Price Setting During a Currency Changeover*

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September 2024
PRELIMINARY AND INCOMPLETE

Abstract

A currency changeover is an ideal natural experiment to investigate alternative models of price setting behavior. We analyze price setting behavior before and after the 2002 adoption of the euro in Austria, Slovakia, Finland, and Greece, as well as for the recent changeover of Croatia (January 2023). We use micro price data from various sources to inspect price setting behavior since the time of the changeover announcement and during the transition period. Several moments of the distribution of price changes are recorded, e.g. frequency of price adjustment, size of price changes, cross sectional dispersion of relative prices. We develop a tractable theoretical model of the price setting decisions and highlight testable implications. We compare the data with the predictions of different price setting models.

JEL Classification Numbers: E5
Keywords: Sticky Prices, Menu Cost, Price Dispersion.

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1 Introduction

We study price setting behavior around the time of the euro changeover. This critical time, when all price tags were replaced, provides a natural experiment enabling us to discriminate among different price-setting models. Gaiotti and Lippi (2004); Hobijn et al. (2006); Berardi et al. (2013), for example, studied this event using data for restaurant prices, with the aim to explain the higher-than-average inflation recorded by these prices around the changeover time.

We use micro price data underlying the CPI of several European countries to compute statistics of the distribution of price changes (e.g. frequency of price adjustment, size of price changes, cross sectional dispersion) since the time of the announcement of the policy and during the transition period. We compare them to the predictions of the different price setting models, using data at the finest level of disaggregation.[1]

This paper uses CPI micro-price data going back to the euro changeover for four countries: Austria, Slovakia, Finland and Greece. These data are complemented with CPI micro data for the UK, a country that did not experience the euro changeover. Furthermore, we collected scraped data for Croatia, a country that in January 1st 2023 replaced its official currency for the euro. We began the collection of price information a year before the changeover. Scraped data is complemented with price information collected at the same time for other European countries not experiencing a currency changeover. These data allow us to construct counterfactuals using difference-in-difference techniques and synthetic control methods.

We show evidence of time varying decision rules that are evident as the Euro transition approaches. The frequency of price adjustment (both including and excluding items on sale) increases in the months before and after the changeover This pattern is robust to accounting for changes in the relative price of products due to potential rounding effects after the euro transition. In addition, we observe an important decline in the variance of price changes, that starts at least two years in advance of the Euro transition and that persists until the Euro becomes the sole legal tender. The interquartile range of price changes decreases leading up to the euro transition and reaches a minimum on the changeover date. Viewed through the lenses of our theoretical framework, the observed patterns on the frequency of price changes and on the distribution of price changes, allow us to discriminate across several price-setting theories, especially since the patterns are documented in the context of a large natural experiment that involved the prices of all the items of the countries studied.

We then present a simple menu cost model to model the pricing decisions as the changover approaches. Differently from canonical (steady state) model, the firm problems includes time

[1]The data is confidential and available only at the statistical agency of each country.

variation, since the price setting policy should depend on the proximity to the date at which prices should be set in euros. In particular we consider the optimal price setting problem of the firm subject to a menu cost and an additional cost (a "fine") that is borne by the firm if the euro adoption occurs too late relative to the mandated changeover date, similar to Hobijn et al. (2006). Solving this problem is involved because the time dependence implies that the value functions that encode the optimal decisionsare described by partial differential equations. We use the results by Alvarez et al. (2022) and solve the partial differential equation (pde) that characterizes the time evolution of the value functions and the cross sectional distribution of price gaps (desired adjustments) using a perturbation method.

Here is a literature review paragraph that can be adjusted. Francesco since yo know the literature way better maybe add/substract relevant papers?: Several studies have utilized micro pricing data to examine inflation related to the euro changeover. For example, Meriküll and Rõõm (2015) analyzed Estonia's experience, while Baumgartner et al. (2005) focused on Austria, and Kurri (2007) investigated Finland. Other research has leveraged disaggregated data from the harmonized indices of consumer prices provided by Eurostat. Notable examples include Hüfner and Koske (2008), which studied Slovakia, and Ercolani and Dutta (2006), which used Denmark, Sweden, and the UK—countries that did not adopt the euro—as a control group. These studies consistently found that while the overall inflationary effect of euro adoption was modest.

2 Data

We use data from five countries that experienced the euro changeover: Austria, Finland, Greece, Slovakia, and Croatia. For Austria, Finland, Greece, and Slovakia, we rely on confidential datasets gathered by their respective statistical offices, containing product-level data used to construct each country's Consumer Price Index (CPI).[2] Nonetheless, with the exception of Finland, we were able to access data outside the statistical agencies of the respective countries, enabling us to perform panel-level regressions across countries and sectors. The datasets complement one another due to the diversity of the countries' sizes, the characteristics of each dataset, and the differing approaches to the euro transition: Austria, Finland, and Greece transitioned gradually, with both the euro and the legacy currencies in circulation for an extended period to allow for adaptation by consumers and businesses (Madrid-style

[2]We inquired about the availability of CPI micro-data across all statistical agencies and central banks in countries that experienced the euro changeover. Unfortunately, this data is often either not disseminated outside the statistical systems of these countries, no longer technically accessible due to changes in automation systems, or has been destroyed (e.g., retention periods for data are typically tied to the time frame during which previously published results might need reconsideration or recalculation).

approach). Slovakia, on the other hand, implemented a more abrupt changeover (Big-bang approach), where the euro rapidly replaced the previous currency, minimizing overlap and transition time (see Appendix B for more details).

Additionally, we collected daily posted prices for Croatia, which recently experienced a euro changeover on January 1, 2023. Similar to Slovakia, Croatia adopted a relatively fast transition, with a dual circulation period lasting only two weeks, during which both the euro and the kuna were accepted. Our data collection for Croatia began six months before the changeover event and continued for two years, allowing us to comprehensively capture the transition dynamics.[3]

2.1 CPI Data: Austria, Finland, Greece, Slovakia, and the UK

Austria: We obtained the micro-data underlying the Austrian CPI from 1996 to 2004, covering the years surrounding the euro changeover, which took place in January 2002. The data is collected by Statistics Austria every month and covers approximately 90% of the total CPI (Baumgartner et al., 2005). Each observation includes the following information: product, product category (five-digit ECOICOP), date, outlet, region, price, unit, currency, and indicators for whether the price is missing or whether a product was "on sale" when its price was sampled in a particular month. We use this sales flag to calculate statistics on the frequency and size of price changes, excluding sales. The dataset also contains CPI expenditure weights for each product category. Overall, the dataset includes a total of 63,267 products across 842 product categories and 21 cities.

Finland: We use the micro-data underlying the Finnish CPI from 1997 to 2008. In Finland, the euro changeover took place in January 2002. The data can only be accessed at Statistics Finland and comprise 55% of the items in the CPI basket (Kurri, 2007). Each observation includes information on the product, price, product category (four-digit COICOP), collection method (e.g., interview), geographic area (e.g., south, east), outlet, type of outlet, and currency (one euro is equivalent to 5.94573 markka). The dataset also includes indicators for whether the price is missing, imputed, or whether the product is on sale in a particular month. The number of collected products is approximately 498 items and 2,600 outlets. The

[3]The implementation of dual pricing–displaying prices in both the national currency and the euro–also varied across countries in terms of duration and enforcement strictness. For instance, Austria, Finland, and Greece mandated dual pricing from October 1, 2001, to February 28, 2002, covering a five-month period leading up to and following the euro's introduction. Slovakia required dual pricing from August 24, 2008, to December 31, 2009, resulting in a longer duration of over a year. Croatia implemented dual pricing from September 5, 2022, to January 15, 2023, spanning approximately four months. Enforcement strictness also differed; while some countries imposed stringent regulations to ensure compliance, others were more lenient, leading to variations in how consistently dual pricing was applied across businesses.

coverage of the data varies significantly around the euro changeover. The data include about 50,000 observations per month. Three months before the euro changeover, the number of observations increased to 90,000, and from January 2002 to May 2002, it remained around 65,000 observations before dropping back to 50,000. These changes in coverage occurred for two reasons. First, Statistics Finland collected more price information to measure the CPI as accurately as possible during the euro changeover. Second, before 2012, Statistics Finland resampled products every five years, leading to a large update of products around the euro changeover. As a result, only 30% of the product-outlet pairs present in the data before the changeover continued after the introduction of the euro. Since we are interested in the pricing behavior of continuing products, we use this sample to compute the relevant pricing moments.

Greece: The CPI micro-data for Greece cover approximately 95% of the CPI items. We obtained the data from the Hellenic Statistical Authority (ELSTAT) for the years 1998–2005 (the euro changeover took place in January 2002). The dataset contains information on products, product categories (two-digit COICOP), regions (cities or towns), and outlets. However, it does not include information on product substitutions or products on sale. Before 2002, all prices were quoted in drachma (one euro is equivalent to 340.750 drachma). Starting in 2001, there was a significant increase in price collection in cities and outlets, resulting in approximately 80% more observations. In total, the dataset contains information on 726 distinct products, 26 cities, and 44 outlets. However, it does not provide data on product substitutions or sales.

Slovakia: We obtained CPI pricing data for Slovakia from the Slovak Statistical Office (SSO). The data are monthly and cover the period from 2007 to 2012 (the euro changeover took place in January 2009). Each observation includes information on the product, price, product category (five-digit COICOP), period, outlet, and district. The dataset contains prices for 754 products collected across 39 districts and 1,752 outlets. Before 2009, all prices were quoted in korunas (one euro is equivalent to 30.1260 korunas). The data also include an indicator for whether the price is missing or imputed. However, the dataset does not contain information on whether a product was on sale in a particular month.

United Kingdom: We use the CPI micro-data from the UK's Office for National Statistics (ONS). Since the UK did not experience a euro changeover, we use this dataset primarily as a reference. Although the dataset is available from 1996 to 2020, we focus on the periods surrounding the euro changeover. The dataset is publicly available and includes prices for approximately 1,000 individual goods or services each month, collected locally from more

than 14,000 retail stores across the UK. It also includes indicators for whether the price is missing, out of stock, or if a product substitution occurred. Additionally, the dataset provides information on whether a product was on sale in a particular month, product categories (two-digit COICOP), and covers about 57% of the UK CPI basket.

Table 1 summarizes our datasets. The main advantage of the Austrian data is that, unlike the Greek data, it includes indicators for sales and product substitutions. Moreover, the dataset was not subject to a large resampling phase around the euro changeover, as was the case with the Finnish data. The Slovak data complements the other datasets, as the country experienced a different changeover approach during a different time period. Only the Slovak and Finnish datasets contain price quotes in their original currencies before the changeover.

Table 1: Data Description: CPI Price

Note: The table summarizes each of the five CPI micro-data sets we use. Since the UK did not experience a changeover, the UK micro-data–which is the only dataset that is publicly available–is primarily used as a reference.

	Austria	Finland	Greece	Slovakia	UK
Dates	1996-2004	1997-2008	1998-2005	2007-2012	1996-2004
Outlets	84	2,600	44	1,752	1,336
Item-Outlet	63,267	78,158	42,264	110,369	431,752
COICOP	Class	Subclass	Division	Subclass	Subclass
Sales	✓	✓	✗	✗	✓
Subs.	✓	✓	✗	✓	✓
Imputation	✗	✗	✗	✓	✓
Currency	✗	✓	✗	✓	✓
Obs.	4,086,209	7,274,026	3,171,673	6,213,783	10,573,127
Changeover	Jan 2002	Jan 2002	Jan 2002	Jan 2009	N/A
Approach	Madrid	Madrid	Madrid	Big-bang	N/A
Exchange	13.7603 ATS	5.94573 FIM	340.75 GRD	30.126 SKK	N/A
Access	Restricted Remote	Restricted On-Site	Restricted Remote	Restricted Remote	Public Remote

Pricing Moments: We begin computing the frequency of price changes in each data set by constructing an indicator that equals one if the price for a given item×store is different than that of the previous month. We exclude product substitutions and imputations whenever

that information is available.[4]

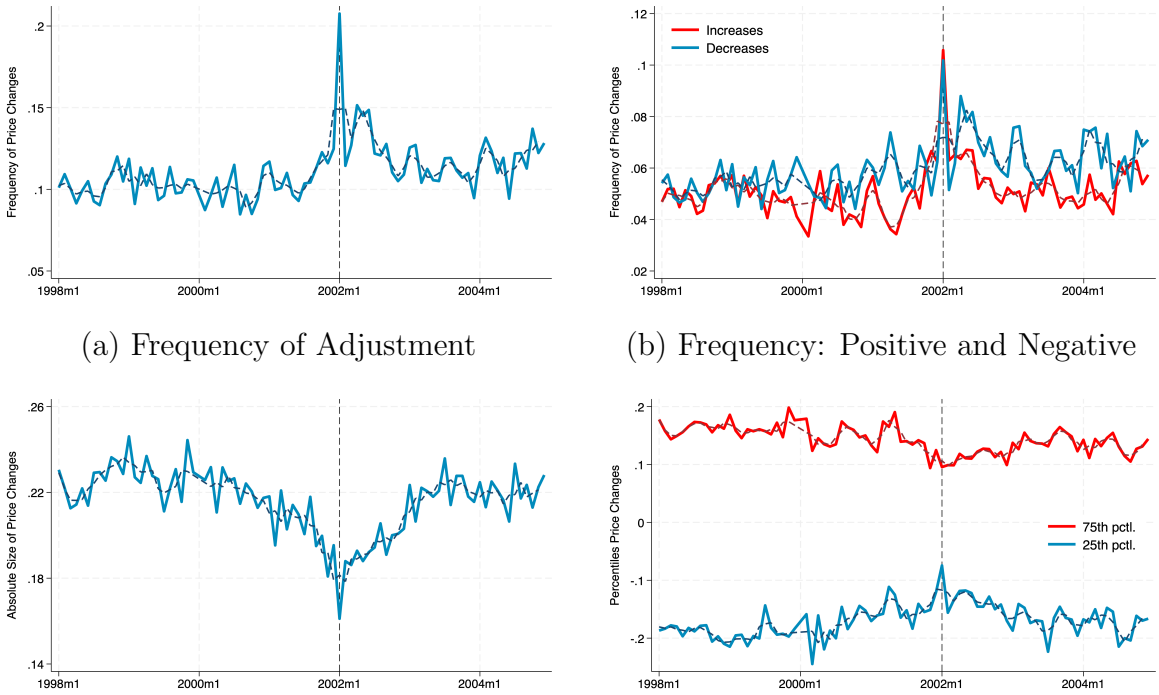
An important issue during the euro changeover was rounding, as in many countries, the minimum unit of account in the national currency was smaller than the lowest euro denomination. To address this, the European Commission issued guidelines stating that after conversion from the national currency, the euro amount could be rounded to the nearest euro cent (two decimal places). For instance, the European Central Bank (ECB) set a fixed exchange rate for Croatia at 1 euro = 7.53450 kuna. Following this rate, HRK 100 would convert to EUR 13.2722808, which would then be rounded to EUR 13.27. These guidelines ensured consistency and minimized discrepancies during the transition. Since we are interested in studying price changes beyond those caused by rounding, we compute the frequency of price changes while excluding those smaller than 0.1%. Because the Slovak data include price quotes in their original currency (Slovak koruna), we can exclude price changes attributable to rounding. Figure A4 demonstrates that excluding price changes smaller than 0.1% is a stricter criterion than merely removing those caused by rounding. Similarly, as shown later using high-frequency data for Croatia, this restriction also eliminates most price changes resulting from rounding in that country.

We begin by presenting the patterns of pricing moments for Austria. Panel (a) in Figure 1 illustrates the frequency of price changes in Austria before and after the euro changeover, controlling for seasonality. The figure highlights a significant increase in the probability of price adjustments in the period leading up to the changeover.[5] The frequency of price changes increases for both price increases and price decreases, as shown in panel (b). The distribution of relative price changes also evolves as the changeover approaches. There is a significant reduction in the cross-sectional dispersion of price changes that begins a few months before the introduction of the euro and persists for a few months after the euro becomes the sole legal tender. Panel (c) shows that the absolute size of price changes decreases. Panel (d) depicts the patterns of the first and third quartiles of the distribution of price changes, indicating that the interquartile range of price changes–the difference between the third and first quartiles–reached its minimum at the time of the changeover and recovered in subsequent periods. Figure A3 presents similar patterns observed in Slovakia, Greece, and Finland during the euro transition. For Austria, we have data on sales and product substitutions, while for Slovakia, the dataset includes information on imputations and product substitutions. Figure A1 shows that the observed pricing patterns around the changeover are not influenced by sales, substitutions, or imputations, as these factors remain largely constant throughout

[4]Panel (a) Figure A1 shows that in the Austrian case, we do not find evidence that sales or product substitutions increase around the changeover. Panel (b) shows similar evidence for product substitutions and imputations in Slovakia.

[5]Figure A2 shows that these patterns are robust for price changes larger than 1% as well.

