International Equity Flows, Monetary Policy, and Time Consistency *

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Abstract

This paper studies optimal monetary policy in an environment with international equity flows. We show that foreign ownership of domestic equity introduces a time inconsistency problem for the monetary authority. When foreign investors own domestic equity, the monetary authority faces an incentive to depart from full employment to reduce foreign investors' profits. Anticipating low returns, foreign investors buy equity at reduced prices, leading to a reduction in domestic households' wealth. We show that households over-divest equity, leading to multiple Pareto-ranked equilibria, where the bad equilibrium is marked by depressed equity values and elevated unemployment. Unlike existing theories on capital flow management, our model highlights the desirability of capital controls on equity inflows.

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1 Introduction

Over the past two decades, international equity flows have steadily expanded, driven by the rapid deepening and development of financial markets (Figure 1). A widely held view is that the increase in cross-border equity flows can contribute to international risk-sharing and raise welfare. In line with this view, the literature on international real business cycles has shown that allowing households to trade equity and other securities with foreign investors contributes to diversifying income risk and stabilizing consumption. However, there is a question of whether international equity flows remain desirable once we depart from a frictionless environment.

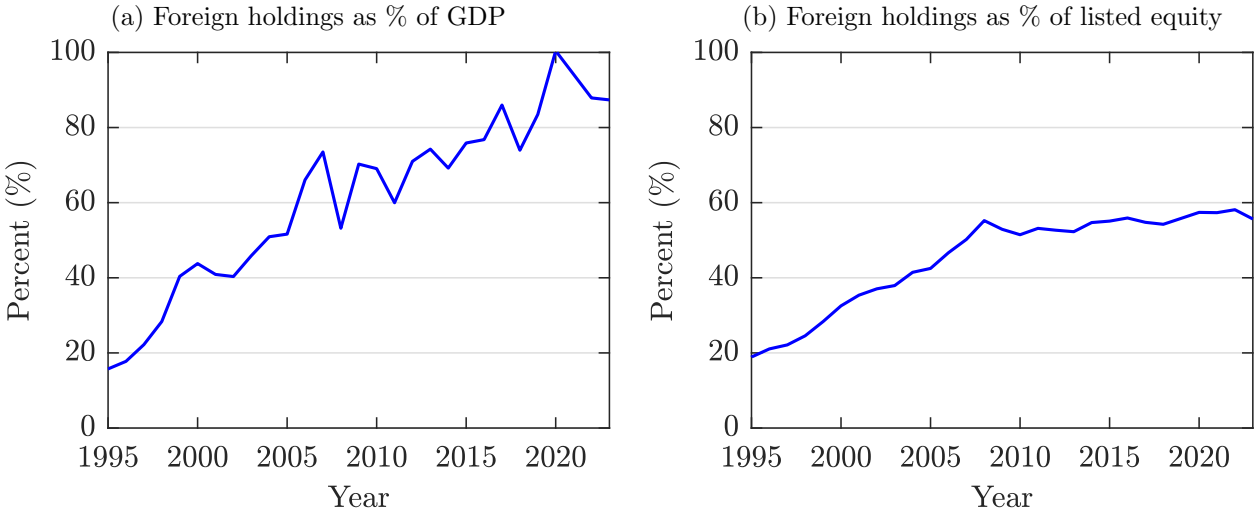


Figure 1: Foreign ownership of domestic equity

Notes: The figure shows the stock of domestic equity owned by foreigners as a share of gross domestic product (GDP) and as a share of domestic equity stocks—panels (a) and (b), respectively. Foreign ownership of domestic equity is measured as the sum of foreign direct investment (FDI) and foreign ownership of non-financial and of bank domestic equity stocks (FOE). The plotted series is a simple average across the following countries: Austria, Belgium, Brazil, Bulgaria, Canada, Chile, Colombia, Croatia, Czech Republic, Denmark, Finland, France, Germany, Greece, Hungary, Israel, Italy, Japan, South Korea, Mexico, Norway, Poland, Portugal, Slovak Republic, Slovenia, Spain, Sweden, Turkey, United Kingdom, United States. The time period is 1995-2023. Data are taken from the External Wealth of Nations (EWN) database (Lane and Milesi-Ferretti, 2018) and the OECD National Accounts Statistics.

This paper explores the implications of international equity flows in a model featuring nominal frictions. Our analysis reveals that international equity flows introduce a time inconsistency problem for monetary policy, resulting in a coordination failure where households sell an excessive share of domestic equity to foreign investors. We show that when domestic agents sell their domestic equity, they create a temptation for the central bank to use monetary policy to diminish future profits, lowering equity prices and reducing household wealth. Our

theory underscores that international equity flows can introduce an inefficiency where none had existed.

We formalize these insights in a simple open-economy New Keynesian model that allows households to trade domestic equity with deep-pocket foreign investors. To transparently illustrate our results, the baseline model abstracts from uncertainty, imperfections in capital markets, and domestic heterogeneity. To understand our results, consider first the case where households own the entire share of domestic equity. As is standard in the literature, the optimal monetary policy induces full employment (or, in other words, a zero labor wedge or zero output gap). That is, the government raises employment until the point at which the marginal value of consumption resulting from one additional unit of labor compensates for the disutility of providing one more unit of labor.

In an economy where households own all domestic equity, an increase in real wages, given a level of employment, has no effects on the resource constraint. Changes in labor income and profits cancel out in the households' budget constraint. The split between labor income and profits is irrelevant. However, this is not the case when foreign investors own domestic equity. When foreign investors own domestic equity, higher real wages now translate into lower profits for foreign investors while the budget constraint of domestic households is relaxed. Although domestic households also face lower profits, this is more than compensated by higher salaries. The split between labor income and profits is no longer irrelevant.

This observation has important implications for monetary policy. To the extent that monetary policy affects real wages, a central bank has incentives to target higher real wages when foreign investors own a share of domestic equity. In our baseline model with nominal wage rigidites and a single final good, assumed to be tradable internationally, we show that the central bank finds it optimal to follow a contractionary monetary policy. Starting from an allocation with a zero labor wedge, the central bank perceives a first-order gain from appreciating the exchange rate and increasing real wages to reduce equity payouts for foreign investors. That is, when foreign investors own a fraction of the domestic equity, the central induces a positive labor wedge to shift resources from foreign investors to domestic households.

The time inconsistency problem emerges because when foreign investors buy equity, they anticipate the central bank's incentives to erode their profits ex-post and, therefore, buy equity at a lower price. As a consequence, the inability of the central bank to commit translates into lower wealth for domestic households. When households sell equity, they do not internalize that a higher share of foreign-owned equity will prompt the central bank to lower profits, resulting in a fall in the price of equity today and lower welfare. Collectively, they therefore sell too much equity. Sales of domestic equity end up distorting output ex-post,

leaving no benefits to households ex-ante.

Moreover, we show that multiple equilibria arise quite naturally. When households sell more assets, this lowers future dividends and reduces the price of equity shares, to the extent that deep-pocket foreign investors take the other side in the market. While individual investors are indifferent between holding or not the equity, collectively, when they sell more equity, this lowers their wealth. Interestingly, the presence of multiple equilibria is not a result of fire sales and amplification through financial constraints, as it has been explored in the literature.

The policy implication is that restricting purchases of equity by foreigners is optimal. In particular, when domestic households own initially the entire stock of equity, the optimal policy takes a simple form: banning all equity inflows is optimal. When foreign investors initially own some equity, the optimal capital control faces a tradeoff. On one hand, repurchasing equity from foreign investors ensure that output is closer to the full employment. On the other hand, doing so maximizes the capital gains by foreign investors at the expense of domestic households. As a result, we show that it is optimal to repurchase only a small share of the domestic equity from foreign investors.

In one of the extensions of the model, we incorporate non-tradable goods. Our result that monetary policy departs from full employment remains unchanged, but now an important consideration is the impact of the relative price of non-tradables on foreigners' profits. While an appreciation lowers profits in firms in the non-tradable sector in *units of domestic currency*, the lower output of non-tradables raises its relative price and may increase the profits received by foreign investors (*in units of tradables*). We show that the implications for monetary policy depend on the the elasticity of substitution between tradables and non-tradables. When the elasticity is below one, the increase in the relative price of non-tradables may outweigh the benefits of higher real wages. Therefore, an appreciation is not necessarily optimal. On the other hand, when the elasticity is above one, appreciating the exchange rate remains optimal because the increase the increase in the relative price of non-tradables is small relative to the increase in wages.

Related literature. Our paper is related to an extensive literature on monetary policy in open economies. A key theme in this literature, common with closed economy models, is about macro-stabilization. When prices or wages are rigid, monetary policy can help align relative prices toward the full-employment allocation. However, when a country fixes the exchange rate or joins a monetary union, it loses the ability to use the exchange rate as a shock absorber (Mundell, 1960; Friedman, 1953; Schmitt-Grohé and Uribe, 2001).

A vast number of studies have also examined channels by which a departure from full employment may be optimal. One strand of this literature is concerned with terms of trade manipulation. When countries have market power in the tradable goods they produce, central banks can use monetary policy to tilt the terms of trade on their favor (e.g., Obstfeld and Rogoff, 1995 and Corsetti and Pesenti, 2005). Another strand of this literature considers the case when foreign assets and liabilities are in domestic currency and analyzes how variations in exchange rates can generate valuation effects on the net foreign asset position. In an incomplete market economy with endogenous portfolios, Fanelli (2023) shows how the optimal monetary policy under commitment balances macro-stabilization and insurance while Ottonello and Perez (2019), Du, Pflueger and Schreger (2020) and Engel and Park (2022) study the tradeoffs for the optimal monetary policy without commitment. In contrast to these contributions, we consider a model where agents trade equity with foreign investors and show how this gives rise to a novel source of time inconsistency.

Our paper is also related to the literature on international portfolio choice.[1] A central theme in this literature is on understanding the diversification of equity portfolios. Our paper takes a normative perspective and highlight a cost from such diversification. In addition, most of these studies are in the c the context of real models, thus abstracting from the role of monetary policy. A notable exception is Engel and Matsumoto (2009) who show how a portfolio of home and foreign equities and a forward position in foreign exchange can achieve to a first-order the complete market allocations. However, they do not consider optimal monetary policy.

Our work also relates to recent studies examining the implications of the rise in cross-country asset ownership. For instance, Quadrini and Ríos-Rull (2024) analyze optimal taxation in a setting where multinationals engage in profit shifting, while Atkeson, Heathcote and Perri (2022) show how rising asset values in the U.S. have adversely impacted welfare. By contrast, our focus is on studying the implications of the rise in cross-country asset ownership for monetary policy.

Our paper is also related to the literature on monetary unions, in particular, those emphasizing how joining a monetary union can deal with time inconsistency problems in monetary policy. In Alesina and Barro (2002), joining a monetary reduces the inflationary bias generated by the time inconsistency problem of monetary policy, as stressed by Barro and Gordon (1983). Fornaro (2022) presents an environment where the central bank has incentives to depreciate the exchange rate to reduce the real value of non-tradable collateral that foreign

[1] See, for example, Engel and Matsumoto (2009), Coeurdacier, Kollmann and Martin (2010), Devereux and Sutherland (2006), Tille and Van Wincoop (2010).

investors can recover ex post. Because of this time inconsistency problem, he shows that joining a monetary union can help increase bond inflows. Different from these studies, the time inconsistency problem of monetary policy in this paper is related to international equity flows. We show that this channel provides distinctive implications for monetary policy and capital controls.

Our paper is also related to the literature on monetary policy with heterogenous agents in closed economies. Much of the HANK literature assumes an equal share of profits between households or considers setups where equilibrium profits are zero, muting redistributive motives through equity holdings (e.g., Dávila and Schaab, 2023). Two exceptions are Bhandari, Evans, Golosov and Sargent (2021) and Acharya, Challe and Dogra (2020). However, these studies examine optimal policy under commitment. Moreover, they assume that fiscal policies are in place that effectively equalize marginal utilities of consumption at the steady state. Thus, their focus is on how monetary policy can provide insurance against aggregate shocks. In our open economy model, we examine a redistribution motive that emerges naturally because the government is unable to tax foreign investors and does not place any weight on their welfare.

