Change in Average Prices: Inflation, Quality Change, or Market Frictions?*

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Abstract

Growing access to item-level transaction data has led to widespread use of average sales-price metrics for both narrow and broad product classes. Such Unit Value Indices (UVI) conflate pure price changes with shifts in product mix so mismeasure inflation for heterogeneous goods. This paper derives an exact decomposition of the UVI into (i) a within component reflecting the matched-model arithmetic Laspeyres index and (ii) a product-mix component that captures changing mix among continuing, entering and exiting products. The mix term reflects price dispersion driven in part by quality variation. A quality-adjusted UVI (QUVI) can correct or at least reduce this bias, but estimating quality adjustment is challenging. Using hedonic procedures for adjustment factors, we derive the conditions when QUVI measures coincide with hedonic indices that take into account product turnover and quality. The framework also considers the identification challenges of disentangling quality effects from market frictions.

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1 Introduction

The increasing availability of digitized item-level transaction data enables the calculation of average sales prices across the full universe of items within narrowly or broadly defined product groups. Such measures are frequently used by retailers and market analysts as indicators of price trends.[1] The appeal of the average sales price lies in its simplicity—it provides a direct decomposition of total revenue as the product of average price and units sold. In academic and statistical communities, this measure is recognized as the *unit value index* (UVI).

The UVI has been widely studied, and both its strengths and limitations are well documented (Balk, 1998; Silver, 2010). On the one hand, for homogeneous products that can be treated as perfect substitutes, the UVI serves as an ideal price index with desirable properties. It has also been proposed as a solution to outlet substitution bias (Reinsdorf, 1998). On the other hand, even within narrowly defined product categories, items frequently vary in quality. In such cases, changes in the average sales price reflect not just price changes, but also shifts in product quality and mix. This issue has challenged researchers using unit prices from administrative trade data to estimate inflation for product classes that are not fully homogeneous (see, e.g., Broda and Weinstein (2006); Fast and Fleck (2019)).

To address these limitations, the literature has introduced the concept of a *quality-adjusted unit value index* (QUVI), which aims to account for variation in quality across goods by deflating observed prices by estimated quality-adjustment factors. Conceptually, the goal in much of the literature is to convert heterogeneous goods into quality-adjusted units that are perfect substitutes so that the QUVI is an exact price index. A number of studies have developed methods to estimate such quality adjustments (de Haan, 2015; de Haan and Krsinich, 2018; Byrne et al., 2017; Aizcorbe and Chen, 2022). Estimating quality adjustments is challenging and requires strong identifying assumptions. These challenges are even greater when it is recognized, as is likely, that observed price

[1] For example, in a recent earnings call, Walmart reported trends in its average sales price—referred to as the AUR (average unit retail); see https://corporate.walmart.com/content/dam/corporate/documents/newsroom/2025/05/15/walmart-releases-q1-fy26-earnings/q1-fy26-earnings-call-transcript.pdf. Amazon similarly discusses ASP (average sales price) on earnings calls; see https://s2.q4cdn.com/299287126/files/doc_financials/2023/q4/Amazon-Q4_2023_Transcript.pdf. Circana, a leading provider of retail transaction data (and the source of our data), regularly reports on average price patterns; see https://www.circana.com/post/us-office-supplies-sales-revenue-declined-2-in-2022. CIRP conducts equity securities research to the investment community tracks the average sales price for numerous firms including Apple (see, https://cirpapple.substack.com/p/latest-iphone-average-selling-price). It should be emphasized that retailers and industry analysts are well aware the product mix impacts the average sales price even in narrowly defined product groups.

dispersion reflects not only quality differences, but also market frictions. Such frictions may arise from information asymmetries, search costs, or imperfect buyer-seller matching. These may be especially relevant for newly introduced products. Byrne et al. (2017), for example, incorporate these frictions in their analysis of a QUVI index for semiconductors. More generally, taking into account such frictions is important if the objective is to identify quality-adjustment factors from price dispersion.

We build on this literature by investigating the theoretical and empirical properties of the UVI and QUVI. Our first contribution is an exact decomposition of the UVI into two components: a within-product term and a product mix term. The novelty of our decomposition is that the within term is shown to be largely a function of an matched-model arithmetic Laspeyres index. Consequently, the deviation of the UVI from this standard Laspeyres benchmark is entirely attributable to product mix effects. These include changes among continuing products as well as the entry and exit of items. These product mix terms reflect price dispersion across products, which, in the absence of perfect homogeneity, is typically associated with differences in quality. We extend the same decomposition to the QUVI. In this case, the within term corresponds to a quality-adjusted Laspeyres index, and the mix terms capture residual price dispersion even after controlling for quality, including dispersion induced by market frictions.

Our second contribution is to derive conditions under which the measured QUVI provides bounds on the exact price index. This bounding approach represents a more modest and achievable objective compared to the more ambitious goal of computing a QUVI that directly serves as a proxy for an exact price index. Our approach builds on the existing literature (see, e.g., von Auer (2014) and de Haan (2015)), which derives conditions under which the measured QUVI yields bounds in the absence of product turnover. We also extend the insights of de Haan (2015), who considers product turnover and derives conditions under which the QUVI equals a Paasche index inclusive of entry and a Laspeyres index inclusive of exit.[2]

We extend these insights to environments characterized by both product turnover and associated quality change, using a hedonic methodology that adjusts for both observable

[2] de Haan (2015) explores this extension conceptually using a single imputation method—imputing the missing price for entering and exiting goods. This approach closely resembles what we refer to as our basic method, although we apply a double imputation procedure that separately imputes price relatives for entering and exiting goods. Notably, de Haan (2015) does not implement his version of the basic method empirically. Our main contribution is to extend this logic by incorporating the methodology of Erickson and Pakes (2011), which enables hedonic imputation for both observable and unobservable characteristics. Moreover, by using full imputation hedonics in our preferred time-varying adjustment approach, the QUVI bounds we generate are consistent with characteristics-adjusted price indices. In addition, we implement this method empirically and examine its relationship with the hedonic Tornqvist and Fisher indices.

and unobservable characteristics. We show that combining the hedonic estimation approach of Erickson and Pakes (2011) with the earlier literature enables us to derive conditions under which the QUVI yields bounds that account for product entry and exit. Specifically, we derive adjustment factors that produce: (i) a QUVI equal to a hedonic arithmetic Laspeyres index inclusive of product exit; (ii) a QUVI equal to a hedonic arithmetic Paasche index inclusive of product entry; and (iii) our preferred QUVI measure, which closely approximates a hedonic Tornqvist index inclusive of both product entry and exit.[3]

These equivalences are useful for a variety of reasons. These bounds are generally tighter (see Erickson and Pakes (2011) and Ehrlich et al. (2025)) than matched-model bounds when using this Erickson and Pakes (2011) methodology. We argue that these hedonic index equivalent QUVI indices are interpretable as characteristic-adjusted (constant characteristic) unit value indices. Importantly, it is by using full imputation hedonics that the price indices are interpretable as characteristic-adjusted price indices.Mapping hedonic indices to the QUVI enables taking advantage of the relationship between the UVI and QUVI as we demonstrate in our analysis.

The methodology in Erickson and Pakes (2011) as implemented by Ehrlich et al. (2025) is well suited for this purpose. This methodology has the attractive feature of capturing changing valuations of both observable and unobservable characteristics in the context of product turnover and quality change. This approach has an advantage relative to other hedonic approaches. Advocates for the QUVI that use price dispersion to measure quality-adjustment factors without direct information on product attributes highlight that such an approach avoids the challenges of unobserved characteristics in hedonic estimation. The Erickson and Pakes (2011) methodology overcomes this limitation of hedonics.

The more ambitious exact price index approach to the QUVI requires quality-adjustment factors that make goods after adjustment perfect substitutes. In the absence of market frictions quality-adjustment factors account for all within-period price dispersion. These stark properties and implications seem unlikely – some component of within-period price dispersion likely reflects market frictions. Disentangling quality-adjustment factors and market frictions yields an inherent identification problem. We characterize and explore these identification problems with what we denote as the frictional QUVI. In

[3]The relationship between the hedonic Tornqvist index and the midpoint of the QUVI bounds is not exact, but as we show, they are very similar empirically. Appendix Section A.4.2 provides further details on this connection between the hedonic Tornqvist index and the QUVI. The equivalence results we establish do not rely specifically on the Erickson and Pakes (2011) method; in principle, any hedonic imputation method can be used. However, the desirable properties of the Erickson and Pakes (2011) method carry over to the resulting QUVI indices.

particular, we pursue approaches to the identification problem by drawing on life cycle-based approaches akin to Byrne et al. (2017) (hereafter BKM). BKM show that the price dispersion for semi-conductors dissipates over the life cycle. They use this evidence to identify quality-adjustment factors based on price dispersion several periods after entry. In the spirit of this approach, we consider two alternative polar cases that normalize market frictions at different points of the life cycle. While illustrative of the issues, without further product group specific analyses of the type done by BKM, these polar cases don't provide a generic solution to the inherent identification problem of disentangling price dispersion from market frictions.

To examine these issues empirically, we use rich, item-level transaction data from Circana (formerly NPD) covering four narrowly defined consumer electronics categories: desktop computers, notebook computers, stereo headphones, and streaming audio speakers. These product groups exhibit rapid quality change and high product turnover over our sample period from 2017:Q1 to 2020:Q4. The data include quarterly SKU/UPC-level prices and quantities as well as detailed product attributes. These attributes allow us to apply hedonic methods to estimate quality-adjustment factors.

Our results for our consumer tech product groups show that the UVI yields positive inflation rates, while the matched-model arithmetic Laspeyres index indicates deflation. According to our decomposition, the difference is driven by product mix effects. For our preferred hedonic-based Tornqvist approximation to the QUVI, we show that this yields more negative inflation than the standard matched-model arithmetic Laspeyres. We also find the measure of the QUVI that is equivalent to the hedonic arithmetic Laspeyres inclusive of product exit yields lower inflation than the standard matched-model Laspeyres. These findings are consistent with the Erickson and Pakes (2011) and Ehrlich et al. (2025) arguments that the hedonic arithmetic Laspeyres inclusive of product exit is a tighter upper bound than the matched-model arithmetic Laspeyres.

For our exploratory frictional QUVI using polar cases, we find a lower implied inflation rate than our hedonic equivalent indices in most cases. We find that for two of our four product groups that the frictional QUVI falls below the lower bound hedonic Paasche index – raising questions about the properties and interpretation of the frictional QUVI using the polar cases we consider.

We also benchmark our QUVI indices against alternative quality-adjusted price indices that fall outside the UVI framework. In particular, we compare the QUVI indices to the CES unified price index (CUPI) from Redding and Weinstein (2020). The CUPI is, under strong but distinct assumptions, an exact price index that accounts for product turnover and quality change. Both the CUPI and frictional QUVI are related distinct

attempts at exact price indices. The CUPI explicitly assumes imperfect substitution in preferences and the frictional QUVI uses market frictions as an alternative approach to capture imperfect substitutability. Over our sample period, the deflation implied by the CUPI is substantially larger—between 5 and 22 percentage points more than the hedonic Tornqvist which aligns closely with the midpoint of our bounded approach for the QUVI. The CUPI also falls much more than the frictional QUVI – for the polar exploratory cases we consider.

We also exploit this structure of the QUVI to quantify the extent to which observed changes in the UVI reflect quality growth. For our sample, we find substantial implied quality growth across all product categories, driven largely by shifts in the product mix. We put more confidence in the results of these measures using our bounding approach.

The remainder of the paper proceeds as follows. Section 2 outlines the conceptual framework for decomposing both the UVI and the QUVI, the underpinnings of the QUVI and empirical implementation approaches for the QUVI. Section 3 describes the point-of-sale data used in the analysis. Section 4 presents empirical results, including index construction and decompositions. Section 5 concludes.

2 Conceptual Framework

2.1 Decompositions of the UVI and QUVI

In this section, we consider the properties of the UVI and QUVI relative to standard matched-model price indices. This analysis has implications for understanding the residual price dispersion across products – specifically, the extent to which such variation reflects product quality and market frictions amongst other factors. This analysis highlights that it is potentially important to take into account other sources of price dispersion than product quality – both to interpret the UVI and also to develop empirical methods for quality adjustment.

The standard UVI measures the relative change in quantity weighted average prices, which combines average price changes, quality growth, and changes in product mix. The formula for the standard UVI for a basket of goods (Ω) across adjacent period pairs $\{t-1,\ t\}$ is:

$$\frac{\mathbb{P}_t}{\mathbb{P}_{t-1}} = \frac{\sum_{k\in\Omega_t} p_{kt}q_{kt} / \sum_{k\in\Omega_{t-1}} p_{kt-1}q_{kt-1}}{\sum_{k\in\Omega_t} q_{kt} / \sum_{k\in\Omega_{t-1}} q_{kt-1}} \tag{1}$$

where p_{kt} is the price of item k in period t, q_{kt} is the quantity of item k in period t and $\mathbb{P}_t$

is the aggregate unit price in t for the product group. It is instructive to consider a simple accounting decomposition of the UVI. As shown in Appendix Section A.3.1, the UVI can be decomposed exactly as follows:

$$\frac{\mathbb{P}_t}{\mathbb{P}_{t-1}} = 1 + s^c_{t-1}\sum_{k\in C} s^c_{kt-1}\left(\frac{p_{kt}}{p_{kt-1}} - 1\right) + \sum_{k\in C}\Delta w_k\left(\frac{p_{kt}}{\mathbb{P}_{t-1}}\right) + \sum_{k\in N} w_{kt}\left(\frac{p_{kt}}{\mathbb{P}_{t-1}}\right) - \sum_{k\in X} w_{kt-1}\left(\frac{p_{kt-1}}{\mathbb{P}_{t-1}}\right) \tag{2}$$

where $s^c_{kt-1} = p_{kt}q_{kt}/\sum_{k\in C} p_{kt}q_{kt}$ is the expenditure share for k in $t-1$ for goods that continue into t, $s^c_{t-1} = \sum_{k\in C} p_{kt}q_{kt}/\sum_{k\in\Omega_t} p_{kt}q_{kt}$ is the share of expenditures in $t-1$ for total expenditures in $t-1$ inclusive of exit accounted for by goods that continue into t and $w_{kt} = q_{kt}/\sum_{k\in\Omega_t} q_{kt}$ is the quantity share for item k in period t. The first term is a within term that interestingly we show is the growth rate from a standard matched-model arithmetic Laspeyres weighted by the share of expenditures for continuing products.[4] The next set of terms reflect changes in the product mix from both continuing products, entering and exiting products. It is these product mix terms that yield the gap between the UVI and this standard matched-model index. These product mix terms will in general cumulatively be non-zero. Moreover, it is likely that these product mix terms in part reflect dispersion in prices from product quality. This simplicity of this decomposition is a virtue in that it can be easily implemented to provide an initial benchmarking of UVI measures.[5]

While this decomposition of the UVI is of interest, the limitation of the UVI in the presence of product quality variation is well known in the price index literature (see, e.g., Balk (1998), de Haan (2015), de Haan and Krsinich (2018), Byrne et al. (2017), Aizcorbe and Chen (2022)). This recognition has led to the development of a quality-adjusted UVI (QUVI) given by:

$$\frac{P_t}{P_{t-1}} = \frac{\sum_{k\in\Omega_t} p_{kt}q_{kt}/\sum_{k\in\Omega_{t-1}} p_{kt-1}q_{kt-1}}{\sum_{k\in\Omega_t} \lambda_{kt}q_{kt}/\sum_{k\in\Omega_{t-1}} \lambda_{kt-1}q_{kt-1}} \tag{3}$$

where λ_{kt} is a quality-adjustment factor, which standardizes units of quantity in terms of product quality for item k in period t, and P_t denotes the quality-adjusted average price

[4]The within term encompasses the matched-model arithmetic Laspeyres, but is distinct given the multiplicative term taking into account the share of expenditures for continuing products relative to all products in $t-1$. In practice, their magnitude and variation is very similar. Often for the sake of brevity we refer to the within term as the arithmetic Laspeyres.

[5]There are also antecedents to this decomposition in the literature. Balk (1998) relates the UVI to the Fisher index in an environment without product turnover.

in period t. These adjustment factors can reflect simple differences in package size and weight, differences in observable characteristics, and differences in unobservable characteristics that relate to quality. A challenge of course is how to measure the quality-adjustment factors and conditional on measurement how to interpret.

Before proceeding to the challenging measurement and interpretation issues, it is instructive to perform the same type of decomposition for the QUVI:

$$\begin{aligned}\frac{P_t}{P_{t-1}} = 1 + s^c_{t-1} \sum_{k \in C} s^c_{kt-1} \left(\frac{p_{kt}/\lambda_{kt}}{p_{kt-1}/\lambda_{kt-1}} - 1 \right) + \sum_{k \in C} \Delta\omega_k \left(\frac{p_{kt}/\lambda_{kt}}{P_{t-1}} \right) \\ + \sum_{k \in N} \omega_{kt} \left(\frac{p_{kt}/\lambda_{kt}}{P_{t-1}} \right) - \sum_{k \in X} \omega_{kt-1} \left(\frac{p_{kt-1}/\lambda_{kt-1}}{P_{t-1}} \right)\end{aligned} \tag{4}$$

where ω_{kt} denotes the quality-adjusted quantity share of item k given by $\omega_{kt} = \lambda_{kt} q_{kt} / \sum_{k \in \Omega_t} \lambda_{kt} q_{kt}$. The within term is now the growth of the quality-adjusted arithmetic Laspeyres (weighted by the share of expenditures for continuing goods). The remaining product mix terms reflect quality-adjusted changes. An open question is whether these quality-adjusted product mix terms are components of inflation.[6]

Lastly, the formulas for the UVI and QUVI have interesting connections as well. Combining the UVI with the QUVI yields the following expression:

$$\mathbb{P}_t / \mathbb{P}_{t-1} = \left[P_t / P_{t-1} \right] \left[\Lambda_t / \Lambda_{t-1} \right] \tag{5}$$

where $\Lambda_t = \sum_{k \in \Omega_t} \lambda_{kt} q_{kt} / \sum_{k \in \Omega_t} q_{kt}$ measures the quantity weighted average product "quality" in period t. That is the UVI (LHS) is equal to the product of the QUVI (first term RHS) and quantity weighted average product quality (second term). This quality index has natural appeal – it is a measure of real output growth explained by the observed changes in quality/characteristics. Moreover, the product of quality and quantity indices would also provide the full measure of real output growth for a given product group. As will become clear below, this quality index is of interest even if the QUVI measure is a bound rather than an exact price index.

2.2 Underpinnings and Interpretation of the UVI and QUVI

To understand the UVI and QUVI, it is useful to consider the sources of the price dispersion with and without adjustment factors – and in turn to think about how to take into

[6] Byrne et al. (2017) in their appendix have an approximation of a decomposition of the QUVI exploring a special case with no exit and only entry.

account market frictions as a source of price dispersion. As a starting point, it is useful to review the logic underlying the circumstances under which the UVI is an exact price index and in turn the circumstances in which the QUVI is an exact price index. If the goods in question are perfect substitutes without any quality adjustment, then the UVI is an exact price index (see formal discussion in Appendix Section A.3.1). With perfect substitutes and no frictions in the market, the law of one price will hold:

$$p_{kt} = P_t \; \forall \; k \in \Omega_t \tag{6}$$

While the conditions under which the law of one price holds are very strong, it is interesting to consider the implications of the law of one price in terms of the above decomposition. As we show in Appendix Section A.3.1, if equation (6) holds, the UVI collapses to the arithmetic Laspeyres with the within term as the cumulative product mix terms are zero. This inference highlights that the product mix terms in equation (2) reflect the presence of price dispersion. For the UVI, such price dispersion will typically reflect at least to some extent uncontrolled for product quality. As such, the product mix terms in the UVI decomposition do not reflect components of inflation.

Now consider the QUVI. If the quality-adjustment factors λ_{kt} yield that the quality-adjusted units are perfect substitutes, this implies a quality-adjusted law of one price and the QUVI is an exact price index. It is instructive to briefly review the assumptions for these implications to hold (following the insights of Balk (1998), Byrne et al. (2017), Diewert (2020)). With linear utility in quality-adjusted quantities that make goods perfect substitutes the utility maximization is given by:

$$\max_{\{q_{kt}\}} \sum_{k \in \Omega_t} \lambda_{kt} q_{kt} \text{ such that } E_t = \sum_{k \in \Omega_t} p_{kt} q_{kt}$$

where E_t is expenditures. Solving the utility maximization problem, the quality-adjusted law of one price emerges:[7]

$$p_{kt} = \lambda_{kt} P_t \; \forall \; k \in \Omega_t \tag{7}$$

Under these conditions the QUVI is an exact price index:

$$\lambda_{kt} P_t q_{kt} = p_{kt} q_{kt}$$

[7]The formal arguments for the UVI being an exact price index with the law of one price follows similar logic. With identical goods $\lambda_{kt} = 1$.

$$P_t = \sum_{k \in \Omega_t} p_{kt} q_{kt} / \sum_{k \in \Omega_t} \lambda_{kt} q_{kt}$$

$$\frac{P_t}{P_{t-1}} = \frac{\sum_{k \in \Omega_t} p_{kt} q_{kt} / \sum_{k \in \Omega_t} \lambda_{kt} q_{kt}}{\sum_{k \in \Omega_{t-1}} p_{kt-1} q_{kt-1} / \sum_{k \in \Omega_{t-1}} \lambda_{kt-1} q_{kt-1}}$$

These conditions for the QUVI to be an exact price index are very strong requiring perfect substitutes of quality-adjusted units. In addition, equation (7) implies that all price dispersion is accounted for by quality – an equally strong implication. Under these strong conditions, the QUVI will collapse to a quality-adjusted Laspeyres without any contribution from the product mix terms when equation (7) holds.

