# Ownership Structure and Economic Growth: A Network Approach

Koki Okumura*
UCLA

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**Abstract**

We develop an endogenous growth model that incorporates three inter-firm networks: ownership structures, product market competition, and technology spillovers. Using this framework, we examine the impact of ownership structures on economic growth and social welfare. To quantify these effects, we estimate the model using data from approximately 1,000 U.S. publicly traded firms with patent holdings and solve the dynamic oligopoly problems among these firms. Our numerical analysis suggests that in 2017, the observed level of common ownership reduced the expected economic growth rate by 0.08 percentage points compared to a counterfactual scenario with a fully dispersed ownership structure in R&D decision. This result implies that, under common ownership, the internalization of business-stealing effects dominates that of technological spillovers. Furthermore, when a benevolent social planner optimally allocates R&D investment, the socially optimal level of R&D is more than twice the equilibrium level, resulting in a 0.25 percentage point increase in the economic growth rate. Finally, we demonstrate that an optimal uniform R&D subsidy of 32% reduces the consumption-equivalent welfare loss from 0.7% to 0.1%, relative to the social welfare achieved under the optimal R&D allocation.

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# 1 Introduction

Over the past few decades, corporate ownership in the United States has increasingly concentrated among a small number of institutional investors. This trend is referred to as the rise of common ownership. Under common ownership, firms' strategic decisions to maximize returns to their investors may (partially) internalize the impact on the profits of other firms. As a result, common ownership not only influences competition in product markets but also reshapes firms' incentives to invest in R&D, drawing increasing attention and concern from economists and policymakers.

Existing endogenous growth models implicitly assume that corporate ownership is entirely dispersed, but this unrealistic assumption could significantly alter firms' R&D investment decisions. For example, consider the problem of how much OpenAI should invest in R&D to enhance the performance of its generative AI, ChatGPT. If both OpenAI and Microsoft are owned by the same owners, OpenAI may take into account the loss of common owners' returns due to ChatGPT's performance improvements reducing the market share of Microsoft's generative AI, Copilot (internalization of the business-stealing effect). Conversely, if ChatGPT's performance improvements result in technological spillovers that eventually enhance Copilot's performance, this could further increase the common owners' returns (internalization of the technology spillover effect).

In this paper, we develop an endogenous economic growth model incorporating three inter-firm networks: ownership structure, product market rivalry, and technology spillover. We incorporate the product market rivalry and technology spillover networks into the model because they determine the magnitude of inter-firm externalities arising from the business-stealing effect and technology spillovers. Furthermore, the overlap between these two networks and the ownership structure networks determines the extent to which these externalities are internalized. We estimate these three networks using data on approximately 1,000 publicly traded U.S. firms that hold patents. By solving the dynamic oligopoly problem among these firms, we analyze the impact of the rise of common ownership and optimal R&D policy on economic growth.

We extend Pellegrino (2024)'s static framework of product market competition into an endogenous growth model that incorporates technology spillovers and strategic R&D decisions. Pellegrino (2024) generalizes the widely used hedonic demand system from the empirical IO literature to a general equilibrium setting and proposes an estimation method for this framework. Building on his work, we develop an endogenous growth model in which firms accumulate knowledge capital through their own R&D investments

and technology spillovers from other firms. This knowledge capital enhances product quality and labor productivity, driving economic growth.

We incorporate corporate ownership structure into our framework to analyze how common ownership influences economic growth. In our model, following Rosen (1974) and Ederer and Pellegrino (2024), owners receive profits proportional to their ownership shares, while firms strategically determine production and dynamic R&D investments to maximize their owners' values. This approach implies that each firm maximizes a linear combination of its own value and those of other commonly owned firms. Consequently, firms partially internalize both the business-stealing effect on commonly owned firms with similar products and the technological spillover effect on commonly owned firms with similar technologies.

Our model retains tractability despite incorporating forward-looking and dynamic R&D decisions by a large number of oligopolistic firms. This tractability arises from formulating the firms' dynamic R&D problem as a linear-quadratic differential game, avoiding the curse of dimensionality. This approach facilitates the analysis of dynamic oligopolies, even when involving approximately 1,000 U.S. publicly listed firms with patent holdings. Furthermore, the framework offers another computational advantage when calculating the expected growth rate and social welfare. Once the equilibrium is solved, the expected growth rate and social welfare can be calculated with minimal additional computational effort for any initial distribution of knowledge capital.

We begin by examining the theoretical properties of the model in an economy without shocks. In this deterministic economy, a matrix termed the "technology transition matrix," which captures the dynamics of technological spillovers and endogenous R&D strategies, plays a key role in describing the conditions for the existence of a balanced growth path, the economic growth rate, and the stationary distribution of productivity. Specifically, the irreducibility of the technology transition matrix serves as a sufficient condition for the existence of a globally stable balanced growth path. Intuitively, the irreducibility of the technology transition matrix implies that all firms are technologically connected, leading to all firms growing at the same rate. On the balanced growth path, the growth rate of all firms is given by the dominant eigenvalue of the technology transition matrix, and the distribution of productivity across firms corresponds to a scalar multiple of the associated eigenvector.

We can estimate our framework using Compustat and publicly available data without relying on confidential data. Following Pellegrino (2024), we estimate product market

competition for U.S. firms holding patents based on the product similarity measures of Hoberg and Phillips (2016). To estimate the technological spillover network, we follow Bloom et al. (2013) and estimate the technological similarity between firms by utilizing information from patent classifications. The ownership structure of firms is estimated using the dataset on institutional ownership constructed by Backus et al. (2021) from 13f filings. Using these estimated networks and Compustat financial statement data, we estimate the distribution of firm-level knowledge capital implied by the model.

We estimate the relationship between the magnitude of technological spillovers among firms and the observed technological similarity to quantify our model. The main challenge in this estimation is to empirically distinguish between common shocks to the knowledge capital of multiple firms and technological spillover effects. For instance, when new research opportunities arise in a specific technological field, all firms in that field may expand their R&D efforts and enhance their knowledge capital. This effect could be mistakenly interpreted as being driven by technological spillovers. Following Bloom et al. (2013) and Lucking et al. (2019), we address this issue by constructing an instrumental variable for knowledge capital using changes in the user cost of R&D, driven by state-level tax variations developed by Wilson (2009). This approach allows us to estimate the causal effects of knowledge capital on technology spillovers.

Using the estimated model, we first examine the characteristics of the competitive equilibrium. The model's predicted R&D investment shows a strong correlation (0.89) with the observed values, indicating that our framework is well-suited for counterfactual analysis. Next, we calculate the social return and private return of R&D for each firm. While the social return and private return of R&D in competitive equilibrium show a very high correlation, the social return exhibits greater variance. We also decompose the drivers of economic growth. In our framework, the expected economic growth rate can be decomposed into three factors: growth from knowledge spillovers, growth from R&D, and growth arising from the idiosyncratic shock and the convexity of the aggregator based on Ito's lemma. We find that the contribution of knowledge spillovers to economic growth is approximately 1.5 times greater than that of R&D.

After examining the characteristics of the baseline competitive equilibrium, we analyze various counterfactual scenarios. In the framework of this study, it is possible to examine counterfactual scenarios in which the ownership structure differs between the static production decision and the dynamic R&D decision. For both production and R&D, we consider three cases: fully dispersed ownership, common ownership as observed in

the data, and monopoly by a single investor (which, in this framework, is equivalent to the case in which all investors hold the same portfolio). Regarding production, we find that as ownership becomes more concentrated—from dispersed to common to monopoly—the economic growth rate increases. Specifically, as of 2017, replacing dispersed ownership with common ownership in production increases the economic growth rate by 0.027 percentage points. This occurs because reduced competition in the product market raises the private returns from additional innovation, thereby enhancing the incentives for R&D. In contrast, when the ownership structure in R&D is more concentrated, the economic growth rate declines. Specifically, for R&D, replacing dispersed ownership with common ownership decreases the economic growth rate by 0.084 percentage points. This suggests that the negative impact on R&D incentives, due to the internalization of the effect of appropriating business, outweighs the positive effect from internalizing technological spillovers. Furthermore, we find that when the ownership structure is more concentrated in either production or R&D, social welfare declines.

Next, we quantify the impact of changes in ownership structure over time on equilibrium outcomes such as economic growth and social welfare using data from 1999 to 2017. For each year, we measure the three inter-firm networks—ownership structure, production linkages, and technology spillovers—as well as initial distribution of knowledge capital to compute the expected economic growth rate and other equilibrium outcomes under different ownership scenarios. Our findings suggest that, over the sample period, the spread of common ownership has led the equilibrium outcomes under common ownership observed in the data to gradually diverge from those under fully dispersed ownership and move closer to those under monopolistic ownership. Specifically, in 1999, the economic growth rate under common ownership was only 0.01 percentage points lower than that under dispersed ownership. However, by 2017, the economic growth rate under common ownership had declined by 0.04 percentage points relative to dispersed ownership.

We compare the allocation chosen by a benevolent social planner who maximizes social welfare with the allocation observed under competitive equilibrium. When the planner takes the production level determined by the competitive equilibrium as given, the socially optimal level of R&D investment is more than twice as high as that observed in competitive equilibrium. When the planner can also choose the production level, the socially optimal R&D investment increases further, reaching roughly four times the competitive equilibrium level. As a result, in 2017, the economic growth rate under the

planner who optimizes only R&D was 0.03 percentage points higher than that under competitive equilibrium, and the economic growth rate under the planner who chooses both production and R&D was 0.08 percentage points higher than that under competitive equilibrium.

We also analyze the effects of a uniform subsidy for R&D. The results indicate that social welfare is maximized when the R&D subsidy rate is set at 32%. Under this optimal rate, the consumption-equivalent welfare loss is 0.1% compared to the scenario in which a social planner selects the optimal level of R&D. In contrast, without any R&D subsidy, the consumption-equivalent welfare loss relative to the optimal R&D allocation is 0.7%. This suggests that a uniform subsidy for R&D can mitigate most of the welfare losses associated with R&D allocation.

Furthermore, following Ederer and Pellegrino (2024), we investigate how alternative assumptions about corporate governance affect our findings. Rather than assuming that investors influence a firm's decision-making strictly in proportion to their ownership stakes, we explore the possibility that large investors may exert influence beyond their proportional shareholdings. Under this "super-proportional influence" assumption, the effects of common ownership are similar to those under the proportional influence model, reducing the economic growth rate by roughly 0.05 percentage points in 2017. If we assume that only blockholders with at least a 5% equity stake can exert influence, the growth rate declines by about 0.03 percentage points in 2017. Similarly, even when we assume that investors have limited attention, the results remain largely unchanged relative to the proportional influence model. Finally, under the most conservative corporate governance model which accounts for frictions arising from managerial entrenchment, the estimated impact on economic growth in 2017 is a reduction of approximately 0.01 to 0.015 percentage points.

Although this paper analyzes the impact of the rise in common ownership in the United States on economic growth, our framework can be more broadly applied to study the effects of ownership structures on economic growth. The concentration of corporate ownership is not unique to the United States or the 21st century. For instance, in South Korea during the mid-1990s, the value added by five chaebols accounted for approximately 10% of the GDP (Chang, 2003). Similarly, in Japan during the first half of the 20th century, three zaibatsu held around 30% of the total assets of the top 100 manufacturing firms (Takeda, 2020). These zaibatsu were dissolved by the Allied forces at the end of World War II. With appropriate data, our framework can also be used to analyze the impact of

Korean chaebols on economic growth and the effects of Japan's zaibatsu dissolution on subsequent economic growth in Japan.

This study makes several contributions to the literature. First, it builds on the extensive research on innovation and product market competition (d'Aspremont and Jacquemin, 1988; Aghion et al., 2001, 2005; Acemoglu and Akcigit, 2012; Bloom et al., 2013; Peters, 2020; Akcigit and Ates, 2021, 2023; Liu et al., 2022; Cavenaile et al., 2023; De Ridder, 2024). Bloom et al. (2013) empirically examine how product market and technological proximity influence innovation. They show that higher product market proximity intensifies the business-stealing effect, whereas greater technological proximity enhances technology spillovers. Cavenaile et al. (2023) develop an oligopolistic general equilibrium Schumpeterian growth model that accommodates competition among a small number of firms within each industry. However, such models involving oligopolistic dynamic games encounter significant computational challenges, with computation time increasing exponentially with the number of firms. To overcome this limitation, Hopenhayn and Okumura (2024) incorporate a linear-quadratic network game into the Schumpeterian framework. Unlike existing endogenous growth models, our approach directly matches firm-level and firm-pair-level data with the model. In this paper, we extend our framework to include networks of ownership structures, allowing us to examine their impact on economic growth.

Our research contributes to the rapidly growing literature on common ownership and innovation (Gutierrez and Philippon, 2017; He and Huang, 2017; Lopez and Vives, 2019; Anton et al., 2023, 2024; Kini et al., 2024). Lopez and Vives (2019) examine how symmetric common ownership affects innovation among symmetric oligopolistic firms in a static framework. Anton et al. (2024) show that the effect of common ownership on innovation decreases with product market proximity and increases with technological proximity. This result suggests that greater product proximity leads to larger business-stealing effects, while greater technological proximity enhances technology spillovers, and these externalities are partially internalized under common ownership. Furthermore, similar to Lopez and Vives (2019), Anton et al. (2024) provide a static model where oligopolistic firms conduct R&D under common ownership. Our study differs from the existing literature in two respects. First, we develop a dynamic general equilibrium framework in which the economic growth rate is determined endogenously and the economy evolves along a (stochastic) balanced growth path. Second, we introduce a method for estimating heterogeneous inter-firm networks using data, enabling quantitative analysis to address

macroeconomic questions.

The remainder of the paper proceeds as follows. Section 2 constructs an endogenous growth model incorporating common ownership. Section 3 describes the data and the estimation method of the model. Section 4 reports the characteristics of the baseline model and the results of various counterfactual analyses. Section 5 presents the results based on different corporate governance models. Section 6 concludes.

# 2 Model

We develop an endogenous economic growth model that incorporates ownership structure to examine its impact on firms' dynamic R&D investment, economic growth, and social welfare.

## 2.1 Environment

Our environment incorporates three networks: ownership structure, product market rivalry, and technology spillover. There are $n$ oligopolistic firms, indexed by $i \in \{1, 2, \ldots, n\}$, and they determine production and dynamic R&D investment under strategic interaction through these networks. Time is infinite and continuous, $t \in [0, \infty)$.

### 2.1.1 Demand

We introduce the Generalized-Hedonic-Linear demand, developed by Pellegrino (2024), to model product market rivalry among a large number of oligopolists. We assume that products have two types of characteristics. The first type of characteristics is a common characteristic shared by all products and indexed by $k \in \{1, 2, \ldots, m\}$. The second type of characteristics is an idiosyncratic characteristic unique to each product and share the same index $i$ as the product they correspond to. The scalar $\psi_{ki} \geq 0$ denotes the quantity of the common characteristic $k$ provided by product $i$. Each product $i$ is described by an $m$-dimensional, strictly positive column vector $\boldsymbol{\psi}_i$:

$$\boldsymbol{\psi}_i = \begin{bmatrix} \psi_{1i} & \psi_{2i} & \ldots & \psi_{mi} \end{bmatrix}^T$$

The length of the common characteristic of each product is normalized to 1 without loss of generality:

$$\sum_{k=1}^{m} \psi_{ki}^2 = 1 \quad \forall i \in \{1, 2, \ldots, n\}$$

We stack all the vectors $\boldsymbol{\psi}_i$ that captures the common characteristics of product $i$ and define the $m \times n$ matrix $\Psi$:

$$\Psi \equiv \begin{bmatrix} \boldsymbol{\psi}_1 & \boldsymbol{\psi}_2 & \cdots & \boldsymbol{\psi}_n \end{bmatrix} \equiv \begin{bmatrix} \psi_{11} & \psi_{12} & \cdots & \psi_{1n} \\ \psi_{21} & \psi_{22} & \cdots & \psi_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \psi_{m1} & \psi_{m2} & \cdots & \psi_{mn} \end{bmatrix}$$

We aggregate the characteristics of all products and obtain the production function of the final goods producer. Let $q_{i,t}$ be the number of units produced by firm $i$ and consumed by households at time $t$, and define the $n$-dimensional vector $\mathbf{q}_t$ :

$$\mathbf{q}_t \equiv \begin{bmatrix} q_{1,t} & q_{2,t} & \cdots & q_{n,t} \end{bmatrix}^T$$

Let $y_{kt}^C$ be the total units of common characteristic $k$ aggregated over all products:

$$y_{k,t}^C = \sum_{i=1}^{n} \psi_{k,i} q_{i,t}$$

Therefore, the matrix $\Psi$ transforms units of goods into units of common characteristics:

$$\mathbf{y}_t^C = \Psi \mathbf{q}_t \tag{1}$$

Let $y_{it}^I$ be the number of units of each characteristics, and we assume that each unit of good $i$ provides one unit of its corresponding idiosyncratic characteristic:

$$\mathbf{y}_t^I = \mathbf{q}_t \tag{2}$$

A representative final goods producer produces final goods by aggregating common and idiosyncratic characteristics using a linear quadratic aggregator:

$$Y_t = \alpha \sum_{k=1}^{m} \left( b_{k,t}^C y_{k,t}^C - \frac{1}{2} \left( y_{k,t}^C \right)^2 \right) + (1 - \alpha) \sum_{i=1}^{n} \left( b_{i,t}^I y_{i,t}^I - \frac{1}{2} \left( y_{i,t}^I \right)^2 \right), \tag{3}$$

where $b^C_{k,t} > 0$ and $b^I_{i,t} > 0$ are characteristic-specific quality, and $\alpha \in [0,1]$ is the weight that is assigned to common characteristics. Inserting equations (1) and(2) into equation (3), we obtain a vector form of aggregate output represented by the quantity vector:

$$Y_t = \mathbf{q}_t^T \mathbf{b}_t - \frac{1}{2}\mathbf{q}_t^T \Sigma \mathbf{q}_t \tag{4}$$

where

$$\mathbf{b}_t \equiv \alpha \Psi' \mathbf{b}^C_t + (1-\alpha)\mathbf{b}^I_t,$$

$$\Sigma \equiv \begin{bmatrix} \sigma_{11} & \sigma_{12} & \cdots & \sigma_{1n} \\ \sigma_{21} & \sigma_{22} & \cdots & \sigma_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_{n1} & \sigma_{n2} & \cdots & \sigma_{nn} \end{bmatrix} \equiv \alpha \Psi^T \Psi + (1-\alpha)\, I, \tag{5}$$

and $I$ is the $n \times n$ identity matrix.

$\Sigma$ is a matrix representing the network of product market rivalry, where each element $\sigma_{ij}$ captures the substitutability between two products $i$ and $j$. This is because, from (4), a larger value of $\sigma_{ij}$ lead to a higher penalty in final good production when both $q_i$ and $q_j$ are large. Note that the diagonal element of the product market rivalry matrix $\Sigma$ is 1 by construction: $\sigma_{ii} = 1$ for all $i$. The product market rivalry matrix $\Sigma$ consists of the weighted sum of the matrix $\Psi^T\Psi$ and identity matrix. The matrix $\Psi^T\Psi$ is representing the network of product similarity, encompassing the similarity between all pairs of products. Each element of the matrix $[\Psi^T\Psi]_{ij} = \boldsymbol{\psi}_i^T \boldsymbol{\psi}_j$ represents the cosine similarity between the common characteristics of product $i$ and product $j$. The term "cosine similarity" is used because, geometrically, $\boldsymbol{\psi}_i^T \boldsymbol{\psi}_j$ measures the cosine of the angle between vectors $\boldsymbol{\psi}_i$ and $\boldsymbol{\psi}_j$ in the space of common characteristics $\mathbb{R}^m$. From (5), the substitutability between two goods $\sigma_{ij}$ is proportional to the cosine similarity of their common characteristics $\boldsymbol{\psi}_i^T \boldsymbol{\psi}_j$ for all $i \neq j$, and the common characteristics weight $\alpha$ translates the magnitude of similarity to the magnitude of substitutability. As demonstrated by Pellegrino (2024), these characteristics of the demand system allow for the estimation of substitutability between any two products using product similarity data.

