Decentralized Collective Action: Theory and Evidence from Protests in Hong Kong

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According to Tullock's paradox of revolution, mass movements cannot be explained by rational choice models with strategic substitutability, which is inconsistent with the recent field evidence. Motivated by the fact that many protests and movements are decentralized and leaderless nowadays, we devise a search-and-matching model of protest and voting which features endogenous strategic interaction and multiple threshold equilibria. In the "quiet" equilibrium, the comparative statics behave like the standard global games featuring strategic complementarity. In this case, protest turnouts are small, crackdown reduces turnouts with ampliflying marginal effects, and regime changes are rare. The opposite occurs in the "unrest" equilibrium that features strategic substitutability, which is plausible when the search frictions are low (e.g. via social media) and the goal of decentralized protests is modest. To test the model's implications, we construct a new database detailing the distribution of Facebook users and the deployment of tear gas during the 2019 protests in Hong Kong. We use the location of police stations to instrument crackdown. Results from the instrumental variable estimation support our model's implications and provide strong evidence of unrest equilibrium featuring strategic substitutability. Our paper provides a new resolution to Tullock's paradox.

JEL Classification: D72, D74, D82, D83

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I. INTRODUCTION

Protests involve confrontation and are often perceived as deviant. Though dissidents almost always exist, they hide among the silent majority and face a constant challenge of finding linkminded people to spark a movement. Thus, a major barrier to collective actions is finding ways to overcome this 'frictional' environment. Historically, successful collective actions often benefited from skilled and charismatic organizers who mobilized people to join a movement (Olson 1965). Benefited from communication technology, protesters nowadays are easier to find one another through the internet. This can explain why protests are increasingly decentralized, in which telecommunication technologies have played an increasingly significant role in more recent movements (Castells 2015; Enikolopov et al. 2020; Manacorda and Tesei 2020). Social media has significantly contributed to the sucess of many contemporary movements. Acemoglu et al. (2018), for example, showed that the number of Twitter posts containing "Tahrir hashtags" predicted the protests in Tahrir Square, Egypt, during the Arab Spring.

In political economy literature, there is broad consensus that large-scale collective actions often emerge under conditions of strategic complementarities, and social media is thought to play a crucial role in coordinating participants' actions (Manacorda and Tesei 2020; Zhuravskaya et al. 2020). However, evidence of strategic substitutability has also been reported in some mass movements (Cantoni et al 2019; Hager et al 2022). This raises Tullock's paradox of revolution once again: if communication facilitates coordination, yet people tend to free ride under conditions of strategic substitutability, why are massive turnouts still observed in equilibrium, especially when the risks of repression are heightened, making an environment increasingly frictional and large-scale events more difficult to emerge?

To analyze the dynamics between search frictions and decentralized social movements, we introduce the first search-theoretic model of decentralized collective actions. The advantage of decentralized movements is that protesters can find and mobilize one another without the need for leaders. However, the downside lies in search frictions and strategic uncertainty among protesters, challenges that were mitigated by organizers before the age of social media. Similar to traditional models that characterize protests as a global game, individuals in our model observe a private signal about the fundamental and adopt a threshold strategy in equilibrium (Morris and Shin 1998; Shadmehr 2018). Unlike traditional models, however, search friction now endogenizes strategic interactions. Before the rise of social media, the flow of information was more limited, and search frictions were high. Under these circumstances, mobilization heavily depended on social movement organizations, and mass protests occurred only under strategic complementarity. The advent of social media reduces search frictions and introduces an additional equilibrium. When individuals perceive their participation as pivotal (Myatt 2017), they would join protests strategically, balanc-

ing the odds of achieveing a successful movement against the risk of triggering costly repression. We prove that in a low search-friction environment, mass mobilization can arise under strategic substitutability.

Our new model also yields distinct predictions regarding protest turnout in the presence of government crackdowns. The comparative statics enable us to derive testable implications, facilitating equilibrium identification and empirical testing of the model using real-world data. Specifically, our model predicts that when governments repress, turnout will decrease under strategic complementarity but increase under strategic substitutability. It also predicts that the marginal impact of crackdowns on turnout is positive but diminishing.

Using hand-collected data from social media and news reports, we test our model by examining the impact of tear gas deployment on support for pro-protest candidates during a local election held amid the 2019-2020 mass protests in Hong Kong. Since teat gas could be more likely to be deployed in pro-protester areas and thus potentially endogenous, we instrument tear gas deployment using the geographical distance between a constituency and its nearest police station. The instrument is relevant because tear gas deployment is more likely to occur in constituencies closer to police stations. It is also exogenous, as the location of police stations is historically predetermined and unrelated to the political alignment of an area. Estimates based on instrumental regressions reveal that tear gas deployment widens support for pro-protest candidates by 57 log points and raises their probability of winning in constituency where tear-gas was deployed by 33%. Results from a placebo test using data from the 2015 election data corroborate predictions of our model. Our findings align with a conclusion from Cantoni et al. (2019), who used data from a lab-in-the-field experiment during the 2016 Umbrella Movement and found that protest participation acted as a strategic substitute among university students in Hong Kong.

The model offers a solution to the paradox of revolution (Tullock 1971). A key assumption underlying this paradox is about people's belief about their marginal contribution to the success of an event is negligible. Similar to protest participation, the significance of a single vote to a candidate's victory is also marginal. If people hold this belief, they are unlikely to vote or participate. However, this assumption has been challenged by some empirical studies. For example, a laboratory experiment by Duffy and Tavits (2008) estimated that individuals' subjective probability of their votes being decisive can be as high as 0.33. This belief in being pivotal to the success of an endeavor provides a plausible explaination for why citizens choose to join a protest despite the possibility of free-riding. The belief also explains why repression may backfire. In the presence of strategic substitutability, the expectation that repression will reduce turnout can motivate individuals to show up, effecreduces countering the intended repression effect.

Our paper also contributes to two strands of literature on protests. First, the debate over whether massive turnout results from strategic substitutability or strategic complementarity re-

mains a focus in recent research. Since Olson (1965) and Tullock (1971), political activists have often been thought of as free-riders. While no empirical evidence have been found in the context of protests, this idea has recently been validated using data from a natural field experiment conducted among political activists during a political campaign in Western Europe (Hager et al., 2023). Evidence of strategic consideration has also been identified in the Umbrella Movement in Hong Kong (Cantoni et al., 2019; Bursztyn et al., 2021). Nevertheless, the assumption of strategic complementarity has often been made in some theoretical contributions (Bueno de Mesquita 2010; Edmond 2013; Passarelli and Tabellini 2017). Drawing on our model and data from a more recent protest in Hong Kong, we provide both theoretical support and empirical evidence for the role of strategic substitutability in this debate. Although our original data are observational, findings from our instrumental variable estimation encompass a broader population spectrum, addressing a limitation often in experimental studies.

Second, our paper is among the first studies in the economics literature to provide direct empirical evidence on the counter-productiveness of repression. Discussions on the relationship between repression and protest dynamics remain scant in the economics literature (Cantoni et al., 2024). To date, repression has primarily been studied for its role in maintaining regime stability through censorship and political terror (Shadmehr and Bernhardt, 2015), without a focus on its relationship with protests (Besley and Persson, 2011). Existing contributions are often theoretical and largely situated within political science (e.g. Shadmehr and Bernhardt, 2011). The backfire effect can be explained by the logic of strategic substitution and the perception of being pivotal. Citizens are more likely to protest against an incumbent regime when they believe that repression will deter participation and cause turnout to fall short of the threshold needed for success. Repression works when participation is strategic complement, as citizens believe they can hide among the crowd to reduce their chances of being caught. Conversely, when individuals think the outcome hinges on their involvement, especially if the required threshold is relatively low, they are more likely to show up. Our model captures this dynamics, which is further supported by additional anecdotal evidence. A central question is that if governments know that repression can backfire, why would they choose to do so in the first place? The existence of multiple equilibria in our model provides a plausible explanation. Governments do not know *ex ante* which equilibrium they are in until the first bullet is fired. The plausibility of strategic complementarity and substitution puts governments in a dilemma. Our study challenges the deterrence assumption and offers a novel mechanism to explain why repression may ultimately undermine the goals of an oppressive government

II. BASELINE MODEL

Decentralized movement. There is a large number, n, of citizens with heterogeneous payoffs from the outcomes of a decentralized movement opposing the incumbent government. The payoff

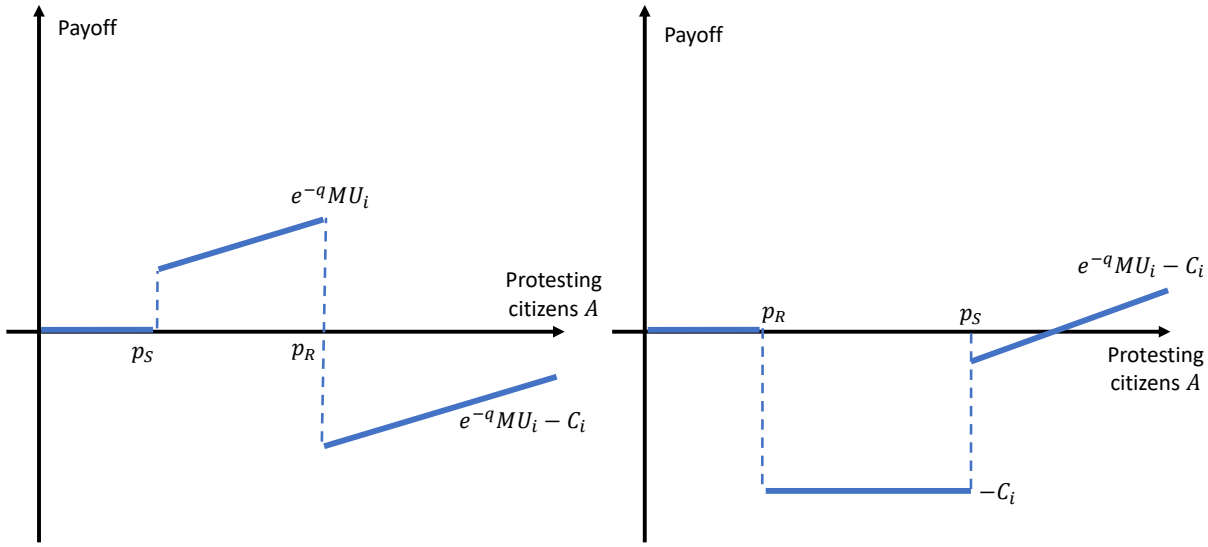


FIGURE 1: Individual Payoff.

for citizen i is given by

$$\mathcal{U}_i(a_i) = e^{-q} \Pr_i \left\{ \frac{A}{n} > p_S \right\} M_i U_i - \Pr_i \left\{ \frac{A}{n} > p_R \right\} C_i, \text{ where } A = a_i + \sum_{j \neq i} a_j,$$

where $a_i \in \{0, 1\}$ indicates that citizen i's decision to protest. A decentralized movement is composed of a social network of protests, which can form through various channels such as schools, workplaces, neigbhorhoods, and online communities. Let M_i represent citizen i's expected effective fraction of protesting citizens within his social network (excluding himself). If the decentralized movement succeeds, citizen i receives a payoff of $M_i U_i$, which is increasing M_i, reflecting the idea that a higher expected effective fraction of protesters in his social network amplifies his marginal benefit of protest U_i.

A decentralized movement succeeds if the *total* number of protesters across social groups, $A \equiv \sum_i a_i$, exceeds the threshold $p_S \in (0, 1)$ (i.e., $A/n > p_S$). However, the government represses when the size of the protest becomes sufficiently large, such that $A/n > p_R$, where $p_R \in (0, 1)$. In this case, citizen i's incurs a loss of C_i, which varies across citizens. The left panel of Figure 1 illustrates the payoff when $p_S < p_R$, where the government tolerates some successful protests without repression. The contrasting case $p_S > p_R$ is illustrated in the right panel of Figure 1.

We model a decentralized movement using a random matching model. Each citizen engages with his social groups randomly with probability e^{-q}, where $q > 0$ represents the search friction, capturing the difficulty of meeting and networking with other citizens. The social network of citizen

$i$is represented by the adjacency matrix $G_{ij}^n \in [0,1]$, where G_{ij}^n measures the importance of citizen j in citizen i's social network, where $G_{ii}^n = 0$ and $\sum_j G_{ij}^n = 1$. The expected effective fraction of protesting citizens within citizen i's social network is

$$M_i = \mathbb{E}_i \sum_j G_{ij}^n 1_{a_j=1}.$$

Dispersed information. Although citizens are heterogenous, the "sufficient statistics" for a citizen's action u_i, as we will see later, is given by

$$u_i \equiv \log\left(\frac{U_i}{C_i}\right),$$

which is referred to as citizen i's private interest in protesting. A high level of private interest indicates that the citizen derives greater utility from participating in protests compared to abstaining, after accouting for the potential consequences of repression. Citizens' private interest are heterogenous, private, and follow a normal distribution:

$$u_i = \theta + \epsilon_i,\ \epsilon_i \sim N\left(0, \sigma^2\right),$$

where ϵ_i is i.i.d. normal among citizens and σ captures the dispersion of private interests among citizens. Additionally, the social interest in the protest is captured by θ, which reflects the aggregate state of this society. However, there is an aggregate uncertainty and a lack of common knowledge about the social interest, leading to strategic uncertainty among citizens. This encapsulates, for instance, the situation where citizen i is enthusiatic about protesting, but he is uncertain whether other individuals share the same enthusiatism. This uncertainty may cause hestitation to participate. The lack of common knowledge about social saliencies is particularly significant for decentralized movements. In particular, θ is given by

$$\theta = \bar{\theta} + \varepsilon,\ \varepsilon \sim N\left(0, \frac{\sigma^2}{\psi}\right),$$

where $\bar{\theta}$ is the prior interest. Unknown to citizens, the error term ε catpures the aggregate uncertainty regarding the social interest, which is common for all citizens. Specifically, ψ measures the confidence in the social interest relative to the private interest.

After observing his private interest u_i, citizen i's posterior belief about the social interest is

$$\theta|_{u_i} \sim N\left(\theta_i, \frac{\sigma^2}{1+\psi}\right),\ \text{where} \tag{1}$$

$$\theta_i \equiv \frac{\psi\bar{\theta} + u_i}{\psi + 1}. \tag{2}$$

Essentially, θ_i is citizen i's posterior on θ, which puts a weight of $\psi/(1+\psi)$ on the prior interest $\bar{\theta}$ and a weight of $1/(1+\psi)$ on his private interest u_i. The weight on private interest is smaller when a citizen has greater confidence ψ on the prior interest $\bar{\theta}$.

Pivotal protester. Citizens are pivotal, which means that they internalize the impact of their choice of a_i on A as $A = a_i + \sum_{j \neq i} a_j$. Rearraning the expectation operator, citizen i's payoff becomes[1]

$$\mathcal{U}_i(a_i) = e^{-q} \Pr_i \{A_{-i} + a_i > \lceil p_S n \rceil\} M_i U_i - \Pr_i \{A_{-i} + a_i > \lceil p_R n \rceil\} C_i$$

where A_{-i} is the random variable of the number of protesting citizens excluding citizen i. Denote citizen i's gain from protest as $\Delta_i \equiv \mathcal{U}_i(1) - \mathcal{U}_i(0)$, which is given by

$$\Delta_i \equiv \mathcal{U}_i(1) - \mathcal{U}_i(0) = e^{-q} P_i^S M_i U_i - P_i^R C_i, \tag{3}$$

where

$$P_i^S \equiv \Pr_i \{A_{-i} + 1 > \lceil p_S n \rceil\} - \Pr_i \{A_{-i} > \lceil p_S n \rceil\} = \Pr_i \{A_{-i} = \lceil p_S n \rceil\}, \tag{4}$$

$$P_i^R \equiv \Pr_i \{A_{-i} + 1 > \lceil p_R n \rceil\} - \Pr_i \{A_{-i} > \lceil p_R n \rceil\} = \Pr_i \{A_{-i} = \lceil p_R n \rceil\}. \tag{5}$$

We refer P_i^S and P_i^R as to the citizen i's *pivotal* probability of success and of repression respectively. In particular, P_i^S is the probability that citizen i is the marginal participant whose involvement determines the success of the protest. Similarly P_i^R is the probability that citizen i is the marginal participant whose involvement triggers government repression. Arranging terms, it is strictly optimal for a citizen i to protest upon observing u_i iff

$$\Delta_i > 0 \Leftrightarrow u_i > \log\left(\frac{P_i^R}{P_i^S}\right) - \log M_i + q. \tag{6}$$

Given the protest interest θ, the probability that a citizen will protest is given by[2]

$$P(\theta) \equiv \Pr\{a_j = 1 | \theta\}. \tag{7}$$

After observing his private interest u_i, citizen i has a different posterior of θ, which is denoted as $f_i(\theta)$ following (1). We are going to solve $P(\theta)$ in an equilibrium shortly. Given $P(\theta)$ and $f_i(\theta)$, the pivotal probabilities and expected size of protesting citizens are given by the following combinatorics:

$$P_i^S = \frac{(n-1)!}{\lceil p_S n \rceil! (n - 1 - \lceil p_S n \rceil)!} \int_0^1 P(\theta)^{\lceil p_S n \rceil} [1 - P(\theta)]^{n-1-\lceil p_S n \rceil} f_i(\theta) \, d\theta, \tag{8}$$

$$P_i^R = \frac{(n-1)!}{\lceil p_R n \rceil! (n - 1 - \lceil p_R n \rceil)!} \int P(\theta)^{\lceil p_R n \rceil} [1 - P(\theta)]^{n-1-\lceil p_R n \rceil} f_i(\theta) \, d\theta, \tag{9}$$

$$M_i \equiv \sum_j G_{ij}^n \Pr_i \{a_j = 1\} = \int P(\theta) f_i(\theta) \, d\theta. \tag{10}$$

[1] Recall that $\lceil x \rceil$ is the smallest integer that is weakly greater than x.

