Risk-taking over the Life Cycle: Aggregate and Distributive Implications of Entrepreneurial Risk*

Dejanir H. Silva† Robert M. Townsend‡

October 2024

Abstract

We study the risk-taking behavior over the life cycle of entrepreneurs subject to partial insurance against idiosyncratic shocks. The model quantitatively accounts for the aggregate and idiosyncratic risk premia, the life-cycle profiles of consumption and risk-taking, and the patterns of wealth inequality observed in the data. A reform that relaxes the risk constraints reduces the idiosyncratic risk premium and induces an investment boom. Consistent with a Kuznets curve, inequality increases in the short run and declines in the long run. The initial generation of entrepreneurs benefits from better insurance, but future generations will be worse off after the reform.

KEYWORDS: Entrepreneurship, risk-taking, risk premium, insurance, inequality

JEL CLASSIFICATION: G11, G51, E44.

*We would like to thank Antonio Antunes, Francisco Buera, Pedro Brinca (discussant), Sylvain Catherine (discussant), David Lagakos, Juan Passadore, and seminar participants at Barcelona GSE Summer Forum, Econometric Society Meeting, FGV-EPGE, LACEA-LAMES, Lubramacro, SED, UFABC, WFA, and Yale for their helpful comments and suggestions.

†Purdue University, Krannert School of Management. Email: dejanir@purdue.edu.

‡Massachusetts Institute of Technology, Department of Economics. Email: rtownsen@mit.edu.

1 Introduction

Entrepreneurship is inherently a risky activity. Entrepreneurial risk is particularly important in developing economies, where the bulk of production occurs in privately owned businesses, and risk-sharing opportunities are limited. Given that entrepreneurs hold under-diversified portfolios, such risks have far-reaching implications.[1] At the individual level, entrepreneurial risk distorts investment and savings decisions. The importance of business income is highly heterogeneous and varies substantially over the life cycle, shaping wealth inequality patterns. At the aggregate level, imperfect insurance leads to an inefficient risk premium, which depresses the capital stock and hinders economic development. The pervasive effects of limited risk-sharing highlight the relevance of policy interventions that alleviate the consequences of entrepreneurs' lack of diversification.

This paper studies the aggregate and distributive implications of entrepreneurial risk in a quantitative life-cycle model with imperfect idiosyncratic insurance. We discipline the model using a rich dataset on small business in Thailand, which includes information on the returns of entrepreneurial activity and entrepreneurs' risk-taking and consumption.[2] The model captures several salient features of the data, such as the risk premium on business returns, the life-cycle profile of risk-taking and consumption, and the patterns of inequality between- and within-age groups. Combining a theory of the idiosyncratic risk premium with detailed information on portfolios and business returns, we assess the limits to risk sharing faced by entrepreneurs and, ultimately, the impact of relaxing these constraints on investment, inequality, and economic development.

We start by studying empirically the determinants of expected entrepreneurial returns. Expected returns are tightly connected to the marginal product of capital (MPK), so the required return on a project is potentially a key determinant of its scale.[3] Imperfect risk sharing implies that entrepreneurs require compensation for holding idiosyncratic risk. In contrast, expected returns should depend only on the exposure to aggregate risk under perfect risk sharing. To assess the importance of risk-sharing frictions, we consider a parsimonious two-factor model where expected returns depend on the exposure to aggregate and idiosyncratic risk. We find that entrepreneurs with higher exposure to either source of risk receive higher expected returns. More importantly, and perhaps surprisingly, a two-factor model explains most of the observed variation in entrepreneurial re-

[1] For evidence on the under-diversification of entrepreneurs see, e.g., Moskowitz and Vissing-Jørgensen (2002), and Herranz et al. (2009) for a study focused on small businesses.

[2] A large literature studies the macroeconomic and asset-pricing implications of firm-level risk (see, e.g., Christiano et al. 2014, Gârleanu et al. 2015, Dou 2016, and Herskovic et al. 2018), but these studies usually rely on data of public companies to discipline their quantitative exercises.

[3] See David et al. (2022) on how differences in (aggregate) risk premium lead to dispersion in MPK.

turns. Differences in idiosyncratic volatility and exposure to aggregate risk explain almost 70% of the cross-sectional variation in average returns. Given idiosyncratic volatility and aggregate betas are measured with error, this captures only a lower bound on how much these two factors explain the variation in returns.[4] Therefore, a risk-based explanation can account for nearly all cross-section differences in expected entrepreneurial returns.

While more than 90% of the variance is explained by idiosyncratic shocks, less than half of expected returns are compensation for such shocks. Sharpe ratios then vary by source of risk. This is inconsistent with an autarky allocation, where only total variance would matter, and also inconsistent with perfect risk sharing, as the idiosyncratic risk is priced. This evidence suggests entrepreneurs partially insure idiosyncratic shocks.

Next, we consider the determinants of entrepreneurs' risk-taking decisions, as measured by the share of net worth invested in the business. Risk-taking depends not only on the project's risk and return but also on attitudes towards risk. Households' characteristics potentially shape their risk preferences. In particular, we find substantial variation in risk-taking over the life cycle. Young entrepreneurs are nearly 40% more exposed to the business than old entrepreneurs. Differences in risk-taking cannot be explained by differences in expected returns, suggesting heterogeneity in risk tolerance across age groups.

To capture these motivating facts, we propose a general equilibrium model with two main ingredients: limited idiosyncratic insurance and finite lives with imperfect altruism. Entrepreneurs operate a technology exposed to aggregate and idiosyncratic shocks. Due to a moral hazard problem, entrepreneurs must bear a fraction of the idiosyncratic risk in equilibrium. The moral hazard parameter controls the degree of partial insurance. We identify the moral hazard parameter by matching the decomposition of expected returns into aggregate and idiosyncratic components performed in the data. Intuitively, if entrepreneurs can diversify a significant fraction of the risk, expected returns would reflect primarily compensation for holding aggregate risk. We find that entrepreneurs can diversify roughly 40% of the idiosyncratic volatility. Therefore, entrepreneurs face relevant limitations in diversifying risks, significantly affecting their investment decisions.

Importantly, we assume that entrepreneurial households also receive some labor income from household members working outside the business, consistent with what we observe in the data. This implies that entrepreneurs' effective risk aversion depends inversely on the ratio of human wealth (present discounted value of future labor income) to financial wealth (investment in safe assets and the business). The human-financial wealth ratio declines over the life cycle in the data. This mechanism endogenously creates het-

[4]When we sort entrepreneurs by their exposure to aggregate and idiosyncratic risk, as is standard in empirical asset pricing, we obtain that nearly 90% of the variation can be explained by these two factors.

erogeneity in risk aversion, which is essential to replicate the empirical life-cycle profiles.[5]

Entrepreneurial risk has a significant effect on inequality. On average, wealth inequality initially increases with age and then declines later in life. Heterogeneity in returns drives the initial increase in inequality, while an increase in the marginal propensity to consume (MPC) as entrepreneurs approach the end of the life cycle brings inequality down. The model roughly captures the wealth inequality patterns observed in the data.

The model is also able to generate the heterogeneity in risk premia found in the data. To capture heterogeneity in expected returns, we extend the model in two dimensions. We introduce differences in idiosyncratic volatility and decreasing returns to scale (DRS) in production. DRS is essential to generate differences in expected returns. Without DRS, the capital-labor ratio is equalized across entrepreneurs, and so it is expected returns.

Generating the patterns of expected returns with standard ingredients, such as skill heterogeneity and collateral constraints, can be challenging. A version of the model with skill heterogeneity, but no differences in idiosyncratic risk, is unable to generate heterogeneity in expected returns. More productive entrepreneurs end up with larger business, while expected returns are determined by the risk exposure. Introducing collateral constraints does sever the link between risk and return, so expected returns are instead driven by differences in net worth for constrained entrepreneurs. However, we do not find a significant relationship between expected returns and net worth empirically. Risk exposure actually explains a larger fraction of the variation in expected returns for entrepreneurs with relatively low net worth, who are more likely to be constrained. In combination, these results point towards a risk-based explanation of entrepreneurial returns.

Having shown the model captures the salient features of our data, we study a counterfactual where entrepreneurs have access to better idiosyncratic risk. Improving risk sharing leads to a decline in the idiosyncratic risk premium and an increase in the capital stock in the long run. The effect is quantitatively significant, where a reform that reduces the risk premium by 140 basis points leads to an increase of the capital stock of 13%. Financial development, captured here by the extent of idiosyncratic risk sharing, is then an important determinant of the level of economic development.[6]

We also consider the dynamic implications of relaxing risk constraints. Improving idiosyncratic insurance leads to an investment boom that lasts for a decade, accompanied by a sharp increase in the value of private businesses in the short run. Inequality falls in the long run, as entrepreneurs are less exposed to risk after the reform. In contrast,

[5]The mechanism is reminiscent of work on portfolio choice with labor income (see, e.g., Bodie et al. 1992, Heaton and Lucas 1997, and Viceira 2001).

[6]See Levine (2005) for a review of the connections between financial and economic development.

inequality increases in the short run due to a revaluation effect. The increase in the value of the business benefits entrepreneurs with larger initial investments, which are relatively richer before the reform. It takes a long time for inequality to converge to its new long-run level due to intergenerational links. This pattern is consistent with Kuznets's (1955) hypothesis over the relationship between inequality and economic development.[7]

Considering the transitional dynamics is relevant to assess the welfare implications. Entrepreneurs are worse off in the long run after the reform, despite the benefits of better diversification, as entrepreneurs accumulate less wealth over time. In contrast, the initial generation's welfare improves with the reform. They received their bequest before the intervention, and the value of their businesses increased substantially in the short run. Therefore, most of the gains of the reform are reaped by the initial generation of entrepreneurs and wage earners, who receive higher wages given the higher capital stock.

Related literature. This paper relates to several strands of literature in macroeconomics and finance. First, the work studying how firm-level uncertainty affects asset prices and the real economy (Herskovic et al. 2016, Iachan et al. 2021).[8] While this literature focuses primarily on business-cycle fluctuations, we study how firm-level uncertainty affects the economy in the long run. Second, the work on how the entrepreneurs' lack of diversification affects real investment (Panousi and Papanikolaou 2012), capital structure (Chen et al. 2010, Herranz et al. 2015), and risk-taking (Chen and Strebulaev 2019). This work is mainly in partial equilibrium and abstracts from the aggregate implications of entrepreneurial risk.

We also contribute to recent work on the importance of heterogeneous returns for inequality. This literature documents substantial heterogeneity in portfolio returns (Fagereng et al. 2019, Bach et al. 2020); it finds that private business wealth is one the main sources of wealth at the top (Smith et al. 2019, Smith et al. 2020), and that return heterogeneity is important to quantitatively match the observed levels of inequality (Gomez 2017, Hubmer et al. 2021).[9] We study how the idiosyncratic risk premium in private business affects inequality and how changes in this premium affect the dynamics of inequality. Greenwald et al. (2021) show that financial wealth inequality increases after a decline in interest rates. We find similar effects for reducing the idiosyncratic risk premium in the short run, but inequality actually falls in the long run.

[7] Greenwood and Jovanovic (1990) provides an early treatment of the Kuznets dynamics and its connection with financial development.

[8] A related literature studies the asset-pricing implications of labor income risk in infinite-horizon (e.g. Constantinides and Duffie 1996) and life-cycle models (e.g. Storesletten et al. 2007).

[9] See De Nardi and Fella (2017) for a survey of quantitative Bewley models of wealth inequality.

A related literature studies the extent to which households partially insure *labor income* shocks (Blundell et al. 2008 and Kaplan and Violante 2010). Krueger and Perri (2006) and Heathcote et al. (2014) show how the degree of partial insurance can be inferred with data on consumption and labor income. We focus on entrepreneurial risk and show that the degree of partial insurance can be identified the idiosyncratic risk premium.

An extensive micro-development literature studies risk sharing (Townsend 1994, Morduch 1995) and the risk and return of production activities in developing economies (Udry and Anagol 2006, De Mel et al. 2008). Karlan et al. (2014) conducted a randomized control trial extending credit and insurance to farmers and found that the lack of insurance is the binding constraint to investment. Their results are consistent with our findings that relaxing risk constraints significantly impact investment. The macro-development literature studies the aggregate implications of credit constraints (Buera and Shin 2013, Midrigan and Xu 2014, Moll 2014). Our approach is complementary to theirs, as we focus instead on the role of risk constraints. Our work is closer to the original model of uninsurable investment risk by Angeletos (2007), which we extend to allow for partial idiosyncratic insurance, a rich demographic structure, and aggregate risk. These extensions are crucial to capture the patterns of consumption and risk-taking observed in the micro-data and derive the dynamics of inequality in response to a relaxation of risk constraints.

2 Motivating evidence

In this section, we provide motivating evidence on entrepreneurial activity in the context of a developing country. First, we study the determinants of entrepreneurial returns. Second, we consider how risk-taking and consumption decisions evolve over the life cycle.

Data. We use data from the Townsend Thai Monthly Survey, an intensive monthly survey initiated in 1998 in four provinces of Thailand. Two provinces, Chachoengsao and Lopburi, are semi-urban in a more developed central region near the capital, Bangkok. The other two provinces are rural, Buriram and Srisaket, and are located in the less developed northeastern region by the border of Cambodia. In each province, the survey is conducted in four villages, chosen at random within a given township. A detailed discussion of the survey can be found in Samphantharak and Townsend (2010).

Our sample covers 710 households and 14 years of monthly data, starting in January 1999. These economies were subject to various aggregate and idiosyncratic shocks during this time. Rice cultivation is affected by seasonal variations in rainfall and temperature. Restrictions on exports to the EU affected shrimp ponds. Milk cows' productivity varies

substantially over time for a given animal and over the herd. The exposure to this rich set of shocks highlights the importance of entrepreneurial risk, enabling us to disentangle the role of aggregate and idiosyncratic shocks.

The data collected in the Townsend Thai Monthly Survey includes information on the net income generated by the business and household total assets and liabilities. The data is rich enough to construct a detailed balance sheet for these businesses. We can compute the return on assets (ROA), measured as net profits income over business assets. We also measure the fraction of entrepreneurs' wealth invested in the business and the fraction invested in safe (real or financial) assets, providing us with a measure of risk-taking. Finally, the data includes information on labor income and consumption. This allows us to characterize the savings behavior and the importance of non-business income for entrepreneurial households. See Appendix B for a detailed description of these variables.[10]

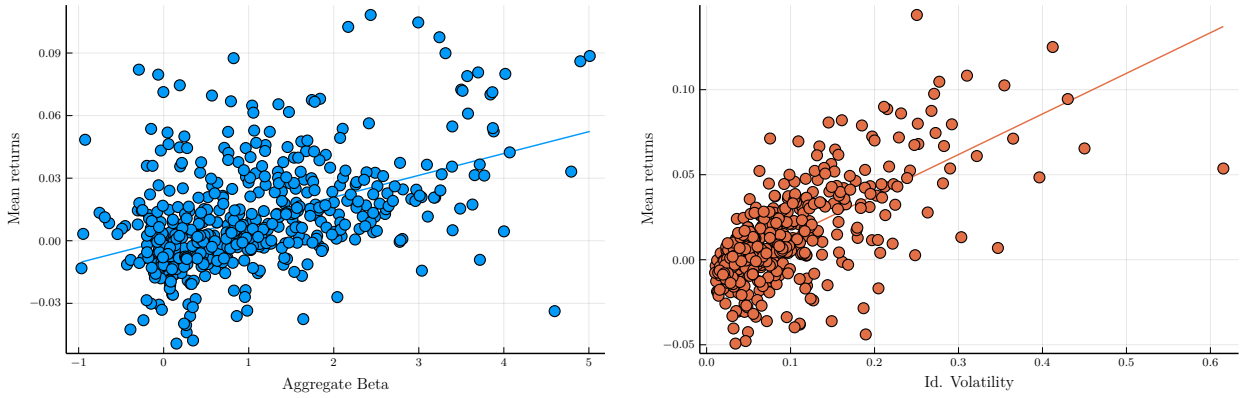
2.1 The determinants of expected entrepreneurial returns

Whether and how much to invest in a business depends on its risk/return trade-off. We study this trade-off by considering the required risk compensation implicit in entrepreneurial returns. Perfect risk sharing provides a natural benchmark. Under this assumption, differences in expected returns are entirely driven by differences in the exposure to aggregate risk, as idiosyncratic shocks can be perfectly insured or diversified. On the other extreme, entrepreneurs can be in financial autarky, without access to any insurance. In this case, only the total amount of risk is relevant to entrepreneurs, causing expected returns to vary with both aggregate and idiosyncratic volatility. When idiosyncratic shocks can be partially insured or diversified, expected returns depend not only on the total variance of returns but also on the relative importance of each source of risk.

This discussion motivates using a two-factor model to explain the cross-section of expected returns of private businesses based on the exposure to aggregate and idiosyncratic shocks. This approach allows us to evaluate whether perfect risk sharing or autarky holds and, more importantly, whether a risk-based explanation can quantitatively account for the differences in entrepreneurial returns observed in the data. We estimate our factor model following the standard two-pass regression methodology developed by Fama and MacBeth (1973). In the first stage, we estimate entrepreneurs' exposure to aggregate risk by the slope of a time-series regression of returns for entrepreneur i, $R_{i,t}$, on the cross-sectional average return across all entrepreneurs in a given province, R_t^{agg}. Formally, we

[10]The household is the unit of measurement, even though households typically consist of multiple members doing separate or partially overlapping activities. We treat the household as a whole as unitary (see, e.g., Doepke and Tertilt (2016) for a discussion of unitary models of the household).

Figure 1: Average returns vs. aggregate and idiosyncratic risk



Note: The left (right) panel shows a scatter plot of average time-series returns for each entrepreneur against aggregate beta (idiosyncratic volatility). Aggregate beta is measured as the slope of the time-series regression of individual returns on the leave-one-out average return in the entrepreneur's province. Idiosyncratic volatility is calculated as the volatility of residuals from the same regression. To limit the influence of outliers, we trim 1% of the observations in the left and right tails.

run the regression:

$$R_{i,t} = \alpha_i + \beta_i R_t^{agg} + \epsilon_{i,t},$$

for each entrepreneur $i \in \{1, \ldots, N\}$. Notice that R_t^{agg} plays the role of the market portfolio in standard tests of the capital asset pricing model (CAPM). By averaging across entrepreneurs in a given region, idiosyncratic risk gets diversified, so β_i captures the exposure to aggregate risk for entrepreneur i.[11] We measure the exposure to idiosyncratic risk by the variance of the residuals in the above regression, $\sigma_i^2 = var[\epsilon_{i,t}]$.

Figure 1 shows how the exposure to each source of risk is related to entrepreneurs' average return. We find a strong association between average time-series returns and exposure to aggregate and idiosyncratic risk. The left panel on Figure 1 shows that entrepreneurs more exposed to aggregate risk have higher average returns. Similarly, the right panel shows a positive and significant relationship between average returns and idiosyncratic volatility, consistent with entrepreneurs' limited access to insurance.

One possible concern is that idiosyncratic volatility may be correlated with the aggregate beta, e.g., riskier projects could be more exposed to both sources of risk. The positive association between idiosyncratic risk and expected returns could still be consistent with perfect risk sharing. To address this point, we run a second-stage cross-sectional regression of average return for entrepreneur i, $\overline{R}_i = \frac{1}{T}\sum_{t=1}^{T} R_{i,t}$, on the estimated exposure to

[11] Following Samphantharak and Townsend (2018), we adopt the province as the relevant geographic unit. Moreover, we compute a "leave-one-out" mean of the returns, so we regress the return of entrepreneur i on the average return across all entrepreneurs other than i on a given province.

aggregate risk, $\hat{\beta}_i$, and idiosyncratic risk $\hat{\sigma}_i^2$:

$$\overline{R}_i = \lambda_0 + \lambda_{ag}\hat{\beta}_i + \lambda_{id}\hat{\sigma}_i^2 + u_i.$$

λ_{ag} and λ_{id} correspond to the *price of aggregate and idiosyncratic risk*, respectively. They capture the required compensation for one unit of exposure to each source of risk.

Table 1 shows the results for the second-stage regression. We find a positive and significant price of risk for β_i and σ_i^2. These two factors account for a significant fraction of the cross-sectional variation in expected returns. Column 1 shows that the CAPM-inspired one-factor model explains a sizeable fraction of the variation in average portfolio returns, with an adjusted R^2 of 34%. Given the limited empirical success of the CAPM (see, e.g., Fama and French 1992), it is surprising that a single aggregate factor plays a significant role in explaining entrepreneurial returns. Column 2 shows that idiosyncratic risk explains an even larger fraction of the variation, with an adjusted R^2 of 58%. Together, these two factors explain most of the variation in expected returns, with an adjusted R^2 of 68%. To put this result in perspective, the R^2 for the three-factor Fama-French model in the cross-section of 25 equity portfolios sorted by size and book to market is 73%, comparable to the R^2 for our two-factor model in a cross-section of more than 500 entrepreneurs.

An important issue is that our risk exposure measures are noisy estimates of the actual beta and idiosyncratic volatility, so measurement error may bias our results. To deal with this issue, we embed our two-stage procedure into a generalized method of moments (GMM) framework and compute standard errors that account for the uncertainty in the estimation of $\hat{\beta}_i$ and $\hat{\sigma}_i^2$.[12] We also follow the standard procedure in empirical asset pricing (see, e.g., Black et al. 1972) and group entrepreneurs into bins, or portfolios, according to a 5×5 double sort based on aggregate beta and idiosyncratic risk.[13] The group-level betas and idiosyncratic variance are arguably better measured than their individual-level counterparts, which limits the impact of measurement error. Column 4 shows the results. We obtain a similar price of aggregate risk and a larger price of idiosyncratic risk, consistent with some attenuation bias in this case. The two risk factors account for 88% of the variation in group-level returns. Figure 2 shows the fit of the two-factor model by comparing predicted and realized average returns. The model successfully accounts for the significant variation in expected returns observed in our data. Both factors are necessary to achieve this success. We show in Appendix B.3 that a single-factor model, based on ei-

[12] See Cochrane (2009) for a discussion on how to correct the standard errors in two-pass regressions.

[13] Even though these portfolios are not tradeable, forming portfolios allows us to reduce the noise in the estimation and better capture the risk/return relationship. To avoid within-portfolio diversification, we assign to the portfolio the average idiosyncratic volatility instead of the volatility of the average.

Dependent Variable:	mean_roa			
Model:	(1)	(2)	(3)	(4)
Variables				
(Intercept)	-0.0071	0.0093***	-0.0026	-0.0036
	(0.0045)	(0.0015)	(0.0037)	(0.0023)
beta	0.0195***		0.0117***	0.0090**
	(0.0044)		(0.0039)	(0.0032)
sigma2		0.1246***	0.1031***	0.2127***
		(0.0280)	(0.0292)	(0.0718)
Fit statistics				
Observations	541	541	541	24
R^2	0.344	0.578	0.685	0.892
Adjusted R^2	0.343	0.577	0.684	0.882

*Signif. Codes: ***: 0.01, **: 0.05, *: 0.1*

Table 1: Estimation of two-factor model

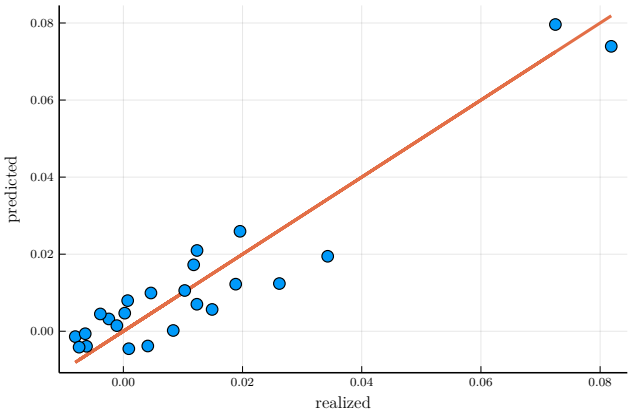


Figure 2: Realized vs. predicted returns

Note: The left panel shows a cross-sectional regression of average returns on the aggregate beta or idiosyncratic variance (or both). Standard errors account for the uncertainty in estimating the regressors, and they are robust to contemporaneous correlations between error terms across entrepreneurs. The right panel shows the 45° degree line and a scatter plot of the predicted returns of the two-factor model and actual average returns for the portfolio-level analysis.

ther an aggregate or idiosyncratic factor, performs substantially worse than the two-factor model. In particular, we can reject the hypothesis of a single-factor model based on either source of risk. Therefore, this evidence suggests that a two-factor model is necessary to explain the cross-section of entrepreneurial returns.

Risk and return decomposition. To better grasp the role of each risk factor, we provide a decomposition of entrepreneurial returns into an aggregate and idiosyncratic component. Table 2 decomposes the risk premium and variance of returns using the results from Table 1. Most risk is idiosyncratic, accounting for 94% of the variance, while idiosyncratic shocks account for slightly less than half of the risk premium. This implies that the Sharpe ratio of the aggregate component is nearly five times larger than the Sharpe ratio of the idiosyncratic component.[14]

The results from Table 2 are inconsistent with the perfect risk sharing and autarky benchmarks. The fact that the idiosyncratic risk premium explains almost half of total expected returns indicates that limits to diversification are substantial, which allows us to reject the perfect risk-sharing benchmark. Financial autarky is also inconsistent with the observed pattern in the Sharpe ratio. As shown in Appendix E.3, the share of variance of the idiosyncratic component should coincide with its share of expected returns under autarky. Equivalently, the Sharpe ratio should be proportional to the volatility.[15] This

[14] Samphantharak and Townsend (2018) computes a similar decomposition based on entrepreneurs' total portfolio returns, including risky and safe assets. The decomposition of the portfolio's risky component, which is our focus, is the relevant one to map into the structural model in Section 3.

[15] In a two-factor model, the risk premium is given by $p^{ag}\sigma_{ag} + p^{id}\sigma_I$, where (p^{ag}, p^{id}) is the return per

Table 2: Aggregate and idiosyncratic components of risk and return

	Risk premium	% of returns	Volatility	% of variance	Sharpe ratio
Total returns	4.4%	100%	21.0%	100%	0.21
Aggregate component	2.4%	54.7%	5.1%	6.0%	0.47
Idiosyncratic component	2.0%	45.3%	20.3%	94.0%	0.10

Note: The aggregate component is measured as the coefficient on the aggregate beta on the cross-sectional regression times the average beta across entrepreneurs. The idiosyncratic component is measured as the cross-sectional regression coefficient on idiosyncratic risk times the idiosyncratic variance averaged across entrepreneurs. The variance decomposition is computed at the individual level based on the results from the first-stage regression and then averaged across all entrepreneurs.

implies that the Sharpe ratio for the idiosyncratic component should be four times *larger* than the one for the aggregate component, but we observe the opposite pattern in the data.[16] These results suggest that partial insurance is relevant to explain the empirical patterns involving risk and return. We revisit this decomposition through the lenses of our structural model in Section 6.

2.2 Risk-taking and savings over the life cycle

Next, we consider entrepreneurs' risk-taking and savings decisions. The decision of how much to invest depends on the risk and return of the entrepreneurial activity and households' attitude towards risk. Behavior towards risk is potentially shaped by households' characteristics, such as age and household size. We study how risk-taking and consumption decisions depend on expected returns, age, and a range of demographic controls.

