Risk-taking with Financing Constraints*

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Abstract

We analyze how financing constraints affect non-financial firms' risk-taking behaviors, shaped by leveraged returns and associated risks. We show that improved credit conditions can either promote or deter socially productive risk-taking. Following an interest rate cut, firms are less likely to take risks in a low-interest-rate environment but more inclined to do so in a high-interest-rate setting. This result leads to a non-monotonic, hump-shaped relationship between interest rates and the dispersion of firm returns at the aggregate level. When risk-taking enhances overall productivity, total factor productivity and the "IS" curve also display a hump shape. An optimal interest rate may exist, determined by the extent of return associations and credit limits. Financial regulation, targeting asset pledgeability and the interdependence of firms' risky projects, can thus shape the effectiveness of interest rate policy.

Key Words: financing constraints; risk-taking; valuation effect; substitution effect; optimal interest rate

JEL code: E22; E44; E61; G32

1 Introduction

Socially productive risk-taking is pivotal in driving opportunities and progress within society. It fuels the exploration of new ideas and ventures, essential for aggregate productivity. Yet, the propensity for individuals to take risks varies based on numerous factors, such as

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the nature of a project's risk, the individual's risk tolerance, leverage limits, and prevailing interest rates. In the context of non-financial firms, how do financing constraints influence their risk-taking behavior given persistent low (real) interest rates observed since the global financial crisis? Does a reduction in interest rates encourage or deter risk-taking? Will financial regulations have material impacts on the effectiveness of interest rate policy, especially when firms' successes are correlated?

This paper answers these questions by studying the joint impact of interest rates and leverage restrictions on firms' risk-taking incentives. Financial conditions exhibit non-linear/non-monotonic effects on risk-taking, indicating the existence of an optimal interest rate under different degrees of financing constraints and return associations among firms. In short, although a high interest rate hurts capital accumulation and output, an overly low rate deters risk-taking, reducing overall productivity and negatively impacting the economy as well.

Using multiple proxies for risk-taking, we begin with an empirical analysis of how changes in the U.S. interest rates influence risk-taking. We study different periods marked by varying interest rates and leverage conditions. Our findings indicate a strong, non-linear, and non-monotonic relationship between the interest rates and the risk-taking measures. For instance, although interest-rate levels and policy cuts were similar in 2001 and 2007, risk-taking measures decreased in 2001 but rose in 2007, when firms held higher leverage. This underscores the critical role that financial conditions can play in shaping risk-taking behavior.

To understand the impact of financial conditions on incentives for risk-taking, we explore a model scenario featuring risk-averse entrepreneurs with three investment options: risk-free savings, a costlier yet risk-free project yielding a higher return, and a riskier project that can be either successful or unsuccessful and promises the highest expected return. Apart from savings, firms can borrow funds for either project type. Undertaking a risky project incurs idiosyncratic costs, such as those associated with testing and prototyping in research and development, which can vary significantly across industries and project types. The availability of a risk-free project provides an alternative option, affecting entrepreneurs' choice to pursue riskier ventures based on borrowing capacity and idiosyncratic costs. As a result, some firms choose to take risks while others do not.

We obtain two key effects. First, easing financial conditions through higher leverage or lower interest rates raises firm profitability or value; this "valuation effect" reduces the incentive to pursue risky, high-return projects, as the need for higher expected growth through risk-taking decreases when firm value increases. However, the easing liquidity also triggers a traditional "searching for yield" effect, making riskier projects with a higher expected return more attractive—this is sometimes understood as the substitution effect. In particular,

the model establishes a critical threshold of the idiosyncratic costs for executing the risky project, below which firms choose this option. We analyze how leverage constraints and interest rates influence this threshold via the valuation and searching-for-yield effects.

Suppose firms are financially constrained if they implement the risky project. We also consider cases in which they are not. Reducing interest rates primarily boosts firm value, leading to fewer firms taking risks. This happens when firms' equity is only marginally leveraged or low debt servicing costs make the safer project more attractive. Therefore, lowering the policy rate may not incentivize firms to pursue socially beneficial but individually risky projects under strict leverage constraints. In contrast, when firms can leverage extensively or when debt servicing costs are high, the leveraged valuation effect is outweighed by the substitution effect, driven by an improved leveraged return relative to leveraged risk. In this case, lowering the policy rate encourages greater risk-taking. As a result, the "searching-for-yield" behavior (e.g., Rajan (2006)) is more pronounced in a low interest rate environment only if firms or financial institutions are highly leveraged.

Therefore, our theory suggests that further rate reductions in an environment with low interest rates may discourage firms from pursuing socially beneficial but risky projects. This dynamic could cause aggregate economic indicators to display hump-shaped patterns. For example, the "IS" curve—which illustrates the relationship between output and the real interest rate—might slope upward when rates fall to very low levels. Consequently, an overly accommodative monetary (or fiscal) policy could be contractionary.

In summary, although a lower interest rate generally stimulates investment and output constrained by financial frictions, it can discourage productive risk-taking, diminish productivity, and ultimately decrease output if it drops too much. In this sense, financial frictions bring about the usual endogenous capital misallocation and selection in the existing exogenous productivity distribution, such as in Midrigan and Xu (2014), Moll (2014), and Buera and Moll (2014);[1] but more importantly, our model shows that the frictions can influence the productivity distribution itself through risk-taking, creating the possibility of an optimal interest rate level because many aggregate variables are hump-shaped when we vary the target interest rate.

Calibrating to macro- and micro- data, we indeed find a hump-shaped total factor productivity (TFP) curve and a hump-shaped IS curve by varying the interest rate. Additionally, the calibrated model replicates the decline in risk-taking measures observed in 2001 and the increase in 2007, despite similar interest rate levels and cuts. This pattern is closely tied to higher firm leverage in 2007 than in 2001. Thus, the stricter leverage requirements in-

[1] Gopinath et al. (2017) found substantial capital misallocation in southern Europe, showing that the decline in real interest rates—often linked to euro convergence—widened the dispersion of capital returns and reduced total factor productivity (TFP) by channeling capital to less productive firms in underdeveloped financial markets. See also recent work on the counterproductive general-equilibrium effect of low interest rate by Asriyan et al. (2024).

troduced after the global financial crisis, together with persistently low real interest rates, could lead to sustained reductions in risk-taking and lower equilibrium marginal products of capital, further reinforcing the low interest rate environment.

Finally, our calibration identifies an optimal target interest rate based on the extent of the interdependence of firms' risky projects and specific leverage constraints. Although this paper leaves aside long-term influences like demographic changes or technological shifts on the long-run interest rate, it focuses on the real rate component that can be targeted by monetary-fiscal policies at least in the medium run. The interdependence of firm production and credit limits shapes this optimal target rate.

In modeling interdependence, we borrow from Statistics literature by using a generalized Binomial distribution that considers the interdependence of the Bernoulli trials. Specifically, firms' risk-taking behaviors can influence the success probabilities of other firms. However, each entrepreneur takes their probability of success as given, so the aggregate economy contains externality. Positive association captures knowledge spillover (e.g., Jones and Williams (2000)) when the project is successful and risk spillover (e.g., conditional value at risk along the line of Adrian and Brunnermeier (2016)). In contrast, negative association captures crowding out and learning from others' mistakes.

When project outcomes are positively associated, interest rates should be lowered to direct resources toward firms engaged in productive projects, which overall feature beneficial knowledge spillover. In contrast, raising interest rates can be advantageous when project outcomes are negatively associated. However, a lower rate may also be optimal if the negative association is particularly severe. Given the hump-shaped relationship between risk-taking and interest rates, both very low and very high rates can deter risk-taking, but only low rates effectively encourage investment. Credit limits also influence the optimal interest rate. This suggests that financial regulation policies (e.g., targeting asset collaterability) significantly impact monetary or fiscal policy effectiveness. The coordination should balance efficiency and risk in the economy. For realistic credit limits in our model, e.g. within 10% above or below our calibrated benchmark, and a broad range of association scenarios, the optimal real lending rate falls between 2.2% and 3.1%.

Related literature. Our theory of risk choices in the context of financial conditions closely aligns with literature on entrepreneurship with financial frictions, as seen in works such as Cagetti and De Nardi (2006), Buera and Shin (2013), and Buera et al. (2015). In these studies, agents face the decision of becoming either an entrepreneur or a worker (or a serial entrepreneur in Brandt et al. (2022)), and financial frictions influence this choice. Notably, agents' optimization problems often exhibit non-convexity due to discrete choices, which can intertwine risk-taking with career decisions. For instance, in Vereshchagina and Hopenhayn (2009), where agents make occupational and risky project choices under no-

borrowing constraints, those in the intermediate wealth range who opt to be entrepreneurs may exhibit risk-seeking behavior as this "lottery" mechanism can help smooth value functions. Compared to this literature, our emphasis lies in illustrating how varying financial conditions impact firms' risk-taking behaviour, the related macroeconomic effects, and the optimal interest rate.

Our framework connects naturally to financial frictions[2] and risks in general. Building on Bernanke et al. (1999), Christiano et al. (2014) show that risk shocks in a quantitative business-cycle model with financing constraints generate counter-cyclical spreads and account for a large fraction of macroeconomic fluctuations. Miao and Wang (2010) include long-term risky defaultable debt in a macroeconomic model with financial shocks to the recovery rate. Cui and Kaas (2020) develop a framework to analyze belief risks in credit contracts. We add to this literature by examining the effect of financing constraint/interest rates on *endogenous* risk-taking behaviors.

Our work also adds to the literature on active risk management: the seminal paper by Froot et al. (1993) in the static case and Bolton et al. (2011) in the dynamic case show that financing constraints generate the rationale for active risk management. In studying risk-taking with banks, Dell Ariccia et al. (2014) find the trade-off between the risk-shifting and searching-for-yield effects in supplying credit. When the borrowing cost falls, risk-shifting can occur because the marginal return for monitoring is higher, inducing banks to monitor more and reduce loan risk. In subsequent empirical studies, however, Dell Ariccia et al. (2017) and Jiménez et al. (2014) (and the reference therein) find that loan yield data mostly suggests the searching-for-yield effect. Unlike banks, non-financial firms may have more curvature in their preferences because they are more risk-averse or face more dividend smoothing requirements (Jermann and Quadrini, 2012); they may also face tighter financing constraints as part of the demand side for credit. This makes the valuation effect more salient, and consequently, in the data, we observe a non-monotonic relationship between interest rate and risk-taking. The model shows the non-monotonic effect via a project selection problem different from the usual bank's risk-monitoring problem.

To illustrate the underlying mechanism, we use a highly tractable framework that yields closed-form individual risk-taking policy functions. At the aggregate level, project associations are accounted for without complicating individual firm decisions, thanks to the flexible generalized Binomial distribution with a dependence structure of each Bernoulli trial. While this technical feature may be of independent interest, its primary value is simplifying the optimal target interest rate analysis in response to return associations among risky projects.

The rest of the paper is organized as follows: We first present stylized facts about the non-monotonic risk-taking measures to variations in interest rates. Section 3 presents a

[2]The literature on financial frictions is vast, and the seminal contribution at least includes Kiyotaki and Moore (1997), Bernanke et al. (1999), Mendoza (2010), and Brunnermeier and Sannikov (2014).

simple two-period model, illustrating the key effects of interest rate and leverage on risk-taking. Section 4 extends the two-period model into an infinite-horizon economy in which the correlation between project returns is considered. Section 5 shows quantitatively how the target interest rate and borrowing constraint shape the macroeconomic outcomes, followed by the optimal interest rate discussion. Section 6 concludes.

2 Motivating Evidence

We start with some evidence about the non-monotonic relationship between firms' risk-taking behavior and interest rates.

As a proxy for firm risk-taking behavior, we first utilize a metric that captures the variation in individual stock returns (in log) within a given year. When firms undertake riskier ventures, the volatility of stock returns tends to increase (assuming all else remains constant) due to heightened uncertainty. We extract daily stock returns for non-financial firms from CRSP (see Appendix A for data description) from 1984 to 2020. The inter-quartile range (IQR) of daily returns is then employed to approximate the extent of firms' risk-taking. Unlike standard deviation, IQR offers the advantage of being less influenced by extreme volatility. However, the findings below are robust even when employing standard deviation as the measure.

The left panel of Figure 1 shows the median inter-quartile range (IQR) of individual stock returns plotted against the logarithmic level of the (gross) Cleveland rate, a widely used gauge of real interest rates provided by the Federal Reserve Bank of Cleveland. A quadratic fit between these variables, accompanied by a 95% confidence interval, is also shown. At higher real interest rates, there appears to be a negative correlation between the interest rate and stock return variability. Conversely, this relationship transitions to a positive one as the interest rate declines to lower levels. Notice that this non-monotonic relationship remains robust when we use a non-parametric fit, and it persists across sectors classified by the one-digit SIC code (see Appendix A).

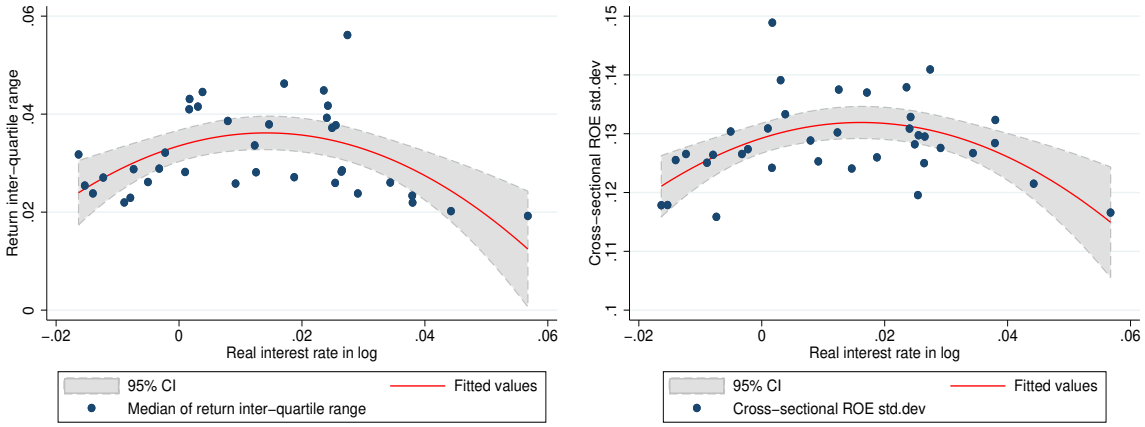
We also look at the cross-sectional standard deviation of return on equity (ROE) as an alternative measure of risk-taking. The right panel of Figure 1 shows the measure plotted against the real rate with a quadratic fit. It shows a similar non-monotonic pattern.

Further, the firm-level data supports the macro evidence above. We adopt the following empirical framework:

$$y_{ij,t} = \beta_0 + \beta_1(R_t)\Delta R_t + X_{ij,t-1} + f_i + h_j + \Gamma_t, \tag{1}$$

where $y_{ij,t}$ is the IQR of the stock return of firm i in sector j at year t in the baseline regression, R_t is the level of interest rate, ΔR_t is a measure of interest-rate shock, Γ_t is an

Figure 1: Risk-taking Measures and Real Interest Rate



Note: The left panel uses the median of each year's inter-quartile range (IQR) of individual stock returns. The right panel uses the cross-sectional standard deviation of ROE each year. The real interest rate measure is from the Federal Reserve Bank of Cleveland.

aggregate control which includes the current and lagged terms of inflation, unemployment rate, real GDP growth rate, and the lagged interest rate.[3] The control vector $X_{ij,t-1}$ encompasses various factors, including the firm leverage ratio (measured by long-term debt over fixed assets such as property, plant, and equipment), logarithmic size (total assets), market-to-book ratio, ROE, return on assets (ROA), return on investment (ROI), Tobin's Q, and cash holdings (measured by cash and short-term investments over total assets). These firm-level financial data are sourced from US COMPUSTAT covering 1984 to 2020. f_i represents the firm fixed effect, while h_j accounts for the sector effect. We cluster standard errors two ways to account for correlations within firms and within years.

Our primary coefficient of interest, β_1, indicates how the interest-rate level influences firms' risk-taking behavior measured by $y_{ij,t}$. Notably, $\beta_1(R_t)$ can vary based on the level of interest rates. Inspired by Figure 1, we partition the sample periods into two: one characterized by low interest rate levels and the other by high interest rate levels. We use the median interest rate level as the threshold to determine whether a specific period is associated with low or high interest rates. The period featuring higher (lower) interest rates than the median is designated as the high (low) rate period. Various measures of the real rate are employed, including the federal funds rate deflated by past inflation ("ffr" in Table 1) and by expected inflation ("ffr2"), the Cleveland real rate, and the real prime rate (prime rate in Table 1) as a measure of the actual interest rate charged on creditworthy borrowers. For exogenous interest rate fluctuations, we utilize the monetary policy shock proposed by Jarociński and Karadi (2020) ("mpshock") as the source of exogenous interest rate changes.[4]

Table 1 presents the findings derived from estimating the specification outlined in equa-

[3]The results remain robust when we use current aggregate controls only or lagged aggregate controls only.
[4]Other measures, such as Swanson (2021), give similar results.

Table 1: Variation in IQR with respect to Changes in Interest Rate

VARIABLES	(1) return iqr	(2) return iqr	(3) return iqr	(4) return iqr	(5) return iqr	(6) return iqr	(7) return iqr	(8) return iqr
mpshock	0.024***	-0.014*	0.019***	-0.008	0.009*	-0.011	0.018***	-0.012
	(0.005)	(0.007)	(0.005)	(0.014)	(0.004)	(0.010)	(0.004)	(0.008)
Observations	47,214	52,038	46,738	52,772	44,869	53,266	47,214	52,038
R-squared	0.563	0.557	0.502	0.587	0.432	0.623	0.560	0.556
Firm FE	YES	YES	YES	YES	YES	YES	YES	YES
Interest use	ffr	ffr	ffr2	ffr2	Cleveland rate	Cleveland rate	prime rate	prime rate
Interest level	low	high	low	high	low	high	low	high

Robust standard errors in parentheses. *** p<0.01, ** p<0.05, * p<0.1.

Note: Results from estimating equation (1). Alternative measures of the real interest rate are employed to classify the rate regime: the real federal funds rates, using the federal funds rate deflated by the past inflation (ffr, columns 1 and 2) and deflated by the expected inflation (ffr2, columns 3 and 4); the Cleveland real rate (columns 5 and 6); and the real prime rate, using the prime rate deflated by the past inflation (columns 7 and 8).

tion (1), employing firms' return IQR as a measure of risk-taking behaviors. The coefficient β_1 is significantly positive when the interest rate is low and, conversely, negative when the interest rate is high. An unexpected unit of interest rate cut, estimated to range from 3 to 6 basis points according to Jarociński and Karadi (2020), results in a reduction of the firm's IQR by approximately 90-240 basis points in a low-interest-rate environment. Conversely, in a high-interest-rate environment, the IQR increases by 80-140 basis points across different measures of the real rate.

In line with John et al. (2008) and Boubakri et al. (2013), we also examined variations in return on equity (ROE) and return on assets (ROA) as alternative measures of firm-level risk-taking. We observe a similar sign switch of β_1, transitioning from positive in a low-interest-rate environment to negative in a high-interest-rate environment.[5]

What might contribute to the interest rate's non-monotone effect? Firm leverage can be an important factor. To see this, we examine 2001 and 2007, marking the onset of respective recessions. During both periods, the Cleveland rate was around $1.7 - 1.8\%$, yet the median firm leverage ratio in 2007 surpassed that of 2001 by approximately 19%. Despite encountering similar interest rate reductions (about 150 basis points) in both years, they triggered differing responses in risk-taking measures. The 5-year moving-average IQR declines 7.5% after 2001, whereas it rises by 2% after 2007. Using the cross-sectional standard deviation of ROE as the risk-taking measure leads to similar results (a 7% fall after 2001 and a 3.24% rise after 2007), and our model below will shed light on these two periods.

