Investment Demand Shocks and Procyclical Inflation: A Theory*

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Abstract

This paper attributes the origin of aggregate fluctuations to the stochastic synchronization of firms' investment spikes, which we call investment avalanches. In a monopolistic competition model with idiosyncratic productivity shocks, firms' decisions on lumpy investments exhibit complementarity in equilibrium due to aggregate demand externality. A firm's investment can cause other firms' investments, which in turn causes other firms' and so on. The size of this investment avalanche depends on the distribution of firms' states on productivity and capital. We show that the variation of avalanche size generates significant fluctuations in aggregate investment demand in a model with no exogenous aggregate shocks. We demonstrate that a New Keynesian business cycle model with investment demand shocks naturally produces procyclical inflation. Real wage rigidity and diminishing returns to labor due to the presence of capital are essential in our propagation mechanism, in which a positive investment demand shock shifts out labor demand, deteriorating the marginal product of labor and pushing up the marginal cost of final goods production. The model implies that an optimal monetary policy accommodates some inflation caused by the

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investment demand shock. In contrast, the optimal policy delivers disinflation when a positive productivity shock hits investments.

1 Introduction

"Animal spirits" haunt business cycle discussions. In regular business ups and downs, investors' collective mood appears to drive fluctuations in aggregate investment demand. However, a properly founded mechanism that generates animal spirits is yet to be found. In standard business cycle literature, this origin of aggregate fluctuations is captured by exogenous shocks on preference or technology, such as a time preference shock, a productivity shock in the investment goods sector, or a shock on the volatility of underlying uncertainty. Attributing the cause of animal spirits to an exogenous aggregate fundamental shock is consequential. It implies that the welfare loss of business cycles does not directly originate from animal spirits but from market imperfections that hinder the economy's response to preferences, technology, or endowment shocks.

In this paper, we develop a model of business cycles driven by aggregate investment demand without appealing to exogenous aggregate shocks. We consider a monopolistic competition model, in which firms' investment decisions are complementary in equilibrium due to aggregate demand externality: a capital increase in a sector raises demand for goods and leads to investments in other sectors. We assume that capital is indivisible up to a lumpiness parameter. Thus, how many firms invest in a period, i.e., the extensive margin of capital adjustments, determines aggregate investment demand. When the economy consists of a finite large number n of firms, the aggregate investment demand deviates from the steady-state level due to the finiteness. The deviation is amplified by the aggregate demand externality in equilibrium. We will show that this amplification effect, an investment avalanche, leads to a qualitatively different distribution of the number of investing firms from what the central limit theorem predicts and is quantitatively significant even when n is very large.

We demonstrate that the investment demand shocks originating from investment avalanches cause a procyclical response in inflation rates. Our model incorporates capital and sticky real wages in a standard New Keynesian business cycle model. The model generates a pro-

cyclical impulse response of inflation rates and consumption to investment demand shocks with these two elements. The propagation mechanism hinges on labor demand. When a positive investment demand shock hits an economy, an increased capital in the next period induces households to consume more in the next and current period through wealth effects. An increase in investment demand also tightens the final goods market and shifts out labor demand. Under sticky real wages and diminishing returns to labor due to the presence of capital, an increased labor input deteriorates the marginal product of labor. Hence, intermediate producers pass on the increased production costs to final goods producers, leading to cost-push inflation.

This inflationary response to an investment demand shock contrasts with the disinflationary reaction of investment to a productivity shock in our model. Exogenous shocks on the marginal efficiency of investments or investment-specific technological shocks have been used in the New Keynesian literature to incorporate an autonomous fluctuation in aggregate investment. However, we find that these shocks do not generate procyclical inflation in our model. Labor demand explains the difference. A positive productivity shock on future investments does not necessarily raise demand for labor upon impact. In contrast, a positive investment demand shock raises the demand for final goods and, thus, labor demand in the final goods sector. Inflation in our model results from increased labor demand under real wage rigidities that lead to an inefficient over-employment during a labor demand boom.

Related literature This paper builds on a long list of literature on the origin of business cycles and the role of investment demand. Our model is closest to the business cycle literature on investment shocks. Fisher (2006) found empirical evidence for technological shocks in the investment goods sector and explored its implications in business cycles. Justiniano, Primiceri, and Tambalotti (2010) established an important role played by an investment technological shock in an estimated business cycle model and interpreted it as a shock originating from the financial sector that transforms investment into effective capital. Christiano, Motto, and Rostagno (2014) took the view further and found a shock on the volatility of underlying risk, such as in Bloom (2009), plays an essential role in an estimated New Keynesian model.

While these studies attribute the aggregate fluctuation to an exogenous aggregate shock to production technology or the financial environment that converts investment expenditures to capital increases, we focus on interactions of many firms' simultaneous investment decisions. Whereas an investment technological shock expands capital without costing a resource, an investment demand shock in our model uses resources and crowds out consumption. In this sense, our model is related to Galí and Rabanal (2004), who took a more direct approach in which a shock on time preference affects the representative agents' demand toward current or future consumption. However, the welfare implications of the model differ since investment fluctuations due to time preference shocks reflect efficient responses of an economy. In contrast, investment avalanche shocks in our model embody a coordination failure among investors who fail to control aggregate investment amounts.

Our model is in the tradition of sectoral business cycles of Long and Plosser (1983), which feature realistic technological shocks at the sectoral level. It has been pointed out that the aggregate fluctuations generated from idiosyncratic shocks suffer a diversification effect by which the volatility of aggregate fluctuations decreases fast as the number of sectors increases, as discussed by Dupor (1999) and Horvath (2000). An introduction of non-linear behavior at the micro level, such as in (S,s) models, may open the possibility of avoiding the diversification effect. However, previous research has concentrated on the case where non-linear behaviors generate weak aggregate fluctuations (Caplin and Spulber, 1987; Caballero and Engel, 1991; Thomas, 2002; Khan and Thomas, 2003, 2008).

This paper instead illustrates that the (S,s) behavior can cause aggregate fluctuations. Our attempt is related to recent studies on the role of interest elasticity of investment demand in achieving the diversification effect (Auclert, Rognlie, and Straub, 2020; Koby and Wolf, 2020; Winberry, 2021; Zwick and Mahon, 2017). In our model, firms' investments exhibit strategic complementarity in equilibrium as defined in Caballero and Engel (1993). In an environment with a finite number of firms rather than a continuum of firms, we show that the complementarity in lumpy investments can generate sizable aggregate fluctuations.

Our model shares the spirit of endogenous business cycle studies, such as sunspot models pursued by Galí (1994) and Wang and Wen (2008) among others. Our model includes micro-level independent shocks but no exogenous aggregate shocks. In that sense, our model

is capable of generating aggregate fluctuations endogenously. Our model differs from the sunspot and indeterminacy models in that our model produces a locally unique equilibrium rather than a continuum of equilibria. We select a locally unique equilibrium closest to an equilibrium of a "smooth" economy in which the lumpiness of investments is irrelevant to aggregates and show that the equilibrium exhibits significant fluctuations. The point of the local uniqueness is that the equilibrium of a smooth economy cannot be generically realized as an equilibrium of an (S,s) economy with finite firms, and the equilibrium of the (S,s) economy deviates from the smooth counterpart quantitatively significantly.

Such authors have studied the departure from the diversification effect as Durlauf (1993), Gabaix (2011), and Acemoglu, Carvalho, Ozdaglar, and Tahbaz-Salehi (2012), among others. Our paper's underlying analytics of the fluctuation mechanism are based on power-law distributions. Unlike the granular hypothesis of Gabaix (2011), our model features a stochastic synchronization called avalanches. The so-called self-organized criticality model that generates avalanches has been explored by Scheinkman and Woodford (1994), Nirei (2006), and Nirei, Stachurski, and Watanabe (2020). The investment avalanche employed in this paper has been proposed by Nirei (2015) in an environment of real business cycles, and a mathematically complete treatment of avalanches in an (S,s) model has been recently provided by Nirei and Scheinkman (2024).

Lumpy investments Authors such as Cooper and Haltiwanger (1996) and Gourio and Kashyap (2007) have emphasized the vital role that lumpy investments play in business cycles. We quickly recapitulate this point using a Japanese business survey (BSJBSA).[1] We define a lumpy investment as an incident of a firm's annual gross investment exceeding 20% of outstanding capital. In our dataset, the lumpy investment occurs at a rate of 13.5% with a standard deviation of 21.6%.

The left panel of Figure 1 shows the sizable portion of lumpy investments in aggregate investments. Total investment indicates the aggregate investments (tangible and intangible)

[1]The Ministry of Economy, Trade and Industry conducts the Basic Survey of Japanese Business Structure and Activities. It covers firms with 50 or more employees and 30 million yen or more in capital. The response rate in 2022 was 90.2%. In our database, the survey consists of an unbalanced panel of 31197 firms from 2007 to 2021. See Nirei (2024) for details.

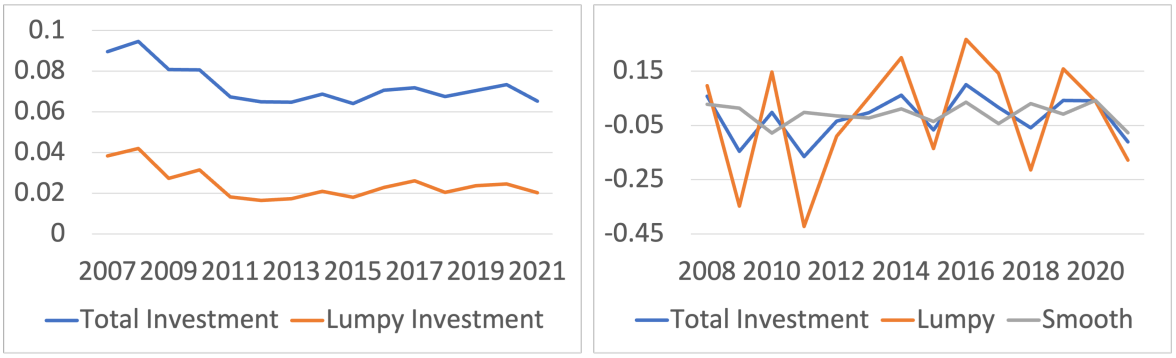


Figure 1: Aggregate total investments and lumpy investments in Japanese business survey. The left panel plots the ratio of the aggregate investments to aggregate assets held by the firms. The right panel plots growth rates. Source: BSJBSA; Nirei (2024).

divided by the total assets held by the firms in each year. The lumpy investment shows the investments aggregated over firms that exhibited an investment spike in the year, divided by the total assets. The average ratio of lumpy investment to total investment is 33%. In the right panel, we observe a clear comovement of the lumpy and total investments growth rates with the coefficient of correlation exceeding 95%. When we define the total minus the lumpy investment as smooth investments, the correlation coefficient between the smooth and total growth rates is only 29%. In what follows, we will explicitly incorporate the fluctuations of lumpy investments in a business cycle model.

The rest of the paper is organized as follows. Section 2 presents the model. Section 3 shows the impulse-response analysis of an aggregate investment demand shock. Section 4 analyzes the mechanism of an investment avalanche that causes the aggregate investment demand shock. Section 5 concludes. The appendix contains proofs and extensions of the model with heterogeneous time-to-build.

2 Model

The model consists of representative final good producers, wholesalers, intermediate good producers, the central bank, and representative households. Households consume final goods, supply labor, own firms, and have access to risk-free assets. Intermediate producers mo-

nopolistically produce differentiated goods using capital and labor. Wholesalers aggregate intermediate goods, incur price adjustment costs, and sell them to final goods producers. Final goods are used for consumption and investment. Intermediate producers own capital and make investment decisions. Our model features indivisible capital where intermediate producers can choose the capital level only discretely. In the remainder of this section, we present parametric specifications and define an equilibrium.

2.1 Production

Final goods Final good Y_t is produced using a CES aggregator function,

$$Y_t = \left(\int y_{it}^{\frac{\epsilon_c - 1}{\epsilon_c}} di \right)^{\frac{\epsilon_c}{\epsilon_c - 1}},$$

where $\epsilon_c > 1$, and competitively supplied. Denoting the price of y_{it} by p_{it}, competitive price of Y_t is $P_t = \left(\int p_{it}^{1-\epsilon_c} di \right)^{\frac{1}{1-\epsilon_c}}$ and minimized cost is $\int p_{it} y_{it} di = P_t Y_t$. Derived demand for y_{it} is $y_{it} = \left(\frac{p_{it}}{P_t} \right)^{-\epsilon_c} Y_t$.

Wholesaler $i \in [0, 1]$ is a monopolistic supplier of y_i with quadratic price adjustment costs à la Rotemberg. Wholesalers are symmetric. Production function of wholesaler i is $y_{it} = Y_{it}^m$ where Y_{it}^m is a CES aggregate of intermediate goods, $Y_{it}^m = \left(\int (y_{ijt}^m)^{\frac{\eta-1}{\eta}} dj \right)^{\frac{\eta}{\eta-1}}$ with $\eta > 1$.

The real value of wholesaler i is the expected discounted sum of its future dividends, $\max \mathbb{E}\left[\sum_t \Lambda_t \Omega_{it} \right]$, where Λ_t denotes the stochastic discount factor and

$$\Omega_{it} := \frac{p_{it} Y_{it}^m - \int p_{jt}^m y_{ijt}^m dj}{P_t} - \frac{\psi_P}{2} \left(\frac{p_{it}}{p_{i,t-1}} - 1 \right)^2 Y_t,$$

subject to production and demand functions. Minimized unit cost is $P_t^m := \left(\int (p_{jt}^m)^{1-\eta} dj \right)^{\frac{1}{1-\eta}}$. Derived demand for intermediate good j is $y_{ijt}^m = \left(\frac{p_{jt}^m}{P_t^m} \right)^{-\eta} Y_{it}^m$.

We consider a symmetric equilibrium where $p_{it} = P_t$ and $p_{it}^m = P_t^m$ for all i. The relative price of the intermediate composite is denoted by $m_t := P_t^m / P_t$. The markup charged by the wholesale sector is $1/m_t$. The inflation rate is $\pi_t := P_t / P_{t-1} - 1$. Using the notations, the equilibrium aggregate profits of wholesale are $\Omega_t := \int \Omega_{it} di = (1 - m_t - (\psi_P/2)\pi_t^2) Y_t$.

