Sufficient Statistics for Measuring Forward-Looking Welfare

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Measuring Welfare

- Standard welfare measure converts income at τ into equivalent income in τ_0 dollars.
- For static decision makers, this is income at τ deflated by average price change.
- Foundation for real quantities in national accounts (e.g. real income) & cost of living.
- Dynamic stochastic decision maker's welfare depends on future uncertain outcomes.
- Requires future state-contingent constraints (prices & incomes), probabilities, plans.
- Such welfare measures typically calculated using fully specified structural models.

Theory Ahead of Measurement

- Sufficient statistics formulas of index number theory unusable for dynamic problems.
- This tension is a classic problem dating back to Fisher & Pigou.
- For example, Samuelson (1961) concludes with:

> *"The futures prices needed for making the requisite wealth-like comparisons are simply unavailable. So it would be difficult to make operational the theorists' desired measures."*

What We Do: Theory

- Develop sufficient-statistics for welfare in three steps:
  - For unconstrained household:
    1. use changes in savings to learn about expectations about future;
    2. obtain compensated savings by matching different households over time.
  - For constrained households:
    3. use non-homotheticity to match constrained and unconstrained by budget shares.
- Key assumption: preferences are “separable” between the present and future.
- Non-parametric method allowing for life-cycle motives & non-exponential discounting.
- Do not specify beliefs or preferences about state-contingent prices and returns.

What We Do: Empirical Illustration

- Apply our methodology to the US using the PSID.
- Compute dynamic welfare and cost-of-living measures between 2005 – 2019:
  - Dynamic welfare measures can be very different to static ones.
  - Dynamic non-homotheticities an order of magnitude more important than static ones.
- Welfare measure can be used as outcome in reduced-form work:
  - Job loss associated with 20% reduction in welfare.

Related Literature

- Static literature: Feenstra (1994), Hamilton (2001), Costa (2001), Almås (2012), Atkin et al. (2024), Jaravel and Lashkari (2024), Baqaee et al. (2024).

- Older dynamic literature: Samuelson (1961), Alchian and Klein (1973), Pollack (1975), Hulten (1979).

- More recent literature: Reis (2005), Aoki and Kitahara (2010), Jones and Klenow (2016), Basu et al. (2022), Del Canto et al. (2023), Fagereng et al. (2022).

Agenda

- Stripped-down Example
- More General Environment
- Solving Money Metric for Rentiers
- Solving Money Metric for Non-Rentiers
- Empirical Illustration Using PSID data
- Conclusion

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Complete markets with CRRA homothetic preferences

- Intertemporal preferences for planning horizon J:

$$\mathcal{U}(\mathbf{c}, \boldsymbol{\pi}) = \sum_{j=0}^{J} \sum_{s_j} \beta(s^j)\pi(s^j) \frac{c(s^j)^{1-1/\sigma}}{1-1/\sigma}, \qquad \text{where} \qquad \sigma \neq 1.$$

- Subject to

$$\begin{aligned} p(s^0)c(s^0) + \sum_{k \in S_1} a_k(s^0) &= w, \\ p(s^j)c(s^j) + \sum_{k \in S_{t+1}} a_k(s^j) &= R_k(s^{j-1})a_k(s^{j-1}), \\ p(s^J)c(s^J) &\leq R_k(s^{J-1})a_k(s^{J-1}). \end{aligned}$$

- Value function $V(\{\mathbf{p}, \mathbf{R}, \boldsymbol{\pi}\}, w)$ given preference parameters J, $\boldsymbol{\beta}$, and σ.

Money Metric Utility

- Cohorts with horizon J at each date τ facing $p(\cdot|\tau)$, $R(\cdot|\tau)$, and $\pi(\cdot|\tau)$.
- Index each cohort's problem by start date τ, i.e. value function is $V(\tau, w)$.
- Money metric utility $m(\tau, w|\tau_0)$ is wealth in τ_0 that gives same utility:

$$V(\tau, w) = V(\tau_0, m(\tau, w|\tau_0)).$$

- Use $m(\tau, w|\tau_0)$ to measure growth (by varying τ & w) or cost-of-living (by varying τ_0).
- Write $m(\tau, w)$ instead of $m(\tau, w|\tau_0)$ from now (hold base year τ_0 constant).

