Monetary Policy, the Yield Curve, and the Repo Market

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ABSTRACT

The paper develops a general equilibrium model in which bonds serve both as investment opportunities and as collateral for loans. The model integrates the yield curve with the repo market. Preferred-habitat investors and arbitrageurs generate downward-sloping demand for bonds and upward-sloping supply of collateral. The results show that quantitative easing increases the relative collateral value of targeted bonds in the repo market, (i) strengthening local supply and (ii) dampening term premium effects of asset purchases. Academics and policymakers should consider the bond and the repo markets in combination since traded assets and economic agents are the same in both markets.

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1 Introduction

The recent experience with quantitative easing (QE) programs around the world has starkly demonstrated the importance of demand factors in fixed-income markets, highlighting two aspects that are worthy of analytical examination. First, the price of nearly riskless bonds delivering known streams of payments rises persistently with large purchases by central banks (Bernanke, 2020). Hence, a large demand shock affects the yield curve. This casts doubts on the assumption of inelastic demand underlying the standard consumption-based model used in macroeconomics and finance. Second, these asset purchases induce a scarcity of high-quality collateral and exert downward pressure on the rates at which the targeted securities trade in the repurchase agreements (repo) market, the main secured money market.[1] Thus, a large demand shock also affects the secured money market. This calls into question the assumption of an inelastic short-term interest rate underlying the standard models in macroeconomics and finance. Given the essential role of both repo markets and the yield curve in the financial and economic system, these important stylized facts require an understanding of fixed-income markets in which durable assets such as bonds not only serve as investment vehicles but also as collateral for short-term loans, in the spirit of Kiyotaki and Moore (1997).

Does the yield curve interact with secured money markets, where investors use bonds to collateralize loans? Intuitively, it should. For instance, in the traditional models of the yield curve, bond prices reflect the current realizations of money market rates and a premium attached to the risk that these rates might change in the future. Generally, the money market is summarized by the stochastic behavior of a unique, exogenous short interest rate. However, this partial equilibrium approach does not allow for shifts in the bond market itself to affect borrowing and lending rates, since the latter are presumed to be exogenous. Such a restriction is at odds with the growing recognition that demand and supply forces, particularly QE, affect *both* the prices in the bond market (D'Amico and King, 2013; Greenwood and Vayanos, 2010, 2014; Vayanos and Vila, 2021) and the repo rates associated with bonds in the secured money market (D'Amico et al., 2018; Arrata et al., 2020; Corradin and Maddaloni, 2020; Pelizzon et al., 2024). These two robustly documented empirical facts have generally been considered in isolation, even though the demand for bonds generated by QE is the common driving force behind both of them. Motivated by this evidence, our paper is the first attempt to offer a comprehensive model that integrates downward-sloping demand in the bond market with upward-sloping supply of collateral in the repo market, generating important monetary policy implications. The proposed model differs from the standard consumption-based model used in macroeconomics and finance along two dimensions.

First, the standard consumption-based model assumes that bonds are priced by a representative agent, thus inelastic to quantity changes. This assumption is, however, inconsistent with the effect of

[1] The repo contract is a secured loan that consists of the spot sale of a bond combined with a forward agreement to repurchase the bond on a future day. The counterparty enters a reverse repo by buying the collateral on the spot market and stipulating a forward contract to sell the security. The average daily volume of repo transactions outstanding globally, according to the Bank for International Settlements, is about $12 trillion, which hovers around 12% of the world's gross domestic product.

asset purchases on the yield curve, which has been explained by using heterogeneous agents. Vayanos and Vila (2021) (VV) develop a model where preferred-habitat investors demand bonds of certain maturities and risk-averse arbitrageurs ensure that bonds are priced consistently with each other. This approach induces an effect of bond demand on the stochastic discount factor that lowers the slope of the yield curve, but abstracts from money market considerations such as the effects of QE on repo rates.

A second aspect that the consumption-based model shares with the VV model is that bonds with identical cash flows have the same price. In reality, however, bonds with identical cash flows have different prices.[2] Duffie (1996)'s approach offers a simple explanation: as a result of demand, bonds with identical cash flows may have different prices, provided their value as collateral also differs. This approach induces an effect of bond demand on their collateral value in the repo market, but abstracts from term structure considerations such as the effects of bond demand on the stochastic discount factor and thus the term premium. Empirically, however, Hu et al. (2013) find that the yield curve fitting errors are positively correlated with both the value of bonds as collateral and the term premium.

This paper helps to fill this gap in the literature. We consider two markets, the bond and the money markets, and two types of agents, preferred-habitat investors and risk-averse arbitrageurs. Preferred-habitat investors demand specific bonds, rather than bonds with particular maturities as in VV. As a consequence, bonds with equivalent cash flows may have different prices. For example, bonds of the same maturity may either be targeted or not targeted by asset purchases. Arbitrageurs are risk-averse and intervene to ensure that bonds are priced consistently, transmitting the demand for specific bonds across the term structure of interest rates. Arbitrageurs trade two bonds of the same tenor when demand forces induce relative price differences, a strategy commonly referred to as a *convergence trade*. Moreover, arbitrageurs engage in *carry trades*, investments that are profitable when the interest rate does not change. The holdings of arbitrageurs thus influence the stochastic discount factor and the slope of the yield curve. To supply the bond to the preferred-habitat investors, arbitrageurs borrow it on the repo market, transmitting the demand for specific bonds from the bond to the repo market. In equilibrium, bond demand in either the repo market or the bond market affects both markets simultaneously. Moreover, the equilibrium in the repo market affects the portfolios of arbitrageurs, and hence the stochastic discount factor and the slope of the entire yield curve. This mechanism affects the transmission of conventional and unconventional monetary policy.

Our undertaking offers two main results. First, we show that the presence of bonds that are relatively costly to borrow on the repo market limits the arbitrage activity and thus hinders the transmission of bond demand on the stochastic discount factor and the slope of the yield curve. Second, the presence of bonds that are relatively costly to borrow creates relative price anomalies between bonds with identical cash flows. In the context of QE, these two effects result in a weaker duration extraction

[2] Cornell and Shapiro (1989) were among the first to document mispricing between bonds with identical cash flows. Such mispricing is precisely the reason to smooth the fit of the Treasury yield curve on the data. Hu et al. (2013) use yield curve fitting errors to measure the severity of funding constraints on arbitrage capital.

channel and a stronger local supply effect of asset purchases. Both effects come into play when preferred-habitat investors purchase bonds and refrain from lending them on the repo market, for example when central banks do not lend on the repo market the bonds purchased via QE. Overall, this paper makes the point that the downward-sloping demand in the bond market must be considered in combination with the upward-sloping supply of collateral in the repo market. Thus, a broader message of our paper is that term structure movements cannot always be understood in terms of macroeconomic activity and the stance of monetary policy, but that the availability of bonds as collateral in the repo market is also a contributing element.

The paper shows that the bond and repo markets feature non-trivial interactions with each other, over and above their known arbitrage connection, warranting interest in our two-market general equilibrium exchange economy approach. In fact, the expected return-risk ratio in the bond market is consistent with the absence of arbitrage only when the interest rate accounts for the collateral value of the bond, requiring empirical studies of the bond market to consider a broader picture that includes the rates at which securities are financed (consistent with the evidence in Fontaine and Garcia, 2012). Special repo rates measure the collateral value of bonds and refer to the rates at which borrowers obtain financing when posting high-demand bonds as collateral. The repo specialness of a bond is the difference between the general level of interest rates and its repo rate. Our analysis thus suggests that the yield curve should be estimated by dropping highly special securities from the pool of bonds used to fit the yield curve – a practice currently followed by the Fed (Gürkaynak et al., 2007) but not by the ECB, even though specialness is quantitatively larger in Europe.

Our paper has important implications for the optimal conduct of monetary policy. The presence of bonds that are relatively costly to borrow distorts the effects of both conventional and unconventional monetary policy. Bonds that are relatively costly to borrow impair the pass-through of conventional monetary policy, such as rate hikes, to repo rates and bond yields. Moreover, the paper shows that QE itself increases the collateral values of specific bonds by reducing their availability in the repo market. This endogenous consequence of QE on the collateral value of bonds has two effects. First, it hinders the ability of quantitative policies such as QE and QT to affect the stochastic discount factor and the term premium by absorbing duration risk (duration extraction channel); second, it strengthens the mispricing of targeted bonds relative to non-targeted bonds (local supply channel). Thus, our paper uncovers a general trade-off between the duration extraction and the local supply effects of quantitative policies. For example, Lucca and Wright (2024) document that in Australia, narrow asset purchases known as yield curve control (YCC) affected only the class of bonds targeted by the central bank, generating local supply effects without duration extraction effects on the term premium. Importantly, our model suggests that policymakers can control the relative strength of local supply and duration extraction channels by appropriately using the securities lending facilities (SLF), a monetary policy tool that enables market participants to borrow the securities portfolio of the central bank in the repo market and thus facilitates short selling. Collectively, the results of this paper indicate

that monetary policy tools for the bond market, such as rate setting, YCC, and QE/QT, should be designed in combination with policy tools that influence the repo market, such as the SLF, whereby central banks make their bonds available for market participants in the repo market. Academics and policymakers should consider the bond and the repo markets in combination with each other, since traded assets and economic agents are the same in both markets.

The remainder of the paper is organized as follows. Section 2 surveys the related literature. Section 3 presents a theory of the term structure of interest rates that integrates capital and money markets. Section 4 contains a calibration of the model to market data. Section 5 offers concluding remarks. All proofs are available in the Appendix.

2 Literature Review

2.1 The Related Literature

The literature on the yield curve, the relationship between bond tenors and their yields, is vast. Unconventional monetary policies have renewed efforts by researchers to explain the effects of demand pressure on the yield curve (see, e.g., D'Amico and King, 2013; Greenwood and Vayanos, 2014; Lucca and Wright, 2024). The canonical framework for this recent literature is found in Vayanos and Vila (2021); it provides the analytical structure to harmonize recent empirical findings with the received preferred-habitat theory (pioneered by Culbertson, 1957 and Modigliani and Sutch, 1966), which accounts for the differences in investment horizons across investors. This approach considers the yield curve as endogenous to the portfolio holdings of the investors (see also Du et al., 2022). Recently, the preferred-habitat theory has been extended to study the foreign exchange market in Greenwood et al. (2023) and Gourinchas et al. (2022), to the credit risk market in Costain et al. (2022), and the interest rate swaps market in Hanson et al. (2024). Related papers include Ray et al. (2023), who develop a macroeconomic model with preferred-habitat investors, and He et al. (2022), who study Treasury bond prices in times of crisis using a preferred-habitat framework where dealers subject to regulatory constraints provide general collateral repo financing. However, none of the papers above focuses on the dual role of bonds as both investment opportunities and collateral, as we do. Our point of departure from the recent literature on habitat preferences is concentrating on investors' preferences for special bonds within maturity buckets, rather than for bonds with specific maturities.[3] Preferences for special bonds can be motivated by investment mandates and liquidity considerations (Pasquariello and Vega, 2009) and induce price differences between bonds with identical cash flows (Krishnamurthy, 2002).[4]

[3]Naturally, bonds differ on many dimensions other than maturity. For example, Chen et al. (2022) study clientele effects on bond prices and repo rates in the context of Islamic bonds, and D'Amico et al. (2022) focus on Green premia, the yield differences between maturity-matched conventional and Green bonds.

[4]Other papers on price differences between securities with equivalent cash flows include Vayanos and Weill (2008) and Gârleanu et al. (2021). We complement their stationary search-based contribution from a term structure perspective,

In a foundational paper, Duffie (1996) shows that bond prices and the interest rate on the loans they collateralize are connected by an arbitrage restriction and shows that the collateral value of bonds increases as arbitrageurs intensify their demand for collateral to sell a bond short. Other contributions in this area include Jordan and Jordan (1997), Buraschi and Menini (2002), and Fisher (2002). We contribute to this literature by considering the effects of the collateral value of bonds on the stochastic discount factor and the slope of the yield curve. To our knowledge, our paper is the first general equilibrium model formalizing this idea.

2.2 Motivating Evidence

Collectively, the consensus view in the literature on the effects of QE in fixed-income markets has highlighted stable empirical patterns that have proven robust across countries and over time.

2.2.1 The Effects of QE on the Yield Curve

QE significantly affects the yield curve. To put this into perspective, Christensen and Rudebusch (2012) use data around policy announcements in the US Treasury market to estimate a reduction of the term premium on the order of 29 basis points (bps) (see also Gagnon et al., 2018). Moreover, QE generates local supply effects. D'Amico and King (2013) quantify this effect during the first large-scale asset purchase program in the US at around 30 bps. Altavilla et al. (2021) and Koijen et al. (2021) study QE in the EU market. For a discussion on the theory and evidence of QE, see Joyce et al. (2012).

2.2.2 The Effects of QE on Repo Specialness

QE significantly affects repo specialness. In the context of US markets, D'Amico et al. (2018) document an effect of - 1.8 bps per billion dollars purchased by the Fed, with a stronger impact at the short end of the curve. Today, the Fed holds around $5 trillion in US Treasury securities. In the EU, Arrata et al. (2020) document that large-scale asset purchases affect repo specialness through the collateral scarcity channel and estimate that purchasing 1% of the outstanding bond increases its specialness by 0.78 bps. The ECB held 39% of German government bond (Bundesanleihen or bunds) at the end of 2017.[5] The estimates in Corradin and Maddaloni (2020) are even larger. Online Appendix Figure OA.I illustrates that the volume-weigthed average repo specialness rises with the proportion of bonds held by the ECB (top panel), and so does the noise measure of Hu et al. (2013) capturing yield curve fitting errors (bottom panel). In January 2017, the ECB introduced a SLF

which has the advantage of allowing for time series analyses. Copeland et al. (2014) and Mancini et al. (2016) present extensive descriptions of the institutional aspects of the US and European markets for repurchase agreements. Consistent with our model, Graveline and McBrady (2011) and Maddaloni and Roh (2021) show that inelastic investors participate in the repo market substantially less than in the secondary market, increasing the scarcity of collateral.

[5]The specialness of German bonds is sizeable (see https://www.bis.org/publ/mktc11.pdf, Figure IV.13).

that enabled investors to borrow its securities portfolio against cash. The ECB, however, imposed a borrowing rate equal to 30 bps below the GC rate. Both repo specialness and yield curve fitting errors ceased to increase after the introduction of the SLF.

