (b) Non-Electric

Notes: These plots show the lower and upper bound changes $\Delta\underline{H}(\tau)$ and $\Delta\overline{H}(\tau)$ in the probability that an electric (panel (a)) and non-electric (panel (b)) product is offered in several countries/regions under different US consumer subsidy rates on electric products. The probability is computed by calculating the average number of 1's per electric (non-electric) product. The bounds reflect both uncertainty arising from unsolved binary decisions due to computational infeasibility, as well as from statistical uncertainty regarding the fixed cost parameter vector.

Figure 32: Change in Log Variable Profits (Destination) - Consumer Subsidy

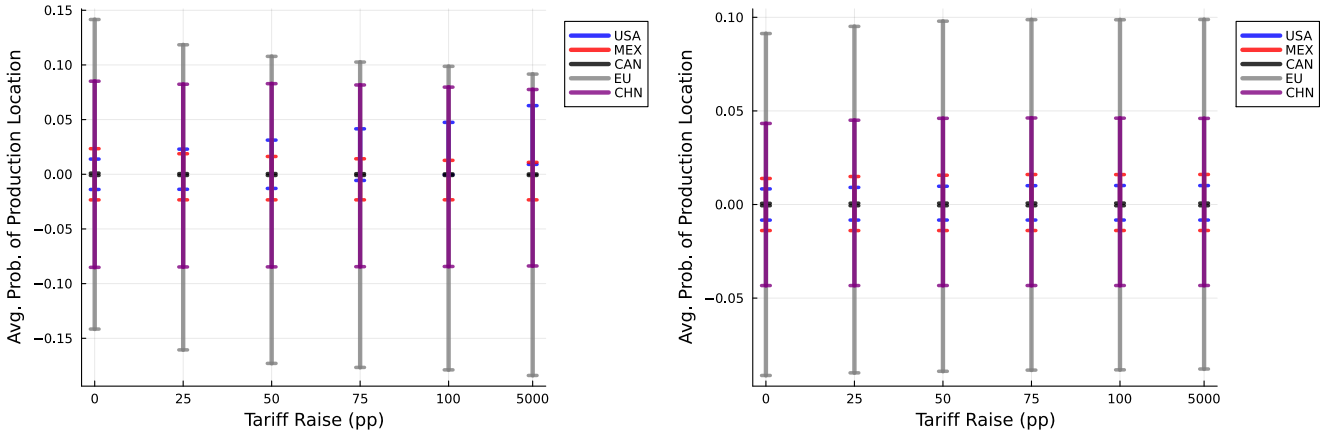


(a) Electric (b) Non-Electric

Notes: These plots show the lower and upper bound changes $\Delta\underline{H}(\tau)$ and $\Delta\overline{H}(\tau)$, respectively, in the (log) of total variable profits across electric (panel (a)) and non-electric (panel (b)) products at the origin-level for several production countries/regions under different US consumer subsidy rates on electric products. The bounds reflect both uncertainty arising from unsolved binary decisions due to computational infeasibility, as well as from statistical uncertainty regarding the fixed cost parameter vector.

D.3 Tariffs on Electric Vehicles Not Produced in the United States

Figure 33: Change in Probability of Production Location - Tariffs on ROW

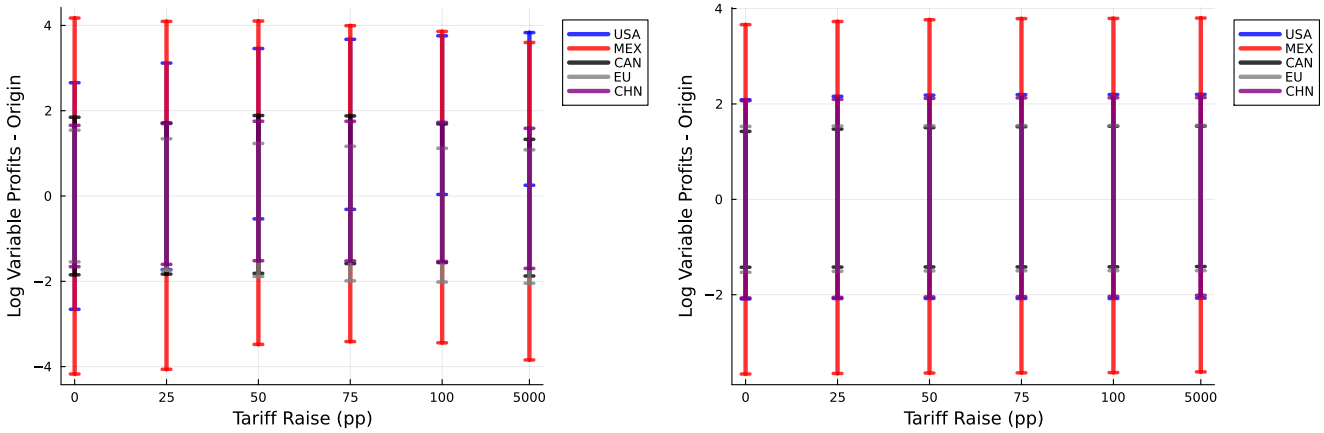


(a) Electric

(b) Non-Electric

Notes: These plots show the lower and upper bound changes $\Delta\underline{H}(\tau)$ and $\Delta\overline{H}(\tau)$ in the probability that an electric (panel (a)) and non-electric (panel (b)) product is produced in several countries/regions under different US tariff raises on electric products. The probability is computed by calculating the average number of 1's per electric (non-electric) product. The bounds reflect both uncertainty arising from unsolved binary decisions due to computational infeasibility, as well as from statistical uncertainty regarding the fixed cost parameter vector.