Finally, we are also related to the literature on capital controls and capital flow management policies. This literature has developed a rationale for taxes on debt flows based on pecuniary externalities (e.g., Bianchi, 2011) and aggregate demand externalities (e.g., Schmitt-Grohé and Uribe, 2016; Farhi and Werning, 2016).[2] Here we provide a rationale for capital controls on equity flows based on a time inconsistency problem of monetary policy.

Outline. The remainder of the paper is organized as follows. Section 2 introduces the environment. Section 3 presents the theoretical analysis and Section 4 presents quantitative results. Section 5 analyzes extensions of the baseline framework. Section 6 concludes.

2 The Model

Consider a small open economy (SOE) populated by a continuum of identical households and firms. The rest of the world economy is populated by a foreign investors, who trade international bonds and firms' shares of the SOE. There is no uncertainty and time is discrete and indexed by $t \in \{0, 1...\}$.

We proceed to describe the decision problems of households and firms and then turn to analyze the competitive equilibrium.

[2]See Bianchi and Lorenzoni (2021) for a review of the literature.

Firms. A unit mass of competitive firms produce a final good which is tradable in international goods market. Firms use labor, h, as an input with a standard production function $F(\cdot)$ that satisfies the following properties: $F(0) = 0$, $F'(\cdot) > 0$, and $F''(\cdot) < 0$, $\lim_{h\to 0} F'(h) = \infty$, and $\lim_{h\to\infty} F'(h) = 0$.

We assume the law of one price holds, and without loss of generality, we normalize the price of the good in foreign currency to 1. The firm's problem consists of choosing labor hours to maximize profits:

$$\Pi_t \equiv \max_{h_t \geq 0} \left\{ E_t F(h_t) - W_t h_t \right\}, \tag{1}$$

where E_t is the nominal exchange rate, defined as the price of foreign currency in terms of domestic currency, and W_t is the nominal wage in domestic currency.

Profit maximization implies

$$F'(h_t) = \frac{W_t}{E_t}, \ \forall t \geq 0. \tag{2}$$

Households. There is a continuum of households of unit mass with preferences represented by

$$\sum_{t=0}^{\infty} \beta^t \left[u(c_t) - v(l_t) \right], \tag{3}$$

where $\beta \in (0,1)$ is the discount factor, c_t denotes consumption, l_t denotes hours worked, and the functions $u(\cdot)$ and $v(\cdot)$ satisfy the following standard properties: $u'(\cdot) > 0$, $u''(\cdot) < 0$, $v'(\cdot) \geq 0$, and $v''(\cdot) \geq 0$. We also assume Inada conditions, $\lim_{c\to 0}, u'(c) = \infty$, $\lim_{c\to\infty} u'(c) = 0$ $\lim_{l\to 0} v'(l) = 0$, and $\lim_{l\to\infty} v'(l) = \infty$.

Households trade domestic firms' shares and bonds with foreign investors in international financial markets. Let $b_t \in \mathbb{R}$ and $\theta_t \in \mathbb{R}^+$ respectively denote the holdings of bonds and shares at the beginning of the period of period t. Firms' shares entitle their owner to the counterpart fraction of firm profits. Bonds are assumed to be one-period and denominated in foreign currency.[3] Then, the households' budget constraint is

$$c_t + b_{t+1} + Q_t \theta_{t+1} = \frac{W_t}{E_t} l_t + R b_t + \left(\frac{\Pi_t}{E_t} + Q_t \right) \theta_t, \tag{4}$$

For our baseline model, we assume that households do not face borrowing restrictions; they are only subject to a no-Ponzi game constraint.

[3] Our results would be unchanged if households were to hold foreign equities instead of bonds.

At period 1 the nominal wage is rigid. We assume that the quantity of hours is determined by firms; thus households must supply the labor that firms demand. For simplicity, hours are evenly distributed across households, and therefore, using (2), we have that the number of hours worked for each household satisfy

$$l_1 = F'^{-1}\left(\frac{\overline{W}_1}{E_1}\right), \tag{5}$$

where $\overline{W}_1$ denotes an exogenous rigid wage.

At every period, households optimize their consumption, asset portfolio, and labor supply (except at $t = 1$, where the number of hours is determined by firms' labor demand). Let $V_t(b_t, \theta_t)$ denote the value at the beginning of the period for a household with an initial asset portfolio (b_t, θ_t). The households' optimization problem can then be represented recursively as follows:

$$\begin{aligned} V_t\left(b_t, \theta_t\right) = & \max_{c_t, l_t, b_{t+1}, \theta_{t+1} \geq 0} \left\{u\left(c_t\right) - v\left(l_t\right) + \beta V_{t+1}\left(b_{t+1}, \theta_{t+1}\right)\right\} \\ & \text{subject to:} \\ & c_t + b_{t+1} + Q_t\theta_{t+1} = \frac{W_t}{E_t} l_t + Rb_t + \left(\frac{\Pi_t}{E_t} + Q_t\right)\theta_t \end{aligned} \tag{6}$$

and (5) if $t = 1$. It is worth highlighting that if the return on bonds equals the return of firms' shares, as will be the case in equilibrium, households are indifferent between different portfolios, provided they yield the same net foreign asset position and, consequently, the same level of savings. Underlying this result is the absence of uncertainty, which will allows us to clearly illustrate our mechanism.

Foreign investors. We assume that foreign investors have deep pockets. By a no-arbitrage condition, this implies that in equilibrium, the return on bonds must equal the return on firms' shares. That is, the price of firms' shares is given by

$$RQ_t = \frac{\Pi_{t+1}}{E_{t+1}} + Q_{t+1} \ , \ \forall t \geq 0. \tag{7}$$

Competitive Equilibrium. We can now define the competitive equilibrium for a given exchange rate policy. We restrict attention to a symmetric equilibrium, in which all of the households always take the same consumption, labor, and portfolio decisions. In what follows, we use capital letters to denote the corresponding equilibrium allocations.

Definition 1. Given initial conditions (B_0, Θ_0), a rigid wage $\overline{W}_1$, and an exchange rate policy, $\{E_t\}_{t=0}^{\infty}$, a *competitive equilibrium* is a sequence of allocations $\{C_t, H_t\}_{t=0}^{\infty}$, portfolios $\{B_{t+1}, \Theta_{t+1}\}_{t=0}^{\infty}$ and prices $\{W_t, Q_t\}_{t=0}^{\infty}$ such that:

(i) Households optimize. That is, consumption, labor, and portfolios solve problem (6).

(ii) Firms choose employment optimally. That is, (2) holds.

(iii) The price for firms' shares satisfies condition (7).

Disposable income and balance of payments. As we will see, a key equilibrium object for our analysis is the *disposable income net of foreign-owned firms' profits*. That is, formally,

$$I(H_t, \Theta_t) \equiv F(H_t) - (1 - \Theta_t) \underbrace{[F(H_t) - F'(H_t) H_t]}_{=\Pi_t/E_t}, \tag{8}$$

This expression follows from substituting labor demand (2) and firms' profits (1) into households' aggregate income $(W_t/E_t) H_t + (\Pi_t/E_t) \Theta_t$. Combining the income with households' budget constraint (4), one obtains a standard *balance of payment condition*,

$$I(H_t, \Theta_t) + (R-1)B_t - C_t = [B_{t+1} - (1 - \Theta_{t+1})Q_t] - [B_t - (1 - \Theta_t)Q_t], \tag{9}$$

which states that the current account (LHS) is given by the change in the net foreign asset position (RHS). Equations (8) and (9) then make clear that foreign holdings of domestic equity constitute a liability for the domestic economy. Specifically, to run a current account deficit, the economy must either reduce its bond positions or increase its liabilities by selling domestic equities.

3 International Equity Flows and Exchange Rate Policy

In this section, we characterize the equilibrium and analyze the implications of international equity flows for the optimal exchange rate policy. We assume $\beta R = 1$, which guarantees a stationary continuation equilibrium from period $t = 2$ onwards and simplifies the analytical results.

Roadmap. We proceed with our analysis as follows: First, we characterize the steady-state equilibrium for $t \geq 2$. Second, we analyze the optimal exchange rate policy for period $t = 1$. Finally, we examine the scope for capital controls in period $t = 0$.

3.1 Steady-state equilibrium for $t \geq 2$

The assumption that nominal wages are flexible after $t \geq 2$ ensures monetary neutrality in this continuation equilibrium. Moreover, given that $\beta R = 1$, we have that all allocations are constant. We use the subscript "ss" to denote steady-state values. The following lemma characterizes this continuation equilibrium for any given exchange rate policy and for any initial portfolio (B_2, Θ_2).

Lemma 1 (Steady-state equilibrium)**.** *Given an initial portfolio (B_2, Θ_2), the continuation equilibrium at period $t = 2$ features constant consumption and labor. In particular, their steady-state values, H_{ss} and C_{ss}, are given by*

$$\frac{v'(H_{ss})}{u'(C_{ss})} = F'(H_{ss}) \tag{10}$$

$$C_{ss} = I(H_{ss}, \Theta_2) + (R-1)B_2 \tag{11}$$

and the sequence of portfolio positions $\{B_{t+1}, \Theta_{t+1}\}_{t\geq 2}$ satisfy

$$B_{t+1} - Q_{ss}(1 - \Theta_{t+1}) = B_2 - Q_{ss}(1 - \Theta_2) \tag{12}$$

for all $t \geq 2$ where

$$Q_{ss} = \frac{F(H_{ss}) - F'(H_{ss})H_{ss}}{R-1}. \tag{13}$$

Condition (10) is the standard intra-temporal condition between consumption and labor, which equates their marginal rate of substitution to their marginal rate of transformation. The expression for consumption, (11), results from a balanced current account in steady state. As equation (8) makes clear, the steady-state level of consumption is lower the larger is the share of the domestic equity owned by foreign investors. Related to this point, substituting disponsable income (8) and firms' share price (13) into consumption (11), one obtains the relatively more familiar expression for consumption,

$$C_{ss} = F(H_{ss}) + (R-1)[B_2 - (1 - \Theta_2)Q_{ss}], \tag{14}$$

which highlights that consumption equals output plus the annuity value of the net foreign asset position. As indicated by (12), any portfolio that keeps the same net foreign asset position is consistent with an equilibrium.

3.2 Continuation equilibrium at period $t = 1$

In period $t = 1$ the nominal wage is rigid and consequently the exchange rate does affect the equilibrium allocation. The below lemma describes the equilibrium outcome for any given exchange rate E_1 and initial portfolio (B_1, Θ_1).

Lemma 2 (Equilibrium at $t = 1$)**.** *Given an initial portfolio (B_1, Θ_1) and an exchange rate E_1, the continuation equilibrium at period $t = 1$ is such that*

i) H_1 is given by

$$\frac{\overline{W}_1}{E_1} = F'(H_1) \tag{15}$$

ii) $C_t = C_{ss}$ for all $t \geq 1$, $H_t = H_{ss}$ for all $t \geq 2$, where H_{ss} and C_{ss} are given by (10) and (11), with the portfolio (B_2, Θ_2) satisfying

$$B_2 - (1 - \Theta_2)Q_{ss} = B_1 - (1 - \Theta_1)Q_{ss} + \frac{I(H_1, \Theta_1) - I(H_{ss}, \Theta_1)}{R} \tag{16}$$

and $\{B_{t+1}, \Theta_{t+1}\}_{t \geq 2}$ satisfy (12).

