Economies of Density and Congestion in Equipment Rental Markets*

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Abstract

Equipment rental markets are growing and improving access to technology to small-scale producers, which are prevalent in developing countries. Yet, the implications of this expansion for aggregate productivity as well as its distributional implications are not well understood. We propose a novel theory of equipment rental markets where demand varies by farm size and location; and supply varies by service dispatch technologies. An efficient allocation prioritizes large-scale demand, because the cost of moving equipment in space dilutes with scale; as well as small-scale demand in dense locations, because it maximizes machine-capacity utilization. We assess the merits of different market arrangements quantitatively, leveraging unique transaction level data from a rental market in India to calibrate our model. We show that market provision improves service finding rates for small-scale farmers while aggregate productivity raises by 2%. Importantly, we show that increases in the supply of capital have non-linear effects on aggregate productivity gains, because those depend on the joint spatial and productivity distribution of demand.

JEL codes: O13, O18, O47, R41.
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1 Introduction

Low agricultural productivity and smallholder farming are prevalent in poor economies Adamopoulos and Restuccia (2014). Technology embodied in capital accounts for a third of productivity differentials between poor and rich economies, Caunedo and Keller (2020). There are two major hurdles to improve their productivity: first, the presence of increasing returns to scale in land size and potential gains from agglomeration; second, large fixed costs that limit investment in technology. Rental markets for equipment directly overcome fixed costs of adoption, Bassi et al. (2022), but the gains from adopting technology depend on market and land structure, because of returns to agglomeration. Hence, the spatial concentration of demand interacts with the cost of adopting technology and the market structure to determine equilibrium returns to adoption. This interaction is fundamental to agricultural productivity and often overlooked.

Is the expansion of equipment rental markets beneficial to aggregate productivity in agriculture and welfare? Earlier evidence finds mixed effects, Pingali (2007); Daum and Birner (2020). We argue that the joint spatial, productivity and size distribution of farms is a key feature to understand these mixed results. We start from the observation that agricultural production is highly time-sensitive and delays in service provision are costly for farm output. Delays are equilibrium outcomes that depend on farms distribution in space, because equipment travels in space to provide service, as well as the magnitude of demand, because those costs are diluted the larger the service hours contracted. At the same time, unused machine-service capacity is costly for providers, and hence, servicing demand of small magnitude in spatially dense areas, maximizes utilization. These features generate economies of density.

We propose a frictional model of search and matching in the allocation of capital services (rentals). Such a model is the natural framework to study queueing, as well as sorting and rental rate dispersion, which are empirically relevant features of these markets. As in the labor search tradition, the main friction built into the model is that it takes time for an agent to find an equipment provider willing to provide service at a price that is acceptable for both parties.[1] Formally, we model directed search with two sided heterogeneity: demand

[1] As highlighted by Lagos (2000) and Sattinger (2002), queueing models are powerful to micro-found a

varies by hours requested, location, and the returns from service; while supply varies by the technology for service provision. Providers set prices with commitment and agents build expectations about the queue lengths when deciding where to stand in line. Providers can accommodate multiple services per period and face service capacity constraints in terms of machine hours, the two main generalization relative to Shi (2002). The first feature allows us to discuss compositional changes in serviced orders across demand size (hours) as well as to optimize service provision in space. The second feature, paired with discreteness in hours demanded, speaks directly to the role of small-scale orders in maximizing capacity utilization within a period. Farmers understand that providers offering lower equipment rental rates are those where waiting times for service provision are longer, and they trade-off the probability of service, with the cost of service. Once they commit to a provider, they stand in line for a period.

We model providers' dispatch technologies following two prevalent methods in the markets that we study: profit-maximizing dispatchers ("market" providers), and first-come-first-served dispatchers ("fcfs" providers). While the former ranks orders by profitability, the latter does not and is closer to provisions often seen in markets subsidized by local governments, where small holder access to subsidized services is a salient concern.[2] The main predictions of the model are that when small- and large-scale farmers are equally distributed in space, small-scale farmers are more likely to approach the fcfs providers than the *mkt* provider: travel costs imply that smaller orders are more costly to serve so profit-maximizing dispatchers would delay their service. Large-scale farmers are more likely to approach *mkt* providers, where they are more likely to get served early. Sorting induces disparities in service finding rates between small and large-scale farmers but in equilibrium, rental rates adjust so that farmers face identical expected profits from either provider.

We then use our model economy to study the equilibrium implications of alternative market arrangements for aggregate productivity and service findings rates to small-holder farmers. Before doing so, we augment our stylized theoretical framework to more heterogeneity in equipment demand, spatial location and productivity distributions, and work with

matching process between, in this case, farmers' orders and service providers.

[2] *fcfs* providers induce a matching technology that resembles an urn draw with multiples draws/vacancies per period. For *mkt* providers, the easiest way to characterize the matching technology is to work with queue lengths and its composition, since there is no closed-form counterpart.

simulated outcomes. We do so consistently with the empirical distribution of demand for rental services that we observe in the state of Karnataka India. This is a useful environment for our assessment because equipment ownership rates are low, small-scale producers are prevalent, and travel times could be large across locations. Perhaps most importantly, rental markets for equipment are active.

We discipline the model in two stages: in the first stage we target the queue lengths for small-scale farmers, as observed in the transaction level data from hubs; the share of large scale farmers in each market, as inferred from the Census data; and the observed average profitability of hub providers. Hub providers are have a technology for provision that resemble the FCFS. The reason is that the set up of these hubs was partially subsidized by the government with the intent of granting access to technologies to small-scale producers. With these targets, we jointly calibrate the composition of farmers in each market along large and small scales, the ratio of farmers per provider and the cost of service provision as parameterized by the wage of equipment drivers. The main outcome of this calibration exercise are the endogenous queues by provider and farmer type, and the equilibrium rental prices per hour of equipment.

In the second stage, we augment the model to accommodate further heterogeneity. We bootstrap queues from the empirical joint distribution of service-hours demanded, productivity and plot location, and the assess service finding rates, and equilibrium productivity costs. In the simulated economy, we also test the implications for efficiency and distributional outcomes of alternative market arrangements, as well as technologies to optimize service routes given the realization of queues.

First, we ask how does the current equilibrium where FCFS and *mkt* providers coexist compare to allocations where only equipment owners are allowed to supply rental services. One could interpret this counterfactual as measuring the effect of a government induced increase in supply consistent with the creation of service hubs in the locations under analysis. We find that service capacity increased dramatically relative to ownership rates, and that lead to two-to-three fold increase in service findings rates, and declines in the cost of service provision of more than 20% per hour serviced.

Second, we investigate alternative market deregulation scenarios, where we allow providers in hub to maximize profits, irrespective of its distributional consequences. The short run

response of the economy (with no endogenous entry or exit of providers) implies higher service finding rates for large-scale farmers relative to the FCFS provision. However, service finding rates for large-scale farmers are below those of small scale farmers, suggesting no detrimental effect on market access to small scale access. This small scale producers are drawn to the market in response to the increased supply of services. While service findings rates for small-holder farmers do not change relative to the baseline, the rental costs increase for all farmers in the short run.

In the long run, once the number of providers can adjusts through entry and exit, rental costs fall again towards their baseline levels, and service findings rates are still higher for small holder farmers than large holder farmers. Importantly, we show that aggregate productivity costs from delays are 2% lower than in the benchmark equilibrium, suggesting that the deregulation improves allocations. To model entry and exit into the market, we assume that in the deregulated markets providers pay an operating cost equal to an annuity of the expected profitability of a provider. Hence, in equilibrium, the measure of providers in the market is such that they exactly cover the operating cost.

Third, we ask what happens to allocations as we increase the overall supply of equipment services. We can show that service finding rates for small scale farmers are non-linear in the supply of services and that they demand on the joint spatial and size distribution of demand. Only large enough increases in supply benefit small-holder farmers relatively to large-scale farmers. Indeed, we show that market suppliers have comparable wait times to a fcfs service dispatch for plots that entail lower travel costs, irrespective of their size. The largest differences in service wait times, and productivity costs associated to delays across providers are concentrated in farms operating farther away from service hubs, i.e. where spatial density is lower.

Literature Review. Delays are often overlooked as a barrier to technology adoption, yet they are a potentially important mechanism in sectors like agriculture, where returns are extremely sensitive to the timing of activities.[3] The role of barriers to technology adoption in

[3]Other markets with similar features include the market for perishable products, or the allocation of personal shoppers. There is an extensive literature that studies frictional meetings between customers and providers in taxi markets Frechette et al. (2019), but there is little discussion of consumer costs from delays in service provision.

agriculture as a source of low productivity in poor countries has received extensive attention (see Suri and Udry, 2022 for a review). We contribute to a growing literature that highlights the importance of adopting mechanized practices, Caunedo and Keller (2020); Caunedo and Kala (2021), by drawing attention to rental markets, which could be disruptive in lowering adoption costs, (Yang et al., 2013; Manuelli and Seshadri, 2014; Yamauchi, 2016).

Daum and Birner (2020) highlights how mixed evidence on the impact of mechanization in the 70s and 80s may have delayed mechanization all together in the developing world. One usual misconception is that small-holder farmers do not benefit from mechanization. Daum and Birner (2020) suggests that smallholder farmers could benefit with the appropriate technologies and institutional setting. A key question that arises is what is the best contractor model and how can it be supported.

Rental markets were foundational to the transformation of the agriculture sector in many countries, Binswanger (1986) and Olmstead and Rhode (1995). While there are studies of land rental markets Chen et al. (2022) Acampora et al. (2025), much less is known about rental markets for equipment, Caunedo and Kala (2021) being one exception. Distinctively from the experimental evidence, in this paper we focus on the market structure. We highlight that working rental markets for inputs could benefit agglomeration even if land consolidation is not achievable.

The notion that there might be scale economies associated to concentrating production in certain locations goes back to Marshall (1890). Holmes and Lee (2012) explore it in the context of crop choices of adjacent plots, where agglomeration economies rely on economies of scale in output. Duranton and Puga (2004) review the micro-foundations for agglomeration economies and classifies them into three mechanisms: "sharing", "matching" and "learning". In our framework the first two mechanisms are at play. A paper that studies the "sharing" mechanism is Bassi et al. (2022), with an application to rental markets for door producers in urban Uganda, where they argue frictions are relatively limited. In contrast, we document substantial price dispersion in rental rates paired with unused service capacity, a common symptom of matching frictions. We are explicit about the role of "matching" in generating service transactions between providers and input demanders, and how they affect rental prices and queueing behaviour.

2 Agricultural equipment rental markets: an empirical exploration

We inform our theory by four main characteristics of these markets that distinguish them from other durable rental markets.

First, in areas where small-holder farmers are prevalent, equipment ownership is not cost effective, and most of the service capacity is determined by equipment suppliers that specialize in equipment rentals rather than their own cultivation.

Second, since agricultural activities are highly time sensitive, the timing of demand in a location is synchronous. This synchronisity leads to endogenous delays in service provision as a function of service capacity. Delays in land-preparation have been documented to generate productivity costs for farmers, Paudel et al. (2019); Liu et al. (2023) among others. In addition, since equipment travels for transactions to take place, the joint distribution between travel time and the scale of demand, i.e. service-hours per request, determine access to equipment.

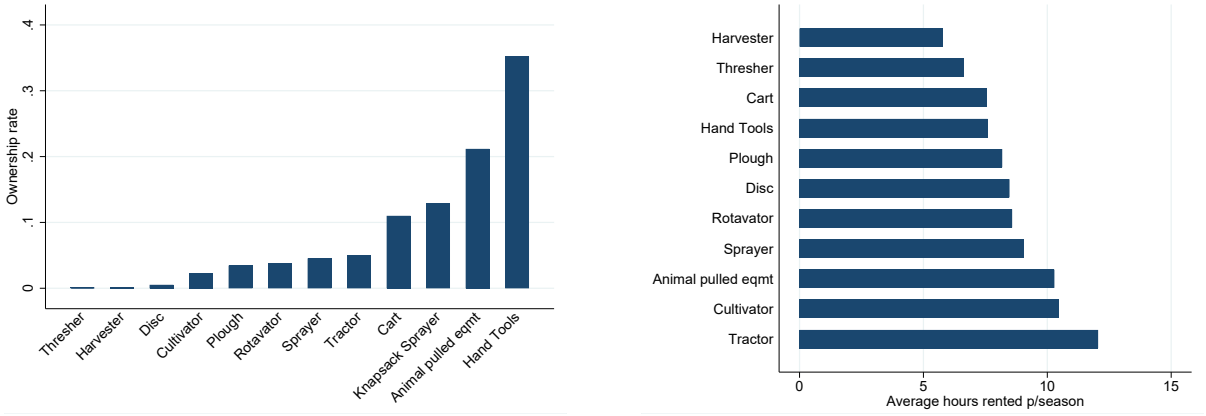
Third, we observe price dispersion in rental rates after controlling for observable household characteristics and market characteristics. Most importantly, service capacity often exceeds demand despite farmers facing delays. Both these features are consistent with frictional rental markets.

2.1 Data description

We combine four sources of data from agricultural rental markets in the state of Karnataka in the south of India. First, we use our own census of farming households covering 40,000 households across 150 villages, including information on equipment ownership and rental market engagement. This data allow us to characterize the universe of equipment supply and farmer demand. Second, we use transaction level data from the universe of equipment rentals engaged through a public-private platform during the Kharif season of 2018 (May-October).[4]

[4]This platform was created in the context of a broader mechanization program subsidizing equipment purchases in the state of Karnataka since 2016, but transactions occurring outside of the platform are not recorded. We do check that transactions in our sample cover the full range of farm sizes as well as the type of crops in our Census.

Figure 1: Ownership and rentals by implement.



Owned Rentals

The ownership rate is the share of farmers that report owning a given implement relative to the total population surveyed. Rental hours correspond to the average hours reported for the whole season. Source: Own survey of farmers.

Transactions include information on the number of hours requested, acreage, implement type, as well as farmer identifiers (such as their name, village, and phone number). Importantly, we observe queuing behavior across the season for farmers of different characteristics. To obtain farmer characteristics we use farmer identifiers to match transaction with detailed survey data that we collected over 5500 farming households with information on rental market engagement, delays in service provision; as well as output and input expenses, crop choices and land ownership. This data allow us to report characteristics of the joint distribution of farm productivity, size and location.

2.2 Equipment supply and demand

Less than 5% of the farmers report owning larger equipment such as tractors, or rotavators and cultivators, see Figure 1.[5] We find the same ownership rates for these types of equipment

[5]Given land holdings, ownership of equipment is not cost-effective for most farmers. For instance, the rental price of a rotavator is between ₹750 and ₹1,000 per hour (including tractor, a driver and fuel) and the average farmer demands about 6 hours of rotavator services in the season or between ₹4500 and ₹6000 in services. The purchase price of a new rotavator is over ₹110,000 which means that, even absent maintenance costs, the average farmer needs 19 years to amortize the investment. The rental rate for an inferior technology that serves a similar purpose, i.e. a harrow, is half of the rental rate of the rotavator (₹360) and the cost of purchase is about ₹50000.

when analyzing census data (less than 5%), see Figure 11 in Appendix E.

Farmers rely on rental markets to access mechanization services. Since equipment ownership is low, equipment is mostly available through custom-hiring-centers or providers that travel to these locations seasonally. The average hours rented in a season per farmer is 12 hours for tractors, 10 hours for cultivators and 8 hours for Rotavators. With the exception of sprayers, which are used for plant protection, equipment rentals are mostly concentrated in implements related to the land-preparation stage. These include rotavators, ploughs and cultivators on the higher end, and animal pulled equipment (e.g. harrows) and disc which are typically cheaper to rent, Table 8 in Appendix E. Mechanization is most prevalent during land-preparation in our sample, a common first stage of mechanization in agriculture (Caunedo and Kala, 2021).

2.3 Queuing and Delays by Location and Scale

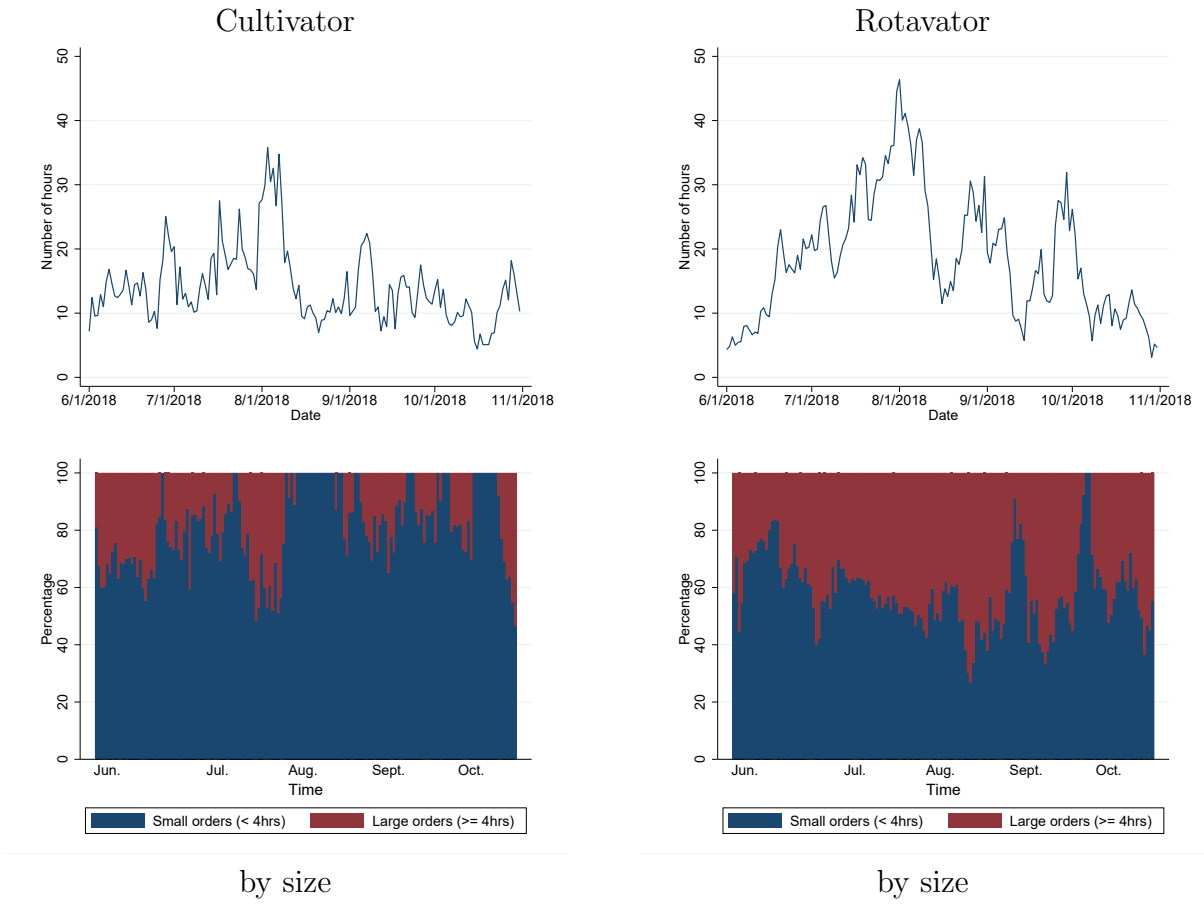
Demand for equipment rental services vary by agricultural process, e.g. land preparation; and farm outcomes are sensitive to the timing of completion of such a process. The synchronous nature of many of these processes induces queuing in the market.

