Optimal Asset Market Operations

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Abstract

We characterize governments' optimal responses to asset market disturbances across a broad class of models with financial frictions. We show that the Ramsey plan can be achieved by a policy rule targeting a specific relationship between asset returns, regardless of the underlying disturbances. This relationship is determined by asset supply and demand elasticities that can be estimated empirically with standard identification strategies. Absent financial frictions, the optimal policy stabilizes spreads across all assets. However, in canonical financial intermediation models, the optimal rule prescribes a time-varying spread that increases with expected returns on capital to facilitate intermediation. We apply our framework to study the optimal design of asset purchase programs, as implied by key empirical estimates of asset supply and demand elasticities.

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1 Introduction

How should governments respond to disturbances in asset markets? Government intervention in asset markets has become increasingly complex, expanding the traditional tools to include policies such as discount window lending facilities, quantitative easing, Operation Twist, and the Troubled Asset Relief Program. These "asset market operations" became commonplace in response to economic disturbances during the Great Financial Crisis. However, existing models of financial intermediation used to study these policies are mainly based on detailed microfoundations and analyze policy responses to a fully specified shock. Policy prescriptions based on such analysis inevitably rely on the structure of particular microfoundations, leading to concerns about their broader applicability. Since the deep parameters of these microfoundations are hard to identify and measure empirically and shocks difficult to observe, implementing these prescriptions is challenging in practice. This paper aims to provide a policy framework that overcomes these challenges.

We show that in a large class of financial intermediation models and for arbitrary disturbances, optimal asset market operations entail (up to first order) following a policy rule that specifies a certain target relationship between expected returns on assets. The relationship is governed by economic agents' asset supply and demand elasticities with respect to returns, with the levels of returns fluctuating to reflect the underlying disturbances. To the extent that the elasticities can be identified empirically, conditional on the subset of relevant returns they contain all information relevant to policy design. Since implementing such a policy rule only relies on observing holdings and expected returns on assets traded in the market, and does not require observing disturbances themselves, we refer to it as a *market-implementable rule*.

What makes such an optimal policy rule possible is a combination of the core idea behind the sufficient statistics literature (Harberger, 1964; Chetty, 2009) and the design of robust policy rules in the New Keynesian literature (Giannoni and Woodford, 2003). Building on the sufficient statistics approach, we summarize preferences, technology, and frictions with a set of asset supply and demand elasticities. These elasticities are particularly suitable for studying the optimal responses to disturbances around a Ramsey steady state. Moreover, as is well known in the New Keynesian

literature, optimal responses can often be achieved by a policy rule that targets a certain relationship between endogenous variables, regardless of the sources of disturbances. In combination with the sufficient statistics approach, we show that the optimal policy design boils down to a targeting rule where relevant trade-offs are reflected in observable asset returns and governed by agents' asset supply and demand elasticities.

Our framework features a production economy with a general form of financial intermediation. A representative household has preferences over various forms of financial assets (of an arbitrary number), while the financial sector issues (or holds) these financial assets and holds claims on physical capital. Financial frictions are described by a generic mapping between expected returns and the issuance (or holdings) of financial assets, allowing for not only canonical frictions due to agency problems (Bernanke et al., 1999; Gertler and Karadi, 2011; Gertler and Kiyotaki, 2010) but also frictions due to search and bargaining in the interbank market in state-of-the-art models such as Afonso and Lagos (2015); Bianchi and Bigio (2022). The government can implement long-run macroprudential policy in the form of taxes on assets. However, in response to shocks, it can only intervene in asset markets by trading financial assets—performing asset market operations. We study a Ramsey problem for the government and characterize its optimal asset market operations around the Ramsey steady state in response to a one-time unexpected shock that shifts preferences, production technology, and financial frictions in arbitrary ways.

We show that the Ramsey plan can be achieved by a market-implementable rule, and we provide a formula for the rule. The formula describes how expected returns on financial assets should be linked to a set of "base returns," including, for example, the expected returns on capital and financial sector equity. The rule depends on three sets of elasticities: household's asset demand elasticities, which encode information about their preferences; the financial sector's asset supply elasticities, which encapsulate all relevant details about financial frictions; and capital supply elasticities determined by the technology of the economy. To implement the rule, the government can commit to issuing or purchasing financial assets perfectly elastically, promising financial returns contingent on the base returns as prescribed by the rule. By making such a commitment, the government achieves the Ramsey plan for any underlying disturbances, as the base returns fluctuate to incorporate information about the disturbances, while

the financial returns adjust to address the presence of externality in the economy resulting from financial frictions.

We use our formula to isolate the key trade-off the Ramsey planner confronts. The planner aims to satiate households' demand for assets. However, a pecuniary externality exists in the class of models we study: expected returns on assets affect the financial sector's ability to intermediate capital and alter the production possibility frontier. Absent financial frictions, we show that the planner equalizes returns on all assets—effectively, the planner follows a version of the Friedman rule. However, financial frictions give the planner a motive to deviate from this simple rule: When expected returns on capital are high, the planner can lower the funding cost so that the financial sector can maintain a certain level of intermediation. The optimal policy rule thus prescribes time-varying spreads, in contrast to the Friedman rule in a neoclassical setup. By maintaining the optimal target relationship of one asset return relative to another, the planner may moderate or amplify the spread relative to a laissez-faire outcome, depending on the underlying disturbances. We illustrate this motive in an example where financial frictions take a form similar to originating from microfoundations such as costly state verification (Bernanke et al., 1999) and asset diversion (Gertler and Kiyotaki, 2010).

To illustrate the generality of our approach, we apply our framework to study the optimal government asset purchase program in a state-of-the-art macro-finance model with search and bargaining in interbank market (Bianchi and Bigio, 2022). While the model features an interbank market with liquidity shocks and detailed settlement arrangement, we argue that the microfoundation in this model does not allow for enough flexibility: When the model is calibrated to match steady-state moments, asset supply elasticities of the financial sector are around a few thousand, which we argue is implausibly large. This is reminiscent of the large discrepancy between asset demand elasticities implied by standard macro-finance models and the typical empirical estimates, as highlighted by Gabaix and Koijen (2021) This example not only demonstrates the use of our approach in identifying features most relevant for policy design but also emphasizes the need for empirical disciplines on these elasticities.

We demonstrate that the key ingredients for our optimal target rule can be readily recovered from the data with existing identification strategies developed by empirical finance literature on asset demand estimation. We use asset holdings from the Flow

of Fund (FoF) data for different entities in the U.S. economy. We interpret "the aggregate financial sector" in our model broadly to include banks, money market funds, firms, pension funds, etc., allowing for heterogeneity in their asset supply and demand structure, where we categorize their asset holdings assets into liquid, illiquid, and productive assets. We construct *granular instrumental variables* following Gabaix and Koijen (2024) by isolating idiosyncratic shocks in each sector and we use them to identify the asset supply and demand elasticities of other sectors. We show that the elasticities of asset supply and demand for the aggregate financial sector are less than 5, which is broadly consistent with evidence from the empirical finance literature.

Using our estimated model, we study the government's optimal response to a shock to the financial sector's net worth. In response to a 5% decrease in financial sector net worth, the optimal policy prescribes an increase in returns on liquid assets by 1% and a decrease in returns on illiquid assets by 2%. Implementing this policy involves issuing liquid assets and purchasing illiquid assets—a policy similar to the Troubled Asset Relief Program carried out during the Great Recession.

Our result complements the work by Farhi and Werning (2016), which studies how state-contingent macroprudential taxes can address aggregate demand externality under nominal rigidity. In this paper, we focus on the government's optimal asset market operations when macroprudential taxes cannot respond swiftly to disturbances and the main source of externality originates from intermediation frictions. Our aim to understand how the government can best operate in asset markets is related to works that study the management of government debt in public finance, such as Angeletos (2002), Aparisi de Lannoy et al. (2022) and Angeletos et al. (2023). These works focus on the design of optimal debt structures with distortionary taxes. In this paper, we highlight the central role of financial frictions in shaping the government's best responses to disturbances in asset markets. Finally, our analysis formulates the Ramsey problem as a planner facing an intertemporal supply and demand system as Auclert et al. (2024). This formulation allows us to build on the sufficient statistics result in Chiang and Zoch (2023) and derive optimal policy rules for a large class of financial intermediation models.

2 Model

2.1 Setup

Time is discrete, with $t \in \{0, \ldots, \infty\}$. The economy comprises a representative household, a financial sector, and the government. There are three classes of assets: productive assets, financial assets, and the net worth of the financial sector. Productive assets consist of the value of physical capital, which can only be held by the financial sector, yielding returns R_t^A. Financial assets, denoted by set $\mathcal{B}$, are in zero net supply. They can be traded by all agents and generate returns $\boldsymbol{R}_t^B$. The net worth of the financial sector can be held only by the household, and its returns are denoted by R_t^N.

We study how the government can best respond to a one-time unexpected shock at time 0, $\{\boldsymbol{\omega}_t\}$, which is a sequence of aggregate states that moves preferences, technology, and the financial sector's ability to intermediate assets: With probability one, $\boldsymbol{\omega}_t = \bar{\boldsymbol{\omega}}$; with probability zero, $\boldsymbol{\omega}_t$ follows a known path given by

$$\boldsymbol{\omega}_t = \bar{\boldsymbol{\omega}} + \hat{\boldsymbol{\omega}}_t, \quad \forall t \geq 0.$$

Throughout the paper, we use the convention that a generic variable $\boldsymbol{x}_t$ is a column vector and braces $\{x_s\}$ denote sequences from $s = 0, \ldots, \infty$. We use $\mathbf{0}_{\mathcal{B}}$ and $\mathbf{1}_{\mathcal{B}}$ to represent vectors of zeros and ones of dimension $|\mathcal{B}|$.

Household

The representative household derives utility from consumption and holdings of financial assets and equity, $\{c_t, \boldsymbol{b}_t, n_t\}$, where asset holdings are denoted in real values with the consumption goods as numeraire. The household solves the following maximization problem:

$$\max_{\{\boldsymbol{b}_t, n_t, c_t\}_{t=0}^{\infty}} \mathbb{E} \sum_{t=0}^{\infty} \beta^t u\left(\{c_s, \boldsymbol{b}_s, n_s\}_{s \leq t}; \boldsymbol{\omega}_t\right) \tag{1}$$

subject to budget constraints

$$c_t + \mathbf{1}_{\mathcal{B}}^{\intercal} \boldsymbol{b}_t + n_t + \Psi\left(\{\boldsymbol{b}_s, n_s\}_{s \leq t}; \boldsymbol{\omega}_t\right) = w_t + (\boldsymbol{R}_t^B - \boldsymbol{\tau}_t^B)^{\intercal} \boldsymbol{b}_{t-1} + (R_t^N - \tau_t^N) n_{t-1} - T_t.$$

The household supplies one unit of labor inelastically for income w_t and pays lump-sum taxes T_t. The household uses income for consumption and for savings in various assets. Asset holdings are taxed at rate $\boldsymbol{\tau}_t^B$ and τ_t^N, and they can potentially incur a portfolio adjustment cost $\Psi\left(\{\boldsymbol{b}_s, n_s\}_{s\leq t}; \boldsymbol{\omega}_t\right)$. We allow the aggregate state $\boldsymbol{\omega}_t$ to shift the household's preferences and costs of holding assets. These shifts can generate "asset demand shocks" such as a flight to liquidity, which allows us to study policy responses to those scenarios. We use $\mathbb{E}$ to denote expectation before the realization of $\boldsymbol{\omega}_t$.

Production

Production of output uses labor h_t and capital k_{t-1} subject to a constant-return-to-scale production function $F(h_t, k_{t-1}; \boldsymbol{\omega}_t)$. The aggregate state $\boldsymbol{\omega}_t$ shifts the production of output, allowing disturbances such as TFP shocks. A representative firm chooses labor input and capital rental to maximize profit, taking wage w_t and the rental rate of capital ϱ_t as given:

$$\max_{h_t, k_{t-1}} F(h_t, k_{t-1}; \boldsymbol{\omega}_t) - w_t h_t - \varrho_t k_{t-1}. \tag{2}$$

The accumulation of capital is governed by a capital production function $\Phi(\cdot)$:

$$k_t = \Phi(\iota_t; \boldsymbol{\omega}_t) k_{t-1}, \tag{3}$$

where ι_{t+1} is the investment rate per unit of capital and the aggregate state $\boldsymbol{\omega}_t$ shifts capital production, allowing for disturbances such as capital destruction shocks. Holding capital from period $t-1$ to period t is taxed at rate τ_t^A and generates returns R_t^A:

$$R_{t+1}^A + \tau_{t+1}^A \coloneqq \frac{1}{q_t}\Big[\varrho_{t+1} + \max_{\iota_{t+1}} \left\{q_{t+1}\Phi(\iota_{t+1}; \boldsymbol{\omega}_{t+1}) - \iota_{t+1}\right\}\Big]. \tag{4}$$

We denote the value of capital in period t by

$$a_t^K \coloneqq q_t k_t \tag{5}$$

which comprises the supply of productive assets in the economy. Tax τ_t^A allows the government to control the supply of capital in the economy.

The Financial Sector

The financial sector holds productive assets, a_t^F. It can issue or hold financial assets, $\boldsymbol{b}_t^F$. We adopt the convention that positive entries of $\boldsymbol{b}_t^F$ indicate issuance and, therefore, constitute the supply of financial assets to the rest of the economy and liabilities of the financial sector, for example, deposits issued by banks. Negative entries correspond to financial assets held by the financial sector, for example, Treasury bills or reserves at the central bank. Given the equity of the financial sector, n_t, the balance sheet is given by:

$$a_t^F = \mathbf{1}_{\mathcal{B}}^\intercal \boldsymbol{b}_t^F + n_t.$$

Frictions in the financial sector are described by how its asset holdings and issuance depend on returns $\{R_{s+1}^A, \boldsymbol{R}_{s+1}^B\}$ per unit of equity:

$$a_t^F = \theta_t n_t, \quad \boldsymbol{b}_t^F = \boldsymbol{\phi}_t n_t, \tag{6}$$

where

$$\boldsymbol{\phi}_t \coloneqq \boldsymbol{\phi}(\{R_{s+1}^A, \boldsymbol{R}_{s+1}^B\}, \boldsymbol{\omega}_t)$$

is a function of returns sequences. The financial sector balance sheet implies that $\theta_t = \mathbf{1}_{\mathcal{B}}^\intercal \boldsymbol{\phi}_t + 1$.

The function $\boldsymbol{\phi}_t$ captures how the financial sector's ability to intermediate and supply assets depend on expected returns in any equilibrium. As we discuss in Section 2.2, this formulation of the financial sector allows us to nest a large class of financial intermediation models. The aggregate state $\boldsymbol{\omega}_t$ represents disturbances affecting the financial sector's ability to supply different assets.

Financial sector equity n_t represents the residual claim on holdings on its balance sheet, and the return on equity R_t^N satisfies:

$$R_t^N = R_t^A \theta_{t-1} - \boldsymbol{R}_t^{B\intercal} \boldsymbol{\phi}_{t-1}. \tag{7}$$

Government

The government can issue or hold financial assets with positions $\boldsymbol{b}_t^G$, where a positive entry stands for issuance and a negative entry for holdings. The total resource the government receives from the (net) issuance of assets is $\mathbf{1}_{\mathcal{B}}^\intercal \boldsymbol{b}_t^G$. The government budget

constraints are given by:

$$\mathbf{1}_{\mathcal{B}}^{\intercal}\boldsymbol{b}_t^G = \boldsymbol{R}_t^{B\intercal}\boldsymbol{b}_{t-1}^G - \tau_t^A a_{t-1}^K - \boldsymbol{\tau}_t^{B\intercal}\boldsymbol{b}_{t-1} - \tau_t^N n_{t-1} - T_t. \tag{8}$$

We assume that the government can operate in the asset markets by adjusting $\boldsymbol{b}_t^G$ contingent on the realizations of $\{\hat{\boldsymbol{\omega}}_t\}$. These operations include traditional tools, like open market operations, and unconventional tools such as quantitative easing and tightening, the Troubled Asset Relief Program, and Operation Twist. We allow the government to adjust T_t contingent on $\{\hat{\boldsymbol{\omega}}_t\}$, but restrict the taxes $\tau_t^A, \boldsymbol{\tau}_t^B, \tau_t^N$ so that they cannot respond to the shocks. Taxes $\tau_t^A, \boldsymbol{\tau}_t^B, \tau_t^N$ act as macroprudential policies as they allow the government to influence the incentive to issue and hold assets, confronting frictions in the financial sector. The restriction that these taxes are not state-contingent captures the fact that many macroprudential regulations might not be adjusted timely in response to unexpected shocks. The restriction is analogous to this on product market subsidies present in the New Keynesian framework – these subsidies correct monopolistic distortions in the steady state but cannot respond to shocks, thus making room for monetary policy.

Definition of Equilibrium

An equilibrium consists of allocations $\{c_t, \boldsymbol{b}_t, n_t, h_t, k_t, \iota_t, a_t^K, a_t^F, \boldsymbol{b}_t^F\}$, prices $\{w_t, \varrho_t, q_t\}$, returns $\{R_t^A, \boldsymbol{R}_t^B, R_t^N\}$, and policies $\{\boldsymbol{b}_t^G, T_t, \tau_t^A, \boldsymbol{\tau}_t^B, \tau_t^N\}$ such that: $\{c_t, \boldsymbol{b}_t, n_t\}$ solves the households' problem (1) given returns and policies; $\{h_t, k_t, \iota_t\}$ solves the firm's problem (2) given wage, rental rate, and capital price; supply of productive asset is given by (5); the financial sector's supply and demand of assets, $\{a_t^F, \boldsymbol{b}_t^F\}$, is given by (6); the labor and capital rental market clear; returns on productive assets and equity are (4) and (7); asset markets clear:

$$a_t^K = a_t^F, \quad \boldsymbol{b}_t^G + \boldsymbol{b}_t^F = \boldsymbol{b}_t;$$

and the government budget constraint (8) holds. The goods market clears as a result of the Walras' law.

2.2 Nested Models of Financial Intermediation

In Section 2.1, we describe the financial sector with the function $\boldsymbol{\phi}(\cdot)$ that depends on returns. This formulation allows us to nest a large class of financial intermediation models with various microfoundations. The nesting result is useful because, for policies concerning quantities and prices of assets, all relevant information about the financial sector is summarized by its asset supply and demand. The exact microfoundation is irrelevant as long as we can directly measure the shape of asset supply and demand.

For canonical macroeconomic models in which financial intermediaries issue only one asset ($|\mathcal{B}| = 1$), Chiang and Zoch (2023) show that the formulation contains intermediation frictions originating from asset diversion (Gertler and Karadi, 2011), costly state verification (Bernanke et al., 1999), costly leverage (Uribe and Yue, 2006), and collateral constraints similar to Kiyotaki and Moore (1997) and Bianchi and Mendoza (2018).

This paper extends the framework to allow for an arbitrary number of financial assets $\mathcal{B}$. This extension provides a general policy framework to study how the government can best operate in various asset markets. In Appendix C, we show that our formulation contains some recent frameworks developed to study these policies. For example, by specializing to $|\mathcal{B}| = 3$, our formulation of the financial sector nests intermediation frictions in Sims and Wu (2021), Bianchi and Bigio (2022), Piazzesi et al. (2019), where the assets in $\mathcal{B}$ contain reserves, deposits, and government bonds. Similarly, by specializing to $|\mathcal{B}| = 2$, our framework nests the type of financial intermediation studied in Gertler et al. (2012) where assets in $\mathcal{B}$ contain deposits and outside equity.

Function $\boldsymbol{\phi}(\cdot)$ in the class of models discussed above share a similar structure: Given returns $\{R^A, \boldsymbol{R}^B\}$ in any steady state, the responses follow a geometric structure across the time horizon:

$$\boldsymbol{\phi}_{s,\boldsymbol{R}^j_{t+1}}(\{R^A, \boldsymbol{R}^B\}) = \boldsymbol{\varphi}_j(R^A, \boldsymbol{R}^B) \times \boldsymbol{\Xi}(R^A, \boldsymbol{R}^B)^{t-s}.$$

Although our theoretical results do not rely on this property, the structure can be a useful restriction for empirical estimation, as in Chiang and Zoch (2023).

3 The Ramsey Problem

We consider the policy problem of the government under full commitment. The government is assumed to have the same objective function as the household, and it sets policies to choose an equilibrium that maximizes the value of the objective function. Our goal is to characterize the solution to the planner's problem in terms of observable sufficient statistics. To do so, we consider an alternative formulation of the standard Ramsey problem, building on the sufficient statistics literature as in Chetty (2009), which traces back to Harberger (1964).

3.1 A Sequence Space Representation

Our first step is to represent any equilibrium as an intertemporal system of supply and demand for assets.

Lemma 1 *Given* $\{R^A_{s+1}, \tau^A_{s+1}; \boldsymbol{\omega}_s\}$*, there exist functions* $w_t(\cdot), \varrho_t(\cdot), k_t(\cdot)$*, and* $q_t(\cdot)$ *that satisfy production and investment optimality and clear the labor and capital rental markets, with capital supply given by* $a^K_t(\cdot) \coloneqq q_t(\cdot)k_t(\cdot)$*.*

Similarly, given $\{R^A_{s+1}, \boldsymbol{R}^B_{s+1}, \tau^A_{s+1}, \boldsymbol{\tau}^B_{s+1}, \tau^N_{s+1}, T_s; \boldsymbol{\omega}_s\}$*, there exist asset demand functions* $\boldsymbol{b}_t(\cdot), n_t(\cdot)$*, and Lagrange multipliers* $\lambda_t(\cdot)$ *that solve the household's problem with welfare* $v(\cdot)$*.*

Lemma 1 shows that given $\{R^A_{s+1}, \tau^A_{s+1}; \boldsymbol{\omega}_s\}$ and initial conditions, the optimality of production and market clearing for the factor markets determines all relevant allocations and prices in the production block. Similarly, the household's problem (1) allows us to derive how their optimal decisions depend on returns, wage, and taxes

$$\{\boldsymbol{R}^B_{s+1} - \boldsymbol{\tau}^B_{s+1}, R^N_{s+1} - \tau^N_{s+1}, w_s, T_s\}.$$

Using the expression for wages $w_t(\cdot)$ and returns $R^N_t(\cdot)$ from the financial sector in Equation (7), we can express the household's asset demand, Lagrange multipliers, and welfare as functions of $\{R^A_{s+1}, \boldsymbol{R}^B_{s+1}, \tau^A_{s+1}, \boldsymbol{\tau}^B_{s+1}, \tau^N_{s+1}, T_s, \boldsymbol{\omega}_s\}$.[1]

[1]From Equation 7, we can define returns $R^N_{t+1}(\cdot)$ as functions of $\{R^A_{s+1}, \boldsymbol{R}^B_{s+1}; \boldsymbol{\omega}_s\}$ for $t \geq 0$. Initial return $R^N_0(\cdot)$ is a function of $\{R^A_{s+1} - \tau^A_{s+1}, \boldsymbol{R}^B_{s+1} - \boldsymbol{\tau}^B_{s+1}, \boldsymbol{\omega}_s\}$, as it depends on $q_0(\cdot)$ and $R^B_0(\cdot)$.

With these aggregate functions, we define the *excess asset supply* as:

$$\boldsymbol{\epsilon}_t \coloneqq \boldsymbol{b}_t^G + \boldsymbol{\phi}_t(\cdot)n_t(\cdot) - \boldsymbol{b}_t(\cdot), \quad \kappa_t \coloneqq a_t^K(\cdot) - \theta_t(\cdot)n_t(\cdot). \tag{9}$$

In any equilibrium, returns and policies must be that the excess asset supply is zero in all markets:

$$\boldsymbol{\epsilon}_t = \mathbf{0}_{\mathcal{B}}, \quad \kappa_t = 0,$$

and the government budget constraint holds. A Ramsey planner chooses policies subject to these equilibrium conditions as implementability constraints. These market clearing conditions, together with the government budget constraint, guarantee the feasibility of the goods market.

3.2 The Planner's Problem

At time $t = 0$, before observing the realization of $\{\boldsymbol{\omega}_t\}$, the planner announces policies $\{\boldsymbol{b}_t^{\boldsymbol{G}}, T_t, \tau_t^A, \boldsymbol{\tau}_t^B, \tau_t^N\}$. As discussed in Section 2.1, we allow $\boldsymbol{b}_t^G, T_t$ to be stage-contingent—they can depend on realizations of shocks $\{\hat{\boldsymbol{\omega}}_t\}$, and we restrict $\tau_t^A, \boldsymbol{\tau}_t^B, \tau_t^N$ to be the same across all possible realizations of shocks.

Optimal policies and returns on assets in a Ramsey equilibrium solve:

$$\min_{\boldsymbol{\gamma}_t, \eta_t, \xi_t} \max_{R_{t+1}^A, \boldsymbol{R}_{t+1}^B, \tau_{t+1}^A, \boldsymbol{\tau}_{t+1}^B, \tau_{t+1}^N, \boldsymbol{b}_t^G, T_t} v(\{R_{s+1}^A, \boldsymbol{R}_{s+1}^B, \tau_{s+1}^A, \boldsymbol{\tau}_{s+1}^B, \tau_{s+1}^N, T_s; \boldsymbol{\omega}_s\}) \tag{10}$$

$$+ \mathbb{E}\sum_{t=0}^{\infty} \beta^t \Big\{ \underbrace{\boldsymbol{\gamma}_t^\intercal \boldsymbol{\epsilon}_t + \eta_t \kappa_t}_{\text{market clearing}} + \underbrace{\xi_t\big(\mathbf{1}_{\mathcal{B}}^\intercal \boldsymbol{b}_t^G + T_t + \tau_t^A a_{t-1}^K + \boldsymbol{\tau}_t^{B\intercal} \boldsymbol{b}_{t-1} + \tau_t^N n_{t-1} - \boldsymbol{R}_t^{B\intercal} \boldsymbol{b}_{t-1}^G\big)}_{\text{government budget constraint}} \Big\},$$

given initial conditions $\{\boldsymbol{R}_0^B, \boldsymbol{b}_{-1}^G, \boldsymbol{b}_{-1}, n_{-1}, \theta_{-1}, \boldsymbol{\phi}_{-1}, k_{-1}\}$.

We use ξ_t, $\boldsymbol{\gamma}_t$ and η_t to denote the Lagrange multipliers of the asset market clearing conditions and the government budget constraints. The first-order condition with respect to $\boldsymbol{b}^G$ represents the planner's trade-off in supplying or absorbing assets from the market, which can be written in the following form:

$$\beta\xi_{t+1}(\boldsymbol{R}_{t+1}^B - \boldsymbol{\tau}_{t+1}^B) - \xi_t \mathbf{1}_{\mathcal{B}} = \boldsymbol{\gamma}_t - \beta\xi_{t+1}\boldsymbol{\tau}_{t+1}^B. \tag{11}$$

The left-hand side represents the fiscal cost the government incurs from issuing the assets, including the payment in period $t+1$ and the proceeds from issuance in

period t; the right-hand side represents the social value of excess asset supply, capturing how a unit of extra asset supply affects the household's welfare through returns $\{R^A_{s+1}, \boldsymbol{R}^B_{s+1}\}$. Both sides are presented as net of the impact of asset issuance on tax revenue, as captured by $\beta\xi_{t+1}\boldsymbol{\tau}^B_{t+1}$.

The optimality condition in Equation (11) provides a foundation for designing an optimal policy rule: returns on assets should reflect the social value of excess asset supply. In particular, if the social values of two assets are the same, returns on the two assets should equalize, illustrating the logic behind the Friedman rule. As we will see, this logic behind the Friedman rule will prevail when (1) the government is equipped with all macro-prudential taxes, which happens when the shock $\{\hat{\boldsymbol{\omega}}_s\}$ does not occur, or (2) there is no *pecuniary externality* in the financial sector—when $\boldsymbol{\phi}$ is not a function of returns, and household's net worth choice results in a standard Euler equation, with no terms reflecting adjustment costs or non-pecuniary benefits. However, with the presence of financial frictions, the planner will generally want to deviate from the Friedman rule in response to shock $\{\hat{\boldsymbol{\omega}}_s\}$, as a policy that generates spreads between returns will allow the planner to correct for the externality in the financial sector. The goal of our analysis is to characterize the appropriate corrections with a set of observable elasticities that contain sufficient information about the underlying preferences, technology, and frictions for policy design.

