The roots of self-fulfilling debt crises*

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Abstract

Self-fulfilling crises are typically conceived, following the seminal work of Cole and Kehoe (2000), as a coordination problem among small lenders. We show eliminating coordination problems by adding a deep-pocketed large lender does *not* eliminate self-fulfilling crises. The reason is a dynamic hold-up problem. If the large lender buys debt at depressed prices in a crisis, he has incentive to overpay next period to prevent default. The sovereign can exploit this by structuring its debt auction to fail if the large lender does not intervene at an exorbitant prices. This ex-post rent extraction eliminates the ex-ante incentives of the large lender to intervene in a crisis, making self-fulfilling crises possible. Model extensions show the extent to which this hold-up problem manifests depends on the functioning of secondary markets, large lender's funding costs, and the presence of syndicated lending. Adverse shocks to secondary market liquidity and large lender funding costs propagate into self-fulfilling crises, the latter creating scope for a contagion-like crises across multiple countries. Syndicated lending, a means of increasing illiquid exposure for large lenders, is expensive but acts as insurance against crises. We find empirical support for all these predictions.

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1 Introduction

Self-fulfilling crises are a defining feature of sovereign debt markets. When small lenders consider lending to a sovereign, they must take into account what other lenders will do. If other lenders pay relatively high prices for sovereign bonds, the sovereign can rollover existing debt and remain solvent. If not, the sovereign may not be able to pay for all the debt that comes due, resulting in default. As a small lender, one's own investment decisions are immaterial and prices hinge entirely on what others do. When everyone thinks in this way, expectations of default can be self-fulfilling: If lenders expect default, it is optimal not to lend and the sovereign defaults; if lenders expect repayment, it is optimal to lend and the sovereign repays. But is lack of coordination the only fundamental cause of rollover crises? Our analysis answers this question with a resounding "no."

In the classic self-fulfilling setup described above, the introduction of large lenders—who are not subject to any coordination problem—can potentially prevent a crisis. And, large lenders do prevent a crisis whenever they hold debt at risk of default and have sufficient resources. However, across periods, a holdup problem arises: A large investor who purchases bonds in the current period will have incentives to prevent a default in the next period and be willing to pay more than the actuarially-fair price. The sovereign knows this and will intend to exploit the large lender's incentives. If the sovereign can extract that surplus ex post, then the large investor has no incentive ex ante to intervene in the crisis.

How much surplus the sovereign can extract depends on the extent to which the sovereign can credibly threaten to default and whether the large lender can offload default risk exposure in secondary markets. Hence, conditional on a sunspot crisis occurring, the sharp prediction of our model is that a large investor should intervene in the primary auction and immediately resell that debt in secondary markets—if it has enough resources and if these markets exist and are frictionless.[1] These results show the fundamental problem in self-fulfilling crises extend beyond coordination problems

[1]Secondary markets for bonds are one of several ways to circumvent the sovereign's effective bargaining power. If the large lender can purchase anonymously or can reduce its exposure through credit default swaps, then we obtain similar results.

to the market's micro-structure: Who owns the debt, how much they can afford to buy, how much they hold, and whether they able to trade away default risk are all relevant factors determining the likelihood of a self-fulfilling crisis.

To establish these results, we begin with a two-period $t = 1, 2$ model featuring a sovereign who issues one-period debt in an auction. This is bid on by a continuum of measure-zero small lenders and a deep-pocketed large lender.[2] If the auction price exceeds a minimum price, chosen by the sovereign, the issuance is distributed to the highest bidders; otherwise, no debt is issued. In this context a sunspot crises presents an arbitrage opportunity: Whenever the sunspot would result in default, the large lender can bid up the bond price close to the fundamental value—that is, the price when the sunspot would result in no-default. In doing so, the large lender prevents default and earns a return. Because of this arbitrage opportunity, any expectation-driven-default equilibrium cannot be sustained, leaving the repayment equilibrium as the unique outcome.

This result, however, does not carry over when there are two or more auctions. This can be seen by adding a period $t = 0$ (and corresponding auction). In the $t = 1$ auction, there is the unique equilibrium outlined above. But in the $t = 0$ auction, a self-fulfilling debt crisis can arise. To see why, consider a deep-pocketed large lender executing the "arbitrage" trade to profit from a hypothetical crisis. After intervening, the large lender will subsequently hold all the newly-issued bonds. In the following period, the large lender's reservation price incorporate both the actuarially-fair price for the bonds and the value of its *existing* bond holdings. If the sovereign defaults in a failed auction, he can induce the large lender to bid its reservation value by designing the auction to fail if the large lender does not bid its reservation value. He does this by choosing a minimum auction price equal to the reservation value, which is necessarily higher than what small lenders are willing to pay. This ex-post rent extraction reduces or eliminates the sovereign's incentive to intervene ex-ante in the crisis, making a self-fulfilling crisis possible.

We extend this model along several dimensions revealing a number of important

[2]The analysis carries over to this case if large lenders take the actions of other large lenders as given.

implications and testable predictions. First, we incorporate secondary markets. The hold-up problem arises because the large investor is unable to buy bonds and then reallocate that risk. When there is a secondary market for default risk, the large investor can intervene in the primary auction and immediately sell exposure to default risk to small investors. This eliminates the pessimistic equilibrium. There is no crisis when the large investor resells bonds to small investors because they know that next period the large investor will intervene if there is a crisis. One implication is that, if a crisis occurs, a large lender should buy debt in the primary auction and unload it immediately on secondary markets. This result is consistent with our previous work (Chaumont et al., 2023), where we found large bank holding companies hold almost zero exposure to Argentinean sovereign debt despite being dealers. Another implication is that when liquidity in secondary markets worsens, self-fulfilling risk is more probable. Empirically, we find a positive relationship between idiosyncratic innovations in credit default swap (CDS) bid-ask spreads and measures of default risk and default risk volatility.

Second, we relax the assumption that large lenders are deep pocketed. Recall that to intervene in a crisis, the large lender must individually purchase the entire issuance, which could require substantial resources. As a consequence, a shock to bank balance sheets should make crises more likely. Empirically we show negative shocks to large banks' distance to default lead to larger default risk and default risk volatility as measured in CDS spreads. Moreover, this should occur in many countries at the same time, looking like contagion. We show the common factor in an estimated factor model of CDS-based default risk is strongly positively correlated with bank distance to default, suggesting bank balance sheets as a source of contagion. A third implication is that larger countries—who thereby require more lender investment to purchase debt issuance—should be more likely to experience sunspot crises. This country-size effect is evident in regressions of default risk on country fundamentals.

Last, we extend the model to give the sovereign a choice between auctioning debt or using syndicated issuance. A syndicated loan places debt directly onto the books of a counterparty, and existing empirical work suggests these loans are highly illiquid compared to bonds. Given this choice, we show (1) syndicated issuance is weakly more ex-

pensive for the sovereign than an auction and strictly so in the presence of self-fulfilling risk and (2) syndicated issuance should be used by the sovereign as insurance against sunspot risk. The first prediction is established by a substantial empirical literature—syndicated loans carry a spread over bonds. The second agrees with the Greek pattern of debt issuance leading up to its debt crisis that commenced in late 2008. When Greek debt was trading at near-German bond rates prior to 2008, Greece used a mixture of syndicated and auction loans that traded at similar yields. At the eve of the crisis, it switched entirely to auctions until spreads rise sharply. From 2009 on, auctions are not used at all, but debt is still placed (at elevated spreads) using syndicated loans.

Our results sheds light on proposed solutions for resolving self-fulfilling crises. Calvo (1988) thought the solution to multiplicity would be to impose a floor on the price, and Ayres et al. (2018) showed formally that (with small lenders) his conjecture was correct. Our results show a minimum price actually is a key *cause* of indeterminancy, not a solution. Michael Mussa, the former Chief Economist of the IMF, has written (Mussa, 2002) that debt in hands of a few banks would result in less of a sunspot equilibrium problem. For this reason, in the 1990s Argentina had a "megacanje" that concentrated debt into a few banks hands. This restructuring essentially converted debt into syndicated loans, which would reduce self-fulfilling risk ex post. The IMF's "catalytic approach" to dealing with crises, where the IMF provides just enough money to get private lenders to lend, should reduce the risk of crises in a given period while minimizing its own risk of being held up. Last, the European Union banned naked CDS during the crisis in an attempt to eliminate "speculative" bets against countries. While generally this would create more scope for indeterminacy, dealers were explicitly exempt from these restrictions.

Although we establish our main results using a simplified Cole and Kehoe (2000) model, we show that our results hold in a broader class of sovereign debt models. In particular, we show that the result holds (i) in models like Calvo (1988) where multiple equilibria arise because the amount of resources to be raised is decided by the sovereign before the market decides the price for newly issued bonds; and (ii) in state-of-the art default models with equilibrium multiplicity where uncertainty about default costs can generate a variety of dynamics in default risk, debt issuance and bond prices as in Aguiar

et al. (2020).

Related Literature. The sovereign debt literature, since its conception in Eaton and Gersovitz (1981), has had a focus on determining possible equilibria in sovereign debt markets. In some particular environments it has been shown that there is a unique equilibrium (see Auclert and Rognlie (2016) and Aguiar and Amador (2019)). However, uniqueness does not hold in general and agents' expectations can lead to multiple equilibria in models of sovereign default.

The seminal work by Cole and Kehoe (1996, 2000) shows that sovereign's inability to commit to repay its debt obligations can generate self-fulfilling defaults. When lenders expect a default they are unwilling to buy newly issued debt, which reduces sovereign's ability to repay its current debt obligations and may trigger a default. Self-fulfilling defaults are studied by a large body of recent work including Chamon (2007), Chatterjee and Eyigungor (2012), Conesa and Kehoe (2017), Roch and Uhlig (2018), Aguiar et al. (2016), Bianchi et al. (2018), Aguiar and Amador (2020), Aguiar et al. (2020) and Bianchi and Mondragon (2022), among others. The possibility of a self-fulfilling debt crisis has improved the literature's ability to explain the movements in interest rate spreads and why sovereigns may prefer to issue long-term debt over short-term debt. Bocola and Dovis (2019) quantify the role of rollover risk in driving Italian bonds' yields during the period of 2008-2012 and find that on average 13% of the spread is explain by the possibility of a self-fulfilling debt crisis.

A substantial literature exists on how international financial institutions, such as the IMF, interact with self-fulfilling crises. Papers here include Jeanne (2000), Kumar et al. (2000), Huang and Goodhart (1999), and Spiegel (2005).

We contribute to the literature on self-fulfilling defaults in the following ways. Firstly, we show that, with short-term debt, self-fulfilling default equilibria do not survive the existence of a large lender with access to sufficient resources. We emphasize that funding costs at the core of these kind of equilibria in a realistic environment where a lenders have positive mass. Secondly, we show how long-term debt contracts together with frictional markets to trade default risk give rise to a hold-up problem that restores self-

fulfilling debt crises. Thirdly, we provide empirical evidence consistent with the predictions of our theory. We find that the probability of default increases with the size of the issuing country, after controlling for fundamentals and country fixed effects.

Another strand of the literature following Calvo (1988) shows how the feedback between debt accumulation and interest rates can also generate multiple equilibria. Multiplicity arises because the sovereign chooses an amount to finance before the market chooses the interest rate of sovereign bonds. In the simplest version of the model, there are two equilibria. A first equilibrium in which interest rates, debt issuances and default risk are low. And, another equilibrium in which a higher expectation of default leads to lower bond prices and the need to issue more debt to finance a given fiscal deficit. Larger debt accumulation is then consistent with more default risk and lenders' expectations are validated in equilibrium. This kind of multiplicity is also present in Lorenzoni and Werning (2019) and Ayres et al. (2019). In Section 5.1 we show that our results extend to this framework as well.

2 Simple model

In this Section, we provide an explanation of our results in a simple model without a complete formalization. We provide a formal analysis in the online Appendix.

2.1 Environment

Consider an economy with three periods, 0, 1 and 2, and one general-purpose consumption good per period. A sovereign government has an endowment $0 < Y_0 \leq Y_1 \leq Y_2$ and an initial debt burden $B_0 > 0$. The sovereign derives utility

$$\mathbb{E}\left[u(G_0) + \beta u(G_1) + \beta^2 u(G_2)\right]$$

from a spending stream $\{G_0, G_1, G_2\}$, where $\beta \in (0,1)$, and $u(\cdot)$ is a continuously differentiable and strictly concave function satisfying $\lim_{x \searrow 0} u'(x) = \infty$ and $\lim_{x \nearrow \infty} u(x) = 0$.

The sovereign issues debt to lenders in order to smooth consumption. We consider

only short-term debt. That is, debt B_1 is issued in period 0 to be paid in period 1, and debt B_2 is issued in period 1 to be paid in period 2. Before repaying the debt in a period t, the sovereign has the option to default on B_t. If the sovereign defaults in a period t, we assume that it stays in default in the periods after t. Let $\delta_t = 1$ denote if the sovereign defaulted in period t or earlier, and $\delta_t = 0$ denote if it has not defaulted. When the sovereign defaults, it keeps $Y - h(Y) \in [0, Y]$ of the endowment Y, where $h(\cdot)$ is continuous, and both $h(Y)$ and $Y - h(Y)$ are weakly increasing in Y.

There are two types of lenders: a set $I = [0, 1]$ of small lenders without a mass point and a lender l with a mass point. For simplicity, we assume that the small lenders hold all of the initial debt B_0. All lenders are risky neutral, discount the future at rate $1/R$ and have deep pockets — that is, they can spend any amount purchasing bonds and have negative consumption in any period. As a result, lenders derive the utility

$$\mathbb{E}\left[x_0 + R^{-1}x_1 + R^{-2}x_2\right]$$

from a consumption stream $\{x_0, x_1, x_2\}$. We assume that $0 < \beta R \leq 1$, which, combined with the assumption $0 < Y_0 \leq Y_1 \leq Y_2$, implies that the sovereign doesn't save.

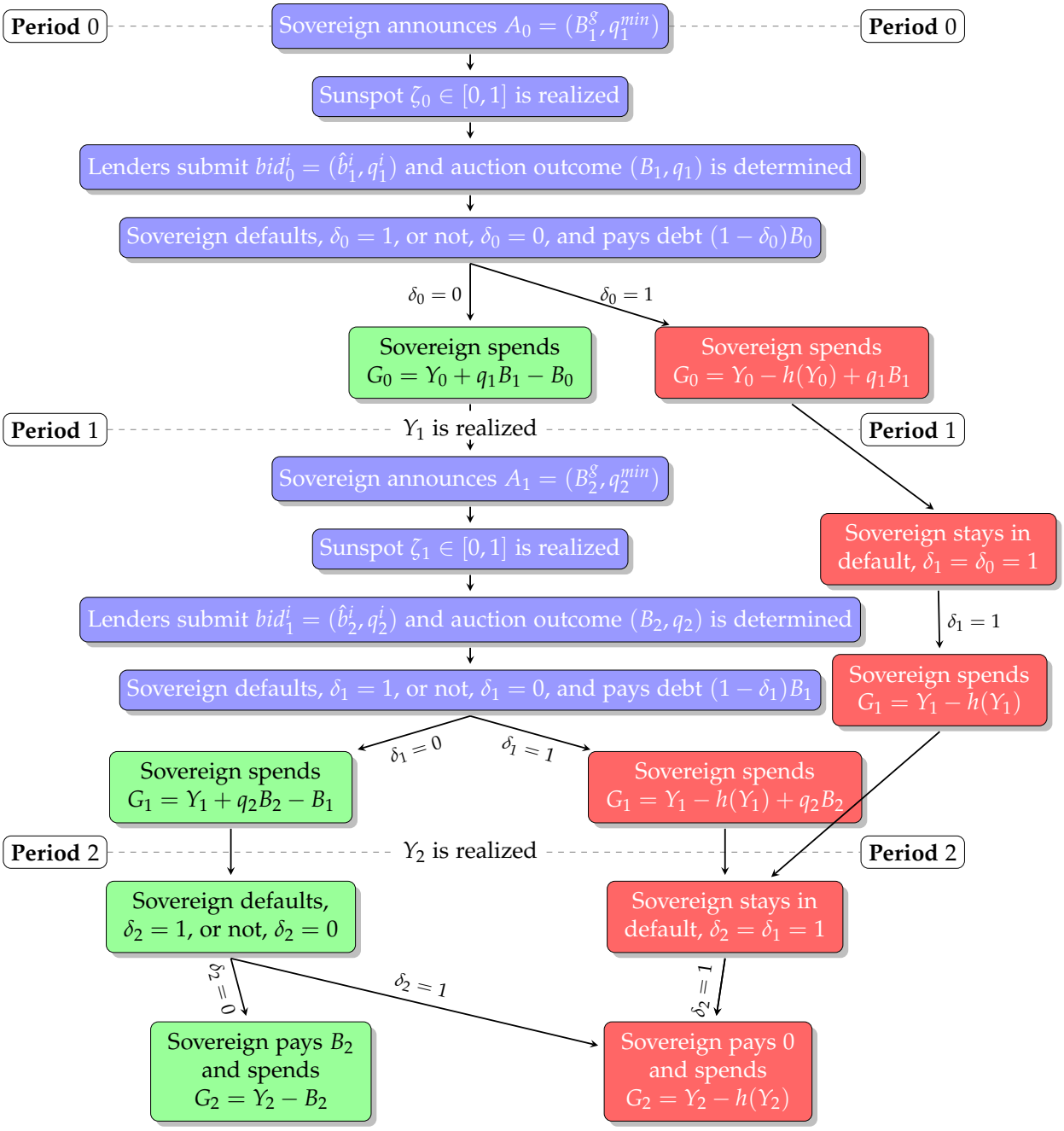
The sovereign issues bond in a primary auction characterized by a bond issuance and a reservation value. For now, we do not allow the lenders to trade bonds in a secondary bond market. We later extend the model to consider this possibility. There are two publicly observed and independent sunspot variables, ζ_0 and ζ_1, realized in periods 0 and 1, which follow a uniform distribution in the interval $[0, 1]$. The sunspot variables are realized after the sovereign announces the auction, but before the auction itself. Figure 1 depicts the sequence of actions.

