Identifying Firm-Level Financial Frictions using Theory-Informed Restrictions*

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Abstract

We propose a theory-based method for identifying financially constrained firms using firm-level balance sheet data. We separately identify firm level productivity, liquidity and financial frictions shocks from observed output, debt and production inputs using sign and magnitude restrictions. These restrictions are proven consistent with a wide range of structural dynamic models, and valid regardless of the type of financing imperfection. Based on the identified shocks we construct indicators for identifying financially constrained firms. The method is validated for large US manufacturing firms —documenting consistency with narrative methods—, and for the quasi-universe of Spanish firms —highlighting its broad applicability—.

Keywords: financial frictions, dynamic corporate finance theory, sign restrictions.

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1 Introduction

A large theoretical literature has shown that financial market imperfections generate a wedge between the cost of internal and external funds, and imply that firms might face financing constraints which distort their investment and employment decisions. These firm-level frictions matter for the amplification and persistence of aggregate shocks, and they explain a substantial part of the productivity gap between developing and developed countries.[1] However, despite their importance, we still have limited understanding on how to empirically identify financially constrained firms.

Existing methods can be broadly classified into three categories, which we label as theory-based methods, narrative methods, or fixed-criteria methods. Regarding the first, several works solve and estimate dynamic firm-level models where the latent intensity of financial frictions is an endogenous variable.[2] A limitation of this approach is that the mapping from observables to this latent variable is inherently model-specific and generally only consistent for a specific type of financial frictions. In reality, different types of firms might be subject to different types of financial constraints, and hence such theory-based methods might not be able to detect all financially constrained firms.[3]

Narrative methods exploit that large public companies are often required to publish reports on their financial position, and the information in these reports can be used to identify firms that face financial frictions.[4] A limitation of this approach is that these reports are typically only available for public firms.[5] Therefore, in practice researchers often rely on ad hoc "fixed criteria", such as age, size or sector level indicators, to identify financially constrained firms. Such indicators are often only loosely related to the underlying finance theory and will generally mix identification based on financial constraints with other firm or sector level characteristics.

In this work, we propose a new theory-based approach for identifying financially constrained firms that is consistent with a wide class of models with financial imperfections and only requires commonly available balance sheet data.

The key idea underlying our method is that modern dynamic investment models with financial frictions imply optimality conditions at the firm level that come with sign and mag-

[1] For reviews of this literature, see Brunnermeier et al. (2012) and Buera et al. (2015).

[2] Among others, see Whited (1992), Hubbard et al. (1995), Love (2003), and Whited and Wu (2006).

[3] Heterogeneity in financial frictions is emphasized both by Nikolov et al. (2021), who show that different firms (small versus large, and private versus public) are affected by different types of financial frictions, and by the recent literature that highlights the importance of asset based and earning based borrowing constraints (e.g. Kermani and Ma 2020, Lian and Ma 2021, Caglio et al. 2021, Drechsel 2022).

[4] See for example Hoberg and Maksimovic (2014) and Buehlmaier and Whited (2018).

[5] In general, attempts to extrapolate theory-based or narrative indicators to construct quantitative financial frictions measures for the whole population (e.g. Lamont et al. 2001, Hadlock and Pierce 2010, Whited and Wu 2006) have been found to be largely unsuccessful (see Farre-Mensa and Ljungqvist 2015).

nitude restrictions informed by canonical economic mechanisms, such as decreasing marginal returns of variable inputs and increasing cost of debt in leverage. We show that we can exploit these restrictions to identify the underlying financial frictions, liquidity and productivity shocks in a structural panel vector autoregressive model, using similar methods as used in the macroeconometric sign-restriction literature for aggregate time series (e.g. Uhlig 2005). With the identified financial frictions shocks we can assess the effects of exogenous changes in financial frictions on firm level variables and construct indicators for identifying financially constrained firms.

The key advantage of our methodology is that, because it relies on a limited set of identifying restrictions, it is consistent with a wide class of theoretical models. More specifically, we show that the derived restrictions hold in the presence of frictions that determine a wedge between the internal and external cost of funds, as well as asset-based and earning-based borrowing constraints. In addition, the methodology addresses the other limitations of existing methods: only observed values for output, debt and production factors are required, and the resulting financial frictions shocks are firm specific and not sector level indicators, for instance.

Methodology We start by developing our method for a stylized dynamic optimization problem of a firm that uses a variable input to produce and faces financial frictions that imply that the premium in the cost of external finance increases in leverage. The firm is subject to productivity shocks (which could also be interpreted as demand shocks), to liquidity shocks, that affect profits and wealth but are unrelated to productivity, and to financial shocks, which affect the premium cost of external finance. We derive the first order conditions of the model and linearize them to obtain a system of three equations in output, debt, and variable input, with the three exogenous shocks.

We show that, by imposing a minimal set of assumptions - decreasing marginal returns in the variable input, and excess cost of debt increasing and convex in leverage - it is possible to derive a set of firm-level sign and magnitude restrictions that can separately identify the three structural shocks. Importantly, these identifying restrictions continue to be sufficient in extensions of the model, e.g. in the presence of other inputs potentially subject to adjustment costs, and of asset-based and earning-based borrowing constraints.

With the identified financial frictions shocks we construct a new indicator for identifying financially constrained firms. We show that we can use the financial friction shock directly as an indicator, or we may threshold the shock to obtain a binary indicator that is more comparable to the existing literature.

To operationalize our identification approach we embed our linearized structural model in a flexible firm level structural panel vector autoregressive model. The reduced form

parameters of this model can be consistently estimated using the panel GMM methods of Cao and Sun (2011). To conduct inference on the structural elements of interest, e.g. the structural shocks, impulse responses or financial friction indicators, we follow the econometric literature on sign-based inference, and show how these methods can be adopted to estimate and conduct inference on the identified sets for the structural elements (e.g. Gafarov et al. 2018, Granziera et al. 2018).

Empirical validation The second part of the paper highlights the advantages of our methodology using several empirical studies. First, we embed the baseline structural model into a realistically calibrated industry equilibrium framework, and we use it to simulate a panel of firm-level balance sheet data. We show that the structural shocks identified with our methodology provide a good approximation of the true shocks, and that our financial frictions indicator can accurately identify the financially constrained firms.

Next, we consider a panel of US Compustat manufacturing firms. For this sample we show that our theory-based indicator for financial frictions is closely related to the narrative indicator proposed by Hoberg and Maksimovic (2014). The approaches label similar firms as financially constrained. Additionally, we exploit the 2008-2009 financial crisis as a quasi-natural experiment to verify that our indicator is able to predict real outcomes consistently. Specifically, we use data up to 2007, before the onset of the crisis, to identify more financially constrained firms. We show that conditional on other observable characteristics in 2007 such as size, productivity, leverage and industrial sector, these firms reduced employment much more than less constrained firms at the beginning of the crisis in 2008.

Last, we apply our indicator to the quasi-universe of Spanish manufacturing firms above 5 employees. This is important because while the analysis on Compustat is useful for comparison and verification purposes, it is a sample of large public companies which are arguably less exposed to financial frictions. After confirming on this dataset the results of the test based on the 2008-2009 financial crisis, we tested two hypotheses. First, our estimated financial frictions shock should explain a larger part of employment variability for small than for large firms. Intuitively, smaller firms are more likely to be subject to credit shocks due to a "flight to quality" effect. When a lender is constrained in its funding, it will prioritize lending to its larger customers, insulating them from fluctuations in the availability of credit, and leaving the smaller customers more exposed. Second, we also expect financial shocks to be more persistent for small firms than for large firms. The intuition for the second hypothesis is that large firms have better access not only to their main lender but also to relations with multiple banks. If in a given period the firm is forced to reduce its employment level because of unexpected problems in accessing external financing, it is expected that it will be able to find alternative sources relatively quickly. In contrast, many smaller firms typically rely only

on one main lender. If they face a larger number of financing problems, they will be much less likely to quickly find suitable alternatives. Our empirical evidence strongly supports both hypotheses.

Relation to literature This paper broadly relates to two strands of literature: the identification and quantification of firm-level financial frictions, and the sign-based identification literature in macroeconomics and econometrics.

With respect to the financial frictions literature, we are motivated by the lack of reliable measures based on balance sheet data (e.g. Farre-Mensa and Ljungqvist 2015). Our proposed methodology is related to the large literature that uses the insights from dynamic corporate finance theory to identify financial frictions. In particular, since we focus on the optimality conditions of a dynamic model of firms' financing and investment decisions, our paper is related to earlier approaches based on the estimation of Euler equations for fixed capital investment (e.g., among others, Whited 1992, Hubbard et al. 1995, Love 2003, Whited and Wu 2006).[6] As we show in the context of the models analyzed in this paper, a limitation of this approach is that in general the mapping between the latent intensity of financial frictions and observables is model specific, and hence not suitable to detect different types of financing imperfections.

Our approach differs in two main aspects. First, we use multiple equations simultaneously to identify financial frictions. We show that, because these frictions have different effects on real and financial variables, using different equations leads to sufficient identifying restrictions to disentangle financial frictions shocks from the other shocks. Second, our identification strategy is consistent with several different models and types of financial frictions.[7]

Our paper is also related to the empirical literature that uses quasi natural experiments to identify the causal effect of financial frictions. Among others, see for example Chaney et al. (2012), who use variations in local house prices and show that firms exposed to an exogenous increase in the collateral value of their assets increase their investment relative to the other firms. Another example is Chodorow-Reich (2014), who exploits the fact that bank-firm relations are sticky, and that the 2008-2009 financial crisis affected asymmetrically

[6] In related theory-based approaches, Caggese (2007) uses the Euler Equation for variable capital investment, while Cherchye et al. (2020) measure financial frictions as the difference between the firm's profitability with its actual inputs and the firm's achievable profitability with optimal inputs.

[7] More generally, our theory-based approach is related to the literature that develops and estimates firm investment models with financial frictions, as in Hennessy and Whited (2007). More recently, Nikolov et al. (2021) use the empirical policy function estimation technique introduced by Bazdresch et al. (2017), and Catherine et al. (2022) use a structural model to quantify the aggregate effects of financing constraints using well identified reduced form evidence on collateral constraints. These papers estimate the structural parameters of these models, and therefore provide useful information on the nature of financial frictions and their average intensity in the industry, but do not provide an identification strategy to identify time varying financial frictions at the firm-level, which is the objective of this paper.

lenders depending on their exposure to the subprime market. He shows that firms borrowing from these exposed lenders reduced employment more than the other firms. Our objective is fundamentally different from these papers. We provide a novel-theory based methodology to measure the intensity of financial shocks at the firm level using only commonly available balance sheet data and without the need of an external shock such as the 2008 banking crisis or exogenous variations in property prices. Furthermore, we show that our estimated shocks can be used to construct reliable indicators to select financially constrained firms.

Moreover, our emphasis is on developing a financial constraints indicator that is consistent with different types of financial frictions, which is empirically important as shown in Lian and Ma (2021), Kermani and Ma (2020), Caglio et al. (2021) and Drechsel (2022), among others.

The sign restriction based identification approach that we use stems from the pioneering work of Faust (1998) and Uhlig (2005), and has become a popular identification strategy for aggregate time series applications, see Uhlig (2017) and Kilian and Lütkepohl (2017) more discussion.[8] By now a large number of papers have worked out robust inference methods for structural vector autoregressive models that are identified using sign restrictions (e.g. Baumeister and Hamilton 2015, Gafarov et al. 2018, Granziera et al. 2018, Arias et al. 2018, Giacomini and Kitagawa 2020). We adopt a frequentist approach, where in the first step the reduced form panel VAR coefficients are estimated using a panel GMM approach (e.g. Arellano and Bond 1991, Holtz-Eakin et al. 1988, Cao and Sun 2011) and in the second step the sets of structural impulse responses are recovered using optimization methods (e.g. Gafarov et al. 2018, Granziera et al. 2018). Robust confidence intervals, that have correct frequentist coverage, are obtained by similar optimization routines, then also taking into account the estimation uncertainty from the reduced form estimates. We refer to Canova (2007) and Canova and Ciccarelli (2013) for a more elaborate discussion regarding panel VAR models.

The remainder of this paper is organized as follows. The next section describes a simple but generic model from which we derive our sign restrictions that allow to identify the real and financial shocks. Section 3 provides the estimation methodology. The empirical exercises are presented in Section 4. Any references to sections, equations, lemmas etc. which start with "S" refer to the supplementary material.

[8] We note that aggregate time series methods have been used for studying the effects of aggregate financial shocks (e.g. Caldara et al. 2016, Furlanetto et al. 2017), which is distinct from our objective of identifying firm level financial shocks.

2 Firm optimization and identification

In this section, we describe a benchmark dynamic model of a firm subject to financial frictions. The model is used to derive restrictions for identifying firm-specific shocks which are subsequently used for constructing indicators for identifying financially constrained firms. In addition, we discuss how our identification strategy is consistent with several extensions of the model that allow for alternative ways to model financial frictions.

2.1 Baseline model

The firm produces a consumption good, with price normalized to one, using the decreasing returns to scale production function

$$y_t = z_t l_t^{\alpha} \ , \tag{1}$$

where y_t denotes output, l_t is the production input, z_t is an exogenous productivity process and the constant α satisfies $0 < \alpha < 1$. The production input is non-durable and not subject to adjustment costs.[9]

The production input l_t summarizes variable inputs that need to be paid at least partially in advance and are potentially subject to financial frictions. Prominent examples include labour and other variable production inputs such as materials. For convenience, we refer to l_t as labour and the price of one unit of l_t is the wage w. In this model, we assume that l_t needs to be financed in advance because labour hired in one period generates revenues at the beginning of the next period. At the beginning of period t the firm has financial resources equal to

$$s_t = y_{t-1} - b_{t-1} \ ,$$

where b_{t-1} is the face value of one-period debt borrowed in the previous period. The firm observes the new values of productivity z_t, financial frictions ξ_t, and θ_t, the latter being a process related to overhead costs of production. The latent stochastic processes $\log z_t$, $\log \xi_t$ and $\log \theta_t$ are allowed to be persistent. We denote their innovations by ε_t^z, ε_t^{ξ} and ε_t^{θ} which we refer to as productivity, financial frictions, and liquidity shocks, respectively. The budget constraint is given by

$$\text{div}_t = s_t - \theta_t F - w l_t + \Theta(b_t, \xi_t) \ , \qquad \text{with} \qquad \Theta(b_t, \xi_t) \equiv \frac{b_t}{1+r} - c_t \ , \tag{2}$$

where div_t denotes dividends, and $\theta_t F$ are overhead costs of production, that vary over time with θ_t, while F is a positive constant. Since the purpose of θ_t is to cause fluctuations in

[9] While we think about the innovations to $\log z_t$ as productivity shocks, in the context of this model an equally plausible interpretation is that they capture demand shocks. Moreover, below we consider an extension of the model with additional inputs that are durable and subject to adjustment costs.

profits that generate additional liquidity needs, we denote the shock ε_t^θ as liquidity shock.[10] $\Theta(b_t, \xi_t)$ captures the funds obtained in period t from borrowing a face value of debt b_t to be repaid the next period, and c_t is the excess cost of external finance as described below. We assume that the firm is subject to financial frictions. First, the firm cannot issue equity while in operation, i.e.

$$\text{div}_t \geq 0 \ . \tag{3}$$

Second, we assume that b_t is subject to financial frictions that increase its cost. In the literature, researchers have considered a variety of asymmetric information or enforceability problems that limit the amount of debt firms can borrow. These different approaches imply limits to the access to external finance that can be classified in two broad categories: (i) frictions that determine a maximum borrowing limit and (ii) frictions that determine a premium in the risk-adjusted cost of credit, increasing in debt or leverage. In both cases, these frictions imply levels of borrowing and investment lower than it would be optimal in the absence of financial frictions. In the benchmark model of this section, we only consider the second approach and introduce a reduced-form excess cost cost of credit as a function of the amount borrowed. Importantly, in Section 2.3, we show that our results are confirmed in richer models that include also a borrowing limit and the presence of both asset-based and earnings-based collateral.

Specifically, here we assume that borrowing implies the additional cost

$$c_t = \begin{cases} \xi_t b_t^\gamma & \text{if} \quad b_t > 0 \\ 0 & \text{if} \quad b_t \leq 0 \end{cases} \ , \tag{4}$$

where $\gamma > 1$. Since in this model there is no capital and firms are ex ante identical, this formulation is equivalent to assuming that c_t is increasing and convex in leverage. In Appendix S2.1 we provide a microfoundation for this assumption. To illustrate, suppose that b_t is banking debt, and the bank expects that higher debt increases the probability that the firm refuses ex post to repay it, because it prefers an outside option to default on the debt and steal some of the assets purchased with it. The additional fee c_t ensures that the bank receives the required expected return from the loan.

Furthermore, notice that (4) potentially implies a kink in the cost of credit around $b_t = 0$. However, empirically we nearly exclusively observe positive values of debt, because while financial frictions reduce borrowing, other important factors such as the debt tax shield, or debt as an incentive device for managers, increase it. As shown below, in the model we include this feature by assuming the firm has a relatively high discount factor, so that has a

[10] Empirically these types of costs are significant. Arellano et al. (2019) show, in a model similar to the one considered in this paper, that shocks to fixed production costs are key to generate a dispersion in financial spreads as large as the ones observed in the data.

preference for distributing dividends and is a net borrower in the steady state. Therefore in the following analysis we focus on the identification of the shocks conditional on observing positive debt.

We interpret ξ_t in (4) as a latent process driving the excess cost of external finance, and its innovation ε_t^{ξ} as a financial frictions shock. An increase in ξ_t could be for example interpreted as a credit crunch that reduces a lender's ability to obtain funding and hence increases the return it requires on its loans. Notice the difference between ξ_t, which is an exogenous stochastic process, and c_t, which is an endogenous quantity that depends on the firm's decisions.

The objective of the firm is to maximise its intertemporal value $V_t(S_t)$, which is defined as the net present value of dividends. S_t is the vector of state variables.[11] We have:

$$V_t(S_t) = \max_{l_t, b_t} (1 + \phi_t) \operatorname{div}_t + \frac{1}{1+r} \frac{1}{\mu} \mathbb{E}_t \left[V_{t+1} \left(S_{t+1} \right) \right] \ , \tag{5}$$

where ϕ_t is the Lagrange multiplier associated to the no dividends constraint (3). Therefore, $1 + \phi_t$ represents the shadow value of having one additional unit of wealth in period t.[12] The presence of financial frictions implies firms have a preference for not distributing dividends and accumulating savings, which would make all firms net savers in equilibrium. To counteract this, in the literature it is often assumed either the presence of tax distortions that reduce the cost of debt, or that firms have a preference for distributing than retaining dividends, because their discount rate is higher than the interest rate. We take the latter option and assume that firms discount dividends at the rate $\frac{1}{1+r}\frac{1}{\mu}$, where $\mu > 1$.[13]

Assumptions

To separately identify the productivity, financial friction and liquidity shocks from observed output, debt and productivity series alone we impose the following basic assumptions.

[11] If the stochastic processes are AR(1), then $S_t = (s_t, \theta_t, z_t, \xi_t)$.

[12] Note that the maximization problem (5) implies that the firm is infinitely lived. That is, we implicitly restrict parameter values to a subset such that: i) The firm always finds it optimal to continue, because the net present value of its future profits is always positive in all states. ii) The firm is never forced to exit because is unable to borrow to repay its maturing debt. This restriction simplifies the analysis and derivations, but it is not essential for our results. In Appendix S2 we illustrate a slightly modified version of the model which includes endogenous exit (either voluntary or because of bankruptcy) and we use it to simulate the artificial panel data analyzed in Section 4.1, which confirm all of our results.

[13] An alternative way to limit the accumulation of savings is to assume that constrained agents have finite lives, as in Bernanke et al. (1999) and Gertler and Kiyotaki (2015). Our assumption of impatient entrepreneurs follows Kiyotaki and Moore (1997). Alternatively, tax benefits of debt are introduced, for example, in Jermann and Quadrini (2012), while agency frictions between managers and shareholders are assumed in Arellano et al. (2019). These alternative ways to generate firm borrowing in the steady state would not change the main results of our analysis.

Assumption 1. *Decreasing marginal returns in the inputs. There exist constants* $\underline{\alpha}$ *and* $\overline{\alpha}$*, such that* $0 < \underline{\alpha} \leq \alpha \leq \overline{\alpha} < 1$.

Assumption 2. *Marginal excess cost of debt* c_t *is increasing in leverage:* $\gamma > 1$.

Assumption 1 continues to hold if we assume more than one input and constant returns to scale technology in all inputs, as long as at least one input is predetermined (see Section 2.3). The lower $\underline{\alpha}$ and upper $\overline{\alpha}$ bounds can be application specific.[14]

Assumption 2 is consistent with different types of financial imperfections usually considered in financial frictions models (see Section S2.1 for a microfoundation). Moreover, in our model extensions in Section 2.3, we relax it allowing for the presence of collateralized credit.

2.2 Identifying restrictions

Having described our baseline model we now show how its key economic mechanisms, i.e. decreasing returns to scale and increasing marginal costs of debt in leverage, can be exploited to identify the productivity, liquidity and financial friction shocks. To show this we proceed in three steps: (i) we derive the first order conditions for the firm, (ii) log linearize these, and then (iii) we show that the economic mechanisms yield sufficient sign and magnitude restrictions to partially identify the structural parameters of the log linearized model, which in turn allows to recover measures for the structural shocks without committing to the specific functional form assumptions of the model.

First order conditions

The first order condition for debt b_{t+1} is given by

$$(1+\phi_t)\left(\frac{1}{1+r} - \gamma\xi_t b_t^{\gamma-1}\right) + \frac{1}{1+r}\frac{1}{\mu}\left[\mathbb{E}_t\left(\frac{\partial V_{t+1}}{\partial s_{t+1}}\right)\frac{\partial s_{t+1}}{\partial b_t}\right] = 0 \ ,$$

where $\frac{\partial s_{t+1}}{\partial b_t} = -1$ and, for the envelope theorem, $\mathbb{E}_t(\frac{\partial V_{t+1}}{\partial s_{t+1}}) = \mathbb{E}_t(1+\phi_{t+1})$. It follows that

$$\frac{1}{1+r} = \psi_t\left(\frac{1}{1+r} - \gamma\xi_t b_t^{\gamma-1}\right) \ , \tag{6}$$

and

$$\psi_t \equiv \mu\frac{1+\phi_t}{\mathbb{E}_t(1+\phi_{t+1})} \ . \tag{7}$$

[14]For example, in our main empirical application in Section 4 which considers labour as the variable input, α is the elasticity of output to labour, and is a well known quantity, remarkably stable both over time and across countries, and often found in the range between $\underline{\alpha} = 0.4$ and $\overline{\alpha} = 0.8$.

