Population Aging, Firms, and Worker Composition

Santiago Caicedo, Cheng Chen, Aspen Gorry and Takahiro Hattori*

March 19, 2025

Abstract

Population aging changes the *composition of workers* by experience and human capital over time. How these changes matter for firms has important implications for many policy choices, such as immigration, education, and retirement programs. To understand the impact of worker composition on the economy, we develop a framework to study the allocation of workers who differ in experience and human capital to heterogeneous firms. We then apply it using Japanese establishment-employee matched data. In a static model with no costs to reallocating workers, the Japanese GDP would be 5.3% higher with the optimal experience and skill composition of workers. We later extend the model to a dynamic setting by adding the life-cycle dimension of the worker and a firing cost paid by the firm. A calibration exercise substantiates a *strong complementarity* between workers' human capital and firm productivity, which implies differential hiring/firing strategies used by firms and *distributional effects* of the firing cost/population aging on workers and firms. Finally, we use the Japanese establishment level to provide evidence supporting these predictions.

Keywords: Population Aging, Employment Protection, Composition of the Labor Force, Worker-Firm Matching, Multi-Dimensional Heterogeneity of Workers, Comparative Advantage of Productive Firms

JEL-Codes: D21, D22, E23, E24, J11, J23, J58

*Caicedo: Northeastern University, sa.caicedo@northeastern.edu; Chen: Clemson University, cchen9@clemson.edu; Gorry: Clemson University, cgorry@clemson.edu; Hattori: The University of Tokyo, takahiro.hattori@pp.u-tokyo.ac.jp, hattori0819@gmail.com. We thank Isabella Becerra for her excellent research assistance.

1 Introduction

An important difference in economies is the composition of workers by observable demographic groups such as experience and human capital. Recently, the composition of workers has been evolving rapidly due to the aging of the population, declines in fertility, and immigration globally (see Toossi and Elka, 2017; Cardona, 2023; Frey, 2025; Qi, 2025). Early studies by Welch (1979) and Berger (1985) show that increases in cohort sized of the baby boom generation reduced wages and wage growth for these larger cohorts, highlighting that workers of different experience levels are imperfect substitutes in the labor market. Given large differences in demographic patterns across countries and over time it is important to understand how changes in the composition of labor supply across demographic groups matter for firm-level and aggregate outcomes.

To understand how demographic changes matter for firm distributions and aggregate outcomes, we develop a framework that characterizes the optimal allocation of workers to firms. Our primary interest in this paper is understanding the allocation of workers of different experiences and human capital levels to firms in a market with and without frictions. Given our interest in changes in these worker demographics, we use Japanese establishment data to study the employment of different *groups* of workers at heterogeneous firms. Japan is an interesting example as it has a particularly old workforce and a frictional labor market.[1] Understanding these optimal portfolios of heterogeneous establishments provides evidence on the optimal mix of workers in production and hence informs a variety of policy questions such as immigration, education, retirement, etc.

We first develop a static frictionless framework to understand the efficient allocation of heterogeneous (e.g., experience and skill) workers to heterogeneous firms. Using Japanese establishment-employee matched data, we parameterize group-level productivity by firm size bin (i.e., worker group-firm-specific productivity) and find that both the supply of workers by experience and human capital are important for observed wages.[2] Using these group-level productivities, we can use the model to compute the optimal composition of workers across

[1] The median experience of the Japanese population was 48.4 years in 2020 and has been increasing over time (IMF, 2020). Moreover, the growth rate of the working-age population has declined from 0.24% in 2001 to −0.45% in 2021 (World Bank). Labor market regulations in Japan include high firing costs and the lifetime employment system (see Kawaguchi and Murao (2014) and Fujimoto (2017) for details).