3 The Model

3.1 The Bond Market

The model is cast in continuous time $t \in (0,...,\infty)$ and features a market for default-free zero coupon Treasury bonds. Bonds are indexed by their tenor τ and by their status $i = \{\text{g, s}\}$; that is, as general as opposed to special bonds. General and special bonds of the same tenor have equivalent cash flows, but their prices can differ because of the demand effects of preferred-habitat investors detailed below. At time t, a bond with tenor τ has a price $P^{\tau}_{i,t}$ expressed in dollars per unit of notional principal. The continuously compounded yield to maturity is

$$y^{\tau}_{i,t} = -\frac{1}{\tau}\log P^{\tau}_{i,t}. \tag{1}$$

The short rate r_t is the limit of the yield to maturity when τ goes to zero. We take r_t as exogenous, and assume it follows a Vasicek process whose parameters have the usual interpretation.[6]

$$dr_t = \kappa_r(\bar{r} - r_t)dt + \sigma_r dv^r_t. \tag{2}$$

3.2 The Repo Market

Bonds can be used as collateral to obtain overnight secured financing in the repo market.[7] A repo contract is a loan collateralized by a bond, and can be classified into two types. In General Collateral (GC) contracts, the primary purpose is to secure funding, and multiple bonds can serve interchangeably as collateral. Special Collateral (SC) contracts are instead secured by a specific bond and often motivated by the willingness to borrow that special bond, which can be used to cover short selling commitments. Competition to borrow a special security causes lenders to accept a lower interest rate in exchange for cash. The owners of special bonds can thus borrow cash at cheaper conditions, and enjoy a "*repo specialness*" premium that varies with the demand for that particular bond. Repo rates are collateralized and hence risk free.[8]

[6] The choice of a Gaussian model is standard. For a discussion of non-Gaussian models, see Berardi et al. (2021).

[7] We focus on overnight repo transactions for simplicity because the modeling of term repos would require an additional index. Empirically, overnight contracts attract by far the dominant proportion of volume.

[8] As is standard in modeling repurchase agreements, we abstract from collateral rehypothecation and credit risk and assume that the repo market clears once a day (see, e.g., Duffie, 1996). We consider unlimited borrowing without default risk. The results hold under re-use of collateral as long as the pass-through of the rehypothecated collateral is less than one, as is well understood empirically. A study of repo haircuts is available upon request.

Let $r^{\tau}_{i,t}$ denote the repo rate of a bond with status i and maturity τ. As noted above, the repo market is segmented, and offers two distinct opportunities to lend money against either general or special bonds at their respective overnight market rates, namely:

1. Reverse any of a basket of generic bonds ($i=g$) in the GC market by entering an agreement that earns the GC repo rate $r^{\tau}_{g,t}=r_t$. The short-rate process in Equation (2) must thus be interpreted as describing the GC repo rate dynamics (e.g., the SOFR rate).

2. Reverse a specific bond ($i=s$), which is elastic in supply, and earn the SC repo rate $r^{\tau}_{s,t}<r_t$. The SC rate $r^{\tau}_{s,t}$ will be determined in equilibrium.

While the GC secures higher interest rates, arbitrageurs might want to forgo returns on their cash to borrow the special bonds needed to meet any pending short selling commitments. The spread between GC and SC repo rates does not result in any arbitrage opportunities, as we demonstrate below.

As an illustration, Online Appendix Figure OA.II shows the average of the daily rates on repo transactions collateralized by German treasury bonds ranging from 2012 to 2018, using volume-weighted monthly trailing average high-resolution data. We distinguish between GC and SC transactions and plot the latter for benchmark time-to-maturity buckets. The figure clarifies that SC rates are lower than GC rates, indicating that the owner of a special bond can borrow cash at cheaper conditions. Moreover, SC rates vary stochastically across tenors and over time. Repo specialness is the difference between GC and SC rates. In Europe, where bonds are issued "on tap," repo specialness persists for the entire lifetime of a bond.

3.3 Preferred-Habitat Investors

As a group, preferred-habitat investors such as central banks have a demand for the *special* bond of a certain tenor. These investors have habitat preferences, which we allow to be a function of tenor, toward bonds with specific characteristics. We define as special those bonds that are targeted by preferred-habitat investors and index them through $i=s$; for clarification, think of securities eligible for QE as obvious candidates for specialness. Conversely, we refer to bonds of all maturities for which the excess demand is permanently zero as general and index them through their status $i=g$; one example is bonds that are not targeted by asset purchases. The demand of preferred-habitat investors is expressed net of the size of the issue supplied by the government, which is normalized to zero, without loss of generality. Borrowing the specification from VV, we define the excess demand $Z^{\tau}_{i,t}$ for bond i with tenor τ by

$$Z^{\tau}_{i,t}=\begin{cases} -\alpha_{\tau}\log P^{\tau}_{i,t}-\beta^{\tau}_{t} & i=s, \\ 0 & i=g. \end{cases} \tag{3}$$

In the above, $\beta_t^\tau = \theta_\tau + \omega_\tau q_t$ is composed of a term θ_τ that is constant over time but can depend on maturity and a loading ω_τ on the stochastic demand factor q_t, which evolves as

$$dq_t = \kappa_q(\overline{q} - q_t)dt + \sigma_q dv_t^q. \tag{4}$$

Equation (3) is a definition of segmented markets according to which the demand of preferred-habitat investors affects only special bonds. The process for demand risk in Equation (4) is autoregressive and mean-reverting. The parameters κ_q, $\overline{q}$, and σ_q have the usual interpretation of speed of mean reversion, long-run mean, and standard deviation of a process that has normal innovations. Under normal market conditions, Equation (3) describes preferences of private investors for specific securities.[9] In the context of QE, this formulation captures the purchases by central banks of targeted bonds relative to non-targeted bonds.

3.4 The Arbitrageurs

Arbitrageurs resort to short-term repo financing and engage in term structure trades to smooth out price differences that would otherwise arise in a segmented equilibrium. For example, arbitrageurs such as hedge funds would short-sell a bond that is overpriced as a result of demand pressure. To this end, they would borrow the bond, earning the associated repo rate, and simultaneously sell it outright on the bond market. The reverse repo contract would then be rolled over until the bond matures or the position is closed. The portfolio holdings of arbitrageurs are denoted through $X_{i,t}^\tau$. The next expression is the dynamics of arbitrageurs' wealth W_t.

$$dW_t = r_t W_t dt + \underbrace{\int_0^\infty X_{g,t}^\tau \left(\frac{dP_{g,t}^\tau}{P_{g,t}^\tau} - r_t \right) d\tau}_{\text{General bonds}} + \underbrace{\int_0^\infty X_{s,t}^\tau \left(\frac{dP_{s,t}^\tau}{P_{s,t}^\tau} - r_{s,t}^\tau \right) d\tau}_{\text{Special bonds}}. \tag{5}$$

Arbitrageurs' wealth W_t achieves the remuneration r_t offered by the GC rate, the highest among short rates. The first integral on the right side of the equation is the marked-to-market value of the portfolio of general bonds net of their financing cost, the GC rate r_t. The second integral is the marked-to-market value of the portfolio of special bonds net of their financing costs, each represented by the respective SC repo rate $r_{s,t}^\tau$. Arbitrageurs establish a long (short) position by buying (selling) the bond outright in the spot market and finance that purchase (sale) by using the bond as collateral to enter an overnight (reverse) repo agreement. The next trading day, arbitrageurs must either close the outright position or roll over the short-term collateralized financing.

Equation (5) is *not* a standard law of motion of wealth, even if the restriction $r_{i,t}^\tau = r_t \; \forall \; (i,\tau)$ will be

[9] For example, private investors such as bond mutual funds often express demand for *on-the-run*, Green, and Islamic bonds. Consistently, Fleckenstein and Longstaff (2021) document that Treasury convenience premia have discontinuities at specific annual maturities induced by clientele effects unrelated to fundamentals.

shown to correspond to the VV case in which the short rate is constant in the cross-section of bonds. Notably, our approach departs from the textbook portfolio allocation problem where a sets of risk assets is financed by a single short rate. Here, the holdings of arbitrageurs are financed on the repo market for collateralized lending. Unlike an opportunity cost interpretation, $r^{\tau}_{s,t}$ thus denotes the cost of the collateralized loan (which repos the bond) to finance the position, in the spirit of Tuckman and Vila (1992).[10]

Why are repo rates more relevant than a simple exogenous process for the short rate? Market considerations aside, the hallmark of special repo rates is the exposure to demand forces (Duffie, 1996). From a theoretical asset pricing perspective, there is simply no room for demand pressure to impact the exogenously specified short rate process in Equation (2). In the model we propose, the demand forces that affect bond prices contribute to the endogenous determination of special repo rates $r^{\tau}_{s,t}$. Special repo rates are important from a quantitative viewpoint. For example, using data from the New York Fed, Copeland et al. (2014) estimate SC repo transactions to be about 60% of the daily volume in the US market, with the remaining 40% constituted by GC transactions. The SC daily volume share of the EU repo market is even larger; for instance, Arrata et al. (2020) report an average value of 87%. Thus, the models that exogenously specify the process for the short rate are suitable for describing the GC repo market but completely abstract from the relatively larger SC segment of the repo market.

3.5 Bond Market Clearing

The bond market clearing condition is

$$Z^{\tau}_{i,t}+X^{\tau}_{i,t}=0. \tag{6}$$

Due to market clearing, and since the demand for general bonds does not exceed their supply from the government, arbitrageurs are only active in special bonds in equilibrium. Of course, nothing prevents arbitrageurs from trading general bonds as well, so that in equilibrium these securities would be as profitable as special bonds from their perspective. Effectively, arbitrageurs issue synthetic τ-maturity special bonds by accepting the rollover risk associated with short sales financed through SC repurchase agreements. Thus, higher activity from preferred-habitat investors increases repo specialness by locking up the bond and symmetrically increasing the demand for collateral to short the bond by arbitrageurs.[11]

[10] For details on how institutional investors finance Treasury trades, see Fisher (2002). A similar insight on their budget constraint can be found in He et al. (2022), where the GC rate results from regulatory frictions. We complement their approach by focusing on SC rates that vary across bonds, induced by exceptional demand.

[11] Our approach is consistent with Banerjee and Graveline (2013), who decompose the premium of special Treasury bonds into higher prices encountered by long investors and larger borrowing costs for short sellers.

3.6 General Bonds, Special Bonds

Two issues of the same tenor may differ in terms of collateral value: as discussed, asset purchases increase the collateral value of targeted bonds relative to that of non-targeted bonds with the same characteristics. While both are exposed to the same interest rate risk (duration), only the former is targeted by preferred-habitat investors and thus affected by demand pressure. To highlight this distinction in our model, we define as special those bonds that are exposed to two risk factors and as general those bonds exposed to one risk factor. Formally, let us conjecture that the price process is exponentially affine in the short rate and, conditionally on the bond status, in demand forces.

$$-\log P_{i,t}^{\tau} = \begin{cases} A_\tau r_t + B_\tau X_{s,t}^{\tau} + C_\tau & i = s, \\ A_\tau r_t + C_\tau & i = g. \end{cases} \tag{7}$$

Specific to our framework, bonds with identical cash flows can trade at different prices because of demand pressure. This feature adds a layer of realism to the models of the term structure and arises because the exposure of general bonds to demand forces is restricted to zero (by construction), so that the price of these bonds reflects only the risk of changes in the short GC rate r_t and the associated price of risk. Equation (7) reflects a market segmentation, as the compensation for the general interest rate risk r_t is common to general and special bonds, while demand forces only exert direct pressure on the price of bonds targeted by preferred-habitat investors. In Treasury markets, we regularly observe that bonds on special are overpriced with respect to general bonds with identical cash flows. Let $B_{i,\tau} = B_\tau \mathbb{1}_{[i=s]}$, where $\mathbb{1}_{[i=s]} = 1$ if $i = s$ and 0 otherwise, and write Equation (7) more compactly as

$$-\log P_{i,t}^{\tau} = a_{i,\tau} r_t + b_{i,\tau} \beta_t^{\tau} + c_{i,\tau}. \tag{8}$$

In the above, we have used the bond market clearing condition $X_{i,t}^{\tau} = \alpha_\tau \log P_{i,t}^{\tau} + \beta_\tau$ and we have defined $a_{i,\tau} = A_\tau (1 + \alpha_\tau B_{i,\tau})^{-1}$, $b_{i,\tau} = B_{i,\tau}(1 + \alpha_\tau B_{i,\tau})^{-1}$, and $c_{i,\tau} = C_\tau (1 + \alpha_\tau B_{i,\tau})^{-1}$. For simplicity, we consider the model without demand risk ($\omega_\tau = 0$). This scenario is relevant, since policymakers often specify the volume of quantitative interventions.

3.7 Equilibrium in the Bond Market

The equilibrium in the bond market is a set of bond prices such that the market clears and arbitrageurs behave optimally given the demand of preferred-habitat investors. Replace Equations (2) and (6) into

Equation (8) to derive the dynamics of bond prices.[12]

$$\frac{dP^{\tau}_{i,t}}{P^{\tau}_{i,t}} = \mu^{\tau}_{i,t}dt - a_{i,\tau}\sigma_r dv^r_t, \tag{9}$$

$$\mu^{\tau}_{i,t} \equiv \dot{a}_{i,\tau}r_t + a_{i,\tau}\kappa_r(r_t - \bar{r}) + \frac{1}{2}a^2_{i,\tau}\sigma^2_r + \dot{b}_{i,\tau}\theta_\tau + b_{i,\tau}\dot{\theta}_\tau + \dot{c}_{i,\tau}.$$

The notation $\dot{a}_{i,\tau}$ represents the partial derivative of $a_{i,\tau}$ with respect to time t. In the expression above, $\mu^{\tau}_{i,t}$ is the expected return from a bond. The volatility of bond returns depends on the innovations in the short rate dv^r_t, whose effects vary across maturities. We note that Equation (9) describes the returns of both general and special bonds, since the coefficients $(a_{i,\tau}, b_{i,\tau}, c_{i,\tau})$ depend on the bond status. In equilibrium, through market clearing, arbitrageurs' net exposures are only short positions in special bonds. Arbitrageurs set their portfolio holdings as the solution of

$$\max_{\{X^{\tau}_{i,t}\}} \quad \frac{\mathbb{E}_t\left[dW_t\right]}{dt} - \frac{\gamma}{2}\frac{\mathbb{V}_t\left[dW_t\right]}{dt}. \tag{10}$$

Substituting Equation (9) into the arbitrageurs' wealth dynamics in Equation (5), we obtain

$$dW_t = \left[W_t r_t + \int_0^{\infty} X^{\tau}_{g,t}\left(\mu^{\tau}_{g,t} - r_t\right) + X^{\tau}_{s,t}\left(\mu^{\tau}_{s,t} - r^{\tau}_{s,t}\right)d\tau\right]dt - \left[\int_0^{\infty} a_{g,\tau}X^{\tau}_{g,t} + a_{s,\tau}X^{\tau}_{s,t}d\tau\right]\sigma_r dv^r_t.$$

Replacing the above expression into Equation (10), we obtain

$$\max_{\{X^{\tau}_{i,t}\}} W_t r_t + \int_0^{\infty} X^{\tau}_{g,t}\left(\mu^{\tau}_{g,t} - r_t\right) + X^{\tau}_{s,t}\left(\mu^{\tau}_{s,t} - r^{\tau}_{s,t}\right)d\tau - \frac{\gamma}{2}\left[\sigma^2_r\int_0^{\infty} a_{g,\tau}X^{\tau}_{g,t} + a_{s,\tau}X^{\tau}_{s,t}d\tau\right]^2.$$

The first-order condition with respect to the position in the bond with tenor τ and status i is

$$\mu^{\tau}_{i,t} - r^{\tau}_{i,t} = -a_{i,\tau}\lambda_t, \tag{11}$$

where

$$\lambda_t = -\gamma\sigma^2_r\int_0^{\infty} a_{g,\tau}X^{\tau}_{g,t} + a_{s,\tau}X^{\tau}_{s,t}d\tau. \tag{12}$$

Equation (11) is an equilibrium term structure equation where the drift of the bond price $\mu^{\tau}_{i,t}$ is compared against the rate at which arbitrageurs can exchange cash for the bond $r^{\tau}_{i,t}$ and the duration of the position times the market price of interest rate risk $-a_{i,\tau}\lambda_t$. This equilibrium condition closely resembles the familiar term structure arbitrage equation, with one difference that is our first important contribution: the riskless rate r_t is replaced by the cross-section of overnight general and special repo rates, $r^{\tau}_{i,t}$. Since the foundational paper by Vasicek (1977), the characterization of the term structure

[12] To improve readability, only the equations to which there is a subsequent reference are numbered.

by the absence of arbitrage is routinely based on the restriction $r_{i,t}^{\tau} = r_t \ \forall \tau$. In practice, however, financing costs differ across bonds, since bonds can be used for collateralized borrowing at a variety of special rates. Hence, we relax this assumption and propose a generalized equilibrium condition that allows the short rate to vary with the collateral value the bond grants to its holder.

