Executive-Pay Caps, Efficiency and Inequality

Francesco Caselli*

Abstract

I study the introduction of caps on managerial pay in a span-of-control economy with heterogenous managerial talent. The introduction of pay caps transforms managers in equalitarian cooperators, and leads them to hire fewer workers than they would in the absence of pay caps. This implies that the marginal product of labor is not equalized among firms, and that agents who would be of too low ability to manage firms in the Laissez-Faire Economy do become managers in the Pay-Cap economy. The pay cap does reduce inequality between managers and workers, and among managers, but increases inequality among workers: the "lucky" ones who get to work for a high-ability manager earn more than those stuck with low-ability managers. Quantitatively, overall inequality does decrease substantially, but the social planner would have to be more inequality averse than is usually assumed by economists to choose to impose the pay cap.

*London School of Economics, Centre for Macroeconomics (CFM), and Centre for Economic Policy Research (CEPR). Thanks: Thomas Brzustowski, Russell Cooper, Wouter Den Haan, Pedro Gomes, Johannes Matt, Chris Pissarides, Martin Weale, and especially Shengxing Zhang (for helping me to clarify the equilibrium definition in the pay-cap economy) and seminar participants at the LSE, ETH Zürich, Singapore Management University. I acknowledge financial support from CFM and declare that I have no financial relationships or other potential conflicts of interest in undertaking or circulating this research.

1 Introduction

In recent years and in multiple countries there have been calls for the introduction of legislated caps on the pay of top corporate officers. Congresspersons in the United States have submitted legislation limiting the ratio of CEO pay to median pay in the company to 50. The British Labour Party's most recent manifesto calls for a similar cap set at 20.[1] There already exist caps on certain bonuses in the EU and in Australia. Calls for pay caps from civil society and public intellectuals are widespread.

Proponents of executive pay caps have a variety of objectives, but certainly the chief goal is to limit the degree of inequality both within the firm and in society at large.[2] Opponents of the proposal argue that it would be distortionary and lead to inefficiencies.

Despite the earnestness of proponents and the confident dismissal of opponents there is little detailed discussion of the effectiveness of pay caps as a check on inequality or, respectively, of the precise mechanisms through which they would adversely affect efficiency, much less a quantitative assessment of either. In this paper, I attempt a macroeconomic, general equilibrium formalization of pay-cap policies and present a first-pass assessment of its quantitative consequences for inequality, GDP, and welfare.

My chosen framework for this analysis is a span-of-control economy à la Lucas (1978). Agents in the model make a worker-v-manager occupational choice decision, with full knowledge of their own managerial abilities (which are heterogeneous) and perfect foresight about the financial consequences of their choices. The fact that the model features exclusively labour income and the two stark categories of "managers" and "workers" make it ideally suited to study the consequences of policies which constrain the relative labour income of the former relative to the latter. Additional advantages of the framework include its simplicity and its being well understood in

[1] For comparison, the average CEO-median worker *base* pay ratio is 159 in Russell 3000 companies, 263 in SP 500 companies, and 144 in FTSE 100 companies (Wharton's WRDS database). Ratios inclusive of bonuses, pensions, options and shares, and other perks are much larger, but harder to compute. Typically, proposed pay caps are for total compensation and would therefore be highly binding – and most likely reverberate well outside these "elite" companies (for which CEO compensation is available).

[2] Several authors have identified in CEO compensation a leading driver of the enormous rise in inequality in capitalist societies since the 1980s (e.g. Piketty (2017).

its "laissez faire" version (i.e. without pay caps) - the benchmark against which I will evaluate the "pay-cap" economy. Last but not least, the framework allows me to focus exclusively on labor-income inequality without having to track the ways in which labour-income inequality is amplified by or interacts with capital income inequality.

It turns out that imposing pay caps on managerial income transforms the workings of the labour market. In the Laissez-Faire scenario, as is well known, all workers earn the same wage.[3] The reason is that if worker i is paid less by manager j than worker i' is paid by manager j', worker i can successfully approach manager j' with a proposal which undercuts worker i'. In a Pay Cap economy no such undercutting offer will be accepted. Because the manager's pay is tied to the pay of workers, a manager who takes on a worker at a lower wage than what he currently pays is cutting his own income. More generally, while in a Laissez Faire economy managers welcome every opportunity to lower wages, in a Pay-Cap economy managers seek to maximize the average pay because their own income is tied to it.

The difference in managerial objective function induced by the Pay Cap causes the Pay-Cap economy to be inefficient. In the (efficient) Laissez-Faire economy the marginal productivity of workers is equalized across firms, with more productive managers presiding over larger firms. In the Pay-Cap economy, instead, managers do not wish to expand employment beyond a certain level, because doing so (given the diminishing returns induced by the span-of-control mechanism) would drag down income per worker in the firm. Indeed the model shows that in the Pay-Cap economy all managers choose the same firm size, irrespective of their managerial abilities. Essentially, the Pay-Cap economy is inefficient because talented managers refuse to take on enough workers.

Perhaps surprisingly, the model's implications for the effects of Pay-Caps on inequality are ambiguous. Obviously inequality *within firms*, i.e. between managers and workers, declines relative to the Laissez-Faire case. And since managers of different ability now operate firms

[3]For simplicity in this paper I assume that all agents have the same productivity as workers – i.e. they only differ in their managerial skills. However the insight developed in this paragraph applies equally if agents are heterogenous in their productivity as workers. In this case all workers *with the same productivity* earn the same wage in the Laissez-Faire economy.

of the same size, inequality among managers also falls. On the other hand, inequality among workers will tend to increase. The reason is that the (lucky) workers employed by high-ability managers earn more than the ones employed by lower ability managers. Theoretically, depending on parameter values (and specific inequality measure) this latter effect could dominate. I also note a corollary of the fact that workers earn more when working for better managers: for the Pay-Cap system not to unravel side payments from workers to managers must be prohibited.[4]

The quantitative part of the paper begins with a calibration in which parameters of the model are chosen so that its Laissez-Faire version matches moments of the US firm size distribution - the idea being that the Laissez-Faire set up more closely approximates the US status quo. In the Laissez-Faire economy the distribution of firm sizes is closely related to the parameters of the production function and to the distribution of managerial talent, and it is therefore a good guide to the quantification of the model.

Having quantified the model, I solve for key equilibrium macroeconomic aggregates both under laissez faire and under pay caps. The results confront us with a classic efficiency-inequality trade off: the Laissez-Faire economy exhibits a considerably higher GDP, at the cost of much greater inequality. As a crude way to adjudicate between these two options I compute welfare in the two economies as the sum of all individual utilities, using an isoelastic utility function with a coefficient of relative risk aversion equal to 2. The Laissez-Faire economy exhibits greater welfare under this criterion.

A final contribution of the paper is to relate the theory of pay caps to the theory of labor management (e.g. Ward, 1958; Vanek, 1970; Dréze, 1986; Brzuwstowski and Caselli, 2021). The key observation is that the pay cap induces on the manager an objective function (average income per person involved with the firm) which is identical to the objective function typically associated with equalitarian producer cooperatives. Building on this observation, I establish formal equivalence results between the Laissez-Faire and Pay-Cap economies and two versions of an economy populated by producer cooperatives. In particular, the Laissez-Faire (Pay-Cap)

[4]Conversely side payments would restore efficiency, as is well known about models with nontrasferable utility, such as mine.

economy is isomoprphic to the cooperative economy when workers can (cannot) commit to a certain compensation level for managers.

The mechanisms at the centre of my model are somewhat related to similar mechanisms highlighted by Farrell and Scotchmer (1988), who also find that equalitarian pay (can) lead to inefficiently small firms. However, in their model this results entirely by sorting of workers by ability: high ability workers choose to eschew the benefits of increasing returns to scale to avoid subsidizing low-ability workers. My mechanism is different because I deliberately rule out sorting by making agents homogeneous as workers.

Section 2 describes the basic physical set up of the economy. Section 3 briefly reviews the (well-know) equilibrium solution for the Laissez-Faire economy. Section 4 is where the paper's novel contribution begins. In particular, it proposes an equilibrium concept for a Pay-Cap economy and introduces an algorithm to solve for it. Section 5 calibrates the model and presents the quantitative comparison between the Laissez-Faire economy and the Pay-Cap economy. Finally Section 6 returns to theory and establishes equivalence results between the economies studies in this paper and cooperative economies with the without commitment.

2 Environment

Consider a static economy with a population of mass 1. The basic physical framework is a Lucas span-of-control environment. Each agent can be either a manager or a worker. Each agent has a managerial ability z, where the CDF of z is $G(z)$ and is defined on $[\underline{z}, \infty)$. Everyone has the same ability when working as a worker.

A production unit can only function if it has a manager. Conditional on having a manager of ability z, output is $zF(l)$, where l is the mass of workers. The manager has a limited span of control and therefore $F'(l) > 0$ and $F''(l) < 0$.

The assumption that agents are homegeneous as workers is quite restrictive. In the more realistic case in which individuals have heterogeneous abilities as workers, the Pay-Cap economy

will exhibit very strong tendencies towards sorting by ability - i.e. the best managers will tend to match with the best workers. On the other hand I believe there is considerable value in presenting a preliminary exploration of the theoretical mechanisms triggered by Pay-Caps in a simpler framework which abstracts from sorting issues. Another possible way to think about the omission of heterogeneity among workers is that it makes the reduction in inequality achieved by the Pay-Cap economy as computed here an upper bound on what can be achieved under a more realistic description.[5]

3 Laissez-Faire Economy

In a laissez-faire economy individuals decide whether to be managers or workers. If they are managers they choose how many workers to hire. The labor market is perfectly competitive. This implies that all workers receive the same wage w.

Managers are residual claimants of output after paying the workers, hence the income of a manager of ability z is

$$zF(l) - wl.$$

An equilibrium for this economy is completely standard and well known. As a summary, it is defined by the following requirements.

- Managers maximize their own income taking the wage w as given. This implies that, for

[5]Yet another way of thinking about the omission of worker heterogeneity is that the model describes a combined policy of Pay-Caps *and* prohibitions on segregation by ability. In practice, the way segregation would be expected to occur is by the outsourcing of low-productivity labor, e.g. technology firms purchasing security and cleaning services from other firms. Practices like this are indeed already present in many economies, and are aimed at circumventing internal wage norms as well as norms regarding health insurance and pensions, but they would be exacerbated in a Pay-Cap economy. They can be countered (and in some countries they already are) by mechanisms that force a legal redefinition of the firm boundaries for the purposes of pay-cap calculations. If a cleaning-service provider sells 80% of its services to a single client the pay-cap rule could be applied over a composite firm including the cleaning service workers. This simple mechanism can be generalized using each firm's input-output structure. If a firm z sells x% of its output to firm y, the average wage of firm z could contributes x% to the average wage of firm y. Economies where the Pay-Cap rule is applied on wages defined according to criteria such as these will behave very similarly to the Pay-Cap economy with no worker heterogeneity described here.

every z such that some agents of type z are managers,

$$zF'(l) = w. \tag{1}$$

We define $l_z(w)$ the solution to this equation for given z. Note that $l_z(w)$ is increasing in z. We also define $\pi_z(w)$ the corresponding maximized values of managerial income. Note that $\pi_z(w)$ is also increasing in z, as higher-ability managers always have the option of producing at the same level of employment as lower-productivity managers.

- If in equilibrium an agent with ability $\tilde{z}$ is indifferent between managing and being a worker, then all agents with ability $z > \tilde{z}$ ($z < \tilde{z}$) are managers (workers). This reflects the fact that there is both free entry in the managerial occupation and the labor market is competitive. $\tilde{z}$ is defined by

$$\tilde{z}F(l_{\tilde{z}}(w)) - wl_{\tilde{z}}(w) = w \tag{2}$$

- The labor market must clear. The market-clearing condition is

$$\int_{\tilde{z}}^{\infty} l_z(w)dG(z) = G(\tilde{z}) \tag{3}$$

Equations (2) and (3) pin down the equilibrium wage and the cutoff $\tilde{z}$.

