Idea Rents and Firm Growth

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Abstract

Which firms drive aggregate productivity growth? Price-earnings ratios differ markedly across publicly-listed firms. Large differences remain after netting out proxies for firm-specific discount factors. We find that high P/E firms tend to see increases in their earnings relative to sales, which we interpret as rents from ideas. We construct an endogenous growth model with shocks to firm innovation step-sizes and R&D efficiency and calibrate it to match patterns in the data. The model can be used to infer expected growth contributions of individual firms (such as members of the Magnificent Seven) and individual sectors (such as AI firms). We find that the share of growth coming from the smallest listed firms exceeds their 10% sales share, whereas the largest firms account for less than their 10% sales share.

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1 Introduction

The ratio of stock prices to earnings is well-known to be related to earnings growth prospects, as captured simply in the Gordon growth model equation $\frac{P}{E} = 1/(r-g)$, where r is the stochastic discount factor and g is the growth rate of earnings. To the extent that high P/E ratios predict high earnings growth, as found for example by Vuolteenaho (2002), such high P/E ratio may reflect heteregeneous "growth options" as in Berk, Green and Naik (1999) and Kogan and Papanikolaou (2014). Thus P/E ratios may shed light on the dynamics of firm innovation and how they relate to aggregate growth.

In this paper we do three things. First, we document patterns relating to firm-level P/E ratios using publicly-listed U.S. firms from 1976–2023. Second, we construct an endogenous growth model and calibrate it to match the empirical patterns. Third, we glean insights from our calibrated model about the contributions of different firms to aggregate productivity growth.

We confirm that high P/E ratios predict faster earnings growth in the cross-section of firms. Just as important, we find that high P/E ratios mostly predict faster growth in earnings *relative* to sales rather than simply parallel growth in sales and earnings. Firms with high earnings tend to have lower P/E ratios, and R&D-intensive firms tend to have higher P/E ratios.[1]

Motivated by the facts we document, we construct an endogenous growth model in the spirit of Klette and Kortum (2004). Like their model, ours features multiproduct firms engaging in creative destruction through quality innovations. We deviate in two ways that are critical for mimicking the data: (i) stochastic idiosyncratic innovation step sizes and R&D efficiency across firms, and (ii) innovation arrival rates that are a convex function of a firm's R&D expenditures alone.

In particular, heterogeneous step sizes induce heterogeneity in firm P/E

[1] We also find that R&D intensity is declining in firm sales across firms within years. R&D coverage is much better within manufacturing than other sectors, so all of our results using R&D data are for manufacturing only.

ratio, as price-cost markups and the profitability of innovation are increasing in step sizes (Aghion and Howitt, 1992). Permanent differences in step sizes, however, would not generate heterogeneous P/E ratios in our model along a balanced growth path. Stochastic firm step sizes, in contrast, generate persistent but not permanent differences in P/E ratios across firms. High P/E ratio firms enjoy *temporarily* high step sizes, leading their sales to grow quickly for awhile. This is akin to Luttmer (2011), who concluded that temporarily good innovation opportunities were essential for understanding why many large U.S. firms ascended in just a few decades (e.g., Wal-Mart, Apple, Amazon). In our model, high P/E ratios portend faster growth in earnings *relative* to sales. As firms add high-markup products, they increase the average markup in their portfolio of products. On top of this, the firm's future R&D spending will revert back down along with its innovation step size, boosting earnings net of R&D.

Klette and Kortum (2004) assumed that R&D costs are increasing in a firm's innovation arrival rate, but decreasing in the firm's stock of products. We drop the stock of products from the firm R&D cost function, akin to Akcigit and Kerr (2018), which makes it harder for larger firms to innovate. Doing so renders the model more tractable as a firm's number of products are no longer a state variable affecting its R&D decision. More importantly, it helps us to explain why R&D/Sales ratios are falling in firm size among U.S. listed firms.

We calibrate our model to match nine moments from the Compustat data: earnings growth per effective worker (a proxy for TFP growth); the average stochastic discount rate across firms in Compustat from David, Schmid and Zeke (2022); the share of predictable earnings growth from P/E ratios that is due to earnings/sales growth across firms; the share of predictable earnings/sales growth from P/E ratios that is due to pre R&D earnings/sales growth across firms; total earnings divided by total sales; the gap between the 90th and 10th percentiles of predicted earnings growth based on P/E differences across firms; the gap between the 90th and 10th percentiles of

predicted pre-R&D earnings growth based on P/E differences across firms; the projection of R&D/Sales on log sales across manufacturing firms; and the 90/50 ratio of sales.

We choose values for nine parameters to match the nine moments: the curvature of the R&D cost function (R&D spending as a function of the innovation arrival rate); the household's discount factor; plus the drift, persistence, and volatility of firm-specific innovation step sizes and R&D cost. The dispersion in innovation step sizes contributes to growth in that innovative firms are endogenously more research intensive. We find that the largest firms, who collectively account for 10% of sales, contribute only 1% of aggregate TFP growth over the subsequent five years. Conversely, the smallest firms accounting for 10% of sales are responsible for 60% of TFP aggregate growth.

Related Literature

Our approach complements research using patents and R&D. This literature has uncovered rich insights, but patenting and R&D are less common outside manufacturing and software. Even within manufacturing, the propensity to patent may be low and vary across firms—see Argente, Baslandze, Hanley and Moreira (2020) for a comparison of products to patents in U.S. consumer packaged goods, and Aghion, Bergeaud, Boppart, Klenow and Li (2024) for the propensity to patent reported innovations in France. The methodology in the current paper can be applied to all firms with earnings and valuations, regardless of sector, patenting, or R&D reporting.

Like us, Kogan, Papanikolaou, Seru and Stoffman (2017) use stock prices to gauge the value of firm innovations. They examine changes in firm stock market capitalization in response to the announcement of firm patent grants. Our effort is distinct in not relying on patents and in focusing on the expected future growth contributions of firms rather than individual innovations. Our paper also relates to studies of Tobin's Q, such as Hall (1993), Hall (2001) and

Crouzet and Eberly (2023). Tobin's Q has a backward-looking component (rents from past innovation), whereas the P/E ratio highlights the discounted value of future innovations that are not reflected in current earnings.

A vast literature explores the empirics of firm dynamics, in particular employment. Recent examples include Decker, Haltiwanger, Jarmin and Miranda (2016), Garcia-Macia, Hsieh and Klenow (2019), Sterk, Sedláček and Pugsley (2021), and Klenow and Li (2021). Our approach emphasizes the need to look at earnings data as well, to get at innovation rates and hence growth contributions. We find that fast-growing firms, also known as gazelles and Luttmer Rockets, are associated with rising earnings relative to sales in addition to growing sales and employment.

Recent precursors to our quantitative modeling of firm dynamics and growth are Cao, Hyatt, Mukoyama and Sager (2017), Akcigit and Kerr (2018), Acemoglu, Akcigit, Alp, Bloom and Kerr (2018), Akcigit and Ates (2023), and Peters and Walsh (2025). Cao et al. model stochastic R&D efficiency (but not stochastic step sizes) in a model with variety creation and own-product innovation by incumbents. The middle three studies focus on firms who conduct R&D and who patent. Peters and Walsh aim to understand the impact of declining population growth on firm dynamism and productivity growth.