### 2.1.2 Technology

Each firm has a linear production technology in labor:

$$q_{i,t} = a_{i,t} l_{i,t} \tag{6}$$

where $a_{i,t}$ is the labor productivity of firm $i$ at time $t$ and $l_{i,t}$ is the production labor employed by firm $i$ at time $t$.

Firms possess their own knowledge capital and utilize it to improve their labor productivity and product quality. As shown in (4), $b_{it}$ represents the value, measured in final good units, generated by the first unit of product $i$. Therefore, $b_{it}$ can be interpreted as the quality of the product. Firms allocate their knowledge capital to improve labor productivity $a_{i,t}$ and product quality $b_{i,t}$:

$$\zeta a_{i,t} + b_{i,t} = z_{i,t} \tag{7}$$

where $\zeta > 0$ is a parameter that determines the transformation rate between improvements in labor productivity and product quality.

Now, we introduce the law of motion for knowledge capital, which increases through technology spillovers from technologically similar firms and the firm's own R&D investment. Let $\tilde{\Omega}$ denote the technological proximity between firms:

$$\tilde{\Omega} = \begin{bmatrix} \tilde{\omega}_{11} & \tilde{\omega}_{12} & \cdots & \tilde{\omega}_{1n} \\ \tilde{\omega}_{21} & \tilde{\omega}_{22} & \cdots & \tilde{\omega}_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{\omega}_{m1} & \tilde{\omega}_{m2} & \cdots & \tilde{\omega}_{mn} \end{bmatrix}$$

where each element $\tilde{\omega}_{ij} \geq 0$ denote the technological proximity between firm $i$ and firm $j$. We assume that all the diagonal elements of the matrix are equal to 0: $\tilde{\omega}_{ii} = 0$ for all $i$. Let $\mathbf{x}_t$denote the vector of R&D efforts:

$$\mathbf{x}_t \equiv \begin{bmatrix} x_{1,t} & x_{2,t} & \dots & x_{n,t} \end{bmatrix}^T,$$

and $\mathbf{W}_t$ the vector of Wiener processes which are independent across firms:

$$\mathbf{W}_t \equiv \begin{bmatrix} W_{1,t} & W_{2,t} & \dots & W_{n,t} \end{bmatrix}^T$$

We assume that firms' knowledge capital follows a multi-dimensional geometric Brownian motion:

$$d\mathbf{z}_t = (\Omega \mathbf{z}_t + \mu \mathbf{x}_t)\, dt + \gamma \mathbf{z}_t \circ d\mathbf{W}_t \tag{8}$$

where $\Omega = \beta\tilde{\Omega}$, and $\beta$, $\mu$, and $\gamma$ are positive scalars. We assume that the magnitude of technology spillovers between two firms is proportional to the technological proximity $\tilde{\Omega}$ , and $\beta$ is the parameter that converts technological proximity into the magnitude of technology spillovers. $\mu$ is a parameter that determines the efficiency of R&D and is common to all firms. $\gamma$ is a parameter that determines the magnitude of shocks in the geometric Brownian motion, which is also common to all firms.

### 2.1.3 Ownership Structure and Firm Objective Function

In our model, investors own multiple firms, and firms compete to maximize the weighted expected returns of their owners, with the weights determined by the proportion of shares. There are $Z$ investment funds indexed by $z$. Let $\tilde{V}^z$ represent the value of fund $z$ and $\hat{V}^i$ represents the value of firm $i$. The the value of fund $\tilde{V}^z$ is given as the weighted sum of the value of firms $\hat{V}^i$ , where the weights correspond to the shareholding ratio of each firm held by the fund:

$$\tilde{V}^z \equiv \sum_{i=1}^{n} s_{iz} \hat{V}^i \tag{9}$$

where $s_{iz}$ is the proportion of shares of firm $i$ owned by fund $z$ such that $\sum_{z=1}^{Z} s_{iz} = 1$. Following Ederer and Pellegrino (2024), we assume that firm $i$ maximizes the value $V^i$, which is the weighted sum of investment funds' values $\tilde{V}^z$, weighted by their respective ownership shares in firm $i$:

$$V^i \equiv \sum_{z=1}^{Z} s_{iz} \tilde{V}^z$$

By substituting the value of the fund (9) into the above equation, we obtain the expression for the objective value of the firm, $V^i$, as a linear combination of the firm's value $\hat{V}^j$:

$$V^i = \sum_{j=1}^{n} \hat{V}^j \sum_{z=1}^{Z} s_{iz} s_{jz}$$

Note that firm $i$'s value $\hat{V}^i$ is distinct from the objective value function $V^i$ that the firm maximizes. The firm $i$'s value $\hat{V}^i$ represents the expected discounted present value of the

cash flows it generates. In contrast, the firm maximizes the weighted average of the value of its owners, which implies that the firm maximizes a linear combination of the values of firms, including those of other firms.

Let define the common ownership weight matrix $K$ as

$$K \equiv \begin{bmatrix} \kappa_{11} & \kappa_{12} & \cdots & \kappa_{1n} \\ \kappa_{21} & \kappa_{22} & \cdots & \kappa_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \kappa_{n1} & \kappa_{n2} & \cdots & \kappa_{nn} \end{bmatrix}$$

where

$$\kappa_{ij} \equiv \frac{\mathbf{s}_i' \mathbf{s}_j}{\mathbf{s}_i' \mathbf{s}_i} \tag{10}$$

and

$$\mathbf{s}_i \equiv \begin{bmatrix} s_{i1} & s_{i2} & \dots & s_{iZ} \end{bmatrix}'.$$

Note that , by construction, the diagonal elements $\kappa_{ii} = 1$ for all $i$. This allows us to rewrite firm $i$'s objective value as

$$V^i \propto \sum_j \kappa_{ij} \hat{V}^j.$$

Common ownership weight $\kappa_{ij}$ represents the weight that each firm $i$'s objective value function assigns to the value of other firms relative to its own value function. Note that the objective value function nests standard profit maximization ($\kappa_{ij} = 0$ for $i \neq j$) and monopoly ($\kappa_{ij} = 1$ for $i \neq j$) as its limit cases.

### 2.1.4 Resources and Household Preference

We close the model with resource constraints and representative household preference. Production labor is supplied inelastically:

$$L = \sum_i l_{i,t}. \tag{11}$$

The final goods are used for consumption and R&D investment. Following the endogenous growth literature (e.g. Acemoglu et al. 2016), we assume that the innovation elasticity for each firm is 0.5. In other words, as the firm-level R&D outcome increases, the R&D

costs imposed on the firm increases quadratically:

$$C_t + \sum_i x_{i,t}^2 = Y_t$$

where $C_t$ is consumption of the final good at time $t$. Finally, we assume that the representative household is risk-neutral and discounts the future at rate $\rho > 0$:

$$\mathbb{E}_0 \left[ \int_0^\infty \exp(-\rho t) C_t dt \right].$$

## 2.2 Equilibrium

We focus on the Markov Perfect Equilibrium, and therefore, we can separate the problem into static and dynamic components. First, we analyze the static problem of competition in the product market, and then we address the dynamic problem of R&D investments.

### 2.2.1 Problem of Final Good Producer

The final good is produced competitively. In each period, the representative final good producer chooses the quantity of each product which maximizes profit:

$$\max_{\mathbf{q}_t} P_t Y_t - \mathbf{q}_t^T \mathbf{p}_t$$

where $\mathbf{p}_t$ is the vector of product prices at time $t$ and $P_t$ is the final good price at time $t$. The profit maximization problem of the final goods producer results in the following linear inverse demand function:

$$\frac{\mathbf{p}_t}{P_t} = \mathbf{b}_t - \Sigma \mathbf{q}_t \tag{12}$$

### 2.2.2 Static Problem of Firms: Production Decision and Technology Choice

We assume that firms maximize the real returns of their owners to ensure the independence of the allocation from the choice of numeraire (Azar and Vives, 2021). Thus, in the static problem, firms maximize the weighted real profits of the firms. Each firm's real gross profit *before* subtracting R&D cost, $\pi_{i,t}$, is given by

$$\pi_{i,t} = \frac{p_{i,t}}{P_t} q_{i,t} - \frac{w_t}{P_t} l_{i,t}$$

$$= \left( b_{i,t} - \frac{1}{a_{i,t}} \frac{w_t}{P_t} - \sum_j \sigma_{ij} q_{j,t} \right) q_{i,t} \tag{13}$$

In the first line on the right-hand side, the labor cost is subtracted from the real revenue. The second line is obtained by substituting the inverse demand (12) and the production function (6).

In the static problem, the objective function of firm $i$ is given by the weighted sum of real gross profit, weighted by the common ownership weights:

$$\sum_j \kappa_{ij} \pi_{j,t}$$

Each firm $i$ simultaneously decides the product quantity $q_{i,t}$ and how to allocate its knowledge capital between improving labor productivity $a_{i,t}$ and product quality $b_{i,t}$, taking as given the output of other firms $\left\{q_{j,t}\right\}_{j \neq i}$, real wage $w_t/P_t$, and its own knowledge capital $z_{i,t}$ under the knowledge capital constraint (7)[1]. The FOC w.r.t. $q_{i,t}$ gives

$$0 = b_{i,t} - \frac{1}{a_{i,t}} \frac{w_t}{P_t} - 2q_{i,t} - \sum_{j \neq i} \sigma_{ij} q_{j,t} - \sum_{j \neq i} \kappa_{ij} \sigma_{ji} q_{j,t} \tag{14}$$

Also, from the optimality conditions for $a_{i,t}$ and $b_{i,t}$, we obtain

$$a_{i,t} = \sqrt{\frac{w_t}{\zeta P_t}} \tag{15}$$

$$b_{i,t} = z_{i,t} - \sqrt{\frac{\zeta w_t}{P_t}}. \tag{16}$$

Intuitively, firms are more inclined to improve labor productivity when labor costs $w_t/P_t$ are higher. Inserting equation (15) into the labor market clearing condition (11), we obtain

$$\sqrt{\zeta \frac{w_t}{P_t}} = \frac{\zeta}{L} \sum_i q_{i,t}. \tag{17}$$

Combining equation (14), (15), (16), and (17), we obtain the expression for the quantities

[1] In our model, the results are unaffected by whether each firm chooses $q_{i,t}$ sequentially after $a_{i,t}$ and $b_{i,t}$, or chooses $a_{i,t}$, $b_{i,t}$,, and $q_{i,t}$ simultaneously.

as a linear function of knowledge capital:

$$\mathbf{q}_t = N\mathbf{z}_t \tag{18}$$

where $N$ is an $n \times n$ matrix

$$N \equiv \left(2\frac{\zeta}{L}J + \Sigma + K \circ \Sigma\right)^{-1},$$

$J$ is an $n \times n$ matrix where all elements are 1, and $I$ is an $n \times n$ identity matrix (see Appendix A.1 for the derivation). The first term inside the blanket, $2\frac{\zeta}{L}J$, captures the labor cost, while the second term $\Sigma$ represents the network of product market rivalry. The third term $K \circ \Sigma$ represents the element-wise product of the common ownership weight $K$ and the network of product market rivalry $\Sigma$. This indicates that, in the static problem, the ownership structure of firms influences their production decisions only through its overlap with the network of product market rivalry.

Given the linear relationship between the quantity $\mathbf{q}_t$ and knowledge capital $\mathbf{z}_t$, we can express the firm's static objective in a quadratic form of the firm's productivity vector $\mathbf{z}_t$:

$$\sum_j \kappa_{ij}\pi_{j,t} = \mathbf{z}^T Q^i \mathbf{z}$$

where

$$Q^i \equiv \sum_h \kappa_{ih} \sum_j \kappa_{hj}\sigma_{hj}\frac{N_h^T N_j + N_j^T N_h}{2}$$

and $N_i$ denotes the $i$ th row of $N$ (see Appendix A.2 for the derivation).

### 2.2.3 Dynamic Problem of Firms: R&D Decision

Given a static production strategy profile, we consider a dynamic R&D game. In the dynamic game, given other players' strategy $\left\{x_{j,t}\right\}_{j\neq i, t\geq 0}$, firm $i$ chooses R&D effort $\{x_{i,t}\}_{t\geq 0}$ to maximize the expected discounted present value of profits weighted by ownership weights:

$$\max_{\{x_{i,t}\}_{t\geq 0}} \quad V^i(\mathbf{z}_0) \equiv \mathbb{E}_0\left[\int_0^\infty \exp(-\rho t)\left\{\sum_j \kappa_{ij}\left(\pi_{j,t} - x_{j,t}^2\right)\right\}dt\right]$$

while the state evolves according to (8). Here, we utilize the fact that the interest rate is derived from the household's Euler equation $r_t = \rho$. The dynamic problem gives the

following HJB equation:

$$\rho V^i(\mathbf{z}) = \max_{x_i} \left\{ \mathbf{z}^T Q^i \mathbf{z} - \sum_j \kappa_{ij} x_{j,t}^2 + V_{\mathbf{z}}^i(\mathbf{z}) \left[ \Omega \mathbf{z} + \mu \mathbf{x} \right] + \frac{\gamma^2}{2} \mathbf{z}^T V_{\mathbf{zz}}^i(\mathbf{z}) \mathbf{z} \right\}. \tag{19}$$

The first term on the right-hand side of the HJB equation represents gross profit. The second term represents R&D costs. Therefore, the first two terms represent the firm's net instantaneous profit. The third term reflects the impact of changes in knowledge capital due to technology spillovers and R&D on firm $i$'s value. The fourth term represents the expected effect of all firms' idiosyncratic shocks on firm $i$'s value.

This dynamic game is a linear quadratic differential game, which significantly simplifies the analysis. This simplification is achieved because the following guess and verify approach holds. Guess that the value can be expressed as a quadratic form of knowledge capital:

$$V^i(\mathbf{z}) = \mathbf{z}^T X^i \mathbf{z}. \tag{20}$$

The first and second derivatives of the value function are then given by:

$$V_{\mathbf{z}}^i(\mathbf{z}) = 2\mathbf{z}^T X^i \tag{21}$$

$$V_{\mathbf{zz}}^i(\mathbf{z}) = 2X^i \tag{22}$$

In a linear quadratic game, firms' strategies are expressed as linear functions of the state variables. Let $X_i^i$ denote the $i$-th column of $X^i$. The first order condition with respect to $x_i$ gives

$$x_i = \left( \mu X_i^i \right)^T \mathbf{z}. \tag{23}$$

Given firms' linear strategies, the law of motion for knowledge capital $\mathbf{z}_t$ can be rewritten as a geometric Brownian motion with constant coefficients. Let define $\tilde{X}$ as the matrix obtained by stacking $X_i^i$:

$$\tilde{X} \equiv \begin{bmatrix} X_1^1 & \cdots & X_n^n \end{bmatrix}^T \tag{24}$$

Then, the law of motion is rewritten as

$$d\mathbf{z}_t = \Phi \mathbf{z}_t dt + \gamma \mathbf{z}_t dW_t \tag{25}$$

where the matrix $\Phi$ that capture the drift of the stochastic process is an $n \times n$ matrix such that

$$\Phi \equiv \begin{bmatrix} \phi_{11} & \phi_{12} & \cdots & \phi_{1n} \\ \phi_{21} & \phi_{22} & \cdots & \phi_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \phi_{n1} & \phi_{n2} & \cdots & \phi_{nn} \end{bmatrix} \equiv \Omega + \mu^2 \tilde{X}.$$

We refer to the matrix $\Phi$ as the *technology transition matrix*, which consists of exogenous technology spillover matrix $\Omega$ and endogenous R&D matrix $\mu^2 \tilde{X}$.

The HJB equation can be transformed into an algebraic Riccati equation, enabling the dynamic oligopoly problem with a number of firms to be solved in a computationally feasible timeframe. Inserting equations(20), (21), (22), and (23) into the HJB equation (19), and rearranging, we obtain stacked algebraic Riccati equations

$$0 = Q_i - \mu^2 \sum_j \kappa_{ij} X_j^j \left(X_j^j\right)^T + \left(\Phi - \frac{1}{2}\left(\rho - \gamma^2\right) I\right)^T X^i + X^i \left(\Phi - \frac{1}{2}\left(\rho - \gamma^2\right) I\right) \tag{26}$$

for $i = 1, 2, ..., n$. Note that the terms $\mu^2 \sum_j \kappa_{ij} X_j^j \left(X_j^j\right)^T$ and $\Phi$ depend on the $X^j$ of other firms $j \neq i$. Therefore, to solve for $X^i$, we need to simultaneously solve the algebraic Riccati equations for all firms.

Here, we define the stability of the solution.

**Definition.** The solution of the stacked algebraic Riccati equations (26) is *stable* if all the eigenvalues of $\Phi - \frac{1}{2}(\rho - \gamma^2) I$ have negative values.[2]

The requirement for stability of the solution corresponds to the condition commonly imposed in growth theory, which requires the discount rate to be sufficiently large to ensure that the utility over an infinite horizon is finite.

In the following, we assume the existence of such a solution.

**Assumption 1.** *There exists a stabilizing solution* $\left(X^1, X^2, \ldots, X^n\right)$ *of the stacked algebraic Riccati equations (26).*

### 2.2.4 Balanced Growth Path in Deterministic Economy

In this subsection, we characterize the existence of a balanced growth path and its equilibrium in a deterministic economy using the technology transition matrix, $\Phi$.

[2]Because matrix $\Phi - \frac{1}{2}\left(\rho - \gamma^2\right) I$ is a symmetric matrix, all of the eigenvalues are real.

We impose the following assumption on $\Phi$, which turns out to be the sufficient condition for the existence of the balanced growth path equilibrium.

**Assumption 2.** *$\Phi$ is irreducible.*

One definition of irreducible matrix is as follows. Consider a directed graph that has $n$ vertices labeled $1, ..., n$ and there is an edge from vertex $i$ to vertex $j$ when $\phi_{ij} > 0$. Then, the matrix $\Phi$ is irreducible if and only if its associated graph is strongly connected (i.e. every vertex is reachable from every other vertex). Intuitively, this assumes that all firms have direct or indirect technology spillover connections each other.

Here, we define the balanced growth path equilibrium in our economy.

**Definition 1. Balanced Growth Path Equilibrium**: A *balanced growth path equilibrium* is a linear Markov perfect equilibrium such that the productivity of all firms grow at the same constant rate.

In the following proposition, we present sufficient conditions for the existence of a balanced growth path and characterize the equilibrium.

**Proposition 1.** *Let Assumption 1 and 2 hold. Then, there exists a largest and positive eigenvalue of $\Phi$, $g$, and strictly positive eigenvector, $\mathbf{z}$, associated with $g$. Moreover, there exists a globally stable balanced growth path equilibrium such that the productivity growth rate of all firms is given by $g$, and the productivity distribution is given by a scalar multiple of $\mathbf{z}$.*

*Proof.* The proposition follows from the Perron-Frobenius theorem. □

Proposition 1 indicates that there exists a unique and globally stable balanced growth path equilibrium when the technology transition matrix $\Phi$ is irreducible. Also, Proposition 1 indicates that the equilibrium productivity growth rate is given by the eigenvalue of matrix $\Phi$ and the stationary distribution of the equilibrium is characterized by the associated eigenvector. Remember that the technology transition matrix consists of an exogenous technology spillover matrix and an endogenous R&D strategy profile matrix. Therefore, this result indicates that our model is an endogenous economic growth model that achieve a balanced growth path. When the knowledge capital of all firms grow at the same constant rate $g$, the growth rate of variables are summarized in Table 1.