Canonical models are based on the standard arbitrage restriction: since a portfolio consisting of the appropriate combination of bond exposures achieves perfect immunization against interest rate risk, such a portfolio should realize the same return as an investment remunerated at the short rate. Therefore, one should observe a constant ratio between mean return and standard deviation across all traded instruments. Building on the idea of a constant excess return to risk (Sharpe) ratio, we note that in practice borrowing is often collateralized. Thus, we must adjust the Sharpe ratio, since the risk-free rate is not constant in the cross-section of bonds.[13] That is natural once we recognize that special bonds generate an implicit stream of cash flows on the repo market. We thus propose a paradigm shift from a focus on arbitrage to one on *executable* arbitrage. In the standard model, investors allocate their wealth between a riskless spot rate and risky bonds. In our interpretation, however, investors also use their positions as collateral to borrow cash. For market participants, differences in the collateral value between bonds are crucial determinants of portfolio choices. Note that the first order condition holds for both general and special bonds.

Remark 1. *Arbitrageurs consider general and special bonds equally profitable in equilibrium.*

Imposing the market clearing condition, we conclude that the market price of interest rate risk is

$$\lambda_t = -\gamma\sigma_r^2 \int_0^{\infty} 0 + a_{s,\tau}\left[\theta_\tau - \alpha_\tau\left(a_{s,\tau} r_t + b_{s,\tau}\theta_\tau + c_{s,\tau}\right)\right] d\tau.$$

Intuitively, the compensation for interest rate risk required by arbitrageurs depends on their holdings of both general and special bonds. Therefore, the market price of interest rate risk λ_t depends on the entire bond portfolio of arbitrageurs, delivering the *transmission* of the duration extraction channel of QE from special bonds to the general term structure of bonds ineligible for asset purchases. Importantly, the market price of interest risk is common to general and special bonds. The portfolio duration of arbitrageurs determines the market price of interest rate risk λ_t, influencing the term premium of both the general and the special yield curves.

3.8 Equilibrium in the Repo Market

The equilibrium in the repo market is characterized by a set of repo rates such that the market clears given the demand of collateral by arbitrageurs and the supply of collateral by preferred-habitat investors. Define the repo specialness $l_{i,t}^{\tau}$ as the difference between GC and SC rates conditional on

[13] The equilibrium concept extends to equities by replacing special repo rates with securities lending rebate rates.

time to maturity:

$$l_{i,t}^{\tau} = r_t - r_{i,t}^{\tau}. \tag{13}$$

Repo specialness is an indicator of the collateral value of bonds, determined by demand and supply forces. Demand and supply of collateral are defined in terms of repo specialness $l_{i,t}^{\tau}$.

First, consider the repo market for general bonds. The supply of general collateral is perfectly elastic with slope $\mathcal{E}_g = 0$, since bonds can be substituted among each other. Moreover, there is no excess demand for general bonds on the repo market, since preferred-habitat investors do not target them on the bond market and arbitrageurs do not need to borrow and sell these bonds short. For general bonds, repo specialness is therefore equal to zero; namely, $l_{g,t}^{\tau} = 0$.

Second, consider the repo market for special bonds. The supply of general collateral is *elastic* to quantities, since bonds cannot be substituted among each other.[14] Thus, incremental quantities of special bonds grant financing at progressively lower repo rates. With rightward shifts in the demand curve for SC bonds in the repo market, equilibrium specialness increases because collateral holders require greater compensation to pledge additional units of the special security (Duffie, 1996). We assume that the supply curve of SC is linear in specialness, with slope $\mathcal{E}_s$.[15] Moreover, special bonds are in high demand on the repo market. Arbitrageurs have entered the commitment to deliver the specific bond in the repo market, and their demand for collateral $-X_{s,t}^{\tau} = Z_{s,t}^{\tau}$ is inelastic to its price. Preferred-habitat investors *may or may not* supply their bonds in the repo market. In the context of QE, central banks may either lend the bonds purchased through the SLF or may not lend their bonds against cash. In general, let $Q_{i,t}^{\tau}$ denote the quantity of bonds that preferred-habitat investors offer as collateral on the repo market without demanding a specialness premium. The demand $D_{i,t}^{\tau}$ and supply $S_{i,t}^{\tau}$ of collateral in the repo market are given by

$$\begin{cases} D_{i,t}^{\tau} = Z_{i,t}^{\tau} & \text{Demand for collateral,} \\ S_{i,t}^{\tau} = Q_{i,t}^{\tau} + l_{i,t}^{\tau}/\mathcal{E}_s & \text{Supply of collateral.} \end{cases}$$

In equilibrium, the demand of collateral equals its supply. Repo specialness is thus given by

$$l_{i,t}^{\tau} = \mathcal{E}_i(Z_{i,t}^{\tau} - Q_{i,t}^{\tau})$$

The level of repo specialness $l_{i,t}^{\tau}$ resulting from asset purchases $Z_{i,t}^{\tau}$ depends by the stance of preferred-habitat investors on the repo market $Q_{i,t}^{\tau}$. We thus study the effects of QE without and with the SLF.

[14] The amount of bonds outstanding is assumed to be fixed, and the repo activity of buy-and-hold investors such as pension funds and insurance companies is limited (Maddaloni and Roh, 2021; Ballensiefen et al., 2023; Coen et al., 2024).

[15] The first order relation between specialness and demand that captures a linear SC supply curve results from the affine specification and can be generalized. For example, a second-degree polynomial would result from a quadratic TSM.

3.8.1 *Equilibrium without Securities Lending Facility*

In the absence of the SLF, the central bank does not lend its portfolio of asset purchases and $Q^{\tau}_{i,t} = 0$. The central bank is not willing to meet the excess demand for special bonds by arbitrageurs, and QE generates repo specialness. In this case, $l^{\tau}_{i,t} = \mathcal{E}_i Z^{\tau}_{i,t}$.

3.8.2 *Equilibrium with Securities Lending Facility*

In the presence of the SLF, the central bank lends its portfolio of asset purchases and $Q^{\tau}_{i,t} = Z^{\tau}_{i,t}$. The simplifying assumption is that the SLF eliminates specialness induced by asset purchases, by lending the entire securities portfolio of the central bank at the GC rate. Through the SLF, the central bank is willing to meet the excess demand for special bonds by arbitrageurs, and QE does not create repo specialness. In this case, $l^{\tau}_{i,t} = \mathcal{E}_i(Z^{\tau}_{i,t} - Q^{\tau}_{i,t}) = 0$.

Summarizing, repo specialness is zero for GC bonds that are in inelastic supply. Repo specialness arises when investors buy SC bonds without posting them as collateral. Our model can thus describe the effects of new monetary policy tools affecting the repo market such as the SLF. Figure 1 illustrates the equilibrium in the SC segment of the repo market as a function of the policy of preferred-habitat investors on the repo market.

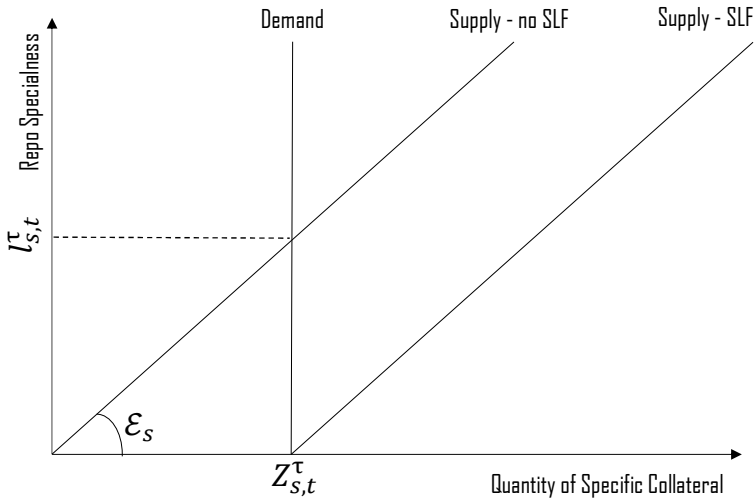


FIGURE 1: **Demand and supply of special collateral.** This figure illustrates the functioning of the market for repurchase agreements collateralized by special bonds. The horizontal axis shows the quantity of collateral, and the vertical axis shows repo specialness. The demand curve is inelastic because of the commitment of short sellers to deliver the specific issue. The supply, by contrast, is elastic, as holders of special collateral bonds require greater compensation to pledge additional units of the security on the repo market. The SLF corresponds to a rightward shift of the supply of collateral.

3.9 *Relative Bond Prices and Repo Rates*

In this section, we derive the equilibrium relationship between bond prices and repo rates. The price of a bond is given by

$$P_{i,t}^{\tau} = \exp\left(-A_{\tau} r_t - B_{i,\tau} X_{i,t}^{\tau} - C_{\tau}\right) = \mathbb{E}_t^* \left[\exp\left(-\int_0^{\tau} r_{t+u} - l_{i,t+u}^{\tau-u} du\right)\right], \tag{14}$$

In the first equality of Equation (14) above, we use the affine representation of the price we had formulated above, which is expressed under the physical measure. In the second equality, we denote by $\mathbb{E}_t^*$ the expectations taken under the risk-neutral measure and equate the bond price to the risk-adjusted present discounted value of its appropriate interest rates (Duffie and Kan, 1996).[16] Thus, the notional principal at maturity is priced using the appropriate bond-specific discount rate $r_{i,t}^{\tau} = r_t - l_{i,t}^{\tau}$.

The relationship between bond prices and repo rates can be derived by considering the relative prices of general and special bonds with identical cash flows. As opposed to the risky carry trade arbitrage conducted between bonds of different tenors, the static arbitrage between general and special bonds is riskless, since both their prices and repo rates are known. Hence, the differential demand pressure in the cash market has its mirror image in the repo specialness generated by arbitrageurs' demand for collateral in the money market.[17] Exploiting this idea, the next result characterizes the effect of demand pressure on the relative price of general and special bonds with identical cash flows.

Lemma 1. *In equilibrium, the price ratio of general to maturity-matched special bonds is*

$$\exp\left(B_{s,\tau} X_{s,t}^{\tau}\right) = \frac{\mathbb{E}_t^*\left[\exp\left(-\int_0^{\tau} r_{t+u} du\right)\right]}{\mathbb{E}_t^*\left[\exp\left(-\int_0^{\tau} r_{t+u} du\right)\exp\left(-\mathcal{E}_s \int_0^{\tau} X_{s,t+u}^{\tau-u} du\right)\right]}.$$

Proof. See Appendix A.

Both general and special bonds promise the payment of equivalent cash flows at maturity. Therefore, their relative price (on the left side of the expression above) in equilibrium must be equal to the ratio of the holding cost of replicating the two bonds through a series of overnight repo contracts, in expected risk-adjusted terms (on the right side of the equation).

Demand pressure induces different valuations between bonds with equivalent cash flows. Such price differences equal the risk-adjusted present discounted value of repo specialness from the pricing date until the bond matures. When $\mathcal{E}_s = 0$, there is no repo specialness and $B_{s,\tau} = 0$. For given levels of demand pressure, repo specialness increases in the elasticity of collateral supply. As a result, the relative price anomaly between special and general bonds on the cash market must also widen. We

[16] The coefficients A_{τ}, $B_{i,\tau}$, and C_{τ} project the current value of the risk factors on the risk-adjusted rational expectations forecast of their future conditional realizations, incorporating their information and persistence into market quotes.

[17] Empirically, D'Amico et al. (2018) use the volume of reverse repos to measure excess demand for bonds.

conclude that $B_{s,\tau}$ rises in $\mathcal{E}_s$. Among others, Duffie (1996), Buraschi and Menini (2002), and Cherian et al. (2004) suggest that repo specialness must be included in the pricing of special bonds. However, these papers are silent on what determines repo specialness and what are the implications of specialness on the market price of interest rate risk and thus on the stochastic discount factor and the term premium.

3.10 Equilibrium in the Bond and in the Repo Market

In this section, we derive in closed form the equilibrium bond prices and repo rates. First, we derive a benchmark equilibrium in the absence of repo specialness, which achieves when the central bank lends its securities portfolio on the repo market using the SLF.

