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INFLATION DISAGREEMENT WEAKENS THE POWER OF MONETARY POLICY

DING DONG, ZHENG LIU, PENGFEI WANG, AND MIN WEI

Abstract. Household inflation disagreement weakens the impact of forward guidance and monetary policy shocks, especially when inflation forecasts are positively skewed. This attenuation effect is not driven by endogenous responses of inflation disagreement to contemporaneous shocks. A model with heterogeneous beliefs about the central bank's inflation target explains these observations. Agents expecting higher future inflation perceive lower real interest rates and borrow more, constrained by borrowing limits. Increased inflation disagreement results in more borrowing-constrained agents, leading to slower aggregate consumption responses to interest rate changes. This mechanism also provides a microeconomic foundation for Euler equation discounting, helping to resolve the forward guidance puzzle.

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Key words and phrases. Inflation disagreement; inflation expectations; heterogeneous beliefs; borrowing constraints; monetary policy transmission; forward guidance puzzle.

JEL classification: E21, E31, E52, E71.

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I. Introduction

Households often disagree in their inflation outlooks (Mankiw et al., 2003; Andrade et al., 2016; Weber et al., 2022; Fofana et al., 2024). For example, Figure 1 shows the cross-sectional distribution of households' expectations of one-year ahead changes in the consumer price index, or CPI, from the University of Michigan Survey of Consumers in June 2023 (left panel). Those inflation expectations vary from 0% to over 20%, with a median of 3.3%. Consumers disagree not just in their inflation forecasts, but also in their perceived inflation target of the Federal Reserve. The right panel of Figure 1 shows the distribution of consumers' perceived inflation target of the FOMC.[1] Despite the Fed's frequent communications of its 2% inflation goal, consumers' perceived inflation target ranges from 0% to over 9%, with a median of 3.0%. These observations illustrate pervasive inflation disagreement among consumers.

Households' inflation disagreement is also time-varying. Figure 2 shows the inter-quartile range (IQR)—the difference between the 75th percentile and the 25th percentile of the distribution—of inflation expectations over both the one-year (in red) and the five-to-ten-year (in blue) horizons from the Michigan survey. Inflation disagreement fluctuates over time, with occasional spikes such as those during the 2008-2009 global financial crisis and in the post-pandemic period.

Studies have shown that inflation expectations are important for the transmission of monetary policy (Orphanides and Williams, 2004; Galí, 2015; Gargiulo et al., 2024). However, less is known about the role of inflation disagreement. This paper examines how inflation disagreement affects the transmission of monetary policy, both empirically and theoretically, focusing on households' inflation disagreement.

We examine the empirical importance of inflation disagreement for the transmission of monetary policy using the method of local projections *à la* Jordà (2005). In particular, we estimate the effects of a monetary policy shock on real activity and inflation, both on average and during periods with high inflation disagreement. We measure inflation disagreement using the IQR of inflation forecasts over the one-year horizon from the Michigan survey, normalized by the median of inflation forecasts.[2] We consider two types of monetary policy shocks, a forward guidance (FG) shock and a shock to the federal funds rate (FFR),

[1] The underlying data for the distribution of consumers' perceived inflation target are provided by Pfajfar and Winkler (2024), who conducted a special survey in June 2023 as a part of the Survey of Consumer Expectations (SCE) of the Federal Reserve Bank of New York.

[2] We normalize by median inflation expectations to control for a mechanical rise in inflation disagreement due to rising inflation expectations. The results are even stronger when we use the raw IQR of inflation forecasts (without normalization) to measure inflation disagreement.

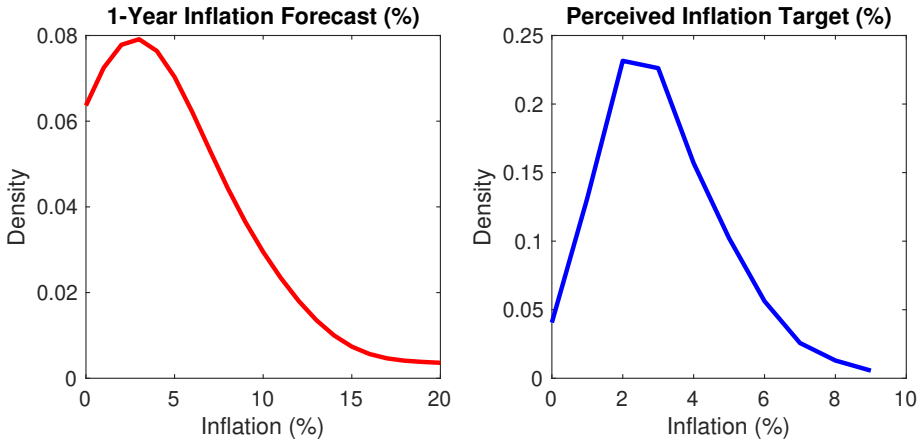


FIGURE 1. Cross-sectional distribution of inflation beliefs.

Note: This figure shows the cross-sectional distribution of inflation beliefs. In each panel, the horizontal axis shows the inflation rate (in percent). The left panel (red line) plots the kernel density of one-year ahead CPI inflation expectations from the Michigan Survey of Consumers in June 2023. The mean, the median, and the IQR of the inflation expectations are 5.2, 3.3, and 6.1 percent, respectively. The right panel (blue line) plots the kernel density of the consumers' perceived inflation target of the FOMC (with values of perceived inflation target above 9% trimmed). The mean, the median, and the standard deviation of the perceived inflation target are 2.9, 3.0, and 1.2 respectively. The data are taken from a special survey conducted by Pfajfar and Winkler (2024) in June 2023 as a part of the Survey of Consumer Expectations of the Federal Reserve Bank of New York.

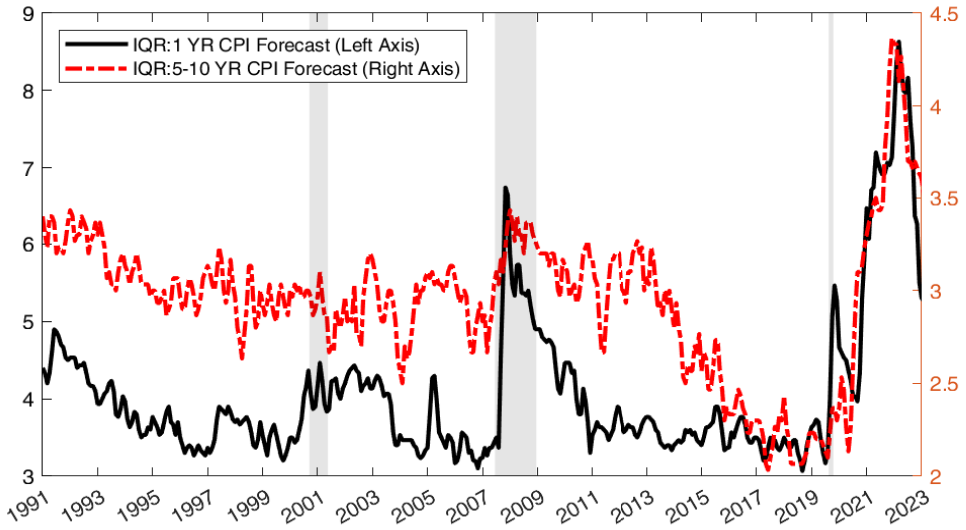


FIGURE 2. Time-varying inflation forecast dispersion from the Michigan Survey of Consumers.

Note: This figure shows the time series of inflation disagreement, measured by the IQR (i.e., the differences between the 75th percentile and the 25th percentile) of CPI inflation forecasts over the one-year horizon from Michigan Survey of Consumers (red line, right axis). The mean, persistence and standard deviation of this time series from July 1991 to December 2023 are 4.09, 0.89 and 1.05 respectively. The series is highly correlated with IQR of inflation forecasts over the five-to-ten-year horizon (blue line, left axis), with a correlation coefficient of 0.60.

both constructed and updated by Swanson (2021) based on high-frequency changes in interest rates around FOMC announcements. Following the approach of Bauer and Swanson (2023b) and Swanson (2024), we isolate the exogenous component of these policy surprises by orthogonalizing them with respect to publicly available information before the FOMC announcements.[3] Our sample covers the period from July 1991 to December 2023.

We find that, absent inflation disagreement, a forward guidance shock that signals a future tightening of monetary policy leads to persistent declines in both consumption and inflation. However, these effects are substantially attenuated in periods with high inflation disagreement. We find similar attenuating effects of inflation disagreement on the power of federal funds rate shocks. The attenuation effects of inflation disagreement are robust to alternative measures of real activity, inflation, monetary policy shocks, and inflation disagreement. They are also robust to controlling for other potential confounding factors (one at a time) and their interactions with the policy shocks.[4]

We conduct a few exercises to shed light on what might be driving the attenuation effect of inflation disagreement. The first exercise shows that, for a given level of inflation disagreement, the attenuation effect is stronger when the distribution of inflation forecasts is more positively skewed. This evidence points to an outsize role of inflation forecasts in the upper tail, a feature we try to capture in our theoretical model. We also try to isolate the exogenous effect of inflation disagreement on monetary policy transmission in two ways. First, we examine how common monetary policy shocks are transmitted differently to geographic regions with different levels of inflation disagreement. We find that a tightening of monetary policy (through either FG or FFR) leads to smaller increases in unemployment and smaller declines in inflation in regions with higher inflation disagreement. Second, we zoom in on a component of inflation disagreement that arises from the different lifetime inflation experiences of different age cohorts among Michigan survey respondents, a component that is arguably orthogonal to contemporaneous shocks (Malmendier and Nagel, 2016; Nagel, 2024).[5] Repeating our baseline analysis using this experience-based measure of inflation disagreement, we obtain impulse responses that are similar to those obtained from our baseline estimation.

[3]Following the earlier studies of Kuttner (2001) and Gürkaynak et al. (2005), many studies have used high-frequency changes in interest rates around the FOMC policy announcements to identify the effects of monetary policy. Examples include Cochrane and Piazzesi (2002); Faust et al. (2004); Gertler and Karadi (2015); Ramey (2016); Nakamura and Steinsson (2018); Stock and Watson (2018); Bauer and Swanson (2023a,b).

[4]The additional controls that we considered include mean inflation expectations, inflation uncertainty, income growth expectations, income growth disagreement, consumer uncertainty, interest rate disagreement, and inflation disagreement of professional forecasters.

[5]The data is from Nagel (2024), kindly shared by Stefan Nagel.

Taken together, this set of evidence suggests that inflation disagreement, especially if driven by the upper tail of inflation forecasts, weakens monetary policy transmission and that this effect is not driven by endogenous responses of inflation disagreement to other shocks to the economy.

To further explore the mechanism behind our findings that inflation disagreement weakens the power of monetary policy, we generalize a standard New Keynesian model to incorporate belief heterogeneity and borrowing constraints. In the model economy, the central bank has a particular inflation target. However, different agents hold different beliefs about that target, reflecting, for example, imperfect credibility of the central bank or individual inattention to monetary policy. With a commonly observed nominal interest rate, an agent who perceives a higher inflation target in turn perceives a lower real interest rate, causing their marginal propensity to consume (MPC) to rise. High-MPC agents finance consumption using both internal funds and external debt, subject to a borrowing constraint. When inflation beliefs become more dispersed, a greater mass of agents would hold beliefs that lie at the upper tail of the belief distribution. Those high-MPC agents borrow to consume and, once they hit the borrowing limit, they can no longer adjust consumption spending freely in response to exogenous shocks.[6] Thus, with greater dispersion of inflation beliefs—or equivalently, with greater inflation disagreement—more agents will become borrowing-constrained, causing aggregate consumption to adjust less than one-for-one to changes in expected future consumption or the real interest rate, akin to models with a discounted Euler equation (e.g. Del Negro et al. (2023)).

In line with the empirical evidence, our model predicts that inflation disagreement attenuates the effects of forward guidance policy on consumption spending. Absent inflation disagreement, the Euler equation in our model coincides with that in the standard model with no discounting. In that case, a decline in the real interest rate in arbitrarily distant future would have the same stimulative effect on current consumption as does a decline in the current real interest rate, giving rise to the forward-guidance puzzle (Del Negro et al., 2023; McKay et al., 2016). In the more general case with inflation disagreement, however, current consumption responds to expected future consumption less than one-for-one. Furthermore, since agents with inflation beliefs on the upper tail face binding borrowing constraints, positive skewness of the inflation beliefs strengthens the attenuation effect, consistent with our empirical evidence.

[6]Higher inflation disagreement also implies a larger share of low-MPC agents, whose beliefs about the inflation target lie in the lower tail of the belief distribution. However, those agents can adjust consumption optimally in response to shocks because they are unconstrained.

In our model, inflation disagreement not only reduces the sensitivity of current consumption to changes in future consumption, but also reduces the sensitivity of consumption to changes in the contemporaneous interest rate. Thus, consistent with the empirical evidence, our model predicts that higher inflation disagreement also leads to more muted effects of conventional interest rate policy shocks.

II. Related Literature

Our paper contributes to the literature on heterogeneity in household inflation expectations (Mankiw et al., 2003; Andrade et al., 2016; Coibion et al., 2020; Ropele et al., 2024). Pervasive inflation disagreement has stimulated much interest in recent empirical studies, most of which focus on understanding potential drivers of such disagreement. For example, Lahiri and Sheng (2008) and Ahn and Farmer (2024) decompose disagreement about inflation expectations into various sources: prior beliefs, responses to common information, or idiosyncratic information. Fofana et al. (2024) find that inflation disagreement can be driven by demographic factors, such as age, sex, marital status, income, and education, but also responds to aggregate shocks to monetary policy and to supply and demand conditions.

Our paper has a different focus. We are interested in studying how household inflation disagreement affects the transmission of monetary policy, including forward guidance and the conventional interest rate policy. In this aspect, our study is closely related to Falck et al. (2021), who examine the implications of inflation disagreement among professional forecasters for the transmission of conventional monetary policy shocks. Our work is also related to Barbera et al. (2023), who decompose professional inflation disagreement into disagreement about trend inflation and about cyclical inflation and find that cyclical inflation disagreement weakens the responses of asset prices to conventional monetary policy shocks, while disagreement about trend inflation does not.[7] We differ from those papers along several dimensions. First, we focus on household rather than professional inflation disagreement. Second, we show that households' inflation disagreement attenuates the responses of macro variables not only to conventional interest rate policy shocks but also to forward guidance shocks, and this is true even after we control for professional inflation disagreement. Third, we show that the attenuation effect holds not just in the aggregate but also across geographic areas. Importantly, we obtain similar attenuation effects when we measure inflation disagreement based on individual forecasters' life experiences, suggesting that the effects are not driven by endogenous responses of inflation disagreement to contemporaneous shocks to

[7]Kwak et al. (2024) use Korean data and find that inflation disagreement among professional forecasters weakens the effects of monetary policy in that country.

the economy. Lastly, our paper develops a theoretical framework that can rationalize those empirical findings.

Our work also contributes to the literature on the forward guidance puzzle, which has been a challenge to the standard New Keynesian framework. In the standard New Keynesian models with rational expectations, forward guidance policy that promises changes in interest rates in the distant future would have implausibly large effects on output and inflation relative to the effects of shocks to the current interest rate (Del Negro et al., 2023; Hagedorn et al., 2019). Previous studies have proposed potential resolutions of the forward guidance puzzle in a representative-agent framework by introducing information frictions (Angeletos and Lian, 2018), bounded rationality (Farhi and Werning, 2019; Gabaix, 2020), imperfect central bank credibility (Andrade et al., 2019; Campbell et al., 2019), or the extensive margin of durable goods purchases (McKay and Wieland, 2022). In our model, the presence of heterogeneous beliefs about the central bank's inflation target, together with borrowing constraints, provides an alternative mechanism for Euler equation discounting that helps resolve the forward guidance puzzle.

Our model mechanism is complementary to that in the heterogeneous-agent New Keyesian (HANK) framework. In an important contribution, McKay et al. (2016) study a HANK model with incomplete markets, where agents face uninsurable income risks and liquidity constraints. They show that a precautionary-savings effect partially offsets the intertemporal substitution effects, dampening the responses of current consumption to changes in future interest rates and therefore helps resolve the forward guidance puzzle (see also McKay et al. (2017)). Werning (2015) argues that the precautionary-savings channel may depend on the assumptions about the cyclicality of idiosyncratic income risks and liquidity. If idiosyncratic income risks are countercyclical or if liquidity relative to income is procyclical, forward guidance policies would be as powerful as in representative agent models. Our model generates heterogeneity in MPCs and Euler-equation discounting through a different mechanism. In our model with inflation disagreement, agents with higher inflation expectations have lower perceived real interest rates and thus are more likely to be borrowing constrained. Greater inflation disagreement results in a larger share of borrowing-constrained agents and thus more sluggish adjustments in aggregate consumption in response to forward-guidance shocks.

Our model highlights the importance of households' debt capacity for the transmission of monetary policy. A monetary policy easing can effectively stimulate consumption spending only if households with high MPC have access to unused debt capacity. However, as pointed out by Sufi (2015), this credit extension channel was extraordinarily weak after the 2008-09 global financial crisis, rendering monetary policy ineffective during that period (see also Beraja et al. (2019)). Our model captures the essence of this "limited credit access" channel

in accounting for the ineffectiveness of monetary policy. In our model, the households with higher inflation expectations have higher MPC and they are more likely to face binding borrowing constraints. They cannot further adjust their borrowing or spending upward even when monetary policy reduces the current or expected interest rates.[8] We show that limited credit access—measured by the net percentage of tightening of lending standards in consumer loans from the Federal Reserve's Senior Loan Officer Opinion Survey (SLOOS)—does reinforce the attenuation effects of inflation disagreement.

Our model implies a positive relation between inflation expectations and current consumption spending at the individual household level. This implication is consistent with empirical evidence. One strand of this literature looks at household survey responses and shows that there is a positive correlation between household inflation expectations and their willingness to spend, at least for highly educated respondents or respondents with high cognitive skills (Vellekoop and Wiederholt (2019), Bachmann et al. (2015), D'Acunto et al. (2023) and Andrade et al. (2023)). However, it is difficult to establish causal effects of changes in inflation expectations on consumption spending using survey data alone. By exploiting a quasi-natural experiment in Germany and using a difference-in-differences approach, D'Acunto et al. (2021) document evidence that the announcement of value-added tax increases in 2005, to be implemented in 2007, raised German consumers' inflation expectations, leading to an immediate increase in consumers' readiness to buy durable goods. Coibion et al. (2022) use a range of randomized information treatments in a large-scale survey of U.S. households to study how different types of communications affect consumers' inflation expectations and ultimately their spending decisions. They find that higher inflation expectations arising from information treatments lead to a rise in household spending on non-durable goods, although not on durable goods, over the next 6 months.

III. Empirical Evidence

III.1. **Baseline empirical model and results.** We examine how inflation disagreement affects the transmission of monetary policy shocks by estimating the following local projections specification in the spirit of Jordà (2005)

$$\log(y^j_{t+h}) - \log(y^j_{t-1}) = \alpha^h_0 + \alpha^h_1 MP_t + \alpha^h_2 IQR^{\pi}_{t-1} + \alpha^h_3 IQR^{\pi}_{t-1} * MP_t + \alpha^h_4 \Gamma_{t-1} + \varepsilon_{t+h}, \quad (1)$$

[8]Some empirical studies find that consumption spending of more indebted households is more responsive to interest rate changes (Cloyne et al., 2020; Cumming and Hubert, 2023). This evidence does not necessarily contradict our model's implication or the empirical evidence in Sufi (2015) and Beraja et al. (2019) that limited debt capacity can dampen the effects of monetary policy shocks. If indebted households have unused debt capacity, lowering interest rates might relax their borrowing constraints and boost their consumption spending. However, such effects would be muted if these households have limited debt capacity.

where $h = 0, 1, 2, ..., 48$ denotes the projection horizon (in months). The dependent variable $\log(y^j_{t+h}) - \log(y^j_{t-1})$ measures cumulative changes in the log level of real personal consumption expenditures (PCE, $j = 1$) or the PCE price index ($j = 2$) from the pre-shock period $(t-1)$ to h-periods after the shock $(t+h)$. The term MP_t denotes a monetary policy shock, which can be a shock to either forward guidance (FG) or the federal funds rate (FFR), which are constructed by Swanson (2021) based on high-frequency changes in interest rates around FOMC announcements. Evidence suggests that raw measures of high-frequency monetary policy surprises, such as those constructed by Swanson (2021), are predictable by lagged macroeconomic news or financial variables that are observed prior to FOMC announcements, reflecting the "Fed information effect" (Nakamura and Steinsson, 2018; Miranda-Agrippino and Ricco, 2021) or the "Fed response to news effect" (Bauer and Swanson, 2023a). To purge those effects, we follow the approach of Swanson (2024), first regressing the policy surprises (FFR or FG) on the same set of predicting variables used in his study and then using the regression residuals as a measure of orthogonalized policy shocks.[9] Inflation disagreement, denoted as IQR^{π}_{t-1}, is the interquartile range of one-year ahead forecasts of CPI inflation from the Michigan Survey of Consumers, normalized by the median of the inflation forecasts. We lag inflation disagreement by one month in the regression to avoid complications from potential endogeneity of inflation forecast dispersion.[10] The term Γ_{t-1} denotes a set of lagged macroeconomic control variables, including the log growth rates of real PCE and industrial production, the PCE inflation rate (12-month percentage changes in the PCE price index), the unemployment rate, and the shadow federal funds rate constructed by Wu and

[9]The set of predictors for monetary policy surprises includes four macroeconomic variables (the most recent surprise in the nonfarm payrolls release, the unemployment rate release, the GDP release, and the core CPI release), nine financial variables (the percent change in the S&P 500 stock index from 3 months before the monetary policy announcement to the day before the monetary policy announcement, the change in the Wu-Xia (2016) shadow federal funds rate, 2-year Treasury yield and 10-year Treasury yield over the same 3-month window, the log change in the Bloomberg Commodity Spot Price index from three months before the FOMC announcement to the day before the announcement, the implied skewness of the ten-year Treasury yield, and the change in the Baa-Treasury spread and percent change in commodity prices over the same 3-month window, and the one-month change in the Chicago Fed's National Financial Conditions Index), and two lagged values of the left-hand side variables. We are grateful to Michael Bauer and Eric Swanson for sharing these data.