Condition (15) captures that when the nominal wage is rigid aggregate labor is entirely determined by aggregate demand. As a consequence of the rigidity, the current disposable income $I(H_1, \Theta_1)$ will differ in general from the future one $I(H_{ss}, \Theta_1)$, which implies that at $t = 1$ the current account may not be balanced. Because preferences are separable between consumption and labor, houesholds maintain a constant path for consumption. As expression (16) indicates, the current account is thus negative if the current income is below the future one. As noted earlier, while net flows are determined, the portfolio composition remains indeterminate.

Next we turn to the problem of a central bank that chooses the exchange rate to maximize households' welfare.

3.3 Optimal exchange rate policy

Using the equilibrium characterization in Lemma 2, we let $\{\mathcal{H}_1, \mathcal{H}_{ss}, \mathcal{C}_{ss}\}$ denote the unique equilibrium policies for aggregate labor and consumption, which are a function of the period 1 exchange rate and the initial portfolio positions, that is, $(E_1; B_1, \Theta_1)$. The problem of the

central bank can then be represented in dual form as follows:

$$\max_{E_1} \{u\left(\mathcal{C}_{ss}(E_1; B_1, \Theta_1)\right) - v\left(\mathcal{H}_1(E_1; B_1, \Theta_1)\right) + \frac{1}{R-1}\left[u\left(\mathcal{C}_{ss}(E_1; B_1, \Theta_1)\right) - v\left(\mathcal{H}_{ss}(E_1; B_1, \Theta_1)\right)\right]\Big\}, \quad (17)$$

where the first term in the objective is the period 1 utility flow and the second term is the present discount value of utility. Notice that this problem is dynamic because the exchange rate affects the continuation equilibrium at period $t = 2$ through its effect on portfolio position (B_2, Θ_2).

The following proposition provides a necessary condition for optimality.

Proposition 1 (Necessary condition for optimality)**.** *Let $\pi(H_t) \equiv F(H_t) - F'(H_t)H_t$ denote the real profits of firms as a function of aggregate labor H_t. Then, the optimal exchange rate policy satisfies*

$$F'\left(\mathcal{H}_1\right) - \frac{v'\left(\mathcal{H}_1\right)}{u'\left(\mathcal{C}_{ss}\right)} - \left(1 - \Theta_1\right)\left[\pi'\left(\mathcal{H}_1\right) + \frac{1}{R-1}\pi'\left(\mathcal{H}_{ss}\right)\frac{\partial \mathcal{H}_{ss}/\partial E_1}{\partial \mathcal{H}_1/\partial E_1}\right] = 0\,, \quad (18)$$

where

$$\frac{\partial \mathcal{H}_{ss}/\partial E_1}{\partial \mathcal{H}_1/\partial E_1} = \frac{u''\left(\mathcal{C}_{ss}\right)F'\left(\mathcal{H}_{ss}\right)\frac{R-1}{R}I_H\left(\mathcal{H}_1, \Theta_1\right)}{v''\left(\mathcal{H}_{ss}\right) - F''\left(\mathcal{H}_{ss}\right)u'\left(\mathcal{C}_{ss}\right) - u''\left(\mathcal{C}_{ss}\right)F'\left(\mathcal{H}_{ss}\right)\frac{1}{R}I_H\left(\mathcal{H}_{ss}, \Theta_1\right)}. \quad (19)$$

A first important observation is that if domestic households own all the firms' shares, i.e., $\Theta_1 = 1$, we arrive at the standard condition that a central bank would depreciate the exchange rate until the marginal rate of transformation equals the marginal rate of substitution. In this scenario, the central bank implements an allocation with a zero labor wedge. Conversely, when foreign investors hold shares in domestic firms, the central bank may opt to deviate from a zero labor wedge to diminish the profits earned by foreigners This mechanism is captured by the last term on the left-hand side of equation (19), which represents the change in the present discounted value of profits paid to foreigners resulting from a marginal increase in the exchange rate. At the optimum, the central bank equates the labor wedge to the expected marginal increase in profits to foreigners.

The sign of the optimal labor wedge depends on how the present discounted value of profits responds to a depreciation. This response results from the composite effect of how labor influences profits and how a depreciation affects labor. The first effect, represented by $\pi'(H_t) = -F''(H_t)H_t > 0$, is positive due to the strict concavity of the production function.

Moreover, the labor response in period 1 to a depreciation of the exchange rate $\partial \mathcal{H}_1/\partial E_1$ is positive, as indicated by (15). That is, a depreciation of the exchange rate depresses the real wage and consequently boosts the aggregate labor demand. By contrast, the sign of future labor response $\partial \mathcal{H}_{ss}/\partial E_1$ is ambiguous, as a consequence of the ambiguity in the sign of the response of labor income $F'(H_t)H_t$ to labor hours H_t.[4] In a case with a positive labor income response, that is, $F'(H_t) + F''(H_t)H_t > 0$, the future labor response is negative, because a depreciation of the exchange rate creates a positive wealth effect on impact that induces households to reduce their future labor supply. More importantly, in that case, starting from a situation without labor wedge [i.e., E_1 such that $\mathcal{H}_1(E_1; B_1, \Theta_1) = \mathcal{H}_{ss}(E_1; B_1, \Theta_1)$], marginally appreciating the exchange rate uniquivocally improves social welfare, which then at the optimal implies a positive labor wedge, as shown by the below corollary.

Corollary 1 (Positive labor wedge)**.** *Suppose that necessary condition* (18) *is also sufficient for characterizing the optimal exchange rate policy. If labor income $F'(H_t)H_t$ is increasing in labor hours H_t, then the optimally induced labor wedge is positive when foreigners own domestic equities.*

We note that if functions $\{u(\cdot), v(\cdot), F(\cdot)\}$ are isoelastic then condition (18) is sufficient for exchange rate optimality and that a production function with diminishing returns to scale guarantees a labor income $F'(H_t)H_t$ increasing in H_t. A case with an isoelastic production function is illustrative because it sheds further insights on the mechanism. Specifically, under $F(h) = h^\alpha$, with $\alpha \in (0, 1)$, disposable income is given by

$$I(H_t, \Theta_1) = [\alpha + (1 - \alpha)\Theta_1] F(H_t),$$

which implies that from the perspective of the SOE *foreign ownership of domestic equities is akin to a reduction in aggregate productivity.* One can then interpret the reduction in aggregate labor induced by the optimal exchange rate through this lens.

Takeaway. This section highlighted the implications of foreign ownership of domestic equities for the determination of the optimal exchange rate. A key insight is that the central bank has incentives to over-appreciate the exchange rate relative to what is required to eliminate the distortions from the rigid nominal wage to hence boost real wages and reduce the profits of firms accrued by foreign investors.

[4] See equation (19) for details. Note that the response of disposable income $I(H_t, \Theta_1)$ to labor hours H_t is $I_H(H_t, \Theta_1) = F'(H_t) + (1 - \Theta_1) F''(H_t) H_t$.

3.4 Equilibrium at period $t = 0$

So far, we have taken as given portfolio position $\{B_1, \Theta_1\}$, and given the portfolio, we have characterized the continuation equilibrium after $t \geq 1$. We will turn now to the determination of the initial portfolio $\{B_1, \Theta_1\}$. Crucially, the portfolio and the price of the firms' shares depend on the expected exchange rate policy.

The following lemma describes the equilibrium at the initial period for any given future exchange rate policy.

Lemma 3 (Equilibrium at $t = 0$)**.** *Given initial portfolios (B_0, Θ_0) and a period-one exchange rate policy $E_1 = \mathcal{E}(B_1, \Theta_1)$, the competitive equilibrium is characterized as follows:*

i) $H_t = H_{ss}$ for all $t \neq 1$ and $C_t = C_{ss}$ for all t. Moreover, C_{ss} H_{ss} and H_1 satisfy (10), (11), (15) given that the portfolio position $\{B_1, \Theta_1\}$ satisfies

$$B_1 - (1 - \Theta_1)Q_0 = B_0 - (1 - \Theta_0)Q_0 + \frac{R-1}{R}\frac{I(H_{ss}, \Theta_0) - I(H_1, \Theta_0)}{R}, \tag{20}$$

where Q_0 is given by

$$Q_0 = \frac{1}{R}\left[\pi(H_1) + \frac{1}{R-1}\pi(H_{ss})\right]. \tag{21}$$

In addition, (B_2, Θ_2) satisfy (16) and $\{B_{t+1}, \Theta_{t+1}\}_{t\geq 2}$ satisfy (12).

As before, in equilibrium, we have perfect consumption smoothing for households. Moreover, because the marginal utility of consumption is constant over time, labor hours are constant except in period 1, where the nominal wage is sticky and the hours are determined by firms' labor demand. As a result disposable incomes at $t = 0$ and after $t \geq 2$ coincide as well.

Also as before we have an indeterminacy of the portfolio. Before, the indeterminacy did not have any implications for real allocations, but now it does have. The reason is that now the portfolios chosen in period 0 affect the exchange rate set by the government in period 1. As we saw above, when households choose to sell more of their firms' shares, the central bank will choose a more appreciated exchange rate. In turn, this has two effects. First, households' will earn higher real wages in period 1, while working fewer hours and obtaining lower profits. Second, as investors anticipate that they will obtain lower profits, households are able to sell their shares at a lower price, which translates into a lower level of consumption.

Intuitively, the existence of multiple equilibria with different allocations reflects a coordination failure within the SOE. Specifically, at the initial period, households do not internalize

the collective effect of their individual portfolio positions on the determination of the future exchange rate. When households sell more of their firms' shares, however, they generate a temptation for the central bank to distort output and reduce profits in period 1. Because foreign investors perfectly anticipate the future exchange rate policy, households end up selling the shares at lower prices. The outcome is that output is distorted and households' wealth is reduced.

3.5 Optimal capital control on international equity flows

The coordination failure uncovered in the previous section suggests a role for government policy. We show next how a policy of capital controls on equity inflows at period 0 can improve welfare.

Specifically, we consider a government that can directly set the equity holdings held by domestic households at $t = 0$. We denote by $\hat{\Theta}_1$ the choice for the government. Formally, households at $t = 0$ solve (6) with the key difference that Θ_{t+1} is restricted to be $\hat{\Theta}_{t+1}$.

To write the optimal policy problem we use that in line with Lemma 3, consumption is constant for all $t \geq 0$ and employment is constant for all $t \neq 1$. Let us denote by $\mathcal{B}_1(\hat{\Theta}_1)$ the equilibrium level of borrowing in period 0 as a function of the capital control policy $\hat{\Theta}_1$. It should be understood that $\mathcal{B}_1(\cdot)$ is also a function of the initial portfolio in the initial period (B_0, Θ_0) but we do not include it to avoid clutter. With some abuse of notation, we denote the equilibrium consumption as $\mathcal{C}_{ss}(\hat{\Theta}_1) = \mathcal{C}_{ss}(\mathcal{E}(\mathcal{B}_1(\hat{\Theta}_1), \hat{\Theta}_1); \mathcal{B}_1(\hat{\Theta}_1), \hat{\Theta}_1)$ and define analogously $\mathcal{H}_1$ and $\mathcal{H}_{ss}$. This notation reflects that a capital control policy $\hat{\Theta}_1$ determines the bond issuance B_1, and the two jointly determine next period exchange rate E_1.

The optimal policy can then be formulated in dual form as follows:

$$\max_{\hat{\Theta}_1 \in [0,1]} \left\{ \frac{R}{R-1} u\left(\mathcal{C}_{ss}(\hat{\Theta}_1)\right) - \left[\left(\frac{R}{R-1} - \frac{1}{R} \right) v\left(\mathcal{H}_{ss}(\hat{\Theta}_1)\right) + \frac{1}{R} v\left(\mathcal{H}_1(\hat{\Theta}_1)\right) \right] \right\}. \tag{22}$$

Crucially, the government takes the decision in period 0 and lets the central bank choose the exchange rate policy in period 1.