### 2.2.5 Social Welfare

Now, we demonstrate that once the equilibrium is solved, the utility of households can be calculated for any initial distribution with minimal additional computational cost. To

Table 1: Growth Rate of Variables in Deterministic Economy

| Growth Rate | Variable |
|---|---|
| 0 | $l_{i,t}$ |
| $g$ | $a_{i,t}$, $b_{i,t}$, $z_{i,t}$, $q_{i,t}$, $x_{i,t}$, $p_{i,t}/P_t$ |
| $2g$ | $\pi_{i,t}$, $C_t$, $Y_t$, $w_t/P_t$ |

characterize the household's value, we introduce the household's HJB equation. Let $V(\mathbf{z}_0)$ denote the expected value of the representative household in the competitive equilibrium given the initial distribution $\mathbf{z}_0$, which is defined as

$$V(\mathbf{z}_0) = \mathbb{E}_0\left[\int_0^\infty \exp(-\rho t)\left\{Y_t - \sum_i x_{i,t}^2\right\} dt \middle| \mathbf{z}_0\right].$$

Note that the total output is given by

$$Y_t = \mathbf{z}_t^T Q \mathbf{z}_t \tag{27}$$

where

$$Q = \frac{1}{2} N^T \left(2\frac{\zeta}{L} J + \Sigma + 2\left(K^T \circ \Sigma + K \circ \Sigma\right)\right) N$$

(see Appendix A.3 for the derivation). The HJB equation for the representative household is given by

$$\rho V(\mathbf{z}) = \mathbf{z}^T Q \mathbf{z} - \mu^2 \tilde{X}^T \tilde{X} + V_{\mathbf{z}}(\mathbf{z}) \Phi \mathbf{z} + \frac{\gamma^2}{2} \mathbf{z}^T V_{\mathbf{zz}}(\mathbf{z}) \mathbf{z},$$

The first term on the right-hand side of the HJB equation represents output. The second term represents the total R&D cost. Therefore, the first two terms represent the household's instantaneous consumption. The third and fourth terms represent the expected change in the household's value due to the stochastic process of knowledge capital. Note that the level of consumption is determined as the difference between total output $Y_t$ and the total R&D expenditure $\sum_i x_{i,t}^2$, which result from firms behavior under the Euler equation $r_t = \rho$. Therefore, in the representative household's HJB equation, which is solved after the firm's problem, there are no choice variables for the representative household.

Since the household's problem is linear quadratic, the household's value can be expressed in a quadratic form of knowledge capital $\mathbf{z}$. Guess that the value is expressed as a quadratic form of knowledge capital:

$$V(\mathbf{z}) = \mathbf{z}^T X \mathbf{z}.$$

We obtain the following Lyapunov equation by substituting $V(z)$ and its derivatives:

$$0 = Q - \mu^2 \tilde{X}^T \tilde{X} + X\left(\Phi - \frac{1}{2}\left(\rho - \gamma^2\right) I\right) + \left(\Phi - \frac{1}{2}\left(\rho - \gamma^2\right) I\right)^T X \tag{28}$$

Therefore, $X$ is obtained as the solution of the Lyapunov equation (28), and the welfare in the competitive equilibrium with initial productivity vector $\mathbf{z}_0$ is given by $V(\mathbf{z}_0) = \mathbf{z}_0^T X \mathbf{z}_0$. This result indicates that once the equilibrium is solved, we can easily compute the expected utility of the representative household for any given initial distribution.

### 2.2.6 Total Value of Firms

When competition in product markets and R&D competition is relaxed, producer surplus increases relative to total surplus. To capture the dynamic producer surplus, this section characterizes the total value of the firms.

Let $V^P$ denote the total value of firms in the competitive equilibrium, which is defined as

$$V^P(\mathbf{z}_0) = \mathbb{E}_0\left[\int_0^\infty \exp(-\rho t)\left(\sum_i \pi_{i,t} - \sum_i x_{i,t}^2\right) dt \,\middle|\, \mathbf{z}_0\right]$$

Note that aggregate real profit of firms is written in vector form as

$$\begin{aligned}\sum_i \pi_{i,t} &= \sum_i \sum_j \kappa_{ij} \sigma_{ij} q_i q_j \\ &= \mathbf{q}_t^T (K \circ \Sigma) \mathbf{q}_t \\ &= \mathbf{z}_t^T P \mathbf{z}_t\end{aligned}$$

where

$$P = \frac{1}{2} N^T \left(\Sigma \circ \left(K + K^T\right)\right) N$$

Comparing with the problem of calculating social welfare in Section 2.2.5, it becomes clear that solving the modified version of the Lyapunov equation (28), in which $Q$ is replaced with $P$. When we express the solution of the Riccati equation with $X^P$, the total value of firms is given by $V^P(\mathbf{z}_0) = \mathbf{z}_0^T X^P \mathbf{z}_0$.

## 2.3 Counterfactual Scenarios

We examine several counterfactual scenarios to compare with the allocation under the baseline setting of common ownership. First, we analyze the allocation chosen by a benevolent social planner aiming to maximize social welfare. Additionally, we consider the allocation resulting from the profit-maximization problem of firms with fully dispersed ownership. Lastly, we explore the "Monopoly" case, where a single investor owns all firms, or equivalently in our setting, where all investors hold identical portfolios of firms.

### 2.3.1 Social Planner

To analyze the socially optimal allocation of R&D and its impact on economic growth and social welfare, we formulate the social planner's problem. The social planner aims to maximize the representative household's lifetime utility, subject to technological constraints. Similar to the competitive equilibrium, the social planner's problem can be separated into a static problem and a dynamic problem.

**Static Problem** First, we characterize the static allocation in the social planner's problem. The social planner maximizes the final good production given the vector of knowledge capital $\mathbf{z}_t$ in each period:

$$\max_{\mathbf{a}_t, \mathbf{b}_t, \mathbf{l}_t, \mathbf{q}_t} \quad Y_t = \mathbf{q}_t^T \mathbf{b}_t - \frac{1}{2} \mathbf{q}_t^T \Sigma \mathbf{q}_t$$

subject to

$$\begin{aligned} q_{i,t} &= a_{i,t} l_{i,t} \quad i = \{1, 2, ..., n\} \\ z_{i,t} &= \zeta a_{i,t} + b_{i,t} \quad i = \{1, 2, ..., n\} \\ L &= \sum_i l_{i,t} \end{aligned} \tag{29}$$

The first constraint represents the production technology of each firm, the second governs the allocation of each firm's knowledge capital between labor productivity and product quality, and the third ensures the labor market clears. By solving the planner's static problem, we obtain a solution of quantities as a linear function of knowledge capital:

$$\mathbf{q}_t^* = N^* \mathbf{z}_t$$

and a quadratic form for output of knowledge capital

$$Y_t^* = \frac{1}{2}\mathbf{z}_t^T N^* \mathbf{z}_t$$

where

$$N^* = \left(2\frac{\zeta}{L}J + \Sigma\right)^{-1}$$

(See Appendix A.4).

**Dynamic Problem** Next, we characterize the dynamic allocation in the social planner's problem. The social planner's dynamic problem is formulated as follows:

$$V^*(\mathbf{z}_0) \equiv \max_{\{\mathbf{x}_t\}_{t\geq 0}} \mathbb{E}_0\left[\int_0^\infty \exp(-\rho t)\left\{\frac{1}{2}\mathbf{z}_t^T N^* \mathbf{z}_t - \mathbf{x}_t^T \mathbf{x}_t\right\} dt\right],$$

subject to the law of motion of knowledge capital (8). The HJB equation of the social planner's value $V^*$ derived from the dynamic problem is given by

$$\rho V^*(\mathbf{z}) = \max_{\mathbf{x}\geq 0}\left\{\frac{1}{2}\mathbf{z}^T N^* \mathbf{z} - \mathbf{x}^T\mathbf{x} + V_{\mathbf{z}}^*(\mathbf{z})\left[\Omega\mathbf{z} + \mu\mathbf{x}\right] + \frac{\gamma^2}{2}\mathbf{z}^T V_{\mathbf{zz}}^*(\mathbf{z})\,\mathbf{z}\right\} \tag{30}$$

The first term on the right-hand side of the HJB equation represents output. The second term represents the total R&D cost. Therefore, the first two terms represent the household's instantaneous consumption. The third and fourth terms represent the expected change in the household's value due to the stochastic process of knowledge capital.

Since the social planner's problem is also linear quadratic, household utility can be expressed in a quadratic form of knowledge capital $\mathbf{z}$. Guess that the value is expressed as a quadratic form of knowledge capital:

$$V^*(\mathbf{z}) = \mathbf{z}^T X^* \mathbf{z} \tag{31}$$

The socially optimal allocation of R&D is a linear function of knowledge capital $\mathbf{z}$, as in the competitive equilibrium:

$$\mathbf{x} = \mu X^* \mathbf{z} \tag{32}$$

Therefore, again, the law of motion for knowledge capital $\mathbf{z}_t$ is rewritten as a geometric

Brownian motion with constant coefficients:

$$d\mathbf{z}_t = \Phi^* \mathbf{z}_t dt + \gamma \mathbf{z}_t \circ d\mathbf{W}_t$$

where the optimal technology transition $\Phi^*$ is given by

$$\Phi^* \equiv \Omega + \mu^2 X^*$$

Inserting (31) and its first and second derivatives with respect to $\mathbf{z}$ and (32) into (30), we obtain a single algebraic Riccati equation

$$0 = \frac{1}{2} N^* - \mu^2 (X^*)^2 + X^* \left( \Phi^* - \frac{1}{2} \left( \rho - \gamma^2 \right) I \right) + \left( \Phi^* - \frac{1}{2} \left( \rho - \gamma^2 \right) I \right) X^* \tag{33}$$

Therefore, $X^*$ is obtained as the solution of the single algebraic Riccati equation (33). Remember that, in the competitive equilibrium, a Riccati equation exists for each firm, requiring simultaneous solutions of these equations. In contrast, for the social planner problem, we only need to solve a single Riccati equation, making the computation much simpler.

The social planner's problem yields a quadratic form for social welfare under the optimal allocation: $V^*(\mathbf{z}_0) = \mathbf{z}_0^T X^* \mathbf{z}_0$. This result allows us to efficiently calculate the expected utility of the representative household for any initial distribution $\mathbf{z}_0$ , requiring minimal additional computation once $X^*$ is obtained.

### 2.3.2 Dispersed Ownership

As the second counterfactual scenario, we consider the case where there is no overlap in firm ownership among investors. In this case, maximizing the returns for investors is equivalent to firms maximizing their own profits. This scenario corresponds to the case where the common ownership weight matrix $K$ is an identity matrix.

### 2.3.3 Monopoly

As the third counterfactual scenario, we consider the "monopoly" case, where all firms are owned by a single investor, to analyze the allocation of R&D, its impact on economic growth, and social welfare under an extremely concentrated ownership structure. Alternatively, in our setting, where firms maximize the weighted sum of investors' returns,

the monopoly case is equivalent to a situation where multiple investors exist but all hold identical portfolios of firm ownership. This corresponds to a common ownership matrix where all elements are equal to 1. In the monopoly case, since all firms share the same objective function, the equilibrium can be determined by solving a single Riccati equation. For details on the characterization and solution of the equilibrium, see the Appendix A.5.

### 2.3.4 Different Ownership Structures for Production and R&D

We consider counterfactual scenarios where the ownership structure for production differs from that for R&D to deepen our understanding of how ownership structure impacts firms' R&D decisions. For instance, we analyze a scenario where a static competitive equilibrium is determined under the baseline common ownership structure, and given this static competitive equilibrium, a benevolent social planner selects the optimal allocation of R&D. This problem can be solved as follows: starting from an initial productivity distribution $\mathbf{z}_0$, the social planner maximizes the value

$$V^{**}(\mathbf{z}_0) \equiv \max_{\{\mathbf{x}_t\}_{t\geq 0}} \mathbb{E}_0 \left[ \int_0^\infty \exp(-\rho t) \left\{ \mathbf{z}_t^T Q \mathbf{z}_t - \mathbf{x}_t^T \mathbf{x}_t \right\} dt \right],$$

subject to

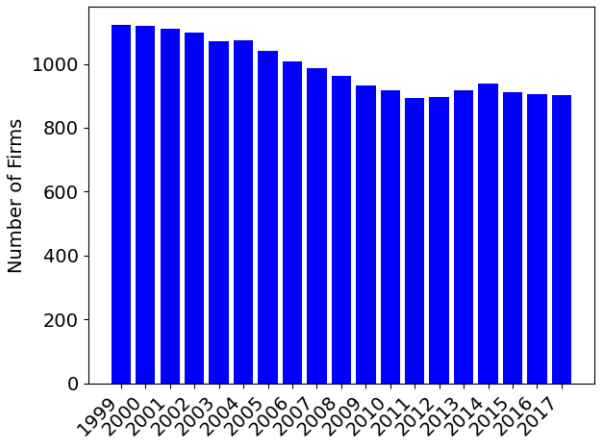
$$d\mathbf{z}_t = (\Omega \mathbf{z}_t + \mu \mathbf{x}_t)\, dt + \gamma \mathbf{z}_t \circ dW_t.$$

Note that, unlike in Section 2.3.1, $N^*/2$ in the social planner's objective function has been replaced by $Q$. Therefore, we obtain the constrained social planner's allocation by solving the modified version of the Riccati equation (33), in which $N^*/2$ is replaced with $Q$. When we express the solution of the modified Riccati equation with $X^{**}$, the social welfare under the constrained optimal allocation is given by $V^{**}(\mathbf{z}_0) = \mathbf{z}_0^T X^{**} \mathbf{z}_0$.

# 3 Data and Estimation

We map model parameters, including three networks—product proximity, technology proximity, and ownership structure—to the observed data. In this section, we explain the data used and the method for estimating the model. The period of data used for our analysis is from 1999 to 2017. Figure 1 plots the number of sample firms by year, while Table 2 summarizes the estimated parameters and their sources.

Figure 1: Number of Firms

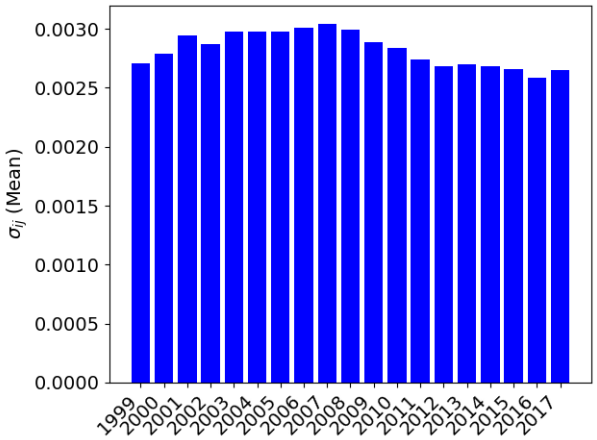



Note: This figure shows the trend in the number of sample firms. The potential set of firms are U.S. publicly traded companies included in DISCERN 2, a dataset linking patents and public companies. Among these, we select firms with positive profit (revenue minus the cost of goods sold) and holding at least one patent during the year or within two years before or after the year (a total of five years).

Table 2: Parameters

| Notation | Description | Value | Source |
|---|---|---|---|
| $\Psi^T\Psi$ | Product proximity | | Form 10-K, Hoberg and Phillips (2016) |
| $\widetilde{\Omega}$ | Technological proximity | | USPTO, Patent classification |
| $K$ | Common ownership weights | | Form 13F, Backus et al. (2021) |
| $\alpha$ | Product proximity $\rightarrow$ Substitutability | .12 | Pellegrino (2024) |
| $\beta$ | Technological proximity $\rightarrow$ Spillover | .00014 | Estimate the law of motion |
| $\gamma$ | St.d. of idiosyncratic shocks | .027 | Estimate the law of motion |
| $\zeta/L$ | Labor augmentation efficiency | .0063 | Compustat, Cost of goods sold |
| $\rho$ | Discount rate | .10 | |
| $\mu$ | R&D efficiency | .05 | 1.6% economic growth rate |

Figure 2: Mean $\sigma_{ij}$




Note: This figure plots the average value of the product proximity $\sigma_{ij}$ for all pairs of sample firms from 1999 to 2017.

## 3.1 Product Market Proximity and Substitutability

Following Pellegrino (2024), we measure product market proximity, denoted as $\Psi^T\Psi$ , using the dataset developed by Hoberg and Phillips (2016). This measure is based on the cosine similarity of the words found in the Business Description sections of 10-K filings. Hoberg and Phillips (2016) constructed a vocabulary comprising 61,146 words that firms use to describe the features and characteristics of their products. For each firm $i$, they generated a vector of word frequencies, where each element of the vector represents the number of times a specific word appears in the firm's product description. Their analysis demonstrates that cosine similarity effectively identifies industry groupings and predicts competitive relationships between firms, outperforming other industry classification methods. Figure 2 plots the average product proximity across all pairs of firms from 1999 to 2017. The figure indicates no clear trend in product proximity over this period.

We adopt the value of $\alpha$, which governs the degree of horizontal differentiation between good, from Pellegrino (2024). Pellegrino (2024) estimates $\alpha$ to align with the inverse cross-price elasticity of demand observed by Nevo (2001), as this parameter links the cosine similarity between products to the magnitude of cross-price elasticity. Pellegrino (2024) shows that the estimated price elasticities of demand derived from GHL demand are well aligned with those estimated in the industrial organization literature. Specifically, Pellegrino (2024) compares them to the estimated price elasticities of demand in the markets for automobiles Berry et al. (1995), ready-to-eat cereal Nevo (2001), and computers Goeree (2008). Table 2 in Pellegrino (2024) shows that the magnitude of the own-price

elasticities of demand for each firm and the cross-price elasticities between firms, derived from GHL, are similar to the values estimated in the literature.

## 3.2 Common Ownership Weight

To calculate the matrix of common ownership profit weights $K$, we first need each firm's ownership shares vector $\mathbf{s}_i$ for each firm $i$. We obtain $\mathbf{s}_i$ from a dataset compiled by Backus et al. (2021), who constructed data using 13(f) filings. Form 13(f) is a mandatory filing with the Securities and Exchange Commission (SEC) filing that requires institutional investors with over $100 million in assets under management to disclose their holdings of U.S. securities, including those of all U.S. public corporations. securities, including those of all U.S. public corporations.

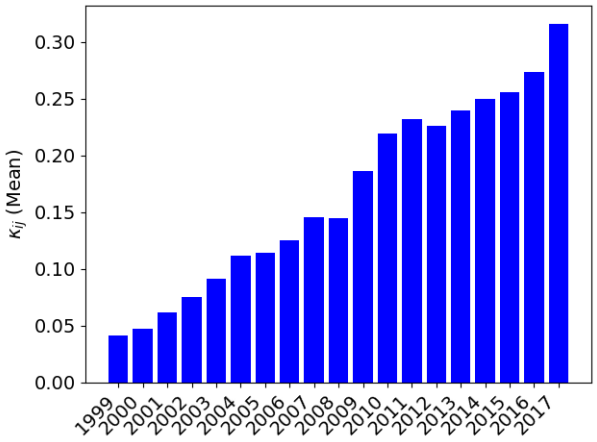
Using CUSIP codes, we merge this dataset with Compustat's total shares outstanding for each firm. We then divide each investor's holdings in a firm by that firm's total shares outstanding to obtain the normalized ownership share for each investor-firm pair. Aggregating these values across all investors for a given firm produces the normalized ownership shares vector $\mathbf{s}_i$. Finally, we apply equation (10) to compute the common ownership profit weights matrix $K$.

Following Ederer and Pellegrino (2024), we incorporate a statistical correction to account for unobserved investors. When corporate ownership is not fully observed, the denominator in equation (10) are biased downwards, leading to unrealistically large $\kappa_{ij}$ values for certain firm pairs. To address this, we assume that unobserved ownership has similar characteristics to observed ownership. Specifically, let $S_i^O$ denote the sum of firm $i$'s shares held by the observed investors, and $\left(\mathbf{s}_i'\mathbf{s}_j\right)^O$ denote the inner product of the ownership share vectors computed solely from observed ownership. When calculating the denominator of common ownership weight $\kappa_{ij}$ where $j \neq i$, we make the following adjustments:

$$\mathbf{s}_i'\mathbf{s}_i = \left[1 + \left(\frac{1 - S_i^O}{S_i^O}\right)^2\right]\left(\mathbf{s}_i'\mathbf{s}_i\right)^O$$

Figure 3 illustrates the trend in the average common ownership weight $\kappa_{ij}$, calculated using the aforementioned method. The value of $\kappa_{ij}$ increased approximately eightfold from 1999 to 2017 as a result of the rise of index funds and the concentration of investors. This indicates that, within our framework, firms increasingly have taken into account the profits of other firms when determining their production levels and R&D activities during

Figure 3: Mean $\kappa_{ij}$




Note: This figure plots the average value of the common ownership weight $\kappa_{ij}$ for all pairs of sample firms from 1999 to 2017. Following Ederer and Pellegrino (2024), a statistical adjustment is applied in the calculation of the denominator of $\kappa_{ij}$ to address the bias caused by unobserved ownership.

this period.