To ensure stationarity, we consider the behavior of the ratio of the value invested in the business to the entrepreneur's financial wealth, our measure of risk-taking, and the ratio of consumption to financial wealth. By looking at ratios instead of levels, we limit the influence of aggregate shocks and focus on potentially more stable relationships. We consider five age groups, ranging from 25 to 80 years old, where group 1 is the youngest and group 5 is the oldest. The cutoffs for each group are chosen such that we have roughly the same number of households in each group. We cluster standard errors by household and year to account for shocks potentially correlated over time or across households. We aggregate the data to the annual level to better isolate the age effects. To limit the influence of outliers, we trim the data at the 1% level at both tails. The details of the empirical methodology are described in Appendix B.2.

unit of risk (the prices of risk) and (σ_{ag}, σ_I) the volatilities. Perfect risk sharing corresponds to $p^{id} = 0$. In autarky, the prices of risk (or Sharpe ratio) are $p^{ag} = \gamma\sigma_{ag}$ and $p^{id} = \gamma\sigma_I$, so the risk premium is $\gamma(\sigma_{ag}^2 + \sigma_I^2)$.

[16] Financial autarky implies that the idiosyncratic Sharpe ratio relative to the aggregate Sharpe ratio should be $20.3/5.1 \approx 4.0$, while we observed a ratio of $0.10/0.47 \approx 0.21$, almost a twentyfold difference.

Table 3: Risk-taking and consumption behavior over the life cycle

	Risk-taking				Consumption-wealth ratio			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Age group: 1	0.30***	0.31***	0.30***	0.27***	0.14***	0.14***	0.12***	0.10***
	(0.01)	(0.01)	(0.02)	(0.02)	(0.009)	(0.008)	(0.008)	(0.007)
Age group: 2	0.27***	0.28***	0.27***	0.25***	0.13***	0.13***	0.12***	0.11***
	(0.01)	(0.01)	(0.01)	(0.01)	(0.009)	(0.007)	(0.007)	(0.007)
Age group: 3	0.27***	0.27***	0.26***	0.25***	0.12***	0.12***	0.11***	0.11***
	(0.01)	(0.01)	(0.01)	(0.01)	(0.008)	(0.007)	(0.008)	(0.007)
Age group: 4	0.23***	0.23***	0.23***	0.22***	0.10***	0.10***	0.09***	0.08***
	(0.02)	(0.01)	(0.01)	(0.01)	(0.007)	(0.006)	(0.006)	(0.006)
Age group: 5	0.22***	0.22***	0.23***	0.21***	0.11***	0.11***	0.10***	0.09***
	(0.01)	(0.01)	(0.01)	(0.01)	(0.008)	(0.008)	(0.008)	(0.007)
Year FE		Yes	Yes	Yes		Yes	Yes	Yes
Demographic controls			Yes	Yes			Yes	Yes
Risk and return controls				Yes				Yes
N	8,680	8,680	7,713	6,499	8,548	8,548	7,693	6,495
Adjusted R^2	0.015	0.025	0.107	0.139	0.010	0.020	0.062	0.070

Clustered (year & household) standard-errors in parentheses
*Signif. Codes: ***: 0.01, **: 0.05, *: 0.1*

Note: Age effects correspond to the conditional expectation of the outcome variable evaluated at the mean of the continuous controls and averaged over the year, province, and sector fixed-effects (when included). Demographic controls include dummies for the province, household size, the number of kids, and a set of sector affiliation dummies. Risk and return controls include the entrepreneur's average return on the business and the exposure to aggregate and idiosyncratic risk.

We compute age effects controlling for year fixed-effects and additional controls following the methodology of Kaplan (2012). Table 3 presents the results. We find that risk-taking has a strong life-cycle pattern. Column 1, the case without any additional controls, shows that the amount invested in the business declines sharply with age, with the coefficient for the oldest group being roughly 30% smaller than for the youngest group. A similar pattern holds as we add more controls. Time fixed-effects do not change the results, as shown in column 2, which suggests that aggregate shocks affect the amount invested in the business and financial wealth but leave their ratio roughly unchanged. Column 3 includes demographic and geographic controls, including the household size, the number of children, the province the household is located in, and her sector affiliation, with similar patterns. Column 4 adds the average return on the business and the exposure to aggregate and idiosyncratic risk as controls. Once again, we find substantial variation in risk-taking over the life cycle, indicating these patterns are driven by differences in attitudes toward risk instead of differences in risk or expected returns.

The consumption-wealth ratio also has a strong life-cycle pattern. However, instead of declining monotonically with age as our risk-taking measure, we find a U-shaped pattern:

consumption-wealth initially decreases with age but increases for the oldest group.

We derive three main conclusions from these results. First, the cross-sectional variation in risk-taking and consumption has a substantial life-cycle component. Second, the life-cycle patterns in risk-taking are mainly driven by differences in risk appetite instead of differences in risk or average return. Third, year fixed-effects do not significantly affect the life-cycle patterns. This suggests a relatively stationary environment despite the presence of aggregate shocks. These facts, in conjunction with the importance of partial insurance discussed in Section 2.1, motivate the ingredients of our theoretical model.

3 A life-cycle model of entrepreneurial risk taking

We consider a model of entrepreneurial activity with two main ingredients: i) imperfect idiosyncratic insurance and ii) finite lives with imperfect altruism. These two ingredients play a crucial role in capturing the empirical patterns from Section 2.

3.1 Environment

Time is continuous, and the economy is populated by two types of households: entrepreneurs and wage earners. Population grows at rate g, and the share of entrepreneurs in the population is constant and given by χ_e. The set of entrepreneurs and wage earners alive at period t are denoted by $\mathcal{E}_t$ and $\mathcal{W}_t$, respectively. Entrepreneurs live for T periods and leave bequests to their offspring. For simplicity, we assume that wage earners have an infinite horizon. While all households receive labor income, only entrepreneurs can access a production technology. Production is exposed to both aggregate and idiosyncratic shocks. Households buy and sell aggregate insurance in a frictionless market, but they have access only to imperfect idiosyncratic insurance. Households borrow and lend at a riskless rate r_t. We now describe in detail the technology, preferences, and financial frictions both types of households face.

Technology. Entrepreneur i combines capital $k_{i,t}$ and hired labor $l_{i,t}$ to produce a final homogeneous good $\tilde{y}_{i,t}$, the numeraire in this economy, using the technology:

$$\tilde{y}_{i,t} = A_t k_{i,t}^{\alpha} l_{i,t}^{1-\alpha}, \tag{1}$$

and we denote scaled output by $y_t = \tilde{y}_t / A_t$. We adopt this convention throughout the paper: variables that grow with aggregate productivity A_t are denoted with a tilde, and

the corresponding scaled variable is denoted without a tilde.

Productivity is subject to aggregate shocks and follows a geometric Brownian motion:

$$\frac{dA_t}{A_t} = \mu_A dt + \sigma_A dZ_t, \tag{2}$$

where Z_t is a standard Brownian motion.

Entrepreneurs can adjust their capital stock by investing in new capital or buying capital from another entrepreneur. The investment technology is risky and subject to adjustment costs. Given a total investment of $\iota_{i,t} A_t k_{i,t}$, capital evolves according to

$$\frac{dk_{i,t}}{k_{i,t}} = \left(\Phi(\iota_{i,t}) - \delta\right) dt + \sigma_I dZ_{i,t}. \tag{3}$$

$Z_{i,t}$ is an idiosyncratic Brownian motion for entrepreneur i, which is independent across entrepreneurs. The investment function $\Phi(\cdot)$ satisfies $\Phi(0) = 0$, $\Phi'(\cdot) > 0$, and $\Phi''(\cdot) < 0$. The concavity of $\Phi(\cdot)$ captures the presence of adjustment costs. Notably, investment is risky and subject to idiosyncratic shocks. Entrepreneurs can also adjust their capital stock by buying capital from other entrepreneurs at the price $\tilde{q}_t = q_t A_t$. The market value of the business is given by $\tilde{q}_t k_{i,t}$. In equilibrium, q_t will be non-stochastic, so the relative price of capital $\tilde{q}_t$ moves proportionally with aggregate productivity shocks.

The evolution of the aggregate capital stock, $k_t = \int_{\mathcal{E}_t} k_{i,t} di$, is not directly affected by idiosyncratic shocks, as these shocks get diversified in the aggregate:

$$dk_t = \left[\int_{\mathcal{E}_t} \left(\Phi(\iota_{i,t}) - \delta\right) k_{i,t} di\right] dt. \tag{4}$$

The return on investing in the project is the sum of the dividend yield and capital gains:

$$dR_{i,t} = \frac{\tilde{y}_{i,t} - \tilde{w}_t l_{i,t} - \iota_{i,t} A_t k_{i,t}}{\tilde{q}_t k_{i,t}} dt + \frac{d(\tilde{q}_t k_{i,t})}{\tilde{q}_t k_{i,t}} \equiv \mu^R_{i,t} dt + \sigma_A dZ_t + \sigma_I dZ_{i,t},$$

where $\tilde{w}_t = w_t A_t$ denotes the wage rate and $\mu^R_{i,t}$ is the expected return. Using Ito's lemma to compute expected capital gains, the expected return on the project is

$$\mu^R_{i,t} \equiv \underbrace{\frac{y_{i,t} - w_t l_{i,t} - \iota_{i,t} k_{i,t}}{q_t k_{i,t}}}_{\text{dividend yield}} + \underbrace{\frac{\dot{q}_t}{q_t} + \mu_A + \Phi(\iota_{i,t}) - \delta}_{\text{expected capital gains}}. \tag{5}$$

Preferences. Entrepreneurs live for T periods. They have isoelastic preferences over consumption $\tilde{c}_{i,t}$ with curvature parameter γ and derive utility of leaving bequests:

$$\mathbb{E}_{s_i}\left[\int_{s_i}^{s_i+T} e^{-\rho(t-s_i)}\frac{\tilde{c}_{i,t}^{1-\gamma}}{1-\gamma}dt + e^{-\rho(T-s_i)}(1-\psi)^{\gamma}V^{*}\frac{\tilde{n}_{i,s_i+T}^{1-\gamma}}{1-\gamma}\right], \tag{6}$$

$\tilde{n}_{i,t}$ denotes financial wealth (or net worth), and s_i denotes the entrepreneur's birthdate.

The parameter ψ measures the strength of the bequest motive. If $\psi = 1$, entrepreneurs give no weight to their offspring. If $\psi = 0$, entrepreneurs behave as if they have infinite horizon.[17] The case $0 < \psi < 1$ captures imperfect altruism.

Entrepreneurial households receive labor income in addition to business income, consistent with the observation that households have multiple sources of income in our data. Labor supply is denoted by $\bar{l}_{i,t}$, and it varies deterministically over the life cycle.[18]

Financial friction. Entrepreneurs face a moral hazard problem, similar to the one in He and Krishnamurthy (2012) and Di Tella (2017). We follow Di Tella (2017) and restrict attention to short-term contracts. Aggregate shocks are perfectly observable by all households, while idiosyncratic shocks are only observed by the entrepreneur. An entrepreneur can divert capital, but a fraction $1-\phi$ of the diverted capital is lost in the process. The parameter $\phi \in (0,1)$ controls the severity of the moral hazard problem.

Following the literature on dynamic moral hazard, the solution to the contracting problem can be implemented by a market structure where entrepreneurs have access to both aggregate and idiosyncratic insurance as well as a riskless asset with return r_t. There is no limit on aggregate insurance, as aggregate shocks are perfectly observable. The quantity of idiosyncratic insurance is limited for incentive purposes. Formally, entrepreneur i pays $p_t^{ag}\tilde{\theta}_{i,t}^{ag}$ to reduce aggregate volatility by $\tilde{\theta}_{i,t}^{ag}$, where p_t^{ag} denotes the price of aggregate insurance. Entrepreneur i can buy idiosyncratic insurance $\tilde{\theta}_{i,t}^{id}$ at zero cost in equilibrium, as insurance providers perfectly diversify across entrepreneurs. However, the amount of idiosyncratic insurance is limited by the *skin-in-the-game* constraint:

$$\tilde{\theta}_{i,t}^{id} \leq (1-\phi)\tilde{q}_t k_{i,t}\sigma_I. \tag{7}$$

This particular market structure represents one possible implementation of the optimal contract allocation. For instance, instead of formal insurance contracts, this implemen-

[17] V^{*}, given in Appendix A.1, equals the value-function coefficient for an infinite-horizon agent.

[18] Notice that $\bar{l}_{i,t}$ denotes the amount of labor exogenously supplied by the household, while $l_{i,t}$ is the amount of labor demanded by the entrepreneur to run her project.

tation may capture informal insurance arrangements. As documented by, e.g., Kinnan and Townsend (2012), kinship networks play an important role in allowing households to share idiosyncratic risk. The insurance constraint can also be interpreted as an equity constraint, where entrepreneurs are unable to freely sell claims on their business to a diversified set of investors, as in e.g. Chen et al. (2010) or Panousi and Papanikolaou (2012).

Constraint (7) is binding in equilibrium, so entrepreneurs are *insurance-constrained*. To focus on entrepreneurial risk, we initially abstract from borrowing constraints and discuss their role in Section 5.2. Households face only a natural borrowing limit:

$$\tilde{n}_{i,t} \geq -\tilde{h}_{i,t}, \tag{8}$$

where $\tilde{h}_{i,t} \equiv \mathbb{E}_t \left[\int_t^{s+T} \frac{\pi_z}{\pi_t} \tilde{w}_z \bar{l}_{i,z} dz \right]$ denotes human wealth. π_t denotes the stochastic discount factor (SDF), which evolves according to $d\pi_t = -r_t \pi_t dt - p_t^{ag} \pi_t dZ_t$.[19]

Entrepreneurs' problem. Entrepreneur i with age a_i chooses the stochastic processes $(\tilde{c}_i, \tilde{\theta}_i^{ag}, \tilde{\theta}_i^{id}, k_i, l_i, \iota_i)$, taking prices $(\tilde{q}, \tilde{w}, r, p^{ag})$ as given, to solve the following problem:

$$\tilde{V}_t(\tilde{n}_i, a_i) = \max_{\tilde{c}_i, \tilde{\theta}_i^{ag}, \tilde{\theta}_i^{id}, k_i, l_i, \iota_i} \mathbb{E}_t \left[\int_0^{T-a_i} e^{-\rho z} \frac{\tilde{c}_{i,t+z}^{1-\gamma}}{1-\gamma} dz + e^{-\rho(T-a_i)} (1-\psi)^\gamma V^* \frac{\tilde{n}_{i,t+T-a_i}^{1-\gamma}}{1-\gamma} \right], \tag{9}$$

subject to (7), (8), non-negativity constraints $c_{i,t}, k_{i,t} \geq 0$, and the law of motion of $\tilde{n}_{i,t}$

$$d\tilde{n}_{i,t} = \left[(\tilde{n}_{i,t} - \tilde{q}_t k_{i,t}) r_t + \tilde{q}_t k_{i,t} \mu_{i,t}^R - p_t^{ag} \tilde{\theta}_{i,t}^{ag} + \tilde{w}_t \bar{l}_{i,t} - \tilde{c}_{i,t} \right] dt + \left(\tilde{q}_t k_{i,t} \sigma_A - \tilde{\theta}_{i,t}^{ag} \right) dZ_t + \left(\tilde{q}_t k_{i,t} \sigma_I - \tilde{\theta}_{i,t}^{id} \right) dZ_{i,t},$$

given initial financial wealth $\tilde{n}_{i,t} = \tilde{n}_i > -\tilde{h}_{i,t}$.

The term in brackets in the expression above is the expected change in financial wealth. The entrepreneur invests $\tilde{n}_{i,t} - \tilde{q}_t k_{i,t}$ in the riskless asset, with rate of return r_t, and she invests the amount $\tilde{q}_t k_{i,t}$ in the risky business technology, with expected rate of return $\mu_{i,t}^R$. The cost of aggregate insurance is $p_t^{ag} \tilde{\theta}_{i,t}^{ag}$. The entrepreneur receives labor income $\tilde{w}_t \bar{l}_{i,t}$ and consumes $\tilde{c}_{i,t}$. The last two terms represent the exposure to aggregate and idiosyncratic risk, which equals the net of insurance risk exposure from the business.

Wage earners' problem. Wage earners do not have access to a production technology. To simplify exposition, we assume they have an infinite horizon. We show in Appendix E.2 that allowing for finite lives and a bequest motive, or allowing an endogenous choice

[19] Even though markets are incomplete, households agree on the valuation of variables not exposed to idiosyncratic risk. Therefore, there is no ambiguity in defining the relevant SDF for aggregate payoffs.

to become an entrepreneur, does not change our main results. Wage earners and entrepreneurs share a per-period isoelastic utility function with curvature parameter γ.[20] As often assumed in models with heterogeneous returns, as e.g. Kiyotaki and Moore (1997), wage earners and entrepreneurs have different discount rates.

The problem of wage earner $j \in \mathcal{W}_t$ is given by

$$\tilde{V}_t^w(\tilde{n}_j) = \max_{\tilde{c}_j, \tilde{\theta}_j^{ag}} \mathbb{E}_t \left[\int_t^\infty e^{-\rho_w(z-t)} \frac{\tilde{c}_{j,z}^{1-\gamma}}{1-\gamma} dz \right], \tag{10}$$

subject to non-negativity constraint $\tilde{c}_{j,t} \geq 0$, $\tilde{n}_{j,t} \geq \tilde{h}_{j,t}$, where $\tilde{h}_{j,t}$ denotes their human wealth, and the law of motion of financial wealth $\tilde{n}_{j,t}$ is

$$d\tilde{n}_{j,t} = \left[\tilde{n}_{j,t} r_t - p_t^{ag} \tilde{\theta}_{j,t}^{ag} + \tilde{w}_t \bar{l}_{j,t} - \tilde{c}_{j,t} \right] dt - \tilde{\theta}_{j,t}^{ag} dZ_t,$$

given initial financial wealth $\tilde{n}_{j,t} = \tilde{n}_j > -\tilde{h}_{j,t}$. $\tilde{\theta}_{j,t}^{ag}$ can take positive or negative values, so wage earners can choose to either buy or provide aggregate insurance to entrepreneurs.[21]

Equilibrium. We provide below a definition of the competitive equilibrium.

Definition 1. *The competitive equilibrium is a set of aggregate stochastic processes: the capital stock k, the interest rate r, the wage rate $\tilde{w}$, the relative price of capital $\tilde{q}$, and the price of aggregate insurance p^{ag}; a set of stochastic processes for each entrepreneur $i \in \mathcal{E}_t$ and wage earner $j \in \mathcal{W}_t$: consumption $\tilde{c}_i$, financial wealth $\tilde{n}_i$, capital k_i, labor l_i, aggregate insurance $\tilde{\theta}_i^{ag}$, and idiosyncratic insurance $\tilde{\theta}_i^{id}$ for $i \in \mathcal{E}_t$; consumption $\tilde{c}_j$ and aggregate insurance $\tilde{\theta}_j^{ag}$ for $j \in \mathcal{W}_t$ such that:*

(a) *Aggregate capital stock satisfies the law of motion (4), given the initial capital stock k_0.*

(b) *$(\tilde{c}_i, \tilde{\theta}_i^{ag}, \tilde{\theta}_i^{id}, k_i, l_i, \iota_i)$ solve entrepreneurs' problem (9), given $(\tilde{q}, \tilde{w}, r, p^{ag})$.*

(c) *$(\tilde{c}_j, \tilde{\theta}_j^{ag})$ solve wage earners' problem (10), given $(\tilde{w}, r, p^{ag})$.*

(d) *Markets clear for all $t \geq 0$:*

i. Goods: $\int_{\mathcal{E}_t} \tilde{c}_{i,t} di + \int_{\mathcal{W}_t} \tilde{c}_{j,t} dj + \int_{\mathcal{E}_t} \iota_{i,t} A_t k_{i,t} di = \int_{\mathcal{E}_t} \tilde{y}_{i,t} di$.

ii. Capital and labor: $\int_{\mathcal{E}_t} k_{i,t} di = k_t$ and $\int_{\mathcal{E}_t} l_{i,t} di = \int_{\mathcal{E}_t} \bar{l}_{i,t} di + \int_{\mathcal{W}_t} \bar{l}_{j,t} dj$.

[20] As shown by Swanson (2012), and consistent with Lemma 1 below, the coefficient of relative risk aversion in the presence of labor is affected by, but is not equal to, γ.

[21] Wage earners also provide idiosyncratic insurance. We have already imposed that they can diversify the exposure to idiosyncratic risk, so their financial wealth is only exposed to aggregate risk.

iii. Aggregate insurance: $\int_{\mathcal{E}_t} \tilde{\theta}^{ag}_{i,t} di + \int_{\mathcal{W}_t} \tilde{\theta}^{ag}_{j,t} dj = 0$.

iv. Riskless bonds: $\int_{\mathcal{E}_t} [\tilde{n}_{i,t} - \tilde{q}_t k_{i,t}] di + \int_{\mathcal{W}_t} \tilde{n}_{j,t} dj = 0$.

3.2 Solution to entrepreneurs' problem

We describe next the solution to the entrepreneurs' problem. We focus on a stationary equilibrium, where scaled aggregate variables are constant, that is, $w_t = w$ and $q_t = q$. Variables in levels then move one-to-one with productivity, e.g., $\tilde{w}_t = wA_t$ and $\tilde{q}_t = qA_t$.

Maximizing expected returns. As $(l_{i,t}, \iota_{i,t})$ enters entrepreneurs' maximization problem only through the expected return on the business, given in Equation (5), they choose these variables to maximize expected returns. Labor demand takes the usual form:

$$w = (1 - \alpha) \left(\frac{k_{i,t}}{l_{i,t}} \right)^{\alpha}. \tag{11}$$

The capital-labor ratio is equalized across entrepreneurs and coincides with the aggregate capital-labor ratio $K_t \equiv k_t / \bar{l}_t$, where k_t denotes the aggregate capital stock and $\bar{l}_t$ denotes the aggregate labor supply. In a stationary equilibrium, capital grows at the same rate as labor supply, which grows with the population at a rate g.

The investment rate $\iota_{i,t}$ satisfies the optimality condition

$$\Phi'(\iota_{i,t}) = \frac{1}{q} \Rightarrow \iota_{i,t} = (\Phi')^{-1} \left(\frac{1}{q} \right) \equiv \iota(q), \tag{12}$$

where $\iota(q)$ is increasing in q, given the concavity of $\Phi(\cdot)$. Substituting Equations (11) and (12) into Equation (5), we obtain

$$\mu^R = \frac{\alpha K^{\alpha - 1} - \iota(q)}{q} + \mu_A + \Phi(\iota(q)) - \delta, \tag{13}$$

where $\mu^R_{i,t} = \mu^R$ in a stationary equilibrium, so expected returns are equalized across entrepreneurs. Realized returns are, of course, still heterogeneous.

Human and total wealth. The lemma below shows that the relevant notion of wealth is *total wealth*, $\omega_{i,t} \equiv n_{i,t} + h_{i,t}$, the sum of financial wealth and human wealth. In particular, the entrepreneur's value function depends on total wealth $\omega_{i,t}$ and on age $a_{i,t} = t - s_i$.

Lemma 1. *Suppose the economy is in a stationary equilibrium. Human wealth satisfies*

$$\frac{\partial h(a)}{\partial a} = \left(r + p^{ag}\sigma_A - \mu_A\right) h(a) - w\bar{l}(a), \tag{14}$$

given $h(T) = 0$. *The (scaled) value function is given by*[22]

$$V(n,a) = \zeta(a)^{-\frac{1}{\gamma}} \frac{\left(n + h(a)\right)^{1-\gamma}}{1-\gamma}, \tag{15}$$

where $\zeta(a)$ *is the consumption to total wealth ratio. The effective risk aversion is given by*

$$-\frac{V_{nn}n}{V_n} = \frac{\gamma}{1 + \frac{h(a)}{n}}. \tag{16}$$

Proof. See Appendix A.1. □

The first part of Lemma 1 implies that human wealth is the present discounted value of future labor income, where the discount rate incorporates the aggregate risk premium $p^{ag}\sigma_A$. This extra discount is required as wages move with aggregate productivity. Human wealth being a risky asset is consistent with Benzoni et al. (2007), which shows that human wealth becomes highly correlated with stocks when aggregate output and labor income are cointegrated, as in our model. As human wealth depends only on the entrepreneur's age, we drop the dependence on the household, i.e., $h_{i,t} = h(a_{i,t})$.

The second part of Lemma 1 gives the value function, an age-dependent CRRA function of total wealth. Notably, the entrepreneur's effective risk aversion depends on $h_{i,t}/n_{i,t}$, the *human-financial wealth* ratio, which varies substantially over the life cycle in the data.

Policy functions. The next proposition provides the entrepreneurs' policy functions.

Proposition 1. *Suppose the economy is in a stationary equilibrium. Then,*
i) *Demand for capital is given by*

$$\frac{qk_{i,t}}{n_{i,t}} = \frac{1 + \frac{h_{i,t}}{n_{i,t}}}{\gamma} \frac{p^{id}}{\phi\sigma_I}, \tag{17}$$

where p^{id} *is the shadow price of idiosyncratic insurance, which is given by*

$$p^{id} = \frac{\mu^R - r - p^{ag}\sigma_A}{\phi\sigma_I}. \tag{18}$$

[22] The scaled value function is related to the original value function by $\tilde{V}_t(\tilde{n}, a) = A_t^{1-\gamma} V\left(\frac{\tilde{n}}{A_t}, a\right)$.

ii) *The price of aggregate insurance is* $p^{ag} = \gamma\sigma_A$*, and the quantity of aggregate insurance is*

$$\theta_{i,t}^{ag} = (qk_{i,t} - n_{i,t})\sigma_A. \tag{19}$$

iii) *The consumption-wealth ratio is given by*

$$\frac{c_{i,t}}{n_{i,t}} = \frac{\bar{r}}{1 - \psi e^{-\bar{r}(T-a_i)}} \left(1 + \frac{h_{i,t}}{n_{i,t}}\right), \tag{20}$$

where $\bar{r} \equiv \frac{1}{\gamma}\rho + \left(1 - \frac{1}{\gamma}\right)\left[r + \frac{(p^{ag})^2 + (p^{id})^2}{2\gamma}\right]$.

Proof. See Appendix A.1. □

The demand for capital has three components: entrepreneur's effective risk aversion, $\gamma/(1 + \frac{h_{i,t}}{n_{i,t}})$, the price of idiosyncratic insurance, p^{id}, and the quantity of non-diversified risk, $\phi\sigma_I$. The human-financial wealth ratio drives differences in effective risk aversion, creating cross-sectional dispersion in risk-taking.

Equation (18) shows that the shadow price of idiosyncratic insurance, the Lagrange multiplier on the skin-in-the-game constraint, corresponds to the return per unit of risk (the Sharpe ratio) of an investor who fully insures the project against aggregate risk. In equilibrium, this Sharpe ratio is positive, so the skin-in-the-game constraint is always binding, that is, $\theta_{i,t}^{id} = (1 - \phi)q_t k_{i,t}\sigma_I$. Entrepreneurs purchase as much idiosyncratic insurance as possible, given it has zero cost in equilibrium.

Rearranging Equation (18), we can express expected excess returns as follows:

$$\mu^R - r = \underbrace{p^{ag}\sigma_A}_{\text{agg. risk premium}} + \underbrace{p^{id}\phi\sigma_I}_{\text{id. risk premium}}.$$

Hence, we obtain a two-factor model for expected returns, corresponding to the compensation for aggregate and idiosyncratic risk, consistent with the evidence in Section 2. In the absence of aggregate risk, growth, and adjustment costs, this expression simplifies to $\alpha K^{\alpha-1} - r = p^{id}\phi\sigma_I$, so the MPK is not equal to the interest rate when $\phi > 0$, even though entrepreneurs face no borrowing constraints. Entrepreneurs do not expand the business, despite $\mu^R > r$, to limit their risk exposure. In equilibrium, the wedge $\mu^R - r$ depends on entrepreneurs' risk aversion and their ability to diversify these risks, as captured by ϕ.

