Firm Entry and Market Interdependence: Evidence from anti-dumping tariffs on Chinese exports*

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Abstract

Using Chinese Customs data, we study how unilateral anti-dumping (AD) tariffs on Chinese products affect their exports to different destination countries. We first document that investigated products experience a decline in their total sales in both the countries that impose tariffs (focal countries) and those that do not (third countries). A decomposition exercise further suggests that the simultaneous declines in exports across countries are primarily driven by a reduction in the number of entrants. In contrast, individual exporters' sales decrease in the focal countries but increase in the third countries. To rationalize these facts, We develop an industry equilibrium framework based on firms' interdependent entry decisions to multiple markets. We show that the entry margin alone can generate cross-market policy spillovers and heterogeneous tariff responses consistent with the data. Finally, we calibrate our model and quantify the global impact of a unilateral tariff on trade and welfare.

Keywords. Market interdependence, Firm entry, Anti-dumping tariffs, HDIA and HSA Preferences

JEL Classification. D12; F12; F13; F14

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1 Introduction

The use of country-specific tariffs such as anti-dumping/countervailing duties has been soaring in recent years. Among these, the U.S.-China trade war has garnered significant attention in both policy circles and academic discussions over the past decade. Although these tariffs and trade conflicts are typically country-specific or country-pair-specific, their impacts can extend to "bystanders," or third countries—nations that neither impose nor directly face the tariffs. This occurs because export markets are intrinsically interconnected. However, existing literature tends to examine third-market effects either at the aggregate level *or* at the firm level. This segmented approach frequently leads to conflicting empirical findings and divergent policy recommendations, underscoring the need for a more holistic perspective.[1]

In this paper, we provide a unified analysis for the impact of anti-dumping (AD) tariffs, combining both firm-level and aggregate-level evidence. Our study makes both empirical and theoretical contributions to this issue and offers policy recommendations based on a rich quantitative model. Empirically, we investigate how AD tariffs impact Chinese exports in both imposing (focal) and third countries, analyzing these effects at both the aggregate and firm levels. A key finding is that responses to AD tariffs in the third markets are opposite at the aggregate and the firm levels. Theoretically, we rationalize these facts using an industry equilibrium model that features firms' interdependent entry decisions across multiple markets and variable markups. The model allows us to derive closed-form formulas for tariff responses at both the aggregate and the firm levels in all markets. Quantitatively, we calibrate the model with a general demand function and entry technology, matching the observed responses at both levels. Finally, we apply the model to evaluate various trade policies.

Our study focuses on the case of AD tariffs imposed on China, as AD tariffs are the most prevalent trade barrier, and China is the most frequently targeted country. Between 2000 and 2011, 11% of Chinese exports to the United States were subject to AD investigations, with tariffs ultimately applied in the majority of these cases. Notably, these AD tariffs are typically both prohibitive—most exceed 50%—and long-lasting, often remaining in effect

[1] On anti-dumping tariffs/countervailing duties, Bown and Crowley (2007) find that tariffs imposed by the U.S. on Japanese exports deflected Japanese exports to third countries (i.e., trade deflection). In contrast, both Bown and Crowley (2010) and Crowley et al. (2018) document reduced Chinese exports to third markets (i.e., a chilling effect) after temporary tariffs are imposed on Chinese goods. Albornoz-Crespo et al. (2021) find a similar chilling effect of Argentinian exports into third markets after U.S. removed its preferential tariffs applied on Argentine imports. In the context of the U.S.-China trade war, Fajgelbaum et al. (2024) find that (positive) co-movements in aggregate exports from "bystanders" to both the U.S. and the rest of the world (instead of trade diversion).

for more than eight years. Furthermore, the availability of extensive and detailed export transaction data allow us to analyze tariff responses at the firm-product-country level. In summary, China is an ideal context for studying the multifaceted impacts of AD tariffs.

We begin our analysis by identifying four stylized facts about the impact of AD tariffs on Chinese exports. First, the initiation of AD investigations leads to a decline in the total export value and quantity of the investigated products in both the focal and third markets. Second, these reductions in total exports are primarily driven by changes in the number of exporters, especially for the third markets. Third, the decline in the number of exporting firms is largely explained by reduced firm entries rather than firm exits. Fourth, at the firm level, individual firms experience a decrease in export value to focal markets but an *increase* in export value to third markets, accompanied by price increases of the investigated product in both types of markets.

The divergence between firm-level and aggregate-level tariff responses in third markets highlights important directions for modeling the effects of tariffs. On the one hand, aggregation plays a critical role: models that focus solely on individual firm decision-making struggle to capture the aggregate implications of tariffs. On the other hand, firm-level responses and the decomposition results concerning firm entries indicate that firms' entry decisions provide a micro-foundation for the observed co-movement in total exports across markets.

We then propose an industry equilibrium model based on firms' simultaneous entry decisions into multiple markets. Our model extends the framework of Melitz (2003) along two important dimensions. First, we introduce a two-layer nested demand structure for exported goods. Foreign consumers exhibit demand for country-specific imported composites (the outer nest), which aggregate all varieties sourced from the same country (the inner nest). For the inner nest, we adopt a generalized demand system known as homothetic demand with direct implicit additivity (HDIA) preferences, introduced by Matsuyama and Ushchev (2017). Compared to the CES demand system, the additional flexibility of HDIA demand allows the model not only to match price responses to tariffs but also to analyze the entry margin in the presence of variable markups. For the outer nest, we assume that the demand for country-specific imported composites is a decreasing function of the ideal price index of the nested varieties.

Second, we generalize the entry technology in Melitz (2003), where firms pay a common entry cost to gain access to all export markets. Specifically, we introduce market-specific entry investments: after incurring an initial entry cost, firms choose—prior to the realization of their productivity—the ex ante entry intensity (rate) for each export market, subject to a

constraint.[2] Our entry technology assumes a constant elasticity of substitution across market entries, which is given by β, to capture the degree of market interdependence. When $\beta = 1$, market entries are perfect complements. Namely, entering one markets grants free access to other markets, as in the Melitz model. Whereas, $\beta = 0$ represents that market entries are perfect substitutes. We do not impose any restrictions on β and estimate it from the data.

To illustrate the key mechanism of the model, we consider a special case of our model with homogeneous firms. This simplification allows us to approximate, to a first order, changes in each variable—whether at the firm or the market level—as a function of changes in the number of exporters in each market (N_i) and the market-specific tariff change. We then focus on the changes in N_i and demonstrate that these changes are weighted averages of changes in the total entrants into export markets and the market-specific tariff shocks. These formulas reveal that, even in markets without tariff changes, the number of exporters can still be affected through the adjustment in the total number of export entrants, i.e., the entry margin.

The intuition behind the entry margin is straightforward. Individual firms make export decisions based on the expected profits received from serving *all* markets. With free entry, the expected profits from exporting are tied to the entry cost, which is constant. When a unilateral tariff reduces profits in the focal market, firms must get compensated by increasing profits in the third markets in order to break even. This explains why individual exporters tend to experience reduced sales in the focal market but increased sales in third markets.

With constant consumer demand, the increase in individual exporters' profits in the third markets arises solely from reduced competition, which necessitates a decline in the number of exporters in those markets. Reduction in the number of exporters in one market implies fewer export entrants overall, leading to a decrease in the number of exporters across all markets. In this way, entrants transmit tariff shocks from the focal market to the third markets, ultimately causing a decline in total export sales across all markets.

Importantly, the entry margin operates independently of industry-level increasing returns to scale (IRS). While declines in both focal and third market sales might suggest the presence of increasing returns to scale, we demonstrate that the entry margin alone can produce the observed tariff responses even when the industry-level production function exhibits constant returns to scale. The underlying mechanism is straightforward: a decline in third market

[2]This entry investment decision can be interpreted as a managerial allocation problem, where managers distribute limited attention across multiple tasks. Such problems are common in span-of-control models, e.g., Lucas (1978), Rosen (1982), Antràs et al. (2006), Garicano and Rossi-Hansberg (2006), Caliendo and Rossi-Hansberg (2012), Chen (2017), Gumpert (2018), and Chen and Suen (2019).

sales necessitates a higher industry-level price in those markets. IRS is just one possibility to associate a reduced number of exporters to higher industry-level prices. Under constant returns to scale, a reduced number of exporters can still drive up prices through higher markups or reduced supply.

The generality of our model assumptions ensures that the tariff implications predicted by our model remain robust under variable markups and firm-level endogenous choice of market-specific entry rates. We further show that, with additional assumptions on the demand function, these results extend to the case of heterogeneous firms, where firms' productivity can follow an arbitrary distribution. Taking advantage of these generality, we follow Baqaee et al. (2023) to estimate the demand function and productivity distribution non-parametrically. We then calibrate the model to match observed tariff responses in both focal and third markets, as well as at the firm and market levels.

The full calibration and counterfactual analysis are still *in progress*. We plan to use the calibrated model to quantify the global impact of a unilateral tariff on trade and welfare. In particular, we will study how the parameter that governs the degree of market complementarity (β) affects the transmission of the tariff shock across various markets quantitatively. Moreover, we plan to calibrate β for different sets of countries/periods to validate this exercise. Finally, we plan to study how the inclusion of the variable markups (i.e., the HDIA demand) affects the responses of export values/quantities with respect to the tariff shock in the globe. In order to achieve this goal, we will study the tariff responses in an alternative world where the demand system is CES and the average markup across firms is the same as in our calibrated model. We will compare the responses between these two model economies.

Literature Review

Our paper contributes to the following four strands of literature.

Firstly, our paper contributes to the extensive literature on market interdependence in the context of international trade. One strand of this research examines market interdependence through the lens of firms' decision-making processes.[3] These studies highlight how market interdependence influences firms' export behavior and how firms' production decisions (the intensive margin) drive policy spillovers in partial equilibrium settings. In contrast, our focus lies on how firms' entry decisions (the extensive margin) generate market interdependence in an industry equilibrium framework. We further demonstrate how the resulting general

[3]Notable references include Antràs et al. (2017), Morales et al. (2019), Albornoz-Crespo et al. (2021), Almunia et al. (2021), and Alfaro-Ureña et al. (2024).

equilibrium effects, specifically changes in the number of exporters, reversely shape firms' intensive margin responses.

Alternatively, another strand of research on market interdependence examines the heterogeneous effects of unilateral trade policies across countries, as highlighted by Breinlich et al. (2022) and Fajgelbaum et al. (2024). This work also connects to the broader literature on trade models with IRS, pioneered by Krugman (1980). Notably, Venables (1987) and Ossa (2011) explore the firm-delocation effects of import tariffs using Krugman-style models—specifically, how import tariffs reduce the number of foreign exporters and the associated welfare implications. Our contribution to this literature lies in accounting for the aggregate effects of tariffs through an alternative mechanism: the entry margin *even without the existence of the increasing returns to scale*. We provide firm-level evidence supporting this mechanism and demonstrate its robustness within a theoretical framework.

In addition, our paper is related to a burgeoning literature on variable markups and their implications for trade (Matsuyama and Ushchev (2017), Mrázová and Neary (2017), Arkolakis et al. (2019), Dhingra and Morrow (2019), Baqaee et al. (2023) etc.) In particular, our paper is closely related to Baqaee et al. (2023). We extend the single-market model studied in Baqaee et al. (2023) to a model with multiple markets and interdependence in market entries. Furthermore, we focus on the distributional effects of demand shocks (tariffs) across markets.

Lastly, our paper is tightly linked to a large literature on temporary trade barriers and especially the anti-dumping tariffs (e.g., Prusa (2001), Bown and Crowley (2007), Blonigen and Prusa (2008), Bown and Crowley (2010), Lu et al. (2013), Blonigen and Prusa (2016), Crowley et al. (2018), Felbermayr and Sandkamp (2020), Sandkamp (2020), Steinbach and Khederlarian (2022) etc.). These papers in general focus on reduced-form firm-level evidence and argue how individual firms' decision making shapes the implications of tariffs. We identify and explain the so-called "missing intercept" problem in the context of AD tariffs: without introducing complicated firm-level decisions (at the intensive margin), changes in the extensive margin *alone* generate empirically consistent firm-level responses to tariffs. Therefore, caution should be taken, when we review documented firm-level evidence. Moreover, our study highlights the importance of unified analyses for evaluating trade policies.

2 The Impact of Anti-dumping Investigations

In this section, we estimate the impact of anti-dumping investigations on Chinese exports. We first introduce our data. We then describe our empirical specifications and present results. We examine the impact on exports to countries that investigate a particular product ("focal" and "focal controls") as well as to those that do not investigate the product ("third" and "third controls"). We decompose the changes in aggregate exports into quantities and prices, as well as into the extensive margin (the number of firms) and the intensive margin (average exports per firm). Finally, we present firm-level analysis and contrast the firm-level responses to those at the market-level.

2.1 Data

Our analysis utilizes several key data sources. First, we use Chinese customs data from 2000 to 2011, which cover the universe of Chinese exporters' transactions. The original data is at the firm-HS8 (Harmonized System)-country level, and we aggregate the data to the HS6-country or firm-HS6 levels. This dataset has been widely used in the literature to study exporter behaviors (Manova and Zhang, 2012). In our online appendix, we show that the aggregate exports in the Chinese customs data are almost identical to the data reported by UN Comtrade, and the market-level responses to AD are also similar if we use the Comtrade data instead.

Second, we use the Temporary Trade Barriers Database (TTBD) from Bown et al. (2020), which provides detailed information on anti-dumping (AD) cases, including the initiation country and date, the targeting country, and the affected products. Bown et al. (2020) classify products using the finest level of the HS codes available for each country, and we use the first six digits (HS6) because only the classification of the first six digits are consistent across countries. We focus on AD cases initiated between 2000 and 2011 against China.

Table 1: Top 15 investigating countries against China, 2000-2011

		Cases initiated between 2000 and 2011			
Country	% of Chinese exports	# cases	# HS6	% of HS6	% of exports to destination
India	1.97	107	181	3.75	15.99
United States	18.95	92	266	4.84	11.20
European Union	19.32	75	172	3.05	8.19
Turkey	0.70	61	124	2.83	7.41
Argentina	0.30	56	125	3.20	17.93
Brazil	1.12	42	72	1.65	5.72
Canada	1.45	27	85	1.73	4.73
Australia	1.59	26	36	0.73	4.39
Mexico	0.99	25	41	0.96	3.14
Colombia	0.20	25	71	1.86	5.41
South Africa	0.60	22	31	0.68	3.00
Peru	0.16	13	54	1.43	3.69
Korea	4.60	13	27	0.49	0.59
Indonesia	1.22	11	34	0.69	1.70
Thailand	1.11	10	35	0.68	2.01

Our empirical findings in this section use only the Chinese customs data and TTBD. Combining these two datasets, we identify HS6 products that are subject to AD investigations from 2000 and 2011 and the relative importance of these products among Chinese exports. In Table 1, we list the top 15 countries that initiated AD cases against China between 2000 and 2011. By the number of cases (products) investigated, India, the United States and the European Union (EU) are the three largest initiators of AD cases against China. The United States and EU are also the two largest destination markets for Chinese exports, each making up around 20% of total export values between 2000 and 2011. The products subject to AD investigations account for a significant share of Chinese exports to these countries. For example, the products subject to U.S. investigations account for 11% of Chinese exports to the U.S.

Finally, in our quantitative section, we estimate the demand system using the passthrough approach studied in Baqaee et al. (2023) and need information on firms' imported inputs (values and prices) and production costs (including the purchase of intermediate goods) . We therefore merge the Chinese customs data with the Annual Survey of Industrial Firms (ASIF). The ASIF covers all industrial firms in China with sales above a certain threshold (5,000,000 RMB, approximately 600,000 USD), and it provides information on firms' material inputs and wage bills from 2000 and 2007. We construct input cost shocks using foreign input prices and the firms' shares of input purchased from particular foreign countries, and use these shocks to estimate marginal cost passthroughs.

2.2 Empirical specifications

We want to study the impact of AD investigations on Chinese exports in different markets. For each product subject to AD investigations, we first define focal and third markets, distinguishing between treated and control products. In our analysis, a market is a country-HS6 product pair. When studying the impact of AD on Chinese exports in the country that initiated the investigation, we define the "focal treated market" as the product being treated and the "focal control market" as the products with the same first four digit code (HS4) but different HS6 in the same country. For the same treated products, we define the "third treated market" as the same HS6 product in countries that do not initiate such an investigation. The corresponding control group includes products with the same HS4 code but different HS6 codes sold in the same set of countries. We then compare the changes in exports to the focal and third markets with those in their corresponding control groups (after the initiation of the AD investigation).

In Table 2, we illustrate our definitions of treated and control groups using an actual example. In Jan./2004, the U.S. initiated an AD case on "certain frozen and canned warm water shrimp and prawns" (HS6 code 160520). The case received preliminary and final determinations in July 2004 and Feb 2005, respectively. We define the focal treated market as frozen and canned shrimps and prawns (160520) sold in the U.S. Within the HS4 code of 1605, there are two other six-digit products, preserved crabs (160510) and lobsters (160530). We define the focal control market as those two products sold in the U.S. Other countries that do not initiate AD investigations against Chinese preserved shrimps and prawns belong to "third markets", such as Japan. A typical third treated market is frozen and canned shrimps and prawns (160520) sold in Japan, while a typical third control markets are preserved crabs (160510) and lobsters (160530) sold in Japan.

Table 2: Defining focal (third) markets and control groups: an example

Market	Country, i	HS6, h	HS6 Description
Focal treated	United States	160520	shrimps and prawns
Focal control	United States	160510	crab
		160530	lobster
Third treated	Japan	160520	shrimps and prawns
Third control	Japan	160510	crab
		160530	lobster

Notes: focal/third treated and control markets in the example of the U.S. AD case on shrimps and prawns.

We first employ an event-study dynamic specification to analyze the impact of AD investigations on Chinese exports. For either focal or third markets, the specification is given by the following equation:

$$y_{iht} = \sum_{k=-K}^{K} \beta_k \text{Treated}_{ih} \mathbb{1}\{t - E_{ih} = k\} + \delta_{ih} + \delta_{is(h)t} + \varepsilon_{iht}. \tag{1}$$

In this equation, E_{ih} represents the year of the investigation. We set $K = 10$ as the largest time-to-treat, grouping $k \geq K$ and $k \leq -K$ together. Here, i denotes the destination country, h is an HS6 product, and $s(h)$ is the HS4 category of h. The δ terms are fixed effects, and standard errors are clustered at the HS6-product (h) level.

Our main dependent variable is the export value, denoted as $\ln R$. We further decompose $\ln R$ into the number of firms ($\ln N$) and average revenue per firm ($\ln \bar{r}$). Additionally, we decompose $\ln R$ into total quantity ($\ln \tilde{Q}$) and unit value ($\ln p^m$).

As we show in the next section, we find that, relative to the control groups, Chinese exports of the treated products present very strong pre-trends, and many of these pre-trends are linear in time. To address this issue, we estimate a linear pre-trend and assume that it would have continued without the treatment, following the approach of Dobkin et al. (2018) and Khederlarian and Steinbach (2022). The modified specification is given by:

$$\begin{aligned} y_{iht} =& \beta_{-K} \text{Treated}_{ih} \mathbb{1}\{t - E_{ih} = -K\} + \sum_{k=0}^{K} \beta_k \text{Treated}_{ih} \mathbb{1}\{t - E_{ih} = k\} \\ &+ \gamma \text{Treated}_{ih} \times (t - E_{ih}) \times \mathbb{1}\{t < E_{ih}\} + \delta_{ih} + \delta_{is(h)t} + \varepsilon_{iht}. \end{aligned} \tag{2}$$

In this specification, $\gamma \text{Treated}_{ih} \times (t - E_{ih}) \times \mathbb{1}\{t < E_{ih}\}$ represents the linear pre-trend for the treated group. The treatment effect at time $k \geq 0$ equals $\beta_k - \gamma(k-1)$, consistent with our normalization that the treatment effect at time $t = E_{ih} - 1$ is zero.

In addition to the level regressions (1) and (2), we also consider the corresponding first-difference regressions. For example, when the dependent variable is the log number of exporters, N_{iht}, the first-difference regression related to (1) is given by:

$$\Delta \ln N_{iht} = \sum_{k=-K}^{K} \beta_k \text{Treated}_{ih} \Delta \mathbb{1}\{t - \text{EventTime}_{ih} = k\} + \delta_{is(h)t} + \varepsilon_{iht}. \tag{3}$$

This specification is useful when we further approximate $\Delta \ln N_{iht}$ with percentage growth rates and decompose the growth rates into different margins. For example, when the growth rate of N_{iht} is small, we can approximate $\Delta \ln N_{iht}$ with the mid-point growth rate as

$$\Delta \ln N_{iht} \approx \frac{N_{iht} - N_{iht-1}}{(N_{iht} + N_{iht-1})/2}.$$

Because the change in the number of exporters must equal the difference between the number of entrants and exiters, we can further decompose the mid-point growth rates into the entry and exit margins as

$$\frac{N_{iht} - N_{iht-1}}{(N_{iht} + N_{iht-1})/2} = \frac{E_{iht}}{(N_{iht} + N_{iht-1})/2} + \frac{-X_{iht-1}}{(N_{iht} + N_{iht-1})/2},$$

where E_{iht} denotes the number of entrants (firms that do not export at time $t-1$ but at time t) and X_{iht-1} denotes the number of exiters (firms that export at time $t-1$ but not at time t). When components have trends, we can also apply the method of extrapolating the pretrends as in equation (2).

When analyzing the impact of AD at the firm level, we aggregate the firm-country-product-year level data at the firm-market-product-year level, where a market can be either focal or third. For either market, we estimate a similar dynamic specification as in equation (1):

$$y_{fht} = \sum_{k=-K}^{K} \beta_k \text{Treated}_h \mathbb{1}\{t - E_h = k\} + \delta_{fh} + \delta_{s(h)t} + \varepsilon_{fht}, \tag{4}$$

where f is the firm subscript, and the fixed effects are firm-HS6 and HS4-year. At the firm level, we focus on three outcome variables: export values, quantities and prices. Since the treatment is at the product-year level, we cluster standard errors at the HS6 product level.

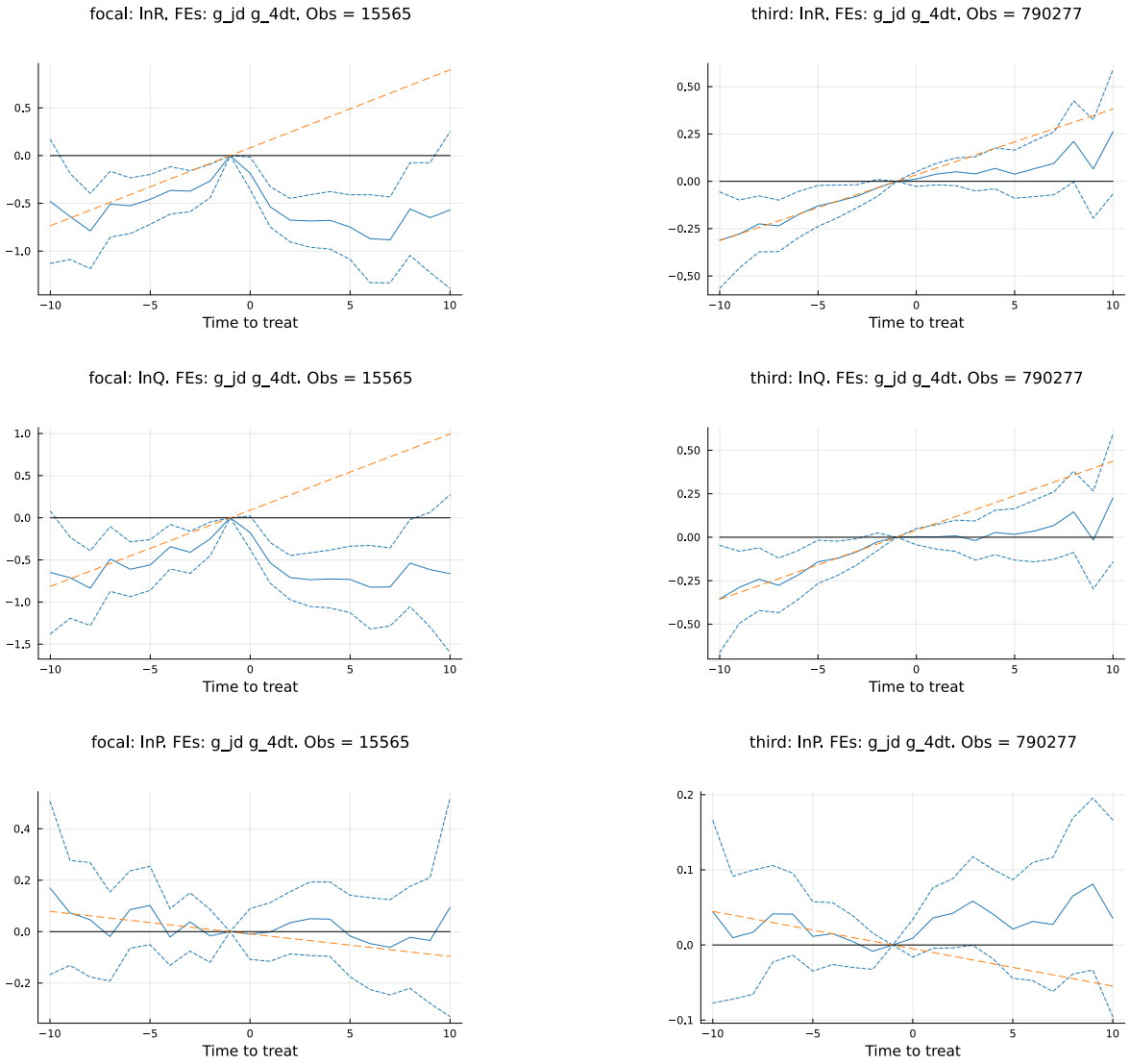
2.3 Market-level responses to AD investigations

We document three key stylized facts in our market-level analysis. Several points are worth mentioning before proceeding. First, we use EU as a single "destination economy" which has a time-varying set of countries in all regressions, as all countries in EU share the same AD and trade policies. Second, we require the AD cases to satisfy the following three restrictions to exclude unusual cases: (1) lasting for at least 5 years; (2) being revoked later than 2011 (or not being revoked at all); (3) having a unique initiation year. We call these AD cases the restricted AD cases.

Fact 1. With the initiation of anti-dumping, export value and quantity drop in both the focal and *third* markets.

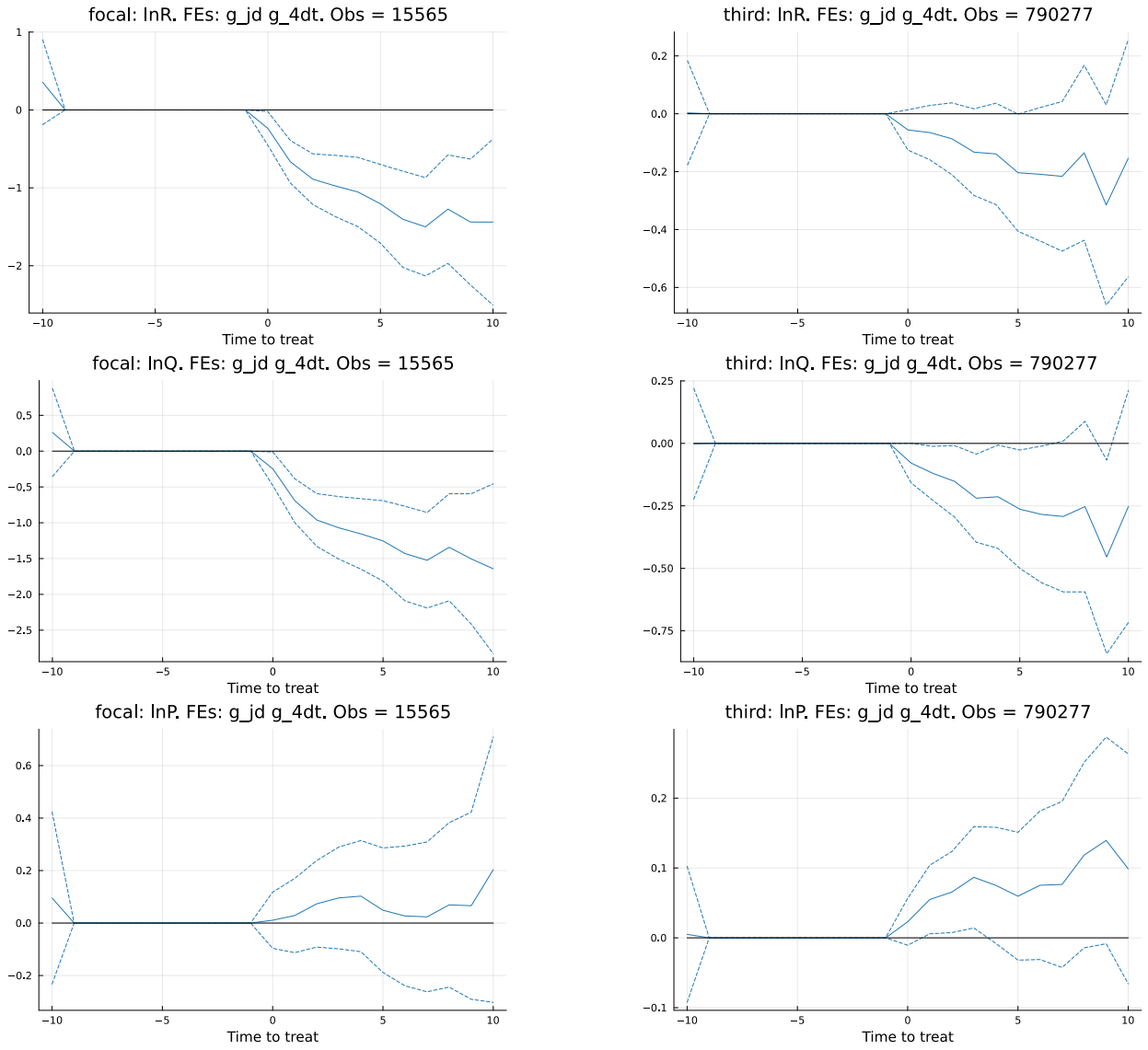
This stylized fact is evident in the first four panels of Figure 1 and continue to hold in the first four panels of Figure 2 when we restrict our AD cases and remove the pretrends. This finding is interesting, as it hints the existence of market interdependence (i.e., complementary markets) after the AD investigations are initiated. Specifically, export value and quantity drop for about 120% and 125% in the focal market (around the fifth year after the initiation) while they drop for about 20% and 30% in the third market. These magnitudes combined imply that the increase in the unit price is small (5%) in the focal market and about 10% in the third market.

Figure 1: Effect of AD investigation on revenue, quantities and prices. Event-study dynamic specifications.



Notes: All specifications include country-HS6-product and country-HS4-product-year fixed effects. Standard errors are clustered at the HS6 product level.

Figure 2: Effect of AD investigation on revenue, quantities and prices. Extrapolating pre-trends.



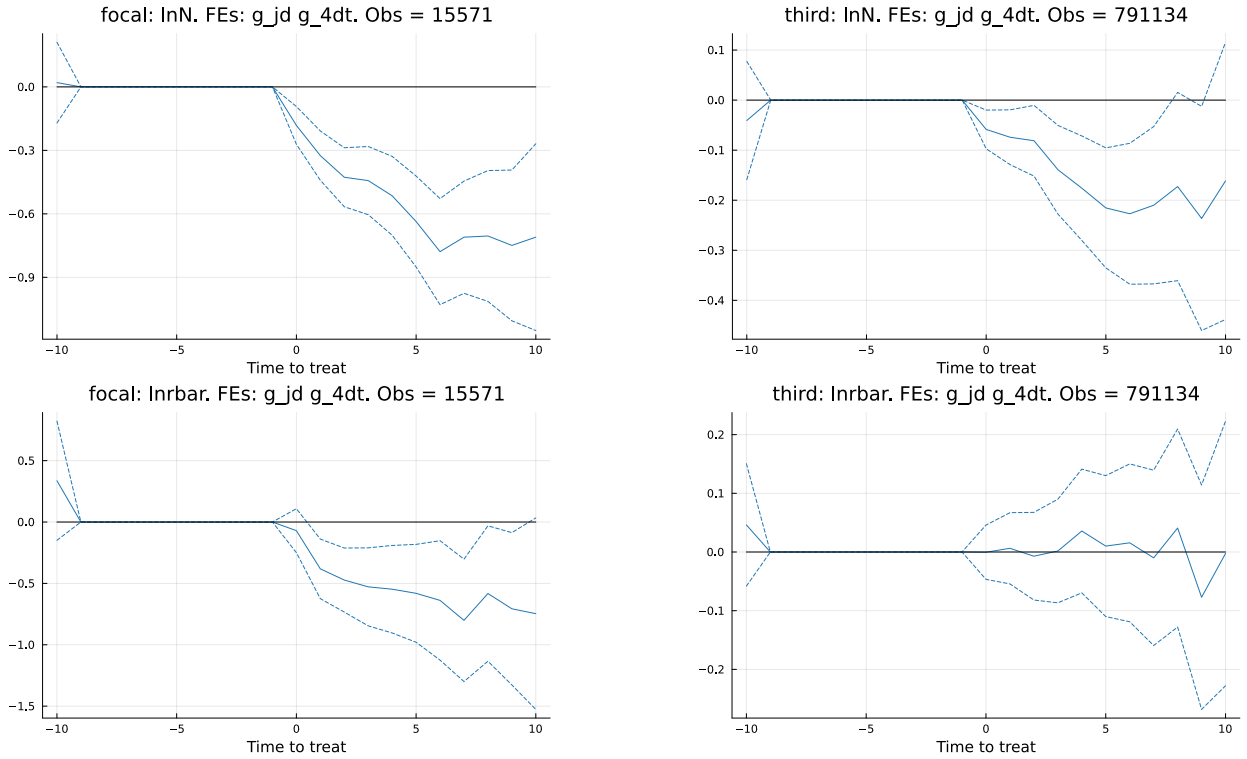
Notes: All specifications include country-HS6-product and country-HS4-product-year fixed effects. Standard errors are clustered at the HS6 product level.

Fact 2. Firms' entry and exit (i.e., the extensive margin) are the main driver of the simultaneous changes in aggregate exports *across markets*. With the initiation of the AD investigation, average revenue decreases in the focal markets but *barely changes* in the third market.

This stylized fact is evident in Figure 3 and Figure 10 in the appendix. This finding

is interesting, as it hints the importance of the extensive margin for explaining changes in aggregate trade values/volumes in a world with *interdependent markets*. Specifically, the number of exporting firms in the focal and third market drop simultaneously after the initiation of the AD investigation, while the magnitude of the drop is reasonably large in the focal market (65%) than in the third market (20%). For the focal market, both the intensive margin (the 55% drop in the firm-level average exports) and the extensive margin (the 65% drop in the number of firms) contribute to the overall drop in the total exported value. However, the drop at the extensive margin accounts for the entire drop in the total exported value to the third market. In a nutshell, the interdependence of drops in exported values between the two types of markets is entirely driven by the extensive margin. Moreover, the nil change in the firm-level exports in the third markets also exemplify the difference between the change in the total exported value and that that in the firm-level exports.

Figure 3: Effect of AD investigation at the market level: number of firms and average revenue.

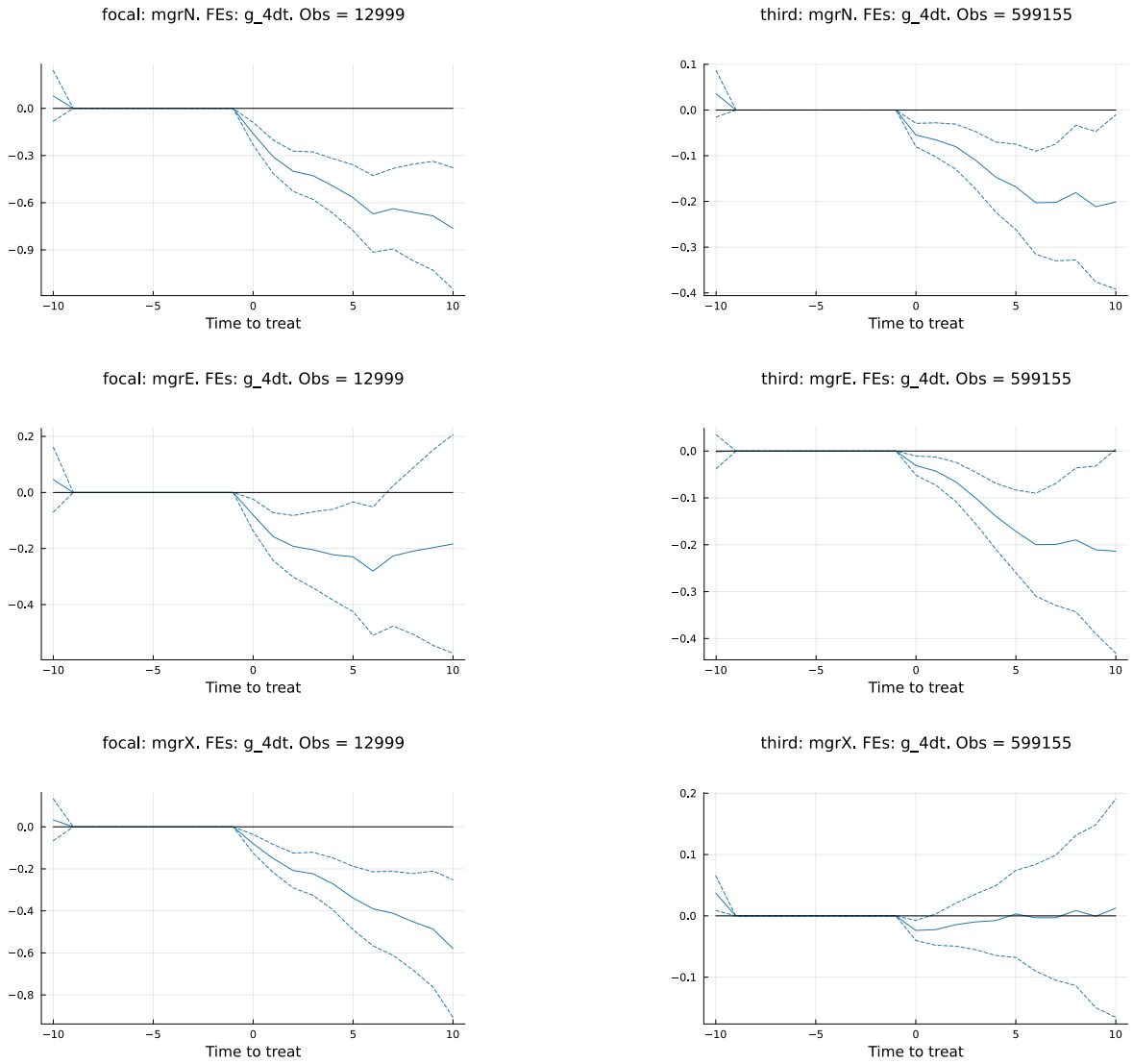


Notes: All specifications include country-HS6-product and country-HS4-product-year fixed effects. Standard errors are clustered at the HS6 product level.

Fact 3. The drop in firms' entry rate is the dominant driver of the growth rate at the extensive margin (i.e., the number of exporters).

This stylize fact is evident in Figure 4, as the 60% drop in the growth rate of the number of exporters in the focal market is explained by the drop in the entry rate (25%) and that in the exit rate (35%) . In the third markets, the 20% drop in the growth rate of the number of exporters is entirely explained by the same level of the drop in the entry rate. In total, the entry margin seems to be the key to explaining the interdependence at both the extensive margin and at the level of aggregate exports between the focal and the third markets.

Figure 4: Effect of AD investigation at the market level: mid-point growth in number of firms and decomposition into entrants and exiters.



Notes: All specifications include country-HS4-product-year fixed effects. Standard errors are clustered at the HS6 product level.