Our transaction level data allows us to measure demand fluctuations measuring outstanding service hours for land preparation implements at a daily frequency, a measure of the "queue".

Figure 2 shows that queuing peaks by the end of July for rotavators and beginning of August for cultivators. At peak, the average provider faces 40 hours of demanded services in queue, which account for over 12 orders on average at a point in time.

Demand moves distinctively between large and small requests, measured as those with service hour requests of more or less than 4 hours. Figure 2 bottom panel shows that Cultivators, for which the hourly rental price is 2/3th of the price of a Rotavator, are more likely to be used by small farmers, see Table 8 in Appendix E. In addition, there are compositional shifts in the queue, with the share of hours accounted for by large farmers increasing at peak.

Figure 2: Hours outstanding in the queue.



Notes: Averages hours outstanding in the queue across hubs in Kharif 2018, overall (top panel) and by order size (bottom panel).

2.4 Delays in rental services

We survey farmers on the most important issues in accessing equipment services. The most prevalent answer is "delays in provision", with 78% of farmers reporting it as an issue. This issue is more prevalent than lack of cash for example. Importantly, larger farmers, i.e. those cultivating at the 75th percentile of the land size distribution, are nearly 5 percentage points less likely to report delays as an issue. In general, service delays are negatively associated with cultivated area, columns (1) and (3) in Table 1.

Since equipment moves in space to service demand, it is possible that these delays are explained by the geographical location of plots, and the correlation between location and

Table 1: Delays as a Function of Land Area and Location Fixed Effects

	Delays (Sum of Average Delays Over the Season)			
Log(Area)	-0.215*	-0.144	-0.319**	-0.128
	(0.115)	(0.0926)	(0.145)	(0.108)
Observations	5,615	5,615	4,345	4,345
R-squared	0.002	0.182	0.003	0.252
Village Fixed Effects	No	Yes	No	Yes
Mean Delays	2.158	2.158	2.789	2.789

Estimated coefficients from a regression of reported delays in service provision and the log(area) owned. The first two columns include those that report zero delays whereas the last two columns only focus on those that report positive delays.

farm size. Columns (2) and (4) in Table 1 show that delays have an important spatial dimension: adding village fixed effects substantially attenuates the correlation between land-size and delays, and increases the r-squared by eight or nine times (depending on whether only positive delays are considered, or all delays are included in the regression). That is, in the surroundings to a particular village, small and large farmers face similar delays. But the negative correlation between delays and size suggest that small farmers may also be located in less dense areas on average, so longer delays are a feature of remoteness rather than size.[6]

How long are these delays? and do they vary over the season? To investigate the nature of delays we rely on our transaction level dataset that includes higher frequency (daily) and more detailed data than survey information.

As demand fluctuates over the season in a somehow predictable manner, it is expected that service supply adjusts accordingly. If supply expands proportionally to increased demand, delays could be constant. Define the service rates as the ratio of hours serviced within a day to the total number of hours outstanding. Service rates fluctuate during the season, and positively correlate with hours serviced suggesting incomplete adjustment in supply (see Appendix E Figure 12). At peak queue hours, service rates are 35% on average for rotavators, and 30% for cultivators. In other words, it takes 2.9 and 3.3 days on average to complete all hours outstanding in the queue, respectively.[7]

[6]The costs of remoteness and market access have been studied in the development context in a variety of settings including Atkin and Donaldson (2015).

[7]One reason to complement this data with our own survey is that we only observe delays for orders that are already in the queue, i.e. where the farmer has accepted to wait in line.

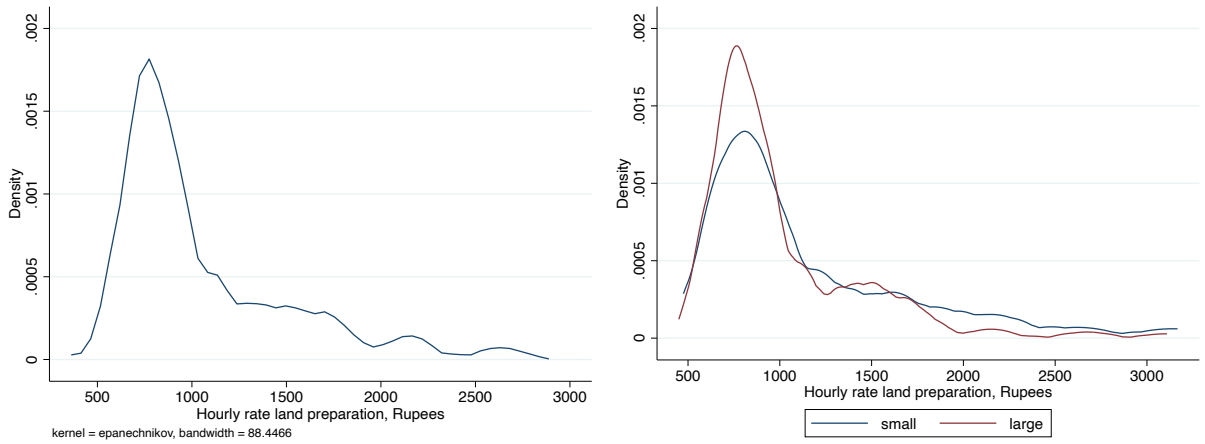


Figure 3: Rental rates

Notes: Hourly rental rates for equipment at land-preparation, including Cultivators and Rotavators. Panel (a) displays the residual rental rate dispersion after controlling for village fixed effects. Panel (b) presents these residuals for farmers with less than the average area cropped (small, < 3.3 acres); and for farmers with more than that area cropped (large, $>= 3.3$). Source: Own survey data.

2.5 Frictional rental markets

But why are there delays to begin with? Is this a consequence of low ownership rates and service capacity, or rather the consequence of frictions in the rental market that prevent farmers and providers to contract services when desired? There are two features of the market that indicate the presence of frictions in the rental market.

We start by building catchment areas of 10km around each piece of equipment either reported owned in our census or in the inventory of the private-public enterprise.[8] We assume a six-week plant preparation season and that each piece of equipment serves three orders a day.[9] The latter is consistent with serviced orders per equipment per driver at peak utilization in our transactions dataset. We estimate that the number of available cultivators can serve up to 2016 orders per season, while average demand in these catchment areas is roughly 1190 orders, or half of the estimated supply. We estimate that the number of available rotavators can serve up to 1008 orders in the season while market demand is 450 orders.

[8]This 10km cutoff correspond to maximum travel distance of the private-public enterprise, but results are robust to enlarging the catchment area to 20km radius.

[9]Our results are robust to assuming a plant preparation season of four weeks, and therefore lower equipment-order supply.

Hence, these estimates suggest that queuing may not be related to supply shortages necessarily unless farmers attempt to access equipment over a shorter time span for the land-preparation stage, i.e. all in the same week.

The second, and perhaps most important feature, is the presence of price dispersion in rental rates within a 10km catchment area of each village.

As part of our survey, we ask farmers how much did they pay for land-preparation equipment rentals during the season prior to Kharif 2018. Figure 3 panel A shows the distribution of rental rates paid per hour, controlling for village fixed effects. That is, the variation in rental rates per hour serviced across farmers within a village.[10] The interquartile range is 1.71 while the coefficient of variation is 0.67.

Importantly, we find systematic disparities between farmers operating above and below the mean farm size in the sample, i.e. 3.3 acres, see Figure 3 panel B. Smaller farmers face more dispersion than their larger counterparts, consistently with queuing risk.

Burdett and Judd (1983) were the first to show that price dispersion could arise in an environment with identical agents where consumers/farmers found it costly to search for providers. Price dispersion can also be related to informational asymmetries (Varian, 1980) or to consumer preferences for certain providers over others (Rosenthal, 1980). The exchange of identical goods for heterogeneous prices is typically a sign of market frictions.

3 A model of capital rental services in space

Before describing the model economy we discuss key features of agricultural equipment rental markets that distinguish them from other durable rental markets, and that inform our modeling choices. First, since agricultural activities are highly time sensitive, the timing of demand in a location is synchronous. This synchronisity leads to endogenous waiting times as a function of service capacity and productivity costs for farmers that wait longer to obtain machine services.[11] Second, since equipment travels for transactions to take place, farmers

[10] Observed dispersion is similar even after we account for total area. Indeed, hourly rental rates are lower on average for farmers with larger area. These results are available upon request.

[11] Using data from rural India we estimate costs of delays in the arrival of mechanization services. At the same time, we show that farmers list delays in service provision as the most prevalent problem in accessing equipment rental services, a more prevalent barrier than lack of credit or financing. See Section 2 for

need to find available providers and providers can charge different prices to farmers showing up in different queues, consistently with frictional rental markets.

We build a model of capital rental services where farmers of different plot sizes and locations search for equipment providers with different technologies for service provision. Some providers prioritize high value requests whereas others simply use a first-come-first-serve dispatch system. The latter, simpler dispatch system, may service requests that would otherwise be rationed out from provision. Both these types of service provision are prevalent in agricultural equipment rental markets across the world.

Formally, we propose a two-sided heterogeneity directed-search model, Shi (2002), where farmers request different service hours, and providers use alternative technologies for service provision. We further extend this framework along two dimensions: (a) providers can serve multiple orders within a period, enabling the study of optimal service routes and the role of travel time in assessing value across orders; and (b) providers face service-hours capacity constraints, enabling the study of congestion and equilibrium service delays.

3.1 Environment

Consider an economy populated by F farmers, heterogeneous in their service-hours demand and location; and H service providers (machines), heterogeneous in their dispatch technology and location. A market is a catchment area around any of these providers, and locations are exogenously given. A fraction $\mathfrak{s}$ of farmers are large-scale farmers and demand k_s hours, while the remaining $(1-\mathfrak{s})$ fraction are small-scale farmers, and demand k_{s^-} hours. Hours demanded are determined by land holdings, and plots are either mechanized or not, bringing discreteness in demand.[12] A fraction $\mathfrak{h}$ of providers use a first-come-first-served ($fcfs$) dispatch technology, while the remaining fraction $1-\mathfrak{h}$ has access to a selection technology that allows them to prioritize high value service requests (mkt).[13] For simplicity, we assume no depreciation or capital accumulation and no maintenance costs. Service providers have a machine-hours capacity constraint $\bar{k}$ per day.

additional details.

[12]The capital demand for a farmer is given exogenous. This is analogous to taking land holdings as given and model capital demand through a Leontief production technology.

[13]Albeit h and H are assumed exogenous, both of them can be easily endogenized with a costly set up of providers and an associated free-entry condition.

Denote the ratio of farmers to service providers, $f = \frac{F}{H}$, and focus on the case where the market is large, i.e. $F, H \to \infty$ and neither side is infinitely larger than the other, $f \in (0, \infty)$. Providers post prices r_{ij} indexed by the scale of demand i and the type of provider j; and a selection rule (with commitment) $\chi_j \in [0, 1]$ simultaneously at the beginning of each period. A rule is a technology available only to *mkt* providers, and allows them to arrange orders in the queue. A provider prefers the large scale order if $\chi_j = 1$, prefers a small scale order if $\chi_j = 0$, and he is indifferent between them for $\chi_j \in (0, 1)$. When the *mkt* provider receives requests from a single farmer type, he randomly selects one farmer for service. The *fcfs* provider serves orders as they arrive in the queue.

The opportunity cost of moving equipment from a provider to the plot, entails the value of time for the equipment driver, i.e. his wage; as well as the value of the foregone services that could have been provided if the equipment would have not traveled, i.e. the shadow value of time given the providers' capacity constraint.

Farmers decide whether and which provider to approach (with commitment) generating queues for each available provider. Providers decide which orders to serve given their selection criteria and capacity constraint. Service provision takes place and farmers produce. Delays in provision occur in equilibrium inducing productivity costs for certain farmers. Given the large number of providers and farmers we focus on a symmetric mixed-strategy equilibrium where ex ante identical providers and farmers use the same strategy and farmers randomize over the set of preferable providers.

A type i-farmer's strategy is a vector of probabilities $P_i \equiv \left(p_{i,\text{fcfs}}, ...; p_{i\text{mkt}}, ...\right)$ where p_{ij} is the probability of applying to each type j-provider. Each farmer maximizes expected profits from farming trading off the probability of obtaining a rental service and the cost of such a service. There is no aggregate uncertainty in the economy and factor prices are time independent.

3.2 Queue lengths as strategies.

A convenient object for analysis is the *queue length*, i.e., the expected number of farmers requesting a service from a given provider.[14] Let q_{ij} be the queue length of type i farmers that apply to a type j provider, where $i \in \{s, s^-\}$ and $j \in \{\text{fcfs}, \text{mkt}\}$. Then, $q_{sj} = \mathfrak{s} F p_{sj}$ and $q_{s^-j} = (1-\mathfrak{s}) F p_{s^-j}$. The farmer type i is determined by its service-hours demanded and location.

Assumption 1. *Service-hours demanded satisfy $k_s > k_{s^-}$. The expected travel time to servicing small-holder farmers is weakly higher than that for large-scale farmers, $d_s \leq d_{s^-}$.*[15]

The probability of approaching different providers for a single farmer should add up to one, which leads to the following feasibility constraints

$$H\left(\mathfrak{h} q_{s,\text{fcfs}} + (1-\mathfrak{h}) q_{s,\text{mkt}}\right) = F\mathfrak{s} \tag{1}$$

$$H\left(\mathfrak{h} q_{s^-,\text{fcfs}} + (1-\mathfrak{h}) q_{s^-,\text{mkt}}\right) = F(1-\mathfrak{s}) \tag{2}$$

A farmer of scale i that requests service from provider j gets served with probability Δ_{ij}.[16] This probability depends on the provider's selection criteria, its capacity, machine-hours demanded k_i and the expected travel time for service d_i. Hence, Δ_{ij} is the sum of the probability of servicing $\bar{o}_i$ type i farmers, across all possible number of orders of type i being served, $\bar{o}_i$, i.e. $\phi_{ij}(\bar{o}_i)$; times the probability that a certain farmer of type i is chosen, $\tilde{\Delta}_{ij}(\bar{o}_i)$,

$$\Delta_{ij} = \sum_{\bar{o}_i} \phi_{ij}(\bar{o}_i) \tilde{\Delta}_{ij}(\bar{o}_i). \tag{3}$$

The probability of type i being served (weakly) declines in the queue length of type $i' \neq i$ farmers. For the *fcfs* provider the result is straightforward because service probabilities decline with the number of machine-hours in the queue, irrespective of their type. For the *mkt* provider with a selection criteria that favors type i' farmers, the decline in the probability

[14] When the number of providers and firms grow large, the probability of requesting a service to a given provider approaches zero and it is inconvenient to work with. Empirically, queues are observable to us.

[15] These features are born in the empirical evidence we discuss in Section 2.

[16] Appendix A contains a full derivation of these probabilities for the case where at most three orders are serviced within a period, the empirically relevant case.

of service for type $i \neq i'$ is strict as the number of type i' farmers in the queue increases. The service probability for type i' farmers is independent of the queue length of type $i \neq i'$ due to the selection criteria.

3.3 Farmer's decisions.

We follow Burdett et al. (2001) and describe a farmer's decision as a function of the market price it would get for the rental service, r_{ij}, which in turn determines its expected "market" profit, U_i. Farmers take the value of the market profit as given when the number of agents in the economy is large, $F, H \to \infty$. Each farmer chooses a service provider to minimize costs given U_i and the production technology:

$$\min_j r_{ij} k_i$$

subject to

$$\pi_{ij}(z_{ij}, k_i, r_{ij}) \equiv \Delta_{ij} \left(z_{ij} k_i^{\alpha} - r_{ij} k_i \right) \geq U_i,$$

where π_{ij} are the expected profits of the farm when requesting service from provider j and $z_{ij} \equiv E(z(\Delta_{ij}))$ is the expected productivity in the farm, which is a function of expected service probability, through equilibrium delays.