Wholesalers maximize their value by choosing their prices. We obtain the following New

Keynesian Phillips curve after usual derivations[2]

$$\pi_t(1+\pi_t) = \frac{1-\epsilon_c+\epsilon_c m_t}{\psi_P} + \mathbb{E}_t\left[\frac{\Lambda_{t+1}}{\Lambda_t}\frac{Y_{t+1}}{Y_t}\pi_{t+1}(1+\pi_{t+1})\right]. \tag{1}$$

We set the stationary inflation rate at zero. The steady-state intermediate goods price is $m = 1 - 1/\epsilon_c$. Symmetry across wholesaler i implies $y^m_{jt} = y^m_{ijt}$ and $Y^m_t := \int Y^m_{it} di = Y^m_{it}$ for any i.

Intermediate goods Intermediate goods are differentiated, and each intermediate good $j \in [0,1]$ is supplied by a monopolist firm j using production function $y^m_{jt} = a_{jt}(k^\alpha_{jt} l^{1-\alpha}_{jt})^\theta$, where $\theta \leq 1$ denotes the returns to scale.

Our model features a lumpy investment behavior. For simplicity, we assume that the capital is indivisible up to a lumpiness parameter λ. Specifically, we assume that an intermediate producer is subject to discrete capital constraint $k_{jt} \in \{k_{j,t-1}(1-\delta)\lambda^s\}_{s=0,\pm1,\ldots}$, where the lumpiness parameter satisfies $(1-\delta)\lambda > 1$. While it is possible to endogenize λ by considering a non-convex cost of adjustments,[3] we adopt the indivisibility assumption for the sake of expositional simplicity.

Intermediate firm j faces demand from wholesales as $y^m_{jt} = \int y^m_{ijt} di = \left(\frac{p^m_{jt}}{P^m_t}\right)^{-\eta} Y^m_t$. The real value of intermediate producer j is $\max \mathbb{E}\left[\sum_t \Lambda_t(\varphi_{jt} - x_{jt})\right]$ where φ_{jt} denotes real operating surplus $\varphi_{jt} := \frac{p^m_{jt}}{P_t} y^m_{jt} - w_t l_{jt}$ and w_t is real wages. Firm j's demand for investment is denoted by $x_{j,t} := k_{j,t+1} - (1-\delta)k_{j,t}$. Solving for labor demand and aggregating, we obtain

[2]The first-order condition with respect to p_{it}:

$$0 = \Lambda_t Y_t\left[\frac{(1-\epsilon_c)p_{it}^{-\epsilon_c}}{P_t^{1-\epsilon_c}} + \epsilon_c \frac{P^m_t}{P_t}\frac{p_{it}^{-\epsilon_c-1}}{P_t^{-\epsilon_c}} - \psi_P\left(\frac{p_{it}}{p_{i,t-1}} - 1\right)\frac{1}{p_{i,t-1}}\right] - \mathbb{E}_t\left[\Lambda_{t+1}Y_{t+1}\psi_P\left(\frac{p_{i,t+1}}{p_{i,t}} - 1\right)\frac{-p_{i,t+1}}{p^2_{i,t}}\right].$$

We imposed the symmetry $p_{it} = P_t$ and denoted $\pi_t = p_{it}/p_{i,t-1} - 1$. Then, we have

$$0 = \Lambda_t Y_t\left[\frac{1-\epsilon_c}{P_t} + \epsilon_c\frac{P^m_t}{P_t^2} - \psi_P\frac{\pi_t}{P_{t-1}}\right] + \mathbb{E}_t\left[\Lambda_{t+1}Y_{t+1}\psi_P\pi_{t+1}\frac{1+\pi_{t+1}}{P_t}\right].$$

[3]For example, see Nirei and Scheinkman (2024) in the context of menu-cost pricing.

aggregate goods supply and labor demand functions as follows:[4]

$$Y_t = Y_t^m = ((1-1/\eta)(1-\alpha)\theta m_t/w_t)^{\frac{(1-\alpha)\theta}{1-(1-\alpha)\theta}} K_t^{\frac{\alpha\theta}{1-(1-\alpha)\theta}}, \tag{2}$$

$$L_t := \int l_{jt}dj = ((1-1/\eta)(1-\alpha)\theta m_t/w_t)^{\frac{1}{1-(1-\alpha)\theta}} K_t^{\frac{\alpha\theta}{1-(1-\alpha)\theta}}, \tag{3}$$

where $K_t := \left(\int (a_{jt}^{\frac{1}{\alpha\theta}} k_{jt})^\rho dj\right)^{1/\rho}$ and $\rho := \frac{(1-1/\eta)\alpha\theta}{1-(1-1/\eta)(1-\alpha)\theta}$.

At the optimal factor demands, the operating surplus is written as a function of productivity and capital $\varphi_t(a_{jt}, k_{jt})$ as

$$\varphi_t(a_{jt}, k_{jt}) = (a_{jt}^{1/(\alpha\theta)} k_{jt})^\rho c_\varphi \left(m_t/w_t^{(1-\alpha)\theta}\right)^{\frac{1}{1-(1-\alpha)\theta}} K_t^{\frac{(\alpha\theta/\eta)/(1-(1-\alpha)\theta)}{1-(1-1/\eta)(1-\alpha)\theta}} \tag{4}$$

where $c_\varphi := (1-(1-1/\eta)(1-\alpha)\theta)\left((1-1/\eta)(1-\alpha)\theta\right)^{\frac{(1-\alpha)\theta}{1-(1-\alpha)\theta}}$. Aggregating (4) yields an expression for aggregate operating surplus:

$$\int \varphi(a_i, k_i)di = c_\varphi \left(m_t/w_t^{(1-\alpha)\theta}\right)^{\frac{1}{1-(1-\alpha)\theta}} K_t^{\frac{\alpha\theta}{1-(1-\alpha)\theta}}.$$

2.2 Lumpy investment

An intermediate producer's capital choice is subject to indivisibility constraint, which plays a central role in our analysis. The operating surplus function φ_t in (4) is strictly concave in k_{jt}. Since the capital choice of an intermediate firm is constrained by a discrete set, the firm optimally chooses a threshold of capital for an investment spike by which the firm's capital increases by a factor of λ. The firm can choose an investment spike in t and no spike in $t+1$, or no spike in t and a spike in $t+1$, which leaves the firm at the same level of capital in $t+2$. At the threshold, firm j is indifferent between the two alternative plans. Thus, the optimal threshold of investment spike, $k^* = k_{j,t+1}$, satisfies the following indifference condition:

$$\mathbb{E}_t\Lambda_{t+1}(\varphi_{t+1}(a, k^*) + (1-\delta)k^*) - \Lambda_t k^* = \mathbb{E}_t\Lambda_{t+1}(\varphi_{t+1}(a, \lambda k^*) + (1-\delta)\lambda k^*) - \Lambda_t \lambda k^*.$$

[4]The derivation is straightforward. The nominal sales is written as $p_{jt}^m y_{jt}^m = (y_{jt}^m)^{1-1/\eta}(Y_t^m)^{1/\eta} P_t^m$ using the demand function for y_{jt}^m. The first-order condition for cost minimization with respect to l_{jt} is $(1-1/\eta)(y_{jt}^m/Y_t^m)^{-1/\eta} m_t(\partial y_{jt}^m/\partial l_{jt}) = w_t$. Thus, the labor cost share is constant: $w_t l_{jt}/(y_{jt}^m p_{jt}^m/P_t) = (1-1/\eta)(1-\alpha)\theta$. Using this and the production function to substitute out l_t and aggregating across j yields (2). Aggregate labor demand is $L_t := \int l_{jt}dj = \int (1-1/\eta)(1-\alpha)\theta(y_{jt}^m)^{1-1/\eta}(Y_t^m)^{1/\eta} m_t/w_t = (1-1/\eta)(1-\alpha)\theta Y_t^m m_t/w_t$ and hence (3) obtains.

Solving for k^*, we obtain

$$k^*_{j,t+1} = \tilde{a}_{j,t+1}\Phi_t K^{\tilde{\theta}}_{t+1} \tag{5}$$

where $\tilde{a}_{j,t} := a_{j,t}^{\frac{1-1/\eta}{1-\theta+\theta/\eta}}$, $\tilde{\theta} := \frac{(\alpha\theta/\eta)/(1-(1-\alpha)\theta)}{1-\theta+\theta/\eta}$, and Φ_t summarizes the expected factor prices

$$\Phi_t := \left(c_\varphi \frac{\lambda^\rho - 1}{\lambda - 1} \mathbb{E}_t \left[\Lambda_{t+1} \left(m_{t+1}/w_{t+1}^{(1-\alpha)\theta} \right)^{\frac{1}{1-(1-\alpha)\theta}} \right] \mathbb{E}_t \left[\Lambda_t - \Lambda_{t+1}(1-\delta) \right]^{-1} \right)^{\frac{1}{1-\rho}}.$$

The optimal inaction region for firm j in t is $k_{j,t} \in [k^*_{j,t+1}/(1-\delta), \lambda k^*_{j,t+1}/(1-\delta))$. The support of a stationary distribution is $k_{j,t} \in [k^*_{j,t}, \lambda k^*_{j,t})$.

We define a state variable s_{jt} of firm j that denotes the distance of log capital from the threshold level as:

$$s_{jt} := \frac{\log k_{jt} - \log k^*_{jt}}{\log \lambda}.$$

Let F_t denote a joint distribution of (a_{jt}, s_{jt}) and F the stationary distribution. Aggregate capital is written by using F_t as $K_t = \left(\int (a_{jt}^{1/(\alpha\theta)} k_{jt})^\rho dj \right)^{1/\rho} = \mathbb{E}^{F_t} \left[(a_{jt}^{1/(\alpha\theta)} k^*_{jt} \lambda^{s_{jt}})^\rho \right]^{1/\rho}$. Substituting $k^*_{j,t+1}$ into this expression for K_{t+1} yields an equilibrium condition for factor prices Φ_t:

$$1 = \mathbb{E}^{F_{t+1}} \left[\tilde{a} \lambda^{\rho s} \right]^{1/\rho} \Phi_t K^{\tilde{\theta}-1}_{t+1}. \tag{6}$$

2.3 Households

Representative households' utility function is $\mathbb{E}_0 \left[\sum_{t=0}^\infty \beta^t (u(c_t) - v(N_t)) \right]$ where u is strictly concave and v is convex. Households own monopolistic firms and receive profits of wholesale Ω_t and intermediate producers $\int \varphi(a_{it}, k_{it}) - x_{it}\, di$. They have access to risk-free nominal asset A_t at risk-free nominal rate i_t. Households' budget constraint in nominal terms is written as

$$A_t + P_t c_t = (1 + i_{t-1}) A_{t-1} + P_t \left(w_t N_t + \Omega_t + \int \varphi(a_{it}, k_{it}) - x_{it}\, di \right). \tag{7}$$

The choice of A_{t+1} leads to the Euler equation

$$u'(c_t) = \beta \mathbb{E}_t \left[\frac{1 + i_t}{1 + \pi_{t+1}} u'(c_{t+1}) \right]. \tag{8}$$

Households instruct their stochastic discount factor $\Lambda_{t+\tau} = \beta^{t+\tau} u'(c_{t+\tau})$ to the firms they own and take the dividends and investments X_t as given and exogenous for their decision.

2.4 Sticky real wage

We introduce various degrees of real wage stickiness to the model. Let $g \in [0, 1]$ denote an exogenous parameter that indicates the flexibility of real wages. The labor supply function is written as

$$w_t = (w_t^f)^g (w_{ss})^{1-g} \tag{9}$$

where

$$w_t^f := v'(N_t)/u'(c_t)$$

denotes a frictionless real wage. This labor supply function nests the polar cases of perfectly flexible real wage $g = 1$ and of constant real wage $g = 0$.

2.5 Monetary policy and market clearing

We assume that monetary policy sets a risk-free nominal rate at

$$1 + i_t = (1 + r_{ss})(1 + \pi_t)^{\phi} \tag{10}$$

with Taylor principle $\phi > 1$. A risk-free asset is supplied at net zero: $A_t = 0$.

The market-clearing conditions for labor and final goods are

$$N_t = L_t \tag{11}$$

$$Y_t \left(1 - \frac{\psi_P}{2}\pi_t^2\right) = c_t + X_t. \tag{12}$$

Aggregate capital is accumulated as

$$K_{t+1} = (1 - \delta)K_t + A_X X_t. \tag{13}$$

Parameter A_X adjusts the difference between aggregate investments X_t and productivity-weighted average of capital K_t, and depends only on the productivity profile and exogenous parameters.[5]

[5] A_X is derived under the premise of s following a uniform distribution (Nirei, 2015). Let s_{it}^* denote the

2.6 Recursive equilibrium

A recursive equilibrium of the above model in a general form involves a dynamic mapping of firms' state distribution $F_t(a, s)$. To maintain tractability, we assume the following for the firm's state variables.

Assumption 1 *(i) $a_{i,t}$ is i.i.d. across i and t and has a finite support $\mathcal{A} = \{a(1), a(2), \ldots, a(H)\}$ with $\max(a) - \min(a) < -(1-\delta)/\log\lambda$,*

(ii) Initial value $s_{i,0}$ conditional on every value of $a_{i,0} \in \mathcal{A}$ is uniformly distributed over $[0, 1)$.

Under Assumption 1, $s_{i,t}$ continues to follow the uniform distribution for any t, since adding to s_{it} independent random shocks or aggregate shocks leaves the uniform distribution of s_{it} unchanged as s_{it} rotates on a circle (see, e.g., Caballero and Engel (1991)). The limited support of a imposed by (i) ensures that firms do not choose to divest in a stationary equilibrium. This simplifies the analysis because firms in a stationary equilibrium either increase their capital by λ or choose inaction.

The recursive equilibrium of $(Y_t, K_{t+1}, X_t, L_t, N_t, c_t, w_t, m_t, i_t, \pi_t)$ with a continuum of intermediate firms $j \in [0, 1]$ is determined by (1,2,3,6,7,8,9,10,11,12,13) under Assumption 1. We write $K_{t+1} = \Xi(K_t)$ for a mapping of aggregate capital which the recursive equilibrium determines.