The findings above prompt us to investigate theoretically the influence of interest rates and leverage, or broader financial conditions, on firm risk-taking.

[5]Further empirical findings are available upon request.

3 A Simple Leveraged Risk-taking Model

In this section, we examine the effect of financing constraints on risk-taking behaviors in a simple two-period model. The model shows that the relationship between risk-taking incentives and financial conditions is non-monotonic.

3.1 An Entrepreneur's Problem

There are two periods and two types of projects. An entrepreneur discounts between two periods with a discount factor β and picks one of the two types of projects. The risky type has two possible returns: with probability p, the return is Π^h; with probability $1-p$, the return is Π^l. The expected return of the risky project is denoted by

$$\Pi \equiv p\Pi^h + (1-p)\Pi^l.$$

The entrepreneur can also choose the safe type with return Π^f. The entrepreneur can borrow at a gross interest rate R with a financing constraint specified later.

For exposition reasons, we assume that

$$\Pi^h > \Pi > \Pi^f > R > \Pi^l. \tag{A1}$$

The risky project's expected return Π is higher than the return Π^f of a safe project. Unlike Vereshchagina and Hopenhayn (2009), who study choices of projects with the same expected return,[6] we impose $\Pi > \Pi^f$ so that socially it may be optimal to take risks while it may not be the case at individual levels. The return of the risky project is higher than Π^f when the project turns out to be successful (i.e., with the return Π^h), while the return is lower than Π^f if the project is unsuccessful (i.e., with the return Π^l). Also, $\Pi^f > R$ so that the safe project has a higher return than the risk-free saving rate. This might be natural as running a firm generally should have a higher return than the lending rate. Finally, our analysis abstracts from saving in risk-free assets (instead of projects) since they are dominated by running safe projects. In the macroeconomic model later, we will consider a case in which an entrepreneur could be indifferent between a safe project and a safe deposit. In addition to A1, we assume that

$$\Pi^f\hat{\Pi} < \Pi^h\Pi^l, \text{ where } \hat{\Pi} \equiv p\Pi^l + (1-p)\Pi^h \tag{A2}$$

so that the risk-free project return is not too high, which captures reasonable features in practice and turns out to be useful in simplifying the analysis.

[6] They study why entrepreneurs take risks even when expected returns are the same as the safe return.

Let ω denote the wealth level of the entrepreneur each period after repaying previous debt, and let s be the amount invested in a project which can be used as collateral. We assume the utility function of the entrepreneur as

$$u(c) = \log c,$$

where consumption/dividends $c = \omega - s$. It may initially seem that the entrepreneur owns the firm, but the entrepreneur can also be its manager, so c could also be interpreted as a dividend payout. Besides obtaining closed-form solutions, there are several advantages of assuming $u(c) = \log c$. The curvature in $u(.)$ captures the manager's preference for dividend smoothing. Lintner (1956) first showed that managers consider dividend smoothing over time, a fact further confirmed by subsequent studies. In addition, increasing c can be interpreted as equity share repurchases while reducing c can be thought of as sales of new shares. Putting curvature in $u(.)$ is thus a simple way of modeling the speed with which firms can vary the funding source when financial conditions change.[7] Notice that what is crucial is the concavity that generates the evaluation effect below, but log utility simplifies the algebra and delivers reasonable quantitative results in the macro model later.[8]

There is a running (utility) cost from implementing the risky project, denoted by utility $\eta \in [\underline{\eta}, +\infty)$, drawn from an i.i.d. distribution (note: the lower bound does not have to be positive or even finite). The value of an entrepreneur with cost η and wealth ω is

$$V(\omega, \eta) = \max\{V^r(\omega, \eta), V^f(\omega)\}.$$

Here, we use the realization of η to control whether the firm chooses a risky or a risk-free project in period t. Using utility cost as in the firm default problem of Cui and Kaas (2020) permits closed-form solutions, and it is equivalent to costs in terms of goods proportional to entrepreneurs' wealth (given the utility and technology in this environment). We abstract from the default issue to focus on the project choices, and firms are required and able to repay all their debt regardless of the realization of the project return. Appendix D introduces the potential of default.

Let b denote the level of borrowing. The value of implementing the risky project,

[7] Jermann and Quadrini (2012) also assume dividend adjustment costs, supported by evidence cited therein.

[8] In the infinite horizon model, it implies $(1 - \beta)$ fraction of net worth is used for paying dividends and rewarding managers. The calibrated β and log utility deliver reasonable quantitative results (see Section 5).

$V^r(\omega, \eta)$, can be written as

$$\begin{aligned} V^r(\omega,\eta) = \max_{s,b} \Big\{ & \log(\omega - s) - \eta + \beta p \log(\Pi^h(s+b) - Rb) \\ & + \beta(1-p)\log(\Pi^l(s+b) - Rb) \Big\} \\ \text{s.t. } & 0 \le b \le \bar{\theta} s, \end{aligned}$$

where $\bar{\theta}$ is a parameter that governs the tightness of the borrowing constraint. The entrepreneur can put in internal savings s, together with the borrowing b; next period, the entrepreneur earns either $\Pi^h(s+b)$ if the project turns out to be productive, or $\Pi^l(s+b)$ if the project turns out to be unproductive. The interest payment is naturally Rb. The amount of borrowing, b, is limited due to financial frictions. The entrepreneur can pledge some of the capital s as collateral, i.e., borrow up to $\bar{\theta}$ fraction of its capital at the time of borrowing.

The value function of implementing the safe project, $V^f(\omega)$, can be written as

$$\begin{aligned} V^f(\omega) = \max_{s,b} & \Big\{ \log(\omega - s) + \beta \log(\Pi^f(s+b) - Rb) \Big\} \\ \text{s.t. } & 0 \le b \le \bar{\theta} s. \end{aligned}$$

Compared to the value function $V^r(\omega, \eta)$, the cost of implementing the risk-free project is normalized to be 0, and the return next period is certain.

The optimization shows that s is linear in the wealth level ω independent of choices of projects. That is, $s = \varphi\omega$, with $\varphi = \frac{\beta}{1+\beta}$. When entrepreneurs choose the risk-free project, their firms will borrow up to the limit $\theta = \bar{\theta}$ since $\Pi^f > R$. When the entrepreneur chooses the risky project, depending on the interest rate R, θ can be one of the three: 0, $\bar{\theta}$, and θ^* as the level of the leverage when the firm is unconstrained:

$$\theta^* \equiv -\frac{\Pi^h \Pi^l - R\hat{\Pi}}{(\Pi^l - R)(\Pi^h - R)} = -\left[p\frac{\Pi^l}{\Pi^l - R} + (1-p)\frac{\Pi^h}{\Pi^h - R} \right]. \tag{2}$$

Assumptions A1 and A2 imply that $\theta^* > 0$. The following proposition determines the threshold cost of taking the risky project. In simple words, the threshold is the difference in log leveraged returns.

Proposition 1. *Suppose A1 and A2 hold. The leverage of a firm that implements the risky project satisfies*

$$\theta = \min\{\theta^*, \bar{\theta}\},$$

where θ^* is defined in (2) and θ^* decreases in the interest R. Those entrepreneurs with

$\eta \leq \max\{\underline{\eta}, \tilde{\eta}\}$ *choose the risky project and the threshold level* $\tilde{\eta}$ *satisfies*

$$\tilde{\eta} = \beta p \log\left(\frac{\Pi^h + (\Pi^h - R)\theta}{\Pi^f + (\Pi^f - R)\bar{\theta}}\right) + \beta(1-p)\log\left(\frac{\Pi^l + (\Pi^l - R)\theta}{\Pi^f + (\Pi^f - R)\bar{\theta}}\right). \tag{3}$$

Proof. See Appendix B. □

3.2 Non-monotonic Risk-takings

How does the lending interest rate (or, more generally, financial conditions) affect risk-taking decisions? Specifically, we investigate how the lending interest rate R affects the risk-taking behavior characterized by $\tilde{\eta}$. First, we show that risk-taking may not be monotone to the interest rate (borrowing cost). A falling borrowing cost does not necessarily induce more risk-taking. Then, we provide a heuristic approach to understand the results.

To simplify the discussion, let us first focus on the case where the financing constraint is binding if a risky project is implemented, and we will relax this assumption later.[9] Then,

$$\frac{\partial \tilde{\eta}}{\partial R} = \beta\left[\frac{\bar{\theta}}{\Pi^f + (\Pi^f - R)\bar{\theta}} - p\frac{\bar{\theta}}{\Pi^h + (\Pi^h - R)\bar{\theta}} - (1-p)\frac{\bar{\theta}}{\Pi^l + (\Pi^l - R)\bar{\theta}}\right], \tag{4}$$

and as will become clear the sign of $\partial\tilde{\eta}/\partial R$ depends on the leverage upper bound $\bar{\theta}$. A higher interest rate will reduce the leveraged return of both projects; however, whether the safe project or the risky project is more attractive depends on the maximum leverage $\bar{\theta}$. If $\bar{\theta}$ is low (high) enough, the threshold $\tilde{\eta}$ increases (decreases) in the interest rate R, and the risky (safe) project is thus preferred. Overall, $\tilde{\eta}$ has a hump-shaped relationship with R.

Proposition 2. *Suppose A1 and A2 hold. Assuming the financing constraint is binding when the risky project is implemented (i.e.,* $\bar{\theta} < \theta^*$*). There exists cutoff levels* $\bar{\theta}^s_L < \bar{\theta}^s_H$ *such that the threshold level* $\tilde{\eta}$ *increases in* R *when* $0 < \bar{\theta} \leq \bar{\theta}^s_L$*, decreases in* R *when* $\bar{\theta}^s_H \leq \bar{\theta} < \theta^*$*, and is hump-shaped in* R *when* $\bar{\theta}^s_L < \bar{\theta} < \bar{\theta}^s_H$*.*

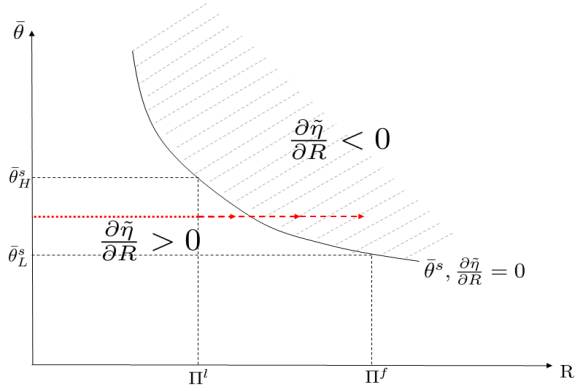
Proof. See Appendix B. □

The proposition can be illustrated by Figure 2. For a given leverage $\bar{\theta}$, the downward-sloping curve is denoted by $R^s(\bar{\theta})$, corresponding to the interest rate such that $\partial\tilde{\eta}/\partial R = 0$ and separates the two regions with different signs of the derivative $\partial\tilde{\eta}/\partial R$.[10] The shaded region above the curve has the feature of $\partial\tilde{\eta}/\partial R < 0$, and the blank area under the curve has the feature of $\partial\tilde{\eta}/\partial R > 0$. When $\bar{\theta} < \bar{\theta}^s_L$, as R increases from the low return realization Π^l

[9] Proposition 1 implies as the interest rate increases, θ^* falls so that the financing constraints for entrepreneurs who choose the risky project might change from binding to non-binding.

[10] Alternatively, the downward-sloping curve also represents $\bar{\theta}^s(R)$, which corresponds to the leverage upper bound such that $\partial\tilde{\eta}/\partial\bar{\theta} = 0$ for a given interest rate R.

Figure 2: Hyperplane of $\frac{\partial \tilde{\eta}}{\partial R} = 0$ in $(\bar{\theta}, R)$ Space



to the safe return Π^f, $\partial\tilde{\eta}/\partial R$ is always positive. When $\bar{\theta} > \bar{\theta}^s_H$, as R increases from the low return realization Π^l to the safe return Π^f, $\partial\tilde{\eta}/\partial R$ is always negative. In these two cases, the interest rate change has a monotone effect on the threshold $\tilde{\eta}$.

However, if $\bar{\theta}$ is middle-ranged i.e. $\bar{\theta} \in (\bar{\theta}^s_L, \bar{\theta}^s_H)$, the effect of interest rate on risk-taking is non-monotone. To see this key result, for the points $(R, \bar{\theta})$ on the red dashed line below the separating curve, $\partial\tilde{\eta}/\partial R$ is positive since $\Pi^l < R < R^s(\bar{\theta})$; for the points $(R, \bar{\theta})$ on the red dashed line above the separating curve, $\partial\tilde{\eta}/\partial R$ turns negative since $R^s(\bar{\theta}) < R < \Pi^f$. In this middle range case, an increase of the interest rate R first encourages more risk-taking behavior and then discourages risk-taking once the interest rate R exceeds $R^s(\bar{\theta})$. This is because an increase in R will not only raise the borrowing cost but also affect the trade-off between leveraged return and volatility.

Notice that the non-monotonic effect is not unique to the interest rate; we also find the non-monotonic effect of leverage as in Appendix B.2. As illustrated below, the key behind the non-monotonicity is how debt servicing costs influence risk-taking incentives.

We approximate the threshold $\tilde{\eta}$ to the second order around a point $\tilde{\eta}_s$ (see below) so we can obtain further analytical insights. First, if we replace the risky project with a (hypothetical) safe project with return Π (the same as the expected return of the risky project), the entrepreneur borrows up to the credit limit with this hypothetical safe project (i.e., $\theta = \bar{\theta}$) because the return $\Pi > \Pi^f > R$. Then, the threshold below which an entrepreneur chooses this hypothetical project can be defined as $\tilde{\eta}_s$:

$$\begin{aligned}
\tilde{\eta}_s &\equiv \beta \log(\Pi(1+\theta) - R\theta) - \beta \log(\Pi^f(1+\theta) - R\theta) \\
&= \beta \log\left(\Pi - R\frac{\bar{\theta}}{1+\bar{\theta}}\right) - \beta \log\left(\Pi^f - R\frac{\bar{\theta}}{1+\bar{\theta}}\right) \\
&= \beta \log\left(\frac{\Pi - \frac{\bar{\theta}R}{1+\bar{\theta}}}{\Pi^f - \frac{\bar{\theta}R}{1+\bar{\theta}}}\right).
\end{aligned}$$

$\tilde{\eta}_s$ is associated with the ratio between the leveraged returns on the risky and risk-free projects; this term represents the "valuation effect" of leverage or interest rate to be further illustrated. Now, we are ready to expand $\tilde{\eta}$ to the second order around $\tilde{\eta}_s$ by using the cut-off (3):

$$\begin{aligned}\tilde{\eta} \approx \quad \tilde{\eta}_s \quad & +\beta p\left[\frac{1+\bar{\theta}}{\Pi(1+\bar{\theta})-R\bar{\theta}}(\Pi^h-\Pi)-\frac{1}{2}\frac{(1+\bar{\theta})^2}{[\Pi(1+\bar{\theta})-R\bar{\theta}]^2}(\Pi^h-\Pi)^2\right] \\ & +\beta(1-p)\left[\frac{1+\bar{\theta}}{\Pi(1+\bar{\theta})-R\bar{\theta}}(\Pi^l-\Pi)-\frac{1}{2}\frac{(1+\bar{\theta})^2}{[\Pi(1+\bar{\theta})-R\bar{\theta}]^2}(\Pi^l-\Pi)^2\right] \\ = \quad \tilde{\eta}_s \quad & \underbrace{-\frac{1}{2}\frac{\beta\sigma^2}{\left[\Pi-R\frac{\bar{\theta}}{1+\bar{\theta}}\right]^2}}_{\tilde{\eta}_\sigma},\end{aligned} \tag{5}$$

where $\sigma^2 \equiv p(\Pi^h-\Pi)^2+(1-p)(\Pi^l-\Pi)^2$ is the variance of the risky project itself. After we adjust for discounting, $\tilde{\eta}_\sigma$ is a negative half of the inverse of the squared (leveraged) Sharpe ratio; this term reflects the disutility related to the volatility of the leveraged return. As $\tilde{\eta}_\sigma$ is negative, $\tilde{\eta}_s$ is above $\tilde{\eta}$. This is intuitive because entrepreneurs will be more cautious (reflected by the lower threshold $\tilde{\eta}$ relative to $\tilde{\eta}_s$) for a riskier project with the same expected return, thanks to risk-aversion or risk-management needs. It represents the "searching for yield effect" or the substitution effect. A higher Sharpe ratio increases the term $\tilde{\eta}_\sigma$, raising the probability of risk-taking. The higher Sharpe ratio increases the opportunity cost of the safe project, making entrepreneurs more likely to opt for the risky project.

If the firm can borrow, i.e., $\bar{\theta} > 0$, the terms above immediately imply:

$$\frac{\partial\tilde{\eta}_s}{\partial R} = \frac{\beta\bar{\theta}(1+\bar{\theta})(\Pi-\Pi^f)}{[\Pi^f(1+\bar{\theta})-R\bar{\theta}]^2} > 0 \text{ and } \frac{\partial\tilde{\eta}_\sigma}{\partial R} = \frac{-\beta\bar{\theta}(1+\bar{\theta})^2}{[\Pi(1+\bar{\theta})-R\bar{\theta}]^3}\sigma^2 < 0.$$

When R increases, financing costs increase, and the entrepreneurs' firm will be less valued regardless of the project chosen. As a result, the higher expected return becomes marginally more attractive because of the curvature in the preferences[11]. This valuation effect makes the risky project with a higher expected return more preferred, raising $\tilde{\eta}_s$, which explains $\frac{\partial\tilde{\eta}_s}{\partial R} > 0$. Meanwhile, the adjusted Sharpe ratio falls, and the searching for yield effect makes the risky project less attractive, which explains $\frac{\partial\tilde{\eta}_\sigma}{\partial R} < 0$.

Depending on parameters, we thus know that a rise in the interest rate R can have a non-monotonic effect on $\tilde{\eta}$ because the valuation effect and the substitution effect work in the opposite directions (Proposition 2). In fact, for the interest rate $R \in (\Pi^l, \Pi^h)$, a trade-

[11] Notice that $\tilde{\eta}_s \approx \beta\frac{\Pi-\Pi^f}{\Pi^f-\frac{\bar{\theta}R}{1+\bar{\theta}}}$, which is the product of the discount factor β, the expected excess return of the risky project $\Pi-\Pi^f$ and the marginal value of lowering the interest rate for the safe project $\left(\Pi^f-R\frac{\bar{\theta}}{1+\bar{\theta}}\right)^{-1}$.

off between the two effects arises when $\bar{\theta}$ is middle ranged. More generally, the higher the debt servicing cost per unit of equity, $\bar{\theta}R/(1+\bar{\theta})$, the weaker the valuation effect and the stronger the substitution effect. The valuation effect, however, dominates if the equity value is high relative to liability. Intuitively, the safer option should be more attractive when the entrepreneur finances most of the firm internally.

Notice that the effect of interest rate on risk-taking depends on the leverage constraint measured by $\bar{\theta}$. Rearranging the terms in (5) and $\tilde{\eta}_s$, we obtain

$$\Pi - \Pi^f = \left(\Pi^f - R\frac{\bar{\theta}}{1+\bar{\theta}}\right)\left[\frac{\tilde{\eta}}{\beta} + \frac{1}{2}\left(\frac{\sigma}{\Pi - R\frac{\bar{\theta}}{1+\bar{\theta}}}\right)^2\right],$$

where $\Pi - \Pi^f$ is the expected excess return of the risky project. Therefore, we have a natural asset pricing interpretation of risk management. For entrepreneurs indifferent between a safe project and a risky project, the excess (leveraged) return compensates for the risk and the cost of implementing the risky project. That is, on top of the safe leveraged return $\left(\Pi^f - R\frac{\bar{\theta}}{1+\bar{\theta}}\right)$, the compensation for the risk is $0.5\sigma^2/\left[\Pi - R\bar{\theta}/(1+\bar{\theta})\right]^2$, while the compensation for the cost is simply $\tilde{\eta}/\beta$.