Proposition 1. Equilibrium with Securities Lending Facility. *In the presence of the SLF, the coefficients governing bond returns in Equation (9) satisfy*

$$\begin{aligned} a_{i,\tau} &= \frac{1-e^{-\kappa_r^*\tau}}{\kappa_r^*} \\ b_{i,\tau} &= 0 \\ c_{i,\tau} &= \kappa_r^*\bar{r}^* \int_0^\tau a_{i,\tau} d\tau - \frac{\sigma_r^2}{2}\int_0^\tau a_{i,\tau}^2 d\tau \end{aligned}$$

where the scalars $(\kappa_r^*, \bar{r}^*)$*, which are the counterparts of* $(\kappa_r, \bar{r})$ *under the risk-neutral measure, are defined by*

$$\kappa_r^* = \kappa_r + \gamma\sigma_r^2 \int_0^\infty \alpha_\tau a_{s,\tau}^2 d\tau, \qquad \kappa_r^*\bar{r}^* = \kappa_r\bar{r} + \gamma\sigma_r^2 \int_0^\infty a_{s,\tau}\left[\theta_\tau - \alpha_\tau c_{s,\tau}\right] d\tau.$$

The short rate r_t *is unique and there is no repo specialness.*

Proof. Special case of Proposition 2.

This equilibrium with money markets corresponds to Vayanos and Vila (2021), where the interest rate is unique and exogenous. This occurs because preferred-habitat investors lend their bonds on the repo market at the GC rate, eliminating repo specialness. Since there is no repo specialness, there are no price differences between bonds with equivalent cash flows and no difference between GC and SC interest rates. This equilibrium cannot thus explain the widespread presence of specialness in the repo market as well as the differential pricing between bonds with identical cash flows. Moreover, this proposition is silent on the effects of demand on special repo rates and bond specialness.

Next, consider the equilibrium that is achieved when the supply of collateral on the repo market is upward sloping. This occurs when the central bank does not use the SLF in combination with its asset purchases. In this case, the central bank does not lend its securities portfolio. Therefore, the repo rates are endogenous to quantities, leading to price differences between bonds with equivalent cash flows and aggregate effects on the market price of risk formalized in the next result.

Proposition 2. Equilibrium without Securities Lending Facility. *In the absence of the SLF, the coefficients governing bond returns in Equation (9) satisfy*

$$
\begin{aligned}
a_{i,\tau} &= \frac{1-e^{-\kappa_r^*\tau}}{\kappa_r^*} \\
b_{i,\tau} &= \frac{\mathcal{E}_i(1-g_\tau)(1-e^{-\int\bar{\theta}_\tau d\tau})}{\bar{\theta}_\tau} \\
c_{i,\tau} &= \kappa_r^*\bar{r}^* \int_0^\tau a_{i,\tau} d\tau - \frac{\sigma_r^2}{2}\int_0^\tau a_{i,\tau}^2 d\tau
\end{aligned}
$$

where $(\kappa_r^*, \bar{r}^*)$*, the counterparts of* $(\kappa_r, \bar{r})$ *under the risk-neutral measure, are defined by*

$$
\kappa_r^* = \kappa_r + \alpha_\tau \mathcal{E}_i + \gamma\sigma_r^2 \int_0^\infty \alpha_\tau a_{s,\tau}^2 d\tau,\ \kappa_r^*\bar{r}^* = \kappa_r\bar{r} + \gamma\sigma_r^2 \int_0^\infty a_{s,\tau}\left[\theta_\tau - \alpha_\tau(b_{s,\tau}\theta_\tau + c_{s,\tau})\right] d\tau.
$$

Moreover, $\bar{\theta}_\tau = \dot{\theta}_\tau(1+\alpha_\tau\mathcal{E}_i)/\theta_\tau$ *and* $g_\tau = e^{-\int\bar{\theta}_\tau d\tau}\int \frac{\alpha_\tau c_{i,\tau}}{\theta_\tau} e^{\int\bar{\theta}_\tau d\tau} d\tau$*. There is a cross-section of repo market rates* $r_{i,t}^\tau = r_t - l_{i,t}^\tau$ *whose repo specialness is given by*

$$
l_{i,t}^\tau = \mathcal{E}_i Z_{i,t}^\tau.
$$

Proof. See Appendix A.

Proposition 2 is the key result of the paper. First, note that the coefficients $a_{i,t}$ and $c_{i,t}$ have the same functional form as in the previous Proposition 1, even if the underlying parameters $(\kappa_r^*, \bar{r}^*)$ differ, as explained below. Importantly, Proposition 2 shows that the absence of the SLF introduces price distortions between securities with equivalent cash flows on the bond market, captured by the $b_{i,\tau}$ coefficients. These price differences on the bond market are reflected in special repo rates $l_{s,t}^\tau$ in the repo market, and are thus not exploitable. The equilibrium jointly describes *two yield curves*: the yield curve of bonds that are not subject to demand pressure, $(i = g)$, as well as the yield curve of bonds that are subject to demand pressure $(i = s)$.

Remark 2. *The* $b_{g,\tau}$ *coefficients are equal to zero for general bonds, as their repo supply is inelastic. Conversely, for special bonds the* $b_{s,\tau}$ *coefficients are positive, leading to higher bond prices because the securities are in elastic supply on the repo market. That is,*

$$
\begin{cases}
\mathcal{E}_g = 0 \Longleftrightarrow b_{g,\tau} = 0 \quad \forall\tau & i = g, \\
\mathcal{E}_s > 0 \Longleftrightarrow b_{s,\tau} > 0 \quad \forall\tau & i = s.
\end{cases}
$$

Remark 2 is intuitive: The $b_{i,\tau}$ coefficients switch off to zero for general bonds, which are not subject to demand pressure and symmetrically are in inelastic supply on the repo market. As an immediate consequence, for general bonds we have $a_{g,\tau} = A_\tau$ and $c_{g,\tau} = C_\tau$. Conversely, special

bonds are overpriced relative to those that are not subject to demand pressure. The sign restriction $b_{i,\tau} \geq 0$ generates the known result that bond prices (repo rates) are higher (lower) for issues on special that guarantee cheaper cash equivalence. Thus, unless the demand pressure $Z^{\tau}_{s,t}$ is negative, SC repo rates are below the GC rate, $r^{\tau}_{s,t} \leq r_t$, specialness is always positive, and special bond prices are above general bond prices, $P^{\tau}_{s,t} \geq P^{\tau}_{g,t}$. The economic reasoning follows Duffie (1996).[18]

The recursion for the $b_{i,\tau}$ coefficients in Proposition 2 is parametrized by $\mathcal{E}_i$, which captures the elasticity of collateral supply. We nest more traditional theories as special cases which obtain by setting $\mathcal{E}_i = 0$, a case corresponding to models where bonds serve only as investment opportunities, the lending rate is exogenous, and the collateral is general.

Remark 3. *Repo specialness $l^{\tau}_{s,t}$ generates price distortions between securities with equivalent cash flows, as captured by $b_{s,\tau}$. Therefore, repo specialness influences the portfolio of arbitrageurs and affects the market price of interest rate risk $\lambda_t = -\gamma\sigma_r^2 \int_0^{\infty} a_{s,\tau}\left[\theta_\tau - \alpha_\tau\left(a_{s,\tau} r_t + b_{s,\tau}\theta_\tau + c_{s,\tau}\right)\right] d\tau$, common for both general and special bonds.*

Remark 3 makes the point that repo specialness affects the exposure of arbitrageurs to the risk that the general interest rate r_t might change, and therefore their required compensation for such risk. Intuitively, arbitrageurs' holdings of special bonds influence the interest rate risk of their portfolio. When repo specialness rises, arbitrageurs reduce their exposure to interest rate risk, affecting its price λ_t.

Remark 4. *Repo specialness $l^{\tau}_{i,t}$ affects the stochastic discount factor M_T, which contributes with the parameters guiding the interest rate process in determining the term premium and is given by*

$$M_T = e^{-\int_t^T r_u du} e^{-\int_t^T \lambda_u dv_t^r - \frac{1}{2}\int_t^T \lambda_u^2 du}.$$

To appreciate the importance of the repo market for the yield curve, consider the risk-adjusted dynamics of the short rate, governed by κ_r^* and $\overline{r}^*$. The risk-adjusted speed of mean reversion κ_r^*, is now a function of the status of the bond through the collateral elasticity of supply $\mathcal{E}_i$, and of the tenor, through the preferred-habitat demand elasticity α_τ. The risk-adjusted long-run mean $\overline{r}^*$ directly depends on the market price of of interest rate risk λ_t and thus on the duration of arbitrageurs' portfolio, composed of bonds that are special in the repo market. Crucially, repo specialness affects the pricing coefficients of *general* bonds $(a_{g,\tau}, b_{g,\tau}, c_{g,\tau})$ and thus the *entire* yield curve, since the holdings of arbitrageurs influence the market price of interest rate risk, which governs the slope of the yield curve.

In sum, Proposition 2 is novel because it gives rise to endogenous price and repo rate differences between bonds targeted by demand pressure and non-targeted securities with identical cash flows

[18] Assume by contradiction $b_{i,\tau} < 0$ for some tenor τ that would occur if net demand pressure were to reduce some equilibrium price. Since the general borrowing rate r_t is not sensitive to quantity, arbitrageurs would want to buy an infinite amount of the relatively underpriced special issue and short sell the general one in order to create a portfolio that achieves a perfect hedge against the financing costs of the position (i.e., its short rate risk) and generates riskless profits when both bonds reach maturity, thus contradicting the concept of equilibrium that requires market clearing, i.e., finite quantities.

that in equilibrium affect the entire yield curve. The first element of novelty is that our setup does not restrict the repo rate of all securities to a common exogenous short rate. The second element of novelty is that the level of special repo rates influences the market price of risk and the stochastic discount factor, and thus the slope of both the general and special bonds' yield curves. The theoretical framework that we have developed offers detailed guidance to understand the transmission of both conventional and unconventional monetary policy to the yield curve and the repo market.

3.11 Bond Specialness and Conventional Monetary Policy

Conventional monetary policy affects the yield curve by changes in the level or dynamics of the GC rate r_t. The empirical evidence in Nguyen et al. (2023) suggests that the scarcity of special bonds impairs the pass-through of rate hikes to the repo market and hence to the yield curve. Our model explains the economic channel underlying this evidence. Recall that special repo rates are given by

$$r^{\tau}_{s,t} = r_t + \mathcal{E}_s X^{\tau}_{s,t} = r_t + \mathcal{E}_s \left[\theta_\tau - \alpha_\tau \left(a_{s,\tau} r_t + b_{s,\tau}\theta_\tau + c_{s,\tau}\right)\right]. \tag{15}$$

Therefore, while the pass-through of monetary policy to the GC rate is complete, SC rates only respond to changes in r_t by a factor of $1-\alpha_\tau a_{s,\tau}\mathcal{E}_s$. Intuitively, GC rate hikes lower the price of special bonds and thus increase their excess demand from the preferred-habitat investors, impairing the transmission of conventional monetary policy on special repo money market rates.

On the bond market, the sensitivity of the yields of general bonds to short rate shocks is A_τ, and the sensitivity of yields of special bonds to short rate shocks is $a_{s,\tau} = A_\tau(1+\alpha_\tau B_{s,\tau})^{-1}$. In our model, the transmission of short rate shocks across the yield curve is stronger for GC bonds than it is for SC bonds, since $A_\tau > A_\tau(1+\alpha_\tau B_{s,\tau})^{-1}$. Therefore, special bond yields react less than general bond yields to rate hikes.

3.12 Bond Specialness and Unconventional Monetary Policy

The literature on QE suggests that asset purchases affect the term structure by influencing the term premium and by inducing local supply effects (see, e.g., D'Amico et al., 2012). In Section 3.12.1, we derive in closed form the effect of special repo rates on the transmission of asset purchases to the term premium, one of the elements of novelty of this paper. In Section 3.12.2, we show by calibration that our term structure model with endogenous repo specialness generates strongly localized supply effects, a feature that, to our knowledge, is new in the literature. Section 3.12.4 discusses QT.

3.12.1 *Bond Specialness and the Term Premium*

The term premium of general bonds with maturity τ is the difference between their yield and the short rate, $TP_g(\tau) = y^{\tau}_{g,t} - r_t$. The yield of general bonds is

$$\begin{aligned} y^{\tau}_{g,t} &= \frac{1}{\tau}\left(a_{g,\tau} r_t + c_{g,\tau}\right) \\ &= \frac{1}{\tau}\left(\frac{1-e^{-\kappa_r^*\tau}}{\kappa_r^*} r_t + \kappa_r^* \overline{r}^* \int_0^{\tau}\left(\frac{1-e^{-\kappa_r^*\tau}}{\kappa_r^*}\right) d\tau - \frac{\sigma_r^2}{2}\int_0^{\tau}\left(\frac{1-e^{-\kappa_r^*\tau}}{\kappa_r^*}\right)^2 d\tau\right). \end{aligned} \tag{16}$$

Equation (16) shows that the yield of general bonds, and thus the term premium, increases with κ_r^* and $\overline{r}^*$, the counterparts of κ_r and $\overline{r}$ under the risk-neutral measure. By Proposition 2, the risk-adjusted speed of mean reversion $\kappa_r^* > \kappa_r$ is influenced by the preferred-habitat demand elasticity α_τ, the elasticity of collateral supply $\mathcal{E}_i$, the risk aversion of arbitrageurs γ, and the volatility of the interest rate σ_r, but does not respond to the preferred-habitat demand intercept θ_τ. Quantitative policies control θ_τ and affect the term premium by exerting an effect on $\overline{r}^*$, the risk-adjusted long-run mean of the interest rate. QE is an exogenous shock to demand $\Delta\theta_\tau < 0$ that does not affect the short rate r_t (recall that θ_τ is the opposite of the intercept of the preferred-habitat demand). At the margin, the effect of θ_τ on the term premium is

$$\frac{\partial TP_g(\tau)}{\partial \theta_\tau} = \frac{1}{\tau}\frac{\partial c_{g,\tau}}{\partial \theta_\tau} = \frac{1}{\tau}\gamma a_{s,\tau}\sigma_r^2(1-\alpha_\tau b_{s,\tau})\int_0^{\tau} A_u du \geq 0. \tag{17}$$

Equation (17) shows the magnitude by which QE ($\Delta\theta_\tau < 0$) compresses the slope of the yield curve. High levels of repo specialness dampen the effect of QE on the term premium, since:

- $a_{i,\tau}$is decreasing in κ_r^* and thus in $\mathcal{E}_i$, implying $a_{s,\tau} \leq a_{g,\tau} = A_\tau$.
- From Lemma 1, $b_{i,\tau}$ is increasing in $\mathcal{E}_i$.

These two effects reinforce each other, as formalized in the next result.