[2] We assume a tie-breaking rule where citizens choose not to protest if they are indifferent.

Counting the combinatorics for the event of $\{A_{-i} = \lceil p_S n \rceil\}$ for P_i^S, there are $C_{\lceil p_S n \rceil}^{n-1}$ combinations among $n-1$ citizens (excluding citizen i himself) where $\lceil p_S n \rceil$ of them protest but $n-1-\lceil p_S n \rceil$ of them do not. Each combination has probability $P(\theta)^{\lceil p_S n \rceil} [1-P(\theta)]^{n-1-\lceil p_S n \rceil}$ for a given θ. The same logic applies to P_i^R. It is important to note that, up to this point, we have not imposed any assumptions about normal distributions. Therefore, the above formulation of the model holds for any distribution of the preference parameters.

III. EQUILIBRIUM

In a perfect Baynesian equilbrium, $P(\theta)$ is the fixed point such that for any θ,

$$P(\theta) = \Pr\{\Delta_j > 0 | \theta\},$$

where Δ_j follows (6) given $P(\theta)$. Consider a threshold equilibrium (to be verified later) in which there exists a marginal citizen such that any citizen j will protest iff their private interest u_j exceeds marginal citizen's u^*:

$$\Delta_j > 0 \Leftrightarrow u_j > u^*. \tag{11}$$

Then the probability that a citizen protests, given the social interest, becomes

$$P(\theta) = \Pr\{u_j > u^* | \theta\} = \Phi\left(\frac{\theta - u^*}{\sigma}\right), \tag{12}$$

where Φ is the cumulative distribution function of the standard normal distribution. By symmetry, $1 - \Phi(x) = \Phi(-x)$.

Lemma 1 *Under a threshold equilibrium, if exists, the log odds ratio of pivotal probabilities of citizen i at the limit is*

$$\lim_{n\to\infty} \log\left(\frac{P_i^R}{P_i^S}\right) = \frac{1+\psi}{\sigma}(z_R - z_S)(\theta_i - u^*) - \frac{1+\psi}{2}\left(z_R^2 - z_S^2\right), \tag{13}$$

where the posterior θ_i is given by (2), $z_S \equiv \Phi^{-1}(p_S)$ and $z_R \equiv \Phi^{-1}(p_R)$. The expected fraction of protesters within citizen i's social group is

$$M_i = \Phi\left(\frac{\theta_i - u^*}{\sigma\sqrt{1 + (1+\psi)^{-1}}}\right). \tag{14}$$

According to Lemma 1, after observing his private interest u_i, citizen i expects that the fraction of protesting citizens in a threshold equilibrium is $M_i = \Phi(\tau_i)$, where

$$\tau_i \equiv \frac{\theta_i - u^*}{\sigma\sqrt{1 + (1+\psi)^{-1}}} = \frac{u_i + \psi\bar{\theta} - (1+\psi)u^*}{\sigma\sqrt{(1+\psi)(2+\psi)}}. \tag{15}$$

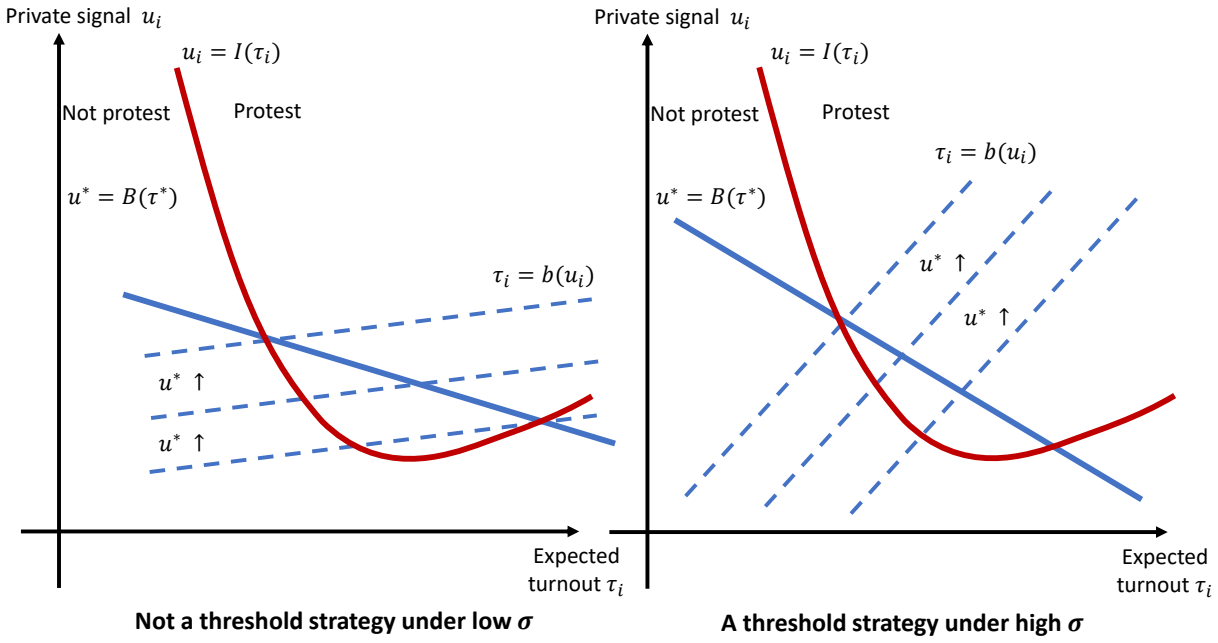


FIGURE 2: Protest Strategy.

We simply refer to τ_i as citizen i's expected turnout, which increases with his private interest in protesting (u_i). This is because a higher private interest raises his posterior belief about θ_i, leading him to expect that his peers have similarly higher private interest in protesting. As a result, more of them are epxected to join the protest. Based on Lemma 1, citizen i chooses to protest in a threshold equilbrium iff (6) is satisfied, i.e.,

$$u_i > \mathcal{I}(\tau_i) \equiv (z_R - z_S)\sqrt{(1+\psi)(2+\psi)}\tau_i - \log\Phi(\tau_i) + \xi, \tag{16}$$

where

$$\xi \equiv q - \frac{1+\psi}{2}\left(z_R^2 - z_S^2\right). \tag{17}$$

The log odds ratio of repression at the limit is $\log\left(P_i^R/P_i^S\right) = (z_R - z_S)\sqrt{(1+\psi)(2+\psi)}\tau_i - (1+\psi)\left(z_R^2 - z_S^2\right)/2$, which is increasing linearly in the expected turnout τ_i and is equal to zero when $\tau_i = (z_R + z_S)/\left[2\sqrt{1+(1+\psi)^{-1}}\right]$. The linearity of the log odds ratio of repression is the result of normal noises, ϵ and ε.

Figure 2 illustrates the individual decision to protest given the marginal interest u^*. $\mathcal{I}(\tau_i)$ represents the indifference curve: citizen i chooses to protest iff his private interest u_i and expected turnout τ_i are realized on the RHS to $\mathcal{I}(\tau_i)$, not to protest on the LHS of $\mathcal{I}(\tau_i)$, and indifferent on $\mathcal{I}(\tau_i)$.

However, not all combinations of u_i and τ_i are rational; citizens update their expected turnout τ_i with respect to u_i according (2). Combining (2) and (15), citizen i's private interest u_i and

expected turnout τ_i are realized on the locus given by

$$\tau_i = b\left(u_i; u^*\right) \equiv \frac{u_i + \psi\bar{\theta} - (1+\psi)\, u^*}{\sigma\sqrt{(1+\psi)\,(2+\psi)}}, \tag{18}$$

It is referred to as the individual expectation condition, illustrated by the dashed lines in Figure 2. According to (16), citizens with u_i and τ_i on the individual expectation curve, and to the RHS (LHS) of the indifference curve will (not) protest. A higher marginal interest u^* or a lower prior $\bar{\theta}$ shifts the individual expectation curve upward, while a higher dispersion of private interest steepens the curve.

What remains to be verified is the threshold strategy. It is not obvious because u_i enters both the LHS of (16) (directly as he has higher benefit from protest) and the RHS [indirectly via updating his belief from u_i in $\mathcal{I}(\tau_i)$]. Citizen i would not follow a threshold strategy if the individual expectation curve (18) crosses the indifference curve (16) more than once. Nevertheless, the adoption of the threshold strategy is guaranteed by the following lemma.

Lemma 2 *Under a threshold equilibrium, citizen i follows a threshold strategy iff*

$$z_S \geq z_R - \sigma. \tag{19}$$

The adoption of the threshold strategy requires that the private interest is sufficiently dispersed (i.e. σ is high). If σ is low, the individual expectation curve (18) in Figure (2) becomes very flat such that it intersects the bending portion of the indifference curve (16) more than once. In the rest of this paper (except in Section IV.), we focus on analyzing the threshold equilibrium and assume that (19) is always satisfied.

Having characterized the individual choice to protest given the marginal interest in protest, we are now ready to solve for the threshold equilibria. Obviously, there always exists a trivial threshold equilibrium with $u^* = \infty$, as not protesting is self-fulfilling. We solve for the non-trivial threshold equilibrium as the fixed point of u^*. The threshold u^* in the threhold equilibrium represents the private interest of the marginal citizen who is indifferent between protesting or not.

In the equilibrium, the marginal citizen evaulates his posterior (2) with $u_i = u^*$, his expected turnout $\tau_i = \tau^*$ according to (15) is given by the following *equilibrium expectation condition*

$$u^* = \mathcal{B}\left(\tau^*\right) = \bar{\theta} - \frac{\sigma}{\psi}\sqrt{(1+\psi)\,(2+\psi)}\tau^*. \tag{20}$$

In the threshold equilibrium, the marginal citizen is indifferent between protesting and not protesting, such that (16) holds at equality at τ^* and u^*, which is referred to as the *indifference condition*. A threshold equilibrium is the fixed point of

$$\begin{aligned} u^* &= \mathcal{I}\left(\tau^*\right); \\ u^* &= \mathcal{B}\left(\tau^*\right). \end{aligned}$$

The following proposition completely characterizes the typology of the threshold equilibrium.

Proposition 1 *Suppose (19) is satisfied. Define two conditions:*

$$z_S < z_R + \frac{\sigma}{\psi}, \tag{21}$$

$$q < q_c, \tag{22}$$

where q_c and τ_c are uniquely given by

$$q_c \equiv \bar{\theta} + \frac{1+\psi}{2}\left(z_R^2 - z_S^2\right) - \phi\left(\tau_c\right)\tau_c + \log\Phi\left(\tau_c\right),$$

$$\phi\left(\tau_c\right) = \sqrt{\left(1+\psi\right)\left(2+\psi\right)}\left(z_R - z_S + \frac{\sigma}{\psi}\right),$$

where $\phi(z) \equiv \Phi'(z)/\Phi(z)$.

a. There always exists a trivial threshold equilibrium with $u^* = \infty$.

b. Suppose condition (21) is not satisfied. There exists a unique non-trivial threshold equilibrium.

Suppose condition (21) is satisfied.

c.i. If condition (22) is satisfied, then there exist two non-trivial threshold equilibria.

c.ii. If condition (22) is not satisfied with equality, then there exists a unique non-trivial threshold equilibrium.

c.iii If condition (22) is not satisfied with inequality, then there does not exist any non-trivial threshold equilibrium.

Figure 3 illustrates Proposition 1. The marginal citizen's indifference curve, $u^* = \mathcal{I}(\tau^*)$, is illustrated by the red line in Figure 3. The equilibrium expectation condition, $u^* = \mathcal{B}(\tau^*)$, is illustrated by the blue line in Figure 3. The left panel illustrates the equilibrium of Proposition 1b. The right panel illustrates the equilibrium of Proposition 1ci. The situation of Proposition 1cii is the case when the indifference curve (red line), $u^* = \mathcal{I}(\tau^*)$, does not intersect with the equilibrium expectation condition (blue line), $u^* = \mathcal{B}(\tau^*)$, in the right panel.

III..A. Endogenous Strategic Interaction

Do the equilibria found in Proposition 1 feature strategic complementarity or strategic substitutability? To address this, we first define the concept of strategic interaction. Formally, consider a scenario where every citizen expects turnouts higher than the equilibrium turnout by δ to $\tau^* + \delta$. As a result, the corresponding threshold for protest is lower to $u_\delta = \mathcal{B}(\tau^* + \delta) < u^*$. Expecting that citizens observing $u_j > u_\delta^*$ will protest, citizen i's expected turnout becomes $\tau_i^\delta = b(u_i; u_\delta)$. He will protest iff his private interest satisifies $u_i > \mathcal{I}\left(\tau_i^\delta\right)$, which is equivalent to $u_i > u_\delta^*$, where

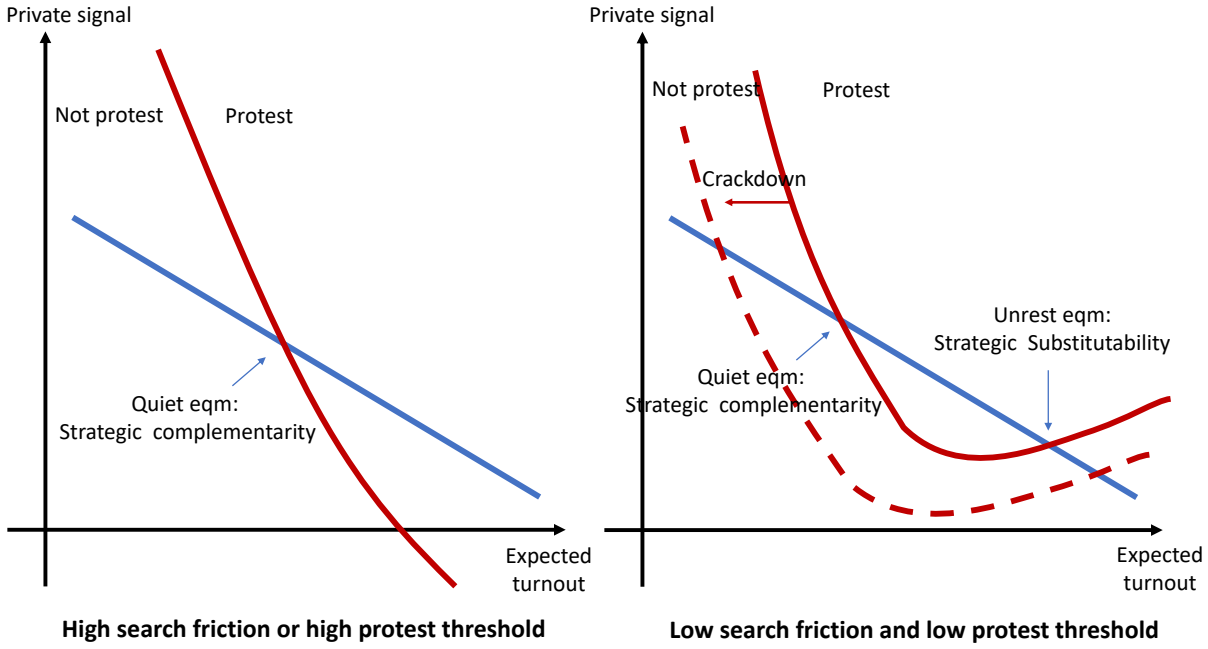


FIGURE 3: Left: Unique Equilibrium of Stratergic Complementarity. Right: Multiple Equilibria with Stratergic Substitutability.

u_δ^* solves $u_\delta^* = \mathcal{I}\left[b\left(u_\delta^*; u_\delta\right)\right]$. A threshold equilibrium features strategic complementarity if there is a higher fraction of protesting citizens than expected; strategic substitutability if there is a lower faction; and strategic neutrality if the faction is the same, which occurs when the protest threshold is lower, i.e., $u_\delta^* < u^*$. The above concept is formalized by the following definition.