Figure 1: Pricing Moments: Austria



(a) Frequency of Adjustment

(b) Frequency: Positive and Negative

(c) Absolute Size of Price Changes

(d) IQR

Note: The figure illustrates pricing moments for Austria before and after the euro changeover. Panel (a) shows the frequency of price changes after controlling for seasonality, with the dashed line representing the three-month moving average. Panel (b) depicts the frequency of price increases and decreases. Panel (c) reports the absolute size of price changes, while panel (d) presents the 25th and 75th percentiles of the price change distribution.

the transition period.

Table 2 summarize these patterns and shows they are similar across all countries transitioning to the euro. Column (1) in Table 2 reports the frequency of price adjustments for all countries during the months of December-January a year before the changeover and the same months a year after, i.e., December 2001 and January 2002 for Austria, Finland, Greece, and the UK (for reference), and December 2008 and January 2009 for Slovakia. The table shows that, with the exception of the UK which did not transition to the euro, the frequency of price changes is higher during the changeover. The average frequency of price changes for identical items during the changeover, which ranges from 20% per month to 40%, is substantially higher than the frequency a year before, which ranges from 15% to 30%. Columns (6) and (7) measure the dispersion of price changes, including the absolute size of log price changes and the interquartile range of price changes, computed as the difference between the 75th and 25th percentiles. Both measures are smaller during the changeover.

Table 2: Pricing Moments at Changeover

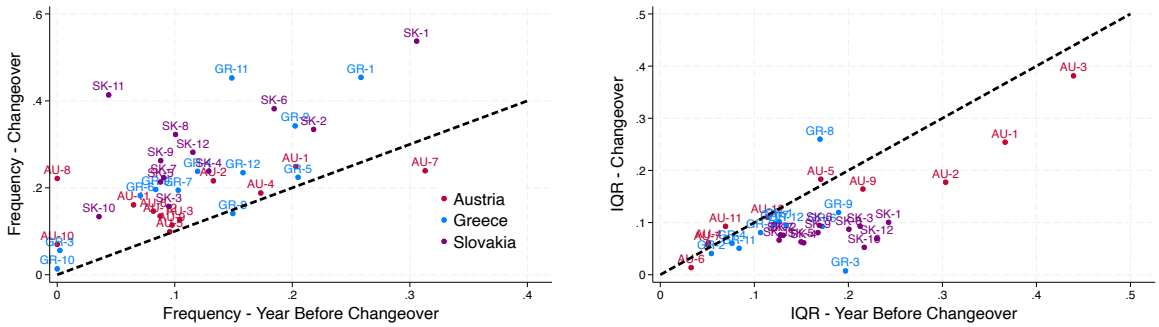
Note: The table shows pricing moments for each country before and after the changeover date. For Austria, Finland, Greece, and the UK, the table shows the moments for December-January of 2002 and the same months a year before. For Slovakia, the table shows the moments for December-January of 2009 and the same months a year before. All statistics are computed inclusive of sales, given that only the data for Austria and the UK include a sales flag. The statistics reported exclude prices changes smaller than 0.1%. Frequency is the probability of price adjustment, excluding price substitution and imputation whenever such information is available. Size positive and size negative are the sizes of price change in logarithms. IQR is the difference between price changes in the 75th percentile and those in the 25th percentile.

Country	Period	(1) Freq.	(2) Freq. Pos.	(3) Freq. Neg.	(4) Size Pos.	(5) Size Neg.	(6) Abs. Size	(7) IQR
Austria	$T-1$	0.14	0.08	0.06	0.18	-0.24	0.21	0.27
	Changeover	0.19	0.10	0.09	0.18	-0.16	0.17	0.17
Finland	$T-1$	0.24	0.13	0.10	0.12	-0.22	0.16	0.11
	Changeover	0.30	0.16	0.14	0.15	-0.17	0.16	0.06
Greece	$T-1$	0.14	0.08	0.06	0.13	-0.13	0.13	0.16
	Changeover	0.29	0.17	0.12	0.12	-0.07	0.10	0.08
Slovakia	$T-1$	0.27	0.15	0.09	0.16	-0.14	0.15	0.16
	Changeover	0.38	0.19	0.16	0.11	-0.08	0.10	0.08
UK	Dec/Jan 2001	0.17	0.07	0.10	0.16	-0.28	0.23	0.33
Placebo	Dec/Jan 2002	0.17	0.07	0.09	0.17	-0.27	0.22	0.32

Sectors: The patterns of pricing moments we uncover across countries also hold across sectors. Although the data sets for each country differ in their sectoral aggregation, all provide information at the COICOP division level, which corresponds to the 2-digit classification. Some examples include Division 01 for "Food and Non-Alcoholic Beverages," Division 03 for "Clothing and Footwear," Division 07 for "Recreation and culture," and Division 08 for "Education." Panel (a) in Figure 2 shows the frequency of price adjustments in each of the countries and sectors that experience a changeover relative to the previous year. All sectors in all countries have a higher frequency of adjustment during the changeover, with the exception of Division 07 in Austria and Division 08 in Slovakia. Similarly, Panel (b) shows that the IQR in most sectors and countries is smaller.

To systematically leverage the sectoral variation in our data, we next examine how the introduction of the euro affected pricing moments using an event-study approach. Importantly, the euro changeover was not a random event; in fact, its announcement, as emphasized in the next section, serves as a potential driver of the patterns we have uncovered regarding the pricing moments. The event-study approach is particularly useful because it allows us to control for country-sector fixed effects, thereby enabling us to illustrate what occurred both before and after the euro changeover while accounting for observed heterogeneity. As

Figure 2: Pricing Moments by Sector



(a) Frequency (b) IQR

Note: Panel (a) shows the frequency of price changes for each COICOP division level for Austria (AU), Greece (GR), and Slovakia (SK) a year before the changeover and during the changeover. Panel (b) uses the same time periods but depicts the interquartile range, the percent difference between the 25th and 75th percentiles of the price change distribution.

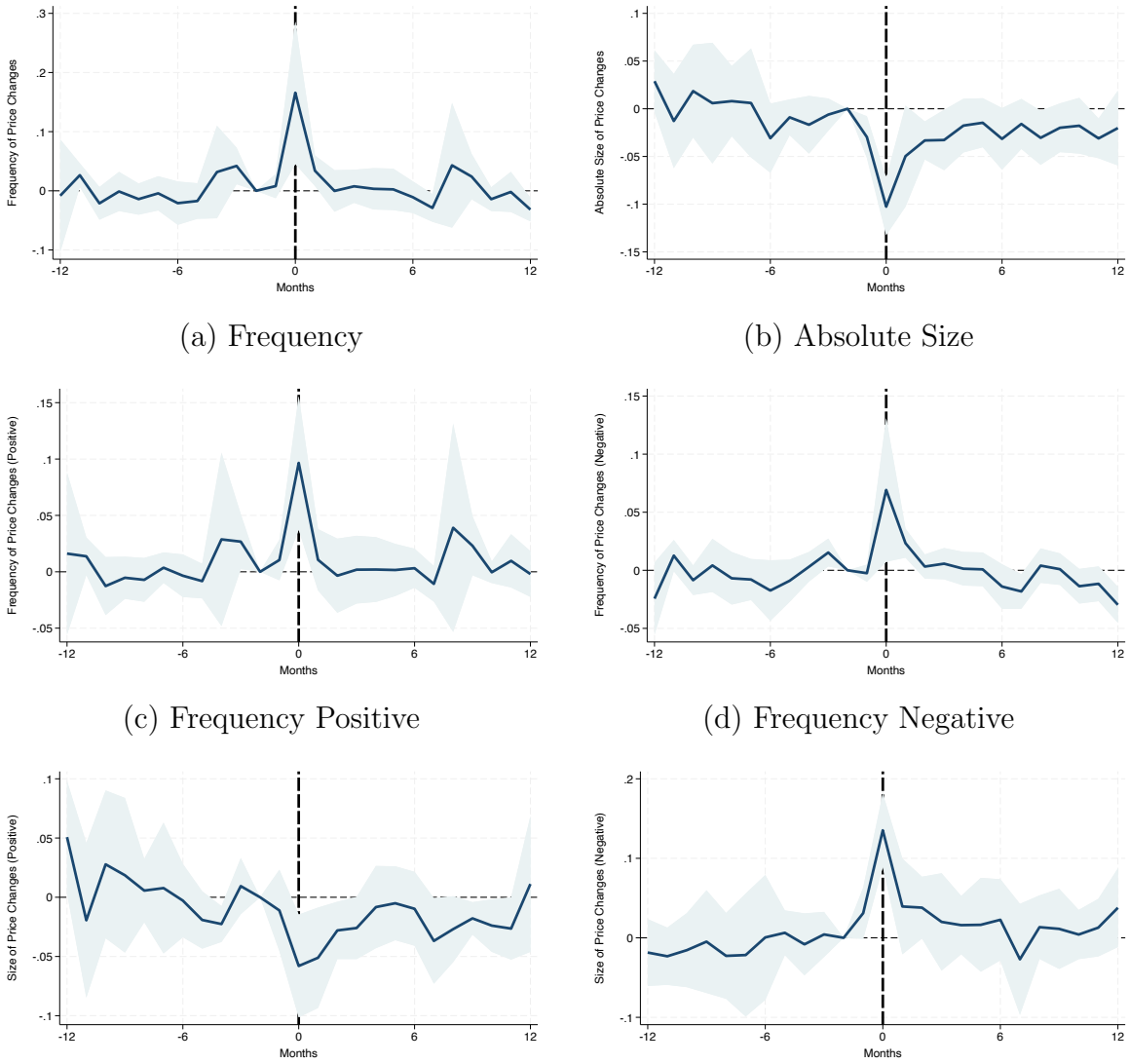
a control, we include data from sectors in the UK, a country that did not experience a changeover. For this analysis, our sample period spans two years before and three years after the euro changeover across all countries, including Slovakia, whose changeover occurred at a different time compared to the other countries. Let Y_{ijt} represent an outcome variable for sector i, country j, and time t (e.g., frequency of price changes, interquartile range, etc.). The specification for our event study is as follows:

$$Y_{ijt} = \alpha + \sum_{k=-\infty}^{\infty} \gamma_k \mathbb{1}\{K_{ijt} = k\} + \theta_{ij} + \sum_{m=2}^{12} \delta_m D_{mt} + \epsilon_{ijt} \tag{1}$$

where θ_{ij} represents sector $\times$ country fixed effects. We considered two-digit COICOP codes as the relevant sectors. K_{it} denotes the number of months relative to the introduction of the euro, such that γ_k for $k < 0$ corresponds to pre-trends and $k \geq 0$ corresponds to dynamic effects k periods after the introduction of the euro. We control for seasonality using month effects and double-cluster the standard errors at both the country and sector levels.

Figure 3 shows our outcome variables before and after the introduction of the euro. Panel (a) shows the frequency of price adjustments, which does not exhibit strong pre-trends or significant changes immediately following the euro changeover. However, after excluding small adjustments, there is a notable increase in the frequency of adjustments during the changeover period. Panels (c) and (d) show that this increase is driven by both price increases and decreases. Panel (b) shows that, conditional on sector $\times$ country effects,

Figure 3: Event Study: Pricing Moments



(a) Frequency

(b) Absolute Size

(c) Frequency Positive

(d) Frequency Negative

(e) Size Positive

(f) Size Negative

Note: The graph shows the evolution of several pricing moments before and after the introduction of the euro. The panels plot the coefficients of γ_k after estimating equation (1). The vertical dashed line indicates the month when the euro was introduced. Panel (a) depicts the frequency of price adjustments. Panel (b) shows the absolute size of price changes. Panels (c) and (d) present the frequency of price increases and decreases, respectively. Panels (e) and (f) display the size of price increases and decreases, respectively. The gray area represents the 95% confidence interval, calculated by clustering the standard errors at the country and sector levels.

there is a significant decrease in the absolute size of price changes at the time of the euro's implementation. Panels (e) and (f) further indicate that this decrease is driven by the smaller sizes of both price increases and decreases around the euro changeover.

2.2 Scraped Data: Croatia

Next, we study the most recent euro transition, which took place in Croatia on January 1, 2023. We collected daily posted prices from Croatia before and after the implementation of the euro as the sole legal tender. Using scraped pricing data offers several advantages compared to CPI pricing data. First, it allows for the calculation of pricing moments for the same barcode across time. Second, it is available at a high frequency, enabling us to capture the exact timing of the changeover. Lastly, scraped price tags provide precise information on how prices in euros and kunas were displayed before and after the dual pricing period.

We collect daily posted prices from Croatia for 125,108 products (item-outlet) sold by 9 major retailers. This information was gathered using web-scraping methods and includes product details such as IDs, prices, and categories. The categories cover a wide range, including recreation and culture, health, food and non-alcoholic beverages, and furnishing, household equipment, and routine maintenance. A pilot phase ran from January 2022 to May 2022, with the final data spanning from May 2022 to July 2024. Importantly, the web-scraped prices closely align with those found in the physical stores of the same retailers, ensuring the accuracy and reliability of the data used in the analysis.[6]

In Croatia, a period of compulsory dual pricing began in September 2022 to help consumers understand the new pricing system and verify the accuracy of conversions. Retailers were required to display the exchange rate clearly and visibly alongside prices. From that date onward, all retailers displayed prices in both kunas and euros. Most gradually transitioned to prioritizing the euro, initially giving prominence to the kuna price before making the euro price more prominent after the changeover.[7]

Furthermore, the conversion of prices from Croatian kuna (HRK) to euro (EUR) was governed by a fixed exchange rate set by the European Central Bank (ECB), which was 1 euro = 7.53450 kuna. After applying this fixed rate, prices were generally rounded to the

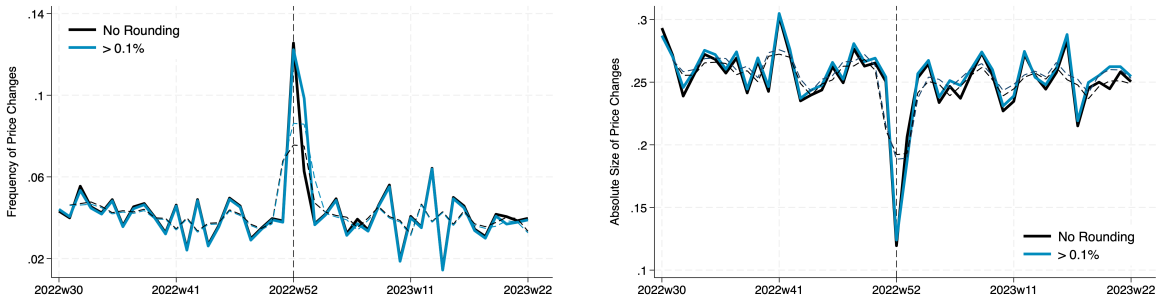
[6] To ensure the accuracy of our web-scraped pricing data, we conducted a thorough comparison with offline data. We collected offline data in two phases: one in December 2022 and another in January 2023. This task was carried out by freelancers hired through the Upwork platform, who were instructed to visit physical stores and collect data on thirty items using the Mobile Inventory PRO free app. Each freelancer received $20 for each store they visited. Immediately after receiving the data from the freelancers, we searched for the same products online to compare the prices between the physical stores and the online listings. This immediate comparison minimized the likelihood of price changes affecting the results. We find that prices in the offline stores closely aligned with those found online. Both before and after the euro changeover, offline stores displayed prices in both euros and kunas, similar to the online listings. However, offline stores were more likely to display the two prices in the same size, while online, the euro price was typically larger after the changeover. The majority of products from offline stores were available online, with an average match rate higher than 75%, and remained consistent before and after the changeover. Importantly, when a product was available in both places, the prices were almost always identical.

[7] An example is shown in Figure A5, which features an online listing from Decathlon in Croatia. The figure displays the same item listed on December 30, 2022, and January 2, 2023, after the euro transition, highlighting a price change that occurred after the changeover.

nearest cent (two decimal places). For instance, an amount of HRK 15.00 would convert to EUR 1.99156 and then be rounded to EUR 1.99. Given that we observe prices at a high frequency, we distinguish between changes in prices due to rounding and those due to real adjustments during the changeover period.

Panel (a) in Figure 4 shows the frequency of price adjustments in Croatia, excluding changes due to rounding and those smaller than 0.1%. As discussed earlier, this threshold eliminates most price changes caused by rounding. The figure highlights that redenomination largely occurred close to the euro changeover date. In the Croatian case, stores redenominated the prices of most of their products simultaneously, indicating a high level of synchronization. Importantly, the figure shows an increase in the frequency of price changes around the changeover, suggesting that relative price adjustments also took place during Croatia's euro transition. Similar to the other countries we study, Panel (b) shows that Croatia also experienced a decrease in the absolute size of price changes.[8]

Figure 4: Pricing Moments: Croatia



(a) Frequency (b) Absolute Size

Note: The figure shows pricing movements for Croatia before and after the euro changeover. Panel (a) presents the frequency of price changes, excluding those due to rounding or smaller than 0.1%. Panel (b) displays the absolute size of price changes using the same definition.

