A Dynamic Analysis of Parental Beliefs and Investments

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Abstract

This paper seeks to quantify the role of parental beliefs about child skill in explaining the socio-economic status skill gap. To achieve this, using U.S. longitudinal data on parental beliefs, parent investments and child skills, I estimate a dynamic model of parent investment decisions and belief updating. The model indicates that there is feedback between beliefs and investments, which acts in the following way. Parents who hold higher beliefs will invest more because there is a higher return to investment, due to complementarity between skills and investments in the production function of skills. When they invest more, they will hold higher beliefs in the following period, which encourages them to invest once again, for the same reasons. Using the estimated model, I decompose the SES skill gap. I find that differences in initial beliefs do not contribute much to the SES skill gap. Instead, differences in initial parent resources (income, assets) and initial skills of children play a larger role. Finally, to understand the importance of information frictions, I conduct a counterfactual where parents know the skills of their children and where they do not update their beliefs. As high SES (low SES) parents tend to over-estimate (under-estimate) the skills of their children, their beliefs decrease (increase). This eventually leads to lower (higher) average skills of children for the high SES (low SES). In this way, removing belief biases serves to reduce the SES skill gap.

JEL codes: D83, I24, J13, J24, O15

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1 Introduction

There is a socio-economic gap in parent investments, with poor parents investing significantly less than rich parents in their children (Caucutt & Lochner, 2020; Bolt, French, Hentall-MacCuish, & O'Dea, 2024; Carneiro, Reis, & Toppeta, 2024). This is concerning as these differences in investments translate into differences in children's human capital and life outcomes, including educational attainment and earnings (Becker & Tomes, 1979, 1986; Solon, 1999; O. Attanasio, Cattan, & Meghir, 2022). The socio-economic status (SES) investment gap could arise because of differences in the resources of parents, skills of children, which affect the marginal returns to investment and/or preferences of parents for the skills of their child (Cunha, 2014; Caucutt, Lochner, & Park, 2017).

Another possible reason for the socio-economic status (SES) investment gap is SES differences in parental beliefs and belief updating. Experimental studies indicate that parental beliefs affect parent investment decisions (Dizon-Ross, 2019; List, Pernaudet, & Suskind, 2021). Furthermore, there is evidence of SES differences in parental beliefs. For example, Dizon-Ross (2019) shows that low SES parents update beliefs differently and are more likely to hold inaccurate beliefs about their child's skill. Experimental studies indicate that there are short-term effects of beliefs on investments and skills, but it is unknown whether there are long-term effects. Long-term effects could arise, for example, if there are ripple effects of earlier beliefs on later beliefs and investments. For instance, it could be that holding higher beliefs leads to higher investments, which in turn leads to higher beliefs in the following period, and so on. That is, there is feedback between beliefs and investments across time periods.

I propose a dynamic model of parent investments with belief updating to (1) explore the feedback between beliefs and investments and to (2) quantify the role of parental beliefs about the skill of their child in explaining the SES skill gap. The model is disciplined with longitudinal data on parental beliefs, investments and child skills from the National Longitudinal Survey of Youth. Parental beliefs are estimated from a factor model based on responses by the mother of the child about (1) the child's ranking in class; (2) the child's future educational attainment and (3) the child's future prospects. Parent investments are also derived from a factor model based on measures of goods and time investments. In addition, child skills are estimated from a factor model based on achievement test scores.

I begin by presenting patterns in the data which motivate the research question and the model. First, there are socio-economic status (SES) differences in parental beliefs and investments. Parents may not know the skill level of their child and low SES parents hold more inaccurate beliefs on average. This is seen as the correlation between beliefs and skills is less than 1 and the correlation is lower for low SES parents. Moreover, on average, high SES parents (those with above median family income) hold higher beliefs, even conditional on child

skills, and invest more in their children. Second, there is suggestive evidence that parental beliefs change when the skills of their child change — in a belief updating regression model, within-child changes in beliefs move together with within-child changes in child skill measures. Third, parent investments are correlated with parental beliefs, even after controlling for time-invariant characteristics specific to the child with child fixed effects. This suggests that there is a strong relationship between the parental beliefs and investments, and it could be that beliefs are influencing investment decisions.

In the next section of the paper, I introduce a dynamic model of parent investments with belief updating to (1) investigate the dynamic relationship between beliefs and investments through connecting belief updating with how beliefs affect investments and with how investments affect skills and to (2) explore the mechanisms behind the socio-economic status investment gap.

The model has the following features. Parents have imperfect information: they do not observe the cognitive skill of their child. Instead, they hold a belief about their child's cognitive skill. Parents make investments that influence the evolution of the unobserved skill. Parents learn about the skill of the child. In every period, they receive a signal (noisy measure of the skill) and update their belief. The model features feedback between beliefs and investments. The beliefs which parents have influence the investments which they make in their children. Parent investments influence beliefs directly — parents who invest more in their child will also hold higher beliefs about next period skill, since they know that investments raise next period skills. Parent investments also influence beliefs indirectly. This is because if parents invest more, the skills of their child will be higher in the following period. This means that in the next period, parents will be more likely to receive a higher signal, which could lead to an upward revision of beliefs. The beliefs will then affect the investments, and so on and so forth.

The model has three periods which span childhood to early adolescence. The parent is forward looking and is an expected utility maximiser. In every period, the parent chooses consumption, investment and next period assets to maximise their objective function, based on their beliefs about their child's skill and their monetary resources. The parent makes these choices subject to the child skill production function (which informs them how investments produce skills), the belief evolution process, the budget constraint and income process. Their utility depends on consumption and the perceived skill of their child. They face a trade-off between consumption and investing in their child.

The model features both observed and unobserved heterogeneity. Observed heterogeneity is present as the belief updating process, child skill production technology and income process depend on mother's education. High education mothers (proxy for SES) could update beliefs differently from low education mothers. Moreover, high education mothers may be more productive in producing skills from investment. In addition, families with mothers with higher

education are more likely to be on higher income trajectories. The model also incorporates unobserved preference heterogeneity — each parent has a probability of being one of two discrete types, with one type holding a higher relative preference for child skill over consumption. The unobserved heterogeneity serves two main functions. First, it controls for unobservables which jointly affect both parental beliefs and investments. Second, it addresses the problem of endogenous initial conditions.

Model estimates shed light on the dynamic interplay between beliefs and investments. There is a positive reinforcing effect of initial beliefs on later beliefs and investments. Parents who hold higher beliefs in their child will invest more, because they perceive a higher marginal return to investment, due to complementarity between skills and investments in the production function. If they invest more, in the following period, on average, they will hold higher beliefs about the skill level of their child, which will incentivise them to invest more for the same reasons, and so on. Furthermore, model estimates suggest that belief updating does not depend on SES: belief updating parameters of low and high education mothers are similar.

Using the estimated model, I evaluate the relative contribution of parental beliefs about child cognitive skill in explaining the socio-economic status (SES) skill gap. The SES skill gap is defined as the difference in the average terminal skills of the high SES children from the average terminal skills of the low SES children. The terminal skill refers to the skill which the child has after the parent makes the investment decision in the third and final period. Although the beliefs of parents affect investments, I find that differences in the initial beliefs which parents have explain less of the SES skill gap than differences in the initial resources of parents (income, assets) and differences in the initial skills of children. Equalising first period beliefs of parents decreases the SES skill gap by less than 1%. In contrast, equalising initial skills of children or initial resources of parents will decrease the SES skill gap by a greater percentage. In general, these results suggest that even though experimental evidence indicates that beliefs affect investments (and skills) in the short-run, the long-term effects of beliefs may be limited.

Finally, to understand the importance of information frictions about the skills of children, I estimate a counterfactual where there are no information frictions (parental beliefs match the skills of their child) and parents do not update their beliefs. Since it is the case that on average, high SES parents over-estimate the skills of their child and low-SES under-estimate the skills of their child, high SES (low SES) now have lower (higher) beliefs in the first period than the baseline. This lowers (raises) their perception of the marginal return to investment (because of the complementarity between skills and investments in the production function), which encourages them to invest less (more). This in turn lowers (raises) their beliefs for the following period, which provides an incentive to invest less (more) in the next period, for the

same reasons, and so on. Overall, there are lower (higher) investments and therefore, lower (higher) terminal skills of children than the baseline. Therefore, the SES skill gap decreases. Moreover, most models assume that parents have perfect information about the skills of their child. This exercise shows that doing so will lead to errors in the predicted investments of parents who have under-estimated or over-estimated the skill levels of their child.

This paper relates to the literature on subjective beliefs/expectations and information frictions within Economics of Education. These studies have established that subjective beliefs of parents are predictive of parent investments and children's outcomes (O. P. Attanasio & Kaufmann, 2014; O. Attanasio, Cunha, & Jervis, 2019; O. Attanasio, Boneva, & Rauh, 2019; Dizon-Ross, 2019; List et al., 2021; Cunha, Elo, & Culhane, 2022; Lekfuangfu & Odermatt, 2022; Conti, Giannola, & Toppeta, 2022; Page & Ruebeck, 2022). In addition, they provide evidence that parents may not know their child's skill level (Dizon-Ross, 2019; Nicoletti, Sevilla, & Tonei, 2022; Bergman, 2021) and there is uncertainty in parental beliefs (Lee, 2023). Also, some papers show that parents revise beliefs in response to information (Nicoletti et al., 2022; Greaves, Hussain, Rabe, & Rasul, 2023). This paper focuses on the beliefs which parents have about their child's skill. Most papers have examined the relationship between beliefs and investments in a static setting. In a dynamic setting, there may be feedback between parental beliefs about the skill level of their child and investments. The key contribution of this paper is to provide a framework which incorporates these dynamics and then use it to explore them.

This paper also relates to the literature on belief formation, learning and belief dynamics (Zafar, 2011; Sanders, 2012; Stinebrickner & Stinebrickner, 2014; Arcidiacono, Aucejo, Ransom, & Maurel, 2016). For instance, the literature on bayesian learning models in marketing and industrial organisation (Ackerberg, 2003; Erdem & Keane, 1996; Crawford & Shum, 2005). These papers model belief updating and provide a framework for how beliefs influence decisions made by agents. In labour economics, most of these papers focus on decisions during young adulthood and later. For example, learning about one's own ability may influence whether one decides to drop out of college. In contrast, this paper focuses on an earlier time period (childhood to early adolescence) and models the belief updating of parents and how beliefs influence parent investments. Furthermore, to the best of my knowledge, with the exception of Sanders (2012), the literature focuses on agents learning about an unobserved state which is fixed over time (e.g. ability). Differently from them, in this paper, parents are learning about an unobserved state (the child's skill) which evolves endogenously based on their investment choices.

A closely related paper is Chikhale (2024), which has a dynamic model where parents hold beliefs about the return to investment and update beliefs after receiving signals from social connections. This paper has several key differences. Firstly, we focus on different dimensions of

parental beliefs. In Chikhale (2024), parents hold beliefs about the return to investment, while in this paper, I focus on parental beliefs about the skill of their child. There is evidence that parents learn on both of these dimensions, and perhaps in the future, our approaches could be combined to investigate the interactions between these two dimensions of beliefs. Secondly, the mechanism of learning is different. In Chikhale (2024), parents update beliefs when they receive signals from their social interactions. However, in my framework, parents update their beliefs based on information which they receive about their child's skill. It is likely that both of these learning mechanisms are important, and therefore, our approaches are complementary. Thirdly, while Chikhale (2024) assumes parents update beliefs in a Bayesian way, I leverage longitudinal data on parental beliefs, investments and skills to estimate SES differences in belief updating.

The rest of this paper is organised as follows. Section 2 introduces the data. Section 3 documents patterns in the data which motivate the research question and the model. Section 4 presents the dynamic model of parent investments and belief updating. Section 5 describes the identification and estimation. Section 6 presents the model estimates and features of the baseline. Section 7 presents a decomposition of the socio-economic status skill gap and an information intervention policy experiment. Section 8 concludes the paper.

2 Data

I use data from the National Longitudinal Survey of Youth 1979 (NLSY79) along with the NLSY Child and Young Adult (CYA). The NLSY79 is a U.S. longitudinal survey of individuals who were aged 14-22 years in 1979. The children of females in the NLSY79 were surveyed in the CYA. By merging the NLSY79 and the CYA, I have information on the mother of the child, the family and the child. This includes household demographics (marital status, employment), mother's characteristics, mother's family background and children's characteristics. In this study, I focus on children of females in the cross-section sample of the NLSY79.

Importantly for the analysis, there are repeated measures of parental beliefs, parent investments and children's skills, which facilitates the study of (1) the evolution of these beliefs and (2) the dynamic relationship between parent beliefs and parent investments.

2.1 Parental Beliefs

The parental belief is extracted from a factor model, based on responses of the child's mother to 3 questions in the survey. I assume that these 3 questions are capturing an underlying belief that the mother has about the child. The factor model is employed to adjust for the measurement error which is embedded in the responses to each of the 3 questions.

The first question is an expectation of the child's future educational attainment. Specifically,

the mother is asked "How far do you think your child will go in school?". Possible options are: (1) Leave high school before graduation; (2) Graduate from high school; (3) Get some college or other training; (4) Graduate from college; (5) Get more than 4 years of college/further training after college; and (6) Something else. In practice, very few mothers choose option 1. Thus, I group option 1 together with option 2, and will henceforth refer to this merged option as "Up to High School". In addition, option 6 (something else) is vague and few mothers chose it. I remove that option from the analysis. Effectively, I treat those who chose option 6 as missing responses. From this point onwards, there are four categories of the expectation: (1) Up to high school; (2) Some college or other training; (3) Graduate from college; and (4) More than college.

The second question is the rating of the child's academic standing in class. She indicates whether she thinks that the child is (1) near the bottom of the class; (2) below the middle; (3) in the middle; (4) above the middle; or (5) one of the best students in class. Most children are rated at being in the middle of the class or above. This is a relative ranking rather than an absolute ranking, and there is evidence that relative rankings matter for parent beliefs and child achievement (Kinsler & Pavan, 2021; Elsner, Isphording, & Zölitz, 2021).

The third question is the mother's rating of the child's future prospects. Possible options are: (1) Poor; (2) Fair; (3) Good; and (4) Excellent.

Overall, mothers tend to hold good opinions about their children. More than 50% of children are expected to attain college education and above. More than 50% of the children are rated as having an academic standing above the middle or one of the top in the class. Also, more than 50% of children are rated as having excellent future prospects. Detailed proportions of the responses to each of the above measures are provided in the appendix.

Two of these measures are about expectations about the child's future. Therefore, aside from capturing the mother's perception about the child's skills, they may also reflect the mother's expectations about planned future investments and future shocks. I am still thinking of a good way to address this issue.

There is variation in the three measures over time. In the sub-sample of children born to females in the cross-section of the NLSY79 who had 3 belief reports between the ages of 9 and 14, (1) around 57% reported at least one change in the expected educational attainment; (2) around 61% made at least one change in the child's academic standing and (3) around 45% made at least one change to the rating of child's future prospects.

Since the latent factor of belief has no natural location or scale, I anchor it to the years of education of the child by age 24 and above. Therefore, a one unit increase in the belief corresponds to a one unit increase in the conditional expectation of the years of education by age 24 and above. I assume that this latent belief is the mean of the distribution of the

parent's belief (a single moment). This differs from other papers which define or elicit the belief distribution. In the following descriptive analysis, what I refer to as the parental belief is this predicted factor score, which is taken as the mean belief.

2.1.1 Are Beliefs Determined by Parent Background and Child Skills?

It is natural to expect that parents who have higher levels of resources will hold higher beliefs in their children, as they know that they can support their children (for example, by financing their education). Parents with higher education or those who have children with higher skill levels may also hold higher beliefs. To investigate the percentage of the variation in parent beliefs explained by these factors, I look at the value of the R^2 in a regression of standardised parental beliefs on these factors. In these regressions, I control for log family income, age of the child, age of the youngest child in the household, gender and birth order.

Parental beliefs are strongly associated with parental background and children's skill levels. Around 67% of the variation in parental beliefs can be explained by these factors, according to a regression of the parental beliefs on parental background (captured by mother/family fixed effects), contemporaneous cognitive skill measures of the child (percentile scores in achievement tests in mathematics, reading recognition, reading comprehension) and contemporaneous non-cognitive skill measures of the child (percentile score of bad behaviour). However, around 33% of the variation in beliefs remains unexplained, suggesting that there are other factors at play.

Table 1: Dependent variable is Standardised Parent Belief

	(1)	(2)	(3)	(4)
Constant	-0.510**	-0.699***	0.421***	-0.248*
	(0.199)	(0.140)	(0.153)	(0.149)
Observations	8269	8269	8269	8269
R^2	0.106	0.373	0.610	0.675
Family FE	No	No	Yes	Yes
Child Skills	No	Yes	No	Yes

Notes: This table presents the constant term and the R^2 value in a regression of the parent belief on a constant term and parental background characteristics and/or measures of children's skill levels. See Section 2.2 for explanation of beliefs and skills.

2.2 Parent Investments

I construct a latent factor of investment from components of the Home Observation Measurement of the Environment (HOME), which proxy goods and time investments. Some components include how often the mother reads to the child, how many books the child has and how often

the child is brought to the museum. A complete list of the component measures used to construct the investment measure are provided in the appendix. The measure of how frequently the child is brought to the museum is used to link the factor over time, so that it can be compared across ages. The investment factor does not have a natural location or scale. I standardise it so that it has a mean 0 and standard deviation of 1 over the entire sample.

