Optimal Fertilizer Policy After a Global Shock*

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Abstract

Fertilizer is central to industrial policy in developing countries and exposed to international shocks because it is almost entirely imported. We study the design of optimal fertilizer subsidy policy and how it should respond to global shocks that affect domestic fertilizer prices in a general equilibrium model with financial frictions, heterogeneous farming technologies, and sectoral labor choices. If village production is sufficiently substitutable, the Ramsey planner "leans into" the shock by decreasing the subsidy to redistribute production toward less affected villages. We quantify the model in Rwanda by collecting household-level data from 444 rural villages from 2020 - 2024 and use the Russian invasion of Ukraine – which caused the real price of fertilizer to double in Rwanda – to estimate key elasticities. These elasticities determine both the qualitative and quantitative features of the optimal subsidy schedule. We quantify the welfare consequences of alternative policy responses and highlight under what conditions other (equally blunt) instruments can be used more effectively than fertilizer subsidies.

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1 Introduction

Industrial policy in developing countries takes many forms, but one sector that plays a central role is agriculture. For example, many Sub-Saharan Africa countries spend up to 25 percent of their agricultural budgets on fertilizer subsidies (Jayne et al., 2018; Holden, 2019). Such policies are often justified by the fact that the agricultural sector is characterized by a high share of employment but relatively low productivity (Gollin et al., 2014), due in large part to the lack of input intensification, including critical intermediate inputs like fertilizer (Restuccia et al., 2008; Boppart et al., 2023).

Yet at the same time, few developing countries produce their own fertilizer. *All* inorganic, nitrogen based fertilizer is imported into Sub-Saharan Africa. Our goal in this paper is to study optimal fertilizer policy against this backdrop. Specifically, we ask how optimal fertilizer policy, central to governments' attack on inefficiently unproductive agriculture, should respond to exogenous changes to global fertilizer prices. Recent evidence suggests this margin is practically relevant too. Global fertilizer prices doubled after Russia invaded Ukraine, generating over 700 new policy announcements (Amaglobeli et al., 2023) and a renewed push for understanding imported fertilizer's role in the industrial policy of developing countries (Shah, 2022; Malpass, 2022).

We characterize the main trade-offs for a Ramsey planner who manages fertilizer policy in a developing country and responds to an exogenous shock to its country's fertilizer price. We highlight elasticities that determine both the qualitative and quantitative properties of the optimal fertilizer subsidy schedule. We then bring the model to the data in Rwanda by collecting household-level data from 2020 - 2024 across 444 rural villages and using the Russian invasion of Ukraine in late 2021 as exogenous variation. This shock manifested locally in Rwanda as doubling of the real domestic fertilizer prices between two consecutive seasons. We show how to use these causal empirical estimates to measure the relevant elasticities and then use the model to quantify the welfare consequences of optimal (and sub-optimal) policy.

Our model is one of household-farmers subject to limited borrowing and financial frictions, similar to canonical models reviewed in Buera et al. (2015) and Kaboski (2023). The model has three core features. First, farmers live in villages with different working capital constraints. This financial friction makes it difficult to borrow for pay inputs before harvest payoffs are realized. Second, villages also differ in their underlying technologies, varying specifically in the technological intensity of fertilizer relative to labor. Third, village production may not be perfectly substitutable in the consumption market.

These assumptions create scope for policy. We show that the Ramsey planner's role is to help achieve the productive redistribution that would have occurred in a frictionless world via general equilibrium price movements. At issue is that financially constrained villages cannot respond fully to output price changes, thus muting equilibrium price effects. Policy then attempts to do the rest. Under certain assumptions, the characterization is stark. For example, with Cobb-Douglas village production, the planner "leans into" the shock and lowers the fertilizer subsidy if villages are sufficiently substitutable with one another. Both financial frictions and technological heterogeneity are critical to this result: if either is removed, the subsidy does not vary with the price. Once we relax the Cobb-Douglas assumption to more general CES settings, the decision to raise or lower the subsidy is a complex interaction between the aggregation elasticity, the CES production elasticity between fertilizer and labor, and the covariance of financial frictions and technology across villages. In a more general sense, these results highlight under what economic conditions a planner would in fact wish to lower the optimal subsidy in a distorted economy when faced with a rising input price.

The basic intuition follows from the planner's incentives redistribution toward more labor-intensive production. All else equal, when villages are sufficiently substitutable in consumption, the planner pushes production shares toward less affected, labor-intensive villages. There are two countervailing forces. First, this redistribution may come at a productivity cost if labor-intensive villages are also the most distorted. Second, the planner may not need to use policy for redistribution if re-optimizing farmers can easily substitute between fertilizer and labor. The first introduces the importance of covariance of technology and frictions matters, the second the elasticity of substitution between fertilizer and labor. A quantitative evaluation of optimal policy requires balancing these competing effects and demands careful measurement of them.

To do so, we show that these elasticities can be mapped to causal empirical estimates from an exposure regression design, where exposure is baseline village-level fertilizer expenditures per acre. Doing so allows us to directly target changes in the shadow value of fertilizer (i.e., the magnitude of the distortion) so that our model is consistent with observed changes to distortions, their relation to baseline fertilizer use, and the relevant village production and final good aggregation elasticities. Our approach shares an intuition with the exact hat techniques used in trade (see Caliendo and Parro, 2022, for a recent review), where specifying the model in differences clarifies the relationship between between model predictions, parameters, and data.

Implementing it requires micro-level evidence and an exogenous shock. We focus on Rwanda, an interesting context for a few reasons. First, agriculture is relatively unproductive, with value added per worker only half of aggregate value added per worker (Kruse et al., 2023), as is true in many other developing countries (Herrendorf et al., 2022). Second, like other Sub-Saharan African countries, Rwanda imports all of their inorganic fertilizer. Finally, the government plays an important role in subsidizing fertilizer for farmers and we show that its policy responds directly to the global shock we study here.

We exploit Russia's invasion of Ukraine as an exogenous shock, the impact of which was felt immediately in Rwanda. Global fertilizer prices doubled and forced the Rwandan government, which controls the domestic price of fertilizer via their national subsidy scheme, into an unprecedented off-schedule price increase of about 40 percent in January 2022, despite a historical precedent of adjusting prices annually each July. They simultaneously increased their per-unit subsidy from 30 to 45 percent for most fertilizers, causing the program's cost to increase from 4 to 19 percent of the Agricultural Ministry's budget (from 5 billion Rwandan francs in 2020 to 13 billion francs in 2022). Taking the global shock and government response in tandem, Rwandan farmers paid 70 percent more per kilogram of fertilizer in 2022 than they did one season before. We show via professional price forecasts that this change was unexpected.

Our data comes from a household-level panel we collected around most of Rwanda between 2020 - 2024, covering about 45,000 individuals from 15,000 households across 444 villages. We create plausibly exogenous variation in the impact of the fertilizer price shock with a shift-share approach as guided by the model. We use baseline village-level fertilizer intensity (total fertilizer expenditures per acre of farmland in production) as a measure of exposure. The rationale for village-level exposure instead of individual-level exposure is due to general equilibrium effects we document empirically and play an important role in our model.

We document the reduced-form empirical impact of this shock, which we view as independently interesting given that this shock induced the largest increase in global fertilizer prices since the 1970s oil crisis. These results add causal, micro evidence on the long reach of conflict via trade flows (Martin et al., 2008), especially in the short-run when substituting to alternative technologies is limited (Barrot and Sauvagnat, 2016; Boehm et al., 2019).[1] We find that farmers in more exposed villages decrease fertilizer use substantially. This translates into harvests, with a gap of 19 percentage points between the most and least exposed villages. These output changes require a

[1] On this point, see also Adamopoulos and Leibovici (2024), who use Russia's invasion to highlight the importance of food import risk in Ethiopia.

reallocation of inputs in the economy. They do not come from land, as we observe no changes in land rental, either in or out, suggesting difficulty of adjusting input structure in the short run due to land market frictions highlighted elsewhere (Chen et al., 2022; Acampora et al., 2023). Instead, the reallocation operates primarily through labor. High fertilizer villages begin to more seriously specialize in agriculture. Total wage earnings decrease by 17 percent, driven entirely by a 33 percent decline in earnings outside the village, which is dominated by non-agricultural jobs. Price changes are consistent with these patterns of pulling employment back into agriculture. Both village agricultural wages and crop sales prices rise in fertilizer intensive villages, the latter of which is concentrated among fertilizer-intensive crops.

These effects imply both a large average impact and substantial heterogeneity across space and we estimate the model using these empirical results. Our main quantitative exercise then starts the economy in its stationary equilibrium, a baseline motivated by the stability of fertilizer prices in the decade leading up to this 2021 shock. We then introduce a shock where the fertilizer price unexpectedly rises and falls back quickly to its baseline level. We trace the impulse response of the economy back to steady state and compute the optimal path of the subsidy. The optimal subsidy falls on impact by 13 percent, from 15 percent to 13 percent, before quickly returning to its baseline level.

The decline follows because we estimate that village production is substitutable for consumption, meaning that less-affected labor intensive villages are reasonable consumption substitutes for those fertilizer-intensive affected villages. The relatively small quantitative magnitude comes from additional parameters. First, we estimate that fertilizer and labor are substitutable in production, with a CES parameter of 1.5. The induced intra-village substitution implies less need for the planner to conduct her own inter-village substitution. Decreasing this parameter to 0.25 causes the subsidy to decline by 53 percent instead of 13 percent. The result joins Boppart et al. (2023) in emphasizing the quantitative importance of production function parameters in models with agricultural sectors, but from a normative perspective instead of a positive one.

Second, we estimate the co-variation of technology and financial tightness and find that labor-intensive villages are the most distorted. Thus, the value of any redistribution toward labor-intensive villages is dampened by the productivity cost of doing so. Making them negative correlated (so that low fertilizer-intensive villages are the least financially constrained) causes the optimal subsidy to decline by 50 percent instead of 13 percent.

Finally, motivated by the underlying mechanisms that drive changes to the optimal fertilizer subsidy, our final exercise explores other policies may be better suited

to respond to higher fertilizer prices. We consider two: subsidies to production and subsidies to labor. As in our baseline case, we assume these subsidies are uniform and cannot target specific villages.[2] We find that increasing labor subsidies are the best response to the fertilizer price shock. Because we find that villages are substitutes, the policy maker wants to shift production toward low-fertilizer-intensity or, identically, high-labor-intensity villages. Thus, increasing a labor subsidy has the same qualitative properties as lowering the fertilizer subsidy, but can achieve the same goal more cost-effectively. On the other hand, because financial frictions dampen the output price elasticity of input choices, price subsidies offer limited benefit.

The rest of the paper proceeds as follows. After a brief detour to talk about additional related literature, we start with the reduced-form impact of the global shock in Rwanda in Sections 2 and 3. Some of the particulars will be used to inform modeling decisions, and thus are useful to see beforehand. Section 4 lays out the details of our model and highlights analytic results that clarify the key features of the model. Section 5 links these key model parameters to the earlier reduced-form results. Finally, 6 provides our quantitative results from the model.

1.1 Related Literature

This paper relates to a number of different literatures. It relates to a substantial literature that has documented the importance of intermediate inputs in the development process (Ciccone, 2002; Jones, 2011; Casal and Caunedo, 2023, for its relation to investment) and specifically their importance in agriculture (Restuccia et al., 2008; Boppart et al., 2023).

We also connect to a literature on government industrial policy, especially those papers concerned with various forms of fertilizer programs (Diop, 2023; Garg and Saxena, 2023; Ghose et al., 2024) and agriculture more broadly (Manysheva, 2022; Hsiao et al., 2024), in addition to others like energy (Ryan, 2022). See Juhász et al. (2024) for a recent review. Most closely related in this dimensions is interesting recent work by Mazur and Tetenyi (2024), who highlight that existing fertilizer programs in Malawi over-subsidize farmers, and emphasize the role of high transaction costs and financial frictions. Our focus on understanding how a subsidy should evolve with its corresponding input price is new to this literature and shows that financial frictions alone are not generally the key driver of the optimal subsidy price schedule. However, like many of these papers, we find that taking seriously the role of general equilibrium

[2] In Rwanda, this is consistent with realized policy and likely due to political economy constraints. District mayors sign *imihigo* contracts with the national government, which are essentially *ex ante* development performance contracts. Each year the results are posted publicly and districts ranked. Thus, subsidizing some villages at the expense of others after an aggregate shock creates a myriad of political issues.

effects, including their distributional consequences, are central to understanding the quantitative implications of policies that subsidize some of the population, as studied in Rotemberg (2019) for small-firm industrial policy. These gains may not be enough to overcome the cost of financing the program, a point recently highlighted in the education market by Fujimoto et al. (2023).

We connect this literature to that on spatial redistribution, which has almost entirely focused on developed countries (Donald et al., 2024; Gaubert et al., 2025; O'Connor, 2025). Our work highlights relevant trade-offs when governments have a limited set of instruments as is common in a developing country context. See also Annan and Ndiaye (2025) on oil policy in Ghana.

Finally, we note that the model developed here is not inherently restricted to agriculture and therefore connects to a broader literature on the evolution of policy away from steady state. See, for example, Acemoglu et al. (2006) and Itskhoki and Moll (2019), though these papers are mostly interested in longer-term growth paths. Relatedly, Buera and Shin (2013) and Buera and Shin (2017) trace the dynamics of reforms in similar models to the one we use here, though without a specific focus on optimal policy. Our model shares core features with this work, but we add to it in a number of ways. First, we show how technological heterogeneity can drive policy responses. Second, we link our quantitative results to causal empirical estimates in a critical market for development policy.

2 Setting, Data, and Fertilizer Price Shock

Global inorganic fertilizer production is concentrated in a small number of countries, because of the importance of comparative advantage in key inputs like natural gas. Russia is the largest exporter globally, implying that shocks to their ability to export play an important role in global price fluctuations. In this section, we detail how global fertilizer prices translated into changes in Rwandan policy, after a brief detour to discuss the data we use in this paper.

2.1 Data

Our main data sources for the empirical results are primary data. It includes household- and vendor-level data, which we discuss in turn. We collect household data from 444 villages across most of Rwanda. This dataset was originally designed to measure the impact of an RCT on last-mile infrastructure and as such covers all of Rwanda except the relatively flat east part of the country.[3] The dataset began in 2020 with a

[3]See Macharia et al. (2022) for the pre-analysis plan and other details about the RCT.

baseline and has been collected annually since, implying 5 visits per household (2020 – 2024, inclusive). We exclude all treated villages from the RCT and focus exclusively on control villages. Despite our description as a household-level panel, these data are an individual-level panel. We collect household rosters and ask individual-level questions on demographics, earnings, and health and education outcomes, along with household-level questions about farming practices, savings, and other related decisions.

Starting in 2021, we began collecting data from traders in the markets associated with these villages. In the 2020 baseline we asked households to tell us about the markets that they bought and sold goods. Any market mentioned by more than 2 households is included in the study, though in practice there are rarely more than one or two markets per village. These data are collected collected monthly by in-person enumeration. This high-frequency collection requires surveys to be relatively short so that enumerators can travel to many markets over the short time horizon. This survey asks questions about goods stocked, inventory, and prices.

2.2 Fertilizer in Rwanda and the Fertilizer Price Shock

The agricultural calendar in Rwanda is made up of two main cropping seasons. Season A begins with planting in September and ends with harvesting in January and February, while Season B runs from March to June. These seasons are generally referenced by the year of Season A's harvest, so that the 2022 season consists of Season A from September 2021 – February 2022 and Season B from March 2022 – June 2022.

Types of Fertilizer and Link to Global Markets The main fertilizers used in Rwanda all include nitrogen. These include DAP (a combination of nitrogen and phosphorus used primarily for beans, maize and wheat), a balanced nitrogen-phosphorus-potassium (NPK) 17-17-17 (used for rice and potatoes), and urea (primarily nitogren) which is used as a topdressing for all of the above crops. These three fertilizers make up 90 percent of the value of all fertilizers used in Rwanda in 2020 (National Institute of Statistics Rwanda, 2021). Nitrogen-based fertilizers use ammonia to deliver the nitrogen to plants, which is produced primarily with natural gas.[4] Synthesizing ammonia accounts for 3-5% of global natural gas consumption (Song et al., 2018). The centrality of natural gas in the production process implies two relevant features. First, shocks to natural gas prices will affect fertilizer prices. Second, countries with

[4]Ammonia (NH_3) is easier for plants to break down than the nitrogen in the air (N_2) due to the triple-covalent bond between the two nitrogen atoms. The Haber-Bosch process subjects natural gas to extremely high pressure, causing its hydrogen atoms to bind with nitrogen atoms in the air. The induced reaction, $N_2 + 3H_2 \rightleftharpoons 2\ NH_3$, creates the ammonia used in fertilizer.

natural gas have a comparative advantage in production. Because of this, production is concentrated in a handful of countries, none of which are in Sub-Saharan Africa.