Campbell and Shiller (1988) and Cochrane (2011) decompose variation in stock market-wide P/E ratios into time variation in the stochastic discount factor versus growth in earnings (or cash flow). Similarly, Atkeson, Heathcote and Perri (2022) and Atkeson, Heathcote and Perri (2024) stress that fluctuating rents from innovation show up in aggregate P/E ratio movements. They explore the implications of this for time-varying U.S. stock market valuations and the U.S. net international asset position. In contrast to these studies, we investigate the implications of cross-firm dispersion in P/E ratios for firm growth contributions.

The rest of the paper is organized as follows. Section 2 documents empirical patterns surrounding firm P/E ratios and subsequent earnings growth. Section

3 builds an endogenous growth model to help interpret these patterns. Section 4 calibrates the model and characterizes firm growth contributions. Section 5 concludes.

2 Empirical patterns

2.1 Data source

Our main variables are stock prices from the Center for Research in Security Prices (CRSP), "street-level" earnings from the Institutional Brokers' Estimate System (I/B/E/S), and sales from Compustat. Our data cover 15,627 unique firms and 162,075 firm-year observations from 1976 to 2023. While the data covers only publicly-listed firms, these firms account from roughly one-fourth of total private non-farm employment.[2]

We source the security-level share price on the last trading date of each month from CRSP. We multiply each firm's stock price by its number of outstanding shares to arrive at each firm's market capitalization. Our data covers securities listed on the NYSE, NYSE MKT, NASDAQ, and Arca (ETF) exchanges. We extract security-level fiscal year-end earnings per share from I/B/E/S. The earnings per share (EPS) is "street-level" in that I/B/E/S adjusts the accounting EPS for items that the majority of analysts include or exclude from their valuation calculations. We calculate the P/E ratio by dividing firm-level share prices by firm-level earnings per share. We construct firm-level earnings by multiplying EPS by the number of shares outstanding. Sales are from Compustat and for each firm's fiscal year.

We construct the growth of earnings h years ahead g_E^h and growth of the

[2]Employment reported in Compustat for firms in our sample relative to nonfarm payroll employment from the BLS.

earnings-to-sales ratio $g^h_{E/S}$ as

$$g^h_E = \frac{1}{h}\log\left(\frac{E_{t+h}}{E_h}\right) \qquad g^h_{E/S} = \frac{1}{h}\log\left(\frac{E_{t+h}/S_{t+h}}{E_t/S_t}\right).$$

We drop firms with negative earnings when calculating the growth rate of earnings. Later on, we do the same in the model for purposes of comparing data and model moments.

2.2 Motivating facts

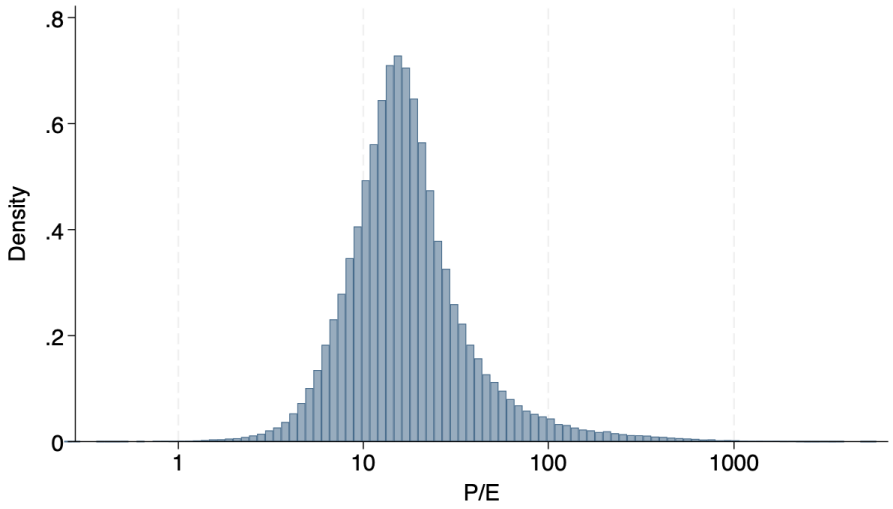
Here we document several patterns in the data that will motivate our modeling choices. First, there is significant dispersion of P/E ratios across firms (in a given year and industry). Second, firms with high P/E ratios tend to have high subsequent earnings growth—even relative to sales growth. And, within manufacturing where R&D data is available, high P/E ratio firms exhibit higher R&D/Sales ratios). Third, firms with higher earnings tend have low P/E ratios and low subsequent earnings growth.

Significant dispersion in P/E ratios across firms. Figure 1 shows the distribution of raw P/E ratios pooled across firm-years. Appendix Figure A1 shows the standard deviation and interquartile range across firms within each year of our sample. The degree of dispersion fluctuates noticeably, but is nontrivial in all years. Appendix Figure A2 and A3 show significant dispersion after taking out year effects and fiscal-year end month effects or 4-digit SIC industry fixed effects.

Differences in P/E ratios across firms could be due to firm-specific discount rates. However, the cross-sectional dispersion is not just about dispersion in firm-specific risks. We calculate the residual of $\frac{P}{E}_{it} - \frac{1}{r_{it}}$ for each firm i and year t, where r_{it} is the firm-specific risk measured by David, Schmid and Zeke (2022) using Fama-French factor models. According to the Gordon growth formula, the $\frac{1}{r_{it}}$ is a proxy for variation in the $\frac{P}{E}_{it}$ ratio if all firms have the same constant

earnings growth rate. Dispersion in the residual thus reflects variation in the expected growth rate of rents. We find significant dispersion in the residual (see Appendix Figure A4). This is consistent with the findings of Vuolteenaho (2002) and De La O, Han and Myers (2025) that differences in firm-level stock returns are partly driven by changes in cash-flow expectations as opposed to entirely reflecting changes in the discount rate.

Figure 1: Distribution of P/E

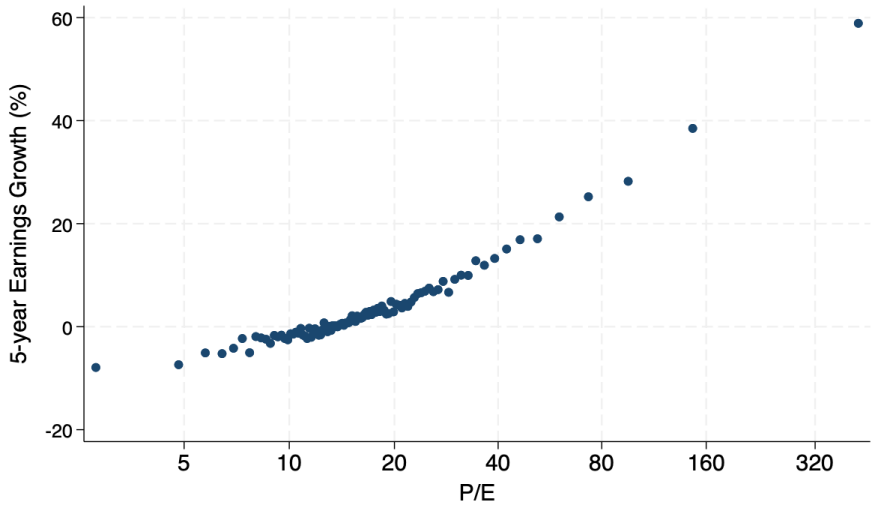


Source: CRSP and I/B/E/S. Figure trims the top and bottom 0.1% observations each year.