Hence, when the interest rate rises, the risk of holding a portfolio consisting of long positions in risky projects and short positions in bonds increases. Assuming all other factors such as Π, Π^f, σ, and $\bar{\theta}$ remain constant, this rise in interest rates tends to dampen the willingness to take risk, thereby exerting downward pressure on the threshold cost $\tilde{\eta}$. Nonetheless, the rise in interest rates also diminishes the safe leveraged return. To justify the surplus return $\Pi - \Pi^f$ for the "marginal" entrepreneur, it becomes necessary for $\tilde{\eta}$, the threshold cost of undertaking the risky project, to go up.

Remark: we have found the non-monotonicity of risk-taking when the liquidity constraint is binding. That is, $\bar{\theta} < \theta^*_{min}$, where

$$\theta^*_{min} \equiv \frac{\Pi^h\Pi^l - \Pi^f\hat{\Pi}}{(\Pi^h - \Pi^f)(\Pi^f - \Pi^l)}$$

is the minimal leverage when entrepreneurs are not constrained, which is obtained by assuming $R = \Pi^f$ in (2). We further generalize that the non-monotonicity can persist even when the liquidity constraint is not binding (i.e. $\bar{\theta} \geq \theta^*_{min}$).

Proposition 3. *Suppose A1 and A2 hold. Let $\bar{R}(\bar{\theta})$ be such that $-\left[p\frac{\Pi^l}{\Pi^l - \bar{R}} + (1-p)\frac{\Pi^h}{\Pi^h - \bar{R}}\right] = \bar{\theta}$ when entrepreneurs just become unconstrained. If $\bar{\theta} \geq \theta^*_{min}$, there exist an interest rate level $R^u(\bar{\theta})$ such that the risk-taking threshold $\tilde{\eta}$ for unconstrained entrepreneurs decreases in R when $\bar{R}(\bar{\theta}) \leq R < R^u(\bar{\theta})$ and increases in R when $R^u(\bar{\theta}) \leq R < \Pi^f$.*

Proof. See Appendix B. □

3.3 Examples

The entrepreneur may or may not be financially constrained when implementing the risky project because of the project's risk profile and the entrepreneur's risk preference. Figure 3 is a numerical illustration where the parameterization is as follows: $\Pi^f = 1.1$, $\Pi^h = 1.3$, $\Pi^l = 0.95$, $p = 0.65$, and $\beta = 0.96$. Note that this specification satisfies A2. We do not have to specify $F(\cdot)$ for individual decision problems given the interest rate R. However, it will be used in the analysis later with credit market equilibrium.

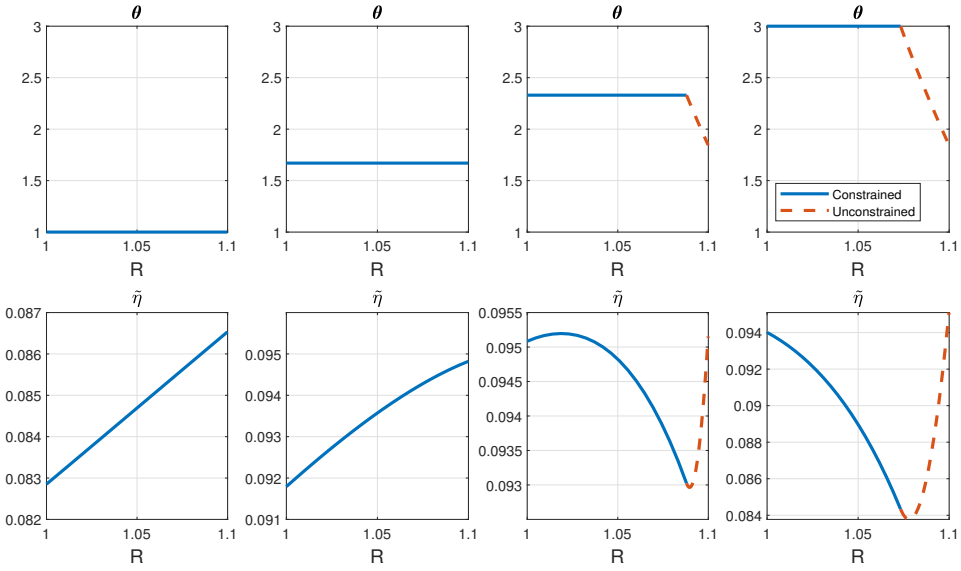
The figure delivers the key message of the model: a cut in the interest rate reduces risk-taking when the leverage limit $\bar{\theta}$ is low, while it encourages risk-taking when $\bar{\theta}$ is high. When the leverage is low, a cut in the interest rate R mainly works through the "valuation effect" since the debt servicing cost is low; the entrepreneur prefers taking less risk. When the leverage is high, however, the valuation effect is dominated by the substitution effect with the same interest rate cut since the high leverage amplifies the rise in the Sharpe ratio of the portfolio with the leveraged risky project. "Searching for yield" behaviors in a low-interest rate environment are thus more likely when the firm is highly leveraged. In this case, reducing the external funding cost encourages risk-taking, as it increases the Sharpe ratio of the leveraged risky project.

Note that the financing constraint might become slack for high leverage limits and interest rates, as shown in the third and fourth columns of Figure 3. Consistent with Proposition 3, when the borrowing constraint ceases to be binding, risk-taking first decreases and then increases with R. Furthermore, we show that the non-monotonic relationship is also robust if we endogenize the credit limit $\bar{\theta}$ (see Appendix D).

Finally, Figure 4 further shows that the effect of borrowing cost R on the risk-taking behavior also depends on the profile of the risky project return Π^l and Π^h. In this experiment, we fix $\bar{\theta} = 1.5$. The first column uses the value of Π^h and Π^l previously specified. Now, we increase Π^h and decrease Π^l simultaneously to keep the mean return of the risky project Π unchanged. For simplicity, let's first look at the case when the financing constraint is binding. If the project becomes riskier, as the interest rate R increases, the Sharpe ratio falls more, and the risky project becomes less attractive. In other words, the substitution effect dominates when the risky project becomes riskier. This result is intuitive because when risk goes up, the leveraged volatility and the Sharpe ratio are sensitive to interest rate changes.

We also note that as the project becomes riskier, the interest-rate range in which the leverage constraint ceases binding expands. As risks increase, the financially unconstrained interest-rate region with a dominating valuation effect also expands.

Figure 3: Effect of R on $\tilde{\eta}$ for Alternative $\bar{\theta}$



Note: The top panel of each column shows the leverage associated with the risky project; the bottom panel of each column shows the threshold cost for risk-taking. Each column corresponds to a different leverage upper bound (from the left to the right: $\bar{\theta} = 1.00, 1.67, 2.33,$ and 3.00).

4 Infinite-horizon Model with Project Return Associations

Now, we extend the previous two-period model into an infinite-horizon general equilibrium model. We also introduce associations of project successes/failures without changing the key ingredients of the model. The next section will use the equilibrium model for policy discussion.

4.1 The Environment

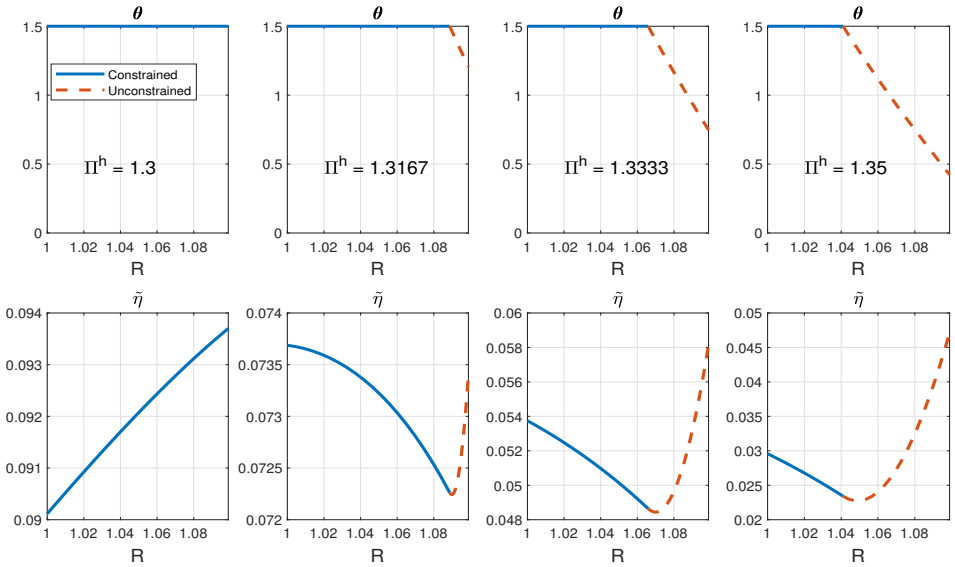
Time is discrete and infinite. We use recursive notations, i.e., let variable x denote x_t and let x_+ denote x_{t+1}. The economy is populated by a unit measure of entrepreneurs who may run firms. The economy also has a household sector and a government which runs flat-rate value-added tax policy[12] and credit policy.

Entrepreneurs. Entrepreneurs can choose among the risk-free project, the risky project, and risk-free savings. The firm technology is represented by

$$y = (zk)^{\alpha} (\ell)^{1-\alpha},$$

[12]VAT tax policy is equivalent to taxing labor income and capital income at the same rate. We also experimented with a lump-sum tax policy; the qualitative results are similar.

Figure 4: Effect of R on $\tilde{\eta}$ under Different Risk Profiles



Note: The top panel of each column shows the leverage associated with the risky project; the bottom panel of each column shows the threshold cost for risk-taking. Each column corresponds to a different risk profile (from the left to the right: $\Pi^h = 1.3, 1.32, 1.33,$ and 1.35), while the expected return $\Pi = p\Pi^h + (1-p)\Pi^l$ is kept the same across all panels.

where k is capital input, ℓ is labor input, z is risky if an entrepreneur chooses a risky project, and z is a constant number if the entrepreneur chooses the safe project.

Consider a firm with capital k. In the labor market, the firm hires workers at the competitive wage rate w, which leads to labor demand proportional to the firm's effective capital input. The firm maximizes its capital return by solving

$$\max_{\ell}\{(1-\tau^y)\,(zk)^{\alpha}\,(\ell)^{1-\alpha} - w\ell\}.$$

where τ^y is the VAT tax rate set by the government. The optimal labor choice is:

$$\ell = zk\left(\frac{(1-\alpha)}{w/(1-\tau^y)}\right)^{\frac{1}{\alpha}}. \tag{6}$$

Therefore, the firm's capital return (before debt repayment) is rzk, where r can be shown as

$$r \equiv (1-\tau^y)\alpha\left[\frac{(1-\alpha)}{w/(1-\tau^y)}\right]^{\frac{1-\alpha}{\alpha}}. \tag{7}$$

Denote the depreciation rate of capital as δ, and we specify $\{z^f, z^h, z^l\}$ as

$$\Pi^f = rz^f + 1 - \delta, \quad \Pi^h = rz^h + 1 - \delta, \text{ and } \Pi^l = rz^l + 1 - \delta. \tag{8}$$

In this way, the decision problem introduced below will be similar to the problems discussed in the simple model. We retain the one-period project and one-period debt structure from the previous two-period model. While it would be interesting to expand the model to incorporate long-term projects and long-term debt, doing so would introduce complexities like the influence of long-term interest rates on the valuation effect through discounting and the search-for-yield effect by selecting different asset classes and refinancing with new debt in the portfolio. To keep the analysis straightforward, we will continue with the one-period projects and credit contracts, as both effects have already been captured. Additionally, we view that the idiosyncratic implementation costs can capture the projects' varying terms and credit risks, as perceived by entrepreneurs.

Let $V(\omega)$ be the expected value of an entrepreneur with wealth ω before knowing the cost of implementing the risky project. We let $V^r(\omega, \eta)$ and $V^f(\omega)$ as before denote the value for an entrepreneur who chooses the risky and risk-free projects, respectively. Let $V^d(\omega)$ be the value of an entrepreneur who chooses to save in deposits with a return R^d. We have the following relationship:

$$V(\omega) = \max\{V^d(\omega), \mathbb{E}\left[\max\{V^r(\omega, \eta), V^f(\omega)\}\right]\},$$

where the expectation is taken over the distribution of η. Entrepreneurs decide first whether to save or become active entrepreneurs. If they decide to become active entrepreneurs, they can choose between the safe and the risky project. The value function of an entrepreneur who takes the risky project can be rewritten as

$$\begin{aligned} V^r(\omega, \eta) = \max_{s,b} \Big\{ & (1-\beta)\log(\omega - s) - \eta + \beta p V(\Pi^h(s+b) - Rb) \\ & + \beta(1-p)V(\Pi^l(s+b) - Rb) \Big\} \\ \text{s.t. } & 0 \leq b \leq \bar{\theta} s, \end{aligned}$$

where $(1-\beta)$ serves as a normalization. As before, the entrepreneur can invest with internal savings s, together with the borrowing b; next period, the entrepreneur earns either $\Pi^h(s+b)$ (if the project turns out to be productive) or $\Pi^l(s+b)$ (if the project turns out to be less productive). The interest payment is naturally Rb.

For flexibility in the model calibration, we assume a fixed (utility) cost η^f associated with running the risk-free project. The value function of an entrepreneur who takes a safe

project or saves in deposits can be rewritten as

$$
\begin{aligned}
V^f(\omega) &= \max_{s,b} \Big\{ (1-\beta)\log(\omega - s) - \eta^f + \beta V(\Pi^f(s+b) - Rb) \Big\} \\
&\quad \text{s.t. } 0 \le b \le \bar{\theta} s; \\
V^d(\omega) &= \max_{s} \Big\{ (1-\beta)\log(\omega - s) + \beta V(R^d s) \Big\},
\end{aligned}
$$

respectively. Naturally, when entrepreneurs choose the risk-free project, they will borrow up to the limit $\theta = \bar{\theta}$ if $\Pi^f > R$. The deposit rate R^d may differ from the lending rate R. We allow a constant $\tau \ge 0$ to be the difference between the deposit rate R^d and the lending rate R:

$$
R = R^d (1 + \tau). \tag{9}
$$

The parameter τ represents the intermediation costs or interest rate markup, making calibration below also more flexible.

The proof to the following Proposition 4 shows that the value functions satisfy the following forms $V^r(\omega, \eta) = \log(\omega) - \eta + v^r$, $V^f(\omega) = \log(\omega) - \eta^f + v^f$, and $V^d(\omega) = \log(\omega) + v^d$ for some endogenous v^r, v^f, and v^d. As in the two-period model, the saving function is linear in wealth $s = \beta\omega$, and we have a similar characterization of the threshold cost:

Proposition 4. *Define* $l^d \equiv \log R^d$, $l^r \equiv p \log\big(\Pi^h + \theta(\Pi^h - R)\big) + (1-p)\log\big(\Pi^l + \theta(\Pi^l - R)\big)$ *and* $l^f \equiv \log\big(\Pi^f + \bar{\theta}\big(\Pi^f - R\big)\big) - \beta^{-1}\eta^f$. *Those entrepreneurs with* $\eta \le \max\{\underline{\eta}, \tilde{\eta}\}$ *choose the risky project, where* $\tilde{\eta}$ *satisfies,*

$$
\tilde{\eta} = \beta(l^r - l^f), \tag{10}
$$

and the leverage $\theta = \min\{\theta^*, \bar{\theta}\}$. *Additionally, let* ϕ *be the fraction of entrepreneurs who choose to save in deposits, we have the following condition*

$$
\begin{cases}
\phi = 1 & \text{if } l^d > \mathbb{E}\left[\max\{l^r - \frac{\eta}{\beta}, l^f\}\right] \\
\phi \in (0,1) & \text{if } l^d = \mathbb{E}\left[\max\{l^r - \frac{\eta}{\beta}, l^f\}\right] \\
\phi = 0 & \text{if } l^d < \mathbb{E}\left[\max\{l^r - \frac{\eta}{\beta}, l^f\}\right]
\end{cases}. \tag{11}
$$

Proof. See Appendix B. □

The first branch of (11) with $\phi = 1$ should be excluded in equilibrium as there is no production in the economy. In equilibrium with $\phi \in (0,1)$, those entrepreneurs should be indifferent between saving via the risk-free deposits and becoming active entrepreneurs who

choose the two types of projects. That is, $l^d = \mathbb{E}\left[\max\{l^r - \frac{\eta}{\beta}, l^f\}\right]$, or more specifically

$$l^d = (1-F)l^f + Fl^r\beta^{-1}\int_{\underline{\eta}}^{\tilde{\eta}} \eta dF(\eta),$$

which governs how capital return rate r varies with R. Entrepreneurs who save in risk-free deposits can be considered unconstrained firms in practice. In the model, we can allow them to operate a different technology, which, in equilibrium, features the same rate of return as safe deposits.[13]

Households. Households are hand-to-mouth consumers and they supply labors to firms.[14] To focus on the firm side, we assume a Greenwood-Hercowitz-Huffman CRRA utility function and a representative household maximizes

$$(1-\beta)\sum_{t=0}^{\infty}\beta^t U\left(C_t - \frac{\kappa L_t^{1+\gamma}}{1+\gamma}\right),$$

where $U(.)$ is an increasing and concave function, C_t is households' consumption level, L_t is the labor supply, $\kappa > 0$ is disutility parameter, and γ is the inverse of the Frisch labor elasticity. The household chooses consumption and labor supply according to (where again we ignore the time subscript):

$$C = wL; \tag{12}$$

$$w = \kappa L^{\gamma}. \tag{13}$$

Project outcomes' association. The economy has infinitely many similar risky projects. Once a project is selected, each risk-taker is randomly assigned to one of these projects. The number of risky projects that achieve high returns at the aggregate level may display associations. These associations represent externalities when individual firms treat market prices (such as the wage and lending rates) and the success probability p as given. However, all market participants jointly determine these factors.

A positive association suggests that when one entrepreneur's risky project succeeds, it increases the likelihood of another entrepreneur's project succeeding. This implies that successful projects create knowledge spillovers, where one entrepreneur's success enhances

[13] With this modelling choice, $\phi = 1$ is possible in equilibrium. However, with $\phi = 1$, there is no risk-taking, and the model becomes a standard neo-classical model.

[14] The assumption is less strong than it seems. In incomplete-market models, the interest rate is lower than the time-preference rate. Households would like to borrow, but (unlike entrepreneurs) households do not have collaterals, so they will not borrow in equilibrium.

the chances of favorable outcomes for others. Similarly, a positive association means that if one entrepreneur's project fails, others are also more likely to face failures, aligning with the concepts of adverse risk spillover and conditional value at risk as discussed in Adrian and Brunnermeier (2016). Conversely, a negative association indicates the opposite relationship: the success of one project may crowd out opportunities for another, while the failure of one project provides lessons that help reduce the likelihood of failures in others.