[10]In our baseline regressions, we focus on inflation disagreement based on one-year ahead inflation forecasts. In the data, these short-term inflation forecasts are highly correlated with longer-term forecasts (Andrade et al., 2016; Weber et al., 2022). Our results are robust to using long-term inflation forecasts to measure inflation disagreement, as we show in Appendix A.1.1.

TABLE 1. Summary statistics

Variable	Mean	SD	AutoCorr
Inflation Disagreement	1.39	0.50	0.55
Real PCE Growth (month-over-month, log changes, %)	0.23	0.94	0.08
PCE Inflation (month-over-month, log changes, %)	0.17	0.20	0.47
Orthogonalized FG Shocks	0.00	0.87	-0.05
Orthogonalized FFR Shocks	0.02	0.75	-0.07
Shadow Federal Funds Rate	2.18	2.62	0.99

Note: The monthly sample covers the period from July 1991 to December 2023.

Xia (2016). The term ε_{t+h} denotes the regression residuals. The monthly sample covers the period from July 1991 to December 2023.[11]

The key parameters of interest in the local projections specification (1) are α_1^h and α_3^h. The coefficient α_1^h captures the effects of a monetary policy shock on the macroeconomic variable of interest in the absence of inflation disagreement. The coefficient α_3^h captures the marginal effects of the monetary policy shock as inflation disagreement increases. If α_1^h and α_3^h have opposite signs, high inflation disagreement would weaken the effect of monetary policy shocks.

Table 1 presents the summary statistics of the variables used in our regressions. There is substantial inflation disagreement in our sample. Our measure of inflation disagreement has a mean of 1.39, implying that the IQR of one-year ahead CPI inflation forecasts is modestly above the median forecasts. Inflation disagreement is also modestly persistent and volatile, with a first-order autocorrelation of 0.55 and a standard deviation of 0.50. The FG and FFR shocks from Swanson (2024) are both highly volatile, close to being i.i.d., and have little persistence. By construction, those two types of shocks are uncorrelated.

We first consider the impulse responses to a one-standard-deviation FG shock that signals a future tightening of monetary policy.[12] The upper panels of Figure 3 show that such a shock would reduce real consumption absent inflation disagreement ($\alpha_1^h < 0$), but the effect is attenuated when inflation disagreement increases ($\alpha_3^h > 0$). The point estimates imply that, absent inflation disagreement, a one-standard-deviation tightening FG shock would

[11]The results are qualitatively and quantitatively similar when we use the pre-COVID sample (not reported).

[12]Swanson (2021) shows that a one-standard-deviation tightening FG shock would raise the two-year Treasury yield by about 4.6 basis points on average.

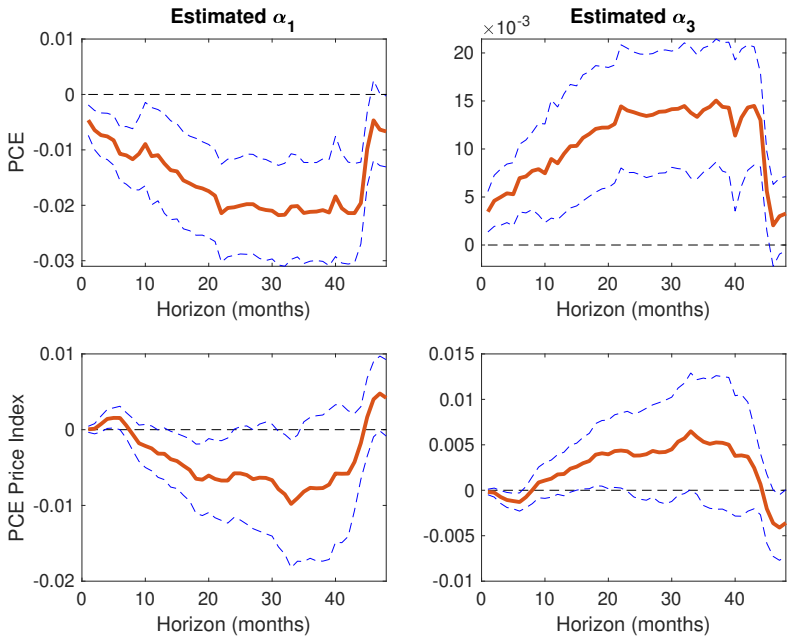


FIGURE 3. Cumulative impulse responses to a forward guidance shock

Note: This figure shows the cumulative impulse responses of real personal consumption expenditure (PCE, top panels) and the PCE price index (bottom panels) to a forward guidance shock estimated using local projections (1). The solid lines show the point estimates of the impulse responses. The dashed lines show the 68% confidence intervals based on a Newey-West heteroskedasticity and autocorrelation consistent (HAC) estimator.

reduce real consumption by 1.74 percent cumulatively over a period of two years, but this effect would be 35% smaller if inflation disagreement increases by one standard deviation.[13]

The lower panels of Figure 3 shows that the FG shock—in the absence of inflation disagreement—also reduces the price level (i.e., $\alpha_1^h < 0$), but the effects are partially blunted by positive inflation disagreement (i.e. $\alpha_3^h > 0$). The point estimates imply that, without inflation disagreement, a one-standard-deviation FG shock would lead to a cumulative decline in the PCE price index of about 0.5 percent over a two-year period ($h = 24$), but the effect would be about 33% smaller with a one-standard-deviation increase in inflation disagreement.[14]

[13]From the summary statistics presented in Table 1, the FG shock has a standard deviation of 0.87 and the inflation disagreement measure has a standard deviation of 0.50. The point estimate of $\alpha_1 = -0.020$ at the two-year horizon (i.e., $h = 24$) implies that a one-standard-deviation shock to FG reduces consumption by $0.020 \times 0.87 \times 100 = 1.74\%$ absent inflation disagreement. The point estimate of $\alpha_3 = 0.014$ at the two-year horizon implies that a one-standard-deviation increase in inflation disagreement attenuates the negative effect of FG on consumption by $(0.014 \times 0.50)/0.020 \times 100 = 35\%$.

[14]The point estimate of $\alpha_1^{h=24} = -0.0058$ implies that a one-standard-deviation shock to FG would reduce the PCE price index by $-0.0058 \times 0.87 \times 100 = 0.50\%$ in periods with no inflation disagreement. The estimated

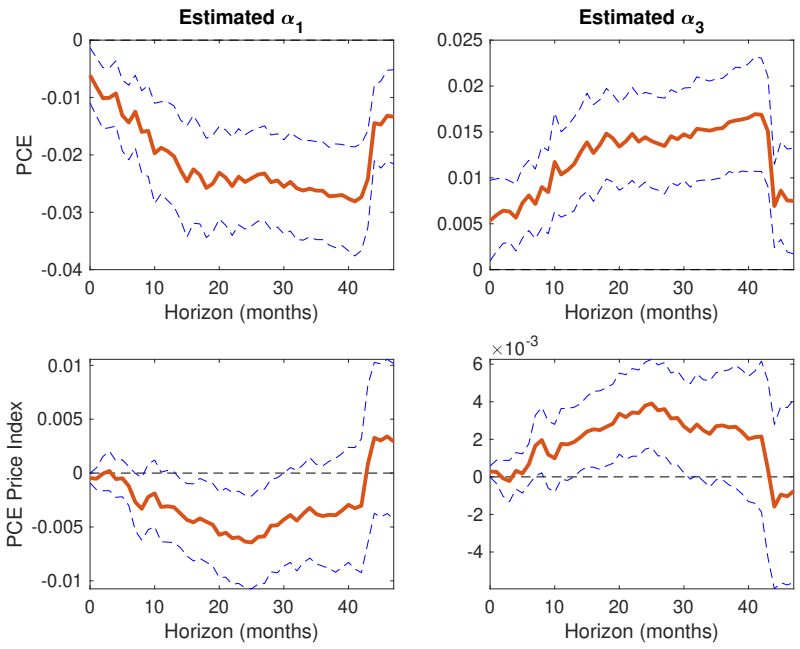


FIGURE 4. Cumulative impulse responses to a federal fund rate shock

Note: This figure shows the cumulative impulse responses of real personal consumption expenditure (PCE), and the PCE price index following a federal fund rate shock estimated from the local projections model (1). The solid lines show the point estimates of the impulse responses. The dashed lines show the 68% confidence intervals based on a Newey-West heteroscedasticity and autocorrelation consistent (HAC) estimator.

Inflation disagreement has a similar attenuation effect on the FFR shocks, as shown in Figure 4. A tightening FFR shock in the absence of inflation disagreement reduces consumption and the price level ($\alpha_1^h < 0$); when inflation disagreement is higher, the contractionary effects are weaker ($\alpha_3^h > 0$). Quantitatively, a one-standard-deviation increase in inflation disagreement would weaken the cumulative effects of an FFR shock on consumption and the price level by 29.1% and 29.7%, respectively.[15]

III.2. **Robustness.** The attenuation effects of inflation disagreement for monetary policy shocks are robust to alternative measures of real activity, inflation, and inflation disagreement. They are also robust to the inclusion of additional control variables and their interactions with the policy shocks.

$\alpha_3^{h=24} = 0.0038$ implies that, in periods with inflation disagreement one standard deviation above zero, the effects of the FG shock on the PCE price index would be weakened by $0.0038 \times 0.50/0.0058 \times 100 = 32.8\%$.

[15]The estimated $\alpha_1^{h=24}$ and $\alpha_3^{h=24}$ for cumulative PCE changes are -0.0247 and 0.0144 respectively. A one-standard-deviation inflation disagreement above zero would thus reduce the effects of a fed funds rate shock by $(0.0144 \times 0.50)/0.0247 \times 100 = 29.1\%$. The estimated $\alpha_1^{h=24}$ and $\alpha_3^{h=24}$ for cumulative PCE price level changes are -0.0064 and 0.0038 respectively, implying an attenuation effect of $(0.0038 \times 0.50)/0.0064 \times 100 = 29.7\%$.

III.2.1. *Alternative measures of economic activity and inflation.* The baseline results are robust to alternative measures of activity and inflation. Instead of real PCE and the PCE price index, we consider industrial production or unemployment as an alternative measure of real activity and the consumer price index as an alternative measure of inflation. We show that the responses of those variables to monetary policy shocks are similarly attenuated by inflation disagreement (see Appendix A.4).

III.2.2. *Removing the effects of demographic factors and aggregate shocks from inflation expectations.* Individual inflation expectations can be affected by both demographic factors (such as age, income, the region of residence, education, marital status, and home ownership status) and aggregate shocks (Fofana et al., 2024). To examine whether the attenuation effect is driven by those factors, we use the cross-sectional archives of individual responses in the Michigan survey to construct measures of inflation expectations that are purged of the effect of demographic factors as well as aggregate shocks (via a time fixed effect). We then construct a "purified" measure of inflation disagreement using the IQR of those inflation expectations.

Inflation disagreement may also directly respond to monetary policy shocks and oil supply shocks (Fofana et al., 2024). To further purge these effects, we construct an "orthogonalized" measure of inflation disagreement, by regressing the purified measure constructed above on current and lagged values (for up to 12 months) of the FFR and FG surprises constructed by Swanson (2021) and oil supply news shocks constructed by Känzig (2021) and then taking the residuals.

Both the purified and the orthogonalized measures are highly correlated with our baseline measure of inflation disagreement. Using either measure (scaled by the median of corresponding inflation expectations) to re-estimate the baseline local projections model in Eq. (1), we obtain impulse responses of real PCE and inflation that are similar – both qualitatively and quantitatively – to those obtained in our baseline regressions. More details can be found in Appendix A.1.2.

III.2.3. *Other potential confounding factors.* Our baseline empirical results are robust to including additional control variables. In particular, we re-estimate the baseline local projections by adding one of the following control variables and its interaction with our measure of monetary policy shock: (1) the median inflation expectations, (2) inflation uncertainty constructed by Binder (2017), (3) the median income growth expectations, (4) income growth

disagreement,[16] (5) consumer uncertainty, (6) disagreement about future two-year Treasury yield from Blue Chip Financial Forecasts (Blue Chip), and (7) professional forecasters' disagreement about future inflation from Blue Chip, all measured at the one-year horizon.[17]

Figure 5 reports the cumulative effects of a one-standard-deviation tightening shock to FG (upper panels) and to FFR (lower panels) at the one-year horizon estimated from the baseline local projections (model 0) and the 7 alternative models, with the red circles representing the estimates of α_1 and the blue diamonds those of α_3. The figure shows that the attenuation effects of inflation disagreement are robust across all alternative model specifications. In almost all the alternative models, the estimates of α_1 are significantly positive while those of α_3 are significantly negative, as in our baseline model. Notably, Model (7) shows that household inflation disagreement further weakens the transmission of monetary policy even after controlling for the attenuating effects of professional inflation disagreement that has been documented by Falck et al. (2021) and Barbera et al. (2023). The full results for all horizons can be found in Appendix A.3.

IV. What drives the attenuation effects of inflation disagreement?

In our baseline regressions, we measure inflation disagreement using the IQR of consumers' inflation forecasts, which could mask important heterogeneity of inflation forecasts within the middle two quartiles (e.g., skewness of the forecast distribution). We also treat inflation disagreement as a given regime or state of the economy, analogous to business cycle booms or recessions. However, individual inflation forecasts and the resulting inflation disagreement are in general endogenous, and the attenuation effects of inflation disagreement might reflect endogenous responses of both inflation disagreement and macroeconomic variables to some omitted shocks. We now address each of these concerns in turn.

IV.1. **Positive skewness of inflation expectations.** A spike in inflation disagreement can be driven by a rising concentration on either the left or the right tail of the inflation forecast distribution (or both). To provide additional insight into which tail drives the attenuation effects of inflation disagreement, we modify the baseline empirical specification (1) by including a measure of the positive skewness of inflation expectations (denoted by $Skew^{\pi}_{t-1}$)

[16]We measure income disagreement based on Q8 in the Michigan survey following the approach of Bachmann et al. (2013). In particular, we define

$$IncomeDisagreement_t = \sqrt{Favorable_t + Unfavorable_t - (Favorable_t - Unfavorable_t)^2}$$

where $Favorable_t$ ($Unfavorable_t$) denotes the share of households reporting that their personal financial conditions will be better off (worse off) in the next year.

[17]The first 5 control variables are all constructed based on the Michigan survey data. The last 2 control variables are based on the Blue Chip data.

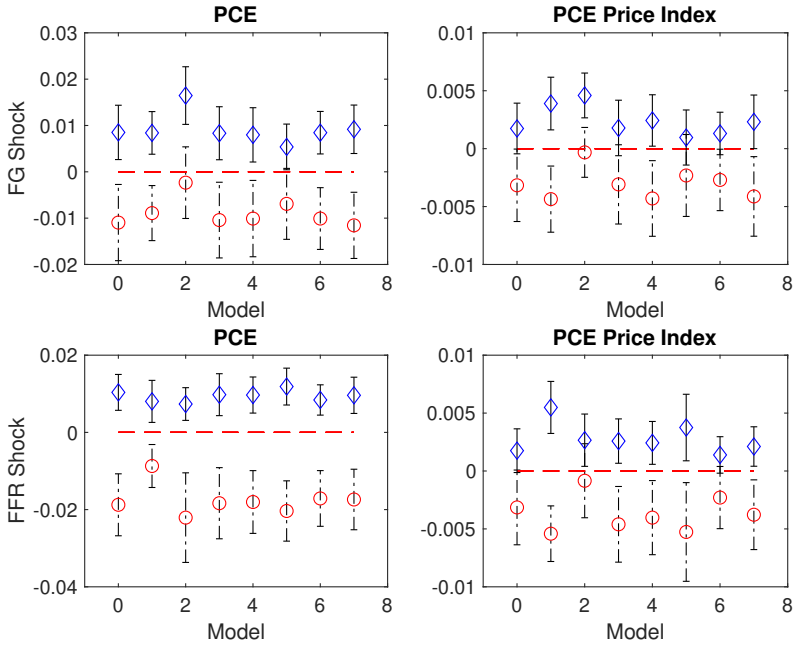


FIGURE 5. Cumulative impulse responses to monetary policy shocks at the one-year horizon: Alternative models with different control variables

Note: This figure shows the cumulative responses of real personal consumption expenditure (PCE; left panels) and the PCE price index (right panels) at the one-year horizon ($h = 12$) following a one-standard-deviation tightening shock to forward guidance (FG, top panels) or to the federal fund rate (FFR, bottom panels) under alternative specifications of the local projections model. For each model, the red circles represent the point estimates of α_1, the blue diamonds represent the point estimates of α_3, and the whiskers represent the 68% confidence bands (with Newey-West standard errors). Model 0 represents the baseline specification. The other model specifications differ in the set of control variables. These include (1) the median one-year ahead inflation expectations from the Michigan survey (model 1); (2) one-year-head inflation uncertainty index constructed by Binder (2017) based on the Michigan survey (model 2); (3) one-year-ahead median income growth expectation of consumers from the Michigan survey (model 3); (4) disagreement about one-year-ahead income growth from the Michigan survey (model 4); (5) consumers' perceived uncertainty concerning vehicle purchases from the Michigan survey (model 5); (6) one-year-ahead disagreement (top 10 average minus bottom 10 average) about two-year Treasury yields from the Blue Chip (model 6); and (7) one-year-ahead disagreement (top 10 average minus bottom 10 average) about CPI inflation from the Blue Chip.

and its interactions with the monetary policy shocks as two additional explanatory variables. The modified empirical specification is given by

$$\begin{aligned}\log(y^j_{t+h}) - \log(y^j_{t-1}) = \alpha^h_0 + \alpha^h_1 MP_t + \alpha^h_2 IQR^\pi_{t-1} + \alpha^h_3 IQR^\pi_{t-1} * MP_t \\ + \alpha^h_4 Skew^\pi_{t-1} + \alpha^h_5 Skew^\pi_{t-1} * MP_t + \alpha^h_6 \Gamma_{t-1} + \varepsilon_{t+h}\end{aligned} \tag{2}$$

where we measure the positive skewness of inflation expectations by the difference between the upper IQR (the 75^{th} percentile minus the median) and the lower IQR (the median minus the 25^{th} percentile) of the one-year ahead inflation forecast distribution in the Michigan survey, scaled by the IQR of the inflation forecasts.

Figure 6 shows the results: Inflation disagreement weakens the effects of the FG shock ($\alpha_3 > 0$), and the attenuation effects are stronger with a more positively skewed distribution of inflation forecasts ($\alpha_5 > 0$). These effects are statistically significant and economically important. In Figure A.11 in Appendix A.2, we show that a positive skewness also strengthens the attenuation effect of inflation disagreement for the FFR shock.