High P/E firms have higher subsequent earnings growth and higher R&D intensity. One may be concerned that the dispersion in P/E is simply noise. However, this is not the case. Figure 2 plots the 5-year ahead earnings growth against current year P/E. A high P/E ratio today predicts high subsequent earnings growth. Among manufacturing firms for whom R&D is reliably reported, high P/E firms have higher R&D intensity, as shown in Figure 3.

Markups may play an important role in the relationship between earnings growth and P/E ratios. Row 1 of Table 1 shows the coefficient from regressing h-year ahead annual earnings growth for $h = 1, 2, \ldots, 5$ on current P/E ratio. At all horizons, higher P/E ratios today predict higher subsequent earnings growth.

Figure 2: Earnings growth vs P/E

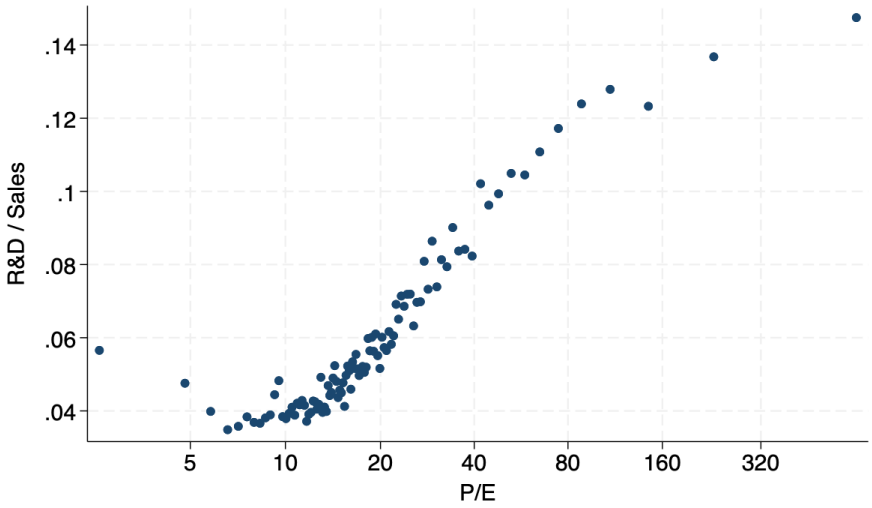


Source: Compustat. The y-axis is average yearly earnings growth from year t to $t+5$. The x-axis is P/E ratio in t.

Rows 2 and 3 decompose this relationship into growth of the earnings-to-sales ratio versus growth in sales, respectively. The bulk of the relationship (77 to 84%) between earnings growth and P/E is accounted for by higher P/E ratios predicting higher growth in earnings *relative* to sales.

Firms with higher earnings have lower P/E ratios and earnings growth. While high P/E ratio predicts high earnings growth, high earnings firms have lower P/E ratios and earnings growth. Figure 4 plots the P/E of a firm against its share of total earnings and Figure 5 plots the 5-year ahead earnings growth of a firm against its share of total earnings. Both the P/E ratio and earnings growth decline with earnings. Figure 6 shows that sales growth also declines with sales. Table 2 shows that at all horizons the negative relationship between earnings growth and earnings is predominantly (64 to 77%) through a negative relationship between growth in E/S and current earnings.

Figure 3: R&D/Sales vs P/E



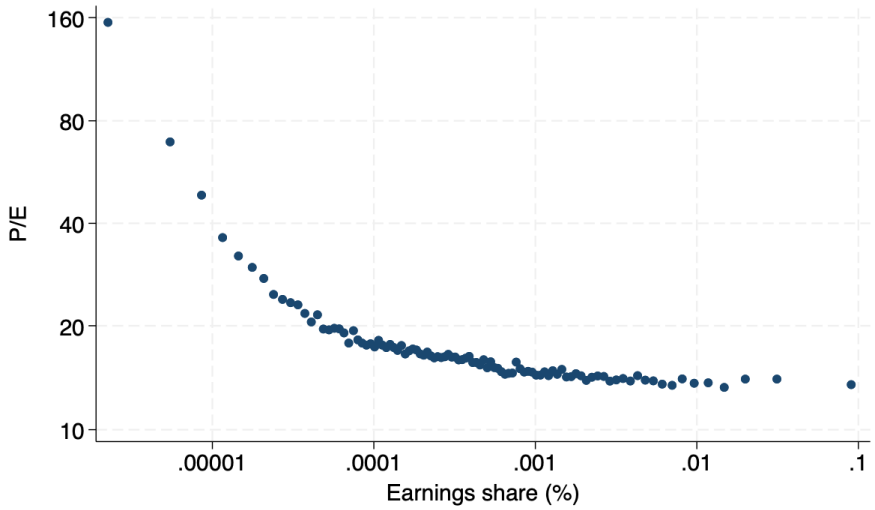
Source: Compustat. The y-axis is R&D over sales in year t. The x-axis is P/E ratio in t.

Table 1: Growth of earnings and earnings/sales vs P/E ratio

	$h=1$	$h=2$	$h=3$	$h=4$	$h=5$
g_E on $\log(P/E)$	0.422	0.262	0.189	0.150	0.127
	(0.005)	(0.003)	(0.002)	(0.002)	(0.002)
$g_{E/S}$	0.355	0.213	0.151	0.117	0.098
	(0.005)	(0.003)	(0.003)	(0.002)	(0.002)
g_S	0.067	0.049	0.039	0.033	0.029
	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)

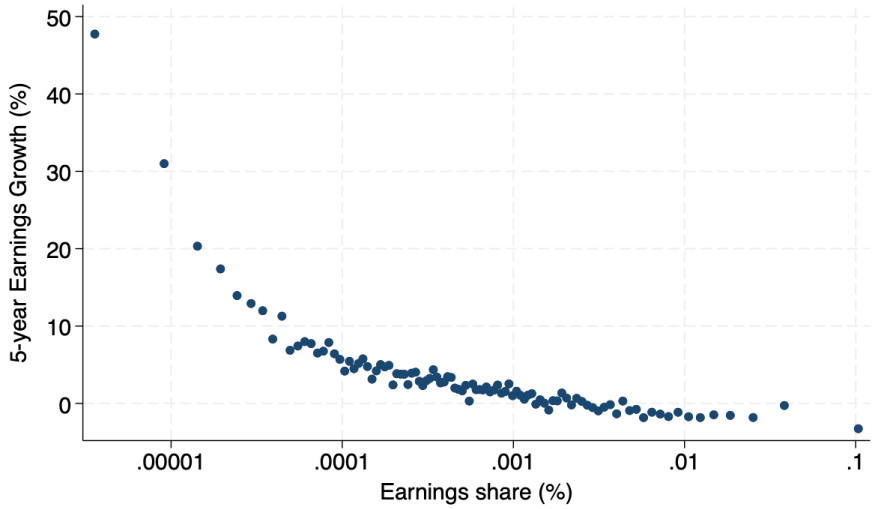
Source: Compustat. The first row displays the coefficient when regressing average yearly earnings growth from year t to $t+h$ on P/E ratio in t. The second and third row decomposes the coefficient into growth in the ratio of earnings to sales and sales.