2.3 Child Skill Measures

This paper uses the following measures of skills provided in the data. The cognitive skill measures are objective, while the rest are subjective measures reported by the child's mother.

Cognitive skill measures are the percentile scores on Peabody Individual Achievement Tests in mathematics, reading recognition and reading comprehension. Higher percentile scores imply that the child performs better relative to children of the same age. Most mothers do not observe the scores on these achievement tests — I assume that these scores are correlated with the child's school grades, which I do not observe. In one of the survey years, the survey collected school transcript information. In this year, the achievement test percentile scores are correlated with the national percentile ranks in school achievement tests (see appendix).

The non-cognitive skill measure is the Behaviour Problems Index (BPI) percentile score. It is a measure of the level and frequency of problem behaviours exhibited by the child. It is based on the mother's responses to a set of questions about the child's behaviour. A higher percentile on the BPI corresponds to a higher level of behaviour problems. More information about the BPI is provided in the appendix. Since the BPI is worded to capture the mother's observations of the child exhibiting specific problem behaviours, I interpret the BPI as being something distinct from the mother's belief about the child skill.

The health measure is a subjective rating of child's health provided by the child's mother. Specifically, she rates the child's health as one of the following: poor, fair, good or excellent. In practice, few mothers rate their children's health as being poor. Therefore, I merge the poor and fair options into a single category.

2.4 Summary Statistics and Sample

Table 2 presents descriptive statistics of children born to females in the cross-section of the NLSY79 who have parent beliefs reported at three time points: age 9-10, age 11-12 and age 13-14. The average child has a mother who attained 13 years of schooling (12 years corresponds to graduating from high school, assuming no grades were repeated). The average percentile score of mothers on the Armed Forces Qualification Test (AFQT), which is a measure of cognitive skills, is 44.62. The average age of the mother at birth is around 25.6 years. Around 78% of the sample are Whites (non-Black non-Hispanic), 15% are Black and 7% are Hispanic. 51% of

the children are male, and the rest are female.

Table 2: Summary Statistics

	Mean	Std. Dev.	Min	Max.
Mother's Years of Education	13.51	2.3	3.0	20.0
Mother's AFQT Percentile	46.62	28.3	0.2	100.0
Mother's Age at Birth	25.64	4.0	14.0	34.0
White	0.78	0.4	0.0	1.0
Black	0.15	0.4	0.0	1.0
Hispanic	0.07	0.3	0.0	1.0
Male	0.51	0.5	0.0	1.0

Notes: This table presents the sample statistics of the children who have parental beliefs reported at ages 9-10, 11-12 and 13-14. The Armed Forces Qualification Test (AFQT) percentile is a measure of cognitive skill.

For the data patterns provided in the following section, to maximise the sample, I include all children in the cross-section sample who had the relevant variables non-missing between the ages of 5 and 14. This is generally larger than the sample with the summary statistics in Table 2. For the model estimation, there is a different sample selection which is described in Section 5.

3 Patterns in the Data

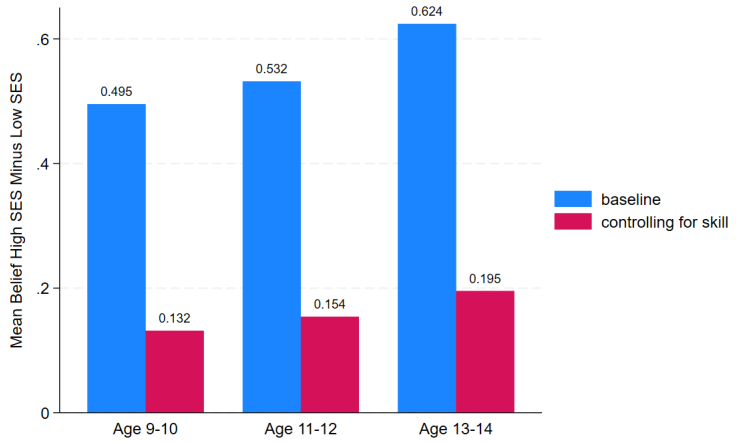
In this section, I present patterns in the data which motivate the research question and the model.

3.1 Socio-economic Status Differences in Beliefs and Investments

On average, high socio-economic status (SES) parents hold higher beliefs in their children, even conditional on skills. Figure 1 shows the average difference in the standardised belief factor score between the high SES and the low SES parents at different ages of the child. The blue bar is the unconditional average difference, while the red bar is the average difference conditional on contemporaneous skill measures of the child (percentile scores in achievement test scores in mathematics, reading recognition and reading comprehension, percentile score of behaviour problems). At all ages, both the blue and the red bars are positive, indicating that the high SES parents hold higher beliefs about their child, even conditional on skill measures. For example, the unconditional difference in beliefs at age 9-10 is 0.495 standard deviations of the belief factor score.

Furthermore, the cross-section correlation between belief factor scores (anchored to years of education at age 24 and above) and skill factor scores (anchored to years of education at

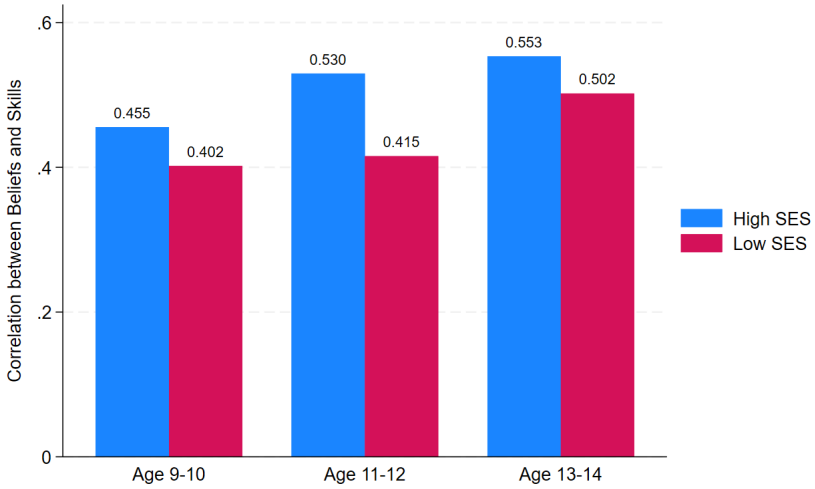
Figure 1: Average Difference in Beliefs Between High SES and Low SES



Notes: This figure presents the mean difference in the belief factor score (standardised) between high SES and low SES parents at different ages of the child. The blue bar is the unconditional difference in beliefs, while the red bar is the difference conditional on contemporaneous skill measures. The parental belief factor score has been standardised (mean 0 and standard deviation of 1). High SES parents are defined as parents who have above median family income. Otherwise, parents are low SES.

age 24 and above) is less than 1, indicating that parents may not know the skill of their child. Figure 2 shows the correlation between beliefs and skills for high SES and low SES parents. The correlation between skills and beliefs is lower for low SES parents, suggesting that low SES parents hold more inaccurate beliefs on average.

Figure 2: Correlation between Belief Factor Score (Anchored) and Skills (Anchored)



Notes: This figure presents the cross-section correlation between belief factor score (anchored to years of education at age 24 and above) and the skill factor score (anchored to years of education at age 24 and above). High SES parents are defined as parents who have above median family income. Otherwise, parents are low SES.

In addition, on average, high SES parents invest more in their children. Table 3 shows the average difference in the standardised investment factor score between the high SES and the low SES parents. The average difference in investment factor scores between high SES and low SES parents is 0.56 standard deviations at age 9-10.

Table 3: Average Difference in Investment between High SES and Low SES

	Mean Investment Gap
9-10 years	0.56
11-12 years	0.62
13-14 years	0.58

Notes: This table displays the average difference in investment factor score (standardised) between the high and low SES families. High SES parents are defined as parents who have above median family income, otherwise they are low SES.

3.2 Parents Update Beliefs when Skills Change

In this section, I provide suggestive evidence that parents revise their beliefs when they receive new information. The way I achieve this is by estimating a belief updating model and showing that the beliefs are correlated with measures of child skills (I refer to these as signals). These have been described in the data section.

Empirical Strategy: I model belief updating in the following way. I assume that the belief for child j at period t (μ_{jt}) depends linearly on the belief in period $t-1$ ($\mu_{j,t-1}$) and the signals at period t (similar to Stinebrickner and Stinebrickner (2014)). The signals are measures of child skills — achievement test scores in mathematics (math), reading recognition (rrecog) and reading comprehension (rcomp), the behaviour problems index (badbehaviour) and child's health (healthexcellent dummy variable). I also include a set of controls denoted by X_{it}. This contains dummies for region of residence (north, south, east, west), urban/rural residence, SMSA status (not in SMSA, SMSA but not in central city, SMSA in central city, SMSA in unknown central city), an indicator for whether mother of the child is employed, an indicator for whether the child's biological father lives with the child, the education of mother's spouse, mother's marital status and the child's age at time t. I also include a child fixed effect δ_i to account for time-invariant characteristics of the child which parents use to form beliefs. This means that the coefficients are estimated based on within-child variation in the lag belief and signals.

$$\mu_{jt} = \alpha + \beta_1 \mu_{j,t-1} + \beta_2 math_{jt} + \beta_3 rrecog_{jt} + \beta_4 rcomp_{jt} + \beta_5 badbehaviour_{jt} + \beta_6 healthexcellent_{jt} + \gamma X_{jt} + \delta_i + \epsilon_{jt} \quad (1)$$

Given the short T panel, since Equation 1 contains both a lag dependent variable and a

fixed effect, estimating this equation via fixed effects or first differences will yield inconsistent estimates of the coefficient on the lag dependent variable. To obtain consistent estimates, I employ the Blundell-Bond dynamic panel data estimator, which is a system generalised method of moments estimator. I do not assume that the contemporaneous skill measures are uncorrelated with the error term (the contemporaneous skill measures may be endogenous), therefore I do not include moment condtiions relating the contemporaneous signals to the error term. More details on the estimator and moment conditions are provided in the appendix.

Estimates: Table 4 presents estimates of the coefficients on the lag belief and the signals in the linear belief updating model. First, estimates suggest that parents are using the previous information which they had about the child to form their current belief — the lag belief is a positive and significant predictor of current belief. Second, the beliefs covary with the skill measures in the expected direction. When achievement test scores are higher, beliefs may be revised upwards. When the child exhibits more behaviour problems, beliefs may be revised downwards. When the child is perceived to be in better health, belief may be revised upwards. Third, some skill measures are statistically significant predictors of current belief, which indicates that beliefs change when skills change. From column 1 of Table 4, a one percentile increase in the mathematics achievement test score percentile is associated with a 0.013 standard deviation increase in the parental belief factor score. A one percentile increase in the behaviour problems percentile (which means a higher level of behaviour problems) is associated with a 0.008 standard deviation decrease in the parental belief factor score. The sensitivity of the beliefs to skill measures does not appear to depend much on socio-economic status.

3.3 Investments are Correlated with Beliefs

In this section, I explore the correlation between parental beliefs and parent investments using the belief factor score and the investment factor score. The construction of both factor scores are explained in the data section of this paper.

Empirical Strategy: To understand the relationship between beliefs and investments, I regress the investment factor score I_{jt} of child j at time t on the belief factor score μ_{jt}. I include child fixed effects to control for time-invariant unobserved characteristics specific to the child which jointly influence beliefs and investments. Furthermore, in one specification, I include the lag standardised investment factor score to capture the persistent preferences of parents which may influence their investment decisions. I interpret the coefficient on the belief as a correlation, because despite including these controls, there may still be unobserved shocks to the child which affect both beliefs and investments. For example, there may be an unobserved shock to child skills which affects both beliefs and investments. The purpose of this exercise is to examine whether beliefs are related to investments, even after controlling for observables

Table 4: Dependent variable is Standardised Parental Belief Factor Score

	(1) All	(2) High SES	(3) Low SES
Lag Belief	0.243***	0.192***	0.256***
	(0.047)	(0.057)	(0.062)
Math	0.013***	0.011***	0.008***
	(0.002)	(0.003)	(0.003)
Reading Recognition	0.002	0.005	0.001
	(0.002)	(0.003)	(0.003)
Reading Comprehension	0.002	0.003	-0.001
	(0.003)	(0.003)	(0.003)
Bad Behaviour	-0.008***	-0.009***	-0.006***
	(0.002)	(0.002)	(0.002)
Health: excellent	0.117	0.147	0.353***
	(0.124)	(0.121)	(0.131)
Observations	5379	3186	2193
Time-Varying Controls	Yes	Yes	Yes
Child FE	Yes	Yes	Yes
Signals Treated as Endogenous	Yes	Yes	Yes

Notes: Regression of the standardised parental belief factor score on lag standardised belief factor score and measures of child skills — achievement test percentile scores in mathematics, reading recognition, reading comprehension, behaviour problems index percentile and child health (dummy variable for excellent). The omitted category for health is poor/fair/good. Refer to earlier section for how parental belief is constructed. Time-varying controls refer to dummies for region of residence (north, south, east, west), urban/rural residence, SMSA status (not in SMSA, SMSA but not in central city, SMSA in central city, SMSA in unknown central city), an indicator for whether mother of the child is employed, an indicator for whether the child's biological father lives with the child, the education of mother's spouse and mother's marital status. All regressions control for the age of the child at the time when the belief was reported. Standard errors in parentheses (* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$). Standard errors are clustered at the family level in the OLS model. Robust standard errors are used in the child FE models. I use all available observations for the child between ages 5 and 14.

and time-invariant unobservables.

Specification with child FE and lag investment for child j at time t:

$$\underbrace{I_{jt}}_{\text{investment}} = \alpha + \beta \underbrace{\mu_{jt}}_{\text{belief}} + \gamma X_{jt} + \psi I_{j,t-1} + \underbrace{\delta_j}_{\text{child FE}} + \eta_{jt} \tag{2}$$

X_{jt}: log(family income), age of child, age of youngest child in household

The regression equation has a lag dependent variable and a fixed effect. Given the short T panel, the coefficient on the lag dependent variable will be inconsistently estimated with fixed effects or first difference estimators. Therefore, I use the Blundell-Bond dynamic panel data estimator (system GMM). This estimator relies on the following assumptions: X_{jt} are pre-determined (weakly exogenous) and η_{jt} have limited time dependence.

Estimates: Table 5 presents estimates of the coefficient on the standardised belief factor score in a regression of the standardised investment factor score on the standardised belief factor score. The four columns represent four different specifications. All specifications suggest that investments are positively associated with beliefs. The first column is the ordinary least squares estimate. The child fixed effects model estimates are presented in columns 2 and 3. In column 3, lag investment is included to capture persistent preferences which parents may have over investment decisions. The child fixed effect estimate (column 3) suggests that a 1 unit increase in the parent belief is associated with a 0.063 standard deviation increase in the investment. The strong correlation between beliefs and investments suggests that there is an important relationship between these variables — it could be that beliefs of parents are influencing their investments. Furthermore, the positive correlation suggests that on average, parents invest more when they hold higher beliefs, as in Dizon-Ross (2019). Despite this finding, in the model, I will not impose that parents invest more when they hold higher beliefs. Instead, the model will be flexible enough to inform whether parents invest more or less when they hold higher beliefs in their child.

3.4 Summary and Implications of Data Patterns

Before moving to the model, I summarise the data patterns and relate them to the research question and the model. First, there are SES differences in beliefs and investments. High socio-economic status parents hold higher beliefs, even conditional on skills. High SES parents also hold more accurate beliefs, on average. In addition, there is a SES investment gap — on average, high SES parents invest more in their children. Second, there is suggestive evidence that parent beliefs change when the skills of their child change. Third, beliefs and investments are correlated, even after controlling for time-invariant characteristics of children. This indicates that there is an important relationship between beliefs and investments, and it could be that

Table 5: Dependent variable is Investment Factor Score

	(1)	(2)	(3)
Belief	0.254***	0.063***	0.069***
	(0.014)	(0.015)	(0.018)
Observations	9526	9526	9526
Child FE	No	Yes	Yes
Lag Investment	No	No	Yes

Notes: This table reports coefficients on the standardised parental belief factor score in a regression of the standardised investment factor score on the standardised belief factor score. Column 3 is estimated with the Blundell-Bond dynamic panel data estimator. The construction of the belief factor score and the investment factor score is provided in the data section of this paper. All regressions control for log(family income), the age of the youngest child and the age of the child at the time when the belief was reported. Standard errors in parentheses (* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$). Standard errors are clustered at the family level in the OLS model. Standard errors are clustered at the child level in the child FE model. Robust standard errors are used in the model with lag investment. I use all available observations for the child between ages 5 and 14.

beliefs are influencing investment choices.

In the next section of the paper, I introduce a dynamic model of parent investments with belief updating to (1) connect belief updating with how beliefs affect investments and how investments affect skills, (2) explore the mechanisms behind the socio-economic status investment gap and (3) conduct a policy experiment. In particular, the model will be used to investigate whether differences in beliefs can explain the socio-economic status skill gap. It will also be used to explore SES differences in beliefs and belief updating. It can also be used to understand the contribution of early beliefs versus later beliefs towards the SES investment gap.

4 Model

4.1 Model Overview

In this section, I introduce a dynamic model of parent investments and belief updating. The model connects parent belief updating with how beliefs influence investments and how investments in turn affect child human capital outcomes (cognitive skill).

In this model, each parent has a single child. The model has 3 periods which are each 2 years in length (corresponding to the frequency of the data used in the estimation). These correspond to the child being aged between 9-10, 11-12 and 13-14.