The following observation will also prove very useful when comparing the LF to the PC economy. Using (1) and (2) we find the employment level in the marginal firm, l^*, which solves

$$\frac{F(l^*)}{1+l^*} = F'(l^*).$$

Hence, in the LF economy the marginal firm sets the average income accruing to all those involved in it (one manager and l workers) equal to the marginal product of an additional worker.[6] This is a reflection of the LF economy's efficiency (and indeed it is the core observation to prove it,

[6] I deliberately eschew the more rigorous lexicon involving mass and measure in order to focus on the intuitive content of the discussion here and elsewhere.

which is easily done). The marginal manager is one who is equally useful as a worker or as an employer, and the two sides of the equation represent her contribution to output under the two choices.

Finally, notice that a corollary of the preceding discussion is that all non-marginal firms in the LF economy will have employment l larger than l^*.

Having solved for l^* and $\tilde{z}$ we can compute some standard macroeconomic aggregates, later to be compared to the analogous aggregates in the PC Economy. GDP is the sum of the output produced by all firms:

$$Y^{LF} = \int_{\tilde{z}}^{\infty} z(l_z(w))^{\alpha} dG(z).$$

Assuming that individual utility is a function U of individual income, we can also define the utilitarian welfare measure

$$W^{LF} = U(w)G(\tilde{z}) + \int_{\tilde{z}}^{\infty} U(\pi_z(w))dG(z). \tag{4}$$

We can also compute a number of measures of inequality. A "global" inequality measure would treat all incomes equally irrespective of occupation. But it would also be interesting for comparative purposes to produce "within" inequality measures (i.e. inequality among workers, and inequality among managers), as well "between" (inequality between managers and workers).

Because of their intuitive appeal, for overall and within inequality I focus on percentile ratios. When looking at the global inequality measure, we are interested in y_p/y'_p, where y_p is the p_{th} percentile of the personal income distribution, and p and p' are the percentiles chosen for comparison (e.g. 90_{th} and 10_{th}). In the LF model income is increasing in z, so the agent occupying the p_{th} percentile of the distribution of *personal* income has ability z_p identified by

$$G(z_p)100 = p.$$

Then we have

$$y_p^{LF} = \begin{cases} w & \text{if } z_p \leq \tilde{z} \\ \pi_{z_p}(w) & \text{if } z_p > \tilde{z} \end{cases} \tag{5}$$

As we will see in the calibration section my best assessment is that less than 5% of US workers can be classified as managers, so we need to pick a "high" percentile greater than the 95_{th} to get a percentile ratio different from 1.

The "within" inequality measure for workers is trivially 1, as all workers earn the same wage. This will not be true in the PC economy. Inequality within managers can be recovered analogously to overall inequality by an appropriate rescaling.[7]

For inequality between managers and workers one has, in principle, two choices that seem most natural. One is the average managerial income divided by the average worker income. the other is the average among firms of the ratio of managerial income to worker income. In the present model, these two measures give identical results. In the LF economy this is because all workers earn the same wage. In the PC economy this is because workers and managers earn the same income and because, as we shall see, the ratio of workers to managers is constant across levels of ability. In the LF economy the measure of between inequality is thus calculable as

$$\overline{\pi}^{LF} = \frac{\int_{\tilde{z}}^{\infty} \pi_z(w) dz}{[1 - G(\tilde{z})]w}.$$

4 Economy with Pay Caps

We now study what happens if the government imposes a rule that managers and workers within a firm must be paid the same amount.[8] I proceed in three stages. First, I discuss informally how the introduction of a pay cap causes the "labour market" as traditionally modelled to break

[7] The agent occupying the p_{th} percentile of the *managerial* income distribution has ability z_p identified by $[G(z_p) - G(\tilde{z})]/[1 - G(\tilde{z})]100 = p$. Her income is $\pi_{z_p}(w)$.

[8] In future versions of this paper I will re-write the model (or add an extension) for a generic maximum allowable pay ratio, as well as solve for the welfare maximizing pay ratio. I will also explore alternative pay-caps, particularly in terms of the average wage in the economy - instead of the within-firm wage.

down, and the consequent economic logic that will dominate the allocation of individuals to managerial and worker jobs, respectively. Second, on the basis of that logic, I set out a compact definition of equilibrium based on a minimum set of rationality requirements. Third, I develop a simple recursive algorithm to solve for the equilibrium of the PC economy. This construction also doubles as a proof of existence (uniqueness is established in the appendix).

We begin the exploration of the PC economy by defining the income $y(z, l)$ earned by the manager of a firm in which the manager has ability z and she hires l workers. Because of the constraint that everyone earns the same, we have

$$y(z, l) = \frac{zF(l)}{1 + l}.$$

The numerator is the total revenue generated by the firm. The numerator is the number of people sharing (equally) this pie: the manager and the l workers.

Just as in the LF economy, it is plausible to assume that any manager finding herself at the helm of a firm will try to choose the number of workers which maximizes her income. However, in light of the different objective function, this translates into very different labour-demand behaviour. In particular, maximization of $y(z, l)$ with respect to l implies that all managers in the PC economy wish to set employment at l^*, the same employment level chosen by the marginal firm in the LF economy.

The intuition for this result is well understood from the literature on equalitarian cooperatives. Recall that l^* equalizes average to marginal revenue. If marginal revenue exceeds average revenue the manager can enhance her objective (which is to maximize average revenue, just as in a coop) by hiring more workers - and viceversa.

An immediate consequence of this observation is that firm size in the PC economy is invariant with the ability of the manager. Intuitively, an increase in z increases average and marginal income by the same proportion, and thus cancels out in the choice of optimnal employment level. It is easy to foresee that this insensitivity of firm size to managerial skill will be one important

source of inefficiency in the PC economy. In particular, it means that the marginal product of labour is not equalized across firms.

Up to now in this discussion I have treated the occupational choice of agents as given, but of course a full account of the PC economy must identify not only which agents become managers but also how workers are allocated to managers with different ability. The key problem here is that – even though agents are homogenous in their productivity as workers – they enjoy different incomes depending on the manager they work for. In particular, workers earn $y(z, l^*)$, just like their managers, so the lucky ones who work for a higher-z manager earn more.

As discussed in the Introduction, there is no plausible mechanism which forces worker incomes to equalize, as in the LF economy. The standard argument for workers receiving a common wage is that a worker receiving a lower wage than another worker can always offer to replace the higher-wage worker for a slightly lower salary. No such under-cutting can exist in a pay-ratio economy. A worker approaching a manager offering to work for less than $y(z, l)$ will be rebuffed, because the manager would then have to pay herself this lower income as well.

Despite the absence of the coordinating mechanism of wages, it is easy to agree on certain minimal rationality properties we would expect a PC economy to feature. First of all, since anyone would always prefer to manage than to being workers in firms managed by someone of lower ability than themselves, PC-economy equilibria should not have lower ability agents manage higher ability agents. If they did, both the lower-ability agent and the higher-ability agent would *both* be better off by switching roles within the organization - and it is difficult to see what could prevent them from doing so.

It is also the case that *everyone* would always prefer to be a worker in a firm managed by someone with higher ability than being managers themselves. This creates a form of "excess supply" of labour to managers with high ability. The (hypothetical) person with highest ability in the economy has literally the entire population vying to work for her. As we move down the ability distribution, fewer and fewer workers cue up for a job. This means that we need to identify an assignment criterion which determines, for each level of managerial ability, who will be picked

as worker from the set of those who would welcome an offer from that employer. Since agents are homogeneous as workers, managers are indifferent as to the composition of their worker pool. This suggests that a notion of equilibrium consistent with rationality involves managers choosing their workers randomly from the "applicant pool," i.e. that all workers who would prefer to work for a given manager rather than manage themselves should be equally likely to end up working for her.[9]

We can now summarize and make more precise the set of rationality properties we wish the PC economy should feature, by collecting them in the following formal definition of an equilibrium.

Definition 1. *Define*

$$y_z = \max_l \frac{zF(l)}{1+l} = \frac{zF(l^*)}{1+l^*}.$$

An equilibrium for the PC Economy is a function $\mu(z)$ which represents the measure of agents of type z who become managers, and a function $\omega(z, z')$ which represents the fraction of agents of type z who work for a manager of type z', where

1. *Every agent of type z who is a manager hires l^* workers and earns y_z*
2. *Workers who work for a manager of type z earn y_z*
3. *An agent of type z is either a manager or works for a manager of type z', such that $y_{z'} \geq y_z$*
4. *If $A_{z'} = \{z; y_{z'} > y_z\}$, then $\omega(z, z') = \omega(z')$ for all $z \in A_{z'}$*
5. *The labor market clears, or*

$$(1+l^*)\int_{\underline{z}}^{\infty} \mu(z)dz = 1$$

[9]This is obviously where the assumption that all agents are equally productive as workers has a big impact. It is easy to see that adding heterogeneous productivity as workers can inject a strong impulse for assortative matching in the PC economy, and this can further deteriorate the PC economy's performance both from the point of view of efficiency (if ability as worker is positively correlated with ability as manager, because in this case even more agents who *should* be managers will be workers instead) and inequality. As discussed in Section 2, the model with homogeneous ability should therefore be interpreted either as an upper bound on the welfare benefits of a PC policy, or as a model of a *combination* of PC policies and policies which limit firms' abilities to use outsourcing to get around the PC constraint.

Equilibrium conditions (1-3) and (5) are straightforward and follow immediately from the discussion in the preceding paragraphs without the need for further commentary. Condition (4) is the formalization of the randomization criterion for hiring which follows from managers' indifference as to the composition of their pool of workers. The set $A_{z'}$ can heuristically interpreted as the set of applicants for jobs with managers of ability z', and it simply says that all applicants have equal probability to be offered a job. In other words if both agents with managerial ability z_1 and agents with managerial ability z_2 would rather work with a manager of ability z' than manage themselves, then the proportion of all agents of ability z_1 which works for agents of ability z' equals the proportions of all agents of ability z_2 which does the same.

While I think that the compact enumeration of equilibrium conditions above should be satisfactory for most readers, I recognize that some may prefer a more vivid and richer account of the environment to get fully comfortable that my equilibrium requirements are plausible. For this reason, in the appendix I provide color with a description of a more specific environment, loosely inspired by search-theoretic ideas, where the idea of an "applicant pool" is taken quite literally. There I show that Definition 1 is a reduced form of the equilibrium definition in that (narratively more satisfying) extended version of the model. For example, condition (3) can be reinterpreted as saying that managers are agents who applied to work for a better manager but were unsuccessful.

It is useful to linger slightly longer on Definition 1 with some further commentary. Note that the equilibrium conditions are fully consistent with the existence of a threshold $\tilde{z}^{PC}$ such that $\mu(z) = 0$ for $z < \tilde{z}^{PC}$. However, unlike in the LF economy where a threshold clearly divides entrepreneurs from workers, here for all $z \geq \tilde{z}^{PC}$ we have that some agents are workers (for managers with higher ability than theirs) and some are managers (hiring workers not as good as them). Below $\tilde{z}^{PC}$ everyone is a worker. For this reason, and also because each managers hire fewer workers in the PC economy compared to the LF economy, we have $\tilde{z}^{PC} < \tilde{z}$ (the threshold for entrepreneurship in the LF economy).

This discussion of $\tilde{z}^{PC}$ also helps to clarify why the inequality in equilibrium condition (3) is

weak. For all $z > \tilde{z}^{PC}$ managers hire workers who have strictly lower ability (this is guaranteed by the way we have written condition (4), with a $>$). But when we get to $\tilde{z}^{PC}$ there will typically not be left sufficiently many "applicants" of strictly lower ability left so that each potential manager can hire l^* workers. Hence, at (and only at) ability level $\tilde{z}^{PC}$ we have managers hiring workers with their same ability.