Where ψ_t is the shadow value of finance in the firm today relative to the next period. The right hand side of (6) is the net present value of borrowing one additional unit of debt due in period $t+1$. The left hand side is the net present value of funds saved to obtain 1 unit of wealth in period $t+1$. The productivity of the firm determines the shadow cost ϕ_t of the non-negativity constraint on dividends. When productivity increases, such cost becomes higher relative to its expected value next period, thus increasing also ψ_t, and (6) is satisfied with equality by an increase in borrowing b_t. The first order condition for labour is

$$\frac{1}{1+r}\mathbb{E}_t\left(\frac{\partial V_{t+1}}{\partial s_{t+1}}\frac{\partial s_{t+1}}{\partial l_t}\right) - (1+\phi_t)w = 0 \ , \tag{8}$$

where $\frac{\partial s_{t+1}}{\partial l_t} = \alpha z_t l_t^{\alpha-1}$. Substituting we get

$$\frac{\alpha z_t}{l_t^{1-\alpha}} = (1+r)\,\psi_t w \ . \tag{9}$$

Intuitively, a firm with little wealth today finds it expensive to borrow, and chooses a sub-optimally low value of labour input l_t, which is reflected in high marginal product $\frac{\alpha z_t}{l_t^{1-\alpha}}$ and an high value of ψ_t from equation (9).

Linearized model

Next, we log linearize the first order conditions around the steady state with respect to debt and labour and combine these with the output equation. Specifically, we solve (6) with respect to b_t and (9) with respect to l_t to obtain

$$\begin{aligned}
\log b_t &= c_b + \frac{\psi}{(\gamma-1)(\psi-1)}\log\psi_t - \frac{1}{\gamma-1}\log\xi_t \\
\log l_t &= c_l + \frac{1}{1-\alpha}\log z_t - \frac{1}{1-\alpha}\log\psi_t \\
\log y_t &= \log z_t + \alpha\log l_t
\end{aligned} \tag{10}$$

where c_b and c_l are constants and ψ is the steady state value of ψ_t.

The system (10) depends on the endogenous latent intensity of frictions ψ_t, which positively affects $\log b_t$. Intuitively, the tighter are financial frictions today relative to tomorrow, the higher is the shadow value of resources, the more the firm wants to borrow. The same intuition explains why l_t is negatively related to ψ_t. As it is clear from the previous discussion, ψ_t is an endogenous object. We claim, and later prove in Section S1, that in the log linearized model $\log\psi_t$ is equal to

$$\log\psi_t = c_\psi - \pi_1\log s_t + \pi_2\log\xi_t + \pi_3\log\theta_t + \pi_4\log z_t \ , \tag{11}$$

where $\pi_1, \ldots, \pi_4$ are positive coefficients. Substituting (11) into (10) and rearranging provides a linear mapping from the stochastic processes $g_t = \log(\xi_t, \theta_t, z_t)'$ to the observable variables $Y_t = \log(b_t, l_t, y_t)'$. Specifically we have

$$Y_t = c + D_s \log s_t + B g_t \ , \tag{12}$$

where c is a vector of constants and

$$D_s = \pi_1 \begin{bmatrix} -\frac{1}{\gamma-1}\frac{\psi}{\psi-1} \\ \frac{1}{1-\alpha} \\ \frac{\alpha}{1-\alpha} \end{bmatrix} \ , \quad B = \begin{bmatrix} -\left(\frac{1}{\gamma-1} - \frac{\psi\pi_2}{(\gamma-1)(\psi-1)}\right) & \frac{\psi\pi_3}{(\gamma-1)(\psi-1)} & \frac{\psi\pi_4}{(\gamma-1)(\psi-1)} \\ -\frac{\pi_2}{1-\alpha} & -\frac{\pi_3}{1-\alpha} & \frac{1-\pi_4}{1-\alpha} \\ -\frac{\alpha\pi_2}{1-\alpha} & -\frac{\alpha\pi_3}{1-\alpha} & \frac{1-\alpha\pi_4}{1-\alpha} \end{bmatrix} \ . \tag{13}$$

The linearized structural model (12) relates the firm output and production variables to the firms financial resources s_t and the different stochastic processes. In practice, the latent variables g_t are correlated over time. To capture this we write

$$\tilde{A}(L) g_t = \varepsilon_t \ , \qquad \text{with} \qquad \tilde{A}(L) = I - \tilde{A}_1 L - \ldots - \tilde{A}_p L^p \ , \tag{14}$$

where L is the lag operator and $\varepsilon_t = (\varepsilon_t^{\xi}, \varepsilon_t^{\theta}, \varepsilon_t^{z})'$ captures the serially uncorrelated innovations to the latent financial frictions, liquidity and productivity processes. We assume that the polynomial $\tilde{A}(L)$ is invertible.[15]

Substituting g_t into (12) and reordering the terms gives

$$Y_t = c + D W_t + A_1 Y_{t-1} + \ldots + A_p Y_{t-p} + B\varepsilon_t \ , \tag{15}$$

where the coefficients satisfy $A_j = B\tilde{A}_j B^{-1}$, for $j = 1, \ldots, p$, $W_t = (\log s_t, \ldots, \log s_{t-p})'$ and $D = (D_s, A_1 D_s, \ldots, A_p D_s)$. The model (15) can be recognized as a structural vector autoregressive model for a given firm with predetermined explanatory variables W_t. Without loss of generality we assume that the structural shocks are normalized to be mutually uncorrelated and have mean zero and unit variance: $\mathbb{E}(\varepsilon_t) = 0$ and $\mathbb{V}(\varepsilon_t) = I_K$.

Sign and magnitude restrictions

The second moments of the observable variables in the linear system (15) only allow to recover the structural shocks up to orthogonal transformations (e.g. Kilian and Lütkepohl 2017). In contrast, if the matrix B is parameterized as in (13), all parameters in B and the structural shocks can be recovered from the second moments of the observables.[16]

[15] Formally, $\tilde{A}(z)$ satisfies $\det|\Theta(z)| \neq 0$ for all $z \in \mathbb{C}$ such that $|z| \leq 1$.

[16] This is easy to verify as in (13) there are five free parameters that determine the nine elements of B. These five parameters can all be recovered from the variance of the process $\{B\varepsilon_t\}$.

We can view these two scenarios as two extremes: on the one hand it seems excessive to assume that the model is completely uninformative about B, yet at the same time the specific parameterization of B in (13) is undoubtedly due to some of the specific modeling choices made for the baseline model. Indeed, below we show that reasonable changes in the baseline model lead to different parametrizations for B.

To this extent, the approach in this paper for identifying B and the structural shocks takes a middle ground between these two extremes. Specifically, we only use the parametrization of B in (13) to recover a set of sign and magnitude restrictions that allow us to set-identify B. The key benefit of this approach is that, as shown below in Section 2.3, these sign and magnitude restrictions hold under a wide set of alternative models, implying that even if the baseline model is misspecified (as it likely is in practice) the sign and magnitude restrictions are plausible.

Proposition 1. *For the baseline model defined in Section 2.1 we have that under Assumptions 1-2 the matrix B in* (13) *satisfies the following sign restrictions*

$$B = \begin{bmatrix} - & + & + \\ - & - & + \\ - & - & + \end{bmatrix} . \tag{16}$$

In addition, the following magnitude restrictions hold

$$\underline{\alpha} \leq \left\{ \frac{B_{31}}{B_{21}}, \frac{B_{32}}{B_{22}} \right\} \leq \overline{\alpha} \qquad \textit{and} \qquad \frac{B_{33}}{B_{23}} > 1 \ . \tag{17}$$

The proof is given in Appendix S1.

The economic intuition for the sign restrictions in (16) is as follows. A positive financial shock ε_t^{ξ} raises the cost of external finance ξ_t. The firm reduces debt b_t and therefore is also forced to reduce labour input l_t, which implies lower output y_t. A positive productivity shock ε_t^{z} increases the marginal return on labour and incentivises the firm to increase borrowing b_t to increase l_t. y_t also increases, both because of the direct effect of the productivity shock (more output y_t for given labour input) and because of the indirect effect of increasing l_t. A liquidity shock (that is, a positive value of ε_t^{θ}) increases θ_t and increases borrowing b_t. Moreover, by tightening financial frictions and increasing the marginal cost of debt, it implies the firm desires to hire less labour l_t and produces less output.

Notice that the financial shock ε_t^{ξ} and the productivity shock ε_t^{z} cannot be disentangled using the sign restrictions alone, because both shocks affect the 3 variables in the same direction. However, they can be separately identified thanks to the magnitude restrictions in (17). The economic intuition for the first magnitude restriction in (17) is that both a

positive ε_t^θ shock and a positive ε_t^ξ shock, all else equal, increase financial frictions. Since the marginal productivity of labour is decreasing, when the firm is forced to reduce the labour input because of stronger financial frictions, then labour productivity increases and the fall in output is mitigated relative to the fall in labour.[17] Regarding the second inequality in (17), the economic intuition of why the elasticity of output to the productivity shock (relative to the elasticity of labour) is larger than for the other shocks is also simple. An increase in z_t increases output $\log y_t$ both directly in the production function and indirectly because it also increases labour input. Conversely the other shocks only affect output indirectly through their effect on labour input.

2.3 Robustness to alternative model specifications

As the general discussion surrounding the identifying assumptions eluded to, the derived sign and magnitude restrictions rely on simple economic mechanisms, which are likely to hold in many other dynamic models with possibly different types of financial frictions. Here we show that this is indeed the case: the restrictions implied by the baseline model are robust to several model extensions and different ways of micro-founding the financial frictions. We keep the exposition brief, merely stating the types of deviations for which our results hold, but in Appendix S2 we provide detailed derivations and the formal results that effectively extend Proposition 1.

2.3.1 Capital in the production function

We first extend the baseline model by adding capital in the production function

$$y_t = z_t l_t^\alpha k_t^\beta \ . \tag{18}$$

With $\beta > 0$ and $0 < \alpha + \beta \leq 1$. We assume that capital takes one period to install, and therefore k_t is pre-determined at time $t-1$.[18]

The optimization problem becomes:

$$V_t(S_t, k_t) = \max_{l_t, b_t, k_{t+1}} (1+\phi_t)\text{div}_t + \frac{1}{1+r} E_t \left[V_{t+1}(S_{t+1}, k_{t+1})\right], \tag{19}$$

[17] Models that include customer capital and/or inventories are also consistent with this prediction. Financially constrained firms are known to maximize current revenues while controlling input costs, either by running down inventories, or by increasing prices.(e.g. Gilchrist et al. 2017)

[18] To be precise, capital takes one period to install and two periods to generate revenues. At the beginning of period t, the firm has installed capital k_t, and chooses l_t and k_{t+1}. Labour l_t generates revenues at the end of period t, while capital k_{t+1} generates revenues at the end of period $t+1$.

subject to the budget constraint:

$$\Theta(b_t, \xi_t) = -s_t + \theta_t F + wl_t + \text{div}_t + f(i_t), \tag{20}$$

where i_t is net investment,

$$i_t \equiv k_{t+1} - (1-\delta)k_t,$$

and $0 < \delta < 1$ is the depreciation rate of capital. The function $f(i_t)$ represents the total expenditure to invest, which includes the direct cost i_t as well as capital adjustment costs. Since we do not plan to use the first order condition for k_{t+1} to identify the shocks, we do not need to fully specify capital adjustment costs and the investment policy function. More specifically, we only impose that, conditional on current wealth s_t and installed capital k_t, productivity z_t affects investment i_t directly, while the latent variables ξ_t and θ_t affect it only indirectly, by changing the latent shadow cost ψ_t, same as it happens for labour input (see the system 10). Hence $f(i_t)$ can be represented as a function of s_t, k_t, z_t and ψ_t, and log-linearising yields:

$$\log f(i_t) = \epsilon^i_s \log s_t - \epsilon^i_\psi \log \psi_t + \epsilon^i_z \log z_t + \epsilon^i_k \log k_t, \tag{21}$$

where we do not need to impose any restriction on ϵ^i_s and ϵ^i_k. Regarding the elasticities with respect to the productivity shock z_t and to ψ_t, we only impose that investment weakly decreases in financial frictions, and weakly increases in productivity. hence:

$$\epsilon^i_\psi \leq 0; \epsilon^i_z \geq 0 \tag{22}$$

These restrictions are very mild and consistent with all the types of capital adjustment costs commonly considered in the literature, such as convex adjustment costs, fixed adjustment costs, and irreversibility. In particular, we allow ϵ^i_ψ and ϵ^i_z to be equal to zero to be consistent with the potential presence of non-convex adjustment costs which create inaction regions in the optimal investment policy. For example, if fixed capital is irreversible, ϵ^i_ψ might be equal to zero when, after an increase in financial frictions which reduces funds available to purchase capital and labour, the irreversibility constraint binds. Likewise, ϵ^i_z might be also equal to zero when the firm has an inefficiently high level of installed capital k_t. In Appendix S2 we derive and log-linearize the first order conditions for b_t and l_t and show that this model yields the same system (12) and set of identifying restrictions (16) and (17) of the benchmark model, with the only difference that the vector of predetermined variables is now $W_t = (\log s_t, \log k_t)'$ instead of $\log s_t$.

2.3.2 Models with collateralized borrowing

We consider extensions of the baseline model that allow for collateralized credit, where collateral can be both assets and revenues. While a large literature in macro-finance has emphasised constraints of the first type, a recent literature points out the larger empirical relevance of the second type (e.g. Lian and Ma 2021). For clarity of exposition we consider the two forms of borrowing separately, even though to consider them jointly would not affect the results.

Importantly, we not only show that our identification restrictions are valid in these alternative models, but also that they are able to capture financial frictions shocks that affect both the price of credit (the shock to the cost of credit ξ_t for a given loan to value ratio) and its quantity (a shock to the loan to value ratio).

Asset based borrowing

We consider the model with capital analysed in Section 2.3.1 and modify equation (4) as follows

$$c_t = \begin{cases} \xi_t \left(\frac{b_t}{\left(1-\lambda_t^1\right)(1-\delta)k_t} \right)^{\gamma} & \text{if} \quad b_t > \left(1-\lambda_t^1\right)(1-\delta)k_t \\ 0 & \text{if} \quad b_t \leq \left(1-\lambda_t^1\right)(1-\delta)k_t \end{cases}, \tag{23}$$

where $\left(1-\lambda_t^1\right)(1-\delta)k_t$ is the collateral value of the firm's current physical assets and $0 < \lambda_t^1 < 1$ is the fraction of value of capital that *cannot* be used as collateral. One way to micro-found this assumption is to assume that the lender, in case the debt is not repaid, is able to liquidate the firm and the residual value of current assets is $\left(1-\lambda_t^1\right)(1-\delta)k_t$.[19] In other words, the firm is able to borrow at the risk free rate as long as debt is below the value of the firm's collateral. Any borrowing above such level incurs in additional costs proportional to the debt to collateral assets ratio.[20]

Notice that in this formulation a shock that increases ξ_t has the same interpretation as in the benchmark model, is a shock that increases the cost of credit. Conversely, a shock that increases λ_t^1 reduces the collateral value of capital, and hence the availability of low cost borrowing. The firm maximises the value function (19) subject to (20), (21), (22) and (23). In Appendix S2 we show that this model yields the same system and set of identifying restrictions described above, also in this case with $W_t = (\log s_t, \log k_t)'$. In particular, we also show that the same set of sign and inequality restrictions apply whether we consider

[19]Therefore, current capital remains in place until debt is repaid at the beginning of period $t+1$. Conditional on repayment of the debt, the new capital is installed. In case of no repayment, the firm is liquidated.

[20]In the benchmark model we were focusing on firms with positive values of debt, so that $c_t > 0$. Likewise, for the models in this section, we focus on firms that have positive uncollateralised debt. In the data, firms that borrow collateralised debt usually also have positive uncollateralised borrowing such as short term banking debt, lines of credit, or trade credit.

the shock to affect the price of credit (ξ_t) or the the quantity of credit (λ_t^1). Therefore, our estimated financial shock is able to capture both an increase in the price of credit as well as tighter quantity constraints.

Earnings based borrowing

We finally consider the possibility of earnings based borrowing. We go back to the benchmark model without capital of Section 2.1, and we follow Drechsel (2022) by assuming that borrowing capacity is proportional to profits. Therefore, we modify equation (4) as follows:

$$c_t = \begin{cases} \xi_t \left(\frac{b_t}{(1-\lambda_t^2)\pi_t} \right)^{\gamma} & \text{if} \quad b_t > \pi_t \\ 0 & \text{if} \quad b_t \leq \pi_t \end{cases} . \tag{24}$$

Where profits π_t are the net present value of revenues net of variable costs:

$$\pi_t \equiv \frac{y_t}{1+r} - wl_t \tag{25}$$

One way to micro-found this assumption is to assume that the lender, in case the debt is not repaid, is able to seize the fraction $1 - \lambda_t^2$ of firms' profits to repay the debt. Apart from equation (24), we follow here the benchmark model. The derivation of this model is more complicated because of the presence of a feedback effect between input demand and the collateral constraint. Notice that the first order condition (9) implies that the effective cost of labour is higher than the wage bill, because it includes the relative shadow cost of resouces ψ_t. Therefore optimal labour input is below the level that maximises profits, and with an earning-based borrowing constraint, there is an additional motive to expand the labour input to increase revenues and relax the constraint. The details of the derivations are in Appendix S2. We show that even though this constraint changes the magnitude of several reduced form coefficients, all our sign and inequality restrictions remain valid.[21]

2.4 Identifying financially constrained firms

Proposition 1 enables the partial identification of the contemporaneous effects of the structural shocks on debt, labour and output. Moreover, given a set for B we can identify other structural functions of interest. Specifically, since D and $A_1, \ldots, A_p$ are identifiable from the autocovariances of the process $\{Y_t, W_t\}$ we can set identify any function of B, D,

[21]Notice that, in the case of earning based borrowing, a positive shock to z_t has potentially ambiguous effects on financial frictions because it implies additional demand for funds to invest, but also additional revenues generated that relax the borrowing constraint. In the Appendix, we show that the first effect dominates under very mild assumptions.

$A_1, \ldots, A_p$ and the data. For instance, this enables the recovery of the structural shocks $\varepsilon_t = (\varepsilon_t^{\xi}, \varepsilon_t^{\theta}, \varepsilon_t^{z})'$, impulse responses and forecast error variance decompositions (e.g. Kilian and Lütkepohl 2017, Chapter 4). Here we will be mainly interested in constructing indicators for identifying financially constrained firms.

For this several authors have considered the Lagrange multiplier representing the relative shadow cost of external finance, i.e. ψ_t, as a theory-based indicator of financial frictions. Since this shadow cost is unobservable, previous literature has approximated it with a function of observable firm characteristics (e.g., among others, Whited 1992, Hubbard et al. 1995, Love 2003, Whited and Wu 2006). However, there is no guarantee that such approximations have general validity, and Farre-Mensa and Ljungqvist (2015) have shown that financial frictions indicators based on extrapolations of this approach have limited ability to identify financially constrained firms in practice. In our framework, we also have face a similar limitation. Equation (11) represents $\log \psi_t$ as a linear function of the three latent stochastic processes g_t. However, as explained in Appendix S2, the coefficients π_1-π_4 are model specific, and therefore we cannot obtain an estimate of ψ_t that has the same general validity of our estimated shocks ε_t.

Therefore, our preferred indicator is based directly on the financial friction shock ε_t^{ξ}. According to our identification strategy, a positive ε_t^{ξ} shock increases external financing costs, reduces borrowing, and reduces firm inputs below the optimal choice of the firm if it were financially unconstrained, so that the firm operates with an inefficiently high marginal product of labour. As such, firms with a larger value of this shock are likely more financially constrained than the other firms.

Furthermore, notice that ε_t^{ξ} measures the shock to financial frictions costs, rather than their level. This is an advantage rather than a limitation, because in our general framework it is not the *absolute* level of financial frictions (the shadow value of external finance ϕ_t) that affect real decisions of firms, but rather their *relative* value ψ_t.[22] This result is common to most dynamic investment models with financial frictions, and well known in the literature, as also highlighted by Gomes et al. (2006) and Whited and Wu (2006), who notice that "*finance constraints can only affect investment if they are time varying. It is the shadow value of the constraint today, relative to tomorrow, that is important*".

In other words, the financial shock ε_t^{ξ} is a valid financial frictions indicator because by

[22]Intuitively, on the one hand current financial frictions increase funding costs, and thus reduce optimal investment. On the other hand future expected financial constraints increase the value for the firms of the earnings generated by current investment, and hence increase optimal desired investment. If these two forces are equally strong, they compensate each other and the firm chooses the same optimal investment of an unconstrained firm. This can be easily seen in our model by substituting (7) in the first order condition of labor to obtain $\mathbb{E}_t(1+\phi_{t+1})\frac{\alpha z_t}{l_t^{1-\alpha}} = \mu(1+r)(1+\phi_t)w$. A higher value of ϕ_t increases the cost of labor on the right hand side, while higher expected value $\mathbb{E}_t(1+\phi_{t+1})$ increases the return to labour on the left hand side.

causing unexpected changes in the cost of external finance, it is the main driver of the fluctuations in ψ_t. In Appendix S4, using data simulated from a realistically calibrated version of our benchmark model, we show that ε_t^ξ explains around 70% of the variation in in ψ_t.[23] This finding is consistent with the theoretical result that since financial frictions are costly, firms self finance to mitigate them over time, and are able to do so effectively since financially constrained firms are also very productive at the margin. It follows that in the long run firms tend to keep financial frictions low, and hence periods of high values of ψ_t are mainly driven by unexpected shocks (among others, see Moll 2014).