Our formulation of the Ramsey problem is close to the dual approach in public finance but differs in a subtle way. A standard procedure in the dual approach is to solve for equilibrium prices as a function of policy instruments using market clearing conditions and to substitute the resulting expressions into the objective function. Our formulation retains market clearing conditions as constraints for the planner and keeps equilibrium returns as choice variables, representing the Ramsey problem as a saddle-point problem. The advantage of our formulation is that the saddle-point can be characterized in terms of elasticities of functions such as $\boldsymbol{b}_t$, θ_t, and $\boldsymbol{\phi}_t$ with respect to returns on assets (through functions $\boldsymbol{\epsilon}_t$ and κ_t). These elasticities can be measured in the data, allowing us to describe optimal policies in terms of empirically observable objects.

3.3 Ramsey Steady State

The Ramsey planner takes into account the social value of asset supply and optimally designs the issuance of financial assets $\boldsymbol{b}_t^G$ along with taxes τ_t^A, $\boldsymbol{\tau}_t^B$ and $\boldsymbol{\tau}_t^N$.

Absent aggregate shocks, we define a Ramsey steady state as follows:

Definition 1 *Given $\boldsymbol{\omega}_t = \bar{\boldsymbol{\omega}}$ for all $t \geq 0$, a Ramsey steady state consists of returns $\bar{R}^A, \bar{\boldsymbol{R}}^B$, policies $\bar{\boldsymbol{b}}^G, \bar{T}, \bar{\tau}^A, \bar{\boldsymbol{\tau}}^B, \bar{\tau}^N$, and multipliers $\bar{\boldsymbol{\gamma}}, \bar{\eta}$, and $\bar{\xi}$, which solve the saddle-point problem (10) for all periods t, with the initial condition given by the steady-state returns, policies, and allocations.*

With the planner's problem described in (10), the Ramsey steady state is characterized by the following lemma:

Lemma 2 *A Ramsey steady state satisfies*

$$\beta(\bar{\boldsymbol{R}}^B - \bar{\boldsymbol{\tau}}^B) = \mathbf{1}_{\mathcal{B}}, \quad \beta(\bar{R}^A + \bar{\tau}^A) = 1, \quad \beta(\bar{R}^N - \bar{\tau}^N) = 1, \quad \textit{(Friedman)} \tag{12}$$

with market clearing conditions: $\bar{\boldsymbol{b}}^G + \bar{\boldsymbol{\phi}}\bar{n} = \bar{\boldsymbol{b}}$, $\bar{a}^K = \bar{\theta}\bar{n}$, and the government budget constraint

$$\mathbf{1}_B^\intercal \bar{\boldsymbol{b}}^G + \bar{T} + \bar{\tau}^A \bar{a}^K + \bar{\boldsymbol{\tau}}^{B\intercal} \bar{\boldsymbol{b}} + \bar{\tau}^N \bar{n} = \bar{\boldsymbol{R}}^{B\intercal} \bar{\boldsymbol{b}}^G.$$

The multipliers are such that $\bar{\boldsymbol{\gamma}} - \beta\bar{\xi}\bar{\boldsymbol{\tau}}^B = \mathbf{0}_{\mathcal{B}}$, $\bar{\eta} + \beta\bar{\xi}\bar{\tau}^A = 0$, and $\bar{\xi} = \bar{\lambda}$.

Proof. See Appendix A. □

In the Ramsey steady state, all returns are effectively equalized with β^{-1} after adjusting for taxes, featuring a generalized version of the Friedman rule. In particular, the multipliers are such that the social value of asset supply is zero.

In the steady state, the government meets the household's demand for financial assets with the issuance of $\boldsymbol{b}^G$. While this impact returns and distorts the financial sector's holdings of capital due to pecuniary externality through $\boldsymbol{\phi}_t$, the government uses macro-prudential tools τ^A and $\boldsymbol{\tau}^B$ to restore the level of capital to the optimal level. Taxes $\bar{\tau}^A$ and $\bar{\boldsymbol{\tau}}^B$ are designed so that the private sector internalizes the pecuniary externality. Tax on net worth, $\bar{\tau}^N$, on the other hand, ensures that the household's holding of net worth is optimal. These taxes, along with lump-sum tax $\bar{T}$, guarantee

that the values of resources are equalized between the household and the government: $\bar{\xi} = \bar{\lambda}$.

4 Optimal Asset Market Operations

We analyze the government's optimal responses to shock $\hat{\boldsymbol{\omega}}_t$ around the Ramsey steady state, focusing on the first-order expansion of the Ramsey plan. For any variable, we denote its deviation from the steady state as $\hat{\boldsymbol{x}}_t \coloneqq \boldsymbol{x}_t - \bar{\boldsymbol{x}}_t$. We use $\boldsymbol{f}_{t,\boldsymbol{x}_s}$ to denote the Jacobian of function $\boldsymbol{f}_t(\boldsymbol{x}_s)$ with respect to $\boldsymbol{x}_s$, evaluated at the steady state. Given a shock realization $\{\hat{\boldsymbol{\omega}}_s\}$, an equilibrium consists of asset returns $\hat{R}^A_{s+1}, \hat{\boldsymbol{R}}^B_{s+1}$ that satisfy the asset market clearing conditions for each period t:

$$\hat{\boldsymbol{b}}^G_t + \sum_{s=0}^{\infty} \left(\boldsymbol{\epsilon}_{t,R^A_{s+1}} \hat{R}^A_{s+1} + \boldsymbol{\epsilon}_{t,\boldsymbol{R}^B_{s+1}} \hat{\boldsymbol{R}}^B_{s+1} + \boldsymbol{\epsilon}_{t,T_s} \hat{T}_s + \boldsymbol{\epsilon}_{t,\boldsymbol{\omega}_s} \hat{\boldsymbol{\omega}}_s \right) = 0,$$

$$\sum_{s=0}^{\infty} \left(\kappa_{t,R^A_{s+1}} \hat{R}^A_{s+1} + \kappa_{t,\boldsymbol{R}^B_{s+1}} \hat{\boldsymbol{R}}^B_{s+1} + \kappa_{t,T_s} \hat{T}_s + \kappa_{t,\boldsymbol{\omega}_s} \hat{\boldsymbol{\omega}}_s \right) = 0,$$

and government policies $\hat{\boldsymbol{b}}^G_t$ and $\hat{\boldsymbol{T}}_t$ that satisfy the government budget constraints.

The optimal state-contingent policy requires specifying $\hat{\boldsymbol{b}}^G_t$ and $\hat{\boldsymbol{T}}_t$ as (linear) functions of $\{\hat{\boldsymbol{\omega}}_s\}$ to maximize the household's welfare function $v(\cdot)$. However, a policy prescription stated in this form is impractical: Implementing such a policy requires measurements of unobserved asset market disturbances $\kappa_{t,\boldsymbol{\omega}_s} \hat{\boldsymbol{\omega}}_s$ and $\boldsymbol{\epsilon}_{t,\boldsymbol{\omega}_s} \hat{\boldsymbol{\omega}}_s$. Since these disturbances can arise from various sources and affect the economy in arbitrary ways, measuring them and identifying their transmission mechanisms are infeasible. To circumvent these challenges, we seek an alternative design of the optimal policy that achieves the Ramsey plan.

4.1 Market-Implementable Rule

We seek to find a *policy rule* that specifies what the government should implement to achieve the Ramsey plan. For such a rule to be useful, we require that it satisfies two criteria: (i) The rule specifies relationships between prices (or quantities) readily observable in the markets, such as expected returns of assets. (ii) The relationship between these variables should only depend on a set of "market elasticities":

Definition 2 *A market elasticity χ_{s,ϱ_t} is either a price elasticity or an income elasticity of allocation $\chi_s \in \{a_s^K, a_s^F, \boldsymbol{b}_s, \boldsymbol{b}_s^F, n_s, h_s\}$ with respect to $\varrho_t \in \{R_{t+1}^A, \boldsymbol{R}_{t+1}^B, R_{t+1}^N, w_t, T_t\}$.*

These elasticities can be estimated using standard identification strategies. For asset markets, these elasticities are closely related to active research in empirical finance on asset demand estimation, following Gabaix and Koijen (2021) and Gabaix and Koijen (2024).

Definition 3 *If a policy rule satisfies criteria (i) and (ii), we refer to it as a "market-implementable rule."*

A market-implementable rule has two desirable properties. First, it shares the strength of the sufficient statistics approach (Chetty, 2009), which summarizes relevant information about preferences, technology, and frictions through estimable elasticities. While this literature typically avoids prescribing optimal policy, the approach is particularly valuable for our purpose as we focus on optimal responses to disturbances around a steady state where the relevant elasticities can be measured. Second, such a rule shares the virtues of "robust policy rules" in the New Keynesian literature, developed by Giannoni and Woodford (2003) and Woodford (2011), allowing the government to implement the Ramsey plan without specific knowledge of underlying shocks. Our criteria impose an additional requirement that variables and their target relationships must be readily observable and estimable in markets—a property often absent in New Keynesian policy rules but crucial for practical implementation. This emphasis on market-implementability helps overcome common policy design challenges, such as difficulties in measuring deep parameters of a model or variables such as the output gap or natural interest rate.

4.2 Social Value Accounting

To describe our result, we adopt the following notation: For multipliers and returns, we use vectors such as $\hat{\boldsymbol{\xi}} := [\beta^t \hat{\xi}_t]$ and $\hat{\boldsymbol{R}}^B := [\beta^{t+1} \hat{\boldsymbol{R}}_{t+1}^B]$ to denote the first-order expansion of the sequences, discounted at rate β; a matrix such as $\boldsymbol{\epsilon}_{\boldsymbol{R}^j} := [\boldsymbol{\epsilon}_{t,\boldsymbol{R}_{s+1}^j}]$ describes how $\boldsymbol{\epsilon}_t$ in period t responds to expected return $\boldsymbol{R}_{s+1}^j$.

From Equation (11), an optimal choice of asset issuance $\boldsymbol{b}^G$ satisfies the following

Euler equation up to first-order expansion:

$$\Delta\hat{\boldsymbol{\xi}} \otimes \mathbf{1}_{\mathcal{B}} + \beta\bar{\xi}\hat{\boldsymbol{R}}^{B} = \tilde{\boldsymbol{\gamma}}, \tag{13}$$

where $\Delta\hat{\boldsymbol{\xi}} \coloneqq [\beta^t(\hat{\xi}_{t+1} - \hat{\xi}_t)]$, $\tilde{\boldsymbol{\gamma}} \coloneqq \left[\beta^t(\hat{\boldsymbol{\gamma}}_t - \beta\hat{\xi}_{t+1}\bar{\boldsymbol{\tau}}^{B})\right]$ and $\otimes$ denotes the Kronecker product. The optimality condition trades off the fiscal cost for the government of such an arrangement on the left-hand side and the social value of asset supply on the right-hand side.

Our goal is to show that regardless of the underlying shocks, we can express $\tilde{\boldsymbol{\gamma}}$ and $\Delta\hat{\boldsymbol{\xi}}$ as (linear) functions of returns, where the functions consist of only estimable market elasticities. Therefore, we can transform the optimality condition in Equation (13) into a market-implementable rule.

Lemma 3 *Under the Ramsey plan:*

(i) The social value of asset supply is given by

$$\tilde{\boldsymbol{\gamma}}^{\intercal} = \bar{\lambda}\Big[\hat{\boldsymbol{R}}^{A\intercal}\boldsymbol{\Gamma}^{A}_{\boldsymbol{\epsilon}} - \hat{\boldsymbol{R}}^{B\intercal}\boldsymbol{\Gamma}^{B}_{\boldsymbol{\epsilon}} - \hat{\boldsymbol{R}}^{N\intercal}\boldsymbol{\Gamma}^{N}_{\boldsymbol{\epsilon}}\Big],$$

(ii) The marginal value of governmental resources satisfies

$$\Delta\hat{\boldsymbol{\xi}}^{\intercal} = \Delta\hat{\boldsymbol{\lambda}}^{\intercal} - \bar{\lambda}\Big[\hat{\boldsymbol{R}}^{A\intercal}\boldsymbol{\Gamma}^{A}_{\Delta\boldsymbol{T}} - \hat{\boldsymbol{R}}^{B\intercal}\boldsymbol{\Gamma}^{B}_{\Delta\boldsymbol{T}} - \hat{\boldsymbol{R}}^{N\intercal}\boldsymbol{\Gamma}^{N}_{\Delta\boldsymbol{T}}\Big],$$

where matrices $\boldsymbol{\Gamma}^{j}_{\boldsymbol{\epsilon}}$ and $\boldsymbol{\Gamma}^{j}_{\Delta\boldsymbol{T}}, \forall j \in \{A, B, N\}$, consist of market elasticities evaluated at the Ramsey steady state, given in Appendix A.5.

Proof. See Appendix A. □

Intuitively, matrices $\boldsymbol{\Gamma}^{j}_{\boldsymbol{\epsilon}}$ and $\boldsymbol{\Gamma}^{j}_{\Delta\boldsymbol{T}}$ capture how government policies affect households' exposure to return disturbances through various channels. Consider first an increase in government asset supply: it affects asset returns directly in the financial asset markets and indirectly through spillovers in the capital market. These changes then influence households' portfolio decisions, altering their exposure to return disturbances in both markets. The matrices $\boldsymbol{\Gamma}^{j}_{\boldsymbol{\epsilon}}$ account for these interconnected effects through asset demand and supply elasticities while compensating appropriately for the income effect from changes in returns. Similarly, when the government redistributes resources intertemporally through taxes and transfers, $\boldsymbol{\Gamma}^{j}_{\Delta\boldsymbol{T}}$ captures how this affects

household exposure to return disturbances, both directly through changes in household asset demand and indirectly through the asset return adjustments triggered by these changes in household asset demand. Under the Ramsey plan, this effect on exposure $\mathbf{\Gamma}^j_{\Delta \boldsymbol{T}}$ is taken into account when equating the government's intertemporal valuation of resources $\Delta\hat{\boldsymbol{\xi}}$ with the household's valuation $\Delta\hat{\boldsymbol{\lambda}}$.

The key observation from Lemma 3 is that both $\tilde{\boldsymbol{\gamma}}$ and $\Delta\hat{\boldsymbol{\xi}}$ can be determined without knowing the exact realizations of $\hat{\boldsymbol{\omega}}$, conditional on asset returns and $\Delta\hat{\boldsymbol{\lambda}}$. This feature reflects that around a Ramsey steady state, matrices $\mathbf{\Gamma}^j_{\boldsymbol{\epsilon}}$ and $\mathbf{\Gamma}^j_{\Delta \boldsymbol{T}}$ together with return disturbances appropriately capture the effects of policy, as the effects of $\hat{\boldsymbol{\omega}}$ on the transmission of policy do not have a first-order impact on welfare. However, this does not mean that shocks are irrelevant. Rather, all relevant information about the underlying shocks is absorbed in asset return disturbances $\hat{\boldsymbol{R}}^A$, $\hat{\boldsymbol{R}}^B$, and $\hat{\boldsymbol{R}}^N$, as these returns adjust to clear the asset markets.

Lemma 3 expresses both the social value of asset supply ($\tilde{\boldsymbol{\gamma}}$) and the government's intertemporal valuation ($\Delta\hat{\boldsymbol{\xi}}$) in terms of observable asset returns. While $\tilde{\boldsymbol{\gamma}}$ depends solely on returns, $\Delta\hat{\boldsymbol{\xi}}$ also incorporates the household's intertemporal valuation $\Delta\hat{\boldsymbol{\lambda}}$. Although $\Delta\hat{\boldsymbol{\lambda}}$ is not directly observable in markets, there are many instances under which there exists an asset (or a set of assets) that allows us to "span" the household's intertemporal valuation, as stated in the following assumption.

Assumption 1 *There exists a linear function $\boldsymbol{\ell}(\cdot)$ such that*

$$\Delta\hat{\boldsymbol{\lambda}} = -\bar{\lambda}\ \boldsymbol{\ell}(\hat{\boldsymbol{R}}^B, \hat{\boldsymbol{R}}^N).$$

Intuitively, Assumption 1 holds when a "standard" Euler equation exists for the household with respect to some return $\hat{R}^*_{t+1}$:

$$\hat{\lambda}_t = \hat{\lambda}_{t+1} + \bar{\lambda}\hat{R}^*_{t+1}.$$

This can occur in several cases. First, if holding a particular asset (say, B_0) incurs no utility or portfolio adjustment costs, then $\boldsymbol{\ell}(\hat{\boldsymbol{R}}^B, \hat{\boldsymbol{R}}^N) = \hat{\boldsymbol{R}}^{B_0}$. Similarly, if holding equity is costless, then $\boldsymbol{\ell}(\hat{\boldsymbol{R}}^B, \hat{\boldsymbol{R}}^N) = \hat{\boldsymbol{R}}^N$. Even when no single asset satisfies a standard Euler equation, one may exist for a "synthetic" asset with return $\hat{R}^*_{t+1} = \boldsymbol{\alpha}^\intercal \hat{\boldsymbol{R}}^B_{t+1}$ for some portfolio weight $\boldsymbol{\alpha}$, as we demonstrate in Appendix B.3. Finally, a

useful special case is when the household has a quasi-linear preference:

$$u\left(\{c_s, \boldsymbol{b}_s, n_s\}_{s\leq t}; \boldsymbol{\omega}_t\right) = c_t + \nu(\boldsymbol{b}_t; \boldsymbol{\omega}_t). \tag{14}$$

In this case, $\lambda_t \equiv 1$ and $\boldsymbol{\ell}(\hat{\boldsymbol{R}}^B, \hat{\boldsymbol{R}}^N) \equiv \mathbf{0}$, which satisfies Assumption 1.

4.3 The Optimal Market-Implementable Rule

Given Lemma 3 and Assumption 1, we derive a market-implementable rule that implements the Ramsey plan.

Theorem 1 *Under Assumption 1, the Ramsey plan can be achieved by a market-implementable rule of the following form:*

$$-\boldsymbol{\ell}(\hat{\boldsymbol{R}}^B, \hat{\boldsymbol{R}}^N) \otimes \mathbf{1}_{\mathcal{B}} + \hat{\boldsymbol{R}}^B = \tilde{\boldsymbol{\Gamma}}_A^\intercal \hat{\boldsymbol{R}}^A - \tilde{\boldsymbol{\Gamma}}_B^\intercal \hat{\boldsymbol{R}}^B - \tilde{\boldsymbol{\Gamma}}_N^\intercal \hat{\boldsymbol{R}}^N,$$

where $\tilde{\boldsymbol{\Gamma}}_j \coloneqq \boldsymbol{\Gamma}_{\boldsymbol{\epsilon}}^j + \boldsymbol{\Gamma}_{\Delta \boldsymbol{T}}^j \otimes \mathbf{1}_{\mathcal{B}}, \forall j \in \{A, B, N\}$ *are matrices that consist of market elasticities given in Appendix A.5, evaluated at the Ramsey steady state.*

Proof. See Appendix A. □

Theorem 1 expresses the planner's optimality condition in Equation (13) as restrictions on asset returns that a Ramsey plan needs to satisfy. The left-hand side represents the value of a sequence of operations: receiving one unit of goods at time t, investing in one of the assets in $\mathcal{B}$ until $t+1$, and paying back β^{-1} units of goods at $t+1$. The transfers of goods are valued at $\Delta\boldsymbol{\lambda}$ while holding the asset generates a return $\hat{\boldsymbol{R}}^B$. On the right-hand side, $\tilde{\boldsymbol{\Gamma}}_j$ captures the total effect of such operation on the household's exposure to return disturbances, with $\boldsymbol{\Gamma}_{\boldsymbol{\epsilon}}^j$ and $\boldsymbol{\Gamma}_{\Delta \boldsymbol{T}}^j$ from Lemma 3 accounting for changes in exposure due to asset supply and the government's intertemporal redistribution through taxes and transfers.

To implement the Ramsey plan with the policy rule, the government can commit to target returns $\hat{\boldsymbol{R}}^B$ based on the realizations of $\hat{\boldsymbol{R}}^A$ and $\hat{\boldsymbol{R}}^N$. This commitment can be supported by a perfectly elastic supply or demand schedule of $\boldsymbol{b}^G$ given $\hat{\boldsymbol{R}}^B$, $\hat{\boldsymbol{R}}^A$, and $\hat{\boldsymbol{R}}^N$. Because the rule does not depend on $\hat{\boldsymbol{\omega}}$, the government maintains the same relationship between returns, even as different realizations of $\hat{\boldsymbol{\omega}}$ lead to different equilibrium returns and asset trades.

This policy rule depends on the underlying structure of the model only through matrices $\tilde{\mathbf{\Gamma}}_j$. The exact microfoundations are irrelevant to the optimal policy rule as long as they imply the same market elasticities. Since these elasticities can be estimated using standard identification strategies, Theorem 1 allows us to prescribe optimal policy within our framework without relying on specific functional forms or microfoundations.

5 The Role of Financial Frictions

We now apply Theorem 1 to a series of examples to illustrate how financial frictions shape the design of the optimal policy rule.

5.1 Neoclassical Growth Benchmark

First, we provide a benchmark case where key features of the financial sector are absent. Specifically, we consider an economy where (1) financial intermediation do not respond to changes in returns: $\boldsymbol{\phi}(\cdot) \equiv \bar{\boldsymbol{\phi}}$ for a constant vector $\bar{\boldsymbol{\phi}}$, and (2) the household derives no utility and faces no cost in holding net worth, and returns on net worth equals the intertemporal price.

Corollary 1 *If $\boldsymbol{\phi}(\cdot) \equiv \bar{\boldsymbol{\phi}}$ and $\Delta\hat{\boldsymbol{\lambda}} = -\bar{\lambda}\hat{\boldsymbol{R}}^N$, then*

$$\hat{\boldsymbol{R}}^B = \hat{\boldsymbol{R}}^A \otimes \mathbf{1}_{\mathcal{B}}$$

constitute a market-implementable rule that achieves the Ramsey plan, and $\hat{\boldsymbol{R}}^N = \hat{\boldsymbol{R}}^A$ in equilibrium.

The optimal policy rule prescribes that the planner maintains a constant spread between all returns, following a version of the Friedman rule for any realizations of shocks $\hat{\boldsymbol{\omega}}$. Since all returns are equalized, we have $\hat{\boldsymbol{R}}^N = \hat{\boldsymbol{R}}^A$. Through their holdings of net worth, the household effectively controls the capital stock as in a neoclassical growth model. At the same time, the planner issues the necessary amount for each type of asset to satiate any additional motives for the household to hold these assets.

Corollary 1 allows us to isolate the role of financial frictions in designing the optimal policy rule: In the absence of pecuniary externality in $\boldsymbol{\phi}(\cdot)$, the planner maintains

constant spreads between all assets. This benchmark clarifies that in our framework, the optimal policy rule would maintain variations in spreads solely due to frictions in the financial sector.

5.2 Frictions in Financial Intermediation

To isolate the role of financial frictions, consider a special case where the household has a quasi-linear preference as in Equation (14):

$$u\left(\{c_s, \boldsymbol{b}_s, n_s\}_{s\leq t}; \boldsymbol{\omega}_t\right) = c_t + \nu(\boldsymbol{b}_t; \boldsymbol{\omega}_t).$$

The financial sector net worth is fixed exogenously: $n_t = \bar{n}$, and there is no portfolio and capital adjustment cost: $\Psi(\cdot) \equiv 0$ and $k_t = (1 + \iota_t - \delta)k_{t-1}$.

Corollary 2 *With quasi-linear preference, fixed net worth, and no portfolio and capital adjustment cost, the optimal market-implementable rule is given by*

$$\hat{\boldsymbol{R}}^B = \left(\bar{n}\check{\boldsymbol{\theta}}_{\boldsymbol{R}^B}\boldsymbol{b}^{-1}_{\boldsymbol{R}^B}\right)^{\intercal}\hat{\boldsymbol{R}}^A,$$

where

$$\check{\boldsymbol{\theta}}_{\boldsymbol{R}^B} \coloneqq \boldsymbol{\theta}_{\boldsymbol{R}^B} + \boldsymbol{\theta}_{\boldsymbol{R}^A}(-\boldsymbol{\kappa}_{\boldsymbol{R}^A})^{-1}\boldsymbol{\kappa}_{\boldsymbol{R}^B}.$$

To build intuition, we simplify the setup to the case where $|\mathcal{B}| = 1$ with the level of intermediation per net worth is given by:

$$\vartheta(R^A_{t+1} - R^B_{t+1}), \quad \vartheta'(\cdot) > 0.$$

This form of financial friction arises from microfoundations such as costly state verification (Bernanke et al., 1999) or costly leverage (Cúrdia and Woodford, 2011).

With these simplifications, asset supply and demand reduce to a static system where holdings and issuance respond only to returns one period ahead:

$$\epsilon_{s,\boldsymbol{R}^j_{t+1}} = b^F_{s,\boldsymbol{R}^j_{t+1}} = \mathbf{0}, \quad \kappa_{s,\boldsymbol{R}^j_{t+1}} = a^F_{s,\boldsymbol{R}^j_{t+1}} = \mathbf{0}, \quad \forall s \neq t.$$

The optimal policy rule is given by

$$\hat{R}^B_{t+1} = -\frac{\bar{n}\bar{\vartheta}'}{\bar{b}_{R^B}}\left[1 - \frac{\bar{n}\bar{\vartheta}'}{\bar{n}\bar{\vartheta}' - \bar{a}^K_{R^A}}\right]\hat{R}^A_{t+1}. \tag{15}$$

Because $\bar{\vartheta}', \bar{b}_{R^B} > 0$ and $\bar{a}^K_{R^A} < 0$, the policy rule prescribes that the planner targets a negative relationship between $\hat{R}^A_{t+1}$ and $\hat{R}^B_{t+1}$. When $\hat{R}^A_{t+1} > 0$, the planner aims to increase exposure to $\hat{R}^A_{t+1}$. Given that $\theta_{R^B} = -\bar{\vartheta}' < 0$, this can be achieved by decreasing excess asset supply and lowering the financial sector's funding cost. This effect is magnified when the financial sector's intermediation capacity is highly sensitive to its funding cost (large $\bar{\vartheta}'$) and when household asset demand is inelastic (small $\bar{b}_{R^B}$). However, reducing asset supply has costs since assets provide utility to households, as captured by $v(\cdot)$. From the household's Euler equation:

$$1 = \beta R^B_{t+1} + \nu_b(b_t),$$

the welfare cost of reduced government asset supply is reflected in the premium on the asset, given the household's response $\hat{b}_t$ and disturbance $\hat{\omega}_t$:

$$\hat{R}^B_{t+1} = -(\bar{\nu}_{bb}\hat{b}_t + \bar{\nu}_{b\omega}\hat{\omega}_t).$$

Optimal Policy with Forward-Looking Frictions

A similar logic extends to a setup where financial frictions generate an intertemporal asset supply system. Consider frictions of the form

$$\theta_{s,R^A_{t+1}} = -\theta_{s,R^B_{t+1}} = \bar{\vartheta}' \times \Xi^{t-s}\mathbf{1}_{\{t \geq s\}},$$

with $\bar{\vartheta}' > 0$. This forward-looking friction resembles those in asset diversion models as in Gertler and Kiyotaki (2010), as shown in Chiang and Zoch (2023).

For simplicity, assume that capital supply is perfectly elastic ($a^K_{R^A} \to -\infty$) and return on capital $\hat{R}^A$ is exogenous. The intertemporal asset supply system implies the following optimal rule:

$$\hat{R}^B_{t+1} = -\frac{\bar{n}\bar{\vartheta}'}{\bar{b}_{R^B}} \sum_{s=0}^{t} \Xi^{t-s}\hat{R}^A_{s+1}, \forall t \geq s,$$

and zero for all $t < s$. Unlike the static case in Equation (15), when the optimal target for $\hat{R}^B_{t+1}$ reflects return disturbances $\hat{R}^A_{s+1}$ for all $s \leq t$, since lowering R^B_{t+1} enhances the financial sector's intermediation capacity across these periods. The weight Ξ^{t-s} on returns in prior periods diminishes with time horizons as their impact on the financial sector decays.

Optimal Policy v.s. Laissez Faire

The optimal policy rule in Equation (15) prescribes a negative relationship between $\hat{R}^A_{t+1}$ and $\hat{R}^B_{t+1}$. However, depending on the underlying disturbances, the government may need to trade assets to either stabilize or amplify the spread between returns compared to a laissez-faire economy to maintain the relationship given by the optimal rule.