[2] Note that this approach takes this measured technology as given when considering changes in demographics. Modeling endogenous technological changes that could arise in response to changes in the composition of workers is beyond the scope of our framework.

groups. We find that the optimal composition has a larger share of experienced workers (21-35 years of experience or 41-55 years old) workers than Japan's current population. We also find a strong complementarity between firm size and worker's human capital in determining the worker group-firm-specific productivity. Finally, we find Japanese GDP would be 5.3% higher with the optimal experience and skill composition of workers.

The optimal allocations described by the static model are consistent with the data if there are no costs of reallocating workers across firms. However, labor market frictions, such as firing costs that are prevalent in Japan, mean that there are likely significant costs for workers to move across firms. In the presence of such friction, it is necessary to model both the cost of worker reallocation and how workers transition across demographic groups over time. Modeling the demographic transitions allows us to better understand the feasibility of the optimal allocations of workers that the model describes. Finally, the static model is not able to capture some important features of the Japanese data. For instance, we find that larger establishments have, on average, younger (i.e., less experienced) workers but also have longer average tenure. Understanding why experience-tenure profiles of workers differ by establishment is crucial to explaining this pattern.

We extend the model to include these dynamic features to understand these worker transitions across groups and be able to reconcile these facts on how worker tenure and experience differ across firms. The dynamic model features workers whose experience evolves deterministically and human capital that evolves stochastically over their life cycle. Heterogeneous firms that differ in their productivity then hire a portfolio of workers in each bin and must plan for how that portfolio evolves over time. Finally, we include a firing cost that makes it costly to reallocate workers and distinguishes internal and external labor markets so the model has a meaningful definition of tenure.

Using this dynamic framework, we revisit the optimal composition of labor supply for production. To provide evidence on group-specific labor productivity, parameters of the model are chosen to match key moments obtained from the microdata, including wages and the percentage of workers in each group. Moreover, the parameter for the complementarity between firm productivity and human capital is informed by matching the differential experience-tenure profiles across firms. This calibrated parameter implies a strong (log) complementarity and leads to differential hiring/firing strategies used by the establishments. High-productivity establishments hire more young workers and fire middle-aged and old workers who have not upgraded their human capital enough. Low-productivity establishments hire fewer young

workers and fill the vacancies by hiring experienced workers separated from high-productivity establishments.

Having calibrated the model, we implement two counterfactual analyses. First, we increase the firing cost, which dampens productive establishments' incentives to hire young workers and incentivizes them to keep more old workers. As a result, young (old) workers end up with lower (higher) wages, leading to higher wage inequality.[3] Various establishments are also affected differently. As productive establishments' firing of young workers drops, their employment/output shares in the market shrink. Interestingly, their profitability actually decreases less (relative to the unproductive ones), thanks to lower wages of young workers whom they hire disproportionately more. In total, lower (product) market concentration due to a higher firing cost leads to fewer worker movements (i.e., less firing) and increasing wage inequality in the model. Next, we shift the distribution of workers toward the older ones to study the impact of population aging. This change hurts high-productivity establishments, as their comparative advantage is to use workers' human capital as opposed to the experience. As a result, their market shares go down. In summary, both analyses emphasize the *distributional effects* of labor market policies/changes in labor supply.

Finally, we provide evidence for some key predictions of our dynamic model. First, we obtain information on hiring at the establishment-year level to show that larger establishments hire disproportionately more job-market entrants and are more likely to hire job-market entrants (as opposed to job-market incumbents). These findings are consistent with the model's predictions of firms' hiring strategies. Next, we provide evidence for our model's prediction of the effects of population aging on firms. Specifically, we exploit differential changes in the growth rate of the working-age population across Japanese prefectures to investigate how they affect the employment/sales shares of the largest establishments. We find that these shares of the top establishments increase more when the growth rate of the working-age population in that prefecture is higher. Moreover, this finding continues to hold, when we use the 20-year lagged birth rate as an instrument variable (IV) for the growth rate of the working-age population or the average age of the prefecture-level population as the independent variable.[4] In summary, our model's key predictions at both the establishment level and the market level are consistent with the empirical findings.

