Skewed Fluctuations and Propagation Through Production Networks

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Abstract

Skewness is a prevalent feature of macroeconomic time series and may arise exogenously, because common shocks are asymmetrically distributed, or endogenously, as symmetric sectoral shocks propagate through production networks. This paper quantifies the importance of asymmetric network propagation in generating skewness at the aggregate level. Previous theoretical work often studies these two possibilities in isolation and precludes the interaction of local and global co-movements. We nest all possible sources of skewness in a network-augmented factor model (NAFM) where output has a network, a common, and an idiosyncratic component. In this model, skewness can arise not only from skewness in the three components, but also from coskewness due to the higher order covariation between components. An analysis of real gross output growth of 43 U.S. sectors over the sample 1957-2019 finds that while the common and network components both exhibit skewness, the dominant source of asymmetry are the co-skewness terms. To help interpret our results, we construct and estimate a micro-founded multi-sector general equilibrium model and show that it can generate skewness and co-skewness consistent with the data.

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1 Introduction

Skewness is a pervasive feature of the U.S. data. Aggregate skewness can arise because of skewed common shocks such as rare disasters, or skewed sector-specific shocks to granular sectors.[1] A recent theoretical literature focusing on multi-sector macroeconomic models with production networks shows that asymmetric network propagation of negative and positive shocks can also generate skewness in aggregate economic activity. Network connections arise because each sector produces output by purchasing its inputs from other sectors which are complementary (i.e., inputs are not easily substitutable).[2] While all these sources—skewed common shocks, skewed sector-specific shocks, and asymmetric network propagation—can contribute to aggregate skewness in theoretical network models, their relative importance is uncertain because of the lack of empirical evidence. Understanding the relative importance of these sources of skewness is helpful for policymakers in guiding targeted responses during downturns and in attenuating the welfare costs of business cycles (see, for example, La'O and Tahbaz-Salehi (2022)).

In this paper, we quantify the relative importance of asymmetric network propagation and skewed shocks in generating skewness at the aggregate level. Our analysis is based on a (reduced-form) network-augmented factor model (NAFM) that is an approximation to the various multi-sector network models considered in the literature. The model is used to express sector-level output growth as the sum of three components—a network propagation effect, common shocks and sector-level shocks—. Importantly, our model allows for different degrees of heterogeneity in how sectors are connected to each other.

To assess the relative importance of the sources of skewness, we focus on Fisher's coefficient of skewness,

$$\kappa_3 = \frac{\mu_3}{\sigma^3} = \frac{E[(X-\mu)^2]}{\sigma^3},$$

where μ_k is the k-th central moment of the random variable X and $\sigma^2 = \mu_2$ is the variance.[3] Our point of departure is that if X is the sum of components, the skewness in X is not the sum of the skewness of the components. For example, if $X_1 \sim (\mu_1, \sigma_1^2)$ and $X_2 \sim (\mu_2, \sigma_2^2)$, then the coefficient

[1] For skewness due to disasters, see Rietz (1988), Barro (2006), and Nakamura, Steinsson, Barro, and Ursua (2013). For skewed sector-specific shocks to granular sectors, see Gabaix (2011) and Acemoglu, Carvalho, Ozdaglar, and Tahbaz-Salehi (2012).

[2] For example, Acemoglu, Ozdaglar, and Tahbaz-Salehi (2017), and Baqaee and Farhi (2019) emphasize that complementarities in production lead to asymmetric network propagation of shocks (that is, the amplification of negative shocks is higher than the positive shocks) that can generate skewness in the aggregate output growth distribution.

[3] Note that time-series tests for skewness need to account for weak dependence in the data. See Bai and Ng (2001), Bai and Ng (2005), and Bontemps and Meddahi (2005).

of skewness for $X = X_1 + X_2$ is

$$\begin{aligned}\kappa_3 &= E\left(\frac{(X_1-\mu_1)+(X_2-\mu_2)}{\sigma}\right)^3 \\ &= \frac{\sigma_1^3}{\sigma^3}\kappa_1 + \frac{\sigma_2^3}{\sigma^3}\kappa_2 + \frac{3\sigma_1^2\sigma_2}{\sigma^3}E(X_1-\mu_1)^2(X_2-\mu_2) + \frac{3\sigma_2^2\sigma_1}{\sigma^3}E(X_2-\mu_2)^2(X_1-\mu_1).\end{aligned}$$

Notably, the skewness of X is not simply the (variance-weighted) sum of skewness of X_1 and X_2, but include two terms coming from the higher order covariability between X_1 and X_2. These two terms, known as co-skewness, will not be zero unless X_1 and X_2 are independent.

As an illustration of the quantitative importance of the co-skewness terms that are the focus of this paper, consider the decomposition of the skewness of nominal GDP growth. The data is quarterly from 1950Q1 to 2024Q4 and, by construction, it is the sum of real GDP growth and the growth rate of the implicit price deflator.[4] The Fisher's coefficient of skewness of nominal GDP growth is -0.99, and, hence, this variable is negatively skewed as documented in previous literature. The terms of the decomposition above are -0.75, 0.16, -0.26, and -0.14, respectively.[5] The first and second terms are respectively the contributions from the skewness of real GDP growth and the growth rate of the deflator, and the latter two terms are the co-skewness terms. Note that the contribution of the co-skewness terms is of the same order of magnitude as that of the first two terms. Their net effect is to overcome the positive skewness of inflation (as measured by the growth rate of the deflator) and increase the negative skewness of nominal GDP growth so that it is actually larger in absolute value than that of its real counterpart.

Our problem is more complicated because we have three components—a network (local), a common (global), and idiosyncratic—, the dynamics are possibility involved, and the relation must be consistent with structural constraints based on the observed input-output matrix. Despite these complications, we show that the factor-augmented network model provides a framework to segregate the network propagation effect from that of the asymmetric shocks. In this sense, our paper extends previous literature (e.g., Acemoglu, Ozdaglar, and Tahbaz-Salehi (2017)) that shows that network effects and asymmetric shocks jointly deliver skewed aggregate series, by decomposing their relative contributions.[6]

We analyze data on sector-level output growth at the NAICS 3-digit level. To that end, we construct network connections based on yearly input-output tables produced by the Bureau of

[4]The raw data were taken from FRED at the Federal Reserve Bank of St. Louis. Real GDP is computed as the ratio of nominal GDP and the implicit price deflator, both of which are seasonally adjusted.

[5]The diligent reader can verify that the sum of these four terms is indeed -0.99.

[6]Baqaee and Farhi (2019) and Ruge-Murcia (2024) argue that a nonlinear propagation mechanism in a production network can also deliver skewed aggregate series . In this paper, we work exclusively in a linear framework, and leave the extension to a nonlinear framework for future research.

Economic Analysis (BEA), and estimate the parameters of the factor-augmented network model underlying the three components from the sectoral data. We find that the skewness attributed to the common shocks are much diminished once the production network is modeled. The network contributes to skewness directly through the production channel, and also indirectly through co-skewness with the common factors. Even without heterogeneity, introducing the network reduces skewness contributed by the common factors from -1.36 to -0.59. The direct effect is larger the more heterogeneous are the spillovers. We then construct and estimate a micro-founded model and show that it can generate skewness and co-skewness consistent with the U.S. data. We also show that the relative sparsity of the I-O table (which features large diagonal terms) need not imply low connectivity across sectors, and that skewed sectoral shocks propagated thorough the network can account for the skewness in the data, as conjectured in Acemoglu, Ozdaglar, and Tahbaz-Salehi (2017). In contrast, skewed aggregate shocks propagated thorough the network induce limited skewness in the data. As an additional contribution to the literature, we report method of moments estimates of the elasticity of substitution between material inputs and between consumption goods. The estimates lie between 0 and 1 and, hence, imply gross complementarity. This result is important because some of the most interesting results in network analysis are based on the assumption of gross complementarity.

A final contribution of the paper is the use of our economic model to decompose the skewness of real sectoral output growth, along the lines of the decomposition we performed above for GDP growth. We argue that absent a sectoral price index that coincides exactly with the sectoral definition, it is not possible to compute directly from the data the contribution of quantity, price, and their co-skewness to the skewness of real sectoral output growth. Thus, we deploy our estimated structural model for this task and show that, as in the case of GDP growth, the contribution of quantity growth (resp. price growth) is negative (resp. positive), that the co-skewness terms are quantitatively large, and that the net effect is negative so that real sectoral output growth is negatively skewed, as observed in the U.S. sectoral data.

The paper is organized as follows. Section 2 presents the data and econometric framework. Section 3 discusses the estimation of the network-augmented factor model and reports reduced-form estimates. Section 4 constructs a multi-sector general equilibrium model with a production network, reports its structural estimates, and uses counter-factual experiments and decompositions to shed additional light on the sources of skewness observed in the U.S. data. Section 5 conludes.

2 Data and Econometric Framework

From early work by Neftci (1984), De Long and Summers (1988), Hamilton (1989) and Sichel (1993) to the more recent work by Salgado, Guvenen, and Bloom (2019), evidence of aggregate skewness is well documented.[7] To get a glimpse of skewness in macroeconomic aggregates, we consider the output gap, the unemployment rate minus its natural rate (i.e., the unemployment gap), and real output growth for 1960Q1 to 2019Q4 in the December 2024 vintage produced by the Congressional Budget Office (CBO). We also take the 1960M1-2019M12 data from the December 2024 vintage of FRED-MD. As explained in McCracken and Ng (2016), this panel of monthly macroeconomic variables is known to have a factor structure, meaning that it is well represented by the factor model $X = F\Lambda + e$.[8] We estimate three factors by principal components, denoted respectively by $\hat{F}_1$, $\hat{F}_2$, and $\hat{F}_3$, and find that these estimates explain about 30% of the variation in the data and load heavily on real activity, interest rate, and price variables, respectively. The top panel of Table 1 shows that whether we consider quarterly or monthly data, these macro aggregates appear non-normal over the sample from 1960 to 2019, displaying negative skewness and/or excess kurtosis. Large negative values of $\hat{F}_1$ are recorded in 1980M5, 1974M12, and 2008M12, which correspond to NBER recession dates. Even excluding the pandemic observations from the analysis, both measures of skewness show stronger evidence of asymmetry in the 1984-2019 sample than in the pre-1984 sample.

It is also possible to define aggregate skewness from the bottom up by analyzing the GDP-by-industry database produced by the BEA. Since this is the primary data to be used in the rest of the paper, we describe it now in more detail. First, note that there have been changes to the definition of sectors over the years: At the 3-digit level, which is our base case, the number of sectors increased from 43 for the sample 1947-1962 to 61 for the sample 1963-1996 and further increased to 66 for 1997-2022. To strike a balance between the span of the data (T) and the number of sectors (N), we use the 1947 classification to aggregate data in subsequent years (i.e., 1947-2022) into 43 sectors. The sectors and their respective NAICS code are listed in Table A1 in Appendix A. Dropping the observations before the Korean War and after the pandemic leave us with data for $N = 43$ sectors for the sample 1957-2019 with $T = 63$. Real gross output per capita for sector i is obtained upon dividing total industry output (i.e., nominal gross output) from the Make Table of the Input-Output (I-O) accounts by the annual GDP deflator (from NIPA Table 1.1.9.) and the

[7] The density of a symmetrically distributed variable should have equal mass on the left and right of its mean, which is also its median and mode. In contrast, a skewed variable has an unequal mass around the center of its density. For instance, a left-skewed variable has a long left tail, and its density is right-leaning.

[8] The data are downloaded from `https://www.cbo.gov/data/budget-economic-data` and `https://research.stlouisfed.org/econ/mccracken/fred-databases`.

estimate of the U.S. population produced by the BEA. The former series was obtained from the BEA website, and the latter two series were obtained from FRED at the Federal Reserve Bank of St. Louis. An important feature of these data is that nominal gross output in the I-O table is measured as the product of the quantity of sectoral output times its price. Thus, a measure of real industry output – obtained by dividing nominal output by a price index – will unavoidably include both the output quantity and a relative price component. In the case where the price index is the GDP deflator, this relative price will be the real price of the sectoral good. The growth rate of real output in sector i at time t, y_{it}, is computed as 100 times the log difference of real gross output per capita.

We estimate three factors F and the corresponding idiosyncratic errors e from the $T \times N$ panel of data for real sectoral output growth rates. Summary statistics are reported in the bottom panel of Table 1. The three factors explain about 60% of the variation in the data. Most of the sectoral output growth rates and $\hat{e}$ series are not very persistent, as only around 30% have a first-order autocorrelation coefficient exceeding 0.3. The mean and median of estimates of σ_i^y and $\sigma_i^{\hat{e}}$ are stable over the sub-samples. The value of the first factor $\hat{F}_1$ is most negative in 2009, 1982, and 1975, which are years of economic downturns. Although the average (across units) of the skewness $\kappa_3^{\hat{e}}$ is close to zero, the average of the skewness κ_3^y is negative. Figure 1 plots the sample coefficient of skewness for each sector along with the sectoral weight. Though there is substantial heterogeneity, 33 of the 43 series are negatively skewed. Utilities, Apparel, and Air Transport (corresponding to Sectors 6, 21, 29) have the most negative κ_3^y while Farming, and Food/Tobacco (corresponding to sectors 1 and 19) are the most positively skewed.