$F_t(a, s)$ stays at the stationary distribution F under Assumption 1. Writing $B := \mathbb{E}^F[\tilde{a}\lambda^{\rho s}]^{-1/\rho}$, (6) implies $\Phi_t = BK_{t+1}^{1-\tilde{\theta}}$, leading to the New Keynesian IS relationship:

$$u'(c_t) = \beta\mathbb{E}_t\left[u'(c_{t+1})\left(\frac{c_\varphi}{B^{1-\rho}}\frac{\lambda^\rho - 1}{\lambda - 1}\left(\frac{w_{t+1}^{(1-\alpha)\theta}}{m_{t+1}}\right)^{\frac{-1}{1-(1-\alpha)\theta}} K_{t+1}^{-\frac{1-\theta}{1-(1-\alpha)\theta}} + 1 - \delta\right)\right].$$

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state corresponding to $k^*_{i,t+1}$. Under $\theta = 1$, we obtain $A_X = \left(\frac{\lambda-1}{\log\lambda}B\mathbb{E}[a_{it}^{\eta-1}]\right)^{-1}$ as follows.

$$X_t = \int\int_0^{s^*_{it}} (\lambda-1)(1-\delta)k_{it}ds_{it}di = (1-\delta)(\lambda-1)\int\int_0^{s^*_{it}} \lambda^{s_{it}}k^*_{it}ds_{it}di = \frac{(1-\delta)(\lambda-1)}{\log\lambda}\int(\lambda^{s^*_{it}} - 1)k^*_{it}di$$
$$= \frac{(1-\delta)(\lambda-1)}{\log\lambda}\int((1-\delta)^{-1}k^*_{i,t+1} - k^*_{it})di = \frac{\lambda-1}{\log\lambda}B\mathbb{E}[a_{it}^{\eta-1}](K_{t+1} - (1-\delta)K_t)$$

Equilibrium dynamics when $\theta = 1$ From now on, we focus on the constant-returns-to-scale case, $\theta = 1$. The recursive equilibrium system is written as follows.[6]

$$
\begin{aligned}
Y_t &= K_t^{\alpha} L_t^{1-\alpha} && \text{(GDP)} \\
L_t &= \left((1-1/\eta)(1-\alpha) m_t / w_t\right)^{\frac{1}{\alpha}} K_t && \text{(Labor Demand)} \\
w_t &= \left(v'(N_t)/u'(c_t)\right)^{g} w_{ss}^{1-g} && \text{(Labor supply)} \\
N_t &= L_t && \text{(Labor market clearing)} \\
c_t + X_t &= \left(1 - \frac{\psi_P}{2}\pi_t^2\right) Y_t && \text{(Goods market clearing)} \\
K_{t+1} &= (1-\delta) K_t + A_X X_t && \text{(Capital accumulation)} \\
u'(c_t) &= \beta \mathbb{E}_t \left[\frac{(1+r_{ss})(1+\pi_t)^{\phi}}{1+\pi_{t+1}} u'(c_{t+1}) \right] && \text{(Euler equation and Taylor rule)} \\
1 &= c_{\varphi} B^{\rho-1} \frac{\lambda^{\rho}-1}{\lambda-1} \mathbb{E}_t \left[\frac{\Lambda_{t+1}}{\Lambda_t} \left(m_{t+1}^{1/\alpha} w_{t+1}^{1-1/\alpha} + 1 - \delta \right) \right] && \text{(Factor prices)} \\
\pi_t(1+\pi_t) &= \frac{1-\epsilon_c+\epsilon_c m_t}{\psi_P} + \mathbb{E}_t \left[\frac{\Lambda_{t+1}}{\Lambda_t} \frac{Y_{t+1}}{Y_t} \pi_{t+1}(1+\pi_{t+1}) \right] && \text{(Phillips curve)}
\end{aligned}
$$

Steady state The steady state of the above system of equations satisfies $\pi_t = 0$, $\Lambda_{t+1}/\Lambda_t = \beta$, and $X_t = K_t \delta / A_X$. Under flexible real wages $g = 1$, the steady-state wage is determined by:

$$
\left(\frac{1}{\beta} - 1 + \delta\right)^{\alpha} w^{1-\alpha} = \mathbb{E}^F \left[\lambda^{s\rho} a^{\eta-1}\right]^{1/(\eta-1)} \left(\frac{\lambda^{\rho}-1}{\lambda-1} c_{\varphi}\right)^{\alpha} \frac{\epsilon_c - 1}{\epsilon_c}.
$$

It is straightforward to see that our model is continuous with the limit case of divisible capital when $\lambda \to 1$. By solving the model with divisible capital, we obtain the steady-state wage w_{smooth} to be determined by

$$
(1/\beta - 1 + \delta)^{\alpha} w_{smooth}^{1-\alpha} = \mathbb{E}[a^{\eta-1}]^{1/(\eta-1)} \alpha^{\alpha} (1-\alpha)^{1-\alpha} (1-1/\eta)(1-1/\epsilon_c),
$$

which coincides with the equation above when $\lambda \to 1$.

[6] In the equations, the stochastic discount factor satisfies $\Lambda_{t+1}/\Lambda_t = \beta u'(c_{t+1})/u'(c_t)$. Constants are $\rho = \frac{(1-1/\eta)\alpha}{1-(1-1/\eta)(1-\alpha)}$, $c_{\varphi} = \left(1-(1-1/\eta)(1-\alpha)\right)\left((1-1/\eta)(1-\alpha)\right)^{\frac{1-\alpha}{\alpha}}$, and $B = \mathbb{E}^F[\lambda^{s\rho} a^{\eta-1}]^{-1/\rho}$. To evaluate B and A_X, we normalize the productivity by $\mathbb{E}[a^{\eta-1}] = 1$ and use uniformly distributed s, resulting in $\mathbb{E}[\lambda^{s\rho}] = (\lambda^{\rho}-1)/(\rho \log \lambda)$ and $A_X = \left(\frac{\lambda-1}{\log \lambda} \frac{\lambda^{\rho}-1}{\rho \log \lambda}\right)^{-1}$. Households' budget constraint is omitted by Walras's law.

Approximated dynamics around the steady state We consider a first-order perturbation of the equilibrium system around the steady state to analyze the dynamics. We denote as $\tilde{X}_t$ the proportional deviations of the variable X_t to its steady state value. The preference is specified as $u(c) = (c^{1-\sigma} - 1)/(1-\sigma)$ and $v(N) = \chi N^{1+\gamma}/(1+\gamma)$ with $\sigma > 0$ and $\gamma \geq 0$.

$$
\begin{aligned}
\tilde{K}_{t+1} &= (1-\delta)\tilde{K}_t + \delta \tilde{X}_t \\
\alpha \tilde{K}_t + (1-\alpha)\tilde{L}_t &= \frac{c_{ss}}{Y_{ss}}\tilde{c}_t + \frac{X_{ss}}{Y_{ss}}\tilde{X}_t \\
\tilde{m}_t - \tilde{w}_t &= \alpha(\tilde{L}_t - \tilde{K}_t) \\
\tilde{w}_t &= g(\sigma \tilde{c}_t + \gamma \tilde{L}_t) \\
\sigma(\tilde{c}_{t+1} - \tilde{c}_t) &= \frac{\alpha(K_{ss}/L_{ss})^{\alpha-1}}{\alpha(K_{ss}/L_{ss})^{\alpha-1} + 1 - \delta}\left((1-\alpha)(\tilde{L}_{t+1} - \tilde{K}_{t+1}) + \tilde{m}_{t+1}\right) \\
\pi_t &= \frac{\epsilon_c - 1}{\psi_P}\tilde{m}_t + \beta \pi_{t+1} \\
\sigma(\tilde{c}_{t+1} - \tilde{c}_t) &= \phi \pi_t - \pi_{t+1}
\end{aligned}
$$

2.7 Complementarity of lumpy investment decisions

In our analysis, the complementarity exhibited by firms' investment decisions in equilibrium plays a key role. We define the degree of complementarity using the model developed in this section.

Consider a steady-state equilibrium of the model with a continuum of firms. Let Δ denote a forward difference of a variable concerning t. The state variable $s_{it} = (\log k_{it} - \log k^*_{it})/\log \lambda$ evolves as

$$
s_{i,t+1} = s_{it} + \frac{\log(1-\delta) - \Delta \log K_t - (\eta - 1)\Delta \log a_{it}}{\log \lambda}
$$

if $s_{i,t+1} > 0$. In an avalanche process, depreciation $1-\delta$ and productivity shocks $\Delta \log a_{it}$ are realized first. At the steady state of a continuum model, $\Delta \log K_t = 0$. Thus, firms with $s_{it} \leq -\frac{\log(1-\delta) - (\eta-1)\Delta \log a_{it}}{\log \lambda}$ invest and transit to $s_{i,t+1} = 1 + s_{it} + \frac{\log(1-\delta) - (\eta-1)\Delta \log a_{it}}{\log \lambda}$, whereas other firms' $s_{i,t+1}$ is positive and determined by the equation above. We denote the threshold state for investment as $s^*_{it} := -\frac{\log(1-\delta) - (\eta-1)\Delta \log a_{it}}{\log \lambda}$.

Since $a_{i,t}$ has a finite support $\mathcal{A} = \{a(1), \ldots, a(H)\}$ by Assumption 1, firms are classified

into H^2 groups according to the realization of $(a_{i,t}, a_{i,t+1})$. In particular, firms experiencing $a_{i,t} = a(h_0)$ and $a_{i,t+1} = a(h_1)$ belong to group $h = h_0 + (h_1 - 1) * H$. Write $\Delta \log a(h) = \log a(h_1) - \log a(h_0)$, $s^*(h) = -(\log(1-\delta) - (\eta - 1)\Delta \log a(h))/\log \lambda$, and let $\mu(h)$ denote the measure of firms in group h.

Now we suppose that firms in group h and located in $[s^*(h), s^*(h) + \nu(h)]$ for a small value $\nu(h) > 0$ also invest. Since s is uniformly distributed, $\mu(h)\nu(h)$ is the measure of firms located in this interval. Their investments increase K_{t+1} and ΔK_{t+1}, reducing s_{jt} of other firms by ν'. The measure of firms induced to invest because of the reduction in s_{jt} is ν'. Thus, we define the degree of complementarity as $\vartheta(h) := \lim_{\nu(h)\to 0} \nu'/(\mu(h)\nu(h))$. The complementarity $\vartheta(h)$ measures the number of firms induced to invest by an investment of a firm in group h. Then, we obtain the following property.

Proposition 1 *The degree of complementarity when a firm in the group h invests is*

$$\vartheta(h) = \tilde{\theta}\frac{\tilde{a}(h_1)}{\mathbb{E}^F[\tilde{a}]},$$

where $\tilde{\theta} = \frac{(\alpha\theta/\eta)/(1-(1-\alpha)\theta)}{1-\theta+\theta/\eta}$. The expected value $\vartheta = \sum_h \mu(h)\vartheta(h) = \tilde{\theta}$ is increasing in θ for $\theta \leq 1$ and converges to 1 as $\theta \to 1$.

The proof is provided in the appendix.

The measure of firms induced to invest by an investment of a firm which is chosen randomly across groups is denoted by ϑ. Since ϑ is an expected value of $\vartheta(h)$ across h, an immediate consequence of Proposition 1 is $\vartheta = 1$ under constant-returns-to-scale technology $\theta = 1$. This implies that, when an extra firm located at s^*_{it} invests, one more firm, on average, is induced to invest through the positive dependence of threshold k^* on aggregate capital K.

3 Investment demand shocks in a finite economy

In the previous section, we present a model with a continuum of firms. We showed that the firms' investment decisions exhibit complementarity in equilibrium. However, this complementarity cannot generate stochastic fluctuations because, with a continuum of firms, any idiosyncratic stochastic variables are aggregated to a deterministic value. This section introduces a model with a finite number of firms n. The model with a continuum of firms in the

previous section approximates the finite economy model when $n \to \infty$. The finite economy model provides us with a microfoundation for investment demand shocks, whereas the continuum model pins down the expectation of model agents and determines the propagation mechanism of shocks, including investment demand shocks.

Due to the finiteness of firms, each of which is constrained by indivisible capital, aggregate investments generically deviate from those of the continuum model. We will show in Section 4 that this deviation is quantitatively non-negligible even when n is large. The non-negligible deviation is generated by an amplification effect of the number of firms engaging in investment spikes, which we call an *investment avalanche*. In Section 3, we start with defining an investment demand shock. We then demonstrate impluse-response functions in a simple case of the model, which indicate that the shock generates contemporaneous comovement of investment, consumption, and inflation.

3.1 Investment demand shocks: definition

In Section 2, we present a model with a continuum of intermediate producers and derive a recursive equilibrium that pins down the aggregate capital path as $K_{t+1} = \Xi(K_t)$. The intermediate producers incur idiosyncratic productivity shocks but cannot generate aggregate stochastic fluctuations because a continuum of independent shocks always integrates to a deterministic value.

This section considers a large but finite n number of intermediate producers. Due to the finiteness, the idiosyncratic shocks on producers can generate aggregate stochastic fluctuations around the deterministic path determined by Ξ. Let $X_t^e = (K_{t+1} - \Xi(K_t))/A_X$ denote the aggregate investment expected in the continuum model. Then, an *investment demand shock* ϵ_t is defined as a deviation of actual aggregate investment from X_t^e:

$$\epsilon_t := X_t^n / X_t^e - 1,$$

where X_t^n is the aggregate investment determined in the model with n firms, which shares the predetermined aggregate capital with the continuum model, $K_t^n = K_t$.

The precise model with n firms is shown in Section 4.1. Firms' behaviors in the model with n firms are aggregated so that their limits in $n \to \infty$ correspond to those in a continuum

model. We need two assumptions to facilitate the analysis of a finite economy model: a behavioral assumption to manage the dimensionality issue arising from a finite number of firms and an equilibrium selection to handle a multiplicity of equilibria. For a potential multiplicity problem, we simply choose an equilibrium aggregate capital K^n_{t+1} close to a continuum model's equilibrium aggregate capital $K_{t+1} = \Xi(K_t)$, as specified in Section 4.