Farm's productivity depends on the realization of a random shock determining the timing of agricultural activities. We summarize the optimal timing for agricultural activities by the "optimal" land preparation date, $\theta^{\star}$, and relate deviations from this optimal timing to productivity costs. The realization of the land preparation date is a random draw, θ, from a known distribution $G(\bar{\theta}(\Delta_{ij}))$ with mean $\bar{\theta}(\Delta_{ij})$ that depends on provider j's probability of service. If the realization of the preparation date differs from the optimal, the farmer faces a productivity cost proportional to the delay relative to the optimal date as follows,

$$z_{ij} \equiv E(z(\Delta_{ij})) = \bar{z}_i (1 - \eta(\bar{\theta}(\Delta_{ij}) - \theta^{\star}) I_{\theta^{\star} \leq \bar{\theta}(\Delta_{ij})}).$$

where η is the productivity cost per delayed service day in percentage points. The expected productivity is independent of the choice of provider whenever the expected wait time is relatively low, i.e. the probability of service is high.

Assumption 2. *Let* $e\bar{t}a(\Delta_{ij}) = -\ln(\Delta_{ij})$*, a strictly decreasing function of the probability of service.*

The expected productivity z_{ij} is a function of an exogenous component $\bar{z}_i$ and an endogenous component $1 - \frac{\partial z_{ij}}{\partial \Delta_{ij}} \frac{\Delta_{ij}}{z_{ij}}$ which depends on the elasticity of productivity to the probability of service. Under Assumption 2 this elasticity is constant an equal to η and hence, the surplus from transactions is independent of the probability of service.

A type i farmer requests a service from a type j firm with positive probability if the expected profits are weakly larger than U_i. The strict inequality cannot hold because then a type i farmer would apply to that provider with probability 1, yielding $q_{ij} \to \infty$ as the number of farmers grows large. Then, $\Delta_{ij} \to 0$ contradicting that $\pi_{ij}(z_{ij}, r_{ij}, k_i) > U_i$. The farmers' strategy is therefore

$$\begin{aligned} q_{ij} \in (0, \infty) \quad &\text{if} \quad \pi_i(z_{ij}, r_{ij}, k_i) = U_i, \\ q_{ij} = 0 \quad &\text{if} \quad \pi_i(z_{ij}, r_{ij}, k_i) < U_i. \end{aligned} \tag{4}$$

This expression summarizes the trade off between lower provision cost and higher farming profits; against a lower probability of service. Given the shape of the probability function (which enters into expected profits, π) there exist a unique queue $q(r_{ij}, U_i)$ that satisfies the problem of the farmer. The farmer decides his queuing strategy as a function of his capital demand, k_i, expected productivity z_{ij} and market prices r_{ij}.

3.4 Service provider's decisions.

A service provider j maximizes expected returns. The cost of servicing a farmer depends on its location relative to the provider. For a vector $\{U_i, k_i, d_i\}_{i=s,s^-}$, which providers take as given, they choose queue lengths by picking the cost of service of farmers of different scale r_{ij} and service strategy. The queue length is reset at the end of each period and therefore the service provision problem is static.[17] In making pricing and service strategy decisions, providers take capacity constraints as given.

[17] This feature allow us to handle the high dimensionality of the combinatorics problem when providers are allowed to prioritize certain farmer types.

Assumption 3. *Providers capacity satisfies*

$$o(k_{s^-} + d_{s^-}) <= \bar{k} \quad \text{and} \quad o(k_s + d_s) > \bar{k},$$

$$(o-1)(k_s + d_s) + k_{s^-} + d_{s^-} <= \bar{k},$$

$$(o-1)(k_{s^-} + d_{s^-}) + k_s + d_s <= \bar{k}.$$

Hence, if the provider serves only large-scale orders, it can serve $(o-1)$ orders, or it can instead combine those $(o-1)$ orders with one-small scale order. Service capacity is also enough to serve $o-1$ small scale orders and one large scale order. In either case, the provider serves up to o orders within a period. In computing service probabilities, we assume an empirically relevant upper bound for the number of orders per machine of $o = 3$.[18]

First-come-first-served (*fcfs*) provider. Consider the problem of a *fcfs* provider. His value is the expected return from servicing at most $o = 3$ orders within each period. Let $\bar{o}_i \leq o$ be the number of orders of type i being served within the period. The per period return $\tilde{V}$ from facing queue q_{fcfs} depends on the number of orders of each type being served, $\{\bar{o}_s, \bar{o}_{s^-}\}$ and the revenue per order type net of labor and transportation costs, $\{r_{i,\text{fcfs}}k_i - wk_i - wd_i\}$.[19] The value for a first-come-first-served provider is

$$V_{\text{fcfs}}(\bar{k}) = \max_{\{r_{i,\text{fcfs}}\}_{i=s,s^-}} \tilde{V}\left(\{\bar{o}_s, \bar{o}_{s^-}\}_{q_{\text{fcfs}}}, \{r_{i,\text{fcfs}}k_i - wk_i - wd_i\}_{i=s,s^-}\right), \tag{5}$$

subject to farmers' strategies, equation 4, and feasibility

$$\sum_{i \in q_{\text{fcfs}}} k_i + d_i \leq \bar{k}. \tag{6}$$

Hence, the cost of travel time includes the foregone services that could have been provided if the equipment was not traveling, as well as the opportunity cost of the driver, which commands a wage w per hour.

Market (*mkt*) provider. Consider now the problem of a *mkt* provider who, in addition

[18] This corresponds to the median number of daily orders serviced in our data.

[19] Optimal travel routes could potentially make these revenues non-separable across serviced orders. To keep the model tractable, we assume travel time associated to each order and relax this assumption in the quantitative exercise when allowing for optimal travel routes.

to choosing the cost of provision, $r_{i\text{mkt}}$, chooses a selection criteria χ. This choice in turn determines the type of orders being served and their quantity, given service capacity. The value of a *mkt* provider is

$$V_{\text{mkt}}(\bar{k}) = \max_{\chi, \{r_{i,\text{mkt}}\}_{i=s,s^-}} \tilde{V}\left(\{\bar{o}_s, \bar{o}_{s^-}\}_{(q_{\text{mkt}},\chi)}, \{r_{i,\text{mkt}}k_i - wk_i - wd_i\}_{i=s,s^-}\right), \tag{7}$$

subject to farmers' strategies, equation 4, and feasibility

$$\sum_{i \in q_{\text{mkt}}} k_i + d_i \leq \bar{k}. \tag{8}$$

The full description of the value of these providers, $\tilde{V}$, can be found in Appendix B.

Entry. Providers pay an entry cost I_j, so free entry assures that their expected profitability equals the entry cost,

$$I_j = V_j(\bar{k}). \tag{9}$$

4 Symmetric Equilibrium

Definition 1 (Equilibrium)**.** A symmetric equilibrium consists of farmers expected profits U_s, U_{s^-}, provider strategies r_{ij}, χ, and farmer strategies, q_{ij} for $i = \{s, s^-\}$ and $j = \{\text{fcfs}, \text{mkt}\}$, that satisfy:

1. given U_s, U_{s^-} and other providers' strategies, each type provider maximizes value, equations 5 and 7;
2. observing the providers' decisions, farmers choose who to queue with, equation 4;
3. the values U_s, U_{s^-}, through q_{ij}, are consistent with feasibility, equations 1 and 2; and
4. providers values $V_{\text{fcfs}}, V_{\text{mkt}}$ satisfy free-entry, equation 9.

Proposition 1. *In all symmetric equilibria where providers serve both types of farmers, the selection process is* $\chi = 1$ *and the per period profit of servicing farmers of type* i *is* V_i^j*:*

$$V_i^j = \gamma_{1i}^j(\tilde{z}_{ij}k_s^\alpha - wk_s - wd_s) + \gamma_{2i}^j(\tilde{z}_{ij}k_{s^-}^\alpha - wk_{s^-} - wd_{s^-}),$$

where $\gamma^j_{1,i}, \gamma^j_{2,i}$ are non-linear functions of the queue lengths and the elasticity of the service probabilities with respect to the length of the queue; and where $\tilde{z}_{ij} \equiv \bar{z}_i(1-\eta)$.

The expected per period value of servicing large-scale farmers is higher than for low-scale farmers, $V^j_s > V^j_{s^-}$. If the surplus from large-scale orders is sufficiently larger than from small-scale orders, the expected profit for large-scale farmers is greater than for small-holder farmers, $U_s > U_{s^-}$.[20]

A few characteristics are worth highlighting. First, differences in location and the cost of travel explain disparities in the incentives to serve farmers operating different scales. For two plots located at the same distance to the provider, the marginal cost of service is lower for large scale farmers. Second, small-scale farmers are useful in terms of capacity utilization (Assumption 3) and therefore, even providers that prioritize large-scale farmers have incentives to serve them. Third, the *fcfs* provider manages to attract some large-scale farmers by lowering their rental costs relative to the *mkt* provider. These lower costs for both large and small farmers compensate them for higher expected queues. Finally, the farmers' expected profit from equipment services depends on the return to his own demand for services and on the equilibrium rental rates. In equilibrium, farmers served by both providers are indifferent between them. Hence, the product between the probability of service and the cost of service equalizes across providers.

5 A quantitative assessment of the market for agricultural equipment rentals

In this section, we bring the model to the data to characterize equilibrium allocations and study the impact of counterfactual provider dispatch systems and levels of equipment supply on aggregate productivity and welfare.

We proceed in two steps. First, we solve the model in Section 3 for equilibrium market rental rates and queue lengths across providers and farmers of different production scale. Second, we simulate queues and service provision strategies for farmers with different pro-

[20] The ratio of the surpluses $\frac{\tilde{z}k_s^{\alpha} - wk_s - wd_s}{\tilde{z}k_{s^-}^{\alpha} - wk_{s^-} - wd_{s^-}}$ must be larger than a constant that depends on the elasticity of the probability of service, see equation 24 in Appendix B.

duction scales and geographical locations within each market.

In the first step, we take a stand on the heterogeneity in machine-hours demanded. We construct two groups of farmers following their average machine-hours requests in the transaction data: those with requests of more than 3.5 machine-hours per order are denominated large-scale while those with requests of less than 3.5 machine-hours are denominated small-scale. Then, we solve for an equilibrium in which both types of farmers are served by both types of providers, as in the data. We call this equilibrium the "status quo".

In the second step, we use equilibrium rental rates and queues to simulate 1000 realizations of the queue $q^\star$, with equilibrium composition $(q_s^\star, q_{s^-}^\star)$. The sample paths for queues are drawn from the joint empirical distribution of productivity, machine-hours and geographical location. Hence, while farmers sort based on their scales and rental rates following the first step, the simulation exercise allows for more heterogeneity in space, productivity and hours requested across farmers.

For each of these realizations, we compute the expected delay in service provision and subsequent productivity costs for farmers. In theory, having obtained equilibrium queue lengths and composition allows one to compute expected wait time by farm type and the productivity costs associated to those, given the dispatch technology. However, providers can only price based on scale and not on distance traveled (as we observe in the data). Therefore, the distribution of delays would respond to the geographical distribution of demand in each of these locations. In addition, within each size bin of farm sizes, we observe variation in productivity across farmers, which in turns feeds into the aggregate productivity costs of different equilibrium dispatch strategies.

At the same time, we compute provider's profitability under alternative dispatch systems. Our benchmark results follow a "hub and spoke" pattern, i.e. the equipment must return to the CHC between two consecutive orders. But we also study allocations where providers optimize service provision in space, i.e. they solve a traveling salesman problem. The value function for each provider maps any queue to their service value, conditional on the dispatch system and the delivery route. The value of an order for these providers depends on the density of orders around them, and the size of the order relative to its serving capacity.[21]

[21] As we explain in Appendix D, this is a high dimensional problem, and the number of possible combinations of orders to be served within a period grows exponentially with the number of orders in the queue and

5.1 Calibration

There are 11 parameters to calibrate, as shown in Table 2. We calibrate eight of these parameters directly from data while the remaining two are calibrated internally by solving the model. Consistently with the evidence in Section 2 we use data for the Kharif season (May to October) in year 2018.

There are 5 parameter that are common across markets: the providers' discount factor β, their opportunity cost of moving equipment in space w, the productivity cost of delays η, the curvature of the farming profit function α and the maximum service capacity for each machine $\bar{k}$. We set the discount factor to $\beta = 0.999$ with an implied daily discount rate of 0.1%. The opportunity cost of travel time equals the hourly wage for drivers from our transaction data, at $w =$₹75. The curvature of the profit function is set to $\alpha = 0.6$, as estimated from our own survey data on farm profitability and the structural relationship, i.e. $\pi_i = (1-\alpha)y_i$. We set α to the average ratio of profits to value added as reported by farming households. To discipline the productivity costs of delays, $\eta = 3.4\%$, we use high frequency (daily) data on farm profitability and planting decisions from ICRISAT, see Appendix C for a detailed explanation. We should also calibrate the optimal planting date, $\theta^\star$. We assume enough service capacity such that farmers choose providers in a manner that on expectation, there are no productivity losses from using mechanized services, i.e. $E(z(\Delta_{ij})) = \bar{z}$.[22] Finally, we need a mapping between the probability of service and the realization of the service date, $\theta(\Delta_{ij})$. We assume $\theta(\Delta_{ij}) \equiv -\ln(\Delta_{it})$, so the elasticity of farm productivity to the probability of service is constant equal to η, see Appendix B.1.

The remaining 6 parameters are market specific, as well as the joint-distribution of order sizes and travel time. The first 4 parameters and the distribution are estimated without solving the model. We use our census to compute equipment ownership and using information on the inventory of implements in hubs we compute the share of first-come-first-serve providers (machines) relative to the total supply of equipment in each market h. To characterize the productivity of farmers requesting different machine-hours we use the subsample

its characteristics (including hours serviced and location, i.e., latitude and longitude). Details of the value computation are in the Online Appendix

[22] This is analogous to an outside option that entails no usage of mechanization services and that is large enough so that farmers participate in this market only if on expectation, they face no productivity costs. An example of this outside option is hired/family labor.

Table 2: Parameterization

Parameter	Description	Value	Source/Moment
Measured directly in the data			
common across hubs			
α	Curvature of the profits function	0.6	Survey data
β	Discount factor	0.999	Interest rate
w	Travel/op. cost (INR/hr)	75	Platform data
η	Productivity loss/day	3.4%	ICRISAT sample
$\bar{k}$	Hub-capacity (hours)	12	Platform data, peak
hub specific		method/value	
h	Share of fcfs providers	0.86	Census data
μ	Log-normal mean of productivity	8.89	Survey data
σ	Log-normal s.d. of productivity	1.33	Survey data
ρ	Correlation order size and productivity	-0.12	Survey + Platform data
k_i, d_i	Joint-distribution of order size and travel time	B-splines	Platform data
Calibrated using the model (hub-specific)			
s	Share of large farmers	0.3	Census data
f	No. of farmers/No. of equipment	5.1	Small-scale queue, fcfs

Notes: Benchmark model parameterization. Productivity is measured as output per acre. Hub-capacity corresponds to the hours serviced per machine within a day at the peak of service demand, i.e. the maximum number of hours outstanding in the queue during the season.

of transactions that overlaps with the survey data (approximately, 1,300 observations) and compute the underlying correlation between farm productivity, measured as output per acre, and machine-hours requested. For our benchmark market, this correlation is negative at -0.12.[23] We assume that the distribution of productivity is log-normal, $\ln(\bar{z}) \sim N(\mu, \sigma)$ and fit the empirical distribution of value-added per acre for survey farmers in the catchment area of each hub via maximum likelihood. The estimated mean is 8.89 and its variance is 1.3.[24] Finally, we fit the joint distribution of machine-hours demanded and travel time to services from our transactions dataset via B-splines, see Figure **??** in Appendix E.[25] On the travel dimension, the distribution is typically bimodal, with orders bunching at less than 10-minutes travel time from the hub and 30-minutes travel time.

[23]Their correlation ranges from -0.16 to 0.16 displaying the wide-heterogeneity in demand characteristics across markets, see Table 9 in the Appendix.

[24]Productivity distributions vary across markets with estimates of average productivity from 7.42 to 9.83 log points, and a log-variance ranging from 1.0 to 2.9, columns (3-4) in Table 9.

[25]We could have alternatively calibrated a joint distribution of productivity, machine-hours requested and travel time. However, the overlap between the survey and transaction level data is 20% of the survey data.

Table 3: Moments

	Share of large scale s		Queue q_{s-fcfs}		Queue *untargeted* q_{sfcfs}/q_{s-fcfs}	
	data	model	data	model	data	model
benchmark	0.2	0.3	3.3	3.3	3.3	2.1

Notes: Calibration moments, data and model counterparts, Columns (2-5); untargeted queue length for large-scale farmers relative to small-holder farmers, Columns (6-7).

There are two parameters calibrated computing the equilibrium of the model. The ratio of providers to farmers minimizes the distance between the model predicted queue of small-holder farmers at the *fcfs* provider and the data. This requires a ratio of 5.1 farmers per provider in our benchmark market to match exactly the queue of 3.3 orders on average, see Table 3. We also pick the share of large-scale requests that its closest to its empirical counterpart while generating an equilibrium allocation that displays service request from both types of farmers to both type of providers (as in the data). For our benchmark market, the share of large-scale orders is 30% in the model (20% in the data). A smaller share of large scale farmers would generate full sorting of small farmers into the *fcfs* which is counterfactual.