These stark assumptions and implications raise questions about the applicability of the QUVI empirically. However, there are ways to relax the assumptions on the explicit causes of price dispersion and still retain the QUVI as an exact price index. One approach is to modify the environment to introduce market frictions that relax the implication that all price dispersion reflects quality variation. Such frictions may arise from many factors including information asymmetries, search costs, or imperfect buyer-seller matching. In Appendix Section A.3.3, we introduce frictions in the above consumer's problem showing that this implies a frictional quality-adjusted law of one price:

$$z_{kt} p_{kt} = \lambda_{kt} P_t \; \forall \; k \in \Omega_t \tag{8}$$

where z_{kt} are the frictions that reflect residual quality-adjusted price dispersion.[8] Equation (8) relaxes the stark implication that all price dispersion is due to quality variation. An interesting implication of introducing such market frictions is that the formula for the QUVI does not change.[9] Instead, the presence of market frictions has implications for the source of price dispersion – importantly the recognition that price dispersion likely reflects more than quality variation. This implies there are inherent identification challenges for disentangling market frictions from quality-adjustment factors if price dispersion is used to measure those factors.

[8] Our approach in appendix section A.3.3 is in the spirit of Byrne et al. (2017). They formally introduce adjustment costs into the consumer's problem for newly introduced items. We take a simpler approach imposing quantity constraints or Hsieh and Klenow (2009) like distortions into the consumer's problem. We discuss how their empirical implementation can be interpreted in our framework below.

[9] This point is important as it implies that if one has a method for identifying λ_{kt} factors independent of price dispersion then one can compute the QUVI. For identical goods subject to market frictions where the $\lambda_{kt} = 1$ this implies the UVI is still an appropriate price index.

2.3 A Bounding Approach to the QUVI

A different approach is to pursue a more modest objective for the QUVI measures and in turn the interpretation of the QUVI. Rather than implementing a measurement method that that seeks to make the QUVI a proxy for an exact price index, instead seek to construct measures of the QUVI that bound the exact price index. This approach is consistent with the insights of von Auer (2014) and de Haan (2015) who characterized conditions under which adjustment factors for the QUVI in the absence of product turnover directly yield an upper bound arithmetic Laspeyres and a lower bound arithmetic Paasche. Consider the specification of an adjustment factor for $\tau = \{t-1, t\}$ given by:[10]

$$\bar{\lambda}_{k\tau} = \left(\frac{p_{kt}}{P_t}\right)^{\alpha} \left(\frac{p_{kt-1}}{P_{t-1}}\right)^{1-\alpha} \tag{9}$$

For now we assume away product turnover. The parameter α is a timing/mix parameter that enables using base $(t-1)$ or alternatively current t period weights for the price index. Specifically, if $\alpha = 1$ using this adjustment factor yields the QUVI to be equal to the matched-model arithmetic Laspeyres:

$$\begin{aligned}
\frac{P_t}{P_{t-1}} &= \frac{\sum_{k\in\Omega} p_{kt}q_{kt} / \sum_{k\in\Omega} p_{kt}q_{kt}}{\sum_{k\in\Omega} p_{kt-1}q_{kt-1} / \sum_{k\in\Omega} p_{kt}q_{kt-1}} \\
&= \frac{\sum_{k\in\Omega} p_{kt}q_{kt-1}}{\sum_{k\in\Omega} p_{kt-1}q_{kt-1}} \\
&= \sum_{k\in\Omega} s_{kt-1}\left[\frac{p_{kt}}{p_{kt-1}}\right]
\end{aligned}$$

In the $\alpha = 1$ case, the price dispersion in t is assumed to capture all of the $\lambda_{k\tau}$ dispersion. The resulting price index is the matched-model arithmetic Laspeyres with well-known upper bound properties of the exact price index.

For $\alpha = 0$, this adjustment factor yields the QUVI to be equal to the matched-model arithmetic Paasche:

$$\begin{aligned}
\frac{P_t}{P_{t-1}} &= \frac{\sum_{k\in\Omega} p_{kt}q_{kt} / \sum_{k\in\Omega} p_{kt-1}q_{kt}}{\sum_{k\in\Omega} p_{kt-1}q_{kt-1} / \sum_{k\in\Omega} p_{kt-1}q_{kt-1}} \\
&= \frac{\sum_{k\in\Omega} p_{kt}q_{kt}}{\sum_{k\in\Omega} p_{kt-1}q_{kt}}
\end{aligned}$$

[10] Under strong assumptions, this equation is consistent with the quality-adjusted law of one price as in equation (7). However, implementation of this equation does not require these strong assumptions. Instead, as we show, polar cases of this implementation yield well-known price indices that are bounds of the true cost of living.

$$= \left(\sum_{k \in \Omega} s_{kt} \left[\frac{p_{kt}}{p_{kt-1}} \right]^{-1} \right)^{-1}$$

In these two insightful bounding cases, the QUVI ultimately just depends on price relatives. Of course, a limitation of this approach is that product turnover has been neglected. A clear advantage of the QUVI is that the set of goods in Ω_{t-1} and Ω_t can be different with different characteristics. We show below that with hedonic imputation that the above logic can be extended to derive conditions for the QUVI to be equivalent to the hedonic Laspeyres inclusive of exit, and alternatively the hedonic Paasche inclusive of entry. As discussed by Pakes (2003), Erickson and Pakes (2011), and Ehrlich et al. (2025), such hedonic upper and lower bounds inclusive of product exit and entry have desirable properties.

Moreover, with or without product turnover, intermediate cases of these bounds are interesting. The geometric mean of the arithmetic Laspeyres and arithmetic Paasche (the Fisher) is readily computed. Moreover, $\alpha = 0.5$ yields a QUVI that is a close approximation of the Tornqvist (see Appendix Section A.4.2). In our implementation below, we derive and implement with hedonics a QUVI that is a close approximation of the hedonic Tornqvist inclusive of both product entry and exit. The hedonic Tornqvist (and Fisher) are interpretable as useful midpoints of well defined upper and lower bounds (the Tornqvist is the average of the geometric Laspeyres and geometric Paasche).[11]

For this bounding approach, the objective and interpretation of the adjustment factors and the QUVI are different. The adjustment factors are intended to make the implementation of equation (3) bounds (or averages of bounds). It might be more appropriate with the bounding approach to denote the QUVI as a *generalized unit value index* as in von Auer (2014).

In the next section, we explore the details of the empirical implementation issues. We review methods that have been used in the literature, show how with hedonics we can generate equivalent QUVI's to alternative hedonic indices inclusive of product entry and exit, and discuss the potential role of market frictions if one seeks to estimate quality-adjustment factors from price dispersion.

[11] von Auer (2014) also discussed the Fisher QUVI and the case of $\alpha = 0.5$. He noted that they both have desirable index properties and the $\alpha = 0.5$ case had been suggested by Davies (1924). Importantly, this earlier work focused on an environment without product turnover.

2.4 Empirical Implementation of Alternative Approaches to the QUVI

The approach taken here is to consider what de Haan (2015) denotes as a bilateral approach – specifically a method for computing the adjustment factors for the pair of periods $t-1$ and t. This approach has the advantage of computing the full set of factors for all of the goods present in $t-1$ and t inclusive of entry and exit. The relative adjustment factors are thus interpreted as the relative factors for this set of products that will inherently be different for any other pair of periods. In the environment of item-level transactions data, this approach is important given the high product turnover of goods. As will become clear below, a second distinct related advantage is that when we consider the time-varying adjustment factor approach with hedonics, the resulting QUVI measures are tightly linked to chained hedonic indices inclusive of entry and exit.

Empirical implementation of measuring the quality-adjustment factors is a challenge – especially if using price dispersion to estimate quality-adjustment factors since in this case it is important to take into account market frictions. We consider several alternatives including those that have been used in the literature. In particular, one method we use is that proposed by de Haan and Krsinich (2018) (DHK). They measure quality adjustment factors indirectly from a time dummy hedonic regression (see Appendix Section A.5.1). This time dummy approach yields time-invariant quality adjustment factors for the pair of periods $t-1$ and t.

A second approach we consider builds on the bounds approach discussed in the prior section – now extending it to be inclusive of entry and exit. For this purpose, we use the Erickson and Pakes (2011) approach as implemented by Ehrlich et al. (2025). We refer to this method as EP-TV (TV represents the time-varying valuations of both observable and unobservable characteristics). The EP-TV method uses a log first difference approach to control for unobserved fixed effects and also captures time-varying valuations of unobservable characteristics.

The implementation of the log difference hedonic model from EP-TV takes multiple steps. First, we estimate a log level hedonic model in the base period by regressing log price levels on a vector of observed product characteristics denoted as X_k':

$$\ln p_{kt} = X_k' \beta_t + e_{kt}$$

Using the log level model, the EP-TV method extracts the estimated residual for $t-1$: $\hat{e}_{kt-1}$. Using this lagged residual estimate as a proxy for unobserved characteristics, we

include it as an additional control variable in a log difference hedonic model:

$$\Delta \ln p_{kt} = X_k' \, \Omega_t + \kappa_t \hat{e}_{kt-1} + u_{kt}$$

This approach immediately yields predicted price relatives for continuing goods and exiting goods. We implement the procedures used by Ehrlich et al. (2025) to compute the predicted price relative for entering goods.[12]

A critically important feature of the EP-TV approach is that it is designed to take into account both observable and unobservable characteristics. Advocates of the QUVI that use price dispersion for identification of quality-adjustment factors highlight that hedonic methods typically face the challenge of unobservable characteristics. The EP-TV method overcomes this challenge.

With the estimates from the EP-TV approach we can extend the bounds approach of the prior section to be inclusive of entry and exit. Our preferred approach is to compute adjustment factors using hedonic estimates for continuing, entering and exiting goods. We refer to this as the time-varying adjustment method.

As a starting point, we implement the following based on the EP-TV estimates:

$$\Delta \ln p_k/\lambda_k = \Delta\widehat{\ln p_k} \tag{10}$$

where $\Delta\widehat{\ln p_k}$ is the predicted price relatives from the EP-TV method. With these estimates in hand we implement the following extension of the approach taken in the prior section for continuing goods:

$$\begin{aligned} \ln \lambda_{kt} &= \ln \bar{\lambda}_k + (1-\alpha)\Delta \ln \lambda_k \\ \ln \lambda_{kt-1} &= \ln \bar{\lambda}_k - \alpha \Delta \ln \lambda_k \end{aligned} \tag{11}$$

where $\Delta \ln \lambda_k = \Delta \ln p_k - \Delta\widehat{\ln p_k}$ and $\bar{\lambda}_k$ is consistent with the adjustment factors in equation (9). For entering and exiting goods, we use the following approach:

$$\begin{aligned} \ln \lambda_{kt} &= \ln p_{kt} - (1-\alpha)\Delta\widehat{\ln p_k} \quad \text{for entry} \\ \ln \lambda_{kt-1} &= \ln p_{kt-1} + \alpha\Delta\widehat{\ln p_k} \quad \text{for exit} \end{aligned} \tag{12}$$

This time-varying adjustment approach yields several interesting bounds for the

[12] Ehrlich et al. (2025) consider several alternatives for entering goods. Here we use their approach of assuming the expenditure weighted mean of the initial (missing) residual is zero.

QUVI.[13] As shown in Appendix Section A.2, for the two polar cases of $\alpha \in \{0, 1\}$ the QUVI is equal to:[14]

$$\frac{P_t}{P_{t-1}} = \begin{cases} \left(\sum_{k \in C} s_{kt} \left[\widehat{\frac{p_{kt}}{p_{kt-1}}} \right]^{-1} + \sum_{k \in N} s_{kt} \left[\widehat{\frac{p_{kt}}{p_{kt-1}}} \right]^{-1} \right)^{-1} & \text{for } \alpha = 0 \\ \sum_{k \in C} s_{kt-1} \left[\widehat{\frac{p_{kt}}{p_{kt-1}}} \right] + \sum_{k \in X} s_{kt-1} \left[\widehat{\frac{p_{kt}}{p_{kt-1}}} \right] & \text{for } \alpha = 1 \end{cases} \tag{13}$$

The first equation with $\alpha = 0$ is the hedonic arithmetic Paasche inclusive of entry and the second equation with $\alpha = 1$ is the hedonic arithmetic Laspeyres inclusive of exit. Thus, under the conditions stated the QUVI yields an upper and lower bound of the true cost of living. As emphasized by Pakes (2003), Erickson and Pakes (2011), and Ehrlich et al. (2025) these bounds are generally tighter than the matched-model arithmetic Laspeyres and Paasche indices. These are full-imputation hedonic price indices using predicted price relatives for continuing, entering and exiting goods. As emphasized by Crawford and Neary (2023), this makes these characteristics-based price indices.

Alternative midpoint indices with desirable properties using the averaging of the bounds are readily computable. The geometric mean of the above two indices is an hedonic Fisher index inclusive of product entry and exit. A related distinct case is to set $\alpha = 0.5$. As we show in Appendix Section A.4.2, when $\alpha = 0.5$ there is an equivalence between the hedonic Tornqvist used in Ehrlich et al. (2025) and a geometric version of a quality-adjusted unit value index defined (in log form) as $\ln \text{GQUVI} = \sum_{\Omega_t} s_{kt} \ln p_{kt}/\lambda_{kt} - \sum_{\Omega_{t-1}} s_{kt-1} \ln p_{kt-1}/\lambda_{kt-1}$.

An intermediate version of extending the bounding approach in the prior section is to use hedonics only to fill in the missing prices for entering and exiting goods. This approach combines equation (9) with equation (12). In this case, the QUVI bounds are given by:

$$\frac{P_t}{P_{t-1}} = \begin{cases} \left(\sum_{k \in C} s_{kt} \left[\frac{p_{kt}}{p_{kt-1}} \right]^{-1} + \sum_{k \in N} s_{kt} \left[\widehat{\frac{p_{kt}}{p_{kt-1}}} \right]^{-1} \right)^{-1} & \text{for } \alpha = 0 \\ \sum_{k \in C} s_{kt-1} \left[\frac{p_{kt}}{p_{kt-1}} \right] + \sum_{k \in X} s_{kt-1} \left[\widehat{\frac{p_{kt}}{p_{kt-1}}} \right] & \text{for } \alpha = 1 \end{cases} \tag{14}$$

These are hybrid hedonic Laspeyres and Paasche indices using actual price relatives for continuing goods and using the EP-TV hedonics for the price relatives for entering and

[13] We take exponentials to generate the required level λ_{kt} factors. There is some approximation error here given the use of estimated values.

[14] Importantly, in equation (13) the $s_{kt} = p_{kt}q_{kt}/\sum_{k \in \Omega_t} p_{kt}q_{kt}$. That is, the shares are for all goods in t inclusive of C and N. For s_{kt-1} these are shares inclusive of C and X. The same remarks apply to equation (14).

exiting goods respectively.[15] Our preferred approach is the time-varying adjustment method above, but for completeness in results presented below we also consider this case – which we refer to as the basic method.

If the objective is the more ambitious one of identifying true quality-adjustment factors, taking into account the potential role of market frictions in observed price dispersion is important. This yields inherent identification issues as is evident from equation (8) since price dispersion inherently involves both quality and market frictions.

Byrne et al. (2017) (BKM) overcome such identification issues resolving them based upon detailed empirical analysis of the evolution of price dispersion of semiconductors over the course of the life cycle. They find price dispersion is especially high for semiconductors in the first several periods after entry. Based on their empirical analysis, they assume frictions have dissipated after $T = 20$ quarters. This enables them to identify the quality-adjustment factors based on the price dispersion across products in period T. Importantly they assume truly time invariant quality-adjustment factors so they can use adjustment factors based on the period T price dispersion for all periods. In the face of item-level product turnover, BKM are able to implement this approach with aggregation. In particular, the authors determine relative quality of goods at a supplier level (i.e. the next level of aggregation above the item level) to account for products exiting before a long run price is observed. This process requires calculating a supplier's average long run price relative after controlling for characteristic differences at the item level. They apply the supplier level adjustment factors to the individual products.

Following a similar approach as BKM at scale is potentially challenging. For one, this would seemingly require the type of detailed life cycle analysis of price dispersion they conduct for every product group. Second, even with such an analysis, there is no guarantee the patterns they found for semiconductors would hold more generally (i.e., there is some T after which price dispersion dissipates). Third, their aggregation approach is limiting for the type of item-level data we are using with a very high pace of high frequency product turnover.

To help illustrate and explore these issues further, we consider an approach that mod-

[15] This hybrid approach is equivalent to what is often referred to as a double imputation method. That is, imputing price relatives for entering and exiting goods respectively. Our basic method is very similar to that of de Haan (2015) who derives a version of equation (14) using a single imputation method. Importantly, neither the single or double imputation approach yields bounds that are interpretable as characteristics-adjusted price indices. It is only the full imputation indices that can be interpreted in this manner.

ifies equation (9) to incorporate market frictions:

$$\bar{\lambda}_{k\tau} = \left(\frac{z_{kt}p_{kt}}{P_t}\right)^{\alpha}\left(\frac{z_{kt-1}p_{kt-1}}{P_{t-1}}\right)^{1-\alpha} \tag{15}$$

This equation is consistent with the frictional law of one price in equation (8) where residual quality-adjusted price dispersion is captured by z_{kt}. Interestingly, the identification problem can be overcome for the polar cases $\alpha \in (0, 1)$. To see this, consider the recursive relationship between z_{kt} and z_{kt-1} implied by equation (8) given by:

$$z_{kt} = \left(\frac{p_{kt}/\lambda_{kt}}{p_{kt-1}/\lambda_{kt-1}}\right)^{-1}\left(\frac{P_t}{P_{t-1}}\right) z_{kt-1} \tag{16}$$

Combining equation (16) with equation (15), permits us to consider two polar cases associated with $\alpha = 0$ (1). Specifically, we consider a case where $z_{kt} = 1$ at the beginning vs the end of the life cycle. For the former case and letting $\alpha = 0$, quality adjustment and residual price dispersion (z_{kt}) can be jointly determined going forward from the start of the life cycle. For the latter case and letting $\alpha = 1$, quality adjustment and residual price dispersion (z_{kt-1}) can be jointly determined going backwards from the end of the life cycle. We consider both cases in the empirical analysis that follows.

While this approach (especially for $\alpha = 1$ and $z_{kt} = 1$ at the end of the life cycle) is similar in spirit to BKM, our implementation is not based on any product group specific analysis of price dispersion over the life cycle.[16] We regard our implementation as exploratory helping to provide guidance on the identification challenges.

In practice, we implement this by combining the time-varying adjustment approach with these with these alternative ways of normalizing z_{kt} (to either one at the beginning or end of life cycle). We note that using equations (15) and (16) does not require using the time-varying adjustment approach. Instead it can be applied to the basic approach.

2.5 Taking Stock

Before proceeding to the empirical analysis, it is useful to discuss the implications of the limitations of the measurement of the "quality-adjusted" prices critically important for the measurement of the QUVI. Our view is that the bounding approach for the QUVI has more promise than approaches with the objective for producing a QUVI that is an exact price index.

[16] In a future draft of this paper, we plan on providing an analysis of the evolution of price dispersion over the course of the life cycle for our product groups.

The time dummy approach advocated by de Haan and Krsinich (2018) has the virtue of imposing structure on the measurement of quality-adjustment factors. In addition, it is a method that can incorporate product turnover. However, the time dummy approach has limitations on multiple dimensions. The method does not address unobservable characteristics. Moreover, the imposition of time invariant relationships between prices and characteristics is problematic even between t and $t-1$ (see Pakes (2003)). Relatedly, Pakes (2003) emphasizes hedonic characteristic estimates should not be interpreted as capturing quality alone. It is these limitations that are related to the bounding approach for hedonics.

If the objective is the true quality-adjustment factors then sorting out the contribution of market frictions and quality is required. BKM make important progress on this dimension, but it is unclear how to implement their approach at scale. We provide analysis of some polar cases that overcome the identification problem in the manner described, but these cases are not generic generic solutions to the identification problem. Unlike the bounding approach, it is unclear the properties of the frictional QUVI estimates we present. In contrast, the bounding approach for the QUVI based on hedonic imputation enables computation of empircal QUVI with well known and desirable properties.

The bounding approach builds on the insights of our core original decomposition in equation (2). In this equation, the UVI is decomposed exactly into the matched-model arithmetic Laspeyres and product mix terms without any other identifying assumptions. This provides a benchmark to evaluate the UVI. The bounding approach takes this several steps further. Rather than the benchmark being just the matched-model arithmetic Laspeyres, the bounding approach yields estimates of the QUVI that are alternatively the upper bound hedonic arithmetic Laspeyres inclusive of exit, the lower bound hedonic arithmetic Paasche inclusive of entry and midpoints such as the QUVI that is an approximation of an hedonic Tornqvist inclusive of product entry and exit.

A related advantage of the bounding approach is that the bounds and their average cases depend in practice on only price relatives (either actual or predicted).[17] Price indices that depend on price relatives are not relying on decomposing price dispersion into quality and market frictions to produce the index. Consistent with this discussion, a broader interpretation of the QUVI using EP-TV hedonics for "quality"-adjustment factors is that this reflects the QUVI with characteristic-adjusted prices.