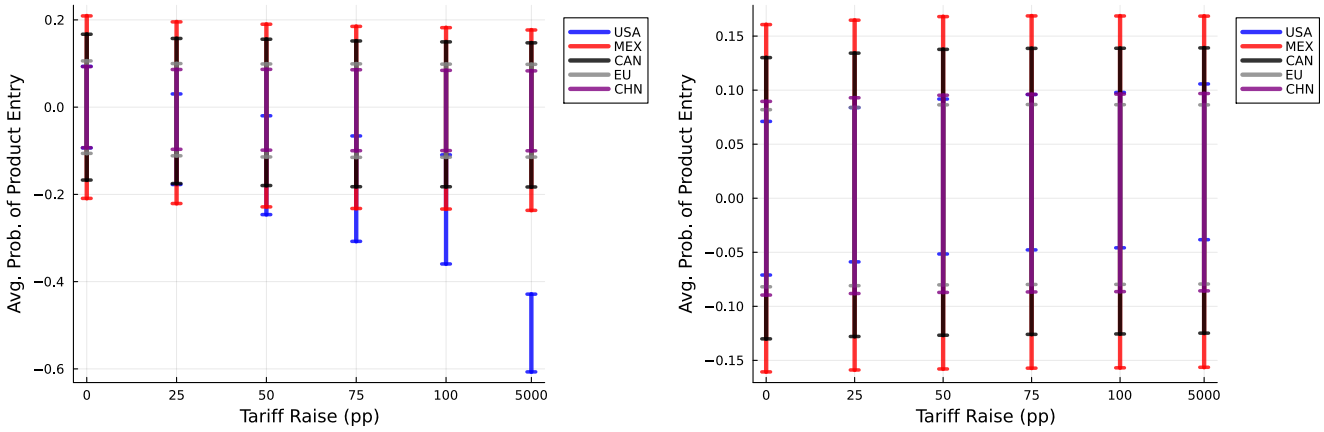
Figure 34: Change in Log Variable Profits (Origin) - Tariffs on ROW



(a) Electric (b) Non-Electric

Notes: These plots show the lower and upper bound changes $\Delta\underline{H}(\tau)$ and $\Delta\overline{H}(\tau)$, respectively, in the (log) of total variable profits across electric (panel (a)) and non-electric (panel (b)) products at the destination level for several production countries/regions under different US tariff raises on electric products. The bounds reflect both uncertainty arising from unsolved binary decisions due to computational infeasibility, as well as from statistical uncertainty regarding the fixed cost parameter vector.

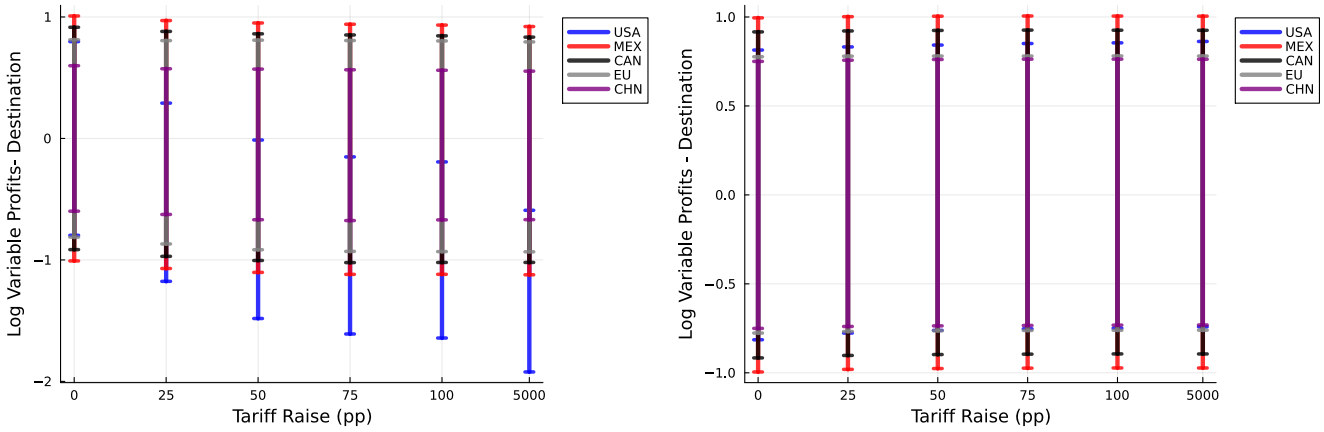
Figure 35: Change in Probability of Product Offering - Tariffs on ROW



(a) Electric (b) Non-Electric

Notes: These plots show the lower and upper bound changes $\Delta\underline{H}(\tau)$ and $\Delta\overline{H}(\tau)$ in the probability that an electric (panel (a)) and non-electric (panel (b)) product is offered in several countries/regions under different US tariff raises on electric products. The probability is computed by calculating the average number of 1's per electric (non-electric) product. The bounds reflect both uncertainty arising from unsolved binary decisions due to computational infeasibility, as well as from statistical uncertainty regarding the fixed cost parameter vector.

Figure 36: Change in Log Variable Profits (Destination) - Tariffs on ROW



(a) Electric

(b) Non-Electric

Notes: These plots show the lower and upper bound changes $\Delta\underline{H}(\tau)$ and $\Delta\overline{H}(\tau)$, respectively, in the (log) of total variable profits across electric (panel (a)) and non-electric (panel (b)) products at the destination level for several production countries/regions under different US tariff raises on electric products. The bounds reflect both uncertainty arising from unsolved binary decisions due to computational infeasibility, as well as from statistical uncertainty regarding the fixed cost parameter vector.