## 3.3 Technological Proximity

For technological proximity, we use patent data and follow the methodologies of Bloom et al. (2013) and Lucking et al. (2019) to construct measures of technological proximity between firms. Specifically, we utilize the DISCERN 2 dataset (Arora et al., 2024), which links data on U.S. publicly listed firms from the Compustat database to their patents. Following Bloom et al. (2013), we calculate technological proximity between firms using both the Jaffe and Mahalanobis measures, based on the Cooperative Patent Classification.

Denote the vector of the share of patents of firm $i$ in any given technology class by $T_i$. When we use the Jaffe measure (Jaffe, 1986), the technological proximity $\tilde{\omega}_{ij}$ is measured by

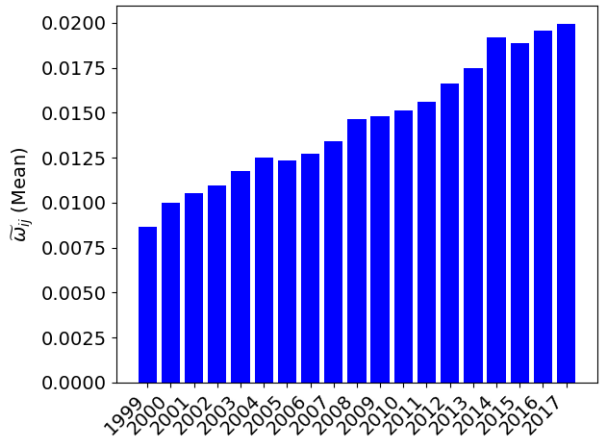
$$\tilde{\omega}_{ij} = \frac{T_i T_j'}{\left(T_i T_i'\right)^{1/2} \left(T_j T_j'\right)^{1/2}}$$

We measure $T_i$ using both subclass-level and group-level classifications[3].

Building on the approaches of Bloom et al. (2013) and Lucking et al. (2019), we con-

[3]When subclass-level classifications are used to calculate $T_i$, the granularity becomes excessively fine, leaving most elements of $\tilde{\Omega}$ equal to zero. Consequently, we avoid using subclass-level classifications for this purpose.

Figure 4: Mean $\widetilde{\omega}_{ij}$




Note: This figure plots the average value of the technology proximity $\omega_{ij}$ for all pairs of sample firms from 1999 to 2017. The technology proximity is calculated using the Jaffe measure based on group-level patent classification.

struct an alternative version of $\tilde{\omega}_{ij}$ using the Mahalanobis proximity metric. Unlike the Jaffe metric, which assumes spillovers within the same technology class or industry and no spillovers across different classes, the Mahalanobis metric accounts for spillovers between technology classes. Specifically, it assigns higher proximity to pairs of technologies that frequently co-occur within the same firm. Detailed definitions and the construction methodology for the Mahalanobis measures can be found in the online appendices of Bloom et al. (2013) and Lucking et al. (2019).

Figure 4 plots the average value of the technology proximity $\omega_{ij}$ for all pairs of sample firms. The technological proximity is calculated using the Jaffe measure based on group-level patent classifications. According to the figure, technological proximity between firms has been increasing over time. This suggests that firms are innovating in closer technological fields, facilitating easier diffusion of technology.

## 3.4 Knowledge Capital Distribution and Labor Augmentation Efficiency

Next, we identify firms' profits $\{\pi_{i,t}\}$, quantity $\mathbf{q}_t$, labor-augmenting efficiency $\zeta/L$, and knowledge capital $\mathbf{z}_t$ step-by-step. The identified knowledge capital $\mathbf{z}_t$ is used for estimating its law of motion and as an initial distribution for numerical analysis.

The firm-level profit $\pi_{i,t}$ is measured as "Revenue -- Total" (revt) minus the production costs represented by "Costs of Goods Sold" (cogs) in the Compustat database. Nominal variables such as revenue and the cost of goods sold are deflated using the GDP deflator.

Table 3: Estimation of the Law of Motion – Jaffe Measure, Subclass

| | (1) | (2) | (3) |
|---|---|---|---|
| Dependent Variable: | $z_{i,t+1} - z_{i,t}$ | $z_{i,t+1} - z_{i,t}$ | $z_{i,t+1} - z_{i,t}$ |
| $\beta$ | 0.000053*** | 0.000039*** | 0.000037*** |
| | (0.000010) | (0.000010) | (0.000012) |
| $\hat{\mu}$ | | .053** | |
| | | (0.021) | |
| Year Fixed Effects | ✓ | ✓ | ✓ |
| IV | | | ✓ |
| IV 1st Stage F-statistics | | | 5007 |
| Patent Classification | Subclass | Subclass | Subclass |
| Technological Proximity | Jaffe | Jaffe | Jaffe |
| No. observations | 16,324 | 15,173 | 14,181 |

SEs clustered by years and 4-digit naics industries are reported in parentheses.
* $p < 0.1$, **$p < 0.05$, *** $p < 0.01$.

Then, given the product market rivalry $\sigma_{ij}$ and ownership structure $\kappa_{ij}$, we identify the quantity vector $\mathbf{q}_t$ such that it satisfies the following profit formula for each firm derived from the model:

$$\pi_{i,t} = \sum_j \kappa_{ij}\sigma_{ij}q_{i,t}q_{j,t}$$

Next, we identify $\zeta/L$ using the following relationship in our model:

$$\frac{\zeta}{L}\mathbf{q}_t^T J \mathbf{q}_t = \text{total production cost at time } t.$$

We calculate the total production cost as the sum of the production costs for all firms in our sample. Note that in our model, $\zeta$ and $L$ cannot be separately identified. Finally, we identify the knowledge capital of firms $\mathbf{z}_t$ using (18).

## 3.5 Law of Motion of Knowledge Capital

Using each firm's identified knowledge capital, we estimate the law of motion for knowledge capital in our model and obtain the parameter values associated with the level of technology spillover, $\beta$, and its volatility $\gamma$. Specifically, we estimate the following regression equation.

$$z_{i,t+1} - z_{i,t} = \beta \sum_{j \neq i} \tilde{\omega}_{ij,t} z_{j,t} + \text{Year FE}_t + \epsilon_{i,t} \tag{34}$$

where $z_{i,t}$ is the knowledge capital of firm $i$ at time $t$, $\tilde{\omega}_{ij,t}$ is the technology proximity between firm $i$ and firm $j$ at time $t$. Note that the term $\sum_{j \neq i} \tilde{\omega}_{ij,t} z_{j,t}$ does not include firm $i$'s

Table 4: Estimation of the Law of Motion – Jaffe Measure, Group

| | (1) | (2) | (3) |
|---|---|---|---|
| Dependent Variable: | $z_{i,t+1} - z_{i,t}$ | $z_{i,t+1} - z_{i,t}$ | $z_{i,t+1} - z_{i,t}$ |
| $\beta$ | 0.000191*** | 0.0000152*** | 0.000140*** |
| | (0.000035) | (0.000035) | (0.000039) |
| $\hat{\mu}$ | | .037** | |
| | | (0.021) | |
| Year Fixed Effects | ✓ | ✓ | ✓ |
| IV | | | ✓ |
| IV 1st Stage F-statistics | | | 4176 |
| Patent Classification | Group | Group | Group |
| Technological Proximity | Jaffe | Jaffe | Jaffe |
| No. observations | 16,324 | 15,173 | 14,181 |

SEs clustered by years and 4-digit naics industries are reported in parentheses.
* $p < 0.1$, **$p < 0.05$, *** $p < 0.01$.

Table 5: Estimation of the Law of Motion – Mahalanobis Measure, Subclass

| | (1) | (2) | (3) |
|---|---|---|---|
| Dependent Variable: | $z_{i,t+1} - z_{i,t}$ | $z_{i,t+1} - z_{i,t}$ | $z_{i,t+1} - z_{i,t}$ |
| $\beta$ | 0.000030*** | 0.000019*** | 0.000019*** |
| | (0.000006) | (0.000006) | (.000007) |
| $\hat{\mu}$ | | 0.055104*** | |
| | | (0.020619) | |
| Year Fixed Effects | ✓ | ✓ | ✓ |
| IV | | | ✓ |
| IV 1st Stage F-statistics | | | 6402 |
| Patent Classification | Subclass | Subclass | Subclass |
| Technological Proximity | Mahalanobis | Mahalanobis | Mahalanobis |
| No. observations | 16,324 | 15,173 | 14,181 |

SEs clustered by years and 4-digit naics industries are reported in parentheses.
* $p < 0.1$, **$p < 0.05$, *** $p < 0.01$.

Table 6: Estimation of the Law of Motion – Mahalanobis Measure, Group

| | (1) | (2) | (3) |
|---|---|---|---|
| Dependent Variable: | $z_{i,t+1} - z_{i,t}$ | $z_{i,t+1} - z_{i,t}$ | $z_{i,t+1} - z_{i,t}$ |
| $\beta$ | 0.000038*** | 0.000020** | 0.000026*** |
| | (0.000010) | (0.000010) | (.000010) |
| $\hat{\mu}$ | | 0.053** | |
| | | (0.022) | |
| Year Fixed Effects | ✓ | ✓ | ✓ |
| IV | | | ✓ |
| IV 1st Stage F-statistics | | | 7301 |
| Patent Classification | Group | Group | Group |
| Technological Proximity | Mahalanobis | Mahalanobis | Mahalanobis |
| No. observations | 16,324 | 15,173 | 14,181 |

SEs clustered by years and 4-digit naics industries are reported in parentheses.
* $p < 0.1$, **$p < 0.05$, *** $p < 0.01$.

Table 7: Predicting Knowledge Capital Using R&D Tax Credits

| | (1) |
|---|---|
| Dependent Variable: | $z_{i,t}$ |
| User cost of R&D | −39.495*** |
| | (4.7044) |
| Year Fixed Effects | ✓ |
| No. observations | 12,947 |

SEs clustered by years and 4-digit naics industries are reported in parentheses.
* $p < 0.1$, **$p < 0.05$, *** $p < 0.01$.

own knowledge capital. Therefore, the serial correlation of knowledge capital does not affect the coefficient $\beta$.

In Table 3, column (1) presents the results of regression (34). The high knowledge capital of technologically proximate firms significantly contributes to growth in their own knowledge capital. Column (2) shows that the same result holds even after controlling for firms' R&D investments. Columns (1) and (2) of Table 4 present the results when the technological proximity is constructed using group-level patent classification, which is more granular than subclass-level classification. Columns (1) and (2) of Table 5 show the results when technological proximity is measured using the Mahalanobis Measure, which accounts for technological spillovers across different patent subclasses. Finally, Columns (1) and (2) of Table 6 present the results when the Mahalanobis Measure is constructed based on group-level patent classifications. For all specifications, we obtain robust results indicating the existence of technology spillovers from technologically proximate firms.

To address the endogeneity concern, we adopt an approach similar to Bloom et al. (2013) and Lucking et al. (2019), using tax-induced changes in the effective cost of R&D to create exogenous variations in innovation activities. This method builds upon Wilson (2009), who constructs state-by-year R&D tax-price data using state-level tax credits, depreciation allowances, and corporate income taxes. We implement an IV approach by first projecting knowledge capital onto the user cost of R&D. Table 7 demonstrates that the user cost of R&D are strong predictors of knowledge capital. In this first-stage regression, we control for year fixed effects. From this regression, we obtain the predicted knowledge capital from tax credits, denoted as $z_{j,t}^{TAX}$. In the 2SLS analysis, we construct an instrument for the endogenous variable $\sum_{j \neq i} \tilde{\omega}_{ij,t} z_{j,t}$ by replacing actual knowledge capital with the fitted values$z_{j,t}^{TAX}$. Column (3) of Table 3 displays the results when using the instrument $\sum_{j \neq i} \tilde{\omega}_{ij,t} z_{j,t}^{TAX}$. The IV strategy indicates that the main results remain qualitatively and quantitatively robust. As seen in column (3) of Tables 4 through 6, we obtain robust results even when using the Mahalanobis measure instead of the Jaffe measure or employing group-level patent classifications instead of subclass-level classifications.

We also estimate $\gamma$, the parameter that determines the magnitude of the idiosyncratic shock, using the results of this regression analysis. Specifically, we estimate $\gamma$ by regressing the squared residuals $\hat{\epsilon}_{i,t}^2$, which are considered an approximation of the variance of the error term, on the square of knowledge capital $z_{j,t}^2$. We select column 3 of Table 4, which presents the estimation results using the instrument constructed with group-level patent classification and the Jaffe measure, as our baseline, and set the model parameters to

$\beta = 0.00014$ and $\gamma = 0.027$.

## 3.6 Discount Rate and R&D Coefficient

The remaining parameters that need to be estimated are the discount rate $\rho$ and R&D coefficient $\mu$. We set the discount rate $\rho$ to 10%. While this value is high compared to the observed real return on safe assets (often less than 5%), it is low compared to the private return on R&D estimated in the literature. For example, Bloom et al. (2013) find the private return on R&D to be more than 20%.

We internally estimate the R&D coefficient $\mu$ to align with the observed economic growth rate. Specifically, we set $\mu$ such that the economic growth rate generated by the model matches the average growth rate of GDP per capita in the United States between 1988 and 2015, which is 1.6%.

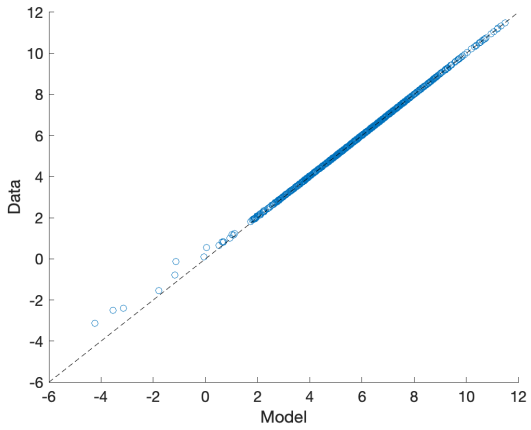
Another way to estimate $\mu$ is to simultaneously estimate it with $\beta$ by estimating the law of motion. However, if only a certain proportion of the actual expenses used for R&D is observed in financial statements, the coefficient estimated using the above regression equation would have an upward bias. Therefore, we do not use the regression coefficient to estimate the model parameters, but rather estimate $\mu$ to match the economic growth rate. In fact, we find that the estimated value of mu obtained from the estimated law of motion (0.053 to 0.056) is slightly higher than the estimated value of $\mu$ obtained through internal estimation to match the economic growth rate (0.045).

## 3.7 Validation

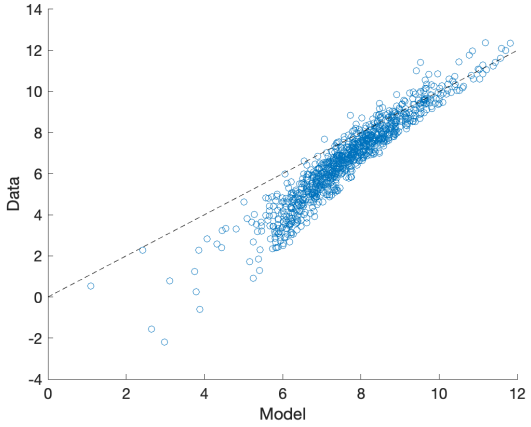
Next, we discuss the goodness of fit of the estimated model for both target and non-target moments. Figure 5 plots firm-level profit, sales, and R&D expenditure from the data and the model. As previously mentioned, the model is calibrated to match the firm-level profit observed in the data. As shown in Figure 5 (a), the model-generated firm-level profit almost perfectly matches the data. Figure 5 (b) plots firm-level sales from both the data and the model, while Figure 5 (c) plots firm-level R&D expenditure from both sources. Note that no information regarding firms' R&D investments was used in the model estimation. The correlation between model-generated sales and Compustat sales is 0.88, while the correlation between model-generated R&D expenditure and Compustat R&D expenditure is 0.86.

Figure 5: Firm-Level Profits, Sales and R&D Investments: Model vs. Data

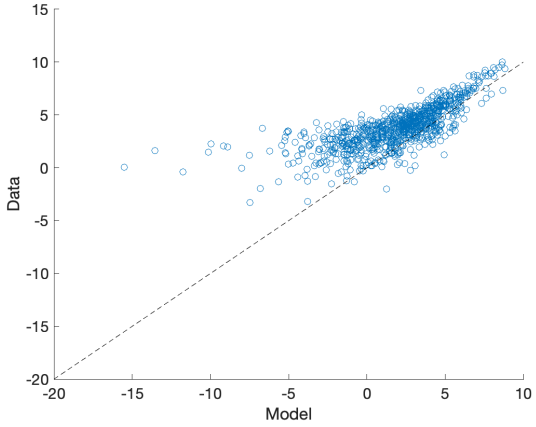
(a) log (Profits)




(b) log (Sales)




(c) log (R&D Expenditures)




Note: These figures plot firm-level profit, sales, and R&D expenditure from the model and Compustat data. Firms' profits in the data are measured as "Revenue – Total" (revt) minus production costs, represented by "Cost of Goods Sold" (cogs) in the Compustat database. The model is calibrated so that firm-level profits in the model matches those in the data.

Table 8: Growth Decomposition

| Oligopolistic Schumpeterian growth model | Computation time ($n$ firms) | Number of firms | Space of productivity |
|---|---|---|---|
| Cavenaile et al. (2023) | $O(2^n)$ | 4 | Six grids |
| Our model | $O\left(n^4\right)$ | Around 1000 | Continuous |

# 4 Numerical Analysis

We conduct several numerical exercises using the estimated model. First, we decompose the contributions of R&D and knowledge spillovers to economic growth. Next, in the counterfactual analysis, we find that a more concentrated ownership structure reduces both the economic growth rate and social welfare.

## 4.1 Algorithms and Computation Speed

To solve the competitive equilibrium, we solve the system of the Riccati equations backwards. Specifically, given $\begin{bmatrix} X_\tau^1 & \cdots & X_\tau^n \end{bmatrix}$, update $\begin{bmatrix} X_{\tau-\Delta}^1 & \cdots & X_{\tau-\Delta}^n \end{bmatrix}$ by

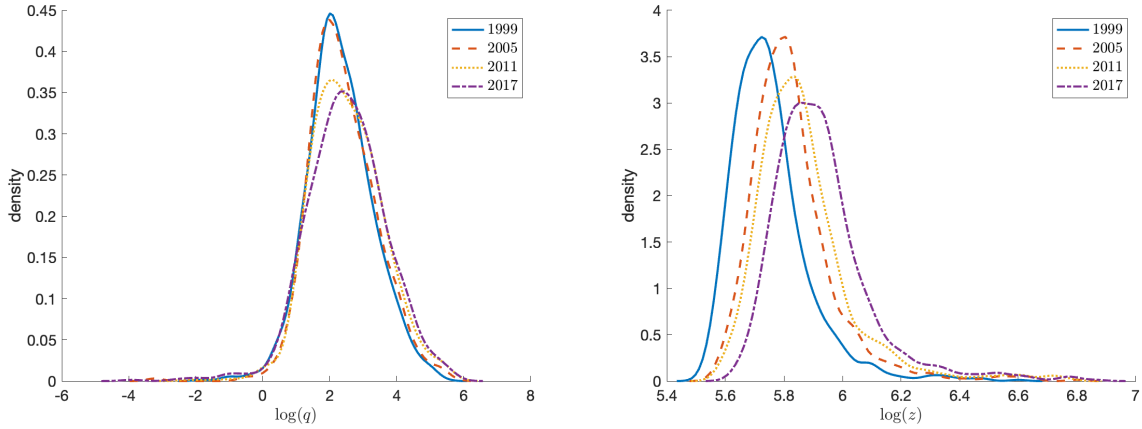
$$-\frac{X_\tau^i - X_{\tau-\Delta}^i}{\Delta} = Q^i - \mu^2 \sum_j \kappa_{ij} X_{j,\tau}^j \left(X_{j,\tau}^j\right)^T + \left(\Phi_\tau - \frac{1}{2}\left(\rho - \gamma^2\right) I\right)^T X_\tau^i + X_\tau^i \left(\Phi_\tau - \frac{1}{2}\left(\rho - \gamma^2\right) I\right)$$

When the number of firms is $n$, each of the $n$ firms has an $n \times n$ matrix of undetermined coefficients $X^i$. The number of our sample of publicly traded patenting firms in the US ranges from 903 to 1,123, depending on the year. When $n = 1000$, the total number of undetermined coefficients becomes $1000^3 \simeq 1$ billion. Nevertheless, by utilizing an efficient vectorized algorithm, we can solve the problem in approximately a few minute on a laptop computer.