The below proposition provides a necessary condition for optimality.

Proposition 2 (Necessary Condition)**.** *The optimal capital control satisfies*

$$F'\left(\mathcal{H}_1\right) - \frac{v'\left(\mathcal{H}_1\right)}{u'\left(\mathcal{C}_{ss}\right)} - (1 - \Theta_0) \left[\pi'\left(\mathcal{H}_1\right) + R \left(\frac{R}{R-1} - \frac{1}{R} \right) \pi'\left(\mathcal{H}_{ss}\right) \frac{\partial \mathcal{H}_{ss} / \partial \hat{\Theta}_1}{\partial \mathcal{H}_1 / \partial \hat{\Theta}_1} \right] = 0, \tag{23}$$

where

$$\frac{\partial \mathcal{H}_{ss}/\partial \hat{\Theta}_1}{\partial \mathcal{H}_1/\partial \hat{\Theta}_1} = \frac{u''\left(\mathcal{C}_{ss}\right) F'\left(\mathcal{H}_{ss}\right) \frac{R-1}{R} \frac{1}{R} I_H(\mathcal{H}_1, \Theta_0)}{v''\left(\mathcal{H}_{ss}\right) - F''\left(\mathcal{H}_{ss}\right) u'\left(\mathcal{C}_{ss}\right) - u''\left(\mathcal{C}_{ss}\right) F'\left(\mathcal{H}_{ss}\right) \left(1 - \frac{R-1}{R} \frac{1}{R}\right) I_H(\mathcal{H}_{ss}, \Theta_0)}. \quad (24)$$

As was the case for the optimal exchange rate policy, if households own all of the domestic firms' shares (i.e., $\Theta_0 = 1$), the optimal capital control implements an equilibrium with a null labor wedge. This requires $\hat{\Theta}_1 = 1$ to rule out distortionary incentives to the future exchange rate. In this case, the optimal capital control bans households from selling domestic equities to foreign investors. This is beneficial for the SOE because otherwise, before buying, the investors would correctly anticipate the future distortionary exchange rate, which would then only lead to a deterioration of the initial households' wealth.

As it was also the case for the optimal exchange rate policy, if households do not own all of the domestic equity, the optimal capital control is distortionary. This is because in this case foreign investors appropriate for themselves part of the efficiency gains from eliminating future distortionary inventives to the exchange rate via higher equity prices today. As the below corollary states, moreover, if condition (23) is both necessary and sufficient for optimality and if labor income $F'\left(H_t\right) H_t$ is increasing in labor hours H_t, then the optimal capital control satisfies $\hat{\Theta}_1 \in (\Theta_0, 1)$. That is, under those conditions, the optimal policy commands households to repurchase part—but not all—of the equity owned by foreigners.

Corollary 2. *Suppose that necessary condition (23) is also sufficient for characterizing the optimal capital control. If labor income $F'\left(H_t\right) H_t$ is increasing in labor hours H_t, then if $\Theta_0 < 1$, the optimal capital control satisfies $\hat{\Theta}_1 \in (\Theta_0, 1)$.*

3.6 Discussion: The Role of Commitment

Consider the monetary authority problem at $t = 0$ when it has the ability to commit to future policies. We can write this problem as follows:

$$\max_{\{H_{t+1}, Q_0, E_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t [u(C) - v(H_t)] \quad (25)$$

subject to

$$\sum_{t=0}^{\infty} \frac{C - \frac{W_t}{E_t} l_t}{R^t} \leq Rb_0 + \theta_0 \sum_{t=0}^{\infty} \frac{F(H_t) - \frac{W_t}{E_t} H_t}{R^t} \tag{26}$$

$$\frac{W_t}{E_t} u'(C) = v'(H_t) \quad \forall t \neq 1 \tag{27}$$

$$H_1 = F'^{-1}\left(\frac{\overline{W}_1}{E_1}\right) \tag{28}$$

where we have used that consumption must be constant in equilibrium and we have replaced the price of equity in the intertemporal budget constraint.

We argue that this problem with commitment is equivalent to the problem without commitment but where the monetary authority can use capital controls. That is, given any solution of the Markov equilibrium with capital controls, the monetary authority with commitment (and no capital controls) can replicate that solution. That is, let Θ_1^* the portfolio chosen at $t = 0$ in the Markov equilibrium and $H_1^M = \mathcal{H}(\Theta_1^*)$ the asscoiated level of employment. With commitment, this is a feasible choice for the central bank.

Conversely, let H_1^* the optimal employment under commitment. In the Markov equilibrium l_1 is a function of Θ. That is, $l_1 = \mathcal{H}(\Theta)$. I could pick Θ_1^* such that $H_1^* = \mathcal{H}(\Theta_1^*)$

These findings highlight the central role that the absence of commitment plays for the desirability of capital controls in equity flows. When the central bank cannot commit to an exchange rate policy at $t = 1$, the economy may be stuck in an equilibrium where all households sell the equity and this induces the central bank to distort the level of employment.

4 Quantitative results

In this section, we provide a simple quantification to gauge the importance of equity flows in shaping the optimal exchange rate policy and the welfare implications of capital controls.

Parameter values. The time period is a year. We set the intertemporal elasticity of substitution to $\sigma = 1$, the inverse labor supply elasticity to $\psi = 3$, the labor share to $\alpha = 0.65$, and the gross interest rate $R = 1.04$. All values are standard in the literature.[5]

Exchange rate policy. Figure 2 presents the results for the optimal exchange rate policy at $t = 1$. The figure shows the exchange rate, the labor wedge, firms' profits, output, consumption, and welfare all as a function of the share of foreign-owned equity implied by

[5]For the elasticity of labor supply, see in particular, Keane and Rogerson (2012)

the capital control. The values in the y-axis are all expressed in percentage deviation from an economy with flexible prices (or, equivalently, one where the central bank were to implement an economy with a zero labor wedge). To keep the net foreign asset position constant, we vary B_1 so that $B_1 = (1 - \Theta_1) Q^*_{ss}$, where Q^*_{ss} denotes the price of firms' shares under flexible nominal wages and full domestic equity ownership (i.e., $\Theta_1 = 1$).[6]

If foreigners do not own domestic firms' shares, we can see that the labor wedge is null under the optimal policy (panel [a]). Given that domestic households own all the firms' shares the central bank simply chooses the exchange rate to eliminate the distortions induced by the rigid wage. Namely, it chooses the exchange rate to align the real wage with the one corresponding to the flexible price allocation. In line with this, the figure shows that all allocations coincide with the flexible price benchmark for $1 - \Theta_1 = 0$.

As foreign investors' holdings of domestic firms' shares increase, the benefits for the SOE of boosting real wages and squeezing firms' profits are larger. Thus, when $1 - \Theta_1 > 0$, the central bank over appreciates the exchange rate relative to what is required to achieve the flexible price allocation and it induces a positive labor wedge. Both aggregate labor and aggregate output fall. Despite the increase in real wages, consumption falls as well, because labor income and firms' profits do so. For example, when foreign investors own half of the firms' shares, the central bank over appreciates the exchange rate by around 2%, which reduces both labor income and firms' profits by around 3%. Aggregate labor and aggregate output fall by 5.75% and 3.15%, respectively, while consumption does so by 0.1%. In addition, the appreciation of the nominal exchange rate generates an increase in real wages of about 2%. As highlighted above, foreign holdings of firms' shares act like a negative productivity shock on the SOE, which then induces the central bank to implement a lower output level to save on disutility costs from supplying labor hours.

Panel (f) reports the welfare gains from implementing the optimal exchange rate policy relative to a policy that would keep a zero labor wedge. As the figure shows, there are sizable gains from departing from a zero-labor wedge allocation. When foreign investors own the entire share of firms, households are willing to give up 1% of period 1 consumption to shift to the optimal policy.

[6] Proceeding in this way ensures that as we change Θ_1 all allocations would remain constant under flexible prices.

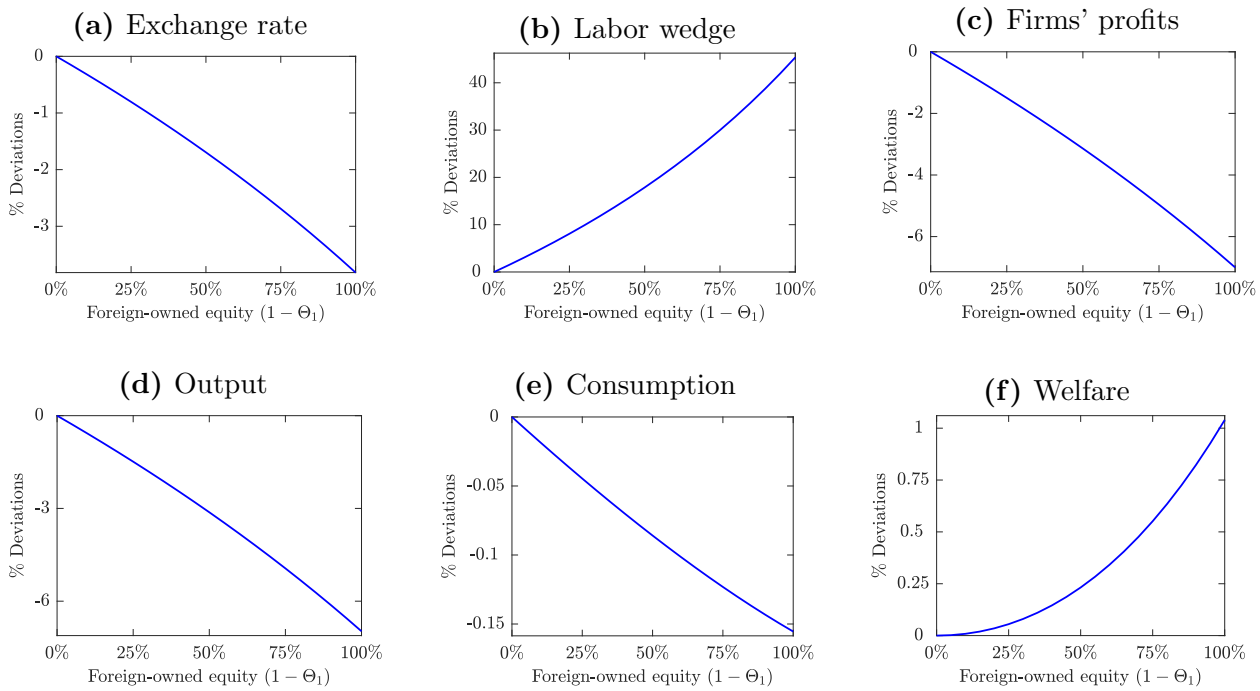


Figure 2: Continuation equilibrium at period $t = 1$ under the optimal exchange rate

Notes: The figure plots the continuation equilibrium at period $t = 1$ under the optimal exchange rate as a function of the foreign-owned fraction of domestic firms' shares implied by the capital control. For any given $1 - \Theta_1$ external debt is such that $B_1 = (1 - \Theta_1) Q^*_{ss}$, where Q^*_{ss} is the price of firms' shares under flexible nominal wages and full domestic equity ownership, so that under those wages the net foreign asset position of the SOE is independent of the composition of the initial portfolio. Variables are expressed as percentage deviations from the benchmark continuation equilibrium with flexible nominal wages. Welfare is expressed in terms of period 1 consumption equivalence.

Capital controls. Figure 3 presents the equity price, the households' wealth, and their welfare as a function of the foreign-owned share of domestic equities implied by the capital control. It does so for a case in which initially foreign investors do not own domestic equities and for a case in which the investors own a fraction of the equities.

If initially foreigners do not own domestic equities, we can see that banning households from internationally selling the equity maximizes social welfare in the SOE. Any other capital control would induce distortionary incentives to the future exchange rate and consequently it would depress the equity price today. This price fall would indeed be socially inefficient for the SOE because its households initially own all of the equity. A worst-case scenario with a full sale of the equities to the foreigners would have a quantitatively large impact on the equity price and the households' well-being. Specifically, the price would fall by around 20 basis points below its level under the optimal policy, while welfare would do so by around 1 percentage point in terms of consumption equivalent units.