We don't regard these challenges as implying the empirical QUVI under alternative

[17]For the case of $\alpha = 0.5$ this is only approximately true. It holds for the geometric version of the UVI which we denoted GQUVI above. The GQUVI and QUVI under time-varying adjustment at $\alpha = 0.5$ are very similar in practice.

approaches is not of interest. The UVI has natural appeal given its simplicity. Moreover, the decomposition in equation (5) is of interest even if the QUVI is a bound (or average of bounds). The resulting quantity index can in the latter cases be interpreted as bounds for quality growth/real output growth adjusted for quality. More generally, the empirical QUVI under different approaches are efforts to control for the role of heterogeneous characteristics that yield product mix effects that reflect multiple factors. The methods proposed are efforts to sort these product mix effects out and exploring their implications are insightful.

3 Data and Estimation

We use proprietary data that the NPD Group (now part of Circana) provided to the U.S. Census Bureau. The data consists of monthly sales and quantity data at the item-store level from 2017 through 2020. The database consists of point-of-sales (POS) information provided by thousands of stores belonging to both brick-and-mortar and online retailers. We aggregate the item-by-store-level observations to the national item-level by quarter level and calculate total quantity sold and average price for each item-level-quarter. For our analysis, we produce measures for four product groups in the broader category of consumer electronics: desktop computers, notebook computers, stereo headphones, and audio streaming speakers. Within each product group, the aggregated item-quarter level data covers thousands of observations in our sample. We follow previous work such as Ehrlich et al. (2025) and Hottman et al. (2016) when processing the raw POS data.

When calculating UVIs and other price indices, we take a period of time to correspond to a quarter (using the National Retail Federation definition of months which adjusts for weekends and holidays). When measuring the rates of change for an index, we calculate the annual average quarterly rate on a Q4-to-Q4 basis. This measure differs slightly for 2017 as our sample period begins in 2017Q1, which introduces a seasonality issues for measures covers our first year of data. For this reason, we primarily present results using annual rates of change for 2017Q4 to 2020Q4. Other measures presented in the following section consists of various decompositions that are aggregated to a sample average across all quarters.

In addition to the information on item-level sales and quantities, the database also contains information on item characteristics, which we utilize to produce hedonic indices. For example, with notebook computers, these attributes include descriptions for amount of RAM, storage capacity, screen size, and battery life. For each product group, we use an

elastic net approach on a quarter-by-quarter basis regressing item-by-quarter price levels or relatives on the attributes to produce hedonic estimates according to either the hedonic model from Erickson and Pakes (2011) or the time-dummy hedonic model that underlies the quality-adjustment procedure from de Haan and Krsinich (2018).[18]

An issue we do not directly address in this paper is chain drift. de Haan (2015) amongst others have noted that chain drift is an issue with scanner data. In Ehrlich et al. (2025), we show that chain drift is more of an issue for matched-model indices than hedonic indices for the type of consumer tech goods we consider in this paper.[19] While we do not address, the methods we are using are readily adaptable to multi-lateral methods.

4 Empirical Results

In this section, we begin with the standard UVI, which commingles price and quality growth. We compare the UVI to standard matched-model indices and use the decomposition from equation (2) to provide insights about the patterns. Analysis of the QUVI and the frictional QUVI follow with again comparison to alternative price indices including matched-model, the CUPI and the hedonic indices (e.g., hedonic Tornqvist) that takes into account product turnover and quality change.

4.1 Standard UVI

To begin our analysis, we start with a comparison of the standard UVI with matched-model indices such as the arithmetic Laspeyres and Tornqvist in Figure 1. The standard UVI is positive for nearly all cases, indicating that the average price of goods are increasing across all four products in our sample. In stark contrast, matched-model indices indicate there is a consistent decline in the average price change for our sample of consumer electronics over the same period. The arithmetic Laspeyres and the Tornqvist are much lower than the UVI and also exhibit distinct time series patterns relative to the UVI – especially for Desktop computers and Stereo Headphones. The UVI declines for Desktop Computers in 2020 while the matched-model indices rise. The UVI declines very sharply in 2020 for Stereo Headphones while the matched-model indices exhibit a continuation of the trend decline from earlier years. The very different time series patterns of the UVI and matched-model indices highlights that the UVI needs to be used with caution even

[18] A complete description of the EP-TV hedonic model is provided in Appendix Section A.6.2 and the QUVI following DHK is provided in Appendix Section A.5.1.

[19] It has also been found that chain drift is more of an issue with high frequency scanner data at a local level – e.g., store-level.

for measuring the changing pace of inflation (and not just the level). We also observe that the Tornqvist is below the arithmetic Laspeyres consistent with the presence of substitution bias.

The decomposition in equation (2) permits us to provide further insights into the contribution of the within term (the matched-model Laspeyres) and the product mix terms. Table 1 presents the decomposition for the average quarterly change in the standard UVI based from equation (2). The product mix terms cumulatively are very positive generating the large gap between the UVI and the Laspeyres.[20] The product mix contribution is especially large for Stereo Headphones. Apparently, there is considerable shifting towards higher price items for all of these product groups.

4.2 QUVI: Bounding Approach

In this section, we focus on the QUVI using the bounding approach inclusive of product entry and exit. We also compare results using this approach with the time-dummy hedonic adjustment based on DHK as well as matched-model and other price indices.

Figure 2 plots the average quarterly rate of inflation using the QUVI with time-varying adjustment under alternative values of the timing parameter α. As noted above, the QUVI with time-varying adjustment is equal to a hedonic arithmetic Paasche inclusive of entry when $\alpha = 0$ and a hedonic arithmetic Laspeyres inclusive of exit when $\alpha = 1$. These polar bounding versions of the QUVI correctly reflect the general case where the Laspeyres is above the Paasche. Furthermore, the hedonic Fisher, the geo-mean of these two bounds, and the QUVI with $\alpha = 0.5$ are incredibly similar as well.

Figure 3 plots the average quarterly rate of inflation from the QUVI with time-varying adjustment with $\alpha = 0.5$ and compares it to the Tornqvist and EP-TV hedonic Tornqvist price index inclusive of product entry and exit. We find that the hedonic Tornqvist index falls below the matched-model Tornqvist index. The QUVI with time-varying adjustment

[20] For the product mix terms, we focus on the combined effect since caution must be used in interpreting the separate contribution of the product mix terms for continuers vs net entry. As emphasized in Foster et al. (2001), the between terms for continuers vs net entry are challenging to interpret individually without taking into account a reference point. In their application of decomposing productivity, they used a reference point of period $t-1$ aggregate productivity. This permitted the between term to be interpretable as whether changes in shares of activity among continuers were moving resources to firms with higher (lower) than the reference point productivity. Likewise, the net entry terms become interpretable as whether the entering firms have productivity above (below) the reference point productivity and exiting firms have productivity above (below) reference point productivity. We note that consideration of such reference points is not relevant for the combined contribution of the product mix terms. In our context, wWithout such reference points the continuer (net entry) component will tend to be negative (positive) even in the absence of price dispersion if the share of continuing goods (net entry) is declining (rising). In the absence of price dispersion, the terms will be offsetting so their sum is zero.

closely tracks the EP-TV index. As we have noted above, the EP-TV hedonic Tornqvist is equivalent to a geometric version of the quality-adjusted price index (denoted GQUVI) when the latter uses the same quality-adjustment factors as those we use in the time-varying adjustment version of the QUVI. Thus, it is not surprising that the time-varying adjustment QUVI with $\alpha = 0.5$ and EP-TV hedonic Tornqvist track each other so closely. The QUVI is inherently an arithmetic mean compared to the GQUVI so that can yield some modest discrepancy between the two measures. While the GQUVI has some natural appeal given its properties, the QUVI remains of interest since it can be more readily compared to the UVI.

The decomposition of the time-varying adjusted version of the QUVI is reported in Table 2 for the case of $\alpha = 0.5$. In stark contrast to the decomposition of the UVI, the product mix terms are now negative. The very large positive product mix terms in the UVI decomposition become negative even for Stereo Headphones which has an especially large positive product mix contribution to the UVI. Comparing the UVI and QUVI decompositions is consistent with the view that the UVI positive mix terms are due to uncontrolled for quality effects driving the positive UVI. Also, interestingly but not surprisingly, the QUVI with $\alpha = 0.5$ is lower than the matched-model arithmetic Laspeyres with quality adjustment. In turn, the hedonic arithmetic Laspeyres inclusive of exit is lower than this matched-model Laspeyres. And the QUVI at $\alpha = 0.5$ (and its close cousin the hedonic Tornqvist) is lower than the QUVI that is equivalent to the hedonic arithmetic Laspeyres inclusive of product exit.

To provide further perspective, Figure 4 plots the average quarterly rate of inflation based on the QUVI under three alternative forms of adjustment. One is the time-varying adjustment method with $\alpha = 0.5$. The second is the basic method with $\alpha = 0.5$. The third is the DHK method. We find that the QUVI with either time-varying or basic adjustment with $\alpha = 0.5$ falls below the DHK series. Comparing the the QUVI with basic adjustment to the time-varying adjustment, we find that accounting for time-varying quality changes in product quality results in a steeper decline in the measured QUVI.

4.3 Price Dispersion

Before we consider the frictional QUVI, we explore the variation in price levels and relatives in our sample consumer electronics. We do so for raw and "quality" adjusted prices using methods that are consistent with quality-adjustment procedures following DHK, basic adjustment, and time-varying adjustment approaches. Table 3 displays the average quarterly variance of log prices and shows that the degree of residual price dispersion de-

pends on the choice of quality adjustment. As shown in columns (1) through (4), we find that the variance of log price levels decreases when comparing raw to quality-adjusted prices following DHK, basic adjustment, and time-varying adjustment. time-varying adjustment produces the steepest decline in price dispersion. For this case, the variance of residual log prices is 1% or less of the raw dispersion.

Next, we show that there are differences in dispersion in log price relatives as well albeit the variance in raw price relatives is two orders of magnitude smaller than raw price levels. The (pairwise) time-invariant methods do not impact the dispersion in price relatives. The time varying method yields the hedonic price relatives taken from the EP-TV model. The latter produces an even lower variance than raw price relatives as expected. Overall, we find that incorporating hedonic information from a first difference hedonic regression such as the EP-TV model produces the lowest variance in both log price levels and relatives.

4.4 Frictional QUVI

Figure 5 presents the FQUVI for the two polar cases. The lower FQUVI tends to be the $\alpha = 1$ case where we normalize $z_t = 1$ to the end of the life cycle. Our prior is that this is the more reasonable case. An exception to this quantitative pattern is streaming audio speakers. This is also the product group where the FQUVI is actually higher than the time-varying adjustment QUVI.

It is instructive to compare the FQUVI at $\alpha = 0$ and $\alpha = 1$ to the time-varying QUVI at the $\alpha = 0$ and $\alpha = 1$. For $\alpha = 1$, the time-varying QUVI is the hedonic Laspeyres inclusive of exit and an upper bound to the true cost of living. For $\alpha = 0$, the time-varying QUVI is the hedonic Paasche inclusive of entry and a lower bound to the true cost of living. The FQUVI falls outside these bounds for desktop computers regardless of α, falls outside these bounds for $\alpha = 1$ for notebook computers, and lies within these bounds regardless of α for stereo headphones and audio speakers.

Figure 6 plots the average quarterly rate of inflation according to the frictional QUVI when taking the average of the two polar cases. We compare this to the QUVI with time-varying adjustment with $\alpha = 0.5$, and the quality-adjusted Laspeyres. In terms of the decomposition in equation (4), the latter is the same component for both the time-varying QUVI and frictional QUVI.

We find that the average frictional QUVI produces a lower rate inflation than the time-varying QUVI at $\alpha = 0.5$ for 3 of the 4 product groups. For these cases the gaps are notable. From Table 5, the product-mix component in the frictional QUVI accounts for

an additional 0.7-2.2 percentage point decline in prices relative to the quality-adjusted Laspeyres for Desktop Computers, Notebook Computers, and Stereo Headphones. For the case of Streaming Audio Speakers, the frictional QUVI is again below the quality-adjusted Laspeyres, but above the QUVI with time-varying adjustment.

In comparison to quality-adjusted indices outside of the UVI framework, Table 4 displays the cumulative rate of inflation for the frictional QUVI, the time-varying adjusted QUVI, the EP-TV hedonic Tornqvist index, and the CUPI over our sample period. We calculate the cumulative rate in part to display the aggressive decline in prices across all methods. The time-varying adjusted QUVI and the EP-TV hedonic Tornqvist are closely aligned in magnitude as noted above. The FQUVI tends to be lower than the time-varying adjusted QUVI (and by construction the EP-TV hedonic Tornqvist) except for Stereo Headphones. The CUPI is dramatically lower especially without imposing what Ehrlich et al. (2025) denote as a common goods rule. Even with the latter, the CUPI tends to be substantially lower.[21]

4.5 Price and Quality Growth in the Standard UVI

We now return to the standard UVI for a complete decomposition of the change in average prices. As noted above, in principle the QUVI is the quality-adjusted inflation component within the standard UVI: $\mathbb{P}_t/\mathbb{P}_{t-1} = \left[P_t/P_{t-1}\right]\left[\Lambda_t/\Lambda_{t-1}\right]$ where $\Lambda_t = \sum_{k\in\Omega_t}\lambda_{kt}q_{kt}/\sum_{k\in\Omega_t}q_{kt}$ measures the quantity weighted average product quality in period t. We can further decompose the change in average quality growth using the following equation:

$$\frac{\Lambda_t}{\Lambda_{t-1}} = 1 + \sum_{k\in C} w_{kt-1}\left(\frac{\Delta\lambda_k}{\Lambda_{t-1}}\right) + \sum_{k\in C}\Delta w_k\left(\frac{\lambda_{kt}}{\Lambda_{t-1}}\right) + \sum_{k\in N} w_{kt}\left(\frac{\lambda_{kt}}{\Lambda_{t-1}}\right) - \sum_{k\in X} w_{kt-1}\left(\frac{\lambda_{kt-1}}{\Lambda_{t-1}}\right)$$

where we define the three components of the decomposition in an analogous fashion to those in equation (2) or (4). We now use these relationships below to quantify product quality growth. These calculations could be done with any of the measures of the QUVI. However, the interpretation here is subject to the issues above. When using the time-varying QUVI this approach is providing bounds (and relatedly the average of a lower and upper bound). For the FQUVI, the polar cases considered imply results should be treated with appropriate caution.

Figure 7 first plots three series in tandem: (1) the standard UVI, (2) the time-varying

[21] The common goods rule defines the threshold for when products should be considered common goods for the computation of the terms in the CUPI (see Appendix Section A.6.2).

QUVI, and (3) the implied measure of quality growth in the standard UVI when using the time-varying QUVI. Taken together, the three series show that the standard UVI differs significantly from matched-model indices as a result of persistently high quality growth. The average quarterly rate of quality growth is above 4.75 percent for all product groups except Desktop Computers, which has a slightly lower rate of quality growth around 3 percent. Given this is the bounding approach, this should be interpreted as providing guidance about the midpoint of the upper and lower bounds.

Figure 8 repeats this exercise using the FQUVI. In this case, the implied average quarterly rate of quality growth is above five percent for all product groups except Desktop Computers, which has a slightly lower rate of quality growth around 4 percent.

Table 6 displays the decomposition of the average quarterly rate of quality growth in the standard UVI. We compare the measure of quality growth using the QUVI with time-varying adjustment and the frictional QUVI. The following points are consistent across either measure of the QUVI. The within component of average quality growth is non-zero as a result of the inclusion of time-varying adjustment and indicates that the first difference hedonic model captures positive quality growth for the set of continuing goods. The component's magnitude is nearly an order of magnitude smaller than the product mix term. The latter accounts for almost all of this index of quality growth. The difference between the two sets of results are with respect to their magnitudes. For all but streaming audio speakers, the implied measure of quality growth using frictional QUVI captures an additional 0.4 to 1 percentage point increase in average quality. This relative difference comes almost entirely from the change in product mix.

5 Conclusion

In this paper, we examine the properties of the Unit Value Index (UVI) and explore alternative approaches for adjusting for quality—or at least observable and unobservable product characteristics—through the construction of a Quality-Adjusted Unit Value Index (QUVI). We develop an exact decomposition showing that the UVI can be expressed as the sum of growth in a standard matched-model arithmetic Laspeyres index and a set of product mix effects arising from continuing, entering, and exiting products.

For the product groups we study, the product mix component contributes significantly and positively to the UVI, making it appear inflationary, whereas the arithmetic Laspeyres index—an upper bound on the true cost of living—indicates deflation. As a result, the UVI substantially overstates true cost-of-living inflation. Moreover, we find

that the time series behavior of the UVI differs markedly from the arithmetic Laspeyres and other matched-model indices, not just in levels, but in dynamic patterns.

The QUVI adjusts the UVI by applying "quality" adjustment factors. Identifying these factors is inherently difficult. Our preferred approach is to identify adjustment factors that generate bounds to the true cost of living. Building on the existing literature, we extend and implement methods to estimate these adjustment factors using hedonics for the bounding approach. The resulting QUVI indices (both the bounds and their averages) diverge meaningfully from the UVI. They exhibit behavior consistent with quality adjustment: they are lower than the matched-model arithmetic Laspeyres.

The more ambitious objective of computing adjustment factors that make the empirical QUVI an approximation of the true cost of living faces fundamental identification challenges. We are sympathetic to the view that if this is the objective then market frictions need to be taken into account in estimating the quality-adjustment factors. However, the identification challenges of disentangling quality from market frictions are substantial. We have explored alternative approaches cognizant of these challenges. Our findings yield that when we adjust the QUVI to account for market frictions, we find that measured inflation rates tend to be lower than the average of of those from our bounding approach. For two of the four product groups, the implied frictional QUVI lie outside the bounds.

An alternative approach to measuring the exact price index is the CES Unified Price Index (CUPI). The CUPI has considerable appeal theoretically as it incorporates product turnover and quality change in an integrated utility maximizing framework. The frictional QUVI and CUPI are conceptually related in the sense both seek to produce exact price indices in the presence of imperfect substitutability – in the CUPI this is captured directly through preferences and in the frictional QUVI through market frictions. Interestingly, we find the CUPI yields even lower rates of inflation than the frictional QUVI. However, as highlighted by Ehrlich et al. (2025), implementation of the CUPI has its own challenges.

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Figures and Tables

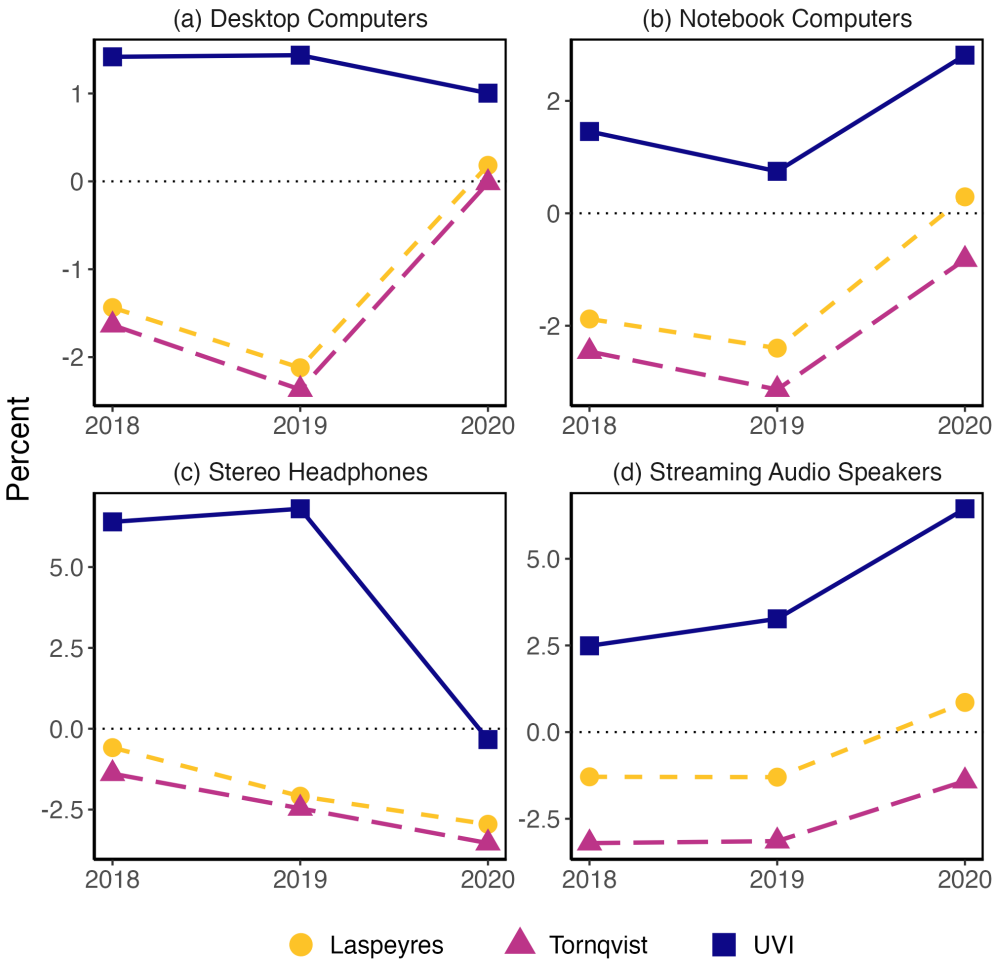


Figure 1: Comparison to Matched-Model Indices - the Standard UVI

Note: Values equal the average quarterly percent change, aggregated from chained quarterly price indices on a Q4-to-Q4 basis.