Towards Solution

- Define $c\left(s^j|\tau, w\right)$ to be consumption of cohort τ with initial wealth w in state s^j.
- Money metric is also Lucas-number relative to consumption in τ_0 with unit wealth:

$$\mathcal{U}(m(\tau, w) \times \mathbf{c}(\cdot|\tau_0, 1)) = \mathcal{U}(\mathbf{c}(\cdot|\tau, w)).$$

- Combine with Euler equations and intertemporal budget constraint:

$$m(\tau, w) = w \Bigg/ \left[\frac{\sum_{j=0}^{J}\sum_{s_j}\left(\beta(s^j)\pi(s^j|\tau)\right)^{\sigma} p\left(s^j|\tau\right)^{1-\sigma}\prod_{l=0}^{j} R_{s_{l+1}}\left(s^l|\tau\right)^{\sigma-1}}{\sum_{j=0}^{J}\sum_{s_j}\left(\beta(s^j)\pi(s^j|\tau_0)\right)^{\sigma} p\left(s^j|\tau_0\right)^{1-\sigma}\prod_{l=0}^{j} R_{s_{l+1}}\left(s^l|\tau_0\right)^{\sigma-1}}\right]^{\frac{1}{1-\sigma}}.$$

- Need beliefs about discounting, probabilities, prices & returns.

Using Consumption-Savings Decisions to Back-Out Future Prices

- Denote consumption-wealth ratio by

$$B^P(\tau, w) = B^P(\tau) = \frac{p\left(s^0|\tau\right) c\left(s^0|\tau, w\right)}{w}.$$

- Consumption-wealth ratio satisfies

$$B^P(\tau) = \frac{p(s^0)^{1-\sigma}}{\sum_{j=0}^{J} \sum_{s_j} \left(\beta(s^j)\pi(s^j|\tau)\right)^{\sigma} p(s^j|\tau)^{1-\sigma} \prod_{l=0}^{j} R_{s_{l+1}}(s^l|\tau)^{\sigma-1}}.$$

- Plug into previous expression to get

$$\log m(\tau, w) = \underbrace{\log w - \log \frac{p\left(s^0|\tau\right)}{p(s^0|\tau_0)}}_{\text{“real wealth”}} + \underbrace{\frac{1}{1-\sigma} \log \frac{B^p(\tau)}{B^P(\tau_0)}}_{\substack{\text{adjustment for} \\ \text{future}}}.$$

- If savings rate falls with $\sigma < 1 \Rightarrow$ more optimistic.

Stripping down even more

- Consumption savings problem

$$U = \frac{1}{1-1/\sigma}\left(c_0^{1-1/\sigma} + \beta c_1^{1-1/\sigma}\right), \qquad \text{and} \qquad p_0 c_0 + p_1 c_1 / R = w.$$

- Consider change in wealth, prices, and returns from $(w, \mathbf{p}, R)$ to $(w', \mathbf{p}', R')$.
- Consumption equivalent is λ that solves:

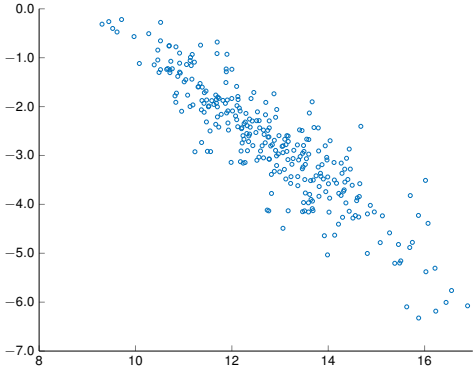
$$(\lambda c_0)^{1-1/\sigma} + \beta(\lambda c_1)^{1-1/\sigma} = (c_0')^{1-1/\sigma} + \beta(c_1')^{1-1/\sigma}.$$

- Using Euler equation $c_0^{-1/\sigma}/p_0 = \beta R c_1^{-1/\sigma}/p_1$ and budget constraint,

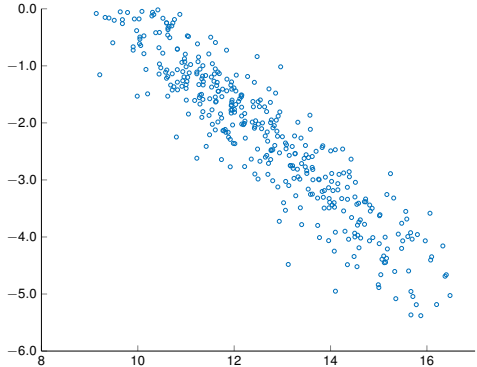
$$\lambda = \frac{w'}{w} \times \frac{p_0}{p_0'} \times \left(\frac{1 + \beta^\sigma \left(\frac{p_1/R}{p_0}\right)^{1-\sigma}}{1 + \beta^\sigma \left(\frac{p_1'/R'}{p_0'}\right)^{1-\sigma}} \right)^{\frac{1}{1-\sigma}} = \frac{w'}{w} \times \frac{p_0}{p_0'} \times \left(\frac{p_0' c_0' / w'}{p_0 c_0 / w} \right)^{\frac{1}{1-\sigma}}.$$

- Money metric is λw. Do not need to know R, p_1,R', p_1' or β.