Notice that we follow the Statistics literature in labelling the interdependence as an association instead of a correlation since the relationship between the outcomes of two entrepreneurs may not be linear. To characterize the association structure in Bernoulli distribution, we follow the approach of Conway-Maxwell Binomial/Poisson (CMB/CMP) distribution as in Shmueli et al. (2005) and Kadane (2016). To the best of our knowledge, this specification is new in a macroeconomic setup, and it modifies the baseline model with minimal departure because the individual problem stays the same. The specification also allows a straightforward aggregation. Specifically, we characterize the overall dependence structure through an exogenous parameter $\nu \in (0, +\infty)$. For CMB distribution:

$$Prob\{m = k\} = \frac{\binom{n}{k}^{\nu} p^k (1-p)^{n-k}}{\sum_{j=0}^{n} \binom{n}{j}^{\nu} p^j (1-p)^{n-j}} \equiv \frac{\binom{n}{k}^{\nu} p^k (1-p)^{n-k}}{D(\nu, p, n)},$$

where m is the total number of successes in n trials, $\binom{n}{k} \equiv \frac{n!}{k!(n-k)!}$, and we have used

$$D(\nu, p, n) \equiv \sum_{j=0}^{n} \binom{n}{j}^{\nu} p^j (1-p)^{n-j}.$$

The expression stands for the probability of k realization of high returns among n risky projects where the probability of realizing high return Π^h for each project is p. If $\nu = 1$, it becomes the standard binomial distribution. When $0 < \nu < 1$, risky projects' returns are positively associated, and when $\nu > 1$, risky projects' returns are negatively associated.[15] To see this, denote X_i as the random variable of result at i^{th} position. Consider $P(X_2 = z^l | X_1 = z^l)$, the conditional probability of the second project achieving a low

[15] See Kadane (2016) for more detailed discussions.

return conditional on the low return of the first project is[16]

$$\begin{aligned}\frac{P(X_2 = z^l, X_1 = z^l)}{P(X_1 = z^l)} &= \frac{P(X_2 = z^l, X_1 = z^l)}{P(X_2 = z^l, X_1 = z^l) + P(X_2 = z^h, X_1 = z^l)} \\ &= \frac{\frac{1}{D(\nu,p,2)}(1-p)^2}{\frac{1}{D(\nu,p,2)}(1-p)^2 + \frac{1}{2}\frac{1}{D(\nu,p,2)}\binom{2}{1}^{\nu} p(1-p)} \\ &= \frac{(1-p)}{(1-p) + 2^{\nu-1}p}. \end{aligned} \tag{14}$$

If $\nu < 1$, we have $2^{\nu-1} < 1$, then $P(X_2 = z^l | X_1 = z^l) > 1 - p$, implying a positive association. If $\nu > 1$, we have $2^{\nu-1} > 1$, then $P(X_2 = z^l | X_1 = z^l) < 1 - p$, implying a negative association. Similarly, $P(X_2 = z^h | X_1 = z^h) > p$ if $\nu < 1$ and $P(X_2 = z^h | X_1 = z^h) < p$ if $\nu > 1$.

The association of risky outcomes influences the macroeconomy. Denote the overall proportion of success to be $p^s(\nu)$ when the number of projects n goes to infinity. When $n \to \infty$ and p is small/large enough, we know that the CMB distribution converges to the CMP distribution (see Shmueli et al. (2005) and Daly and Gaunt (2016)) for $\nu > 0$:

$$Prob\{m = k\} = \frac{\frac{(\lambda)^k}{(k!)^\nu}}{\sum_{j=0}^{j=n} \frac{(\lambda)^j}{(j!)^\nu}},$$

where $\lambda = n^\nu p$ when p is small and $\lambda = n^\nu(1-p)$ when p is large. Additionally, the overall success probability $p^s(\nu)$ satisfies the following (see Appendix C.1 for more details):

$$p^s(\nu) = \begin{cases} p^{\frac{1}{\nu}} & \text{if } p < p^* \\ (1 - (1-p)^{\frac{1}{\nu}}) & \text{if } p \geq p^{**}, \end{cases} \tag{15}$$

for some $p^* > 0$ and close to zero and $p^{**} < 1$ and close to one, respectively. When p is small, high returns are relatively more scarce. The number of high-return realizations converges to CMP distribution. Considering the association, the social planner realizes that the overall probability of Π^h is $p^{1/\nu}$. When p is large, $1 - p$ is small and counting the low realizations Π^l will converge to the CMP distribution with $\lambda = n^\nu(1-p)$. The social planner realizes that the overall probability of Π^l is $(1-p)^{1/\nu}$. Finding p^* and p^{**} is a numerical question, which might be unnecessary; we obtain $p^s(\nu)$ via simulating CMB samples with a large n.

The central implication of CMB/CMP is that when individuals choose a project, they perceive the *ex-ante probability* of achieving a high return, Π^h, as p, accurately reflecting their own project's success likelihood. Each entrepreneur considers their firm to be small

[16]The result uses the property of the exchangeability: $P(X_2 = z^h, X_1 = z^l) = P(X_2 = z^l, X_1 = z^h)$.

and, therefore, does not believe its return is correlated with those of others. This assumption preserves the solution to each entrepreneur's decision problem. However, while project returns are correlated at the aggregate level, individuals understand this correlation/association and treat $p^s(\nu)$ (the average probability of success) as given when making their choices, assuming only others' projects are correlated/associated. In rational-expectation equilibrium, after we aggregate all individual choices, $p^s(\nu)$ is the consistent average probability of success across all projects. This modeling approach effectively captures a pecuniary externality: individuals overlook the impact of their own choices on the overall correlation/association of project returns. This externality influences aggregate credit demand and, consequently, the interest rate that clears the credit market.

We can write the aggregate expected return of risky projects $\Pi(\nu)$ as

$$\Pi(\nu) = p^s(\nu)\Pi^h + (1 - p^s(\nu))\Pi^l.$$

The aggregate expected return affects the social welfare as we aggregate entrepreneurs' choices. Therefore, (15) implies that $\Pi(\nu)$ becomes

$$\Pi(\nu) = \begin{cases} p^{\frac{1}{\nu}}\Pi^h + (1 - p^{\frac{1}{\nu}})\Pi^l & \text{if } p < p^* \\ (1 - (1-p)^{\frac{1}{\nu}})\Pi^h + (1-p)^{\frac{1}{\nu}}\Pi^l & \text{if } p \geq p^{**} \end{cases}. \tag{16}$$

To illustrate, first note that $\Pi = \Pi(1)$ represents no association effect. Now suppose there is a positive association, i.e., $\nu < 1$. When $p > p^{**}$, $p^s(\nu)$ takes the second branch of (15). That is, $p^s(\nu) = 1 - (1-p)^{\frac{1}{\nu}} > p$ and thus $\Pi(\nu) > \Pi$. When $p < p^*$, $\Pi(\nu) < \Pi$ holds similarly. Intuitively, when p is large, high returns are relatively more abundant, and the knowledge spillover dominates the adverse risk spillover. Therefore, the overall probability of success viewed by the social planner becomes higher. That is why the overall aggregate expected return is above $\Pi(1)$. Note that one can verify that when p is large, $\nu \to 0$ means $p^s(\nu) \to 1$ because of the strong positive association, and $\nu \to +\infty$ means $p^s(\nu) \to 0$ because of the strong negative association.

Our calibration below suggests a relatively large p, leading to a social return $\Pi(\nu)$ exceeding the private return $\Pi(1)$. This finding aligns with a large literature on R&D, demonstrating that the social return rate to R&D is typically higher than the private rate of return (see Griffith (2000)).

The (consolidated) government agency. We consider a consolidated government which conducts joint monetary and fiscal policies with the following budget constraint:

$$G + R^d B_{-1} = T + B. \tag{17}$$

The expenditure side includes government spending G and debt repayment $R^d B_{-1}$ accumulated previously. For simplicity, we assume that the government can borrow without the intermediation cost so that the repaying interest rate is also R^d because of no-arbitrage. The revenue side includes tax $T = \tau^y Y$ (where Y is the aggregate output shown below) and the newly issued debt B.

B represents a broad liquidity policy, as we consider the government agency as a consolidated identity that includes a monetary authority, a fiscal authority, and financial intermediaries, which lend to the firm sector. Therefore, B is the net debt position of the joint agency; when $B < 0$, the consolidated identity lends to the firm sector. Alternatively, if we interpret the consolidated agency with only monetary and fiscal authorities, B can be considered an outcome of the quantitative easing (QE) policy. That is, we simplify the institutional details of credit/liquidity policy to focus on the equilibrium effect of the interest rate on entrepreneurs' risk-taking and how the association of projects affects the optimal interest rate.

The government agency sets (G, R) every period. Then, $R^d = R/(1+\tau)$ and B will be determined in the credit market, while τ^y will be pinned down by the government budget constraint.

4.2 Equilibrium

Denote the total wealth of private agents as Ω. Wealth accumulation implies:

$$\Omega_+ = \beta\Omega\left\{\phi R^d + (1-\phi)\left\{ \left[1-F(\tilde{\eta})\right]\left[\Pi^f(1+\bar{\theta}) - R\bar{\theta}\right] + F(\tilde{\eta})\left[\Pi(\nu)(1+\theta) - R\theta\right] \right\}\right\}, \tag{18}$$

where again ϕ is the probability of saving entrepreneurs (note: they save with the rate R^d). The next period's wealth Ω_+ will come from three sources. All entrepreneurs put a β fraction of their wealth aside, which explains $\beta\Omega$. The group of entrepreneurs have a ϕ fraction of savers (who have a return R^d) and $(1-\phi)(1-F(\tilde{\eta}))$ fraction implementing the safe project with a return $\Pi^f(1+\bar{\theta}) - R\bar{\theta}$.[17] The remaining entrepreneurs take risks, and the project return after leverage is $\Pi(\nu)(1+\theta) - R\theta$.

Capital productivity is endogenous in the aggregate because of credit-induced investment in different technologies. That is, capital reallocation linked with credit conditions determines the endogenous productivity of capital. Define Z as the endogenous capital efficiency unit that takes into account the technology choices of entrepreneurs:

$$Z = z^f(1-F)(1+\bar{\theta}) + \bar{z}F(1+\theta), \text{ where } \bar{z} = p^s(\nu)z^h + (1-p^s(\nu))z^l. \tag{19}$$

[17] Here, we focus on the case of $\Pi_f > R$. The numerical exercises below allow $\Pi_f \leq R$, in which case $\theta = 0$.

Then, the aggregate output can be written as

$$Y = \int (z_i k_i)^{\alpha} \ell_i^{1-\alpha} di = \left((1-\phi) Z \beta \Omega\right)^{\alpha} L^{1-\alpha}. \tag{20}$$

Notice that each firm produces $z_i k_i / \alpha$ in which $z_i k_i$ is retained. Additionally, $(1-\phi) Z \beta \Omega$ can be regarded as the effective capital stock used in production,

To define equilibrium, we look at market clearing conditions for labor and for credits. For the labor market, hours of work L should be the labor supply from households, so that the "rental rate" r can be written in a conventional way, which is the marginal product of effective capital stock (using L in the product)

$$r = \alpha \left((1-\phi) Z \beta \Omega\right)^{\alpha-1} L^{1-\alpha}. \tag{21}$$

For the credit market, the consolidated agency conducts the credit/liquidity policy, determining the market-clearing interest rate. As mentioned, we can think of the consolidated policy maker conducting an interest rate policy by choosing R. An interest rate R corresponds to a particular debt/liquidity policy B according to the market clearing condition. The total liquidity savers provide at period t is $\phi \beta \Omega$. The total credit demanded by entrepreneurs who borrow is $\left[(1-F(\tilde{\eta}))\bar{\theta} + F(\tilde{\eta})\theta\right](1-\phi)\beta\Omega$: a fraction $[1-F(\tilde{\eta})](1-\phi)$ of whom use the safe technology and borrow to the credit limit implied by $\bar{\theta}$ and a fraction $F(\tilde{\eta})(1-\phi)$ of whom use the risky technology and leverage to $\theta \leq \bar{\theta}$. Therefore, the clearing of credit market implies

$$\left[(1-F(\tilde{\eta}))\bar{\theta} + F(\tilde{\eta})\theta\right](1-\phi)\beta\Omega = \phi\beta\Omega - B. \tag{22}$$

Definition. Given states including the entrepreneurs' wealth Ω, a government spending G, and an interest-rate policy target R, a recursive competitive equilibrium is a collection of variables $\{L, C, \tilde{\eta}, \theta, \phi, \Pi^h, \Pi^l, \Pi^f, r, w, \tau^y, B, R^d, \Omega_+\}$ such that

- Households supply labor according to (13) ; their budget constraint (12) holds;
- $\tilde{\eta}$ and θ solve the entrepreneurs' problem with risk-taking choice and leverage choice as shown in Proposition 4;
- project returns (8) are satisfied, with r given by (7);
- The wealth dynamics (18) holds;
- The consolidated government budget constraint is satisfied, i.e., (17) holds with tax revenue $T = \tau^y Y$ where output is defined in (20);
- The credit market clears, i.e., (22) holds with (11) satisfied, and the deposit rate R^d is determined by (9);

- The labor market clears, i.e., (21) holds.

One can immediately see that the measure of association, ν, matters. It affects the market demand for credit/liquidity and, therefore, the tax revenue T. For example, when $v < 1$ and p is small, according to (16) we know that $\Pi(\nu) < \Pi(1) = \Pi$ and the tax revenue T is lower. We will also show how the outcome association affects other equilibrium variables and social welfare. We focus on steady-state analysis in the quantitative exercises below, as they are simple and already contain many non-trivial results. The steady-state social welfare of entrepreneurs $V(\Omega)$ and households W can be seen as follows. Given the value functional form and the nature of the steady state economy, the social welfare measure, which is related to the present value of utility of consumption from all groups, thus becomes (see Appendix C.3 for details) :

$$V(\Omega) + W = \log(\Omega) + \tilde{V} + U\left(C^h - \frac{\kappa L^{1+\gamma}}{1+\gamma}\right) + \text{constants},$$

where Ω, $\tilde{V}$, C^h, and L are all endogenous. $\tilde{V}$ includes the base return (on the risk-free project or the safe deposits) and the relative value gain from choosing the risky project.

5 Macroeconomic Effects of Target Interest-rate Policy

Using the macroeconomic environment introduced above, we now analyze the aggregate steady-state effects of varying the (real) interest rate target. Note that the consolidated government agency implements the interest rate policy (with government-debt management in the background), and it can thus be considered a joint monetary-fiscal policy outcome in practice.

To clarify, we do not consider external factors (such as demographic trends) that may affect the real interest rate, potentially through impacts on the subjective discount factor, β. Here, we treat the long-term interest rate target as given. Instead, our focus is on the interest rate component, which can be influenced by monetary and fiscal policy. For instance, this can be managed through medium-term targets for inflation and nominal interest rates. Our analysis includes questions such as whether policy should encourage risk-taking and how the optimal policy is shaped by factors like the credit limit and the degree of outcome association, ν.

5.1 Parameterization

It becomes too complex to study the optimal interest rate fully analytically. We choose to calibrate the model and assess the policy effects. Table 2 reports the calibrated parameters.

Some parameters are exogenous. The capital share is set to a conventional value $\alpha = 0.33$. The inverse of the Frisch elasticity of labor supply γ is set to $1/1.5$, also a conventional number for macroeconomic models. We assume a CRRA utility for U, and the relative risk aversion is set to 2, so households are slightly more risk-averse than entrepreneurs. Note this number does not affect macroeconomic variables; it only slightly influences the aggregate welfare comparison. The discussion below illustrates the key steps in calibrating other parameters.

Table 2: Calibration

	Value	Explanation/Target		Value	Explanation/Target
β	0.9437	Discount factor	z^f	1	Normalization
γ	1/1.5	Inverse Frisch elasticity	z^h	1.5989	ROE top 80th per.
κ	1.6179	Hours 0.33	z^l	0.0785	ROE bottom 20th per.
α	0.33	Capital share	p	0.6803	ROE std. dev.
δ	0.0938	Investment-to-output 17%	μ	-3.1785	Debt-to-output ratio: 65.2%
τ	0.02	Interest-rate spread	σ	0.0031	Elasticity of ROE: 3.71
$\bar{\theta}$	0.5600	Median leverage ratio	G	0.0828	Gov-spending-to-output 18%
η^f	0.0305	Median ROE	R	1.0370	Prime rate

κ, which governs the disutility of labor, is calibrated to hit the labor hours such that $L = 0.33$ after normalizing total hours to unity. The model targets government spending to GDP ratio G/Y as 18%, which pins down G. The capital depreciation rate δ is calibrated so that the investment-to-output ratio is 17%.

We choose the real prime rate to be the status quo interest rate R, since there is no default in the model and the prime rate is a benchmark interest rate used for high-quality borrowers. We use the period 1984-2020 (see Appendix A for data description). The average annualized real (gross) prime rate during this period is 1.037, while the average annualized gross federal funds rate during this period is 1.017. Therefore, the interest-rate markup τ is set to 2%. Also, we set the discount factor as $\beta = 0.9437$ such that the gap between $1/\beta$ and R is 2.2%, which targets the average gap between BAA corporate bonds and 10-year Treasury bonds in the sample period. This gap reflects reasonable liquidity and risk premia not modelled in the economy. Also, the model implies that $(1 - \beta) \approx 5.6\%$ of net worth is used for entrepreneurs' consumption, which should be interpreted as paying dividends or rewarding managers in practice. Notice that the average S&P 500 dividend yield in the sample period is 2.4%, which implies the rest (3.2%) could be considered as being used to incentivize managers, which seems conservative.[18]

[18]The estimates of CEO pay to company equity ratios are around 3% to 12% according to the literature about CEO compensation and equity performance.

The baseline sets $\nu = 1$ (no association among projects). In practice, both the positive and negative associations explained above are likely in different industries. But we will illustrate the macro effects of different degrees of association.

Next, we use the top $80th$ percentile value (Π^{80th}) and the bottom $20th$ percentile value (Π^{20th}) of cross-sectional ROE to represent the net leveraged return for high realization and low realization, i.e.,

$$\Pi^{80th} \equiv (1+\bar{\theta})\Pi^h - R\bar{\theta}; \quad \Pi^{20th} \equiv (1+\bar{\theta})\Pi^l - R\bar{\theta}.$$

As a normalization, we map the ROE's median return to the net leveraged risk-free return $\Pi^m \equiv (1+\bar{\theta})\Pi^f - R\bar{\theta}$. The top $80th$ percentile, bottom $20th$ percentile, and median net returns for the period of 1984-2020 are 18%, -15%, and 5%, respectively, so that $\Pi^{80th} = 1.18$, $\Pi^{20th} = 0.85$, and $\Pi^m = 1.05$. The benchmark exercise normalizes $z^f = 1$ and assumes that some entrepreneurs save. Then, we obtain η^f from the indifference condition, and we obtain r as a function of $\bar{\theta}$:

$$r = \frac{\Pi^m + R\bar{\theta}}{1+\bar{\theta}} - (1-\delta).$$

Additionally, z^h and z^l are also functions of $\bar{\theta}$ by substituting the above expression for r to the expression for Π^{80th} and Π^{20th}:

$$z^h = \frac{1}{r}\left[\frac{\Pi^{80th} + R\bar{\theta}}{1+\bar{\theta}} - (1-\delta)\right]; \quad z^l = \frac{1}{r}\left[\frac{\Pi^{20th} + R\bar{\theta}}{1+\bar{\theta}} - (1-\delta)\right].$$

Notice that the leverage parameter $\bar{\theta}$ is needed for r, z^h, and z^l. It is set according to the firm leverage ratio used in Section 2. The median of the leverage ratio is 0.56. Then, r, z^h, and z^l are identified immediately as shown above.

We assume the cost distribution follows a log-normal distribution with the mean as μ and variance σ^2 after log transformation. We calibrated other distributions with positive support and found it is not crucial for the qualitative conclusion below. Therefore, the remaining key parameters are: $\{\mu, \sigma, p\}$. They are chosen to match the three moments below.

(1). The model targets the debt-to-output ratio 0.652 as in the data[19] after we use the total debts of the non-financial businesses.