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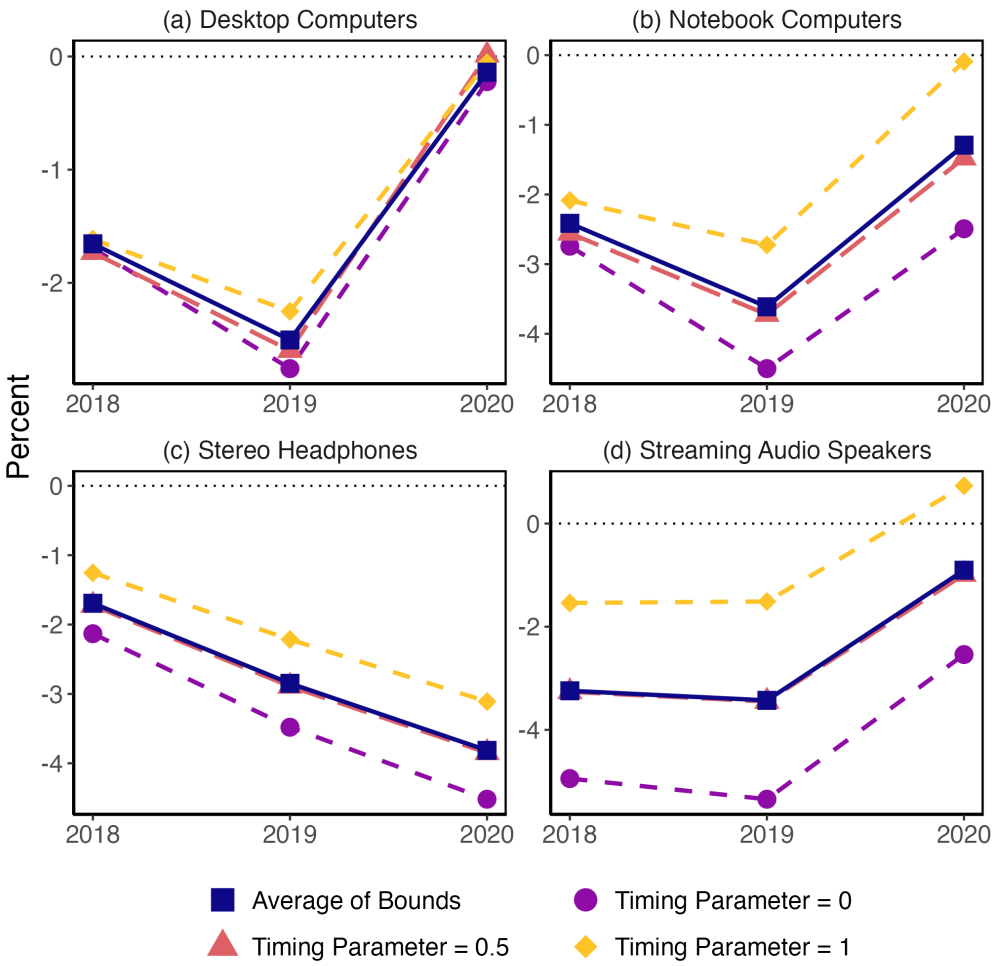


Figure 2: Variation in Time-Varying Adjustment

Note: Values equal the average quarterly percent change, aggregated from chained quarterly price indices on a Q4-to-Q4 basis. The different series plots the average quarterly percent change for the QUVI with time-varying adjustment where the timing parameter α varies between 0 and 1. The series titled Average of Bounds plots the geo-mean of the series when the timing parameter $\alpha \in \{0, 1\}$.

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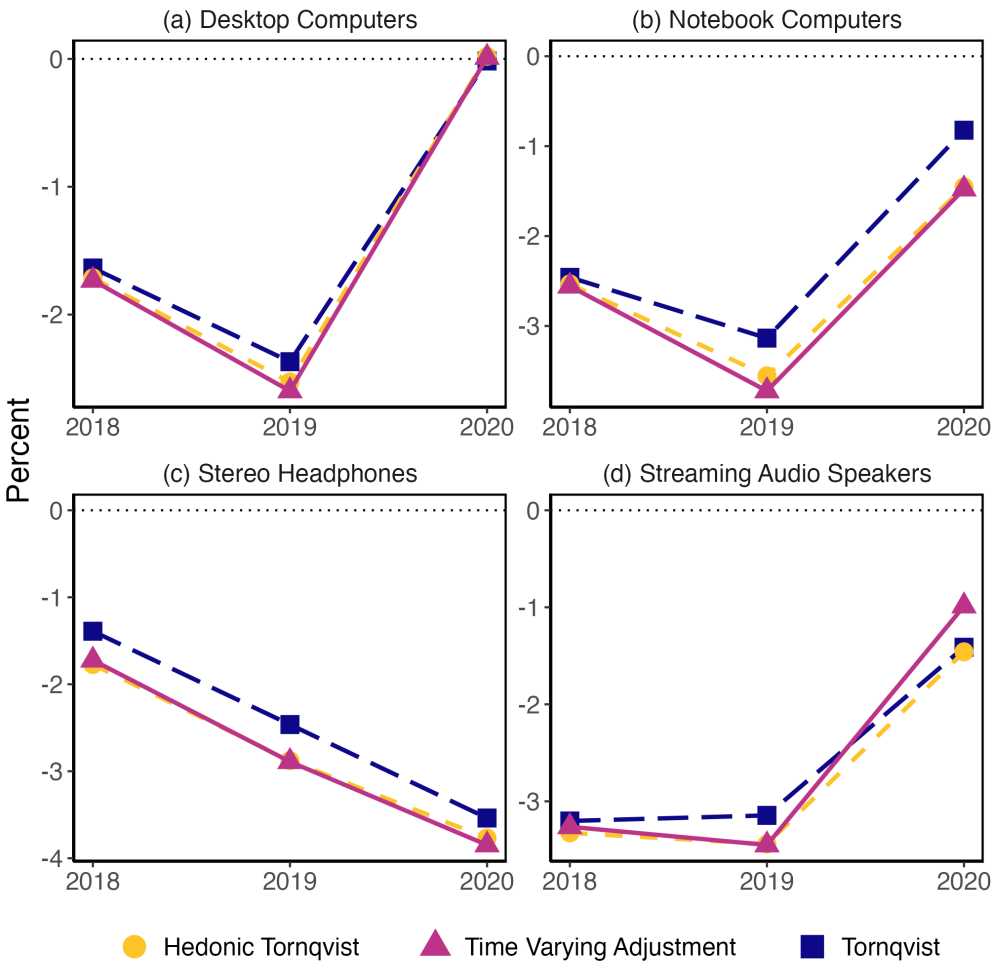


Figure 3: Comparison to Superlative and Hedonic Indices - the QUVI

Note: Values equal the average quarterly percent change, aggregated from chained quarterly price indices on a Q4-to-Q4 basis. The EP-TV series refers to the hedonic Tornqvist index constructed using methods from Erickson and Pakes (2011)

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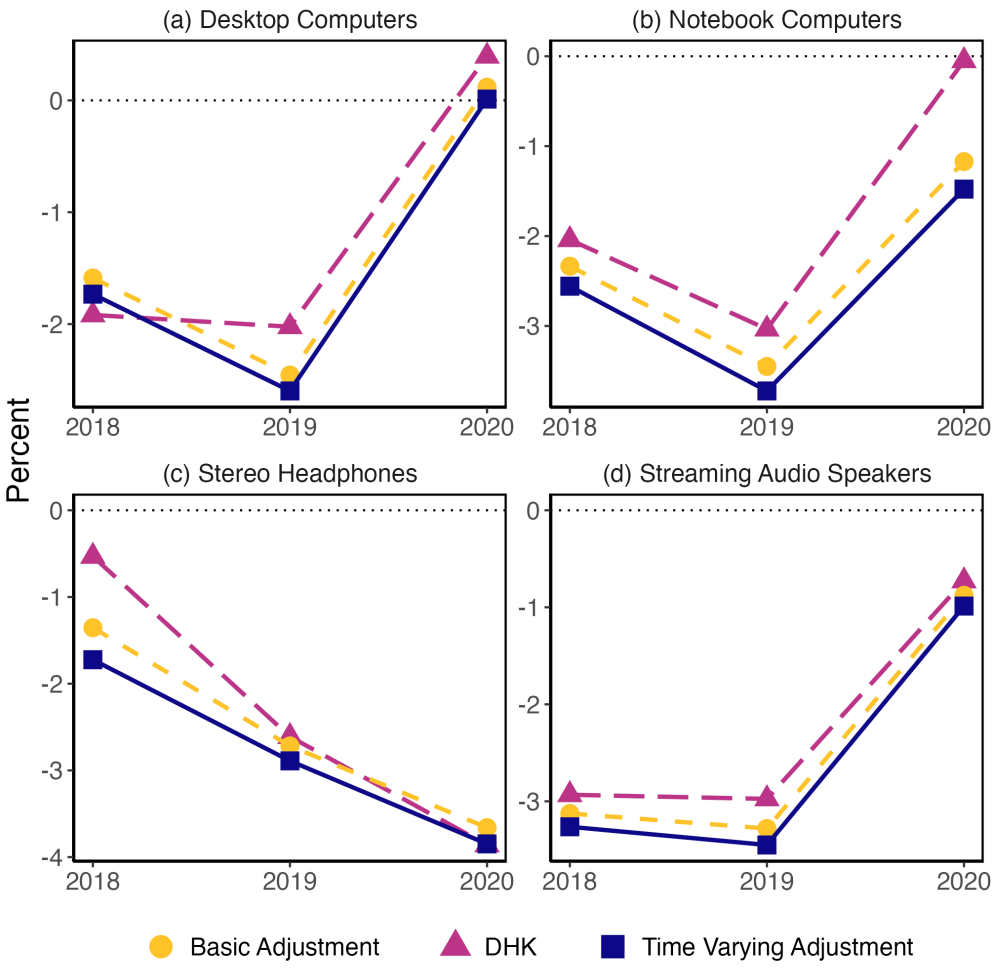


Figure 4: Comparison of Specifications for Quality-Adjustment factors

Note: Values equal the average quarterly percent change, aggregated from chained quarterly price indices on a Q4-to-Q4 basis. The Basic Adjustment series refers to the QUVI adjusted using average prices and no time-varying adjustment with a timing parameter equal to 0.5. The DHK series refers to the QUVI adjusted using hedonic information from a time dummy hedonic model as in de Haan and Krsinich (2018). The Time-Varying Adjustment series refers to the QUVI adjusted using average prices and hedonic information from the EP-TV hedonic model with a timing parameter equal to 0.5.

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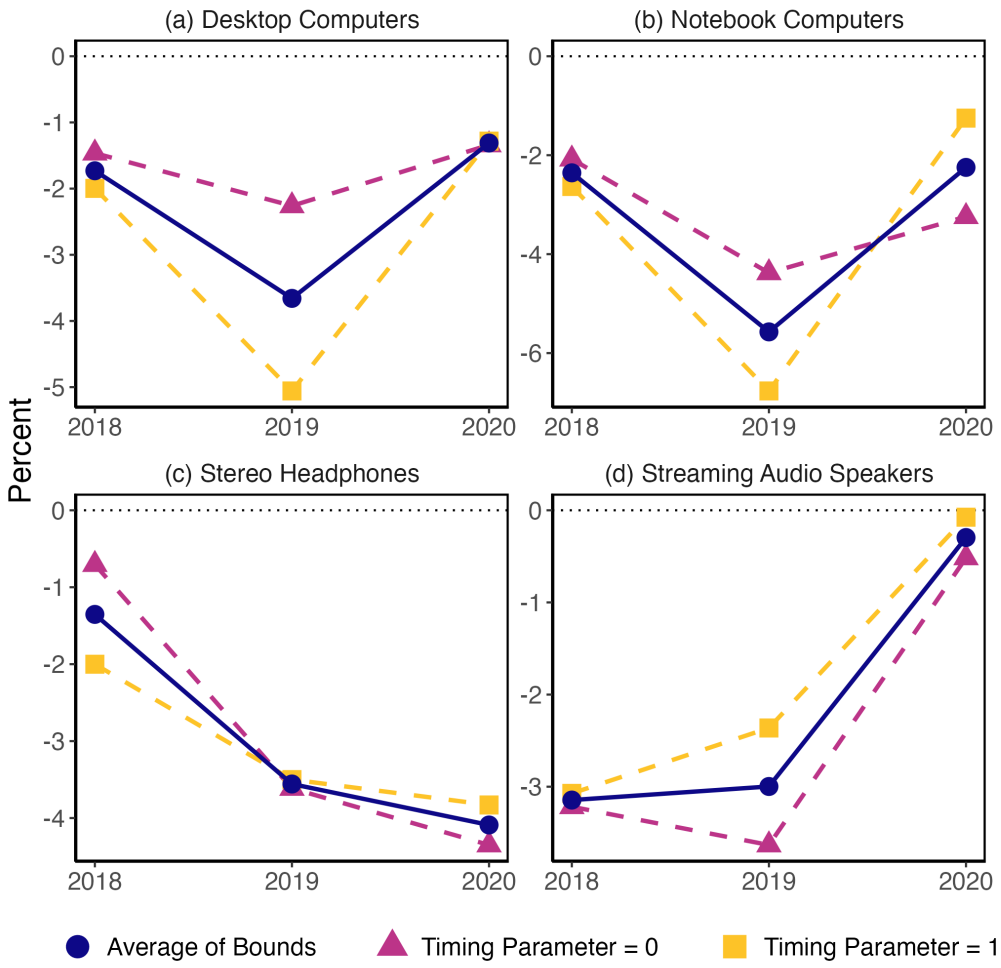


Figure 5: Variation in the Frictional QUVI

Note: Values equal the average quarterly percent change, aggregated from chained quarterly price indices on a Q4-to-Q4 basis. The different series plots the average quarterly percent change for the frictional QUVI where the timing parameter $\alpha \in \{0, 1\}$. The series titled Average of Bounds plots the simple arithmetic average of the series when the timing parameter $\alpha \in \{0, 1\}$.

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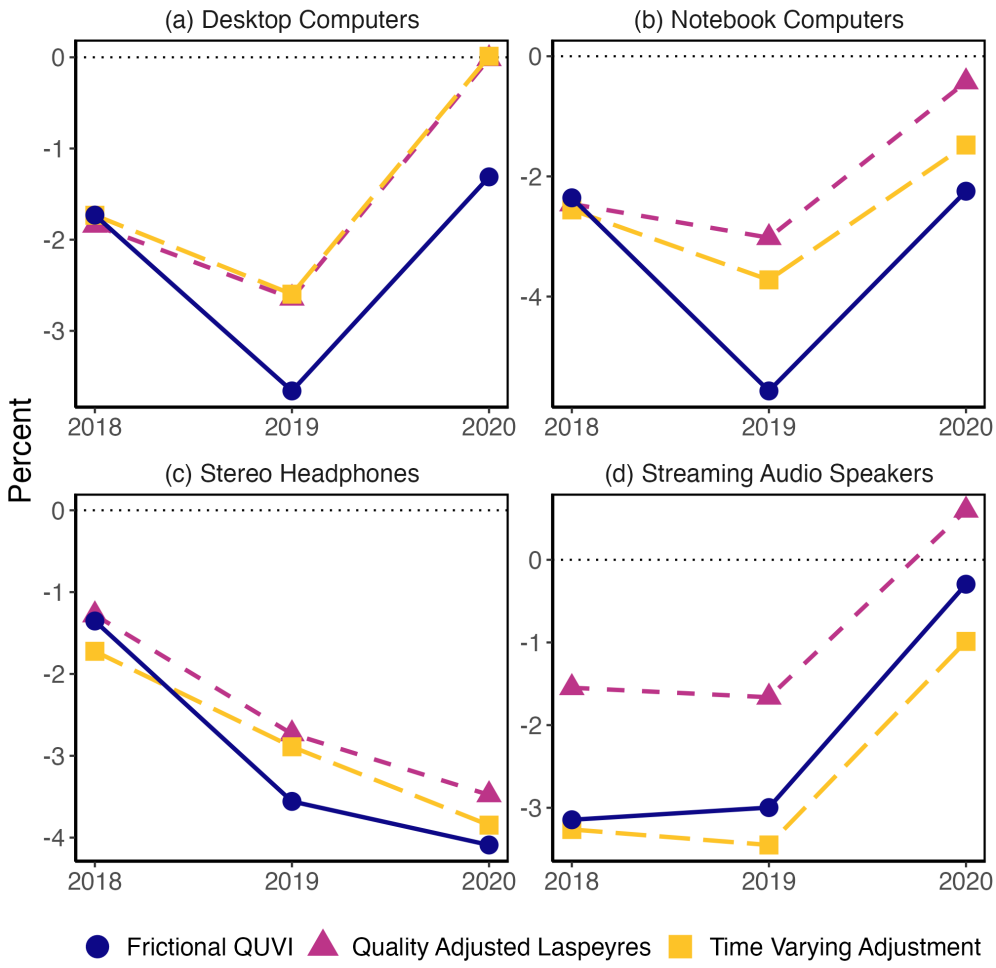


Figure 6: The Frictional QUVI

Note: Values equal the average quarterly percent change, aggregated from chained quarterly price indices on a Q4-to-Q4 basis. The Frictional QUVI refers to the average QUVI series which are adjusted using average prices, hedonic information from the EP-TV model, and the estimated measure of residual price dispersion. The average QUVI is the simple arithmetic average of two frictional QUVI series calculated with the timing parameter set to 0 and 1. The Quality-Adjusted Laspeyres series refers to an arithmetic Laspeyres index using estimated price relatives calculated using the EP-TV method.

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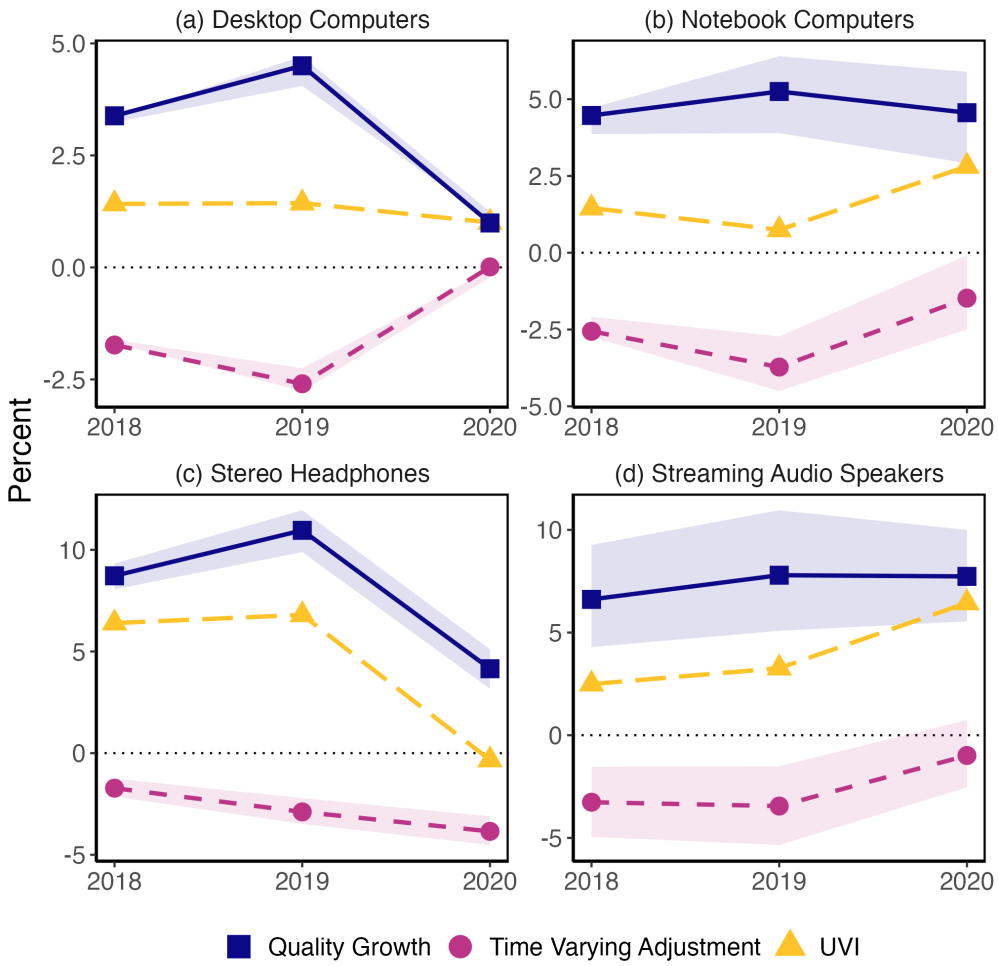


Figure 7: Price and Quality Growth in the Standard UVI: Using Time-Varying QUVI with $\alpha = 0.5$

Note: Values equal the average quarterly percent change, aggregated from chained quarterly price indices on a Q4-to-Q4 basis. The Quality Growth series refers to the difference between the UVI and time-varying QUVI series.