Figure 4: P/E ratio vs. earnings



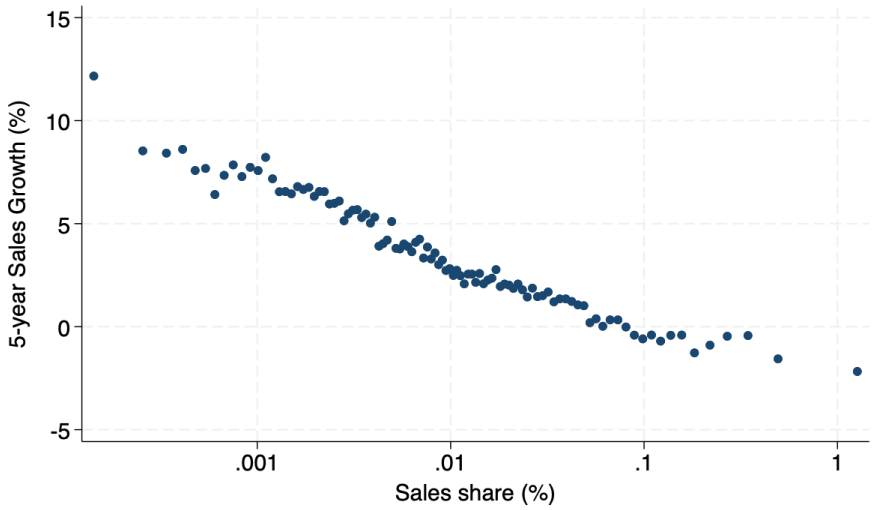
Source: Compustat. The y-axis is P/E ratio in year t. The x-axis is share of total earnings in t.

Figure 5: Earnings growth vs earnings



Source: Compustat. The y-axis is average yearly growth of earnings from year t to $t+5$. The x-axis is share of total earnings in t.

Figure 6: Sales growth vs sales



Source: Compustat. The y-axis is average yearly growth of sales from year t to $t+5$. The x-axis is share of total sales in t.

Table 2: Growth of earnings and earnings/sales vs earnings

	$h=1$	$h=2$	$h=3$	$h=4$	$h=5$
g_E on $\log(E)$	-0.066	-0.047	-0.036	-0.031	-0.027
	(0.002)	(0.001)	(0.001)	(0.001)	(0.001)
$g_{E/S}$	-0.051	-0.033	-0.024	-0.020	-0.017
	(0.002)	(0.001)	(0.001)	(0.001)	(0.001)
g_S	-0.016	-0.014	-0.012	-0.011	-0.011
	(0.0005)	(0.0004)	(0.0004)	(0.0004)	(0.0004)

Source: Compustat. The first row displays the coefficient when regressing average yearly earnings growth from year t to $t+h$ on earnings in t. The second and third row decomposes the coefficient into growth in the ratio of earnings to sales and sales.

3 Model

Based on the facts we documented in the previous section, in this section we formulate an endogenous growth model with stochastic but persistent differences in innovation step sizes and R&D efficiency across firms. Growth in earnings/sales (especially earnings gross of R&D expenditures) is suggestive of rising markups at high P/E firms. Thus high P/E ratio firms may have opportunities to invest in innovations with high quality steps that will command high markups, fueling their fast earnings growth. A high P/E ratio could then suggest that investors have information about the firm's investment opportunities that they price into firm valuations. If firms have information about their opportunities as well, this could account for the high R&D intensity of high P/E ratio firms.

Our model builds on Klette and Kortum (2004), which features endogenous quality growth and creative destruction by multiproduct firms. The key difference is that we allow each firm's innovation step size to evolve stochastically (and idiosyncratically) rather than being common or fixed over time. This will create markup dynamics and in turn heterogeneous P/E ratios to help the model match the patterns we found in the data.

3.1 Set up

Household A representative household inelastically supplies L units of production labor and maximizes its discounted utility

$$\max_{\{C_t\}_{t=1}^{\infty}} \sum_{t=0}^{\infty} \beta^t \log(C_t),$$

subject to

$$A_{t+1} = A_t(1 + r_t) + w_t L - C_t,$$

and a standard no-Ponzi game condition. Here C denotes consumption, A the household's wealth from owning all of the firms in the economy, r the real interest rate, w the real wage, and $\beta \in (0,1)$ the discount factor. The household's Euler equation links along a balanced growth path the equilibrium growth rate and interest rate to the discount factor:

$$1 + g = \beta(1 + r). \tag{1}$$

Final goods production Final output Y is a Cobb-Douglas bundle of a unit interval of intermediate goods with qualities $q(i)$:

$$Y = \exp\left(\int_0^1 \log\left(q(i)y(i)\right) di\right).$$

Final output can be used for consumption C or research expenditures R.

We assume the final goods market is competitive. So final output production can be characterized as the behavior of a representative firm solving

$$\max_{\{y(i)\}_{i=0}^1} P \cdot \exp\left(\int_0^1 \log\left(q(i)y(i)\right) di\right) - \int_0^1 p(i)y(i)\, di, \tag{2}$$

taking prices P and $p(i)$ as given. Due to the Cobb-Douglas structure, the solution to the final goods producer's problem features equal spending on each product

$$p(i)y(i) = PY. \tag{3}$$

In the following, we normalize the price of the final good $P = \exp\left(\int_0^1 \log\left(p(i)/q(i)\right) di\right)$ to 1 in all periods.

Intermediate goods production The intermediate goods are produced by a large fixed J number of firms indexed by $j \in \{1, \cdots, J\}$. Each firm j has a blueprint to produce each variety i with quality $q(i,j)$. The firms needs one unit of production labor to produce one unit of the variety. More specifically,

firm j can produce one unit of good i at quality $q(i,j)$ with technology $y(i,j) = l(i,j)$ where $l(i,j)$ is the labor input of firm j into producing good i.

The quality levels at which a firm produces change endogenously over time as a result of R&D activity. Each firm has access to a R&D technology with heterogeneous and stochastic step sizes and R&D efficiency. That is, a firm j can randomly draw x_j number of lines to innovate upon by spending

$$R_{jt} = \phi_{jt} x_j^{\nu} Y \tag{4}$$

units of final output. ϕ_{jt} captures the R&D cost of firm j at time t. $1/\phi_{jt}$ is the R&D efficiency of the firm. For the lines drawn by firm j in period t, the firms can improve the highest quality in the line by a factor of $\gamma_{jt} > 1$. The firms can also lose products at rate $\bar{x}$ that is endogenous but is taken as given by the firms.

The step size and R&D cost of a firm (γ_{jt}, ϕ_{jt}) is stochastic and follows a N-state Markov chain. It can take on N values: $(\gamma_1, \phi_2), ..., (\gamma_N, \phi_N)$. The transition probability is given by

$$M = \begin{bmatrix} m_{11} & m_{12} & \cdots & m_{1N} \\ \vdots & \vdots & \vdots & \vdots \\ m_{N1} & m_{N2} & \cdot & m_{NN} \end{bmatrix}, \tag{5}$$

where $m_{kk'}$ denotes the probability of a firm with step size γ_k and R&D cost ϕ_k in t moving to step size $\gamma_{k'}$ and R&D cost $\phi_{k'}$ in $t+1$. The Markov chain is ergodic.