Table 3: Impact of AD in Focal and Third Markets, EU combined

	Level regressions					First differences		
	$\ln R$	$\ln Q$	$\ln P$	$\ln N$	$\ln \bar{r}$	$\frac{N_{iht}-N_{iht-1}}{(N_{iht}+N_{iht-1})/2}$	$\frac{E_{iht}}{(N_{iht}+N_{iht-1})/2}$	$\frac{-X_{iht-1}}{(N_{iht}+N_{iht-1})/2}$
focal:	-1.411	-1.489	0.078	-0.731	-0.695	-0.683	-0.219	-0.464
	(0.351)	(0.379)	(0.154)	(0.151)	(0.264)	(0.148)	(0.149)	(0.117)
third:	-0.206	-0.307	0.102	-0.202	-0.007	-0.200	-0.203	0.003
	(0.147)	(0.169)	(0.064)	(0.094)	(0.082)	(0.073)	(0.079)	(0.064)

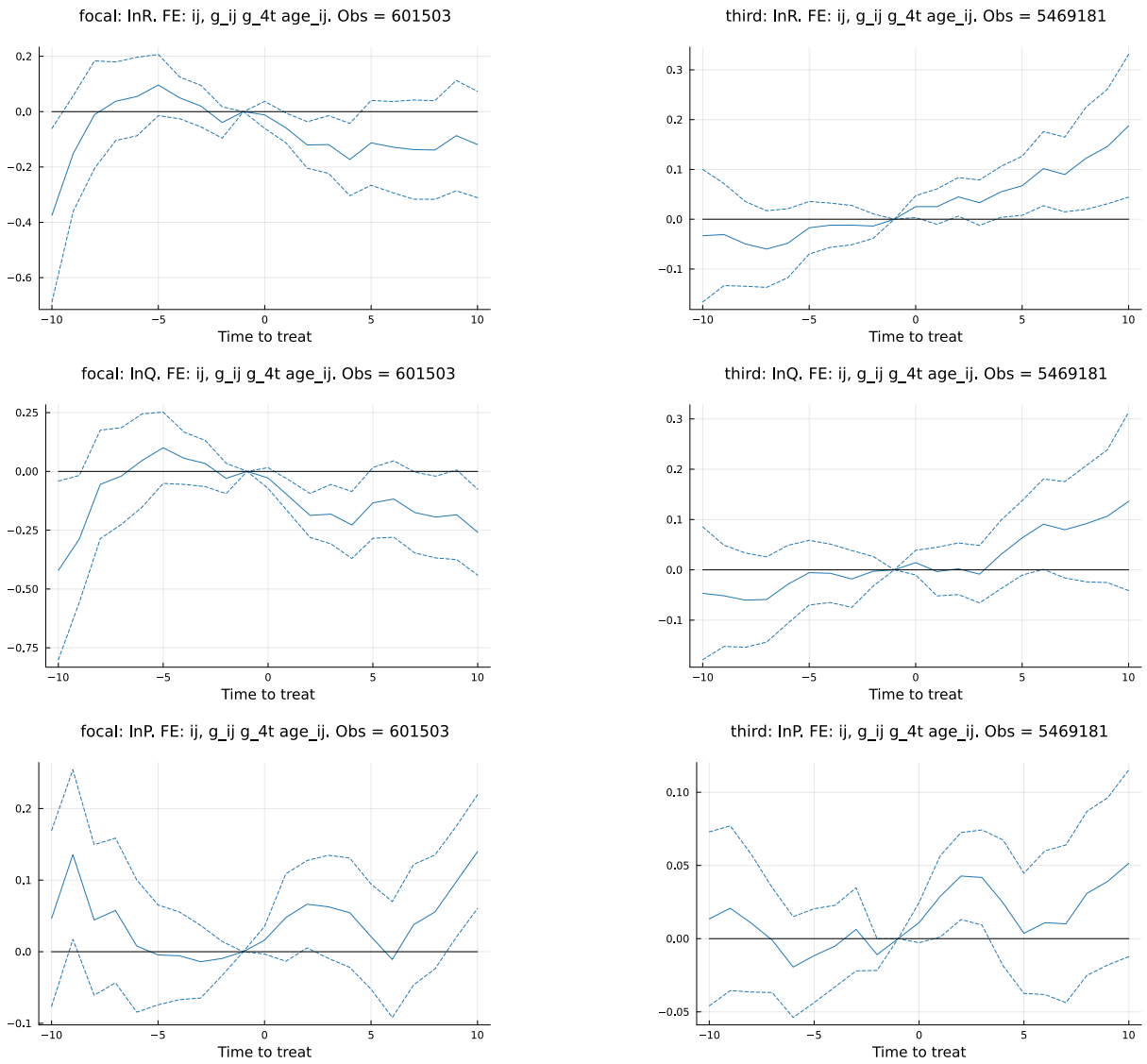
Notes: we calculate the average impact for at least six years after the treatment, i.e., $t \geq E_{ih} + 6$. It is calculated as $\frac{1}{K-5}\sum_{k\geq 6}^{K}\left(\hat{\beta}_k - \hat{\gamma}(k-1)\right)$ where $\hat{\beta}$ and γ are estimated from regression (2).

2.4 Firm-level responses to AD investigations

Fact 4. With the initiation of anti-dumping, firms' export value drops in the focal markets but *increases* in the third markets.

We now turn to the firm-level analysis. As discussed in Section 2.2, we aggregate the firm-country-product-year level data at the firm-market-product-year level and estimate the dynamic specification (4). The top two panels in Firgure 5 present the results for the export value. Similar to the market-level results in Figure 1, we find a decline in the export value in the focal market, though the magnitude of the decline is much smaller at the firm level. However, the impact of AD on firms' export value in the third market is qualitatively different from the market-level analysis. We find that firm-level exports actually *increase* in the third market. Another difference is that we do not find much pretrends at the firm level, both in the focal and third markets. As is shown in the bottom two panels of Figure 5, AD investigations seem to raise the export prices in both markets. Though the estimates are only significantly different from zero at the 5% in certain years after the treatment, almost all of the point esimates are positive.

Figure 5: Effect of AD investigation at the firm level: Revenue, Quantity, and Price



Notes: All specifications include firm-HS6-product and HS4-product-year fixed effects. Standard errors are clustered at the HS6 product level.

3 Model

In this section, we rationalize the stylized fact using an industry equilibrium framework based on firms' simultaneous entry decisions to multiple markets. In Section 3.1, we describe the economic environment and highlight key model assumptions. In Section 3.2, we present our main analytical results in a special case with homogeneous firms. In Section 3.4, we

extend these results into the case of heterogeneous firms, under additional assumptions on the demand function.

3.1 Setup

There are $N+1$ countries. Each of the $N+1$ countries is indexed by i, and $i \in \{0, 1, \ldots, N\}$ in which country 0 is the home country. We abstract our analysis from the home economy and focus on the N foreign economies which include one focal market (that changes its import tariff) and $N-1$ third markets (that do not change import tariffs). We choose to do so, as anti-dumping tariffs are unlikely to generate general equilibrium effects in the domestic market and firm-product level data of firms' domestic sales (in China) are unavailable to us. In addition, we focus on how firms from *the home country* (i.e., China) respond to the tariff shock, as firm-product level data of countries other than China are also unavailable to us.

Households In each foreign market i, goods are supplied by firms from all $N+1$ countries in the Armington fashion. We focus on an industry equilibrium of one sector where goods are all produced by firms from the home economy. Specifically, the sectoral composite good of Q_i for goods that are supplied by home exporters is pinned down by the following inverse demand function:

$$P_i^I = D_i(Q_i) \Leftrightarrow E_i \equiv E_i(Q_i) = Q_i D_i(Q_i), \tag{5}$$

where D_i a demand shifter, and P_i^I and Q_i are the sectoral price and quantity respectively. Additionally, $\eta_i \equiv -\frac{\mathrm{d}\log Q_i}{\mathrm{d}\log P_i^I}$ is the elasticity of substitution between sectors (i.e., between goods produced by home exporters and those supplied by other N foreign countries). We assume that $\eta_i > 1$, so that $E_i'(Q_i) > 0$. A useful special case for the sectoral demand is the CES demand function:

$$P_i^I = D_i Q_i^{-\frac{1}{\eta_i}} \Leftrightarrow E_i = D_i Q_i^{1-\frac{1}{\eta_i}}, \quad \eta_i > 1,$$

where P_i^I is the *ideal price index* that are relevant for sectoral expenditure (or welfare) since $P^I Q = E$.

Firms from the home country produce various varieties in this sector which are denoted by ω. We use Ω_i to denote the set of all varieties that are exported from the home country to foreign market i (i.e., exporting firms to country i), and Ω is the set of all exported varieties

(all exporting firms from home) from the home country.

For products produced by firms from the same country, consumers have a demand function that features homotheticity with direct implicit additivity (HDIA).[4] Specifically, we assume that consumers have the Kimball demand towards exported varieties ω given a sectoral composite Q_i (of varieties produced by home firms):

$$\int_{\Omega_i} \Upsilon\left(\frac{q_\omega^i}{Q_i}\right) \mathrm{d}\omega = 1, \tag{6}$$

The Kimball aggregator Υ satisfies that $\Upsilon' > 0, \Upsilon'' < 0$, and $\Upsilon(0) = 0$. The CES perference is a special case with $\Upsilon(x) = x^{\frac{\sigma-1}{\sigma}}$. We derive the inverse demand function for variety ω as[5]

$$\frac{p_\omega^i}{P_i} = \Upsilon'\left(\frac{q_\omega^i}{Q_i}\right), \tag{7}$$

in which the *price aggregator* P_i is defined as

$$P_i Q_i = \bar{\delta}_i \int_{\Omega_i} p_\omega^i q_\omega^i \mathrm{d}\omega \equiv \bar{\delta}_i E_i, \tag{8}$$

and the *demand index* $\bar{\delta}_i$ is defined as

$$\frac{1}{\bar{\delta}_i} = \int_{\Omega_i} \Upsilon'\left(\frac{q_\omega^i}{Q_i}\right) \frac{q_\omega^i}{Q_i} \mathrm{d}\omega. \tag{9}$$

Note that $P_i = P_i^I \bar{\delta}$ in the CES case. However, the ideal price index P_i^I does not coincide with the price aggregator P_i (up to a constant) in general, since $\bar{\delta}$ is also endogenous. In the special case, the demand system considered features a CES preference over all country-sector (or exporting-country specific) goods in the outer nest and a Kimball preference over varieties exported by the home country in the inner nest.

Entry and Exit In each period, firms make entry decisions in two steps. First, they pay an entry cost and choose an entry intensity (i.e., entry rate) for each foreign market subject to a managerial capacity constraint. Then, firms draw productivities from an exogenous distribution, and the entry opportunity realizes. Finally, the firms decide whether to export to markets that grant them access.

[4] See Matsuyama and Ushchev (2017) for details.

[5] See Appendix B for details.

The entry cost is given by f_e. Once the entry cost is paid, firms maximize the expected profits subject to the entry technology[6]:

$$\begin{aligned} \max_{J_i \geq 0} \quad & \sum_{i=0}^{N} J_i \bar{\pi}_i \\ \text{s.t.} \quad & \left[\sum_{i=0}^{N} \left(\frac{J_i}{h_i} \right)^{\frac{1}{1-\beta}} \right]^{1-\beta} = 1, \end{aligned}$$

in which h_i denoted the easiness of entering country i, $\bar{\pi}_i$ is the expected profit in country i, and $\beta \in [0, 1)$. Variable $\{J_i\}$ is the endogenous entry intensity chosen by the firm.

Solving this optimization problem gives the entry rate

$$J_i = h_i \left[\frac{(h_i \bar{\pi}_i)^{\frac{1}{\beta}}}{\sum_k (h_k \bar{\pi}_k)^{\frac{1}{\beta}}} \right]^{1-\beta} \tag{10}$$

The free entry condition implies that

$$\sum_i J_i \bar{\pi}_i = f_e \Leftrightarrow \left[\sum_k (h_k \bar{\pi}_k)^{\frac{1}{\beta}} \right]^{\beta} = f_e.$$

The parameter β is of key interest, since it measures the degree of market interdependence through the entry condition. First, let us consider two extreme cases with $\beta = 0$ and $\beta \nearrow 1$. When $\beta = 0$, entry into different markets are perfect substitutes. Firms only enter into the market(s) with the highest post-entry return, i.e., $i \in \arg\max_k \{h_k \bar{\pi}_k\}$. Since firms must be indifferent across all markets when thinking of entry, the entry rate is pinned down by other conditions in equilibrium. The free entry condition implies that markets with positive entry

[6] Boehm and Oberfield (2020) consider a model extension with this entry technology in their online appendix. In our understanding, this constraint is fundamentally related to the *fixed* amount of managerial resources (e.g., time or capacity) that is use to promote successful entries into various export markets. Specifically, we normalize this amount to one and treat J_i as the endogenous entry probably into market i, and thus $J_i \in [0, 1]$. $\left(\frac{J_i}{h_i} \right)^{\frac{1}{1-\beta}}$ is the required convex managerial cost (capacity) that generates the entry probability of J_i in market i, as $\beta \in [0, 1)$ and $\left(\frac{1}{h_i} \right)^{\frac{1}{1-\beta}}$ is the cost shifter. As a result, the overall managerial cost across all markets is $\sum_{i=0}^{N} \left(\frac{J_i}{h_i} \right)^{\frac{1}{1-\beta}} = 1$ or $\left[\sum_{i=0}^{N} \left(\frac{J_i}{h_i} \right)^{\frac{1}{1-\beta}} \right]^{1-\beta} = 1$.

have constant expected profit $\bar{\pi}_i = \frac{f_e}{h_i}$.[7] This property ensures complete insulation in the third markets when (tariff) shocks happen in the focal markets, i.e., markets are independent.

When $\beta \nearrow 1$, entry into different markets are perfect complements. In particular, firms enter into all markets at exogenous rates, $J_i = h_i$. This is the standard entry technology in Melitz (2003): once the entry fee is paid, firms are able to export to any foreign countries, though they may choose not to. The free entry condition then boils down to the familiar one in which the sum of expected profits over all markets equals the entry cost: $\sum_i h_i \bar{\pi}_i = f_e$.

After entry, firms make a draw of productivity z from an exogenous distribution G, and entry opportunity realizes. We use O to denote the set of markets with realized entry opportunities facing firms. Each market features a fixed cost f_i, and firms export to this market if they make positive profit there. Incumbent firms exit the market randomly with a probability of ν_i. Both fixed cost and entry cost are denominated in units of labor, which is chosen to be the numerarie.

Production Each firm supplies a distinct variety ω and competes monopolistically. Labor is the only factor in the economy. The production function is linear: firms with productivity z produces zl units of goods using l labor. Each country sets its own tariff rate, and t_i describes the ratio between consumer and producer price in country i. Moreover, τ_i captures the iceberg shipping cost from home country to country i, and $\tau_i \geq 1$.

Taking the residual demand given in equation (7), firms set prices in each market to maximize their profits there. Specifically, each firm solves the following profit maximization problem in each market:

$$\max_p \quad (p - \frac{\tau_i}{z}) q_i \left(t_i p\right),$$

where the explicit form of demand function is

$$q_i(p) = (\Upsilon')^{-1}\left(\frac{p}{P_i}\right) Q_i = \Psi\left(\frac{p}{P_i}\right) Q_i,$$

and $\Psi \equiv (\Upsilon')^{-1}$. We assume that Kimball aggregator ensures a unique interior solution.[8]

[7] To see this, let $k = h_i \bar{\pi}_i$ if $i \in \arg\max_k \{h_k \bar{\pi}_k\}$. Then,

$$k = k \sum_i \frac{J_i}{h_i} = \sum_i J_i \bar{\pi}_i = f_e.$$

[8] This requires $\Upsilon'''(x)x + 2\Upsilon''(x) < 0$. That Υ''' exists will ensure that the profit function π_z^i is continuous

The usual Lerner formula gives

$$\frac{p-\tau/z}{p} = -\frac{q_i(t_i p)}{q_i'(t_i p)t_i p} \equiv \frac{1}{\sigma_i(t_i p)},$$

where $\sigma_i \equiv -\frac{\mathrm{d}\log q_i}{\mathrm{d}\log p}$ is the price elasticity. Noticing that σ_i could be written as a function of p/P_i or q/Q_i, i.e.,

$$\sigma_i(p) = \underbrace{-\frac{\Psi'\left(\frac{p}{P_i}\right)\frac{p}{P_i}}{\Psi\left(\frac{p}{P_i}\right)}}_{\sigma\left(\frac{p}{P_i}\right)} = \underbrace{-\frac{\Upsilon'\left(\frac{q_i}{Q_i}\right)}{\Upsilon''\left(\frac{q_i}{Q_i}\right)\frac{q_i}{Q_i}}}_{\sigma\left(\frac{q_i}{Q_i}\right)}$$

with $q_i \equiv q_i(p)$. In addition, let μ be the markup, i.e., the ratio between price and marginal cost. Then,

$$\mu_i = \frac{\sigma_i}{\sigma_i - 1},$$

and we define $\mu\left(\frac{p}{P_i}\right)$ and $\mu\left(\frac{q}{Q_i}\right)$ accordingly. As usual, we assume that $\sigma_i > 1$ so markup is positive.

We now summarize the set of variables related to firms' production. For a firm with productivity z, its price, markup, and quantity in country i are determined implicitly by

$$p_z^i = \frac{\mu_z^i \tau_i}{z}, \quad q_z^i = q_i(t_i p_z^i), \quad \text{and } \mu_z^i = \mu\left(\frac{t_i p_z^i}{P_i}\right) = \mu\left(\frac{q_z^i}{Q_i}\right),$$

Let $\tilde{P}_i = P_i/t_i$. Then, $\tilde{P}_i$ is the *producer price index* while P_i is the *consumer price index*. We have $\tilde{P}_i Q_i = R_i\bar{\delta}_i$, where $R_i = E_i/t_i$ is the aggregate export revenue in country i. Comparing to the CES case, markup is no longer constant, so the pass-through of tariff will be incomplete.

The per period variable profits and sales are as follows:

$$\pi_z^i = r_z^i\left(1 - \frac{1}{\mu_z^i}\right), \quad r_z^i = p_z^i q_z^i = \mu_z^i l_z^i.$$

Then, firms export to market i if $\pi_z^i \geq f_i$. Since the profit function is increasing and continuous, there exists a threshold z_i^* such that firms export to country i if and only if $z \geq z_i^*$. In the rest of the paper, we only consider the case in which z_i^* is within the range of possible productivity draws (i.e., within the interior of the distribution), i.e., $\pi_{z_i^*}^i = f_i$.

in z.

Finally, there is an exogenous exit shock ν_i in each country. Firms discount future profits at a rate of ρ. Then, the expected entry profit of country i satisfies that

$$\bar{\pi}_i = \frac{1}{\rho + \nu_i} \int_{z_i^*}^{\infty} [\pi_z^i - f_i] dG(z). \tag{11}$$

Equilibrium We consider a perfect foresight industry equilibrium in which the wage wage and tariffs $\{t_i\}$ are exogenous. In equilibrium, firms decide optimally on entry and production taking as given the time series of sectoral prices and composites, which also clear all $N+1$ goods markets. For most of the time, we focus on a stationary equilibrium in which all aggregate variables are constant. Let M be the number of entrants and N_i be the number of exporters to country i, then the law of motion on N_i is given by:

$$\dot{N}_i = J_i M[1 - G(z_i^*)] - \nu_i N_i.$$

In a stationary equilibrium, $\dot{N}_i = 0$ for all i.

Infra-marginal ratio of consumer surplus and cost pass-through Before, proceeding, we introduce two key endogenous variables that will be used intensively in our following analyses. First, the infra-marginal surplus ratio (as in Baqaee et al. (2023)) is given by:

$$\delta_i = \delta\left(\frac{q_i}{Q_i}\right) = \frac{\Upsilon\left(\frac{q_i}{Q_i}\right)}{\Upsilon'\left(\frac{q_i}{Q_i}\right)\frac{q_i}{Q_i}}$$

δ_i is one plus the ratio of consumer surplus to revenue evaluated at (p_i, q_i). Since Υ is concave, $\delta_i > 1$. Next, the firm-level pass-through rate of a cost on th price is defined as

$$\rho = -\frac{\mathrm{d}\log p}{\mathrm{d}\log z} = 1 - \frac{\mathrm{d}\log \mu}{\mathrm{d}\log z},$$

where the latter equality holds fixing τ_i and t_i. $\rho > 0$ due to the supermodularity of the profit function. Solving this equation renders that

$$\rho_i = \rho\left(\frac{q_i}{Q_i}\right) = \frac{1}{1 + \frac{\mu'\left(\frac{q_i}{Q_i}\right)\frac{q_i}{Q_i}}{\mu\left(\frac{q_i}{Q_i}\right)}\sigma\left(\frac{q_i}{Q_i}\right)},$$

which shows that the cost pass-through is related to the price elasticity and the elasticity of the markup (i.e., the elasticity of the price elasticity). Evidently, $\mu'\left(\frac{q_i}{Q_i}\right) = 0$ when the demand is CES, which implies that $\rho_i = 1$ (i.e., a complete passthough). Intuitively, if the markup increases with the normalized quantity $\frac{q_i}{Q_i}$ faster (i.e., a larger elasticity), the cost passthorugh will be smaller implying that a given reduction in the marginal cost leads to a smaller reduction in the price. Accordingly, the producer's profit margin increases.

Finally,, we impose several conditions on the two elasticities before proceeding:

Assumption 1. $\rho_i \leq 1$, $\sigma_i > \eta_i$ *and* $\delta_i < \frac{\eta_i}{\eta_i - 1}$

Baqaee et al. (2023) argue that the first condition is the *weak* version of Marshall's second law of demand, which states that markups are increasing with productivity. It is equivalent to $\mu_i' \geq 0$. The second condition follows Atkeson and Burstein (2008) that substitution between products is stronger within sectors than across sectors. In our context, it means that the substitution between goods exported from the same country is higher than that between goods from different countries of origin. The third condition imposes a diminishing return with respect to firms' entry, i.e., there shall not be infinitely many firms in equilibrium. In general, a firm's entry generates a negative externality and a positive one to its competitors in the industry. On the one hand, it intensifies the market competition and reduces other firms' profit. On the other hand, it increases the industry demand when the variety effect is present. This condition implies that the competition effect dominates the aggregate demand effect, so entrants' post-entry profit decreases with the number of existing firms. Note that with nested CES demand as in Atkeson and Burstein (2008), the second condition above is equivalent to the third condition. However, these two conditions do not align with each other in general.

3.2 Homogeneous firms

We start with the case in which firms are homogeneous. As a result, the productivity distribution G degenerates to a single mass point at a given productivity level z. We assume that in equilibrium, all markets have strictly positive measure of exporters.[9] To simplified the notation, we suppress the notation of the variety (ω) and use p_i and q_i to denote firm-level price and quantity in the case of homogeneous firms.

In what follows, we lay down three sets of equilibrium conditions. First, the Kimball

[9] A sufficient condition could be $\liminf_{q/Q \to \infty} \mu(q/Q) = \underline{\mu} > 1$.

preference with homogeneous firms implies that

$$N_i \Upsilon \left(\frac{q_i}{Q_i} \right) = 1. \tag{12}$$

The income accounting condition (or the balanced budget equation) implies that

$$E_i(Q_i) = N_i t_i p_i q_i \Leftrightarrow p_i^I(Q_i) = N_i t_i \mu \left(\frac{q_i}{Q_i} \right) \frac{\tau_i}{z} \frac{q_i}{Q_i}. \tag{13}$$

Equations (12) and (13) pin down $\frac{q_i}{Q_i}$ and Q_i as functions of N_i and model parameters such as t_i, z and τ_i. Therefore, we can write σ_i, η_i, ρ_i, δ_i as functions of N_i.

The next condition expresses the per period variable profits as a function of N_i, Q_i and model parameters:

$$\pi_i(N_i) = \left(1 - \frac{1}{\mu_i} \right) p_i q_i = \left(1 - \frac{1}{\mu_i} \right) \frac{E_i(Q_i)}{N_i t_i}. \tag{14}$$

equation (11) implies that $\bar{\pi}_i = \frac{\pi_i - f_i}{\rho + \nu_i}$. As the ratio of entry intensities is proportional to the ratio of expected post-entry profits:

$$\frac{J_i}{J_j} = \frac{h_i^{\frac{1}{\beta}} \bar{\pi}_i^{\frac{1-\beta}{\beta}}}{h_j^{\frac{1}{\beta}} \bar{\pi}_j^{\frac{1-\beta}{\beta}}},$$

we can use equation (11) to calculate the series of j_i.

Finally, we have two sets of conditions for $(M, \{N_i\})$:

$$\sum_{i=0}^{N} (h_i \bar{\pi}_i)^{\frac{1}{\beta}} = f_e^{\frac{1}{\beta}}, \quad \forall\, i$$

$$J_i M = \nu_i N_i,$$

where the first condition above is the free entry condition and the second set of conditions is the balanced flow of firms in each market. In total, there are $N + 2$ equations for $N + 2$ unknowns among all markets. Notice that with equation (10), the above equations imply that

$$J_i = h_i \left(\frac{h_i \bar{\pi}_i}{f_e} \right)^{\frac{1-\beta}{\beta}} = \frac{\nu_i N_i}{M} \quad \Rightarrow \quad \pi_i(N_i) = (\rho + \nu_i) \left(\frac{\nu_i N_i}{h_i M} \right)^{\frac{\beta}{1-\beta}} \frac{f_e}{h_i} + f_i, \tag{15}$$

in which $\pi_i(N_i)$ is given implicitly in equation (14). Therefore, the second part of the above equations provides us an implicit function N_i in terms of M. We define such a mapping using $N_i = N_i(M)$. Inserting it back into the free entry condition, we pin down the equilibrium mass of entrants M as follows:

$$\sum_{i=0}^{N} \left(h_i \bar{\pi}_i(N_i(M))\right)^{\frac{1}{\beta}} = f_e^{\frac{1}{\beta}}. \tag{16}$$

In this way, we have solved the stationary equilibrium. We show that a stationary equilibrium exists and is unique under Assumption 1:

Proposition 1. *Suppose Assumption 1 holds for all N_i and $\liminf_{N_i \to k} \frac{\eta_i}{\eta_i - 1} \frac{1}{\delta_i} > 1$ for $k \in \{0, +\infty\}$. Then, there exists a unique stationary equilibrium.*

Proof. See Appendix C.3. ■

3.2.1 Comparative statics: tariff shocks and market interdependence

We now analyze market-specific responses to tariff shocks $\{t_i\}$. Before presenting our results, it is useful to clarify the model counterparts of the empirical patterns and to define several variables that are used in the empirical analyses. First, we refer aggregate variables (in the model) to those at the product-country level (in the data) and firm variables to those at the firm-product-country level. To streamline notations, we use the expectation operator $\mathbb{E}_x[y]$ defined as follows:

$$\mathbb{E}_x[y] = \frac{\int x(v) y(v) \mathrm{d}v}{\int y(v) \mathrm{d}v},$$

in which x and y are functions of v. Note that v could be both discrete and continuous. At the aggregate level in market i, we have total revenue R_i, total quantity $\tilde{Q}_i$, unit value p_i^m, the number of exporters N_i, and average revenue r_i^m where

$$\tilde{Q}_i = \int_{\Omega_i} q_i \mathrm{d}\omega; \;\; p_i^m = \frac{R_i}{\tilde{Q}_i} = \frac{\int_{\Omega_i} p_i q_i \mathrm{d}\omega}{\int_{\Omega_i} q_i \mathrm{d}\omega} = \mathbb{E}_q[p_i]; \;\; r_i^m \equiv R_i / N_i.$$

At the firm level, we have revenue r_i, profit π_i, price p_i, and quantity q_i. In the case of homogeneous firms, unit price coincides with firm price, and average revenue coincides with firm revenue. In addition, we have the market-specific utility Q_i (which matters for welfare)

and the total number of entrants M, both of which are unobservable from the data.[10]

We discuss how various price measures are related. There are four variables related to prices: firm price p_i, aggregate unit value p_i^m, (Kimball) price aggregator P_i, and the ideal price index P_i^I. First, the unit price p_i^m is the quantity weighted average of firm prices (p_i). In particular, $p_i = p_i^m$ in the case of homogeneous firms. Next, the Kimball price aggregator P_i follows the definition in equation (8), and the ideal price index P_i^I is defined in equation (5). They satisfy that $P_i = P_i^I \bar{\delta}_i$, where $\bar{\delta}_i$ is defined in equation (9). P_i^I is the aggregation of p_i adjusted by the number of varieties, while P_i is further adjusted by the infra-marginal surplus ratio. In the case of CES demand with homogeneous firms, we have $\bar{\delta}_i = \frac{\sigma_i}{\sigma_i - 1}$, $P_i^I = N_i^{\frac{1}{1-\sigma_i}} p_i$, and $P_i = \frac{\sigma_i}{\sigma_i - 1} N_i^{\frac{1}{1-\sigma_i}} p_i$.

The implications of tariffs We first present the following proposition that characterizes the response of M to a set of tariff shocks $\mathrm{d}\log t_i$. This is a central result since the transmission of tariff shocks across market works *solely* through firms' entry. All the results assume that the economy was in a stationary equilibrium before tariff changes.

Proposition 2. *Suppose assumption 1 holds, and firms have the same productivity z. In response to a set of tariff shocks $\mathrm{d}\log t_i$ where $i \in \{1, 2, 3, ..., N\}$, changes in the total number of entrants are given by*

$$\mathrm{d}\log M = -\frac{\sum_{i=0}^{N} s_i \xi_i \frac{\mathrm{d}\log t_i}{\theta_i}}{\sum_{i=0}^{N} s_i \xi_i} = -\mathbb{E}_{s\xi}\left[\frac{\mathrm{d}\log t}{\theta_i}\right], \tag{17}$$

in which

$$\theta_i = \left(\frac{1}{\eta_i} - \frac{1}{\sigma_i}\right)\left(\frac{1}{\rho_i} - 1\right)\delta_i + 1 - \frac{\eta_i - 1}{\eta_i}\delta_i > 0, \tag{18}$$

$$s_i = \frac{J_i \bar{\pi}_i}{\sum_k J_k \bar{\pi}_k}, \qquad \textit{and} \qquad \xi_i = \frac{\frac{\pi_i}{\pi_i - f_i}\eta_i\theta_i}{\frac{\beta}{1-\beta} + \frac{\pi_i}{\pi_i - f_i}\eta_i\theta_i} \leq 1. \tag{19}$$

Proof. See Appendix C.2 ■

A tariff increase affects the firm's profit through a direct (negative) demand effect and an indirect (positive) competition effect, which has to be compensated by the adjustment in the

[10] As we focus on a representative sector, we use the change in real consumption, $\mathrm{d}\log Q_i$, to measure the welfare impact in country i.

firm mass due to the free entry condition. θ_i captures the required adjustment in the firm mass related to these two effects, which is reflected by the two items in the right-hand-side (RHS) of equation (18). The first item corresponds to the pro-competitive (i.e., competition) effect originating from variable markups, which vanishes when markups are constant (i.e., $\rho_i = 1$).[11] The second term (i.e., $1 - \frac{\eta_i - 1}{\eta_i}\delta_i$) gauges the required reduction in the firm mass due to the direct demand effect. It becomes smaller (i.e., a bigger $1 - \frac{\eta_i - 1}{\eta_i}\delta_i$), when expected post-entry profits feature a *stronger* decreasing return with respect to firms' entry. This is the case, as a *stronger* decreasing return implies a smaller required reduction in the firm mass needed to compensate the profit loss and to reestablish the free entry condition.[12]

When the free entry condition holds, the expected profit of entry is constant. Entry is then reduced to maintain that level of profit. Proposition 2 shows that the required reduction in total entry is equal to the weighted averaged of required entry reduction in each market. The weights, $s_i\xi_i$, are the product of the share of expected profits in market i, s_i, and a wedge ξ_i, which varies with the level of entry interdependence.

In this model, markets are independent from each other except that they may import from the same firm. Firms' entry is then the only channel that transmits shocks from market to market. In the following proposition, We show that the change in the number of total entrants, $\mathrm{d}\log M$, is a sufficient statistics for all changes in all markets.

Proposition 3. *Suppose that Assumption 1 holds, and firms have the same productivity z. In response to a set of tariff shocks $\mathrm{d}\log t_i$ where $i \in \{1, 2, ..., N\}$, the following equations characterize changes of endogenous variables in equilibrium:*

i) At the aggregate level,

$$\mathrm{d}\log N_i = -\left[(1-\xi_i)\,\mathbb{E}_{s\xi}\left[\frac{\mathrm{d}\log t}{\theta}\right] + \xi_i \frac{\mathrm{d}\log t_i}{\theta_i}\right] = (1-\xi_i)\mathrm{d}\log M + \xi_i \frac{-\mathrm{d}\log t_i}{\theta_i}, \quad (20)$$

$$\mathrm{d}\log \tilde{Q}_i = \underbrace{\left(\delta_i\left(\frac{1}{\rho_i} - 1\right) - \eta_i\theta_i + 1\right)}_{>0} \mathrm{d}\log N_i - \eta_i \mathrm{d}\log t_i,$$

[11] Surviving firms increase markups after the tariff shock as there are fewer of them. As a result, the competition becomes softer. This effect is stronger and thus the required reduction in the firm mass is smaller, when the passthrough rate ρ_i is lower.

[12] An example is the case of nested CES demand:

$$\pi_i \propto N_i^{\frac{\sigma-\eta}{1-\sigma}}, \quad \text{and} \quad \frac{\sigma_i - \eta_i}{1-\sigma_i} = -\eta_i\left(1 - \frac{\eta_i - 1}{\eta_i}\frac{\sigma_i}{\sigma_i - 1}\right) \equiv -\eta_i\theta_i$$

, where the pro-competitive effect vanishes.

$$\mathrm{d}\log R_i = \underbrace{\left[\left(\frac{1}{\rho_i}-1\right)\left(1-\frac{1}{\sigma_i}\right)\delta_i - \eta_i\theta_i + 1\right]}_{>0}\mathrm{d}\log N_i - \eta_i \mathrm{d}\log t_i;$$

ii) At the firm level,

$$\mathrm{d}\log \pi_i = -\eta_i\theta_i \mathrm{d}\log N_i - \eta_i \mathrm{d}\log t_i,$$

$$\mathrm{d}\log p_i = -\left(\frac{1}{\rho_i}-1\right)\frac{\delta_i}{\sigma_i}\mathrm{d}\log N_i,$$

$$\mathrm{d}\log q_i = \left(\delta_i\left(\frac{1}{\rho_i}-1\right) - \eta_i\theta_i\right)\mathrm{d}\log N_i - \eta_i \mathrm{d}\log t_i,$$

$$\mathrm{d}\log r_i = \left[\left(\frac{1}{\rho_i}-1\right)\left(1-\frac{1}{\sigma_i}\right)\delta_i - \eta_i\theta_i\right]\mathrm{d}\log N_i - \eta_i \mathrm{d}\log t_i;$$

$$\mathrm{d}\log Q_i = \underbrace{\left(\frac{\delta_i}{\rho_i} - \eta_i\theta_i\right)}_{>0}\mathrm{d}\log N_i - \eta_i \mathrm{d}\log t_i.$$

Proof. See Appendix C.2. ■

Proposition 3 presents a complete characterization of the changes in each market. The number of firms in market i, N_i, is a sufficient statistic for our comparative statics analyses in that market. In particular, changes in firm-level and aggregate level variables come from either the local tariff shock or the change in N_i. The change of N_i, $\mathrm{d}\log N_i$, is a weighted average of the change in the mass of all entrants ($\mathrm{d}\log M$) and the rescaled local tariff shock, $-\mathrm{d}\log t_i/\theta_i$, where the latter measures $\mathrm{d}\log N_i$ absent the market interdependence. To obtain a direct comparison with the empirical results, we consider a case in which only one (focal) market raises its tariff and obtain the following result.

Corollary 1. *Suppose that assumption 1 holds, and firms have the same productivity z. Country j ("the focal market") raises its tariff, i.e., $\mathrm{d}\log t_j > 0$, while all other countries*

("the third markets") retain previous tariffs, i.e., $\mathrm{d}\log t_i = 0$ *for* $i \neq j$*. We have the following qualitative equilibrium changes:*

i) In the focal market j*,*

$$\mathrm{d}\log N_j < 0, \mathrm{d}\log \tilde{Q}_j < 0, \mathrm{d}\log R_j < 0, \mathrm{d}\log p_j > 0, \mathrm{d}\log q_j < 0, \mathrm{d}\log r_j < 0, \mathrm{d}\log Q_j < 0;$$

ii) In the third market $i \neq s$*,*

$$\mathrm{d}\log N_i < 0, \mathrm{d}\log \tilde{Q}_j < 0, \mathrm{d}\log R_i < 0, \mathrm{d}\log p_i > 0, \mathrm{d}\log Q_i < 0,$$

$$\mathrm{d}\log q_i > 0 \text{ if and only if } \frac{\eta_i}{\eta_i - 1} > \frac{\delta_i}{1 - \frac{\delta_i}{\sigma_i}\left(\frac{1}{\rho_i} - 1\right)},$$

$$\mathrm{d}\log r_i > 0 \text{ if and only if } \frac{\eta_i}{\eta_i - 1} > \delta_i \left[1 + \left(\frac{1}{\rho_i} - 1\right)\frac{1}{\sigma_i}\right].$$

Proof. See Appendix C.2 ■

It is clear that predictions in Corollary 1 are consistent with the empirical facts. In particular, the above results rationalize simultaneous drops in aggregate trade value/quantity *and* the number of firms in both the focal and the third markets. The corollary also predicts simultaneous increases in firm-level and unit prices in both markets, which are consistent with the empirical results. Importantly, Corollary 1 can yield opposite effects on export value/quantity at the firm and the aggregate levels in the third markets: firm-level export quantity/revenue can increase while aggregate export quantity/revenue must drop in the third markets. This has to be the case when the demand is next CES where $\rho = 1$.

The simultaneous drops in the number of exporters in both markets deserve more explanations. The tariff increase in the focal market makes the number of exporters go down there, as both the total number of exporters and the entry intensity go down. For the third markets, the number of exporters goes down, as the decrease in the total number of exporters *dominates* the increasing entry intensity allocated to the third markets. In total, the market complementarity linked through the *shared* pool of exporting entrants (always) dominates the market substitutability that operates through the endogenous choice of entry intensities.[13]

[13] I.e., Increasing expected profits in one market leads to higher entry intensity into that market and lower

3.2.2 Some special cases

Nested CES demand : First, it is instructive to study the special case with nested CES preference. Namely, we have $\Upsilon_i(x) = x^{\frac{\sigma_i-1}{\sigma_i}}$, $D_i(x) = D_i x_i^{-\frac{1}{\eta_i}}$. Assumption 1 is equivalent to $\sigma_i > \eta_i$. Furthermore,

$$\mu_i = \delta_i = \frac{\sigma_i}{\sigma_i - 1}, \quad \eta_i\theta_i = \frac{\sigma_i - \eta_i}{\sigma_i - 1}, \quad \rho_i = 1.$$

We then restate proposition 3 as

Corollary 2. *Suppose Assumption 1 holds, and firms have the same productivity z. Furthermore, the preference is assumed to be nested CES. In response to a set of tariff shocks $\mathrm{d}\log t_i$ where $i \in \{1, 2, ..., N\}$, the following equations characterize changes in the equilibrium:*

i) At the aggregate level,

$$\mathrm{d}\log R_i = \mathrm{d}\log \tilde{Q}_i = \frac{\eta_i - 1}{\sigma_i - 1}\mathrm{d}\log N_i - \eta_i \mathrm{d}\log t_i;$$

ii) At the firm level,

$$\mathrm{d}\log p_i = 0, \quad \mathrm{d}\log \pi_i = \mathrm{d}\log r_i = \mathrm{d}\log q_i = -\frac{\sigma_i - \eta_i}{\sigma_i - 1}\mathrm{d}\log N_i - \eta_i \mathrm{d}\log t_i;$$

$$\mathrm{d}\log Q_i = \frac{\eta_i}{\sigma_i - 1}\mathrm{d}\log N_i - \eta_i \mathrm{d}\log t_i.$$

The model with nested CES preference inherits all the properties of our general model except for the pro-competitive effect. With constant markups, profit, revenue, quantity co-move perfectly, and firm-level quantity and revenue always decrease with the number of firms in the third markets.