[3] Young workers are paid less than old workers on average, both in the data and the model.

[4] When we use the average experience of the prefecture-level population, which is strongly and negatively correlated with the growth rate of the working-age population, the model predicts that employment/sales shares accounted for by top firms are lower in markets with older workers.

Understanding how firms choose a portfolio of workers contributes to a number of important areas of research. First, we relate most closely to a literature that seeks to understand the impact of population aging on economic outcomes. Most closely related to our paper are papers that consider the effect of population aging on firm dynamics including Karahan et al. (2019), Peters and Walsh (2021) and Hopenhayn et al. (2022).[5] These papers study how the growth rate of the size of the labor force matters for firm entry/exit (i.e., dynamism), innovation, and labor share. Our paper complements this work by abstracting from overall changes in the size of the working-age population but focusing on how changes in the composition of the labor force are important for firm productivity and inequality. Specifically, we show how an increase in the relative supplies of experienced and inexperienced workers affects the output of heterogeneous firms and market concentration.

Second, we relate to the literature on firm dynamics with worker heterogeneity. Recent papers in this literature including Engbom et al. (2019), Gregory (2020), Bagga (2022), and Kim (2023) focus on workers who are heterogeneous in a single dimension: skill. We enrich this framework by emphasizing the importance of worker experience in the portfolio of workers hired by firms. Experience is interesting as it ties firm portfolios to deterministic patterns of demographic change driven by population aging. Our paper complements this literature by showing that firms have *heterogeneous* exposure to worker skills and experience because more productive firms hire relatively more skilled workers. This heterogeneous exposure has important implications for how population aging and firing costs influence aggregate productivity, product market concentration, and wage inequality between experienced and inexperienced workers.

Finally, our paper contributes to the literature on the impact of firing costs on firm outcomes. Bentolila and Bertola (1990) and Hopenhayn and Rogerson (1993) are the two key early contributions. Bentolila and Bertola (1990) find that the firing cost has larger effects on the propensity to fire than to hire, increasing the average employment of a single firm in a partial equilibrium setting. In a general equilibrium setting, Hopenhayn and Rogerson (1993) shows that firing taxes distorts firms' hiring and firing decisions, decreasing output and productivity for the aggregate economy. Subsequent research extends Hopenhayn and Rogerson (1993) by introducing capital into the production function (Veracierto, 2001) or search and matching frictions into the labor market (Alvarez and Veracierto, 2001) or innovation activities into the

[5]Other papers on the aggregate implications of population aging that do not focus on firm dynamics include Maestas et al. (2016), Hayashi (2023), and Bianchi and Paradisi (2024).

firm's problem (Poschke, 2009). We introduce the portfolio choice of workers into the model and show that a firing cost that applies equally to every firm leads to strong distributional effects on workers and firms.

The paper is structured as follows. Section 2 describes the Japanese establishment-worker data and key empirical findings. Section 3 presents the static model, quantifies the gains from labor reallocation, and extends it to a dynamic framework, which we calibrate to microdata (work-in-progress) and contrast to additional empirical evidence in Section 4. Finally, Section 5 concludes.

2 Empirical Findings

In this section, we describe the Japanese establishment work dataset and present our main empirical finding, documenting that larger establishments have younger workers but with longer tenure.

Japanese Establishment Data

We use the basic survey of wage structure (BSWS), which is a survey of wage structure at the establishment level. The BSWS is conducted annually by the Japanese Ministry of Health, Labor, and Welfare (MHLW). The universe of the survey is private establishments with five or more regular employees and public establishments with 10 or more regular employees in Japan, except those classified in agriculture, forestry, fishery, and the legislative, administrative, and judicial branches of local and national governments with a new random sample of workers each year. The survey has been conducted continuously since 1948; however, we have access to data for the period 1998–2017. Approximately 55,000 establishments provide valid responses annually, and those surveyed include approximately 1,280,000 employees each year. Although board members are not included, all types of workers (regular and temporary workers and full- and part- time workers) are surveyed. Detailed information on hours and wages, including overtime hours and annual bonus pay, and information on worker's experience, education, tenure, and work type (manufacturing/management/administrative/R&D) are available. There is neither top nor bottom coding of wage records.[6]