We assume the intermediate inputs market features Bertrand competition. Under this, the producer in each line is the highest quality producer who sets the price to the quality-adjusted marginal cost of the second-highest quality producer $p(i) = \frac{w}{q(i,j'(i))}$ while its own marginal cost is $\frac{w}{q(i,j(i))}$. Hence, markup in each line is determined by the relative quality of the best and second best

producers

$$\mu(i) = \frac{q(i, j(i))}{q(i, j'(i))}. \tag{6}$$

Since the highest-quality producer is the firm that last innovated on the line, the markup in a line is the quality step size of the best producer *at the time it acquired the line*. This is in general different from the current step size of the producer. Going forward, we will use $\gamma(i)$ to denote the markup on the line i. Thus the profit of the firm producing in line i relative to aggregate output Y is

$$\pi(\gamma(i)) = \left(1 - \frac{1}{\gamma(i)}\right). \tag{7}$$

Let $\mathcal{N}_j$ be the portfolio of lines where firm j is the highest-quality producer. The measure of $\mathcal{N}_j$ is n_j. Firm j's total earnings before expensing R&D relative to output Y in a period is

$$\int_{i \in \mathcal{N}_j} \pi(\gamma(i))\, di.$$

Since there are N types of step size, we can write the earnings before R&D as

$$\tilde{E}(\{\omega_k^j\}, n_j) = n_j \sum_k \omega_k^j \pi(\gamma_k), \tag{8}$$

where ω_k^j is the share of firm j's portfolio where the firm had step size γ_k when it acquired the line and n_j is the number of lines in the portfolio. Earnings after R&D is (again expressed relative to aggregate output)

$$E(\{\omega_k^j\}, n_j, x_j, \phi_j) = \tilde{E}(\{\omega_k^j\}, n_j) - \phi_j x_j^{\nu}. \tag{9}$$

In each period, the firm sees its current step size γ_{jt} and R&D cost ϕ_{jt} then chooses the number of lines to draw x_j to maximize the present discounted value of the firm. The value of the firm (before dividend) is the present

discounted value of its earnings stream

$$V_{j,t} = \max_{\{x_{j,t+\tau}, n_{j,t+\tau+1}\}_{\tau=0}^{\infty}} \Big\{ E(\{\omega_{k,t}^{j}\}, n_{j,t}, x_{j,t}) Y_t \tag{10}$$

$$+ \sum_{\tau=1}^{\infty} \left(\prod_{s=1}^{\tau} \frac{1}{1+r_{t+s}} \right) E(\{\omega_{k,t+\tau}^{j}\}, n_{j,t+\tau}, x_{j,t+\tau}) Y_{t+\tau} \Big\}, \tag{11}$$

subject to

$$n_{j,\tau+1} = n_{j,\tau}(1-\bar{x}) + x_{j,\tau}, \ \forall \tau.$$

The earnings stream in turn depends on the history of the firm's step size and R&D cost draws, on how fast the firm loses its products to creative destruction, and on its innovation decisions.

$$E(\{\omega_{k,t+1}^{j}\}, n_{j,t+1}, x_{j,t+1}) = \tilde{E}(\{\omega_{kt}^{j}\}, n_{jt})(1-\bar{x}_t) + x_{jt}\pi(\gamma_{jt}) - \phi x_{j,t+1}^{\nu}. \tag{12}$$

We consider a balanced growth path with a stationary equilibrium on which the interest rate r, the rate of creative destruction $\bar{x}$ and the growth rate of $g_t = Y_t/Y_{t-1} - 1 = g$ are constant $\forall t$. Then the problem of the firm simplifies to solving

$$V(\{\omega_{kt}^{j}\}, n_{jt}, \gamma_{jt}, \phi_{jt}) = \tilde{E}(\{\omega_{kt}^{j}\}, n_{jt}) \frac{1+r}{1+r-(1+g)(1-\bar{x})} \tag{13}$$

$$+ \max_{\{x_{j,t+\tau}\}_{\tau=0}^{\infty}} \mathbb{E}_{\gamma_{jt},\phi_{jt}} \left\{ \sum_{\tau=0}^{\infty} \left(\frac{1+g}{1+r} \right)^{\tau} \left(x_{j,t+\tau} v(\gamma_{j,t+\tau}) \frac{1+g}{1+r} - \phi_{j,t+\tau} x_{j,t+\tau}^{\nu} \right) \right\}.$$

The first line is firm value from the portfolio of products that the firm acquired from past innovation. The second line is firm value from expect earnings from future innovation. Here $v(\gamma)$ denotes the present discounted value relative to output Y of a line with step size γ between the best and second best producer

$$v(\gamma(i)) = \pi(\gamma(i)) \frac{1+r}{1+r-(1+g)(1-\bar{x})}. \tag{14}$$

From the above problem, the optimal choice of x_{jt} only depends on γ_{jt} and

ϕ_{jt}. Hence we can write the R&D decision as choosing a policy function $x(\gamma,\phi)$ that solves

$$\max_{x\geq 0}\left\{xv(\gamma)\frac{1+g}{1+r}-\phi x^{\nu}\right\}. \tag{15}$$

The solution to this is

$$x(\gamma,\phi)=\left(\frac{1+g}{1+r}v(\gamma)\frac{1}{\nu\phi}\right)^{1/(\nu-1)}, \tag{16}$$

which says that firms with higher current step sizes and higher R&D efficiency (lower ϕ) draw more lines because it has a higher markup on any line it acquires or the cost of R&D is lower. The R&D spending is

$$\phi x(\gamma,\phi)^{\nu}=\left(\frac{1}{\phi}\right)^{\frac{1}{\nu-1}}\left(\frac{1+g}{1+r}v(\gamma)\frac{1}{\nu}\right)^{\nu/(\nu-1)} \tag{17}$$

which increases with R&D efficiency and step size. The additional firm value generated by today's R&D net of the R&D cost is

$$\left(\frac{1+g}{1+r}v(\gamma)\right)^{\nu/(\nu-1)}\cdot\left(1-\frac{1}{\nu}\right)\left(\frac{1}{\nu\phi}\right)^{1/(\nu-1)} \tag{18}$$

Hence, firms with higher step sizes and R&D efficiency spend more on R&D and add more value.