The demand for capital depends on the price and quantity of *idiosyncratic* risk. Access to aggregate insurance leads to a separation between entrepreneurs' choice of business scale and the amount of aggregate risk they are willing to hold. Investing in the business becomes then a decision about how much idiosyncratic risk to bear.

Equation (19) gives the demand for aggregate insurance. Poor entrepreneurs buy aggregate insurance, $\theta_{i,t}^{ag} > 0$, while rich entrepreneurs provide insurance, $\theta_{i,t}^{ag} < 0$. This arrangement can be implemented by having richer entrepreneurs send transfers to poor entrepreneurs as an indemnity after a negative aggregate shock, with transfers in the opposite direction after a positive aggregate shock.

The consumption-wealth ratio is given in Equation (20). The first term $\bar{r}/(1-\psi e^{-\bar{r}(T-a)})$ represents the marginal propensity to consume (MPC). It is increasing in age, as it is typical in finite-horizon problems. The bequest motive parameter ψ controls the strength of this effect. If $\psi = 0$, the MPC is constant. If $\psi = 1$, the MPC gets arbitrarily large as the entrepreneur approaches the end of life, so the stock of wealth is fully consumed at the final age T, as in Merton (1969). Finally, the consumption-wealth ratio depends on the level of the human-financial wealth ratio.

Taking stock. We derive three main lessons from the results in Proposition 1. First, entrepreneurs' risk-taking and consumption decisions depend on the human-financial ratio, $h_{i,t}/n_{i,t}$, which varies over the life cycle. Second, the expected excess return on the business is comprised of an aggregate risk premium and an idiosyncratic risk premium. Third, the idiosyncratic risk premium affects the business scale. These three facts play an important role in the model's ability to match the facts discussed in Section 2.

4 Life-cycle patterns

We consider next the quantitative implications of the model for the life-cycle behavior of entrepreneurs. We first describe the model's calibration and then the life-cycle profiles for risk-taking, consumption, and financial wealth.

4.1 Risk-taking and savings over the life cycle

Technology, preferences, and demographics. We adopt the following calibration, summarized in Table 4. The capital share is set to $\alpha = 0.33$, and the depreciation rate is set to $\delta = 0.10$, common values found in the literature (see e.g. Campbell 1994). Expected productivity growth is set to $\mu = 0.003$, following the evidence in Jeong and Townsend (2007) for Thailand. The investment function assumes the functional form $\Phi(\iota) = \sqrt{\overline{\Phi}^2 + 2\iota} - \overline{\Phi}$. This corresponds to the case of quadratic adjustment costs, as the investment rate required for capital to grow at rate g is $\iota = \overline{\Phi}(g+\delta) + 0.5(g+\delta)^2$. The coefficient of the investment function is chosen to match a long-run relative price of capital

Table 4. Calibrated parameters

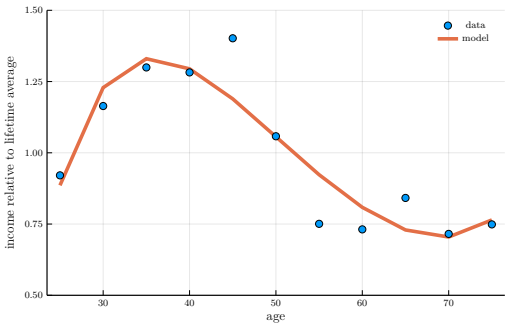
	Parameter	Choice
	Preferences	
ρ	Entrepreneur's rate of time preference	0.141
ρ_w	Wage earner's rate of time preference	0.127
ψ	Bequest motive	0.37
γ	Risk aversion	9.23
	Technology & financial friction	
μ_A	Average productivity growth rate	0.003
σ_A	Aggregate volatility	0.051
σ_{id}	Idiosyncratic volatility	0.203
ϕ	Moral hazard parameter	0.571
$\overline{\Phi}$	Adjustment cost parameter	0.90
α	Capital share in production function	0.33
δ	Depreciation rate	0.10
	Demographics	
g	Population growth	0.003
χ_e	Share of entrepreneurs in the population	0.46
T	Life span (adult life)	55

q of one. The discount rate of wage earners is chosen to match a risk-free rate of $r = 3.5\%$, consistent with the average real rate for Thailand over the last two and half decades. The discount rate of entrepreneurs and the bequest motive parameters are chosen to match the consumption-wealth ratio at the beginning and end of life. The life horizon is set to $T = 55$, so it covers the life span from 25 to 80 years old, and the population growth is set to $g = 0.3\%$, the most recent value for population growth in Thailand. The parameter χ_e is chosen to match the average share of business wealth to financial wealth.

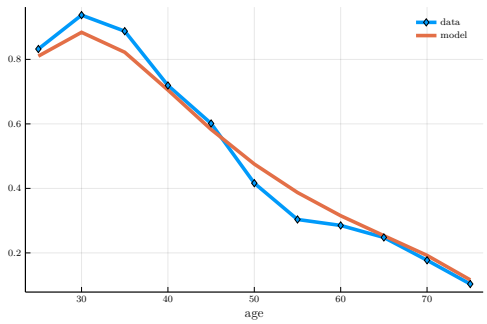
Risk, return, and the moral hazard parameter. The volatility parameters, σ_A and σ_I, are chosen to match the aggregate and idiosyncratic components of total volatility. The aggregate risk premium is given by $p^{ag}\sigma_A = \gamma\sigma_A^2$, so we choose γ to match the aggregate risk premium. The idiosyncratic risk premium is informative about ϕ. If $\phi = 0$, the idiosyncratic risk premium equals zero. As we raise ϕ, the importance of the idiosyncratic risk premium increases. Using expression (17) to solve for p^{id}, we obtain the idiosyncratic risk premium in terms of the moral hazard parameter ϕ and observable quantities, namely the level of idiosyncratic volatility and the exposure of entrepreneurs' total wealth to the private business.[23] Given these quantities, we can identify ϕ.

[23] See Section 6 for a detailed discussion of the determination of p^{id} in equilibrium.

Figure 3: Life Cycle Profiles: Labor Income and Human-Financial Ratio

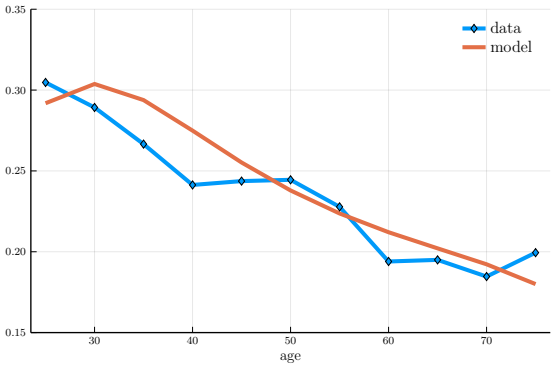


(a) Labor Income

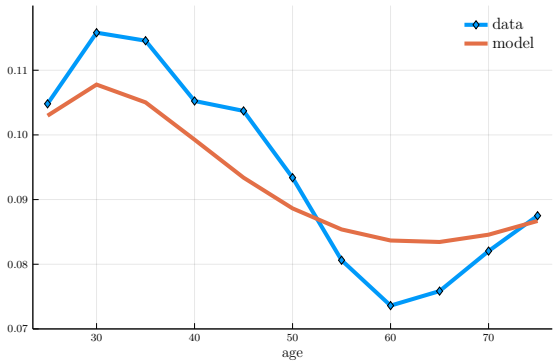


(b) Human-financial wealth ratio

Figure 4: Life Cycle Profiles: Risk-taking and Consumption-Wealth Ratio



(a) Share of wealth invested in the business



(b) Consumption-wealth ratio

Measuring human wealth. We follow Gârleanu and Panageas (2015) and assume that $\bar{l}_{i,t}$ is a function of age: $\bar{l}_{i,t} = \sum_{l=1}^{L} \Gamma_l e^{\varphi_l a_i}$, given the normalization $\frac{\int_{\mathcal{E}_t} \bar{l}_{i,t} di}{\int_{\mathcal{E}_t} di} = 1$. This functional form is flexible enough to capture the empirical labor income dynamics while being analytically tractable. We estimate the parameters $(\Gamma_l, \varphi_l)_{l=1}^{L}$ by non-linear least squares and set the number of exponential terms to $L = 3$. The left panel of Figure 3 shows that the functional form approximates well the empirical labor income profile.

We consider next the human-financial wealth $h_{i,t}/n_{i,t}$. For simplicity, we use the empirical age effects without any additional controls. In Appendix B, we show that the empirical life-cycle profiles are very similar after we control for year fixed-effects or additional demographic variables. The right panel of Figure 3 shows that the human-financial wealth ratio declines over the life cycle. This is the result of labor income being relatively high at the beginning of the life cycle and that households have fewer years of future income as time goes by. Quantitatively, human wealth is nearly as important as financial wealth at the beginning of life, and nearly half financial wealth by age 50.

Risk-taking and savings. The left panel of Figure 4 shows that our measure of risk-taking, the share of wealth invested in the business, declines with age, consistent with the evidence in Table 3. The model generates this pattern by having the effective risk aversion decrease with the human-financial wealth ratio. Young entrepreneurs are endogenously less risk-averse than older entrepreneurs, so they invest a larger fraction of their wealth in the business. Notice that the ratio of the risk-taking measure at the beginning and the end of life is entirely determined by the human-financial wealth ratio, which was calibrated independently of any information on the cross-section of entrepreneurs' risk-taking.

The consumption-wealth ratio is roughly U-shaped as a function of age. This non-monotonic behavior is the result of two forces. First, the human-financial wealth ratio declines with age, which induces households to reduce consumption. Second, the MPC increases with age, which induces households to consume more. The first effect initially dominates, and the second effect dominates as the entrepreneur gets older.

4.2 Distributive implications of entrepreneurial risk

We consider next how entrepreneurial risk affects wealth inequality. We focus on the joint distribution of (scaled) financial wealth and age, denoted by $f(n,a)$. Given this joint distribution, we obtain the average wealth by age, $n(a) = \int_{-h(a)}^{\infty} nf(n|a)dn$, and average entrepreneurial wealth, $n_e = \int_0^T n(a)f(a)da$, where $f(a)$ is the age distribution.

Between-group inequality. The next proposition characterizes average wealth by age.

Proposition 2. *Suppose the economy is in a stationary equilibrium. Then,*
i) Between-group inequality: *The wealth share of entrepreneurs of age a, $\frac{f(a)n(a)}{n_e}$, satisfies*

$$\log\frac{f(a)n(a)}{n_e} = \log\frac{f(0)n(0)}{n_e} + \underbrace{\log\left(\frac{1+\frac{h(0)}{n(0)}}{1+\frac{h(a)}{n(a)}}\right)}_{\substack{\text{human-to-financial}\\ \text{wealth effect}}} + \underbrace{\left[r + \frac{(p^{ag})^2}{\gamma} + \frac{(p^{id})^2}{\gamma} - (g+\mu_A)\right]a}_{\text{generalized "r-g" effect}} - \underbrace{\int_0^a \frac{\bar{r}}{1-\psi e^{-\bar{r}(T-a')}}da'}_{\text{average MPC effect}}. \tag{21}$$

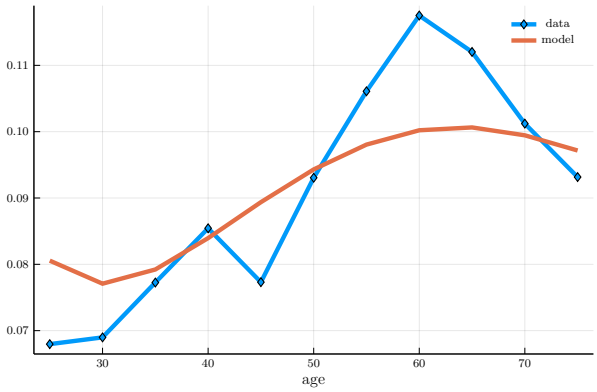
$$\text{where } n(0) = \left[e^{-\left(r+\frac{(p^{ag})^2}{\gamma}+\frac{(p^{id})^2}{\gamma}-(g+\mu_A)-mpc_e\right)T} - 1\right]^{-1} h(0), \text{and } mpc_e = \frac{1}{T}\int_0^T \frac{\bar{r}}{1-\psi e^{-\bar{r}(T-a)}}da.$$

ii) Average financial wealth: *The average financial wealth of entrepreneurs is given by*

$$n_e = f(0)(n(0)+h(0))\int_0^T e^{\left(r+\frac{(p^{ag})^2}{\gamma}+\frac{(p^{id})^2}{\gamma}-(g+\mu_A)\right)a}\frac{e^{-\bar{r}a} - \psi e^{-\bar{r}T}}{1-\psi e^{-\bar{r}T}}da - h_e, \tag{22}$$

where $h_e = \int_0^T f(a)h(a)da$.

Figure 5: Financial wealth distribution across age groups



Proof. See Appendix A.2. □

Proposition 2 decomposes the wealth distribution across age groups into three effects. The human-to-financial wealth effect, as labor income accelerates the accumulation of financial wealth. The generalized "$r - g$" effect, where the correct rate of return includes the aggregate and idiosyncratic risk premium. The average MPC effect, capturing how wealth accumulated at a given age depends on past consumption decisions. The second part of Proposition 2 characterizes average entrepreneurial wealth, which reflects the distribution of wealth between entrepreneurs and workers.

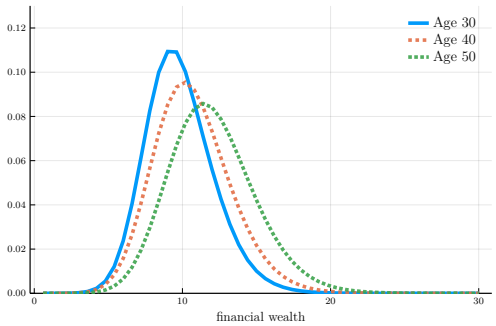
Figure 5 shows the share of wealth among entrepreneurs held by each age group. The model captures the inverted U-pattern of financial wealth observed in the data. The human-financial wealth effect and the "$r - g$" effect dominate the average MPC effect for young entrepreneurs, so the wealth share initially increases with age. The average MPC effect dominates later in life, bringing down the wealth share.

This hump-shaped behavior of wealth is typical of life-cycle models, see e.g. Gomes (2020). The fact that wealth accumulation responds to the price of idiosyncratic risk through the channels discussed above will be particularly important when we consider counterfactual changes in insurance in Section 6.

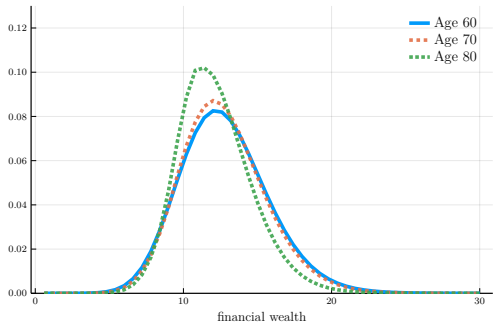
Within-group inequality. We characterize next the wealth distribution by age. Let $\mu_{n,t}(n,a)$ denote the expected change and $\sigma_{n,t}(n,a)$ the instantaneous volatility of financial wealth. The evolution of $f(n|a)$ is given by the Kolmogorov Forward Equation.

Lemma 2 (Kolmogorov Forward Equation)**.** *The conditional distribution of financial wealth*

Figure 6: Stationary distribution of financial wealth conditional on age



(a) Ages 30, 40, and 50



(b) Ages 60, 70, and 80

$f_t(n|a)$ satisfies the partial differential equation

$$\frac{\partial f_t(n|a)}{\partial t} + \frac{\partial f_t(n|a)}{\partial a} = -\frac{\partial [f_t(n|a)\mu_{n,t}(n,a)]}{\partial n} + \frac{1}{2}\frac{\partial^2 \left[f_t(n|a)\sigma^2_{n,t}(n,a)\right]}{\partial n^2}, \tag{23}$$

and the boundary condition $f_t(e^{-gT}n|0) = f_t(n|T)$, given an initial condition $f_0(n|a)$.

Proof. See Appendix A.3. □

It is possible to solve for the conditional distribution of financial wealth in closed form for the special case where entrepreneurs leave no bequests, that is, $\psi = 1$.

Proposition 3 (Within-group inequality). *Suppose $\psi = 1$ and $r + \frac{(p^{ag})^2}{\gamma} + \frac{(p^{id})^2}{\gamma} > \mu_A$.*
i) Shifted log-normal distribution. *Conditional on age, financial wealth follows a shifted log-normal distribution with support $(-h(a), \infty)$, i.e., $n + h(a)$ follows a log-normal distribution.*
ii) Variance by age. *The variance of n conditional on age a is given by*

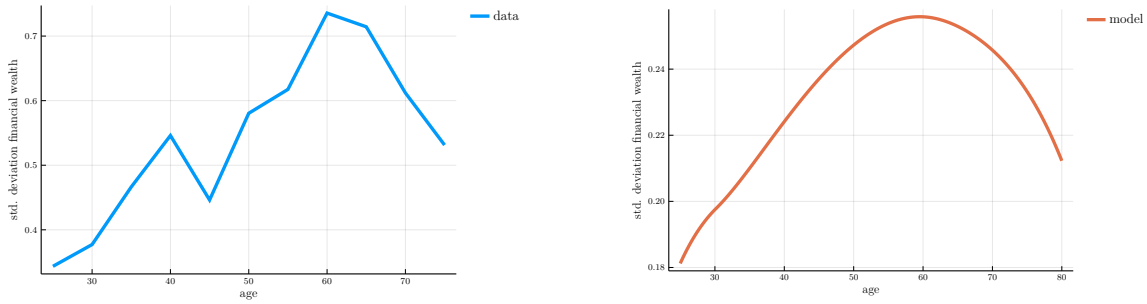
$$\mathbb{V}[n|a] = \left[e^{\left(\frac{p^{id}}{\gamma}\right)^2 a} - 1\right]\left[h(0)e^{\left(r+\frac{(p^{ag})^2}{\gamma}+\frac{(p^{id})^2}{\gamma}-\mu_A\right)a}\frac{e^{-\bar{r}a} - e^{-\bar{r}T}}{1 - e^{-\bar{r}T}}\right]^2. \tag{24}$$

iii) Inverted-U shape of inequality over the life cycle. *There exists $0 < \hat{a} < T$ such that $\mathbb{V}[n|a]$ is increasing in a for $a < \hat{a}$ and decreasing for $a > \hat{a}$.*

Proof. See Appendix A.4. □

Proposition 3 shows that wealth follows a *shifted log-normal* distribution, with an age-dependent shifter $-h(a)$. Expression (24) shows how the variance of wealth evolves over the life cycle. Wealth dispersion increases at the beginning of life, as some entrepreneurs receive a series of positive shocks while others suffer adverse shocks. The increasing MPC provides a countervailing force, bringing dispersion down at the end of the life cycle.

Figure 7: Standard deviation of financial wealth within age groups



(a) Within-group inequality: data

(b) Within-group inequality: model

Note: Panel (a) shows the standard deviation of financial wealth by age in the data. Panel (b) shows the standard deviation of financial wealth by age in the model economy. In both cases, we normalize the standard deviation by entrepreneurs' average wealth.

The results in Proposition 3 assume no bequests. In the general case, we must use numerical methods. Figure 6 shows the stationary distribution of financial wealth for selected ages. The mean and the dispersion of the distribution initially increase with age, then eventually both start to decline as entrepreneurs get older.

The same inverted U-pattern can be found in the data, as shown in Figure 7. The standard deviation of n/n_e increases sharply until roughly age 60 and then declines by the end of the life cycle. Quantitatively, the model generates a substantial increase in inequality, over 40% from early in life until the peak, even though idiosyncratic return shocks are the only source of heterogeneity across entrepreneurs. The increase in inequality is even more pronounced in the data. Introducing differences in preferences or labor income could potentially bring the within-group inequality closer to the one in the data.

Consumption inequality and risk sharing. As shown in Appendix A.4, the variance of log total wealth, and ultimately log consumption, increases with age: $\mathbb{V}[\log \omega_{i,t}|a] = \mathbb{V}[\log c_{i,t}|a] = \left(\frac{p^{id}}{\gamma}\right)^2 a$. The steepness of the variance life-cycle profile depends on the degree of risk sharing in the economy, in line with work using the consumption dispersion over the life cycle to infer how imperfect risk sharing is (see, e.g., Storesletten et al. 2004).

5 Heterogeneous risk premia

In the previous section, we showed that the model captures the main life-cycle patterns observed in our data. In this section, we show that an extension of the baseline model can also capture the heterogeneity in expected returns documented in Section 2.

5.1 Risk heterogeneity and decreasing returns to scale

Proposition 1 shows that the expected return on the business includes compensation for idiosyncratic risk, consistent with the evidence in Table 2. However, the risk premium is equalized across entrepreneurs, contrasting with the rich heterogeneity in expected returns observed in the data. This will be the case even if we introduce heterogeneity in idiosyncratic volatility. From Equation (11), the capital-labor ratio would be equalized across entrepreneurs, so they would still have the same expected return by Equation (13).

Introducing decreasing returns to scale. To capture heterogeneity in risk premia, the model needs a combination of heterogeneous exposure to idiosyncratic risk and *decreasing returns to scale* (DRS) in production.[24] To see this fact, consider the following extension of the baseline model. Production depends not only on the amount of capital and labor but also on a fixed entrepreneurial ability e_i:

$$\tilde{y}_{i,t} = A_t k_{i,t}^{\alpha} l_{i,t}^{\beta} e_i^{1-\alpha-\beta}.$$

For simplicity, we focus initially on the case where e_i is common among entrepreneurs, and we normalize e_i to 1. This formulation captures a form of span-of-control problem (Lucas 1978). We also assume that entrepreneurs differ in their exposure to idiosyncratic risk, which is denoted by $\sigma_{I,i} \in \{\sigma_I^1, \ldots, \sigma_I^n\}$. Finally, to focus on the role of return heterogeneity, we abstract from the overlapping generations feature in this section, so we set $g = 0$ and $T \to \infty$. The rest of the environment is the same as discussed in Section 3.

Lemma 3 (Model with DRS). *In a stationary equilibrium, the investment rate is given by* (12)*, the price of aggregate risk is* $p^{ag} = \gamma\sigma_A$*, and aggregate insurance is given by* (19)*. Labor demand is given by* $w_t = \beta k_{i,t}^{\alpha} l_{i,t}^{\beta-1}$*, and the conditional expectation of marginal returns is given by*

$$\mu_{i,t}^R = \frac{\alpha \left(\frac{\beta}{w}\right)^{\frac{\beta}{1-\beta}} k_{i,t}^{\frac{\alpha+\beta-1}{1-\beta}} - \iota(q)}{q} + \mu_A + \Phi(\iota(q)) - \delta. \tag{25}$$

Given value function $V_i(\omega)$*, the optimality conditions for consumption and capital are given by*

$$c_{i,t}^{-\gamma} = V_{i,\omega}, \qquad qk_{i,t} = -\frac{V_{i,\omega}}{V_{i,\omega\omega}} \frac{p_{i,t}^{id}}{\phi\sigma_{I,i}}.$$

Proof. See Appendix A.5. □

[24] We focus on heterogeneity in idiosyncratic risk. David et al. (2022) study differences in aggregate risk.

Lemma 3 describes the model solution for the case with DRS. Expected returns are *scale-dependent* when $\alpha + \beta < 1$, as they vary with capital. Notice that heterogeneity in expected returns is driven by differences in the dividend yield, given that expected capital gains are the same across entrepreneurs. This is consistent with how we measure returns in the data, which captures mainly differences in the dividend yield.

Scale-dependent returns, however, imply that there is no closed-form solution to the model. In this case, policy functions will not be linear in total wealth anymore. To obtain analytical results, we adapt the perturbation methods used by Viceira (2001). Consider the following log-linear approximation of the policy functions:

$$\log c_j(\omega_{i,t}) = \log \bar{c}_j + \psi^j_{c,\omega}\hat{\omega}_{i,t} + \mathcal{O}(\hat{\omega}^2_{i,t}), \qquad \log k_j(\omega_{i,t}) = \log \bar{k}_j + \psi^j_{k,\omega}\hat{\omega}_{i,t} + \mathcal{O}(\hat{\omega}^2_{i,t}),$$

where $\hat{\omega}_{i,t} = \log \omega_{i,t} - \log \overline{\omega}_j$, given $\overline{\omega}_j = \exp \mathbb{E}[\log \omega_{i,t} | \sigma_{I,i} = \sigma^j_I]$, denotes the deviations of log total wealth from its mean. Notice that the approximation point $\overline{\omega}_j$ and the intercepts $(\bar{c}_j, \bar{k}_j)$ are endogenous, as in a risky steady state approximation (see e.g. Coeurdacier et al. 2011). The following proposition provides the main result of this section.

Proposition 4 (Heterogeneous risk premia)**.** *The price of idiosyncratic risk is given by*

$$p^{id}_{i,t} = \overline{p}^{id} - \psi^j_{p^{id},\omega}\hat{\omega}_{i,t}, \tag{26}$$

where $\mathbb{E}[p^{id}_{i,t}] = \overline{p}^{id}$ is equalized across entrepreneurs, and $\psi_{p^{id},\omega}$ is a positive constant. Therefore, the unconditional expectation of returns is given by

$$\mathbb{E}[\mu^R_{i,t}] = r + p^{ag}\sigma_A + \overline{p}^{id}\phi\sigma_{I,i}. \tag{27}$$

Proof. See Appendix A.6. □

Proposition 4 shows that risk premia are heterogeneous in the economy with DRS and differences in idiosyncratic volatility. Entrepreneurs operating riskier technologies have higher expected returns in equilibrium, consistent with the evidence in Section 2. From Equation (26), there is variation in the conditional Sharpe ratio, as entrepreneurs experience shocks to total wealth $\hat{\omega}_{i,t}$. Still, the unconditional Sharpe ratio is the same for everyone. Therefore, unconditional expected returns vary with idiosyncratic volatility.

Skill heterogeneity. We abstracted from skill heterogeneity to focus on differences in risk. Consider now the polar opposite assumptions: $e_i \in \{e^1, \ldots, e^n\}$ and $\sigma_{I,i} = \sigma_I$. The following proposition shows that there is no heterogeneity in risk premia in this case.

Proposition 5 (Skill heterogeneity)**.** *The price of idiosyncratic risk is still given by Equation* (26)*, so* $\mathbb{E}[p^{id}_{i,t}] = \overline{p}^{id}$ *is equalized across entrepreneurs. Capital is given by* $\overline{k}_j = e_j K$*. Moreover, the unconditional expectation of returns is equalized across entrepreneurs.*

Proposition 5 shows that differences in skill alone cannot explain the heterogeneity in risk premia observed in the data. It is perhaps surprising that more skilled entrepreneurs do not experience higher returns on average. The explanation is reminiscent of the logic in Berk and Green (2004) who study how skill affects mutual fund returns. In a setting with DRS in investment returns, they argue that more skilled managers end up with *larger* funds, and expected returns are explained by their exposure to risk. The same intuition applies in our setting: more skilled entrepreneurs end up with larger businesses, but expected returns are driven by their exposure to risk, consistent with Equation (27).