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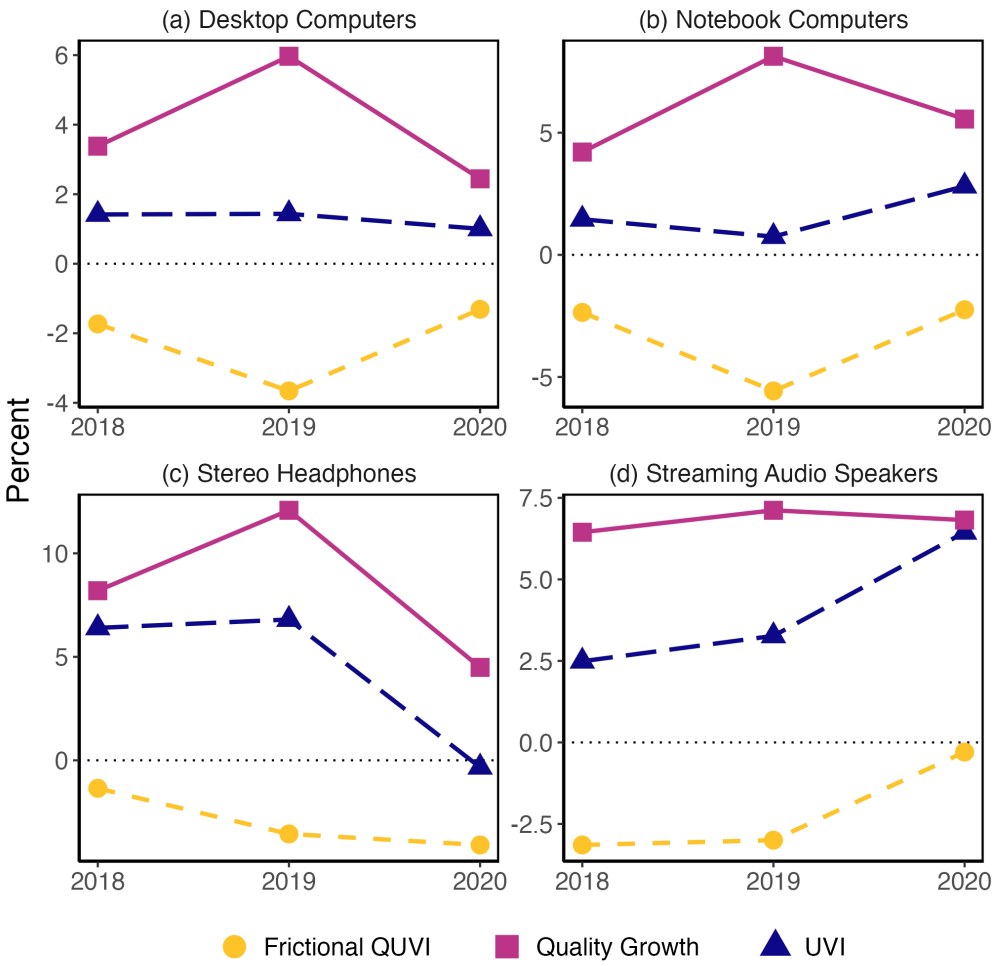


Figure 8: Price and Quality Growth in the Standard UVI: Using FQUVI

Note: Values equal the average quarterly percent change, aggregated from chained quarterly price indices on a Q4-to-Q4 basis. The Quality Growth series refers to the difference between the UVI and frictional QUVI series.

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Table 1: Decomposing the Standard UVI

	Laspeyres (1)	Within (2)	Product-Mix (3)	UVI (4)
Desktop Computers	−1.59	−1.51	3.27	1.77
Notebook Computers	−1.65	−1.57	2.08	0.51
Stereo Headphones	−1.49	−1.49	7.36	5.87
Streaming Audio Speakers	−1.47	−1.47	2.92	1.46

Note: This table presents the average quarterly decomposition of the standard UVI over our sample period. The formula for the decomposition of the standard UVI is provided in equation (2). Product-mix terms in the decomposition are summed and reported in column 3.

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Table 2: Decomposing the QUVI with Time-Varying Adjustment

	QA Laspeyres (1)	Within (2)	Product-Mix (3)	QUVI (4)
Desktop Computers	−1.96	−1.86	−0.05	−1.90
Notebook Computers	−2.26	−2.17	−0.57	−2.74
Stereo Headphones	−2.49	−2.49	−0.94	−3.43
Streaming Audio Speakers	−1.80	−1.79	−1.58	−3.37

Note: This table presents the average quarterly decomposition of the QUVI with time-varying adjustment where the timing parameter $\alpha = 0.5$. The formula for the decomposition of the QUVI is provided in equation (4). Product-mix terms in the decomposition are summed together and reported in column 3.

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Table 3: Average Quarterly Variance of Prices

	Levels (logs)				Relatives (logs)	
	Raw	DHK	Basic Adjustment	time-varying adjustment	Raw	EP-TV
	(1)	(2)	(3)	(4)	(5)	(6)
Desktop Computers	1.113	0.973	0.003	0.002	0.010	0.007
Notebook Computers	0.778	0.697	0.004	0.003	0.017	0.009
Stereo Headphones	1.446	0.928	0.013	0.004	0.055	0.017
Streaming Audio Speakers	1.180	0.968	0.015	0.012	0.058	0.046

Note: Values equal the average quarterly variance of log prices. The variance is calculated for price levels and relative changes which are unadjusted or quality-adjusted using several alternative methods. Quality-adjusted price levels in columns 2 through 4 correspond to methods of quality adjustment for the QUVI and correspond to the methods presented in Figure 4 Quality-adjusted price relatives in column 6 corresponds to the EP-TV method discussed in Section 2.4

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Table 4: Comparison to Alternative Quality-Adjusted Indices

				CUPI	
	QUVI	FQUVI	Hedonic Tornqvist	No CGR	30p CGR
	(1)	(2)	(3)	(4)	(5)
Desktop Computers	−16.6	−24.7	−16.2	−25.0	−22.2
Notebook Computers	−28.1	−36.1	−27.4	−49.1	−43.6
Stereo Headphones	−30.3	−32.2	−30.2	−57.4	−52.2
Streaming Audio Speakers	−28.0	−23.9	−29.6	−63.8	−52.0

Note: Values equal the cumulative rate of inflation measured using the QUVI with time-varying adjustment, the frictional QUVI, the hedonic Tornqvist, and CUPI. Columns 4 through 5 presents estimates for the CUPI under varying Common Goods Rules (CGRs), which requires that items are considered common or continuing goods only when their expenditure shares are above a certain percentile in both period $t-1$ and t. Appendix Section A.6.2 provides a discussion of CGRs and their effect on the CUPI.

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Table 5: Decomposing the Frictional QUVI

	QA Laspeyres	Within	Product-Mix	FQUVI
	(1)	(2)	(3)	(4)
Desktop Computers	−1.96	−1.86	−0.86	−2.72
Notebook Computers	−2.26	−2.17	−1.00	−3.18
Stereo Headphones	−2.49	−2.49	−1.16	−3.65
Streaming Audio Speakers	−1.80	−1.79	−1.29	−3.08

Note: This table presents the average quarterly decomposition of the frictional QUVI over our sample period. The formula for the decomposition of the frictional QUVI is provided in equation (4). Values in columns 2 and 3 sums to the value presented in column 4.

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Table 6: Decomposing Quality Growth in the Standard UVI

	Within	Product-Mix	Quality Growth
	(1)	(2)	(3)
Panel A: QUVI with Time-Varying Adjustment			
Desktop Computers	0.38	3.37	4.04
Notebook Computers	0.64	2.74	3.63
Stereo Headphones	1.01	8.99	10.74
Streaming Audio Speakers	0.39	3.75	5.32
Panel B: Frictional QUVI			
Desktop Computers	0.35	4.32	5.08
Notebook Computers	0.60	3.26	4.38
Stereo Headphones	1.05	9.24	11.15
Streaming Audio Speakers	0.49	3.35	4.90

Note: This table presents the average quarterly decomposition of the implied quality growth in the standard UVI over our sample period using the QUVI with time-varying adjustment and the frictional QUVI. Values in columns 1 and 2 sums to the value presented in column 3.

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A Technical Appendix

In this appendix, we expand on relevant materials not included in the main body of text. First, we decompose the UVI and QUVI. Second, we cover how the QUVI provides bounds on the rate of inflation. Third, we review the case when the UVI and QUVI act as the cost-of-living index for consumer demand. Fourth, we discuss a geo-mean price index, a geometric equivalent to the UVI. Fifth, we discuss alternative forms of quality adjustment for the QUVI from the literature. Lastly, we conclude with a discussion of alternative indices that provide a measure of inflation using the change in average prices including matched-model, hedonic, and demand based indices.

A.1 Decomposing Unit Value Indices

The formula for the unit value index (UVI) for a basket of goods (Ω) across adjacent period pairs $\{t-1,\ t\}$ is:

$$\begin{aligned} \text{UVI} &= \frac{\sum_{k\in\Omega_t} p_{kt}q_{kt} / \sum_{k\in\Omega_t} q_{kt}}{\sum_{k\in\Omega_{t-1}} p_{kt-1}q_{kt-1} / \sum_{k\in\Omega_{t-1}} q_{kt-1}} \\ &= \frac{\sum_{k\in\Omega_t} w_{kt}p_{kt}}{\sum_{k\in\Omega_{t-1}} w_{kt-1}p_{kt-1}} \end{aligned} \tag{A1}$$

where w_{kt} denotes the quantity share of good k in period t.[22] To write the UVI as an exact decomposition of within-item price changes and changes in product-mix, we decompose the level change in average prices following from equation (2) in Foster et al. (2001) (FHK):

$$\Delta P = \sum_{k\in C} w_{kt-1}\Delta p_k + \sum_{k\in C} \Delta w_k p_{kt-1} + \sum_{k\in C} \Delta w_k \Delta p_k + \sum_{k\in N} w_{kt}p_{kt} - \sum_{k\in X} w_{kt-1}p_{kt-1}$$

where C denotes the basket of continuing goods, N denotes the basket of entering goods, and X denotes the basket of exiting goods. We rewrite this decomposition in terms of the unit value index:

$$\text{UVI} = 1 + \sum_{k\in C} w_{kt-1}\left(\frac{\Delta p_k}{P_{t-1}}\right) + \sum_{k\in C} \Delta w_k \left(\frac{p_{kt}}{P_{t-1}}\right) + \sum_{k\in N} w_{kt}\left(\frac{p_{kt}}{P_{t-1}}\right) - \sum_{k\in X} w_{kt-1}\left(\frac{p_{kt-1}}{P_{t-1}}\right)$$

This expression has three terms:

[22] We also refer to the UVI as the standard UVI to differentiate it from the quality-adjusted UVI (or QUVI). Second, the additional equality shows the UVI is equal to the relative change in quantity share weighted average prices.

1. The Within term, $\sum_{k\in C} w_{kt-1}\left(\frac{\Delta p_k}{P_{t-1}}\right)$
2. The Continuers term, $\sum_{k\in C} \Delta w_k \left(\frac{p_{kt}}{P_{t-1}}\right)$
3. The Net-Entry term, $\sum_{k\in N} w_{kt}\left(\frac{p_{kt}}{P_{t-1}}\right) - \sum_{k\in X} w_{kt-1}\left(\frac{p_{kt-1}}{P_{t-1}}\right)$

We can rewrite the Within term to connect it to a traditional matched-model index:

$$\begin{aligned}\sum_{k\in C} w_{kt-1}\left(\frac{\Delta p_k}{P_{t-1}}\right) &= \sum_{k\in C}\left(\frac{p_{kt-1}q_{kt-1}}{\sum_{k\in\Omega_{t-1}} p_{kt-1}q_{kt-1}}\right)\left(\frac{p_{kt}}{p_{kt-1}}-1\right)\\ &= \left(\frac{\sum_{k\in C} p_{kt-1}q_{kt-1}}{\sum_{k\in\Omega_{t-1}} p_{kt-1}q_{kt-1}}\right)\sum_{k\in C}\left(\frac{p_{kt-1}q_{kt-1}}{\sum_{k\in C} p_{kt-1}q_{kt-1}}\right)\left(\frac{p_{kt}}{p_{kt-1}}-1\right)\\ &= s^c_{t-1}\sum_{k\in C} s^c_{kt-1}\left(\frac{p_{kt}}{p_{kt-1}}-1\right)\end{aligned}$$

where s^c denotes the expenditure share for the basket of continuing goods and s^c_k denotes the expenditure share on good k within the basket of continuing goods. Assuming that a good gains market share over the course of its life-cycle (i.e. $s^c_{t-1} \Rightarrow 1$), the Within term will approximate the traditional arithmetic Laspeyres price index. Taken together, the UVI is:

$$\begin{aligned}\text{UVI} = 1 + s^c_{t-1}\sum_{k\in C} s^c_{kt-1}\left(\frac{p_{kt}}{p_{kt-1}}-1\right) + \sum_{k\in C}\Delta w_k\left(\frac{p_{kt}}{P_{t-1}}\right)\\ + \sum_{k\in N} w_{kt}\left(\frac{p_{kt}}{P_{t-1}}\right) - \sum_{k\in X} w_{kt-1}\left(\frac{p_{kt-1}}{P_{t-1}}\right)\end{aligned} \tag{A2}$$

This set-up can also be applied to the quality-adjusted UVI (QUVI), whose formula is:

$$\begin{aligned}\text{QUVI} &= \frac{\sum_{k\in\Omega_t} p_{kt}q_{kt} / \sum_{k\in\Omega_t}\lambda_{kt}q_{kt}}{\sum_{k\in\Omega_{t-1}} p_{kt-1}q_{kt-1} / \sum_{k\in\Omega_{t-1}}\lambda_{kt-1}q_{kt-1}}\\ &= \frac{\sum_{k\in\Omega_t}\omega_{kt}(p_{kt}/\lambda_{kt})}{\sum_{k\in\Omega_{t-1}}\omega_{kt-1}(p_{kt-1}/\lambda_{kt-1})}\end{aligned} \tag{A3}$$

where ω_{kt} denotes the quality-adjusted quantity weight for good k in period t:

$$\omega_{kt} = \frac{\lambda_{kt}q_{kt}}{\sum_{k\in\Omega_t}\lambda_{kt}q_{kt}}$$

Given the structure of the UVI and QUVI are similar, the steps to decompose the QUVI are the same as the UVI. Hence, the QUVI is the sum of the same three terms that hold

now on a quality-adjusted basis:

$$\text{QUVI} = 1 + s^c_{t-1} \sum_{k \in C} s^c_{kt-1} \left(\frac{p_{kt}/\lambda_{kt}}{p_{kt-1}/\lambda_{kt-1}} - 1 \right) + \sum_{k \in C} \Delta\omega_k \left(\frac{p_{kt}/\lambda_{kt}}{P_{t-1}} \right) + \sum_{k \in N} \omega_{kt} \left(\frac{p_{kt}/\lambda_{kt}}{P_{t-1}} \right) - \sum_{k \in X} \omega_{kt-1} \left(\frac{p_{kt-1}/\lambda_{kt-1}}{P_{t-1}} \right) \tag{A4}$$

A.2 Quality Unit Value Indices as Bounds on Inflation

In this section, we build on work such as von Auer (2014) and de Haan (2015) to show how the QUVI with information from a first difference hedonic model can provide bounds on the rate of inflation which coincide with full-imputation hedonic indices such as the Laspeyres and Paasche indices. This bounding property requires a specific definition for quality adjustment factors, which we refer to as time-varying adjustment for the set of continuing goods.

In a scenario without product turnover, we begin by defining quality adjustment factors under the assumption that relative differences are pairwise time-invariant:

$$\lambda_{kt-1} = \lambda_{kt} \; \forall \; k \in \Omega_t$$

We express the common component to this set of adjustment factors as a function of average price dispersion:

$$\lambda_{k\tau} = \bar{\lambda}_k = \left(\frac{p_{kt}}{P_t} \right)^{\alpha} \left(\frac{p_{kt-1}}{P_{t-1}} \right)^{1-\alpha} \text{ for } \tau \in \{t-1,\ t\} \tag{A5}$$

where the timing parameter $\alpha \in [0, 1]$ determines the weight put on prices in period $t-1$ versus t. In an index, average prices in the quality adjustment factors will cancel out, meaning that the average price of a good is sufficient to normalize units in terms of each product's value. Note that de Haan (2015) would refer to equation (A5) as a form of basic adjustment.

To provide intuition on how the choice of α potentially impacts the QUVI, consider the cases where $\alpha \in \{0,\ 1\}$. When $\alpha = 0$, quality adjustment is based on the price level observed in the base period, and the QUVI simplifies to a Paasche index:

$$\begin{aligned} \frac{P_t}{P_{t-1}} &= \frac{\sum_{k \in \Omega} p_{kt} q_{kt} / \sum_{k \in \Omega} p_{kt-1} q_{kt}}{\sum_{k \in \Omega} p_{kt-1} q_{kt-1} / \sum_{k \in \Omega} p_{kt-1} q_{kt-1}} \\ &= \frac{\sum_{k \in \Omega} p_{kt} q_{kt}}{\sum_{k \in \Omega} p_{kt-1} q_{kt}} \end{aligned}$$

$$= \left(\sum_{k \in \Omega} s_{kt} \left[\frac{p_{kt}}{p_{kt-1}} \right]^{-1} \right)^{-1}$$

In the other extreme case where $\alpha = 1$, the QUVI simplifies to a Laspeyres index:

$$\begin{aligned} \frac{P_t}{P_{t-1}} &= \frac{\sum_{k \in \Omega} p_{kt} q_{kt} / \sum_{k \in \Omega} p_{kt} q_{kt}}{\sum_{k \in \Omega} p_{kt-1} q_{kt-1} / \sum_{k \in \Omega} p_{kt} q_{kt-1}} \\ &= \frac{\sum_{k \in \Omega} p_{kt} q_{kt-1}}{\sum_{k \in \Omega} p_{kt-1} q_{kt-1}} \\ &= \sum_{k \in \Omega} s_{kt-1} \left[\frac{p_{kt}}{p_{kt-1}} \right] \end{aligned}$$

For values of $\alpha \in (0,\ 1)$, the QUVI will fall somewhere between these two extreme cases.

The examples without product turnover are interesting but limited since product turnover and quality change is ubiquitous with item-level transactions data. Moreover, the QUVI approach has the appeal of addressing product turnover in the context of quality change. To extend the above to an environment with product turnover we use hedonics. Details of the EP-TV hedonic estimation are provided in the main paper in section 2.3 with further discussion in Appendix A.6.2. In this section, we provide more detail about how using hedonics enables the bounding results in section 2.3.

The use of hedonics via the EP-TV method enables incorporating product entry and exit as well as time-varying adjustment for continuing goods. In this time-varying environment, quality adjustment factors for a good k can be written as a combination of factors' average value as defined in the previous point and the change in factors:

$$\begin{aligned} \ln \lambda_{kt} &= \ln \bar{\lambda}_k + (1 - \alpha) \Delta \ln \lambda_k \\ \ln \lambda_{kt-1} &= \ln \bar{\lambda}_k - \alpha \Delta \ln \lambda_k \end{aligned} \tag{A6}$$

To recover the change in adjustment factors, we use the predicted value from the EP-TV first difference hedonic model:

$$\Delta \ln p_k / \lambda_k = \Delta \widehat{\ln p_k} \Rightarrow \Delta \ln \lambda_k = \Delta \ln p_k - \Delta \widehat{\ln p_k} \ \forall \ k \in C \tag{A7}$$

Taken together, quality-adjusted price relatives with time-varying adjustment match "characteristic-adjusted" price relatives for the first difference hedonic model. The EP-TV first difference model also permits us to measure the quality-adjustment factors for

entering and exiting products:

$$\text{(A8)} \qquad \begin{aligned} \ln \lambda_{kt} &= \ln p_{kt} - (1-\alpha)\widehat{\Delta \ln p_k} \quad \text{for entry} \\ \ln \lambda_{kt-1} &= \ln p_{kt-1} + \alpha \widehat{\Delta \ln p_k} \quad \text{for exit} \end{aligned}$$

where the definition for adjustment factors are consistent with our method of basic adjustment as in equation (A5).

For the case of $\alpha \in \{0,\ 1\}$, adding time-varying adjustment for the set of continuing goods changes the value of $\{\lambda_{kt-1},\ \lambda_{kt}\}$ as follows:

$$\text{(A9)} \qquad \lambda_{kt-1} = \begin{cases} p_{kt-1} & \text{when } \alpha = 0 \\ \exp(-\Delta \ln \lambda_k) \times p_{kt} & \text{when } \alpha = 1 \end{cases}$$

$$\lambda_{kt} = \begin{cases} \exp(\Delta \ln \lambda_k) \times p_{kt-1} & \text{when } \alpha = 0 \\ p_{kt} & \text{when } \alpha = 1 \end{cases}$$

When $\alpha = 0$, the QUVI with time-varying adjustment is equivalent to the full-imputation hedonic Paasche index:

$$\begin{aligned} \frac{P_t}{P_{t-1}} &= \frac{\sum_{k\in\Omega_t} p_{kt}q_{kt} / \sum_{k\in\Omega_{t-1}} p_{kt-1}q_{kt-1}}{\sum_{k\in\Omega_t} \lambda_{kt}q_{kt} / \sum_{k\in\Omega_{t-1}} \lambda_{kt-1}q_{kt-1}} \\ &= \frac{\sum_{k\in\Omega_t} p_{kt}q_{kt}}{\sum_{k\in C} \exp(\Delta \ln \lambda_k) p_{kt-1}q_{kt} + \sum_{k\in N} \tilde{p}_{kt-1}q_{kt}} \\ &= \left(\frac{\sum_{k\in C} \exp(\Delta \ln \lambda_k) p_{kt-1}q_{kt} + \sum_{k\in N} \tilde{p}_{kt-1}q_{kt}}{\sum_{k\in\Omega_t} p_{kt}q_{kt}} \right)^{-1} \\ &= \left(\sum_{k\in C} \frac{p_{kt}q_{kt}}{\sum_{k\in\Omega_t} p_{kt}q_{kt}} \left[\frac{\exp(\Delta \ln \lambda_k) p_{kt-1}}{p_{kt}} \right] + \sum_{k\in N} \frac{p_{kt}q_{kt}}{\sum_{k\in\Omega_t} p_{kt}q_{kt}} \left[\frac{\tilde{p}_{kt-1}}{p_{kt}} \right] \right)^{-1} \\ &= \left(\sum_{k\in C} s_{kt} \left[\widehat{\frac{p_{kt}}{p_{kt-1}}} \right]^{-1} + \sum_{k\in N} s_{kt} \left[\widehat{\frac{p_{kt}}{p_{kt-1}}} \right]^{-1} \right)^{-1} \end{aligned}$$

where the predicted price relative comes from the hedonic model: $\widehat{p_{kt}/p_{kt-1}} = \exp(\widehat{\Delta \ln p_k})$, the predicted price level uses the observed price level and the predicted price relative $\tilde{p}_{kt-1} = p_{kt}/\widehat{(p_{kt}/p_{kt-1})}$, and expenditure share s_{kt} are defined over the total basket of goods Ω_t that are available in period t. Note the expression $\tilde{p}_{kt-1}$ is consistent with the definition for quality adjustment factors in equation (A8) for entering goods when $\alpha = 0$.