[6]The survey was redesigned in 2005 for the purpose of obtaining better information on non-regular workers and workers with fixed-term contracts. Before 2005, a surveyed worker only reported information on whether

We link establishments over time using the identifiers provided by the Ministry of Internal Affairs and Communications. Specifically, the sample of BSWS comes from the Economic Census, which contains all establishments in Japan and has a unique identifier for each establishment.[7] By linking the observations (i.e., establishments) in the BSWS to those in the Economic Census, we can link an establishment that appears in the BSWS for multiple years over time. This generates a panel dataset of establishments for the period of 1998-2011. Additionally, we are able to obtain information on establishment age for two sub-periods of our sample: 1998-2001 and 2004-2011.

Summary Statistics

Although the BSWS provides information at the worker level, we aggregate worker-level information to the establishment-year level, as workers cannot be linked over time. Specifically, for a variety of worker-level information ($tenure_{mean}$, age_{mean}, edu_{mean}, $salary_{mean}$, $bonus_{mean}$ and $wage_{mean}$), we calculate establishment-level averages of these variables and present their summary statistics for the entire sample period (1998-2017) in Table 1.[8] On average, an establishment in our sample employs one hundred workers and has employees that are roughly 42 years old. Japanese workers have longer average tenure in our sample than typical U.S. workers, while the average tenure has had a downward trend in the data (from around 11 years in the late 1990s to around 10 in the 2010s). The average years of schooling had increased from around 12.6 in 1998 to around 13.5 in 2017.

he/she is a temporary (or permanent) worker, where being temporary means the length of employment is less than a month. From 2005 onward, a permanent worker who is surveyed reports whether he/she is a regular worker and whether he/she is on a fixed or open-ended contract.

[7] The unique identifiers of establishments in the Economic Census are revised every three to five years. We have obtained the concordance between the current and previous identifiers each time the identifiers are revised.

[8] Annual salary is defined as the sum of the wage and the bonus.

Table 1: Summary Statistics

	mean	count	sd	p10	p25	p50	p75	p90
Employment	101.1	1094653	321.3	8	13	28	71	193
Share of non-regular workers	0.350	723674	0.340	0	0.0323	0.233	0.642	0.908
Tenure	10.24	1094653	6.052	3	5.455	9.417	14.11	18.57
Age	41.65	1094653	7.649	32.09	36.83	41.50	46.35	51.43
Years of Schooling	13.14	1073209	1.408	11.57	12	12.95	14.14	15.20
Salary	36967.9	1094653	18845.4	14028	22861.1	34968.4	48072.9	62441.2
Bonus	6324.0	1094653	6490.8	0	1100	4474.9	9756.5	15424.1
Wage	30643.9	1094653	13365.5	13399.5	20938.9	30164.6	38911.7	47977.7

Time period: 1998-2017. For all the variables, we calculate establishment-level averages and report the summary statistics across establishments and over the years. Unit for salary/wage/bonus: one hundred yen (or one USD)

Main Fact: Larger Establishments Have Younger Workers But With Longer Tenure

Larger establishments have workers who are younger but with longer tenure on average, as shown by Table 2. In this table, establishments in the highest quintile (employment ≥ 116) have workers who are about three years younger (or four years less experienced) than those in establishments that belong to the lowest quintile (employment ≤ 11). Experience is defined as experience minus the sum of six and the number of schooling years.[9] To the contrary, they have about four more years in terms of the tenure (i.e., with the current employer). This finding is puzzling. As younger workers tend to have shorter tenure, workers in larger establishments should have shorter tenure given that that they are younger then those in small establishments.