5.2 The role of borrowing constraints

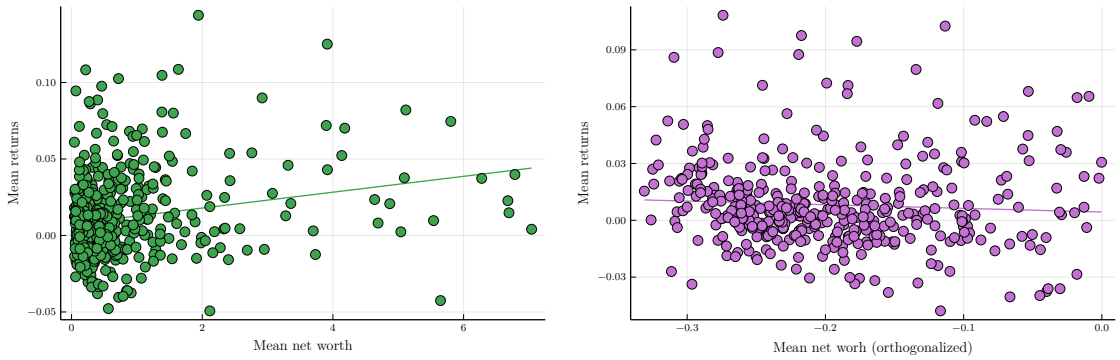
We have seen that differences in expected returns are entirely driven by differences in risk exposure. However, this result relies on the absence of borrowing constraints. Consider the case with limited pledgeability of physical capital, where entrepreneurs face collateral constraints. Differences in expected returns are then driven by differences in net worth among constrained entrepreneurs. Hence, if collateral constraints are a major factor limiting the entrepreneurial activity, we should observe a strong negative association between financial wealth and expected returns.

We test this prediction in our data. The left panel of Figure 8 shows the (cross-sectional) scatter plot of average returns and average net worth. We find a non-significant positive relationship between returns and net worth. However, the theory predicts the association between wealth and expected returns should hold only for entrepreneurs with relatively low net worth, after controlling for differences in risk exposure, so a sharper test would focus only on poorer entrepreneurs. The right panel shows the relationship between average returns and the component of net worth that is orthogonal to aggregate beta and idiosyncratic volatility for entrepreneurs with below-average (orthogonalized) net worth. We observe a non-significant association between net worth and expected returns. In both cases, net worth explains only a negligible fraction of the variation in the cross-section of entrepreneurial returns, with an adjusted R^2 of less than half a percentage point.

These findings are consistent with the results in Proposition 4, which predict no association between expected returns and net worth after controlling for risk.[25] In contrast,

[25]For instance, in the version with skill heterogeneity, a skilled entrepreneur has a larger net worth but the same expected return compared to a less skilled entrepreneur, conditional on the same risk exposure.

Figure 8: Average returns vs. average net worth



Note: The left panel shows a scatter plot of the average net worth against the average return for each entrepreneur. Net worth is normalized by its cross-sectional mean. The right panel shows a scatter plot of the residuals of a cross-sectional regression of average net worth on aggregate beta and idiosyncratic variance against average time-series returns for a sample of entrepreneurs with below-average orthogonalized net worth. To limit the influence of outliers, we trim 1% of the observations in the left and right tails.

the model with leverage constraints predicts no association between expected returns and risk exposure for entrepreneurs with sufficiently low net worth. To test this prediction, we run the same cross-sectional regressions from Section 2, but restricted to a sample of entrepreneurs with below-average net worth. The association between risk and return is actually *stronger* for this subsample, where the R^2 of the cross-sectional regression is 0.82, compared to $R^2 = 0.68$ for the whole sample. These results suggest that collateral constraints are not the main driver of expected entrepreneurial returns in our sample.

6 Counterfactuals

Having shown that the model captures the salient features of our data, we study next the effects of counterfactual changes in idiosyncratic insurance. We find that financial development, captured by changes in the moral hazard parameter ϕ, is tightly linked to economic development. We study first the long-run implications of a reform that improves insurance, and then we consider the transitional dynamics associated with the reform.

6.1 Aggregate capital and idiosyncratic risk premium

We revert to the baseline case of constant returns to scale production and $\sigma_{I,i} = \sigma_I$. The interest rate is given by the standard condition $r = \rho_w + \gamma\mu_A - \frac{\gamma(\gamma+1)}{2}\sigma_A^2$, see Appendix C.1. From Equation (12), we obtain $q = \overline{\Phi} + g + \delta$, where we choose $\overline{\Phi}$ such that $q = 1$.

From Proposition 1, we have $p^{ag} = \gamma\sigma_A$. From Equation (18), we obtain

$$r + p^{ag}\sigma_A + p^{id}\phi\sigma_I = \frac{\alpha K^{\alpha-1} - \iota(q)}{q} + g + \mu_A. \tag{28}$$

The left-hand side equals the required rate of return to invest in the business. The right-hand side gives the actual expected return, a function of the MPK. Equation (28) generalizes the standard textbook relation between MPK and interest rates, as $r = \alpha K^{\alpha-1} - \delta$ in the absence of growth and risk.

Expression (28) gives an inverse relation between the idiosyncratic risk premium $p^{id}\phi\sigma_I$ and the capital-labor ratio K. This downward-sloping relationship is represented by the solid blue line in the left panel of Figure 9, which we call the MPK schedule. This inverse relationship is analogous to the one in the model with DRS from Section 5, as there are DRS in production at the aggregate level, given the exogenous labor supply.

We need another condition relating K and p^{id}. Aggregating the demand for capital (17) across all entrepreneurs, we obtain

$$p^{id} = \underbrace{\gamma}_{\text{risk aversion}} \quad \underbrace{\phi\sigma_I}_{\text{effective risk}} \quad \underbrace{\frac{qK}{\chi_e(n_e + h_e)}}_{\text{id. risk exposure}}, \tag{29}$$

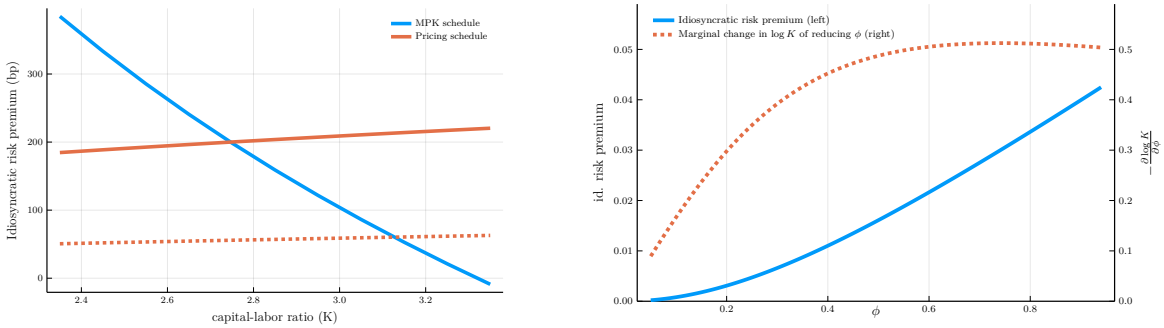
where n_e and h_e denote the average financial and human wealth of entrepreneurs.

The price of idiosyncratic risk depends on the product of the risk aversion γ and the idiosyncratic risk (net of insurance) $\phi\sigma_I$. p^{id} also depends on the *idiosyncratic risk exposure*, that is, the ratio of physical assets to total wealth of entrepreneurs. Entrepreneurs require a higher idiosyncratic risk premium when they are more exposed to the business. Using Equation (22) to eliminate $n_e + h_e$, we obtain an implicit relationship between p^{id} and K. The left panel of Figure 9 plots this relationship as the solid upward-sloping curve, which we call the *pricing schedule*. The intersection of these schedules determines the idiosyncratic risk premium and the capital stock.

Price of idiosyncratic risk in the data The model matches the risk premium and volatility for both aggregate and idiosyncratic risk given in Table 2. A striking fact is that the Sharpe ratio for aggregate risk is nearly five times larger than the one for idiosyncratic risk, despite risk being mostly idiosyncratic. Expression (29) sheds light on this pattern.

The Sharpe ratio of aggregate risk corresponds to $p^{ag} = \gamma\sigma_A$. The Sharpe ratio of idiosyncratic risk is given by $p^{id}\phi$, as the volatility computed in the data does not consider any insurance available to entrepreneurs. If we were to naively price the idiosyncratic risk

Figure 9: Idiosyncratic Risk Premium and Capital Stock



(a) MPK schedule vs. Pricing schedule

(b) Idiosyncratic risk premium and $-\frac{\partial \log K}{\partial \phi}$

Note: In the left panel, the solid (dashed) upward-sloping curve shows the pricing schedule in the initial (new) stationary equilibrium for $\phi_0 = 0.571$ ($\phi_1 = 0.286$). The right panel shows the equilibrium idiosyncratic risk premium and the marginal increase in the capital stock by reducing the moral hazard parameter for different values of ϕ.

by analogy with the aggregate risk, the Sharpe ratio would be $\gamma\sigma_I$, that is, more than four times larger than the one for aggregate risk. However, from the pricing equation,

$$p^{id}\phi = \gamma\sigma_I\phi^2\frac{qK/\chi_e}{n_e + h_e}.$$

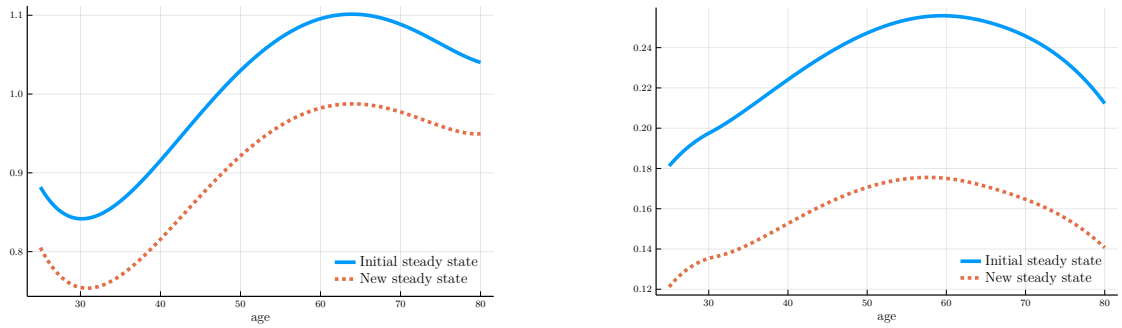
The moral hazard parameter is $\phi = 0.57$, which reduces p^{id} by $1 - \phi^2 = 67\%$. The rest of the adjustment comes from the risk exposure factor: $qK/\chi_e(n_e + h_e) \approx 0.16$. The price of idiosyncratic is low either due to insurance mechanisms or because only a fraction of their wealth is exposed to this risk. Without human wealth and heterogeneous agents, one would incorrectly attribute the low Sharpe ratio to a high degree of insurance. The economy would appear to have better insurance than it has.

6.2 Long-run effects of relaxing insurance constraints

We consider next the aggregate implications of relaxing insurance constraints. High values of ϕ capture situations where access to insurance arrangements, formal or informal, is rather limited. As institutional arrangements improve, e.g., mechanisms to monitor entrepreneurs' activities, such frictions are expected to be reduced, and entrepreneurs would bear less risk. Financial development reduces the moral hazard parameter ϕ.

Panel (a) of Figure 9 shows the impact of reducing ϕ. The MPK schedule is unchanged, but the pricing schedule is shifted down. In the long run, the reduction in expected returns leads to a reduction in the MPK and higher capital stock. Reducing the moral hazard parameter in half, from $\phi_0 = 0.57$ to $\phi_1 = 0.29$, the idiosyncratic risk premium falls by

Figure 10: Financial development and inequality in the long-run



(a) Between-group inequality

(b) Within-group inequality

Note: Inequality in a stationary equilibrium for $\phi_0 = 0.571$ (initial steady state) and $\phi_1 = 0.286$ (new steady state). Panel (a) shows average financial wealth by age. Panel (b) shows the standard deviation of financial wealth by age. All variables are normalized by entrepreneurs' average wealth in the initial steady state.

about 140 basis point, raising the capital stock by roughly 13%.

Panel (b) of Figure 9 shows that the economy's response to changes in ϕ is highly non-linear. The idiosyncratic risk premium (solid line) is convex in ϕ, so changes in the moral hazard parameter lead to stronger effects in economies with low financial development. The dashed line shows the marginal change in capital due to a reduction in the moral hazard parameter for different values of ϕ. The same reduction in ϕ leads to an increase in the capital stock that is five times larger in an economy with low financial development (high ϕ) compared to an economy with high financial development (low ϕ).

Financial development also has important implications for inequality. The left panel of Figure 10 shows the average financial wealth for each age group relative to average entrepreneurial wealth on the initial stationary equilibrium. Financial wealth falls for all age groups in the new equilibrium, as it is harder to accumulate wealth with lower expected returns. In the long run, inequality falls after a reduction in ϕ, as shown in the right panel of Figure 10, given that entrepreneurs are less exposed to risk.

6.3 Dynamic effects of relaxing risk constraints

So far, we have compared two different stationary equilibria. To compute the welfare implications of relaxing risk constraints, it is important to explicitly take into account what happens during the transition. We turn next to the dynamic effects of the reform.

Computing the transitional dynamics. We consider a "small open economy" version of the model, where the interest rate stays at the level of the original stationary equilib-

rium.[26] This allows us to focus on the dynamic implications of fluctuations in the idiosyncratic risk premium. The next proposition characterizes the evolution of aggregate variables during the transition.

Proposition 6. *The price of aggregate risk is* $p_t^{ag} = \gamma\sigma_A$. *The evolution of* $(q_t, K_t, \{\zeta_t(a), h_t(a), \omega_t(a)\})$ *is characterized by a pair of ordinary differential equations (ODEs)*

$$\begin{aligned}
\dot{K}_t &= \left[\Phi(\iota(q_t)) - \delta - g\right] K_t \\
\dot{q}_t &= \left[r + \gamma\sigma_A^2 + \gamma\phi^2\sigma_I^2 \frac{q_t K_t}{\chi_e \omega_{e,t}} + \delta - \mu_A - \Phi(\iota(q_t)) - \frac{\alpha K_t^{\alpha-1} - \iota(q_t)}{q_t}\right] q_t
\end{aligned}$$

and three partial differential equations (PDEs)

$$\begin{aligned}
\frac{\partial \zeta_t(a)}{\partial t} &= -\frac{\partial \zeta_t(a)}{\partial a} + \zeta_t^2(a) - \bar{r}_t \zeta_t(a) \\
\frac{\partial h_t(a)}{\partial t} &= -\frac{\partial h_t(a)}{\partial a} + (r + \gamma\sigma_A^2 - \mu_A) h_t(a) - (1-\alpha) K_t^{\alpha} \bar{l}(a) \\
\frac{\partial \omega_t(a)}{\partial t} &= -\frac{\partial \omega_t(a)}{\partial a} + \left[r + \gamma\sigma_A^2 - \mu_A + \gamma\phi^2\sigma_I^2 \left(\frac{q_t K_t}{\chi_e \omega_{e,t}}\right)^2 - \zeta_t(a)\right] \omega_t(a),
\end{aligned}$$

subject to the boundary conditions described in the appendix.

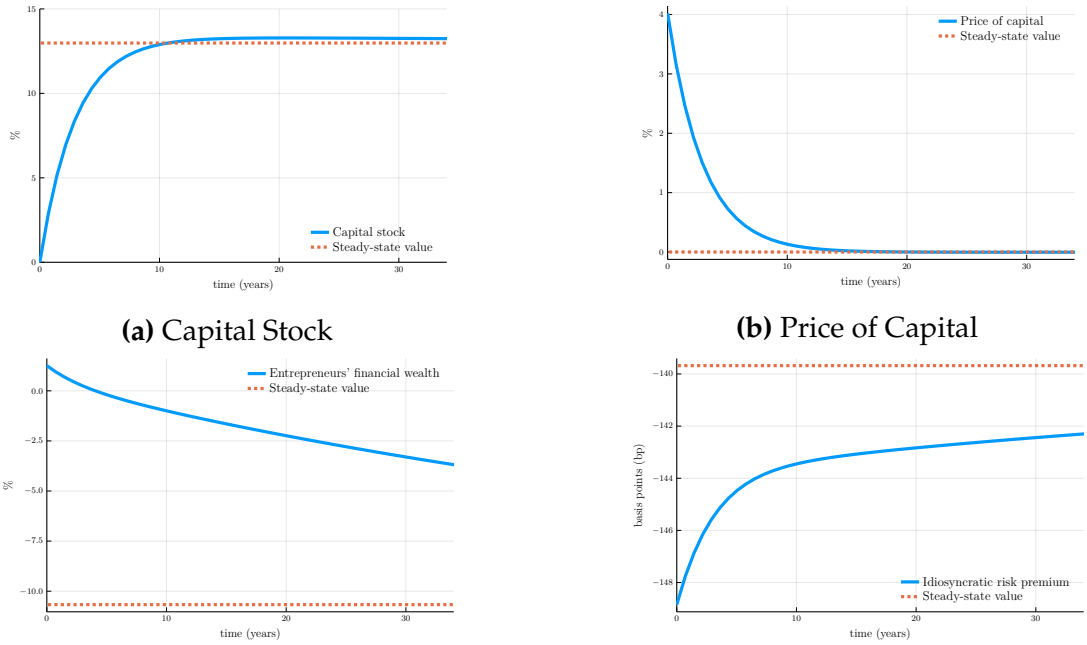
Proof. See Appendix A.7. □

We obtain the first ODE by aggregating (3) and the second one by the time-dependent version of (28). The first PDE, for the consumption-wealth ratio $\zeta_t(a)$, comes from the HJB equation; the second one corresponds to the time-dependent version of (14), and the last PDE can be obtained by averaging the budget constraint of entrepreneurs.

The transitional dynamics for heterogeneous agents models are often computed using a shooting algorithm, as in, e.g., Guerrieri and Lorenzoni (2017) or Achdou et al. (2017). As such an algorithm is impractical in our setting, we adopt a combination of perturbation and finite-difference methods. First, we use finite differences to discretize the system of ODE/PDEs. We then linearize the system around the new stationary equilibrium. In contrast to the approach in e.g. Ahn et al. (2018), which linearizes around the economy without aggregate shocks, we do not assume that shocks are small. The economy continues to be exposed to aggregate shocks, but variables in levels are proportional to the

[26] We show in Appendix D that the interest rate is constant during the transitional dynamics when wage earners have Epstein-Zin utility with linear intertemporal preferences.

Figure 11: Transitional dynamics: aggregate variables



(a) Capital Stock

(b) Price of Capital

(c) Entrepreneurs' Financial Wealth

(d) Idiosyncratic Risk Premium

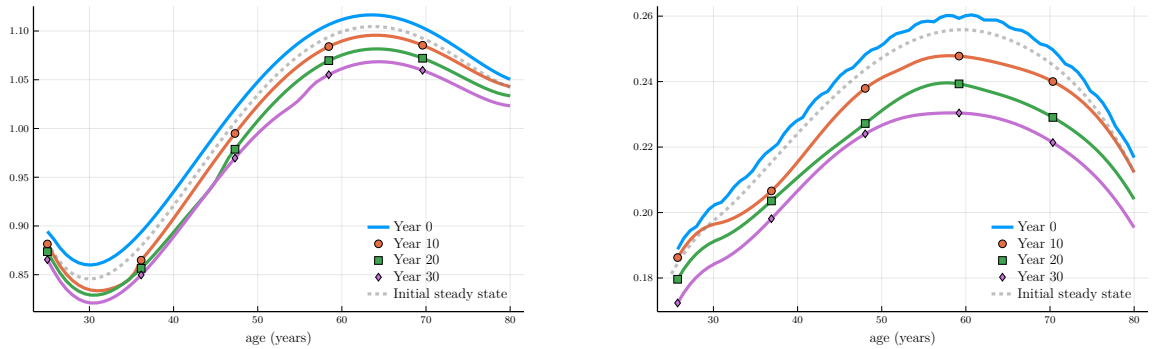
Note: Transitional dynamics from a stationary equilibrium with $\phi_0 = 0.571$ to a stationary equilibrium with $\phi_1 = 0.286$. Capital stock, the relative price of capital, and entrepreneurs' financial wealth are expressed as percentage deviations from the initial steady state. The idiosyncratic risk premium is expressed as absolute deviation from the initial steady state in basis points.

aggregate shock, such as $\tilde{q}_t = q_t A_t$. By avoiding small-risk approximations, we capture time-varying risk premia and precautionary savings effects using this method.[27]

Short-run dynamics: the overshooting effect. Figure 11 shows the transitional dynamics for aggregate variables. We consider a reform that reduces the moral hazard parameter by half, from $\phi_0 = 0.571$ under our calibration to $\phi_1 = 0.286$. We observe an investment boom and a sharp increase in business valuation that lasts for roughly a decade, as shown in Panels (a) and (b). The short-run response of q exceeds by a large margin its long-run level, i.e. there is an *overshooting* effect. As entrepreneurs bear less risk, they also require a smaller premium. In the long run, this leads to a larger capital stock. However, capital is fixed in the short run, so expected returns can only decrease through expected capital losses. The price of capital then jumps on impact and slowly reverts to its long-run level. This logic is reminiscent of Dornbusch's (1976) overshooting model, where exchange rates react more strongly to shocks in the short run to create expected capital losses to investors. The overshooting effect has important implications for wealth dynamics and inequality.

[27] See Winberry (2018) on perturbation methods for heterogeneous agent models with aggregate risk.

Figure 12: Transitional dynamics: inequality



(a) Between-group inequality

(b) Within-group inequality

Note: Transitional dynamics from a stationary equilibrium with $\phi_0 = 0.571$ to a stationary equilibrium with $\phi_1 = 0.286$. Panel (a) shows average financial wealth by age. Panel (b) shows the standard deviation of financial wealth by age.

While entrepreneurs' wealth goes down in the long run, their wealth actually increases in the short run. Wealth jumps up on impact due to a revaluation effect. However, given lower expected returns, they now accumulate wealth at a slower pace. This effect takes a long time to materialize, as it is partially transmitted through lower bequests. Even thirty years after the shock, the reduction in financial wealth is only 25% of the long-run effect.

Similarly, the short-run response of the idiosyncratic risk premium exceeds its long-run level by nearly ten basis points, as shown in Panel (d). After a decade, the difference between the risk premium to the new steady state level is cut by more than half, and then the risk premium increases slowly as entrepreneurs' wealth declines.

Kuznets dynamics. The left panel of Figure 12 shows average financial wealth for each age group at different points in time, normalized by average entrepreneurial wealth in the initial equilibrium. Due to the revaluation effect, financial wealth increases on impact. The effect is stronger for younger entrepreneurs, as they are more exposed to the business. Over time, financial wealth goes down due to the reduction in expected returns.

The right panel on Figure 12 shows the standard deviation of financial wealth by age. Again the short-run and long-run responses are different. While wealth inequality goes down in the long run, wealth inequality increases in the short run. Rich entrepreneurs hold more capital, so wealthier entrepreneurs benefit the most from the reform.

Inequality interacts in interesting ways with the demographic structure. After the initial increase, wealth dispersion goes down for all age groups, but entrepreneurs starting their professional lives just after the reform are the ones most affected. Ten years after the reform, the drop in inequality is two times larger for 35-year-old entrepreneurs, who lived

their entire professional life under the new regime, relative to 80-year-old entrepreneurs, who lived most of their lives under the old regime. Similarly, the drop in inequality is more pronounced for 35-year-old entrepreneurs than for 25-year-old entrepreneurs.

Taking stock. The reform initially leads to an investment boom and more wealth inequality. As the economy approaches its new output level, inequality starts to recede, reaching a lower level in the long run. Inequality increases as the economy enters a high growth phase, and inequality goes down as the economy reaches a higher level of development, consistent with a Kuznets's (1955) curve.[28]

Welfare implications. We turn next to the welfare implications of insurance constraints. Remember that the value function of an entrepreneur of age a at time t is given by $V_t(n,a) = \zeta_t^{-\frac{1}{\gamma}}(a)\frac{(n+h_t(a))^{1-\gamma}}{1-\gamma}$. The welfare of an entrepreneur depends then on financial wealth n, human wealth $h_t(a)$, and the consumption-wealth ratio $\zeta_t(a)$, which captures the path of expected future returns. We evaluate financial wealth at the average level of the age group, $n = n_t(a)$. Finally, we take a monotonic transformation of the value function to measure welfare in consumption units. Hence, our measure of welfare will be

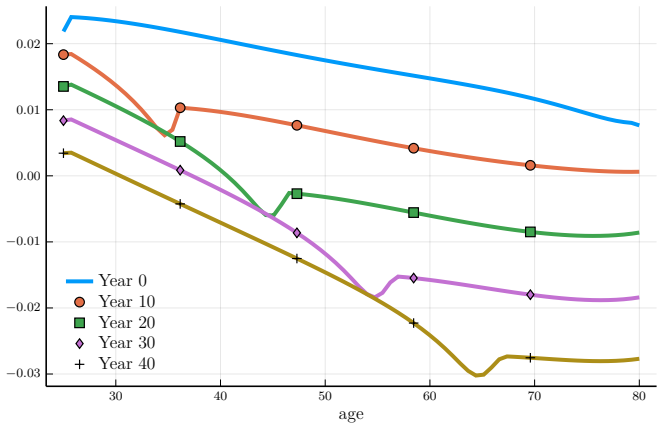
$$\mathcal{W}_t(a) = \log\left[u^{-1}\left(V_t(n_t(a),a)\right)\right] - \log\left[u^{-1}\left(V^*(n^*(a),a)\right)\right] = \frac{\hat{\zeta}_t(a)}{\gamma(\gamma-1)} + \hat{\omega}_t(a),$$

where $u(c) = \frac{c^{1-\gamma}}{1-\gamma}$, and a hat denotes log deviations from the initial steady state.

Figure 13 shows the welfare gains for each age group at different points in time. The generation that is alive at the moment of the reform benefits the most, with gains concentrated on younger entrepreneurs. This is the result of the revaluation effect. However, the negative impact on wealth accumulation affects future generations of entrepreneurs. As they receive smaller bequests and it becomes harder to accumulate wealth, their welfare is adversely impacted. Figure 13 shows how demographics affect welfare gains. Ten years after the intervention, the welfare gains for entrepreneurs who started their professional life after the reform, the ones with age between 25 to 35 years old, have welfare gains that are smaller than the entrepreneurs of the same age at the time of the intervention. Thirty years after the reform, entrepreneurs aged between 35 and 55 are worse off compared to an equilibrium without the reform, with larger welfare losses for the older entrepreneurs. Therefore, the initial generation of entrepreneurs reaps most of the benefits of the reform.

[28] Moll (2012) derived a Kuznet's curve by showing the steady-state top wealth share is hump-shaped in financial development. In contrast, we focus on the transitional dynamics instead of long-run comparisons.

Figure 13: Transitional dynamics: welfare



7 Conclusion

We study the aggregate and distributive implications of entrepreneurial risk. We propose a life-cycle model of entrepreneurship with aggregate and idiosyncratic risk under limited insurance. Entrepreneurial returns command a positive idiosyncratic risk premium, which accounts for a large fraction of total returns. The model captures quantitatively the empirical patterns of risk-taking and savings over the life cycle, the inverted-U shape of wealth inequality, and the level of aggregate and idiosyncratic risk premia. An extension with decreasing returns to scale and differences in idiosyncratic volatility also generates the heterogeneity in risk premia observed in the data.

We also study the impact of relaxing insurance constraints. An improvement in idiosyncratic insurance increases output, reduces inequality in the long run, and generates rich transitional dynamics. There is an investment boom in the short run, accompanied by an increase in inequality. Inequality goes eventually down in the long run with important intergenerational effects. Most of the welfare gains are concentrated in the generations that are alive at the time of the change, and future generations of entrepreneurs are actually worse off.

References

Achdou, Yves, Jiequn Han, Jean-Michel Lasry, Pierre-Louis Lions, and Benjamin Moll, "Income and wealth distribution in macroeconomics: A continuous-time approach," Technical Report, National Bureau of Economic Research 2017.

Ahn, SeHyoun, Greg Kaplan, Benjamin Moll, Thomas Winberry, and Christian Wolf, "When inequality matters for macro and macro matters for inequality," *NBER Macroeconomics Annual*, 2018, *32* (1), 1–75.

Angeletos, George-Marios, "Uninsured idiosyncratic investment risk and aggregate saving," *Review of Economic Dynamics*, 2007, *10* (1), 1–30.