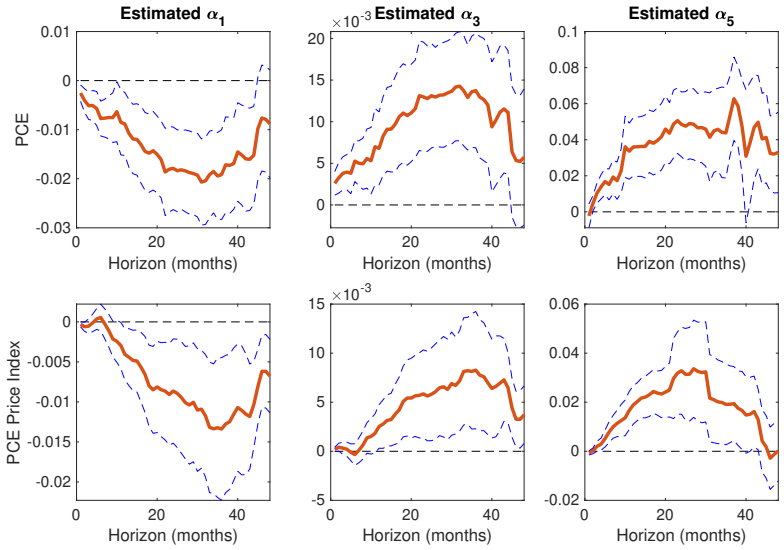


FIGURE 6. Cumulative impulse responses to a forward guidance shock: Effects of positive skewness of the inflation forecast distribution.

Note: This figure shows estimated cumulative responses of real personal consumption expenditure (PCE) and the PCE price index to a forward guidance shock from the local projections model (2). The solid lines show the point estimates of the impulse responses. The dashed lines show the 68% confidence intervals based on a Newey-West heteroskedasticity and autocorrelation consistent (HAC) estimator.

IV.2. **State-level inflation disagreement.** One potential concern with our empirical findings is that a high level of inflation disagreement could be capturing some omitted aspects of monetary policy, such as poor policy communications, which could raise inflation disagreement and make monetary policy less effective at the same time. If so, our baseline findings would not necessarily capture an independent attenuating effect of inflation disagreement on monetary policy transmission.

To address this concern, we exploit the cross-sectional variations in the effects of inflation disagreement on the transmission of monetary policy shocks using state-level data. We make use of the fact that monetary policy is common to all regions in the U.S., but households in different regions might have different degrees of inflation disagreement. We examine whether the common monetary policy shocks have more muted effects on macro outcomes in regions with higher inflation disagreement.

To implement this approach, we construct a monthly measure of state-level inflation disagreement using data from the SCE of the New York Fed, which records the inflation expectations of consumers residing in each of the 50 states.[18] Using the state-level measures, we estimate the panel-data local projections

$$\log(UR_{s,t+h}) - \log(UR_{s,t-1}) = \beta_1^h IQR_{s,t-1}^{\pi} + \beta_2^h IQR_{s,t-1}^{\pi} * MP_t + \mu_s^h + \gamma_t^h + \varepsilon_{s,t+h}. \quad (3)$$

with $h = 0, 1, \ldots, 24$. In this specification, $\log(UR_{s,t+h}) - \log(UR_{s,t-1})$ denotes the cumulative changes in the unemployment rate in state s from $t-1$ to $t+h$, $IQR_{s,t-1}^{\pi}$ denotes the state-level inflation disagreement, measured by the IQR of one-year ahead inflation forecasts among consumers from state s and scaled by the state-level median inflation forecast, and MP_t denotes the monetary policy shocks (either FG or FFR). The average effects of monetary policy shocks (as well as those of other shocks that are common to all states) are absorbed by the time fixed effect γ_t^h. The term μ_s^h denotes the state fixed effects. The monthly sample covers June 2013 to September 2023. The key parameter of interest is β_2^h, which captures the differential impact of the monetary policy shock in states with high inflation disagreement. A negative value of β_2^h would imply an attenuation effect of inflation disagreement: in a state with higher inflation disagreement, a tightening of monetary policy leads to a smaller increase in state-level unemployment.

Figure 7 plots the estimated cumulative responses of state-level unemployment (i.e., β_2^h) following a contractionary FG shock (left panel) and FFR shock (right panel). These estimates are significantly negative beyond the first few months, indicating that the contractionary effects of monetary policy shocks are indeed weaker in states with higher inflation disagreement. The fact that the attenuating effect of inflation disagreement holds not only in the aggregate but also across regions that face a common set of monetary policy shocks shows that this effect is unlikely to be driven by endogenous responses of inflation disagreement to monetary policy.[19]

[18] For a plot of the state-level disagreement measures, see Figure A.6 in the online appendix.

[19] We focus on state-level unemployment as the dependent variable in our regressions. The Bureau of Labor Statistics (BLS) does not publish state-level price indices. Hazell et al. (2022) construct state-level price indices based on the micro-price data that the BLS collects for constructing the CPI. Their sample overlaps with the SCE sample only for the periods from 2013:Q3 to 2017:Q4, which is too short for our analysis. The BLS does publish CPI data for 23 metropolitan areas (MSA), which we merge with the state-level inflation forecasts data in SCE for estimating the attenuation effects of inflation disagreement. To deal with missing monthly observations in the MSA-level CPI data, we convert the monthly series to quarterly by taking the within-quarter average of non-missing entries. We use the information about the states in which an MSA is located to do a crosswalk between the MSA-level CPI data and the SCE state-level inflation forecasts data. We then estimate a set of local projections similar to Eq. (3) using the MSA-level data.

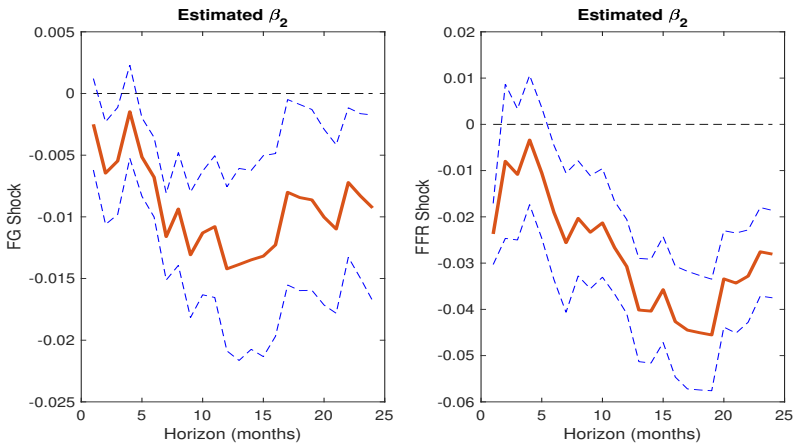


FIGURE 7. Cumulative impulse responses of state-level unemployment to a tightening monetary policy shock: The marginal effects of inflation disagreement.

Note: This figure shows the cumulative impulse responses of state-level unemployment following a contractionary monetary policy shock in states with high inflation disagreement (i.e., β_2^h in Eq. (3)). The left panel shows the responses to an FG shock and the right panel show those to an FFR shock. The solid lines show the point estimates of the impulse responses. The dashed lines show the 68% confidence intervals with standard errors clustered at the state level.

IV.3. **Experience-based inflation disagreement measure.** To further differentiate between an exogenous attenuating effect from inflation disagreement and endogenous responses of both inflation disagreement and macroeconomic variables to omitted shocks, we examine a component of inflation disagreement that is arguably orthogonal to contemporaneous shocks. In particular, we examine inflation disagreement that arises from individual forecasters' lifetime inflation experience, which has been shown to have important effects on individuals' inflation forecasts (Malmendier and Nagel, 2016; Nagel, 2024). These authors assume that individuals perceive inflation as following an AR(1) process and learn about the parameters over time using inflation realized during their lifetime based on a recursive updating rule with gain parameters that decrease with age. Individuals then form expectations about future inflation using their current parameter estimates. Notably, these forecasts are formed only based on inflation observed during each individual's lifetime and will not react to contemporaneous shocks that are not yet reflected in realized inflation.

In this exercise, we calculate the IQR of the experience-based one-year-ahead inflation forecasts across age cohorts constructed by Nagel (2024), normalized by the median Michigan

Consistent with our baseline findings, we find that inflation disagreement weakens the effects of monetary policy shocks on MSA-level consumer prices (see Appendix A.1.4).

one-year-head inflation forecast.[20] We replace our baseline inflation disagreement measure by this experience-based measure and re-estimate the local projections specification (1) over the sample of July 1991 to Dec 2023.[21] Despite the differences of the experience-based inflation disagreement from our baseline measure (see Figure A.9 in the appendix), the impulse responses of consumption and inflation, shown in Figures 8 for an FG shock and Figure A.10 in the online appendix for an FFR shock, are qualitatively similar to those obtained from our baseline estimation. The fact that a rise in inflation disagreement largely unrelated to contemporaneous developments also dampens the effectiveness of monetary policy supports a causal interpretation of the attenuating effect of inflation disagreement.

V. A New Keynesian model with heterogeneous beliefs

In the previous section, we document the stylized facts that higher disagreement in inflation forecast attenuates the effectiveness of conventional and unconventional monetary policy, and the attenuation effects are stronger if spikes in disagreement are driven by more forecasters with high inflation expectation (positive skewness). To understand the mechanism through which disagreement (and positive skewness) in inflation forecast can weaken the power of monetary policy shocks, we generalize the standard New Keynesian model to incorporate heterogeneous beliefs and borrowing constraints.

V.1. The forward guidance puzzle in a representative-agent model.

We first illustrate the forward guidance (FG) puzzle, a challenge facing the standard representative-agent New Keynesian models. In those models, news about future real interest rates at any horizon——however distant in the future——has an equally powerful effect on current consumption as a change in the current interest rate. This implication seems implausible, and hence the FG puzzle (Del Negro et al., 2023).

To put the FG puzzle into context, consider the intertemporal Euler equation derived from the standard model with a logarithmic utility function:

$$\frac{1}{C_t} = \beta R_{ft} E_t \frac{1}{C_{t+1}} \frac{1}{\Pi_{t+1}}, \tag{4}$$

where C_t denotes real consumption in period t, R_{ft} denotes the risk-free nominal interest rate, Π_{t+1} denotes the inflation rate from t to $t+1$, $\beta \in (0,1)$ is a subjective discount factor,

[20]The results are very similar if we normalize the experience-based IQR by the median experience-based inflation forecast.

[21]The original data of cohort-specific components in inflation forecasts constructed by Nagel (2024) is available only at the quarter frequency. We interpolate the data into monthly, assuming that the experience-based components in inflation expectations remain the same within each quarter.

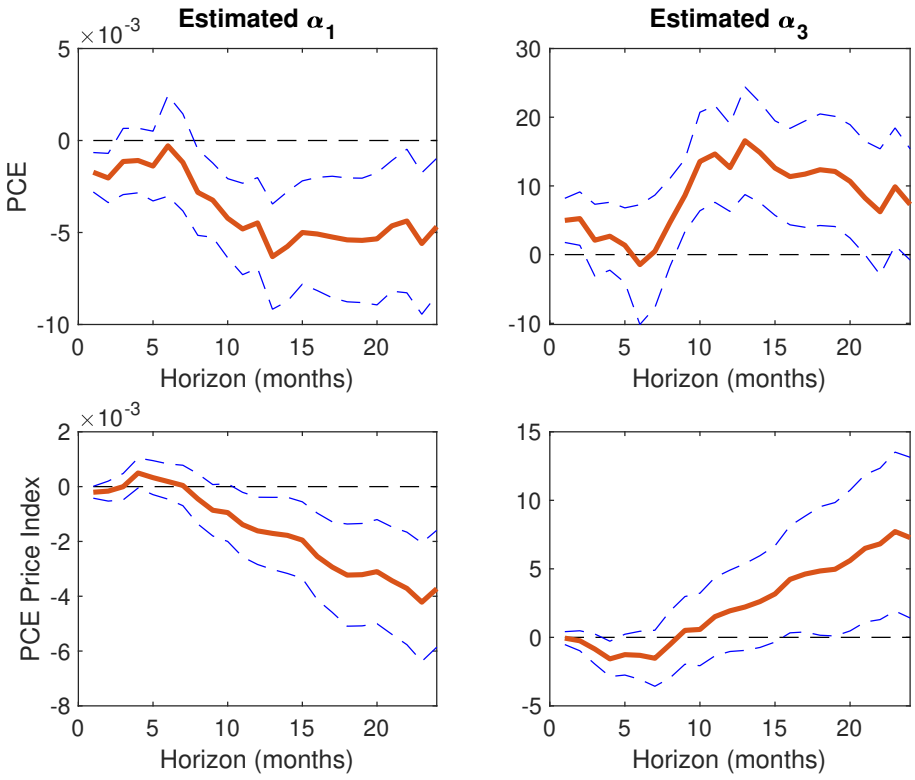


FIGURE 8. Cumulative impulse responses to a forward guidance shock: Experience-based measure of inflation disagreement

Note: This figure shows the cumulative responses of real personal consumption expenditure (PCE) and the PCE price index following a forward guidance shock using the experience-based inflation disagreement measure (i.e., the IQR of cohort-specific components in inflation forecast, normalized by the median one-year ahead inflation forecast in the Michigan survey). The solid lines show the point estimates of the impulse responses. The dashed lines show the 68% confidence intervals based on a Newey-West heteroskedasticity and autocorrelation consistent (HAC) estimator. Each period represents a month.

and E_t is a conditional expectation operator. Log-linearizing the Euler equation around the steady state and iterating forward leads to

$$\hat{C}_t = -\sum_{j=0}^{\infty} E_t(\hat{R}_{ft+j} - E_t\hat{\Pi}_{t+j+1}), \tag{5}$$

where the hatted variables denote the log-deviations the corresponding variables from their steady-state levels.

Since there is no discounting on the right-hand of Eq (5), expected policy rate changes in the future—no matter how distant it is from the present—have equally powerful effects on current consumption as does a change in the current interest rate, an implication that seems implausible.

One way to attenuate the power of forward guidance within the representative-agent framework is to introduce a time-varying discount factor (denoted by β_t) in the Euler equation.

For example, consider a log-linearized Euler equation given by

$$\hat{C}_t = -\hat{\beta}_t + E_t\hat{C}_{t+1} - (\hat{R}_{ft} - E_t\hat{\Pi}_{t+1}),$$

where

$$\hat{\beta}_t \equiv \frac{1-\rho}{\rho}\hat{C}_t, \quad \rho \in (0,1). \tag{6}$$

This modification results in a "discounted Euler equation" given by

$$\hat{C}_t = \rho E_t\hat{C}_{t+1} - \rho(\hat{R}_{ft} - E_t\hat{\Pi}_{t+1}). \tag{7}$$

If $\rho \in (0,1)$, a future interest rate change has a smaller effect on current consumption than does a current interest rate change of the same magnitude.[22] This can be seen more directly by iterating Eq. (7) forward to obtain

$$\hat{C}_t = -\rho\sum_{j=0}^{\infty} E_t\rho^j(\hat{R}_{ft+j} - E_t\hat{\Pi}_{t+j+1}), \quad \rho \in (0,1).$$

So, in principle, a discounted Euler equation can resolve the forward guidance puzzle. But the question remains: What is behind the discounting of the Euler equation?

V.2. **A model with heterogeneous beliefs.** We now provide a micro-foundation for the discounted Euler equation by introducing heterogeneous beliefs about the central bank's inflation target.

Assume that the central bank follows a Taylor rule

$$R_{ft} = r^*\Pi_t^*\left(\frac{\Pi_t}{\Pi_t^*}\right)^{\varphi}\exp(\xi_t), \qquad \varphi > 1, \tag{8}$$

where r^* denotes the natural real interest rate, Π_t^* denotes the inflation target, and ξ_t denotes a monetary policy shock. The parameter $\varphi > 1$ measures the responsiveness of the policy rate to deviations of inflation from the target.[23]

Assume that the true process of the inflation target is a random walk such that

$$\Pi_{t+1}^* = \Pi_t^*\exp(\varepsilon_{t+1}), \tag{9}$$

where ε_{t+1} is an i.i.d. random variable with a mean of $-\frac{1}{2}\sigma_r^2$ and a variance of σ_r^2. In the special case with no fluctuations in ε_{t+1} (i.e., with $\sigma_r = 0$), the inflation target would be a constant (e.g. a 2 percent annual rate).

[22]The discounting of the Euler equation can arise from bounded rationality, such as myopia of agents toward future surprises in the economy (Gabaix, 2020).

[23]For analytical tractability, we do not include output gap in the Taylor rule. Putting output gap in the Taylor rule would not affect the main results, which depend mainly on households' heterogeneous beliefs about the inflation target.

The household family consists of a large number of members indexed by $j \in [0,1]$. Each member has her own belief about the stochastic process of the inflation target, which might be different from the true process specified in Eq. (9). We assume that member j's conditional expectation of the growth rate of the inflation target is given by

$$E_t^j \frac{\Pi_{t+1}^*}{\Pi_t^*} = e_{jt}, \quad j \in [0,1], \tag{10}$$

where E_t^j is a conditional expectations operator for member j and e_{jt} is a random variable drawn from a time-varying distribution with the cumulative density function $G_t(e)$. Absent belief heterogeneity, rational expectations would imply that $e_{jt} = 1$ for all j and t. In general, however, e_{jt} is a random variable that might differ across members and across time.[24]

In the beginning of each period t, the household receives labor income, dividends, and returns from savings. The household makes equal lump-sum transfers of the net worth to all members of the family. The family members then make individual consumption-saving decisions in decentralized markets. As we shall see, household members with higher inflation expectations will choose to consume more today, financed by both the internal net worth and external debt, subject to a borrowing constraint. By contrast, household members with lower inflation expectations will prefer to save more today and consume more in the future, and they make interior optimal choices because they do not face a binding borrowing constraint.

The expected utility function of the household family is given by

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\int_0^1 \log C_{jt} dj - \psi \frac{N_t^{1+\gamma}}{1+\gamma} \right]$$

where C_{jt} is consumption by agent j, and N_t is a homogeneous labor supply. The family has a beginning-of-period net worth (denoted by A_t) given by

$$A_t = \frac{\int_0^1 B_{jt} dj}{P_t} + \frac{W_t}{P_t} N_t + D_t, \tag{11}$$

where B_{jt} is value of member j's net savings (i.e., bond holdings) from period $t-1$ to t, P_t is the aggregate price level, W_t is the nominal wage rate, and D_t is the real dividend income from the firms that the household owns.

[24]Since the inflation target follows a random-walk process, our assumption about the individual belief process in Eq. (10) implies that inflation disagreement is highly persistent. This model feature is consistent with the empirical evidence in Andrade et al. (2016), who find that the term structure of inflation disagreement is almost flat, meaning that inflation disagreement for long horizons (such as 5 to 10 years ahead) is almost as large as that for shorter horizons (such as 1 year ahead). The model feature is also consistent with the evidence in Weber et al. (2022), who find a strong positive correlation between short-term and long-term inflation expectations for households, firms, and professional forecasters. In the Michigan Survey data that we use, the correlation between the 1-year ahead inflation expectations and the 5-to-10 year ahead expectations is also positive, at 0.60.

Each individual member receives a lump-sum transfer A_t from the household family. She then chooses her consumption and savings based on her own inflation expectations, subject to the flow-of-funds constraint

$$C_{jt} + \frac{B_{jt+1}/R_{ft}}{P_t} \leq A_t, \tag{12}$$

and the borrowing constraint

$$\frac{B_{jt+1}/R_{ft}}{P_t} \geq -\bar{B} \tag{13}$$

where $\bar{B}$ is an exogenous borrowing limit that cannot exceed A_t.

Denote by $w_t \equiv W_t/P_t$ the real wage rate. The optimizing labor supply decision is given by

$$\Lambda_t w_t = \psi N_t^{\gamma}, \quad where \quad \Lambda_t = \int_0^1 \Lambda_{jt} dj = \int_0^1 \frac{1}{C_{jt}} dj, \tag{14}$$

and Λ_{jt} is the multiplier associated with the budget constraint (12) and equals the marginal utility from consumption.