2.2 Solution

We solve the model backwards starting in period 2.

Period 2: If the sovereign is already in default, it stays in default. Otherwise, the cost of defaulting on outstanding bonds B_2 is given by $h(Y_2)$. The value function of the

Figure 1: Sequence of actions



sovereign in period 2 satisfies

$$V_2(B_2, \delta_1) = \begin{cases} u\big(Y_2 - B_2\big) & \text{if } \delta_1 = 0 \text{ and } B_2 \leq h(Y_2) \\ u\big(Y_2 - h(Y_2)\big) & \text{otherwise} \end{cases}. \tag{1}$$

Here we have assumed that the sovereign repays when indifferent, eliminating the possibility of a mixed strategy equilibrium analyzed in Aguiar et al. (2020). We will return to this case later. Associated with the value function V_2, we have the default policy function

$$\hat{\delta}_2^*(B_2, \delta_1) = \begin{cases} 0 & \text{if } \delta_1 = 0 \text{ and } B_2 \leq h(Y_2) \\ 1 & \text{if } \delta_1 = 1 \text{ or } B_2 > h(Y_2) \end{cases}. \tag{2}$$

The value function V_2 and default policy $\hat{\delta}_2^*$ are functions of the total outstanding debt B_2 and are independent of who holds the debt: the small lenders or the large lender. That is because period 2 is the last period, so the sovereign welfare is independent of how the debt is distributed. This is not the case in period 1.

Period 1: Let the total outstanding period-1 debt be B_1, and the period-1 debt held by the large lender be $B_1^l \leq B_1$. We can easily obtain the period-1 debt held by the small lenders using the formula $B_1^s = B_1 - B_1^l$ so we do not keep it as a state variable.

How much are small lenders willing to pay in period 1 for period-2 bonds? What about the large lender? Before answering these questions, we need to establish the sovereign default decision at the end of period 1 as a function of the period-2 bond price q_2 and the amount sold in the auction B_2. We know that if the issuance B_2 is strictly greater than $h(Y_2)$, the sovereign defaults in period 2 and therefore the lenders are not willing to buy the bonds at any positive price. So, let us assume $0 \leq B_2 \leq h(Y_2)$.

The sovereign default at a price q_2 and bond issuance B_2 if, and only if,

$$\begin{aligned} W_1^r(B_1, q_2, B_2) &= u(Y_1 - B_1 + q_2 B_2) + \beta u(Y_2 - B_2) \\ &< u(Y_1 - h(Y_1) + q_2 B_2) + \beta u(Y_2 - h(Y_2)) = W_1^d(B_1, q_2, B_2). \end{aligned}$$

Which implies that

$$\hat{\delta}_1^*(B_1, q_2, B_2, \delta_0) = \begin{cases} 0 & \text{if } \delta_0 = 0 \text{ and } W_1^r(B_1, q_2, B_2) \geq W_1^d(B_1, q_2, B_2) \\ 1 & \text{if } \delta_0 = 1 \text{ or } W_1^r(B_1, q_2, B_2) < W_1^d(B_1, q_2, B_2) \end{cases}. \tag{3}$$

The above equations highlight the key feature of the classic Cole-Kehoe model: the

possibility of multiple equilibria. When small lenders believe that the sovereign will not default, the price is $1/R$. In this case, if $W_1^r(B_1, 1/R, B_2) \geq W_1^d(B_1, 1/R, B_2)$, the sovereign does not default. When small lenders believe the sovereign will default, they are indifferent between bidding zero or not bidding in the auction. In this case, we assume they do not bid. If the large lender also does not bid, the primary auction fails, the price is zero, no bonds are sold, and if $W_1^r(B_1, 0, 0) < W_1^d(B_1, 0, 0)$, the sovereign does default.[3]

To summarize, the price small lenders are willing to pay is given by

$$q_2^s(B_1, B_2, \delta_0) = \begin{cases} 0 & \text{if } \underbrace{W_1^r(B_1, 1/R, B_2) < W_1^d(B_1, 1/R, B_2)}_{\text{Default in a successful auction}} \text{ or } \delta_0 = 1 \\ 1/R & \text{if } \underbrace{W_1^r(B_1, 0, 0) \geq W_1^d(B_1, 0, 0)}_{\text{Repay in a failed auction}} \text{ and } \delta_0 = 0 \\ q_2 \in \{0, 1/R\} & \text{otherwise} \end{cases}.$$

The impact of the price on the default decision is not internalized by the small lenders because, being small, they do not influence the auction price. However, the large lender understands how his bid influences the auction price. To see how this works, let us open the cases. If $W_1^r(B_1, 1/R, B_2) \geq W_1^d(B_1, 1/R, B_2)$ and $W_1^r(B_1, 0, 0) \geq W_1^d(B_1, 0, 0)$, the large lender knows that the sovereign will not default even if there is no demand for its bonds. In this case, the large lender does not pay for the bonds anything more than what small lenders would pay: $1/R$.

What about when the sovereign defaults in a failed auction? This happens when $W_1^r(B_1, 0, 0) < W_1^d(B_1, 0, 0)$. In this case, the large lender is willing to pay a higher price if it prevents a default. The large lender utility of paying the price q_2 for the period-2 bond if the government does not default is

$$B_1^l - q_2 B_2 + \frac{B_2}{R}.$$

If pricing the bonds at zero leads to default, the large lender payoff is zero. Thus, the

[3]The assumption that small lenders do not bid when they believe the sovereign will default limits the set of equilibrium outcomes, but it is not important to derive our main results.

highest price that the large lender is willing to pay to prevent a default satisfies

$$B_1^l - q_2 B_2 + \frac{B_2}{R} \geq 0 \quad \Longleftrightarrow \quad q_2 \leq \frac{1}{R} + \frac{B_1^l}{B_2}.$$

To summarize, the price the large lender is willing to pay is given by

$$q_2^l(B_1, B_1^l, B_2, \delta_0) = \begin{cases} 0 & \text{if } W_1^r\left(B_1, \frac{1}{R} + \frac{B_1^l}{B_2}, B_2\right) < W_1^d\left(B_1, \frac{1}{R} + \frac{B_1^l}{B_2}, B_2\right) \text{ or } \delta_0 = 1 \\ 1/R & \text{if } W_1^r(B_1, 0, 0) > W_1^d(B_1, 0, 0) \text{ and } \delta_0 = 0 \\ \frac{1}{R} + \frac{B_1^l}{B_2} & \text{otherwise} \end{cases}.$$

When comparing the case with a continuum of small lenders, and the one with one large lender, it is interesting to notice two important results. First, we can have multiple prices that are optimal for the small lenders depending on their beliefs. That is the usual sunspot region discussed in the literature. The price that the large lender is willing to pay, generically, is unique. That is because the large lender, being able to buy all the bonds, does not face the type of coordination problem faced by small lenders. The second important aspect of the above prices is that the large lender is always willing to pay for the bonds at least the same that small lenders are willing to pay — and strictly more in some cases. That is, $q_2^l(B_1, B_1^l, B_2, \delta_0) \geq q_2^s(B_1, B_2, \delta_0)$.

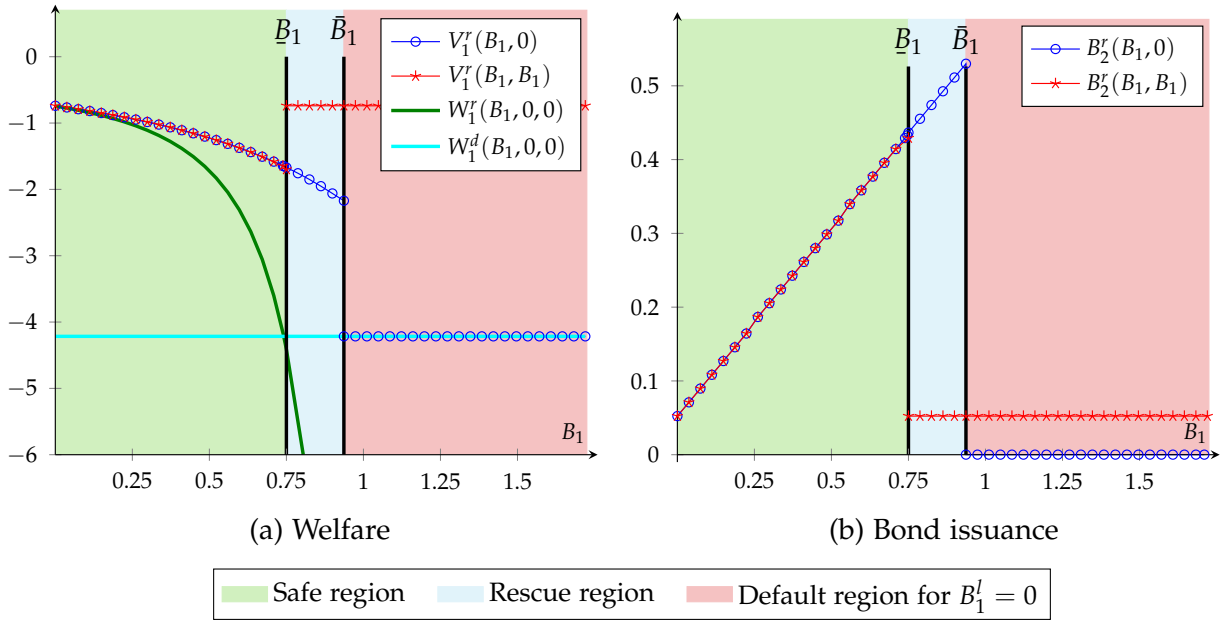
Remark 1. *Generically, in period 1 forward, all equilibria are payoff equivalent, the bond price is determined by the price the large lender is willing to pay and only fundamental default occurs.*

The above result implies that the optimal bond issuance and default decision solves

$$\max_{B_2, \delta_1} \left\{ u\Big(Y_1 - B_1 + \delta_1[B_1 - h(Y_1)] + q_2^l(B_1, B_1^l, B_2, \delta_0)B_2\Big) + \beta V_2(B_2, \delta_1) \right\} \tag{4}$$
$$\text{subject to} \quad \delta_1 = \hat{\delta}_1^*(B_1, q_2, B_2, \delta_0) \quad \text{and} \quad 0 \leq B_2 \leq h(Y_2).$$

We denote by $V_1^r(B_1, B_1^l)$ the maximum, and $B_2^r(B_1, B_1^l)$ and $\delta_1^r(B_1, B_1^l)$ the argmax, of problem (4) in the case where $\delta_0 = 0$; and $V_1^d(B_1, B_1^l)$ the maximum, and $B_2^d(B_1, B_1^l)$ and $\delta_1^d(B_1, B_1^l)$ the argmax, of the problem (4) in the case where $\delta_0 = 1$. Finally, denote by $V_1(B_1, B_1^l, \delta_0) = (1 - \delta_0)V_1^r(B_1, B_1^l) + \delta_0 V_1^d(B_1, B_1^l)$ the maximum as a function of δ_0.

Figure 2: Period-1 Outcomes



(a) Welfare (b) Bond issuance

Note: The parameters for this numerical illustration are $Y_0 = 0.8, Y_1 = 1.1, Y_2 = 1.2, R = 1.05, \beta = 1/R, h(\cdot) = 0.62$, and $u(G) = G^{1-\sigma}/(1-\sigma)$ with $\sigma = 3$.

Figure 2 depicts the period-1 outcomes for different levels of B_1 and two cases of lenders' bond holdings: small lenders holdings all the bonds, $B_1^l = 0$, and the large lender holding all the bonds, $B_1^l = B_1$. Panel 2a shows that for lower levels of B_1, when $W_1^r(B_1,0,0) > W_1^d(B_1,0,0)$, the sovereign welfare is independent of the holdings of the large lender. This region, shaded green in the graphs, is delimited by the first black line at $B_1 = \underline{B}_1$. In this region, the large lender does not pay for the bonds anything above the price small lenders would pay. As a result, $V_1^r(B_1,0) = V_1^r(B_1,B_1^l)$ and, as we can see in Figure 2b, $B_2^r(B_1,0) = B_2^r(B_1,B_1^l)$.

For higher levels of B_1, when $W_1^r(B_1,0,0) < W_1^d(B_1,0,0)$, the outcomes for $B_1^l = 0$ and $B_1^l = B_1$ are different. The large lender is willing to overpay for the period-2 bonds in order to prevent a default. The sovereign can set the reservation price in the auction to be the large lender highest valuation, extracting all the surplus from the large lender. When $B_1^l = B_1$, that means wiping out all the sovereign debt since

$$Y_1 - B_1 + q_2^l(B_1, B_1^l, B_2, \delta_0)B_2 = Y_1 - B_1 + \left[\frac{1}{R} + \frac{B_1}{B_2}\right] B_2 = Y_1 + \frac{B_2}{R}.$$

As a result, both $V_1^r(B_1, B_1^l)$ and $B_2^r(B_1, B_1^l)$ are constant in B_1 in the region satisfying $W_1^r(B_1, 0, 0) < W_1^d(B_1, 0, 0)$ if there is no default.

For the above parametric specification, the sovereign does not default for any value of $B_1 \leq \bar{B}_1$. However, there is a stark difference in how it happens depending on whether $B_1^l = B_1$ or $B_1^l = 0$. If $B_1^l = B_1$, the large lender always buy the bonds in the auction paying its highest valuation: $\frac{1}{R} + \frac{B_1}{B_2}$. If $B_1^l = 0$, there are a continuum of equilibria, which are all payoff equivalent, where the small and large lenders buy different shares of the bond supplied in the auction at the price $1/R$. For values of $B_1 > \bar{B}_1$ the sovereign also does not default when $B_1^l = B_1$, while it does default if $B_1^l = 0$. It is still possible, for some parametric specifications, that the sovereign defaults on B_1 when $B_1^l = B_1$ if

$$\begin{aligned} W_1^d\left(B_1, \frac{1}{R} + \frac{B_1}{B_2}, B_2\right) &= u\left(Y_1 - h(Y_1) + B_1 + \frac{B_2}{R}\right) + \beta u\left(Y_2 - h(Y_2)\right) \\ &> u\left(Y_1 + \frac{B_2}{R}\right) + \beta u\left(Y_2 - B_2\right) = W_1^r\left(B_1, \frac{1}{R} + \frac{B_1}{B_2}, B_2\right). \end{aligned}$$

This tends to happen for large enough B_1, and will always happen for some B_1 if $u(\cdot)$ is unbounded. However, in the numerical example above the utility is $u(x) = -\frac{1}{2x^2}$, which is bounded above by zero. Given the other parameters, that implies that the sovereign never defaults in this case for any level of B_1 if $B_1^l = B_1$.

In the case where $B_1 = B_1^s$, due to concavity, the level of B_1 leading to default always exists and is lower than the one leading to default when the large lender hold all the debt since the analogous condition for default in this case is

$$\begin{aligned} W_1^d\left(B_1, \frac{1}{R}, B_2\right) &= u\left(Y_1 - h(Y_1) + \frac{B_2}{R}\right) + \beta u\left(Y_2 - h(Y_2)\right) \\ &> u\left(Y_1 - B_1 + \frac{B_2}{R}\right) + \beta u\left(Y_2 - B_2\right) = W_1^r\left(B_1, \frac{1}{R}, B_2\right). \end{aligned}$$

The red-shaded region in Figure 2, delimited by $B_1 = \bar{B}_1$, marks the level of B_1 that leads to default when small lenders hold all of the period-1 debt. Note that the blue region is the region that, absent the large lender, there would be multiple equilibria.

Period 0: Now we can turn to the choices in period 0. First, consider the willingness to pay for the bond of the large lender. We know from the above analysis that the sovereign default decision in period 1, $\delta_1^r(B_1, B_1^l)$, depends on the bond holdings of the large lender. It is also worth noting that, even when $W_1^r(B_1, 0, 0) < W_1^d(B_1, 0, 0)$ and the sovereign does not default, the payoff of the large lender in period 1 is zero. That is because the sovereign in this case can induce the large lender to overpay for the period 2 bonds to prevent a default in period 1. Specifically, when $W_1^r(B_1, 0, 0) < W_1^d(B_1, 0, 0)$ and $\delta_1^r(B_1, B_1^l) = 0$, the payoff of the large lender of paying any price $q_1 > 0$ on any quantity $B_1^l > 0$ of bonds is given by

$$-q_1 B_1^l + R^{-1}\left[B_1^l - \left(\frac{1}{R} + \frac{B_1^l}{B_2}\right) B_2\right] + R^{-2} B_2 = -q_1 B_1^l < 0.$$

This is in essence a holdup problem that allows the sovereign to extract surplus from the large lender in period 1, regardless of any investment the large lender made to purchase bonds in period 0. As a result, the large lender is unwilling to purchase the period-1 bonds at any price.