Independent markets: $\beta = 0$ Next, we consider the case where $\beta = 0$. In this case, $\xi_i = 1$. Then,

$$\mathrm{d}\log N_i = -\frac{\mathrm{d}\log t_i}{\theta_i}, \quad \mathrm{d}\log M = -s_i\frac{\mathrm{d}\log t_i}{\theta_i}.$$

As suggested by the name, only the focal market is affected by the tariff shock since firms make orthogonal entry decisions for each market. Comparing to equation (20), the focal

entry intensities into all other market

market bears the full cost without any diversion into the third markets. The total number of entrants changes, only because the number of entrants into the focal market changes.

Common entry: $\beta \to 1$ In this case, entry is common across all markets up to some exogenous constants as in Melitz (2003). With exogenous exit, N_i is linear in M in the stationary equilibrium. Then, all markets witness the same change in the number of firms, which is given by the change in the number of total entrants. Formally,

$$\mathrm{d}\log N_i = \mathrm{d}\log M = -\,\mathbb{E}_{s\xi}\left[\frac{\mathrm{d}\log t}{\theta}\right]; \quad \forall i \in \{1,2,...,N\},$$

in which $\xi_i = \frac{\pi_i \eta_i \theta_i}{\pi_i - f_i}$. In this way, the common entry technology standard in the literature implies a strong diversion force such that while tariff shocks may be asymmetric across markets, their impact on entry is always fully diversified across markets. All markets bear the same reduction in competition to jointly buffer a unilateral tariff shock.

3.2.3 Entry interdependence

We now discuss the role of entry interdependence (β) in the transmission of shocks. Note that the change in the number of active firms in market i ($\mathrm{d}\log N_i$) is a *sufficient statistic* to measure how β influences the shock transmission. As before, we consider only one country ("j") increases the tariff. Simple calculation shows

$$\frac{\mathrm{d}\log N_i}{\mathrm{d}\log t_j} = -\left[(1-\xi_i)\tilde{s}_j + \xi_i \mathbb{1}\{i=j\}\right]\frac{1}{\theta_j}$$

where $\tilde{s}_j = \frac{s_j \xi_j}{\sum_i s_i \xi_i}$ and $\xi_i = \frac{v_i}{v_i + \tilde{\beta}}$, in which $v_i = \frac{\pi_i}{\pi_i - f_i}\eta_i \theta_i$ and $\tilde{\beta} = \beta/(1-\beta)$. Since $\tilde{\beta}$ is a monotonic transformation of β, we consider the cross derivative of $\mathrm{d}\log N_i$ with respect to $\mathrm{d}\log t_j$ and $\tilde{\beta}$:

$$\frac{\partial}{\partial \tilde{\beta}}\frac{\mathrm{d}\log N_i}{\mathrm{d}\log t_j} = -\left[(1-\xi_i)\,\tilde{s}_j\left[-\frac{\xi_j}{v_j} + \sum_i \tilde{s}_i \frac{\xi_i}{v_i}\right] - \frac{\xi_i^2}{v_i}\left(\mathbb{1}\{i=j\} - \tilde{s}_j\right)\right]\frac{1}{\theta_j}.$$

In general, the sign is ambiguous (i.e., not necessarily negative for the third markets), as an increase in β can either reduce or increase the importance of the focal market (i.e., $\tilde{s}_j$) in shock

transmissions.[14] Nevertheless, we show in below that a larger β leads to more transmission of shocks unambiguously in cases where markets are either independent initially or symmetric.

Independent markets Markets are independent when $\beta = 0$, which implies $\xi_i = 1$ and $\tilde{s}_j = s_j$. Then, we have

$$\frac{\partial}{\partial \tilde{\beta}} \frac{\mathrm{d} \log N_i}{\mathrm{d} \log t_j} = \frac{1}{v_i} \left(\mathbb{1}\{i = j\} - s_j \right) \frac{1}{\theta_j} \quad \begin{cases} < 0, & \text{if } i \neq j, \\ > 0, & \text{if } i = j. \end{cases}$$

Now changes in $\tilde{s}_j$ do not matter (to the first order) when we vary β from the initial equilibrium. A tariff increase in market j leads to a larger (smaller) drop of the number of firm in the third (focal) market, when β increases. As a result, the tariff shock is more diversified with a bigger β which implies more interdependence across countries. In terms of the real consumption Q, the focal market suffers less while all the third markets suffer more when β is bigger.

Symmetric markets We consider another scenario in which all markets are identical in the initial equilibrium, i.e., $\xi_i = \xi$, $v_i = v$, $\theta_i = \theta$, $\tilde{s}_j = \frac{1}{N+1}$. Then, we have

$$\frac{\partial}{\partial \tilde{\beta}} \frac{\mathrm{d} \log N_i}{\mathrm{d} \log t_j} = \frac{\xi^2}{v} \left(\mathbb{1}\{i = j\} - \frac{1}{N+1} \right) \frac{1}{\theta} \quad \begin{cases} < 0, & \text{if } i \neq j, \\ > 0, & \text{if } i = j. \end{cases}$$

With symmetric markets, there is no first order change in the influence of shock $\tilde{s}_j$ for all ξ_i from the initial equilibrium. Therefore, the diversification force becomes the only equilibrium force. The distributional effect of a market-specific tariff shock across market becomes more pronounced with a larger β and is maximized when β approaches to 1.

3.3 Discussion of the entry margin

The delocation effect has been used traditionally as evidence for IRS. While our framework also features industry-level IRS for generality, it is important to note that the entry margin

[14] When β is larger, markets are more interdependent but the influence of the shock $\tilde{s}_j$ can become smaller (the term of $-\frac{\xi_j}{v_j}$ in this equation). That is, even if all markets are more interdependent when β increases, the focal market can become relatively less important (among all markets) than before. For the focal market, the influence of shock $\tilde{s}_j$ can become *larger* (the term of $\sum_i \tilde{s}_i \frac{\xi_i}{v_i}$ in this equation) when β increases, which leads to an ambiguous sign for the focal market as well.

operates independently from the assumption og IRS. Based on Hopenhayn (1992), we can present an industry equilibrium model featuring firm entries into multiple markets, which assumes perfect competition and a decreasing returns to scale production function at the firm level. This alternative setup also generates qualitatively the same tariff responses in both the focal and the third markets as the ones derived above.[15] In this way, we show that the delocation effect does not necessarily imply the presence of IRS. Whereas, the entry margin alone can account for the delocation effect.

3.4 Heterogeneous firms

We extend our benchmark model into the environment of heterogeneous firms now for two purposes. First, we will use the heterogeneous firms model to implement our calibration. Second, we will show that our key qualitative results obtained the homogeneous firms case also hold in the heterogeneous firms case.

3.4.1 Equilibrium conditions

We follow the same strategy to solve the equilibrium with heterogeneous firms. Compared to the case with homogeneous firms, there is one more equilibrium condition that determines the exit cutoff on firm productivity. This is the selection channel popularized by models of heterogeneous firms. Specifically, the zero-profit condition can be written as

$$\left(\mu_i(z_i^*)-1\right)\frac{\tau_i}{z_i^*}q_i(z_i^*)=\left(1-\frac{1}{\mu_i(z_i^*)}\right)p_i(z_i^*)q_i(z_i^*)=f_i, \tag{21}$$

where z_i^* is the exit cutoff and firms are indexed by their productivity draws and their export destination. In addition, the markup rule is pinned down by the pricing rule:

$$\frac{t_i}{P_i}\mu\left(\frac{q_i(z)}{Q_i}\right)\frac{\tau_i}{z}=\Upsilon'\left(\frac{q_i(z)}{Q_i}\right). \tag{22}$$

Given the price aggregator P_i and the real consumption Q_i, we can solve for $q_i(z)$ (and thus $\mu_i(z)$ and $\pi_i(z)$) as functions of P_i and Q_i.

The second set of equilibrium conditions uses the definition of the Kimball preference and the balance budget (or income accounting) equation to solve for P_i and Q_i. Specifically, we

[15] The materials are available upon requests.

have

$$\frac{N_i}{1-G(z_i^*)}\int_{z_i^*}^{\infty}\Upsilon\left(\frac{q_i(z;P_i,Q_i)}{Q_i}\right)\mathrm{d}G(z)=1. \tag{23}$$

and

$$P_i=\bar{\delta}_i(P_i,Q_i)P_i^I=\bar{\delta}_i(P_i,Q_i)D_i(Q_i). \tag{24}$$

Together, they implicitly pin down P_i and Q_i (given N_i).

Third, the per-period variable profits can be expressed as a function of N_i, P_i and Q_i:

$$\bar{\pi}_i(N_i)=\frac{1}{\rho+\nu_i}\int_{z_i^*(N_i)}^{\infty}[\pi_i(z;N_i)-f_i]\,\mathrm{d}G(z), \tag{25}$$

as firm-level price and quantity depend on these three variables.

Finally, we have the two equilibrium conditions, the free entry condition and the balanced flow of firms, to determine the total number of entrants and the number of firms that export to each market $(M,\{N_i\})$:

$$\sum_{i=0}^{N}(h_i\bar{\pi}_i)^{\frac{1}{\beta}}=f_e^{\frac{1}{\beta}},$$
$$J_i[1-G(z_i^*)]M=\nu_iN_i.$$

There are $N+2$ equations for $N+2$ unknowns. Similar to the case of homogeneous firms, we can derive that

$$J_i=h_i\left(\frac{h_i\bar{\pi}_i}{f_e}\right)^{\frac{1-\beta}{\beta}}=\frac{\nu_iN_i}{M[1-G(z_i^*)]}\quad\Rightarrow\quad\bar{\pi}_i(N_i)=\left(\frac{\nu_iN_i}{h_iM[1-G(z_i^*)]}\right)^{\frac{\beta}{1-\beta}}\frac{f_e}{h_i}, \tag{26}$$

Combined with equation (25), the above equation provides us an implicit function N_i in terms of M. We define such mapping by $N_i=N_i(M)$. Inserting it back into the free entry equation, we can pin down the equilibrium mass of total entrants M as in the case of homogeneous firms:

$$\sum_{i=0}^{N}\left(h_i\bar{\pi}_i(N_i(M))\right)^{\frac{1}{\beta}}=f_e^{\frac{1}{\beta}},$$

3.4.2 Comparative Statics

We define $\lambda_i(z) = \frac{N_i t_i p_i(z) q_i(z)}{E_i}$, the sales density of firms with productivity z in country i. Then, $\mathbb{E}^i[\lambda_i] = 1$. We use $\mathbb{E}^i_x[y]$ to denote the x-weighted average of variable y:

$$\mathbb{E}^i_x[y] = \frac{\int_{z_i^*}^{\infty} x(z) y(z) \frac{g(z)}{1-G(z_i^*)} \mathrm{d}z}{\int_{z_i^*}^{\infty} x(z) \frac{g(z)}{1-G(z_i^*)} \mathrm{d}z}.$$

For example, a useful and easily verifiable identity is $\mathbb{E}^i_{\lambda_i}[\delta_i] = \bar{\delta}_i$. Additionally, we denote the number of potential entrants in market i by M_i^p, i.e., $M_i^p = J_i M$. With the balanced flow condition, $\mathrm{d}\log M_i^p = \mathrm{d}\log N + \varphi_i^* \mathrm{d}\log z_i^*$, in which $\varphi_i^* = \frac{z_i^* g(z_i^*)}{1-G(z_i^*)}$. In other words, φ_i^* is the elasticity of entrant share $1 - G(z_i^*)$ to threshold z_i^*, i.e., $\varphi_i^* = -\frac{\mathrm{d}\log[1-G(z_i^*)]}{\mathrm{d}\log z_i^*}$.

We discuss equilibrium changes in key variables in what follows. In the homogeneous firms case, the change in the number of firms (i.e., $\mathrm{d}\log N_i$) is a sufficient statistic that characterizes various changes in the equilibrium. Now, either $\mathrm{d}\log N_i$ or $\mathrm{d}\log z_i^*$ can be a sufficient statistic. We decide to focus on changes in z_i^* rather than N_i for cleaner expositions, which leads to the following proposition.

Proposition 4. *In response to a set of tariff shocks* $\mathrm{d}\log t_i$*, the following equations characterize changes in the equilibrium:*

$$\begin{aligned}
\mathrm{d}\log M &= -\frac{\sum_i s_i \xi_i^t \mathrm{d}\log t_i}{\sum_i s_i \xi_i^M} \\
\mathrm{d}\log z_i^* &= z_i^M \mathrm{d}\log M + z_i^t \mathrm{d}\log t_i \\
\mathrm{d}\log M_i^p &= (1 - \xi_i^M)\mathrm{d}\log M - \xi_i^t \mathrm{d}\log t_i,
\end{aligned}$$

where

$$z_i^M = -\frac{1}{\frac{1-\beta}{\beta}\bar{\pi}_i^z - (N_i^z + \varphi_i^*)}, \quad z_i^t = \frac{N_i^t - \frac{1-\beta}{\beta}\bar{\pi}_i^t}{\frac{1-\beta}{\beta}\bar{\pi}_i^z - (N_i^z + \varphi_i^*)},$$

$$\xi_i^M = \frac{\frac{1-\beta}{\beta}\bar{\pi}_i^z}{\frac{1-\beta}{\beta}\bar{\pi}_i^z - (N_i^z + \varphi_i^*)}, \quad \xi_i^t = -\frac{\frac{1-\beta}{\beta}\left[\bar{\pi}_i^z N_i^t - (\varphi_i^* + N_i^z)\bar{\pi}_i^t\right]}{\frac{1-\beta}{\beta}\bar{\pi}_i^z - (N_i^z + \varphi_i^*)}, \tag{27}$$

and s_i *is the expected profit share as in equation* (19)*. We define* $\bar{\pi}_i^z$*,* $\bar{\pi}_i^t$*,* N_i^z*,* N_i^t *in Appendix D.2.*

Compared to the formula in equation (19) in the homogeneous firms case, the weights ξ_i^M and ξ_i^t contain additional terms due to selection and its interaction with variable markups. We illustrate this difference using the nested CES demand. In such a case, the difference comes solely from the selection effect, and ξ_i^t and ξ_i^M take the following forms:

$$\xi_i^M = \frac{\frac{\hat{\pi}_i}{\hat{\pi}_i - f_i}\eta_i\theta_i}{\frac{\hat{\pi}_i}{\hat{\pi}_i - f_i}\eta_i\theta_i + \frac{\beta}{1-\beta}\left(1 + \frac{\eta_i\theta_i}{\sigma_i - 1}\lambda_i^*\varphi_i^*\right)}, \quad \text{and} \quad \xi_i^t = \xi_i^M \frac{1}{\theta_i},$$

where θ_i is defined in equation (18) and $\hat{\pi}_i \equiv \int_{z_i^*}^{\infty} \pi_i \frac{g(z)}{1-G(z_i^*)} \mathrm{d}z$ is the average profit. Relative to equation (19), the weight of entry interdependence, $\frac{\beta}{1-\beta}$, is augmented by the weight of selection cutoff $\lambda_i^*\varphi_i^*$. Intuitively, the impact on number of total entrants is smaller, when the productivity thresholds are more responsive to the tariff shock(s). In other words, the movement of thresholds absorbs the impact of the tariff shock(s) to some extent. Unfortunately, we are unable to prove the results that we have obtained in the homogeneous firms case in general. In order to obtain sharper predictions on the comparative statics, we introduce the following additional assumption.

Assumption 2. *We assume the following conditions hold:*

i) The strong second Marshall law of demand: $\rho_i \leq 1$, $\rho_i' \leq 0$;

ii) Excessive entry: $\bar{\delta}_i \leq \mathbb{E}_{\lambda_i}^i[\mu_i^{-1}]^{-1}$;

iii) Under-selection: $\bar{\delta}_i \geq \delta_i^*$.

iv) Firms' profit decreases on entry threshold z_i^*: $\frac{\mathrm{d}\log \pi_i}{\mathrm{d}\log z_i^*} < 0$.

Comparing with Assumption 1, the first assumption above presents a stronger version of Marshall's second law of demand: not only is there incomplete passthrough, the passthrough also decreases on firm productivity. The second and third assumptions come from Baqaee et al. (2023). The second assumption implies that consumer welfare improves, if resources used in firm entry are reallocated to production. Similarly, the third one indicates that consumer welfare increases, if the exit threshold goes up. The final assumption regulates the relationship between the exit threshold and firm-level profitability.[16] Mathematically, the above assumption assures that both ξ_i^M and ξ_i^t defined in equation (27) are strictly positive.

[16]This assumption is trivially true when demand is CES, as the profit ratio of two firms is proportional to the productivity ratio the two firms (e.g., z_i^* and z) and the firm on the cutoff makes the profit that equals the fixed operation cost. However, with variable markup, the reduction in profit from intensified competition (i.e., an increase in z_i^*) is mitigated by markup changes, due to changes in firm-level market shares. Therefore, this assumption is not trivially true in the general case.

The following proposition summarizes our comparative statics results in the case of heterogeneous firms:

Proposition 5. *Suppose that assumption 1 and 2 hold. Country j ("the focal market") raises its tariff, i.e., $\mathrm{d}\log t_j > 0$, while all other countries ("the third markets") do not change tariffs, i.e., $\mathrm{d}\log t_i = 0$ for $i \neq j$. Then, $\xi_i^t > 0$, $\xi_i^M > 0$, $z_i^t > 0$, $z_i^M > 0$. We have the following equilibrium changes:*

i) In all markets:

$$\mathrm{d}\log M < 0, \quad \mathrm{d}\log Q_i < 0, \quad \mathrm{d}\log M_i^p < 0, \quad \mathrm{d}\log p_i > 0, \quad \mathrm{d}\log \tilde{Q}_i < 0, \quad \log R_i < 0$$

ii) In the third market $i \neq j$:

$$\mathrm{d}\log z_i^* < 0, \quad \mathrm{d}\log p_i^m > 0$$

iii) In the focal market:

$$\mathrm{d}\log z_j^* = \underbrace{z_j^M}_{>0}\Bigg[\underbrace{\left(\frac{s_j\xi_j^M}{\sum_i s_i\xi_i^M} - 1\right)}_{<0}\underbrace{N_j^t}_{<0} + \underbrace{\bar{\pi}_j^t}_{<0}\underbrace{\left(\frac{1-\beta}{\beta} - \frac{s_j\xi_j^M}{\sum_i s_i\xi_i^M}\frac{\varphi_j^* + N_j^z}{\bar{\pi}_j^z}\right)}_{>0}\Bigg]\mathrm{d}\log t_j, \tag{28}$$

where we define $\bar{\pi}_i^z$, $\bar{\pi}_i^t$, N_i^z, N_i^t in Appendix D.2.

iv) Across firms in the same market:

$$\frac{\partial\,\mathrm{d}\log\mu_i}{\partial\log z} > 0, \quad \frac{\partial\,\mathrm{d}\log\pi_i}{\partial\log z} < 0, \quad \frac{\partial\,\mathrm{d}\log r_i}{\partial\log z} < 0, \quad \frac{\partial\,\mathrm{d}\log q_i}{\partial\log z} < 0$$

Proposition 5 shows that most results obtained from the case of homogeneous firms continue to hold in the case of heterogeneous firms under Assumption 2. In particular, the number of total entrants and potential entrants to each market all decrease with the tariff. Notably, we are *unable* to show that the number of actual entrants and thus the number of active firms in each market go down with the tariff due to the selection effect (i.e., the decreasing cutoff)

Turing to the selection effect, we show that threshold z_i^* must decrease in third markets while change ambiguously in the focal market, as shown in equation (28). In the third market,

market competition becomes softer, as all surviving firms have increase profits thanks to fewer entrants and higher firm-level market shares (fixing the exit cutoff). As a result, the exit cutoff decreases. In the focal market, surviving firms increase prices and markups, and more productive ones increase markups disproportionately more. Thus, this (anti-)competitive effect favor unproductive firms disproportionately more and reduces the exit cutoff in the focal market. In the focal market, however, there is a direct negative impact on firm profit coming from the tariff shock. As a result, the marginal firm can either make more or less profit after the tariff shock, depending on whether the (anti-)competitive effect dominates the direct tariff effect. As a result, the ambiguity arises in the focal market. When the demand is nested CES, the exit cutoff increases in the focal market unambiguously, as the the (anti-)competitive effect is gone.[17]

Proposition 5 also shows that the consumption of composite good Q_i decrease in all markets. The loss in the third market is obvious with less varieties, lower aggregate productivity, and higher markup. In the focal market, the result suggests that the likely increase in the exit threshold cannot fully offset the negative effect on varieties and markups. A similar logic applies to the result associated with the total quantity $\tilde{Q}_i$. While the number of actual entrants may increase, they sell much less to achieve gain markups.

With heterogeneous firms, the model makes no definite predictions on changes in firm-level profit, revenue, or quantity. Nevertheless, two unambiguous predictions on firm-level changes are derived. First, all firms raise their markups and prices, in both focal and third markets. This is true due to reduced firm entry which results in increasing market power of incumbent firms. In the third markets, as the exit cutoff always goes down and thus firms are less productive on average (after the tariff shock). Thus, the increase the unit price is larger than the average increase of continuing firm-level price increases due to the compositional effect. In the focal market, however, entrants can be more productive than incumbents after the tariff shock. As a result, the unit price may increase less than the average increase of continuing firm-level prices. Second, conditional on survival, low productivity firms gain relative to high productivity firms in profit, sales, and quantity. As a result, less productive firms gain relative to more productive firms, conditioning on survival. Thanks to the strong second Marshall law of demand, (surviving) more productive firms increase their prices and markups more (in both markets) after the tariff shock. In other words, *firm-specific*

[17]There are two cases where the exit cutoff is likely to increase in the focal market: 1) the focal market has a large weight $\frac{s_j \xi_j^M}{\sum_i s_i \xi_i^M}$, which may due to a large profit share s_j; 2) markets are very independent: β is close to zero.

exposure to the negative tariff shock is larger for more productive firms. In total, (surviving) unproductive firms benefit more (or lose less) as they increase markups/prices less and thus benefit from the anti-competitive effect more after the tariff shock. This intuition provides another explanation for why the exit cutoff can go down in the focal market: the marginal firms can have increasing profits while the more productive ones have shrinking profits (after the tariff shock in the focal market).

CES demand We relegate a complete equilibrium analysis with CES demand to Appendix D.2. Evidently, CES demand satisfies Assumption 2. Comparing to the general case, this special case yields the definitive prediction that $\mathrm{d}\log z_j^* > 0$ in the focal market. This assumption also simplifies the comparative statics results. However, the advantages in analytical tractability is not enough to warrant CES demand an ideal modeling choice for the quantification. With constant markup, it restricts any firm-level price adjustment and thus implies that changes in quantity and sales are always equal. However, our empirical results show significant changes in firm-level ass well as aggregate-level price changes.

Entry Interdependence As in the homogeneous case, it suffices to study the impact of β on equilibrium thresholds z_i^*, which are sufficient statistics for measuring changes in other endogenous variables. For the same reason as in the homogeneous case, we focus on the symmetric case and suppress the country index. The following analysis confirms that the implication of entry interdependence obtained in the homogeneous case extends into the environment of heterogeneous firms.

First, $\mathrm{d}\log M$ is independent of β, as

$$\mathrm{d}\log M = -\frac{1}{N+1}\frac{\xi^t}{\xi^M}\mathrm{d}\log t_j = -\frac{1}{N+1}\left((\varphi^* + N^z)\frac{\bar{\pi}^t}{\bar{\pi}^z} - N^t\right)\mathrm{d}\log t_j < 0,$$

in which $\bar{\pi}^z < 0$, $\bar{\pi}^t < 0$, $N^z + \varphi^* > 0$, $N^t < 0$, and they are independent of β. In third markets, we have

$$\mathrm{d}\log z_i^* = z^M \mathrm{d}\log M = \frac{1}{(N^z + \varphi^*) - \frac{1-\beta}{\beta}\bar{\pi}^z}\mathrm{d}\log M, \tag{29}$$

where z^M is the exit cutoff in the initial equilibrium. Thus, $\mathrm{d}\log z_i^*$ decreases with β. We show in Appendix D.2 that $\mathrm{d}\log Q_i$ increases with $\mathrm{d}\log z_i^*$ in both the focal and third markets. Therefore, there is a larger reduction in the consumption of the sectoral composites in the

third markets when β increases. For the focal market,

$$\mathrm{d}\log z_j^* = \frac{\frac{N}{N+1}N^t\frac{1}{N+1}\frac{\bar{\pi}^t}{\bar{\pi}^z}(\varphi^* + N^z) - \bar{\pi}^t\frac{1-\beta}{\beta}}{\frac{1-\beta}{\beta}\bar{\pi}^z - (N^z + \varphi^*)}\mathrm{d}\log t_j.$$

We show in Appendix D.2 that $\frac{\partial \mathrm{d}\log z_j^*}{\partial \beta} > 0$ and $\mathrm{d}\log Q_j$ increases with $\mathrm{d}\log z_j^*$. Therefore, the impact of a bigger β on the consumption of the sectoral composites in the focal market is always positive, despite that $\mathrm{d}\log z_j^*$ can be either positive or negative. Namely, the focal market suffers less from the tariff shock, when markets are more interdependent via firms' entry.

4 Model Parameterization

4.1 Estimating Pass-through

We follow Amiti et al. (2019) and write the log change in firm f's marginal cost from $t-1$ to t as

$$\Delta mc_{ft} = \underbrace{\varphi_{ft-1}\sum_h \omega_{fht-1}^{imp}\Delta p_{fht}^{imp}}_{\Delta mc_{ft}^*} + (1-\varphi_{ft-1})\Delta w_{ft} + (p_{ft-1}^{imp} - w_{ft-1})\Delta\varphi_{ft} - \Delta\log z_{ft},$$

where the first term, Δmc_{ft}^*, is the weighted change in the prices of imported inputs, where the weights depend on the lagged import shares ω_{fht-1}^{imp} of each product h and the overall share of imported inputs φ_{ft-1}. If the firm does not import any foreign inputs, both ω_{fht-1}^{imp} and $\Delta mc_{ft}^* = 0$ equal zero. The other terms represent the change in workers' wages, the change in foreign input shares, and the change in productivity. Amiti et al. (2019) observe both Δmc_{ft} and $\Delta mc_{ft}^* = 0$, so they use the latter as an IV for the former. We do not observe the former. However, in the Belgium data, regressing Δmc_{ft} on Δmc_{ft}^* yields a coefficient of 0.97. Therefore, we believe that using the observed change in marginal costs, Δmc_{ft}^*, will not introduce much biases.

To estimate the pass-through, we regress the change in export prices in destination i and HS6 product h, Δp_{fiht}, on Δmc_{ft}^*. We interact the latter with dummies indicating the exporter's lagged market share in the same market, $\lambda_{f,i,h,t-1}$, in the previous year. We group observations based on their market shares and use the following six intervals, (0,0.01], (0.01,0.03], (0.03,0.1], (0.1,0.3], (0.3,0.5], (0.5,1], which we further denote with

$\Lambda^k, k = 1, \ldots, 6$. We aim to estimate the following equation:

$$\Delta p_{fiht} = \sum_k \rho^k \mathbb{1}\left(\lambda_{f,i,h,t-1} \in \Lambda^k\right) \times \Delta mc^*_{ft} + \delta_{hs4,d} + \delta_t + \delta_k \varepsilon_{fiht},$$

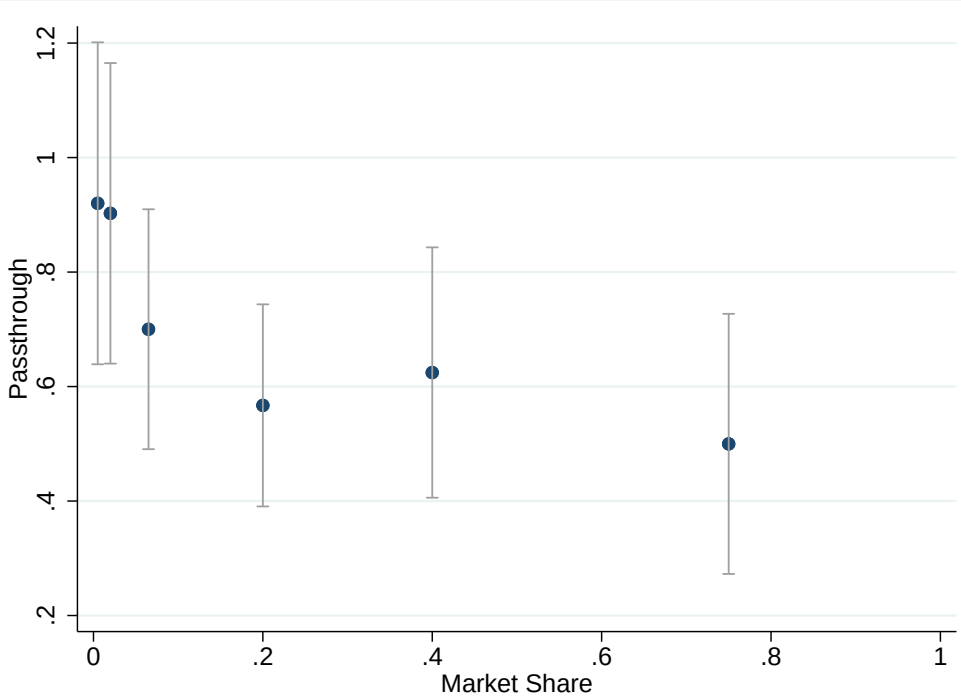
where we include HS4-destination, time and market share group fixed effects to control for potential confounding factors. We weigh the observations by their lagged market shares, so each group-specific pass-through is a weighted average of the pass-throughs of the firms in that group. Standard errors are clustered at the firm level.

One potential endogeneity problem is that the changes in input prices also reflect changes in input quality, and they may be correlated with output quality and prices. To address this issue, we introduce an IV for Δmc^*_{ft} using export prices to the Rest of the World (RoW, $\neq$ CHN) as follows:

$$IV_{ft} \equiv \varphi_{ft-1} \sum_{h,o} \omega^{imp}_{fhot-1} \Delta p^{d=RoW}_{fhot},$$

where o denotes a source country and d denotes the destination country. We then interact IV_{ft} with market share dummies $\mathbb{1}\left(\lambda_{f,i,h,t-1} \in \Lambda^k\right)$ and use the interaction terms as IVs for the corresponding pass-through estimates. Figure 6 shows the estimates of the pass-throughs by the six market share groups. Pass-throughs drop from close to one for the smallest first to around 0.5 for the largest firms, consistent with the findings of Amiti et al. (2019).

Figure 6: Estimates of Marginal Cost Pass-throughs by Firm Market Shares



Notes: The figure shows the estimates of the pass-throughs by firm market shares. The x-axis shows the market share group, and the y-axis shows the pass-through estimates. The dots represent the point estimates, and the lines represent the 95% confidence intervals.

4.2 Estimating Demand Curves

In this section, we use passthrough estimates from the previous section and follow Baqaee and Farhi (2021) to estimate the demand curves non-parametrically. Similar to their application, we have two cumulative sales distributions: (1) the full sample of all exporters, denoted by $\Lambda(\theta)$ and (2) the NBS exporters, denoted by $\Lambda^N(\theta)$ where θ is the pertentile rank of the firm's productivity, and $\Lambda(\theta)$ and $\Lambda^N(\theta)$ denote the sales share of firms with the bottom $100 \times \theta$ percent productivities in the two datasets. To better infer the corresponding pass-thoughs later, we group observations in each market (destination-HS6-product) based on their market shares and use twelve market share cutoffs, 0.001, 0.005, 0.01, 0.02, 0.03, 0.05, 0.1, 0.2, 0.3, 0.5, 0.8 and 1. Note that these market shares are calculated using the common denominator, the total exports in a market in the full sample, so they are consistent indicators of firm size across the two samples. We calculate the cumulative observation and sales shares below each cutoff. These two shares correspond to θ_i and $\Lambda(\theta_i)$ in the full sample,

and θ_i^N and $\Lambda(\theta_i^N)$ in the NBS-matched data.

We fit a smooth curve to third-order polynomial to $\Lambda(\theta)$ and $\Lambda^N(\theta)$ of the form $e^{c_0+c_1\theta+c_2\theta^{c_3}}$. We then use the fitted curves to estimate the density functions $\lambda(\theta) \equiv \mathrm{d}\Lambda(x)/\mathrm{d}x$ and $\lambda^N(\theta) \equiv \mathrm{d}\Lambda^N(x)/\mathrm{d}x$, respectively. The results are shown in Panel (a) of Figure 7.

We have estimated market-share-weighted pass-throughs for different segments in the NBS data:

$$\alpha^N(\theta_1, \theta_2) = \frac{\int_{\theta_1}^{\theta_2} \lambda^N(x)\rho(x)\mathrm{d}x}{\int_{\theta_1}^{\theta_2} \lambda^N(x)\mathrm{d}x}.$$

We follow Baqaee and Farhi (2021) and calculate the cumulative pass-through $\alpha^N(0, \theta)$. Given that we know $\alpha^N(0, \theta_1)$ and $\alpha^N(\theta_1, \theta_2)$ for two cutoffs θ_1, θ_2, we can compute the sales share weighted average as

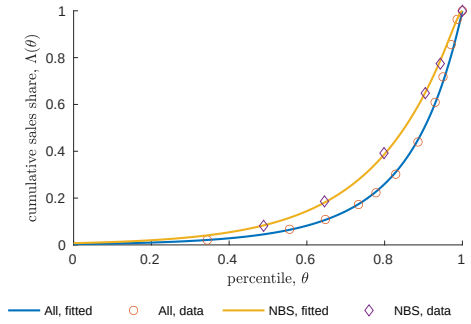
$$\alpha^N(0, \theta_2) = \frac{\alpha^N(0, \theta_1) \times \int_0^{\theta_1} \lambda^N(x)\mathrm{d}x + \alpha^N(\theta_1, \theta_2) \times \int_{\theta_1}^{\theta_2} \lambda^N(x)\mathrm{d}x}{\int_0^{\theta_1} \lambda^N(x)\mathrm{d}x + \int_{\theta_1}^{\theta_2} \lambda^N(x)\mathrm{d}x}.$$

Having obtained $\alpha^N(0, \theta)$ for all the cutoffs, we fit a spline function to $\alpha^N(0, \theta)$, which is shown in Panel (b) of Figure 7. We then use the relationship

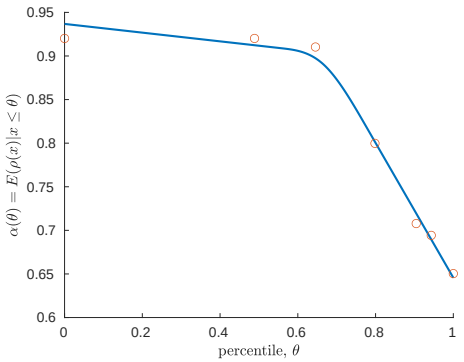
$$\rho^N(\theta) = \frac{\left(\int_0^{\theta} \lambda^N(x)dx\right)}{\lambda^N(\theta)} \frac{d\alpha^N(0, \theta)}{d\theta} + \alpha^N(0, \theta),$$

and recover the pass-throughs for all values of θ for exporters showing up in the NBS data. The result is shown in Panel (c) of Figure 7.

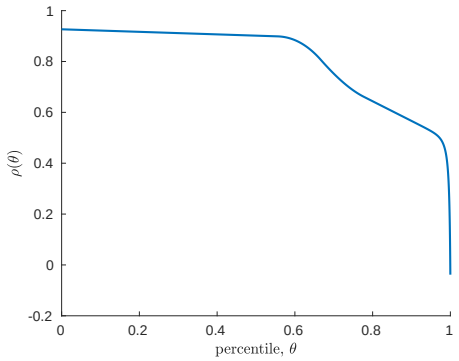
Figure 7: Fitting $\lambda(\theta), \rho(\theta)$



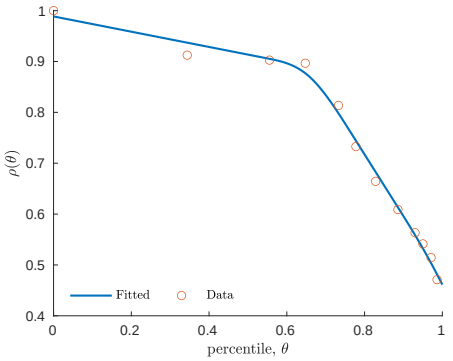
(a) Fitting $\Lambda(\theta), \Lambda^N(\theta)$



(b) Fitting $\alpha(0, \theta)$ in NBS



(c) Implied $\rho(\theta)$ in NBS



(d) Fitting $\rho(\theta)$ for all firms

Notes: The figure shows the fitted curves for the cumulative market shares, cumulative average pass-throughs and firm-specific pass-throughs. The x-axis shows the percentile cutoff θ, and the y-axis shows the corresponding curves.

Next, we combine $\rho^N(\theta)$ together with the sales density from the full Customs data to recover $\rho(\theta)$ for all firms. Given the sampled points θ_i and θ_i^N earlier, we impose $\rho(\theta_i) = \rho^N(\theta_i^N)$ for all i because firms at the $100\theta_i$-th percentile in the full sample are constructed to have similar firm sizes as those at the $100\theta_i^N$-th percentile in the NBS sample. We then fit a smooth spline to $\{\theta_i, \rho(\theta_i)\}_i$ from [0,1] assuming that the pass-through for the smallest firm is 1. The fitted line $\rho(\theta)$ is illustrated in Panel (d) of Figure 7.

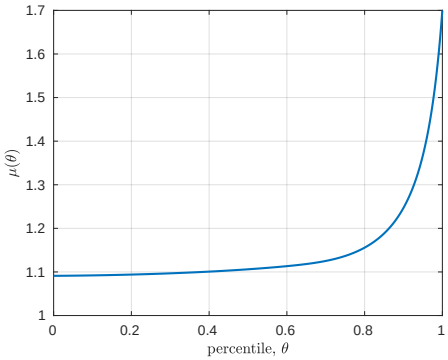
Equipped with $\rho(\theta)$, we are ready to solve the markups μ_θ, firm productivities A_θ and demand parameters B_θ. Note that we can only identify the product of $A_\theta B_\theta$ and refer to it

as firm productivity. We solve the following system of differential equations:

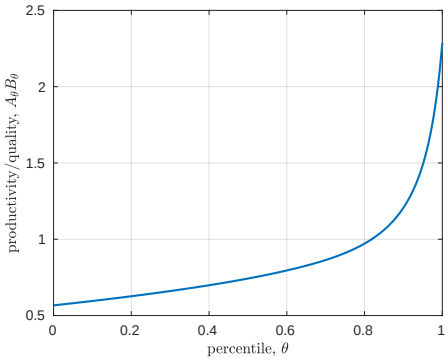
$$\begin{aligned}
\frac{\mathrm{d}\log\mu_\theta}{\mathrm{d}\theta} &= (\mu_\theta - 1)\,\frac{1-\rho_\theta}{\rho_\theta}\frac{\mathrm{d}\log\lambda_\theta}{\mathrm{d}\theta},\\
\frac{\mathrm{d}\log(A_\theta B_\theta)}{\mathrm{d}\theta} &= \frac{\mu_\theta - 1}{\rho_\theta}\frac{\mathrm{d}\log\lambda_\theta}{\mathrm{d}\theta},\\
\frac{\mathrm{d}\log\delta_\theta}{\mathrm{d}\theta} &= \frac{\mu_\theta - \delta_\theta}{\delta_\theta}\frac{\mathrm{d}\log\lambda_\theta}{\mathrm{d}\theta},
\end{aligned}$$

where we need three boundary conditions for the three curves. We set the markup of the largest firm, $\mu_1 = 1.7$, normalize $A_0 B_0 = 1$ and set the consumer surplus ratio of the smallest firm, $\delta_0 = 1.5$. The solutions are presented in Panels (a) to (c) of Figure 8, where we have further renormalized $A_\theta B_\theta$ such that $p_{\theta_0} = 1$ for θ_0 such that μ_{θ_0} coincides with the average markup. The implied average markup is 1.27 and the implied average consumer surplus ratio is 1.18. In addition to these three curves, we also plot the implied prices $p_\theta \equiv \frac{\mu_\theta}{A_\theta B_\theta}$ in Panel (d) and the implied relative quantities q_θ/Q in Panel (e).

