Stock-Generated Externalities:
The Principle of Targeting and Foregone Environmental Benefits

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Abstract

Conventional wisdom states that targeting either stocks of capital or flows of production could have equivalent effects on externalities correction. However, this may not hold when externalities are generated from stocks that exhibit exogenous growth, such as atmospheric CO_2 removal by trees and water purification by shellfish. I present a dynamic optimization model of privately-owned renewable resource extraction to explore how different incentive targets affect the provision of social benefits from natural resources over time. I find that while stock-targeted incentives can yield socially optimal levels of the natural resource in steady state, levels under flow-targeting (i.e. extraction or harvest) converge to those under no policy or business-as-usual (BAU). This is likely driven by revenue sources for the firm. Stock-targeting allows the firm to flexibly maximize profits over two sources of revenue: the natural resource stock and the private good. By contrast, flow-targeting solely changes the price of the private good, leading to similar profit maximization and extraction behavior as BAU. I then explore the importance of other policy design choices for environmental quality over time by calibrating the model to scientific parameters from the Chesapeake Bay, where Eastern oyster (*C. virginica*) cultivation yields water filtration benefits. Both subsidies on stocks and taxes on harvest can increase environmental benefits, but harvest taxes at scales that produce minor increases also yield substantial losses in producer surplus, suggesting that the costs of such a policy may outweigh its benefits. The model provides a theoretical framework for program design aimed at minimizing environmental damages through the management of privately-owned living natural resources.

Keywords: externalities, natural resources, incentives, targeting, optimal control
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1 Introduction

Conventional wisdom states that externalities scale with flows of production (Pigou, 1920; Ayres and Kneese, 1969). If a firm increases factory output, then greenhouse gas (GHG) emissions increase in tandem. The classic problems of CO_2 accumulation in the atmosphere, overfished species, and the external benefits from the stock of knowledge are all generated from productive activity (Adams, 1990; Kolstad, 1996; Kroetz et al., 2022). This implies that externalities can be fully internalized by incentives directed at flows of private goods production or its inputs (Holtermann, 1976). However, there is an increasingly important category of externalities that are generated by stocks of capital, rather than flows. A question remains as to whether flow-targeted incentives can allow us to fully internalize externalities from stocks of physical assets, which I term "stock-generated externalities," across time. I will attempt to shed light on the issue of optimal policy design for stock-generated externalities in this paper.

A common source of stock-generated externalities are renewable natural resources, otherwise known as natural capital. In terms of positive externalities provision, not only can stocks of natural resources absorb and sequester pollutants, but they can actively filter pollutants at every instant through physiological mechanisms. For example, stocks of trees and shellfish provide CO_2 sequestration and water purification benefits, respectively, in this vein. Analogously, stocks of ecosystems such as wetlands and mangrove forests provide benefits through biodiversity and habitat provision for local species in addition to shoreline defense of coastal real estate across time. On the negative externalities side, herds of livestock (e.g. cows) emit methane whether they are intended for meat production or not. Given the increasing necessity to mitigate the negative impacts of human activity on the environment, a growing body of policies and programs have emerged partnering with industries that privately hold these types of stocks in order to internalize the environmental benefits or damages produced.

One prevalent feature of industries that utilize privately-owned living natural resources is their extractive nature, otherwise known as harvest. Given that the resources are renewable, extraction or harvest leads to a tension in dynamic externalities production. On the one hand, extraction means that the subset of the stock used for private production can no longer produce the externality. On the other, this may leave more resources for the remaining stock in the form of food and crucial micronutrients. Increased availability of these resources can lead to higher growth of the natural resource stock in future periods in the form of larger, already-existing biomass and/or reproduction. One channel may dominate the other depending on the profit maximizing levels of extraction and the presence of other revenue-generating alternatives for the firm both instantaneously and across time. As a result, there is an unclear link between private goods production and externalities in these industries.

Does the policy target, either the stock of natural resources or the flow of production, matter when designing an incentive to internalize a stock-generated externality? To answer this question,

I present a dynamic optimization model of exhaustible resource extraction with exogenous stock growth and joint production of private goods and external benefits.[1] The model draws heavily from the seminal work of Dasgupta and Heal (1974).[2] I focus on a representative consumer with preferences over consumption in addition to the stock of a natural resource, as proposed in Beltratti et al. (1993) and Heal (1998), where the stock grows logistically over time. I present a collection of both centralized and decentralized equilibria in order to explore realizations of the following policies on a representative firm that extracts the natural resource: 1) the Social Planner's benchmark, 2) business-as-usual (BAU) or no policy benchmark, 3) stock-targeted incentives, and 4) flow-targeted incentives. I focus on a setting where the firm maintains sole ownership over the resource within a fixed production area in order to isolate the effects of different policy targets along the intensive margin. Further, I focus on time-varying external benefits provision that scale with the total stock of the natural resource, which captures the general functional forms of ecosystem services provision in mathematical biology, ecology and forestry (Navar, 2009; Jordan-Cooley et al., 2011; Assmuth and Tahvonen, 2018). Despite the simplicity of the model presented in this paper, the conclusions can help inform what firm behavior may underlie the performance of policies that include the extensive margin channel. The policy states highlighted in this paper are informed by existing programs and policies in forestry, fisheries and livestock, which target the accumulation or harvest of biomass in general terms (Mercer et al., 2010; Jack, 2013; Alix-Garcia et al., 2015; Jayachandran et al., 2017; U.S. Department of Agriculture, 2020, 2021; Frey et al., 2021; Sandi, 2023; Kampmann, 2024; Khalil, 2024).[3]

I present both theoretical and numerical calibration results as part of the analysis. The theoretical results show that in equilibrium, a stock-targeted incentive that yields socially optimal levels of biomass stock in the long run is feasible. However, the level of the natural resource under a flow incentive eventually converges to BAU, implying that environmental benefits provision under flow-targeting may be no better than the absence of a policy in the long run.

There are a couple of mechanisms that can explain the lack of firm biomass conservation under flow-targeting. First, a stock incentive generates two sources of revenue for the firm: the stock of the natural resource and the flow of production. This allows flexibility to forego revenues from private goods production in favor of those from the stock, analogous to the behavior of a multi-product firm. By contrast, a flow incentive alters the price of the private good. Given that the firm already engages in profit maximization behavior along the private good dimension under BAU, the flow incentive does little to incentivize them to deviate. Therefore, the level of the stock under a

[1] The model can be easily altered to account for external damages.

[2] The model is also referred to as the Dasgupta-Heal-Solow-Stiglitz (DHSS) model, after pioneering work by Dasgupta and Heal (1974), Solow (1974) and Stiglitz (1974), with existence and characterization results by Mitra (1980) and Mitra et al. (2013). The models used in this paper are also heavily influenced by bioeconomics models in fisheries and natural resources, though I do not explore common resource externalities in this framework (Gordon, 1954; Clark and Munro, 1975; Schaefer, 1991).

[3] There are also programs underway to develop and incentivize the adoption of new technologies that minimize externalities produced from these industries (Quinton, 2019; Watson, 2021). The topic of technical change along the externality, population growth, and private production dimensions will be the focus of future work.

flow incentive will coincide with BAU in the long run. Further, there is a tension in the way a flow incentive alters the choices of the firm dynamically. While increases in the marginal incentive over time may encourage a forward-looking firm to conserve now in order to obtain higher returns in the future, these increases can attentuate these conservation effects by encouraging progressively higher current harvest.

Based on the theoretical results, this begs the question of what a policymaker can *practically* implement? While the principle of targeting states that the first-best policy targets the externality directly, the task of administration is a major issue (Karp, 2011). To put things into perspective, suppose there is a stand of trees. A first-best incentive targeting CO_2 sequestration would require knowledge of not only the total biomass of trees in the stand, but the exact amount of carbon absorbed and retained in biomass and sequestered into soil as opposed to respired out at every instant, specific to the species. Unsurprisingly, such policies are often infeasible due to costly measurement and necessitate the use policies that target proxies or deviate from the first-best along other dimensions, such as scale and timing of policy updates (Segerson, 1988; Schmutzler and Goulder, 1997; Jack et al., 2008; Jacobsen et al., 2020). However, it is unclear what dimension matters most for the design of policies to internalize stock-generated externalities across time. For instance, the theoretical results suggest that a flow policy may perform better than the no policy benchmark in some instants if it is an subsidy that increases over time *or* if it is a tax.

I therefore expand the model to account for stock pollution reduction through the accumulation of natural capital in order to understand what policy design elements matter for environmental benefits provision over time. I calibrate the model using scientific parameters of nitrogen emissions and Eastern oyster (*C. virginica*) production in the Chesapeake Bay, USA. Water pollution due to agricultural runoff is a major issue in the region and oyster aquaculture is a mitigation tool that is being explored by policymakers at both the state and federal levels (National Oceanic and Atmospheric Administration, 2021; Sandi, 2023; National Oceanic and Atmospheric Administration, 2024). I allow the incentives to vary not only by target but by the scale, sign, and time variation of policy updates. These choices are made to mimic administrative or political economy constraints of a hypothetical policymaker and to understand what the most important lever is with respect to policy design.

I find that the sign and scale of an incentive, conditional on target, can matter substantially for natural resource accumulation, while time variation plays a minor role regardless of target. First, the monetary value of incentives may remain constant for a given period of time due to budgetary approval processes or political economy constraints, so I explore how the length of these delays may impact stock accumulation behavior by firms. I find that large increases in the monetary value of incentives generated from longer delays in the policy induce a forward-looking firm to conserve the natural resource more than it would under instantaneous updates, particularly under flow-targeting. However, the level differences in conservation are minimal, with a less than 0.003 – 0.05% increase in biomass relative to instantaneous updates depending on the policy target.

Second, taxes and subsidies may lead to different stock accumulation effects, and the scale of those incentives can play a larger or smaller role conditional on policy target. Targeting the natural resource stock with a positive incentive yields increased environmental benefits relative to flow-targeting and BAU across the different counterfactural policies, consistent with the theoretical results. Larger subsidies on stock accumulation result in noticeable increases in the natural resource stock across time. A tax on harvest or flows also encourages small increases in natural resource conservation and reductions in pollution. However, a harvest tax that is large enough to induce changes in stock accumulation may not be practical to implement. The scale of such an incentive yields negative producer surplus across time with little increase in social surplus. Thus, the costs of implementing a flow tax may outweigh its benefits.

This paper contributes to three major strands of literature. The first is body of work focused on second-best, or imperfect, market-based instruments to control externalities. While much of the core literature has focused on optimal policies (Farzin, 1996; Nordhaus, 1992; Acemoglu et al., 2012; Golosov et al., 2014; Shapiro, 2016), the choice over policy design elements that are most important for internalizing externalities can differ based on the reasons why second-best policies are needed in the first place. Previous literature has focused on deviations in the presence of distortionary taxation (Bovenberg and Goulder, 2002; Fullerton and Wolverton, 2005; Barrage, 2020), uncertainty or opaqueness of externality measurement (Shortle and Dunn, 1986; Schmutzler and Goulder, 1997; Cremer and Gahvari, 2002; Jacobsen et al., 2020), and heterogeneous emitters (Claassen and Horan, 2001; Knittel and Sandler, 2018). This paper takes inspiration from the measurement literature, where different policy targets are more or less imperfect proxies for externalities production. Where this paper differs is by focusing on the problem of stock-generated externalities, rather than those generated from flows, as is standard. Further, this paper incorporates other deviations from the first best in terms of time variation, sign, and scale, in order to mimic the administrative constraints faced by policymakers.

The second are models of renewable natural resources and externalities. The literature stems from both natural resources economics and macroeconomics. The macroeconomic literature in this vein has focused primarily on how pollution negatively affects socially valuable renewable natural resources (Tahvonen and Kuuluvainen, 1991; Arrow et al., 2000). These models emphasize the importance of conservation and the downstream impacts of uncontrolled pollution emissions, but do not highlight that some resources may be sources of pollution mitigation in and of themselves.[4] By contrast, both classic and recent work in natural resources economics, particularly in forestry, has given attention to extractive firm behavior when joint production of private goods and public benefits occurs (van Kooten et al., 1995; Xabadia et al., 2008; DePiper et al., 2017; Assmuth et al., 2021). However, a common feature of these models is the omission of a consumer, partially due

[4]There is also an increasing body of work that highlights the importance of biodiversity both socially and economically. For a comprehensive review see Dasgupta (2021), as well as recent work by Giglio et al. (2024). While I do not focus my analysis on the value of groups of species, the model can serve as a framework to evaluate policies for single species conservation in extractive industries and can therefore be extended to include inter-species interactions within ecosystems.

to the fact that these industries are commodity-based. This prevents analysis of markets where demand may not be perfectly elastic, as in the case of the oyster industry or other specialty agricultural products. As a result, the models may not account for more comprehensive price dynamics post-policy. I help bridge this gap by explicitly modeling social benefits from stocks of natural resources using a general functional form for consumer preferences.

Finally, I contribute to empirical and policy work on programs to internalize stock-generated externalities. In particular, there is a growing set of randomized agricultural programs aimed at internalizing the ecosystem benefits provided by natural resources either within or buffering privately-owned agricultural plots, otherwise known as Payments for Ecosystem Services (PES) (Jack et al., 2008; Alix-Garcia et al., 2015; Jayachandran et al., 2017; Jayachandran, 2023). There are also a number of government and privately-implemented programs incentivizing land conservation and water purification in lieu of (or in conjunction with) farm or aquacultural production, such as carbon credit generation and nutrient credit trading programs (U.S. Department of Agriculture, 2020, 2021; Frey et al., 2021; Sandi, 2023; Taylor et al., 2023). For example, recent legislation passed by the government of Denmark will tax livestock producers of cows and pigs starting in the year 2030. The tax is levied based on the total stock held by producers, which proxies for the total amount of methane emissions generated from the animals' physiological processes (Graham-McLay, 2024; Kampmann, 2024; Khalil, 2024; Sengupta and Fuente, 2024). The authors and designers of these policies generally characterize their programs and policies as separate, particularly with respect to other natural resources. My point of view is that at their core, these programs attempt to address the same underlying problem highlighted in this paper. The hope is that the general framework presented here may help inform researchers and policymakers focused on internalizing stock-generated externalities going forward.

2 Model

I begin with a stylized model of renewable natural resource extraction with costless harvest. Time is continuous and infinite. A representative consumer has preferences over both consumption of a private good C_t and the stock of natural capital S_t, where subscript t represent that the variables change over, but are not explicit functions of, time.

$$U(C_t, S_t) \tag{1}$$

Utility is assumed to be well-behaved ($U_C, U_S > 0$, $U_{CC}, U_{SS} \leq 0$). Preferences are assumed to be separable in C_t and S_t.[5][6] Preferences in S_t are general enough to capture benefits generated from

[5]Separability is a common functional form assumption in environment and natural resources contexts (Tahvonen and Kuuluvainen, 1991; Bovenberg and Smulders, 1995; Heal, 1998).

[6]The version of the model with preferences over the stock of pollution is assumed to decrease in pollution stock and therefore implicitly increase in the stock of the natural resource. See Section 3.1.

both physiological mechanisms from the stock at every instant t, as well as those from the change in the stock, which is a function of the stock at every instant.

Production consists of harvesting H_t amount of the natural resource at every instant. The production function is denoted

$$F(H_t) \tag{2}$$

where $F_H > 0$ and $F_{HH} \leq 0$. The generality of these functional forms allows for modeling flexibility over different preferences and technology assumptions, such as under markets for commodities with decreasing returns to scale.[7]

Meanwhile, the natural resource changes exogenously through function

$$G(S_t) \tag{3}$$

which is logistic in time. This type of function is often parabolic in S_t such that $G(S_t) > 0$ for all $S_t \in [0, \bar{S}]$, with upper bound $\bar{S}$ denoting the population carrying capacity within a given geographic area.[8] Other versions may oscillate if there is some critical population $S_0 \geq 0$ such that growth is either negative or tends quickly towards zero, also called an Allee effect (Dennis, 1989).[9] The model results do not fundamentally change with the presence of Allee effects, so I assume a parabolic function in S_t for ease of exposition.

The stock of the natural resource evolves through the differential equation

$$\dot{S} = G(S_t) - H_t \tag{4}$$

where $\dot{S}$ denotes the change in the natural resource stock with respect to time. Note that Equation 4 is nearly identical to classic wild capture fisheries models in bioeconomics (Gordon, 1954; Clark and Munro, 1975; Schaefer, 1991). Here I explicitly focus on a situation where the firm solely owns the stock of the natural resource within a fixed production area in order to capture the decisions by agents in industries where property rights are more secure, such as the lumber or aquaculture industries. Further, I do not focus on extensive margin effects due to the complexity of environmental characteristics of neighboring geographic areas, such as the presence of wild natural capital stock, water flow, and sediment porosity and permeability, among others. Depending on these characteristics and the production methods of the firm, expansion and contraction of production areas can lead to unclear effects on the levels of the total natural resource stock (both cultivated

[7] Note that increasing returns to scale technologies are not generally found in extractive natural resources-based industries.

[8] I do not consider time-varying carrying capacities in this paper, but time variance may play a role in determining the optimal harvest under the different policy scenarios (Carson et al., 2009).

[9] The parabolic form is called a pure compensation growth function, while a growth function that is both convex and concave but positive up to $\bar{S}$, i.e. a function with weak Allee effects such that $S_0 = 0$, is referred to as a depensation growth function. The functional form with strong Allee effects ($S_0 > 0$) is called a critical depensation growth function. See Appendix Figure A1 for graphical illustrations.

and wild).

2.1 The Planning Problem

The Social Planner's objective is to maximize the discounted present value of social welfare given choices in consumption C_t and the state of the natural capital stock S_t at every instant, where ρ represents the social discount rate and private goods production $F(H_t)$ is equal to consumption for all t.

$$U_0 = \max_{\{C_t, S_t\}_{t=0}^{\infty}} \int_0^{\infty} e^{-\rho t} U(C_t, S_t) dt \tag{5}$$

$$\dot{S} = G(S_t) - H_t$$
$$C_t = F(H_t)$$
$$S_0 > 0$$

The first order necessary conditions (FONCs) yield the following Euler equation, which is also the condition that equates marginal benefits and marginal costs of natural resource stock accumulation for each t.

$$G_S + \frac{U_S}{U_C} \frac{1}{F_H} = \rho - \frac{\dot{U}_C}{U_C} - \frac{\dot{F}_H}{F_H} \tag{6}$$

Y_X denotes the derivative of a function $Y(X_t)$ with respect to X_t, where t is suppressed for clarity. On the left hand side, the marginal benefits of stock accumulation include both private returns in the form of the marginal growth of the stock, as well as public returns, which is represented by the marginal rate of substitution between stock and consumption, scaled by the marginal product of harvest. On the right hand side is the marginal cost of stock accumulation, which includes the change in the household's consumption choices as well as the change in production.