2.3 Discussion

Three important remarks are in order. First, the evidence presented in this section indicates that the changeover is not neutral with respect to pricing moments. Firms take advantage of this period to adjust their relative prices, and the presence of real price changes suggests that frictions exist in the price adjustment processes of firms.

[8]The composition of price changes during the changeover was strongly influenced by Croatian regulations: any price increase tied to the changeover, including during the dual pricing period, was explicitly forbidden. As shown in Figure A6, a decomposition of the frequency of adjustments reveals a much lower frequency of price increases around the euro changeover.

Second, the increase in pricing activity around the changeover across all countries is surprising, particularly given the observed decrease in the cross-sectional dispersion of price changes. This is unexpected because one might anticipate fewer price changes (i.e., larger inaction bands) leading up to the euro transition if firms were mandated to adjust their prices by the changeover date. In other words, if firms are required to adjust their prices at a specific time, they are unlikely to make changes in the preceding periods, regardless of whether price adjustment frictions exist. We formalize this insight in the next section.

Third, the start of the dual pricing period does not appear to impact pricing moments in any country. As previously noted, the enforcement and implementation of dual pricing during the euro transition varied across countries. While most required businesses to display prices in both the national currency and the euro, enforcement was sometimes lenient, focusing on encouraging compliance rather than imposing strict penalties. This occasionally led to instances of non-compliance. Even when prices for each product were required to be displayed in both currencies, regulations often explicitly prohibited real price changes during the dual pricing period. For example, in Croatia, price increases during the dual pricing implementation or the euro changeover were strictly forbidden under consumer protection measures. As a result, there was no unusual pricing activity during this period, as the regulatory framework effectively prevented substantial price adjustments.

In the next section, we develop an optimal price-setting model with time-varying decision rules that is consistent with the empirical evidence presented in this section.

3 A tractable model of price setting with a changeover

Let t denote calendar time and T the date of the euro changeover, by which time each firm must set price tags in euros. For convenience, we will label as kronas the national currency before the changeover, which will merge into the euro at time T.

A baseline price setting model. We consider a menu cost model where price changes are subject to a fixed menu cost ψ. We model the firm price setting decision using a standard menu cost model where the firm has a quadratic cost. Let x denote the firm's desired markup adjustment, following the law of motion $dx = \sigma dW$. The firm value function $\tilde{u}(x)$ solves

$$\rho\, \tilde{u}(x) = Bx^2 + \tilde{u}_{xx}(x)\frac{\sigma^2}{2} \quad \text{for} \quad \underline{x} < x < \bar{x}$$

with boundary condition $\tilde{u}(\underline{x}) = \tilde{u}(\bar{x}) = \psi + \tilde{u}(0)$, and smooth pasting conditions $\tilde{u}_x(0) = \tilde{u}_x(\bar{x}) = \tilde{u}_x(\underline{x}) = 0$. It is easily seen that the function $\tilde{u}(x)$ is symmetric and has a minimum

at $x = 0$ (when the ideal markup is used) and that $\underline{x} = -\bar{x}$. The optimally chosen threshold $\bar{x}$ denotes the maximum deviation from the ideal markup, such that if $|x| < \bar{x}$ the firm is inactive while a price adjustment is implemented if $|x| \geq \bar{x}$. Notice that in this problem the policy rule, namely the optimal threshold $\bar{x}$, is constant in time.[9]

The euro changeover. Let $t = 0$ denote the data in which the changeover is announced. We assume that if a price change is made at any $t < T$ the firm pays the menu cost ψ and is also able to renominate prices from kronas into euro (at no additional cost).[10] A firm setting prices in krona after time T is subject to a once and for all fine (a fixed cost) ϕ.

Let $u(x,t)$ be the value function of the firm setting prices in kronas. The value function $u(x,t)$ now also features a time argument since the problem depends on the distance from the date T, when a price renomination is mandatory (or else a fine is paid). We have

$$\rho\, u(x,t) = Bx^2 + u_t(x,t) + u_{xx}(x,t)\frac{\sigma^2}{2} \quad \text{for } \underline{x}(t) < x < \bar{x}(t) \text{ and } t < T \tag{2}$$

The value function must satisfy value matching condition (a continuity requirement)

$$u(\underline{x}(t),t) = u(\bar{x}(t),t) = \psi + \tilde{u}(0) \quad \text{for } t < T \tag{3}$$

Note that upon the first price adjustment the firm adopts the euro and the continuation value is given by the steady state value function, evaluated at the optimal after-rest gap $x^* = 0$. The value function $u(x,t)$ must satisfy the smooth pasting condition (optimal thresholds)

$$u_x(\underline{x}(t),t) = u_x(\bar{x}(t),t) = 0 \quad \text{for } t < T \tag{4}$$

Finally the value function has a boundary condition at $t = T$ given by

We adjusted the terminal condition, see slides

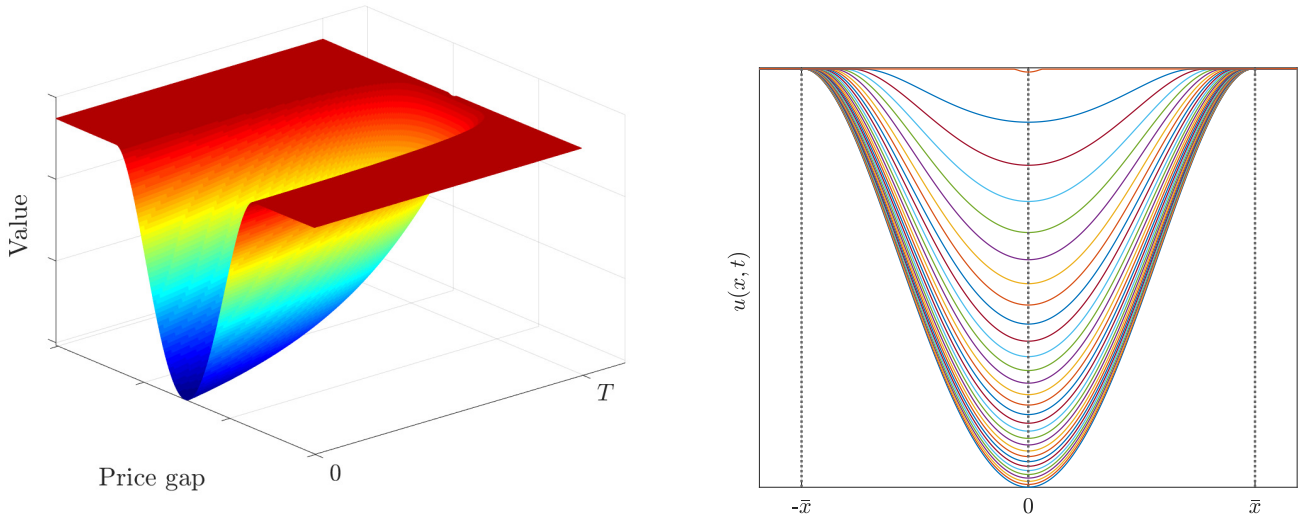
$$u(x,T) = \phi + \tilde{u}(x) \quad \text{for all } x \tag{5}$$

This means that firms which have not yet adopted the euro prices at T, will have to pay a (once and for all) fine, ϕ, and continue with the steady state problem maintaining the gap at x, with value $\tilde{u}(x)$.

[9] Given $\bar{x}$ the steady state value function solving the ode is $\tilde{u}(x) = \frac{2B}{\rho^2} + \frac{B}{\rho}x^2 + C\left(e^{\xi x} + e^{-\xi x}\right)$ where $\xi \equiv \sqrt{\frac{2\rho}{\sigma^2}}$ and smooth pasting at $\bar{x}$ implies $C \equiv \frac{-2B\bar{x}}{\rho\xi(e^{\xi\bar{x}} - e^{-\xi\bar{x}})}$. The second derivative $\tilde{u}_{xx}(\bar{x}) = -\frac{2B}{\rho}\left(\bar{x}\xi\, \coth(\bar{x}\xi) - 1\right) < 0$.

[10] An alternative model might separate the price-change decisions and the curency renomination decision, each with its own cost

Figure 5: Value Function $t \leq T$



Note:

Alternative terminal condition. We could alternatively assume that the firms *must* adopt the euro *and* reset their price gap at T, in which case the terminal condition would read $u(x,T) = \phi + \psi + \tilde{u}(0)$. This alternative assumption does not affect the dynamics of the problem for $t < T$, but does affect the prediction for the price changes observed at $t = T$. We return to this discussion later.

Linearization Consider the value function $u(x,t;\phi)$, where the dependence on ϕ is explicitly emphasized. Clearly $u(x,t;0) = \tilde{u}(x)$, since absent the fine ϕ the horizon T is irrelevant for the firm's policy. Consider the following

$$v(x,t) \equiv \frac{\partial u(x,t;\phi)}{\partial \phi}\Big|_{\phi=0} \quad \text{for all } x,t \tag{6}$$

The function $v(x,t)$ is informative about how the value function changes after a small perturbation of the original problem. It has the great advantage that it can be evaluated in closed form around $\phi = 0$, so that an analytic solution can be obtained. Likewise, let us define the derivative of the optimal policy threshold

$$\underline{z}(t) \equiv \frac{\partial \underline{x}(t;\phi)}{\partial \phi}\Big|_{\phi=0} \quad , \quad \bar{z}(t) \equiv \frac{\partial \bar{x}(t;\phi)}{\partial \phi}\Big|_{\phi=0} \quad \text{for all } t < T \tag{7}$$

Differentiating equation (2) with respect to ϕ and evaluating it at $\phi = 0$ gives

$$\rho\, v(x,t) = v_t(x,t) + v_{xx}(x,t)\frac{\sigma^2}{2} \quad \text{for } \underline{x} < x < \bar{x} \text{ and } t < T \tag{8}$$

This PDE for $v(x,t)$ holds on the time-invariant domain $\underline{x} < x < \bar{x}$. Such invariance provides a consistent simplification of the original problem where the thresholds of the inaction region, $\underline{x}(t), \bar{x}(t)$ depend on time.

Differentiating the value matching condition equation (3) with respect to ϕ gives $u_x(\underline{x}(t),t)\underline{z}(t) + v(\underline{x}(t),t) = 0$. An equivalent expression holds at the upper boundary. Evaluating the expressions at $\phi = 0$ and noting that $u_x(\underline{x}(t),t) = 0$ gives the boundary conditions

$$v(\underline{x},t) = 0 \quad \text{and} \quad v(\bar{x},t) = 0 \quad \text{for all } t < T \tag{9}$$

We stress that these boundary conditions hold on the time invariant domain $\underline{x} < x < \bar{x}$.

Equivalently, differentiating the smooth pasting condition equation (4) with respect to ϕ gives $v_x(\underline{x}(t),t) + u_{xx}(\underline{x}(t),t)z(t) = 0$. An equivalent expression holds at the upper boundary. Evaluating the expressions at $\phi = 0$ gives the smooth pasting conditions

$$v_x(\underline{x},t) + \tilde{u}_{xx}(\underline{x})\,\underline{z}(t) = 0 \quad \text{and} \quad v_x(\bar{x},t) + \tilde{u}_{xx}(\bar{x})\,\bar{z}(t) = 0 \quad \text{for all } t < T \tag{10}$$

which are informative on the time evolution of the policy rules, i.e. the thresholds $\underline{x}(t), \bar{x}(t)$ in the original problem as indicated in equation (7). The steady state value function $\tilde{u}(x)$ is symmetric and analysis of the steady state problems gives $\tilde{u}_{xx}(\bar{x}) = \tilde{u}_{xx}(-\bar{x})$ where the analytic expression is readily computed (see footnote 9). Finally differentiating the boundary condition at $t = T$ in equation (5) gives

$$v(x,T) = 1 \tag{11}$$

We state the following Lemma (proof in appendix).

LEMMA 1. The function $v(x,t)$ that solves equation (8), equation (9), and equation (11) satisfies $v(x,t) = v(-x,t)$ for $\underline{x} < x < \bar{x}$ and all t, i.e. $v(x,t)$ is symmetric in x. By equation (10) it then follows that $\bar{z}(t) = -\underline{z}(t)$.

Proof. (of Lemma 1). Let $\hat{v}(x,t) \equiv v(x,t) - v(-x,t)$. Note that if $v(x,t)$ solves equation (8), equation (9), and equation (11) then, by the linearity of the equations, $\hat{v}(x,t)$ solves: (i) $\rho\,\hat{v}(x,t) = \hat{v}_t(x,t) + \hat{v}_{xx}(x,t)\frac{\sigma^2}{2}$, (ii) $\hat{v}(\bar{x},t) = 0$, and (iii) $\hat{v}(x,T) = 0$. Note that $\hat{v}(x,t) = 0$ solves equations (i)-(iii). By the uniqueness of the solution to the heat equation in (i) then

this is the only solution and thus $v(x,t) = v(-x,t)$ for $\underline{x} < x < \bar{x}$ and all t.

Solving for the value function. Next we give a closed form solution for $v(x,t)$. For $j = 1, 2, 3....$ consider the eigenfunctions $\varphi_j(x) = \frac{\cos(\frac{j\pi}{2\bar{x}}x)}{\sqrt{\bar{x}}}$ and the eigenvalues $\lambda_j = -\rho - \frac{(j\pi)^2}{8}N$ where $N = \frac{\sigma^2}{\bar{x}^2}$ is the frequency of price changes per period. Note that the eigenfunctions are orthonormal as the inner product $\langle \varphi_i, \varphi_j \rangle = 0$ for $i \neq j$ and $\langle \varphi_j, \varphi_j \rangle = 1$.[11]

These eigenvalue eigenfunction pairs solve the pde in equation (8) and the boundary condition equation (9). We use the projection of the terminal condition $v(x,T) = 1$ in equation (11) on the orthonormal basis to get the closed form solution

$$v(x,t) = \sum_{j=1}^{\infty} \langle \varphi_j, 1 \rangle \ \varphi_j(x) e^{\lambda_j (T-t)} \quad \text{for} \quad x \in (-\bar{x}, \bar{x}) \tag{12}$$

where we note that $\langle \varphi_j, 1 \rangle = \frac{4\sqrt{\bar{x}}}{j\pi} \sin\left(\frac{j\pi}{2}\right)$ and $\varphi_j(x) = \frac{\cos\left(\frac{j\pi}{2\bar{x}}x\right)}{\sqrt{\bar{x}}}$. This gives

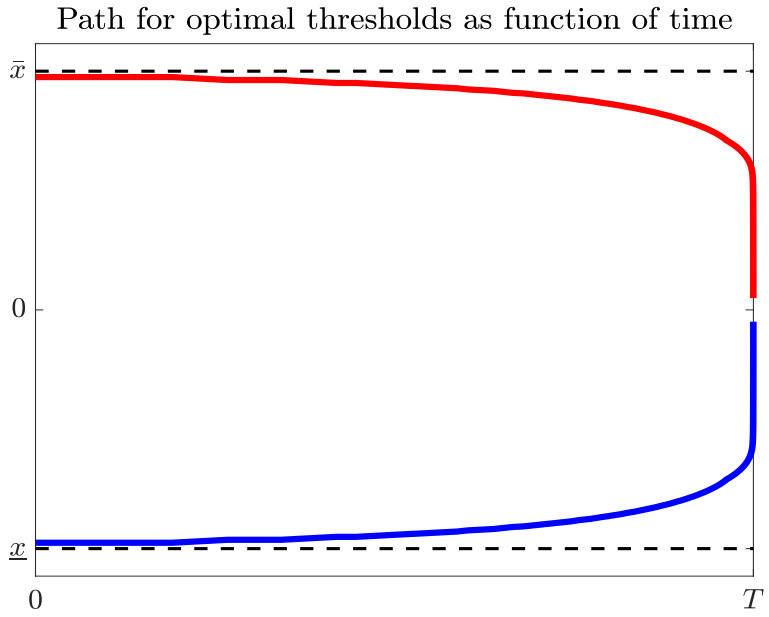
$$v(x,t) = \sum_{j=1}^{\infty} \frac{4 \sin\left(\frac{j\pi}{2}\right) \cos\left(\frac{j\pi}{2\bar{x}}x\right)}{j\pi} e^{\lambda_j (T-t)} \quad \text{for} \quad x \in (-\bar{x}, \bar{x}) \tag{13}$$

Computing the derivative $v_x(x,t) = -\sum_{j=1}^{\infty} \frac{2}{\bar{x}} \sin\left(\frac{j\pi}{2}\right) \ \sin\left(\frac{j\pi}{2\bar{x}}x\right) e^{\lambda_j (T-t)}$. Using equation (10) we write

$$\bar{z}(t) = -\frac{v_x(x,t)}{\tilde{u}_{xx}(\bar{x})} = 2\frac{\sum_{j=1}^{\infty} \left(\sin\left(\frac{j\pi}{2}\right)\right)^2 e^{\lambda_j (T-t)}}{\bar{x} \ \tilde{u}_{xx}(\bar{x})} = 2\frac{\sum_{j=1,3,5,...} e^{\lambda_j (T-t)}}{\bar{x} \ \tilde{u}_{xx}(\bar{x})} \ . \tag{14}$$

[11] Simple analysis gives $\bar{x} = \int_{-\bar{x}}^{\bar{x}} \left(\cos(\frac{j\pi}{2\bar{x}}x)\right)^2 dx$.