In the empirical applications, ε_t^ξ can be used directly as indicator of the intensity of financial frictions at the firm level. Alternatively, one can follow the strategy normally adopted in the finance literature, of separating a given group of firms in likely constrained and likely unconstrained. In our case, this corresponds to a binary variable equal to one for the firms with the highest value of ε_t^ξ, and zero otherwise:

$$\mathcal{I}_t^\xi = \begin{cases} 1 & \text{if} \quad \varepsilon_t^\xi > \tau \\ 0 & \text{else} \end{cases} , \tag{26}$$

where τ is a user chosen threshold. $\mathcal{I}_t^\xi$, since it captures firms with a large financial friction shock ε_t^ξ, is a good candidate to identify firms with an higher *relative* shadow cost of external finance than the other firms. In theory, the choice of τ is not crucial for the test since the intensity of financial frictions is monotonously increasing in the shock. That is, $\mathcal{I}_t^\xi = 1$ observations are more likely financially constrained than $\mathcal{I}_t^\xi = 0$ observations regardless of the specific value of τ used. In practice, external information can be used to select an appropriate value of τ. For example, if the researcher believes that only a relatively small fraction of firms faces significant financial frictions, she can choose τ such that it corresponds to the cross-sectional quantile that implements this requirement.

We also consider alternative financial frictions indicators based on the historical decomposition of debt and labour productivity in terms of the financial frictions shocks.[24] Specifically, let $\lambda_{d,\xi,h}$ denote the horizon h impulse response of ε_t^ξ on debt. The part of debt that is due to financial frictions is then given by

$$\log b_t^\xi = \sum_{h=0}^{t-1} \lambda_{b,\xi,h} \varepsilon_{t-h}^\xi \ .$$

[23]The liquidity shock only explains around 0.1%, and the productivity shock explains around 10%. Productivity shocks explain part of the fluctuations in ψ_t in the benchmark model because when productivity increases, investment needs increase more than the increase in current revenues. This effect is however reduced in the model with earning based constraints, since in that case productivity shocks also increase borrowing directly (see the discussion in Appendix S2.4 for details).

[24]See Kilian and Lütkepohl (2017, page 115) for details on historical decompositions.

The advantage of using the historical decomposition is that it takes into account lagged financial friction shocks. Similar as above we can also threshold the historical decomposition to construct a binary indicator

$$\mathcal{I}_t^b = \begin{cases} 1 & \text{if} \quad \log b_t^\xi < \tau \\ 0 & \text{else} \end{cases} . \tag{27}$$

Firms identified with $\mathcal{I}_t^b = 1$ received a sequence of shocks that increased the cost of external finance and reduced their debt, relative to the other firms. We also perform the same decomposition on the other endogenous variables to obtain $\log y_t^\xi - \log l_t^\xi$, which is the part of average product of labour due to the financial frictions shock, and compute:

$$\mathcal{I}_t^{\frac{y}{l}} = \begin{cases} 1 & \text{if} \quad \log y_t^\xi - \log l_t^\xi > \tau \\ 0 & \text{else} \end{cases} . \tag{28}$$

While the $\mathcal{I}_t^b$ indicator focuses on the financial consequences of the shock (firms are forced to reduce their debt), the indicator $\mathcal{I}^{\frac{y_t}{l_t}}$ focuses on its real consequences. Firms identified with $\mathcal{I}^{\frac{y_t}{l_t}} = 1$ are those that reduce the most their labour input relative to their output because of the current and past financial frictions shocks.

In our empirical work below we use both our benchmark measure $\mathcal{I}_t^\xi$, as well as the alternative measures $\mathcal{I}_t^b$ and $\mathcal{I}^{\frac{y_t}{l_t}}$, to identify financially constrained firms.

2.5 Revisiting the financial frictions literature

In the previous sections, we showed that our indicator for financial frictions is consistent with several extensions of our benchmark model. In this section, we discuss its relation to the broader theoretical and empirical financial frictions literature.

Relation to other approaches to model financial frictions.

In the benchmark model and in its extensions, we assume that investment is an inter temporal decision, that is, variable inputs acquired in period t generate revenues in period $t+1$. This assumption is plausible, since there is usually a time lag between when variable inputs are acquired and revenues generated. Nonetheless, many authors in the financial frictions literature consider intra-period constraints, in which financial frictions affect production inputs that generate revenues in the same period in which they are acquired. In Appendix S2.4, we discuss the validity of our identification strategy in the context of such models, in which usually borrowing is limited by the value of firms' assets, similarly to our extended model in section 2.3.2. In the appendix we consider two examples of this approach, from Jermann and Quadrini (2012) and Buera and Shin (2013), and we show that in this case

the identification restrictions are still able to identify financial frictions shocks, while they might fail to identify the productivity or liquidity shocks when the firm is permanently at the borrowing limit. However, we also show that these problems arise not because of the intra-period nature of the constraint, but because of the unrealistic assumption that uncollateralised credit is completely excluded, because the interest rate on debt becomes infinite for any borrowing above the collateral value of the assets. Relaxing this strong assumption, and allowing some uncollateralised borrowing at higher interest rates, would allow our approach to identify also the other shocks.

More generally, another important aspect of our indicator is that is based on shocks identified using within-firm variation.[25] We discussed in the previous section that this is without loss of generality, since fluctuations in the relative cost of credit are mostly driven by unexpected shocks. Moreover, this approach has the advantage to make our indicator robust to permanent cross-firms differences. For example, many empirical studies find that large firms have on average higher leverage than small firms. This feature could be easily included in our model by allowing for permanent cross firms differences in productivity, and by assuming that larger firms have lower *average* values of ξ than smaller firms. Importantly, in this setting, our identification strategy and financial friction indicator would continue to be valid, because as explained in Section 2.4 it is not the absolute level of financial frictions, but rather their relative intertemporal value ψ_t, that matters for the real decisions of firms. In Appendix S2.5 we discuss other possible extensions of the model.

Finally, one limitation of our approach is that our procedure is designed to identify financial frictions on the "intensive margin" of active firms, and therefore is unable to measure the extensive margin of these frictions, such as firms forced to exit production, or unable to enter. This limitation is related to the nature to the data (we only observe the borrowing, investment and production decisions of active firms) rather than to the model itself.

Relation to other financial frictions indicators used in the literature

As mentioned in the introduction, many researchers often rely on ad hoc "criteria" to identify financially constrained firms. On the one hand, both theory and empirical evidence show that younger, smaller, and zero dividends firms are on average more financially constrained than the complementary groups of older, larger, and dividends paying firms. This is why these characteristics are often used as financial frictions indicators. While this approach can be useful for testing specific types of model-based predictions, it is not generally applicable, and has clear limitations that are overcome by our approach.

First, firms classified according to size, age or tangibility of assets likely differ along other

[25]For instance, the constant c in (15) captures the firm fixed effect, and in the panel data model introduced below this becomes c_i indicating the firm.

dimensions besides access to finance, and therefore one cannot infer that different behavior is evidence of financing constraints. For example, since Gertler and Gilchrist (1994) many authors have used size as proxy of financial frictions and interpreted the fact that smaller firms are more procyclical than large firms as evidence of a financial accelerator effect. However, Crouzet and Mehrotra (2020) have shown that such higher procyclicality does not seem related to indicators of financial strength and is likely driven by other factors.

Second, these are indicators of "potentially facing financial imperfections", rather than of "being actually financially constrained". Note that, precisely because small and young firms are more likely to face financing imperfections, it is important to identify which of them are currently constrained and what are the real effect of these constraints, which is what our indicator allows to do, since it requires only commonly available balance sheet data. In other words, our approach allows the researcher to identify within a given group of firms which ones are more financially constrained, without making strong assumptions on the specific type of financial imperfections they face.

Similar considerations can be provided regarding dividend status. On the one hand, not paying dividends is certainly correlated to the likelihood of facing financing imperfections. On the other hand, the correlation is far from perfect, with many firms paying dividends even though their financing costs are relatively high, while many cash rich and unconstrained firms pay no dividends.[26]

3 Empirical methodology

In this section we show how the set of sign and magnitude restrictions in Proposition 1 can be exploited to identify and estimate real and financial shocks at the firm level, and construct indicators for financially constrained firms.

We assume that we observe outcome variables for N firms for a total of T time periods. The $K = 3$ observable variables —debt, labour and output— are summarized in the vector $Y_{i,t} = (\log b_{i,t}, \log l_{i,t}, \log y_{i,t})'$ where i indexes the firm and t the time period. Additionally, the $L \times 1$ vector $W_{i,t}$ denotes the predetermined variables, which may include lagged savings and capital. In the linearized baseline model (12) only savings are included, but the extensions from Section 2.3 and in the empirical applications, we also include capital in $W_{i,t}$.

As derived in (15) the firm level model can be written as

$$Y_{i,t} = c_i + d_t + DW_{i,t} + A_1 Y_{i,t-1} + \ldots + A_p Y_{i,t-p} + B\varepsilon_{i,t} \ , \tag{29}$$

[26]For example, Farre-Mensa and Ljungqvist (2015) notice that unconstrained firms should react to exogenous increases in tax rates by increasing leverage, to take advantage of the debt tax shield, more so than constrained firms. They find that firms not paying dividends do not seem more constrained than dividend paying firms according to this criteria. They actually seem, if anything, less financially constrained.

where the only difference is that we include c_i —a firm level fixed effect— and d_t —a time fixed effect—. The fixed effects are intended to capture any time- or firm-invariant differences between the firms over time that are not explained in the model. We can view (29) as a K-dimensional Structural Panel Vector Autoregressive (SPVAR) model (e.g. Holtz-Eakin et al. 1988, Cao and Sun 2011).

The $K \times K$ coefficient matrix B is restricted by the sign and inequality restrictions derived in Proposition 1. Importantly, as explained in more detail below these restrictions are generally not sufficient to point identify B, but will merely shrink the identified set of admissible B matrices.

We proceed as follows. First, we briefly recall the estimation of the reduced form coefficients of the panel VAR following the approach of Cao and Sun (2011). Second, we show how to construct identified sets and confidence regions for different elements of the structural model. For the latter we adopt a frequentist approach based on projection arguments, see also Granziera et al. (2018).

3.1 Reduced model inference

The common reduced form parameters of model (29) are summarized in the vector μ which includes the regression coefficients D, the autoregressive coefficient matrices A_j and the variance matrix of the reduced form residuals denoted by Σ. We have that

$$\mu \equiv (\text{vec}(\Phi)', \text{vech}(\Sigma)')' \ , \qquad \Phi = (D, A_1, \ldots, A_p) \ , \qquad \Sigma = BB' \ . \tag{30}$$

We estimate the reduced form coefficients using the panel GMM approach discussed in Cao and Sun (2011). To illustrate this approach let $u_{i,t} = B\varepsilon_{i,t}$ denote the reduced form shocks.

We first remove the time fixed effects by subtracting the cross-sectional mean from each variables, i.e. we compute $Y_{i,t,j} - \frac{1}{N}\sum_{i=1}^{N} Y_{i,t,j}$ and $W_{i,t,k} - \frac{1}{N}\sum_{i=1}^{N} W_{i,t,k}$ for all i, j, k, t. Subsequently we take first differences of the demeaned variables to remove the firm-level fixed effects.

The resulting cross-sectionally demeaned SPVAR model in first differences is given by

$$\Delta Y_{i,t} = D\Delta W_{i,t} + A_1 \Delta Y_{i,t-1} + \ldots + A_p \Delta Y_{i,t-p} + \Delta u_{i,t} \ , \tag{31}$$

which effectively eliminates the fixed effects. The GMM estimator for μ relies on moment conditions that identify these parameters. Following Arellano and Bond (1991) a suitable set of moment conditions is given by

$$\mathbb{E}_F(\Delta u_{i,t} Y'_{i,t-1-l}) = 0 \ , \quad \mathbb{E}_F(\Delta u_{i,t} W'_{i,t-l}) = 0 \ , \quad l = 1, 2, \ldots, t+p-1 \quad t = 1, \ldots, T \ . \tag{32}$$

which requires the reduced form shocks to be serially uncorrelated. Exact assumptions are spelled out in Assumption S1 as stated in Appendix S3. To define the estimator for μ it is convenient to write the model in vector form. First, we define the first difference vectors

$$\underbrace{\Delta Y_i}_{T\times K} = \begin{bmatrix} \Delta Y_{i,1}' \\ \vdots \\ \Delta Y_{i,T}' \end{bmatrix} \quad \underbrace{\Delta u_i}_{T\times K} = \begin{bmatrix} \Delta u_{i,1}' \\ \vdots \\ \Delta u_{i,T}' \end{bmatrix} \quad \underbrace{\Delta X_{i,t}}_{(L+Kp)\times 1} = \begin{bmatrix} \Delta W_{i,t} \\ \Delta Y_{i,t-1} \\ \vdots \\ \Delta Y_{i,t-p} \end{bmatrix} \quad \underbrace{\Delta X_i}_{T\times(L+Kp)} = \begin{bmatrix} \Delta X_{i,1}' \\ \vdots \\ \Delta X_{i,T}' \end{bmatrix}.$$

These imply that the model for ΔY_i is given by

$$\Delta Y_i = (I_K \otimes \Delta X_i)\text{vec}(\Phi) + \Delta u_i \ .$$

The moment conditions (32) imply that the following instruments can be used for estimation.

$$Z_i = \begin{bmatrix} \tilde{Z}_{i,1}' & 0 & \dots & 0 \\ 0 & \tilde{Z}_{i,2}' & 0 & \dots \\ \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & \tilde{Z}_{i,T}' \end{bmatrix} = \begin{bmatrix} Z_{i,1}' \\ Z_{i,1}' \\ \vdots \\ Z_{i,T}' \end{bmatrix}$$

where $\tilde{Z}_{i,t} = (W_{i,-p+1}', \dots W_{i,t-1}', Y_{i,-p}', \dots Y_{i,t-2}')'$. With these instruments the efficient GMM estimator for $\phi = \text{vec}(\Phi)$ is given by

$$\hat{\phi} = \text{vec}\left\{ \left[S_{ZX}' S_{ZZ}^{-1} S_{ZX}\right]^{-1} S_{ZX}' S_{ZZ}^{-1} S_{ZY} \right\} \tag{33}$$

where $S_{ZX} = \frac{1}{N}\sum_{i=1}^N Z_i'\Delta X_i$, $S_{ZY} = \frac{1}{N}\sum_{i=1}^N Z_i'\Delta Y_i$ and $S_{ZZ} = \frac{1}{N}\sum_{i=1}^N Z_i' G Z_i$, with G a tri-diagonal matrix with two on the main diagonal and minus one of the first sub-diagonals, see Cao and Sun (2011). The variance matrix Σ is estimated by

$$\widehat{\Sigma} = \frac{1}{NT}\sum_{t=1}^T\sum_{i=1}^N \hat{u}_{i,t}\hat{u}_{i,t}' \qquad \hat{u}_{i,t} = (y_{i,t} - y_{i,\cdot}) - \widehat{\Phi}'(X_{i,t} - X_{i,\cdot}) \tag{34}$$

where $y_{i,\cdot}$ and $X_{i,\cdot}$ denote the time averages.

Cao and Sun (2011) show that, under mild regularity conditions (see Theorem S1 in the appendix for a formal statement), the estimates $\hat{\phi}$ and $\text{vech}(\widehat{\Sigma})$ are $\sqrt{N}$-consistent and have a normal limiting distribution. Recalling the definition of the reduced form parameters in (30), we have that for $N \to \infty$ the reduced form parameters satisfy

$$\sqrt{N}(\hat{\mu} - \mu) \xrightarrow{d} N(0, \Omega) \ . \tag{35}$$

To save space we provide the details for this result in Appendix S3. There we also provide an estimator for the asymptotic variance that we denote by $\widehat{\Omega}$, which satisfies $\widehat{\Omega} \xrightarrow{p} \Omega$ as $N \to \infty$.

3.2 Structural model inference

Having described the estimation of the reduced form model parameters we discuss inference for several elements of the structural model. Specifically, we show how the sign and magnitude restrictions of Proposition 1 can be used to recover: (i) structural impulse responses and (ii) the structural shocks ε_t. By combining them we can estimate the indicators for financially constrained firms in (26)-(28).

Structural impulse responses

Since the parameters $A = (A_1, \ldots, A_p)$ and B are common, the structural impulse responses are independent of i. In particular, for any given variable l, shock k and horizon h the corresponding structural impulse response is given by

$$\lambda_{l,k,h}(A, B) = e_l' C_h(A) B e_k \ ,$$

where e_l and e_k denote the l-th and k-th column of the identity matrix I_K and the $K \times K$ matrix $C_h(A)$ is defined recursively for $h = 0, \ldots, H$ by

$$C_h(A) = \sum_{m=1}^{h} C_{h-m}(A) A_m$$

with $C_0(A) = I_K$ (e.g. Kilian and Lütkepohl 2017, Chapter 4). We note that the elements of $\lambda_{l,k,0}$ correspond to the elements of B.

The matrix B is such that $\Sigma = BB'$ and it satisfies the collection of sign and magnitude restrictions given in Proposition 1. We summarize these restrictions in $\mathcal{R}(\mu)$. These restrictions are not sufficient to uniquely recover B from μ. Instead for any μ there exists a set of matrices B that satisfy the restrictions. Formally, the identified set for the structural impulse responses $\lambda_{l,k,h}(A, B)$, for $h = 0, \ldots, H$, is defined as

$$\mathcal{I}_{l,k}^{\mathcal{R}}(\mu) = \left\{ v \in \mathbb{R}^H : v_h = \lambda_{l,k,h}(A, B),\ \Sigma = BB',\ B \in \mathcal{R}(\mu) \right\} \ . \tag{36}$$

The upper and lower bounds of the elements of this set are defined by

$$\begin{array}{lll} \overline{v}(\mu) & = \sup_{B \in \mathbb{R}^{K \times K}} \lambda_{l,k,h}(A, B) & s.t. \quad \Sigma = BB',\ B \in \mathcal{R}(\mu) \\ \underline{v}(\mu) & = \inf_{B \in \mathbb{R}^{K \times K}} \lambda_{l,k,h}(A, B) & s.t. \quad \Sigma = BB',\ B \in \mathcal{R}(\mu) \end{array} \ . \tag{37}$$

Given estimates for A and Σ (as obtained in the previous section), the upper and lower bounds, for each l, k, h can be computed using numerical optimization methods (e.g. Gafarov et al. 2018). In practice, we use the SQP algorithm that is discussed in detail in Nocedal and Wright (2006). This gives an estimate for the identified sets of impulse responses.

Importantly, simply plugging in $\widehat{A}$ and $\widehat{\Sigma}$ only yields estimates for the upper and lower bounds of the set and does not take into account the estimation uncertainty of the reduced form parameters. To adjust the sets for estimation uncertainty note that the limiting distribution (35) implies the following estimate for the $1-\alpha$ confidence region for the reduced form parameters.

$$CS_N(1-\alpha,\mu) = \left\{\mu \in \mathbb{R}^{d_\mu} : N(\hat{\mu}-\mu)'\widehat{\Omega}^{-1}(\hat{\mu}-\mu) \leq \chi^2_{1-\alpha,d_\mu}\right\} \tag{38}$$

where $\chi^2_{1-\alpha,d_\mu}$ denotes the $1-\alpha$ critical value of the χ-squared distribution with d_μ degrees of freedom, with d_μ the dimension of μ. Under the assumptions stated in Appendix S3 this Wald ellipsoid covers the true reduced form parameters with probability $1-\alpha$ as $N \to \infty$.

Using the confidence region for the reduced form parameters we can construct confidence intervals around the structural impulse responses. In particular, for the impulse response $\lambda_{l,k,h}$ we have

$$CS_N(1-\alpha,\lambda_{l,k,h}) = \left[\inf_{\mu \in CS_N(1-\alpha,\mu)} \underline{v}(\mu), \sup_{\mu \in CS_N(1-\alpha,\mu)} \overline{v}(\mu)\right] , \tag{39}$$

where $\underline{v}(\mu)$ and $\overline{v}(\mu)$ are defined in (37) as the lower and upper bounds of the identified set for a given vector of reduced form parameters μ. The confidence interval (39) is based on the "worst case" values of the reduced form parameters that lie within the confidence region of the structural shocks. The computation of these confidence intervals is again done by numerical methods, the difference with the *estimates* for the identified sets is that we now also optimize over the admissible set of reduced form parameters, instead of only over B. Gafarov et al. (2018) discuss several approaches for making the optimization feasible and we follow their approach by jointly optimizing over B and μ using the aforementioned SQP algorithm.

Based on these individual confidence intervals we may define the confidence region for the vector of impulse responses $\lambda_{l,k} = (\lambda_{l,k,1}, \ldots, \lambda_{l,k,H})$, where H is the largest horizon considered, by

$$CS_N(1-\alpha,\lambda_{l,k}) = CS_N(1-\alpha,\lambda_{l,k,1}) \times \ldots \times CS_N(1-\alpha,\lambda_{l,k,H}) \subseteq \mathbb{R}^H . \tag{40}$$

Similar as in Gafarov et al. (2018) this confidence region has correct frequentist coverage

under mild assumptions. We note that this set is effectively based on a Bonferroni correction and as such it is conservative, see also Granziera et al. (2018) and Hoesch et al. (2022) who adopt similar methods to construct confidence sets in the context of SVAR models.

Structural shocks and indicators

Next, we discuss how the structural shocks can be recovered. Since, $u_{i,t} = B\varepsilon_{i,t}$, it follows that if B is known, or estimable, we may consider $\hat{\varepsilon}_{i,t} = B^{-1}\hat{u}_{i,t}$ and thus infer the structural shocks from the reduced form shocks as defined in (34). In our setting however there exists a range of matrices B that satisfy the sign and zero restrictions and thus there are multiple candidates for the structural shocks. In practice, when we wish to use the shocks for further analysis we rely on the median shock estimates and conduct robustness checks using the minimum and maximum values of the identified set, see also Baumeister and Hamilton (2019).