Market Structure Fertilizer is not freely traded in Rwanda, instead managed by the Ministry of Agriculture and Animal Resources (MINAGRI). The government offers licenses for organizations to import fertilizer from abroad (usually 4 – 10 private firms and large NGOs), who then sell fertilizer via a set of licensed retailers scattered throughout the country. The government sets two prices: the market price and subsidy level. The market price is the price received by an importer, and the consumer pays market price net subsidy. Technically, this involves importers selling to retail vendors at the subsidized price. The importers are then reimbursed the difference between their negotiated price and the subsidized price by the government.[5] The normal price-setting involves the government announcing prices on or around July 1 preceding fertilizer purchases for Season A, then held fixed for the crop year and reset only in the next July. We say "normal" here because we will shortly show how price announcements adjust after the 2021 fertilizer price shock.

One natural question when dealing with non-market prices is the equilibrium possibility of quantity restrictions. In practice this seems to be of limited concern, as farmers can almost always access the fertilizer they demand. By law, any fertilizer importer is required to service all agro-dealers in the country. Thus, the fertilizer market is national instead of a collection of closed local fertilizer markets, limiting the possibility of local shocks driving a local mis-match of supply and demand. Second, even when there is the possibility of national under-supply, the government induces imports to correct the projected imbalance. Both our farmer survey evidence and discussions with importers and agro-dealers support the view that this margin is relatively unimportant, though could play an indirect role through the government's budget.

Fertilizer Use and Finance Across Villages Finally, we note that there is a strong correlation between fertilizer use and external finance across villages in Rwanda. We use our baseline data across all villages to measure external debt by village, which is all debt net of that held by others in the village (including local savings groups), and includes almost exclusively bank and microfinance-held debt. Figure 1a plots a bin scatter of fertilizer expenditures as a share of harvest and its relationship to external debt in the village. Figure 1b shows that villages with higher debt are also closer to

[5]This process is managed by the Rwandan government's Smart Nkunganire System (link here). There are no subsidized products sold outside this system which facilitates repayment.

banks. These are of course equilibrium relationships, but offer some baseline evidence on heterogeneity in financial access and its relation to agricultural input intensity.

Figure 1: Fertilizer and External Finance

(a) External Debt and Fertilizer's Share of Harvest

(b) Average Walking Time to Bank

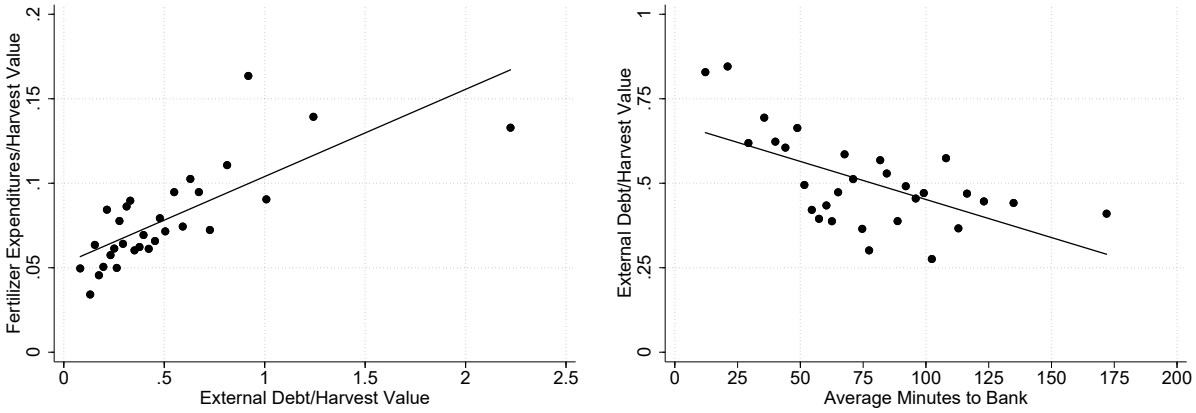


Figure Notes: Panel (a) plots fertilizer expenditures as a share of harvest value for all 444 villages and its relationship to total external debt held by the village. Panel (b) relates external finance levels to average walking time to the bank. Both sub-figures use bin scatters with 30 bins.

2.2.1 Shock to Global Fertilizer Prices

One of the most important nitrogen-based fertilizer producers is Russia. In 2020, they produced 32 percent of the world's NPK and 10 percent of urea, contributing 13 percent of all nitrogen exports in the world (FAOSTAT, 2024).

In 2022, the Russia-Ukraine war created two main issues that caused fertilizer prices to spike. First, fertilizer and ammonia exports declined, the latter of which made it difficult to produce fertilizer in other, non-natural gas producing countries like Morocco. Recent estimates suggest Russian ammonia exports fell 63 percent between 2021 and 2022, and urea fell 23 percent (Glauber and Laborde, 2022). Belarusian potash exports also collapsed, which is the "K" in NPK fertilizer. While Rwanda primarily imports fertilizer from Morocco and Saudi Arabia, the decline in global supply impacted global prices.[6]

The second implication of the war was its impact on the natural gas market. Russian natural gas exports fell dramatically in 2022, while Europe simultaneously attempted to replace Russian oil demand with U.S. natural gas (Gil Terte, 2023). This redirected American gas exports toward Europe. Thus, not only was fertilizer

[6]In 2020, 58 percent of Rwanda DAP imports came from Morocco, 28 percent from Saudi Arabia and 14 percent from Russia. By 2022, Russian DAP fell to zero and was partially made up for by an increase in Saudia Arabian fertilizer. The total quantity of Rwandan DAP imports fell from 24 million tons in 2020 to 15.6 million tons in 2022.

is relatively short supply globally, its most important input was increasing in price for non-agricultural reasons. In the Appendix, we show that monthly global fertilizer prices closely track natural gas prices, with correlations of 0.67 and 0.69 for DAP and urea.

The relative (but not total) importance of these shocks remains subject to debate, but clearly none are related to Rwandan policy decisions. Figure 2 shows they were also unanticipated. It plots the global prices of DAP and urea from 2015 to 2023 using the World Bank Commodity Price Outlook (World Bank, 2024). The nominal price shows the sharp rise from a period of relative stability, with DAP and urea rising to 2.5 and 3 times their 2020 level by 2022.[7] The dashed lines in Figure 2 plot the forecasted prices from each October release from 2018 to 2022. The 2018 - 2020 forecasts of continued stable prices were not realized.

2.2.2 Response in Rwanda

As mentioned above, the Rwandan government had historically updated fertilizer prices one per year. This policy-induced price stickiness was irrelevant when global fertilizer prices were stable during most of the 2000s. But the Rwandan government was forced to respond to the higher prices with an unprecedented off-schedule price change in January 2022 before the start of Season B. The market price for DAP was adjusted upward by 40 percent. For some sense of scale, the average annual price change between 2018 and 2021 was 4 percent. Similar adjustments were made for urea and NPK.

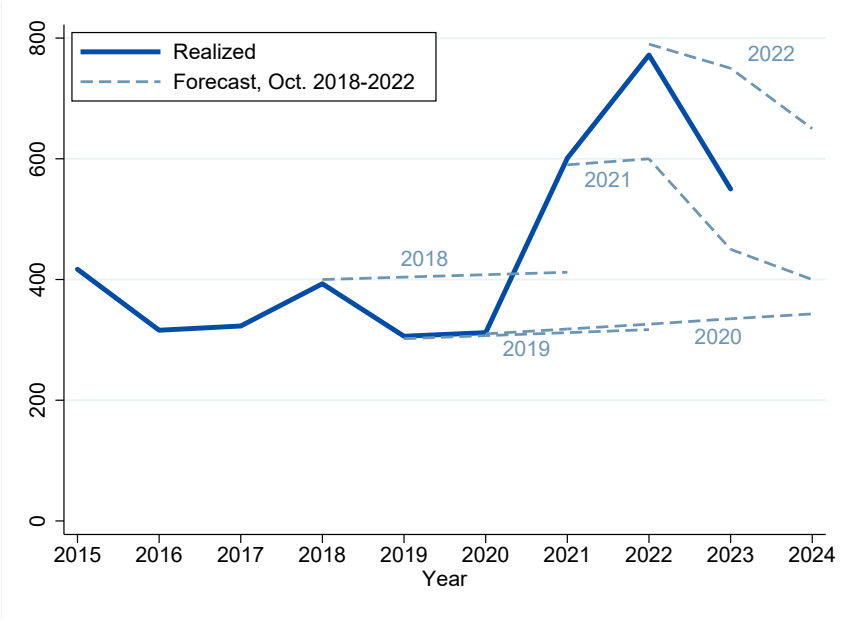
In Figure 3, we use government documents to plot the government-defined market price for 3 main fertilizers along with the subsidy rate. The farmers then pay (1-subsidy) $\times$ the market price. In addition, the figure includes markers at any point when the government releases an updated price (even if that price is identical to the previous one). The discussion above is reflected in these figures. First, prices and subsidy rates remain stable until 2022. Second, through 2021, all price adjustments occur in Season A and are held fixed in B. However, in 2022-B, both the market price and subsidy rates are updated due to the stark rise in international prices.

Figure 3 shows that the government made two adjustments in 2022. The first was an increase in the market price (Figure 3a). Second, they limited pass-through to farmers by increasing the subsidy rate (Figure 3b). As such, program jumped from 4 to 19 percent of MINAGRI's total budget. In raw values the cost of the

[7]The fertilizer price actually starts to rise in mid-2021. This was due to an issue of low U.S. coal inventories putting upward pressure on natural gas prices in 2021. That pressure was released at the end of 2021 on the backs of higher gas production and a mild winter (EIA, 2022). Thus, the proper non-war counterfactual would have been a temporary and relatively minor increase in natural gas prices (and thus fertilizer prices) in 2021 only. This is consistent with the 2021 forecast in Figure 2.

Figure 2: Global Fertilizer Prices and Projected Prices (Current USD/metric ton)

(a) DAP



(b) Urea

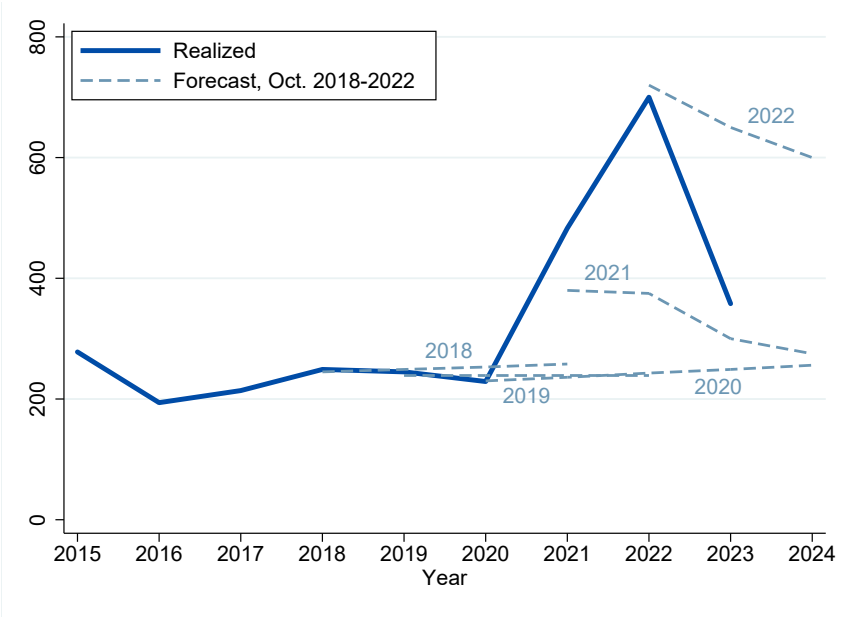


Figure Notes: The solid line plots the realized price for DAP and urea from 2015 - 2024. The dashed lines plot the projected prices each year. Both realized and projected prices are derived from the World Bank Commodity Price Outlook (World Bank, 2024).

program rose from 5 billion francs in 2019 to 13 billion in 2021. Figure 3c then plots the implied nominal prices for farmers. To summarize, Figure 3d plots the relevant shock: the farmer fertilizer price deflated by the rural Rwandan CPI, normalized at

Figure 3: Rwandan Fertilizer Prices

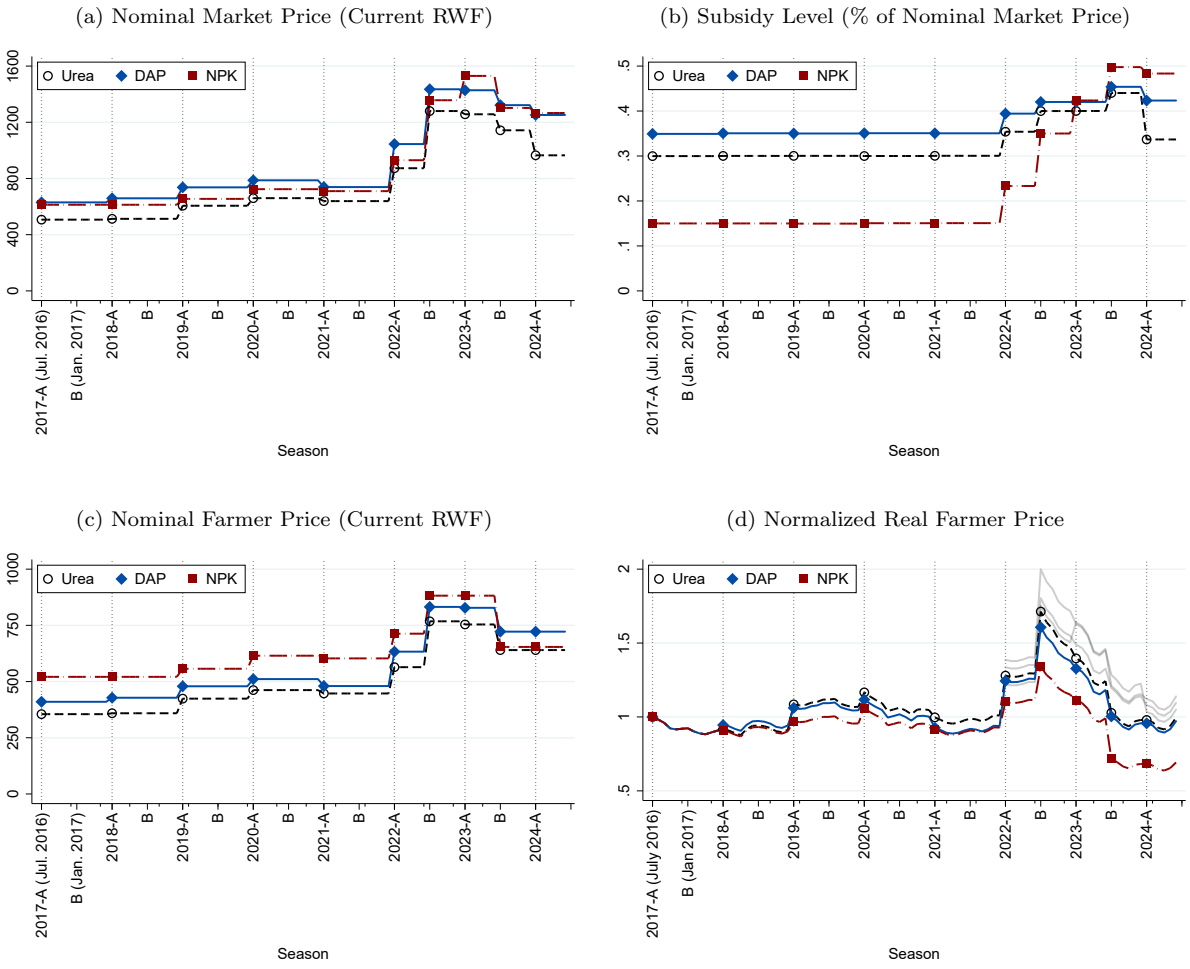


Figure Notes: Panel (a) plots the nominal market price defined by the Rwandan government in current Rwandan francs. Panel (b) plots the subsidy level. Panel (c) is the nominal farmer price, equal to (1-Subsidy) × Nominal Market Price. Panel (d) plots the nominal farmer fertilizer price deflated by the rural Rwandan CPI and normalizes by the 2017-A prices. The gray bars are the counterfactual price paths if the government kept the subsidy rates equal to their 2017-A level. In all figures, a marker is included when new prices are released even if the price is not updated. Notice that before 2022, all price updates were in Season A.