A sector-based measure of aggregate output growth can be obtained by taking a weighted sum of sectoral output growth,

$$Y_t = \omega_t' \boldsymbol{y}_t$$

where $\boldsymbol{y}_t$ is the $N \times 1$ vector of sectoral output growth rates, y_{it}. The simplest approach is to use an equal weight of $1/N$. An alternative is to use the share of sector i's output in total output at time t, $\omega_{it} = \frac{z_{it}}{\sum_{i=1}^N z_{it}}$. However, this measure can be sensitive to time variations. A more stable measure of weights is $\omega_{it} = \bar{\omega}_i = (1/T)\sum_{t=1}^T \omega_{it}$ for all t. The coefficient of skewness corresponding to the three weighting schemes are -1.257, -1.608, and -1.680. Regardless of the choice of weights ω_t, aggregate output defined as above also exhibits negative skewness, consistent with the measures of skewness found for aggregate data reported earlier.

The input-output (I-O) matrix Γ will play an important role in our analysis. We can write the output as the sum of primary inputs and intermediate inputs, so that $Y = V + \sum_{j=1}^N \Gamma_{ji} Y_j$.[9]

[9]This relation holds by definition for the level of the variables, and for their growth rates under additional

Thus $\boldsymbol{y}_t = (I - \Gamma')^{-1} V_t$, where V_t stands for primary inputs (like, labor). The Use table of the I-O accounts is employed to construct an empirical counterpart, say Γ_{BEA}, of the theoretical input-output matrix, Γ. I-O tables are produced annually by the BEA. The Use table records total purchases of inputs to industry production and the commodities that are consumed by final users in billions of current dollars at producer prices, with rows containing commodities and columns containing users. That is, the rows of column j of Γ_{BEA} are payments by sector j to all sectors while the columns of row i are the amount supplied by i to all sectors. Following the literature, we normalize Γ_{BEA} to Γ so that Γ_{ij} is the payment from sector j to i as a fraction of the value of production of sector j, and $\sum_{i=1}^{N} \Gamma_{ij} = 1$.

Figure 2 plots the estimated Γ_{BEA} using a heatmap with sector numbers listed in Table A1 in Appendix A. The figure shows intuitive interactions between sectors. For instance, 66.5% of material expenditures by the petroleum and coal products (#24) goes to goods produced by the oil and gas extraction (#3), and 44.3% of material expenditures by food services and drinking places (#42) goes to goods produced by food and beverage and tobacco products (#19). An important feature of the table at this level of disaggregation is the large values along the main diagonal, meaning that a substantial proportion of material expenditures by sector j goes to goods produced in the same sector. However, we show below that even if the I-O table is relatively sparse and features substantially larger diagonal than off-diagonal entries, it still implies strong connectivity across sectors. Finally, given the fact that I-O tables evolve (albeit slowly) over time and respond to changes in technology and prices, a concern that we address below is whether the Γ constructed from the data is a correct measure of the theoretical concept in the model.[10]

2.1 A Factor-Augmented Network Model When Γ is Known

Foerster, Sarte, and Watson (2011) (hereafter, FSW) show that the representation

$$(I_N - \ddot{\Phi}_1 L)\boldsymbol{y}_t = (\ddot{\Pi}_0 + \ddot{\Pi}_1 L)\ddot{\epsilon}_t,$$

where I_N is the $N \times N$ identity matrix, L is the lag operator, and $\ddot{\Phi}_1$, $\ddot{\Pi}_0$, and $\ddot{\Pi}_1$ are conformable matrices, encompasses the first-order approximate solution of many production network models previously considered in the literature. In VAR terminology, this is a B model where the contemporaneous effects occur through the shocks. If $\ddot{\Phi}_1 = \Gamma'$ and $\ddot{\Pi}_0 = I$, (1) is the model in Long and Plosser (1983); if $\ddot{\Phi}_1 = 0$ and $\ddot{\Pi}_0 = (I - \Gamma')^{-1}$, it is the model in Carvalho (2010); and if $\ddot{\Pi}_0 = (I - \Gamma')^{-1}$ and $\ddot{\Phi}_1$ is a known diagonal matrix, it is the model in Hovarth (1998). All these

assumptions like Cobb-Douglas production functions.

[10] This is of particular importance for the recent literature that assumes strong complementarity of materials inputs because this specification implies that the Input-Output Table is an endogenous object that responds to shocks.

models impose $\ddot{\Pi}_1 = 0$. FSW further assume that $\ddot{\epsilon}_t = \ddot{\Lambda}\dot{F}_t + \ddot{e}_t$ has a factor structure. In particular, $\dot{\Lambda}$ is a $N \times r$ matrix of factor loadings corresponding to the $r \times 1$ vector of common factors $\ddot{F}_t$, and $\ddot{e}_t$ is a $N \times 1$ vector of idiosyncratic errors. FSW estimate the common component $\ddot{C}_t = \ddot{\Lambda}\ddot{F}_t$ after filtering out the network effects of idiosyncratic shocks from their data. Precisely, taking the model matrices $\ddot{\Pi}_0$ to be Γ'_{BEA}, the transpose of the input-output matrix, they study the relative importance of common and sector-specific shocks.

2.2 Decomposition of Variance and Skewness

Our goal is to study the relative contribution of common shocks, sectoral shocks, and the production network to skewness. Our starting point is the model,

$$\boldsymbol{y}_t = \dot{\Phi}_0 \boldsymbol{y}_t + \dot{\Phi}_1 y_{t-1} + \dot{\epsilon}_t, \tag{1}$$

where $\dot{\Phi}_0$ and $\dot{\Phi}_1$ are $N \times N$ matrices. In SVAR terminology, this is an A model. With $\dot{\epsilon}_t = \dot{\Lambda}\dot{F}_t + \dot{e}_t$, the baseline model is

$$\boldsymbol{y}_t = \dot{\Phi}_0 \boldsymbol{y}_t + \dot{\Phi}_1 y_{t-1} + \dot{\Lambda}\dot{F}_t + \dot{e}_t.$$

Let the inverse Leontief matrix be

$$\mathcal{L} = (I_N - \dot{\Phi}_0)^{-1}.$$

Then, the reduced form $\boldsymbol{y}_t = \mathcal{L}\left(\dot{\Phi}_1 \boldsymbol{y}_{t-1} + \dot{\Lambda}\dot{F}_t + \dot{e}_t\right)$ implies

$$\boldsymbol{y}_t = \underbrace{\mathcal{L}\dot{\Phi}_1 \boldsymbol{y}_{t-1}}_{\text{network+dynamics}} + \underbrace{\mathcal{L}\dot{\Lambda}\dot{F}_t}_{\text{network+common}} + \underbrace{\mathcal{L}\dot{e}_t}_{\text{network+idiosyncratic}}.$$

We can see that the network effects are confounded with the common and idiosyncratic components. However, by adding and subtracting terms, we can separate the terms:

$$\begin{aligned} \boldsymbol{y}_t &= \underbrace{(\mathcal{L} - I_N)\dot{\Lambda}\dot{F}_t + (\mathcal{L} - I_N)\dot{e}_t}_{\text{current network}} + \underbrace{\mathcal{L}\Phi_1 \boldsymbol{y}_{t-1}}_{\text{dynamic network}} + \underbrace{\dot{\Lambda}\dot{F}_t}_{\text{common}} + \underbrace{\dot{e}_t}_{\text{idiosyncratic}} \\ &= \underbrace{(\mathcal{L} - I_N)\dot{\Lambda}\dot{F}_t + (\mathcal{L} - I_N)\dot{e}_t + \mathcal{L}\Phi_1 \boldsymbol{y}_{t-1}}_{\text{network}} + \underbrace{\dot{\Lambda}\dot{F}_t}_{\text{common}} + \underbrace{\dot{e}_t}_{\text{idiosyncratic}}. \end{aligned}$$

A decomposition of aggregate output growth, $Y_t = \omega' \boldsymbol{y}_t$, is

$$\begin{aligned} Y_t &= \underbrace{\omega'(\mathcal{L} - I_N)\dot{\Lambda}\dot{F}_t + \omega'(\mathcal{L} - I_N)\dot{e}_t + \omega'\mathcal{L}\Phi_1 \boldsymbol{y}_{t-1}}_{\text{network}} + \underbrace{\omega'\dot{\Lambda}\dot{F}_t}_{\text{common}} + \underbrace{\omega'\dot{e}_t}_{\text{idiosyncratic}} \\ &= \mathcal{N}_t + \mathcal{C}_t + \mathcal{E}_t. \end{aligned} \tag{2}$$

Thus, aggregate output growth has three components: local connectivity through the network, global connectivity through the common component, and idiosyncratic shocks. This leads to the following decomposition of variance,

$$\text{var}(Y_t) = \text{var}(\mathcal{N}_t) + \text{var}(\mathcal{C}_t) + \text{var}(\mathcal{E}_t) + 2\text{cov}(\mathcal{N}_t, C_t) + 2\text{cov}(\mathcal{N}_t, \mathcal{E}_t) + 2\text{cov}(\mathcal{C}_t, \mathcal{E}_t).$$

A decomposition of skewness is more involved. As noted above, if y_t is the sum of two components, the skewness of the sum is not the sum of the skewness of the two components, but also depends on the higher order covariation of the components. Given that Y_t has three primitive components, a decomposition of the coefficient of skewness is

$$\begin{aligned}
\kappa_3^Y &= \frac{1}{T}\sum_{t=1}^{T}\left(\frac{Y_{it}-\mu_i}{\sigma_i}\right)^3 \\
&= s_{\mathcal{N}}^3\kappa_3^{\mathcal{N}} + + s_{\mathcal{C}}^3\kappa_3^{\mathcal{C}} + + s_{\mathcal{E}}^3\kappa_3^{\mathcal{E}} \\
&\quad + 3s_{\mathcal{C}}^2 s_{\mathcal{E}}\text{CoS}(\mathcal{C},\mathcal{C},\mathcal{E}) + 3s_{\mathcal{C}}^2 s_{\mathcal{N}}\text{CoS}(\mathcal{C},\mathcal{C},\mathcal{N}) \\
&\quad + 3s_{\mathcal{N}}^2 s_{\mathcal{E}}\text{CoS}(\mathcal{N},\mathcal{N},\mathcal{E}) + 3s_{\mathcal{N}}^2 s_{\mathcal{C}}\text{CoS}(\mathcal{N},\mathcal{N},\mathcal{C}) \\
&\quad + 3s_{\mathcal{E}}^2 s_{\mathcal{C}}\text{CoS}(\mathcal{E},\mathcal{E},\mathcal{C}) + 3s_{\mathcal{E}}^2 s_{\mathcal{N}}\text{CoS}(\mathcal{E},\mathcal{E},\mathcal{N}) \\
&\quad + 6s_{\mathcal{N}} s_{\mathcal{C}} s_{\mathcal{E}}\text{CoS}(\mathcal{N},\mathcal{C},\mathcal{E}),
\end{aligned}$$

where $s_H = \sigma_H/\sigma_Y$, and $H, J = \mathcal{N}, \mathcal{C}$, or $\mathcal{N}$,

$$\text{CoS}(H,H,J) = \frac{E\left((H-\mu_H)^2(J-\mu_J)\right)}{\sigma^2 H \sigma_J}.$$

A system of k components has $k!(k-2)!/2$ co-skewness terms. Since $k = 3$ in our case, there are six possible sources that can contribute to the skewness of Y. The decomposition takes σ_i^2 (i.e., the variance of Y_i) constant but allows for other types of time-varying second moments to contribute to skewness. For example, $\text{CoS}(\mathcal{C},\mathcal{C},\mathcal{N})$, is the the covariance between the volatility (squared variations of the the common component $\mathcal{C}$) and the network component.

We take the view that the I-O table produced by the BEA is a good approximation to, but not an exact proxy of, the theoretical object in our multi-sector models. Possible reasons for discrepancies between the theoretical and the empirical objects are the slow evolution of the I-O table as a result of technological progress and long-term trends in relative prices to which firms will respond.[11]. A more flexible approach is to assume that the network effect is represented by Γ' only up to scale ϕ_0, and this leads to the inverse Leontieff matrix

$$\mathcal{L} = (I_N - \phi_0\Gamma)^{-1}.$$

[11]For example, comparing I-O tables over time shows the steady substitution of metal (notably aluminum) for plastics in car manufacturing. Long-term trends in relative sectoral prices are documented by Ruge-Murcia and Wolman (2022)

where $\phi_0\Gamma$ now stands for $\dot{\Phi}_0$ in equation (1). Similar inverse Leontieff matrices are specified by von Lehn and Winberrry (2022) and di Giovanni and Hale (2022). Foerster, Sarte, and Watson (2011) and Atalay (2017) adjust the input-output matrix for capital goods by incorporating maintenance expenditures into its diagonal entries. The relative importance of the network now depends on ϕ_0. When $\phi_0 > 0$, the production network and common factors co-exist. When $\phi = 1$, Γ from the U.S. I-O table is the appropriate measure of spillovers, and $\mathcal{C}_t$ is the common component of $Y_t - \mathcal{N}_t$ obtained by principal components.