More interestingly, we finally note that the equilibrium as outlined features a degree of imperfect sorting. Even though the highest quality managers hire workers from the entire ability distribution, the pool of applicants worsens as we move down the managerial ability distribution. As a result, the percentage of high ability workers increases in the ability of the manager.

I now present a simple algorithm which allows me to construct the equilibrium of the PC economy. I present the mechanism as a *sequential* assignment mechanism. It is important to note that in doing so I am not surreptitiously introducing dynamic elements. The dynamic-sounding language is useful to understand the logic of the construction, but the point is to arrive at an allocation which satisfies Definition 1. Having said that, the logic here will be that we start from the agents with the highest abilities, and assign to each of them l^* randomly-chosen workers from the entire pool of agents who are willing to work for them (which is everybody else). At this first iteration, then, all the highest ability managers and a representative sample of the rest of the population drop out of the "unattached" population. At the next iteration, we proceed in the same fashion with the highest ability managers remaining in the unattached population. And so on until everyone is attached.[10]

More formally, at any iteration t there is a distribution $G_t(z)$ of *unattached workers,* workers who are neither been assigned to manage a firm nor have they yet been assigned to work in a firm. (This is not a CDF as it does not integrate to 1). In this set workers with ability t are assigned to be managers. A fair lottery is held among everyone else to assign them to these manages as workers. This then defines a new set of unattached workers, and so on. Because of the sequential nature of the allocation mechanism, we can trace how $G_t(z)$ evolves with t, i.e. as the allocation

[10]See Farrell and Scotchmer (1988) for a similar recursive strategy to construction (and existence proof) of a Nash equilibrium for an equalitarian economy.

plays itself out.

More specifically, consider a random variable ζ, with distribution $G(-\zeta)$. This is clearly the mirror-image of the distribution of z, only with high values of ability coming first. Now suppose that the allocation mechanism has reached the point where t is the lowest ζ (highest ability) in the set of unattached workers. Suppose furthermore that $\tilde{g}_\zeta(t)$ is the measure of workers of ability $-\zeta$ who are still unattached "at t," then

$$\dot{\tilde{g}}_\zeta(t) = -\frac{\tilde{g}_t(t)l^*}{\int_t^{-\underline{z}} \tilde{g}_v(t)dv}\tilde{g}_\zeta(t). \tag{6}$$

This equation describes the rate at which workers of various abilities become attached. The numerator is the measure of workers becoming attached, i.e. the number of agents becoming managers times the number of workers they hire. The denominator is the total measure of unattached workers. So the ratio is the fraction of workers becoming attached. Since attachment occurs randomly this is also the fraction of workers with ability $-\zeta$ who become attached. This equation constitutes a whole system of infinite differential equations, one for each value of z. For each of them, the boundary condition is

$$\tilde{g}(-\infty) = g(\infty).$$

Labor market clearing tells us when the sequential assignment procedure stops. In particular, there will be a threshold $\tilde{\zeta}$ such that $\tilde{\zeta}$ is the lowest (or first) ζ for which

$$\int_{\tilde{\zeta}}^{-\underline{z}} \tilde{g}_v(\tilde{\zeta})dv \leq \tilde{g}_{\tilde{\zeta}}(\tilde{\zeta})l^*$$

I.e. managers of type $\tilde{z}$ hover up all residual unattached workers. As discussed above, if the inequality is strict then some agents of type $\tilde{\zeta}$ have to become workers even at this iteration.

The allocation mechanism just described pins down the measure of agents of each type z, $\mu(z)$

who are managers in equilibrium. In particular, we have

$$\mu(z) = \begin{cases} \tilde{g}_{-z}(-z) & \text{if } -z < \tilde{\zeta} \\ \frac{\int_{\tilde{\zeta}}^{-z} \tilde{g}_v(\tilde{\zeta})dv}{l^* + \tilde{g}_{\tilde{\zeta}}(\tilde{\zeta})} & \text{if } z = -\tilde{\zeta} \\ 0 & \text{if } z > \tilde{\zeta}. \end{cases}$$

In words, for $z > -\tilde{\zeta}$ the measure of managers of ability z is the measure of agents of ability z who are still unattached when the sequential allocation mechanism reaches them. The allocation mechanism also pins down $\omega(z, z')$. For $z > -\tilde{\zeta}$ we have

$$\omega(z, z') = \frac{\tilde{g}_{-z'}(-z')l^*}{\int_{-z'}^{-z} \tilde{g}_v(-z')dv} \frac{\tilde{g}_{-z'}(-z)}{g(-z)}.$$

It should be clear from the way we have constructed the equilibrium that all of the conditions in Definition 1 are met by the resulting allocation and, in particular, by the functions $\mu(z)$ and $\omega(z, z')$. Conditions 1 and 2 have been used explicitly. Condition 3 is satisfied because we started from the highest ability managers and worked "down" from them. Condition 4 follows from the equation for ω we have just written coupled with the fact that, at each iteration, we have randomized over the unattached population. Condition 5 has also been used explicitly.

It should also be clear that the procedure outlined above is always feasible. Hence, the description of the algorithm for constructing an equilibrium doubles as an equilibrium-existence proof. In the Appendix, I complete the picture by also presenting a proof of uniqueness. Therefore,

Proposition 1. *An equilibrium of the PC Economy exists and is unique.*

Using the distribution $\mu(z)$ we can compute various macroeconomic aggregates which can usefully be compared to the Laissez-Faire analogues. In particular, GDP is again the sum of the outputs of all operating firms

$$Y^{PC} = \int_{\underline{z}}^{\infty} \mu(z) z F(l^*) dz,$$

and the utilitarian welfare measure is

$$W^{PC} = \int_{\underline{z}}^{\infty} \mu(z)(1+l^*)U\left(\frac{zF(l^*)}{1+l^*}\right)dz. \tag{7}$$

In order to compute percentiles of the overall personal income distribution, we use the fact that income (of managers and their workers) is increasing in managerial ability z, coupled with the fact that every income y_z is enjoyed by $\mu(z)(1+l^*)$ agents. This implies that the ability z_p^{PC} associated with income y_p^{PC} at the p_{th} percentile of the income distribution in the PC economy is identified by

$$\int_{\underline{z}}^{z_p^{PC}} \mu(z)(1+l^*)100dz = p.$$

The constant firm-size property also implies that inequality within workers and within managers are identical to overall inequality. Finally, the average ratio of manager to worker income (as well as the ratio of average manager incomes to average worker incomes) is 1.

5 Quantification

5.1 Calibration

We quantify this model under the assumption that $F(l)$ is Cobb-Douglas and $G(z)$ is Pareto:

$$F(l) = l^{\alpha}, \tag{8}$$

$$G(z) = 1 - \left(\frac{\underline{z}}{z}\right)^{\gamma} \text{ for } z \in [\underline{z}, \infty), \text{ and } \gamma > 1. \tag{9}$$

$\gamma > 1$ ensures the distribution has finite mean, which seems a sensible requirement in this context.

One advantage of the minimal parametrization we have imposed is that *all* standard aggregate and distributional equilibrium values of the LF economy can be computed in closed-form - lending considerable nimbleness and transparency to the calibration. The calculation details and closed-form formulas for GDP, welfare, and inequality statistics computed below (for the LF economy)

are collected in an Appendix not for publication. The PC economy is computed numerically following discretization (and using the sequential assignment mechanism described in the previous section).

The model has three parameters: α, γ, and $\underline{z}$. The last of these three has no bearing on ratios of aggregates from the two economies (e.g. Y^{LF}/Y^{PR} is invariant to $\underline{z}$) or on comparisons of inequality measures. Hence we only have to calibrate α and γ. we presume that the USA economy resembles the LF economy and hence we calibrate the parameters by picking two moments from the data and matching them to the LF version of the model.

Both of the parameters we need to pin down are "production side" coefficients which are closely linked to the firm-size distribution. α governs the speed at which decreasing returns set in, while γ governs the dispersion of productivity, which in turn has a first-order impact on the dispersion of firm sizes. Two available moments from the firm size distribution are average firm size, and the fraction of firms which have at least 10,000 employees. These will be our calibration targets.[11]

In the LF economy, the assumption on F implies, from (1),

$$l_z(w) = \left(\frac{\alpha z}{w}\right)^{\frac{1}{1-\alpha}}$$

Then from (2)

$$\tilde{z} = \frac{1}{\alpha}\left(\frac{\alpha}{1-\alpha}\right)^{1-\alpha} w \equiv Aw$$

Plugging the last two expression into the market clearing condition (3) we can solve for w as

$$w = \frac{1}{A}\left(\frac{\gamma - 1}{\gamma - \alpha\gamma - 1}\right)^{1/\gamma} \underline{z}.$$

We can use these results to produce closed-form formulas for our calibration targets. Average

[11] I report below on an alternative calibration strategy which uses a moment from the managerial income distribution.

firm size is

$$s = \frac{1}{1 - G(\tilde{z})} = \left(\frac{\tilde{z}}{\underline{z}}\right)^{\gamma} = \frac{\gamma - 1}{(1-\alpha)\gamma - 1}. \tag{10}$$

This is increasing in α (the more slowly marginal productivity declines, the larger the span of control) and decreasing in γ (the thinner the tails of the productivity distribution, the smaller the average firm size).

As for the second calibration target, define λ_q as the share of aggregate employment in firms at least as large as q.[12] We can invert our labor-demand equation to identify the z of the manager who hires q workers, z_q:

$$z_q = (q-1)^{1-\alpha} \left(\frac{w}{\alpha}\right).$$

Then

$$\lambda_q = \int_{z_q}^{\infty} (1 + l_z) dG(z),$$

which after calculations reduces to

$$\lambda_q = \frac{\gamma(1-\alpha)q - 1}{\gamma - 1} \left(\frac{\alpha}{(1-\alpha)(q-1)}\right)^{\gamma(1-\alpha)}. \tag{11}$$

Equations (10) and (11) can be jointly solved for α and γ as functions of s and λ_q (for a given choice of q among those available from the data). More details on the joint solution of these equations in the unpublished appendix.

We measure s in the US economy as the total number of employees (including the self-employed), divided by the total number of firms in the economy. Using the 2019 US Census's Annual Business Survey this is 22.4. (The data do not include public sector workers, agriculture, religious services, or private household work). Note that $1/s$ in our model is the percentage of managers in the labour force, so this figure implies that less than 5% are managers.

[12] Here, and also in the definition of average firms size above, we consistently treat the manager as part of the firm's employment. Hence,

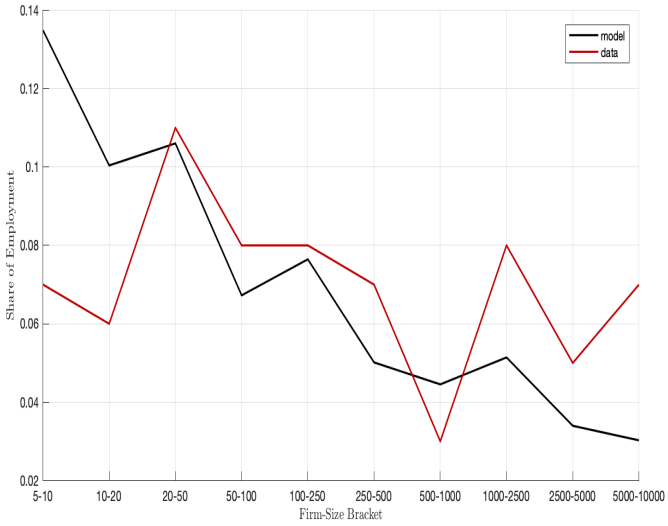
$$q = \left(\frac{\alpha}{w}\right)^{\frac{1}{1-\alpha}} z_q^{\frac{1}{1-\alpha}} + 1$$

and aggregate employment in the formula for λ_q below is 1. I believe this choice is the most faithful to the way the data moments are computed.