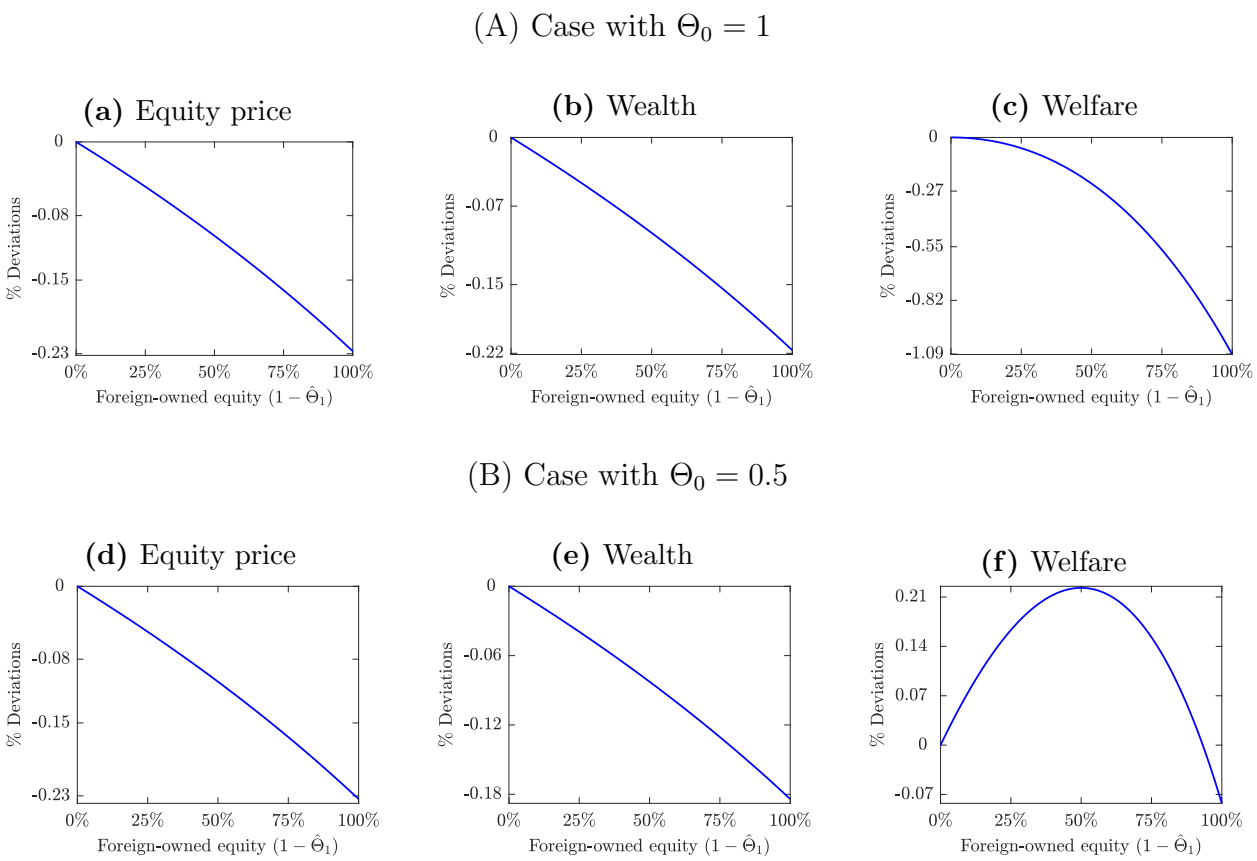


Figure 3: Equilibrium at period $t = 0$ as a function of the capital control

Notes: The figure plots the equilibrium at period $t = 0$ as a function of $\hat{\Theta}_1$. Panels A and B present respectively the case where the initial domestic equity holdings are $\Theta_0 = 1$ and $\Theta_0 = 0.5$ while B_0 is such that $B_0 = (1 - \Theta_0) Q^*_{ss}$ Welfare is measured in period 1 consumption equivalent.

By contrast, if foreigners initially own some of the equities, we can see that inducing a small equity repurchase slightly improves social welfare in the SOE relative to not repurchasing. Repurchasing a large share is not optimal for the SOE because the resulting increase in the equity price plays against its households. In effect, an scenario with a full equity repurchase would significantly increase the equity price, which would strongly reduce the households' welfare.

5 Extensions

In this section, we extend the theory in two directions. First, we allow for binding financial constraints. Second, we incorporate a non-tradable good in the model.

5.1 The case with a binding debt limit

In our baseline model, the SOE does not face financial constraints. This implies that households are able to perfectly smooth consumption over time independently of the equity inflows they receive. We now consider a situation where the economy may face a binding borrowing limit.

Specifically, we assume that households borrowing must be such that $b_{t+1} \geq -\underline{b}$ and focus on a situation where the initial debt level is $B_0 < -\underline{b}$. We interpret this situation as a sudden unwillingness of foreign investors of rolling over the bonds. To repay its debt obligations and smooth consumption, the small open economy therefore has incentives to sell some of its domestic equity.

In this context, the optimal capital control policy is no longer to prevent all equity inflows. The government, however, still imposes controls on equity inflows, because households could end up selling too much equity from a social point of view. Indeed, by selling a sufficiently large amount of equity, households may still be able to perfectly smooth consumption.

Figure 4 presents the optimal capital control and the resulting equilibrium outcome as a function of external debt $-B_0$ (for the case of $\Theta_0 = 1$). We can see in the figure (panel [a]) that for $B_0 \geq -\underline{b}$), the government still prevents any sales of domestic equity—all equity inflows are banned. As B_0 falls strictly below $\underline{b}$, the borrowing constraint becomes binding (panel [b]) and the government allows for some equity inflows. In this case, the central bank faces a trade-off between smoothing consumption, on the one hand, and preventing the emergence of an output gap at $t = 1$ and lower equity prices at $t = 0$, on the other. Panel (c) shows the resulting fall in output when the government chooses optimally the level of capital controls.[7]

The key takeaway is that even in a situation where equity inflows are socially desirable, the government still has incentives to reduce the amount of inflows relative to the unregulated competitive equilibrium. Notice that the optimal equity inflows restriction helps support equity prices, which indeed reduces the SOE's needs to sell domestic equity as well as mitigates a vicious cycle—usually referred to as "fire sales"—between drops in equity prices and further equity selling needs. Importantly, in this environment, equity prices fall not because of inherent differences in valuations between domestic and foreign investors, but because of distortionary incentives induced by equity inflows to the future exchange rate policy.

[7]While we do not plot it here, the worst equilibrium in this parameterization is such that households sell all the equity, accumulate bonds and keep constant consumption. For larger amounts of debt, households would become constrained in the worst equilibrium.

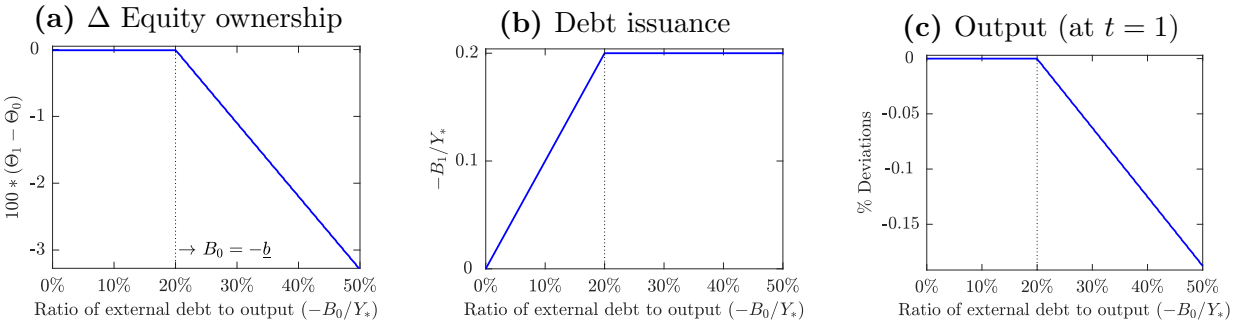


Figure 4: Optimal Capital Control when the SOE can be financially constrained

Notes: The figure plots the equilibrium outcome under the constrained optimal capital control as a function of external debt $-B_0$ for the case of $\Theta_0 = 1$. Domestic production at period $t = 1$ (Panel c) is expressed as a percentage deviation from a benchmark continuation equilibrium with flexible nominal wages and without foreign ownership of the equity. The debt limit is such that $\underline{b} = -0.2Y_*$, where Y_* is the output level in the benchmark equilibrium. All other parameters are as in the baseline calibration.

5.2 Model with tradable and non-tradable goods

So far, we have considered an environment with a single final good, assumed to be tradable internationally. In this section, we extend the baseline model by introducing a non-tradable good and we analyze the implications of international equity flows of non-tradable firms for monetary policy.

5.2.1 Environment with tradables and non-tradables

Households. Preferences are given again by

$$\sum_{t=0}^{\infty} \beta^t \left[u(c_t) - v\left(l_t\right) \right], \tag{29}$$

but now consumption $c_t = c(c_t^T, c_t^N)$ is a linearly homogenous aggregator of the tradable good, c_t^T, and the non-tradable good, c_t^N. Equivalently, we will use $U(c_t^T, c_t^N)$ to denote the utility as a function of the two consumption goods, and γ_t the elasticity of substitution between the two goods. In addition, l_t is the total number of hours households work in the tradable and non-tradable sector. That is, $l_t = l_t^T + l_t^N$.

The budget constraint expressed in units of tradables is given by

$$c^T + \frac{P_t^N}{E_t} c_t^N + b_{t+1} + \sum_j Q_t^j \theta_{t+1}^j = \frac{W_t}{E_t} l_t + b_t R + \sum_j \left(\frac{\Pi_t^j}{E_t} + Q_t^j \right) \theta_t^j$$

where the sums are indexed by $j \in \{N, T\}$, Q_t^j is the price of equity of firms in sector j in units of foreign currency, and $P_t^N > 0$ is price of the non-tradable good in domestic currency. In the budget constraint, we have assumed implicitly that the nominal wage is the same in both sectors, a result that follows because labor hours in both sectors are perfect substitutes for households.

In addition to the optimality conditions presented in Section 2, we have a condition equating the marginal rate of substitution between the goods to their relative price:

$$\frac{P_t^N}{E_t} = \frac{U_N(c^T, c_t^N)}{U_T(c_t^T, c_t^N)}.$$

where U_T and U_N respectively denote the marginal utility with respect to the tradable and the non-tradable good.

We continue to assume that the nominal wage in units of domestic currency is fixed at $\bar{W}$ at $t = 1$.[8] This implies that hours worked in both sectors are determined by firms' labor demand.

Firms. Firms in each sector choose labor hours to maximize profits. That is, firms solve

$$\Pi_t^j \equiv \max_{h_t^j \geq 0} \left\{ P_t^j F^j \left(h_t^j \right) - W_t h_t^j \right\} \ , \text{ for } j \in \{N, T\} \ . \tag{30}$$

where recall that by the law of one price and the fact that the foreign price is normalized to one, we have $P_t^T = E_t$.

Competitive equilibrium. We define now a competitive equilibrium in this environment.

Definition 2. Given initial conditions (B_0, Θ_0), a rigid wage $\overline{W}_1$, and an exchange rate policy, $\{E_t\}_{t=0}^{\infty}$, a *competitive equilibrium* is a sequence of allocations $\{C_t^T, C_t^N, H_t^T, H_t^N\}_{t=0}^{\infty}$, portfolios $\{B_{t+1}, \Theta_{t+1}^T, \Theta_{t+1}^N\}_{t=0}^{\infty}$ and prices $\left\{W_t, P_t^T, Q_t^T, Q_t^N\right\}_{t=0}^{\infty}$ such that:

(i) Households and firms optimize.