A steady state is defined as a set of allocations $\{C^*, H^*, S^*\}$ such that $\dot{S} = 0$ and $\dot{U}_C = \dot{F}_H = 0$. This gives us the following steady state condition.

$$G_{S^*} + \frac{U_{S^*}}{U_{C^*}} \frac{1}{F_{H^*}} = \rho \tag{7}$$

The social discount rate ρ plays a major role in determining the optimal level of the natural resource stock in a steady state equilibrium. Under constant discount rates, a myopic Social Planner implies $\rho \to \infty$ and a natural resource stock in steady state that is lower than when the population is at its maximum growth, also called its maximum sustainable yield (MSY). In other words, the Social Planner may make choices that deplete the stock in steady state in the presence of myopia. By contrast, a Social Planner that values the utility of future generations the same as those in the present is represented by $\rho = 0$. Under a discount rate of zero, S^* is higher than its maximum sustainable yield, which is the optimal result obtained under the Green Golden Rule (Beltratti et al.,

1993). The choice of discount rates in the presence of environmental externalities is a topic of active debate (see Heal (2017) and Polasky and Dampha (2021) for comprehensive reviews). Given the scope of the problem modeled here, where the representative consumer maintains positive utility in stock of the natural resource, a discount rate closer to zero appears to be more appropriate (Clark, 1973). Therefore, for the remainder of the Section 2, I will assume a social discount rate ρ at or near zero.

Note that the Planning Problem is strongly reminiscent of Beltratti et al. (1993) and Heal (1998). I will further break this model apart to highlight different policy states and how they perform along their equilibrium paths as well as in their respective steady states.

2.2 The Decentralized Equilibrium Under Business-As-Usual

Let there be a world with a representative consumer and a representative firm, where the firm does not internalize the social benefits generated from a stock of natural capital that it owns, i.e. no policy exists. I refer to this world as business-as-usual (BAU).

First, the firm's objective is to maximize the present discounted value of profits from private good production subject to the growth of the natural resource, where θ_t is the firm's internal rate of return at time t.[10]

$$\Pi_0 = \max_{\{H_t, S_t\}_{t=0}^{\infty}} \int_{t=0}^{\infty} e^{-\int_0^t \theta(\tau) d\tau} F(H_t) dt \tag{8}$$

$$\dot{S} = G(S_t) - H_t$$
$$S_0 > 0$$

Market prices are normalized to 1. The first order necessary conditions for the representative firm yield

$$G_S = \theta_t - \frac{\dot{F}_H}{F_H} \tag{9}$$

where the left hand side represents the marginal benefits with respect to stock accumulation, and only consist of the firm's private benefits. The right hand side represent its marginal costs, which consist of its forgone internal rate of return and the growth in the marginal product of harvest.

Second, the consumer must now take into account the evolution of assets they hold in order to make choices over consumption. The consumer owns claims A_t on firm profits π_t, where choices in consumption in a given instant help determine the evolution of the value of future claims $q_t \dot{A}$,

[10] The choice of a time-varying internal rate of return is due to the no arbitrage condition along the equilibrium path, but is also a feature that is commonly observed in firms. See Gormsen and Huber (2023) for a recent overview of time-varying corporate discount rates.

reminiscent of (Lucas, 1978).[11]

$$q_t \dot{A} = A_t \pi_t - C_t \tag{10}$$

In this setting, consumers have little say over how the firm manages their stock of natural resources, but rather influences firm behavior strictly through the consumption of the private good. Under the current model, regulation must be aimed at the producer in order to gain traction.

The consumer therefore maximizes their present discounted value of utility

$$U_0 = \max_{\{C_t, A_t\}_{t=0}^{\infty}} \int_0^{\infty} e^{-\rho t} U(C_t, S_t) dt \tag{11}$$

subject to Equation 10, where claims on assets in the initial period is positive ($A_0 > 0$). Notice that even though the consumer has preferences over the stock of the natural resource, it does not play a role in the consumer's problem directly and, therefore, is taken as given. The FONCs give us the Euler equation

$$\frac{\dot{q}}{q_t} + \frac{\pi_t}{q_t} = \rho - \frac{\dot{U}_C}{U_C} \tag{12}$$

where the left hand side represents the marginal benefits of accumulating assets, and consists of the growth rate of the price of future claims as well as firm profits normalized by the future claims price.

An equilibrium transition path exists if there are allocations $\{C_t, H_t, A_t, S_t\}_{t=0}^{\infty}$ and profits $\{\pi_t\}_{t=0}^{\infty}$ such that

1. The consumer and producer maximize their own objectives subject to the evolution of claims on profits and the natural resource stock, respectively, and
2. Private good and asset markets clear, i.e.
   (a) $C_t = F(H_t) = \pi_t$
   (b) $A_t = 1$

for all t. Note that in order to ensure $2(b)$, the following no arbitrage condition must hold where the consumer's marginal benefit of asset accumulation is equal to the firm's internal rate of return.

$$\theta_t = \frac{\dot{q}}{q_t} + \frac{\pi_t}{q_t} \tag{13}$$

Thus, Equation 9 can be rewritten as

$$G_S = \rho - \frac{\dot{U}_C}{U_C} - \frac{\dot{F}_H}{F_H} \tag{14}$$

which resembles the Social Planner's Euler Equation (Equation 6). However, the marginal benefits from stock accumulation only consist of its private returns in the form of the marginal stock growth

[11] This is conceptually similar to a Lucas tree model under homogeneous agents without uncertainty and, potentially, with actual trees.

because the firm does not internalize the social benefits generated from the natural capital stock.

A steady state equilibrium is defined as a set of allocations $\{C^*, H^*, S^*\}$ and profits $\{\pi^*\}$ such that $\dot{S} = 0$. Under BAU, the following steady state equilibrium condition must hold.

$$G_{S^*} = \rho \tag{15}$$

Given $G(S^*) = H^*$ and a consumer that is as patient as the Social Planner, Equation 15 implies that the firm will aim to harvest at a level that very nearly maximizes the growth of the natural resource stock in order to maximize profits, i.e. harvests at MSY, which I will denote H_{MSY}.[12] This profit maximizing behavior at MSY echoes the classic steady state result in sole-owner fisheries bioeconomic models (Gordon, 1954; Clark and Munro, 1975).

Now that we have both the allocations under the Social Planner's problem and BAU as conceptual benchmarks, we can then begin to explore how social benefits under different policy targets may converge to or differ from either.

2.3 Stock Incentives

Suppose the firm is provided an incentive at time $t = 0$ based on the total stock of the natural resource that it owns. There are no administration costs to policy implementation. The firm's objective function now becomes to maximize the present discounted value of

$$F(H_t) + \sigma(S_t) \tag{16}$$

where $\sigma(S_t)$ is the policy function is differentiable with respect to S_t and is autonomous with respect to t. The consumer's budget constraint is adjusted to include lump sum payments γ_t to fund the program.

$$q_t \dot{A} = A_t \pi_t - C_t - \gamma_t \tag{17}$$

An equilibrium transition path exists if there are allocations $\{C_t, H_t, A_t, S_t\}_{t=0}^{\infty}$, profits and taxes $\{\pi_t, \gamma_t\}_{t=0}^{\infty}$ such that

1. The consumer and producer maximize their own objectives subject to the evolution of claims on profits and the natural resource stock, respectively
2. Private good and asset markets clear, i.e.
   (a) $C_t = F(H_t) = \pi_t$
   (b) $A_t = 1$, and
3. The government balanced budget condition $\gamma_t = \sigma(S_t)$ holds.

[12] The value may be just below MSY due to impatience ($\rho \approx 0$).

The FONCs give us the following condition for the equilibrium transition path.

$$G_S + \frac{\sigma_S}{F_H} = \rho - \frac{\dot{U}_C}{U_C} - \frac{\dot{F}_H}{F_H} \tag{18}$$

Compared to Equation 6, a stock incentive $\sigma(S_t)$ that will yield the Social Planner's allocations for all t exists if the marginal incentive σ_S is equal to the marginal rate of substitution between the natural resource stock and private good consumption, which implies that C_t must be expressed as a function of S_t. Recalling Equation 4, this is not necessarily possible given $\dot{S}$ is endogenously determined by C_t. Therefore, knowledge of S_t may not be sufficient to fully determine C_t. However, we may be able to derive what an optimal policy is in steady state.

Suppose we are in steady state as defined in Section 2.2. Equation 18 reduces to

$$G_{S^*} + \frac{\sigma_{S^*}}{F_{H^*}} = \rho \tag{19}$$

Given $\dot{S} = 0$, we can now express C^* as a function of S^* where

$$C^* = F(G(S^*)) \tag{20}$$

Equation 20 implies that a policy that takes the following functional form

$$\sigma_{S^*} = \frac{U_{S^*}}{U_{C^*}} \tag{21}$$

can lead to the socially optimal levels of the natural resource stock in steady state, as well as the following proposition.

Proposition 1 *Suppose there is an incentive $\sigma(S_t)$ that is differentiable in S_t. If the marginal incentive in steady state takes the form*

$$\sigma_{S^*} = \frac{U_{S^*}}{U_{C^*}}$$

such that $C^* = F(G(S^*))$, then the level of the natural resource stock will converge to the social optimum.

Proposition 1 provides some clarity with respect to policy design. First, the policy must eventually converge to the steady state marginal value under the Social Planner. Trivially, a policymaker may set a constant marginal policy such that Equation 21 holds for all t in order to obtain levels of the natural resource stock that eventually converge to the social optimum. However, there may also be time-varying policies that achieve the same steady state level of the stock.

For illustration, suppose that we have the following consumer preferences

$$U(C_t, S_t) = \ln C_t + \ln S_t \tag{22}$$

with a linear production function $F(H_t) = H_t$, and a Pearl-Verhulst equation with an intrinsic population growth rate $g = 1$ and a population carrying capacity $\bar{S}$.

$$G(S_t) = S_t \left(1 - \frac{S_t}{\bar{S}}\right) \tag{23}$$

Taken together, a steady state marginal policy that satisfies Proposition 1 will take the form

$$\sigma_{S^*} = \frac{U_{S^*}}{U_{C^*}} = \left(1 - \frac{S^*}{\bar{S}}\right) \tag{24}$$

When expressed in aggregate terms, this implies that the following functional form of $\sigma(S_t)$ will yield the socially optimal levels of the natural resource stock in steady state.

$$\sigma(S_t) = S_t \left(1 - \frac{S_t}{2\bar{S}}\right) \tag{25}$$

Assuming $S_t \leq \bar{S}$ for all t, Equation 25 suggests that a time-varying incentive that converges to the socially optimum may closely map to the growth of the natural resource.[13] This is encouraging for policymakers, as many natural resource management strategies already rely on biological stock assessments and estimations of the growth of the stock, updated at regular intervals (U.S. Department of Agriculture, 2022; National Oceanic and Atmospheric Administration, 2023). Note that a marginal policy function σ_S that yields the socially optimal level of the stock in steady state may not have an elementary antiderivative with respect to S_t. However, policies that are defined on a marginal basis are often implemented in practice, so it may be sufficient from a policymaker's perspective to strictly obtain the functional form of the marginal incentive.

A broader implication of stock-targeting is that given $U_C, U_S > 0$ and by Equations 6 and 19, any positive marginal incentive $\sigma_S > 0$ will yield a natural capital stock that is higher than BAU across time. This means that as long as the stock-targeted policy is a subsidy, a policymaker stands to obtain some extra social benefit relative to doing nothing.[14]

The increase in stock accumulation in steady state relative to BAU is likely driven by differences in firm incentives. Under a policy that subsidizes accumulation of the stock, the firm now has two sources of revenue – private goods production and its inventory of the natural resource. This allows the firm flexibility to forego revenue from private production in favor of stock accumulation, which is analogous to the behavior of a multi-product firm. Increases in stock can now yield higher returns

[13] See Appendix Figure A2 for the equilibrium time paths of the natural resource stock under the social optimum and the stock incentive for this example policy.

[14] At least in this stylized world with no policy administration costs. Costs of administration can be accounted for in this model with an explicit policymaker's objective, and is the subject of future work.

with respect to either source of revenue in the future. This then begs the question of whether a policy that incentivizes harvest may be able to achieve similar results. For that, we turn to our last policy.

2.4 Flow Incentives

Now consider a world where the firm is incentivized at $t = 0$ based on some transformation of the flow of production, in this case the total harvest of the natural resource. The firm's objective is to now maximize the present discounted value of

$$F(H_t) + \tau(H_t) \tag{26}$$

where $\tau(H_t)$ is a twice differentiable function of H_t and is autonomous with respect to t. $\tau(H_t)$ may be either an increasing or decreasing function of H_t. As in Section 2.3, the consumer pays for the policy in a lump sum manner and the equilibrium path is defined as before, which yields the following condition along the equilibrium path.

$$G_S = \rho - \frac{\dot{U}_C}{U_C} - \frac{\dot{F}_H + \dot{\tau}_H}{F_H + \tau_H} \tag{27}$$

Equation 27 differs substantially from the Social Planner's transition path (Equation 6) in a couple of key ways. For one thing, S_t does not appear in any term other than the marginal growth of the stock, similar to BAU. Further, $\dot{\tau}_H$ implies that the incentive may have to be time-varying in order for the policy to have an effect on stock accumulation, whereas U_S/U_C could be constant across time.[15] This is likely because an incentive on harvest alters the price faced by producers for private good production, similar to the effect of a tax or subsidy on intertemporal consumption choices.

In order for a flow policy to produce the socially optimal levels of the natural resource stock across time, it must be the case that such an incentive yields the Social Planner's allocations in steady state. Suppose that we are in a steady state as defined in previous sections. The Euler equation is then

$$G_{S^*} = \rho + \frac{\dot{\tau}_{H^*}}{F_{H^*} + \tau_{H^*}} \tag{28}$$

where $\dot{\tau}_{H^*} \neq 0$. Given that the policy is a function of harvest H_t and not explicitly of t, harvest itself must also be time-varying. Note that the change in the marginal incentive with respect to time can also be expressed as

$$\dot{\tau}_H = \tau_{HH}\dot{H} \tag{29}$$

H^* in steady state implies $\dot{H} = 0$, which contradicts $\dot{\tau}_{H^*} \neq 0$. Equation 28 therefore reduces to

$$G_{S^*} = \rho \tag{30}$$

[15] Note that $\tau_H \neq -F_H$ otherwise the path becomes undefined.

which is the steady state Euler equation under BAU.[16] This brings us to the following proposition.

Proposition 2 *There does not exist an autonomous flow incentive $\tau(H_t)$ that can achieve the socially optimal allocations for all t. Further, the stock of the natural resource under such a policy will eventually converge to that under BAU, or no policy.*

Given that the model highlights the most ideal case where the extensive margin channel is shut down and the firm is at its technological frontier, there is only one pathway for the policymaker to alter the choices of the firm. If the policymaker structures an incentive to target harvest, this changes the price firms receive for the private good. The firm's incentive will be to maximize their yields in order to maximize profits in the long run, which is the same behavior we observe under BAU. In other words, the steady state level of benefits provided to society from the natural resource will be no better than the in absence of a policy in the long run. Unlike what we see under stock policies, this implies that a flow policy cannot fully internalize a stock-generated externality across time. Note that the choice of an incentive $\tau(H_t)$ that is autonomous with respect to t drives this result. There may exist a non-autonomous flow policy that can satisfy Equation 28 for all t. However, time-dependence that does not otherwise violate other assumptions may also lead to a degenerate policy trajectory where $\tau_H \to \infty$.

Proposition 2 does not rule out that there may be some t in which flow-targeting can yield improvements in stock accumulation over BAU. Comparing Equations 14 and 27, it must be the case that

$$\frac{\dot{F}_H + \dot{\tau}_H}{F_H + \tau_H} > \frac{\dot{F}_H}{F_H} \tag{31}$$

for some t in order for the firm to accumulate a higher amount of the natural resource stock relative to BAU. This is driven by two components – the marginal incentive τ_H and its change over time $\dot{\tau}_H$. Suppose for illustration that $\dot{F}_H > 0$ along the path. On the one hand, if $\dot{\tau}_H > 0$, then the firm may be incentivized to reduce harvest today if it anticipates that it can obtain higher returns from harvest in the future. However, an increase in the marginal incentive can simultaneously have a dampening effect on stock accumulation through progressively increasing current returns to harvest, so changes in stock accumulation relative to BAU may be minimal. On the other hand, if the incentive is a tax on harvest $\tau_H < 0$, then the firm is incentivized to reduce harvest at each instant as long as the tax remains in place.

In order to gain clarity about how tenuous these improvements may be and to explore other possible policy structures with respect to both flow and stock incentives, I apply more specific functional forms and pollution dynamics to the model. The choices are informed by the inorganic nitrogen emissions problem in the Chesapeake Bay, where cultivation of oysters for food consumption may provide water purification benefits.

[16] The result trivially reduces to BAU in steady state if the policy $\sigma_2(H_t)$ is not twice differentiable.

3 Nutrient Pollution and Oyster Aquaculture in the Chesapeake Bay

Nutrient pollution in water bodies is an increasingly important problem faced by many coastal regions. Nutrient loads, defined as the total amount of substances such as nitrogen and phosphorus measured in a body of water passing a specific geographic location at a given point in time, often come from both point and non-point sources, though the main contributor has been from non-point fertilizer use in agriculture (Randall and Mulla, 2001; Yang et al., 2022). Excess nutrient loads in surface water can lead to eutrophication, or the excess supply of organic matter such as algae and cyanobacteria (Pinckney et al., 2001; Taylor and Heal, 2023). Though high nutrient loads do not often cause direct physical harm to humans, eutrophication can lead to hypoxia (low dissolved oxygen) through the decay of excess organic matter and can increase the prevalence of toxic algae blooms (Anderson et al., 2008; Rabalais et al., 2009; Rabotyagov et al., 2014). Both of these channels can increase mortality in species that are both commercially important and necessary for a functioning aquatic ecosystem. While requirements to set standards for nutrient levels in ground and surface water were established in the Clean Water Act of 1972 and federal criteria were set through the Environmental Protection Agency in the 2000s and 2010s, enforcement at the state level has remained a challenge and efforts to control nutrient emissions from non-point sources has been under-explored and generally ineffective (U.S. Environmental Protection Agency, 2007, 2009; Tomczyk et al., 2023).