For completeness, we report the (untargeted) ratio of queue lengths of large-scale and small-holder farmers. This ratio is higher in the data than in the model (3.3 in the data vs. 2.1 in the model). Thus, the model predicts that small scale farmers should be more strongly sorted into *fcfs* providers than observed in the data for our benchmark market.[26]

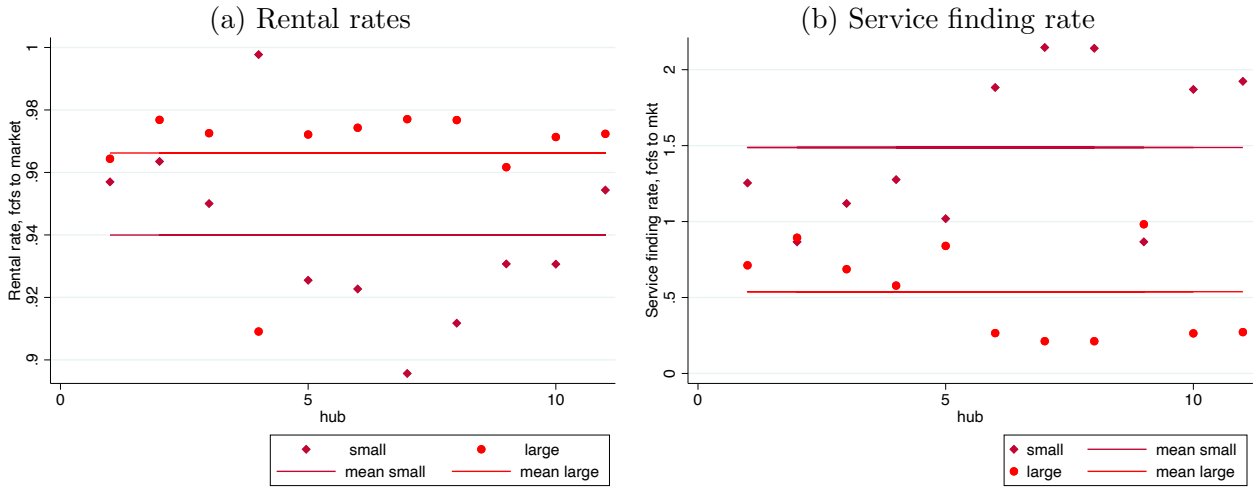
5.2 Status quo equilibrium

We solve for the rental rates and queue lengths when both types of farmers have access to both types of providers, *the status quo* equilibrium henceforth.

We find that rental rates for both types of farmers are lower for the *fcfs* provider than for the *mkt* provider, Figure 4 panel (a), compensating for longer queues on average. Rental rates are particularly lower for small farmers queueing with *fcfs* providers because despite

[26]Outcomes for all other markets can be found in Table 10 in Appendix E.

Figure 4: Equilibrium



Notes: Equilibrium rental rates and service finding rates, ratio between fcfs and market outcomes for small and large scale farmers. Markets ordered by the share of large-scale farmers and relative demand per provider, f as in Table 9.

facing longer queues on average, they do not face the risk of being allocates to the end of the queue, as with the *mkt* provider.[27] Thus, the service finding rate for small-scale farmers is larger with the *fcfs* than with the *mkt* providers, while the opposite is true for large-scale farmers, see Figure 4 panel (b). Service finding rates are the ratio between the number of serviced orders per period and the number of farms searching for a service, $\frac{q_{ij}\Delta_{ij}H}{F} = \frac{q_{ij}\Delta_{it}}{f}$, as commonly used in the job search literature (Barnichon and Figura, 2015).

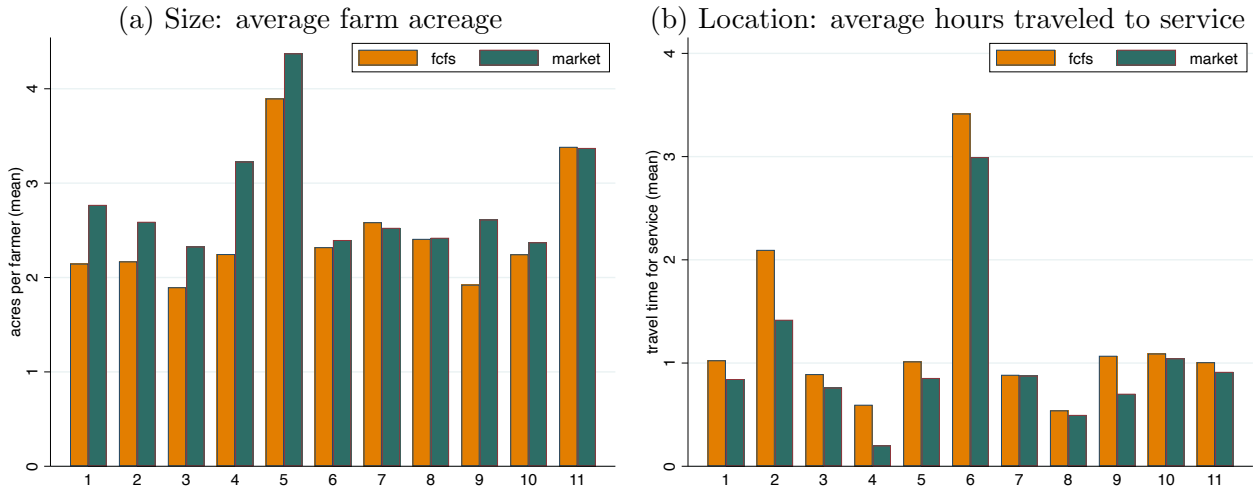
5.2.1 Farmer allocation across providers

We start by describing the sorting of farmers into different providers, classifying farmers by the acreage of the plot posted for service (size); and by travel time for service (location), see Figure 5.

The average order size served by a *mkt* provider is 3 acres while the average order size served by a *fcfs* provider is 2.2 acres. This difference is a consequence of the disparities in the queue composition discussed before. There are also systematic differences in the travel time

[27] In Market 4 the relative rental rates for small holder farmer are similar across dispatch system and more similar than for large-scale farmers. The reason is that there are relatively small differences in the measure of large farmers in the queue.

Figure 5: Demand characteristics by provider



Notes: Panel (a) shows the sorting of farmers across providers by average farm acreage per order and Panel (b) shows the travel time to the service location, in hours.

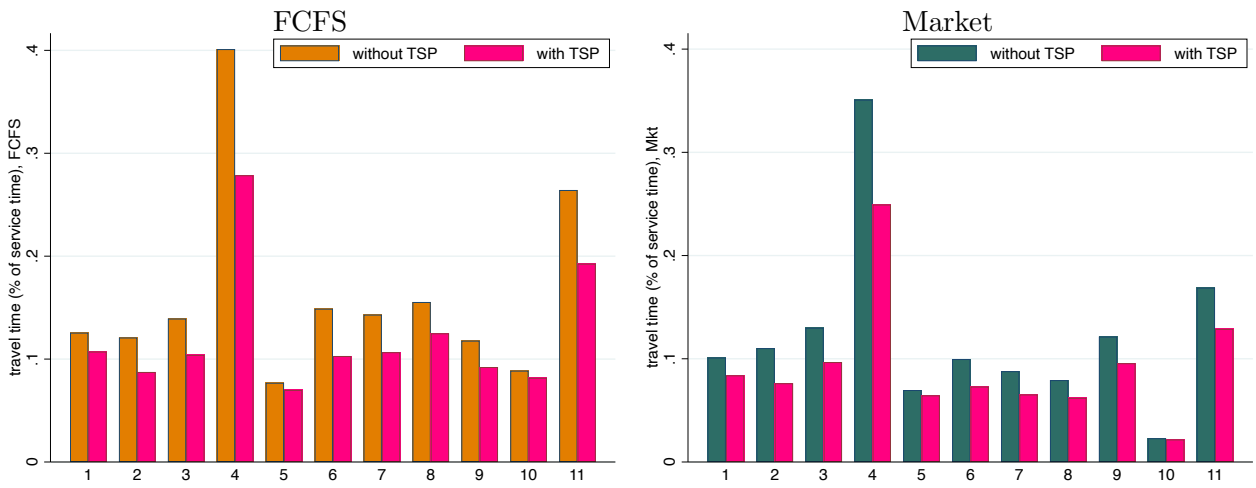
to locations. During a service day, *mkt* providers travel on average .8 hours (48 minutes), while *fcfs* providers travel twice as much. This differential travel time reflects not only differences in the queue composition along the location dimension, but also disparities in the ability to prioritize orders.

While equal-access-concerns may favor a fcfs service arrangement, it is possible to improve upon this allocation by optimizing service delivery within a day. Figure 6 displays travel times when providers are allowed to solve a TSP among the orders served within each day. This option is particularly beneficial to *fcfs* providers that are not allowed to prioritize order sizes. The average travel time (as % of service time) is 9.5% for the *mkt* provider and it declines to 6.4% once optimizing routes. The average travel time (as % of service time) is 14% for the *fcfs* provider and it declines to 10.6% once solving the TSP.

5.2.2 Effectiveness in service delivery

Figure 7 panel (a) compares waiting times across the bootstrapped samples for different dispatch system and demand characteristics. On average, the median wait time faced by farmers queueing with *fcfs* providers is longer that with *mkt* providers. This is the outcome of longer queues as well as composition effects, where *fcfs* are more likely to attract small-

Figure 6: Travel times



Notes: Travel time by provider and dispatch system, "with TSP" corresponds to allocations where providers solve a traveling salesman problem within a day. Travel time is expressed as a ratio of the total service within a day.

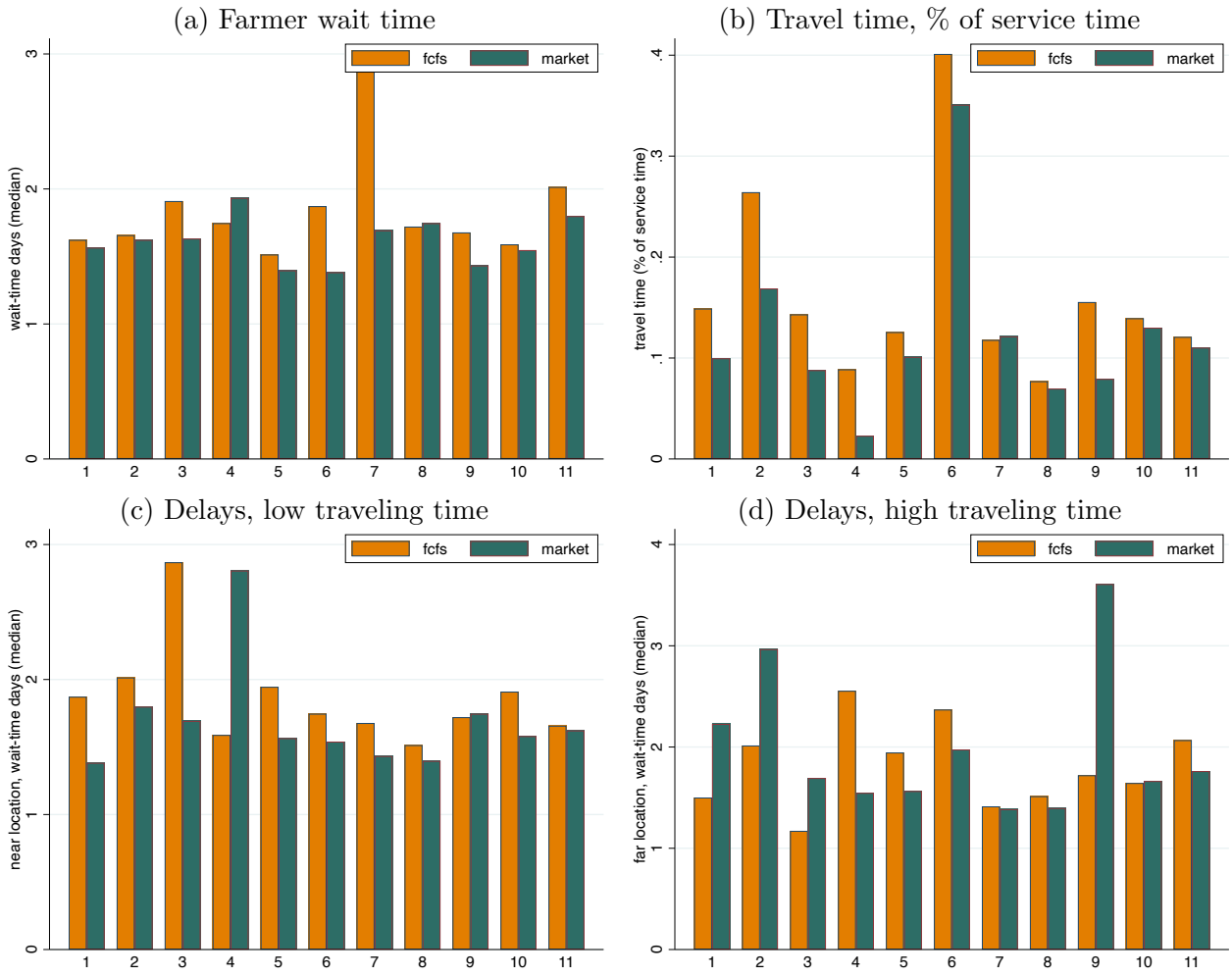
scale orders. *fcfs* providers also face larger equipment transportation costs (in terms of the opportunity cost of time). These costs can double those of the *mkt* providers in certain markets, depending on the joint allocation of demand in space and farm-size. In other words, *fcfs* providers' inability to prioritize orders also induces traveling "too much" relative to their market counterparts, see Figure 7 panel (b). It is not surprising then that the value of optimizing service routes is higher on average for *fcfs* providers as we showed before.

We need to enlarge here what we have to say about the TSP problem, Ref1.

Delays in space The bottom panels in Figure 7 display the distribution of waiting time for service time across farmers located less than 30 minutes of travel time from the equipment hub (panel (c)), and those located more than 30 minutes (panel (d)). Most of the excess delay in near locations observed for *mkt* providers has to do with the sorting of demand by size, i.e. the *mkt* providers attract relatively more large-scale farmers, which induces delays in service provision toward smaller farmers. The difference in delays between providers is maximized for far away locations, because *mkt* providers systematically avoid servicing those.

It would be useful to know what is the share of farmers "close" and "far" in each location

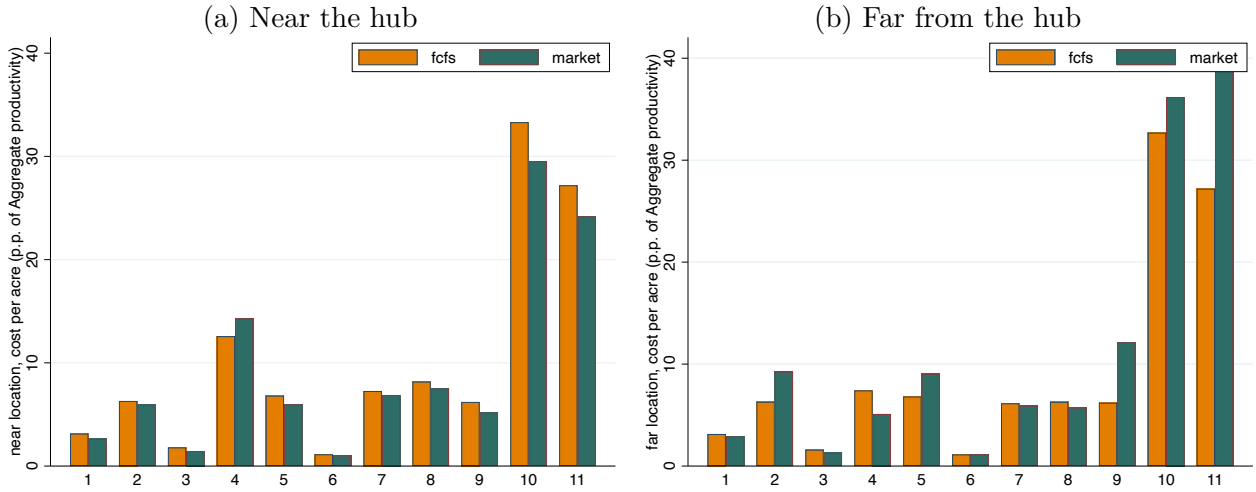
Figure 7: Time management by hub



Notes: Panel (a) shows average days waiting for service across providers for the average farm in the catchment area of each hub. Panel (b) shows the average travel time as a ratio of the service time by provider and hub. Panel (c) shows the average days waiting for service across providers for the average farm in locations within 30 minutes of travel of the centroid of each hub. Panel (d) shows the same measure for the average farm in locations more than 30 minutes away from the centroid of each hub. Wait times are conditional on queueing with a provider.

Figure 8: Productivity costs in space

This picture can go to the appendix



Notes: Panel (a) shows the cost in revenue per acre for the average farm in locations within 30 minutes of travel of the centroid of each hub. Panel (b) shows the same measure for the average farm in locations more than 30 minutes away from the centroid of each hub. The cost is measured in p.p. of the average revenue per acre in each market.

5.2.3 The productivity costs of delays.

These delays induce costs in revenue per acre that depend on the joint distribution of location, size and productivity. Figure 8 reports the productivity cost per acre across providers for the bootstrapped samples. These are measured as the decline in revenue per acre relative to the average revenue per acre in each market. Productivity costs per acre for the *fcfs* providers are higher for locations nearby, in part due to the sorting of farmers in their own queue. Hence, despite improving finding rates for small-farmers unconditionally, improvements in finding rates are disproportionally driven by those farther away from equipment hubs. In addition, this average productivity cost depends on the average productivity of the farms affected by delays. Conditional on servicing far away locations, the disparities in delays are larger across providers, with those queueing with the *mkt* providers facing the larger costs. These costs are largest for markets where the relatively supply of equipment is lower (higher f given the share of large-scale farmers).

6 The interplay between demand characteristics, service capacity and aggregate productivity

In this section we take the calibrated economy and characterize how allocations and aggregate outcomes change across alternative counterfactual economies. First, we implement a deregulation of the market that allows *fcfs* providers to have access to a technology to prioritize the most profitable orders. We study this policy in the short-run, with a fixed number of suppliers; and in the long-run, once we allow for providers entry and exit so that equilibrium profitability resembles that of the baseline economy with two dispatch systems. We show that given the calibrated equipment supply, service probabilities for small-holder farmers are higher than that of large-scale ones, despite the latter being prioritized after a market deregulation.