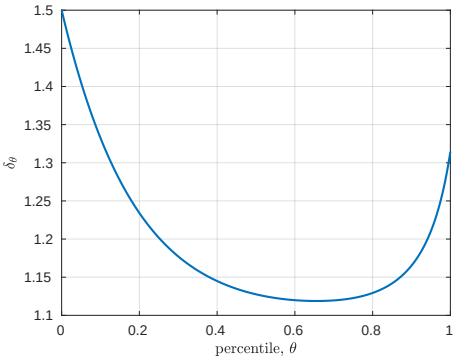
Figure 8: Other curves wrt θ



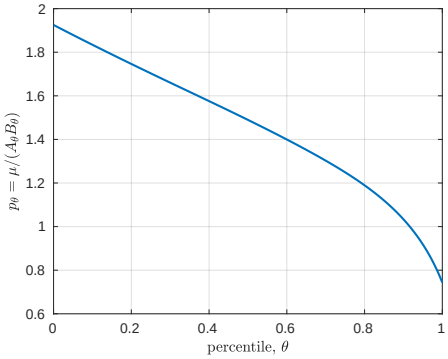
(a) μ_θ, targeting $\mu_1 = 1.7$



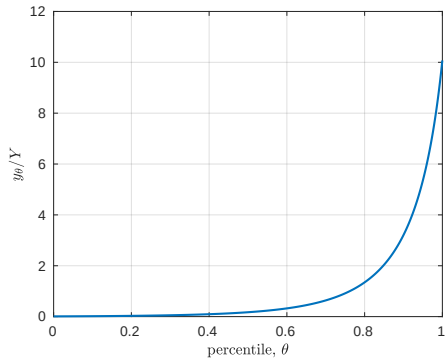
(b) $A_\theta B_\theta$



(c) δ_θ, initial condition $\delta_0 = 1.5$



(d) $p_\theta \equiv \frac{\mu_\theta}{A_\theta B_\theta}$, normalized to 1 for θ_0 s.t. $\mu(\theta_0) = \bar{\mu}$



(e) $q(\theta)/Q$, or $y(\theta)/Y$

Notes: The figure shows the solutions to the differential equations for the demand system. The x-axis shows the percentile cutoff θ, and the y-axis shows the corresponding values of the curves.

4.3 Other model parameters

We now provide a rough discussion on how to calibrate the remaining parameters of the model in the homogeneous-firm, symmetric-market case. Three parameters are left to be calibrated, the relative quantity q/Q, the elasticity of substitution η and the parameter that governs market interdependence β. In the homogeneous-firm case, we can back out these three parameters using three moments sequentially. First, consider the impact of a change in the number of firms on the price of the firms in a typical third market, i. Simplifying the results in Section 3, we obtain

$$\mathrm{d}\log p_i/\mathrm{d}\log N_i = -\left(\frac{1}{\rho}-1\right)\frac{\delta}{\sigma}.$$

In our generalized Kimball demand, ρ, δ and σ are functions of q/Q, as we have illustrated in Figures 7 and 8. We can then solve for q/Q using the observed price change and the observed change in the number of firms. Intuitively, a larger q/Q implies that the demand curve is less elastic, and the price is more (less) responsive to a change in the number of firms (an increase in the marginal cost).

Second, consider the impact of a change in the number of firms on the export quantities in a typical third market, i. Simplifying the results in Section 3, we obtain

$$\mathrm{d}\log \tilde{Q}_i \Big/ \mathrm{d}\log N_i = d\log \bar{q}_i = \delta\left(\frac{1}{\rho}-1\right) - \eta\theta + 1,$$

where θ is defined in equation (18) and is a function of q/Q. This equation pins down the (log) change in the average firm-level export quantity. At this step, all parameters on the right hand side are known except for η. We can find η to match the observed change in the export quantities relative to the change of the number of firms. Note that $\eta\theta$ *decreases* with η (as θ is a function of η). A larger η implies that the demand for the composite of Chinese exports (in each market) is more elastic and thus the transmission of the export drop is stronger after the tariff shock. As a result, the drop in the average firm-level export quantity ($d\log \bar{q}_i$) also becomes larger, which absorbs a part of the larger drop in the aggregated export quantity.

Finally, consider the change in the number of firms in a third market i relative to that in the focal market j, which can be considered as the "transmission" of shocks across markets.

Simplifying the results in Section 3, we obtain

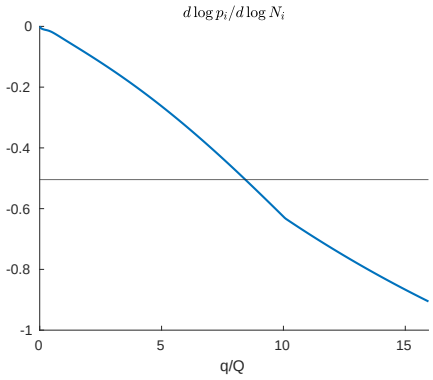
$$\mathrm{d}\log N_i/\mathrm{d}\log N_j = \frac{s_j}{s_j + \eta\theta\frac{1-\beta}{\beta}} \leq 1,$$

where s_j denotes the focal market's share in China's global exports of the investigated product. We measure s_j in the data, and the only remaining parameter is β, which we choose to match the observed $\mathrm{d}\log N_i/\mathrm{d}\log N_j$. Intuitively, the more complementary the markets are (i.e., a larger β), the more co-movements at the extensive margin (i.e., a higher $\mathrm{d}\log N_i/\mathrm{d}\log N_j$). In fact, the (log) reduction in the number of exporting firms is the same across all markets when $\beta = 1$.

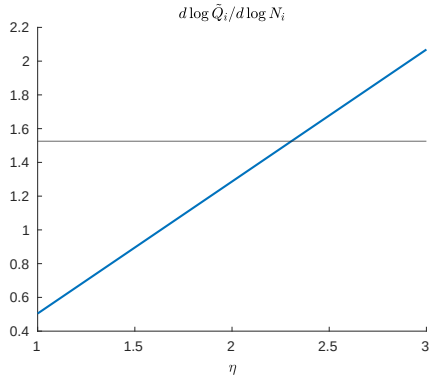
Table 4: Calibration of $x \equiv q/Q, \eta, \beta$

Moment	Expression	Target	Related Parameter	Value
$\mathrm{d}\log p_i/\mathrm{d}\log N_i$	$-\left(\frac{1}{\rho}-1\right)\frac{\delta}{\sigma}$	$\frac{0.1}{-0.2} = -0.5$	$x \equiv \frac{q}{Q}$	8.38
$\mathrm{d}\log \tilde{Q}_i \Big/ \mathrm{d}\log N_i$	$\delta\left(\frac{1}{\rho}-1\right) - \eta\theta + 1$	$\frac{-0.3}{-0.2} = 1.5$	η	2.29
$\mathrm{d}\log N_i/\mathrm{d}\log N_j$	$\frac{s_j}{s_j+\eta\theta\frac{1-\beta}{\beta}}$	$\frac{-0.2}{-0.73} = 0.27$	β	0.62

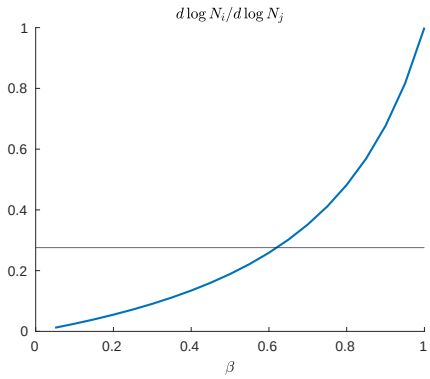
Figure 9: Identification of $x = q/Q, \eta, \beta$



(a) Identification of $x \equiv q/Q$



(b) Identification of η



(c) Identification of β

Notes: The figure shows how the targeted moments change as we vary the model parameters.

5 The global impact of AD Tariffs: TBA

We plan to study how β affects the transmission of the tariff shock across markets quantitatively. Moreover, we plan to calibrate β for different sets of countries/periods.

We also plan to study how the inclusion of the variable markups affects the responses of export values/quantities with respect to the tariff shock in various markets.

6 Conclusions

TBA

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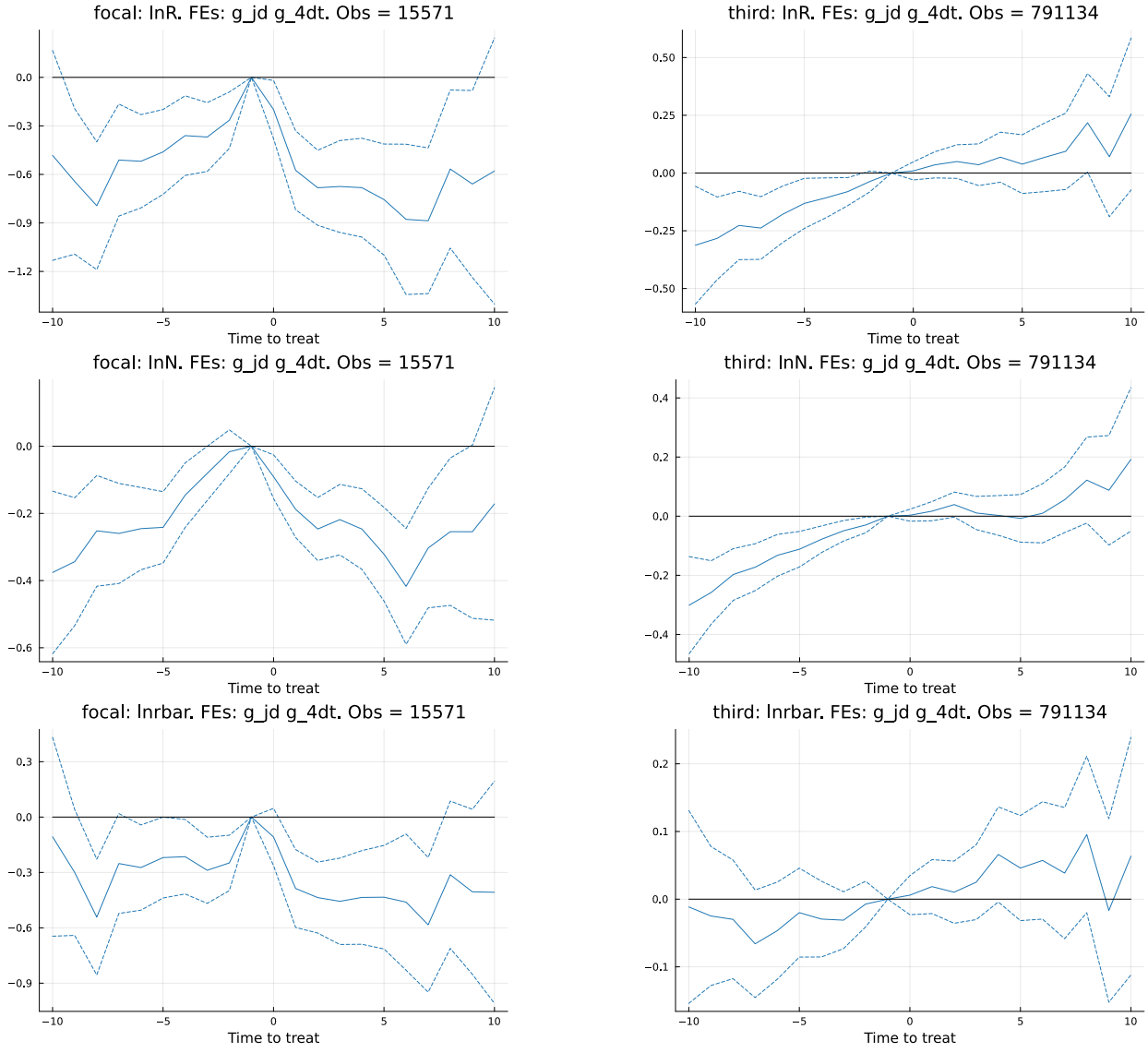
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A Additional Empirical Results

A.1 Market-level Analysis

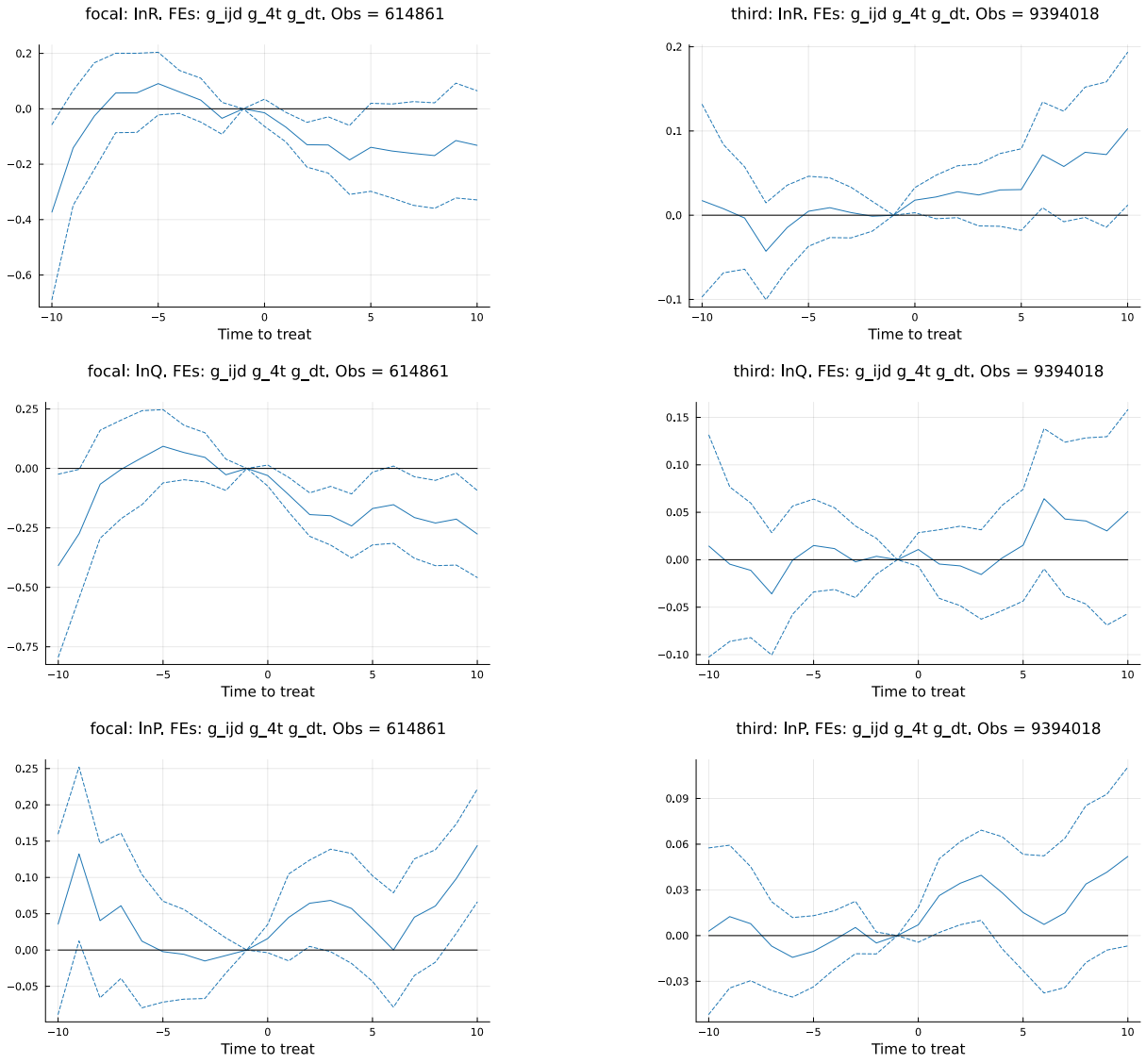
Figure 10: Effect of AD investigation at the market level: number of firms and average revenue.

Notes: All specifications include country-HS6-product and country-HS4-product-year fixed effects. Standard errors are clustered at the HS6 product level.



A.2 Firm-level Analysis

Figure 11: Effect of AD investigation at the firm-country-product-year level.



B Derivations

Derivation of demand function equation (7) Households solve the following expenditure minimization problem holding Q:

$$\begin{aligned} E = \min_{q_\omega} \quad & \int_\Omega p_\omega q_\omega \mathrm{d}\omega \\ \text{s.t.} \quad & \int_\Omega \Upsilon\left(\frac{q_\omega}{Q}\right) \mathrm{d}\omega = 1. \end{aligned}$$

Let λ be the Lagrange multiplier of the utility constraint. The first order condition is then

$$p_w = \lambda \Upsilon'\left(\frac{q_w}{Q}\right)\frac{1}{Q} \Rightarrow \frac{p_w}{P} = \Upsilon'\left(\frac{q_w}{Q}\right),$$

in which P is defined such that $PQ \equiv \lambda$. Multiplying both sides with $\frac{q_w}{Q}$, and summing them over ω, we obtain that

$$\frac{1}{PQ}\int_\Omega p_\omega q_\omega \mathrm{d}\omega = \int_\Omega \Upsilon'\left(\frac{q_\omega}{Q}\right)\frac{q_w}{Q}\mathrm{d}\omega \equiv \frac{1}{\bar{\delta}} \Rightarrow \lambda = \bar{\delta}E.$$

C Propagation and aggregation equations with homogeneous firms

C.1 Equilibrium conditions

We expand the equilibrium equations presented in Section 3.2 to the first order in the shocks.

Welfare

$$\delta_i \mathrm{d}\log N_i + \mathrm{d}\log\frac{q_i}{Q_i} = 0 \tag{30}$$

Budget balance

$$\begin{aligned} \mathrm{d}\log P_i^I =& \mathrm{d}\log N_i + \mathrm{d}\log t_i + \mathrm{d}\log\mu_i + \mathrm{d}\log\frac{\tau_i}{z} + \mathrm{d}\log\frac{q_i}{Q_i} \\ \Rightarrow \quad -\frac{1}{\eta_i}\mathrm{d}\log Q_i =& \mathrm{d}\log N_i + \mathrm{d}\log t_i + \mathrm{d}\log\frac{\tau_i}{z} + \left[\left(\frac{1}{\rho_i}-1\right)\frac{1}{\sigma_i}+1\right]\mathrm{d}\log\frac{q_i}{Q_i} \end{aligned} \tag{31}$$

Markup

$$\mathrm{d}\log\mu = \left(\frac{1}{\rho_i} - 1\right)\frac{1}{\sigma_i}\mathrm{d}\log\frac{q_i}{Q_i}, \tag{32}$$

in which

$$\rho_i = -\frac{\partial \log p_i}{\partial \log z} = \frac{1}{1 + \frac{\mu'\left(\frac{q_i}{Q_i}\right)\frac{q_i}{Q_i}}{\mu\left(\frac{q_i}{Q_i}\right)}\sigma_i}.$$

Variable profit

$$\mathrm{d}\log\pi_i = \frac{1}{\mu_i - 1}\mathrm{d}\log\mu_i + \mathrm{d}\log P_i^I + \mathrm{d}\log Q_i - \mathrm{d}\log N_i - \mathrm{d}\log t_i \tag{33}$$

Balanced flow

$$\frac{\pi_i \mathrm{d}\log\pi_i - f_i \mathrm{d}\log f_i}{\pi_i - f_i} = \mathrm{d}\log(\rho + \nu_i) + \frac{\beta}{1-\beta}\left[\mathrm{d}\log\nu_i + \mathrm{d}\log N_i - \mathrm{d}\log h_i - \mathrm{d}\log M\right] + \mathrm{d}\log f_e - \mathrm{d}\log h_i \tag{34}$$

Free entry

$$\frac{1}{1-\beta}\sum_{i=0}^{N}\frac{\left(\frac{\nu_i N_i}{h_i M}\right)^{\frac{1}{1-\beta}}}{\sum_k\left(\frac{\nu_k N_k}{h_k M}\right)^{\frac{1}{1-\beta}}}\left[\mathrm{d}\log N_i + \mathrm{d}\log\nu_i - \mathrm{d}\log h_i - \mathrm{d}\log M\right] = 0$$

$$\Rightarrow \quad \sum_{i=0}^{N} s_i\left[\mathrm{d}\log N_i + \mathrm{d}\log\nu_i - \mathrm{d}\log h_i\right] = \mathrm{d}\log M, \tag{35}$$

in which $s_i = \frac{J_i\bar{\pi}_i}{\sum_k J_k\bar{\pi}_k} = \frac{\left(\frac{\nu_i N_i}{h_i M}\right)^{\frac{1}{1-\beta}}}{\sum_k\left(\frac{\nu_k N_k}{h_k M}\right)^{\frac{1}{1-\beta}}}$.

C.2 Proof of Proposition 2, 3 and Corollary 1

Changes in M and N_i We consider only $\mathrm{d}\log t_i$ and set $\mathrm{d}\log\frac{\tau_i}{z} = \mathrm{d}\log\nu_i = \mathrm{d}\log h_i = \mathrm{d}\log f_i = \mathrm{d}\log f_e = 0$. Then, we obtain the following system of equations

$$\begin{aligned}
\delta_i \mathrm{d}\log N_i + \mathrm{d}\log\frac{q_i}{Q_i} &= 0 \\
-\frac{1}{\eta_i}\mathrm{d}\log Q_i = \mathrm{d}\log N_i + \mathrm{d}\log t_i + \left[\left(\frac{1}{\rho_i}-1\right)\frac{1}{\sigma_i}+1\right]\mathrm{d}\log\frac{q_i}{Q_i} \\
\mathrm{d}\log\pi_i = \frac{1}{\rho_i}\mathrm{d}\log\frac{q_i}{Q_i} + \mathrm{d}\log Q_i \\
\frac{\pi_i}{\pi_i - f_i}\mathrm{d}\log\pi_i = \frac{\beta}{1-\beta}\left[\mathrm{d}\log N_i - \mathrm{d}\log M\right] \\
\sum_{i=0}^{N} s_i \mathrm{d}\log N_i = \mathrm{d}\log M
\end{aligned}$$

Combining the first three equations, we obtain that

$$\mathrm{d}\log\pi_i = -\eta_i\theta_i \mathrm{d}\log N_i - \eta_i \mathrm{d}\log t_i,$$

in which

$$\theta_i = \underbrace{\left(\frac{1}{\eta_i}-\frac{1}{\sigma_i}\right)\left(\frac{1}{\rho_i}-1\right)\delta_i}_{\geq 0} + \underbrace{1-\frac{\eta_i-1}{\eta_i}\delta_i}_{>0} > 0.$$

The first bracket is non-negative since $\sigma_i > \eta_i$ and $\rho_i \leq 1$. The second bracket is positive since $\delta_i < \frac{\eta_i}{\eta_i - 1}$.

Then, we obtain that

$$\begin{aligned}
&\frac{\pi_i}{\pi_i - f_i}\left[-\eta_i\theta_i \mathrm{d}\log N_i - \eta_i \log t_i\right] = \frac{\beta}{1-\beta}\left[\mathrm{d}\log N_i - \mathrm{d}\log M\right] \\
\Rightarrow\ &\mathrm{d}\log N_i = \frac{\frac{\beta}{1-\beta}}{\frac{\beta}{1-\beta} + \frac{\pi_i}{\pi_i - f_i}\eta_i\theta_i}\mathrm{d}\log M - \frac{\eta_i\frac{\pi_i}{\pi_i - f_i}}{\frac{\beta}{1-\beta} + \frac{\pi_i}{\pi_i - f_i}\eta_i\theta_i}\mathrm{d}\log t_i
\end{aligned}$$

Using the free entry condition,

$$\mathrm{d}\log N_i - \mathrm{d}\log M = -\xi_i \mathrm{d}\log M - \xi_i\frac{\mathrm{d}\log t_i}{\theta_i},$$

where

$$\xi_i = \frac{\frac{\pi_i}{\pi_i - f_i}\eta_i\theta_i}{\frac{\beta}{1-\beta} + \frac{\pi_i}{\pi_i - f_i}\eta_i\theta_i} \leq 1,$$

then

$$\sum_{i=0}^{N} s_i \mathrm{d}\log N_i = \mathrm{d}\log M \Rightarrow \mathrm{d}\log M = -\frac{\sum_{i=0}^{N} s_i\xi_i\frac{\mathrm{d}\log t_i}{\theta_i}}{\sum_{i=0}^{N} s_i\xi_i} = -\mathbb{E}_{s\xi}\left[\frac{\mathrm{d}\log t}{\theta}\right].$$

Hence, we can re-write $\mathrm{d}\log N_i$ as

$$\mathrm{d}\log N_i = (1-\xi_i)\mathrm{d}\log M - \xi_i\frac{\mathrm{d}\log t_i}{\theta_i} = -\left[(1-\xi_i)\,\mathbb{E}_{s\xi}\left[\frac{\mathrm{d}\log t}{\theta}\right] + \xi_i\frac{\mathrm{d}\log t_i}{\theta_i}\right]$$

Changes in other variables Similarly,

$$\begin{aligned}
\mathrm{d}\log\pi_i &= -\eta_i\theta_i\mathrm{d}\log N_i - \eta_i\mathrm{d}\log t_i \\
&= \eta_i\theta_i\left[(1-\xi_i)\,\mathbb{E}_{s\xi}\left[\frac{\mathrm{d}\log t}{\theta}\right] + \xi_i\frac{\mathrm{d}\log t_i}{\theta_i}\right] - \eta_i\mathrm{d}\log t_i \\
&= \eta_i(1-\xi_i)\theta_i\left\{\mathbb{E}_{s\xi}\left[\frac{\mathrm{d}\log t}{\theta}\right] - \frac{\mathrm{d}\log t_i}{\theta_i}\right\}
\end{aligned}$$

The relative quantity has the following expression:

$$\mathrm{d}\log\frac{q_i}{Q_i} = -\delta_i\mathrm{d}\log N_i = \delta_i\left[(1-\xi_i)\,\mathbb{E}_{s\xi}\left[\frac{\mathrm{d}\log t}{\theta}\right] + \xi_i\frac{\mathrm{d}\log t_i}{\theta_i}\right]$$

The utility change:

$$\begin{aligned}
\mathrm{d}\log Q_i &= -\eta_i\left(\mathrm{d}\log N_i + \mathrm{d}\log t_i + \left[\left(\frac{1}{\rho_i}-1\right)\frac{1}{\sigma_i}+1\right]\mathrm{d}\log\frac{q_i}{Q_i}\right) \\
&= -\eta_i\left\{1 - \left[\left(\frac{1}{\rho_i}-1\right)\frac{1}{\sigma_i}+1\right]\delta_i\right\}\mathrm{d}\log N_i - \eta_i\mathrm{d}\log t_i \\
&= -\eta_i\left[\underbrace{(1-\delta_i)}_{<0} - \underbrace{\delta_i\left(\frac{1}{\rho_i}-1\right)\frac{1}{\sigma_i}}_{>0}\right]\mathrm{d}\log N - \eta_i\mathrm{d}\log t_i
\end{aligned}$$

Notice that

$$\begin{aligned}\eta_i\theta_i &= \left(1-\frac{\eta_i}{\sigma_i}\right)\left(\frac{1}{\rho_i}-1\right)\delta_i+\eta_i-(\eta_i-1)\delta_i\\ &= -\frac{\eta_i}{\sigma_i}\left(\frac{1}{\rho_i}-1\right)\delta_i+\eta_i(1-\delta_i)+\frac{\delta_i}{\rho_i},\end{aligned}$$

then $\eta_i\theta_i-\frac{\delta_i}{\rho_i}<0$. Plugging it into the formula for $\mathrm{d}\log Q_i$ and using the expression for $\mathrm{d}\log N_i$,

$$\begin{aligned}\mathrm{d}\log Q_i &= \left(\eta_i\theta_i-\frac{\delta_i}{\rho_i}\right)\left[(1-\xi_i)\,\mathbb{E}_{s\xi}\left[\frac{\mathrm{d}\log t}{\theta}\right]+\xi_i\frac{\mathrm{d}\log t_i}{\theta_i}\right]-\eta_i\theta_i\frac{\mathrm{d}\log t_i}{\theta_i}\\ &= \underbrace{-\left(\frac{\delta_i}{\rho_i}-\eta_i\theta_i\right)(1-\xi_i)}_{<0}\,\mathbb{E}_{s\xi}\left[\frac{\mathrm{d}\log t}{\theta}\right]\underbrace{-\left[\xi_i\frac{\delta_i}{\rho_i}+(1-\xi_i)\,\eta_i\theta_i\right]}_{<0}\frac{\mathrm{d}\log t_i}{\theta_i}\end{aligned}$$

To obtain the individual quantity change, we have

$$\begin{aligned}\mathrm{d}\log q_i =&\mathrm{d}\log\frac{q_i}{Q_i}+\mathrm{d}\log Q_i\\ =&\delta_i\left[(1-\xi_i)\,\mathbb{E}_{s\xi}\left[\frac{\mathrm{d}\log t}{\theta}\right]+\xi_i\frac{\mathrm{d}\log t_i}{\theta_i}\right]-\left(\frac{\delta_i}{\rho_i}-\eta_i\theta_i\right)(1-\xi_i)\,\mathbb{E}_{s\xi}\left[\frac{\mathrm{d}\log t}{\theta}\right]\\ &-\left[\xi_i\frac{\delta_i}{\rho_i}+(1-\xi_i)\,\eta_i\theta_i\right]\frac{\mathrm{d}\log t_i}{\theta_i}\\ =&\left(\delta_i\left(1-\frac{1}{\rho_i}\right)+\eta_i\theta_i\right)(1-\xi_i)\,\mathbb{E}_{s\xi}\left[\frac{\mathrm{d}\log t}{\theta}\right]+\left[\xi_i\delta_i\left(1-\frac{1}{\rho_i}\right)-(1-\xi_i)\eta_i\theta_i\right]\frac{\mathrm{d}\log t_i}{\theta_i}\\ =&\left(\delta_i\left(1-\frac{1}{\rho_i}\right)+\eta_i\theta_i\right)(1-\xi_i)\left(\mathbb{E}_{s\xi}\left[\frac{\mathrm{d}\log t}{\theta}\right]-\frac{\mathrm{d}\log t_i}{\theta_i}\right)-\underbrace{\delta_i\left(\frac{1}{\rho_i}-1\right)}_{>0}\frac{\mathrm{d}\log t_i}{\theta_i}.\end{aligned}$$

Observe that

$$\begin{aligned}\delta_i\left(1-\frac{1}{\rho_i}\right)+\eta_i\theta_i &= \delta_i\left(1-\frac{1}{\rho_i}\right)+\left(1-\frac{\eta_i}{\sigma_i}\right)\left(\frac{1}{\rho_i}-1\right)\delta_i+\eta_i-(\eta_i-1)\delta_i\\ &= -\frac{\eta_i}{\sigma_i}\left(\frac{1}{\rho_i}-1\right)\delta_i+\eta_i-(\eta_i-1)\delta_i\\ &= \eta_i\delta_i\Bigg[\underbrace{-\frac{1}{\sigma_i}\left(\frac{1}{\rho_i}-1\right)}_{<0}+\underbrace{\frac{1}{\delta_i}-\frac{\eta_i-1}{\eta_i}}_{>0}.\Bigg]\end{aligned}$$

Hence, it is negative with ρ_i is small[18] and $\delta_i \approx \frac{\eta_i}{\eta_i - 1}$ It becomes positive when pass-through is close to complete. For example, with CES demand, $\rho = 1$, and this term is always positive. The following condition gives the precise signing condition:

$$\delta_i \left(1 - \frac{1}{\rho_i}\right) + \eta_i \theta_i > 0 \iff \frac{\eta_i}{\eta_i - 1} > \frac{\delta_i}{1 - \frac{\delta_i}{\sigma_i}\left(\frac{1}{\rho_i} - 1\right)}.$$

The GE effect on firms' quantity is ambiguous. This is intuitive: while market share increases with firms' exit ($\mathrm{d}\log \frac{q_i}{Q_i} > 0$), the aggregate demand also goes down with less variety ($\mathrm{d}\log Q_i < 0$). However, if $\mathbb{E}_{s\xi}\left[\frac{\mathrm{d}\log t}{\theta}\right] - \frac{\mathrm{d}\log t_i}{\theta_i} < 0$ and $\mathbb{E}_{s\xi}\left[\frac{\mathrm{d}\log t}{\theta}\right] > 0$, $\mathrm{d}\log q_i < 0$.

The total quantity $\tilde{Q}_i = N_i q_i$. Then,

$$\begin{aligned}
\mathrm{d}\log \tilde{Q}_i &= \mathrm{d}\log N_i + \mathrm{d}\log q_i \\
&= \underbrace{\left(\delta_i \left(1 - \frac{1}{\rho_i}\right) + \eta_i \theta_i - 1\right)}_{<0} (1 - \xi_i)\, \mathbb{E}_{s\xi}\left[\frac{\mathrm{d}\log t}{\theta}\right] + \underbrace{\left[\xi_i \delta_i \left(1 - \frac{1}{\rho_i}\right) - (1 - \xi_i)\eta_i \theta_i - \xi_i\right]}_{<0} \frac{\mathrm{d}\log t_i}{\theta_i}.
\end{aligned}$$

The first coefficient is negative since

$$\begin{aligned}
\delta_i \left(1 - \frac{1}{\rho_i}\right) + \eta_i \theta_i - 1 &= -\frac{\eta_i}{\sigma_i}\left(\frac{1}{\rho_i} - 1\right)\delta_i + \eta_i - (\eta_i - 1)\delta_i - 1 \\
&= \underbrace{-\frac{\eta_i}{\sigma_i}\left(\frac{1}{\rho_i} - 1\right)\delta_i}_{\leq 0} + \underbrace{(\eta_i - 1)(1 - \delta_i)}_{<0}
\end{aligned}$$

We then turn to prices. With homogeneous firms, the aggregate unit value, $\frac{N_i p_i q_i}{N_i q_i}$, is the same as individual price, p_i. Furthermore, the price change is equivalent to the markup change with only tariff shocks. Therefore,

$$\begin{aligned}
\mathrm{d}\log p_i = \mathrm{d}\log \mu_i &= \left(\frac{1}{\rho_i} - 1\right)\frac{1}{\sigma_i}\mathrm{d}\log \frac{q_i}{Q_i} \\
&= \left(\frac{1}{\rho_i} - 1\right)\frac{\delta_i}{\sigma_i}\left[(1 - \xi_i)\, \mathbb{E}_{s\xi}\left[\frac{\mathrm{d}\log t}{\theta}\right] + \xi_i \frac{\mathrm{d}\log t_i}{\theta_i}\right].
\end{aligned}$$

[18] If the strong version of Marshall's second law of demand in Baqaee et al. (2023) holds, pass-through is decreasing with productivity. Hence, a lower ρ is associated with a lower σ (higher markup μ), making this term more negative.

Finally, we discuss individual sales, $r_i = p_i q_i$ and aggregate sales, $R_i = p_i \tilde{Q}_i$.

$$
\begin{aligned}
\mathrm{d}\log r_i =& \mathrm{d}\log p_i + \mathrm{d}\log q_i \\
=& \left(\delta_i\left(1-\frac{1}{\rho_i}\right)+\eta_i\theta_i\right)(1-\xi_i)\,\mathbb{E}_{s\xi}\left[\frac{\mathrm{d}\log t}{\theta}\right] + \left[\xi_i\delta_i\left(1-\frac{1}{\rho_i}\right)-(1-\xi_i)\eta_i\theta_i\right]\frac{\mathrm{d}\log t_i}{\theta_i} \\
&+\left(\frac{1}{\rho_i}-1\right)\frac{\delta_i}{\sigma_i}\left[(1-\xi_i)\,\mathbb{E}_{s\xi}\left[\frac{\mathrm{d}\log t}{\theta}\right]+\xi_i\frac{\mathrm{d}\log t_i}{\theta_i}\right] \\
=& \left[\left(1-\frac{1}{\rho_i}\right)\left(1-\frac{1}{\sigma_i}\right)\delta_i+\eta_i\theta_i\right](1-\xi_i)\,\mathbb{E}_{s\xi}\left[\frac{\mathrm{d}\log t}{\theta}\right] \\
&+\underbrace{\left[\xi_i\delta_i\left(1-\frac{1}{\rho_i}\right)\left(1-\frac{1}{\sigma_i}\right)-(1-\xi_i)\eta_i\theta_i\right]}_{<0}\frac{\mathrm{d}\log t_i}{\theta_i} \\
=& \left[\left(1-\frac{1}{\rho_i}\right)\left(1-\frac{1}{\sigma_i}\right)\delta_i+\eta_i\theta_i\right](1-\xi_i)\,\mathbb{E}_{s\xi}\left(\left[\frac{\mathrm{d}\log t}{\theta}\right]-\frac{\mathrm{d}\log t_i}{\theta_i}\right) \\
&-\delta_i\left(\frac{1}{\rho_i}-1\right)\left(1-\frac{1}{\sigma_i}\right)\frac{\mathrm{d}\log t_i}{\theta_i}
\end{aligned}
$$

Even though $\mathrm{d}\log p_i$ is always positive to tariff shocks, the impact of $\mathbb{E}_{s\xi}[\mathrm{d}\log t/\mathrm{d}\theta]$ on $\mathrm{d}\log r_i$ is similar to that in $\mathrm{d}\log q_i$ and remains ambiguous. Note that

$$
\left(1-\frac{1}{\rho_i}\right)\left(1-\frac{1}{\sigma_i}\right)\delta_i+\eta_i\theta_i = (\eta_i-1)\delta_i\left[-\left(\frac{1}{\rho_i}-1\right)\frac{1}{\sigma_i}+\frac{\eta_i}{\eta_i-1}\frac{1}{\delta_i}-1\right],
$$

then we obtain the following sign condition:

$$
\left(1-\frac{1}{\rho_i}\right)\left(1-\frac{1}{\sigma_i}\right)\delta_i+\eta_i\theta_i>0 \iff \frac{\eta_i}{\eta_i-1}>\delta_i\left[1+\left(\frac{1}{\rho_i}-1\right)\frac{1}{\sigma_i}\right].
$$

We do similar exercises for the aggregate revenue

$$
\begin{aligned}
\mathrm{d}\log R_i =& \mathrm{d}\log r_i + \mathrm{d}\log N_i \\
=& \left[\left(1-\frac{1}{\rho_i}\right)\left(1-\frac{1}{\sigma_i}\right)\delta_i + \eta_i\theta_i\right](1-\xi_i)\,\mathbb{E}_{s\xi}\left[\frac{\mathrm{d}\log t}{\theta}\right] \\
&+ \left[\xi_i\delta_i\left(1-\frac{1}{\rho_i}\right)\left(1-\frac{1}{\sigma_i}\right) - (1-\xi_i)\eta_i\theta_i\right]\frac{\mathrm{d}\log t_i}{\theta_i} - \left[(1-\xi_i)\,\mathbb{E}_{s\xi}\left[\frac{\mathrm{d}\log t}{\theta}\right] + \xi_i\frac{\mathrm{d}\log t_i}{\theta_i}\right] \\
=& \underbrace{\left[\left(1-\frac{1}{\rho_i}\right)\left(1-\frac{1}{\sigma_i}\right)\delta_i + \eta_i\theta_i - 1\right]}_{<0}(1-\xi_i)\,\mathbb{E}_{s\xi}\left[\frac{\mathrm{d}\log t}{\theta}\right] \\
&+ \underbrace{\left[\xi_i\delta_i\left(1-\frac{1}{\rho_i}\right)\left(1-\frac{1}{\sigma_i}\right) - (1-\xi_i)\eta_i\theta_i - \xi_i\right]}_{<0}\frac{\mathrm{d}\log t_i}{\theta_i} \\
=& \left[\left(1-\frac{1}{\rho_i}\right)\left(1-\frac{1}{\sigma_i}\right)\delta_i + \eta_i\theta_i - 1\right](1-\xi_i)\,\mathbb{E}_{s\xi}\left(\left[\frac{\mathrm{d}\log t}{\theta}\right] - \frac{\mathrm{d}\log t_i}{\theta_i}\right) \\
&- \left[\delta_i\left(\frac{1}{\rho_i}-1\right)\left(1-\frac{1}{\sigma_i}\right) + 1\right]\frac{\mathrm{d}\log t_i}{\theta_i}
\end{aligned}
$$

The signing follows that

$$
\begin{aligned}
\left(1-\frac{1}{\rho_i}\right)\left(1-\frac{1}{\sigma_i}\right)\delta_i + \eta_i\theta_i - 1 &= -\frac{\eta_i-1}{\sigma_i}\left(\frac{1}{\rho_i}-1\right)\delta_i + \eta_i - (\eta_i-1)\delta_i - 1 \\
&= \underbrace{-\frac{\eta_i-1}{\sigma_i}\left(\frac{1}{\rho_i}-1\right)\delta_i}_{\leq 0} + \underbrace{(\eta_i-1)(1-\delta_i)}_{<0} < 0.
\end{aligned}
$$

As the aggregate quantity, the aggregate revenue must decline in all markets despite that prices go up.