One approach to reducing ambient excess nutrients in estuaries and bays, which are typically downstream of agricultural areas, is by encouraging shellfish aquaculture. Shellfish cultivation jointly produces not only private goods (i.e. the harvested shellfish) but positive environmental benefits in the form of nutrient and algae removal services. The mechanisms that link shellfish, such as oysters, to eutrophication mitigation are feeding and absorption. Bivalve shellfish consume particulate organic matter (POM) through filter-feeding and release them as inert biodeposits, as well as absorb nutrients directly into shell and tissue (Dumbauld et al., 2009; Kellogg et al., 2013, 2014; Dalrymple and Carmichael, 2015; Darrow et al., 2017; Bricker et al., 2018; Fulweiler, 2019; Rullens et al., 2019; van der Schatte et al., 2020). There have been many small-scale studies and reviews of nutrient sequestration and biodeposition from oyster and other shellfish cultivation, and the value of the services provided have ranged from $5,500 to $99,000 (2011 USD) on an annual per hectare basis (Songsangjinda et al., 2000; Porter et al., 2004; Grabowski et al., 2012; Rose et al., 2015; Humphries et al., 2016; Darrow et al., 2017; Yu and Gan, 2021).[17] For example, a study of Chesapeake Bay Eastern oyster (*C. virginica*) aquaculture estimated that a farm deploying a floating rack system with 286 oysters/m^2 per raft may be able to remove 331 kg/hectare of nitrogen

[17] Note that improper management and/or design of oyster farms may have detrimental impacts on the surrounding ecosystem, though there is a broad consensus that properly-managed farms provide net overall ecosystem benefits (Porter et al., 2004; Dumbauld et al., 2009; Oo and Oo, 2016; Gentry et al., 2020; Barrett et al., 2022).

and 47 kg/hectare of phosphorous per 12 to 24-month period (Higgins et al., 2011).[18] Based on a review by Rose et al. (2015), the costs of shellfish farming as a eutrophication mitigation approach tend to be far lower than comparable mechanical methods such as urban stormwater treatment facility upgrades.

There are a few existing government programs to incentivize the production of nutrient removal services by oyster farmers. The Chesapeake Bay Nutrient Credit Trading Program is one such example. A total maximum daily load for nitrogen, phosphorus and sediment was initially set in 2010 for the Chesapeake Bay Watershed to limit the use of fertilizers by land-based farms upstream of the Bay.[19] The Maryland Departments of Environment and Agriculture implemented their nutrient credit trading program in 2018 (Sandi, 2023). In 2019, oyster aquaculture operations were included as voluntary participants under the Maryland program. Credits from oyster farming can be generated from harvest of up to four different size classes, where size is positively correlated with time held in the water. However, the majority of the oysters cultivated in the region are intended for the fresh half-shell market and, based on industry accounts, a vast majority of half-shell consumers prefer oysters that are within the small or extra small size grades (approximately 2 to 3 inches in length). Therefore, incentives to harvest at larger size classes may be weak. In addition, though more credits are generated from larger oysters, the per unit difference in the generation of each credit is small (Maryland Department of the Environment, 2023). Finally, denitrification has been more strongly associated with unharvested oyster reefs rather than harvested oyster aquaculture (Kellogg et al., 2014; Bruce et al., 2021). Taken together, evidence suggests that policies aimed at reducing nutrient pollution through oyster aquaculture may do better to incentivize oyster biomass accumulation rather than harvesting activities, as is currently implemented in the Chesapeake Bay Nutrient Credit Trading Program.

I add more specificity to the model from Section 2 to include the functional forms and parameters from the theoretical and scientific literature, as well as more specific pollution dynamics informed by this regional and regulatory context. There are other benefits to using this setting for calibration purposes. First, oysters are filter feeders, which means oyster farmers do not incur feed costs for their stock, similar to lumber and unlike the livestock and fin fish aquaculture industries. Given that the model abstracts away from input costs, the lack of feed costs and minimization of labor costs through production methods that utilize tidal energy for this industry allows for the parameters used to be more directly applicable. Second, farms are typically of limited size and additional leases are notoriously hard to obtain (Metcalf, 2023). Therefore, limiting the scope of the problem

[18] Common growing methods in the region are rack and bag systems, where the oysters are flipped or turned using manual labor or, primarily, tidal energy (University of Maryland, 2024). These methods prevent Eastern oysters from "cementing" to each other to form complex reef systems. The formation of oyster reefs, while ecologically valuable, can lead to both mortality through sedimentation and misshapen oyster meat, which farmers are generally unable to sell (Jordan-Cooley et al., 2011; Bruce et al., 2021). While other commercially-valuable oyster species like Pacific oysters (*C. gigas*) also form reef structures, some species such as Olympia oysters (*O. lurida*), which are native to the West Coast, do not use other individuals as substrate (Escapa et al., 2004; Brumbaugh and Coen, 2009).

[19] The jurisdictions for the watershed include Delaware, Maryland, New York, Pennsylvania, Virginia, West Virginia and the District of Columbia (U.S. Environmental Protection Agency, 2015).

to a fixed production area is appropriate based on the industry's regulatory constraints. Finally, though Maryland's Nutrient Credit Trading Program is currently active, oyster farm participation has been low in terms of total credits generated, which allows for a relatively clean interpretation of the calibration results (Maryland Department of the Environment, 2023).

3.1 Model Characterization Under Nutrient Pollution

Given the generality of the model presented in Section 2, here I provide the functional forms fed into the numerical calibration as well as the parameters used as inputs to the model. First, let the consumer have preferences over consumption C_t and the stock of pollution D_t, where pollution is taken as given. I apply a CRRA utility function for the consumer that is separable in both consumption and the stock of pollution. The utility function for the representative consumer is defined as

$$U(C_t, D_t) = \log(C_t) - \frac{D_t^{1-\eta} - 1}{1-\eta} \tag{32}$$

where $\eta < 1$. For simplicity, the representative consumer is modeled as having risk-neutral preferences over consumption C_t.

Turning to the firm, private goods production is specified as a decreasing returns to scale Cobb-Douglas production function

$$F(H_t) = H_t^{\alpha} \tag{33}$$

such that $\alpha \in (0, 1)$. This is to reflect the findings in the production function estimation literature, where extractive renewable natural resources industries, such as the oyster industry, generally exhibit decreasing returns to scale (Stier, 1980; Nautiyal and Singh, 1985; Scuderi and Chen, 2019).[20][21]

The growth function of the natural resource is a Pearl-Verhulst logistic function

$$G(S_t) = gS_t\left(1 - \frac{S_t}{\bar{S}}\right) \tag{34}$$

where $g = r - m$ is the intrinsic population growth rate, equal to the recruitment rate r (total spat divided by total adult oysters) less the mortality rate m, and $\bar{S}$ is the environmental carrying capacity within the cultivation area (Schaefer, 1991). While the Pearl-Verhulst growth function abstracts away from other environmental factors that can affect population growth such as interspecies interactions and exogenous environmental pressures, the model contains enough of the key features of population growth to serve as an initial benchmark for modeling and is thus considered

[20] This can be due to factors external to the firm. For example, a lumber company that fells a tree may not observe ex ante that the lumber is low quality due to parasite infestation, adverse weather events, etc. They may therefore need to discard timber that would otherwise have been sold with better quality wood.

[21] Technical change is not explored in this analysis.

the workhorse in mathematical ecology (Berryman, 2003; Shorrocks, 2024).[22] Based on Equation 34, the level of the stock that attains maximum sustainable yield is $S_{MSY} = \bar{S}/2$ and the steady state level of harvest that attains maximum growth, or MSY, is $H_{MSY} = G(S_{MSY})$.

The primary addition to the model is the evolution of the the stock of pollution $\dot{D}$

$$\dot{D} = ne - B(S_t) \tag{35}$$

where ne represents emissions net of decay, and $B(S_t)$ is the total amount of pollution removed by the stock of the natural resource at every instant.[23] Net emissions are defined as

$$ne = e_{enter} - e_{exit} \tag{36}$$

e_{enter} represents the total amount of exogenous emissions of inorganic nitrogen (N) that flow into the Chesapeake Bay watershed, measured in kilograms (kg) per year, and e_{exit} is the estimated total amount of inorganic N exiting the watershed through the surface water network (i.e. rivers and streams). Given that the watershed system is complex and unstable, the equation below is used as a stylized back-of-the-envelope calculation for e_{exit} based on information reported in Lindsey et al. (2003), Sanford and Pope (2013), Lee et al. (2014) and Clune et al. (2021).

$$e_{exit} = SW \cdot RE \cdot e_{enter} \tag{37}$$

SW is the proportion of water that enters the surface water network from either the groundwater system or from soil runoff, and RE is the proportion of surface water per year that exits the Bay via river export. ne therefore represents the total amount of inorganic N that remains in the watershed per year.

Finally, nitrogen filtration benefits are based on Jordan-Cooley et al. (2011), which presents a model of oyster reef population growth with filtration and sedimentation. Given that I focus on non-reef oyster aquaculture, I simplify their filtration function to account for the fact that sedimentation is no longer a factor obstructing oyster filtration. This is due to the prevailing aquaculture techniques of rack and bag culture, which prevent oysters from using other individuals as substrate.

$$B(S_t) = F_{max} S_t \tag{38}$$

F_{max} is the maximum filtration rate.[24]

[22] Other functions can be substituted in that account for other population growth characteristics, such as Allee effects, as mentioned in Section 2 (Liermann and Hilborn, 2001).

[23] This implies that D_t is a function of S_t. The model in Section 2 is therefore a generalization of the one presented here.

[24] Filtration with sedimentation in Jordan-Cooley et al. (2011) is a function of oyster biomass

$$F(S_t) = \frac{F_{max}}{[Cg]_{max}} [Cg(S_t)] e^{\frac{[Cg]_{max} - Cg(S_t)}{[Cg]_{max}}}$$

The complete model with nutrient pollution dynamics under the Social Planner's problem can therefore be written as

$$U_0 = \max_{\{C_t, S_t\}_{t=0}^{\infty}} \int_0^{\infty} e^{-\rho t} \left(\log(C_t) - \frac{D_t^{1-\eta} - 1}{1-\eta} \right) dt \tag{39}$$

$$\dot{S} = g S_t \left(1 - \frac{S_t}{\bar{S}} \right) - H_t$$
$$\dot{D} = ne - F_{max} S_t$$
$$C_t = H_t^{\alpha}$$
$$S_0 > 0, D_0 > 0$$

where the Euler Equation is

$$g \left(1 - \frac{2S_t}{\bar{S}} \right) - \phi_t F_{max} \frac{C_t}{\alpha H_t^{\alpha-1}} = \rho - \left(\frac{\dot{C}}{C_t} \right)^{-1} - \frac{[\dot{H_t^{\alpha-1}}]}{H_t^{\alpha-1}} \tag{40}$$

$\phi_t < 0$ represents the shadow cost of nitrogen pollution stock accumulation. ϕ_0 is calibrated to \$10/kg N at $t = 0$ based on the upper bound of the social cost of nitrogen reported in Keeler et al. (2016). $-\phi_t F_{max} = U_S$ is therefore the marginal value to society of an additional kg of oysters accumulated with respect to the nitrogen filtration benefit it provides at time t. Note that the trajectory of the stock of pollution may not reach a steady state. This is due to fact that, first, the level of pollution is taken as given by the consumer where the only method of control in this setting is natural resource stock accumulation, and second, there are biological constraints on stock accumulation that can prevent a steady state level of pollution to be reached, even under the Social Planner. The rest of the results follow as in Section 2.[25]

Equation 40 also suggests use of the following time-varying marginal subsidy with respect to the stock.

$$\sigma_S = -\phi_t F_{max} \left(g S_t \left(1 - \frac{S_t}{\bar{S}} \right) \right)^{\alpha} \tag{41}$$

As mentioned in Section 2.3, Equation 41 does not yield an elementary antiderivative with respect to the stock of natural resources due to the presence of a decreasing returns to scale production function. However, given that the marginal policy function is a monotonic transformation of the stock's growth function, the marginal incentive may still be implementable given S_t is known.

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where C is the maximum possible deposition rate, $g(S_t)$ represents sedimentation on deposition ($g(S_t) < 0 \ \forall \ t$), and $[Cg]_{max}$ is the maximum possible deposition given sedimentation for a given t. Without sedimentation (setting $g(S_t) = g_{max}$) yields

$$F(S_t) = F_{max}$$

The filtration function notation used above was reported in Lipcius et al. (2019).

[25] Other additions to this model can include costs to natural resource accumulation or harvest. The addition of these costs do not fundamentally alter the conclusions of this paper.

The parameters used for numerical calibration are presented in Table 1, along with the sources for each parameter where applicable. Reports of individual oysters of commercial harvest size (76 mm or approximately 3 inches shell length) are converted to kg of wet weight biomass using Mann et al. (2009).[26] Carrying capacity is approximated as the maximum commercial harvest of Eastern oysters between 1993 and 2022, and serves as a lower bound of the true environmental carrying capacity. Filtration rate F_{max} was converted from a meta-analysis reported in Cornwell et al. (2023) for annual N per acre per year removed by 300 grams of oyster dry weight per square meter.

Table 1: Parameters and Sources

Parameters		Estimate	Source
Recruitment rate	r	0.4349	Maryland Department of Natural Resources (2020)
Mortality rate	m	0.1	Doering et al. (2021)
Carrying capacity	$\bar{S}$	20,411,640 kg	Tarnowski (2023)
Nitrogen loadings	e_{enter}	131,541,680 kg/yr	Clune et al. (2021)
Surface water	SW	0.5	Clune et al. (2021)
River export	RE	0.72	Lee et al. (2014)
Filtration rate	F_{max}	5.29 kg/kg/yr	Cornwell et al. (2023)
Social discount rate	ρ	2%	Council of Economic Advisers (2024)
Output elasticity	α	0.8	Industry Sources
Pollution risk aversion	η	0.75	
Initial harvest	H_0	$H_{MSY} \cdot 0.75$	
Initial stock	S_0	$S_{MSY} \cdot 0.45$	
Initial pollution	D_0	10,000,000 kg	

The harvest technology employed in this model does not provide a one-to-one comparison with the production inputs reported in previous production function estimation studies of the oyster industry, so I choose $\alpha = 0.8$ as an informed starting point based on conversations with industry professionals.[27] The pollution risk aversion parameter and initial values are chosen to illustrate the theoretical results graphically.[28]

3.2 Numerical Analysis

All numerical calibration results are generated in Matlab using `ode45`. I use the equilibrium conditions and functional forms stated above as inputs. The system is allowed to optimize over 100 years

[26] All dry weight measures are converted to wet weight by multiplying dry weight in g by 100. One standard bushel of oysters weighs approximately 45 to 60 lbs and contains between 100 to 150 oysters. Conservatively, the smallest average market size oyster (76 mm shell length) weighs 4.8 oz unshucked. Given that an oyster shell weighs 1 to 2.5 oz, 1 g of oyster dry weight is equivalent to 3.5274 oz wet weight, which is the approximate wet weight of shucked oyster of market size (Grizzle et al., 2017).

[27] The value of α can also vary with environmental and regulatory conditions, which I do not explore in this paper.

[28] For example, choosing $H_0 > G(S_0)$ will lead to population collapse and, therefore, uninformative transition paths.

(i.e. $T = 100$) for ease of graphical display, though is expected to continue along the steady state trajectories as $t \to \infty$. Initial guesses for the transition path of harvest are made under each policy scenario. The guesses are flexibly chosen based on total surplus maximization in present values over the following functional forms and start at the same initial harvest H_0: linear, exponential, and sigmoidal. Given that BAU and the flow incentive exhibit a closed form solution for the stock in steady state where $S^* \approx S_{MSY} = 10,205,820$ kg, harvest at the end of the defined time period is $H_T = H_{MSY} = 1,708,855$ kg. The time paths under the Social Planner do not have closed form solutions in steady state. However, the results suggest that S^* under the Social Planner is greater than under BAU or the flow incentive, so I set $H_T = H_{MSY} \cdot 0.8$ for illustrative purposes.[29] The specific functional forms and total surplus results are presented in Appendix Section B. The transition paths are adjusted from their initial guesses over 25 iterations with a step size of 0.000001.

3.2.1 Theoretical Benchmarks

It is important to first understand how the stocks of the natural resource, pollution, harvest and further, welfare evolve across time under the theoretical benchmarks in order to glean how policy deviations may help or hinder environmental benefits provision. Figures 1–4 present the time paths of the aformentioned states, choices, and the utility of a representative consumer.

Figure 1 presents the evolution of the natural resource stock across time, where the blue line represents the trajectory under the social optimum and the green line is the equilibrium path under no policy, or BAU. The level of the natural resource stock under the social optimum diverges away from the BAU benchmark quickly and converges to the steady state level of the stock at approximately 14.7 million kg of biomass within 40 years. The pollution stock (Figure 2) similarly diverges between the two benchmarks, where the stock of pollution under the social optimum is lower than BAU for all instants except $t = 0$, though the trajectories more noticeably diverge at $t = 8$. This is driven by differences in harvest (Figure 3), where the level of harvest under BAU is much higher across time than under the social optimum. The trajectories align with the results of Sections 2.1 and 2.2, where the level of harvest in steady state under the social optimum will be less than the profit-maximizing level under BAU. Lower levels of harvest result in a natural resource stock that is higher than under the firm's maximum sustainable yield $S^* > S_{MSY}$ in steady state. Figure 4 shows the trajectories of utility in both current (Panel (a)) and present values (Panel (b)). In both figures, the level of utility for a representative consumer is consistently higher across time under the social optimum due to the avoided disutility pollution, in spite of the lower levels of consumption across time.

The following sections explore how proxy policies, either incentives on stock accumulation or harvest, along with deviations in terms of sign, scale, and time variation, can affect dynamic environ-

[29] The trajectories are allowed to adjust depending on values set for H_0 and H_T and do not fundamentally change the results, though the trajectories generated from some starting points are less informative than those highlighted here.

mental benefits provision. Given the correspondence between pollution removal, the stock of the natural resource, and harvest, I primarily focus on the trajectories of the natural resource stock. The time paths for pollution stock, harvest, and surplus under each policy scenario are presented in Appendix Figures A3–A26.

3.2.2 Policies that Vary on Sign and Scale

Based on Equation 18, a stock-targeting policy must be a subsidy in order to influence the firm to conserve more of the natural resource than under BAU across time. Figure 5 presents the time paths of the natural resource stock under different constant per unit subsidies targeting stock accumulation relative to the BAU and socially optimal benchmarks. The policies vary by size of the subsidy to illustrate how scale can affect stock accumulation. The yellow dotted line ("Small") represents the natural resource stock trajectory under a constant per unit policy set at 1000 USD. The orange dotted line ("Medium") are the levels under a constant per unit subsidy at 10,000 USD, while the red dotted line ("Large") are the levels under a constant per unit subsidy at 445,000 USD. Finally, the purple dotted line ("Very Large") are the levels under a constant per unit subsidy at 8,092,113.71 USD, which is the value of stock accumulation at $t = 0$ under the social optimum multiplied by a factor of two. The values of per unit subsidies are chosen for illustrative purposes.

Consistent with the predictions in Section 2.3, increasing the scale of a stock subsidy increases accumulation of the stock of biomass by the firm. However, the relationship appears to be non-linear, given that order of magnitude increases in the value of the subsidy yield lower relative increases in biomass accumulation. While a "very large" subsidy may lead to gains in the natural resource stock in excess of the social optimum, the gains in consumer surplus (Figure 6 Panel (a)) combined with the gains in producer surplus (Figure 6 Panel (b)) indicate that total welfare may be lower relative to a policy that yields a time path of producer surplus that more closely mimics that under the Social Planner (see Section 3.2.3).