Using the Business Dynamics Statistics of the US Census Luttmer (2010) constructs λ_q for a large number of values of q - and shows that they are remarkably stable over time. For calibration purposes I (arbitrarily) pick $q = 10000$, for which $\lambda_q = 0.25$. The non-targeted λ_q's can be used to assess how well the calibrated model approximates the overall firm-size distribution.

The values of α and γ implied by the calibration described above are 0.75 and 4.67. As mentioned these values can be used in (11) to generate other model-implied λ_q's and compare them to non-targeted λ_q's from the data. This is done in Figure 1. For ease of interpretation, the figure uses the data and model-generated λ_qs to compute the fraction of employment accounted by firms in consecutive q brackets, i.e. it plots against q the series $\lambda_{q-1} - \lambda_q$. The calibration

Figure 1: Data and Model-implied share of employment by firm-size bracket.



forces the sum of the values of the two series to be the same (or 0.85) but the distribution of employment shares across size brackets in unconstrained. We see that the model does well for most intermediate size brackets but overestimates the share of employment accounted for by firms in the 5-10-worker and (to a lesser extent) in the 10-20-worker brackets, while underestimating employment in firms with more than 1000 workers.

5.2 Quantitative Results

5.2.1 GDP

Having calibrated the model, we can compute GDP in the two economies and compute the ratio to assess the magnitude of the efficiency loss of going to a PC economy.[13] I find

$$\frac{Y^{LF}}{Y^{PC}} = 1.53.$$

In other words, the efficiency loss from a pay-cap policy is huge, with the economy losing approximately one half of its output.

In the Introduction we have already discussed the most intuitive reasons why the PC economy is inefficient. Conditional on being a manager in both economies, any (non-marginal) manager employs fewer people, so some lower ability agents (who did not manage in the LF Economy) must now manage firms. Furthermore, in the LF economy employment was increasing in managerial ability, while in the PC economy all managers manage the same number of workers: so the more talented the manager, the more workers she loses in the PC economy. We can call the inefficiency arising from the smaller and uniform employment level the "firm-size" effect.

These effects can be made vivid by comparing the average firm size in the LF Economy and in the PC economy: the ratio of the former to the latter is 5.50. We can also compare how "low" the PC economy ends up recruiting workers: if we take the ratio of the lowest ability among managers in the LF economy and in the PC economy we get 1.90.

However, we must now discuss a more subtle additional source of inefficiency which arises from the assignment mechanism we have (plausibly, one hopes) imposed in the definition of the PC equilibrium. In the LF economy *all* agents with ability $z > \tilde{z}$ choose to be managers, so there is a clear partition in the population based on ability. In the PC economy, instead managers draw workers from the entire ability distribution (truncated at their own level of ability). This

[13] In fat recall that we can compute GDP only up to a proportional constant $\underline{z}$, which however cancels out in Y^{LF}/Y^{PC}.

means that not only many low-ability individuals who would been workers under LF become managers under PC, but also that many high-ability individuals who were managers under LF become workers under PC. We can call this inefficiency the "wasted manager" effect.

One way to assess the relative importance of the firm-size effect and the wasted-manager effect is to compute a counterfactual in which all firms employ l^* workers (as in the PC economy), but the labour force is forcibly partitioned into managers and workers strictly according to ability. In other words, we choose a counterfactual z, $\tilde{z}^{CF}$, such that

$$(1 + l^*) \int_{\tilde{z}^{CF}}^{\infty} dG(z) = 1,$$

and compute the corresponding counterfactual aggrgeate GDP as

$$Y^{CF} = F(l^*) \int_{\tilde{z}^{CF}}^{\infty} z dG(z).$$

Comparing Y^{CF} to Y^{LF} provides us with a quantitative measure of the firm-size effect. Using our calibration, $Y^{LF}/Y^{CF} = 1.45$, which is pretty close to the total effect of 1.53. We can conclude that the vast majority of the efficiency loss from pay caps is associated with the firm-size effect and only a relatively small component is due to the wasted-manager effect.

5.2.2 Inequality

Table 1 reports the model's implications for inequality.

As expected, the LF economy generates considerable inequality between workers and managers, with the average managerial income being seven times larger than the wage (first column). In contrast the PC economy features no between inequality by design.

Inequality among managers, as captured by the 90-10 percentile range, is also vastly larger in the LF economy (second column). This is because high-ability managers leverage their skills to operate larger firms, thereby amplifying their advantage over lower ability managers. In the PC economy all firms have the same size, so there is no amplification of the differences in ability.

Table 1: Inequality statistics

	Between	Managers	Workers	Overall 90_{th}	Overall 99_{th}	Overall 99.9_{th}
LF economy	7	6.58	1	1	3.66	26.35
PC economy	1	1.84	1.84	1.84	3.03	4.97
LF/PC Ratio	7	3.58	0.54	0.54	1.21	5.30

Note. Between: Average manager income divided by average worker income. Managers: Ratio of 90_{th} percentile of managers income distribution to the 10_{th} percentile. Workers: Ratio of 90_{th} percentile of worker income distribution to the 10_{th} percentile. Overall p_{th}: ratio of p_{th} percentile of the overall personal income distribution to the 10_{th} percentile.

Inequality among workers, instead, is higher under the PC arrangement - which is perhaps a novel insight this paper brings to the debate over pay-cap proposals (third column). As already discussed, this descends from the fact that workers now share equally with their managers, so the more fortunate workers who are assigned to a better manager earn more. Indeed inequality among workers is identical to inequality among managers.

The last three columns look at the overall income distribution. recall from the calibration that firs size is 22, implying than less than 5 percent of agents are managers in the US economy. This implies that the person at the 90_{th} percentile of the personal income distribution is a worker in the LF economy, and thus the 90-10 percentile ratio is 1. In contrast, in the PC economy the managerial and worker income distributions are identical, and thus so must be the personal income distribution. This implies more inequality by the 90-10 range criterion in the PC economy than in the LF economy (fourth column). We need to climb to the 99-10 percentile ratio to reverse the ordering of inequality between LF and PC economy. We also see that inequality really explodes in the LF economy beyond the 99_{th} percentile - while remaining subdued in the PC economy. Hence, despite the complexity of the inequality picture, we can say that PC policies *are* effective at curbing so-called "top-1 percent" inequality, which is arguably the main motivating factor for these proposals.

5.2.3 Winners, Losers, Welfare

A first step in assessing the welfare consequences of the PC policy is to ask who wins and who loses. In particular, since agents only differ in their managerial ability z, we can compare the income and utility experienced by an agent of ability z in the LF economy with the income and utility such an agent can expect to experience in the PC economy.

In the LF economy there is a simple (weakly) monotonic relationship between z and income, so the utility of an agent of ability z can simply be computed by plugging w or π_z into the utility function, depending on whether z is greater or less than $\tilde{z}$. We can also compute income and utility as a function of the percentile of the LF income distribution, using (5).

In the PC economy an agent with ability z could become either a manager, and earn y_z, or be a worker in a coop managed by someone with some ability $z' > z$ - and earning a correspondingly higher income. In particular, if we define y_z^{PC} the expected income of an agent of ability z in the PC economy, we have

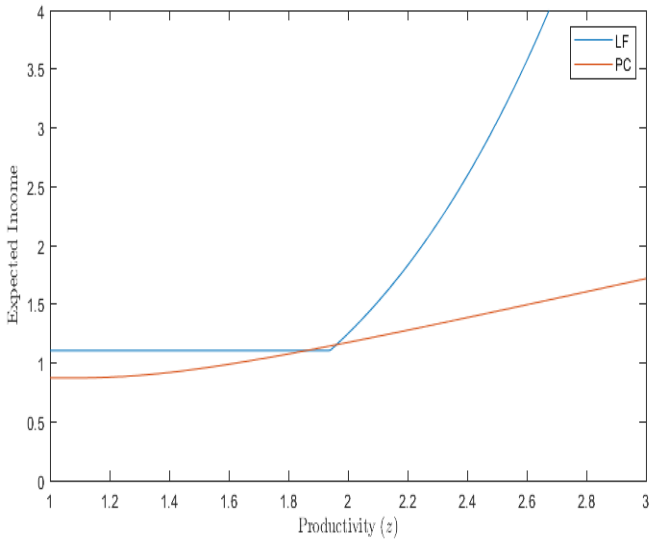
$$y_z^{PC} = \begin{cases} y_{z'} & \text{with probability } \omega(z') \text{ (for } z' > z) \\ y_z & \text{with probability } 1 - \int_z^{\infty} \omega(z')dz' \end{cases}$$

Figure 2 shows individual (expected) incomes against ability z in the two economies. LF incomes are obviously constant at w up to the threshold $\tilde{z}$ and then steeply rising with managerial ability for those who become managers. In the PC Economy expected income is monotonically increasing in z.[14] This reflects the fact that agents have a lower bound of income represented by the option of becoming managers, i.e. an agent with ability z cannot have income less than y_z. If they are not managers, they sample from a distribution which is bounded-below by this income. The further to the right, the higher the probability that the agent will be a manager. Yet income for managers increases much less steeply than in the LF Economy because managers

[14]To be more precise the profile of y_z^{PC} is not monotonic over the entire support of z. Agents with ability $z < \tilde{z}^{PC}$ can only be workers and hence all have the same income. In our calibration, however, $z < \tilde{z}^{PC}$ is extremely close to the lower bound of the domain of z so this flat region is undetectable in the figure.

do not leverage their ability by operating larger firms.

Figure 2: Expected income by ability in LF and PC Economy.



Clearly the most striking feature of Figure 2 is that, quantitative, expected income is lower in the PC Economy for almost all values of z. This is not surprising for the larger zs: it was clear that most LF managers would not welcome a transition to a PC economy. But the transition is bad for most workers, too, and the lower their level of ability, the worse they are hurt by the PC policy. The reason follows from the previous discussion: the lower a worker's ability, the "worse" the distribution of managers they sample from in the PC Economy, and hence the lower their expected income.

There is a small group of agents clustered around $\tilde{z}$ who gain from the PC policy. These agents sample from a very high quality pool of managers and hence their expected income under PC exceed the wage in the LF Economy. Indeed even a few agents who were managers in the LF Economy prefer to switch to the PC Economy. These are agents whose managerial income was very close to the wage under LF, and who are therefore also better off when given the chance to share equally with a higher-quality manager - albeit the output of this manager's firm is much reduced. Quantitatively, this group of beneficiaries from transitioning to a PC Economy is minuscule: less than 1 percent of the population. (In looking at 2 it may be well to remember

that the distribution of z is Pareto, and that more than 95 per cent of the population has z below the kink in the LF income profile.)

The income profiles in 2 tells pretty much all we need to know about comparative welfare in the two economies, but, for completeness, we can introduce a more theoretically-rigorous welfare comparison by choosing a utility measure and a social-welfare criterion. Given a utility function $U(y)$, the utility of an agent with ability z in the LF economy is simply $U(y_z^{LF})$. Her expected utility is computed as

$$W_z^{PC} = U(y_z)\left[1 - \int_z^{\infty} \omega(z')dz'\right] + \int_z^{\infty} U(y_{z'})\omega(z')dz'.$$

We can also use an utilitarian welfare comparison a la Lucas (1987, not 1978), by computing t which solves

$$\int U(ty_z^{LF})dG(z) = \int W_z^{PC}dG(z).$$

If $t < 1$ the utilitarian social planner would rather impose a proportional tax equal to $1 - t$ than move to the PC economy. If $t > 1$ it perceives moving to the PC economy as as beneficial as giving everyone a proportional subsidy pf (t-1) percent.