Lemma 2. ***Repo markets and the transmission of QE.*** *The term premium compression effect exerted by QE in combination with the SLF is larger than that achieved by QE without the SLF.*

$$\underbrace{\frac{1}{\tau}\gamma\sigma_r^2 A_\tau \int_0^{\tau} A_u du}_{\textit{Effect of QE with the SLF}} \geq \underbrace{\frac{1}{\tau}\gamma\sigma_r^2 a_{s,\tau}(1-\alpha_\tau b_{s,\tau})\int_0^{\tau} A_u du}_{\textit{Effect of QE without the SLF}}.$$

Proof. Follows by comparing the term premium effects of QE ($\Delta\theta_\tau < 0$) in Propositions 1 and 2. *Q.E.D.*

We view this result as one of our key contributions and one that has a natural interpretation. When the short rate is exogenous, QE lowers the term premium by inducing arbitrageurs to increase their

short-selling activity. However, when the short rate is endogenous, QE in the absence of the SLF endogenously generates repo specialness, which acts in the *opposite* direction: Repo specialness induces local supply effects and partly offsets the impact of QE on the term premium. The transmission of QE becomes weaker when the special collateral is supplied with higher elasticity $\mathcal{E}_s$, raising the difference between the interest rate risk of general and special bonds A_τ and $a_{s,\tau}$ and their relative mispricing $b_{s,\tau}$.

The market price of risk λ_t governs the slope of the yield curve. Thus, a larger magnitude of arbitrageurs' exposures reduces the slope of the yield curve (Vayanos and Vila, 2021). When we also endogenize the effects of asset purchases in the repo market, repo specialness dampens the transmission of QE by reducing the magnitude of arbitrageurs' portfolio through two interrelated channels.

The first channel operates through market clearing. Repo specialness increases the price of bonds in the cash market, reducing the demand of preferred-habitat investors and, symmetrically, the exposure of arbitrageurs and their required compensation for risk. The second channel operates through the optimal choice of arbitrageurs. Recall that arbitrageurs employ a reverse carry trade strategy that consists of short-selling long-term bonds and investing the proceeds in the series of overnight interest rates. The reverse carry trade of arbitrageurs has a negative duration, and hence its profitability decreases in the term premium. The arbitrageurs require a risk compensation λ_t that is proportional to their exposure to the reverse carry trade strategy, and thus their activity constitutes the link between an increase in the demand of preferred-habitat investors and the reduction of the term premium. However, consider that repo specialness is the cost of reverse carry trade strategies hedged against interest rate risk. As a limit to arbitrage, repo specialness thus reduces the exposure of arbitrageurs to the reverse carry trade strategy, moving their negative portfolio duration closer to zero. With higher repo specialness, arbitrageurs scale down their reverse carry trade portfolio, *ceteris paribus*. Repo specialness affects the profitability of reverse carry trades, in turn influencing the market price of risk and the term premium.

As a result, asset purchases reduce the term premium by less than they would absent their endogenous effect on repo specialness. In sum, since short-selling long-term bonds is an activity with a negative duration, the cost of short-selling limits the reduction of the portfolio duration of arbitrageurs and the term premium compression of asset purchases. In aggregate, repo specialness thus increases the portfolio duration of arbitrageurs and hinders their market activity, partially impeding the transmission of QE, which generally aims to reduce the duration of arbitrageurs and flatten the yield curve.

3.12.2 Bond Specialness and Local Supply Effects

Local supply effects refer to relative price anomalies between bonds with similar duration risks (D'Amico and King, 2013). Consider two neighboring maturity buckets, τ_1 and τ_2, which are exposed to similar duration risk. In the context of our model, $P_{g,t}^{\tau_1} \approx P_{g,t}^{\tau_2}$. Now, suppose the central bank purchases bonds with maturity τ_1. QE is partially transmitted to the general term structure of interest rates, maintaining the relationship between bonds of similar maturity. However, part of QE's

transmission endogenously affects the repo market, resulting in repo specialness $l_{s,t}^{\tau_1}$ for the security directly purchased. This introduces a mispricing between general and special bonds of the same maturity, which is not executable and thus consistent with the absence of arbitrage. Thus, QE increases the price of targeted bonds, leading to an increase in $P_{s,t}^{\tau_1}$. Consequently, QE widens the difference between the average prices of bonds with neighboring maturities, τ_1 and τ_2. In sum, bond specialness generates strongly localized supply effects. Local supply effects induced by special repo rates are a unique feature of our theory that places this paper in a comfortable position to examine YCC, where the central bank targets specific groups of bonds and local supply effects are particularly pronounced.

3.12.3 The Trade-off between Local Supply and Duration Extraction Effects

A related contribution of our undertaking is to uncover a novel trade-off between the local supply and duration extraction channels of asset purchases. This trade-off is best illustrated by inspecting the optimality condition of arbitrageurs in Equation (11), which requires that $(\mu_{i,t}^{\tau} - r_t + l_{i,t}^{\tau})/a_{i,\tau} = -\lambda_t$. Now, consider an exogenous change in repo specialness, which policymakers can control. If repo specialness $l_{i,t}^{\tau}$ rises, for given levels of interest rates r_t, only two things (or a combination thereof) can occur:

1. the expected returns from shorting the specific bond $-\mu_{i,t}^{\tau}$ must rise relative to its duration risk $a_{i,\tau}$, lowering the specific bond's yield and strengthening the local supply effect of asset purchases;

2. the market price of interest rate risk λ_t must decrease, steepening the entire yield curve and partially impeding the duration extraction channel of asset purchases.

Section 4 illustrates these scenarios by calibration. Scenario 1) corresponds to the YCC, which induces local supply effects without affecting the term premium. Scenario 2) corresponds to QE without the SLF, which induces repo specialness that dampens the transmission of asset purchases on the term premium. *Ceteris paribus*, an increase in the repo specialness of a bond is either reflected in a lower yield of the specific bond or distributed across the entire term structure through a change in the market price of risk which flattens the yield curve. Conversely, a reduction in specialness acts in the opposite direction. Therefore, a third scenario of QE with the SLF generates less pronounced local supply effects and results in a stronger compression of the term premium.

3.12.4 Bond Specialness and Quantitative Tightening

Our model offers some implications for QT, an exogenous shock to demand $\Delta\theta_\tau > 0$ that does not affect the short rate r_t (recall that θ_τ is the negative of the intercept of the preferred-habitat demand). On the repo market, QT reduces the repo specialness generated by QE. (By construction, QT also reduces the amount of bonds supplied on the SLF. However, the central bank can still control how much of its

remaining portfolio of bonds to supply on the repo market). While QT reduces specialness in the repo market, its effect is nonlinear since repo specialness has a lower bound at zero. This feature highlights that, even abstracting from its broad macroeconomic implications, QT is not simply a reversal of QE.

We have highlighted a trade-off between the duration extraction and the local supply channels of asset purchases. The same trade-off is at work with QT, which achieves stronger term premium effects on the bond market when repo specialness is lower. On the other hand, QT achieves stronger local supply effects on the bond market when repo specialness is higher. In some cases, policymakers utilize QT to reduce the size of the portfolio of the central bank without aiming to influence the slope of the yield curve. In this case, our findings suggests that the central bank should not supply its securities portfolio as collateral on the repo market. In most cases, however, policymakers utilize QT to generate duration effects and influence the slope of the yield curve. Central banks with this objective should supply their remaining portfolio of bonds as collateral on the repo market.

3.13 Empirical Implications

Our model yields general predictions on the effects of repo specialness on the yield curve. To summarize, our analysis predicts that high levels of repo specialness (a) are associated with lower returns on bonds in excess of the risk-free rate (Equation (11)) and (b) affect the slope of the yield curve (Remark 4). With respect to (a), Fontaine and Garcia (2012) document that repo specialness lowers bond returns in excess of the risk-free rate, concluding that the impact of funding conditions on interest rates is large and pervasive. With regard to (b), Hu et al. (2013) report that repo specialness correlates positively with both the yield curve fitting errors and the term premium. Overall, our theory extends the classical term structure equilibrium equation suggesting that the excess return to risk ratio should be constant in the cross-section of bonds only after taking into account the financing rate of the asset. Buraschi and Menini (2002) and D'Amico and Pancost (2022) offer empirical support to this notion.

Moreover, our model yields important predictions on the effects of monetary policy. To recap, our theory predicts that high levels of repo specialness result into (c) lower pass-through of policy rate changes to the repo market and the yield curve (Subsection 3.11). In the presence of high levels of repo specialness, (d) quantitative policies have weaker duration extraction effect of on the term premium and stronger local supply effects on the yields on targeted bonds (Subsection 3.12). A number of recent empirical papers have studied related predictions. With regard to (c), Nguyen et al. (2023) document that repo specialness impairs the pass through of rate hikes to the repo market and hence to the yield curve. Moreover, Eisenschmidt et al. (2024) show that repo rate dispersion impairs the transmission of monetary policy. With respect to (d), Lucca and Wright (2024) document that targeted asset purchases in Australia had very limited effects on the term premium and strongly localized supply effects, in a context where the Royal Bank of Australia limited the extent to which investors could borrow securities from its portfolio. Consistent with our theory, D'Amico et al. (2022) document

that repo specialness strenghtens the effects of QE on the yield of the specific targeted securities.

4 Calibration

The aim of this calibration is to evaluate the model's capacity to accurately represent market data and to assess the impact of counterfactual scenarios determined by monetary policy, especially those affecting demand pressures on bond and repo markets.

4.1 Estimation

The structural estimation of our model involves the simultaneous fit of the general and special yield curves in the bond market. Our estimation is centered on European Treasury market data, where specialness is quantitatively large and economically meaningful. We utilize Euro overnight index swaps (OIS) data to estimate the general yield curve ($i = g$) and German Treasury data to estimate the special yield curve ($i = s$), with a maximum maturity set at 30 years. The reason is that OIS contracts are exposed to interest rate risk, but to little demand pressure, while German Treasury bonds are exposed to both interest rate risk and substantial demand pressure. As a result, German Treasury bonds are highly special on the repo market.

The equilibrium of the model is characterized by seven key parameters: the interest rate parameters $\overline{r}$,κ_r,σ_r; the risk-aversion coefficient of arbitrageurs γ; the functions α_τ and θ_τ, which describe the slope and intercept of preferred-habitat investor demand; and the elasticity of supply in the repo market $\mathcal{E}_s$. Since habitats are defined over bonds with equivalent cash flows, we henceforth assume that the preferred-habitat demand intercept and slope are constant across maturities and equal to the scalars θ and α. We estimate these seven parameters using the Generalized Method of Moments (GMM). The targeted moments are the average yields of OIS and German Treasury bonds at maturities of 1, 5, 7, 10, 15, 20, and 30 years. The estimation procedure is detailed in Appendix C.

The baseline calibration employs daily OIS and German Treasury data from January 1, 2017, to December 31, 2018, sourced from Bloomberg. All rates are expressed on an annual basis. The choice of this sample period is motivated by the pronounced levels of specialness observed in German Treasury bonds during that period, and the consequent large spread between maturity-matched OIS rates and German Treasury yields. To check for the robustness of our results, we conduct model estimation over a longer time series, detailed in Online Appendix B, obtaining remarkably similar estimates and fit of the data. Table I presents the baseline parameter estimates.

The point estimates for the short-rate process r_t show that a high long-run mean, a low diffusion, and high persistence of shocks yield the best fit of the data. The somewhat high estimate for the long-run mean $\overline{r}$ is balanced by a low speed of mean reversion κ_r, which implies a significant expected passage of time until the process reaches its long-run mean. The value of $\mathcal{E}_s$ is around

TABLE I: **Calibration**

Parameter	Interpretation	Value	Standard Error
$\bar{r}$	Long-run mean of r_t	4.4418	0.2014
κ_r	Mean reversion of r_t	0.0957	0.1400
σ_r	Diffusion of r_t	0.0100	0.0090
$\mathcal{E}_s$	Special collateral supply slope	0.0851	0.0053
θ	Preferred-habitat demand intercept	-2.5933	0.5210
α	Preferred-habitat demand slope	0.0202	0.2851
γ	Arbitrageur risk aversion	3.5399	0.7441

0.085 and estimated with high confidence, suggesting that the importance of the upward sloping supply of collateral is quantitatively large. The estimates of the preferred-habitat demand slope are in line with the related literature, considering that our specification focuses on a constant demand intercept and slope rather than exponential functions.[19] One advantage of our calibration relative to the preferred-habitat literature is that we estimate a negative value for the intercept θ of the demand of preferred-habitat investors. (Recall that $\theta < 0$ is consistent with a positive level of price pressure for bonds targeted by preferred-habitat investors). The arbitrageur risk aversion γ that best matches the data is lower than the estimates in the related literature. However, this value is fully in line with the macro-finance literature, which suggests $2 \leq \gamma \leq 10$, when interpreted as the absolute risk aversion coefficient of arbitrageurs with constant absolute risk aversion utility. Figure 2 presents the model fit of the data. The model does a good job in simultaneously fitting both a general yield curve to the OIS data and a special yield curve to the German Treasury yield curve.

Given the capacity of the model to accurately fit the data, Figure 3 presents our baseline calibration, with the top panel showing the bond market, and the bottom panel showing the repo market. The figure shows the salient characteristics of the model. First, as the top panel shows, two yield curves – general and special – co-exist simultaneously. For each tenor, the yield to maturity of the special bond exposed to demand pressure is lower than that of the general bond. Thus, the yield curve of general bonds (black solid line) lies above the yield curve constructed by interpolating the prices of special bonds (grey dashed line). That is intuitive, given that the two curves can be perfectly replicated by rolling over GC and SC repo contracts and that GC repo rates are generally above SC ones.

Second, unlike previous studies in the literature, the joint modeling of the GC and SC yield curves in the bond market is only possible in the context of our theory, because we account for differentials in the special repo rates induced by these bonds. In the bottom panel of Figure 3, we show the repo rate for GCs (black solid line), which was assumed to be constant across time to maturity, and the repo rate for SC bonds (grey dashed line), which decreases with maturity. The SC rate is implied

[19] For instance, VV specify $\alpha(\tau) = \alpha e^{-\tau\delta_\alpha}$ and estimate the implied habitat demand slope for 15-year bonds to be 0.06.