Within the framework of existing endogenous economic growth models, solving the dynamic oligopoly problem involving all publicly traded companies with patents in the United States—approximately 1,000 firms—within a realistic timeframe is challenging. When considering a dynamic oligopoly with $n$ firms, the state variables typically include the productivity of each firm, leading to a computational complexity of $O(2^n)$. For instance, Cavenaile et al. (2023) introduce dynamic oligopoly into Schumpeterian growth model, similar to ours. However, due to the curse of dimensionality, they analyze oligopolies with a maximum of four firms. In contrast, our model has a computational complexity of order $O\left(n^4\right)$, allowing us to analyze markets with approximately 1,000 firms.

Figure 6: Distributions of Knowledge Capital and Quantity

(a) Log of Quantity $\log \mathbf{q}_t$ (b) Log of Knowledge Capital $\log \mathbf{z}_t$




Note: This figure illustrates the distributions of the estimated quantity $\log \mathbf{q}_t$ and knowledge capital $\log \mathbf{z}_t$. The distributions for every six years (1999, 2005, 2011, and 2017) are shown.

Furthermore, while Cavenaile et al. (2023)'s model uses only six discrete grids for productivity, our model allows for a continuous state space.

## 4.2 Distribution of Knowledge Capital and Quantity

Using the estimated model, we first examine the distribution of firms' knowledge capital, $\mathbf{z}_t$, and output, $\mathbf{q}_t$, as well as how these distributions change over time.

Figure 6(a) shows the distribution of output. We observe that the variance of this distribution tends to increase over time, suggesting that market concentration has been rising within our sample. This observation is consistent with Autor et al. (2020), who report increasing firm concentration across various industries.

Figure 6(b) presents the estimated distribution of knowledge capital. The distribution shifts to the right over time, which aligns with technological progress. Moreover, the variance of the knowledge capital distribution also tends to increase, indicating that knowledge capital has become more concentrated among certain firms[4].

[4]Akcigit and Ates (2021, 2023) provide evidence of a decline in business dynamism and suggest that the diffusion of knowledge from technologically advanced firms to follower firms has become less effective.

## 4.3 Social and Private Marginal Return on R&D

Next, we compare the private marginal returns on R&D investment with the social marginal returns on R&D investment. The marginal return on private R&D investment is measured by the extent to which a firm's R&D investment increases the weighted value of its owner. This is given by the following equation[5]:

$$\text{Private marginal return on R\&D investment by firm } i = \mu \times \frac{d}{dz_i} V_{\mathbf{z}}^i(\mathbf{z}) = 2\mu \left(X_i^i\right)^T \mathbf{z}$$

Expressing this in vector form, we obtain:

$$\text{Private marginal return on R\&D investment } = 2\mu \tilde{X}\mathbf{z}.$$

Similarly, the social marginal return on R&D investment is measured by the increase in social welfare resulting from an additional unit of R&D investment by firm $i$:

$$\text{Social marginal return on R\&D investment } \mu \times \frac{d}{d\mathbf{z}} V_{\mathbf{z}}(\mathbf{z}) = 2\mu X\mathbf{z}.$$

The figure 7 plots the private marginal returns on R&D and the social marginal returns on R&D for each firm. The correlation between the predicted private marginal return and social marginal return on R&D investment from the model is high at 0.995. However, the social marginal return is more concentrated than the private one: for firms with high social and private returns on R&D investment, the social returns tend to exceed the private returns, whereas for firms with low social and private returns on R&D investment, the social returns tend to fall below the private returns.

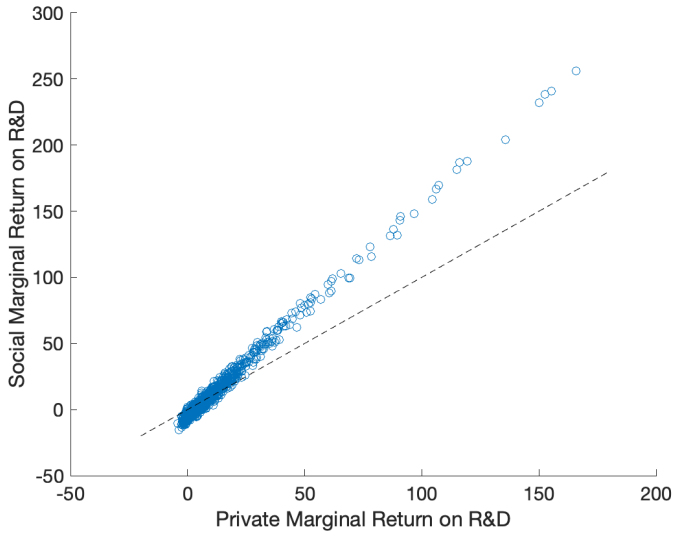
## 4.4 Growth Decomposition

We derive the formula that allows us to break it down into three terms: (i) knowledge spillovers, (ii) R&D, and (iii) a term related to shocks and the concavity of the production function derived from Ito's lemma.

Given the quadratic form of output (27) and the law of motion of knowledge capital

[5] In our model, where R&D investment entails a quadratic cost, the private marginal return to R&D, $2\mu \left(X_i^i\right)^T \mathbf{z}$ is equal to twice the amount of investment $2x_i$.

Figure 7: Private and Social Marginal Return on R&D Investment




Note: This figure plots the private and social marginal return on R&D for each firm.

Table 9: Growth Decomposition

| Description | Growth rate in 2017 |
|---|---|
| Tech Spillover | 0.66% |
| R&D | 0.48% |
| Ito | 0.61% |
| Total (Expected) | 1.75% |

Note: In this table, the contribution to economic growth is decomposed into three components—technology spillover, R&D, and the term arising from Itô's lemma—using equations (36) to (38). This calculation is based on the estimated three networks (ownership structure, technology spillover, and product market rivalry) as well as knowledge capital in 2017.

(25), we apply Ito's lemma to obtain the stochastic process of log of output:

$$d\log Y_t = \left[\frac{\mathbf{z}_t^T\left(Q\Phi+\Phi^TQ\right)\mathbf{z}_t}{\mathbf{z}_t^TQ\mathbf{z}_t}+\gamma^2\left\{\frac{\sum_i z_{i,t}^2Q_{ii}}{\mathbf{z}_t^TQ\mathbf{z}_t}-2\frac{\mathbf{z}_t^TQ\text{diag}\left(\mathbf{z}_t^2\right)Q\mathbf{z}_t}{\left(\mathbf{z}_t^TQ\mathbf{z}_t\right)^2}\right\}\right]dt+2\gamma\frac{\mathbf{z}_t^TQ\text{diag}\left(\mathbf{z}_t\right)}{\mathbf{z}_t^TQ\mathbf{z}_t}dW_t \tag{35}$$

(see Appendix A.8 for the derivation). Because technology transition matrix $\Phi$ consists of exogenous technology spillover matrix $\Omega$ and endogenous R&D term $\mu^2\tilde{X}$, we can decompose the first drift term further:

$$Q\Phi+\Phi^TQ=\left(Q\Omega+\Omega Q\right)+\mu^2\left(Q\tilde{X}+\tilde{X}^TQ\right)$$

Given this observation, let define the contribution of technology spillover, R&D, and Ito term on expected economic growth rate as:

$$\frac{d\log Y_t|_{\text{spillover}}}{dt}=\frac{\mathbf{z}_t^T\left(Q\Omega+\Omega Q\right)\mathbf{z}_t}{\mathbf{z}_t^TQ\mathbf{z}_t} \tag{36}$$

$$\frac{d\log Y_t|_{\text{R\&D}}}{dt}=\mu^2\frac{\mathbf{z}_t^T\left(Q\tilde{X}+\tilde{X}^TQ\right)\mathbf{z}_t}{\mathbf{z}_t^TQ\mathbf{z}_t} \tag{37}$$

$$\frac{d\log Y_t|_{\text{Ito}}}{dt}=\gamma^2\left\{\frac{\sum_i z_{i,t}^2Q_{ii}}{\mathbf{z}_t^TQ\mathbf{z}_t}-2\frac{\mathbf{z}_t^TQ\text{diag}\left(\mathbf{z}_t^2\right)Q\mathbf{z}_t}{\left(\mathbf{z}_t^TQ\mathbf{z}_t\right)^2}\right\} \tag{38}$$

In Table 9, we show the results of the growth decomposition. According to this decomposition, technology spillover contributes the most to economic growth, driving a 0.66% increase. Next, the impact of the term related to the variance of firms' idiosyncratic shocks, which can be derived from Ito's formula, is the second largest, driving a 0.60% increase in economic growth. This is because, in our model, aggregate output is a concave function of knowledge capital, and by Jensen's inequality, higher idiosyncratic shocks faced by firms promote greater economic growth. Finally, R&D contributes to a 0.48% increase in economic growth.

## 4.5 Impact of Different Ownership Structure

In this section, we compare the values of total R&D investment, total output, expected growth rate, and expected social welfare under different ownership structures. The estimation of the three networks—ownership structure, technology spillover, and product

market rivalry—in this section is based on data in the United States in 2017. The expected economic growth rate is derived from the drift term in equation (35) using the observed distribution of knowledge capital. Similarly, expected social welfare is computed based on the observed distribution of knowledge capital.

In the counterfactual analysis, we examine three ownership structure: monopoly, common ownership (baseline) and dispersed ownership. Furthermore, we consider not only cases where production and R&D investments are conducted under the same ownership structure, but also cases where production and R&D are conducted under different ownership structures. In any cases, the static problem is solved given the distribution of knowledge capital, and the dynamic problem of R&D is solved given the static strategy profiles. This comparison allows us to deepen our understanding of the impact of differences in the competitive environments of production and R&D.

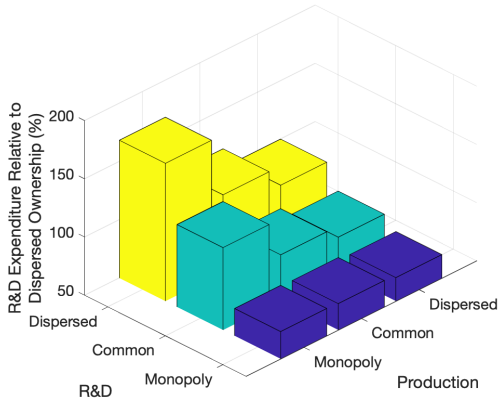
Figure 8(a) shows the relationship between firms' ownership structures and the total amount of R&D investment. As the ownership structure in production becomes more concentrated—that is, moving from dispersed ownership to common ownership and then to monopoly—R&D investment increases. This occurs because the appropriability of returns from innovation improves, thereby encouraging more R&D spending. In contrast, as the ownership structure in R&D becomes more concentrated, R&D investment declines. When R&D ownership is concentrated, firms tend to internalize inter-firm externalities—specifically, the business stealing effect and the technology spillover effect—more effectively. Our results suggest that the impact of internalizing the business stealing effect is greater than that of internalizing the technology spillover effect.

Since an increase in the total amount of R&D investment raises the expected economic growth rate, Figures 8(a) and 8(c) exhibit similar trends. In other words, as the ownership structure in production becomes more concentrated, the expected economic growth rate increases; conversely, as the ownership structure in R&D becomes more concentrated, the economic growth rate decreases. When the ownership structure of production changes from dispersed ownership to common ownership, the economic growth rate increases by 0.03 percentage points. On the other hand, when the ownership structure of R&D changes to common ownership, the economic growth rate decreases by 0.08 percentage points.

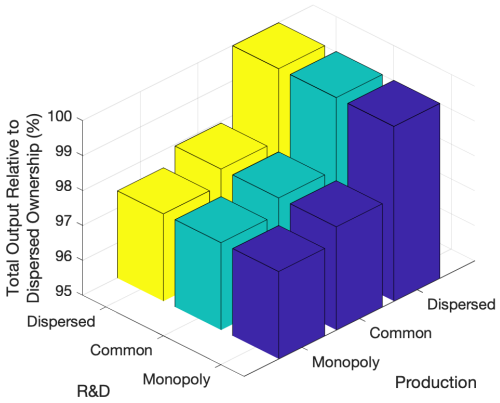
Figure 8(b) shows the total output under various ownership structures. Assuming the distribution of knowledge capital is given, output does not depend on the ownership structure of R&D. When the ownership structure of production changes from dispersed ownership to common ownership, production decreases by approximately 2.3%. In 2017

Figure 8: Impact of Different Ownership Structures for Production and R&D Investment.

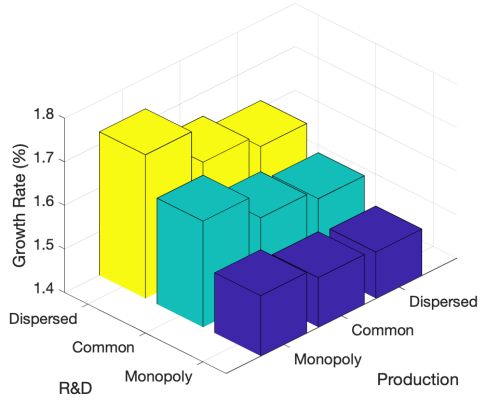
(a) R&D Expenditure



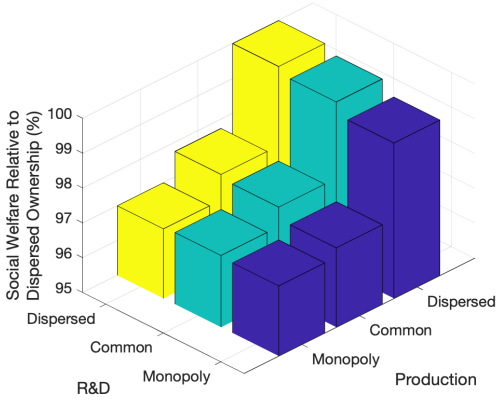

(b) Total Output




(c) Expected Economic Growth Rate




(d) Expected Social Welfare




Note: The top-left figure plots the expected economic growth rate. The top-right figure shows the economic growth rate driven by R&D. The bottom-left figure illustrates the economic growth rate attributable to technology spillover. The bottom-right figure depicts the transition of the expected growth rate arising from idiosyncratic shocks and the convexity of the aggregate production function. Each figure presents different cases for the ownership structures of production and R&D: monopoly, common ownership, and dispersed ownership.

economy, where common ownership had become widespread, there was little difference in output between production under common ownership and production under monopoly. Note that in our model, the supply of production labor is inelastic, and the aggregate market power itself does not affect production. The variation in production due to different ownership structures arises from misallocation caused by heterogeneity in market power across firms.

Figure 8(b), which shows total output, and Figure 8(d), which shows expected social welfare, exhibit similar patterns. This indicates that social welfare largely depends on the static misallocation in the product market resulting from the ownership structure of products. On the other hand, the impact of the R&D ownership structure on social welfare is ambiguous. The concentration of ownership in R&D firms promotes the internalization of both the negative externality of R&D (the business-stealing effect) and the positive externality (the technology spillover effect). This improves the allocation of R&D investment across firms, generating a positive effect on social welfare. On the other hand, when the ownership structure in R&D becomes concentrated, internalizing the business stealing effect leads to an overall underinvestment in R&D. Since these two effects offset each other, the impact of the ownership structure in R&D on social welfare remains ambiguous.

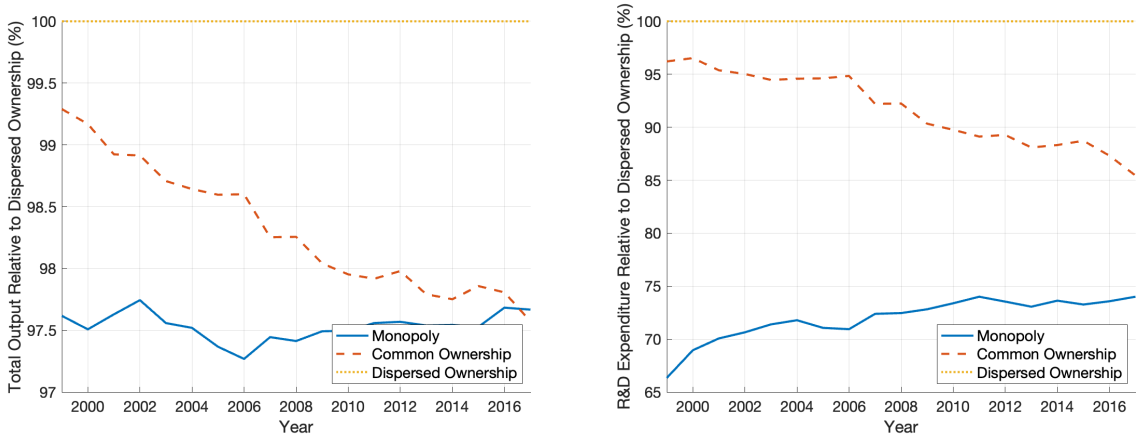
## 4.6 Time Trends under Different Ownership Structure

Using our dataset, we estimate firm networks—including ownership structure, production networks, and technology spillovers—and measure each firm's knowledge capital for every year from 1999 to 2017. Incorporating these estimates into our framework, we compute the equilibrium outcomes such as the expected economic growth rate and social welfare annually. This approach enables us to quantify the impact that changes in ownership structure over time have had on both economic growth and social welfare. Figure 9(a) illustrates the evolution of aggregate output under three scenarios: monopoly, common ownership, and dispersed ownership. In most periods, aggregate output is highest under dispersed ownership, followed by common ownership and then monopoly. Moreover, although production under common ownership initially resembles that under dispersed ownership, it gradually converges toward the production level observed under monopoly by the end of the sample period, reflecting the increasing prevalence of common ownership.

Figure 9(b) shows the total R&D investment across the same three scenarios. Throughout the sample period, the total R&D investment is highest under dispersed ownership,

Figure 9: Total Output and R&D Expenditure

(a) Total Output $Y$ (b) Total R&D Expenditure $\mathbf{x}_t^T \mathbf{x}_t$




Note: Figure (a) shows the evolution of output $Y$ from 1999 to 2017 as defined in the model in Section 2, while Figure (b) illustrates the evolution of the total R&D expenditure $\mathbf{x}_t^T \mathbf{x}_t$ over the same period. Each figure presents different cases for the ownership structures of production and R&D: monopoly, common ownership, dispersed ownership. In both graphs, the values are normalized to 1 under the dispersed ownership structure.

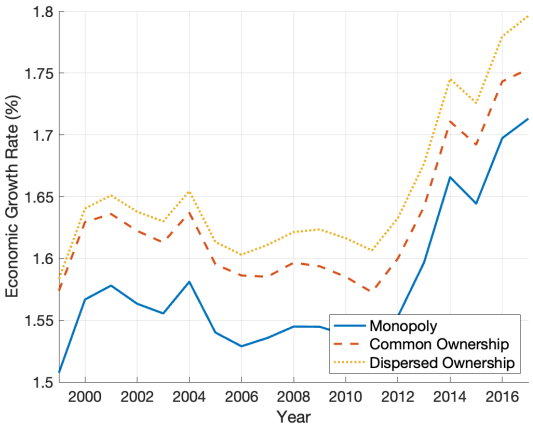
followed by common ownership and then monopoly. Similar to total output, the total R&D expenditure under common ownership starts off similar to that under dispersed ownership but shifts to an intermediate level between monopoly and dispersed ownership by the end of the sample period.

Figure 10 (a) plots the evolution of the expected economic growth rate, while Figures 10 (b), (c), and (d) respectively decompose this rate into contributions from R&D, technology spillovers, and the component arising from Ito's lemma, respectively, as described in Section 4.4. Across the entire period, the expected economic growth rate and its components are highest under dispersed ownership, followed by common ownership and then monopoly. Under common ownership, these values tend to converge toward those observed under monopoly, deviating from the levels under dispersed ownership. By 2017, the economic growth rate under common ownership was 0.04 percentage points lower than that under dispersed ownership, while the rate under monopoly was 0.08 percentage points lower than that under dispersed ownership.