Definition 1 *For all infinitestimal δ, define the following strategic interaction of given a threshold equilibrium*

$$\textit{strategic complementarity} \qquad \textit{if } u_\delta^* < u^*; \tag{23}$$

$$\textit{strategic substitutability} \qquad \textit{if } u_\delta^* > u^*; \tag{24}$$

$$\textit{strategic neutrality} \qquad \textit{if } u_\delta^* = u^*. \tag{25}$$

The following lemma simplifies the classification of strategic interaction by comparing the slopes of the indifference curve and the equilibrium expectation condition.

Lemma 3 *Strategic interaction of given a threshold equilibrium is equivalent to the following slope comparison:*

$$\textit{strategic complementarity} \quad : \quad \mathcal{B}'\left(\tau^*\right) > \mathcal{I}'\left(\tau^*\right); \tag{26}$$

$$\textit{strategic substitutability} \quad : \quad \mathcal{B}'\left(\tau^*\right) < \mathcal{I}'\left(\tau^*\right); \tag{27}$$

$$\textit{strategic neutrality} \quad : \quad \mathcal{B}'\left(\tau^*\right) = \mathcal{I}'\left(\tau^*\right). \tag{28}$$

Note that in equilibrium every combination of u^* and τ^* must lie on the locus of (20), $u^* = \mathcal{B}(\tau^*)$. When the marginal citizen expects slitghtly higher turnouts from other citizens, the equilibrium marginal interest of protest changes by $\mathcal{B}'(\tau^*)$, and the individual expectation curve to shift downward, as shown in Figure 2. Recall that, by following a threshold strategy, citizens strictly prefer to protest if their (u_i, τ_i) lies on the right of the indifference curve, such that $u_i > \mathcal{I}(\tau_i)$. Thus, after expecting higher turnouts from other citizens, more citizens on the individual expectation curve will find themselves in the region of $u_i > \mathcal{I}(\tau_i)$ if $\mathcal{B}'(\tau^*) > \mathcal{I}'(\tau^*)$, i.e. strategic complementarity. Conversely, this is the case of strategic substitutability.

Proposition 2 *a. The threshold equilibrium in Proposition 1b features strategic complementarity.*

b. The threshold equilibrium in Proposition 1ci with a higher expected turnout features strategic substitutability, while the other threshold equilibrium with a lower expected turnout features strategic complementarity.

c. The threshold equilibrium in Proposition 1cii features strategic neutrality.

Proposition 2b offers a novel resolution to Tullock's paradox. In this case, a larger turnout still occurs in the equilibrium featuring strategic substitutability. The intuition behind this is that, in our framework, strategic interaction is endogenous. Specifically, as citizen's turnout is large, the marginal benefit from encountering other protesting citizens in a decentralized mass movement diminishes due to the search-and-matching nature of the process. On the other hand, the marginal cost of protest, in terms of triggering repression, becomes higher. Thus, as turnout grows, the choice to protest become strategic substitution among citizens. In Tullock's paradox, the nature of strategic substitutability is exogeneously given, always leading to low turnouts because protesting citizens crowd each other out. In our model, however, there is a feedback casuality driven by the search-and-matching mechanism.

Proposition 3 *Condition (22) in Proposition 1ci, i.e., the necessary condition for multiple threshold equilibria is more likely to be satisfied when the search friction q is low, the goal of protest p_S is low, the prior interest $\bar{\theta}$ is high, or the information dispersion σ is low.*

The source of multiplicity in our model lies in the search-and-matching nature of the decentralized movement. As illustrated in Figure 3, multiple equilibria arise because the indifference curve $u^* = \mathcal{I}(\tau^*)$ bends backward and intersects the equilibrium expectation condition $u^* = \mathcal{B}(\tau^*)$. It is clearly illsutrated in the equation that the bending is due to the presence of $\log M_i$ in $\mathcal{I}(\tau_i)$. Without this factor, the indifference curve $u^* = \mathcal{I}(\tau^*)$ is monotonic and cuts the equilibrium expectation condition $u^* = \mathcal{B}(\tau^*)$ only once. The effect of search-and-matching is stronger when the search friction q is low. The mechanism by which search-and-matching creates multiplicity is

through strategic uncertainty: citizens are uncertain about the faction of protesting citizens they will encounter (M_i), but their payoff increases with that fraction. As a resut, they can coordinate in two different ways, either with high or low thresholds and turnouts. This effect is more pronounced when citizens are more certain about achieving the protest payoff without triggering repression, which is more likely when the goal of the protest p_S is low, the prior interest $\bar{\theta}$ is high, or the information dispersion σ is low.

III..B. Comparative Statics

To characterize the implications of different strategic interactions, we are interested in deriving the comparative statics across various threshold equilibria in the case of Propositon 1c-i.

Proposition 4 *The comparative statics for the threshold equilibria, featuring strategic substitutability and strategic complementarity, are given by*

		dq	$d\bar{\theta}$
Strategic	$d\tau^*$	$-$	$+$
Substitutability	du^*	$+$	$\pm$
Strategic	$d\tau^*$	$+$	$-$
Complementarity	du^*	$-$	$+$

As illustrated in Figure 3, a higher search friction q shifts the indifference curve $u^* = \mathcal{I}(\tau^*)$ downard, which reduces the expected turnout τ^* and increases the marginal interest u^* in the high-τ equilibrium (which also features strategic substitutability for the reason of Proposition 2) but increases the expected turnout τ^* and decreases the marginal interest u^* in the low-τ equilibrium (strategic complementarity).

A higher prior $\bar{\theta}$ shifts the equilibrium expectation condition $u^* = \mathcal{B}(\tau^*)$ in Figure 3 upward. It reduces the expected turnout τ^* and increases the marginal interest u^* in the low-τ equilibrium (strategic complementarity), but increases the expected turnout τ^* in the high-τ equilibrium (strategic substitutability). The comparative statics on the marginal interest u^* depends on the location of (τ^*, u^*) on the indifference curve. If (τ^*, u^*) is located on the upward (downward) sloping part of the indifference curve, then we have $du^*/d\bar{\theta} > (<)0$, as delineated in Figure 3.

III..C. Implications of Unpopular Crackdowns

Consider an unpopular crackdown that makes it more difficult for citizens to meet each other for a decentralized protest but also raises the prior interest in protest. Specifically, the search friction q increases with the intensity of an unpopular crackdown x, while the prior $\bar{\theta}$ increases even more with x, such that

$$\theta_x > q_x.$$

Proposition 5 *The comparative statics of a unpopular crackdown are given by*

	Own effects		Marginal effects		
		dx		dq	$d\bar{\theta}$
Strategic Substitutability	$d\tau^*$	$+$	$d\frac{d\tau^*}{dx}$	$+$	$-$
Strategic Complementarity	$d\tau^*$	$-$	$d\frac{d\tau^*}{dx}$	$-$	$+$

Proposition 5 states that the consequences of an unpopular crackdown are opposite in equilibria with different strategic interactions, as illustrated in Figure 3. Furthermore, an unpopular crackdown will polarize the two equilibria, as the large turnout in the strategic substitutability equilibrium will become even larger, while the small turnout in the strategic complementaritye equilibrium will become even smaller. This highlighs an important policy implication of distinguishing between these mechanisms, as otherwise opposite predictions could be made. Specifically, an unpopular crackdown actually increases protest turnout, although the marginal effects of the crackdown through reducing the search friction q and higher prior $\bar{\theta}$ are diminishing.

III..D. Strategic Uncertainty at the Limit

In this section we are interested in the equilibrium selection when strategic uncertainty goes to the limit. We ignore the uninteresting case $\sigma = 0$, since (19) and (21) imply $z_R \leq z_S$, hence there is always unique non-trivial threshold.

Proposition 6 *a. If $\sigma \to \infty$, then $\tau^* \to 0$ and $u^* \to q - \frac{1+\psi}{2}\left(z_R^2 - z_S^2\right) + \log 2$. The threshold equilibrium features strategic substitutability.*

b. If $\psi \to \infty$, then $\tau^* \to \frac{z_R+z_S}{2}$ and $u^* \to \bar{\theta} - \frac{\sigma}{2}(z_R + z_S)$. The threshold equilibrium features strategic substitutability if $z_R < z_S$.

c. If $\psi = 0$, then $\tau^* \to 0$ and $u^* \to q - \frac{1}{2}\left(z_R^2 - z_S^2\right) + \log 2$. The threshold equilibrium features strategic substitutability.

When there are multiple threshold equilibrium and strategic uncertainty approaches its limit, the equilibrium featuring strategic substitutability is always selected (except in the unusual case $z_R < z_S$ in Proposition 6b). This result suggests that, theoretically, the equilibrium featuring strategic substitutability is more robust.

IV. INVERTED-THRESHOLD EQUILIBRIUM

So far we have maintained condition (19), ensuring the existence of a threshold equilibrium. To complete our analysis, in this section we solve for the equilibrium when condition (19) is violated.

In this case, the choice to protest follows an inverted threshold strategy, rather than a threshold strategy.

Consider an inverted threshold equilibrium (to be verified later) where a citizen j will protest iff his private interest is sufficiently *lower* than the marginal interest u^*:

$$\Delta_j > 0 \Leftrightarrow u_j < u^*. \tag{29}$$

Then the probability that a citizen protests given social interest becomes

$$P(\theta) = \Pr\{u_j < u^* | \theta\} = \Phi\left(\frac{u^* - \theta}{\sigma}\right). \tag{30}$$

Following the proof of Lemma 1, we can show that under an inverted threshold equilibrium, if exists, the log odds ratio of pivotal probabilities of citizen i at the limit is

$$\lim_{n\to\infty} \log\left(\frac{P_i^R}{P_i^S}\right) = -(z_R - z_S)\sqrt{(1+\psi)(2+\psi)}\tau_i - \frac{1+\psi}{2}\left(z_R^2 - z_S^2\right), \tag{31}$$

where the expected turnout becomes

$$\tau_i = \frac{u^* - \theta_i}{\sigma\sqrt{1 + (1+\psi)^{-1}}}. \tag{32}$$

Citizen i's expected fraction of protesters in his social group is $M_i = \Phi(\tau_i)$.

Citizen i chooses to protest in an inverted threshold equilbrium iff

$$u_i > \tilde{I}(\tau_i) \equiv -(z_R - z_S)\sqrt{(1+\psi)(2+\psi)}\tau_i - \log\Phi(\tau_i) + \xi. \tag{33}$$

The following lemma verifies the the adoption of inverted threshold strategy in this case.

Lemma 4 *Under an inverted threshold equilibrium, citizen i follows an inverted threshold strategy iff (19) is violated.*

The equilibrium expectation condition *becomes*

$$u^* = \tilde{\mathcal{B}}(\tau^*) = \bar{\theta} + \frac{\sigma}{\psi}\sqrt{(1+\psi)(2+\psi)}\tau^*, \tag{34}$$

which becomes a *upward-sloping* curve. Like before, an inverted threshold equilibrium is the fixed point of $u^* = \tilde{I}(\tau^*) = \tilde{\mathcal{B}}(\tau^*)$. The following proposition completely characterizes the typology in the remaining region.

Proposition 7 *Suppose (19) is violated. Define the condition*

$$q < \tilde{q}_c, \tag{35}$$

where $\tilde{q}_c$ and $\tilde{\tau}_c$ are uniquely given by

$$\tilde{q}_c \equiv \bar{\theta} + \frac{1+\psi}{2}\left(z_R^2 - z_S^2\right) - \phi\left(\tilde{\tau}_c\right)\tilde{\tau}_c + \log\Phi\left(\tilde{\tau}_c\right),$$

$$\phi\left(\tilde{\tau}_c\right) = -\sqrt{\left(1+\psi\right)\left(2+\psi\right)}\left(z_R - z_S + \frac{\sigma}{\psi}\right).$$

a. There always exists a trivial inverted threshold equilibrium with $u^* = -\infty$.

b. Suppose condition (21) is satisfied. There exists a unique non-trivial inverted-threshold equilibrium, which features strategic complementarity.

Suppose condition (21) is not satisfied.

c.i. If condition (35) is satisfied, then there exist two non-trivial inverted-threshold equilibria. The one with a higher expected turnout features strategic substitutability; the other threshold equilibrium with a lower expected turnout features strategic complementarity.

c.ii. If condition (35) is not satisfied with equality, then there exists a unique non-trivial inverted-threshold equilibrium, which features strategic neutrality.

c.iii If condition (35) is not satisfied with inequality, then there does not exist any non-trivial inverted-threshold equilibrium.

V. EXTENSION WITH VOTING

In most situations, individual-level protest participation data is unavailable (with notable exceptions such as field experiments), but voting data is typically more accessible at the constituency level. In this section, we extend the model to incorporate the decision of voting against the incumbent, allowing the model to generate testable implications using voting data.

Voting. Now, a typical citizen must decide not only whether to vote against the incumbent but also whether to protest against him. The payoff of citizen i is

$$\mathcal{U}_i\left(a_i, b_i\right) = \mathbb{E}_i\left\{\begin{array}{l}\Pr_i\left\{\frac{B}{n} \leq p_W\right\} W_i + \Pr_i\left\{\frac{B}{n} > p_W\right\} L_i \\ +e^{-q}\Pr_i\left\{\frac{A}{n} > p_S\right\} M_i U_i - \Pr_i\left\{\frac{A}{n} > p_R\right\} C_i\end{array}\right\},$$

where citizen i votes against (for) the incumbent if $b_i = 1$ ($b_i = 0$). The incumbent wins the election if the number of protest votes $B \equiv \sum b_i$ is below the threshold $p_W \in (0,1)$, i.e., $B/n \leq p_W$. If the incumbent wins (loses), then citizen i enjoys the payoff W_i (L_i).

Assumptions. In this paper, we focus on the scenario where $W_i > L_i$, i.e., the incumbent's victory is beneficial to all citizens and should be considered optimal in the first-best outcome. However, preferences regarding protests differ between two types of citizens. A *conservative* citizen j prefers avoiding protests, i.e., $U_j < 0$. Therefore, a conservative citizen always chooses $a_j = b_j = 0$. On the other hand, an *activist* citizen i favors engaging in protests, i.e., $U_i > 0$. However, activist citizens, acting with consistent principles, can not simultaneously choose $a_i = 1$ and $b_i = 0$, i.e., they cannot protest on the street but vote for the incumbent. This behaviorial assumption

simiplifies the dimensionality of the action space that needs to be considered when solving for the threshold equilibrium. To further reduce the dimensionality of heterogeneity, we assume that the cost of repression is proportional to the differential benefit of electing the incumbent, i.e., $C_i = k^{-1}(W_i - L_i)$ for some constant $k > 0$. In the rest of this paper, we focus on the decisions of activist citizens. Denote π as the share of activist citizens. We assume that if all activists vote and protest against the incumbent, the protest will succeed and incumbent will lose, i.e.,

$$\max\{p_W, p_S\} < \pi. \tag{36}$$

Incorporating the choice of voting, modify citizen i's gain from protest (both in action and in the ballot) as

$$\Delta_i \equiv U_i(1,1) - U_i(0,0) = -P_i^W(W_i - L_i) - P_i^R C_i + e^{-q}\Pr_i\{a_j = 1\} P_i^S U_i, \tag{37}$$

where

$$P_i^W \equiv \Pr_i\{B_{-i} \leq \lfloor p_W n \rfloor\} - \Pr_i\{B_{-i} + 1 \leq \lfloor p_W n \rfloor\} = \Pr_i\{B_{-i} = \lfloor p_W n \rfloor\}. \tag{38}$$

We refer to P_i^W as citizen i's *pivotal* probability of electing the incumbent. In particular, P_i^W is the probability that citizen i is the marginal citizen whose vote will be critical to electing the incumbent. Arranging the terms, it is strictly optimal for citizen i to protest after observing u_i iff

$$\Delta_i > 0 \Leftrightarrow u_i > \log\left(\frac{kP_i^W + P_i^R}{P_i^S}\right) - \log \mathbb{E}_i P(\theta) + q. \tag{39}$$

$$u^* = \mathcal{I}(\tau^*) = \log\{\exp[(1+\psi)(z_R - z_S)\tau^*] + \zeta_W \exp[(1+\psi)(z_W - z_S)\tau^*]\} \tag{40}$$

$$- \log\Phi\left(\frac{\tau^*}{\sqrt{1 + (1+\psi)^{-1}}}\right) + \xi \tag{41}$$

Lemma 5 *Under a threshold equilibrium in the extended model, citizen i follows a threshold strategy iff*

$$z_S > \max\{z_R, z_W\} - \sigma. \tag{42}$$

Proposition 8 *In the extended model, suppose the following conditions are satisfied:*

$$z_S < \min\{z_R, z_W\} + \frac{\sigma}{\psi}, \tag{43}$$

$$q < q_c, \tag{44}$$

where q_c is given in Appendix A5. There exist two non-trivial threshold equilibria. Proposition 2b, Proposition 4 and Proposition 5 hold in the extended model.

VI. HYPOTHESES

Our theoretical results provide a number of testable implications. The equlibria of our model exhibit opposite mechanisms. Formally, to test the equilibrium strategic interaction predicted by the model, we propose the following hypotheses according to Proposition 4.