To illustrate the importance of this parameter, Table 2 reports the decomposition of variance and skewness as ϕ_0 varies. The top panel shows that the variance due to the network component increases monotonically with ϕ_0, while the covariance between $\mathcal{C}$ and $\mathcal{N}$ increases for values of ϕ_0 up to about 0.3 and then decreases. The bottom panel shows that the direct contribution of the network component $\mathcal{N}$ to skewness increases sharply with ϕ_0 at the expense of the common component $\mathcal{C}$. However, $\mathcal{C}$ still contributes to skewness, albeit indirectly through its higher order covariation with $\mathcal{N}$. For many values of ϕ_0, the two coskewness terms NNC and NCC contribute more to skewness than $\mathcal{N}$, with a combined contribution to skewness that is maximized at values of ϕ_0 between 0.3 and 0.4. The point to highlight from Table 2 is that the production network contributes directly and indirectly to skewness, and both effects depend on ϕ_0 via the inverse Leontief matrix in a non-linear way. Though helpful in gaining insight, ϕ_0 is unknown. In the following subsection, we suggest ways to allow for a more general inverse Leontief matrix.

2.3 Allowing for Heterogeneity

Our goal is to understand the relative importance of the different sources of skewness, while entertaining the possibility that the input-output matrix Γ obtained from the BEA may be an imperfect measure of Γ_0 that generates the data. To this end, let $\odot$ denote element-by-element multiplication, $\Phi_0 = A_0 \odot \Gamma'$, $\Phi_1 = A_1 \odot \Gamma'$, where A_0 and A_1 are $N \times N$ matrices with 1's on the diagonals. These matrices will be used to accommodate measurement issues (such as difficulty in distinguishing between materials and investment goods as discussed in FSW) and also allow for heterogeneous spillovers. We consider the factor-augmented network model model for the $N \times 1$ vector $\mathbf{y_t} = (\mathbf{y_{1t}}, \ldots, \mathbf{y_{Nt}})'$,

$$\boldsymbol{y}_t = \Phi_0 \boldsymbol{y}_t + \Phi_1 \boldsymbol{y}_{t-1} + \Theta \boldsymbol{c}_t + e_t \tag{3}$$

where $\boldsymbol{c}_t = \Lambda F_t$ is a $N \times 1$ vector of common component, and $\Theta = \theta 1_N'$, $\theta = (\theta_1, \ldots, \theta_N)'$ is a $N \times 1$ vector to be jointly determined with the unknowns in Φ_0 and Φ_1. Hence, while the inverse Leontief matrix in FSW is known and given by $(I_N - \dot{\Phi}_0)^{-1} = (I_N - \Gamma')^{-1}$, our adjusted inverse Leontief

matrix is now given by

$$\mathcal{L} = (I_N - \Phi_0)^{-1} = (I_N - A_0 \odot \Gamma')^{-1}.$$

This matrix depends on A_0 which has to be estimated. Since we take Γ to be the normalized input-output matrix from the BEA, the estimation of Φ_0 and Φ_1 reduces to estimation of A_0 and A_1 jointly with θ. But $A_0 = [a_{ij}^0]$ and $A_1 = [a_{ij}^1]$ each has $N(N-1)$, and unrestricted estimation is not possible.

To motivate the restrictions that we impose, let $W = \Gamma'$ to simplify notation. The equation for sector i for the model defined by (3) is

$$y_{it} = \sum_{j \neq i} a_{ij}^0 W_{ij} y_{jt} + \sum_{j \neq i} a_{ij}^1 W_{ij} y_{jt-1} + \theta_i c_{it} + e_{it}.$$

We consider three models that makes a^0 and a^1 increasingly less restrictive,[12]

$$y_{it} = a^0 \sum_{j \neq i} W_{ij} y_{jt} + a^1 \sum_{j \neq i} W_{ij} y_{jt-1} + \theta_i c_{it} + e_{it} \tag{Model I}$$

$$y_{it} = a_i^0 \sum_{j \neq i} W_{ij} y_{jt} + \sum_{j \neq i} a_i^1 W_{ij} y_{jt-1} + \theta_i c_{it} + e_{it} \tag{Model II}$$

$$y_{it} = a_{i\mathcal{G}}^0 \sum_{j \in \mathcal{G}_i} W_{ij} y_{jt} + a_i^0 \sum_{j \notin \mathcal{G}_i} W_{ij} y_{jt} + a_{i\mathcal{G}}^1 \sum_{j \in \mathcal{G}_i} W_{ij} y_{jt-1} + a_i^1 \sum_{j \notin \mathcal{G}_i} W_{ij} y_{jt-1} + \theta_i c_{it} + e_{it}. \tag{Model III}$$

Model I is the homogeneous case where $\Phi_0 = a_0 W$ and $\Phi_1 = a_1 W$. Model II has $A_0 = a_L^0 1_N'$ where $a_L^0 = (a_1^0, \ldots, a_N^0)'$ is heterogeneous. Model III allows some within-row difference depending on (known) group membership $\mathcal{G}_i$,

$$A_{ij}^0 = \begin{cases} a_{i\mathcal{G}}^0 & j \in \mathcal{G}_i \\ a_i^0 & j \neq \mathcal{G}_i \end{cases}.$$

Given sector i's group membership, we estimate two parameters $\{a_{i\mathcal{G}}^0, a_i^0\}$. The parameter $a_{i\mathcal{G}}^0$ corresponds to all the sectors that are in the same group as i, and the parameter a_i^0 to all the sectors in other groups. We use both economic and statistical restrictions to determine membership $\mathcal{G}_i$ as follows:

1) Economic restrictions: $\mathcal{G}_i$ is a list of all 3 digit sectors that are in the same 1-digit classification as i. Since each of the N sectors at the 3-digit level must belong to one of the five sectors at the 1 digit level, and $\cap_i \mathcal{G}_i = \emptyset$, there are $2N$ free parameters in A^0, and similarly for A^1.

2) Statistical restrictions: $\mathcal{G}_i$ is a list of the K sectors most correlated with i according to LASSO. The difference with (1) above is that a sector can belong to more than one $\mathcal{G}_i$. Hence, with the

[12] In implementation, we let $A_0 = a_L^0 a_R^{0'} + I_N - \text{diag}(a_L^0 a_R^{0'})$ and $A_1 = a_L a_R^{0'} + I_N - \text{diag}(a_L^1 a_R^{1'})$, where 1_N is a $N \times 1$ vector. We impose restrictions on the $N \times 1$ vectors a_L^j and a_R^j for $j = 0, 1$, while ensuring that $\text{diag}(\Phi_0) = 1$.

statistical approach, the number of unknowns in A^0 and A^1 are still $2N$, but the sectors in each group are data determined.

In summary, there is no variation in the rows and columns of A_0 and A_1 in Model I. The rows of A_0 and A_1 in Model II differ, but each row's column entries are the same. We use the static version of Model II (without y_{t-1}) as our base model. Model III further relaxes the structure by allowing within-row heterogeneity.

3 Estimation

Models I and II resemble a spatial dynamic panel model with interactive fixed effects. While spatial analysis estimates the spillover parameters taking W as given, network analysis typically takes the estimation of the connectivity matrix W as the goal. de Paula, Rasul, and Souza (2024) use the exogenous covariates to identify the parameters, while Lewbel, Qu, and Tan (2023) recover the structural parameters from the reduced form coefficients. In the psychometrics literature, the W matrix is estimated from the covariance structure of the data but uses regularization to prevent overfitting (see Epskamp, Rhemtulla, and Borsboom (2017)). In between the extremes of completely latent and completely observed W is the case that treats W as imperfectly observed. For example, Lam and Souza (2016, 2019) assume that the true connectivity matrix W^* is related to the observed W via $W^* = B_0 + \beta_0 W$. Then B_0 is estimated when there is no information about W, but β_0 is estimated when W is observed up to scale. Our use of A_0 is in the spirit of allowing for possible misalignment between the data matrix Γ and model matrix Γ_0,

Spatial autoregressive models are often estimated by maximum likelihood or by instrumental variables. Maximum likelihood is efficient but requires computation of the determinant of a $N \times N$ matrix with unknown parameters, which can be computationally costly when N is large. Less appealing in our context is that we want to learn about the possible asymmetry of the errors, so we refrain from specifying parametric distributions. Estimation by GMM-IV is also possible if there are covariates. These are absent in our analysis, but as a robustness check, we also consider IV estimation using estimates of the common component in lieu of X as instruments.

Let the parameters of the model be collected into a vector $\beta = (a^{0'}, a^{1'}, \theta)$, and let β^* denote their true value. We will estimate β based on the covariance structure of the idiosyncratic errors. The following assumption will be used.

Assumption A

1) $E[e^*_{it}e^*_{jt}] = E[e_{it}(\beta^*)e_{jt}(\beta^*)] = 0$ for $i \neq t$;

2) $W = \Gamma'$ satisfies $\|W\|_\infty < \delta_W < \infty$ and is such that for all i, $\sum_{j=1}^N W_{ij} = 1$;

3) (i) $\max(\|\Phi_0\|_1, \|\Phi_0\|_\infty) \leq \delta_0 < 1$, (ii) $\max(\|\Phi_1\|_1, \|\Phi_1\|_\infty) \leq \delta_1 < 1$, and (iii) $\delta_C = \delta_1/(1-\delta_0) < 1$.

Assumption A.2 is due to the summing up constraint induced by the production network structure. To ensure stationarity, A.3(i) controls contemporaneous dependence and A.3(ii) controls persistence. Bounding the maximum row sum $\|\Phi_0\|$ and column sum of $\|\Phi_0\|_1$ ensure that the adjusted Leontief $\mathcal{L} = (I_N - \Phi_0)^{-1}$ exists. Given that W is known, the assumptions on Φ_0, Φ_1 are implicit restrictions on A_0 and A_1. Together with A.3(ii) and (iii), a stable reduced form $\boldsymbol{y}_t = \mathcal{L}\Phi_1 \boldsymbol{y}_{t-1} + \mathcal{L}\epsilon_t$ is obtained.

Assumption A.1 is key to identification. Specifically, it requires that the idiosyncratic errors e^*_{it} and e^*_{jt} are cross-sectionally uncorrelated when evaluated at the true β^*. Consider Model I without common factors and dynamics so that $\beta = a^0 = (a^0_1, \ldots, a^0_N)'$. Then $y_i = a^0 W_i \mathbf{y} + \mathbf{e_i}$ for some a^0 not necessarily equal the true value a^* and $e_{it} = e^*_{it} - \sum_{k=1}^N (a^0_{ik} - a^{0*}_{ik}) w_{ik} y_{kt}$. Under the maintained assumption that $\text{cov}(e^*_i, e^*_j) = 0$, the pairwise covariance of the errorscov$(e_i, e_j) = E[(y_i - a^0 W_i \boldsymbol{y})(y_j - a^0 W_j \boldsymbol{y})]$ satisfies

$$\text{cov}(e_i, e_j) = (a^0 - a^{0*}) \Big[(a^0 - a^{0*}) E(W_i \boldsymbol{y} W_j \boldsymbol{y}) - E(W_i \boldsymbol{y} e_j + W_j \boldsymbol{y} e_i) \Big]$$

The above can be zero only if $a^0 = a^{0*}$. A similar idea is used in Liu, Liu, Li, and Lin (2023) to estimate homogeneous spatial models.[13] Our problem is a bit more involved but conceptually similar. Given our economic definition of $\mathcal{G}_i$ in Model III, each β_i is a $2N \times 1$ vector. Let $g(\beta) = \text{vech}(E[e_t(\beta) e_t(\beta)'])$ be the $N(N-1)/2$ unique covariances of the idiosyncratic errors satisfying $E[g(\beta^*)] = 0$. The order condition for identification is $N(N-1)/2 > \dim(\theta) = (p+1) \times 2N$. When $p = 1$, the above condition requires $N > 5$, which is satisfied in our application where $N = 43$.

In implementation, we iterate between $a_0 = (a^0_1, \ldots, a^0_N)$ and the parameters in the common component, given initial values $(a^{(k)}_0, \theta^{(k)})$, for $k = 1$. To estimate the base case (static Model II), the following two steps are repeated till convergence:
1) Let $(F^{(k)}, \Lambda^{(k)})$ be the principal components of $\epsilon^{(k)}_t = y_t - \Phi^{(1)}_0 \boldsymbol{y}_t$. Let $\boldsymbol{c}^{(k)}_t = \Lambda^{(k)} F^{(k)}_t$.
2) With $e^{(k)}_t = \boldsymbol{y}_t - \Phi^0 \mathbf{y_t} - \boldsymbol{\Theta} \mathbf{c_t}$, let $g^{(k)}(a, \theta) = \text{vech}(\Sigma^{(k)}_e(a, \theta))$ be the $N(N-1)/2$ unique sample covariance of idiosyncratic errors. Let

$$(a^{(k+1)}, \theta^{(k+1)}) = \min_{a,\theta} g^{(k)}(a, \theta) g'^{(k)}(a, \theta).$$

Estimation of the dynamic model and Model III only require changing the definition of e_t. Because N and T are small, we compute bootstrap standard errors.