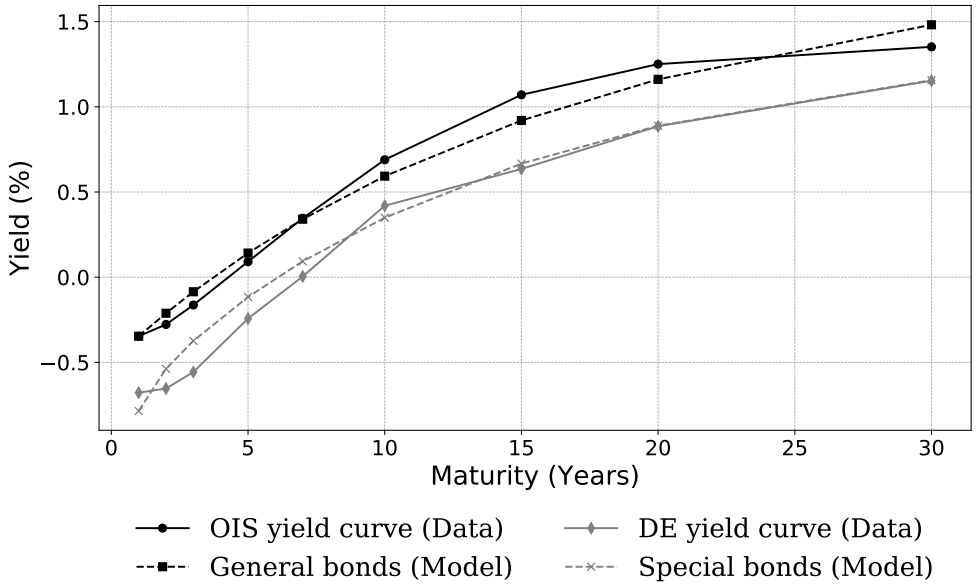


FIGURE 2: **Model Fit.** This figure compares the yields implied by the model against the observed data.

by the relationship $r_t + \mathcal{E}_s X_s^\tau$ and captures the average special repo rates conditional on maturity. The average magnitude of the repo rates implied by the model is consistent with the data shown in Online Appendix Figure OA.I. Setting $\mathcal{E}_s = 0$ returns the equilibrium to the benchmark VV model in Proposition 1, where the interest rate is unique and there are no price differences between general and special bonds. Indeed, the vertical distance between the GC and the SC curve reflects the elasticity $\mathcal{E}_s$ of the repo market supply of SC, and so does the difference between GC and SC repo rates.

4.2 Monetary Policy Instruments

We next utilize the calibrated model to perform counterfactual analyses and assess the transmission of two monetary policy tools aimed at influencing the bond market: YCC, and QE. To appreciate the importance of the repo market, we then evaluate the effect of QE in combination with a third monetary policy tool aimed at influencing the repo market, the SLF.

4.2.1 Yield Curve Control

YCC is a policy tool that targets bond yields at specific maturities. Figure 4 presents a calibration where the demand pressure directed toward the 10-year maturity special bond, $\theta_{10} = -4.75$, is relatively stronger than the demand pressure toward other maturities $\theta = -2.59$. Relative to the baseline calibration where $y_s^{10} = 0.42$, this scenario results into $y_s^{10} = 0.22$. This targeted demand pressure captures the structural intervention of central banks to control yields at specific maturities. Targeted net excess demand may also reflect institutional constraints on investors, the reopening of a Treasury auction, or short squeezes, as diverse positions may induce a spike in valuations in otherwise common

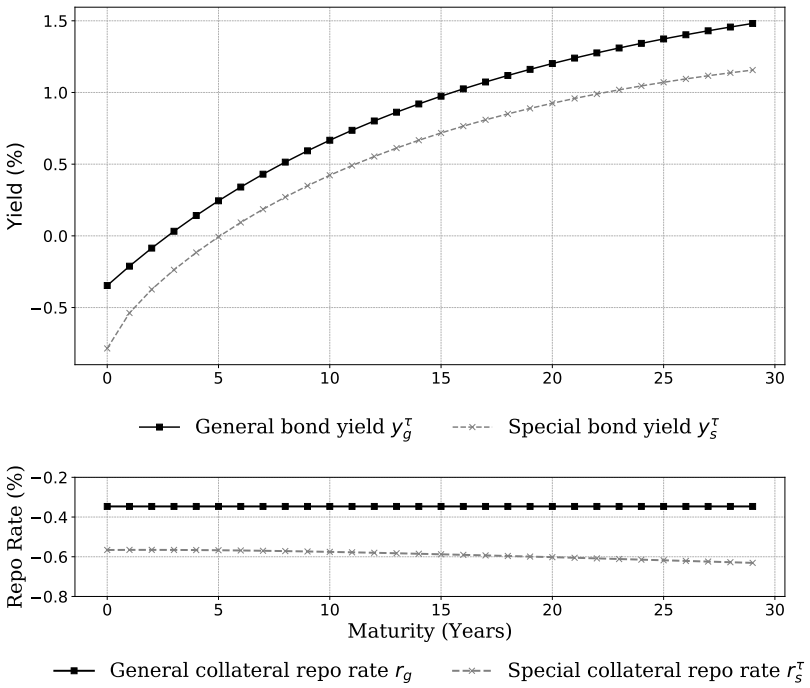


FIGURE 3: **Baseline Calibration.** The top panel of the figure shows the bond market and the bottom panel shows the repo market. Black solid lines represent general bonds and grey dashed lines represent special bonds.

value settings (Nyborg and Strebulaev, 2003).

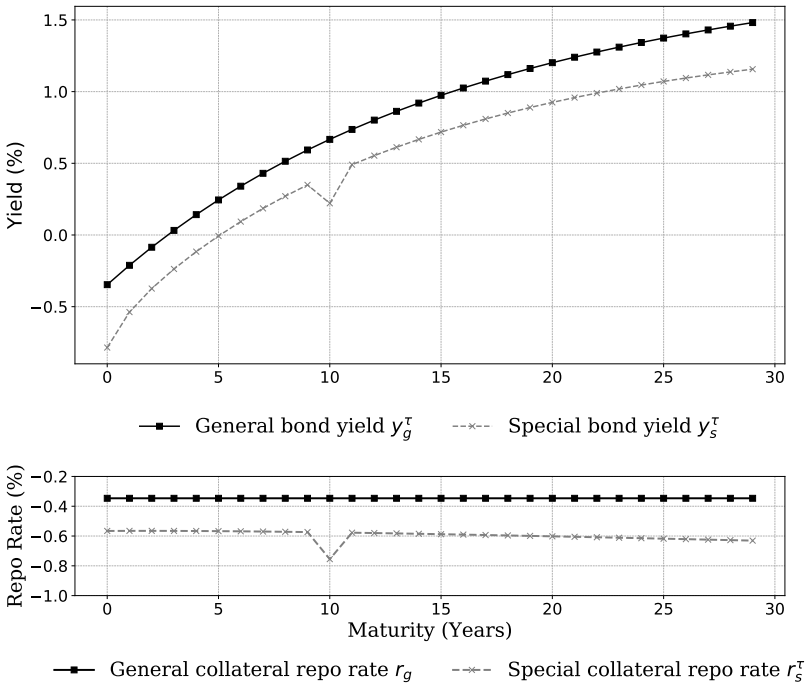


FIGURE 4: **Yield Curve Control.** This figure shows the effects of purchases targeted to specific bonds.

In the top panel of Figure 4, excess demand induces a proportional kink in the yield curve of special

bonds. This pattern is empirically relevant (see, e.g., Gürkaynak et al. (2007), Figure 4, and Lucca and Wright (2024), Figure 6). Thus, from a modeling perspective, the flexibility of our framework allows for nonmonotonicity and bridges the gap between equilibrium models of the term structure of interest rates and econometric interpolation techniques (in the spirit of Nelson and Siegel, 1987). The mirror image of the intervention by the central bank is represented in the bottom panel of Figure 4, where the cross-section of special repo rates reaches a trough for the 10-year tenor SC that is more aggressively targeted, illustrating the endogeneity of repo rates. Intuitively, when YCC exerts significant demand pressure that raises a bond's price and lowers its yield, arbitrageurs borrow the bond in the repo market to respond to the large demand for this bond created by the central bank, thus increasing its repo specialness. Since the SC bond cannot be replaced with bonds of similar tenor on the repo market, the net supply effects on both prices and special repo rates are strongly localized. The magnitude of the special repo rates for 10-year bonds $r_s^{10} = -0.76$ implied by the model is consistent with the data presented in Online Appendix Figure OA.II.

This calibration exercise generates important policy implications. In our model, policymakers can fine-tune their asset purchases to be impactful for the yield of long-term bonds, while minimizing distortions on the repo market. This is generally consonant with the practice of the major central banks, including the ECB, the Bank of Japan, and the Fed, over the past decade.[20] The reason is that bond prices have a forward-looking outlook while special repo rates reflect the *existing* stock of collateral. As the rate of decay of exceptional demand pressure diminishes, the bond price immediately increases, thus reflecting expectations that its future specialness will decline. On the other hand, what matters for the degree of collateral specialness is the quantity of bonds available on the repo market at each point in time. Thus, by fixing the overall amount purchased of a bond and the effect of the purchase on its yield, predictable repeated interventions smooth the distortions in the repo market over time when compared to a one-time operation. As noted, we expect the SLF policies to reduce repo specialness and allow the resulting kinks to be arbitraged away in the yield curve, affecting the localization of supply effects and inducing market forces to smooth them.

4.2.2 Quantitative Easing

QE is a policy tool that involves the purchase of bonds along the entire yield curve. Figure 5 presents a scenario with $\theta = -3.59$, where the demand intercept of preferred-habitat investors is equally shocked along the yield curve. Thus, we consider a counterfactual change of the baseline calibration where $\Delta\theta = -1$. (Recall that QE corresponds to $\Delta\theta < 0$). The top panel of the figure shows the yield change relative to the baseline calibration induced by QE, for both general and special bonds. The bottom

[20] From the FAQ on the Public Sector Purchase Program available on the ECB website: "The need to preserve smooth market functioning calls for the necessary amount of purchases at yields below the Deposit Facility Rate [special bonds] to be distributed over time, rather than abruptly changing the sectors of the yield curve where asset purchases take place." Thus, the ECB distributes its bond purchases to smooth distortions, as we argue it should. We are not aware of other models of QE and the term structure that generate this implication.

panel of the figure instead shows the change in the repo rate relative to the baseline calibration induced by QE, for both general and special bonds. Thus, our model has the advantageous feature that QE impacts both the bond market and the repo market, as documented by the literature surveyed in Section 2.2. Differently from the YCC, QE exerts a broad impact on the entire term structure of both general

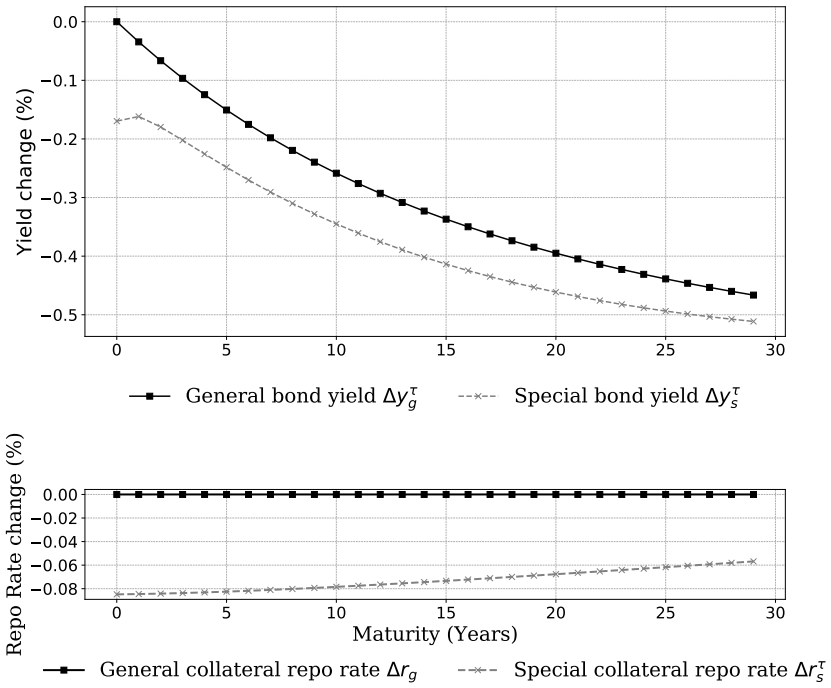


FIGURE 5: **Quantitative Easing.** This figure shows the effects of quantitative easing.

and special bonds. That is, the effects of purchases of special bonds $(i=s)$ extend to the general term structure of bonds that are not directly targeted $(i=g)$. The reason is that QE influences the market price of interest rate risk, common to both general and special bonds. As discussed in Section 3.12.1, QE is partly transmitted to the market price of risk λ_t, and partly to the special repo rates of the bonds directly purchased $r^\tau_{s,t}$. Special bond yields respond more strongly than general bond yields to asset purchases, which also increase their repo specialness. By contrast, QE does not exert any effect on the general collateral rate r_t, which follows the exogenous process specified in Equation (2).

Along the yield curve, QE exerts its strongest impact on long-term bonds, which are more sensitive to changes in the market price of interest rate risk, λ_t. The effect of QE on yields peaks at -0.45% for 30-year bonds. Conversely, the impact of QE on special repo rates $r^\tau_{s,t}$ is more pronounced at shorter maturities. The reason is that price-elastic preferred-habitat investors reduce their demand of long-term bonds as their yields declines. Indeed, Equation (15) implies $r^\tau_{s,t}=r_t+\mathcal{E}_s(\alpha_\tau \log P^\tau_{s,t}+\theta)$. Therefore, the lower special repo rates implied by a decrease in θ are partly offset by the increase in the bond price $P^\tau_{s,t}$, particularly at longer maturities. The effect of QE on special repo rates peaks at -0.08% for nearly maturing bonds. For clarity of exposition, we have muted local supply effects in this scenario.

4.2.3 *Securities Lending Facility*

The SLF is a policy tool that involves lending on the repo market part or all of the securities portfolio of the central bank on the repo market. We hereby consider the effects of QE in combination with the SLF. In this scenario, the central bank purchases bonds and makes its securities portfolio available on the repo market as collateral. As we next illustrate, QE in combination with the SLF influences the term premium without inducing repo specialness, thus inducing a stronger impact on the term premium than does QE in the absence of the SLF.

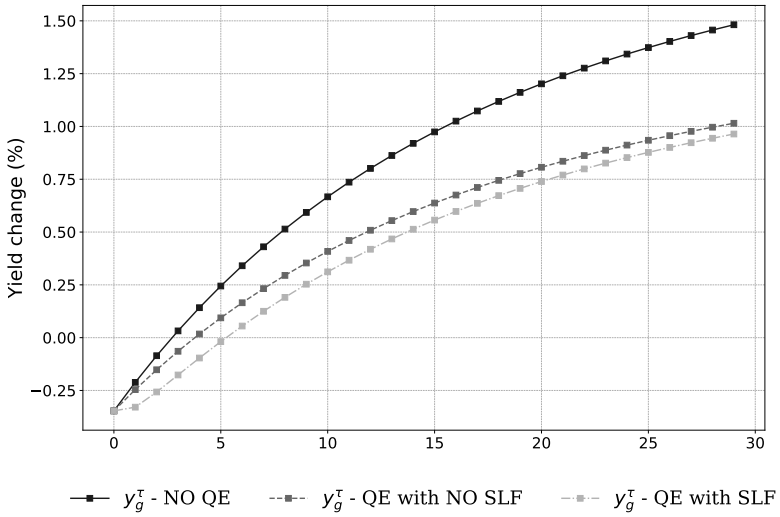


FIGURE 6: **Quantitative Easing and the Securities Lending Facility.** This figure shows the effects of alternative scenarios involving quantitative easing and the securities lending facility on the yields of general bonds.