The case with $\beta \to 1$ With fixed entry h_i, the balanced flow condition is now $h_i M = \nu_i N_i$. Then

$$\mathrm{d}\log N_i = \mathrm{d}\log M.$$

Totally differentiating the free entry condition $\sum_i h_i\bar{\pi}_i = f_e$,

$$\sum_i \frac{h_i}{\rho+\nu_i}\mathrm{d}\pi_i = 0 \Rightarrow \sum_i \frac{h_i\bar{\pi}_i}{\sum_k h_k\bar{\pi}_k}\frac{\pi_i}{\pi_i - f_i}\mathrm{d}\log\pi_i = 0.$$

Recall that $\mathrm{d}\log\pi_i = -\eta_i\theta_i \mathrm{d}\log N_i - \eta_i \mathrm{d}\log t_i$,

$$\sum_i s_i\xi_i \mathrm{d}\log M = -\sum_i s_i\xi_i \frac{\mathrm{d}\log t_i}{\theta_i} \Rightarrow \mathrm{d}\log M = -\mathbb{E}_{s\xi}\left[\frac{\mathrm{d}\log t}{\theta}\right],$$

in which $\xi_i = \frac{\pi_i\eta_i\theta_i}{\pi_i - f_i}$.

C.3 Proof of proposition 1

Totally differentiating the system, we have shown that

$$\frac{\mathrm{d}\log\pi_i}{\mathrm{d}\log N_i} = -\eta_i\theta_i < 0 \quad \text{and} \quad \frac{\mathrm{d}\log N_i}{\mathrm{d}\log M} = 1 - \xi_i \geq 0,$$

where that assumption 1 holds for all N_i ensures $\theta_i > 0$.

Therefore, $\pi_i(M)$ is decreasing in M, and so is $\frac{N_i(M)}{M}$. The left hand side of equation (16) is also decreasing in M. In that sense, the equilibrium will be unique if it exists. From equation (15), we must have $N_i(M) \to \infty$ if $M \to \infty$ and $N_i(M) \to 0$ if $M \to 0$. It suffices to have $\pi_i(N_i)$ satisfy the Inada condition that

$$\lim_{N_i\to 0} \pi_i(N_i) = \infty, \quad \lim_{N_i\to\infty} \pi_i(N_i) = 0.$$

Note that for any $N_1 > N_0 > 0$,

$$\frac{\pi_i(N_1)}{\pi_i(N_0)} = \exp\left(-\int_{N_0}^{N_1} \frac{\eta_i(x)\theta_i(x)}{x}\mathrm{d}x\right).$$

The integral diverges at $N_1 = \infty$ or $N_0 = 0$ since the limit conditions ensures that

$$\min\left[\liminf_{N_i\to 0} \eta_i\theta_i, \liminf_{N_i\to\infty} \eta_i\theta_i\right] > 0.$$

Hence, we have verified the Inada condition and established the existence of the equilibrium.

D Propagation and aggregation equations with heterogeneous firms

D.1 Equilibrium conditions

We expand the equilibrium equations presented in Section 3.4 to the first order in the shocks.

Pricing Totally differentiating the pricing equation (22)

$$\mathrm{d}\log\mu_i - \mathrm{d}\log P_i + \mathrm{d}\log\tau_i + \mathrm{d}\log t_i = -\frac{1}{\sigma_i}\mathrm{d}\log\frac{q_i}{Q_i}. \tag{36}$$

We suppress firm productivity z for brevity. Similarly, μ_i is an abbreviation for $\mu\left(\frac{q_i(z)}{Q_i}\right)$.

Selection Totally differentiating the selection equation (21),

$$\left.\frac{\partial\log\left(\mu_i(z)-1\right)\frac{\tau_i}{z}q_i(z)\right)}{\partial\log z}\right|_{z=z^*}\mathrm{d}\log z_i^* + \mathrm{d}\left(\log(\mu_i(z_i^*)-1)\frac{\tau_i}{z_i^*}q_i(z_i^*)\right) = \mathrm{d}\log f_i.$$

The first partial derivative has the following form:

$$\frac{\partial\log\left(\mu_i(z)-1\right)\frac{\tau_i}{z}q_i(z)\right)}{\partial\log z} = \frac{\mu_i(z)}{\mu_i(z)-1}\frac{\partial\log\mu_i(z)}{\partial\log z} - 1 + \frac{\partial\log q_i(z)}{\partial\log z} = \sigma_i(z) - 1.$$

Since $\frac{\mu_i\tau_i}{z} = P_i\Upsilon'\left(\frac{q_i}{Q_i}\right)$,

$$\frac{\partial\log\mu_i(z)}{\partial\log z} - 1 = \frac{\Upsilon''\left(\frac{q_i(z)}{Q_i}\right)\frac{q_i(z)}{Q_i}}{\Upsilon'\left(\frac{q_i(z)}{Q_i}\right)}\frac{\partial\log q_i(z)}{\partial\log z} = -\frac{1}{\sigma_i(z)}\frac{\partial\log q_i(z)}{\partial\log z}.$$

Plugging it into the equation, we obtain the RHS equals to $\sigma_i - 1$ with $\frac{\mu_i}{\mu_i-1} = \sigma_i$.[19] Finally, we rewrite the selection condition as follows:[20]

$$(\sigma_i(z_i^*)-1)\mathrm{d}\log z_i^* + \mathrm{d}\log\pi_i(z_i^*) = \mathrm{d}\log f_i. \tag{37}$$

[19] This is straightforward with the CES preference: $\pi \propto z^{\sigma-1} \Rightarrow \frac{\partial\log\pi}{\partial\log z} = \sigma - 1$,

[20] The notation $\mathrm{d}\log x_i \equiv \mathrm{d}\log x_i(z)$, the log change in x_i holding z constant.

Welfare Totally differentiating equation (23) and using that $\Upsilon'\left(\frac{q_i}{Q_i}\right)\frac{q_i}{Q_i} = \frac{t_i p_i q_i}{E_i \bar{\delta}_i}$, we obtain that

$$\bar{\delta}_i \mathrm{d}\log N_i + \left(\bar{\delta}_i - \delta_i(z_i^*)\lambda_i(z_i^*)\right)\frac{g(z_i^*)}{1-G(z_i^*)}\mathrm{d}z^* + \mathbb{E}^i_{\lambda_i}\left[\mathrm{d}\log\frac{q_i}{Q_i}\right] = 0 \tag{38}$$

Price index (budget balance) Note that

$$\frac{1}{\bar{\delta}_i} = \frac{N_i}{1-G(z_i^*)}\int_{z^*}^{\infty}\Upsilon'\left(\frac{q_i(z)}{Q}\right)\frac{q_i(z)}{Q}g(z)\mathrm{d}z$$

Totally differentiating $\bar{\delta}_i$,

$$-\mathrm{d}\log\bar{\delta}_i = \mathrm{d}\log N_i + (1-\lambda_i(z_i^*))\frac{g(z_i^*)}{1-G(z_i^*)}\mathrm{d}z_i^* + \mathbb{E}^i_{\lambda_i}\left[\left(1-\frac{1}{\sigma_i}\right)\mathrm{d}\log\frac{q_i}{Q_i}\right]. \tag{39}$$

Then, we have

$$\mathrm{d}\log P_i + \frac{1}{\eta_i}\mathrm{d}\log Q_i = \mathrm{d}\log\bar{\delta}_i. \tag{40}$$

Markup

$$\mathrm{d}\log\mu_i = \left(\frac{1}{\rho_i}-1\right)\frac{1}{\sigma_i}\mathrm{d}\log\frac{q_i}{Q_i}, \tag{41}$$

in which

$$\rho_i = -\frac{\partial\log p_i}{\partial\log z} = \frac{1}{1+\frac{\mu'\left(\frac{q_i}{Q_i}\right)\frac{q_i}{Q_i}}{\mu\left(\frac{q_i}{Q_i}\right)}\sigma_i}.$$

Variable and expected profit

$$\mathrm{d}\log\pi_i = \sigma_i\mathrm{d}\log\mu_i + \mathrm{d}\log\tau_i + \mathrm{d}\log q_i. \tag{42}$$

$$\mathrm{d}\log\bar{\pi}_i = -\mathrm{d}\log(\rho+\nu_i) + \mathbb{E}^i_{\pi_i - f_i}\left[\frac{\pi_i\mathrm{d}\log\pi_i - f_i\mathrm{d}\log f_i}{\pi_i - f_i}\right]. \tag{43}$$

Balanced flow Totally differentiating equation (26),

$$\frac{1}{\beta}\log h_i + \frac{1-\beta}{\beta}\left[\log \bar{\pi}_i - \log f_e\right] = \log \nu_i + \log N_i - \log M + \frac{g(z_i^*)}{1-G(z_i^*)}\mathrm{d}z_i^*. \tag{44}$$

Free entry

$$\sum_{i=0}^{N} s_i \left[\mathrm{d}\log h_i + \mathrm{d}\log \bar{\pi}_i\right] = \mathrm{d}\log f_e, \tag{45}$$

in which $s_i = \left(\frac{h_i \bar{\pi}_i}{f_e}\right)^{\frac{1}{\beta}} = \frac{J_i \bar{\pi}_i}{\sum_k J_k \bar{\pi}_k}$.

D.2 Proof of proposition 4 and 5

D.2.1 Solving for $\mathrm{d}\log M$ and $\mathrm{d}\log z_i^*$

We consider only $\mathrm{d}\log t_i$ and set $\mathrm{d}\log \tau_i = \mathrm{d}\log \nu_i = \mathrm{d}\log h_i = \mathrm{d}\log f_i = \mathrm{d}\log f_e = 0$. Also, let $\sigma_i^* = \sigma_i(z_i^*)$, $\varphi_i^* = \frac{z_i^* g(z_i^*)}{1-G(z_i^*)}$, $\pi_i^* = \pi_i(z_i^*)$, $\delta_i^* = \delta_i(z_i^*)$, and $\lambda_i^* = \lambda_i(z_i^*)$. Then, we obtain the following system of equations:

$$\begin{aligned}
\mathrm{d}\log P_i &= \frac{1}{\rho_i \sigma_i}\mathrm{d}\log\frac{q_i}{Q_i} + \mathrm{d}\log t_i, \\
(\sigma_i^* - 1)\mathrm{d}\log z_i^* + \mathrm{d}\log \pi_i^* &= 0, \\
\bar{\delta}_i \mathrm{d}\log N_i + \left(\bar{\delta}_i - \delta_i^* \lambda_i^*\right)\varphi_i^* \mathrm{d}\log z_i^* + \mathbb{E}^i_{\lambda_i}\left[\mathrm{d}\log\frac{q_i}{Q_i}\right] &= 0, \\
-\mathrm{d}\log P_i - \frac{1}{\eta_i}\mathrm{d}\log Q_i = \mathrm{d}\log N_i + (1-\lambda_i^*)\varphi_i^* \mathrm{d}\log z_i^* + \mathbb{E}^i_{\lambda_i}&\left[\left(1 - \frac{1}{\sigma_i}\right)\mathrm{d}\log\frac{q_i}{Q_i}\right], \\
\mathrm{d}\log \pi_i &= \frac{1}{\rho_i}\mathrm{d}\log\frac{q_i}{Q_i} + \mathrm{d}\log Q_i, \\
\mathbb{E}^i_{\pi_i - f_i}\left[\frac{\pi_i \mathrm{d}\log \pi_i}{\pi_i - f_i}\right] &= \frac{\beta}{1-\beta}\left[\mathrm{d}\log N_i - \mathrm{d}\log M + \varphi_i^* \mathrm{d}\log z_i^*\right], \\
\sum_{i=0}^{N} s_i \left[\mathrm{d}\log N_i + \varphi_i^* \mathrm{d}\log z_i^*\right] &= \mathrm{d}\log M.
\end{aligned}$$

First, we write $\mathrm{d}\log\frac{q_i}{Q_i} = \rho_i\sigma_i[\mathrm{d}\log P_i - \mathrm{d}\log t_i]$. Then

$$\mathrm{d}\log \pi_i = \sigma_i[\mathrm{d}\log P_i - \mathrm{d}\log t_i] + \mathrm{d}\log Q_i \Longrightarrow (\sigma_i^* - 1)\mathrm{d}\log z_i^* = -\sigma_i^*[\mathrm{d}\log P_i - \mathrm{d}\log t_i] - \mathrm{d}\log Q_i.$$

Combining conditions on welfare and price index,

$$
\begin{aligned}
-\frac{\bar{\delta}_i}{\eta_i}\mathrm{d}\log Q_i =&\bar{\delta}_i\mathrm{d}\log P_i - \left(\bar{\delta}_i - \delta_i^*\lambda_i^*\right)\varphi_i^*\mathrm{d}\log z_i^* - \mathbb{E}^i_{\lambda_i}\left[\mathrm{d}\log\frac{q_i}{Q_i}\right]\\
&+\bar{\delta}_i\left(1-\lambda_i^*\right)\varphi_i^*\mathrm{d}\log z_i^* + \bar{\delta}_i\,\mathbb{E}^i_{\lambda_i}\left[\left(1-\frac{1}{\sigma_i}\right)\mathrm{d}\log\frac{q_i}{Q_i}\right],\\
-\frac{\bar{\delta}_i}{\eta_i}\mathrm{d}\log Q_i =&\bar{\delta}_i\mathrm{d}\log P_i - (\bar{\delta}_i-\delta_i^*)\lambda_i^*\varphi_i^*\mathrm{d}\log z_i^* + \mathbb{E}^i_{\lambda_i}\left[\left(\bar{\delta}_i(\sigma_i-1)-\sigma_i\right)\rho_i\right]\left[\mathrm{d}\log P_i - \mathrm{d}\log t_i\right].
\end{aligned}
$$

We obtain the following relationship between $\mathrm{d}\log Q_i$ and $\mathrm{d}\log z_i^*$:

$$
\begin{aligned}
&\left\{\frac{1}{\sigma_i^*}\left(\bar{\delta}_i + \mathbb{E}^i_{\lambda_i}\left[\left(\bar{\delta}_i(\sigma_i-1)-\sigma_i\right)\rho_i\right]\right) - \frac{\bar{\delta}_i}{\eta_i}\right\}\mathrm{d}\log Q_i\\
=&-\left\{\frac{\sigma_i^*-1}{\sigma_i^*}\left(\bar{\delta}_i + \mathbb{E}^i_{\lambda_i}\left[\left(\bar{\delta}_i(\sigma_i-1)-\sigma_i\right)\rho_i\right]\right) + (\bar{\delta}_i-\delta_i^*)\lambda_i^*\varphi_i^*\right\}\mathrm{d}\log z_i^* + \bar{\delta}_i\mathrm{d}\log t_i\\
\Rightarrow\quad \mathrm{d}\log Q_i =&-\left[(\sigma_i^*-1)\left(1+\frac{\bar{\delta}_i}{\eta_i\gamma_i}\right) + (\bar{\delta}_i-\delta_i^*)\frac{\lambda_i^*\varphi_i^*}{\gamma_i}\right]\mathrm{d}\log z_i^* + \frac{\bar{\delta}_i}{\gamma_i}\mathrm{d}\log t_i,
\end{aligned}
$$

in which $\gamma_i = \frac{1}{\sigma_i^*}\left(\bar{\delta}_i + \mathbb{E}^i_{\lambda_i}\left[\left(\bar{\delta}_i(\sigma_i-1)-\sigma_i\right)\rho_i\right]\right) - \frac{\bar{\delta}_i}{\eta_i}$. It is helpful to take a pause here and comment on the signs of these coefficients. Before provoking Assumption 2, we have

$$
\begin{aligned}
\bar{\delta}_i + \mathbb{E}^i_{\lambda_i}\left[\left(\bar{\delta}_i(\sigma_i-1)-\sigma_i\right)\rho_i\right] =&\bar{\delta}_i\left(1+\mathbb{E}^i_{\lambda_i}[(\sigma_i-1)\rho_i]\right) - \mathbb{E}^i_{\lambda_i}[\sigma_i\rho_i]\\
&\geq 1+\mathbb{E}^i_{\lambda_i}[(\sigma_i-1)\rho_i] - \mathbb{E}^i_{\lambda_i}[\sigma_i\rho_i] = 1-\mathbb{E}^i_{\lambda_i}[\rho_i]\geq 0,
\end{aligned}
$$

where we have used that $\rho_i \leq 1$, the weak second Marshall law of demand, and $\bar{\delta}_i \geq 1$. Thus, we have $\gamma_i \geq -\frac{\bar{\delta}_i}{\eta_i}$. Under Assumption 2, we further show that

$$
\begin{aligned}
\mathbb{E}^i_{\lambda_i}\left[\left(\bar{\delta}_i(\sigma_i-1)-\sigma_i\right)\rho_i\right] \leq&\,\mathbb{E}^i_{\lambda_i}[\mu_i^{-1}]^{-1}\,\mathbb{E}^i_{\lambda_i}[(\sigma_i-1)\rho_i] - \mathbb{E}^i_{\lambda_i}[\sigma_i\rho_i]\\
=&\,\mathbb{E}^i_{\lambda_i}[\mu_i^{-1}]^{-1}\left(\mathbb{E}^i_{\lambda_i}[(\sigma_i-1)\rho_i] - \mathbb{E}^i_{\lambda_i}[\mu_i^{-1}]\,\mathbb{E}^i_{\lambda_i}[\sigma_i\rho_i]\right)\\
=&-\mathbb{E}^i_{\lambda_i}[\mu_i^{-1}]^{-1}\mathrm{Cov}^i_{\lambda_i}\left(\sigma_i\rho_i, \frac{1}{\mu_i}\right) < 0.
\end{aligned}
$$

The second equality uses that $\sigma_i - 1 = \frac{\sigma_i}{\mu_i}$. With the strong second Marshall law of demand, both $\sigma_i\rho_i$ and $1/\mu_i$ decrease on z. Then, $\mathrm{Cov}^i_{\lambda_i}\left(\sigma_i\rho_i, \frac{1}{\mu_i}\right) > 0$. Since $\sigma_i^* > \eta_i$ and $\bar{\delta}_i > \delta_i^*$,

$$
-\frac{\bar{\delta}_i}{\eta_i} \leq \gamma_i < 0, \quad Q_i^z \equiv -\left[(\sigma_i^*-1)\left(1+\frac{\bar{\delta}_i}{\eta_i\gamma_i}\right) + (\bar{\delta}_i-\delta_i^*)\frac{\lambda_i^*\varphi_i^*}{\gamma_i}\right] > 0.
$$

That is, holding tariffs, welfare increases with productivity threshold. With the CES demand, we have

$$\gamma_i = \frac{\eta_i - \sigma_i}{\eta_i(\sigma_i - 1)} < 0, \quad Q_i^z = \frac{\eta_i(\sigma_i - 1)}{\sigma_i - \eta_i} > \eta_i > 1.$$

We then turn to the welfare condition. Note that

$$\begin{aligned}
&\bar{\delta}_i \mathrm{d}\log N_i + \left[\left(\bar{\delta}_i - \delta_i^* \lambda_i^*\right)\varphi_i^* - \frac{\sigma_i^* - 1}{\sigma_i^*}\mathbb{E}_{\lambda_i}^i[\rho_i \sigma_i]\right]\mathrm{d}\log z_i^* = \frac{1}{\sigma_i^*}\mathbb{E}_{\lambda_i}^i[\rho_i\sigma_i]\mathrm{d}\log Q_i \\
\Rightarrow\quad &\bar{\delta}_i \mathrm{d}\log N_i + \left[\left(\bar{\delta}_i - \delta_i^* \lambda_i^*\right)\varphi_i^* - \frac{\sigma_i^* - 1}{\sigma_i^*}\mathbb{E}_{\lambda_i}^i[\rho_i \sigma_i]\right]\mathrm{d}\log z_i^* \\
&= \frac{1}{\sigma_i^*}\mathbb{E}_{\lambda_i}^i[\rho_i\sigma_i]\left\{-\left[(\sigma_i^* - 1)\left(1 + \frac{\bar{\delta}_i}{\eta_i\gamma_i}\right) + (\bar{\delta}_i - \delta_i^*)\frac{\lambda_i^*\varphi_i^*}{\gamma_i}\right]\mathrm{d}\log z_i^* + \frac{\bar{\delta}_i}{\gamma_i}\mathrm{d}\log t_i\right\} \\
\Rightarrow\quad &\bar{\delta}_i \mathrm{d}\log N_i = \left\{-\frac{\mathbb{E}_{\lambda_i}^i[\rho_i\sigma_i]}{\sigma_i^*\gamma_i}\left[(\sigma_i^* - 1)\frac{\bar{\delta}_i}{\eta_i} + (\bar{\delta}_i - \delta_i^*)\lambda_i^*\varphi_i^*\right] - \left(\bar{\delta}_i - \delta_i^*\lambda_i^*\right)\varphi_i^*\right\}\mathrm{d}\log z_i^* + \frac{\bar{\delta}_i\,\mathbb{E}_{\lambda_i}^i[\rho_i\sigma_i]}{\sigma_i^*\gamma_i}\mathrm{d}\log t_i \\
\Rightarrow\quad &\mathrm{d}\log N_i = \left\{-\frac{\mathbb{E}_{\lambda_i}^i[\rho_i\sigma_i]}{\sigma_i^*\gamma_i}\left[(\sigma_i^* - 1)\frac{1}{\eta_i} + (1 - \frac{\delta_i^*}{\bar{\delta}_i})\lambda_i^*\varphi_i^*\right] - \left(1 - \frac{\delta_i^*}{\bar{\delta}_i}\lambda_i^*\right)\varphi_i^*\right\}\mathrm{d}\log z_i^* + \frac{\mathbb{E}_{\lambda_i}^i[\rho_i\sigma_i]}{\sigma_i^*\gamma_i}\mathrm{d}\log t_i
\end{aligned}$$

Also,

$$\begin{aligned}
\mathrm{d}\log \pi_i &= \frac{\sigma_i}{\sigma_i^*}\left(-(\sigma_i^* - 1)\mathrm{d}\log z_i^* - \mathrm{d}\log Q_i\right) + \mathrm{d}\log Q_i \\
&= -\sigma_i\frac{\sigma_i^* - 1}{\sigma_i^*}\mathrm{d}\log z_i^* + \frac{\sigma_i^* - \sigma_i}{\sigma_i^*}\mathrm{d}\log Q_i \\
&= -\sigma_i\frac{\sigma_i^* - 1}{\sigma_i^*}\mathrm{d}\log z_i^* + \frac{\sigma_i^* - \sigma_i}{\sigma_i^*}\left\{-\left[(\sigma_i^* - 1)\left(1 + \frac{\bar{\delta}_i}{\eta_i\gamma_i}\right) + (\bar{\delta}_i - \delta_i^*)\frac{\lambda_i^*\varphi_i^*}{\gamma_i}\right]\mathrm{d}\log z_i^* + \frac{\bar{\delta}_i}{\gamma_i}\mathrm{d}\log t_i\right\} \\
&= -\sigma_i\frac{\sigma_i^* - 1}{\sigma_i^*}\mathrm{d}\log z_i^* - (\sigma_i^* - \sigma_i)\left[\frac{(\sigma_i^* - 1)}{\sigma_i^*}\left(1 + \frac{\bar{\delta}_i}{\eta_i\gamma_i}\right) + \frac{\bar{\delta}_i - \delta_i^*}{\sigma_i^*}\frac{\lambda_i^*\varphi_i^*}{\gamma_i}\right]\mathrm{d}\log z_i^* + \frac{\sigma_i^* - \sigma_i}{\sigma_i^*}\frac{\bar{\delta}_i}{\gamma_i}\mathrm{d}\log t_i \\
&= -(\sigma_i^* - 1)\mathrm{d}\log z_i^* - (\sigma_i^* - \sigma_i)\left[\frac{(\sigma_i^* - 1)}{\sigma_i^*}\frac{\bar{\delta}_i}{\eta_i\gamma_i} + \frac{\bar{\delta}_i - \delta_i^*}{\sigma_i^*}\frac{\lambda_i^*\varphi_i^*}{\gamma_i}\right]\mathrm{d}\log z_i^* + \frac{\sigma_i^* - \sigma_i}{\sigma_i^*}\frac{\bar{\delta}_i}{\gamma_i}\mathrm{d}\log t_i.
\end{aligned}$$

Comparing to the CES case, tariffs t_i have direct effect on firms' profit since markups are distorted. While aggregate demand shock (the third term) has direct impact on firms' market power, the redistribution of firms (the second term) imposes an additional one. Notice that average variable profit $\bar{\pi}_i$ and revenue $\hat{r}_i$ have the following form:

$$\hat{\pi}_i = \int_{z_i^*}^{\infty}\pi_i\frac{g(z)}{1 - G(z_i^*)}\mathrm{d}z, \quad \hat{r}_i = \int_{z_i^*}^{\infty}p_iq_i\frac{g(z)}{1 - G(z_i^*)}\mathrm{d}z = \int_{z_i^*}^{\infty}\sigma_i\pi_i\frac{g(z)}{1 - G(z_i^*)}\mathrm{d}z$$

Then,

$$\begin{aligned}
\mathbb{E}^i_{\pi_i-f_i}\left[\frac{\pi_i \mathrm{d}\log\pi_i}{\pi_i-f_i}\right] =& -(\sigma_i^*-1)\,\mathbb{E}^i_{\pi_i-f_i}\left[\frac{\pi_i}{\pi_i-f_i}\right]\mathrm{d}\log z_i^* \\
&- \mathbb{E}^i_{\pi_i-f_i}\left[\frac{\pi_i(\sigma_i^*-\sigma_i)}{\pi_i-f_i}\right]\left[\frac{(\sigma_i^*-1)}{\sigma_i^*}\frac{\bar{\delta}_i}{\eta_i\gamma_i}+\frac{\bar{\delta}_i-\delta_i^*}{\sigma_i^*}\frac{\lambda_i^*\varphi_i^*}{\gamma_i}\right]\mathrm{d}\log z_i^* \\
&+ \mathbb{E}^i_{\pi_i-f_i}\left[\frac{\pi_i(\sigma_i^*-\sigma_i)}{\pi_i-f_i}\right]\frac{\bar{\delta}_i}{\sigma_i^*\gamma_i}\mathrm{d}\log t_i \\
=& -\left\{\frac{\hat{\pi}_i(\sigma_i^*-1)}{\hat{\pi}_i-f_i}+\frac{\sigma_i^*\hat{\pi}_i-\hat{r}_i}{\hat{\pi}_i-f_i}\left[\frac{(\sigma_i^*-1)}{\sigma_i^*}\frac{\bar{\delta}_i}{\eta_i\gamma_i}+\frac{\bar{\delta}_i-\delta_i^*}{\sigma_i^*}\frac{\lambda_i^*\varphi_i^*}{\gamma_i}\right]\right\}\mathrm{d}\log z_i^* \\
&+ \frac{\sigma_i^*\hat{\pi}_i-\hat{r}_i}{\hat{\pi}_i-f_i}\frac{\bar{\delta}_i}{\sigma_i^*\gamma_i}\mathrm{d}\log t_i
\end{aligned}$$

We define that

$$\begin{aligned}
\bar{\pi}_i^z =& -\left\{\frac{\hat{\pi}_i(\sigma_i^*-1)}{\hat{\pi}_i-f_i}+\bar{\pi}_i^t\left[\frac{\sigma_i^*-1}{\eta_i}+\left(1-\frac{\delta_i^*}{\bar{\delta}_i}\right)\lambda_i^*\varphi_i^*\right]\right\}<0, \quad \bar{\pi}_i^t=\frac{\sigma_i^*\hat{\pi}_i-\hat{r}_i}{\hat{\pi}_i-f_i}\frac{\bar{\delta}_i}{\sigma_i^*\gamma_i}\leq 0 \\
N_i^z =& -N_i^t\left[\frac{\sigma_i^*-1}{\eta_i}+(1-\frac{\delta_i^*}{\bar{\delta}_i})\lambda_i^*\varphi_i^*\right]-\left(1-\frac{\delta_i^*}{\bar{\delta}_i}\lambda_i^*\right)\varphi_i^*, \quad N_i^t=\frac{\mathbb{E}^i_{\lambda_i}[\rho_i\sigma_i]}{\sigma_i^*\gamma_i}<0,
\end{aligned}$$

as $\gamma_i < 0$ and Assumption 2 holds. Moreover, since $\gamma_i < 0$ and $N_i^t < 0$, we must have $\varphi_i^* + N_i^z > 0$. Under the CES demand, the above terms become

$$\bar{\pi}_i^z = -\frac{\hat{\pi}_i(\sigma_i-1)}{\hat{\pi}_i-f_i}, \quad \bar{\pi}_i^t=0, \quad N_i^z=\frac{(\sigma_i-1)^2}{\sigma_i-\eta_i}-(1-\lambda_i^*)\,\varphi_i^* \quad N_i^t=\frac{1}{\gamma_i}=\frac{\eta_i(\sigma_i-1)}{\eta_i-\sigma_i}$$

Then, we write the balanced flow condition,

$$\begin{aligned}
&\frac{1-\beta}{\beta}\left[\bar{\pi}_i^z\mathrm{d}\log z_i^*+\bar{\pi}_i^t\mathrm{d}\log t_i\right]=(N_i^z+\varphi_i^*)\,\mathrm{d}\log z_i^*+N_i^t\mathrm{d}\log t_i-\mathrm{d}\log M \\
\Rightarrow\quad &\left[\frac{1-\beta}{\beta}\bar{\pi}_i^z-(N_i^z+\varphi_i^*)\right]\mathrm{d}\log z_i^*=\left[N_i^t-\frac{1-\beta}{\beta}\bar{\pi}_i^t\right]\mathrm{d}\log t_i-\mathrm{d}\log M \\
\Rightarrow\quad &\mathrm{d}\log z_i^*=z_i^M\mathrm{d}\log M+z_i^t\mathrm{d}\log t_i,
\end{aligned}$$

where

$$z_i^M=-\frac{1}{\frac{1-\beta}{\beta}\bar{\pi}_i^z-(N_i^z+\varphi_i^*)}, \quad \text{and } z_i^t=\frac{N_i^t-\frac{1-\beta}{\beta}\bar{\pi}_i^t}{\frac{1-\beta}{\beta}\bar{\pi}_i^z-(N_i^z+\varphi_i^*)}.$$

Therefore,

$$\begin{aligned}\mathrm{d}\log N_i - \mathrm{d}\log M + \varphi_i^* \mathrm{d}\log z_i^* =& \frac{1-\beta}{\beta}\left[\bar{\pi}_i^z \mathrm{d}\log z_i^* + \bar{\pi}_i^t \mathrm{d}\log t_i\right] \\ =& -\frac{\frac{1-\beta}{\beta}\bar{\pi}_i^z}{\frac{1-\beta}{\beta}\bar{\pi}_i^z - (N_i^z + \varphi_i^*)}\mathrm{d}\log M \\ &+ \frac{\frac{1-\beta}{\beta}\left[\bar{\pi}_i^z N_i^t - (\varphi_i^* + N_i^z)\bar{\pi}_i^t\right]}{\frac{1-\beta}{\beta}\bar{\pi}_i^z - (N_i^z + \varphi_i^*)}\mathrm{d}\log t_i \\ \equiv& -\xi_i^M \mathrm{d}\log M - \xi_i^t \mathrm{d}\log t_i.\end{aligned}$$

Then, we have solved for $\mathrm{d}\log M$:

$$\mathrm{d}\log M = -\frac{\sum_i s_i \xi_i^t \mathrm{d}\log t_i}{\sum_i s_i \xi_i^M}.$$

Note that $\varphi_i^* + N_i^z > 0$, $\bar{\pi}_i^z < 0$, $\bar{\pi}_i^t \leq 0$ and $N_i^t < 0$, we must have

$$\bar{\pi}_i^z N_i^t - (\varphi_i^* + N_i^z)\bar{\pi}_i^t = -\frac{\hat{\pi}_i(\sigma_i^* - 1)}{\hat{\pi}_i - f_i} N_i^t - \frac{\delta_i^*}{\bar{\delta}_i}\lambda_i^* \varphi_i^* \bar{\pi}_i^t > 0 \tag{46}$$

which implies that $\xi_i^t > 0$. Moreover, since $\bar{\pi}_i^z < 0$ and $\varphi_i^* + N_i^z > 0$, we must have $0 < \xi_i^M < 1$. Therefore, $\mathrm{d}\log M < 0$ when the tariff in market i increases.

With the CES demand, ξ_i^t and ξ_i^M take the following expressions:

$$\begin{aligned}\xi_i^M =& \frac{\frac{1-\beta}{\beta}\bar{\pi}_i^z}{\frac{1-\beta}{\beta}\bar{\pi}_i^z - (N_i^z + \varphi_i^*)} = \frac{\frac{\hat{\pi}_i}{\hat{\pi}_i - f_i}\eta_i\theta_i}{\frac{\hat{\pi}_i}{\hat{\pi}_i - f_i}\eta_i\theta_i + \frac{\beta}{1-\beta}\left(1 + \frac{\eta_i\theta_i}{\sigma_i - 1}\lambda_i^*\varphi_i^*\right)}, \\ \xi_i^t =& -\frac{\frac{1-\beta}{\beta}\left[\bar{\pi}_i^z N_i^t - (\varphi_i^* + N_i^z)\bar{\pi}_i^t\right]}{\frac{1-\beta}{\beta}\bar{\pi}_i^z - (N_i^z + \varphi_i^*)} = -\xi_i^M N_i^t = \xi_i^M \frac{1}{\theta_i},\end{aligned}$$

where we have used that $\eta_i\theta_i = \frac{\sigma_i - \eta_i}{\sigma_i - 1}$ from Section 3.2.2.

Signs of $\mathrm{d}\log z_i^*$ We then go back to the case with $\mathrm{d}\log z_i^*$. Since $\bar{\pi}_i^z < 0$ and thus $z_i^M > 0$. Then, $\mathrm{d}\log z_i^* < 0$ in the third market unambiguously. The case with the focal market is more complicated since z_i^t can be either positive (when β is close to one) or negative (when β is close to zero). In general, we cannot derive an unambiguous result on the sign of $\mathrm{d}\log z_j^*$.

In other words, the productivity threshold of exiting may fall or increase in the focal market.

$$\mathrm{d}\log z_j^* = z_j^M \left[\left(\frac{s_j \xi_j^M}{\sum_i s_i \xi_i^M} - 1 \right) N_j^t + \bar{\pi}_j^t \left(\frac{1-\beta}{\beta} - \frac{s_j \xi_j^M}{\sum_i s_i \xi_i^M} \frac{\varphi_j^* + N_j^z}{\bar{\pi}_j^z} \right) \right] \mathrm{d}\log t_j \tag{47}$$

With the CES demand (and thus $\bar{\pi}_j^t = 0$), $\mathrm{d}\log z_j^* = z_j^M \left(\frac{s_j \xi_j^M}{\sum_i s_i \xi_i^M} - 1 \right) N_j^t \log t_j > 0$, as $z_j^M > 0$, $N_j^t < 0$ and $\mathrm{d}\log t_j$. In other words, the exit cutoff increases in the focal market unambiguously.

Entry Interdependence in the focal market Let $\tilde{\beta} = \frac{1-\beta}{\beta}$. Partially differentiating equation (29) with respect to $\tilde{\beta}$, we obtain that

$$\begin{aligned} \frac{\partial \mathrm{d}\log z_j^*}{\partial \tilde{\beta}} &= \frac{\bar{\pi}^t (N^z + \varphi^*) + \bar{\pi}^z \left(-\frac{N}{N+1} N^t - \frac{1}{N+1} \frac{\bar{\pi}^t}{\bar{\pi}^z} (\varphi^* + N^z) \right)}{\left[(N^z + \varphi^*) - \frac{1-\beta}{\beta} \bar{\pi}^z \right]^2} \\ &= \frac{\frac{N}{N+1} \left(\bar{\pi}^t (N^z + \varphi^*) - \bar{\pi}^z N^t \right)}{\left[(N^z + \varphi^*) - \frac{1-\beta}{\beta} \bar{\pi}^z \right]^2} < 0, \end{aligned}$$

as $\varphi_i^* + N_i^z > 0$, $\bar{\pi}_i^z < 0$, $\bar{\pi}_i^t \leq 0$ and $N_i^t < 0$. Since $\frac{\mathrm{d}\tilde{\beta}}{\mathrm{d}\beta} < 0$, we obtain $\frac{\partial \mathrm{d}\log z_j^*}{\partial \beta} > 0$.

Changes in M_i^p For the potential entrants M_i^p,

$$\mathrm{d}\log M_i^p = (1 - \xi_i^M) \mathrm{d}\log M - \xi_i^t \mathrm{d}\log t_i.$$

Therefore, $\mathrm{d}\log M_i^p < 0$ in both focal and third markets.

Changes in Q_i Finally, we consider the welfare change:

$$\mathrm{d}\log Q_i = Q_i^z \mathrm{d}\log z_i^* + \frac{\bar{\delta}_i}{\gamma_i} \mathrm{d}\log t_i.$$

In the third market, $\mathrm{d}\log Q_i < 0$ since $\mathrm{d}\log z_i^* < 0$ and $Q_i^z > 0$. In the focal market j,

$$\mathrm{d}\log Q_j = \left\{ Q_j^z z_j^M \left[\left(\frac{s_j \xi_j^M}{\sum_i s_i \xi_i^M} - 1 \right) N_j^t + \bar{\pi}_j^t \left(\frac{1-\beta}{\beta} - \frac{s_j \xi_j^M}{\sum_i s_i \xi_i^M} \frac{\varphi_j^* + N_j^z}{\bar{\pi}_j^z} \right) \right] + \frac{\bar{\delta}_j}{\gamma_j} \right\} \mathrm{d}\log t_j.$$

We show that the bracket is negative by showing that

$$Q_j^z z_j^M \left(\frac{s_j \xi_j^M}{\sum_i s_i \xi_i^M} - 1 \right) N_j^t + \frac{\bar{\delta}_j}{\gamma_j} = \frac{1}{\gamma_j} \left[Q_j^z z_j^M \left(\frac{s_j \xi_j^M}{\sum_i s_i \xi_i^M} - 1 \right) \frac{\mathbb{E}_{\lambda_j}^j [\rho_j \sigma_j]}{\sigma_j^*} + \bar{\delta}_j \right] < 0,$$

since the left term is negative. Note that $z_j^M \leq \frac{1}{N_i^z + \varphi_i^*}$,

$$\bar{\delta}_i + Q_j^z z_j^M \left(\frac{s_j \xi_j^M}{\sum_i s_i \xi_i^M} - 1 \right) \frac{\mathbb{E}_{\lambda_j}^j [\rho_j \sigma_j]}{\sigma_j^*} \geq \bar{\delta}_i - Q_j^z z_j^m \frac{\mathbb{E}_{\lambda_j}^j [\rho_j \sigma_j]}{\sigma_j^*} \geq \bar{\delta}_i - \frac{Q_j^z}{N_i^z + \varphi_i^*} \frac{\mathbb{E}_{\lambda_j}^j [\rho_j \sigma_j]}{\sigma_j^*}.$$

With $\gamma_j < 0$, it suffices to show that $\bar{\delta}_j > \frac{Q_j^z}{N_i^z + \varphi_i^*} \frac{\mathbb{E}_{\lambda_j}^j [\rho_j \sigma_j]}{\sigma_j^*}$. We can rewrite $N_j^z + \varphi_j^*$ as follows

$$N_j^z + \varphi_j^* = \frac{1}{\bar{\delta}_j} \left(\frac{\mathbb{E}_{\lambda_j}^j [\rho_i \sigma_j]}{\sigma_j^*} \left[(\sigma_j^* - 1) + Q_j^z \right] + \delta_j^* \lambda_j^* \varphi_j^* \right)$$

Then,

$$\frac{Q_j^z}{N_i^z + \varphi_i^*} \frac{\mathbb{E}_{\lambda_j}^j [\rho_j \sigma_j]}{\sigma_j^*} = \bar{\delta}_j \frac{\frac{\mathbb{E}_{\lambda_j}^j [\rho_j \sigma_j]}{\sigma_j^*} Q_j^z}{\frac{\mathbb{E}_{\lambda_j}^j [\rho_i \sigma_j]}{\sigma_j^*} \left[(\sigma_j^* - 1) + Q_j^z \right] + \delta_j^* \lambda_j^* \varphi_j^*} < \bar{\delta}_j.$$

We then complete the proof that $\mathrm{d} \log Q_j < 0$. Regardless of threshold changes, welfare loss in the focal market is certain.