Given the structural shocks and the impulse responses we can estimate our indicators for identifying financially constrained firms. Specifically, the empirical counterparts of (26)-(28) are given by

$$\widehat{\mathcal{I}}^{\xi}_{i,t} = \begin{cases} 1 & \text{if} \quad \hat{\varepsilon}^{\xi}_{i,t} > \tau \\ 0 & \text{else} \end{cases} \tag{41}$$

$$\widehat{\mathcal{I}}^{b}_{i,t} = \begin{cases} 1 & \text{if} \quad \log \hat{b}^{\xi}_{i,t} > \tau \\ 0 & \text{else} \end{cases} \tag{42}$$

$$\widehat{\mathcal{I}}^{\frac{y}{l}}_{i,t} = \begin{cases} 1 & \text{if} \quad \log \hat{y}^{\xi}_{i,t} - \log \hat{l}^{\xi}_{i,t} > \tau \\ 0 & \text{else} \end{cases} \tag{43}$$

where $\log \hat{x}^{\xi}_{i,t} = \sum_{h=0}^{t-1} \lambda_{x,\xi,h} \hat{\varepsilon}^{\xi}_{i,t-h}$, with $x = b, l, y$, and the impulse responses $\lambda_{x,\xi,h}$ are typically taken as the median impulse responses of the estimated identified sets.

4 Empirical studies

In this section, after verifying the validity of our methodology on simulated data, we evaluate it on empirical firm level panel data. We focus on manufacturing firms to reduce cross industry heterogeneity. Nonetheless, the methodology could be also applied to other sectors. We first analyse a sample of US firms from Compustat. This is a useful benchmark to evaluate our methodology, because we can compare our results on the effects of financial frictions with those from other studies using the same data and period. Importantly, we can compare our financial frictions indicator with one based on narrative information. Then we apply our methodology to a large sample of firms from SABI, comprising the quasi universe

of Spanish firms. We consider an estimation procedure consistent with all the alternative models described in Section 2.3, which in practice implies adding the beginning of period stock of fixed capital as predetermined explanatory variable.

4.1 Simulated data

In the supplementary material section S4 we show the results from a simulation study. We use the structural model described in Section 2 to simulate an artificial industry, and draw a panel of $N = 10000$ firms for $T = 10$ periods. The parameters of the model are calibrated to match key moments related to the dynamics of productivity and financial variables at the firm level. In other words, we make sure that the simulated panel of firms is as much as possible realistic along the main dimensions of interest. With the simulated data we obtain three main findings: First, we show that the structural shocks identified with our methodology are good approximation of the true shocks. Second, we show that, in a realistically calibrated industry, our financial frictions indicator $\mathcal{I}_t^{\xi}$ defined in (26) is very accurate in identifying financially constrained firms. Third, these results apply regardless of whether we choose lower bound, median or upper bound structural shocks from our identified set. For a detailed illustration of these results, see Section S4.

4.2 Large US manufacturing firms

We consider manufacturing firms (SIC codes between 2000 and 3999), and we include all firm-year observations with at least 20 employees. We measure output $y_{i,t}$ with total revenues (Compustat variable *"sale"*), debt $b_{i,t}$ with the sum of short term (*"dlc"*) and long term debt (*"dltt"*), variable input $l_{i,t}$ with the cost of goods sold (*"cogs"*), fixed capital with property, plant and equipment (*"ppent"*), and financial wealth $a_{i,t}$ with cash and short term investments (*"che"*). We deflate these variables using the GDP deflator, and we winsorise the 1st and 99th percentiles of their distribution. Furthermore, we exclude outliers in the production technology by censoring the 1% tails of the distribution of ratio of cost of good sold to revenues.

We estimate the upper, median and lower bounds of the structural shocks $\hat{\varepsilon}_{i,t}$ using the following procedure. We consider the SPVAR model (29) with $p = 2$ lags and including savings and fixed capital as pre-determined variables. First, we estimate the reduced form model parameters using equations (33) and (34). Second, given the estimates we solve the programs (37) for $h = 0$ and all $l, k = 1, \ldots, K$, which corresponds to the recovery of B. We then recover the structural shocks $\hat{\varepsilon}_{i,t} = B^{-1}\hat{u}_{i,t}$ corrsponding to the median B as well as the upper and lower bounds for B. The restrictions that we impose are those derived in

Proposition 1 with $\underline{\alpha} = 0.4$ and $\overline{\alpha} = 0.8$.[27] Then we determine whether firms are financially constrained. Regarding our benchmark indicator defined in Equation (41), we select our benchmark threshold τ such that we classify as constrained the firms with the 25% highest values of $\hat{\varepsilon}^{\xi}_{i,t}$. This is based on the assumption that at any given time only a relatively small fraction of Compustat firms is financially constrained. Nonetheless, we analyse the robustness of the results to values of τ that select the 50% and 90% highest values of $\hat{\varepsilon}^{\xi}_{i,t}$. We proceed in a similar way to construct the alternative indicators defined in equations (42) and (43). Finally, notice for some empirical applications it might be more appropriate to select a threshold that is year-specific, so that τ_t selects the most constrained firms in a given year t. This is the case for example of the great recession experiment explained in the next section.

4.2.1 The great recession as a natural experiment

In this section, we verify the validity of our indicators of financial frictions with a quasi natural experiment setting, using the 2008-2009 financial crisis. One notable feature of that crisis, emphasised in a wide empirical literature, is that it was sudden and unexpected. Initial problems in financial markets became apparent during the second half of 2007, but only during 2008 financial and economic conditions began to deteriorate rapidly. The fact that the crisis was so sudden and unexpected has been exploited by many researchers to construct quasi-natural experiments where the treated sample is a group of firms more likely to be affected by the crisis because of exogenous reasons.

An example is Chodorow-Reich (2014), who exploits the fact that the financial crisis affected asymmetrically lenders depending on their exposure to the subprime market. The author argues that firms chose lenders in the pre-crisis period without knowing or evaluating the danger of such exposure. In his case, the treated sample are firms that were borrowing from lenders which during the 2008-2009 crisis were more adversely affected because of such exposure.

We propose here a pseudo-natural experiment based on the following identification strategy. Define $\hat{\varepsilon}^{\xi}_{i,2007}$ as the financial friction shock for firm i in year 2007 (computed using only balance sheet data up to 2007), and $\widehat{\mathcal{I}}^{\xi}_{i,2007}$ as the associated financial frictions indicator. Firms selected with $\widehat{\mathcal{I}}^{\xi}_{i,2007} = 1$ are facing difficulties in accessing external finance *before* the crisis, and hence we expect them to be more negatively affected by the onset of the financial crisis in 2008 than the other firms, because in that period, given the sudden freeze of financial markets, it was practically impossible to find alternative sources of financing.

[27] In practice this means imposing the very mild assumption that the output elasticity to the variable input is between 0.4 and 0.8. In our sample, the firm-level median value of the ratio between cost of goods sold and sales is 0.66, and the 25th and 75th percentiles are 0.48 and 0.79, respectively.

More specifically, our hypothesis is that firms identified at the end of 2007 as financially constrained will reduce their employment in 2008 relatively more than the other firms, more so than the difference in employment decisions between constrained and unconstrained firms for any year t before 2007.

To implement this test, we run a regression where the dependent variable is the log of employment in period t. Among the regressors, we include the lagged dependent variable, several lagged control variables, and the lagged financial constraints indicator $\widehat{\mathcal{I}}^{\xi}_{i,t-1}$. All regressors are also interacted with the dummy Gr, equal to one for the year 2008 and equal to zero for the previous years. Therefore, the coefficient of $\widehat{\mathcal{I}}^{\xi}_{i,t-1}$ indicates whether the most constrained firms in year t-1 have an employment policy in year t different from the other firms, and the coefficient of $\widehat{\mathcal{I}}^{\xi}_{i,t-1} * Gr$ indicates whether this relation is different when $t = 2008$ than when $t < 2008$.

Our hypothesis is that the coefficient of $\widehat{\mathcal{I}}^{\xi}_{i,t-1} * Gr$ is negative and significant. We run these regressions on a balanced sample starting in 1992, and therefore comprising all firms that were continuously in operation in the 16 years before 2008. This excludes the youngest firms and thus likely weakens our test, but makes the estimation of the $\widehat{\mathcal{I}}^{\xi}_{i,t-1} * Gr$ coefficient more meaningful, because it compares the year 2008 with a relatively long period which includes both past recessionary and expansionary periods.[28] Importantly, for the estimation of $\widehat{\mathcal{I}}^{\xi}_{i,t-1}$ we only consider information up to 2007.

Table 1 shows the estimation results. Column 1 includes as independent variables those described above plus sector-year and firm fixed effect. The coefficient of $\widehat{\mathcal{I}}^{\xi}_{i,t-1}$ is not significant and very close to zero, indicating that over the whole period there is no systematic relation between the financial friction indicator in period $t-1$ and employment growth in the next period. This result is consistent with the widely held view that, in normal times, large public US companies are subject to few financial frictions. Conversely, $\widehat{\mathcal{I}}^{\xi}_{i,t-1} * Gr$ is quantitatively large, negative, and significant, indicating that the group of most financially constrained firms in 2007 contracted their employment in 2008 by around 6.7% more than the other firms. This result is robust to sector-year fixed effect. In the other columns of this table we introduce additional control variables that rule out alternative explanations. Since firms with $\widehat{\mathcal{I}}^{\xi}_{i,t-1} = 1$ are in part identified with declines in output and inputs, the indicator might simply capture unproductive firms. Therefore, in Column 2 we include an alternative indicator $\widehat{\mathcal{I}}^{z}_{i,t-1}$ which is equal to one for the 25% of firms with lowest productivity shock $\hat{\varepsilon}^{z}_{i,t-1}$ and zero otherwise. The coefficient of $\widehat{\mathcal{I}}^{z}_{i,t-1}$ is negative, indicating that less productive firms in year $t-1$ tend to reduce employment in the next period. Its interaction with the Gr dummy is also negative, but more importantly the coefficient of $\widehat{\mathcal{I}}^{\xi}_{i,t-1} * Gr$ remains large,

[28] Furthermore, to limit the possibility that results are driven by abnormal changes in employment caused by exceptional events, we exclude the 1% tails of the distribution of yearly employment changes.

negative, and significant, consistently with our hypothesis that it captures financially constrained rather than unproductive firms. In Column 3, we control for other characteristics that are normally related to financial frictions, such as dummies selecting the 25% smallest, least productive and with highest leverage firms in period $t-1$. In Column 4 we also include labour productivity and leverage in period $t-1$ in levels. All these variables are also interacted with the dummy Gr. We find that controlling for all these characteristics changes little the magnitude of the coefficient of $\widehat{\mathcal{I}}^{\xi}_{i,t-1} * Gr$. In other words, our measure of financial frictions does not simply capture factors such as leverage, size and productivity. Finally, in the last column of Table 1 we substitute the indicator $\widehat{\mathcal{I}}^{\xi}_{i,t-1} * Gr$ with the actual value of $\hat{\varepsilon}^{\xi}_{i,t-1}$. Its interaction with the Gr dummy is also negative and significant, confirming the previous result.

In order to address the concern that the results show in Table 1 might have been obtained by chance, Table 2 performs a placebo experiment. We repeat the estimation shown above in column (2) of Table 1, but in a different time frame. The first column, denoted with "2008", is the experiment described above, in which the coefficient of $\widehat{\mathcal{I}}^{\xi}_{i,t-1} * Gr$ measures the effects of financial frictions in 2007 for employment growth in 2008. The next column (2) is a "2007" placebo experiment in which we repeat the same procedure described above on a balanced sample of firms with data up to 2007. In this case the coefficient of $\widehat{\mathcal{I}}^{\xi}_{i,t-1} * Gr$ measures the effects of financial frictions in 2006 for employment growth in 2007. In the following columns (3) to (8) we consider placebo experiments from 2001 to 2006. Since none of these years witnessed a financial shock as large as the one in 2008, we should find the coefficient of $\widehat{\mathcal{I}}^{\xi}_{i,t-1} * Gr$ to be smaller with respect to the one in column 1, and indeed we find that this is the case. In columns (2)-(8), the coefficient is either not statistically significant, or marginally significant, and always much smaller than in the first column. This evidence reinforces our conclusion that $\widehat{\mathcal{I}}^{\xi}_{t}$ is a valid measure of financial frictions.

Table 3 considers the alternative indicators described in equations (42) and (43). Columns (1) and (3) are analogous to column 1 in Table 1, but using the alternative indicators $\widehat{\mathcal{I}}^{\frac{y}{l}}_{i,t-1}$ and $\widehat{\mathcal{I}}^{b}_{i,t-1}$, respectively, as indicators of financial frictions. Furthermore Columns (2) and (4) consider the continuous values $\hat{b}^{\xi}_{i,t-1}$ and $(\frac{\hat{y}}{l})^{\xi}_{i,t-1}$ and are therefore analogous to column (5) of Table 1. We find that these alternative indicators generate qualitatively similar results than our benchmark indicator $\widehat{\mathcal{I}}^{\xi}_{i,t-1}$.

Finally, in Appendix S5 we provide some additional robustness checks. Table S4 considers alternative values of the threshold τ to compute the financial frictions indicator (see equation 41), and we find that the coefficient of $\widehat{\mathcal{I}}^{\xi}_{i,t-1} * Gr$ is always negative and significant, and is larger the higher the threshold is. Table S5 considers alternative regressions in which $\widehat{\mathcal{I}}^{\xi}_{i,t}$ is computed using the lower and upper bound values of the identified set of $\hat{\varepsilon}^{\xi}_{i,t-1}$, rather than the median value used for Table 1, and finds similar results, with the coefficient of $\widehat{\mathcal{I}}^{\xi}_{i,t-1} * Gr$

Table 1: Financial frictions in 2007 and employment contraction in 2008

	(1)	(2)	(3)	(4)	(5)
$\log(l_{t-1})$	0.896***	0.888***	0.888***	0.891***	0.894***
	(99.962)	(97.533)	(100.554)	(102.903)	(101.970)
$\log(l_{t-1}) * Gr$	-0.006	-0.005	0.006	0.005	0.007
	(-1.577)	(-1.451)	(1.230)	(1.131)	(1.533)
$\mathcal{I}^{\xi}_{t-1}$	0.003	0.006	0.002	-0.004	
	(0.575)	(1.087)	(0.451)	(-0.680)	
$\mathcal{I}^{\xi}_{t-1} * Gr$	-0.067***	-0.062***	-0.061***	-0.059***	
	(-3.967)	(-3.579)	(-3.432)	(-3.142)	
$\mathcal{I}^{Z}_{t-1}$		-0.035***	-0.033***	-0.030***	
		(-6.510)	(-6.271)	(-5.705)	
$\mathcal{I}^{Z}_{t-1} * Gr$		-0.016	-0.018	-0.016	
		(-0.862)	(-0.961)	(-0.861)	
$Small_{t-1}$			-0.043	-0.039	-0.026
			(-1.260)	(-1.201)	(-0.929)
$Small_{t-1} * Gr$			0.062***	0.060***	0.066***
			(2.713)	(2.621)	(2.798)
$Highlev_{t-1}$			-0.025***	-0.014*	-0.014*
			(-3.443)	(-1.743)	(-1.752)
$Highlev_{t-1} * Gr$			0.003	-0.008	0.002
			(0.200)	(-0.425)	(0.090)
$Lowprod_{t-1}$			-0.021**	-0.014	-0.015
			(-2.256)	(-1.371)	(-1.543)
$Lowprod_{t-1} * Gr$			0.021	0.046**	0.034*
			(1.242)	(2.362)	(1.700)
$labp_{t-1}$				0.022*	0.036***
				(1.730)	(2.766)
$labp_{t-1} * GR$				0.028*	0.025*
				(1.788)	(1.673)
lev_{t-1}				-0.010**	-0.016***
				(-2.357)	(-3.512)
$lev_{t-1} * GR$				0.007	0.005
				(1.147)	(0.677)
$\hat{\varepsilon}^{\xi}_{t-1}$					-0.011***
					(-3.548)
$\hat{\varepsilon}^{\xi}_{t-1} * Gr$					-0.022***
					(-3.002)
Obs.	6,710	6,710	6,710	6,710	6,710
R^2	0.858	0.859	0.860	0.860	0.860
Number of firm	506	506	506	506	506

Notes: The table shows the differential effects of the great recession on financially constrained firms. Firm fixed effects and sector-year fixed effects are included in all specifications. The dependent variable is the logarithm of the number of employees in year t, $\log(l_t)$. Among the regressors, $\widehat{\mathcal{I}}^{\xi}_{t-1}$ is a dummy variable equal to one for the upper quartile of the financial friction shock $\hat{\varepsilon}^{\xi}_{t-1}$ in year $t-1$ and zero otherwise. $\widehat{\mathcal{I}}^{Z}_{t-1}$ is a dummy variable equal to one for the lower quartile of the productivity shock $\hat{z}_{t-1}$ in year $t-1$ and zero otherwise. Gr is a dummy variable equal to 1 for year 2008 and equal to zero otherwise. lev_{t-1} is total debt over fixed assets, and $labp_{t-1}$ is labour productivity (measured as real output divided by number of employees). The variables $highlev_{t-1}$ and $lowprod_{t-1}$, are equal to one for the 25% firm-year observations with highest leverage and lower labour productivity in year $t-1$, respectively, and equal to zero otherwise. $Small_{t-1}$ is a dummy variable if the firm belongs to the quartile of smallest firms in period $t-1$, and equal to zero otherwise. Standard errors are clustered at the firm level. Robust t-statistics are given in parentheses and *** p<0.01, ** p<0.05, * p<0.1.

Table 2: Placebo test

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
VARIABLES	2008	2007	2006	2005	2004	2003	2002	2001
$\mathcal{I}^{\xi}_{t-1}$	0.006	0.004	0.009**	0.009**	0.008*	0.004	0.003	0.002
	(1.087)	(0.892)	(2.110)	(2.135)	(1.805)	(0.925)	(0.706)	(0.356)
$\mathcal{I}^{\xi}_{t-1} * Gr$	-0.062***	-0.010	-0.022	-0.021	-0.010	-0.030*	-0.005	0.005
	(-3.579)	(-0.600)	(-1.277)	(-1.305)	(-0.748)	(-1.827)	(-0.285)	(0.239)
Obs.	6,710	6,880	7,125	7,435	7,632	7,802	7,957	8,090
R^2	0.859	0.858	0.857	0.856	0.860	0.853	0.850	0.847
Number of firm	506	519	537	563	577	591	603	613

Notes: The table shows the differential effects of the great recession on financially constrained firms. Firm fixed effects and sector-year fixed effects are included in all specifications. Column (1) replicates the results from Column (1) in Table 1. See the footnote of Table 1 for details. Column (2) is a "2007" placebo experiment in which we repeat the same procedure described above on a balanced sample of firms with data from from 1990 to 2007, instead of from 1991 to 2008. In this case the coefficient of $\widehat{\mathcal{I}}^{\xi}_{t-1} * Gr$ measures the effects of financial frictions in 2006 for employment growth in 2007. Columns (3)-(8) are placebo experiments in which the coefficient of $\widehat{\mathcal{I}}^{\xi}_{t-1} * Gr$ measures the effects of financial frictions in 2000-2005 for employment growth in 2001-2006, respectively. Standard errors are clustered at the firm level. Robust t-statistics are given in parentheses and *** p<0.01, ** p<0.05, * p<0.1.

being negative and statistically significant, even though slightly smaller in magnitude.

4.2.2 Consistency with narrative indicators

In this section, we compare our indicators of financial frictions with those constructed using narrative methods. In particular, we consider the indicator proposed by Hoberg and Maksimovic (2014). These authors perform an automatic text analysis of the "Liquidity and Capital Resources" section of the 10-K reports of public US firms. They look for a set of keywords that are related to "delay", and that are relatively close in the text to another set of keywords related to investment plans, and derive a quantitative index, called *delaycon*, that measures the likelihood firms signal the intention to delay their investment plans because of liquidity issues. The index measures the severity of this problem in the cardinal sense, meaning that it can be used to rank firms according to the severity of their constraints (both cross sectionally and over time), and it is publicly available.[29]

Further, the authors refine this indicator with additional information on the firms financing plans. The indicator *debtdelaycon* is high if additional keywords indicate the firms will likely address these liquidity problems by issuing debt. Conversely, the indicator *equitydelaycon* signals the firms will likely address them by issuing equity.

In terms of the power of the *delaycon* indicator to detect financially constrained firms, the authors argue that because the keywords related to delay and investment are mentioned

[29]See http://faculty.marshall.usc.edu/Gerard-Hoberg/MaxDataSite/index.html. The authors initially define a set of firms identified as financially constrained while the other firms are considered not financially constrained. then they construct a continuous version of this indicator using cosine similarity methods.

Table 3: Financial frictions in 2007 and employment contraction in 2008 - Alternative indicators

	(1)	(2)	(3)	(4)
$\log(l_{t-1})$	0.904***	0.906***	0.902***	0.906***
	(98.981)	(90.676)	(98.012)	(90.411)
$\log(l_{t-1}) * Gr$	-0.005	-0.005	-0.005	-0.006
	(-1.347)	(-1.461)	(-1.319)	(-1.476)
$\mathcal{I}_{t-1}^{\frac{y}{l}}$	0.020***			
	(3.549)			
$\mathcal{I}_{t-1}^{\frac{y}{l}} * Gr$	-0.041**			
	(-2.541)			
$(\frac{\hat{y}}{l})_{t-1}^{\xi}$		0.131***		
		(3.265)		
$(\frac{\hat{y}}{l})_{t-1}^{\xi} * Gr$		-0.272***		
		(-4.032)		
$\mathcal{I}_{t-1}^{b}$			0.014**	
			(2.476)	
$\mathcal{I}_{t-1}^{b} * Gr$			-0.038**	
			(-2.317)	
$\hat{b}_{t-1}^{\xi}$				0.014***
				(3.225)
$\hat{b}_{t-1}^{\xi} * Gr$				-0.030***
				(-3.898)
Obs.	6,710	6,710	6,710	6,710
R^2	0.858	0.858	0.858	0.858
Number of firm	506	506	506	506

Notes: The table shows the differential effects of the great recession on financially constrained firms. Firm fixed effects and sector-year fixed effects are included in all specifications. The dependent variable is the logarithm of the number of employees in year t, $\log(l_t)$. Among the regressors, $\widehat{\mathcal{I}}_{t-1}^{\frac{y}{l}}$ is equal to one if the value of $(\frac{\hat{y}}{l})_{t-1}^{\xi}$, which is the part of labour productivity explained by past financial frictions shocks, is larger than the 75th percentile and zero otherwise. $\widehat{\mathcal{I}}_{t-1}^{b}$ is equal to one if $\hat{b}_{t-1}^{\xi}$, which is the part of debt explained by the past financial frictions shocks is smaller than the 25th percentile, and zero otherwise. Standard errors are clustered at the firm level. Robust t-statistics are given in parentheses and *** p<0.01, ** p<0.05, * p<0.1.

in the "Capitalization and Liquidity" subsection, it follows by context that these firms are delaying investment due to liquidity problems. However lacking an explicit causal link from liquidity issues to capital investment decisions, the concern remains that the liquidity problems might be related to timing issues rather than financial constraints that significantly raise the cost of external finance.