To understand how the spread between returns behaves under the optimal policy in contrast to laissez-faire, consider the market clearing conditions:

$$\begin{aligned}
&\bar{a}^K_{R^A}\hat{R}^A_{t+1} + \kappa_\omega\hat{\omega}_t = n\bar{\vartheta}'(\hat{R}^A_{t+1} - \hat{R}^B_{t+1}) \\
&\hat{b}^G_t + n\bar{\vartheta}'(\hat{R}^A_{t+1} - \hat{R}^B_{t+1}) + \epsilon_\omega\hat{\omega}_t = \bar{b}_{R^B}\hat{R}^B_{t+1}.
\end{aligned}$$

Under the optimal policy rule, the government commits to an asset supply schedule that targets Equation (15) with resulting trades $\hat{b}^G_t$ that clear the market. Under laissez-faire, the government remains inactive with $\hat{b}^G_t = 0$.

The equilibrium spreads under laissez-faire and optimal policy are respectively:

$$spread^{LF} = \frac{\bar{a}^K_{R^A}\epsilon_\omega + \bar{b}_{R^B}\kappa_\omega}{\bar{b}_{R^B}(n\bar{\vartheta}' - \bar{a}^K_{R^A}) - \bar{a}^K_{R^A}n\bar{\vartheta}'}\hat{\omega}_t,$$

$$spread^{OP} = \frac{(\bar{b}_{R^B}(n\bar{\vartheta}' - \bar{a}^K_{R^A}) - \bar{a}^K_{R^A}n\bar{\vartheta}')\kappa_\omega}{\bar{b}_{R^B}(n\bar{\vartheta}' - \bar{a}^K_{R^A})^2 - \bar{a}^K_{R^A}(n\bar{\vartheta}')^2}\hat{\omega}_t.$$

To understand the equilibrium outcomes under optimal policy and laissez-faire, we consider two conceptually distinct scenarios. First, consider a disturbance that causes a decrease in excess financial asset supply ($\epsilon_\omega\hat{\omega}_t < 0$ and $\kappa_\omega\hat{\omega}_t = 0$). In this case, the spread increases under laissez-faire. However, by committing to the optimal policy rule, the government would perfectly stabilize the spread by supplying $\hat{b}^G_t$ necessary to offset the decrease in financial asset supply (or increase in demand). Second, consider a disturbance that causes an increase in excess capital supply ($\kappa_\omega\hat{\omega}_t > 0$ and $\epsilon_\omega\hat{\omega}_t = 0$). In this case, the spread increases under laissez-faire. Yet, from the formula, we can see that the optimal policy rule prescribes the government to increase the spread above and beyond the level of laissez-faire to facilitate intermediation.

In practice, economic disturbances typically affect both financial assets and capital

supply simultaneously. For instance, an exogenous decrease in the household financial sector n_t leads to both decreased excess financial asset supply ($\epsilon_\omega \hat{\omega}_t < 0$) and increased excess capital supply ($\kappa_\omega \hat{\omega}_t > 0$). The resulting spread movements and optimal policy reflect the balance between these two forces.

5.3 Interbank Market with Search and Bargaining

Bianchi and Bigio (2022) develop a state-of-the-art model of the banking sector that issues deposits, holds reserves and government debt, as well as some productive capital. Inside the banking sector, banks face idiosyncratic liquidity shocks and need to settle balances with reserves in an interbank market with search frictions. The presence of interbank market frictions allows the government to affect relative returns on assets by changing the relative supply of bonds and reserves, if banks are not satiated with reserves. This enables the government to influence aggregate holdings of productive assets: open market operations have real effects.

Through the lens of our framework, their banking sector corresponds to a specific function $\theta, \boldsymbol{\phi}$, as we show in Appendix C. We can thus apply our result in Theorem 1 that describes how frictions in the financial sector determine optimal open market operations through the sensitivities of θ and $\boldsymbol{\phi}$ with respect to returns. Because returns farther in the future do not affect the financial sector's choices in this setup, we show how the financial sector's holdings and issuance of assets and liabilities respond to next-period expected returns in Table 1.

	R^A_{loan}	R^B_{reserve}	R^B_{deposit}	R^B_{govbnd}
θ_{loan}	4800	-470	-9.2	-3900
ϕ_{reserve}	-315	-5.5	0.5	310
ϕ_{deposit}	0	0	0	0
ϕ_{govbnd}	5100	-460	-9.8	-4200

Table 1: The financial sector's responses to returns in Bianchi and Bigio (2022). We use the extended version of the model calibrated to match 2006 as the steady state. See Section 5 of Bianchi and Bigio (2022) for details. Note: positive values for sensitivities of ϕ mean that liabilities of this type increase (holdings of this type decrease). Columns do not sum up to up due to zero due to rounding.

The most prominent feature of Table 1 is large sensitivities, indicating a high degree

of substitutability between certain assets. Similarly, large sensitivities emerge in other models featuring reserves and government bonds, e.g., Piazzesi et al. (2019) and Sims and Wu (2021). Assuming $n_t = \bar{n}$, and a standard calibration for the household asset demand, these values imply extreme responses in $\hat{\boldsymbol{R}}^B$ and sizes of open market operations in response to $\hat{R}^A$.

Given the central role of the sensitivities of θ and $\boldsymbol{\phi}$ in shaping optimal policies, we need empirical discipline on these key objects. As we show in Section 6, the large sensitivities implied by these models are at odds with the empirical measures of their counterparts. This message echoes that from Gabaix and Koijen (2021), who argue that standard portfolio choice models predict virtually flat demand curves for bonds and equity, at odds with the empirical evidence in favor of low elasticities.

6 Taking the Model to the Data

Our goal for this section is to demonstrate that elasticities relevant to the optimal target rule in Theorem 1 can be obtained by established empirical methods.

We measure the sensitivities of the aggregate functions describing the financial sector θ and $\boldsymbol{\phi}$, using data on their balance sheets and expected returns. To address the simultaneity bias, we identify relevant elasticities with the Granular Instrumental Variables (GIV) approach proposed by (Gabaix and Koijen, 2024). Finally, we estimate the capital supply using the instruments we construct for the financial sector, and we calibrate the household sector.

We estimate a disaggregated version of the financial sector, allowing for the presence of multiple entities with different sensitivities, and aggregate them to obtain the aggregate function θ and $\boldsymbol{\phi}$. We discuss details of our aggregation procedure in Appendix D.

6.1 Asset Supply and Demand Estimation

Empirical Specification

The financial sector consists of J entities. Let $b^F_{j,b,t}$ denote financial asset $b \in \mathcal{B}$ issued by the entity j in period j. The net worth of entity j (less the holdings of other

entities) is denoted by $n_{j,t}$. Holdings and issuance of assets per unit of net worth are defined as $\boldsymbol{\phi}_{b,t} \coloneqq \{\phi_{j,b,t}\}_{j=1}^{J}$ where $\phi_{j,b,t} \coloneqq b_{j,b,t}^{F}/n_{j,t}$.

Each $\phi_{j,b,t}$ is parameterized by a vector of characteristics $\boldsymbol{f}_j$ of dimension F:

$$\phi_{j,b,t} = \bar{\phi}_{j,b}(t) - \sum_{m=1}^{|\mathcal{B}|} \phi_{b,R_m^B}\left(\boldsymbol{f}_j\right)\left(\hat{R}_{t+1}^{A} - \hat{R}_{m,t+1}^{B}\right) + \eta_{b,t} + \varepsilon_{j,b,t},$$

where $\bar{\phi}_{j,b}(t)$ denotes a deterministic trend of $\phi_{j,b,t}$. We use $\hat{R}_{m,t+1}^{B}$ and $\hat{R}_{t+1}^{A}$ to denote deviations of returns from their trends, $\eta_{b,t}$ are aggregate shocks, and $\varepsilon_{j,b,t}$ are idiosyncratic shocks i.i.d. by entity-asset-time.

We rewrite our specification of $\boldsymbol{\phi}_{b,t}$ in terms of its deviations from trends, $\hat{\boldsymbol{\phi}}_{b,t} \coloneqq \{\hat{\phi}_{j,b,t}\}_{j=1}^{J}$:

$$\hat{\phi}_{j,b,t} = -\sum_{m=1}^{|\mathcal{B}|} \phi_{b,R_m^B}\left(\boldsymbol{f}_j\right)\left(\hat{R}_{t+1}^{A} - \hat{R}_{m,t+1}^{B}\right) + \eta_{b,t} + \varepsilon_{j,b,t}. \tag{16}$$

The specification above imposes several types of restrictions. First, to apply the standard granular instrumental variable approach, we assume entities in the financial sector respond only to expected returns one period ahead. Second, they respond to spreads between R_{t+1}^{A} and $\boldsymbol{R}_{t+1}^{B}$, instead of the returns separately. Therefore, the response of $\phi_{j,b,t}$ to $\hat{R}_{t+1}^{A}$ is equal to $-\sum_{m=1}^{|B|}\phi_{b,R_m^B}\left(\boldsymbol{f}_j\right)$. Finally, we assume that sensitivities $\phi_{b,R_m^B}\left(\boldsymbol{f}_j\right)$ have a factor structure, where $\breve{\boldsymbol{\phi}}_{b,R_m^B} \coloneqq \{\breve{\phi}_{\ell,b,R_m^B}\}_{\ell=1}^{F}$ denote the loadings:

$$\phi_{b,R_m^B}\left(\boldsymbol{f}_j\right) = \breve{\boldsymbol{\phi}}_{b,R_m^B}^{\intercal}\boldsymbol{f}_j$$

Given a choice of factors, our task is to recover matrices $\breve{\boldsymbol{\phi}}_m = [\breve{\boldsymbol{\phi}}_{1,R_m^B}, \cdots, \breve{\boldsymbol{\phi}}_{|\mathcal{B}|,R_m^B}]_{b=1}^{|\mathcal{B}|}$, $\forall m \in \mathcal{B}$. To obtain consistent estimates in the presence of unobservable shocks $\eta_{b,t}$ and $\varepsilon_{j,b,t}$ that determine equilibrium values of spreads, we need to use instrumental variables.

For each asset $b \in \mathcal{B}$, we follow the GIV approach by Gabaix and Koijen (2024) to construct instrumental variables $Z_{b,t}$, which we detailed in Appendix E.2. The idea of the GIV approach is to identify parameters of interest by exploiting idiosyncratic shocks $\varepsilon_{j,b,t}$ when each of these idiosyncratic shocks is "large enough" to generate

non-trivial movements in equilibrium spreads.

$$\mathbb{E}\left[\left(\begin{bmatrix}\mathbf{P}\hat{\boldsymbol{\phi}}_{1,t}\\ \vdots \\ \mathbf{P}\hat{\boldsymbol{\phi}}_{|\mathcal{B}|,t}\end{bmatrix} - \begin{bmatrix}\breve{\boldsymbol{\phi}}_{1}\\ \vdots \\ \breve{\boldsymbol{\phi}}_{|\mathcal{B}|}\end{bmatrix}\hat{S}_t\right)\begin{bmatrix}Z_{1,t} & \cdots & Z_{|\mathcal{B}|,t}\end{bmatrix}\right] = \mathbf{0}_{F\times|\mathcal{B}|} \tag{17}$$

where $\mathbf{P} \coloneqq (\mathbf{f}^\intercal\mathbf{f})^{-1}\mathbf{f}^\intercal$ is a projection matrix with $\boldsymbol{f} = [\boldsymbol{f}_1 \cdots \boldsymbol{f}_J]^\intercal$ and $\hat{S}_t \coloneqq \{\hat{R}^A_{t+1} - \hat{R}^B_{b,t+1}\}_{b=1}^{|\mathcal{B}|}$ denotes the spreads.

Data

We use quarterly Flow of Funds data, with the sample period 1993Q1-2023Q4 to obtain information about asset holdings of various entities. We give an overview below and provide details in Appendix E.1.

Entities: We work with $J = 6$. We group sectors in the Flow of Funds into the following broad categories: "firms", "banks," "mutual funds," "pension funds", "insurance companies," and "other finance."

Financial assets: We set $|\mathcal{B}| = 2$ and call the two financial assets "liquid" and "illiquid". We group checkable deposits, currency, federal funds and security repurchase agreements, money market fund shares, time and savings deposits, Treasuries, and municipal securities as a liquid asset. The illiquid asset is comprised of agency- and GSE-backed securities, corporate and foreign bonds, mortgages, consumer credit, and loans. We calculate the net worth $n_{j,t}$ of entity j by subtracting its holdings of equity of other entities from the net worth of entity j reported in the FoF.

Returns: We use real 5-year Treasury yield as the rate of return on the liquid asset and the real BAA yield as the rate of return on the illiquid asset. To calculate R^A_t we proceed in two steps. We measure the expected rate of return on equity R^N_t by regressing the realized rate of return on dividend yield on S&P500 (Fama and French, 1988). We then recover R^A_t by applying Equation (7), using data on asset holdings of nonfinancial business.

Factors: For factors $\boldsymbol{f}_j$, we adopt a two-factor structure ($F = 2$). The first factor is simply a vector of ones. For the second factor, we use the first principal component of a matrix containing time averages of each entity's portfolio shares.

Estimation Results

We estimate matrices $\{\breve{\boldsymbol{\phi}}_m\}_{m=1}^{|\mathcal{B}|}$, using the generalized method of moments with the moment conditions in Equation (17). Given $\{\breve{\boldsymbol{\phi}}_m\}_{m=1}^{|\mathcal{B}|}$, we calculate $\phi_{b,R_m^B}(\boldsymbol{f}_j)$ for each entity, and aggregate them using the average net worth shares of each entity:

$$\phi_{b,R_m^B} \coloneqq \sum_{j=1}^{J} \frac{\bar{n}_j}{\bar{n}} \phi_{b,R_m^B}(\boldsymbol{f}_j),$$

where $\bar{n}_j \coloneqq \frac{1}{T}\sum_{t=1}^{T} n_{j,t}$ and $\bar{n} \coloneqq \frac{1}{\sum_{j=1}^{J} \bar{n}_j}$. We obtain ϕ_{b,R^A} as $-\sum_{m=1}^{|\mathcal{B}|} \phi_{b,R_m^B}$ and use the balance sheet condition to recover $\theta_{R^A} = \sum_{b}^{|\mathcal{B}|} \phi_{b,R^A}$ and $\theta_{R_m^B} = \sum_{b}^{|\mathcal{B}|} \phi_{b,R_m^B}$.

Table 2 shows the resulting estimates of sensitivities.

	R^A	R^B_{liquid}	R^B_{illiquid}
θ	0.035	0.06	-0.09
ϕ_{liquid}	0.03	-0.31	0.28
ϕ_{illiquid}	0.005	0.37	-0.38

Table 2: The estimated responses to returns. Note: positive values for sensitivities of ϕ mean that liabilities of this type increase (holdings of this type decrease).

The signs of the estimated sensitivities are intuitive: higher R^A increases demand for productive assets. To finance holdings of productive assets, the financial sector issues liquid assets and reduces holdings of illiquid assets. Similarly, for liquid and illiquid assets, when the expected return for an asset is higher, the financial sector reduces its issuance (or increases its holdings) of this asset. The size of our estimates for these sensitivities is low. This finding is consistent with the evidence in Gabaix et al. (2024) and in contrast to those in Table 1 that we obtain from a calibrated structural model.

6.2 Household and Production

Capital Supply Estimation

We use the GIV constructed in Section 6.1 as sources of exogenous variations in capital demand and use them to estimate the elasticity of capital supply.

To construct a series of capital k_t in the economy, we use the balance sheets of $J = 6$ entities and the capital market clearing condition to calculate their value of their capital holdings as:

$$k_{j,t} = (\mathbf{1}_B^\intercal \boldsymbol{\phi}_{j,t} + 1) n_{j,t}.$$

We use the shocks recovered in the estimation of the parameters describing the financial sector in 2SLS regression of k_t on R^A_{t+1} to estimate the sensitivity of k_t with respect to R^A_{t+1}. The point estimate equals 1.12.

Households Calibration

We assume the household's utility function has the following form

$$u\left(\{c_s, \boldsymbol{b}_s, n_s\}_{s\leq t}; \boldsymbol{\omega}_t\right) = c_t - \sum_{i=1}^{|\mathcal{B}|} \varsigma_i (b_{i,t} - \bar{b}_i)^2 - \varsigma_n (n_t - \bar{n} - \omega_t)^2,$$

and we assume that there is no portfolio adjustment cost: $\Gamma(\cdot) \equiv 0$. Given these assumption, the functions $\boldsymbol{b}_t(\cdot)$ and $n_t(\cdot)$ depend only on expected returns next period and all their cross-price elasticities are zero. The disturbance ω_t shifts demand for net worth. We normalize $\bar{\omega}$ to 0. We set $\beta = 0.99$ and calibrate parameters of the utility function such that at the steady state the semi-elasticities with respect to expected returns are equal to 1.

7 Policy Implications

7.1 Optimal Target Rule

The formula in Theorem 1 implies that the optimal policy maintains a tight relationship between returns on financial and productive assets. We evaluate the formula using the estimated sensitivities of the financial sector and parameterization of the

households and production sector.

$$\begin{bmatrix} \hat{R}^{B^*}_{\text{liquid},t+1} \\ \hat{R}^{B^*}_{\text{illiquid},t+1} \end{bmatrix} = \mathcal{L}\hat{R}^{A}_{t+1}, \quad \mathcal{L} = \begin{bmatrix} 0.16 \\ -0.36 \end{bmatrix}.$$

The optimal policy involves widening spreads when returns on productive assets are high. The planner encourages the financial sector to increase holdings of productive assets. This logic closely resembles the one presented in Section 5, with $|\mathcal{B}| = 1$.

The sign of comovement between returns on productive assets and various types of financial assets reflects the degree of substitutability. Table 2 shows that illiquid assets are substitutes for productive assets. The target returns set by the planner act to encourage the financial sector to rebalance its portfolio from illiquid financial assets to productive assets.

7.2 Responses to Financial Net Worth Shocks

To further understand the implications of the optimal policy, we study responses to a shock that reduces the net worth of the financial sector. This shock, labeled "equity shock" by Christiano et al. (2014) is often considered to be one of the main drivers of the Great Financial Crisis (Gertler and Kiyotaki, 2010; Gertler and Karadi, 2011).[2] We implement the equity shock as a transitory level shift in the household sector's willingness to hold financial net worth. Given our parameterization of the utility function, we have

$$n_t(\cdot;\omega_t) = n_t(\cdot;\bar{\omega}) + \frac{1}{\varsigma_n}\hat{\omega}_t.$$

To parameterize the net worth shock, we assume that it follows $\hat{\omega}_t = \rho_\omega \hat{\omega}_{t-1}$. The value of $\hat{\omega}_0$ corresponds to a destruction of 5% of steady-state net worth, and we set the persistence parameter $\rho_\omega = 0.75$. From Equation (9), we can calculate the impact of net worth shocks on the excess supply of productive and financial assets:

$$\boldsymbol{\kappa}_{\boldsymbol{\omega}}\hat{\boldsymbol{\omega}} = \{-\bar{\theta}\hat{\omega}_t\}, \quad \hat{\boldsymbol{\epsilon}}_{\boldsymbol{\omega}}\hat{\boldsymbol{\omega}} = \{\bar{\boldsymbol{\phi}}\hat{\omega}_t\}.$$

To illustrate the effect of the government's asset market operations, we compare two government policies: (1) the economy's response to these disturbances under

[2]The shock is usually modeled as an event that increases the rate of capital depreciation or reduces capital quality as in Merton (1973).

the optimal target rule and (2) the government does not intervene in asset markets, setting $\boldsymbol{b}_t^G = \mathbf{0}_{\mathcal{B}},\ \forall t$.

Figure 1 shows how the economy responds to the disturbances in the asset markets. The left column shows the response under the optimal target rule, and the right column shows the responses with no asset market intervention.

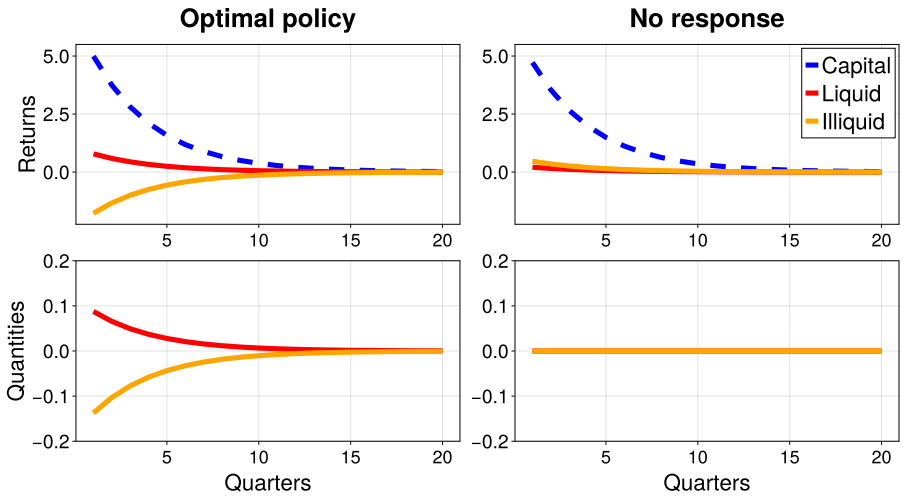


Figure 1: Equilibrium returns and quantities of assets traded by the government under optimal policy and under no asset market operations ($\hat{\boldsymbol{b}}^G = \mathbf{0}$), Responses of returns expressed in percentage points. Responses of quantities expressed as % of steady state quarterly GDP.

To achieve the optimal levels of returns, the planner increases the issuance of liquid assets and purchases illiquid assets—a policy reminiscent of the Trouble Asset Relief Program carried out during the 2008-2009 financial crisis. The size of optimal asset market operations is small, with amounts of assets issued or purchased by the government being well below 0.2% of quarterly GDP. The potency of asset market operations is a consequence of low supply and demand elasticities and echoes the findings of Gabaix and Koijen (2021).

Figure 2 shows the holdings of assets under the two government policies. Comparing the amount of capital under the two policies, the figure shows that the amount of capital follows similar paths, but the level is slightly lower under the optimal policy. This implies a lower level of aggregate output under optimal policy. However, the

holdings of financial assets under the two policies are in stark contrast: Without government interventions (right panel), holdings of both financial assets increase slightly as the household shifts away from holding financial net worth. Under optimal policy, the household holds a significantly higher amount of liquid assets and is able to increase their illiquid liabilities (e.g., mortgage). The equilibrium under optimal policy results in the path of net worth holdings closer to the shifted bliss point than the in the absence of response.

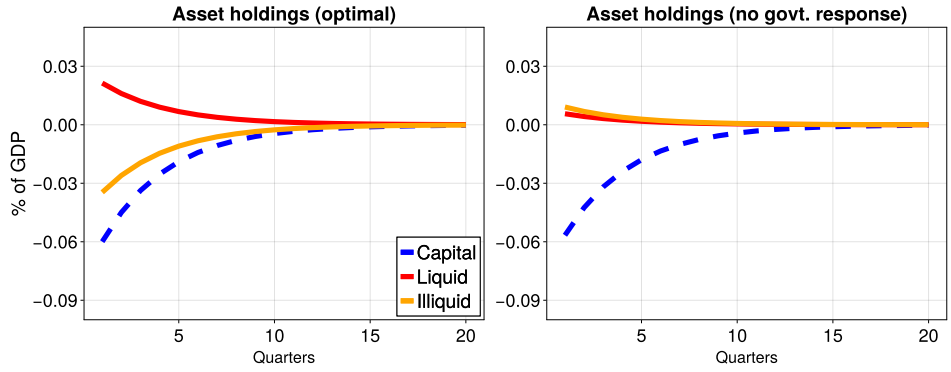


Figure 2: Household holdings of assets and quantity of capital under the optimal policy (left) and under no asset market operations (right).

8 Conclusion

In this paper, we characterize governments' optimal responses to asset market disturbances. We show that in a large class of models with financial, the Ramsey plan entails a policy rule that targets a specific relationship between expected returns between different assets, regardless of the underlying disturbances. The policy rule is determined by asset supply and demand elasticities, which can be empirically estimated with existing identification strategies in empirical finance. We refer to a policy rule of this form as a "market-implementable rule." The policy rule we characterize only possesses the virtue of the sufficient statistics approach—microfoundations are unnecessary for implementing the optimal policy rule but also insights from the New Keynesian literature, where the optimal policy rule can be designed without the need to specify the underlying disturbances. We show these elasticities implied by state-of-the-art financial intermediation models can be of orders of magnitude larger than our

estimates for the U.S. economy. In response to a shock that decreases the financial sector's net worth, the government purchases illiquid assets and issues liquid assets, targeting higher returns on liquid and lower returns on illiquid assets.

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A Proofs and Derivations

A.1 Aggregate Functions

The Production Sector

From the optimality of capital rental of the firms' problem, we have

$$\varrho_t = F_k(h_t, k_{t-1}, \boldsymbol{\omega}_t),$$

which defines a function $\mathring{\varrho}(h_t, k_{t-1}, \boldsymbol{\omega}_t)$.

The optimality of investment gives us

$$q_t \Phi'(\iota_t, \boldsymbol{\omega}_t) = 1,$$

which allows us to define function $\tilde{\iota}(q_t, \boldsymbol{\omega}_t)$.