The relatively higher service probability for small-holder farmers is however not warranted for every level of equipment supply. So as a second set of counterfactuals we study economies where we vary the relative supply of equipment and show that small-holder farmers are indeed rationed out of the market when the relative supply of equipment is low. The level of equipment supply that assures access to services to small-holder farmers depends on the spatial distribution of demand.

6.1 From *fcfs* providers to *mkt* providers: Market deregulation

One of the findings in Section 5.2.2 is that *fcfs* providers could benefit from operating a technology that allows them to optimize equipment in space as well as optimize the type of orders being served. We study the effect of a market deregulation through counterfactuals. Because prioritizing large-scale farmers is costless and the fcfs providers are at least as well off as before (i.e. they can now prioritize the high marginal return orders), a profit driven fcfs provider would choose to adopt the technology, leading to a shift in provider composition towards $h = 0$.[28] In other words, there is no longer any differentiation between these two types of providers. The nature of the equilibrium may however change due to the endogenous

[28] As we pointed out before, the value of service for *mkt* providers is always above the one for first-come-first-served providers, and therefore each provider has incentives to adopt a dispatch system that prioritizes large orders.

farmer sorting, and pricing of services.

Table 4: Effect of Market Deregulation

	Benchmark		Deregulation	
	fcfs	mkt	short-run	long-run
Service Finding Rate				
average farmer	0.41	0.56	0.49	0.5
small-scale	0.28	0.22	0.29	0.3
large-scale	0.13	0.34	0.2	0.21
Rental Rate				
small-scale	121	125	122	120
large-scale	93	94	92	92

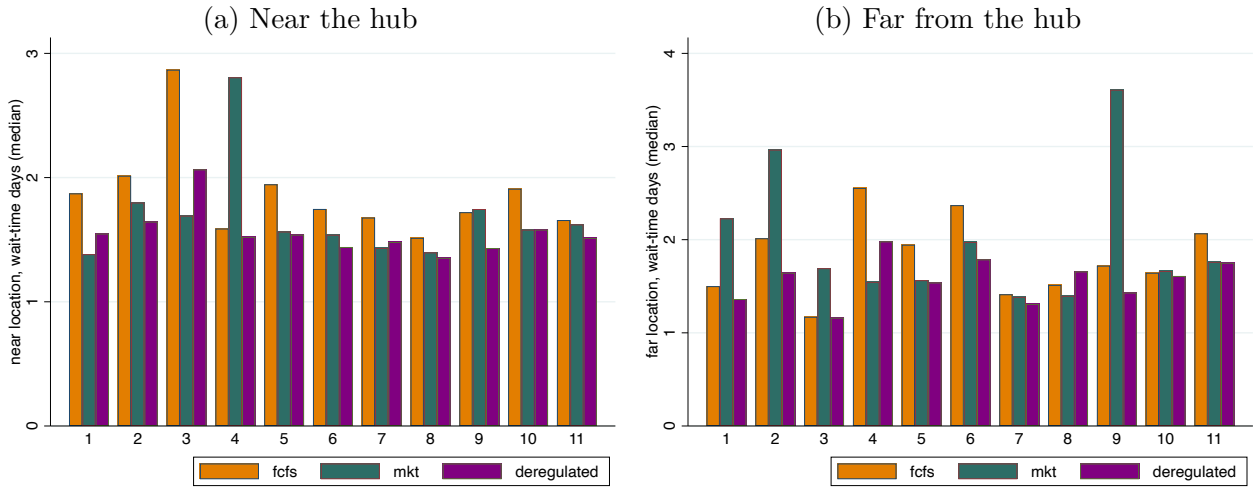
Notes: This table shows the service findings rates and hourly rental rates (in rupees) across different equilibrium allocations. These results are presented by farmer scale of production and by provider. The "benchmark allocation" corresponds to the calibrate economy, whereas the "deregulated" economy corresponds to economies where *fcfs* providers are allowed to prioritize orders as their market counterparts. The "short-run" is an equilibrium without providers' entry-and-exit whereas the "long-run" allows for it.

The short run response of the economy (without endogenous entry or exit of providers) implies higher service finding rates for large-scale farmers relative to the fcfs provision, consistently with the profit maximizing strategy of *mkt* providers (see Table 4). However, their service finding rates are below those of small scale farmers that are drawn to the market in response to the increased supply of services. While service findings rates for small-holder farmers do not change relative to the baseline, the rental costs increase for all farmers. The long run response of the economy (with endogenous entry and exit) restores rental costs to their baseline levels, and generates allocations where service findings rates are higher for small holder farmers than large holder farmers, despite *mkt* providers prioritizing the latter. The reason is that small-scale requests are valuable when there are service capacity constraints because they improve capacity utilization.

Figure **??** plots the change in average farm sizes served and the travel time for service, i.e. the change in the distribution of served location, as the market deregulates. Exit of providers induces an increase in the average size of the farm served by each provider, and a reduction in the travel time to service, consistently with providers prioritizing services with low marginal cost of provision. Travel time as a proportion of the service time declines by 20%, on average across hubs.

It is possible to rank allocations based on the costs in profitability incurred by farmers,

Figure 9: Impact of Deregulation, delays



as well as the travel costs incurred by providers to service those slots.

$$\iota_j = \int_{k_i,d_i} \Delta_{ij}(\bar{z}_i\eta(\theta_i - \theta^\star)k_i^\alpha + d_i w)\zeta_i$$

where k_i indexes the size of the order (in acres/machine-hours), d_i indexes the travel time to a location and ζ_i is the joint distribution of farms in space and size as calibrated in the model. This cost ι can also be computed per acre

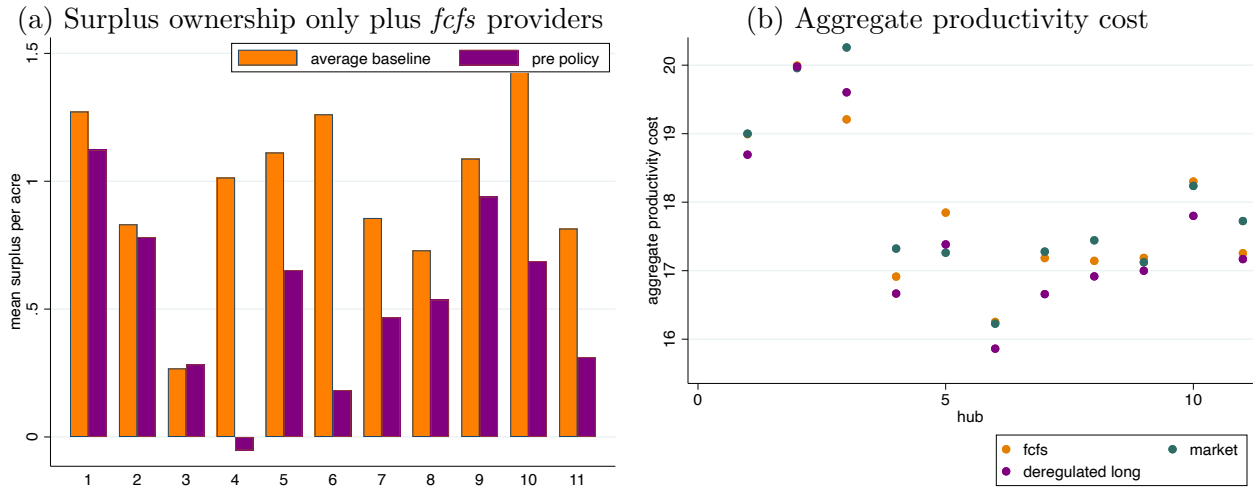
$$\iota_j = \int_{k_i,d_i} \Delta_{ij} \frac{\bar{z}_i\eta(\theta_i - \theta^\star)k_i^\alpha + d_i w}{k_i}\zeta_i$$

6.2 Alternative relative supply of equipment

An extensive literature in agricultural economies has studied the effect of government subsidies for equipment. Two conclusions arise: (a) equipment subsidies are regressive because they benefit relatively wealthier farmers; (b) the impact of these subsidies in mechanization is widely heterogeneous, with only a handful of successful cases (see review in Pingali, 2007).

Our results suggest a different interpretation of the heterogeneity in the effectiveness of subsidies. First, small-holder farmers can benefit disproportionally more from these subsidies

Figure 10: Impact of Deregulation, surplus



Notes: Panel (a) shows the surplus per acre. Panel (b) shows the average productivity costs across different providers, and in the counterfactual where all providers behave as *mkt* providers.

than large-scale farmers (at least in terms of service findings rates). Their ability to do so depends on their value in terms of capacity utilization through their spatial; and the increase in machine-hours supply. Indeed, if the supply increase is relatively low, service finding rates for small-holder farmers are below that of large-scale farmers as we show in Table 5. One could erroneously interpret such an effect to be regressive, benefiting disproportionally large scale farmers.

Table 5: Service finding rates across supply of machine-hours, H.

	H_{ps}	H relative to H_{ps}			
	1	1.4	2	3.3	5
average farm	0.15	0.21	0.28	0.39	0.47
small-scale	0.05	0.08	0.12	0.19	0.26
large-scale	0.10	0.13	0.16	0.19	0.20

Note: Column H_{ps} corresponds to the level of pre-subsidy supply of machine-hours. The pre-subsidy supply level is estimated as the equipment supply absent CHC's machine hours, see Section **??**. Service finding rates are computed as described in the text.

7 Conclusion

Rental markets hold considerable promise in expanding mechanization access and increasing productivity in the farming sector. However, the spatial distribution of demand in space and its synchronous nature, as well as the fixed supply capacity, poses important policy-relevant trade offs for service provision. The returns to these rental markets depend crucially on factors such as spatial density, i.e. the proximity of suppliers to farmers, the overall supply capacity, and the ability to optimize traveling equipment time. In this paper, we document and quantify how these factors determine the distributional effects of rental markets.

We find that when the government increases service capacity by subsidizing the purchase of equipment for rental service provision, and at the same time imposes a first-come-first-serve dispatch system to allocate services, it induces misallocation in service provision. When equipment owners are allowed to maximize their profits by prioritizing large scale orders, the equilibrium allocation may induce higher finding rates for small-scale producers. The reason is that small-scale orders are valuable in terms of service capacity-utilization, particularly when located in high-density areas.

We provide a parsimonious framework to study the allocation of shared services across heterogenous demand and spatial allocations. The framework can be readily extended to study similar markets where capacity utilization and geography are important determinants in service provision. Finally, while we take the location of service providers as given, a natural step forward would be to study the properties of the endogenous location of providers in space, as in Oberfield et al. (2020).

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A Characterization of the probability of service.

A.1 Probability of service for a particular farmer

The probability that a provider chooses a particular farmer given that he chooses one of type i is $\tilde{\Delta}_{ij}(1)$, i.e.

$$\tilde{\Delta}_{ij}(1) = \sum_{n=0}^{f_i-1} \binom{f_i - 1}{n} p_{ij}^n (1-p_{ij})^{f_i-1-n} \frac{1}{n+1},$$

where f_i is the number of farmers of type i searching for a provider, $f_s = \mathfrak{s}F$ and $f_{s^-} = (1-\mathfrak{s})F$ and $\binom{f_i-1}{n} = \frac{f_i-1!}{n!(f_i-1-n)!}$. Hence,

$$\tilde{\Delta}_{ij}(1) = \frac{1-(1-p_{ij})^{f_i}}{f_i p_{ij}}.$$

As the number of agents in the economy gets large, and using the definition of queue lengths above, the service probability simplifies to

$$\tilde{\Delta}_{ij}(1) = \frac{1-e^{-q_{ij}}}{q_{ij}}.$$

That is, the probability that at least one farmer of type i has requested a service, $1-e^{-q_{ij}}$, divided by the number of requests of a given type, q_{ij}.

Next, consider the probability of a particular farmer being served when the provider serves $\bar{o} = 2$ orders of type i, $\tilde{\Delta}_{ij}(2)$. Similar computations to those above yield a service probability as follows

$$\tilde{\Delta}_{ij}(2) = 2(\frac{1-e^{-q_{ij}}}{q_{ij}}) - e^{-q_{ij}}.$$

Finally, consider the probability of a particular farmer being served when the provider serves $\bar{o} = 3$ orders of type i, $\tilde{\Delta}_{ij}(3)$, which follows

$$\tilde{\Delta}_{ij}(3) = 3(\frac{1-e^{-q_{ij}}}{q_{ij}}) - 2e^{-q_{ij}} - e^{-q_{ij}} q_{ij}.$$

A.2 Probability of service of a farmer of type i

Next, we characterize the probability that a provider of type j services a farmer of type i given that a farmer of type i is standing in the queue.

First-come-first-served. The *fcfs* provider only considers feasibility and the position of the farmer in the queue. Let the probability of serving $\bar{o}$ farmersof type i be $\phi_{\text{ifcfs}}(\bar{o})$. Given

the queue lengths at this provider, there are ${}^{q_s+q_{s^-}}P_o = \frac{q_s+q_{s^-}!}{(q_s+q_{s^-}-o)!}$ possible permutations for the o-tuple, (the provider identifier has been dropped for notational convenience). Under Assumption 1, a *fcfs* provider serves a single large-scale farmer if one of the large-scale farmers are among the first three positions in the queue, which occurs with probability $\hat{\phi}_{s\text{fcfs}}(1) \equiv 3q_s \frac{{}^{q_{s^-}}P_2}{{}^{q_s+q_{s^-}}P_3}$; and at least one has applied.

$$\phi_{s\text{fcfs}}(1) = \psi_{s\text{fcfs}}(1)\hat{\phi}_{s\text{fcfs}}(1),$$ [29]

where $\psi_{s\text{fcfs}}(1) \equiv (1 - e^{-q_{s^-\text{fcfs}}} - q_{s^-\text{fcfs}}e^{-q_{s^-\text{fcfs}}})$ is the probability of having at least three orders in the queue of which at least two are of type s^-, when a single farmer of type s has requested service. To this probability we should add the probability of service when less than 3 farmers apply for service. The latter is $\hat{\psi}_{s\text{fcfs}}(1) \equiv \left(e^{-q_{s\text{fcfs}}}(e^{-q_{s^-\text{fcfs}}} + q_{s^-\text{fcfs}}e^{-q_{s^-\text{fcfs}}})\right)$, i.e. the probability of service of large scale order when there are no other service request or there is exactly one additional order requested.

A *fcfs* provider services 2 large-scale farmers if there are two or more large-scale orders in the first o positions of the queue and at least two large scale farmers have requested service. Let the first probability be $\hat{\phi}_{s\text{fcfs}}(2) \equiv 3q_s(q_s - 1)\frac{{}^{q_s-2+q_{s^-}}P_1}{{}^{q_s+q_{s^-}}P_3}$

$$\phi_{s\text{fcfs}}(2) = \psi_{s\text{fcfs}}(2)\hat{\phi}_{s\text{fcfs}}(2),$$

where $\psi_{s\text{fcfs}}(2) = (1 - e^{-q_{s\text{fcfs}}} - e^{-q_{s^-\text{fcfs}}}q_s e^{-q_{s\text{fcfs}}})$ is the probability that there are at least three orders in the queue conditional of a farmer of type s requesting service, of which at least two are of type s (including the one requesting service).[30] To this probability we should add the probability that there are only two large-scale farmers in the queue $\hat{\psi}_{s\text{fcfs}}(2) \equiv \left(q_{s\text{fcfs}}e^{-q_{s\text{fcfs}}}e^{-q_{s^-\text{fcfs}}}\right)$.

Given feasibility, the *fcfs* provider never serves 3 large-scale orders, $\phi_{s\text{fcfs}}(3) = 0$.

A *fcfs* provider serves a single small-holder farmer if there is one of them in the first o positions of the queue. This probability is defined analogously to its counterpart for large scale orders,

$$\phi_{s^-\text{fcfs}}(1) = \psi_{s^-\text{fcfs}}(1)\hat{\phi}_{s^-\text{fcfs}}(1),$$

and adding the probability $\hat{\psi}_{s^-\text{fcfs}}(1)$ when there are less than three orders.

A *fcfs* provider services 2 small-holder farmers if at least two small-scale orders in the

[29] Note that $\hat{\phi}_{s\text{fcfs}}(i)$ are not the expected probabilities, but rather the probability conditional on the observed queue length. We can numerically show that when $F, H \to \infty$ these two are arbitrarily close.

[30] This is the probability that at least another large scale and at least one small scale farmer request service, or at least two other large scale farmers request service.

first o positions of the queue, $\hat{\phi}_{s^-\text{fcfs}}(2) \equiv 3\frac{q_{s^-}(q_{s^-}-1)q_s}{q_s+q_{s^-}-P_3}$

$$\phi_{s^-\text{fcfs}}(2) = \psi_{s^-\text{fcfs}}(2)\hat{\phi}_{s^-\text{fcfs}}(2),$$

where $\psi_{s^-\text{fcfs}}(2) = (1-e^{-q_{s\text{fcfs}}})(1-e^{-q_{s^-\text{fcfs}}})$ is the probability that there are at least three orders in the queue conditional of a farmer of type s^- requesting service, of which at least one is of type s and at least two are of type s^- (including the one requesting the service). To this probability we should add the probability that there are only two orders in the queue, $\hat{\psi}_{s^-\text{fcfs}}(2)$ defined analogously than for large-scale farmers.