Log consumption-wealth against log wealth



(a) 2005



(b) 2019

- Consumption-wealth ratio strongly declining in wealth (as in Straub 2019).
- Whose changes in consumption-wealth ratio do we use?

Agenda

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Time Separable Preferences

- Preferences rank consumption streams over consumption stream $\mathbf{c}$ with beliefs $\boldsymbol{\pi}$.
- Assume preferences can be represented as

$$U = D(c(s^0), F(\{c(s^j)\}_{j>0}, \{\pi(s^j)\}_{j>0}, U)),$$

  where F is homogeneous function in $\{c(s^j)\}_{j>0}$.
- Homothetic, then F and D homog. of degree 1 and F independent of U.

Some Examples Satisfying Our Assumption

- Non-homothetic patience

$$U = \frac{c(s^0)^{1-\sigma}}{1-\sigma} + \sum_{j=1}^{J} \beta_j(U) \frac{c(s^j)^{1-\sigma}}{1-\sigma}.$$

- Non-homothetic risk-aversion

$$U = \frac{c(s^0)^{1-\sigma}}{1-\sigma} + \sum_{j=1}^{J} \beta^t \frac{\left[\sum_{s^j} \pi(s^j) c(s^j)^{\gamma(U)}\right]^{\frac{1-\sigma}{\gamma(U)}}}{1-\sigma}.$$

- Non-homothetic intertemporal elasticity of substitution

$$U = \frac{c(s^0)^{1-\sigma(U)}}{1-\sigma(U)} + \sum_{j=1}^{J} \beta^j \frac{c(s^j)^{1-\sigma(U)}}{1-\sigma(U)}.$$

- Key: relative future spending shares not function of consumption today (given U).

Some Examples Violating Our Assumption

- Dynamic Stone-Geary:

$$U = \frac{(c(s^0) + a_0)^{1-\sigma}}{1-\sigma} + \sum_{j=1}^{J} \beta^j \frac{(c(s^j) + a_j)^{1-\sigma}}{1-\sigma}.$$

- Dynamic Addilog:

$$U = \frac{c(s^0)^{1-\sigma_0}}{1-\sigma_0} + \sum_{j=1}^{J} \beta^j \frac{c(s^j)^{1-\sigma_j}}{1-\sigma_j}.$$

Adjusting to Satisfy Our Assumption

- However, versions of these preferences can be nested.
- e.g. Pseudo-Stone-Geary works:

$$U = \frac{(c(s^0) + a_0)^{1-\sigma}}{1-\sigma} + \frac{\left[\left(\sum_{j=1}^{J} \beta_j c(s^j)^{1-\sigma}\right)^{\frac{1}{1-\sigma}} + a_1\right]^{1-\sigma}}{1-\sigma}.$$

- Pseudo-Addilog works:

$$U = \frac{c(s^0)^{1-\sigma_0}}{1-\sigma_0} + \frac{F\left(\left\{c(s^j)\right\}_{j>0}\right)^{1-\sigma_1}}{1-\sigma_1},$$

where F is H.O.D 1. (implicit addilog also works).

Environment

- First period budget constraint:

$$p(s^0|\tau)c(s^0|\tau) + \sum_{k \in K} a_k(s^0|\tau) = w.$$

- Each subsequent history s^j:

$$p(s^j|\tau)c_n(s^j|\tau) + \sum_{k \in K} a_k(s^j|\tau) = \sum_{k \in K} R_k(s^j|\tau)a_k(s^{j-1}|\tau) + y(s^j|\tau),$$

  where $y(s^j|\tau)$ is payment from assets that cannot be traded.

- Borrowing constraints

$$\sum_k a_k(s^j|\tau) \geq -X(s^j|\tau).$$

  No-ponzi requires $X(s^J) = 0$ for every s^J.

- *Rentiers*: $y(s^j|\tau) = 0$ for all s^j. (Extends to risk free $y(s^j|\tau)$ with more assumptions.)

Dynamic Money Metric Utility

- Value function is

$$V(\underbrace{\{\mathbf{p},\mathbf{R},\boldsymbol{\pi},\mathbf{X}\}}_{\text{indexed by } \tau}, w, \mathbf{y}) = \max_{\mathbf{c},\mathbf{a}} \{\mathcal{U}(\{\mathbf{c},\boldsymbol{\pi}\}) : \text{constraints satisfied}\}.$$

- With incomplete markets, no single budget constraint, many possible compensations.

- Money metric of decision problem $(\tau, w, \mathbf{y})$ in τ_0 dollars is $m(\tau, w, \mathbf{y})$ solving

$$V(\tau, w, \mathbf{y}) = V(\tau_0, m(\tau, w, \mathbf{y}), \mathbf{0}).$$

Equivalent lump sum in τ_0, rentier thereafter, to make agent indifferent.