The sequence of events in periods 2 and 3 are as follows. Parents do not observe their child's cognitive skill. Instead, they enter each period with a prior belief about the cognitive skill (distribution over the log skill). Then they receive a signal (noisy measure of the child's cognitive skill) and update their belief, obtaining a posterior belief about the cognitive skill. Using this posterior belief, parents make their choices for the period: consumption, investment and next period assets. After this, based on the child skill production function, their posterior belief about the current skill and their investment, parents form a belief of the child's skill in the next period. This is the prior belief which they start the next period with.

In the first period, parents do not update their beliefs. They enter directly with the posterior belief about the child's cognitive skill. And make their choices for the period: consumption, investment and next period assets.

Parents are forward looking and are expected utility maximisers. Their utility depends on consumption and the perceived skill of their child (since skill is unobserved). Parents have the incentive to invest in their children, because they know that investing will increase skills, and will raise their own utility (Solon, 1999). Parents make their choices of consumption, investments and next period assets to maximise their objective function, subject to the following processes. Firstly, the child skill production technology, which informs them how their investments influence the next period skills of the child. Secondly, their belief updating process, which informs them how their actions will influence the evolution of future beliefs. Thirdly, the resource constraint and family income process. Parents face uncertainty in the signal generation process, the skill production process and the income process.

The model features observed heterogeneity. The skill production function parameters depend on mother's education — this allows for high education mothers to be more productive in converting investment to child skills. In addition, the family income process depends on mother's education — families with mothers who have higher education are on higher income trajectories.

The model also features unobserved preference heterogeneity. This is introduced to control for unobserved factors which affect both parental beliefs and investments. Parents can be one of two discrete types. One type has a higher relative value for child skill over consumption and is more likely to hold a higher prior belief in the first period. The probability of being either type depends on the initial conditions of the parent-child pair (initial skills of child).

Parent-child pairs are heterogeneous in terms of their beliefs about child cognitive skill, income, assets, mother's education and the unobserved discrete type.

Next, I will explain the value function, the utility, the skill production function, belief updating, resource constraint and income process in greater detail. In the following notation, the parent-child pair will be indexed by j. Time t is indexed by the age of the child.

4.1.1 Utility

The utility of the parent j depends on consumption c_{jt} and the perceived skill of their child θ_{jt} (since they do not observe the skill). This is why parents are motivated to invest — when they invest more, the perceived skill of the child will increase and the parent's utility will increase. That parent utility depends directly on the skills of the child follows Del Boca, Flinn, and Wiswall (2014) and Del Boca, Flinn, Verriest, and Wiswall (2019).

I assume that the utility is additive and separable in terms of logs in both consumption and the perceived skill of the child. There is a weight κ_h on the utility from the perceived skill, which captures the relative value of child skill over consumption. The relative value depends on the unobserved type h — one type of parent will have a higher relative value for child skill over consumption. The expectation operator with the tilde represents that the expectation of the log skills is based on the parent's (subjective) belief about the skill of their child. The belief will be introduced in greater detail later — it is a distribution over the skills of the child.

$$u\left(\underbrace{c_{jt}}_{\text{consumption}}, \underbrace{\tilde{\theta_{jt}}}_{\text{perceived skill}}\right) = \ln c_{jt} + \underbrace{\kappa_h}_{\substack{\text{relative preference for} \\ \text{child skill over consumption}}} \times \underbrace{\tilde{E}_t \ln \theta_{jt}}_{\substack{\text{utility from} \\ \text{perceived skill}}} \tag{3}$$

4.1.2 Child Skill Production Technology

Next period log skills of child j $\ln \theta_{j,t+1}$ will be produced by the current period log skills $\ln \theta_{jt}$ and log investments $\ln i_{jt}$ according to a translog production function (equation 4). The interaction term between the log current skills and the log investment allows for the productivity of investment to depend on the current skill of the child. For example, if $\gamma_{s,3}$ is greater than 0, then investments are more productive when children have higher skill levels. There is an age-specific total factor productivity A_t which captures exogenous time-varying factors which influence child development aside from parent investment (e.g. school education). This A_t is assumed to be equal at age 11-12 and age 13-14. The production of skills is subject to an idiosyncratic shock η_{t+1}, which is normally distributed with mean 0 and variance $\sigma^2_{s,\eta}$. The parameters of the skill production function depend on mother's education $s \in \{1, 2\}$, as mothers with different levels of education may have different levels of productivity.

$$\ln \theta_{j,t+1} = A_t + \gamma_{s,1} \ln \theta_{jt} + \gamma_{s,2} \ln i_{jt} + \gamma_{s,3} \ln \theta_{jt} \times \ln i_{jt} + \eta_{j,t+1} \tag{4}$$

Though parents know the skill production technology (including the return to investment), they do not observe the skill of the child, which is an input in the production function. Therefore, they need to take expectations with respect to the current log skill ($\ln \theta_{jt}$) to determine how

their investment will affect the next period log skill.

In this model, because of the log utility and translog production function, the sign of the $\gamma_{s,3}$ parameter will determine whether parents invest more or less when they hold higher beliefs about their child. If $\gamma_{s,3}$ is positive (negative), parents will invest more (less) in their child when they hold higher beliefs. It is because a positive (negative) $\gamma_{s,3}$ increases (decreases) the marginal return to investment in the child when skills are higher. This can be obtained from the first order condition with respect to investment in a simplified version of this model.

4.1.3 Belief about Child Skill and Belief Updating

Parents do not observe the log skill of the child $\ln \theta_{jt}$. They enter each period with a belief about the log skill, which is a distribution over the log skill of their child j $\ln \theta_{jt} \sim \mathcal{N}(\mu_{jt}, \Delta_{jt})$. Then they receive a signal g_{jt}, which is a noisy measure of the log skill $\ln \theta_{jt}$.

$$g_{j,t+1} = \ln \theta_{j,t+1} + \epsilon_{j,t+1}, \epsilon_{j,t+1} \sim \mathcal{N}(0,\, \sigma_\epsilon^2) \tag{5}$$

After they receive the signal, they update their belief and obtain a posterior belief, which is a posterior distribution over the log skill of the child $\ln \theta_{jt} \sim \mathcal{N}(\mu_{jt}^{int}, \Delta_{jt}^{int})$, which influences the parent's choices of consumption, investment and next period assets. Because the posterior belief is the belief which parents use to make their decisions in that period, this is what we will discuss and keep track of.

$$\mu_{jt}^{int} = \psi_s \mu_{jt} + (1 - \psi_s)\, g_{jt} \tag{6}$$

The mean posterior belief is a weighted average of the mean prior belief and the signal. The weights on the mean prior belief and the signal depend on mother's education category s. This allows for socio-economic status differences in terms of belief updating and accuracy of beliefs about cognitive skills.

Given their posterior belief about the cognitive skill, the skill production function and their investment choice, parents then form their prior belief about the next period log skill for the following period. The mean of the distribution of next period log skill $\mu_{j,t+1}$ equals the parent's expected value of log skill ($\ln \theta_{j,t+1}$), which follows from the skill production function, as in Equation 7 below. The tilde over the expectation operator represents that the expectation is subjective (based on posterior belief of the parent), rather than rational. From Equation 7, parents will hold a higher next period prior mean $\mu_{j,t+1}^{int}$ if they (1) hold higher beliefs about their child's current skill and/or (2) invest more in their child.

$$\begin{aligned}
\mu_{j,t+1} &\equiv \tilde{E}\left[\ln \theta_{j,t+1} | \mu_{jt}, g_{jt}, i_{jt}\right] \\
&= \tilde{E}\left[A_t + \gamma_{s,1} \ln \theta_{jt} | \mu_{jt}, g_{jt}, i_{jt}\right] + \tilde{E}\left[\gamma_{s,2} \ln i_{jt} + \gamma_{s,3} \ln \theta_{jt} \times \ln i_{jt} + \eta_{j,t+1} | \mu_{jt}, g_{jt}, i_{jt}\right] \\
&= A_t + \gamma_{s,1} \tilde{E}\left[\ln \theta_{jt} | \mu_{jt}, g_{jt}, i_{jt}\right] + \gamma_{s,3} \ln i_{jt} \tilde{E}\left[\ln \theta_{jt} | \mu_{jt}, g_{jt}, i_{jt}\right] + E\left[\gamma_{s,2} \ln i_{jt} + \eta_{j,t+1} | \mu_{jt}, g_{jt}, i_{jt}\right] \\
&= A_t + \left(\gamma_{s,1} + \gamma_{s,3} \ln i_{jt}\right) \tilde{E}\left[\ln \theta_{jt} | \mu_{jt}, g_{jt}, i_{jt}\right] + \gamma_{s,2} \ln i_{jt} \quad (7)
\end{aligned}$$

The variance Δ_{jt}^{int} is assumed to be homogeneous for all parents, conditional on mother's education. The parents know the variance they will have in future periods when they make their decisions.

4.1.4 Resource Constraint and Income Process

In every period t, parents have a budget constraint. The sum of consumption, investment and savings (in terms of assets in the next period) cannot exceed the available resources:

$$c_{jt} + i_{jt} + a_{j,t+1} = (1 + r)\, a_{jt} + y_{jt} \quad (8)$$

Log family income is a cubic function of the age of the child. The residuals of this process follow an AR(1). These equations describe two-year income processes, given that each period in the model is two years long. The parameters of the family income process depend on mother's education category $s \in \{1, 2\}$. This is because families where mothers have higher education tend to be on higher income trajectories.

$$\ln(y_{j,t+1}) = \tau_{s,0} + \tau_{s,1} age_{j,t+1} + \tau_{s,2} age_{j,t+1}^2 + \tau_{s,3} age_{j,t+1}^3 + \nu_{j,t+1} \quad (9)$$

$$\nu_{j,t+1} = \rho_s \nu_{jt} + u_{y,jt}, u_{y,jt} \sim \mathcal{N}(0,\, \sigma_u^2) \quad (10)$$

The combination of concave utility and income uncertainty will generate a motive for precautionary savings.

4.1.5 Value Function

The state variables of the parent are (1) the belief about the child cognitive skill after receiving the signal — this is a distribution with mean μ_t^{int} and variance Δ_t^{int}, (2) income y_t, (3) assets a_t, (4) mother's education category $s \in \{1, 2\}$, where $s = 1$ denotes up to median education (up to 12 years education) while $s = 2$ denotes above median education (above 12 years education) and (5) unobserved discrete type $h \in \{1, 2\}$. Type $h = 2$ is what I refer to as the "high" type

parent. This type holds a higher relative preference for child skill over consumption.

In the first and second period, the value function of the parent is given by Equation (11). Parents choose a sequence of consumption, investment and next period assets to maximise the sum of their flow utility and the discounted expected value of the next period value function. There is a tilde over the expectations operator to indicate that the expectation of the parent depends on their subjective belief about their child's skill.

$$V_t\left(\mu_t^{int}, \Delta_t^{int}, a_t, y_t, s, h\right) = \max_{c_t, i_t, a_{t+1}} \quad \ln c_t + \kappa_h \tilde{E}_t \ln \theta_t + \beta \tilde{E}_t V_{t+1}\left(\mu_{t+1}^{int}, \Delta_{t+1}^{int}, a_{t+1}, y_{t+1}, s, h\right) \tag{11}$$

The third and final period value function is given by Equation (12). The final investment decision determines the child's final human capital θ_{T+1}. As before, there is a tilde over the expectations operator to indicate that the expectation of the parent depends on their subjective belief about their child's skill. Different from the previous periods, the parent no longer faces uncertainty in the family income process and signal generation process. I assume that the resources which have not been consumed by the final period will be consumed and discounted by factor β.

$$V_T\left(\mu_T^{int}, \Delta_T^{int}, a_T, y_T, s, h\right) = \max_{c_T, i_T, a_{T+1}} \quad \ln c_T + \kappa_h \tilde{E}_T \ln \theta_T + \beta \kappa_h \tilde{E}_T \ln \theta_{T+1} + \beta \ln\left((1+r)\, a_{T+1}\right) \tag{12}$$

4.1.6 Initial Conditions and Unobserved Heterogeneity in Preferences

To understand the role of parental beliefs, I will compare the investments and skills of children whose parents hold higher beliefs to those with lower beliefs. One concern is that families in which parents hold higher beliefs may be systematically different from those who hold lower beliefs. For instance, parents who hold higher beliefs may also hold higher income and assets. They may also have children with higher skills. I control for these systematic differences in the following ways.

Firstly, following French and Jones (2011), the initial conditions of the parent-child pair (income of parents, assets of parents and initial skills of children) are obtained directly from the data. This means that if parents who hold higher income and assets are also more likely to have children with higher initial skills, this will also be the case in the distribution of simulated individuals used to estimate the model.

Secondly, I introduce unobserved preference heterogeneity in the style of Heckman and Singer (1984) and built on by Keane and Wolpin (1997) along with van der Klaauw and Wolpin (2008). It may be that differences in unobservable preferences are driving differences in both

the beliefs and investments. Furthermore, these underlying preferences are likely to be initial conditions. This is because parents who held different preferences would have invested differently in earlier periods (before the model begins at age 9-10). A way to adjust for this selection is to solve the optimisation problem from an earlier age (before age 9-10 when the model in this paper begins) and then check whether the model can replicate the correlations between preferences and beliefs in the first period of the model. However, as French and Jones (2011) mention, this approach is computationally intensive. Furthermore, this approach would require making assumptions such as the skill production function being identical at earlier ages (before age 9-10), which may be difficult to accept, given findings in the literature. Therefore, instead, I follow the approach of French and Jones (2011) and model the correlation between preferences and a subset of initial conditions. These correlations will inform us about whether parents who hold higher beliefs are also more likely to have a higher relative value for child skill over consumption (and therefore invest more).

The unobservable heterogeneity enters in the form of discrete types. A parent can be one of two unobserved types. One type has a higher relative preference for child skill over consumption and is more likely to start with a higher prior mean belief in the first period. The probability of being either type is determined by a logistic function of the first period value of the log skill of the child (predicted value from factor model)[1]. The first period log skills of the child is informative of the type — the idea is that parents who have higher preferences would have invested more in earlier periods, and this would affect the log skills of the child. The parameter in this type probability function is estimated together with other parameters in the method of simulated moments step explained in the next section.

$$Pr\left(\text{Unobserved Type } h = 2 | \ln \theta_{j1}\right) = \frac{exp\left(\pi_0 \ln \theta_{j1}\right)}{1 + exp\left(\pi_0 \ln \theta_{j1}\right)} \tag{13}$$

5 Identification and Estimation

In this section, I discuss identification of the model, data preparation followed by the estimation of the model.

5.1 Discussion of Identification

The family income process can be identified directly from moments in the data. Next, I discuss the identification of the distribution of the latent factors (beliefs, log skills and investment) and the skill production function parameters.

[1] In an earlier version of the paper, the type probability depended on the first period beliefs, income of parents, assets of parents and the skill of the child. But the coefficients on the variables aside from the skill of the child were very small, and it appeared that some coefficients were not well identified.

At each time period t, there are $L_t^I \geq 3$ noisy measurements of investments for child j. Each measurement b_{jk} for child j at time period t is assumed to be related to the latent investment according to the following equation:

$$b_{jkt} = \alpha_{kt} + \lambda_{kt}\Phi_{jt} + v_{jkt}, k \in \left\{1, 2, ...L_t^I\right\} \tag{14}$$

The errors v_{jkt} are assumed to be independent across individuals and measures. In a similar way, there are $L_t^\theta \geq 3$ measurements of the latent log skills $\ln \theta_{jt}$ and $L_t^\mu \geq 3$ measurements of the latent parental belief μ_{jt}^{obs} for $t = 1, 2, 3$. Given this, we can identify the distribution of $\left(\left\{\Phi_{jt}\right\}_{t=1}^3, \left\{\ln \theta_{jt}\right\}_{t=1}^3, \left\{\mu_{jt}^{obs}\right\}_{t=1}^3\right)$ according to arguments in Cunha, Heckman, and Schennach (2010).

The belief factors and log skill factors do not have natural location and scale. I follow Cunha et al. (2010) and anchor these factors in terms of years of education by age 24 and above. In addition, the investment factor score Φ_{jt} is not in monetary units, while the investment in the model is in monetary units. To make both measures comparable, I adopt the following method from Caucutt and Lochner (2020). I assume that there is a parametric mapping function $\phi(.)$ which connects the investment factor Φ_{jt} to the monetary investment i_{jt} each period. That is, $\Phi_{jt} = \phi(i_{jt})$, where the function $\phi(.)$ is constrained to be monotone, so that higher values of the investment factor correspond to higher values of monetary investment. Consequently, given the parametric assumption, the function can be inverted to obtain investment in monetary units i_t. And therefore we can retrieve $\ln i_t$ and its associated distribution.