From the definition of R^A_{t+1}, the law of motion for capital, and the optimality condition

with respect to labor, we have

$$
\begin{aligned}
&(R^A_{t+1} + \tau^A_{t+1})q_t = \varrho(h_{t+1}, k_t, \boldsymbol{\omega}_{t+1}) + [q_{t+1}\Phi(\tilde{\imath}(q_{t+1}, \boldsymbol{\omega}_{t+1}), \boldsymbol{\omega}_{t+1}) - \tilde{\imath}(q_{t+1}, \boldsymbol{\omega}_{t+1})],\\
&k_t = \Phi(\tilde{\imath}(q_t, \boldsymbol{\omega}_t), \boldsymbol{\omega}_t)k_{t-1},\\
&w_t = F_h(h_t, k_{t-1}, \boldsymbol{\omega}_t), \quad \forall t \geq 0.
\end{aligned}
$$

Given initial condition k_{-1} and terminal condition $\lim_{T\to\infty}\prod_{s=0}^{T}(R^A_{s+1})^{-1}q_T = 0$, we can obtain the following functions as the solution of the system:

$$
\mathring{k}_t(\{R^A_{s+1} + \tau^A_{s+1}, w_s; \boldsymbol{\omega}_s\}), \quad \mathring{q}_t(\{R^A_{s+1} + \tau^A_{s+1}, w_s; \boldsymbol{\omega}_s\}), \quad \mathring{h}_t(\{R^A_{s+1} + \tau^A_{s+1}, w_s; \boldsymbol{\omega}_s\}).
$$

From labor market clearing

$$
\mathring{h}_t(\{R^A_{s+1} + \tau^A_{s+1}, w_s; \boldsymbol{\omega}_s\}) = 1, \tag{18}
$$

we can solve for a wage function

$$
w_t(\{R^A_{s+1} + \tau^A_{s+1}; \boldsymbol{\omega}_s\}).
$$

Define

$$
\begin{aligned}
q_t(\{R^A_{s+1} + \tau^A_{s+1}; \boldsymbol{\omega}_s\}) &= \mathring{q}_t(\{R^A_{s+1} + \tau^A_{s+1}, w_s(\{R^A_{\ell+1} + \tau^A_{\ell+1}, \boldsymbol{\omega}_\ell\}); \boldsymbol{\omega}_s\}),\\
k_t(\{R^A_{s+1} + \tau^A_{s+1}; \boldsymbol{\omega}_s\}) &= \mathring{k}_t(\{R^A_{s+1} + \tau^A_{s+1}, w_s(\{R^A_{\ell+1} + \tau^A_{\ell+1}, \boldsymbol{\omega}_\ell\}); \boldsymbol{\omega}_s\}),
\end{aligned}
$$

then we have the rental rate of capital

$$
\varrho_t(\{R^A_{s+1} + \tau^A_{s+1}; \boldsymbol{\omega}_s\}) = \mathring{\varrho}(1, k_{t-1}(\{R^A_{s+1} + \tau^A_{s+1}; \boldsymbol{\omega}_s\})(\cdot), \boldsymbol{\omega}_t),
$$

and the value of capital supply

$$
a^K_t(\{R^A_{s+1} + \tau^A_{s+1}; \boldsymbol{\omega}_s\}) \coloneqq q_t(\{R^A_{s+1} + \tau^A_{s+1}; \boldsymbol{\omega}_s\})k_t(\{R^A_{s+1} + \tau^A_{s+1}; \boldsymbol{\omega}_s\}).
$$

The Financial Sector

For $t > 0$, from the definition

$$
R^N_t \coloneqq R^A_t\theta_{t-1} - \boldsymbol{R}^{B\intercal}_t\boldsymbol{\phi}_{t-1},
$$

together with the definition $\boldsymbol{\phi}_t \coloneqq \boldsymbol{\phi}(\{R^A_{s+1}, \boldsymbol{R}^B_{s+1}\}, \boldsymbol{\omega}_t)$, and that $\theta_t = \mathbf{1}^\intercal_{\mathcal{B}}\boldsymbol{\phi}_t + 1$, we can write

$$R^N_{t+1} = R^N_{t+1}(\{R^A_{s+1}, \boldsymbol{R}^B_{s+1}\}, \boldsymbol{\omega}_{t+1}).$$

For $t = 0$, we have

$$(R^A_0 + \tau^A_0)q_{-1} = \varrho(1, k_{-1}, \boldsymbol{\omega}_0) + \big[q_0\Phi(\mathring{\imath}(q_0, \boldsymbol{\omega}_0), \boldsymbol{\omega}_0) - \mathring{\imath}(q_0, \boldsymbol{\omega}_0)\big],$$

Using function $q_t(\{R^A_{s+1} + \tau^A_{s+1}; \boldsymbol{\omega}_s\})$, we can obtain

$$R^A_0(\{R^A_{s+1} + \tau^A_{s+1}; \boldsymbol{\omega}_s\}).$$

Suppose that the initial returns for asset $\mathcal{B}$ are given by $R^B_0(\{R^B_{s+1} - \tau^B_{s+1}\})$, then we have

$$R^N_0(\{R^A_{s+1} + \tau^A_{s+1}, \boldsymbol{R}^B_{s+1} - \boldsymbol{\tau}^B_{s+1}; \boldsymbol{\omega}_s\}).$$

The Household Sector

Directly from the household problem, we can solve for the welfare function

$$\mathring{v}(\{\boldsymbol{R}^B_s - \boldsymbol{\tau}^B_s, R^N_s - \tau^N_s, w_s, T_s, \boldsymbol{\omega}_s\}),$$

and their asset demand functions

$$\mathring{\boldsymbol{b}}_t(\{\boldsymbol{R}^B_s - \boldsymbol{\tau}^B_s, R^N_s - \tau^N_s, w_s, T_s, \boldsymbol{\omega}_s\}), \quad \mathring{n}_t(\{\boldsymbol{R}^B_s - \boldsymbol{\tau}^B_s, R^N_s - \tau^N_s, w_s, T_s, \boldsymbol{\omega}_s\}),$$

the Lagrange multipliers on their budget constraints:

$$\mathring{\lambda}_t(\{\boldsymbol{R}^B_s - \boldsymbol{\tau}^B_s, R^N_s - \tau^N_s, w_s, T_s, \boldsymbol{\omega}_s\}).$$

Using the wage function $w_t(\cdot)$ and the return function $R^N_{t+1}(\cdot)$ and $R^N_0(\cdot)$, we can obtain

$$v(\{R^A_s, \boldsymbol{R}^B_s, \tau^A_s, \boldsymbol{\tau}^B_s, \tau^N_s, T_s, \boldsymbol{\omega}_s\}) = \mathring{v}(\{\boldsymbol{R}^B_s - \boldsymbol{\tau}^B_s, R^N_s(\cdot) - \tau^N_s, w_s(\cdot), T_s, \boldsymbol{\omega}_s\}),$$

and similarly, we can obtain functions $\boldsymbol{b}_t(\cdot)$ and $n_t(\cdot)$, and $\lambda_t(\cdot)$.

A.2 Optimality Conditions

The FOC for returns $\boldsymbol{R}_{t+1}^j, \forall j \in \{\mathcal{A}, \mathcal{B}\}$ and $\boldsymbol{b}_t^G$ are given by:

$$
\begin{aligned}
[\boldsymbol{R}_{t+1}^j]: \quad & v_{\boldsymbol{R}_{t+1}^j} - (\beta^{t+1}\xi_{t+1}\boldsymbol{b}_t^{G\intercal} + \xi_0 \boldsymbol{b}_{-1}^{G\ \intercal}\boldsymbol{R}_{0,\boldsymbol{R}_{t+1}^B}^B)\mathbf{1}_{\{j\in\mathcal{B}\}} \\
& + \sum_{s=0}^{\infty}\beta^s\Big[\boldsymbol{\gamma}_s^\intercal \boldsymbol{\epsilon}_{s,\boldsymbol{R}_{t+1}^j} + \eta_s \kappa_{s,\boldsymbol{R}_{t+1}^j} + \beta\xi_{s+1}(\boldsymbol{\tau}_{s+1}^{B\ \intercal}\boldsymbol{b}_{s,\boldsymbol{R}_{t+1}^j} + \tau_{s+1}^N n_{s,\boldsymbol{R}_{t+1}^j} + \tau_{s+1}^A a_{s,\boldsymbol{R}_{t+1}^j}^K)\Big] = 0, \\
[b_t^G]: \quad & \xi_t \mathbf{1}_{\mathcal{B}}^\intercal - \beta\xi_{t+1}(\boldsymbol{R}_{t+1}^B)^\intercal + \boldsymbol{\gamma}_t^\intercal = 0.
\end{aligned}
$$

By the envelope theorem, the effects of policies on the household's welfare are:

$$
\begin{aligned}
& v_{T_t} = -\beta^t\lambda_t, \quad v_{\tau_t^A} = \sum_{s=0}^{\infty}\beta^s\lambda_s w_{s,\tau_t^A} + \lambda_0 a_{-1}^K R_{0,\tau_t^A}^A, \\
& v_{\boldsymbol{\tau}_t^B} = -\beta^t\lambda_t \boldsymbol{b}_{t-1}^\intercal + \lambda_0 \boldsymbol{b}_{-1}^{G\ \intercal}\boldsymbol{R}_{0,\boldsymbol{\tau}_t^B}^B, \quad v_{\tau_t^N} = -\beta^t\lambda_t n_{t-1},
\end{aligned}
$$

and the effects of returns on the household's welfare are given by:

$$
v_{\boldsymbol{R}_{t+1}^j} = \big(\beta^{t+1}\lambda_{t+1}\boldsymbol{b}_t^\intercal + \lambda_0 \boldsymbol{b}_{-1}^{\intercal}\boldsymbol{R}_{0,\boldsymbol{R}_{t+1}^j}^B\big)\mathbf{1}_{\{j\in\mathcal{B}\}} + \sum_{s=0}^{\infty}\beta^s\lambda_s\big(R_{s,\boldsymbol{R}_{t+1}^j}^N n_{s-1} + w_{s,R_{t+1}^A}\mathbf{1}_{\{j\in\mathcal{A}\}}\big),
$$

where

$$
R_{s,\boldsymbol{R}_{t+1}^j}^N = \tilde{R}_{s,\boldsymbol{R}_{t+1}^j}^N + \Big[\theta_t \mathbf{1}_{\{j\in\mathcal{A}\}} - \boldsymbol{\phi}_t^\intercal \mathbf{1}_{\{j\in\mathcal{B}\}}\Big]\mathbf{1}_{\{s=t+1\}} + \Big[\theta_{-1}R_{0,R_{t+1}^A}^A\mathbf{1}_{\{j\in\mathcal{A}\}} - \boldsymbol{\phi}_{-1}^\intercal \boldsymbol{R}_{0,\boldsymbol{R}_{t+1}^B}^B\mathbf{1}_{\{j\in\mathcal{B}\}}\Big]\mathbf{1}_{\{s=0\}},
$$

and

$$
\tilde{R}_{s,\boldsymbol{R}_{t+1}^j}^N \coloneqq R_s^A\theta_{s-1,\boldsymbol{R}_{t+1}^j} - \boldsymbol{R}_s^{B\intercal}\boldsymbol{\phi}_{s-1,\boldsymbol{R}_{t+1}^j}.
$$

The FOC for taxes $\mathcal{T}_t \in \{T_t, \tau_t^A, \boldsymbol{\tau}_t^B, \tau_t^N\}$ are given by:

$$
\begin{aligned}
[\mathcal{T}_t]: \quad & v_{\mathcal{T}_t} + \beta^t\xi_t(\mathbf{1}_{\{\mathcal{T}=T\}} + \mathbf{1}_{\{\mathcal{T}=\boldsymbol{\tau}_t^B\}}\boldsymbol{b}_{t-1}^\intercal + \mathbf{1}_{\{\mathcal{T}=\tau^A\}}a_{t-1}^K + \mathbf{1}_{\{\mathcal{T}=\tau^N\}}n_{t-1}) - \xi_0 \boldsymbol{b}_{-1}^{G\ \intercal}\boldsymbol{R}_{0,\boldsymbol{\tau}_t^B}^B\mathbf{1}_{\{\mathcal{T}=\boldsymbol{\tau}_t^B\}} \\
& + \sum_{s=0}^{\infty}\beta^s\Big[\boldsymbol{\gamma}_s^\intercal\boldsymbol{\epsilon}_{s,\mathcal{T}_t} + \eta_s\kappa_{s,\mathcal{T}_t} + \beta\xi_{s+1}(\boldsymbol{\tau}_{s+1}^{B\ \intercal}\boldsymbol{b}_{s,\mathcal{T}_t} + \tau_{s+1}^N n_{s,\mathcal{T}_t} + \tau_{s+1}^A a_{s,\mathcal{T}_t}^K)\Big] = 0.
\end{aligned}
$$

From the household and production problem, we have

$$\boldsymbol{b}_{s,\boldsymbol{R}^j_{t+1}} = -\boldsymbol{b}_{s,\boldsymbol{\tau}^B_{t+1}}\mathbf{1}_{\{j\in\mathcal{B}\}} + \boldsymbol{b}_{s,\tau^A_{t+1}}\mathbf{1}_{\{j\in\mathcal{A}\}} - \sum_{\ell=0}^{\infty}\boldsymbol{b}_{s,\tau^N_{\ell+1}}R^N_{\ell+1,\boldsymbol{R}^j_{t+1}},$$

$$n_{s,\boldsymbol{R}^j_{t+1}} = -n_{s,\boldsymbol{\tau}^B_{t+1}}\mathbf{1}_{\{j\in\mathcal{B}\}} + n_{s,\tau^A_{t+1}}\mathbf{1}_{\{j\in\mathcal{A}\}} - \sum_{\ell=0}^{\infty}n_{s,\tau^N_{\ell+1}}R^N_{\ell+1,\boldsymbol{R}^j_{t+1}}.$$

Similarly, for the financial sector,

$$\boldsymbol{b}^F_{s,\boldsymbol{R}^j_{t+1}} = -\boldsymbol{b}^F_{s,\boldsymbol{\tau}^B_{t+1}}\mathbf{1}_{\{j\in\mathcal{B}\}} + \boldsymbol{b}^F_{s,\tau^A_{t+1}}\mathbf{1}_{\{j\in\mathcal{A}\}} + \boldsymbol{\phi}_{s,\boldsymbol{R}^j_{t+1}}n_s - \sum_{\ell=0}^{\infty}\boldsymbol{b}^F_{s,\tau^N_{\ell+1}}R^N_{\ell+1,\boldsymbol{R}^j_{t+1}},$$

$$a^F_{s,\boldsymbol{R}^j_{t+1}} = -a^F_{s,\boldsymbol{\tau}^B_{t+1}}\mathbf{1}_{\{j\in\mathcal{B}\}} + a^F_{s,\tau^A_{t+1}}\mathbf{1}_{\{j\in\mathcal{A}\}} + \theta_{s,\boldsymbol{R}^j_{t+1}}n_s - \sum_{\ell=0}^{\infty}a^F_{s,\tau^N_{\ell+1}}R^N_{\ell+1,\boldsymbol{R}^j_{t+1}}.$$

Together, we have

$$\boldsymbol{\epsilon}_{s,\boldsymbol{R}^j_{t+1}} = -\boldsymbol{\epsilon}_{s,\boldsymbol{\tau}^B_{t+1}}\mathbf{1}_{\{j\in\mathcal{B}\}} + \boldsymbol{\epsilon}_{s,\tau^A_{t+1}}\mathbf{1}_{\{j\in\mathcal{A}\}} + \boldsymbol{\phi}_{s,\boldsymbol{R}^j_{t+1}}n_s - \sum_{\ell=0}^{\infty}\boldsymbol{\epsilon}_{s,\tau^N_{\ell+1}}R^N_{\ell+1,\boldsymbol{R}^j_{t+1}},$$

$$\kappa_{s,\boldsymbol{R}^j_{t+1}} = -\kappa_{s,\boldsymbol{\tau}^B_{t+1}}\mathbf{1}_{\{j\in\mathcal{B}\}} + \kappa_{s,\tau^A_{t+1}}\mathbf{1}_{\{j\in\mathcal{A}\}} - \theta_{s,\boldsymbol{R}^j_{t+1}}n_s - \sum_{\ell=0}^{\infty}\kappa_{s,\tau^N_{\ell+1}}R^N_{\ell+1,\boldsymbol{R}^j_{t+1}}.$$

Substituting the elasticities with respect to returns, the FOCs for taxes, and the expressions for $v_{\tau^A_{t+1}}, v_{\boldsymbol{\tau}^B_{t+1}}, v_{\tau^N_{t+1}}$, we have

$$\begin{aligned}[\boldsymbol{R}^j_{t+1}]: \quad & \lambda_0\big[n_{-1}R^N_{0,\boldsymbol{R}^j_{t+1}} - \boldsymbol{b}^{F\intercal}_{-1}\boldsymbol{R}^B_{0,\boldsymbol{\tau}^B_{t+1}}\mathbf{1}_{\{j\in\mathcal{B}\}} - a^K_{-1}R^A_{0,\tau^A_{t+1}}\mathbf{1}_{\{j\in\mathcal{A}\}}\big] \\ & + \sum_{s=0}^{\infty}\beta^s\Big[\boldsymbol{\gamma}^\intercal_s(\boldsymbol{\phi}_{s,\boldsymbol{R}^j_{t+1}}n_s) - \eta_s(\theta_{s,\boldsymbol{R}^j_{t+1}}n_s)\Big] \\ & + \beta^{t+1}\xi_{t+1}(\boldsymbol{b}^{F\intercal}_t\mathbf{1}_{\{j\in\mathcal{B}\}} - a^K_t\mathbf{1}_{\{j\in\mathcal{A}\}}) + \sum_{\ell=0}^{\infty}\beta^{\ell+1}\xi_{\ell+1}n_\ell R^N_{\ell+1,\boldsymbol{R}^j_{t+1}} = 0.\end{aligned}$$

Because $R^N_{0,\boldsymbol{R}^j_{t+1}} = a^K_{-1}R^A_{0,R^A_{t+1}} - \boldsymbol{b}^{F\intercal}_{-1}\boldsymbol{R}^B_{0,R^A_{t+1}}$ and

$$\beta^{t+1}\xi_{t+1}(\boldsymbol{b}^{F\intercal}_t\mathbf{1}_{\{j\in\mathcal{B}\}} - a^K_t\mathbf{1}_{\{j\in\mathcal{A}\}}) + \sum_{\ell=0}^{\infty}\beta^{\ell+1}\xi_{\ell+1}n_\ell R^N_{\ell+1,\boldsymbol{R}^j_{t+1}} = \sum_{\ell=0}^{\infty}\beta^{\ell+1}\xi_{\ell+1}n_\ell\tilde{R}^N_{\ell+1,\boldsymbol{R}^j_{t+1}},$$

we have

$$[\boldsymbol{R}^j_{t+1}]: \quad \sum_{s=0}^{\infty}\beta^s\Big[\boldsymbol{\gamma}_s^\intercal(\boldsymbol{\phi}_{s,\boldsymbol{R}^j_{t+1}}n_s)-\eta_s(\theta_{s,\boldsymbol{R}^j_{t+1}}n_s)\Big]+\sum_{\ell=0}^{\infty}\beta^{\ell+1}\xi_{\ell+1}n_\ell\tilde{R}^N_{\ell+1,\boldsymbol{R}^j_{t+1}}=0.$$

Use the expression for $\tilde{R}^N_{\ell,\boldsymbol{R}^j_{t+1}}$,

$$[\boldsymbol{R}^j_{t+1}]: \quad \sum_{s=0}^{\infty}\beta^s\Big[\boldsymbol{\gamma}_s^\intercal\boldsymbol{\phi}_{s,\boldsymbol{R}^j_{t+1}}-\eta_s\theta_{s,\boldsymbol{R}^j_{t+1}}+\beta\xi_{s+1}(R^A_{s+1}\theta_{s,\boldsymbol{R}^j_{t+1}}-\boldsymbol{R}^{B}_{s+1}{}^\intercal\boldsymbol{\phi}_{s,\boldsymbol{R}^j_{t+1}})\Big]n_s=0.$$

From FOC $[b^G_t]$, we have

$$\beta\xi_{t+1}\boldsymbol{R}^{B}_{t+1}{}^\intercal-\boldsymbol{\gamma}_t^\intercal=\xi_t\mathbf{1}_{\mathcal{B}}^\intercal,$$

and

$$[R^A_{t+1}]: \quad \sum_{s=0}^{\infty}\beta^s\Big[-\xi_s\mathbf{1}_{\mathcal{B}}^\intercal\boldsymbol{\phi}_{s,R^A_{t+1}}+(\beta\xi_{s+1}R^A_{s+1}-\eta_s)\theta_{s,R^A_{t+1}}\Big]n_s=0.$$

From the financial sector's balance sheet, we have $\mathbf{1}_{\mathcal{B}}^\intercal\boldsymbol{\phi}_{s,R^A_{t+1}}=\theta_{s,R^A_{t+1}}$ and

$$[R^A_{t+1}]: \quad \sum_{s=0}^{\infty}\beta^s\Big[-\xi_s+(\beta\xi_{s+1}R^A_{s+1}-\eta_s)\Big]\theta_{s,R^A_{t+1}}n_s=0.$$

Suppose that $\boldsymbol{\theta}_{\boldsymbol{R}^A}=[\theta_{s,R^A_{t+1}}]$ is full-rank, then we have

$$\beta\xi_{s+1}R^A_{s+1}-\eta_s=\xi_s, \tag{19}$$

and FOC $[\boldsymbol{R}^B_{t+1}]$ holds trivially.

From the FOCs for taxes and (19), we have

$$\xi_t=\lambda_t,\ \boldsymbol{\tau}^B_{t+1}=(\beta\xi_{t+1})^{-1}\boldsymbol{\gamma}_t,\ \tau^A_{t+1}=-(\beta\xi_{t+1})^{-1}\eta_t,\ \tau^N_{t+1}=(\beta\xi_{t+1})^{-1}(\eta_t^\intercal\theta_t-\boldsymbol{\gamma}_t^\intercal\boldsymbol{\phi}_t).$$

A.3 Ramsey steady state

The Ramsey steady state satisfies:

$$\bar{\xi} = \bar{\lambda},\ \bar{\boldsymbol{\tau}}^B = (\beta\bar{\xi})^{-1}\bar{\boldsymbol{\gamma}},\ \bar{\tau}^A = -(\beta\bar{\xi})^{-1}\bar{\eta},\ \bar{\tau}^N = (\beta\bar{\xi})^{-1}(\bar{\eta}\bar{\theta} - \bar{\boldsymbol{\gamma}}^\intercal\bar{\boldsymbol{\phi}}),$$
$$\beta(\bar{R}^A + \bar{\tau}^A) = 1, \quad \beta(\bar{\boldsymbol{R}}^B - \bar{\boldsymbol{\tau}}^B) = \mathbf{1}_{\mathcal{B}},$$

and

$$\begin{aligned}
[fin\ mkt]: &\quad \bar{\boldsymbol{b}}^G + \bar{\boldsymbol{b}}^F = \bar{\boldsymbol{b}},\\
[cap\ mkt]: &\quad \bar{a}^K = \bar{a}^F,\\
[gov\ bgt]: &\quad \mathbf{1}_{\mathcal{B}}^\intercal\bar{\boldsymbol{b}}^G + \bar{T}_t + \bar{\boldsymbol{\tau}}^{B\intercal}\bar{\boldsymbol{b}} + \bar{\tau}^N\bar{n} + \bar{\tau}^A\bar{a}^K = \bar{\boldsymbol{R}}^{B\intercal}\bar{\boldsymbol{b}}^G.
\end{aligned}$$

A.4 First Order Responses

$$[b_t^G]: \quad \hat{\xi}_t\mathbf{1}_{\mathcal{B}}^\intercal - \beta\hat{\xi}_{t+1}\bar{\boldsymbol{R}}^{B\intercal} - \beta\bar{\xi}\hat{\boldsymbol{R}}_{t+1}^{B\ \intercal} + \hat{\boldsymbol{\gamma}}_t^\intercal = 0$$

$$\begin{aligned}
[T_t]: \quad & \beta^t(-\hat{\lambda}_t + \hat{\xi}_t) + \sum_{s=0}^{\infty}\beta^s\left[\hat{\boldsymbol{\gamma}}_s^\intercal\boldsymbol{\epsilon}_{s,T_t} + \hat{\eta}_s\kappa_{s,T_t} + \boldsymbol{\gamma}^\intercal\hat{\boldsymbol{\epsilon}}_{s,T_t} + \eta\hat{\kappa}_{s,T_t}\right]\\
& + \sum_{s=0}^{\infty}\beta^{s+1}\left[\hat{\xi}_{s+1}(\boldsymbol{\tau}^{B\intercal}\boldsymbol{b}_{s,T_t} + \tau^N n_{s,T_t} + \tau^A a_{s,T_t}^K) + \xi(\boldsymbol{\tau}^{B\intercal}\hat{\boldsymbol{b}}_{s,T_t} + \tau^N\hat{n}_{s,T_t} + \tau_{s+1}^A\hat{a}_{s,T_t}^K)\right] = 0.
\end{aligned}$$

$$\begin{aligned}
[\boldsymbol{R}_{t+1}^j]: \quad & \hat{v}_{\boldsymbol{R}_{t+1}^j} - \beta^{t+1}(\hat{\xi}_{t+1}\bar{\boldsymbol{b}}_t^{G\intercal} + \hat{\xi}_0\boldsymbol{b}_{-1}^{G\ \intercal}\boldsymbol{R}_{0,\boldsymbol{R}_{t+1}^B}^B + \bar{\xi}_{t+1}\hat{\boldsymbol{b}}_t^{G\intercal} + \bar{\xi}_0\boldsymbol{b}_{-1}^{G\ \intercal}\hat{\boldsymbol{R}}_{0,\boldsymbol{R}_{t+1}^B}^B)\mathbf{1}_{\{j\in\mathcal{B}\}}\\
& + \sum_{s=0}^{\infty}\beta^s\left[\hat{\boldsymbol{\gamma}}_s^\intercal\boldsymbol{\epsilon}_{s,\boldsymbol{R}_{t+1}^j} + \hat{\eta}_s\kappa_{s,\boldsymbol{R}_{t+1}^j} + \boldsymbol{\gamma}^\intercal\hat{\boldsymbol{\epsilon}}_{s,\boldsymbol{R}_{t+1}^j} + \eta\hat{\kappa}_{s,\boldsymbol{R}_{t+1}^j}\right]\\
& + \sum_{s=0}^{\infty}\beta^{s+1}\hat{\xi}_{s+1}(\boldsymbol{\tau}^{B\intercal}\boldsymbol{b}_{s,\boldsymbol{R}_{t+1}^j} + \tau^N n_{s,\boldsymbol{R}_{t+1}^j} + \tau^A a_{s,\boldsymbol{R}_{t+1}^j}^K)\\
& + \sum_{s=0}^{\infty}\beta^{s+1}\xi(\boldsymbol{\tau}^{B\intercal}\hat{\boldsymbol{b}}_{s,\boldsymbol{R}_{t+1}^j} + \tau^N\hat{n}_{s,\boldsymbol{R}_{t+1}^j} + \tau^A\hat{a}_{s,\boldsymbol{R}_{t+1}^j}^K) = 0,
\end{aligned}$$

where

$$\hat{v}_{\boldsymbol{R}^j_{t+1}} = \beta^{t+1}\Big[\Big(\hat{\lambda}_{t+1}\boldsymbol{b}^{G\intercal} + \lambda\hat{\boldsymbol{b}}^{G\intercal}_t\Big)\mathbf{1}_{\{j\in\mathcal{B}\}} + \Big(\hat{\lambda}_{t+1}a^K + \lambda\hat{a}^K_t + \sum_{s=0}^{\infty}\beta^s(\hat{\lambda}_s w_{s,R^A_{t+1}} + \lambda\hat{w}_{s,R^A_{t+1}})\Big)\mathbf{1}_{\{j\in\mathcal{A}\}}\Big]$$
$$+ \Big(\lambda\boldsymbol{b}^{G\intercal}\hat{\boldsymbol{R}}^B_{0,\boldsymbol{R}^B_{t+1}} + \hat{\lambda}_0\boldsymbol{b}^{G\intercal}\boldsymbol{R}^B_{0,\boldsymbol{R}^B_{t+1}}\Big)\mathbf{1}_{\{j\in\mathcal{B}\}} + \Big(\lambda a^K\hat{R}^A_{0,R^A_{t+1}} + a^K R^A_{0,R^A_{t+1}}\Big)\mathbf{1}_{\{j\in\mathcal{A}\}}$$
$$+ \sum_{s=0}^{\infty}\beta^{s+1}\Big(\hat{\lambda}_{s+1}\tilde{R}^N_{s+1,\boldsymbol{R}^j_{t+1}}n + \lambda\hat{\tilde{R}}^N_{s+1,\boldsymbol{R}^j_{t+1}}n + \lambda\tilde{R}^N_{s+1,\boldsymbol{R}^j_{t+1}}\hat{n}_s\Big).$$

Using $\beta(R^A+\tau^A)=1, \quad \beta(\boldsymbol{R}^B-\boldsymbol{\tau}^B)=\mathbf{1}_{\mathcal{B}}$ and

$$\boldsymbol{\tau}^{B\intercal}_{s+1}\boldsymbol{b}_{s,T_t} + \tau^N_{s+1}n_{s,T_t} + \tau^A_{s+1}a^K_{s,T_t} = \tau^A_s\kappa_{s,T_t} - \boldsymbol{\tau}^{B\intercal}_s\boldsymbol{\epsilon}_{s,T_t},$$
$$\boldsymbol{\tau}^{B\intercal}\boldsymbol{b}_{s,\boldsymbol{R}^j_{t+1}} + \tau^N n_{s,\boldsymbol{R}^j_{t+1}} + \tau^A a^K_{s,\boldsymbol{R}^j_{t+1}} = \tau^A_{s+1}\kappa_{s,\boldsymbol{R}^j_{t+1}} - \boldsymbol{\tau}^{B\intercal}_{s+1}\boldsymbol{\epsilon}_{s,\boldsymbol{R}^j_{t+1}} - \tilde{R}^N_{s+1,\boldsymbol{R}^j_{t+1}}n_s,$$

we have

$$[T_t]: \quad \beta^t(-\hat{\lambda}_t + \hat{\xi}_t)$$
$$+ \sum_{s=0}^{\infty}\beta^s\Big[(\hat{\boldsymbol{\gamma}}_s - \beta\hat{\xi}_{s+1}\boldsymbol{\tau}^B)^{\intercal}\boldsymbol{\epsilon}_{s,T_t} + (\hat{\eta}_s + \beta\hat{\xi}_{s+1}\tau^A)\kappa_{s,T_t} + (\boldsymbol{\gamma}^{\intercal}\hat{\boldsymbol{\phi}}_s - \eta\hat{\theta}_s)n_{s,T_t}\Big] = 0,$$

and

$$[\boldsymbol{R}^j_{t+1}]: \quad \hat{v}_{\boldsymbol{R}^j_{t+1}} - \beta^{t+1}(\hat{\xi}_{t+1}\boldsymbol{b}^{G\intercal} + \hat{\xi}_0\boldsymbol{b}^{G\intercal}\boldsymbol{R}^B_{0,\boldsymbol{R}^B_{t+1}} + \xi\hat{\boldsymbol{b}}^{G\intercal}_t + \bar{\xi}_0\boldsymbol{b}^{G\intercal}\hat{\boldsymbol{R}}^B_{0,\boldsymbol{R}^B_{t+1}})\mathbf{1}_{\{j\in\mathcal{B}\}}$$
$$+ \sum_{s=0}^{\infty}\beta^s\Big[(\hat{\boldsymbol{\gamma}}_s - \beta\hat{\xi}_{s+1}\boldsymbol{\tau}^B)^{\intercal}\boldsymbol{\epsilon}_{s,\boldsymbol{R}^j_{t+1}} + (\hat{\eta}_s + \beta\hat{\xi}_{s+1}\tau^A)\kappa_{s,\boldsymbol{R}^j_{t+1}} - \beta\hat{\xi}_{s+1}\tilde{R}^N_{s+1,\boldsymbol{R}^j_{t+1}}n_s\Big]$$
$$+ \sum_{s=0}^{\infty}\beta^s\Big[\boldsymbol{\gamma}^{\intercal}(\hat{\boldsymbol{\phi}}_{s,\boldsymbol{R}^j_{t+1}}n + \boldsymbol{\phi}_{s,\boldsymbol{R}^j_{t+1}}\hat{n}_s + \hat{\boldsymbol{\phi}}_s n_{s,\boldsymbol{R}^j_{t+1}}) + \eta(-\hat{\theta}_{s,\boldsymbol{R}^j_{t+1}}n - \theta_{s,\boldsymbol{R}^j_{t+1}}\hat{n}_s - \hat{\theta}_s n_{s,\boldsymbol{R}^j_{t+1}})\Big]$$
$$+ \sum_{s=0}^{\infty}\beta^{s+1}\hat{\xi}_{s+1}(\boldsymbol{\tau}^{B\intercal}\boldsymbol{b}_{s,\boldsymbol{R}^j_{t+1}} + \tau^N n_{s,\boldsymbol{R}^j_{t+1}} + \tau^A a^K_{s,\boldsymbol{R}^j_{t+1}}) = 0.$$