Changes in P_i It is useful to note that in the focal market,

$$\begin{aligned} &(\sigma_j^* - 1) \mathrm{d} \log z_j^* + \mathrm{d} \log Q_j \\ &= \left\{ \left((\sigma_j^* - 1) + Q_j^z \right) z_j^M \left[\left(\frac{s_j \xi_j^M}{\sum_i s_i \xi_i^M} - 1 \right) N_j^t + \bar{\pi}_j^t \left(\frac{1 - \beta}{\beta} - \frac{s_j \xi_j^M}{\sum_i s_i \xi_i^M} \frac{\varphi_j^* + N_j^z}{\bar{\pi}_j^z} \right) \right] + \frac{\bar{\delta}_j}{\gamma_j} \right\} \mathrm{d} \log t_j < 0, \end{aligned}$$

where the negative sign comes from the same arguments that prove $\mathrm{d} \log Q_j < 0$. Then, $(\sigma_i^* - 1) \mathrm{d} \log z_i^* + \mathrm{d} \log Q_i < 0$ for all country i. Recall that

$$\mathrm{d} \log P_i - \mathrm{d} \log t_i = -\frac{1}{\sigma_i^*} \left[(\sigma_i^* - 1) \mathrm{d} \log z_i^* + \mathrm{d} \log Q_i \right].$$

Then, we observe $\mathrm{d} \log P_i > 0$ in both focal and third markets.

Aside: Conditions on the sign of $\bar{\pi}_i^z$ Note that

$$\bar{\pi}_i^z = -\frac{\hat{\pi}_i(\sigma_i^* - 1)}{\hat{\pi}_i - f_i} + \frac{\sigma_i^*\hat{\pi}_i - \hat{r}_i}{\sigma_i^*(\hat{\pi}_i - f_i)}\left[(\sigma_i^* - 1) + Q_i^z\right] = -\frac{\hat{r}_i(\sigma_i^* - 1)}{\sigma_i^*(\hat{\pi}_i - f_i)} + \frac{(\sigma_i^*\hat{\pi}_i - \hat{r}_i)Q_i^z}{\sigma_i^*(\hat{\pi}_i - f_i)}.$$

Then

$$\bar{\pi}_i^z < 0 \iff \frac{\frac{\sigma_i^*\hat{\pi}_i}{\hat{r}_i} - 1}{\sigma_i^* - 1}Q_i^z < 1.$$

That is, whenever $\frac{\sigma_i^*\hat{\pi}_i}{\hat{r}_i}$ is small (close to one), $\bar{\pi}_i^z < 0$. In the CES case, $\frac{\sigma_i^*\hat{\pi}_i}{\hat{r}_i} = 1$.

D.2.2 Changes in firm-level variables

Changes in π_i and $\bar{\pi}_i$ For profit,

$$\begin{aligned}\mathrm{d}\log\pi_i &= \sigma_i\left[\mathrm{d}\log P_i - \mathrm{d}\log t_i\right] + \mathrm{d}\log Q_i \\ &= -\frac{\sigma_i}{\sigma_i^*}\left[(\sigma_i^* - 1)\mathrm{d}\log z_i^* + \mathrm{d}\log Q_i\right] + \mathrm{d}\log Q_i.\end{aligned}$$

Then, $\mathrm{d}\log\pi_i$ always increases with σ_i. That is, conditional on survival, small firms (high σ_i) gain relative to large firms (low σ_i) in log term. We now consider the expected profit $\bar{\pi}_i$:

$$\mathrm{d}\log\bar{\pi}_i = \frac{\beta}{1-\beta}\left[\mathrm{d}\log N_i - \mathrm{d}\log M + \varphi_i^*\mathrm{d}\log z_i^*\right] = \frac{\beta}{1-\beta}\xi_j^t\left[\frac{s_j\xi_i^M}{\sum_k s_k\xi_k^M} - \mathbb{1}\{i=j\}\right]\mathrm{d}\log t_j$$

The expected profit drops in the focal market while increases in the third market. Whereas, the sign of average profit $\hat{\pi}_i$ is rather ambiguous. Observe that

$$\mathrm{d}\log\hat{\pi}_i = \frac{\hat{\pi}_i - f_i}{\hat{\pi}_i}\left(\varphi_i^*\mathrm{d}\log z_i^* + \mathrm{d}\log\bar{\pi}_i\right) = \frac{\hat{\pi}_i - f_i}{\hat{\pi}_i}\left((\varphi_i^* + \bar{\pi}_i^z)\mathrm{d}\log z_i^* + \bar{\pi}_i^t\mathrm{d}\log t_i\right),$$

where the sign of $\varphi_i^* + \bar{\pi}_i^z$ is undetermined. Thus, the average profit may fall or increase even with the CES demand.

Changes in p_i and $\frac{q_i}{Q_i}$ We can discuss the implication on average revenue and quantity. Note that

$$\begin{aligned}
\mathrm{d}\log\frac{q_i}{Q_i} &= \rho_i\sigma_i\left[\mathrm{d}\log P_i - \mathrm{d}\log t_i\right] > 0, \\
\mathrm{d}\log\mu_i = \left(\frac{1}{\rho_i}-1\right)\frac{1}{\sigma_i}\mathrm{d}\log\frac{q_i}{Q_i} &= (1-\rho_i)\left[\mathrm{d}\log P_i - \mathrm{d}\log t_i\right] > 0.
\end{aligned}$$

Therefore, markup μ_i increases among all firms and in both focal and third markets. So does the price p_i, and larger firms respond more in prices.

Changes in r_i For sales,

$$\begin{aligned}
\mathrm{d}\log r_i &= -\frac{1}{\mu_i - 1}\mathrm{d}\log\mu_i + \mathrm{d}\log\pi_i \\
&= \left[1+\rho_i(\sigma_i-1)\right]\left[\mathrm{d}\log P_i - \mathrm{d}\log t_i\right] + \mathrm{d}\log Q_i \\
&= -\frac{1+\rho_i(\sigma_i-1)}{\sigma_i^*}\left[(\sigma_i^*-1)\mathrm{d}\log z_i^* + \mathrm{d}\log Q_i\right] + \mathrm{d}\log Q_i \\
&= -\frac{1+\rho_i(\sigma_i-1)}{\sigma_i^*}(\sigma_i^*-1)\mathrm{d}\log z_i^* + \frac{(\sigma_i^*-1)-\rho_i(\sigma_i-1)}{\sigma_i^*}\mathrm{d}\log Q_i
\end{aligned}$$

In third market, the first term is positive and the second negative. In the focal market, $\mathrm{d}\log r_j < 0$ if $\mathrm{d}\log z_j^* > 0$. But we do not know the sign when $\mathrm{d}\log z_j^* < 0$. With the CES demand, $\mathrm{d}\log r_i = -(\sigma_i^*-1)\mathrm{d}\log z_i^*$. That is, sales increases in third markets while decreases in the focal market.

Changes in q_i For quantity,

$$\begin{aligned}
\mathrm{d}\log q_i &= \rho_i\sigma_i\left[\mathrm{d}\log P_i - \mathrm{d}\log t_i\right] + \mathrm{d}\log Q_i \\
&= -\frac{\rho_i\sigma_i}{\sigma_i^*}(\sigma_i^*-1)\mathrm{d}\log z_i^* + \frac{\sigma_i^*-\rho_i\sigma_i}{\sigma_i^*}\mathrm{d}\log Q_i
\end{aligned}$$

The analysis is identical to that on sales. With the CES demand, $\mathrm{d}\log q_i = -(\sigma_i^*-1)\mathrm{d}\log z_i^*$. In both quantity and sales, small firms gain relative to large firms.

D.2.3 Changes in other aggregate variables

We use f_x^i to denote the density function of variables in market i, i.e., $f_x^i(z) = \frac{x(z)\frac{g(z)}{1-G(z)}}{\int_{z_i^*}^{\infty} x(z)\frac{g(z)}{1-G(z)}\mathrm{d}z}$. In particular, the welfare density of firms with productivity z in market i is given by

$$f_\Upsilon^i(z) = \frac{\Upsilon\left(\frac{q_i(z)}{Q_i}\right)\frac{g(z)}{1-G(z)}}{\int_{z_i^*}^{\infty}\Upsilon\left(\frac{q_i(z)}{Q_i}\right)\frac{g(z)}{1-G(z)}\mathrm{d}z} = N_i\Upsilon\left(\frac{q_i(z)}{Q_i}\right),$$

where we have used the definition of Kimball aggregation. Furthermore, we define that $f_x \succcurlyeq f_y$ if the distribution of x first-order stochastically dominates that of y. Finally, we let $\bar{x}_i = \int_{z_i^*}^{\infty} x(z)\frac{g(z)}{1-G(z)}\mathrm{d}z$. The following lemmas turn out to be useful in later analysis.

Lemma 1. *In any market i, $f_q^i \succcurlyeq f_r^i$, and $f_q^i \succcurlyeq f_\Upsilon^i$.*

Proof. It is well known that the monotone likelihood ratio property (MLRP) implies first order stochastic dominance. Namely, if $\frac{f_x}{f_y}$ increases in z, then $f_x \succcurlyeq f_y$. Note that $\frac{r_i(z)}{q_i(z)} = p_i(z)$ and p is decreasing on z, then $f_q^i \succcurlyeq f_r^i$. Since Υ is concave, $\Upsilon\left(\frac{q_i}{Q_i}\right)/q_i$ decreases on z, $f_q^i \succcurlyeq f_\Upsilon^i$. ■

Lemma 2. *In any market i, $\lambda_i^* = \frac{r_i^*}{\bar{r}_i} > \frac{q_i^*}{\bar{q}_i}$, and $\frac{\delta_i^*}{\bar{\delta}_i}\lambda_i^* > \frac{q_i^*}{\bar{q}_i}$.*

Proof. Consider x and y such that $\frac{f_x}{f_y}$ satisfies the MLRP. Then,

$$\frac{x_i^*}{\bar{x}_i} - \frac{y_i^*}{\bar{y}_i} = \frac{1}{\bar{x}_i\bar{y}_i}\left(\int_{z_i^*}^{\infty}(x_i^* y(z) - y_i^* x(z))\frac{g(z)}{1-G(z)}\mathrm{d}z\right) < 0$$

since $\frac{x_i^*}{y_i^*} < \frac{x(z)}{y(z)}$ for $z > z^*$. Using this relationship, we obtain $\lambda_i^* > \frac{q_i^*}{\bar{q}_i}$, and $N_i\Upsilon\left(\frac{q_i(z)}{Q_i}\right) > \frac{q_i^*}{\bar{q}_i}$. Also,

$$N_i\Upsilon\left(\frac{q_i(z)}{Q_i}\right) = N_i\delta_i\Upsilon'\left(\frac{q_i^*}{Q_i}\right)\frac{q_i^*}{Q_i} = \frac{N_i\delta_i t_i r_i^*}{\bar{\delta}_i E_i} = \frac{\delta_i^*}{\bar{\delta}_i}\lambda_i^*$$

■

Changes in R_i Aggregate sales drop in all markets. To see this,

$$\mathrm{d}\log R_i = \left(1 - \frac{1}{\eta_i}\right)\mathrm{d}\log Q_i - \mathrm{d}\log t_i < 0.$$

Since N_i may become larger or smaller, the sign average revenue r_i is ambiguous. This will also be true even with the CES demand. In the third market, while firms may export more value, small exporters also enter.

Changes in $\tilde{Q}_i$ The total quantity $\tilde{Q}_i = N_i \int_{z_i^*}^{\infty} q_i \frac{g(z)}{1-G(z_i^*)} \mathrm{d}z$. Then,

$$\mathrm{d}\log \tilde{Q}_i = \mathrm{d}\log N_i + \varphi_i^* \mathrm{d}\log z_i^* - \frac{q_i^*}{\bar{q}_i}\varphi_i^* \mathrm{d}\log z_i^* + \mathbb{E}_{q_i}^i[\mathrm{d}\log q_i],$$

where

$$\mathbb{E}_{q_i}^i[\mathrm{d}\log q_i] = -\mathbb{E}_{q_i}^i[\rho_i\sigma_i]\frac{\sigma_i^*-1}{\sigma_i^*}\mathrm{d}\log z_i^* + \left(1 - \frac{\mathbb{E}_{q_i}^i[\rho_i\sigma_i]}{\sigma_i^*}\right)\mathrm{d}\log Q_i,$$

and $\bar{q}_i$ is the average quantity. Using the welfare condition and the expression of $\mathrm{d}\log \frac{q_i}{Q_i}$, we obtain

$$\begin{aligned}
\mathrm{d}\log \tilde{Q}_i &= \left(\frac{\delta_i^*}{\bar{\delta}_i}\lambda_i^* - \frac{q_i^*}{\bar{q}_i}\right)\varphi_i^* \mathrm{d}\log z_i^* + \left(-\frac{1}{\bar{\delta}_i}\mathbb{E}_{\lambda_i}^i[\rho_i\sigma_i] + \mathbb{E}_{q_i}^i[\rho_i\sigma_i]\right)[\mathrm{d}\log P_i - \mathrm{d}\log t_i] + \mathrm{d}\log Q_i \\
&\leq \left(\frac{\delta_i^*}{\bar{\delta}_i}\lambda_i^* - \frac{q_i^*}{\bar{q}_i}\right)\varphi_i^* \mathrm{d}\log z_i^* - \left(1 - \frac{1}{\bar{\delta}_i}\right)\frac{\mathbb{E}_{\lambda_i}^i[\rho_i\sigma_i]}{\sigma_i^*}[(\sigma_i^*-1)\mathrm{d}\log z_i^* + \mathrm{d}\log Q_i] + \mathrm{d}\log Q_i \\
&= \left(\frac{\delta_i^*}{\bar{\delta}_i}\lambda_i^* - \frac{q_i^*}{\bar{q}_i}\right)\varphi_i^* \mathrm{d}\log z_i^* + \left\{-\left(1 - \frac{1}{\bar{\delta}_i}\right)\frac{\mathbb{E}_{\lambda_i}^i[\rho_i\sigma_i]}{\sigma_i^*}[(\sigma_i^*-1) + Q_i^z] + Q_i^z\right\}\mathrm{d}\log z_i^* \\
&\quad + \left\{-\left(1 - \frac{1}{\bar{\delta}_i}\right)\frac{\mathbb{E}_{\lambda_i}^i[\rho_i\sigma_i]}{\sigma_i^*} + 1\right\}\frac{\bar{\delta}_i}{\gamma_i}\mathrm{d}\log t_i,
\end{aligned}$$

where the first inequality uses that $\mathbb{E}_{q_i}^i[\rho_i\sigma_i] \leq \mathbb{E}_{\lambda_i}^i[\rho_i\sigma_i]$, a result of Lemma 1, and the second equality uses the expression of $\mathrm{d}\log Q_i$ in $\mathrm{d}\log z_i^*$.

We show first that in third market, $\mathrm{d}\log \tilde{Q}_i < 0$. It suffices to show that the coefficient of $\mathrm{d}\log z_i^*$ is positive. As Lemma 2 shows that $\frac{\delta_i^*}{\bar{\delta}_i}\lambda_i^* > \frac{q_i^*}{\bar{q}_i}$, we only need that

$$f(\bar{\delta}_i) \equiv -\left(1 - \frac{1}{\bar{\delta}_i}\right)\frac{\mathbb{E}_{\lambda_i}^i[\rho_i\sigma_i]}{\sigma_i^*}[(\sigma_i^*-1) + Q_i^z] + Q_i^z > 0.$$

Using the definition of Q_i^z,

$$\begin{aligned}\gamma_i f(\bar{\delta}_i) =& \left(1-\frac{1}{\bar{\delta}_i}\right)\frac{\mathbb{E}^i_{\lambda_i}[\rho_i\sigma_i]}{\sigma_i^*}\left((\sigma_i^*-1)\bar{\delta}_i+(\bar{\delta}_i-\delta_i^*)\lambda_i^*\varphi_i^*\right)\\ &-\frac{\sigma_i^*-1}{\sigma_i^*}\left(\bar{\delta}_i+\bar{\delta}_i\,\mathbb{E}^i_{\lambda_i}[(\sigma_i-1)\rho_i]-\mathbb{E}^i_{\lambda_i}[\rho_i\sigma_i]\right)-(\bar{\delta}_i-\delta_i^*)\lambda_i^*\varphi_i^*\\ =&-\frac{\sigma_i^*-1}{\sigma_i^*}\left[\bar{\delta}_i\left(1+\mathbb{E}^i_{\lambda_i}[(\sigma_i-1)\rho_i]-\left(1-\frac{1}{\bar{\delta}_i}\right)\mathbb{E}^i_{\lambda_i}[\rho_i\sigma_i]\right)-\mathbb{E}^i_{\lambda_i}[\rho_i\sigma_i]\right]\\ &-\left[1-\left(1-\frac{1}{\bar{\delta}_i}\right)\frac{\mathbb{E}^i_{\lambda_i}[\rho_i\sigma_i]}{\sigma_i^*}\right](\bar{\delta}_i-\delta_i^*)\lambda_i^*\varphi_i^*.\end{aligned}$$

Note that

$$\begin{aligned}&-\frac{\sigma_i^*-1}{\sigma_i^*}\left[\bar{\delta}_i\left(1+\mathbb{E}^i_{\lambda_i}[(\sigma_i-1)\rho_i]-\left(1-\frac{1}{\bar{\delta}_i}\right)\mathbb{E}^i_{\lambda_i}[\rho_i\sigma_i]\right)-\mathbb{E}^i_{\lambda_i}[\rho_i\sigma_i]\right]\\ =&-\frac{\sigma_i^*-1}{\sigma_i^*}\bar{\delta}_i\left(1+\mathbb{E}^i_{\lambda_i}[(\rho_i-)\sigma_i]-\mathbb{E}^i_{\lambda_i}[\rho_i\sigma_i]\right)\\ =&-\frac{\sigma_i^*-1}{\sigma_i^*}\bar{\delta}_i\left(1-\mathbb{E}^i_{\lambda_i}[\rho_i]\right)<0,\end{aligned}$$

and

$$-\underbrace{\left[1-\left(1-\frac{1}{\bar{\delta}_i}\right)\frac{\mathbb{E}^i_{\lambda_i}[\rho_i\sigma_i]}{\sigma_i^*}\right]}_{>0}\underbrace{(\bar{\delta}_i-\delta_i^*)}_{>0}\lambda_i^*\varphi_i^*<0.$$

Then, $\gamma_i f(\bar{\delta}_i)<0$, and $f(\bar{\delta}_i)>0$.

Next, we show that $\mathrm{d}\log\tilde{Q}_j<0$ in the focal market. From the last step, we obtain

$$\mathrm{d}\log\tilde{Q}_j\leq A_j\mathrm{d}\log z_j^*+B_j\mathrm{d}\log t_j,$$

where

$$\begin{aligned}A_j=&\left(\frac{\delta_j^*}{\bar{\delta}_j}\lambda_j^*-\frac{q_j^*}{\bar{q}_j}\right)\varphi_j^*-\left(1-\frac{1}{\bar{\delta}_j}\right)\frac{\mathbb{E}^j_{\lambda_j}[\rho_j\sigma_j]}{\sigma_j^*}\left[(\sigma_j^*-1)+Q_j^z\right]+Q_j^z\\ B_j=&\left\{-\left(1-\frac{1}{\bar{\delta}_j}\right)\frac{\mathbb{E}^i_{\lambda_j}[\rho_i\sigma_j]}{\sigma_j^*}+1\right\}\frac{\bar{\delta}_j}{\gamma_j}.\end{aligned}$$

We shown in the third market case that $A_j \geq 0$. Using equation (47),

$$A_j \mathrm{d}\log z_j^* + B_j \mathrm{d}\log t_j = N_j^t \mathrm{d}\log t_j \left[A_j z_j^M \left[\left(\frac{s_j \xi_j^M}{\sum_i s_i \xi_i^M} - 1 \right) N_j^t + \bar{\pi}_j^t \left(\frac{1-\beta}{\beta} - \frac{s_j \xi_j^M}{\sum_i s_i \xi_i^M} \frac{\varphi_j^* + N_j^z}{\bar{\pi}_j^z} \right) \right] + \frac{B_j}{N_j} \right.$$

Since $\frac{\bar{\pi}_j^t}{N_j^t} \left(\frac{1-\beta}{\beta} - \frac{s_j \xi_j^M}{\sum_i s_i \xi_i^M} \frac{\varphi_j^* + N_j^z}{\bar{\pi}_j^z} \right) > 0$, it suffices to show that $-A_j z_j^M + \frac{B_j}{N_j} > 0$. Note that

$$\begin{aligned} -A_j z_j^M + \frac{B_j}{N_j} &= - \frac{(N_j^z + \varphi_j^*) - \frac{q_j^*}{\bar{q}_j}\varphi_j^* - (\sigma_j^* - 1) + \left(1 - \frac{\mathbb{E}_{\lambda_j}^j[\rho_j \sigma_j]}{\sigma_j^*}\right) \left[(\sigma_j^* - 1) + Q_j^z\right]}{(N_j^z + \varphi_j^*) - \frac{1-\beta}{\beta}\bar{\pi}_i^z} + 1 + \left(\frac{\sigma_j^*}{\mathbb{E}_{\lambda_j}^j[\rho_i \sigma_i]} - 1 \right) \bar{\delta} \\ &> \frac{\frac{q_j^*}{\bar{q}_j}\varphi_j^* + (\sigma_j^* - 1) - \left(1 - \frac{\mathbb{E}_{\lambda_j}^j[\rho_j \sigma_j]}{\sigma_j^*}\right) \left[(\sigma_j^* - 1) + Q_j^z\right]}{N_j^z + \varphi_j^*} + \left(\frac{\sigma_j^*}{\mathbb{E}_{\lambda_j}^j[\rho_i \sigma_i]} - 1 \right) \bar{\delta}_j \end{aligned}$$

Using the expression of $N_j^z + \varphi_j^*$ in $(\sigma_j^* - 1) + Q_j^z$,

$$\begin{aligned} \left(\frac{\sigma_j^*}{\mathbb{E}_{\lambda_j}^j[\rho_j \sigma_j]} - 1 \right) \bar{\delta}_j (N_j^z + \varphi_j^*) &= \left(\frac{\sigma_j^*}{\mathbb{E}_{\lambda_j}^j[\rho_j \sigma_j]} - 1 \right) \bar{\delta}_j \frac{1}{\bar{\delta}_j} \left(\frac{\mathbb{E}_{\lambda_j}^j[\rho_j \sigma_j]}{\sigma_j^*} \left[(\sigma_j^* - 1) + Q_j^z\right] + \delta_j^* \lambda_j^* \varphi_j^* \right) \\ &> \left(1 - \frac{\mathbb{E}_{\lambda_j}^j[\rho_j \sigma_j]}{\sigma_j^*}\right) \left[(\sigma_j^* - 1) + Q_j^z\right]. \end{aligned}$$

Therefore,

$$-A_j z_j^M + \frac{B_j}{N_j} > \frac{\frac{q_j^*}{\bar{q}_j}\varphi_j^* + (\sigma_j^* - 1)}{N_j^z + \varphi_i^*} > 0.$$

We then complete the proof that $\mathrm{d}\log \tilde{Q}_j < 0$.

Changes in p_i^m Finally, we consider the aggregate price level. Recall that $p_i^m = \frac{R_i}{Q_i} = \frac{\int_{z_i^*}^{\infty} r_i g(z) \mathrm{d}z}{\int_{z_i^*}^{\infty} q_i g(z) \mathrm{d}z}$, then

$$\begin{aligned} \mathrm{d}\log p_i^m &= \left(-\frac{r_i^*}{\bar{r}_i} + \frac{q_i^*}{\bar{q}_i} \right) \varphi_i^* \mathrm{d}\log z_i^* + \mathbb{E}_{r_i}^i[\mathrm{d}\log r_i] - \mathbb{E}_q[\mathrm{d}\log q_i] \\ &= \left(-\frac{r_i^*}{\bar{r}_i} + \frac{q_i^*}{\bar{q}_i} \right) \varphi_i^* \mathrm{d}\log z_i^* + \left(1 - \mathbb{E}_{r_i}^i[\rho_i] + \mathbb{E}_{r_i}^i[\rho_i \sigma_i] - \mathbb{E}_{q_i}^i[\rho_i \sigma_i]\right) (\mathrm{d}\log P_i - \mathrm{d}\log t_i) \end{aligned}$$

Lemma 2 shows that $-\frac{r_i^*}{\bar{r}_i} + \frac{q_i^*}{\bar{q}_i} < 0$, and Lemma 1 implies that $\mathbb{E}^i_{r_i}[\rho_i \sigma_i] > \mathbb{E}^i_{q_i}[\rho_i \sigma_i]$.

With $\mathbb{E}^i_{r_i}[\rho_i] \leq 1$ and $\mathrm{d}\log P_i - \mathrm{d}\log t_i > 0$, we proved that $\mathrm{d}\log p_i^m > 0$ if $\mathrm{d}\log z_i^* < 0$. That is, average price rises at the aggregate level in third markets. In the focal market, the sign is ambiguous. For example, we have shown that the average price will go up if goods are homogeneous. Whereas, it is immediate that the average price will go down with the CES demand ($\mathrm{d}\log z_i^* > 0$).

E Homothetic demand with a single aggregator (HAS)

In this section, we rewrite our model using a homothetic demand function with a single aggregator (HAS) which is introduced by Matsuyama and Ushchev (2017). We will show that all the qualitative results we have obtained using the Kimball demand continue to hold in the HSA case.

E.1 Setup

Households There are $N+1$ countries indexed by i, and $i \in \{0, 1, \ldots, N\}$ in which country 0 is the home country. There are also many sectors in the economy, but our analysis focuses on an industry equilibrium of one sector. The home country produces many varieties in this sector which are denoted by ω. We use Ω_i to denote the set of all varieties that are exported to country i, and Ω the set of all varieties in the home country.

In country i, consumers have HSA preference towards exported varieties ω given a sectoral composite Q_i:

$$\int_{\Omega_i} s\left(\frac{p^i_\omega}{A_i}\right) \mathrm{d}\omega = 1,$$

where A_i is price aggregate that differs from the ideal price and $s(.)$ is the (revenue) market share function. Note that

$$\frac{d\log P_i^I}{d\log p^i_\omega} = s\left(\frac{p^i_\omega}{A_i}\right).$$

The ideal price index can be calculated

$$\log\left(P_i^I\right) = C + \log(A_i) - \int_{\omega\in\Omega_i}\int_{\frac{p^i_\omega}{A_i}}^{\infty} \frac{s(\zeta)}{\zeta} d\zeta d\omega, \tag{48}$$

See Matsuyama and Ushchev (2017) for the detail.

Production We first solve the firm's optimal pricing. First, the industry-level demand function is

$$P_i^I = D_i(Q_i) = D_i Q_i^{-\frac{1}{\eta_i}} \Leftrightarrow E_i \equiv E_i(Q_i) = Q_i D_i(Q_i) = D_i Q_i^{1-\frac{1}{\eta_i}} = D_i^{\eta_i} P_i^{I\,1-\eta_i}.$$

Thus, the objective function of the firm can be written as

$$\max_{p_\omega^i} D_i^{\eta_i} P_i^{1-\eta_i} s\left(\frac{t_i p_\omega^i}{A_i}\right) \frac{p_\omega^i - \frac{\tau_i}{z}}{t_i p_\omega^i},$$

where p_ω^i is the FOB price and $t_i p_\omega^i$ is the CIF price. Note that $\frac{D_i^{\eta_i} P_i^{1-\eta_i} s\left(\frac{t_i p_\omega^i}{A_i}\right)}{t_i p_\omega^i}$ is the output supplied by firm ω. The FOC is

$$s^{'}\left(\frac{t_i p_\omega^i}{A_i}\right) \frac{t_i}{A_i} \frac{p_\omega^i - \frac{\tau_i}{z}}{p_\omega^i} + s\left(\frac{t_i p_\omega^i}{A_i}\right) \frac{\frac{\tau_i}{z}}{\left(p_\omega^i\right)^2} = 0,$$

which is equivalent to

$$s^{'}\left(\frac{t_i p_\omega^i}{A_i}\right) \frac{t_i p_\omega^i}{A_i} \left(p_\omega^i - \frac{\tau_i}{z}\right) + s\left(\frac{t_i p_\omega^i}{A_i}\right) \frac{\tau_i}{z} = 0,$$

leads to the optimal pricing rule as

$$t_i p_\omega^i = \frac{\sigma(\upsilon)}{\sigma(\upsilon) - 1} \frac{t_i \tau}{z}$$

where $\upsilon \equiv \frac{t_i p_\omega^i}{A_i}$. Note that we have used the definition of the elasticity of substitution:

$$\sigma(\upsilon) = 1 - \frac{\upsilon s^{'}(\upsilon)}{s(\upsilon)}.$$

In addition, let μ be the markup, i.e., the ratio between price and marginal cost. Then,

$$\mu_i = \frac{\sigma_i}{\sigma_i - 1},$$

and we define $\mu\left(\frac{p_\omega^i}{A}\right)$ accordingly. As usual, we assume that $\sigma_i > 1$ so markup is positive.

We now summarize the set of variables related to firms' production. For a firm with

productivity z, its price, markup, and quantity in country i are determined implicitly by

$$t_i p_z^i = \frac{\mu_z^i t_i \tau_i}{z}, \quad q_z^i = q_i\left(\frac{t_i p_z^i}{A_i}\right) \equiv q_i\left(\frac{p_z^i}{P_i}\right), \quad \text{and } \mu_z^i = \mu\left(\frac{t_i p_z^i}{A_i}\right) \equiv \mu\left(\frac{p_z^i}{P_i}\right).$$

where $P_i \equiv \frac{A_i}{t_i}$ is the (producer) price aggregator while A_i is the (consumer) price aggregator. $R_i = E_i/t_i$ is the aggregate export revenue in country i. Comparing to the CES case, markup is no longer constant, so the pass-through of tariff will be incomplete.

The per period variable profits and sales are as follows

$$\pi_z^i = r_z^i\left(1 - \frac{1}{\mu_z^i}\right), \quad r_z^i = p_z^i q_z^i = \mu_z^i l_z^i,$$

as the wage rate is normalized to a constant in various markets. Then, firms export to market i if

$$\pi_z^i \geq f_i$$

Since the profit function is increasing and continuous, there exists a threshold z_i^* such that firms export to country i if and only if $z \geq z_i^*$. In the rest of the paper, we only consider the case in which z_i^* is interior, i.e., $\pi_{z_i^*}^i = f_i$.

Finally, there is an exogenous exit shock ν_i in each country. Firms discount future profits at a rate of ρ. Then, the expected entry profit of country i is defined as in equation (11), which is the same as in the HDIA case.

Entry and Exit This part is the same as in the Kimball case.

Equilibrium This part is the same as in the Kimball case.

Infra-marginal surplus ratio and pass-throughs The infra-marginal surplus ratio in the HSA case is defined as:

$$\delta_i = \delta\left(\frac{p_i}{P_i}\right) = 1 + \frac{\int_{\frac{p_i}{P_i}}^{\infty} \frac{s(\zeta)}{\zeta} d\zeta}{s\left(\frac{p_i}{P_i}\right)} > 1,$$

where $P_i = \frac{A_i}{t_i}$ and A_i is the (consumer) price aggregator. δ_i is the ratio of consumer surplus to revenue evaluated at (p_i, q_i). In the case of homogeneous firms where $s\left(\frac{p_i}{P_i}\right) = \frac{1}{N_i}$, we

have

$$\delta_i = 1 + \int_{\omega \in \Omega_i} \int_{\frac{p_i}{P_i}}^{\infty} \frac{s(\zeta)}{\zeta} d\zeta d\omega = 1 + C + \log(A_i) - \log\left(P_i^I\right),$$

where we have used equation (48). We define the individual pass-through of a variety as the elasticity of its price to its productivity:

$$\rho = -\frac{\mathrm{d}\log p}{\mathrm{d}\log z} = 1 - \frac{\mathrm{d}\log \mu}{\mathrm{d}\log z},$$

in which the latter equality holds fixing τ_i and t_i. $\rho > 0$ due to the supermodularity of the profit function. Note that

$$\mu_i = 1 + \frac{s(\upsilon)}{-\upsilon s'(\upsilon)}.$$

Therefore, we have

$$\rho_i = \rho\left(\frac{p_i}{P_i}\right) = \frac{1}{1 - \frac{\mu'\left(\frac{p_i}{P_i}\right)\frac{p_i}{P_i}}{\mu\left(\frac{p_i}{P_i}\right)}}.$$

In addition, we continue to assume Assumption 1 holds.

E.2 Homogeneous firms

E.2.1 Solving the equilibrium

We start with the case of homogeneous firms. In such a case, the productivity distribution G degenerates into a single mass point at z. We assume that in equilibrium, all markets have strictly positive measure of exporters.

First, we compute the profit in each market π_i. The HSA preference implies that

$$N_i s\left(\frac{p_i}{P_i}\right) = 1, \tag{49}$$

where $P_i \equiv \frac{A_i}{t_i}$. Note that

$$E_i = D_i^{\eta_i} {P_i^I}^{1-\eta_i}$$

and

$$\log\left(P_i^I\right) = C + \log(P_i) + \log(t_i) - N_i \int_{\frac{p^i}{P_i}}^{\infty} \frac{s(\zeta)}{\zeta} d\zeta. \tag{50}$$

Balanced budget in country i implies

$$E_i = D_i^{\eta_i} P_i^{I\,1-\eta_i} = N_i t_i p_i q_i \Leftrightarrow D_i^{\eta_i} P_i^{I\,1-\eta_i} = N_i t_i \mu \left(\frac{t_i p_i}{A_i}\right) \frac{s\left(\frac{t_i p_i}{A_i}\right) D_i^{\eta_i} P_i^{I\,1-\eta_i}}{t_i p_i} \frac{\tau_i}{z} \Leftrightarrow 1 = \mu \left(\frac{p_i}{P_i}\right) \frac{\frac{\tau_i}{z}}{p_i}. \tag{51}$$

Equations (49), (50) and (51) explicitly determine $\frac{p_i}{P_i}$, P_i^I and p_i as functions of N_i. Therefore, we may write σ_i, η_i, ρ_i, δ_i as functions of N_i. Similarly, we express profit π_i as a function of N_i with the following equation

$$\pi_i(N_i) = \left(1 - \frac{1}{\mu_i}\right) p_i q_i = \left(1 - \frac{1}{\mu_i}\right) \frac{E_i}{N_i t_i}. \tag{52}$$

(11) implies that $\bar{\pi}_i = \frac{\pi_i - f_i}{\rho + \nu_i}$.

Next, we solve for the equilibrium $(M, \{N_i\})$ using the free entry condition and balanced flow of firms. Namely,

$$\sum_{i=0}^{N} (h_i \bar{\pi}_i)^{\frac{1}{\beta}} = f_e^{\frac{1}{\beta}},$$

$$J_i M = \nu_i N_i.$$

There are $N+2$ equations for $N+2$ unknowns. Noticing that with (10), the above equations imply that

$$J_i = h_i \left(\frac{h_i \bar{\pi}_i}{f_e}\right)^{\frac{1-\beta}{\beta}} = \frac{\nu_i N_i}{M} \quad \Rightarrow \quad \pi_i(N_i) = (\rho + \nu_i) \left(\frac{\nu_i N_i}{h_i M}\right)^{\frac{\beta}{1-\beta}} \frac{f_e}{h_i} + f_i, \tag{53}$$

in which $\pi_i(N_i)$ is given implicitly in (52). Therefore, this equation gives us an implicit function N_i of M. We define such mapping by $N_i = N_i(M)$. Inserting it into the free entry equation, we pin down the equilibrium mass of entrants M as follows:

$$\sum_{i=0}^{N} \left(h_i \bar{\pi}_i(N_i(M))\right)^{\frac{1}{\beta}} = f_e^{\frac{1}{\beta}}. \tag{54}$$

In this way, we have solved the stationary equilibrium. We can show that a stationary equilibrium exists and is unique under Assumption 1 as in the Kimball case.

E.2.2 Propagation and aggregation equations with homogeneous firms

We expand the equilibrium equations presented in Section E.2 to the first order in the shocks.

Welfare

$$\mathrm{d}\log N_i - \left(\sigma_i\left(\frac{p_i}{P_i}\right) - 1\right)\mathrm{d}\log\frac{p_i}{P_i} = 0. \tag{55}$$

Definition of price aggregator

$$\mathrm{d}\log P_i^I = \mathrm{d}\log P_i + \mathrm{d}\log t_i + N_i s\left(\frac{p_i}{P_i}\right)\mathrm{d}\log\frac{p_i}{P_i} - \mathrm{d}\log N_i\left(\delta_i - 1\right).$$

Note that

$$\frac{\mathrm{d}\log\mu_i}{\mathrm{d}\log\frac{p_i}{P_i}} = -\left(\frac{1}{\rho_i} - 1\right),$$

$$N_i s\left(\frac{p_i}{P_i}\right) = 1,$$

and

$$\frac{\sigma_i - 1}{\frac{p_i}{P_i}} = -\frac{s^{'}(\upsilon)}{s(\upsilon)}$$

Therefore, the above equation can be simplified to

$$\mathrm{d}\log P_i^I = \mathrm{d}\log P_i + \mathrm{d}\log t_i + \mathrm{d}\log\frac{p_i}{P_i} - \mathrm{d}\log N_i\left(\delta_i - 1\right) \Rightarrow \mathrm{d}\log P_i^I = \mathrm{d}\log p_i + \mathrm{d}\log t_i - \mathrm{d}\log N_i\left(\delta_i - 1\right). \tag{56}$$

Markup

$$\mathrm{d}\log\mu_i = -\left(\frac{1}{\rho_i} - 1\right)\mathrm{d}\log\frac{p_i}{P_i}, \tag{57}$$

in which

$$\rho_i = \rho\left(\frac{p_i}{P_i}\right) = \frac{1}{1 - \frac{\mu'\left(\frac{p_i}{P_i}\right)\frac{p_i}{P_i}}{\mu\left(\frac{p_i}{P_i}\right)}}.$$

Pricing Pricing implies that

$$\mathrm{d}\log p_i = \mathrm{d}\log \mu_i = -\left(\frac{1}{\rho_i} - 1\right)\mathrm{d}\log\frac{p_i}{P_i},$$

where the fist part comes from the pricing rule and the second part comes from the definition of the passthrough.