Figure 6: Dynamics of decision rules before changeover



Here is where what we had ends. What follows are the properties of the problem at T and the description of the KFEs (fromo notes and the slides)

Properties of the problem at T

Recall value matching and the terminal condition

$$\begin{aligned} u(\bar{x}(t), t) &= \psi + \tilde{u}(0) \qquad \text{for: } t < T \\ u(x, T) &= \phi + \tilde{u}(x) \qquad \text{for all } x \end{aligned}$$

taking the limit of value matching from $t \uparrow T$ we obtain

$$u(\bar{x}(T), T) = \psi + \tilde{u}(0)$$

and evaluating the terminal condition at $x = \bar{x}(T)$

$$u(\bar{x}(T), T) = \phi + \tilde{u}(\bar{x}(T))$$

combining these two equations

$$\tilde{u}(\bar{x}(T)) = \psi - \phi + \tilde{u}(0)$$

since $\tilde{u}(\cdot)$ is a known function we can solve for $\bar{x}(T)$.

Derivative of $\bar{x}(T,\phi)$ with respect to ϕ is infinity: Approximating $\bar{x}(T,\phi)$ for a small ϕ, we first take the derivative with respect to ϕ:

$$\tilde{u}_x(\bar{x}(T,\phi))\bar{x}_\phi(T,\phi) = -1$$

we know that $\bar{x}(T,0) = \bar{x}$ and $\tilde{u}_x(\bar{x}(T,0)) = 0$ from smooth pasting. Then evaluating the derivative at zero:

$$\tilde{u}_x(\bar{x}(T,0))\bar{x}_\phi(T,0) = \tilde{u}_x(\bar{x})\bar{z}(T) = -1$$

hence $\bar{z}(T) = \infty$.

$\bar{x}(T,\phi)$ is proportional to $\sqrt{\phi}$: Let $\phi = s^2$, then

$$\tilde{u}(\bar{x}(T,s)) = \psi - s^2 + \tilde{u}(0)$$

we approximate this function with a taylor expansion around $s = 0$

$$\tilde{u}(\bar{x}(T,s)) = \tilde{u}(\bar{x}) + \tilde{u}_x(\bar{x})(\bar{x}(T,s) - \bar{x}) + \frac{1}{2}\tilde{u}_{xx}(\bar{x})(\bar{x}(T,s) - \bar{x})^2 + o((\bar{x}(T,s) - \bar{x})^2)$$

using $\bar{x}(T,0) = \bar{x}$. Now using smooth pasting $\tilde{u}_x(\bar{x}) = 0$

$$\tilde{u}(\bar{x}(T,s)) = \tilde{u}(\bar{x}) + \frac{1}{2}\tilde{u}_{xx}(\bar{x})(\bar{x}(T,s) - \bar{x})^2 + o((\bar{x}(T,s) - \bar{x})^2)$$

and using value matching

$$\tilde{u}(\bar{x}(T,s)) = \psi + \tilde{u}(0) + \frac{1}{2}\tilde{u}_{xx}(\bar{x})(\bar{x}(T,s) - \bar{x})^2 + o((\bar{x}(T,s) - \bar{x})^2)$$

using $\tilde{u}(\bar{x}(T,s)) = \psi - s^2 + \tilde{u}(0)$ and rearranging, we get

$$\bar{x}(T,s) \approx \bar{x} \pm \left[\frac{s^2}{-\tilde{u}_{xx}(\bar{x})/2}\right]^{1/2}$$

or

$$\bar{x}(T,\phi) \approx \bar{x} \pm \frac{\sqrt{\phi}}{[-\tilde{u}_{xx}(\bar{x})/2]^{1/2}}$$

Elasticity of $\bar{z}(t)$ Recall that

$$\bar{z}(t) = 2\frac{\sum_{j=1,3,5,\ldots} e^{\lambda_j(T-t)}}{\bar{x}\ \tilde{u}_{xx}(\bar{x})}$$

taking the derivative with respect to t

$$\bar{z}'(t) = 2\frac{\sum_{j=1,3,5,\ldots} -\lambda_j e^{\lambda_j(T-t)}}{\bar{x}\ \tilde{u}_{xx}(\bar{x})}$$

evaluating it at $t = T$

$$\bar{z}'(T) = 2\frac{\sum_{j=1,3,5,\ldots} -\lambda_j}{\bar{x}\ \tilde{u}_{xx}(\bar{x})} \to -\infty$$

The semi-elasticity is

$$-\frac{\bar{z}'(t)}{\bar{z}(t)} = \sum_{j=1,3,5,\ldots} \lambda_j \frac{e^{\lambda_j(T-t)}}{\sum_{i=1,3,5,\ldots} e^{\lambda_i(T-t)}} \to -\infty$$

and as t goes to T

$$-\frac{\bar{z}'(T)}{\bar{z}(T)} \to -\infty$$

Further deriving $\bar{z}(t)$ as $\rho \to 0$

$$\bar{z}(t) = -\frac{v_x(x,t)}{\tilde{u}_{xx}(\bar{x})} = 2\frac{\sum_{j=1,3,5,\ldots} e^{\lambda_j(T-t)}}{\bar{x}\ \tilde{u}_{xx}(\bar{x})}$$

where $\lambda_j = -\rho - \frac{(j\pi)^2}{8}N$. We know that $\tilde{u}_{xx}(\bar{x}) = -\frac{2B}{\rho}\left(\bar{x}\xi\ \coth(\bar{x}\xi) - 1\right)$ and, as $\rho \to 0$, $\tilde{u}_{xx}(\bar{x}) \approx \frac{-4B\bar{x}^2}{3\sigma^2} = \frac{-4B}{3N}$

$$\bar{z}(t) = -\frac{v_x(x,t)}{\tilde{u}_{xx}(\bar{x})} = 3N\frac{\sum_{j=1,3,5,\ldots} e^{\frac{-N(j\pi)^2}{8}(T-t)}}{-2B\bar{x}}$$

then

$$\frac{\bar{z}(t)}{\bar{x}} = 3N \frac{\sum_{j=1,3,5,\ldots} e^{\frac{-N(j\pi)^2}{8}(T-t)}}{-2B\bar{x}^2}$$

using $N = \frac{\sigma^2}{\bar{x}^2}$ to substitute for $\bar{x}^2$

$$\frac{\bar{z}(t)}{\bar{x}} = 3N^2 \frac{\sum_{j=1,3,5,\ldots} e^{\frac{-N(j\pi)^2}{8}(T-t)}}{-2B\sigma^2}$$

Recall also that

$$\bar{x}(T,\phi) \approx \bar{x} \pm \frac{\sqrt{\phi}}{[-\tilde{u}_{xx}(\bar{x})/2]^{1/2}}$$

using $\tilde{u}_{xx}(\bar{x}) \approx \frac{-4B}{3N}$

$$\bar{x}(T,\phi) \approx \bar{x} \pm \left(\frac{3N\phi}{2B}\right)^{\frac{1}{2}}$$

If needed in the presentation we have a couple of figures showing how $\bar{x}(T,\phi)$ changes as a function of N

4 Time Varying Density of Price Gaps

Distribution of price gaps for all $0 \leq t \leq T$

- Time-varying density of price gaps denominated in kuna

$$\begin{aligned}
k_t(x,t) &= k_{xx}(x,t)\frac{\sigma^2}{2} \text{ for all } x \in [\underline{x}(t), \bar{x}(t)] && \text{KFE} \\
k(\underline{x}(t),t) &= k(\bar{x}(t),t) = 0 && \text{Exit} \\
k_x(0,t) &= 0 && \text{Symmetry/Differentiability} \\
k(x,0) &= \tilde{k}(x) \text{ for all } x \in [\underline{x}(0), \bar{x}(0)] && \text{Initial Condition}
\end{aligned}$$

- Time-varying density of price gaps denominated in euro

$$e_t(x,t) = e_{xx}(x,t)\frac{\sigma^2}{2} \qquad \text{for } x \neq 0, x \in [\underline{x}, \bar{x}] \qquad \text{KFE}$$

$$e(\underline{x},t) = e(\bar{x},t) = 0 \qquad \text{Exit}$$

$$\int_{\underline{x}(t)}^{\bar{x}(t)} k(x,t)dx = 1 - \int_{\underline{x}}^{\bar{x}} e(x,t)dx \qquad \text{Mass Preservation}$$

$$\boxed{-\frac{\sigma^2}{2}[k_x(\bar{x}(t),t) - k_x(\underline{x}(t),t)] = \frac{\sigma^2}{2}[e_x(0^-,t) - e_x(\underline{x},t) + e_x(\bar{x},t) - e_x(0^+,t)]}$$

By symmetry: $\boxed{-\sigma^2 k_x(\bar{x}(t),t) + \sigma^2[e_x(\bar{x},t) - e_x(0^+,t)] = 0}$

where the frequency of redenomination is $K(t) = -\sigma^2 k_x(\bar{x}(t),t)$ and the frequency of price change at $t < T$ is $N(t) = -\sigma^2[k_x(\bar{x}(t),t) + e_x(\bar{x},t)]$.

Distribution of price gap at $t > T$

- Time-varying density of price gaps denominated in <u>kuna</u>

$$k_t(x,t) = k_{xx}(x,t)\frac{\sigma^2}{2} \text{ for all } x \in [\underline{x}, \bar{x}] \qquad \text{KFE}$$

$$k(\underline{x},t) = k(\bar{x},t) = 0 \qquad \text{Exit}$$

$$k_x(0,t) = 0 \qquad \text{Symmetry/Differentiability}$$

$$k(x,T^+) = k(x,T^-) \text{ for all } x \in [\underline{x}(T^-), \bar{x}(T^-)] \qquad \text{Initial Condition}$$

- Time varying density of price gaps denominated in <u>euro</u>

$$e_t(x,t) = e_{xx}(x,t)\frac{\sigma^2}{2} \text{ for all } x \in [\underline{x}, 0) \cup (0, \bar{x}] \qquad \text{KFE}$$

$$e(\underline{x},t) = e(\bar{x},t) = 0 \qquad \text{Exit}$$

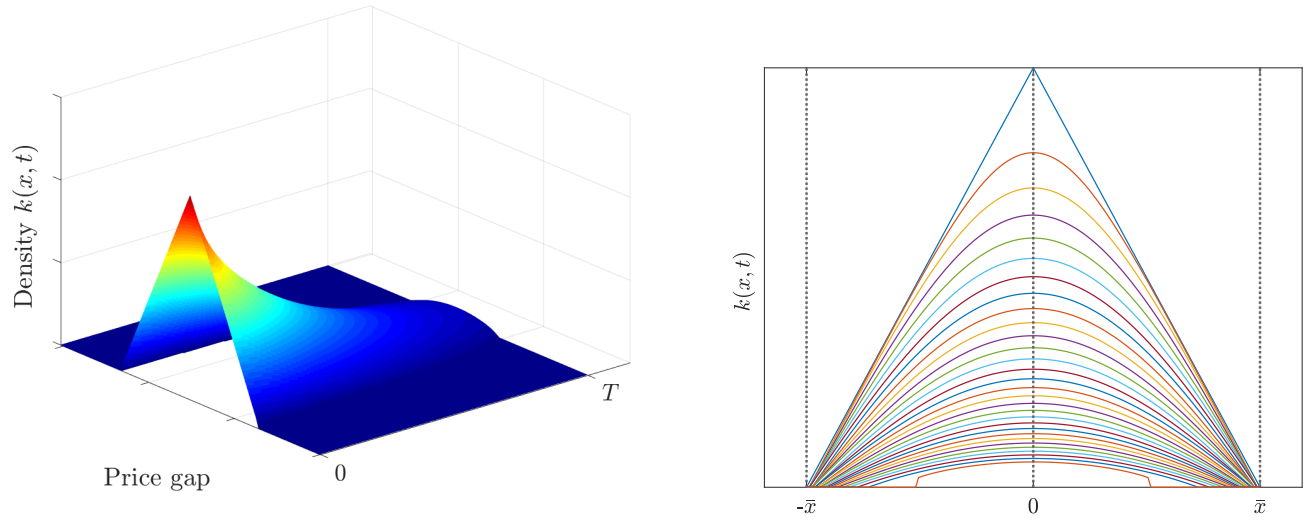
$$1 = \int_{\underline{x}}^{\bar{x}} [e(x,t) + k(x,t)]dx \qquad \text{Mass Preservation}$$

$$e(x,T^+) = e(x,T^-) \text{ for } x \in [\underline{x}, \bar{x}] \qquad \text{Initial Condition}$$

- Converges to a symmetric linear tent map: $\tilde{e}(x) = \frac{\bar{x}-|x|}{\bar{x}^2}$

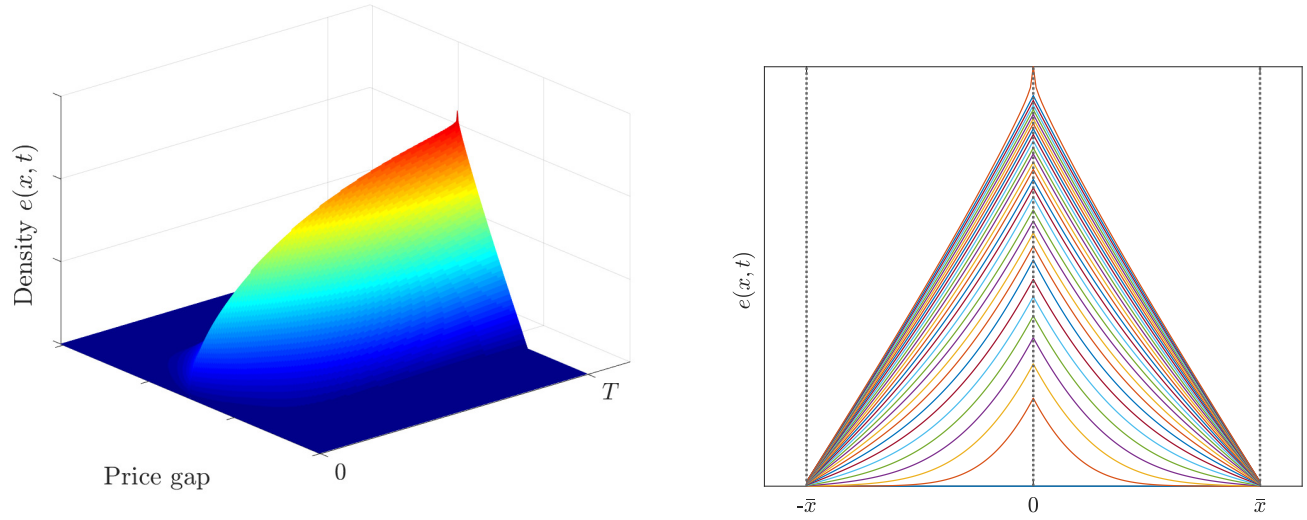
where the frequency of price adjustment is at $t > T$ is $N(t) = -\sigma^2[e_x(\bar{x},t)]$.

Figure 7: Distribution $k(x,t)$



Note:

Figure 8: Distribution $e(x,t)$



Note:

Figure 9: Redenomination

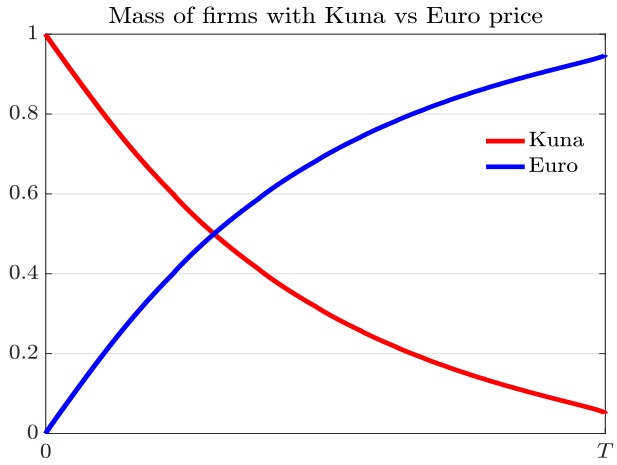
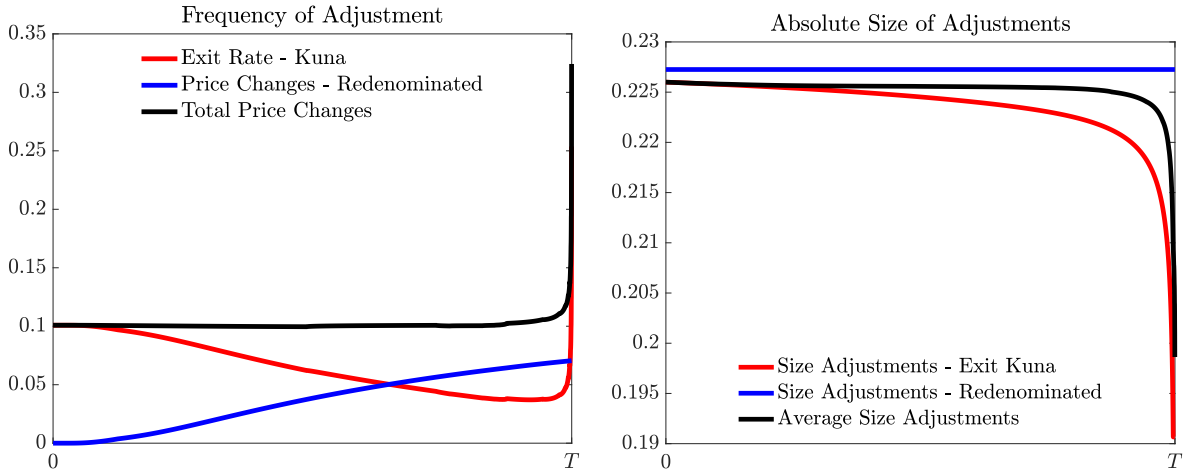


Figure 10: Optimal Pricing Before Changeover: $t < T$



Note:

5 Flow Penalty Model

The problem of a firm that must decide when to switch form the local currency price, say Kuna, to the new currency price, say the Euro, which will become the effective new currency at time T.