With n firms and an aggregate shock ϵ_t, the model economy features the profile of $(a_i, s_i)_i$ as a state, which causes the curse of dimensionality in the sense of Krusell and Smith, Jr. (1998). To deal with the issue, we make a behavioral assumption that agents use F instead of the actual profile of (a, s) to form expectations. Consequently, households expect the economy's future path to be determined by Ξ. Furthermore, we assume that intermediate firm j follows their threshold policy under the stationary distribution F in a continuum economy. Namely, firm j follows a threshold rule (5) with $\Phi_t = \Phi$. Thus, intermediate firms respond to realizations of K^n_{t+1} and $a_{i,t+1}$, but not to $(a_{i,t+1}, s_{i,t+1})_i$.

It is commonly thought that idiosyncratic productivity shocks would generate little aggregate fluctuation in ϵ_t due to the diversification effect. According to the central limit theorem, the standard deviation of an average of independent n shocks declines as fast as $1/\sqrt{n}$. Thus, for example, one million firms' idiosyncratic shocks of 10% standard deviation generate aggregate volatility of $10\%/\sqrt{10^6} = 0.01\%$ standard deviation, which does not amount to the order of magnitude for business cycle fluctuations.

The diversification argument for ϵ_t does not hold in our model with n firms. First, a firm's investment is a non-linear function of aggregate capital due to indivisibility. Second, firms' investment decisions are complementary to each other in equilibrium. A firm's decision to invest increases the aggregate capital, which raises the demand for all firms and pushes up the threshold of their investment decisions. Hence, a firm's lumpy investment increases the likelihood of the other firms' investments and causes an avalanche of investment spikes within a period. The aggregate investment demand shock ϵ_t is determined by the size of the investment avalanche.

We will show in Section 4 that the investment demand shock caused by an avalanche exhibits quantitatively substantial variation. Analytically, we will show that the convergence speed of the volatility of ϵ_t to zero is significantly slower than $1/\sqrt{n}$. Thus, Section 4 provides

a microfoundation for the aggregate investment demand shock ϵ_t.

We set the timing of the investment demand shock as follows. At the end of $t-1$, aggregate capital K_t is already installed, and the recursive equilibrium pins down the expected capital in $t+1$ as $K^e_{t+1} = \Xi(K_t)$. Thus, the investment of t expected at the end of $t-1$ is $X^e_t = (K^e_{t+1} - (1-\delta)K_t)/A_X$.

At the beginning of t, the actual aggregate investment X^n_t is determined in a model with n firms, which depends on n firms' capital profile and the realization of idiosyncratic productivity shocks. The deviation of X^n_t from X^e_t constitutes an investment demand shock ϵ_t. After ϵ_t is realized, production and consumption in t occur.

3.2 Inflationary response to an investment demand shock: A simple case of constant real wages

We obtain an analytical solution for the dynamics in the simple setup with constant real wages, i.e., $g = 0$. Suppose that the economy was at its steady state in $t = 0$, and an investment shock $\epsilon_1 > 0$ hits in $t = 1$. In period 2, the economy starts with a higher capital stock $\tilde{K}_2 > 0$. Since the economy has more wealth and higher labor productivity that the steady state, the dynamics after $t = 2$ exhibit a positive supply shock compared to the steady-state, generating a dynamics of consumption boom $\tilde{c}_t > 0$ and deflation $\pi_t < 0$ for $t = 2, 3, \ldots$.

We are interested in the response on impact, $t = 1$, before the capital increases. The linearized dynamics in Section 2.6 can be used to determine the response analytically.

$$\frac{K^\alpha_{ss} L^{1-\alpha}_{ss}}{c_{ss}}(1-\alpha)\tilde{L}_1 = \tilde{c}_1 + \frac{X_{ss}}{c_{ss}}\epsilon_1 \tag{14}$$

$$\tilde{m}_1 = \alpha \tilde{L}_1 \tag{15}$$

$$\pi_1 = \frac{\epsilon_c - 1}{\psi_P}\tilde{m}_1 + \beta\pi_2 \tag{16}$$

$$\tilde{c}_2 - \tilde{c}_1 = \frac{1}{\sigma}\left(\phi\pi_1 - \pi_2\right). \tag{17}$$

The investment demand shock propagates to inflation in this four-equation system (14–17) intuitively. The increase in investment demand ϵ_1 shifts out labor demand in (14) for fixed consumption. The increase in labor input reduces the marginal product of labor in the

intermediate sector and pushes up the intermediate goods price given the constant real wage in (15). The increase in $\tilde{m}_t$ leads to cost-push inflation in the Phillips curve (16).

The procyclical effect on inflation may be reversed if $\tilde{c}_1$ decreases significantly. However, this possibility can be ruled out if the deflation π_2 is sufficiently small. We obtain $\pi_1 > 0$ when $\pi_2 = 0$ and $\tilde{c}_2 > 0$ after some algebra of the four equations.

Diminishing returns to labor and real wage rigidity provide a key environment for the propagation of an investment demand shock to inflation. If returns to labor inputs are constant, we do not have a channel (15) where an increase in labor inputs raises marginal costs of final goods. Capital plays an essential role here, as can be seen by $\alpha > 0$ in (15). The sticky real wage is also important. In a general case $g > 0$, labor demand function is $\tilde{m}_t - \tilde{w}_t = \alpha(\tilde{L}_t - \tilde{K}_t)$ as in Section 2.6. Thus, a decrease in real wages may facilitate an increase in labor input if wages are sufficiently flexible, mitigating the cost pressure on inflation.

3.3 Impulse-response functions

We solve for impulse-response functions numerically for a general case of $g > 0$ when real wages are sticky but not constant. We will contrast the investment demand shock to a marginal efficiency of investment (MEI) concerning the inflation responses. To do so, we allow A_X in the capital accumulation equation (13) to be stochastic. This modification is implemented simply by introducing an i.i.d. random variable a_t^X that denotes the logarithmic difference from the steady state value of A_X. Since A_X determines the efficiency of conversion from investment to capital, a_t^X represents a marginal efficiency of investment (MEI) shock highlighted by Justiniano, Primiceri, and Tambalotti (2010).

We consider two versions of the IST shock. In the first version, we follow Justiniano, Primiceri, and Tambalotti (2010) to define the MEI shock as a transformation ratio of newly installed capital to purchased investment goods. The MEI shock is revealed to agents when the investment materializes as capital, i.e., in the next period of the investment purchase. An alternative version of MEI shock features a news shock in which MEI shocks that expand investment in $t = 2$ are revealed to agents in $t = 1$. The first version generates a straightforward lack of response in $t = 1$, whereas the second version generates actions in $t = 1$ caused

by the arrival of the news.

Parameter values are calibrated as follows. We calibrate the model to annual data and set $\beta = 0.98$ and $\delta = 0.1$. We assume constant returns to scale production technology, $\theta = 1$. The elasticity of substitution for the final wholesale sector is set at $\epsilon_c = 6$. We set χ so that the steady-state labor supply equals 1. The degree of wage flexibility is set at $g = 0.12$. The lumpiness parameter is $\lambda = 1.2$, corresponding to the benchmark value of 20% used in the literature on lumpy investments (Cooper, Haltiwanger, and Power, 1999; Gourio and Kashyap, 2007). The rest of the parameter values are calibrated as shown in Table 1.

α	β	δ	γ	ϵ_c	η	σ	ψ_P	λ	ϕ
0.36	0.98	0.1	0.5	6	30	3	20	1.2	1.2

Table 1: Calibrated values of parameters at the annual frequency

To compute the impulse-response of an investment demand shock, we set that an investment demand X increases by 1%, causing capital in the next period increases by 0.1% from the steady state level. When we compute the impulse-response of an IST shock without news, we set that A_X increases by 1% so that the capital in the next period is 0.1% higher than the steady state. For the case of IST shock with news, we set an increase of A_X so that the increase in the next period increases by 0.1%.[7]

Figure 2 shows the impulse responses for an investment demand shock (avalanche) in solid lines and those for an IST news shock in dashed lines. The impulse responses for an IST shock without news are identical to that for an IST news shock on and after $t = 2$, whereas no responses occur in $t = 1$.

The top panel shows that the capital transition is identical between the two experiments: capital increases by 0.1% in the first period after the shock, and it slowly converges back to the steady state. The bottom left figure shows that the consumption transition is identical after $t = 2$. The consumption responses differ in $t = 1$. Consistently with our analysis of $g = 0$, an investment demand shock causes a positive reaction in contemporaneous consumption.

[7]When an IST shock is revealed to agents in $t = 1$, choice X_t changes. Thus, in this experiment, we set the value of capital increase (0.1%) to be comparable to the avalanche shock and computed the IST shock accordingly.

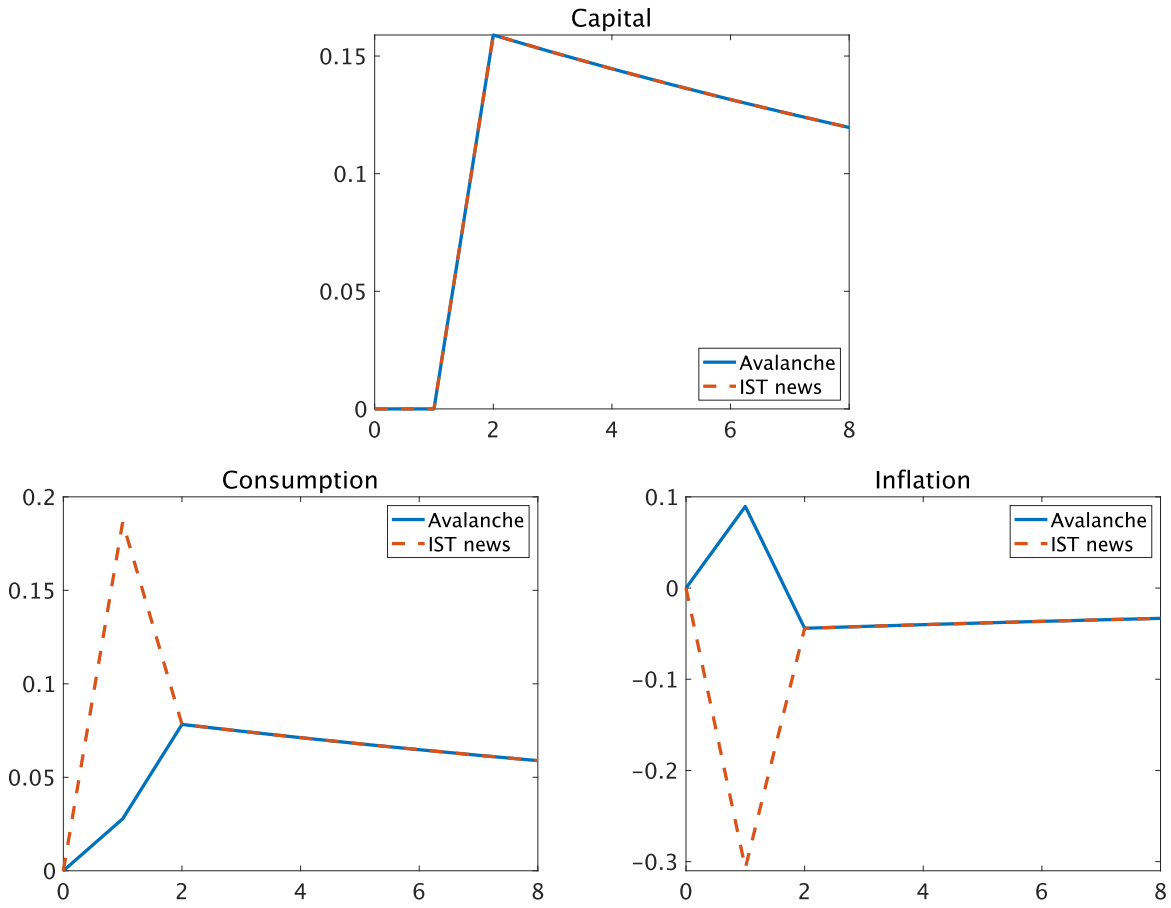


Figure 2: Impulse response functions (annual) for capital, consumption, and inflation rates (%)

The bottom-right plot shows inflation's response. The inflation rate dips below the steady state level in all experiments after $t = 2$. This reflects the convergence of capital toward the steady state. After the capital jumps up, the slow decline of capital is associated with a slow decrease in consumption. Hence, during the transition, the real interest rate must increase toward the steady-state level, leading to inflation that drops below the steady-state level when the capital is increased and increases toward the steady state afterward.

A stark contrast emerges in the inflation response between an investment demand shock and an IST news shock. Consistently with our previous analysis for $g = 0$, the inflation rate reacts positively to an investment demand shock in the period when the shock hits. This procyclical effect of the investment demand shock on inflation is understood by inspecting

the equilibrium conditions. The investment demand shock affects consumption negatively in the goods market clearing condition and positively in the labor demand condition. The first direct effect on consumption is mitigated by the wealth effect in which households, knowing that the capital is increased in the future, wish to increase the consumption level for a prolonged time. The second direct effect reduces the marginal product of labor and raises the real price of intermediate goods when the real wage is rigid. An increase in marginal unit labor costs is passed through to an intermediate price. The increase in intermediate cost pushes up the inflation rate.

The contemporaneous response to an IST shock was completely muted as an artifact of the specification that the productivity shock was realized one period after the shock. An alternative specification is that the investment-specific productivity shock that takes effect in the next period is known in this period. In this specification, IST features an element of news shocks.

We note that the positive consumption response and the negative inflation response occur by the IST news shock. This is easy to interpret. The wealth effect caused by the positive shock on the investment sector results in an increase in consumption when the shock is revealed. The immediate response in consumption is more potent than that of an investment demand shock because the future capital increase arrives without costing any investment resources. The sizeable immediate increase in consumption leads to negative consumption growth in the subsequent period, depressing the real interest rate and thus lowering the inflation rate under the Taylor principle of the monetary policy response function.