(ii) Market for non-tradable clears. That is, $C_t^N = F^N \left(H_t^N \right)$.

(iii) Prices for firms' shares satisfy

$$RQ_t^j = \frac{\Pi_{t+1}^j}{E_{t+1}} + Q_{t+1}^j \ , \text{for } j \in \{T, N\} \ . \tag{31}$$

[8] An analysis with sticky prices for non-tradables is available upon request.

Crucially, relative to the definition of competitive equilibrium in Section 2, an equilibrium also requires the market for the non-tradable good to clear domestically.

Profits for non-tradable firms. For simplicity, assume that the composition consumption follows a CES, with relative share ω for the tradable good and elasticity γ between the goods. Moreover, assume that the production function for non-tradables is given by $F^N(H) = H^{\alpha^N}$, with $\alpha^N \in (0,1)$. Then, in any continuation equilibrium at period $t = 1$, profits in the non-tradable sector are given by

$$\frac{\Pi_t^N}{E_t} = \underbrace{\frac{1-\omega}{\omega}\left(\frac{F^N\left(H_t^N\right)}{C_{ss}^T}\right)^{-1/\gamma}}_{=P_t^N/E_t}\left(1-\alpha^N\right)F^N\left(H_t^N\right), \tag{32}$$

Equation (32) suggests that whether the central bank will seek to target a higher or lower employment in non-tradable sector depends on the elasticity of substitution γ.

If $\gamma = 1$, non-tradable employment does not affect firm profits in the sector. This is because lower employment lower wages but the price of non-tradables raises in the same proportion, muting any effects on profits in units of tradables. In a situation where households own all shares in the tradable sector, that is, $\Theta^T = 1$, the above indicates that the central bank will not depart from a zero labor wedge when $\gamma = 1$.

If $\gamma < 1$ (i.e., tradable and non-tradable goods are complements), the relative price responds more than proportionally to a change in the ratio of the quantities consumed. Thus, everything else the same, declines in non-tradable production boosts firm profits in the sector, whereas increases depress them. For given labor hours in the tradable sector H_1^T and H_{ss}^T, it may be optimal for the central bank to depreciate the currency relative to the case where the small own economy own all shares in the non-tradable sector.

Finally, if $\gamma > 1$ (i.e., the goods are substitutes), the relative price responds less than proportionally to a change in the ratio of the quantities consumed. In this situation, the central bank has incentives to over-appreciate the currency, as in the baseline model with only tradables.

5.2.2 Optimal exchange rate policy

In what follows, we assume that $U(c^T, c^N) = \omega^T u(c^T) + \omega^N u(c^N)$, with $\omega^T + \omega^N = 1$. Together with the assumption that $\beta R = 1$, this ensures stationarity in the continuation equilibrium

at period $t = 2$ and constant tradable consumption over time.

The proposition below provides a necessary condition for exchange rate optimality at period 1.

Proposition 3 (Necessary condition for optimality)**.** *The optimal exchange rate policy satisfies*

$$\sum_j \frac{\mathcal{P}_1^j}{E_1}\left[F'^j\left(\mathcal{H}_1^j\right) - \frac{v'\left(\mathcal{H}_1^j + \mathcal{H}_1^{-j}\right)}{\omega^j u'\left(\mathcal{C}_1^j\right)}\right]\frac{\partial \mathcal{H}_1^j}{\partial E_1} = \sum_j \left(1-\Theta_1^j\right)\left[\frac{\partial\left(\mathbf{\Pi}_1^j/E_1\right)}{\partial E_1} + \frac{1}{R-1}\frac{\partial\left(\mathbf{\Pi}_{ss}^j/E_{ss}\right)}{\partial E_1}\right]. \tag{33}$$

As in Section 2, the optimal exchange rate induces a labor wedge that depends on how changes in labor affects profits received by foreign investors. The difference is that now the analysis considers how profits in both sectors change.

As we saw above, a depreciation may indeed affect profits in the two sectors in different directions. An implication of this result is that there may exist multiple combinations of Θ^T and Θ^N such that the government does not have incentives to depart from a zero labor wedge. This set is formally given by

$$\Gamma \equiv \left\{\left(\Theta_1^N, \Theta_1^T\right) \in [0,1]^2 : \sum_j \left(1-\Theta_1^j\right)\left[\frac{\partial\left(\mathbf{\Pi}_1^j/E_1\right)}{\partial E_1} + \frac{1}{R-1}\frac{\partial\left(\mathbf{\Pi}_{ss}^j/E_{ss}\right)}{\partial E_1}\right] = 0\right\}.$$

Figure 5 illustrates the set for different values for the elasticity. In all cases, we consider the case where the two goods are complements. We can see that as the share of equity increases in one sector, the share of equity in the other sector needs to increase to mute the incentives to deviate from a zero labor wedge.

An important takeaway from this section is that the composition of equity flows across sectors is crucial for monetary policy. In particular, a central bank may want to depreciate or appreciate depending on which equity shares investors hold. Moreover, a central bank that seeks to reduce its incentives to reduce investors' rents ex-post can still stimulate equity inflows to the extent that they are balanced appropriately across sectors.

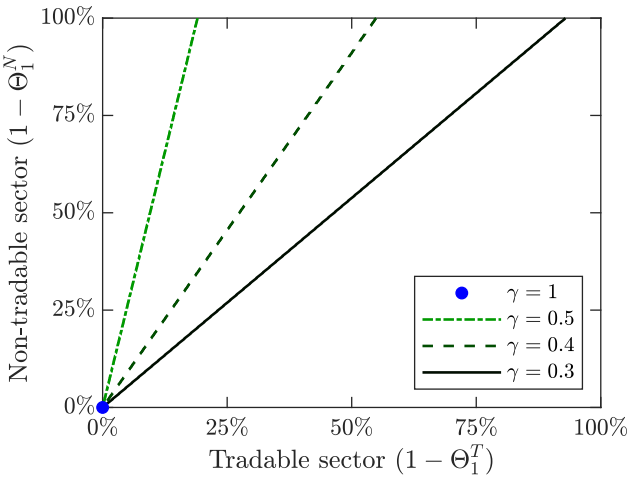


Figure 5: International equity portfolios that induce a non-distortionary exchange rate

Notes: The figure plots the set of international equity portfolios that induce a non-distortionary exchange. Parameter values are $\chi = \alpha^N$, $\psi = 0$, $\alpha^T = 0.25$, $\alpha^N = 0.75$, $R = 1.04$, $\gamma = \{0.3, 0.4, 0.5, 1\}$ and $\omega^T = \{0.06, 0.1, 0.13, 0.25\}$. Condition $\chi = \alpha^N$ is set to normalize non-tradable output to 1 under flexible nominal wages. Parameter ω^T is adjusted so that under flexible nominal wages the share of tradable output in total output equals 0.20.

6 Conclusion

In this paper, we have examined the implications of international equity flows for monetary policy. We present a model where foreign investors own a share of domestic equity and characterize the optimal monetary policy. We show that international equity flows generate a time inconsistency for monetary policy: when domestic agents sell their domestic equity, they create a temptation for the central bank to use monetary policy to diminish future profits, which in turn leads to lower stock prices and reduced household wealth. In contrast to existing theory of prudential capital flow management policies, the model calls for capital controls on equity flows and on the composition of the flows between the tradable and the non-tradable sector.

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A Proofs

A.1 Proof of Lemma 1

Proof. Consider the problem (6). Together with the envelope conditions, the first-order conditions with respect to $c_t, l_t, b_{t+1}, \theta_{t+1}$ yield

$$w_t = \frac{v'(l_t)}{u'(c_t)}, \tag{A.1}$$

$$u'(c_t) = \beta R u'(c_{t+1}) \tag{A.2}$$

$$u'(c_t)Q_t = \beta(\Pi_{t+1} + Q_{t+1})u'(c_{t+1}) \tag{A.3}$$

where the latter implicitly assumes that $\theta_{t+1} > 0$, a result that will be immediate from (A.2) and (7). Using that $\beta R = 1$ we obtain from (A.2) that $c_t = c_{t+1} = c_{ss}$ for all t. From (A.2),(A.3) and (7), it is also immediate that the portfolio between bonds and equity is undetermined.

Given $c_t = c_{ss}$, equations (A.1) and (2), imply that $l_t = h_t = h_{ss}$ and $w_t = w_{ss}$ for all $t \geq 2$. We thus have that profits are constant as well and given by $\Pi_{ss} = F(h_{ss}) - F'(h_{ss})h_{ss}$. The expression (13) in the lemma then follows by iterating forward on (7) and using a no-bubble condition.

Iterating forward on the budget constraint (4) and using No-Ponzi, we obtain

$$c_{ss} = w_{ss}l_{ss} + \pi_{ss}\theta_{ss} + (R-1)b_{ss},$$

Combining this expression with (2) with market clearing $h_t = l_t$ and using the definition of $I(\Theta, H)$ we arrive to (11). Finally, replacing c_{ss} in the budget constraint (4), we obtain (12).

□

A.2 Proof of Lemma 2

Proof. First, note that for $t \geq 2$, the economy is in a steady-state equilibrium. Therefore, we have labor, wages, profits, and prices for $t \geq 2$, as characterized in Lemma 1.

At $t = 1$, households solve

$$\begin{aligned} V_1(b_1, \theta_1) \quad = \quad & \max_{\{c_1, b_2, \theta_2 \geq 0\}} \left\{ u(c_1) - v(l_1) + \frac{1}{R} V_{ss}(b_2, \theta_2) \right\} \\ & \text{subject to:} \\ & c_1 + \frac{1}{R} b_2 + Q_{ss}\theta_2 = \frac{\bar{W}_1}{E_1} l_1 + b_1 + \left(\frac{\Pi_1}{E_1} + Q_{ss} \right) \theta_1 \,, \end{aligned} \tag{A.4}$$

where we have used that $Q_1 = Q_{ss}$ and that the value function is constant for $t \geq 2$, a result that follows because in a steady-state equilibrium, wages, profits, and equity prices are constant. The value for $t \geq 2$ is given by

$$V_{ss}(b_2, \theta_2) = \frac{R}{R-1} \left[u(c_{ss}) - v(l_{ss}) \right]$$

with $w_{ss} = v'(l_{ss}) / u'(c_{ss})$ and $c_{ss} = w_{ss} l_{ss} + \pi_{ss}\theta_2 + (R-1)b_2$.

From optimality with respect to b_2 and using the envelope condition, we obtain $c_1 = c_{ss}$. Using that consumption, wages, and hours are constant and that Q_{ss} satisfies (7), we obtain by iterating forward on the budget constraint that

$$c_{ss} = \frac{R-1}{R} \left(\frac{\bar{W}_1}{E_1} l_1 + \frac{\Pi_1}{E_1} \theta_1 \right) + \frac{1}{R} \left(w_{ss} l_{ss} + \pi_{ss} \theta_1 \right) + (R-1) b_1 \,,$$

Using (2) and the definition of $I(H, \Theta)$ together with market clearing $l_t = h_t$, we obtain

$$C_{ss} = I(H_1, \Theta_1) + \frac{I(H_{ss}, \Theta_1) - I(H_1, \Theta_1)}{R} + (R-1) B_1 \,, \tag{A.5}$$

where $F'(H_1) = \bar{W}_1 / E_1$. Using $c_1 = c_{ss}$ and replacing (A.5) in the budget constraint (A.4), we obtain

$$B_2 - B_1 + Q_{ss}(\Theta_2 - \Theta_1) = \frac{I(H_1, \Theta_1) - I(H_{ss}, \Theta_1)}{R} \,.$$

which is (12) in the lemma.