Since the mapping between Φ_{jt} and i_{jt} is unique as $\phi(.)$ is monotone, we can substitute $i_{jt} = \phi^{-1}\left(\Phi_{jt}\right)$ into the production function. Given the distribution $\left(\left\{\Phi_{jt}\right\}_{t=1}^3, \left\{\ln \theta_{jt}\right\}_{t=1}^3, \left\{\mu_{jt}^{obs}\right\}_{t=1}^3\right)$ and the parametric form of the production function, we can identify the parameters in the skill production function and investment mapping function. Because the parameters of the $\phi(.)$ function are known, we can identify the coefficients in the production function.

$$E\left[\ln \theta_{j,t+1} | \ln \theta_{jt}, \Phi_{jt}\right] = E\left[A_t + \gamma_{s,1} \ln \theta_{jt} + \gamma_{s,2} \ln \phi^{-1}(\Phi_{jt}) + \gamma_{s,3} \ln \theta_{jt} \times \ln \phi^{-1}(\Phi_{jt}) + \eta_{jt}\right]$$

The variance of the error term in the skill production technology can be identified as $\sigma_{s,\eta}^2 = Var(\ln \theta_{j,t+1}) - Var(\gamma_{s,1} \ln \theta_{jt} + \gamma_{s,2} \ln \phi^{-1}(\Phi_{jt}) + \gamma_{s,3} \ln \theta_{jt} \times \ln \phi^{-1}(\Phi_{jt}))$. The variances on the right hand side of the equation are identified from the distribution of the latent factors of log skills and investments, along with the mapping from the investment factor to monetary investment (which enables us to retrieve the distribution of the log investment).

However, identification will fail if the error term in the production function is correlated with any of the input arguments. This may occur, for example, if parent investments respond

to a shock to the child skill which is unobserved to the researcher.

Table 6: Summary of Identification of Estimated Parameters

Parameter	Identifying Data Features
$\kappa_{h=1}, \kappa_{h=2}$	Ratio of next period assets to investment factor score, moments of investment factor score conditional on observables
$A_1, A_2, \gamma_{s=1,1}, \gamma_{s=1,2}, \gamma_{s=1,3}, \gamma_{s=2,1}, \gamma_{s=2,2}, \gamma_{s=2,3}, \sigma^2_{s=1,\eta}, \sigma^2_{s=2,\eta}$	Coefficients and residuals of regression of log skill factor score on lag log skill factor score, lag investment factor score and interaction between lag log skill factor score and lag investment factor score
$\psi_{s=1}, \psi_{s=2}, \sigma^2_\epsilon$	Coefficients and residuals of regression of the posterior belief on lag log skills, lag investment and the current log skills
π_0	Residuals of regression of investment factor score on belief factor score and the correlation of investment factor score conditional on quartile of skill factor score

Notes: This table provides a summary of the moments which are particularly informative of the parameters estimated in the method of simulated moments step. For ease of exposition, I refer to the belief factor score, log skills and investment factor scores as data, even though they are quantities estimated from the raw data with factor models. In practice, after I estimate the factor models, I use the predicted factor scores of the belief, log skills and investment factor scores as data in the two-step estimation procedure.

Now, I discuss the remaining parameters to be identified. In theory, all parameters are jointly identified by the moments and statistics from the auxiliary models. However, some moments and statistics from the auxiliary models may be particularly informative for certain parameters (parameters have first order effects on these moments, rather than only second order effects).

The relative preference for child skill over consumption $\{\kappa_1, \kappa_2\}$ will be particularly informed by moments of the ratio of next period assets to the current period investment factor score. This is because parents who have a higher preference for skill over consumption will invest more in the current period and save less (lower next period assets). This can be seen in the optimal solution to a simplified static version of the model. It is also because I assume that the price of monetary investment is equal to the price of consumption (both normalised to 1), so that the relative price of investment will not enter the first order condition.

The weight on the prior belief in belief updating $\{\psi_{s=1}, \psi_{s=2}\}$ and the variance of the error term in the signal σ^2_ϵ are particularly informed by the coefficients and residuals of a regression of the belief factor score on the lag skill factor score, lag investment factor score and the skill factor score.

The type probability parameter π_0 will be particularly informed by two categories of moments. First, the standard deviation of the investment factor score conditional on the belief factor score. The idea is that conditional on observables, the variation in investment is driven by the unobserved heterogeneity. Second, the autocorrelation of the investment factor score conditional on log skills of the child, which provides information on the "fixed effect" of unobserved preferences.

In Table 6, I summarise the data features which are most informative for the estimation of particular parameters. For ease of exposition, I refer to the belief factor score, log skills factor scores and investment factor scores as data, even though they are quantities estimated from the raw data with factor models. In practice, after I estimate the factor models, I use the predicted factor scores of the belief, log skills and investment as data in the two-step estimation procedure.

5.2 Data Preparation

Sample selection for model estimation and data moments: I focus on the children of the females in the cross-section sample of the National Longitudinal Survey of Youth 1979 (NLSY79). In the estimation, I include all the observations with non-missing values of these variables in the first period of the model (child aged 9-10): parental belief (estimated from the factor model), income, assets and child skill. Parental beliefs, income, assets, child skills, investments can be missing in other periods. For the time being, I assume that these values are missing at random and compute the data moments from the relevant non-missing values. For example, the average parental beliefs when child is age 11-12 is computed as the average of all non-missing parental beliefs for children aged 11-12. Furthermore, because the model does not allow parents to borrow (assets must be weakly positive), I exclude observations with negative values of assets in any of the 3 periods. On top of this, I only include children whose mothers reported that they were married when the child was aged 9-10, 11-12 and 13-14. This leaves a sample size of 635 parent-child pairs for the estimation.

Factor models and anchoring: Factor models are used to extract the underlying latent beliefs, investments and log skills of the child from multiple noisy measures of these latent variables. Factor models are useful because they adjust for measurement error in the noisy measures of these latent variables. The measures used to estimate the factor models for beliefs and investments have been described in the earlier section on data. The measures used to estimate the factor model for log skills of the child are achievement test scores in mathematics, reading recognition and reading comprehension.

I interpret the belief extracted from the factor model as the mean posterior belief of the parent. This is because by the time the mother reports the belief measures, I expect that she would

already have received signals about the current skill level of the child.

I require the belief factor to be comparable with the log skills factor. To achieve this, I set the latent factors of beliefs and log skills to be in the same units, by anchoring both latent factors to years of schooling at age 24 and above. This means that a 1 unit increase in the latent factor of belief (or log skill) corresponds to a 1 unit increase in the conditional expectation of the years of education by age 24 and above. One challenge is that the measures used for the log skills latent factor grow over time, while the measures used for the belief latent factor do not. As a result, if I use these measures to estimate latent factor models for log skills and beliefs directly, mean log skills will grow over time while mean beliefs will not, so the levels of the two latent factors cannot be compared. To address this, I interpret the measures of belief as capturing the perceived deviation of the log child skills from the mean log skills. Using the belief measures, I estimate a latent factor, which I interpret as the deviation of the belief of skill from the mean. Then I correct for the deviation by adding the mean skill of the child to the factor score at the corresponding age. This corrected measure is what I call the belief factor score.

More information on the factor models for belief and skill are provided in the appendix. The factor models are estimated and the predicted factor scores are treated as data in the two-step estimation procedure.

Converting investment factor scores to investment in monetary units: The data contains multiple noisy measures of investment, which are not in monetary units. As discussed above, we can identify the monetary investment by assuming a functional form for the $\phi(.)$ mapping function between the factor and the investment in monetary units. In practice, as in Caucutt and Lochner (2020), I assume a quadratic mapping function between investment factors Φ_{jt} and the investment in monetary units i_{jt} for child j at time t. That is, $\Phi_{jt} = \phi(i_{jt}) = \phi_0 + \phi_1 i_{jt} + \phi_2 i_{jt}^2$. The function is monotone, so that $\phi'(.) > 0$, which means higher factor scores correspond to higher monetary investment. For the time being, the parameters of this function are set at $\phi_0 = -18.23 \times 10^{-2}$, $\phi_1 = -4.46 \times 10^{-6}$ and $\phi_2 = 5.62 \times 10^{-14}$.

5.3 Estimation

Estimation proceeds in two steps. In the first step, some parameters are calibrated or estimated directly from the data. These parameters are then used as inputs in the second step, where the remaining parameters are estimated via method of simulated moments.

5.3.1 Step One: Calibration and External Estimation

Following Del Boca et al. (2014), the discount factor for households is set to $\beta = 0.95$. The corresponding interest rate is $r = \frac{1}{\beta} - 1$. The family income process is estimated directly from the data. I obtain estimates of the following parameters which are used as inputs in the step

two of the estimation.

$$\left\{\tau_{s=1,0}, \tau_{s=1,1}, \tau_{s=1,2}, \tau_{s=1,3}, \rho_{s=1}, \sigma^2_{s=1,u}, \tau_{s=2,0}, \tau_{s=2,1}, \tau_{s=2,2}, \tau_{s=2,3}, \rho_{s=2}, \sigma^2_{s=2,u}\right\}$$

5.3.2 Step Two: Method of Simulated Moments

In step two, I estimate the remaining parameters via method of simulated moments: the relative preference for child skill over consumption, the parameters in the skill production function, the parameters relating to parental beliefs and belief updating and the parameter in the type probability function. The set of parameters to estimate is Γ defined below.

$$\begin{aligned}\Gamma \equiv \big\{&\kappa_{h=1}, \kappa_{h=2}, A_1, A_2, \gamma_{s=1,1}, \gamma_{s=1,2}, \gamma_{s=1,3}, \sigma^2_{s=1,\eta}, \dots \\ &\gamma_{s=2,1}, \gamma_{s=2,2}, \gamma_{s=2,3}, \sigma^2_{s=2,\eta}, \sigma^2_{\epsilon}, \psi_{s=1}, \psi_{s=2}, \pi_0\big\}\end{aligned}$$

Given a guess of parameters, the optimisation problem of the parent is solved numerically by backward induction, beginning from the final period T. This provides the policy functions (choices of investment, consumption and next period assets given the values of the state variables) and value function. Next, the trajectories of 5 $\times$ N parent-child pairs are forward simulated. Given the type probability parameter, in every simulation, a type is drawn for each parent. Then shocks are drawn and the policy functions are used to determine the choice variables and the values of state variables in the following period. After that, moments and statistics from auxiliary models are computed from the simulated data ($M(\Omega)$). Finally, the GMM criterion function (15) is computed with the distance between the moments and statistics from the auxiliary models from the simulated data ($M(\Omega)$) and the corresponding statistics from the sample data (M_D). A new guess of the parameters is made and the abovementioned steps are repeated until we find the parameter values which minimise the GMM criterion function.

The method of simulated moments estimator is given by Equation (15), where M_D are the data moments and $M(\Omega)$ are the corresponding moments derived from the simulated data, which are generated from the parameters Ω.

$$\hat{\Omega} = \arg\min_{\Omega} \left(M_D - M(\Omega)\right)' W \left(M_D - M(\Omega)\right) \tag{15}$$

I compute the variance-covariance matrix of the moments and statistics from auxiliary models S by bootstrap with $K = 200$ replications according to the formula (16), using the moments and statistics from auxiliary models computed from the data (M_D) and the corresponding statistics computed from K resamples of the data ($M_1, M_2, ...M_K$).

$$S = \frac{1}{K} \sum_{K}^{k=1} (M_k - M_D) (M_k - M_D)' \tag{16}$$

The asymptotically optimal weighting matrix would be the inverse of the variance-covariance matrix of the moments. However, in small samples, using the optimal weighting matrix may not be ideal (Altonji & Segal, 1996). Therefore, instead of using the optimal weighting matrix, the weighting matrix W which I use is the matrix which retains the diagonal entries of the inverse of S and has zeros on the off-diagonal elements. Standard errors for the parameter estimates are computed using the formula derived by Gourieroux, Monfort, and Renault (1993), which adjust for the simulation noise.

I use a combination of moments and statistics from auxiliary models to estimate the model. For example, mean and standard deviation of skill factor scores, investment factor scores and assets. Both unconditional and conditional moments (conditional on family income quartile, quartile of skills in the previous period) are included. In addition, I include contemporaneous covariances or correlations between variables (skills, investment factor score, belief factor score) and correlations between variables over time. To estimate the skill production function parameters, I include the coefficients and residuals of a regression of the skill factor score on the lag skill factor score, lag investment factor score and an interaction between the lag skill factor score and the lag investment factor score. A comprehensive list of the moments used in the estimation is provided in the appendix.

6 Model Estimates

In this section, I present the parameter estimates, discuss the model fit and explore features of the baseline.

6.1 Parameter Estimates

The parameters in the income process were estimated directly from the data (step one). The remaining parameters were estimated by method of simulated moments (step two). Standard errors for the estimates will be computed using the formulas derived by Gourieroux et al. (1993), which adjusts for simulation noise (in progress).

Preferences and Type Probabilities: There are two unobserved discrete types of parents. One type has a higher relative preference for skill of the child over consumption. I refer to this type as the "high" type. Parents who have children with higher log skills in the first period are more likely to be the "high" type. This makes sense as parents who hold higher relative preferences for skills would be more likely to invest more in earlier periods, which means that their children have higher log skills by the first period of the model. In the simulated data,

parents are more likely to be the "high" type (around 63%).

Table 7: Preference Parameters and Type Probability Parameters

Parameter	Description	Estimate	S.E.
$\kappa_{h=1}$	Relative preference for skill over consumption: Type 1	1.991	0.00341
$\kappa_{h=2}$	Relative preference for skill over consumption: Type 2	3.123	1.50579
π_0	Type probability logistic: coefficient on log skill	0.498	0.25463

Skill Production Function: The weight on the current log skills in the production function is around 0.69 and above, which suggests that cognitive skills are persistent over time. This is in line with the literature on child skill production functions (see for example Cunha et al. (2010) and O. Attanasio, Meghir, and Nix (2020)). Investments by mothers with higher education are more productive — the weight on log investment in the production function is higher. The variance of idiosyncratic shocks in the production function is low for mothers with low education (up to median education) and mothers with high education (above median education).

Table 8: Child Skill Production Function Parameters

Parameter	Description	Estimate	S.E.
A_1	Age 9-10 total factor productivity	0.328	0.16026
A_2	Age 11-12, 13-14 total factor productivity	0.216	0.16097
Up to Median Education			
$\gamma_{s=1,1}$	Weight on log skill	0.701	0.01212
$\gamma_{s=1,2}$	Weight on log investment	0.038	0.01029
$\gamma_{s=1,3}$	Weight on interaction between log skill and log investment	0.025	0.00088
$\sigma^2_{s=1,\eta}$	Variance of idiosyncratic shock	0.103	0.01623
Above Median Education			
$\gamma_{s=2,1}$	Weight on log skill	0.795	0.02430
$\gamma_{s=2,2}$	Weight on log investment	0.053	0.01920
$\gamma_{s=2,3}$	Weight on interaction between log skill and log investment	0.014	0.00042
$\sigma^2_{s=2,\eta}$	Variance of idiosyncratic shock	0.117	0.01853

The coefficient on the interaction term between log skills and log investment ($\gamma_{s,3}$) is positive for both mothers with high education ($s = 2$) and low education ($s = 1$). With a log utility function and the translog production function, the sign of this coefficient determines whether parents invest more/less when they hold higher beliefs in their child. As the interaction is positive, parents who hold higher beliefs will invest more in their child.

Belief Updating: The weight on the prior belief are not statistically different for both high education and low education mothers. This suggests belief updating does not depend on SES — parents place similar weights on the prior and the new information (signal) when revising their beliefs.

Income Process: The income process depends on mother's education (categories: up to median education, above median education). Families with high education mothers ($s = 2$) are on higher income trajectories.

Table 9: Belief Parameters

Parameter	Description	Estimate	S.E.
σ^2_ϵ	Variance of error in signal	2.456	0.50047
$\psi_{s=1}$	Weight on prior (Up to median education)	0.678	0.04528
$\psi_{s=2}$	Weight on prior (Above median education)	0.662	0.04405

Table 10: Family Income Process Parameters

Parameter	Description	Estimate	S.E.
Up to Median Education			
$\tau_{s=1,0}$	Intercept term	11.052	0.52939
$\tau_{s=1,1}$	Coefficient on age	-0.024	0.18055
$\tau_{s=1,2}$	Coefficient on age squared	0.004	0.01951
$\tau_{s=1,3}$	Coefficient on age cubed	-1.008×10^{-4}	0.00067
$\rho_{s=1}$	AR(1) residual persistence parameter	0.605	0.04065
$\sigma^2_{s=1,u}$	AR(1) residual variance	0.257	0.04754
Above Median Education			
$\tau_{s=2,0}$	Intercept term	11.204	0.49423
$\tau_{s=2,1}$	Coefficient on age	0.038	0.16958
$\tau_{s=2,2}$	Coefficient on age squared	-1.373×10^{-4}	0.01840
$\tau_{s=2,3}$	Coefficient on age cubed	-4.320×10^{-5}	0.00064
$\rho_{s=2}$	AR(1) residual persistence parameter	0.518	0.04556
$\sigma^2_{s=2,u}$	AR(1) residual variance	0.413	0.05994

6.2 Model Fit

I present a subset of the data moments and the model moments in Table 11 — the mean beliefs by family income quartile at ages 11-12 and 13-14 and the mean log skills by family income quartile at ages 11-12 and 13-14. A complete list of moments used in the estimation and the values of the data moments and model moments are provided in the appendix.