Given that Section 2.4 supposes that either a tax on harvest or a subsidy that increases with respect to time may incentivize a firm to accumulate more natural resource stock, I employ constant per unit harvest taxes that mimic the constant per unit stock subsidies exercise. Figure 7 presents the time paths of the natural resource stock under different constant per unit taxes targeting harvest relative to BAU. The policies take on the negative values of the policies utilized in Figures 5 and 6 except for the "very large" tax, which is equal to 20,230,284.25 USD, or the negative of the value of stock accumulation at $t = 0$ under the social optimum multiplied by a factor of five. The purpose of the different "very large" monetary value under flow-targeting is to yield more meaningful deviations in the time path relative to BAU which, as shown in Figure 7, still tracks closely with the BAU path of stock accumulation despite the increased size of the tax.

In general, a flow-targeting policy does little to change firm behavior, which supports the mechanism proposed in Section 2.4 where given a fixed production area, firms may not deviate from their

profit-maximizing level of harvest even in the presence of disincentives to produce the private good. Taxes generally push the firm to harvest slightly less and accumulate higher amounts of the natural resource across time, with larger taxes encouraging more conservation. This supports the interpretation of the results in Section 2.4, where the sign of τ_H can play a role in the total benefits provided to society from the natural resource stock. Such a policy comes at a major cost to firms, which experience large negative profits across time without substantial gains in consumer surplus, especially at values of taxes that would yield even somewhat noticeable increases in biomass relative to no policy (Figure 8).

3.2.3 Time-Varying Policies

Figures 9 – 12 present the time paths of the natural resource stock under both instantaneously updating incentives and policy deviations through time variation. First, Proposition 1 proposes that for a stock policy, any marginal incentive that equals the marginal rate of substitution between the stock and consumption in steady state can yield time paths of the natural resource stock that converge to the social optimum. Figure 9 shows these trajectories under two such policies – one that varies with respect to time ("Time-Varying") and one that is set at the steady state value of stock accumulation for all t ("Steady State"). The time paths under each policy coincide closely with the social optimum, where the steady state marginal incentive yields a time path that is within 0.002% of the social optimum. The higher initial levels of biomass under a time-varying stock subsidy is primarily from the approximation of the marginal rate of substitution, where the initial values of the policy do not fully account for the effects of harvest on the growth of the natural resource (Appendix Figure A11). However, the difference between stock accumulation under a time-varying stock subsidy and the social optimum is small (within 0.015% of the social optimum).

Second, I implement marginal incentives with "chunky" subsidies, which are discontinuously updated every five, ten, 25, or 50 years (the purple, red, green, and dark blue dashed lines, respectively), highlighted in Figure 10. The firm has perfect foresight and expects the policies to be updated at these intervals from time $t = 0$. The lags in monetary values of incentives are to mimic potential political economy or administrative constraints that could prevent instantaneous updating by a policymaker. Similar to the trajectories in Figure 9, holding policies constant for a given number of years does not make a notable difference in stock accumulation by the firm, though lags can somewhat affect the level of biomass held by the firm. The small but progressively increasing levels of the stock relative to the social optimum by policy lag are driven by the fact that the value of the marginal incentive is higher than under an instantaneously-updating policy for each time step, where the value of the policy that updates every 50 years is the largest for the longest amount of time (Appendix Figure A16). However, the differences in stock accumulation are minimal, ranging from within 0.02% to 0.06% relative to the social optimum.

I further explore how time variation in flow policies may affect biomass accumulation. While the

model does not yield a functional form for an optimal flow incentive, Equation 31 suggests a flow policy that increases over time may affect firm behavior more than a static alternative. However, the size of these changes in the policy with respect to time plays an unclear role in natural resource stock accumulation. Therefore, I assume that a hypothetical policymaker has made a "best guess" at policy structure by setting it to the ideal policy under the Social Planner ($\tau_H = U_S/U_C$), which is analogous to the oyster harvest instrument used in the Chesapeake Bay Nutrient Credit Trading Program.[30] This choice does not imply that optimal allocations will be attained, based on Proposition 2. As shown in Figure 11, the transition paths of stock accumulation under BAU and the flow incentive tightly coincide, as expected in theory. It appears that a flow subsidy produces lower biomass accumulation relative to BAU (within 0.05%) across time, supporting the hypothesis in Section 2.4 that small instantaneous increases in the subsidy may induce a firm to harvest more today if it does not foresee large returns to conserving biomass for future production.[31] This is further supported by the results in Figure 12, which implement the "chunky" policy updates employed in Figure 10. Small instantaneous increases in a flow policy from the five and 10 year policy updates result in lower levels of biomass accumulation relative to larger increases under the 25 and 50 year lags in policy adjustments, though the paths coincide closely with the time-varying flow policy with instantaneous updates. The results together suggest that time variation in either stock- or flow-targeted policies may not play a strong role in changing firm accumulation of natural resources and, therefore, environmental benefits provision.

4 Conclusion

Typically, economists have assumed that internalizing environmental externalities can be adequately done by targeting flows of production. However, this does not seem to be the case when externalities are generated from stocks of living natural resources. I employ a dynamic partial equilibrium model of renewable natural resources extraction using a continuous time framework to evaluate how differences in policy targets contribute to the efficacy of such programs. I find that policies that do not target the stock of the natural resource may provide little additional social benefit in the long run. Subsidies that target accumulation of the natural resource stock directly provide higher benefits relative to BAU, where benefits can converge to the social optimum under appropriately-designed policies.

Numerically, this conclusion is explored through counterfactual policies to address inorganic nitrogen pollution in surface and groundwater. The model is calibrated to inorganic nitrogen pollution and hydrological parameters from the Chesapeake Bay, in addition to biological and industry-

[30] H_T can be expressed as a function of S_T given $F(H_T)$ is monotonic. Therefore $\tau(H_t)$ can be expressed as a function of strictly H_T.

[31] Note that the trajectory of the stock under BAU and flow-targeting have not fully reached a steady state. However, the trajectories tend towards the profit maximizing level of the stock as t increases, and deviations away from BAU under flow-targeting are estimated to converge back to BAU within 140 years.

informed parameters with respect to Eastern oyster production and the water purification benefits provided. I vary these policies by sign, scale, and time variation, conditional on policy target. The calibration results support those derived in theory with respect to the targets of each policy. Further, I find that the scale and sign of the policies matter substantially for the trajectory of allocations under stock-targeting, while the time variation of policy updates matters little regardless of target. In particular, taxes incentivize firms to conserve the natural resource under flow-targeting. However, taxes may not be practical due to negative profits across time relative to gains in consumer surplus, which appear to be minimal.

The results can be explained by the source of revenues for the firm under different targets. First, under a stock-targeting, the producer effectively operates as a multi-product firm. This allows for the firm to forego production of the private good in favor of maximizing revenues from the stock of the natural resource, which leads to increases in natural resource conservation. Second, under flow-targeting, the firm continues to obtain revenues from one source, which is private goods production. Under BAU, the firm's incentives are to maximize their sustainable yield with respect to the stock, which is at a level where the growth of the natural resource is at its highest. Thus, with a flow policy, a firm has little incentive to deviate from this behavior, particularly in steady state. Though a flow policy may have some bite if it is time varying, the scale and sign of the policy seem to matter more for stock accumulation than how it changes over time. This is because increases in natural resource stock in anticipation of higher future returns to harvest can be simultaneously attenuated by increases in current returns. Therefore, policymakers partnering with industries that extract renewable natural resources would be well advised to implement policies that directly target the accumulation of those stocks, as in many PES programs, in contrast to those that target harvest.

There are several limitations to this work, as well as potential avenues for model extension. First, the model does not account for the costs of policy administration, which can influence the relative attactiveness of some policy targets versus others. Based on the numerical exercise, results suggest that in the presence of positive program implementation costs, the benefits of a flow subsidy may not be enough to warrant program implementation. Further, the costs of stock subsidy can also outweigh the benefits if a policymaker wants to obtain levels of the natural resource that are higher than the social optimum.

Second, I assume one large "plot" over which production occurs, which strictly captures intensive margin effects. It is important to recognize that flow incentives may affect natural resource accumulation in ambiguous ways along the extensive margin. These policies may, in fact, allow for an increase in natural capital stock if the geographic areas over which firm production occurs has sparse or no natural capital stock prior to firm entrance. However, the effects of any flow-targeted policies aimed at increasing natural capital stock through the extensive margin will be attenuated through the firm profit maximization via harvest, as highlighted in this paper. Further, the analysis does not capture effects on the total stock of the natural resource (i.e. those that are both wild and cultivated). While firm activity may increase the stock of natural capital in places where the

stock is sparse ex ante, it may also lead to a decrease in areas that have an abundance of natural capital through production practices that effectively "spread out" the stock in order to maximize private goods production along cost minimization or quality dimensions. This is potentially the case in the oyster industry, where rack and bag aquaculture may displace densely-populated naturally occurring reef structures if incentives to encourage additional aquaculture leases are present.

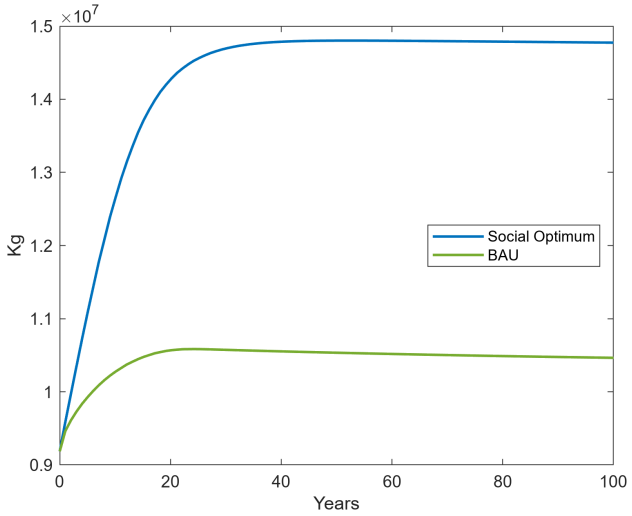
Third, the analysis focuses on a class of policy functions that are not an explicit function of time. This restriction is particularly important for the flow incentive, where it may be the case that a policy that is a function of time can achieve the socially optimal transition paths. Such a policy may need to be monotonically increasing in time to affect the firm harvest choices in the socially optimal direction. In addition to the reasons stated in Section 2.4, considering again the problem of government administrative costs and budget constraints, a subsidy that is an increasing function of time would be fiscally impractical, and is therefore not a class of incentives featured in this paper.

Fourth, the model abstracts away from firm heterogeneity. The introduction of heterogenous firms on either the pollution or land quality dimension may provide insight over how payment schemes need to adjust to account for differences across firms, in line with Xabadia et al. (2008). Fifth, the numerical calibration currently employs a linear environmental benefits function based on common growing methods in the case study of focus. Given the model of water filtration presented in Jordan-Cooley et al. (2011), these functions can be highly nonlinear in the stock of biomass, and the potential non-convexities may contribute to different policy performance conclusions (Starrett, 1972; Sudit and Whitcomb, 1976). Sixth, this is a setting in which only one good exists. Possible extensions could include the introduction of substitutes or complements to the natural resource-based good presented in this model.

Finally, technological change in either production, the growth of the natural resource stock or environmental benefits provision is absent. Agricultural research institutions often involve themselves in the development of innovations on all three dimensions (Nell, 2002; Dégremont et al., 2015; Villanueva-Fonseca et al., 2017; Quinton, 2019; Wadsworth et al., 2019; Watson, 2021). In future work, I plan to incorporate technical change along both of these dimensions in order to understand the role it may play in optimal policy design to internalize stock-generated externalities. In addition, though the issues highlighted here highlight the importance of cultivated living natural resources as sources of external benefits or damages, the model can be extended to include other natural resources management problems such as single or multi-species conservation, natural shoreline protection, and groundwater management. In addition, though the issues highlighted here highlight the importance of cultivated living natural resources as sources of external benefits or damages, the model can be extended to include other natural resources management problems such as single or multi-species conservation, natural shoreline protection, and groundwater management. The broader hope is that the model presented in this paper serves as useful tool for both policymakers and researchers as we continue to explore both market-based and regulatory tools to minimize damages to the environment.

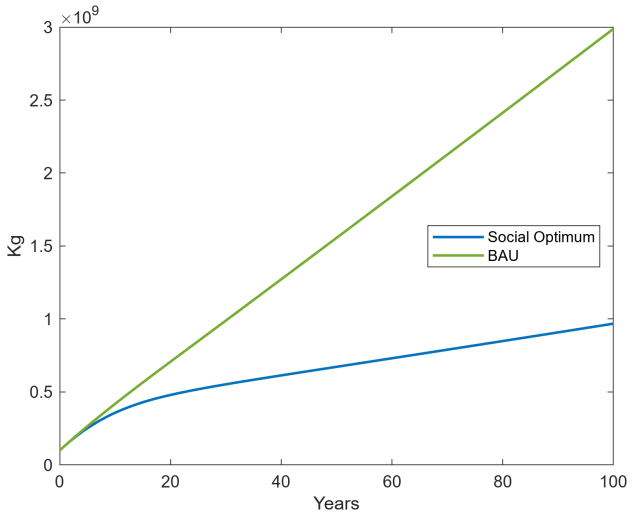
5 Figures

Figure 1: Natural Resource Stock Trajectories Under Theoretical Benchmarks



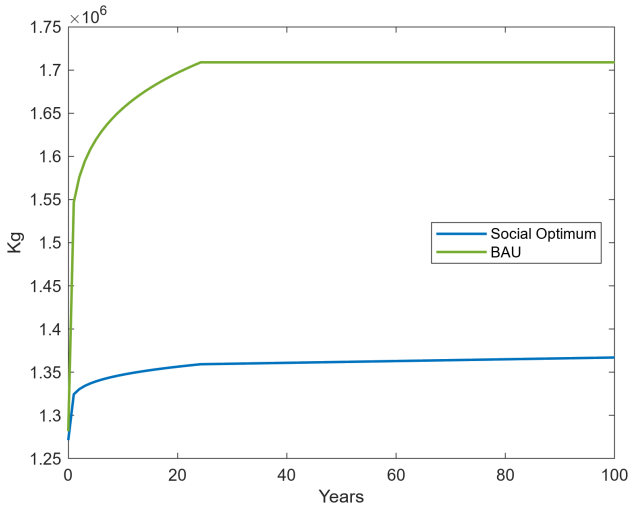
The lines represent the levels of the natural resource stock over time in kilograms under the two theoretical benchmarks. The trajectories are generated from the model specification with pollution dynamics (Section 3.1) and t is in years. The blue solid line represents the trajectory under the Social Optimum, and the green line represents the trajectory under the no policy, or BAU, benchmark.

Figure 2: Pollution Stock Trajectories Under Theoretical Benchmarks



The lines represent the levels of inorganic nitrogen stock over time in kilograms under the two theoretical benchmarks. The trajectories are generated from the model specification with pollution dynamics (Section 3.1) and t is in years. The blue solid line represents the trajectory under the Social Optimum, and the green line represents the trajectory under the no policy, or BAU, benchmark.

Figure 3: Harvest Trajectories Under Theoretical Benchmarks

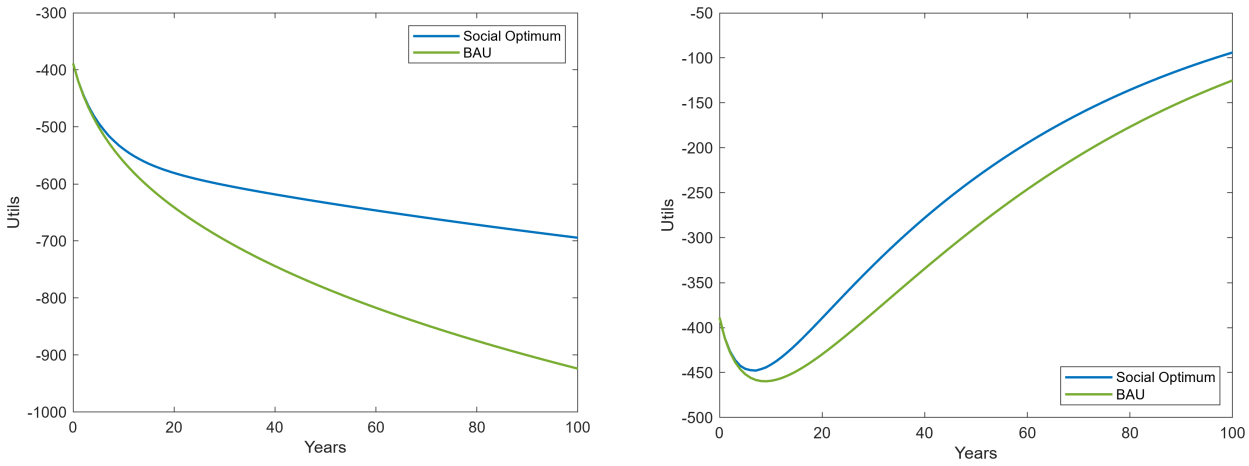


The lines represent the levels of harvest over time in kilograms under the two theoretical benchmarks. The trajectories are generated from the model specification with pollution dynamics (Section 3.1) and t is in years. The blue solid line represents the trajectory under the Social Optimum, and the green line represents the trajectory under the no policy, or BAU, benchmark.

Figure 4: Utility Trajectories Under Theoretical Benchmarks

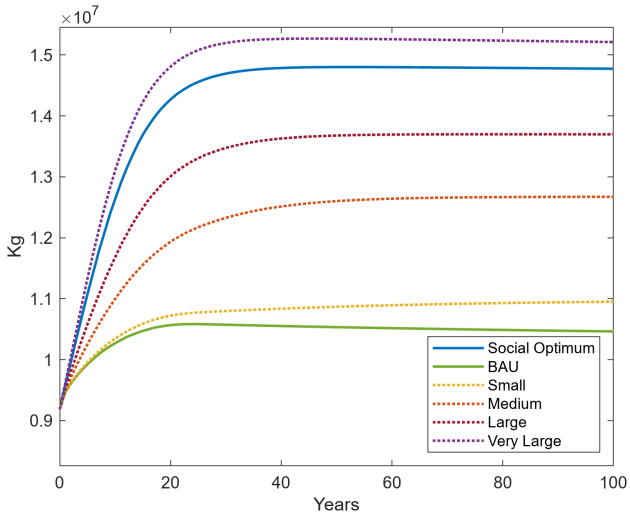
(a) Current Values

(b) Present Values



The lines represent the levels of utility over time for a representative consumer under the two theoretical benchmarks. The trajectories are generated from the model specification with pollution dynamics (Section 3.1) and t is in years. The blue solid line represents the trajectory under the Social Optimum, and the green line represents the trajectory under the no policy, or BAU, benchmark.