Note that $\beta(R^A+\tau^A)=1$ and $\beta(\boldsymbol{R}^B-\boldsymbol{\tau}^B)=\mathbf{1}_{\mathcal{B}}$ imply

$$(\boldsymbol{\gamma}^{\intercal}\boldsymbol{\phi}_{s,\boldsymbol{R}^j_{t+1}} - \eta\theta_{s,\boldsymbol{R}^j_{t+1}} + \beta\lambda\tilde{R}^N_{s+1,\boldsymbol{R}^j_{t+1}})\hat{n}_s = 0,$$
$$\boldsymbol{\gamma}^{\intercal}\hat{\boldsymbol{\phi}}_{s,\boldsymbol{R}^j_{t+1}} - \eta\hat{\theta}_{s,\boldsymbol{R}^j_{t+1}} + \beta\lambda\hat{\tilde{R}}^N_{s+1,\boldsymbol{R}^j_{t+1}} = \beta\lambda(\hat{R}^A_{s+1}\theta_{s,\boldsymbol{R}^j_{t+1}} - \hat{\boldsymbol{R}}^{B\intercal}_{s+1}\boldsymbol{\phi}_{s,\boldsymbol{R}^j_{t+1}}).$$

Therefore,

$$
\begin{aligned}
[\boldsymbol{R}^j_{t+1}]: \quad & \Big[\beta^{t+1}(\hat{\lambda}_{t+1}-\hat{\xi}_{t+1})\boldsymbol{b}^{G\intercal}+(\hat{\lambda}_0-\hat{\xi}_0)\boldsymbol{b}^{G\intercal}\boldsymbol{R}^B_{0,\boldsymbol{R}^B_{t+1}}\Big]\mathbf{1}_{\{j\in\mathcal{B}\}} \\
& + \Big[\beta^{t+1}(\hat{\lambda}_{t+1}a^K+\lambda\hat{a}^K_t)+(\hat{\lambda}_0 R^A_{0,R^A_{t+1}}+\lambda\hat{R}^A_{0,R^A_{t+1}})a^K+\sum_{s=0}^{\infty}\beta^s(\hat{\lambda}_s w_{s,R^A_{t+1}}+\lambda\hat{w}_{s,R^A_{t+1}})\Big]\mathbf{1}_{\{j\in\mathcal{A}\}} \\
& + \sum_{s=0}^{\infty}\beta^{s+1}\Big[(\hat{\lambda}_{s+1}-\hat{\xi}_{s+1})\tilde{R}^N_{s+1,\boldsymbol{R}^j_{t+1}}n+\beta^{s+1}\lambda(\hat{R}^A_{s+1}\theta_{s,\boldsymbol{R}^j_{t+1}}-\hat{\boldsymbol{R}}^{B\intercal}_{s+1}\boldsymbol{\phi}_{s,\boldsymbol{R}^j_{t+1}})n\Big] \\
& + \sum_{s=0}^{\infty}\beta^s\Big[(\hat{\boldsymbol{\gamma}}_s-\beta\hat{\xi}_{s+1}\boldsymbol{\tau}^B_{s+1})^\intercal\boldsymbol{\epsilon}_{s,\boldsymbol{R}^j_{t+1}}+(\hat{\eta}_s-\beta\hat{\xi}_{s+1}\tau^A_{s+1})\kappa_{s,\boldsymbol{R}^j_{t+1}}+(\boldsymbol{\gamma}^\intercal_s\hat{\boldsymbol{\phi}}_s-\eta_s\hat{\theta}_s)n_{s,\boldsymbol{R}^j_{t+1}}\Big]=0.
\end{aligned}
$$

Because $R^A\hat{\theta}_s-\boldsymbol{R}^{B\intercal}\hat{\boldsymbol{\phi}}_s=(\beta\xi)^{-1}(\eta\hat{\theta}_s-\boldsymbol{\gamma}^\intercal\hat{\boldsymbol{\phi}}_s)$, we have

$$
\hat{R}^N_{s+1}=\hat{R}^A_{s+1}\theta-\hat{\boldsymbol{R}}^{B\intercal}_{s+1}\boldsymbol{\phi}+R^A\hat{\theta}_s-\boldsymbol{R}^{B\intercal}\hat{\boldsymbol{\phi}}_s=\hat{R}^A_{s+1}\theta-\hat{\boldsymbol{R}}^{B\intercal}_{s+1}\boldsymbol{\phi}+(\beta\xi)^{-1}(\eta\hat{\theta}_s-\boldsymbol{\gamma}^\intercal\hat{\boldsymbol{\phi}}_s),
$$

and

$$
\boldsymbol{\gamma}^\intercal\hat{\boldsymbol{\phi}}_s-\eta\hat{\theta}_s=\beta\xi\Big[\hat{R}^A_{s+1}\theta-\hat{\boldsymbol{R}}^{B\intercal}_{s+1}\boldsymbol{\phi}-\hat{R}^N_{s+1}\Big].
$$

Substitute $\boldsymbol{\gamma}^\intercal_s\hat{\boldsymbol{\phi}}_s-\eta_s\hat{\theta}_s$, note that

$$
a^F_{s,\boldsymbol{R}^j_{t+1}}=\theta_{s,\boldsymbol{R}^j_{t+1}}n_s+\theta_s n_{s,\boldsymbol{R}^j_{t+1}},\ \boldsymbol{b}^F_{s,\boldsymbol{R}^j_{t+1}}=\boldsymbol{\phi}_{s,\boldsymbol{R}^j_{t+1}}n_s+\boldsymbol{\phi}_s n_{s,\boldsymbol{R}^j_{t+1}},\ a^F_{s,T_t}=\theta_s n_{s,T_t},\ \boldsymbol{b}^F_{s,T_t}=\boldsymbol{\phi}_s n_{s,T_t},
$$

and define

$$
\tilde{\boldsymbol{\gamma}}_s\coloneqq\hat{\boldsymbol{\gamma}}_s-\beta\hat{\xi}_{s+1}\boldsymbol{\tau}^B,\quad \tilde{\eta}_s\coloneqq\hat{\eta}_s+\beta\hat{\xi}_{s+1}\tau^A,\quad \tilde{\xi}_t=\hat{\xi}_t-\hat{\lambda}_t.
$$

The FOC for returns can be written as

$$
\begin{aligned}
[\boldsymbol{R}^j_{t+1}]: \quad & -[\beta^{t+1}\tilde{\xi}_{t+1}\boldsymbol{b}^{G\intercal}+\tilde{\xi}_0\boldsymbol{b}^{G\intercal}\boldsymbol{R}^B_{0,\boldsymbol{R}^B_{t+1}}]\mathbf{1}_{\{j\in\mathcal{B}\}}-\sum_{s=0}^{\infty}\beta^{s+1}\tilde{\xi}_{s+1}\tilde{R}^N_{s+1,\boldsymbol{R}^j_{t+1}}n \\
& + \Big[\beta^{t+1}(\hat{\lambda}_{t+1}a^K+\lambda\hat{a}^K_t)+(\hat{\lambda}_0 R^A_{0,R^A_{t+1}}+\lambda\hat{R}^A_{0,R^A_{t+1}})a^K+\sum_{s=0}^{\infty}\beta^s(\hat{\lambda}_s w_{s,R^A_{t+1}}+\lambda\hat{w}_{s,R^A_{t+1}})\Big]\mathbf{1}_{\{j\in\mathcal{A}\}} \\
& + \sum_{s=0}^{\infty}\beta^{s+1}\lambda\Big[\hat{R}^A_{s+1}a^F_{s,\boldsymbol{R}^j_{t+1}}-\hat{\boldsymbol{R}}^{B\intercal}_{s+1}\boldsymbol{b}^F_{s,\boldsymbol{R}^j_{t+1}}-\hat{R}^N_{s+1}n_{s,\boldsymbol{R}^j_{t+1}}\Big] \\
& + \sum_{s=0}^{\infty}\beta^s\Big[\tilde{\boldsymbol{\gamma}}^\intercal_s\boldsymbol{\epsilon}_{s,\boldsymbol{R}^j_{t+1}}+\tilde{\eta}_s\kappa_{s,\boldsymbol{R}^j_{t+1}}\Big]=0.
\end{aligned}
$$

Similarly, we have

$$[T_t]: \quad \beta^t \tilde{\xi}_t + \sum_{s=0}^{\infty} \beta^s \left[\tilde{\boldsymbol{\gamma}}_s^{\intercal} \boldsymbol{\epsilon}_{s,T_t} + \tilde{\eta}_s \kappa_{s,T_t} \right] + \beta^{s+1} \xi \left[\hat{R}_{s+1}^A a_{s,T_t}^F - \hat{\boldsymbol{R}}_{s+1}^{B\ \intercal} \boldsymbol{b}_{s,T_t}^F - \hat{R}_{s+1}^N n_{s,T_t} \right] = 0,$$

and

$$[b_t^G]: \quad \tilde{\boldsymbol{\gamma}}_t^{\intercal} - \beta \bar{\xi} \hat{\boldsymbol{R}}_{t+1}^{B\ \intercal} - (\tilde{\xi}_{t+1} - \tilde{\xi}_t) \mathbf{1}_{\mathcal{B}}^{\intercal} = (\hat{\lambda}_{t+1} - \hat{\lambda}_t) \mathbf{1}_{\mathcal{B}}^{\intercal}.$$

We now simplify the expression for

$$\beta^{t+1}(\hat{\lambda}_{t+1} a^K + \lambda \hat{a}_t^K) + (\hat{\lambda}_0 R_{0,R_{t+1}^A}^A + \lambda \hat{R}_{0,R_{t+1}^A}^A) a^K + \sum_{s=0}^{\infty} \beta^s (\hat{\lambda}_s w_{s,R_{t+1}^A} + \lambda \hat{w}_{s,R_{t+1}^A})$$

First, for $a_{-1}^K R_{0,R_{t+1}^A}^A$, note that because capital is predetermined in the initial period and labor is inelastic, we have $\varrho_{0,R_{t+1}^A} = 0$ and

$$q_{-1} R_{0,R_{t+1}^A}^A = \boldsymbol{\phi}(\iota_0) q_{0,R_{t+1}^A},$$

which implies

$$a_{-1}^K R_{0,R_{t+1}^A}^A = k_0 q_{0,R_{t+1}^A}.$$

In order to link q_{0,R_{t+1}^A} to future capital income, we use equation (4) and take derivatives with respect to R_{t+1}^A:

$$q_s \mathbf{1}_{\{s=t\}} + (R_{s+1}^A + \tau_{s+1}^A) q_{s,R_{t+1}^A} = \varrho_{s+1,R_{t+1}^A} + \Phi(\iota_{s+1}) q_{s+1,R_{t+1}^A}, \forall s \geq 0.$$

Multiply both sides by $(R_{s+1}^A + \tau_{s+1}^A)^{-1} k_s$ and use $\Phi(\iota_{s+1}) k_s = k_{s+1}$, we have

$$(R_{s+1}^A + \tau_{s+1}^A)^{-1} k_s q_s \mathbf{1}_{\{s=t\}} + k_s q_{s,R_{t+1}^A} = (R_{s+1}^A + \tau_{s+1}^A)^{-1} (k_s \varrho_{s+1,R_{t+1}^A} + k_{s+1} q_{s+1,R_{t+1}^A}).$$

Let $R_{0,l+1}^A = \prod_{s=0}^{l} \left(R_{s+1}^A + \tau_{s+1}^A \right)$ and use $k_l \varrho_{l,R_{t+1}^A} = -w_{l,R_{t+1}^A}$ from the optimality of

production,[3] we have

$$a^K_{-1}R^A_{0,R^A_{t+1}} = -\sum_{s=0}^{\infty}(R^A_{0,s+1})^{-1}\Big(w_{s+1,R^A_{t+1}} + a^K_t \mathbf{1}_{\{s=t\}}\Big),$$

and the following first-order expansion:

$$a^K_{-1}\hat{R}^A_{0,R^A_{t+1}} = \sum_{s=0}^{\infty}\beta^{s+2}\sum_{l=0}^{s}\hat{R}^A_{l+1}\Big(w_{s+1,R^A_{t+1}} + a^K_t \mathbf{1}_{\{s=t\}}\Big) - \sum_{s=0}^{\infty}\beta^{s+1}\Big(\hat{w}_{s+1,R^A_{t+1}} + \hat{a}^K_t \mathbf{1}_{\{s=t\}}\Big).$$

Using these expressions in the FOC for returns, we have:

$$\begin{aligned}
[\boldsymbol{R}^j_{t+1}]: \quad & -[\beta^{t+1}\tilde{\xi}_{t+1}\boldsymbol{b}^{G\intercal}_t + \tilde{\xi}_0 \boldsymbol{b}^{G\ \intercal}_{-1}\boldsymbol{R}^B_{0,\boldsymbol{R}^B_{t+1}}]\mathbf{1}_{\{j\in\mathcal{B}\}} - \sum_{s=0}^{\infty}\beta^s\tilde{\xi}_s\tilde{R}^N_{s,\boldsymbol{R}^j_{t+1}}n_{s-1} \\
& + \sum_{s=0}^{\infty}\beta^{s+1}\Big[\Big(\hat{\lambda}_{s+1} - \hat{\lambda}_0 + \beta\lambda\sum_{l=0}^{s}\hat{R}^A_{l+1}\Big)\Big(w_{s+1,R^A_{t+1}} + a^K_t\mathbf{1}_{\{s=t\}}\Big)\Big]\mathbf{1}_{\{j\in\mathcal{A}\}} \\
& + \sum_{s=0}^{\infty}\beta^s\Big[\tilde{\boldsymbol{\gamma}}^{\intercal}_s\boldsymbol{\epsilon}_{s,\boldsymbol{R}^j_{t+1}} + \tilde{\eta}_s\kappa_{s,\boldsymbol{R}^j_{t+1}} + \beta\xi_{s+1}\Big(\hat{R}^A_{s+1}a^F_{s,\boldsymbol{R}^j_{t+1}} - \hat{\boldsymbol{R}}^{B\ \intercal}_{s+1}\boldsymbol{b}^F_{s,\boldsymbol{R}^j_{t+1}} - \hat{R}^N_{s+1}n_{s,\boldsymbol{R}^j_{t+1}}\Big)\Big] = 0.
\end{aligned}$$

Let $\tilde{\boldsymbol{\iota}}_{\boldsymbol{b}}$ be a vector of size $|\mathcal{B}|$ that sums to one, then the FOC for $\boldsymbol{b}^G$ implies

$$[b^G_t]: \quad (\tilde{\boldsymbol{\gamma}}_t - \beta\bar{\xi}\hat{\boldsymbol{R}}^B_{t+1})^{\intercal}\tilde{\boldsymbol{\iota}}_{\boldsymbol{b}} - (\tilde{\xi}_{t+1} - \tilde{\xi}_t) = \hat{\lambda}_{t+1} - \hat{\lambda}_t,$$

and therefore

$$\begin{aligned}
[\boldsymbol{R}^j_{t+1}]: \quad & -[\beta^{t+1}\tilde{\xi}_{t+1}\boldsymbol{b}^{G\intercal}_t + \tilde{\xi}_0 \boldsymbol{b}^{G\ \intercal}_{-1}\boldsymbol{R}^B_{0,\boldsymbol{R}^B_{t+1}}]\mathbf{1}_{\{j\in\mathcal{B}\}} - \sum_{s=0}^{\infty}\beta^s\tilde{\xi}_s\tilde{R}^N_{s,\boldsymbol{R}^j_{t+1}}n_{s-1} \\
& - \Big[\sum_{s=0}^{\infty}\beta^{s+1}(\tilde{\xi}_{s+1} - \tilde{\xi}_0)\Big(w_{s+1,R^A_{t+1}} + a^K_t\mathbf{1}_{\{s=t\}}\Big)\Big]\mathbf{1}_{\{j\in\mathcal{A}\}} \\
& + \Big[\sum_{s=0}^{\infty}\beta^{s+1}\sum_{l=0}^{s}\Big(\beta\lambda\hat{R}^A_{l+1} - (\beta\lambda\hat{\boldsymbol{R}}^B_{l+1} - \tilde{\boldsymbol{\gamma}}_l)^{\intercal}\tilde{\boldsymbol{\iota}}_{\boldsymbol{b}}\Big)\Big(w_{s+1,R^A_{t+1}} + a^K_t\mathbf{1}_{\{s=t\}}\Big)\Big]\mathbf{1}_{\{j\in\mathcal{A}\}} \\
& + \sum_{s=0}^{\infty}\beta^s\Big[\tilde{\boldsymbol{\gamma}}^{\intercal}_s\boldsymbol{\epsilon}_{s,\boldsymbol{R}^j_{t+1}} + \tilde{\eta}_s\kappa_{s,\boldsymbol{R}^j_{t+1}} + \beta\xi_{s+1}\Big(\hat{R}^A_{s+1}a^F_{s,\boldsymbol{R}^j_{t+1}} - \hat{\boldsymbol{R}}^{B\ \intercal}_{s+1}\boldsymbol{b}^F_{s,\boldsymbol{R}^j_{t+1}} - \hat{R}^N_{s+1}n_{s,\boldsymbol{R}^j_{t+1}}\Big)\Big] = 0.
\end{aligned}$$

[3] Because F is constant-return-to-scale, we have $w_t = F_t - \varrho_t k_t$ and $w_{s,R^A_{t+1}} = -\varrho_{s,R^A_{t+1}}k_s$ where we use $F_{t,k} = \varrho_t$.

Note that

$$\sum_{s=0}^{\infty}\beta^{s+1}\sum_{l=0}^{s}(\beta\lambda\hat{R}^A_{l+1}-(\beta\lambda\hat{\boldsymbol{R}}^B_{l+1}-\tilde{\boldsymbol{\gamma}}_l)^\intercal\tilde{\boldsymbol{\iota}}_{\boldsymbol{b}})(w_{s+1,R^A_{t+1}}+a^K_t\mathbf{1}_{\{s=t\}})$$
$$=\sum_{s=0}^{\infty}(\beta\lambda\hat{R}^A_{s+1}-(\beta\lambda\hat{\boldsymbol{R}}^B_{s+1}-\tilde{\boldsymbol{\gamma}}_s)^\intercal\tilde{\boldsymbol{\iota}}_{\boldsymbol{b}})\sum_{l=s}^{\infty}\beta^{l+1}(w_{l+1,R^A_{t+1}}+a^K_t\mathbf{1}_{\{l=t\}}).$$

Therefore, we have

$$\begin{aligned}[\boldsymbol{R}^j_{t+1}]:\quad &-[\beta^{t+1}\tilde{\xi}_{t+1}\boldsymbol{b}^{G\intercal}_t+\tilde{\xi}_0\boldsymbol{b}^{G\ \intercal}_{-1}\boldsymbol{R}^B_{0,\boldsymbol{R}^B_{t+1}}]\mathbf{1}_{\{j\in\mathcal{B}\}}-\sum_{s=0}^{\infty}\beta^s\tilde{\xi}_s\tilde{R}^N_{s,\boldsymbol{R}^j_{t+1}}n_{s-1}\\
&-\sum_{s=0}^{\infty}\beta^{s+1}\Big[(\tilde{\xi}_{s+1}-\tilde{\xi}_0)(w_{s+1,R^A_{t+1}}+a^K_t\mathbf{1}_{\{s=t\}})\\
&\qquad+(\beta\lambda\hat{R}^A_{s+1}-(\beta\lambda\hat{\boldsymbol{R}}^B_{s+1}-\tilde{\boldsymbol{\gamma}}_s)^\intercal\tilde{\boldsymbol{\iota}}_{\boldsymbol{b}})\boldsymbol{z}^K_{s+1,R^A_{t+1}}\Big]\mathbf{1}_{\{j\in\mathcal{A}\}}\\
&+\sum_{s=0}^{\infty}\beta^s\Big[\tilde{\boldsymbol{\gamma}}^\intercal_s\boldsymbol{\epsilon}_{s,\boldsymbol{R}^j_{t+1}}+\tilde{\eta}_s\kappa_{s,\boldsymbol{R}^j_{t+1}}+\beta\xi_{s+1}\Big(\hat{R}^A_{s+1}a^F_{s,\boldsymbol{R}^j_{t+1}}-\hat{\boldsymbol{R}}^{B\ \intercal}_{s+1}\boldsymbol{b}^F_{s,\boldsymbol{R}^j_{t+1}}-\hat{R}^N_{s+1}n_{s,\boldsymbol{R}^j_{t+1}}\Big)\Big]=0,\end{aligned}$$

where

$$\boldsymbol{z}^K_{s+1,R^A_{t+1}}\coloneqq-\sum_{l=s}^{\infty}\beta^{l-s}\Big(w_{l+1,R^A_{t+1}}+a^K_t\mathbf{1}_{\{l=t\}}\Big).$$

Using FOC T_t to solve for $\tilde{\xi}_t$ and to reduce the expression of FOCs for returns and $\boldsymbol{b}^G$, we have

$$\begin{aligned}[\boldsymbol{R}^j_{t+1}]:\quad &\sum_{s=0}^{\infty}\beta^s\Big[\tilde{\boldsymbol{\gamma}}^\intercal_s\tilde{\boldsymbol{\epsilon}}_{s,\boldsymbol{R}^j_{t+1}}+\tilde{\eta}_s\tilde{\kappa}_{s,\boldsymbol{R}^j_{t+1}}+\beta\xi\Big(\hat{R}^A_{s+1}\tilde{a}^F_{s,\boldsymbol{R}^j_{t+1}}-\hat{\boldsymbol{R}}^{B\ \intercal}_{s+1}\tilde{\boldsymbol{b}}^F_{s,\boldsymbol{R}^j_{t+1}}-\hat{R}^N_{s+1}\tilde{n}_{s,\boldsymbol{R}^j_{t+1}}\Big)\Big]\\
&=\Big[\sum_{s=0}^{\infty}\beta^{s+1}(\beta\xi\hat{R}^A_{s+1}-(\beta\lambda\hat{\boldsymbol{R}}^B_{s+1}-\tilde{\boldsymbol{\gamma}}_s)^\intercal\tilde{\boldsymbol{\iota}}_{\boldsymbol{b}})\boldsymbol{z}^K_{s+1,R^A_{t+1}}\Big]\mathbf{1}_{\{j\in\mathcal{A}\}},\end{aligned}\tag{20}$$

and

$$\begin{aligned}[b^G_t]:\quad &\sum_{s=0}^{\infty}\beta^s\Big[\tilde{\boldsymbol{\gamma}}^\intercal_s\boldsymbol{\epsilon}_{s,\Delta T_{t+1}}+\tilde{\eta}_s\kappa_{s,\Delta T_{t+1}}+\beta\xi\Big(\hat{R}^A_{s+1}a^F_{s,\Delta T_{t+1}}-\hat{\boldsymbol{R}}^{B\ \intercal}_{s+1}\boldsymbol{b}^F_{s,\Delta T_{t+1}}-\hat{R}^N_{s+1}n_{s,\Delta T_{t+1}}\Big)\Big]\mathbf{1}^\intercal_{\mathcal{B}}\\
&=\beta^{t+1}\bar{\xi}\hat{\boldsymbol{R}}^{B\ \intercal}_{t+1}-\beta^t\tilde{\boldsymbol{\gamma}}^\intercal_t+\beta^t(\hat{\lambda}_{t+1}-\hat{\lambda}_t)\mathbf{1}^\intercal_{\mathcal{B}},\end{aligned}\tag{21}$$

where

$$\begin{aligned}\tilde{\boldsymbol{\epsilon}}_{s,\boldsymbol{R}^j_{t+1}} \coloneqq\ & \boldsymbol{\epsilon}_{s,\boldsymbol{R}^j_{t+1}} + \sum_{\ell=0}^{\infty} \boldsymbol{\epsilon}_{s,T_\ell} \tilde{R}^N_{\ell,\boldsymbol{R}^j_{t+1}} n_{\ell-1} + \Big[\boldsymbol{\epsilon}_{s,T_{t+1}} \boldsymbol{b}^{G\intercal} + \boldsymbol{\epsilon}_{s,T_0} \boldsymbol{b}^{G\ \intercal}_{-1} \boldsymbol{R}^B_{0,\boldsymbol{R}^B_{t+1}}\Big] \mathbf{1}_{\{j\in\mathcal{B}\}} \\ & + \Big[\sum_{\ell=0}^{\infty} (\boldsymbol{\epsilon}_{s,T_{\ell+1}} - \beta^{\ell+1}\boldsymbol{\epsilon}_{s,T_0})(w_{\ell+1,R^A_{t+1}} + a^K_t \mathbf{1}_{\{\ell=t\}})\Big] \mathbf{1}_{\{j\in\mathcal{A}\}}, \\ \boldsymbol{\epsilon}_{s,\Delta T_{t+1}} \coloneqq\ & \beta^{-1}\boldsymbol{\epsilon}_{s,T_{t+1}} - \boldsymbol{\epsilon}_{s,T_t},\end{aligned}$$

and similarly for other objects. Note that these objects are comprised of market elasticities as defined in Definition 2, along with along with steady state holdings of assets.[4]

A.5 Matrix Form

Define

$$\tilde{\boldsymbol{\gamma}} \coloneqq [\beta^t \tilde{\boldsymbol{\gamma}}_t], \quad \hat{\boldsymbol{R}}^j \coloneqq [\beta^{t+1} \hat{\boldsymbol{R}}^j_{t+1}], \quad \tilde{\boldsymbol{\epsilon}}_{\boldsymbol{R}^j} = [\tilde{\boldsymbol{\epsilon}}_{s,\boldsymbol{R}^j_{t+1}}], \quad \boldsymbol{\epsilon}_{\Delta \boldsymbol{T}} = [\boldsymbol{\epsilon}_{s,\Delta T_{t+1}}]$$

and similarly for other matrices, and let

$$\Delta\hat{\boldsymbol{\lambda}} \coloneqq [\beta^t(\hat{\lambda}_{t+1} - \hat{\lambda}_t)], \quad \Delta\hat{\boldsymbol{\xi}} \coloneqq [\beta^t(\hat{\xi}_{t+1} - \hat{\xi}_t)].$$

We have

$$[R^A_{t+1}]: \quad \tilde{\boldsymbol{\gamma}}^\intercal(\tilde{\boldsymbol{\epsilon}}_{\boldsymbol{R}^A} - \mathcal{J}\boldsymbol{z}^K_{\boldsymbol{R}^A}) + \tilde{\boldsymbol{\eta}}^\intercal \tilde{\boldsymbol{\kappa}}_{\boldsymbol{R}^A} + \xi\big[\hat{\boldsymbol{R}}^{A\intercal}(\tilde{\boldsymbol{a}}^F_{\boldsymbol{R}^A} - \boldsymbol{z}^K_{\boldsymbol{R}^A}) - \hat{\boldsymbol{R}}^{B\intercal}(\tilde{\boldsymbol{b}}^F_{\boldsymbol{R}^A} - \mathcal{J}\boldsymbol{z}^K_{\boldsymbol{R}^A}) - \hat{\boldsymbol{R}}^{N\intercal}\tilde{\boldsymbol{n}}^F_{\boldsymbol{R}^A}\big] = \mathbf{0},$$

$$[\boldsymbol{R}^B_{t+1}]: \quad \tilde{\boldsymbol{\gamma}}^\intercal \tilde{\boldsymbol{\epsilon}}_{\boldsymbol{R}^B} + \tilde{\boldsymbol{\eta}}^\intercal \tilde{\boldsymbol{\kappa}}_{\boldsymbol{R}^B} + \xi\big[\hat{\boldsymbol{R}}^{A\intercal}\tilde{\boldsymbol{a}}^F_{\boldsymbol{R}^B} - \hat{\boldsymbol{R}}^{B\intercal}\tilde{\boldsymbol{b}}^F_{\boldsymbol{R}^B} - \hat{\boldsymbol{R}}^{N\intercal}\tilde{\boldsymbol{n}}_{\boldsymbol{R}^B}\big] = \mathbf{0},$$

$$[\boldsymbol{b}^G_t]: \quad (\Delta\hat{\boldsymbol{\xi}} \otimes \mathbf{1}_{\mathcal{B}})^\intercal + \xi\hat{\boldsymbol{R}}^{B\intercal} = \tilde{\boldsymbol{\gamma}}^\intercal.$$

where

$$\Delta\hat{\boldsymbol{\xi}}^\intercal = \Delta\hat{\boldsymbol{\lambda}}^\intercal - \big[\tilde{\boldsymbol{\gamma}}^\intercal \boldsymbol{\epsilon}_{\Delta\boldsymbol{T}} + \tilde{\boldsymbol{\eta}}^\intercal \boldsymbol{\kappa}_{\Delta\boldsymbol{T}} + \xi\big(\hat{\boldsymbol{R}}^{A\intercal}\boldsymbol{a}^F_{\Delta\boldsymbol{T}} - \hat{\boldsymbol{R}}^{B\intercal}\boldsymbol{b}^F_{\Delta\boldsymbol{T}} - \hat{\boldsymbol{R}}^{N\intercal}\boldsymbol{n}_{\Delta\boldsymbol{T}}\big)\big].$$

$\mathcal{J} \coloneqq \boldsymbol{I} \otimes \tilde{\boldsymbol{\iota}}_{\boldsymbol{b}}$, and $\boldsymbol{I}$ is an identity matrix of the size of the time horizon.