Table 11: Selected Moments

Moment	Data	Model
Mean log skill age 11-12: Family income Q1	13.454	13.523
Mean log skill age 11-12: Family income Q2	13.684	13.689
Mean log skill age 11-12: Family income Q3	13.755	13.754
Mean log skill age 11-12: Family income Q4	14.050	13.863
Mean log skill age 13-14: Family income Q1	13.801	13.741
Mean log skill age 13-14: Family income Q2	13.959	13.935
Mean log skill age 13-14: Family income Q3	14.107	14.016
Mean log skill age 13-14: Family income Q4	14.324	14.174
Mean belief age 11-12: Family income Q1	13.329	13.481
Mean belief age 11-12: Family income Q2	13.643	13.638
Mean belief age 11-12: Family income Q3	13.889	13.803
Mean belief age 11-12: Family income Q4	14.242	13.924
Mean belief age 13-14: Family income Q1	13.775	13.722
Mean belief age 13-14: Family income Q2	13.755	13.919
Mean belief age 13-14: Family income Q3	14.170	13.988
Mean belief age 13-14: Family income Q4	14.509	14.183

7 Counterfactuals

7.1 Decomposition of Socio-economic Status Skill Gap

In this section, to investigate the relative contribution of parental beliefs about child cognitive skill towards the socio-economic status (SES) skill gap, I decompose the SES skill gap by conducting several counterfactuals where I shut down different sources which drive the SES skill gap. The SES skill gap is defined in Equation (17), where the terminal skill refers to the skill which the child has after the parent makes the final investment decision (in third and last period).

$$\text{SES skill gap} = \text{Mean(high SES terminal skill)} - \text{Mean(low SES terminal skill)} \tag{17}$$

Various factors contribute towards the SES skill gap in the model. These include differences in beliefs of parents, skill levels of children, resources of parents (income, assets), unobserved preferences (relative preference for child skill over consumption). They also include SES differences in belief updating (different weights placed on the prior versus the signal), income trajectories and skill production function.

To determine the contribution of each of these factors, I conduct several counterfactuals where I eliminate each of these sources of inequality and explore how much the SES skill gap changes. For each counterfactual, Table 12 presents the sources of inequality, the skill gap and the corresponding percentage change in the SES skill gap. Appendix Table A.2 presents the SES investment gaps in period 1, 2 and 3 in the same counterfactuals. In Table 12, an X in the column represents that there is inequality in that factor across parent-child pairs. In the counterfactuals, equalising the belief updating, equalising the income trajectory and skill production functions means assigning the high SES belief updating weight on the prior, income trajectory and skill production function parameters to all parent-child pairs respectively. Equalising initial beliefs, initial income, initial assets and initial skills of children is achieved by assigning all parent-child pairs the median value of these variables (computed from the baseline) in the first period, respectively. Equalising preferences denotes assigning all parent-child pairs the "high" type preference — all parents are the discrete type which has the higher relative value for child skill over consumption.

The first counterfactual (C1) is the model baseline. Parent-child pairs are heterogeneous in terms of initial beliefs about their child's cognitive skill, income, assets and children's skills. These values are obtained directly from the data. There are socio-economic status differences in the belief updating process, income trajectory and the skill production function. In subsequent counterfactuals, sources of inequality are progressively removed.

When inequality in beliefs is removed (parents have the same beliefs in period 1) in counterfactual C1.a, the SES skill gap hardly changes. In counterfactuals C1.b to C1.d, when inequality in initial skills, initial income or initial assets are removed, the SES skill gap decreases by a greater percentage from the baseline. From counterfactual C2 onwards, SES differences in belief updating, income trajectories and skill production function parameters are eliminated. We see a similar pattern in the SES skill gap when moving from counterfactuals C2 to C2.a to C2.d. When inequality in beliefs is removed in counterfactual C2.a, the SES skill gap hardly decreases from C2. In counterfactuals C2.b to C2.d, when the inequality in child skills, income and assets are removed, the SES skill gap decreases.

Overall, by themselves, differences in initial parental beliefs do not contribute much to the SES skill gap. When moving from counterfactuals C1 to C1.a or from C2 to C2.a, the SES skill gap decreases at most by a few percentage points from the baseline. In contrast, differences in initial skills of children and resources of parents (income, assets) contribute more to the SES skill gap. Eliminating SES differences in belief updating, income process and skill production function parameters raises the SES skill gap. This is primarily driven by assigning the low SES families the high SES production function, which has lower complementarity between the skills and investments, reducing the force to invest more when there are higher beliefs.

Table 12: SES Skill Gap in Counterfactual Scenarios

Scenario	Belief Updating	Income Trajectory	Skill Production Function	Initial Belief	Initial Skill	Initial Income	Initial Assets	Preferences	SES Skill Gap	Percentage of Baseline SES Skill Gap
C1	X	X	X	X	X	X	X	X	0.748	100.000
D1.a	X	X	X		X	X	X	X	0.743	99.332
D1.b	X	X	X	X		X	X	X	0.591	79.038
D1.c	X	X	X	X	X		X	X	0.591	78.998
D1.d	X	X	X	X	X	X		X	0.311	41.558
D1.e	X	X	X	X	X	X	X		0.737	98.605
C2				X	X	X	X	X	0.737	98.571
D2.a					X	X	X	X	0.733	98.032
D2.b				X		X	X	X	0.586	78.289
D2.c				X	X		X	X	0.642	85.805
D2.d				X	X	X		X	0.412	55.129
D2.e				X	X	X	X		0.728	97.361

Notes: This table presents the SES skill gap and the corresponding percentage of the baseline SES skill gap for different counterfactual scenarios. An "X' in the belief updating, income trajectory and skill production function means that belief updating, income trajectories and/or skill production functions differ by SES. When the "X" is absent, the belief updating, income trajectories and/or skill production function have been set at the high SES values for all families. An "X" in the initial beliefs, initial skill, initial income and/or initial assets means that these are heterogeneous across families. When the "X" is absent, these values have been set at the median value across all families respectively. An "X" in the preferences means that there is unobserved heterogeneity in terms of preferences. The absence of an "X" means that all families have the "high" type relative preference for child skill over consumption. Low SES is defined as families with up to median income when the child is age 9-10, otherwise families are high SES.

In addition, I examine whether there may be any interactions between parental beliefs and other sources of inequality: children's skills, parent resources and unobserved preferences. I assess this by estimating counterfactuals where inequality in initial beliefs is removed together with inequality in one another factor in the first period. The SES skill gaps in these counterfactuals are presented in Appendix Table A.1 (corresponding SES investment gaps in periods 1, 2 and 3 are presented in Appendix Table A.3). Then I compare the SES skill gap in these counterfactuals with the counterfactual where the inequality in only a single factor has been eliminated. For example, to determine whether beliefs interact with skills, I compare the SES skill gap in counterfactual C1.a (inequality in initial beliefs removed) and counterfactual C1.b (inequality in initial skills removed) with counterfactual D1.b (inequality in beliefs and initial skills removed). If the sum of the decrease in the SES skill gap in counterfactuals C1.a and C1.b is equal to the decrease in the SES skill gap in counterfactual D1.b, it suggests that there are no interactions between beliefs and skills. Analogous comparisons can be made to assess whether there are interactions between beliefs and other factors. The computations suggest that beliefs interact with initial skills, income, assets and preferences. However, the differences are not large, suggesting that the strength of the interaction is low.

On balance, the results in this section suggest that even though the experimental evidence indicates that there are short-term effects of beliefs on investments and skills, long-term effects of beliefs may be limited.

7.2 Eliminating Belief Updating and Imperfect Information about Child Cognitive Skill

To understand the importance of incorporating belief updating and imperfect information about the child's cognitive skill into models of parent investments, I estimate a counterfactual which is akin to "perfect information", where I shut down belief updating. In this counterfactual, the posterior mean beliefs of parents match the log skills of their child in every period. In addition, parents do not update their beliefs. All other components of the model are the same as before.

In practice, this is achieved by simulating parents' decisions when (1) posterior mean beliefs are set equal to the log skills of the child, (2) posterior belief variances are set close to 0 (in practice it is 0.05) and (3) the weight on the prior mean belief in belief updating is 1 (this means that the weight on the signal is 0). Parents still face uncertainty in the evolution of skills, but the uncertainty only arises from the idiosyncratic error term in the production function, instead of also because they do not observe the current skills of their child. They still face uncertainty in the income process. But at the point when they make their investment, consumption and next period asset decisions, they observe the cognitive skill level of their child. And they also know that in future periods, they will observe the skill of their child before making their decisions.

Table 13 presents the percentage change in the log skills, beliefs (which are now equal to the skills of the child by construction), investments, consumption and assets in the high SES and low SES groups from the baseline, which features information frictions about the child skill and belief updating.

Table 13: Percentage Change from Baseline

	Low SES	High SES
Mean log skill age 11-12	0.009	-0.003
Mean log skill age 13-14	0.023	-0.005
Mean log skill terminal	0.024	-0.008
Mean posterior belief age 9-10	1.058	-1.310
Mean posterior belief age 11-12	0.659	-0.714
Mean posterior belief age 13-14	0.465	-0.420
Mean investment age 9-10	0.136	0.083
Mean investment age 11-12	0.218	-0.035
Mean investment age 13-14	0.078	-0.146
Mean consumption age 9-10	-0.284	0.802
Mean consumption age 11-12	-0.322	-0.509
Mean consumption age 13-14	-0.051	-0.067
Mean next period assets choice age 9-10	0.014	-0.263
Mean next period assets choice age 11-12	0.006	-0.143
Mean next period assets choice age 13-14	-0.005	-0.084

Notes: This table presents the percentage change from the baseline in the mean log skills, mean prior beliefs, mean investment, mean consumption and mean next period assets choice in the counterfactual which eliminates belief updating and imperfect information about the skill level of children. Low SES is defined as families with up to median income at age 9-10, otherwise families are high SES.

On average, high SES over-estimate the skills of their child and low SES under-estimate the skills of their child. Therefore, when beliefs are changed to match the skills of children, low SES (high SES) will raise (lower) beliefs in the first period, which is age 9-10. Higher (lower) beliefs in turn lead to higher (lower) investments, because of higher (lower) perceived returns to investments, due to complementarity between skills and investments in the production function. Higher (lower) investments in turn lead to higher (lower) beliefs in the second period, and so on. Eventually, the average skills of low SES increase and the average skills of high SES decrease. Therefore, removing belief biases will work to lower the SES skill gap.

The feedback between beliefs and investments does not appear to be strong on the average. Next, we zoom into the subgroups which are the most affected by the change from imperfect information to perfect information. These are the groups which had beliefs which were furthest away from the skills of their child. Specifically, we focus on the following subgroups: (1) At or below median of those who under-estimated and (2) At or above median of those who over-estimated. Table 14 presents the percentage change in the log skills, beliefs (which are now equal to the skills of the child by construction), investments, consumption and assets in these subgroups from the baseline.

These results highlight the dynamic relationship between beliefs and investments, which arise because of the complementarity between skills and investments in the production function.

Table 14: Percentage Change from Baseline

	Under-estimate	Over-estimate
Mean log skill age 11-12	0.063	0.019
Mean log skill age 13-14	0.165	0.055
Mean log skill terminal	0.131	0.044
Mean posterior belief age 9-10	11.392	3.317
Mean posterior belief age 11-12	6.526	2.032
Mean posterior belief age 13-14	4.264	1.311
Mean investment age 9-10	2.876	0.707
Mean investment age 11-12	3.902	1.169
Mean investment age 13-14	-0.798	-0.181
Mean consumption age 9-10	-4.935	-1.293
Mean consumption age 11-12	-2.175	-0.849
Mean consumption age 13-14	-2.049	-0.516
Mean next period assets choice age 9-10	-0.085	-0.021
Mean next period assets choice age 11-12	-2.272	-0.546
Mean next period assets choice age 13-14	-0.795	-0.194

Notes: This table presents the percentage change from the baseline in the mean log skills, mean prior beliefs, mean investment, mean consumption and mean next period assets choice in the counterfactual which eliminates belief updating and imperfect information about the skill level of children. Low SES is defined as families with up to median income at age 9-10, otherwise families are high SES.

Parents who under-estimated (over-estimated) the skills of their child would want to invest more in earlier periods (invest more in later periods) because they perceive the marginal returns to investment to be higher (lower) than before. Investments are higher (lower) in the first two periods for those who under-estimated (over-estimated) the skills of their child. Consumption and assets are generally lower (higher) than the baseline.

Most models assume that parents perfectly observe the skills of their child. In such models, the predicted investments of parents who under-estimate (over-estimate) the skills of their child would be too high (low). This counterfactual suggests that omitting belief updating and assuming perfect information about the child skill could lead to wrong predictions of the investments made by parents who have over-estimated or under-estimated the skills of their children. Given that many parents do not know the skills of their child, this could be significant.

8 Conclusion

Parent investments are important determinants of children's human capital outcomes. Gaps in investments between rich and poor families translate into gaps in the skills of children. Various factors contribute towards the socio-economic status investment gap, including differences in parental resources, initial skills of children, preferences of parents for investing in their children and beliefs of parents. The aim of this project is to quantify the role of parent beliefs about child cognitive skill in explaining the SES skill gap.

To assess this, I estimate a dynamic model of parent investments and belief updating.

The model has the following features. Parents do not observe the skill level of their child. Instead, they hold a belief about their child's skill. Parents make investments which influence the evolution of the unobserved skill. In addition, they learn about the skill over time. They receive signals (noisy measures of the child skill) which they use to update their belief. In the model, there is feedback between beliefs and investments. The beliefs which parents have affect their investment choices. Moreover, the investment of parents also affects their beliefs. Parents who invest more will hold higher beliefs about their child's skills in the next period. Through this channel, beliefs in earlier periods can influence beliefs in later periods.

Model estimates highlight that the feedback between beliefs and investments acts to reinforce the effect of initial beliefs on later beliefs and investments. Parents who hold higher beliefs invest more, because the marginal return to investment is higher due to complementarity between skills and investments in the skill production function. Parents who invest more will hold higher beliefs about the skill of their child in the following period, which will incentivise them to invest more again for the same reasons, and so on. In addition, the estimates indicate that there are no differences in belief updating by SES.

Using the estimated model, I decompose the socio-economic status (SES) skill gap. I find that differences in initial parental beliefs about their child's cognitive skill do not explain much of the SES skill gap. Instead, initial differences in parental resources (income, assets) and skills of children explain a larger proportion of the gap. This result indicates that although experimental evidence suggests that beliefs affect investments and skills in the short-term, the long-term effect of beliefs may be limited.

To understand the implications of information frictions, I conduct a counterfactual where I eliminate belief updating and imperfect information about the skills of the child. Parents beliefs match the skills of their children and they do not update their beliefs. Because high SES parents over-estimated the skills of their child and low SES under-estimated the skills of their child on average, beliefs of high SES (low SES) will decrease (increase) on average. Because the dynamic relationship between beliefs and investments indicates that higher beliefs lead to higher investments, high SES (low SES) parents will invest less (more) on average. Overall, the feedback between the beliefs and investments has an equalising effect on the skills between the two groups. Most models assume that parents observe the skills of their child. This counterfactual suggests that assuming perfect information will lead to wrong predictions of investments for parents who have under-estimated or over-estimated the skills of their children.

It could be that the effect of parental beliefs about the child's cognitive skill is larger than what I have estimated. First, the model abstracts from parental beliefs and skills of the child on alternative dimensions. It may be that parental beliefs about the child's cognitive skill are being counteracted by parental beliefs about returns to investment or non-cognitive skills. Second,

the model omits school inputs and teacher beliefs. Suppose school inputs and home investment are substitutes, schools provide more resources to high potential children and parental beliefs are highly correlated with teacher beliefs. Then when parents hold higher beliefs, teachers also hold higher beliefs and provide more resources to children. Since school and home inputs are substitutes, parents may invest less even though they hold higher beliefs.

In the future, it would be interesting to determine whether these results also apply to other contexts, such as developing countries and younger children. In addition, extending the model framework to incorporate multiple dimensions of skills, parental beliefs on other dimensions and/or school and neighbourhood inputs would be informative.

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A Decomposition of SES Skill Gap: Additional Counterfactuals

Table A.1: SES Skill Gap in Counterfactual Scenarios

Scenario	Belief Updating	Income Trajectory	Skill Production Function	Initial Belief	Initial Skill	Initial Income	Initial Assets	Preferences	SES Skill Gap	Percentage of Baseline SES Skill Gap
D1.b	X	X	X			X	X	X	0.586	78.395
D1.c	X	X	X		X		X	X	0.583	77.981
D1.d	X	X	X		X	X		X	0.305	40.820
D1.e	X	X	X		X	X	X		0.732	97.847
D2.b						X	X	X	0.582	77.758
D2.c					X		X	X	0.636	85.000
D2.d					X	X		X	0.408	54.614
D2.e					X	X	X		0.723	96.728

Notes: This table presents the SES skill gap and the corresponding percentage of the baseline SES skill gap for different counterfactual scenarios. An "X' in the belief updating, income trajectory and skill production function means that belief updating, income trajectories and/or skill production functions differ by SES. When the "X" is absent, the belief updating, income trajectories and/or skill production function have been set at the high SES values for all families. An "X" in the initial beliefs, initial skill, initial income and/or initial assets means that these are heterogeneous across families. When the "X" is absent, these values have been set at the median value across all families respectively. An "X" in the preferences means that there is unobserved heterogeneity in terms of preferences. The absence of an "X" means that all families have the "high" type relative preference for child skill over consumption. Low SES is defined as families with up to median income when the child is age 9-10, otherwise families are high SES.