Substitute the optimal R&D decision into the firm's value function yields the value of firm

$$\begin{aligned} V\left(\{\omega_{kt}^{j}\},n_{jt},\gamma_{jt},\phi_{jt}\right) &= \tilde{E}(\{\omega_{kt}^{j}\},n_{jt})\frac{1+r}{1+r-(1+g)(1-\bar{x})} \\ &+ \left(1-\frac{1}{\nu}\right)\cdot\mathbb{E}_{\gamma_{jt},\phi_{jt}}\sum_{\tau=0}^{\infty}\left(\frac{1+g}{1+r}\right)^{\tau}\left(\frac{1+g}{1+r}v(\gamma_{j,t+\tau})\right)^{\nu/(\nu-1)}\left(\frac{1}{\nu\phi_{j,t+\tau}}\right)^{1/(\nu-1)}. \end{aligned} \tag{19}$$

The ratio of price to earnings before R&D is then

$$\begin{aligned}\frac{V(\{\omega_{kt}^j\},n_{jt},\gamma_{jt},\phi_{jt})}{\tilde{E}(\{\omega_{kt}^j\},n_{jt})} &= \frac{1+r}{1+r-(1+g)(1-\bar{x})} \\ &+ \left(1-\frac{1}{\nu}\right)\cdot\frac{\left(\frac{1+g}{1+r}v(\gamma_{j,t})\right)^{\nu/(\nu-1)}\left(\frac{1}{\nu\phi_{j,t}}\right)^{1/(\nu-1)}}{\tilde{E}(\{\omega_{kt}^j\},n_{jt})} \\ &+ \left(1-\frac{1}{\nu}\right)\cdot\mathbb{E}_{\gamma_{jt},\phi_{jt}}\frac{\sum_{\tau=1}^{\infty}\left(\frac{1+g}{1+r}\right)^{\tau}\left(\frac{1+g}{1+r}v(\gamma_{j,t+\tau})\right)^{\nu/(\nu-1)}\left(\frac{1}{\nu\phi_{j,t+\tau}}\right)^{1/(\nu-1)}}{\tilde{E}(\{\omega_{kt}^j\},n_{jt})}.\end{aligned}$$

The first term does not depend on firm size because all firms face the same rate of creative destruction. The second term depends on the current step size and R&D efficiency of the firm. Firms with higher step size and R&D efficiency add more value in the next period and raise the value of the firm relative to current earnings. The third term depends on the stochastic process of the step size and R&D efficiency. For example, when step sizes and R&D efficiency are iid, expected future earnings is the same across firms and hence this second term declines with today's earnings. On the other hand, the relationship is ambiguous if step size and/or R&D efficiency are positively correlated.

Market clearing and aggregates Since we have a unit measure of lines, the aggregate rate of creative destruction is equal to

$$\bar{x}=\sum_{j=1}^{J}x(\gamma_{jt},\phi_{jt})=\sum_{k=1}^{N}Js_kx(\gamma_k,\phi_k), \tag{20}$$

where s_k is the stationary share of firms (γ_k,ϕ_k) according the the Markov transition probabilities M. It is the share of firms with step size γ_k and R&D cost ϕ_k in each period.

The equilibrium growth rate of aggregate output (holding fixed aggregate labor input) is

$$1+g=\prod_{k=1}^{N}\left(\gamma_k,\phi_k\right)^{x(\gamma_k)\cdot s_kJ}. \tag{21}$$

Equations (20), (21), and the household's Euler equation (1) jointly pins down equilibrium $\bar{x}$, r and g.

We also have the labor market clearing condition

$$\int_0^1 l(i,j)di = L,$$

the asset market clearing condition

$$\sum_{j=1}^{J} V_{j,t} = A_t$$

and goods market clearing

$$\sum_j R_j = R = Y - C. \tag{22}$$

Finally, we have the number of lines operated by the firms summing to 1

$$1 = \sum_{j=1}^{J} n_j.$$

Equilibrium definition, BGP definition A decentralized equilibrium consists of a sequence of quantities and prices that jointly solve the problems of the final producer, the intermediate producer, the household and is consistent with the market clearing conditions and the aggregate constraints. As mentioned above, we define a balanced growth path (BGP) in the standard way, i.e., as a path along which all quantities grow at constant rates.

4 Calibration and consequences

In this section, we calibrate the model and calculated the growth contribution of firms. To keep the model parsimonious we approximate the Markov chain in the model with a VAR(1) process using the method described in Gospodinov

and Lkhagvasuren (2014). Namely, we calibrate parameters of the process

$$\begin{pmatrix} \log(\gamma_t - 1) \\ \log \phi_t \end{pmatrix} = \begin{pmatrix} \rho_\gamma \log(\gamma_{t-1} - 1) \\ \rho_\phi \log \phi_{t-1} \end{pmatrix} + \begin{pmatrix} \varepsilon_{\gamma,t} \\ \varepsilon_{\phi,t} \end{pmatrix}$$

$$\begin{pmatrix} \epsilon_{\gamma,t} \\ \epsilon_{\phi,t} \end{pmatrix} \overset{iid}{\sim} N \left(\begin{pmatrix} \mu_\gamma \\ \mu_\phi \end{pmatrix}, \begin{bmatrix} \sigma_\gamma^2 & \sigma_{\gamma,\phi} \\ \sigma_{\gamma,\phi} & \sigma_\phi^2 \end{bmatrix} \right).$$

We use this method to generate $N = 625$ discrete states and the associated transition matrix for the Markov process of step size and R&D cost.

We calibrate 9 parameters to 9 moments. The parameters are the curvature ν of the R&D cost function, the drift (μ_γ, μ_ϕ), persistence (ρ_γ, ρ_ϕ) and volatility $(\sigma_\gamma, \sigma_\phi, \sigma_{\gamma,\phi})$ of the step size and R&D cost VAR(1) process, and the discount factor β. The nine moments are the growth of aggregate earnings net of labor input growth and growth in the number of firms, the average stochastic discount factor from David et al. (2022), the contribution of earnings-to-sales ratio to the relationship between the average earnings growth over the next 5 years and current $P/(E + R\&D)$, the contribution of earnings before R&D to sales ratio to the relationship between 5 year earnings growth and $P/(E + R\&D)$ aggregate earnings to aggregate sales ratio, the dispersion of predicted average 5 year growth in earnings and pre-R&D earnings, the relationship between R&D intensity and sales, and sales dispersion.

The aggregate earnings/sales and growth in aggregate earnings together are informative of the drift μ_γ and μ_ϕ. Given the aggregate growth moment, the discount rate pins down β. The dispersion in pre-R&D earnings growth is informative of the volatility parameter σ_γ while the dispersion of sales is informative of σ_ϕ. The growth of net earnings is less dispersed than the growth of gross earnings, indicating that high step draws incur higher R&D cost and hence a positive covariance between ϕ and γ. The contribution of

earnings/sales is informative of the curvature parameter. When curvature is higher, firms R&D choice and hence the number of products they innovate upon vary less with the step size and R&D cost draws. As a result, a firm's earnings growth comes less from variation in the number of product lines it adds and more from growth in earnings/sales via different step sizes or R&D cost. The contribution of pre-R&D earnings to sales ratio is informative of the persistence of step size draws ρ_γ. The R&D intensity relationship with sales is informative of the persistence ρ_ϕ. All else equal, higher ρ_ϕ generates more dispersion in sales and a flatter relationship between R&D/sales with sales.

Table 3 displays the target and data moments while Table 4 displays the calibrated parameters. The overall fit is good, although we generate less aggregate earnings/sales and sales dispersion than in the data. The implied cost weighted price-cost markup is 1.22. We find that the step size is persistent with $\rho_\gamma = 0.93$ while the R&D cost is also persistent with $\rho_\phi = 0.82$. Step size and R&D cost have a correlation of 0.54. The model needs high variance of R&D cost generate the empirical sales dispersion.