2017-A. The gray lines are counterfactual price paths that would have resulted from the government holding its subsidy rates fixed at the 2017-A levels.

Our data show the same patterns, as should be expected given that we survey these same retailers. For some preliminary comparisons, Figure 4 plots the raw log per-unit prices for various intermediate inputs and agricultural outputs over time derived from the vendor survey. The 95 confidence intervals are constructed via standard errors clustered at the market level. Each contains a dashed line at January 2022, where prices are normalized to zero. The top row plots two inputs: the most common type of fertilizer (DAP) and fungicide (Detan) purchased from markets in

our study. Fertilizer prices rise by almost 40 percent, consistent with the evolution of the price set by the Rwandan government. Fungicide prices remain relatively stable. The latter is more easily sourced locally and thus not subject to the same shocks. The bottom row of Figure 4 plots the same log prices but for two main crops produced in Rwanda, beans and sweet potatoes. Neither of these crops are imported from abroad but see their prices rise with a few months lag, suggesting an important equilibrium spillover onto food prices from the fertilizer price shock. We return to this issue in the next section.

Figure 4: Raw Prices for Inputs (2021-2024)

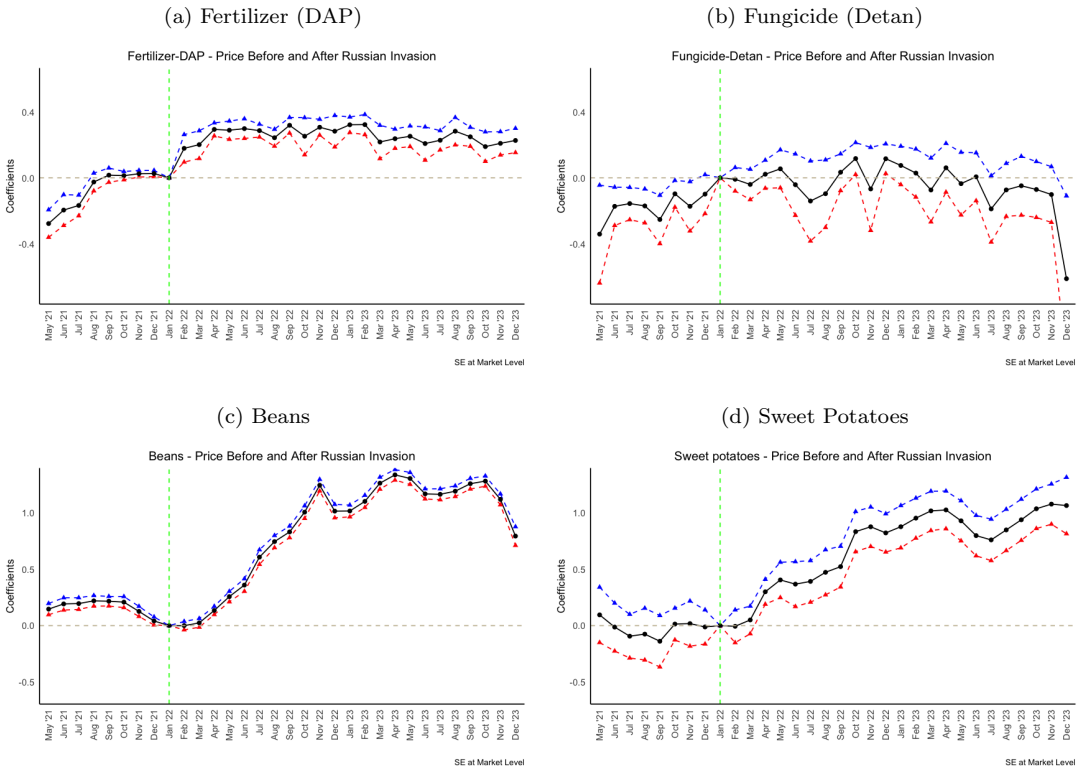


Figure Notes: This figure plots log prices for listed item over time as averages over all vendors. Dashed lines indicate 95 percent confidence interval with standard errors clustered by market.

To study these patterns more systematically we exploit the panel dimension of the data and run a series of regressions

$$\log(p^j_{imt}) = \alpha + \beta Post_t + \theta_i + \eta_m + \nu \log(t) + \varepsilon_{it} \tag{2.1}$$

where, for each product j, we regress the log price on a dummy $Post_t = 1$ if post-January 2022. We control for vendor i's fixed effect θ_i, month effects γ_m for seasonality, and $\log(t)$ is a linear time trend. The results are in Table 1 and confirm the

patterns from the raw data in Figure 4.

Table 1: Price Changes Post-January 2022 for Intermediate Inputs

Inputs	Fertilizer			Other Intermediates		
	DAP	NPK	Urea	Roket	Detan	Ridomil
Post-January 2022	0.35***	0.39***	0.33***	0.41	0.14*	-0.18
	(0.03)	(0.05)	(0.08)	(0.26)	(0.07)	(0.24)
Observations	1,658	1,227	1,605	3,172	2,623	1,192
R-squared	0.44	0.43	0.14	0.61	0.27	0.41

Table notes: Standard errors clustered by market are in parentheses. Ridomil is a fungicide and Rocket and Detan are insecticides used in Rwanda. All regressions include trader and month fixed effects and a linear time trend. Statistical significance at 0.10, 0.05, and 0.01 is denoted by *, **, and, ***.

3 Empirical Results

To estimate the impact of this shock across villages, we take advantage of the fact that villages differ substantially in their fertilizer intensity, measured here as baseline fertilizer expenditure per acre of land in agricultural production.[8] We focus on village-level intensity instead of farmer-level intensity due to general equilibrium effects that will become clear below. The fertilizer intensity of village v at survey wave t is

$$\widetilde{f}_{vt} = \log\left(\frac{\sum_{h\in v} f_{hvt}}{\sum_{h\in v} \ell_{hvt}}\right).$$

where f_{hvt} is the fertilizer use of household h in village v at year t. ℓ_{hvt} is land used in production. We then consider the following regression

$$y_{hvmt} = \alpha + \beta Post_t + \eta\left(Post_t \times \tilde{f}_{v0}\right) + \delta_m + \theta_v + \zeta_t + \varepsilon_{hvmt}, \tag{3.1}$$

Here, y is the outcome for household h in village v in month m in year t. $Post_t$ equals one after January 2022, and we include village, month, and survey wave fixed effects.[9] Our interest here is the parameter η, which measures how outcomes vary with baseline fertilizer intensity $\widetilde{f}_{v0}$ after January 2022.

We also include the simpler "single difference" regression to measure the average change. This is somewhat more tenuously interpreted causally, but given the unexpected nature of the shock, offers a helpful benchmark that we will use when

[8] Because prices do not vary across space, expenditures here do not pick up any spatial price variation. We use expenditures only to aggregate different fertilizers via their prices.

[9] Because of the number of villages covered by our study, survey waves are conducted over multiple months.

calibrating the model. This regression takes the form

$$y_{hvmt} = \alpha + \beta Post_t + \delta_m + \theta_v + \varepsilon_{hvmt}. \tag{3.2}$$

In all cases, we trim continuous outcomes at 1 percent and run the regressions as a poisson regressions to interpret as percentages with zeros (e.g. Chen and Roth, 2024). Outcomes in Rwandan francs are deflated by the monthly Rwandan CPI for rural consumers.

3.1 Headline Household Earnings Channels

We begin in Table 2 with farming outcomes, including fertilizer expenditures and harvests.

Table 2: Agricultural Production

			Harvest Value					
	Fertilizer Expenditures		Total		Cash Crops		Staple Crops	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Post	-0.067***	-0.052*	-0.149***	-0.059**	-0.013	-0.192***	-0.198***	-0.003
	(0.021)	(0.031)	(0.021)	(0.029)	(0.028)	(0.049)	(0.022)	(0.029)
Post x Village Fertilizer Intensity		-0.089***		-0.060***		-0.050**		-0.065***
		(0.020)		(0.015)		(0.021)		(0.016)
Observations	40,070	40,003	40,456	40,389	40,456	40,389	40,456	40,389

Table notes: Standard errors clustered by village are in parentheses. The regression is given in (3.1) and is run as a Poisson regression. Statistical significance at 0.10, 0.05, and 0.01 is denoted by *, **, and, ***.

Column (1) shows that fertilizer use declines more in places with higher baseline usage, showing that these villages are indeed more exposed to the shock (along with the mechanical aspect of the result that it bounded by zero below). For some sense of scale of this and the other results below, note that the difference between the fifth and ninety-fifth percentile village in terms of baseline fertilizer intensity is 3.47 log points (from -1.75 to 1.72). Thus, the decline in fertilizer intensity is about $-0.08 \times 3.47 = 28$ percentage points larger between the top and bottom of the distribution.

This decline in fertilizer impacts harvest. We value crops at the median sale price at baseline, so columns 2 – 4 do not include price changes induced by the shock. Column (2) shows that a one log point increase in baseline fertilizer intensity causes a 5 percent decline in harvests. Both cash and staple crops exhibit similar variation across villages.

Table 3 covers total household labor market earnings, a second critical income source. Again, fertilizer intensive villages exhibit a much larger decline in wage income, entirely accounted for by a decline in wage earnings outside the village. Thus,

the shock seems to draw workers back into a relatively less profitable farming sector.

Table 3: Labor Market Earnings

	Total		Outside Village		Inside Village	
	(1)	(2)	(3)	(4)	(5)	(6)
Post	-0.228***	-0.184***	-0.253***	-0.258***	-0.238***	-0.121**
	(0.020)	(0.034)	(0.035)	(0.060)	(0.030)	(0.051)
Post x Village Fertilizer Intensity		-0.049***		-0.108***		0.014
		(0.017)		(0.028)		(0.026)
Observations	40,060	39,997	40,040	39,978	40,061	39,999

Table notes: Standard errors clustered by village are in parentheses. The regression is given in (3.1) and is run as a Poisson regression. Statistical significance at 0.10, 0.05, and 0.01 is denoted by *, **, and, ***.

3.2 Output Price Changes

The household results above suggest a retrenchment of farm activity after the shock. We show that this is consistent with general equilibrium forces. Table 4 shows that the price of agricultural output rises after the shock. To show this, we run a crop-household regression

$$p_{chvmt} = \alpha + \beta Post_t + \eta \left(Post_t \times \tilde{f}_{v0} \right) + \delta_{cm} + \theta_v + \zeta_t + \varepsilon_{chvmt},$$

where c is the crop and the remaining subscripts are as before. Unlike our previous regression, we include crop-month fixed effects to take into account crop-specific seasonal price patterns. Table 4, Column (2) shows that the price of agricultural output rises after the shock.

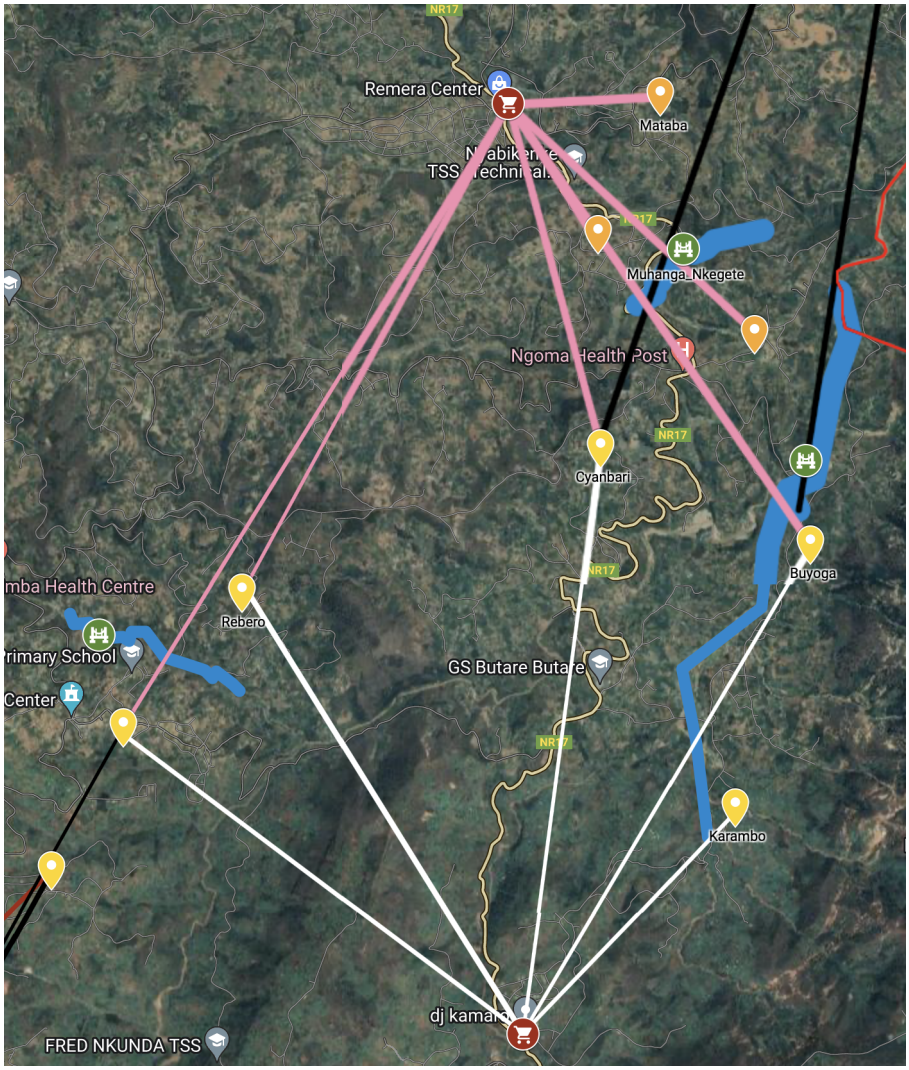
Table 4: Crop Prices

VARIABLES	All	All	Low Market	High Market
	(1)	(2)	(3)	(4)
Post	0.023***	-0.074***	-0.078***	-0.061***
	(0.009)	(0.014)	(0.020)	(0.023)
Post x Village Fertilizer Intensity		0.032***	0.032***	0.012
		(0.006)	(0.009)	(0.010)
Observations	59,150	59,145	30,524	28,621

Table notes: Standard errors clustered by village are in parentheses. All outcome variables are in logs and regressions are run as OLS. Statistical significance at 0.10, 0.05, and 0.01 is denoted by *, **, and, ***.

Villages do not each sell to unique markets. Figure 5 shows 9 villages (the yellow pins) that interact in two different markets (the red shopping carts). We would only expect the market price to respond to a village's own fertilizer intensity if it made up an important share of the market-level fertilizer intensity. As a robustness check, we therefore measure the residual fertilizer intensity of the market by computing the

Figure 5: Example of Village Links to Markets



average fertilizer intensity of market-connected villages, given by (3.3), and test how prices adjust.

$$\tilde{f}_{Mv0} = \log\left(\frac{\overbrace{\sum_{\hat{v}\in M_v}\sum_{h\in\hat{v}} p_{x0} f_{h\hat{v}0}}^{\text{fert. for hhs that trade in same market}} - \overbrace{\sum_{h\in v} p_{x0} f_{hv0}}^{\text{own fert.}}}{\underbrace{\sum_{\hat{v}\in M_v}\sum_{h\in\hat{v}} \ell_{h\hat{v}0}}_{\text{land for hhs that trade in same market}} - \underbrace{\sum_{h\in v} \ell_{hv0}}_{\text{own land}}}\right) / |M_v| \tag{3.3}$$

(3.3) takes total baseline fertilizer for all households that trade in the same market as village v net of that fertilizer from v, and similarly for land, to compute a net

fertilizer intensity for the market. If $\tilde{f}_{Mv0}$ is high, then village v makes up only a small portion of the market's fertilizer intensity. We would not expect output prices to respond to village v's fertilizer intensity in this case. On the other hand, if $\tilde{f}_{Mv0}$ is low, village v plays an important role in market M.

Columns (3) and (4) replicate the same regression as Column (2) of Table 4, but breaks the sample into those with high market-connected fertilizer intensity and those with low market-connected fertilizer intensity. The output price in v responds to v's fertilizer intensity only when v is a sufficiently large share of the market.

3.3 Wages

We next study the impact on wages. We break them into two groups: inside-village agricultural jobs and all others. We run these regressions at the individual level and include individual fixed effects. We also continue to use the Poisson regression for consistency with our earlier results, though it is not critical given that wages are positive for all workers. Regardless, these decisions have no impact on the results. The results are in Table 5.