Table A.2: SES Investment Gap in Counterfactual Scenarios

Scenario	P1 SES Investment Gap	P1 Percentage of Baseline	P2 SES Investment Gap	P2 Percentage of Baseline	P3 SES Investment Gap	P3 Percentage of Baseline
C1	119232.31	100.000	106384.21	100.000	59701.07	100.000
C1.a	118878.53	99.703	106338.80	99.957	59327.87	99.375
C1.b	119232.31	100.000	106497.66	100.107	59615.24	99.856
C1.c	76425.10	64.098	97927.26	92.051	54475.90	91.248
C1.d	43400.95	36.400	36847.95	34.637	17541.06	29.381
C1.e	125131.41	104.948	114345.41	107.483	65443.41	109.618
C2	115318.03	96.717	100082.78	94.077	59871.50	100.285
C2.a	114767.46	96.255	99336.29	93.375	59740.89	100.067
C2.b	115318.03	96.717	100136.08	94.127	59788.46	100.146
C2.c	81181.01	68.086	90132.78	84.724	53314.35	89.302
C2.d	41320.14	34.655	38084.78	35.799	17276.16	28.938
C2.e	118605.40	99.474	107856.53	101.384	65624.18	109.921

Notes: This table presents the SES investment gap and the corresponding percentage of the baseline SES investment gap for different counterfactual scenarios. Low SES is defined as families with up to median income when the child is age 9-10, otherwise families are high SES.

Table A.3: SES Investment Gap in Counterfactual Scenarios

Scenario	P1 SES Investment Gap	P1 Percentage of Baseline	P2 SES Investment Gap	P2 Percentage of Baseline	P3 SES Investment Gap	P3 Percentage of Baseline
D1.b	118878.53	99.703	106259.30	99.883	59299.46	99.327
D1.c	75375.09	63.217	97811.85	91.942	54161.30	90.721
D1.d	43293.58	36.310	35008.96	32.908	17867.61	29.928
D1.e	124948.63	104.794	113729.02	106.904	64971.88	108.829
D2.b	114767.46	96.255	99281.23	93.323	59686.41	99.975
D2.c	80392.71	67.425	88882.65	83.549	53250.98	89.196
D2.d	40864.02	34.273	36607.72	34.411	17526.86	29.358
D2.e	117711.52	98.725	106862.00	100.449	65371.13	109.497

Notes: This table presents the SES investment gap and the corresponding percentage of the baseline SES investment gap for different counterfactual scenarios. Low SES is defined as families with up to median income when the child is age 9-10, otherwise families are high SES.

B Data Appendix

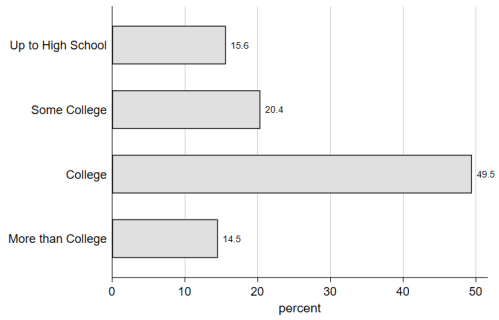
B.1 Measures of Latent Belief

B.1.1 Expectations of Educational Attainment

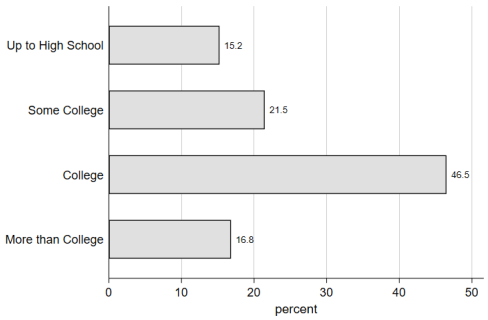
I examine the responses to parent expectations of educational attainment by the age of the child. Focusing on the subsample of children who have 3 reported expectations (at 9-10 years, 11-12 years and 13-14 years), I compute the percentages in each expectations category (up to high school, some college, college and more than college) and plot them below. Overall, parents become less optimistic about their children's educational attainment as their children grow older. A higher proportion of children are expected to attain up to high school education and some college at 13-14 years compared to when the children were 9-10 years. Though the overall percentages in each expectations category do not change drastically from age 9-10 years to age 13-14 years, there are within-child changes in expectations.

Figure B.1: Parent Expectations of Educational Attainment

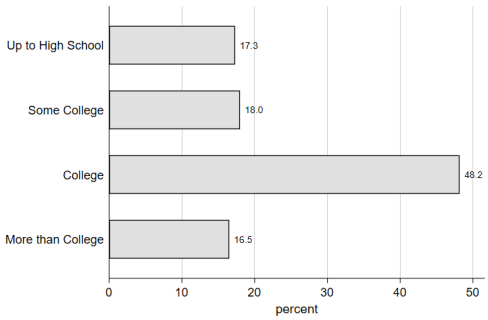
(a) 9-10 years



(b) 11-12 years



(c) 13-14 years



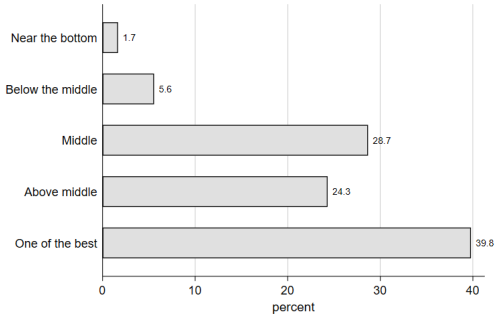
Notes: These figures present the percentage of responses in 4 categories (up to high school, some college, college and more than college) to the question "How far do you think your child will go in school?". The sample is the children who had their educational attainment expectations reported at age 9-10, 11-12 and 13-14.

B.1.2 Evaluation of Child's Academic Standing in Class

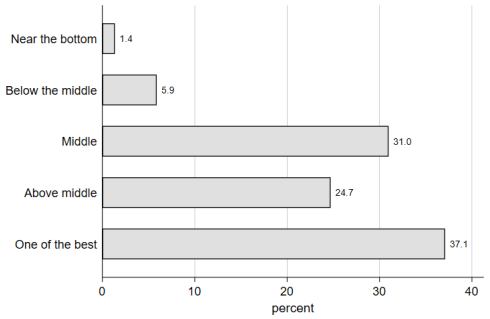
I plot the responses to the evaluation of child's academic standing in class for children who have 3 reports (at 9-10 years, 11-12 years and 13-14 years). There is a noticeable decline in the percentage of mothers who believe that their child is "one of the best" from ages 11-12 to ages 13-14.

Figure B.2: Rating of Child's Academic Standing in Class

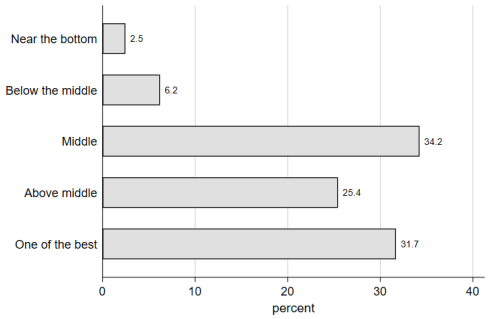
(a) 9-10 years



(b) 11-12 years



(c) 13-14 years



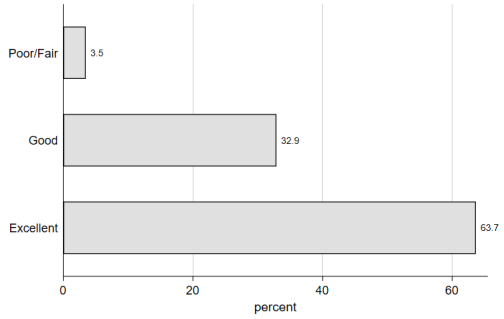
Notes: These figures present the percentage of responses in 5 categories (near the bottom, below the middle, middle, above the middle, one of the best) to the rating of the child's academic standing in class. The sample is the children who had their rating of academic standing in class reported at age 9-10, 11-12 and 13-14.

B.1.3 Evaluation of Child's Future Prospects

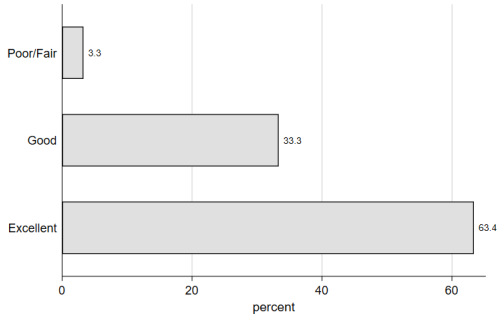
I plot the responses to the evaluation of child's future prospects for children who have 3 reports (at 9-10 years, 11-12 years and 13-14 years). There is a drop in the percentage of "Excellent" future prospects ratings from ages 11-12 to ages 13-14.

Figure B.3: Rating of the Child's Future Prospects

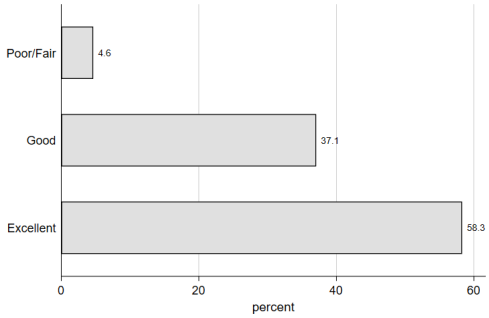
(a) 9-10 years



(b) 11-12 years



(c) 13-14 years



Notes: These figures present the percentage of responses in 3 categories (poor/fair, good, excellent) to the rating of the child's future prospects. In the survey, poor and fair were separate options. As few mothers chose the option of poor, I combined the poor and fair options into a single category. The sample is the children who had their ratings reported at age 9-10, 11-12 and 13-14.

B.2 Measures of Parent Investments

Components of the Home Observation Measurement of the Environment (HOME) used to construct the parent investment measure are as follows:

- How many magazines the family gets regularly
- Whether the child has the use of a record player, tape deck or CD player at home and at least 5 children's records or tapes
- The number of books the child has
- How often the mother reads to child
- Whether the family has a musical instrument which the child can use at home
- Whether the family receives a daily newspaper
- Whether the family encourages the child to pursue hobbies
- Whether the child attends special lessons

- How often a family member gets a chance to take the child on any kind of outing
- How often the child is brought to the museum
- How often the child is brought to music/theatre performance
- How often the family gets together
- How often the child eats with mom and dad

B.3 Cognitive Skill Measure: PIAT Achievement Tests

The Peabody Individual Achievement Test (PIAT) is a well-established assessment of children's academic achievement. The NLSY Child and Young Adult administered three subtests of the PIAT to children aged between 5 and 14 years: Mathematics, Reading Recognition and Reading Comprehension.

B.4 Non-Cognitive Skill Measure: Behaviour Problems Index

Mothers of children aged 4 and above answer a series of questions on the frequency of the child exhibiting problem behaviours. Some examples of the components which are used to construct the index are: cheats or tells lies, has sudden changes in mood or feeling, is disobedient at home and is not liked by other children. This has been treated as a measure of children's non-cognitive skills by other studies.

B.5 Correlation Between PIAT Scores and School Transcript

In most survey years, the NLSY Child and Young Adult does not collect information on school grades. However, school transcript information was collected for around 3,000 children in the year 1995-1996. In that year, the NLSYCYA elicited additional information on school characteristics and characteristics of the child at school. Transcripts were available for a subset of students, providing information on the national percentile ranks of the child in vocabulary, reading comprehension, reading, language and math. I find that these percentile scores are correlated with PIAT achievement test percentile scores in survey years 1994 and/or 1996.

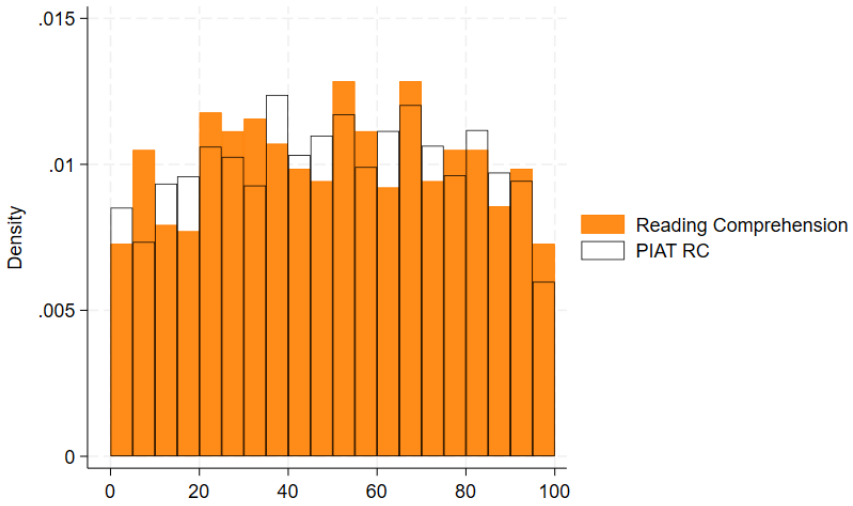
Table B.1: Correlation between PIAT percentiles and national percentiles on transcript tests

	(1) Vocabulary	(2) Reading Comprehension	(3) Reading	(4) Language	(5) Math
PIAT Mathematics Percentile	0.199***	0.185***	0.235***	0.270***	0.483***
	(0.037)	(0.041)	(0.035)	(0.039)	(0.036)
PIAT Reading Comprehension Percentile	0.262***	0.314***	0.265***	0.203***	0.134***
	(0.034)	(0.039)	(0.033)	(0.038)	(0.035)
PIAT Reading Recognition Percentile	0.390***	0.357***	0.398***	0.382***	0.184***
	(0.037)	(0.040)	(0.036)	(0.045)	(0.041)
Observations	997	881	1026	911	1077
R^2	0.525	0.546	0.579	0.529	0.445

Notes: This table presents the coefficients on the percentile scores on the PIAT mathematics test, PIAT reading comprehension test and the PIAT reading recognition test in a regression of the dependent variable on these variables. There are several dependent variables: the national percentile ranks of the child in vocabulary, reading comprehension, reading, language and math. Standard errors are clustered at the family level in all regressions. Standard errors in parentheses (* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$). The sample is the children who had both achievement test score percentiles in 1994/1996 and the school transcript information in 1995-1996.

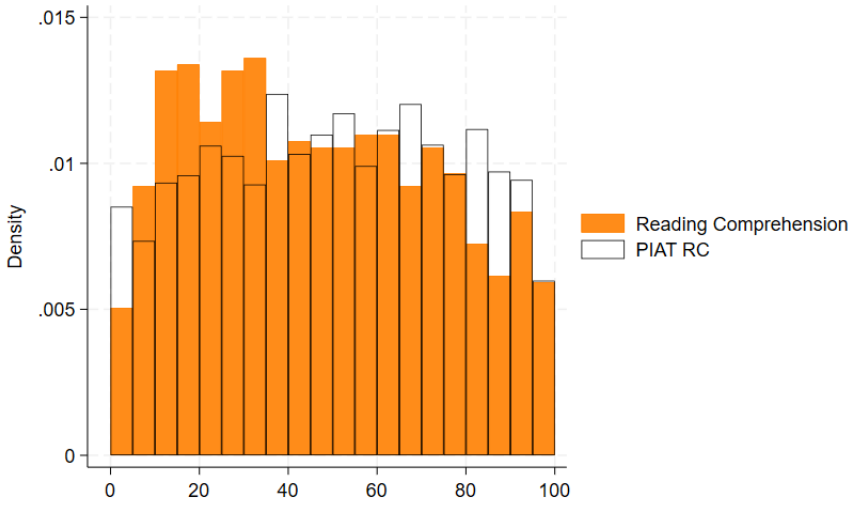
I provide several histograms which showcase the extent of overlap between the national percentile scores in the tests and a relevant PIAT test percentile score.

Figure B.4: Language Test Transcript Score



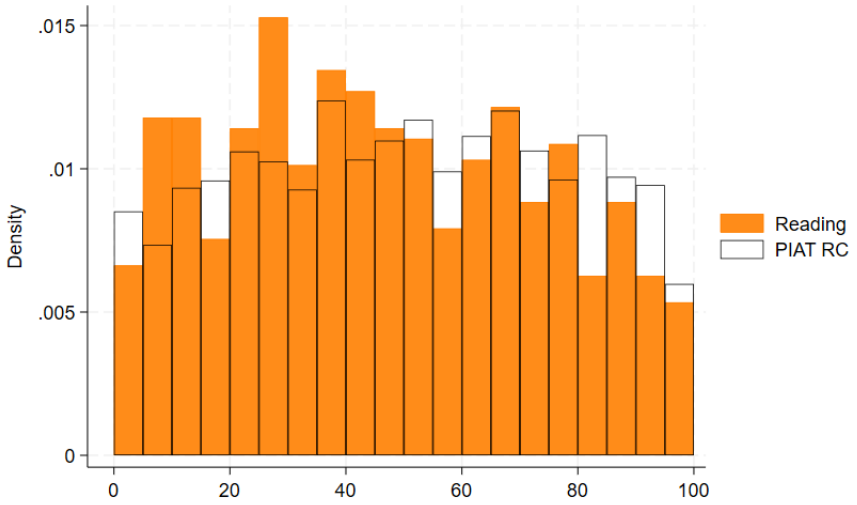
Notes: This figure presents an overlapping histogram of the national percentiles in the language test and the percentiles in the PIAT reading comprehension test (PIAT RC).