A *fcfs* provider services 3 small-holder farmers if there are three small-scale orders in the first o positions of the queue. This probability is defined as $\hat{\phi}_{s^-\text{fcfs}}(3) = \frac{q_{s^-}P_3}{q_s+q_{s^-}P_3}$

$$\phi_{s^-\text{fcfs}}(3) = \psi_{s^-\text{fcfs}}(3)\hat{\phi}_{s^-\text{fcfs}}(3),$$

where $\psi_{s^-\text{fcfs}}(3)$ is the probability of having at least two other small scale requests, i.e. $\psi_{s^-\text{fcfs}}(3) = (1-e^{-q_{s^-\text{fcfs}}} - q_{s^-\text{fcfs}}e^{-q_{s^-\text{fcfs}}})$.

The general form for the probability of service is,

$$\Delta_{i\text{fcfs}} = \sum_{\bar{o}=1}^{3} \hat{\psi}_{i\text{fcfs}}(\bar{o}) + \phi_{i\text{fcfs}}(\bar{o})\tilde{\Delta}_{i\text{fcfs}}(\bar{o}), \tag{10}$$

where we have defined $\hat{\psi}_{s\text{fcfs}}(3) \equiv 0$ to ease notation.

The main difference in the probability of service for large and small relies on the queue lengths. If the queue lengths are identical, then a first-come-first-served provider serves both types of farmers with the same probability, $\sum_{\bar{o}=2}^{3}\phi_{s^-\text{fcfs}}(\bar{o}) = \sum_{\bar{o}=2}^{3}\phi_{s\text{fcfs}}(\bar{o})$.

Market. The *mkt* provider has a technology that allows him to prioritize farmers of either type. The probability of interest is the probability that exactly $\bar{o}$ farmers of type i are served conditional on the farmer under consideration having applied.

Conditional on a large farmer having applied, a single large-scale farmer is served by a *mkt* provider if the provider does not prioritize large scale farmers and there is one large-scale order among the first o available positions, which happens with probability $(1-\chi)\tilde{\phi}_{s\text{mkt}}(1) = (1-\chi)\phi_{s\text{fcfs}}(1)$; or if the provider prioritizes large scale farmers and no other large-scale farmer requested service, $\chi\psi_{s\text{mkt}}(1) = \chi e^{-q_{s\text{mkt}}}(1-e^{-q_{s^-\text{mkt}}} - q_{s^-\text{mkt}}e^{-q_{s^-\text{mkt}}})$. These service probabilities add up to,

$$\phi_{s\text{mkt}}(1) = \chi(\psi_{s\text{mkt}}(1)) + (1-\chi)\tilde{\phi}_{s\text{mkt}}(1),$$

the latter assumes that at least three orders have been requested. We should add to these the event that there are less than three orders, which happens with probability $\hat{\psi}_{i\text{mkt}}(1) = \hat{\psi}_{i\text{fcfs}}(1)$ for any i=s, s^- by definition.

Two large-scale farmers are served by a *mkt* provider if he does not prioritize large orders and they stand in the first 3 positions, which happens with probability $(1-\chi)\tilde{\phi}_{s\text{mkt}}(2) = (1-\chi)\phi_{s\text{fcfs}}(2)$; or if the provider prioritizes those orders and there is at least one additional large-scale service request, which happens with probability $\chi\psi_{s\text{mkt}}(2) = \chi(1 - e^{-q_{s\text{mkt}}} - e^{-q_{s^-\text{mkt}}} q_s e^{-q_{s\text{mkt}}})$.[31] These service probabilities add up to

$$\phi_{s\text{mkt}}(2) = \chi(\psi_{s\text{mkt}}(2)) + (1-\chi)\tilde{\phi}_{s\text{mkt}}(2).$$

We then add the probability when less than three orders are in the queue, $\hat{\psi}_{s\text{mkt}}(2)$ defined analogously to the *fcfs* provider.

Feasibility prevents three large-scale orders to be served within the period and therefore, $\phi_{s\text{mkt}}(3) = 0$.

Analogous arguments can be used to describe the probabilities of service of small scale farmers. A single small-holder farmer is always served by a *mkt* provider (conditional on a request) if it prioritizes high-scale requests and at least two large scale farmers have requested service, which occurs with probability $\chi\psi_{s^-\text{mkt}}(1) = \chi(1 - e^{-q_{s\text{mkt}}} - q_{s\text{mkt}} e^{-q_{s\text{mkt}}}$; or if the provider does not prioritize high-scale requests and there is a single small-scale order among the first three orders in the queue, $(1-\chi)\tilde{\phi}_{s^-\text{mkt}}(1)$, where $\tilde{\phi}_{s^-\text{mkt}}(1) = \phi_{s^-\text{fcfs}}(1)$. The reason for always serving a small scale order even when prioritizing large scale is that capacity constraints allow the provider to serve at most $o-1$ orders leaving always and idle slot. Finally, we add the probability that there are less than two large-scale orders in the queue.

$$\phi_{s^-\text{mkt}}(1) = \chi(\psi_{s^-\text{mkt}}(1)) + (1-\chi)\tilde{\phi}_{s^-\text{mkt}}(1),$$

and $\hat{\psi}_{s^-\text{mkt}}(1)$.

Two small-holder farmers are served by a *mkt* provider if it prioritizes high-scale requests and exactly one large-scale farmer requests service and at least another small scale farmer requests service, which occurs with probability $\chi\psi_{s^-\text{mkt}}(2) = \chi q_{s\text{mkt}} e^{-q_{s\text{mkt}}}(1 - e^{-q_{s^-\text{mkt}}})$. Alternatively, two small-holder farmers are served if the provider does not prioritize large-scale orders and there are two small-scale orders among the first three orders in the queue.

$$\phi_{s^-\text{mkt}}(2) = \chi(\psi_{s^-\text{mkt}}(2)) + (1-\chi)\tilde{\phi}_{s^-\text{mkt}}(2).$$

[31] If there are more large-scale orders the provider still serves two because of its capacity constraints.

To these probabilities we add those associated to the event when there are strictly less than two orders in the queue, $\hat{\psi}_{s^-\text{mkt}}(2)$.

Three small-holder farmers are served by the *mkt* provider if it prioritizes high-scale requests and no large-scale farmer requests service and there are at least three small requests, which occurs with probability $\chi\psi_{s^-\text{mkt}}(3) = \chi e^{-q_{s\text{mkt}}}(1 - e^{-q_{s^-\text{mkt}}} - q_{s^-\text{mkt}}e^{-q_{s^-\text{mkt}}})$, or if it does not prioritize them and there are three small-scale orders among the first three in the queue,

$$\phi_{s^-\text{mkt}}(3) = \chi\psi_{s^-\text{mkt}}(3) + (1-\chi)\tilde{\phi}_{s^-\text{mkt}}(3).$$

In sum, the probability of service for a *mkt* provider follows

$$\Delta_{i\text{mkt}} = \sum_{\bar{o}=1}^{3} \hat{\psi}_{i\text{mkt}}(\bar{o}) + \phi_{i\text{mkt}}(\bar{o})\tilde{\Delta}_{i\text{mkt}}(\bar{o}). \tag{11}$$

Following equations 10 and 11, the probability of being served for a given type i (weakly) declines in the queue length of the other type of farmers. In the first-come-first-served provider the result is straightforward. For the *mkt* provider, the decline in the probability of service is strict for the small scale farmers and independent of the queue length of small-scale orders when the provider prioritizes large-scale orders.

A.3 Unconditional probabilities of service.

The unconditional probabilities of service are important in characterizing the value of service for each provider. We consider alternative scenarios, i.e. when the provider serves at capacity ($o = 3$ orders) and when the provider serves less than capacity.

The first-come-first-served provider can serve three orders of small scale (given feasibility) with a service probability of

$$\Phi_{s^-\text{fcfs}}(3) = \left(1 - e^{-q_{s^-j}}(1 + q_{s^-j} + \frac{1}{2}q_{s^-j}^2)\right)\frac{q_{s^-j}P_3}{q_{sj}+q_{s^-j}P_3};$$

or to serve two orders of one type and one of another, with probability

$$\bar{\Phi}_{i\text{fcfs}}(1) = (1 - e^{-q_{ij}})(1 - e^{-q_{i'j}} - q_{i'j}e^{-q_{i'j}})\frac{3q_{ij}q_{i'j}(q_{i'j}-1)}{q_{ij}+q_{i'j}P_3},$$

and $\bar{\Phi}_{i'\text{fcfs}}(2) = \bar{\Phi}_{i\text{fcfs}}(1)$ for $i' \neq i$.

The provider can also serve two orders of large size, (either because he received only two

orders, or because all orders in the queue are of large scale)

$$\tilde{\Phi}_{s\text{fcfs}}(2) = \left(1 - e^{-q_{s\text{fcfs}}} - q_{s\text{fcfs}} e^{-q_{s\text{fcfs}}}\right) e^{-q_{s-,fcfs}},$$

or it can receive exactly two orders of small size and serve those,

$$\tilde{\Phi}_{s^-\text{fcfs}}(2) = \frac{1}{2} q_{s^-j}^2 e^{-q_{s^-j}} (e^{-q_{sj}}).$$

Finally, the provider can serve two orders, one of each type

$$\tilde{\Phi}_{i\text{fcfs}}(1_2) = \left(q_{ij} e^{-q_{ij}} q_{i'j} e^{-q_{i'j}}\right).$$

or only one order, with occurs with probability

$$\tilde{\Phi}_{i\text{fcfs}}(1_1) = \left(q_{ij} e^{-q_{ij}} e^{-q_{i'j}}\right).$$

The probabilities for the *mkt* provider are similar to the ones above, except that we need to account for the *mkt* provider's ability to select large scale orders.

The *mkt* provider can serve three orders of small scale,

$$\bar{\Phi}_{s^-\text{mkt}}(3) = \chi e^{-q_{s\text{mkt}}}(1 - e^{-q_{s^-\text{mkt}}} - q_{s^-\text{mkt}} e^{-q_{s^-\text{mkt}}} - \frac{1}{2} q_{s^-\text{mkt}}^2 e^{-q_{s^-\text{mkt}}}) + (1-\chi)\Phi_{s^-\text{fcfs}}(3), \quad (12)$$

or the orders of large scale and one small,

$$\bar{\Phi}_{s\text{mkt}}(2) = \chi((1 - e^{-q_{s\text{mkt}}} - q_{s\text{mkt}} e^{-q_{s\text{mkt}}})(1 - e^{-q_{s^-\text{mkt}}}) + (1-\chi)\bar{\Phi}_{s\text{fcfs}}(2), \quad (13)$$

or two orders of small scale and on large,

$$\bar{\Phi}_{s^-\text{mkt}}(2) = \chi q_{s\text{mkt}} e^{-q_{s\text{mkt}}}(1 - e^{-q_{s^-\text{mkt}}} - q_{s^-\text{mkt}} e^{-q_{s^-\text{mkt}}}) + (1-\chi)\bar{\Phi}_{s^-\text{fcfs}}(2). \quad (14)$$

When there are less than three orders in the queue there is no need to prioritize orders, and therefore the probabilities of service are identical to those characterized for the FCFS problem, i.e. $\tilde{\Phi}_{i\text{mkt}} = \tilde{\Phi}_{i\text{fcfs}}$.

Expected value of service provision The characterization of the probability allows

us to compute the expected value of service provision:

$$\begin{aligned}\tilde{V}\left(\{\bar{o}_s,\bar{o}_{s^-}\}_{\mathbf{q}_{\text{fcfs}}},\{(r_{i,\text{fcfs}}-w)k_i-wd_i\}_{i=s,s^-}\right)\equiv\\ \sum_{i=s,s^-}\bar{\Phi}_{i\text{fcfs}}(2)\left[2\left((r_{i\text{fcfs}}-w)k_i-wd_i\right)+(r_{i'\text{fcfs}}-w)k_{i'}-wE(d_{i'})\right]+\\ \tilde{\Phi}_{i\text{fcfs}}(1_1)\left((r_{i\text{fcfs}}-w)k_i-wd_i\right)+\\ +\tilde{\Phi}_{i\text{fcfs}}(1_2)\left((r_{i\text{fcfs}}-w)k_i-wd_i+(r_{i'\text{fcfs}}-w)k_{i'}-wE(d_{i'})\right)+\\ \tilde{\Phi}_{i\text{fcfs}}(2)2\left((r_{i\text{fcfs}}-w)k_i-wd_i\right)\\ \Phi_{s^-\text{fcfs}}(3)3((r_{s^-\text{fcfs}}-w)k_{s^-}-wd_{s^-}),\end{aligned}\tag{15}$$

$$\begin{aligned}\tilde{V}\left(\{\bar{o}_s,\bar{o}_{s^-}\}_{(\mathbf{q}_{\text{mkt}},\chi)},\{(r_{i,\text{mkt}}-w)k_i-wd_i\}_{i=s,s^-}\right)\equiv\\ \sum_{i=s,s^-}\bar{\Phi}_{i\text{mkt}}(2)\left[2\left((r_{i\text{mkt}}-w)k_i-wd_i\right)+(r_{i',mkt}-w)k_{i'}-wE(d_{i'})\right]+\\ \tilde{\Phi}_{i\text{mkt}}(1_1)\left((r_{i\text{mkt}}-w)k_i-wd_i\right)+\\ +\tilde{\Phi}_{i\text{mkt}}(1_2)\left((r_{i\text{mkt}}-w)k_i-wd_i+(r_{i'\text{fcfs}}-w)k_{i'}-wE(d_{i'})\right)+\\ \tilde{\Phi}_{i\text{mkt}}(2)2\left((r_{i\text{mkt}}-w)k_i-wd_i\right)\\ \Phi_{s^-\text{mkt}}(3)3((r_{s^-\text{mkt}}-w)k_{s^-}-wd_{s^-}).\end{aligned}\tag{16}$$

where the first two terms in either expression correspond to the expected value of serving three orders or different types, while the remaining terms correspond to the expected returns of serving strictly less than three orders.

B Proofs

B.1 Proposition 1

First, we solve for the equilibrium value of service when both farmers queue with both providers. Then, we show the value when the service provider serves a single type of farmers. Then we show that the guess that the selection criteria for the *mkt* provider should be to prioritize large scale orders. Finally, we show that the expected value of service is higher for large scale farmers.

Value of service when serving both type of farmers.

Proof. Using the definition of the expected value of service provision (equations 15 and 16) and rearranging terms, as well as the participation constraint for the farmers, equation 4,

the problem of the provider is

$$\max_{q_{sj},q_{s^-j}} \sum_{i=s,s^-} \Phi_{ij}(2)[z_{ij}k_i^\alpha - \frac{U_i}{\Delta_{ij}} - w(k_i+d_i)] + \sum_{i=s,s^-} \Phi_{ij}(1)[z_{ij}k_{i'}^\alpha - \frac{U_{i'}}{\Delta_{i'j}} - w(k_{i'}+E(d_{i'}))] + \Phi_{s^-j}(3)3[z_{ij}k_{s^-}^\alpha - \frac{U_{s^-}}{\Delta_{s^-j}} - w(k_{s^-}+d_{s^-})],$$

where $\Phi_{ij}(2) = (\bar{\Phi}_{ij}(2)2 + \tilde{\Phi}_{ij}(2)2 + \tilde{\Phi}_{ij}(1_1) + \tilde{\Phi}_{ij}(1_2))$ and $\Phi_{ij}(1) = (\bar{\Phi}_{ij}(2) + \tilde{\Phi}_{ij}(1_2))$.