In the next subsection, we will first show that estimates suggest substantial heterogeneity in A_0. We will then decompose variance and skewness based on Models I, II, IId (dynamic), and III.

[13] More precisely, Liu, Liu, Li, and Lin (2023) consider the model $y_i = \rho W_i \mathbf{Y} + \mathbf{X_i}'\beta + \epsilon_\mathbf{i}$ and show that $E[(Y_i - \rho W_i \mathbf{Y} - \mathbf{X_i}'\beta)(\mathbf{Y_j} - \rho \mathbf{W_j Y} - \mathbf{X_j}'\beta] = \mathbf{0}$ only if ρ and β are at their true values. Their objective function is $Q(\rho, \beta) = \frac{1}{N(N-1)} \sum_{i \neq j} \epsilon_i) \rho, \beta) \epsilon_j) \rho, \beta)^2$.

In all cases, we assume that the number of factors is one. Thus, the total number of parameters to be estimated is N plus the number of free parameters in A_0 and A_1.

3.1 Results

Our base case Model II is static, which allows for heterogeneity in the rows of A^0, collected into a $N \times 1$ vector denoted a_L^0. Focusing on the static Model II helps focus on the main results as we only find weak time dependence in the data. As shown in Figure 3, most of these estimates are significantly different from one. The sectors with the largest $\hat{a}_{L,i}^0$ are oil and gas, mining, and primary metals, while the values of $\hat{a}_{L,i}^0$ are smallest for education and health care. Figure 3 also shows the sectoral output shares of each sector (plotted on the right axis). It is clear that the observed heterogeneity in the estimates of A^0 cannot be attributed solely to sector size. In particular, the sectors with the largest estimates of $\hat{a}_{L,i}^0$ are not those with the highest output shares. This parameter heterogeneity plays a critical role in the skewness decomposition. Before proceeding to decomposition results, we first verify the properties of the estimator using a Monte Carlo experiment.

We use a calibrated Monte Carlo experiment to assess the performance of the GMM estimator based on the covariance structure. Specifically, for each $i = 1, \ldots, N$, we generate data y_{it} of different length T using estimates from Model II as 'true values', and the simulated data are used to estimate the model. The exercise is repeated 1000 times. The estimator is more efficient for a larger T, but the estimates are reasonably accurate even with $T = 63$ as in the application. Figure 4 plots the mean of $\hat{a}_L^0$ in 1000 replications, along with the one-standard error confidence band. We see that the 'true' value is often close to the mean estimate, and the mean estimates are within one standard error of the 'true' values in 42 of the 43 cases. These findings highlight the reliability of our GMM estimator based on the covariance structure even when the sample size (T) is small.

Summarizing heterogeneity in the dynamic version of Models II and in Model III is more challenging as we estimate Λ and A^1 in addition to A^0. Instead of looking at the individual estimates, we focus on the variance and skewness implied by each model. The top panel of Table 3 reports the share of the variance attributed to the components, where for all models considered, the total variance to be explained is always 10.088. In the 'factor-only' model (column labeled F) where the network component $\mathcal{N}$ is absent, the variability in the data can only be attributed to the common component $\mathcal{C}$ or the idiosyncratic error $\mathcal{E}$, and they are orthogonal by construction. Absent the production network, the data find $\mathcal{C}$ to contribute significantly more to variance than $\mathcal{E}$.[14] However, once the network component is allowed, the importance of $\mathcal{C}$ falls to give way

[14] Foerster, Sarte, and Watson (2011) show that the common component explains almost all of the variation in

to $\mathcal{N}$.[15] The precise importance of $\mathcal{N}$ depends on heterogeneity allowed in A^0. When A^0 is homogeneous (in Model I), the importance shifts from $\mathcal{C}$ to the covariance terms between $\mathcal{C}$ and $\mathcal{N}$. The more heterogeneous is A^0 (in Models II, II(d), and III), the more important are the network and covariance terms. In our base case of Model II, the indirect contribution of $\mathcal{N}$ through covariances is as important as its direct contribution.

The lower panel of Table 3 shows the contribution of the components to total skewness, which for this data is -1.687. Absent the network component, skewed common shocks directly contribute an amount of -1.363, with the remaining skewness coming from coskewness between $\mathcal{E}$ and $\mathcal{C}$. There is little evidence of skewed idiosyncratic shocks. Introducing the network component drastically changes the decomposition, especially when A^0 is heterogeneous. As we increase heterogeneity going from Model I to III, skewness due to $\mathcal{N}$ directly and indirectly via the coskewness terms NNC and NCC increase at the expense of $\mathcal{C}$. Evaluated at our base case Model II estimates, the dominant source of skewness comes from NNC and NCC (-0.488 and -0.248), which are larger than the direct contribution of $\mathcal{N}$, being -0.327. Thus, while the common and sectoral shocks do not directly contribute to skewness in a significant way, their indirect contribution is non-trivial. The most robust contribution is NNC, accounting for about one-quarter of the total skewness of -1.687.

Our decomposition thus suggests that the direct contribution of skewed shocks to observed skewness in aggregate output is trivial once network connectedness is allowed. The production network contributes directly to the network, but the primary source of skewness is the coskewness arising from higher-order covariation between the three components. The production network thus propagates skewness, even though the observed Γ^{BEA} matrix seems relatively sparse, as seen in Figure 2. In Section 4, we introduce a micro-founded macroeconomic model that incorporates network linkages across sectors to help interpret our decomposition results.

3.2 Alternative Estimation Results

Our primary estimation strategy employs a GMM estimator using the moments implied by the assumption that idiosyncratic shocks across sectors are uncorrelated. This approach does not require additional distributional assumptions on the idiosyncratic shocks. However, more efficient estimators based on maximum likelihood estimation (MLE) impose a normality assumption, which can significantly alter the decomposition results obtained via GMM. To examine how the relative importance of each model component changes under the normality assumption, we re-estimate the model using a quasi-MLE approach that follows Aquaro, Bailey, and Pesaran (2020).

industrial production growth absent a production network.

[15] Foerster, Sarte, and Watson (2011) and Atalay (2017) report a similar result whereby the share of variance explained by the common component reduces drastically when one accounts for the network connections.

Table B1 in Appendix B compares the variance and skewness decompositions obtained from GMM and MLE estimations. The contribution of the network $\mathcal{N}$ component to the overall variance rises to 72% under MLE, compared to approximately 33% under GMM. This increase comes at the expense of both the idiosyncratic and common components. Figure B1 plots the GMM and MLE estimates of A^0, indicating that the MLE-based estimates differ substantially from those obtained via GMM. The bottom panel of Table B1 depicts a similar picture when we compare the skewness decompositions. The dominant source of skewness comes from the network $\mathcal{N}$ component, which directly contributes 65% of skewness in the case of MLE compared to around 19% in the case of GMM. The indirect contribution of the network component through NNC is similar in both cases, but the contribution due to NCC and NNE is negligible in the case of MLE.

The bottom panel of Table B1 presents a similar pattern when comparing the skewness decompositions. The network $\mathcal{N}$ component is the dominant source of skewness, which directly accounts for 65% under MLE compared to about 19% under GMM. Although the indirect contribution of the network component through NNC is similar in both cases, the contributions from NCC and NNE are negligible under MLE. These findings highlight how the distributional assumptions regarding idiosyncratic shocks affect their contribution to aggregate skewness.

3.3 Alternative Sectoral Classification

Our main results use data from 43 sectors, as shown in Table A1. Here, we provide the decomposition of variance and skewness under a broader 2-digit classification comprising 14 sectors. Table A2 in Appendix A lists the 14 sectors included in this analysis. We estimate our baseline model II with heterogeneous A^0 using the data from these 14 sectors. Table C1 presents the decomposition of variance and skewness using both GMM and MLE estimates along with factors-only decomposition.

The variance decomposition in the top panel of table C1 shows the importance of network $\mathcal{N}$ component in explaining variance in both GMM and MLE estimates. However, the difference between the two decompositions is in the importance of the covariance between network and sector-specific component, NE. The decomposition based on GMM estimates gives more importance to sector-specific components, whereas the MLE's decomposition gives more weight to common components.

The top panel of Table C1 in Appendix C highlights the significance of the network component, $\mathcal{N}$, in explaining variance for both GMM- and MLE-based decompositions. However, the two approaches diverge in the role of the covariance between the network and sector-specific components (NE). While the GMM-based decomposition attributes greater importance to the sector-specific component, the MLE-based decomposition places more weight on the common component. A

similar pattern emerges in the skewness decomposition, where the network $\mathcal{N}$ component primarily accounts for aggregate skewness. Under GMM, the coskewness term NNE plays a larger role, whereas, under MLE, the coskewness term NNC is more influential.

Comparing these results to the ones obtained using 43 sectors shows that the direct contribution of the network component increases from 19% to 49% when using the 14-sector classification. The coskewness terms, NNC and NNE, remain similarly important in both classifications. To understand why, we examine the estimates of A^0 for the 14-sector setting and find that they exhibit less heterogeneity than those from the 43-sector dataset. Figure C1 in Appendix C displays the GMM-based estimates of A^0, which, compared to Figure 3, are notably more homogeneous. Consequently, the skewness decomposition becomes more concentrated in fewer components.

4 An Economic Model

In order to help us interpret the results above, this section constructs a multi-sector general equilibrium model with a production network.[16] The model follows closely that in Ruge-Murcia (2024) but relaxes the assumption of normally distributed shocks and allows instead for skewed shocks from a generalized extreme value (GEV) distribution. The latter modeling choice is based on Acemoglu, Ozdaglar, and Tahbaz-Salehi (2017), who argue that for microeconomic shocks to translate into macroeconomic tail risk, the former need to exhibit some minimal degree of tail risk relative to the normal distribution. The economy consists of infinitely-lived households and perfectly-competitive firms in N heterogeneous sectors whose output may be used as materials input by other firms or as final goods by households. A period in the model is one year.

4.1 Producers

The representative firm in sector $i \in N$ produces the output quantity $q_{i,t}$ using the technology[17]

$$q_{i,t} = z_{i,t}(z_t h_{i,t})^{\eta}(m_{i,t})^{1-\eta}, \tag{5}$$

where $z_{i,t}$ and z_t are productivity shocks, $h_{i,t}$ is labor input, $m_{i,t}$ is materials, and $\eta \in (0,1)$ is a parameter. The shock $z_{i,t}$ is idiosyncratic and affects only firms in sector i, while the shock z_t is aggregate and affects all firms in all sectors. Labor is purchased from households in a frictionless

[16] Early multi-sector models with an input-output structure include Long and Plosser (1983), Hovarth (1998), Hovarth (2000), Bouakez, Cardia, and Ruge-Murcia (2009), Foerster, Sarte, and Watson (2011), and Acemoglu, Carvalho, Ozdaglar, and Tahbaz-Salehi (2012). Some of the literature that followed is surveyed, for instance, in Carvalho and Tahbaz-Salehi (2019) and is cited elsewhere in this paper.

[17] Since the firm is representative, we abstract from keeping separate indices for firms within the sector and with a slight abuse of notation equate a sector with its representative firm.

market at the wage $w_{i,t}$ per period. Labor is completely mobile across sectors and, hence, wages will be the same in all sectors in equilibrium. Materials are purchased from all sectors and combined according to

$$m_{i,t} = \left(\sum_{j=1}^{N} (\omega_{ji})^{1/\psi} (m_{j,i,t})^{(\psi-1)/\psi} \right)^{\psi/(\psi-1)}, \tag{6}$$

where $m_{j,i,t}$ is the quantity of good purchased from sector j, $\omega_{ji} \geqslant 0$ are weights that add up to 1, and $\psi > 0$ is a parameter that determines the elasticity of substitution between goods from different sectors. In the case where $\omega_{ji} = 0$, good j is not a material input in the production of good i. As it is well known, (6) includes as a special cases the Leontief function when $\psi \to 0$ and the Cobb-Douglas function when $\psi = 1$. Baqaee and Farhi (2019) show that relaxing the Cobb-Douglas assumption and adopting nonlinear solution methods allows the Input-Output Table to react endogenously to shocks and give rise to nonlinearities to depend on the production structure. For example, if material inputs are gross complements (that is, $\psi < 1$), aggregate output is a concave function of sectoral productivity and, consequently, is negatively skewed. This implication of the model is studied by Dew-Becker and Vedolin (2021) using U.S. data and by Miranda-Pinto, Silva, and Young (2023) using cross-country data.