The analysis in this section demonstrates that an investment demand shock can cause a positive inflationary response contemporaneously. An investment demand shock increases capital in the future, leading to a wealth effect that increases consumption. Moreover, the increase in investment and consumption shifts the labor demand outward. Under a sticky real wage, the increased labor demand raises the unit labor cost and the cost of intermediate goods for final goods producers. This marginal cost effect causes contemporaneous inflation.

3.4 Policy analysis

The investment demand shock contrasts with an IST news shock in inflation responses on impact. In addition, the demand shock has policy implications that differ from the IST shocks. The investment demand shock leads to inefficiently large working hours under real wage rigidity. This leads to an increase in inflation for households to work and consume more, resulting in distortions in the labor supply that should be reduced. This increase in inflation is much smaller in the IST case.

We illustrate this point by an exercise to consider an optimal inflation rate set by a discretionary monetary policy with no concern for future commitment issues. In period t, the monetary authority faces an investment demand shock or an IST shock. Capital in $t+1$ will increase above the steady-state level, and the recursive equilibrium pins down all $t+1$ variables given to the monetary authority. Consider a choice of π_t that maximizes representative households' utility:

$$\max_{c_t, L_t} u(c_t) - v(L_t) + \beta V\left(K_{t+1}, Y_{t+1}, \pi_{t+1}\right).$$

subject to all equilibrium conditions, including

$$K_{ss}^{\alpha} L_t^{1-\alpha} = c_t + \frac{\psi_P}{2}\pi_t^2 Y_{ss} + X_{ss}\left(1+\epsilon_t\right)$$

in the constant real wage case, $g = 0$.

Since all $t+1$ variables are given, only the allocation in t is a matter of concern. Then,

$$\pi_t(1+\pi_t) = \frac{\varepsilon_P - 1}{\psi_P}\Big(\frac{w_{ss}}{(1-\alpha)\,K_{ss}^{\alpha}} L_t^{\alpha} - 1\Big) + \beta \mathbb{E}_t\Big(\pi_{t+1}(1+\pi_{t+1})\frac{Y_{t+1}}{Y_t}\Big).$$

This equation implicitly defines an equilibrium relationship Ψ such that:

$$L_t := \Psi(\pi_t).$$

Using Ψ, the maximization problem for the monetary policymaker is:

$$\max_{\pi_t} u\left(K_{ss}^{\alpha}\Psi(\pi_t)^{1-\alpha} - \frac{\psi_P}{2}\pi_t^2 Y_{ss} - X_{ss}(1+\epsilon_t)\right) - v(\Psi(\pi_t)) + \beta V(K_{t+1}, Y_{t+1}, \pi_{t+1}).$$

With writing $\mathrm{MPL}_t := (1-\alpha)\,K_{ss}^{\alpha} L_t^{-\alpha}$, the first-order condition for optimal π_t is:

$$u'(c_t) \times \mathrm{MPL}_t - v'(L_t) = \frac{u'(c_t)\psi_P \pi_t Y_{ss}}{\Psi'(\pi_t)}.$$

If $\pi_t = 0$, then (up to the forward term), the equilibrium allocation yields $L_t = L_{ss}$ and $\text{MPL}_t = \text{MPL}_{ss}$,but $c_t < c_{ss}$. As a consequence, we obtain:

$$\begin{aligned} & u'\left(c_t\left(\pi_t = 0\right)\right) \times \text{MPL}_t\left(\pi_t = 0\right) - v'\left(L_t\left(\pi_t = 0\right)\right) \\ & = u'\left(c_t\left(\pi_t = 0\right)\right) \times \text{MPL}_{ss} - v'(L_{ss}) > 0. \end{aligned}$$

Thus, the policymaker wants the agent to consume and work more. To achieve this allocation, the inflation rate must be positive $\pi_t > 0$.

Indeed, if the inflation rate is 0 in the first period, then labor and output are very close to their steady-state values due to the New-Keynesian Phillips curve. As investment increases and output is fixed, consumption falls. As a consequence, the first-order condition of households according to their labor supply is not fulfilled: $\text{MPL}_t \times u'(c_t) - v'(L_t) > 0$. In other words, agents would like to work more to consume more. A positive inflation stimulates the economy, as the marginal cost of labor has to increase. Since the real wage is fixed, this implies that the productivity of labor decreases, and thus labor demand increases. In other words, positive inflation allows the monetary authority to partially undo the effect of an investment demand shock on consumption.

In the case of an IST shock, the investment amount that generates the same capital stock as an investment demand shock is smaller since agents correctly expect an increase in the productivity of investment. Hence, the distortionary effects caused by the real wage rigidity are quantitatively much smaller than the case of an investment demand shock.

3.5 Time to build

Previous results on impulse-response functions in Figure 2 showed a short-lived response for inflation. A richer response is obtained in a more realistic setup where capital increases only gradually after making a decision on an investment spike. To this end, we extend the model to include time-to-build. A detailed analysis of the model with time-to-build is presented in the next section. Here, we only show the results of impulse-response functions in this extended model.

Figure 3 shows the impulse-response functions when an investment demand shock hits $t = 0$. Time is quarterly in this setup. Parameter values are adjusted to the quarterly setup.

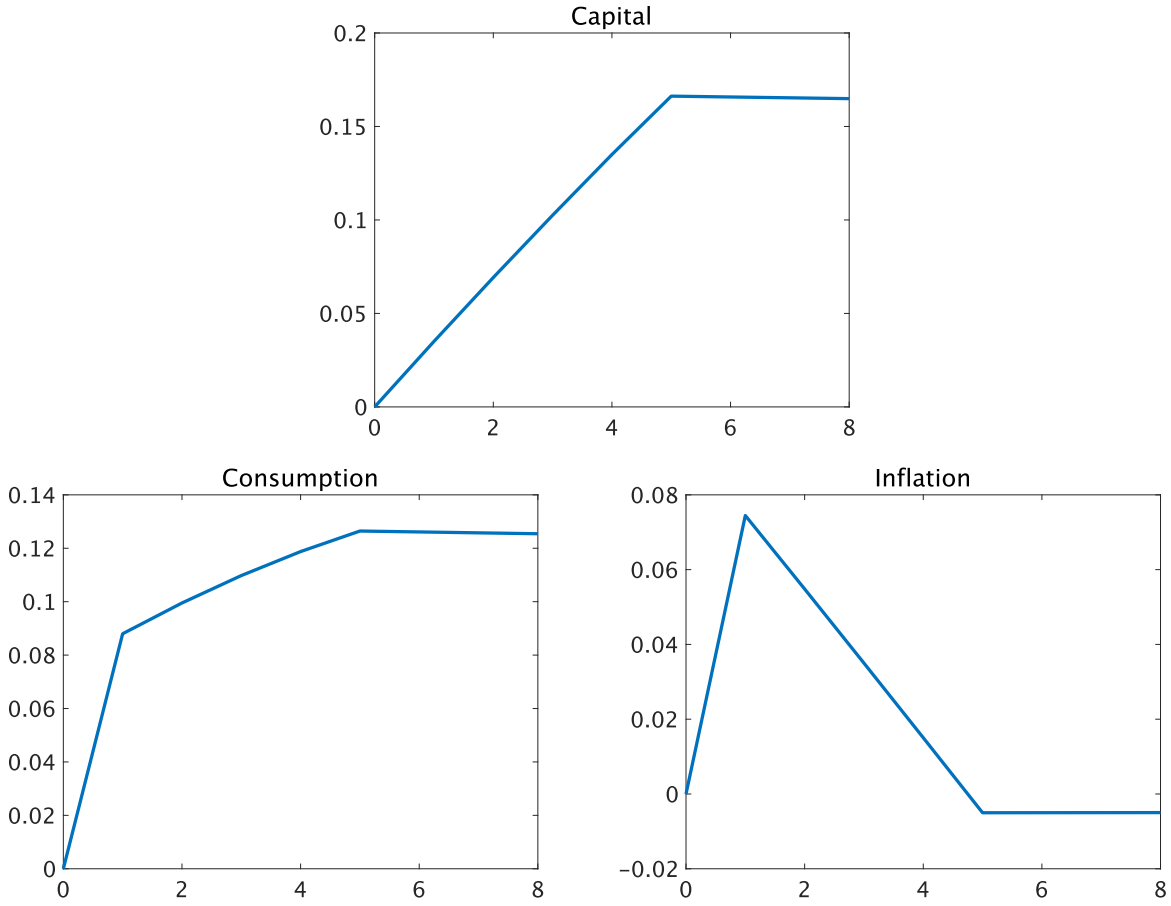


Figure 3: Impulse response functions (quarterly) for capital, consumption, and inflation rates (%) with four quarters time-to-build.

The changed values from the previous model are $\beta = 0.995$, $\delta = 0.025$, $g = 0.03$, and $\psi = 80$. Other parameter values are not changed. It is assumed that capital increases linearly for four quarters until completion in $t = 5$.

The top panel of Figure 3 shows a gradual increase in capital that takes one year to build up. The bottom left panel shows a gradual increase in consumption, which reflects the increase in production capacity of this economy during the time-to-build. The bottom right panel presents an inflation path. The inflation jumps on impact and gradually slows toward the slightly deflationary level in $t = 5$, after which it returns to the stationary level. The disinflation during time-to-build is the consequence of increasing consumption during the time. Hence, an investment demand shock positively impacts both consumption and

inflation, but the inflation effect comes early, and the consumption effect comes later in this model.

4 Microfoundation of investment demand shocks

This section analyzes an investment demand shock ϵ_t. The investment demand shock is defined as a gap between the equilibrium aggregate investment X_t^n in an economy with n firms and that in an economy with a continuum of firms, $\epsilon_t := X_t^n/X_t^e - 1$. The aggregate investment in a continuum economy is determined by the recursive equilibrium as $X_t^e = (\Xi(K_t) - (1-\delta)K_t)/A_X$. This section presents a model with n firms and characterizes non-negligible fluctuations of ϵ_t analytically and numerically.

4.1 Model with a finite number of firms

This section presents a model with n intermediate producers. The model converges to that with a continuum of producers used in Section 2 as $n \to \infty$.[8]

We introduce a finite number of intermediate goods but keep a continuum of symmetric wholesales $i \in [0,1]$. Thus, we have $Y_t = (\int y_{it}^{(\epsilon_c-1)/\epsilon_c} di)^{\epsilon_c/(\epsilon_c-1)}$ and $P_t = (\int p_{it}^{1-\epsilon_c} di)^{1/(1-\epsilon_c)}$ as in Section 2.

Wholesaler i's production function is $y_{it} = Y_{it}^m$ as in Section 2, but Y_{it}^m is now a composite good produced by a finite number of intermediate good producers:

$$Y_{it}^m = \left(n^{-1/\eta} \sum_{j=1}^{n} (y_{ijt}^m)^{(\eta-1)/\eta} \right)^{\eta/(\eta-1)}.$$

The right-hand side converges as $n \to \infty$ to its counterpart in Section 2, $(\int (y_{ijt}^m)^{(\eta-1)/\eta} dj)^{\eta/(\eta-1)}$ (Nirei and Scheinkman, 2024). The linear production function and symmetry imply $Y_t = Y_t^m = Y_{it}^m$ for all i. A wholesaler's minimized unit cost of production is symmetric across wholesalers and equal to $P_t^m = \left(\sum_{j=1}^n (p_{jt}^m)^{1-\eta}/n \right)^{1/(1-\eta)}$. Derived demand of wholesaler i for an intermediate producer j is $y_{ijt}^m = (p_{jt}^m/P_t^m)^{-\eta} Y_{it}^m/n$. Aggregating across symmetric i, we obtain the total demand for j as $y_{jt}^m = (p_{jt}^m/P_t^m)^{-\eta} Y_t/n$.

[8] See Nirei and Scheinkman (2024) for the convergence results.

We assume constant returns to scale of intermediate good productions, $\theta = 1$. The production function of intermediate firm j is $y_{jt} = a_{jt}k_{jt}^{\alpha}l_{jt}^{1-\alpha}$ and the real value of operating surplus is $\varphi = (p_{jt}/P_t)m_t y_{jt} - w_t l_{jt}$. Facing the demand function derived above, j chooses labor demand as $l_{jt} = (1-1/\eta)(1-\alpha)m_t p_{jt} y_{jt}/(w_t P_t)$. Aggregating across j, we obtain:

$$L_t = \sum_{j=1}^{n} l_{jt} = (1-1/\eta)(1-\alpha)(m_t/w_t)Y_t.$$

Also, by substituting the labor demand function in the production function and aggregating it, we obtain

$$Y_t = \left((1-1/\eta)(1-\alpha)m_t/w_t\right)^{(1-\alpha)/\alpha} K_t^n,$$

where

$$K_t^n := \left(n^{\rho-1}\sum_{j=1}^{n}(a_{jt}^{1/\alpha}k_{jt})^{\rho}\right)^{1/\rho}.$$

Thus, we recover the exact relationships between aggregate variables as in Section 2 for $\theta = 1$. The operating surplus is written as a function of capital and prices as

$$\varphi_t(a_{jt}, k_{jt}) = (a_{jt}^{1/\alpha}k_{jt})^{\rho} c_{\varphi} m_t^{1/\alpha} w_t^{1-1/\alpha} (K_t^n/n)^{1-\rho},$$

which is equivalent to (4) evaluated at $\theta = 1$ except for K_t^n/n which replaces K_t in (4). The optimal threshold for an investment spike is also obtained as in Section 2 by replacing K_t with K_t^n/n. We note that the threshold is linear in aggregate capital with $\theta = 1$:

$$k_{j,t+1}^{*} = a_{j,t+1}^{\eta-1}\Phi_t K_{t+1}^n/n.$$

4.2 Investment avalanche

Due to the nonlinearity arising from the indivisible capital, the model can have multiple equilibria. We select an equilibrium close to the equilibrium determined by the model with a continuum of firms. To do so, we use the best response dynamics of firms following the threshold policy $k_i^* = \gamma(a_i, K^e) := a_i^{\eta-1}BK^e$. The selection algorithm is the following.