Agenda

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Notation and Data Requirements

- Consumption to wealth ratio:

$$B^P(\tau, w, y) = \frac{p(s^0|\tau)c(s^0|\tau, w, y)}{w}.$$

- EIS — log change in $\frac{B^P}{1-B^P}$ to uniform change in present prices: $\sigma(\tau, w, y)$.
- We observe *cross-section* of w, B^P, and $p(s^0|t)$ for each $t \in [\tau_0, \tau]$.
- Subset of households are rentiers, denoted by $(t, w, \mathbf{0})$.
- Consider special cases that build to general result.

Special Case: Homothetic Rentiers

Proposition

For homothetic rentiers with constant EIS,

$$\log m(\tau, w, \mathbf{0}) = \underbrace{\log w - \log \frac{p(s^0|\tau)}{p(s^0|\tau_0)}}_{\text{“real wealth”}} + \underbrace{\frac{\log\left(B^P(\tau, \mathbf{0})/B^P(\tau_0, \mathbf{0})\right)}{1-\sigma}}_{\substack{\text{adjustment for} \\ \text{future}}}.$$

- Generalizes simple example for incomplete markets & more general preferences.

Non-Homothetic Rentiers with Constant EIS

Proposition

Money metric is solution to the fixed point problem:

$$\log m(\tau, w, \mathbf{0}) = \underbrace{\log w - \log \frac{p(s^0|\tau)}{p(s^0|\tau_0)}}_{\text{real wealth}} + \underbrace{\frac{\log\left(B^P(\tau, w_\tau, \mathbf{0})/B^P(\tau_0, w^*_{\tau_0}, \mathbf{0})\right)}{1-\sigma}}_{\substack{\text{adjustment for} \\ \text{future}}},$$

where $w^*_{\tau_0}$ satisfies the equation

$$m(\tau_0, w^*_{\tau_0}, \mathbf{0}) = m(\tau, w, \mathbf{0}).$$

- Must use compensated $\Delta \log B^P$.
- Infer from observed B^P of matched household w^*_t (Baqaee et al., 2024).

Non-Homothetic Rentiers with non-Constant EIS

Proposition

Money metric is solution to the fixed point problem:

$$\log m(\tau, w, \mathbf{0}) = \underbrace{\log w - \log \frac{p(s^0|\tau)}{p(s^0|\tau_0)}}_{\text{real wealth}} + \underbrace{\int_{\tau_0}^{\tau} \frac{d \log B^P(t, w_t^*, \mathbf{0})/dt}{1 - \sigma(t, w_t^*, \mathbf{0})} dt}_{\text{adjustment for future}},$$

where for each $t \in [\tau_0, \tau]$, w_t^* *satisfies the equation*

$$m(t, w_t^*, \mathbf{0}) = m(\tau, w, \mathbf{0}).$$

- Must use compensated $\frac{d \log B^P(t, w_t^*, \mathbf{0})}{dt}$ and EIS, obtained from matched household w_t^*.

Agenda

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Introduce Multiple Goods within Period

- Extend analysis to allow for multiple goods within each period.
- Assume present spending shares depend *only* on today's prices and utility,

$$b_n(p_1(s^0), \ldots, p_N(s^0), U).$$

- e.g.: non-homothetic CES:

$$\frac{p_n c_n}{\sum_{n'} p'_n c'_n} = \frac{\omega_n^\gamma U^{\varepsilon_n \gamma} p_n^{1-\gamma}}{\sum \omega_{n'}^\gamma U^{\varepsilon_{n'} \gamma} p_{n'}^{1-\gamma}}.$$

- Violated if budget shares depend also on present spending (conditional on U).

Money Metric for Rentiers with Multiple Goods

Proposition

Money metric is solution to the fixed point problem:

$$\log m(\tau, w, \mathbf{0}) = \underbrace{\log w - \int_{\tau_0}^{\tau} \sum_{n \in N} B_n(t, w_t^*, \mathbf{0}) \frac{d \log p_n}{dt} dt}_{\substack{\text{wealth deflated} \\ \text{w/ compensated spending}}} + \underbrace{\int_{\tau_0}^{\tau} \frac{d \log B^P(t, w_t^*, \mathbf{0})/dt}{1 - \sigma(t, w_t^*, \mathbf{0})} dt}_{\substack{\text{adjustment for} \\ \text{future}}},$$

where for each $t \in [\tau_0, \tau]$, w_t^* *satisfies the equation*

$$m(t, w_t^*, \mathbf{0}) = m(\tau, w, \mathbf{0}).$$

Money Metric for Non-Rentiers

Proposition

If every U maps to a different vector of budget shares (holding prices constant), then there is some function from budget shares and time into money metric utility.