The first-order condition with respect to nominal savings is given by

$$\Lambda_{jt} = \beta R_{ft} \mathbb{E}_t^j \frac{\Lambda_{t+1}}{\Pi_{t+1}} + \Omega_{jt} \quad \forall j, \tag{15}$$

where Ω_{jt} is the Lagrangian multiplier associated with the borrowing constraint (13). Define $r_{ft} = R_{ft}/\Pi_t^*$ and $\pi_t = \Pi_t/\Pi_t^*$. The Euler equation of individual j, who believes that $\frac{\Pi_{t+1}^*}{\Pi_t^*} = e_{jt}$ in expectations, can be written as

$$\Lambda_{jt} = \beta r_{ft} \mathbb{E}_t^j \left[\frac{\Lambda_{t+1}}{\pi_{t+1}} \frac{\Pi_t^*}{\Pi_{t+1}^*} \right] + \Omega_{jt} \tag{16}$$

Since aggregate normalized inflation π_{t+1} (derived from the firms' decisions) and the aggregate Lagrangian multiplier Λ_{t+1} are both uncorrelated with individual beliefs e_{jt}, we can integrate out the individual beliefs such that $\mathbb{E}_t^j \frac{\Lambda_{t+1}}{\pi_{t+1}} = \mathbb{E}_t \frac{\Lambda_{t+1}}{\pi_{t+1}}$ for all j. Thus, the Euler equation (16) can be rewritten as

$$\Lambda_{jt} = \beta \frac{1}{e_{jt}} r_{ft} \mathbb{E}_t \left[\frac{\Lambda_{t+1}}{\pi_{t+1}} \right] + \Omega_{jt}. \tag{17}$$

Denote by e_t^* the belief of the marginal agent, who is indifferent between borrowing or saving. Since the marginal agent's borrowing constraint is not binding (i.e., $\Omega_t^* = 0$), her Euler equation is given by

$$\frac{1}{\bar{C}_t} = \frac{\beta}{e_t^*} r_{ft} \mathbb{E}_t \left[\frac{\Lambda_{t+1}}{\pi_{t+1}} \right] \tag{18}$$

where $\bar{C}_t = A_t + \bar{B}$ is the maximum consumption attainable with internal funds A_t and external debt up to the borrowing limit $\bar{B}$.

For an agent j with a perceived inflation target higher than that of the marginal agent (i.e., with $e_{jt} > e_t^*$), her perceived real interest rate would be lower. Accordingly, she would

choose to consume more, partly financed by external debt. Since the agent faces a binding borrowing constraint, her maximum amount of consumption is given by $\bar{C}_t$. For an agent who has a belief lower than that of the marginal agent (i.e., $e_{jt} \leq e_t^*$), his optimal consumption choice is not constrained by the borrowing limit, and the level of consumption would depend on his perceived inflation target relative that of the marginal agent. For those unconstrained agents, a higher perceived inflation target (relative to that of the marginal agent) implies a lower perceived real interest rate and thus a higher level of consumption.

The consumption decision rule is given by

$$C_{jt} = \begin{cases} C_t + \bar{B}, & \text{for } e_{jt} > e_t^* \\ \frac{e_{jt}}{e_t^*}(C_t + \bar{B}), & \text{for } e_{jt} \leq e_t^* \end{cases}, \tag{19}$$

where $C_t \equiv \int_0^1 C_{jt} dj$ denotes aggregate consumption.[25]

Given the consumption decision rule (19), we can express the average marginal utility Λ_t as a function of aggregate consumption and the distribution of individual beliefs. We can then rewrite the Euler equation (18) for the marginal agent as

$$1 = \beta r_{ft} \mathbb{E}_t \frac{C_t + \bar{B}}{C_{t+1} + \bar{B}} \frac{1}{\pi_{t+1}} \frac{e_{t+1}^*}{e_t^*} F(e_{t+1}^*), \tag{20}$$

where $F(e^*)$ is a function of the belief distribution and is given by

$$F(e^*) = \left[\frac{1 - G(e^*)}{e^*} + \int_{e_{\min}}^{e^*} \frac{1}{e} dG(e) \right]. \tag{21}$$

The optimal labor supply decisions imply that

$$\Lambda_t W_t = \psi N_t^{\gamma}, \qquad \text{where} \quad \Lambda_t = \frac{1}{C_t + \bar{B}} e_t^* F(e_t^*),$$

Aggregate production function is given by

$$Y_t = Z_t N_t, \tag{22}$$

where Y_t denotes aggregate output and Z_t denotes labor productivity.

As in the standard New Keynesian model, we assume that firms producing differentiated intermediate goods face monopolistic competition in the product markets and each firm sets a price for its own product, taking as given the demand schedule derived under a CES aggregation technology, and price adjustment incurs a quadratic cost in the spirit of

[25] In deriving the consumption decision rule, we have used the relation $A_t = C_t$, which is obtained by aggregating the flow-of-funds constraint (12) over all consumers and imposing the bond market clearing condition that $\int_0^1 B_{jt} dj = 0$.

Rotemberg (1982).[26] Firms' optimizing decisions lead to the Phillips curve relation (in log-linearized form)

$$\hat{\pi}_t = \varphi_y[\hat{w}_t - \hat{Z}_t] + \beta \mathbb{E}_t \hat{\pi}_{t+1}, \tag{23}$$

where a hatted variable denotes the log-deviations of the variable from its steady-state value and the slope parameter φ_y is a function of the fundamental parameters including the relative risk aversion, the Frish elasticity of labor supply, the elasticity of substitution between differentiated products, and the size of price adjustment costs.

The log-linearized monetary policy rule in equation (8) implies that

$$\hat{r}_{ft} = \varphi \hat{\pi}_t + \xi_t \tag{24}$$

In an equilibrium, the markets for final goods, bonds, and labor all clear. Final goods market clearing implies that

$$C_t + \frac{\chi_p}{2} \left[\pi_t - 1\right]^2 Y_t = Y_t, \tag{25}$$

where χ_p measures the size of the price adjustment costs. Bond market clearing implies that

$$\int_0^1 B_{jt} dj = 0. \tag{26}$$

An equilibrium consists of allocations $\{C_t, N_t, Y_t\}$ and prices $\{w_t, r_{ft}, \pi_t\}$ such that (i) taking all prices as given, the allocations solve the households' utility maximization problem; (ii) taking all prices but its own as given, the allocations and each firm's price solve its profit maximizing problems; (iii) final goods market, bond market, and labor market all clear.

V.3. **Analytical results from the model with heterogeneous beliefs.** Log-linearizing the Euler equation (20) around the deterministic steady state, we obtain

$$\hat{C}_t = \underbrace{\frac{\mu + (1-\theta)\kappa}{\mu + \kappa}}_{\equiv \beta_1} \mathbb{E}_t \hat{C}_{t+1} - \underbrace{\frac{(1+\kappa)\mu}{\mu + \kappa}}_{\equiv \beta_2} \left(\hat{r}_{ft} - \mathbf{E}_t \hat{\pi}_{t+1}\right), \tag{27}$$

[26]We drive the Phillips curve relation in Appendix B.2. For analytical tractability, we assume that firms are fully rational and do not disagree about the future inflation target. In reality, of course, firms may also disagree about their inflation expectations. For example, the Survey of Firms' Inflation Expectations conducted by the Cleveland Fed shows that firms' one-year ahead inflation expectations have a mean of 5% and a cross-sectional standard deviation of 1.6% in the second quarter of 2023. In comparison, in the same quarter, consumers' one-year ahead inflation expectations from the Michigan survey have a mean of about 6% and a standard deviation of about 9.6%. This observation suggests that inflation disagreement among firms, while substantive, may not be as pervasive as that among consumers. In our view, allowing for firms' inflation disagreement would make the model more realistic and would likely introduce additional frictions in the Phillips curve. However, it would unlikely alter our model's mechanism through which belief heterogeneity on the consumer side leads to a discounted Euler equation and thus attenuating the power of forward guidance.

where a hatted variable denotes the log-deviations of that variable from its steady-state value. The parameter $\kappa \equiv \frac{\bar{B}}{A} = \frac{\bar{B}}{C} \in (0,1)$ denotes the steady-state loan-to-value ratio; the parameter $\theta \equiv -\frac{F'(e^*)e^*}{F(e^*)} \in [0,1)$ denotes the inverse elasticity of $F(\cdot)$ with respect to the cutoff belief e^*, evaluated at the steady state equilibrium; and $\mu \in (0,1]$ denotes the inverse elasticity of the leverage ratio $\frac{C_t}{C_t+\bar{B}}$ with respect to the cutoff belief e^*, also evaluated at the steady state.[27]

In the special case with $\theta = 0$ and $\mu = 1$, the model nests the standard Euler equation in the representative-agent models, such that $\beta_1 = \beta_2 = 1$, and there is no "discounting" of the Euler equation. In the more general case with heterogeneous beliefs about the central bank's inflation target, we have $\theta \in (0,1)$ and $\mu \in (0,1)$. In this case, the Euler equation would be discounted in the sense that $\beta_1 < 1$, such that aggregate consumption in the current period changes less than one-for-one with expected future consumption. This result is formally stated in Proposition V.3 below.

Proposition V.1. *(Euler-equation discounting) With belief heterogeneity, aggregate current consumption responds less than one-for-one to changes in future consumption.*

Proof. In Appendix B.1, we show that $\theta \in (0,1)$, $\mu \in (0,1)$, and $\kappa \in (0,1)$ with heterogeneous beliefs. It follows immediately that

$$\beta_1 \equiv \frac{\mu + (1-\theta)\kappa}{\mu + \kappa} = 1 - \frac{\theta\kappa}{\mu + k} \in (0,1). \tag{28}$$

□

Recall that in the standard New Keynesian framework *a la* section V.1, the intertemporal discount factor in linearized equation (5) equals 1. In our framework with heterogeneous expectation about future inflation, by contrast, the coefficient β_1 is less than 1.

To obtain sharper characterizations of how the magnitude of Euler equation discounting (i.e., the size of β_1) depends on the dispersions in inflation beliefs, we assume that the households' idiosyncratic beliefs follow a Pareto distribution with the cumulative density function (cdf)

$$G(e) = \begin{cases} 1 - (\frac{e_{min}}{e})^{\alpha} & \text{if } e \geq e_{min} \\ 0 & \text{if } e < e_{min} \end{cases} \tag{29}$$

We fix the scale parameter at $e_{min} \equiv \frac{\alpha-1}{\alpha}$ such that the mean stays constant at $E(e) = 1$, implying that the agents' expectations on average are rational. The shape parameter α measures the thickness of the tail, with a smaller α corresponding to more dispersed beliefs.

[27] We derive the log-linearized Euler equation (20) in Appendix B.1.

Proposition V.2. *Assuming that individual beliefs of the inflation target follow the Pareto distribution* (29)*. Then, β_1 increases with α, implying that more dispersed beliefs lead to greater Euler equation discounting.*

Proof. In Appendix B.3, we prove in Lemma B.2 and B.3 that θ decreases with α and that μ increases with α. It follows that $\beta_1 \equiv 1 - \frac{\theta\kappa}{\mu+k}$ increases with α. □

Heterogeneous beliefs about the inflation target and the resulting inflation disagreement can also weaken the elasticity of aggregate consumption to the contemporaneous real interest rate (i.e., $\beta_2 < 1$). Furthermore, more dispersed beliefs lead to smaller responses of aggregate consumption to changes in the real interest rate (i.e., β_2 increases with α). These results are formally established in Proposition V.3 below.

Proposition V.3. *Assuming that individual beliefs of the inflation target follow the Pareto distribution* (29)*. Then, β_2 increases with α, implying that an increase in the mean-preserving dispersion in beliefs lead to more muted responses of aggregate consumption to changes in the contemporaneous real interest rate.*

Proof. The parameter β_2 is given by

$$\beta_2 = \frac{(1+\kappa)(\mu+\kappa-\kappa)}{\mu+\kappa} = 1+\kappa-\frac{(1+\kappa)\kappa}{\mu+\kappa}, \qquad \kappa \in (0,1).$$

In Appendix B.3 we prove that μ increases with α. It immediately follows that β_2 also increases with α. □

Proposition V.3 also implies that inflation disagreement weakens the response of real activity to other demand shocks, such as a shock to the natural real interest rate.[28]

Finally, we evaluate the effectiveness of monetary policy in stabilizing inflation. We show that an increase in inflation disagreement reduces the sensitivity of inflation to changes in the output gap. Or equivalently, inflation disagreement flattens the Phillips curve. This result is formally stated in Proposition V.4.

Proposition V.4. *The effectiveness of contemporaneous monetary policy shocks on inflation decreases with inflation disagreement.*

[28]Introducing stochastic natural real interest rate (denoted as r_t^n) into the model obtains

$$\hat{C}_t = \underbrace{\frac{\mu+(1-\theta)\kappa}{\mu+\kappa}}_{\equiv\beta_1}\mathbb{E}_t\hat{C}_{t+1} - \underbrace{\frac{(1+\kappa)\mu}{\mu+\kappa}}_{\equiv\beta_2}(\hat{r}_{ft} - \mathbf{E}_t\hat{\pi}_{t+1} - \hat{r}_t^n) \tag{30}$$

where $\hat{r}_t^n$ denotes deviation of natural real interest rate from steady state. According to Prop. V.3 that β_2 is an increasing function of α, it is implied that higher inflation disagreement weakens the effects of shocks to r_t^n.

Proof. Using Equations (23), (24), and (27), we can derive the following inflation response to a monetary policy shock (assuming that $\hat{Z}_t = 0$)

$$\begin{aligned}
\hat{\pi}_t &= \varphi_y \hat{w}_t + \beta E_t \hat{\pi}_{t+1} \\
&= \varphi_y \left[\frac{\mu + \kappa(1-\theta)}{(1+\kappa)\,\mu} \hat{C}_t + \gamma \hat{N}_t \right] + \beta E_t \hat{\pi}_{t+1} \\
&= \varphi_y \left[\frac{\mu + \kappa(1-\theta)}{(1+\kappa)\,\mu} \hat{C}_t + \gamma \hat{C}_t \right] + \beta E_t \hat{\pi}_{t+1} \\
&= \varphi_y (\gamma + \frac{\mu + \kappa(1-\theta)}{(1+\kappa)\,\mu}) \hat{C}_t + \beta E_t \hat{\pi}_{t+1} \qquad (31)
\end{aligned}$$

where $\beta_3 \equiv \gamma + \frac{\mu+\kappa(1-\theta)}{(1+\kappa)\mu}$. From Propositions V.2 and V.3, greater inflation disagreement implies smaller sensitivity of aggregate consumption to monetary policy shocks (either to FG or to FFR). It follows immediately from Eq. (31) that inflation disagreement also reduces the sensitivity of inflation to monetary policy shocks. □

Overall, our model predicts that inflation disagreement attenuates the power of both forward guidance and conventional monetary policy. These predictions are consistent with our empirical evidence.

VI. Supporting evidence for the model mechanism

The model mechanism relies on the interactions between inflation disagreement and borrowing constraints. Borrowing constraints introduce an asymmetry in the responses of agents with different inflation beliefs. An agent with higher perceived future inflation has also a lower perceived real interest rate; as such, the agent has a higher MPC and is thus more likely to face binding borrowing constraints. In contrast, an agent who has a lower perceived future inflation rate is unconstrained.

The interactions between belief heterogeneity and borrowing constraints lead to three testable implications of the model: First, agents with higher inflation expectations are more likely to experience binding borrowing constraints. Second, inflation disagreement should have a stronger attenuation effect on the transmission of monetary policy when inflation expectations are more skewed to the upside. Third, the attenuating effect should also be stronger when aggregate credit availability is more stringent (i.e., the borrowing limit is lower for all agents). We have presented evidence supporting the second implication that positive skewness in inflation forecast amplifies the attenuation effects of inflation disagreement. We now present some evidence that supports the model mechanism along the other two dimensions.

VI.1. **Subjective inflation expectations and perceived borrowing constraints.** To investigate the linkage between individual inflation expectations and their perceived borrowing constraints, we use the micro-level data from the SCE, which contains information about individual expectations of future inflation, current and future perceived borrowing conditions, and their consumption spending plans. It also includes the respondents' socioeconomic and demographic characteristics such as age and employment status.

We use a monthly panel from June 2013 to December 2023 to estimate the regression

$$y_{jt} = b_0 + b_1 Exp_Inflation_{jt} + b_2 \Phi_{jt} + \mu_j + \gamma_t + \varepsilon_{jt}, \tag{32}$$

where we consider three alternative dependent variables (y_{jt}), including (1) an indicator of household j's willingness to increase consumption; (2) an indicator of the household's perceived difficulty in credit access (i.e., obtaining credit and loans) in the current period; or (3) an indicator of perceived difficulty in credit access over the next 12 months.[29] The main independent variable is $Exp_Inflation_{jt}$, which denotes the household's inflation expectations over the next year. The term Φ_{jt} denotes a set of socioeconomic and demographic control variables that include the respondents' current income, expected income growth, employment status, age, and survey tenure. We also control for the individual fixed effect (μ_j) and the time fixed effect (γ_t).

Table 2 shows the estimation results. Consistent with the implications of our model, individuals with higher subjective inflation expectations are more willing to increase consumption spending. Higher inflation expectations are also associated with perceptions of harder credit access, both in the current month and over the next 12 months, corroborating our model's key mechanism that agents with higher inflation expectations expect tighter borrowing constraints.

VI.2. **Aggregate conditions for credit access.** Our model implies that the attenuation effects of inflation disagreement depend crucially on aggregate conditions for credit access. When credit is easier to obtain (i.e., when the borrowing limits are expanded), inflation disagreement can still attenuate the power of monetary policy shocks, but the attenuation effects would be weaker. We now provide some empirical evidence supporting this model implication.

To implement this idea, we estimate a variation of our baseline local projection specification by including a tipple interaction term between lending standards, inflation disagreement,

[29]The indicator of willingness to increase consumption is based on answers to Q26 in the survey and equals 100 if the respondent plans to increase their spending and zero otherwise. The indicator of perceived credit access is based on the responses to Q28 and Q29 in the survey and uses a five-level scale. For example, we assign a value of 5 if the respondent reports "much harder" and 1 if "much easier."

TABLE 2. Relations of subjective inflation expectations with consumers' willingness to spend and perceived credit access conditions

Dep. Var.	Willing to Increase Spending	Harder Credit Access Current period	Harder Credit Access Next 12 months
	(1)	(2)	(3)
Exp_Inflation	0.084***	0.003***	0.006***
	(0.017)	(0.001)	(0.001)
Income	-0.081	-0.011***	-0.014***
	(0.137)	(0.004)	(0.004)
Exp_IncomeGrowth	0.245***	-0.003***	-0.004***
	(0.059)	(0.001)	(0.001)
Employed	2.679***	-0.047	-0.011
	(0.850)	(0.030)	(0.028)
Individual Fixed Effect	Yes	Yes	Yes
Time Fixed Effect	Yes	Yes	Yes
Observations	104,626	104,643	104,639
No. of Unique I.D.	15,497	15,501	15,501

Note: This table shows the estimation results from the empirical specification (32). The three columns correspond to the regressions with the three different dependent variables: (1) individual willing to increase spending, which equals 100 if the household plans to increase consumption spending in the current month and zero otherwise; (2) perceived harder credit access in the current period; and (3) perceived harder credit access over the next 12 monthson one-year ahead inflation expectations The values of the perceived credit access indicators range from 5 to 1, corresponding to the individuals' perceived credit access as much harder, somewhat harder, the same, somewhat easier, or much easier. All regressions use household-level data from the Survey of Consumer Expectations by the New York Fed, with the sample covering the periods from June 2013 to December 2023. The standard errors shown in the parentheses are clustered by respondent. The stars denote the p-values: $^{*}\, p < 0.1$; $^{**}\, p < 0.05$; $^{***}\, p < 0.01$.

and monetary policy shocks. Specifically, we estimate the following local projections:

$$\log(y^j_{t+h}) - \log(y^j_{t-1}) = \alpha^h_0 + \alpha^h_1 MP_t + \alpha^h_2 IQR^{\pi}_{t-1} + \alpha^h_3 IQR^{\pi}_{t-1} * MP_t + \alpha^h_4 LoanStd_t * IQR^{\pi}_{t-1}$$
$$+\alpha^h_5 LoanStd_t * FG_t + \alpha^h_6 IQR^{\pi}_{t-1} * LoanStd_t * MP_t + \alpha^h_7 \Gamma_{t-1} + \varepsilon_{t+h}. \quad (33)$$

Here, the variable *LoanStd* denotes the lending standards measured by the net percentage of domestic banks reporting increased willingness to make consumer installment loans, with data obtained from the Federal Reserve's Senior Loan Officer Opinion Survey on Bank Lending Practices (SLOOS).[30] A higher value of *LoanStd* indicates more favorable lending standards. The other variables are defined in the baseline specification. According to our theory, easing lending standards should mitigate the attenuation effects of inflation disagreement for monetary policy shocks (i.e. $\alpha_6 < 0$).

The impulse responses to an FG shock shown in Figures 9 are consistent with our model's mechanism. The figures show that inflation disagreement attenuates the power of forward guidance ($\alpha_3 > 0$), but the attenuation effects are partly mitigated by more favorable lending

[30]The time series of lending standards is available from FRED (series ID: DRIWCIL) at the quarterly frequency from the second quarter of 1982. Since the survey is conducted at quarterly frequency, we interpolate the data into monthly, assuming that the lending standards remain the same within each quarter. We include $LoanStd_{t-1}$ in the set of macroeconomic controls (Γ_{t-1}).