1. Capital is depreciated as $k_i^o = (1-\delta)k_{i,t}$.

2. Productivity profile $(a_i)_i$ realizes. K^e is set at $K_{t+1}^e = \Xi(K_t)$.

3. Firms apply the threshold rule $\gamma(a_i, K^e)$ to set (k_i).

4. Compute the aggregate capital K^n using k_i. Stop if $K^n/n = K^e$ or if $k_i \neq k_i^o$ for all i. Otherwise, reset K^e to K^n/n and repeat step 3.

This process stops in finite time by construction in step 4. The selected equilibrium is close to K^e in the sense that there is no equilibrium aggregate capital with $k_i \in [k_i^*, \lambda k_i^*)$ between the selected K and K_{t+1}^e. This property holds because of the characteristic of the best response dynamics (Vives, 1990). We will show in the next section that the case of all firms adjusting (i.e., $k_i \neq k_i^o$ for all i) occurs with probability zero when $n \to \infty$ if a and s are orthogonal.

The best response dynamic starts at realizing idiosyncratic productivity shocks and capital depreciation. Initial expectation of future capital K_{t+1}^e sets the extensive margin of firms who adjust capital due to productivity shocks and depreciation. The equilibrium is reached if the resulting aggregate capital equals K_{t+1}^e. Otherwise, successive adjustments of capital follow until an equilibrium is reached. We call this successive adjustment within a period an investment avalanche.

4.3 Non-negligible fluctuations of investment avalanches

In this section, we demonstrate non-negligible fluctuations of the investment avalanche analytically and numerically. We use ϑ defined in the continuum model to characterize the asymptotic distribution of the number of firms involved in an investment avalanche. We consider the following experiment in a model with a finite number n of firms. As in Proposition 1, we impose Assumption 1 that s_i follows the uniform distribution over $[0, 1)$ and a_i and s_i are orthogonal at F. First, we draw a profile $(a_i, s_i)_{i=1}^n$ from F^n. This profile corresponds to a capital profile $(k_i)_i$ where $k_i = \lambda^{s_i} k_i^*$ where $k_i^* = a_i^{\eta-1} B K^n/n$ and K^n solves $K^n/n = (\sum_{i=1}^n (a_i^{1/\alpha} k_i)^\rho/n)^{1/\rho}$. Firms expect K^e, the next period aggregate capital, to be the same as K.

Then, we let capital be depreciated at rate δ, and find a next-period capital profile by the selection algorithm. In the first round of step 3, firms with state $s_i \leq s^*$ hit the threshold and decide to invest. In step 4, suppose that $\log(K/n) = \log K^e + \epsilon_0^n$ with $\epsilon_0^n > 0$ occurs.

That is, the gross investments of firms that go below the threshold by the direct effect of depreciation are greater than expected. This increases the threshold, and firms with state with $s^* < s_i \leq s^* + \epsilon_0/\log\lambda =: s_0^n$ go below the threshold in the second round of step 3. Let Z_0^n denote the set of those firms and $z_0^n := |Z_0^n|$ the number of those firms. If $z_0^n > 0$ occurs, the aggregate capital is increased in step 4. The increment is denoted by ϵ_1^n. Then, a new set of firms with $s_0^n < s_i \leq s_0^n + \epsilon_1^n =: s_1^n$ are induced to invest in the third round of step 3. Let Z_1^n denote the sef of those firms and $z_1^n := |Z_1^n|$. We repeat this process by defining ϵ_u^n, s_u^n, Z_u^n, and z_u^n for $u = 0, 1, \ldots, U$ accordingly, where U denotes the iteration times where the algorithm stops. The size of an investment avalanche is defined as $L^n := \sum_{u=0}^{U} z_u^n$. When $\epsilon_0^n < 0$, we can similarly define a process of retracting investment decisions. We are interested in the distribution of L^n, which characterizes the distribution of $\Delta \log K$ in this experiment.

Since s_i are i.i.d. random variables and the sets of newly investing firms Z_u^n are mutually exclusive, z_u^n is a stochastic process. A probability function of z_{u+1}^n conditional on z_u^n is given by a binomial distribution with probability ϵ_u^n and population $n - \sum_{v=1}^{u} z_v^n$. Thus, $(z_u^n)_u$ is a branching process in which each of "parents" z_u^n bears a "child" with probability $(n - \sum_{v=1}^{u} z_v^n)\epsilon_u^n / z_u^n$. Thus, L^n is a cumulative sum of the branching process with initial parents z_0^n until no children are born.

We show in appendix B that the mean of z_{u+1}^n, which follows the binomial with mean $(n - \sum_{v=1}^{u} z_v)\epsilon_u^n$, converges to ϑz_u^n as $n \to \infty$. Thus, as $n \to \infty$, the binomial distribution $Bin(\epsilon_u^n, n - \sum_{v=1}^{u} z_v^n)$ converges to a Poisson distribution with mean ϑz_u, which is equivalent to z_u-convolution of a Poisson distribution with mean ϑ. Let L denote a cumulative sum of a Poisson branching process $(z_u)_u$ in which each parent bears a random number of children following a Poisson distribution with mean ϑ. Thus, we obtain the following properties using the results of Nirei and Scheinkman (2024): (i) L^n converges to L in total variation (Proposition 6), (ii) the limit distribution of L conditional on z_0 follows the Borel-Tanner distribution with parameter ϑ (Proposition 8a), (iii) Since z_0 also follows a Poisson distribution, an unconditional distribution of L is represented as a Generalized Poisson Distribution (Proposition 8b), (iv) Complementarity ϑ determines the shape of the tail part of distributions. In particular, when $\vartheta \to 1$, the dispersion index of L diverges to infinity (Proposition 8d).

Unlike L, the distribution of z_0^n obeys the diversification effect. In the initial step of an avalanche, z_0^u follows a binomial distribution with probability $s^* = -(1-\delta)/\log\lambda$ and population n. By the central limit theorem, $z_0^n/\sqrt{n}$ asymptotically follows a normal distribution with finite variance $s^*(1-s^*)$. From this observation, we can draw a heuristic inspection of the diversification effect for L^n. Conditional on $z_0^n = 1$, the asymptotic distribution of L^n follows a power-law-tailed distribution with tail index 0.5 (Nirei and Scheinkman, 2024, Corollary 5). Thus, the conditional variance $L^n/n \mid z_0^n = 1$ declines as $\int_1^n (L^n/n)^2 (L^n)^{-1.5} dL^n \sim 1/\sqrt{n}$. Since the distribution of $z_0^n/\sqrt{n}$ is normal with a finite variance, the mean of $|z_0^n|$ scales as $\sqrt{n}$. This implies that unconditional variance of L^n/n scales as n^0, namely a constant (Nirei, 2015).

4.4 Time to build

It is straightforward to introduce time-to-build, i.e., an investment gestation lag is more than 1 period. Time-to-build helps the model to generate an autocorrelated series of aggregate investment. This is for a simple reason. Let $J \geq 1$ denote the time to build. We suppose that the investment is decided in J periods in advance, and the investment purchase is spread out over the $J-1$ periods. In each period, the investment decisions among firms cause an investment avalanche regarding the J-period ahead capital. Thus, an aggregate investment is the weighted average of the past investment avalanches, generating an autocorrelation.

Let $s_{it} := k_{i,t+J} - (1-\delta)k_{i,t+J-1}$ denote the increment of capital that materializes in J periods later. The investment s_{it} is spent over J periods from t to $t+J-1$ with weight for each period ζ_τ, $\tau = 0, 1, \ldots, J-1$. Thus, firm i's total investments x_{it} in period t is a weighted sum of past s_{it} as $x_{it} = \sum_{\tau=0}^{J-1} \zeta_\tau s_{i,t-\tau}$. The real value of intermediate firm i is $\mathbb{E}_0 \sum_{t=0}^{\infty} \Lambda_t(\varphi_t(a_{it}, k_{it}) - x_{it})$.

Let Λ_t^J denote a J-period rolling average as $\Lambda_t^J := \sum_{\tau=0}^{J-1} \zeta_\tau \Lambda_{t+\tau}$. Then, the terms relevant to the choice of k_{t+J} is:

$$\mathbb{E}_t \Lambda_{t+J} \varphi_{t+J}(a_{i,t+J}, k_{i,t+J}) + (1-\delta)\Lambda_{t+1}^J k_{t+J} - \Lambda_t^J k_{t+J}$$

The firm's threshold for lumpy investment is solved as in the previous section. First, the

indifference condition between an investment spike in $t+J$ or $t+J+1$ yields

$$k^*_{i,t+J} = \tilde{a}_{i,t+J} K^{\tilde{\theta}}_{t+J} \Phi^J_t$$

where Φ^J_t summarizes the expected factor prices

$$\Phi^J_t := \left(c_\varphi \frac{\lambda^\rho - 1}{\lambda - 1} \mathbb{E}_t \left[\Lambda_{t+J} \left(m_{t+J} / w^{(1-\alpha)\theta}_{t+J} \right)^{\frac{1}{1-(1-\alpha)\theta}} \right] \mathbb{E}_t \left[\Lambda^J_t - \Lambda^J_{t+1}(1-\delta) \right]^{-1} \right)^{\frac{1}{1-\rho}}. \quad (18)$$

The steady-state values are affected by the change in stochastic discount factors in (18). At the steady state, $\Lambda^J_t / \Lambda_{t+J}$ becomes $\sum_{\tau=0}^{J-1} \zeta_\tau \Lambda_{t+\tau} / \Lambda_{t+J} = \sum_{\tau=0}^{J-1} \zeta_\tau \beta^{\tau - J} =: (\beta_J)^{-1}$. Also, at the steady state, $\Lambda^J_t / \Lambda^J_{t+1} = \beta^{-1}$. The equation at the steady state becomes:

$$\Phi^J := \left(c_\varphi \frac{\lambda^\rho - 1}{\lambda - 1} \left[\frac{\beta_J}{\beta} \left(w^{(1-\alpha)\theta} \frac{\epsilon_c}{\epsilon_c - 1} \right)^{\frac{-1}{1-(1-\alpha)\theta}} \right] \left[\frac{1}{\beta} - (1-\delta) \right]^{-1} \right)^{\frac{1}{1-\rho}}. \quad (19)$$

4.5 Numerical analysis of investment avalanches

We investigate the property of investment avalanches by numerical simulations. We consider a flexible real wage case $g = 1$. First, we compute the steady state of an economy with a continuum of firms. Then, we simulate the path of an economy with a finite n number of firms. The unit of time is a quarter. A recursive equilibrium $K^e_{t+1} = \Xi(K_t)$ determines the expected aggregate capital. Using a computation result of the recursive equilibrium $K_{t+1} = \Xi(K_t)$ and applying a first-order approximation of the dynamics, we set that $\log K^e_{t+1}$ follows an AR(1) process with persistent 0.975, i.e., the half-life period of a deviation from the steady state is 20 quarters. Time-to-build is set to $J = 8$ quarters. The firm-level productivity process is a logarithmic AR(1) process with persistence $\rho_a = 0.9$ and an i.i.d. shock of standard deviation $\sigma_a = 0.03$. The number of firms is set at $n = 30000$, the number of firms covered by the Japanese business survey in Figure 1.

Figure 4 shows the simulated time series of aggregate investments. The standard deviation of the fluctuation is 2.4%, and the autocorrelation coefficient is 0.83. Note that the exogenous shocks given to the model are firm-level independent productivity shocks with $\sigma_a = 0.03$. Thus, the simple average of productivity shocks yields $\sigma_a / \sqrt{n} \approx 0.017\%$ standard deviation, which cannot generate the 2.4% standard deviation of aggregate investment. The

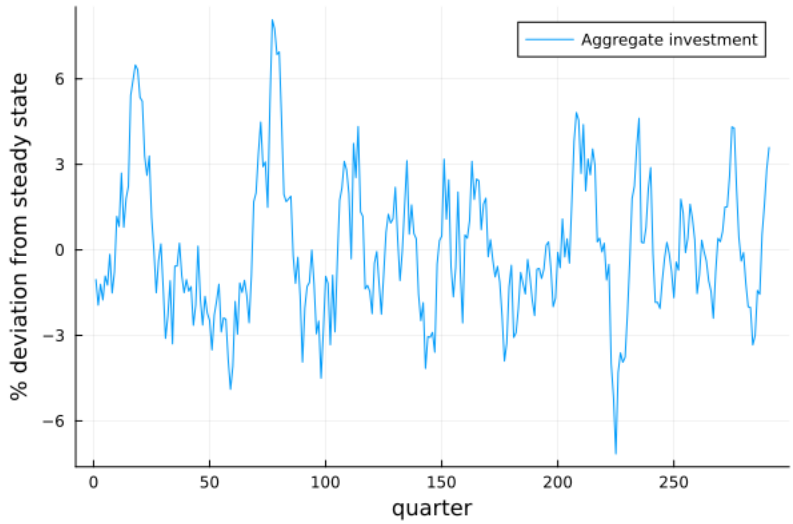


Figure 4:

true source of aggregate fluctuations is the evolution of capital profile, which determines the size of investment avalanches.

The size of investment avalanches is measured by the number of firms involved in the avalanche after the initial response to the capital depreciation and productivity shocks occurred. Figure 5 shows a distribution of the investment avalanche measured by the number of firms in this way. The plot shows a counter-cumulative distribution conditional on whether the avalanche resulted in a positive or negative growth of aggregate capital. We observe that the size of investment avalanches exhibits a sizable fluctuation.

The number of firms in an investment avalanche and the lumpiness λ determines the magnitude of aggregate investment decided by firms in a period. In this simulation, the standard deviation of the aggregate investment amount decided is 10.5%. The decided amount is equally spread over the time-to-build periods J. The aggregate investment in a period is a sum of the overlapping investments decided in the current and the past $J-1$ periods. Hence, by diversification, the volatility of aggregate investments is smaller than that of aggregate investment amounts decided in a period.