In this respect, we think the subindex *equitydelaycon* is a more precise indicator, because the well-known pecking order theory implies that issuing equity is on average more costly than issuing debt. Therefore a firm identified as having liquidity problems in financing investment, and planning to issue equity, is likely to be constrained in issuing debt, which should be correlated to a positive financial frictions shock $\varepsilon^{\xi}_{i,t}$. Conversely, the subindex *debtdelaycon* is potentially problematic for our purpose, since a firm that is able to issue debt to address liquidity problems is not likely to be substantially financially constrained. The validity of the subindex *equitydelaycon* is also highlighted by the authors who report that it is more strongly correlated to their narrative evidence on investment delay than *debtdelaycon*. Furthermore, *equitydelaycon* is positively correlated to the Whited and Wu (2006) index of financial frictions, more so than the main *delaycon* index, while *debtdelaycon* is negative correlated (see Table 2 in Hoberg and Maksimovic (2014)).

Another issue with this indicator is that by construction it might mostly capture firms constrained in fixed capital investment, because of the type of keywords used by the authors, which focus on expansion plans and capital investment (see Hoberg and Maksimovic 2014, page 1321). Therefore, firms that do not have expansion plans but are constrained and forced to reduce their variable inputs such as wages and materials, might not be captured by this indicator.

This is not to say that the narrative indicator *equitydelaycon* is flawed, rather that by design it likely focuses on constraints to fixed capital expansion, differently from our measure, which can potentially capture financial frictions affecting both expanding and contracting firms. We control for this potential difference by performing regressions in which we only include firm-year observations in which firms did not reduce their stock of fixed capital.

In Table 4 we show regression results where the dependent variable is *equitydelaycon* and the main explanatory variable is our benchmark financial frictions indicator $\widehat{\mathcal{I}}^{\xi}_{i,t}$. Standard errors are clustered at the firm level, and all regressions include Sector-Year fixed effects. In column 1, we consider the full sample, which comprises all manufacturing firms with at least 3 observations in the 1997-2015 period (the years for which the narrative data is available). In column 2 we consider the sub-sample that excludes firm-year observations in which firms reduced their stock of fixed capital. And in column 3 we consider the complementary sub-sample. The coefficient of $\widehat{\mathcal{I}}^{\xi}_{i,t}$ is positive and significant for the whole sample, and columns 2 and 3 show that the correlation is entirely driven by periods in which the firms did not

reduce their stock of fixed assets, which is consistent with the view that *equitydelaycon* is mainly capturing expansion constraints. Columns 3-6 repeat the same regressions in columns 1-3, adding the same control variables added in Table 1 plus a variable that measures firms size (log of fixed assets). The coefficient of $\widehat{\mathcal{I}}^{\xi}_{i,t}$ becomes smaller in magnitude but it is still statistically significant for the non-contracting firms in column 5.

Table 4: Consistency with narrative indicators of financial frictions.

	(1)	(2)	(3)	(4)	(5)	(6)
$\widehat{\mathcal{I}}^{\xi}_{t}$	0.003*	0.007***	0.000	0.002	0.006**	-0.001
	(1.663)	(2.630)	(0.077)	(1.023)	(2.260)	(-0.473)
$small_t$				0.028***	0.026***	0.031***
				(5.202)	(4.650)	(4.486)
$highlev_t$				0.000	-0.000	0.006
				(0.107)	(-0.100)	(1.201)
$lowprod_t$				0.018***	0.017***	0.021***
				(4.958)	(4.308)	(4.201)
$log(fixedassets_t)$				0.002	0.001	0.002
				(1.224)	(0.778)	(1.137)
lev_t				0.001	-0.000	0.001
				(1.120)	(-0.265)	(1.278)
$labp_t$				-0.000	-0.000	-0.000
				(-0.616)	(-0.356)	(-0.715)
Obs.	13,697	8,147	5,550	13,557	8,063	5,494
R^2	0.072	0.097	0.078	0.096	0.117	0.112

Notes: The table shows the results from regressing a narrative indicator on financial frictions on the indicator computed using our procedure. The dependent variable *equitydelaycon* is a quantitative indicator which is higher the more likely the firm is to delay investment decisions because of liquidity problems, which the firm plan to address them by issuing equity (see Hoberg and Maksimovic (2014) for details). Among the regressors, $\widehat{\mathcal{I}}^{\xi}_{t}$ is a dummy variable equal to one for the upper quartile of the financial friction shock $\hat{\varepsilon}^{\xi}_{t}$ in year t and zero otherwise. lev_t is total debt over fixed assets, and $labp_t$ is labour productivity (measured as real output divided by number of employees). The variables $highlev_t$ and $lowprod_t$, are equal to one for the 25% firm-year observations with highest leverage and lower labour productivity in year $t-1$, respectively, and equal to zero otherwise. $Small_t$ is a dummy variable if the firm belongs to the quartile of smallest firms in period t, and equal to zero otherwise. All specifications include a constant and sector-year fixed effects. Standard errors are clustered at the firm level. Robust t-statistics are given in parentheses and *** p<0.01, ** p<0.05, * p<0.1.

Table 5 repeats the analysis of the first three columns of Table 4 for different thresholds to compute $\widehat{\mathcal{I}}^{\xi}_{i,t}$. Its coefficient more than doubles when moving from the benchmark indicator (middle panel) to the one that identifies financially constrained firms with those with a financial frictions shock above the 90th percentile (lower panel). One plausible explanation is

that in normal times few Compustat firms are financially constrained, and the 90th percentile is a more precise threshold to identify them. In terms of magnitude, the interquartile range of the dependent variable *equitydelaycon* is equal to 0.1. Therefore, with respect to this range, belonging to the most constrained firms according to column 2 (lower panel), increases the value of the narrative indicator by 16%.

Table 5: Consistency with narrative indicators of financial frictions, alternative thresholds.

$\widehat{\mathcal{I}}_t^{\xi} = 1$ if $\varepsilon_t^{\xi} > 50\%$			
	(1)	(2)	(3)
$\widehat{\mathcal{I}}_t^{\xi}$	0.001	0.003	-0.001
	(0.815)	(1.501)	(-0.424)
Obs.	13,697	8,147	5,550
R^2	0.072	0.097	0.078
$\widehat{\mathcal{I}}_t^{\xi} = 1$ if $\varepsilon_t^{\xi} > 75\%$(Benchmark)			
	(1)	(2)	(3)
$\widehat{\mathcal{I}}_t^{\xi}$	0.003*	0.007***	0.000
	(1.663)	(2.630)	(0.077)
Obs.	13,697	8,147	5,550
R^2	0.072	0.097	0.078
$\widehat{\mathcal{I}}_t^{\xi} = 1$ if $\varepsilon_t^{\xi} > 90\%$			
	(1)	(2)	(3)
$\widehat{\mathcal{I}}_t^{\xi}$	0.012***	0.016***	0.009**
	(3.829)	(3.506)	(2.173)
Obs.	13,697	8,147	5,550
R^2	0.073	0.099	0.080

Notes: The table shows the results from regressing a narrative indicator on financial frictions on the indicator computed using our procedure. The dependent variable *equitydelaycon* is a quantitative indicator which is higher the more likely the firm is to delay investment decisions because of liquidity problems, which the firm plan to address them by issuing equity (see Hoberg and Maksimovic (2014) for details). $\widehat{\mathcal{I}}_t^{\xi}$ is a dummy variable equal to one for the upper quartile of the financial friction shock $\hat{\varepsilon}_t^{\xi}$ in year $t-1$ and zero otherwise. All specifications include a constant and sector-year fixed effects. Standard errors are clustered at the firm level. Robust t-statistics are given in parentheses and *** p<0.01, ** p<0.05, * p<0.1.

4.3 Quasi-universe of Spanish manufacturing firms

The evidence in the previous sections, based on Compustat data, has shown that our financial frictions indicator is consistent with alternative ways to identify financially constrained firms.

Importantly, it has also shown that it is able to identify both constraints that cause firms to contract their activity, as in the great recession experiment, and those that cause firms to limit their expansion, as those identified by the narrative indicator of Hoberg and Maksimovic (2014).

While useful for comparison purposes, it must be noted that Compustat is a sample of large public companies, and therefore it is interesting to verify our methodology also on more comprehensive data that include smaller firms. Indeed one of the main motivations of this paper is to provide a new indicator that can be computed for the latter group, since it requires only balance sheet data. Therefore, in this section we apply our methodology on a sample of Spanish manufacturing firms from SABI (Sistema de Análisis de Balances Ibéricos), which includes the quasi-universe of Spanish firms with more than 5 employees. After applying some basic filtering (dropping firms-year observations with missing assets and revenues and employment data), we obtain a sample of around 45.000 observations every year for the 2000-2018 period, for a total of more than 860.000 observations. 67% of all the observations are on firms smaller than 20 employees, 31% on firms between 20 and 250 employees, and 1.6% on firms larger than 250 employees.

We first perform the Great Recession Exercise in Table 1 also on the Spanish sample, obtaining similar results to those of the Compustat sample. Details are in the Appendix, see Table S6 in Section S5. Here we are interested in describing two additional tests that take advantage of the larger heterogeneity of the Spanish sample. In particular, we focus on the magnitude and persistence of the effects of financial frictions shocks, verifying the following hypotheses:

Hypothesis 1: If $\hat{\varepsilon}^{\xi}_{i,t}$ measures financial frictions shocks, we expect it to explain a larger part of employment volatility for small relative to large firms.

Hypothesis 2: If $\hat{\varepsilon}^{\xi}_{i,t}$ measures financial frictions shocks, we expect its effect to be more persistent for small relative to large firms.

The intuition for the first hypothesis is that smaller firms are more likely to be subject to credit shocks, because of a "flight to quality" effect. When a lender is constrained in its funding, it will prioritize lending to its larger customers, insulating them from fluctuations in the availability of credit, and leaving the smaller customers more exposed.

The intuition for the second hypothesis is that large firms not only have better access to their main lender, but also relations with multiple banks. If in a given period the firm is forced to reduce its employment level because of unexpected problems in accessing external financing, it is expected that it will be able to find alternative sources relatively quickly. Conversely, many smaller firms typically rely only on one main lender. If they face an increase in financing problems, they will be much less likely to find quickly suitable alternatives.

To test Hypotheses 1 and 2, we measure to what extent financial frictions shocks $\hat{\varepsilon}^{\xi}_{i,t}$

explain employment variations at different time horizons. In practice we run several panel regressions with firm fixed effects in which $\hat{\varepsilon}^{\xi}_{i,t}$ is the main explanatory variable, and the dependent variable is the log of employment at different time horizons. The statistic we use to verify the hypotheses is the *within* R^2 statistic, which measures the share of within-firm variability in the dependent variable explained by $\hat{\varepsilon}^{\xi}_{i,t}$. For this purpose, both dependent and independent variables are are deviations from 2digit sector-year averages, to ensure the within R2 statistic is not affected by sector-year factors.

The regressions results are shown in Panel A of Table 6. For completeness, in addition to the *within* R^2 statistic we also report the standardized regression coefficient of $\hat{\varepsilon}^{\xi}_{i,t}$.[30] For comparison, Panel B of Table 6 shows the same statistics for the productivity shock $\hat{\varepsilon}^{z}_{i,t}$. In both cases we report regression results for all firms and for different size groups.

The results relative to all firms show that on impact (first column) the financial frictions shock $\hat{\varepsilon}^{\xi}_{i,t}$ explains a larger fraction of the variation in employment than the productivity shock $\hat{\varepsilon}^{z}_{i,t}$ (14.2% versus 6.5%). This difference is entirely driven by the smaller group of firms (14.2% versus 6.4%), while the opposite is true for larger firms. Looking at the effect on impact for firms larger than 500 employees, the productivity shocks explains a larger fraction of the variation in employment than the financial frictions shocks (21% versus 10%), confirming Hypothesis 1. Notice that this does not imply that productivity or demand improvements are not important for the employment growth of small firms. In our procedure for estimating the shocks we control for year fixed effects, thus abstracting from common long run growth factors.

Looking at the other columns, we see that on average for the whole sample the productivity shocks tend to be more persistent and to explain a larger fraction of the variation in employment at longer horizons than the financial shock. Intuitively, productivity shocks have, by their nature, persistent effects. Think for example about a firm that introduces a new line of products. This boosts revenues in the short term and the effect is expected to be persistent, as the firm builds a customer base for the new products. Conversely, financial shocks are less persistent because generate costly distortions, and the firms act (by finding alternative financing sources, or increasing retained earnings) to mitigate their long run effects. Importantly, Panel A shows that the effects of financial friction shocks are more persistent for smaller than for larger firms, consistently with Hypothesis 2.

[30] We standardize the coefficients by subtracting firm-level means from both dependent and independent variables and then dividing by the group level standard deviations, where the groups are the three size categories of firms. This standardization procedure is irrelevant for the within R^2 statistic. In a robustness exercise, not reported here, we also subtract from both dependent and independent variables the sector-year effects, and then we repeat the analysis, finding very similar results to those reported in Table 6.

Table 6: Local Projections of the effect of shocks on small and large firms.

	(1) $\log(l_t)$	(2) $\log(l_{t+1})$	(3) $\log(l_{t+2})$	(4) $\log(l_{t+3})$	(5) $\log(l_{t+4})$	(6) $\log(l_{t+5})$
Panel A, independent variable is $\hat{\varepsilon}^{\xi}_{i,t}$						
All firms	-0.316***	-0.269***	-0.199***	-0.143***	-0.095***	-0.055***
	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)
within R^2	0.142	0.106	0.0595	0.0311	0.0143	0.00496
Less than 250 employees	-0.317***	-0.270***	-0.200***	-0.143***	-0.095***	-0.055***
	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)
within R^2	0.142	0.106	0.0597	0.0311	0.0144	0.00499
between 250 and 500 empl.	-0.246***	-0.219***	-0.151***	-0.128***	-0.076***	-0.034**
	(0.028)	(0.024)	(0.021)	(0.022)	(0.016)	(0.013)
within R^2	0.112	0.0894	0.0473	0.0336	0.0125	0.00253
Larger than 500 empl.	-0.239***	-0.172***	-0.106***	-0.037	-0.012	0.030
	(0.042)	(0.041)	(0.033)	(0.025)	(0.026)	(0.023)
within R^2	0.100	0.0510	0.0212	0.00277	0.000299	0.00202
Panel B, independent variable is $\hat{\varepsilon}^{z}_{i,t}$						
All firms	0.217***	0.221***	0.191***	0.158***	0.125***	0.091***
	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)
within R^2	0.0648	0.0676	0.0511	0.0352	0.0227	0.0124
Less than 250 employees	0.217***	0.221***	0.191***	0.158***	0.125***	0.091***
	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)
within	0.0644	0.0673	0.0509	0.0351	0.0227	0.0124
between 250 and 500 empl.	0.251***	0.229***	0.187***	0.149***	0.101***	0.069***
	(0.030)	(0.028)	(0.023)	(0.022)	(0.016)	(0.015)
withinR^2	0.112	0.0953	0.0701	0.0435	0.0218	0.0100
Larger than 500 empl.	0.345***	0.328***	0.249***	0.177***	0.090***	0.044*
	(0.043)	(0.047)	(0.044)	(0.032)	(0.027)	(0.026)
withinR^2	0.210	0.185	0.111	0.061	0.016	0.004

Notes: All variables are deviations from 2digit sector-year averages. The dependent variable is the logarithm of the number of employees. From the first to last column, the time horizon increases from 0 years, with $\log(l_t)$ as dependent variable, to 5 years, with $\log(l_{t+5})$ as dependent variable. Firm fixed effects are also included. Standard errors are clustered at the firm level. Robust t-statistics are given in parentheses and *** p<0.01, ** p<0.05, * p<0.1.

5 Conclusion

We develop a novel approach to identify and estimate financial shocks and the intensity of financial frictions at the firm level. Our methodology is inspired by the sign-based identification literature for aggregate time series (e.g. Uhlig 2005). Its key advantage is that it is easy to implement, and that it relies on a limited set of identifying restrictions. In particular we show that, imposing only mild assumptions, which are consistent with a wide class of theoretical models, we can simultaneously recover firm's productivity, profitability and financial shocks and the latent intensity of financial frictions. We validate our methodology on simulated data from the structural model. Furthermore, we apply it on both Compustat data and on the quasi universe of Spanish manufacturing firms above 5 employees, providing several empirical tests of its validity.

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Supplementary Material for: Identifying Firm-Level Financial Frictions using Theory-Informed Restrictions (For Online Publication)

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Abstract

This supplement contains:

S1: Proof of Proposition 1

S2: Extended model derivations

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S1 Proofs

Proof of Proposition 1

Here we prove the sign restrictions in (16) and the magnitude restrictions in (17). From the budget constraint (2) we obtain:

$$\Theta(b_t, \xi_t) = wl_t + \text{div}_t - (s_t - \theta_t F) \tag{S1}$$

The net funds borrowed, $\Theta(b_t, \xi_t)$, cover the difference between financial wealth net of the overhead costs, $s_t - \theta_t F$, and the expenditures for wages wl_t and dividends div_t. As discussed in Section 2.1, the discount term $\mu > 1$ is an incentive for the firm to borrow to distribute dividends. It follows that in the steady state the firm is a net borrower, and $\Theta(b_t, \xi_t)$ is strictly positive. To see this, consider a firm that is unconstrained, and is able to finance its wage bill with internal finance, without borrowing. In this case, keeping one unit of resources as savings generates $1 + r$ units next period, and hence its net present value is $(1 + r)(\frac{1}{1+r}\frac{1}{\mu}) = \frac{1}{\mu} < 1$. Since the value to distribute this unit as dividends in the current period t is equal to 1, it follows that the firm finds it optimal to distribute all its savings as dividends. Then the firm will start to borrow to increase dividends further, and in doing so its cost of borrowing increases above $1 + r$, because of financing frictions, until it becomes so large that the firm will stop increasing dividends.

Furthermore, notice that the functional form in (4) implies that the elasticity of the amount borrowed $\Theta(b_t, \xi_t)$ with respect to the financial frictions process ξ_t is negative. Moreover, the function $\Theta(b_t, \xi_t)$ is concave and increasing in b_t, and reaches the maximum value for $b_t = b_t^M \equiv \left[\frac{1}{1+r}\frac{1}{\gamma\xi_t}\right]^{\frac{1}{\gamma-1}}$. The firm will never optimally choose a face value of debt larger than b_t^M because it would imply increasing the promised payment next period to the creditors while actually receiving less funds today. Therefore, a necessary condition for the optimal choice of debt is:

$$\frac{\partial \Theta_t}{\partial b_t}\frac{b_t}{\Theta_t} > 0 \ . \tag{S2}$$

We log-linearize $\Theta(b_t, \xi_t)$ and obtain

$$\log \Theta_t = \pi_b \log b_t - \pi_\xi \log \xi_t \ , \tag{S3}$$

where the above discussion clarifies that that $\pi_b > 0$ and $\pi_\xi > 0$.

Before log-linearising the budget constraint, we need to define the optimal dividend policy.

In equilibrium the firm decides dividends balancing the need to distribute earnings early, because of the high discount factor, with the need to save to accumulate wealth and reduce future financial frictions. Conditional on current wealth s_t, a sufficient statistic for the opportunity cost to distribute dividends rather than saving is the shadow cost of finance ψ_t. For example, if the firm is suddenly more productive and needs to increase production inputs, this will increase ψ_t in equilibrium signalling an higher cost of distributing dividends and hence reducing available resources to invest. Therefore, conditional on current financial wealth s_t, an increase in the shadow cost of finance ψ_t implies the firm desires to reduce dividends. Hence a log-linear approximation of the dividend policy can be derived as follows:

$$\log \mathrm{div}_t = \pi^s_{div} \log s_t - \pi_{div} \log \psi_t \ , \tag{S4}$$

where $\pi_{div} \geq 0$ and $\pi^s_{div} \geq 0$.These elasticities are equal to zero when the dividend constraint (3) is binding, otherwise they are positive. Log-linearising (S1) around the steady state and using (S3) and (S4) yields:

$$\pi_b \log b_t - \pi_\xi \log \xi_t = -\left(\frac{s - \overline{div}\pi^s_{div}}{\Theta}\right) \log s_t + \frac{F}{\Theta} \log \theta_t + \frac{wl}{\Theta} \log l_t - \overline{div}\pi_{div} \log \psi_t \ ,$$

where $\overline{div}$ is the (positive) steady state value of dividends, and as discussed above, Θ is positive. We substitute $\log l_t$ and $\log b_t$ using (10) and solve for $\log \psi_t$. The comparison with Equation (11) delivers the solution for the coefficients π_2, π_3 and π_4:

$$\pi_2 = \frac{\pi_b \frac{1}{\gamma - 1} + \pi_\xi}{\pi_b \ \frac{1}{\gamma-1}\frac{\psi}{\psi-1} + \frac{wl}{\Theta}\frac{1}{1-\alpha} + \frac{\overline{div}\pi_{div}}{\Theta}} \ , \qquad \pi_3 = \frac{\frac{F}{\Theta}}{\pi_b \ \frac{1}{\gamma-1}\frac{\psi}{\psi-1} + \frac{wl}{\Theta}\frac{1}{1-\alpha} + \frac{\overline{div}\pi_{div}}{\Theta}}$$

$$\text{and} \qquad \pi_4 = \frac{\frac{wl}{\Theta}\frac{1}{1-\alpha}}{\pi_b \frac{1}{\gamma-1}\frac{\psi}{\psi-1} + \frac{wl}{\Theta}\frac{1}{1-\alpha} + \frac{\overline{div}\pi_{div}}{\Theta}} \tag{S5}$$

Assumptions 1 and 2 together with the fact that π_ξ and π_b are positive constants, that π_{div} is non-negative, and that $\psi > 1$, imply that π_2 and π_3 are positive coefficients, while π_4 is between 0 and 1. This proves all the magnitude restrictions in (17) and all the sign restrictions in (16) except the fact that the element B_{11} in the matrix B is negative.