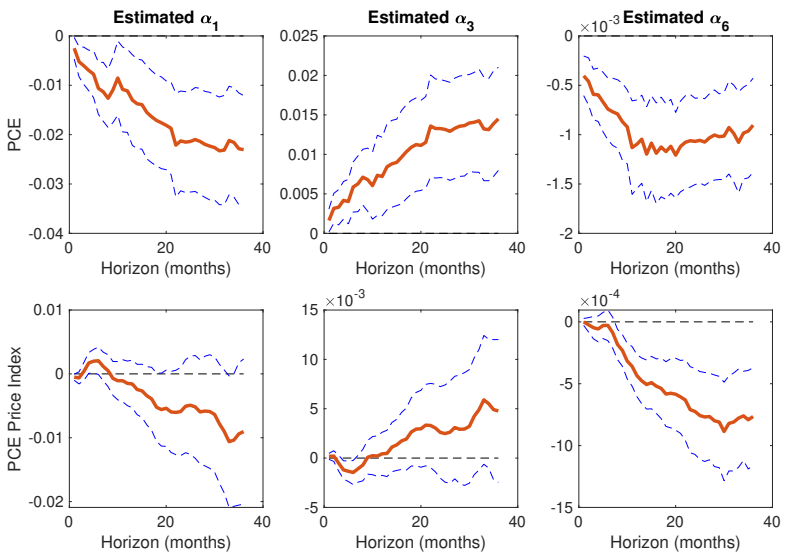


FIGURE 9. Cumulative impulse responses to a forward guidance shock: Effects of lending standard

Note: This figure shows the cumulative impulse responses of real personal consumption expenditure (PCE) and PCE price index to a forward guidance shock from the local projections model (33). The dashed lines show the 68% confidence intervals based on a Newey-West heteroscedasticity and autocorrelation consistent (HAC) estimator.

conditions to consumers ($\alpha_6 < 0$). We obtain similar results for the FFR shock (see Appendix A.5).

VII. Conclusion

Survey data shows pervasive and time-varying disagreement among consumers in their inflation expectations. We provide empirical evidence that inflation disagreement weakens the power of both forward guidance and conventional monetary policy. Absent inflation disagreement, a surprise tightening of monetary policy—either through forward guidance or the federal funds rate—would lead to persistent and significant declines in consumer spending and inflation. However, in periods with high inflation disagreement, the recessionary effects of the monetary policy shock would be significantly attenuated. These empirical findings are robust. Furthermore, positive skewness of inflation forecasts strengthens the attenuation effect of inflation disagreement. Importantly, our evidence suggests that the attenuation effect is not driven by endogenous responses of inflation disagreement to other contemporaneous shocks.

These empirical observations can be rationalized by a simple theoretical New Keynesian model featuring belief heterogeneity and borrowing constraints. In the model, those households who believe that the central bank has a higher inflation target have a lower perceived

real interest rate, resulting in a high propensity to consume. Consumption spending can be partly financed by external debt, subject to borrowing constraints. Higher inflation disagreement results in a greater mass of households who face binding borrowing constraints, resulting in sluggish adjustments in consumption spending and aggregate inflation following a monetary policy shock. This model mechanism is supported by empirical evidence and it provides a microeconomic foundation for Euler equation discounting that helps resolve the forward guidance puzzle.

Our findings have important policy implications. For example, in response to the post-pandemic surge in inflation, the Federal Reserve aggressively tightened monetary policy by rapidly raising the federal funds rate target from near zero to a range between 5.25 percent and 5.5 percent. In addition, the Fed signaled that policy would remain tight until substantial progress has been made toward the 2 percent inflation goal. Despite these policy actions, consumer spending remained resilient and inflation remained persistently above 2 percent. Our findings suggest that elevated inflation disagreement during much of the post-pandemic period may have weakened the power of monetary policy.

To maintain analytical tractability, we have intentionally kept the theoretical model simple by abstracting from many realistic features of the economy. For example, the agents in our model are extremely naive, with their beliefs following an i.i.d. distribution. In a more realistic environment with persistent beliefs, agents could learn from their past mistakes, and such learning may have important consequences for the transmission of monetary policy. Our model also abstracts from other plausible sources of heterogeneity, such as heterogeneity in income or wealth that may give rise to precautionary savings, a crucial feature of the HANK models that helps alleviate the forward guidance puzzle (McKay et al., 2016). We conjecture that incorporating those realistic features into our model would provide a richer framework for studying the quantitative importance of belief heterogeneity in explaining the empirical observations. It would also make the framework more useful for evaluating alternative policies, such as the role of fiscal and monetary policy coordination in stabilizing inflation and macroeconomic fluctuations. Our work represents a small first step toward that promising avenue for future research.

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Online Appendix

Inflation Disagreement Weakens the Power of Monetary Policy

Ding Dong, Zheng Liu, Pengfei Wang, and Min Wei

Appendix A. Robustness of empirical results

A.1. **Alternative measures of inflation disagreement.**

A.1.1. *Disagreement about long-term inflation.* Figures A.1 and A.2 show the impulse responses to a forward guidance shock and a federal funds rate shock, respectively, from the estimated local projections with inflation disagreement measured by the IQR of 5-10 year ahead inflation forecasts in the Michigan survey (normalized by the median of the 5-10 year ahead inflation expectations). Evidently, the attenuation effects of long-term inflation disagreement are similar to those of the short-term disagreement in our baseline empirical model. Quantitatively, a one-standard-deviation increase in long-term inflation disagreement would weaken the cumulative effects of a forward guidance shock on consumption expenditures and the price level by 12.4% and 11.25% respectively[1], and those of a federal fund rate shock by 12.6% and 11.9% respectively[2].

A.1.2. *Alternative measure of inflation disagreement.* We re-estimate the baseline local projections specification (1) using either a purified measure or an othogonalized measure of inflation disagreement.

[1]The FG shock has a standard deviation of 0.87 and the long-term inflation disagreement measure has a standard deviation of 0.125. The estimated $\alpha_1^{h=24}$ and $\alpha_3^{h=24}$ on cumulative PCE change are -0.030 and 0.0298 respectively. A one-standard-deviation higher inflation disagreement would thus reduce the effects of a forward guidance shock by $0.125 * 0.0298/0.030 * 100 = 12.4\%$. The estimated $\alpha_1^{h=24}$ and $\alpha_3^{h=24}$ on cumulative PCE price level change are -0.010 and 0.009 respectively, implying an attenuation effect of $0.125 * 0.0/0.0 * 100 = 11.25\%$.

[2]The federal fund rate shock has a standard deviation of 0.75 and the long-term inflation disagreement measure has a standard deviation of 0.125. The estimated $\alpha_1^{h=24}$ and $\alpha_3^{h=24}$ on cumulative PCE change are -0.051 and 0.052 respectively. A one-standard-deviation higher inflation disagreement would thus reduce the effects of a forward guidance shock by $0.125 * 0.052/0.051 * 100 = 12.6\%$. The estimated $\alpha_1^{h=24}$ and $\alpha_3^{h=24}$ on cumulative PCE price level change are -0.0154 and 0.0146 respectively, implying an attenuation effect of $0.125 * 0.0146/0.0154 * 100 = 11.9\%$.

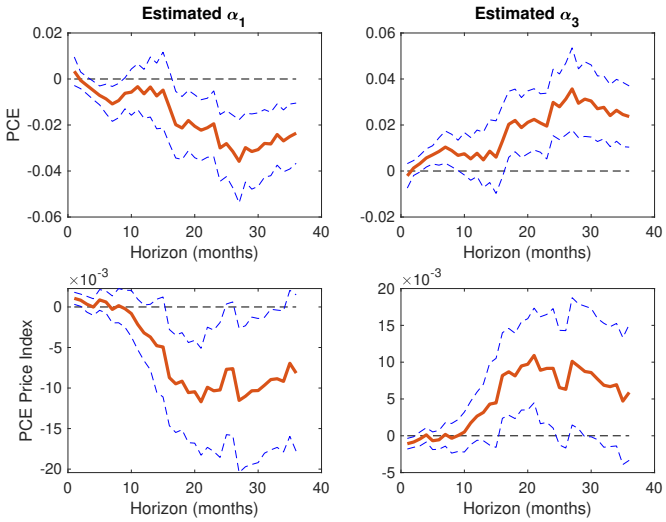


FIGURE A.1. Estimated impulse response to a forward guidance shock (disagreement about long-term inflation)

Note: This figure shows the cumulative responses of real personal consumption expenditure (PCE) and the PCE price index following a forward guidance shock using disagreement measure based on 5-10 year ahead inflation forecast. The solid lines show the point estimates of the impulse responses. The dashed lines show the 68% confidence intervals based on a Newey-West heteroscedasticity and autocorrelation consistent (HAC) estimator.

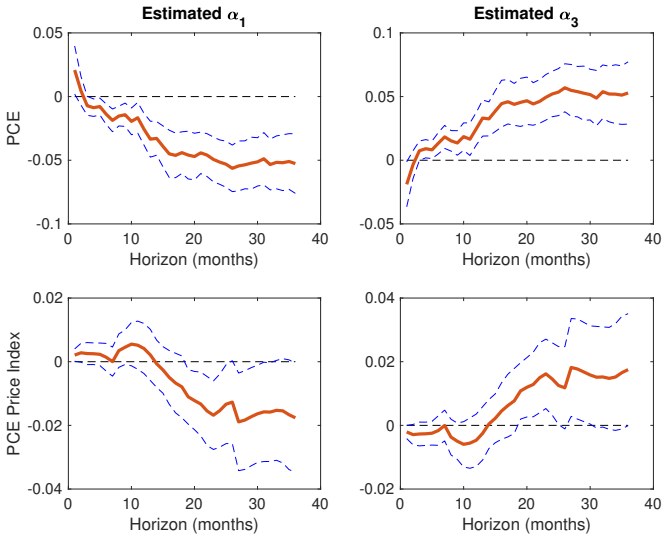


FIGURE A.2. Estimated impulse response to federal fund rate shocks (disagreement about long-term inflation)

Note: This figure shows the cumulative responses of real personal consumption expenditure (PCE) and the PCE price index following a forward guidance shock using disagreement measure based on 5-10 year ahead inflation forecast. The solid lines show the point estimates of the impulse responses. The dashed lines show the 68% confidence intervals based on a Newey-West heteroscedasticity and autocorrelation consistent (HAC) estimator.

To examine whether the attenuation effect of inflation disagreement is driven by various demographic factors, we use the cross-sectional archives of individual responses in the Michigan survey from July 1991 to December 2023 to estimate the panel data specification[3]

$$\pi^e_{jt} = \gamma_t + \beta Z_{jt} + \varepsilon_{jt}, \tag{A.1}$$

where π^e_{jt} denotes one-year ahead CPI inflation expectations of individual j in period t; γ_t denotes a time fixed effect, capturing the responses of individual inflation expectations to changes in aggregate economic conditions; Z_{jt} is a vector of individual demographic characteristics, including income (in log units), home ownership status, region of residence, education, sex, and marital status; and ε_{jt} denotes the regression residual, which measures the individual inflation expectations that are not explained by the demographic factors and aggregate shocks.

Figure A.3 plots the purified measure (blue dashed line) and the orthogonalized measure (black dashed line). Evidently, those measures are both highly correlated with the raw measure of inflation disagreement used in our baseline regressions (red solid line).

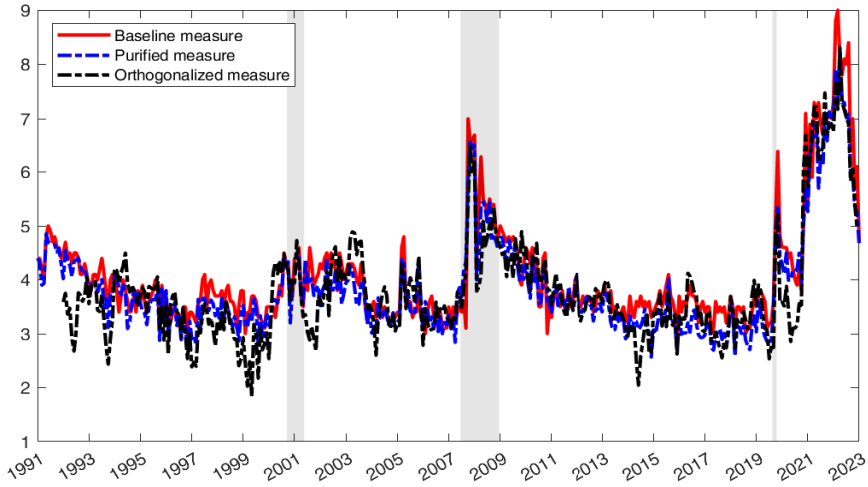


FIGURE A.3. Alternative measures of inflation disagreement.

Note: This figure shows three measures of inflation disagreement. The red solid line shows the baseline measure of inflation disagreement, defined as the IQR of the one-year ahead CPI inflation forecasts from the Michigan Survey of Consumers. The blue dashed line shows the purified measure of inflation disagreement, defined as the IQR of the one-year ahead inflation forecasts that are unexplained by demographics and aggregate shocks (i.e., the IQR of ε_{jt} in Eq. (A.1)). The black dashed line shows the orthogonalized inflation disagreement, defined as the components of the purified inflation disagreement that are unexplained by current and lagged values of monetary policy surprises and oil supply shocks.

[3]The data is accessible via https://data.sca.isr.umich.edu/findings/findings.php. Information on homeownership is not available for surveys in 1991 and 1992, so we drop the regressants for these two years. We exclude individual responses with recorded inflation expectation above 20% or below -20%.

Figures A.4 and A.5 show that the impulse responses of real PCE and inflation to an FG shock and a federal fund rate shock, respectively, are also similar to those obtained using the raw measure of inflation disagreement.

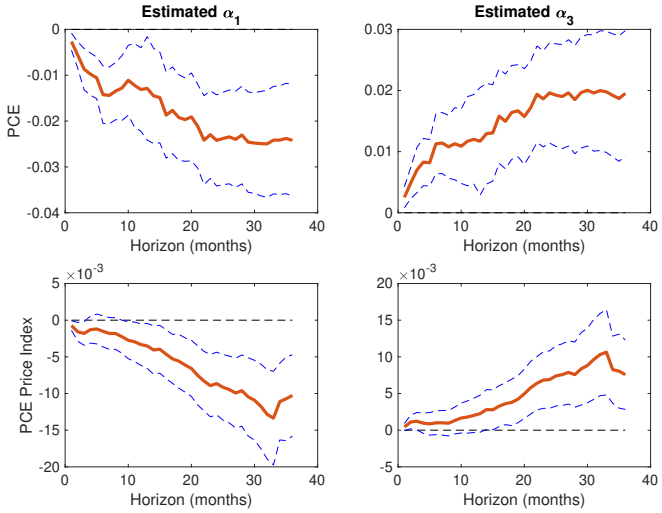


FIGURE A.4. Estimated impulse response to forward guidance shocks (orthogonalized measure of inflation disagreement)

Note: This figure shows the cumulative responses of real personal consumption expenditure (PCE) and the PCE price index following a forward guidance shock using the orthogonalized inflation disagreement measure based on the residuals from the regression of the purified measure (i.e., the IQR of ε_{jt} in Eq. (A.1)) on current and lagged values of monetary policy surprises and oil supply shocks. The solid lines show the point estimates of the impulse responses. The dashed lines show the 68% confidence intervals based on a Newey-West heteroscedasticity and autocorrelation consistent (HAC) estimator.

A.1.3. *State-level inflation disagreement.* The heat map in Figure A.6 illustrates the variations in inflation disagreement across selected U.S. States from 2013 to 2023, with darker shades indicating higher levels of disagreement. Some states consistently report higher levels of inflation disagreement, likely reflecting divergences in economic and other local factors within the state that might influence residents' inflation views.

A.1.4. *MSA-level inflation disagreement and the price response.* In Section IV.2, we present state-level evidence demonstrating that inflation disagreement diminishes the impact of monetary policy on unemployment. The BLS does not publish state-level price indice, but it does publish the CPI data for 23 metropolitan statistical areas (MSAs).[4]. Consequently, we study the attenuating effect of inflation disagreement on the price level responses using the MSA-level data to by first constructing the MSA-level inflation disagreement based on

[4]We do not use the state-level price indices constructed by Hazell et al. (2022) becasue their sample overlaps with that of the Survey of Consumer Expectations only for the periods from 2013:Q2 to 2017:Q4, which is too short for our analysis.

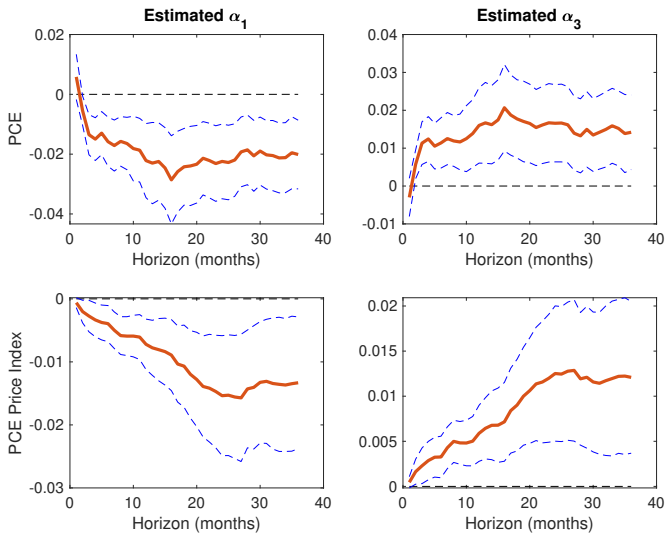


FIGURE A.5. Estimated impulse response to federal fund rate shocks (orthogonalized measure of inflation disagreement)

Note: This figure shows the cumulative responses of real personal consumption expenditure (PCE) and the PCE price index following a federal fund rate shocks using the orthogonalized inflation disagreement measure based on the residuals from the regression of the purified measure (i.e., the IQR of ε_{jt} in Eq. (A.1)) on current and lagged values of monetary policy surprises and oil supply shocks. The solid lines show the point estimates of the impulse responses. The dashed lines show the 68% confidence intervals based on a Newey-West heteroscedasticity and autocorrelation consistent (HAC) estimator.

the state(s) where the MSA resides and then matching them with the state-level inflation disagreement data from the SCE. We estimate the panel local projections using the following equation at quarterly frequency

$$\log(P_{m,t+h}) - \log(P_{m,t-1}) = \beta_1^h IQR_{m,t-1}^{\pi} + \beta_2^h IQR_{m,t-1}^{\pi} * MP_t + \mu_m^h + \gamma_t^h + \varepsilon_{m,t+h}. \quad \text{(A.2)}$$

with h = 0, 1, ..., 8. In this specification $\log(P_{m,t+h}) - \log(P_{m,t-1})$ denotes the cumulative change in the consumer price index in MSA m, $IQR_{m,t}^{\pi}$ is the MSA-level inflation disagreement in quarter t, measured by the IQR of inflation expectations within the MSA, scaled by the median inflation forecast in the corresponding MSA,[5] and MP_t is the aggregate monetary policy shock (either an FG shock or an FFR shock). The time fixed effect γ_t^h captures the average effects from monetary policy and other aggregate shocks across all MSAs, while the coefficient β_2^h captures the differential effect in MSAs with higher inflation disagreement: A positive value of β_2^h implies that falls rises less in response to a policy tightening in a MSA with a higher level of inflation disagreement. We also control for the MSA-fixed effect μ_m^h. The sample covers 2013Q2 to 2023Q3.

[5]For MSAs spanning multiple states, we take the simple average of inflation disagreement across those states.