Table 5: Log Daily Wages

	Inside Agr.		Other Jobs	
	(1)	(2)	(3)	(4)
Post	0.192***	-0.032***	0.259***	-0.045
	(0.009)	(0.012)	(0.015)	(0.028)
Post x Village Fertilizer Intensity		0.012*		-0.003
		(0.006)		(0.012)
Observations	13,970	13,968	16,875	16,848

Table notes: Standard errors clustered by village are in parentheses. Regressions include individual fixed effects. Statistical significance at 0.10, 0.05, and 0.01 is denoted by *, **, and, ***.

Agricultural wages inside the village change as well. They fall by 7 percent in low fertilizer intensity villages and exhibit no change on net in high fertilizer intensity villages. There is no change to non-agricultural wages.[10,11] To summarize, low fertilizer intensive villages see output prices and village wages fall. High fertilizer intensive villages see output prices rise and agricultural wages remain approximately the same.

To recap, villages using more baseline fertilizer (1) decrease fertilizer use and harvests, (2) see higher sales prices for crops, (3) see within-village agricultural wages

[10]While we do not ask households for a detailed price listing of non-agricultural purchases (only aggregated expenditures), we do collect those data from market traders. The two main non-agricultural products sold in local markets are cooking oil and salt. There is no change in the price of either. We similarly see no adjustment in fungicide prices. So to that extent, one should interpret the changes reported here as changes to the price of these inputs or goods relative to the non-agricultural output price as would be required in a general equilibrium model.

[11]Another possibility is that households are selling less of their harvest to the market and thus the average sold unit is of higher quality. We find no difference in the share of crops sold to the market, stored for own consumption, or gifted to others.

rise, and (4) pull workers from other jobs into agricultural employment.

4 Model

The intra-village heterogeneity observed in the empirical results suggests that the shock has significant distributional consequences in addition to its average effect. As discussed in Section 2, the Rwandan government responded to the exogenous fertilizer price shock with an increase to their subsidy level. However, that subsidy level is fixed across space and thus cannot directly target issues related to spatial heterogeneity or inequality. Our goal now is to understand the underlying economics of such a restricted subsidy, and how the optimal policy balances dueling welfare concerns of maximizing output and helping the poor. To do so, we develop a general equilibrium model that we can use as a laboratory to study policy and link it to the empirical results covered in the last section.

Sectors and Village Structure The model is cast in discrete time with two sectors: non-agriculture, or "manufacturing" m for short, and agriculture a. Within the economy are a continuum of villages, denoted $j \in \mathcal{J}$. Each village contains a measure one of *ex ante* identical households that live forever, but will be heterogeneous *ex post* due to idiosyncratic productivity shocks and the corresponding savings decisions. Each household derives utility from each of the two sectors' final goods, and utility flow is Cobb-Douglas with discount rate β,

$$\sum_{t=0}^{\infty} \beta^t \Big[\zeta \log(c_{at}) + (1-\zeta) \log(c_{mt}) \Big].$$

Households save s denominated in the agricultural final good with exogenous gross return R (which may be less than one due to storage costs or depreciation) and are restricted to savings, $s \geq 0$.

Villages differ in two ways: their technological fertilizer intensity α_j and the severity of their working capital constraint ϕ_j, both of which we will formalize below. The c.d.f. $G(\alpha, \phi)$ summarizes the distribution of villages across these two characteristics.

Manufacturing Production The manufacturing sector produces its final consumption good according to the production function $Y_{mt} = AN_{mt}$ where N_{mt} is efficiency units of labor hired in a centralized, competitive market that draws from all villages. We normalize the output price of the manufacturing sector to $p_m = 1$. In equilibrium, the wage $w_{mt} = A$ for all t.

Agricultural Production Each household i in village j produces a village-specific crop with the production function

$$y_{ij} = z_i \left(\alpha_j^{\frac{1}{\sigma}} x^{\frac{\sigma-1}{\sigma}} + (1-\alpha_j)^{\frac{1}{\sigma}} n^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\gamma\sigma}{\sigma-1}}$$

where x is fertilizer and n is labor. α_j is sub-scripted, meaning that it differs across villages. Note that the returns to scale are equal to γ everywhere. Input choices are made after the realization of z, an household-level i.i.d. productivity shock drawn from a distribution with c.d.f $F(z)$.

Input choices are constrained by a working capital constraint that varies across villages,

$$p_x x + w_{aj} n \leq \phi_j s$$

where p_x is the price of fertilizer and w_{aj} is the village-specific equilibrium wage per unit of labor. This constraint says that a household can pay for inputs up to a multiple ϕ_j of its savings s. Thus, a village is defined by its technological fertilizer intensity α_j and the tightness of its working capital constraint ϕ_j.

The village-specific crops are transformed into the final agricultural consumption good by a CES aggregator

$$Y_a = \left(\int_j y_j^{\frac{\nu-1}{\nu}} dj \right)^{\frac{\nu}{\nu-1}}$$

so that a stand-in firm has the profit function

$$\max_{y_{aj}} \; p_{ac} \left(\int_j y_j^{\frac{\nu-1}{\nu}} dj \right)^{\frac{\nu}{\nu-1}} - \int_j p_{aj} y_{aj} dj.$$

Here, p_{ac} is the price of the final agricultural consumption good paid by consumers and p_{aj} is the output price received by households in village j.

Notice that the continuum of households within any village implies no household market power for the differentiated products.

Household Labor Allocation Each worker in village j can work inside the village at the local market clearing wage w_{aj} or in the centralized manufacturing market for wage w_m. Each individual gets productivity draws (ξ_a, ξ_m) which map into earnings with a parameter θ. The earnings of a worker are

$$\text{Earnings}(\xi_a, \xi_m) = \max\{\xi_a^{\frac{-1}{\theta}} w_a, \xi_m^{\frac{-1}{\theta}} w_m\}$$

We assume throughout that $\xi_a \sim Exp(T_a)$ and $\xi_m \sim Exp(T_m)$ which when combined with the continuum of individuals within a household lets us characterize the total

household i labor earnings as

$$\mathcal{W}_{ij} = (T_a w_{aj}^{\theta} + T_m w_m^{\theta})^{\frac{1}{\theta}} \Gamma\left(\frac{\theta - 1}{\theta}\right). \tag{4.1}$$

where Γ is the gamma function. This is identical for all households in village j, so we drop the i subscript and write $\mathcal{W}_j$ going forward.

Fertilizer, Subsidies, and Government Budgets Fertilizer is imported and farmers pay a price p_x for it. p_x is made up of an exogenous market price p_x^{market} and the government subsidy τ_x, so that $p_x := (1 - \tau_x)p_x^{\text{market}}$. The government buys at p_x^{market} and sells to farmers at $(1 - \tau_x)p_x^{\text{market}}$ incurring a cost of $\tau_x p_x^{\text{market}}$ on each unit.

That subsidy is financed by taxes, subject to a balanced budget constraint. We design the tax system to mimic the relevant features of the Rwandan code (though experiment with other designs in the quantitative results). Expenditures on agricultural goods – both inputs and outputs – are exempt from VAT. And while there is in principle income taxes on agricultural profits, the thresholds exempt nearly all non-commercial-scale farming. Farm incomes must be above 12 million RWF (9,200 USD in 2024, though the franc has steadily depreciated over the study period) to incur any tax burden. Other taxes are designed to capture revenue from the agricultural sector, but play a minor role.[12] Instead, the main tax faced by rural Rwandans is non-agricultural transactions subject to VAT.

We therefore model government revenue as a tax τ_m on manufacturing consumption. The government earns τ_m on each unit given the normalized manufacturing output price.

Household Problem in Village j The timing of the model works as follows. At $t - 1$, households choose savings s_t to bring into period t. Some of that depreciates they enter t with Rs_t. Then the shock z is realized. Households then make decisions on farm inputs and the allocation of labor, income is realized, and consumption and savings decisions are made.

The individual state of the household is therefore (z, s), and the aggregate state is the distribution μ across individual states and villages. Recursively, the problem is

[12] For example, market fees are paid by traders who operate stalls in markets, but these are small revenue generators and we exclude them for simplicity. Similarly, non-agricultural earnings are subject to income taxes (above a threshold), but given that much of this work is informal in our setting, we refrain from modeling this tax on non-agricultural wages.

$$
\begin{aligned}
v_j(z,s,\mu) \;=\; & \max_{c_a,c_m,x,n_a,s'} \; \zeta\log(c_a)+(1-\zeta)\log(c_m)+\beta\int_{z'} v(z',s',\mu')dF(z') \\
s.t. \quad & p_{ac}c_a+(1+\tau_m)c_m+p_{ac}(s'-Rs)=p_{aj}z\left(\alpha_j^{\frac{1}{\sigma}}x^{\frac{\sigma-1}{\sigma}}+(1-\alpha_j)^{\frac{1}{\sigma}}n^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\gamma\sigma}{\sigma-1}}-p_x x-w_{aj}n+\mathcal{W}_j \\
& p_x x+w_{aj}n\leq\phi_j Rs \\
& s'\geq 0
\end{aligned}
$$

The first constraint is the budget constraint, the second is the financial friction, and the third is the no-borrowing constraint. Recall that $p_x=(1-\tau_x)p_x^{\text{market}}$ so that the subsidy enters directly into the budget constraint and financial friction. A household's income is made up of agricultural profits $\pi_j(z,s)$ and wage earnings $\mathcal{W}_j$.

We note here that when we introduce a shock to the fertilizer price p_x^{market}, we do so with a so-called "MIT shock." That is, we unexpectedly shock the stationary equilibrium with some baseline price p_{x0}^{market} with a deterministic sequence of prices $\{p_{xt}^{\text{market}}\}_{t=1}^{t=\infty}$. Hence, the recursive program above includes perfect foresight of the evolution of the aggregate state μ.

4.1 Recursive Competitive Equilibrium

Given a subsidy level τ_x and market price of fertilizer p_x^{market}, a competitive equilibrium of this model is a set of decision rules for households $(c_a, c_{mj}, x_j, n_{aj}, s_j')$, the manufacturing firm N_m, and the final agricultural good producer y_{aj}, prices $\{p_{aj}\}, p_{ac}, w_a$ and tax rate τ_m such that (i) the household's decision rules are consistent with its optimization problem given prices and taxes, (ii) N_m solves the manufacturing firm's decision problem, (iii), y_{aj} solves the agricultural final goods firm decision problem, (iv) the law of motion for the aggregate state, $\mu' := \Lambda(\mu)$, is consistent with the decision rules, and (v) the government balances its budget and markets clear:

1. Government budget balance:

$$\tau_x p_x^{market}\int_j\int_{s,z} x_j(s,z)d\mu_j dj=\tau_m\int_j\int_{s,z} c_{mj}(s,z)d\mu_j dj$$

2. Market clearing:

   (a) Agricultural intermediate market: for each j,

$$y_{aj}=\int_{s,z} z\left(\alpha_j^{\frac{1}{\sigma}}x_j(s,z)^{\frac{\sigma-1}{\sigma}}+(1-\alpha_j)^{\frac{1}{\sigma}}n_j(s,z)^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\gamma\sigma}{\sigma-1}}d\mu_j$$

(b) Agricultural final goods market:

$$\left(\int_j y_{aj}^{\frac{\nu-1}{\nu}} dj\right)^{\frac{\nu}{\nu-1}} = \int_j \int_{(s,z)} c_{aj}(s,z) d\mu_j dj$$

(c) The agricultural labor market: for each j,

$$\int_{s,z} n_j(s,z) d\mu_j = T_a w_{aj}^{\theta-1} \Gamma \left(\frac{\theta-1}{\theta}\right) \left(T_a w_{aj}^{\theta} + T_m w_m^{\theta}\right)^{\frac{1-\theta}{\theta}}$$

(d) The manufacturing labor market:

$$N_m = T_m w_m^{\theta-1} \Gamma \left(\frac{\theta-1}{\theta}\right) \int_j \left(T_a w_{aj}^{\theta} + T_m w_m^{\theta}\right)^{\frac{1-\theta}{\theta}} dj$$

which amounts to the restriction that $w_m = A$.

4.2 Analytic Results

The goal of the model is to understand how the optimal subsidy τ_x responds to changes in the market fertilizer price p_x^{market}. To clarify the underlying economics of the problem, we first provide some analytic results in the static version of the model (or, identically, $R = 0$). Doing so necessitates a small change in the financial friction which we re-write as

$$p_x x + w_a n \leq \phi_j$$

where now ϕ_j is a maximum cost level in village j instead of a multiple of savings (with zero savings, this would imply the agricultural sector would not exist).

We make three additional assumptions. The first is purely for notational convenience: we make the idiosyncratic shocks degenerate with $Pr(z = 0) = 1$. This eliminates the need to carry distributions of idiosyncratic shocks through the analysis and has no bearing on the results. The latter two will look familiar to readers of standard macro-development models with an agricultural sector, such as Restuccia et al. (2008). We let $\sigma \to 1$ so that the production function becomes Cobb-Douglas, which we write as $y_j = z^{\alpha_j} n^{\eta_j}$, where $\eta_j = \gamma - \alpha_j$. We also assume $\theta \to \infty$, which makes labor supply infinitely elastic and guarantees that local agricultural wages are pinned down by the manufacturing sector in equilibrium, $w \equiv w_{aj} = w_m = A$ for all j.

In this simplified model, a household in village j solves

$$\begin{aligned} & \max_{c_a, c_m, x, n_a} \quad \zeta \log(c_a) + (1-\zeta)\log(c_m) \\ s.t. \quad & p_{ac} c_a + (1+\tau_m) c_m = p_{aj} x^{\alpha_j} n_a^{\eta_j} - p_x x - wn + w \\ & p_x x + wn \leq \phi_j \end{aligned}$$

We define the competitive equilibrium of this simplified economy in the Appendix, though it differs only slightly from the one defined above.

4.3 The Frictionless Equilibrium

To understand how τ_x should respond to fertilizer price changes, it is helpful to start from the frictionless economy with $\phi_j = \infty$ for all villages. Here, $\tau_x = 0$, so $p_x = p_x^{\text{market}}$. Characterizing this economy allows us to show how equilibrium forces work to reallocate production across heterogeneous villages. We then use this as a benchmark for the economy with binding financial frictions.

Because we are interested in changes between two equilibria, our strategy is to characterize the new equilibrium (with higher p_x^{market}) in terms of the baseline (with low p_x^{market}). To start, Lemma 1 characterizes the new equilibrium in this way. All proofs are in the Appendix.

Lemma 1. *Assume that the market fertilizer price rises to $p_{x2}^{market} > p_{x1}^{market}$. Define*

$$\mathcal{Y} = \frac{\int_j y_{j1}^{\frac{\nu-1}{\nu}} dj}{\int_j \left(\frac{p_{x2}}{p_{x1}}\right)^{\frac{-\alpha_j(\nu-1)}{\nu-(\nu-1)\gamma}} y_{j1}^{\frac{\nu-1}{\nu}} dj}.$$

This value $\mathcal{Y}$, combined with exogenous parameters, fully characterizes the p_{x2}^{market} equilibrium relative to the p_{x1}^{market} equilibrium. For example, the equilibrium price ratios are

$$\frac{p_{ac2}}{p_{ac1}} = \mathcal{Y}^{\frac{\nu-(\nu-1)\gamma}{\nu-1}} \qquad \textit{and} \qquad \frac{p_{aj2}}{p_{aj1}} = \mathcal{Y}^{\frac{\nu-(\nu-1)\gamma-1}{\nu-1}} \left(\frac{p_{x2}}{p_{x1}}\right)^{\frac{\alpha_j}{\nu-(\nu-1)\gamma}}$$

The term $\mathcal{Y}$ determines the ratio of baseline aggregate output an adjusted version of itself, where the adjustment depends on the change to the fertilizer price p_x.[13] Thus, it summarizes the aggregate decline in production from the higher fertilizer price. The key feature of $\mathcal{Y}$ is that it depends on no endogenous values from the new p_{x2} equilibrium so that it fully characterizes the new equilibrium in terms of the baseline

[13] In the proof of Lemma 1, we show that $Y_{a2}/Y_{a1} = 1/\mathcal{Y}^{\frac{\nu}{\nu-1}-\gamma}$.

one.