(2). Assuming risk-taking firms are financially constrained in the status quo, we have the standard deviation of the overall return in the model

$$\sigma_I = \sqrt{F(\tilde{\eta})}(1+\bar{\theta})\sqrt{p(1-p)}\left(\Pi^h - \Pi^l\right) = \sqrt{F(\tilde{\eta})}\sqrt{p(1-p)}\left(\Pi^{80th} - \Pi^{20th}\right),$$

[19] In the model, debt-to-output ratio is $\frac{D}{Y} = \frac{\left[(1-F(\tilde{\eta}))\bar{\theta} + F(\tilde{\eta})\bar{\theta}\right]}{\frac{r}{\alpha}\left[z^f(1-F)(1+\bar{\theta}) + \bar{z}F(1+\bar{\theta})\right]}$. In the data, we use the total debt of the non-financial businesses over the total output of the non-financial businesses. Note that $\tilde{\eta}$ is solved by (10).

corresponding to the average cross-sectional standard deviation (0.13) of ROE in the sample.

(3). Its sensitivity to the interest rate becomes

$$\frac{\partial \sigma_I}{\partial R}\frac{R}{\sigma_I} = \left[\frac{1}{2}\frac{f(\tilde{\eta})}{F(\tilde{\eta})}\frac{\partial \tilde{\eta}}{\partial R} + \frac{1}{\Pi^h - \Pi^l}\left(\frac{\partial \Pi^h}{\partial R} - \frac{\partial \Pi^l}{\partial R}\right)\right] R, \tag{23}$$

corresponding to 3.71 in the data. Note that when the financing constraint is binding, the derivative $\partial\tilde{\eta}/\partial R$ can be computed as

$$\frac{\partial \tilde{\eta}}{\partial R} = \begin{cases} \beta p \frac{1}{\Pi^{80th}}\left(\frac{\partial \Pi^h}{\partial R} + \left(\frac{\partial \Pi^h}{\partial R} - 1\right)\bar{\theta}\right) \\ +\beta\left(1-p\right)\frac{1}{\Pi^{20th}}\left(\frac{\partial \Pi^l}{\partial R} + \left(\frac{\partial \Pi^l}{\partial R} - 1\right)\bar{\theta}\right) \\ -\beta\frac{1}{\Pi^m}\left(\frac{\partial \Pi^f}{\partial R} + \left(\frac{\partial \Pi^f}{\partial R} - 1\right)\bar{\theta}\right) \end{cases}.$$

Since this outcome is unique to our model, we elaborate the procedure. First, as in the calibration above, entrepreneurs are indifferent between saving in safe assets and implementing projects when $\phi \in (0,1)$. That is, we have the second branch of (11), and with some algebra we obtain

$$\frac{\partial \Pi^f}{\partial R} = \frac{\partial r}{\partial R} = \frac{\frac{1}{R^d}\frac{1}{1+\tau} + \bar{\theta}\left[(1-F)\frac{1}{\Pi^m} + F\left(p\frac{1}{\Pi^{80th}} + (1-p)\frac{1}{\Pi^{20th}}\right)\right]}{(1+\bar{\theta})\left[(1-F)\frac{1}{\Pi^m} + F\left(p\frac{z^h}{\Pi^{80th}} + (1-p)\frac{z^l}{\Pi^{20th}}\right)\right]}$$

and $\partial\Pi^h/\partial R$ and $\partial\Pi^l/\partial R$ follow immediately. Second, we examine when risk-taking entrepreneurs may not be financing-constrained, but the result points to the constrained scenario. It turns out that p is about 0.68, meaning that the high return is more likely observed, so being financially constrained is more likely when entrepreneurs take risks.

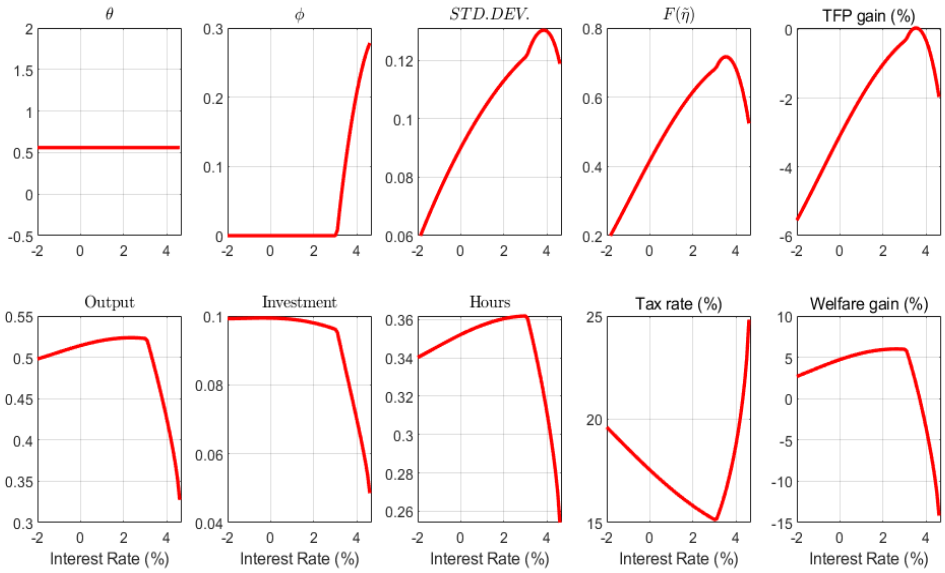
We have $\phi = 0.153$ in the calibrated economy, implying that 84.7% of firms are financing-constrained. The fraction of constrained firms is not crucial for at least two reasons. First, the consolidated "government agency" in equilibrium saves, which can include savings from unconstrained firms and households in reality. Second, we tried adding another group of entrepreneurs with a different success probability p, so their firms are unconstrained if they choose the risky project. By varying the relative population of this group, we can hit a certain target fraction of unconstrained firms in the economy. Still, we have the non-monotonic result, as illustrated in the examples of the two-period model. The optimal interest rate levels shown below differ, but the qualitative conclusions obtained below are robust.

5.2 The Effects of Interest Rate Policy

Suppose the policymaker implements different interest rate levels by adjusting B (Figure 5).

This exercise reveals a curved relationship between the risk-taking threshold $\tilde{\eta}$ and the

Figure 5: The Effects of Interest Rate Policy



Note: This figure plots equilibrium variables as functions of the (net) interest rate $R-1$ at the calibrated parameters. The variables shown are the optimal leverage θ, the share of saving firms ϕ, the standard deviation of project returns, the share of risk-taking firms among active firms $F(\tilde{\eta})$, TFP, output, investment, hours of work, tax rate, and the consumption-equivalent (in the calibrated economy) welfare gain.

interest rate. The hump-shaped relationship also applies to the standard deviation of investment returns as the interest rate fluctuates. As the interest rate increases, saving in secure deposits becomes more appealing. The proportion (ϕ) of savers who provide liquidity weakly rises with the interest rate. It's worth noting that the total factor productivity (TFP), represented as

$$TFP \equiv Z^{\alpha} = \left[z^{f}(1-F(\tilde{\eta}))(1+\bar{\theta}) + \bar{z}F(\tilde{\eta})(1+\theta)\right]^{\alpha},$$

also inherits the humped shape of risk-taking thanks to the presence of $F(\tilde{\eta})$. At the calibrated interest rate level (3.70%), TFP is close to its peak, but TFP is maximized at the interest rate level of 3.55%.

A lower interest rate relaxes borrowing conditions and encourages investment. Still, it may also lead to inefficiency. The fall in productivity at the aggregate level is caused by less risk-taking and falling work hours, which eventually reduces output. In this sense, expansionary monetary policy can eventually become contractionary. In other words, the conventional "IS" curve slopes downward when the interest rate is not too low, but once the interest rate falls below 2%, the output starts to fall quickly, and the IS curve becomes upward-sloping. For example, compared to the highest output (roughly when the rate is around 2.5%), a -2% rate implies almost a 5% fall in output.

To be more precise, when the interest rate falls in the range considered, investment goes up because it is easier to invest when financing costs drop. The issue is that productivity-enhancing risk-taking may also fall when financing costs drop, leading to a fall in hours and output. The second effect can only dominate when productivity distribution is endogenous, thanks to endogenous risk-taking. This effect is related to the usual capital misallocation, but the endogenous productivity distribution is essential.

When assessing the policy effect on the social welfare of a target economy, we compute the corresponding consumption equivalence measure ψ of the baseline calibrated economy. Given that entrepreneurs' consumption is $(1-\beta)$ fraction of their wealth, we calculate the gain/loss of wealth of entrepreneurs (Ω_{base}) and the consumption of households (C^h_{base}) such that the baseline economy with $(1+\psi)\Omega_{base}$ and $(1+\psi)C^h_{base}$ (and everything else stays as their calibrated level) has the same welfare measure as in the target economy.

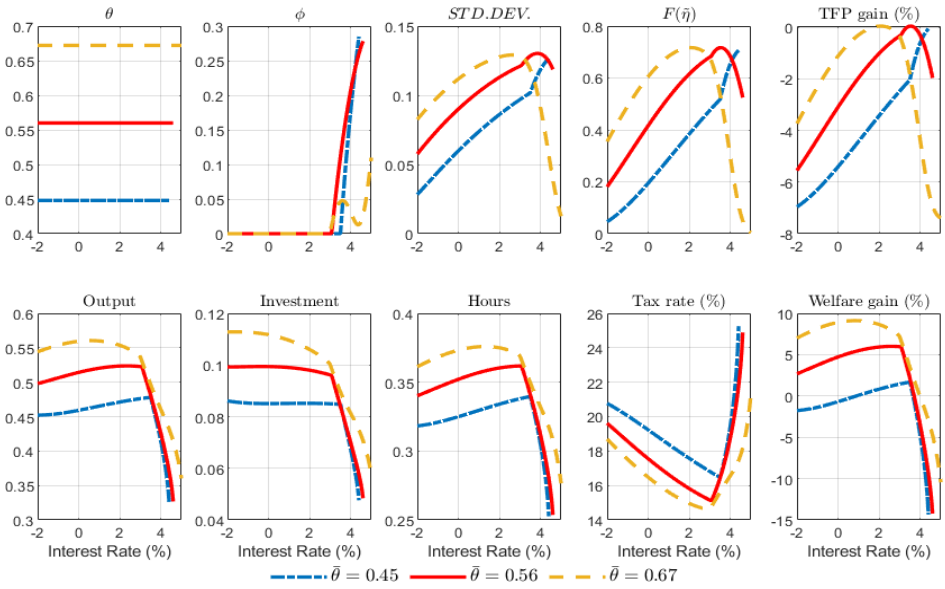
Figure 5 shows that welfare closely tracks output. When the interest rate rises, the government borrows more (or saves less) since firms borrow less and/or save more. As explained above, initially, risk-taking behaviour increases with the interest rate. Productivity rises, and the economy expands, so the government can reduce the tax rate given a larger tax base. Eventually, the government has to raise more taxes from private agents to satisfy the government budget constraints. Risk-taking also falls significantly when the interest rate becomes high enough. Therefore, the social welfare eventually falls, and the social welfare also displays a hump shape, suggesting the optimal interest rate being around 2.7%.

Remark: Our theory, therefore, implies that excessively low interest rates can be detrimental to output, productivity, and overall welfare. While the traditional "Keynesian" mechanism remains present (especially for investment), a sharp decline in interest rates can quickly dampen risk-taking incentives, ultimately harming productivity and social welfare.

The role of leverage limit. Notice that financial regulation (e.g., macro-prudential policy and systematic risk oversight) can significantly affect the level of credit limit. The borrowing limit influences the interest rate policy's effectiveness, which can be seen by assessing interest rate policy with different $\bar{\theta}$.

When $\bar{\theta}$ declines by 20% from the calibrated value of $\bar{\theta} = 0.56$ to $\bar{\theta} = 0.45$ (illustrated by the blue dash-dotted lines in Figure 6), the proportion of risk-taking entrepreneurs, $F(\tilde{\eta})$, monotonically increases with the interest rate. This finding reaffirms the earlier observation that the valuation effect dominates, particularly when leverage is low. As a result, the equilibrium proportion of savers, ϕ, decreases for most R as the demand for liquidity diminishes due to the reduction in external financing limit, $\bar{\theta}$. Only when the interest rate is above 4.2%, ϕ increases above the case when $\bar{\theta} = 0.56$. Simultaneously, TFP mostly decreases with declining leverage, resulting in a downward shift in output.

Figure 6: Interest Rate Effects under Alternative Financing Constraints



Note: This figure shows the equilibrium outcomes under three possible leverage limits of $\bar{\theta}$. Variables are the same as in Figure 5.

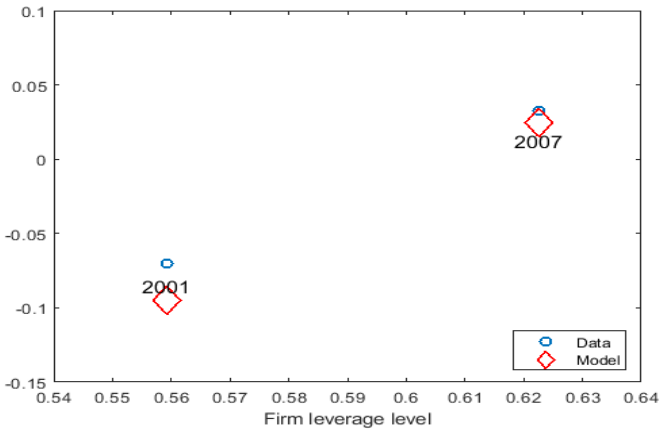
For comparison, the yellow dashed lines show a 20% increase in $\bar{\theta}$ to 0.67. With this adjustment, $F(\tilde{\eta})$ now displays a downward slope over a broader interest rate range, reflecting the substitution effect's dominance when leverage is high. Specifically, when $\bar{\theta}$ is sufficiently elevated, a rise in the interest rate discourages risk-taking, consistent with Proposition 2. Additionally, at this higher level of $\bar{\theta}$, the risk-taking incentive (captured by $F(\tilde{\eta})$) decreases when the interest rate is high and increases when the interest rate is low. This shift is illustrated by the leftward movement of the $F(\tilde{\eta})$ curve. Intuitively, at a low interest rate, the debt servicing costs are low, so increased leverage significantly impacts the leveraged Sharpe ratio, greatly raising the opportunity cost of choosing the safe project and thereby promoting risk-taking. This observation is in line with the corollary of Proposition 2 (see Corollary B.2 in the Appendix).

TFP generally rises for those R levels under which increased risk-taking accompanies higher leverage, enabling more resources to be allocated to entrepreneurs. While higher leverage itself can potentially boost TFP, the overall shape of TFP is primarily influenced by F, which is highly sensitive to changes in the interest rate. Therefore, even if leverage constraints are relaxed, TFP can still decline at a given interest rate (as described for F above). Notice that higher leverage still positively impacts output, investment, hours worked, and public finance. As a result, the need for entrepreneurs to save decreases, leading to a reduction in ϕ.

Remark: As in the 2-period model, we have attempted to endogenize $\bar{\theta}$ by introducing, for example, a limited commitment framework where entrepreneurs can abscond with their firm's value. The fundamental hump-shaped relationship between risk-taking and the interest rate remains intact. This is expected, as even if a decrease in the interest rate causes $\bar{\theta}$ to rise, Figure 6 already contains the message that the additional leverage effect shifts the risk-taking curve $F(\tilde{\eta})$ (and the TFP curve) leftward, thus preserving the overall hump shape in the relationship.

Cross validation. The model's quantitative implications regarding varying leverage exhibit alignment with the data. Figure 7 illustrates this comparison for 2001 and 2007 with comparative-static analysis. Given that we only fit the model with relevant interest rates and leverages, keeping other parameters fixed, a perfect fit should not be expected. The association of project returns, which will be discussed later, can improve the fit. However, the results below already demonstrate that the impact of an interest rate cut significantly relies on the borrowing constraint.

Figure 7: Changes of Standard Deviations of Cross-sectional ROE



Note: The model results are calculated with leverage and interest rates from the data, keeping other parameters fixed. The left points are for 2001 (with a lower leverage ratio), and the right points are for 2007 (with a higher leverage ratio). The data counterparts are 5-year moving averages.

We focus on the model's steady-state outcomes by using medium-term inputs and comparing them to medium-term outputs. Specifically, we fit the model using 5-year moving averages of leverage and compare the model's cross-sectional standard deviation of ROE to the 5-year moving average observed in the data. Additionally, as noted in Section 2, both the levels and changes in interest rates were similar in 2001 and 2007. These two years are thus natural main targets.

The Cleveland rate was approximately 1.7% in 2001, and between 2001 and 2002, it

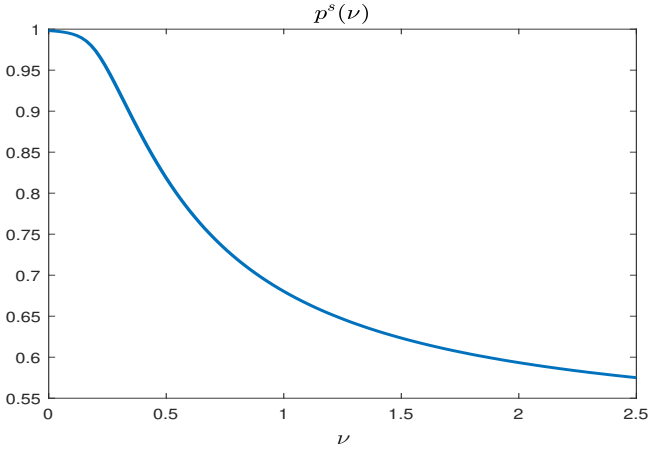
dropped by 142 basis points. The average leverage during the 2001-2006 period was around 0.559, with the cross-sectional standard deviation of ROE being 7% lower than in 2000. Using the steady-state model with the same leverage, interest rate levels, and the observed rate cut predicts a 9.55% decrease in standard deviation. While the model slightly overshoots, it is directionally consistent with the data.

In 2007, the Cleveland rate was approximately 1.8%. Between 2007 and 2008, it fell by 150 basis points, making the interest rate levels and cuts in 2001 and 2007 comparable. However, the 5-year moving average of leverage in 2007 was around 0.623, higher than in 2001. Using these values in the steady-state model predicts a 2.43% increase in the cross-sectional standard deviation of ROE. In the data, the moving average of the cross-sectional ROE standard deviation rose by 3.24% compared to 2006, showing that the model's prediction closely aligns with the observed data.

The role of association of project outcomes. We now examine scenarios involving positive association (e.g., $\nu = 0.85$) and negative association (e.g., $\nu = 1.15$). We interpret changes in ν as exogenous shocks to the economy.

Notice that when projects exhibit positive correlation and p is relative high, as discussed in Section 4, we know that $\Pi(\nu) > \Pi$. The reason is that the knowledge spillover of the higher return dominates the risk spillover effect of the low return when the high return is relatively more abundant. The opposite is true if project outcomes are negatively associated. The "social" probability of success, $p^s(\nu)$ is thus a decreasing function of ν, as shown by the simulation result in Figure 8 using the benchmark $p = 0.6803$.

Figure 8: "Social" Probability of Success under Different ν.



Note: The simulated probability of success $p^s(\nu)$ from CMB as a function of ν.

As explained before, we follow Kadane (2016) and refer to dependence as "positive

associations" and "negative associations". To further understand this observation, we obtain the correlation of two consecutive draws and compute the correlation of the two draws being z^l (see Appendix C.2 for more details). Under benchmark parameters, $\nu = 0.85$ corresponds to a correlation of 0.045, and $\nu = 1.15$ corresponds to a correlation of -0.045.[20] For $\nu = 0.85$, the pair-wise correlation (0.045) may appear small initially, but the cumulative effect (or the association of project returns) across millions of projects cannot be seen from that. The "social" probability of success thus better reflects this cumulative effect and $p^s(0.85) = 0.71$, about 4.4% more than the independent probability p. Another way to see this is to examine the conditional probability. When $\nu = 0.85$, (14) implies that the probability of the failing second project conditioning on the failing first project is about 0.34, about 7.2% more than the independent probability of failing $1-p$. Therefore, pair-wise correlation does not serve the purpose of measuring the overall association effect. Thus, we use ν, and notice that the social probability $p^s(\nu)$ is close to unity when ν approaches zero according to Figure 8.