Let $\bar{V}_{ij}$ be the profit per order of type i for provider j, i.e. $\bar{V}_{ij} \equiv \big((r_{ij}-w)k_i - wd_i\big)$. The optimality condition swith respect to the queue length of large-scale and small-holder farmers are

$$\sum_{i=s,s^-}\left(\frac{\partial\Phi_{ij}(2)}{\partial q_{sj}}\bar{V}_i + \frac{\partial\Phi_{ij}(1)}{\partial q_{sj}}\bar{V}_{i'}\right) + 3\frac{\partial\Phi_{s^-j}(3)}{\partial q_{sj}}\bar{V}_{s^-} + \sum_{i=s,s^-}\left(\Phi_{ij}(2)\frac{\partial\bar{V}_i}{\partial q_{sj}} + \Phi_{ij}(1)\frac{\partial\bar{V}_{i'}}{\partial q_{sj}}\right) + \Phi_{s^-j}(3)3\frac{\partial\bar{V}_{s^-}}{\partial q_{sj}} = 0, \tag{17}$$

$$\sum_{i=s,s^-}\left(\frac{\partial\Phi_{ij}(2)}{\partial q_{s^-j}}\bar{V}_i + \frac{\partial\Phi_{ij}(1)}{\partial q_{s^-j}}\bar{V}_{i'}\right) + 3\frac{\partial\Phi_{s^-j}(3)}{\partial q_{s^-j}}\bar{V}_{s^-} + \sum_{i=s,s^-}\left(\Phi_{ij}(2)\frac{\partial\bar{V}_i}{\partial q_{s^-j}} + \Phi_{ij}(1)\frac{\partial\bar{V}_{i'}}{\partial q_{s^-j}}\right) + \Phi_{s^-j}(3)3\frac{\partial\bar{V}_{s^-}}{\partial q_{s^-j}} = 0, \tag{18}$$

where

$$\frac{\partial\bar{V}_i}{\partial q_{ij}} = -\left[\frac{\bar{V}_i + w(k_i+d_i) - z_{ij}k_i^\alpha(1-\frac{\partial z_{ij}}{\partial\Delta_{ij}}\frac{\Delta_{ij}}{z_{ij}})}{\Delta_{ij}}\right]\left(\frac{\partial\Delta_{ij}}{\partial q_{ij}}\right).$$

Given Assumption 2 the elasticity of productivity to the probability of service equals $\frac{\partial z_{ij}}{\partial\Delta_{ij}}\frac{\Delta_{ij}}{z_{ij}} \equiv \eta$. We can define $\tilde{z}_i \equiv \bar{z}_i(1-\eta)$, as the "adjusted" productivity, and the envelope condition reads

$$\frac{\partial\bar{V}_i}{\partial q_{ij}} = -(\bar{V}_i + w(k_i+d_i) - \tilde{z}_i k_i^\alpha))\frac{1}{\Delta_{ij}}\left(\frac{\partial\Delta_{ij}}{\partial q_{ij}}\right).$$

Let the elasticity of the probability of service with respect to the queue length be $\zeta_{q\Delta}(o) \equiv -\frac{\partial \Delta_{sj}}{\partial q}\frac{q_{sj}}{\Delta_{sj}(o)}$, let the elasticity of the value of service to the queue length be $\zeta_{\bar{V}q} \equiv \frac{\partial \bar{V}}{\partial q_{ij}}\frac{q_{ij}}{\bar{V}}$ and that of the probability of arrival of o orders to the queue length be $\zeta_{q\psi}(o) \equiv \frac{\partial \psi}{\partial q}\frac{q}{\psi(o)}$. The envelope condition indicates that the elasticity of the value of service to the queue length is an inversely proportional function of the elasticity of the probability of service to the queue length, $\zeta_{\Delta ij}$.

$$\zeta_{\bar{V}_i} = \left(1 + \frac{w(k_i + d_i) - \tilde{z}k_i^{\alpha}}{\bar{V}_i}\right)\zeta_{\Delta ij}.$$

Equations 17 and 18 form a system of linear equations that can be solved for the two unknowns $\bar{V}_{s^-}, \bar{V}_s$ as a function of the queue lengths.[32]

$$\Gamma \begin{bmatrix} \bar{V}_s \\ \bar{V}_{s^-} \end{bmatrix} = a \begin{bmatrix} \tilde{z}k_s^{\alpha} - w(k_s + d_s) \\ \tilde{z}k_{s^-}^{\alpha} - w(k_{s^-} + d_{s^-}) \end{bmatrix}$$

where $\Gamma = \begin{bmatrix} \Gamma_1 \Gamma_2 \\ \Gamma_3 \Gamma_4 \end{bmatrix}$, for

$$\Gamma_1 = \frac{\partial \Phi_{sj}(2)}{\partial q_{sj}} + \frac{\partial \Phi_{s^-j}(1)}{\partial q_{sj}} + \zeta_{\Delta_s q_s}\left(\frac{\Phi_{sj}(2)}{q_{sj}} + \frac{\Phi_{s^-j}(1)}{q_{sj}}\right)$$

$$\begin{aligned} \Gamma_2 \quad = \quad & \frac{\partial \Phi_{s^-j}(2)}{\partial q_{sj}} + \frac{\partial \Phi_{sj}(1)}{\partial q_{sj}} + 3\frac{\partial \Phi_{s^-j}(3)}{\partial q_{sj}} + \\ & \zeta_{\Delta_{s^-} q_s}\left(\frac{\Phi_{sj}(1)}{q_{sj}} + \frac{\Phi_{s^-j}(2)}{q_{sj}} + 3\frac{\Phi_{s^-j}(3)}{q_{sj}}\right) \end{aligned}$$

$$\Gamma_3 = \frac{\partial \Phi_{sj}(2)}{\partial q_{s^-j}} + \frac{\partial \Phi_{s^-j}(1)}{\partial q_{s^-j}} + \zeta_{\Delta_s q_{s^-}}\left(\frac{\Phi_{s^-j}(1)}{q_{s^-j}} + \frac{\Phi_{sj}(2)}{q_{s^-j}}\right)$$

$$\begin{aligned} \Gamma_4 \quad = \quad & \frac{\partial \Phi_{s^-j}(2)}{\partial q_{s^-j}} + \frac{\partial \Phi_{sj}(1)}{\partial q_{s^-j}} + 3\frac{\partial \Phi_{s^-j}(3)}{\partial q_{s^-j}} + \\ & \zeta_{\Delta_{s^-} q_{s^-}}\left(\frac{\Phi_{sj}(1)}{q_{s^-j}} + \frac{\Phi_{s^-j}(2)}{q_{s^-j}} + 3\frac{\Phi_{s^-j}(3)}{q_{s^-j}}\right) \end{aligned}$$

[32] Note that equation 17 reduces to 21 when there are no small-scale orders.

and in the LHS

$$a = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = \begin{bmatrix} \zeta_{\Delta_s q_s}(\frac{\Phi_{sj}(2)}{q_{sj}} + \frac{\Phi_{s^-j}(1)}{q_{sj}}) & \zeta_{\Delta_{s^-} q_s}(\frac{\Phi_{sj}(1)}{q_{sj}} + \frac{\Phi_{s^-j}(2)}{q_{sj}} + 3\frac{\Phi_{s^-j}(3)}{q_{sj}}) \\ \zeta_{\Delta_s q_{s^-}}(\frac{\Phi_{s^-j}(1)}{q_{s^-j}} + \frac{\Phi_{sj}(2)}{q_{s^-j}}) & \zeta_{\Delta_{s^-} q_{s^-}}(\frac{\Phi_{sj}(1)}{q_{s^-j}} + \frac{\Phi_{s^-j}(2)}{q_{s^-j}} + 3\frac{\Phi_{s^-j}(3)}{q_{s^-j}}) \end{bmatrix}$$

The last vector on the LHS corresponds to the surplus from trade for each farmer type.

Standard matrix algebra implies that expected value to the providers satisfies

$$\bar{V}_s^j = \gamma_{1s}^j(\tilde{z}k_s^\alpha - wk_s - wd_s) + \gamma_{2s}^j(\tilde{z}k_{s^-}^\alpha - wk_{s^-} - wd_{s^-}) \tag{19}$$

$$\bar{V}_{s^-}^j = \gamma_{1s^-}^j(\tilde{z}k_{s^-}^\alpha - wk_{s^-} - d_{s^-}) + \gamma_{2s^-}^j(\tilde{z}k_s^\alpha - wk_s - wd_s) \tag{20}$$

for $\gamma_{1s^-}^j = \frac{\Gamma_1 a_{22} - a_{12}\Gamma_3}{\Gamma_1\Gamma_4 - \Gamma_2\Gamma_3}$ and $\gamma_{2,s^-}^j = \frac{a_{21}\Gamma_1 - a_{11}\Gamma_3}{\Gamma_1\Gamma_4 - \Gamma_2\Gamma_3}$ while $\gamma_{1,s}^j = \frac{a_{11}\Gamma_4 - \Gamma_2 a_{21}}{\Gamma_1\Gamma_4 - \Gamma_2\Gamma_3}$ and $\gamma_{2s}^j = \frac{a_{12}\Gamma_4 - \Gamma_2 a_{22}}{\Gamma_1\Gamma_4 - \Gamma_2\Gamma_3}$. Notice that the denominator of each of the γ parameters shifts depending on the provider as a function of the probability of service. This heterogeneity changes the value for the derivatives in Γ. □

Value of service when serving only large scale farmers.

*If a **provider j attracts only large-scale farmers**, i.e. $q_{s^-j} = 0$, then the expected per period profit of the provider satisfies*

$$\bar{V}_s = \gamma(q_{sj}, \zeta_{\Delta q}, \zeta_{\psi q}, \alpha)(z_{sj}k_s^\alpha - w(k_s + d_s))$$

where the second term corresponds to the surplus associated to the transaction and $\gamma \in (0, 1)$ is a non-linear function of the queue length, the elasticity of the service probability with respect to the length of the queue, ζ, and the share of capital in farming production.

Proof. The problem of the supplier when it only receives large scale orders is

$$\max_{q_{sj}, r_{sj}} \psi \bar{V}_s$$

subject to

$$\Delta_{sj}\pi_s(r_{sj}, k_s) \geq U_s$$

$$\sum_{i \in \hat{q}_j} k_s(i) + E(d_s(i)) \leq \bar{k}$$

where $\psi = 2\left(1 - e^{-q_{s,\text{mkt}}}(1 + q_{s,\text{mkt}})\right) + e^{-q_{s,\text{mkt}}}q_{s,\text{mkt}}$ because there are no small-scale orders and either the supplier serves one or two orders of large scale.

Using the definition of profits to the farmers, equation 4, we can replace the cost of capital into the objective function. Replacing the rental price of capital as a function of the expected profits, the provider solves

$$\max_{q_{sj}} \psi \left[z_{sj} k_s^{\alpha} - \frac{U_s}{\Delta_{sj}} - w(k_s + d_s) \right]$$

Note that the properties of the probabilities ψ and Δ (decreasing and convex in the queue length) imply that the first order conditions to the problem are necessary and sufficient for an optimum. The optimality condition for the queue length is

$$\frac{\partial \psi}{\partial q} \bar{V}_s - \psi \left[\frac{\bar{V}_s + w(k_s + d_s) - \tilde{z} k_s^{\alpha}}{\tilde{\Delta}_{sj}(2)} \right] \frac{\partial \tilde{\Delta}_{sj}(2)}{\partial q} = 0. \tag{21}$$

Let the elasticity of the probability of service with respect to the queue length be $\zeta_{q\Delta}(o) \equiv -\frac{\partial \Delta_{sj}}{\partial q} \frac{q_{sj}}{\Delta_{sj}(o)}$ and let the elasticity of the probability of arrival of o orders to the queue length be $\zeta_{q\psi}(o) \equiv \frac{\partial \psi}{\partial q} \frac{q}{\psi(o)}$. Finally, let $\gamma(q_{sj}, \zeta_{\Delta q}, \zeta_{\psi q}, \alpha) \equiv \frac{\zeta_{q\Delta}}{\zeta_{q\psi} + \zeta_{q\Delta}}$,

$$\bar{V}_s = \gamma(q_{sj}, \zeta_{\Delta q}, \zeta_{\psi q}, \alpha)(\tilde{z} k_s^{\alpha} - w(k_s + d_s)) \tag{22}$$

which proves the result. For the value to be positive we require $\gamma > 0$ which is true by construction. The provider takes a fraction of the surplus from the transaction.

□

*If a **provider j attracts only small-holder farmers**, i.e. $q_{sj} = 0$, then the expected per period profit of the provider satisfies*

$$\bar{V}_{s^-} = \gamma(q_{s^-j}, \zeta_{\Delta q}, \zeta_{\psi q}, \alpha)(\tilde{z} k_{s^-}^{\alpha} - w(k_{s^-} + d_{s^-}))$$

where $\gamma \in (0, 1)$ is a non-linear function of the queue length, the elasticity of the service probability with respect to the length of the queue, ζ, and the share of capital in farming production.

The derivations when the provider serves only small-scale providers follow the same steps as the ones above, so we omit them for brevity.

The *mkt* provider wants to prioritize large scale orders: Compute $\frac{\partial \Pi_{\text{mkt}}}{\partial \chi}$, which are strictly positive, given the definition for the unconditional probabilities of service, 12 to 14, and the value of the provider, equations 7 and 16. Then, the optimal selection rule is at the corner, $\chi = 1$.

Expected profits to the farmers The expected profits to the farmers depend on the equilibrium being realized, i.e. whether providers serve both type of farmers or providers specialize in a single type. The reason is that the expected profits to the farmer depend on the cost of service, which can be in turn expressed as a function of the value of service using the definition of the value per period, $z_{ij}k_i^\alpha - \frac{U_i}{\Delta_{ij}} - w(k_i + d_i) = \bar{V}_i^j$

$$U_i = (z_{ij}k_i^\alpha - w(k_i + d_i) - \bar{V}_i^j)\Delta_{ij} \tag{23}$$

Replacing the values of expected profits for the providers we obtain

1. If providers serve both type of farms,

$$U_s = \Delta_{sj}((1 - \gamma_{1s}^j)(z_{s^-j}k_s^\alpha - wk_s + wd_s) - \gamma_{2s}^j(z_{s^-j}k_{s^-}^\alpha - wk_{s^-} + wd_{s^-}))$$

$$U_{s^-} = \Delta_{s^-j}((1 - \gamma_{1s^-}^j)('z_{sj}k_{s^-}^\alpha - wk_{s^-} + wd_{s^-}) - \gamma_{2s^-}^j(z_{sj}k_s^\alpha - wk_s + wd_s))$$

2. If a provider serves only large scale farmers,

$$U_s = \tilde{\Delta}_{sj}(1 - \gamma(q_{sj}, \zeta_{\Delta q}, \zeta_{\psi q}, \alpha))(\tilde{z}k_s^\alpha - wk_s + wd_s)$$

3. If a provider serves only small scale farmers,

$$U_{s^-} = \tilde{\Delta}_{s^-j}(1 - \gamma(q_{s^-j}, \zeta_{\Delta q}, \zeta_{\psi q}, \alpha))(\tilde{z}k_{s^-}^\alpha - wk_{s^-} + wd_{s^-})$$

   When the providers specialize in service provision, they determine the expected profits to the farmer.

Equilibrium queue lengths. In an equilibrium where farmers reach out to both providers, they should be indifferent across them and the feasibility constraints of the economy should be satisfied.[33] We describe the indifference condition for large scale farmers, the ones for small scale farmers are analogous.

$$\frac{\Delta_{s\text{mkt}}}{\Delta_{s\text{fcfs}}} = \frac{(1 - \gamma_{1s}^{\text{fcfs}})(\tilde{z}k_s^\alpha - wk_s - wd_s) - \gamma_{2s}^{\text{fcfs}}(\tilde{z}k_{s^-}^\alpha - wk_{s^-} - wd_{s^-})}{(1 - \gamma_{1s}^{\text{mkt}})(\tilde{z}k_s^\alpha - wk_s - wd_s) - \gamma_{2s}^{\text{mkt}}(\tilde{z}k_{s^-}^\alpha - wk_{s^-} - wd_{s^-})}$$

When small scale farmers queue only with *fcfs* providers, the indifference condition for

[33] If they choose to reach out to a single provider, then the equilibrium queue length is determined by feasibility only, which in turn determines the expected value for farmers.

the large farmer is

$$\frac{\Delta_{s\text{mkt}}}{\Delta_{s\text{fcfs}}} = \frac{(1-\gamma_{1s}^{\text{fcfs}})(\tilde{z}k_s^\alpha - wk_s - wd_s) - \gamma_{2s}^{\text{fcfs}}(\tilde{z}k_{s^-}^\alpha - wk_{s^-} - wd_{s^-})}{(1-\gamma_s^{\text{mkt}})(\tilde{z}k_s^\alpha - wk_s - wd_s)}$$

These indifference conditions jointly with the feasibility constraints of the economy, equations 1 and 2, yield the optimal queue lengths by provider and type.

Rental rates The rental rates can be computed from the definition of U once the optimal queues have been solved for.

Value of service for farmers $U_s \geq U_{s^-}$ whenever the different in the surplus of service for large-scale providers is large enough. Because in equilibrium the market value of service is the same irrespective of the provider, it is w.l.o.g. to use the values from the *fcfs* providers.

$$\begin{aligned} U_s - U_{s^-} &\geq 0 \\ (\Delta_{sj}(1-\gamma_{1s}^j) + \Delta_{s^-j}\gamma_{2s^-}^j)(\tilde{z}k_s^\alpha - wk_s - wd_s) - \\ (\Delta_{sj}\gamma_{2s}^j + \Delta_{s^-j}(1-\gamma_{1s^-}^j))(\tilde{z}k_{s^-}^\alpha - wk_{s^-} - wd_{s^-})) &\geq 0 \end{aligned}$$

If the surplus is weakly higher for large scale farmers, $\tilde{z}k_s^\alpha - wk_s - wd_s \geq \tilde{z}k_{s^-}^\alpha - wk_{s^-} - wd_{s^-}$, then it is sufficient that

$$\frac{(\Delta_{sj}(1-\gamma_{1s}^j) + \Delta_{s^-j}\gamma_{2s^-}^j)}{(\Delta_{sj}\gamma_{2s}^j + \Delta_{s^-j}(1-\gamma_{1s^-}^j))} \geq \frac{(\tilde{z}k_{s^-}^\alpha - wk_{s^-} - wd_{s^-})}{(\tilde{z}k_s^\alpha - wk_s - wd_s)}. \tag{24}$$

The above is a condition on the elasticities of the probability of service to the queue lengths (in γ) relative to the values of the surplus.

Value of service for providers The value of serving large farmers is higher than the value of serving small-farmers for any provider For $\bar{V}_s > \bar{V}_{s^-}$ it is sufficient that $(\gamma_{1s} - \gamma_{2s^-}) > 0$ and $(\gamma_{2s} - \gamma_{1s^-}) > 0$, see equations 19 and . This is the same as

$$a_{11}(\Gamma_4 + \Gamma_3) - a_{21}(\Gamma_2 + \Gamma_1)$$

$$a_{12}(\Gamma_4 + \Gamma_3) - a_{22}(\Gamma_2 + \Gamma_1)$$

If the queue lengths are the same $\Gamma_4 + \Gamma_3 = \Gamma_2 + \Gamma_1$ and also $a_{11} > a_{21}$ and $a_{12} > a_{22}$ because the elasticity of the probability of service to the queue large farmers is higher than for the queue of small farmers. By continuity, if the queue lengths are not too different the above result holds. Intuitively, the reason is that if there are no systematic differences in travel time across farmers, then the provider's marginal cost of provision is higher for the smaller

farmers and therefore the provider finds them less valuable.

C Costs of delays

Simple correlations between farm productivity and service delays cannot recover the cost of delays if ex-ante productivity is correlated with the timing of farmers' engagement in the rental market. To overcome this hurdle we exploit high-frequency (daily) data from ICRISAT and estimate the whole profile of profitability for farmers preparing their land at different dates during the agricultural season, using plots of comparable characteristics to those in our sample.