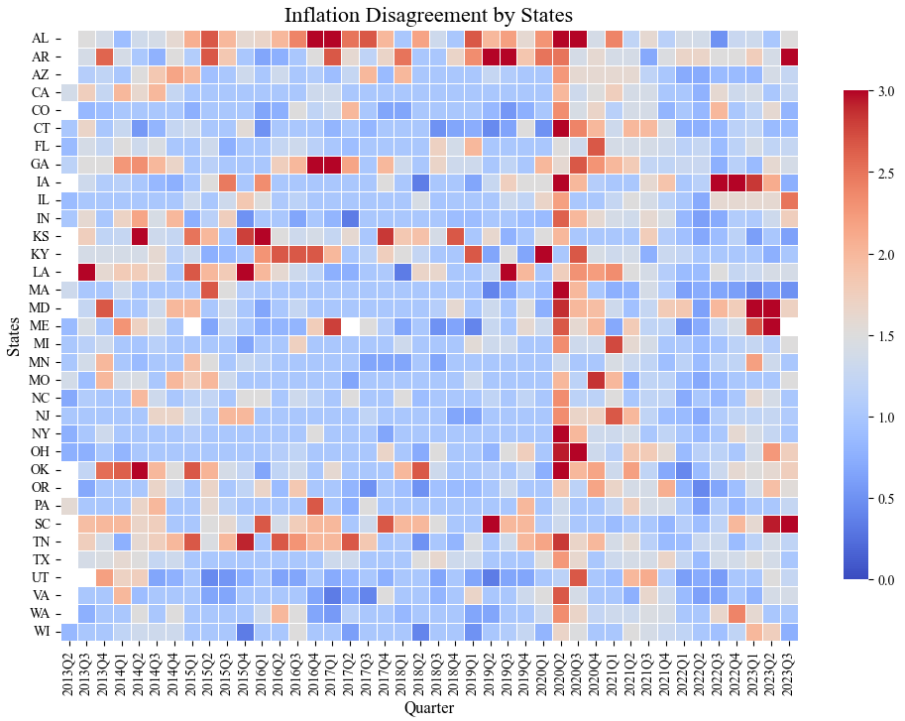


FIGURE A.6. Inflation disagreement across selected states over time

Note: The heat map plots inflation disagreement for 34 U.S. states with 20 observations or more in each quarter over our sample period of 2013Q2 to 2023Q3. Inflation disagreement is measured by the IQR of inflation expectations among Survey of Consumer Expectations respondents in each State and each quarter, scaled by the median inflation forecast from that state in the quarter.

The heat map in Figure A.7 illustrates the variation in inflation disagreement across the 23 MSAs from 2013 to 2023, with darker shades indicating higher levels of disagreement. Similar to states, some MSAs also consistently report higher levels of disagreement.

Figure A.8 plots the estimated cumulative responses of MSA-level CPI following a contractionary FG shock (left panel) and FFR shock (right panel). These estimates are significantly positive beyond the first few months, indicating that the contractionary effects of monetary policy shocks on CPI are indeed weaker in states with higher inflation disagreement.

A.1.5. *Experience-based measure of inflation disagreement.* We re-estimate the baseline local projections specification in Eq. (1) with an experience-based measure of inflation disagreement, which is constructed using the cohort-specific inflation forecasts from Nagel (2024). Figure A.9 plots the experience-based measure (red dashed line) against the baseline measure (black solid line). Notably, the experience-based measure was lower and more stable than the baseline measure until around 2004 and rose much less post-pandemic, the latter reflecting the slow-moving nature of the experience-based inflation forecasts.

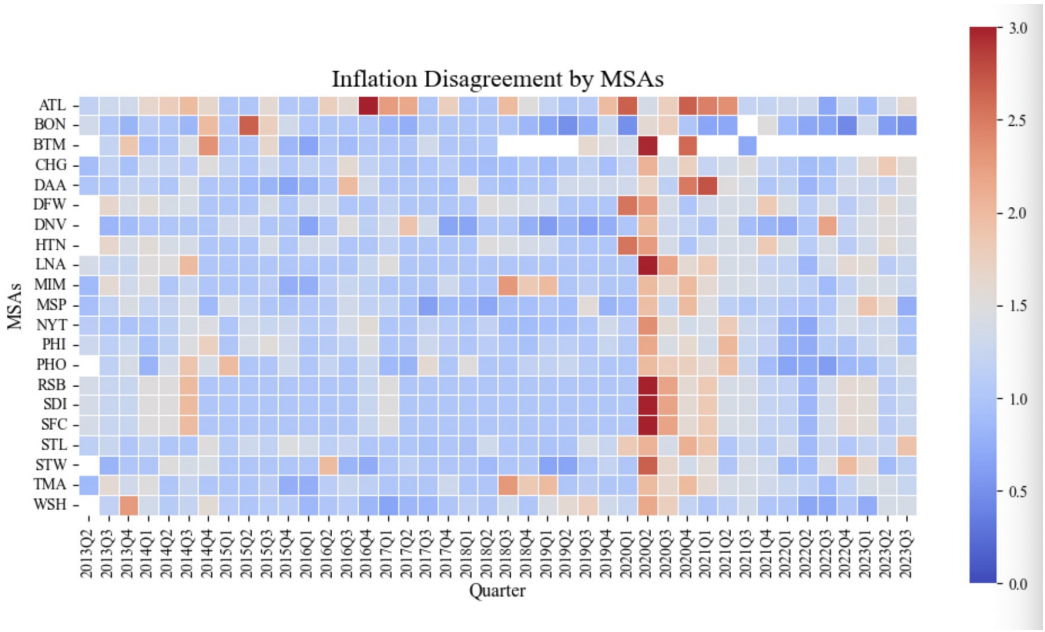


FIGURE A.7. Inflation disagreement across MSAs and over time

Note: The heat maps plots inflation disagreement from selected MSAs over the quarterly sample period of 2013Q2 to 2023Q3. Inflation disagreement is measured by the IQR of inflation expectations among the Survey of Consumer Expectations respondents in each MSA and each quarter, scaled by the median inflation forecast from that MSA in the quarter. For MSAs spanning multiple states, we take the simple average of inflation disagreement across those states.

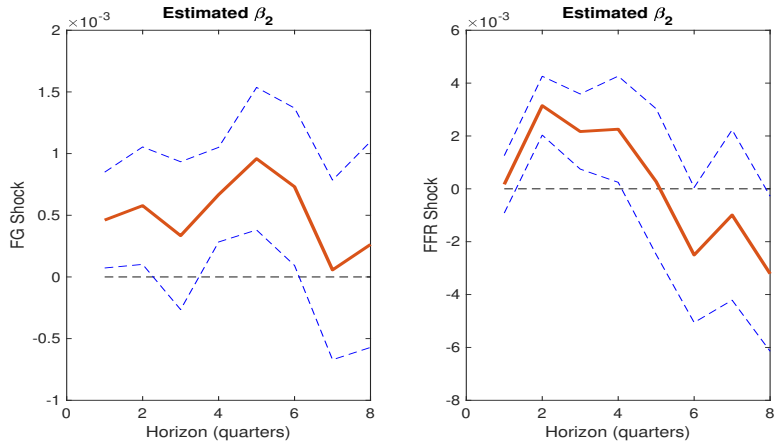


FIGURE A.8. Cumulative impulse responses of CPI to a monetary policy shock: MSA-level data

Note: This figure shows the cumulative impulse responses of MSA-level CPI index following a contractionary shock to forward guidance (left panel) or to the federal fund rate (right panel) estimated from the local projections model (3). The solid lines show the point estimates of the impulse responses. The dashed lines show the 68% confidence intervals with standard errors clustered at the MSA-level.

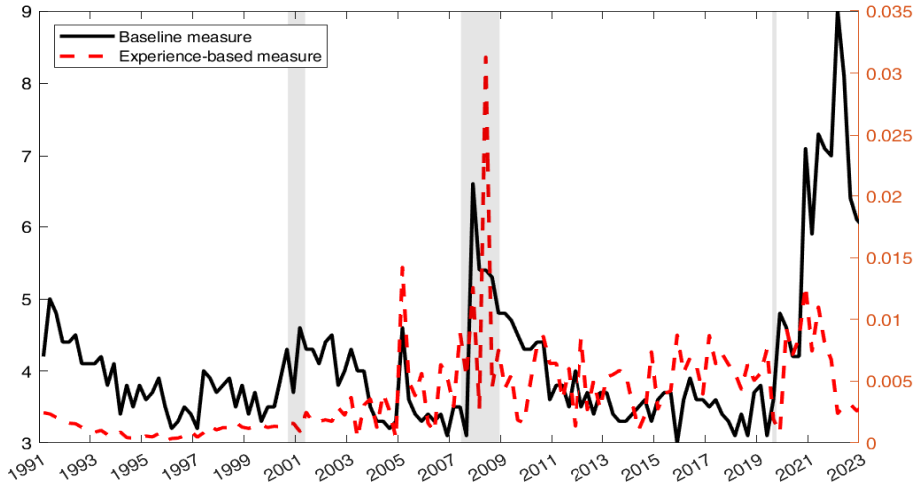


FIGURE A.9. Experience-based measures of inflation disagreement.

Note: This figure shows two measures of inflation disagreement. The black solid line shows the baseline measure of inflation disagreement, defined as the IQR of the one-year ahead CPI inflation forecasts from the Michigan Survey of Consumers. The red dashed line shows the experience-based measure of inflation disagreement, defined as the IQR of the cohort-specific components in one-year ahead inflation forecasts constructed by Nagel (2024). The correlation between two quarterly series is 0.25.

Figure A.10 shows that the impulse responses of real PCE and inflation to federal fund rate shocks are also similar to those obtained using the raw measure of inflation disagreement.

A.2. **Positive skewness of inflation expectations and the FFR shock.** As in the case of forward guidance shocks, Figure A.11 shows that the attenuation effects of inflation disagreement for federal fund rate shocks are also amplified by positive skewness of the inflation forecast distribution.

A.3. **Other potential confounding factors.** According to the standard Euler equation, household's consumption-saving decisions are affected by many factors, including the nominal saving rate, expected inflation, the discount factor, and expected future income changes. While this paper focuses on the effect of inflation disagreement on aggregate consumption and its response to monetary policy shocks, the expectation and disagreement about other variables could confound the attenuating effect of inflation disagreement. For example, the households with high income expectation may increase current spending to smooth consumption. Leduc and Liu (2016) shows a negative effect of consumer uncertainty on aggregate consumption and price level through a real option-value channel.

To test the robustness of our empirical results, we include various additional potential confounding factors in the baseline specification (1). In particular, we consider the empirical

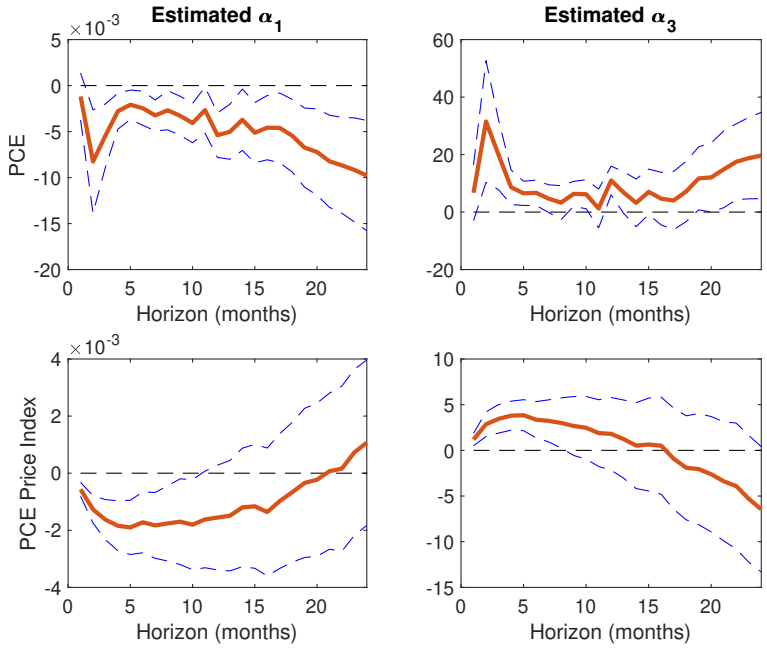


FIGURE A.10. Estimated impulse response to a federal fund rate shock (experience-based measure of inflation disagreement)

Note: This figure shows the cumulative responses of real personal consumption expenditure (PCE) and the PCE price index following a federal fund rate shock using the experience-based inflation disagreement measure (i.e., the IQR of cohort-specific components in inflation forecast, normalized by the median one-year ahead inflation forecast in the Michigan survey). The solid lines show the point estimates of the impulse responses. The dashed lines show the 68% confidence intervals based on a Newey-West heteroskedasticity and autocorrelation consistent (HAC) estimator. Each period represents a month.

specification

$$\log(y^j_{t+h}) - \log(y^j_{t-1}) = \alpha^h_0 + \alpha^h_1 FG_t + \alpha^h_2 IQR^\pi_{t-1} + \alpha^h_3 IQR^\pi_{t-1} * FG_t + \alpha^h_4 X_{t-1} + \alpha^h_5 X_{t-1} * FG_t + \alpha^h_6 \Gamma_{t-1} + \varepsilon_{t+h}, \tag{A.3}$$

where X_t denotes the additional control variable. The set of those control variables that we consider include (1) the median of one-year ahead inflation expectations from the Michigan survey (model 1); (2) short-term inflation uncertainty index constructed by Binder (2017) from the Michigan survey (model 2); (3) the median of one-year ahead income growth expectation of consumers from the Michigan survey (model 3); (4) (one-year ahead) income disagreement from the Michigan survey (model 4); (5) consumers' perceived uncertainty concerning vehicle purchases from the Michigan survey (model 5); (6) forecast disagreement in the private sector about future two-year Treasury rates from the Blue Chip, and (7) forecast disagreement about one-year ahead CPI from the Blue Chip. We include an additional control variable and its interaction with the policy shock one at a time.

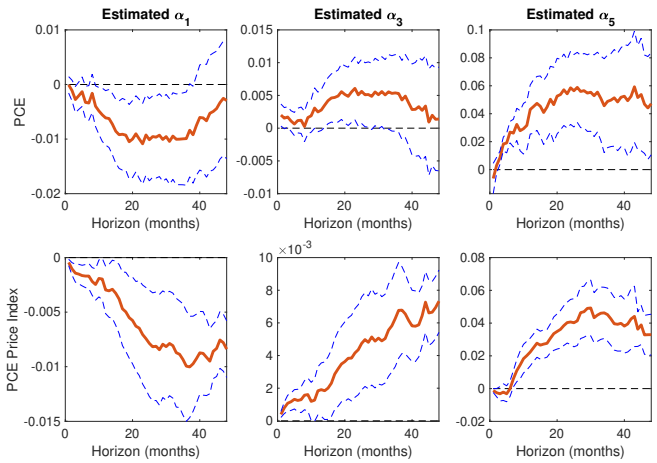


FIGURE A.11. Cumulative impulse responses to a federal funds rate shock: effects of positive skewness of the inflation forecast distribution.

Note: This figure shows estimated impulse responses of monthly real personal consumption expenditure (PCE), and PCE price index to federal fund rate shocks from the local projections model (2) (where FG is replaced by MP). The solid lines show the point estimates of the impulse responses. The dashed lines show the 68% confidence intervals based on a Newey-West heteroscedasticity and autocorrelation consistent (HAC) estimator.

The impulse responses to a forward guidance shock in these 7 alternative specifications are shown in Figures A.12 through A.19 The impulse responses to a federal funds rate shock in these alternative specifications are shown in Figures A.20 through A.27.

In each case, we find that inflation disagreement attenuates the power of forward guidance shocks and the fed funds rate shocks.

A.4. **Alternative measures of real activity and inflation.** We replace the LHS of Eq. (1) by cumulative growth rates of unemployment, industrial production and CPI. Figure A.28 shows the impulse responses of monthly unemployment rate (upper panel), industrial production (mid panel) and consumer price index (lower panel) to a forward guidance shock.

The upper panel of Figure A.28 shows that an identified forward guidance shock in absence of disagreement is followed by a rise in the unemployment rate ($\alpha_1^h > 0$), but the effect is mitigated if the current state is characterized by a rise in inflation disagreement ($\alpha_3^h < 0$). Similar results are obtained from regressions for industrial production (mid panel), indicating that a positive forward guidance shock predicts a decline in output ($\alpha_1^h < 0$), but the effect is again mitigated in states with high inflation disagreement ($\alpha_3^h > 0$). For example, a one-standard-deviation higher inflation forecast disagreement will reduce the effects of forward guidance shocks on 2-year ahead unemployment and industrial production by 34.7% and 34.2% respectively. Forward guidance policy is also less effective in stabilizing inflation when it is carried out during times of high disagreement in inflation expectations *a la* Prop.

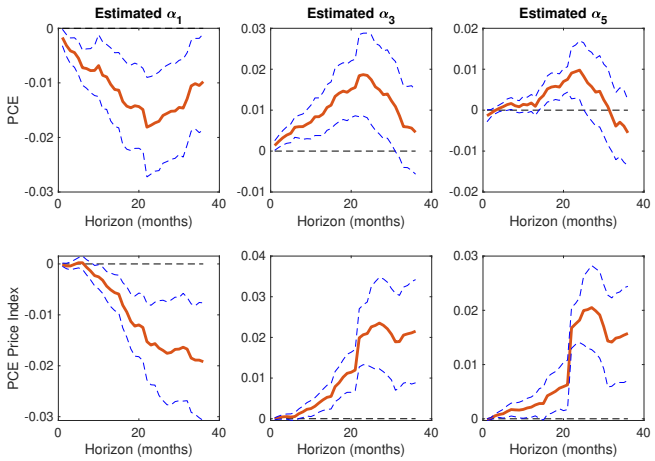


FIGURE A.12. Estimated response to forward guidance shocks (controlling for inflation expectation)

Note: This figure shows estimated impulse responses of monthly real personal consumption expenditure (PCE), and PCE price index to identified forward guidance shock from the local projections model (A.3), where we include the median inflation expectation and its interactions with the forward guidance shock. The solid lines show the point estimates of the impulse responses. The dashed lines show the 68% confidence intervals based on a Newey-West heteroscedasticity and autocorrelation consistent (HAC) estimator.

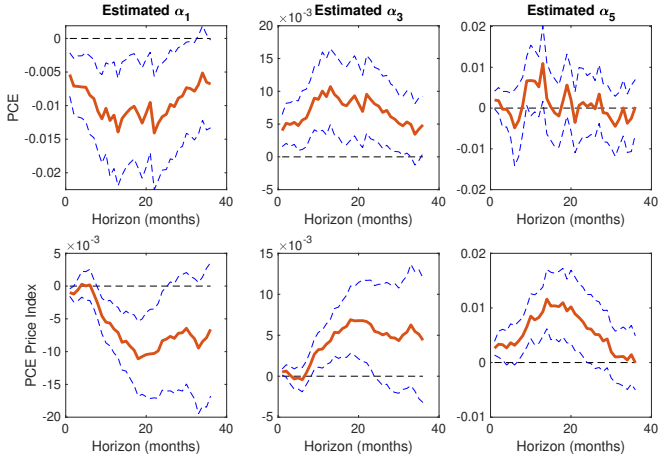


FIGURE A.13. Cumulative impulse responses to forward guidance shocks (controlling for consumer sentiment)

Note: This figure shows estimated impulse responses of monthly real personal consumption expenditure (PCE), and PCE price index to identified forward guidance shock from the local projections model (A.3), where the additional control variable X_t is the consumer sentiment index from Michigan Survey of Consumers. The solid lines show the point estimates of the impulse responses. The dashed lines show the 68% confidence intervals based on a Newey-West heteroscedasticity and autocorrelation consistent (HAC) estimator.

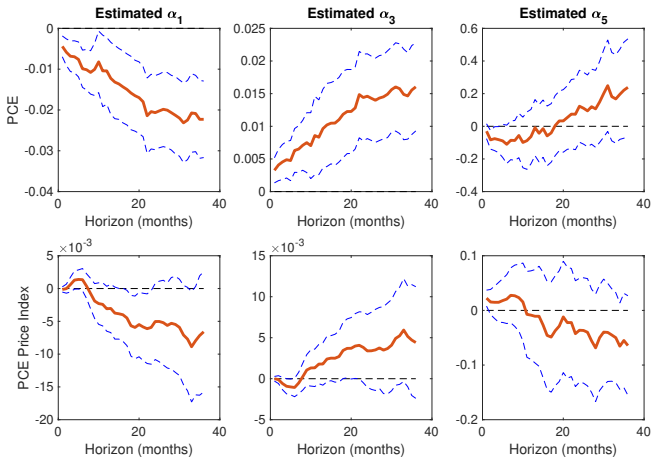


FIGURE A.14. Cumulative impulse responses to forward guidance shocks (controlling for expected income growth)

Note: This figure shows estimated impulse responses of monthly real personal consumption expenditure (PCE), and PCE price index to identified forward guidance shock from the local projections model (A.3), where the additional control variable X_t is the median income expectation from Michigan Survey of Consumers. The solid lines show the point estimates of the impulse responses. The dashed lines show the 68% confidence intervals based on a Newey-West heteroscedasticity and autocorrelation consistent (HAC) estimator.