□

A.3 Proof of Proposition 1

Proof. The first-order condition of the problem of the central bank is

$$\frac{R}{R-1}u'\left(\mathcal{C}_{ss}\right)\frac{\partial}{\partial E_1}\mathcal{C}_{ss}-v'\left(\mathcal{H}_1\right)\frac{\partial}{\partial E_1}\mathcal{H}_1-\frac{1}{R-1}v'\left(\mathcal{H}_{ss}\right)\frac{\partial}{\partial E_1}\mathcal{H}_{ss}=0\ .$$

From (A.5) it follows that

$$\begin{aligned}\frac{\partial}{\partial E_1}\mathcal{C}_{ss} &= \frac{R-1}{R}\left[F'\left(\mathcal{H}_1\right)-\left(1-\Theta_1\right)\pi'\left(\mathcal{H}_1\right)\right]\frac{\partial}{\partial E_1}\mathcal{H}_1+\\ &\quad+\frac{1}{R}\left[F'\left(\mathcal{H}_{ss}\right)-\left(1-\Theta_1\right)\pi'\left(\mathcal{H}_{ss}\right)\right]\frac{\partial}{\partial E_1}\mathcal{H}_{ss}\ .\end{aligned}$$

Condition (18) follows from substituting the above derivative for $\frac{\partial}{\partial E_1}\mathcal{C}_{ss}$ and equation (10) into the above first-order condition. Condition (19) follows from applying the Implicit Function Theorem to equation (10) with respect to E_1 and combining the resulting equation with the above condition for $\frac{\partial}{\partial E_1}\mathcal{C}_{ss}$.

□

A.4 Proof of Corollary 1

Proof. If condition (18) is both neccesary and sufficient for characterizing the optimal exchange rate policy, then the optimal exchange rate is unique. Moreover, if labor income $F'(H_t)H_t$ is increasing in labor hours H_t, then starting from E_1 such that $\mathcal{H}_1(E_1;B_1,\Theta_1)=\mathcal{H}_{ss}(E_1;B_1,\Theta_1)$, marginally appreciating the exchange rate is welfare improving. From these results directly follows that the optimally induced labor wedge when foreigners own domestic equities is always positive. □

A.5 Proof of Lemma 3

Proof. At $t = 0$, households solve

$$\begin{aligned} V_0(b_0, \theta_0) \quad \equiv \quad & \max_{\{c_0, l_0, b_1, \theta_1\}} \left\{ u(c_0) - v(l_0) + \frac{1}{R} V_1(b_1, \theta_1) \right\}, \\ & \text{subject to:} \\ & c_0 + \frac{1}{R} b_1 + Q_0 \theta_1 = \frac{W_0}{E_0} l_0 + b_0 + \left(\frac{\Pi_0}{E_0} + Q_0 \right) \theta_0 , \end{aligned} \tag{A.6}$$

where we have already imposed that price $Q_0 = \frac{1}{R}(\Pi_1/E_1 + Q_{ss})$ is consistent with condition (7) and where initial portfolio positions $\{b_0, \theta_0\}$ are exogenous and thus taken as given. The value of households at $t = 1$ is given by

$$V_1(b_1, \theta_1) = u(c_1) - v(l_1) + \frac{1}{R-1} \left[u(c_{ss}) - v(l_{ss}) \right]$$

with $\bar{W}_1/E_1 = F'(l_1)$, $w_{ss} = v'(l_{ss})/u'(c_{ss})$ and

$$c_1 = c_{ss} = \frac{R-1}{R} \left(\frac{\bar{W}_1}{E_1} l_1 + \frac{\Pi_1}{E_1} \theta_1 \right) + \frac{1}{R} \left(w_{ss} l_{ss} + \pi_{ss} \theta_1 \right) + (R-1) b_1 .$$

From optimality with respect to c_0 and l_0, we obtain $W_0/E_0 = v'(l_0)/u'(c_0)$. Moreover, from optimality with respect to b_1 and using the envelope condition, we obtain $c_0 = c_1 = c_{ss}$. By iterating forward on the budget constraint it then follows that

$$c_0 = \frac{R-1}{R} \left(\frac{W_0}{E_0} l_0 + \frac{\Pi_0}{E_0} \theta_0 + b_0 \right) + \frac{1}{R} \left[\frac{R-1}{R} \left(\frac{\bar{W}_1}{E_1} l_1 + \frac{\Pi_1}{E_1} \theta_0 \right) + \frac{1}{R} \left(w_{ss} l_{ss} + \pi_{ss} \theta_0 \right) \right],$$

with $W_0/E_0 = v'(l_0)/u'(c_0)$, $\bar{W}_1/E_1 = F'(l_1)$, and $w_{ss} = v'(l_{ss})/u'(c_0)$.

Using (2) and the definition of $I(H, \Theta)$ together with market clearing $l_t = h_t$, we obtain $H_0 = H_{ss}$ and

$$C_{ss} = I(H_{ss}, \Theta_0) + \frac{R-1}{R} \frac{I(H_1, \Theta_0) - I(H_{ss}, \Theta_0)}{R} + (R-1) B_0, \tag{A.7}$$

where $F'(H_{ss}) = v'(H_{ss})/u'(C_{ss})$ and $F'(H_1) = \bar{W}_1/E_1$. Using $c_1 = c_{ss}$ and replacing (A.7) in the budget constraint (A.6), we obtain

$$B_1 - (1 - \Theta_1) Q_0 = B_0 - (1 - \Theta_0) Q_0 + \frac{R-1}{R} \frac{I(H_{ss}, \Theta_0) - I(H_1, \Theta_0)}{R}, \tag{A.8}$$

which is (20) in the lemma.

□

A.6 Proof of Proposition 2.

Proof. The first-order condition of the problem is

$$\frac{R}{R-1}u'\left(\mathcal{C}_{ss}\right)\frac{\partial}{\partial\Theta_1}\mathcal{C}_{ss}-\left[\left(\frac{R}{R-1}-\frac{1}{R}\right)v'\left(\mathcal{H}_{ss}\right)\frac{\partial}{\partial\Theta_1}\mathcal{H}_{ss}+\frac{1}{R}v'\left(\mathcal{H}_1\right)\frac{\partial}{\partial\Theta_1}\mathcal{H}_1\right]=0.$$

From (A.7) it follows that

$$\frac{\partial}{\partial\Theta_1}\mathcal{C}_{ss}=I_H\left(\mathcal{H}_{ss},\Theta_0\right)\frac{\partial}{\partial\Theta_1}\mathcal{H}_{ss}+\frac{1}{R}\frac{R-1}{R}\left[I_H\left(\mathcal{H}_1,\Theta_0\right)\frac{\partial}{\partial\Theta_1}\mathcal{H}_1-I_H\left(\mathcal{H}_{ss},\Theta_0\right)\frac{\partial}{\partial\Theta_1}\mathcal{H}_{ss}\right],$$

where recall that

$$I_H\left(H_t,\Theta_t\right)=F'\left(H_t\right)-\left(1-\Theta_t\right)\pi'\left(H_t\right).$$

Condition (23) follows from substituting the above derivative for $\frac{\partial}{\partial\Theta_1}\mathcal{C}_{ss}$ and equation (10) into the above first-order condition. Condition (24) follows from applying the Implicit Function Theorem to equation (10) with respect to Θ_1 and combining the resulting equation with the above condition for $\frac{\partial}{\partial\Theta_1}\mathcal{C}_{ss}$. □

A.7 Proof of Corollary 2.

Proof. Assume condition (23) is both necessary and sufficient for optimality and labor income $F'\left(H_t\right)H_t$ is increasing in labor hours H_t. Consider $\Theta_0<1$. Then,

1. $\Theta_1=1$ is not optimal, because otherwise conditions (18) and (23) would not simultaneously hold.

2. $\Theta_1\leq\Theta_0$ is not optimal either, because from combining conditions (18) and (23), it follows that
$$\frac{1-\Theta_1}{1-\Theta_0}<1\Leftrightarrow R>1.$$

□

A.8 Equilibrium derivation in Extension 5.1

First, we derive the continuation equilibrium at period $t = 1$. If at $t = 1$ given $B_1 = -\underline{b}$ and Θ_1 the debt limit is slack, then allocation $\{H_1, H_{ss}, C_1\}$ and exchange rate E_1 are jointly determined by the conditions in lemma 2 and proposition 1. The debt limit is slack if there exists B_{t+1} such that $B_{t+1} \leq -\underline{b}$. Otherwise, if that condition does not hold, allocation $\{H_1, H_{ss}, C_1, C_{ss}\}$ and exchange rate E_1 are jointly determined

$$\begin{aligned}
C_1 &= I\left(H_1, \Theta_1\right) - \frac{R-1}{R}\underline{b} + Q_{ss}\left(\Theta_1 - \underline{\theta}\right) < C_{ss}, \\
C_{ss} &= I\left(H_{ss}, \underline{\theta}\right) - \frac{R-1}{R}\underline{b}, \\
\frac{\overline{W}_1}{E_1} &= F'\left(H_1\right), \\
\frac{v'\left(H_{ss}\right)}{u'\left(C_{ss}\right)} &= F'\left(H_{ss}\right), \\
0 &= F'\left(H_1\right) - \frac{v'\left(H_1\right)}{u'\left(C_1\right)} - \left(1 - \Theta_1\right)\pi'\left(H_1\right),
\end{aligned}$$

where the first-order condition associated with the determination of the optimal exchange rate policy (i.e., the last equation above) is static, because allocation $\{H_{ss}, C_{ss}\}$ does not depend on Θ_1.

We now turn to the equilibrium at period $t = 0$. The characterization of the equilibrium allocation in the lemma follows directly from imposing a binding debt limit $B_1 = -\underline{b}$.

A.9 Proof of Proposition 3

The proofs proceeds according to the following three steps:

1. Solve the problem of households in the continuation equilibrium at period $t = 2$ and derive that continuation equilibrium.

2. Do the same as in Step 1. but for the continuation equilibrium at period $t = 1$.

3. Solve the problem of the central bank in the continuation equilibrium at period $t = 1$.

Step 1. Let $V_{ss}\left(b_2;\theta_2^N,\theta_2^T\right)$ be the value at the beginning of the period of a household with bonds b_2 and pair of firms' shares $\left(\theta_2^N,\theta_2^T\right)$. This value satisfies

$$\begin{aligned} V_{ss}\left(b_2;\theta_2^N,\theta_2^T\right) \equiv & \max_{\left\{c_N,c_T,l,b',\theta'_N,\theta'_T\right\}} \left\{u_N\left(c_N\right)+u_T\left(c_T\right)-v\left(l\right)+\frac{1}{R}V_{ss}\left(b';\theta'_N,\theta'_T\right)\right\} \\ & \text{subject to:} \\ & \sum_j p_{ss}^j c_j+\frac{1}{R}b'+\sum_j Q_{ss}^j\theta'_j = w_{ss}l+b_2+\sum_j\left(\pi_{ss}^j+Q_{ss}^j\right)\theta_2^j \quad \text{(A.9)} \end{aligned}$$

where $P_t^j/E_t = p_{ss}^j$, $W_t/E_t = w_{ss}$, $\Pi_t/E_t = \pi_{ss}$ and $Q_t = Q_{ss}$ $\forall t \geq 2$ hold because of stationarity and where $V_t\left(\cdot;\cdot\right) = V_{ss}\left(\cdot;\cdot\right)$ $\forall t \geq 2$ is for the moment conjectured but later verified. Price p_{ss}^T is already given by $p_{ss}^T = 1$. Value $V_{ss}\left(\cdot;\cdot\right)$ already incorporates assumption $\beta R = 1$ and prices $Q_{ss}^j = \frac{1}{R-1}\pi_{ss}^j$ are already consistent with pricing condition (7). For convenience the notation is such that $u_j\left(\cdot\right) = \omega^j u\left(\cdot\right)$.