Steady state (or Late) euro ($t > T$): for $x \in (-\bar{x}, \bar{x})$ and the Bellman equation is

$$\rho \tilde{u}^L(x) = Bx^2 + \tilde{u}^L_{xx}(x)\frac{\sigma^2}{2}$$

with boundaries $\tilde{u}^L(\bar{x}) = \tilde{u}^L(0) + \psi$ (value matching) and $\tilde{u}^L_x(\bar{x}) = 0$ (smooth pasting), where $\text{argmin}_x \tilde{u}(x) = 0$, so $x = 0$ is the optimal return point.

We have

$$\tilde{u}(x) = \frac{\left(\psi - \frac{B}{\rho}\bar{x}^2\right)\cosh(\xi x)}{\cosh(\xi \bar{x}) - 1} + \frac{\sigma^2 B}{\rho^2} + \frac{Bx^2}{\rho}$$

where $\xi \equiv \sqrt{\frac{2\rho}{\sigma^2}}$ and $\tilde{u}_x(\bar{x}) = 0$ is the smooth pasting condition.

Early euro: $t \leq T$ value function (superscript E) Early adoption entails a flow cost ϕ of using the wrong currency, the Bellman equation is

$$\rho \tilde{u}^E(x,t,\phi) = \phi + Bx^2 + \tilde{u}^E_{xx}(x,t,\phi)\frac{\sigma^2}{2} + \tilde{u}^E_t(x,t,\phi)$$

with boundaries $\tilde{u}^E_x(\bar{x},t,\phi) = 0$ (smooth pasting) and $\tilde{u}^E(x,T,\phi) = \tilde{u}^L(x)$ (terminal condition). We have

$$\tilde{u}^E(x,t) = \tilde{u}(x) + \frac{\phi\left(1 - e^{-\rho(T-t)}\right)}{\rho}$$

Late Kuna value function (for $t > T$): Using the Kuna after T involves a cost (per unit of time) equal to ϕ. The value function solves a steady state problem (does not depend on t), it solves the ODE (indexed by the fixed cost ϕ)

$$\rho u^L(x,\phi) = \phi + Bx^2 + u^L_{xx}(x,\phi)\frac{\sigma^2}{2} \quad \text{for} \quad x \in \left(-\bar{x}^L, \bar{x}^L\right)$$

with boundaries

$$u^L(\bar{x}^L,\phi) = \psi + \tilde{u}(0) \quad , \quad u^L_x(\bar{x}^L,\phi) = 0$$

where $\tilde{u}(x)$ is the late euro value function. This boundary means that upon the first price change after T it is profitable for the firm to adopt the euro and stop paying the penalty ϕ. That is why the decision rule for this problem, the threshold $\bar{x}^L$, is smaller than the steady state threshold $\bar{x}$ (because there is one additional advantage of changing prices if kuna is used for $t > T$). The boundary of the inaction region is an implicit function of the cost ϕ, that is $\bar{x}^L(\phi)$, and it is immediate that $\bar{x}^L(0) = \bar{x}$ that is the boundary of the steady state problem.

Perturbation. Define the perturbation for $u^L(x,\phi)$ as

$$v^L(x) \equiv \left.\frac{\partial u^L(x,\phi)}{\partial \phi}\right|_{\phi=0}$$

that is the derivative of the u^L function w.r.t. ϕ, evaluated at $\phi = 0$. By inspecting the above ode's for u, one can see that the function v solves

$$\rho v^L(x) = 1 + v^L_{xx}(x)\frac{\sigma^2}{2} \quad \text{for} \quad x \in (-\bar{x}, \bar{x})$$

with boundary

$$v^L(\bar{x}) = 0$$

The solution for v^L is

$$v^L(x) = \frac{1}{\rho}\left(1 - \frac{e^{\xi x} + e^{-\xi x}}{e^{\xi \bar{x}} + e^{-\xi \bar{x}}}\right) \text{where} \quad \xi \equiv \sqrt{\frac{2\rho}{\sigma^2}}$$

Early Kuna value function for $T_0 < t < T$: the value function solves the PDE (indexed by the fixed cost ϕ)

$$\rho u^E(x,t,\phi) = Bx^2 + u^E_{xx}(x,t,\phi)\frac{\sigma^2}{2} + u^E_t(x,t,\phi) \quad \text{for} \quad x \in \Big(-\bar{x}(t), \bar{x}(t)\Big)$$

with boundaries (value matching and smooth pasting respectively)

$$u^E(\bar{x}(t),t,\phi) = \psi + \tilde{u}^E(0,t) \quad , \quad u^E_x(\bar{x}(t),t,\phi) = 0$$

where $\tilde{u}^E(x,t,\phi)$ is the early euro value function, $x = 0$ is the optimal return point (given symmetry) and $\tilde{u}^E(0,t,\phi) = \tilde{u}(0) + \phi\Big(\frac{1-e^{-\rho(T-t)}}{\rho}\Big)$ (known in closed form, see XXX). The value function u^E also has the terminal boundary condition at $t = T$

$$u^E(x,T,\phi) = u^L(x,\phi) \qquad \forall x$$

Perturbation. Define the perturbation for $u^E(x,t,\phi)$ and $\bar{x}(t,\phi)$ as

$$v^E(x,t) \equiv \frac{\partial u^E(x,t,\phi)}{\partial \phi}\Big|_{\phi=0} \quad , \quad z(t) \equiv \frac{\partial \bar{x}(t,\phi)}{\partial \phi}\Big|_{\phi=0}$$

that is the derivative of the u^E function w.r.t. ϕ, evaluated at $\phi = 0$.

By inspecting the above pde's for u^E, one can see that the function v solves

$$\rho v^E(x,t) = v^E_{xx}(x,t)\frac{\sigma^2}{2} + v^E_t(x,t) \quad \text{for} \quad x \in (-\bar{x}, \bar{x})$$

with boundaries at $x = \bar{x}$ and $t = T$ given respectively by

$$v^E(\bar{x}, t) = \frac{1 - e^{-\rho(T-t)}}{\rho} \quad \text{and} \quad v^E(x, T) = v^L(x) \quad \forall x$$

Likewise the perturbation of the smooth pasting condition gives

$$u^E_{xx}(\bar{x}, t, 0)\ z(t) + v_x(\bar{x}, t) = 0$$

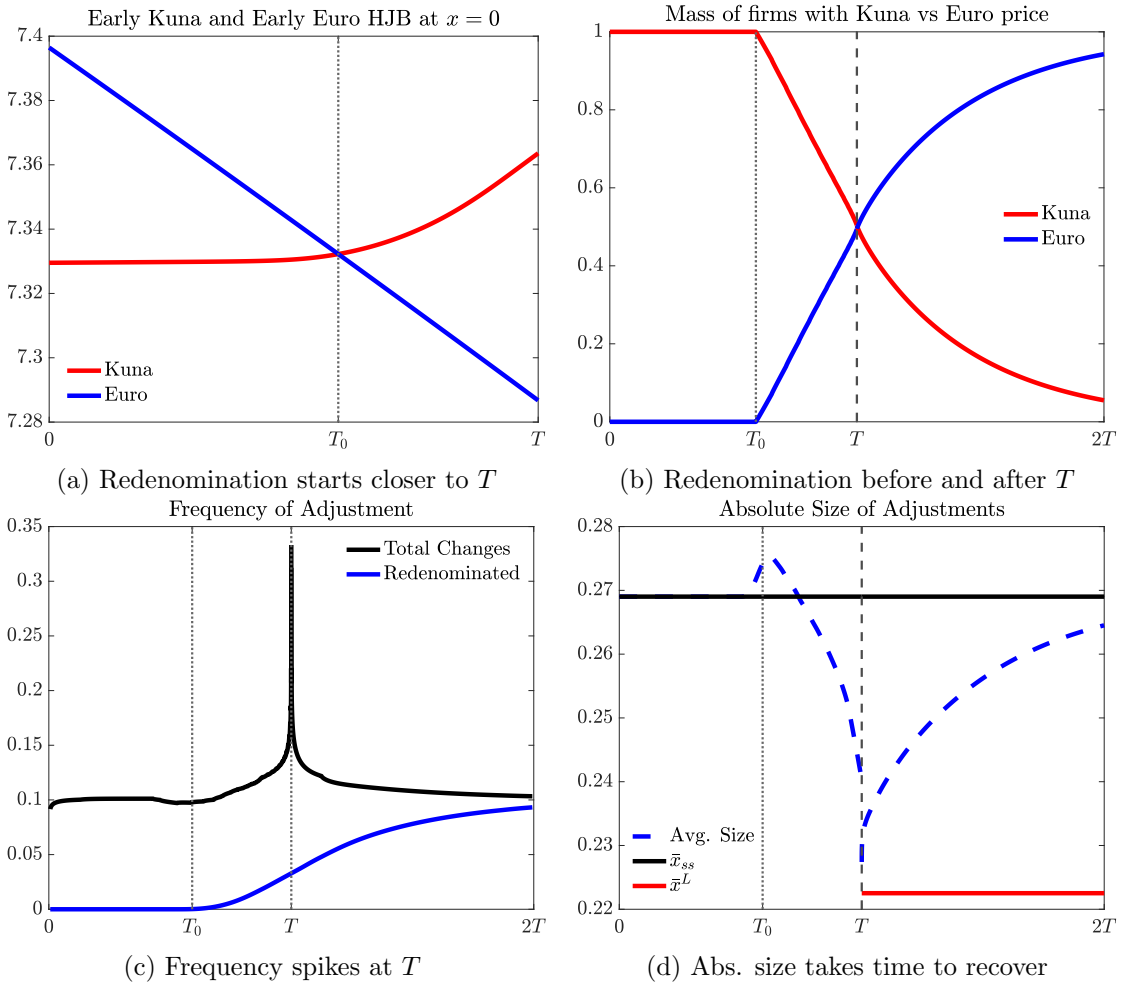
where $u^E_{xx}(\bar{x}, t, 0) = \tilde{u}_{xx}(x)$ since it is evaluated at $\phi = 0$. This condition will be used to study the evolution of the decision rule $\bar{x}(t) = \bar{x} + \phi\, z(t)$

First adoption time: T_0 the model encodes the first time when euro adoption occurs. This occurs at time T_0 such that

$$u^E(0, T_0) = \tilde{u}^E(0, T_0) = \tilde{u}(0) + \phi\Big(\frac{1 - e^{-\rho(T-T_0)}}{\rho}\Big)$$

where the last equality uses the solution for $\tilde{u}^E(x, t)$, and both expression are conditional on adjusting prices, i.e. on returning to $x = 0$. No adoption occurs for $t < T_0$, and every price change corresponds to an adoption for $t > T_0$.

Figure 11: Optimal Pricing Before and After Changeover



(a) Redenomination starts closer to T

(b) Redenomination before and after T

(c) Frequency spikes at T

(d) Abs. size takes time to recover

Note:

6 Observation cost problem

Let θ be the observation cost and $dx = \sigma dW$. There is a cost for using the "wrong currency", i.e. euro for $t < T$ and kune for $t > T$.

The flow cost is Bx^2 is supplemented by another flow cost ϕ per unit of time when the wrong currency is used.

Steady state problem. Upon adjusting, and settgin $x = 0$, the problem is to choose the best duration:

$$v(0) = \min_{\tau} \mathbb{E}\Big(\int_0^{\tau} e^{-\rho s} Bx(s)^2 ds + e^{-\rho \tau}(\theta + v(0)) \Big)$$

which gives

$$v(0) = \min_{\tau} \frac{B\sigma^2 \int_0^{\tau} e^{-\rho s} s \, ds + e^{-\rho\tau}\theta}{1 - e^{-\rho\tau}}$$

Integrating

$$v(0) = \min_{\tau} \frac{\frac{1-e^{-\rho\tau}(\rho\tau+1)}{\rho^2} B\sigma^2 + e^{-\rho\tau}\theta}{1 - e^{-\rho\tau}}$$

FOC with respect to τ

$$0 = \frac{(B\sigma^2 e^{-\rho\tau}\tau - \rho e^{-\rho\tau}\theta)\left[1 - e^{-\rho\tau}\right] - \rho e^{-\rho\tau}\left(\frac{1-e^{-\rho\tau}(\rho\tau+1)}{\rho^2} B\sigma^2 + e^{-\rho\tau}\theta\right)}{[1 - e^{-\rho\tau}]^2}$$

or

$$0 = \left(B\sigma^2\tau - \rho\theta\right)\left[1 - e^{-\rho\tau}\right] - \rho\left(\frac{1 - e^{-\rho\tau}(\rho\tau+1)}{\rho^2} B\sigma^2 + e^{-\rho\tau}\theta\right)$$

or

$$\frac{\theta}{B\sigma^2} = \frac{\rho\tau(1 - e^{-\rho\tau}) - (1 - e^{-\rho\tau}(\rho\tau+1)}{\rho^2}$$

or

$$\frac{\theta}{B\sigma^2} = \frac{\rho\tau - 1 + e^{-\rho\tau}}{\rho^2}$$

approx exponential for small $\rho \to 0$

$$\frac{\theta}{B\sigma^2} = \frac{\rho\tau - 1 + 1 - \rho\tau + (\rho\tau)^2/2 + o(\rho^2)}{\rho^2}$$

gives

$$\tau^* = \sqrt{\frac{2\theta}{B\sigma^2}}$$

let v^* be the steady state value when $\tau = \tau^*$. Let us try to approximate v^* for small ρ

$$v^* = \frac{\frac{1-e^{-\rho\tau^*}(\rho\tau^*+1)}{\rho^2} B\sigma^2 + e^{-\rho\tau^*}\theta}{1 - e^{-\rho\tau^*}}$$

Expand to second order

$$v^* = \frac{\frac{1-(1-\rho\tau^*+\frac{(\rho\tau^*)^2}{2}+o(\rho^2))(\rho\tau^*+1)}{\rho^2} B\sigma^2 + (1 - \rho\tau^* + \frac{(\rho\tau^*)^2}{2} + o(\rho^2))\theta}{\rho\tau^* + \frac{(\rho\tau^*)^2}{2} + o(\rho^2)}$$

collect

$$v^* = \frac{\frac{1-(1-\frac{(\rho\tau^*)^2}{2}+o(\rho^2))}{\rho^2} B\sigma^2 + (1 - \rho\tau^* + \frac{(\rho\tau^*)^2}{2} + o(\rho^2))\theta}{\rho\tau^* + \frac{(\rho\tau^*)^2}{2} + o(\rho^2)}$$

or, using the approx expression for τ^*

$$\lim_{\rho\to 0}\rho\, v^* = \frac{\frac{(\tau^*)^2}{2}B\sigma^2+\theta}{\tau^*} = \frac{2\theta}{\tau^*}$$

Price changes after T. Obviously For any $t \geq T$ the problem for an agent who adjusts is to immediately adopt the euro and get value $v(0) = v^*$.

6.1 Pricing decisions before the changeover date ($t < T$) .

We discuss two possible policies before for $t < T$

Early euro decisions for $t \in (T-\tau^*, T)$. The value for the early Euro adoption (i.e. abandoning kuna at $t < T$) is

$$v^E(t) = \min_{\tau}\left(B\sigma^2\int_0^{\tau} s\, e^{-\rho s}ds + \phi\int_t^T e^{-\rho s}ds + e^{-\rho\tau}(\theta+v^*)\right)$$

This policy only makes sense if $t+\tau > T$, otherwise it is dominated by a policy of not adopting the euro so early which saves the flow cost of wrong currency pricing ϕ.