Notice also that α_j affects how village j's output price p_{aj2}/p_{aj1} changes after the shock. This alone should not be too surprising: villages have different fertilizer cost shares in production, and thus are affected differently. But *where* the new equilibrium wants to redistribute lost production is at the heart of this problem. To make this more explicit, we combine Lemma 1 with a bit of algebra to characterize the change in household farm profit (we derive this explicitly in the proof of Lemma 1),

$$\pi_{j2} = \underbrace{\mathcal{Y}\left(\frac{p_{x2}}{p_{x1}}\right)^{\frac{\alpha_j(1-\nu)}{\nu-(\nu-1)\gamma}}}_{\equiv \text{ adjustment to } \pi_{j1}} \underbrace{(1-\gamma)p_{j1}y_{j1}}_{\equiv\ \pi_{j1}} \tag{4.2}$$

which implies that profit rises in village j if and only if

$$\pi_{j2} > \pi_{j1} \Leftrightarrow \mathcal{Y}^{\frac{\nu-(\nu-1)\gamma}{(\nu-1)(1-\gamma)}} > \left(\frac{p_{x2}}{p_{x1}}\right)^{\frac{\alpha_j}{1-\gamma}}.$$

The right-hand side is the partial equilibrium decline in village j profit from the higher fertilizer price. Naturally, this effect is larger for villages with high α: these villages use more fertilizer and thus their costs are more affected. The left-hand side is the equilibrium increase in profit that results from the change to j's output price. This GE response depends not on α_j, but on how that village production is aggregated, summarized by the parameter ν and adjusted by the returns to scale γ. Thus, understanding the total effect requires understanding how the direct, PE effect of the shock compares to the GE price response across villages. Lemma 1 lets us characterize these effects. Re-writing (4.2) in log differences (and recalling that $p_x = p_x^{\text{market}}$ here) gives

$$\Delta \log(\pi_j) = \log(\mathcal{Y}) + \frac{\alpha_j(1-\nu)}{\nu-(\nu-1)\gamma}\Delta \log(p_x^{\text{market}})$$

or as a derivative measuring its response to a fertilizer price shock,

$$\frac{\partial \Delta \log(\pi_j)}{\partial \Delta \log(p_x^{\text{market}})} = \frac{\partial \log(\mathcal{Y})}{\partial \Delta \log(p_x^{\text{market}})} + \frac{\alpha_j(1-\nu)}{\nu-(\nu-1)\gamma}.$$

This leads immediately to a summary measure of how profit changes across all villages after a fertilizer price shock. We specify this in Proposition 1.

Proposition 1. *In the frictionless economy, the cross-village variation in profit after*

a change in the market fertilizer price is

$$\frac{\partial^2 \Delta_t \log(\pi_j)}{\partial \Delta_t \log(p_x^{market}) \partial \alpha_j} = \frac{(1-\nu)}{\nu - (\nu-1)\gamma} \begin{cases} < 0, & \text{if } \nu > 1 \\ = 0, & \text{if } \nu = 1 \\ > 0, & \text{if } \nu < 1 \end{cases}$$

Proposition 1 characterizes the distributional consequences of the fertilizer price shock. Low-α villages see the largest changes in profit when villages are substitutes ($\nu > 1$), with the opposite pattern holding when villages are complements ($\nu < 1$). The intuition relies on understanding how village substitutability affects patterns of equilibrium redistribution. After the shock, equilibrium forces work to redistribute that lost production to other villages. This is why ν matters. If villages are substitutes ($\nu > 1$), the equilibrium tries to source production from the relatively unscathed (in the PE sense) low-α villages. On the other hand, with complementarity ($\nu < 1$) this is not viable. The decline in production from high-α villages needs to be replaced with production from those same villages. Thus, the equilibrium shifts the output price changes toward higher-α villages to cushion the blow of the higher input price.

These results offer an efficient benchmark for the evolution of equilibrium outcomes after a fertilizer price shock.[14] As we will show, the inability of the frictional economy to replicate the redistribution highlighted in Proposition 1 will be central to the government's ability to affect welfare with a fertilizer subsidy.

4.4 Frictional Equilibrium

We now return to the frictional equilibrium, where each village is subject to $p_x x + wn \leq \phi_j$. The baseline economy is characterized by fertilizer state $(p_{x1}^{\text{market}}, \tau_{x1})$, implying farmer price $p_{x1} = (1-\tau_{x1})p_{x1}^{\text{market}}$. To make the results as sharp as possible, we assume that ϕ_j is tight enough that it binds with equality for all j at baseline farmer price p_{x1} (which also guarantees it binds at $p_{x2}^{\text{market}} > p_{x1}^{\text{market}}$ for a fixed subsidy level). Thus, we study an economy in which financial frictions are extremely tight, but still heterogenous across villages. This implies that input choices are

$$x_j^c = \frac{\alpha_j \phi_j}{\gamma p_x} \qquad \text{and} \qquad n_j^c = \frac{\eta_j \phi_j}{\gamma w} \tag{4.3}$$

where the superscript c is a reminder that these come from the "constrained" frictional equilibrium (and we will sometimes use superscript u to denote the frictionless or "unconstrained" economy).

[14] Technically, this is constrained efficient because households cannot trade *ex ante* insurance contracts, but our interest here is on the impact of financial frictions.

It will be useful to define the village j deviation of fertilizer (x) and output (y) from its frictionless benchmark,

$$\kappa_j^x := \frac{x_j^u}{x_j^c} \quad , \quad \kappa_j^y := \frac{y_j^u}{y_j^c}$$

A high κ_j^x, for example, implies that x_j^c is far below its unconstrained or frictionless level x_j^u. We focus our discussion in terms of the fertilizer but note that κ_j^x and κ_j^y move one-for-one (see footnote 15). Plugging in the relevant input choices yields

$$\kappa_j^x = p_x^{\frac{-\alpha_j}{1-\gamma}} (p_j^u)^{\frac{1}{1-\gamma}} \left(\frac{\gamma \left(\frac{\eta_j}{w}\right)^{\frac{\eta_j}{1-\gamma}} \alpha_j^{\frac{\alpha_j}{1-\gamma}}}{\phi_j} \right) \tag{4.4}$$

κ_j^x offers a summary variable we can use to measure how well policy does at moving village j's fertilizer choice closer to its frictionless benchmark, inclusive of price differences between the two equilibria. We start with the following question: if the government does not adjust its subsidy after the market fertilizer price rises from $p_{x2}^{\text{market}} > p_{x1}^{\text{market}}$, what happens to the set $\{\kappa_j^x\}_{j \in \mathcal{J}}$? We will then use this information to show how the optimal fertilizer subsidy should evolve.

Characterizing Production After the Fertilizer Price Shock One approach to answering this question would be to characterize κ^x directly. This unfortunately depends not only on (α_j, ϕ_j) but on the distribution $G(\alpha, \phi)$ and various other parameters that determine equilibrium prices, making characterization difficult or at least a complicated non-linear combination of model features. But our interest is in changes to κ^x, not its level. This offers a way to sidestep many of these issues. Defining $\Delta_t x = x_2 - x_1$ as the difference between the p_{x2} and p_{x1} equilibria, (4.4) becomes

$$\Delta_t \log(\kappa_j^x) = \frac{-\alpha_j}{1-\gamma} \Delta_t \log(p_x) + \frac{1}{1-\gamma} \Delta_t \log(p_{aj}^u) \tag{4.5}$$

Equation (4.5) measures evolution of fertilizer in village j relative to its frictionless benchmark. Applying Lemma 1 to compute village j's frictionless equilibrium price p_{aj}^u then taking a second difference with village k yields

$$\Delta_t \log(\kappa_j^x) - \Delta_t \log(\kappa_k^x) = (\alpha_k - \alpha_j)(\nu - 1) \Delta_t \log(p_x). \tag{4.6}$$

(4.6) offers a "diff-in-diff" characterization of how the fertilizer gap evolves between any two villages.[15] It shows that the frictional equilibrium always under-delivers the

[15] (4.5) and (4.6) link directly to the output because $\Delta_t \log(\kappa_j^x) = (1/\gamma) \Delta_t \log(\kappa_j^y)$. Thus, discussing the results in terms of inputs is equivalent to discussing in terms of output.

necessary redistribution, except in the knife-edge Cobb-Douglas case ($\nu = 1$). To see this more clearly, rewrite $\kappa_j^x = x_j^u / x_j^c$ as the components of the fraction. For two villages with $\alpha_H > \alpha_L$, writing (4.6) in this manner implies

$$\overbrace{\Delta_t \log(x_H^c) - \Delta_t \log(x_L^c)}^{\equiv\ \Delta \text{ frictional equilibrium}} > \overbrace{\Delta_t \log(x_H^u) - \Delta_t \log(x_L^u)}^{\equiv\ \Delta \text{ frictionless benchmark}} \quad \text{if } \nu > 1 \tag{4.7}$$

$$\Delta_t \log(x_H^u) - \Delta_t \log(x_L^u) = \Delta_t \log(x_H^c) - \Delta_t \log(x_L^c) \quad \text{if } \nu = 1 \tag{4.8}$$

$$\Delta_t \log(x_H^c) - \Delta_t \log(x_L^c) < \Delta_t \log(x_H^u) - \Delta_t \log(x_L^u) \quad \text{if } \nu < 1. \tag{4.9}$$

The left-hand side measures the relative fertilizer changes between H and L in the frictional equilibrium and compares them to their frictionless counterparts on the right, which were characterized in Proposition 1. What differs across different ν is where the efficient economy wants to shift production: toward low-α villages if $\nu > 1$ and high-α villages if $\nu < 1$. This is accomplished in the frictionless equilibrium via general equilibrium output price changes. In either case, however, the frictional equilibrium fails to deliver this redistribution. It keeps too much fertilizer in high-α villages when $\nu > 1$. That is, equation (4.7) shows that the frictional equilibrium does not sufficiently close the fertilizer gap between high- and low-α villages. The opposite is true with $\nu < 1$: the gap does not grow large enough relative to its frictionless benchmark (equation 4.9). Together, this pattern manifests in changing village-level distortions, which we summarize in Proposition 2.

Proposition 2. *Assume the market fertilizer price rises from p_{x1}^{market} to p_{x2}^{market} for a fixed subsidy τ_x. The equilibrium fertilizer shadow prices $\{\lambda_{jt}^c\}$ evolve across villages as*

$$\frac{\partial^2 \Delta_t \log(1 + \lambda_j^c)}{\partial \Delta_t \log(p_x^{market}) \partial \alpha_j} = \frac{1 - \nu}{\nu}$$

for any non-degenerate distribution $G(\alpha, \phi)$.

Proposition 2 summarizes the frictional counterpart to the frictionless baseline in Proposition 1. The term $1 + \lambda_j^c$ is the Restuccia and Rogerson (2008) distortion faced by village j. Thus, Proposition 2 summarizes how distortions evolve across villages. With complementarity ($\nu < 1$), Proposition 1 showed that the frictionless equilibrium tries to compensate high-α villages for its higher fertilizer costs. Proposition 2 shows that this is not accomplished here: high-α villages see the largest change in shadow price due to the insufficient reallocation documented above. The same is true with substitutable villages ($\nu > 1$). Here, the frictionless equilibrium shifts resources from high-α to low-α villages. Proposition 2 shows the frictional equilibrium fails to do so because prices cannot sufficiently redistribute (equation 4.7). Taken together, the

frictional economy always under-delivers the necessary redistribution.[16]

The economic intuition for this result is that financial frictions dampen the output price elasticity of farmer input choices. Notice, for instance, that the output price p_{aj} does not enter village j's input choices (4.3). The intuition is simple: if j constrained away from profit maximizing input choices at its baseline output price, then the output price rises, there is little that can be done to respond to it. But by eliminating these responses, the reallocation demanded by the frictionless equilibrium is impossible to achieve. This opens scope for a subsidy.

Implications for the Optimal Subsidy Our next step, then, is to characterize how the optimal subsidy should react to deliver this unrealized reallocation. A seemingly reasonable intuition is that a planner should increase its subsidy to counter tighter financial frictions. But this intuition fails to account for the fact that cross-village technological heterogeneity offers the government an alternative to recoup lost production: shift production to other villages. Proposition 3 shows that the subsidy response depends only on how easily the government can utilize other, less exposed villages to replace lost production.

Proposition 3. *Assume a Ramsey planner chooses a fertilizer subsidy (or tax if negative) τ_x to maximize utilitarian welfare,*

$$\max_{\tau_x} \quad \int_j \Big[\zeta \log(c_{aj}) + (1-\zeta)\log(c_{mj})\Big] dj$$

subject to τ_x being consistent with the resulting competitive equilibrium of the frictional economy, including government budget balance. For any non-degenerate distribution of $G(\alpha, \phi)$, the optimal subsidy is such that

$$\frac{\partial \tau_x^*}{\partial p_x^{market}} \begin{cases} < 0, & \text{if } \nu > 1 \\ = 0, & \text{if } \nu = 1 \\ > 0, & \text{if } \nu < 1 \end{cases}$$

If the marginal distribution of α is degenerate so that all villages have identical α, then $\frac{\partial \tau_x^*}{\partial p_x^{market}} = 0$ for all values of ν.

[16] The same result generalizes to the case in which farmers have CES production technologies instead of Cobb-Douglas, but with some loss of clarity. In this case, the cross-partial derivative in Proposition 2 is

$$\frac{\partial^2 \Delta_t \log(1+\lambda_j^c)}{\partial \Delta_t \log(p_x^{market}) \partial \alpha_j} = \left(\frac{1-\nu}{\nu}\right)\left(\frac{1}{\sigma}\right) g(p_{x1}, \Delta_t \log(p_x), \alpha_j),$$

where $g > 0$ always and σ is the CES parameter. Thus, the sign of the derivative remains entirely dependent on ν. Notice also that the derivate is modulated by σ: if $\sigma \to \infty$, there is no change in the shadow price, where the change is largest when $\sigma \to 0$. As discussed in the introduction, σ does not change the qualitative properties but is critical to the quantitative results, as will become clear shortly. See the Appendix for derivation of this result.

The level of $\tau_x^*(p_x^{\text{market}})$ is a complicated object to characterize, even in this simplified model, making it difficult to derive any useful economic intuition from it. The derivative, on the other hand, has a simple characterization that highlights the central economic trade-off. Specifically, the planner increases the subsidy in response to higher fertilizer prices only when $\nu < 1$. The intuition is entirely based on our earlier results that highlight where the planner needs to incentivize production. When high-α villages decrease production, substitutability ($\nu > 1$) implies that low-α villages can replace that production. But financial frictions limit the ability of prices to properly incentivize this redistribution. The planner therefore lowers its subsidy, shifting production toward less affected villages.

Attempting the same policy with complementarity ($\nu < 1$) would imply a large negative impact on the aggregate availability of the agricultural consumption good. Thus, the planner seeks to subsidize the same affected villages to increase production. While doing so simultaneously increases subsidization to low-α villages, the cost to the government's budget is relatively small given the technological irrelevance of fertilizer in these places, and thus still worthwhile.[17] That the subsidy remains constant when all villages have the same α emphasizes the centrality of heterogeneity in the argument.

4.5 Relaxing Assumptions Used Here

These results come from a simplified model. Allowing villages to more easily substitute labor for capital, or inducing wage effects via a finite labor supply elasticity will affect the magnitude of the planner's chosen subsidy level and allow us to study the role of alternative policy tools rendered trivially uninteresting by our simplifications here (e.g., wage subsidies).

As we show in the Appendix, relaxing the Cobb-Douglas assumption alone to a CES production function eliminates the clarity of theoretical results provided above. Proposition 3, on the sign of the change in subsidy, instead depends on ν, the CES production function parameter σ, and the co-movement of frictions and technology (ϕ_j, α_j), specifically the covariance of the unit cost and financial friction across villages.

However, the main message remains the same: the government uses its subsidy to induce reallocation across heterogeneous production units that is otherwise limited by financial frictions. Parameters other than just the tightness of financial friction affect both the sign and magnitude of how the subsidy evolves with fertilizer prices. Thus,

[17] Proposition 3 does not depend on how the subsidy is financed. In the Appendix, we provide the proof of under alternative financing schemes in which all income is taxed and the case in which only labor market income is taxed.

answering this question both quantitatively *and* qualitatively depends on credibly measuring them. Our goal now is to link these model parameters back to the causal empirical estimates in the full dynamic model, to quantify how the subsidy should change in response to the fertilizer price shock.

5 Quantitative Exericse and Model Estimation

5.1 Quantitative Exercise

Our main quantitative exercise works as follows. We start the model with a baseline market fertilizer price p_{x0}^{market} and subsidy level τ_{x0} that all agents assume will remain fixed forever. This generates a baseline stationary equilibrium. We then shock the economy with a deterministic sequence $\{p_{xt}^{\text{market}}, \tau_{xt}\}_{t=1}^{t=\infty}$ that eventually converges back to its baseline values $(p_{x0}^{\text{market}}, \tau_{x0})$ and trace out the transition path of the economy.[18]

Under this assumption, we calibrate the model using the empirical counterparts $\{\widehat{p}_{xt}^{\text{market}}, \widehat{\tau}_{xt}\}_{t=0}^{t=\infty}$ that were realized in Rwanda and restrict ourselves to model outcomes from $t \in \{0, 1, 2, 3\}$ to be consistent with our empirics. Once calibrated, we will feed in different time paths $\{p_{xt}^{\text{market}}, \tau_{xt}\}_{t=0}^{t=\infty}$ to study counterfactual outcomes.