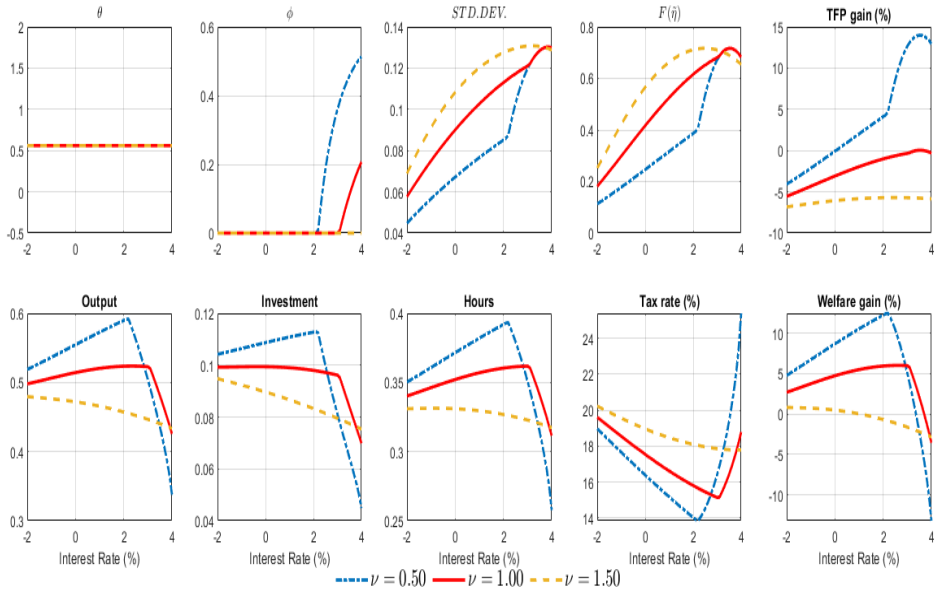
When project outcomes become positively associated (the blue dashed-dotted lines in Figure 9), the social return assigns a higher probability $p^s(\nu) > p$ to Π^h. However, individuals still maintain belief in the probability p for their own project, and they understand that aggregate capital efficiency unit is higher because of (19), putting downward pressure on the marginal product r according to (21). This channel shrinks the expected return gap between the risky project and risk-free project. As a result, the risk-taking measure $\tilde{\eta}$ decreases. With a lower r, the gross return of the safe project Π^f also diminishes, making it also less appealing. We thus observe a weakly higher ϕ across all interest rate levels. Still, TFP rises because of the positive association and the relatively large p, leading to higher investment, output, and hours for low values of interest rate.

Positive associations boost entrepreneurs' welfare compared to the baseline (the solid lines Figure 9). As more firms save in safe deposits, greater government borrowing is required, leading to higher taxes and reduced social welfare at higher interest rates. This results in a lower optimal interest rate than in the baseline scenario. The optimal rate must balance the welfare of entrepreneurs with projects and those saving in deposits. Note that the optimal welfare with a positive association exceeds the optimal welfare under baseline parameters due to the higher social return.

As ν increases, the projects negatively associate with each other, and we observe the opposite reaction compared to the above. Notably, more risk-taking is observed for most levels of interest rate. This excessive risk-taking comes from lower capital efficiency units Z, putting upward pressure on the marginal product r. Since individuals believe in the same probability of success p, the risky project becomes more attractive as ν increases. Also,

[20] The correlation further increases as ν decreases. For instance, $\nu = 0.4$ yields a correlation of 0.18, $\nu = 0.2$ yields a correlation of 0.24, and $\nu = 0$ results in a correlation of 0.29.

Figure 9: Macroeconomy under Different ν.



Note: This figure plots the equilibrium outcomes for different levels of project correlation. $\nu = 1$ is the benchmark case (no correlation) as shown in Figure 5. $\nu < 1$ means project outcomes are positively correlated; $\nu > 1$ means project outcomes are negatively correlated.

notice that since the interest rate cannot exceed β^{-1}, ϕ eventually remains at zero across all interest rate levels, reducing savings and investment, which puts another upward pressure on r. As a result, the risk-taking measure $\tilde{\eta}$ increases. This effect is similar to reducing R while keeping r constant: for any given R, the values of Π^h, Π^l, and Π^f all increase. Consequently, the substitution effect dominates for large interest rate R, as Proposition 2 illustrates. This dominance widens the downward-sloping part of $F(.)$ (the dashed yellow line), representing the fraction of risk-takers.

With the projects negatively associated, the above discussion suggests excessive risk-taking in the private sector because of the externality. Therefore, given the hump-shaped risk-taking behavior when we vary R, the interest rate level can be raised or cut to reduce excessive risk-taking. Because of continuity, for the small negative association, we should expect the optimal interest rate to rise (as the opposite of the case of a positive association) to deter excess risk-taking. However, when the negative association is more severe, the benefit of correction in externality by raising interest rate is outweighed by the fall in investment and output because of the high financing cost. A low rate is optimal. Therefore, for $\nu = 1.5$, both the output-maximizing and welfare-maximizing interest rates are significantly lower than the rate that maximizes the risk-taking fraction of entrepreneurs. We find that $R = 1.026$ maximizes $F(\tilde{\eta})$ and $R = 1.032$ maximizes the standard deviation. However, even with an interest rate as low as -2%, output still does not reach its peak.

To further illustrate, note that in cases of no association and positive association (repre-

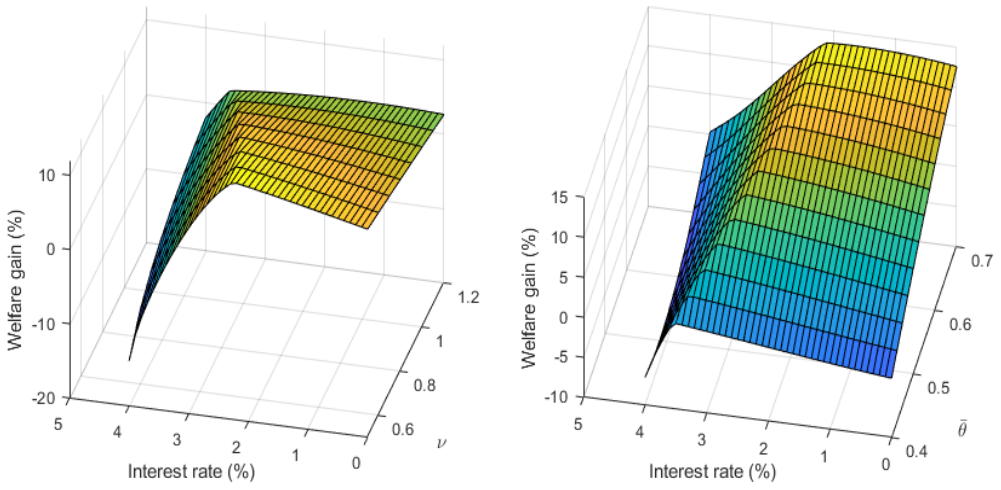
sented by the solid and dashed-dotted lines), risk-taking probability $F(\tilde{\eta}) > 0$ at the optimal interest rate level. The optimal rate should place the economy at the intersection of the flatter section and the steeper section of the $F(\tilde{\eta})$ curve. The flatter section features $\phi = 0$, and the steeper section features $\phi > 0$. In this way, risk-taking is highly responsive to interest rate changes. Of course, when project outcomes are only slightly negatively associated, the policymaker should raise the interest rate. However, if the negative association is severe (e.g., $\nu = 1.5$), the flatter section of $F(.)$ spans over all possible interest rates below $1/\beta$. Avoiding risk-taking altogether and focusing on the level of investment thus may be better for society, as the yellow dashed line shows. We examine the implication of a continuum of ν below.

5.3 Further on Optimal Interest Rate

Now, we expand the range of project associations and credit limits.

The left panel of Figure 10 illustrates the welfare gain as we vary both the interest rate and the association measure, with the credit limit fixed at the calibrated level $\bar{\theta} = 0.56$. Welfare remains similar when the interest rate is below 3%. Although the optimal rate is 2.7% for $\nu = 1$, lowering it to 2% reduces the consumption-equivalent measure by less than 0.1%. In contrast, higher rates, especially above 3%, significantly reduce welfare. For instance, a 4% rate lowers welfare by nearly 11%.

Figure 10: Welfare under various credit limits and associations

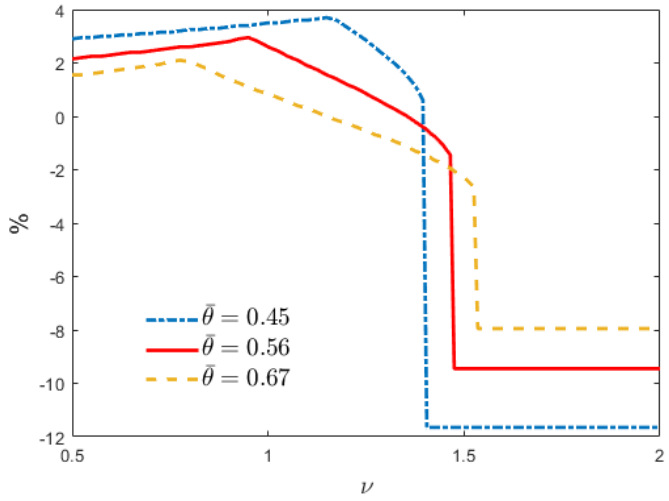


Note: Welfare gains (consumption equivalent measure) relative to the calibrated economy.

The right panel shows the welfare gain when $\nu = 1$ (no association among project returns) for different interest rate levels and credit limits. For each credit limit $\bar{\theta}$, the cor-

Figure 11: Optimal Interest Rate under Different Associations and Credit Limits

.



responding optimal interest rate ranges from 2.2% to 3.5%. Recall Figure 6 shows that a higher credit limit requires a lower optimal interest rate. When $\bar{\theta}$ is sufficiently large, welfare varies little at lower interest rates. For each credit limit, welfare drops sharply when the rate exceeds the corresponding optimal level. This exercise suggests that coordinating interest rate policy (targeting R) with financial regulations (e.g., macro-prudential policy targeting $\bar{\theta}$) is important for welfare and output. In economies with risk-taking, relaxing the credit constraint lowers the optimal interest rate, even though this may seem counter-intuitive. However, relaxed credit constraints increase risk-taking on the intensive margin, regardless of the interest rate. Therefore, the optimal interest rate should be lowered to trigger the valuation effect and curb excessive risk-taking. This can be explained by the fact that risk-taking eventually declines at sufficiently low interest rates, as shown by the hump-shaped relationship between $F(\tilde{\eta})$ and R in previous discussion.

Next, we examine the combined effect of the credit limit and the project association. Figure 11 shows the optimal interest rate as a function of the association measures ν across different $\bar{\theta}$.

First, in a risk-taking economy (e.g., for $\nu \leq 1.4$), the optimal rate is non-monotonic to the association measure for any given credit limit $\bar{\theta}$. This result mirrors the hump-shaped welfare in Figure 9. The underlying reason is the tradeoff between promoting risk-taking for productivity gains and stimulating investment to grow the economy after a rate cut. When risky project returns are highly positively correlated, the economy sees a much higher social return, $\Pi(\nu)$, compared to Π. In response, the optimal interest rate should be lower to channel funds to entrepreneurs. Conversely, the optimal interest rate should rise when project returns are negatively correlated. But if the negative association is too severe, as

explained above, the planner may prefer little to no risk-taking; in fact, the planner may choose a lower interest rate to focus on stimulating investment. These two extreme scenarios lead to the optimal rate peaking at a certain value of ν.

The peak of optimal interest rate depends on the credit limit: approximately 3.7% for $\bar{\theta} = 0.45$ at around $\nu = 1.15$; about 2.12% for $\bar{\theta} = 0.67$ at around $\nu = 0.79$. As shown in Figure 6, a more relaxed credit limit tends to lower the optimal interest rate. However, Figure 9 suggests that this relationship can become ambiguous with varying associations of project outcomes. Thus, credit constraints play a dominant role in our quantitative analysis. When we restrict the credit limit to be within 10% of the calibrated level of $\bar{\theta} = 0.56$, which aligns with the sample data, the optimal interest rate peaks between 2.2% and 3.1%. Notably, these peaks occur close to $\nu = 1$, suggesting a social return that is about 5% above or below the private return.

Second, a shift from an economy with risk-taking to non-risk-taking is possible. For high enough ν (i.e., when project returns are sufficiently negatively associated), the optimal interest rate always falls to a certain level. To see this, notice that as the interest rate falls, eventually, no entrepreneurs will implement risky projects. Investment demand (in safe projects) thus dominates when the interest rate is sufficiently low. The government needs to save to achieve a very low interest rate, and the resources come from distortionary taxation. Therefore, the government may achieve a local optimum in an economy without risk-taking, and the optimal rate is no longer affected by ν. As there is another local optimal interest rate under risk-taking, we thus observe the discrete jump. Notice that when the leverage increases, the interest rate range that generates the substitution effect (i.e., when a cut of interest rate leads to more risk-taking) is larger, so the discrete jump occurs at a higher ν (with more negatively associated projects). The optimal level also goes up to counterbalance the relaxation of credit constraints.

6 Conclusion

In this paper, we show that the effect of interest rates on risk-taking could be non-monotonic overall, which depends on leverage conditions influenced by financial regulations. Specifically, when firm leverage is low (high), firm value effect is strong (weak), and an interest-rate cut discourages (encourages) risk-taking. When the leverage condition is moderate, an interest rate cut encourages risk-taking only when the interest rate level is high and discourages risk-taking otherwise. Our analysis may shed light on why earlier studies found mixed results regarding the relationship between liquidity and firm volatility. Therefore, with a further cut of interest rate in a low-interest-rate environment, firms may not pursue risky but socially desirable projects.

Apart from leverage conditions, we also show that whether an interest rate policy incentivizes firm risk-taking depends crucially on the association of project returns among risk-takers. The analysis highlights the need for a careful mix of interest rate policy and financial regulations, such as macro-prudential policies. A natural implication is that monetary policy could have a medium-run growth effect via this risk-taking channel, although technological progress still largely determines long-term growth. Future research can explore optimal capital taxation that alters projects' risk profiles which seem crucial for productivity and welfare.

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Online Appendix

A Data Source

We use daily stock returns between 1984 and 2020 from CRSP to calculate the inter-quartile range (IQR) of the daily returns and the standard deviations of annualized stock returns. CRSP database maintains the most comprehensive collection of security price, return, and volume data for the NYSE, AMEX, and NASDAQ stock markets. Additional financial data (mentioned in the main text) for the same period is sourced from COMPUSTAT. Firms from the financial sector are excluded from the sample. Return on equity (ROE) is measured as net income over total equity (total assets minus total liabilities). We exclude extreme observations by dropping the firm observation if any of its illiquid ratio, size, Tobin's Q, cash holdings, ROA, ROE, etc, are below the 1st or above the 99th percentile.

For interest-rate measures, we obtain the federal funds rate, inflation, the Federal Reserve Bank of Cleveland's real interest rate and expected inflation, and the prime rate from FRED, maintained by the Federal Reserve Bank of St. Louis.

The relationship between the risk-taking measure using IQR and the real rate at the industry level is in Figure 12. The relationship is robust if we use the cross-sectional standard deviation of ROEs.

B Derivations and Proofs

B.1 Proof of Proposition 1

If the entrepreneur chooses a safe project, we have the value function:

$$\begin{aligned} V^f(\omega) &= \max_{s, 0\le b\le \bar{\theta}s} \{\log(\omega - s) + \beta \log(\Pi^f(s+b) - Rb)\} \\ &= \max_{s, 0\le \theta\le \bar{\theta}} \{\log(\omega - s) + \beta \log s + \beta \log(\Pi^f(1+\theta) - R\theta)\}. \end{aligned}$$

The optimal saving is $s = \varphi\omega$ with $\varphi = \frac{\beta}{1+\beta}$. Since $\Pi^f > R$, the credit constraint is binding: $\theta = \bar{\theta}$. If the entrepreneur chooses a risky project, the value function becomes:

$$\begin{aligned} V^r(\omega, \eta) &= \max_{s, 0\le b\le \bar{\theta}s} \Big\{ \log(\omega - s) - \eta + \beta p \log(\Pi^h(s+b) - Rb)\} + \beta(1-p)\log(\Pi^h(s+b) - Rb) \Big\} \\ &= \max_{s, 0\le \theta\le \bar{\theta}} \Big\{ \log(\omega - s) + \beta \log s - \eta + \beta p \log\Big(\Pi^h(1+\theta) - R\theta\Big) \\ &\quad + \beta(1-p)\log\Big(\Pi^l(1+\theta) - R\theta\Big) \Big\}. \end{aligned}$$

The optimal saving is $s = \varphi\omega$ again. Let θ^* denote the level of unconstrained optimal leverage that a firm would choose in the absence of financing constraints. Then, θ^* solves the first-order condition (ignoring $\theta > 0$, but we will come back to that):

$$p\frac{\Pi^h - R}{\Pi^h(1+\theta) - R\theta} + (1-p)\frac{\Pi^l - R}{\Pi^l(1+\theta) - R\theta} = 0.$$

Therefore, the cutoff interest rate level $\Pi^h\Pi^l/\hat{\Pi}$ (above which $\theta^* < 0$) is obtained by setting $\theta = 0$ above and it is straightforward to verify that $\Pi^h\Pi^l/\hat{\Pi} \in \big(\Pi^l, \Pi^h\big)$. If $R > \Pi^h\Pi^l/\hat{\Pi}$, then the interest rate is too high to justify borrowing. However, under Assumption A2, $\theta^* > 0$ because

$R < \Pi^f \leq \Pi^h\Pi^l/\hat{\Pi}$. Then we can express $\theta^* > 0$ as

$$\theta^* \equiv -\frac{\Pi^h\Pi^l - R(p\Pi^l + (1-p)\Pi^h)}{(\Pi^l - R)(\Pi^h - R)} = -\left[p\frac{\Pi^l}{\Pi^l - R} + (1-p)\frac{\Pi^h}{\Pi^h - R}\right]. \tag{24}$$

The optimal leverage $\theta = \min\{\theta^*, \bar{\theta}\}$. We now show that θ^* decreases in R. Notice that

$$\begin{aligned}\frac{\partial\theta^*}{\partial R} &= \frac{\hat{\Pi}(\Pi^l - R)(\Pi^h - R) + (-\Pi^h\Pi^l + R\hat{\Pi})(+\Pi^h + \Pi^l - 2R)}{(\Pi^l - R)^2(\Pi^h - R)^2} \\ &= \frac{\Pi^l\Pi^h(R - \Pi) + (\Pi^h\Pi^l - \hat{\Pi}R)R}{(\Pi^l - R)^2(\Pi^h - R)^2}.\end{aligned}$$

The numerator above is quadratic and concave in R. With some algebra, the maximum value of the numerator is $\frac{\Pi^h\Pi^l(\Pi^h\Pi^l - \hat{\Pi}\Pi)}{\hat{\Pi}} < 0$. To verify this, we can use the convexity feature, $\frac{p}{\Pi^h} + \frac{1-p}{\Pi^l} > \frac{1}{p\Pi^h + (1-p)\Pi^l} = \frac{1}{\Pi}$ which implies $\Pi^h\Pi^l < \hat{\Pi}\Pi$. Thus, the numerator is negative, which implies that $\frac{\partial\theta^*}{\partial R} < 0$. The optimal leverage $\theta = \min\{\theta^*, \bar{\theta}\}$.