In the other regions, the large lender is willing to pay q_1 if $\delta_1^r(B_1, B_1^l) = 0$ and

$$\begin{aligned} W_0^r\left(B_0, q_1, B_1, B_1^l\right) &= u\left(Y_0 - B_0 + q_1 B_1\right) + \beta V_1^r(B_1, B_1^l) \\ &\geq u\left(Y_0 - h(Y_0) + q_1 B_1\right) + \beta V_1^d(B_1, B_1^l) = W_0^d\left(B_0, q_1, B_1, B_1^l\right). \end{aligned}$$

Which implies that

$$q_1^l(B_0, B_1, B_1^l) = \begin{cases} 0 & \text{if either} \quad \begin{array}{l} \delta_1^r(B_1, B_1^l) = 1 \ \text{ or} \\ W_1^r(B_1, 0, 0) < W_1^d(B_1, 0, 0) \ \text{ or} \\ W_0^r\left(B_0, 1/R, B_1, B_1^l\right) < W_0^d\left(B_0, 1/R, B_1, B_1^l\right) \end{array} \\ 1/R & \text{otherwise} \end{cases},$$

Consider now the willingness to pay of small lenders. When the large lender is willing to pay $\frac{1}{R}$, the valuation of small lenders is also $1/R$. However, in the region where $W_1^r(B_1, 0, 0) \leq W_1^d(B_1, 0, 0)$ and $\delta_1^r(B_1, B_1^l) = 0$, the large lender is not willing to pay

anything for the bonds because it will be exploited in period 1. The small lenders do not face this issue and are willing to pay $\frac{1}{R}$ for the bonds if the sovereign does not default on them. That is, if $W_0^r\left(B_0, \frac{1}{R}, B_1, 0\right) \geq W_0^d\left(B_0, \frac{1}{R}, B_1, 0\right)$.

To summarize, the price small lenders are willing to pay is given by

$$q_1^s(B_0, B_1, 0) = \begin{cases} 0 & \text{if either} \quad \delta_1^r(B_1, 0) = 1 \text{ or} \\ & \quad W_0^r\left(B_0, 1/R, B_1, 0\right) < W_0^d\left(B_0, 1/R, B_1, 0\right) \\ 1/R & \text{if either} \quad \delta_1^r(B_1, 0) = 0 \text{ and} \\ & \quad W_0^r\left(B_0, 0, 0, 0\right) \geq W_0^d\left(B_0, 0, 0, 0\right) \\ q_1 \in \{0, 1/R\} & \text{otherwise} \end{cases} .$$

We argued that in period 1 all equilibria are payoff equivalent. Multiplicity of equilibria can exist only because lenders are indifferent between holding or not bonds in some states of the world, leading to multiple distribution of bond holdings consistent with equilibrium. The reason is that the large lender is always willing to step in to prevent a sovereign default if it is efficient to do so. In contrast to period 1, the presence of a large lender does not guarantee the elimination of sunspot equilibria in period 0. This is due to a hold-up problem that arises. A large lender holding previously issued debt is incentivized to overpay for new bonds to avert a sovereign default. The sovereign, by designing the auction, can exploit this situation to extract surplus from the large lender. Anticipating this, the large lender may be reluctant to rescue the sovereign in the first place, leading to the reemergence of sunspot equilibria if $W_1^r(B_1, 0, 0) < W_1^d(B_1, 0, 0)$.

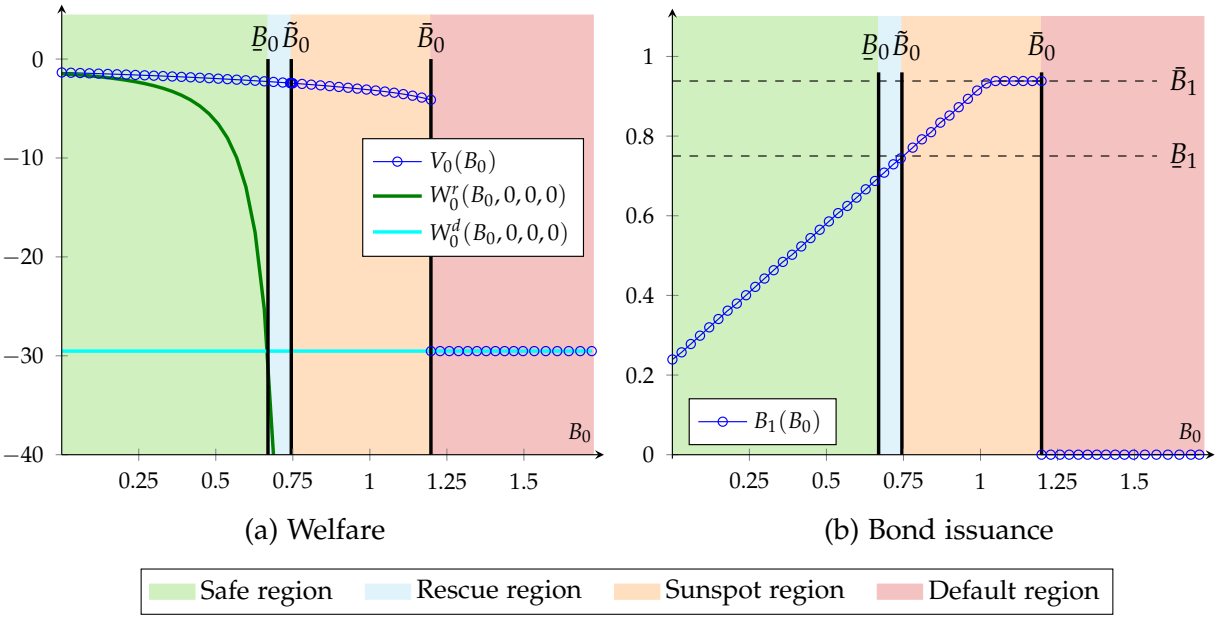
Remark 2. *There exists parameter region with sunspot equilibria in period 0.*

To illustrate this point, consider the sovereign problem in the equilibrium with the highest period-0 equilibrium price. That is, the sovereign solves

$$\max_{B_1, \delta_0} \left\{ u\Big(Y_0 - B_0 + \delta_0[B_0 - h(Y_0)] + \bar{q}_1^s(B_0, B_1, 0)B_1\Big) + \beta V_1(B_1, 0, \delta_0) \right\} \tag{5}$$
$$\text{subject to} \quad \delta_0 \in \{0, 1\} \quad \text{and} \quad B_1 \geq 0.$$

We denote by $V_0(B_0)$ the maximum, and $B_1(B_0)$ and δ_0 the argmax, of problem (5).

Figure 3: Period-0 Outcomes



Note: The parameters for this numerical illustration are the same as in Figure 2.

Figure 3 illustrates the period-zero outcomes for varying levels of initial debt (B_0). Four distinct regions emerge: *Safe*, *Rescue*, *Sunspot* and *Default*. In the *Safe* region, since $W_0^r(B_0,0,0,0) \geq W_0^d(B_0,0,0,0)$, the sovereign would not default in a failed auction, and only no-default equilibria exist. In the *Rescue* region, since $W_0^r(B_0,0,0,0) < W_0^d(B_0,0,0,0)$, the sovereign defaults in a failed auction. However, there is no hold-up problem for the large lender, as it does not have to overpay to prevent default in period 1 because $W_1^r(B_1,0,0) \geq W_1^d(B_1,0,0)$ for $B_1 = B_1(B_0)$. Thus, the large lender intervenes, and only no-default equilibria occur. In the *Sunspot* region, the sovereign defaults in a failed auction ($W_0^r(B_0,0,0,0) < W_0^d(B_0,0,0,0)$), and a hold-up problem exists for the large lender ($W_1^r(B_1,0,0) < W_1^d(B_1,0,0)$ for $B_1 = B_1(B_0)$). Due to this, the large lender does not participate in the auction, and the equilibrium outcome depends on the beliefs of small lenders. If they believe the sovereign will not default, they roll over the debt at the risk-free price, and no default occurs. Otherwise, small lenders do not bid, the auction fails, and the sovereign defaults. In the *Default* region, even at the highest price lenders are willing to pay the sovereign defaults, and only default equilibria exist.

3 Extensions

In the previous section, we saw that a holdup problem could generate self-fulfilling crises in our benchmark model. In this section, we analyze three extensions. First, we discuss how a secondary market for bonds can eliminate self-fulfilling crises in period 0 if it has a low degree of friction. Second, we highlight how funding costs can ultimately prevent the large lender from intervening, generating self-fulfilling crises even in period 1. Third, while we have so far highlighted how the holdup problem creates self-fulfilling crises, it can also be used to prevent crisis, self-fulfilling or otherwise, when the sovereign sell bonds directly to large lenders in a syndicated loan.

3.1 Secondary markets

The results presented so far were established without considering a market for real-locating credit risk, such as a secondary market for existing bonds or a credit-default swap (CDS) market. The holdup problem that generates sunspot crises in the bench-mark model can be resolved in the presence of such markets. To illustrate this result, we modify the environment by incorporating a market for trading credit risk. Specifically, we introduce a secondary market for bonds at the end of period 0, allowing bonds B_1 to be traded at the endogeneous price $q_1(B_0, B_1, B_1^l)$.

Can sunspot equilibria exist with this secondary market? When the secondary market opens, the large investor has the opportunity to sell its bond holdings to small investors at the valuation determined by the small investors. If a default has occurred ($\delta_0 = 1$), all investors value the bonds at zero. If no default has occurred ($\delta_0 = 0$), small investors recognize that, due to the presence of the large lender, there is a unique equilibrium moving forward where only fundamental default can happen. Consequently, the large lender can always sell its holdings to small investors at the valuation associated with only fundamental default, $1/R$, which aligns with the valuation of small investors. This implies that the price the large lender is willing to bid in the auction must converge to the price small lenders bid, effectively eliminating self-fulfilling crisis.

Remark 3. *In the economy with a frictionless secondary market for sovereign bonds all equilibria are payoff equivalent and only fundamental default occurs.*

3.1.1 Trading frictions

The introduction of perfectly liquid secondary markets completely eliminates the possibility of sunspot crises within the baseline model. This naturally leads to the conjecture that markets with higher levels of friction would be more susceptible to sunspot-driven crises. To formalize this intuition, we introduce a limit α on the fraction of bonds that can be sold in the secondary market. The idea behind is that to unload large amounts of bonds in the secondary market is expensive due to frictions. Here we take a simple formulation that to sell a fraction α of bond holdings is costless, but to trade any amount beyond that is prohibitively expensive. This friction can be interpreted as either a cost of transacting in the bond itself or a cost associated with trading CDSs.

The presence of trading frictions creates a wedge between the price the large lender is willing to pay in the auction and the price in the secondary market. In the example above, instead of the price the large lender is willing to bid coinciding with the maximum price small lenders are willing to bid, $1/R$, we have it equal to α/R. The reason is that the large lender knows that, of a purchase of B_1^l bonds, αB_1^l can be sold in the secondary market at the price $1/R$, with the remaining bond holdings, $(1-\alpha)B_1^l$, being subject to the holdup problem discussed previously. When the limit α is low, a bond issuance at the price α/R may not be enough to prevent the sovereign from defaulting. The large lender will then choose to abstain from the primary auction entirely, thereby generating self-fulfilling crisis driven by small lenders' pessimistic beliefs.

Remark 4. *There exists parameter region and $\bar{\alpha} \in (0,1)$ such that sunspot equilibria in period 0 exist if, and only if, $\alpha < \bar{\alpha}$.*

3.1.2 Evidence

The theoretical framework suggests that periods of heightened illiquidity are associated with larger regions of indeterminacy and a greater vulnerability to crises. This notion

finds suggestive support in the time series data for bid-ask spreads on sovereign debt, which tend to widen during periods of financial stress. We empirically examine this relationship by investigating the correlation between bid-ask spreads and sovereign debt risk. Our analysis reveals a positive association between periods of elevated bid-ask spreads and higher sovereign bond yields, even after controlling for other relevant factors.

Furthermore, the theory predicts that, If assets can be readily moved off-balance sheet at a low cost, banks should use them to minimize their sovereign debt exposure. We will later explore this prediction in the context of syndicated loans, which, unlike bonds, are inherently less liquid and more challenging to resell.

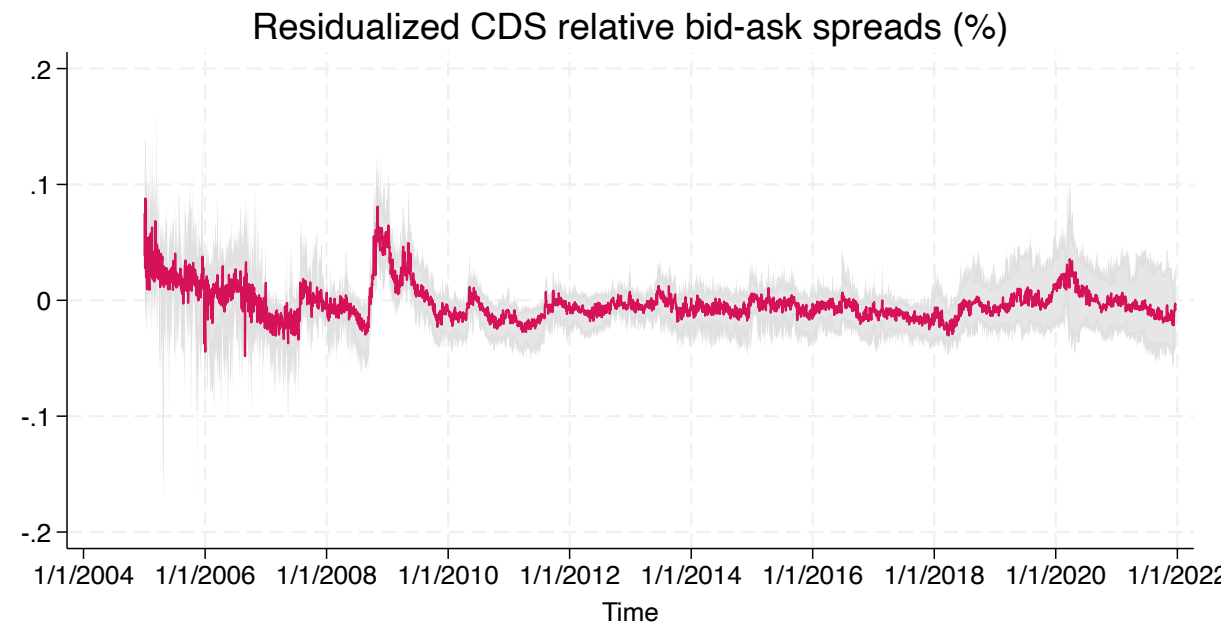
Our empirical approach exploits daily information coming from credit default swap (CDS) data with its associated implied default probabilities (IDP). We identify a shock to bid-ask spreads by projecting them on country, year fixed effects, month effects to capture seasonality, and a cubic in log IDP. Bid-ask spreads are measured as the percent deviation of the bid-ask spread in levels deflated by the mid price. The residual, call it $u_{k,t}$, is at a country-date level, and figure 4 displays the median and interquartile range over time. Ignoring noise early in the sample,[4] the median of the series captures liquidity shocks that occurred in the GFC and the pandemic. The largest median spreads in 2010 occur in 5/2010, which is when Greece received its EU-IMF bailout package. Covid in 2020 is another period where illiquidity worsens for most countries. But there is also substantial variation across countries at most points in time and not just in the context of these two global cycles.

We then examine how the shock $u_{k,t}$ propagates to a measure of sunspot risk, which is log IDP volatility over the last two-weeks. Let $\text{Vol IDP}_{k,t} = \sigma(\{\log \text{IDP}_{k,t-h} | h = 0, \ldots, 13\})$ give the rolling standard deviation for log IDP. Let $\bar{u}_{k,t} = E[\{u_{k,t-h} | h = 0, \ldots, 13\}]$ give the rolling mean u.[5] Figure 5 binscatters $\text{Vol IDP}_{k,t}$ and $u_{k,t-H}$ for $H = 14$ (so there is no overlap on time periods). A strong positive relationship between bid-ask residuals and Vol IDP exists, consistent with the model's prediction that illiquidity in

[4]For 2004 and 2005, we have data for 36 and 46 countries respectively. From 2006 on, coverage varies between 51 and 87 countries, growing over time.