The firm maximizes

$$E_\tau \sum_{t=\tau}^{\infty} \beta^{t-\tau} \Lambda_{\tau,t} d_{i,t}, \tag{7}$$

where $\Lambda_{\tau,t}$ is the household's marginal rate of substitution of consumption between periods τ and t, and $d_{i,t}$ is profits. Profits are total revenue minus total costs,

$$d_{i,t} = p_{i,t} \left(c_{i,t} + \sum_{j=1}^{N} m_{i,j,t} \right) - \left(w_{i,t} h_{i,t} + \sum_{j=1}^{N} p_{j,t} m_{j,i,t} \right). \tag{8}$$

Revenue is the sale of goods for final and intermediate consumption (respectively denoted by $c_{i,t}$ and $m_{i,j,t}$) at the unit price $p_{i,t}$. Costs are the wage bill and total expenditures on materials inputs. Since final and intermediate goods are physically the same good and the firm is unable to price-discriminate across customers, all output $y_{i,t}$ is sold at the same price. As usual the optimal choice of labor equates the marginal product of labor to the real wage, and the optimal choice of $m_{j,i,t}$ is

$$m_{j,i,t} = \omega_{ji} m_{i,t} \left(p_t^j / Q_t^{m_{i,t}} \right)^{-\psi}. \tag{9}$$

where $P_t^{m_{i,t}}$ is the sector-specific price index

$$P_t^{m_{i,t}} = \left(\sum_{j=1}^{N} \omega_{ji} p_{j,t}^{1-\psi} \right)^{1/(1-\psi)}. \tag{10}$$

The sector-specific shock follows the process,

$$\ln z_{i,t} = \rho \ln z_{i,t-1} + \zeta_{i,t}, \tag{11}$$

where $\rho \in (-1, 1)$ is the autocorrelation coefficient and $\zeta_{i,t}$ is an independent and identically distributed (i.i.d.) innovation drawn from a generalized extreme value (GEV) distribution with mean zero. As in Eusepi, Hobijn, and Tambalotti (2011), innovations are drawn from a common distribution but draws are independent across sectors. The aggregate shock follows the process

$$\ln(z_t/z_{t-1}) = \mu + \varrho \ln(z_{t-1}/z_{t-2}) + \epsilon_t, \tag{12}$$

where μ is the mean gross rate of economic growth, $\varrho \in (-1, 1)$ is the autocorrelation coefficient, and ϵ_t is an i.i.d. innovation drawn from a GEV distribution with mean zero. Because there is long-run growth in this economy, the model is rendered stationary by rescaling all variables by z_{t-1}.

The GEV distribution is due to Jenkinson (1955) and represents in a unified and flexible way the Type I, Type II, and Type III extreme value distributions, also known respectively as the Gumbel, Fréchet, and Weibull distributions. The GEV distribution is characterized by three parameters—a location, a scale, and a shape parameter—and it allows positive and negative skewness. The Gumbel, Fréchet, and Weibull distributions are the especial cases of the GEV distribution when the shape parameter is zero, larger than zero, or smaller than zero, respectively. The probability density function (PDF) of the GEV distribution is

$$f(x) = (1/\kappa) g(x)^{\vartheta+1} \exp(-g(x)), \tag{13}$$

where $g(x) = ((1 + (x - \iota)\vartheta/\kappa))^{-1/\vartheta}$ when $\vartheta \neq 0$ and $g(x) = \exp(-(x - \iota)/\kappa)$ when $\vartheta = 0$, ι is the location parameter, κ is the scale parameter, and ϑ is the shape parameter (see Jenkinson (1955)). Coles (2001) and Embrechts, Klüppelberg, and Mikosch (2011) present applications of the GEV distribution to insurance and finance, and Kim and Ruge-Murcia (2019) study the implications of extreme GEV shocks for optimal monetary policy.

4.2 Households

Households are identical and their total number is normalized to be one. The representative household maximizes

$$E_\tau \sum_{t=\tau}^{\infty} \beta^{t-\tau} \left(\ln(c_t) - \gamma h_t \right), \tag{14}$$

where E_τ is expectation conditional on information available at time τ, $\beta \in (0, 1)$ is the subjective discount factor, c_t is final consumption, and h_t is total hours worked. Final consumption is a bundle of goods produced in all sectors,

$$c_t = \left(\sum_{i=1}^{N} (\xi_i)^{1/\chi} (c_{i,t})^{(\chi-1)/\chi} \right)^{\chi/(\chi-1)}, \tag{15}$$

where c_t^i is consumption of good i, $\xi^i \geqslant 0$ are weights that add up to 1, and $\chi > 0$ is a parameter that determines the elasticity of substitution between goods from different sectors. The case where $\xi^i = 0$ describes the situation where good i is not consumed directly by households. Notice, however, that the good may still be consumed indirectly by households if i is a material input in the production of another good j for which $\xi^j > 0$ (think, for instance, of the lithium used in the production of electric car batteries). Total hours worked is the sum of the hours supplied to each sector,

$$h_t = \sum_{j=1}^{N} h_{j,t}. \tag{16}$$

The only financial asset in this economy is a one-period bond. The bond is real, non-contingent, and risk-less, meaning that it pays with certainty one unit of consumption at maturity regardless of the state of nature. The household's budget constraint is

$$\sum_{i=1}^{N} p_{i,t} c_{i,t} + b_t = \sum_{i=1}^{N} w_{i,t} h_{i,t} + (1 + r_{t-1}) b_{t-1}, \tag{17}$$

where b_t is the number of bonds purchased at time t and r_{t-1} is the real interest rate. The Euler equation that describes the household's utility maximization is

$$1/c_t = \beta(1 + r_t) E_t \left(1/c_{t+1}\right), \tag{18}$$

where the left-hand side is the marginal cost (in terms of utility) of acquiring one additional bond and the right-hand-side is the discounted expected marginal benefit of keeping the bond till next period. The optimal choice of good n is given by the isoelastic demand function

$$c_{i,t} = \xi_i c_t (p_{i,t})^{-\chi}. \tag{19}$$

The solution to the household's problem defines an aggregate price index that will serve as the numeraire in this model. That is,

$$\left(\sum_{i=1}^{N} \xi_i p_{i,t}^{1-\chi} \right)^{1/(1-\chi)} = 1. \tag{20}$$

Using the facts that bond holdings are zero (because agents are identical) and wages are the same in all sectors (because labor is completely mobile across sectors), the aggregate counterpart

of the household's budget constraint is

$$\sum_{i=1}^{N} p_{i,t} c_{i,t} = w_t \sum_{i=1}^{N} h_{i,t}, \tag{21}$$

where $w_t = w_{i,t}$ for all i is the aggregate wage. Write profits in sector i as

$$d_{i,t} = v_{i,t} - w_t h_{i,t}, \tag{22}$$

where

$$v_{i,t} = p_{i,t} \left(c_{i,t} + \sum_{j=1}^{N} m_{i,j,t} \right) - \sum_{j=1}^{N} p_{j,t} m_{j,i,t},$$

is the value added in sector i (i.e., gross output minus the cost of materials inputs). Since $d_{i,t} = 0$ because firms are perfectly competitive, use (22) to write $h_{i,t} = v_{i,t}/w_t$ and substitute into (21) to obtain the aggregate resource constraint,

$$c_t = \sum_{i=1}^{N} v_{i,t}, \tag{23}$$

whereby aggregate consumption equals aggregate output, measured in terms of value added as in the National Income and Product Accounts.

The model is solved using a first-order perturbation with the rational-expectation solution of the linearized system found using the approach in Klein (2000).

4.3 Estimation of Structural Model

We estimate the multi-sector by the simulated method of moments (SMM) originally developed by McFadden (1989) for the estimation of discrete response models. Lee and Ingram (1991) and Duffie and Singleton (1993) extend SMM to time-series models and show that the estimator is consistent and asymptotically normal under standard regularity conditions. In this project, the moments used to estimate the model are the variance shares and the components of skewness of real sectoral output computed under our benchmark Model II (see Table 3). The estimator minimizes the weighted distance between the moments computed from the U.S. data and from artificial data simulated from the model. The weighting matrix is the identity matrix. Note that the definition of real sectoral output from the model used to construct the moments is exactly the same as that based on the I-O data, that is $q_{i,t} p_{i,t}$, where $q_{i,t}$ is the quantity of output and $p_{i,t}$ is the real price of good i (see the discussion in Section 2). The total number of moments is 17. The length of the simulated series is equal to that of the actual data, that is 63 observations. In order

to limit the effect of starting values on the results, the simulated sample contains 100 additional "training" observations that are discarded when computing the moments.

The estimated parameters are the autoregressive coefficients of the aggregate and sectoral productivity shocks, the parameters of their GEV innovations, and the parameters χ and ψ that determine the elasticity of substitution between goods from different sectors in consumption production, respectively. The total number of estimated parameters is 8 and the number of degrees of freedom is 9. Monte-Carlo results reported in Ruge-Murcia (2007) indicate that asymptotic standard errors are not always a good approximation to the true variability of SMM estimates in small samples. For this reason, we compute standard errors using a parametric k-step bootstrap with 199 replications. This bootstrap was proposed by Davidson and MacKinnon (1999) and is attractive because it reduces the computational costs of the standard bootstrap by performing a fixed number of quasi-Newton steps for each bootstrap sample. Andrews (2002) examines the higher-order properties of the k-step bootstrap for extremum estimators and finds that it delivers higher-order improvements over procedures based on first-order asymptotics. We use five steps of the Levenberg–Marquardt (LM) algorithm, which is a robust Gauss–Newton algorithm designed to solve non-linear least squares problems like ours. Preliminary experimentation showed basically no gain from employing more than five steps.

We use additional information to fix some parameters to economically plausible values as follows. The discount rate (β) is set to 0.99, which is the mean of the inverse real interest rate during the sample period (about 1% per year). The gross rate of aggregate productivity growth in steady state (μ) is set to 1.012. This value implies a net growth rate of 1.2% per year, which is the average annual growth rate of total factor productivity (TFP) in the series constructed by John Fernald (see Fernald (2012). The elasticity parameter (η) is set to 0.3 based on KLEM data from the Bureau of Economic Analysis (BEA) that record payments to all factors production in all sectors. The weight of the disutility of work in the utility function (γ) is normalized to one. The production weights ω_{ji} were computed using the Use tables of the Input-Output (I-O) accounts for the period 1957 to 2019. The consumption weights (ξ_i) for the same 43 sectors used above were computed using data from the column "Personal Consumption Expenditures" in the final-user part of the Use tables for the same period.

4.4 SMM Estimates

SMM estimates are reported in Table 4. Both the aggregate and sectoral shocks are persistent with relatively high and statistically significant autoregressive coefficients. The scale parameters of the innovations are not precisely estimated but both shape parameters are negative and significant.

That is, the null hypothesis that the shape parameter is zero (i.e., that innovations follow a Gumbel distribution) can be rejected at the 5% level in favor of the alternative that the parameter is less than zero. Hence, we conclude that aggregate and sectoral productivity innovations are better characterized by Weibull distributions. The estimated probability density functions are plotted in Figure 3. The median of the aggregate and sectoral productivity innovations are 0.045 and 0.130, respectively, but in both cases the distribution features long left tails meaning that very large negative productivity innovations can occur with a small but non-negligible probability.

The elasticity of substitution between consumption goods is $\chi = 0.748$. This estimate is statistically different from 0, which corresponds to the Leontief function, and from 1, which corresponds to the Cobb-Douglas function, at the 5% level. This estimate is roughly in line with the value of 0.85 reported by Herrendorf, Rogerson, and Valentinyi (2013) based on a three-sector model and somewhat higher than the value of 0.445 reported by Ruge-Murcia (2024) based on asset prices from ten sectors. However, all these estimates are consistent in that they are less than 1, indicating that consumption goods are complements. The elasticity of substitution between materials is $\psi = 0.834$ and statistically different from 0 at the 5% level and from 1 at the 10% level. This is higher than the estimates reported by Atalay (2017), which range between 0.07 and 0.13, and the values used in calibrations by Baqaee and Farhi (2019) and Dew-Becker and Vedolin (2021), who employ 0.001 and 0.1, respectively, and comparable the value of 0.728 reported by Ruge-Murcia (2024). These results are important because several results in the network literature are based on the assumption that material inputs are gross complements, i.e., their elasticity of substitution is less than 1.

Figure 4 compares the moments predicted by our model (vertical axis) and those computed using the U.S. sectoral data (horizontal axis). The continuous line is the 45 degree line, and should the model match exactly the moments from the data, all dots would lie on this line. Table 10 reports the numerical values used to construct Figure 4. Overall, these results show that the model generates moments that are roughly in line with those of the data. The correlation between the two set of moments (0.978) is positive, high, and statistically significant, and the R^2 is 0.952, meaning that the model explains about 95% of variance of the data moments.

4.5 Sources of Skewness

In this section we use our estimated model to shed additional light on the sources of skewness observed in the data. This analysis complements that based on the reduced-form factor model reported above. To that end, we perform five counterfactual experiments where we simulate data from the estimated model using counterfactual I-O tables or symmetric shock innovations, and then perform on the simulated data the same decomposition we performed on the U.S. data under our

Model II.

The first experiment assumes that the I-O table is the identity matrix. Thus, each sector uses exclusively its own output as materials input and there are no input-output interactions in the market for intermediate goods. The second experiment assumes that the I-O table is completely symmetric in that each entry equals $1/N$, where N is the number of sectors. This means that each sector buys materials from all sectors of the economy in equal proportions and thus, input-output interactions are maximized. In both of these experiments productivity shocks follow the estimated GEV distributions reported in Table 4. Conversely, the remaining experiments assume different distributions for the productivity shocks but impose the actual I-O table computed from the U.S. data. More precisely, the third experiment assumes that both aggregate and sectoral productivity shocks are drawn from Normal (and, hence, symmetric) distributions with same standard deviations as the corresponding GEV distributions they replace. The fourth experiment assumes that aggregate productivity shocks are normally distributed while sectoral productivity shocks follows a GEV distribution (and, hence, are negatively skewed). Finally, the fifth experiment reverses the latter design and assumes that aggregate productivity shocks follow a GEV distribution while sectoral productivity shocks are normally distributed.