Finally, we inspect the distributions of s_{it} over periods. In an analytical characterization, we assume that the distribution of s deviates little from the uniform distribution. Simulation results support this assumption. We computed the first four non-centered moments of $(s_{it})_i$

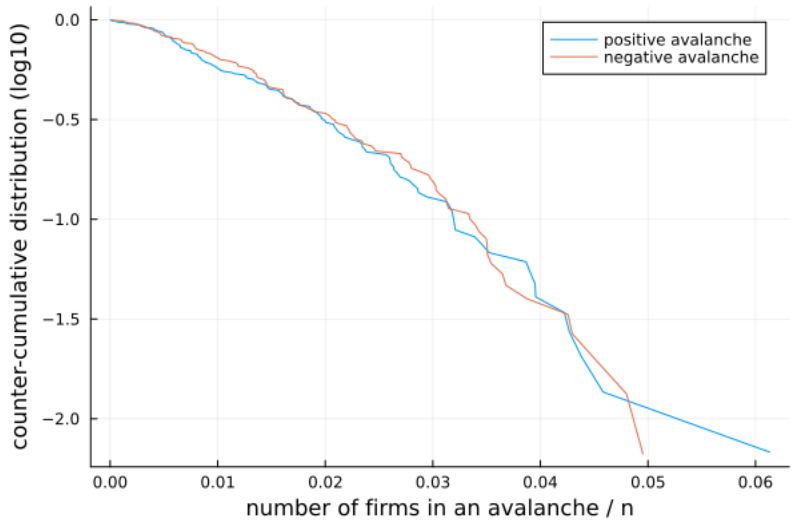


Figure 5:

for each t. In a run for 300 quarters, the maximum deviation of the first moment from that of the uniform distribution is 1.1%, and less than 2.5% for all four moments. To check the convergence of moments, Figure 6 inspects if the processes of simulated moments are stationary and converging to their true moments. The simulation shows that the deviation process is stationary and that the time averages of the simulated moments seem to show convergence to the true moments of the uniform distribution and the assumed orthogonality with a_{it}.

4.6 Why does an avalanche occur? An intuition

To understand the investment avalanche intuitively, we contrast with an economy with smoothly adjusted capital without indivisibility. An intermediate good is monopolistically supplied with production function $y_i^m = k_i^\alpha l_i^{1-\alpha}$. The monopolist faces demand function $y_i^m = (p_i^m)^{-\eta} Y$ where $Y = (\int (y_i^m)^{(\eta-1)/\eta})^{\eta/(\eta-1)}$. Aggregate labor demand function is $L = (1-1/\eta)(1-\alpha)Y/w$, and labor supply $v'(L) = wu'(c)$. The goods market clearing condition is $Y = c + X$. Operating profit is $(1-1/\eta)(1-\alpha) y_i^{1-1/\eta} Y^{1/\eta}$. Substituting l, the profit function is $\varphi(k_i, K, w)$. Maximizing monopolist's value leads to $\Lambda_t/\Lambda_{t+1} - 1 + \delta = \partial\varphi/\partial k_i$. By aggregating this expression across i, k_i and K cancel out and we are left with the following

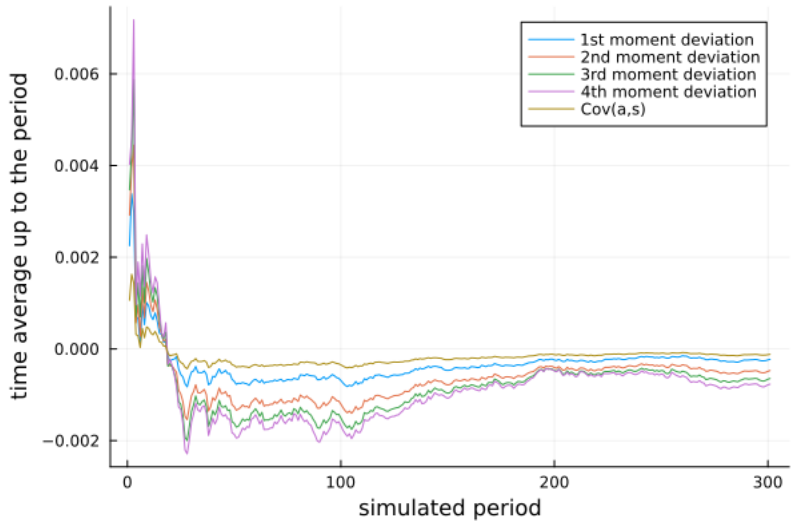


Figure 6: Time-averages of simulated moments $\mathbb{E}[a_{it}]-1/2$, $\mathbb{E}[a_{it}^2]-1/3$, $\mathbb{E}[a_{it}^3]-1/4$, $\mathbb{E}[a_{it}^4]-1/5$, and $Cov(a_{it}, s_{it})$.

factor price condition[9]

$$w_t^{1-\alpha}(R_t - 1 + \delta)^{\alpha} = \alpha^{\alpha}(1-\alpha)^{1-\alpha}(1-1/\eta)\left(\int a_i^{\eta-1}di\right)^{\frac{1}{\eta-1}}.$$

As long as the factor prices must satisfy this equation, the optimal k_i is linear in K; hence, the equilibrium level of K is unaffected. As is often the case in a constant-returns-to-scale economy, the level of aggregate capital is determined by the households' decisions. Firms' decisions do constrain the factor prices, but they do not put a constraint on aggregate capital.

As in our model, the aggregate capital adjustment is made at the firm's extensive margin when capital is discrete. In this case, it no longer holds that any level of aggregate capital is consistent with firms' decisions. In other words, firms' decisions do constrain equilibrium aggregate capital. As a result, a socially efficient level of aggregate capital may not be

[9]The derivation is straightforward.

$$\left(\int a_i^{\eta-1}di\right)^{\frac{1}{\eta-1}}\left(\frac{\Lambda_t}{\Lambda_{t+1}} - 1 + \delta\right) = \rho(1-(1-1/\eta)(1-\alpha))\left(\frac{(1-1/\eta)(1-\alpha)}{w_t}\right)^{\frac{(1-\alpha)(1-1/\eta)+(1-\alpha)/(\alpha\eta)}{1-(1-1/\eta)(1-\alpha)}}$$
$$= \alpha(1-1/\eta)\left(\frac{(1-1/\eta)(1-\alpha)}{w_t}\right)^{\frac{1-\alpha}{\alpha}}$$

where $\rho = \alpha(1-1/\eta)/(1-(1-1/\eta)(1-\alpha))$.

supported as an equilibrium since it may violate the firm's decision at the extensive margin. When the firm at the extensive margin invests, it raises aggregate capital and incentivizes other firms to follow the firm in the smoothly adjusting model. The same incentive operates in a discrete adjusting model, which results in an avalanche.

5 Conclusion

This paper presents a model in which aggregate investment demand shocks arise. We consider that the firm's capital is indivisible, so the investment at the firm level is lumpy. We also consider a monopolistically competitive economy where an increase in aggregate demand incentivizes firms' investments. In this setup, a firm's lumpy investment induces another firm's lumpy investment, inducing an investment avalanche. How far the investment avalanche continues depends on firms' capital profile and idiosyncratic productivity. Hence, as the capital and productivity profile evolves, the avalanche size also varies. This generates fluctuations in investment avalanche size, which we call investment demand shocks.

The paper analyzes the fluctuations of the investment avalanche analytically and numerically. Our analysis shows that the investment demand shocks generated by the avalanche are quantitatively substantial. Under some time-to-build, the investment demand shocks can exhibit autocorrelation. Thus, the investment avalanche provides a micro-foundation for the animal spirits that drive investment demands in business cycles.

We incorporate the mechanism of investment avalanches in an otherwise standard New Keynesian business cycle model. Under the rigidity of real wages, the model allows for analytic solutions and interpretation of the propagation mechanism of the micro-founded investment demand shocks. The analysis of impulse-response dynamics of an investment demand shock demonstrates that the investment demand shock can generate a positive comovement of inflation and output in the business cycle frequency under a reasonable parameter alignment.

Appendix

A Proof of Proposition 1

We write the threshold rule (5) as $k^*_{i,t} = \Phi_t \tilde{a}_{i,t} K^{\tilde{\theta}}_{t+1}$ with $\tilde{a} = a^{(1-1/\eta)/(1-\theta+\theta/\eta)}$ and $\tilde{\theta} = (\alpha\theta/\eta)/(1-(1-\alpha)\theta)/(1-\theta+\theta/\eta)$. Using the threshold, firm i's capital is written as $k_{i,t} = \lambda^{s_{i,t}} k^*_{i,t}$. Thus, we obtain

$$K_t = \mathbb{E}^F\left[(a_{it}^{1/(\alpha\theta)}\lambda^{s_{it}}k^*_{it})^\rho\right]^{1/\rho} = \mathbb{E}^F\left[(a_{it}^{1/(\alpha\theta)}\lambda^{s_{it}}\tilde{a}_{i,t})^\rho\right]^{1/\rho}\Phi_t K_t^{\tilde{\theta}} = \mathbb{E}^F\left[\tilde{a}_{it}\lambda^{\rho s_{it}}\right]^{1/\rho}\Phi_t K_t^{\tilde{\theta}}. \quad (20)$$

Consider a perturbation $\nu(h)$ from the stationary threshold $s^*(h)$. Then, a perturbed aggregate capital evaluated at a stationary equilibrium is written as follows.

$$K^\rho_{t+1} = \sum_h \mu(h)\left[\int_0^{s^*(h)+\nu(h)} (a^{1/(\alpha\theta)}(h_1)\lambda(1-\delta)\lambda^{s_{it}}\tilde{a}(h_0)\Phi_t K_t^{\tilde{\theta}})^\rho ds \right.$$
$$\left. + \int_{s^*(h)+\nu(h)}^1 (a^{1/(\alpha\theta)}(h_1)(1-\delta)\lambda^{s_{it}}\tilde{a}(h_0)\Phi_t K_t^{\tilde{\theta}})^\rho ds\right]$$

Now consider a pertubation $\nu(h) > 0$ and $\nu(h') = 0$ for $h' \neq h$. Then, we have $\partial\rho\log K_{t+1}/\partial\nu(h) = (\lambda^\rho - 1)\mu(h)(a^{1/(\alpha\theta)}(h_1)(1-\delta)\lambda^{s^*(h)+\nu(h)}\tilde{a}(h_0)\Phi_t K_t)^\rho / K^\rho_{t+1}$. This expression is evaluated at $\nu(h) = 0$ by using $s^*(h) = (-\log(1-\delta) + \Delta\log\tilde{a}(h))/\log\lambda$ and (20) as

$$\left.\frac{\partial(\rho\log K_{t+1})}{\partial\nu(h)}\right|_{\nu=0} = \frac{(\lambda^\rho-1)\mu(h)(a^{1/(\alpha\theta)}(h_1)(1-\delta)\lambda^{s^*}\tilde{a}(h_0)\Phi_t K_t^{\tilde{\theta}})^\rho}{K^\rho_{t+1}\big|_{\nu=0}}$$
$$= (\lambda^\rho-1)\mu(h)((1-\delta)\lambda^{-\log(1-\delta)/\log\lambda}))^\rho\frac{\tilde{a}(h_1)}{\mathbb{E}^F[\tilde{a}\lambda^{\rho s}]}$$
$$= \mu(h)\rho(\log\lambda)\frac{\tilde{a}(h_1)}{\mathbb{E}^F[\tilde{a}]},$$

where the last equality used Assumption 1 that s is uniformly distributed conditional on every a.

Under the uniform distribution of s_{it}, an increase in $\log K_{t+1}$ increases the threshold by $\tilde{\theta}d\log K_{t+1}/\log\lambda$. Hence, we obtain

$$\vartheta(h) := \lim_{\nu(h)\to 0}\frac{\nu'}{\mu(h)\nu(h)} = \left.\frac{d\nu'}{\mu(h)d\nu(h)}\right|_{\nu=0} = \frac{\tilde{\theta}}{\mu(h)\log\lambda}\left.\frac{\partial\log K_{t+1}}{\partial\nu}\right|_{\nu(h)=0} = \frac{\tilde{\theta}\tilde{a}(h_1)}{\mathbb{E}^F[\tilde{a}]}.$$

When we perturb ν measure of firms unconditional on h, we obtain the degree of complementarity as the average of $\vartheta(h)$,

$$\vartheta = \sum_h \mu(h)\vartheta(h) = \tilde{\theta} = \frac{(\alpha\theta/\eta)/(1-(1-\alpha)\theta)}{1-\theta+\theta/\eta}.$$

We note that the right-hand side function is increasing in θ for $\theta \leq 1$ and converges to 1 as $\theta \to 1$.

B Proofs for statements in Section 4.3

B.1 Distributions of ϵ_0^n and n_δ

In step 1 of the equilibrium selection algorithm, capital is depreciated by δ. In step 2, productivity a_{it} is updated. In the first round of step 3, firms with $s_{it} \leq s_{it}^* := -(\log(1-\delta) - (\eta-1)\Delta \log a_{it})/\log\lambda$ invest.

The net increase of $\log K_t^n/n$ due to their investments is denoted by ϵ_0^n. Hence,

$$\begin{aligned}\epsilon_0^n &= \frac{1}{\rho}\left(\log\left(\sum_{i:s_{it}\leq s_{it}^*}\frac{(a_{i,t+1}^{1/\alpha}\lambda(1-\delta)k_{it})^\rho}{n} + \sum_{i:s_{it}>s_{it}^*}\frac{(a_{i,t+1}^{1/\alpha}(1-\delta)k_{it})^\rho}{n}\right) - \log\sum_i \frac{(a_{it}^{1/\alpha}k_{it})^\rho}{n}\right) \\ &= \frac{1}{\rho}\left(\log\left((\lambda^\rho-1)\sum_{i:s_{it}\leq s_{it}^*}\frac{(a_{i,t+1}^{1/\alpha}\lambda^{s_{it}})^\rho}{n} + \sum_i \frac{(a_{i,t+1}^{1/\alpha}\lambda^{s_{it}})^\rho}{n}\right) - \log\sum_i \frac{(a_{it}^{1/\alpha}\lambda^{s_{it}})^\rho}{n}\right) + \log(1-\delta)\end{aligned}$$

As $n \to \infty$, ϵ_0^n degenerates to zero, and $\sqrt{n}\epsilon_0^n$ converges to a non-degenerate random variable with a finite variance by the central limit theorem.