Finally, we determine the threshold for taking risky project $\tilde{\eta}$. By taking the difference between $V^r(\omega, \eta)$ and $V^f(\omega)$ above, we obtain (3).

B.2 Proof of Proposition 2

Assuming $\theta^* > \bar{\theta}$, we have $\theta = \bar{\theta}$ when an entrepreneur chooses the risky project. Define $x \equiv R\frac{\bar{\theta}}{1+\bar{\theta}}$ as the debt servicing cost per unit of capital used in production, then $\tilde{\eta}$ in (3) can be rewritten as

$$\tilde{\eta} = \beta p \log\left(\frac{\Pi^h - x}{\Pi^f - x}\right) + \beta(1-p)\log\left(\frac{\Pi^l - x}{\Pi^f - x}\right).$$

We then obtain

$$\frac{\partial\tilde{\eta}}{\partial x} = \beta\frac{\Pi^h\Pi^l - \Pi^f\hat{\Pi} + (\Pi^f - \Pi)x}{(\Pi^h - x)(\Pi^l - x)},$$

where we use $\hat{\Pi} \equiv (1-p)\Pi^h + p\Pi^l$ and $\Pi + \hat{\Pi} = \Pi^h + \Pi^l$. Under Assumptions A1 and A2, one can verify

$$\Pi^h > \Pi^l > R\frac{\theta^*}{1+\theta^*} > R\frac{\bar{\theta}}{1+\bar{\theta}}.$$

Then, the denominator of $\partial\tilde{\eta}/\partial x$ is positive. Additionally, since $(\Pi^f - \Pi)$ is negative, we can thus reach $\partial\tilde{\eta}/\partial x = 0$ if $x = x^s$, where

$$x^s \equiv \frac{\Pi^h\Pi^l - \Pi^f\hat{\Pi}}{\Pi - \Pi^f}.$$

And $\partial\tilde{\eta}/\partial x > 0$ if $x < x^s$, and $\partial\tilde{\eta}/\partial x < 0$ if $x > x^s$.

Notice that

$$\frac{\partial\tilde{\eta}}{\partial R} = \frac{\partial\tilde{\eta}}{\partial x}\frac{\partial x}{\partial R}.$$

Since $\partial x/\partial R > 0$, the sign of $\partial\tilde{\eta}/\partial R$ depends on the sign of $\partial\tilde{\eta}/\partial x$. Let R^s be the interest rate such that $x = x^s$, i.e.,

$$R^s = R^s(\bar{\theta}) \equiv x^s(1/\bar{\theta} + 1).$$

Note that R^s depends on the leverage upper bound $\bar{\theta}$ and the superscript s indicates that the debt

servicing cost is x^s (a parameter determined by the project returns as shown above). Therefore, we define two boundary values

$$\bar{\theta}_H^s \equiv \frac{\Pi^h\Pi^l - \Pi^f\hat{\Pi}}{(\Pi - \Pi^f)\Pi^l - (\Pi^h\Pi^l - \Pi^f\hat{\Pi})}, \quad \bar{\theta}_L^s \equiv \frac{\Pi^h\Pi^l - \Pi^f\hat{\Pi}}{(\Pi - \Pi^f)\Pi^f - (\Pi^h\Pi^l - \Pi^f\hat{\Pi})}$$

determined by setting $R^s(\bar{\theta}_H^s) = \Pi^l$ and $R^s(\bar{\theta}_L^s) = \Pi^f$. Based on the two boundary values, we have

1). If $\bar{\theta} > \bar{\theta}_H^s$, then $R^s(\bar{\theta}) < \Pi^l$. For the range of interest rate under assumptions, $R > \Pi^l > R^s(\bar{\theta})$, the debt servicing cost x is above x^s. In this case, $\partial\tilde{\eta}/\partial x < 0$ and thus $\partial\tilde{\eta}/\partial R < 0$;

2). If $\bar{\theta} < \bar{\theta}_L^s$, then $R^s(\bar{\theta}) > \Pi^f$. For the range of interest rate under assumptions, $R < \Pi^f < R^s(\bar{\theta})$, the debt servicing cost x is below x^s. In this case, $\partial\tilde{\eta}/\partial x > 0$ and thus $\partial\tilde{\eta}/\partial R > 0$;

3). Otherwise, $\tilde{\eta}$ is hump-shaped in R and $\Pi^l < R^s(\bar{\theta}) < \Pi^f$ when $\bar{\theta}_L^s < \bar{\theta} < \bar{\theta}_H^s$.

The above completes the proof. However, we can extend the reasoning by varying the leverage limit while holding the interest rate R fixed. To see this, notice that

$$\frac{\partial\tilde{\eta}}{\partial\bar{\theta}} = \frac{\partial\tilde{\eta}}{\partial x}\frac{\partial x}{\partial\bar{\theta}}.$$

Since $\partial x/\partial\bar{\theta} > 0$, the sign of $\partial\tilde{\eta}/\partial\bar{\theta}$ depends on the sign of $\partial\tilde{\eta}/\partial x$. Let $\bar{\theta}^s$ be the leverage upper bound such that $x = x^s$, i.e.,$\bar{\theta}^s \equiv \bar{\theta}^s(R) = \frac{x^s}{R - x^s}$. One can show that $0 < \bar{\theta}^s < \theta^*$. Therefore, $\tilde{\eta}$ is hump-shaped in $\bar{\theta}$, and $\bar{\theta}^s$ decreases in R. Finally, we have the following corollary:

Corollary. *Suppose A1 and A2 hold and suppose the financing constraint is binding when the risky project is implemented (i.e., $\bar{\theta} < \theta^*$). We have that $\bar{\theta}^s(R) \in (0, \theta^*)$, the leverage upper bound* such that $\partial\tilde{\eta}/\partial R = 0$, *decreases in R. When $0 < \bar{\theta} < \bar{\theta}^s(R)$, the cutoff $\tilde{\eta}$ is increasing in $\bar{\theta}$; when $\bar{\theta}^s(R) < \bar{\theta} < \theta^*$, the cutoff $\tilde{\eta}$ is decreasing in $\bar{\theta}$.*

Unlike Proposition 2 with three scenarios based on the level of $\bar{\theta}$, there are no multiple cases here. This is because R^s can be lower than the lower bound Π^l, higher than the upper bound Π^f, or in between, depending on the level of $\bar{\theta}$. However, $\bar{\theta}^s$ is always between the lower and upper bounds, i.e., $0 < \bar{\theta}^s < \theta^*$.

B.3 Proof of Proposition 3

When the borrowing constraint is slack under the entrepreneur's risky choice, we have

$$\tilde{\eta} = \beta p \log\left(\frac{\Pi^h + (\Pi^h - R)\theta^*}{\Pi^f + (\Pi^f - R)\bar{\theta}}\right) + \beta(1-p)\log\left(\frac{\Pi^l + (\Pi^l - R)\theta^*}{\Pi^f + (\Pi^f - R)\bar{\theta}}\right),$$

where $\theta^* = -\left[p\frac{\Pi^l}{\Pi^l - R} + (1-p)\frac{\Pi^h}{\Pi^h - R}\right]$. Then, we can express $\tilde{\eta}$ in this case as

$$\begin{aligned}\tilde{\eta} =& \beta[p\log p + (1-p)\log(1-p) + \log R + \log(\Pi^h - \Pi^l) \\ & - p\log(R - \Pi^l) - (1-p)\log(\Pi^h - R) - \log(\Pi^f + (\Pi^f - R)\bar{\theta})].\end{aligned}$$

Taking the derivative with respect to R, we obtain

$$\frac{\partial\tilde{\eta}}{\partial R} = \beta\left[\frac{\bar{\theta}}{\Pi^f + (\Pi^f - R)\bar{\theta}} - \frac{\theta^*}{R}\right]. \tag{25}$$

To prove the non-monotonic cutoff level for an unconstrained entrepreneur in Proposition 3 we proceed in two steps, each with a Lemma: 1) we show that $\tilde{\eta}$ is convex in the interest rate R when

the financing constraint is slack; 2) if $\bar{\theta}$ is large enough we will see $\tilde{\eta}$ decreases and then increases in R when the credit constraint is slack.

Lemma 5. *For an financially unconstrained entrepreneur, the cutoff level $\tilde{\eta}$ is convex in R.*

Proof. With the expression for θ^* equation (24), we can rewrite $\frac{\partial\theta^*}{\partial R}$ as

$$\frac{\partial\theta^*}{\partial R} = \frac{\theta^*}{R\hat{\Pi} - \Pi^h\Pi^l}\left[\frac{\Pi^h\Pi^l + \theta^*(\Pi^h\Pi^l - R^2)}{R}\right].$$

From Proposition 1, we know $\frac{\partial\theta^*}{\partial R} < 0$. With this inequality and Assumption A2 ($\Pi^h\Pi^l > \Pi^f\hat{\Pi}$), we have $\Pi^h\Pi^l - R\hat{\Pi} > \Pi^h\Pi^l - \Pi^f\hat{\Pi} > 0$, which implies $\Pi^h\Pi^l + \theta^*(\Pi^h\Pi^l - R^2) > 0$. Then, it follows that

$$-\frac{\partial\frac{\theta^*}{R}}{\partial R} = \frac{\theta^*\left[\Pi^l\Pi^h + \theta^*(\Pi^h\Pi^l - R^2)\right] + \theta^*(\Pi^h\Pi^l - R\hat{\Pi})}{R^2(\Pi^h\Pi^l - R\hat{\Pi})} > 0 \text{ and } \frac{\partial^2\tilde{\eta}}{\partial R^2} = \frac{\partial\frac{\bar{\theta}}{\Pi^f + (\Pi^f - R)\bar{\theta}}}{\partial R} - \frac{\partial\frac{\theta^*}{R}}{\partial R} > 0.$$

Thus, the cutoff level $\tilde{\eta}$ is convex in the interest rate R for an unconstrained entrepreneur. □

Lemma 6. *At the interest rate $\bar{R}(\bar{\theta})$, where the borrowing constraint shifts from binding to non-binding, $\tilde{\eta}$ decreases in the interest rate R, i.e., $\frac{\partial\tilde{\eta}}{\partial R}|_{R=\bar{R}(\bar{\theta})} < 0$.*

Proof. We use the non-monotonic result from the binding case in Proposition 2. Since when the interest rate is higher than $R^s(\bar{\theta})$, the rate at which $\tilde{\eta}$ reaches its maximum under a binding borrowing constraint, we have $\frac{\partial\tilde{\eta}}{\partial R} < 0$. Therefore, if $\bar{R}(\bar{\theta}) > R^s(\bar{\theta})$, it follows that $\frac{\partial\tilde{\eta}}{\partial R}|_{R=\bar{R}(\bar{\theta})} < 0$. We need to show $\bar{R}(\bar{\theta}) > R^s(\bar{\theta})$. It turns out more convenient to compare the debt servicing cost, $R\frac{\bar{\theta}}{1+\bar{\theta}}$ in both cases. From the proof of Proposition 2 in Appendix B.2

$$R^s(\bar{\theta})\frac{\bar{\theta}}{1+\bar{\theta}} = x^s = \frac{\Pi^h\Pi^l - \Pi^f\hat{\Pi}}{\Pi - \Pi^f} = g(\Pi^f),$$

where $g(s) \equiv \frac{\Pi^h\Pi^l - s\hat{\Pi}}{\Pi - s}$. Meanwhile

$$\bar{R}(\bar{\theta})\frac{\bar{\theta}}{1+\bar{\theta}} = \frac{-p\Pi^l(\Pi^h - \bar{R}(\bar{\theta})) - (1-p)\Pi^h(\Pi^l - \bar{R}(\bar{\theta}))}{-p(\Pi^h - \bar{R}(\bar{\theta})) - (1-p)(\Pi^l - \bar{R}(\bar{\theta}))} = \frac{\Pi^h\Pi^l - \bar{R}(\bar{\theta})\hat{\Pi}}{\Pi - \bar{R}(\bar{\theta})} = g(\bar{R}(\bar{\theta})).$$

The rest of the proof shows $g(.)$ is decreasing and $\bar{R}(\bar{\theta}) < \Pi^f$, implying $\bar{R}(\bar{\theta}) > R^s(\bar{\theta})$.

By convexity, $\frac{p}{\Pi^h} + \frac{1-p}{\Pi^l} > \frac{1}{p\Pi^h + (1-p)\Pi^l} = \frac{1}{\Pi}$ which implies $\Pi^h\Pi^l < \hat{\Pi}\Pi$. Then, $g(s)$ decreases in s, since the first order condition of $g(s)$ with respect to s is $\frac{-\hat{\Pi}(\Pi - s) + \Pi^h\Pi^l - s\hat{\Pi}}{(\Pi - s)^2} = \frac{\Pi^h\Pi^l - \hat{\Pi}\Pi}{(\Pi - s)^2} < 0$.

Notice that if the entrepreneur is unconstrained, then it must be that $\bar{R}(\bar{\theta}) < \Pi^f$. To see this claim, first observe that θ^* is a decreasing function of R. By the definition of $\bar{R}(\bar{\theta})$, we have $\theta^*(R) = \bar{\theta}$ when $R = \bar{R}(\bar{\theta})$. By the definition of θ^*_{min}, we have $\theta^*(R) = \theta^*_{min}$ when $R = \Pi^f$. Second, if $\bar{R}(\bar{\theta}) \geq \Pi^f$ that is opposite of the claim, then $\bar{\theta} = \theta^*(\bar{R}(\bar{\theta})) \leq \theta^*(\Pi^f) = \theta^*_{min}$ as $\theta^*(R)$ decreases in R, which contradicts with our assumption that $\bar{\theta} > \theta^*_{min}$, and thus we prove the claim. □

Finally, using Lemma 5 and Lemma 6, we can finish the proof of Proposition 3. To show $\tilde{\eta}$ changes from decreasing to increasing in R, we need to find further an interest rate level such that $\frac{\partial\tilde{\eta}}{\partial R} > 0$.

In the proof of (B.2), the derivative of $\tilde{\eta}$ with respect to the debt servicing cost x is zero when $x = x^s$; we see that $\frac{\partial\tilde{\eta}}{\partial R}|_{R=\Pi^f} = 0$ when $\bar{\theta}$ takes the value of $\bar{\theta}^s_L$, since the debt servicing cost is indeed $x = x^s$ when $\bar{\theta} = \bar{\theta}^s_L$ and $R = \Pi^f$. It is straightforward to verify that $\frac{\partial^2\tilde{\eta}}{\partial R\partial\bar{\theta}} > 0$ according

to (25), which means that $\frac{\partial\tilde{\eta}}{\partial R}$ increases in $\bar{\theta}$. Thus $\frac{\partial\tilde{\eta}}{\partial R}|_{R=\Pi^f} > 0$ when $\bar{\theta} > \theta^*_{min}$ since we already know that $\theta^*_{min} = \bar{\theta}^s_L$. Then by Lemma 5, Lemma 6, and the mean value theorem, when $\bar{\theta} > \theta^*_{min}$ there exists an interest rate $R^u(\bar{\theta})$ where $\bar{R}(\bar{\theta}) < R^u(\bar{\theta}) < \Pi^f$ such that $\frac{\partial\tilde{\eta}}{\partial R}|_{R=R^u(\bar{\theta})} = 0$, since we already know that $\frac{\partial\tilde{\eta}}{\partial R}|_{R=\bar{R}(\bar{\theta})} < 0$ (from Lemma 6), $\frac{\partial\tilde{\eta}}{\partial R}|_{R=\Pi^f} > 0$, and $\frac{\partial\frac{\partial\tilde{\eta}}{\partial R}}{\partial R} > 0$ (from Lemma 5).

Thus, the implied risk-taking threshold for unconstrained entrepreneurs first decreases with R and then increases with R later. That is, $\frac{\partial\tilde{\eta}}{\partial R} < 0$ if $\bar{R}(\bar{\theta}) \leq R \leq R^u(\bar{\theta})$ and $\frac{\partial\tilde{\eta}}{\partial R} > 0$ if $R^u(\bar{\theta}) < R(\bar{\theta}) \leq \Pi^f$.

B.4 Proof of Proposition 4

Guess that $V^r(\omega,\eta) = \log(\omega) + v^r - \eta$, $V^f(\omega) = \log(\omega) + v^f - \eta^f$, $V^d(\omega) = \log(\omega) + v^d$. Plugging these guessed forms into the entrepreneur's Bellman equation, we have the following

$$\begin{aligned} v^r =& \mathcal{B} + \beta p \log\Big(\Pi^h(1+\theta) - R\theta\Big) + \beta(1-p)\log\Big(\Pi^l(1+\theta) - R\theta\Big) \\ &+ \beta \max\{v^d, \mathbb{E}\Big[\max\{v^r - \eta', v^f - \eta^f\}\Big]\}; \qquad (26) \\ v^f =& \mathcal{B} + \beta\Big[\log(\Pi^f(1+\bar{\theta}) - R\bar{\theta})\Big] + \beta\max\{v^d, \mathbb{E}\Big[\max\{v^r - \eta', v^f - \eta^f\}\Big]\}; \qquad (27) \\ v^d =& \mathcal{B} + \beta\log(R^d) + \beta\max\{v^d, \mathbb{E}\Big[\max\{v^r - \eta', v^f - \eta^f\}\Big]\}, \qquad (28) \end{aligned}$$

where $\mathcal{B} \equiv (1-\beta)\log(1-\beta) + \beta\log\beta$.

One can solve for the three unknowns v^r, v^f, and v^d from the above three equations, and verify the guess. Their exact solutions are, however, not essential for our purposes. The choice (between saving in deposits or being an active entrepreneur) depends on the maximal value of the two, $\mathbb{E}\left[\max\{v^r - \eta, v^f - \eta^f\}\right]$. As for the share of firms saving in deposits, we have

$$\phi = \begin{cases} 0 & \text{if } v^d < \mathbb{E}\left[\max\{v^r - \eta, v^f - \eta^f\}\right] \\ (0,1) & \text{if } v^d = \mathbb{E}\left[\max\{v^r - \eta, v^f - \eta^f\}\right] \\ 1 & \text{if } v^d > \mathbb{E}\left[\max\{v^r - \eta, v^f - \eta^f\}\right] \end{cases}.$$

Using the expressions for v^r, v^f, and v^d , we have ϕ as follows:

$$\phi = \begin{cases} 0 & \text{if } l^d < \mathbb{E}\left[\max\{l^r - \beta^{-1}\eta, l^f\}\right] \\ (0,1) & \text{if } l^d = \mathbb{E}\left[\max\{l^r - \beta^{-1}\eta, l^f\}\right] \\ 1 & \text{if } l^d > \mathbb{E}\left[\max\{l^r - \beta^{-1}\eta, l^f\}\right] \end{cases}.$$

Notice that when $\phi = 0$, the active projects strictly dominate; when $\phi = 1$, the risk-free deposits strictly dominate; when $0 < \phi < 1$, an entrepreneur is indifferent between saving via the risk-free deposits and becoming an active entrepreneur implementing projects.

The choice of θ follows the same reasoning as in the two-period model, for which the proof has already been provided. For the cutoff, there exists a level below which entrepreneurs choose the risky project. Similar to the proof before, we have $\tilde{\eta} = v^r - (v^f - \eta^f)$ as:

$$\tilde{\eta} = \beta p\log\Big(\Pi^h + (\Pi^h - R)\theta\Big) + \beta(1-p)\log\Big(\Pi^l + (\Pi^l - R)\theta\Big) - \beta(\log(\Pi^f(1+\bar{\theta}) - R\bar{\theta}) - \eta^f/\beta),$$

where we have used the value functions and $\theta = \min\{\theta^*, \bar{\theta}\}$ (with θ^* being the unconstrained solution as given in (24)). If the right-hand side is below $\underline{\eta}$, we set $\tilde{\eta} = \underline{\eta}$. Therefore, entrepreneurs with $\eta \leq \max\{\underline{\eta}, \tilde{\eta}\}$ choose the risky project.