Figure B.5: Reading Comprehension Test Transcript Score



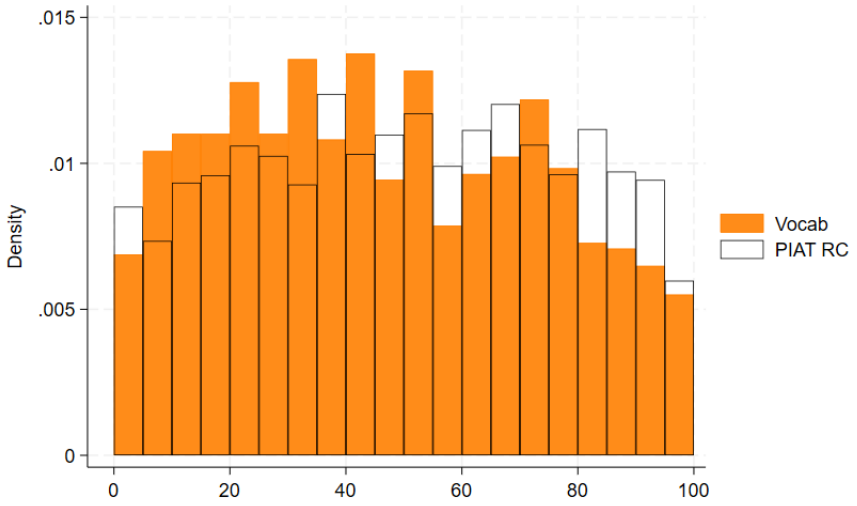
Notes: This figure presents an overlapping histogram of the national percentiles in the reading comprehension test and the percentiles in the PIAT reading comprehension test (PIAT RC).

Figure B.6: Reading Total Test Transcript Score



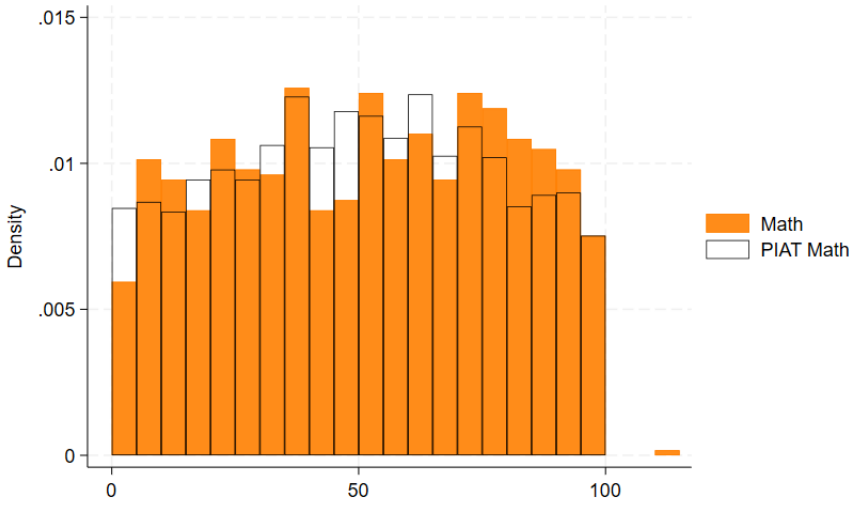
Notes: This figure presents an overlapping histogram of the national percentiles in the reading total test and the percentiles in the PIAT reading comprehension test (PIAT RC).

Figure B.7: Vocabulary Test Transcript Score



Notes: This figure presents an overlapping histogram of the national percentiles in the vocabulary total test and the percentiles in the PIAT reading comprehension test (PIAT RC).

Figure B.8: Math Test Transcript Score



Notes: This figure presents an overlapping histogram of the national percentiles in the mathematics test and the percentiles in the PIAT mathematics test (PIAT Math).

B.6 How Accurate are Expectations of Educational Attainment?

Earlier, I examined the gap between parental beliefs and skills (both anchored to the years of education). In this section, I assess the accuracy of a component measure of the parent belief — expectations of educational attainment — by matching it to the realised educational attainment of the child.

I focus on the sub-sample of children whose education outcomes were observed at age 27 and above. I compare the expectation which parents held about their child's educational attainment at age 13-14 to the child's realised educational attainment. On average, parents are overly optimistic. When their children were aged 13-14 years, 55.9% were expected to graduate with college education and above. However, only 60.65% out of this 55.9% attained college education and above by age 27 (see Figure B.9). If parent investment decisions are influenced by expectations of educational attainment, investment decisions may be inefficient. That parents tend to be over-optimistic about their children is consistent with other studies (O. Attanasio et al., 2020). Parents tend to perceive that their children have a higher level of skills than they actually have (Dizon-Ross, 2019; Nicoletti et al., 2022; Kinsler & Pavan, 2021; Bergman, 2021).

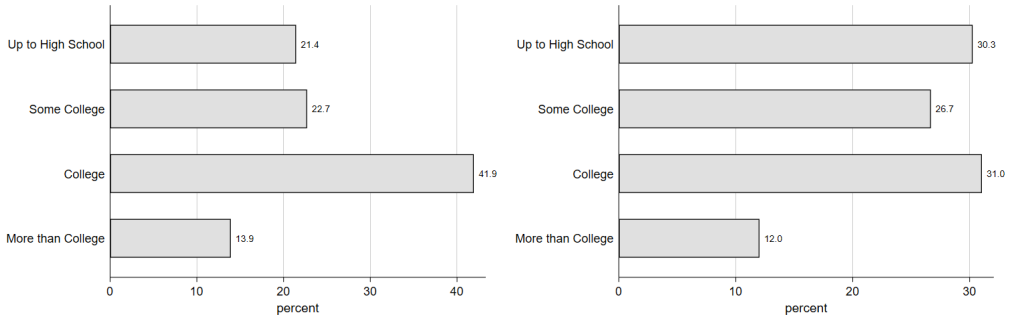
Table B.2: Percentage Accuracy by Demographics

	Up to Median	Above Median
Education	49.07	67.93
Income	45.25	70.22

Figure B.9: Expectations at 13-14 years versus Realised Educational Attainment

(a) Expectations at age 13-14 yr

(b) Educational Attainment at Age 27 and above



The sub-sample of children who have education observed at age 27 and above is a selected sample (children were born to younger mothers). To mitigate the concern that the over-optimism is due to the use of a selected sample, I expand the sample of children to those who have education reported at age 24 and above and focus instead on whether parents correctly predicted that their child would graduate from college (parents who expected college education or more than college education are counted as expecting college graduation). In this sample, I also find evidence of inaccurate predictions. When their children were aged 13-14, 58.93% were expected to attain college education and above. However, only 59.93% out of the 58.93% actually graduated from college.

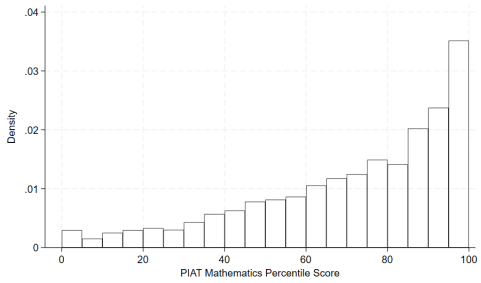
Furthermore, there are socio-economic status differences in prediction accuracy. Table B.2 presents the percentages by education and family income level of the child's mother. On average, mothers with above median level of education (more than 12 years of schooling, which corresponds to high school) made more accurate predictions. Mothers of families with above median family income made substantially better predictions than those in families with up to median family income.

B.7 How Accurate are Current Ratings of Child Academic Standing?

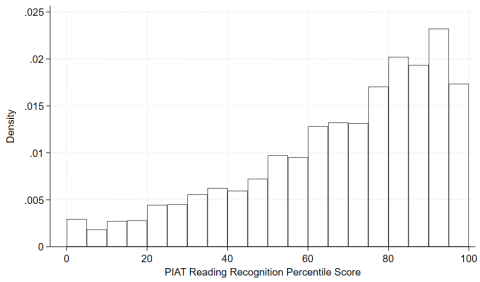
I compare the mother's rating of child's academic standing in class with the child's percentile rank in the Peabody Individual Achievement Tests in mathematics, reading recognition and reading comprehension. I concentrate on assessing the accuracy of the ratings at the extreme ends (top of the class and bottom of the class). If the child is rated as being at the best in the

Figure B.10: Ranked Among Top of the Class: Histograms of Achievement Test Percentile Scores

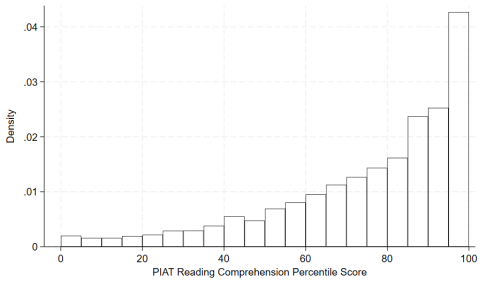
(a) PIAT Mathematics



(b) PIAT Reading Recognition



(c) PIAT Reading Comprehension



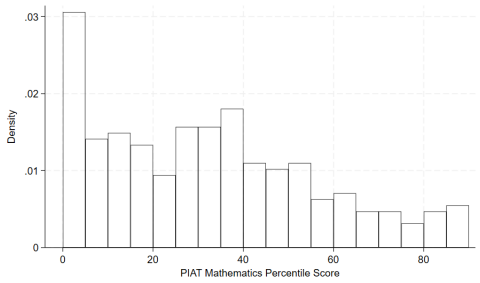
Notes: This figure presents histograms of the contemporaneous percentile scores on (a) PIAT mathematics, (b) PIAT reading recognition and (c) PIAT reading comprehension. The sample is children who were ranked at the top of the class by their mothers. The histograms span all periods, so there can be multiple observations of the same child if the child was ranked as being among the top of the class in more than one period.

class (bottom of the class), it seems likely that the child would have higher (lower) percentile scores on the achievement tests. In contrast, it may be difficult to gauge whether rankings in the middle are objective, since this may depend on the child's ability relative to others in the class.

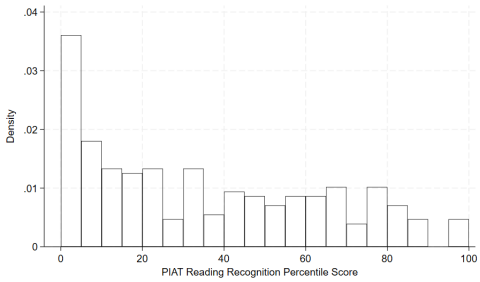
Children who are ranked at the bottom of the class are indeed more likely to have lower percentile scores on the PIAT achievement tests in the same period. However, there are some children who perform relatively well (score 80 percentiles and above), even though they were ranked at the bottom of the class. Children who were ranked among the best in the class are more likely to have attained high percentile scores on the PIAT achievement tests. But there are some children who scored significantly worse than the average (20 percentiles and below). If the achievement test scores are objective measures of the child ability, the discrepancy between the objective ranking and the mother's subjective ranking could indicate that some mothers hold imperfect information about the child. Yet, depending on the class and quality of the school which the child attends, if peer quality is high (low), a child who is ranked at the top (bottom) of the class may not be ranked at the top (bottom) in national rankings. Therefore, the evidence presented in this section should be taken as suggestive, rather than conclusive.

Figure B.11: Ranked Near Bottom of the Class: Histograms of Achievement Test Percentile Scores

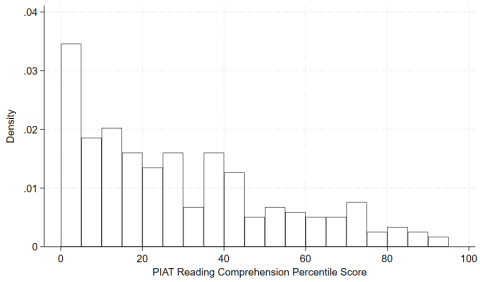
(a) PIAT Mathematics



(b) PIAT Reading Recognition



(c) PIAT Reading Comprehension



Notes: This figure presents histograms of the contemporaneous percentile scores on (a) PIAT mathematics, (b) PIAT reading recognition and (c) PIAT reading comprehension. The sample is children who were ranked near the bottom of the class by their mothers. The histograms span all periods, so there can be multiple observations of the same child if the child was ranked near the botton of the class in more than one period.

C Further Details on Empirical Strategy for Parent Belief Updating Data Pattern

This system generalised method of moments (system GMM) estimator employs moment conditions from the error term in the level equations (errors in Equation 1) and moment conditions from the first-differenced errors (errors in Equation 18 below). Instruments for the endogenous lag dependent variable are as follows. In the first differenced specification, the instrument is the second lag of the dependent variable. Where available, earlier lags are also used as instruments in a similar way. In the level equations, the instrument is the difference between the lag dependent variable and the second lag of the dependent variable.

$$\Delta\mu_{jt} = \beta_1\Delta\mu_{j,t-1} + \beta_2\Delta math_{jt} + \beta_3\Delta readingrecog_{jt} + \beta_4\Delta readingcomp_{jt} + \beta_5\Delta badbehaviour_{jt} + \beta_6\Delta healthexcellent_{jt} + \gamma\Delta X_{jt} + \Delta\epsilon_{jt} \quad (18)$$

Δ: first difference operator

One may also be concerned that the contemporaneous signals are correlated with the error

term. There may be unobserved shocks (to the econometrician) which jointly influence both the beliefs and the signals. To address this potential issue, I do not include moment conditions relating the contemporaneous signals to the error term. The signals are treated in a similar way to the lag dependent variable. Specifically, the difference between the first and second lags is used as an instrument in the level equation. And the second lag is used as an instrument in the first differenced equation. Where available, later lag differences and later lags are also used as instruments.

The updating model (Equation 1) is not Bayesian. In Bayesian updating of a normal random variable, the posterior mean is a weighted average of the prior mean and the new information, and the weights depend on the precision of the prior relative to the informativeness of the signal. These weights would change with time. The specification in Equation 1 could be compatible with Bayesian updating if coefficients in the regression are allowed to vary with time.

D Further Details of Estimation

D.1 Sample Selection

I focus on the children of the females in the cross-section sample of the National Longitudinal Survey of Youth 1979 (NLSY79). In the estimation, I include all the observations with non-missing values of these variables in the first period: parental belief factor score, income, assets and child skill. Parental belief factor scores, income, assets, child skills, investments can be missing in other periods. For the time being, I assume that these values are missing at random and compute the data moments from the relevant non-missing values. For example, the average belief factor score when child is age 11-12 is computed as the average of all non-missing belief factor scores for children aged 11-12. Furthermore, because the model does not allow parents to borrow (assets must be weakly positive), I exclude observations with negative values of assets in any of the 3 periods. On top of this, I only include families where mothers were married during the interviews taken at age 9-10, 11-12 and 13-14.

D.2 Method of Simulated Moments

This section provides additional details of the method of simulated moments estimation step.

Discretisation: There are 5 continuous state variables: mean in prior belief, variance in prior belief, assets and income. I use 4 point grids for the mean of prior belief, variance of prior belief and the signal. I use 7 point grids for the assets and family income. There are more grid points at lower levels of assets, where responses may be more non-linear. The grid of family income and the family income transition matrix are constructed from a normal distribution truncated at 3 standard deviations. Consumption and investment grids are endogenous, where

the maximum value is determined by the total resources available (assets plus family income) given that value of the state space. The endogenous consumption grid has 50 points and the endogenous investment grid has 10 points. Since parents do not observe the skill of the child, there is also a 5-point discretised grid of values of log skill. The discretisation depends on the belief distribution which the parent has over the child skill. This will be used by the parent to take expectations over the child's skill in the computation of the next period value function.

Interpolation/Extrapolation: Linear interpolation is used to evaluate points within the grid (between grid points). Linear extrapolation is used to evaluate points outside of the grid. Interpolation/extrapolation is used to compute the expected value of next period value function given the current state when solving for the value functions and the policy functions. It is also used to obtain optimal choices in the simulation (since state variables of individuals may not fall on the grid).

Integration: 5-node Gauss-Hermite quadrature is used to integrate over the log skill, shock variables (shock in signal generation process, shock in skill production process).

Solving for Value Functions and Policy Functions by Backward Induction: Begin from the final period. There are three choice variables: consumption, investment in the child and assets next period (or savings). Given consumption and investment choices, assets are determined from the resource constraint. In every period, I loop over possible consumption and investment choices and choose the combination which maximises the objective function. The consumption loop is the outer loop and the investment loop is in the inner loop. The consumption and investment grids are endogenous. The maximum possible values of consumption is equal to the resources in that period (return on asset state variable plus income in that period). Further, the maximum value of the inner loop investment grid will be the resources minus the consumption choice.

The value functions and policy functions are first computed for the last period, period T. Then, I proceed to period $T-1$ and perform a similar computation, but now taking expectations over the next period value function given possible values of choice variables (expectation depends on the expected next period income, error terms in the signal generating process, discretised error term in the skill production process, belief about the child's skill)

Optimisation Algorithm: The minimisation algorithm for the method of simulated moments procedure is Bound Optimisation by Quadratic Approximation (BOBYQA) by Powell (2009). This is a gradient-free algorithm which approximates the value function by a quadratic function. Then it minimises the Frobenius norm of the change to the second derivative matrix of the quadratic approximation. I use the implementation provided in the NLopt package in Julia.

Programming Language: The program for the estimation is written and implemented

in Julia.

D.3 Moment Conditions and Auxiliary Models in Estimation

A total of 112 moment conditions and/or statistics from auxiliary moments are used to estimate the 16 parameters. The values of these in the data and in the model are provided in the section on Model Fit in the appendix. I provide information on some moments and auxiliary models here.