- We can invert budget shares to get back money metric utility.
- e.g. dentist and landlord with same budget shares at same time are on same indifference curve.
- To obtain this inverse:
  1. Regress wealth on budget shares and time for rentiers.
  2. Use fitted relationship to impute money metric wealth for non-rentiers.

Assumption on Common Preferences, Prices, and Probabilities.

- Conditional on observables, rentiers and non-rentiers have same preferences relation both in the cross-section within each period and across cohorts at different times.
- Prices w/in a period can only vary as a function of observable characteristics.
- Future prices, returns and beliefs can change over time, but:
  - across rentiers, can only vary as a function of observable characteristics,
  - between non-rentiers and rentiers, can vary even conditional on characteristics.
- We do not require that households' beliefs about the future be "objective" in any sense.

Extension 1: Pseudo rentiers

- Consider a subset of households with risk-free cash flow $y(s^j|\tau) = y(j|\tau)$.
- For example, public sector employees, teachers, pensioners on defined benefits, etc.
- For these households, assume no ad-hoc borrowing constraints & access to bonds of maturities $\{1, \ldots, J\}$.
- These households' problem is isomorphic to rentier with augmented wealth

$$w(s^0|\tau) + \sum_{j=0}^{J} \frac{y(j|\tau)}{R(j|\tau)},$$

  where $R(j|\tau)$ is return on bond with maturity j purchased in τ.

- Do not pursue this in empirical application (for now).

Extension 2: Changes in mortality

- Let λ_P and λ_F be prob. of reaching P and F.
- Marginal willingness to pay for increasing survival probabilities: $\Phi_P(\tau, w)$, $\Phi_F(\tau, w)$.
- Money metric solves:

$$\log u(\tau, w, \mathbf{0}) = \underbrace{\log w - \int_{\tau_0}^{\tau} \left(\sum_{n \in N} B_n(t, w_t^*) \frac{d \log p_n}{dt} - \frac{1}{1-\sigma} \frac{d \log B^P(t, w_t^*)}{dt} \right) dt}_{\text{what we had before}}$$
$$+ \underbrace{\int_{\tau_0}^{\tau} \left(\Phi_P(t, w_t^*) \frac{d \log \lambda_P(t)}{dt} + \Phi_F(t, w_t^*) \frac{d \log \lambda_F(t)}{dt} \right) dt}_{\text{compensated value of increased survival}}$$
$$+ \underbrace{\int_{\tau_0}^{\tau} \frac{\sigma}{1-\sigma} \left(1 - B^P(t, w_t^*)\right) \frac{d \log \lambda_F(t)}{dt} dt}_{\text{changes in consmption/wealth ratio due to } d\lambda_F}.$$

Extension 3: Leisure

- Results unchanged for rentiers if, conditional on observables, leisure choices do not change as a function of calendar time (e.g. labor productivity = 0, or 9-to-5 job).
- Results unchanged for non-rentiers if relative static budget shares only depend on utility and static prices of goods and services.
- Rules out non-separabilities between consumption choices and leisure.
- Example:

$$U^{\frac{\sigma-1}{\sigma}} = \tilde{P}\left(c\left(s^0\right), U\right)^{\frac{\sigma-1}{\sigma}} + \tilde{F}\left(\left\{c(s^j)\right\}_{j>0}, \boldsymbol{\pi}, U\right)^{\frac{\sigma-1}{\sigma}} + \tilde{H}\left(\left\{l(s^j)\right\}_{j\geq 0}, \boldsymbol{\pi}, U\right).$$

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Data Requirements

- Household survey with financial net worth, age, consumption survey, subset of rentiers.
  (We use PSID, bi-annual from $2005 - 2019$).
  (Group age into decade of life.)

- Prices of goods and services.
  (CPI prices).

- Elasticity of intertemporal substitution $\sigma(\tau, w)$
  (use Best, Cloyne, Ilzetzki, and Kleven 2020 of $\sigma = 0.1$.)
  (if consumption is a normal good, then compensated EIS $<$ uncompensated EIS.)

Classifying Rentiers

- Proxy wealth = net assets (including DC) + discounted labor income + transfers.
- Forecast income using cross-section + CBO forecast of NGDP.
- Discount future labor income and transfers by 4% real rate (Catherine et al., 2022).
- Rentiers: Net financial assets $\geq 90\%$ of total wealth & not unemployed.
- Drop from rentier set if net assets are in the top and bottom 2.5%.