We divide firms into H^2 groups according to their profile $(a_{it}, a_{i,t+1})$ as in Section 2. Let $n(h)$ denote the number of firms in group h. Then, the number of firms in h that invest in step 3, denoted by $n_\delta(h)$, follows a binomial distribution with population $n(h)$ and probability $s^*(h)$. The total number of firms investing in this step is determined by the sum of the binomials over h, $n_\delta = \sum_h n_\delta(h)$.

B.2 Distribution of z_0^n

Suppose $\epsilon_0^n > 0$. Step 4 of the equilibrium selection algorithm is repeated. z_0^n denotes the number of firms in $[s_{it}^*, s_{it}^* + \epsilon_0^n/\log\lambda)$. If $z_0^n = 0$, the algorithm stops, and an equilibrium

capital in t is determined. If $z_0^n > 0$, the algorithm continues.

Let $z_0^n(h)$ denote the number of firms that belong to group h and are located in $[s^*(h), s^*(h)+\epsilon_0^n/\log\lambda)$. The firms that do not invest in step 3 and belong to group h are in $(s^*(h), 1]$ uniformly. Hence, $z_0^n(h)$ follows a binomial distribution with population $n(h) - n_\delta(h)$ and probability $\epsilon_0^n/(\log\lambda(1-s^*(h)))$. We note that the mean of $n_\delta(h)/n(h)$ is $s^*(h)$. Hence, the mean of $z_0^n(h)$ is $n(h)\epsilon_0^n/\log\lambda$. Therefore, the mean of $z_0^n = \sum_h z_0^n(h)$ is $n\epsilon_0^n/\log\lambda$.

If $\epsilon_0^n < 0$, step 4 of the equilibrium selection algorithm searches for firms in $(s_{it}^* + \epsilon_0^n, s_{it}^*]$ that "retract" the investment decision they made in step 3. We denote the number of retracted firms by a negative of z_0^n.

B.3 Mean of z_u^n

In a model with n firms, the average capital level satisfies $(K^n/n)^\rho = \sum_{i=1}^n (a_i^{1/\alpha}k_i)^\rho/n$. If k_i^ρ incerases to $(\lambda k_i)^\rho$, $(K^n/n)^\rho$ increases by $(\lambda^\rho - 1)(a_i^{1/\alpha}k_i)^\rho/n$. Hence, firm i's lumpy investment increases $\log(K^n/n)$ by

$$\Delta\log(K^n/n) := \frac{\lambda^\rho - 1}{\rho}\frac{a_i^{\rho/\alpha}k_i^\rho}{\sum_{j=1}^n (a_j^{1/\alpha}k_j)^\rho} + o\left(\frac{a_i^{\rho/\alpha}k_i^\rho}{\sum_{j=1}^n (a_j^{1/\alpha}k_j)^\rho}\right).$$

Using normalized capital $s_i = (\log k_i - \log k_i^*)/\log\lambda$, we have $k_i = \lambda^{s_i}k_i^*$. We consider a stationary equilibrium of a continuum model where s_i is distributed uniformly over $(0,1]$ and independent of a_i. We also use the threshold rule for a stationary equilibrium of a continuum model: $k_j^* = B\tilde{a}_j K^n/n$. Then, i's investment decreases s_j for $j \neq i$ by $\Delta s_j := \Delta\log(K^n/n)/\log\lambda$. Since s_j is uniformly distributed, the number of firms (in the group h) that hit the threshold because of this decrease in s_j follows a binomial distribution with population $n - n_\delta h - \sum_{\tau=0}^{u-1} z_\tau^n(h)$ and probability $\Delta s_j/(1-s^*(h))$.

We use $a_i^{\rho/\alpha}k_i^\rho = a_i^{\rho/\alpha}(\lambda^{s_i}B\tilde{a}_i K^n/n)^\rho = \tilde{a}_i\lambda^{s_i}BK^n/n$. Thus, for a given sequence of $(z_\tau^n(h))_{\tau=0}^{u-1}$, the mean of $z_u^n = \sum_h z_u^n(h)$ is

$$\operatorname*{plim}_{n\to\infty}\sum_h \sum_{i\in\mathcal{Z}_{u-1}(h)} \frac{\Delta\log(K^n/n)}{(1-s^*(h))\log\lambda}\left(n - n_\delta(h) - \sum_{\tau=0}^{u-1} z_\tau^n(h)\right) = \operatorname*{plim}_{n\to\infty}\sum_{i\in\mathcal{Z}_{u-1}} \frac{\lambda^\rho - 1}{\rho\log\lambda}\frac{\tilde{a}_i\lambda^{\rho s_i}}{\sum_{j=1}^n \tilde{a}_j\lambda^{\rho s_j}/n}.$$

By the law of large numbers, $\sum_{j=1}^n \tilde{a}_j\lambda^{\rho s_j}/n$ converges in probability as $n\to\infty$ to $\mathbb{E}^F[\tilde{a}](\lambda^\rho - 1)/(\rho\log\lambda)$. Also, note that $s_i \to 0$ as $n\to\infty$ for $i \in \mathcal{Z}_{u-1}$. Hence, the mean of

z_u^n conditional on $\mathcal{Z}_{u-1}$ is asymptotically equal to $\frac{\sum_{i\in\mathcal{Z}_{u-1}}\tilde{a}_i}{\mathbb{E}^F[\tilde{a}]}$. This implies that the mean of z_u conditionally on z_{u-1}, but unconditionally on $\mathcal{Z}_{u-1}$, converges in probability to z_{u-1}.

In particular, the mean of z_u^n conditional on that $\mathcal{Z}_{u-1}$ contains a single firm in group h converges as $n \to \infty$ to $\vartheta(h) = \tilde{a}(h_1)/\mathbb{E}[\tilde{a}]$, which is the degree of complementarity we obtained in Proposition 1 in the model with a continuum of firms. Thus, the mean number of firms induced to invest due to an investing firm (unconditional on h), ϑ, converges in probability to $\sum_h \mu(h)\vartheta(h) = 1$ as $n \to \infty$. In this way, the degree of complementarity ϑ defined in the continuum model characterizes the avalanche effect of investments in a model of a finite number of firms.

Since Poisson distribution is an asymptotic distribution of a binomial distribution with a finite mean, z_u converges in law to a Poisson distribution with mean ϑz_{u-1}. Furthermore, since a Poisson distribution is infinitely divisible and z_{u-1} is an integer, the asymptotic distribution of z_u is equivalent to a z_{u-1}-times convolution of a Poisson distribution with mean ϑ.

C Gradual capital build-up

In this section, we specify that a lumpy investment takes J periods to complete and capital increases gradually during the J periods. Suppose that a firm decides a lumpy investment in period t. Then, the firm commits to a series of investment purchases $(s_t, s_{t+1}, \ldots, s_{t+J-1})$. Capital develops as

$$k_{t+j} = (1-\delta)^j k_t + (1-\delta)^{j-1} s_t + (1-\delta)^{j-2} s_{t+1} + \cdots + s_{t+j-1}.$$

We impose that the cumulated capital increase corresponds to the lumpiness parameter λ. Thus,

$$k_{t+J} = (1-\delta)^J k_t + (1-\delta)^{J-1} s_t + (1-\delta)^{J-2} s_{t+1} + \cdots + s_{t+J-1} = \lambda(1-\delta)k_t.$$

Moreover, we impose that j-th period investment s_{t+j} is proportional to k_t:

$$s_{t+j} = \zeta_j k_t.$$

Hence $\zeta_j \geq 0$, $j = 0, 1, \ldots, J-1$, satisfies a condition

$$(1-\delta)^J + (1-\delta)^{J-1}\zeta_0 + (1-\delta)^{J-2}\zeta_1 + \cdots + \zeta_{J-1} = \lambda(1-\delta).$$

There always exists a series $(\zeta_j)_{j=0}^{J-1}$ which satisfies the condition. In particular, when $J = 1$, the model is reduced to the simplest case with $\zeta_0 = (\lambda - 1)(1-\delta)$.

Substituting $s_{t+j} = \zeta_j k_t$, we have

$$k_{t+j} = \xi_j k_t \quad \text{for } j = 1, 2, \ldots, J,$$

where $\xi_j := (1-\delta)^j + (1-\delta)^{j-1}\zeta_0 + (1-\delta)^{j-2}\zeta_1 + \cdots + \zeta_{j-1}$.

At the threshold k^* of investment decision, a firm is indifferent between starting investment in t and $t+1$. The two paths differ in capital between $t+1$ and $t+J$ and follow the same path after $t+J+1$. Using (4), firm i's operating surplus is

$$\varphi_t(a_{it}, k_{it}) = k_{it}^{\rho} B(a_{it}, w_t, m_t, K_t) \tag{21}$$

where

$$B(a_{it}, w_t, m_t, K_t) := a_{it}^{\rho/(\alpha\theta)} c_\varphi \left(m_t / w_t^{(1-\alpha)\theta} \right)^{\frac{1}{1-(1-\alpha)\theta}} K_t^{\frac{(\alpha\theta/\eta)/(1-(1-\alpha)\theta)}{1-(1-1/\eta)(1-\alpha)\theta}}$$

and $c_\varphi := (1-(1-1/\eta)(1-\alpha)\theta)\left((1-1/\eta)(1-\alpha)\theta\right)^{\frac{(1-\alpha)\theta}{1-(1-\alpha)\theta}}$.

If we write the difference in operating surplus in t between the two paths as $\Delta\varphi_t$, we have

$$\Delta\varphi_{t+j} = B(a_{i,t+j}, w_{t+j}, m_{t+j}, K_{t+j})(\xi_j^{\rho} - (\xi_{j-1}(1-\delta))^{\rho}) k_{i,t+j}^{\rho} \quad \text{for } j = 1, \ldots, J.$$

The difference in investment cost in $t+j$ is $\zeta_j k_t - \zeta_{j-1}(1-\delta)k_t$ for $j = 0, \ldots, J-1$ where $\zeta_{-1} = 0$ by convention. Let k^* denote the capital level in $t+1$ at which investment in t is indifferent. Thus, in t, a firm is indifferent in investing if $k_t = k^*/(1-\delta)$. Hence, $k^*/(1-\delta)$ satisfies

$$\begin{aligned}
&\mathbb{E}_t \left[\sum_{j=1}^{J} \Lambda_{t+j} B(a_{i,t+j}, w_{t+j}, m_{t+j}, K_{t+j})(\xi_j^{\rho} - (\xi_{j-1}(1-\delta))^{\rho}) \left(\frac{k^*}{1-\delta} \right)^{\rho} \right] \\
&= \mathbb{E}_t \left[\sum_{j=0}^{J} \Lambda_{t+j}(\zeta_j - \zeta_{j-1}(1-\delta)) \right] \frac{k^*}{1-\delta}
\end{aligned}$$

where $\zeta_{-1} = \zeta_J = 0$ by convention. The optimal threshold is solved as

$$\frac{k^*}{1-\delta} = \left(\frac{\mathbb{E}_t \left[\sum_{j=1}^{J} \Lambda_{t+j} B(a_{i,t+j}, w_{t+j}, m_{t+j}, K_{t+j}) (\xi_j^\rho - (\xi_{j-1}(1-\delta))^\rho) \right]}{\mathbb{E}_t \left[\sum_{j=0}^{J} \Lambda_{t+j} (\zeta_j - \zeta_{j-1}(1-\delta)) \right]} \right)^{1/(1-\rho)}$$
$$= \mathbb{E}_t \left[\sum_{j=1}^{J} \tilde{\Lambda}_{t+j} B(a_{i,t+j}, w_{t+j}, m_{t+j}, K_{t+j}) \right]^{1/(1-\rho)}$$

where

$$\tilde{\Lambda}_{t+j} := \frac{\Lambda_{t+j}(\xi_j^\rho - (\xi_{j-1}(1-\delta))^\rho)}{\mathbb{E}_t \left[\sum_{j=0}^{J} \Lambda_{t+j} (\zeta_j - \zeta_{j-1}(1-\delta)) \right]}.$$

We focus on the case $\theta = 1$. Also, in this section, we assume that the idiosyncratic productivity $a_{i,t}$ is i.i.d. across i and t. Let $\tilde{c}_\varphi = c_\varphi \mathbb{E}_t[(a_{t+1}/a_t)^{\rho/\alpha}]$. Then,

$$\left(\frac{k^*}{1-\delta} \right)^{1-\rho} = \mathbb{E}_t \left[\sum_{j=1}^{J} \tilde{\Lambda}_{t+j} B(a_{i,t+j}, w_{t+j}, m_{t+j}, K_{t+j}) \right] = \mathbb{E}_t \left[\sum_{j=1}^{J} \tilde{\Lambda}_{t+j} a_{i,t+j}^{\rho/\alpha} c_\varphi (m_{t+j}/w_{t+j}^{1-\alpha})^{1/\alpha} K_{t+j}^{1-\rho} \right]$$
$$= a_{i,t}^{\rho/\alpha} \mathbb{E}_t \left[\sum_{j=1}^{J} \tilde{\Lambda}_{t+j} \tilde{c}_\varphi (m_{t+j}/w_{t+j}^{1-\alpha})^{1/\alpha} K_{t+j}^{1-\rho} \right]$$

By definition of s_{it}, we have $k_{i,t+1} = \lambda^{s_{i,t+1}}(1-\delta)k_{i,t}^*$. Recall the assumption that $a_{i,t+J}$ is known in t. Thus,

$$K_{t+1}^\rho = \int (a_{i,t+1}^{1/\alpha} k_{i,t+1})^\rho di$$
$$= (1-\delta)^\rho \int \left(a_{i,t+1}^{\eta-1+1/\alpha} \lambda^{s_{i,t+1}} \right)^\rho di \mathbb{E}_t \left[\sum_{j=1}^{J} \tilde{\Lambda}_{t+j} \tilde{c}_\varphi (m_{t+j}/w_{t+j}^{1-\alpha})^{1/\alpha} K_{t+j}^{1-\rho} \right]^{\rho/(1-\rho)}$$

Hence, under the assumption that (a, s) follows the stationary distribution F,

$$k^* = a_{i,t}^{\eta-1} B K_{t+1}$$

where $B = \mathbb{E}^F[\left(a^{\eta-1+1/\alpha}\lambda^s\right)^\rho]^{-1/\rho}$. Thus, we recover the same formula for the investment threshold.

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