Figure 5: Natural Resource Stock Trajectories Under Constant Marginal Stock Subsidies

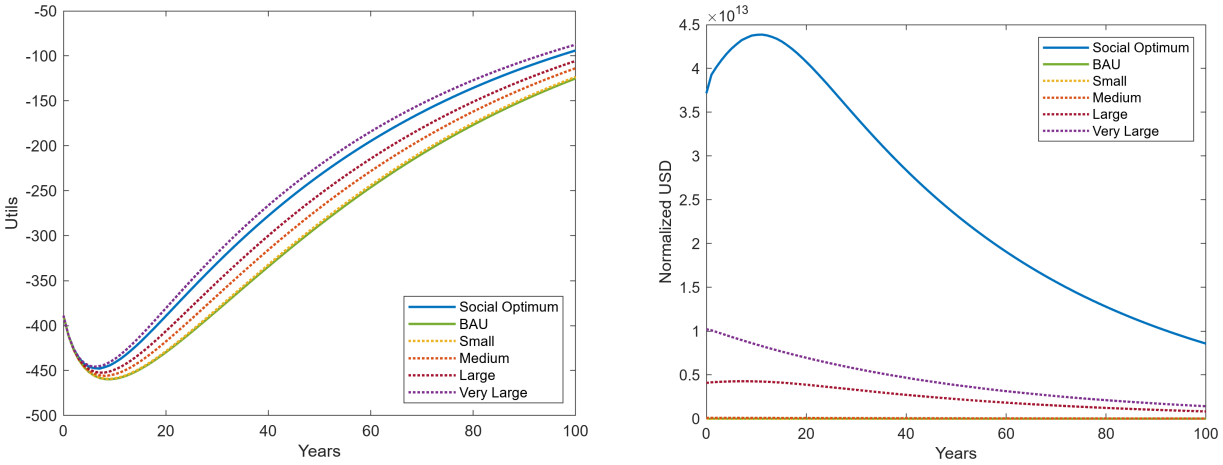


The lines represent the levels of utility of a representative consumer over time under the two theoretical benchmarks. The trajectories are generated from the model specification with pollution dynamics (Section 3.1) and t is in years. The blue solid line represents the trajectory under the Social Optimum, and the green line represents the trajectory under the no policy, or BAU, benchmark. The yellow dotted line ("Small") represents the trajectory under a constant per unit subsidy set at 1000 USD. The orange dotted line ("Medium") are the levels under a constant per unit subsidy at 10,000 USD. The red dotted line ("Large") are the levels under a constant per unit subsidy at 445,000 USD. The purple dotted line ("Very Large") are the levels under a constant per unit subsidy at 8,092,113.71 USD, which is the value of stock accumulation at $t = 0$ under the Social Optimum multiplied by a factor of two.

Figure 6: Surplus Trajectories in Present Values Under Constant Marginal Stock Subsidies

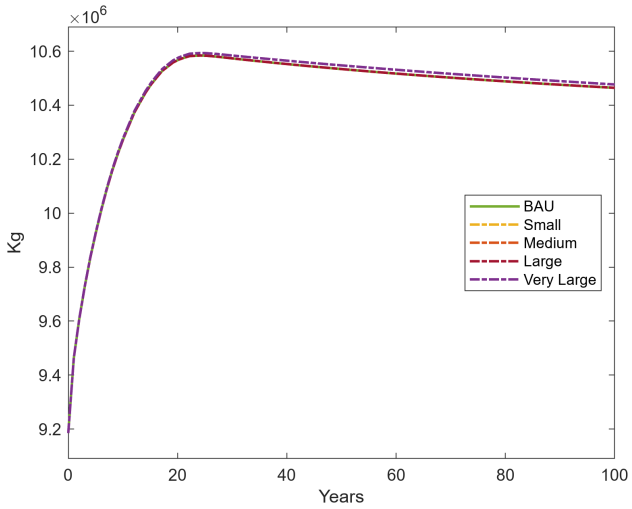
(a) Consumer Surplus

(b) Producer Surplus



The lines represent the levels of consumer and producer surplus in present values under the two theoretical benchmarks in addition to different constant per-unit incentives. The trajectories are generated from the model specification with pollution dynamics (Section 3.1) and t is in years. The blue solid line represents the trajectory under the Social Optimum, and the green line represents the trajectory under the no policy, or BAU, benchmark. The yellow dotted line ("Small") represents the trajectory under a constant per unit subsidy set at 1000 USD. The orange dotted line ("Medium") are the levels under a constant per unit subsidy at 10,000 USD. The red dotted line ("Large") are the levels under a constant per unit subsidy at 445,000 USD. The purple dotted line ("Very Large") are the levels under a constant per unit subsidy at 8,092,113.71 USD, which is the value of stock accumulation at $t = 0$ under the Social Optimum multiplied by a factor of two.

Figure 7: Natural Resource Stock Trajectories Under Constant Marginal Flow Taxes

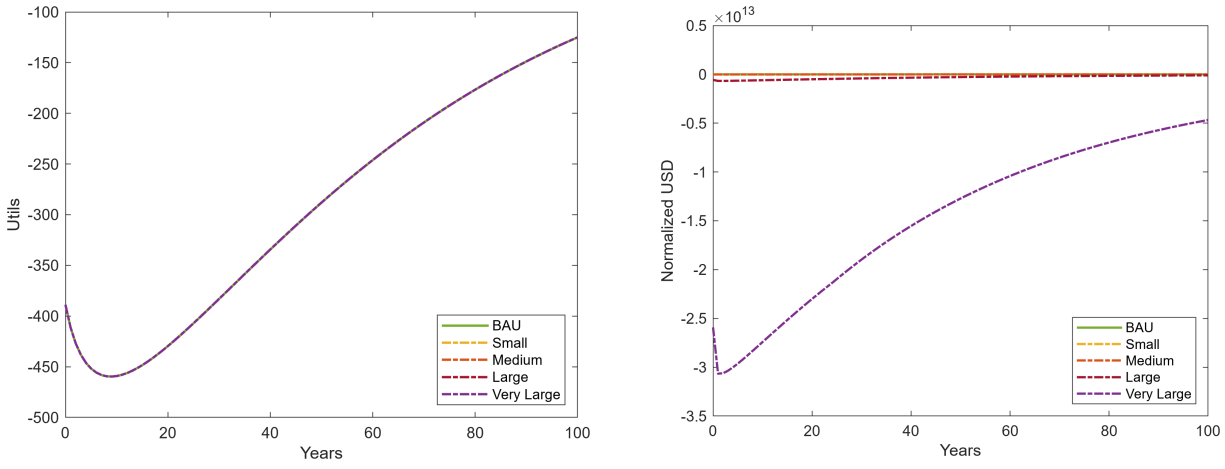


The lines represent the levels of the natural resource in kilograms over time under the no policy benchmark in addition to different constant per-unit incentives. The trajectories are generated from the model specification with pollution dynamics (Section 3.1) and t is in years. The green line represents the trajectory under the no policy, or BAU benchmark. The yellow dot-dashed line ("Small") represents the trajectory under a constant per unit tax set at 1000 USD. The orange dot-dashed line ("Medium") are the levels under a constant per unit tax at 10,000 USD. The red dot-dashed line ("Large") are the levels under a constant per unit tax at 445,000 USD. The purple dot-dashed line ("Very Large") are the levels under a constant per unit tax at 20,230,284.25 USD, which is the value of stock accumulation at $t = 0$ under the Social Optimum multiplied by a factor of five.

Figure 8: Surplus Trajectories in Present Values Under Constant Marginal Flow Taxes

(a) Consumer Surplus

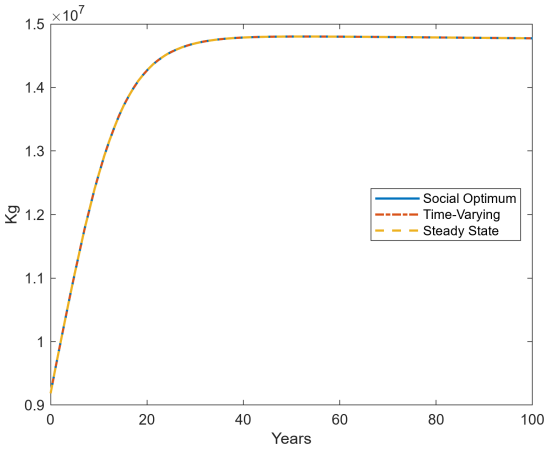
(b) Producer Surplus



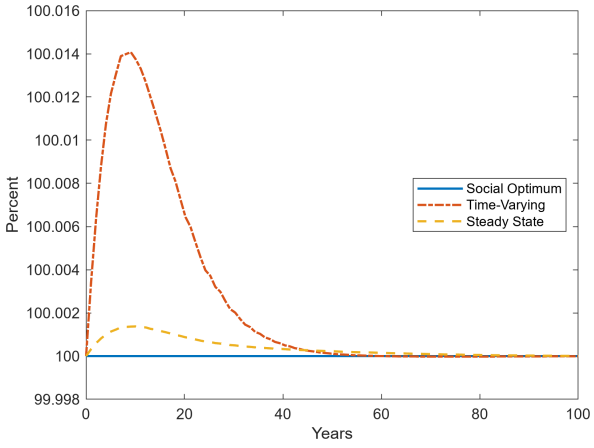
The lines represent the levels of consumer and producer surplus in present values under the no policy benchmark in addition to different constant per-unit incentives. The trajectories are generated from the model specification with pollution dynamics (Section 3.1) and t is in years. The green line represents the trajectory under the no policy, or BAU benchmark. The yellow dot-dashed line ("Small") represents the trajectory under a constant per unit tax set at 1000 USD. The orange dot-dashed line ("Medium") are the levels under a constant per unit tax at 10,000 USD. The red dot-dashed line ("Large") are the levels under a constant per unit tax at 445,000 USD. The purple dot-dashed line ("Very Large") are the levels under a constant per unit tax at 20,230,284.25 USD, which is the value of stock accumulation at $t = 0$ under the Social Optimum multiplied by a factor of five.

Figure 9: Natural Resource Stock Under Stock Subsidies that Converge to Social Optimum

(a) Natural Resource Stock Trajectories



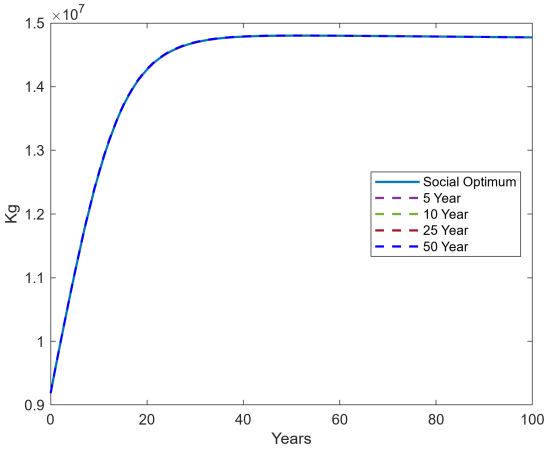
(b) Deviations from the Social Optimum



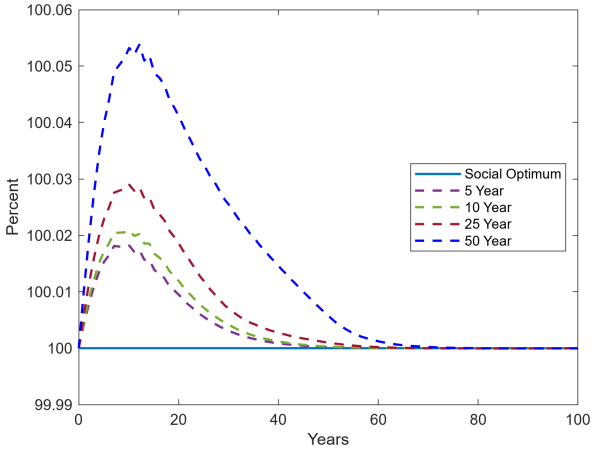
The lines represent the levels of the natural resource stock (Panel (a)) and the Deviations (Panel (b)) from the Social Optimum under policies that satisfy Proposition 1. The trajectories are generated from the model specification with pollution dynamics (Section 3.1) and t is in years. The blue solid line represents the trajectory under the Social Optimum. The orange dot-dashed line ("Time-Varying") represents the trajectory under a time-varying stock subsidy. The yellow dashed line ("Steady State") represents the trajectory under a constant per unit subsidy set at the steady state value of natural resource stock accumulation under the Social Optimum.

Figure 10: Natural Resource Stock Under Stock Subsidies with Periodic Updates

(a) Natural Resource Stock Trajectories



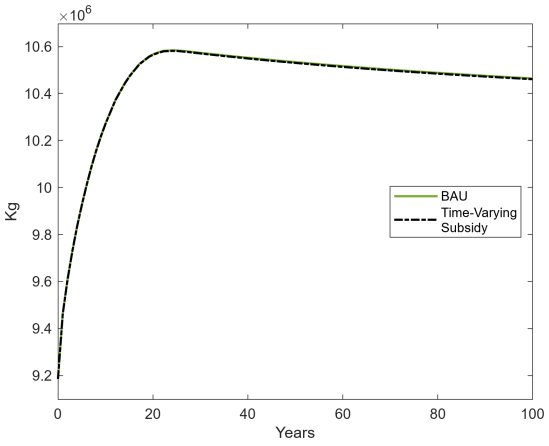
(b) Deviations from the Social Optimum



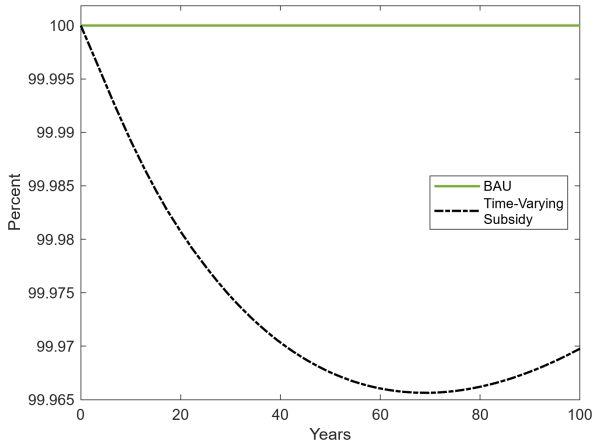
The lines represent the levels of the natural resource stock (Panel (a)) and the percentage deviations (Panel (b)) under a stock policy over time against policies with different periodic updates. The trajectories are generated from the model specification with pollution dynamics (Section 3.1) and t is in years. The blue solid line represents the trajectory under the Social Optimum. The purple dashed line ("5 Year") is the trajectory under a policy that updates every five years. The green dashed line ("10 Year") is the trajectory under a policy that updates every ten years. The red dashed line ("25 Year") is the trajectory under a policy that updates every 25 years. The blue dashed line ("50 Year") is the trajectory under a policy that updates every 50 years.

Figure 11: Natural Resource Stock Under Time-Varying Flow Subsidies

(a) Natural Resource Stock Trajectories



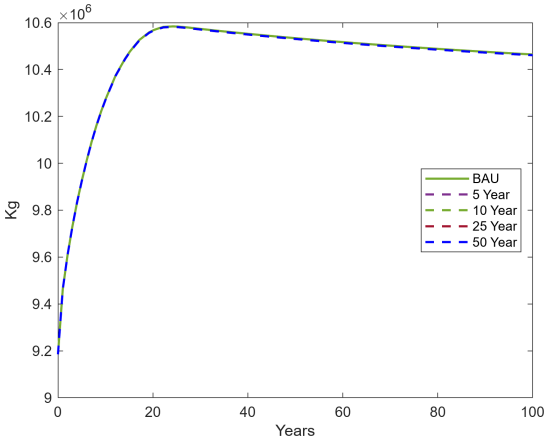
(b) Deviations from BAU



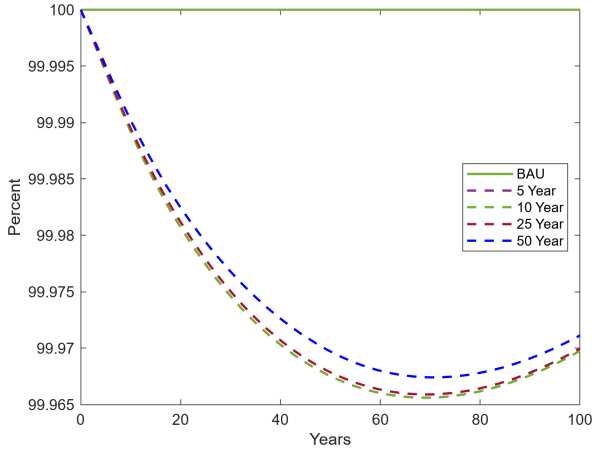
The lines represent the levels of the natural resource stock (Panel (a)) and the Deviations (Panel (b)) from no policy, or BAU. The trajectories are generated from the model specification with pollution dynamics (Section 3.1) and t is in years. The green solid line represents the trajectory under the no policy, or BAU. The black dot-dashed line ("Time-Varying Subsidy") represents the trajectory under a time-varying subsidy set at the value of stock accumulation as if under the Social Planner.

Figure 12: Natural Resource Stock Under Flow Subsidies with Periodic Updates

(a) Natural Resource Stock Trajectories



(b) Deviations from BAU



The lines represent the levels of the natural resource stock (Panel (a)) and the percentage deviations (Panel (b)) under a stock policy over time against subsidies that are updated periodically to to the value of stock accumulation as if under the Social Planner. The trajectories are generated from the model specification with pollution dynamics (Section 3.1) and t is in years. The green solid line represents the trajectory under no policy, or BAU. The purple dashed line ("5 Year") is the trajectory under a policy that updates every five years. The green dashed line ("10 Year") is the trajectory under a policy that updates every ten years. The red dashed line ("25 Year") is the trajectory under a policy that updates every 25 years. The blue dashed line ("50 Year") is the trajectory under a policy that updates every 50 years.