I choose a standard CRRA function, such that an individual with income y experiences utility

$$U(y) = \frac{y^{1-\sigma}}{1-\sigma}.$$

Following wide-spread practice motivated by observed consumption-saving behaviour I set $\sigma = 2$. With that, a plot of (expected) utility in the two economies against z looks virtually indistinguishable from 2. The solution for t is 0.76 implying that the social planner would be willing to impose a tax up to 24 percent of income in the LF economy to prevent a switch to the PC Economy.

5.3 Alternative Calibration

As an alternative to the calibration based on the employment share in firms larger than 10,000 employees I now briefly report on an alternative using a moment of the managerial income distribution. The BLS National Occupational Employment and Wage Estimates provide selected percentiles of the earnings distribution by occupation. Unfortunately for "Chief Executives" both the 90_{th} and the 75_{th} percentiles are censored, which means that we cannot construct meaningful inequality measures for the category which best approximates the managers in the model. However, for the broader "Top Executives" category we do observe the 75_{th} percentile (the 90_{th} is still censored). If we are willing to assume that percentile ratios among top executives are similar to percentile ratios among chief executives we can use the ratio between the 75_{th} percentile and the 25_{th} percentile as a calibration target.[15] The value of this ratio in the data is 2.48. The formulas for the percentiles of the managerial income distribution in the LF Economy are in the unpublished appendix.

When calibrating on firm size and on the 75-25 percentile ratio of the managerial income distribution we obtain $\alpha = 0.78$ and $\gamma = 5.46$. The "out of sample" ability of the model to predict the employment share of various firm-size brackets is quite similar to the one with the baseline calibration, except for a worsening for the smallest size bracket. The efficiency and welfare losses from going to a pay-cap system are more modest than under the benchmark calibration, but still sizable. LF income is 34 percent larger than PC income and the multiplier of LF income t which achieves social welfare indifference with respect to the PC is 0.81.

[15] A more daring alternative could use the Russell 3000 data I mentioned in the Introduction. According to the US Census's Annual Business Survey there were 5,722,143 firms in the US in 2018. Hence, assuming (as predicted by the model) that managerial income is strictly increasing in firm size, the lowest earning CEO in the Russell 3000 occupies the $[1 - 3000/5,722,143]_{th}$, or 0.9995_{th} percentile of the managerial income distribution. Since we observe her income (from the Russell 3000 data set) this could be a possible calibration target.

6 Equivalence with Cooperative Economies

So far we have looked at what may be termed "capitalist economies," with or without pay-caps on managers. In this section I study an alternative institutional setting (for the same physical environment): I assume that all production takes place in producer cooperatives. In Cooperatives, or *labor-managed firms*, decisions are taken collectively and democratically by the entire work force. Since workers outnumber managers, as a general principle this implies that firm goals and decisions (including about the income split between workers and managers) are made with the intent to maximize the income of the workers.

As before, I discuss a minimal set of rationality constraints which it seems plausible to expect of an equilibrium for an economy made-up of worker-managed firms. These constraints have to cover the way in which workers (and managers) get together to form cooperatives, and the way that individual incomes are decided.

A first constraint I will impose is that *within each cooperative*, equal work will attract equal pay - or in other words all the non-managerial workers of a given cooperative will receive the same income. Admittedly this is more a tractability requirement - akin to focusing on symmetric equilibria – than a strict rationality constraint. One could imagine situations in which an arbitrary majority of, say, 51 percent of workers votes for a compensation scheme which gives them the entire non-managerial share of income. In a richer model such equilibria could probably be ruled out endogenously - for example by allowing the disenfranchised among the workers to withdraw effort. Real-world worker cooperatives certainly keep to the equal-pay for equal-work principle very strictly.

The key rationality assumptions are based on the construct in Brzustowsky and Caselli (2021). First, in equilibrium it should not be possible to reassign a worker or a manager from the co-operative she is assigned to, in such a way that both the reassigned worker and the workers in the receiving cooperative experience an increase in income. If such an assignment was possible, then either party would approach the other in order to effect this mutually beneficial transition. Implicit in this construct is that coops cannot keep their workers captive: the coop which loses

the agent cannot stop her from leaving.

Second, the process of cooperative formation should be one in which workers internalize the effect of firm size on income. Hence, in equilibrium the mass of workers in a cooperative cannot exceed the mass of workers which maximizes the income of the cooperative's workers (given the ability of the manager). If that was the case, we would have to conclude that workers irrationally accepted too many workers in the cooperative. An equivalent argument to reach the same equilibrium condition is to simply say that, should the firm size exceed the worker-income maximizing size, a majority of workers can vote to expel the excess workers.

The next equilibrium condition we state concerns the remuneration of managers. Here we will look at two alternatives. In the first scenario all managers of a given level of ability earn the same income, irrespective of which coop they run. In the alternative scenario we require that managers receive the same pay as the workers that they manage, irrespective of their ability or number of workers managed.

The assumption in the first scenario is associated with an environment in which there is a market for managers, and worker cooperatives compete for the services of the best managers. This then implies that managers of a given level of ability cannot be paid differently by different coops, because if they were the lesser paid could approach the high-paying cooperative and offer to undercut their existing manager. Essentially, the median worker of each cooperative is an employer and the manager is an employee hired on a competitive market. On reflection, however, this implies an element of commitment. Just as in the "capitalist" version the manager credibly commits to pay the promised wage out of the firm's cash flow, so in this scenario the worker collective must credibly promise the payment π to the manager. For this reason, I will call equilibria constructed with the condition of equal managerial pay per equal ability *equilibria with commitment*.

In the context of a cooperative economy, commitment about managerial pay may be somewhat less realistic than commitment over worker wages in a capitalist economy. After all, the essence of a cooperative is worker democracy, and it is legitimate to ask what would happen if workers

voted on the distribution of income between themselves and management *after* production has taken place - i.e. when the manager can no longer respond to any pay cut by threatening to migrate to another coop. A reasonable benchmark for this case is that workers will impose an equalitarian pay structure in which the manager receives the same pay as the workers, which is our second scenario. This is why I will describe equilibria based on the condition of equalitarian pay within firms as *equilibria without commitment.*[16]

In the next two subsection I move from the informal discussion above to more formal equilibrium definitions, and derive the equivalence results. In the first subsection I look at equilibria with commitment, and in the second at equilibria without commitment.

6.1 Cooperative Economy with Commitment

Definition 2. *Define*

$$y_z^{cc} = \max_l \frac{zF(l) - \pi_z}{l}$$

and l_z^{cc} the corresponding argmax (and where the superscript "cc" is a mnemonic for "cooperatives with commitment"). An equilibrium for the CC Economy is a function π_z^{cc}, a function $\mu^{cc}(z)$ which represents the measure of agents of type z who become managers, and a function $\omega^{cc}(z, z')$ which represents the fraction of agents of type z who work in cooperatives managed by a manager with ability z', such that

1. *Every agent of type z who is a manager earns the same income π_z and operates a firm with l_z^{cc} workers*

2. *Workers who work in a coop managed by a manager of ability z earn y_z^{cc}*

3. *There are agents of ability z who choose to be managers (or $\mu^{cc}(z) > 0$) if and only if*

[16]Strictly speaking, worker-income maximization without commitment might be thought to imply that the managers receive zero pay. In assuming that, instead, workers treat managers as themselves I am implicitly imposing an additional assumption of basic human decency on the workers - collectively if not individually. Economists who think this is going too far may prefer to reinterpret the no-commitment scenario as one in which cooperatives have a constitutional rule prescribing strictly equalitarian division of income among all members of the coop (which of course includes management).

$\pi_z \geq y^{cc}_{z'}$ *for every* z'

4. *The labor market clears, or*

$$\int_{\underline{z}}^{\infty} \mu(z)(1 + l^{cc}_z)dz = 1$$

Equilibrium conditions 1 and 2 follow directly from the discussion of minimal rationality constraints for a cooperative equilibrium above. Condition 3 was not discussed but is obviously right ("if" part: if I am of type z and a worker, I can always approach a coop currently using a type z manager and offer to manage for $\pi_z - \epsilon$; "only if" part: even more obvious). Condition 4 needs no comment.

Perusal of Definition 2 leads inescapably to the following

Proposition 2. *An equilibrium of the CC Economy exists, is unique, and is isomorphic to the equilibrium of the LF Economy.*

6.2 Cooperative Economy without Commitment

When coops cannot (credibly) promise to pay managers more than they pay workers, an agent occupational choice problem is identical to the situation in the PC economy. Each agent would prefer to be a worker in a coop managed by an agent with higher ability than himself, but would rather be a manager than being managed by an agent of inferior ability. Just as in the PC economy, therefore, the most plausible mechanism to assign agents to occupations is to randomize the pool of agents who are assigned to be workers in coops managed by agents of higher ability than them. Together with the discussion in the first part of this section, this then leads to the following definition:

Definition 3. *Define*

$$y^{cn}_z = \max_l \frac{zF(l)}{1+l} = \frac{zF(l^*)}{1+l^*}$$

(where the superscript "cn" is a mnemonic for "cooperatives with no commitment"). An equilibrium for the CN Economy is a function $\mu(z)$ *which represents the measure of agents of type* z

who become managers, and a function $\omega(z, z')$ which represents the fraction of agents of type z who work in a cooperative where the manager has ability z', where

1. *Every agent of type z who is a manager manages l^* workers and earns y_z^{cn}*
2. *Workers who work in a coop managed by an agent of ability z earn y_z^{cn}*
3. *An agent of type z is either a manager or works for a manager of type z', such that $y_{z'} \geq y_z$*
4. *If $A_{z'} = \{z; y_{z'} > y_z\}$, then $\omega(z, z') = \omega(z')$ for all $z \in A_{z'}$*
5. *The labor market clears, or*

$$(1 + l^*) \int_{\underline{z}}^{\infty} \mu(z)dz = 1$$

To see why Condition 1 applies to a CN Economy because the objective function in an equalitarian cooperative is to maximize average income $zF(l)/(1 + l)$. To see this more formally, note that the objective function of the workers is to maximize $[zF(l) - \pi]/l$ (non-managerial share of income divided by non-managerial staff). Then substitute the equal-pay constraint $\pi = [zF(l) - \pi]/l$. The other conditions do not seem to require further commentary. Since Definition 3 is formally identical to the definition of the PC Economy we trivially have

Proposition 3. *An equilibrium of the CN Economy exists, is unique, and is isomorphic to the equilibrium of the PC Economy.*

7 Conclusions

I have studied the introduction of caps on managerial pay in a span-of-control economy with heterogenous managerial talent. The introduction of pay caps transforms managers in equalitarian cooperators, and leads them to hire fewer workers than they would in the absence of pay caps. This implies that the marginal product of labor is not equalized among firms, and that agents who would be of too low ability to manage firms in the Laissez-Faire Economy do become managers in the PC economy. The pay cap does reduce inequality between managers and

workers, and among managers, but increases inequality among workers: the "lucky" ones who get to work for a high-ability manager earn more than those stuck with low-ability managers. Quantitatively, overall inequality does decrease substantially, but the social planner would have to be more inequality averse than is usually assumed by economists to choose to impose the pay cap.

The quantitative results are liable to change, in favour of pay-cap policies, in three extensions that I plan to pursue in future work.

First, while this paper only looks at a pay cap of one, proposed pay caps are obviously larger than that. Future work will seek to characterize the effect of a pay cap for a generic cap greater of equal to one. Such work will then open the door to the identification of the optimal pay cap (if any). There is also scope to explore alternative forms of the pay-cap policy, for example expressed as a ratio to the economy-wide average wage rather than the within-firm average wage.

Second, in a way the present framework stacks the deck against pay caps as the LF Economy is fully efficient. Many inefficiencies have been identified which imply that LF economies operate inside the production frontier. In our framework, a fairly natural avenue to explore is monopsony power in labor markets. Monopsony by employers has become a major theme in the debate on the causes of increasing inequality, with increasingly string evidence of its importance in compressing worker incomes, especially at the bottom. Clearly introducing monopsony power in the model would make the LF economy both inefficient and more unequal, leading to an improvement of the relative attractiveness of pay caps.