We can write it as

$$v^E(t) = \min_{\tau}\left(\frac{1-e^{-\rho\tau}(\rho\tau+1)}{\rho^2}B\sigma^2 + \phi\frac{e^{-\rho t}-e^{-\rho T}}{\rho} + e^{-\rho\tau}(\theta+v^*)\right) \approx v^* + \phi(T-t)$$

Because the flow cost does not depend on τ, since it is only paid until $T < t+\tau$, then the optimal choice conditional on early adoption is $\tau = \tau^*$, as in the steady state problem.

Early adoption will not occur if $v^E(t) > v^L(t)$ (late adoption preferable), or if $v^E(t) > v^*$ (keep Kuna pricing). The latter implies that for $t < T-\tau^*$ the firm will not choose early adoption (might it choose a late one?)

It is also immediate that at $t = T-\tau^*$ the optimal choice is to keep kuna pricing and plan early adoption at the next occasion, when $t = T$!

Late Kuna decisions for $t < T$. For an agent with Kuna, adjusting prices at $t < T$ the problem is choosing between keep using kunas or using early euro. The value for keep using kunas and possibly ending up with late Kuna if $t+\tau > T$ (hence the superscript L), is:

$$v^L(t) = \min_{\tau}\left(B\sigma^2\int_0^{\tau} s\, e^{-\rho s}ds + \phi\int_T^{\max(T,t+\tau)} e^{-\rho(s-t)}ds + e^{-\rho\tau}(\theta+\min\{v^L(t+\tau), v^E(t+\tau), v^*\})\right)$$

more precisely, depending on whether $t+\tau$ is smaller or larger than T we have

$$v^L(t) = \begin{cases} \min_\tau \left(B\sigma^2 \int_0^\tau s\, e^{-\rho s} ds + e^{-\rho\tau}(\theta + \min\{v^L(t+\tau), v^E(t+\tau)\}) \right) & \text{if } \ t+\tau < T \\ \min_\tau \left(B\sigma^2 \int_0^\tau s\, e^{-\rho s} ds + \phi \int_T^{t+\tau} e^{-\rho(s-t)} ds + e^{-\rho\tau}(\theta + v^*) \right) & \text{if } \ t+\tau > T \end{cases}$$

We can rewrite it as

$$v^L(t) = \begin{cases} \min_\tau \left(\frac{1-e^{-\rho\tau}(\rho\tau+1)}{\rho^2} B\sigma^2 + e^{-\rho\tau}(\theta + \min\{v^L(t+\tau), v^E(t+\tau)\}) \right) & \text{if } \ t+\tau < T \\ \min_\tau \left(\frac{1-e^{-\rho\tau}(\rho\tau+1)}{\rho^2} B\sigma^2 + \phi \frac{e^{-\rho(T-t)} - e^{-\rho\tau}}{\rho} + e^{-\rho\tau}(\theta + v^*) \right) & \text{if } \ t+\tau > T \end{cases}$$

The first order condition when $\tau + t > T$ is

$$\rho e^{-\rho\tau}(\theta + v^*) = B\sigma^2 e^{-\rho\tau}\tau + \phi e^{-\rho\tau}$$

or

$$\rho(\theta + v^*) = B\sigma^2\tau + \phi$$

or

$$\tau^L = \frac{\rho(\theta + v^*) - \phi}{B\sigma^2}$$

so the policy does NOT depend on t. Considering the limit for $\rho \to 0$ and the expression for ρv^* derived above

$$\lim_{\rho\to 0} \tau^L = \frac{\frac{2\theta}{\tau^*} - \phi}{B\sigma^2}$$

This is cool! it shows that if $\theta \to 0$ then $\tau^L = \tau^*$, i.e. no change if there is no cost to mispricing. Otherwise, duration is smaller than the steady state i.e. $\tau^L < \tau^*$

Let us compute the approximate value for v^L when $t + \tau > T$ i.e. when $t > T - \tau^L$ assuming $\rho \to 0$

$$v^L(t) = \frac{1 - e^{-\rho\tau^L}(\rho\tau^L + 1)}{\rho^2} B\sigma^2 + \phi \frac{(e^{-\rho(T-t)} - e^{-\rho\tau^L})}{\rho} + e^{-\rho\tau^L}(\theta + v^*)$$

$$\lim_{\rho\to 0} v^L(t) - v^* = \frac{1 - 1 + \rho\tau^L - (\rho\tau^L)^2/2 - (\rho\tau^L - (\rho\tau^L)^2 + (\rho\tau^L)^3/2)}{\rho^2} B\sigma^2 + \theta + \phi(t + \tau^L - T)$$

$$\lim_{\rho\to 0} v^L(t) - v^* = \frac{(\tau^L)^2}{2} B\sigma^2 + \theta + \phi(t + \tau^L - T)$$

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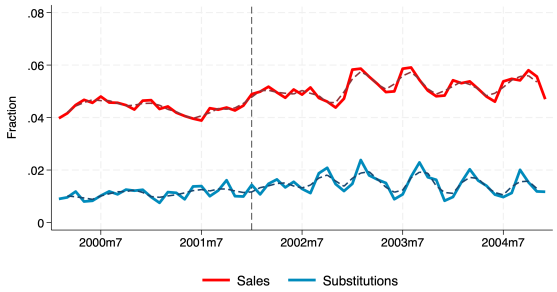
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APPENDIX

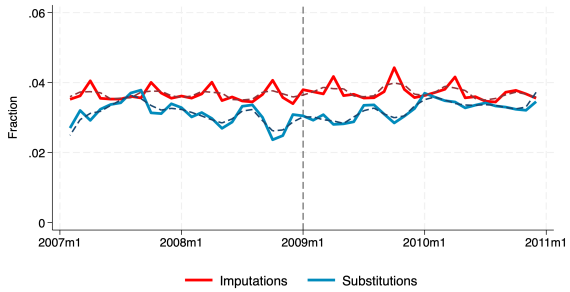
A Tables and Figures

Figure A1: Sales, Imputations, and Substitutions: Austria and Slovakia

(a) Austria

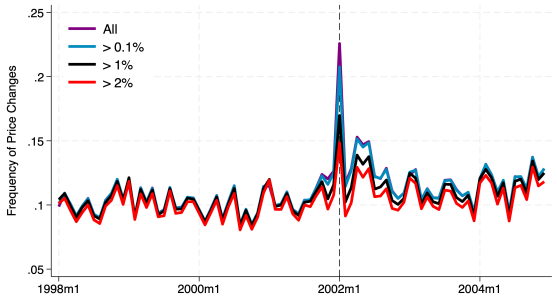


(b) Slovakia

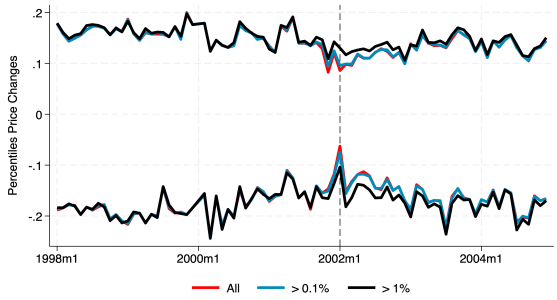


Note: Panel (a) shows the frequency of sales and product substitutions in Austria around the euro changeover. Panel (b) shows the frequency of price imputations and product substitutions in Slovakia around the euro changeover. These indicators are provided by each data set respectively. Observations are aggregated using equal weights for each item.

Figure A2: Pricing Moments: Austria



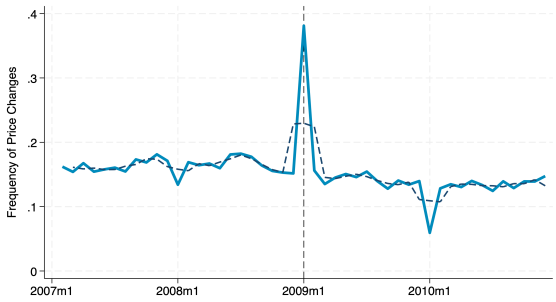
(a) Frequency of Adjustment



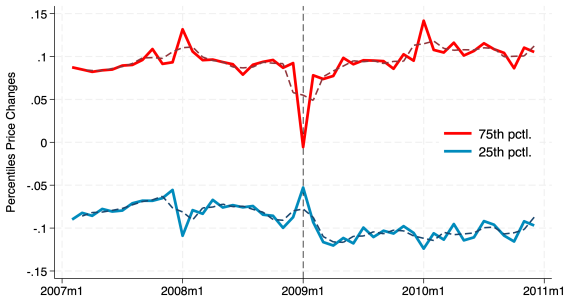
(d) IQR

Note: The figure shows pricing moments for Austria before and after the euro changeover. Panel (a) presents the frequency of price changes after controlling for seasonality, with the dashed line representing the three-month moving average. Panel (b) shows the 25th and 75th percentiles of the price change distribution. The purple line depicts the frequency of all price changes, the blue line reports price adjustments larger than 0.1%, and the black line represents price changes larger than 1%.

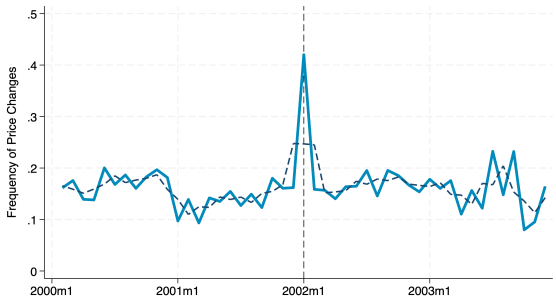
Figure A3: Pricing Moments: Slovakia, Greece, and Finland



(a) Frequency: Slovakia



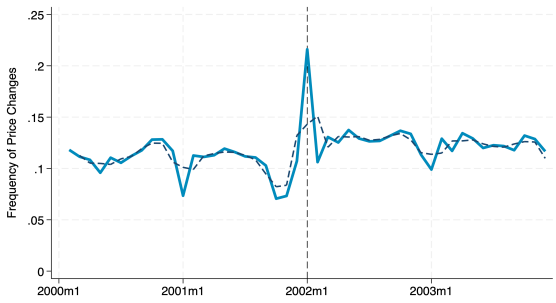
(b) IQR: Slovakia



(a) Frequency: Greece



(b) IQR: Greece



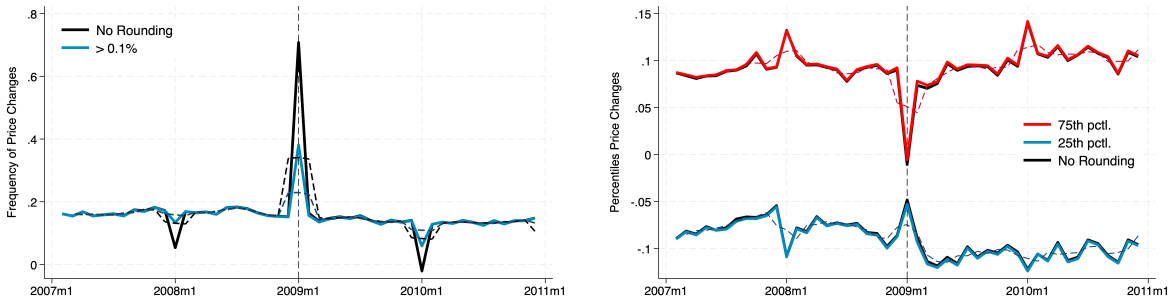
(a) Frequency: Finland



(b) IQR: Finland

Note: The figure shows pricing moments for Slovakia, Greece, and Finland before and after the euro changeover. Panel (a) presents the frequency of price changes after controlling for seasonality, with the dashed line representing the three-month moving average. Panel (b) displays the frequency of price increases and decreases. Panel (c) reports the absolute size of price changes, while panel (d) shows the 25th and 75th percentiles of the price change distribution.

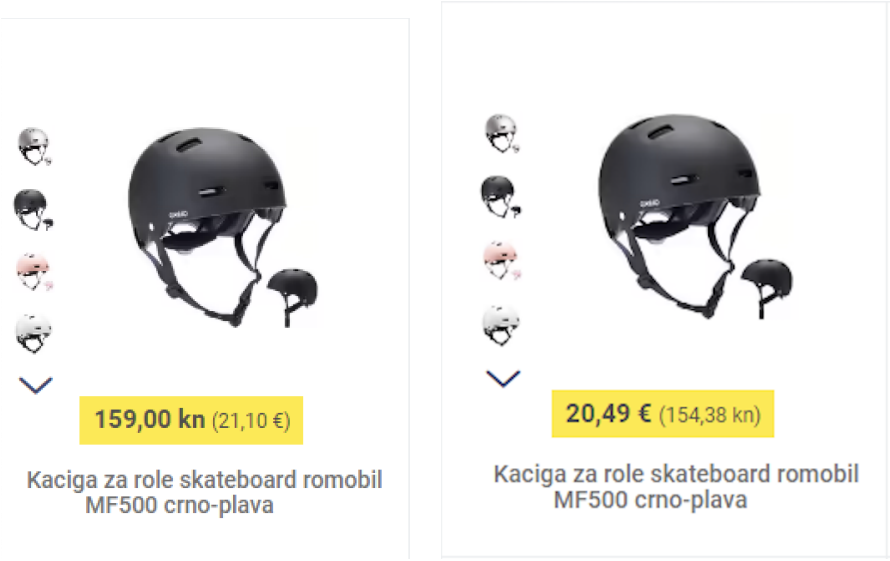
Figure A4: Pricing Moments: Slovakia (No Rounding)



(a) Frequency of Adjustment (d) IQR

Note: The figure shows pricing trends for Slovakia before and after the euro changeover. Panel (a) presents the frequency of price changes as a three-month moving average after controlling for seasonality. Panel (b) displays the 25th and 75th percentiles of the price change distribution. The red line depicts the frequency of all price changes excluding those due to rounding, while the blue line represents price adjustments larger than 0.1%.

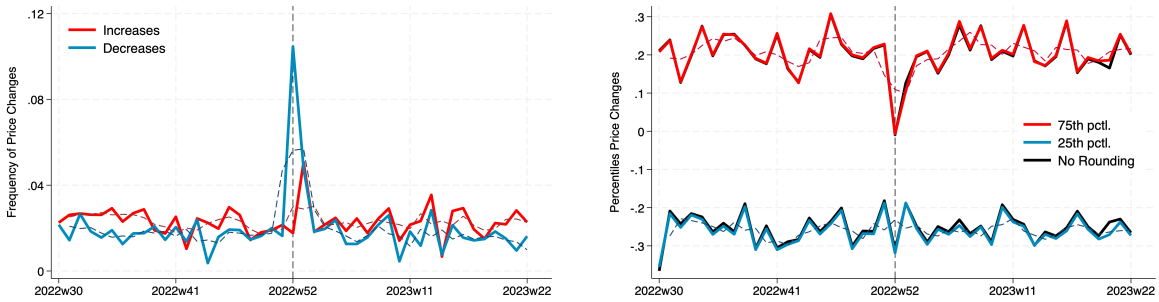
Figure A5: Croatia: Example of a Listing



(a) Dec 30, 2022 (b) Jan 2, 2023

Note: The figure shows an example of an online listing for a roller skate helmet at Decathlon in Croatia on December 30, 2022, and January 2, 2023, i.e., before and after the euro transition.

Figure A6: Pricing Moments: Croatia



(a) Frequency (b) IQR

Note: The figure shows pricing movements for Croatia before and after the euro changeover. Panel (a) presents the frequency of price increases and decreases, excluding price changes due to rounding and those smaller than 0.1%. Panel (b) displays the 25th and 75th percentiles of the price change distribution using the same definition of price changes.

B Introduction of the Euro: Timetable

The euro adoption patterns for Austria, Greece, and Finland are quite different from the one for Slovakia. The three countries used the "Madrid-style approach", gradual changeover during a three-year transitional period. The euro was introduced as the official currency on January 1, 1999 in Austria and Finland and on January 1, 2001 in Greece. The euro banknotes and coins were introduced in the three countries on January 1, 2002, after a transitional period of three years (or one year for Greece) when the euro was the official currency but only existed as "book money". The dual circulation lasted for two months in all the four countries from January 1, 2002 to February 28, 2002. There was a mandatory dual display of prices from October 1, 2001 to February 28, 2002. The euro was introduced in Slovakia on 1 January 2009, both cash and non-cash circulation without a transitional period. This is the so called - "Big-Bang" approach, where euro banknotes and coins enter use on the same day as the euro officially becomes the country's new currency. Dual display of prices was mandatory from 24 August 2008 until 31 December 2009 (i.e. one year after the euro introduction date). Though obligatory dual pricing period finished in December 2009, it was recommended to continue in dual prices display by June 2010.

B.1 Austria

- 1995: EU membership.
- 1995: ERM(European Exchange Rate Mechanism) membership.