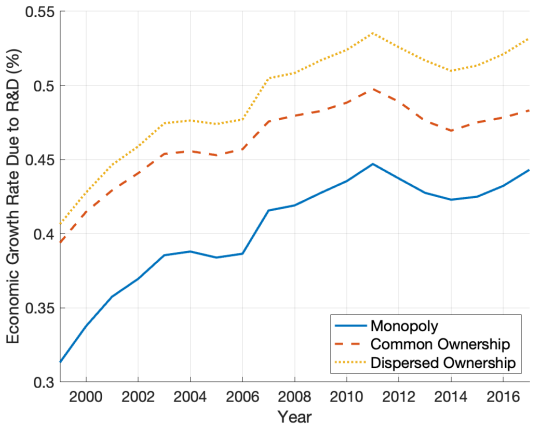
Figure 11(a) depicts the evolution of social welfare, which consistently follows the ranking: dispersed ownership, common ownership, then monopoly. As of 2017, social welfare under common ownership is approximately 2.8% lower than under the equilibrium associated with dispersed ownership. Figure 11(b) displays the proportion of total

Figure 10: Growth Rates

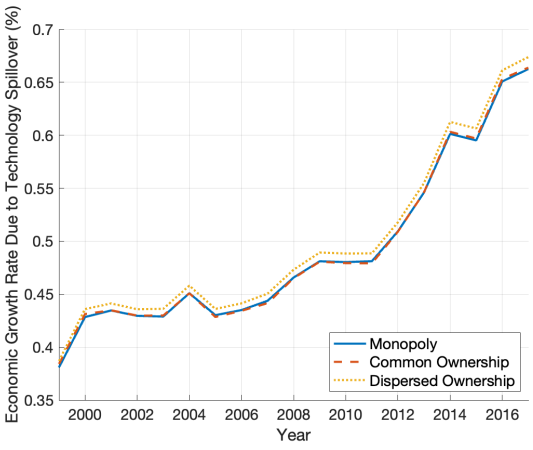
(a) Expected Economic Growth Rates



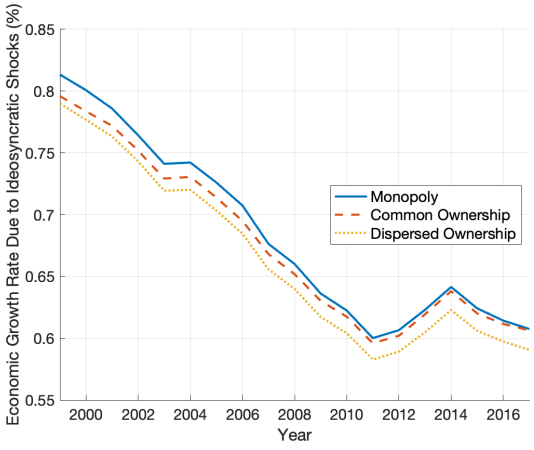

(b) Growth Rates due to R&D




(c) Growth Rates due to Technology Spillover



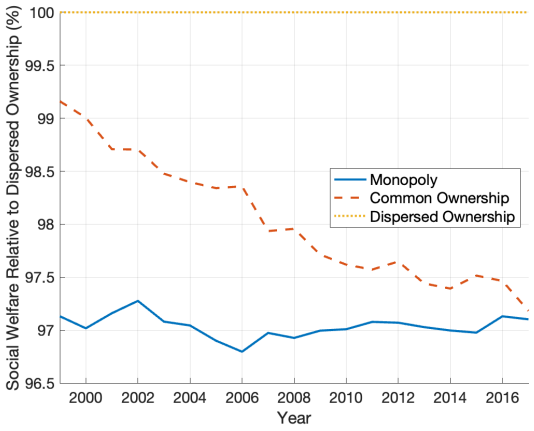

(d) Expected Growth Rates due to Ito Term



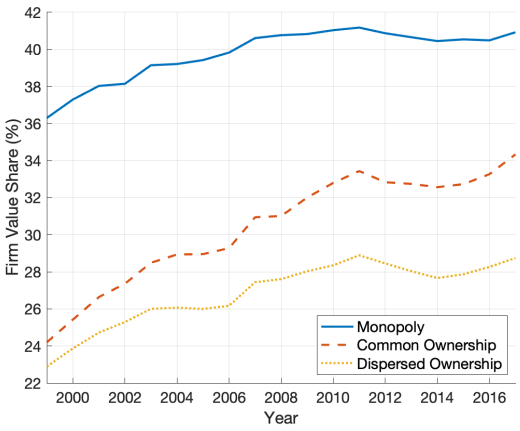

Note: Figure (a) plots the expected economic growth rate. Figure (b) shows the economic growth rate driven by R&D. Figure (c) illustrates the economic growth rate attributable to technology spillover. Figure (d) depicts the transition of the expected growth rate arising from idiosyncratic shocks and the convexity of the aggregate production function. Each figure presents different cases for the ownership structures of production and R&D: monopoly, common ownership, and dispersed ownership.

Figure 11: Expected Social Welfare and Firm Value Share

(a) Expected Social Welfare




(b) Firm Value Share




Note: Figure (a) plots the trend of social welfare from 1999 to 2017, while Figure (b) shows the trend of the total value of firms as a proportion of social welfare. Each figure presents different cases for the ownership structures of production and R&D: monopoly, common ownership, and dispersed ownership. In Figure (a), the values are normalized to 1 under the dispersed ownership structure.

firm value relative to overall social value. For instance, in 2017 the firm value share is about 40% under monopoly, 32% under common ownership, and 26% under dispersed ownership. These differences indicate that the ownership structure significantly affects the distribution of surplus.

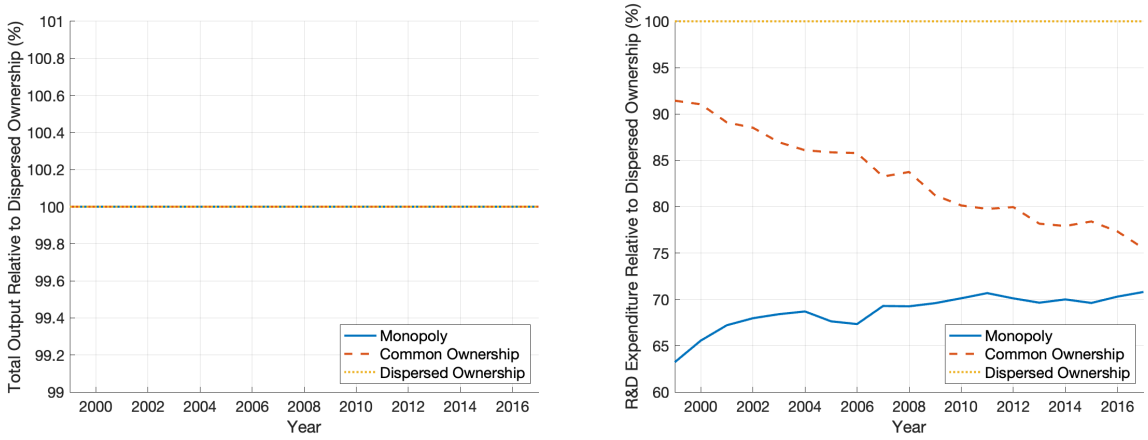
## 4.7 Time Trends under Different Ownership Structure for R&D

In the previous section, we considered the case where ownership of production and ownership of R&D were identical. In this section, to analyze the impact of ownership of R&D, we fix the ownership of production at dispersed ownership and focus on cases where only the ownership of R&D differs. Regarding the ownership structure of R&D, we again consider the three cases: monopoly, common ownership, and decentralized ownership.

Figure 12 illustrates the evolution of output from 1999 to 2017. Since the ownership structure of production is identical across all scenarios and the same knowledge capital, estimated annually, is applied, aggregate output remains consistent across scenarios. In contrast, the R&D expenditure differs among scenarios. During the sample period discussed in the previous section, the R&D expenditure under common ownership gradually diverges from that under dispersed ownership and incrementally approaches the R&D

Figure 12: Total Output and R&D Expenditure – Different Ownership Structure for R&D

(a) Total Output $Y$ (b) Total R&D Expenditure $\mathbf{x}_t^T \mathbf{x}_t$



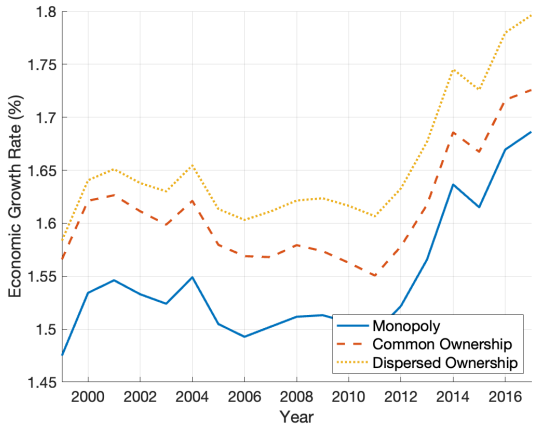

Note: Figure (a) shows the evolution of output $Y$ from 1999 to 2017 as defined in the model in Section 2, while Figure (b) illustrates the evolution of the total R&D expenditure $\mathbf{x}_t^T \mathbf{x}_t$ over the same period. Each line represents a case where the ownership structure of production is fixed as dispersed ownership, while the ownership structure of R&D varies. In both graphs, the values are normalized to 1 under the dispersed ownership structure.

expenditure under monopoly.

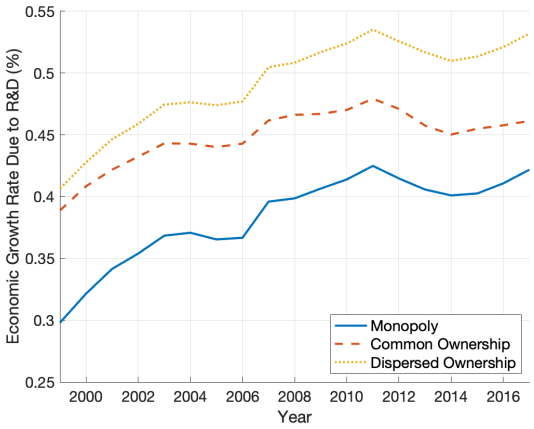
Figure 13 (a) plots the evolution of the expected economic growth rate. Figures 13 (b), (c), and (d) respectively plot the growth rates attributed to R&D, technology spillovers, and the component arising from Ito's lemma, decomposed according to the formula in Section 4.4. When the ownership structure of production is fixed, the contributions of technology spillovers to the growth rate and the contribution of the term related to Ito's lemma to the growth rate remain the same across all scenarios. This is because, as seen in the equations (36) and (38) , these terms depend solely on the knowledge capital $\mathbf{z}_t$, the matrix $Q$ determined by the static problem, and the parameter values. Therefore, differences in the economic growth rate arise only from the contribution to the growth rate of R&D. According to Figure 13 (a), when the ownership structure of production is fixed at dispersed ownership and the ownership structure of R&D changes from dispersed ownership to common ownership, the economic growth rate declines by 0.07 percentage points. When the ownership structure of R&D changes from dispersed ownership to monopoly, the economic growth rate declines by 0.12 percentage points. The magnitude of these changes is larger than the results in the previous section, where both the ownership structure of production and R&D were altered. This is because concentrating ownership structure over production increases the appropriability of returns on innovation and

Figure 13: Growth Rates – Different Ownership Structure for R&D

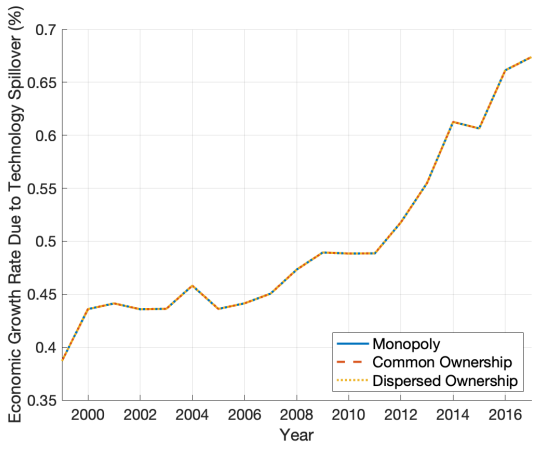
(a) Total Economic Growth Rates



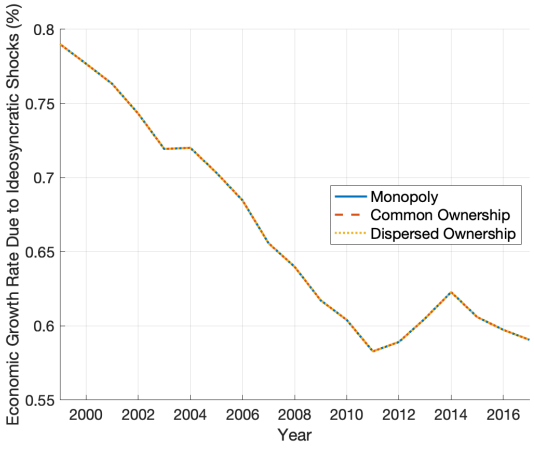

(b) Growth Rates due to R&D




(c) Growth Rates due to Technology Spillover



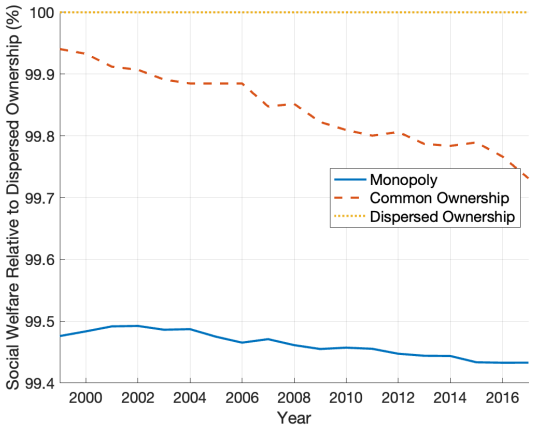

(d) Expected Growth Rates due to Ito Term



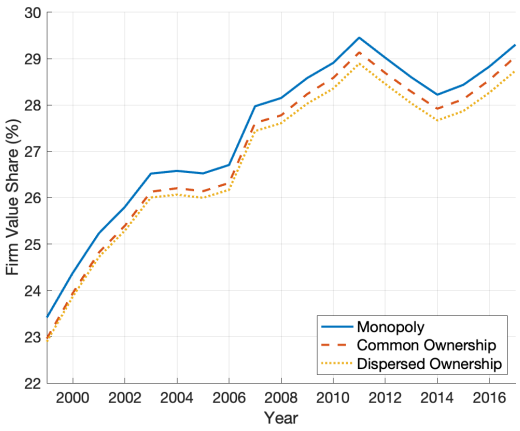

Note: Figure (a) plots the expected economic growth rate. Figure (b) shows the economic growth rate driven by R&D. Figure (c) illustrates the economic growth rate attributable to technology spillover. Figure (d) depicts the transition of the expected growth rate arising from idiosyncratic shocks and the convexity of the aggregate production function. Each line represents a case where the ownership structure of production is fixed as dispersed ownership, while the ownership structure of R&D varies.

Figure 14: Social Welfare and Firm Value Share – Different Ownership Structure for R&D

(a) Expected Social Welfare




(b) Firm Value Share




Note: Figure (a) plots the trend of social welfare from 1999 to 2017, while Figure (b) shows the trend of the total value of firms as a proportion of social welfare. Each line represents a case where the ownership structure of production is fixed as dispersed ownership, while the ownership structure of R&D varies. In Figure (a), the values are normalized to 1 under the dispersed ownership structure.

thereby it has the opposite effect of concentrating ownership structure over R&D on firms' R&D investment.

Figure 14(a) plots the social welfare under each scenario. As of 2017, the welfare loss under common ownership, compared to the equilibrium under dispersed ownership, is approximately 0.28%. Figure 11(b) shows the proportion of the total value of firms relative to social value. As the concentration of the ownership structure increases, the total value of firms becomes larger.

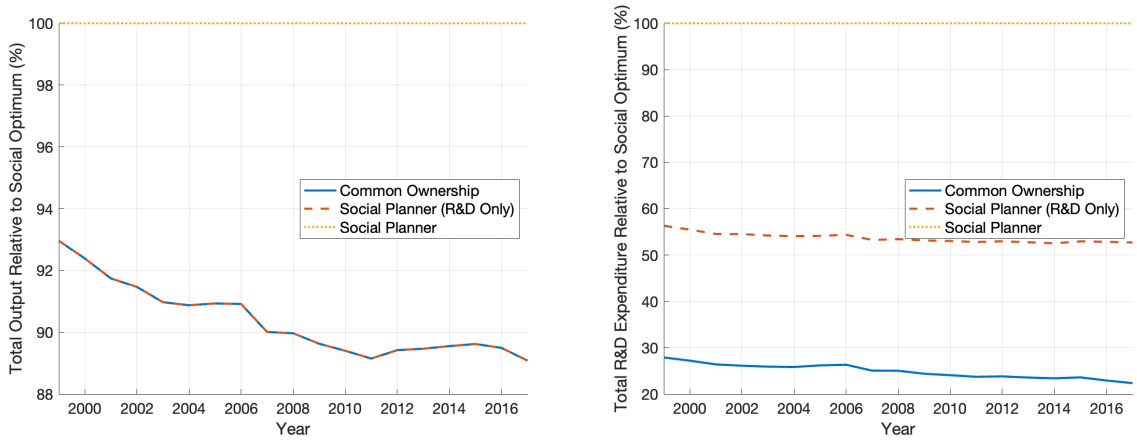
## 4.8 Social Optimum

In this section, we compare the allocation chosen by a benevolent social planner who maximizes social welfare with the allocation in a competitive equilibrium. We consider two types of social planner problems. In the first case, given a static competitive equilibrium in the product market, the social planner determines only the dynamic R&D choices. The second case allows the planner to choose both the production quantity of goods and the level of R&D investment.

Figure 15(a) illustrates the evolution of total output under competitive equilibrium and the social planner's allocation from 1999 to 2017. Given the observed identical distribution of knowledge capital, the output under the planner's problem where production takes

Figure 15: Total Output and R&D Expenditure – Social Optimum

(a) Total Output $Y$ (b) Total R&D Expenditure $\mathbf{x}_t^T \mathbf{x}_t$




Note: Figure (a) shows the evolution of output $Y$ from 1999 to 2017 as defined in the model in Section 2, while Figure (b) illustrates the evolution of the total R&D expenditure $\mathbf{x}_t^T \mathbf{x}_t$ over the same period. These figures plot three cases: (1) the case where the ownership structure of production and R&D follows common ownership, (2) the case where the social planner chooses the optimal R&D given production under common ownership, and (3) the case where the social planner selects both the optimal production and R&D. In both graphs, the values are normalized to 1 under the dispersed ownership structure.

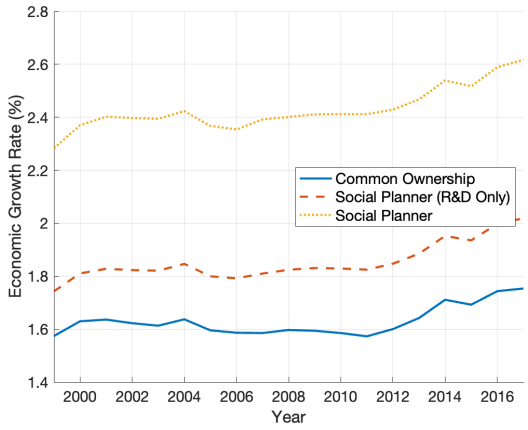
place under competitive equilibrium is identical to the output when both production and R&D occur under competitive equilibrium. When the planner can also determine production levels, the resolution of misallocation in production leads to an approximately 11% increase in output.

Figure 15(b) shows the evolution of total R&D expenditure. In a setting where production takes place under competitive equilibrium and the social planner chooses only the R&D allocation, the level of R&D expenditure is more than twice as high as in the case where both production and R&D are determined under competitive equilibrium.. When the planner also determine production levels, the socially optimal level of R&D increases further, reaching approximately four times the R&D expenditure observed under competitive equilibrium for both production and R&D.