5.2 Estimating Model

The model includes several parameters. On the production side, we require the CES aggregator for agricultural production ν. For village production, we need to estimate the joint distribution of fertilizer intensity α and financial frictions ϕ across villages, $G(\alpha, \phi)$ along with the elasticity between fertilizer and labor σ and returns to scale γ, along with the idiosyncratic shock distribution $F(z)$. In the labor market, we need values for the exponential distributions (T_a, T_m) and the parameter θ that governs the returns to skill. Finally, we require the return to savings R and utility function parameters including the discount rate β.

While we set some of these directly outside the model, many will be estimated via the empirical results from Section 3. These results were derived from regressions of the form given in (3.1),

$$y_{hvmt} = \alpha + \beta Post_t + \eta \left(Post_t \times \tilde{f}_{v0} \right) + \delta_m + \theta_v + \zeta_t + \varepsilon_{hvmt}.$$

[18] The international fertilizer price and Rwandan subsidy level were extremely stable in the decade leading up to 2021 (see Section 2). While this is of course just one realization of a random variable, the expert forecasts in Figure 2 are consistent with beliefs that the price stability would continue. Beliefs consistent with this result would closely approximate our assumption. This comes at the quantitative cost of missing any increased uncertainty on the evolution of fertilizer prices after the shock.

We will use of the usual exposure intuition, where the relevant parameter is

$$\frac{\partial \mathbb{E}[\Delta_t y_{hvmt}]}{\partial \tilde{f}_{v0}} = \hat{\eta}.$$

When we derive estimating equations below, we will generally do so in the same derivative terms. To economize on notation, we write $\overline{\Delta}_t y := \Delta_t \mathbb{E}[y_t]$. It will also sometimes be helpful to match the post outcomes for the average village, which we match to the average effect regressions provided in Section 3.

5.2.1 Parameters Set Exogenously

We normalize manufacturing productivity $A = 1$ and skill shifter $T_m = 1$, then set two parameters outside the model, the discount rate $\beta = 0.96$ and the returns to scale $\gamma = 0.8$.

5.2.2 Parameters that Match to Empirical Results Directly

We then match three critical parameters directly to the empirical results. These are the complementarity between village production, ν, the skill parameter θ that governs the labor supply elasticity, and the CES parameter in the production technology σ. We discuss them in turn.

Substitutability Between Village Production, ν The market clearing condition for the final agricultural good producer implies that that village j output price is a function of its production share and the agricultural consumption price,

$$p_{ajt} = \left(\frac{Y_{at}}{y_{ajt}}\right)^{\frac{1}{\nu}} p_{act}.$$

Some algebra implies

$$(1-\nu)\Delta_t \log(p_{ajt}) = \Delta_t \log(p_{ajt} y_{ajt}) - (\Delta_t \log(Y_{at}) + \nu \Delta_t \log(p_{act}))$$

Notice that the last term in parenthesis is constant across villages, which means that

$$(1-\nu)\underset{[+0.030]}{\frac{\partial \overline{\Delta}_t \log(p_{jt})}{\partial x_{j0}}} = \underset{[-0.047]}{\frac{\partial \overline{\Delta}_t \log(p_{jt} y_{jt})}{\partial x_{j0}}}.$$

The parameter ν depends on the relative movements of village j production to its output price across villages. The relevant values from Section 3 (in brackets below the equation) give $\nu = 2.57$.

Our theoretical results in Section 4 highlight the importance of this parameter. The natural experiment combined with micro data allows us to infer it independently of the remaining model structure. This is our first, and most important, result suggesting that the model will imply a decline in the optimal fertilizer subsidy after the fertilizer price rises.

Joint Estimation of Production Elasticity σ and Labor Supply Elasticity θ Each household i in village j has a cost ratio of

$$\frac{p_{xt}x_{ijt}}{w_{ajt}n_{ijt}} = \frac{\alpha_j}{1-\alpha_j}\left(\frac{w_{ajt}}{p_{xt}}\right)^{\sigma-1} \tag{5.1}$$

First, note that because the cost ratio is identical for every household, this also represents the village-level cost ratio. Second, because internal agricultural labor markets clear village-by-village, $w_{ajt}\int_i n_{ijt}di$ is equal to total agricultural wage earnings for households in village j. This implies that we can re-write (5.1) as

$$\underset{\text{[-0.097]}}{\overline{\Delta}_t \log(p_{xt}x_{jt})} = \overline{\Delta}_t \log(\text{Agr. Earnings}_j) + (\sigma-1)[\underset{\text{[+0.193]}}{\overline{\Delta}_t \log(w_{ajt})} - \underset{\text{[+0.67]}}{\overline{\Delta}_t \log(p_x)}] \tag{5.2}$$

and noting that the fertilizer price p_x does not vary across villages,

$$\underset{\text{[-0.059]}}{\frac{\partial\overline{\Delta}_t \log(p_{xt}x_{jt})}{\partial x_{j0}}} = \frac{\partial\overline{\Delta}_t \log(\text{Agr. Earnings}_j)}{\partial x_{j0}} + (\sigma-1)\underset{\text{[+0.013]}}{\frac{\partial\overline{\Delta}_t \log(w_{ajt})}{\partial x_{j0}}}. \tag{5.3}$$

A brief look at these equations might suggest that σ can be estimated directly from our reduced-form empirical results on agricultural wages, agricultural earnings, and fertilizer expenditures. The issue is that model-consistent agricultural earnings include own time on farm and the unobserved labor productivity that come with it, so are not directly comparable to the reduced-form results in Section 3. Instead, we take advantage of the properties of the exponential distribution to note that the earnings ratio in village j is

$$\frac{\text{Total Manufacturing Earnings}}{\text{Total Agricultural Earnings}} = \frac{T_m}{T_a}\left(\frac{w_{mt}}{w_{ajt}}\right)^{\theta}.$$

which means that the relevant terms are

$$\begin{aligned}
\overline{\Delta}_t \log(\text{Agr. Earnings}) &= \underset{[-0.285]}{\overline{\Delta}_t \log(\text{Man. Earnings})} + \underset{[+0.193]}{\theta \overline{\Delta}_t \log(w_{ajt})} \\
\frac{\partial \overline{\Delta}_t \log(\text{Agr. Earnings})}{\partial x_{j0}} &= \underset{[-0.096]}{\frac{\partial \overline{\Delta}_t \log(\text{Man. Earnings})}{\partial x_{j0}}} + \underset{[+0.013]}{\theta \frac{\partial \overline{\Delta}_t \log(w_{ajt})}{\partial x_{j0}}}
\end{aligned}$$

Plugging this back into (5.2) and (5.3) gives two equations and two unknowns.

$$\begin{aligned}
0.19 &= 0.19\theta - 0.48(\sigma - 1) \\
0.04 &= 0.01\theta + 0.01(\sigma - 1)
\end{aligned}$$

Thus, using both the average effect of the shock and the variation with baseline profit allows us to jointly determine σ and θ. Together, these imply $\sigma = 1.54$ and $\theta = 2.31$, implying fertilizer and labor are slightly more substitutable than Cobb-Douglas. That σ is near one is a useful sanity check on the results. We observe a large increase in fertilizer prices but a small change in the fertilizer cost share of revenue, a result intuitively consistent with approximately Cobb-Douglas production.

5.2.3 Parameters Set to Match Equilibrium Moments

What remains are parameters that require solving the impulse response of the model to match moments. There are the shape parameter from the exponential distribution of agricultural skills, T_a, the gross interest rate R, the standard deviation of the log-normal idiosyncratic shocks σ_z, and the joint distribution of technology and financial frictions across villages, $G(\alpha, \phi)$ which has 5 parameters (discussed below). Together, we have 8 parameters to jointly choose. While they are jointly determined, we discuss the link between parameters and moments heuristically below.

T_a, R, σ_z to Match Baseline Cross-Sectional Moments We can normalize $T_m = 1$ as well, which leaves T_m to match the average baseline wage premium in the manufacturing sector. To measure the wage premium, we run the cross-sectional regression on baseline data,

$$\log(w_{i\ell v0}) = \beta_0 + \beta_1 \mathbb{1}[\ell = \text{outside}] + \beta_2 X_{i\ell v0} + \theta_v + \varepsilon_{i\ell v0}$$

for individual i working in location ℓ in village v at baseline. We control for individual characteristics like age and education in X and village fixed effects in θ. The outcome variable $\log(w)$ is the log daily wage rate. This yields $\hat{\beta}_1 = 0.23$ (s.e. $= 0.009$).

We set the interest rate R such that we match the total savings/harvest ratio of the average village. The standard deviation of the idiosyncratic shocks σ_z is set to match the average within-village variation in log yields.

Joint Estimation of Technological and Financial Friction Heterogeneity, $G(\alpha, \phi)$ We parameterize the joint distribution by assuming the marginal distribution of $\alpha \sim \text{Beta}(p_\alpha, q_\alpha)$ and $\phi \sim \text{Fréchet}(\theta_\phi, s_\phi, 1)$ where the parameters are shape, scale and location equal to 1 (so that villages can always self-finance). We then combine the two with a Frank copula, which imposes the joint c.d.f.

$$G(\alpha, \phi) = \frac{-1}{\rho} \log \left[1 + \frac{(e^{-\rho H_\alpha} - 1)(e^{-\rho H_\phi} - 1)}{e^{-\rho} - 1} \right]$$

where H_α and H_ϕ are the marginal c.d.f.s for technology and the financial friction. This leaves us with five parameters. Two from each of the marginals and ρ, which governs co-movement between the two.

Critical for quantifying the model is to make sure the changes in distortions we derive are consistent with the data. If the model predicts (overly) large changes to distortions, the model will also want to generate large swings in fertilizer subsidies. This is a particularly complicated issue here, because we do not know whether villages using little fertilizer do so because of technology α or frictions ϕ.

We therefore re-cast the theoretical distortions as functions of our empirical results and use this to target changes in distortions directly. We write this term as $(1 + \lambda)$ in the equation

$$(1 + \lambda_{ijt})^\sigma p_{xt} x_{ijt} = \gamma^\sigma \alpha_j z_{ijt}^{\frac{\sigma-1}{\gamma}} (p_{ajt} y_{aijt})^{\sigma - \frac{\sigma-1}{\gamma}} p_{ajt}^{\frac{\sigma-1}{\gamma}} p_x^{1-\sigma}. \tag{5.4}$$

λ_{ijt} is the shadow value of fertilizer for household i in village j at time t or, equivalently, $1 + \lambda_{ijt}$ the reduced-form distortion (in a Restuccia and Rogerson, 2008, sense) faced by the household.[19] This is the same moment whose evolution across villages was formalized in the simpler static model of Section 4 (see Proposition 2). While it does not admit such a straightforward characterization here, we can take a similar approach as above to link to empirical estimates. Log differencing and averaging yields

[19] This equation follows from manipulating the first order condition for fertilizer. While the CES assumption makes it somewhat less intelligible, note that with $\sigma = 1$ it simplifies to

$$\frac{p_{xt} x_{ijt}}{p_{ajt} y_{aijt}} = \frac{\gamma \alpha_j}{1 + \lambda_{ijt}}$$

where $\gamma \alpha_j$ is the exponent on fertilizer in the production function $y = z(x^\alpha n^{1-\alpha})^\gamma$. (5.4) is just the CES generalization of this familiar Cobb-Douglas result.

$$\overline{\Delta}_t \log(1+\lambda_{ij}) = \underset{[-0.097]}{-\frac{1}{\sigma}\overline{\Delta}_t \log(p_x x_{ij})} + \underset{[+0.025]}{\frac{1-\sigma}{\sigma\gamma}\overline{\Delta}_t \log(p_{aj})} + \underset{[+0.207]}{\left(1+\frac{1-\sigma}{\sigma\gamma}\right)\overline{\Delta}_t \log(p_{aj}y_{aj})} + \underset{[+0.67]}{\left(\frac{1-\sigma}{\sigma}\right)\Delta_t \log(p_x)} \tag{5.5}$$

(5.5) tells us how the average distortion changes after the price shock. Notice that we can use the variation induced by the empirical price shock, combined with our previously-estimated values of σ and γ to measure the average value of (5.5). With $\sigma = 1.54$ and $\gamma = 0.8$ this implies $\overline{\Delta}_t \log(1+\lambda_{ij}) = -0.044$. Thus, the average distortion falls by 4 percent.[20] The inference here is that households are constrained financially. A higher fertilizer price causes the unconstrained fertilizer level to fall, thus bringing the distortion down. This is the first sense in which we target the change in distortions directly.

The second aspect of distortions we target is their inter-village evolution. Taking the derivative of (5.5) noting that p_x is constant across villages implies

$$\frac{\partial\overline{\Delta}_t \log(1+\lambda_{ij})}{\partial x_{j0}} = \underset{[-0.059]}{-\frac{1}{\sigma}\frac{\partial\overline{\Delta}_t \log(p_x x_{ij})}{\partial x_{j0}}} \underset{[+0.030]}{-\frac{\sigma-1}{\sigma\gamma}\frac{\partial\overline{\Delta}_t p_{aj}}{\partial x_{j0}}} + \underset{[-0.047]}{\left(1-\frac{\sigma-1}{\sigma\gamma}\right)\frac{\partial\overline{\Delta}_t \log(p_{aj}y_{aj})}{\partial x_{j0}}}. \tag{5.6}$$

Intuitively, this moment determines the co-movement of technology α and financial frictions ϕ across villages. Roughly, the larger this value, the more negative correlated α and ϕ (that is, high α villages also have the tightest financial constraints). With $\sigma = 1.54$ and $\gamma = 0.8$ we find positive co-movement, with $\partial\overline{\Delta}_t \log(1+\lambda_{ij})/\partial x_{j0} = 0.025$.

We require three additional moments after matching these two distortion-related moments. We therefore target the average and variance of household-level fertilizer shares of harvest value. These are 9.5 percent and 0.03 respectively. The third moment is the share of that variance that is within villages. Literally, this is the R^2 from a regressing household-level fertilizer shares on village fixed effects. Our motivation for this moment comes from the centrality of inter-village heterogeneity highlighted in Proposition 3. To the extent that cross-sectional household variation is primarily within-village, the model will imply limited quantitative scope for the fertilizer subsidy to respond to prices. We find that 21 percent of the total variance is

[20] The average change in distortion is increasing in γ. At $\gamma = 0.99$, $\overline{\Delta}_t \log(1+\lambda_{ij}) = -0.03$ and at $\gamma = 0.5$ $\overline{\Delta}_t \log(1+\lambda_{ij}) = -0.09$. Thus, it is negative for any value of γ.

across villages, with the remaining 79 percent coming from within-village variation.

5.2.4 Parameters and Discussion

There are a few parameters worth discussing in more detail.

First, the gross interest rate R is low, implying a large fraction of savings depreciates between periods. While a common assumption (Donovan, 2021; Lagakos et al., 2023), it is often argued as a function of physical crop storage depreciation or inflation. The large value we find is inconsistent with empirical post-harvest loss estimates on the order of 5-10 percent or inflation rates of similar magnitudes. We show in the Appendix that this can be rationalized as a stand-in for a lean season where household run down savings with limited income-earning opportunities (Bryan et al., 2014; Brooks and Donovan, 2020; Fink et al., 2020; Lagakos et al., 2023).[21] Regardless of the rationale, at the annual level we jointly target limited savings and the change in distortions. Matching them jointly takes the alternative rationale for limited savings off the table: that frictions simply do not bind.

6 Quantitative Results

6.1 Optimal Policy and Welfare

In the baseline steady state equilibrium, the optimal fertilizer subsidy is 15 percent. This is about half the Rwandan government's 30 percent subsidy in 2020. There are of course a number of reasons for this discrepancy, but one we take up in the Appendix is the potentially different welfare weights placed on poorer households by the government than our utilitarian Ramsey planner.[22] We save that discussion for the Appendix but note that the quantitative results are similar.