Third, the model assumes that both worker and managerial effort (per hour of work) are given, and invariant to the institutional setup. In economics, a long tradition on efficiency wages suggests that worker effort is increasing in pay. There is also evidence, for example in organizational behaviour, that workers respond positively to lower earnings gaps between themselves and management. Allowing for these effects should make the PC economy less inefficient compared to the LF economy.[17]

[17] In the opposite direction, many economists believe that managerial performance would decline with pay caps, though there is virtually no evidence for this.

A Appendix

A.1 A more descriptive PC Economy

In this section I sketch a more descriptive version of the PC economy, loosely inspired on a search-theoretic interpretation.[18] As already mentioned in the text, the equilibrium definition in the text can be interpreted as the reduced form of this more elaborate description of the environment and optimality conditions.

The environment is one in which workers can submit applications to all firms they would like to work for, taking as given the probability that their applications will be accepted. Applications do not incur any costs. If none of their job applications are accepted, workers can open their own business. If multiple job applications are accepted, workers can accept the best offer.

Entrepreneurs have labour demand l^*. If they receive more applications than their labor demand, they select employees from the applicant pool.

The revised equilibrium definition is as follows

Definition 4. *An equilibrium for the search-theoretic PC economy is a set of agents' occupational choices, entrepreneurs' labor demand decision, workers' job application decisions, entrepreneurs' selection decision over job applicants, the probability that a worker is assigned a certain job, and an assignment between workers and entrepreneurs, such that (1) there is no unemployment; (2) those decisions are individually optimal.*

In this large economy, workers take as given the probability that they will be assigned a job. And because applications do not incur any costs, they apply to all firms that promise a payoff greater than the payoff for them to open their own business. Because entrepreneurs are indifferent between applicants, they select employees from applicants at random. Because workers can always start their own business, the labour market clears. Because the job application is costless, all firms receive more job applicants than the number of workers they demand.

[18] I reiterate my gratitude to Shenxing Zhang for suggesting this interpretation. The first draft of this appendix literally begun with a cut-and-paste from an email of his.

A.2 Uniqueness of PC Economy Equilibrium

Suppose that there are two pairs of functions, $\{\mu(z), \omega(z)\}$ and $\{\mu'(z), \omega'(z)\}$, with $\mu(z) \neq \mu'(z)$ and/or $\omega(z) \neq \omega'(z)$ for at least some z, which both satisfy conditions (1-5) of Definition 1 - and we derive a contradiction.

Define z^* as the highest value of z such that one or both the functions differ across the two equilibria, i.e. $\mu(z) = \mu'(z)$ and $\omega(z) = \omega'(z)$ for all $z > z^*$. Suppose first that (without loss of generality) $\mu(z^*) > \mu'(z^*)$. Hence under μ more agents of type z^* are managers than under μ'. Since $\omega(z) = \omega'(z)$ for $z > z'$, however, the same measure of agents of type z^* works for managers with ability greater than z^* in both equilibria. Hence, if $\mu(z^*) > \mu'(z^*)$ some agents of ability z^* must necessarily work for agents of ability inferior to z^*, which contradicts equilibrium condition (3).

So the only way that the two equilibria can begin to differ at z^* is if (without loss of generality) we had $\omega(z^*) > \omega'(z^*)$. But note that, since $\mu(z^*) = \mu'(z^*)$ and every manager hires l^* workers, the same overall measure of workers must be hired by managers of type z^* in both equilibria. Furthermore, since $\mu(z) = \mu'(z)$ and $\omega(z) = \omega'(z)$ for $z > z^*$, the measure of agents that could possibly be hired by managers of type z^* is the same in both equilibria, so once again we reach a contradiction.

B Appendix not for Publication

B.1 Solving for w

$$\int_{\tilde{z}}^{\infty} l_z(w) dG(z) = G(\tilde{z})$$

$$\int_{Aw}^{\infty} \left(\frac{\alpha z}{w}\right)^{\frac{1}{1-\alpha}} \frac{\gamma \underline{z}^{\gamma}}{z^{\gamma+1}} dz = 1 - \left(\frac{\underline{z}}{Aw}\right)^{\gamma}$$

$$\left(\frac{\alpha}{w}\right)^{\frac{1}{1-\alpha}} \gamma \underline{z}^{\gamma} \int_{Aw}^{\infty} \frac{1}{z^{\gamma+1-\frac{1}{1-\alpha}}} dz = 1 - \left(\frac{\underline{z}}{Aw}\right)^{\gamma}$$

$$\left(\frac{\alpha}{w}\right)^{\frac{1}{1-\alpha}} \frac{\gamma \underline{z}^{\gamma}}{\gamma - \frac{1}{1-\alpha}} \int_{Aw}^{\infty} \frac{\left(\gamma - \frac{1}{1-\alpha}\right)}{z^{\gamma+1-\frac{1}{1-\alpha}}} dz = 1 - \left(\frac{\underline{z}}{Aw}\right)^{\gamma}$$

The expression in the integral is a Pareto with scale 1 and shape $\gamma - 1/(1-\alpha)$. Hence

$$\left(\frac{\alpha}{w}\right)^{\frac{1}{1-\alpha}} \frac{\gamma \underline{z}^{\gamma}}{\gamma - \frac{1}{1-\alpha}} \left(\frac{1}{Aw}\right)^{\gamma - \frac{1}{1-\alpha}} = 1 - \left(\frac{\underline{z}}{Aw}\right)^{\gamma}$$

$$\alpha^{\frac{1}{1-\alpha}} \frac{\gamma \underline{z}^{\gamma}}{\gamma - \frac{1}{1-\alpha}} A^{\frac{1}{1-\alpha}} \left(\frac{1}{Aw}\right)^{\gamma} = 1 - \left(\frac{\underline{z}}{Aw}\right)^{\gamma}$$

$$\left(\frac{\alpha^{\frac{1}{1-\alpha}} A^{\frac{1}{1-\alpha}} \gamma \underline{z}^{\gamma}}{\gamma - \frac{1}{1-\alpha}} + \underline{z}^{\gamma}\right) \left(\frac{1}{Aw}\right)^{\gamma} = 1$$

$$(Aw)^{\gamma} = \left(\frac{\alpha^{\frac{1}{1-\alpha}} A^{\frac{1}{1-\alpha}} \gamma \underline{z}^{\gamma} + \gamma \underline{z}^{\gamma} - \frac{1}{1-\alpha} \underline{z}^{\gamma}}{\gamma - \frac{1}{1-\alpha}}\right)$$

$$(Aw)^{\gamma} = \left(\frac{\alpha^{\frac{1}{1-\alpha}} A^{\frac{1}{1-\alpha}} \gamma + \gamma - \frac{1}{1-\alpha}}{\gamma - \frac{1}{1-\alpha}}\right) \underline{z}^{\gamma}$$

$$\tilde{z} = Aw = \left(\frac{\alpha^{\frac{1}{1-\alpha}} A^{\frac{1}{1-\alpha}} \gamma + \gamma - \frac{1}{1-\alpha}}{\gamma - \frac{1}{1-\alpha}}\right)^{1/\gamma} \underline{z}$$

$$\tilde{z} = Aw = \left(\frac{\alpha^{\frac{1}{1-\alpha}} \left(\frac{1}{\alpha} \left(\frac{\alpha}{1-\alpha}\right)^{1-\alpha}\right)^{\frac{1}{1-\alpha}} \gamma + \gamma - \frac{1}{1-\alpha}}{\gamma - \frac{1}{1-\alpha}}\right)^{1/\gamma} \underline{z}$$

$$\tilde{z} = Aw = \left(\frac{\frac{\alpha}{1-\alpha}\gamma + \gamma - \frac{1}{1-\alpha}}{\gamma - \frac{1}{1-\alpha}}\right)^{1/\gamma} \underline{z}$$

$$\tilde{z} = Aw = \left(\frac{\alpha\gamma + \gamma - \alpha\gamma - 1}{\gamma - \alpha\gamma - 1}\right)^{1/\gamma} \underline{z}$$

$$\tilde{z} = Aw = \left(\frac{\gamma - 1}{\gamma - \alpha\gamma - 1}\right)^{1/\gamma} \underline{z}$$

B.2 Derivation of λ_q

$$\lambda_q = \int_{z_q}^{\infty} (1 + l_z) dG(z)$$

After calculations, this reduces to

$$\lambda_q = 1 - G(z_q) + \int_{z_q}^{\infty} \left(\frac{\alpha z}{w}\right)^{\frac{1}{1-\alpha}} \frac{\gamma \underline{z}^{\gamma}}{z^{\gamma+1}} dz$$

$$\lambda_q = 1 - G(z_q) + \left(\frac{\alpha}{w}\right)^{\frac{1}{1-\alpha}} \gamma \underline{z}^{\gamma} \int_{z_q}^{\infty} \frac{1}{z^{\gamma+1-\frac{1}{1-\alpha}}} dz$$

$$\lambda_q = 1 - G(z_q) + \left(\frac{\alpha}{w}\right)^{\frac{1}{1-\alpha}} \frac{\gamma \underline{z}^{\gamma}}{\gamma - \frac{1}{1-\alpha}} \int_{z_q}^{\infty} \frac{\left(\gamma - \frac{1}{1-\alpha}\right)}{z^{\gamma+1-\frac{1}{1-\alpha}}} dz$$

The expression in the integral is a Pareto with scale 1 and shape $\gamma - 1/(1-\alpha)$. Hence

$$\lambda_q = \left(\frac{\underline{z}}{z_q}\right)^{\gamma} + \left(\frac{\alpha}{w}\right)^{\frac{1}{1-\alpha}} \frac{\gamma \underline{z}^{\gamma}}{\gamma - \frac{1}{1-\alpha}} \left(\frac{1}{z_q}\right)^{\gamma - \frac{1}{1-\alpha}}$$

$$\lambda_q = \left(\frac{\underline{z}}{z_q}\right)^{\gamma} \left[1 + \left(\frac{\alpha}{w}\right)^{\frac{1}{1-\alpha}} \frac{\gamma(1-\alpha)}{\gamma(1-\alpha) - 1} z_q^{\frac{1}{1-\alpha}}\right]$$

$$\lambda_q = \left(\frac{\underline{z}}{(q-1)^{1-\alpha}\left(\frac{w}{\alpha}\right)}\right)^{\gamma} \left[1 + \left(\frac{\alpha}{w}\right)^{\frac{1}{1-\alpha}} \frac{\gamma(1-\alpha)}{\gamma(1-\alpha) - 1} (q-1) \left(\frac{w}{\alpha}\right)^{\frac{1}{1-\alpha}}\right]$$

$$\lambda_q = \left(\frac{\underline{z}}{(q-1)^{1-\alpha}\left(\frac{w}{\alpha}\right)}\right)^{\gamma} \left[1 + \frac{\gamma(1-\alpha)}{\gamma(1-\alpha) - 1} (q-1)\right]$$

$$\lambda_q = \left(\frac{\underline{z}}{(q-1)^{1-\alpha}\left(\frac{w}{\alpha}\right)}\right)^{\gamma} \frac{\gamma(1-\alpha) - 1 + \gamma(1-\alpha)q - \gamma(1-\alpha)}{\gamma(1-\alpha) - 1}$$

$$\lambda_q = \left(\frac{\underline{z}}{(q-1)^{1-\alpha}\left(\frac{w}{\alpha}\right)}\right)^{\gamma} \frac{\gamma(1-\alpha)q - 1}{\gamma(1-\alpha) - 1}$$

$$\lambda_q = \left(\frac{\alpha}{w}\right)^{\gamma} \frac{\gamma(1-\alpha)q - 1}{\gamma(1-\alpha) - 1} \left(\frac{\underline{z}}{(q-1)^{1-\alpha}}\right)^{\gamma}$$

$$\lambda_q = \left(\frac{\alpha}{1-\alpha}\right)^{\gamma(1-\alpha)} \frac{(1-\alpha)\gamma - 1}{\gamma - 1} \frac{\gamma(1-\alpha)q - 1}{\gamma(1-\alpha) - 1} \left(\frac{1}{(q-1)^{1-\alpha}}\right)^{\gamma}$$

$$\lambda_q = \left(\frac{\alpha}{1-\alpha}\right)^{\gamma(1-\alpha)} \frac{\gamma(1-\alpha)q - 1}{\gamma - 1} \left(\frac{1}{q-1}\right)^{\gamma(1-\alpha)}$$

$$\lambda_q = \frac{\gamma(1-\alpha)q-1}{\gamma-1}\left(\frac{\alpha}{(1-\alpha)(q-1)}\right)^{\gamma(1-\alpha)}$$