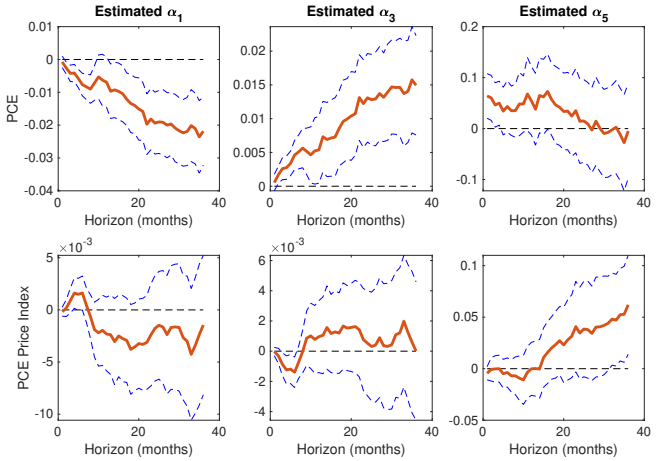


FIGURE A.15. Cumulative impulse responses to forward guidance shocks (controlling for consumer uncertainty)

Note: This figure shows estimated impulse responses of monthly real personal consumption expenditure (PCE), and PCE price index to identified forward guidance shock from the local projections model (A.3), where the additional control variable X_t is consumer uncertainty from Michigan Survey of Consumers. The solid lines show the point estimates of the impulse responses. The dashed lines show the 68% confidence intervals based on a Newey-West heteroscedasticity and autocorrelation consistent (HAC) estimator.

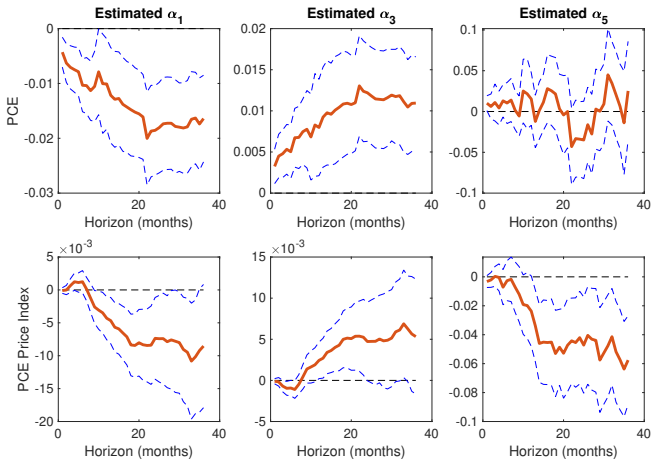


FIGURE A.16. Cumulative impulse responses to forward guidance shocks (controlling for income disagreement)

Note: This figure shows estimated impulse responses of monthly real personal consumption expenditure (PCE), and PCE price index to identified forward guidance shock from the local projections model (A.3), where the additional control variable X_t is consumer's income disagreement from Michigan Survey of Consumers. The solid lines show the point estimates of the impulse responses. The dashed lines show the 68% confidence intervals based on a Newey-West heteroscedasticity and autocorrelation consistent (HAC) estimator.

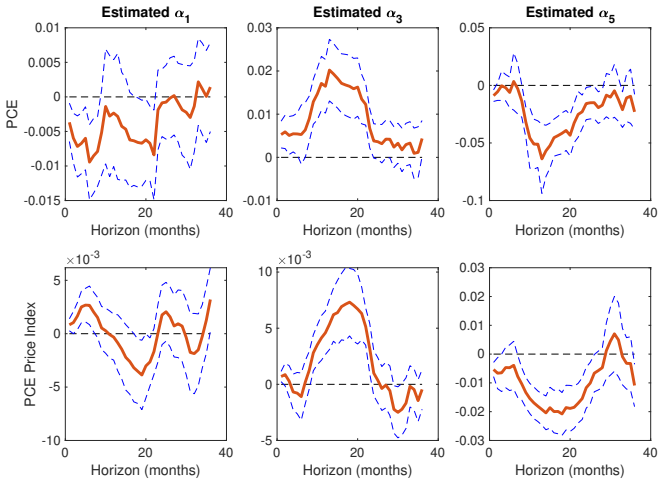


FIGURE A.17. Cumulative impulse responses to forward guidance shocks (controlling for inflation uncertainty)

Note: This figure shows estimated impulse responses of monthly real personal consumption expenditure (PCE), and PCE price index to identified forward guidance shock from the local projections model (A.3), where the additional control variable X_t is the inflation uncertainty measure from the Survey of Professional Forecasters. The solid lines show the point estimates of the impulse responses. The dashed lines show the 68% confidence intervals based on a Newey-West heteroscedasticity and autocorrelation consistent (HAC) estimator.

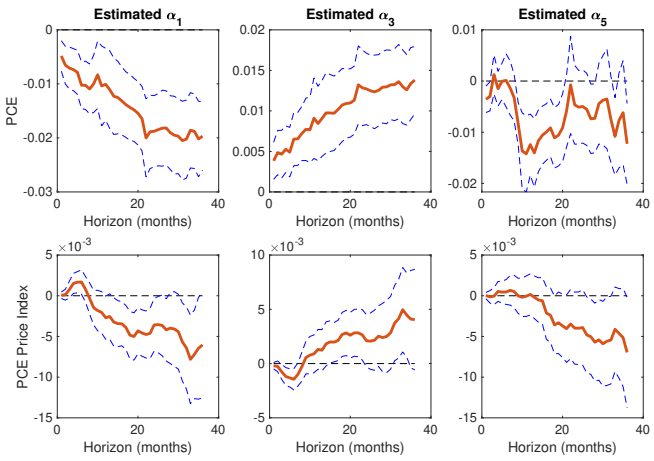


FIGURE A.18. Cumulative impulse responses to forward guidance shocks (controlling for forecast disagreement of 2-year Treasury yield)

Note: This figure shows estimated impulse responses of monthly real personal consumption expenditure (PCE), and PCE price index to identified forward guidance shock from the local projections model (A.3), where the additional control variable X_t is the forecast dispersion of the 2-year Treasury yields from the Blue Chip Financial Forecasts Database. The solid lines show the point estimates of the impulse responses. The dashed lines show the 68% confidence intervals based on a Newey-West heteroscedasticity and autocorrelation consistent (HAC) estimator.

V.4. The bottom panel of Figure A.28 supports this prediction: news about future monetary tightening helps stabilize the price level ($\alpha_1^h < 0$), but the effect is mitigated if the current economy features high inflation disagreement (i.e. $\alpha_3^h > 0$).

Similarly, we replace the LHS of Eq. (1) by cumulative growth rate of unemployment, industrial production and CPI, and estimate their responses to federal fund rate shocks.

The upper panel of Figure A.29 shows that a positive policy rate shock predicts a rise in the unemployment rate ($\alpha_1^h > 0$), but the effect is mitigated in a state with high inflation disagreement ($\alpha_3^h < 0$). Similar results are obtained from regression on output (middle panel): a positive policy rate shock predicts a decline in industrial production ($\alpha_1^h < 0$), but the effect is mitigated in state with high inflation disagreement ($\alpha_3^h > 0$). The attenuating effects on stabilizing inflation (lower panel) are also consistent with previous results on PCE price level.

A.5. **Liquidity constraint and the FFR shock.** As in the case of forward guidance shocks, Figure A.30 shows that the attenuation effects of inflation disagreement for federal fund rate shocks are also mitigated by the loosing of lending standard.

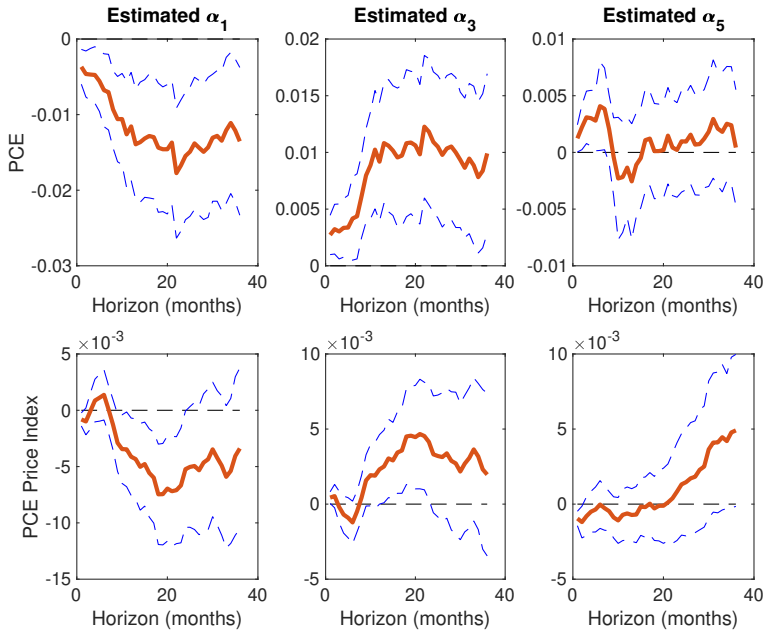


FIGURE A.19. Cumulative impulse responses to forward guidance shocks (controlling for forecast disagreement of 1-year ahead CPI)

Note: This figure shows estimated impulse responses of monthly real personal consumption expenditure (PCE), and PCE price index to identified forward guidance shock from the local projections model (A.3), where the additional control variable X_t is the forecast dispersion of one-year ahead CPI from the Blue Chip Financial Forecasts Database. The solid lines show the point estimates of the impulse responses. The dashed lines show the 68% confidence intervals based on a Newey-West heteroscedasticity and autocorrelation consistent (HAC) estimator.

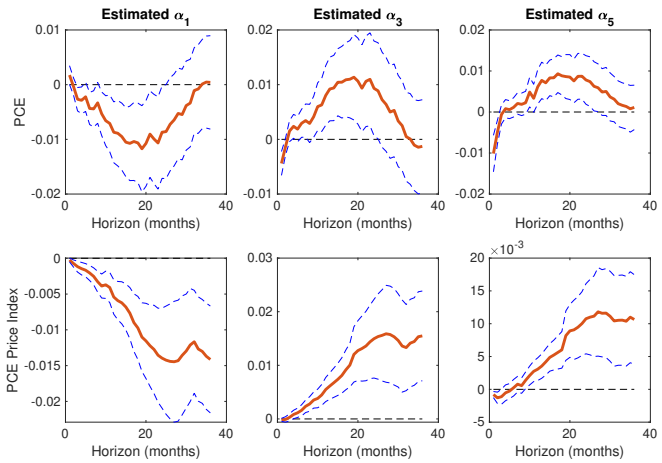


FIGURE A.20. Cumulative impulse responses to federal fund rate shocks (controlling for inflation expectation)

Note: This figure shows estimated impulse responses of monthly real personal consumption expenditure (PCE) and PCE price index to a federal funds rate shock from the local projections model (A.3) (with FG replaced by MP), where we include the median inflation expectation and its interactions with the federal funds rate shock. The solid lines show the point estimates of the impulse responses. The dashed lines show the 68% confidence intervals based on a Newey-West heteroscedasticity and autocorrelation consistent (HAC) estimator.

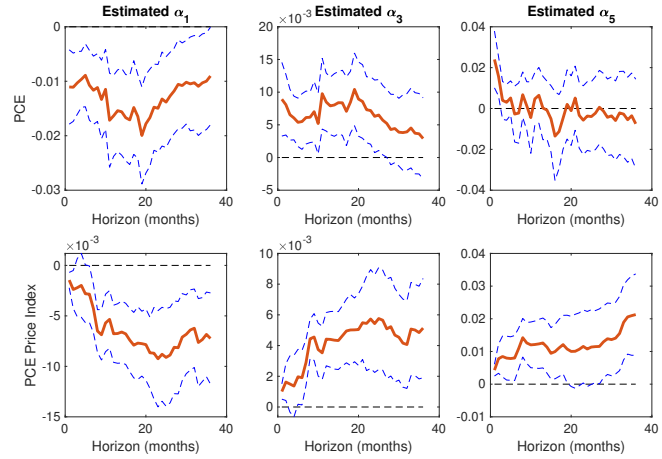


FIGURE A.21. Cumulative impulse responses to federal fund rate shocks (controlling for consumer sentiment)

Note: This figure shows estimated impulse responses of monthly real personal consumption expenditure (PCE) and PCE price index to a federal funds rate shock from the local projections model (A.3) (with FG replaced by MP), where we include the consumer sentiment index from the Michigan Survey of Consumers and its interactions with the federal funds rate shock. The solid lines show the point estimates of the impulse responses. The dashed lines show the 68% confidence intervals based on a Newey-West heteroscedasticity and autocorrelation consistent (HAC) estimator.

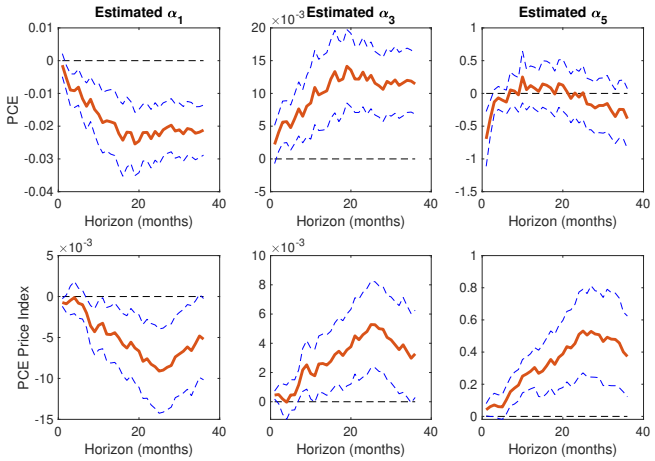


FIGURE A.22. Cumulative impulse responses to federal fund rate shocks (controlling for income growth expectation)

Note: This figure shows estimated impulse responses of monthly real personal consumption expenditure (PCE) and PCE price index to a federal funds rate shock from the local projections model (A.3) (with FG replaced by MP), where we include the median income expectation from the Michigan Survey of Consumers and its interactions with the federal funds rate shock. The solid lines show the point estimates of the impulse responses. The dashed lines show the 68% confidence intervals based on a Newey-West heteroscedasticity and autocorrelation consistent (HAC) estimator.

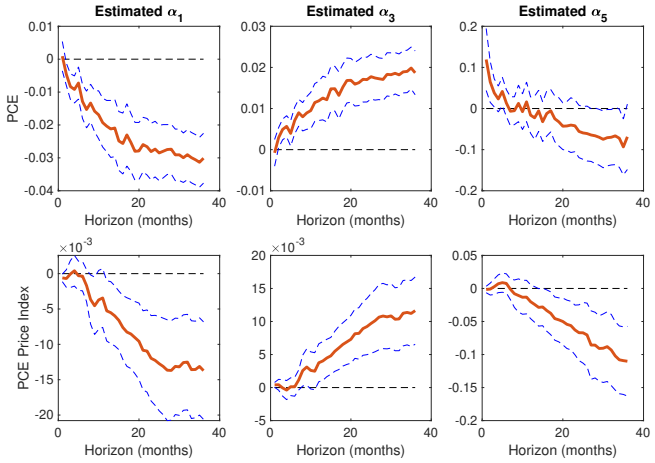


FIGURE A.23. Estimated response to federal fund rate shocks (controlling for consumer uncertainty)

Note: This figure shows estimated impulse responses of monthly real personal consumption expenditure (PCE) and PCE price index to a federal funds rate shock from the local projections model (A.3) (with FG replaced by MP), where we include consumer uncertainty from the Michigan Survey of Consumers and its interactions with the federal funds rate shock. The solid lines show the point estimates of the impulse responses. The dashed lines show the 68% confidence intervals based on a Newey-West heteroscedasticity and autocorrelation consistent (HAC) estimator.

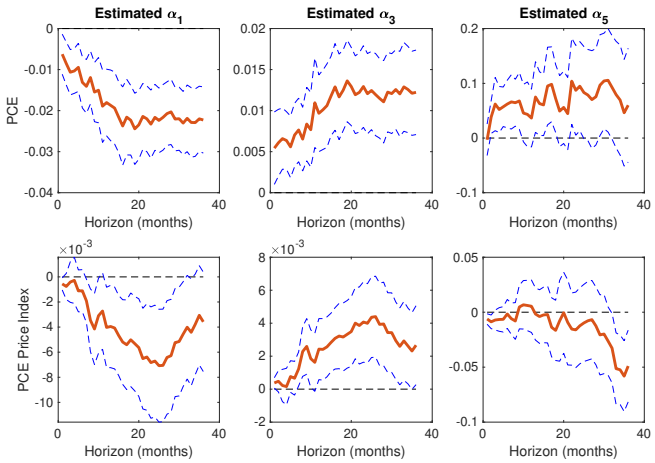


FIGURE A.24. Estimated response to federal fund rate shocks (controlling for income disagreement)

Note: This figure shows estimated impulse responses of monthly real personal consumption expenditure (PCE) and PCE price index to a federal funds rate shock from the local projections model (A.3) (with FG replaced by MP), where we include consumer's income disagreement from the Michigan Survey of Consumers and its interactions with the federal funds rate shock. The solid lines show the point estimates of the impulse responses. The dashed lines show the 68% confidence intervals based on a Newey-West heteroscedasticity and autocorrelation consistent (HAC) estimator.

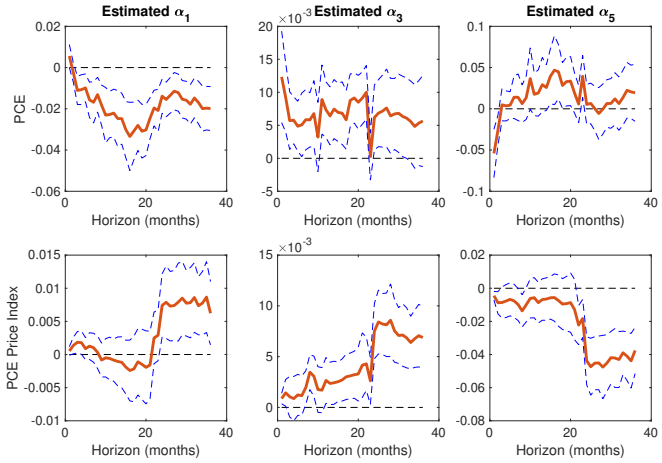


FIGURE A.25. Cumulative impulse responses to federal fund rate shocks (controlling for inflation uncertainty)

Note: This figure shows estimated impulse responses of monthly real personal consumption expenditure (PCE) and PCE price index to a federal funds rate shock from the local projections model (A.3) (with FG replaced by MP), where we include the inflation uncertainty measure from the Survey of Professional Forecasters and its interactions with the federal funds rate shock. The solid lines show the point estimates of the impulse responses. The dashed lines show the 68% confidence intervals based on a Newey-West heteroscedasticity and autocorrelation consistent (HAC) estimator.

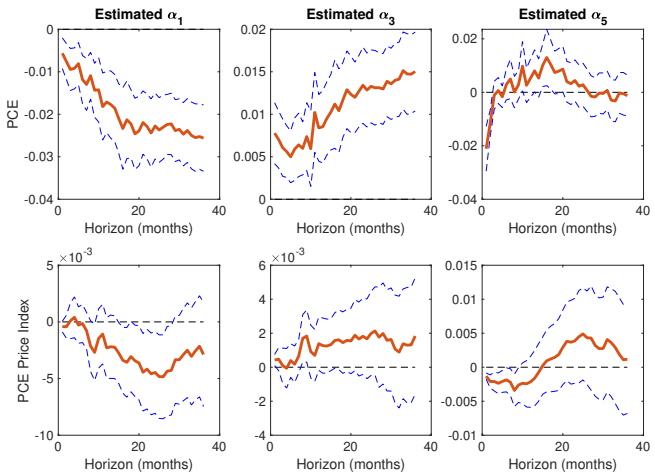


FIGURE A.26. Cumulative impulse responses to federal fund rate shocks (controlling for forecast disagreement of 2-year Treasury yield)

Note: This figure shows estimated impulse responses of monthly real personal consumption expenditure (PCE) and PCE price index to a federal funds rate shock from the local projections model (A.3) (with FG replaced by MP), where we include the one-year ahead forecast dispersion of 2-year Treasury yields from the Blue Chip Financial Forecasts Database and its interactions with the federal funds rate shock. The solid lines show the point estimates of the impulse responses. The dashed lines show the 68% confidence intervals based on a Newey-West heteroscedasticity and autocorrelation consistent (HAC) estimator.