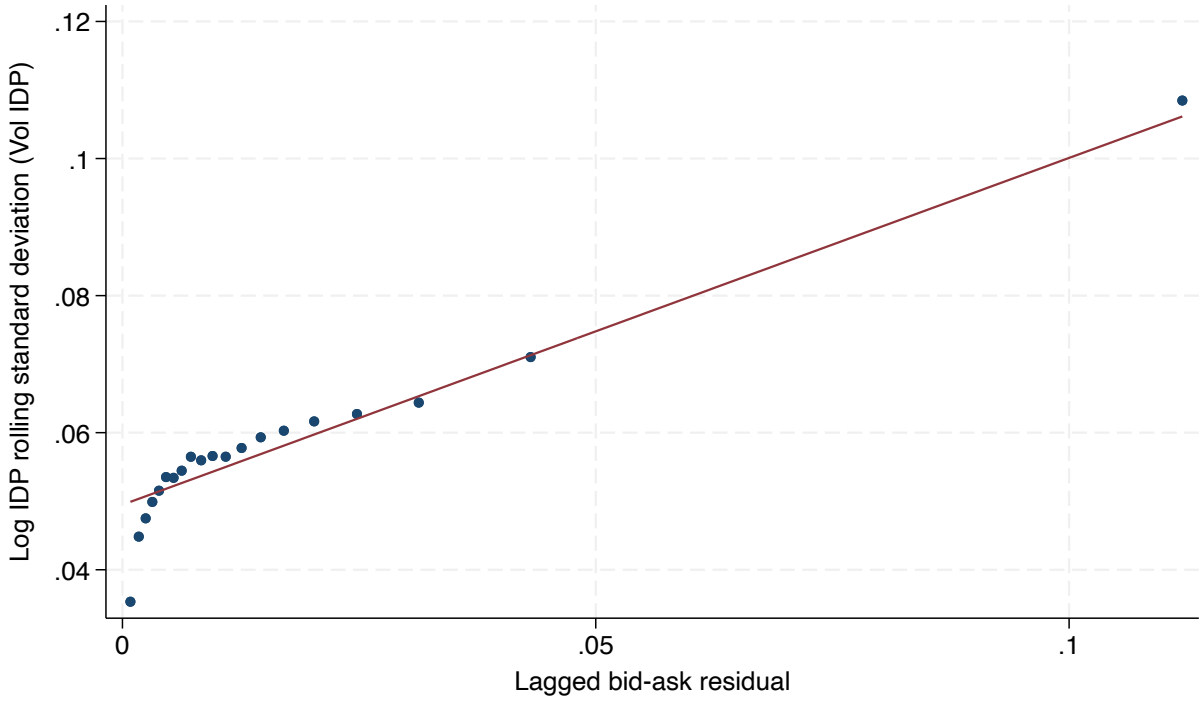
[5]We exclude dates with missing data from the calculations.

Figure 4: Residual bid-ask CDS spreads over time



Note: underlying series is residual from a regression of relative bid-ask spreads on log price, date, and fixed effects for country and calendar month; line and shaded region are median and IQR across countries, respectively; raw data from Bloomberg.

Figure 5: Binscatter of residual bid-ask CDS spreads and volatility of log IDP



secondary markets propagates into sunspot risk. The regressions in table 1 provide similar empirical evidence, linking $\bar{u}_{k,t}$ or its innovations to greater IDP volatility.

Table 1: The effect of shocks to bid-ask spreads on IDP volatility

	(1)	(2)	(3)	(4)
Lagged bid-ask residual	0.409		0.136	
	(127.7)		(43.1)	
Innovation to lagged bid-ask residual		0.274		0.0287
		(68.7)		(7.4)
Twice-lagged bid-ask residual		0.462		0.200
		(126.5)		(55.8)
Lagged log IDP rolling SD			0.405	0.389
			(236.5)	(218.5)
R^2-within	0.19	0.18	0.32	0.30
R^2-between	0.63	0.59	0.92	0.91
R^2-overall	0.21	0.20	0.35	0.32
N	285339	282139	285339	282139

Note: dependent variable is the log IDP rolling standard deviation; data is daily; a corresponds to 14 calendar days; all specifications include a constant and year dummies; t-statistics are in parentheses.

3.2 Cost of funds

The baseline model assumes the large lender has unlimited resources, highlighting the importance of the holdup problem in determining the presence of a rollover crisis. However, raising large amounts of funds is expensive in reality. Intervention might require borrowing from other institutions or issuing equity, especially during times of financial stress. Risk managers or regulators may also impose limits on the absolute exposure a sovereign-debt trading desk can hold.

To formalize this, we introduce a general funding cost for the large lender that can encompass different microfoundations. Specifically, we assume that the larger lender faces a funding cost ϕa for spending $a \geq 0$ in the auction, where $\phi > 0$ is a constant.[6]

[6]The more general formulation is that the funding cost is a function $cost(\cdot)$ of the spending amount a. Here we assume $cost(a) = \phi a$ for $a \geq 0$ and $cost(a) = 0$ for $a < 0$, but our results are robust to alternative formulations.

As a result, in a period t small lenders derive utility $\mathbb{E}[x_t - a_t]$ from an income x_t and spending a_t in the auction, while the large lender derives utility $\mathbb{E}[x_t - \phi a_t]$ from an income x_t and spending a_t in the auction.

The funding cost $\phi > 1$ captures additional costs that are unique to large lenders, such as more stringent regulations or difficult to diversify a large portfolio due to price impact in illiquid markets. The additional funding cost can prevent the large lender from bidding strategically. This case is interesting because it can lead to equilibrium multiplicity in the absence of a holdup problem, and provides economic insights on how self-fulfilling debt crisis are generated in equilibrium. The parameter ϕ can also capture better access to funding when $0 < \phi < 1$. In this case, the large lender has lower cost than the small lenders, and can profit from buying bonds below its valuation. This could be the case, for example, if too-big-to-fail policies give large banks a competitive advantage to attract deposits in periods of turmoil.

The way the funding cost changes the problem is straightforward. Since it costs ϕ to bid in the auction, the large lender now discounts the return by the factor ϕ. As a result, the analysis from the previous section remains the same but now the price the large lender is willing to pay becomes $\frac{q_1^l(B_0,B_1,B_1^l)}{\phi}$ for period-1 bonds, and $\frac{q_2^l(B_1,B_1^l,B_2,\delta_0)}{\phi}$ for period-2 bonds. For $\phi > 1$, this factor reduces the price the large lender is willing to pay, reducing the *rescue* region and, potentially, generating self-fulfilling crisis. That happens even in period 1, when for $\phi = 1$ a self-fulfilling crisis does not exist for $\phi = 1$.

Remark 5. *For any $\phi > 1$, there exists parameter region such that sunspot equilibria exist.*

3.2.1 Evidence

We now investigate whether this implication holds empirically. Our measure of sunspot risk is the rolling 10-day standard deviation of the 1-year implied default probability (IDP), which we label Vol IDP. We compute IDP using CDS spreads at a daily frequency. Using a tight window limits the potential for fundamental risk (such as the release of quarterly GDP numbers) to explain a large share of IDP volatility. We use two measures for country's size: Debt and GDP.

Our preferred way of controlling for fundamental risk is to use the IDP level as its frequency is higher than traditional measures like Debt/GDP (daily rather than quarterly) and has considerably more explanatory power than them. Because the theory is about what happens before default, we also restrict the sample to have an IDP of less than 40%. We focus on emerging economies by also requiring the IDP to be greater than 1%. Table 5 in the appendix lists summary statistics.

Our benchmark specification is

$$\text{Vol IDP}_{k,q,t} = \alpha_q + \beta \log \text{Debt}_{k,q} + \gamma_1 \text{IDP}_{k,q,t} + \gamma_2 \text{IDP}^2_{k,q,t} + \epsilon_{k,q,t} \tag{6}$$

where k indexes countries, q time at a quarterly frequency, and t time at a daily frequency. This specification controls for fundamental risk by using the implied default probability and time fixed effects. We expect $\beta > 0$ as countries with more debt require the large bank to have more resources to execute the arbitrage opportunity.

The result is in model 1 of Table 6. As expected the coefficient is positive, and it is also statistically significant at conventional confidence levels.[7] The magnitude is of modest size, with a one standard deviation increase in log debt implying an increase in volatility equal to 6% of Vol IDP's standard deviation. Consistent with our expectation that the IDP absorbs all the fundamental default risk, including debt/GDP and its square (as is done in model 2) contributes almost no explanatory power, increasing the R^2 by a negligible amount.

While the amount of debt is the appropriate measure of size viewed through the model lens, a possible worry is it may be correlated with some other omitted variable that is producing $\beta > 0$. To alleviate this concern, we replace debt as a measure of size with GDP in models 3 and 4. In both models, the coefficient is still positive and similar to when using debt. Since more debt else equal would tend to worsen the country's situation while more GDP else equal would improve it, it is reassuring to have the same sign coefficient in both specifications.[8]

[7] We use Newey-West standard errors to correct for the autocorrelation in $\epsilon_{k,q,t}$ induced by the rolling window construction of Vol $\text{IDP}_{k,q,t}$.

[8] Most of our variation and statistical power comes from cross-sectional differences in size across coun-

Table 2: Volatility controlling for fundamental risk

	(1) Vol IDP	(2) Vol IDP	(3) Vol IDP	(4) Vol IDP
Log debt	0.0102	0.0110		
	(10.88)	(9.67)		
Log GDP			0.0113	0.0111
			(9.68)	(9.95)
IDP	0.0433	0.0437	0.0449	0.0438
	(10.19)	(9.80)	(10.25)	(9.82)
IDP^2	0.000338	0.000320	0.000293	0.000317
	(1.56)	(1.43)	(1.34)	(1.41)
Debt/GDP		-0.0671		-0.0311
		(-3.68)		(-1.83)
$Debt/GDP^2$		0.0332		0.0224
		(2.85)		(1.95)
Quarter FEs	Yes	Yes	Yes	Yes
Observations	58412	58412	58412	58412
R^2	0.39	0.39	0.39	0.39

Note: Newey-West standards errors with 10 lags have been used; t-stats are in parentheses; all regressions include a non reported constant.

Another implication from the model is that we should expect the increased susceptibility to sunspots only when the country is in the self-fulfilling region. In general, identifying whether a country is in the self-fulfilling region is very difficult. However, at very low levels of the IDP, the model would predict the country is not in the crisis region. Consequently, the effect of size on IDP volatility should be smaller at small IDP levels and larger at high IDP levels. To test this, we consider a semi-parametric specification where the response of IDP volatility to size-dependence is allowed to vary by IDP bin. Specifically, we use

$$\text{Vol IDP}_{k,q,t} = \alpha_q + \sum_j \beta_j \mathbf{1}_{[\text{IDP}_{k,q,t} \in [p_j, p_{j+1})]} \log \text{Debt}_{k,q} + \gamma_1 \text{IDP}_{k,q,t} + \gamma_2 \text{IDP}^2_{k,q,t} + \epsilon_{k,q,t} \quad (7)$$

for $\{p_j\}$ the IDP quintiles.

As shown in model 1 of Table 7, the results are consistent with theory. At low levels of the IDP, the elasticity of IDP volatility to size is small and positive while at high IDP levels the elasticity is much larger. At the highest IDP level, a one standard deviation in size implies a volatility increase equal to 43% of Vol IDP's standard deviation. The estimates are also much more statistically significant at the higher IDP quantiles. For brevity, we do not report the estimates using log GDP in place of log debt, but the results are almost identical.

3.2.2 Bank-specific shocks and global contagion

The theoretical finding that increased funding costs increase indeterminancy provides a natural mechanism through which shocks to globally systemically important banks (GSIBs) propagate to self-fulfilling sovereign debt crises *in multiple countries simultaneously*. This suggests that financial crises, in which GSIB funding costs increase, should be associated with contagion, loosely speaking. It is not actually contagion, however, because the crisis is not spreading from one country to another but from GSIBs—predominantly located in the the U.S.—to all countries at the same time.

tries (such as comparing Argentina and Uruguay). Including country fixed effects, the estimates become statistically insignificant.

Table 3: Level and volatility controlling for fundamental risk nonparametrically

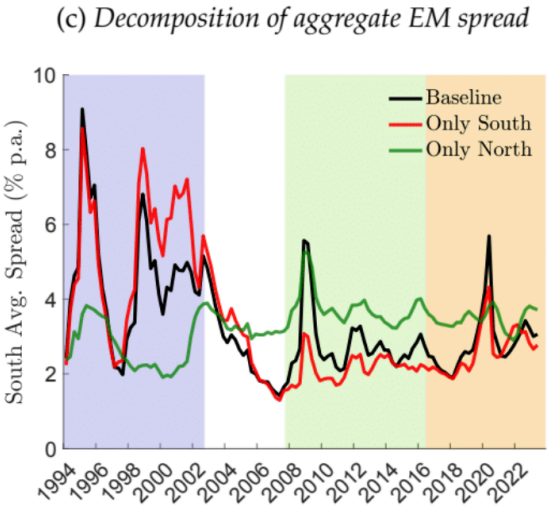
	(1) Vol IDP		(2) Vol IDP	
IDP Q=2	-0.0700	(-1.53)	-0.0824	(-1.71)
IDP Q=3	-0.0584	(-1.27)	-0.0874	(-1.82)
IDP Q=4	-0.111	(-1.99)	-0.0978	(-1.70)
IDP Q=5	-1.459	(-8.59)	-1.328	(-8.69)
IDP Q=1 $\times$ Log debt	0.00330	(2.10)	0.00372	(2.06)
IDP Q=2 $\times$ Log debt	0.00653	(6.82)	0.00738	(6.90)
IDP Q=3 $\times$ Log debt	0.00717	(7.89)	0.00874	(8.40)
IDP Q=4 $\times$ Log debt	0.00998	(7.47)	0.00991	(6.71)
IDP Q=5 $\times$ Log debt	0.0709	(10.41)	0.0660	(10.84)
Debt/GDP			-0.0966	(-4.45)
Debt/GDP^2			0.0572	(3.91)
Quarter FEs	Yes		Yes	
Observations	58412		58412	
R^2	0.17		0.17	

Note: Newey-West standards errors with 10 lags have been used; t-stats are in parentheses; all regressions include a non reported constant.

The testable prediction of the theory, then, is that financial crises in the U.S. should show up as international sovereign debt crises. One piece of evidence in favor of this prediction comes from Bai et al. (2024), which estimates time-varying contribution of developed economy shocks to emerging markets (Figure 6). They find that the Global Financial Crisis (GFC) period beginning in 2007 had by far the highest contribution of U.S. shocks to emerging markets. Comparing this series with the distance to default for large banks in figure 7, there was only one substantial period where default became much more likely, and that was in the GFC. For every unit decrease in distance to default, estimates from Gilchrist and Zakrajsek indicate credit spreads increase by 0.19pp. So these fluctuations in distance to default correspond to significant swings in credit spreads.

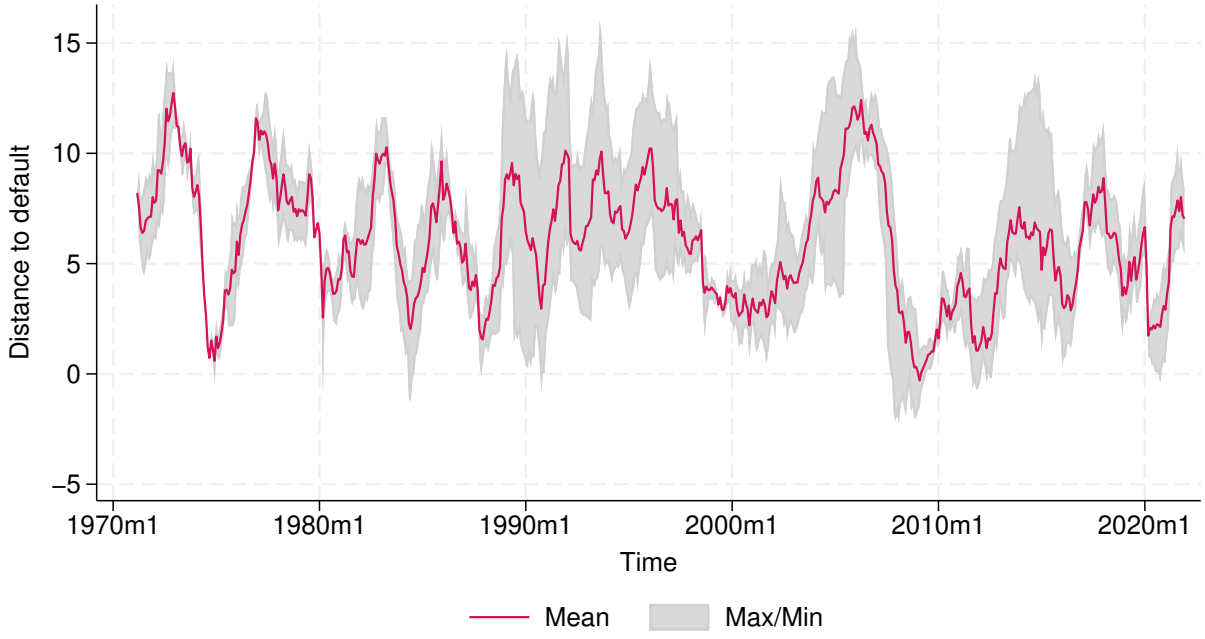
While the GFC is the period with largest financial stress for banks, figure 7 shows there are many other periods where banks were under pressure. Through the lens of the model, this bank stress should propagate into greater default risk for sovereigns and greater volatility of default risk. And these testable implications are borne out in figure

Figure 6: Bai et al. contribution of North shocks to Southern spreads



Source: Bai, Kehoe, Lopez, and Perri (2024)