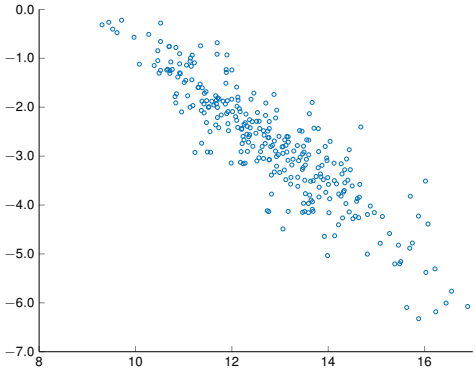
Expenditures

- Total consumption is expenditures on seven consumption categories.
  - (food, housing, transportation, education, health, clothing, and recreation)
- For home-owners, impute housing by matching owners to similar renters.
- In 2019, survey includes new question on owner-occupied housing costs.
- Regress surveyed housing costs on imputed housing costs, $\beta = 1.03$, $R^2 = 0.59$.
- Non-parametric kernel regression consumption-wealth ratio for rentiers:

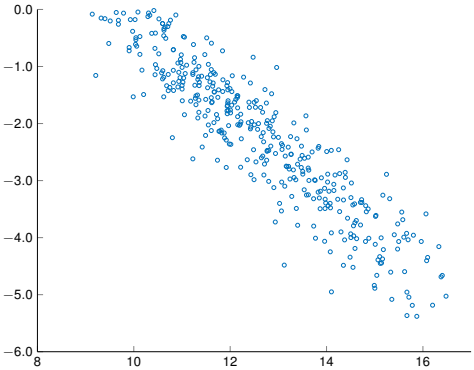
$$\log B^P_{h,\tau} = \hat{B}(\log \text{wealth}_{h,\tau}, \tau, age_{h,\tau}) + \text{error}_{h,\tau}.$$

- Static budget shares regressed on flexible function of age, wealth, and date.

Log consumption-wealth against log wealth



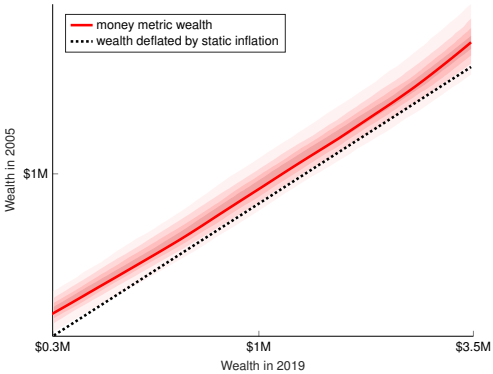
(a) 2005



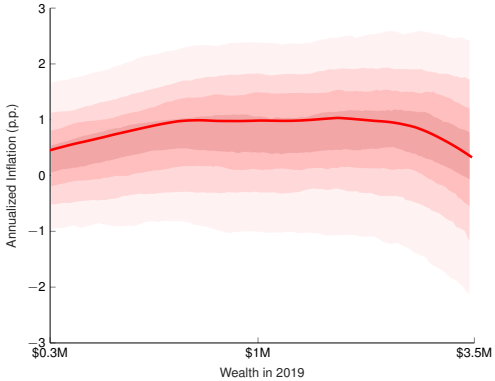
(b) 2019

- Consumption-wealth ratio strongly declining in wealth (as in Straub 2019).

Money metric wealth in 2005 base prices for 60-69 year olds



(a) Money metric

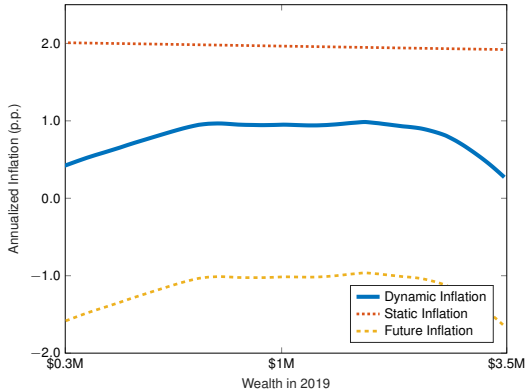


(b) Annualized inflation

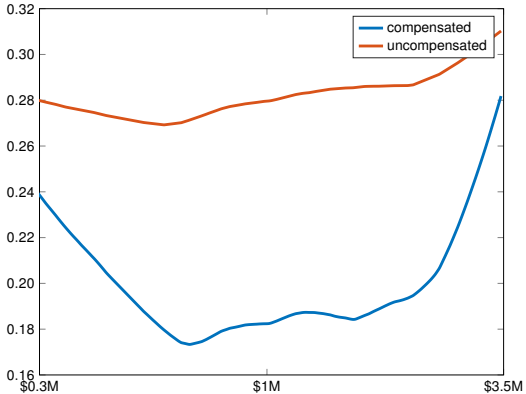
- Money metric converts wealth in 2019 into equivalent in 2005 and vice versa.
- Black line deflates nominal wealth in 2019 by the static CPI between 2005 and 2019.