We define an optimal planting time as the date that maximizes the profits per acre in a given village year. Then, we define the cost of the delay as the difference in average value added per acre or profit per acre (depending on the variable of interest) as we move away from the optimal planting date.

Formally, we estimate

$$Y_{i,year} = \beta_0 + \beta_1^{+}\left(\text{Planting Date-Optimal}\right)_{>0} + \beta_1^{-}\left(\text{Planting Date-Optimal}\right)_{\leq 0} + \alpha X_{i,year} + \epsilon_{i,year}$$

where X are controls for plot characteristics, farmer, village and time fixed effects. Standard errors are clustered at the village level.

Our estimates for the costs in value added per acre are reported in Table 6. They indicate that within a 5-day windows, each additional day away from the profit maximizing date entails a cost of 3.4% in terms of value added per acre. In the 5-day window, for farmers that plant too early relative to the optimal date, moving *closer* to the optimal date by one day increases value added per acre by INR 391 per acre. Conversely, for farmers that plant too late relative to the optimal date, moving *away* from the optimal date by one day reduces value added per acre by INR 215 per acre. Therefore, moving closer to optimal date increases returns. This result is robust to enlarging the window around the optimal planting date, with an estimate cost of delay in the planting date of 8.5% per day.

As a robustness check, we also estimate the cost of deviation with sowing time estimated at the weekly rather than daily level. These estimates are reported in Table 7. They indicate a cost of delay of between 5.6% to 11.5% for an additional week's delay in terms of value added per acre, or between 0.8% and 1.6% per day of delay (depending on whether we restrict delay to be within 5 or 3 weeks of optimal sowing time, respectively).

These estimates are also consistent with agronomic estimates that estimate yield losses

from deviating from the optimal planting time in other contexts - for instance, Liu et al. (2023) estimate yield losses of about 13% for a 10-day deviation from the optimal sowing time for winter wheat in China, or about 1.3% per day.[34]

Table 6: Costs of Delays Relative to Optimal Planting Time, Value Added per Acre

	Cost per day, value added per acre				
	Whole Sample	5-day around optimal		10-day around optimal	
		Before	After	Before	After
β_1	-41.97	391.1**	-215.7	1,166***	-931.1***
	(26.33)	(140.2)	(146.9)	(338.7)	(298.5)
Observations	6,034	1,461	1,882	1,010	1,221
R-squared	0.408	0.625	0.584	0.706	0.659
Mean of Value Added per acre	10228	11425	10694	11921	10998

Notes: The optimal date is a village-year measure, and is the sowing date that maximized mean value added per acre in the village in a given year. Standard errors clustered at the village-level in parentheses.
* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$, **** $p < 0.001$.

Table 7: Costs of Delays Relative to Optimal Planting Time by Week, Value Added per Acre

	Cost per Week, value added per acre				
	Whole Sample	5 weeks around optimal		3 weeks around optimal	
		Before	After	Before	After
β_1	-164.5	-115.7	-665.7**	171.3	-1,313**
	(146.6)	(301.8)	(275.9)	(292.7)	(470.4)
Observations	6,034	2,799	4,049	2,333	3,416
R-squared	0.405	0.518	0.432	0.535	0.453
Mean of Value Added per acre	10228	11810	9926	11344	10155

Notes: The optimal date is a village-year measure, and is the week of sowing that maximized mean value added per acre in the village in a given year. Standard errors clustered at the village-level in parentheses.
* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$, **** $p < 0.001$.

[34]ICRISAT's sample sizes after controlling for crops are too small to reliably estimate the effects on yields. The point estimates for yield costs in crops such as pea are 5.3% over a five day window of the optimal planting date.

D Numerical Solution and Output

D.1 Value function computation

The value function maps an ordered queue to the expected present value of this queue. Each order i in the queue comprises two dimensions: h_i, the number of hours demanded discretized to 6 bins, and d_i the travel hours to and from the hub that represents a variable cost of service. For a queue length equal to 3, the value function is a mapping from R^6 to R^1.

$$V(\{(k_1, \nu_1), (k_2, \nu_2), (k_3, \nu_3)\}) : R^6 \to [0, \infty]$$

The relatively high dimensionality of the problem prompts us to implement the sparse-grid method proposed by Smolyak (1963) (see Judd et al. (2014) for details). The grid points are selected for an approximation level of 2, which results in 85 grid points. We then construct a Smolyak polynomial consisting 85 orthogonal basis functions, which belong to the Chebyshev family. The integration nodes are selected by applying the tensor product rule to the one-dimensional Smolyak grid points at the approximation level of 2. Integration is carried out using Newton-Cotes quadrature.

D.2 Simulations

We simulate the expected wait time and productivity cost under the fcfs arrangement and the market arrangement respectively for three cases: when productivity is uncorrelated, negatively correlated or positively correlated to the number of hours demanded. Productivity, measured in revenue per acre, is simulated and assigned to each order observed in the actual data. We make a large number of draws of productivity sequences, each with length equal to the number of actual orders, from a log normal distribution where the parameters are obtained by fitting the actual productivity information to a log normal distribution by hub. We then choose a sequence for each simulation case that produces a correlation with hours demanded in the data that is the closest to a target correlation for that case, and assign that sequence to the actual orders. In the uncorrelated case, the target is zero; in the negatively correlated case, the target is the actual correlation for that hub; in the positively correlated case, the target is symmetric to the negative correlation.

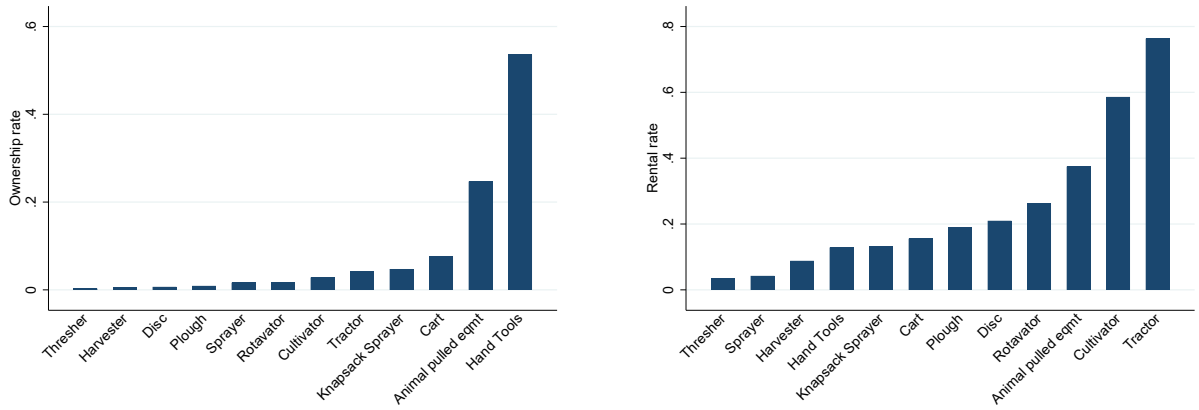
We use bootstrap sampling of the actual orders for the simulation and assume each bootstrap sample represents an actual queue.

We compute the wait time for the first three orders in each bootstrap sample under the fcfs arrangement and the market arrangement respectively. In any period if one or more

out of these three orders are not served, the queue is filled going through the bootstrap sample. The productivity cost is calculated by multiplying the simulated productivity by the wait-time and the percentage productivity loss per day as described in the table 2.

E Additional Tables and Figures

Figure 11: Ownership and rentals by implement.



Owned Rentals

Notes: The ownership (rental) rate is the share of farmers that report to own a given implement relative to the total population surveyed. Sources: Own census of farmers

Table 8: Summary Statistics of Commonly Rented Implements

	Commonly Rented Implements		
	Number of Transactions	Median Price Per Hour	Peak Month
Rotavator 6 Feet	11,239	770	July
Cultivator Duckfoot	7,287	550	April
Cultivator 9 Tyne	5,245	525	May
Plough 2MB Hydraulic Reversible	3,716	450	February
Trolley 2 WD	2,436	250	January
Harvester Tangential Axial Flow (TAF)-Trac	2,048	1800	May
Rotavator 5 Feet	1,811	700	September
Blade Harrow Cross	1,793	360	March
Knapsack Sprayer 20 Liters	1,688	22.5	October
Blade Harrow 5 Blade	1,600	360	June

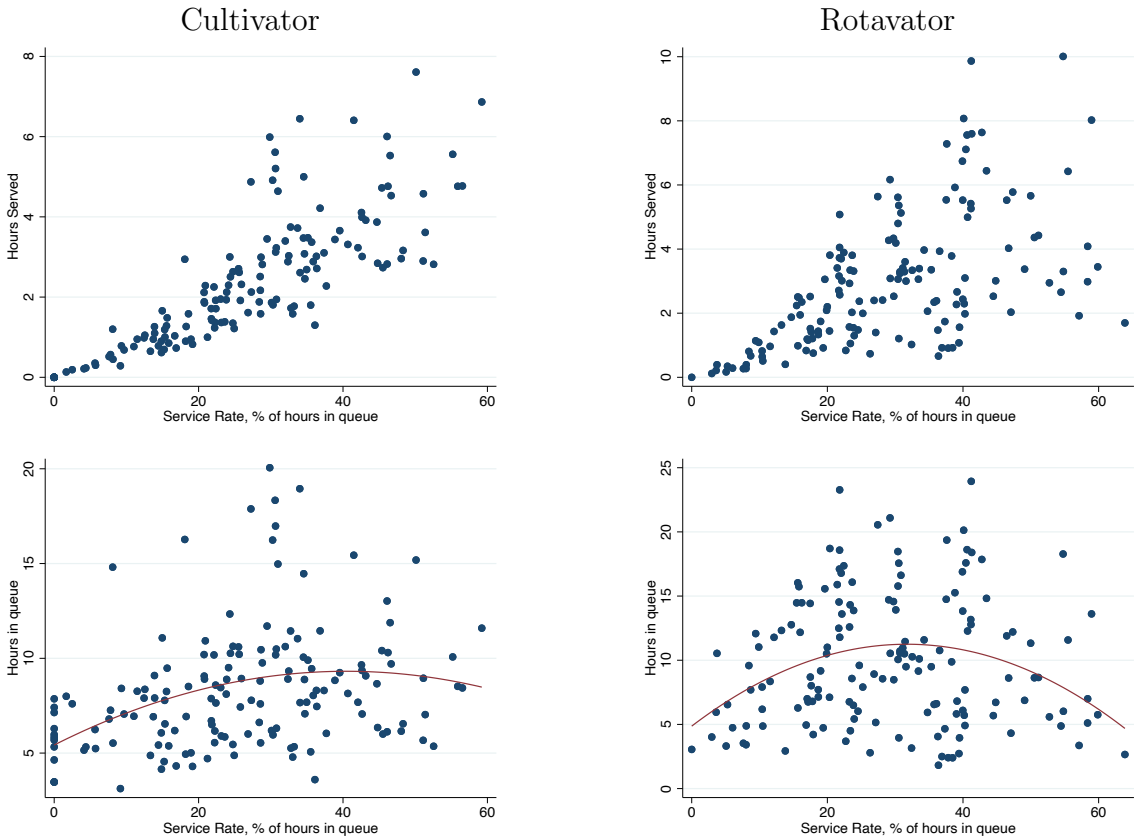
Prices are in Rupees and Peak Month corresponds to the month with the largest number of completed transactions. Source: Transaction level data.

Table 9: Market specific characteristics.

Measured Directly					Calibrated	
Supply		Demand				*Farmers*
Market	sh. fcfs h	Productivity mean	Productivity variance	Correlation prod - hours	*sh. large* s	*per provider* f
(1)	(2)	(3)	(4)	(5)	(6)	(7)
1	0.50	9.83	1.12	-0.01	*0.5*	*5.2*
2	0.63	8.85	2.56	0.16	*0.5*	*4.9*
3	0.67	8.15	2.89	0.01	*0.3*	*4.9*
4	0.83	7.43	1.57	-0.16	*0.4*	*6.4*
5	0.86	9.08	1.52	0.07	*0.3*	*4.8*
6	0.86	7.43	1.57	-0.16	*0.3*	*4.9*
7	0.86	8.89	1.33	-0.12	*0.2*	*4.9*
8	0.86	8.89	1.33	-0.12	*0.2*	*4.9*
9	0.86	8.89	1.33	-0.12	*0.3*	*5.1*
10	0.86	9.08	1.52	0.07	*0.3*	*5.6*
11	0.86	8.85	1.07	0.00	*0.3*	*5.6*
average	0.78	8.67	1.62	-0.04	0.3	5.2

Notes: Market-specific parameters for each market-implement combination. Markets in Column (1) ordered by the share of large scale farmers and demand per provider, f. Markets labeled 5, 7 and 9 correspond to Cultivators while the rest correspond to Rotavators. Markets 7 to 9 are associated with a common location but different implement. Column (2) reports the share of first-come-first-serve providers in the total equipment supply within each market. Columns (3)-(5) report demand characteristics while Columns (6)-(7) report parameters calibrated jointly.

Figure 12: Service rates.



Notes: Service rates and queuing behavior. Each data point corresponds to the average queue or hours served across hubs per day in the season. The bottom panels include a polynomial fit of the series. Source: Own computations and transaction data.

Table 10: Moments

	Share of large scale s		Queue q_{s^-fcfs}		Queue *untargeted* q_{sfcfs}/q_{s^-fcfs}	
	data	model	data	model	data	model
(1)	(2)	(3)	(4)	(5)	(6)	(7)
1	0.5	0.5	2.3	2.3	2.3	0.9
2	0.3	0.5	1.0	2.0	1.0	0.7
3	0.3	0.3	2.0	2.0	2.0	1.6
4	0.4	0.4	3.2	3.2	3.2	1.4
5	0.3	0.3	2.3	2.3	2.3	1.6
6	0.4	0.3	2.7	2.7	2.7	2.7
7	0.2	0.2	4.3	4.3	4.3	6.0
8	0.2	0.2	7.3	4.3	4.4	6.0
9	0.2	0.3	3.3	3.3	3.3	2.1
10	0.3	0.3	4.0	4.0	4.0	2.7
11	0.3	0.3	4.3	4.3	4.3	2.9

Notes: Calibration moments, data and model counterparts, Columns (2-5). Untargeted queue length for large-scale farmers relative to small-holder farmers, Columns (6-7).

Table 11: Status Quo Equilibrium Outcomes

(a) Average queue length

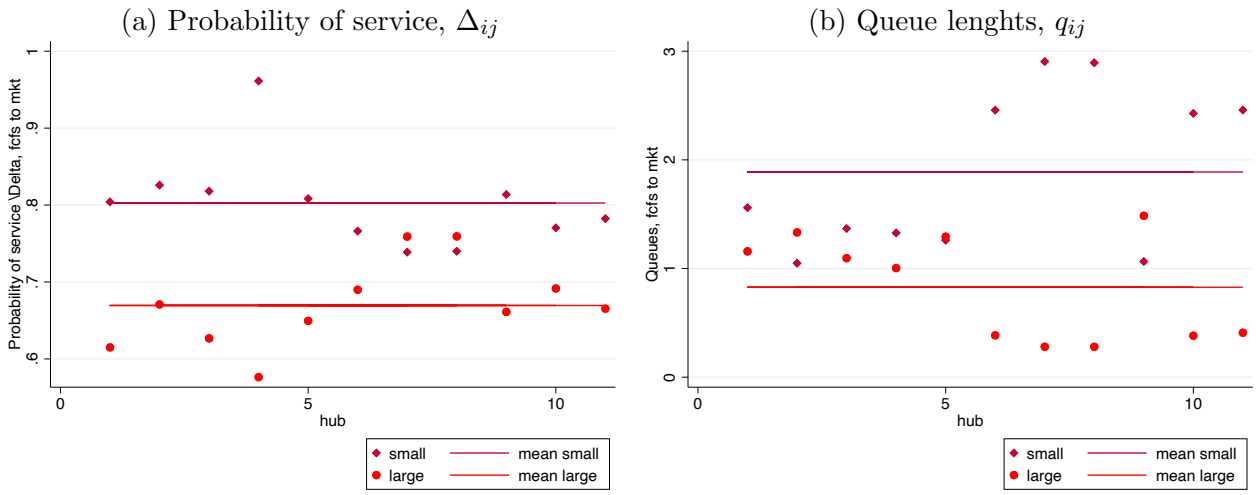
	small		large	
	fcfs	mkt	fcfs	mkt
1	2.0	2.0	1.1	2.6
2	3.0	1.7	2.1	2.7
3	3.5	1.5	1.1	3.0
4	3.3	1.8	1.2	2.4
5	4.3	2.0	1.1	2.5
6	2.0	2.2	1.1	2.2
7	2.0	2.8	1.0	1.7
8	2.0	1.9	1.1	2.6
9	2.5	1.3	1.1	3.6
10	4.0	1.6	1.2	3.9
11	4.0	1.9	1.0	2.3

(b) Rental rates

	small		large	
	fcfs	mkt	fcfs	mkt
1	1.00	1.01	0.76	0.76
2	0.84	0.89	0.73	0.74
3	1.04	1.09	0.82	0.82
4	0.92	0.97	0.73	0.74
5	0.95	0.98	0.74	0.74
6	1.10	1.11	0.81	0.81
7	1.22	1.23	0.85	0.86
8	1.04	1.06	0.75	0.76
9	0.92	0.99	0.74	0.74
10	0.90	0.93	0.68	0.69
11	0.96	1.02	0.75	0.78

Notes: Panel (a) reports equilibrium average queue lengths by hub for farmers of different scale and different providers. Panel (b) reports the equilibrium rental rates per hour for each provider and service type within each market. Rental rates are normalized to the rental rate charged by the *fcfs* provider to small-scale farmers in hub 1.

Figure 13: Service finding rate decomposition



Notes: Conditional probability of service and queue lengths by hub for the calibrated economy.