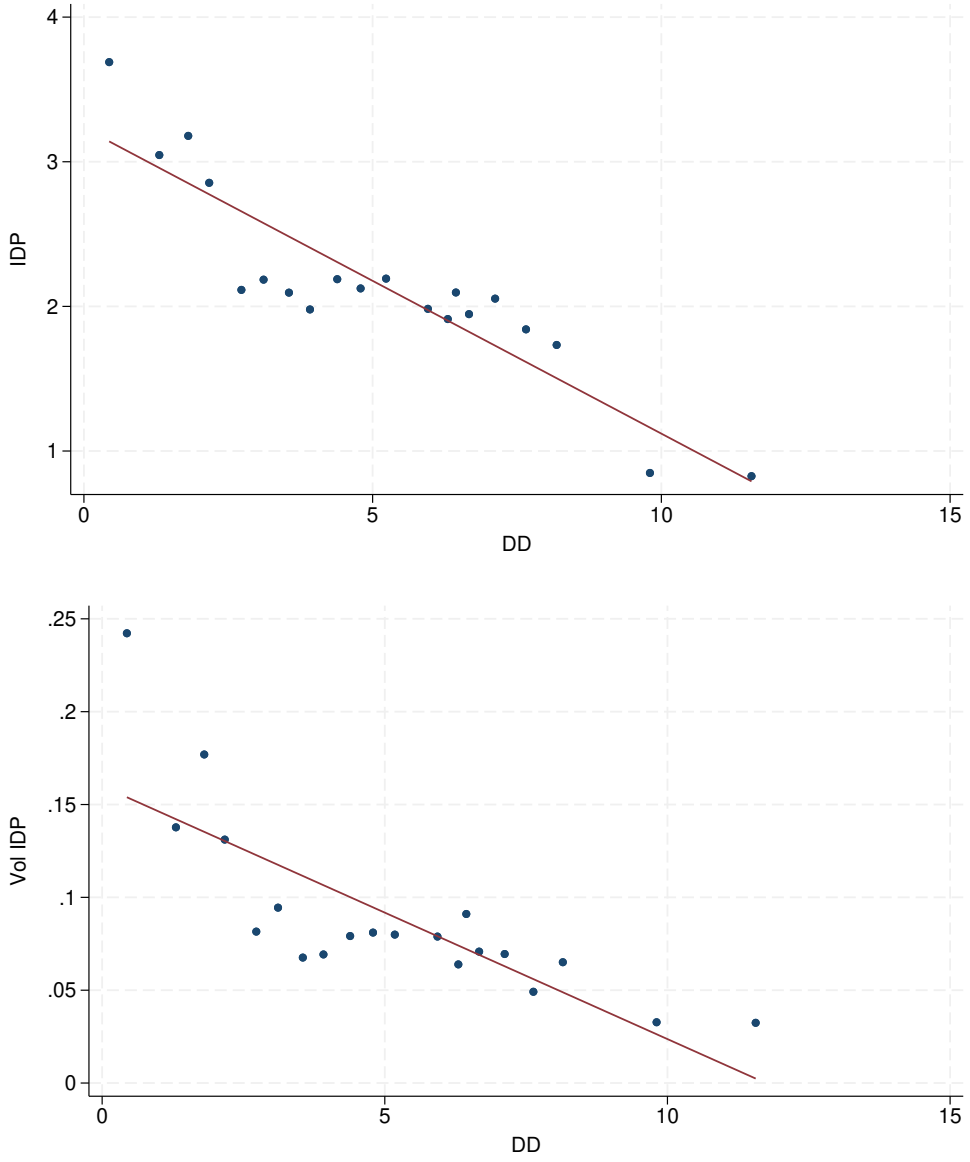
Figure 7: Distance to default for large banks



Note: The sample includes JP Morgan and Wells Fargo from 1971 on, Bank of America from 1980 on, Morgan Stanley from 1987 on and Goldman Sachs from 2000 on.

8, which plots bin scatters of IDP and Vol IDP against bank distance to default (DD). As distance to default decreases, and especially as it gets close to zero indicating severe stress, the IDP and Vol IDP rise.

Figure 8: IDP, Vol IDP, and distance to default for large banks



These graphical results are confirmed in table 4. As DD gets closer to zero, IDP and Vol IDP tend to rise, even controlling for year and country fixed effects. The effects are strongest at low DD, but even for relatively high DD (thus effectively dropping the GFC) the connection remains.

Table 4: Distance to default and implied default probabilities

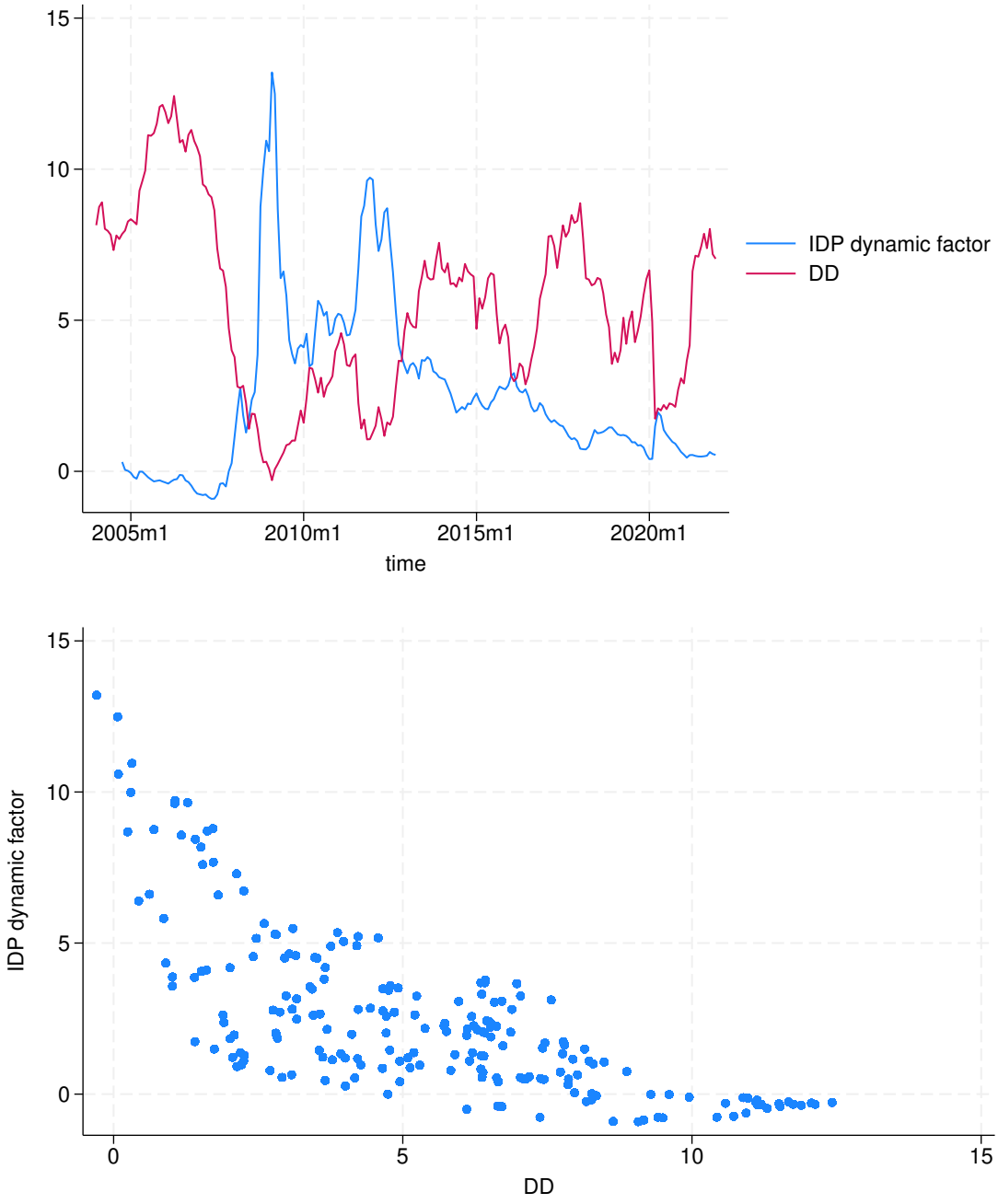
	(1) IDP	(2) IDP	(3) IDP	(4) Vol IDP	(5) Vol IDP	(6) Vol IDP
DD * $\mathbf{1}[DD < 4]$	-0.442	-0.469	-0.469	-0.00915	-0.0165	-0.0122
	(-10.3)	(-7.8)	(-10.1)	(-4.5)	(-5.9)	(-4.4)
DD * $\mathbf{1}[DD \geq 4]$	-0.258	-0.255	-0.268	-0.00420	-0.00848	-0.00585
	(-17.2)	(-7.6)	(-10.2)	(-5.8)	(-5.4)	(-3.7)
IDP				0.0402	0.0406	0.0534
				(54.7)	(55.6)	(48.1)
IDP^2				0.000540	0.000534	0.000358
				(36.2)	(36.0)	(19.3)
Year FEs	No	Yes	Yes	No	Yes	Yes
Country FEs	No	No	Yes	No	No	Yes
R^2	0.02	0.03	0.03	0.66	0.66	0.66
N	13729	13729	13729	13705	13705	13705

Note: dependent variable is the IDP rolling standard deviation; t-statistics are in parentheses.

The model also admits a true contagion mechanism. An increase in sovereign risk in one country reduces the market value of long positions for the large lender. This reduction in net worth serves to increase funding costs. Hence the financial linkages of countries to the U.S. can create contagion, like in Arellano et al. (2018). However, the mechanism here includes not just fundamental increases sovereign risk pricing, as in Arellano et al. (2018), but also increased sunspot risk.

We empirically investigate the contagion-like mechanisms by using a factor model. We specify that the IDP for each country loads on a common factor but otherwise evolves for each country according to a random walk with a country-specific variance. The common factor, which captures a cross-country component of default risk, evolves according to an AR1. After estimating this with maximum likelihood, we examine the relationship between the "contagion" factor and distance to default for large banks. The time series and scatter for these two variables is shown in figure 9. The plots suggest the model's predicted relationship that large lender distress should result in greater default risk for multiple countries simultaneously.

Figure 9: Global default risk factor and distance to default for large banks



One less than obvious multi-country aspect occurs when the large lender has a maximal amount of resources that can be used to prevent a crisis. Imagine there are two identical countries, A and B, and both are in the crisis region. Suppose the large lender has enough resources to bail out exactly one country. If a crisis occurs in A, intervention in A is desirable. But that intervention would prevent a crisis in A in equilibrium. But then in equilibrium a crisis would occur in B, where the lender would then like to intervene instead. This suggests there cannot be a pure strategy equilibrium. Another implication is that in a multi-country environment where a large lender has enough resources to rescue only one country, crises–if they occur—must occur simultaneously in multiple countries.

3.3 Syndicated loans

The main mechanism driving the presence or absence of sunspot crises is a holdup problem. The availability of secondary market resale in section 3.1 eliminated sunspot crises to the extent resell was costless by removing exposure from the large lender's portfolio. However, the holdup problem can actually be an advantage for both the large lender and the sovereign in certain states—if the large lender can commit to maintaining exposure on its balance sheet. This is where the distinction between syndicated loans and bonds becomes important. Syndicated loans are less liquid than bonds, especially when held by a single lender (Santos and Shao, 2023). This allows a large lender to credibly commit to maintaining exposure on its balance sheet. This commitment is essential, because if the large lender can unload the debt in a Walrasian auction, the holdup problem would not exist.

In the equilibrium of the baseline model, the large lender is always weakly outbid by small lenders and so, wlog, never obtains exposure in equilibrium. We now suppose that the sovereign has a discrete choice of running an auction or making a take-it-or-leave-it (TIOLI) offer to the large lender. To simplify the analysis, assume we are in the same economy as before but for one difference. In period 0 the sovereign revenue can take two values, Y_0^l and Y_0^h, such that if Y_0^h is realized, the sovereign is in the safe region

and if Y_0^l is realized, the sovereign is in the sunspot region. The revenue Y_0^l occurs with probability $\epsilon > 0$ and Y_0^h occurs with probability $1 - \epsilon$.

Suppose that before knowing the realization of Y_0 the sovereign can sell additional B_0^l bonds to the large lender in a syndicated loan. How much would the large lender be willing to pay for such bonds? If the revenue Y_0^h is realized the sovereign repays the debt fully. If the revenue Y_0^l is realized either there is a default or the holdup problem occurs and the large lender overpays for B_1 bonds. In either case, when Y_0^l is realized the large lender payoff is zero. As a result, the large lender is willing to pay a price $(1 - \epsilon)B_0^l$ for the B_0^l bonds in a syndicated loan.

Why would the sovereign want to do that? For two reasons. First, there is a direct motive for insurance. This operation reduces the sovereign revenue by ϵB_0^l when Y_0^h is realized and increases it by $(1 - \epsilon)B_0^l$ when Y_0^l is realized. Since $Y_0^l < Y_0^h$ the sovereign has a direct increase in welfare. Moreover, the additional revenue $(1 - \epsilon)B_0^l$ when Y_0^l is realized can, for some parameters, move the sovereign away from the sunspot region, eliminating self-fulfilling crisis.

Remark 6. *There exist parameter regions and revenue processes such that sunspot equilibria are eliminated in the presence of syndicated loans.*

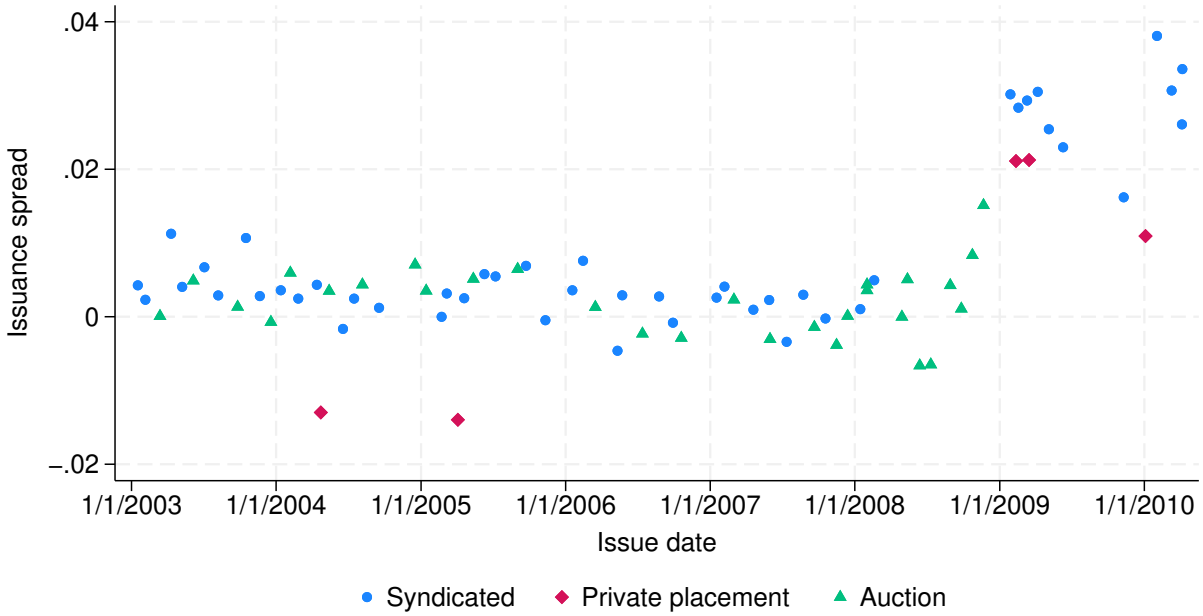
3.3.1 Evidence

This theory gives some testable predictions. First, syndicated loans transact at higher spreads / lower prices at origination to compensate the large lender for being held up later. Gupta et al. (2008) find precisely this: The less liquid the syndicated loan is, the greater the associated spread.

Second, syndicated loan activity should occur in response to increased periods of sunspot risk. This is supported by the pattern of Greek sovereign borrowing over the GFC.

The Greek borrowing pattern complements the mechanism in Bocola and Dovis (2019): Not only did the sovereign go longer in maturity, they also placed larger fractions of debt with syndicated loans. The lengthened maturity reduces debt-service obligations,

Figure 10: Greek issuance over the Global Financial Crisis



Source: Issuance data from Bank of Greece, authors' calculations.
Notes: (1) private issuances are floating rate, others are fixed rate.
(2) the type of one issuance on 5/21/2009 could not be determined and is dropped.