- 1999: EMU (Economic and Monetary Union) membership.
- January 1, 1999: The euro was introduced as the official currency but only existed as "book money".
- September 1, 2001: Euro information campaign. Start of euro frontloading to credit institutes, possible start of euro sub-frontloading of companies.
- October 1, 2001: Dual pricing starts.
- December 15, 2001: Euro starter-kits for consumers.
- January 1, 2002: The euro banknotes and coins were introduced in Austria.
- January 1, 2002: Dual circulation beginning date - Austrian schillings and euro are both legal tender.
- February 28, 2002: Dual circulation ending date - Euro is the sole legal tender, exchange of schillings to euro only possible at OeNB (The Oesterreichische Nationalbank) without amount and time limit.
- February 28, 2002: End of dual pricing.

B.2 Finland

- 1995: EU membership.
- 1996: ERM (European Exchange Rate Mechanism) membership.
- December 1997: The final changeover plan was published.
- 1999: EMU (Economic and Monetary Union) membership.
- January 1 1999: The euro was introduced as the official currency but only existed as "book money".
- December 1999: Bank of Finland initiated information efforts for the cash-changeover and started regular cooperation with the national campaign.
- October 1, 2001: Dual pricing starts.
- January 1, 2002: The euro banknotes and coins were introduced in Finland.
- January 1, 2002: Dual circulation beginning date.

- February 28, 2002: Dual circulation ending date - Euro is the sole legal tender.
- February 28, 2002: End of dual pricing.

B.3 Greece

- 1981: EU membership
- January 1 1999 –Dec. 31 2000: ERM II
- January 2001: EMU (Economic and Monetary Union) membership
- January 1, 2001: The euro was introduced as the official currency but only existed as "book money". The adoption of the single currency by Greece (January 1, 2001) marked the start of the "transitional period" in which the euro could only be used in scriptural form and the domestic market formed part of the integrated euro area market. During the "transitional period" monetary policy was conducted in euro, credit institutions' current accounts with the Bank of Greece were denominated in euro, and all transactions in the interbank money market were carried out in euro. The "transitional period" lasted one year and ended on December 31, 2001.
- September 1, 2001: Bank of Greece supplied commercial banks with euro coins.
- October 1, 2001: Bank of Greece supplied commercial banks with euro banknotes. Dual pricing starts.
- November 1, 2001: The banks, to which the Bank of Greece had frontloaded euro coins and banknotes, sub-frontloaded to retailers euro coins.
- December 1, 2001: The banks, to which the Bank of Greece had frontloaded euro coins and banknotes, sub-frontloaded to retailers low-denomination banknotes (€5 and €10).
- December 17, 2001: The National Cash Changeover Plan provided for the frontloading to the public of starter kits containing 45 euro coins of all denominations worth 5,000 drachmas (€14.67).
- January 1, 2002: The euro banknotes and coins were introduced in Greece.
- January 1, 2002: Dual circulation beginning date - euro banknotes and coins circulated in Greece in parallel with drachma banknotes and coins (dual circulation period). Non-cash transactions (cheques, remittances, credit cards etc.) were conducted in euro only.
- February 28, 2002: Dual circulation ending date - Euro is the sole legal tender .

B.4 Slovakia

- May 2004: EU membership.
- 2005: ERM II membership.
- July 2008: Setting conversion foreign exchange rate of SKK/EUR by the EU Council.
- July - December 2008: Ensuring the necessary number of euro banknotes and coins for cash circulation in Slovakia.
- September - December 2008: Supplying of the National Bank of Slovakia and commercial banks with cash euros.
- December 2008: Supplying of retailers with cash euros.
- August 24, 2008: Compulsory dual pricing starts.
- December 31, 2009: End of compulsory dual pricing.
- January 1, 2009: Accession into the eurozone, introduction of the euro.
- January 16, 2009: Dual circulation end date (exchange of crown cash for euros and withdrawal of Slovak crowns from circulation). Euro becomes the sole legal tender.
- June 2010: End of recommended dual pricing.

B.5 Croatia

- July 1, 2013: EU membership.
- July 10, 2020: ERM II (European Exchange Rate Mechanism II) membership.
- July 2021: Setting conversion foreign exchange rate of HRK/EUR by the EU Council.
- 2021 - 2022: Ensuring the necessary number of euro banknotes and coins for cash circulation in Croatia.
- October 2022 - December 2022: Supplying the Croatian National Bank and commercial banks with cash euros.
- December 2022: Supplying retailers with cash euros.
- September 5, 2022: Compulsory dual pricing starts.

- January 1, 2023: Accession into the eurozone, introduction of the euro.
- January 1, 2023: Dual circulation begins.
- January 15, 2023: Dual circulation end date (exchange of kuna cash for euros and withdrawal of Croatian kunas from circulation). Euro becomes the sole legal tender.
- January 15, 2023: End of compulsory dual pricing.

C Price Setting Problem at the Changeover - Special case with Infinite cost

We describe first the stylized problem of a firm in a stationary environment. This environment will correspond to the one once the firm has adapted its pricing to the new currency. Later we will describe the problem of the firm before it has set the prices in the new currency, but after it has been announced that it there is a currency changeover. We will refer to the currency as the Euro.

Stationary problem (post Euro pricing). We let p be the price gap, the difference between the ideal price that maximizes current profits and the price charged by the firm. We assume that the price gap follows a BM without drift $dp = \sigma dW$, when it is uncontrolled –i.e. when prices are not being changed. The changes on the price gap are interpreted as changes in the exogenous cost of production, which follow a brownian motion without drift. The firm period losses, relative to the static profit maximizing level are Bp^2 per unit of time, with $B > 0$. This loss function can be interpreted as a second order approximation to the loss of an arbitrary profit function. We assume that prices can be changed only by paying a menu cost is $\psi > 0$. The firm minimized the expected discounted cost using discount factor $\rho > 0$.

The optimal policy for this is a symmetric sS rule, with an inaction region $[\underline{p}, \bar{p}]$, and an optimal return point $p^* = 0$. The firm changes prices the first time its price gap reached the boundary of the inaction region, at which time it changes its price as to set the price gap to zero, i.e. to the level that maximized static profits. The symmetry of the solution, given by $\underline{p} = -\bar{p}$ and $p^* = \left(\underline{p} + \bar{p}\right)/2$ are due to the symmetry of the loss function and the lack of drift for the uncontrolled price gap. The value function $v(p)$ for a firm with gap p satisfies:

$$\rho\, v(p) = Bp^2 + v''(p)\frac{\sigma^2}{2} \text{ for all } p \in [\underline{p}, \bar{p}] \text{ and } v(p) = v(0) + \psi \text{ otherwise} \tag{15}$$

satisfying the usual boundary conditions: value matching, smooth pasting, and optimal return:

$$v(\bar{p}) = v(\underline{p}) = v(0) + \psi \text{ and } v'(\bar{p}) = v'(\underline{p}) = v'(0) = 0 \tag{16}$$

Pre-Euro pricing problem. Assume that at $t = 0$ there is an announcement that at time $T > 0$ prices has to be set in a new currency. In particular assume that setting pricing in a new currency involves the same cost ψ, and that if prices are set in the new currency since $0 < t_0$ the profits for a firm at times $t > t_0$ with price gat p are the Bp^2, same as before. This

means that if a firm has not changed prices just before T, it must change them at that time, to set then in Euros. Of course, when prices are set in a new currency, the firm can also set them in a way to change its price gap. Indeed the optimal policy will be to set its price gap to zero. After the firm has set prices in the new currency the problem of the firm becomes again the stationary problem studied above. In effect this is equivalent to assume that between announces a mandatory price change has to occur between now $t = 0$ and $t = T$.

We now define a new value function, which gives the expected present value of losses for firms that have not yet adopted the new currency before T, or in other words firms that have not changed their price since $t = 0$. We find it more convenient to keep track of time until the changeover τ, which we define $\tau = T - t$. The optimal policy is also given by a symmetric sS rule, except that in this case the barriers are time varying. We denote the upper barrier by $\tilde{p}(\tau)$ and we let the lower barrier as $\vec{p}(\tau) = -\tilde{p}(\tau)$ and $p^*(\tau) = 0$ for all $\tau \geq 0$. This value function satisfies

$$\rho\ \tilde{v}(p,\tau) = Bp^2 + \tilde{v}_{pp}(p,\tau)\frac{\sigma^2}{2} - \tilde{v}_\tau(p,\tau) \text{ for all } p \in [-\tilde{p}(\tau), \tilde{p}(\tau)] \text{ and for all } \tau > 0 \tag{17}$$

$$\tilde{v}(p,\tau) = v(0) + \psi \text{ for all } |p| > \tilde{p}(\tau) \text{ and for all } \tau > 0 \tag{18}$$

and the boundary conditions:

$$\tilde{v}(\tilde{p}(\tau),\tau) = v(0) + \psi \text{ for all } \tau > 0 \tag{19}$$

$$\tilde{v}_p(\tilde{p}(\tau),\tau) = 0 \text{ for all } \tau > 0 \tag{20}$$

$$\tilde{v}_p(0,\tau) = 0 \text{ for all } \tau > 0 \tag{21}$$

$$\tilde{v}(p,0) = v(0) + \psi \text{ for all } p \tag{22}$$

Differentiating value matching evaluated at $(\tilde{p}(\tau),\tau)$ with respect to time and using value matching we get;

$$\tilde{v}_\tau(\tilde{p}(\tau),\tau) = 0 \text{ for all } \tau > 0 \tag{23}$$

and differentiating with respect to time again, and differentiating smooth pasting evaluated at $(\tilde{p}(\tau),\tau)$ with respect to time we have:

$$\tilde{p}'(\tau) = \frac{\tilde{v}_{p\tau}(\tilde{p}(\tau),\tau)}{\tilde{v}_{pp}(\tilde{p}(\tau),\tau)} = \frac{\tilde{v}_{\tau\tau}(\tilde{p}(\tau),\tau)}{\tilde{v}_{p\tau}(\tilde{p}(\tau),\tau)} \tag{24}$$

The next proposition shows that the range of inaction decreases as the firm gets closer to the changeover date. The limit of the range of inaction is discretely smaller than the one for the stationary case.

PROPOSITION 1. Let $\tilde{p}(\tau)$ and $\vec{p}(\tau)$ be the upper and lower thresholds for the firm which has not changed prices yet: (i) the limit of the threshold as time is approaching the date of the changeover is discretely smaller than the value for the threshold for the stationary case, i.e.: $\tilde{p}(0) < \bar{p}$, and (ii) for all for all $\tau \geq 0$ the derivatives are $\tilde{p}'(\tau) \geq 0$ and $\vec{p}'(\tau) \leq 0$.

The next proposition find a sharp bound for the discrete change in the inaction region just before the changeover date for the undiscounted case, i.e. the case where $\rho \downarrow 0$. The bound is independent of any other parameter. We will show numerically that for strictly positive and reasonable values this ratio is very accurate. In Appendix D.1 we state precisely the form that the firm problem takes when $\rho \downarrow 0$.

PROPOSITION 2. Let $\bar{p}(\rho)$ be the threshold for adjustment in the stationary case with discount rate $\rho > 0$. Let $\tilde{p}(\tau; \rho)$ be the adjustment threshold before pricing in Euros for a problem with discount rate ρ. The threshold just before the changeover is discretely smaller than the stationary case by a factor of $\sqrt{1/3}$, i.e.:

$$\lim_{\rho \downarrow 0} \frac{\tilde{p}(0; \rho)}{\bar{p}(\rho)} = \sqrt{\frac{1}{3}} \tag{25}$$

The next proposition uses the undiscounted case to show that the width of inaction is strictly decreasing in time everywhere and that it has infinite derivative as time gets close to the changeover date.

PROPOSITION 3. Let ρ be the discount rate. Let $\tilde{p}(0; \rho), \vec{p}(0; \rho)$ be the upper and lower threshold for the problem of a firm that as not price in Euros just before the changeover, i.e at date T (or $\tau = 0$), and with a discount rate $\rho > 0$. It is strictly increasing, with infinite derivative just before the change-over date:

$$\lim_{\rho \downarrow 0} \tilde{p}'(\tau; \rho) > 0 \text{ and } \lim_{\rho \downarrow 0} \vec{p}(\tau; \rho) < 0 \text{ for all } \tau > 0, \tag{26}$$

$$\lim_{\rho \downarrow 0} \tilde{p}'(0; \rho) = \infty \text{ and } \lim_{\rho \downarrow 0} \tilde{p}'(0; \rho) = -\infty \tag{27}$$

The next proposition shows that $\tilde{p}(\tau)$ can be taken to be a function of a single scalar, namely $\sigma^2 \frac{\psi}{B}$, if we measure time in normalized units.

PROPOSITION 4. The threshold function $\tilde{p}(\cdot)$ and the threshold $\bar{p}$ are themselves functions of two parameters $\sigma^2 \frac{\psi}{B}$ and ρ/σ^2. The optimal thresholds in the undiscounted case (i.e. as

$\rho \to 0$) can be expressed as a function normalized time, i.e. $\tau\sigma^2$ and $\sigma^2\frac{\psi}{B}$:

$$\lim_{\rho \downarrow 0} \tilde{p}\left(\tau; \rho, \psi, \sigma^2, B\right) = \hat{p}\left(\tau\sigma^2; \sigma^2\frac{\psi}{B}\right) \text{ for all } \tau \geq 0 \tag{28}$$

for some function $\hat{p} : \mathbb{R}_+ \times \mathbb{R}_{++} \to [\bar{p}(0)/\sqrt{3}, \bar{p}(0))$ and $\bar{p}(0) = \left(6\sigma^2\frac{\psi}{B}\right)^{\frac{1}{4}}$.

Three comments are in order for the use of this proposition. First, recall that $\bar{p}(0)$ can be measured in terms of observables for the stationary case, since in the theory it is equal to the average size of absolute price changes, and its square is the variance of price changes $Var(\Delta p)$. Second, the scaling of time is in terms of σ^2. This can also be measured using the equation (reference) that in the stationary case links the variance of price changes $Var(\Delta p)$ with the average number of price changes per unit of time N, namely $\sigma^2 = NVar(\Delta p)$. Third, the usefulness of equation (28) is that one can compute it numerically for a given $\sigma^2\frac{\psi}{B}$, in principle an observable quantity, and for $\sigma^2 = 1$, and obtain its values for any σ^2 by scaling time τ aprropriately.

C.1 Dynamics of cross sectional statistics

To compute statistics such as the frequency and size of price changes as a function of the time before and after the changeover, we first describe the dynamics of the cross section of price gaps.

We define the densities $\tilde{g}$ and g that keep track of the distribution of the price gaps across firms, previous to shifting to Euros and after the shift respectively. To construct them we need the initial distribution $g_0(x)$ as well as the threshold for the firms that have not changed prices $\{\underline{p}(t), \bar{p}(t)\}_{t=0}^{T}$ and the threshold for the stationary case (after the change to euro pricing) $\underline{p}, \bar{p}$. We define $\underline{p}(t), \bar{p}(t)$ as:

$$\bar{p}(t) \equiv \tilde{p}(T-t) \text{ and } \underline{p}(t) \equiv \tilde{p}(T-t) \text{ for all } 0 \leq t \leq T. \tag{29}$$

Their law of motion are:

$$\tilde{g}_t(p,t) = \frac{\sigma^2}{2}\tilde{g}_{pp}(p,t) \text{ for } p \in [\underline{p}(t), \bar{p}(t)] \text{ for all } t \in [0,T]$$

with boundary $\tilde{g}(\bar{p}(t), t) = \tilde{g}(-\bar{p}(t), t)$ for all $t \in [0,T]$, since $\underline{p}(t)$ and $\bar{p}(t)$ are exit points. Let

$$\tilde{G}(t) = \int_{\underline{p}(t)}^{\bar{p}(t)} \tilde{g}(p,t)dp$$

be the total number of firms that has not change price to Euro yet. Then the rate of firm that adjust to price in euro Euro for the first time is given by

$$\tilde{G}'(t) = \int_{\underline{p}(t)}^{\bar{p}(t)} \tilde{g}_t(p,t)dp = \frac{\sigma^2}{2}\left[\tilde{g}_p(\bar{p}(t),t) - \tilde{g}_p(\bar{p}(t),t)\right]$$

were we have replace the Kolmogorov forward and performed the integration. The initial condition is:

$$\tilde{g}(p) = g_0(p) \text{ for } p \in [\underline{p}(0), \bar{p}(0)] \text{ and } g(0,0) = 1 - \int_{\underline{p}(0)}^{\bar{p}(0)} g_0(p)dp$$

After the firms price in euros we have:

$$g_t(p,t) = \frac{\sigma^2}{2} g_{pp}(p,t) \text{ for } p \in [\underline{p}(t)) \cup (\bar{p}(t)] \text{ for all } t \geq 0$$

with boundary $g(\bar{p}(t),t) = g(\bar{p}(t),t)$ for all $t \in [0,T]$. At $p = 0$ the Kolmogorov forward equation does not need to hold since we must include the density of firms that enter after changing prices, the sume of those that price in Euros for the first time and those that change their prices already set in Euros.