The term B_{11} is equal to $-(\frac{1}{\gamma-1} - \frac{\psi\pi_2}{(\gamma-1)(\psi-1)})$. $\frac{1}{\gamma-1}$ is the direct negative effect of the ξ_t shock on borrowing b_t. However, reducing borrowing reduces labour input, and this increases financial frictions ψ_t (see the labour first order condition 9) driving the firm to increase borrowing, and partly dampening the direct effect of the shock. The dampening factor is the term $\frac{\psi\pi_2}{(\gamma-1)(\psi-1)}$. Intuitively, since the marginal product of labour has increased, the firm finds it optimal to borrow more until the higher marginal cost of external finance compensates the higher return on labour. Therefore, this equilibrium implies that borrowing is always lower than before the shock (otherwise labour input would not decrease), meaning that the optimality conditions imply that the dampening factor $\frac{\psi\pi_2}{(\gamma-1)(\psi-1)}$ cannot be larger than the direct effect $\frac{1}{\gamma-1}$, and hence the term B_{11} is negative. This proves Proposition 1.

S2 Model extensions

S2.1 Static model with endogenous convex excess cost of finance

In this section we provide a microfoundation of the convex cost of external finance assumed in (4). Consider the following simplified static version of our benchmark model. The firm borrows $\hat{b} = \dfrac{b}{1+r} - c$ to finance the labour input and produce $y = zl^\alpha$ at the beginning of the second period. r is the real interest rate while we assume the wage is $w = 1$ and as in our main model, the term c measures the excess cost of financial constraints. The timing is as follow. Before observing z, the firm needs to borrow to be able to hire workers. Since borrowing is costly, the firm only borrows the amount necessary to pay wages, so that $\hat{b} = l$,

and profits net of repaying the debt are:

$$\pi = z\ \hat{b}^{\alpha} - b \tag{S6}$$

After borrowing and before actually hiring labour, the firm has $\hat{b}$ amount of cash. It observes z, drawn from a given distribution with $z \in [0, Z]$, and then has two options: i) hire labour, produce, repay the debt and obtaint the profit π. ii) default and steal a fraction λ of the cash. In this case we assume the lenders get zero return, because any additional cash is lost. The firm defaults if the return from defaulting is higher than producing and repaying the debt, namely if $\lambda\hat{b} > \pi$, while it chooses to produce and repay the debt if $\lambda\hat{b} \leq \pi$. Substituting π using Equation S6 and rearranging, we determine that the minimum productivity not to default, denoted with z^* is:

$$z^* = \frac{\lambda\hat{b} + b}{\hat{b}^{\alpha}}. \tag{S7}$$

Ex ante, a risk neutral lender which requires an expected return of $1+r$, will lend the amount $\hat{b}$ under the promise of a repayment of b, such that the expected repayment $p\left(z > z^*\right) b$ is:

$$p\left(z > z^*\right) b = (1 + r)\hat{b}. \tag{S8}$$

The excess cost of borrowing can be defined as:

$$c = \frac{b}{1 + r} - \hat{b}, \tag{S9}$$

that is, the difference between the sum the firm would receive in the absence of financial imperfections $\dfrac{b}{1+r}$ and $\hat{b}$. Substituting $\hat{b}$ in (S8) using (S9) yields:

$$c = \frac{b}{R^F}\left(1 - p\left(z > z^*\right)\right)$$

if $p\left(z > z^*\right)$ was constant, then c would be linear in the face value of debt b. However,

from (S6) and (S7) it is easy to see that an increasing in borrowing $\hat{b}$ increases z^*, reduces $p(z > z^*)$ and increases b, implying that c is increasing and convex in b.

S2.2 Derivation of the model with capital

In this section, we prove that Proposition 1 continues to hold for the linearized model with capital that was discussed in Section 2.3.1. The first order condition for debt is unchanged and the one for labour becomes

$$\frac{\alpha z_t k_t^{\beta}}{l_t^{1-\alpha}} = (1+r)\,\psi_t w \ . \tag{S10}$$

Furthermore, as in the benchmark model, $\log \psi_t$ can be approximated by a linear function:

$$\log \psi_t = \pi_1 \log s_t + \pi_2 \log \xi_t + \pi_3 \log \theta_t + \pi_4 \log z_t - \pi_5 \log k_t \ .^{1} \tag{S11}$$

By substituting it back into the log linearized equations and solving for the reduced form we obtain

$$Y_t = c + DW_t + Bg_t \ ,$$

Where $W_t = (s_t, k_t)'$, while the B matrix is identical to the one in the benchmark model. The main difference is that the budget constraint is now determined by Equation (20), where:

$$\log \Theta_t = \pi_b \log b_t - \pi_\xi \log \xi_t + \pi_k \log k_t$$

[1]Note that the proof of proposition 1 does not require any assumption regarding the coefficient π_5. Such coefficient is likely positive, because more capital increases the assets available to the firm. However, it might be negative in certain situations if capital is subject to substantial disinvestment costs. Consider for example the extreme case that capital is fully irreversible. In this case, it could be that after a negative productivity shock the firm is forced to hold an inefficiently high amount of capital which increases the intensity of financial frictions. On the one hand, this possibility is unlikely to be empirically very relevant, and any bias would directly affect only the D matrix. On the other hand, one could allow for a more flexible relationship between $\log \psi_t$ and k_t, and this would again only affect the matrix D, not the identification of B.

As in the benchmark case, $\pi_b > 0$ and $\pi_\xi > 0$, while we do not need to impose any restriction on π_k. Log-linearising the budget constraint following the same procedure outlined in Appendix A1, using (21), and solving for $\log \psi_t$ yields:

$$\pi_2 = \frac{\pi_b \frac{1}{\gamma-1} + \pi_\xi}{\pi_b \ \frac{1}{\gamma-1}\frac{\psi}{\psi-1} + \frac{wl}{\Theta}\frac{1}{1-\alpha} + \frac{\overline{div}\pi_{div}}{\Theta} + \frac{\epsilon^i_\psi}{\Theta}}; \pi_3 = \frac{\frac{F}{\Theta}}{\pi_b \ \frac{1}{\gamma-1}\frac{\psi}{\psi-1} + \frac{wl}{\Theta}\frac{1}{1-\alpha} + \frac{\overline{div}\pi_{div}}{\Theta} + \frac{\epsilon^i_\psi}{\Theta}};$$
$$\pi_4 = \frac{\frac{wl}{\Theta}\frac{1}{1-\alpha} + \frac{\epsilon^i_z}{\Theta}}{\pi_b \ \frac{1}{\gamma-1}\frac{\psi}{\psi-1} + \frac{wl}{\Theta}\frac{1}{1-\alpha} + \frac{\overline{div}\pi_{div}}{\Theta} + \frac{\epsilon^i_\psi}{\Theta}} \tag{S12}$$

These coefficients are identical to those derived in the benchmark case (see Equation S5) except for the presence of the term $\frac{\epsilon^i_\psi}{\Theta}$ at the denominator and of the term $\frac{\epsilon^i_z}{\Theta}$ at the numerator of π_4. Both terms are weakly positive, as shown in (22), and moreover ϵ^i_z, the elasticity of investment to productivity, is likely to be small if positive, because of capital adjustment costs, so that π_4 continues to be smaller than 1, which is consistent with the fact that it represents a dampening factor. If it was larger than 1, it would imply that positive productivity shocks increase financial frictions so much that the firm actually wants to reduce its variable input l_t, which would violate profit maximization. It is then easy to see that the proof of Proposition 1 is the same as for the benchmark case.

S2.3 Models with collateralised borrowing

Asset based borrowing

Following the specification in Section 2.3.2, in the case of asset based borrowing the firm maximizes (19) subject to (20), (21), (22) and (23). Taking the first order condition for debt, assuming λ^1_t is constant and log-linearising yields:

$$\log b_t = \frac{1}{\gamma-1}\frac{\psi}{\psi-1}\log\psi_t - \frac{1}{\gamma-1}\log\xi_t + \frac{\gamma}{\gamma-1}\log k_t \ .$$

The first order condition for labour is the same as (S10) and $\log \psi_t$ is determined by (S11). We can substitute and solve for $\log b_t, \log l_t$ and $\log y_t$, and verify that the B Matrix is the same as in the benchmark model.

Therefore, it is possible to prove Proposition 1 following the same procedure outlined above. Intuitively, more capital k_t relaxes the borrowing constraint, but since it is predetermined, the only parameter affected is π_5 the other parameters π_2, π_3 and π_4 are identical to those derived above in Section S1.

Financial shock to the collateral value of assets

If we assume that the financial shock is a reduction in $1 - \lambda_t^1$, the collateral value of capital, rather than an increase in ξ_t, then the linearized first order condition for the debt becomes:

$$\log b_t = \frac{1}{\gamma - 1}\frac{\psi}{\psi - 1}\log \psi_t - \frac{\gamma}{\gamma - 1}\log(1 - \lambda_t^1) + \frac{\gamma}{\gamma - 1}\log k_t$$

Following the same procedure outlined above, the system becomes:

$$D_s = \begin{bmatrix} -\frac{1}{\gamma-1}\frac{\psi}{\psi-1}\pi_1 \\ \frac{1}{1-\alpha}\pi_1 \\ \frac{\alpha}{1-\alpha}\pi_1 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} -\left(\frac{\gamma}{\gamma-1} - \frac{1}{\gamma-1}\frac{\psi}{\psi-1}\pi_2\right) & \frac{\pi_3}{(\gamma-1)(\psi-1)} & \frac{\pi_4}{(\gamma-1)(\psi-1)} \\ -\frac{\pi_2}{1-\alpha} & -\frac{\pi_3}{1-\alpha} & \frac{1-\pi_4}{1-\alpha} \\ -\frac{\alpha\pi_2}{1-\alpha} & -\frac{\alpha\pi_3}{1-\alpha} & \frac{1-\alpha\pi_4}{1-\alpha} \end{bmatrix}.$$

Which is identical to the benchmark case except for the B_{11} coefficient, whose first term is $-\frac{\gamma}{\gamma-1}$ instead of $-\frac{1}{\gamma-1}$. Notice that, in this case, the last term in Equation 12 would be $g_t = \log(1 - \lambda_t, \theta_t, z_t)'$. This difference does not alter the properties of the system and the proof of proposition 1 is once again identical to the one outlined above. In other words, our approach is able to capture any type of financial shocks, both those that increase the cost of credit and those that reduce its quantity.

Models with collateralised borrowing: Earnings based borrowing

Using the new definition of financial costs in (24), the firm maximises the value function subject to

$$div_t = s_t - F\theta_t - wl_t + \frac{b_t}{1 + r} - \xi_t \left[\frac{b_t}{(1 - \lambda_t^2)\,\pi_t}\right]^{\gamma}.$$

Where π_t is defined in (25). Note that, also in this case, a positive shock to the cost of

credit ξ_t and a negative shock to quantity of collateral λ_t^2 have similar implications in a linear approximation of this model. Therefore we consider the shock to be ξ_t and we assume λ_t^2 to be constant. Taking the first order condition for labour, and using (7) and the definition of π_t we obtain:

$$\frac{\alpha z_t l_t^{\alpha-1}}{1+r}\frac{1}{\psi_t} - w = -\left(\frac{1}{1-\lambda^2}\right)^{\gamma}\gamma\xi_t b_t^{\gamma}\pi_t^{-\gamma-1}\left(\frac{\alpha z_t l_t^{\alpha-1}}{1+r} - w\right) \tag{S13}$$

We evaluate this first order condition in the steady state:

$$\frac{\alpha z l^{\alpha-1}}{1+r}\frac{1}{\psi} - w = -\left(\frac{1}{1-\lambda^2}\right)^{\gamma}\gamma\xi b^{\gamma}\pi^{-\gamma-1}\left(\frac{\alpha z l^{\alpha-1}}{1+r} - w\right) \tag{S14}$$

In the benchmark case we have shown that $\dfrac{\alpha z l^{\alpha-1}}{1+r}\dfrac{1}{\psi} - w = 0$. Since $\psi > 1$, it follows that the last term on the right hand side of (S14), $\left(\dfrac{\alpha z l^{\alpha-1}}{1+r} - w\right)$, is positive. This term represents the increase in profits obtained by increasing labour input in a financially constrained equilibrium. Hence the right hand side of equation (S14) has negative value, meaning that the earning based borrowing reduces the effective cost of labour relative to the benchmark case, increasing labour demand. Intuitively, labour has an additional value in increasing profits, and the firm chooses optimally a level above the level chosen in the benchmark model, but still below the level in the absence of financial frictions.

We derive the first order condition for debt to obtain:

$$b_t = \frac{1}{1+r}\left(\frac{y_t}{1+r} - w l_t\right)^{\frac{\gamma}{\gamma-1}}\left(1-\lambda_t^2\right)^{\frac{\gamma}{\gamma-1}}\left(\frac{1}{\gamma\xi_t}\right)^{\frac{1}{\gamma-1}}\left(1-\frac{1}{\psi_t}\right)^{\frac{1}{\gamma-1}}.$$

And we substitite it in (S13) and rearrange obtaining:

$$\frac{\alpha z_t l_t^{\alpha-1}}{1+r}\frac{1}{\psi_t} = w - \Psi_t$$

where

$$\Psi_t \equiv (\gamma\xi_t)^{-\frac{1}{\gamma-1}}\left(\frac{\alpha z_t l_t^{\alpha-1}}{1+r}-w\right)\left(\frac{1}{1+r}\right)^{\frac{\gamma}{\gamma-1}}\left(\frac{y_t}{1+r}-wl_t\right)^{\frac{1}{\gamma-1}}\left(1-\lambda_t^2\right)^{\frac{\gamma}{\gamma-1}}\left(1-\frac{1}{\psi_t}\right)^{\frac{\gamma}{\gamma-1}} \tag{S15}$$

We then log linearise the labour first order condition around the steady state:

$$\log z_t-(1-\alpha)l_t-\log\psi_t=-\frac{1}{w-\Psi}\left(\epsilon_l^{\Psi}\log l_t+\epsilon_\xi^{\Psi}\log\xi_t+\epsilon_\psi^{\Psi}\log\psi_t\right) \tag{S16}$$

where $\frac{1}{w-\Psi}$ is positive, being equal to the steady state value of the marginal productivity of labour. The definition of Ψ_t in (S15) implies that, around the steady state, the elasticities of Ψ_t to ξ_t and ψ_t, $\epsilon_\xi^{\Psi}$and ϵ_ψ^{Ψ}, respectively, are positive, while the elasticity of of Ψ_t to l_t, ϵ_l^{Ψ}, can be positive or negative. On the one hand the term $\left(\frac{y_t}{1+r}-wl_t\right)^{\frac{1}{\gamma-1}}$ increases in l_t, because, as mentioned before, profits $\frac{y_t}{1+r}-wl_t$ increase in labour around the steady state. However, then term $\frac{\alpha z_t l_t^{\alpha-1}}{1+r}-w$ decreases in labour, because of decreasing marginal returns. Intuitively the positive term dominates when the firm is very constrained and labour input is substantially below its unconstrained optimal level. However, as l_t increases towards such level, eventually Ψ_t goes to zero, and hence the elasticity ϵ_l^{Ψ} becomes negative.

Rearranging we get:

$$\log l_t=\frac{1}{(1-\alpha)-\frac{1}{w-\Psi}\epsilon_l^{\Psi}}\log z_t+\frac{\frac{1}{w-\Psi}\epsilon_\xi^{\Psi}}{(1-\alpha)-\frac{1}{w-\Psi}\epsilon_l^{\Psi}}\log\xi_t-\frac{1-\frac{1}{w-\Psi}\epsilon_\xi^{\Psi}}{(1-\alpha)-\frac{1}{w-\Psi}\epsilon_l^{\Psi}}\log\psi_t \tag{S17}$$

Consider the elasticity of labour to the productivity shock, $\frac{1}{(1-\alpha)-\frac{1}{w-\Psi}\epsilon_l^{\Psi}}$. As in the benchmark model, it is larger the larger is α, which reduces the degree to which labour productivity falls when labour demand increases. A positive value of ϵ_l^{Ψ} means more labour input reduces its effective cost by relaxing the borrowing constraint. However, this positive effect vanishes, and eventually ϵ_l^{Ψ} becomes negative, as labour increases, as explained above, so that the elasticity of labour to the productivity shock is always positive and finite. Fur-

thermore, consider the elasticity of labour to ψ_t. In the term at the numerator, $1-\frac{1}{w-\Psi}\epsilon_{\psi}^{\Psi}$, "1" is the direct negative effect of tighter financial frictions on labour input. "$-\frac{1}{w-\Psi}\epsilon_{\psi}^{\Psi}$" is the dampening effect representing the fact that, around the steady state, more labour input relaxes the borrowing constraint. As explained above, this dampening factor vanishes as labour approaches the unconstrained level, so that it is never larger than the direct effect. Otherwise, more financial frictions would not affect labour negatively, and hence the firm would not be financially constrained in equilibrium.

Finally, we log-linearise the first order condition for debt, we substitute labour using (S17), and we rearrange obtaining:

$$\log b_t = \left(\frac{\Xi}{1-\frac{1}{w-\Psi}\epsilon_{\xi}^{\Psi}} + \frac{\gamma}{\gamma-1}\frac{\frac{zl^{\alpha}}{1+r}}{\frac{zl^{\alpha}}{1+r}-wl}\right)\log z_t - \left(\frac{1}{\gamma-1}-\Xi\right)\log\xi_t + \left(\frac{1}{\gamma-1}\frac{\psi}{\psi-1}-\Xi\right)\log\psi_t \tag{S18}$$

$$\Xi \equiv \eta\frac{\gamma}{\gamma-1}\frac{1-\frac{1}{w-\Psi}\epsilon_{\xi}^{\Psi}}{(1-\alpha)-\frac{1}{w-\Psi}\epsilon_{l}^{\Psi}}; \eta \equiv \frac{\alpha\frac{zl^{\alpha-1}}{1+r}-w}{\frac{zl^{\alpha-1}}{1+r}-w} \tag{S19}$$

The term η measures the elasticity of profits to labour which, as argued above, is positive in the steady state. Therefore, following the results shown above, the term Ξ is also positive, and it represents the effect labour input has, because it relaxes the borrowing constraint, on borrowing. It is easy to see that it increases the elasticity of b_t to the productivity shock, while it dampens the elasticities to both ξ_t and ψ_t. As explained before, for financial frictions to exist in the linearised equilibrium, this dampening effect cannot be larger than the direct effects. To see this, notice that the term η is only positive when the firm is financially constrained, and goes to zero as labour input approaches the unconstrained level. So the dampening effect Ξ cannot become sufficiently large to completely offset the direct effect and neutralize financial frictions.

It follows that we can represent the linear system (10) as:

$$\log b_t = \epsilon_z^b \log z_t - \epsilon_\xi^b \log \xi_t + \epsilon_\psi^b \log \psi_t \tag{S20}$$

$$\log l_t = \epsilon_z^l \log z_t + \epsilon_\xi^l \log \xi_t - \epsilon_\psi^l \log \psi_t$$

$$\log y_t = \log z_t + \alpha \log l_t$$

Where the coefficients $\epsilon_z^b, \epsilon_\xi^b, \epsilon_\psi^b, \epsilon_z^l, \epsilon_\xi^l$ *and* ϵ_ψ^l are all positive. Substituting $\log \psi_t$ using (11) we can derive the matrix B for the case of earning based borrowing:

$$B = \begin{bmatrix} -\left(\epsilon_\xi^b - \epsilon_\psi^b \pi_2\right) & \epsilon_\psi^b \pi_3 & \left(\epsilon_\psi^b \pi_4 + \epsilon_z^b\right) \\ -\left(\epsilon_\psi^l \pi_2 - \epsilon_\xi^l\right) & -\epsilon_\psi^l \pi_3 & \left(\epsilon_z^l - \epsilon_\psi^l \pi_4\right) \\ -\alpha\left(\epsilon_\psi^l \pi_2 - \epsilon_\xi^l\right) & -\alpha\epsilon_\psi^l \pi_3 & \left(1 + \alpha\left(\epsilon_z^l - \epsilon_\psi^l \pi_4\right)\right) \end{bmatrix} \tag{S21}$$

There are three terms, $\epsilon_\xi^b - \epsilon_\psi^b \pi_2$, $\epsilon_\psi^l \pi_2 - \epsilon_\xi^l$ and $\epsilon_z^l - \epsilon_\psi^l \pi_4$ in which a direct effect (the first term), is dampened by an indirect effect (the second term). It is easy to see that this dampening effect is smaller than the direct one, for financial frictions to exist around the steady state. For the term $\epsilon_\xi^b - \epsilon_\psi^b \pi_2$, the reasoning is analogous to the one illustrated in Section S1. For the other two terms, we start by claiming that both π_2 and π_3 are positive.

For the term $\epsilon_\psi^l \pi_2 - \epsilon_\xi^l$, the first component $\epsilon_\psi^l \pi_2$ implies that a financial friction shock makes borrowing more costly, and this reduces the demand for labour, as in the benchmark model. However, the same shock also increases the relative desirability of labour input to relax the constraint, which is the second term ϵ_ξ^l. This second term is dampening in nature, meaning that it cannot be larger than the direct effect. To see this, notice that if that was the case, then financial frictions would not reduce labour, which would imply $\epsilon_\xi^\Psi = 0$, and therefore also $\epsilon_\xi^l = 0$. Hence it must be that $-\left(\epsilon_\psi^l \pi_2 - \epsilon_\xi^l\right) < 0$. A similar reasoning can be used to argue that the term $\epsilon_z^l - \epsilon_\psi^l \pi_4$ is positive, where ϵ_z^l is the direct positive effect of productivity on labour input, while $-\epsilon_\psi^l \pi_4$ is the dampening effect caused by higher financial frictions ψ_t. Once again this indirect effect cannot dominate, because it would drive the value of ϵ_ψ^l to zero. Given this, to prove Proposition 1 it is sufficient to prove that the

coefficients π_2, π_3 are positive, and that and π_4 is not smaller than $-\frac{\epsilon_z^b}{\epsilon_\psi^b}$.