C Omitted Details in the Macroeconomic Model

C.1 The average return in the society

Let m denote the number of realizations of Π^h in the economy, and let the proportion of success be $p^s(\nu) = \mathbb{E}\left[\frac{m}{n}\right]$ as n goes to infinity, which is the share of realized return Π^h.

It has been shown in Shmueli et al. (2005) that for the Conway-Maxwell Poisson distribution, $\mathbb{E}[m]$ converges to $\lambda^{1/\nu} - \frac{\nu-1}{2\nu}$ when n goes to infinity and p is small i.e. $p < p^*$. This suggests that $p^s(\nu)$ converges to $p^{1/\nu}$ when n goes to infinity after substituting $\lambda = n^\nu p$ as illustrated in Daly and Gaunt (2016). Here, $p^{1/\nu}$ represents the probability of high realizations assessed by the social planner, which differs from the individual's assessment p whenever there is an association in the projects (i.e., $\nu \neq 1$). As a result, the average return from the society's perspective becomes $p^{\frac{1}{\nu}}\Pi^h + (1 - p^{\frac{1}{\nu}})\Pi^h$. The case where p is large, i.e. $p > p^{**}$, follows a similar reasoning but one should replace p by $1 - p$ in the argument above.

C.2 Pairwise correlation for association

Let $\mathbb{I}_{x_1=l}$ be the indicator function for $x_1 = l$ (the first trial fails) and $\mathbb{I}_{x_2=l}$ for $x_2 = l$ (the second trial fails), respectively. The covariance of the indicator functions is

$$\begin{aligned} Cov(\mathbb{I}_{x_1=l}, \mathbb{I}_{x_2=l}) &= \mathbb{E}[\mathbb{I}_{x_1=l}\mathbb{I}_{x_2=l}] - \mathbb{E}[\mathbb{I}_{x_1=l}]\mathbb{E}[\mathbb{I}_{x_2=h}] \\ &= Pr(x_1 = l, x_2 = l) - Pr(x_1 = l)P(x_2 = l) \\ &= Pr(x_1 = h)Pr(x_1 = l)\left[Pr(x_2 = l|x_1 = l) - Pr(x_2 = l|x_1 = h)\right], \end{aligned}$$

With $\sigma(\mathbb{I}_{x_1=l}) = \sqrt{Pr(x_1 = h)Pr(x_1 = l)}$ and $\sigma(\mathbb{I}_{x_2=l}) = \sqrt{Pr(x_2 = h)Pr(x_2 = l)}$ being the standard deviation of these two indicator functions, then the correlation coefficient ρ_ν is

$$\begin{aligned} \rho_\nu &= \frac{Cov(\mathbb{I}_{x_1=l}, \mathbb{I}_{x_2=l})}{\sigma(\mathbb{I}_{x_1=l})\sigma(\mathbb{I}_{x_2=l})} \\ &= \frac{\sigma(\mathbb{I}_{x_1=l})}{\sigma(\mathbb{I}_{x_2=l})}(Pr(x_2 = l|x_1 = l) - Pr(x_2 = l|x_1 = h)) \\ &= \frac{Pr(x_2 = l, x_1 = l)}{Pr(x_1 = l)} - \frac{Pr(x_2 = l, x_1 = h)}{Pr(x_1 = h)} \\ &= \frac{\frac{1}{D(\nu,p,2)}(1-p)^2}{\frac{1}{D(\nu,p,2)}(1-p)^2 + \frac{1}{2}\frac{1}{D(\nu,p,2)}\binom{2}{1}^\nu p(1-p)} - \frac{\frac{1}{2}\frac{1}{D(\nu,p,2)}\binom{2}{1}^\nu p(1-p)}{\frac{1}{D(\nu,p,2)}p^2 + \frac{1}{2}\frac{1}{D(\nu,p,2)}\binom{2}{1}^\nu p(1-p)} \\ &= \frac{1-p}{1-p+2^{\nu-1}p} - \frac{2^{\nu-1}(1-p)}{p+2^{\nu-1}(1-p)}, \end{aligned}$$

where the second line uses the property of exchangeability (which implies $\sigma(\mathbb{I}_{x_1=l}) = \sigma(\mathbb{I}_{x_2=l})$) and the fifth line uses the definition of the CMB distribution and the property of exchangeability.

C.3 Social Welfare

We first derive entrepreneurs' welfare, which consists of the contribution of three types: those who take risks, those who implement the risk-free project, and those who save in safe deposits:

$$V_e(\omega) = (1-\phi)\int^{\tilde{\eta}} V^r(\omega,\eta)dF(\eta) + (1-\phi)\int_{\tilde{\eta}} V^f(\omega,\eta^f)dF(\eta) + \phi\int_{\tilde{\eta}} V^d(\omega)dF(\eta). \quad (29)$$

We have two scenarios below, each of which determines the value $\max\{v^d, \mathbb{E}\left[\max\{v^r - \eta', v^f - \eta^f\}\right]\}$ in (26) - (28) differently. For exposition simplicity, $\tilde{\eta}$ is assumed to be above $\underline{\eta}$, which is always verified in our numerical exercises. In addition, the lower bound $\underline{\eta}$ is set to zero since it simplifies the derivation and does not affect welfare comparison.

Scenario 1: When $\phi = 0$, no one chooses the safe deposit option. That is, $v^d < \mathbb{E}\left[\max\{v^r - \eta', v^f - \eta^f\}\right]$. Then, we can explicitly express $\mathbb{E}\max\{v^r - \eta', v^f - \eta^f, v^d\}$ as follows

$$\begin{aligned}
\max\{v^d, \mathbb{E}\left[\max\{v^r - \eta', v^f - \eta^f\}\right]\} &= \mathbb{E}\max\{v^r - \eta', v^f - \eta^f\} \\
&= (1-F)\left(v^f - \eta^f\right) + Fv^r - \int^{\tilde{\eta}} \eta dF(\eta) \\
&= v^f - \eta^f + \int^{\tilde{\eta}} F(\eta)d\eta,
\end{aligned}$$

since $\tilde{\eta} = v^r - v^f + \eta^f$. This means that we can explicitly solve the values in (26) - (28). It turns out that we only need to express

$$\begin{aligned}
v^f &= \mathcal{B} + \beta\left[\log(\Pi^f(1+\bar{\theta}) - R\bar{\theta})\right] + \beta\left[v^f - \eta^f + \int^{\tilde{\eta}} F(\eta)d\eta\right] \\
&= \frac{\mathcal{B}}{1-\beta} + \frac{\beta}{1-\beta}\left[\log(\Pi^f(1+\bar{\theta}) - R\bar{\theta})\right] + \frac{\beta}{1-\beta}[\int^{\tilde{\eta}} F(\eta)d\eta - \eta^f],
\end{aligned}$$

where $\mathcal{B}$ is defined above in the proof of Proposition 4, the second equality results from rearranging v^f on the right-hand side of the first equation and dividing both sides by 1-β. Therefore, we can rewrite (29) as

$$\begin{aligned}
V_e(\omega) &= \int^{\tilde{\eta}} (V^r(\omega, \eta))dF(\eta) + \int_{\tilde{\eta}} V^f(\omega, \eta^f) \\
&= \log(\omega) + v^r F(\tilde{\eta}) - \int^{\tilde{\eta}} \eta dF(\eta) + [1 - F(\tilde{\eta})]\left(v^f - \eta^f\right) \\
&= \log(\omega) + \tilde{\eta}F(\tilde{\eta}) + (v^f - \eta^f) - F(\tilde{\eta})\tilde{\eta} + \int^{\tilde{\eta}} F(\eta)d\eta \\
&= \log(\omega) + \frac{\mathcal{B}}{1-\beta} + \frac{\beta}{1-\beta}\left[\log(\Pi^f(1+\bar{\theta}) - R\bar{\theta})\right] + \frac{1}{1-\beta}\left[\int^{\tilde{\eta}} F(\eta)d\eta - \eta^f\right],
\end{aligned}$$

where the third equality uses $v^r = v^f - \eta^f + \tilde{\eta}$ and the last equality substitutes the expression for v^f derived above.

Scenario 2: When $0 < \phi < 1$, entrepreneurs are indifferent between investing in risk-free deposits and taking on projects. Then, we can explicitly express $\max\{v^d, \mathbb{E}\left[\max\{v^r - \eta', v^f - \eta^f\}\right]\}$ as

$$\max\{v^d, \mathbb{E}\left[\max\{v^r - \eta', v^f - \eta^f\}\right]\} = v^d.$$

Using the result above and following a similar procedure as in Scenario 1, we can express v^d as

$$v^d = \mathcal{B} + \beta\log R^d + \beta v^d = \frac{\mathcal{B}}{1-\beta} + \frac{\beta\log(R^d)}{1-\beta},$$

where the second equality results from rearranging v^d. Therefore, we can rewrite (29) as

$$V_e(\omega) = V^d(\omega) = \log(\omega) + v^d = \log(\omega) + \frac{\mathcal{B}}{1-\beta} + \frac{\beta \log(R^d)}{1-\beta},$$

where we used the expression for v^d in the last equality.

Remark: In a steady-state economy, project choice is independent of the wealth level. The distribution of the wealth level ω is always preserved. Denote V as the value function of the entrepreneurs. Thus, $\omega = \Omega$ and $V = V_e(\Omega)$. We, therefore, conclude that entrepreneurs' welfare can be represented by $\log(\Omega) + \tilde{V} + constants$ as shown in the main text, since the results in Scenarios 1 and 2 above imply the constant term is $\mathcal{B}/(1-\beta)$ and

$$\tilde{V} = (1-\beta)^{-1}\left(\beta \max\{l^d, l^f + \beta^{-1}\int^{\tilde{\eta}} F(\eta)d\eta\}\right).$$

Note that both Ω and $\tilde{V}$ are endogenous to policy variations.

D Endogenous leverage

This section introduces the concept of endogenous leverage, where the maximum leverage an entrepreneur can sustain depends on the risk of default. Here, we allow entrepreneurs engaged in risky projects to have the option to default upon realizing a return of Π^l. Entrepreneurs with safe projects may also choose to default under certain conditions.

Let $\bar{\theta}^r$ and $\bar{\theta}^f$ represent the maximum allowable leverage for entrepreneurs undertaking risky and safe projects, respectively. We demonstrate how these leverage bounds, $\bar{\theta}^r$ and $\bar{\theta}^f$, can be determined endogenously by incorporating the potential of default. In the event of default, we assume that entrepreneurs retain a fraction $\varphi^r < 1$ of total revenue from risky projects and $\varphi^f < 1$ from safe projects, pocketing these amounts without repaying any debt. However, after default, entrepreneurs face a probability ξ of permanent exclusion from entrepreneurial activities, losing access to both safe and risky project opportunities.

The value of risk-taking entrepreneurs in the low state Π^l, conditional on not defaulting, is denoted as V^{rl}_{expost}, which is given by:

$$\begin{aligned} V^{rl}_{expost} &= \log(\Pi^l(s+b) - Rb) + \mathbb{E}\max\{v^r - \eta', v^f\} \\ &= \log(1+\bar{\theta}^r) + \log\left(\Pi^l - \frac{\bar{\theta}^r R}{1+\bar{\theta}^r}\right) + \log(s) + \mathbb{E}\max\{v^r - \eta', v^f\}. \end{aligned}$$

If they choose to default, the value becomes:

$$\begin{aligned} V^D(\Pi^l(s+b) - Rb)) &= \xi\left[\log\left(\varphi^r \Pi^l(s+b)\right) + \left(\log(1-\beta) + \frac{\beta}{1-\beta}(\log\beta + \log R)\right)\right] \\ &\quad + (1-\xi)\left[\log\left(\varphi^r \Pi^l(s+b)\right) + \mathbb{E}\max\{v^r - \eta', v^f\}\right] \\ &= \log(1+\bar{\theta}^r) + \log(\Pi^l) + \log(\varphi^r) + \log(s) \\ &\quad + \xi\left(\log(1-\beta) + \frac{\beta}{1-\beta}(\log\beta + \log R)\right) + (1-\xi)\mathbb{E}\max\{v^r - \eta', v^f\}, \end{aligned}$$

Therefore, the non-default condition becomes:

$$V^{rl}_{expost} \geq V^D\left(\Pi^l(s+b) - Rb)\right),$$

or

$$\xi\left[\frac{\beta}{1-\beta}\log(\Pi^l(1+\bar{\theta}^r)-R\bar{\theta}^r)+\frac{1}{1-\beta}\int_{\underline{\eta}}^{\tilde{\eta}}F(\eta)d\eta-\frac{\beta}{1-\beta}\log R\right]$$
$$\geq \underbrace{\log(\Pi^l)+\log(\varphi^r)-\log(\Pi^l-\frac{\bar{\theta}^r R}{1+\bar{\theta}^r})}_{\text{immediate gain}}.$$

In this inequality, the left-hand side represents the expected future benefits from not defaulting, weighted by the probability of losing entrepreneurial status ξ, while the right-hand side captures the immediate gain from defaulting under low returns Π^l. This condition ensures that entrepreneurs prefer not to default as long as the expected benefits of maintaining their status exceed the immediate gain from default.

Similarly, the non-default condition for entrepreneurs with safe projects becomes:

$$V^f_{expost} \quad \geq \quad V^D(\Pi^f(s+b)-Rb))$$

or

$$\xi\left[\frac{\beta}{1-\beta}\log(\Pi^f(1+\bar{\theta}^f)-R\bar{\theta}^f)+\frac{1}{1-\beta}\int_{\underline{\eta}}^{\tilde{\eta}}F(\eta)d\eta-\frac{\beta}{1-\beta}\log R\right]$$
$$\geq \underbrace{\log(\Pi^f)+\log(\varphi^f)-\log(\Pi^f-\frac{\bar{\theta}^f R}{1+\bar{\theta}^f})}_{\text{immediate gain}}.$$

The endogenous leverage upper bounds $\bar{\theta}^r$ and $\bar{\theta}^f$ can be determined by setting the non-default conditions derived above to hold as equalities. For numerical illustration, Figure 13 shows how $\tilde{\eta}$ varies with the interest rate R, and a similar non-monotonic relationship remains.

Figure 12: IQR and Real Interest Rate at the Industry Level.

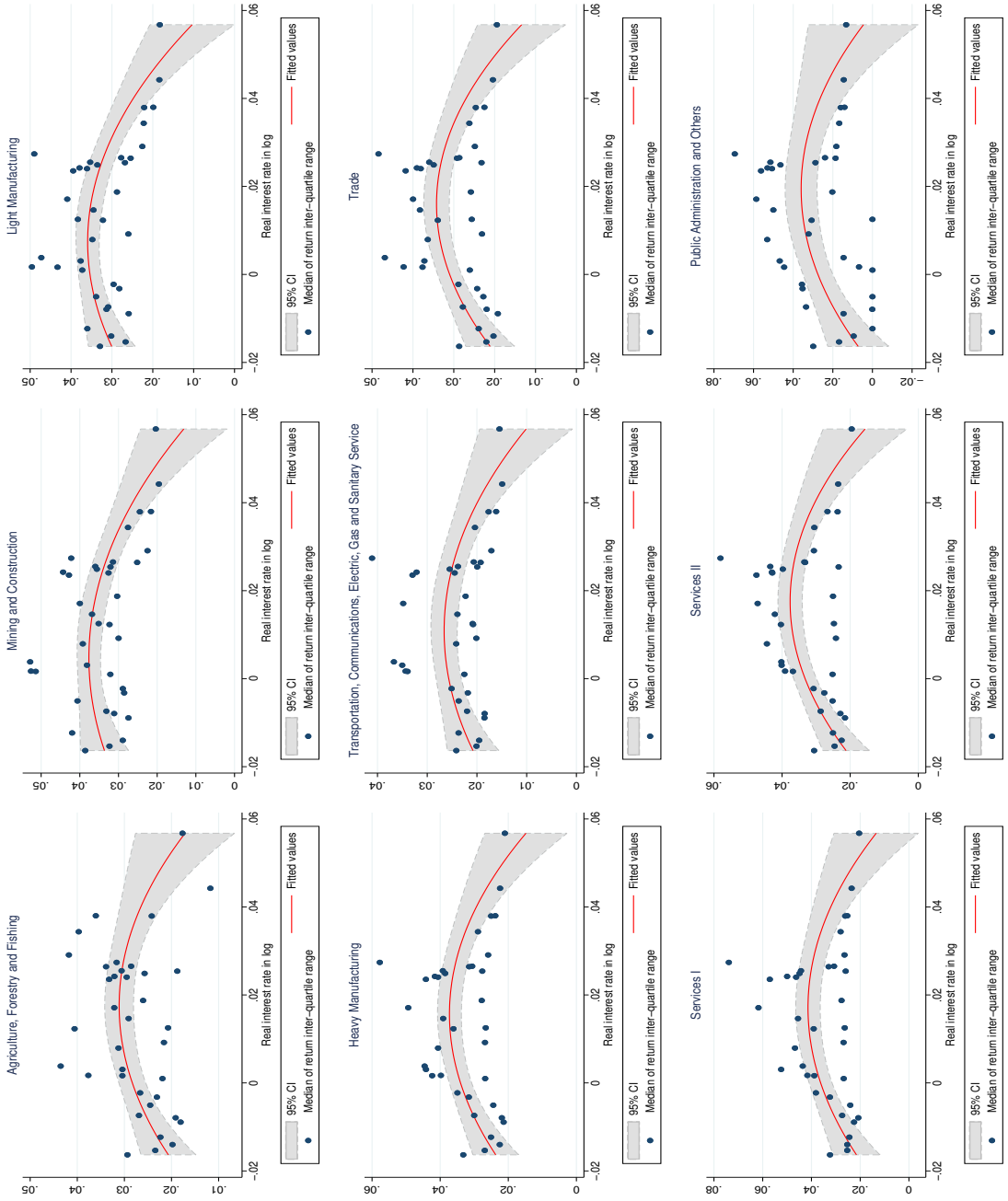
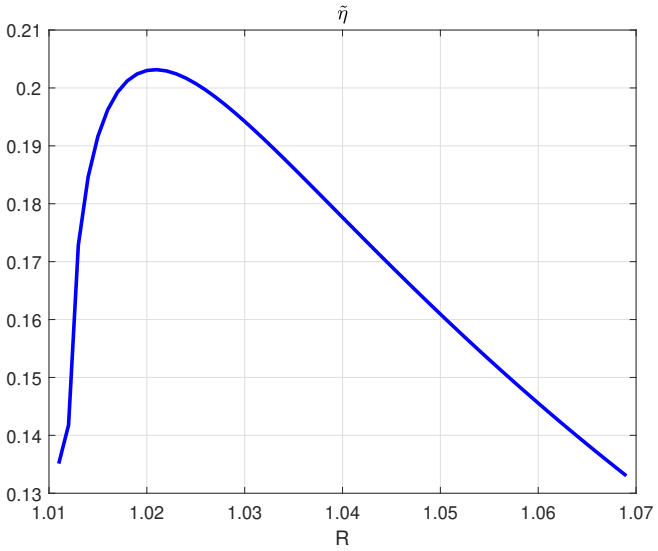


Figure 13: Effect of R on $\tilde{\eta}$ for endogenous $\bar{\theta}$



Note: A numerical illustration with $\beta = 0.96, \Pi^h = 1.3, \Pi^l = 0.95, \Pi^f = 1.07, p = 0.6, \xi = 0.12, \varphi^r = 0.2,$ $\varphi^f = 0.5$, and $F(.)$ being log-normal distribution with $\mu = 0, \sigma = 1.2$.