Figure 6 shows the calibrated yields y_g^τ of general bonds (i.e., the general term structure of interest rates) under three alternative scenarios. The "NO QE" scenario corresponds to a central bank that does not conduct additional asset purchases beyond those in the baseline calibration of Figure 3, where $\theta = -2.59$. The "QE with NO SLF" scenario shows the yields of general bonds when the central bank conducts QE in the absence of the SLF by setting $\theta = -3.59$, i.e., $\Delta\theta = -1$. This case was discussed in the above Section 4.2.2. The "QE with SLF" scenario shows the yields of general bonds when the central bank conducts QE in combination with the SLF. In this case, the central bank again sets $\theta = -3.59$. Moreover, the central bank sets $Z_{s,t}^\tau = Q_{s,t}^\tau$, posting its securities portfolio as collateral on the repo market. The SLF counteracts the effect of QE on bond and repo specialness. This scenario corresponds to the equilibrium in Proposition 1, with price pressure on the bond market but without price pressure on the repo market and the resulting price differences between bonds with equivalent cash flows. In combination with the SLF, the effect of QE on the term premium is larger at all maturities. This result was derived analytically in Lemma 2.

As an important insight, this counterfactual result highlights that dysfunctional repo markets impair the transmission of QE to the term premium. The key mechanism at work is that repo specialness

constitutes a limit to arbitrage that prevents term structure arbitrageurs from entering aggressive short positions over the long term. Substantial levels of repo specialness thus induce marginal agents to hold more conservative positions, reducing their exposure to interest rate changes and their required compensation for bearing negative duration risk in the form of a lower term premium (recall that arbitrageurs are short-sellers and their profits decrease in the term premium). Thus, this analysis suggests that the SLF reinforces QE, rather than undoing it. Indeed, QE involves an exchange of cash for bonds with the private sector. Conversely, the SLF involves an exchange of bonds for cash with the private sector. Therefore, one might think that the SLF acts in the opposite direction of QE. However, the QE with SLF scenario shifts the *duration risk* of the purchased bonds from the private sector to the central bank. On the other hand, by posting its securities as collateral on the repo market, the central bank avoids obstructing the activity of arbitrage. Overall, the SLF maximizes the impact of QE on the term premium. Figure 6 shows that this effect can be substantial and economically sizeable.

5 Conclusion

Standard models used in macroeconomics and finance generally explain the yield curve by considering agents' decisions to invest at different maturities. This approach regards bonds as investment opportunities but abstracts from their important role as collateral.

In this paper, we develop a general equilibrium model in which bonds serve both as investment opportunities and as collateral for short-term loans. The model considers two markets, the bond and the repo markets, and two types of agents, investors with preferences for specific bonds and risk-averse arbitrageurs. This heterogeneity gives rise to price differences between bonds with identical cash flows that affect the relative collateral value of bonds in the repo market. This effect arises when preferred-habitat investors purchase bonds but refrain from offering them on the repo market. The collateral value of bonds influences the wealth dynamics of arbitrageurs, and therefore the stochastic discount factor and the transmission of rate and demand shocks across the yield curve.

The paper offers important conclusions for the optimal conduct of conventional and unconventional monetary policy. To maximize the transmission of monetary policy to the yield curve, policymakers should modulate the quantity of bonds available as collateral in the repo market. The scarcity of high-quality collateral reduces the pass-trough of rate hikes and cuts to bond yields and repo rates. Moreover, the scarcity of high-quality collateral hinders the transmission of QE/QT to the slope of the yield curve and strengthens the localization of their effects. Therefore, the monetary policy authority can modulate the strength of the local supply and duration extraction channels of quantitative policies by controlling the ability of market participants to borrow the securities portfolio of the central bank on the repo market. Typically, policymakers utilize QE/QT to generate duration effects and influence the slope of the yield curve. Central banks with this objective should supply their bonds on the repo market. On occasion, policymakers may want to generate local supply effects. For example, green

QE can only affect the relative cost of capital of green and brown issuers through local supply effects. In this case, the central bank should not supply on the repo market the assets purchased.

Overall, the paper suggests that academics and policymakers should consider the bond and the repo market in combination with each other, since traded assets and economic agents are the same in both markets. Future research could incorporate our model into a macroeconomic setup in which interest rates interact with real economic activity.

A Proof of Lemma 1

By substituting Equation (7) into the affine representation in Equation (14), we obtain the price of general and special bonds, since $X^{\tau}_{g,t}=0$ for general bonds whose status is $i=g$:

$$\begin{aligned} P^{\tau}_{g,t} &= \mathbb{E}^{*}_{t}\left[e^{-\int_0^{\tau} r_{t+u}du}\right] = e^{(-A_{\tau}r_t - C_{\tau})}, \\ P^{\tau}_{s,t} &= \mathbb{E}^{*}_{t}\left[e^{-\int_0^{\tau} r^{\tau-u}_{s,t+u}du}\right] = e^{(-A_{\tau}r_t - B_{\tau}X^{\tau}_{s,t} - C_{\tau})}. \end{aligned}$$

The result follows by taking the price ratio of the general bond $P^{\tau}_{g,t}$ to the special bond $P^{\tau}_{s,t}$, since the repo market equilibrium requires $r^{\tau}_{s,t}=r_t+\mathcal{E}_i X^{\tau}_{i,t}$. *Q.E.D.*

B Proof of Proposition 1

The optimality condition for arbitrageurs on the bond market is

$$\mu^{\tau}_{i,t} - r^{\tau}_{s,t} = a_{i,\tau}\gamma\sigma_r^2 \int_0^{\infty} a_{g,\tau}X^{\tau}_{g,t} + a_{s,\tau}X^{\tau}_{s,t}d\tau. \tag{B.1}$$

Substitute $\mu^{\tau}_{i,t}$ and $r^{\tau}_{s,t}$ to derive the following relationship.

$$\begin{aligned} &\dot{a}_{i,\tau}r_t + a_{i,\tau}\kappa_r(r_t-\overline{r}) + \frac{1}{2}a^2_{i,\tau}\sigma^2_r + \dot{b}_{i,\tau}\theta_{\tau} + b_{i,\tau}\dot{\theta}_{\tau} + \dot{c}_{i,\tau} - r_t + l^{\tau}_{i,t} \\ &= a_{i,\tau}\gamma\sigma_r^2 \int_0^{\infty} a_{g,\tau}X^{\tau}_{g,t} + a_{s,\tau}X^{\tau}_{s,t}d\tau. \end{aligned}$$

From the repo market equilibrium without the SLF, we have $l^{\tau}_{i,t}=\mathcal{E}_s Z^{\tau}_{s,t}$. From the bond market equilibrium, we have $X^{\tau}_{g,t}=0$ and $X^{\tau}_{s,t}=-Z^{\tau}_{s,t}=\theta_{\tau}-\alpha_{\tau}\left(a_{\tau}r_t+b_{\tau}\theta_{\tau}+c_{\tau}\right)$. Therefore,

$$\begin{aligned} &\dot{a}_{i,\tau}r_t + a_{i,\tau}\kappa_r(r_t-\overline{r}) + \frac{1}{2}a^2_{i,\tau}\sigma^2_r + \dot{b}_{i,\tau}\theta_{\tau} + b_{i,\tau}\dot{\theta}_{\tau} + \dot{c}_{i,\tau} - r_t - \mathcal{E}_i\left[\theta_{\tau} - \alpha_{\tau}\left(a_{i,\tau}r_t + b_{i,\tau}\theta_{\tau} + c_{i,\tau}\right)\right] \\ &= a_{i,\tau}\gamma\sigma_r^2 \int_0^{\infty} a_{\tau}\left[\theta_{\tau} - \alpha_{\tau}\left(a_{s,\tau}r_t + b_{s,\tau}\theta_{\tau} + c_{s,\tau}\right)\right]d\tau. \end{aligned} \tag{B.2}$$

Equation (B.2) must hold for all values of r_t. By matching the coefficients in the risk factors, we derive the following ordinary differential equation (ODE).

$$\dot{a}_{i,\tau}+a_{i,\tau}\left(\kappa_r+\alpha_\tau\mathcal{E}_i\right)-1=-a_{i,\tau}\gamma\sigma_r^2\int_0^\infty\alpha_\tau a_{s,\tau}^2 d\tau. \tag{B.3}$$

Moreover, the terms that are independent of the risk factors must add up to zero, implying

$$\begin{aligned}\dot{c}_{i,\tau}=&a_{i,\tau}\kappa_r\bar{r}-\frac{1}{2}a_{i,\tau}^2\sigma_r^2-\dot{b}_{i,\tau}\theta_\tau-b_{i,\tau}\dot{\theta}_\tau+\mathcal{E}_i(\theta_\tau-\alpha_\tau(b_{i,\tau}+c_{i,\tau}))\\&+a_{i,\tau}\gamma\sigma_r^2\int_0^\infty a_{s,\tau}\left[\theta_\tau-\alpha_\tau(b_{s,\tau}\theta_\tau+c_{s,\tau})\right]d\tau.\end{aligned}$$

We further require that the optimality holds for both general and special bonds. Lemma 1 implies $b_{g,\tau}=0$. We obtain

$$\dot{b}_{s,\tau}+b_{s,\tau}\bar{\theta}_\tau=\mathcal{E}_s\left(1-\frac{\alpha_\tau c_\tau}{\theta_\tau}\right) \tag{B.4}$$

$$\dot{c}_{i,\tau}=a_{i,\tau}\kappa_r\bar{r}-\frac{1}{2}a_{i,\tau}^2\sigma_r^2+a_{i,\tau}\gamma\sigma_r^2\int_0^\infty a_{s,\tau}\left[\theta_\tau-\alpha_\tau(b_{s,\tau}\theta_\tau+c_{s,\tau})\right]d\tau, \tag{B.5}$$

where we have defined $\bar{\theta}_\tau=\dot{\theta}_\tau(1+\alpha_\tau\mathcal{E}_s)/\theta_\tau$. We thus have a system of three ODEs. Since at maturity a bond repays the principal notional, we have $A_0=B_0=C_0=0$, implying that the system must be solved with initial conditions $a_{i,0}=b_{i,0}=c_{i,0}=0$. We follow Vayanos and Vila (2021) and proceed in two steps. First, we take the integrals as given and solve the above equations as linear ODEs. Second, we require that the solution is consistent with the value of the integrals. We obtain

$$\begin{aligned}a_{i,\tau}&=\frac{1-e^{-\kappa_r^*\tau}}{\kappa_r^*}\\b_{i,\tau}&=\frac{\mathcal{E}_i(1-g_\tau)(1-e^{-\int\bar{\theta}_\tau d\tau})}{\bar{\theta}_\tau}\\c_{i,\tau}&=\kappa_r^*\bar{r}^*\int_0^\tau a_{i,u}du-\frac{\sigma_r^2}{2}\int_0^\tau a_{i,u}^2du\end{aligned}$$

where $(\kappa_r^*,\bar{r}^*)$ are defined by

$$\kappa_r^*=\kappa_r+\alpha_\tau\mathcal{E}_i+\gamma\sigma_r^2\int_0^\infty\alpha_\tau a_{s,\tau}^2 d\tau \tag{B.6}$$

$$\kappa_r^*\bar{r}^*=\kappa_r\bar{r}+\gamma\sigma_r^2\int_0^\infty a_{s,\tau}\left[\theta_\tau-\alpha_\tau(b_{s,\tau}\theta_\tau+c_{s,\tau})\right]d\tau \tag{B.7}$$

and the function g_τ is given by

$$g_\tau = e^{-\int \bar{\theta}_\tau d\tau} \int \frac{\alpha_\tau c_{i,\tau}}{\theta_\tau} e^{\int \bar{\theta}_\tau d\tau} d\tau.$$

Proposition 1 achieves as a particular case when repo specialness $l^\tau_{i,t} = 0$. *Q.E.D.*

C Calibration

Proposition 2 defines $(\kappa^*_r, \bar{r}^*)$ as

$$\kappa^*_r = \kappa_r + \alpha_\tau \mathcal{E}_i + \gamma\sigma^2_r \int_0^\infty \alpha_\tau \left(\frac{1 - e^{-\kappa^*_r \tau}}{\kappa^*_r} \right)^2 d\tau,$$
$$\kappa^*_r \bar{r}^* = \kappa_r \bar{r} + \gamma\sigma^2_r \int_0^\infty a_{s,\tau} \left[\theta_\tau - \alpha_\tau (b_{s,\tau}\theta_\tau + c_{s,\tau}) \right] d\tau.$$

When intercept and slope of preferred-habitat investors' demand are constant over maturities, $\bar{\theta}_\tau = 0$ and Equation (B.4) implies

$$b_{i,\tau} = \mathcal{E}_i \left(\int_0^\tau 1 - \alpha c_{i,\tau} \theta^{-1} d\tau \right),$$
$$c_{i,\tau} = \kappa^*_r \bar{r}^* \int_0^\tau \frac{1 - e^{-\kappa^*_r \tau}}{\kappa^*_r} d\tau - \frac{\sigma^2_r}{2} \int_0^\tau \left(\frac{1 - e^{-\kappa^*_r u}}{\kappa^*_r} \right)^2 du.$$

Therefore,

$$\kappa^*_r \bar{r}^* = \kappa_r \bar{r} + \gamma\sigma^2_r \int_0^\infty \frac{1 - e^{-\kappa^*_r \tau}}{\kappa^*_r} \left[\theta - \alpha\theta \left(\mathcal{E}_i \int_0^\tau 1 - \alpha c_{i,u} \theta^{-1} du \right) - \alpha c_{i,\tau} \right] d\tau.$$

We estimate the parameters by GMM. The targeted moments are the average yields of OIS and German Treasury bonds at maturities of 1, 5, 7, 10, 15, 20, and 30 years. The yields of general and special bonds are, respectively,

$$y^\tau_{g,t} = \frac{a_{g,\tau} r_t + c_{g,\tau}}{\tau}.$$
$$y^\tau_{s,t} = \frac{a_{s,\tau} r_t + b_{s,\tau} \left[\theta_\tau - \alpha_\tau \left(a_{s,\tau} r_t + b_{s,\tau} \theta_\tau + c_{s,\tau} \right) \right] + c_{s,\tau}}{\tau}.$$

The standard errors are obtained through bootstrapping 1000 random samples from the original one.