Results are reported in Table 5. Consider first, Columns 1 and 2 in this table. Without input-output interactions (i.e., with an identity I-O matrix), the model delivers limited and not statistically significant skewness despite the fact that shocks are negatively skewed, and variance shares that are substantially different from those in the data. In contrast, the model with maximal I-O interactions delivers skewness, skewness decomposition, and variance shares similar to those obtained using the actual I-O table. Hence, we conclude that despite the facts that the U.S. I-O table is sparse, features large diagonal entries and smaller off-diagonal ones, and it superficially resembles an identity matrix (see Figure 2), it does generate strong connectivity between sectors. Put differently, near sparsity needs not imply low connectivity.

Consider now columns 3 through 5. Note that the (linear) model with Normal shocks can account for the second moments of the data but it cannot generate skewness. Allowing for skewed aggregate and sectoral productivity shocks (one at a time) shows that both versions can account for the second moments of the data but only the version with skewed sectoral shocks can generate substantial skewness and a decomposition roughly in line with the data. Put differently, skewed sectoral shocks propagated thorough the network can account for the skewness in the data, as conjectured in Acemoglu, Ozdaglar, and Tahbaz-Salehi (2017). In contrast, skewed aggregate shocks propagated thorough the network induce limited skewness in the data.

4.6 Quantities vs Real Prices

Evidence reported in Section 2 shows that real output growth is negatively skewed in most sectors. But, as we point out above, the measure of real sectoral output that can be constructed from the U.S. I-O accounts is in fact the product of quantity times a real price. Lack of data prevent us from segregating skewness into price and quantity, but we can deploy our model to decompose the skewness of real sectoral output growth into the parts arising from quantities and from real prices separately. As documented above in this paper, coskewness can be a an important contributor to overall skewness and, hence, we write the skewness of $y_{n,t} = \Delta ln(q_{n,t}) + \Delta ln(p_{n,t})$ as

$$\kappa_3^y = s_q^3 \kappa_3^q + + s_p^3 \kappa_3^p + 3 s_q^2 s_p \text{CoS}(q,q,p) + 3 s_p^2 s_q \text{CoS}(p,p,q).$$

Figure 5 plots the model-based estimates of the skewness of sectoral output, κ_3^y, and its components κ_3^q, κ_3^p, $\text{CoS}(q,q,p)$, and $\text{CoS}(p,p,q)$. This figure shows that the skewness of the real price and the coskewness $\text{CoS}(q,q,p)$ are positive, while the skewness of quantity and the coskewness $\text{CoS}(p,p,y)$ are negative. The conclusion of this decomposition is that the the overall negative skewness of the sectoral output as constructed from the I-O accounts is driven by the latter two terms.

5 Conclusions

This paper employs both a factor-augmented network model and a micro-founded structural model to examine the relative contribution of commons shocks, sector-level shocks, and network effects to the skewness of output growth. A key insight is that skewness depends not only on the skewness of these components but also on the co-skewness that arises from their higher-order covariation. Results indicate that while the common and network components exhibit skewness, the dominant source of asymmetry are the co-skewness terms identified here. Taking the open-minded view that the I-O tables constructed by the BEA may be a good approximation to, but not an exact proxy of, the theoretical object in our multi-sector models, we propose models to represent, and methods to estimate, more general network specifications. We show that even if the I-O table is relatively sparse and features substantially larger diagonal than off-diagonal entries, it still implies strong connectivity across sectors.

We also show that a fully-specified structural model can generate skewness and co-skewness in line with the U.S. data, with a key role played by asymmetric sectoral shocks that propagate thorough the network. The model allows us to decompose the contribution of quantities and real prices to the skewness of real output growth and to document that their coskewness is an

important contributor to overall skewness. Finally, this paper provides structural estimates of the elasticity of substitution between material inputs and between consumption goods that may used by future researchers in calibrations. The estimates lie between 0 and 1 and, thus, imply gross complementarity as assumed by most of the theoretical network literature.

Table 1: Skewness in the Data

Aggregate Data.

	κ_3			Kelly			κ_4		
	60-19	60-84	84-19	60-19	60-84	84-19	60-19	60-84	84-19
$y - y^*$	-0.099	-0.145	-0.456	-0.025	0.074	-0.189	3.221	2.679	2.479
$u - u^*$	-0.790	-0.507	-1.250	-0.314	-0.059	-0.470	3.588	2.919	3.923
Δy	-0.289	-0.287	-1.307	0.030	-0.049	0.036	4.770	3.113	7.598
$\hat{F}_1^A$	-1.238	-1.229	-1.739	-0.129	-0.291	-0.047	6.750	5.219	9.718
$\hat{F}_2^A$	0.435	-1.022	0.565	0.006	-0.049	0.044	9.735	16.197	7.571
$\hat{F}_3^A$	-0.333	-0.747	0.623	-0.023	-0.168	0.170	4.711	3.870	3.927

Sectoral Data

	1957-2019	Pre 1984	Post 1984
R^2_C, 3 factors	0.560	0.628	0.611
$\text{AR}_{\hat{e}} > .3$	0.279	0.279	0.209
$\text{AR}_Y > .3$	0.302	0.233	0.349
avg $\sigma_{\hat{e}}$	4.141	3.364	3.923
avg σ_y	6.698	6.019	6.805
med $\sigma_{\hat{e}}$	3.512	2.749	2.960
med σ_y	6.109	5.773	5.471
$\kappa_3(\hat{F}_1)$	-1.457	-0.479	-1.965
$\text{avg}(\kappa_3^{\hat{e}})$	0.128	0.089	0.148
$\text{avg}(\kappa_3^{y})$	-0.464	-0.073	-0.587

Table 2: Decomposition with $\dot{\Phi}_0 = \phi_0 \Gamma'$

	ϕ_0								
	0.010	0.050	0.100	0.200	0.300	0.400	0.500	0.600	0.750
Total Variance = 10.088									
N	0.048	0.065	0.091	0.157	0.240	0.343	0.463	0.601	0.844
C	0.599	0.542	0.475	0.353	0.250	0.165	0.099	0.051	0.011
E	0.020	0.020	0.020	0.019	0.018	0.017	0.016	0.014	0.015
2 NC	0.325	0.364	0.404	0.456	0.470	0.447	0.385	0.287	0.026
2 NE	0.007	0.009	0.011	0.015	0.021	0.029	0.037	0.046	0.104
2 CE	0.000	0.000	0.000	0.000	0.000	0.000	-0.000	0.000	-0.000
Total Skewness=-1.687									
N	-0.016	-0.027	-0.046	-0.107	-0.208	-0.358	-0.567	-0.845	-1.413
C	-0.749	-0.640	-0.520	-0.327	-0.189	-0.097	-0.041	-0.013	-0.000
E	0.001	0.001	0.001	0.001	0.000	0.000	0.000	0.000	-0.001
NNC	-0.172	-0.230	-0.307	-0.464	-0.599	-0.685	-0.692	-0.597	-0.114
NCC	-0.620	-0.663	-0.691	-0.673	-0.581	-0.444	-0.290	-0.150	-0.008
NNE	-0.005	-0.007	-0.011	-0.018	-0.028	-0.039	-0.050	-0.055	-0.137
NEE	-0.003	-0.003	-0.004	-0.005	-0.006	-0.007	-0.007	-0.007	-0.019
CCE	-0.076	-0.066	-0.055	-0.037	-0.024	-0.014	-0.007	-0.003	-0.002
CEE	-0.007	-0.007	-0.006	-0.005	-0.004	-0.003	-0.002	-0.001	0.002
NCE	-0.040	-0.044	-0.048	-0.050	-0.048	-0.042	-0.031	-0.017	0.006

Table 3: Decomposition of Variance and Skewness

Model	F	I	II	II(d)	III	F	I	II	IID	III
κ_2		Share of Variance 10.088						S.E		
N	-	0.105	0.327	0.448	0.494	-	0.075	0.069	0.062	0.066
C	0.954	0.539	0.088	0.133	0.035	0.038	0.047	0.013	0.017	0.006
E	-	0.024	0.036	0.040	0.034	-	0.005	0.008	0.008	0.010
2 NC	-	0.462	0.312	0.152	0.214	-	0.052	0.044	0.035	0.038
2 NE	0.046	-0.031	0.169	0.129	0.186	0.038	0.033	0.028	0.033	0.042
2 CE	0.000	-0.099	0.068	0.099	0.037	0.000	0.013	0.009	0.012	0.007
κ_3		Contribution to $\kappa_3 = -1.687$						S.E.		
N	-	-0.057	-0.327	-0.403	-0.633	-	0.123	0.197	0.182	0.257
C	-1.363	-0.630	-0.041	0.007	-0.007	0.140	0.045	0.005	0.002	0.001
E	0.004	0.001	-0.007	-0.002	-0.003	0.014	0.001	0.002	0.002	0.002
NNC	-	-0.379	-0.488	-0.471	-0.409	-	0.131	0.112	0.110	0.083
NCC	-	-0.844	-0.248	-0.290	-0.089	-	0.112	0.037	0.040	0.015
NNE	-	0.023	-0.223	-0.233	-0.319	-	0.040	0.083	0.077	0.099
NEE	-	-0.006	-0.062	-0.064	-0.053	-	0.007	0.018	0.016	0.018
CCE	-0.303	0.114	-0.056	0.005	-0.017	0.181	0.018	0.007	0.007	0.003
CEE	-0.025	-0.012	-0.024	0.001	-0.013	0.042	0.003	0.004	0.006	0.003
NCE	-	0.103	-0.211	-0.237	-0.144	-	0.047	0.035	0.039	0.025

Table 4: SMM Estimates

Parameter	Estimate	b.s.e
Aggregate shock		
AR coefficient	0.900	0.050
Scale	0.068	0.191
Shape	-1.876	0.120
Sectoral shock		
AR coefficient	0.692	0.144
Scale	0.246	0.203
Shape	-1.587	0.089
Elasticity of substitution		
Consumption	0.748	0.109
Production	0.834	0.098

Table 5: Counterfactuals

	1	2	3	4	5	6	7
				Variance Shares			
N	0.327	0.328	0.540	0.321	0.333	0.346	0.339
C	0.088	0.155	0.047	0.175	0.161	0.157	0.153
E	0.036	0.002	0.090	0.001	0.002	0.002	0.002
2 NC	0.312	0.450	0.117	0.471	0.460	0.464	0.453
2 NE	0.169	0.039	0.206	0.019	0.027	0.020	0.033
2 CE	0.069	0.026	-0.001	0.013	0.017	0.012	0.021
				S.E.			
N	0.067	0.047	0.104	0.051	0.050	0.048	0.050
C	0.005	0.004	0.006	0.005	0.005	0.005	0.005
E	0.011	0.005	0.019	0.005	0.005	0.005	0.005
2 NC	0.021	0.014	0.042	0.015	0.015	0.015	0.015
2 NE	0.025	0.015	0.061	0.015	0.016	0.015	0.016
2 CE	0.013	0.009	0.013	0.011	0.010	0.009	0.010

	1	2	3	4	5	6	7
				Skewness			
Total	-1.687	-1.624	-0.542	-1.023	-0.096	-0.029	-0.318
N	-0.328	-0.297	-0.372	-0.175	-0.020	-0.011	-0.070
C	-0.041	-0.110	-0.018	-0.084	-0.004	-0.005	-0.017
E	-0.007	-0.000	-0.032	0.000	-0.000	-0.000	-0.000
NNC	-0.488	-0.639	-0.044	-0.412	-0.035	-0.026	-0.131
NCC	-0.247	-0.458	-0.011	-0.323	-0.019	-0.019	-0.082
NNE	-0.223	-0.040	0.028	-0.008	-0.007	0.011	-0.007
NEE	-0.062	-0.002	-0.097	-0.000	-0.000	0.001	-0.001
CCE	-0.056	-0.021	0.025	-0.006	-0.003	0.005	-0.002
CEE	-0.024	-0.001	-0.012	-0.000	-0.000	0.001	-0.000
NCE	-0.211	-0.058	-0.010	-0.014	-0.009	0.015	-0.008
				S.E.			
Total	0.361	0.205	1.747	0.157	0.052	0.059	0.122
N	0.174	0.110	1.000	0.078	0.024	0.026	0.051
C	0.001	0.002	0.003	0.001	0.000	0.000	0.000
E	0.004	0.002	0.005	0.002	0.000	0.000	0.001
NNC	0.060	0.044	0.303	0.034	0.009	0.010	0.025
NCC	0.013	0.015	0.041	0.012	0.001	0.001	0.005
NNE	0.096	0.048	0.317	0.036	0.013	0.016	0.028
NEE	0.030	0.017	0.053	0.013	0.003	0.004	0.006
CCE	0.005	0.005	0.008	0.004	0.000	0.000	0.001
CEE	0.008	0.006	0.008	0.004	0.001	0.001	0.002
NCE	0.038	0.032	0.085	0.024	0.004	0.004	0.011