Decomposing Inflation (example for 60 – 69 year olds)

$$\log \frac{w}{u_{2005}(2019, w)} = \underbrace{\int_{2005}^{2019} \sum_{n \in N} B_n(t, w_t^*, \mathbf{0}) \frac{d \log p_n}{dt} dt}_{\text{static inflation}} - \underbrace{\frac{1}{1-\sigma} \log \left(\frac{B^P(2019, w, \mathbf{0})}{B^P(2005, w_{2005}^*, \mathbf{0})} \right)}_{\text{future relative to static inflation}}.$$



Change in log consumption wealth ratios 2005 – 2019 (60 – 69 year olds)



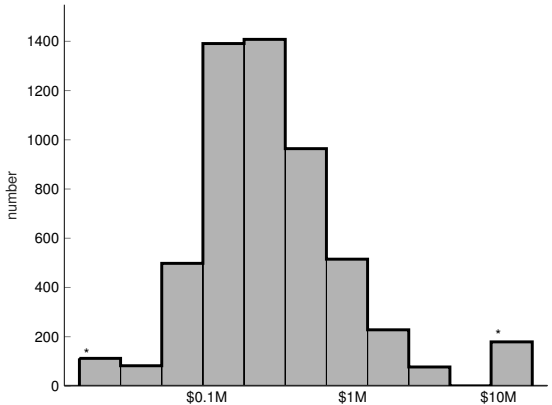
- Consumption-wealth ratios grew — but compensated less than uncompensated.

Imputing Money Metric Wealth for Non-Rentiers

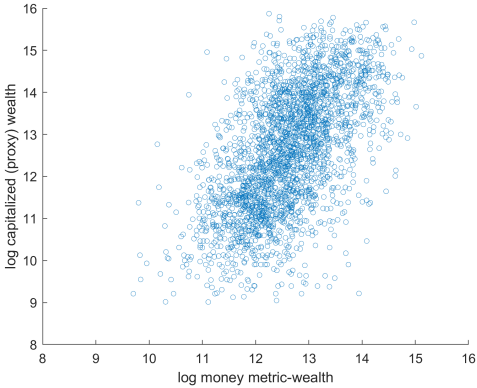
Fit rentiers' wealth to budget shares, squared budget shares, and age bin for each date:

$$\log w_{h,\tau} = \boldsymbol{\alpha}'_{\tau} X_{h,\tau} + \text{error}_{h,\tau}.$$

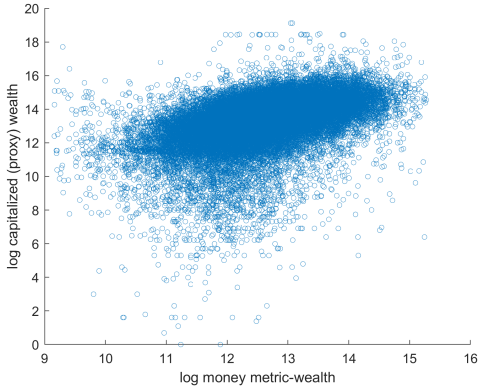
Use fitted relationship to predict money metric for non-rentiers.



Capitalized Wealth against Money Metric Wealth (in logs)



(a) Rentiers



(b) Non-rentiers

Dynamic Welfare Treatment Effect

- Consider a treatment that affects households over time in uncertain ways.
- For example, job training program, educational investment, interest rate policy, etc.
- Our money metric estimates can be used to study welfare treatment effect.
- Illustrate using job loss for head of household using PSID.

Percent change in money metric wealth due to job loss

	log nominal money metric	
	(1)	(2)
Job Loss	-0.197	-0.218
	(0.031)	(0.034)
Job loss $\times$ 1(age $\geq$ 60)		0.180
		(0.083)
Lagged LHS	Yes	Yes
Controls	Yes	Yes
Observations	48,357	48,357
	Full Sample	

Controls: year fixed effects, age group, marital status of HH head, industry, and education level.

- Budget shares map to different rentier-equivalent wealth.
- Estimates identified via difference in budget shares of job-havers vs. -losers.

Agenda

Stripped-down Example

More General Environment

Solving Money Metric for Rentiers

Solving Money Metric for Non-Rentiers

Empirical Illustration Using PSID data

Conclusion

Conclusion

- Generalize money metrics to intertemporal preferences, risk, and incomplete markets.
- Use time-separability to infer it from consumption-savings for rentiers.
- Match rentiers and non-rentiers using budget shares.
- Static and dynamic different, with heterogeneity in wealth & age.
- Ingredient for policy evaluation of shocks that affect future.