At the solution to the optimization problem, households consume the goods according to $u'_N(c_N)/u'_T(c_T) = p_{ss}^N$ and supply labor hours according to $v'(l)/u'_T(c_T) = w_{ss}$. Moreover, households are indifferent between holding bonds b' and holding firms' shares θ'_N or θ'_T, as well as they perfectly smooth consumption of both goods over time. These results follow from the FOCs and the envelope condition of the problem. Thus, the solution is a vector $\{c_{ss}^N, c_{ss}^T, l_{ss}, b_{ss}, \theta_{ss}^N, \theta_{ss}^T\}$, with

$$c_{ss}^T = w_{ss}l_{ss}-p_{ss}^N c_{ss}^N+\sum_j \pi_{ss}^j\theta_2^j+\frac{R-1}{R}b_2\ , \quad \frac{u'_N\left(c_{ss}^N\right)}{u'_T\left(c_{ss}^T\right)} = p_{ss}^N\ , \quad \frac{v'\left(l_{ss}\right)}{u'_T\left(c_{ss}^T\right)} = w_{ss}\ ,$$

where $(b',\theta'_N,\theta'_T) = (b_{ss},\theta_{ss}^N,\theta_{ss}^T)$ is any portfolio position that satisfies budget constraint (A.9) given $c_j = c_{ss}^j$ and $l = l_{ss}$. Value $V_{ss}(\cdot;\cdot)$ is given by

$$V_{ss}\left(b_2;\theta_2^N,\theta_2^T\right) = \frac{R}{R-1}\left[u_N\left(c_{ss}^N\right)+u_T\left(c_{ss}^T\right)-v\left(l_{ss}\right)\right]\ .$$

Combining these optimality conditions with firm optimality conditions $p_{ss}^j F'_j\left(h_{ss}^j\right) = w_{ss}$ and with market clearing conditions $L_{ss} = H_{ss}^N + H_{ss}^T$ and $C_{ss}^N = F_N\left(H_{ss}^N\right)$, one gets that in equilibrium, aggregate consumption of the tradable good is given by

$$C_{ss}^T = Y_{ss}\left(\Theta_2^N,\Theta_2^T\right)+\frac{R-1}{R}B_2\ ,$$

where $Y_t\left(\Theta^N,\Theta^T\right)$ is defined as

$$Y_t\left(\Theta^N,\Theta^T\right)\equiv F_T\left(H_t^T\right)-\sum_j\left(1-\Theta^j\right)\underbrace{\frac{P_t^j}{E_t}\left[F_j\left(H_t^j\right)-F_j'\left(H_t^j\right)H_t^j\right]}_{=\Pi_t^j/E_t},$$

and where the following conditions hold:

$$\begin{aligned}
&p_{ss}^T=1\,,\\
&p_{ss}^N=\frac{u_N'\left[F_N\left(H_{ss}^N\right)\right]}{u_T'\left(C_{ss}^T\right)}\,,\\
&\frac{v'\left(H_{ss}^N+H_{ss}^T\right)}{u_T'\left(C_{ss}^T\right)}=p_{ss}^jF_j'\left(H_{ss}^j\right)\,.
\end{aligned}$$

The set of admissible portfolio positions is characterized by

$$\frac{1}{R}\left(B_{ss}-B_2\right)+\sum_j\underbrace{\frac{1}{R-1}\pi_{ss}^j}_{=Q_{ss}^j}\left(\Theta_{ss}^j-\Theta_2^j\right)=0\,,\ \text{with}\ \ B_{ss}\geq-\underline{b}$$

Step 2. Let $V_1\left(\cdot;\cdot\right)$ be the counterpart value at period $t=1$ to value $V_{ss}\left(\cdot;\cdot\right)$. The former value satisfies

$$\begin{aligned}
V_1\left(b_1;\theta_1^N,\theta_1^T\right)\ \equiv\ &\max_{\left\{c_1^N,c_1^T,b_2,\theta_2^N,\theta_2^T\right\}}\left\{u_N\left(c_1^N\right)+u_T\left(c_1^T\right)-v\left(l_1\right)+\frac{1}{R}V_{ss}\left(b_2;\theta_2^N,\theta_2^T\right)\right\}\\
&\text{subject to:}\\
&\sum_j\frac{P_1^j}{E_1}c_1^j+\frac{1}{R}b_2+\sum_jQ_{ss}^j\theta_2^j=\frac{\bar{W}_1}{E_1}l_1+b_1+\sum_j\left(\frac{\Pi_1^j}{E_1}+Q_{ss}^j\right)\theta_1^j\qquad\text{(A.10)}
\end{aligned}$$

This value already incorporates both results $V_2\left(\cdot;\cdot\right)=V_{ss}\left(\cdot;\cdot\right)$ and $Q_1^j=\frac{1}{R}\left(\pi_{ss}^j+Q_{ss}^j\right)=Q_{ss}^j$ as well as the slackness of the financing limits. Labor hours l_1 are taken as given in the optimization problem on the RHS because nominal wages $\bar{W}_1$ are rigid.

At the solution to the optimization problem, households consume the goods according to $u_N'(c_1^N)/u_T'(c_1^T)=P_1^N/E_1$. Moreover, households are indifferent between holding bonds b_2 and holding firms' shares θ_2^N or θ_2^T, as well as they perfectly smooth consumption of the tradable good over time. Therefore, $c_1^T=c_{ss}^T$, which implies

$$c_1^T=\frac{R-1}{R}\left(\frac{\bar{W}_1}{E_1}l_1-\frac{P_1^N}{E_1}c_1^N+\sum_j\frac{\Pi_1^j}{E_1}\theta_1^j+b_1\right)+\frac{1}{R}\left(w_{ss}n_{ss}-p_{ss}^Nc_{ss}^N+\sum_j\pi_{ss}^j\theta_1^j\right)\,,\qquad\text{(A.11)}$$

where $\{c_1^N, c_{ss}^N, n_{ss}\}$ are given by $u_N'(c_1^N)/u_T'(c_1^T) = P_1^N/E_1$, $u_N'(c_{ss}^N)/u_T'(c_1^T) = p_{ss}^N$, and $v'(n_{ss})/u_T'(c_1^T) = w_{ss}$. Any portfolio position $\{b_2, \theta_2^N, \theta_2^T\}$ that is consistent with both budget constraint (A.10) and consumption rule (A.11) is optimal. Value $V_1(\cdot;\cdot)$ can then be reduced to

$$V_1\left(b_1; \theta_1^N, \theta_1^T\right) = \frac{R}{R-1} u_T\left(c_1^T\right) + u_N\left(c_1^N\right) - v\left(l_1\right) + \frac{1}{R-1}\left[u_N\left(c_{ss}^N\right) - v\left(n_{ss}\right)\right].$$

Combining these optimality conditions with firm optimality conditions $P_1^j F_j'\left(h_1^j\right) = \bar{W}_1$ and market clearing conditions $L_1 = H_1^N + H_1^T$ and $C_1^N = F_N\left(H_1^N\right)$, one gets that in equilibrium aggregate consumption of the tradable good is given by

$$C_1^T = C_{ss}^T = Y_1\left(\Theta_1^N, \Theta_1^T\right) + \frac{Y_{ss}\left(\Theta_1^N, \Theta_1^T\right) - Y_1\left(\Theta_1^N, \Theta_1^T\right)}{R} + \frac{R-1}{R} B_1\,, \tag{A.12}$$

where the following conditions hold:

$$\frac{P_t^T}{E_t} = 1\,, \tag{A.13}$$

$$\frac{P_t^N}{E_t} = \frac{u_N'\left[F_N\left(H_t^N\right)\right]}{u_T'\left(C_1^T\right)}\,, \tag{A.14}$$

$$\left(1 + S_1^j\right)\frac{P_1^j}{E_1} F_j'\left(H_1^j\right) = \frac{\bar{W}_1}{E_1}\,, \tag{A.15}$$

$$\frac{v'\left(H_{ss}^N + H_{ss}^T\right)}{u_T'\left(C_1^T\right)} = p_{ss}^j F_j'\left(H_{ss}^j\right). \tag{A.16}$$

Any aggregate portfolio position $\{B_2, \Theta_2^N, \Theta_2^T\}$ that is consistent with both $B_2 \geq -\underline{b}$ and $\Theta_2^j \geq \underline{\theta}$ and that in addition satisfies the following current-account condition is admissible:

$$\frac{1}{R}\left(B_2 - B_1\right) + \sum_j \underbrace{\frac{1}{R-1}\pi_{ss}^j}_{=Q_{ss}^j}\left(\Theta_2^j - \Theta_1^j\right) = \frac{Y_1\left(\Theta_1^N, \Theta_1^T\right) - Y_{ss}\left(\Theta_1^N, \Theta_1^T\right)}{R}\,. \tag{A.17}$$

For any given exchage rate $E_1 > 0$ this continuation equilibrium admits a unique allocation but multiple portfolio positions.

Step 3. In the continuation equilibrium at period $t = 1$ the central bank solves

$$\max_{E_1}\left\{\frac{R}{R-1} u_T\left(C_1^T\right) + u_N\left[F_N\left(H_1^N\right)\right] - v\left(H_1^N + H_1^T\right) + \frac{u_N\left[F_N\left(H_{ss}^N\right)\right] - v\left(H_{ss}^N + H_{ss}^T\right)}{R-1}\right\},$$

where equilibrium quantities $\{C_1^T, H_1^j, H_{ss}^j\}$ should be regarded here as functions of exchange rate E_1—as specified by formulae (A.12)-(A.17).

The first-order condition of the problem is

$$\begin{aligned} 0 &= \frac{R}{R-1}u_T'\left(C_1^T\right)\frac{\partial C_1^T}{\partial E_1} + u_N'\left[F_N\left(H_1^N\right)\right]F_N'\left(H_1^N\right)\frac{\partial H_1^N}{\partial E_1} - v'\left(H_1^N + H_1^T\right)\left(\frac{\partial H_1^N}{\partial E_1} + \frac{\partial H_1^T}{\partial E_1}\right) \\ &\quad + \frac{1}{R-1}\left\{u_N'\left[F_N\left(H_{ss}^N\right)\right]F_N'\left(H_{ss}^N\right)\frac{\partial H_{ss}^N}{\partial E_1} - v'\left(H_{ss}^N + H_{ss}^T\right)\left(\frac{\partial H_{ss}^N}{\partial E_1} + \frac{\partial H_{ss}^T}{\partial E_1}\right)\right\}, \end{aligned} \tag{A.18}$$

where

$$\begin{aligned} \frac{\partial C_1^T}{\partial E_1} &= \frac{R-1}{R}\left[F_T'\left(H_1^T\right)\frac{\partial H_1^T}{\partial E_1} - \sum_j\left(1-\Theta_1^j\right)\frac{\partial\left(\Pi_1^j/E_1\right)}{\partial E_1}\right] + \\ &\quad + \frac{1}{R}\left[F_T'\left(H_{ss}^T\right)\frac{\partial H_{ss}^T}{\partial E_1} - \sum_j\left(1-\Theta_1^j\right)\frac{\partial\left(\Pi_{ss}^j/E_1\right)}{\partial E_1}\right]. \end{aligned} \tag{A.19}$$

Substituting $\{(A.13), (A.14), (A.16), (A.19)\}$ into (A.18) one gets

$$\sum_j \frac{P_1^j}{E_1}\left[F_j'\left(H_1^j\right) - \frac{v'\left(H_1^j + H_1^{-j}\right)}{u_j'\left(C_1^j\right)}\right]\frac{\partial H_1^j}{\partial E_1} = \sum_j\left(1-\Theta_1^j\right)\left[\frac{\partial\left(\Pi_1^j/E_1\right)}{\partial E_1} + \frac{1}{R-1}\frac{\partial\left(\Pi_{ss}^j/E_{ss}\right)}{\partial E_1}\right].$$