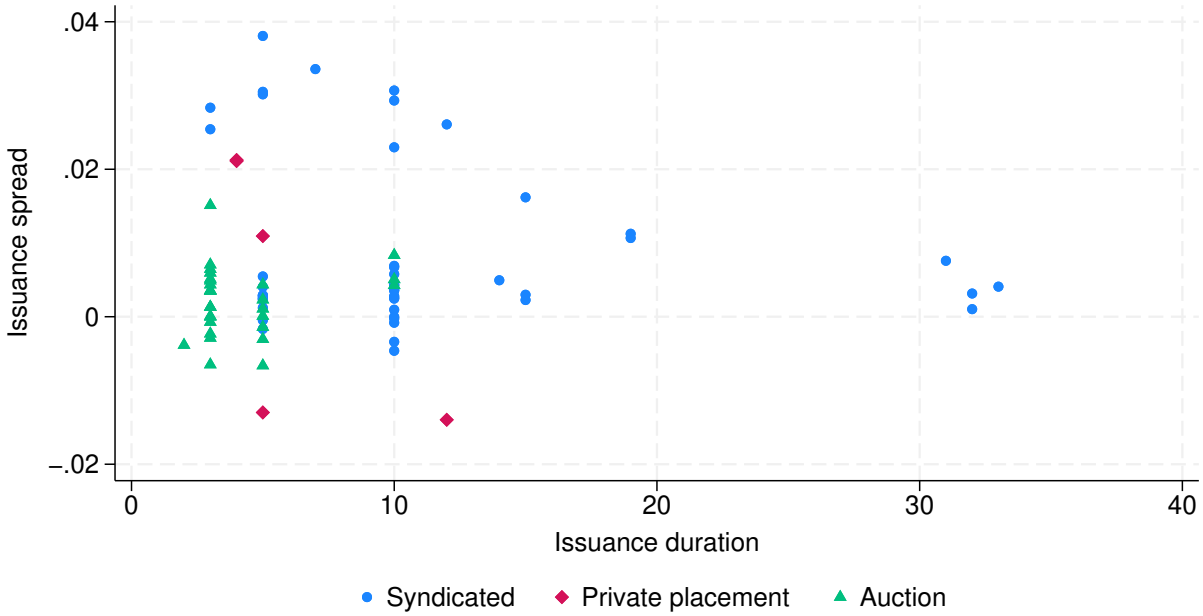
helping to prevent a run, and the syndicated loans provide insurance if a crisis occurs.

4 Infinite horizon model

5 Discussion on alternative models

Our model builds closely on the type of rollover crisis studied by Cole and Kehoe (2000), but our non-arbitrage refinement can be applied more broadly. Here we informally discuss how similar arguments apply to two alternative frameworks where multiple equilibria arise. We first focus on the type of multiplicity introduced by Calvo (1988). In his framework, multiplicity arises because, for a given budget deficit to be financed, multiple combinations of debt issuance and debt price can cover such deficit and be consistent with some expectation about default risk in equilibrium. We then study the framework proposed by Aguiar et al. (2020) which extends Cole and Kehoe (2000) by introducing uncertainty between the time of the auction and the repayment decision.

Figure 11: Greek issuance, spreads and maturity over the Global Financial Crisis



Source: Issuance data from Bank of Greece, authors' calculations.
Notes: (1) private issuances are floating rate, others are fixed rate.
(2) the type of one issuance on 5/21/2009 could not be determined and is dropped.

We leave the formal details and proofs to appendices B.2 and B.3.

5.1 Multiplicity of equilibria à la Calvo (1988)

Calvo (1988) proposes a two periods model where multiple interest rates can arise in equilibrium, depending on lender's (rational) expectations about the probability of a default. As opposed to Cole and Kehoe (2000), a sovereign does not choose to issue a given amount of debt but, instead, it chooses to raise a certain amount in revenues, for example to cover some amount of fiscal deficit. Thus, a deficit d can be covered by a continuum of combinations of debt issuance B' and bond price q such that $qB' = d$. Among all possible combinations of price and quantity, more than one of them can be consistent with rational expectations about default risk. That is, consistent with a no-arbitrage condition determining the price for some expected default probability and an optimal default decision for a given value of B' that supports such expected default risk. This type of multiplicity also arises in Lorenzoni and Werning (2019) and Ayres et al.

(2019).

Following Ayres et al. (2019), we slightly modify our benchmark model of Section **??**. As before there are two periods, $t = 1, 2$. At the beginning of period 1, the country has initial endowment, Y_1, and an initial stock of debt, $B_1 > 0$, with duration larger than one period. We model long-term debt assuming that each unit of debt matures with probability $\lambda \in (0, 1)$ at the beginning of $t = 1$. All the stock of debt outstanding at the beginning of $t = 2$ is assumed to mature in that period.

The main differences compared to our benchmark model are: (a) the timing of actions in period 1; and, (b) that the sovereign chooses an amount of resources to be collected through debt issuance instead of next period's stock of debt. Within period 1, the timing of actions is as follows: (i) the sunspot shock ζ_1 is realized; (ii) the sovereign decides to repay maturing debt or default on the whole stock of debt; (iii) the sovereign announces the size of the deficit d_1 (in units of good) to be covered with newly issued debt, (iv) each lender $i \in I$ submits a bid in the form of a one-step demand function, $bid_1^i = (\hat{b}_2^i, q_2^i)$, where the demand is $\hat{b}_2^i$ for any price $q_2 \leq q_2^i$, and zero otherwise; and (v) the sovereign orders the bid from highest to lowest price and issues enough units of the bond to raise d_1 units of goods when lenders pay the price of the marginal bid as in Section **??**.

In period 2, if the country is not in default, the sovereign observes the realization of the endowment $Y_2 \in y^\ell, y^h$, with $y^\ell < y^h$, and stock of debt B_2 outstanding and decides whether to repay or default. Let $p \in (0, 1)$ be the probability that y^ℓ is realized. Under this framework, we prove the following proposition, analogous to our main result of Section **??**. We provide all the details of the environment and the proof in appendix B.2.

Proposition 1. *Suppose there is a large lender with resources given by $\bar{a} > 0$ such that $d_1^{nd} \leq \bar{a}$, where d_1^{nd} solves the problem of the sovereign at the beginning of period $t = 1$ under the assumption that bonds will be priced at the no default price. Then, the low-price equilibrium does not exist.*

As in our baseline economy, the intuition behind this result is that a large lender has incentives to pay the price for the bond that is consistent with no-default. This is because, even though they make zero profit for the newly purchased bonds, they increase

the value of the bonds that they carried in their balance sheet from the past, as only a fractions λ of their bonds mature at the beginning of the period 1.[9]

5.2 Aguiar, Chatterjee, Cole and Stangebye (2021)

Aguiar et al. (2020) extends the Cole and Kehoe (2000) environment to introduce uncertainty between the time of the auction and the repayment. Specifically, they add a shock to the value function of the sovereign in case of default. Such uncertainty generates another type of equilibria where bonds price are depressed, but strictly positive, and default and repayment occur with positive probability depending of the sovereign shock.[10] This is in contrast with Cole and Kehoe (2000) where, at the time of the auction, based on the sunspot realization lenders know whether the sovereign will default or not, which results in either the bond price being zero or being associated with probability one of repayment in the current period.

Can this new type of equilibrium survive the existence of a large bank? Let us consider this possibility. Aguiar et al. (2020) established conditions for three equilibrium prices: $p_l = 0 < p_m < p_h$. At the price p_l the sovereign defaults for sure, at the price p_m the sovereign defaults with probability $\mu \in (0,1)$ strictly between 0 and 1, and at the price p_h the sovereign does not default. Note that the conditions are such that the sovereign is strictly better of by defaulting when facing p_l, and strictly better off by not defaulting when facing p_h.

Now suppose the large lender joins the primary auction for bonds when lenders are playing the equilibrium with price p_m. Since the sovereign is strictly better off by not defaulting when facing p_h, and indifferent when facing p_m, the large lender can submit a

[9] We setup a model of long-term debt as a way to have lenders holding unmatured debt and who could profit from the non-arbitrage refinement. Alternatively, we could have a model in which the sovereign could issue debt in multiple rounds as in Lorenzoni and Werning (2019) and short term debt. In such a case, if the low price equilibrium were about to be played, the large lender could purchase bonds at a price that is higher than the low price equilibrium but that forces the sovereign to issue bonds in a next round. Then, it could purchase enough bonds in the following round to eliminate the low price equilibrium and make a profit out of the bonds purchased in a previous round.

[10] Another interpretation of their result is that they establish the existence of a mixed strategy equilibrium. A zero price makes the sovereign strictly better of by defaulting, a sufficiently high price makes the sovereign strictly better off by not defaulting, and there is a price in between where the sovereign is indifferent and play a mixed strategy with the exact probabilities that support the price.

bid to buy the whole supply at price $p^* = p_h - \gamma$ for $\gamma \in (0, p_h - p_m)$ and the sovereign will not default. In this case, the price p_h is the fair value of the bonds, and the large bank's expected profit is $(p_h - p^*)B^g = \gamma B^g > 0$. Assuming the large bank has enough resources to buy the whole supply of bonds at a price $p^* > p_m$, the large bank can make strictly positive profit and, as a result, the price p_m is not an equilibrium outcome.

6 Conclusion

Self-fulfilling debt crisis has been an appealing explanation for many sovereign defaults events, and with the recent increase in sovereigns' debt around the world, it will certainly stay in the mind of policy makers and academics for years to come. Traditionally, the recommendation for sovereigns to prevent self-fulfilling debt crisis has been to stay in a *safe* region, where the sovereign is able to rollover its debt regardless of the market conditions for its debt. Going beyond that level would expose the sovereign to default events that are unrelated to sovereign fundamentals.

The above recommendation, however, only reflects the borrower side of the market and misses the actions that can be taken by lenders when coordination problems are overcome by the presence of large market participants. Our results show the causes of self-fulfilling crises extend far beyond coordination problems. A holdup problem makes cost of funds, liquidity of secondary markets, and the ability to issue syndicated loans all relevant for determining the extent of self-fulfilling risk, and we showed empirical support for these predictions. Banking crises, which reduce the amount of resources banks have to intervene in sunspot crises, seem to propagate to sovereign debt crises. In turn, this provides a rationale for the data's cross-country correlations in sudden stops (a rationale that is orthogonal to country fundamentals). Restrictions on credit default swaps (CDS) such as the European Union's ban on naked CDS can allow sovereigns to take advantage of large lenders off-equilibrium—and thereby harm themselves on-equilibrium. Regulations that penalize taking on risky debt may endogenously increase debt's riskiness and expose investors to more risk. Obfuscating the distribution of bond holdings might promote financial stability. The roots of self-fulfilling debt crises extend

far, and thinking carefully about these issues will improve our understanding of why self-fulfilling crises occur and how they can be prevented.

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Online Appendix for "Self-fulfilling debt crises and limits to arbitrage"

Gaston Chaumont, Grey Gordon and Bruno Sultanum

A Proofs

Proof of Lemma ??: Let $\left\{V_1, V_2, \hat{\delta}_1^*, \hat{\delta}_2^*, A_1^*, \boldsymbol{bid}_1^*, \{v_1^{i*}\}_i, \boldsymbol{B}_2^*, q_2^*\right\}$ be an equilibrium and suppose by the way of contradiction that for some vector (ω_1, A_1, ζ_1) and $B_2 > 0$ we have $\int_i B_2^{i*}(A_1; \boldsymbol{bid}_1^*(\omega_1, A_1, \zeta_1)) = B_2$ and $q_2^*(A_1; \boldsymbol{bid}_1^*(\omega_1, A_1, \zeta_1)) > q_2^{nd}(\omega_1, B_2) \geq 0$. We divide the proof into several steps and we omit the arguments ω_1, A_1 and ζ_1 below to keep the notation short.

Step 1 — $\hat{\delta}_1^*(\omega_1, B_2^*, q_2^*) \in [0, 1)$: If $\hat{\delta}_1^*(\omega_1, B_2^*, q_2^*) = 1$, then equation (**??**) is just $-\frac{R^i}{R} q_2 B_2^{i*}$ for all lenders, which is always non-positive. In this case, any lender i buying a positive amount of bonds B_2^{i*} at the price q_2, and getting a strictly negative payoff, would be strictly better off bidding zero and getting at least zero payoff.

Step 2 — Small lenders are allocated zero bonds: If, at the price q_2, a small lender $i \in I$ were allocated bonds $B_2^{i*} > 0$, their profit would be

$$
\begin{aligned}
&\left[1 - \hat{\delta}_1^*\right] B_1^i + \left[\frac{1}{R} \mathbb{E}_{Y_2} \left\{ \left(1 - \hat{\delta}_1^*\right) \left[1 - \hat{\delta}_2^*\left(B_2^*, 0, Y_2\right)\right] \right\} - \frac{R^i}{R} q_2^* \right] B_2^{i*} = \\
&\left[1 - \hat{\delta}_1^*\right] \left\{ B_1^i + \left[\frac{1}{R} \mathbb{E}_{Y_2} \left\{1 - \hat{\delta}_2^*\left(B_2^*, 0, Y_2\right)\right\} - q_2^* \right] B_2^{i*} \right\} = \\
&\left[1 - \hat{\delta}_1^*\right] \left\{ B_1^i + \left[\frac{1 - F\left(h^{-1}(B_2)\right)}{R} - q_2^* \right] B_2^{i*} \right\} = \\
&\left[1 - \hat{\delta}_1^*\right] \left\{ B_1^i + \left[q_2^{nd,s}(\omega_1, B_2) - q_2^* \right] B_2^{i*} \right\}.
\end{aligned}
$$

We established in step 1 that $[1 - \hat{\delta}_1^*] > 0$, and, since $q_2^{nd,s}(\omega_1, B_2) \leq q_2^{nd}(\omega_1, B_2) < q_2^*$, the term $[q_2^{nd,s}(\omega_1, B_2) - q_2^*] B_2^{i*}$ is strictly negative. As a result, lender $i \in I$ would be strictly better off bidding zero and being allocated no bonds.

Step 3 — The large lender is allocated zero bonds, which is a contradiction: We know that small lenders are allocated zero bonds, which implies that the large lender has to be allocated all the bonds $B_2 > 0$, but this can not happen. If, at the price q_2, the large lender were allocated bonds $B_2^{l*} = B_2 > 0$, their profit would be

$$
\begin{aligned}
&\left[1 - \hat{\delta}_1^*\right] B_1^l + \left[\frac{1}{R} \mathbb{E}_{Y_2} \left\{ \left(1 - \hat{\delta}_1^*\right) \left[1 - \hat{\delta}_2^*\left(B_2^*, 0, Y_2\right)\right] \right\} - \frac{R^l}{R} q_2^* \right] B_2 = \\
&\left[1 - \hat{\delta}_1^*\right] \left\{ B_1^l + \left[\frac{1}{R} \mathbb{E}_{Y_2} \left\{1 - \hat{\delta}_2^*\left(B_2^*, 0, Y_2\right)\right\} - \frac{R^l}{R} q_2^* \right] B_2 \right\} =
\end{aligned}
$$

$$\left[1-\hat{\delta}_1^*\right]\left\{\frac{B_1^l}{B_2}+\frac{1-F\left(h^{-1}(B_2)\right)}{R}-\frac{R^l}{R}q_2^*\right\}B_2 =$$
$$\left[1-\hat{\delta}_1^*\right]\left\{\tilde{q}_2^{nd,l}(\omega_1,B_2)-q_2^*\right\}(1+\phi)B_2.$$

From the above we already know that $\tilde{q}_2^{nd,l}(\omega_1,B_2) \geq q_2^*$, otherwise the above term is strictly negative and the large lender would be better off bidding to purchase zero bonds. We had assume by the way of contradiction that $q_2^* > q_2^{nd,l}(\omega_1,B_2)$, but note that by definition we also have that $\tilde{q}_2^{nd,l}(\omega_1,B_2) > q_2^{nd,l}(\omega_1,B_2)$, which lead us to the case where $\tilde{q}_2^{nd,l}(\omega_1,B_2) \geq q_2^* > q_2^{nd,l}(\omega_1,B_2) \geq 0$.

If $W_1(\omega_1,0,0,0) \leq W_1(\omega_1,0,0,1)$, then $q_2^{nd,l}(\omega_1,B_2) = \tilde{q}_2^{nd,l}(\omega_1,B_2)$ and therefore $\tilde{q}_2^{nd,l}(\omega_1,B_2) > q_2^{nd,l}(\omega_1,B_2)$ cannot hold.