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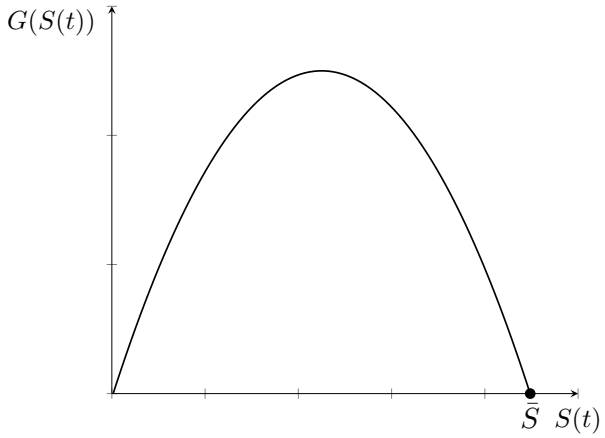
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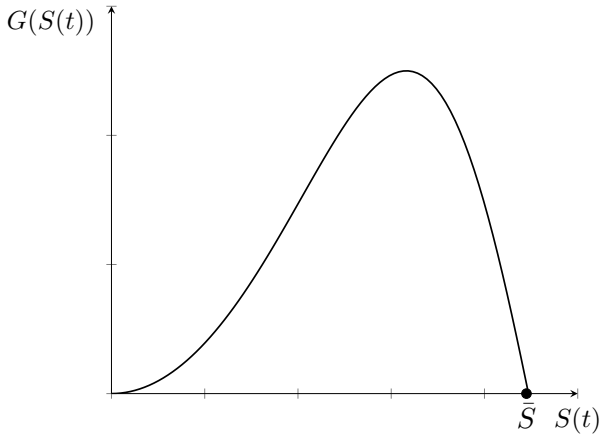
A Figures

Figure A1: Population Growth Functions

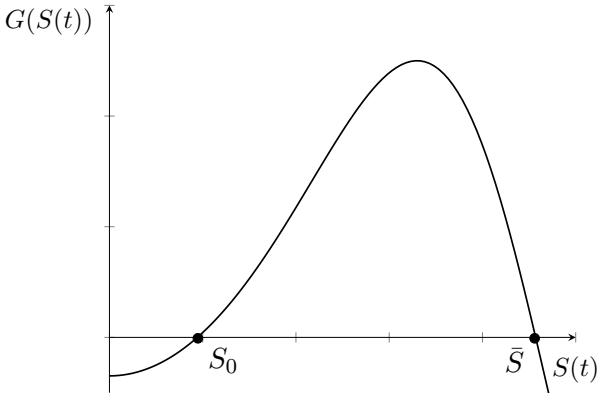
(a) Pure Compensatory Growth



(b) Depensatory Growth

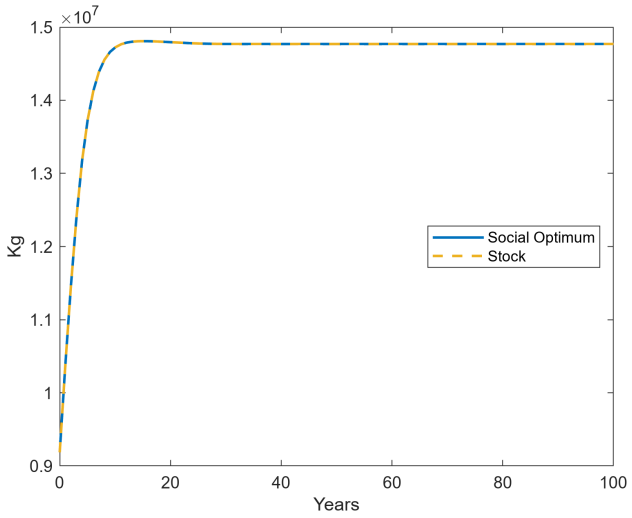


(c) Critical Depensatory Growth



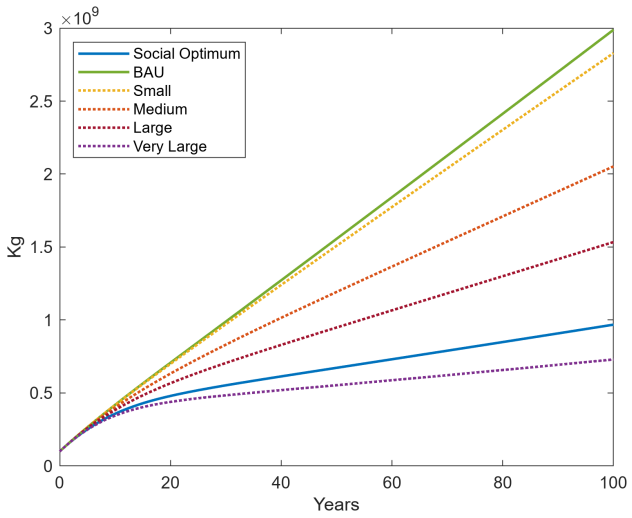
The figures represent different population growth functions for a single species within a fixed geographic area, i.e. habitat. Panel (a) represents pure compensatory growth, which is concave with respect to the total stock of biomass S_t across all levels of biomass, and where population growth converges to zero as the stock reaches the environmental carrying capacity $\bar{S}$, which is time-invariant under the current model. Panel (b) represents depensatory growth, which is concave up to a given level of biomass and then concave, converging to $\bar{S}$. Panel (c) represents critical depensatory growth, where growth is negative up to a critical threshold $S_0 > 0$, also referred to as the "Allee threshold." This indicates that the population may collapse at or below some positive level S_0.

Figure A2: Natural Resource Stock Trajectories Under Log Linear Separable Utility



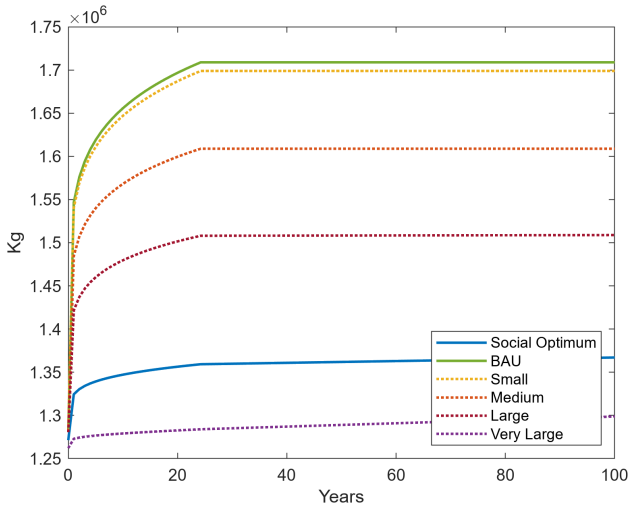
The lines represent the levels of the natural resource stock over time in kilograms under the Social Optimum versus a stock incentive, where the consumer has log linear separable utility with respect to both consumption and the natural resource stock and production is linear in harvest (Section 2.3). The blue solid line represents the trajectory under the Social Planner benchmark. The yellow dashed line represents the trajectory of the under the stock targeting policy in Equation 25.

Figure A3: Pollution Stock Trajectories Under Constant Marginal Stock Subsidies



The lines represent the levels of inorganic nitrogen stock in kilograms over time under the two theoretical benchmarks. The trajectories are generated from the model specification with pollution dynamics (Section 3.1) and t is in years. The blue solid line represents the trajectory under the Social Optimum, and the green line represents the trajectory under the no policy, or BAU, benchmark. The yellow dotted line ("Small") represents the trajectory under a constant per unit subsidy set at 1000 USD. The orange dotted line ("Medium") are the levels under a constant per unit subsidy at 10,000 USD. The red dotted line ("Large") are the levels under a constant per unit subsidy at 445,000 USD. The purple dotted line ("Very Large") are the levels under a constant per unit subsidy at 8,092,113.71 USD, which is the value of stock accumulation at $t = 0$ under the Social Optimum multiplied by a factor of two.

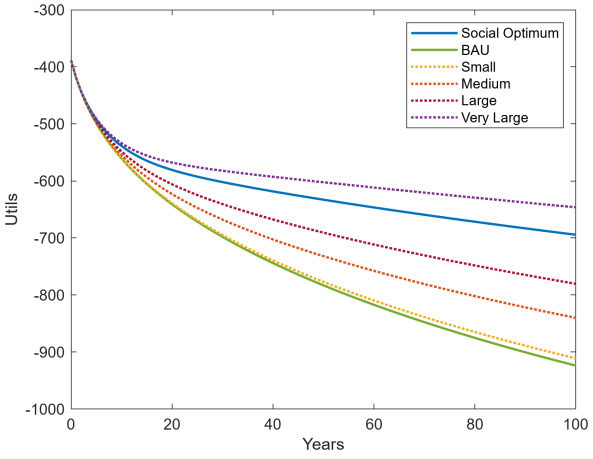
Figure A4: Harvest Trajectories Under Constant Marginal Stock Subsidies



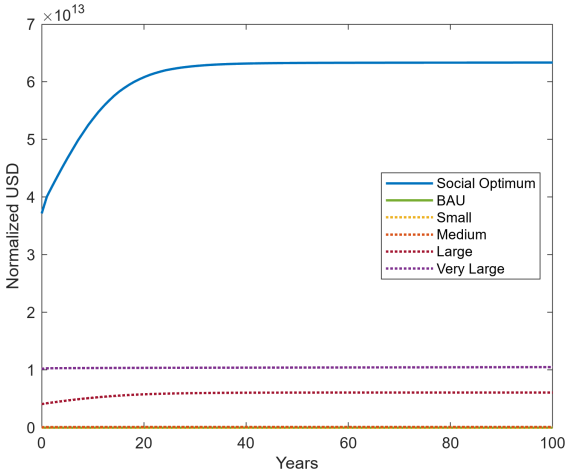
The lines represent the levels of harvest in kilograms over time under the two theoretical benchmarks. The trajectories are generated from the model specification with pollution dynamics (Section 3.1) and t is in years. The blue solid line represents the trajectory under the Social Optimum, and the green line represents the trajectory under the no policy, or BAU, benchmark. The yellow dotted line ("Small") represents the trajectory under a constant per unit subsidy set at 1000 USD. The orange dotted line ("Medium") are the levels under a constant per unit subsidy at 10,000 USD. The red dotted line ("Large") are the levels under a constant per unit subsidy at 445,000 USD. The purple dotted line ("Very Large") are the levels under a constant per unit subsidy at 8,092,113.71 USD, which is the value of stock accumulation at $t = 0$ under the Social Optimum multiplied by a factor of two.

Figure A5: Surplus Trajectories in Current Values Under Constant Marginal Stock Subsidies

(a) Consumer Surplus

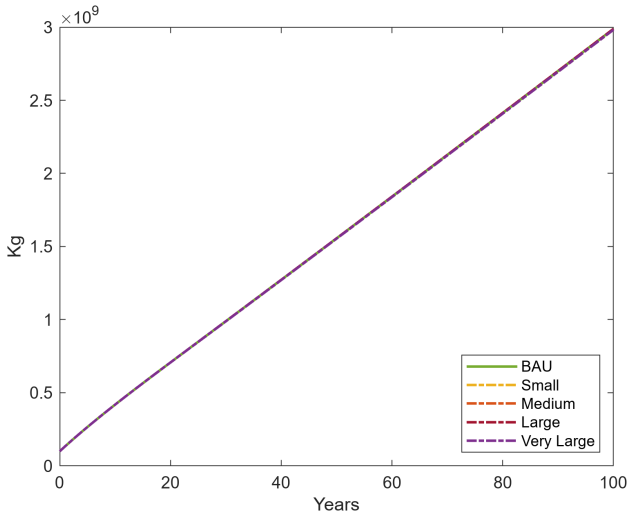


(b) Producer Surplus



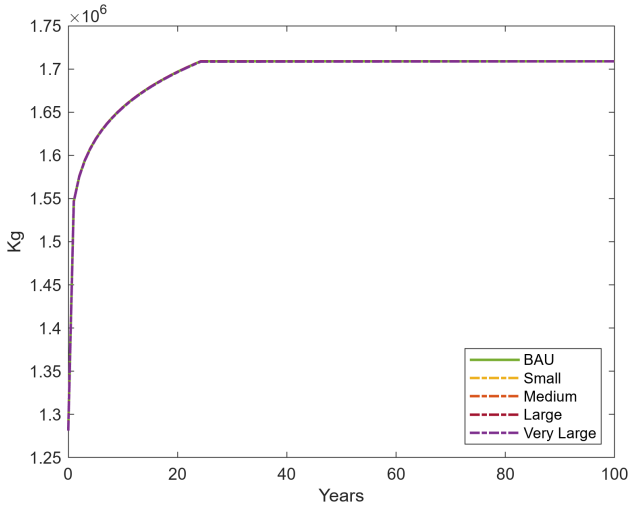
The lines represent the levels of consumer and producer surplus in current values under the two theoretical benchmarks in addition to different constant per-unit incentives. The trajectories are generated from the model specification with pollution dynamics (Section 3.1) and t is in years. The blue solid line represents the trajectory under the Social Optimum, and the green line represents the trajectory under the no policy, or BAU, benchmark. The yellow dotted line ("Small") represents the trajectory under a constant per unit subsidy set at 1000 USD. The orange dotted line ("Medium") are the levels under a constant per unit subsidy at 10,000 USD. The red dotted line ("Large") are the levels under a constant per unit subsidy at 445,000 USD. The purple dotted line ("Very Large") are the levels under a constant per unit subsidy at 8,092,113.71 USD, which is the value of stock accumulation at $t = 0$ under the Social Optimum multiplied by a factor of two.

Figure A6: Pollution Stock Trajectories Under Constant Marginal Flow Taxes



The lines represent the levels of inorganic nitrogen stock in kilograms over time under the no policy benchmark. The trajectories are generated from the model specification with pollution dynamics (Section 3.1) and t is in years. The green line represents the trajectory under the no policy, or BAU benchmark. The yellow dot-dashed line ("Small") represents the trajectory under a constant per unit tax set at 1000 USD. The orange dot-dashed line ("Medium") are the levels under a constant per unit tax at 10,000 USD. The red dot-dashed line ("Large") are the levels under a constant per unit tax at 445,000 USD. The purple dot-dashed line ("Very Large") are the levels under a constant per unit tax at 20,230,284.25 USD, which is the value of stock accumulation at $t = 0$ under the Social Optimum multiplied by a factor of five.

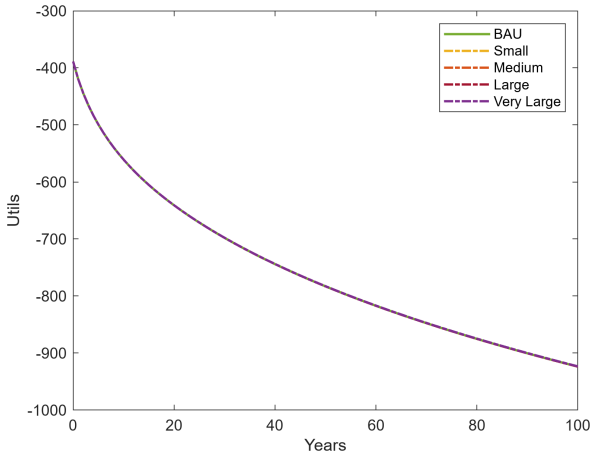
Figure A7: Harvest Trajectories Under Constant Marginal Flow Taxes



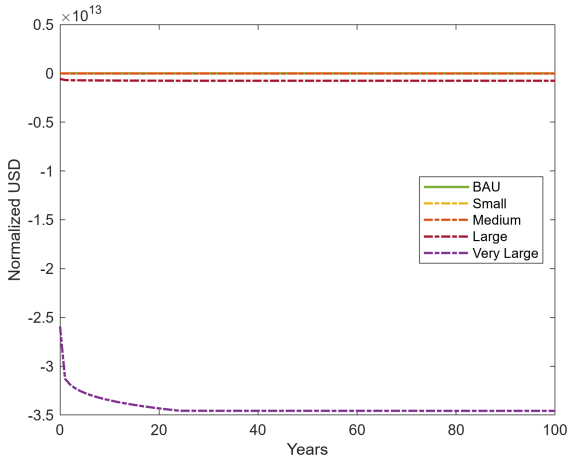
The lines represent the levels of harvest in kilograms over time under the no policy benchmark. The trajectories are generated from the model specification with pollution dynamics (Section 3.1) and t is in years. The green line represents the trajectory under the no policy, or BAU benchmark. The yellow dot-dashed line ("Small") represents the trajectory under a constant per unit tax set at 1000 USD. The orange dot-dashed line ("Medium") are the levels under a constant per unit tax at 10,000 USD. The red dot-dashed line ("Large") are the levels under a constant per unit tax at 445,000 USD. The purple dot-dashed line ("Very Large") are the levels under a constant per unit tax at 20,230,284.25 USD, which is the value of stock accumulation at $t = 0$ under the Social Optimum multiplied by a factor of five.

Figure A8: Surplus Trajectories in Current Values Under Constant Marginal Flow Taxes

(a) Consumer Surplus

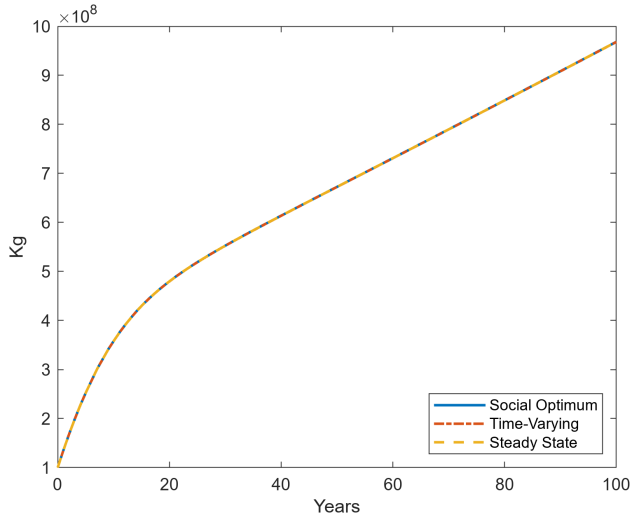


(b) Producer Surplus



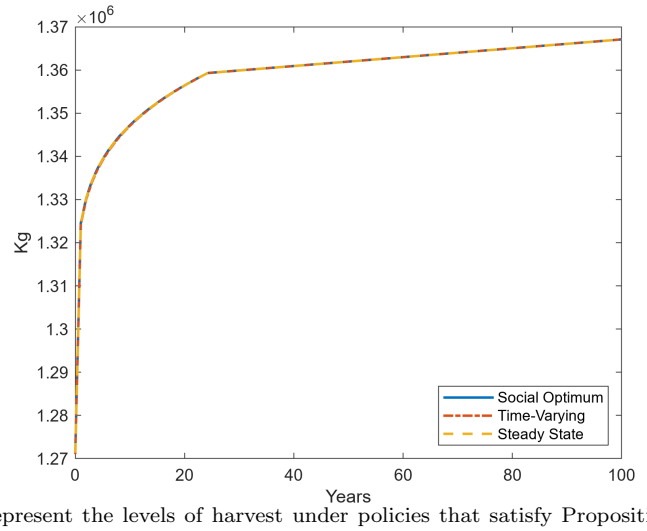
The lines represent the levels of consumer and producer surplus in current values under the no policy benchmark in addition to different constant per-unit incentives. The trajectories are generated from the model specification with pollution dynamics (Section 3.1) and t is in years. The green line represents the trajectory under the no policy, or BAU benchmark. The yellow dot-dashed line ("Small") represents the trajectory under a constant per unit tax set at 1000 USD. The orange dot-dashed line ("Medium") are the levels under a constant per unit tax at 10,000 USD. The red dot-dashed line ("Large") are the levels under a constant per unit tax at 445,000 USD. The purple dot-dashed line ("Very Large") are the levels under a constant per unit tax at 20,230,284.25 USD, which is the value of stock accumulation at $t = 0$ under the Social Optimum multiplied by a factor of five.