[4] From the labor market clearing condition in Equation (18), we can write $\boldsymbol{w}_{\boldsymbol{R}^A} = -\tilde{\boldsymbol{h}}^{-1}_{\boldsymbol{w}} \tilde{\boldsymbol{h}}_{\boldsymbol{R}^A}$, and therefore $w_{l,R^A_{t+1}}$ is also a market elasticity consistent with Definition 2.

We define the following notation for arbitrary syntax $\boldsymbol{\chi}$:

$$\check{\boldsymbol{\chi}}_{\boldsymbol{R}^B} \coloneqq \boldsymbol{\chi}_{\boldsymbol{R}^B} + \boldsymbol{\chi}_{\boldsymbol{R}^A}(-\tilde{\boldsymbol{\kappa}}_{\boldsymbol{R}^A})^{-1}\tilde{\boldsymbol{\kappa}}_{\boldsymbol{R}^B},$$
$$\check{\boldsymbol{\chi}}_{\Delta\boldsymbol{T}} \coloneqq \boldsymbol{\chi}_{\Delta\boldsymbol{T}} + \boldsymbol{\chi}_{\boldsymbol{R}^A}(-\tilde{\boldsymbol{\kappa}}_{\boldsymbol{R}^A})^{-1}\boldsymbol{\kappa}_{\Delta\boldsymbol{T}}.$$

Solving for $\tilde{\boldsymbol{\eta}}$, we have

$$[\boldsymbol{R}^B_{t+1}]: \quad \tilde{\boldsymbol{\gamma}}^\intercal(\check{\tilde{\boldsymbol{\epsilon}}}_{\boldsymbol{R}^B} - \mathcal{J}\check{\boldsymbol{z}}^K_{\boldsymbol{R}^B}) + \xi\big[\hat{\boldsymbol{R}}^{A\intercal}(\check{\tilde{\boldsymbol{a}}}^F_{\boldsymbol{R}^B} - \check{\boldsymbol{z}}^K_{\boldsymbol{R}^B}) - \hat{\boldsymbol{R}}^{B\intercal}(\check{\tilde{\boldsymbol{b}}}^F_{\boldsymbol{R}^B} - \mathcal{J}\check{\boldsymbol{z}}^K_{\boldsymbol{R}^B}) - \hat{\boldsymbol{R}}^{N\intercal}\check{\tilde{\boldsymbol{n}}}_{\boldsymbol{R}^B}\big] = \boldsymbol{0}.$$

Solving for $\tilde{\boldsymbol{\gamma}}$ from FOC $\boldsymbol{R}^B$, we have

$$[\boldsymbol{R}^B_{t+1}]: \quad \tilde{\boldsymbol{\gamma}}^\intercal = \xi\big[\hat{\boldsymbol{R}}^{A\intercal}\boldsymbol{\Gamma}^A_{\boldsymbol{\epsilon}} - \hat{\boldsymbol{R}}^{B\intercal}\boldsymbol{\Gamma}^B_{\boldsymbol{\epsilon}} - \hat{\boldsymbol{R}}^{N\intercal}\boldsymbol{\Gamma}^N_{\boldsymbol{\epsilon}}\big],$$

where

$$\boldsymbol{\Gamma}^A_{\boldsymbol{\epsilon}} \coloneqq (\check{\tilde{\boldsymbol{a}}}^F_{\boldsymbol{R}^B} - \check{\boldsymbol{z}}^K_{\boldsymbol{R}^B})(-\check{\tilde{\boldsymbol{\epsilon}}}_{\boldsymbol{R}^B} + \mathcal{J}\check{\boldsymbol{z}}^K_{\boldsymbol{R}^B})^{-1},$$
$$\boldsymbol{\Gamma}^B_{\boldsymbol{\epsilon}} \coloneqq (\check{\tilde{\boldsymbol{b}}}^F_{\boldsymbol{R}^B} - \mathcal{J}\check{\boldsymbol{z}}^K_{\boldsymbol{R}^B})(-\check{\tilde{\boldsymbol{\epsilon}}}_{\boldsymbol{R}^B} + \mathcal{J}\check{\boldsymbol{z}}^K_{\boldsymbol{R}^B})^{-1},$$
$$\boldsymbol{\Gamma}^N_{\boldsymbol{\epsilon}} \coloneqq \check{\tilde{\boldsymbol{n}}}_{\boldsymbol{R}^B}(-\check{\tilde{\boldsymbol{\epsilon}}}_{\boldsymbol{R}^B} + \mathcal{J}\check{\boldsymbol{z}}^K_{\boldsymbol{R}^B})^{-1}.$$

Define

$$\boldsymbol{\Gamma}^A_{\Delta\boldsymbol{T}} \coloneqq \check{\boldsymbol{a}}^F_{\Delta\boldsymbol{T}} + \boldsymbol{\Gamma}^A_{\boldsymbol{\epsilon}}\check{\boldsymbol{\epsilon}}_{\Delta\boldsymbol{T}}, \quad \boldsymbol{\Gamma}^B_{\Delta\boldsymbol{T}} \coloneqq \check{\boldsymbol{b}}^F_{\Delta\boldsymbol{T}} + \boldsymbol{\Gamma}^B_{\boldsymbol{\epsilon}}\check{\boldsymbol{\epsilon}}_{\Delta\boldsymbol{T}}, \quad \boldsymbol{\Gamma}^N_{\Delta\boldsymbol{T}} \coloneqq \check{\boldsymbol{n}}_{\Delta\boldsymbol{T}} + \boldsymbol{\Gamma}^N_{\boldsymbol{\epsilon}}\check{\boldsymbol{\epsilon}}_{\Delta\boldsymbol{T}},$$

then we have

$$\Delta\hat{\boldsymbol{\xi}}^\intercal = \Delta\hat{\boldsymbol{\lambda}}^\intercal - \xi\big[\hat{\boldsymbol{R}}^{A\intercal}\boldsymbol{\Gamma}^A_{\Delta\boldsymbol{T}} - \hat{\boldsymbol{R}}^{B\intercal}\boldsymbol{\Gamma}^B_{\Delta\boldsymbol{T}} - \hat{\boldsymbol{R}}^{N\intercal}\boldsymbol{\Gamma}^N_{\Delta\boldsymbol{T}}\big].$$

Let

$$\tilde{\boldsymbol{\Gamma}}_j \coloneqq \boldsymbol{\Gamma}^j_{\boldsymbol{\epsilon}} + \boldsymbol{\Gamma}^j_{\Delta\boldsymbol{T}} \otimes \boldsymbol{1}^\intercal_{\mathcal{B}}, \quad \forall j \in \{A, B, N\},$$

then we have

$$[\boldsymbol{b}^G_t]: \quad \lambda^{-1}(\Delta\hat{\boldsymbol{\lambda}} \otimes \boldsymbol{1}_{\mathcal{B}})^\intercal + \hat{\boldsymbol{R}}^{B\intercal} = \hat{\boldsymbol{R}}^{A\intercal}\tilde{\boldsymbol{\Gamma}}_A - \hat{\boldsymbol{R}}^{B\intercal}\tilde{\boldsymbol{\Gamma}}_B - \hat{\boldsymbol{R}}^{N\intercal}\tilde{\boldsymbol{\Gamma}}_N.$$

B Benchmarks and Examples

B.1 Neoclassical Growth Benchmark

Because $\Delta\hat{\boldsymbol{\lambda}} = -\lambda\hat{\boldsymbol{R}}^N$, optimality of the policy rule requires

$$\hat{\boldsymbol{R}}^{B\intercal} - \hat{\boldsymbol{R}}^{N\intercal} \otimes \mathbf{1}_{\mathcal{B}}^{\intercal} = \hat{\boldsymbol{R}}^{A\intercal}\tilde{\boldsymbol{\Gamma}}_A - \hat{\boldsymbol{R}}^{B\intercal}\tilde{\boldsymbol{\Gamma}}_B - \hat{\boldsymbol{R}}^{N\intercal}\tilde{\boldsymbol{\Gamma}}_{N}..$$

When θ and $\boldsymbol{\phi}$ are constants, we have

$$\tilde{\boldsymbol{\Gamma}}_A \coloneqq \theta\tilde{\boldsymbol{\Gamma}}_N - \boldsymbol{\Gamma}_{\boldsymbol{\epsilon}}^z(\boldsymbol{I}_{\mathcal{B}} + \check{\boldsymbol{\epsilon}}_{\Delta\boldsymbol{T}} \otimes \mathbf{1}_{\mathcal{B}}^{\intercal}), \quad \tilde{\boldsymbol{\Gamma}}_B \coloneqq \boldsymbol{\phi}\tilde{\boldsymbol{\Gamma}}_N - \mathcal{J}\boldsymbol{\Gamma}_{\boldsymbol{\epsilon}}^z(\boldsymbol{I}_{\mathcal{B}} + \check{\boldsymbol{\epsilon}}_{\Delta\boldsymbol{T}} \otimes \mathbf{1}_{\mathcal{B}}^{\intercal}),$$

where $\boldsymbol{\Gamma}_{\boldsymbol{\epsilon}}^z \coloneqq \check{\boldsymbol{z}}_{\boldsymbol{R}^B}^K(-\check{\tilde{\boldsymbol{\epsilon}}}_{\boldsymbol{R}^B} + \mathcal{J}\check{\boldsymbol{z}}_{\boldsymbol{R}^B}^K)^{-1}$, $\boldsymbol{I}_{\mathcal{B}} \coloneqq \boldsymbol{I} \otimes I_{\mathcal{B}}$, and $I_{\mathcal{B}}$ is an identity of size $|\mathcal{B}|$.

The formula for the optimal rule reduces to

$$\begin{aligned}\hat{\boldsymbol{R}}^{B\intercal} - \hat{\boldsymbol{R}}^{N\intercal} \otimes \mathbf{1}_{\mathcal{B}}^{\intercal} &= \hat{\boldsymbol{R}}^{A\intercal}\tilde{\boldsymbol{\Gamma}}_A - \hat{\boldsymbol{R}}^{B\intercal}\tilde{\boldsymbol{\Gamma}}_B - \hat{\boldsymbol{R}}^{N\intercal}\tilde{\boldsymbol{\Gamma}}_N \\ &= -(\hat{\boldsymbol{R}}^{A\intercal} - \hat{\boldsymbol{R}}^{B\intercal}\mathcal{J})\boldsymbol{\Gamma}_{\boldsymbol{\epsilon}}^z \times [\boldsymbol{I}_{\mathcal{B}} + \check{\boldsymbol{\epsilon}}_{\Delta\boldsymbol{T}} \otimes \mathbf{1}_{\mathcal{B}}^{\intercal}].\end{aligned}$$

Because θ and $\boldsymbol{\phi}$ are constant, the definition of R^N implies that

$$\hat{\boldsymbol{R}}^{B\intercal} - \hat{\boldsymbol{R}}^{N\intercal} \otimes \mathbf{1}_{\mathcal{B}}^{\intercal} = -\hat{\boldsymbol{R}}^{A\intercal} \otimes (\theta\mathbf{1}_{\mathcal{B}}^{\intercal}) + \hat{\boldsymbol{R}}^{B\intercal}[\boldsymbol{I}_{\mathcal{B}} + \boldsymbol{I} \otimes (\boldsymbol{\phi}\mathbf{1}_{\mathcal{B}}^{\intercal})].$$

Therefore, the optimality condition reduces to

$$\hat{\boldsymbol{R}}^{B\intercal}[\boldsymbol{I}_{\mathcal{B}} + \boldsymbol{I} \otimes (\boldsymbol{\phi}\mathbf{1}_{\mathcal{B}}^{\intercal})] - \hat{\boldsymbol{R}}^{A\intercal} \otimes (\theta\mathbf{1}_{\mathcal{B}}^{\intercal}) = -(\hat{\boldsymbol{R}}^{A\intercal} - \hat{\boldsymbol{R}}^{B\intercal}\mathcal{J})\boldsymbol{\Gamma}_{\boldsymbol{\epsilon}}^z \times [\boldsymbol{I}_{\mathcal{B}} + \check{\boldsymbol{\epsilon}}_{\Delta\boldsymbol{T}} \otimes \mathbf{1}_{\mathcal{B}}^{\intercal}].$$

Note that because $\theta = \boldsymbol{\phi}\mathbf{1}_{\mathcal{B}}^{\intercal} + 1$, one solution to the system is given by

$$\hat{\boldsymbol{R}}^{A\intercal} \otimes \mathbf{1}_{\mathcal{B}}^{\intercal} = \hat{\boldsymbol{R}}^{B\intercal}.$$

To check for uniqueness, we need to check that the following matrix is invertible:

$$[\boldsymbol{I}_{\mathcal{B}} + \boldsymbol{I} \otimes (\boldsymbol{\phi}\mathbf{1}_{\mathcal{B}}^{\intercal})] - \mathcal{J}\boldsymbol{\Gamma}_{\boldsymbol{\epsilon}}^z \times [\boldsymbol{I}_{\mathcal{B}} + \check{\boldsymbol{\epsilon}}_{\Delta\boldsymbol{T}} \otimes \mathbf{1}_{\mathcal{B}}^{\intercal}].$$

Recall that

$$\boldsymbol{\Gamma}_{\boldsymbol{\epsilon}}^z \coloneqq \check{\boldsymbol{z}}_{\boldsymbol{R}^B}^K(-\check{\tilde{\boldsymbol{\epsilon}}}_{\boldsymbol{R}^B} + \mathcal{J}\check{\boldsymbol{z}}_{\boldsymbol{R}^B}^K)^{-1}.$$

Because θ and $\boldsymbol{\phi}$ are constant, we have

$$\check{\tilde{\boldsymbol{\epsilon}}}_{\boldsymbol{R}^B} = -\check{\tilde{\boldsymbol{b}}}_{\boldsymbol{R}^B} + \tilde{\boldsymbol{n}}_{\boldsymbol{R}^B} \otimes \boldsymbol{\phi} + (\tilde{\boldsymbol{n}}_{\boldsymbol{R}^A} \otimes \boldsymbol{\phi})(-\tilde{\boldsymbol{\kappa}}_{\boldsymbol{R}^A})^{-1}\tilde{\boldsymbol{\kappa}}_{\boldsymbol{R}^B},$$

and

$$\tilde{\boldsymbol{\kappa}}_{\boldsymbol{R}^A}^{-1}\tilde{\boldsymbol{\kappa}}_{\boldsymbol{R}^B} = \tilde{\boldsymbol{n}}_{\boldsymbol{R}^A}^{-1}\tilde{\boldsymbol{n}}_{\boldsymbol{R}^B}.$$

From the property of Kronecker products,

$$\tilde{\boldsymbol{n}}_{\boldsymbol{R}^j} \otimes \boldsymbol{\phi} = (\boldsymbol{I} \times \tilde{\boldsymbol{n}}_{\boldsymbol{R}^j}) \otimes (\boldsymbol{\phi} \times 1) = (\boldsymbol{I} \otimes \boldsymbol{\phi})(\tilde{\boldsymbol{n}}_{\boldsymbol{R}^j} \otimes 1).$$

Therefore,

$$\check{\tilde{\boldsymbol{\epsilon}}}_{\boldsymbol{R}^B} = -\check{\tilde{\boldsymbol{b}}}_{\boldsymbol{R}^B} + (\boldsymbol{I} \otimes \boldsymbol{\phi})\tilde{\boldsymbol{n}}_{\boldsymbol{R}^B} + (\boldsymbol{I} \otimes \boldsymbol{\phi})\tilde{\boldsymbol{n}}_{\boldsymbol{R}^A}(-\tilde{\boldsymbol{n}}_{\boldsymbol{R}^A}^{-1})\tilde{\boldsymbol{n}}_{\boldsymbol{R}^B} = -\check{\tilde{\boldsymbol{b}}}_{\boldsymbol{R}^B},$$

and

$$\mathcal{J}\boldsymbol{\Gamma}_{\boldsymbol{\epsilon}}^z \coloneqq \mathcal{J}\check{\boldsymbol{z}}_{\boldsymbol{R}^B}^K(\check{\tilde{\boldsymbol{b}}}_{\boldsymbol{R}^B} + \mathcal{J}\check{\boldsymbol{z}}_{\boldsymbol{R}^B}^K)^{-1} = \boldsymbol{I}_{\mathcal{B}} - \check{\tilde{\boldsymbol{b}}}_{\boldsymbol{R}^B}(\check{\tilde{\boldsymbol{b}}}_{\boldsymbol{R}^B} + \mathcal{J}\check{\boldsymbol{z}}_{\boldsymbol{R}^B}^K)^{-1}.$$

The condition is reduced to

$$[\boldsymbol{I}_{\mathcal{B}} + \boldsymbol{I} \otimes (\boldsymbol{\phi}\mathbf{1}_{\mathcal{B}}^\intercal)] - [\boldsymbol{I}_{\mathcal{B}} - \check{\tilde{\boldsymbol{b}}}_{\boldsymbol{R}^B}(\check{\tilde{\boldsymbol{b}}}_{\boldsymbol{R}^B} + \mathcal{J}\check{\boldsymbol{z}}_{\boldsymbol{R}^B}^K)^{-1}] \times [\boldsymbol{I}_{\mathcal{B}} + \check{\boldsymbol{\epsilon}}_{\Delta \boldsymbol{T}} \otimes \mathbf{1}_{\mathcal{B}}^\intercal].$$

B.2 Quasi-Linear Preference

Suppose that the household has a quasi-linear preference without portfolio adjustment cost:

$$u\left(\{c_s, \boldsymbol{b}_s, n_s\}_{s \le t}\right) = c_t + \nu(\boldsymbol{b}_t),$$

then transfers do not affect asset holdings and we have

$$\boldsymbol{\epsilon}_{\Delta \boldsymbol{T}} = \mathbf{0}, \quad \boldsymbol{a}_{\Delta \boldsymbol{T}}^F = \mathbf{0}, \quad \boldsymbol{b}_{\Delta \boldsymbol{T}}^F = \mathbf{0}, \quad \boldsymbol{n}_{\Delta \boldsymbol{T}} = \mathbf{0}.$$

As a result,

$$\tilde{\boldsymbol{\Gamma}}_j = \boldsymbol{\Gamma}_{\boldsymbol{\epsilon}}^j, \quad \forall j \in \{A, B, N\},$$

and

$$\check{\tilde{\boldsymbol{\epsilon}}}_{\boldsymbol{R}^B} = \check{\boldsymbol{\epsilon}}_{\boldsymbol{R}^B}, \quad \check{\tilde{\boldsymbol{a}}}_{\boldsymbol{R}^B}^F = \check{\boldsymbol{a}}_{\boldsymbol{R}^B}^F, \quad \check{\tilde{\boldsymbol{b}}}_{\boldsymbol{R}^B}^F = \check{\boldsymbol{b}}_{\boldsymbol{R}^B}^F, \quad \check{\tilde{\boldsymbol{n}}}_{\boldsymbol{R}^B} = \check{\boldsymbol{n}}_{\boldsymbol{R}^B}.$$

The formula reduces to

$$[\boldsymbol{b}_t^G]: \quad \lambda^{-1}(\Delta\hat{\boldsymbol{\lambda}} \otimes \mathbf{1}_{\mathcal{B}})^\intercal + \hat{\boldsymbol{R}}^{B\intercal} = \hat{\boldsymbol{R}}^{A\intercal}\tilde{\boldsymbol{\Gamma}}_A - \hat{\boldsymbol{R}}^{B\intercal}\tilde{\boldsymbol{\Gamma}}_B - \hat{\boldsymbol{R}}^{N\intercal}\tilde{\boldsymbol{\Gamma}}_N.$$

where

$$\begin{aligned}
\tilde{\boldsymbol{\Gamma}}_A &= (\check{\boldsymbol{a}}^F_{\boldsymbol{R}^B} - \check{\boldsymbol{z}}^K_{\boldsymbol{R}^B})(-\check{\boldsymbol{\epsilon}}_{\boldsymbol{R}^B} + \mathcal{J}\check{\boldsymbol{z}}^K_{\boldsymbol{R}^B})^{-1}, \\
\tilde{\boldsymbol{\Gamma}}_B &= (\check{\boldsymbol{b}}^F_{\boldsymbol{R}^B} - \mathcal{J}\check{\boldsymbol{z}}^K_{\boldsymbol{R}^B})(-\check{\boldsymbol{\epsilon}}_{\boldsymbol{R}^B} + \mathcal{J}\check{\boldsymbol{z}}^K_{\boldsymbol{R}^B})^{-1}, \\
\tilde{\boldsymbol{\Gamma}}_N &= \check{\boldsymbol{n}}_{\boldsymbol{R}^B}(-\check{\boldsymbol{\epsilon}}_{\boldsymbol{R}^B} + \mathcal{J}\check{\boldsymbol{z}}^K_{\boldsymbol{R}^B})^{-1}.
\end{aligned}$$

Moreover, if there is no capital adjustment cost, then $\boldsymbol{z}^K_{\boldsymbol{R}^A} = \mathbf{0}$ and we have

$$\tilde{\boldsymbol{\Gamma}}_A \coloneqq \check{\boldsymbol{a}}^F_{\boldsymbol{R}^B}(-\check{\boldsymbol{\epsilon}}^{-1}_{\boldsymbol{R}^B}), \quad \tilde{\boldsymbol{\Gamma}}_B \coloneqq \check{\boldsymbol{b}}^F_{\boldsymbol{R}^B}(-\check{\boldsymbol{\epsilon}}^{-1}_{\boldsymbol{R}^B}), \quad \tilde{\boldsymbol{\Gamma}}_N \coloneqq \check{\boldsymbol{n}}_{\boldsymbol{R}^B}(-\check{\boldsymbol{\epsilon}}^{-1}_{\boldsymbol{R}^B}).$$

B.3 Synthetic Intertemporal Price

Consider the case where the household has an additively separable preference over c_t, n_t, and $\boldsymbol{b}_t$:

$$u\left(\{c_s, \boldsymbol{b}_s, n_s\}_{s\le t}; \boldsymbol{\omega}_t\right) = \tilde{u}(c_t; \boldsymbol{\omega}_t) + \tilde{v}(n_t; \boldsymbol{\omega}_t) + \nu(\boldsymbol{b}_t), \tag{22}$$

and there is no portfolio adjustment cost. If $\nu_{\boldsymbol{bb}}$ is not full rank, there exists a vector $\boldsymbol{\alpha}$ such that

$$\boldsymbol{\alpha}^\intercal \nu_{\boldsymbol{bb}} = \mathbf{0}, \quad \boldsymbol{\alpha}^\intercal \mathbf{1}_{\mathcal{B}} = 1.$$

With such a vector $\boldsymbol{\alpha}$, Assumption 1 holds with return $\hat{R}^*_{t+1} = \boldsymbol{\alpha}^\intercal \hat{\boldsymbol{R}}^B_{t+1}$:

$$\Delta\hat{\boldsymbol{\lambda}}^\intercal = -\bar{\lambda}\hat{\boldsymbol{R}}^{*\intercal},$$

where $\hat{\boldsymbol{R}}^* \coloneqq [\beta^{t+1}\hat{R}^*_{t+1}]$.

Let $\boldsymbol{\Omega}$ be a set of orthonormal basis such that $\boldsymbol{\Omega}_1 = \boldsymbol{\alpha}$, and define

$$\hat{\boldsymbol{b}}^Z_t \coloneqq \boldsymbol{\Omega}^\intercal \hat{\boldsymbol{b}}_t, \quad \hat{\boldsymbol{R}}^Z_{t+1} = \boldsymbol{\Omega}^\intercal \hat{\boldsymbol{R}}^B_{t+1},$$

where the first elements are given by $\hat{b}^*_t \coloneqq \boldsymbol{\alpha}^\intercal \hat{\boldsymbol{b}}_t$ and $\hat{R}^*_{t+1}$.

The household budget constraint is:

$$\hat{c}_t + (\boldsymbol{\Omega}^\intercal \mathbf{1}_{\mathcal{B}})^\intercal \hat{\boldsymbol{b}}_t^Z + \hat{n}_t = \hat{w}_t + (\boldsymbol{R}^Z - \boldsymbol{\tau}^Z)^\intercal \hat{\boldsymbol{b}}_{t-1}^Z + \hat{\boldsymbol{R}}_t^Z \boldsymbol{b}^Z + R^N \hat{n}_{t-1} + \hat{R}_t^N n - \hat{T}_t.$$

Solve for $\hat{c}_t$, and substitute it in $\Delta\hat{\lambda}_{t+1} = -\bar{\lambda}\hat{R}_{t+1}^*$ with $\hat{\lambda}_t = u''(\bar{c})\hat{c}_t$ from the household's problem. We have

$$\hat{b}_{t+1}^* - \left[1 + \beta^{-1}\right]\hat{b}_t^* + \beta^{-1}\hat{b}_{t-1}^* = \epsilon_t^*, \tag{23}$$

where

$$\begin{aligned}\epsilon_t^* \coloneqq &- \left[\hat{w}_t + \hat{\boldsymbol{R}}_t^Z \boldsymbol{b}^Z + \hat{R}_t^N n - \hat{T}_t\right] + \left[\hat{w}_{t+1} + \hat{\boldsymbol{R}}_{t+1}^Z \boldsymbol{b}^Z + \hat{R}_{t+1}^N n - \hat{T}_{t+1}\right] - \frac{u'}{u''}\beta\hat{R}_{t+1}^* \\ &+ \sum_{j=2}^{|\mathcal{B}|}\left[\left(\boldsymbol{\Omega}_j^\intercal \mathbf{1}_{\mathcal{B}}\hat{b}_t^{Z_j} - (\boldsymbol{R}^Z - \boldsymbol{\tau}^Z)\hat{b}_{t-1}^{Z_j}\right) - \left(\boldsymbol{\Omega}_j^\intercal \mathbf{1}_{\mathcal{B}}\hat{b}_{t+1}^{Z_j} - (\boldsymbol{R}^Z - \boldsymbol{\tau}^Z)\hat{b}_t^{Z_j}\right)\right].\end{aligned}$$

From the household's problem, we have

$$\Delta\hat{\lambda}_{t+1}\mathbf{1}_{\mathcal{B}} + \bar{\lambda}\hat{\boldsymbol{R}}_{t+1}^B = v_{\boldsymbol{bb}}\hat{\boldsymbol{b}}_t.$$

Because

$$\boldsymbol{\alpha}^\intercal v_{\boldsymbol{bb}}\hat{\boldsymbol{b}}_t = \Delta\hat{\lambda}_{t+1} + \boldsymbol{\alpha}^\intercal\bar{\lambda}\hat{\boldsymbol{R}}_{t+1}^B = 0$$

for any $\hat{\boldsymbol{b}}_t$, it implies that $\boldsymbol{\alpha}^\intercal v_{\boldsymbol{bb}} = \mathbf{0}_{\mathcal{B}}$.