Hypothesis 1a A society features *strategic substitutability* only if the number of turnout *increases* when citizens are

i. easier to connect with each other;

ii. more sympathetic to protests.

Hypothesis 1b A society features *strategic complementarity* only if the number of turnout *decreases* when citizens are

i. easier to connect with one another;

ii. more sympathetic to protests.

To address multiple equilibria in our model, we propose two procedures. One procedure is to assume that a particular equilibrium is always selected for the entire sample (although we do not know which one) and test Hypotheses 1a and 1b directly. Of course, if our model is correct but the data includes a mixture of two equilibria, insiginificant results may arise, as the comparative statics summarized in Proposition 4 differ between the two equilibria.

A more robust, albeit less powerful, procedure is to rely on quantile analysis, in the spirit of Echenique and Komunjer (2009), which we will describe in the next section. Intuitively, if we observe a larger protest, it is more likely that an equilibrium of strategic substitutability is in play, allowing us to test Hypothesis 1a with higher quantiles and vice versa for testing Hypothesis 1b. The model is rejected by the data if both Hypotheses 1a and 1b are rejected by either procedure. In this case, the data exhibits neither mechanisms of strategic substitutability nor of strategic complementarity as predicted by the model.

If either Hypotheses 1a or 1b is not rejected, we can further test the corresponding policy implication predicted by the model. For example, suppose the goverment adopts a policy involving unpopular crackdowns. The model predicts that such a crackdown will polarize the two equilibria. To formally test the policy implications predicted by the model, we propose the following hypotheses according to Proposition 5.

Hypothesis 2a Given a society featuring *strategic substitutability*, a unpoplar crackdown *increases* the number of turnout.

Hypothesis 2b Given a society featuring *strategic complementarity*, a unpoplar crackdown *decreases* the number of turnout.

If either Hypotheses 1a and 2a (for strategic substitutability), or Hypotheses 1b and 2b (for strategic complementarity) is not rejected, then we can finally test the corresponding marginal effects of the policy predicted by the model. Formally, we propose the following hypotheses according to second part of Proposition 5.

Hypothesis 3a Given a society featuring *strategic substitutability*, the effect of a unpoplar crackdown on the number of turnout is *diminishing* when citizens are

i. easier to connect with one another;

ii. more sympathetic to protests.

Hypothesis 3b Given a society featuring *strategic complementarity*, the effect of a unpoplar crackdown on the number of turnout is *amplifying* when citizens are

i. easier to connect with one another;

ii. more sympathetic to protests.

These implications are tested in our empirical analysis performed in the next section.

VII. DATA

The 2019 District Council election in Hong Kong provides a unique opportunity to study the effect of crackdown on collective actions. On February 13, 2019, the Hong Kong government introduced an extradition bill that sought to repeal the existing ban on surrendering fugitives to Mainland China.[3] Since June 2019, large-scale rallies and decentralized protests have proliferated. The police adopted a confrontational approach, including the use of force during the movement. Protests and repression remained active in the leap-up to the District Council election held in November 2019. [4] As election day approached, candidates were frequently asked to state their views on the protests, which influenced many people's voting decisions. This made the election outcomes a valuable test for our theory, as it helps rule out potential confounding factors that could explain the election outcomes.

[3]There has been no extradition agreement between Mainland China and Hong Kong. The extradition ban was established by the British colonial government before the 1997 handover, driven by concerns "about the consequences of an agreement which results in defendants being transferred from the transparent, impartial, legal system found in Hong Kong to the mainland's considerably less transparent system," according to the Tenth Report by the House of Commons Foreign Affairs Committee of the United Kingdom.

[4]All elected members serve a four-year term. District councils hold budgetary power over local projects within their constituencies. They manage public facilities, such as libraries and markets, and provide advice to the government on local policies, including transport and zoning.

VII..A. Tear Gas Deployment

To test our hypotheses, we hand-collected data on tear gas deployment across 452 constituencies (nested within 18 districts) in Hong Kong, from June 1, 2019 (the start of large scale demonstrations) to election day (November 23, 2019). We verified each incident by cross-checking over 22 news sources and media websites, as listed in Appendix XI.. The location of each incident was identified, sometimes using information from videos and photos. We avoided using existing online databases, which rely heavily on self-reports and are prone to significant Type I and Type II errors.[5] For example, we found that a popular online source missed 20% of the tear-gassed constituencies in our dataset, and about 21% of the tear-gassed constituencies in the onlien source could not be verified.[6]

According to our data set, police deployed tear gas in over 25% of the constituencies. Rarely used in the history of Hong Kong, more than 15,000 rounds of tear gas were fired between June and November, right before the election. In contrast, fewer than 100 rounds were used during the Umbrella Movement in 2014.

The use of tear gas is linked to crackdowns in our model. [7] While our data set records tear gas deployment, other invasive crowd control weapons, such as rubber rounds, super socks and 40mm react rounds, were used less frequently. When deployed, they were often used in combination with tear gas. From June to November 2019, the correlation coefficients between tear gas and these crowd control weapons were 97%, 95% and 93% respectively, according to data released by the Hong Kong Police Force. Tear gas deployment is easier to identified, even for those far from the protest sites, as it is visible and leaves a long-lasting smell. It is also easily recognized and widely reported in the the media during the movement. Since tear gas had rarely deployed before the 2019 movement in Hong Kong (with the exception of the Umbrella Movement), media reports are expected to capture geographical variations sufficiently and accurately.

[5] We did use information from groups on platofrms like Telegram to cross-validate some events. However, unless an incident was reported by at least one other independent source, and accompanied by a photo or a video that allows us to identify the location using Google Maps and Google Street View, the event is excluded.

[6] For example, HKMap.live, a popular online mapping of tear gas deployment and protests in Hong Kong, missed 26 out of 130 treated constituencies. Self-reporting tends to amplify Type I errors. According to our dataset, 28 constituencies contained false positives. For example, tear gas was reportedly smelled in a constituency in Tuen Mun but none was fired at the reported location. Sometimes, location names were abbreviated in reporets shared on information groups. For instance, Tuen Mun Tai Hing could refer to Tai Hing Estate, Tai Hing Operations Base, Tai Hing Sports Center, all of which are located in different constituencies. While we found that tear gas was reported to have been fired in only one of these constituencies, other locations were marked in some online databases. Finally, some reports confused tear gas warnings (black flag) with actual firings.

[7] Although the name "tear gas" may sound harmless, its hazardous effects on human health are well documented. Tear gas has been used in high-density residential areas and enclosed space, such as subways and shopping malls, where it exposes people to high concentrations of irritants for prolonged periods. During the Arab Spring, police in Cairo fired tear gas near a van, leading to 37 deaths, according to Reuters. Scientific research reports deaths and permanent disabilities, such as blindness, traumatic brain injury resulting in vegetative states, amputations, loss of limb function, and chronic asthma (Chan et al. 2019). Moreover, its genetic consequences, carcinogenicity, and environmental impacts remain largely unknown. Given its health impacts, the Chemical Weapons Convention explicitly prohibits its use in warfare.

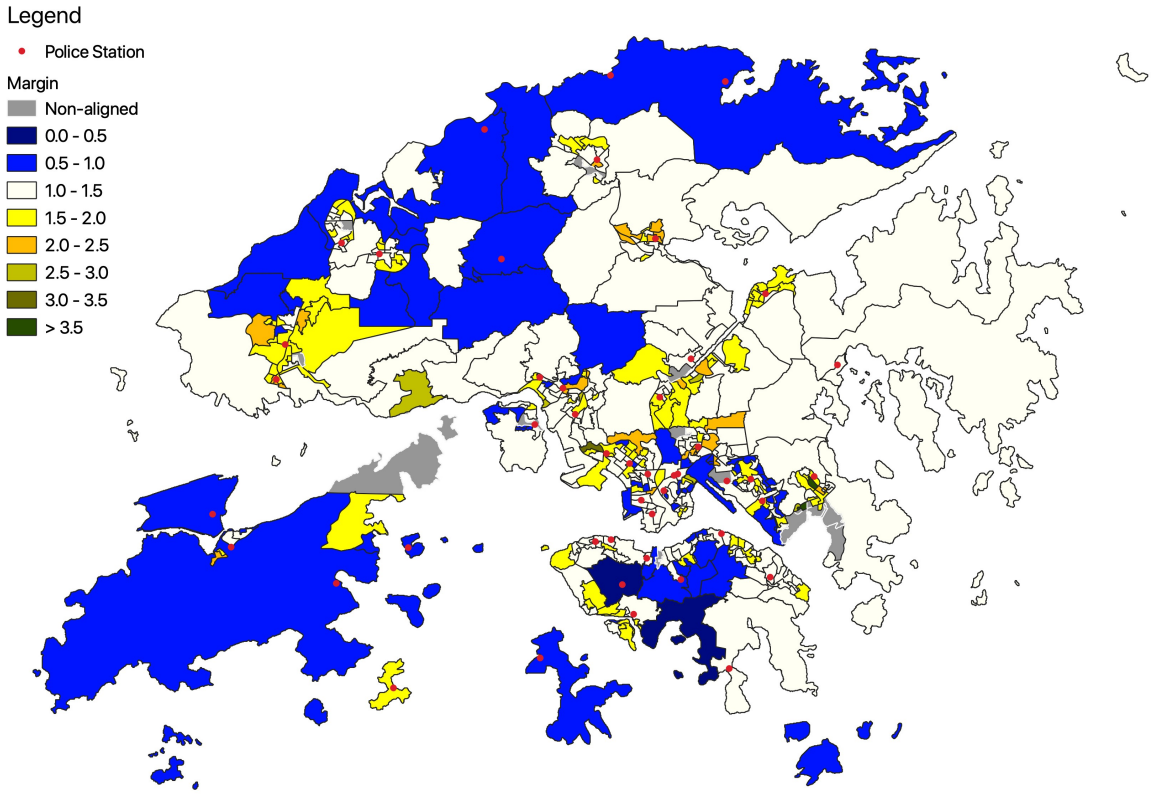


FIGURE 4: Voting Patterns in the 2019 Hong Kong District Council Election.

This map displays the locations of police stations and voting pattern of the 2019 Hong Kong District Council Election. Margins are defined as the ratio of votes for pro-protest candidates to votes for the pro-government candidates.

VII..B. Votes for the Pro-protest Candidates

The variable is defined as the log difference between the votes for pro-protest candidates and those for pro-government candidates. Figure 4 displays the election results of the election. Political alignments are determined based on the party affiliations of the candidates, who may be pro-protest, pro-government or independent. For candidates who identify themselves as independent, their alignments are classified based on their speeches and/or manifestoes.[8] No constituency was uncontested in the election. For candidates without a clear alighment, votes for them are categorized as non-aligned. As a result, when only one political camp has candidates in a constituency, the data for that constituency is treated as missing value. Election results and the number of electors are based on administrative data provided by the Registration and Electoral Office.

[8]Weeks before the election, civil society organizations and media outlets created websites that listed the political alignments of candidates based on past speeches and social media posts. Many voters used these sites as a guide to identify the political alignment of a candidate. We incorporate this information when coding the variable.

VII..C. Social Media

Our model predicts that the effects of repression depend on people's awareness of others' opinions regarding the event. The Chinese University of Hong Kong conducted a series of surveys at protest sites (CCPOS, 2020), asking protesters how they obtained and shared information about the events. Approximately 80% to 90% of respondents reported using Facebook as a source of information.[9] To measure how easily citizens can access information about the protests and learn about others' views, we use the estimated number of Facebook and Instagram users in each constituency. Based on data from January 2019 data from Facebook, Hootsuite estimated the percentage of Facebook and Instagram users in Hong Kong by their gender and age groups. We then estimated the (logged) number of Facebook and Instagramer users in each constituency using these estimates along with demographic statistics from the Census and Statistics Deparment of Hong Kong.

VII..D. Other Data

To control for heterogeneities, we include a number of control variables from various sources in our cross-sectional data set.[10] These control variables include: (1) the (logged) number of college graduates in a constituency, (2) a binary variable indicating whether a constituency is pro-government, which takes the value of 1 if a pro-government candidate (based on their political party affiliations) has won in all four previous district council elections, and 0 otherwise, (3) a similar binary variable for pro-democracy constituencies, with the same coding rule applied; and (4) the income group that a constituency falls under (high, middle, or low), based on the tertile of median income in the constituency. According to a public opinion survey (CCPOS 2020), individuals with higher levels of education and those in higher income groups were more sympathetic towards the protesters. All data were obtained from the Census and Statistics Deparment of Hong Kong and the Registration and Electoral Office websites. We use data from the 2016 population by-census to code variables describing constituency characteristics.[11] Table ?? reports the summary statistics of the variables used in our econometric analysis.

[9] Another popular platform, LIHKG, became increasingly important as the event progressed. The disucssion forum was particularly popular among younger people, who were the main supporters of the movement.

[10] Ideally, if elections were held frequently and coincided with tear gas deployment, a difference-in-differences approach could be a viable option. However, the previous district council election was held in 2015, and given the rapidly changing political landscape in Hong Kong, the parallel trend assumption is likely violated, making difference-in-differences approach unsuitable.

[11] There were boundary changes between 2016 and 2019. Since most of the changes involve dividing more populated constituencies and recombining them with part of nearby smaller constituencies, we use the values of the preceding constituencies as the estimates for the newly established constituencies. As demographic variables such as median income tend to be stable in the short run, we do not expect significant distortions from this treatment.

TABLE 1: SUMMARY STATISTICS: CONSTITUENCY CHARACTERISTICS AND VOTING PATTERNS

	Pro-protest candidates won		Tear gas was deployed		
	Yes	No	Yes	No	All
Number of electors	9372.04	7705.19	8690.19	9309.66	9139.72
Number of pro-protest votes	3907.98	2461.21	3563.30	3760.10	3706.33
Number of pro-government votes	2718.60	2791.83	2600.95	2775.89	2728.65
Median income	27226.89	30437.36	27546.15	27704.05	27660.83
Average age	43.94	43.81	43.50	44.09	43.93
Number of colleage graduates	3400.49	3231.84	3683.65	3253.30	3371.10
Number of social media users	1.46	1.45	1.44	1.47	1.46
Observations	389	63	124	328	452

Notes. The table shows the summary statistics of the main variables by election outcomes and tear gas deployment. The unit is district council constituency. The distributions of votes and electors are obtained from the Registration and Electoral Office. The socioeconomic information in each constituency comes from the Census and Statistics Department. The incidences of tear gas comes from authors' collection based on news reports. The number of social media users are estimated based on demographic data from the Census and Statistics Department and reports from Hootsuite. Details about the tear gas and social media user data contain in the Data Appendix.

VIII. IDENTIFICATION STRATEGY

The outcomes in the theoretical model are operationalized in two ways. First, the turnout in Hypotheses 1-3 is captured by $log\theta^*(g_j)$, which is measured as the difference between the logarithms of the pro-protest votes and the pro-government votes in constituency j. The model abstracts from the counter-collective action by citizens, and we normalize the pro-protest votes by the pro-government votes. However, not every constituency has a pro-government candidates standing for election. Therefore, alternatively, we measure $I_j \equiv 1_{A(g_j)p_j \geq \theta_j}$ as an indicator of whether a pro-protest candidate won the election in constituency j. Although the two cases are isomorphic in the model, in practice they are not equivalent: pro-protest candidates can receive more votes than their pro-government competitors but still lose the election due to votes being diverted to non-affilated candidates. Setting up these two alternatives provide a more robust test of our theory using the available data.

The specification is given by the following potential-outcome framework:

$$\log \theta^*(g_j) = 1\{g_j = H\} y_{H,j} + 1\{g_j = L\} y_{L,j}, \text{ where} \tag{45}$$

$$\begin{aligned} y_{H,j} &= \tau + control_j \times \beta_H + \varepsilon_{H,j}, \\ y_{L,j} &= control_j \times \beta_L + \varepsilon_{L,j}, \\ I_j &= \begin{cases} 1, \text{ if } I_j^* \geq 0, \\ 0, \text{ if } I_j^* < 0, \end{cases} \end{aligned}$$

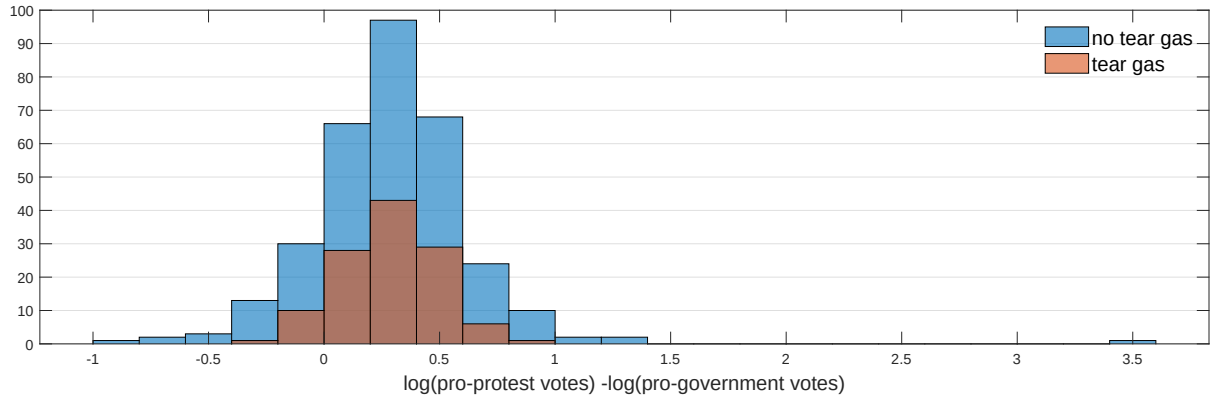


FIGURE 5: Conditional Distributions of Pro-protest Votes.