Figure 1: Skewness and Value Share of Output Growth by Sector

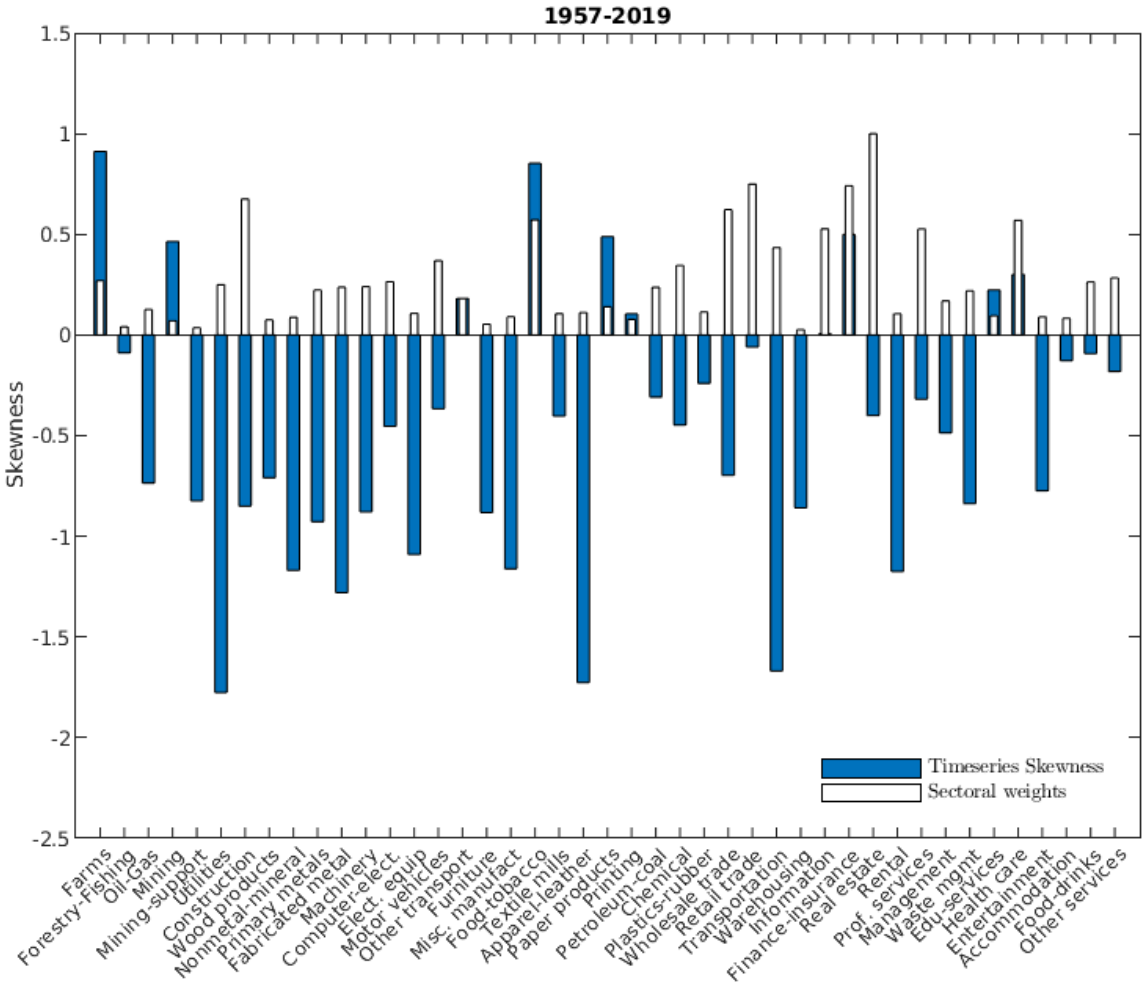


Figure 2: Heatmap of W

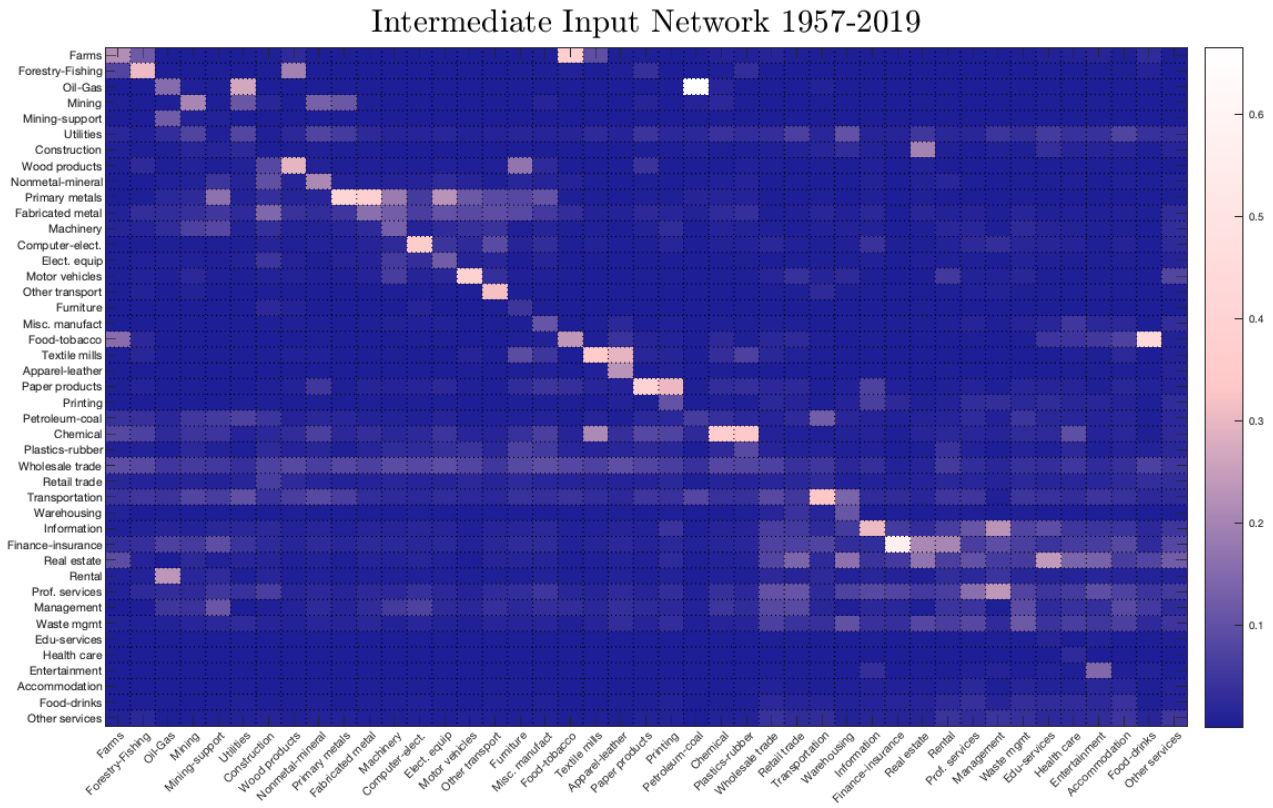


Figure 3: GMM Estimates of A^0 for 43 Sectors

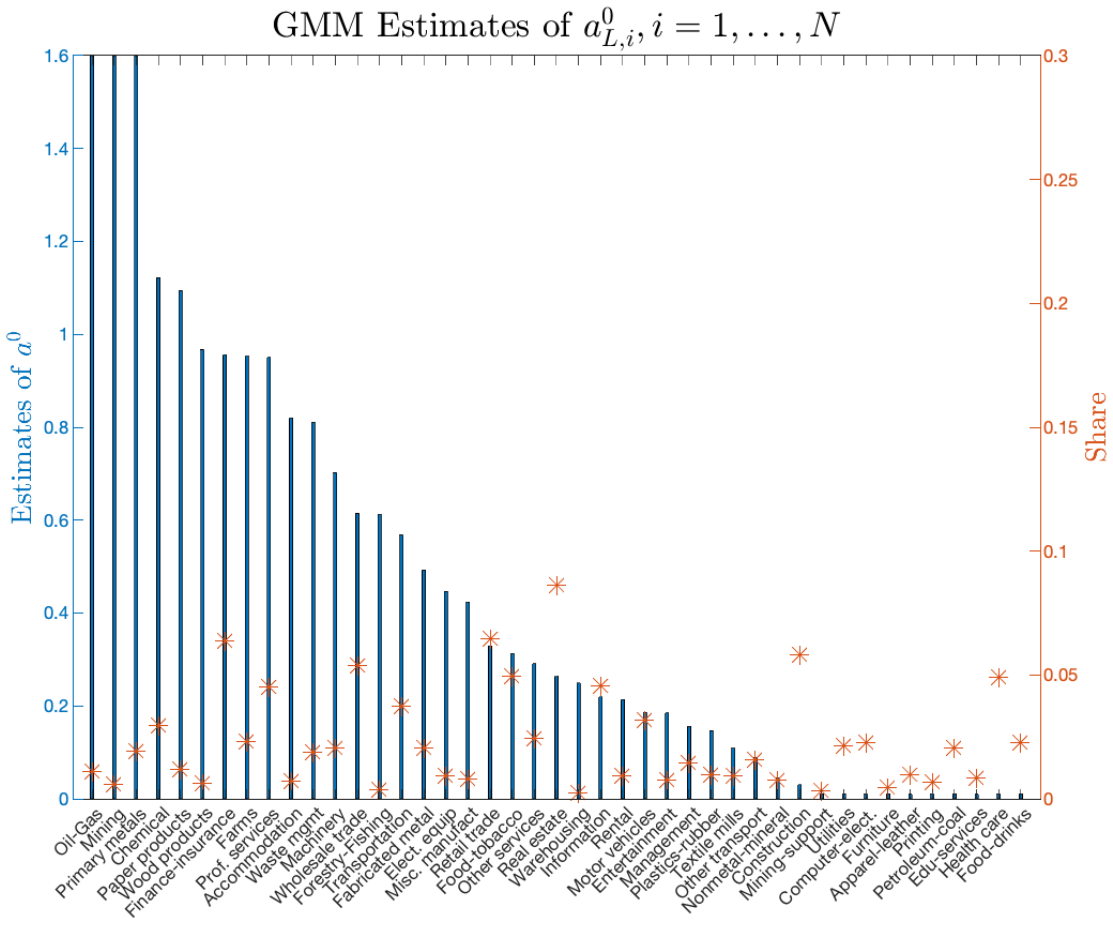
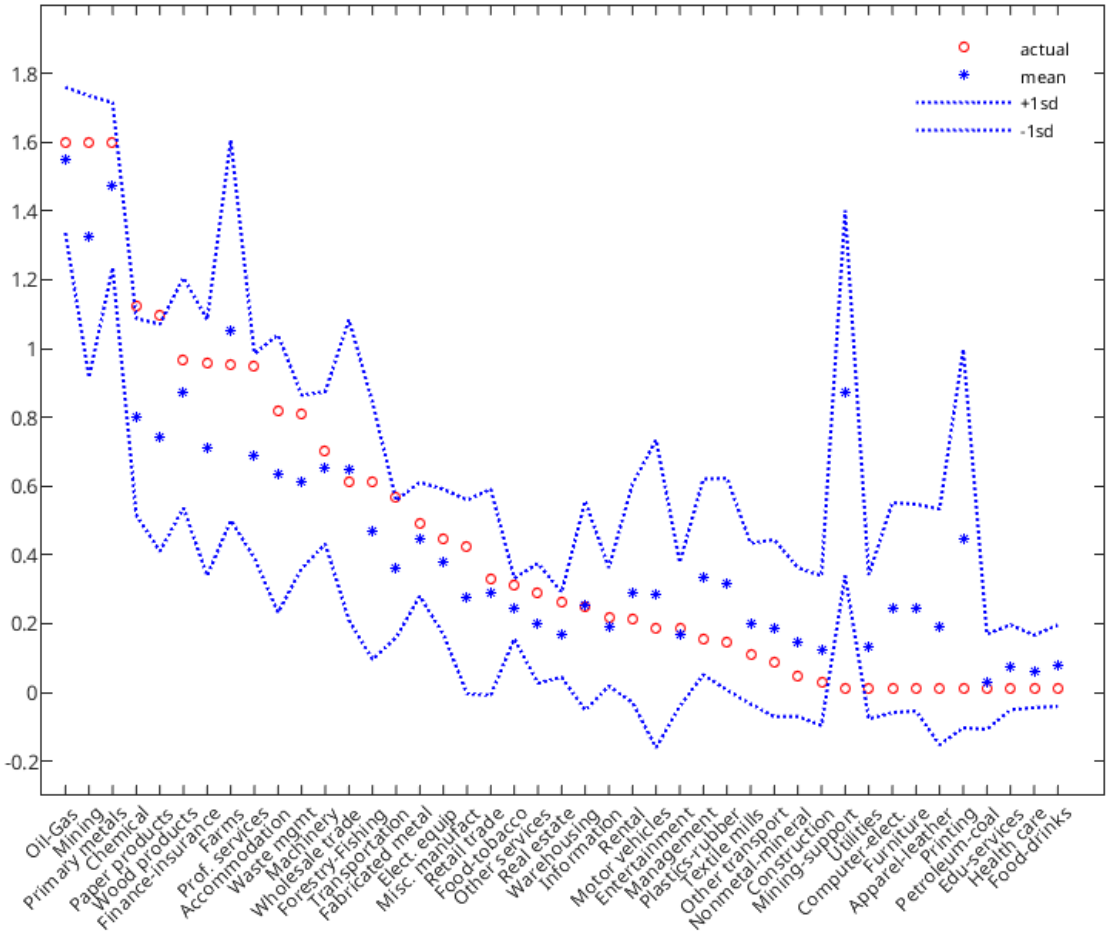


Figure 4: Calibrated Monte Carlo, $T = 60, N = 43$



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A Data Appendix

Our primary dataset comes from the U.S. Bureau of Economic Analysis (BEA) GDP-by-industry database, which provides annual sector-level output data from 1947 to 2019. During this period, the BEA revised its industry classifications several times, increasing the level of detail. In particular, three-digit sectors rose from 43 in 1947–1962 to 61 in 1963–1996 and 66 in 1997–2019. To maintain consistency across the entire time period, we use the 1947 classification. We map all later, more detailed classifications back to this original 43-sector framework. This choice balances having a sufficiently long sample period (maximizing T) without reducing the number of sectors (managing N). As a result, the primary dataset in this paper is aligned with the 43-sector classification from 1947–1962. This approach provides a consistent basis for analyzing sectoral output over the entire sample. Table A1 lists the NAICS codes and names of the 43 sectors used in the analysis.