Variable profit

$$\mathrm{d}\log \pi_i = \frac{1}{\mu_i - 1}\mathrm{d}\log\mu_i + (1-\eta_i)\mathrm{d}\log P_i^I - \mathrm{d}\log N_i - \mathrm{d}\log t_i$$

We can simplify the above expression to

$$\mathrm{d}\log \pi_i = -\left(\frac{1}{\rho_i} - 1\right)(\sigma_i - 1)\mathrm{d}\log\left(\frac{p_i}{P_i}\right) - (\eta_i - 1)\left[\mathrm{d}\log p_i + \mathrm{d}\log t_i - \mathrm{d}\log N_i\,(\delta_i - 1)\right] - \mathrm{d}\log N_i - \mathrm{d}\log t_i \tag{58}$$

Balanced flow

$$\frac{\pi_i \mathrm{d}\log\pi_i - f_i \mathrm{d}\log f_i}{\pi_i - f_i} = \mathrm{d}\log(\rho + \nu_i) + \frac{\beta}{1-\beta}\left[\mathrm{d}\log\nu_i + \mathrm{d}\log N_i - \mathrm{d}\log h_i - \mathrm{d}\log M\right] + \mathrm{d}\log f_e - \mathrm{d}\log h_i. \tag{59}$$

Free entry

$$\begin{aligned} &\frac{1}{1-\beta}\sum_{i=0}^{N}\frac{\left(\frac{\nu_i N_i}{h_i M}\right)^{\frac{1}{1-\beta}}}{\sum_k\left(\frac{\nu_k N_k}{h_k M}\right)^{\frac{1}{1-\beta}}}\left[\mathrm{d}\log N_i + \mathrm{d}\log\nu_i - \mathrm{d}\log h_i - \mathrm{d}\log M\right] = 0 \\ \Rightarrow \quad &\sum_{i=0}^{N} s_i\left[\mathrm{d}\log N_i + \mathrm{d}\log\nu_i - \mathrm{d}\log h_i\right] = \mathrm{d}\log M, \end{aligned} \tag{60}$$

where $s_i = \frac{J_i\bar{\pi}_i}{\sum_k J_k\bar{\pi}_k} = \frac{\left(\frac{\nu_i N_i}{h_i M}\right)^{\frac{1}{1-\beta}}}{\sum_k\left(\frac{\nu_k N_k}{h_k M}\right)^{\frac{1}{1-\beta}}}$.

E.2.3 Proof of comparative statics results when firms are homogeneous

In this subsection, we prove that all the results established in Proposition 2, 3 and Corollary 1 are still true when the demand function features HSA. Specifically, the change in M and

that of N_i take similar forms as in the Kimball case and $\mathrm{d}\log N_i$ is still the sufficient statistic for measuring changes in other variables in market i. The signs of changes established in Corollary 1 continue to hold now.

Changes in M and N_i We consider only $\mathrm{d}\log t_i$ and set $\mathrm{d}\log\frac{\tau_i}{z} = \mathrm{d}\log\nu_i = \mathrm{d}\log h_i = \mathrm{d}\log f_i = \mathrm{d}\log f_e = 0$. Then, we obtain the following system of equations

$$\begin{aligned}
\mathrm{d}\log N_i &= \left(\sigma_i\left(\frac{p_i}{P_i}\right)-1\right)\mathrm{d}\log\frac{p_i}{P_i}\\
\mathrm{d}\log P_i^I &= \mathrm{d}\log P_i + \mathrm{d}\log t_i + \mathrm{d}\log\frac{p_i}{P_i} - \mathrm{d}\log N_i\,(\delta_i-1)\\
\mathrm{d}\log\pi_i &= -\left(\frac{1}{\rho_i}-1\right)\mathrm{d}\log N_i - (\eta_i-1)\left[\mathrm{d}\log p_i + \mathrm{d}\log t_i - \mathrm{d}\log N_i\,(\delta_i-1)\right] - \mathrm{d}\log N_i - \mathrm{d}\log t_i\\
\mathrm{d}\log p_i &= \mathrm{d}\log\mu_i = -\left(\frac{1}{\rho_i}-1\right)\mathrm{d}\log\frac{p_i}{P_i}\\
\frac{\pi_i}{\pi_i-f_i}\mathrm{d}\log\pi_i &= \frac{\beta}{1-\beta}\left[\mathrm{d}\log N_i - \mathrm{d}\log M\right]\\
\sum_{i=0}^{N} s_i\mathrm{d}\log N_i &= \mathrm{d}\log M
\end{aligned}$$

Combining the first, third and fourth equations, we have

$$\mathrm{d}\log\pi_i = -\left(\frac{1}{\rho_i}-1\right)\mathrm{d}\log N_i - (\eta_i-1)\left[-\left(\frac{1}{\rho_i}-1\right)\mathrm{d}\log\frac{p_i}{P_i} + \mathrm{d}\log t_i - \mathrm{d}\log N_i\,(\delta_i-1)\right] - \mathrm{d}\log N_i - \mathrm{d}\log t_i$$

or

$$\mathrm{d}\log\pi_i = -\left(\frac{1}{\rho_i}-1\right)\frac{\sigma_i-\eta_i}{\sigma_i-1}\mathrm{d}\log N_i - \left[1-(\eta_i-1)(\delta_i-1)\right]\mathrm{d}\log N_i - \eta_i\mathrm{d}\log t_i,$$

where $1-(\eta_i-1)(\delta_i-1) > 1-(\eta_i-1)\frac{1}{\eta_i-1} = 0$. Therefore, we have

$$\mathrm{d}\log\pi_i = -\theta_i\mathrm{d}\log N_i - \eta_i\mathrm{d}\log t_i, \tag{61}$$

where

$$\theta_i \equiv \left(\frac{1}{\rho_i}-1\right)\frac{\sigma_i-\eta_i}{\sigma_i-1} + \left[1-(\eta_i-1)(\delta_i-1)\right] > 0.$$

Then, we obtain that

$$\frac{\pi_i}{\pi_i - f_i}\left[-\theta_i \mathrm{d}\log N_i - \eta_i \mathrm{d}\log t_i\right] = \frac{\beta}{1-\beta}\left[\mathrm{d}\log N_i - \mathrm{d}\log M\right]$$
$$\Rightarrow \mathrm{d}\log N_i = \frac{\frac{\beta}{1-\beta}}{\frac{\beta}{1-\beta} + \frac{\pi_i}{\pi_i - f_i}\theta_i}\mathrm{d}\log M - \frac{\eta_i \frac{\pi_i}{\pi_i - f_i}}{\frac{\beta}{1-\beta} + \frac{\pi_i}{\pi_i - f_i}\theta_i}\mathrm{d}\log t_i$$

Using the free entry condition,

$$\mathrm{d}\log N_i - \mathrm{d}\log M = -\xi_i \mathrm{d}\log M - \xi_i \frac{\eta_i}{\theta_i}\mathrm{d}\log t_i,$$

where

$$\xi_i = \frac{\frac{\pi_i}{\pi_i - f_i}\theta_i}{\frac{\beta}{1-\beta} + \frac{\pi_i}{\pi_i - f_i}\theta_i} \leq 1,$$

Then, we have

$$\sum_{i=0}^{N} s_i \mathrm{d}\log N_i = \mathrm{d}\log M \Rightarrow \mathrm{d}\log M = -\frac{\sum_{i=0}^{N} s_i \xi_i \frac{\eta_i}{\theta_i}\mathrm{d}\log t_i}{\sum_{i=0}^{N} s_i \xi_i} = -\mathbb{E}_{s\xi}\left[\frac{\eta}{\theta}\mathrm{d}\log t\right].$$

Hence, we can re-write $\mathrm{d}\log N_i$ as

$$\begin{aligned}\mathrm{d}\log N_i &= (1-\xi_i)\mathrm{d}\log M - \xi_i \frac{\eta_i}{\theta_i}\mathrm{d}\log t_i \\ &= -\left[(1-\xi_i)\,\mathbb{E}_{s\xi}\left[\frac{\eta}{\theta}\mathrm{d}\log t\right] + \xi_i \frac{\eta_i}{\theta_i}\mathrm{d}\log t_i\right]\end{aligned}$$

Therefore, both the total number of entrants and the number of active firms in each market decline.

Changes in other variables

Firm-level profit π_i Similarly, we have

$$\begin{aligned}\mathrm{d}\log \pi_i &= -\theta_i \mathrm{d}\log N_i - \eta_i \mathrm{d}\log t_i, \\ &= \theta_i\left[(1-\xi_i)\,\mathbb{E}_{s\xi}\left[\frac{\eta}{\theta}\mathrm{d}\log t\right] + \xi_i \frac{\eta_i}{\theta_i}\mathrm{d}\log t_i\right] - \eta_i \mathrm{d}\log t_i \\ &= \theta_i(1-\xi_i)\left\{\mathbb{E}_{s\xi}\left[\frac{\eta \mathrm{d}\log t}{\theta}\right] - \frac{\eta_i \mathrm{d}\log t_i}{\theta_i}\right\}\end{aligned}$$

Therefore, firm-level profit drops in the focal market and *increases* in the third market.

Relative price $\frac{p_i}{P_i}$ The relative price has the following expression:

$$\mathrm{d}\log\frac{p_i}{P_i}=\frac{\mathrm{d}\log N_i}{\sigma_i-1},$$

which declines in both the focal and the third markets (as the number of firms decreases). Moreover, $\frac{p_i}{A_i}$ decreases in every market, since $A_i=P_it_i$. Of course, this is consistent with the increasing market share $s(\frac{p_i}{A_i})$ of surviving firms in both the focal and third markets.

Firm-level price p_i Let us look at the mark-up pricing rule:

$$p_i=\mu_i\frac{\tau_i}{z},$$

where $\mu_i\equiv\frac{\sigma_i}{\sigma_i-1}$ and

$$\sigma_i=1-\frac{\upsilon s^{'}(\upsilon)}{s(\upsilon)}$$

We assume that the demand elasticity (or markup) increases (or decreases) with the relative price $\frac{p_i}{P_i}$. Since we have

$$\mathrm{d}\log\frac{p_i}{P_i}=\frac{\mathrm{d}\log N_i}{\sigma_i-1}<0,$$

and

$$\mathrm{d}\log p_i=\mathrm{d}\log\mu_i=-\left(\frac{1}{\rho_i}-1\right)\mathrm{d}\log\frac{p_i}{P_i},$$

we must have

$$\mathrm{d}\log p_i>0.$$

Price aggregator A_i, the ideal price index P_i^I Based on the discussion in the previous step, we must have

$$\mathrm{d}\log\frac{p_i}{P_i}=\frac{\mathrm{d}\log N_i}{\sigma_i-1}<0,\ \ \mathrm{d}\log p_i=\mathrm{d}\log\mu_i>0\Rightarrow\mathrm{d}\log P_i>\mathrm{d}\log p_i>0,\ \ \mathrm{d}\log A_i=\mathrm{d}\log P_i+\mathrm{d}\log t_i>0.$$

Therefore, we must have that the price aggregator increases in all markets. Moreover, we have

$$\mathrm{d}\log P_i^I=\mathrm{d}\log p_i+\mathrm{d}\log t_i-\mathrm{d}\log N_i\left(\delta_i-1\right),$$

and $\mathrm{d}\log t_i \geq 0$ and $\mathrm{d}\log N_i < 0$ and $\mathrm{d}\log p_i > 0$ for all markets. The ideal price index must increase in all markets.

Total export value E_i and real consumption Q_i Since the ideal price index must increase in all markets, the real consumption $Q_i = D_i^{\eta_i} P_i^{I^{-\eta_i}}$ and total expenditure on Chinese goods $E_i = Q_i P_i^I = D_i^{\eta_i} P_i^{I^{1-\eta_i}}$ drop in all markets.

Firm-level output q_i Recall that average firm output equals

$$q_i = \frac{s\left(\frac{p_i}{P_i}\right) D^{\eta_i} P_i^{I^{1-\eta_i}}}{t_i p_i}.$$

Therefore, we must have

$$\mathrm{d}\log q_i = \mathrm{d}\log s_i + (1-\eta_i)\mathrm{d}\log P_i^I - \mathrm{d}\log t_i - \mathrm{d}\log p_i,$$

which is equivalent to

$$\mathrm{d}\log q_i = -\left[1-(\eta_i-1)(\delta_i-1)\right]\mathrm{d}\log N_i - \eta_i\left(\mathrm{d}\log p_i + \mathrm{d}\ \log t_i\right),$$

where $1-(\eta_i-1)(\delta_i-1) > 0$. Substituting $\mathrm{d}\log p_i = -\left(\frac{1}{\rho_i}-1\right)\frac{\mathrm{d}\log N_i}{\sigma_i-1}$ into the above equation, we have

$$\mathrm{d}\log q_i = -\left[1-(\eta_i-1)(\delta_i-1) - \frac{\eta_i}{\sigma_i-1}\left(\frac{1}{\rho_i}-1\right)\right]\mathrm{d}\log N_i - \eta_i \mathrm{d}\log t_i,$$

which leads to

$$\mathrm{d}\log q_i = \left[1-(\eta_i-1)(\delta_i-1) - \frac{\eta_i}{\sigma_i-1}\left(\frac{1}{\rho_i}-1\right)\right]\left[(1-\xi_i)\,\mathbb{E}_{s\xi}\left[\frac{\eta}{\theta}\mathrm{d}\log t\right] + \xi_i\frac{\eta_i}{\theta_i}\mathrm{d}\log t_i\right] - \eta_i \mathrm{d}\log t_i \tag{62}$$

where the first term reflects the GE effect of the change in the number of firms on firm output (which can be positive or negative) while the second term captures the direct effect coming from the tariff change.

There are two cases to consider. First, when ρ_i is very small (i.e., close to zero), output level drops in all markets as $\mathrm{d}\log t_i \geq 0$ in all markets and strictly positive in one market. $\rho \approx 0$ implies that there are substantial increases in *the price level* p_i in all markets (due to the GE feedback from $\mathrm{d}\log N_i$) after a market-specific tariff shock. As a result, output q_i

drops in all markets.

Next, when ρ_i is close to one (meaning that $1-(\eta_i-1)(\delta_i-1)-\frac{\eta_i}{\sigma_i-1}\left(\frac{1}{\rho_i}-1\right)>0$), output level increases in the third market unambiguously. In such a case, we can write out $\mathrm{d}\log q_j$ in the focal market as

$$\begin{aligned}
& \mathrm{d}\log q_j \\
= & \left[1-(\eta_j-1)(\delta_j-1)-\frac{\eta_j}{\sigma_j-1}\left(\frac{1}{\rho_j}-1\right)\right] \\
& \left[(1-\xi_j)\,\mathbb{E}_{s\xi}\left[\frac{\eta}{\theta}\mathrm{d}\log t\right]+\frac{\xi_j\eta_j}{1-(\eta_j-1)(\delta_j-1)+\frac{\sigma_j-\eta_j}{\sigma_j-1}\left(\frac{1}{\rho_j}-1\right)}\mathrm{d}\log t_j\right]-\eta_j\mathrm{d}\log t_j \\
< & \frac{1-(\eta_j-1)(\delta_j-1)-\frac{\eta_j}{\sigma_j-1}\left(\frac{1}{\rho_j}-1\right)}{1-(\eta_j-1)(\delta_j-1)+\frac{\sigma_j-\eta_j}{\sigma_j-1}\left(\frac{1}{\rho_j}-1\right)}\eta_j\mathrm{d}\log t_j-\eta_j\mathrm{d}\log t_j \\
< & 0,
\end{aligned}$$

as $\mathrm{d}\log t_j>0$. This implies that quantity sold in the focal market goes down in all possible cases. When $\rho\approx 1$, there is almost no price (and markup) change after the tariff shock. Moreover, the market share per firm in the third market increases, and there is no tariff shock. Therefore, output has to go up in the third market. For the focal market, the impact is negative, as there is a direct negative impact on the output coming from the tariff shock.

Firm-level revenue r_i Based on the above derivation, we can write out the change in firm-level (tariff-exclusive) revenue as

$$\mathrm{d}\log r_i=\mathrm{d}\log p_i+\mathrm{d}\log q_i=-\left[1-(\eta_i-1)(\delta_i-1)\right]\mathrm{d}\log N_i-\eta_i\mathrm{d}\log t_i+(1-\eta_i)\mathrm{d}\log p_i,$$

where $1-(\eta_i-1)(\delta_i-1)>0$. Since

$$\mathrm{d}\log p_i=\mathrm{d}\log\mu_i=-\left(\frac{1}{\rho_i}-1\right)\mathrm{d}\log\frac{p_i}{P_i}=-\left(\frac{1}{\rho_i}-1\right)\frac{\mathrm{d}\log N_i}{\sigma_i-1},$$

we have

$$\mathrm{d}\log r_i=\left[1-(\eta_i-1)(\delta_i-1)-\frac{\eta_i-1}{\sigma_i-1}\left(\frac{1}{\rho_i}-1\right)\right]\left[(1-\xi_i)\,\mathbb{E}_{s\xi}\left[\frac{\eta}{\theta}\mathrm{d}\log t\right]+\xi_i\frac{\eta_i}{\theta_i}\mathrm{d}\log t_i\right]-\eta_i\mathrm{d}\log t_i \tag{63}$$

Equation (63) resembles equation (62). Therefore, a similar argument applies here. There are two cases to consider. First, when ρ_i is very small (i.e., close to zero), output level drops in all markets as $\mathrm{d}\log t_i \geq 0$ in all markets and strictly positive in one market. $\rho \approx 0$ implies that there are substantial increases in *the price level* p_i in all markets (due to the GE feedback from $\mathrm{d}\log N_i$) after a market-specific tariff shock. As a result, both output q_i and revenue $p_i q_i$ drop in all markets (as the price elasticity is always bigger than unity).

Next, when ρ_i is close to one (meaning that $1-(\eta_i-1)(\delta_i-1)-\frac{\eta_i-1}{\sigma_i-1}\left(\frac{1}{\rho_i}-1\right)>0$), output level increases in the third market unambiguously. In such a case, we can write out $\mathrm{d}\log q_j$ in the focal market as

$$\begin{aligned}
& \mathrm{d}\log r_j \\
= & \left[1-(\eta_j-1)(\delta_j-1)-\frac{\eta_j-1}{\sigma_j-1}\left(\frac{1}{\rho_j}-1\right)\right] \\
& \left[(1-\xi_j)\,\mathbb{E}_{s\xi}\left[\frac{\eta}{\theta}\mathrm{d}\log t\right]+\frac{\xi_j\eta_j}{1-(\eta_j-1)(\delta_j-1)+\frac{\sigma_j-\eta_j}{\sigma_j-1}\left(\frac{1}{\rho_j}-1\right)}\mathrm{d}\log t_j\right]-\eta_j\mathrm{d}\log t_j \\
< & \frac{1-(\eta_j-1)(\delta_j-1)+\frac{1-\eta_j}{\sigma_j-1}\left(\frac{1}{\rho_j}-1\right)}{1-(\eta_j-1)(\delta_j-1)+\frac{\sigma_j-\eta_j}{\sigma_j-1}\left(\frac{1}{\rho_j}-1\right)}\eta_j\mathrm{d}\log t_j-\eta_j\mathrm{d}\log t_j \\
< & 0,
\end{aligned}$$

as $\theta_i \equiv \left(\frac{1}{\rho_i}-1\right)\frac{\sigma_i-\eta_i}{\sigma_i-1}+[1-(\eta_i-1)(\delta_i-1)]>0$ and $\mathrm{d}\log t_j>0$. This implies export revenue in the focal market goes down in all possible cases. When $\rho \approx 1$, there is almost no price (and markup) change after the tariff shock. Moreover, the market share per firm in the third market increases, and there is no tariff shock. Therefore, both output and revenue go up in the third market. For the focal market, the impact is negative, as there is a direct negative impact on revenue coming from the tariff shock.

Total export quantity $\tilde{Q}_i$ Total (physical) quantity of output across firms is $\tilde{Q}_i = N_i q_i$. Thus, we must have

$$\mathrm{d}\log\tilde{Q}_i = \mathrm{d}\log N_i + \mathrm{d}\log q_i,$$

which

$$\mathrm{d}\log\tilde{Q}_i = \mathrm{d}\log N_i \mathrm{d}\log q_i = \left[(\eta_i-1)(\delta_i-1)+\frac{\eta_i}{\sigma_i-1}\left(\frac{1}{\rho_i}-1\right)\right]\mathrm{d}\log N_i-\eta_i\mathrm{d}\log t_i<0, \tag{64}$$

as $\mathrm{d}\log N_i < 0$ and $\mathrm{d}\log t_i \geq 0$ in all markets. In summary, the action at the extensive margin drive the change in aggregated export quantity/value (even if firm-level export value increases).

E.3 Heterogeneous firms

E.3.1 Solving the equilibrium

We follow the same strategy to solve the equilibrium with heterogeneous firms. Comparing to the previous case, there will be a selection cutoff z_i^* on firm productivity. We show that it will be pinned down once N_i (or M_i^P) is known, and thus does not affect the determination of N_i and M.

First, we solve for the equilibrium of each market given N_i. The pricing rule is the same as in the homogeneous case:

$$t_i p^i(z) = \frac{\sigma(\upsilon)}{\sigma(\upsilon)-1}\frac{t_i\tau}{z},$$

where $\upsilon \equiv \frac{t_i p^i(z)}{A_i}$. Then, we may write $p_i(z)$ as a function of the relative price $\frac{p^i(z)}{P_i}$ where $P_i \equiv \frac{A_i}{t_i}$ (or eventually P_i).

The selection condition implies that

$$\left(1-\frac{1}{\mu_i(z_i^*)}\right)\frac{s\left(\frac{p_i(z_i^*)}{P_i}\right)}{t_i}D^{\eta_i}{P_i^I}^{1-\eta_i} = f_i.$$

z_i^* is then a function of P_i and P_i^I.

We have two equilibrium conditions for P_i and P_i^I as follows. First, market clearing implies that

$$M_i^P\int_{z_i^*}^{\infty} s\left(\frac{p_i(z;P_i,P_i^I)}{P_i}\right)\mathrm{d}G(z) = 1.$$

Second, the ideal price index P_i^I still satisfy

$$\log\left(P_i^I\right) = C + \log(t_i) + \log(P_i) - M_i^P\int_{z_i^*}^{\infty}\int_{\frac{p^i(z)}{P_i}}^{\infty}\frac{s(\zeta)}{\zeta}d\zeta\mathrm{d}G(z),$$

Together, they implicitly determine P_i and P_i^I. In this way, we have shown that all variables

can be written as a function of N_i or M_i^P (i.e., the number of potential entrants into market i). The expected profit $\bar{\pi}_i$ is given by equation (65).

$$\bar{\pi}_i(M_i^P) = \frac{1}{\rho + \nu_i} \int_{z_i^*(M_i^P)}^{\infty} [\pi_i(z; N_i) - f_i] \, \mathrm{d}G(z). \tag{65}$$

Next, we solve for the equilibrium $(M, \{N_i\})$ using the free entry condition and balanced flow of firms. Namely,

$$\sum_{i=0}^{N} (h_i \bar{\pi}_i)^{\frac{1}{\beta}} = f_e^{\frac{1}{\beta}},$$
$$J_i[1 - G(z_i^*)]M = \nu_i N_i.$$

There are $N + 2$ equations for $N + 2$ unknowns. Noticing that with equation (10),

$$J_i = h_i \left(\frac{h_i \bar{\pi}_i}{f_e} \right)^{\frac{1-\beta}{\beta}} = \frac{\nu_i N_i}{M[1 - G(z_i^*)]} \quad \Rightarrow \quad \bar{\pi}_i(N_i) = \left(\frac{\nu_i N_i}{h_i M[1 - G(z_i^*)]} \right)^{\frac{\beta}{1-\beta}} \frac{f_e}{h_i},$$

Together with (65), this equation gives us an implicit function N_i of M. We define such mapping by $N_i = N_i(M)$. Inserting it into the free entry equation, we pin down the equilibrium mass of entrants M as follows:

$$\sum_{i=0}^{N} (h_i \bar{\pi}_i(N_i(M)))^{\frac{1}{\beta}} = f_e^{\frac{1}{\beta}},$$

In this way, we have solved the stationary equilibrium. Note that we have $M_i^P = \frac{N_i}{1 - G(z_i^*)}$. Thus, we can derive the value of M_i^P given N_i and z_i^* (which is a function of N_i eventually). We continue to assume Assumption 2 in our analysis of heterogeneous firms with HSA demand.

E.3.2 Propagation and aggregation equations with heterogeneous firms

We expand the equilibrium equations presented in Section E.3.1 to the first order in the shocks.

Definition of the price aggregator

$$\log\left(P_i^I\right) = C + \log(t_i) + \log(P_i) - M_i^P \int_{z_i^*}^{\infty} \int_{\frac{p^i(z)}{P_i}}^{\infty} \frac{s(\zeta)}{\zeta} d\zeta \mathrm{d}G(z),$$

In the case of heterogeneous firms, the sales (share)-weighed consumer surplus ratio is defined

$$\bar{\delta}_i \equiv 1 + M_i^P \int_{z_i^*}^{\infty} \frac{\int_{\frac{p_i}{P_i}}^{\infty} \frac{s(\zeta)}{\zeta} d\zeta}{s\left(\frac{p_i}{P_i}\right)} s\left(\frac{p_i}{P_i}\right) \mathrm{d}G(z).$$

Therefore, we must have

$$\begin{aligned} & \mathrm{d}\log P_i^I \\ = \; & \mathrm{d}\log P_i + \mathrm{d}\log t_i + M_i^P \int_{z_i^*}^{\infty} \frac{s\left(\frac{p_i(z)}{P_i}\right)}{\frac{p_i(z)}{P_i}} \frac{p_i(z)}{P_i} \mathrm{d}\log \frac{p_i(z)}{P_i} \mathrm{d}G(z) \\ & -\mathrm{d}\log M_i^P \left(\bar{\delta}_i - 1\right) + M_i^P s\left(\frac{p_i(z_i^*)}{P_i}\right) \left(\delta_i^* - 1\right) g(z_i^*) dz_i^*, \end{aligned}$$

Since we have

$$M_i^P [1 - G(z_i^*)] s\left(\frac{p_i(z_i^*)}{P_i}\right) = \lambda_i(z_i^*),$$

where $\lambda_i(z_i^*)$ is the sales share (density), the above expression can be simplified to

$$\mathrm{d}\log P_i^I = \mathrm{d}\log t_i - \mathrm{d}\log M_i^P \left(\bar{\delta}_i - 1\right) + \mathbb{E}_\lambda[\mathrm{d}\log p_i(z)] + \lambda_i(z_i^*) \left(\delta_i^* - 1\right) \frac{z_i^* g(z_i^*)}{1 - G(z_i^*)} \mathrm{d}\log(z_i^*). \tag{66}$$

Welfare We have

$$M_i^P \int_{z_i^*}^{\infty} s\left(\frac{p_i(z; P_i, P_i^I)}{P_i}\right) \mathrm{d}G(z) = 1.$$

Total differentiation leads to

$$\mathrm{d}\log M_i^P - \lambda_i(z_i^*) \frac{z_i^* g(z_i^*)}{1 - G(z_i^*)} \mathrm{d}\log(z_i^*) + \mathbb{E}_\lambda \left[(1 - \sigma_i(z)) \mathrm{d}\log \frac{p_i(z)}{P_i}\right] = 0. \tag{67}$$

Zero profit We have

$$\left(1-\frac{1}{\mu_i(z_i^*)}\right)\frac{s\left(\frac{p_i(z_i^*)}{P_i}\right)}{t_i}D^{\eta_i}P_i^{I\,1-\eta_i}=f_i.$$

Note that

$$\begin{aligned}
&\mathrm{d}\log\left(1-\frac{1}{\mu_i(z_i^*)}\right)+\mathrm{d}\log s\left(\frac{p_i(z_i^*)}{P_i}\right)\\
=\;&\left[-\left(\frac{1}{\rho_i^*}-1\right)(\sigma_i(z_i^*)-1)-(\sigma_i(z_i^*)-1)\right]\left[\mathrm{d}\log\left(\frac{p_i(z_i^*)}{P_i}\right)\right.\\
&\left.+\frac{\partial\log\left(\frac{p_i(z_i^*)}{P_i}\right)}{\partial\log z_i^*}\mathrm{d}\log z_i^*\right]\\
=\;&-\frac{1}{\rho_i^*}(\sigma_i(z_i^*)-1)\left[\mathrm{d}\log\left(\frac{p_i(z_i^*)}{P_i}\right)-\mathrm{d}\log z_i^*\right],
\end{aligned}$$

as

$$t_i p_\omega^i=\frac{\sigma(\upsilon)}{\sigma(\upsilon)-1}\frac{t_i\tau}{z},$$

and thus

$$\frac{\partial\log\left(\frac{p_i(z_i^*)}{P_i}\right)}{\partial\log z_i^*}=-1.$$

Therefore, total differentiation leads to

$$-\frac{1}{\rho_i^*}(\sigma_i(z_i^*)-1)\left[\mathrm{d}\log\left(\frac{p_i(z_i^*)}{P_i}\right)-\mathrm{d}\log z_i^*\right]-(\eta_i-1)\mathrm{d}\log P_i^I-\mathrm{d}\log t_i=\mathrm{d}\log f_i, \tag{68}$$

which makes total sense. A higher survival cutoff z_i^* increases the firm profit at the cutoff z_i^*, while lower market share induced by a higher $\frac{p_i(z_i^*)}{P_i}$ reduces the firm profit.

Sales share We have

$$\lambda_i(z)=M_i^P[1-G(z_i^*)]s\left(\frac{p_i(z)}{P_i}\right),$$

which implies that

$$\mathrm{d}\log\lambda_i(z)=(1-\sigma_i(z))\,\mathrm{d}\log\frac{p_i(z)}{P_i}+\mathrm{d}\log M_i^P-\frac{z_i^*g(z_i^*)}{1-G(z_i^*)}\mathrm{d}\log(z_i^*) \tag{69}$$

Markup

$$\mathrm{d}\log\mu_i = -\left(\frac{1}{\rho_i}-1\right)\mathrm{d}\log\frac{p_i}{P_i}, \tag{70}$$

in which

$$\rho_i = \rho\left(\frac{p_i}{P_i}\right) = \frac{1}{1-\frac{\mu'\left(\frac{p_i}{P_i}\right)\frac{p_i}{P_i}}{\mu\left(\frac{p_i}{P_i}\right)}}.$$

Pricing Pricing implies that

$$\mathrm{d}\log p_i = \mathrm{d}\log\mu_i = -\left(\frac{1}{\rho_i}-1\right)\mathrm{d}\log\frac{p_i}{P_i},$$

where the fist part comes from the pricing rule and the second part comes from the definition of the passthrough.

Variable profit Firm's operating profit can be expressed

$$\pi_i(z) = \left(1-\frac{1}{\mu_i(z)}\right)\frac{s\left(\frac{p_i(z)}{P_i}\right)}{t_i}D^{\eta_i}{P_i^I}^{1-\eta_i}.$$

Therefore, we must have

$$\mathrm{d}\log\pi_i(z) = \frac{1}{\mu_i-1}\mathrm{d}\log\mu_i - (\sigma_i(z)-1)\mathrm{d}\log\left(\frac{p_i(z_i^*)}{P_i}\right) - (\eta_i-1)\mathrm{d}\log P_i^I - \mathrm{d}\log t_i,$$

which can be reduced to

$$\mathrm{d}\log\pi_i(z) = -\frac{1}{\rho_i(z)}(\sigma_i(z)-1)\log\left(\frac{p_i(z)}{P_i}\right) - (\eta_i-1)\mathrm{d}\log P_i^I - \mathrm{d}\log t_i. \tag{71}$$

Balanced flow Totally differentiating (53),

$$\frac{1}{\beta}\log h_i + \frac{1-\beta}{\beta}\left[\log\bar{\pi}_i - \log f_e\right] = \log\nu_i + \log M_i^P - \log M. \tag{72}$$

Free entry

$$\sum_{i=0}^{N} s_i \left[\mathrm{d}\log h_i + \mathrm{d}\log \bar{\pi}_i\right] = \mathrm{d}\log f_e, \tag{73}$$

in which $s_i = \left(\frac{h_i \bar{\pi}_i}{f_e}\right)^{\frac{1}{\beta}} = \frac{J_i \bar{\pi}_i}{\sum_k J_k \bar{\pi}_k}$.

E.3.3 Proof of comparative statics results when firms are heterogeneous

In this subsection, we prove that all the results established in Proposition 4 and 5 continue to hold when the demand function features HSA. Specifically, the change in M and that of M_i^P take similar forms as in the Kimball case and $\mathrm{d}\log M_i^P$ is a sufficient statistic for measuring changes in other variables in market i.

Changes in M and M_i^P We consider only $\mathrm{d}\log t_i$ and set $\mathrm{d}\log \frac{\tau_i}{z} = \mathrm{d}\log \nu_i = \mathrm{d}\log h_i = \mathrm{d}\log f_i = \mathrm{d}\log f_e = 0$. Then, we obtain the following system of equations

$$\begin{aligned}
\mathrm{d}\log P_i^I = \mathrm{d}\log t_i - \mathrm{d}\log M_i^P \left(\bar{\delta}_i - 1\right) + \mathbb{E}_\lambda[\mathrm{d}\log p_i(z)] + \lambda_i(z_i^*)\left(\delta_i^* - 1\right)\varphi_i^* \mathrm{d}\log(z_i^*) \\
\mathrm{d}\log M_i^P - \lambda_i(z_i^*)\varphi_i^* \mathrm{d}\log(z_i^*) + \mathbb{E}_\lambda\left[(1-\sigma_i(z))\mathrm{d}\left(\log\frac{p_i(z)}{P_i}\right)\right] = 0 \\
-\frac{1}{\rho_i^*}(\sigma_i(z_i^*)-1)\left[\mathrm{d}\log\left(\frac{p_i(z_i^*)}{P_i}\right) - \mathrm{d}\log z_i^*\right] - (\eta_i - 1)\mathrm{d}\log P_i^I - \mathrm{d}\log t_i = 0 \\
\mathrm{d}\log p_i(z) = \mathrm{d}\log \mu_i = -\left(\frac{1}{\rho_i} - 1\right)\mathrm{d}\log\frac{p_i}{P_i} \\
\mathrm{d}\log \pi_i(z) = -\frac{1}{\rho_i(z)}(\sigma_i(z)-1)\log\left(\frac{p_i(z)}{P_i}\right) - (\eta_i - 1)\mathrm{d}\log P_i^I - \mathrm{d}\log t_i \\
\left[\frac{\mathbb{E}^i \pi_i \mathrm{d}\log \pi_i}{(\rho+\nu_i)\bar{\pi}_i}\right] = \frac{\beta}{1-\beta}\left[\mathrm{d}\log M_i^P - \mathrm{d}\log M\right], \\
\sum_{i=0}^{N} s_i \mathrm{d}\log M_i^P = \mathrm{d}\log M,
\end{aligned}$$

where $\varphi_i^* = \frac{z_i^* g(z_i^*)}{1-G(z_i^*)}$. In summary, we have $6(N+1)+1$ equations and the same number of unknown changes $(\mathrm{d}\log P_i^I,\ \mathrm{d}\log(z_i^*),\ \mathrm{d}\log p_i(z),\ \mathrm{d}\left(\log\frac{p_i(z)}{P_i}\right),\ \mathrm{d}\log\pi_i,\ \mathrm{d}\log M_i^P,\ \mathrm{d}\log M)$. Note that the the following equality hold:

$$\mathrm{d}\log M_i^P = \mathrm{d}\log N_i + \varphi_i^* \mathrm{d}\log z_i^*.$$

The path forward is to express $\mathrm{d}\log P_i^I$, $\mathrm{d}\log(z_i^*)$, $\mathrm{d}\log p_i(z)$ and $\mathrm{d}\left(\log\frac{p_i(z)}{P_i}\right)$ using $\mathrm{d}\log M_i^P$ and $\mathrm{d}\log t_i$ and the first four sets of equations above. First, substituting the second and fourth equations above into the first equation above leads to

$$\begin{aligned}
& \mathrm{d}\log P_i^I \\
= \; & \mathrm{d}\log t_i - \mathrm{d}\log M_i^P\left(\bar{\delta}_i - 1\right) - \mathbb{E}_\lambda\left[\left(\frac{1}{\rho_i}-1\right)\mathrm{d}\log\frac{p_i(z)}{P_i}\right] \\
& + (\delta_i^* - 1)\left[\mathrm{d}\log M_i^P + \mathbb{E}_\lambda\left((1-\sigma_i(z))\mathrm{d}\left(\log\frac{p_i(z)}{P_i}\right)\right)\right],
\end{aligned}$$

where we have successfully eliminated $\mathbb{E}_\lambda[\mathrm{d}\log p_i(z)]$ and $\lambda_i(z_i^*)\varphi_i^*\mathrm{d}\log(z_i^*)$. We can rewrite this equation as

$$\mathrm{d}\log P_i^I = \mathrm{d}\log t_i - \left(\bar{\delta}_i - \delta_i^*\right)\mathrm{d}\log M_i^P - \mathbb{E}_\lambda\left[\left(\left(\frac{1}{\rho_i}-1\right) + (\delta_i^*-1)(\sigma_i(z)-1)\right)\mathrm{d}\left(\log\frac{p_i(z)}{P_i}\right)\right]. \tag{74}$$

The above equation has clear economic interpretations. First, the ideal price index increases, when the tariff goes up and/or the number of firms (that try to enter the market) goes down (as $\bar{\delta}_i > \delta_i^*$). Second, when the expected $\mathrm{d}\left(\log\frac{p_i(z)}{P_i}\right)$ increases, the ideal price index goes down. This is true, as a lower average market share leads to a smaller average markup, which pushes down the ideal price index.

We have one extra pricing condition for the firm at the cutoff:

$$\mathrm{d}\log p_i(z_i^*) = -\left(\frac{1}{\rho_i^*}-1\right)\mathrm{d}\log\frac{p_i(z_i^*)}{P_i},$$

which implies

$$\mathrm{d}\log p_i(z_i^*) = \frac{\frac{1}{\rho_i^*}-1}{\frac{1}{\rho_i^*}}\mathrm{d}\log P_i = (1-\rho_i^*)\mathrm{d}\log P_i.$$

Moreover, we have

$$\mathrm{d}\log p_i(z) = (1-\rho_i(z))\mathrm{d}\log P_i,$$

for any z. Thus, we must have

$$\frac{\mathrm{d}\log p_i(z)}{\mathrm{d}\log p_i(z_i^*)} = \frac{1-\rho_i(z)}{1-\rho_i^*}.$$

From the second equation above, we have

$$\lambda_i(z_i^*)\varphi_i^*\mathrm{d}\log(z_i^*) = \mathrm{d}\log M_i^P + \mathbb{E}_\lambda\left[(1-\sigma_i(z))\mathrm{d}\left(\log\frac{p_i(z)}{P_i}\right)\right].$$

Since

$$\mathrm{d}\log p_i(z) = (1-\rho_i(z))\mathrm{d}\log P_i,$$

we must have

$$\lambda_i(z_i^*)\varphi_i^*\mathrm{d}\log(z_i^*) = \mathrm{d}\log M_i^P + \mathbb{E}_\lambda\left[\rho_i(z)(\sigma_i(z)-1)\right]\mathrm{d}\log P_i$$

and

$$\mathrm{d}\log P_i = \frac{\lambda_i(z_i^*)\varphi_i^*\mathrm{d}\log(z_i^*) - \mathrm{d}\log M_i^P}{\mathbb{E}_\lambda\left[\rho_i(z)(\sigma_i(z)-1)\right]}.$$

Since

$$\mathrm{d}\log P_i^I = \mathrm{d}\log t_i - \left(\bar{\delta}_i - \delta_i^*\right)\mathrm{d}\log M_i^P + \mathbb{E}_\lambda\left[\rho_i(z)\left(\left(\frac{1}{\rho_i}-1\right) + (\delta_i^*-1)(\sigma_i(z)-1)\right)\right]\mathrm{d}\log P_i,$$

we have

$$\mathrm{d}\log P_i^I = \mathrm{d}\log t_i - \left(\bar{\delta}_i - \delta_i^*\right)\mathrm{d}\log M_i^P + \frac{1-\bar{\rho}_i + (\delta_i^*-1)\mathbb{E}_\lambda\left(\rho_i(z)(\sigma_i(z)-1)\right)}{\mathbb{E}_\lambda\left[\rho_i(z)(\sigma_i(z)-1)\right]}\left[\lambda_i(z_i^*)\varphi_i^*\mathrm{d}\log(z_i^*) - \mathrm{d}\log M_i^P\right.$$

or

$$\mathrm{d}\log P_i^I = \mathrm{d}\log t_i - \left(\bar{\delta}_i - \delta_i^*\right)\mathrm{d}\log M_i^P + \left[(\delta_i^*-1) + \frac{1-\bar{\rho}_i}{\mathbb{E}_\lambda\left[\rho_i(z)\sigma_i(z)\right] - \bar{\rho}_i}\right]\left[\lambda_i(z_i^*)\varphi_i^*\mathrm{d}\log(z_i^*) - \mathrm{d}\log M_i^P\right],$$

or

$$\mathrm{d}\log P_i^I = \mathrm{d}\log t_i - \left[\left(\bar{\delta}_i - 1\right) + \gamma_i\right]\mathrm{d}\log M_i^P + \left[\gamma_i + (\delta_i^*-1)\right]\lambda_i(z_i^*)\varphi_i^*\mathrm{d}\log(z_i^*), \qquad (75)$$

where

$$\gamma_i \equiv \frac{1-\bar{\rho}_i}{\mathbb{E}_\lambda\left[\rho_i(z)\sigma_i(z)\right] - \bar{\rho}_i} \geq 0,$$

as $\bar{\sigma}_i > 1$ and $cov(\rho_i(z), \sigma_i(z)) > 0$ (thanks to the strong second law of Marshallian demand).