This figure shows the distributions of votes across 452 contituencies, based on whether tear gas was used in a contituency between June 1, 2019 and November 23, 2019.

where $1\{g_j = H\} = 1$ if tear gas was deployed in unit j and $control_j$ consists of the logarithms of the number of college graduates and the number of social media users, the income level of unit j, historical political allignments, and district fixed effects. The average treatment effect of crackdown is thus given by $\mathbb{E}_j(y_{H,j} - y_{L,j})$.

VIII..A. Instrument

Tear gas deployment is likely to be endogenous. For instance, a constituency with a higher proportion of pro-government citizens is less likely to elect pro-protest candidates, but it may be more prone to violent clashes with protesters. In such casese, ε_j and $1\{g_j = H\}$ are correlated, leading to biased OLS estimates that underestimate the true positive treatment effect. This potential endogeneity may explain why the distributions of pro-protest votes between the treatment and control constitutencies indistinguishable, as shown in Figure 5.

To address this concern, we instrument tear gas deployment using an exogeneous variable: the driving distance between the population center of a constituency and its nearest police station. Both of these are pre-determined before the protests and are unrelated to the political alignment of the constituency. [12] The distance were calculated using Google Maps.

A valid instrument must be both relevant and exogeneous. Our instrument is relevant because it captures variations in the endogenous variable. To maintain public order, police stations are typically located near regional population centers. Given that many protests were organized in population centers to maximize participation, the instrument is correlated with tear gas treatment. During the protest period, it was not uncommon for people to demonstrate outside politice stata-

[12]There are a few outliners at the right tail, mostly in the rural districts. The distance measure is also subject to a non-neligible measurement errors due to potential misidentifications of the population center, traffic direction, and road conditions in urban areas.

tions to express their frustration with the police. (This occurred at about 15 out of 46 police stations, according to our data set.) In most cases, the police dispersed the crowds with tear gas. These factors make our instrument sensitive to the treatment status. The closer a police station is to a population center, the more likely that a constituency was treated. A weak but relevant instrument can lead to biased estimates and misleading test results. As we will show later, the first-stage F-statistics are sufficiently large to avoid the weak instrument problem.

Our instrument also satisfies the exogeneity condition. The location of police stations is unlikley to be correlated with voters' political preferences during the 2019 election. Many of the police stations in use were established up by the British colonial government before 1997, making them unlikely to be linked to political alignment of the population. The location of these stations was influenced by various factors. Some police stations, still in operation today, were built before the Second World War (WWII) to combat piracy, triads and other criminal activities. These police stations were built along the coast, near local settlements, or on higher ground for defensive purposes (Hong Kong Police Force n.d.). Other police stations are located in new towns, which were built to accommodate Hong Kong's booming population. To maintain public order, police stations were generally established in densely populated areas. As the city expanded and evolved, new police stations were built primarily for administrative and economic efficiency, land availability, and urban planning. Givan that the location of police stations is unlikely to be correlated with political alignment after the protests, and that people with certain polical preferences are unlikely to consciously choose to live closer to or farther from a police station, our instrument generates the necessary exogenous variation to explain tear gas deployment and can be used to estimate the impact of crackdowns on voter turnout.

IX. RESULTS

In this section we present our findings from testing the hypotheses derived in Section VI.. The Centre for Communication and Public Opinion Survey (CCPOS) at the Chinese University of Hong Kong conducted six waves of surveys on public opinions regarding the anti-extradition bill movement in Hong Kong (CCPOS 2020). The survey results indicate that citizens with higher levels of education and income tend to be more supportive of the protest. As a result, in addition to testing Hypotheses 1 to 3, we also expect constituencies with higher average income levels and more educated residents to have a greater number of citizens sympathetic to the protests. Finally, following research on the impact of communication technologies on protest participation (e.g. Enikolopov et al. 2020), we hypothesize that constituencies with mroe social media users will experience lower search friction, making it easier for citizens to access information about the protests.

IX..A. Testing Hypothesis 1

Table 2 reports the estimates from OLS and three quantile regressions. In constituencies where the pro-protest margin is higher (e.g. above the 90th percent quantile), the level of education significantly predicts voting patterns and in the expected direction. A similar pattern is observed for our proxy of search frictions. Areas with more social media users saw more votes for pro-protest candidates. Additionally, we found that constituencies with a larger population from the democracy camp experienced a greater margin between the pro-protest and pro-government candidates. Overall, we find significant evidence supporting the equilibrium featuring strategic substitutability. Consequently, Hypothesis 1b is rejected in favor of Hypothesis 1a.

TABLE 2: EFFECTS OF SEARCH FRICTION AND OTHER COVARIATES

	Net log pro-protest votes			
	mean	50th	75th	90th
Not protest-sympathetic				
Middle income constituency	0.017	0.016	-0.056	-0.012
	(0.037)	(0.030)	(0.046)	(0.029)
High income constituency	-0.038	-0.023	-0.060	-0.048
	(0.060)	(0.047)	(0.065)	(0.057)
Solidly pro-gov't	-0.137****	-0.104****	-0.127****	-0.155****
	(0.027)	(0.022)	(0.030)	(0.027)
Protest-sympathetic				
Solidly pro-democracy	0.234****	0.152****	0.216***	0.190*
	(0.068)	(0.042)	(0.072)	(0.111)
Education	0.055	0.034	0.071	0.080*
	(0.048)	(0.035)	(0.049)	(0.045)
Search frictions				
Social media users	0.138	0.074	0.068	0.211**
	(0.099)	(0.073)	(0.102)	(0.101)
District FE	✓	✓	✓	✓
(Pseudo) R^2	0.288	0.153	0.144	0.200
N	437	437	437	437

Notes: The first column reports the OLS results. The second to the last columns report quantile regression results at the 50th, 75th and 90th quantiles. District fixed effects for 18 administrative districts are included in all models. Robust standard errors are reported in parentheses. $*$: p $<$ 0:10, $**$: p $<$ 0:05, $***$: p <0.01, $****$: p $<$ 0:001.

IX..B. Testing Hypotheses 2 and 3

Table 3 reports the findings under various model specifications. We can reject Hypothesis 1b, which posits an equilibrium characterized by strategic complementarity, and find significant results that support the equilibrium characterized by strategic substitutability.

TABLE 3: AVERAGE TREATMENT EFFECTS OF TEAR GAS DEPLOYMENT

	Net log pro-protest votes		
	IV	IV homo	OLS
Average Treatment Effect	0.411**	0.249***	0.027
	(0.168)	(0.076)	(0.029)
Marginal treatment effect of tear gas			
Not protest-sympathetic			
Middel income constituency	-0.040		-0.028
	(0.082)		(0.077)
High income constituency	-0.171		-0.148
	(0.114)		(0.113)
Solidly pro-gov't	-0.107		-0.120*
	(0.067)		(0.065)
Protest-sympathetic			
Solidly pro-democracy	0.176*		0.172
	(0.103)		(0.111)
Education	0.054		0.026
	(0.099)		(0.094)
Search frictions			
Social media users	0.121*		0.068
	(0.068)		(0.059)
Effective F statistic of IV	31.98	31.98	-
District FE	✓	✓	✓
N	437	437	437

Notes: Testing the average treatment effects and the marginal treatment effect on the net protest votes. The instrument is time distance between the population center of constituency and its nearest police station. Robust standard errors clustered at district level are reported in parentheses. $*$: p $<$ 0:10, $**$: p $<$ 0:05, $***$: p $<$0.01, $****$: p $<$ 0:001.

Table 3 reports the findings under different specifications. Columns 3 presents the plain vanilla regressions without the instrument. The treatment effect of repression is either not statistically sigificant (p-value = 0.338) or economically insignificant (0.027). The weak treatment effect may be due to confounders, as illustrated in Figure 5 .

Columns 1 and 2 of Table 3 present our key results based on instrumental variable estimation. To reject the null of a weak instrument, a standard F-statistic should be at least 10. However, a weak instrument problem may arise when the endogeneous variable (tear gas deployment) is binary and the excluded exogenous variable (distance from the nearest police station) is continuous. Following Xu (2021), we also compute the effective F-statistic. The critical value for a 5% test with a 10% worst-case bias is 23.1, and the effective F-statistic for our instrument is 31.98, indicating that the instrument is not weak. Using the location of police station as an instrument, we find that repression has a statistically and economically significant positive effect on support for pro-protest candidates. Specifically, after tear gas deployment, the gap between pro-protest and pro-government votes widens by 0.36 log points on average, implying that pro-protest candidates are 23% more likely to win. This result is consistent with Hypothesis 2a where an unpopular crackdown polarizes the society and increases protest turnout in an equilibrium featuring strategic substitutability.

The marginal treatment effects, reported in the remaining rows of Columns 1 and 2, align with Hypothesis 3a, which posits that the effect of an unpopular crackdown diminishes in an equilibrium with strategic substitutability. In summary, our findings provide empirical support for the model's implications and strong evidence of an equilibrium characterized by strategic substitutability.

IX..C. Robustness Checks

IX..C..1 Placebo Test

We also conduct a placebo test to assess the robustness of our findings. This test examines whether tear gas deployment predicts election outcomes in the 2015 District Council Election. Since tear gas was not deployed prior to the 2015 election, thre treatment variable should have no predictive power on the election outcomes for this year. If it does, this would suggest that tear gas deployment is non-random and endogenous, potentially driven by a confounder that simultaneously influences tear gas locations and election outcomes.

To perform this test, we re-estimate the models specified in Table 3, using the election result from 2015 to code the dependent variable. The results of the placebo test are reported in Table 4. Across all specifications, the treatment variable does not predict election outcomes in 2015. These findings imply that the voting patterns observed in 2019 are driven by the use of tear gas but not pre-existing factors.

IX..C..2 Binary Outcome

As explained earlier, candidates from a particular political camp may secure more votes than they otherwise would but still lose the election if a significant portion of votes are diverted to independent candidates. This is especially relevant for inexperienced candidates running for office. Their

TABLE 4: PLACEBO TESTS

	Net log pro-democracy votes		
	IV	IV-homo	OLS
Average Treatment Effect	0.021	0.247	0.045
	(0.353)	(0.231)	(0.051)
Control variables	✓	✓	✓
District FE	✓	✓	✓
N	300	300	300

Notes: The dependent variable is net log pro-democracy votes in the 2015 District Council Election. Estimations are based on instrumental variable regressions. Control variables include income group, education, whether a constituency were solidly pro-government or pro-democracy, and numbers of social media users. Robust standard errors clustered at district level are reported in parentheses. ∗: p < 0:10, ∗∗: p < 0:05, ∗ ∗ ∗: p <0.01, ∗ ∗ ∗∗: p < 0:001.

victories may often be interprted as protest votes against the pro-government regime. Therefore, instead of focusing solely on vote counts, we also examine whether inexperienced, pro-democracy candidates are more likely to win constituencies where tear gas was deployed. Table 5 reports these findings. Similar to the linear model results, while a naive probit estimate suggests that tear gas has no significant impacts, our IV estimates reveal a positive and significant effect after accounting for potential endogeneity. Constituencies where tear gas was deployed see an approximately 40 percentage point increase in the likelihood of pro-democracy winning. This substantial marginal effect is partly attributable to the overwhelming success of pro-democracy candidates, who secured over 85 per cent of the seats in the 2019 election.

TABLE 5: CONTROL FOR CANDIDATES' EXPERIENCE

	Inexperienced candidates win	
	IV	Probit
Average Treatment Effect	0.414***	0.123
	(0.070)	(0.134)
Control variables	✓	✓
District FE	✓	✓
N	452	452

Notes: The dependent variable is a binary variable taking value 1 if an experienced candidate win in the 2019 District Council Election. Estimations are based on instrumental variable regressions. Control variables include income group, education, whether a constituency were solidly pro-government or pro-democracy, and numbers of social media users. Robust standard errors clustered at district level are reported in parentheses. ∗: p < 0:10, ∗∗: p < 0:05, ∗ ∗ ∗: p <0.01, ∗ ∗ ∗∗: p < 0:001.

X. OTHER EVIDENCE

The previous sections demonstrates that tear gas deployment increases the wining margins of pro-protest candidates. In this section, we present additional evidence from other sources to substantiate our model assumptions and supplement our econometric results.

X..A. Social Networks and Payoff

Our payoff function addresses the importance of social factors in influencing individuals' participation decisions. In our model, payoff from participating increases when individuals' expected effective fraction of protesters M increases, and M_i is a function of social networks G_{ij}^n. According to a survey conducted by the Chinese University of Hong Kong at the protest sites in June 2019 (CCPOS 2020), many participants used private communication softwares such as WhatsApp and Telegram to distribute information about, and probably their views on the protests.[13] If only few people use them, one may question the behavioral assumption behind the payoff function. Figure 6 shows that WhatsApp was commonly used among protesters to distribute information about the protests. About 60% of respondents indicated that they used it quite often or very often. Telegram was used less often, only about 35% people used that quiet often or very often. This is possible because Telegram was less popular than WhatsApp back then. And Telegram was preferred by more radical protests who used the self-destruct function of Telegram for their actions.This partly explains why about 10% of respondents refused to answer questions related to their use of Telegram during the survey.

X..B. Role of Repression

An assumption behind the comparative static results is that mass turnout was related to repression under the unrest equilibrium. In Tullock's model, protesting citizens crowd each other out. In contrast, in our model, while a large turnout increases the chance of experiencing repression and yield a lower payoff, a large turnout can be compensated by a higher M_i. This explains why a larger turnout is possible in the presence of strategic substitutability. Given the model setup, one would expect a strong positive association between repression and turnout. The survey by CCPOS provides yet another chance to test this hypothesis based on individual level data.

Motivations to participate vary. While some people went to the street to raise international awareness, some may simply want to voice our their discontents about the bill. Repression could play no roles in explaining their actions and did not affect their decisions. At the protest sites, CC-

[13]The survey was commissioned by the Independent Police Complaints Council (IPCC), a civilian body of the Hong Kong Government and adminstered by the Centre for Communication and Public Opinion Survey (CCPOS) at the Chinese University of Hong Kong. The cited report was based on data collected from a series of surveys conducted at protest sites between June 2019 and December 2019. These surveys covered six major demonstrations led by the Civil Human Rights Front (CHRF) and 10 other larger gatherings during the period.

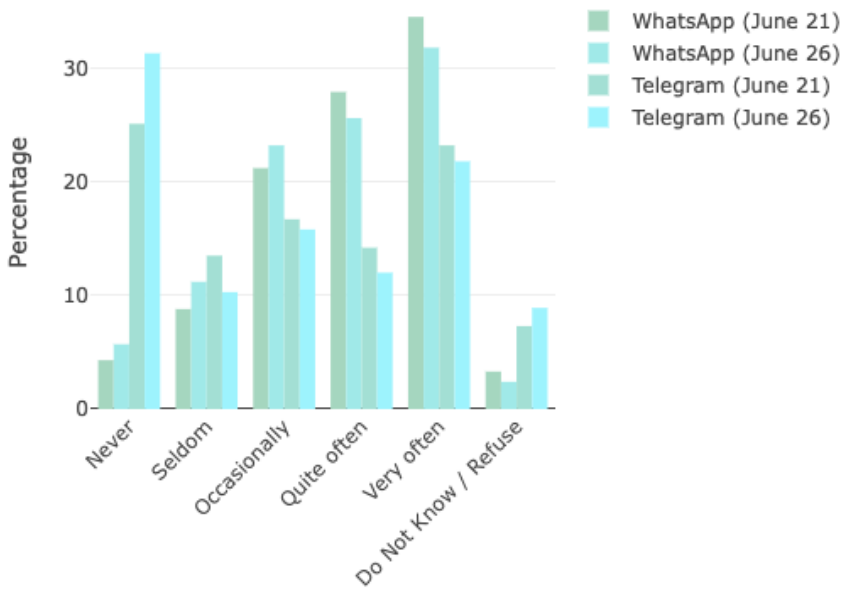


FIGURE 6: Use of Social Media in the Movement.