We begin by plotting the optimal fertilizer subsidy in Figure 6. The optimal subsidy falls from 31 to 28 percent on impact, following by a short rebound to its baseline level. The initial decline is driven by our empirical results: villages are substitutes and therefore the Ramsey planner uses the subsidy to redistribute production toward less affected villages. Also in Figure 6a we include the path observed in the data. Two features are clear: first, the model does a good job of hitting the baseline level, despite not being a calibration moment. Second, clearly, the optimal policy moves in

[21] In fact, this "lean season" extension aggregates to an identical model, except with interest rates that are specific to the level of household savings at the beginning of the season. This adds an additional amplification channel driven by heterogeneous spreads (see, e.g., Cavalcanti et al., 2024).

[22] Specifically, we assume that the policymaker maximizes a generalized mean of individual-level welfare where utilitarian and Rawlsian welfare are special cases. We can then set the weights to guarantee the optimal baseline subsidy is 30 percent. This corresponds to a case in which the planner is slightly more concerned about the poor than the utilitarian planner.

the opposite direction of the data. Table 6 offers the welfare gap between the optimal policy, holding fixed the baseline level, and implement the government policy.

Figure 6: Optimal Fertilizer Subsidy Time Path

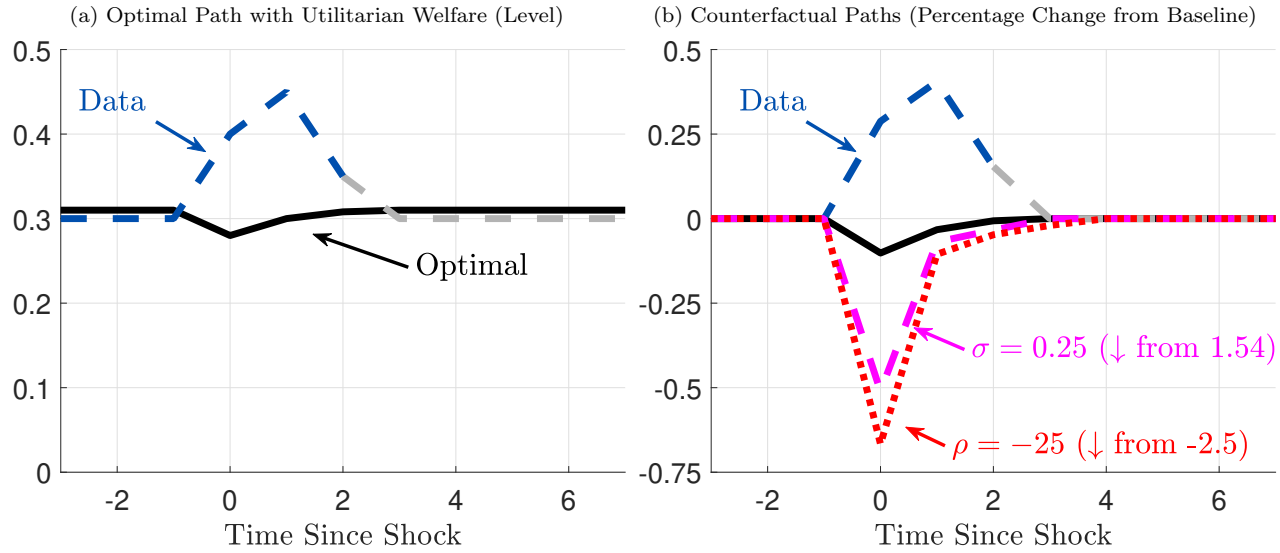


Figure Notes: Panel (a) plots the optimal time path for the fertilizer subsidy, along with the empirically-observed path. Panel (b) overlays two counterfactual time paths for comparison. The part of the "data" curve in gray is extrapolated because the data series ends in calendar year 2024.

The consequences of holding the subsidy fixed are small according to our model, costing only 0.3% of consumption-equivalent welfare, a natural consequence of the limited adjustment in the optimal subsidy. The consequences of increasing the subsidy at the same rate as the government are much larger, at 1.7% of consumption-equivalent welfare. Keeping the subsidy high funds intermediate-intensive production, specifically at a time when doing so is extremely costly in terms of taxes owed by all. Thus, the subsidy at this point is a transfer to specifically unproductive farmers. Note, however, that production rises (as does labor productivity). Taken together, these results emphasize the distinction between maximizing welfare and production in this economic environment.

Table 6: Aggregate Consequences of Different Subsidy Paths

	Welfare	Y_a
Optimal	–	
Keep Fixed	-0.03%	0.07%
Data	-1.7%	2.3%

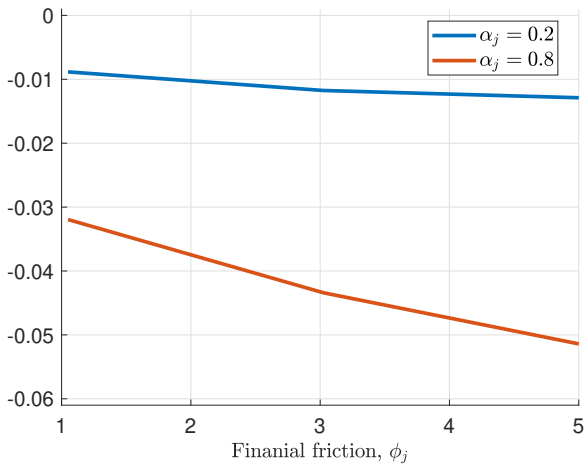
Figure notes: Welfare here is in aggregate consumption-equivalent terms. It is the decline in aggregate expenditures that makes the average person in the optimal world indifferent to the listed counterfactual. Columns (2) and (3) are averages over the first 5 years of the transition.

Figure 6b asks whether the small adjustments in the fertilizer subsidy are pre-ordained by the model, and shows they are not. They are driven by two key features of our estimation. First, fertilizer and labor are relatively substitutable. Second, there is limited correlation between estimated financial frictions and technology across villages. If either of those are eliminated, the subsidy can fall by 50-70 percent on impact.

Finally, figure 7 plots village-level outcomes of the optimal subsidy relative to the baseline of an unchanged subsidy. We report the percentage change between the two. Figure 7a shows that production falls for all villages, a natural consequence of the lower subsidy rate. However, it falls substantially more for high-α villages. The intuition follows from our theoretical results earlier. Because villages are substitutes, the optimal subsidy attempts to re-direct production toward low-α villages after the price shock. Holding the subsidy fixed at its baseline level keeps too much production in high-α villages.

Figure 7: Impact of Optimal Subsidy Level on Village-Level Outcomes

(a) Production



(b) Village Output Price

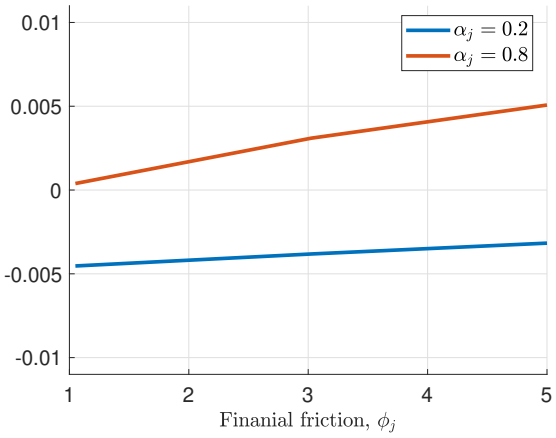


Figure Notes: These figures report the outcome in the period of initial impact, measured as as the log difference between the optimal subsidy and the economy in which the subsidy is held fixed at its baseline level.

7 Conclusion

Agriculture has low labor productivity relative to other sectors in developing countries. This fact, along with its implications for the distribution of employment across sectors, have been well appreciated. Less appreciated is the fact that farmers are subject to global forces affecting the price of one of their most critical inputs, fertilizer.

This paper shows that these fluctuations can be quite damaging for rural households but that the government can offer stability in terms of adjusting already-existing policy levers. We show this in the context of Rwanda.

An interesting angle for future work would be to consider the long-run implications of these shocks. We know, for example, that long run elasticities of substitution for intermediates can be substantially different from short-run elasticities (Peter and Ruane, 2022). In fact, in late 2023 the Rwandan government created a joint venture with a Moroccan fertilizer company to create a fertilizer processing plant in Rwanda, suggesting a similar divergence in this context (MINAGRI, 2023). These types of adjustments are rare, but perhaps more importantly, do not necessarily solve issues related to the procurement of inputs like natural gas. Alternatives include new technologies that deliver nitrogen via different means, such as biofertilizers (Schültz et al., 2018). Thus, future policy will likely require a combination of both short run stability and longer run adaptation.

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A Natural Gas Prices and Global Fertilizer Prices

Figure 8 plots the relationship between natural gas prices and fertilizer prices since 2014 from the World Bank Commodity Price Outlook (World Bank, 2024), normalized by January 2014. The correlation between the gas prices and the two fertilizer prices are 0.67 and 0.69 for DAP and urea. Figure 8 also shows that two key fertilizer prices more than double in just two years between 2020 and 2022.

Figure 8: Fertilizer Prices and Natural Gas Prices

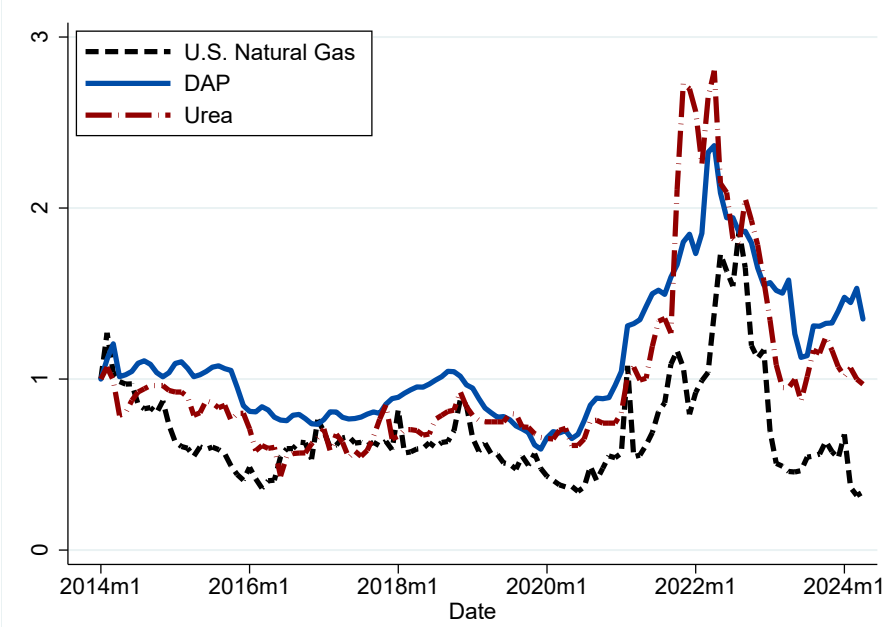


Figure Notes: Nominal monthly prices of DAP, urea, and U.S. natural gas from World Bank (2024). 2014m1 normalized to one.

B Competitive Equilibrium of Simplified Model

Given a subsidy level τ_x and market price of fertilizer p_x^{market}, a competitive equilibrium of this model is a set of decision rules for households $(c_{aj}, c_{mj}, x_j, n_{aj})$, the manufacturing firm N_m, and the final agricultural good producer y_{aj}, prices $\{p_{aj}\}, p_{ac}, w_a$ and tax rate τ_m such that (i) the household's decision rules are consistent with its optimization problem given prices and taxes, (ii) N_m solves the manufacturing firm's decision problem, (iii), y_{aj} solves the agricultural final goods firm decision problem, and (iv) the government balances its budget and markets clear:

1. Government budget balance: $\tau_x p_x^{market} \int_j x_j dj = \tau_m \int_j c_{mj} dj$.
2. Market clearing:
   (a) Agricultural intermediate goods market: for each j, $y_{aj} = x_j^{\alpha_j} n_j^{\eta_j}$
   (b) Agricultural final goods market: $\left(\int_j y_j^{\frac{\nu-1}{\nu}} dj\right)^{\frac{\nu}{\nu-1}} = \int_j c_{aj} dj$
   (c) The manufacturing labor market: $N_m = 1 - \int_j n_j dj$, which amounts to the condition that $w_a = w_m = A$.

C Proofs

C.1 Characterization of Frictionless Equilibrium and Proof of Lemma 1

Proof. The proof of Lemma 1 proceeds essentially by guess-and-verify. Specifically, we hypothesize that there exists some value $\mathcal{Y}$ such that the following results hold:

$$\frac{p^2_{ac}}{p^1_{ac}} = \frac{Y^1_a}{Y^2_a} = \mathcal{Y}$$

We then show that there is only one value for $\mathcal{Y}$ that satisfies the market clearing conditions and optimality conditions.

Constructing the Candidate $\mathcal{Y}$ From market clearing for village production

$$p_{aj} = \left(\frac{Y_a}{y_{aj}}\right)^{\frac{1}{\nu}} p_{ac}$$

Therefore,

$$\frac{p^2_{aj}}{p^1_{aj}} = \left(\frac{Y^2_a}{Y^1_a}\right)^{\frac{1}{\nu}} \left(\frac{y^2_{aj}}{y^1_{aj}}\right)^{\frac{-1}{\nu}} \left(\frac{p^2_{ac}}{p^1_{ac}}\right) = \mathcal{Y}^{\frac{\nu-1}{\nu}} \left(\frac{y^2_{aj}}{y^1_{aj}}\right)^{\frac{-1}{\nu}}. \tag{C.1}$$

Using this price ratio in combination with the village farm first order conditions implies

$$\frac{y^2_{aj}}{y^1_{aj}} = \left(\frac{p^2_{aj}}{p^1_{aj}}\right)^{\frac{\gamma}{1-\gamma}} \left(\frac{p^2_x}{p^1_x}\right)^{\frac{-\alpha_j}{1-\gamma}} = \mathcal{Y}^{\frac{(\nu-1)\gamma}{\nu(1-\gamma)}} \left(\frac{y^2_{aj}}{y^1_{aj}}\right)^{\frac{-\gamma}{\nu(1-\gamma)}} \left(\frac{p^2_x}{p^1_x}\right)^{\frac{-\alpha_j}{1-\gamma}}$$

which, re-arranging, gives

$$\frac{y^2_{aj}}{y^1_{aj}} = \mathcal{Y}^{\frac{(\nu-1)\gamma}{\nu-(\nu-1)\gamma}} \left(\frac{p^2_x}{p^1_x}\right)^{\frac{-\alpha_j\nu}{\nu-(\nu-1)\gamma}}. \tag{C.2}$$

and plugging this result into (C.1) gives the village price ratio of

$$\begin{aligned} \frac{p^2_{aj}}{p^1_{aj}} &= \mathcal{Y}^{\frac{\nu-1}{\nu}} \mathcal{Y}^{\frac{-(\nu-1)\gamma}{\nu(\nu-(\nu-1)\gamma)}} \left(\frac{p^2_x}{p^1_x}\right)^{\frac{\alpha_j}{\nu-(\nu-1)\gamma}} \\ &= \mathcal{Y}^{\frac{\nu-(\nu-1)\gamma-1}{\nu-(\nu-1)\gamma}} \left(\frac{p^2_x}{p^1_x}\right)^{\frac{\alpha_j}{\nu-(\nu-1)\gamma}} \end{aligned} \tag{C.3}$$

Aggregate to get total production

$$Y_2 = \left(\int y_{2j}^{\frac{\nu-1}{\nu}} dj \right)^{\frac{\nu}{\nu-1}} = \mathcal{Y}^{\frac{(\nu-1)\gamma}{\nu-(\nu-1)\gamma}} \left(\int_j \left(\frac{p_x^2}{p_x^1} \right)^{\frac{-\alpha_j(\nu-1)}{\nu-(\nu-1)\gamma}} y_{aj1}^{\frac{\nu-1}{\nu}} dj \right)^{\frac{\nu}{\nu-1}}.$$

Under the maintained hypothesis that $Y_{a2} = Y_{a1}\mathcal{Y}^{-1}$, then

$$\mathcal{Y}^{-1} \left(\int_j y_{a1j}^{\frac{\nu-1}{\nu}} dj \right)^{\frac{\nu}{\nu-1}} = \mathcal{Y}^{\frac{(\nu-1)\gamma}{\nu-(\nu-1)\gamma}} \left(\int_j \left(\frac{p_x^2}{p_x^1} \right)^{\frac{-\alpha_j(\nu-1)}{\nu-(\nu-1)\gamma}} y_{aj1}^{\frac{\nu-1}{\nu}} dj \right)^{\frac{\nu}{\nu-1}}$$

which can be re-arranged to solve for the candidate $\mathcal{Y}$

$$\mathcal{Y} = \left(\frac{\int_j y_{a1j}^{\frac{\nu-1}{\nu}} dj}{\int_j \left(\frac{p_x^2}{p_x^1} \right)^{\frac{-\alpha_j(\nu-1)}{\nu-(\nu-1)\gamma}} y_{aj1}^{\frac{\nu-1}{\nu}} dj} \right)^{\frac{\nu-(\nu-1)\gamma}{\nu-1}}. \tag{C.4}$$

The construction of $\mathcal{Y}$ in (C.4) uses two conditions. First, it uses the relevant farm first order conditions. Thus, it already satisfies the optimality conditions of the village farms. Second, it uses the market clearing condition from the stand-in firm that aggregates village production into the agricultural final good. Thus, it satisfies this market clearing condition by construction. The only remaining non-trivial market clearing condition is that of the demand and supply for the final agricultural good.