Bach, Laurent, Laurent E Calvet, and Paolo Sodini, "Rich pickings? Risk, return, and skill in household wealth," *American Economic Review*, 2020, *110* (9), 2703–47.

Bansal, Ravi and Amir Yaron, "Risks for the long run: A potential resolution of asset pricing puzzles," *The journal of Finance*, 2004, *59* (4), 1481–1509.

Barro, Robert J, "Rare disasters, asset prices, and welfare costs," *American Economic Review*, 2009, *99* (1), 243–64.

Benzoni, Luca, Pierre Collin-Dufresne, and Robert S Goldstein, "Portfolio choice over the life-cycle when the stock and labor markets are cointegrated," *The Journal of Finance*, 2007, *62* (5), 2123–2167.

Berk, Jonathan B and Richard C Green, "Mutual fund flows and performance in rational markets," *Journal of political economy*, 2004, *112* (6), 1269–1295.

Black, Fischer, Michael C Jensen, and Myron S Scholes, "The capital asset pricing model: Some empirical tests," 1972.

Blundell, Richard, Luigi Pistaferri, and Ian Preston, "Consumption inequality and partial insurance," *American Economic Review*, 2008, *98* (5), 1887–1921.

Bodie, Zvi, Robert C Merton, and William F Samuelson, "Labor supply flexibility and portfolio choice in a life cycle model," *Journal of Economic Dynamics and Control*, 1992, *16* (3), 427–449.

Buera, Francisco J and Yongseok Shin, "Financial frictions and the persistence of history: A quantitative exploration," *Journal of Political Economy*, 2013, *121* (2), 221–272.

Campbell, John Y, "Inspecting the mechanism: An analytical approach to the stochastic growth model," *Journal of Monetary Economics*, 1994, *33* (3), 463–506.

Chen, Hui, Jianjun Miao, and Neng Wang, "Entrepreneurial finance and nondiversifiable risk," *The Review of Financial Studies*, 2010, *23* (12), 4348–4388.

Chen, Zhiyao and Ilya A Strebulaev, "Macroeconomic risk and idiosyncratic risk-taking," *The Review of Financial Studies*, 2019, *32* (3), 1148–1187.

Christiano, Lawrence J, Roberto Motto, and Massimo Rostagno, "Risk shocks," *American Economic Review*, 2014, *104* (1), 27–65.

Cochrane, John H, *Asset pricing: Revised edition*, Princeton university press, 2009.

Coeurdacier, Nicolas, Helene Rey, and Pablo Winant, "The risky steady state," *American Economic Review*, 2011, *101* (3), 398–401.

Constantinides, George M and Darrell Duffie, "Asset pricing with heterogeneous consumers," *Journal of Political economy*, 1996, *104* (2), 219–240.

David, Joel M, Lukas Schmid, and David Zeke, "Risk-adjusted capital allocation and misallocation," *Journal of Financial Economics*, 2022, *145* (3), 684–705.

Detemple, Jerome and Shashidhar Murthy, "Equilibrium asset prices and no-arbitrage with portfolio constraints," *The Review of Financial Studies*, 1997, *10* (4), 1133–1174.

Di Tella, Sebastian, "Uncertainty shocks and balance sheet recessions," *Journal of Political Economy*, 2017, *125* (6), 2038–2081.

Doepke, Matthias and Michèle Tertilt, "Families in macroeconomics," in "Handbook of macroeconomics," Vol. 2, Elsevier, 2016, pp. 1789–1891.

Dornbusch, Rudiger, "Expectations and exchange rate dynamics," *Journal of political Economy*, 1976, *84* (6), 1161–1176.

Dou, Winston Wei, "Embrace or fear uncertainty: Growth options, limited risk sharing, and asset prices," *Unpublished working paper, MIT*, 2016.

Fagereng, Andreas, Martin Blomhoff Holm, Benjamin Moll, and Gisle Natvik, "Saving behavior across the wealth distribution: The importance of capital gains," Technical Report, National Bureau of Economic Research 2019.

Fama, Eugene F and James D MacBeth, "Risk, return, and equilibrium: Empirical tests," *Journal of political economy*, 1973, *81* (3), 607–636.

_ **and Kenneth R French**, "The cross-section of expected stock returns," *the Journal of Finance*, 1992, *47* (2), 427–465.

Gârleanu, Nicolae and Stavros Panageas, "Young, old, conservative, and bold: The implications of heterogeneity and finite lives for asset pricing," *Journal of Political Economy*, 2015, *123* (3), 670–685.

_ **,** _ **, and Jianfeng Yu**, "Financial entanglement: A theory of incomplete integration, leverage, crashes, and contagion," *American Economic Review*, 2015, *105* (7), 1979–2010.

Ge, Hong, Kai Xu, and Zoubin Ghahramani, "Turing: a language for flexible probabilistic inference," in "International Conference on Artificial Intelligence and Statistics, AISTATS 2018, 9-11 April 2018, Playa Blanca, Lanzarote, Canary Islands, Spain" 2018, pp. 1682–1690.

Gomes, Francisco, "Portfolio choice over the life cycle: A survey," *Annual Review of Financial Economics*, 2020, *12* (1), 277–304.

Gomez, Matthieu, "Asset prices and wealth inequality," 2017.

Greenwald, Daniel L, Matteo Leombroni, Hanno Lustig, and Stijn Van Nieuwerburgh, "Financial and Total Wealth Inequality with Declining Interest Rates," Technical Report, National Bureau of Economic Research 2021.

Greenwood, Jeremy and Boyan Jovanovic, "Financial development, growth, and the distribution of income," *Journal of Political Economy*, 1990, *98* (5, Part 1), 1076–1107.

Grossman, Sanford J and Jean-Luc Vila, "Optimal dynamic trading with leverage constraints," *Journal of Financial and Quantitative Analysis*, 1992, *27* (2), 151–168.

Guerrieri, Veronica and Guido Lorenzoni, "Credit crises, precautionary savings, and the liquidity trap," *The Quarterly Journal of Economics*, 2017, *132* (3), 1427–1467.

He, Zhiguo and Arvind Krishnamurthy, "A model of capital and crises," *The Review of Economic Studies*, 2012, *79* (2), 735–777.

Heathcote, Jonathan, Kjetil Storesletten, and Giovanni L Violante, "Consumption and labor supply with partial insurance: An analytical framework," *American Economic Review*, 2014, *104* (7), 2075–2126.

Heaton, John and Deborah Lucas, "Market frictions, savings behavior, and portfolio choice," *Macroeconomic Dynamics*, 1997, *1* (01), 76–101.

Herranz, Neus, Stefan Krasa, and Anne P Villamil, "Small firms in the SSBF," *Annals of Finance*, 2009, *5* (3), 341–359.

_ , _ , and _ , "Entrepreneurs, risk aversion, and dynamic firms," *Journal of Political Economy*, 2015, *123* (5), 1133–1176.

Herskovic, Bernard, Bryan Kelly, Hanno Lustig, and Stijn Van Nieuwerburgh, "The common factor in idiosyncratic volatility: Quantitative asset pricing implications," *Journal of Financial Economics*, 2016, *119* (2), 249–283.

_ , Thilo Kind, and Howard Kung, "Micro uncertainty and asset prices," *Available at SSRN 3220825*, 2018.

Hubmer, Joachim, Per Krusell, and Anthony A Smith Jr, "Sources of US wealth inequality: Past, present, and future," *Nber macroeconomics annual*, 2021, *35* (1), 391–455.

Iachan, Felipe S, Dejanir Silva, and Chao Zi, "Under-diversification and idiosyncratic risk externalities," *Journal of Financial Economics*, 2021.

Jeong, Hyeok and Robert M Townsend, "Sources of TFP growth: occupational choice and financial deepening," *Economic Theory*, 2007, *32* (1), 179–221.

Kaplan, Greg, "Inequality and the life cycle," *Quantitative Economics*, 2012, *3* (3), 471–525.

_ **and Giovanni L Violante**, "How much consumption insurance beyond self-insurance?," *American Economic Journal: Macroeconomics*, 2010, *2* (4), 53–87.

Karlan, Dean, Robert Osei, Isaac Osei-Akoto, and Christopher Udry, "Agricultural decisions after relaxing credit and risk constraints," *The Quarterly Journal of Economics*, 2014, *129* (2), 597–652.

Kinnan, Cynthia and Robert Townsend, "Kinship and financial networks, formal financial access, and risk reduction," *American Economic Review*, 2012, *102* (3), 289–93.

Kiyotaki, Nobuhiro and John Moore, "Credit cycles," *Journal of Political economy*, 1997, *105* (2), 211–248.

Krueger, Dirk and Fabrizio Perri, "Does income inequality lead to consumption inequality? Evidence and theory," *The Review of Economic Studies*, 2006, *73* (1), 163–193.

Kuznets, Simon, "Economic growth and income inequality," *The American economic review*, 1955, *45* (1), 1–28.

Levine, Ross, "Finance and growth: theory and evidence," *Handbook of economic growth*, 2005, *1*, 865–934.

Lucas, Robert E, "On the size distribution of business firms," *The Bell Journal of Economics*, 1978, pp. 508–523.

McElreath, Richard, *Statistical rethinking: A Bayesian course with examples in R and Stan*, Chapman and Hall/CRC, 2018.

Mel, Suresh De, David McKenzie, and Christopher Woodruff, "Returns to capital in microenterprises: evidence from a field experiment," *The Quarterly Journal of Economics*, 2008, *123* (4), 1329–1372.

Merton, Robert C, "Lifetime portfolio selection under uncertainty: The continuous-time case," *The Review of Economics and Statistics*, 1969, pp. 247–257.

Midrigan, Virgiliu and Daniel Yi Xu, "Finance and misallocation: Evidence from plant-level data," *American Economic Review*, 2014, *104* (2), 422–58.

Moll, Benjamin, "Inequality and financial development: A power-law kuznets curve," *Princeton University Note*, 2012.

_ , "Productivity losses from financial frictions: Can self-financing undo capital misallocation?," *American Economic Review*, 2014, *104* (10), 3186–3221.

Morduch, Jonathan, "Income smoothing and consumption smoothing," *Journal of Economic Perspectives*, 1995, *9* (3), 103–114.

Moskowitz, Tobias J and Annette Vissing-Jørgensen, "The returns to entrepreneurial investment: A private equity premium puzzle?," *American Economic Review*, 2002, *92* (4), 745–778.

Nardi, Mariacristina De and Giulio Fella, "Saving and wealth inequality," *Review of Economic Dynamics*, 2017, *26*, 280–300.

Panousi, Vasia and Dimitris Papanikolaou, "Investment, idiosyncratic risk, and ownership," *The Journal of Finance*, 2012, *67* (3), 1113–1148.

Samphantharak, Krislert and Robert M Townsend, *Households as corporate firms: an analysis of household finance using integrated household surveys and corporate financial accounting* number 46, Cambridge University Press, 2010.

_ **and** _ , "Risk and return in village economies," *American Economic Journal: Microeconomics*, 2018, *10* (1), 1–40.

Smith, Matthew, Danny Yagan, Owen Zidar, and Eric Zwick, "Capitalists in the Twenty-first Century," *The Quarterly Journal of Economics*, 2019, *134* (4), 1675–1745.

_ **, Owen Zidar, Eric Zwick et al.**, "Top Wealth in America: New Estimates and Implications for Taxing the Rich," Technical Report 2020.

Storesletten, Kjetil, Christopher I Telmer, and Amir Yaron, "Consumption and risk sharing over the life cycle," *Journal of monetary Economics*, 2004, *51* (3), 609–633.

_ **,** _ **, and** _ , "Asset pricing with idiosyncratic risk and overlapping generations," *Review of Economic Dynamics*, 2007, *10* (4), 519–548.

Swanson, Eric T, "Risk aversion and the labor margin in dynamic equilibrium models," *American Economic Review*, 2012, *102* (4), 1663–91.

Townsend, Robert M, "Risk and insurance in village India," *Econometrica*, 1994, pp. 539–591.

Udry, Christopher and Santosh Anagol, "The return to capital in Ghana," *American Economic Review*, 2006, *96* (2), 388–393.

Viceira, Luis M, "Optimal Portfolio Choice for Long-Horizon Investors with Nontradable Labor Income," *The Journal of Finance*, 2001, *56* (2), 433–470.

Winberry, Thomas, "A method for solving and estimating heterogeneous agent macro models," *Quantitative Economics*, 2018, *9* (3), 1123–1151.

Online Appendix

A Proofs

A.1 Proof of Lemma 1 and Proposition 1

Proof. We start by showing part (b) of Lemma 1, that is, we solve for $h_t(a)$ and its dynamics. Then, we proceed to solve for the entrepreneurs' value function and policy functions.

Pricing human wealth. Define the stochastic discount factor (SDF) for this economy as the process π_t satisfying the law of motion

$$\frac{d\pi_t}{\pi_t} = -r_t dt - p_t^{ag} dZ_t. \tag{A.1}$$

Without loss of generality, we assumed that the SDF is not exposed to idiosyncratic risk, as we only use the SDF to price human wealth which is not exposed to idiosyncratic risk. Integrating the process above, we obtain

$$\frac{\pi_z}{\pi_t} = \exp\left(-\int_t^z \left(r_u + \frac{(p_u^{ag})^2}{2}\right) du - \int_t^z p_u^{ag} dZ_u\right). \tag{A.2}$$

Similarly, integrating the process for A_t

$$\frac{A_z}{A_t} = \exp\left(\int_t^z \left(\mu_A - \frac{\sigma_A^2}{2}\right) du + \int_t^z \sigma_A dZ_u\right). \tag{A.3}$$

Hence, we can explicitly compute the following expectation

$$\begin{aligned}\mathbb{E}_t\left[\frac{\pi_z A_z}{\pi_t A_t}\right] &= \mathbb{E}_t\left[\exp\left(-\int_t^z \left(r_u - \mu_A + \frac{(p_u^{ag})^2 + \sigma_A^2}{2}\right) du - \int_t^z \left(p_u^{ag} - \sigma_A\right) dZ_u\right)\right] \\ &= \exp\left(-\int_t^z \left(r_u + p_u^{ag}\sigma_A - \mu_A\right) du\right),\end{aligned} \tag{A.4}$$

where we used Ito's isometry and the fact that p_t^{ag} is deterministic.

Human wealth is given by

$$h_t(a) = \mathbb{E}_t\left[\int_t^{t+T-a} \frac{\pi_z A_z}{\pi_t A_t} w_z \bar{l}(a+z-t)dz\right] = \int_t^{t+T-a} e^{-\int_t^z \left(r_u + p_u^{ag}\sigma_A - \mu_A\right)du} w_z \bar{l}(a+z-t)dz. \tag{A.5}$$

Consider the human wealth for someone born at date s, so $a = t - s$:

$$h_t(t-s) = \int_t^{s+T} e^{-\int_t^z \left(r_u + p_u^{ag}\sigma_A - \mu_A\right)du} w_z \bar{l}(z-s)dz. \tag{A.6}$$

Differentiating the expression above with respect to time yields

$$\frac{\partial h_t(a)}{\partial t} + \frac{\partial h_t(a)}{\partial a} = \left(r_t + p_t^{ag}\sigma_A - \mu_A\right) h_t(a) - w_t \bar{l}(a), \tag{A.7}$$

which gives (14) in a stationary equilibrium.

The HJB equation. The HJB equation for problem (9) is given by

$$\rho \tilde{V}_t(\tilde{n}, t-s; A_t) = \max_{\tilde{c}_t, \tilde{\theta}_t^{ag}, \tilde{\theta}_t^{id}, k_t, l_t, \iota_t} \left\{ \frac{\tilde{c}_t^{1-\gamma}}{1-\gamma} + \frac{\mathbb{E}_t\left[d\tilde{V}_{i,t}\right]}{dt} \right\}, \tag{A.8}$$

subject to (7) as well as the terminal and boundary conditions

$$\tilde{V}_t(\tilde{n}, T) = (1-\psi)^\gamma V^* \frac{\tilde{n}^{1-\gamma}}{1-\gamma}; \qquad \lim_{\tilde{n} \to -\tilde{h}_t(a)} \tilde{V}_t(\tilde{n}, a) = \begin{cases} 0, & \text{if } \gamma < 1 \\ -\infty, & \text{if } \gamma \geq 1 \end{cases}. \tag{A.9}$$

The terminal condition captures the effect of bequests and the boundary condition the fact that consumption is zero if the entrepreneur hits the natural borrowing limit.

Using Ito's lemma, the HJB reduces to a partial differential equation for $\tilde{V}_t(\tilde{n}, a; A_t)$:

$$\rho \tilde{V}_t = \max_{\tilde{c}_t, \tilde{\theta}_t^{ag}, \tilde{\theta}_t^{id}, k_t, l_t, \iota_t} \frac{\tilde{c}_t^{1-\gamma}}{1-\gamma} + \frac{\partial \tilde{V}_t}{\partial t} + \frac{\partial \tilde{V}_t}{\partial a} + \frac{\partial \tilde{V}_t}{\partial \tilde{n}}\mu_{\tilde{n},t} + \frac{\partial \tilde{V}_t}{\partial A_t}\mu_A A_t + \frac{1}{2}\frac{\partial^2 \tilde{V}_t}{\partial \tilde{n}^2}(\sigma_{ag,t}^2 + \sigma_{id,t}^2) + \frac{\partial^2 \tilde{V}_t}{\partial \tilde{n} \partial A_t}\sigma_{ag,t}\sigma_A A_t + \frac{1}{2}\frac{\partial \tilde{V}_i}{\partial A_t^2}\sigma_A^2 A_t^2, \tag{A.10}$$

subject to (7), where $(\mu_{\tilde{n},t}, \sigma_{ag,t}, \sigma_{id,t})$ are the drift and diffusion terms for $\tilde{n}_t$.

First, we verify that the following guess for the value function solves the PDE

$$\tilde{V}_t(\tilde{n}, a; A_t) = \zeta_t(a)^{-\gamma} \frac{\left(\tilde{n} + A_t h_t(a)\right)^{1-\gamma}}{1-\gamma}. \tag{A.11}$$

Plugging the derivatives of the equation above into the HJB equation, we obtain

$$\frac{\rho}{1-\gamma} = \max_{c_{i,t},k_{i,t},l_{i,t},\iota_{i,t},\theta^{ag}_{i,t},\theta^{id}_{i,t}} \left\{ \frac{\zeta_t^{\gamma}(a)}{1-\gamma}\left(\frac{c_{i,t}}{\omega_{i,t}}\right)^{1-\gamma} - \frac{\gamma}{1-\gamma}\frac{1}{\zeta_t(a)}\left(\frac{\partial \zeta_t(a)}{\partial t} + \frac{\partial \zeta_t(a)}{\partial a}\right) + r_t + \frac{q_t k_{i,t}}{\omega_{i,t}}(\mu^R_{i,t} - r_t) \right.$$
$$- \frac{p^{ag}_t \theta^{ag}_{i,t}}{\omega_{i,t}} + \frac{h_{i,t}}{\omega_{i,t}}\sigma_A p^{ag}_t - \frac{c_{i,t}}{\omega_{i,t}} - \frac{\gamma}{2}\left[\left(\frac{q_t k_{i,t} + h_{i,t}}{\omega_{i,t}}\sigma_A - \frac{\theta^{ag}_{i,t}}{\omega_{i,t}}\right)^2 + \left(\frac{q_t k_{i,t}}{\omega_{i,t}}\sigma_I - \frac{\theta^{id}_{i,t}}{\omega_{i,t}}\right)^2\right]$$
$$\left. + p^{id}_t\left[(1-\phi)\frac{q_t k_{i,t}}{\omega_{i,t}}\sigma_I - \frac{\theta^{id}_{i,t}}{\omega_{i,t}}\right]\right\}, \tag{A.12}$$

where $p^{id}_{i,t}$ denotes the Lagrange multiplier on the skin-in-the-game constraint.

From the expression above, it is immediate that the optimal value of $(l_{i,t}, \iota_{i,t})$ maximizes the expected return on the business. The first-order conditions for $(l_{i,t}, \iota_{i,t})$ are given in (11) and (12), respectively. The expected return on the business will be equalized, allowing us to write $\mu^R_{i,t} = \mu^R_t$.

Policy functions. The first order condition for $\theta^{id}_{i,t}$ is given by

$$\gamma\left[\frac{q_t k_{i,t}}{\omega_{i,t}}\sigma_I - \frac{\theta^{id}_{i,t}}{\omega_{i,t}}\right] = p^{id}_{i,t}. \tag{A.13}$$

The equation above implies that the skin-in-the-game constraint is always binding, so $p^{id}_{i,t} > 0$ and $\theta^{id}_t = (1-\phi)q_t k_{i,t}\sigma_I$. If this was not the case, i.e. $p^{id}_{i,t} = 0$, then we would have $\theta^{id}_{i,t} = q_t k_{i,t}\sigma_I$, which violates the skin-in-the-game constraint.

The first-order conditions for capital and aggregate insurance are given by

$$\mu^R_t - r + p^{id}_{i,t}(1-\phi)\sigma_I = \gamma\left[\left(\frac{q_t k_{i,t} + h_{i,t}}{\omega_{i,t}}\sigma_A - \frac{\theta^{ag}_{i,t}}{\omega_{i,t}}\right)\sigma_A + \left(\frac{q_t k_{i,t}}{\omega_{i,t}}\sigma_I - \frac{\theta^{id}_{i,t}}{\omega_{i,t}}\right)\sigma_I\right]$$
$$p^{ag}_t = \gamma\left(\frac{q_t k_{i,t} + h_{i,t}}{\omega_{i,t}}\sigma_A - \frac{\theta^{ag}_{i,t}}{\omega_{i,t}}\right). \tag{A.14}$$

Combining the expressions above, we obtain

$$p^{id}_{i,t} = \frac{\mu^R_t - r_t - p^{ag}_t\sigma_A}{\phi\sigma_I}, \tag{A.15}$$

which coincides with expression (18) after we write $p^{id}_{i,t} = p^{id}_t$.

The demand for capital can be written as

$$\frac{q_t k_{i,t}}{\omega_{i,t}} = \frac{p_t^{id}}{\gamma \phi \sigma_I}. \tag{A.16}$$

Multiplying by $\omega_{i,t}/n_{i,t}$, we obtain expression (17). Solving for $\theta_{i,t}^{ag}$ in the optimality condition for aggregate insurance we obtain (19).

The first-order condition for consumption gives

$$\frac{c_{i,t}}{\omega_{i,t}} = \zeta_t(a). \tag{A.17}$$

Plugging the expressions above back into the HJB, we obtain a PDE for $\zeta_t(a)$

$$\frac{\partial \zeta_t(a)}{\partial t} + \frac{\partial \zeta_t(a)}{\partial a} = \zeta_t^2(a) - \bar{r}_t \zeta_t(a), \tag{A.18}$$

where $\bar{r}_t \equiv \frac{1}{\gamma}\rho + \left(1 - \frac{1}{\gamma}\right)\left[r + \frac{(p_t^{id})^2 + (p_t^{ag})^2}{2\gamma}\right]$.

Define $z_{s,t} \equiv \zeta_t^{-1}(t-s)$ as the wealth-consumption ratio for an entrepreneur born at date s. Differentiating with respect to t, we obtain

$$\dot{z}_{s,t} = -\frac{1}{\zeta_t^2(a)}\left[\frac{\partial \zeta_t}{\partial t} + \frac{\partial \zeta_t}{\partial a}\right] = \bar{r}_t z_{s,t} - 1. \tag{A.19}$$

Solving the above differential equation, we get

$$z_{s,t} = \int_t^{s+T} e^{-\int_t^u \bar{r}_z dz} du + e^{-\int_t^{s+T} \bar{r}_z dz} z_{s,s+T}, \tag{A.20}$$

or in terms of $\zeta_t(a)$, we have

$$\zeta_t(a) = \frac{1}{\int_t^{t+T-a} e^{-\int_t^u \bar{r}_z dz} du + e^{-\int_t^{t+T-a} \bar{r}_z dz}(1-\psi)(V^*)^{\frac{1}{\gamma}}}, \tag{A.21}$$

where we used the boundary condition $\zeta_t^{-1}(T) = (1-\psi)\,(V^*)^{\frac{1}{\gamma}}$.

Assuming $(V^*)^{\frac{1}{\gamma}} = \frac{1}{\bar{r}}$ and a stationary equilibrium, where $\bar{r}_t = \bar{r}$, we obtain

$$\zeta(a) = \frac{\bar{r}}{1 - \psi e^{-\bar{r}(T-a)}}. \tag{A.22}$$

$(V^*)^{\frac{1}{\gamma}} = \frac{1}{\bar{r}}$ ensures $\psi = 0$ gives the corresponding value in an infinite-horizon economy.

Price and quantity of aggregate insurance. The demand for aggregate insurance for entrepreneus is $\theta_{i,t}^{ag} = (q_t k_{i,t} + h_{i,t})\sigma_A - \omega_{i,t}\frac{p_t^{ag}}{\gamma}$. A similar expression holds for wage earners

$$\theta_{j,t}^{ag} = h_{j,t}\sigma_A - (n_{j,t} + h_{j,t})\frac{p^{ag}}{\gamma}, \tag{A.23}$$

Combining the demand for aggregate insurance for entrepreneurs and wage earners with the corresponding market-clearing condition, we obtain

$$\int_{\mathcal{E}_t}\left[(qk_{i,t} + h_{i,t})\sigma_A - (n_{i,t} + h_{i,t})\frac{p^{ag}}{\gamma}\right]di + \int_{\mathcal{W}_t}\left[h_{j,t}\sigma_A - (n_{j,t} + h_{j,t})\frac{p^{ag}}{\gamma}\right]di = 0. \tag{A.24}$$

Rearranging the expression above, we can solve for the price of aggregate insurance p^{ag}

$$p_t^{ag} = \frac{\int_{\mathcal{E}_t}(qk_{i,t} + h_{i,t})di + \int_{\mathcal{W}_t} h_{j,t}dj}{\int_{\mathcal{E}_t}(n_{i,t} + h_{i,t})di + \int_{\mathcal{W}_t}(n_{j,t} + h_{j,t})dj}\gamma\sigma_A = \gamma\sigma_A, \tag{A.25}$$

using the fact that $\int_{\mathcal{E}_t} n_{i,t}di + \int_{\mathcal{W}_t} n_{j,t}dj = \int_{\mathcal{E}_t} qk_{i,t}di$. The demand for aggregate insurance for entrepreneurs if then given by $\theta_{j,t}^{ag} = (q_t k_{i,t} - n_{i,t})\sigma_A$.

□

A.2 Proof of Proposition 2

Proof. We start by deriving the law of motion of financial wealth for an entrepreneur of a given age. Using the value of capital, $k_{i,t}$, aggregate and idiosyncratic insurance, $(\theta_{i,t}^{ag}, \theta_{i,t}^{id})$, and the definition of the price of idiosyncratic risk, p_t^{id}, given in Proposition 1, we can write the law of motion of financial wealth as follows

$$d\tilde{n}_{i,t} = \left[r_t\tilde{\omega}_{i,t} + \frac{(p_t^{id})^2}{\gamma}\tilde{\omega}_{i,t} + \frac{(p_t^{ag})^2}{\gamma}\tilde{\omega}_{i,t} - \tilde{h}_{i,t}(r_t + p_t^{ag}\sigma_A) + \tilde{w}_t\bar{l}_{i,t} - \tilde{c}_{i,t}\right]dt + \left(\tilde{\omega}_{i,t}\frac{p_t^{ag}}{\gamma} - \tilde{h}_{i,t}\sigma_A\right)dZ_t + \frac{p_t^{id}}{\gamma}\tilde{\omega}_{i,t}dZ_{i,t}, \tag{A.26}$$

where $\tilde{\omega}_{i,t} = \tilde{n}_{i,t} + \tilde{h}_{i,t}$.