Left multiply $\boldsymbol{\Omega}^\intercal$, we have

$$\bar{\lambda}(\hat{\boldsymbol{R}}_{t+1}^Z - \hat{R}_{t+1}^*\boldsymbol{\Omega}^\intercal\mathbf{1}_{\mathcal{B}}) = \boldsymbol{\Omega}^\intercal v_{\boldsymbol{bb}}\boldsymbol{\Omega} \times \hat{\boldsymbol{b}}_t^Z,$$

where the first row and column of $\boldsymbol{\Omega}^\intercal v_{\boldsymbol{bb}}\boldsymbol{\Omega}$ are zeros, and its $(1,1)$ minor is non-zero because $\boldsymbol{\alpha}$ is unique.

As a result, $\hat{\boldsymbol{b}}_t^{Z_j}$ can be solved as linear functions of $\hat{\boldsymbol{R}}_{t+1}^Z$ for all $j > 1$, and Equation

(23) implies

$$\boldsymbol{\Omega}^\intercal(\beta^{-1}\boldsymbol{\epsilon}_{s,T_{t+1}} - \boldsymbol{\epsilon}_{s,T_t}) = \begin{pmatrix} -\mathbf{1}_{\{s=t\}} \\ 0 \\ \vdots \\ 0 \end{pmatrix}.$$

Let $\tilde{\boldsymbol{\gamma}}_t^* = \boldsymbol{\alpha}^\intercal \tilde{\boldsymbol{\gamma}}_t$, then

$$\tilde{\boldsymbol{\gamma}}_t^\intercal(\beta^{-1}\boldsymbol{\epsilon}_{s,T_{t+1}} - \boldsymbol{\epsilon}_{s,T_t}) = (\boldsymbol{\Omega}^\intercal\tilde{\boldsymbol{\gamma}}_t)^\intercal(\beta^{-1}\boldsymbol{\Omega}^\intercal\boldsymbol{\epsilon}_{s,T_{t+1}} - \boldsymbol{\Omega}^\intercal\boldsymbol{\epsilon}_{s,T_t}) = -\tilde{\boldsymbol{\gamma}}_t^*\mathbf{1}_{\{s=t\}},$$

and, in matrix form,

$$\tilde{\boldsymbol{\gamma}}^\intercal\boldsymbol{\epsilon}_{\Delta\boldsymbol{T}} = -\tilde{\boldsymbol{\gamma}}^{*\intercal},$$

where $\tilde{\boldsymbol{\gamma}}^* \coloneqq [\beta^t\tilde{\boldsymbol{\gamma}}_t^*]$.

Moreover, because $\hat{n}_t$ satisfies

$$\Delta\hat{\lambda}_{t+1} + \hat{R}_{t+1}^N = \hat{R}_{t+1}^N - \hat{R}_{t+1}^* = \tilde{v}_{nn}\hat{n}_t + \tilde{v}_{n\boldsymbol{\omega}}\hat{\boldsymbol{\omega}}_t,$$

we have $\boldsymbol{n}_{\Delta\boldsymbol{T}} = \mathbf{0}$, which implies that

$$\boldsymbol{a}_{\Delta\boldsymbol{T}}^F = \mathbf{0}, \quad \boldsymbol{b}_{\Delta\boldsymbol{T}}^F = \mathbf{0}, \quad \boldsymbol{\kappa}_{\Delta\boldsymbol{T}} = \mathbf{0}.$$

Therefore, we have $\check{\boldsymbol{\epsilon}}_{\Delta\boldsymbol{T}} = \boldsymbol{\epsilon}_{\Delta\boldsymbol{T}}$ and

$$\boldsymbol{\Gamma}_{\Delta\boldsymbol{T}}^A \coloneqq \boldsymbol{\Gamma}_{\boldsymbol{\epsilon}}^A\boldsymbol{\epsilon}_{\Delta\boldsymbol{T}}, \quad \boldsymbol{\Gamma}_{\Delta\boldsymbol{T}}^B \coloneqq \boldsymbol{\Gamma}_{\boldsymbol{\epsilon}}^B\boldsymbol{\epsilon}_{\Delta\boldsymbol{T}}, \quad \boldsymbol{\Gamma}_{\Delta\boldsymbol{T}}^N \coloneqq \boldsymbol{\Gamma}_{\boldsymbol{\epsilon}}^N\boldsymbol{\epsilon}_{\Delta\boldsymbol{T}}.$$

As a result, Lemma 3 implies

$$\Delta\hat{\boldsymbol{\xi}}^\intercal = \Delta\hat{\boldsymbol{\lambda}}^\intercal - \tilde{\boldsymbol{\gamma}}^\intercal\boldsymbol{\epsilon}_{\Delta\boldsymbol{T}} = -\bar{\lambda}\hat{\boldsymbol{R}}^{*\intercal} + \tilde{\boldsymbol{\gamma}}^{*\intercal}.$$

Use it in the first order condition for $\boldsymbol{b}^G$:

$$[\boldsymbol{b}_t^G]: \quad (\Delta\hat{\boldsymbol{\xi}} \otimes \mathbf{1}_{\mathcal{B}})^\intercal + \xi\hat{\boldsymbol{R}}^{B\intercal} = \tilde{\boldsymbol{\gamma}}^\intercal,$$

we have

$$[\boldsymbol{b}_t^G]: \quad \xi(\hat{\boldsymbol{R}}^B - \hat{\boldsymbol{R}}^* \otimes \mathbf{1}_{\mathcal{B}})^\intercal = (\tilde{\boldsymbol{\gamma}} - \tilde{\boldsymbol{\gamma}}^* \otimes \mathbf{1}_{\mathcal{B}})^\intercal.$$

From Lemma 3, we have

$$[\boldsymbol{R}_{t+1}^B]: \quad \tilde{\boldsymbol{\gamma}}^\intercal = \xi\big[\hat{\boldsymbol{R}}^{A\intercal}\boldsymbol{\Gamma}_{\boldsymbol{\epsilon}}^A - \hat{\boldsymbol{R}}^{B\intercal}\boldsymbol{\Gamma}_{\boldsymbol{\epsilon}}^B - \hat{\boldsymbol{R}}^{N\intercal}\boldsymbol{\Gamma}_{\boldsymbol{\epsilon}}^N\big],$$

and

$$[\boldsymbol{R}_{t+1}^B]: \quad (\tilde{\boldsymbol{\gamma}}^* \otimes \mathbf{1}_{\mathcal{B}})^\intercal = \xi\big[\hat{\boldsymbol{R}}^{A\intercal}\boldsymbol{\Gamma}_{\boldsymbol{\epsilon}}^A - \hat{\boldsymbol{R}}^{B\intercal}\boldsymbol{\Gamma}_{\boldsymbol{\epsilon}}^B - \hat{\boldsymbol{R}}^{N\intercal}\boldsymbol{\Gamma}_{\boldsymbol{\epsilon}}^N\big]\boldsymbol{I}_{\boldsymbol{\alpha}},$$

where $\boldsymbol{I}_{\boldsymbol{\alpha}} \coloneqq \boldsymbol{I} \otimes (\boldsymbol{\alpha}\mathbf{1}_{\mathcal{B}}^\intercal)$.

Using these expressions in the optimality condition for $\boldsymbol{b}^G$, the optimal policy rule is reduced to

$$\hat{\boldsymbol{R}}^{B\intercal}[\boldsymbol{I}_{\mathcal{B}} - \boldsymbol{I}_{\boldsymbol{\alpha}}] = \big[\hat{\boldsymbol{R}}^{A\intercal}\boldsymbol{\Gamma}_{\boldsymbol{\epsilon}}^A - \hat{\boldsymbol{R}}^{B\intercal}\boldsymbol{\Gamma}_{\boldsymbol{\epsilon}}^B - \hat{\boldsymbol{R}}^{N\intercal}\boldsymbol{\Gamma}_{\boldsymbol{\epsilon}}^N\big][\boldsymbol{I}_{\mathcal{B}} - \boldsymbol{I}_{\boldsymbol{\alpha}}].$$

Because $rank(\boldsymbol{\alpha}\mathbf{1}_{\mathcal{B}}^\intercal) = 1$ and has 1 as an eigenvalue (with $\mathbf{1}_{\mathcal{B}}$ as an eigenvector), $\boldsymbol{I}_{\mathcal{B}} - \boldsymbol{I}_{\boldsymbol{\alpha}}$ has rank $(|\mathcal{B}| - 1) \times T$.

C Nested Models of Financial Intermediation

We show how our framework nests some commonly used models of financial frictions by appropriately choosing functions $\boldsymbol{\phi}_t(\{R_{s+1}^A, \boldsymbol{R}_{s+1}^B\}_{s=t}^\infty)$ and $\theta_t(\{R_{s+1}^A, \boldsymbol{R}_{s+1}^B\}_{s=t}^\infty)$.

C.1 Bianchi and Bigio (2022)

There is a continuum of financial intermediaries (banks) with relative risk aversion $\gamma \geq 0$. Financial intermediaries choose portfolio shares of loans $\bar{\theta} \geq 0$, government debt $\bar{\phi}^{\text{gov}} \leq 0$, reserves $\bar{\phi}^{\text{res}} \leq 0$ and deposits $\bar{\phi}^{\text{dep}} \geq 0$.[5] The real gross rates of return are respectively R_{t+1}^A,$R_{\text{gov},t+1}^B$,$R_{\text{res},t+1}^B$, and $R_{\text{dep},t+1}^B$. Intermediaries face idiosyncratic withdrawal shocks $\omega \in [\omega_{\min}, \infty]$, with $\omega_{\min} \geq -1$, $\mathbb{E}[\omega] = 0$, and a cumulative distribution function F with density f. Intermediaries have to raise enough reserves

[5]The quantity of reserves held by an intermediary is thus $-\bar{\phi}^{\text{res}} n \geq 0$. We use the same convention for government debt.

to settle transactions. In case of a negative realization of ω they first sell their holdings of government debt for reserves and if there is still a shortfall they borrow from the discount window at the rate R^W_{t+1} and from the the interbank market. Intermediaries with positive realizations of ω lend in the interbank market. The interbank market rate is determined by the interbank market tightness. Proposition 7 in Bianchi and Bigio (2022) shows the optimal portfolio solves the following problem[6]

$$\max_{\bar\theta,\bar\phi^{\text{gov}},\bar\phi^{\text{res}},\bar\phi^{\text{dep}}} \mathbb{E}_\omega \left[\left(R^A_{t+1}\bar\theta - R^B_{\text{res},t+1}\left(\bar\phi_{\text{gov}} + \bar\phi_{\text{res}}\right) - R^B_{\text{dep},t+1}\bar\phi_{\text{dep}} + \chi_{t+1}(\bar\phi_{\text{gov}} + \bar\phi_{\text{res}}, \bar\phi_{\text{dep}}, \omega)\right)^{1-\gamma}\right]^{\frac{1}{1-\gamma}}$$

s.t. $\bar\theta = 1 + \bar\phi_{\text{gov}} + \bar\phi_{\text{res}} + \bar\phi_{\text{dep}}$, $\bar\phi_{\text{dep}} \leq \kappa$, and

$$\chi_{t+1}(\bar\phi_{\text{gov}} + \bar\phi_{\text{res}}, \bar\phi_{\text{dep}}, \omega) = \begin{cases} \chi^+_{t+1} s, & s > 0 \\ \chi^-_{t+1} s, & s < 0 \end{cases}, \quad s = -\left(\bar\phi_{\text{gov}} + \bar\phi_{\text{res}}\right) + \left(\frac{R^B_{\text{dep},t+1}}{R^B_{\text{res},t+1}}\right)\omega\bar\phi_{\text{dep}}.$$

Let $\bar\phi_{\text{liq}} \coloneqq \bar\phi_{\text{res}} + \bar\phi_{\text{gov}}$. Proposition 8 in Bianchi and Bigio (2022) shows that if $\bar\theta, \bar\phi_{\text{dep}} > 0$ and $\bar\phi_{\text{liq}} < 0$ solve the portfolio problem, we have:

$$R^A_{t+1} - R^B_{\text{res},t+1} = \chi^+_{t+1} + \left(\chi^-_{t+1} - \chi^+_{t+1}\right) \cdot F\left(\omega^*\right) \cdot \frac{\mathbb{E}_\omega\left[\left(R^e_{t+1}\right)^{-\gamma} \mid \omega < \omega^*\right]}{\mathbb{E}_\omega\left[\left(R^e_{t+1}\right)^{-\gamma}\right]},$$

$$R^B_{\text{gov},t+1} - R^B_{\text{res},t+1} = \chi^+_{t+1},$$

$$R^B_{\text{dep},t+1} - R^B_{\text{res},t+1} = R^A_{t+1} - R^B_{\text{res},t+1} + \left(\frac{R^B_{\text{dep},t+1}}{R^B_{\text{res},t+1}}\right)\frac{\text{Cov}_\omega\left[\left(R^e_{t+1}\right)^{-\gamma} \cdot \chi_{t+1}, \omega\right]}{\mathbb{E}_\omega\left[\left(R^e_{t+1}\right)^{-\gamma}\right]} - \mu,$$

where $\mu \geq 0$ is the multiplier on the leverage constraint, and $\mu(\bar\phi_{\text{dep}} - \kappa) = 0$. ω^* is the level of withdrawal shock ω such that $s = 0$:

$$\omega^* = -\frac{-\bar\phi_{\text{liq}}/\bar\phi_{\text{dep}}}{R^B_{\text{dep},t+1}R^B_{\text{res},t+1}}.$$

[6]We focus on a case with no regulatory reserve requirement, i.e. after the realization of withdrawal shocks and trade in the interbank market it is enough if reserve holdings are nonnegative. Our results are not affected by this simplification. Moreover, without a loss of generality we focus on an equilibrium with holdings of government bonds by intermediaries in deficit not exceeding the surplus of reserves. This is the case on which Bianchi and Bigio (2022) focus.

R^e_{t+1} is the gross return on unit of net worth:

$$R^e_{t+1} \coloneqq R^A_{t+1}\bar{\theta} - R^B_{\text{res},t+1}\bar{\phi}_{\text{liq}} - R^B_{\text{dep},t+1}\bar{\phi}_{\text{dep}} + \chi_{t+1}(\bar{\phi}_{\text{liq}}, \bar{\phi}_{\text{dep}}, \omega)$$

Appendix A in Bianchi and Bigio (2022) shows that slopes of the liquidity yield $\chi_{t+1}(\bar{\phi}_{\text{liq}}, \bar{\phi}_{\text{dep}}, \omega)$ are functions of $R^B_{\text{res},t+1}$, interbank market tightness ϑ_t, and discount window rate R^W_{t+1}:

$$\chi^+_{t+1} = \chi^+(R^B_{\text{res},t+1}, \vartheta_t; R^W_{t+1}) \coloneqq (R^W_{t+1} - R^B_{\text{res},t+1}) \left(\frac{\bar{\vartheta}(\vartheta_t)}{\vartheta_t}\right)^{\eta} \left(\frac{\vartheta_t^{\eta}\bar{\vartheta}(\vartheta_t)^{1-\eta} - \vartheta_t}{\bar{\vartheta}(\vartheta_t) - 1}\right),$$

$$\chi^-_{t+1} = \chi^-(R^B_{\text{res},t+1}, \vartheta_t; R^W_{t+1}) \coloneqq (R^W_{t+1} - R^B_{\text{res},t+1}) \left(\frac{\bar{\vartheta}(\vartheta_t)}{\vartheta_t}\right)^{\eta} \left(\frac{\vartheta_t^{\eta}\bar{\vartheta}(\vartheta_t)^{1-\eta} - 1}{\bar{\vartheta}(\vartheta_t) - 1}\right),$$

where

$$\bar{\vartheta}(\vartheta_t) = \begin{cases} 1 + (\vartheta_t - 1)\exp(\lambda), & \vartheta_t \geq 1 \\ \frac{1}{1+(1/\vartheta_t - 1)\exp(\lambda)}, & \vartheta_t < 1. \end{cases}$$

Market tightness satisfies the interbank market clearing condition:

$$\vartheta_t = \vartheta(R^B_{\text{dep},t+1}, R^B_{\text{res},t+1}, \bar{\phi}_{\text{liq}}, \bar{\phi}_{\text{dep}}, \bar{\phi}_{\text{gov}}) \coloneqq -\frac{-F(\omega^*)\bar{\phi}_{\text{liq}} + \int_{\omega_{\min}}^{\omega^*} \omega f(\omega) d\omega \left(\frac{R^B_{\text{dep},t+1}}{R^B_{\text{res},t+1}}\right) \bar{\phi}_{\text{dep}}}{-(1 - F(\omega^*))\bar{\phi}_{\text{liq}} + \int_{\omega^*}^{\infty} \omega f(\omega) d\omega \left(\frac{R^B_{\text{dep},t+1}}{R^B_{\text{res},t+1}}\right) \bar{\phi}_{\text{dep}} + \bar{\phi}_{\text{gov}}},$$

where we already used the fact that in a symmetric equilibrium portfolio choices of all intermediaries are the same.

Define

$$\tilde{\chi}^{+}(R^{B}_{\text{dep},t+1}, R^{B}_{\text{res},t+1}, \bar{\phi}_{\text{liq}}, \bar{\phi}_{\text{dep}}, \bar{\phi}_{\text{gov}}; R^{W}_{t+1}) \coloneqq \chi^{+}(R^{B}_{\text{res},t+1}, \vartheta(R^{B}_{\text{dep},t+1}, R^{B}_{\text{res},t+1}, \bar{\phi}_{\text{liq}}, \bar{\phi}_{\text{dep}}, \bar{\phi}_{\text{gov}}); R^{W}_{t+1})$$
$$\tilde{\chi}^{-}(R^{B}_{\text{dep},t+1}, R^{B}_{\text{res},t+1}, \bar{\phi}_{\text{liq}}, \bar{\phi}_{\text{dep}}, \bar{\phi}_{\text{gov}}; R^{W}_{t+1}) \coloneqq \chi^{-}(R^{B}_{\text{res},t+1}, \vartheta(R^{B}_{\text{dep},t+1}, R^{B}_{\text{res},t+1}, \bar{\phi}_{\text{liq}}, \bar{\phi}_{\text{dep}}, \bar{\phi}_{\text{gov}}); R^{W}_{t+1})$$
$$\tilde{\chi}^{\Delta}(R^{B}_{\text{dep},t+1}, R^{B}_{\text{res},t+1}, \bar{\phi}_{\text{liq}}, \bar{\phi}_{\text{dep}}, \bar{\phi}_{\text{gov}}; R^{W}_{t+1}) \coloneqq \tilde{\chi}^{-}(\cdot) - \tilde{\chi}^{+}(\cdot).$$
$$\tilde{\chi}(R^{B}_{\text{dep},t+1}, R^{B}_{\text{res},t+1}, \bar{\phi}_{\text{liq}}, \bar{\phi}_{\text{dep}}, \bar{\phi}_{\text{gov}}, \omega; R^{W}_{t+1}) = \begin{cases} \tilde{\chi}^{+}(\cdot)s, & s > 0 \\ \tilde{\chi}^{-}(\cdot)s, & s < 0 \end{cases}, \quad s = -\bar{\phi}_{\text{liq}} + \left(\frac{R^{B}_{\text{dep},t+1}}{R^{B}_{\text{res},t+1}}\right)\omega\bar{\phi}_{\text{dep}}.$$
$$\omega^{*}\left(R^{B}_{\text{dep},t+1}, R^{B}_{\text{res},t+1}, \bar{\phi}_{\text{liq}}, \bar{\phi}_{\text{dep}}\right) \coloneqq \frac{\bar{\phi}_{\text{liq}}/\bar{\phi}_{\text{dep}}}{R^{B}_{\text{dep},t+1}R^{B}_{\text{res},t+1}}$$
$$\tilde{R}^{e}\left(R^{A}_{t+1}, R^{B}_{\text{dep},t+1}, R^{B}_{\text{res},t+1}, \bar{\theta}, \bar{\phi}_{\text{liq}}, \bar{\phi}_{\text{dep}}, \bar{\phi}_{\text{gov}}; R^{W}_{t+1}\right) \coloneqq R^{A}_{t+1}\bar{\theta} - R^{B}_{\text{res},t+1}\bar{\phi}_{\text{liq}} - R^{B}_{\text{dep},t+1}\bar{\phi}_{\text{dep}}$$
$$+ \tilde{\chi}^{+}(\cdot)$$

We now use the above formulas in the first order conditions of the intermediary to solve for optimal portfolio shares as functions of returns only.

Binding leverage constraint

If the leverage constraint is binding

$$R^{A}_{t+1} - R^{B}_{\text{res},t+1} = \tilde{\chi}^{+}(\cdot; R^{W}_{t+1}) + \tilde{\chi}^{\Delta}(\cdot; R^{W}_{t+1}) \cdot F\left(\omega^{*}(\cdot)\right) \cdot \frac{\mathbb{E}_{\omega}\left[\left(R^{e}\left(\cdot; R^{W}_{t+1}\right)\right)^{-\gamma} \mid \omega < \omega^{*}(\cdot)\right]}{\mathbb{E}_{\omega}\left[\left(R^{e}\left(\cdot; R^{W}_{t+1}\right)\right)^{-\gamma}\right]},$$

$$R^{B}_{\text{gov},t+1} - R^{B}_{\text{res},t+1} = \tilde{\chi}^{+}(\cdot; R^{W}_{t+1}),$$
$$\bar{\theta} = 1 + \bar{\phi}_{\text{liq}} + \bar{\phi}_{\text{dep}},$$
$$\bar{\phi}_{\text{dep}} = \kappa.$$

Slack leverage constraint

If the leverage constraint is slack

$$R^A_{t+1} - R^B_{\text{res},t+1} = \tilde{\chi}^+(\cdot; R^W_{t+1}) + \tilde{\chi}^\Delta(\cdot; R^W_{t+1}) \cdot F\left(\omega^*\left(\cdot\right)\right) \cdot \frac{\mathbb{E}_\omega\left[\left(R^e\left(\cdot; R^W_{t+1}\right)\right)^{-\gamma} \mid \omega < \omega^*\left(\cdot\right)\right]}{\mathbb{E}_\omega\left[\left(R^e\left(\cdot; R^W_{t+1}\right)\right)^{-\gamma}\right]},$$

$$R^B_{\text{gov},t+1} - R^B_{\text{res},t+1} = \tilde{\chi}^+(\cdot; R^W_{t+1}),$$

$$R^B_{\text{dep},t+1} - R^B_{\text{res},t+1} = R^A_{t+1} - R^B_{\text{res},t+1} + \left(\frac{R^B_{\text{dep},t+1}}{R^B_{\text{res},t+1}}\right) \frac{\text{Cov}_\omega\left[\left(R^e\left(\cdot; R^W_{t+1}\right)\right)^{-\gamma} \cdot \tilde{\chi}\left(\cdot\right), \omega\right]}{\mathbb{E}_\omega\left[\left(R^e\left(\cdot; R^W_{t+1}\right)\right)^{-\gamma}\right]} - \mu,$$

$$\bar{\theta} = 1 + \bar{\phi}_{\text{liq}} + \bar{\phi}_{\text{dep}}.$$

In both cases it is a system of four equations with nine variables: four quantities $\bar{\theta}, \bar{\phi}_{\text{dep}}, \bar{\phi}_{\text{gov}}, \bar{\phi}_{\text{liq}}$ and five returns $R^A_{t+1}, R^B_{\text{dep},t+1}, R^B_{\text{res},t+1}, R^B_{\text{gov},t+1}, R^W_{t+1}$. We use $\bar{\phi}_{\text{res}} = \bar{\phi}_{\text{liq}} - \bar{\phi}_{\text{gov}}$ and solve for $\bar{\theta}, \bar{\phi}_{\text{dep}}, \bar{\phi}_{\text{gov}}, \bar{\phi}_{\text{res}}$ as functions of returns. We denote the solution as

$$\begin{aligned}
\bar{\theta} &= \theta\left(R^A_{t+1}, R^B_{\text{dep},t+1}, R^B_{\text{res},t+1}, R^B_{\text{gov},t+1}; R^W_{t+1}\right), \\
\bar{\phi}_{\text{dep}} &= \phi_{\text{dep}}\left(R^A_{t+1}, R^B_{\text{dep},t+1}, R^B_{\text{res},t+1}, R^B_{\text{gov},t+1}; R^W_{t+1}\right), \\
\bar{\phi}_{\text{res}} &= \phi_{\text{res}}\left(R^A_{t+1}, R^B_{\text{dep},t+1}, R^B_{\text{res},t+1}, R^B_{\text{gov},t+1}; R^W_{t+1}\right), \\
\bar{\phi}_{\text{gov}} &= \phi_{\text{gov}}\left(R^A_{t+1}, R^B_{\text{dep},t+1}, R^B_{\text{res},t+1}, R^B_{\text{gov},t+1}; R^W_{t+1}\right).
\end{aligned}$$

We have

$$\begin{aligned}
\boldsymbol{R}^A_{t+1} &= \begin{bmatrix} R^A_{t+1} \end{bmatrix} \\
\boldsymbol{R}^B_{t+1} &= \begin{bmatrix} R^B_{\text{dep},t+1} \\ R^B_{\text{res},t+1} \\ R^B_{\text{gov},t+1} \end{bmatrix}
\end{aligned}$$

$$\begin{aligned}
\theta_t(\{R^A_{s+1}, \boldsymbol{R}^B_{s+1}\}_{s=t}^\infty) &= \left[\theta\left(R^A_{t+1}, R^B_{\text{dep},t+1}, R^B_{\text{res},t+1}, R^B_{\text{gov},t+1}; R^W_{t+1}\right)\right] \\
\boldsymbol{\phi}_t(\{R^A_{s+1}, \boldsymbol{R}^B_{s+1}\}_{s=t}^\infty) &= \begin{bmatrix} \phi_{\text{dep}}\left(R^A_{t+1}, R^B_{\text{dep},t+1}, R^B_{\text{res},t+1}, R^B_{\text{gov},t+1}; R^W_{t+1}\right) \\ \phi_{\text{res}}\left(R^A_{t+1}, R^B_{\text{dep},t+1}, R^B_{\text{res},t+1}, R^B_{\text{gov},t+1}; R^W_{t+1}\right) \\ \phi_{\text{gov}}\left(R^A_{t+1}, R^B_{\text{dep},t+1}, R^B_{\text{res},t+1}, R^B_{\text{gov},t+1}; R^W_{t+1}\right) \end{bmatrix}
\end{aligned}$$

Note that we treat R^W_{t+1} as exogenous as in Bianchi and Bigio (2022): the government supplies discount window loans perfectly elastically at R^W_{t+1}. The quantity of discount window loans does not appear in the balance sheet of intermediaries at the stage when they decide on their portfolio allocation. It only enters the government budget constraint: we can either treat it as a part of lump-sum tax T_t or model it explicitly by adding an extra term that depends on other returns on assets (because demand for discount window loans is a function of these returns).

C.2 Gertler, Kiyotaki, Queralto (2012)

Gertler et al. (2012) extend the Gertler-Karadi-Kiyotaki (GKK) framework by allowing intermediaries to issue outside equity. Outside equity affects the ability of intermediaries to divert assets: it is more difficult to divert assets funded by deposits than outside equity. Let $\bar{\theta}$ be the portfolio share of loans, $\bar{\phi}_{\text{dep}}$ the porfiolio share of deposits, and $\bar{\phi}_{\text{oeq}}$ the share of outside equity. Intermediary can divert fraction χ of assets and run away, where

$$\chi = \chi_0 \left(1 + \chi_1 \frac{\bar{\phi}_{\text{oeq}}}{\bar{\theta}} + \frac{\chi_2}{2} \left(\frac{\bar{\phi}_{\text{oeq}}}{\bar{\theta}} \right)^2 \right),$$

The rates of return are $R^A_{t+1}, R^B_{\text{dep},t+1}$, and $R^B_{\text{oeq},t+1}$.