Confirming that Agricultural Market Clearing Condition is Satisfied Combining the characterizations of village output quantity and price from equations (C.2) and (C.3) gives farm revenue

$$\frac{p_{a2j} y_{a2j}}{p_{a1j} y_{a1j}} = \mathcal{Y}^{\frac{\nu-1}{\nu-(\nu-1)\gamma}} \left(\frac{p_x^2}{p_x^1} \right)^{\frac{\alpha_j(1-\nu)}{\nu-(\nu-1)\gamma}}$$

and therefore farm profit can be written as (note that this equation is used in the main text)

$$\begin{aligned}
\pi_{2j} &= (1-\gamma)p_{a2}y_{2j} \\
&= (1-\gamma)\mathcal{Y}^{\frac{\nu-1}{\nu-(\nu-1)\gamma}}\left(\frac{p_x^2}{p_x^1}\right)^{\frac{-\alpha_j(\nu-1)}{\nu-(\nu-1)\gamma}} p_{a1}y_{1j} \\
&= \underbrace{\left(\frac{\int_j y_{a1j}^{\frac{\nu-1}{\nu}}dj}{\int_j\left(\frac{p_x^2}{p_x^1}\right)^{\frac{-\alpha_j(\nu-1)}{\nu-(\nu-1)\gamma}} y_{aj1}^{\frac{\nu-1}{\nu}}dj}\right)\left(\frac{p_x^2}{p_x^1}\right)^{\frac{-\alpha_j(\nu-1)}{\nu-(\nu-1)\gamma}}}_{\equiv \text{ adjustment to } \pi_{1j}} \underbrace{(1-\gamma)p_{a1}y_{1j}}_{\equiv\ \pi_{1j}}
\end{aligned}$$

Using the fact that $p_{aj} = (Y_a/y_{aj})^{\frac{1}{\nu}}p_{ac}$, total baseline income is

$$\begin{aligned}
\int_j \pi_{j1}dj + w &= (1-\gamma)\int_j p_{a1j}y_{a1j}dj + w = (1-\gamma)Y_{a1}^{\frac{1}{\nu}}p_{ac1}\int y_{a1j}^{\frac{\nu-1}{\nu}}dj + w \\
&= (1-\gamma)p_{ac1}\Big(\int_j y_{a1j}^{\frac{\nu-1}{\nu}}\Big)^{\frac{\nu}{\nu-1}} + w = (1-\gamma)p_{a1c}Y_{a1} + w
\end{aligned}$$

which remains constant after the price shock

$$\begin{aligned}
\int \pi_{j2}dj + w &= (1-\gamma)\left(\frac{\int_j y_{a1j}^{\frac{\nu-1}{\nu}}dj}{\int_j\left(\frac{p_x^2}{p_x^1}\right)^{\frac{-\alpha_j(\nu-1)}{\nu-(\nu-1)\gamma}} y_{aj1}^{\frac{\nu-1}{\nu}}dj}\right)\left(\int_j\left(\frac{p_x^2}{p_x^1}\right)^{\frac{-\alpha_j(\nu-1)}{\nu-(\nu-1)\gamma}} p_{a1j}y_{1j}dj\right) + w \\
&= (1-\gamma)\left(\frac{\int_j y_{a1j}^{\frac{\nu-1}{\nu}}dj}{\int_j\left(\frac{p_x^2}{p_x^1}\right)^{\frac{-\alpha_j(\nu-1)}{\nu-(\nu-1)\gamma}} y_{aj1}^{\frac{\nu-1}{\nu}}dj}\right)\left(Y_{a1}^{\frac{1}{\nu}}p_{ac1}\int_j\left(\frac{p_x^2}{p_x^1}\right)^{\frac{-\alpha_j(\nu-1)}{\nu-(\nu-1)\gamma}} y_{1j}^{\frac{\nu-1}{\nu}}dj\right) + w \\
&= (1-\gamma)\left(\int_j y_{a1j}^{\frac{\nu-1}{\nu}}dj\right)Y_{a1}^{\frac{1}{\nu}}p_{ac1} + w \\
&= (1-\gamma)p_{a1c}Y_{a1} + w.
\end{aligned}$$

Total agricultural demand is therefore

$$D_{a2} = \frac{\zeta}{p_{ac2}}\left(\int_j \pi_{j2}dj + w\right) = \frac{\zeta}{\mathcal{Y}p_{a1c}}\left(\int_j \pi_{j1}dj + w\right) = \frac{D_{a1}}{\mathcal{Y}}.$$

Since $Y_{a2} = Y_{a1}/\mathcal{Y}$, this implies $Y_{a2} = D_{a2}$ as required. ■

C.2 Proof of Proposition 1

Proof. This result is proved in the text. ■

C.3 Proof of Proposition 2

Proof. The first part of the proof requires characterizing the new equilibrium as a function of the baseline equilibrium, and we therefore proceed in a nearly identical manner to Lemma 1. Specifically, define

$$\mathcal{Y}^c = \left(\frac{\int_j y_{a1j}^{\frac{\nu-1}{\nu}} dj}{\int_j \left(\frac{\frac{p_x^2}{p_x^1}}{}\right)^{\frac{-\alpha_j(\nu-1)}{\nu}} y_{aj1}^{\frac{\nu-1}{\nu}} dj} \right)^{\frac{\nu}{\nu-1}}.$$

Following an identical methodology to the proof of Lemma 1, this term characterizes the new equilibrium as a function of the baseline one. Specifically important for this proof is that

$$\frac{p_{aj}^2}{p_{aj}^1} = \left(\frac{Y_a^2}{Y_a^1}\right)^{\frac{1}{\nu}} \left(\frac{y_{aj}^2}{y_{aj}^1}\right)^{\frac{-1}{\nu}} \left(\frac{p_{ac}^2}{p_{ac}^1}\right) = \mathcal{Y}^{\frac{-1}{\nu}} \mathcal{Y} \left(\frac{y_{aj}^2}{y_{aj}^1}\right)^{\frac{-1}{\nu}} = \mathcal{Y}^{\frac{\nu-1}{\nu}} \left(\frac{y_{aj}^2}{y_{aj}^1}\right)^{\frac{-1}{\nu}}$$

Plugging in

$$y_{aj}^c = \frac{z_j \phi_j^\gamma}{\gamma^\gamma} \left(\frac{\alpha_j}{p_x}\right)^{\alpha_j} \left(\frac{\eta_j}{w}\right)^{\eta_j}$$

gives

$$\frac{p_{aj}^2}{p_{aj}^1} = \mathcal{Y}^{\frac{\nu-1}{\nu}} \left(\frac{p_{x2}}{p_{x1}}\right)^{\frac{\alpha_j}{\nu}}.$$

The village j shadow price of fertilizer at time t is

$$1 + \lambda_{jt} = \frac{\alpha_j p_{aj} z_j x^{\alpha_j - 1} n_a^{\eta_j}}{p_{xt}}.$$

Using the relevant input choices and the prices,

$$\Delta_t \log(1+\lambda_j) = \Delta_t \log(p_{aj}) - \alpha_j \Delta_t \log(p_x) = \frac{\nu-1}{\nu} \log(\mathcal{Y}) + \frac{\alpha_j(1-\nu)}{\nu} \Delta_t \log(p_x).$$

Taking two derivatives gives the result

$$\frac{\partial^2 \Delta_t \log(1+\lambda_j)}{\partial \Delta_t \log(p_x) \partial \alpha_j} = \frac{1-\nu}{\nu}.$$

■

C.4 Proof of Proposition 3

C.4.1 An Additional Lemma

Before proving this result, it is helpful to characterize a few results from the frictional equilibrium. We do those in Lemma 2.

Lemma 2. *Two results are true in any equilibrium of the frictional economy.*

1. *Total economy-wide expenditures* $E := \int_j \pi_j dj + A$ *do not depend on the market price* p_x^{market} *or the subsidy* τ_x.
2. *The budget-balancing manufacturing tax* τ_m *does not depend on market price* p_x^{market}.

Proof.

Claim 1 Denoting $e_j = \pi_j + A$ as total expenditures in village j and $E = \int_j e_j dj$ as total economy-wide expenditures, the market clearing condition for agricultural consumption implies

$$p_{ac} = \frac{\zeta E}{Y_a}.$$

This means the village j output price is

$$p_{aj} = \left(\frac{Y_a}{y_{aj}}\right)^{\frac{1}{\nu}} p_{ac} = Y_a^{\frac{1-\nu}{\nu}} y_{aj}^{\frac{-1}{\nu}} \zeta E.$$

Total economy-wide expenditures are therefore

$$\begin{aligned} E &= \int_j \left[(Y_a^{\frac{1-\nu}{\nu}} y_{aj}^{\frac{-1}{\nu}} \zeta E) y_{aj} - \phi_j + A \right] dj \\ &= Y_a^{\frac{1-\nu}{\nu}} \left(\int_j y_{aj}^{\frac{\nu-1}{\nu}} dj \right) \zeta E - \mathbb{E}[\phi] + A \\ &= Y_a^{\frac{1-\nu}{\nu}} Y_a^{\frac{\nu-1}{\nu}} \zeta E - \mathbb{E}[\phi] + A \\ &= \zeta E - \mathbb{E}[\phi] + A \end{aligned}$$

which implies $E = \frac{A - \mathbb{E}[\phi]}{1-\zeta}$, which is independent of the fertilizer price and subsidy.

Claim 2 The government's budget constraint is

$$\tau_x p_x^{\text{market}} \int_j x_j dj = \tau_m \int_j c_{mj} dj.$$

Optimality conditions imply

$$\begin{aligned} x_j &= \frac{\alpha_j \phi_j}{\gamma p_x^{\text{market}} (1-\tau_x)} \\ c_{mj} &= \frac{(1-\zeta) e_j}{1+\tau_m} \end{aligned}$$

where $e_j := \pi_j + A$ is total expenditures in village j. This implies that the government budget constraint is

$$\left(\frac{\tau_x}{1-\tau_x}\right) \frac{1}{\gamma} \Big(\int_j \alpha_j \phi_j dj \Big) = \frac{\tau_m}{1+\tau_m} (1-\zeta) E$$

where $E = \int_j \pi_j dj + A$ is economy-wide expenditures. By the first result that E does not depend on p_x^{market}, this proves the second claim. ■

C.4.2 Proof of Proposition 3

Proof. Denoting $e_j = \pi_j + w$ as the expenditures of village j, we can write the consumption decisions $p_{ac} c_a = \zeta e_j$ and $(1+\tau_m) c_m = (1-\zeta) e_j$ which gives the indirect utililty function for village j,

$$\Omega - \zeta \log(p_{ac}) + (\zeta - 1) \log(1+\tau_m) + \log(e_j)$$

with constant Ω. Manipulating the agricultural consumption market clearing condition implies

$$p_{ac} = \frac{\zeta E}{Y_a}$$

where $E := \int_j e_j dj$ is total expenditures. The utilitarian government problem is therefore

$$S := \max_{\tau_x} \; e^{\tilde{\Omega}} Y_a^{\zeta} (1+\tau_m)^{\zeta - 1} E^{1-\zeta}$$

or

$$\log(S) = \max_{\tau_x} \; \tilde{\Omega} + \zeta \log(Y_a) + (\zeta - 1) \log(1+\tau_m) + (1-\zeta) \log(E).$$

where $\tilde{\Omega} := \Omega - \zeta \log(\zeta)$. By Lemma 2, E does not respond to τ_x. Therefore, the first order condition is

$$\zeta \frac{\partial \log(Y_a)}{\partial \tau_x} + (\zeta - 1) \frac{\partial \log(1 + \tau_m)}{\partial \tau_x} = 0. \tag{C.5}$$

By the implicit function theorem, we are interested in

$$\frac{\partial \tau_x^*}{\partial p_x^{\text{market}}} = -\frac{\frac{\partial^2 \log(S)}{\partial \tau_x \partial p_x^{\text{market}}}}{\frac{\partial^2 \log(S)}{\partial \tau_x^2}}$$

Since $\log(S)$ is concave,

$$\text{sign}\left(\frac{\partial \tau_x^*}{\partial p_x^{\text{market}}}\right) = \text{sign}\left(\frac{\partial^2 \log(S)}{\partial \tau_x \partial p_x^{\text{market}}}\right). \tag{C.6}$$

Taking the derivative of (C.5) and using the results of Lemma 2 to note that $\partial \tau_m / \partial p_x^{\text{market}} = 0$, the sign restriction in (C.6) is equivalent to

$$\text{sign}\left(\frac{\partial \tau_x^*}{\partial p_x^{\text{market}}}\right) = \text{sign}\left(\frac{\partial^2 \log(Y_a)}{\partial \tau_x \partial p_x^{\text{market}}}\right). \tag{C.7}$$

Since

$$y_{aj} = \frac{\phi_j}{\gamma^\gamma} \left(\frac{\alpha_j}{p_x^{\text{market}}(1 - \tau_x)}\right)^{\alpha_j} \left(\frac{\eta_j}{A}\right)^{\eta_j},$$

then

$$\log(Y) = \left(\frac{\nu}{\nu - 1}\right) \log\left(\int_j \left[\Delta_j (p_x^{\text{market}})^{-\alpha_j} (1 - \tau_x)^{-\alpha_j}\right]^{\frac{\nu-1}{\nu}} dj\right)$$

where Δ_j summarizes the remaining terms from y_{aj} that do not include p_x^{market} or τ_x. Taking the derivative with respect to the market fertilizer price,

$$\frac{\partial Y_a}{\partial p_x} = \frac{-1}{p_x^{\text{market}}} \int_j \alpha_j \frac{\left[\Delta_j (p_x^{\text{market}})^{-\alpha_j} (1 - \tau_x)^{-\alpha_j}\right]^{\frac{\nu-1}{\nu}}}{\int_k \left[\Delta_k (p_x^{\text{market}})^{-\alpha_k} (1 - \tau_x)^{-\alpha_k}\right]^{\frac{\nu-1}{\nu}} dk}.$$

Define the probability measure

$$\Gamma_j = \frac{\left[\Delta_j (p_x^{\text{market}})^{-\alpha_j} (1 - \tau_x)^{-\alpha_j}\right]^{\frac{\nu-1}{\nu}}}{\int_k \left[\Delta_k (p_x^{\text{market}})^{-\alpha_k} (1 - \tau_x)^{-\alpha_k}\right]^{\frac{\nu-1}{\nu}} dk}$$

so that we can re-write

$$\frac{\partial Y_a}{\partial p_x} = \frac{-\mathbb{E}_\Gamma[\alpha; \tau_x]}{p_x^{\text{market}}}$$

where $\mathbb{E}_\Gamma$ is the expectation of α taken using Γ, which depends (in part) on τ_x. This implies

$$\operatorname{sign}\left(\frac{\partial^2 \log(Y_a)}{\partial \tau_x \partial p_x^{\text{market}}}\right) = -\operatorname{sign}\left(\frac{\partial \mathbb{E}_\Gamma[\alpha;\tau_x]}{\partial \tau_x}\right). \tag{C.8}$$

Note that

$$\frac{\partial \log(\Gamma_j/\Gamma_k)}{\partial \tau_x} = \frac{\nu-1}{\nu}(\alpha_j - \alpha_k)\left(\frac{1}{1-\tau_x}\right).$$

If $\nu > 1$, this is positive for $\alpha_j > \alpha_k$, implying that weight is shifting toward higher α, and therefore $\partial \mathbb{E}_\Gamma[\alpha;\tau_x]/\partial \tau_x > 0$. (C.8) therefore implies $\frac{\partial^2 \log(Y_a)}{\partial \tau_x \partial p_x^{\text{market}}} < 0$. The same exercise gives the required result that $\frac{\partial^2 \log(Y_a)}{\partial \tau_x \partial p_x^{\text{market}}} = 0$ when $\nu = 1$ and $\frac{\partial^2 \log(Y_a)}{\partial \tau_x \partial p_x^{\text{market}}} > 0$ when $\nu < 1$. By (C.7), $\partial \tau_x^*/\partial p_x$ shares the same signs.

■