4.1 Firm's contribution to growth

Using the calibrated model, we can calculate firms contribution to growth. For a group of firms I, contribution to growth is

$$m(I) \cdot \frac{\sum_k x(\gamma_k, \phi_k) \cdot s_k^i \cdot \ln(\gamma_k)}{\sum_k x(\gamma_k, \phi_k) \cdot s_k \cdot \ln(\gamma_k)}$$

where s_k^i is the share for firms with step size k and $m(I)$ is the fraction of firms in I. The so-called Gibrat's Law refers to the special case when contribution to growth is equal to the sales share of the firms. In contrast, when step sizes are independent, the contribution is 1% for all size percentiles.

Using this metric, we find that large firms (higher sales) do contribute more to growth but not as much as what Gibrat's Law predicts. Figure 7 ranks the firms by sales and show the contribution of firms in each decile of firms. The

Table 3: Calibration data vs. model

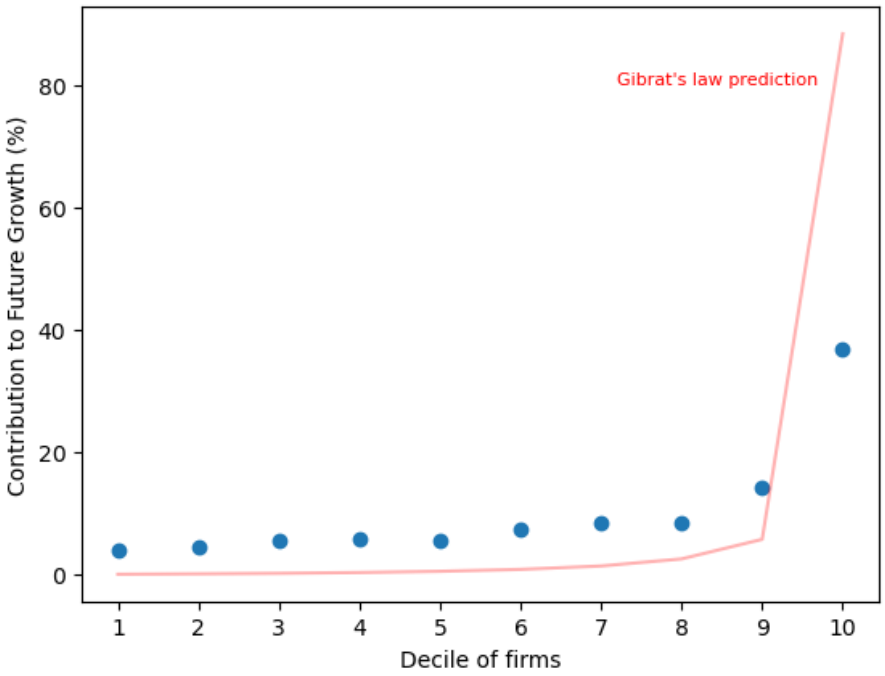
Targets	Data	Model
1. Growth in aggregate earnings, net of labor input growth, $g^\star$	1.3%	1.5%
2. Average stochastic discount factor, $r^\star$	9.7%	9.9%
3. Contribution of $g_{E/S}$ to g_E and $P/(E+R)$ relationship	44%	42%
4. Contribution of $g_{(E+R\&D)/S}$ to $g_{E/S}$ and $P/(E+R)$ relationship	90%	94%
5. Aggregate earnings/aggregate sales	0.28	0.18
6. 90-10 range of fitted g_E	0.09	0.08
7. 90-10 range of fitted $g_{E+R\&D}$	0.15	0.13
8. R&D/sales projected on log sales	-0.028	-0.032
9. 90:50 sales ratio	17.6	14.5

Source: 1. earnings from I/B/E/S and labor input growth from the BLS. 2. average of firm-specific discount rate from David et al. (2022). 3 to 9 are from CRSP, I/B/E/S, and Compustat.

Table 4: Calibrated parameters

Parameter		Value
Curvature of R&D cost function	ν	3.57
Persistence of γ process	ρ_γ	0.93
Mean of shocks to γ process	μ_γ	-0.05
Variance of shocks to γ process	σ^2_γ	1.14
Persistence of ϕ process	ρ_ϕ	0.82
Mean of shocks to ϕ process	μ_ϕ	4.64
Variance of shocks to ϕ process	σ^2_ϕ	41.2
Covariance between shocks	$\sigma_{\gamma\phi}$	3.73
Discount factor	β	0.92

Figure 7: Contribution to growth by firm decile

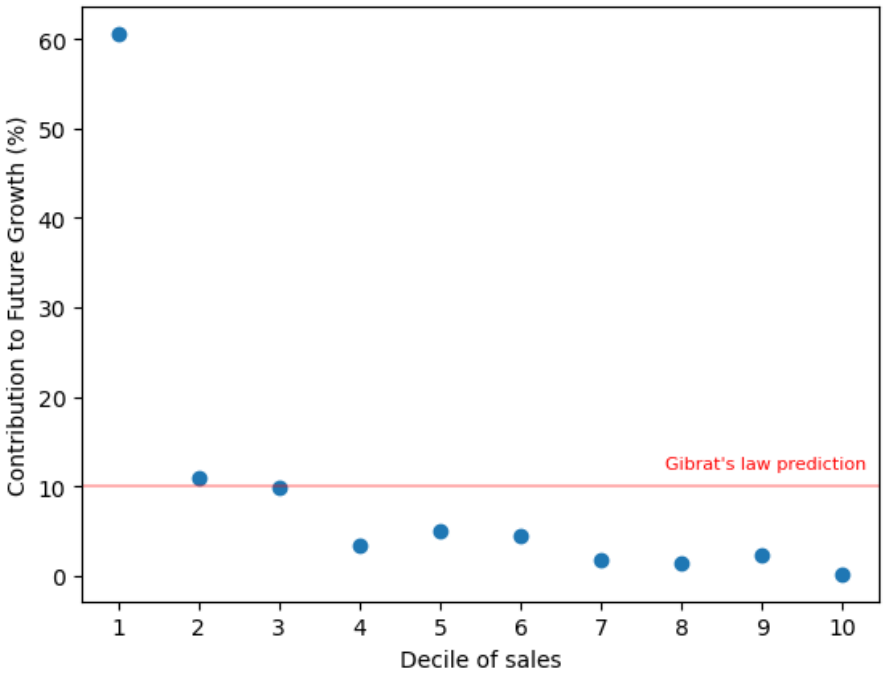


largest 10% of firms contribute to more than 10% of growth. This comes from the high persistence in the shocks. The contribution would be 10% if say the shocks are iid. However, the contribution is smaller than the contribution according to Gibrat's Law. Figure 8 shows this by plotting growth contribution by sales deciles. The largest firms that account for 10% of sales share contribute to much less than 10% of growth. While the smallest firms that account for 10% of sales contribute to 60% of growth.