Using the fact that $p_t^{ag} = \gamma\sigma_A$ in equilibrium, we find that the aggregate risk exposure of entrepreneurs is given $\tilde{n}_{i,t}\sigma_A$. Hence, scaled financial wealth, $n_{i,t} = \tilde{n}_{i,t}/A_t$, does not respond to aggregate shocks. The evolution of $n_{i,t}$ can then be written as

$$dn_{i,t} = \mu_{n,t}(n_{i,t}, a)dt + \sigma_{n,t}(n_{i,t}, a)dZ_{i,t}, \tag{A.27}$$

where

$$\mu_{n,t}(n,a) = \left[r_t + \frac{(p_t^{id})^2}{\gamma} + \frac{(p_t^{ag})^2}{\gamma} - \mu_A - \zeta_t(a) \right] (n + h_t(a)) - \mu_{h,t}(a) \tag{A.28}$$

$$\sigma_{n,t}(n,a) = \frac{p_t^{id}}{\gamma}(n + h_t(a)), \tag{A.29}$$

and $\mu_{h,t}(a)$ is the drift of $h_t(a)$.

Derivation of Equation (21). Notice that total wealth evolves according to

$$\frac{d\omega_{i,t}}{\omega_{i,t}} = \left[r_t + \frac{(p_t^{ag})^2}{\gamma} + \frac{(p_t^{id})^2}{\gamma} - \mu_A - \zeta_t(t - s_i) \right] dt + \frac{p_t^{id}}{\gamma} dZ_{i,t}, \tag{A.30}$$

where s_i is the birthdate of entrepreneur i.

Let $\overline{\omega}_{s,t} \equiv \frac{\int_{\mathcal{E}_t} \mathbb{1}_{s_i=s} \omega_{i,t} di}{\int_{\mathcal{E}_t} \mathbb{1}_{s_i=s} di}$ denote the average total wealth of entrepreneurs born at date s. The law of motion of $\overline{\omega}_{s,t}$ is given by

$$d\overline{\omega}_{s,t} = \left[r_t + \frac{(p_t^{ag})^2}{\gamma} + \frac{(p_t^{id})^2}{\gamma} - \mu_A - \zeta_t(t - s) \right] \overline{\omega}_{s,t} dt \tag{A.31}$$

where the idiosyncratic risk is diversified by averaging out across entrepreneurs of a given cohort.

It is convenient to express total wealth as a function of age instead of the entrepreneurs' birthdate. Let $\omega_t(a)$ denote the average total wealth of investors with age a at period t. Using the fact that $\overline{\omega}_{s,t} = \omega_t(t - s)$, we obtain the following PDE for $\omega_t(a)$:

$$\frac{\partial \omega_t(a)}{\partial t} + \frac{\partial \omega_t(a)}{\partial a} = \left[r_t + \frac{(p_t^{ag})^2}{\gamma} + \frac{(p_t^{id})^2}{\gamma} - \mu_A - \zeta_t(a) \right] \omega_t(a). \tag{A.32}$$

In a stationary equilibrium, $\omega_t(a)$ does not depend on calendar time t, which allow us to write

$$\frac{d \log \omega(a)}{da} = r + \frac{(p^{ag})^2}{\gamma} + \frac{(p^{id})^2}{\gamma} - \mu_A - \zeta(a). \tag{A.33}$$

Integrating the expression above, we obtain

$$\log \omega(a) = \log \omega(0) + \left[r + \frac{(p^{ag})^2}{\gamma} + \frac{(p^{id})^2}{\gamma} - \mu_A \right] a - \int_0^a \zeta(u) du. \tag{A.34}$$

Using the fact $\log \frac{\omega(a)}{\omega(0)} = \log \frac{f(a)\omega(a)}{f(0)\omega(0)} + ga$ and the identity $\omega(a) = n(a)\left(1 + \frac{h(a)}{n(a)}\right)$, we obtain expression (21) after some rearrangement.

Initial wealth. The expression for $\omega(a)$ in levels can be written as

$$\omega(a) = \omega(0)e^{\left(r + \frac{(p^{ag})^2}{\gamma} + \frac{(p^{id})^2}{\gamma} - \mu_A\right)a} \frac{e^{-\bar{r}a} - \psi e^{-\bar{r}T}}{1 - \psi e^{-\bar{r}T}}. \tag{A.35}$$

Evaluating at $a = T$ gives

$$\omega(T) = \omega(0)e^{\left(r + \frac{(p^{ag})^2}{\gamma} + \frac{(p^{id})^2}{\gamma} - \mu_A - mpc_e\right)T}, \tag{A.36}$$

where $mpc_e = \frac{1}{T}\int_0^T \zeta(a)da$.

The boundary condition at age T implies $\omega(0) = e^{-gT}\omega(T) + h(0)$, then

$$\omega(0) = \frac{h(0)}{1 - e^{\left(r + \frac{(p^{ag})^2}{\gamma} + \frac{(p^{id})^2}{\gamma} - \mu_A - g - mpc_e\right)T}}. \tag{A.37}$$

Using $\omega(0) = n(0) + h(0)$ and rearranging the resulting expression for $n(0)$.

Derivation of Equation (22). Multiplying Equation (A.35) by $f(a)$, integrating over age, and using the fact that $f(a) = e^{-ga}f(0)$, we obtain

$$n_e + h_e = f(0)\omega(0)\int_0^T e^{\left(r + \frac{(p^{ag})^2}{\gamma} + \frac{(p^{id})^2}{\gamma} - (g + \mu_A)\right)a} \frac{e^{-\bar{r}a} - \psi e^{-\bar{r}T}}{1 - \psi e^{-\bar{r}T}} da, \tag{A.38}$$

which gives Equation (22) after some rearrangement .

□

A.3 Proof of Lemma 2

Proof. We derive the Kolmogorov Forward Equation as the limit of a discrete-time economy. The discrete-time approximation goes as follows. Time takes values on the discrete set $\{t^1, \ldots, t^L\}$, where $\Delta t = t^{l+1} - t^l$ is the constant time step. Scaled financial wealth $n_{i,t}$ takes values on a discrete grid, $n_{i,t} \in \{n^1, n^2, \ldots, n^J\}$ with a constant step size $\Delta n = n^{j+1} - n^j$. Age is also assumed to take values in a discrete grid $\{a^1, \ldots, a^K\}$, where $\Delta a = a^{k+1} - a^k$, $a^1 = 0$, and $a^K = T$. For simplicity, assume $\Delta a = \Delta t$. The probability

of moving up, down, or staying at the same point of the grid is chosen to approximate (A.27) and are given, respectively, by

$$p_u(n^j, a^k) = \frac{1}{2}\left[\frac{\sigma_n(n^j, a^k)^2}{\overline{\sigma}^2} + \frac{\mu_n(n^j, a^k)}{\overline{\sigma}^2}\Delta n\right] \tag{A.39}$$

$$p_d(n^j, a^k) = \frac{1}{2}\left[\frac{\sigma_n(n^j, a^k)^2}{\overline{\sigma}^2} - \frac{\mu_n(n^j, a^k)}{\overline{\sigma}^2}\Delta n\right] \tag{A.40}$$

$$p_s(n^j, a^k) = 1 - \frac{\sigma_n(n^j, a^k)^2}{\overline{\sigma}^2}. \tag{A.41}$$

where $\overline{\sigma} = \max_{1\leq j\leq J, 1\leq k\leq K} \sigma_n(n^j, a^k)$, $\Delta n = \overline{\sigma}\sqrt{\Delta t}$, and $\Delta a = \Delta t$.

Notice that the expected change in $n_{i,t}$, where $n_{i,t} = n^j$ and $a_i = a^k$, is given by

$$\mathbb{E}\left[n_{i,t+1} - n_{i,t}\right] = p_u(n^j, a^k)\Delta n + p_d(n^j, a^k)(-\Delta n) = \mu_n(n^j, a^k)\Delta t, \tag{A.42}$$

and

$$\mathbb{E}\left[(n_{i,t+1} - n_{i,t})^2\right] = p_u(n^j, a^k)\Delta n^2 + p_d(n^j, a^k)(-\Delta n)^2 = \sigma_n(n^j, a^k)^2\Delta t. \tag{A.43}$$

Let $m(n^j, a^k, t^l)$ denote the mass of agents with financial wealth n^j, age a^k, at period t^l. Summing over n^j, we obtain the mass of agents with age a^k, $M_{k,l} \equiv \sum_{j=1}^{J} m(n^j, a^k, t^l) = e^{g(t^l-(k-1)\Delta t)}$. Summing over (n^j, a^k), we obtain the total population $M_l = \sum_{k=1}^{K} M_{k,l}$, so $M_{l+1} = e^{g\Delta t}M_l$. The law of motion of m, for $k > 1$ and $1 < j < J$, is given by

$$\begin{aligned} m(n^j, a^k, t^l + \Delta t) = {} & p_u(n^j - \Delta n, a^k - \Delta a)m(n^j - \Delta n, a^k - \Delta a, t^l) + p_s(n^j, a^k - \Delta a)m(n^j, a^k - \Delta a, t^l) \\ & + p_d(n^j + \Delta n, a^k - \Delta a)m(n^j + \Delta n, a^k - \Delta a, t^l). \end{aligned} \tag{A.44}$$

The boundary conditions are defined as follows. For $j = 1$ and $j = J$, we will assume a reflecting boundary, that is, if n moves up from n_J or down from n_1, it is immediately reflected back to its initial position

$$\begin{aligned} m(n^J, a^k, t^l + \Delta t) = {} & p_u(n^J - \Delta n, a^k - \Delta a)m(n^J - \Delta n, a^k - \Delta a, t^l) + p_s(n^J, a^k - \Delta a)m(n^J, a^k - \Delta a, t^l) \\ & + p_u(n^j, a^k - \Delta a)m(n^J, a^k - \Delta a, t^l), \end{aligned} \tag{A.45}$$

and analogously for $j = 1$.

Finally, for $k = 1$, we have

$$m(e^{-gT}n^j, a^1, t^l + \Delta t) = e^{gT}\left[p_u\left(n^j - \Delta n, a^K\right) m\left(n^j - \Delta n, a^K, t^l\right) + p_s\left(n^j, a^k\right) m\left(n^j, a^K, t^l\right) + p_d\left(n^j + \Delta n, a^K\right) m\left(n^j + \Delta n, a^K, t^l\right)\right]. \tag{A.46}$$

since each one of the e^{gT} heirs inherit $e^{-gT}n^j$, where we assumed $e^{-gT}n^j$ belongs to the grid.

Let $f(n^j, a^k, t^l) \equiv \frac{m(n^j, a^k, t^l)}{M_l}$ denote the share of agents in state (n^j, a^k) in period t^l. Dividing both sides of (A.44) by M_l and taking a Taylor expansion, we obtain

$$\begin{aligned}(1 + g\Delta t)(f + f_t\Delta t) = {} & \frac{1}{2}\left(\frac{\sigma_n^2 - (\sigma_n^2)_n\Delta n + 0.5(\sigma_n^2)_{nn}\Delta n^2 - (\sigma_n^2)_a\Delta t}{\overline{\sigma}^2} + \frac{\mu_n - (\mu_n)_n\Delta n}{\overline{\sigma}^2}\Delta n\right)\left(f - f_a\Delta t - f_n\Delta n + 0.5f_{nn}\Delta n^2\right) \\ & + \frac{1}{2}\left(\frac{\sigma_n^2 + (\sigma_n^2)_n\Delta n + 0.5(\sigma_n^2)_{nn}\Delta n^2 - (\sigma_n^2)_a\Delta t}{\overline{\sigma}^2} - \frac{\mu_n + (\mu_n)_n\Delta n}{\overline{\sigma}^2}\Delta n\right)\left(f - f_a\Delta t + f_n\Delta n + 0.5f_{nn}\Delta n^2\right) \\ & + \left(1 - \frac{\sigma_n^2 - (\sigma_n^2)_a\Delta t}{\overline{\sigma}^2}\right)(f - f_a\Delta t) + o(\Delta t).\end{aligned} \tag{A.47}$$

Simplifying the expression above and taking the limit $\Delta t \to 0$, we obtain

$$f_t + f_a + gf = \frac{1}{2}(\sigma_n^2)_{nn}f - (\mu_n)_n f + (\sigma_n^2)_n f_n - \mu_n f_n + \frac{1}{2}\sigma_n^2 f_{nn}, \tag{A.48}$$

or, more explicitly, we can write the expression as follows

$$\frac{\partial f(n,a,t)}{\partial t} + \frac{\partial f(n,a,t)}{\partial a} + gf(n,a,t) = -\frac{\partial[f(n,a,t)\mu_n(n,a)]}{\partial n} + \frac{1}{2}\frac{\partial\left[f(n,a,t)\sigma_n^2(n,a)\right]}{\partial n^2}. \tag{A.49}$$

Let $f_t(n|a)$ denote the conditional density at date t, so $f_t(n,a) = f_t(n|a)f(a)$. We can write the Kolmogorov Forward Equation in terms of the conditional density:

$$f(a)\frac{\partial f_t(n|a)}{\partial t} + f(a)\frac{\partial f_t(n|a)}{\partial a} + f_t(n|a)f'(a) = -f(a)\frac{\partial[f_t(n|a)\mu_n(n,a)]}{\partial n} + f(a)\frac{1}{2}\frac{\partial\left[f_t(n|a)\sigma_n^2(n,a)\right]}{\partial n^2} - gf_t(n,a). \tag{A.50}$$

Dividing by $f(a)$ and using the fact that $f'(a) = -gf(a)$, we obtain

$$\frac{\partial f_t(n|a)}{\partial t} + \frac{\partial f_t(n|a)}{\partial a} = -\frac{\partial[f_t(n|a)\mu_n(n,a)]}{\partial n} + \frac{1}{2}\frac{\partial\left[f_t(n|a)\sigma_n^2(n,a)\right]}{\partial n^2}. \tag{A.51}$$

In a stationary equilibrium, we can ignore the dependence on calendar time to obtain

$$\frac{\partial f(n|a)}{\partial a} = -\frac{\partial[f(n|a)\mu_n(n,a)]}{\partial n} + \frac{1}{2}\frac{\partial\left[f(n|a)\sigma_n^2(n,a)\right]}{\partial n^2}. \tag{A.52}$$

□

A.4 Proof of Proposition 3

Proof. The law of motion of (log) total wealth is

$$d\log\omega_{i,t} = \left[r + \frac{(p^{ag})^2}{\gamma} + \frac{(p^{id})^2}{\gamma} - \frac{\bar{r}}{1-e^{-\bar{r}(T-(t-s))}} - \mu_A - \frac{1}{2}\left(\frac{p^{id}}{\gamma}\right)^2\right]dt + \frac{p^{id}}{\gamma}dZ_{i,t}, \tag{A.53}$$

where s denotes the birth date of entrepreneur i.

Integrating the expression above, we obtain

$$\log\omega_{i,t} = \log\omega_{i,s} + \int_s^t \left[r + \frac{(p^{ag})^2}{\gamma} + \frac{(p^{id})^2}{\gamma} - \frac{\bar{r}}{1-e^{-\bar{r}(T-(t'-s))}} - \mu_A - \frac{1}{2}\left(\frac{p^{id}}{\gamma}\right)^2\right]dt' + \frac{p^{id}}{\gamma}(Z_{i,t} - Z_{i,s}), \tag{A.54}$$

where $Z_{i,t} - Z_{i,s} \sim \mathcal{N}(0,a)$ and $a = t - s$.

Hence, $\log\omega_{i,t} \sim \mathcal{N}(m(a), v(a))$, where the mean and variance are given by

$$m(a) = \log h(0) + \left[r + \frac{(p^{ag})^2}{\gamma} + \frac{(p^{id})^2}{\gamma} - \mu_A - \frac{1}{2}\left(\frac{p^{id}}{\gamma}\right)^2 - \bar{r}\right]a + \log\frac{1-e^{-\bar{r}(T-a)}}{1-e^{-\bar{r}T}} \tag{A.55}$$

$$v(a) = \left(\frac{p^{id}}{\gamma}\right)^2 a, \tag{A.56}$$

using the fact that $\omega_{i,s_i} = h(0)$ when $\psi = 1$.

As the ratio of consumption to total wealth is the same for all entrepreneurs of the same age, the variance of log consumption is given by

$$\mathbb{V}[\log c_{i,t}|a] = \mathbb{V}[\log\omega_{i,t}|a] = \left(\frac{p^{id}}{\gamma}\right)^2 a. \tag{A.57}$$

Normalized financial wealth $n_{i,t} = \omega_{i,t} - h_{i,t}$ has a shifted log-normal distribution conditional on $s_i = s$, with support $(-h(a), \infty)$. The expected value and variance of $n_{i,t}$ are

$$\mathbb{E}[n|a] = h(0)e^{\left(r + \frac{(p^{ag})^2}{\gamma} + \frac{(p^{id})^2}{\gamma} - \mu_A - \bar{r}\right)a}\frac{1-e^{-\bar{r}(T-a)}}{1-e^{-\bar{r}T}} - h(a) \tag{A.58}$$

$$\mathbb{V}[n|a] = \left[e^{\left(\frac{p^{id}}{\gamma}\right)^2 a} - 1\right]\left[h(0)e^{\left(r + \frac{(p^{ag})^2}{\gamma} + \frac{(p^{id})^2}{\gamma} - \mu_A\right)a}\frac{e^{-\bar{r}a} - e^{-\bar{r}T}}{1-e^{-\bar{r}T}}\right]^2. \tag{A.59}$$

We show next that $\mathbb{V}[n|a]$ has an inverted U shape. Define the following functions:

$$v_1(a) = \left[e^{\left(\frac{p^{id}}{\gamma}\right)^2 a} - 1\right]^{\frac{1}{2}} e^{\left(r + \frac{(p^{ag})^2}{\gamma} + \frac{(p^{id})^2}{\gamma} - \mu_A\right)a}; \qquad v_2(a) = \frac{e^{-\bar{r}a} - e^{-\bar{r}T}}{1-e^{-\bar{r}T}}. \tag{A.60}$$

The derivative of the product of $v_1(a)$ and $v_2(a)$ will be positive if

$$v_1'(a)v_2(a) + v_1(a)v_2'(a) > 0 \iff \frac{v_1'(a)}{v_1(a)} > -\frac{v_2'(a)}{v_2(a)}, \tag{A.61}$$

for $a \neq 0$ and $a \neq T$. Notice that $-v_2'(a)/v_2(a)$ is positive, monotonically increasing, and approaches ∞ as a approaches T:

$$-\frac{v_2'(a)}{v_2(a)} = \bar{r}\frac{1}{1 - e^{-\bar{r}(T-a)}}. \tag{A.62}$$

The term $v_1'(a)/v_1(a)$ is positive, monotonically decreasing, and approaches $+\infty$ as $a \to 0$:

$$\frac{v_1'(a)}{v_1(a)} = \frac{1}{2}\left(\frac{p^{id}}{\gamma}\right)^2 \frac{e^{\left(\frac{p^{id}}{\gamma}\right)^2 a}}{e^{\left(\frac{p^{id}}{\gamma}\right)^2 a} - 1} + r + \frac{(p^{ag})^2}{\gamma} + \frac{(p^{id})^2}{\gamma} - \mu_A. \tag{A.63}$$

Hence, there exists a unique $0 < \hat{a} < T$ such that $v_1'(a)v_2(a) + v_1(a)v_2'(a) > 0$ for all $a < \hat{a}$ and $v_1'(a)v_2(a) + v_1(a)v_2'(a) < 0$ for all $a > \hat{a}$. Hence, $\mathbb{V}[n|a]$ is equal to zero at $a = 0$, it increases monotonically for $a < \hat{a}$, where it achieves the maximum, and it decreases towards zero for $\hat{a} < a \leq T$.

□

A.5 Proof of Lemma 3

Proof. The HJB for the entrepreneur's problem is given by

$$\rho \tilde{V}_t = \max_{\tilde{c}_i, \tilde{\theta}_i^{ag}, \tilde{\theta}_t^{id}, k_i, l_i, \iota_i} \frac{\tilde{c}_t^{1-\gamma}}{1-\gamma} + \frac{\mathbb{E}_t\left[d\tilde{V}_t\right]}{dt} \tag{A.64}$$

subject to (7). We guess-and-verify that the value function can be written as

$$\tilde{V}^j(\tilde{\omega}; A_t) = A_t^{1-\gamma} V^j\left(\frac{\tilde{\omega}}{A_t}\right). \tag{A.65}$$

where $V_j(\omega)$ is independent of A_t, and $j \in \{1, \ldots, n\}$ index the entrepreneur's type.

Let $\omega_{i,t} \equiv \tilde{\omega}_{i,t}/A_t$ denote scaled total wealth. From the law of motion of $h_{i,t} = \tilde{h}_{i,t}/A_t$ in Lemma 1 and the law of motion of $n_{i,t}$, we obtain

$$d\omega_{i,t} = \mu_{\omega_i,t}dt + \sigma_{i,t}^{ag}dZ_t + \sigma_{i,t}^{id}dZ_{i,t}, \tag{A.66}$$

where $\sigma_{i,t}^{ag} \equiv (qk_{i,t} + h_{i,t} - \omega_{i,t})\,\sigma_A - \theta_{i,t}^{ag}$, $\sigma_{i,t}^{id} \equiv qk_{i,t}\sigma_{I,i} - \theta_{i,t}^{id}$, and

$$\mu_{\omega_i,t} \equiv (r + p^{ag}\sigma_A - \mu_A)\omega_{i,t} + qk_{i,t}(\hat{\mu}_{i,t}^R - r - p^{ag}\sigma_A) + (p^{ag} - \sigma_A)\sigma_{i,t}^{ag} - c_{i,t}. \tag{A.67}$$

The HJB equation for the scaled value function can be written as

$$\hat{\rho}V^j = \max_{c_i,\theta_i^{ag},\theta_i^{id},k_i,l_i,\iota_i} \frac{c_t^{1-\gamma}}{1-\gamma} + V_a^j + V_\omega^j\left[\hat{r}\omega_{i,t} + qk_{i,t}(\hat{\mu}_{i,t}^R - r - p^{ag}\sigma_A) + \hat{p}^{ag}\sigma_{i,t}^{ag} - c_{i,t}\right] + \frac{1}{2}V_{\omega\omega}^j\left((\sigma_{i,t}^{ag})^2 + (\sigma_{i,t}^{id})^2\right). \tag{A.68}$$

subject to $\theta_{i,t}^{id} \leq (1-\phi)qk_{i,t}\sigma_{I,i}$ and $\omega_{i,t} \geq 0$, where $\sigma_{I,i} = \sigma_I^j$ and

$$\hat{\rho} \equiv \rho - (1-\gamma)\left(\mu_A - \frac{\gamma\sigma_A^2}{2}\right), \qquad \hat{r} \equiv r + p^{ag}\sigma_A - \mu_A, \qquad \hat{p}^{ag} \equiv p^{ag} - \gamma\sigma_A. \tag{A.69}$$

It is optimal to choose $l_{i,t}$ and $\iota_{i,t}$ to maximize expected returns. The investment demand is then given by Equation (12) and the labor demand is given by $w = \beta k_{i,t}^{\alpha} l_{i,t}^{\beta-1}$.

The expected return for entrepreneur i is given by

$$\hat{\mu}_{i,t}^R = \frac{(1-\beta)\left(\frac{\beta}{w}\right)^{\frac{\beta}{1-\beta}} k_{i,t}^{\frac{\alpha+\beta-1}{1-\beta}} - \iota(q)}{q} + \mu_A + \Phi(\iota(q)) - \delta. \tag{A.70}$$

The first-order condition for aggregate insurance is given by

$$\hat{p}_t^{ag} = -\frac{V_{\omega\omega}^j}{V_\omega^j}\sigma_{i,t}^{ag} \Rightarrow \theta_{i,t}^{ag} = (qk_{i,t} - n_{i,t})\sigma_A + \frac{V_\omega^j}{V_{\omega\omega}^j}\hat{p}^{ag}. \tag{A.71}$$

In equilibrium, we must have that average $\sigma_{i,t}^{ag}$ across entrepreneurs and workers must be equal to zero, such that scaled wealth is, on average, not exposed to aggregate risk. Then, we have $\hat{p}_t^{ag} = 0$, so $p_t^{ag} = \gamma\sigma_A$, as in the baseline model.

The first-order conditions for $k_{i,t}$ and $\theta_{i,t}^{id}$ are given by

$$\mu_{i,t}^R - r - \gamma\sigma_A^2 + p_i^{id}(1-\phi)\sigma_{I,i} = -\frac{V_{\omega\omega}^j}{V_\omega^j}\left[\sigma_{i,t}^{ag}\sigma_A + \sigma_{i,t}^{id}\sigma_{I,i}\right], \qquad p_{i,t}^{id} = -\frac{V_{\omega\omega}^j}{V_\omega^j}\sigma_{i,t}^{id}, \tag{A.72}$$

where $\mu_{i,t}^R$ denotes the expected *marginal* return on the business:

$$\mu_{i,t}^R = \frac{\alpha\left(\frac{\beta}{w}\right)^{\frac{\beta}{1-\beta}} k_{i,t}^{\frac{\alpha+\beta-1}{1-\beta}} - \iota(q)}{q} + \mu_A + \Phi(\iota(q)) - \delta. \tag{A.73}$$

$\mu^R_{i,t}$ corresponds to the expected profit from an additional unit of capital divided by the cost of one unit of capital, while $\hat{\mu}^R_{i,t}$ corresponds to the expected total profit divided by the cost of the entire capital stock. Under constant returns to scale, the two concepts coincide. With DRS, the marginal concept is the relevant determining the investment decision.

Given that $\sigma^{id}_{i,t} > 0$ by the insurance constraint, and given the concavity of the value function, $V^j_{\omega\omega} < 0$, we have that $p^{id}_{i,t} > 0$. Therefore, the insurance constraint is always binding, that is, $\theta_{i,t} = (1-\phi) q k_{i,t} \sigma_{I,i}$. Rearranging the expressions above, we obtain

$$p^{id}_{i,t} = \frac{\mu^R_{i,t} - r - \gamma\sigma^2_A}{\phi\sigma_{I,i}}, \qquad qk_{i,t} = -\frac{V^j_\omega}{V^j_{\omega\omega}} \frac{p^{id}_{i,t}}{\phi\sigma_{I,i}}. \tag{A.74}$$

Finally, the optimality condition for consumption is given by $c^{-\gamma}_{i,t} = V^j_\omega$. □

A.6 Proof of Propositions 4 and 5

Proof. We consider the case where entrepreneurs have different levels of skill, $e_i \in \{e^1, \ldots, e^n\}$, and different exposures to idiosyncratic risk $\sigma_{I,i} \in \{\sigma^1_I, \ldots, \sigma^n_I\}$. The share of entrepreneurs of type j is denoted by $\chi_{e,j}$. Consider the log-linear approximation of the policy functions:

$$\hat{c}_{i,t} = \psi^j_{c,\omega}\hat{\omega}_{i,t}, \qquad \hat{k}_{i,t} = \psi^j_{k,\omega}\hat{\omega}_{i,t}, \tag{A.75}$$

up to first order in $\hat{w}_{i,t} = \log\omega_{i,t} - \log\overline{\omega}_j$, where $\overline{\omega}_j \equiv \exp\mathbb{E}[\log\omega_{i,t}|\sigma^i_I = \sigma^j_I, e_i = e^j]$, $\hat{c}_{i,t} = \log c_{i,t} - \log c_j(\overline{\omega}_j)$, and $\hat{k}_{i,t} = \log k_{i,t} - \log k_j(\overline{\omega}_j)$.