B.3 More details on solution of (10)-(11)

Specifically, first from (10) we get

$$\gamma = \frac{s-1}{(1-\alpha)s-1}.$$

Plugging this into λ_q and simplifying we get (appendix)

$$\lambda_q = \frac{\frac{s-1}{(1-\alpha)s-1}(1-\alpha)q-1}{\frac{s-1}{(1-\alpha)s-1}-1}\left(\frac{\alpha}{(1-\alpha)(q-1)}\right)^{\frac{s-1}{(1-\alpha)s-1}(1-\alpha)}$$

$$\lambda_q = \frac{(1-\alpha)(s-1)q-(1-\alpha)s+1}{s-1-(1-\alpha)s+1}\left(\frac{\alpha}{(1-\alpha)(q-1)}\right)^{\frac{(1-\alpha)(s-1)}{(1-\alpha)s-1}}$$

$$\lambda_q = \frac{(1-\alpha)[(s-1)q-s]+1}{s-1-s+\alpha s+1}\left(\frac{\alpha}{(1-\alpha)(q-1)}\right)^{\frac{(1-\alpha)(s-1)}{(1-\alpha)s-1}}$$

$$\lambda_q = \frac{(1-\alpha)[(s-1)q-s]+1}{\alpha s}\left(\frac{\alpha}{(1-\alpha)(q-1)}\right)^{\frac{(1-\alpha)(s-1)}{(1-\alpha)s-1}}$$

$$\lambda_q = \frac{(1-\alpha)[(s-1)q-s]+1}{\alpha s}\left(\frac{\alpha}{(1-\alpha)(q-1)}\right)^{\frac{(1-\alpha)(s-1)}{(1-\alpha)s-1}}.$$

This can be solved numerically for α based on a choice of q and data on s and λ_q. The result can be plugged back in the equation for γ.

B.4 GDP in LF economy

Measures of GDP and of welfare ($\overline{\pi}$ is a possible measure of inequality).

$$Y^{LF} = \int_{\tilde{z}}^{\infty} z(l_z)^{\alpha} dG(z)$$

$$Y^{LF} = \int_{\tilde{z}}^{\infty} z(l_z)^{\alpha} dG(z)$$

$$Y^{LF} = \int_{Aw}^{\infty} z \left(\left(\frac{\alpha z}{w} \right)^{\frac{1}{1-\alpha}} \right)^{\alpha} \frac{\gamma \underline{z}^{\gamma}}{z^{\gamma+1}} dz$$

$$Y^{LF} = \int_{Aw}^{\infty} z \left(\left(\frac{\alpha z}{w} \right)^{\frac{1}{1-\alpha}} \right)^{\alpha} \frac{\gamma \underline{z}^{\gamma}}{z^{\gamma+1}} dz$$

$$Y^{LF} = \left(\frac{\alpha}{w} \right)^{\frac{\alpha}{1-\alpha}} \gamma \underline{z}^{\gamma} \int_{Aw}^{\infty} \frac{1}{z^{\gamma+1-\frac{1}{1-\alpha}}} dz$$

$$Y^{LF} = \left(\frac{\alpha}{w} \right)^{\frac{\alpha}{1-\alpha}} \frac{\gamma \underline{z}^{\gamma}}{\gamma - \frac{1}{1-\alpha}} \left(\frac{1}{Aw} \right)^{\gamma - \frac{1}{1-\alpha}}$$

$$Y^{LF} = \frac{(1-\alpha)\alpha^{\frac{\alpha}{1-\alpha}} A^{\frac{1}{1-\alpha}} \gamma \underline{z}^{\gamma}}{(1-\alpha)\gamma - 1} A^{-\gamma} \left(\frac{1}{w} \right)^{\gamma-1}$$

$$Y^{LF} = \frac{(1-\alpha)\alpha^{\frac{\alpha}{1-\alpha}} A^{\frac{1}{1-\alpha}} \gamma \underline{z}^{\gamma}}{(1-\alpha)\gamma - 1} A^{-\gamma} \left(\frac{1}{\frac{1}{A} \left(\frac{\gamma-1}{\gamma-\alpha\gamma-1} \right)^{1/\gamma} \underline{z}} \right)^{\gamma-1}$$

$$Y^{LF} = \frac{(1-\alpha)\alpha^{\frac{\alpha}{1-\alpha}} A^{\frac{\alpha}{1-\alpha}} \gamma}{(1-\alpha)\gamma - 1} \left(\frac{(1-\alpha)\gamma - 1}{\gamma - 1} \right)^{\frac{\gamma-1}{\gamma}} \underline{z}$$

$$Y^{LF} = \frac{(1-\alpha)\alpha^{\frac{\alpha}{1-\alpha}} \left(\frac{1}{\alpha} \left(\frac{\alpha}{1-\alpha} \right)^{1-\alpha} \right)^{\frac{\alpha}{1-\alpha}} \gamma}{((1-\alpha)\gamma - 1)^{\frac{1}{\gamma}}} \left(\frac{1}{\gamma - 1} \right)^{\frac{\gamma-1}{\gamma}} \underline{z}$$

$$Y^{LF} = \frac{(1-\alpha) \left(\frac{\alpha}{1-\alpha} \right)^{\alpha} \gamma}{((1-\alpha)\gamma - 1)^{\frac{1}{\gamma}}} \left(\frac{1}{\gamma - 1} \right)^{\frac{\gamma-1}{\gamma}} \underline{z}$$

$$Y^{LF} = \frac{(1-\alpha)^{1-\alpha}\alpha^{\alpha}\gamma}{((1-\alpha)\gamma - 1)^{\frac{1}{\gamma}}} \left(\frac{1}{\gamma - 1} \right)^{\frac{\gamma-1}{\gamma}} \underline{z}$$

$$Y^{LF} = \frac{(1-\alpha)^{1-\alpha}\alpha^{\alpha}\gamma}{((1-\alpha)\gamma - 1)^{\frac{1}{\gamma}}} \left(\frac{1}{\gamma - 1} \right)^{\frac{\gamma-1}{\gamma}} \underline{z}$$

B.5 Calculation of firm-size effect

Recall that under the chosen functional form

$$l^* = \frac{\alpha}{1-\alpha}$$

Then

$$\begin{aligned}
(1+l^*)\int_{\tilde{z}^{CF}}^{\infty} dG(z) &= 1 \\
\left(1+\frac{\alpha}{1-\alpha}\right)\int_{\tilde{z}^{CF}}^{\infty} \gamma\frac{\underline{z}^{\gamma}}{z^{\gamma+1}}dz &= 1 \\
-\frac{\underline{z}^{\gamma}}{1-\alpha}\left[z^{-\gamma}\right]_{\tilde{z}^{CF}}^{\infty} &= 1 \\
-\frac{\underline{z}^{\gamma}}{1-\alpha}\left[\lim_{x\to\infty}\left(\frac{1}{x}\right)^{\gamma}-\left(\frac{1}{\tilde{z}^{CF}}\right)^{\gamma}\right] &= 1 \\
\tilde{z}^{CF} &= \left(\frac{1}{1-\alpha}\right)^{\frac{1}{\gamma}}\underline{z}.
\end{aligned}$$

Turning to Y^{CF}

$$\begin{aligned}
Y^{CF} &= F\left(l^*\right)\int_{\tilde{z}^{CF}}^{\infty} z dG(z) \\
&= \left(\frac{\alpha}{1-\alpha}\right)^{\alpha}\int_{\tilde{z}^{CF}}^{\infty} z\gamma\frac{\underline{z}^{\gamma}}{z^{\gamma+1}}dz \\
&= \left(\frac{\alpha}{1-\alpha}\right)^{\alpha}\gamma\underline{z}^{\gamma}\int_{\left(\frac{1}{1-\alpha}\right)^{\frac{1}{\gamma}}\underline{z}}^{\infty} z^{-\gamma}dz \\
&= \left(\frac{\alpha}{1-\alpha}\right)^{\alpha}\frac{\gamma\underline{z}^{\gamma}}{1-\gamma}\left[z^{1-\gamma}\right]_{\left(\frac{1}{1-\alpha}\right)^{\frac{1}{\gamma}}\underline{z}}^{\infty} \\
&= \left(\frac{\alpha}{1-\alpha}\right)^{\alpha}\frac{\gamma\underline{z}^{\gamma}}{1-\gamma}\left[-\left(\frac{1}{1-\alpha}\right)^{\frac{1}{\gamma}}\underline{z}\right]^{1-\gamma} \\
&= \alpha^{\alpha}(1-\alpha)^{\frac{\gamma-1}{\gamma}-\alpha}\frac{\gamma}{\gamma-1}\underline{z}
\end{aligned}$$

Output in the laissez-faire economy is

$$Y^{LF} = \frac{(1-\alpha)^{1-\alpha}\alpha^{\alpha}\gamma}{((1-\alpha)\gamma-1)^{\frac{1}{\gamma}}}\left(\frac{1}{\gamma-1}\right)^{\frac{\gamma-1}{\gamma}}\underline{z}$$

Taking the ratio

$$\begin{aligned}\frac{Y^{CF}}{Y^{LF}} &= \frac{\alpha^{\alpha}(1-\alpha)^{\frac{\gamma-1}{\gamma}-\alpha}\frac{\gamma}{\gamma-1}\underline{z}}{\frac{(1-\alpha)^{1-\alpha}\alpha^{\alpha}\gamma}{((1-\alpha)\gamma-1)^{\frac{1}{\gamma}}}\left(\frac{1}{\gamma-1}\right)^{\frac{\gamma-1}{\gamma}}\underline{z}} \\ &= (1-\alpha)^{\frac{\gamma-1}{\gamma}-1}\left[(1-\alpha)\gamma-1\right]^{\frac{1}{\gamma}}(\gamma-1)^{\frac{\gamma-1}{\gamma}-1} \\ &= \left(\frac{(1-\alpha)\gamma-1}{(\gamma-1)(1-\alpha)}\right)^{\frac{1}{\gamma}}\end{aligned}$$

B.6 Between inequality

The income-wage ratio for a manager of ability z is

$$\pi_z = \frac{1-\alpha}{\alpha}\left(\frac{\alpha}{w}\right)^{\frac{1}{1-\alpha}} z^{\frac{1}{1-\alpha}}$$

The average of these ratios is thus

$$\overline{\pi} = \frac{1-\alpha}{\alpha}\left(\frac{\alpha}{w}\right)^{\frac{1}{1-\alpha}} \frac{\int_{\tilde{z}}^{\infty} z^{\frac{1}{1-\alpha}} dG(z)}{1-G(\tilde{z})}.$$

$$\overline{\pi} = \frac{1-\alpha}{\alpha}\left(\frac{\alpha}{w}\right)^{\frac{1}{1-\alpha}} \frac{\gamma\underline{z}^{\gamma}}{\gamma-\frac{1}{1-\alpha}}\left(\frac{1}{Aw}\right)^{\gamma-\frac{1}{1-\alpha}} \frac{(Aw)^{\gamma}}{\underline{z}^{\gamma}};$$

$$\overline{\pi} = \frac{1-\alpha}{\alpha} A^{\frac{1}{1-\alpha}} \alpha^{\frac{1}{1-\alpha}} \frac{\gamma}{\gamma-\frac{1}{1-\alpha}}$$

$$\overline{\pi} = \frac{1-\alpha}{\alpha}\left(\frac{1}{\alpha}\left(\frac{\alpha}{1-\alpha}\right)^{1-\alpha}\right)^{\frac{1}{1-\alpha}} \alpha^{\frac{1}{1-\alpha}} \frac{\gamma}{\gamma-\frac{1}{1-\alpha}}$$

$$\overline{\pi} = \frac{(1-\alpha)\gamma}{(1-\alpha)\gamma-1} \tag{12}$$

A higher γ means a thinner tail and hence lower average managerial relative income. And relative managerial pay is decreasing in the "labor share" α.