Figure 1 shows that the differential experience-tenure profiles across establishments are the keys to understanding the above pattern. The experience-tenure profile is much *steeper* for workers in larger establishments than in smaller establishments. While young workers have similar tenure in all establishments, middle-aged and old workers have longer tenure in larger establishments. For instance, workers with experience of 36-40 years in the largest establishments have about 26 years of tenure, while workers in the smallest establishments have only 16. In summary, middle-aged and old workers drive the difference in average tenure across establishments.

Patterns documented in Table 2 and Figure 1 are robust to the specific measurement we

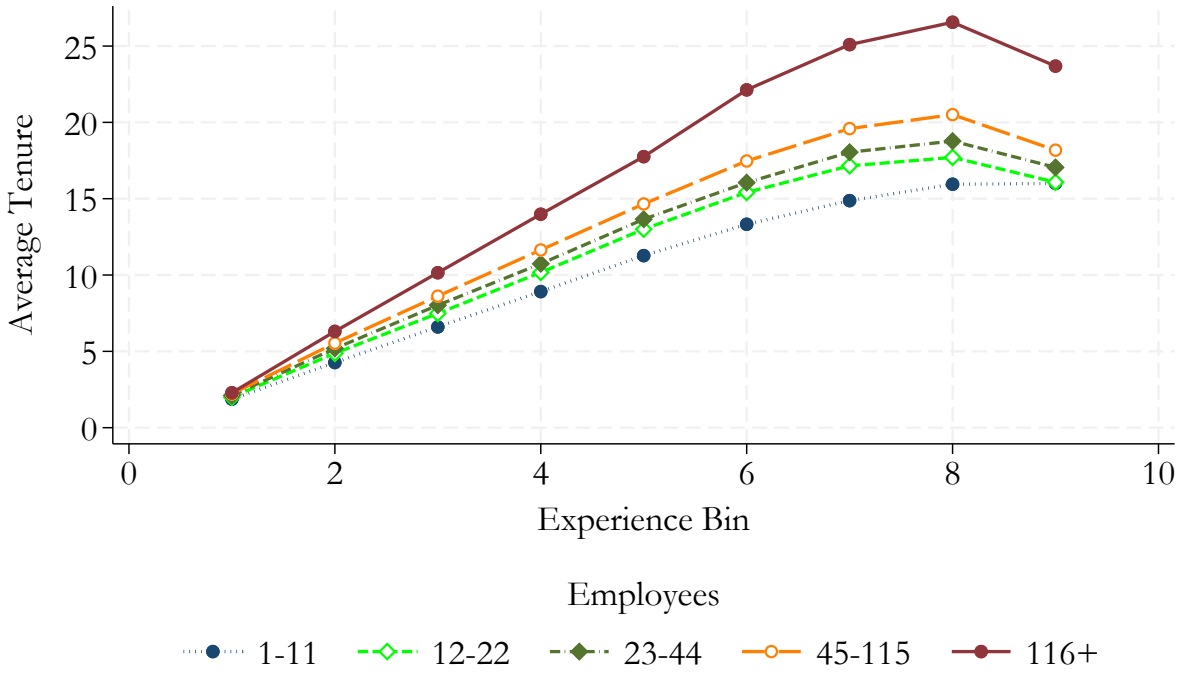
[9]We do not consider the length of unemployment when calculating the experience, as there is no such information at the workers level. This is less of concern, as the unemployment rate is fairly low in Japan.