Since the total mass in conserved, we have:

$$1 = \int_{\underline{p}}^{\bar{p}} g(p,t)dp + \int_{\underline{p}(t)}^{\bar{p}(t)} \tilde{g}(p,t)dp \text{ for all } t \in [0,T]$$

At $t = T$ we have that the mass $\int_{\underline{p}(T)}^{\bar{p}(T)} \tilde{g}(p,T)dp$ of firms transit to $g(0,T)$.

The number of firms that change prices (per unit of time) is given by:

$$\begin{aligned}
N(0) =& G_0(\underline{p}(0)) + 1 - G_0(\bar{p}(0)) \\
n(t) =& \frac{\sigma^2}{2}\left[\tilde{g}_p(\bar{p}(t),t) - \tilde{g}_p(\bar{p}(t),t)\right] + \frac{\sigma^2}{2}\left[g_p(\bar{p},t) - g_p(\bar{p},t)\right] \text{ for all } t \in (0,T) \\
N(T_+) - N(T_-) =& \tilde{G}(T) \\
n(t) =& \frac{\sigma^2}{2}\left[g_p(\bar{p},t) - g_p(\bar{p},t)\right] \text{ for all } t > T
\end{aligned}$$

where $N(t)$ is the cumulative number of price changes, so when N is differentiable $N'(t) = n(t)$.

D Additional material

This appendix gathers material that is used mostly in proofs or computations, and the proofs themselves.

D.1 Undiscounted case

We write the equations that correspond to the undiscounted case, i.e. the case that obtain as the limi $\rho \downarrow 0$. This case is also known as the ergodic control case. The stationary value function solves

$$a = Bp^2 + v''(p)\frac{\sigma^2}{2} \text{ for } p \in [-\bar{p}, \bar{p}], \tag{30}$$

$$v(\bar{p}) = v(-\bar{p}) = v(0) + \psi, \text{ and} \tag{31}$$

$$v'(\bar{p}) = v'(-\bar{p}) = v'(0) = 0 \tag{32}$$

where $a >$ is a constant that must be determined, i.e. it does *not* depend on p.

The value function for the pre-Euro pricing in the undiscounted case, i.e. as $\rho \downarrow 0$ are:

$$a(\tau) = Bp^2 + \tilde{v}''(p,\tau)\frac{\sigma^2}{2} - \tilde{v}_\tau(p,\tau) \text{ for } p \in [-\bar{p}(\tau), \bar{p}(\tau)] \text{ and all } \tau > 0 \tag{33}$$

$$\tilde{v}(0,\tau) = 0 \text{ and all } \tau \geq 0 \tag{34}$$

$$\tilde{v}_p(p,0) = 0 \text{ for all } p \tag{35}$$

$$\tilde{v}(\tilde{p}(\tau),\tau) = \tilde{v}(-\tilde{p}(\tau),\tau) = \tilde{v}(0,\tau) + \psi \text{ and all } \tau \geq 0 \tag{36}$$

$$\tilde{v}'(\tilde{p}(\tau),\tau) = \tilde{v}'(-\tilde{p}(\tau),\tau) = \tilde{v}'(0,\tau) = 0 \text{ and all } \tau \geq 0 \tag{37}$$

$$\tag{38}$$

where $a(\tau) > 0$ is a function of time constant that must be determined, but that it does *not* depend on p.

D.2 Discrete time and state formulation

Let Δ be the length of a time period. The value function $\tilde{v}(\bar{p},\tau)$ is easy to compute in discrete time (we call it $\tilde{v}_\Delta$) solving it backwards from the threshold condition at $\tau = 0$: $\tilde{v}(\bar{p},0) = v_\Delta(0) + \psi$. One period before changeover, at $\tau = \Delta$, we have:

$$\tilde{v}_\Delta(p,\Delta) = \min\left\{ Bp^2\Delta + \frac{v_\Delta(0)}{1+\Delta\rho} + \psi \,,\, v_\Delta(0) + \psi \right\} \tag{39}$$

which, at indifference, gives an expression for the threshold $\tilde{p}_\Delta(\Delta)$ one period before the changeover:

$$Bp^2\Delta + \frac{v_\Delta(0)}{1+\Delta\rho} = v_\Delta(0) \text{ or } \tilde{p}_\Delta(\Delta) = \sqrt{\frac{\rho(v_\Delta(0)+\psi)}{B(1+\Delta\rho)}} \tag{40}$$

Now we state the recursion for value function away from the terminal date. The value at Δ was given above. It is immediate that:

$$\begin{aligned} &\tilde{v}_\Delta(p,(k+1)\Delta) = \\ &\min\left\{ Bp^2\Delta + \frac{\frac{1}{2}\tilde{v}_\Delta(p+\sqrt{\Delta}\sigma,k\Delta) + \frac{1}{2}\tilde{v}_\Delta(p-\sqrt{\Delta}\sigma,k\Delta)}{1+\Delta\rho}\ ,\ v_\Delta(0)+\psi \right\} \\ &\quad \text{for } k = 1,2,3,\ldots \end{aligned} \tag{41}$$

so that $x = \tilde{p}_\Delta((k+1)\Delta)$ solves:

$$\begin{aligned} v_\Delta(0)+\psi = Bx^2\Delta + \frac{\frac{1}{2}\tilde{v}_\Delta(x+\sqrt{\Delta}\sigma,k\Delta) + \frac{1}{2}\tilde{v}_\Delta(x-\sqrt{\Delta}\sigma,k\Delta)}{1+\Delta\rho} \\ \text{for } k = 1,2,3,\ldots \end{aligned} \tag{42}$$

From Kushner and Dupuis (reference) we know that as $\Delta \downarrow 0$ then $\tilde{v}_\Delta(p,t) \to \tilde{v}(p,t)$ weakly for all p,t.

D.3 Undiscounted marginal values

Motivated by the characterization of the undiscounted or ergodic control case we define the p.d.e and boundary for the marginal value, which we denote by u and also by the corresponding boundary $\tilde{p}$. The functions u and $\tilde{p}$ are defined as

$$u(p,\tau) \equiv \lim_{\rho\downarrow 0} \tilde{v}_p(p,\tau;\rho) \tag{43}$$

$$\tilde{p}(\tau) \equiv \lim_{\rho\downarrow 0} \bar{p}(\tau;\rho) \text{ for all } \tau \in \geq 0 \tag{44}$$

The function u and its free boundary $\tilde{p}$ must satisfy:

$$u_\tau(p,\tau) = 2Bp + u_{pp}(p,\tau)\frac{\sigma^2}{2} \text{ for all } p \in [0,\tilde{p}(\tau)] \text{ and } \tau \geq 0 \tag{45}$$

with boundary conditions:

$$u(p,0) = 0 \text{ for all } p \geq 0 \tag{46}$$

$$u(0,\tau) = 0 \text{ for all } \tau \geq 0 \tag{47}$$

$$u(\tilde{p}(\tau),\tau) = 0 \text{ for all } \tau \geq 0 \tag{48}$$

$$u_p(\tilde{p}(\tau),\tau)\tilde{p}'(\tau) = -u_\tau(\tilde{p}(\tau),\tau) \text{ for all } \tau \geq 0 \tag{49}$$

The p.d.e. in (45) follows from differentiating (33) with respect to p, equation (46) follows from differentiating equation (35) with respect to p, the boundary condition equation (47) follows directly from equation (34), the boundary condition equation (48) follows immediately from equation (36), and equation (49) follow directly from equation (37). Note also that we have defined only for positive p, given the symmetry of $\tilde{v}(\cdot,t)$ for all t.

D.4 Proofs

Proof. (of Proposition 1)

First we show (i). Let $\tilde{v}(p,0) = v(0) + \psi$ for al p, but $v(p) < v(0) + \psi$ for all $p \in (0,\bar{p})$. Then $\tilde{v}(p,0) > v(p)$ for for all $p \in (0,\bar{p})$. Also for $p > 0$, both $\tilde{v}(p,0)$ and $v(p)$ are strictly increasing in $(0, \min\{\bar{p}, \tilde{p}(0)\}$. Now, as a contradiction, assume that $\tilde{p}(0) \geq \bar{p}$, then $v(0) + \psi = \tilde{v}(\bar{p},0) > v(\bar{p}) = v(0) + \psi$, verifying the contradiction. Hence $\tilde{p}(0) < \bar{p}$.

Now we turn to the result (ii) on the derivatives. We first establish that for all $0 < \tau_1 < \tau_2$ and any fixed p then $\tilde{v}(p,\tau_1) \geq \tilde{v}(p,\tau_2)$, i.e. that $\tilde{v}(p,\tau)$ is decreasing in τ for a fixed p. To see this, notice that the policy followed starting at τ_1 is feasible for the firm at τ_2 –which essentially will have price change with certainty $\tau_2 - \tau_1$ periods before the changeover– and hence the value of the cost $\tilde{v}(p,\tau_1)$ cannot be larger than the one at $\tilde{v}(p,\tau_2)$.

Likewise, for any $0 > \tau$, the function $\tilde{v}(p,\tau)$ is increasing in p. Again, if this where not the case, the firm at $p_2 > p_1 \geq 0$ could have followed the same policy than at p_1. Now given these two properties notice that if $\tilde{v}(\tilde{p}(\tau_1),\tau_1) = v(0) + \psi$, then $\tilde{v}(\tilde{p}(\tau_1),\tau_2) < v(0) + \psi$ if $\tau_1 < \tau_2$. Since $\tilde{v}(\cdot,\tau_2)$ is increasing in p we have $\tilde{p}(\tau_2) > \tilde{p}(\tau_1)$. An analogous argument follows for $\vec{p}$. □

Proof. (of Proposition 2) For this proof we use the discrete time and state formulation described in Appendix D.2, and then take the appropriate limits. First we use the expression for the threshold $\tilde{p}_\Delta(\Delta)$ in the discrete time-discrete state formulation defined above. Taking the limit as $\Delta \to 0$ in this expression gives

$$\lim_{\Delta \to 0} \tilde{p}_\Delta(\Delta) = \sqrt{\frac{\rho(v_\Delta(0;\rho) + \psi)}{B}}. \tag{50}$$

Now we develop an expression for the continuous time limit as we let $\rho v(p;\rho)$ as $\rho \downarrow 0$. We have $\rho v(p;\rho) \to \psi N(0) + \int_{-\bar{p}_0}^{\bar{p}(0)} p^2 \bar{g}(p;\rho) dp$ where $N(\rho) = \sigma^2/\bar{p}(\rho)^2$ and $g(p;\rho)$ is the density of the invariant distribution given $\bar{p}(\rho)$, which is given by $\bar{g}(p;\rho) = (\bar{p}(\rho) - p)/\bar{p}(\rho)^2$ for $p \in [0, \bar{p}(\rho)]$ and symmetric for $p < [-\bar{p}(\rho), 0]$. Thus, omitting the argument that denotes that this holds as $\rho \downarrow 0$, we have:

$$\begin{aligned}\rho v(p;\rho) \to \psi N + B\int_{-\bar{p}}^{\bar{p}} p^2 \bar{g}(p) dp &= \frac{\psi\sigma^2}{\bar{p}^2} + B2\left[\frac{p^3}{3\bar{p}} - \frac{p^4}{4\bar{p}^2}|_0^{\bar{p}}\right] = \frac{\psi\sigma^2}{\bar{p}^2} + B2\left[\frac{\bar{p}^3}{3\bar{p}} - \frac{\bar{p}^4}{4\bar{p}^2}\right] \\ &= \frac{\psi\sigma^2}{\bar{p}^2} + B2\left[\frac{\bar{p}^2}{3} - \frac{\bar{p}^2}{4}\right] = \frac{\psi\sigma^2}{\bar{p}^2} + B\bar{p}^2\frac{1}{6} = \frac{6\psi\sigma^2 + B\bar{p}^4}{6\bar{p}^2}\end{aligned}$$

where we have omitted the indiction that N and $\bar{p}$ are evaluated at $\rho = 0$ to simplify the notation. Using the expression for $\bar{p} = (6\psi\sigma^2/B)^{\frac{1}{4}}$ or $\bar{p}^4 = 6\psi\sigma^2/B$

$$\rho v(p;\rho) \to \frac{6\psi\sigma^2 + B\bar{p}^4}{6\bar{p}^2} = \frac{6\psi\sigma^2 + 6\psi\sigma^2}{6\bar{p}^2} = 2\frac{\psi\sigma^2}{\bar{p}^2}$$

Thus

$$\frac{\rho\left(v(p;\rho) + \psi\right)}{B} \to 2\frac{\psi\sigma^2}{B\bar{p}^2} = 2\frac{\psi\sigma^2}{B}\frac{\sqrt{B}}{\sqrt{6\psi\sigma^2}} = \sqrt{\frac{4}{6}\frac{\psi\sigma^2}{B}} = \sqrt{\frac{4}{36}\frac{6\psi\sigma^2}{B}} = \sqrt{\frac{1}{9}}\bar{p}^2 = \frac{1}{3}\bar{p}^2$$

Hence

$$\lim_{\rho\downarrow 0}\lim_{\Delta\to 0}\tilde{p}_\Delta(\Delta;\rho) = \lim_{\rho\downarrow 0}\tilde{p}(0;\rho) = \sqrt{\frac{1}{3}}\lim_{\rho\downarrow 0}\bar{p}(\rho)$$

□

Proof. (on Proposition 3) We use the marginal function u described in Appendix D.3 to prove this proposition.

First we note that the maximum of u cannot occur in the boundaries of the u given by $\{(p,\tau)|p = 0, \tau \geq 0\} \cup \{(p,\tau) : p \in [0, \tilde{p}(0)], \tau = 0\} \cup \{(p,\tau)|\tau \geq 0 p = \tilde{p}(\tau)\}$, since $Bp > 0$ in the interior, and at the boundaries $u = 0$. Then we can use Hopf lemma to conclude that in the boundaries the derivatives that are pointing inwards are strictly positive, i.e. :

$$\begin{aligned}u_p(0,\tau) &> 0 \text{ for all } \tau > 0 \\ u_p(\tilde{p}(\tau),\tau) &< 0 \text{ for all } \tau > 0 \\ u_\tau(\tilde{p}(\tau),\tau) &> 0 \text{ for all } \tau > 0\end{aligned}$$

This implies that:

$$\tilde{p}'(\tau) = -\frac{u_\tau(\tilde{p}(\tau), \tau)}{u_p(\tilde{p}(\tau), \tau)} > 0 \text{ for all } \tau > 0\,.$$

Finally, the boundary condition equation (46) is $u(p, 0) = 0$ for all p, hence differentiating it twice with respect to p, and evaluating it at $p = \tilde{p}(0)$ we have $u_{pp}(\tilde{p}(0), 0) = 0$. Also evaluating the p.d.e. equation (45) at $(\tilde{p}(0), 0)$ we have:

$$u_\tau(p, 0) = 2B\tilde{p}(0) + u_{pp}\left(\tilde{p}(0), 0\right)\frac{\sigma^2}{2} > 0$$

since $\tilde{p}(0) > 0$. Hence:

$$\tilde{p}'(0) = -\frac{u_\tau(\tilde{p}(0), 0)}{u_p(\tilde{p}(0), 0)} = \infty\,.$$

□

Proof. (of Proposition 4) The first step shows that the optimal thresholds can be written as follow:

$$\bar{p}\left(\rho, \frac{\psi}{B}, \sigma^2\right) \text{ and } \tilde{p}\left(\tau; \rho, \frac{\psi}{B}, \sigma^2\right) \text{ for all } \tau \geq 0 \tag{51}$$

The reason why the thresholds $\bar{p}$ and $\tilde{p}$ depend only in the ratio of ψ to B is that the objective function of the stationary problem and for the pre-changeover problem are both homogenous of degree one in (ψ, B). This is easily seen by writing the problems in sequence formulation.

The second step is to establish the following homogeneity property for all $\lambda > 0$:

$$\tilde{p}\left(\tau; \rho, \frac{\psi}{B}, \sigma^2\right) = \tilde{p}\left(\tau\lambda; \rho/\lambda, \frac{\psi}{B/\lambda}, \sigma^2/\lambda\right) \tag{52}$$

This must hold because the constants ρ, B and σ^2 are rates per unit of time, and hence τ and (ρ, B, σ^2) are measured in the same units. Setting $\lambda = \sigma^2$ we get:

$$\tilde{p}\left(\tau; \rho, \frac{\psi}{B}, \sigma^2\right) = \tilde{p}\left(\tau\sigma^2; \rho/\lambda, \sigma^2\frac{\psi}{B}, 1\right) \tag{53}$$

Taking $\rho \to 0$ for $\sigma^2 > 0$ we get the desired expression for $\hat{p}$ as follows:

$$\lim_{\rho \downarrow 0} \tilde{p}\left(\tau; \rho, \frac{\psi}{B}, \sigma^2\right) = \tilde{p}\left(\tau\sigma^2; 0, \sigma^2\frac{\psi}{B}, 1\right) \tag{54}$$

□