Wages. Aggregating the labor demand condition, $w_t = \beta k^\alpha_{i,t} l^{\beta-1}_{i,t} e_i^{1-\alpha-\beta}$, we obtain

$$\int_{\mathcal{E}} l_{i,t}di = \left(\frac{\beta}{w_t}\right)^{\frac{1}{1-\beta}} \int_{\mathcal{E}} k_{i,t}^{\frac{\alpha}{1-\beta}} e_{i,t}^{\frac{1-\alpha-\beta}{1-\beta}} di = \left(\frac{\beta}{w_t}\right)^{\frac{1}{1-\beta}} \sum_{j=1}^n \tilde{\chi}_{e,j} e_i^{\frac{1-\alpha-\beta}{1-\beta}} \overline{k}_j^{\frac{\alpha}{1-\beta}} \left[\chi_e + \frac{\alpha\psi_{k,\omega}}{1-\beta}\frac{1}{\tilde{\chi}_{e,j}}\int_{\mathcal{E}_j}\hat{\omega}_{i,t}di\right]. \tag{A.76}$$

Using a law of large numbers to obtain $\int\hat{\omega}_{i,t}di = 0$, we obtain

$$w_t = \beta K^\alpha \left(\frac{\chi_e}{\bar{l}}\right)^{1-\alpha-\beta}, \tag{A.77}$$

where $K = \chi_e \left[\sum_{j=1}^n \tilde{\chi}_{e,j}(\overline{k}^j)^{\frac{\alpha}{1-\beta}}\right]^{\frac{1-\beta}{\alpha}} / \bar{l}$ is the aggregate capital-labor ratio, and $\tilde{\chi}_{e,j} \equiv \chi_{e,j} e_i^{\frac{1-\alpha-\beta}{1-\beta}}$. We choose the units of skill such that $\sum_{j=1}^n \tilde{\chi}_{e,j} = 1$. We also adopt the normalization $\bar{l} = \chi_e$, so $w_t = \beta K^\alpha$.

Expected returns. Expected returns are given by

$$\hat{\mu}_{i,t}^R = \overline{\hat{\mu}}^R + \psi_{\hat{\mu}^R,k}\hat{k}_{i,t}, \tag{A.78}$$

where $\overline{\hat{\mu}}^R \equiv \frac{(1-\beta)B\overline{k}^{\alpha-1}-\iota(q)}{q} + \mu_A + \Phi(\iota(q)) - \delta$ and $\psi_{\hat{\mu}^R,k} \equiv -\frac{(1-\alpha-\beta)B\overline{k}^{\alpha-1}}{q}$, where $B_j = e_j^{\frac{1-\alpha-\beta}{1-\beta}}\left(\frac{\overline{k}_j}{\overline{K}}\right)^{\frac{\alpha\beta}{1-\beta}}$. To ease notation, we dropped the subscript j from above.

Expected marginal returns. The expected marginal return on the business is given by

$$\mu_{i,t}^R = \overline{\mu}^R - \psi_{\mu^R,k}\hat{k}_t, \tag{A.79}$$

where $\overline{\mu}^R \equiv \frac{\alpha B\overline{k}^{\alpha-1}-\iota(q)}{q} + \mu_A + \Phi(\iota(q)) - \delta$ and $\psi_{\mu^R,k} \equiv \frac{1-\alpha-\beta}{1-\alpha}\frac{\alpha B\overline{k}^{\alpha-1}}{q}$.

Log portfolio returns. Define the expected log portfolio returns as

$$r_{i,t}^p \equiv \hat{r} + z_{i,t}(\mu_{i,t}^R - r - \gamma\sigma_A^2) - \frac{1}{2}\left(z_{i,t}\phi\sigma_I\right)^2. \tag{A.80}$$

where $z_{i,t} \equiv \frac{qk_{i,t}}{\omega_{i,t}}$. Log-linearizing the expression above, we obtain

$$r_{i,t}^p = \overline{r}^p + \psi_{r^p,z}\hat{z}_{i,t} + \psi_{r^p,k}k_{i,t}, \tag{A.81}$$

$\overline{r}^p \equiv \hat{r} + \overline{z}(\overline{\hat{\mu}}^R - r - \gamma\sigma_A^2) - \frac{1}{2}\left(\overline{z}\phi\sigma_I\right)^2$, $\psi_{r^p,z} \equiv \overline{z}(\overline{\hat{\mu}}^R - r - \gamma\sigma_A^2) - \left(\overline{z}\phi\sigma_I\right)^2$, $\psi_{r^p,k} \equiv \overline{z}\psi_{\hat{\mu}^R,k}$.

Wealth dynamics. The log of total wealth for entrepreneur i evolves according to

$$d\log\omega_{i,t} = \left[r_{i,t}^p - \frac{c_{i,t}}{\omega_{i,t}}\right]dt + z_{i,t}\phi\sigma_I dZ_{i,t}, \tag{A.82}$$

Log-linearizing the law of motion of $\omega_{i,t}$, we obtain

$$d\hat{\omega}_{i,t} = \left[\psi_{\omega,0} + \psi_{\omega,\omega}\hat{\omega}_{i,t}\right]dt + \phi\sigma_I z_{i,t}dZ_{i,t}, \tag{A.83}$$

where $\psi_{\omega,0} = \overline{r}^p - \frac{\overline{c}}{\overline{\omega}}$, and $\psi_{\omega,\omega} = \psi_{r^p,z}(\psi_{k,\omega} - 1) + \psi_{r^p,k}\psi_{k,\omega} - \frac{\overline{c}}{\overline{\omega}}(\psi_{c,\omega} - 1)$.

Risk taking. The first-order condition for capital and consumption are given by

$$-V_{\omega\omega}\sigma_{i,t}^{id} = V_\omega p_{i,t}^{id}, \qquad V_\omega = c_{i,t}^{-\gamma}. \tag{A.84}$$

Using the fact that $dV_\omega dZ_{i,t} = V_{\omega\omega}\sigma_{i,t}^{id}dt$ and the expressions above, we can express the optimality condition for capital as follows:

$$p_{i,t}^{id}dt = \gamma \frac{dc_{i,t}}{c_{i,t}} dZ_{i,t}. \tag{A.85}$$

Up to first order, we have that $\frac{dc_{i,t}}{c_{i,t}} dZ_{i,t} = d\hat{c}_{i,t} dZ_{i,t} = \psi_{c,\omega} d\hat{\omega}_{i,t} dZ_{i,t}$. We can then write the expression above as follows:

$$\mu_{i,t}^R - r - \gamma\sigma_A^2 = \gamma\psi_{c,\omega}(\phi\sigma_I)^2 z_{i,t}. \tag{A.86}$$

Evaluating the expression above at $\omega_{i,t} = \overline{\omega}$, we obtain

$$\frac{q\overline{k}}{\overline{\omega}} = \frac{\overline{\mu}^R - r - \gamma\sigma_A^2}{\gamma\psi_{c,\omega}(\phi\sigma_I)^2} = \frac{\overline{p}^{id}}{\gamma\psi_{c,\omega}\phi\sigma_I}. \tag{A.87}$$

Linearizing the optimality condition for capital and matching coefficients, we obtain

$$-\psi_{\mu^R,k}\psi_{k,\omega} = (\overline{\mu}^R - r - \gamma\sigma_A^2)(\psi_{k,\omega} - 1). \tag{A.88}$$

Rearranging the expression above, we obtain

$$\psi_{k,\omega} \equiv \frac{\overline{\mu}^R - r - \gamma\sigma_A^2}{\overline{\mu}^R - r - \gamma\sigma_A^2 + \psi_{\mu^R,k}}. \tag{A.89}$$

Consumption. The envelope condition with respect to ω is given by

$$\hat{\rho}V_\omega = V_\omega \hat{r} + \frac{\mathbb{E}\left[dV_\omega\right]}{dt}. \tag{A.90}$$

Using Ito's lemma, we can write the expression above as follows

$$r = \rho + \gamma\left(\mu_A + \frac{1}{dt}\mathbb{E}[d\hat{c}_{i,t}]\right) - \frac{\gamma(\gamma+1)}{2}\sigma_A^2 - \frac{\gamma^2}{2}\mathbb{E}[(d\hat{c}_{i,t})^2]. \tag{A.91}$$

Up to first order, the expression above can be written as

$$r = \rho + \gamma\left[\mu_A + \psi_{c,\omega}\left(\psi_{\omega,0} + \psi_{\omega,\omega}\hat{\omega}_{i,t}\right)\right] - \frac{\gamma(\gamma+1)}{2}\sigma_A^2 - \frac{\gamma^2}{2}\psi_{c,\omega}^2(\phi\sigma_I)^2 z_{i,t}^2. \tag{A.92}$$

By matching coefficients, we obtain the condition

$$\psi_{r^p,z}(\psi_{k,\omega}-1)+\psi_{r^p,k}\psi_{k,\omega}-\frac{\overline{c}}{\overline{\omega}}(\psi_{c,\omega}-1)=\frac{(\overline{p}^{id})^2}{\gamma\psi_{c,\omega}}(\psi_{k,\omega}-1) \tag{A.93}$$

Rearranging the expression above, we obtain

$$\frac{\overline{c}}{\overline{\omega}}\psi_{c,\omega}^2-\psi_{c,\omega}\left[\psi_{r^p,z}(\psi_{k,\omega}-1)+\psi_{r^p,k}\psi_{k,\omega}+\frac{\overline{c}}{\overline{\omega}}\right]-\frac{(\overline{p}^{id})^2}{\gamma}(1-\psi_{k,\omega}). \tag{A.94}$$

Solving the quadratic equation, we obtain

$$\psi_{c,\omega}=\frac{\overline{\omega}}{2\overline{c}}\left[\psi_{r^p,z}(\psi_{k,\omega}-1)+\psi_{r^p,k}\psi_{k,\omega}+\frac{\overline{c}}{\overline{\omega}}+\sqrt{\left(\psi_{r^p,z}(\psi_{k,\omega}-1)+\psi_{r^p,k}\psi_{k,\omega}+\frac{\overline{c}}{\overline{\omega}}\right)^2+4\frac{(\overline{p}^{id})^2}{\gamma}(1-\psi_{k,\omega})\frac{\overline{c}}{\overline{\omega}}}\right]. \tag{A.95}$$

It remains to determine the intercept:

$$r=\rho+\gamma\left[\mu_A+\psi_{c,\omega}\left(\overline{r}^p-\frac{\overline{c}}{\overline{\omega}}\right)\right]-\frac{\gamma(\gamma+1)}{2}\sigma_A^2-\frac{\gamma^2}{2}\psi_{c,\omega}^2(\phi\sigma_I)^2\overline{z}^2. \tag{A.96}$$

We can solve for the consumption-wealth ratio:

$$\frac{\overline{c}}{\overline{\omega}}=\frac{\rho}{\gamma\psi_{c,\omega}}+\left(1-\frac{1}{\gamma\psi_{c,\omega}}\right)\left[r+\frac{(p^{ag})^2}{2\gamma}+\frac{(\overline{p}^{id})^2}{2\gamma\psi_{c,\omega}}\right]+\left[\frac{\gamma\sigma_A^2}{2}-\mu_A\right]\left(1-\psi_{c,\omega}^{-1}\right)+\overline{z}(\overline{\hat{\mu}}^R-\overline{\mu}^R). \tag{A.97}$$

The expression above reduces to $\frac{\overline{c}}{\overline{\omega}}=\overline{r}$ in the case with constant returns to scale.

Determining the approximation point. At the point $\hat{\omega}_{i,t}=0$, the drift of $\hat{\omega}_{i,t}$ is zero, which implies that $\overline{r}^p=\frac{\overline{c}}{\overline{\omega}}$. The triplet $(\overline{c}_j,\overline{k}_j,\overline{\omega}_j)$ satisfies the conditions:

$$r=\rho+\gamma\mu_A-\frac{\gamma(\gamma+1)}{2}\sigma_A^2-\frac{(\overline{p}_j^{id})^2}{2},\qquad \overline{z}_j=\frac{\overline{p}^{id}}{\gamma\psi_{c,\omega}\phi\sigma_I^j},\qquad \overline{r}_j^p=\frac{\overline{c}_j}{\overline{\omega}_j}, \tag{A.98}$$

where $\overline{r}^p=r+\gamma\sigma_A^2-\mu_A+\frac{(\overline{p}^{id})^2}{\gamma\psi_{c,\omega}}-\frac{1}{2}\left(\frac{\overline{p}^{id}}{\gamma\psi_{c,\omega}}\right)^2+\overline{z}(\overline{\hat{\mu}}^R-\overline{\mu}^R)$. These equations give the value of $(\overline{c},\overline{k},\overline{\omega})$ given $\psi_{c,\omega}$. We can then iterate between the computation of $\psi_{c,\omega}$ and $(\overline{c},\overline{k},\overline{\omega})$ until convergence. Using the fact that $r=\rho_w+\gamma\mu_A-\frac{\gamma(\gamma+1)}{2}\sigma_A^2$, we can solve for the price of idiosyncratic risk:

$$\overline{p}_j^{id}=\sqrt{2(\rho-\rho_w)}. \tag{A.99}$$

Hence, the unconditional price of idiosyncratic risk is equalized across entrepreneurs.

Capital for type j satsifies the condition:

$$r + p^{ag}\sigma_A + \overline{p}_j^{id}\phi\sigma_I^j = \frac{\alpha K^{-\frac{\alpha\beta}{1-\beta}}\overline{k}_j^{\frac{\alpha}{1-\beta}-1}e_i^{\frac{1-\alpha-\beta}{1-\beta}} - \iota(q)}{q} + \mu_A + \Phi(\iota(q)) - \delta. \tag{A.100}$$

We can rearrange the expression above as follows:

$$\zeta_j = K^{-\frac{\alpha\beta}{1-\beta}}\overline{k}_j^{\frac{\alpha+\beta-1}{1-\beta}}e_j^{\frac{1-\alpha-\beta}{1-\beta}} \Rightarrow \sum_{j=1}^{n}\tilde{\chi}_{e,j}e_j\zeta_j^{\frac{\alpha}{\alpha+\beta-1}} = K^{-\frac{\alpha\beta}{1-\beta}\frac{\alpha}{\alpha+\beta-1}}\sum_{j=1}^{n}\tilde{\chi}_{e,j}\overline{k}_j^{\frac{\alpha}{1-\beta}}e_j^{\frac{1-\alpha-\beta}{1-\beta}} \tag{A.101}$$

where $\zeta_j \equiv \alpha^{-1}\left\{q\left[r + p^{ag}\sigma_A + \overline{p}^{id}\phi\sigma_I^j - (\mu_A + \Phi(\iota(q)) - \delta)\right] + \iota(q)\right\}$.

Rearranging the expression above, we obtain

$$\zeta = K^{\alpha-1}, \tag{A.102}$$

where $\zeta = \left[\sum_{j=1}^{n}\tilde{\chi}_{e,j}e_j\zeta_j^{\frac{\alpha}{\alpha+\beta-1}}\right]^{\frac{\alpha+\beta-1}{\alpha}}$. The value of $\overline{k}_j$ is then given by $\overline{k}_j = e_jK\left(\frac{\zeta_j}{\zeta}\right)^{\frac{1-\beta}{\alpha+\beta-1}}$.

Price of idiosyncratic risk. The price of idiosyncratic risk is given by

$$p_{i,t}^{id} = \frac{\mu_{i,t}^R - r - \gamma\sigma_A^2}{\phi\sigma_{I,i}} = \overline{p}^{id} - \frac{\psi^j_{\mu^R,k}}{\phi\sigma_I^j}\psi^j_{k,\omega}\hat{\omega}_{i,t}. \tag{A.103}$$

Using the fact that $\mathbb{E}[\hat{\omega}_{i,t}]$ by definition of the approximation point, we obtain $\mathbb{E}[p_{i,t}^{id}] = \overline{p}^{id}$. Unconditional expected marginal returns are then given by

$$\mathbb{E}[\mu_{i,t}^R] = r + p^{ag}\sigma_A + \overline{p}^{id}\phi\sigma_{I,i}. \tag{A.104}$$

□

A.7 Proof of Proposition 6

Proof. Aggregating Equation (3) and using the fact that labor supply grows at rate g, we obtain

$$\dot{K}_t = \left[\Phi(\iota(q_t)) - \delta - g\right]K_t, \tag{A.105}$$

given the initial condition $K_0 = K^*$. From (A.15), we obtain the expression

$$r + p_t^{ag}\sigma_A + p_t^{id}\phi\sigma_I = \frac{\alpha K_t^{\alpha-1} - \iota(q_t)}{q_t} + \frac{\dot{q}_t}{q_t} + \Phi(\iota(q_t)) - \delta + \mu_A. \tag{A.106}$$

Using $p_t^{ag} = \gamma\sigma_A$ and $p^{id} = \gamma\phi\sigma_I \frac{q_t K_t}{\chi_e \omega_{e,t}}$ and solving for $\dot{q}_t$, we obtain

$$\dot{q}_t = \left[r + \gamma\sigma_A^2 + \gamma\phi^2\sigma_I^2 \frac{q_t K_t}{\chi_e \omega_{e,t}} + \delta - \mu_A - \Phi(\iota(q_t)) - \frac{\alpha K_t^{\alpha-1} - \iota(q_t)}{q_t} \right] q_t, \tag{A.107}$$

where $\omega_{e,t} = n_{e,t} + h_{e,t}$.

The ODE above is subject to the terminal condition

$$\lim_{t\to\infty} q_t = q, \tag{A.108}$$

where q is the value of q_t in the new stationary equilibrium.

The first PDE was derived in the proof of Lemma 1 and it was given in (A.18). The boundary conditions are $\zeta_t(T) = (1-\psi)^{-1} (V^*)^{-\frac{1}{\gamma}}$ and

$$\lim_{t\to\infty} \zeta_t(a) = \zeta(a), \tag{A.109}$$

where $\zeta(a)$ is the value in the new stationary equilibrium.

The PDE for the human wealth was given in (A.7). The boundary conditions are $h_t(T) = 0$ and

$$\lim_{t\to\infty} h_t(a) = h(a), \tag{A.110}$$

where $h(a)$ is the value in the new stationary equilibrium.

The PDE for total wealth was derived in the proof of Proposition 2 and it was given in (A.32). The first boundary condition is $\omega_t(0) = e^{-g^T}\omega_t(T) + h_t(0)$. The initial condition for $\omega_t(a)$ is given by

$$\omega_0(a) = n^*(a) + (q_0 - q^*)k^*(a) + h_0(a), \tag{A.111}$$

where variables with an asterisk denote values before the change in ϕ.

□

B Data

We discuss next our empirical measures in more detail. For an extensive discussion of the Townsend Thai Monthly Survey and the derivation of entrepreneurs' balance sheet information from the survey questionnaire, see Samphantharak and Townsend (2010).

B.1 Sample selection and variable definition

The dataset includes information on both economic and demographic variables. Economic variables include households' assets and liabilities, financial wealth, business and labor income, and consumption. Demographic and geographic variables consist of the age of the household's head, year, number of household members, number of children in the household, and province in which the household is located. We focus on a sample of households from age 25 to 80. We drop observations for households without information on age or financial wealth, resulting in an unbalanced panel of 710 households over 14 years, from 1999 to 2012.

Business exposure. The Townsend Thai Monthly Survey contains detailed information on entrepreneurs' assets, including fixed assets, inventories, and financial assets. We classify these assets into business assets and non-business assets, which we treat as safe. The value of the business includes inventories, livestock, agricultural assets, business assets, and household assets. We follow Samphantharak and Townsend (2018) and include the value of household assets (cars, pick-up trucks, fishing boats, and so on) as part of the business. The motivation for this choice is that many of these assets are also used by households in their production activities. The value of safe assets includes cash in hand, account receivables, deposits at financial institutions, ROSCA, other lending, prepaid insurance, and land. Given this breakdown, we can compute the fraction of financial wealth (total asset net of liabilities) invested in the business, which we use as our measure of risk-taking.

Return on business activity. Given estimates for the value of entrepreneurs' businesses and the flow of business income generated in a given period, we can compute the return on assets (ROA) as the ratio of business income over the value of business assets. ROA is a common accounting measure used to capture the profitability of business activities. We compute a time series of returns for a subsample of 541 households with available information on assets and income.

Human wealth. We compute our empirical measure of human wealth in a way analogous to its counterpart in the model as the present discounted value of future expected labor income. This requires us to specify the discount rate and the expected value of future labor. We use the same discount rate used in the model, as shown in Equation (**??**). The expected value of future labor income is age-dependent and computed as the average labor income for households of that age. As in the construction of life-cycle profiles discussed below, we use trimmed means to limit the influence of outliers. Consistent with the model's assumption, human wealth has no idiosyncratic component, as it corresponds to the present discounted value of future labor income across different households conditional on age.

B.2 Life-cycle profiles

When computing life-cycle profiles for a given variable, we aggregate households into 15 age groups. The thresholds determining each group are chosen such that groups have roughly the same number of households. To limit the influence of outliers and reduce the noise in the estimation, we compute trimmed means with a trimming parameter of 7.5% on each side. We trim the data in a similar manner before running the regressions.

The life-cycle profiles presented in the main text are computed without any controls, which we denote by *raw* moments. We show next that controlling for year fixed-effects or demographics variables maintains our results essentially unchanged.

Let $z_{i,k,t}$ denote variable z for household i in age group k at year t. Consider the following process for $z_{i,k,t}$:

$$z_{i,k,t} = \alpha_t + age_k + \delta' x_{i,k,t} + u_{i,k,t}, \tag{B.1}$$

where α_t represents the year fixed effect, age_k is the age-group effect, and $x_{i,k,t}$ is a vector of demographic and geographic controls, which includes the size of the household, the number of children in the household, province dummies, and a set of sector dummies.

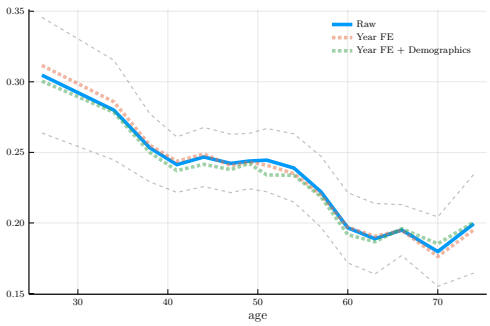
The raw age-group effect is given by

$$age_k^{raw} = \mathbb{E}\left[z_{i,k',t} | k' = k\right]. \tag{B.2}$$

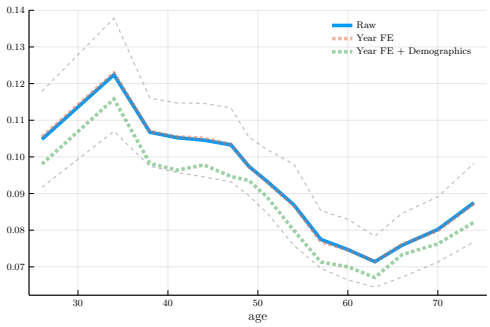
The raw age-group effect can be estimated by taking averages by age or by regressing $z_{i,k,t}$ on a set of dummies for age groups.

We follow Kaplan (2012) and define the age-group effect controlling for year fixed-

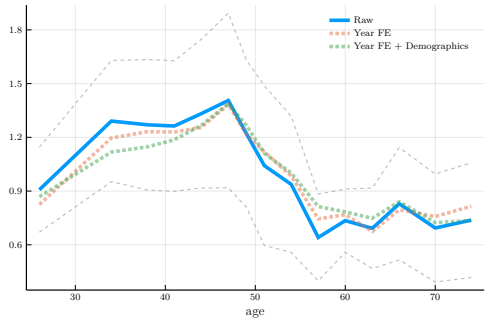
Figure B.1: Life-cycle profiles: raw and additional controls



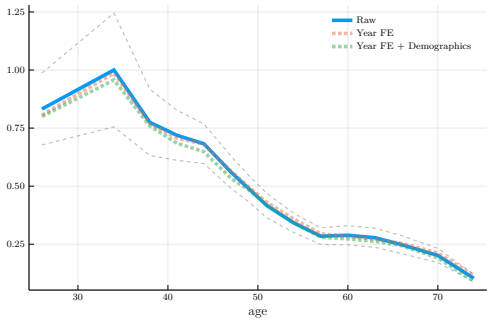
(a) Business exposure



(b) Consumption-wealth ratio



(c) Labor income



(d) Human-financial wealth ratio

effects as follows:

$$age_k^{year-FE} = \frac{1}{\overline{T}} \sum_{t=1}^{\overline{T}} \mathbb{E}\left[z_{i,k',t'} | k' = k, t' = t\right]. \tag{B.3}$$

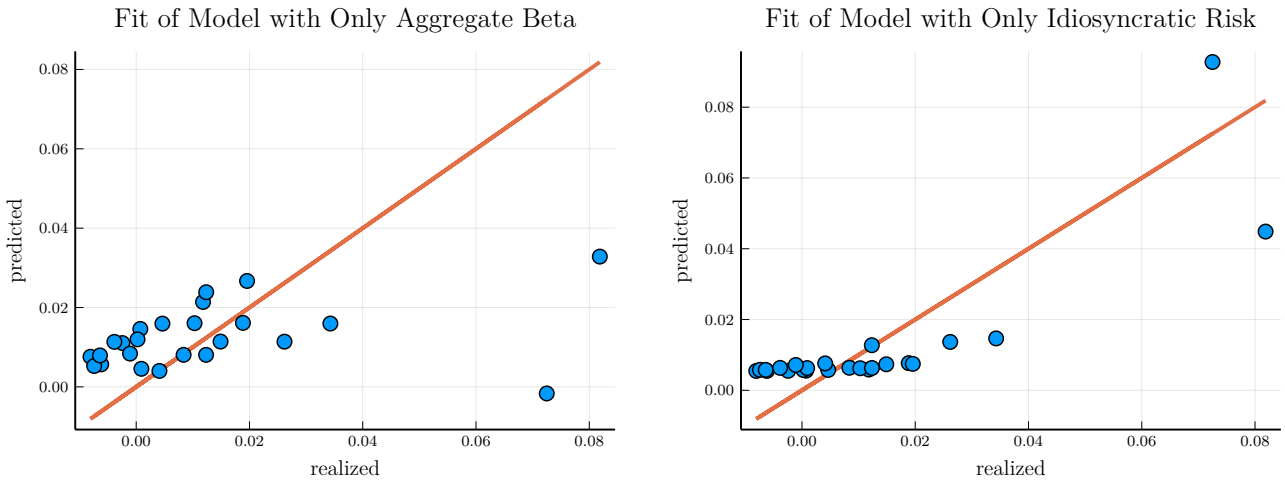
We can estimate $age_k^{year-FE}$ by running a regression on a set of dummy of age-group and year fixed-effects and computing the predicted value of the regression evaluated at age k and at an "average" year. Similarly, we define the age-group effect controlling for year fixed-effects and demographic/geographic controls as follows:

$$age_k^{year-FE+dem} = \frac{1}{\overline{T}} \sum_{t=1}^{\overline{T}} \mathbb{E}\left[z_{i,k',t'} | k' = k, t' = t, x_{i,k',t'} = \overline{x}_{k,t}\right], \tag{B.4}$$

where $\overline{x}_{k,t}$ denotes the average value of the controls $x_{i,k,t}$ conditional on age group k and year t. We can estimate $age_k^{year-FE+dem}$ from the full regression with year and age-group fixed-effects and demographic controls.

Figure B.1 shows our estimates of the different age-group effects for the variables of interest. The grey dashed lines show the 95% confidence interval for the raw estimates. We cluster standard errors by household and year. The life-cycle patterns obtained under the raw measure are very similar to the ones obtained after controlling for year fixed-

Figure B.2: Realized vs. predicted returns: one-factor models



Note: The left (right) panel shows a scatter plot of predicted returns of the single factor with aggregate beta (idiosyncratic variance) as a factor against the realized returns for the portfolio-level analysis.

effects and demographic variables. Moreover, the estimates with additional controls are statistically indistinguishable from the raw estimates.