Figure A9: Pollution Stock Under Stock Subsidies that Converge to Social Optimum



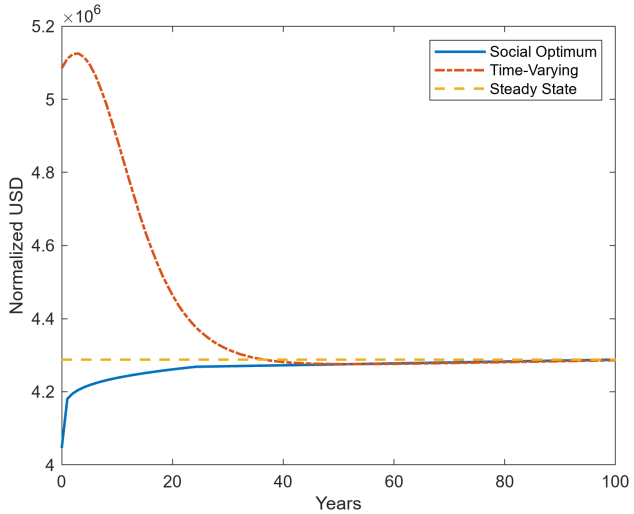
The lines represent the levels of inorganic nitrogen stock under policies that satisfy Proposition 1. The trajectories are generated from the model specification with pollution dynamics (Section 3.1) and t is in years. The blue solid line represents the trajectory under the Social Optimum. The orange dot-dashed line ("Time-Varying") represents the trajectory under a time-varying stock subsidy. The yellow dashed line ("Steady State") represents the trajectory under a constant per unit subsidy set at the steady state value of natural resource stock accumulation under the Social Optimum.

Figure A10: Harvest Stock Under Stock Subsidies that Converge to Social Optimum



The lines represent the levels of harvest under policies that satisfy Proposition 1. The trajectories are generated from the model specification with pollution dynamics (Section 3.1) and t is in years. The blue solid line represents the trajectory under the Social Optimum. The orange dot-dashed line ("Time-Varying") represents the trajectory under a time-varying stock subsidy. The yellow dashed line ("Steady State") represents the trajectory under a constant per unit subsidy set at the steady state value of natural resource stock accumulation under the Social Optimum.

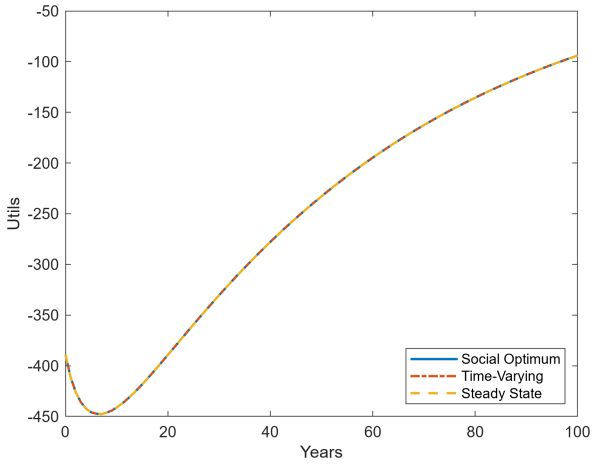
Figure A11: Trajectories of Marginal Stock Subsidies that Converge to Social Optimum



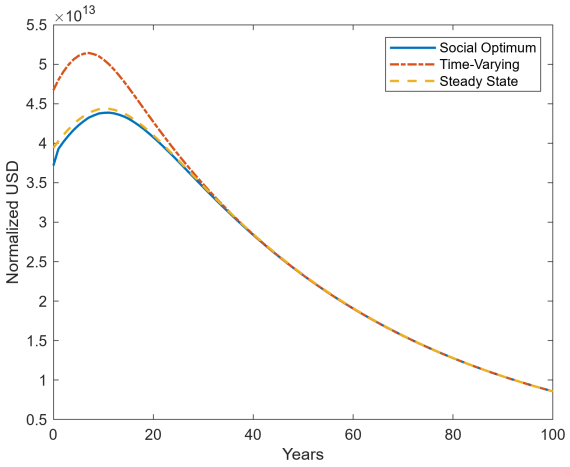
The lines represent the values of marginal stock subsidies that satisfy Proposition 1. The trajectories are generated from the model specification with pollution dynamics (Section 3.1) and t is in years. The blue solid line represents the trajectory under the Social Optimum. The orange dot-dashed line ("Time-Varying") represents the trajectory under a time-varying stock subsidy. The yellow dashed line ("Steady State") represents the trajectory under a constant per unit subsidy set at the steady state value of natural resource stock accumulation under the Social Optimum.

Figure A12: Surplus Trajectories in Present Values Under Stock Subsidies that Converge to Social Optimum

(a) Consumer Surplus



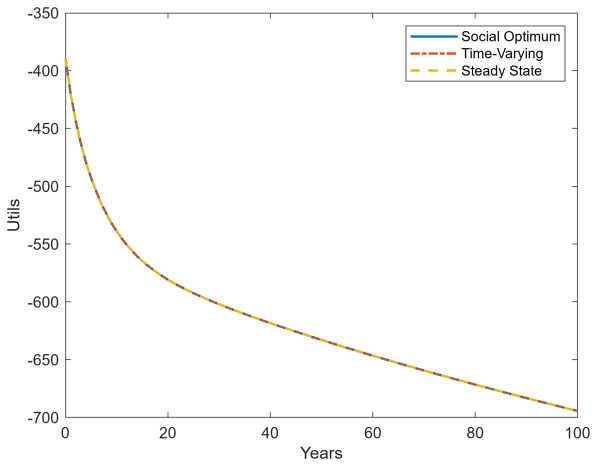
(b) Producer Surplus



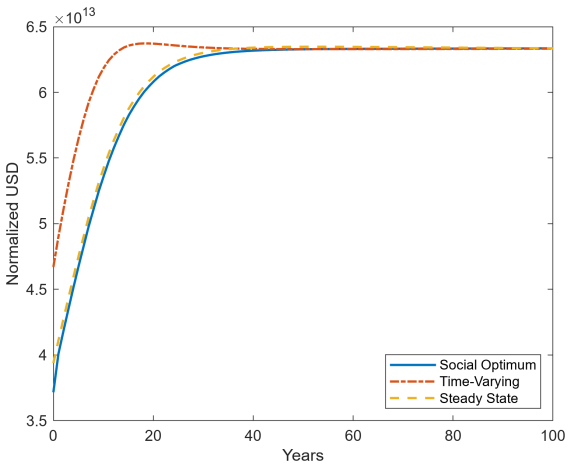
The lines represent the levels of consumer and producer surplus in present values under policies that satisfy Proposition 1. The trajectories are generated from the model specification with pollution dynamics (Section 3.1) and t is in years. The blue solid line represents the trajectory under the Social Optimum. The orange dot-dashed line ("Time-Varying") represents the trajectory under a time-varying stock subsidy. The yellow dashed line ("Steady State") represents the trajectory under a constant per unit subsidy set at the steady state value of natural resource stock accumulation under the Social Optimum.

Figure A13: Surplus Trajectories in Current Values Under Stock Subsidies that Converge to Social Optimum

(a) Consumer Surplus

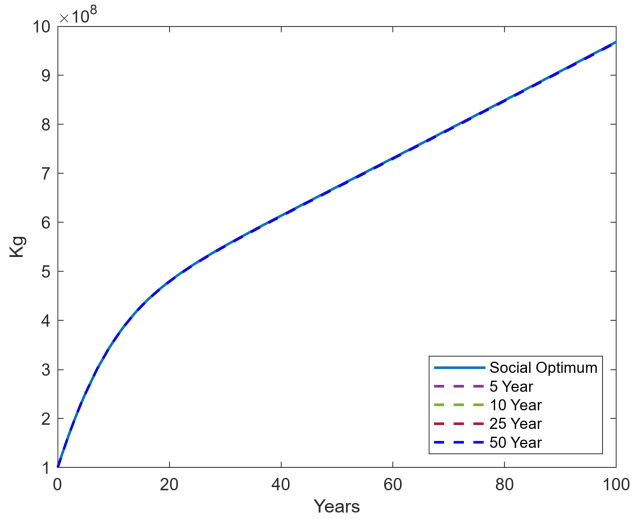


(b) Producer Surplus



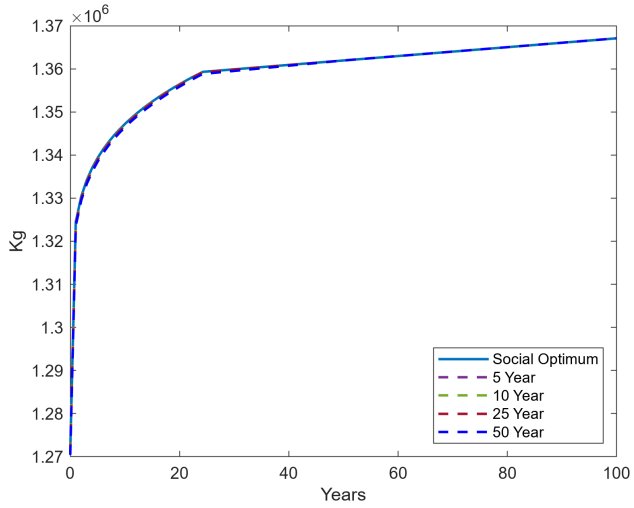
The lines represent the levels of consumer and producer surplus in current values under policies that satisfy Proposition 1. The trajectories are generated from the model specification with pollution dynamics (Section 3.1) and t is in years. The blue solid line represents the trajectory under the Social Optimum. The orange dot-dashed line ("Time-Varying") represents the trajectory under a time-varying stock subsidy. The yellow dashed line ("Steady State") represents the trajectory under a constant per unit subsidy set at the steady state value of natural resource stock accumulation under the Social Optimum.

Figure A14: Pollution Stock Trajectories Under Stock Subsidies with Periodic Updates



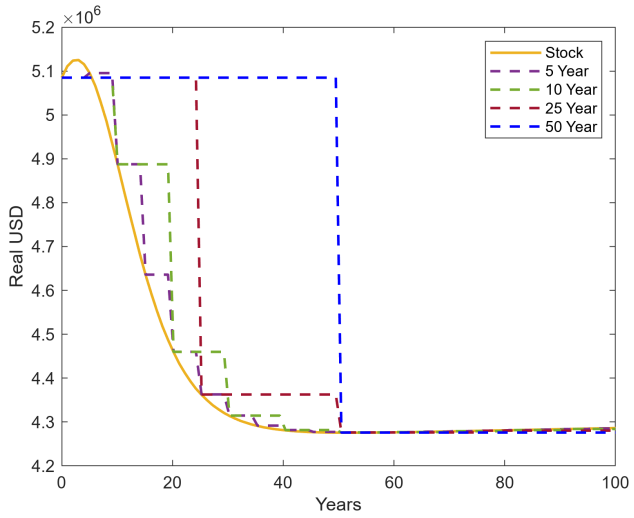
The lines represent the levels of inorganic nitrogen stock (Panel (a)) and the percentage deviations in the levels of inorganic nitrogen stock (Panel (b)) under a stock policy over time against policies with different periodic updates. The trajectories are generated from the model specification with pollution dynamics (Section 3.1) and t is in years. The blue solid line represents the trajectory under the Social Optimum. The purple dashed line ("5 Year") is the trajectory under a policy that updates every five years. The green dashed line ("10 Year") is the trajectory under a policy that updates every ten years. The red dashed line ("25 Year") is the trajectory under a policy that updates every 25 years. The blue dashed line ("50 Year") is the trajectory under a policy that updates every 50 years.

Figure A15: Harvest Trajectories Under Stock Subsidies with Periodic Updates



The lines represent the levels of harvest (Panel (a)) and the percentage deviations in the levels of harvest (Panel (b)) under a stock policy over time against policies with different periodic updates. The trajectories are generated from the model specification with pollution dynamics (Section 3.1) and t is in years. The blue solid line represents the trajectory under the Social Optimum. The purple dashed line ("5 Year") is the trajectory under a policy that updates every five years. The green dashed line ("10 Year") is the trajectory under a policy that updates every ten years. The red dashed line ("25 Year") is the trajectory under a policy that updates every 25 years. The blue dashed line ("50 Year") is the trajectory under a policy that updates every 50 years.

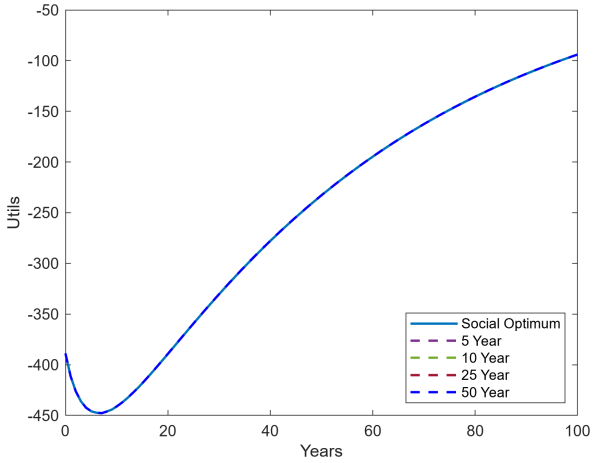
Figure A16: Trajectories of Marginal Stock Subsidies with Periodic Updates



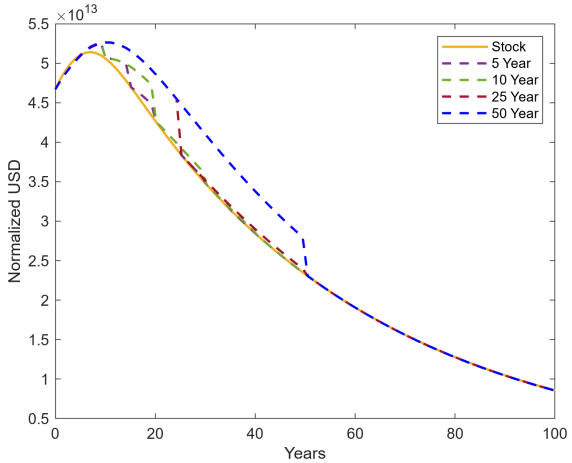
The lines represent the value of the stock subsidy over time with different periodic updates. The trajectories are generated from the model specification with pollution dynamics (Section 3.1) and t is in years. The yellow solid line represents the trajectory under a time-varying subsidy set that satisfies Proposition 1. The purple dashed line ("5 Year") is the trajectory under a policy that updates every five years. The green dashed line ("10 Year") is the trajectory under a policy that updates every ten years. The red dashed line ("25 Year") is the trajectory under a policy that updates every 25 years. The blue dashed line ("50 Year") is the trajectory under a policy that updates every 50 years.

Figure A17: Surplus Trajectories in Present Values Under Stock Subsidies with Periodic Updates

(a) Consumer Surplus



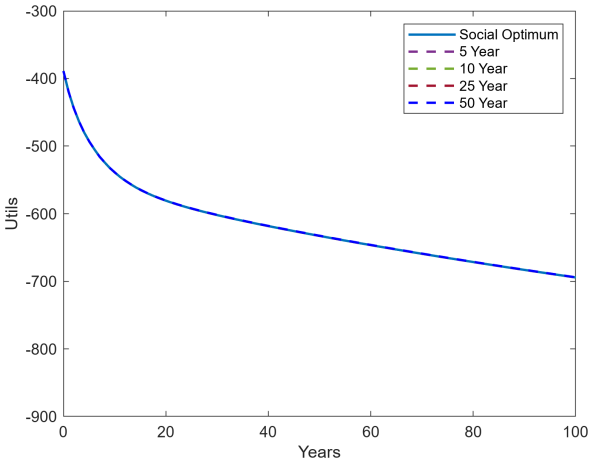
(b) Producer Surplus



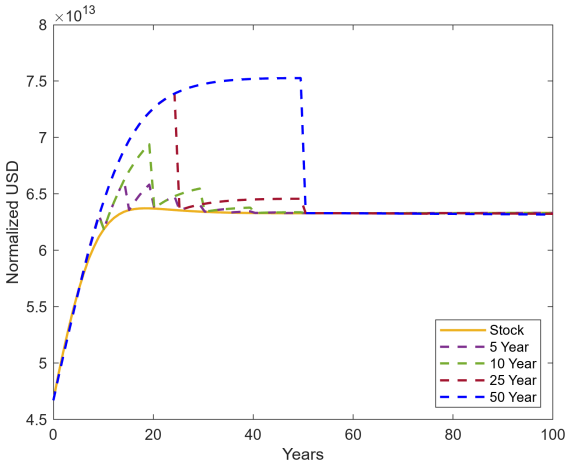
The lines represent the levels of consumer and producer surplus in present values under the two theoretical benchmarks in addition to different constant per-unit incentives. The trajectories are generated from the model specification with pollution dynamics (Section 3.1) and t is in years. The blue solid line in Panel (a) represents the trajectory of consumer surplus under the Social Optimum. The yellow solid line in Panel (b) represents the trajectory of producer surplus under a stock subsidy that satisfies Proposition 1. The purple dashed line ("5 Year") is the trajectory under a policy that updates every five years. The green dashed line ("10 Year") is the trajectory under a policy that updates every ten years. The red dashed line ("25 Year") is the trajectory under a policy that updates every 25 years. The blue dashed line ("50 Year") is the trajectory under a policy that updates every 50 years.

Figure A18: Surplus Trajectories in Current Values Under Stock Subsidies with Periodic Updates

(a) Consumer Surplus

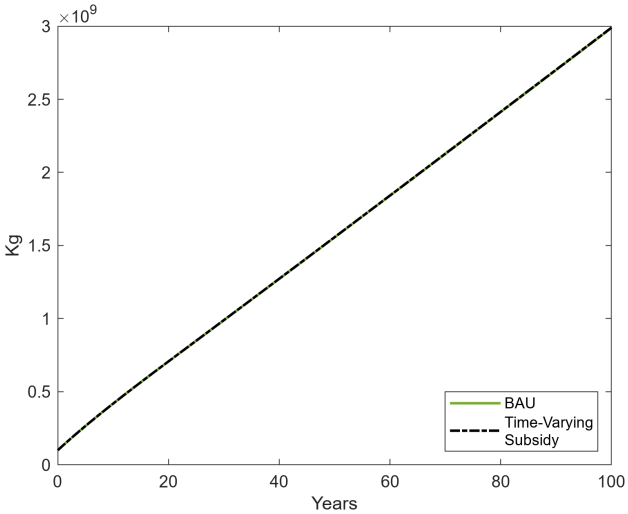


(b) Producer Surplus



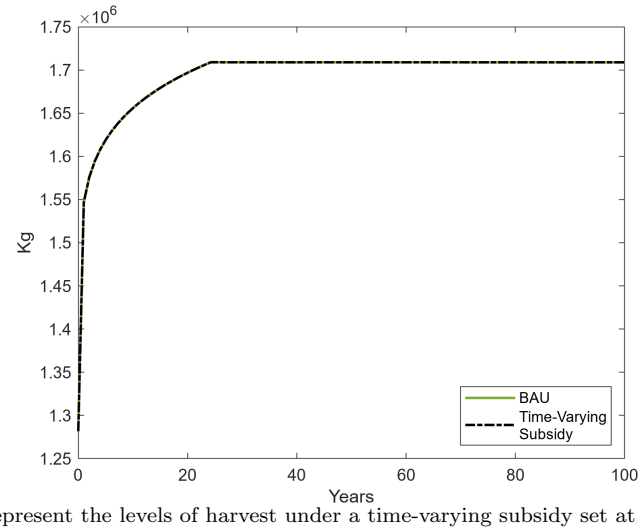
The lines represent the levels of consumer and producer surplus in current values under the two theoretical benchmarks in addition to different constant per-unit incentives. The trajectories are generated from the model specification with pollution dynamics (Section 3.1) and t is in years. The blue solid line in Panel (a) represents the trajectory of consumer surplus under the Social Optimum. The yellow solid line in Panel (b) represents the trajectory of producer surplus under a stock subsidy that satisfies Proposition 1. The purple dashed line ("5 Year") is the trajectory under a policy that updates every five years. The green dashed line ("10 Year") is the trajectory under a policy that updates every ten years. The red dashed line ("25 Year") is the trajectory under a policy that updates every 25 years. The blue dashed line ("50 Year") is the trajectory under a policy that updates every 50 years.