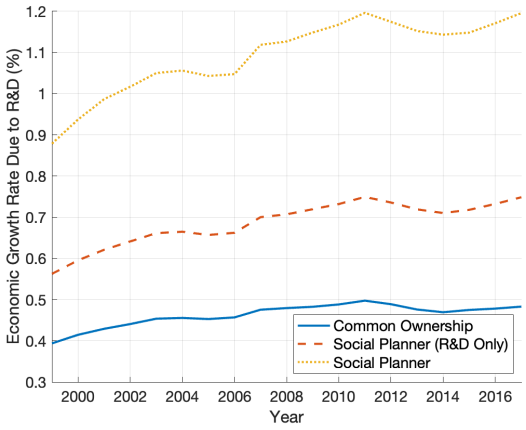
Figure 16 (a) plots the evolution of the expected economic growth rate. Figures 16 (b), (c), and (d) respectively plot the growth rates attributed to R&D, technology spillovers, and the component arising from Ito's lemma, decomposed according to the formula in Section 4.4. When comparing competitive equilibrium with a planner who selects only the allocation of R&D, the contributions of technology spillovers to the growth rate and the contribution of the term related to Ito's lemma to the growth rate remain the same.

Figure 16: Growth Rates – Social Optimum

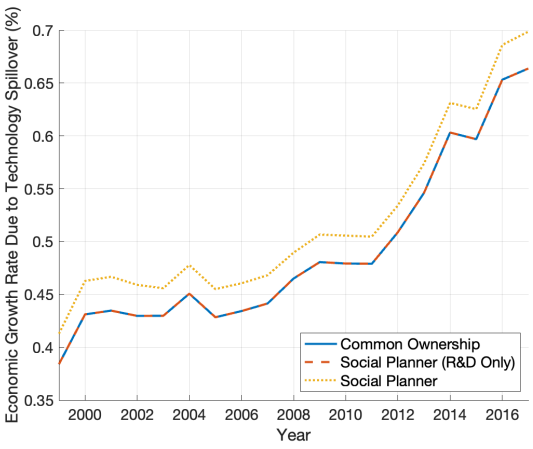
(a) Total Economic Growth Rates



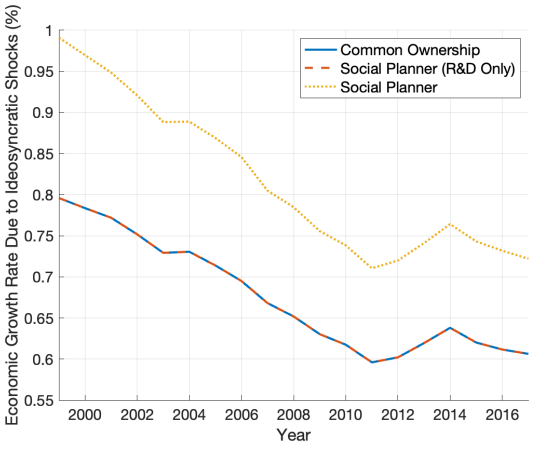

(b) Growth Rates due to R&D




(c) Growth Rates due to Technology Spillover



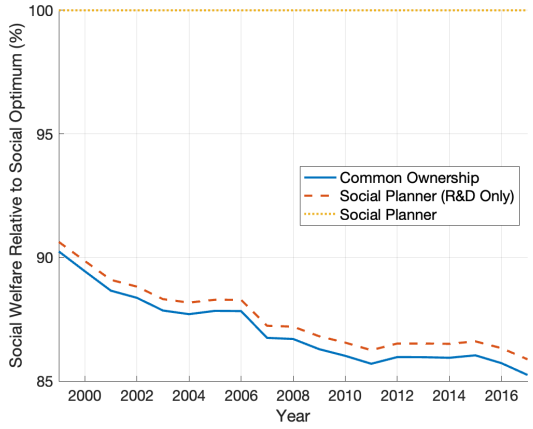

(d) Expected Growth Rates due to Ito Term



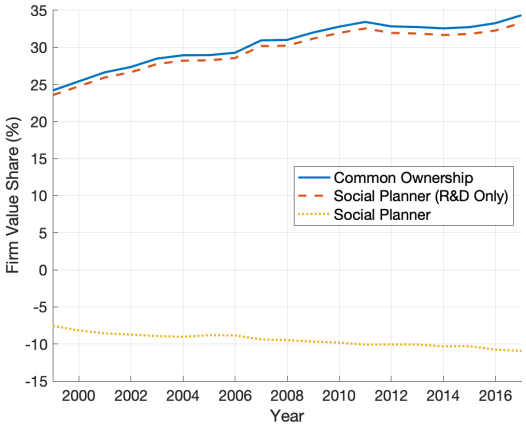

Note: Figure (a) plots the expected economic growth rate. Figure (b) shows the economic growth rate driven by R&D. Figure (c) illustrates the economic growth rate attributable to technology spillover. Figure (d) depicts the transition of the expected growth rate arising from idiosyncratic shocks and the convexity of the aggregate production function. These figures plot three cases: (1) the case where the ownership structure of production and R&D follows common ownership, (2) the case where the social planner chooses the optimal R&D given production under common ownership, and (3) the case where the social planner selects both the optimal production and R&D.

Figure 17: Expected Social Welfare and Firm Value Share – Social Optimum

(a) Expected Social Welfare




(b) Firm Value Share




Note: Figure (a) plots the trend of social welfare from 1999 to 2017, while Figure (b) shows the trend of the total value of firms as a proportion of social welfare. These figures plot three cases: (1) the case where the ownership structure of production and R&D follows common ownership, (2) the case where the social planner chooses the optimal R&D given production under common ownership, and (3) the case where the social planner selects both the optimal production and R&D. In Figure 13(a), the value of social welfare is normalized to 1 when the social planner chooses both the socially optimal level of production and R&D each year.

This is because, as seen in the equations (36) and (38) , these terms depend solely on the knowledge capital $\mathbf{z}_t$, the matrix $Q$ determined by the static problem, and the parameter values. Therefore, the difference in economic growth rates between these two cases arises only from the contribution to the growth rate of R&D. According to Figure 16(a), as of 2017, the difference between the economic growth rate under competitive equilibrium and that under a planner who chooses only R&D is 0.03 percentage points. The optimal economic growth rate when the planner selects both production and R&D is even higher, exceeding the economic growth rate under competitive equilibrium by 0.08 percentage points.

In Figure 17(a), we compare social welfare under competitive equilibrium and the allocations chosen by a social planner. The difference in social welfare between competitive equilibrium and the planner who only optimizes R&D allocation is approximately 0.8%. In contrast, the difference between competitive equilibrium and the planner who optimizes both production and R&D allocation is significantly larger, around 15%. As seen in Figure 15(a), where output increases by approximately 11%. This indicates that most of the welfare gain when the planner optimizes both production and R&D allocation comes

from the improvement in misallocation of production.

In Figure 17(b), we compare the share of firm value in social welfare under competitive equilibrium and the allocations chosen by a social planner. When the planner optimizes only R&D allocation, the share of firm value in social welfare is 1% to 2% lower compared to competitive equilibrium depending on the year. When the social planner also determines the static allocation, we assume that firms set prices at marginal cost. Under this assumption and the assumption of constant returns to scale production technology, the gross profit before deducting R&D expenditures becomes zero. Therefore, when the planner optimizes the allocation of production and R&D, the firm value becomes negative.

### 4.9 Uniform R&D Subsidy

In this section, we analyze the effects of a uniform subsidy for R&D. In addition, we compare the equilibrium under a uniform subsidy for R&D with the allocation chosen by a social planner who selects the optimal level of R&D to examine the effectiveness of a uniform subsidy for R&D.

Figure 18(a) illustrates the expected economic growth rate under various levels of a uniform subsidy for R&D. As the uniform subsidy increases from 0% to 40%, the expected growth rate rises by 0.36%. With a uniform 33% subsidy, the economic growth rate can closely match the level achieved under the social planner's optimal R&D allocation.

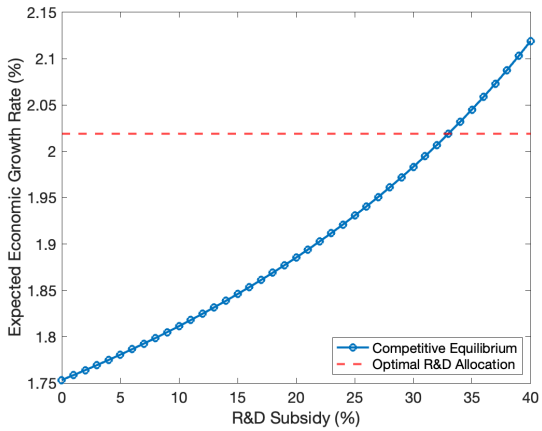
Figure 18(b) shows the expected social welfare under various levels of a uniform subsidy for R&D. In the figure, social welfare is expressed as a ratio relative to the welfare achieved when the planner selects the optimal level of R&D. Social welfare reaches its maximum at an R&D subsidy of approximately 33%. At an R&D subsidy of around 0%, the gap between the social welfare and that achieved by the planner's optimal R&D choice is about 0.73%, whereas at an R&D subsidy of 33%, the gap is less than 0.11%. Although a uniform subsidy cannot completely eliminate the misallocation of R&D, it is an effective policy tool for improving social welfare.

## 5 Alternative Models of Corporate Governance

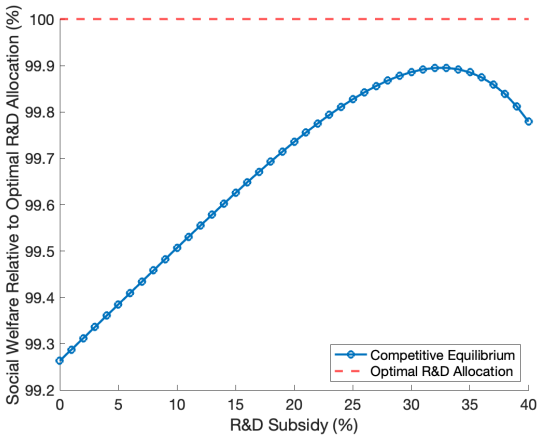
We examines various assumptions about corporate governance and calculates the resulting economic growth rates and social welfare under each assumption. Specifically, following Ederer and Pellegrino (2024), we focuses on four models: (1) super-proportional

Figure 18: Economic Growth Rate and Social Welfare under R&D Subsidy

(a) Expected Growth Rate




(b) Expected Social Welfare




Note: This figure plots the expected economic growth rate and expected social welfare under different levels of R&D subsidy. In the expected economic growth rate figure, the level of the expected economic growth rate under optimal R&D, given the static competitive equilibrium in the product market, is also plotted. In the expected social welfare figure, the consumption equivalent welfare loss, compared to the social welfare under optimal R&D (again, given the static competitive equilibrium in the product market), is plotted. The three networks (ownership structure, technology spillover, product market rivalry) and the distribution of knowledge capital are estimated based on data from 2017.

influence of large investors, (2) blockholder thresholds, (3) rational investor inattention, and (4) governance frictions and managerial entrenchment. By comparing these different scenarios, we analyzes the robustness of ours findings.

## 5.1 Super-Proportional Influence of Large Investors

Following Gilje et al. (2020) and Backus et al. (2021), we assume that the firm $i$ assigns weights $\gamma_{iz}s_{iz}$ to the profits of investor $z$, where $\gamma_{iz}$ captures how a large investor may exert greater influence than smaller ones. Then, the influence-adjusted common ownership weight $\tilde{\kappa}_{ij}$ is:

$$\tilde{\kappa}_{ij} = \frac{\sum_{z=1}^{Z} s_{iz}\gamma_{iz}s_{jz}}{\sum_{z=1}^{Z} s_{iz}\gamma_{iz}s_{iz}}.$$

Following Ederer and Pellegrino (2024), we set $\gamma_{iz} = \sqrt{s_{iz}}$. This adjustment amplifies the impact of large shareholders on firm decisions.

## 5.2 Blockholder Thresholds

Another way to capture the disproportionate control of large investors is to assume that only "blockholders" owning more than a certain amount of a firm's stock actively influence the firm's decisions. Consequently, the firm internalizes the effect on an investor's portfolio if and only if the investor $z$ is a blockholder of firm $i$. In our exercise, we assume a threshold equivalent to 5% of the firm's total stock (i.e., $s_{iz} > 5\%$). The modified common ownership weight is

$$\tilde{\kappa}_{ij} = \frac{\sum_{z=1}^{Z} s_{iz} b_{iz} s_{jz}}{\sum_{z=1}^{Z} s_{iz} s_{jz}} \quad (i \neq j),$$

where $b_{iz} = 1$ if $s_{iz} > 5\%$ and $b_{iz} = 0$ otherwise.

## 5.3 Rational Investor Inattention

Investors may incur monitoring costs when providing strategic advice to corporate management. If a firm constitutes a relatively small portion of an investor's overall portfolio, the investor is less likely to closely monitor the firm's behavior. Gilje et al. (2020) (GGL) estimate the probability that an investor votes against Institutional Shareholders Service (ISS) recommendations as a measure of the investor's level of attention. They find that the larger the weight of firm $i$ in an investor's portfolio, the more frequently the investor deviates from ISS guidance. We follow Ederer and Pellegrino (2024) and construct a common ownership weight adjusted for rational expectations using the estimation results of GGL, available on the WRDS platform. The data estimated by GGL is available up to 2012. For details, please refer to Section 5.3 of Ederer and Pellegrino (2024).

## 5.4 Governance Frictions and Managerial Entrenchment

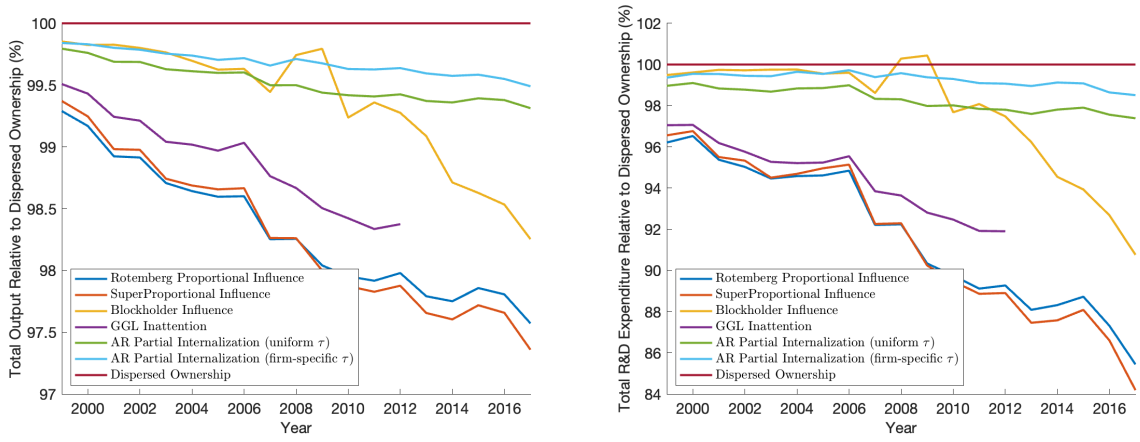
Corporate managers may not fully maximize shareholder value due to governance frictions and entrenchment. Azar and Ribeiro (2021) (AR) propose a flexible objective function in which the manager of firm $i$ discounts the profits of other firms by $\tau_i$ compared to the case of proportional influence:

$$\pi_i + \tau_i \sum_{j \neq i} \kappa_{ij} \pi_j$$

We analyze two specifications highlighted by AR. First, we estimate our model using a constant mitigation factor, $\tau = 0.29$, which is uniform across firms. Second, we employ the

Figure 19: Total Output and R&D – Corporate Governance Models

(a) Total Output $Y$ (b) Total R&D $\mathbf{x}_t^T \mathbf{x}_t$




Note: This figure plots total output and total R&D from 1999 to 2017 based on the proportional, superproportional, and blockholder influence model, the GGL inattention model, and AR partial internalization model. The values under dispersed ownership for each year are normalized to 1.

best-fitting structural specification in AR, where the mitigation factor varies across firms and over time, defined as:

$$\tau_{i,t} = \frac{\exp\left[\theta_0 + \log\left(\text{IHHI}_{i,t}\right)\right]}{1 + \exp\left[\theta_0 + \log\left(\text{IHHI}_{i,t}\right)\right]}$$

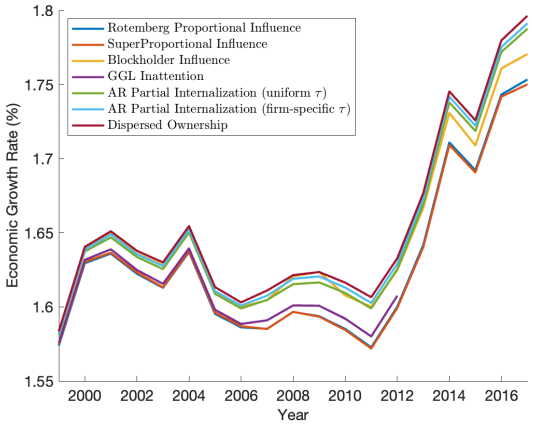
with $\theta_0 = 2.6844$, where $\text{IHHI}_{i,t} = \sum_{z=1}^{Z} s_{iz,t}^2$ is the investor Herfindahl concentration index.

## 5.5 Results

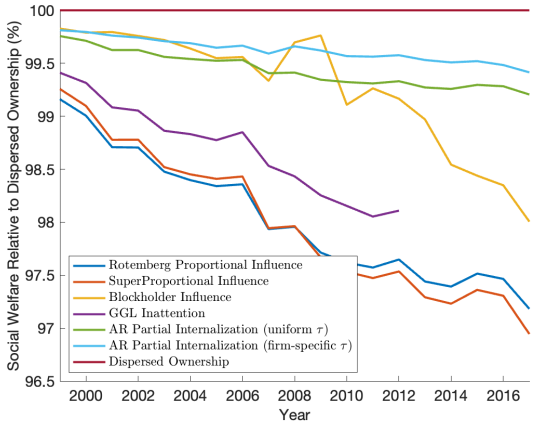
Now, we compare the results under these alternative governance assumptions with the benchmark case that assumes Rotemberg's (proportional) common ownership weights and dispersed ownership structure. Figures 19 through 20 show the time series of total output, total R&D, expected economic growth rates, and expected social welfare under common ownership for each governance assumption and dispersed ownership. Total output, R&D, and expected social welfare in these figures normalize the corresponding values under dispersed ownership to 1 for each year. Overall, the allocations achieved in equilibrium under common ownership with Rotemberg's proportional influence and under common ownership with superproportional influence are quite similar. In contrast, under common ownership where influence is limited to blockholders, the equilibrium

Figure 20: Economic Growth Rate and Social Welfare – Corporate Governance Models

(a) Expected Economic Growth Rate




(b) Expected Social Welfare




Note: This figure plots expected economic growth rate and social welfare from 1999 to 2017 based on the proportional, super-proportional, and blockholder influence model, the GGL inattention model, and AR partial internalization model. The social welfare under dispersed ownership for each year are normalized to 1.

allocation tends to move away from that under dispersed ownership and closer to that under common ownership with Rotemberg's proportional influence in the latter half of the sample period. The main reason for this is that, until around 2010, the largest asset management companies rarely exceeded the 5% blockholder threshold, but, by the end of the sample period, they had become the top shareholders in almost all public companies.

Under common ownership influenced by investors' rational inattention estimated using the GGL approach, the equilibrium allocation generally lies somewhere between (or closer to) the allocation under Rotemberg's proportional influence and that under dispersed ownership. Finally, among all the corporate governance models we examine, the partial-internalization model in AR (using uniform or firm-specific attenuation parameters) produces an equilibrium allocation that is closest to that under dispersed ownership. Compared with dispersed ownership, the expected economic growth rate in 2017 is about 0.01 to 0.02 percentage points lower under this model than under dispersed ownership.

# 6 Conclusion

We develop an endogenous growth model that incorporates three inter-firm networks: ownership structures, product market competition, and technology spillovers. Using this

framework, we examine the impact of ownership structures on firms' R&D investment, economic growth, and social welfare. To quantify these effects, we estimate the model using data from approximately 1,000 U.S. publicly traded firms with patent holdings and solve the dynamic oligopoly problems among these firms. Our numerical analysis suggests that, in 2017, the observed level of common ownership reduced the expected economic growth rate by an estimated 0.08 percentage points compared to a counterfactual scenario with a fully dispersed ownership structure in R&D decisions. This result implies that, under common ownership, the internalization of business-stealing effects dominates that of technological spillovers. Furthermore, when a benevolent social planner takes product market competition as given and optimally allocates R&D investment, the socially optimal level of R&D is more than twice the equilibrium level, resulting in a 0.25 percentage point increase in the economic growth rate. Finally, we show that a uniform R&D subsidy of approximately 30% can reduce the consumption-equivalent welfare loss to below 0.01% relative to the social welfare achieved under the planner's optimal R&D allocation.