We cannot have $W_1(\omega_1,0,0,0) > W_1(\omega_1,0,0,1)$ and $\tilde{q}_2^{nd}(\omega_1,B_2) < q_2^{ind}(\omega_1,B_2)$ because $q_2^* \leq \tilde{q}_2^{nd}(\omega_1,B_2) < q_2^{ind}(\omega_1,B_2)$ would imply $\hat{\delta}_1^*(\omega_1,B_2^*,q_2^*)=1$, which we rule out in step 1. If $W_1(\omega_1,0,0,0) > W_1(\omega_1,0,0,1)$ and $\tilde{q}_2^{nd}(\omega_1,B_2) \geq q_2^{ind}(\omega_1,B_2)$, we have that

$$q_2^{nd,l}(\omega_1,B_2) = \max\left\{\frac{1-F\left(h^{-1}(B_2)\right)}{(1+\phi)R}, q_2^{ind}(\omega_1,B_2)\right\} < q_2^*.$$

In this case, the large lender could reduce its bid slightly below q_2^* maintaining the inequality above. This would either trigger a failed auction, which would increase profits because the sovereign would not default and $q_2^* > \frac{1-F\left(h^{-1}(B_2)\right)}{(1+\phi)R}$, or the default probabilities would not change and profits would increase because the large lender would pay less to buy B_2. In either case, this contradicts large lender buying the bonds B_2 at the price q_2^*. This concludes the proof. □

Proof of Proposition ??: Let $\left\{V_1,V_2,\hat{\delta}_1^*,\hat{\delta}_2^*,A_1^*,\boldsymbol{bid}_1^*,\{v_1^{i*}\}_i,\boldsymbol{B}_2^*,q_2^*\right\}$ be an equilibrium. We must have $q_2^*(A_1;\boldsymbol{bid}_1(\omega_1,A_1,\zeta_1)) \leq q_2^{nd}(\omega_1,B_2^*(A_1;\boldsymbol{bid}_1(\omega_1,A_1,\zeta_1)))$ for every auction A_1 and ζ_1. But then

$$\begin{aligned}
&W(\omega_1,B_2^*,q_2^*,0) - W_1(\omega_1,B_2^*,q_2^*,1)\\
&= u(Y_1+q_2^*B_2^*-B_1) - u(Y_1-h(Y_1)+q_2^*B_2^*) + \beta\mathbb{E}\left[V_2(\omega_1,B_2^*,0,Y_2) - V_2(\omega_1,B_2^*,1,Y_2)\right]\\
&\leq u(Y_1+q_2^{nd}(B_2^*)B_2^*-B_1) - u(Y_1-h(Y_1)+q_2^{nd}(B_2^*)B_2^*)\\
&\quad+\beta\mathbb{E}\left[V_2(\omega_1,B_2^*,0,Y_2) - V_2(\omega_1,B_2^*,1,Y_2)\right]\\
&\leq W_1(\omega_1,B_2^*,q_2^{nd}(\omega_1,B_2^*),0) < W_1(\omega_1,B_2^*,q_2^{nd}(\omega_1,B_2^*),1) < 0,
\end{aligned}$$

where the inequalities come from the concavity of $u(\cdot)$ and equation (**??**), and we omit the argument $(A_1;\boldsymbol{bid}_1(\omega_1,A_1,\zeta_1))$ from q_2^* and B_2^*. The above inequality implies that the sovereign best response has to be to default in period 1. As a result, in any equilibrium, there is probability one of default in period 1, and the sovereign utility is $u\big(Y_1-h(Y_1)\big)+\beta\mathbb{E}\left[u\big(Y_1-h(Y_1)\big)\right]$. This concludes the proof. □

Proof of Proposition ??: First notice that the sovereign's payoff $W_1(\omega_1, B_2, q_2^{nd}(\omega_1, B_2), 0)$ is the same same for any two $\boldsymbol{B}_2$ and $\hat{\boldsymbol{B}}_{\boldsymbol{2}}$ such that

$$\int_{i\in\tilde{I}} B_2^i di = \int_{i\in I} B_2^i di + B_2^l = \int_{i\in\tilde{I}} \hat{B}_2^i di = \int_{i\in I} \hat{B}_2^i di + \hat{B}_2^l.$$

Let us first show that there is a no-default equilibrium. Suppose lenders believe the sovereign will default if, and only if, the auction issuance outcome $\boldsymbol{B}_2$ satisfy

$$W_1(\omega_1, B_2, q_2^{nd}(\omega_1, B_2), 0) < W_1(B_2, q_2^{nd}(B), 1).$$

Note that $W_1(\omega_1, B_2, q_2^{nd}(\omega_1, B_2), 0) < W_1(\omega_1, B_2, q_2^{nd}(\omega_1, B_2), 1)$ implies $W_1(\omega_1, B_2, 0, 0) < W_1(\omega_1, B_2, 0, 1)$ due to the concavity of the utility function. Thus, this belief is consistent with the sovereign's optimal response.

In this case, the optimal choice of the sovereign is any $\boldsymbol{B}_2^*$ that solves problem (**??**), and we know at least one solution exists because of condition (**??**). To see this, first note that such B_2 exists since, by equation (**??**), the set of B_2 satisfying $W(B, q_2^{nd}(B), 0) \geq W(B, q_2^{nd}(B_2), 1)$ is non-empty. We can also show that this set is compact due to the weak inequality and because it has to be bounded above—if B_2 is large enough, it is always optimal to default on period 1. Choosing $B_2^* = B_2^{nd}(Y_1, B_1)$ also has to be better than picking a B_2 satisfying $W_1(\omega_1, B_2, q_2^{nd}(\omega_1, B_2), 0) < W_1(\omega_1, B_2, q_2^{nd}(\omega_1, B_2), 1)$ and defaulting for sure in period 1 because

$$W_1(B_2^*, q_2^{nd}(B_2^*), 0) \geq W_1(B_2^*, q_2^{nd}(B_2^*), 1) \geq W_1(B_2^{nd}(Y_1, B_1), 0, 1) = W_1(B_2, 0, 1)$$

for any B_2 satisfying $W_1(\omega_1, B_2, q_2^{nd}(\omega_1, B_2), 0) < W_1(\omega_1, B_2, q_2^{nd}(\omega_1, B_2), 1)$. The last equality comes simply from the fact that, if the issuance bond price is zero and I default, then issuance volume B_2 does not affect the welfare.

Now let us look at the sunspot equilibrium. Consider the problem

$$\max_{B_2} \left\{ (1-\bar{\zeta}_1) W_1(\omega_1, B_2, q_2^{nd}(\omega_1, B_2), 0) + \bar{\zeta}_1 W_1(\omega_1, B_2, q_2^{nd}(\omega_1, B_2), 1) \right\}$$
$$\text{subject to } W_1(\omega_1, B_2, q_2^{nd}(\omega_1, B_2), 0) \geq W_1(\omega_1, B_2, q_2^{nd}(\omega_1, B_2), 1),$$

where $\bar{\zeta}_1 \in (0, 1)$. From the solution of the above problem—label it $B_2^{\bar{\zeta}_1}$—we can construct an equilibrium where the sovereign issues $B_2^{\bar{\zeta}_1}$ and; if $\zeta_1 \leq \bar{\zeta}_1$: lenders believe the sovereign will default, the realized bond price q_2 is zero, and the sovereign defaults; and if $\zeta_1 > \bar{\zeta}_1$: lenders belief the sovereign will not default, the realized bond price $q_2(B_2^{\bar{\zeta}_1})$ is zero, and the sovereign does not default.

By construction not to default is optimal when $\zeta_1 > \bar{\zeta}_1$ and bond price is $q^{nd}(B_2^{\bar{\zeta}_1})$. That is because $B_2^{\bar{\zeta}_1}$ satisfy the constraint $W_1(\omega_1, B_2, q_2^{nd}(\omega_1, B_2), 0) \geq W_1(\omega_1, B_2, q_2^{nd}(\omega_1, B_2), 1)$ imposed in the above problem. To see the default case note that, for $\bar{\zeta}_1$ small enough, we know $B_2^{\bar{\zeta}_1}$ has to be close to some solution $B_2(Y_1, B_1)$ of problem **??**. But then, because inequality (**??**) is not satisfied, we have that $W_1(B_2^{\bar{\zeta}_1}, 0, 0) < W_1(B_2^{\bar{\zeta}_1}, 0, 1)$ and it is optimal

for the sovereign to default. Since the optimal sovereign decisions are consistent with lenders beliefs

For the last, we need that picking B_2 satisfying the constraint $W_1(\omega_1, B_2, q_2^{nd}(\omega_1, B_2), 0) \geq W_1(\omega_1, B_2, q_2^{nd}(\omega_1, B_2), 1)$ is better than picking one that doesn't satisfy this constraint. For that to be true we assume lenders believe the sovereign will default otherwise. Then we can justify this as an equilibrium outcome in the same way we did for the non-default equilibrium, which also implies that the sovereign highest utility is the maximum in problem (**??**) in the non-default equilibrium. This concludes the proof. □

Proof of Lemma ??: This proof is straightforward. When lenders $i \in I^\epsilon$ replicate the strategies of the equilibrium $\mathcal{E}$, then conditions 1–2 in the equilibrium definition **??** are restatements of conditions 3–4 in the equilibrium definition **??**. Then, since $\mathcal{E}$ is an equilibrium, conditions 3–4 in definition **??** are satisfied, which implies the result. □

Proof of Lemma ??: When $\bar{a}$ does not bind, the conditions imposed in equilibrium definition **??** imply that $q_2^{\epsilon *}(B_2^g, q_2^{min}, \zeta_1) = \frac{1-\hat{\delta}_2^*\left(B_2,(\omega_1,\boldsymbol{B}_2^*,q_2^{\epsilon *}(B_2^*,\zeta_1)),Y_2\right)}{R}$. By replacing this equality in the payoff equation (**??**), we obtain the result. □

Proof of Proposition ?? *(Sketch)*: Suppose we are in a default realization of the sunspot. The lender can bid in the auction to buy all the bonds offered, B_2^*, at any price not larger than the fundamental price of the non-default sunspot realization, $q_2^{nd}(B_2^*)$. This bid creates a floor on the auction price. In this case, it is optimal for the sovereign not to default in period 1 since that was the outcome of the non-default sunspot realizations when the sovereign faces the auction price $q_2^{nd}(B_2^*)$. The lender is indifferent between buying or not buying the bonds, B_2^*, in the auction at the fundamental value $q_2^{nd}(B_2^*)$ because it is the expected payoff of the bonds. The lender is also paid the face value of the period-1 bonds, $B_1 > 0$, which she would otherwise not get paid because the sovereign was going to default. This generates a strictly positive profit—and, therefore, an arbitrage opportunity—that is inconsistent with the equilibrium refinement. □

Proof of Proposition ?? *(Sketch)*: In the subgame associated with issuance $B_2^{nd}(Y_1, B_1)$, it must be the case that the sovereign achieves the utility associated with Problem (**??**) since there cannot be a default—otherwise the coalition would deviate. As a result, the sovereign can always issue $B_2^{nd}(Y_1, B_1)$, and any equilibrium that survives the refinement does not have default in period 1 associated with this issuance level. Then, we cannot have issuance B^* with default in period one. □

B Extensions and Alternative models

B.1 Long term debt

B.2 Calvo (1988)

In this appendix, we provide the formal details for how our non-arbitrage refinement extends to a Calvo (1988) model. Following Ayres et al. (2019), we slightly modify our benchmark model of Section **??**. As before there are two periods, $t = 1, 2$. At the beginning of period 1, the country has initial endowment, Y_1, and an initial stock of debt, $B_1 > 0$, with duration larger than one period. We model long term debt assuming that each unit of debt matures with probability $\lambda \in (0, 1)$ at the beginning of $t = 1$. All the stock of debt outstanding at the beginning of $t = 2$ is assumed to mature in that period.

The main differences compared to our benchmark model are: (a) the timing of actions in period 1; and, (b) that the sovereign chooses an amount of resources to be collected through debt issuance instead of next period's stock of debt. Within period 1, the timing of actions is as follows: (i) the sunspot shock ζ_1 is realized; (ii) the sovereign decides to repay maturing debt or default on the whole stock of debt; (iii) the sovereign announces the size of the deficit d_1 (in units of good) to be covered with newly issued debt, (iv) each lender $i \in I$ submits a bid in the form of a one-step demand function, $bid_1^i = (\hat{b}_2^i, q_2^i)$, where the demand is $\hat{b}_2^i$ for any price $q_2 \leq q_2^i$, and zero otherwise; and (v) the sovereign orders the bid from highest to lowest price and issues enough units of the bond to raise d_1 units of goods when lenders pay the price of the marginal bid as in Section **??**.

In period 2, if the country is not in default, the sovereign observes the realization of the endowment $Y_2 \in y^\ell, y^h$, with $y^\ell < y^h$, and stock of debt B_2 outstanding and decides whether to repay or default. Let $p \in (0, 1)$ be the probability that y^ℓ is realized. The default decision in period 2 is thus characterized by equation (**??**) as in our benchmark model of Section **??**.

In period 1 lenders submit their bids after observing the realization of a sunspot shock, ζ_1, and the announced deficit, d_1. The price submitted by individual lenders has to be consistent with the expected default probability and the no-arbitrage condition equating the expected return of the sovereign bond and the return of the risk free asset, R. That is,

$$R = \rho(q; d_1) \equiv \left[1 - \Pr\left(Y_2(1 - h(1)) < \frac{d_1}{q}\right)\right] \frac{1}{q}, \tag{8}$$

where q is the cutoff price in the auction and $\rho(q; d_1)$ represents the expected return to the lender. Given the optimal default policy of period 2, for each given deficit d_1, the expected return is

$$\rho(q; d_1) = \begin{cases} \frac{1}{q} & \text{if} \quad \frac{1}{q} \leq \frac{y^\ell[1-h(1)]}{d_1} \\ \frac{1-p}{q} & \text{if} \quad \frac{y^\ell[1-h(1)]}{d_1} < \frac{1}{q} \leq \frac{y^h[1-h(1)]}{d_1} \\ 0 & \text{if} \quad \frac{1}{q} > \frac{y^h[1-h(1)]}{d_1} \end{cases}.$$

We can thus define the correspondence relating the deficit d_1 to bond price:

$$\mathcal{Q}(d_1) = \begin{cases} \frac{1}{R} & \text{if} \quad d_1 \leq \frac{y^\ell[1-h(1)]}{R} \\ \frac{1-p}{R} & \text{if} \quad \frac{(1-p)y^\ell[1-h(1)]}{R} < d_1 \leq \frac{(1-p)y^h[1-h(1)]}{R} \\ 0 & \text{if} \quad d_1 > \frac{y^h[1-h(1)]}{R} \end{cases}.$$

It then follows that a necessary condition for multiple equilibrium prices is

$$d_1 \leq \frac{y^\ell[1-h(1)]}{R} < \frac{d_1}{1-p} \leq \frac{y^h[1-h(1)]}{R}.$$

After observing the initial state (Y_1, B_1, ζ_1), and anticipating the market response, the sovereign chooses whether to repay its debt or default. In addition, in case of choosing to repay, the sovereign decides how many resources to raise issuing new debt d_1. We focus on the values of (Y_1, B_1) such that the sovereign does not default at the beginning of period 1, as those are the state in which the non-arbitrage refinement is relevant. After repayment, the sovereign chooses d_1 to solve

$$\begin{aligned} d_1^* = \arg\max\, & u(Y_1 + d_1 - \lambda B_1) + \beta \mathbb{E}_{Y_2} \max\{u(Y_2 - B_2), u(Y_2 - h(Y_2))\} \\ \text{subject to } & B_2 = (1-\lambda)B_1 + d_1/q_2(d_1, \zeta_1). \end{aligned} \tag{9}$$

Definition 1. *An equilibrium is a family* $\left\{\hat{\delta}_1^*, \hat{\delta}_2^*, d_1^*, bid^*, (b_2^{i*})_i, B_2^*, q_2^*\right\}$ *where*

1. *the sovereign period-2 policy function* $\hat{\delta}_2^*(B_2, 0, Y_2)$ *satisfies equation* (**??**)*;*
2. *the sovereign period-1 policy function* d_1^* *satisfies equation* (9)*;*
3. *the lenders bid function is* $bid_1^i(d_1, \zeta_1) = \left(\bar{b}(d_1, \zeta_1), q_2^*(d_1, \zeta_1)\right)$*, where* $q_2^*(d_1, \zeta_1)\bar{b}(d_1, \zeta_1) = \bar{a}$ *if* $q_2^*(d_1, \zeta_1) > 0$*, and the period-2 bond price function* $q_2^*(\cdot)$ *satisfies equation* (8)*; and*
4. *lenders period-2 bond allocation in the auction* $b_2^{i*}(d_1, \zeta_1)$ *solves equation* (**??**) *for all* $i \in I$*.*

Consider now the problem of the sovereign assuming that the price schedule is consistent with no default when there are multiple equilibria, given by

$$\max_{d_1 \geq 0} \left\{ u(h(0)Y_1 + d_1 - \lambda B_1) + \beta \mathbb{E}_{Y_2} \left[V_2\big((1-\lambda)B_1 + d_1/q^{nd}(d_1), 0, Y_2\big) \right] \right\} \tag{10}$$

where $q_2^{nd}(d_1) := \frac{1}{R}$ if $d_1 R < y^h[1-h(1)]$ and $q_2^{nd}(d_1) = 0$ otherwise. For a given deficit d_1, $q_2^{nd}(d_1)$ is the price associated with no default in period 1. This is the highest price the sovereign can get for its debt and induces the highest issuance.

Proposition 2. *Suppose there is a coalition of lenders such that* $d_1^{nd} \leq \epsilon\bar{a}$*, where* d_1^{nd} *solves problem* (10)*. Then, the equilibrium with low bond price does not survive the non-arbitrage refinement.*

The proof is as follows. WRITE PROOF. □

B.3 Aguiar et al (2021)

C Empirical analysis

Our theory predicts that, controlling for fundamental risk, a larger country is more susceptible to self-fulfilling crises. That is because larger financing needs requires a large bank to have more resources to rescue the sovereign during a self-fulfilling crisis—making the crisis more likely to occur. This is in contrast with most models, where homotheticity implies that the a country's size should be irrelevant.

In this section, we investigate whether this implication holds empirically. We use two measures for country's size: Debt and GDP. Our measure of sunspot risk is the rolling 10-day standard deviation of the 1-year implied default probability (IDP), which we label Vol IDP. We compute IDP using CDS spreads at a daily frequency. Using a tight window limits the potential for fundamental risk (such as the release of quarterly GDP numbers) to explain a large share of IDP volatility.