The budget constraint follows equation (S1), where $\Theta(b_t, \xi_t)$ includes the earning based constraint as defined in (24). We linearise $\Theta(b_t, \xi_t)$ obtaining:

$$\log \Theta_t = \pi_b \log b_t - \pi_\xi \log \xi_t + \pi_z \log z_t + \pi_l \log l_t$$

Where π_b and π_ξ are positive, as in the benchmark model, while the previous discussion implies that around the steady state more productivity z_t and more labour input allow more borrowing, and hence also π_z and π_l are positive. Therefore linearising the budget constraint yields:

$$\pi_b \log b_t - \pi_\xi \log \xi_t + \pi_z \log z_t = -(\frac{s}{\Theta} - \overline{div}\pi_{div}^s) \log s_t + \frac{F}{\Theta} \log \theta_t - \frac{\overline{div}\pi_{div}}{\Theta} \log \psi_t + \left(\frac{wl}{\Theta} - \pi_l\right) \log l_t \tag{S22}$$

Where $\dfrac{wl}{\Theta} - \pi_l$ is overall positive, otherwise labour input would not be subject to financial frictions.[2] We then proceed as before, we substitute the labour and debt policy functions in (S22), we solve for ψ_t, and we compare it to (11) to obtain a solution for the undetermined coefficients π_2, π_3 and π_4:

$$\pi_2 = \frac{\pi_b \epsilon_\xi^b + \pi_\xi + \left(\frac{wl}{\Theta} - \pi_l\right) \epsilon_\xi^l}{\pi_b \epsilon_\psi^b + \left(\frac{wl}{\Theta} - \pi_l\right) \epsilon_\psi^l + \frac{\overline{div}\pi_{div}}{\Theta}}; \quad \pi_3 = \frac{\frac{F}{\Theta}}{\pi_b \epsilon_\psi^b + \left(\frac{wl}{\Theta} - \pi_l\right) \epsilon_\psi^l + \frac{\overline{div}\pi_{div}}{\Theta}};$$

$$\pi_4 = \frac{\left(\frac{wl}{\Theta} - \pi_l\right) \epsilon_z^l - \left(\pi_b \epsilon_z^b + \pi_z\right)}{\pi_b \epsilon_\psi^b + \left(\frac{wl}{\Theta} - \pi_l\right) \epsilon_\psi^l + \frac{\overline{div}\pi_{div}}{\Theta}}$$

It is immediate to see that $\pi_2 > 0$ and $\pi_3 > 0$. Regarding π_4, while its denominator is positive, its numerator depends on a positive term, $\left(\frac{wl}{\Theta} - \pi_l\right)$, which reflects the fact that more productivity implies the need for funds to purchase more inputs, and a negative term $-\left(\pi_b \epsilon_z^b + \pi_z\right)$, which represents the additional borrowing capacity generated by higher profits (for given inputs) when z_t increases. Intuitively, the negative term is more likely to

[2]Because the increase in wage bill would be lower than the increase in borrowing generated by relaxing the borrowing constraint.

dominate the smaller is the elasticity α, which reduces the response of inputs demand to z_t. To see this, consider that π_4 is positive at the steady state if and only if a productivity shock increases pledgeable profits more than the wage bill, or $\frac{\partial wl}{\partial z} > (1-\lambda^2)\frac{\partial \pi}{\partial z}$. We assume for simplicity that the firm is not very financially constrained in the steady state, so that Ψ is small and we can approximate to zero $\frac{\partial \Psi}{\partial z}$. Then it is possible to show that $\frac{\partial wl}{\partial z} > \frac{\partial \pi}{\partial z}$, which is a necessary and sufficient condition for $\pi_4 > 0$, requires:

$$\frac{w}{w-\Psi}\left(\frac{\alpha}{\psi}\right)^{\frac{1}{1-\alpha}} > \frac{1-\lambda^2}{2-\lambda^2}\left(\frac{\alpha}{\psi}\right)^{\frac{\alpha}{1-\alpha}} \tag{S23}$$

In the extreme case that Ψ is close to zero and profits are fully collateralisable ($1-\lambda^2 = 1$), π_4 is positive as long as α is larger than 0.5. However, our identifying restrictions are satisfied by much lower values than that. First, because not all profits might be pledged as collateral. For example Drechsel and Kim (2022), in order to match the debt to output ratio observed in the US, using a model with inelastic labour, consider a value of $1-\lambda^2$ equal to 0.53, which would lower the minimum value of α compatible with $\pi_4 > 0$ to 0.35. Second, condition (S23) is satisfied for a lower value of α the tighter financial frictions are, in the form of higher values of Ψ and/or ψ . Third and more importantly, our identifying conditions are satisfied also when also when π_4 is negative, as long as $\pi_4 > -\frac{\epsilon^b_z}{\epsilon^b_\psi}$, where, from (S18), it is easy to see that $-\frac{\epsilon^b_z}{\epsilon^b_\psi} < -1$. In other words, α would have to be so low that a 1% increase in productivity generates so much additional borrowing that it reduces the intensity of financial frictions more than 1% . Summing up, the term B_{13} in (S21) would be negative, thus violating proposition 1, only if the elasticity of variable inputs to output (α in our model) were extremely low, and hence most of the production was done with fixed inputs, which is unrealistic in virtually all industries. Therefore in practice our Assumption 1 is sufficient to satisfy our identification restrictions for realistically chosen values of $\underline{\alpha}$ and $\overline{\alpha}$.

S2.4 Intra-period borrowing constraint

Borrowing limited by current assets, as in Buera and Shin (2013).

Assume that at the beginning of period t the firm has savings s_t. Then the three shocks to z_t, λ_t (the multiplier factor that determines the borrowing limit) and θ_t are realized. The firm needs to borrow if the investment needs for labour l_t and the fixed cost $\theta_t F$ are larger than available wealth s_t. Borrowing is limited by current assets, so that expenditures are constrained to be less than a multiple of s_t:

$$l_t + \theta_t F \leq \lambda_t s_t \tag{S24}$$

with $\lambda_t > 1$ and hence the maximum borrowing is $(\lambda_t - 1)s_t$. For a micro-foundation of this constraint, see Buera and Shin (2013). After producing, the firm decides savings s_{t+1} and dividends div_t to maximize its intertemporal value:

$$V_t(S_t) = \max_{l_t, s_{t+1}} \text{div}_t + \eta_t(\lambda_t s_t - l_t - \theta_t F) + \frac{1}{1+r}\frac{1}{\mu}\mathbb{E}_t[V_{t+1}(S_{t+1})] \tag{S25}$$

subject to the budget constraint:

$$\text{div}_t + \frac{s_{t+1}}{1+r} = s_t + y_t - w l_t \tag{S26}$$

As in the benchmark model, the term $\mu > 0$ increases the desire to distribute dividends and ensures that financial frictions exist in equilibrium. η_t is the lagrange multiplier associated to constraint S24. The first order condition for savings s_{t+1} is given by

$$\frac{1}{1+r} + \frac{1}{1+r}\frac{1}{\mu}\left[\mathbb{E}_t\left(\frac{\partial V_{t+1}}{\partial s_{t+1}}\right)\frac{\partial s_{t+1}}{\partial b_t}\right] = 0 \tag{S27}$$

where $\frac{\partial s_{t+1}}{\partial b_t} = -1$ and, for the envelope theorem, $\mathbb{E}_t(\frac{\partial V_{t+1}}{\partial s_{t+1}}) = \mathbb{E}_t[\eta_{t+1}\lambda_{t+1}]$. Substituting yields:

$$\mu = \mathbb{E}_t(\eta_{t+1}\lambda_{t+1}) \tag{S28}$$

Intuitively, the above condition implies that the gain from distributing dividend early, represented by $\mu > 0$, is compensated by the cost of reducing savings and facing future expected financial frictions on the right hand side. If $\mu > \mathbb{E}_t(\eta_{t+1}\lambda_{t+1})$, the firm reduces savings s_{t+1} to increase dividends, but in doing so it increases future expected financial frictions, and the value of $\mathbb{E}_t(\eta_{t+1}\lambda_{t+1})$ increases until the condition holds with equality. The first order condition for labour yields:

$$\alpha z_t l_t^{\alpha-1} = \eta_t + w \tag{S29}$$

When the constraint is binding and $\eta_t > 0$, then labour demand is determined by $l_t = \lambda a_t$ and the equation above determines the intensity of the constraint, $\eta_t = \alpha z_t l_t^{\alpha-1} - w > 0$.

Equation S28 and the assumption of $\mu > 0$ imply that the constraint is binding in every period with some probability. Equation S24 implies, conditional on savings s_t and on the productivity shock z_t, that this probability is weakly monotonously decreasing in λ_t. In other words, a sufficiently strong negative financial frictions shock (a reduction in λ_t) will cause constraint S24 to bind, reducing debt b_t, the variable input l_t and output y_t. Also, since marginal productivity of labour increases, the reduction in y_t will be lower than the one in l_t. Therefore, a shock to λ_t, conditional to being large enough to cause constraint S24 to bind, generates the same sign and inequality restrictions as in the benchmark model.

Regarding the restrictions to identify the shocks to θ_t and z_t, if they are large enough to make the constraint S24 binding, they also satisfy all the restrictions. A negative liquidity shock will increase debt but reduce labour (because of the binding constraint) and output. A positive productivity shock will increase debt, labour and output, and the inequality restrictions will be satisfied following the same reasoning applied in the benchmark model. The reasoning above clarifies that in this model the identification restrictions are valid when the constraint S24 becomes binding as a consequence of a negative shock. A similar reasoning can be used to show that they are also valid when S24 ceases to bind following a positive shock.

Importantly, if the constraint S24 is already binding in the absence of shocks, the identification restrictions for the financial frictions shock continue to be valid. A reduction in λ_t still reduces, debt, output and labour and increases the output to labour ratio. However, the identification restrictions for the shocks to z_t and θ_t are no longer satisfied. For example, when the constraint binds, a positive productivity shock z_t is unable to affect borrowing because of the extreme assumptions that first, no uncollateralised borrowing is possible, no matter at what rate, and second, the borrowing limit is unaffected by how productive the firm is. If any of these two extreme assumptions is relaxed, then the identification restrictions for z_t are satisfied, and it is easy to see that relaxing the first assumption, and allowing some uncollateralised borrowing at higher rates at the margin, would also imply the identification restrictions for the shocks to θ_t are satisfied.

Intra-period constraint as in Jermann and Quadrini (2012).

We provide here the intuition of why the same conclusions can be obtained with the type of intra-period constraint assumed by Jermann and Quadrini (2012). Adapting to their notation, the firm also uses capital for production as in our extended model with capital, $y_t = z_t l_t^{\alpha} k_t^{\beta}$. The budget constraint then is:

$$\text{div}_t = s_t + y_t - \theta_t F - w l_t + \frac{b_{t+1}}{1+r} - k_{t+1} + (1-\delta)k_t \tag{S30}$$

Jermann and Quadrini (2012) assume that all expenses within the period, including fixed capital investment, the reduction in debt (or increase in savings) and the dividend payments, have to be financed with an intraperiod loan b_t^{IP}, which they show to be equal by construction to revenues, $b_t^{IP} = y_t$. Furthermore, both inter- and intra-period borrowing can be secured with capital, but enforcement is imperfect, giving rise to the constraint $(1-\xi_t)\left(k_{t+1} - \frac{b_{t+1}}{1+r}\right) \geq b_t^{IP}$, where ξ_t is the financial frictions latent variable (see Jermann

and Quadrini 2012 for details).[3] Combining this constraint with the budget constraint yields:

$$\frac{1-\xi_t}{\xi_t}(s_t - \theta_t F - wl_t + (1-\delta)k_t) \geq y_t \tag{S31}$$

Intuitively, a negative financial frictions shock (an increase in ξ_t) reduces borrowing, which is proportional to the left hand side of (S31) and if the shock is large enough to make the constraint (S31) binding, it implies a reduction in output y_t and in the variable input l_t, since the other variables on the left hand side (s_t and k_t) are predetermined. Following the same reasoning adopted above for the case of borrowing limited by the value of assets, one can show that the identification restrictions for the θ_t and z_t shocks are satisfied if these shock cause the constraint to become binding, but not if the constraint was already binding in the absence of shocks. Also in this case the key factor is that this model does not allow the firm to access any amount of more costly uncollateralised borrowing.

S2.5 Other model extensions

In the model we restrict dividends to be positive, and hence we do not allow for equity financing. On the one hand, this assumption is not particularly restrictive especially for small privately owned firms, for which debt is the predominant source of financing. On the other hand, it would be relatively simple to relax it. Since empirically equity issuances are infrequent, one could extend the model to allow for equity financing subject to fixed costs. Then our current identification strategy would likely hold conditional on no equity issuance.

Another characteristic of the model is that the excess cost of credit is the combination of an endogenous variable (debt, or leverage in the model with capital) and a latent variable ξ which mediates the relation between leverage and excess cost of credit. We assume changes in ξ to be exogenous, but in reality they might be related to firm fundamentals. For example, a firm that has a sequence of good results for a number of years might be perceived as high quality and the lender will require a lower spread for given leverage. One could address this

[3] In Jermann and Quadrini (2012) an increase in ξ_t denotes a relaxation of financial frictions. To avoid confusion here we maintain the notation adopted earlier in the paper, in which the opposite is true.

issue in the model by modifying equation 4 so that c_t is a more complicated function of lagged endogenous variables.

Finally, our test is based on variable inputs, but in practice a firm might have different degrees of financial frictions across variable and fixed inputs. As demonstrated in section 2.3.1, our financial frictions indicator remains valid in the presence of fixed capital subject to adjustment costs. However, it might be that fixed capital differentially financially constrained than variable capital, and the researcher is specifically interested in measuring the constraints of the former. In this case, one could in principle expand the System 10 to include also the first order condition for fixed capital and use additional restrictions (for example related to the capital/labour ratio) to identify separate financial frictions for different types of inputs.

S3 Some econometric details

Limiting distribution panel GMM estimates

In this section we discuss the limiting distribution of the reduced form estimates $\hat{\mu}$. We first introduce some additional notation. The commutation matrix $\mathcal{K}_{m,n}$ is defined such that, for any $(m \times n)$ matrix G; $\mathcal{K}_{m,n}\text{vec}(G) = \text{vec}(G')$; and the $m^2 \times (m(m+1))/2$ duplication matrix $\mathcal{D}_m$ is defined such that $\mathcal{D}_m\text{vech}(F) = \text{vec}(F)$ for a symmetric $(m \times m)$ matrix F: Furthermore, $\mathcal{D}_m^+ = (\mathcal{D}_m'\mathcal{D}_m)^{-1}\mathcal{D}_m'$ and $\mathcal{L}_m$ is the $(m(m+1))/2 \times m^2$ elimination matrix defined such that, for any $(m \times m)$ matrix F; $\text{vech}(F) = \mathcal{L}_m\text{vec}(F)$

To derive the joint limiting distribution of $\hat{\phi}$ and $\text{vech}(\widehat{\Sigma})$ we impose the following assumptions on the reduced form errors $u_{i,t} = B\varepsilon_{i,t}$ and the initial conditions.

Assumption S1. *We assume that $u_{i,t}$ is independently and identically distributed across i and t and there exists some $\delta > 0$ such that*

1. $\mathbb{E}_F(u_{i,t}|W_{i,t}, \ldots, W_{i,1}, Y_{i,t-1}, \ldots, Y_{i,t-p}) = 0$
2. $\mathbb{E}_F(u_{i,t}u_{j,s}'|W_{i,t}, , \ldots, W_{i,1}, Y_{i,t-1}, \ldots, Y_{i,t-p}) = \begin{cases} \Sigma & \textit{if} \quad i = j,\ s = t \\ 0 & \textit{else} \end{cases}$.

3. $\mathbb{E}_F\|u_{i,t}\|^{4+2\delta} < \infty$

4. $\max_i \mathbb{E}_F\|X_{i,0}\|^{4+2\delta} < \infty$ *and* $\max_i \mathbb{E}_F\|c_i\|^{4+2\delta} < \infty$

5. $\Sigma_{ZX} = \text{plim}_{N\to\infty} S_{ZX}$ *and* $\Sigma_{ZZ} = \text{plim}_{N\to\infty} S_{ZZ}$ *have full column rank*

6. $\mathbb{E}_F(u_{i,t}u_{i,t}' \otimes u_{i,t}|W_{i,t}, , \ldots, W_{i,1}, Y_{i,t-1}, \ldots, Y_{i,t-p}) = 0$

where $X_{i,0} = (W_{i,-p+1}', \ldots, W_{i,0}', Y_{i,-p}', \ldots, Y_{i,-1}')'$.

To facilitate the characterization of the limiting distribution let

$$Q = \Sigma_{ZX}'\Sigma_{ZZ}^{-1}\Sigma_{ZX} \qquad \text{and} \qquad S = -\text{plim}_{N\to\infty}\frac{1}{NT}\sum_{i=1}^{N}\sum_{t=1}^{T}(u_{i,t} - u_{i,.})(X_{i,t} - X_{i,.})' \ .$$

The asymptotic distribution of the reduced form estimates $\hat{\mu} = (\hat{\phi}', \text{vech}(\widehat{\Sigma})')'$ is summarized in the following Theorem.

Theorem S1. *Assume that the data are generated by model* (29) *and that Assumption S1 holds. We have for* $N \to \infty$ *that*

$$\sqrt{N}(\hat{\mu} - \mu) \xrightarrow{d} N(0, \Omega) \ , \qquad \textit{with} \qquad \Omega = \begin{bmatrix} \Omega_{\phi\phi} & \Omega_{\phi\sigma}' \\ \Omega_{\phi\sigma} & \Omega_{\sigma\sigma} \end{bmatrix} \ ,$$

where

$$\begin{aligned}
\Omega_{\phi\phi} =& \Sigma \otimes Q^{-1} \\
\Omega_{\phi\sigma} =& -\mathcal{D}_K^+(I_K \otimes S)(\Sigma \otimes Q^{-1}) - \mathcal{D}_K^+\mathcal{K}_{K,K}(I_K \otimes S)(\Sigma \otimes Q^{-1}) \\
\Omega_{\sigma\sigma} =& \mathcal{D}_K^+\left[\frac{1}{T+1}\Lambda_{K^2} + \frac{1}{T(T+1)}(\Sigma \otimes \Sigma)(I_{K^2} \otimes \mathcal{K}_{K,K})\right](\mathcal{D}_K^+)' \\
&+ \mathcal{D}_K^+(\Sigma \otimes SQ^{-1}S')(\mathcal{D}_K^+)' + \mathcal{D}_K^+(SQ^{-1}S' \otimes \Sigma)(\mathcal{D}_K^+)' \\
&+ \mathcal{D}_K^+(\Sigma \otimes SQ^{-1}S')\mathcal{K}_{K,K}'(\mathcal{D}_K^+)' + \mathcal{D}\mathcal{K}_{K,K}(\Sigma \otimes SQ^{-1}S')(\mathcal{D}_K^+)'
\end{aligned}$$

and $\Lambda_{K^2} = \mathbb{V}_F(\text{vec}(u_{i,t}u_{i,t}'))$.

Proof. See Theorem 1 in Cao and Sun (2011). □

The important difference with the aggregate time series case is that the limiting covariance between the $\hat{\phi}$ and vech($\widehat{\Sigma}$) is not equal to zero. Therefore we need to take this correlation into account when constructing the confidence sets for the structural coefficients.

The asymptotic variance can be estimated by replacing the population coefficients Σ, Q, S and Λ_{K^2} by their sample counterparts. For Σ the expression is given in equation (34), while for Q, S these are given by

$$\widehat{Q} = S'_{ZX} S_{ZZ}^{-1} S_{ZX} \qquad \text{and} \qquad \widehat{B} = -\frac{1}{NT} \sum_{i=1}^{N} \sum_{t=1}^{T} (u_{i,t} - u_{i,.})(X_{i,t} - X_{i,.})' \ . \tag{S32}$$

For Λ_{K^2}, some more work is required. In general, the variance matrix Λ_{K^2} depends in a complicated way on the fourth-order multivariate cumulants of $u_{i,t}$. Therefore in practice Cao and Sun (2011) suggest to estimate this matrix by

$$\widehat{\Lambda}_{K^2} = (\widehat{\Sigma} \otimes \widehat{\Sigma})(I_{K^2} + \mathcal{K}_{K,K}) \ . \tag{S33}$$

We followed there approach, noting that in our empirical study little differences are obtained when considering a more general estimate for Λ_{K^2}.

S4 Simulation study

We use the structural model described in Section 2 to simulate an artificial industry, and draw a panel of $N = 10000$ firms for $T = 10$ periods. As explained in Fn.10 in the paper, in the model derived in Section 2.1 we abstract for endogenous exit of firms, to simplify the derivations. Instead for these simulations we consider a slightly modified version of the model in which we allow for endogenous exit to happen in equilibrium. More specifically,

the value function can be written as:

$$V_t^{\text{cont}}(S_t) = \max_{l_t, b_t} (1 + \phi_t) \, \text{div}_t + \frac{1}{1+r} \frac{1}{\mu} \mathbb{E}_t \left[V_{t+1} \left(S_{t+1} \right) \right] \ , \tag{S34}$$

where V_t^{cont} is the value conditional on continuing to operate in period t. The beginning of the period value $V_t(S_t)$ is instead given by

$$V_t(S_t) = d\{1(a_t \geq 0) \max \left[V_t^{\text{cont}}(S_t), a_t \right] - 1(a_t < 0) a_t\} + (1-d) a_t \ , \tag{S35}$$

With probability $1-d$, the firm's technology becomes useless, and the firm liquidates its activity.[4] With probability d the firm is still productive, but it might go bankrupt and be forced to liquidate if it has insufficient funds to cover the fixed overhead costs of production. We define as $a \equiv s_t - \theta_t F$ the beginning of the period financial wealth after covering these overhead costs. In (S35), $1(a_t \geq 0)$ is an indicator function that is equal to zero if $a_t < 0$ and the firm goes bankrupt. Note that in the calibrated model the beginning of the period financial wealth $s_t = y_{t-1} - b_{t-1}$ is always positive. That is, the firm always repays the debt. Furthermore, it is also the case that a_t is almost always positive. The firms engage in precautionary saving to avoid inefficient default that would destroy the positive net present value of their business. Therefore inefficient default with $a_t < 0$ only happens in equilibrium with around 2% probability.