Bellman equation of an intermediary is[7]

$$\begin{aligned}\eta_t = &\max_{\bar{\phi}_{\text{oeq}},\bar{\theta}} \Lambda_{t,t+1} \left(f + (1-f)\, \eta_{t+1} \right) \left[\bar{\theta} \left(R^A_{t+1} - R^B_{\text{dep},t+1} \right) - \bar{\phi}_{\text{oeq}} \left(R^B_{\text{oeq},t+1} - R^B_{\text{dep},t+1} \right) + R^B_{\text{dep},t+1} \right] \\ &+ \lambda_t \left[\eta_t - \chi_0 \left(1 + \chi_1 \frac{\bar{\phi}_{\text{oeq}}}{\bar{\theta}} + \frac{\chi_2}{2} \left(\frac{\bar{\phi}_{\text{oeq}}}{\bar{\theta}} \right)^2 \right) \bar{\theta} \right],\end{aligned}$$

where $\Lambda_{t,t+1}$ is the discount factor, η_t is the marginal value of net worth, and λ_t is the Lagrange multiplier on the incentive compatibility constraint. We assume the discount factor is a function of returns only, $\Lambda_{t,t+1} = \Lambda(R^A_{t+1}, R^B_{\text{oeq},t+1}, R^B_{\text{dep},t+1}$.

If the incentive compatibility constraint is binding, first order conditions with respect

[7] See Chiang and Zoch (2023) for a detailed derivation of a similar result in a basic GKK framework.

to portfolio shares result in

$$\frac{R^A_{t+1} - R^B_{\text{dep},t+1}}{R^B_{\text{oeq},t+1} - R^B_{\text{dep},t+1}} = -\frac{1 - \frac{\chi_2}{2}\left(\frac{\bar{\phi}_{\text{oeq}}}{\bar{\theta}}\right)^2}{\chi_1\left(\frac{\bar{\phi}_{\text{oeq}}}{\bar{\theta}}\right) + \chi_2\left(\frac{\bar{\phi}_{\text{oeq}}}{\bar{\theta}}\right)^2}$$

which determines the ratio $\bar{\phi}_{\text{oeq}}/\bar{\theta}$ as a function of returns, $x(R^A_{t+1}, R^B_{\text{oeq},t+1}, R^B_{\text{dep},t+1})$. This allows us to write

$$\bar{\phi}_{\text{oeq}} = x(R^A_{t+1}, R^B_{\text{oeq},t+1}, R^B_{\text{dep},t+1}) \cdot \bar{\theta}.$$

The above result together with $\eta_t = \chi_0\left(1 + \chi_1\frac{\bar{\phi}_{\text{oeq}}}{\bar{\theta}} + \frac{\chi_2}{2}\left(\frac{\bar{\phi}_{\text{oeq}}}{\bar{\theta}}\right)^2\right)\bar{\theta}$ gives us

$$\eta_t = \Lambda_{t,t+1}\left(f + (1-f)\,\eta_{t+1}\right)\left[\frac{\left(R^A_{t+1} - R^B_{\text{dep},t+1} - x\left(\cdot\right)\left(R^B_{\text{oeq},t+1} - R^B_{\text{dep},t+1}\right)\right)}{\chi_0\left(1 + \chi_1 x\left(\cdot\right) + \frac{\chi_2}{2}x\left(\cdot\right)^2\right)}\eta_t + R^B_{\text{dep},t+1}\right].$$

This links η_t to η_{t+1}, $\Lambda_{t,t+1}$ and returns. Given our assumption that $\Lambda_{t,t+1}$ is a function of returns only, we have

$$\eta_t = \eta\left(\left\{R^A_{s+1}, R^B_{\text{oeq},s+1}R^B_{\text{dep},s+1}\right\}_{s\geq t}\right).$$

By using the incentive compatibility constraint, the optimal relationship between $\bar{\theta}$ and $\bar{\phi}_{\text{oeq}}$, and the balance sheet, we have

$$\begin{aligned}
\bar{\theta} &= \frac{1}{\chi_0\left(1 + \chi_1 x\left(\cdot\right) + \frac{\chi_2}{2}x\left(\cdot\right)^2\right)}\eta\left(\left\{R^A_{s+1}, R^B_{\text{oeq},s+1}R^B_{\text{dep},s+1}\right\}_{s\geq t}\right), \\
\bar{\phi}_{\text{oeq}} &= \frac{x\left(\cdot\right)}{\chi_0\left(1 + \chi_1 x\left(\cdot\right) + \frac{\chi_2}{2}x\left(\cdot\right)^2\right)}\eta\left(\left\{R^A_{s+1}, R^B_{\text{oeq},s+1}R^B_{\text{dep},s+1}\right\}_{s\geq t}\right), \\
\bar{\phi}_{\text{dep}} &= \frac{1 - x\left(\cdot\right)}{\chi_0\left(1 + \chi_1 x\left(\cdot\right) + \frac{\chi_2}{2}x\left(\cdot\right)^2\right)}\eta\left(\left\{R^A_{s+1}, R^B_{\text{oeq},s+1}R^B_{\text{dep},s+1}\right\}_{s\geq t}\right) - 1,
\end{aligned}$$

which shows that $\bar{\theta}, \bar{\phi}_{\text{oeq}}, \bar{\phi}_{\text{dep}}$ can be expressed as functions of returns only:

$$\begin{aligned}
\bar{\theta} &= \theta\left(\left\{R^A_{s+1}, R^B_{\text{oeq},s+1}, R^B_{\text{dep},s+1}\right\}_{s\geq t}\right), \\
\bar{\phi}_{\text{oeq}} &= \phi_{\text{oeq}}\left(\left\{R^A_{s+1}, R^B_{\text{oeq},s+1}, R^B_{\text{dep},s+1}\right\}_{s\geq t}\right), \\
\bar{\phi}_{\text{dep}} &= \phi_{\text{dep}}\left(\left\{R^A_{s+1}, R^B_{\text{oeq},s+1}, R^B_{\text{dep},s+1}\right\}_{s\geq t}\right).
\end{aligned}$$

C.3 Sims and Wu (2021)

Sims and Wu (2021) extend the GKK framework by introducing interest-bearing reserves and allowing for a difference between diversion rates of private loans and government bonds. Let $\bar{\theta} \geq 0$ be the portfolio share of private loans and $\bar{\phi}_{\text{gov}}, \bar{\phi}_{\text{res}} \leq 0, \bar{\phi}_{\text{dep}} \geq 0$ the portfolio shares of government bonds, reserves, and deposits. The balance sheet equation is

$$\bar{\theta} = \bar{\phi}_{\text{gov}} + \bar{\phi}_{\text{res}} + \bar{\phi}_{\text{dep}} + 1.$$

The real gross rates of return are respectively R^A_{t+1},$R^B_{\text{gov},t+1}$,$R^B_{\text{res},t+1}$, and $R^B_{\text{dep},t+1}$.

Intermediary can divert a fraction $\chi < 1$ of private loans and $\chi \cdot \Delta$ of government bonds, where $0 \leq \Delta \leq 1$. The reserve requirement is

$$\bar{\phi}_{\text{res}} \leq -\varsigma \bar{\phi}_{\text{dep}}.$$

Bellman equation is

$$\begin{aligned}\eta_t = \max_{\bar{\theta}\geq 0, \bar{\phi}_{\text{gov}} \leq 0, \bar{\phi}_{\text{res}} \leq 0} & \Lambda_{t,t+1} \left(f + (1-f)\,\eta_{t+1}\right) \left[\bar{\theta}\left(R^A_{t+1} - R^B_{\text{dep},t+1}\right)\right. \\ & - \bar{\phi}_{\text{gov}}\left(R^B_{\text{gov},t+1} - R^B_{\text{dep},t+1}\right) - \bar{\phi}_{\text{res}}\left(R^B_{\text{res},t+1} - R^B_{\text{dep},t+1}\right) + R^B_{\text{dep},t+1}\Big] \\ & + \lambda_t \left[\eta_t - \chi\bar{\theta} + \chi\Delta\bar{\phi}_{\text{gov}}\right] - \mu_t \left[\bar{\phi}_{\text{res}} + \varsigma(\bar{\theta} - \bar{\phi}_{\text{gov}} - \bar{\phi}_{\text{res}} - 1)\right]\end{aligned}$$

where $\Lambda_{t,t+1}$ is the discount factor, η_t is the marginal value of net worth, λ_t is the Lagrange multiplier on the incentive compatibility (IC) constraint, and μ_t is the multiplier on the reserve requirement (RR) constraint. We assume the discount factor is a function of returns only, $\Lambda_{t,t+1} = \Lambda(R^A_{t+1}, R^B_{\text{gov},t+1}, R^B_{\text{res},t+1}, R^B_{\text{dep},t+1})$.

The program is linear in portfolio shares. Therefore, unless returns satisfy certain relations, the solution will involve corner solutions.

The reserve requirement constraint is binding whenever $R^B_{\text{dep},t+1} > R^B_{\text{res},t+1}$. The incentive compatibility constraint is binding whenever

$$\max\left\{R^A_{t+1}, R^B_{\text{gov},t+1}\right\} \geq R_{\text{fund},t+1}, \quad R_{\text{fund},t+1} \coloneqq R^B_{\text{dep},t+1} + \frac{\varsigma}{1-\varsigma}\left(R^B_{\text{dep},t+1} - R^B_{\text{res},t+1}\right).$$

We will assume this is the case and also that $R^B_{\text{dep},t+1} \geq R^B_{\text{res},t+1}$. If

$$\frac{R^B_{\text{gov},t+1} - \Delta R^A_{t+1}}{1-\Delta} = R_{\text{fund},t+1}$$

the intermediary is indifferent between holding government debt and loans. Let

$$R^*\left(R^A_{t+1}, R^B_{\text{gov},t+1}, R^B_{\text{dep},t+1}, R^B_{\text{res},t+1}\right) \coloneqq \begin{cases} R^A_{t+1}, & R^B_{\text{gov},t+1} \leq (1-\Delta) R_{\text{fund},t+1} + \Delta R^A_{t+1} \\ R^B_{\text{gov},t+1,} & R^B_{\text{gov},t+1} > (1-\Delta) R_{\text{fund},t+1} + \Delta R^A_{t+1} \end{cases}$$

We can rewrite the Bellman equation as

$$\eta_t = \Lambda_{t,t+1}\left(f + (1-f)\eta_{t+1}\right)\left[\left(R^*\left(R^A_{t+1}, R^B_{\text{gov},t+1}, R^B_{\text{dep},t+1}, R^B_{\text{res},t+1}\right) - R_{\text{fund},t+1}\right)\frac{\eta_t}{\chi} + R_{\text{fund},t+1}\right].$$

Because we assumed $\Lambda_{t,t+1}$ is a function of returns only, we have

$$\eta_t = \eta\left(R^A_{t+1}, R^B_{\text{gov},t+1}, R^B_{\text{res},t+1}, \eta_{t+1}\right).$$

We can therefore use the binding incentive compatibility constraint to get

$$\bar{\theta} - \Delta\bar{\phi}_{\text{gov}} = \frac{1}{\chi}\eta\left(R^A_{t+1}, R^B_{\text{gov},t+1}, R^B_{\text{res},t+1}, \eta_{t+1}\right)$$

The exact portfolio composition depends on returns. For example, if the incentive compatibility constraint is binding and $R_{\text{dep},t+1} \geq R_{\text{res},t+1}$, we can express demand for loans as

$$\theta\left(R^A_{t+1}, R^B_{\text{gov},t+1}, R^B_{\text{dep},t+1}, R^B_{\text{res},t+1}\right) \coloneqq \begin{cases} \frac{1}{\chi}\eta\left(\cdot\right), & (1-\Delta) R_{\text{fund},t+1} + \Delta R^A_{t+1} > R^B_{\text{gov},t+1}, \\ \left[0, \frac{1}{\chi}\eta\left(\cdot\right)\right], & (1-\Delta) R_{\text{fund},t+1} + \Delta R^A_{t+1} = R^B_{\text{gov},t+1}, \\ 0, & (1-\Delta) R_{\text{fund},t+1} + \Delta R^A_{t+1} < R^B_{\text{gov},t+1}. \end{cases}$$

The portfolio share of government debt is

$$\phi_{\text{gov}}\left(R^A_{t+1}, R^B_{\text{gov},t+1}, R^B_{\text{dep},t+1}, R^B_{\text{res},t+1}\right) = \frac{1}{\Delta}\left(\theta\left(\cdot\right) - \frac{1}{\chi}\eta\left(\cdot\right)\right).$$

If $R_{\text{dep},t+1} > R_{\text{res},t+1}$ we can use the reserve requirement together with the balance

sheet to get

$$\phi_{\text{res}}\left(R^A_{t+1}, R^B_{\text{gov},t+1}, R^B_{\text{dep},t+1}, R^B_{\text{res},t+1}\right) = -\frac{\varsigma}{1-\zeta}\left(\theta\left(\cdot\right) - \phi_{\text{gov}}\left(\cdot\right) - 1\right)$$
$$\phi_{\text{dep}}\left(R^A_{t+1}, R^B_{\text{gov},t+1}, R^B_{\text{dep},t+1}, R^B_{\text{res},t+1}\right) = \frac{1}{1-\zeta}\left(\theta\left(\cdot\right) - \phi_{\text{gov}}\left(\cdot\right) - 1\right).$$

If $R_{\text{dep},t+1} = R_{\text{res},t+1}$, then only the sum $\bar{\phi}_{\text{res}} + \bar{\phi}_{\text{dep}}$ is determined and equals

$$\theta\left(\cdot\right) - \phi_{\text{gov}}\left(\cdot\right) - 1.$$

In this model, if rates of return are such that none of the portfolio shares is zero, demand for assets will be perfectly elastic. If rates of return are such that some asset has a portfolio share equal to zero demand for this asset will be inelastic. For example, if the return on loans sufficiently dominates the return on government debt, demand for debt will be inelastic. In this case, demand for loans will have an intermediate elasticity.

D Aggregation

Assume there are J intermediaries in the financial sector, who hold productive assets and net worth of other intermediaries, and issue financial assets. The net worth of an intermediary j is $\mathring{n}_{j,t}$. We denote the intermediary's j holdings of net worth of intermediary j' in period t as $n_{jj',t}$. We normalize $n_{jj,t}$=0. Let

$$n_{j,t} = \mathring{n}_{j,t} - \sum_{j'=1}^{J} n_{jj',t}.$$

We assume that holdings of productive assets and issuance of financial assets are proportional to $n_{j,t}$:

$$a^F_{j,t} = \theta_{j,t}\left(\{\boldsymbol{R}^A_s, \boldsymbol{R}^B_s\}\right) n_{j,t}, \quad \boldsymbol{b}_{j,t} = \boldsymbol{\phi}_{j,t}\left(\{\boldsymbol{R}^A_s, \boldsymbol{R}^B_s\}\right) n_{j,t}$$

The balance sheet of intermediary j is:

$$\mathbf{1}^\intercal \theta_{j,t}\left(\{\boldsymbol{R}^A_s, \boldsymbol{R}^B_s\}\right) n_{j,t} + \sum_{j\prime=1}^{J} n_{jj',t} = \mathbf{1}^\intercal \boldsymbol{\phi}_{j,t}\left(\{\boldsymbol{R}^A_s, \boldsymbol{R}^B_s\}\right) n_{j,t} + \mathring{n}_{j,t}$$

Return on the net worth of intermediary j satisfies

$$R_{j,t}^N = \frac{1}{\mathring{n}_{j,t-1}} \left[\left(\boldsymbol{R}_t^{A\intercal} \boldsymbol{\phi}_{j,t-1} - \boldsymbol{R}_t^{B\intercal} \theta_{j,t-1} \right) n_{j,t-1} + \sum_{j'=1}^{J} R_{j',t}^N n_{jj',t-1} \right]$$

Define:

$$\begin{aligned}
\theta_t \left(\{ \boldsymbol{R}_s^A, \boldsymbol{R}_s^B \} \right) n_t &\coloneqq \sum_{j=1}^{J} \theta_{j,t} \left(\{ \boldsymbol{R}_s^A, \boldsymbol{R}_s^B \} \right) n_{j,t} \\
\boldsymbol{\phi}_t \left(\{ \boldsymbol{R}_s^A, \boldsymbol{R}_s^B \} \right) n_t &\coloneqq \sum_{j=1}^{J} \boldsymbol{\phi}_{j,t} \left(\{ \boldsymbol{R}_s^A, \boldsymbol{R}_s^B \} \right) n_{j,t} \\
n_t &\coloneqq \sum_{j=1}^{J} n_{j,t}
\end{aligned}$$

It implies

$$\begin{aligned}
\theta_t \left(\{ \boldsymbol{R}_s^A, \boldsymbol{R}_s^B \} \right) &= \sum_{j=1}^{J} \theta_{j,t} \left(\{ \boldsymbol{R}_s^A, \boldsymbol{R}_s^B \} \right) \frac{n_{j,t}}{n_t} \\
\boldsymbol{\phi}_t \left(\{ \boldsymbol{R}_s^A, \boldsymbol{R}_s^B \} \right) &= \sum_{j=1}^{J} \boldsymbol{\phi}_{j,t} \left(\{ \boldsymbol{R}_s^A, \boldsymbol{R}_s^B \} \right) \frac{n_{j,t}}{n_t}
\end{aligned}$$

The rate of return on the aggregate financial sector's net worth held by households, R_t^N, is:

$$\begin{aligned}
R_t^N &= \sum_{j=1}^{J} R_{j.t}^N \frac{1}{\sum_{j=1}^{J} n_{j,t-1}} \left(\mathring{n}_{j,t-1} - \sum_{j'}^{J} n_{j'j,t-1} \right) \\
&= \sum_{j=1}^{J} \frac{1}{\sum_{j=1}^{J} n_{j,t}} \left[\left(\boldsymbol{R}_t^{A\intercal} \theta_{j,t-1} - \boldsymbol{R}_t^{B\intercal} \boldsymbol{\phi}_{j,t-1} \right) n_{j,t-1} + \sum_{j'=1}^{J} R_{j',t}^N n_{jj',t-1} \right] \\
&- \sum_{j=1}^{J} R_{j,t}^N \frac{1}{\sum_{j=1}^{J} n_{j,t-1}} \sum_{j'=1}^{J} n_{j'j,t-1} \\
&= \sum_{j=1}^{J} \frac{1}{\sum_{j=1}^{J} n_{j,t}} \left[\left(\boldsymbol{R}_t^{A\intercal} \theta_{j,t-1} - \boldsymbol{R}_t^{B\intercal} \boldsymbol{\phi}_{j,t-1} \right) n_{j,t-1} \right] \\
&= \boldsymbol{R}_t^{A\intercal} \theta_{t-1} - \boldsymbol{R}_t^{B\intercal} \boldsymbol{\phi}_{t-1}.
\end{aligned}$$

Assume the for all j the ratio $n_{j,t}/n_t$ is constant for all j, t, n_j/n. We have

$$\theta_{s,\boldsymbol{R}^B_{t+1}}\left(\{\boldsymbol{R}^A_{k+1},\boldsymbol{R}^B_{k+1}\}_{k\geq s}\right)=\sum_{j=1}^{J}\theta_{j,s,\boldsymbol{R}^B_{t+1}}\left(\{\boldsymbol{R}^A_{k+1},\boldsymbol{R}^B_{k+1}\}_{k\geq s}\right)\cdot\frac{\mathring{n}_j}{\mathring{n}}$$

$$\boldsymbol{\phi}_{s,\boldsymbol{R}^B_{t+1}}\left(\{\boldsymbol{R}^A_{k+1},\boldsymbol{R}^B_{k+1}\}_{k\geq s}\right)=\sum_{j=1}^{J}\boldsymbol{\phi}_{j,s,\boldsymbol{R}^B_{t+1}}\left(\{\boldsymbol{R}^A_{k+1},\boldsymbol{R}^B_{k+1}\}_{k\geq s}\right)\cdot\frac{\mathring{n}_j}{\mathring{n}}$$

The above derivations show that we can aggregate intermediaries using $\mathring{n}_j/\mathring{n}$ as weights. We obtain aggregate θ_t and $\boldsymbol{\phi}_t$. The rate of return R^N_t depends on these aggregate functions in the same way as in the baseline model. The assumption that allocation of net worth between J is constant allows us to obtain sensitivities of the aggregate functions which are weighted sums of sensitivities of these functions at the intermediary level.

E Empirical Appendix

E.1 Data Construction

Our primary data source is the Flow of Funds (FoF) data from the Federal Reserve Board. The FoF includes balance sheets for various entities including households, the federal government, nonprofits, and businesses. This data gives us the supply of various financial instruments in the economy, as well as who owns them. Unfortunately, there are various inconsistencies between the PDF tables available online for viewing and the actual data files available for download. Thus, we adopt the following procedure to correct these discrepancies and consolidate the separate tables into a single dataset of owner holdings over time.

FoF tables containing levels follow two broad structures: tables L100 through L130 correspond to owners, and each table shows the holdings of that owner. Tables L200 through L235 correspond to instruments, and each table shows the owners of that instrument. We loop through the instrument tables (L200-L235) to obtain holdings.

For each instrument, we load in the data table, where each row corresponds to a date, and each column corresponds to each line from PDF tables. Generally, the

first column corresponds to the date, followed by columns for each owner who holds this instrument as an asset or liability on their balance sheet. Each column name has a numeric code corresponding to a total. We use code books provided by the SCF to replace these numeric codes with their names. For example, in table L200, "LM893111005.Q" corresponds to "All sectors; U.S. official reserve assets; liability." The first part before the semicolon denotes the owner (in this case, "All sectors" denotes a total), the second part is the instrument ("U.S. official reserve assets"), and the third part is whether it is an asset or liability ("liability"). Some tables contain a "memo" column. All columns to the right of the memo column contain alternative groupings of the same owners. For example, table L203, "Net Interbank Transactions," is organized by assets and then liabilities. However, after the memo column, the groups are instead "Vault cash", "Depository institution reserves", and "Other transactions with the monetary authority", etc. with assets and liabilities no longer clearly delineated. Including these columns would result in double counting. Consequently, we drop all columns to the right of the memo column. Unfortunately, there is no simple way to identify these, so we must manually code the location of each memo column (if applicable) for each table.

We then proceed to standardize assets/liabilities and names. Generally, the tables have assets followed by liabilities. For these, we simply multiply all the liability columns by -1. Consequently, the totals for most instrument tables should be zero, since for every owner holding an instrument as an asset there is another owner in the economy holding the instrument as a liability. However, some tables contain alternative orderings, for example, liabilities followed by assets, or even assets and liabilities interspersed together. Additionally, some of the column names are incorrect, for example, specifying "asset" when they should read "liability" or even having a completely incorrect name (this can be verified by referencing the FoF z1 PDFs). Thus, we proceed to multiplying all columns denoting liabilities by -1, and then correcting names so that all of those columns correctly specify "liability", while other columns specify "asset". Since most tables are organized to have asset columns followed by liability columns or vice versa, we do this by identifying the column where this switch takes place and then denoting everything to the left or right as assets and liabilities. For the tables where assets and liabilities are interspersed, we manually identify each column as an asset or liability. Finally, some tables do not contain liabilities at all, for example table L207, "Money Market Fund Shares". This would cause our asset

and liability totals to not match, so we create a liability owned fully by the money market fund itself so that the asset and liability totals match.

Next, we drop columns with subtotals that do not add up to another total. For example, table L207 contains a column named "Government-sponsored enterprises", immediately followed by a column called "Of which: FHLB federal funds (3)". This second column is highlighting a portion of the first column, and including it would double count the part of GSEs that are FHLB federal funds. Thus, we manually specify these columns and drop them. Similarly, some columns denote discrepancies between the asset and liability total columns. We drop these as well, since they do not belong to any owner.

Many tables contain aggregate total columns. For example, a total asset column equals the sum of all asset subtotals. Including these would double count the total of assets. Instead, we only want the most disaggregated column totals. In order to drop columns that contain sums of other columns, we employ a simple algorithm: we iterate backward through each column, taking cumulative sums of all previous columns until we encounter a sum that matches the current column (given a small tolerance for rounding errors). This column is thus a total of previous columns and thus is dropped. Since we are iterating backwards through the data table and dynamically dropping certain columns, this approach will identify totals "layer-by-layer", starting with the most granular totals and progressing up to drop totals of previous subtotals.

Next, we reshape the data so that date and owner are indexes, and each column corresponds to a holding. We use the fact that each column name follows the structure "owner; holding; type," for example: "Federal government; other reserves; asset" to pivot the data.

We repeat this entire process for each instrument table, merging them all together on the date variable, as well as any shared columns. This is because multiple tables may contain the same holding, for example "other reserves". We then drop duplicate columns, which leaves us with a dataset that has consolidated owners and instruments from all FoF tables so that each row is a unique date and owner pair, while each column is the holding of an instrument. Finally, we recode owner names for consistency (combining "Banks in U.S.-affiliated areas" with "U.S.-chartered depository institutions" into "Private Depository Institutions").

This now allows us to sum values together for all dates, owners, and tables (where each table corresponds to an instrument). The final product is a dataframe where for each unique combination of date, owner and table, we have a value (representing the sum for this unique combination across all our data), gross holdings (which denotes the positive or asset portion of value), and gross issuance (which denotes the negative, or liability portion of value).

We normalize all quantities by dividing them by the quadratic trend of real GDP. We detrend all series used in the estimation by projecting them on quartic polynomials. We use the average trend level of variables as their steady-state value.

Table 3: steady state values

Variable	R^B_{liquid}	R^B_{illiquid}	R^A	b^H_{liquid}	b^H_{illiquid}	$\bar{n}$	$\bar{\phi}_{\text{liquid}}$	$\bar{\phi}_{\text{illiquid}}$	$\bar{\theta}$
Value	1.0029	1.0097	1.0143	2.728	-1.953	8.364	0.177	-0.218	0.954

E.2 Construction of GIV

We define $\mathbf{f} = [\boldsymbol{f}_1 \cdots \boldsymbol{f}_J]^\intercal$ to be a $J \times F$ matrix of characteristics and

$$\mathbf{P} \coloneqq (\mathbf{f}^\intercal \mathbf{f})^{-1} \mathbf{f}^\intercal, \quad \mathbf{Q} \coloneqq \mathbf{I} - \mathbf{f}\mathbf{P}^\intercal.$$

We use $\boldsymbol{s}_t$ to denote a vector of relative weights that sums to one. In our estimation we use weights $s_{j,t}$ given by the relative net worth in period $t-2$:

$$s_{j,t} \coloneqq \frac{n_{j,t-2}}{\sum_{j=1}^{J} n_{j,t-2}}.$$

We assume these weights are independent of idiosyncratic shocks in period t. Similarly, $\hat{\boldsymbol{\phi}}_{b,t} = \{\hat{\phi}_{j,b,t}\}_{j=1}^{J}$ is a vector containing deviations for all entities. We construct instrumental variables $Z_{b,t}$ as

$$Z_{b,t} \coloneqq \boldsymbol{s}_t^\intercal \mathbf{Q} \hat{\boldsymbol{\phi}}_{b,t}.$$

for each $b \in |\mathcal{B}|$. Because $\sum_{j=1}^{J} s_{j,t} = 1$ we have

$$Z_{b,t} = \boldsymbol{s}_t^\intercal \mathbf{Q} \boldsymbol{u}_{b,t}.$$

where $\boldsymbol{u}_{b,t} = \{u_{j,b,t}\}_{j=1}^{J}$.

Intuitively, in the absence of the factor structure (with $F = 1, \boldsymbol{f}_j = 1$), $Z_{b,t}$ corresponds to differences between two cross-sectional averages of $\hat{\phi}_{j,b,t}$: one weighted with $s_{j,t}$, another equally weighted. As a result, we have

$$\begin{aligned} Z_{b,t} &= \sum_{j=1}^{J} (s_{j,t} - \frac{1}{J}) \hat{\phi}_{j,b,t} \\ &= \sum_{j=1}^{J} (s_{j,t} - \frac{1}{J}) \sum_{m=1}^{|\mathcal{B}|} \left[\breve{\phi}_{b,R_m^B} (R_{m,t+1}^B - R_{t+1}^A) + \eta_{b,t} + u_{j,b,t} \right] \\ &= \sum_{j=1}^{J} (s_{j,t} - \frac{1}{J}) u_{j,b,t} \end{aligned}$$

Since $Z_{b,t}$ is a linear combination of idiosyncratic shocks, it satisfies $\mathbb{E}\left[Z_{b,t}\eta_{m,t}\right] = 0$ for all b, m.