Table A1: NAICS 43 Sectors

No.	Code	Name
1	111	Farms
2	113	Forestry, fishing, and related activities
3	211	Oil and gas extraction
4	212	Mining, except oil and gas
5	213	Support activities for mining
6	22	Utilities
7	23	Construction
8	321	Wood products
9	327	Nonmetallic mineral products
10	331	Primary metals
11	332	Fabricated metal products
12	333	Machinery
13	334	Computer and electronic products
14	335	Electrical equipment, appliances, and components
15	3361	Motor vehicles, bodies and trailers, and parts
16	3364	Other transportation equipment
17	337	Furniture and related products
18	339	Miscellaneous manufacturing
19	311	Food and beverage and tobacco products
20	313	Textile mills and textile product mills
21	315	Apparel and leather and allied products
22	322	Paper products
23	323	Printing and related support activities

No.	Code	Name
24	324	Petroleum and coal products
25	325	Chemical products
26	326	Plastics and rubber products
27	42	Wholesale trade
28	44	Retail trade
29	48	Transportation
30	493	Warehousing and storage
31	51	Information
32	52	Finance and insurance
33	531	Real estate
34	532	Rental and leasing services and lessors of intangible assets
35	54	Professional, scientific, and technical services
36	55	Management of companies and enterprises
37	56	Administrative and waste management services
38	61	Educational services
39	62	Health care and social assistance
40	71	Arts, entertainment, and recreation
41	721	Accommodation
42	722	Food services and drinking places
43	81	Other services, except government

We also provide additional estimation results for an alternative sectoral classification at the NAICS 2-digit classification. Table A2 lists the 14 sectors that we include in the analysis.

Table A2: NAICS 14 Sectors

No.	Code	Name
1	11	Agriculture, forestry, fishing, and hunting
2	21	Mining
3	22	Utilities
4	23	Construction
5	31	Manufacturing
6	42	Wholesale trade
7	44	Retail trade
8	48	Transportation and warehousing
9	51	Information
10	52	Finance, insurance, real estate, rental, and leasing
11	54	Professional and business services
12	6	Educational services, health care, and social assistance
13	7	Arts, entertainment, recreation, accommodation, and food services
14	81	Other services, except government

B Maximum Likelihood Estimation

Figure B1: Comparison of MLE and GMM Estimates

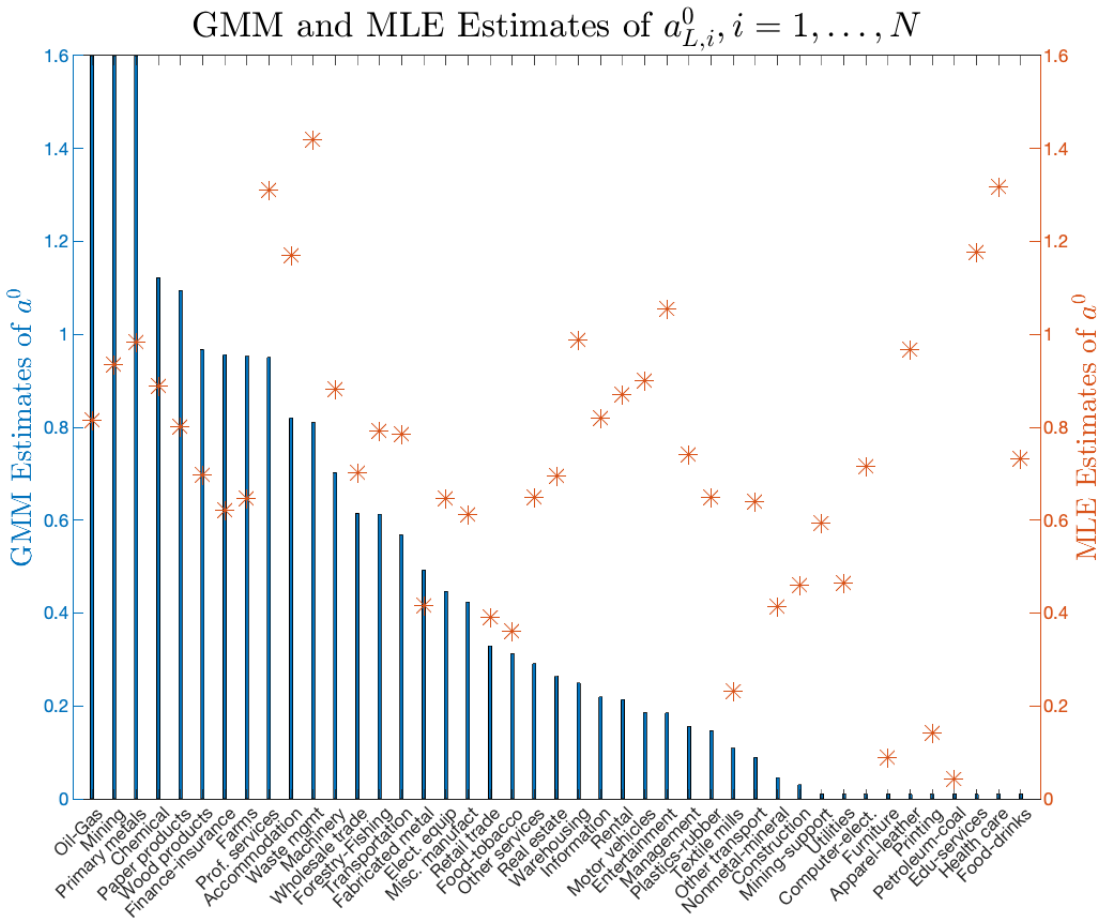


Table B1: GMM vs MLE Decomposition of Variance and Skewness

Model	GMM	MLE	GMM(d)	MLE(d)	GMM	MLE	GMM(d)	MLE(d)
κ_2	Share of Variance 10.088				S.E			
N	0.327	0.720	0.448	0.720	0.069	0.060	0.062	0.060
C	0.088	0.024	0.133	0.024	0.013	0.002	0.017	0.002
E	0.036	0.011	0.040	0.011	0.008	0.005	0.008	0.005
2 NC	0.312	0.226	0.152	0.226	0.044	0.029	0.035	0.029
2 NE	0.169	0.020	0.129	0.020	0.028	0.031	0.033	0.031
2 CE	0.068	-0.002	0.099	-0.002	0.009	0.005	0.012	0.005
κ_3	Contribution to $\kappa_3 = -1.687$				S.E.			
N	-0.327	-1.089	-0.403	-1.089	0.197	0.366	0.182	0.366
C	-0.041	-0.005	0.007	-0.005	0.005	0.000	0.002	0.000
E	-0.007	-0.000	-0.002	-0.000	0.002	0.001	0.002	0.001
NNC	-0.488	-0.548	-0.471	-0.548	0.112	0.082	0.110	0.082
NCC	-0.248	-0.097	-0.290	-0.097	0.037	0.009	0.040	0.009
NNE	-0.223	0.041	-0.233	0.041	0.083	0.095	0.077	0.095
NEE	-0.062	-0.010	-0.064	-0.010	0.018	0.012	0.016	0.012
CCE	-0.056	0.001	0.005	0.001	0.007	0.001	0.007	0.001
CEE	-0.024	-0.000	0.001	-0.000	0.004	0.001	0.006	0.001
NCE	-0.211	0.022	-0.237	0.022	0.035	0.017	0.039	0.017

C Estimation with 14 Sectors

Figure C1: GMM Estimates of A^0 for 14 Sectors

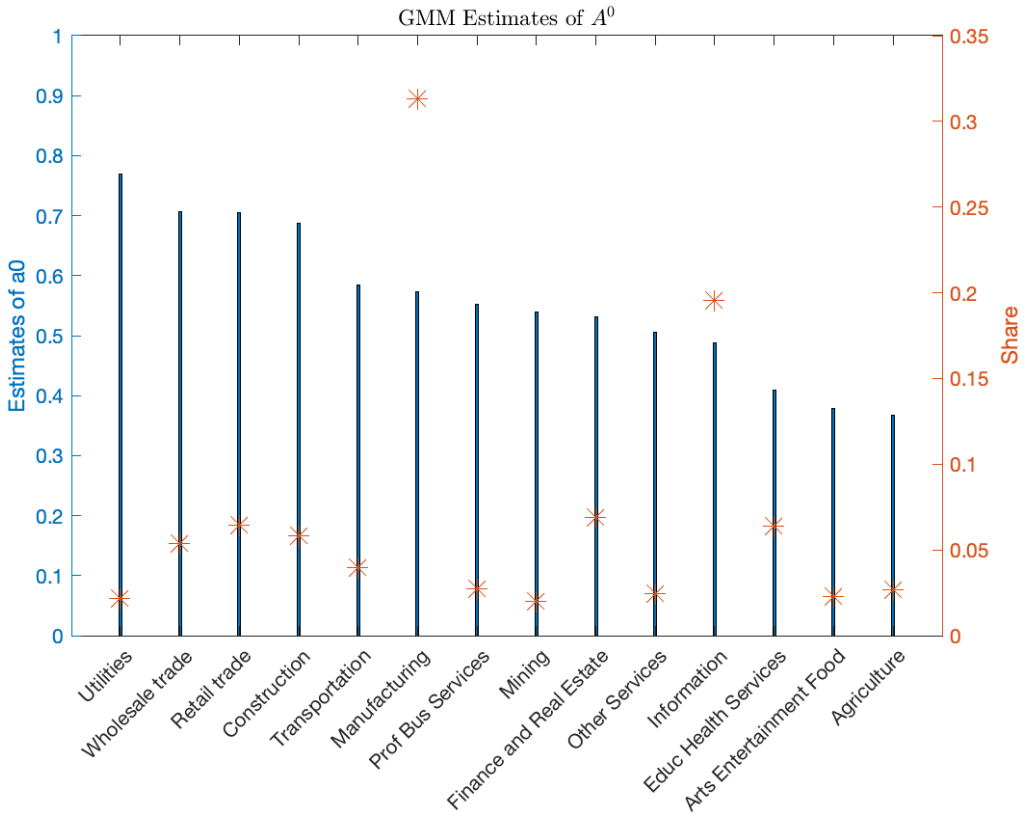


Table C1: Decomposition of Variance and Skewness: 14 sectors

Model	F	GMM	MLE	F	GMM	MLE
κ_2	Share of Variance 10.270				S.E	
N	-	0.615	0.692	-	0.097	0.099
C	0.965	0.022	0.038	0.036	0.018	0.006
E	-	0.045	0.008	-	0.018	0.011
2 NC	-	0.078	0.228	-	0.072	0.040
2 NE	0.035	0.240	0.038	0.036	0.068	0.047
2 CE	-0.000	0.000	-0.004	0.000	0.001	0.012
κ_3	Contribution to $\kappa_3 = -1.746$				S.E.	
N	-	-0.854	-1.027	-	0.330	0.418
C	-1.724	-0.001	-0.009	0.186	0.004	0.001
E	-0.004	-0.003	0.000	0.013	0.004	0.002
NNC	-	-0.165	-0.551	-	0.076	0.083
NCC	-	-0.015	-0.114	-	0.022	0.012
NNE	-	-0.515	-0.036	-	0.140	0.111
NEE	-	-0.086	0.007	-	0.028	0.018
CCE	0.011	-0.006	-0.002	0.083	0.004	0.003
CEE	-0.030	-0.009	0.002	0.042	0.004	0.003
NCE	-	-0.092	-0.016	-	0.032	0.026