References

Altavilla, C., G. Carboni, and R. Motto. 2021. Asset purchase programmes and financial markets: lessons from the euro area. *International Journal of Central Banking* 17:1–48.

Arrata, W., B. Nguyen, I. Rahmouni-Rousseau, and M. Vari. 2020. The scarcity effect of QE on repo rates: Evidence from the euro area. *Journal of Financial Economics* 137:837–856.

Ballensiefen, B., A. Ranaldo, and H. Winterberg. 2023. Money Market Disconnect. *The Review of Financial Studies* forthcoming.

Banerjee, S., and J. J. Graveline. 2013. The cost of short-selling liquid securities. *The Journal of Finance* 68:637–664.

Berardi, A., R. Brown, and S. Schaefer. 2021. Bond risk premia: The information in (really) long term rates. Working Paper.

Bernanke, B. 2020. The new tools of monetary policy. *American Economic Review* 110:943–83.

Buraschi, A., and D. Menini. 2002. Liquidity risk and specialness. *Journal of Financial Economics* 64:243–284.

Chen, M., J. Cherian, Z. Li, Y. Shao, and M. G. Subrahmanyam. 2022. Clientele effect in sovereign bonds: Evidence from Islamic Sukuk bonds in Malaysia. *Critical Finance Review* 11:677–745.

Cherian, J., E. Jacquier, and R. Jarrow. 2004. A model of the convenience yields in on-the-run treasuries. *Review of Derivatives Research* 7:79–97.

Christensen, J. H., and G. D. Rudebusch. 2012. The response of interest rates to US and UK quantitative easing. *The Economic Journal* 122:F385–F414.

Coen, J., P. Coen, and A.-C. Hüser. 2024. Collateral demand in wholesale funding markets. Working paper.

Copeland, A., A. Martin, and M. Walker. 2014. Repo runs: Evidence from the tri-party repo market. *The Journal of Finance* 69:2343–2380.

Cornell, B., and A. Shapiro. 1989. The mispricing of US Treasury bonds: A case study. *The Review of Financial Studies* 2:297–310.

Corradin, S., and A. Maddaloni. 2020. The importance of being special: Repo markets during the crisis. *Journal of Financial Economics* 137:392–429.

Costain, J., G. Nuño, and C. Thomas. 2022. The term structure of interest rates in a heterogeneous monetary union. Working Paper.

Culbertson, J. 1957. The term structure of interest rates. *The Quarterly Journal of Economics* 71:485–517.

D'Amico, S., J. Klausmann, and N. A. Pancost. 2022. The benchmark greenium. Working Paper.

Du, W., B. M. Hébert, and W. Li. 2022. Intermediary balance sheets and the treasury yield curve. Working Paper.

Duffie, D. 1996. Special repo rates. *The Journal of Finance* 51:493–526.

Duffie, D., and R. Kan. 1996. A yield-factor model of interest rates. *Mathematical Finance* 6:379–406.

D'Amico, S., W. English, D. López-Salido, and E. Nelson. 2012. The Federal Reserve's large-scale asset purchase programmes: Rationale and effects. *The Economic Journal* 122:F415–F446.

D'Amico, S., R. Fan, and Y. Kitsul. 2018. The scarcity value of treasury collateral: Repo-market effects of security-specific supply and demand factors. *Journal of Financial and Quantitative Analysis* 53:2103–2129.

D'Amico, S., and T. King. 2013. Flow and stock effects of large-scale treasury purchases: Evidence on the importance of local supply. *Journal of Financial Economics* 108:425–448.

D'Amico, S., and N. Pancost. 2022. Special repo rates and the cross-section of bond prices: The role of the special collateral risk premium. *Review of Finance* 26:117–162.

Eisenschmidt, J., Y. Ma, and A. L. Zhang. 2024. Monetary policy transmission in segmented markets. *Journal of Financial Economics* 151:103738.

Fisher, M. 2002. Special repo rates: An introduction. *Economic Review-Federal Reserve Bank of Atlanta* 87:27–44.

Fleckenstein, M., and F. A. Longstaff. 2021. Treasury richness. Working Paper.

Fontaine, J.-S., and R. Garcia. 2012. Bond liquidity premia. *The Review of Financial Studies* 25:1207–1254.

Gagnon, J., M. Raskin, J. Remache, and B. Sack. 2018. The financial market effects of the Federal Reserve's large-scale asset purchases. *International Journal of Central Banking* 7:45–52.

Gârleanu, N., S. Panageas, and G. Zheng. 2021. A long and a short leg make for a wobbly equilibrium. Working Paper.

Gourinchas, P.-O., W. Ray, and D. Vayanos. 2022. A preferred-habitat model of term premia, exchange rates, and monetary policy spillovers. Working Paper.

Graveline, J., and M. McBrady. 2011. Who makes on-the-run Treasuries special? *Journal of Financial Intermediation* 20:620–632.

Greenwood, R., S. Hanson, J. Stein, and A. Sunderam. 2023. A quantity-driven theory of term premia and exchange rates. *Quarterly Journal of Economics* forthcoming.

Greenwood, R., and D. Vayanos. 2010. Price pressure in the government bond market. *American Economic Review* 100:585–90.

Greenwood, R., and D. Vayanos. 2014. Bond supply and excess bond returns. *The Review of Financial Studies* 27:663–713.

Gürkaynak, R., B. Sack, and J. Wright. 2007. The US Treasury yield curve: 1961 to the present. *Journal of Monetary Economics* 54:2291–2304.

Hanson, S. G., A. Malkhozov, and G. Venter. 2024. Demand-and-supply imbalance risk and long-term swap spreads. *Journal of Financial Economics* 154:103814.

He, Z., S. Nagel, and Z. Song. 2022. Treasury inconvenience yields during the covid-19 crisis. *Journal of Financial Economics* 143:57–79.

Hu, G., J. Pan, and J. Wang. 2013. Noise as information for illiquidity. *The Journal of Finance* 68:2341–2382.

Jordan, B., and S. Jordan. 1997. Special repo rates: An empirical analysis. *The Journal of Finance* 52:2051–2072.

Joyce, M., D. Miles, A. Scott, and D. Vayanos. 2012. Quantitative easing and unconventional monetary policy–an introduction. *The Economic Journal* 122:F271–F288.

Kiyotaki, N., and J. Moore. 1997. Credit cycles. *Journal of Political Economy* 105:211–248.

Koijen, R. S., F. Koulischer, B. Nguyen, and M. Yogo. 2021. Inspecting the mechanism of quantitative easing in the euro area. *Journal of Financial Economics* 140:1–20.

Krishnamurthy, A. 2002. The bond/old-bond spread. *Journal of Financial Economics* 66:463–506.

Lucca, D. O., and J. H. Wright. 2024. The narrow channel of quantitative easing: Evidence from YCC down under. *The Journal of Finance* 79:1055–1085.

Maddaloni, A., and H. Roh. 2021. Not equally special: Collateralised trading of non-banks. Working Paper.

Mancini, L., A. Ranaldo, and J. Wrampelmeyer. 2016. The euro interbank repo market. *The Review of Financial Studies* 29:1747–1779.

Modigliani, F., and R. Sutch. 1966. Innovations in interest rate policy. *The American Economic Review* 56:178–197.

Nelson, C., and A. Siegel. 1987. Parsimonious modeling of yield curves. *Journal of Business* 60:473–489.

Nguyen, B., D. Tomio, and M. Vari. 2023. Safe Asset Scarcity and Monetary Policy Transmission. Working Paper.

Nyborg, K. G., and I. A. Strebulaev. 2003. Multiple unit auctions and short squeezes. *The Review of Financial Studies* 17:545–580.

Pasquariello, P., and C. Vega. 2009. The on-the-run liquidity phenomenon. *Journal of Financial Economics* 92:1–24.

Pelizzon, L., M. Subrahmanyam, and D. Tomio. 2024. Central bank-driven mispricing. Accepted, *Journal of Financial Economics*.

Ray, W., M. Droste, and Y. Gorodnichenko. 2023. Unbundling Quantitative Easing: Taking a Cue from Treasury Auctions. Accepted, *Journal of Political Economy*.

Tuckman, B., and J.-L. Vila. 1992. Arbitrage with holding costs: A utility-based approach. *The Journal of Finance* 47:1283–1302.

Vasicek, O. 1977. An equilibrium characterization of the term structure. *Journal of Financial Economics* 5:177–188.

Vayanos, D., and J.-L. Vila. 2021. A preferred-habitat model of the term structure of interest rates. *Econometrica* 89:77–112.

Vayanos, D., and P.-O. Weill. 2008. A search-based theory of the on-the-run phenomenon. *The Journal of Finance* 63:1361–1398.

Supplemental Appendix

"Monetary Policy, the Yield Curve, and the Repo Market"

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A Asset Purchases and Bond and Repo Markets

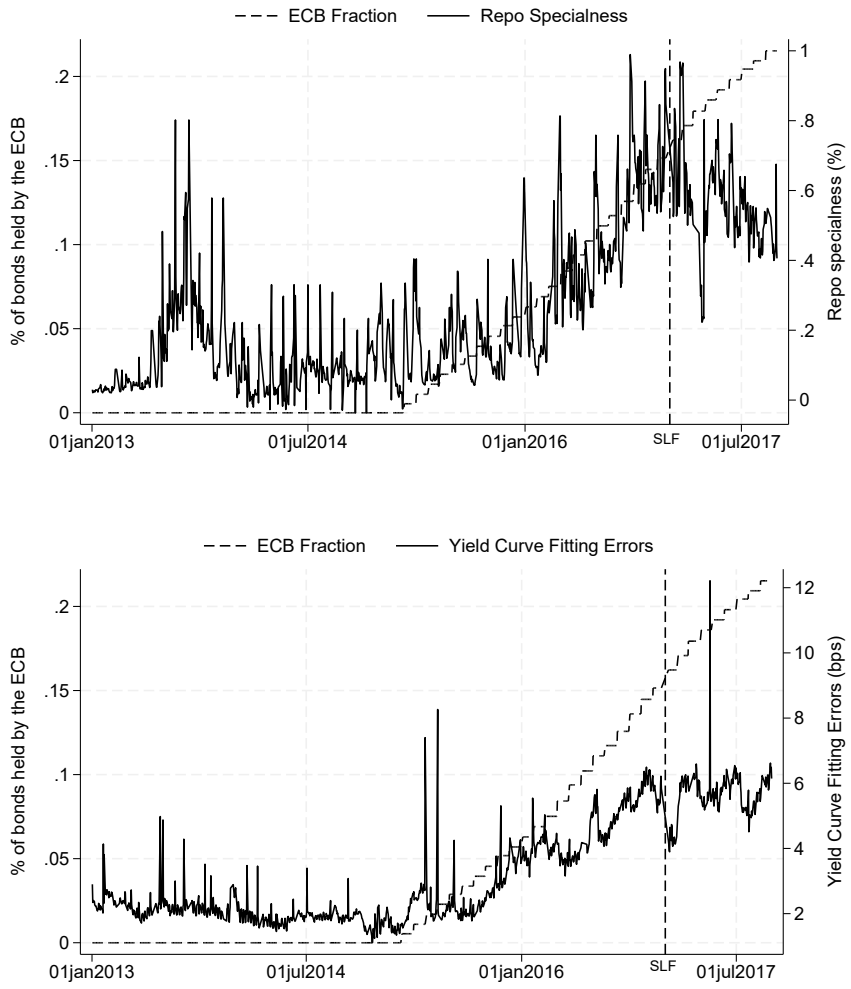


FIGURE OA.I: **Asset Purchases, Repo Specialness, and Yield Curve Fitting Errors**. The top panel shows the proportion of bonds held by the ECB (left y-axis) and the volume-weighted average repo specialness of German treasury bonds (right y-axis). The bottom panel shows the proportion of bonds held by the ECB (left y-axis) and the Hu et al. (2013) German treasury yield curve fitting errors (right y-axis). The dashed vertical line corresponds to the implementation of the cash-collateralized SLF. The data are from Pelizzon et al. (2024).



FIGURE OA.II: **General and Special Repo Rates for German Treasury Bonds**. This figure shows the volume-weighted monthly trailing average of the daily rates on tick-by-tick repo transactions collateralized by German treasury bonds, as recorded by MTS markets from 2012 to 2018. Daily rates are the volume-weighted average of intraday rates. Each trading day, repo transactions for 22 trading days are averaged. We distinguish between general collateral (GC) and special collateral (SC) transactions; the latter are shown for benchmark time-to-maturity buckets.

B Calibration on Extended Sample

We calibrate the model to a sample running from January 1, 2014, to December 31, 2020. Table OA.I presents the parameter estimates. The estimates are remarkably similar to the baseline calibration. Online Appendix Figure OA.III shows the fit of the model to the extended data sample.

TABLE OA.I: **Calibration - Robustness**

Parameter	Interpretation	Value	Standard Error
$\bar{r}$	Long-run mean of r_t	4.5773	0.8241
κ_r	Mean reversion of r_t	0.0814	0.2937
σ_r	Diffusion of r_t	0.0110	0.2803
$\mathcal{E}_s$	Special collateral supply slope	0.0614	0.0204
θ	Preferred-habitat demand intercept	-2.3663	2.0004
α	Preferred-habitat demand slope	0.0148	0.9483
γ	Arbitrageur risk aversion	3.1243	3.1330

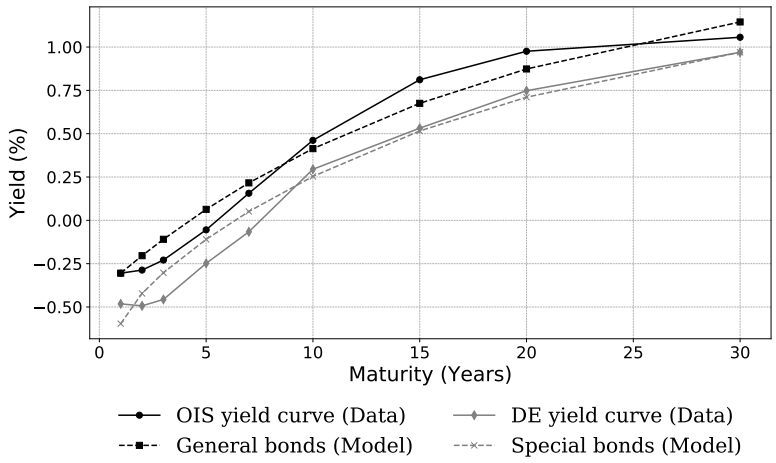


FIGURE OA.III: **Model Fit - Robustness.** This figure compares the yields implied by the model against the data observed over the extended sample.