Our preferred way of controlling for fundamental risk is to use the IDP level as its frequency is higher than traditional measures like Debt/GDP (daily rather than quarterly) and has considerably more explanatory power than them. Because the theory is about what happens before default, we also restrict the sample to have an IDP of less than 40%. We focus on emerging economies by also requiring the IDP to be greater than 1%. Table 5 lists summary statistics. In the appendix we provide a full description of our data and additional summary statistics by country.

Table 5: Summary statistics

Daily variables							
	count	mean	sd	p25	p75	min	max
IDP	58412	2.96	3.23	1.39	3.25	1.00	39.79
Vol IDP	58412	0.11	0.27	0.03	0.11	0.00	15.90
Quarterly variables							
	count	mean	sd	p25	p75	min	max
Log debt	890	25.52	1.63	24.22	26.51	20.98	30.07
Log GDP	890	26.22	1.57	24.81	27.60	22.95	30.13
Debt/GDP	890	0.64	0.46	0.32	0.84	0.06	2.43
Year	890	2014	3.39	2011	2017	2007	2021

Our benchmark specification is

$$\text{Vol IDP}_{k,q,t} = \alpha_q + \beta \log \text{Debt}_{k,q} + \gamma_1 \text{IDP}_{k,q,t} + \gamma_2 \text{IDP}^2_{k,q,t} + \epsilon_{k,q,t} \tag{11}$$

where k indexes countries, q time at a quarterly frequency, and t time at a daily frequency. This specification controls for fundamental risk by using the implied default probability and time fixed effects. We expect $\beta > 0$ as countries with more debt require the large bank to have more resources to execute the arbitrage opportunity.

The result is in model 1 of Table 6. As expected the coefficient is positive, and it is also statistically significant at conventional confidence levels.[11] The magnitude is of modest size, with a one standard deviation increase in log debt implying an increase in volatility equal to 6% of Vol IDP's standard deviation. Consistent with our expectation that the IDP absorbs all the fundamental default risk, including debt/GDP and its square (as is done in model 2) contributes almost no explanatory power, increasing the R^2 by a negligible amount.

Table 6: Volatility controlling for fundamental risk

	(1) Vol IDP	(2) Vol IDP	(3) Vol IDP	(4) Vol IDP
Log debt	0.0102	0.0110		
	(10.88)	(9.67)		
Log GDP			0.0113	0.0111
			(9.68)	(9.95)
IDP	0.0433	0.0437	0.0449	0.0438
	(10.19)	(9.80)	(10.25)	(9.82)
IDP^2	0.000338	0.000320	0.000293	0.000317
	(1.56)	(1.43)	(1.34)	(1.41)
Debt/GDP		-0.0671		-0.0311
		(-3.68)		(-1.83)
$Debt/GDP^2$		0.0332		0.0224
		(2.85)		(1.95)
Quarter FEs	Yes	Yes	Yes	Yes
Observations	58412	58412	58412	58412
R^2	0.39	0.39	0.39	0.39

Note: Newey-West standards errors with 10 lags have been used; t-stats are in parentheses; all regressions include a non reported constant.

While the amount of debt is the appropriate measure of size viewed through the model lens, a possible worry is it may be correlated with some other omitted variable that is producing $\beta > 0$. To alleviate this concern, we replace debt as a measure of size with GDP in models 3 and 4. In both models, the coefficient is still positive and similar to when using debt. Since more debt else equal would tend to worsen the country's situation while more GDP else equal would improve it, it is reassuring to have the same sign coefficient in both specifications.[12]

[11] We use Newey-West standard errors to correct for the autocorrelation in $\epsilon_{k,q,t}$ induced by the rolling window construction of Vol $IDP_{k,q,t}$.

[12] Most of our variation and statistical power comes from cross-sectional differences in size across countries (such as comparing Argentina and Uruguay). Including country fixed effects, the estimates become

Another implication from the model is that we should expect the increased susceptibility to sunspots only when the country is in the self-fulfilling region. In general, identifying whether a country is in the self-fulfilling region is very difficult. However, at very low levels of the IDP, the model would predict the country is not in the crisis region. Consequently, the effect of size on IDP volatility should be smaller at small IDP levels and larger at high IDP levels. To test this, we consider a semi-parametric specification where the response of IDP volatility to size-dependence is allowed to vary by IDP bin. Specifically, we use

$$\text{Vol IDP}_{k,q,t} = \alpha_q + \sum_j \beta_j \mathbf{1}_{[\text{IDP}_{k,q,t} \in [p_j, p_{j+1})]} \log \text{Debt}_{k,q} + \gamma_1 \text{IDP}_{k,q,t} + \gamma_2 \text{IDP}^2_{k,q,t} + \epsilon_{k,q,t} \quad (12)$$

for $\{p_j\}$ the IDP quintiles.

As shown in model 1 of Table 7, the results are consistent with theory. At low levels of the IDP, the elasticity of IDP volatility to size is small and positive while at high IDP levels the elasticity is much larger. At the highest IDP level, a one standard deviation in size implies a volatility increase equal to 43% of Vol IDP's standard deviation. The estimates are also much more statistically significant at the higher IDP quantiles. For brevity, we do not report the estimates using log GDP in place of log debt, but the results are almost identical.

Table 7: Level and volatility controlling for fundamental risk nonparametrically

	(1) Vol IDP		(2) Vol IDP	
IDP Q=2	-0.0700	(-1.53)	-0.0824	(-1.71)
IDP Q=3	-0.0584	(-1.27)	-0.0874	(-1.82)
IDP Q=4	-0.111	(-1.99)	-0.0978	(-1.70)
IDP Q=5	-1.459	(-8.59)	-1.328	(-8.69)
IDP Q=1 × Log debt	0.00330	(2.10)	0.00372	(2.06)
IDP Q=2 × Log debt	0.00653	(6.82)	0.00738	(6.90)
IDP Q=3 × Log debt	0.00717	(7.89)	0.00874	(8.40)
IDP Q=4 × Log debt	0.00998	(7.47)	0.00991	(6.71)
IDP Q=5 × Log debt	0.0709	(10.41)	0.0660	(10.84)
Debt/GDP			-0.0966	(-4.45)
Debt/GDP^2			0.0572	(3.91)
Quarter FEs	Yes		Yes	
Observations	58412		58412	
R^2	0.17		0.17	

Note: Newey-West standards errors with 10 lags have been used; t-stats are in parentheses; all regressions include a non reported constant.

statistically insignificant.

D Data

Our raw data runs from 2004 to 2021 and contains observations of daily Sovereign CDS prices joined with quarterly gross central sovereign debt and quarterly NSA annualized nominal GDP in U.S. dollars as available. We obtain the NY end of day CDS bid and ask prices for the 5-Yr USD Senior Sovereign CDS from CMA DataVision via Bloomberg with the CMAN pricing source setting. We directly source quarterly total, short, and long term gross central sovereign debt in U.S. dollars from the joint World Bank and IMF Quarterly Public Sector Debt (QPSD) database. For GDP, we first source quarterly NSA nominal GDP in local currency from the IMF's International Financial Statistics (IFS) before converting to U.S. dollars with IFS's local currency exchange rates (or Euro exchange rate where applicable) and annualizing. For completeness we further enhance the data by imputing total, short, or long term debt when only two of the three variables are provided.

From the raw data we additionally construct the CDS Midpoint as the average of the bid and ask price, and use it to derive the approximate 1-Yr Implied Default Probability (IDP) in percent. We assume a zero recovery rate and continuous coupon payments such that $IDP = 100 * (1 - e^{-\frac{CDSmidpt.}{10000}})$ following Augustin et al. (2020). Using IDP we calculate the volatility of the 1-Yr Implied Default Probability (IDP Vol) as the trailing 10-day standard deviation of IDP. Additionally, we calculate additionally country fundamentals data permitting.

Given our raw data and constructed variables are only available for some periods, and that we are interested in periods that reflect the possibility of a default occurring, we restrict our daily and quarterly sample to observations with:

1. An average quarterly IDP between 1% and 40%.
2. Least 44 IDP observations (approximately two-thirds of possible observations) within the quarter.
3. Complete information on all variable.
4. No within country Sovereign default resulting in a CDS auction in that quarter.

Table 8 reports summary statistics for IDP by country.

D.1 Measurement of financing rates

We use Gilchrist and Zakrajsek (GZ, 2012) estimates, together with estimates from Spencer (2016), to obtain series for R and $R^l = R(1+\phi)$. First, we note that taking logs, we have $\log(1+r^l) = \log(1+r) + \log(1+\phi)$ where r^l is the net large lender cost of funds and r is the net small lender cost of funds / risk-free rate. Then, we apply the approximation valid for small x that $\log(1+x) \approx x$ to obtain $r^l = r + \phi$ plus approximation error, which we ignore.

In the GZ framework, the relationship between actual spreads $s_{i,t}$ and predicted (via regression) spreads $\hat{s}_{i,t}$ is given by the

$$s_{i,t} = EBP_t + \hat{s}_{i,t}$$

Table 8: IDP summary statistics by country

	count	mean	sd		count	mean	sd
Argentina	2033	11.28	8.38	Japan	191	1.20	0.14
Australia	22	1.26	0.21	Kenya	844	4.12	1.13
Austria	303	1.60	0.33	Korea, Rep.	62	1.59	0.18
Belgium	611	1.93	0.66	Latvia	1311	2.43	1.45
Brazil	2853	1.93	0.81	Lithuania	1202	1.99	0.76
Bulgaria	2002	1.93	0.85	Mexico	2268	1.36	0.40
China	315	1.23	0.16	Netherlands	160	1.16	0.09
Colombia	2389	1.48	0.49	New Zealand	45	1.23	0.30
Costa Rica	2888	2.85	1.10	Philippines	1332	1.37	0.30
Croatia	2075	2.82	0.94	Poland	692	1.72	0.52
Cyprus	1832	4.55	3.52	Portugal	2125	3.74	2.81
Czech Republic	308	1.30	0.23	Romania	2216	2.00	0.98
El Salvador	1968	3.70	0.63	Russian Federation	2655	2.01	0.97
Estonia	502	1.62	1.06	Rwanda	1032	3.91	1.36
France	338	1.69	0.37	Slovak Republic	380	2.06	0.62
Germany	61	1.05	0.04	Slovenia	1282	2.19	1.10
Greece	2450	7.89	6.75	Spain	1259	2.57	1.23
Guatemala	1943	2.64	0.84	Sri Lanka	409	7.59	3.69
Hungary	2011	2.58	1.31	Sweden	16	1.24	0.16
Iceland	1676	2.28	1.21	Switzerland	12	1.40	0.25
Indonesia	2346	1.63	0.40	Thailand	3	1.10	0.10
Ireland	1089	3.81	2.34	Turkey	3097	2.55	1.05
Israel	1065	1.41	0.30	United Kingdom	25	1.19	0.19
Italy	2616	1.99	1.01	Uruguay	98	1.48	0.36

where EBP_t is the excess bond premium (EBP). This means for a predicted zero-credit-spread instrument, $\hat{s}_{i,t} = 0$, the actual spread will be EBP_t. We interpret the small lenders as have access to risk free funding costs, thereby having $\hat{s}_{i,t} = 0$ and $s_{i,t} = EBP_t$. This is the spread over the risk-free rate, which we take to be the real federal funds rate. This gives us r_t. Effectively, we are assuming sovereign bonds are priced like a bond in the GZ sample (which is corporate) rather than like a US treasury, as traditionally the case in the sovereign default literature.

To obtain the large lender costs, we first obtain ϕ_t as an average predicted spread $\hat{s}_{i,t}$ for broker-dealers as defined by GZ. We then add EBP_t and the real federal funds rate to obtain our estimate for r_t^l.

We now describe our procedure for obtaining $\hat{s}_{i,t}$. GZ project bond-level credit spreads $s_{i,t}$ in logs onto a distance-to-default (DD) measure, along with a few other bond characteristics. Distance-to-default is a measure of default risk derived from stock market volatility and continuous-time asset pricing that is related to the one-year default probability via $\Phi(-DD_{i,t})$ for Φ the standard normal cdf. Essentially, it is the number of standard deviations the stock value would need before its debt would exceed its value (implying the firm should go to bankruptcy). Higher distance to default implies safer valuations and generally smaller spreads. GZ's estimates imply log predicted spreads satisfy $\frac{\partial \log \hat{s}_{i,t}}{\partial(-DD_{i,t})} = 0.093$ (Table 4).

We lack access to the raw historical data for GZ. But the reproduction files contains the 25th, 50th, and 75th percentiles of distance to default, allowing time-varying estimates of the mean μ_t and standard deviation σ_t. Assuming $DD_{i,t}$ is normally distributed at any point in time, the mean is the median and the standard deviation can be obtained by inverting $\Phi((q_t(p) - \mu_t)/\sigma_t) = p$ where $q_t(p)$ is the quantile associated with percentile p. Here, p is .25 or .75, giving two estimates for σ_t, which we average to obtain our final estimate for σ_t.

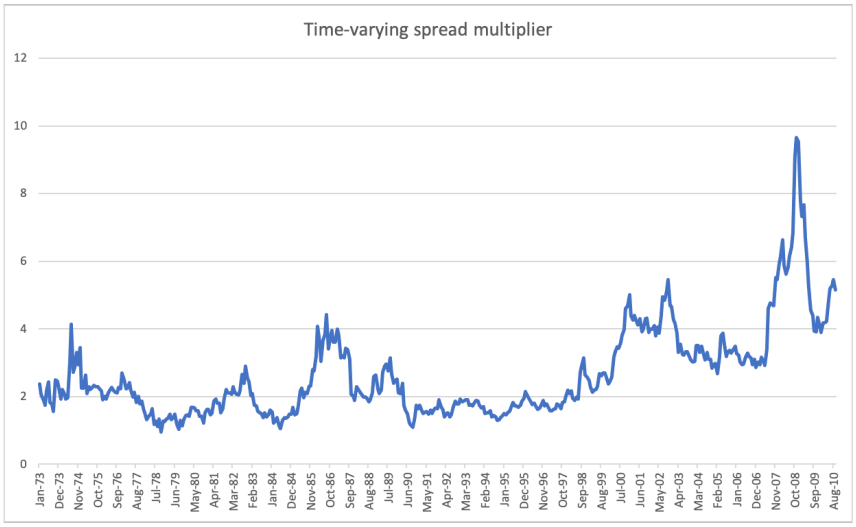
Consistent with the regression framework of GZ, we take

$$\hat{s}_{i,t} = \gamma_t \exp[-.093 DD_{i,t}]. \tag{13}$$

This holds the marginal effect of $DD_{i,t}$ on log spreads at the estimated value and subsumes all contributions from other explanatory variables into a single value, γ_t. With this convention, γ_t is the spread of a firm just at the cusp of default, i.e., $DD_{i,t} = 0$ (such a firm theoretically has a 50% chance of default within one year). To recover γ_t, we first note that, with $DD_{i,t}$ normally distributed $N(\mu_t, \sigma_t^2)$, $\hat{s}_{i,t}$ is log normally distributed $N(\log \gamma_t - .093\mu_t, .093^2\sigma_t^2)$. Consequently, $\mathbb{E}_{i|t}[\hat{s}_{i,t}] = \gamma_t \exp[-.093\mu_t + .093^2\sigma_t^2/2]$. Substituting in our estimates for μ_t and σ_t and the average GZ credit spread (included in their reproduction code), we arrive at a value for γ_t. The numerical average for γ_t is 2.64, but it varies substantially as shown in figure 12, rising sharply higher in the financial crisis.

To get the spread for the large lenders, we use (13) and evaluate it at an estimate of the distance to default for broker-dealers. We obtain a distance to default estimate by using two relationships. First is the one year implied default probability of a CDS spread assuming no recovery is $IDP_{i,t} = 1 - \exp(-u_{i,t}) \approx u_{i,t}$, where $u_{i,t}$ is the running

Figure 12: Time-varying spread multiplier



spread. The running spread we use is an average broker-dealer one year CDS spread from GZ. Second is that the one-year probability of default implied by the distance to default is $\Phi(-DD_{i,t})$. Treating the $IDP_{i,t}$ as the default probability, this gives $DD_{i,t} = -\Phi^{-1}(1-\exp(-u_{i,t}))$. Evaluating (13) gives an average $\hat{s}_{i,t}$ for broker-dealers, which is our ϕ_t estimate. Finally, our r_t^l estimate is $r_t^l = \phi_t + r_t$.

The key series are shown in figure 13. The small lender rate r moves strongly over the cycle, peaking at 3% before collapsing to -1% by 2010. Our ϕ_t variable is given by series labeled "Predicted spread estimate (no EBP)." The CDS spread increases to 3% in 2007, and with it the nearly identical IDP (not shown), which pushes up average $DD_{i,t}$. At the same time, γ_t is rising significantly (figure 12), driving up average $\hat{s}_{i,t}$, our ϕ_t estimate, to above 1%. The large lender rate r_t^l follows the same cycle qualitiatively as r_t, but the wedge ϕ_t between the series grows from close to zero to 1% from 2007-2008 and remains elevated through 2010.

Figure 13: Large lender spread estimates ϕ_t and inputs