If $a_t \geq 0$ and $\max \left[V_t^{\text{cont}}(S_t), a_t \right] = a_t$, the firm voluntarily exits because its value net of current wealth $(V_t^{\text{cont}}(S_t) - a_t)$, is negative. This can happen for financial reasons, after a sequence of negative profits which implies excessively high levels of debt, or for efficiency reasons, if productivity becomes too low.

In order to simulate a panel data of firm, we assume that there are N perfectly competitive firms. We assume the stochastic process for productivity z to be AR(1) while ξ and θ are assumed to be i.i.d. We solve numerically the model to obtain optimal labour choice

[4]Essentially we assume the stochastic process for productivity z_t is persistent with probability d and 0 permanently with probability $1-d$.

$l_t^*(s_t, \theta_t, z_t, \xi_t)$ and optimal debt $b_t^*(s_t, \theta_t, z_t, \xi_t)$. At the beginning of period t there are N firms which were active in the previous period. N^E firms exit after observing the shocks. An equal number of firms enter to keep the total number unchanged. They have identical initial endowment a_0 and initial productivity z_0. They also draw a value of the shock ξ_t. Naturally, the values a_0 and z_0 are such that new firms want to continue regardless of ξ_t.

Calibration

The parameters of the model are calibrated to match key moments related to the dynamics of productivity and financial variables at the firm level. In other words, we make sure that the simulated panel of firms is as much as possible realistic along the main dimensions of interest. We calibrate all the parameters that are important in determining the level and volatility of productivity, liquidity shocks and financial frictions shocks by matching key firm-level empirical moments. This way we ensure that the dispersion in production opportunities, financial wealth and financial frictions in the model are as much as possible realistic (with the obvious caveat that financial frictions are difficult to measure). Among these, the discount term μ affects the desire of firms to distribute dividends and ensures they are borrowers in equilibrium. The chosen value of $\mu = 1.091$ matches a target leverage of around 25%. The implied discount factor of firms is $\frac{1}{1+r}\frac{1}{\mu} = 0.877$. This relatively low value is common in models with financially constrained entrepreneurs that are calibrated to match their distribution of wealth. For example the same parameter is equal to 0.865 in Cagetti and De Nardi (2006). The productivity latent variable z is an AR(1) process with autoregressive coefficient ρ^z, while ξ and θ are i.i.d. The unconditional means of z and θ, denoted as μ^z and μ^θ , respectively, are normalised to one. ρ^z matches the autoregressive coefficient obtained from an estimated measure of revenue TFP at the firm level. The standard deviation of the productivity shock ε^z matches the cross sectional dispersion in the growth rates of labour, and more specifically the 75th percentile. The standard deviation of the liquidity shock ε^θ matches the the 75th percentile of the cross sectional dispersion in the growth rates of profits

over sales. These 3 moments are computed on our dataset of Spanish firms from SABI.[5]

The mean of ξ and the standard deviation of its innovations ε^{ξ} are difficult to measure empirically, which is precisely the motivation for this paper. For those datasets for which interest rate spreads are available, one can use their distribution to proxy for the distribution of the excess cost of external finance. This is of course an imperfect proxy. On the one hand, empirical spreads reflect in large part also debt riskiness, and therefore are larger than the effective expected cost of debt (because with some probability the firm is going to default and not repay it). This implies that they might overestimate the actual excess cost of finance caused by financial frictions. On the other hand, as we explained before, our estimated financial frictions shock captures not only changes in the excess cost, but also changes in the tightness of the borrowing limit. In this respect, the empirical spread is likely to underestimate the cost of financial frictions. Therefore, even though the empirical spreads are poor measures of financial frictions for a given firm in a given point in time, on average these positive and negative biases partly compensate each other, so that the moments in their distribution might be informative of the moments of the distribution of financial frictions. Thus we use these moments to calibrate our parameters related to the level and volatility of the financial frictions latent variable ξ

More specifically, we calibrate the mean of ξ, μ^{ξ}, with the average spread observed on Spanish firm from credit registry data, from Gonzalez and Sy (2023). Furthermore, we calibrate the standard deviation of ε^{ξ}, σ^{ξ}, with the dispersion in spread observed for US Compustat firms from Arellano et al. (2019). Finally, we calibrate the fixed cost parameter F by matching the median ratio between profits and sales for Compustat manufacturing firms, where we measure profits with EBIT (compustat code: ebit).

The other parameters are set as follows: the interest rate r is set to 4.5%. The initial wealth of firms is set to around 80% of median wealth of incumbents. The output elasticity of labour α is set to 0.61. The curvature of the excess cost of debt γ is set to 2.[6]

[5]To reduce in the data some types of firm heterogeneity not represented in the model, we subtract the part of growth rates explained by sector and year fixed effect and we reduce size heterogeneity by including

Table S1: Calibration

	Parameter Value	Target	Data	Model
μ	1.091	Leverage	0.261[1]	0.22[6]
ρ^z	0.5	AR(1) coefficent of estimated RTFP	0.5[2]	0.5
σ^z	0.0275	75% percentile of labour growth	7.4%[2]	8.1%
μ^ξ	0.005	Average spread on bank loans	3.7%[3]	3.51%
σ^ξ	0.75	Cross sectional dispersion in the spread	1.1%[4]	1.16%
σ^θ	0.03	75% percentile of profits growth	3.07%[2]	3.18%
F	14.16	Profits over Sales	6.2%[5]	6.5%

Notes: Untargeted parameters: μ^z and μ^θ are normalisded to one. $\alpha = 0.61$; $\gamma = 2$; $r = 4.5\%$; $s_0 = 25$, corresponding to 80% of the median value for incumbent firms.

1: US firm level data (Source: Giroud and Mueller 2021); 2: our calculations using Spanish firm level data from SABI; 3: Spanish credit registry (Source: Gonzalez and Sy 2023); 4: Compustat (Source: Arellano et al. 2019); 5: our calculation on manufacturing firms from Compustat; 6: Leverage computed as debt over sales.

Simulation Results

Our first exercise is to regress the estimated structural shocks on the outcome variables, and to compare the coefficients to those obtained with the true structural shocks. In all regressions we also include the log of beginning of the period financial wealth as control variable. The results are shown in Table S2. Column 1 shows reduced form regressions using the true shocks. These are normalized in order to have similar mean and variance of the estimated shocks, so that the size of the coefficients in column 1 are comparable to those in columns 2-4. Column 1 shows that the shocks explain a substantial part of the variation in the dependent variables.[7] Columns 2-4 in each table show the same regressions, this time using the median, lower bound and upper bound values of the estimated shocks. In all three cases, the estimated shocks also explain a large part of the variation of the dependent variables. Furthermore, the estimated coefficients in columns 2-4 are quite close to those in column 1, especially for the financial frictions shock ε_t^ξ, which is the main objective of our

firms between 20 and 250 employees.

[6]In practice, μ^ξ , σ^ξ and γ jointly determine the average cost of financial frictions and their dispersion. We chose to set $\gamma = 2$, which helps obtaining a much faster convergence in the solution of the model, and as explained below set the other two parameters to match these two moments.

[7]The unexplained part is due to the fact that we do not include among the regressors the lagged level of the latent productivity process z, and to a lesser extent to the approximation induced by the linearization.

analysis, and for the productivity shock ε_t^z.

Our second exercise is to verify the validity of our financial frictions indicator $\mathcal{I}_t^{\xi}$ defined in (26). From our model simulations we can obtain the value of ψ_t, which measures the relative shadow value of finance and therefore the intensity through which financial frictions affect the real decisions of firms. Therefore, we can measure to what extent, in a realistically calibrated industry, fluctuations in ψ_t are driven by the financial shock ε_t^{ξ}. In Columns 1 and 2 we show, using linear regressions, the share of variation in ψ_t explained by each shock. Column 1 considers the true shocks, and Column 2 the median estimated ones. They show that the most important one is by far the financial shock, which explains 73% of the ψ_t variation in Column 1, and 64.5% in Column 2. The productivity shock only explains around 10% of the variation in ψ_t. Importantly, this is not because the productivity shock is not very volatile. In fact it is the main driver of output, explaining 52% of its variation, while the financial friction shock only explains 19% of its variation.

The third column does a similar exercise, by directly evaluating the financial frictions indicator. We consider indicators based on the 50% highest values of the three shocks ε^{ξ}, ε^{θ}, ε^{z}, and equal to zero otherwise. In the column, we report the percentage of observations that coincide with the 50% highest values of ψ. A value around 50% would indicate that the high shock observations are unable to identify firms with high ψ. This is the case of the z and θ shocks. Conversely, we find that high ξ observations include 85% of the above median observations of ψ, and 97% of the highest quartile observations of ψ (column 4). We also find very similar results if we use the alternative financial frictions indicators described in (42) and (43).

Summing up, these simulation results show that, in the context of the class of models considered, our methodology obtains useful information on the unobservable shocks, and that the indicator $\mathcal{I}_t^{\xi}$ is a reliable indicator of financial frictions. Similar results can be obtained when data is simulated from the more elaborate models discussed in Section 2.3.[8]

[8]In the appendix S2.4 we show that, in a model with earning based constraints, productivity shocks are less positively related to the intensity of financial constraints, because an increase in z increases desired investment, which increases financial frictions as in the benchmark model, but also it increases the collateral value of earnings, and thus partially relaxes financial frictions. Therefore in a model with earning based

Table S2: Simulation results

Output	(1) True	(2) Median	(3) Lower	(4) Upper
$\hat{\varepsilon}^{\xi}_{i,t}$	-0.0350	-0.0321	-0.0313	-0.0324
$\hat{\varepsilon}^{\theta}_{i,t}$	-0.0011	-0.0155	-0.0163	-0.0171
$\hat{\varepsilon}^{z}_{i,t}$	0.0478	0.0500	0.0533	0.0532
Constant	3.3074	3.7280	2.7086	2.7562
Obs.	80,000	80,000	80,000	80,000
R^2	0.7329	0.6114	0.6887	0.6951
Labour	**(1) True**	**(2) Median**	**(3) Lower**	**(4) Upper**
$\hat{\varepsilon}^{\xi}_{i,t}$	-0.0571	-0.0528	-0.0519	-0.0518
$\hat{\varepsilon}^{\theta}_{i,t}$	-0.0017	-0.0242	-0.0224	-0.0241
$\hat{\varepsilon}^{z}_{i,t}$	0.0405	0.0421	0.0465	0.0463
Constant	5.6170	6.1110	5.1886	5.1816
Obs.	80,000	80,000	80,000	80,000
R^2	0.7875	0.7091	0.7586	0.7546
Debt	**(1) True**	**(2) Median**	**(3) Lower**	**(4) Upper**
$\hat{\varepsilon}^{\xi}_{i,t}$	-0.4404	-0.4177	-0.4161	-0.4241
$\hat{\varepsilon}^{\theta}_{i,t}$	0.0054	0.0887	0.1185	0.1171
$\hat{\varepsilon}^{z}_{i,t}$	0.1065	0.1175	0.1244	0.1222
Constant	2.6758	2.1284	2.8343	2.1981
Obs.	80,000	80,000	80,000	80,000
R^2	0.8832	0.8571	0.8253	0.8115

Notes: The table shows the results from regressing output $\log y_t$, labour $\log l_t$ and debt $\log b_t$ on the true shocks (column (1)) and the estimated structural shocks (columns (2)-(4)). The estimates for the shocks correspond to the median (2), lower (3) and upper (4) bound of the identified set. The log of beginning of the period financial wealth is also included in all regressions as a control variable.

Table S3: Variation in ψ explained by the shocks

	% variation explained		Share $\psi > 50\%$	Share $\psi > 75\%$
	True shock	Estimated shock		
ε^{ξ}	73%	65.6%	86%	97%
ε^{θ}	0.1%	10.8%	53%	63%
ε^{z}	9.5%	9.3%	59%	41%

Notes: In Column 1 we display the R^2 obtained from regressing the relative shadow value of finance ψ_t on each of the three true shocks individually. In column 2 we do the same exercise using instead the median estimated shocks. In Column 3 we report the percentage of observations for which the indicators that select the 50% firms with highest ε^{ξ},ε^{θ} and ε^{z} (respectively in rows 1, 2 and 3) include the observations with 50% highest value of ψ. In column 3 we show the same exercise but this time with the 75% highest values of ψ.

S5 Further empirical results

In this appendix, we provide some robustness checks of the main results shown in Section 4.1. Table S4 repeats the estimation in column 1 of Table 1, but varying the threshold τ in 41. Table S5 repeats the same estimations but selecting different values of the identified set. Results are qualitatively similar regardless on whether we compute the indicator $\widehat{\mathcal{I}}^{\xi}_{i,t-1}$ using the upper or lower bound values of the identified set for $\hat{\varepsilon}^{\xi}_{t-1}$.

Table S6 replicates the exercise in Table 1 using the large sample of Spanish firms from SABI, which is representative of the population of firms above 5 employees. One limitation of this sample is that it is only available from 2000 (some years before 2000 are available, but with very limited coverage). Since we use two lags to compute the shocks, we can only include the years from 2002 to 2007 for the independent variables. We focus on a balanced sample of firms present in the whole period, for a total of 26.868 firms. Table S6 shows that the coefficient of $\mathcal{I}^{\xi}_{t-1}$ is negative and significant, indicating that over the sample period, firms that are more financially constrained in year t reduce their employment in year $t+1$ more than the other firms. This result, which is in contrast to what we found for Compustat in Table 1, is likely due to the fact that we are analyzing predominantly small firms for which financial shocks are more persistent than for large ones also in normal times, not only

borrowing, the financial frictions shock ε^{ξ} would be even relatively more important than ε^{z} in driving the fluctuations in ψ, than in the benchmark model.

Table S4: Financial frictions in 2007 and employment contraction in 2008. Alternative thresholds to compute the financial frictions indicator.

	(1)	(2)	(3)
	$> 50\%$	$> 75\%$ (Benchmark)	$> 90\%$
$\log(l_{t-1})$	0.895***	0.896***	0.896***
	(96.684)	(99.962)	(101.229)
$\log(l_{t-1}) * Gr$	-0.003	-0.006	-0.006
	(-0.949)	(-1.577)	(-1.546)
$\mathcal{I}^{\xi}_{t-1}$	-0.002	0.003	-0.001
	(-0.489)	(0.575)	(-0.171)
$\mathcal{I}^{\xi}_{t-1} * Gr$	-0.041***	-0.067***	-0.088***
	(-3.116)	(-3.967)	(-3.441)
Constant	0.151***	0.146***	0.148***
	(9.736)	(10.087)	(10.504)
Obs.	6,710	6,710	6,710
R^2	0.858	0.858	0.858
Number of firm	506	506	506

Notes: The table studies the differential effects of the great recession on financially constrained firms. In column 1, Firm fixed effects and Sector-Year fixed effects are included in all specifications. The dependent variable is the logarithm of the number of employees in year t, $\log(l_t)$. Among the regressors, $\widehat{\mathcal{I}}^{\xi}_{t-1}$ is a dummy variable equal to one for the firms with largest value of the financial friction shock $\hat{\varepsilon}^{\xi}_{t-1}$ in year $t-1$ and zero otherwise. In columns 1, 2 and 3 the threshold value of $\hat{\varepsilon}^{\xi}_{t-1}$ is the 50th, 75th and 90th percentile, respectively. Gr is a dummy variable equal to 1 for year 2008 and equal to zero otherwise. Standard errors are clustered at the firm level. Robust t-statistics are given in parentheses and *** p<0.01, ** p<0.05, * p<0.1.

Table S5: Financial frictions in 2007 and employment contraction in 2008, alternative values of the identified set

	(1)	(2)	(3)
	lower bound value of $\hat{\varepsilon}^{\xi}_{t-1}$	median value of $\hat{\varepsilon}^{\xi}_{t-1}$ (Benchmark)	upper bound value of $\hat{\varepsilon}^{\xi}_{t-1}$
$\log(l_{t-1})$	0.895***	0.896***	0.895***
	(99.422)	(99.962)	(97.772)
$\log(l_{t-1}) * Gr$	-0.003	-0.006	-0.005
	(-0.845)	(-1.577)	(-1.477)
$\mathcal{I}^{\xi}_{t-1}$	-0.002	0.003	-0.004
	(-0.407)	(0.575)	(-0.627)
$\mathcal{I}^{\xi}_{t-1} * Gr$	-0.052***	-0.067***	-0.039**
	(-3.285)	(-3.967)	(-2.103)
Obs.	6,735	6,735	6,735
R^2	0.856	0.857	0.856
Number of firm	508	508	508

Notes: The table studies the differential effects of the great recession on financially constrained firms. In column 1, Firm fixed effects and Sector-Year fixed effects are included in all specifications. The dependent variable is the logarithm of the number of employees in year t, $\log(l_t)$. Among the regressors, $\widehat{\mathcal{I}}^{\xi}_{t-1}$ is a dummy variable equal to one for the firms with largest value of the financial friction shock $\hat{\varepsilon}^{\xi}_{t-1}$ in year $t-1$ and zero otherwise. In columns 1, 2 and 3 the value of $\hat{\varepsilon}^{\xi}_{t-1}$ corresponds to the lower bound, median, and upper bound of the identified set, respectively. Gr is a dummy variable equal to 1 for year 2008 and equal to zero otherwise. Standard errors are clustered at the firm level. Robust t-statistics are given in parentheses and *** p<0.01, ** p<0.05, * p<0.1.

during a financial crisis.

Nonetheless, we also find that the Coefficient of $\mathcal{I}_{t-1}^{\xi} * Gr$ is also negative and significant in all specifications, indicating that this negative relation was even stronger between year 2007 and 2008, thus confirming the results obtained for the Compustat sample.[9]

[9]Note that, due to the data limitation explained above, the SABI sample only starts in 2002, and hence it could be that the coefficient of $\mathcal{I}_{t-1}^{\xi} * Gr$ is in part driven by the fact that we compare 2007 with a limited number of years in which there was fast growth in Spain. Conversely, for the Compustat sample, we avoid this problem by focusing on a much longer time series and by performing several placebo experiments, as explained in Section 4.1.1.

Table S6: Financial frictions in 2007 and employment contraction in 2008 - Small and Medium Spanish Manufacturing Firms.

	(1)	(2)	(3)	(4)	(5)
$\log(l_{t-1})$	0.569***	0.563***	0.576***	0.642***	0.621***
	(96.175)	(94.756)	(96.053)	(107.877)	(103.088)
$\log(l_{t-1}) * Gr$	0.002	0.000	-0.002	-0.006***	-0.007***
	(1.220)	(0.010)	(-1.487)	(-3.923)	(-4.297)
$\mathcal{I}^{\xi}_{t-1}$	-0.019***	-0.021***	-0.020***	-0.018***	
	(-15.770)	(-17.308)	(-16.780)	(-14.434)	
$\mathcal{I}^{\xi}_{t-1} * Gr$	-0.042***	-0.042***	-0.040***	-0.034***	
	(-13.134)	(-13.168)	(-12.620)	(-10.655)	
$\mathcal{I}^{Z}_{t-1}$		-0.021***	-0.016***	0.005***	
		(-17.944)	(-13.493)	(3.959)	
$\mathcal{I}^{Z}_{t-1} * Gr$		-0.037***	-0.029***	-0.011***	
		(-11.467)	(-8.779)	(-3.390)	
$Small_{t-1}$			-0.037	-0.046	-0.039
			(-0.798)	(-0.962)	(-0.803)
$Small_{t-1} * Gr$			0.004	0.002	0.001
			(1.023)	(0.567)	(0.267)
$Highlev_{t-1}$			0.001	0.000	-0.001
			(0.756)	(0.004)	(-0.460)
$Highlev_{t-1} * Gr$			-0.008**	-0.008*	-0.009**
			(-2.457)	(-1.681)	(-2.098)
$Lowprod_{t-1}$			-0.046***	-0.015***	-0.014***
			(-19.216)	(-5.396)	(-5.161)
$Lowprod_{t-1} * Gr$			-0.038***	-0.014***	-0.013***
			(-11.761)	(-3.429)	(-3.334)
$labp_{t-1}$				0.141***	0.138***
				(28.097)	(30.433)
$labp_{t-1} * GR$				0.021***	0.022***
				(7.767)	(8.361)
lev_{t-1}				-0.002	-0.015***
				(-1.221)	(-8.968)
$lev_{t-1} * GR$				-0.003	-0.004**
				(-1.395)	(-2.100)
$\hat{\varepsilon}^{\xi}_{t-1}$					-0.018***
					(-21.214)
$\hat{\varepsilon}^{\xi}_{t-1} * Gr$					-0.021***
					(-12.517)
Obs.	134,272	134,272	134,272	134,272	134,272
R^2	0.355	0.360	0.366	0.380	0.385
Number of firm	24,868	24,868	24,868	24,868	24,868

Notes: The table shows the differential effects of the great recession on financially constrained firms. Firm fixed effects and sector-year fixed effects (2-digit NACE sectors) are included in all specifications. The dependent variable is the logarithm of the number of employees in year t, $\log(l_t)$. Among the regressors, $\widehat{\mathcal{I}}^{\xi}_{t-1}$ is a dummy variable equal to one for the upper quartile of the financial friction shock $\hat{\varepsilon}^{\xi}_{t-1}$ in year $t-1$ and zero otherwise. $\widehat{\mathcal{I}}^{Z}_{t-1}$ is a dummy variable equal to one for the lower quartile of the productivity shock $\hat{z}_{t-1}$ in year $t-1$ and zero otherwise. Gr is a dummy variable equal to 1 for year 2008 and equal to zero otherwise. lev_{t-1} is total debt over fixed assets, and $labp_{t-1}$ is labour productivity (measured as real output divided by number of employees). The variables $highlev_{t-1}$ and $lowprod_{t-1}$, are equal to one for the 25% firm-year observations with highest leverage and lower labour productivity in year $t-1$, respectively, and equal to zero otherwise. $Small_{t-1}$ is a dummy variable if the firm belongs to the quartile of smallest firms in period $t-1$, and equal to zero otherwise. Standard errors are clustered at the firm level. Robust t-statistics are given in parentheses and *** p<0.01, ** p<0.05, * p<0.1.

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