5 Conclusion

This paper constructs an endogenous growth model with shocks to firm innovation step-size and R&D cost and calibrate it to match the relationship between P/E ratios, earnings, earnings growth, and earnings-to-sales growth across firms. Shocks to the step-size and R&D cost of firms generates

Figure 8: Contribution to growth by sales decile

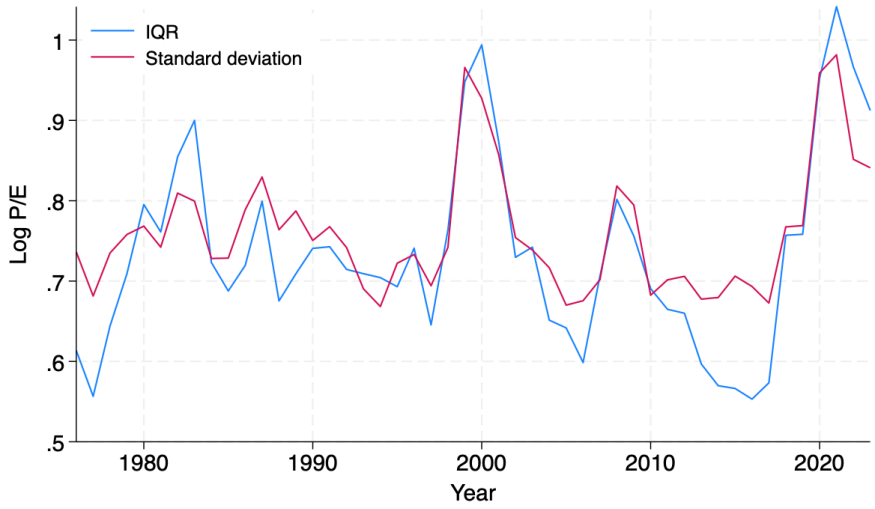


differences in future earnings and hence differences in valuation relative to current earnings. Our calibrate model implies significant deviation from the Gibrat's Law with the largest firms contributing less than their share of sales.

Our model suggests that the P/E ratio could be a good indicator of firm's contribution to growth. There are potential policy uses for real-time estimates of firm contributions to future productivity growth. Because innovations build on each other, large step-size firms confer larger knowledge spillovers. Aghion, Bergeaud, Boppart, Klenow and Li (2024) highlight potential growth gains from reallocation research effort from low to high step-size firms. If governments offer enhanced research subsidies, however, then firms might have an incentive to manipulate their market capitalization or earnings.

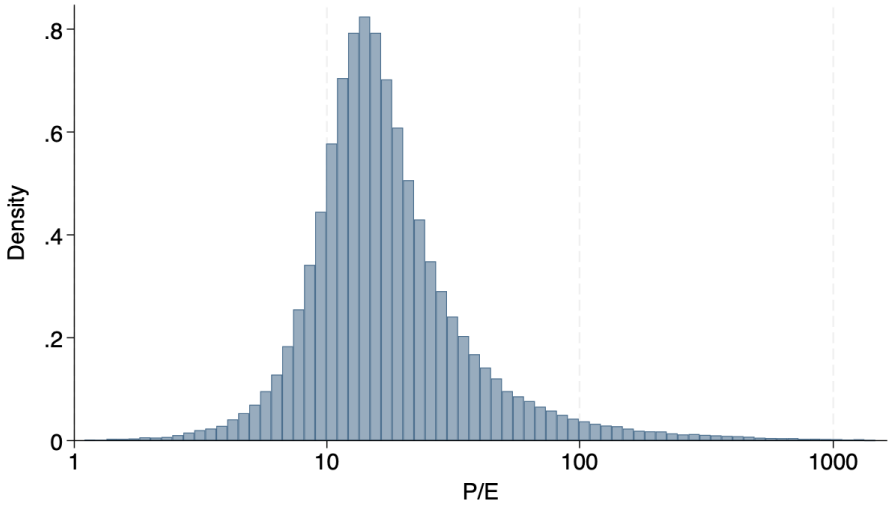
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Figure A1: Cross-firm dispersion of P/E ratios over time



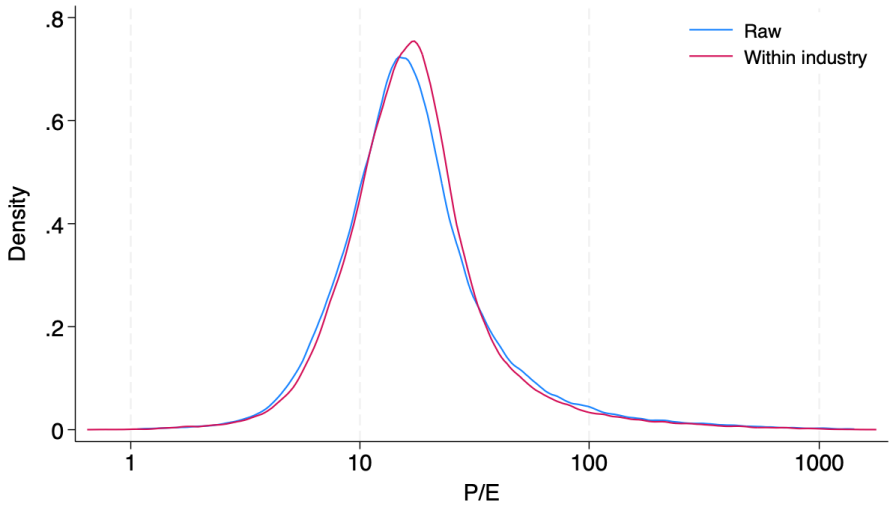
Source: CRSP and I/B/E/S.

Figure A2: Distribution of P/E, removing year and fiscal month FE



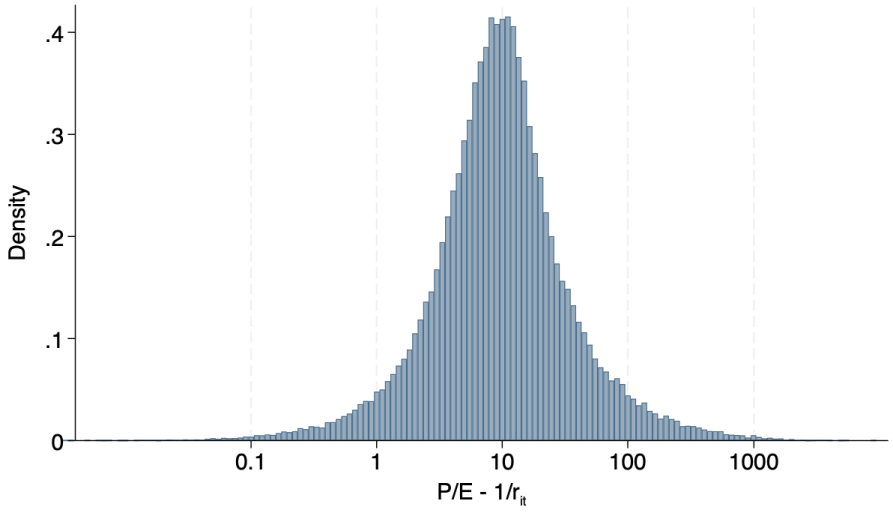
Source: CRSP and I/B/E/S.

Figure A3: Distribution of P/E, removing industry FE



Source: CRSP and I/B/E/S.

Figure A4: Distribution of P/E, removing firm-specific risk



Source: CRSP and I/B/E/S. r_{it} from David et al. (2022)

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