This histogram displays how often protesters used social media platfrom to share information about the movement. The distribution is based on reported answers to the survey question: "How often do you distribute information about the anti-extradition bill movement" through various platforms: (1) Never, (2) Seldom, (3) Occasionally, (4) Quite often, (5) Very often, (6) Don't know / Refuse to answer. The total sample sizes are 316 (June 21) and 418 (June 26). Data source: CCPOS (2020)

POS asked protesters what motivated their actions. Figure 7 presents the distribution of responses in a survey conducted on June 16, four days after the first firing of tear gas in the movement (on June 12). Among four possible responses, almost 90% of protesters stated that they rallied to express dissatisfaction with police's handling of the protests, among other concerns. In a subsequent suvery with a larger sample size, the distribution is comparable. The high percentage fails to refute the hypothesis that the massive turnouts were at least partly due to police crackdowns.

CCPOS also asked participant how would mass-scale arrests of protesters—another form of repression—affect their decisions to participate on August 10, 2019 (n = 2309). Over 75% stated that their motivation had slightly or strongly increased, compared to 7% who reported that their motivation had slightly or strongly decreased (Figure 8). Police mass arrests of protesters had motivated, rather than discouraged, participation. These responses corroborate our previous findings that repression drives collective actions under the equilibirum of strategic substitutability.

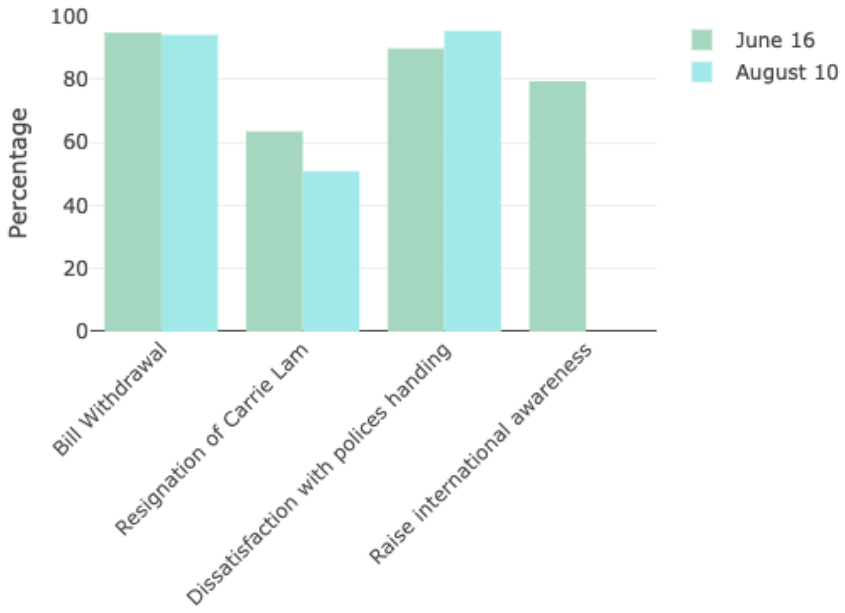


FIGURE 7: Motivations of Participation.

This histogram displays the importance of different protest motivations. The distribution is based on reported answers to the survey question: "How important are the following objectives to your participation in todays protest". In the survey on June 16, 2019, the options were: (1) Call for the withdrawal of the Fugitive Offenders Bill, (2) Call for the resignation of Carrie Lam or major officials, (3) Express dissatisfaction with the police's handling of the protesters, (4) Raise international attention. In the survey on August 10, 2019, the options were: (1) Call for the withdrawal of the Fugitive Offenders Bill, (2) Call for the resignation of Carrie Lam or major officials, (3) Express dissatisfaction with the police's handling of the protesters, (4) Express dissatisfaction with police charging the arrested protesters for rioting, (5) Strive for Hong Kong's democracy / dual universal suffrage, (6) Call for establishment of an independent commission of inquiry. Options (4) to (6) were not available in the June 16 survey, and the option "Raise international attention" was not available in the August 10 survey. Multiple selections were possible in both instances. The total sample sizes are 875 (June 16) and 2309 (August 10). Data source: CCPOS (2020)

X..C. Alignment between Protest and Voting Behaviors

A key assumption behind the extended model in Section V. is that participation and voting pattern is highly correlated. While it is difficult to test the relationship directly as voting data is not available in general, in this section we assess to what extent this assumption is justified given the case at hand.

The search model in Section V. involves two types of citizens, conservatives and activists. CCPOS surveys at the protest sites which we examined before would have limited values due to sample selection bias. Fortunately, the IPCC also tasked the CCPOS with conducting telephone surveys between May 23, 2019 and December 13, 2019, to guage the opinion of the wider public,

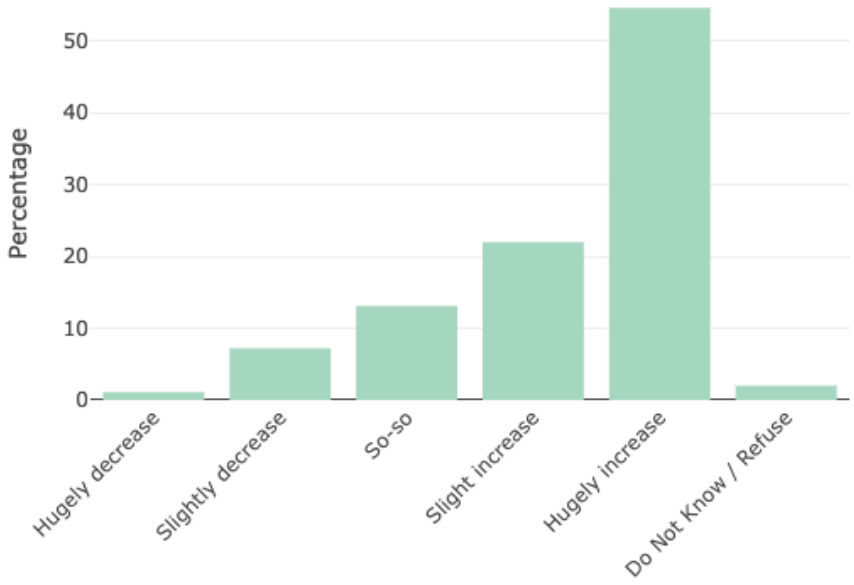


FIGURE 8: Importance of Repressions to Participation.

This histogram displays the importance of repressions to protest participation. The distribution is based on reported answers to the survey question: "How will the polices mass-scale arrests of protesters affect your motivation to participate in protests": (1) Hugely decrease, (2) Slightly decrease, (3) So-so, (4) Slightly increase, (5) Hugely increase, (6) Don't know / Refuse to answer. The survey was conducted on August 10, 2019 at protest sites, with a sample size of 2309. Data source: CCPOS (2020)

including non-protesters. One question in the survey asked respondents whether they were satisfied with police performance 'in conducting daily functions unrelated to public events' and 'during the anti-extradition bill movement?'

"Do you support the anti-extradition bill movement?". Of the 2008 valid responses, 18.0% of respondents were very or somewhat against the movement, while 62.3% were very or somewhat supportive. This pattern aligns with the voting behaviors explored in Section IX..

CCPOS also surveyed opinions on the use of tear gas, rubber bullets and pepper spray against the protesters. As shown in Figure 10, opinions were highly polarized, depending on the political orientation of the respondents. This the mean score on an 11-point Likert scale (0 = very unacceptable, 5 = so-so, 10 = very acceptable) was 2.54. This finding is consistent with responses from the protesters.

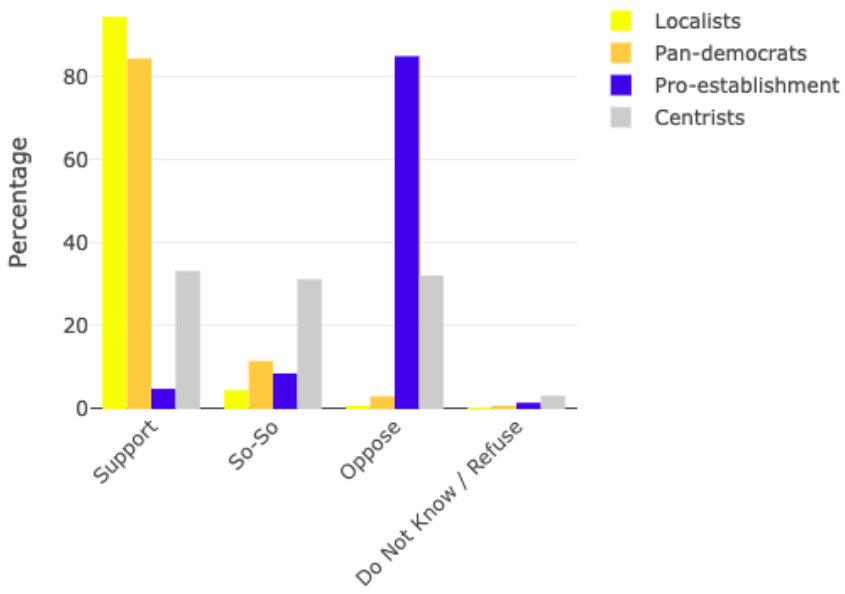


FIGURE 9: Political Orientations and Views on the Anti-extradition Bill Movement

This histogram displays public opinions on the anti-extradition bill movement conditional on political orientation. The distribution is based on reported answers to the survey question: "Do you support the anti-extradition bill movement? Are you very supportive, somewhat supportive, half-half, somewhat against or very against?" Information on political orientation was baesd on another survey question. Data was collected in a telephone survey conducted in November 2019. Sample size = 2008. Data source: CCPOS (2020)

X..D. Google Trends

Our model suggests that people's interest in joining a protest increases in response to repression. However, participation involve risks, and a signal of intent may not always turn into actions. In this subsection, we use time-series data from Google Trends to explore the relationship between tear gas deployment and intentions to protest.

Because Google searches are less subject to social censoring bias, Google Trends data can provide more truthful insights that may not be easily elicited through in-person or telephone interviews (Stephens-Davidowitz 2014). We expect that measures based on Google search data will be more sensitive to the effects of repression. To examine whether repression motivates more people to protest, we combine daily tear gas deployment data from the Hong Kong Police Force with weekly Google Trends data. The variable of interest is the Google Trends popularity index, which ranges from 0 to 100. A value of 100 represents peak search interest for a given region and time interval. We retrieved the indices for various queries containing terms like 'protest schedule', 'protest time', 'hk protest time' and their Chinese-language variants. These indices were averaged to produce a

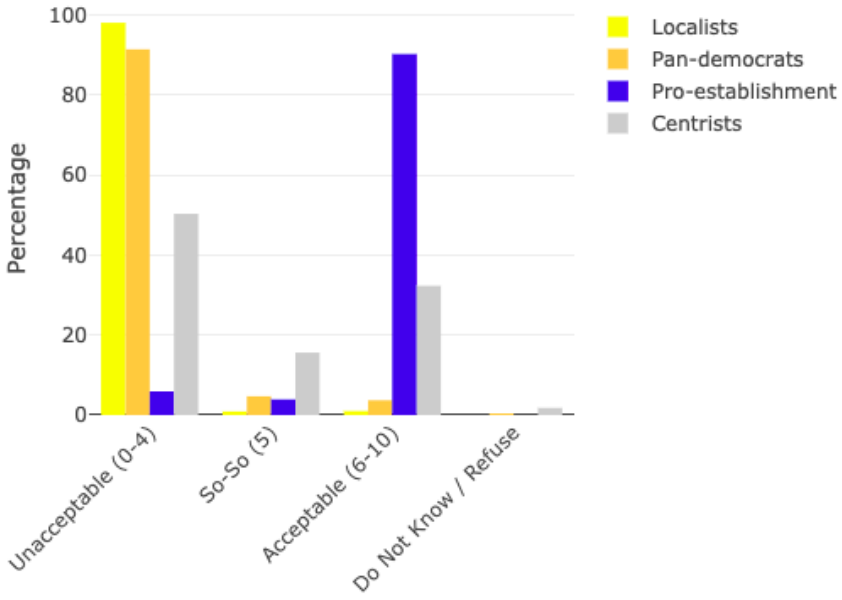


FIGURE 10: Political Orientations and Opinions on Repression

This histogram displays public opinions on the use of violence against the protesters during the movement. The level of acceptance ranges from 0 (unacceptable) to 10 (acceptable). The distribution is based on reported answers to a question asking respodents how acceptable it was to disperse protesters with tear gas, rubber bullet, and pepper spray. Information on political orientation was baesd on another survey question. Data was collected in a telephone survey conducted in November 2019. Sample size = 1990. Data source: CCPOS (2020)

single measure of interest. The resulting average scores, plotted as bars in Figure 11, represent public interest in joining protests in a given week (Google Trends does not provide daily data).

The pattern in Figure 11 indicates that the teat gas deployment and search interests are positively correlated. An exception occurs in mid-November, where the number of tear gas deployment peaked. During this period, police entered the campus of the Chinese University of Hong Kong and the Hong Kong Polytechnic University, clashing with students. As the election date (November 24, 2019) approached, no protests were organized to avoid potential cancellation or postponement. One interpretation of the high voter turnout rate, which increases from 47.01% in 2015 to 71.23% in 2019, is that it was a response to the mid-November campus attacks. In summary, the positive correlation between tear gas deployment and protest-related search interest is consistent with a key prediction of our model: repression backfires when people perceive that the fundamental is sufficiently high.

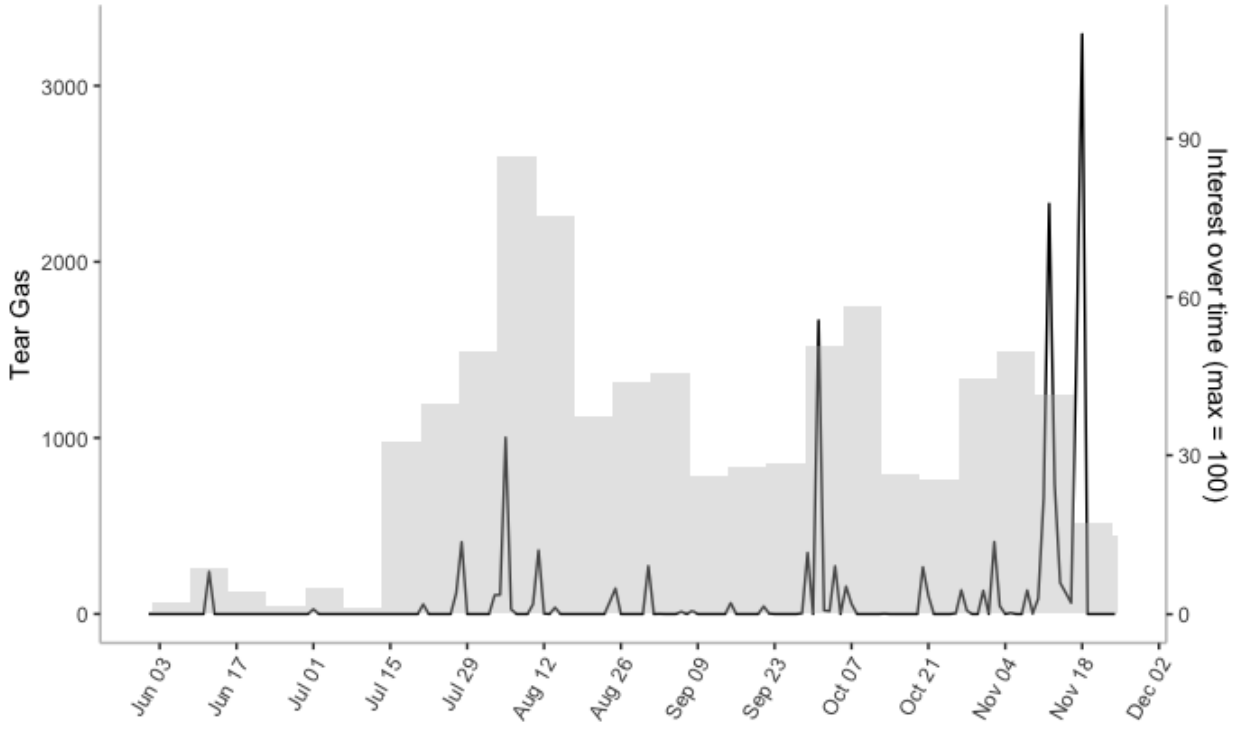


FIGURE 11: Tear gas Deployment and Interest in Protest Participation.

This figure displays the number of tear gas deployed by the police per day during the protests. The number is represented by the line. Data comes from a report by Radio Television Hong Kong (RTHK) based on data provided by the Hong Kong Police Force. Interests in protests are represented by shade. A higher number represents people searched for information about protest schedule more intensively in a particular week. Data was from Google Trends.

X..E. Media Interviews

While the quantitative evidence discussed above provides additional support for our econometric findings, this section presents qualitative evidence from media reports to justifies some behavioural assumptions of the model. Although these reports are subject to media bias, they validate the model's assumptions and demonstrates that it explains the behavior of some protesters.