B.3 The fit of the single-factor models

In Section 2, Figure 2 shows the fit of the two-factor model. We consider here the performance of the one-factor models using either aggregate beta or idiosyncratic variance as the factor. Figure B.2 shows that both models struggle to properly account for the dispersion in entrepreneurial returns, which shows the importance of considering a two-factor model.

B.4 Latent factor model

The latent factor model can be described as follows:

$$\tilde{R}_{it} = \mu_i + \beta_i^\top f_t + \tilde{\epsilon}_{i,t}, \qquad \mu_i = \lambda_0 + \lambda_{ag}^\top \beta_i + \lambda_{id}\tilde{\sigma}_i^2 + u_i, \tag{B.5}$$

where $\epsilon_{i,t} \sim \mathcal{N}(0, \tilde{\sigma}_i^2)$ and $f_t \sim \mathcal{N}(\mathbf{0}_K, I_K)$.

We can write the expression above in matrix form. Let $\tilde{R}_t = [\tilde{R}_{1t}, \ldots, \tilde{R}_{Nt}]^\top$, $\mu = [\mu_1, \ldots, \mu_N]^\top$, $\beta = [\beta_1, \ldots, \beta_N]^\top$, and $\tilde{\epsilon}_t = [\tilde{\epsilon}_{1,t}, \ldots, \tilde{\epsilon}_{N,t}]^\top$. Then,

$$\tilde{R}_t = \mu + \beta f_t + \tilde{\epsilon}_t. \tag{B.6}$$

This implies that $\tilde{R}_t|f_t \sim \mathcal{N}(\mu + \beta f_t, \tilde{\Sigma})$, where $\tilde{\Sigma} = diag(\tilde{\sigma}_1^2, \ldots, \tilde{\sigma}_N^2)$. Marginalizing the distribution above with respect to f_t, we obtain

$$\tilde{R}_t \sim \mathcal{N}(\mu, \beta\beta^\top + \tilde{\Sigma}). \tag{B.7}$$

Hence, the factor model effectively represents the covariance matrix as the sum of a rank K matrix and a diagonal matrix. We can stack the vector of returns to obtain $\tilde{R} = [\tilde{R}_1^\top, \ldots, \tilde{R}_T^\top]^\top$, which follows the distribution

$$\tilde{R} \sim \mathcal{N}\left(\mathbf{1}_T \otimes \mu, I_T \otimes C\right), \tag{B.8}$$

where $\otimes$ denotes the Kronecker product operator and $C \equiv \beta\beta^\top + \tilde{\Sigma}$. This gives us the likelihood for $\tilde{R}$, which depends on the vector of parameters $\Theta = [\{\beta_i^\top, \tilde{\sigma}_i, u_i\}_{i=1}^N, \lambda_0, \lambda_{ag}^\top, \lambda_{id}]^\top$. Notice that we have a total of $(N+1)(K+2)$ parameters.

Priors. Define $\lambda = [\lambda_0, \lambda_{ag}^\top, \lambda_{id}]^\top$. We assume a Gaussian prior for λ:

$$\lambda \sim \mathcal{N}\left(\mathbf{0}_{K+2}, \sigma_\lambda^2 I_{K+2}\right). \tag{B.9}$$

We assume a sufficiently large σ_λ^2, so the priors on λ are relatively uninformative. For the individual-level parameters, we assume a hierarchical prior. The intercept shifter u_i follows the prior

$$u_i \sim \mathcal{N}(0, \sigma_u^2). \tag{B.10}$$

We allow the hyperparameter σ_u to be determined by the data. We then specify the following *hyperprior* for σ_u:[29]

$$\sigma_u \sim Exponential(1). \tag{B.11}$$

The parameter σ_u controls the degree of *regularization*. When σ_u is very large, almost any value of u_i is consistent with the prior, so each u_i can be chosen to match the sample average of returns and there is no regularization. For lower values of σ_u, the estimates of u_i are pulled toward their mean, avoiding overfitting. We let the data determine the appropriate amount of regularization.

We adopt a similar hierarchical prior for β_i:

$$\beta_i \sim \mathcal{N}(\mathbf{0}_K, \sigma_\beta^2 I_K), \tag{B.12}$$

[29] We follow McElreath (2018) and adopt an exponential distribution for the hyperparameter controlling the dispersion of individual effects.

and $\sigma_\beta \sim Exponential(1)$.

Finally, we adopt the following paramerization $\tilde{\sigma}_i = \overline{\sigma} e^{\delta_i}$. The parameter controlling the average volatility follows the distribution

$$\overline{\sigma} \sim Exponential(1). \tag{B.13}$$

The individual level shifters follows the hierarchical prior

$$\delta_i \sim \mathcal{N}(0, \sigma_\delta^2), \tag{B.14}$$

where $\sigma_\delta \sim Exponential(1)$.

Sampling. We sample from the posterior distribution using Hamiltonian Monte Carlo (HMC), a gradient-based sampling method, as implemented in the Julia package Turing.jl (see Ge et al. (2018) for a description of the Turing package).

C Derivations

C.1 Equilibrium prices and capital stock in a stationary equilibrium

Interest rate. The financial wealth of wage earners evolves according to

$$d\tilde{n}_{j,t} = \left[(r + \gamma\sigma_A^2)\tilde{n}_{j,t} + \tilde{w}_t \bar{l}_{j,t} - \tilde{c}_{j,t} \right] dt + \tilde{n}_{j,t}\sigma_A dZ_t. \tag{C.1}$$

using the demand for aggregate insurance $\theta_{j,t} = -n_{j,t}\sigma_A$.

Combining the expression above with the law of motion for human wealth, we obtain the law of motion of total wealth:

$$\frac{d\tilde{\omega}_{i,t}}{\tilde{\omega}_{i,t}} = \left[r + \gamma\sigma_A^2 - \frac{c_{j,t}}{n_{j,t} + h_{j,t}} \right] dt + \sigma_A dZ_t. \tag{C.2}$$

In a stationary equilibrium, wage earners' scaled total wealth, $\omega_{j,t} = \tilde{\omega}_{j,t}/A_t$, is constant. Therefore, the drift of $\omega_{j,t}$ must be zero. Using Ito's lemma, we obtain:

$$\frac{d\omega_{i,t}}{\omega_{i,t}} = \frac{d\tilde{\omega}_{i,t}}{\tilde{\omega}_{i,t}} - \frac{dA_t}{A_t} + \left(\frac{dA_t}{A_t}\right)^2 - \frac{dA_t}{A_t}\frac{d\tilde{\omega}_{i,t}}{\tilde{\omega}_{i,t}} \tag{C.3}$$

$$= \left[r + \gamma\sigma_A^2 - \frac{c_{j,t}}{n_{j,t} + h_{j,t}} - \mu_A \right] dt. \tag{C.4}$$

The interest rate must then satisfy the condition

$$r + \gamma\sigma_A^2 - \left[\frac{1}{\gamma}\rho_w + \left(1 - \frac{1}{\gamma}\right)\left(r + \frac{\gamma\sigma_A^2}{2}\right)\right] - \mu_A = 0, \tag{C.5}$$

using the fact that the consumption-wealth ratio is given by

$$\frac{c_{j,t}}{n_{j,t} + h_{j,t}} = \frac{1}{\gamma}\rho_w + \left(1 - \frac{1}{\gamma}\right)\left(r + \frac{(p^{ag})^2}{2\gamma}\right), \tag{C.6}$$

which is a special case of (20), as we set $p_t^{id} = 0$ and $T \to \infty$.

Rearranging expression (C.5), we obtain

$$r = \rho_w + \gamma\mu_A - \gamma(\gamma + 1)\frac{\sigma_A^2}{2}. \tag{C.7}$$

Relative price of capital. Plugging the expression for $\Phi'(\iota)$ into the first-order condition for ι in equation (12), we obtain

$$\frac{1}{\sqrt{\overline{\Phi}^2 + 2\iota}} = \frac{1}{q} \Rightarrow \iota = \frac{q^2 - \overline{\Phi}^2}{2}. \tag{C.8}$$

In a stationary equilibrium, the capital-labor ratio is constant. Thus, capital grows at the population rate g, which gives us the condition

$$\Phi(\iota) - \delta = g \Rightarrow q = \overline{\Phi} + (g + \delta), \tag{C.9}$$

obtained by plugging the expression for ι into the functional form for $\Phi(\iota)$.

C.1.1 Capital stock and idiosyncratic risk premium

Rearranging the expression for the shadow price of idiosyncratic risk (18), and using the definition of expected returns (13), we obtain the MPK schedule

$$r + p^{ag}\sigma_A + p^{id}\phi\sigma_I = \frac{\alpha K^{\alpha-1} - \iota(q)}{q} + \mu_A + \Phi(\iota(q)) - \delta. \tag{C.10}$$

where r, p^{ag}, and q are functions of parameters, as derived above.

Integrating condition (17) across all entrepreneurs, we obtain

$$p^{id} = \gamma\phi\sigma_I \frac{qK}{\chi_e \omega_e}, \tag{C.11}$$

where χ_e is the fraction of entrepreneurs in the population.

D Transitional dynamics

In this section, we describe the computation of the transitional dynamics. To ensure that that the interest rate is constant, we assume that wage earners have Epstein-Zin preferences and take the limit as the elasticity of intertemporal substitution goes to infinity, that is, workers have linear intertemporal preferences.[30]

D.1 Wage earners with Epstein-Zin preferences

Wage earners have the continuous-time analog of Epstein-Zin preferences with EIS ψ_w and risk aversion γ. The wage earner's problem is given by

$$\tilde{V}_t^w(\tilde{n}_j) = \max_{\tilde{c}_j, \tilde{\theta}_j^{ag}} \mathbb{E}_t \left[\int_t^\infty f_w(\tilde{c}_{j,z}, \tilde{V}_z) dz \right], \tag{D.1}$$

subject to $\tilde{n}_{j,t} \geq \tilde{h}_{j,t}$, where $\tilde{h}_{j,t}$ denotes wage earner j's human wealth, non-negativity constraint $\tilde{c}_{j,t} \geq 0$, and the law of motion of financial wealth $\tilde{n}_{j,t}$

$$d\tilde{n}_{j,t} = \left[\tilde{n}_{j,t} r_t - p_t^{ag} \tilde{\theta}_{j,t}^{ag} + \tilde{w}_t \bar{l}_{j,t} - \tilde{c}_{j,t} \right] dt - \tilde{\theta}_{j,t}^{ag} dZ_t,$$

where $f_w(\tilde{c}, V)$ is the aggregator given by

$$f_w(\tilde{c}, \tilde{V}) = \rho_w \frac{(1-\gamma)\tilde{V}}{1-\psi_w^{-1}} \left\{ \left(\frac{\tilde{c}}{\left((1-\gamma)\tilde{V}\right)^{\frac{1}{1-\gamma}}} \right)^{1-\psi_w^{-1}} - 1 \right\}. \tag{D.2}$$

It is convenient to work with the scaled value function $V_t^w(n)$, which satisfies the condition $\tilde{V}_t^w(\tilde{n}) = A_t^{1-\gamma} V_t^w \left(\frac{\tilde{n}_t}{A_t} \right)$, where $V_t^w(\cdot)$ is independent of A_t. The HJB equation

[30] This assumption is meant to capture, in an extreme form, the essence of the macro-finance literature that assumes a high EIS (see, e.g., Bansal and Yaron 2004 and Barro 2009). In these models, the high EIS dampens movements in interest rates, so risk premia accounts for most of the variation in discount rates.

in terms of the scaled value function is given by

$$\tilde{\rho}_w \frac{(1-\gamma)V^w}{1-\psi_w^{-1}} = \max_{\tilde{c},\tilde{\theta}^{ag}} \rho_w \frac{(1-\gamma)V^w}{1-\psi_w^{-1}} \left(\frac{c}{((1-\gamma)V^w)^{\frac{1}{1-\gamma}}} \right)^{1-\psi_w^{-1}} + V_t^w + V_n^w \left[\tilde{r}n + (\gamma\sigma_A - p^{ag})\theta^{ag} + w\bar{l} - c \right] + \frac{1}{2} V_{nn}^w \left(\theta^{ag} + n\sigma_A \right)^2 . \tag{D.3}$$

where $\tilde{\rho}_w \equiv \rho_w - (1-\psi_w^{-1}) \left(\mu_A - \frac{\gamma\sigma_A^2}{2} \right)$ and $\tilde{r}_t \equiv r_t + \gamma\sigma_A^2 - \mu_A$.

Policy functions. The first-order conditions for this problem are given by

$$\rho_w \left((1-\gamma)V^w \right)^{\frac{\psi_w^{-1}-\gamma}{1-\gamma}} c^{-\psi_w^{-1}} = V_n^w, \qquad \gamma\sigma_A - p^{ag} = -\frac{V_{nn}^w}{V_n^w} \left(\theta^{ag} + n\sigma_A \right). \tag{D.4}$$

We will guess and verify that the scaled value function takes the form

$$V_t^w(n_{j,t}) = \left(\frac{\zeta_{w,t}}{\rho_w^{\psi_w}} \right)^{\frac{1-\gamma}{1-\psi_w}} \frac{(n_{j,t} + h_{j,t})^{1-\gamma}}{1-\gamma}, \tag{D.5}$$

where $\zeta_{w,t}$ and $h_{j,t}$ are potentially time-varying, but they are non-stochastic.

Using the expression for the value function above, we obtain the policy functions:

$$\frac{c_{j,t}}{n_{j,t} + h_{j,t}} = \zeta_{w,t}, \qquad \theta_{j,t}^{ag} = \sigma_A h_{j,t} - \frac{p_t^{ag}}{\gamma}(n_{j,t} + h_{j,t}). \tag{D.6}$$

Inserting the policy functions derived above back into the HJB equation, we obtain

$$\frac{\tilde{\rho}_w}{1-\psi_w^{-1}} = \frac{\psi_w^{-1}}{1-\psi_w^{-1}} \zeta_{w,t} + \frac{1}{1-\psi_w} \frac{\dot{\zeta}_{w,t}}{\zeta_{w,t}} + \tilde{r}_t. \tag{D.7}$$

Rearranging the expression above for the case of a stationary equilibrium, so $\dot{\zeta}_{w,t} = 0$, we obtain the consumption-wealth ratio:

$$\zeta_{w,t} = \psi_w \rho_w + (1-\psi_w) \left(r + \frac{\gamma\sigma_A^2}{2} \right), \tag{D.8}$$

which coincides with the expression for the consumption-wealth ratio for wage earners given in (C.6) in the special case of CRRA preferences, i.e., $\psi_w^{-1} = \gamma$.

Price of aggregate risk and interest rate. The demand for aggregate insurance derived above coincides with the expression for aggregate insurance in the CRRA case (see Equation A.23). Therefore, the same argument used in Section C.1 to solve for the price of aggregate risk can be applied in the case of Epstein-Zin preferences in a non-stationary setting. This implies that the price of aggregate risk is constant and given by $p_t^{ag} = \gamma\sigma_A$ during the transitional dynamics.

We consider next the behavior of the interest rate. A derivation analogous to the one in Section C.1 shows that the interest rate in a stationary equilibrium is given by

$$r = \rho_w + \psi_w^{-1}\mu_A - (1 + \psi_w^{-1})\frac{\gamma\sigma_A^2}{2}, \tag{D.9}$$

which coincides with the expression in Section 6 when $\psi_w^{-1} = \gamma$.

Taking the limit of (D.7) as $\psi_w \to \infty$, we obtain r_t in the case of a non-stationary equilibrium:

$$r_t + \gamma\sigma_A^2 - \mu_A = \rho_w + \frac{\gamma\sigma_A^2}{2} - \mu_A \Rightarrow r_t = \rho_w - \frac{\gamma\sigma_A^2}{2}. \tag{D.10}$$

Therefore, the interest rate is constant when wage earners have linear intertemporal preferences. Moreover, the expression above coincides with the one for the interest rate in the stationary equilibrium (D.9) when specialized to $\psi_w^{-1} = 0$.

E Extensions

E.1 Limited pledgeability and heterogeneous expected returns

In this subsection, we consider a version of the model with limited pledgeability of physical assets and heterogeneous idiosyncratic volatility.

Limited pleadgeability of capital. Let $b_{i,t} \equiv n_{i,t} - q_t k_t$ denote the amount of safe assets held by entrepreneur i. The natural borrowing limit can be written as

$$-b_{i,t} \leq h_{i,t} + qk_{i,t}. \tag{E.1}$$

Entrepreneur can borrow freely against physical assets or human wealth. Let's now assume that there is limited pledgeability of physical assets, that is, entrepreneurs can only borrow a fraction of $1 - \lambda^{-1}$ of the value of physical assets, a form of collateral constraint:

$$-b_{i,t} \geq h_{i,t} + (1 - \lambda^{-1})qk_{i,t} \Rightarrow qk_{i,t} \leq \lambda\omega_{i,t}, \tag{E.2}$$

where $\lambda \geq 1$. Hence, the entrepreneur faces a portfolio problem subject to leverage constraints. The HJB for an entrepreneur can be written as[31]

$$\frac{\rho}{1-\gamma} = \max_{c_{i,t},k_{i,t},l_{i,t},\iota_{i,t},\theta^{ag}_{i,t},\theta^{id}_{i,t}} \left\{ \frac{\zeta^{\gamma}(a)}{1-\gamma}\left(\frac{c_{i,t}}{\omega_{i,t}}\right)^{1-\gamma} + r + \frac{q_t k_{i,t}}{\omega_{i,t}}(\mu^R_{i,t} - r_t) - \frac{p^{ag}\theta^{ag}_{i,t}}{\omega_{i,t}} + \frac{h_{i,t}}{\omega_{i,t}}\sigma_A p^{ag} - \frac{c_{i,t}}{\omega_{i,t}} \right.$$
$$\left. - \frac{\gamma}{1-\gamma}\frac{1}{\zeta(a)}\frac{\partial \zeta(a)}{\partial a} - \frac{\gamma}{2}\left[\left(\frac{q_t k_{i,t} + h_{i,t}}{\omega_{i,t}}\sigma_A - \frac{\theta^{ag}_{i,t}}{\omega_{i,t}}\right)^2 + \left(\frac{q_t k_{i,t}}{\omega_{i,t}}\sigma_{I,i} - \frac{\theta^{id}_{i,t}}{\omega_{i,t}}\right)^2\right] \right\},$$

subject to leverage and insurance constraints: $q_t k_{i,t} \leq \lambda \omega_{i,t}$ and $\theta^{id}_{i,t} \leq (1-\phi) q_t k_{i,t} \sigma_{I,i}$.

The optimal capital demand is given by

$$\frac{q k_{i,t}}{\omega_{i,t}} = \frac{\varphi^{id}_i}{\gamma \phi \sigma_{I,i}}, \tag{E.3}$$

where $\varphi^{id}_{i,t}$ satisfies the condition:

$$\varphi^{id}_i = \min\left\{ \frac{\mu^R_i - r - p^{ag}\sigma_A}{\phi \sigma_{I,i}}, \gamma \phi \sigma_{I,i} \lambda \right\}.$$

φ^{id}_i corresponds to the Lagrange multiplier on the insurance constraint. In this case, this multiplier may not coincide with the price of idiosyncratic risk $p^{id}_i \equiv \frac{\mu^R_i - r - p^{ag}\sigma_A}{\phi \sigma_{I,i}}$.

Equilibrium implications. We only sketch the equilibrium implications. Let $\overline{k}_j$, $\overline{\omega}_j$, and $\overline{\mu}^R_j$ denote the long-run values of capital, total wealth, and expected marginal return for an entrepreneur of type j. We have seen in Section 5 that expected returns are decreasing in the capital stock with DRS, so we can write $\overline{\mu}^R_j = f(\overline{k}_j)$, where $f'(\cdot) < 0$. By analogy with the result in Proposition 2, we can assume that $\overline{\omega}_j$ is an increasing function of the return $\overline{\mu}^R_j$, $\omega_j = g(\overline{\mu}^R_j)$, where $g'(\cdot) > 0$. For a constrained investor, we have $q\overline{k}_j = \lambda \overline{\omega}_j$, so $\overline{k}_j = \frac{\lambda}{q} g_j(\overline{\mu}^R_j)$ The capital stock would be determined by the intersection of $f(\overline{k}_j)$ and $g_j^{-1}(q\overline{k}_j/\lambda)$. In this case, variations in wealth, associated with shifts of the curve $g_j(\cdot)$, will lead to variations in the capital stock and ultimately expected returns. This argument indicates that cross-sectional differences in expected returns, controlling for differences in risk, are driven by differences in net worth for constrained entrepreneurs.

[31] A similar result on the optimal portfolio share with leverage constraints can be found for investors without labor income in, for instance, Grossman and Vila (1992) and Detemple and Murthy (1997).

E.2 Endogenous occupational choice

In this subsection, we introduce an occupational choice into the households' problem. Moreover, we assume that wage earners have a finite horizon and imperfect altruism in the same way as entrepreneurs.

E.2.1 The occupational choice

At the beginning of life, a household can choose to become an entrepreneur or a wage earner. To become an entrepreneur, household i must pay a fixed cost $\varphi_i \tilde{y}_{i,t}$, where φ_i is a cost parameter draw from a distribution $F_\varphi(\cdot)$ with support $[\underline{\varphi}, \overline{\varphi}]$. Let $\tilde{V}_t(\tilde{n}_i, a)$ denote the value function of a household that chose to become an entrepreneur and $\tilde{V}_t^w(\tilde{n}_i, a)$ the value function of a household who chose to become a wage earner. In contrast to the model from Section 3, a wage earner lives for T periods and derives the same utility of bequests as entrepreneurs.

A household that inherits financial wealth $\tilde{n}_i$ will choose to become an entrepreneur if

$$\tilde{V}_t(\tilde{n}_i - \varphi_i \tilde{y}_t, 0) > \tilde{V}_t^w(\tilde{n}_i, 0). \tag{E.4}$$

The value function of an entrepreneur can be written, after normalization, as $V(n, a) = \zeta(a)^{-\frac{1}{\gamma}} \frac{(n+h(a))^{1-\gamma}}{1-\gamma}$. Similarly, the value function of a wage earner can be written as $V^w(n, a) = \zeta_w(a)^{-\frac{1}{\gamma}} \frac{(n+h_w(a))^{1-\gamma}}{1-\gamma}$.

The condition for becoming an entrepreneur can then be written as

$$\zeta(0)^{\frac{1}{\gamma(\gamma-1)}} (n_i + h(0) - \varphi_i y) > \zeta_w(0)^{\frac{1}{\gamma(\gamma-1)}} (n_i + h_w(0)). \tag{E.5}$$

Rearranging the expression above, we obtain that a household becomes an entrepreneur if $\varphi_i < \varphi^*(n_i)$, where the threshold $\varphi^*(n_i)$ is given by

$$\varphi^*(n_i) \equiv \frac{1}{y} \left[\left(\left(\frac{\zeta(0)}{\zeta_w(0)} \right)^{\frac{1}{\gamma(\gamma-1)}} - 1 \right) n_i + \left(\frac{\zeta(0)}{\zeta_w(0)} \right)^{\frac{1}{\gamma(\gamma-1)}} h(0) - h_w(0) \right]. \tag{E.6}$$

It can be shown that $\zeta(0) > \zeta_\omega(0)$ for $\gamma > 1$, so households who received larger bequests are more likely to become entrepreneurs. The difference between $\zeta(0)$ and $\zeta_\omega(0)$ is increasing in p^{id}, the shadow price of idiosyncratic risk.

As the cost parameter is drawn independently of the bequest a household receives,

then the mass of entrepreneurs in a stationary equilibrium is given by

$$\chi_e = F_\varphi\left(\varphi^*(n(0))\right), \tag{E.7}$$

where $n(0)$ is the average financial wealth of newborn entrepreneurs.

In a stationary equilibrium, the mass of entrepreneurs is constant. As $\zeta(0)$, $\zeta_w(0)$, and $n(0)$ depend on the interest rate and the aggregate and idiosyncratic risk premia, then the share of entrepreneurs in the economy depends on the equilibrium expected returns.

E.2.2 Wage earners' problem and equilibrium determination

The optimal consumption-wealth ratio and demand for insurance for wage earners are now given by

$$\frac{c_{j,t}}{\omega_{j,t}} = \zeta_w(a) = \frac{\bar{r}_w}{1-\psi e^{-\bar{r}_w(T-a_j)}}, \qquad \theta^{ag}_{j,t} = h_{j,t}\sigma_A - \frac{p^{ag}}{\gamma}\omega_{j,t}, \tag{E.8}$$

where

$$\bar{r}_w = \frac{1}{\gamma}\rho_w + \left(1-\frac{1}{\gamma}\right)\left(r + \frac{(p^{ag})^2}{2\gamma}\right). \tag{E.9}$$

The price of aggregate insurance, wages, and the relative price of capital are the same as in the baseline model:

$$p^{ag} = \gamma\sigma_A, \qquad w = (1-\alpha)K^\alpha, \qquad q = \Phi_0 + \Phi_1(g+\delta). \tag{E.10}$$

Finite lives for wage earners change the determination of the interest rate. The interest rate is now jointly determined with the capital-labor ratio and the price of idiosyncratic risk by conditions (28), (29), and the market clearing condition for consumption

$$\int_0^T \frac{\bar{r}\omega(a)}{1-\psi e^{-\bar{r}(T-a)}} f(a)da + \int_0^T \frac{\bar{r}_w\omega_w(a)}{1-\psi e^{-\bar{r}_w(T-a)}} f(a)da = \alpha K^\alpha - \iota K, \tag{E.11}$$

where

$$\omega(a) = \omega(0)e^{\left(r+\gamma\sigma_A^2+\frac{(p^{id})^2}{\gamma}-\mu_A-\bar{r}\right)a}\frac{1-\psi e^{-\bar{r}(T-a)}}{1-\psi e^{-\bar{r}T}}, \qquad \omega_w(a) = \omega_w(0)e^{\left(r+\gamma\sigma_A^2-\mu_A-\bar{r}_w\right)a}\frac{1-\psi e^{-\bar{r}_w(T-a)}}{1-\psi e^{-\bar{r}_w T}}.$$

Assuming finite lives for wage earners would change the calibration of ρ_w but otherwise would not affect our main results.

E.3 Financial autarky

We consider next the case of financial autarky, where entrepreneurs have no access to either aggregate or idiosyncratic insurance. To shut down idiosyncratic insurance, we must set $\phi = 1$. To capture the absence of aggregate insurance, we will focus on the special case where there is no demand for aggregate insurance, so the solution would coincide with the case where entrepreneurs have no access to insurance.

Suppose that $h_{i,t} = 0$, so entrepreneurs have no labor income, and assume that $n_{j,t} = 0$ for $j \in \mathcal{W}_t$, so wage earners have no financial wealth. The first assumption implies that $\frac{qk_{i,t}}{n_{i,t}}$ is equalized across entrepreneurs, and the second assumption implies that $qk_{i,t} = n_{i,t}$. As the demand for aggregate insurance is given by $\theta^{ag}_{i,t} = (qk_{i,t} - n_{i,t})\sigma_A$, we obtain that entrepreneurs do not demand aggregate insurance. The solution will then coincide with the case where aggregate insurance is unavailable.

Under these assumptions, the price of idiosyncratic risk, given in Equation (29), specializes to $p^{id} = \gamma\sigma_I$. The risk premium is then given by

$$p^{ag}\sigma_A + p^{id}\sigma_I = \gamma\left[\sigma_A^2 + \sigma_I^2\right]. \tag{E.12}$$

The aggregate Sharpe ratio relative to the idiosyncratic Sharpe ratio is $p^{ag}/p^{id} = \sigma_A/\sigma_I$.