Similarly for the case $\alpha = 1$, the QUVI with time-varying adjustment is equivalent to the full-imputation hedonic Laspeyres index:

$$\begin{aligned}
\frac{P_t}{P_{t-1}} &= \frac{\sum_{k\in\Omega_t} p_{kt}q_{kt} / \sum_{k\in\Omega_{t-1}} p_{kt-1}q_{kt-1}}{\sum_{k\in\Omega_t} \lambda_{kt}q_{kt} / \sum_{k\in\Omega_{t-1}} \lambda_{kt-1}q_{kt-1}} \\
&= \frac{\sum_{k\in C} \exp(-\Delta \ln \lambda_k) p_{kt}q_{kt-1} + \sum_{k\in X} \tilde{p}_{kt}q_{kt-1}}{\sum_{k\in\Omega_{t-1}} p_{kt-1}q_{kt-1}} \\
&= \sum_{k\in C} \frac{p_{kt-1}q_{kt-1}}{\sum_{k\in\Omega_{t-1}} p_{kt-1}q_{kt-1}} \left[\frac{\exp(-\Delta \ln \lambda_k) p_{kt}}{p_{kt-1}}\right] + \sum_{k\in X} \frac{p_{kt-1}q_{kt-1}}{\sum_{k\in\Omega_{t-1}} p_{kt-1}q_{kt-1}} \left[\frac{\tilde{p}_{kt}}{p_{kt-1}}\right] \\
&= \sum_{k\in C} s_{kt-1} \left[\widehat{\frac{p_{kt}}{p_{kt-1}}}\right] + \sum_{k\in X} s_{kt-1} \left[\widehat{\frac{p_{kt}}{p_{kt-1}}}\right]
\end{aligned}$$

where the predicted price level uses the observed price level and the predicted price relative $\tilde{p}_{kt} = \widehat{(p_{kt}/p_{kt-1})} \times p_{kt-1}$ and expenditure share s_{kt-1} are defined over the total basket of goods Ω_{t-1} that are available in period $t-1$. Again, note the expression $\tilde{p}_{kt}$ is consistent with the definition for quality adjustment factors in equation (A8) for exiting goods when $\alpha = 1$. Taken together, the QUVI with time-varying adjustment provides "characteristic-adjusted" bounds on the rate of inflation that are consistent with full-imputation hedonic indices.

Note that there is also an intermediate case that only uses the basic adjustment factors without incorporating time-varying adjustment for common goods. In this case, we show QUVI at $\alpha \in \{0, 1\}$ incorporates unobserved price relatives from either entering or exiting goods. Using the same approach as shown above, the QUVI with basic adjustment takes on the following formula at the extreme values of the timing parameter:

$$\frac{P_t}{P_{t-1}} = \begin{cases} \left(\sum_{k\in C} s_{kt} \left[\frac{p_{kt}}{p_{kt-1}}\right]^{-1} + \sum_{k\in N} s_{kt} \left[\widehat{\frac{p_{kt}}{p_{kt-1}}}\right]^{-1}\right)^{-1} & \text{for } \alpha = 0 \\ \sum_{k\in C} s_{kt-1} \left[\frac{p_{kt}}{p_{kt-1}}\right] + \sum_{k\in X} s_{kt-1} \left[\widehat{\frac{p_{kt}}{p_{kt-1}}}\right] & \text{for } \alpha = 1 \end{cases}$$

Hence, the basic approach at extreme values of the timing parameter equates to the bounds provides by Paasche and Laspeyres indices inclusive of entry and exit. As noted in the main text, there hybrid hedonic indices are equivalent to double imputation hedonic indices where price relatives for continuing goods are used and hedonic based imputed price relatives are used for entering and exiting goods. As discussed in Erickson and Pakes (2011) and Ehrlich et al. (2025), these hybrid indices treat the residual from hedonics in an asymmetric way so we prefer the full hedonic approach in the time-varying adjustment case.

Lastly, how do we reconcile these bounds with the decomposition of the QUVI? Below we show how this works with time-varying adjustment. However, the connection between the bounding properties of the QUVI and its decomposition is not dependent on using time-varying adjustment factors for the set of continuing goods. The same conclusion would hold under the basic approach as well. The primary difference is that the quality-adjusted price relatives would be equivalent to the observed price relative for continuing goods. Starting with the decomposition:

$$\frac{P_t}{P_{t-1}} = 1 + s^c_{t-1} \sum_{k \in C} s^c_{kt-1} \left(\frac{p_{kt}/\lambda_{kt}}{p_{kt-1}/\lambda_{kt-1}} - 1 \right) + \sum_{k \in C} \Delta\omega_k \left(\frac{p_{kt}/\lambda_{kt}}{P_{t-1}} \right) + \sum_{k \in N} \omega_{kt} \left(\frac{p_{kt}/\lambda_{kt}}{P_{t-1}} \right) - \sum_{k \in X} \omega_{kt-1} \left(\frac{p_{kt-1}/\lambda_{kt-1}}{P_{t-1}} \right)$$

For the case when $\alpha = 0$, the decomposition simplifies to the following expression:

$$\frac{P_t}{P_{t-1}} = s^c_{t-1} \sum_{k \in C} s^c_{kt-1} \left(\widehat{\frac{p_{kt}}{p_{kt-1}}} \right) + \sum_{k \in C} \Delta\omega_k \left(\widehat{\frac{p_{kt}}{p_{kt-1}}} \right) + \sum_{k \in N} \omega_{kt} \left(\widehat{\frac{p_{kt}}{p_{kt-1}}} \right)$$

where the changes in product-mix reflect shifts in quality-adjusted quantity shares weighted by the hedonic price relative for entering and continuing goods. Simplifying this expression further brings us back to the full-imputation hedonic Paasche:

$$\begin{aligned}
\frac{P_t}{P_{t-1}} &= s^c_{t-1} \sum_{k \in C} s^c_{kt-1} \left(\widehat{\frac{p_{kt}}{p_{kt-1}}} \right) + \sum_{k \in C} \Delta\omega_k \left(\widehat{\frac{p_{kt}}{p_{kt-1}}} \right) + \sum_{k \in N} \omega_{kt} \left(\widehat{\frac{p_{kt}}{p_{kt-1}}} \right) \\
&= \sum_{k \in C} \omega_{kt} \left(\widehat{\frac{p_{kt}}{p_{kt-1}}} \right) + \sum_{k \in N} \omega_{kt} \left(\widehat{\frac{p_{kt}}{p_{kt-1}}} \right) \\
&= \sum_{k \in C} \omega_{kt} \left(\frac{p_{kt}}{\exp(-\Delta \ln \lambda_k) p_{kt-1}} \right) + \sum_{k \in N} \omega_{kt} \left(\frac{p_{kt}}{\exp(-\Delta \widehat{\ln p_k}) p_{kt}} \right) \\
&= \frac{\sum_{k \in \Omega_t} p_{kt} q_{kt}}{\sum_{k \in C} \exp(-\Delta \ln \lambda_k) p_{kt-1} q_{kt} + \sum_{k \in N} \exp(-\Delta \widehat{\ln p_k}) p_{kt} q_{kt}} \\
&= \left(\frac{\sum_{k \in C} \exp(-\Delta \ln \lambda_k) p_{kt-1} q_{kt} + \sum_{k \in N} \exp(-\Delta \widehat{\ln p_k}) p_{kt} q_{kt}}{\sum_{k \in \Omega_t} p_{kt} q_{kt}} \right)^{-1} \\
&= \left(\sum_{k \in C} s_{kt} \left[\widehat{\frac{p_{kt}}{p_{kt-1}}} \right]^{-1} + \sum_{k \in N} s_{kt} \left[\widehat{\frac{p_{kt}}{p_{kt-1}}} \right]^{-1} \right)^{-1}
\end{aligned}$$

For the case $\alpha = 1$, we begin manipulating the QUVI decomposition:

$$\frac{P_t}{P_{t-1}} = 1 + \sum_{k \in C} \omega_{kt-1} \left(\frac{\Delta p_k / \lambda_k}{P_t} \right) + \sum_{k \in C} \Delta \omega_k \left(\frac{p_{kt} / \lambda_{kt}}{P_{t-1}} \right) + \sum_{k \in N} \omega_{kt} \left(\frac{p_{kt} / \lambda_{kt}}{P_{t-1}} \right) - \sum_{k \in X} \omega_{kt-1} \left(\frac{p_{kt-1} / \lambda_{kt-1}}{P_{t-1}} \right)$$

Assuming time-varying adjustment:

$$\frac{P_t}{P_{t-1}} = 1 + \sum_{k \in C} \omega_{kt-1} \left(\frac{\widehat{1 - p_{kt-1} / p_{kt}}}{P_{t-1}} \right) + \sum_{k \in C} \Delta \omega_k \left(\frac{1}{P_{t-1}} \right) + \sum_{k \in N} \omega_{kt} \left(\frac{1}{P_{t-1}} \right) - \sum_{k \in X} \omega_{kt-1} \left(\frac{p_{kt-1} / \tilde{p}_{kt}}{P_{t-1}} \right)$$

Through algebraic manipulation:

$$\begin{aligned}
\frac{P_t}{P_{t-1}} &= 1 + \left(\frac{1}{P_{t-1}} \right) - \sum_{k \in C} \omega_{kt-1} \left(\frac{\widehat{p_{kt-1} / p_{kt}}}{P_{t-1}} \right) - \sum_{k \in X} \omega_{kt-1} \left(\frac{p_{kt-1} / \tilde{p}_{kt}}{P_{t-1}} \right) \\
&= \frac{1}{P_{t-1}} \pm 1 \\
&= \frac{1}{\sum_{k \in \Omega_{t-1}} p_{kt-1} q_{kt-1} / \sum_{k \in \Omega_{t-1}} \lambda_{kt-1} q_{kt-1}} \\
&= \frac{\sum_{k \in C} \exp(-\Delta \ln \lambda_k) p_{kt} q_{kt-1} + \sum_{k \in X} \tilde{p}_{kt} q_{kt-1}}{\sum_{k \in \Omega_{t-1}} p_{kt-1} q_{kt-1}} \\
&= \sum_{k \in C} \frac{\exp(-\Delta \ln \lambda_k) p_{kt} q_{kt-1}}{\sum_{k \in \Omega_{t-1}} p_{kt-1} q_{kt-1}} + \sum_{k \in X} \frac{\tilde{p}_{kt} q_{kt-1}}{\sum_{k \in \Omega_{t-1}} p_{kt-1} q_{kt-1}} \\
&= \sum_{k \in C} s_{kt-1} \left(\frac{\exp(-\Delta \ln \lambda_k) p_{kt}}{p_{kt-1}} \right) + \sum_{k \in X} s_{kt-1} \left(\frac{\tilde{p}_{kt}}{p_{kt-1}} \right) \\
&= \sum_{k \in C} s_{kt-1} \left(\widehat{\frac{p_{kt}}{p_{kt-1}}} \right) + \sum_{k \in X} s_{kt-1} \left(\widehat{\frac{p_{kt}}{p_{kt-1}}} \right)
\end{aligned}$$

which shows that the QUVI decomposition simplifies to the full-imputation hedonic Laspeyres. Given the first term in the hedonic Laspeyres is equivalent to the Within term in the QUVI decomposition (i.e. $s_{kt-1} = s^c_{t-1} s^c_{kt-1}$), the additional product-mix terms in the decomposition account for the unobserved price changes from the set of exiting goods.

A.3 Unit and Quality-Adjusted Unit Value Indices as Exact Price Indices

A.3.1 Standard Unit Value Indices

Assuming consumer preferences follow a perfect substitutes aggregator across homogeneous varieties for a basket of goods (i.e. $Q = \sum_{\Omega} q_k$), we can show the UVI equals the exact price index under this preference assumption. The utility maximization problem for the representative consumer is:

$$\max_{\{q_{kt}\}} \sum_{k \in \Omega_t} q_{kt} \text{ such that } E_t = \sum_{k \in \Omega_t} p_{kt} q_{kt}$$

where E_t denotes total expenditures. To first derive the law of one price (or LOOP), we state the Lagrangian and take the first order condition with respect to the purchased quantity of good k:

$$\begin{aligned} \mathcal{L} &= \sum_{k \in \Omega_t} q_{kt} - L \left(E_t - \sum_{k \in \Omega_t} p_{kt} q_{kt} \right) \\ \frac{\partial \mathcal{L}}{\partial q_{kt}} &= 1 - p_{kt} L = 0 \; \forall \; k \in \Omega_t \\ p_{kt} &= L^{-1} \; \forall \; k \in \Omega_t \end{aligned}$$

By extension, the LOOP arises from the budget constraint:

$$E_t = \sum_{k \in \Omega_t} \frac{q_{kt}}{L} \Rightarrow L^{-1} = E_t / Q_t|_{Q_t = 1} \equiv P_t \Rightarrow p_{kt} = P_t \; \forall \; k \in \Omega_t$$

where P_t denotes the average price for all goods $k \in \Omega_t$. To derive a formula for the exact price index, we aggregate the LOOP across adjacent period pairs:

$$\begin{aligned} P_t q_{kt} &= p_{kt} q_{kt} \\ P_t &= \sum_{k \in \Omega_t} p_{kt} q_{kt} / \sum_{k \in \Omega_t} q_{kt} \\ \frac{P_t}{P_{t-1}} &= \frac{\sum_{k \in \Omega_t} p_{kt} q_{kt} / \sum_{k \in \Omega_t} q_{kt}}{\sum_{k \in \Omega_{t-1}} p_{kt-1} q_{kt-1} / \sum_{k \in \Omega_{t-1}} q_{kt-1}} \end{aligned}$$

Lastly, note under the law of one price that quantity and expenditure shares equalize:

$$p_{kt} = P_t \; \forall \; k \in \Omega_t \Rightarrow \frac{q_{kt}}{\sum_{k \in \Omega_t} q_{kt}} = \frac{p_{kt} q_{kt}}{\sum_{k \in \Omega_t} p_{kt} q_{kt}}$$

which will prove valuable when we derive results under a quality-adjusted LOOP.

Next, we illustrate how the assumed LOOP impacts the interpretation of the UVI as an exact price index. If we apply the law of one price to the decomposition of the UVI, we can simplify terms:

1. The Continuers term is:

$$\begin{aligned} \sum_{k \in C} \Delta w_k \left(\frac{p_{kt}}{P_{t-1}} \right) &= \left(\frac{P_t}{P_{t-1}} \right) \left(w_t^c - w_{t-1}^c \right) \\ &= \left(\frac{P_t}{P_{t-1}} \right) \left(s_t^c - s_{t-1}^c \right) \end{aligned}$$

   where quantity and expenditure share weights equalize under the law of one price.

2. The Net-Entry term is:

$$\begin{aligned} \sum_{k \in N} w_{kt} \left(\frac{p_{kt}}{P_{t-1}} \right) - \sum_{k \in X} w_{kt-1} \left(\frac{p_{kt-1}}{P_{t-1}} \right) &= \left(\frac{P_t}{P_{t-1}} \right) \left(1 - w_t^c \right) - \left(1 - w_{t-1}^c \right) \\ &= \left(\frac{P_t}{P_{t-1}} \right) \left(1 - s_t^c \right) - \left(1 - s_{t-1}^c \right) \end{aligned}$$

3. Combining terms and simplifying:

$$\begin{aligned} \frac{P_t}{P_{t-1}} &= s_{t-1}^c \sum_{k \in C} s_{kt-1}^c \left(\frac{p_{kt}}{p_{kt-1}} \right) + \left(\frac{P_t}{P_{t-1}} \right) \left(1 - s_{t-1}^c \right) \\ &= \sum_{k \in C} s_{kt-1}^c \left(\frac{p_{kt}}{p_{kt-1}} \right) \end{aligned}$$

Taken together, the UVI simplifies to the arithmetic Laspeyres price index under the LOOP:

(A10) $$p_{k\tau} = P_\tau \; \forall \; k \in \Omega_{t-1} \cup \Omega_t \text{ and } \tau \in \{t-1, \; t\} \; \Rightarrow \; \text{UVI} = \sum_{k \in C} s_{kt-1}^c \left(\frac{p_{kt}}{p_{kt-1}} \right)$$

A.3.2 Quality-Adjusted UVIs without Market Frictions

We now consider a basket of nearly homogeneous goods, which differ in terms of quality. Assuming consumer preferences follow a perfect substitutes aggregator with quality differences (λ) across the relevant set of goods (i.e. $Q = \sum_{\Omega} \lambda_k q_k$), we can show the QUVI equals the exact price index under this preference assumption. The steps to derive this equality essentially repeats the steps for the standard UVI, where the utility maximization problem is:

$$\max_{\{q_{kt}\}} \sum_{k \in \Omega_t} \lambda_{kt} q_{kt} \text{ such that } E_t = \sum_{k \in \Omega_t} p_{kt} q_{kt}$$

Solving the utility maximization problem, the quality-adjusted law of one price (QLOOP) arises:

$$p_{kt} = \lambda_{kt} P_t \; \forall \; k \in \Omega_t$$

where individual prices (p_{kt}) differs from the average price (P_t) as a result of quality differences (λ_{kt}). To derive a formula for the exact price index, we aggregate the law of one price across adjacent period pairs:

$$\begin{aligned} \lambda_{kt} P_t q_{kt} &= p_{kt} q_{kt} \\ P_t &= \sum_{k \in \Omega_t} p_{kt} q_{kt} / \sum_{k \in \Omega_t} \lambda_{kt} q_{kt} \\ \frac{P_t}{P_{t-1}} &= \frac{\sum_{k \in \Omega_t} p_{kt} q_{kt} / \sum_{k \in \Omega_t} \lambda_{kt} q_{kt}}{\sum_{k \in \Omega_{t-1}} p_{kt-1} q_{kt-1} / \sum_{k \in \Omega_{t-1}} \lambda_{kt-1} q_{kt-1}} \end{aligned}$$

Lastly, note the QLOOP produces an equality between quality-adjusted quantity shares and expenditure shares:

$$\frac{\lambda_{kt} q_{kt}}{\sum_{k \in \Omega_t} \lambda_{kt} q_{kt}} = \frac{p_{kt} q_{kt}}{\sum_{k \in \Omega_t} p_{kt} q_{kt}}$$

which departs from the equality of quantity and expenditure share weights under the standard LOOP.

Similar to the case of the UVI, we find that under the QLOOP the QUVI will again collapse to an arithmetic Laspeyres with quality-adjusted price relatives. By applying the QLOOP, we can recast each component in the decomposition of the QUVI to be consistent with theory:

1. The Continuers term is:

$$\sum_{k \in C} \Delta\omega_k \left(\frac{p_{kt}/\lambda_{kt}}{P_{t-1}} \right) = \left(\frac{P_t}{P_{t-1}} \right) \left(\omega_t^c - \omega_{t-1}^c \right)$$
$$= \left(\frac{P_t}{P_{t-1}} \right) \left(s_t^c - s_{t-1}^c \right)$$

   where quality-adjusted quantity and expenditure shares equalize under the QLOOP.

2. The Net Entry term is:

$$\sum_{k \in N} \omega_{kt} \left(\frac{p_{kt}/\lambda_{kt}}{P_{t-1}} \right) - \sum_{k \in X} \omega_{kt-1} \left(\frac{p_{kt-1}/\lambda_{kt-1}}{P_{t-1}} \right) = \left(\frac{P_t}{P_{t-1}} \right) \left(1 - \omega_t^c \right) - \left(1 - \omega_{t-1}^c \right)$$
$$= \left(\frac{P_t}{P_{t-1}} \right) \left(1 - s_t^c \right) - \left(1 - s_{t-1}^c \right)$$

3. Combining terms and simplifying:

$$\frac{P_t}{P_{t-1}} = s_{t-1}^c \sum_{k \in C} s_{kt-1}^c \left(\frac{p_{kt}/\lambda_{kt}}{p_{kt-1}/\lambda_{kt-1}} \right) + \left(1 - s_{t-1}^c \right) \left(\frac{P_t}{P_{t-1}} \right)$$
$$= \sum_{k \in C} s_{kt-1}^c \left(\frac{p_{kt}/\lambda_{kt}}{p_{kt-1}/\lambda_{kt-1}} \right)$$

Taken together, the QUVI simplifies to a quality-adjusted Laspeyres price index under the QLOOP:

$$p_{k\tau} = \lambda_{k\tau} P_\tau \text{ for } \tau \in \{t-1,\ t\} \ \Rightarrow\ \text{QUVI} = \sum_{k \in C} s_{kt-1}^c \left(\frac{p_{kt}/\lambda_{kt}}{p_{kt-1}/\lambda_{kt-1}} \right) \tag{A11}$$

A.3.3 Quality-Adjusted UVIs with Market Frictions

To begin constructing the QUVI with residual price dispersion, we must allow for quality-adjusted price dispersion across goods. That is, equation (8) needs to hold. This can be generated in a number of ways.