One key assumption is that the actions of citizens and activists are complementary in generating the power of collective action. Several media reports reveal that the participation and actions of others often encouraged individual involvement. For example, as documented by a BBC reporter, many protesters believed that their actions could increase pubic awareness and induce further repression. They were aware of the risks involved and continuously balanced these considerations. [14] A female student activist noted that knowing may others shared similar goals is essential to her participation. In such cases, a citizen's payoff is influenced by their ability to find others

[14]BBC (January 1 2020) The stories behind the protest in Hong Kong (in Chinese). https://www.bbc.com/zhongwen/trad/chinese-news-50915865.

with shared beliefs, even within a frictional environment. [15]Furthermore, as a female protester explained, the more people who take to the streets, the less dangerous it becomes to participate. [16] In the language of our model: an individual's actions complement those of others, collectively strengthening the movement. Both sides anticipated that that their actions would reduce the perceived costs of repression and inspire others to join, creating a cascade effect.

XI. CONCLUSION

This paper shows that, in equilibrium, a mass movement can occur in the presence of strategic substitutability. In a protest with high turnout, the addition of another participant increases search friction, but simultaneously, each new participant becomes privotal to repression. The latter effect dominates in the equilibrium characterized by strategic substitutability. The key insight is that strategic interaction is endogenous. This new mechanism rooted in search frictions offers a solution to Tullock's paradox of revolution. Multiple equilibria emerge when search frictions are low or when the goals of protest are modest, situations that are increasingly common in the era of social media. An implication of this is that an unpopular crackdown will polarize a society: it enlarges a large protest under the equilibrium of strategic substitutability, while shrinking a small protest under the equilibrium of strategic complementarity. Our empirical findings provide a strong support to the policy implications of the former, particularly in the case of Hong Kong. The latter channel also explains why some authoritarian regimes adopt confrontational approaches to dissent, hoping that their society being in an equilibrium of strategic complementarity. However, repression has its consequences. By suppressing citizens, a self-interested regime focused solely on its survival may ultimately hasten its own downfall.

The possibility of backlash and its impact on political and social change is well-documented in the literature. Several conditions and mechanisms for backlash have been proposed, with a particular emphasis on leadership and revolutionary entrepreneurs (e.g., Bueno de Mesquita 2010; Dippel and Heblich 2021). Francisco (2004) argues that backlash occurs when (1) news of repression is widely disseminated; (2) leadership is strong; and (3) the cost of joining the backlash is moderate or low. While we share the emphasis on the importance of information, our mechanism highlights the role of decentralization in shaping belief and coordinate collection action - key elements in a global game. Niether strong leadership nor formal organizations are necessary in our model. This feature aligns with the decentralized nature of the Hong Kong protests, where no single organization could claim leadership.

According to classical deterrence arguments, repression should reduce the likelihood of backlash.

[15]BBC (June 15, 2019) The extradition bill controversy. The confession of two protesters (in Chinese). https://www.bbc.com/zhongwen/trad/chinese-news-48634216.

[16]Stand News (July 22, 2019) An interview with a female protester charged with rioting. (In Chinese) http://shorturl.at/fwDH5.

This reasoning is tied to Olson's (1965) selective incentive mechanism, which suggests that monetary or other incentives can internalize social benefits and address the collective action problem. Repression, in this framework, acts as a negative incentive by increasing the cost of participation. However, while the theory predicts that repression strictly reduces the chance of backlash, our model shows that repression itself can serve as a coordination device, a costly signal that is more credible than a cheap talk on the fundamentals. In this sense, a high participation costs can make backlash *more* likely.

Most models of collective action emphasize the role of communication and learning from the regime or peers (e.g., Angeletos et al. 2006, 2007). Barbera and Jackson (2020), for example, examine learning within networks and show how homophilic network can either facilitate or hinder backlash. As they note, crowds can sometimes misinterpret singals. In contrast, our model focuses on learning through the actions of the regime rather than peers or neighbors. We demonstrate that when a government's repression is sensitive to the threat posed by activists, the regime's actions provide new information. This, in turn, helps coordinate collective action.

REFERENCES

Acemoglu, Daron, Tarek Alexander Hassan, and Ahmed Tahoun, "The Power of the Street: Evidence from Egypt's Arab Spring," *Review of Financial Studies*, 31 (2018), 1–42.

Angeletos, George-Marios, Christian Hellwig, and Alessandro Pavan, "Signaling in a Global Game: Coordination and Policy Traps," *Journal of Political Economy*, 114 (2006), 452–484.

Angeletos, George-Marios, Christian Hellwig, and Alessandro Pavan, "Dynamic Global Games of Regime Change: Learning, Multiplicity, and the Timing of Attacks," *Econometrica*, 75 (2007), 711–756.

Barbera, Salvador, and Matthew O. Jackson, "A Model of Protests, Revolution, and Information," *Quarterly Journal of Political Science*, 15 (2020), 297–335.

Besley, Timothy, and Torsten Persson, "The Logic of Political Violence," *Quarterly Journal of Economics*, 126 (2011), 1411–1445.

Bueno de Mesquita, Ethan, "Regime Change and Revolutionary Entrepreneurs," *American Political Science Review*, 104 (2010), 446–466.

Bursztyn, Leonardo, David Cantoni, David Y. Yang, Noam Yuchtman, and Y. Jane Zhang, "Persistent Political Engagement: Social Interactions and the Dynamics of Protest Movements," *American Economic Review: Insights*, 3 (2021), 233–250.

Cantoni, Davide, David Y. Yang, Noam Yuchtman, and Y. Jane Zhang, "Protests as Strategic Games: Experimental Evidence from Hong Kong's Antiauthoritarian Movement," *Quarterly Journal of Economics*, 134 (2019), 1021–1077.

Cantoni, David, Andrew Kao, David Y. Yang, and Noam Yuchtman, "Protests," *Annual Review of Economics*, 16 (2024), 519–543.

Castells, Manuel, *Networks of Outrage and Hope: Social Movements in the Internet Age* (Cambridge: Polity Press, 2015).

CCPOS, "Research Report on Public Opinion during the Anti-Extradition Bill (Fugitive Offenders Bill) Movement in Hong Kong" (The Chinese University of Hong Kong, 2020).

Chan, Emily Ying Yang, Kevin Kei Ching Hung, Heidi Hoi Yi Hung, and Colin A. Graham, "Use of Tear Gas for Crowd Control in Hong Kong," *The Lancet*, 394 (2019), 1517–1518.

Dippel, Christian, and Stephan Heblich, "Leadership in Social Movements: Evidence from the 'Forty-Eighters' in the Civil War," *American Economic Review*, 111 (2021), 472–505.

Duffy, John, and Margit Tavit, "Beliefs and Voting Decisions: A Test of the Pivotal Voter Model," *American Journal of Political Science*, 52 (2008), 603–618.

Echenique, Federico, and Ivana Komunjer, "Testing Models with Multiple Equilibria by Quantile Methods," *Econometrica*, 77 (2009), 1281–1297.

Edmond, Chris, "Information Manipulation, Coordination, and Regime Change," *Review of Economic Studies*, 80 (2013), 1422–1458.

Enikolopov, Ruben, Alexey Makarin, and Maria Petrova, "Social Media and Protest Participation: Evidence from Russia," *Econometrica*, 88 (2020), 1479–1514.

Francisco, Ronald, "After the Massacre: Mobilization in the Wake of Harsh Repression," *Mobilization: An International Quarterly*, 9 (2004), 107–126.

Hager, Anselm, Lukas Hensel, Johannes Hermle, and Christopher Roth, "Group Size and Protest Mobilization Across Movements and Countermovements," *American Political Science Review*, 116 (2022), 1051–1066.

Hager, Anselm, Lukas Hensel, Johannes Hermle, and Christopher Roth, "Political Activists as Free Riders: Evidence from a Natural Field Experiment," *Economic Journal*, 133 (2023), 2068–2084.

Hong Kong Police Force, "Police History" (https://www.police.gov.hk/ppp_en/01_about_us/ph.html, accessed December 8, 2020).

Manacorda, Marco, and Andrea Tesei, "Liberation Technology: Mobile Phones and Political Mobilization in Africa," *Econometrica*, 88 (2020), 533–567.

Morris, Stephen, and Hyun Song Shin, "Unique Equilibrium in a Model of Self-Fulfilling Currency Attacks," *American Economic Review*, 88 (1998), 587–597.

Myatt, David P., "A Theory of Protest Voting," *Economic Journal*, 127 (2017), 1527–1567.

Olson, Mancur, *The Logic of Collective Action* (Cambridge: Harvard University Press, 1965).

Passarelli, Francesco, and Guido Tabellini, "Emotions and Political Unrest," *Journal of Political Economy*, 125 (2017), 903–946.

Shadmehr, Mehdi, "Tullock's Paradox, Pivotal Revolutionaries, and the Strength of Weak States," *Quarterly Journal of Political Science*, 16 (2021), 245–264.

Shadmehr, Mehdi, and Dan Bernhardt, "State Censorship," *American Economic Journal: Microeconomics*, 7 (2015), 280–307.

Shadmehr, Mehdi, and Dan Bernhardt, "Collective Action with Uncertain Payoffs: Coordination, Public Signals, and Punishment Dilemmas," *American Political Science Review*, 105 (2011), 829–851.

Stephens-Davidowitz, Seth, "The Cost of Racial Animus on a Black Candidate: Evidence Using Google Search Data," *Journal of Public Economics*, 118 (2014), 26–40.

Tullock, Gordon, "The Paradox of Revolution," *Public Choice*, 11 (1971), 89–99.

Xu, Ruonan, "On the Instrument Functional Form with a Binary Endogenous Explanatory Variable," *Economics Letters*, 206 (2021), 109993.

Zhuravskaya, Ekaterina, Maria Petrova, and Ruben Enikolopov, "Political Effects of the Internet and Social Media," *Annual Review of Economics*, 12 (2020), 415–438.

APPENDIX: PROOFS

Proof of Lemma 1. Recall the pdf of a beta distribution is given by

$$g_{\alpha,\beta}(x) = \frac{\beta!}{\alpha!(\beta-\alpha-1)!}x^{\alpha}(1-x)^{\beta-\alpha-1},$$

where its mean and variance are given by

$$\mathbb{E}x = \frac{\alpha+1}{\beta+1}, VAR(x) = \frac{(\alpha+1)(\beta-\alpha)}{(\beta+1)^2(\beta+2)^2}.$$

Set $\alpha = \lceil pn \rceil$ and $\beta = n$ for some $p \in (0,1)$ and take n to infinity at the limit. Since the mean tends to p and the variance tends to 0, we must have $g_{\lceil pn \rceil,n}(x)$ tends to a degenerate distribution with point mass at p. Hence, we have

$$\begin{aligned}\lim_{n\to\infty}\log\frac{P_i^R}{P_i^S} &= \lim_{n\to\infty}\log\frac{nP_i^R}{nP_i^S} = \log\lim_{n\to\infty}\frac{\int g_{\lceil p_S n\rceil,n}(\theta) f_i(\theta)\,d\theta}{\int g_{\lceil p_R n\rceil,n}(\theta) f_i(\theta)\,d\theta} = \log\frac{f_i\left[P^{-1}(p_R)\right]}{f_i\left[P^{-1}(p_S)\right]} \\ &= \frac{1+\psi}{\sigma}(z_R - z_S)(\theta_i - u^*) - \frac{1+\psi}{2}\left(z_R^2 - z_S^2\right),\end{aligned}$$

where $P^{-1}(p) = u^* + \sigma\Phi^{-1}(p)$ and

$$f_i(\theta) = \frac{1}{\sigma}\sqrt{\frac{1+\psi}{2\pi}}\exp\left[-\frac{1+\psi}{2\sigma^2}(\theta-\theta_i)^2\right].$$

Finally, note that

$$M_i = \int P(\theta) f_i(\theta)\,d\theta = \int \Pr\{u_j > u^*|\theta\} f_i(\theta)\,d\theta = \Pr\{\theta + \epsilon_j > u^*|_{u_i}\} = \Phi\left(\frac{\theta_i - u^*}{\sigma\sqrt{1+(1+\psi)^{-1}}}\right).$$

QED.

Proof of Lemma 2. Every citizen follows a threshold strategy iff there exists u^* such that $u_j > u^*$ is equivalent to $\Delta_j > 0$. Using Lemma 1, under a threshold equilibrium $\Delta_j > 0$ is equivalent to

$$0 > \mathcal{I}\left[\frac{1}{\sigma\sqrt{1+(1+\psi)^{-1}}}\left(\frac{\psi\bar{\theta}+u_j}{\psi+1} - u^*\right)\right] - u_j.$$

Thus, every citizen follows a threshold strategy iff the RHS of the above has a single crossing at zero with $u_j = u^*$. Expanding the RHS, we have

$$-\left(1 - \frac{z_R - z_S}{\sigma}\right)u_j - \log\Phi\left[\frac{\frac{\psi\bar{\theta}+u_j}{\psi+1} - u^*}{\sigma\sqrt{1+(1+\psi)^{-1}}}\right] + \text{ constant}.$$

Thus, to have a single crossing at zero is necessary and sufficient to have the above monotone in u_j, which implies $1 - \frac{z_R - z_S}{\sigma} > 0$, i.e., condition (19). **QED.**

Proof of Proposition 1. (a) is straightforward to verify. Note that any non-trivial threshold equilibrium τ^* solves $\mathcal{I}(\tau^*) - \mathcal{B}(\tau^*) = 0$. We need to characterize the shape of $\mathcal{I}(\tau) - \mathcal{B}(\tau)$. Note that

$$\begin{aligned}\mathcal{B}'(\tau) &= -\frac{\sigma}{\psi}\sqrt{(1+\psi)(2+\psi)}, \\ \mathcal{I}'(\tau) &= (z_R - z_S)\sqrt{(1+\psi)(2+\psi)} - \phi(\tau).\end{aligned}$$

Also, due to the fact that Φ is log-concave, $\phi(z)$ is positive, strictly decreasing function with $\phi(-\infty) = \infty$ and $\phi(\infty) = 0$. To prove (b), suppose condition (21) is not satisfied, then we have $\mathcal{I}'(\tau) - \mathcal{B}'(\tau)$ is negative, with $\mathcal{I}(-\infty) - \mathcal{B}(-\infty) = \infty$ and $\mathcal{I}(\infty) - \mathcal{B}(\infty) = -\infty$. So there must exist a unique τ^* such that $\mathcal{I}(\tau^*) = \mathcal{B}(\tau^*)$. To prove (c), suppose condition (21) is satisfied, then $\mathcal{I}(\tau) - \mathcal{B}(\tau)$ is a U-shape function with $\mathcal{I}(-\infty) - \mathcal{B}(-\infty) = \mathcal{I}(\infty) - \mathcal{B}(\infty) = \infty$, and its global minimum at τ_c, where τ_c solves the FOC $\mathcal{I}'(\tau_c) = \mathcal{B}'(\tau_c)$. Note that $\mathcal{I}(\tau_c) - \mathcal{B}(\tau_c) = q - q_c$. If $q - q_c < 0$, then $\mathcal{I}(\tau) - \mathcal{B}(\tau)$ must cross zero twice as $\mathcal{I}(\tau) - \mathcal{B}(\tau)$ is U-shape. Thus, there must exist two non-trivial threshold equilibria in this case, i.e., we have proved (c.i). Simillarly, if $\mathcal{I}(\tau_c) - \mathcal{B}(\tau_c) = q - q_c > 0$, then $\mathcal{I}(\tau) - \mathcal{B}(\tau)$ must never cross zero. Thus, there does not exist any non-trivial threshold equilibria in this case, i.e., we have proved (c.iii). The tangential case $\mathcal{I}(\tau_c) - \mathcal{B}(\tau_c) = q - q_c = 0$ is (c.ii), where $\mathcal{I}(\tau) - \mathcal{B}(\tau)$ touches zero at $\tau = \tau_{\min}$. **QED.**

Proof of Proposition 2. According to the proof of Proposition 1, in the case of Proposition 1b, we have $\mathcal{I}'(\tau) - \mathcal{B}'(\tau) < 0$ for all τ, so the unique non-trivial threshold equilibrium features strategic complementarity. In the case of Proposition 1ci, denote τ_1 and τ_2 as the two roots to $\mathcal{I}(\tau) - \mathcal{B}(\tau) = 0$, where $\tau_1 < \tau_2$. Since $\mathcal{I}(\tau) - \mathcal{B}(\tau)$ is U-shape in this case, we must have $\mathcal{I}(\tau_1) - \mathcal{B}(\tau_1) < 0 < \mathcal{I}(\tau_2) - \mathcal{B}(\tau_2)$. Thus, the threshold equilibrium with a higher expected turnout (τ_2) features strategic substitutability; the other threshold equilibrium with a lower expected turnout (τ_1) features strategic complementarity. **QED.**

Proof of Proposition 3. Note that

$$q_c = -\min_{\tau_c}\left\{\left(z_R - z_S + \frac{\sigma}{\psi}\right)\sqrt{(1+\psi)(2+\psi)}\tau_c - \log\Phi(\tau_c) - \frac{1+\psi}{2}\left(z_R^2 - z_S^2\right) - \bar{\theta}\right\}.$$