Figure A19: Pollution Stock Under Flow Subsidies that Converge to Social Optimum



The lines represent the levels of inorganic nitrogen stock under a time-varying subsidy set at the value of stock accumulation as if under the Social Planner. The trajectories are generated from the model specification with pollution dynamics (Section 3.1) and t is in years. The green solid line represents the trajectory under the no policy, or BAU. The black dot-dashed line ("Time-Varying Subsidy") represents the trajectory under a time-varying subsidy set at the value of stock accumulation as if under the Social Planner.

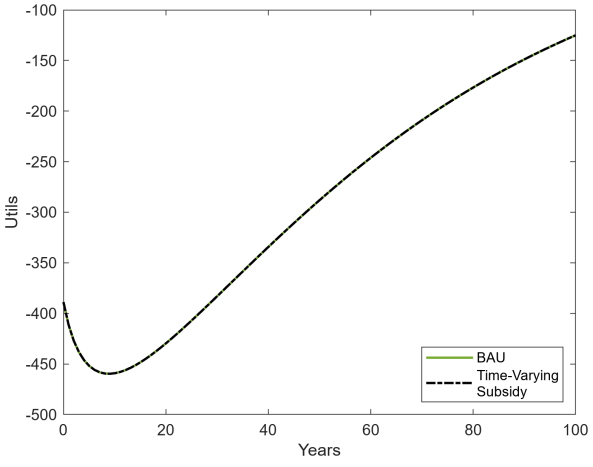
Figure A20: Harvest Stock Under Flow Subsidies that Converge to Social Optimum



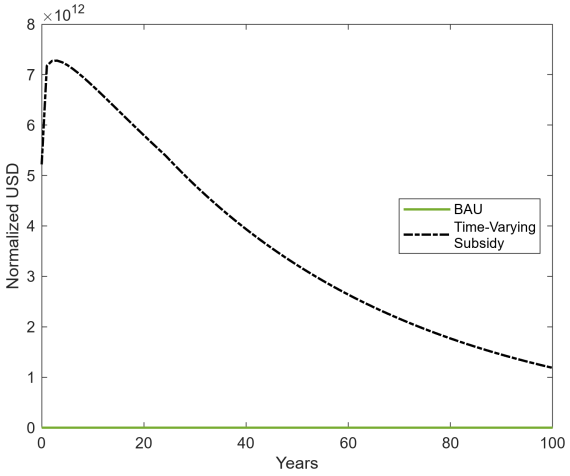
The lines represent the levels of harvest under a time-varying subsidy set at the value of stock accumulation as if under the Social Planner. The trajectories are generated from the model specification with pollution dynamics (Section 3.1) and t is in years. The green solid line represents the trajectory under the no policy, or BAU. The black dot-dashed line ("Time-Varying Subsidy") represents the trajectory under a time-varying subsidy set at the value of stock accumulation as if under the Social Planner.

Figure A21: Surplus Trajectories in Present Values Under Flow Subsidies that Converge to Social Optimum

(a) Consumer Surplus

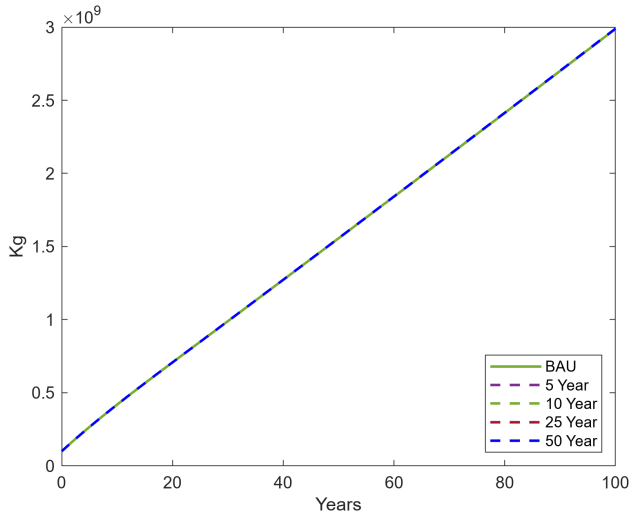


(b) Producer Surplus



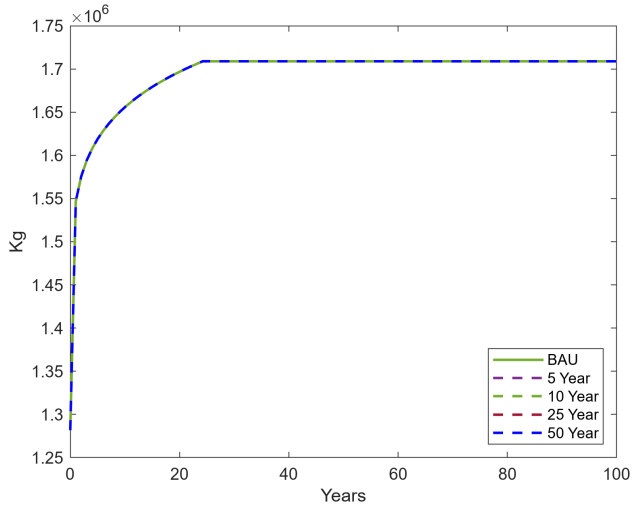
The lines represent the levels of consumer and producer surplus in present values under a time-varying subsidy set at the value of stock accumulation as if under the Social Planner. The trajectories are generated from the model specification with pollution dynamics (Section 3.1) and t is in years. The green solid line represents the trajectory under the no policy, or BAU. The black dot-dashed line ("Time-Varying Subsidy") represents the trajectory under a time-varying subsidy set at the value of stock accumulation as if under the Social Planner.

Figure A22: Pollution Stock Trajectories Under Flow Subsidies with Periodic Updates



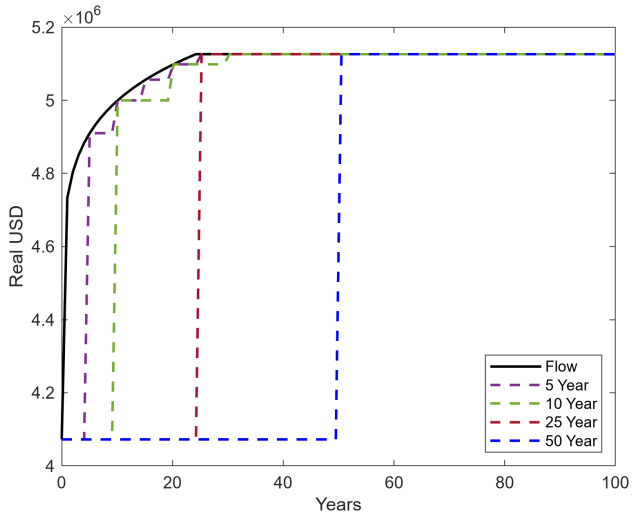
The lines represent the levels of inorganic nitrogen stock under a flow subsidy against policies with different periodic updates. The trajectories are generated from the model specification with pollution dynamics (Section 3.1) and t is in years. The green solid line represents the trajectory under no policy, or BAU. The purple dashed line ("5 Year") is the trajectory under a policy that updates every five years. The green dashed line ("10 Year") is the trajectory under a policy that updates every ten years. The red dashed line ("25 Year") is the trajectory under a policy that updates every 25 years. The blue dashed line ("50 Year") is the trajectory under a policy that updates every 50 years.

Figure A23: Harvest Trajectories Under Flow Subsidies with Periodic Updates



The lines represent the levels of harvest under a flow subsidy against policies with different periodic updates. The trajectories are generated from the model specification with pollution dynamics (Section 3.1) and t is in years. The green solid line represents the trajectory under no policy, or BAU. The purple dashed line ("5 Year") is the trajectory under a policy that updates every five years. The green dashed line ("10 Year") is the trajectory under a policy that updates every ten years. The red dashed line ("25 Year") is the trajectory under a policy that updates every 25 years. The blue dashed line ("50 Year") is the trajectory under a policy that updates every 50 years.

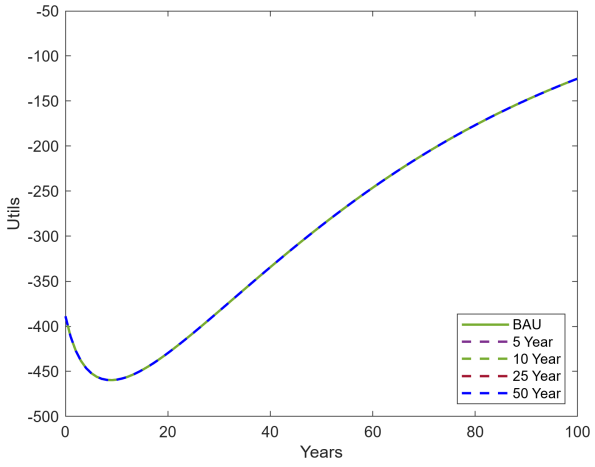
Figure A24: Trajectories of Marginal Flow Subsidies with Periodic Updates



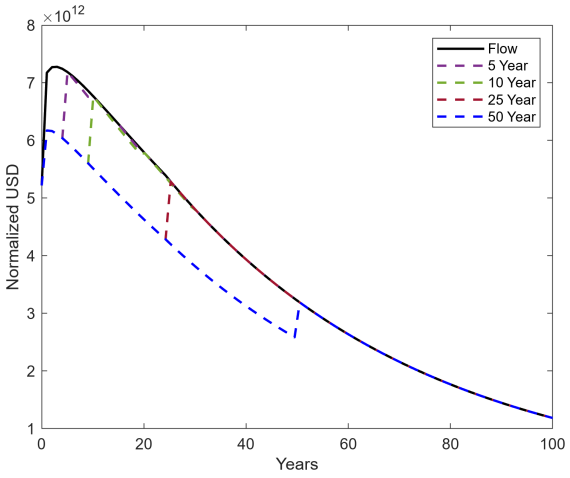
The lines represent the value of the stock subsidy over time with different periodic updates. The trajectories are generated from the model specification with pollution dynamics (Section 3.1) and t is in years. The black solid line represents the trajectory under a time-varying subsidy set at the value of stock accumulation as if under the Social Planner. The purple dashed line ("5 Year") is the trajectory under a policy that updates every five years. The green dashed line ("10 Year") is the trajectory under a policy that updates every ten years. The red dashed line ("25 Year") is the trajectory under a policy that updates every 25 years. The blue dashed line ("50 Year") is the trajectory under a policy that updates every 50 years.

Figure A25: Surplus Trajectories in Present Values Under Flow Subsidies with Periodic Updates

(a) Consumer Surplus



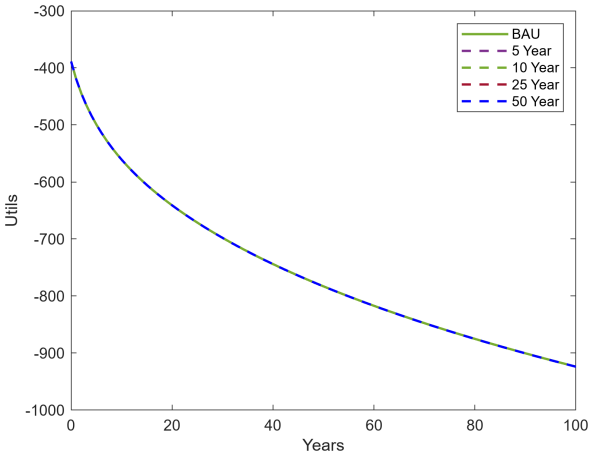
(b) Producer Surplus



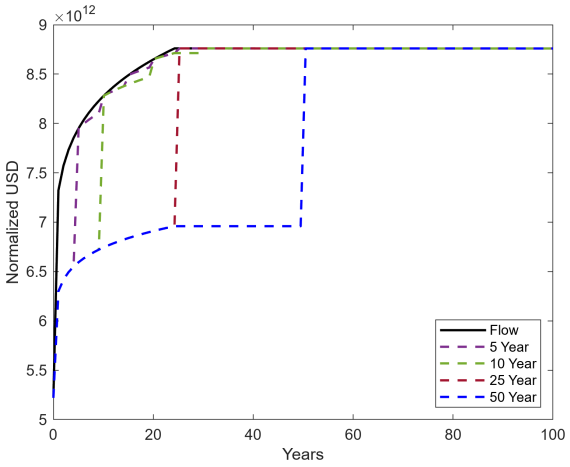
The lines represent the levels of consumer and producer surplus in present values under the two theoretical benchmarks in addition to different constant per-unit incentives. The trajectories are generated from the model specification with pollution dynamics (Section 3.1) and t is in years. The green solid line in Panel (a) represents the trajectory of consumer surplus under no policy, or BAU. The black solid line in Panel (b) represents the trajectory of producer surplus under a time-varying flow subsidy. The purple dashed line ("5 Year") is the trajectory under a policy that updates every five years. The green dashed line ("10 Year") is the trajectory under a policy that updates every ten years. The red dashed line ("25 Year") is the trajectory under a policy that updates every 25 years. The blue dashed line ("50 Year") is the trajectory under a policy that updates every 50 years.

Figure A26: Surplus Trajectories in Current Values Under Flow Subsidies with Periodic Updates

(a) Consumer Surplus



(b) Producer Surplus



The lines represent the levels of consumer and producer surplus in current values under the two theoretical benchmarks in addition to different constant per-unit incentives. The trajectories are generated from the model specification with pollution dynamics (Section 3.1) and t is in years. The green solid line in Panel (a) represents the trajectory of consumer surplus under no policy, or BAU. The black solid line in Panel (b) represents the trajectory of producer surplus under a time-varying flow subsidy. The purple dashed line ("5 Year") is the trajectory under a policy that updates every five years. The green dashed line ("10 Year") is the trajectory under a policy that updates every ten years. The red dashed line ("25 Year") is the trajectory under a policy that updates every 25 years. The blue dashed line ("50 Year") is the trajectory under a policy that updates every 50 years.

B Harvest Trajectories and Total Surplus

An initial guess of the trajectory for harvest must be provided in order to calibrate the model. The system adjusts over subsequent iterations based on the evolution of the state variables (the stocks of the natural resource and pollution) and the costate equations (the Euler equation). However, the model does not provide direct insight into the shape of the time path for the control variable of the system. Therefore, I provide the following guesses of the trajectory: linear, exponential, and sigmoidal. The functional forms of each guess are below.

$$H_t = \begin{cases} H_0 + \left(\frac{|H_t - H_0|}{T}\right) t & \text{if } H_0 < H_t \\ H_0 - \left(\frac{|H_t - H_0|}{T}\right) t & \text{otherwise} \end{cases} \tag{42}$$

$$H_t = \begin{cases} H_0 + \left(\frac{|H_t - H_0|}{(0.25T)^\chi}\right) t^\chi & \text{if } t \leq 0.25T \ \& \ H_0 < H_t \\ H_0 - \left(\frac{|H_t - H_0|}{(0.25T)^\chi}\right) t^\chi & \text{if } t \leq 0.25T \ \& \ H_0 > H_t \\ H_t & \text{if } t > 0.25T \end{cases} \tag{43}$$

$$H_t = \begin{cases} H_0 + |H_t - H_0| \left(\frac{1}{1+\psi e^{\omega t}}\right) & \text{if } H_0 < H_t \\ H_0 - |H_t - H_0| \left(\frac{1}{1+\psi e^{\omega t}}\right) & \text{otherwise} \end{cases} \tag{44}$$

The exponential time path is piecewise based on $t \leqslant 0.25T$ in order to allow flexibility in the time to convergence, which in the current model is set at 25 years after policy implementation. The curvature parameter $\chi = 0.15$ reflects quick convergence to the steady state and is chosen based on conversations with industry professionals. Similarly, $\psi = 50$ and $\omega = -0.4$ determine the speed and timing of convergence under a sigmoidal trajectory.

Each time path is given the same H_0, while the ending values H_t depend on theoretical predictions. BAU and flow incentives are given $H_t = H_{MSY}$, while the system under the Social Planner and stock incentive are given $H_t = 0.8H_{MSY}$. The system is then calibrated to each initial guess, and the present discounted values of consumer and producer surplus are calculated to find the trajectory that maximizes total surplus. Using parameters and initial values in Table 1, the following surpluses were calculated for the main policy calibrations.

Table A1: Harvest Trajectories and Surpluses

Policy	Initial Guess	Producer Surplus	Consumer Surplus	Total Surplus
Social Planner	Linear	3,373,956.37	-24,490.76	3,349,465.61
	Exponential	3,477,056.73	-25,210.19	3,451,846.53
	Sigmoid	3,453.650.69	-24,946.73	3,428,703.96
BAU	Linear	3,626,551.43	-25,493.32	3,601,058.11
	Exponential	4,128,596.12	-29,344.84	4,099,251.28
	Sigmoid	4,014,456.99	-27,801.29	3,986,655.69
Stock	Linear	3,373,860.87	-24,489.49	3,349,371.38
	Exponential	3,476,926.41	-25,208.71	3,451,717.69
	Sigmoid	3,453,532.18	-24,945.39	3,428,586.79
Flow	Linear	3,628,154.99	-25,504.93	3,602,650.05
	Exponential	4,128,750.16	-29,346.82	4,099,403.33
	Sigmoid	4,014,971.70	-27,805.87	3,987,165.83

The exponential trajectory for harvest maximizes total surplus under all scenarios. The calibrations under time varying policy updates and policies under different signs and scales similarly choose exponential paths as the surplus maximizing initial guess. Therefore, I use exponential paths as the primary initial guess for the system in the results presented in Section 3.2.