First, we presuppose there is a level of consumption q_{kt}^* across $k \in \Omega_t$ such that the law of one price must hold:

$$p_{kt} = \lambda_{kt} P_t \iff q_{kt} = q_{kt}^* \ \forall\ k \in \Omega_t$$

Second, consumers are constrained when deviating from this specific level of consumption:

$$\lambda_{kt}\left(q_{kt}-q_{kt}^{*}\right)\leq Q_{kt} \tag{A12}$$

which represents an upper bound on the additional number of units purchased, reflecting a lower bound on the corresponding price paid at the level of consumption. Resolving the utility maximization problem:

$$\begin{aligned}\mathcal{L}=\sum_{k\in\Omega_t}\lambda_{kt}q_{kt}-L\left(E_t-\sum_{k\in\Omega_t}p_{kt}q_{kt}\right)-\sum_{k\in\Omega_t}\tau_{kt}L\left(Q_{kt}/\lambda_{kt}-q_{kt}+q_{kt}^{*}\right)\\ \frac{\partial\mathcal{L}}{\partial q_{kt}}=\lambda_{kt}-p_{kt}L-\tau_{kt}L=0\;\forall\;k\in\Omega_t\\ \frac{p_{kt}+\tau_{kt}}{\lambda_{kt}}=L^{-1}\;\forall\;k\in\Omega_t\end{aligned}$$

where the Lagrange multiplier τ_{kt} represents the relative value of relaxing the budget constraint and purchasing an additional unit of some good k. Following the same procedure as before, we can derive the frictional law of one price (FQLOOP):

$$p_{kt}+\tau_{kt}=\lambda_{kt}\left(P_t+\Gamma_t\right)$$

where $\Gamma_t=\sum_{k\in\Omega_t}\omega_{kt}(\tau_{kt}/\lambda_{kt})$ which denotes the quality-adjusted average wedge between individual good prices and the quality-adjusted average price. For simplicity, we rewrite the FQLOOP where wedges (or residual price dispersion) are multiplicative:

$$z_{kt}p_{kt}=\lambda_{kt}P_t\;\forall\;k\in\Omega_t \tag{A13}$$

where $z_{kt}=1+(\tau_{kt}-\lambda_{kt}\Gamma_t)/p_{kt}$. The primary issues, which is the same as in BKM, is that the definition of quality is now dependent on the degree of quality-adjusted price dispersion.

While previous iterations of the LOOP additionally provided a connection between (quality-adjusted) quantity shares and expenditure share, this does not hold with respect to the FQLOOP. Instead by manipulating the expression for quality-adjusted quantity shares using the formula for average prices and the FQLOOP, we can show instead that

there is a normalization of expenditure share weighted average wedge:

$$\begin{aligned}\frac{\omega_{kt}}{z_{kt}} &= \frac{1}{z_{kt}} \frac{\lambda_{kt} q_{kt}}{\sum_{k \in \Omega_t} \lambda_{kt} q_{kt}} \\ &= \frac{p_{kt}/\lambda_{kt}}{P_t} \frac{\lambda_{kt} q_{kt}}{\sum_{k \in \Omega_t} \lambda_{kt} q_{kt}} \\ &= \frac{p_{kt}/\lambda_{kt}}{\sum_{k \in \Omega_t} p_{kt} q_{kt} / \sum_{k \in \Omega_t} \lambda_{kt} q_{kt}} \frac{\lambda_{kt} q_{kt}}{\sum_{k \in \Omega_t} \lambda_{kt} q_{kt}} \\ &= \frac{p_{kt} q_{kt}}{\sum_{k \in \Omega_t} p_{kt} q_{kt}}\end{aligned}$$

Aggregating implies expenditure share weighted residuals equal one:

$$\sum_{k \in \Omega_t} s_{kt} z_{kt} = 1 \tag{A14}$$

For the QUVI to retain its interpretation as an exact price index, the previous equation must hold. Alternatively, it is equivalent to suppose that the quality-adjusted quantity share weighted harmonic mean is equal to one (i.e. $\left(\sum_k \omega_{kt} z_{kt}^{-1}\right)^{-1} = 1$). Lastly, note that z_{kt} is the multiplicative expression of residual price dispersion, and this normalization is with respect to the quality-adjusted average wedge.

Alternatively, we also note that equation (8) can be motivated in a distinct way. Consider a simple wedge version of the utility maximization problem:

$$\begin{aligned}\mathcal{L} &= \sum_{k \in \Omega_t} \lambda_{kt} q_{kt} - L \left(E_t - \sum_{k \in \Omega_t} p_{kt} z_{kt} q_{kt} \right) \\ \frac{\partial \mathcal{L}}{\partial q_{kt}} &= \lambda_{kt} - p_{kt} z_{kt} L = 0 \; \forall \; k \in \Omega_t \\ \frac{p_{kt} z_{kt}}{\lambda_{kt}} &= L^{-1} \; \forall \; k \in \Omega_t\end{aligned}$$

In this case the wedges z_{kt} are in the spirit of Hsieh and Klenow (2009) as implicit distortions impacting the allocation of consumption. These wedges are placeholders for constraints (such as the type modeled above) that distort allocation. Wedges can be greater or less than one.

Following the same procedure as before, we can derive the frictional law of one price (FQLOOP):

$$p_{kt} z_{kt} = \lambda_{kt} P_t$$

Note in this implicit wedge environment the consumer spends all of its available expen-

ditures E_t but the allocation is distorted by the wedges. This implies

$$E_t = \sum_{k \in \Omega_t} p_{kt} z_{kt} q_{kt} = \sum_{k \in \Omega_t} p_{kt} q_{kt} \tag{A15}$$

In other words, the same relationship of aggregate expenditure weighted z_{kt} holds in this case given by:

$$\sum_{k \in \Omega_t} s_{kt} z_{kt} = 1 \tag{A16}$$

Given residual price dispersion is not explicitly present in the QUVI, its decomposition is equivalent to equation (A4). However, the decomposition under the FQLOOP deviates from the previous results along two important dimensions. To illustrate, we follow similar step in applying the FQLOOP to each term in the FHK decomposition:

- PMC term is:

$$\begin{aligned}
\sum_{k \in C} \Delta\omega_k \left(\frac{p_{kt}/\lambda_{kt}}{P_{t-1}} \right) &= \frac{P_t}{P_{t-1}} \sum_{k \in C} \frac{\Delta\omega_k}{z_{kt}} \\
&= \frac{P_t}{P_{t-1}} \sum_{k \in C} \frac{s_{kt} z_{kt} - s_{kt-1} z_{kt-1}}{z_{kt}} \\
&= \frac{P_t}{P_{t-1}} \sum_{k \in C} \left[s_{kt} z_{kt} - s_{kt} \left(\frac{s_{kt-1} z_{kt-1}}{s_{kt} z_{kt}} \right) \right] \\
&= \frac{P_t}{P_{t-1}} \left[s_t^c - \sum_{k \in C} s_{kt} \left(\frac{\omega_{kt}}{\omega_{kt-1}} \right)^{-1} \right] \\
&= s_t^c \frac{P_t}{P_{t-1}} \left[1 - \sum_{k \in C} s_{kt}^c \left(\frac{\omega_{kt}}{\omega_{kt-1}} \right)^{-1} \right]
\end{aligned}$$

- Net entry term is:

$$\begin{aligned}
\sum_{k \in N} \omega_{kt} \left(\frac{p_{kt}/\lambda_{kt}}{P_{t-1}} \right) - \sum_{k \in X} \omega_{kt-1} \left(\frac{p_{kt-1}/\lambda_{kt-1}}{P_{t-1}} \right) &= \left(\frac{P_t}{P_{t-1}} \right) \sum_{k \in N} \frac{\omega_{kt}}{z_{kt}} - \sum_{k \in X} \frac{\omega_{kt-1}}{z_{kt-1}} \\
&= \left(\frac{P_t}{P_{t-1}} \right) \sum_{k \in N} s_{kt} - \sum_{k \in X} s_{kt-1} \\
&= \left(\frac{P_t}{P_{t-1}} \right) (1 - s_t^c) - (1 - s_{t-1}^c)
\end{aligned}$$

- Combining terms and simplifying:

$$\begin{aligned}\frac{P_t}{P_{t-1}} &= s^c_{t-1}\sum_{k\in C} s^c_{kt-1}\left(\frac{p_{kt}/\lambda_{kt}}{p_{kt-1}/\lambda_{kt-1}}\right) + \frac{P_t}{P_{t-1}}\left[1 - s^c_t\sum_{k\in C} s^c_{kt}\left(\frac{\omega_{kt}}{\omega_{kt-1}}\right)^{-1}\right] \\ &= \left[\frac{s^c_t}{s^c_{t-1}}\right]^{-1}\left[\sum_{k\in C} s^c_{kt}\left(\frac{\omega_{kt}}{\omega_{kt-1}}\right)^{-1}\right]^{-1}\left[\sum_{k\in C} s^c_{kt-1}\left(\frac{p_{kt}/\lambda_{kt}}{p_{kt-1}/\lambda_{kt-1}}\right)\right]\end{aligned}$$

Taken together, the QUVI represents the quality-adjusted Laspeyres plus two additional product mix terms:

$$\frac{P_t}{P_{t-1}} = \left[\frac{s^c_t}{s^c_{t-1}}\right]^{-1}\left[\sum_{k\in C} s^c_{kt}\left(\frac{\omega_{kt}}{\omega_{kt-1}}\right)^{-1}\right]^{-1}\left[\sum_{k\in C} s^c_{kt-1}\left(\frac{p_{kt}/\lambda_{kt}}{p_{kt-1}/\lambda_{kt-1}}\right)\right] \tag{A17}$$

Hence, the different functional form assumptions for the law of one price have an important role on the functional form of the QUVI. The same result would hold with the UVI when all price dispersion is a result of wedges like the set of z's used in the FQLOOP.

A.3.4 Measuring Product Quality with Residual Price Dispersion

With the assumptions from Appendix Section A.3.3, we again focus on the interpretation that quality adjustment factors reflect the true level of product quality and must satisfy the FQLOOP in equation (A13). In this case, we define adjustment factors that build on the basic and time-varying adjustment presented in the previous section but additionally account for residual price dispersion under two alternative values for the timing parameter $\alpha \in \{0, 1\}$. For the set of continuing goods, the adjustment factors account for quality-adjusted price dispersion and time-varying adjustment:

$$\lambda_{kt-1} = \begin{cases} z_{kt-1}p_{kt-1} & \text{when } \alpha = 0 \\ \exp(-\Delta \ln \lambda_k) \times (z_{kt}p_{kt}) & \text{when } \alpha = 1 \end{cases}$$

$$\lambda_{kt} = \begin{cases} \exp(\Delta \ln \lambda_k) \times (z_{kt-1}p_{kt-1}) & \text{when } \alpha = 0 \\ z_{kt}p_{kt} & \text{when } \alpha = 1 \end{cases} \tag{A18}$$

When products are entering and exiting, we assume that wedges are normalized to either the start of their life-cycle when $\alpha = 0$ or the end of their life-cycle when $\alpha = 1$.[23]

[23] This assumption also applies to the beginning or end of our sample period.

From there, we follow the evolution of price dispersion over their life-cycle. In either case, there will be a period where adjustment factors are defined using the unobserved price level for either set of goods. When $\alpha = 0$, the factors will depend on the base period price level, which is unobserved for goods that enter in the current period. We use the EP-TV's hedonic prediction to impute the unobserved price level in these cases. For the set of entering goods, the definition of λ_{kt} is:

$$\lambda_{kt} = \begin{cases} p_{kt} / \exp(\widehat{\Delta \ln p_k}) & \text{when } \alpha = 0 \\ z_{kt} p_{kt} & \text{when } \alpha = 1 \end{cases} \tag{A19}$$

where the expression the imputed price level $\tilde{p}_{kt-1} = p_{kt} / \exp(\widehat{\Delta \ln p_k})$ and residual price dispersion is normalized to one in the case $\alpha = 0$. Similarly for the set of exiting goods, the definition of λ_{kt-1} is:

$$\lambda_{kt-1} = \begin{cases} z_{kt-1} p_{kt-1} & \text{when } \alpha = 0 \\ p_{kt-1} \times \exp(\widehat{\Delta \ln p_k}) & \text{when } \alpha = 1 \end{cases} \tag{A20}$$

where the expression the imputed price level $\tilde{p}_{kt} = p_{kt-1} \times \exp(\widehat{\Delta \ln p_k})$ and residual price dispersion is normalized to one in the case $\alpha = 1$. Note that entry and exit are in reference to the life-cycle process of goods. For goods that temporarily exit and then re-enter due to a stock out, we retain the value of z_k in between periods.

The effect of residual price dispersion on the measured rate of inflation ultimately depends on how weights respond to deviations from the quality-adjusted law of one price. Taking advantage of the frictional QUVI's formula, we find that its derivative with respect to an increase residual price dispersion for a given item is equal to the negative change in that item's quality-adjusted quantity share for either value of $\alpha \in \{0,\ 1\}$:

$$\frac{\partial P_t / P_{t-1}}{\partial z_{k\tau}} \frac{z_{k\tau}}{P_t / P_{t-1}} = -\Delta \omega_k \text{ where } \begin{cases} \tau = t-1 & \text{when } \alpha = 0 \\ \tau = t & \text{when } \alpha = 1 \end{cases}$$

To interpret this effect, first note that an increase in residual price dispersion represents a decrease in an item's relative quality-adjusted price, which may reflect high information frictions or search costs associated with new products as was the case for semi-conductors in in BKM for example. Furthermore, quality-adjusted quantity shares reflect the representativeness of an item when calculating the change in average quality-adjusted prices and change as a result of two factors: (1) a change in a good's quantity shares and (2) a

change in a good's relative quality. At the item level, we can expect the frictional QUVI to fall with respect to an increase in residual price dispersion when new products are of a higher relative quality and increase in popularity over time, which is again consistent with the empirical context of BKM for example.

A.4 An Alternative Geo-Mean Price Index

A.4.1 Geo-Mean Price Indices as an Exact Price Index

Relative to the standard UVI, there is also a geo-mean price index that can be generated from the law of one price:

$$\Delta \ln P = \sum_{\Omega_t} s_{kt} \ln p_{kt} - \sum_{\Omega_{t-1}} s_{kt-1} \ln p_{kt-1}$$

which is aggregated using expenditure shares.[24] Using the decomposition from equation (3) in Foster et al. (2001), the geo-mean price index is:

$$\Delta \ln P = \sum_{k \in C} \bar{s}_k \Delta \ln p_k + \sum_{k \in C} \Delta s_k \ln \bar{p}_k + \sum_{k \in N} s_{kt} \ln p_{kt} - \sum_{k \in X} s_{kt-1} \ln p_{kt-1}$$

Observe that the first term (the within) is similar to a matched model Tornqvist – the difference is the $\bar{s}_k$ reflects averages of expenditure shares in $t-1$ and t using all goods in these periods respectively.

By applying the law of one price, we can simplify the geo-mean price index to a geo-Laspeyres which is similar to the connection between the standard UVI to the arithmetic version of the matched-model index:

1. Within and Continuers terms:

$$\begin{aligned}
\sum_{k \in C} \bar{s}_k \Delta \ln p_k + \sum_{k \in C} \Delta s_k \ln \bar{p}_k &= \sum_{k \in C} s_{kt} \ln p_{kt} - \sum_{k \in C} s_{kt-1} \ln p_{kt-1} \\
&= \sum_{k \in C} s_{kt-1} \Delta \ln p_k + \sum_{k \in C} \Delta s_k \ln p_{kt} \\
&= s^c_{t-1} \sum_{k \in C} s^c_{kt-1} \Delta \ln p_k + \ln P_t \; \Delta s^c
\end{aligned}$$

[24] We use expenditure shares for weights. If the law of one price holds these are equivalent to quantity shares. We don't need that requirement for the decomposition. However, the product mix terms only are equal to zero under this condition.

2. Net Entry:

$$\sum_{k\in N} s_{kt} \ln p_{kt} - \sum_{k\in X} s_{kt-1} \ln p_{kt-1} = \Delta \ln P - \left(\ln P_t s_t^c - \ln P_{t-1} s_{t-1}^c\right)$$

3. Combining terms:

$$\begin{aligned}
\Delta \ln P &= s_{t-1}^c \sum_{k\in C} s_{kt-1}^c \Delta \ln p_k + \ln P_t \, \Delta s^c + \Delta \ln P - \left(\ln P_t s_t^c - \ln P_{t-1} s_{t-1}^c\right) \\
&= s_{t-1}^c \sum_{k\in C} s_{kt-1}^c \Delta \ln p_k + \left(1 - s_{t-1}^c\right) \Delta \ln P \\
&= \sum_{k\in C} s_{kt-1}^c \Delta \ln p_k
\end{aligned}$$

Hence, the geo-mean price will be equivalent to the geo-Laspeyres under the LOOP.

Similarly, a quality-adjusted geo-mean price index can be expressed as follows:[25]

$$\Delta \ln P = \sum_{\Omega_t} s_{kt} \ln p_{kt}/\lambda_{kt} - \sum_{\Omega_{t-1}} s_{kt-1} \ln p_{kt-1}/\lambda_{kt-1}$$

The geo-mean quality-adjusted price index is decomposed in the following fashion:

$$\Delta \ln P = \sum_{k\in C} \bar{s}_k \Delta \ln p_k/\lambda_k + \sum_{k\in C} \Delta s_k \ln \overline{p_k/\lambda_k} + \sum_{k\in N} s_{kt} \ln p_{kt}/\lambda_{kt} - \sum_{k\in X} s_{kt-1} \ln p_{kt-1}/\lambda_{kt-1}$$

Observe that the first term is similar to a quality-adjusted matched-model Tornqvist.

By applying the QLOOP, we can simplify the geo-mean to a quality-adjusted geo-Laspeyres:

1. Within and PMC terms:

$$\begin{aligned}
\sum_{k\in C} \bar{s}_k \Delta \ln p_k/\lambda_k + \sum_{k\in C} \Delta s_k \ln \overline{p_k/\lambda_k} &= \sum_{k\in C} s_{kt} \ln p_{kt}/\lambda_{kt} - \sum_{k\in C} s_{kt-1} \ln p_{kt-1}/\lambda_{kt-1} \\
&= \sum_{k\in C} s_{kt-1} \Delta \ln p_k/\lambda_k + \sum_{k\in C} \Delta s_k \ln p_{kt}/\lambda_{kt} \\
&= s_{t-1}^c \sum_{k\in C} s_{kt-1}^c \Delta \ln p_k/\lambda_k + \ln P_t \Delta s^c
\end{aligned}$$

[25] If QLOOP holds then quality-adjusted qantity and expenditure shares are equivalent.

2. Net entry:

$$\sum_{k\in N} s_{kt} \ln p_{kt}/\lambda_{kt} - \sum_{k\in X} s_{kt-1} \ln p_{kt-1}/\lambda_{kt-1} = \Delta \ln P - (\ln P_t s_t^c - \ln P_{t-1} s_{t-1}^c)$$

3. Combining terms:

$$\begin{aligned}
\Delta \ln P &= s_{t-1}^c \sum_{k\in C} s_{kt-1}^c \Delta \ln p_k/\lambda_k + \ln P_t \Delta s^c + \Delta \ln P - (\ln P_t s_t^c - \ln P_{t-1} s_{t-1}^c) \\
&= s_{t-1}^c \sum_{k\in C} s_{kt-1}^c \Delta \ln p_k/\lambda_k + (1 - s_{t-1}^c)\Delta \ln P \\
&= \sum_{k\in C} s_{kt-1}^c \Delta \ln p_k/\lambda_k
\end{aligned}$$

It is potentially misleading to first apply the decomposition from equation (3) in FHK for changes in geo-means as we immediately rewrite the within and PMC terms to more closely match the initial decomposition based on equation (2) in FHK. However, this exercise does show the geo-mean price index retains the equivalence to the geo-Laspeyres under the law of one price.