Proof Sketch

1. There exist shadow prices q^* that "rationalize" consumer's choices:

$$c_n^*(s^j|q^*, \boldsymbol{\pi}, V(\tau, w, y)) = c_n(s^j|\tau, w, y)$$

   with shadow prices for goods in first period equal to observed prices:

$$q_n^*(s^0|\tau, w, y) = p_n(s^0|\tau).$$

2. Dual shadow prices for rentiers depend on τ and V — not the case for non-rentiers.

3. Money metric is expressible using shadow intertemporal expenditure function:

$$m(\tau, w, \mathbf{0}) = e(q^*(\cdot|\tau_0, m(\tau, w, \mathbf{0})), \boldsymbol{\pi}(\cdot|\tau_0), m(\tau, w, \mathbf{0})).$$

4. Manipulate to get:

$$\log m(\tau, w, \mathbf{0}) = \log w - \log \frac{e(q^*(\cdot|\tau, m(\tau, w, \mathbf{0})), \boldsymbol{\pi}(\cdot|\tau), m(\tau, w, \mathbf{0}))}{e(q^*(\cdot|\tau_0, m(\tau, w, \mathbf{0})), \boldsymbol{\pi}(\cdot|\tau_0), m(\tau, w, \mathbf{0}))}.$$

Proof Sketch

5. Using fundamental theorem of calculus:

$$\log m(\tau, w, \mathbf{0}) = \log w + \int_{\tau}^{\tau_0} \sum_{t=0}^{J} \sum_{s^j} \left(\frac{\partial \log e(q^*\left(s^j|t, m(\tau, w, \mathbf{0})\right), \boldsymbol{\pi}(s^j|t), m(\tau, w, \mathbf{0}))}{\partial \log q^*(s^j|t, m(\tau, w, \mathbf{0}))} \cdot \frac{d \log q^*\left(s^j|t, m(\tau, w, \mathbf{0})\right)}{dt} + \frac{\partial \log e(q^*\left(s^j|t, m(\tau, w, \mathbf{0})\right), \boldsymbol{\pi}(s^j|t), m(\tau, w, \mathbf{0}))}{\partial \boldsymbol{\pi}(s^j|t)} \cdot \frac{d\boldsymbol{\pi}(s^j|t)}{dt} \right) dt.$$

6. Cut through the complexity using time-separability:

$$\frac{\partial \log e(q, \boldsymbol{\pi}, U)}{\partial \log q} \cdot d \log q + \frac{\partial \log e(q, \boldsymbol{\pi}, U)}{\partial \boldsymbol{\pi}} \cdot d\boldsymbol{\pi} = -\frac{d \log b^P(q, \boldsymbol{\pi}, U)}{1 - \sigma^*(q, \boldsymbol{\pi}, U)} + \sum_{n \in N} b_n(q(s^0), U) d \log q_n(s^0).$$

7. Substitute this back in to get desired result.

8. Idea from Baqaee et al. (2024) that compensation is fixed point.

Static non-homotheticity

- Consider non-homothetic CES preferences of the form:

$$U^{\frac{\sigma-1}{\sigma}} = U^{\varepsilon_0} C\left(s^0\right)^{\frac{\sigma-1}{\sigma}} + \sum_{t=1}^{J} \beta^j U^{\varepsilon_j} \sum_{s_j} \pi(s^j) C\left(s^j\right)^{\frac{\sigma-1}{\sigma}}$$

where

$$C\left(s^0\right) = \left(\sum_n \omega_{n0} U^{\varepsilon_n} C\left(s^0\right)^{\zeta_n} c_n(s^0)^{\frac{\gamma-1}{\gamma}} \right)^{\frac{\gamma}{\gamma-1}}$$

and for $t > 0$:

$$C\left(s^j\right) = \left(\sum_n \omega_{nt} U^{\varepsilon_n} c_n(s^j)^{\frac{\gamma-1}{\gamma}} \right)^{\frac{\gamma}{\gamma-1}}.$$

- In period 0, budget shares depend on static relative prices and both wealth & expenditures.

Static non-homotheticity

- For rentiers,

$$\log m(\tau, w, \mathbf{0}) = \log w - \int_{\tau_0}^{\tau} \Bigg[\sum B_n(t, w_t^*) \frac{d \log p_{n0}}{dt} - \frac{1}{(1-\sigma)} \frac{d \log B^P(t, w_t^*)}{dt} - \left(1 - B^P(t, w_t^*)\right) \frac{\sigma}{(1-\sigma)} \frac{d}{dt} \log \left(1 + \frac{\gamma}{1-\gamma} \mathbb{E}_{B_n}[\zeta]\right) + \frac{\gamma \mathbb{E}_{B_n}[\zeta]}{1 - \gamma + \gamma \mathbb{E}_{B_n}[\zeta]} \left(\frac{d}{dt} \log e_0(t, w_t^*) - \sum_n B_n(t, w_t^*) \frac{d}{dt} \log p_n \right) \Bigg] dt.$$

- First line is same as our benchmark.
- The second and third lines are new.
- Requires knowledge of ζ — not identifiable purely from Engel curves.

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