Moment Conditions for Investment: The investment measures in the HOME-SF are not in terms of monetary units. Given the assumed investment mapping function $\phi(.)$, the investment moment conditions are the following:

$$E\left[\Phi_t\right] - \left[\phi_0 + \phi_1 E\left[i_t\right] + \phi_2 E\left[i_t^2\right]\right] = 0, t = 1, 2, 3 \tag{19}$$

Some of the investment moment conditions are conditional on parent income quartile. In those cases, the moment condition (conditional on X, which denotes parent income quartile) will be as follows:

$$E\left[\Phi_t|X\right] - \left[\phi_0 + \phi_1 E\left[i_t|X\right] + \phi_2 E\left[i_t^2|X\right]\right] = 0, t = 1, 2, 3 \tag{20}$$

Auxiliary Model for Belief Parameters: This is motivated from the belief updating equation relating the posterior beliefs to the prior belief (unobserved) and the signal (unobserved). For ease of exposition, I remove the j subscript which indexes the child.

$$\begin{aligned}
\mu_t^{int} &= \psi\mu_t + (1-\psi)g_t \\
&\approx \psi\left[A_{t-1} + \gamma_{s,1}\ln\theta_{t-1} + \gamma_{s,2}\ln i_{t-1} + \gamma_{s,3}\ln\theta_{t-1}\times\ln i_{t-1}\right] + (1-\psi)\left(\ln\theta_t + \epsilon_t\right) \\
&\quad \psi A_{t-1} + \psi\gamma_{s,3}\Phi_{t-1}\ln\theta_{t-1} + \psi\gamma_{s,2}\Phi_{t-1} + (1-\gamma)\ln\theta_t \\
&\quad + (1-\psi)\epsilon_t - \psi\gamma_{s,3}\left(\Phi_{t-1}\ln\theta_t - \ln i_{t-1}\ln\theta_t\right) - \psi\gamma_{s,2}\left(\Phi_{t-1} - \ln i_{t-1}\right)
\end{aligned}$$

D.4 Factor Models for Belief and Skills

Equations for the latent factor model of skills for $t = 1, 2, 3$

$$Q = \alpha + \ln\theta_T + \eta_Q \tag{21}$$

$$m_{1t} = \alpha_1 + \lambda_{m1,t}\ln\theta_t + \epsilon_{m1,t} \tag{22}$$

$$m_{2t} = \alpha_{m2,t} + \lambda_{m2,t}\ln\theta_t + \epsilon_{m2,t} \tag{23}$$

$$m_{3t} = \alpha_{m3,t} + \lambda_{m3,t}\ln\theta_t + \epsilon_{m3,t} \tag{24}$$

The loading on the $\ln \theta_T$ is constrained to be 1 in the anchoring equation so that an increase in the $\ln \theta_T$ by 1 corresponds to a 1 unit increase in the conditional expectation of the adult outcome Q, which is years of education by age 24 and above.

The intercept and the factor loading of one of the repeated measurements (in this case the raw score on PIAT mathematics) ia constrained to be the same at all ages of the child. The growth in that measurement will be informative about the change in the level of the log latent skill as the child ages. The intercepts and the factor loadings on other measurements can be freely chosen. The measurement errors are assumed to be normally distributed. They are also assumed to be uncorrelated across measurements and over time.

Equations for the latent factor model of belief for $t = 1, 2, 3$

$$
\begin{aligned}
Q &= \alpha + \mu + \nu_Q && (25)\\
m^*_{1t} &= \alpha_1 + \lambda_{m1,t}\mu_t + \epsilon_{m1,t} && (26)\\
m^*_{2t} &= \alpha_{m2,t} + \lambda_{m2,t}\mu_t + \epsilon_{m2,t} && (27)\\
m^*_{3t} &= \alpha_{m3,t} + \lambda_{m3,t}\mu_t + \epsilon_{m3,t} && (28)
\end{aligned}
$$

The loading on the $\ln \theta_T$ is constrained to be 1 in the anchoring equation so that an increase in the underlying parent belief μ_t by 1 corresponds to a 1 unit increase in the conditional expectation of the adult outcome Q, which is years of education by age 24 and above. The factor loading and intercept of one of the latent measures is constrained to be the same at all ages of the child. The measurement errors are assumed to be normally distributed. They are also assumed to be uncorrelated across measurements and over time.

The unobserved m^* are mapped to the discrete ordered values in the data according to a threshold model. For example, if the measurement m_{ijt} takes 4 values, the threshold model is like this:

$$
m_{ijt} = \begin{cases}
0 \text{ if } m^*_{ijt} \leq \tau_{1,jt} \\
1 \text{ if } m^*_{ijt} \in [\tau_{1,jt}, \tau_{2,jt}] \\
2 \text{ if } m^*_{ijt} \in [\tau_{2,jt}, \tau_{3,jt}] \\
3 \text{ if } m^*_{ijt} > \tau_{3,jt}
\end{cases}
$$

E Model Fit

Table E.1 to Table E.6 display the moments used in the estimation. Each table provides (1) the data moment, (2) the corresponding moment in the simulated data (model moment), (3) the difference between the data moment and the model moment and (4) the standardised difference between the two (difference divided by standard deviation of data moment). A total of 112

moments were used to estimate the model. These include moments pertaining to the beliefs (Table E.1), the investment factor scores (Table E.2), the log skills of the child (Table E.3), the assets of the family (Table E.4), correlations and covariances (Table E.5) and the ratio of next period assets to investment factor score (Table E.6).

Table E.1: Belief Moments

Moment	Data	Model	SE Data	SE Diff
Mean belief age 11-12: Family income Q1	13.329	13.481	0.097	1.582
Mean belief age 11-12: Family income Q2	13.643	13.638	0.101	0.054
Mean belief age 11-12: Family income Q3	13.889	13.803	0.114	0.756
Mean belief age 11-12: Family income Q4	14.242	13.924	0.113	2.800
Mean belief age 13-14: Family income Q1	13.775	13.722	0.129	0.413
Mean belief age 13-14: Family income Q2	13.755	13.919	0.152	1.077
Mean belief age 13-14: Family income Q3	14.170	13.988	0.143	1.265
Mean belief age 13-14: Family income Q4	14.509	14.183	0.168	1.945
Correlation (low education): Belief age 9-10, Belief age 11-12	0.651	0.817	0.044	3.805
Correlation (high education): Belief age 9-10, Belief age 11-12	0.665	0.813	0.039	3.835
Correlation (low education): Belief age 11-12, Belief age 13-14	0.701	0.837	0.047	2.872
Correlation (high education): Belief age 11-12, Belief age 13-14	0.735	0.816	0.033	2.502
Correlation (low education): Belief age 11-12, Log skill age 11-12	0.499	0.606	0.056	1.900
Correlation (high education): Belief age 11-12, Log skill age 11-12	0.527	0.592	0.050	1.315
Correlation (low education): Belief age 13-14, Log skill age 13-14	0.574	0.764	0.054	3.527
Correlation (high education): Belief age 13-14, Log skill age 13-14	0.532	0.712	0.067	2.684
Regression of belief on lag investment, lag skill and current skill (low education): Constant	3.020	2.724	1.347	0.220
Regression of belief on lag investment, lag skill and current skill (low education): Coefficient on lag investment x lag skill	0.015	-0.157	0.127	1.359
Regression of belief on lag investment, lag skill and current skill (low education): Coefficient on lag investment	0.238	2.444	1.666	1.324
Regression of belief on lag investment, lag skill and current skill (low education): Coefficient on current skill	0.756	0.790	0.100	0.344
Std Residual of Regression of belief on lag investment, lag skill and current skill (low education)	1.023	0.831	0.043	4.474
Regression of belief on lag investment, lag skill and current skill (high education): Constant	2.579	3.450	1.529	0.570
Regression of belief on lag investment, lag skill and current skill (high education): Coefficient on lag investment x lag skill	0.134	0.012	0.098	1.246
Regression of belief on lag investment, lag skill and current skill (high education): Coefficient on lag investment	-1.277	-0.023	1.345	0.932
Regression of belief on lag investment, lag skill and current skill (high education): Coefficient on current skill	0.806	0.751	0.110	0.505
Std Residual of Regression of belief on lag investment, lag skill and current skill (high education)	0.994	0.795	0.039	5.108

Notes: Belief refers to belief factor score. Data is the data moment. Model is the value of the corresponding model moment. SE Data is the standard error of the data moment. SE Diff is difference between the data moment and the simulated moment divided by the standard error of the data moment.

Table E.2: Investment Factor Score Moments

Moment	Data	Model	SE Data	SE Diff
Std investment age 9-10	0.589	0.565	0.018	1.322
Std investment age 11-12	0.566	0.651	0.016	5.213
Std investment factor Age 13-14	0.636	0.365	0.021	13.051
Mean investment age 9-10: Family income Q1	0.257	0.072	0.051	3.639
Mean investment age 9-10: Family income Q2	0.453	0.223	0.046	5.032
Mean investment age 9-10: Family income Q3	0.519	0.428	0.042	2.169
Mean investment age 9-10: Family income Q4	0.673	0.906	0.043	5.436
Mean investment age 11-12: Family income Q1	0.273	0.085	0.046	4.076
Mean investment age 11-12: Family income Q2	0.487	0.239	0.051	4.891
Mean investment age 11-12: Family income Q3	0.491	0.396	0.045	2.100
Mean investment age 11-12: Family income Q4	0.683	0.711	0.042	0.675
Mean investment age 13-14: Family income Q1	0.225	-0.017	0.063	3.842
Mean investment age 13-14: Family income Q2	0.306	0.042	0.059	4.464
Mean investment age 13-14: Family income Q3	0.420	0.119	0.054	5.597
Mean investment age 13-14: Family income Q4	0.626	0.381	0.056	4.385
Std residual of investment conditional on belief age 9-10	0.545	0.555	0.016	0.629
Std residual of investment conditional on belief age 11-12	0.537	0.632	0.018	5.348
Std residual of investment conditional on belief age 13-14	0.577	0.984	0.022	18.296
Correlation (Family income Q1): investment age 9-10, investment age 11-12	0.616	0.852	0.051	4.637
Correlation (Family income Q2): investment age 9-10, investment age 11-12	0.560	0.926	0.058	6.279
Correlation (Family income Q3): investment age 9-10, investment age 11-12	0.609	0.869	0.060	4.318
Correlation (Family income Q4): investment age 9-10, investment age 11-12	0.602	0.771	0.069	2.465
Correlation (Family income Q1): investment age 11-12, investment age 13-14	0.657	0.804	0.055	2.690
Correlation (Family income Q2): investment age 11-12, investment age 13-14	0.596	0.773	0.061	2.913
Correlation (Family income Q3): investment age 11-12, investment age 13-14	0.661	0.907	0.064	3.842
Correlation (Family income Q4): investment age 11-12, investment age 13-14	0.609	0.719	0.065	1.694

Notes: Investment refers to investment factor score. Data is the data moment. Model is the value of the corresponding model moment. SE Data is the standard error of the data moment. SE Diff is difference between the data moment and the simulated moment divided by the standard error of the data moment.

Table E.3: Skill Moments

Moment	Data	Model	SE Data	SE Diff
Std log skill age 11-12	0.734	0.847	0.025	4.443
Std log skill age 13-14	0.787	1.013	0.034	6.572
Mean log skill age 11-12: Family income Q1	13.454	13.523	0.070	0.999
Mean log skill age 11-12: Family income Q2	13.684	13.689	0.059	0.081
Mean log skill age 11-12: Family income Q3	13.755	13.754	0.066	0.010
Mean log skill age 11-12: Family income Q4	14.050	13.863	0.066	2.819
Mean log skill age 13-14: Family income Q1	13.801	13.741	0.077	0.786
Mean log skill age 13-14: Family income Q2	13.959	13.935	0.075	0.322
Mean log skill age 13-14: Family income Q3	14.107	14.016	0.074	1.233
Mean log skill age 13-14: Family income Q4	14.324	14.174	0.076	1.967
Mean log skill age 11-12: Log skill 9-10 Q1	12.921	12.731	0.057	3.299
Mean log skill age 11-12: Log skill 9-10 Q2	13.645	13.516	0.053	2.438
Mean log skill age 11-12: Log skill 9-10 Q3	13.973	13.968	0.037	0.153
Mean log skill age 11-12: Log skill 9-10 Q4	14.436	14.613	0.040	4.422
Mean log skill age 13-14: Log skill 11-12 Q1	13.126	12.806	0.077	4.177
Mean log skill age 13-14: Log skill 11-12 Q2	13.945	13.707	0.055	4.355
Mean log skill age 13-14: Log skill 11-12 Q3	14.289	14.251	0.053	0.715
Mean log skill age 13-14: Log skill 11-12 Q4	14.781	15.103	0.044	7.356
Correlation (low education): Log skill age 9-10, Log skill age 11-12	0.860	0.888	0.021	1.357
Correlation (high education): Log skill age 9-10, Log skill age 11-12	0.790	0.884	0.036	2.596
Correlation (low education): Log skill age 11-12, Log skill age 13-14	0.882	0.903	0.020	1.047
Correlation (high education): Log skill age 11-12, Log skill age 13-14	0.786	0.906	0.037	3.200
Correlation (low education): Log skill age 9-10, Log skill age 13-14	0.817	0.740	0.029	2.671
Correlation (high education): Log skill age 9-10, Log skill age 13-14	0.742	0.779	0.031	1.218
Regression skill production function (low education): Constant 1	1.798	-0.486	0.409	5.584
Regression skill production function (low education): Constant 2	1.663	-0.657	0.419	5.533
Regression skill production function (low education): Coefficient on skill	0.894	1.063	0.031	5.410
Regression skill production function (low education): Coefficient on investment	0.137	2.731	0.435	5.960
Regression skill production function (low education): Coefficient on skill x investment	-0.003	-0.159	0.033	4.685
Std Residual skill production function (low education)	0.387	0.345	0.023	1.807
Regression skill production function (high education): Constant 1	3.481	0.425	1.012	3.018
Regression skill production function (high education): Constant 2	3.420	0.279	1.029	3.053
Regression skill production function (high education): Coefficient on skill	0.775	0.988	0.075	2.818
Regression skill production function (high education): Coefficient on investment	0.300	0.912	0.875	0.699
Regression skill production function (high education): Coefficient on skill x investment	-0.017	-0.047	0.065	0.453
Std Residual skill production function (high education)	0.419	0.346	0.031	2.352

Notes: Log skill refers to log skill factor score. Data is the data moment. Model is the value of the corresponding

Table E.4: Asset Moments

Moment	Data	Model	SE Data	SE Diff
Mean asset age 11-12	0.48	0.44	0.05	0.872
Std asset age 11-12	0.85	0.85	0.11	0.057
Mean asset age 13-14	0.79	0.28	0.06	7.859
Std asset age 13-14	1.23	0.54	0.13	5.088

Notes: Investment refers to investment factor score. Data is the data moment. Model is the value of the corresponding model moment. SE Data is the standard error of the data moment. SE Diff is difference between the data moment and the simulated moment divided by the standard error of the data moment.

Table E.5: Correlations and Covariances Part One

Moment	Data	Model	SE Data	SE Diff
Correlation (low education): Belief age 9-10, Log skill age 11-12	0.523	0.427	0.049	1.951
Correlation (high education): Belief age 9-10, Log skill age 11-12	0.491	0.427	0.044	1.479
Correlation (low education): Belief age 11-12, Log skill age 13-14	0.486	0.572	0.069	1.254
Correlation (high education): Belief age 11-12, Log skill age 13-14	0.487	0.572	0.066	1.296
Covariance: Investment age 9-10, Belief age 9-10	0.254	0.121	0.031	4.353
Covariance: Investment age 11-12, Belief age 11-12	0.223	0.168	0.030	1.814
Covariance: Investment age 13-14, Belief age 13-14	0.339	0.127	0.041	5.146
Covariance: Investment age 9-10, Log skill age 9-10	0.113	0.085	0.020	1.426
Covariance: Investment age 11-12, Log skill age 11-12	0.104	0.166	0.017	3.753
Covariance: Investment age 13-14, Log skill age 13-14	0.124	0.128	0.025	0.187
Covariance: Investment age 9-10, Belief age 11-12	0.278	0.193	0.033	2.598
Covariance: Investment age 11-12, Belief age 13-14	0.297	0.290	0.038	0.179

Notes: Belief, log skill and investment refer to belief factor score, log skill factor score and investment factor score respectively. Data is the data moment. Model is the value of the corresponding model moment. SE Data is the standard error of the data moment. SE Diff is difference between the data moment and the simulated moment divided by the standard error of the data moment.

Table E.6: Ratio of Next Period Assets to Investment Factor Score

Moment	Data	Model	SE Data	SE Diff
Ratio of Assets Age 11-12 to Investment age 9-10: Log skill Q1	-1.04	0.14	1.10	1.078
Ratio of Assets Age 11-12 to Investment age 9-10: Log skill Q2	-2.02	0.60	2.01	1.307
Ratio of Assets Age 11-12 to Investment age 9-10: Log skill Q3	-42.67	0.38	43.28	0.995
Ratio of Assets Age 11-12 to Investment age 9-10: Log skill Q4	0.20	0.25	0.10	0.455
Ratio of Assets Age 13-14 to Investment age 11-12: Log skill Q1	-0.29	-0.11	0.58	0.305
Ratio of Assets Age 13-14 to Investment age 11-12: Log skill Q2	1.12	0.06	1.31	0.812
Ratio of Assets Age 13-14 to Investment age 11-12: Log skill Q3	-0.18	0.16	0.47	0.722
Ratio of Assets Age 13-14 to Investment age 11-12: Log skill Q4	0.48	0.31	0.14	1.220

Notes: Data is the data moment. Model is the value of the corresponding model moment. SE Data is the standard error of the data moment. SE Diff is difference between the data moment and the simulated moment divided by the standard error of the data moment.