A.4.2 Geo-Mean Price Indices as Bounds on Inflation

Similar to the QUVI, the quality-adjusted geo-mean price index with time-varying adjustment will map to hedonic indices such as the geo-Laspeyres, geo-Paasche, and Tornqvist under certain values of the timing parameter α. We show that if $\alpha = 0.5$ as in the baseline case for the time-varying adjustment the geo-mean quality-adjusted price index is equivalent to the full imputation hedonic Tornqvist as in equation (A23).In this analysis, we do not impose QLOOP in any derivation. Beginning with the reduced form decomposition of the geo-mean price index:

$$\Delta \ln P = \sum_{k\in C} \bar{s}_k \Delta \ln p_k/\lambda_k + \sum_{k\in C} \Delta s_k \ln \overline{p_k/\lambda_k} + \sum_{k\in N} s_{kt} \ln p_{kt}/\lambda_{kt} - \sum_{k\in X} s_{kt-1} \ln p_{kt-1}/\lambda_{kt-1}$$

We manipulate each term in the decomposition individually:

1. With time-varying adjustment the price relative in the Within term is equivalent to the hedonic prediction from the EP-TV model:

$$\sum_{k\in C} \bar{s}_k \Delta \ln p_k/\lambda_k = \sum_{k\in C} \bar{s}_k \Delta \widehat{\ln p_k}$$

2. Because the PMC weights changes in expenditure shares using the unweighted geo-mean of quality-adjusted prices, the average of log prices and average of log quality cancel out when $\alpha = 0.5$:

$$\begin{aligned}\sum_{k \in C} \Delta s_k \ln \overline{p_k/\lambda_k} &= \sum_{k \in C} \Delta s_k \times (\ln \overline{p}_k - \ln \overline{\lambda}_k) \\ &= \sum_{k \in C} \Delta s_k \times (\ln \overline{p}_k - \ln \overline{p}_k) \\ &= 0\end{aligned}$$

Note the two additional error terms from the EP-TV model negate when calculating the unweighted geo-mean of quality.

3. We look at the Net-Entry term separately for entering and exiting goods. For entering goods, the quality-adjusted price is:

$$\begin{aligned}\ln p_{kt}/\lambda_{kt} &= \ln p_{kt} - (0.5 \ln p_{kt} + 0.5\widehat{\ln p}_{kt-1}) \\ &= \ln p_{kt} - (\ln p_{kt} - 0.5\Delta\widehat{\ln p}_k) \\ &= 0.5\Delta \ln \widehat{p}_k\end{aligned}$$

By extension, the first term in the Net-Entry term is:

$$\sum_{k \in N} s_{kt} \ln p_{kt}/\lambda_{kt} = \sum_{k \in N} 0.5 s_{kt} \Delta\widehat{\ln p}_k$$

For exiting goods, the quality-adjusted price is:

$$\begin{aligned}\ln p_{kt-1}/\lambda_{kt-1} &= \ln p_{kt-1} - (0.5\widehat{\ln p}_{kt} + 0.5 \ln p_{kt-1}) \\ &= \ln p_{kt-1} - (0.5\Delta\widehat{\ln p}_k + \ln p_{kt-1}) \\ &= -0.5\Delta\widehat{\ln p}_k\end{aligned}$$

Similar to the case for entering goods, the second term in the Net-Entry term is:

$$\sum_{k \in X} s_{kt-1} \ln p_{kt-1}/\lambda_{kt-1} = -\sum_{k \in X} 0.5 s_{kt-1} \Delta\widehat{\ln p}_k$$

Combining terms, the Net-Entry component is:

$$\sum_{k \in N} s_{kt} \ln p_{kt}/\lambda_{kt} - \sum_{k \in X} s_{kt-1} \ln p_{kt-1}/\lambda_{kt-1} = \sum_{k \in N} 0.5 s_{kt} \Delta\widehat{\ln p}_k + \sum_{k \in X} 0.5 s_{kt-1} \Delta\widehat{\ln p}_k$$

4. Taken together, the geo-mean price index under this case is equivalent to a full imputation hedonic Tornqvist:

$$\Delta \ln P = \sum_{k \in C} \bar{s}_k \Delta \widehat{\ln p_k} + \sum_{k \in N} 0.5 s_{kt} \Delta \widehat{\ln p_k} + \sum_{k \in X} 0.5 s_{kt-1} \Delta \widehat{\ln p_k}$$

While we do not empirically implement the geo-mean quality-adjusted price index, this connection implies the QUVI with time-varying adjustment will be approximately equivalent to the EP-TV when the timing parameter $\alpha = 0.5$ as the geo-mean index and the QUVI come from the same quality-adjusted law of one price and use the same definition for quality. There are two reasons that the QUVI will not exactly equal the hedonic Tornqvist. First, the QUVI relies on an arithmetic average whereas the Tornqvist relies on a geo-mean. Second, the equality between expenditure shares and quality-adjusted quantity weights holds under the quality-adjusted law of one price. If the two are highly correlated, the difference between the QUVI and the hedonic Tornqvist will be small as appears to be the case in Figure 3.

There are two additional cases that we will highlight. We show that the geo-mean price index with time-varying adjustment will map to a a hedonic geo-Paasche when the timing parameter $\alpha = 0$ and a hedonic geo-Laspeyres when the timing parameter $\alpha = 1$. Beginning with the case of $\alpha = 0$, the decomposition of the geo-mean price index simplifies to a hedonic Paasche:

$$\begin{aligned}\Delta \ln P &= \sum_{k \in C} 0.5(s_{kt-1} + s_{kt}) \Delta \widehat{\ln p_k} + \sum_{k \in C} 0.5(s_{kt} - s_{kt-1}) \Delta \widehat{\ln p_k} + \sum_{k \in N} s_{kt} \Delta \widehat{\ln p_k} \\ &= \sum_{k \in C} s_{kt} \Delta \widehat{\ln p_k} + \sum_{k \in N} s_{kt} \Delta \widehat{\ln p_k}\end{aligned}$$

Similarly for the case of $\alpha = 1$, the decomposition of the geo-mean price index simplifies to a hedonic Laspeyres:

$$\begin{aligned}\Delta \ln P &= \sum_{k \in C} 0.5(s_{kt-1} + s_{kt}) \Delta \widehat{\ln p_k} - \sum_{k \in C} 0.5(s_{kt} - s_{kt-1}) \Delta \widehat{\ln p_k} + \sum_{k \in X} s_{kt-1} \Delta \widehat{\ln p_k} \\ &= \sum_{k \in C} s_{kt-1} \Delta \widehat{\ln p_k} + \sum_{k \in X} s_{kt-1} \Delta \widehat{\ln p_k}\end{aligned}$$

Taken together, the geo-mean price index with time-varying adjustment has similar properties to the case of the QUVI as it maps to the bounds of a full imputation hedonic Laspeyres and Paasche under extreme values of α.

A.5 Alternative Forms of Quality Adjustment

A.5.1 de Haan and Krsinich (2018)

To implement the quality adjustment procedure in de Haan and Krsinich (2018) (DHK), the authors first run the following time-dummy hedonic model:

$$\ln p_{k\tau} = \alpha_{t-1,t} + \delta_t + X_k' \gamma_{t-1,t} + \epsilon_{k\tau} \text{ for } \tau \in \{t-1,\ t\}$$

where observations are expenditure share weighted. Using information from this model, the authors assume quality adjustment factors are time invariant such that the factor for each good is its predicted price normalized by the prediction for some base good b:

$$\begin{aligned}
\lambda_{kt-1}^{\text{DHK}} &= \frac{\hat{p}_{kt-1}}{\hat{p}_{bt-1}} \\
&= \frac{\exp(\hat{\alpha}_{t-1,t}) \exp\left(X_k' \hat{\gamma}_{t-1,t}\right)}{\exp(\hat{\alpha}_{t-1,t}) \exp\left(X_b' \hat{\gamma}_{t-1,t}\right)} \\
&= \frac{\exp(\hat{\alpha}_{t-1,t}) \exp(\hat{\delta}_t) \exp\left(X_k' \hat{\gamma}_{t-1,t}\right)}{\exp(\hat{\alpha}_{t-1,t}) \exp(\hat{\delta}_t) \exp\left(X_b' \hat{\gamma}_{t-1,t}\right)} \\
&= \frac{\hat{p}_{kt}}{\hat{p}_{bt}} \\
&= \lambda_{kt}^{\text{DHK}}
\end{aligned}$$

Note that the authors additionally extend these results to allow for not just pairwise time-invariance in the definition of adjustment factors as we use here but time-invariance over multiple periods. We use the denominator from the QUVI's formula to derive expressions for the adjustment factors in terms of predicted components from the hedonic model:

$$\begin{aligned}
\sum_{k \in \Omega_t} \lambda_{kt}^{\text{DHK}} q_{kt} \Big/ \sum_{k \in \Omega_{t-1}} \lambda_{kt-1}^{\text{DHK}} q_{kt-1} &= \sum \frac{\hat{p}_{kt}}{\hat{p}_{bt}} q_{kt} \Big/ \sum \frac{\hat{p}_{kt-1}}{\hat{p}_{bt-1}} q_{kt-1} \\
&= \sum_{k \in \Omega_t} \frac{\hat{p}_{bt-1}}{\hat{p}_{bt}} \hat{p}_{kt} q_{kt} \Big/ \sum_{k \in \Omega_{t-1}} \hat{p}_{kt-1} q_{kt-1} \\
&= \sum_{k \in \Omega_t} \frac{\hat{p}_{kt}}{\exp(\hat{\delta}_t)} q_{kt} \Big/ \sum_{k \in \Omega_{t-1}} \hat{p}_{kt-1} q_{kt-1} \\
&= \sum_{k \in \Omega_t} \frac{\hat{p}_{kt}}{\exp(\hat{\delta}_t)} \frac{p_{kt}}{p_{kt}} q_{kt} \Big/ \sum_{k \in \Omega_{t-1}} \hat{p}_{kt-1} \frac{p_{kt-1}}{p_{kt-1}} q_{kt} \\
&= \sum_{k \in \Omega_t} \frac{\exp(\hat{\epsilon}_{kt})}{\exp(\hat{\delta}_t)} p_{kt} q_{kt} \Big/ \sum_{k \in \Omega_{t-1}} \exp(\hat{\epsilon}_{kt-1}) p_{kt-1} q_{kt-1}
\end{aligned}$$

where the authors assume the predicted price relative for the base good b is equal to the time fixed effect in the time dummy hedonic model (i.e. $\hat{p}_{bt}/\hat{p}_{bt-1} = \exp(\hat{\delta}_t)$). Stating the definition of quality adjustment terms explicitly:

$$\lambda_{kt-1}^{DHK} = \exp(\hat{\epsilon}_{kt-1})p_{kt-1} = \exp(\hat{\epsilon}_{kt})p_{kt}/\exp(\hat{\delta}_t) = \lambda_{kt}^{DHK}$$

Quality-adjusted price relatives simplify as follows:

$$\frac{p_{kt}/\lambda_{kt}^{\text{DHK}}}{p_{kt-1}/\lambda_{kt-1}^{\text{DHK}}} = \left[\frac{\exp(\hat{\epsilon}_{kt})}{\exp(\hat{\epsilon}_{kt-1})}\right]^{-1} \exp(\hat{\delta}_t)$$

If we suppose that (1) the time-dummy coefficient approximately equals the true rate of inflation ($P_t/P_{t-1} \approx \exp(\hat{\delta}_t)$) and (2) the error terms approximately equal true residual price dispersion ($z_{k\tau} \approx \exp(\hat{\epsilon}_{k\tau})$), then DHK are mechanically implementing something akin to a FQLOOP:

$$\frac{p_{kt}/\lambda_{kt}^{\text{DHK}}}{p_{kt-1}/\lambda_{kt-1}^{\text{DHK}}} \approx \left[\frac{z_{kt}}{z_{kt-1}}\right]^{-1} \left[\frac{P_t}{P_{t-1}}\right]$$

where the FQLOOP implies:

$$z_{k\tau}p_{k\tau}/\lambda_{k\tau} = P_\tau \text{ for } \tau \in \{t-1,\ t\} \ \Rightarrow\ \frac{p_{kt}/\lambda_{kt}}{p_{kt-1}/\lambda_{kt-1}} = \left[\frac{z_{kt}}{z_{kt-1}}\right]^{-1} \left[\frac{P_t}{P_{t-1}}\right]$$

Lastly, their implementation allows the QUVI to adjust for residual price dispersion:

$$\text{DHK} = \frac{\exp(\hat{\delta}_t)}{\sum_{k\in\Omega_t} s_{kt}\exp(\hat{\epsilon}_{kt})/\sum_{k\in\Omega_{t-1}} s_{kt-1}\exp(\hat{\epsilon}_{kt-1})}$$

However given their hedonic model of choice, they are likely including a greater amount of unobserved product quality in their residual price dispersion relative to our specification.

A.5.2 Byrne, Kovak, and Michaels (2017)

To describe the quality adjustment procedure in Byrne et al. (2017) (BKM), we must first outline their assumptions regarding consumer demand. Consumer preferences follow the perfect substitute aggregator ($\sum \lambda_k q_k$), which corresponds to the same aggregator used to derive the QUVI as the exact price index in a previous portion of these notes. To account for the slow increase in market shares for new goods, BKM additionally assume consumers experience an inter-temporal constraint when adjusting quality-adjusted quantity

shares:

$$\sum_{k \in \Omega_{t-1,t}} A(\omega_{kt},\, \omega_{kt-1}) \leq C \text{ for } C > 0$$

Because consumers face an intertemporal constraint, their expenditure minimization problem is slightly different:

$$\min_{\{q_{kt}\}} \sum_{t=0}^{T} \delta^t \sum_{k \in \Omega_t} p_{kt} q_{kt} \text{ s.t. } Q_t = \sum_{k \in \Omega_t} \lambda_k q_{kt} \text{ and } \sum_{k \in \Omega_{t-1,t}} A(\omega_{kt-1},\, \omega_{kt}) \leq C$$

As a result, the consumer's first order condition is:

$$\frac{p_{kt}}{\lambda_{kt}} - \left[L_t^* A_1(\omega_{kt},\, \omega_{kt-1}) + \delta L_{t+1}^* A_2(\omega_{kt+1},\, \omega_{kt})\right] = L_t \;\forall\; k \in \Omega_t$$

where L_t represents the Lagrange multiplier with respect to the quantity aggregator and L_t^* is the Lagrange multiplier with respect to the intertemporal constraint. Note that if this constraint were not present, we would derive the QLOOP. Assuming in the long-run that quality-adjusted quantity shares have fully adjusted, it is only in the long-run period T when prices equalize:

$$\frac{p_{kT}/\lambda_k}{p_{jT}/\lambda_j} = 1 \;\forall\; j \text{ and } k$$

Assuming that quality adjustment factors are time-invariant, BKM look to use long-run relative prices as measures of quality for the QUVI.

A.6 Average Price Changes

A.6.1 Matched-Model Indices

Traditional matched-model indices are the most common formulas to measure time-series inflation. These indices require only price and quantity data for the set of goods that are present in periods $t-1$ and t. There are two indices that we highlight in this section: an arithmetic Laspeyres and a superlative Tornqvist. First, the arithmetic Laspeyres calculates average price changes using base period weights (e.g. quantity or expenditure shares):

$$\text{Laspeyres} = \sum_{k \in C} s_{kt-1}^c \left(p_{kt}/p_{kt-1}\right) \tag{A21}$$

where s^c_{kt-1} denotes the expenditure share of good k in period $t-1$ within the basket of continuing goods C. It is well-known that using based or current period weights leads to a potential for substitution bias when calculating inflation.

To account for substitution bias across periods, superlative indices such as the Fisher or Tornqvist are preferred formulas as they balance biases in the Laspeyres and Paasche indices. The Tornqvist index is the geo-mean of price changes for continuing goods:

$$\text{Tornqvist} = \prod_{k \in C} \left(p_{kt} / p_{kt-1} \right)^{\bar{s}^c_k} \tag{A22}$$

where weights are the arithmetic average of expenditure shares in period $t-1$ and t: $\bar{s}^c_k = 0.5\ (s^c_{kt-1} + s^c_{kt})$. It is equivalent to the geo-mean of the geometric Laspeyres and Paasche and provides a second-order approximation to changes in the true cost-of-living for a wide class of utility functions (Diewert 1978). However, a persistent critique of matched-model indices is that they do not capture important information on price changes for entering and exiting goods. As we will cover next, hedonic and demand based indices expand the scope of information included in their respective index formula to account for margins of entry and exit.

A.6.2 Quality-Adjusted Indices

Hedonic Indices:

Hedonic indices incorporate information on product characteristics to estimate the rate of inflation on a quality-adjusted basis. To do so, hedonic indices impute price levels or relatives on product characteristics to account for price changes that are on average associated to differences in attributes. In particular, we look at a full-imputation hedonic index using the log first difference hedonic model from Erickson and Pakes (2011) (EP-TV).

The implementation of the log difference hedonic model from EP-TV takes two steps. First, we run a level hedonic model by regressing log price levels on a vector of observed product characteristics denoted as X'_k:

$$\ln p_{kt} = X'_k \beta_t + e_{kt}$$

The first step in this procedure is to use the period $t-1$ estimation of this log level model which provides an estimate $\hat{e}_{kt-1}$. This lagged residual reflects price variation that is unexplained by an observed vector of product characteristics in the base period. Using

this estimate as a proxy for unobserved characteristics, we include it as an additional control variable in a log difference hedonic model:

$$\Delta \ln p_{kt} = X_k' \, \Omega_t + \kappa_t \hat{e}_{kt-1} + u_{kt}$$

Taken together, this approach not only differences out static unobservable using the log first difference, but it also accounts for unobservables that have a dynamic effect on price changes between periods. Using the hedonic model's predicted price relatives, we produce a full-imputation hedonic Tornqvist:

$$\text{(A23) Hedonic Tornqvist} = \prod_{k \in C} \left(\widehat{p_{kt}/p_{kt-1}} \right)^{\bar{s}_k^c} \prod_{k \in N} \left(\widehat{p_{kt}/p_{kt-1}} \right)^{0.5 s_{kt}^c} \prod_{k \in X} \left(\widehat{p_{kt}/p_{kt-1}} \right)^{0.5 s_{kt-1}^c}$$

where N and X denote the basket of entering and exiting goods, respecitvely. This index imputes price changes for continuing goods as well as entering and exiting goods, where the latter two groups' imputed price relatives are weighted in the index formula using half of their observed expenditure shares.[26]

As noted in the main text and appendices above, we use this EP-TV method for the bounding approach to the QUVI.

With respect to measuring the inflation component in the UVI, we incorporate methods of hedonic adjustment following previous work such as de Haan and Krsinich (2018) (DHK). In contrast to DHK, which incorporates the hedonic adjusted price levels from a time dummy hedonic model, we show in a later section how to incorporate information from a log difference hedonic model such as the EP-TV. In contrast to the EP-TV hedonic model, the time dummy hedonic model identifies the aggregate price change conditional on observable characteristics. In doing so, the estimated rate of inflation is susceptible to potential biases when unobserved product characteristics are a driver of individual and average price changes (e.g. omitted variable bias). We compare the estimated rate of inflation using the time dummy hedonic model following DHK versus our approach that incorporates the EP-TV model as part of our empirical results.

Demand Based Indices:

An alternative approach to estimating the time series rate of inflation on a quality-

[26] The imputed price relative for entering goods is calculated under the assumption that the expenditure weighted mean for unobserved product characteristics for entering goods is zero: $\hat{e}_{kt-1} = 0$. Ehrlich et al. (2025) explore the sensitivity of the full imputation hedonic Tornqvist based on the estimated value of that unobserved product characteristics $\hat{e}_{kt-1}$ for entering goods and find the estimated rate of inflation is robust under alternative values.

adjusted basis uses economic theory to derive formulas for an exact price index. The most popular approach in the macroeconomics and international trade literature uses the cost-of-living index derived under a Constant Elasticity of Substitution (CES) preference assumption. Sato (1976) and Vartia (1976) derive the index formula to measure the average price change under the CES preference assumption. The index is similar to a Tornqvist index but uses the logarithmic mean as opposed to the arithmetic mean to calculate average expenditure share weights. Feenstra (1994) adds an additional term to the exact price index to account for changes in entry and exit of goods, and lastly, Redding and Weinstein (2020) accounts for time-varying changes in product appeal.[27]

Among these indices, we focus on the CES unified price index (CUPI) from Redding and Weinstein (2020) as it nests results from prior work and explicitly accounts for time varying product appeal.[28] In contrast to hedonic indices, the CUPI does not rely on additional information on product attributes to estimate quality-adjusted price changes. Instead under the assumption that the unweighted average product appeal is constant for the set of continuing goods, the CUPI measures the quality-adjusted rate of inflation using the following expression:

$$\text{CUPI} = \left[\frac{s_t^c}{s_{t-1}^c}\right]^{\frac{1}{\sigma-1}} \left[\prod_{k\in C}\left(s_{kt}^c/s_{kt-1}^c\right)^{\frac{1}{|C|}}\right]^{\frac{1}{\sigma-1}} \left[\prod_{k\in C}\left(p_{kt}/p_{kt-1}\right)^{\frac{1}{|C|}}\right] \tag{A24}$$

where s_t^c denotes the expenditure shares for continuing goods in period t and σ denotes the elasticity of substitution within the basket of goods. There are three components to the CUPI: (1) the change in expenditure shares for continuing goods, which is often referred to as the the Feenstra adjustment factor, accounts for entry and exit of different varieties. (2) The unweighted geo-mean of changing expenditure shares at the item level represents the change in product appeal for continuing items. (3) The unweighted geo-mean of price relatives for the set of continuing goods measures the change in average prices for continuing goods.

Taken together, the CUPI differs from other indices covered in this section on a number of dimensions. Instead of imputing the unobserved price change for entering and exiting goods, the CUPI identifies the associated effect of entry and exit using the basket of continuing goods. It is the only index covered in this section that relies on unweighted averages to estimate the rate of inflation. Lastly, it also takes into quality-adjusted price

[27] We use the terms "product appeal" and "product quality" interchangeably at the item level with respect to the CUPI.

[28] We use methods presented in Feenstra (1994) to estimate the elasticity of substitution for each product group in our sample.

dispersion that is inherent to the CES expenditure share function. Relative to our approach with the frictional QUVI, the CUPI takes the expression for price dispersion from the CES expenditure share function as given and derives measures of product quality to explain residual price dispersion.