Applying an envelope theorem, we have

$$\frac{dq_c}{dp_S} = \frac{dq_c}{dz_S}\frac{dz_S}{dp_S} = -\sqrt{(1+\psi)(2+\psi)}\left[\tau_c - \frac{z_S}{\sqrt{1+(1+\psi)^{-1}}}\right]\frac{1}{\Phi'(z_s)}$$

which is negative when z_s (i.e., p_s) is low. Similarly, we have $\frac{dq_c}{d\bar{\theta}} = 1 > 0$. Finally, note that $\frac{dq_c}{d\sigma} = -\sqrt{(1+\psi)(2+\psi)}\frac{\tau_c}{\psi}$, which positive when σ is low since τ_c is decreasing in σ. **QED.**

Proof of Proposition 4. Recall that

$$\mathcal{I}(\tau) - \mathcal{B}(\tau) \equiv \left(z_R - z_S + \frac{\sigma}{\psi}\right)\sqrt{(1+\psi)(2+\psi)}\tau - \log\Phi(\tau) + q - \frac{1+\psi}{2}\left(z_R^2 - z_S^2\right) - \bar{\theta}.$$

In general, the comparative statics w.r.t. x at the threshold equilbrium τ^* is given by

$$\begin{aligned}\frac{d\tau^*}{dx} &= \frac{\frac{d}{dx}\left[\mathcal{I}(\tau^*) - \mathcal{B}(\tau^*)\right]}{\mathcal{B}'(\tau^*) - \mathcal{I}'(\tau^*)},\\ \frac{du^*}{dx} &= \frac{d}{dx}\mathcal{B}(\tau^*) + \mathcal{B}'(\tau^*)\frac{d\tau^*}{dx},\end{aligned}$$

where

$$\mathcal{B}'(\tau^*) = -\frac{\sigma}{\psi}\sqrt{(1+\psi)(2+\psi)},$$

$$\begin{aligned}\frac{d}{dq}\left[\mathcal{I}(\tau^*) - \mathcal{B}(\tau^*)\right] &= 1, \frac{d}{dq}\mathcal{B}(\tau^*) = 0,\\ \frac{d}{d\bar{\theta}}\left[\mathcal{I}(\tau^*) - \mathcal{B}(\tau^*)\right] &= -1 = -\frac{d}{d\bar{\theta}}\mathcal{B}(\tau^*)\end{aligned}$$

$$\mathcal{B}'(\tau^*) - \mathcal{I}'(\tau^*) \begin{array}{l} > 0 \text{ if strategic complementarity,}\\ < 0 \text{ if strategic substitutability.}\end{array}$$

Thus we have

$$\begin{aligned}\frac{d\tau^*}{dq} &= -\frac{d\tau^*}{d\bar{\theta}} \begin{array}{l} > 0 \text{ if strategic complementarity,}\\ < 0 \text{ if strategic substitutability.}\end{array}\\ \frac{du^*}{dq} &= -\frac{\sigma}{\psi}\sqrt{(1+\psi)(2+\psi)}\frac{d\tau^*}{dq} \begin{array}{l} < 0 \text{ if strategic complementarity,}\\ > 0 \text{ if strategic substitutability.}\end{array}\end{aligned}$$

$$\frac{du^*}{d\bar{\theta}} = 1 - \frac{1}{1 + \mathcal{I}'(\tau^*)\frac{\psi}{\sigma\sqrt{(1+\psi)(2+\psi)}}} \begin{array}{l} > 0 \text{ if strategic complementarity,}\\ \qquad \text{or strategic substitutability and } \mathcal{I}'(\tau^*) > 0\\ < 0 \text{ if strategic substitutability and } \mathcal{I}'(\tau^*) < 0\end{array}$$

QED.

Proof of Proposition 5. Note that, from the proof of Proposition 4, we have

$$\frac{d\tau^*}{dx} = \theta_x\frac{d\tau^*}{d\theta} + q_x\frac{dq}{dx}\frac{d\tau^*}{dq} = -(\theta_x - q_x)\frac{d\tau^*}{dq} \begin{array}{l} < 0 \text{ if strategic complementarity,}\\ > 0 \text{ if strategic substitutability.}\end{array}$$

Finally,

$$\frac{d^2\tau^*}{dqdx} = -(\theta_x - q_x)\frac{d^2\tau^*}{dq^2} = (\theta_x - q_x)\frac{\mathcal{B}''(\tau^*) - \mathcal{I}''(\tau^*)}{\left[\mathcal{B}'(\tau^*) - \mathcal{I}'(\tau^*)\right]^2}\frac{d\tau^*}{dq} \begin{array}{l} > 0 \text{ if strategic complementarity,}\\ < 0 \text{ if strategic substitutability.}\end{array}$$

$$\frac{d^2\tau^*}{d\theta dx} = -(\theta_x - q_x)\frac{d^2\tau^*}{d\theta dq} = (\theta_x - q_x)\frac{\mathcal{B}''(\tau^*) - \mathcal{I}''(\tau^*)}{\left[\mathcal{B}'(\tau^*) - \mathcal{I}'(\tau^*)\right]^2}\frac{d\tau^*}{d\theta} \begin{array}{l} < 0 \text{ if strategic complementarity,}\\ > 0 \text{ if strategic substitutability.}\end{array}$$

QED.

Proof of Proposition 6. Recall that τ^* solves

$$\begin{aligned} 0 &= \frac{\mathcal{I}(\tau^*) - \mathcal{B}(\tau^*)}{\left(z_R - z_S + \frac{\sigma}{\psi}\right)\sqrt{(1+\psi)(2+\psi)}} \\ \Leftrightarrow \quad \tau^* &= \frac{z_R^2 - z_S^2}{2\left(z_R - z_S + \frac{\sigma}{\psi}\right)\sqrt{1+(1+\psi)^{-1}}} + \frac{\log\Phi(\tau^*) - q + \bar{\theta}}{\left(z_R - z_S + \frac{\sigma}{\psi}\right)\sqrt{(1+\psi)(2+\psi)}}. \end{aligned}$$

Also, we have

$$u^* = \mathcal{B}(\tau^*) = \bar{\theta} - \frac{\sigma}{\psi}\sqrt{(1+\psi)(2+\psi)}\tau^*,$$

or equilivalently,

$$u^* = \mathcal{I}(\tau^*) \equiv (z_R - z_S)\sqrt{(1+\psi)(2+\psi)}\tau_i - \log\Phi(\tau_i) + q - \frac{1+\psi}{2}\left(z_R^2 - z_S^2\right).$$

Thus, we have

$$\begin{aligned} \lim_{\sigma\to\infty}\tau^* &= 0, \ \lim_{\sigma\to\infty}u^* = \lim_{\sigma\to\infty}\mathcal{I}(\tau^*) = \log 2 + q - \frac{1+\psi}{2}\left(z_R^2 - z_S^2\right); \\ \lim_{\psi\to\infty}\tau^* &= \frac{z_R^2 - z_S^2}{2(z_R - z_S)} = \frac{z_R + z_S}{2}, \ \lim_{\sigma\to\infty}u^* = \lim_{\sigma\to\infty}\mathcal{B}(\tau^*) = \bar{\theta} - \sigma\frac{z_R + z_S}{2}; \\ \lim_{\psi\to 0}\tau^* &= 0, \ \lim_{\sigma\to\infty}u^* = \lim_{\sigma\to\infty}\mathcal{I}(\tau^*) = \log 2 + q - \frac{1}{2}\left(z_R^2 - z_S^2\right) \end{aligned}$$

where $-\log\Phi(0) = \log 2$. The slope difference is given by

$$\mathcal{B}'(\tau^*) - \mathcal{I}'(\tau^*) = \left(-z_R + z_S - \frac{\sigma}{\psi}\right)\sqrt{(1+\psi)(2+\psi)} - \phi(\tau^*).$$

Thus, we have

$$\begin{aligned} \lim_{\sigma\to\infty}\mathcal{B}'(\tau^*) - \mathcal{I}'(\tau^*) &= -\infty \Rightarrow \text{strategic substitutability}, \\ \lim_{\psi\to\infty}\mathcal{B}'(\tau^*) - \mathcal{I}'(\tau^*) &= sign(z_R - z_S)\infty \Rightarrow \begin{cases} \text{strategic substitutability if } z_S < z_R, \\ \text{strategic complementarity if } z_S > z_R, \end{cases} \\ \lim_{\psi\to 0}\mathcal{B}'(\tau^*) - \mathcal{I}'(\tau^*) &= -\infty \Rightarrow \text{strategic substitutability}, \end{aligned}$$

where $-\log\Phi(0) = 2$. **QED.**

DATA APPENDIX

Tear Gas Deployment

Procedure

We combine multiple sources to build a constituency level dataset detailing the location of tear gas deployment during the mass protests. Data collection was conducted between August 2020 and September, 2020, when news reports related to the protests were still available online. Several search strategies were employed:

Archieve search We started with the websites of Radio Television Hong Kong (RTHK), the public broadcasting service owned by the Hong Kong SAR Government, Apple Daily and Ming Pao, two traditional news media. We used the search function in the websites with the keywords 'protests' (in Chinese) to retrieve relevant items. We read the reports and saved items when location (i.e. names of streets, schools, and landmarks) was mentioned. When available, we also watched newsclips to help identify exact locations.

Google Search and Snowballing We then used the search terms "Hong Kong" and "protest" (in Chinese) to locate additional news items from other media using Google search engine. Then based on the contents in the reports, we looked for location information to expand the dataset. We snowballed the dataset by reading recommended relevant items.

Additional Search Based on Existing Sources Back in 2020, two tear gas location maps were available online: a map from HKMap.live, which was based on self-reports from members of the civil society, and another produced by researchers from the University of Hong Kong. We Google searched for additional sources to verify an event whenever a protest location was mentioned in these two maps but not in our dataset.

Targeted Search When imprecise information (e.g. only region name) was found in a source, we put region names in Google search to identify additional sources.

In total 161 news items from 22 sources that provide sufficient information were identified.

Coding

Constituency maps from the Hong Kong SAR Governments were used to match with locations identified from news items. Constituencies with tear gas fired are coded as 1 and 0 otherwise. In densely populated regions, the size of affected constituencies could be small. Therefore, a firing event at the intersection of consittutencies is treated as affecting multiple consittutencies simultaneously. A handful of records are based on information from the social media pages of individuals from the civil society. We use mainly videos or photos from non-traditional media for triangulation or coding when other sources are not available.

Sources

Sources include news reports from traditional newspapers, television broadcasting, radio broadcasting (their online news sections), online news. More specifically:

Radio: Commercial Radio Hong Kong, Metro Broadcast, Radio Television Hong Kong (RTHK)

TV: Hong Kong Cable Television, Now TV

Traditional news media: AM730, Apple Daily, Hong Kong Economic Journal, Hong Kong Economic Times, Ming Pao, Oriental Daily News

Online news: HK01, Initium Media, inmediahk.net, Post 852, Stand News, United Social Press

Social media: Facebook pages, Telegram groups or YouTube channels run by the Students' Union Editorial Board of the following universities: City University of Hong Kong, Hang Seng University of Hong Kong, Hong Kong Baptist University, Hong Kong Polytechnic University and Hong Kong Shue Yan University (SYMediaLab by the Department of Journalism and Communication).

Twitter was not commonly used among Hong Kongers. Foreign news sources such as New York Times and BBC News were consulted. However, most events were missing and no additional events were identified from foreign sources.

Google Trends and Tear Gas Deployment

Time series data on rounds of tear gas fired were from a news report from the Radio Television Hong Kong (RTHK), the public broadcasting service owned by the Hong Kong SAR Government. The statistics were based on data provided by the Hong Kong Police Force.

Google Trend data was collected on Aug 21, 2021, May 5, 2021, and May 6, 2021. The score, ranged from 0 to 100, is the average of the popularity indicator of quries made between June 1, 2019 and November 24, 2019 in Hong Kong. The search terms were: "hk protest schedule", "protest schedule", "protest time" "protest timetable" in English and their variants in the Chinese language.

Instrument

The instrumental variable is the driving distance (in km) between the a point in a constituency and its nearest police station. We obtained the list of police stations (with addresses) from the Hong Kong Police Force webpage and Wikipedia. This list includes the headquarter, regional headquarters, district divisions, and several sub-divisions in more remoted areas (the Peak, Stanley, Peng Chau Island, Lamma Island, and Chenug Chau Island). In total 44 stations are identified. Their coordinates were obtained from Google Maps.

To identify popultion centers, we examined the district maps obtained from the official Disctict Council Election webpage. Based on the location of the protest sites and information from Google

Maps, a point was identified for each constituency and their coordinates were obtained from Google Maps. The points are usually protest sites, cross-sections of the busiest streets around the centroid of a constituency, or public area of the nearby residential estates, the last two were likely to be selected for as protest sites given the open space and potential to attract people to join.

To calculate the distance, we first identified the nearest police station for each constituency using Google Maps. The driving distances were then obtained using the Directions function in Google Maps, based on pairs of coordinates. The distance measure takes into account the driving direction of streets, based on the traffic design in early March 2020, when the distance data was obtained.

Political Alignment of Candidates

Political alignments were first determined based on the declared political party affiliations of the candidates, who could be pro-protest, pro-government, or independent.

Affiliations considered pro-protest are 2047 Hong Kong Monitor, Action 18, Civic Party, Civic Passion, Community Alliance, Community March, Community Shatin, DeliberationTW, Democratic Alliance, Democratic Coalition for DC Election, The Democratic Party, Empowering Hong Kong, Fu Sun Generation, Hong Kong Association for Democracy and People's Livelihood, HTTH Env. Concern Group, Labour Party, League of Social Democrats, Luen Wo United, Lung Mun Concern Group, Neighbourhood and Worker's Service Centre, Neo Democrats, North District Blueprint, People Power, Power for Democracy, San Hui Common, Tai Po Democratic Alliance, Tin Shui Wai Connection, Tsz Wan Shan Constructive Power, Tsuen Wan Community Network, Tuen Mun Community Network, and Victoria Social Association.

Affiliations considered pro-government are Alliance for Social and Economic Advancement, Business and Professionals Alliance for Hong Kong, City and Villages Link, Civil Force, Democratic Alliance for the Betterment and Progress of Hong Kong, Federation of Hong Kong and Kowloon Labour Unions, Federation of Public Housing Estates, Hong Kong Federation of Trade Unions, Kowloon West New Dynamic, Liberal Party, New People's Party, New Territories Association of Societies, New Territories Concern Group, and Roundtable.

Then we examined the candidates' manifestos posted on the official district council election website. Candidates who expressed their support for protesters, used popular protest slogans, or have political demands common to people in the democratic camp (e.g. universal suffrage) are classified as pro-protesters. The code is revoked if there were supporting news articles suggesting that the so-called independent candidates had hidden connections with the pro-government camp, which was a common strategy in elections in Hong Kong.

For candidates who did not declared any affiliations or declared themselves as independent,

their alignments were coded based on information from various news articles, archived social media pages of the candidates, websites of political parties, and wikipedia.

Amateur Candidates

We first identified candidates who could be confirmed as new to politics based on news reports. We then identfied winning candidates who are considered as politically connected. This is based on four criteria. Winning candidates are not amateur if they satisfied either one of the following conditions: (1) they were incumbents, (2) they ran for an office in a local election or legislative council election before 2019, (3) they took a high profile position (e.g. secretary and convener) in a political party though they never ran for an office, (4) they worked as community officiers or held similar positions for a political party, or as assistants to politicians, legislators (MP), or local councilors, as these are typical pathways for people aspired to be politicans. We then searched and looked for online news report or Wikipedia pages to further identify candidates who can be considered politically connected (or not) based on the above four criterion.

Social Media Users

Based on data from January 2019 from Facebook, Instagram, and Facebook Messenger, Hootsuite estimated the percentage of social media users in Hong Kong by their gender and age groups. As the Census and Statistics Deparment reported the demographic breakdowns of each district d by gender i and age group j, we estimated the number of social media users (in persons) using the equation:

$$user_d = \Sigma_{ij} population_{dij} x percent_{ij}$$

The variable is log-transformed.

Education

We estimated the number of university degree holders (in persons) in a constituency dc (nested in district d) in 2019 using the equation:

$$log(edu_{19,dc}) = log(edu_{16,dc} + log(edu_{19,d}/edu_{16,d})$$

where $edu_{16,dc}$ is the corresponding figure from the 2016 population by-census, and $edu_{t,d}$ denotes the number of persons with secondary education or above in year t at district d. The variable is log-transformed.

Median Income

We estimated the median household income (in Hong Kong dollars) of a constituency dc (nested in district d) in 2019 using the equation:

$$log(income_{19,dc}) = log(income_{16,dc} + log(income_{19,d}/income_{16,d})$$

where $income_{16,dc}$ is the corresponding figure from the 2016 population by-census, and $income_{t,d}$ denotes the median household income in year t at district d. The variable is log-transformed. $income_{16,dc}$ from the 2016 by-census is defined as the median monthly income (in Hong Kong dollars) from main employment (excluding foreign domestic helpers).