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# Appendix

## Appendix A Theoretical Appendix

### A.1 Derivation of Static Equilibrium

Inserting the optimality condition for technology choice (15) and (16) into the optimality condition for quantity choice (14), we obtain

$$z_{i,t} = 2\sqrt{\zeta \frac{w_t}{P_t}} + \sum_j \left(\sigma_{ij} + \sigma_{ij}\kappa_{ij}\right) q_{j,t}$$

Inserting the expression for real wage (17), we obtain

$$z_{i,t} = \sum_j \left(2\frac{\zeta}{L} + \sigma_{ij} + \sigma_{ij}\kappa_{ij}\right) q_{j,t}.$$

In vector form,

$$\mathbf{z}_t = \left(2\frac{\zeta}{L}J + \Sigma + K \circ \Sigma\right)\mathbf{q}_t.$$

Therefore, the quantities are expressed as linear functions of knowledge diffusion:

$$\mathbf{q}_t = N\mathbf{z}_t$$

where

$$N \equiv \left(2\frac{\zeta}{L}J + \Sigma + K \circ \Sigma\right)^{-1}.$$

### A.2 Derivation of Static Equilibrium

Substituting the FOC with respect to $q_i$ into the expression for the profit (13), we obtain

$$\pi_{i,t} = \sum_j \kappa_{ij}\sigma_{ij} q_{i,t} q_{j,t}$$

Therefore, the static objective function of firm $i$ is rewritten as:

$$\sum_j \kappa_{ij}\pi_{j,t} = \sum_h \kappa_{ih} \sum_j \kappa_{hj}\sigma_{hj}q_{h,t}q_{j,t} \tag{39}$$

From equation (18), we obtain

$$q_{h,t}q_{j,t} = \mathbf{z}_t^T \left( \frac{N_h^T N_j + N_j^T N_h}{2} \right) \mathbf{z}_t$$

Inserting the above equation into the firm's static objective function (39), we express the objective in a quadratic form of the firm's productivity vector $\mathbf{z}_t$:

$$\sum_j \kappa_{ij}\pi_{j,t} = \mathbf{z}^T Q^i \mathbf{z}$$

where

$$Q^i \equiv \sum_h \kappa_{ih} \sum_j \kappa_{hj}\sigma_{hj} \frac{N_h^T N_j + N_j^T N_h}{2}$$

## A.3 Derivation of Output

Let $\mathbf{1}$ denote an $n \times 1$ vector with all elements equal to 1. Rewrite (16) in vector form:

$$\mathbf{b}_t = \mathbf{z}_t - \sqrt{\zeta \frac{w_t}{P_t}} \mathbf{1}$$

Inserting (17), we obtain

$$\begin{aligned} \mathbf{b}_t &= \mathbf{z}_t - \frac{\zeta}{L} \left( \sum_i q_{i,t} \right) \mathbf{1} \\ &= \mathbf{z}_t - \frac{\zeta}{L} J \mathbf{q}_t. \end{aligned}$$

From (18), we obtain

$$\mathbf{b}_t = \left( \frac{\zeta}{L} J + \Sigma + K \circ \Sigma \right) \mathbf{q}_t$$

Therefore, from (4), the total output is given by

$$\begin{aligned}
Y_t &= \mathbf{q}_t^T \left(\frac{\zeta}{L} J + \Sigma + K \circ \Sigma\right) \mathbf{q}_t - \frac{1}{2} \mathbf{q}_t^T \Sigma \mathbf{q}_t \\
&= \mathbf{q}_t^T \left(\frac{\zeta}{L} J + \frac{1}{2}\Sigma + K \circ \Sigma\right) \mathbf{q}_t \\
&= \mathbf{z}_t^T N^T \left(\frac{\zeta}{L} J + \frac{1}{2}\Sigma + K \circ \Sigma\right) N \mathbf{z}_t \\
&= \frac{1}{2} \mathbf{z}_t^T N^T \left(2\frac{\zeta}{L} J + \Sigma + 2\left(K^T \circ \Sigma + K \circ \Sigma^T\right)\right) N \mathbf{z}_t \\
&= \mathbf{z}_t^T Q \mathbf{z}_t
\end{aligned}$$

where

$$Q \equiv \frac{1}{2} N^T \left(2\frac{\zeta}{L} J + \Sigma + 2\left(K^T \circ \Sigma + K \circ \Sigma^T\right)\right) N$$

## A.4 Derivation of the Solution of Social Planner's Static Problem

Defined the Lagrangian as

$$\mathcal{L} = \mathbf{q}_t^T \left(\mathbf{z}_t - \zeta \mathbf{a}_t\right) - \frac{1}{2} \mathbf{q}_t^T \Sigma \mathbf{q}_t - \lambda_t \left\{ \sum_i \frac{q_{i,t}}{a_{i,t}} - L \right\}$$

The FOC w.r.t. $a_{i,t}$ gives

$$a_{i,t} = \sqrt{\frac{\lambda_t}{\zeta}} \tag{40}$$

and therefore,

$$b_{i,t} = z_{i,t} - \sqrt{\zeta \lambda_t} \tag{41}$$

Inserting (40) into (29), we obtain

$$\sqrt{\zeta \lambda_t} = \frac{\zeta}{L} \sum_i q_{i,t} \tag{42}$$

Note that

$$\sqrt{\zeta \lambda_t} \mathbf{1} = \frac{\zeta}{L} \left( \sum_i q_{i,t} \right) \mathbf{1} = \frac{\zeta}{L} J \mathbf{q}_t. \tag{43}$$

The FOC w.r.t. $\mathbf{q}_t$ gives

$$\mathbf{z}_t - \zeta \mathbf{a}_t - \Sigma \mathbf{q}_t = \lambda_t \begin{bmatrix} 1/a_{1,t} \\ 1/a_{2,t} \\ \vdots \\ 1/a_{n,t} \end{bmatrix} \tag{44}$$

Inserting (40), we obtain

$$\mathbf{z}_t = 2\sqrt{\zeta \lambda_t}\mathbf{1} + \Sigma \mathbf{q}_t$$

Inserting (43), we obtain

$$\mathbf{z}_t = \left(2\frac{\zeta}{L}J + \Sigma\right)\mathbf{q}_t$$

Therefore,

$$\mathbf{q}_t = N^* \mathbf{z}_t \tag{45}$$

where

$$N^* = \left(2\frac{\zeta}{L}J + \Sigma\right)^{-1}$$

To obtain the quadratic form of output, inserting (16) into (4),

$$Y_t = \mathbf{q}_t^T \left(\mathbf{z}_t - \sqrt{\zeta \lambda_t}\mathbf{1}\right) - \frac{1}{2}\mathbf{q}_t^T \Sigma \mathbf{q}_t$$

Inserting 43 and using (45),

$$\begin{aligned} Y_t &= \mathbf{q}_t^T \left(\left(2\frac{\zeta}{L}J + \Sigma\right)\mathbf{q}_t - \frac{\zeta}{L}J\mathbf{q}_t\right) - \frac{1}{2}\mathbf{q}_t^T \Sigma \mathbf{q}_t \\ &= \mathbf{q}_t^T \left(\frac{\zeta}{L}J + \frac{1}{2}\Sigma\right)\mathbf{q}_t \\ &= \frac{1}{2}\mathbf{q}_t^T N^{*-1}\mathbf{q}_t \\ &= \frac{1}{2}\mathbf{z}_t^T N^* \mathbf{z}_t \end{aligned}$$

## A.5 Monopoly

As the third counterfactual scenario, we consider the "monopoly" case, where all firms are owned by a single investor, to analyze the allocation of R&D, its impact on economic growth, and social welfare under an extremely concentrated ownership structure. Alternatively, in our setting, where firms maximize the weighted sum of investors' returns,

the monopoly case is equivalent to a situation where multiple investors exist but all hold identical portfolios of firm ownership. This corresponds to a common ownership matrix where all elements are equal to 1. In the monopoly case, since all firms share the same objective function, the equilibrium can be determined by solving a single Riccati equation.

**Static Problem**

First, we characterize the static allocation in the monopolist owner's problem. Like our baseline static Cournot-Nash equilibrium, we assume that the monopolist takes real wage $w_t/P_t$ as given in the static problem. The monopolist chooses $\mathbf{a}_t$, $\mathbf{b}_t$, and $\mathbf{q}_t$ to maximize the producer surplus $\Pi_t$, which is defined as

$$\begin{aligned}\Pi_t &\equiv \frac{\mathbf{q}_t^T \mathbf{p}_t}{P_t} - \frac{w_t}{P_t}\sum_i l_{i,t} \\ &= \mathbf{q}_t^T \mathbf{b}_t - \mathbf{q}_t^T \Sigma \mathbf{q}_t - \frac{w_t}{P_t}\sum_i \frac{q_{i,t}}{a_{i,t}}\end{aligned}$$

subject to the technology choice constraint $z_{i,t} = \zeta a_{i,t} + b_{i,t}$ for each firm. By solving the planner's static problem, we obtain a solution of quantities as a linear function of knowledge capital:

$$\mathbf{q}_t^M = N^M \mathbf{z}_t$$

where

$$N^M \equiv \frac{1}{2}\left(\frac{\zeta}{L}J + \Sigma\right)^{-1}.$$

and a quadratic form for the total profit of knowledge capital:

$$\Pi_t^M = \mathbf{z}_t^T P^M \mathbf{z}_t$$

where

$$P^M = N^M \Sigma N^M$$

(See Appendix A.6).

**Dynamic Problem**

Next, we characterize the static allocation in the monopolist owner's problem. The monopolist's dynamic problem is formulated as follows:

$$V^*(\mathbf{z}_0) \equiv \max_{\{\mathbf{x}_t\}_{t\geq 0}} \mathbb{E}_0 \left[ \int_0^\infty \exp(-\rho t) \left\{ \mathbf{z}_t^T P^M \mathbf{z}_t - \mathbf{x}_t^T \mathbf{x}_t \right\} dt \right],$$

subject to

$$d\mathbf{z}_t = (\Omega \mathbf{z}_t + \mu \mathbf{x}_t)\, dt + \gamma \mathbf{z}_t dW_t$$

We obtain the monopoly allocation by solving the modified version of the Riccati equation (33), in which $Q^*$ is replaced with $P^M$. When we express the solution of the Riccati equation with $\tilde{X}^M$, the R&D allocation is given by

$$\mathbf{x} = \mu \tilde{X}^M \mathbf{z}$$

**Welfare**

In contrast to the social planner's problem, the monopolist maximizes producer surplus, which is distinct from household utility. Therefore, calculating household utility necessitates a further step. The total output is given by

$$Y_t^M = \mathbf{z}_t^T Q^M \mathbf{z}_t$$

where

$$Q^M \equiv \frac{1}{2} N^M \left( 2\frac{\zeta}{L} J + 3\Sigma \right) N^M$$

(See Appendix A.7). Let $V^M$ denote the value of the representative household in the competitive equilibrium, which is defined as

$$V^M(\mathbf{z}_0) = \mathbb{E}_0 \left[ \int_0^\infty \exp(-\rho t) \left( \mathbf{z}^T Q^M \mathbf{z} - \mathbf{x}^T \mathbf{x} \right) dt \middle| \mathbf{z}_0 \right]$$

subject to the law of motion:

$$d\mathbf{z}_t = \Phi^M \mathbf{z}_t dt + \gamma \mathbf{z}_t d\mathbf{W}_t$$

where the technology transition under monopoly allocation $\Phi^M$ is given by

$$\Phi^M \equiv \Omega + \mu^2 \tilde{X}^M.$$

Guess $V^M(\mathbf{z}) = \mathbf{z}^T X^M \mathbf{z}$. Following the same steps as the derivation of welfare in competitive equilibrium, we obtain a single Riccati equation:

$$0 = Q^M - \mu^2 \left(\tilde{X}^M\right)^T \tilde{X}^M + X^M \left(\Phi^M - \frac{1}{2}\left(\rho - \gamma^2\right) I\right) + \left(\Phi^M - \frac{1}{2}\left(\rho - \gamma^2\right) I\right)^T X^M \tag{46}$$

Therefore, $X^M$ is obtained as the solution of the Lyapunov equation (46), and the welfare in monopoly problem is given by $V(\mathbf{z}_0) = \mathbf{z}_0^T X^M \mathbf{z}_0$.

## A.6 Derivation of the Static Solution of Monopolist's Static Problem

The FOC w.r.t. $q_{i,t}$ gives

$$0 = b_{i,t} - \frac{1}{a_{i,t}} \frac{w_t}{P_t} - 2 \sum_j \sigma_{ij} q_{j,t} \tag{47}$$

Therefore, the real profit of each firm is given by:

$$\begin{aligned} \pi_{i,t} &= \left(b_{i,t} - \frac{1}{a_{i,t}} \frac{w_t}{P_t}\right) q_{i,t} - \sum_j \sigma_{ij} q_{i,t} q_{j,t} \\ &= 2 \sum_j \sigma_{ij} q_{j,t} q_{i,t} - \sum_j \sigma_{ij} q_{i,t} q_{j,t} \\ &= \sum_j \sigma_{ij} q_{i,t} q_{j,t} \end{aligned} \tag{48}$$

The FOC w.r.t. $a_{i,t}$ gives

$$a_{i,t} = \sqrt{\frac{1}{\zeta} \frac{w_t}{P_t}}, \tag{49}$$

and therefore,

$$b_{i,t} = z_{i,t} - \sqrt{\zeta \frac{w_t}{P_t}} \tag{50}$$

Inserting (49) into the labor market clearing condition (11), we obtain

$$\sqrt{\zeta \frac{w_t}{P_t}} = \frac{\zeta}{L} \sum_i q_{i,t} \tag{51}$$

Inserting the optimality condition for technology choice (49) and (50) into the optimality condition for quantity choice (47), we obtain

$$z_{i,t} = 2\sqrt{\frac{\zeta w_t}{P_t}} + 2\sum_j \sigma_{ij} q_{j,t}.$$

Inserting the expression for real wage (51), we obtain

$$z_{i,t} = \sum_j \left(2\frac{\zeta}{L} + 2\sigma_{ij}\right) q_{j,t}.$$

In vector form,

$$\mathbf{z}_t = 2\left(\frac{\zeta}{L} J + \Sigma\right) \mathbf{q}_t. \tag{52}$$

Therefore, the quantities are expressed as linear functions of knowledge diffusion:

$$\mathbf{q}_t^M = N^M \mathbf{z}_t \tag{53}$$

where

$$N^M \equiv \frac{1}{2}\left(\frac{\zeta}{L} J + \Sigma\right)^{-1}.$$

From (48), the total profit is given by

$$\Pi_t = \mathbf{q}_t^T \Sigma \mathbf{q}_t$$

Inserting (53) , we obtain

$$\Pi_t = \mathbf{z}_t^T N^M \Sigma N^M \mathbf{z}_t$$

## A.7 Derivation of Output in Monopolist's Problem

Similar to competitive equilibrium, we have

$$\mathbf{b}_t = \mathbf{z}_t - \frac{\zeta}{L} J \mathbf{q}_t.$$

From (52), we obtain

$$\mathbf{b}_t = \left( \frac{\zeta}{L} J + 2\Sigma \right) \mathbf{q}_t$$

Therefore, from (4), the total output is given by

$$\begin{aligned} Y_t &= \mathbf{q}_t^T \left( \frac{\zeta}{L} J + 2\Sigma \right) \mathbf{q}_t - \frac{1}{2} \mathbf{q}_t^T \Sigma \mathbf{q}_t \\ &= \frac{1}{2} \mathbf{q}_t^T \left( 2\frac{\zeta}{L} J + 3\Sigma \right) \mathbf{q}_t \\ &= \mathbf{z}_t^T Q^M \mathbf{z}_t \end{aligned}$$

where

$$Q^M = \frac{1}{2} N^M \left( 2\frac{\zeta}{L} J + 3\Sigma \right) N^M$$

## A.8 Derivation of Stochastic Process of Output

Let $d\mathbf{z}_t = \boldsymbol{\mu}_t dt + \Gamma_t d\mathbf{W}_t$. Ito's lemma states that

$$df(\mathbf{z}_t) = \left\{ \left( f_{\mathbf{z}}(\mathbf{z}_t) \right)^T \boldsymbol{\mu}_t + \frac{1}{2} \mathrm{Tr} \left[ \Gamma_t^T f_{\mathbf{zz}}(\mathbf{z}_t) \Gamma_t \right] \right\} dt + \left( f_{\mathbf{z}}(\mathbf{z}_t) \right)^T \Gamma_t d\mathbf{W}_t$$

We apply this lemma to $d\mathbf{z}_t = \Phi \mathbf{z}_t dt + \gamma \mathrm{diag}(\mathbf{z}_t) d\mathbf{W}_t$ and $f(\mathbf{z}_t) = \mathbf{z}_t^T Q \mathbf{z}_t = Y_t$. Note that

$$\begin{aligned} \boldsymbol{\mu}_t &= \Phi \mathbf{z}_t \\ \Gamma_t &= \gamma \mathrm{diag}(\mathbf{z}_t) \\ f_{\mathbf{z}}(\mathbf{z}_t) &= 2Q\mathbf{z}_t \\ \Gamma_t^T f_{\mathbf{zz}}(\mathbf{z}_t) \Gamma_t &= 2Q \end{aligned}$$

Therefore,

$$
\begin{aligned}
dY_t &= \left\{2\mathbf{z}_t^T Q\Phi\mathbf{z}_t + \frac{1}{2}\mathrm{Tr}\left[\gamma^2 \mathrm{diag}\left(\mathbf{z}_t\right)\left(2Q\right)\mathrm{diag}\left(\mathbf{z}_t\right)\right]\right\} dt + 2\gamma\mathbf{z}_t^T Q\mathrm{diag}\left(\mathbf{z}_t\right) d\mathbf{W}_t \\
&= \left\{\mathbf{z}_t^T\left(Q\Phi + \Phi^T Q\right)\mathbf{z}_t + \gamma^2\sum_i z_i^2 Q_{ii}\right\} dt + 2\gamma\mathbf{z}_t^T Q\mathrm{diag}\left(\mathbf{z}_t\right) d\mathbf{W}_t
\end{aligned}
$$

Next, applying Ito's lemma to $\log Y_t = g\left(Y_t\right)$,

$$
\begin{aligned}
d\log Y_t &= \left\{\frac{1}{Y_t}\left(\mathbf{z}_t^T\left(Q\Phi + \Phi^T Q\right)\mathbf{z}_t + \gamma^2\sum_i z_{i,t}^2 Q_{ii}\right) + \frac{1}{2}\mathrm{Tr}\left[\left(-\frac{1}{Y_t^2}\right)4\gamma^2\mathbf{z}_t^T Q\mathrm{diag}\left(\mathbf{z}_t^2\right)Q\mathbf{z}_t\right]\right\} dt \\
&\quad + \frac{1}{Y_t}2\gamma\mathbf{z}_t^T Q\mathrm{diag}\left(\mathbf{z}_t\right) d\mathbf{W}_t \\
&= \left[\frac{\mathbf{z}_t^T\left(Q\Phi + \Phi^T Q\right)\mathbf{z}_t}{\mathbf{z}_t^T Q\mathbf{z}_t} + \gamma^2\left\{\frac{\sum_i z_{i,t}^2 Q_{ii}}{\mathbf{z}_t^T Q\mathbf{z}_t} - 2\frac{\mathbf{z}_t^T Q\mathrm{diag}\left(\mathbf{z}_t^2\right)Q\mathbf{z}_t}{\left(\mathbf{z}_t^T Q\mathbf{z}_t\right)^2}\right\}\right] dt + 2\gamma\frac{\mathbf{z}_t^T Q\mathrm{diag}\left(\mathbf{z}_t\right)}{\mathbf{z}_t^T Q\mathbf{z}_t} d\mathbf{W}_t
\end{aligned}
$$