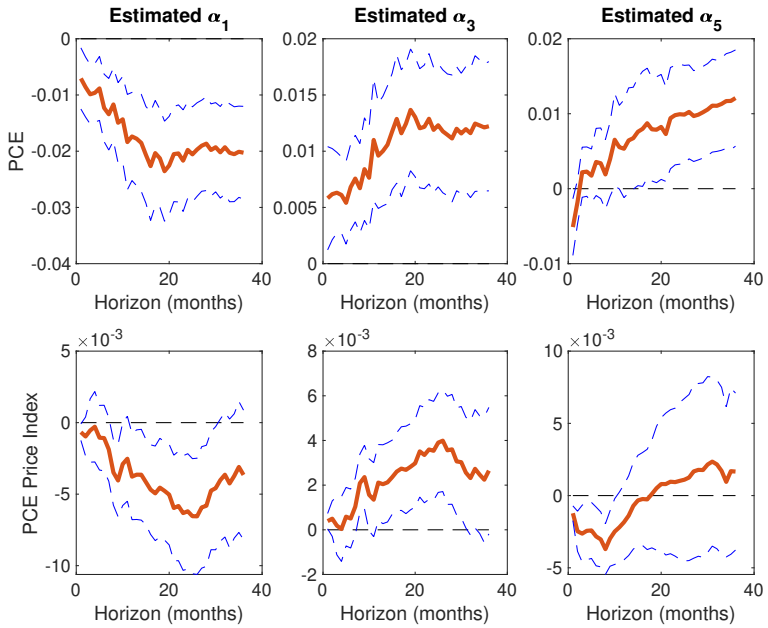


FIGURE A.27. Cumulative impulse responses to federal fund rate shocks (controlling for forecast disagreement of 1-year ahead CPI)

Note: This figure shows estimated impulse responses of monthly real personal consumption expenditure (PCE) and PCE price index to a federal funds rate shock from the local projections model (A.3) (with FG replaced by MP), where we include the forecast dispersion of 1-year ahead CPI from the Blue Chip Financial Forecasts Database and its interactions with the federal funds rate shock. The solid lines show the point estimates of the impulse responses. The dashed lines show the 68% confidence intervals based on a Newey-West heteroscedasticity and autocorrelation consistent (HAC) estimator.

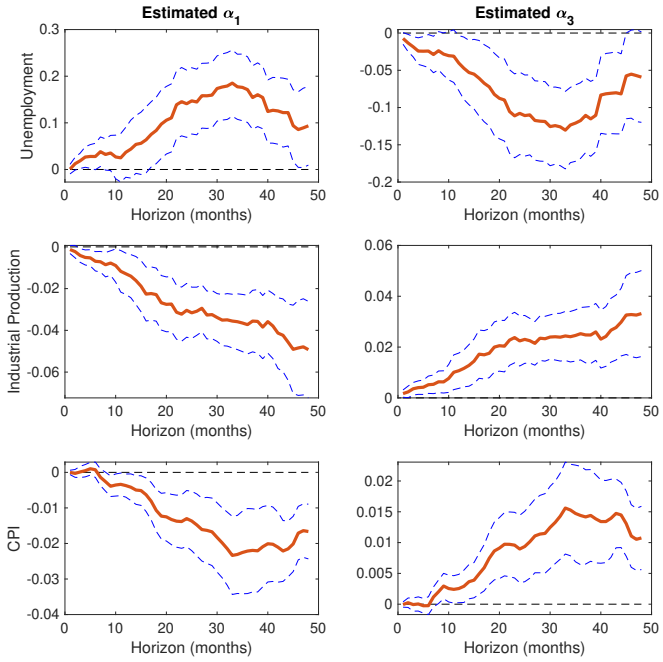


FIGURE A.28. Cumulative impulse responses to forward guidance shocks

Note: This figure shows estimated impulse responses of monthly unemployment rate, industrial production and consumer price index (CPI) to forward guidance shock from the local projections model (1). The solid lines show the point estimates of the impulse responses. The dashed lines show the 68% confidence intervals based on a Newey-West heteroscedasticity and autocorrelation consistent (HAC) estimator.

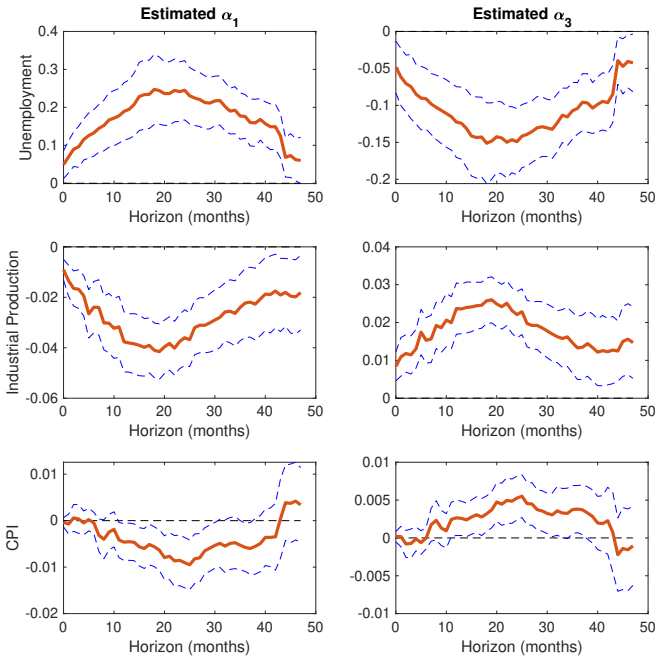


FIGURE A.29. Cumulative impulse responses to federal fund rate shocks

Note: This figure shows estimated impulse responses of monthly unemployment rate, industrial production, and CPI inflation to federal fund rate shock from the local projections model (1). The solid lines show the point estimates of the impulse responses. The dashed lines show the 68% confidence intervals based on a Newey-West heteroscedasticity and autocorrelation consistent (HAC) estimator.

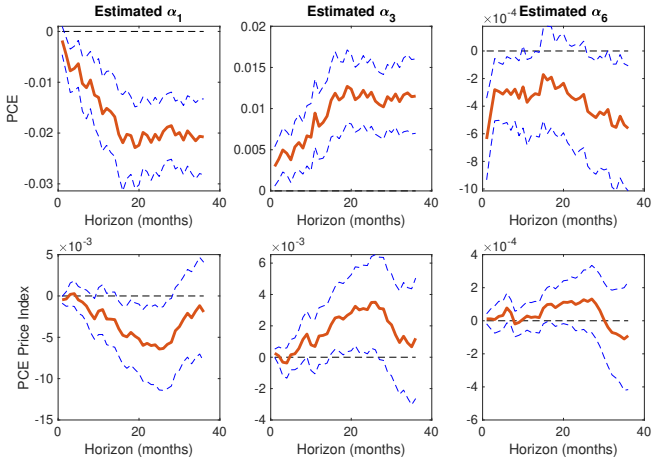


FIGURE A.30. Cumulative impulse responses to a fed funds rate shock: Effects of lending standard

Note: This figure shows the cumulative impulse responses of real personal consumption expenditure (PCE) and PCE price index to a fed funds rate shock from the local projections model (33). The dashed lines show the 68% confidence intervals based on a Newey-West heteroscedasticity and autocorrelation consistent (HAC) estimator.

Appendix B. Derivations and proofs

We derive the log-linearized Euler equation and the Phillips curve in the baseline model and we also provide proofs of some auxiliary results.

B.1. **Derivations of the log-linearized Euler equation.** Log-linearizing equation (20), we obtains

$$-\frac{C}{C+\bar{B}}\hat{C}_t = -\hat{e}^*_t + \hat{r}_{ft} - \mathbf{E}_t\hat{\pi}_{t+1} - \frac{C}{C+\bar{B}}E_t\hat{C}_{t+1} + E_t[1-\theta]\hat{e}^*_{t+1},$$

where θ measures the (inverse) elasticity of $F(\cdot)$ with respect to e^*, such that

$$\theta \equiv -\frac{F'(e^*)e^*}{F(e^*)} = \frac{1-G(e^*)}{1-G(e^*)+e^*\int_{e_{\min}}^{e^*}\frac{1}{e}dG(e)} \in [0,1). \tag{B.1}$$

In the special case with no belief heterogeneity, we would have a degenerate distribution of beliefs, such that $\theta = 0$.

After rearrangement, we have

$$\hat{C}_t - \frac{C+\bar{B}}{C}\hat{e}^*_t = E_t\hat{C}_{t+1} - \frac{C+\bar{B}}{C}E_t[1-\theta]\hat{e}^*_{t+1} - \frac{C+\bar{B}}{C}\left(\hat{r}_{ft} - \mathbf{E}_t\hat{\pi}_{t+1}\right). \tag{B.2}$$

Finally, we replace $\hat{e}^*_t$ with $\hat{C}_t$ using aggregate consumption condition:

$$C_t = (C_t+\bar{B})\left[1-G(e^*_t)+\int_{e_{\min}}^{e^*_t}\frac{e}{e^*_t}dG(e)\right],$$

or equivalently,

$$\frac{C_t}{C_t+\bar{B}} \equiv \Phi(e^*_t), \tag{B.3}$$

where

$$\Phi(e^*_t) \equiv \left[1-G(e^*_t)+\frac{\int_{e_{\min}}^{e^*_t}edG(e)}{e^*_t}\right].$$

By definition, $\Phi(e^*_t)$ is the ratio of average consumption to consumption of the marginal consumer. Since $C_t = A_t$ in equilibrium, $\Phi(e) \equiv \frac{C_t}{C_t+\bar{B}} = \frac{A_t}{A_t+\bar{B}}$ can be interpreted as (the inverse of) the average leverage ratio.

Denote the (inverse) elasticity of $\Phi(\cdot)$ to e^* by μ, such that

$$\mu \equiv -\frac{\Phi'(e^*)e^*}{\Phi(e^*)} = \frac{\int_{e_{\min}}^{e^*}edG(e)}{[1-G(e^*)]e^*+\int_{e_{\min}}^{e^*}edG(e)} \in (0,1]. \tag{B.4}$$

Note have that $\mu = 1$ if and only if inflation expectation is homogeneous. We can derive $\hat{e}^*_t$ as a function of $\hat{C}_t$

$$\frac{\bar{B}}{C+\bar{B}}\hat{C}_t = -\mu\hat{e}^*_t \tag{B.5}$$

Plugging equation (B.5) into the Euler equation (B.2), we have

$$\hat{C}_t\left(1+\frac{\bar{B}}{\mu C}\right)=E_t\hat{C}_{t+1}[1+(1-\theta)\frac{\bar{B}}{\mu C}]-\frac{C+\bar{B}}{C}\left(\hat{r}_{ft}-\mathbf{E}_t\hat{\pi}_{t+1}\right)$$

Denoting the steady state loan-to-value ratio as $\kappa\equiv\frac{\bar{B}}{A}=\frac{\bar{B}}{C}\in(0,1)$, we derive a *discounted Euler equation* as

$$\hat{C}_t=\underbrace{\frac{\mu+(1-\theta)\kappa}{\mu+\kappa}}_{\equiv\beta_1}\mathbb{E}_t\hat{C}_{t+1}-\underbrace{\frac{(1+\kappa)\mu}{\mu+\kappa}}_{\equiv\beta_2}\left(\hat{r}_{ft}-\mathbf{E}_t\hat{\pi}_{t+1}\right). \tag{B.6}$$

Ceteris paribus, a higher θ or lower μ will reduce the responsiveness of current aggregate consumption to future interest rates and future wealth changes. Lower μ will also weaken the effect of contemporaneous interest rate changes on consumption. Intuitively, aggregate consumption is less responsive to shocks when there is a larger mass of constrained household members, who do not adjust sufficiently to changes in wealth (i.e., changes in expected future consumption) or changes in the real interest rate.

B.2. **Derivations of the Phillips curve.** We now derive the Phillips curve.

There is a continuum of intermediate goods producers index by $j\in[0,1]$, each producing a differentiated product Y_{jt}. The final consumption good is a composite of the differentiated intermediate goods, with the aggregation technology

$$Y_t=\left[\int_0^1 Y_{jt}^{\frac{\sigma-1}{\sigma}}dj\right]^{\frac{\sigma}{\sigma-1}},$$

where Y_t denotes the final goods output and $\sigma>1$ denotes the elasticity of substitution between differentiated intermediate goods.

Final goods producers are price takers. Their optimal production decisions lead to the downward-sloping demand schedule for each intermediate product

$$Y_{jt}=\left(\frac{P_{jt}}{P_t}\right)^{-\sigma}Y_t. \tag{B.7}$$

Each variety of intermediate goods is produced using labor as the only input. Intermediate goods producers are price takers in the input market and monopolistic competitors in the product markets. Unlike the households, we assume that firms are perfectly rational, with no heterogeneity in beliefs. Each intermediate goods producer takes as given the price level P_t, the real wage rate w_t, and the demand schedule (B.7), and chooses its own price P_{jt} to maximize the present value of its profit flows, subject to the quadratic price adjustment cost in the spirit of Rotemberg (1982).

The price adjustment cost function is given by

$$\frac{\chi_P}{2}\left[\frac{P_t(i)}{\Pi_t^* P_{t-1}(i)}-1\right]^2 Y_t$$

Define $\Phi_{t,t+\tau}=\Pi_{t+1}^*\times\cdots\times\Pi_{t+\tau}^*$, for $\tau\geq 1$. We can normalize the price as $\tilde{P}_t(i)=\frac{P_t(i)}{\Phi_{0,t}}$, and the cost becomes

$$\frac{\chi_P}{2}\left[\frac{\tilde{P}_t(i)}{\tilde{P}_{t-1}(i)}-1\right]^2 Y_t$$

In a symmetric equilibrium,

$$\frac{P_t(i)}{\Pi_t^* P_{t-1}(i)}=\frac{P_t}{\Pi_t^* P_{t-1}}=\frac{\Pi_t}{\Pi_t^*}\equiv\pi_t.$$

Firm j chooses P_{jt} to maximize the present value of profits

$$\begin{aligned}
& E_t\sum\beta^\tau\frac{\Lambda_{t+\tau}}{\Lambda_t}\left\{\left(\frac{P_{jt+\tau}}{P_{t+\tau}}\right)^{1-\sigma}Y_t-\frac{w_t}{Z_t}\left(\frac{P_{jt+\tau}}{P_{t+\tau}}\right)^{-\sigma}Y_t-\frac{\chi_P}{2}\left[\frac{\tilde{P}_{jt+\tau}}{\tilde{P}_{jt+\tau-1}}-1\right]^2 Y_t\right\}\\
= \;& E_t\sum\beta^\tau\frac{\Lambda_{t+\tau}}{\Lambda_t}\left\{\left(\frac{\tilde{P}_{jt+\tau}}{\tilde{P}_{t+\tau}}\right)^{1-\sigma}Y_t-\frac{w_t}{Z_t}\left(\frac{\tilde{P}_{jt+\tau}}{\tilde{P}_{t+\tau}}\right)^{-\sigma}Y_t-\frac{\chi_P}{2}\left[\frac{\tilde{P}_{jt+\tau}}{\tilde{P}_{jt+\tau-1}}-1\right]^2 Y_t\right\}.
\end{aligned}$$

The firm's optimal pricing decisions is given by

$$\begin{aligned}
(1-\sigma)\left(\frac{\tilde{P}_{jt}}{\tilde{P}_t}\right)^{-\sigma}\frac{Y_t}{\tilde{P}_t} \;&+\; \sigma\frac{w_t}{Z_t}\left(\frac{\tilde{P}_{jt}}{\tilde{P}_t}\right)^{-\sigma-1}\frac{Y_t}{\tilde{P}_t}-\chi_P\left[\frac{\tilde{P}_{jt}}{\tilde{P}_{jt-1}}-1\right]\frac{Y_t}{\tilde{P}_{jt-1}}\\
&+\; \chi_P\beta E_t\frac{\Lambda_{t+1}}{\Lambda_t}\left[\frac{\tilde{P}_{jt+1}}{\tilde{P}_{jt}}-1\right]\frac{\tilde{P}_{jt+1}}{\left(\tilde{P}_{jt}\right)^2}Y_{t+1}=0
\end{aligned}$$

In a symmetric equilibrium with $P_{jt}=P_t$, we have

$$\chi_P\left[\frac{\tilde{P}_t}{\tilde{P}_{t-1}}-1\right]\frac{\tilde{P}_t}{\tilde{P}_{t-1}}=\sigma\frac{w_t}{Z_t}+(1-\sigma)+\chi_P\beta E_t\frac{\Lambda_{t+1}}{\Lambda_t}\left[\frac{\tilde{P}_{t+1}}{\tilde{P}_t}-1\right]\frac{\tilde{P}_{t+1}}{\tilde{P}_t}\frac{Y_{t+1}}{Y_t}$$

Log-linearizing this optimal pricing decision leads to the Phillips curve

$$\hat{\pi}_t=\varphi_y[\hat{w}_t-\hat{Z}_t]+\beta\mathbb{E}_t\hat{\pi}_{t+1} \tag{B.8}$$

B.3. **Proofs of auxiliary results.** Suppose that the idiosyncratic beliefs of households follow a Pareto distribution, such that

$$G(e)=\begin{cases}1-(\frac{e}{e_{min}})^{-\alpha} & \text{if } e\geq e_{min}\\ 0 & \text{if } e<e_{min}\end{cases} \tag{B.9}$$

We fix $E(e) = 1$ by setting $e_{min} = \frac{\alpha-1}{\alpha}$. The variance of inflation expectation is a decreasing function of α:

$$Var(e) = \frac{\alpha}{\alpha-2} \cdot \left(\frac{e_{min}}{\alpha-1}\right)^2 = \frac{1}{\alpha(\alpha-2)}, \quad \alpha > 2. \tag{B.10}$$

We can prove the following Lemmas:

Lemma B.1. *e_t^* is an increasing function of α.*

Proof. Incorporating the distribution function of inflation expectations (e) and the assumption that $e_{min} \equiv \frac{\alpha-1}{\alpha}$ into Eq. (B.5), we obtain

$$\begin{aligned} \frac{1}{1+\kappa} &= (1 - G(e_t^*)) + \frac{\int_{e_{\min}}^{e_t^*} eg(e)de}{e_t^*} = (1 - (1 - (\frac{e_{min}}{e_t^*})^\alpha)) + \frac{\int_{e_{min}}^{e_t^*} e(\frac{\alpha \cdot e_{min}^\alpha}{e^{\alpha+1}})de}{e_t^*} \\ &= -\frac{1}{\alpha-1} \cdot \left(\frac{e_{min}}{e_t^*}\right)^{-\alpha} + \frac{\alpha}{\alpha-1} \cdot \frac{e_{min}}{e_t^*} = -\frac{1}{\alpha-1} \cdot \left(\frac{e_{min}}{e_t^*}\right)^{-\alpha} + \frac{1}{e_t^*}, \end{aligned} \tag{B.11}$$

which implies that e_t^* is an increasing function of α, or a *decreasing* function of inflation disagreement. □

Lemma B.2. *θ is a decreasing function of α.*

Proof. Use α, $e_{min} \equiv \frac{\alpha-1}{\alpha}$, and e_t^* to solve for θ from Eq. (B.1):

$$\begin{aligned} \theta =& \frac{1 - G(e^*)}{1 - G(e^*) + e^* \int_{e_{\min}}^{e^*} \frac{1}{e} g(e)de} = \frac{(\frac{e_{min}}{e_t^*})^\alpha}{(\frac{e_{min}}{e_t^*})^\alpha + e_t^* \int_{e_{min}}^{e_t^*} \frac{1}{e} \frac{\alpha \cdot e_{min}^\alpha}{e^{\alpha+1}} de} \\ =& \frac{1}{\frac{1}{\alpha+1} + \frac{\alpha}{\alpha+1} \cdot \left(\frac{e_t^*}{e_{min}}\right)^{\alpha+1}}, \end{aligned} \tag{B.12}$$

which implies that θ is a decreasing function of α, or an *increasing* function of inflation disagreement. □

Lemma B.3. *μ is an increasing function of α.*

Proof. Use α, $e_{min} \equiv \frac{\alpha-1}{\alpha}$, e_t^* and θ to solve for μ from Eq. (B.4).

$$\begin{aligned} \mu =& \frac{\int_{e_{\min}}^{e^*} eg(e)de}{(1 - G(e^*))e^* + \int_{e_{\min}}^{e^*} eg(e)de} = \frac{\int_{e_{min}}^{e_t^*} e(\frac{\alpha \cdot e_{min}^\alpha}{e^{\alpha+1}})de}{(1 - (1 - (\frac{e_{min}}{e_t^*})^\alpha))e_t^* + \int_{e_{min}}^{e_t^*} e(\frac{\alpha \cdot e_{min}^\alpha}{e^{\alpha+1}})de} \\ =& \frac{\frac{\alpha e_{min}^\alpha}{-\alpha+1}(e_t^{*-\alpha+1} - e_{min}^{-\alpha+1})}{(\frac{e_{min}}{e_t^*})^\alpha e_t^* + \frac{\alpha e_{min}^\alpha}{-\alpha+1}(e_t^{*-\alpha+1} - e_{min}^{-\alpha+1})} = \frac{\frac{\alpha}{-\alpha+1}(1 - (\frac{e_{min}}{e_t^*})^{-\alpha+1})}{1 + \frac{\alpha}{-\alpha+1}(1 - (\frac{e_{min}}{e_t^*})^{-\alpha+1})} \end{aligned} \tag{B.13}$$

which implies that μ is an increasing function of α, or a *decreasing* function of inflation disagreement. □