B.7 Percentiles of the managerial income distribution

The manager occupying the p_{th} percentile of the distribution of *managerial* income, z_p is identified by

$$\frac{1-G(z_p)}{1-G(\tilde{z})} = \frac{100-p}{100},$$

which becomes (using the Pareto CDF)

$$\left(\frac{z_p}{\tilde{z}}\right) = \left(\frac{100}{100-p}\right)^{\frac{1}{\gamma}}.$$

Hence the ratio of productivities of the manager occupying the p_{th} percentile of the managerial income distribution to the median managerial productivity is

$$\left(\frac{z_p}{z_{50}}\right) = \left(\frac{50}{100-p}\right)^{\frac{1}{\gamma}}.$$

Now notice that for any z, z',

$$\frac{\pi_z}{\pi_{z'}} = \left(\frac{z}{z'}\right)^{\frac{1}{1-\alpha}}.$$

Hence the ratio of the income of the manager at the p_{th} percentile to the income of the manager at the median is

$$\frac{\pi_p}{\pi_{50}} = \left(\frac{50}{100-p}\right)^{\frac{1}{(1-\alpha)\gamma}}. \tag{13}$$

B.8 Percentiles of the personal income distribution

We are interested in an *interpercentile range*, y_p/y'_p, where y_p is the p_th percentile of the personal income distribution. In the LF model income is increasing in z, so the agent occupying the p_{th} percentile of the distribution of *personal* income has ability z_p identified by

$$G(z_p)100 = p.$$

Using the Pareto functional form this implies

$$z_p = \left(\frac{100}{100-p}\right)^{\frac{1}{\gamma}}.$$

Also, the agent with ability $\tilde{z}$ occupies the $(G(\tilde{z})100)_{th}$ percentile of the income distribution. We call this percentile $\tilde{p}$ and using the Pareto functional form

$$\tilde{p} = 100 - \left(\frac{1}{\tilde{z}}\right)^{\gamma} 100.$$

Then we have

$$y_p = \begin{cases} w & \text{if } p \leq \tilde{p} \\ (1-\alpha)\left(\frac{\alpha}{w}\right)^{\frac{\alpha}{1-\alpha}} z_p^{\frac{1}{1-\alpha}} & \text{if } p > \tilde{p} \end{cases}$$

B.9 Welfare in LF Economy

$$W^{LF} = U(w)G(\tilde{z}) + \int_{\tilde{z}}^{\infty} U(\pi_z)dG(z)$$

$$W^{LF} = U(w)G(\tilde{z}) + \int_{\tilde{z}}^{\infty} U(\pi_z)dG(z)$$

$$W^{LF} = \frac{w^{1-\sigma}}{1-\sigma}G(\tilde{z}) + \int_{\tilde{z}}^{\infty} \frac{1}{1-\sigma}\left(\frac{\alpha^{\frac{1}{1-\alpha}}\left(\frac{1}{\alpha}-1\right)z^{\frac{1}{1-\alpha}}}{w^{\frac{\alpha}{1-\alpha}}}\right)^{1-\sigma} dG(z)$$

$$W^{LF} = \frac{1}{1-\sigma}\left[w^{1-\sigma}G(\tilde{z}) + \left(\frac{\alpha^{\frac{1}{1-\alpha}}\left(\frac{1}{\alpha}-1\right)}{w^{\frac{\alpha}{1-\alpha}}}\right)^{1-\sigma} \int_{\tilde{z}}^{\infty} z^{\frac{1-\sigma}{1-\alpha}} dG(z)\right]$$

$$W^{LF} = \frac{1}{1-\sigma}\left[w^{1-\sigma}G(\tilde{z}) + \left(\frac{\alpha^{\frac{1}{1-\alpha}}\left(\frac{1}{\alpha}-1\right)}{w^{\frac{\alpha}{1-\alpha}}}\right)^{1-\sigma} \int_{\tilde{z}}^{\infty} z^{\frac{1-\sigma}{1-\alpha}} \frac{\gamma \underline{z}^{\gamma}}{z^{\gamma+1}} dz\right]$$

$$W^{LF} = \frac{1}{1-\sigma}\left[w^{1-\sigma}G(\tilde{z}) + \left(\frac{\alpha^{\frac{1}{1-\alpha}}\left(\frac{1}{\alpha}-1\right)}{w^{\frac{\alpha}{1-\alpha}}}\right)^{1-\sigma} \gamma \underline{z}^{\gamma} \int_{\tilde{z}}^{\infty} \frac{1}{z^{\gamma-\frac{1-\sigma}{1-\alpha}+1}} dz\right]$$

$$W^{LF} = \frac{1}{1-\sigma}\left[w^{1-\sigma}G(\tilde{z}) + \left(\frac{\alpha^{\frac{1}{1-\alpha}}\left(\frac{1}{\alpha}-1\right)}{w^{\frac{\alpha}{1-\alpha}}}\right)^{1-\sigma} \frac{\gamma \underline{z}^{\gamma}}{\gamma - \frac{1-\sigma}{1-\alpha}} \int_{\tilde{z}}^{\infty} \frac{\gamma - \frac{1-\sigma}{1-\alpha}}{z^{\gamma-\frac{1-\sigma}{1-\alpha}+1}} dz\right]$$

$$W^{LF} = \frac{1}{1-\sigma}\left[w^{1-\sigma}\left[1-\left(\frac{\underline{z}}{Aw}\right)^{\gamma}\right] + \left(\frac{\alpha^{\frac{1}{1-\alpha}}\left(\frac{1}{\alpha}-1\right)}{w^{\frac{\alpha}{1-\alpha}}}\right)^{1-\sigma}\frac{\gamma\underline{z}^{\gamma}}{\gamma-\frac{1-\sigma}{1-\alpha}}\left(\frac{1}{Aw}\right)^{\gamma-\frac{1-\sigma}{1-\alpha}}\right]$$

$$W^{LF} = \frac{1}{1-\sigma}\left[w^{1-\sigma}\left[1-\left(\frac{\underline{z}}{Aw}\right)^{\gamma}\right] + \frac{\alpha^{\frac{1-\sigma}{1-\alpha}}A^{\frac{1-\sigma}{1-\alpha}}\left(\frac{1-\alpha}{\alpha}\right)^{1-\sigma}(1-\alpha)\gamma\underline{z}^{\gamma}}{(1-\alpha)\gamma-(1-\sigma)}\left(\frac{1}{Aw}\right)^{\gamma}\left(\frac{1}{w}\right)^{\frac{\alpha(1-\sigma)}{1-\alpha}}\left(\frac{1}{w}\right)^{-\frac{1-\sigma}{1-\alpha}}\right]$$

$$W^{LF} = \frac{1}{1-\sigma}\left[w^{1-\sigma}\left[1-\left(\frac{\underline{z}}{Aw}\right)^{\gamma}\right] + \frac{\alpha^{\frac{1-\sigma}{1-\alpha}}\left(\frac{1}{\alpha}\left(\frac{\alpha}{1-\alpha}\right)^{1-\alpha}\right)^{\frac{1-\sigma}{1-\alpha}}\left(\frac{1-\alpha}{\alpha}\right)^{1-\sigma}(1-\alpha)\gamma\underline{z}^{\gamma}}{(1-\alpha)\gamma-(1-\sigma)}\left(\frac{1}{Aw}\right)^{\gamma}\left(\frac{1}{w}\right)^{-(1-\sigma)}\right]$$

$$W^{LF} = \frac{w^{1-\sigma}}{1-\sigma}\left[1-\left(\frac{\underline{z}}{Aw}\right)^{\gamma} + \frac{(1-\alpha)\gamma}{(1-\alpha)\gamma-(1-\sigma)}\left(\frac{\underline{z}}{Aw}\right)^{\gamma}\right]$$

$$W^{LF} = \frac{w^{1-\sigma}}{1-\sigma}\left[1+\frac{(1-\sigma)}{(1-\alpha)\gamma-(1-\sigma)}\left(\frac{\underline{z}}{Aw}\right)^{\gamma}\right]$$

$$= \frac{w^{1-\sigma}}{1-\sigma}\left[1+\frac{(1-\sigma)}{(1-\alpha)\gamma-(1-\sigma)}\left(\frac{\underline{z}}{Aw}\right)^{\gamma}\right]$$

Since w is linear in $\underline{z}$, $\underline{z}$ will cancel out where it appears inside the square brackets. So it only appears in W as $\underline{z}^{1-\sigma}$.

B.10 Percentiles of the firm size distribution (not used)

The firm occupying the p_{th} percentile of the distribution of firm sizes is managed by a manager with ability z_p, identified by

$$\frac{1-G(z_p)}{1-G(\tilde{z})} = \frac{100-p}{100},$$

which becomes (using the Pareto CDF)

$$z_p = \left(\frac{100}{100-p}\right)^{\frac{1}{\gamma}}\tilde{z}.$$

Hence the p_{th} percentile of the firm size distribution is

$$l_p = \left(\frac{\alpha z_p}{w}\right)^{\frac{1}{1-\alpha}}$$

$$l_p = \left(\frac{100}{100-p}\right)^{\frac{1}{\gamma(1-\alpha)}} \frac{\alpha}{1-\alpha}$$

$$l_p = \left(\frac{\alpha z_p}{w}\right)^{\frac{1}{1-\alpha}}$$

$$l_p = \left(\frac{\alpha \left(\frac{100}{100-p}\right)^{\frac{1}{\gamma}} \tilde{z}}{w}\right)^{\frac{1}{1-\alpha}}$$

$$l_p = \alpha^{\frac{1}{1-\alpha}} \left(\frac{100}{100-p}\right)^{\frac{1}{\gamma(1-\alpha)}} A^{\frac{1}{1-\alpha}}$$

$$l_p = \alpha^{\frac{1}{1-\alpha}} \left(\frac{100}{100-p}\right)^{\frac{1}{\gamma(1-\alpha)}} \left(\frac{1}{\alpha}\left(\frac{\alpha}{1-\alpha}\right)^{1-\alpha}\right)^{\frac{1}{1-\alpha}}$$

$$l_p = \left(\frac{100}{100-p}\right)^{\frac{1}{\gamma(1-\alpha)}} \frac{\alpha}{1-\alpha}$$

References

Brzuwstowski, Thomas and Francesco Caselli (2024), "Economic Growth in a Cooperative Economy," *Journal of the European Economic Association,* forthcoming.

Dréze, Jaques H. (1986), *Labour Management, Contracts and Capital Markets,* Basil Blackwell.

Farrell, Joseph and Suzanne Scotchmer (1988): "Partnerships," *The Quarterly Journal of Economics*, Vol. 103, No. 2 (May, 1988), pp. 279-297.

Vanek, J. (1970), *The General Theory of Labor-Managed Market Economies.* Cornell University Press.

Ward, Benjamin (1958). "The Firm in Illyria: Market Syndicalism." *The American Economic Review*, vol. 48, no. 4, pp. 566–89.