Next, we combine the second and third equations above and obtain

$$
\begin{aligned}
& (\eta_i - 1)\mathrm{d}\log P_i^I \\
= \; & -\frac{1}{\rho_i^*}(\sigma_i(z_i^*) - 1)\Big[\mathrm{d}\log\left(\frac{p_i(z_i^*)}{P_i}\right) - \mathrm{d}\log z_i^*\Big] - \mathrm{d}\log t_i \\
= \; & -\frac{1}{\rho_i^*}(\sigma_i(z_i^*) - 1)\Big(- \rho_i^*\mathrm{d}\log P_i - \mathrm{d}\log z_i^*\Big) - \mathrm{d}\log t_i \\
= \; & (\sigma_i(z_i^*) - 1)\log P_i + \frac{\sigma_i(z_i^*) - 1}{\rho_i^*}\mathrm{d}\log z_i^* - \mathrm{d}\log t_i \\
= \; & (\sigma_i(z_i^*) - 1)\frac{\lambda_i(z_i^*)\varphi_i^*\mathrm{d}\log(z_i^*) - \mathrm{d}\log M_i^P}{\mathbb{E}_\lambda\left[\rho_i(z)(\sigma_i(z) - 1)\right]} + \frac{\sigma_i(z_i^*) - 1}{\rho_i^*}\mathrm{d}\log z_i^* - \mathrm{d}\log t_i \\
= \; & (\sigma_i(z_i^*) - 1)\frac{\rho_i^*\lambda_i(z_i^*)\varphi_i^* + \mathbb{E}_\lambda\left[\rho_i(z)(\sigma_i(z) - 1)\right]}{\mathbb{E}_\lambda\left[\rho_i(z)(\sigma_i(z) - 1)\right]\rho_i^*}\mathrm{d}\log(z_i^*) - \mathrm{d}\log t_i \\
& -\frac{(\sigma_i(z_i^*) - 1)}{\mathbb{E}_\lambda\left[\rho_i(z)(\sigma_i(z) - 1)\right]}\mathrm{d}\log M_i^P. \qquad (76)
\end{aligned}
$$

Combining equations (75) and (76), we can solve for $\mathrm{d}\log z_i^*$:

$$
\begin{aligned}
& (\eta_i - 1)\left[\mathrm{d}\log t_i - \left[\left(\bar{\delta}_i - 1\right) + \gamma_i\right]\mathrm{d}\log M_i^P + \left[\gamma_i + (\delta_i^* - 1)\right]\lambda_i(z_i^*)\varphi_i^*\mathrm{d}\log(z_i^*)\right] \\
= \; & (\sigma_i(z_i^*) - 1)\frac{\rho_i^*\lambda_i(z_i^*)\varphi_i^* + \mathbb{E}_\lambda\left[\rho_i(z)(\sigma_i(z) - 1)\right]}{\mathbb{E}_\lambda\left[\rho_i(z)(\sigma_i(z) - 1)\right]\rho_i^*}\mathrm{d}\log(z_i^*) - \mathrm{d}\log t_i - \frac{(\sigma_i(z_i^*) - 1)}{\mathbb{E}_\lambda\left[\rho_i(z)(\sigma_i(z) - 1)\right]}\mathrm{d}\log M_i^P,
\end{aligned}
$$

which implies

$$
\begin{aligned}
& \eta_i\mathrm{d}\log t_i - \left[(\eta_i - 1)\left(\bar{\delta}_i - 1\right) + \frac{-(\sigma_i^* - \eta_i) - (\eta_i - 1)\bar{\rho}_i}{\mathbb{E}_\lambda\left[\rho_i(z)(\sigma_i(z) - 1)\right]}\right]\mathrm{d}\log M_i^P \\
= \; & \left[\frac{\sigma_i(z_i^*) - 1}{\rho_i^*} - \left[(\eta_i - 1)\left(\delta_i^* - 1\right) + \frac{-(\sigma_i^* - \eta_i) - (\eta_i - 1)\bar{\rho}_i}{\mathbb{E}_\lambda\left[\rho_i(z)(\sigma_i(z) - 1)\right]}\right]\lambda_i(z_i^*)\varphi_i^*\right]\mathrm{d}\log z_i^*,
\end{aligned}
$$

Note that both $(\eta_i - 1)\left(\bar{\delta}_i - 1\right) + \frac{-(\sigma_i^* - \eta_i) - (\eta_i - 1)\bar{\rho}_i}{\mathbb{E}_\lambda[\rho_i(z)(\sigma_i(z) - 1)]}$ and $(\eta_i - 1)\left(\delta_i^* - 1\right) + \frac{-(\sigma_i^* - \eta_i) - (\eta_i - 1)\bar{\rho}_i}{\mathbb{E}_\lambda[\rho_i(z)(\sigma_i(z) - 1)]}$ increase with η_i. Our assumptions assure that

$$
\eta_i - 1 \leq \bar{\sigma}_i - 1; \quad \frac{1}{\bar{\sigma}_i - 1} \geq \bar{\delta}_i - 1 \geq \delta_i^* - 1.
$$

Thus, we must have

$$
1 \geq (\eta_i - 1)\left(\bar{\delta}_i - 1\right) \geq (\eta_i - 1)\left(\delta_i^* - 1\right).
$$

When $\eta = 1$, both terms are clearly negative. When η_i reaches its maximum of $\bar{\sigma}_i$, we have

$$\frac{(\sigma_i^* - \eta_i) + (\eta_i - 1)\bar{\rho}_i}{\mathbb{E}_\lambda \left[\rho_i(z)(\sigma_i(z) - 1)\right]} = \frac{(\sigma_i^* - \bar{\sigma}_i) + \bar{\rho}_i\bar{\sigma}_i - \bar{\rho}_i}{\bar{\rho}_i\bar{\sigma}_i + Cov(\rho_i(z), \sigma_i(z)) - \bar{\rho}_i},$$

which is generally larger than one. To see this, let us write out the expression of $Cov(\rho_i(z), \sigma_i(z))$ as

$$Cov(\rho_i(z), \sigma_i(z)) = \mathbb{E}_\lambda \left[(\rho_i(z) - \bar{\rho}_i)(\sigma_i(z) - \bar{\sigma}_i)\right].$$

Note that $|\rho_i(z) - \bar{\rho}_i| < 1$ as $\rho_i(z) \in [0, 1]$ and σ_i^* takes the largest value among all $\sigma_i(z_i)$s, $\sigma_i^* - \bar{\sigma}_i > Cov(\rho_i(z), \sigma_i(z)) > 0$ and thus $\frac{(\sigma_i^* - \eta_i) + (\eta_i - 1)\bar{\rho}_i}{\mathbb{E}_\lambda[\rho_i(z)(\sigma_i(z) - 1)]} > 1$. Therefore, both $A_i \equiv \frac{(\sigma_i^* - \eta_i) + (\eta_i - 1)\bar{\rho}_i}{\mathbb{E}_\lambda[\rho_i(z)(\sigma_i(z) - 1)]} - (\eta_i - 1)\left(\bar{\delta}_i - 1\right)$ and $B_i \equiv \frac{(\sigma_i^* - \eta_i) + (\eta_i - 1)\bar{\rho}_i}{\mathbb{E}_\lambda[\rho_i(z)(\sigma_i(z) - 1)]} - (\eta_i - 1)\left(\delta_i^* - 1\right)$ are always negative for $\eta_i \in [1, \bar{\sigma}_i]$. In total, we have

$$\left(\frac{\sigma_i^* - 1}{\rho_i^*} + B_i\lambda_i(z_i^*)\varphi_i^*\right) \mathrm{d}\log z_i^* = \eta_i \mathrm{d}\log t_i + A_i \mathrm{d}\log M_i^P,$$

where both $A_i > 0$ and $B_i > 0$. If we ignore the free entry condition, an increase in the tariff and/or in the number of potential entrants is going to increase the survival cutoff due to either the direct effect or the demand constraint (at the industry level).

We derive the comparative statics results. We have the following three equations:

$$\begin{aligned}
&-\frac{1}{\rho_i^*}(\sigma_i(z_i^*) - 1)\left[\mathrm{d}\log\left(\frac{p_i(z_i^*)}{P_i}\right) - \mathrm{d}\log z_i^*\right] - (\eta_i - 1)\mathrm{d}\log P_i^I - \mathrm{d}\log t_i = 0 \\
&\mathrm{d}\log p_i(z) = \mathrm{d}\log \mu_i = -\left(\frac{1}{\rho_i} - 1\right)\mathrm{d}\log\frac{p_i}{P_i} \\
&\mathrm{d}\log \pi_i(z) = -\frac{1}{\rho_i(z)}(\sigma_i(z) - 1)\log\left(\frac{p_i(z)}{P_i}\right) - (\eta_i - 1)\mathrm{d}\log P_i^I - \mathrm{d}\log t_i
\end{aligned}$$

We can substitute $-(\eta_i - 1)\mathrm{d}\log P_i^I - \mathrm{d}\log t_i$ in the last equation using the first equation:

$$\mathrm{d}\log \pi_i(z) = -\frac{1}{\rho_i(z)}(\sigma_i(z) - 1)\log\left(\frac{p_i(z)}{P_i}\right) + \frac{1}{\rho_i^*}(\sigma_i(z_i^*) - 1)\left[\mathrm{d}\log\left(\frac{p_i(z_i^*)}{P_i}\right) - \mathrm{d}\log z_i^*\right].$$

Moreover, since $\mathrm{d}\log p_i(z) = (1 - \rho_i(z))\mathrm{d}\log P_i$, we have

$$\mathrm{d}\log \pi_i(z) = -(\sigma_i^* - \sigma_i(z))\mathrm{d}\log P_i - \frac{1}{\rho_i^*}(\sigma_i(z_i^*) - 1)\mathrm{d}\log z_i^*.$$

Note that

$$\mathrm{d}\log P_i = \frac{\lambda_i(z_i^*)\varphi_i^*\mathrm{d}\log(z_i^*) - \mathrm{d}\log M_i^P}{\mathbb{E}_\lambda\left[\rho_i(z)(\sigma_i(z)-1)\right]}.$$

Therefore, we have

$$\mathrm{d}\log\pi_i(z) = \frac{-\left[(\sigma_i^*-\sigma_i(z))\lambda_i(z_i^*)\varphi_i^* + \left(\frac{\sigma_i^*-1}{\rho_i^*}\right)\mathbb{E}_\lambda\left[\rho_i(z)(\sigma_i(z)-1)\right]\right]\mathrm{d}\log(z_i^*) + (\sigma_i^*-\sigma_i(z))\mathrm{d}\log M_i^P}{\mathbb{E}_\lambda\left[\rho_i(z)(\sigma_i(z)-1)\right]}.$$

Finally, since we have

$$\mathrm{d}\log z_i^* = \frac{\eta_i\mathrm{d}\log t_i + A_i\mathrm{d}\log M_i^P}{\frac{\sigma_i^*-1}{\rho_i^*} + B_i\lambda_i(z_i^*)\varphi_i^*},$$

Therefore, the change in operating profit can be expressed as

$$\mathrm{d}\log\pi_i(z) = -C_i(z)\mathrm{d}\log t_i - D_i(z)\mathrm{d}\log M_i^P.$$

where

$$C_i(z) \equiv \frac{\eta_i\left[(\sigma_i^*-\sigma_i(z))\lambda_i(z_i^*)\varphi_i^* + \left(\frac{\sigma_i^*-1}{\rho_i^*}\right)\mathbb{E}_\lambda\left[\rho_i(z)(\sigma_i(z)-1)\right]\right]}{\mathbb{E}_\lambda\left[\rho_i(z)(\sigma_i(z)-1)\right]\left[\frac{\sigma_i^*-1}{\rho_i^*} + B_i\lambda_i(z_i^*)\varphi_i^*\right]} > 0,$$

and

$$D_i(z) \equiv \frac{A_i\left[(\sigma_i^*-\sigma_i(z))\lambda_i(z_i^*)\varphi_i^* + \left(\frac{\sigma_i^*-1}{\rho_i^*}\right)\mathbb{E}_\lambda\left[\rho_i(z)(\sigma_i(z)-1)\right]\right] - \left[\frac{\sigma_i^*-1}{\rho_i^*} + B_i\lambda_i(z_i^*)\varphi_i^*\right](\sigma_i^*-\sigma_i(z))}{\mathbb{E}_\lambda\left[\rho_i(z)(\sigma_i(z)-1)\right]\left[\frac{\sigma_i^*-1}{\rho_i^*} + B_i\lambda_i(z_i^*)\varphi_i^*\right]}.$$

Note that $D_i(z) > 0$ for sure when z is close to z_i^* (as $\sigma_i(z) \approxeq \sigma_i^*$). Also, $D_i(z) > 0$ for sure when the demand is CES (i.e., a fixed $\sigma_i(z)$). Now, let us recall the fourth part of Assumption 2:

$$\frac{\mathrm{d}\log\pi_i}{\mathrm{d}\log z_i^*} < 0.$$

When the demand is HSA, we can express the change in firm's operating profit as a function

that only contains the change in the survival cutoff and the tariff change:

$$\begin{aligned}
\mathrm{d}\log\pi_i(z) &= \frac{-\left[(\sigma_i^*-\sigma_i(z))\lambda_i(z_i^*)\varphi_i^* + \left(\frac{\sigma_i^*-1}{\rho_i^*}\right)\mathbb{E}_\lambda\left[\rho_i(z)(\sigma_i(z)-1)\right]\right]\mathrm{d}\log(z_i^*)}{\mathbb{E}_\lambda\left[\rho_i(z)(\sigma_i(z)-1)\right]} \\
&\quad \frac{(\sigma_i^*-\sigma_i(z))\left[\left(\frac{\sigma_i^*-1}{\rho_i^*}+B_i\lambda_i(z_i^*)\varphi_i^*\right)\mathrm{d}\log(z_i^*)-\eta_i\mathrm{d}\log t_i\right]}{A_i\mathbb{E}_\lambda\left[\rho_i(z)(\sigma_i(z)-1)\right]} \\
&= \frac{-\frac{D_i(z)}{A_i}}{\mathbb{E}_\lambda\left[\rho_i(z)(\sigma_i(z)-1)\right]}\mathrm{d}\log(z_i^*) \\
&\quad -\frac{\frac{(\sigma_i^*-\sigma_i(z))\eta_i\mathrm{d}\log t_i}{A_i}}{\mathbb{E}_\lambda\left[\rho_i(z)(\sigma_i(z)-1)\right]}.
\end{aligned}$$

Note that when the demand is CES, the change in the survival cutoff is a *sufficient statistic* for measuring the change in firm-level operating profit. This is the case, as the ratio of operating profits is a function of the productivity ratio in the CES case. In general, the tariff change affect firm's operating profits not only through changing the exit cutoff, but also through affecting the profit directly. The assumption that $\frac{\mathrm{d}\log\pi_i}{\mathrm{d}\log z_i^*}<0$ implies that $D_i(z)>0$ for all $z\geq z_i^*$ as $[\rho_i(z)(\sigma_i(z)-1)]>0$ and $A_i>0$. Therefore, the fourth assumption in Assumption 2 ensures that $D_i(z)>0$. It is also trivially true that $C_i(z)>0$.

We now use the balance of flow condition to derive in the change in the total number of potential entrants (into all markets). Recall that

$$\left[\frac{\mathbb{E}^i\,\pi_i(z)\mathrm{d}\log\pi_i(z)}{(\rho+\nu_i)\bar{\pi}_i}\right]=\frac{\beta}{1-\beta}\left[\mathrm{d}\log M_i^P-\mathrm{d}\log M\right],$$

which can be rewritten as

$$\left[\frac{\mathbb{E}^i\,\pi_i\left(-C_i(z)\mathrm{d}\log t_i-D_i(z)\mathrm{d}\log M_i^P\right)}{(\rho+\nu_i)\bar{\pi}_i}\right]=-E_i\mathrm{d}\log t_i-F_i\mathrm{d}\log M_i^P=\frac{\beta}{1-\beta}\left[\mathrm{d}\log M_i^P-\mathrm{d}\log M\right],$$

where

$$E_i\equiv\frac{\mathbb{E}^i\,\pi_i(z)C_i(z)}{(\rho+\nu_i)\bar{\pi}_i},$$

and

$$F_i\equiv\frac{\mathbb{E}^i\,\pi_i(z)D_i(z)}{(\rho+\nu_i)\bar{\pi}_i}.$$

Defining $G_i \equiv \frac{\mathbb{E}^i \pi_i(z) C_i(z)}{\mathbb{E}^i \pi_i(z) D_i(z)}$, we have

$$\mathrm{d}\log M_i^P = \frac{\frac{\beta}{1-\beta}}{\frac{\beta}{1-\beta}+F_i}\mathrm{d}\log M - \frac{F_i}{\frac{\beta}{1-\beta}+E_i}\left(\mathrm{d}\log t_i\right) = \frac{\frac{\beta}{1-\beta}}{\frac{\beta}{1-\beta}+F_i}\mathrm{d}\log M - \frac{F_i}{\frac{\beta}{1-\beta}+F_i}\left(G_i \mathrm{d}\log t_i\right).$$

Using the free entry condition, we can eventually solve for the change in the total number of potential entrants:

$$\sum_{i=0}^{N} s_i \left[\frac{\frac{\beta}{1-\beta}}{\frac{\beta}{1-\beta}+F_i}\mathrm{d}\log M - \frac{F_i}{\frac{\beta}{1-\beta}+F_i}\left(G_i \mathrm{d}\log t_i\right)\right] = \mathrm{d}\log M \Rightarrow \mathrm{d}\log M = -\frac{\sum_{i=0}^{N} \frac{s_i E_i}{\frac{\beta}{1-\beta}+F_i}\left(\mathrm{d}\log t_i\right)}{\sum_{i=0}^{N}\left(\frac{s_i F_i}{\frac{\beta}{1-\beta}+F_i}\right)} < 0.$$

It is clear that the numerator is positive. The denominator is positive when $F_i > 0$. When we focus on the case of symmetric countries, it require $F_i > 0$ in the initial equilibrium.

Number of potential entrants into each market M_i^P The change in the number of potential entrants into each market is derived as

$$\mathrm{d}\log M_i^P = -\frac{E_i}{\frac{\beta}{1-\beta}+F_i}\mathrm{d}\log t_i - \frac{\frac{\beta}{1-\beta}}{\frac{\beta}{1-\beta}+F_i}\frac{\sum_{i=0}^{N} \frac{s_i E_i}{\frac{\beta}{1-\beta}+F_i}\left(\mathrm{d}\log t_i\right)}{\sum_{i=0}^{N}\left(\frac{s_i F_i}{\frac{\beta}{1-\beta}+F_i}\right)} < 0,$$

which is a weighted average of the average change in the tariff and the market-specific tariff change.

Cutoffs z_i^* Since we have

$$\mathrm{d}\log z_i^* = \frac{\eta_i \mathrm{d}\log t_i + A_i \mathrm{d}\log M_i^P}{\frac{\sigma_i^*-1}{\rho_i^*} + B_i \lambda_i(z_i^*)\varphi_i^*},$$

it must be true that $\mathrm{d}\log z_i^* < 0$ for the third market as $\log t_i = 0$ and $\mathrm{d}\log M_i^P < 0$ in the third market. In the focal market, the change in the cutoff is ambiguous, as the tariff increases while the number of potential entrants drops.

Firm-level $p_i(z)$ **and the ideal price index** P_i **and** P_i^I Since

$$\mathrm{d}\log p_i(z) = (1-\rho_i(z))\mathrm{d}\log P_i,$$

we only need to figure out how P_i changes in each market. Let us focus on the following two equations:

$$\lambda_i(z_i^*)\varphi_i^* \mathrm{d}\log(z_i^*) = \mathrm{d}\log M_i^P + \mathbb{E}_\lambda\left[\rho_i(z)(\sigma_i(z)-1)\right]\mathrm{d}\log P_i,$$

and

$$\lambda_i(z_i^*)\varphi_i^* \mathrm{d}\log z_i^* = \frac{\eta_i \mathrm{d}\log t_i + A_i \mathrm{d}\log M_i^P}{\frac{\sigma_i^*-1}{\rho_i^*\lambda_i(z_i^*)\varphi_i^*} + B_i}.$$

Since $B_i > A_i > 0$ and $\mathrm{d}\log t_i = 0$ in the third market, we must have

$$0 > \lambda_i(z_i^*)\varphi_i^* \mathrm{d}\log z_i^* > \mathrm{d}\log M_i^P.$$

Therefore, we must have $\mathrm{d}\log P_i > \mathrm{d}\log p_i(z) > 0$ and $\mathrm{d}\log\left(\frac{p_i(z)}{P_i}\right) < 0$ for all firms in the third market. Moreover, since

$$\mathrm{d}\log P_i^I = \mathrm{d}\log t_i - \left(\bar{\delta}_i - \delta_i^*\right)\mathrm{d}\log M_i^P - \mathbb{E}_\lambda\left[\left(\left(\frac{1}{\rho_i}-1\right) + (\delta_i^*-1)(\sigma_i(z)-1)\right)\mathrm{d}\left(\log\frac{p_i(z)}{P_i}\right)\right],$$

and $\mathrm{d}\log t_i = 0$, $\mathrm{d}\log M_i^P < 0$, $\mathrm{d}\log\left(\frac{p_i(z)}{P_i}\right) < 0$ in the third market, the ideal price index P_i^I drops in the third market.

For the foal market, we have two cases. First, $\mathrm{d}\log z_j^* > 0$. In such a case, we must have $\mathrm{d}\log P_j > \mathrm{d}\log p_j(z) > 0$ and $\mathrm{d}\log\left(\frac{p_j(z)}{P_j}\right) < 0$. As a result, both P_j and P_j^I increase in the focal market (as $\mathrm{d}\log t_j > 0$, $\mathrm{d}\log M_j^P < 0$). Second, $\mathrm{d}\log z_j^* < 0$. In such a case, we still have

$$0 > \lambda_i(z_j^*)\varphi_j^* \mathrm{d}\log z_j^* > \mathrm{d}\log M_j^P,$$

as $\mathrm{d}\log t_j > 0$. Accordingly, $\mathrm{d}\log P_j > \mathrm{d}\log p_j(z) > 0$ and $\mathrm{d}\log\left(\frac{p_j(z)}{P_j}\right) < 0$. As a result, both P_j and P_j^I increase in the focal market (as $\mathrm{d}\log t_j > 0$, $\mathrm{d}\log M_j^P < 0$).

In total, both firm-level price and the ideal price index increase in all markets.

Total export value E_i and real consumption Q_i Since the ideal price index (P_j^I or P_i^I) increases in all markets, the real consumption $Q_i = D_i^{\eta_i} {P_i^I}^{-\eta_i}$ and total expenditure on Chinese goods $E_i = Q_i P_i^I = D_i^{\eta_i} {P_i^I}^{1-\eta_i}$ drop in all markets.

Market share $s_i(z)$ and markup $\mu_i(z)$ Since $\mathrm{d}\log\left(\frac{p_j(z)}{P_j}\right) < 0$ in all markets, the market share of each surviving firms $s\left(\frac{p_j(z)}{P_j}\right)$ increases in all markets. As

$$\sigma_i(z) = 1 - \frac{zs^{'}(z)}{s(z)},$$

where $z \equiv \frac{p_j(z)}{P_j}$, $\sigma_i^{'}(z) > 0$ and $\mathrm{d}\log\left(\frac{p_j(z)}{P_j}\right) < 0$, we must have $d\sigma_i(z) < 0$ and $d\mu_i(z) > 0$ in all markets. In short, the price and markup charged by all surviving firms increase in all markets.

Number of active firms N_i WE have no definitive predictions on how N_i and N_j (i.e., the number of active firms) changes after a market-specific tariff shock. WE have

$$\mathrm{d}\log N_i = \mathrm{d}\log M_i^p - \varphi_i^*\mathrm{d}\log z_i^*.$$

Although we know $0 > \lambda_i(z_i^*)\varphi_i^*\mathrm{d}\log z_i^* > \mathrm{d}\log M_i^P$ in the third market, it can be the case that $0 > \lambda_i(z_i^*)\varphi_i^*\mathrm{d}\log z_i^* > \mathrm{d}\log M_i^P > \varphi_i^*\mathrm{d}\log z_i^*$ in the third market, since $1 > \lambda_i(z_i^*) > 0$. Therefore, $\mathrm{d}\log N_i$ can be either positive or negative in the third market. For the focal market, $\log N_j < 0$ if $\mathrm{d}\log z_j^* > 0$ as $\mathrm{d}\log M_j^p$ is negative (for all markets). If $\mathrm{d}\log z_j^* < 0$, the sign in unclear.

Firm-level output and revenue $q_i(z)$ and $r_i(z)$ In the case of heterogeneous firms, the model make s no clear prediction on the change in firm0level output/revenue in both the focal and the third markets. The model, however, does predict that unproductive reduce (or even increase) output and revenue less relative to productive firms in both the focal and third markets. This comes from heterogeneous responses of markups to the tariff shock. Since

$$\mathrm{d}\log p_i(z) = (1 - \rho_i(z))\mathrm{d}\log P_i,$$

the price and markup of more productive firms increase more after the tariff shock as $\rho_i(z)$ goes down with the productivity draw. Therefore, unproductive firms benefit more (or lose less) thanks to *firm-specific* exposure to the anti-competitive effect after the tariff shock.

We proceed our analysis as follows. Firm-level output is calculated as

$$q_i(z) = \frac{s\left(\frac{p_i(z)}{P_i}\right)}{t_i p_i(z)} D^{\eta_i} {P_i^I}^{1-\eta_i}.$$

Therefore, the change in firm-level output is

$$\mathrm{d}\log q_i(z) = \mathrm{d}\log s\left(\frac{p_i(z)}{P_i}\right) - \mathrm{d}\log t_i - \mathrm{d}\log p_i(z) - (\eta_i - 1)\mathrm{d}\log P_i^I.$$

Since we have

$$-(\sigma_i(z)-1)\mathrm{d}\log\frac{p_i(z)}{P_i} = \mathrm{d}\log s\left(\frac{p_i(z)}{P_i}\right);\ \ \mathrm{d}\log p_i(z) = (1-\rho_i(z))\mathrm{d}\log P_i,$$

we can rewrite the above equation as

$$\mathrm{d}\log q_i(z) = (\sigma_i(z)\rho_i(z) - 1)\mathrm{d}\log P_i - \mathrm{d}\log t_i - (\eta_i - 1)\mathrm{d}\log P_i^I. \tag{77}$$

If $\sigma_i(z)\rho_i(z) < 1$, which is likely to be true for productive firms (as both $\sigma_i(z)$ and $\rho_i(z)$ decrease with the market share), then the firm-level output drops in all markets. However, this argument does not necessarily hold for unproductive firms. For firm-level revenue, we have

$$\mathrm{d}\log r_i(z) = \mathrm{d}\log q_i(z) + \mathrm{d}\log p_i(z) = \rho_i(z)(\sigma_i(z)-1)\mathrm{d}\log P_i - \mathrm{d}\log t_i - (\eta_i - 1)\mathrm{d}\log P_i^I. \tag{78}$$

From both equation (77) and equation (78), we can see that firm-level output/revenue have the same degree of exposure to the tariff change and to the change in the ideal price index (in both the focal and the third markets). However, they also show that firm-level output/revenue are reduced less (or even increase) for unproductive firms, as both $\rho_i(z)$ and $\sigma_i(z)$ decrease with firm productivity and the adjustment price aggregator P_i increases in both markets. Therefore, we have the same key firm-level prediction as in the case with Kimball demand:

$$\frac{\partial\,\mathrm{d}\log\mu_i}{\partial\log z} > 0, \quad \frac{\partial\,\mathrm{d}\log\pi_i}{\partial\log z} < 0, \quad \frac{\partial\,\mathrm{d}\log r_i}{\partial\log z} < 0, \quad \frac{\partial\,\mathrm{d}\log q_i}{\partial\log z} < 0$$

Changes in $\tilde{Q}_i$ The total quantity $\tilde{Q}_i = N_i \int_{z_i^*}^{\infty} q_i \frac{g(z)}{1-G(z_i^*)} \mathrm{d}z$. Then,

$$\mathrm{d}\log \tilde{Q}_i = \mathrm{d}\log N_i + \varphi_i^* \mathrm{d}\log z_i^* - \frac{q_i^*}{\bar{q}_i}\varphi_i^* \mathrm{d}\log z_i^* + \mathbb{E}_{q_i}^i[\mathrm{d}\log q_i] = \mathrm{d}\log M_i^P - \frac{q_i^*}{\bar{q}_i}\varphi_i^* \mathrm{d}\log z_i^* + \mathbb{E}_{q_i}^i[\mathrm{d}\log q_i],$$

where

$$\mathbb{E}_{q_i}^i[\mathrm{d}\log q_i] = \frac{\int_{z_i^*}^{\infty} \mathrm{d}\log q_i \; q_i \frac{g(z)}{1-G(z_i^*)}\mathrm{d}z}{\int_{z_i^*}^{\infty} q_i \frac{g(z)}{1-G(z_i^*)}\mathrm{d}z},$$

where $\bar{q}_i$ is the average quantity. First, we need to solve for $\mathrm{d}\log P_i$. Since we have

$$\lambda_i(z_i^*)\varphi_i^* \mathrm{d}\log(z_i^*) = \mathrm{d}\log M_i^P + \mathbb{E}_\lambda\left[\rho_i(z)(\sigma_i(z)-1)\right]\mathrm{d}\log P_i,$$

and

$$\lambda_i(z_i^*)\varphi_i^* \mathrm{d}\log z_i^* = \frac{\eta_i \mathrm{d}\log t_i + A_i \mathrm{d}\log M_i^P}{\frac{\sigma_i^*-1}{\rho_i^*\lambda_i(z_i^*)\varphi_i^*} + B_i},$$

we must have

$$\mathrm{d}\log P_i = \frac{\frac{\eta_i \mathrm{d}\log t_i + A_i \mathrm{d}\log M_i^P}{\frac{\sigma_i^*-1}{\rho_i^*\lambda_i(z_i^*)\varphi_i^*}+B_i} - \mathrm{d}\log M_i^P}{\mathbb{E}_\lambda\left[\rho_i(z)(\sigma_i(z)-1)\right]} > 0. \tag{79}$$

Next, we have

$$\mathrm{d}\log P_i^I = \left(1 + \frac{\eta_i[\gamma_i + (\delta_i^*-1)]}{\frac{\sigma_i^*-1}{\rho_i^*\lambda_i(z_i^*)\varphi_i^*} + B_i}\right)\mathrm{d}\log t_i - \left[\left(\bar{\delta}_i - 1\right) + \gamma_i - \frac{A_i[\gamma_i + (\delta_i^*-1)]}{\frac{\sigma_i^*-1}{\rho_i^*\lambda_i(z_i^*)\varphi_i^*} + B_i}\right]\mathrm{d}\log M_i^P > 0.$$

Moreover, we have

$$\mathrm{d}\log q_i(z) = (\sigma_i(z)\rho_i(z)-1)\mathrm{d}\log P_i - \mathrm{d}\log t_i - (\eta_i - 1)\mathrm{d}\log P_i^I < (\sigma_i(z)\rho_i(z)-1)\mathrm{d}\log P_i - \mathrm{d}\log t_i,$$

as $\eta_i \geq 1$ and $\mathrm{d}\log P_i^I > 0$. Next, note that

$$\lambda(z_i^*) = \frac{p_i^* q_i^* N_i}{\bar{p}_i \bar{q}_i N_i} > \frac{q_i^*}{\bar{q}_i},$$

where p_i^* and q_i^* are the price and quantity of the firm at the survival cutoff and $\bar{p}_i$ and $\bar{q}_i$ are the average price and quantity. This is true, as the price charged decreases with firm

productivity (i.e., $\bar{p}_i < p_i^*$).[21]

We discuss the third market first. In the third market, $\mathrm{d}\log z_i^*$. Thus, we have

$$-\frac{q_i^*}{\bar{q}_i}\varphi_i^*\mathrm{d}\log z_i^* < -\lambda(z_i^*)\varphi_i^*\mathrm{d}\log z_i^* = -\frac{\eta_i\mathrm{d}\log t_i + A_i\mathrm{d}\log M_i^P}{\frac{\sigma_i^*-1}{\rho_i^*\lambda_i(z_i^*)\varphi_i^*}+B_i}.$$

In total, we have

$$\mathrm{d}\log\tilde{Q}_i < \mathrm{d}\log M_i^P - \frac{\eta_i\mathrm{d}\log t_i + A_i\mathrm{d}\log M_i^P}{\frac{\sigma_i^*-1}{\rho_i^*\lambda_i(z_i^*)\varphi_i^*}+B_i} + \mathbb{E}_{q_i}^i[(\sigma_i(z)\rho_i(z)-1)\mathrm{d}\log P_i - \mathrm{d}\log t_i].$$

In the third market, we have $\mathrm{d}\log t_i$. Therefore, the above expression can be simplified to

$$\mathrm{d}\log\tilde{Q}_i < \left[1 - \frac{A_i}{\frac{\sigma_i^*-1}{\rho_i^*\lambda_i(z_i^*)\varphi_i^*}+B_i}\right]\mathrm{d}\log M_i^P + \mathbb{E}_{q_i}^i[(\sigma_i(z)\rho_i(z)-1)\mathrm{d}\log P_i],$$

where

$$\mathrm{d}\log P_i = \frac{\frac{A_i\mathrm{d}\log M_i^P}{\frac{\sigma_i^*-1}{\rho_i^*\lambda_i(z_i^*)\varphi_i^*}+B_i} - \mathrm{d}\log M_i^P}{\mathbb{E}_\lambda\left[\rho_i(z)(\sigma_i(z)-1)\right]} = -\left[1 - \frac{A_i}{\frac{\sigma_i^*-1}{\rho_i^*\lambda_i(z_i^*)\varphi_i^*}+B_i}\right]\frac{\mathrm{d}\log M_i^P}{\mathbb{E}_\lambda\left[\rho_i(z)(\sigma_i(z)-1)\right]}.$$

Therefore, we have

$$\begin{aligned}
&\mathbb{E}_{q_i}^i[(\sigma_i(z)\rho_i(z)-1)\mathrm{d}\log P_i] \\
= &\left[1 - \frac{A_i}{\frac{\sigma_i^*-1}{\rho_i^*\lambda_i(z_i^*)\varphi_i^*}+B_i}\right]\frac{\mathbb{E}_{q_i}^i(\sigma_i(z)\rho_i(z)-1)}{\mathbb{E}_\lambda\left[\rho_i(z)(\sigma_i(z)-1)\right]}(-\mathrm{d}\log M_i^P) \\
< &\left[1 - \frac{A_i}{\frac{\sigma_i^*-1}{\rho_i^*\lambda_i(z_i^*)\varphi_i^*}+B_i}\right](-\mathrm{d}\log M_i^P),
\end{aligned}$$

as $\rho_i(z) \leq 1$ for all z, $A_i < B_i$ and $-\mathrm{d}\log M_i^P > 0$. Also, note that

$$\mathbb{E}_\lambda\left[\rho_i(z)(\sigma_i(z)-1)\right] > \mathbb{E}_{q_i}^i\left[\rho_i(z)(\sigma_i(z)-1)\right],$$

as the sales share of unproductive firms is larger than the quantity share (as $p_i(z)$ is higher

[21] This is true as the passthrough is always positive.

for unproductive firms) and both $\rho_i(z)$ and $\sigma_i(z)$ increase with z. In total, we have

$$\mathrm{d}\log\tilde{Q}_i < \left[1-\frac{A_i}{\frac{\sigma_i^*-1}{\rho_i^*\lambda_i(z_i^*)\varphi_i^*}+B_i}\right]\mathrm{d}\log M_i^P - \left[1-\frac{A_i}{\frac{\sigma_i^*-1}{\rho_i^*\lambda_i(z_i^*)\varphi_i^*}+B_i}\right]\mathrm{d}\log M_i^P = 0$$

in the third market.

Now, we discuss the focal market. We can rewrite the log change in the total quantity of exports as

$$\mathrm{d}\log\tilde{Q}_j = \mathrm{d}\log M_j^P - \frac{q_j^*}{\bar{q}_j}\frac{1}{\lambda_j(z_j^*)}\frac{\eta_j\mathrm{d}\log t_j + A_j\mathrm{d}\log M_j^P}{\frac{\sigma_j^*-1}{\rho_j^*\lambda_j(z_j^*)\varphi_j^*}+B_j} + \mathbb{E}_{q_j}^i[\mathrm{d}\log q_j].$$

As $\mathrm{d}\log M_j^P < 0$ and $\frac{q_j^*}{\bar{q}_j} < \lambda_j(z_j^*)$, we must have

$$\mathrm{d}\log\tilde{Q}_j < \left[1-\frac{A_j}{\frac{\sigma_j^*-1}{\rho_j^*\lambda_i(z_j^*)\varphi_j^*}+B_j}\right]\mathrm{d}\log M_j^P - \frac{q_j^*}{\bar{q}_j}\frac{1}{\lambda_j(z_j^*)}\frac{\eta_j\mathrm{d}\log t_j}{\frac{\sigma_j^*-1}{\rho_j^*\lambda_j(z_j^*)\varphi_j^*}+B_j} + \mathbb{E}_{q_j}^i[(\sigma_j(z)\rho_j(z)-1)\mathrm{d}\log P_j] - \mathrm{d}\log$$

Since

$$\begin{aligned}&\mathbb{E}_{q_j}^j[(\sigma_j(z)\rho_j(z)-1)\mathrm{d}\log P_j] - \mathrm{d}\log t_j\\ <\ &\left[1-\frac{A_j}{\frac{\sigma_j^*-1}{\rho_j^*\lambda_j(z_j^*)\varphi_j^*}+B_j}\right](-\mathrm{d}\log M_j^P) - \mathrm{d}\log t_j,\end{aligned}$$

we have

$$\mathrm{d}\log\tilde{Q}_j < \left[1-\frac{A_j}{\frac{\sigma_j^*-1}{\rho_j^*\lambda_i(z_j^*)\varphi_j^*}+B_j}\right](\mathrm{d}\log M_j^P - \mathrm{d}\log M_j^P) - \frac{q_j^*}{\bar{q}_j}\frac{1}{\lambda_j(z_j^*)}\frac{\eta_j\mathrm{d}\log t_j}{\frac{\sigma_j^*-1}{\rho_j^*\lambda_j(z_j^*)\varphi_j^*}+B_j} - \mathrm{d}\log t_j < 0,$$

as $\mathrm{d}\log t_j > 0$ in the focal market.