Table 2: Average Worker Characteristics by Establishment Size Bin (1998-2003): full sample

Size Bin	Employment	Age	Experience	Tenure	Wage	Education
1	8.49	42.13	23.83	10.54	32078.94	12.30
2	16.83	40.56	21.91	10.95	35749.17	12.65
3	32.11	40.24	21.51	11.47	36916.25	12.73
4	72.46	39.96	21.11	12.24	38314.43	12.85
5	677.31	39.02	19.72	14.52	44639.36	13.30

Time period: 1998-2003. Unit of average wage: one hundred yen (or one USD). We divide establishments to five size equal-sized bins. Size bin 1: employment ≤ 11; Size bin 2: employment $\in [12, 22]$; Size bin 3: employment $\in [23, 44]$; Size bin 4: employment $\in [45, 115]$; Size bin 5: employment ≥ 116.

Figure 1: Experience-Tenure Profile (1998-2003)



Note: Experience = age - (length of education + 6). Experience bin 1: 1-5 years; ... Experience bin 8: 36-40 years; Experience bin 9: 41-45 years.

use and the specific sample (and time period) we focus on. In Tables A.1 and A.2 in the appendix, we show that the pattern documented in Table 2 continue to hold when we only use workers who are at most 60 years old or use data from the period of 2005-2011.[10] Next, we should that the differential experience-tenure profiles across establishments also hold when we use age instead of the (imputed) experience in Figure A.1 in the appendix. Finally, we show that the differential experience-tenure profiles continue to hold when we use workers who are at most 60 years old or use data from the period of 2005-2011 in Figures A.2, A.3 and A.4 in the appendix.

Establishment Age

establishment age is an important confounding factor for two reasons. First, smaller establishments tend to be younger establishments. Thus, their workers must have shorter tenure (a mechanical correlation). Second, smaller establishments tend to be younger establishments which tend to be matched with younger workers (see Ouimet and Zarutskie (2014)). In order to control for the impact of the establishment age on our documented facts, we implement three exercises.

First, we run regressions to show that the basic facts documented in Table 2 are robust to the inclusion of various fixed effects and establishment age. Appendix Tables A.3 and A.4 present the result that larger establishments have workers who are younger but with longer tenure on average *even after controlling* for the establishment age and various fixed effects. Second, we exclude young establishments (i.e., establishments that are younger than 21 years old) and recalculate average experience and tenure across establishment size bins in two separate periods (1998-2001 and 2008-2011). Results reported in Appendix Table A.5 and Appendix Figure A.5 reconfirm the finding that larger establishments have both younger workers and workers who have longer tenure. Third, we show this finding holds for establishments (with different sizes) in a given establishment-age bin in Appendix Figures A.6 and A.7. Finally, we show our finding on the differential experience/age-tenure profiles across establishments hold for establishments (with different sizes) in a given establishment-age bin in Appendix FiguresA.8 and A.9. In summary, the empirical findings documented in Table 2 and Figure 1 are robust.

The above empirical findings suggest that large and small establishments adopt different hir-

[10]The mandatory retirement experience is 60 in Japan.

ing/firing strategies *over the worker's life cycle*. In the market of young workers, large (i.e., productive) establishments hire disproportionately more of them while small (i.e., unproductive) establishments only hire a few of them. In large establishments, some of the workers get separated from their employers when they become older (maybe due to their inability of upgrading the human capital). In small establishments, most of the workers stay when they get older. Those middle-aged and old workers who leave large establishments are hired by small establishments, as their working experience (in large establishments) is valuable for those small firms. In a nutshell, large establishments *select* a fraction of existing workers and keep them when they become older. To the contrary, small establishments adopt a *gradual* hiring strategy over the worker's life cycle (i.e., hiring more middle-aged and old workers). This conjectured mechanism can explain why the experience-tenure profile is steeper in large establishments.

Firms v.s. Establishments

The above patterns we have shown are related to the size heterogeneity of various *establishments*. Since the modern economy is dominated by multi-establishment firms, it is interesting to know whether these patterns continue to hold at the firm level. In particular, are the differences in average tenure and experience of workers documented above driven by the size heterogeneity across firms?[11] Fortunately, we have a size indicator of the firm that each establishment belongs to (i.e., 5-9 employees, 10-29 employees...above 5,000 employees). In what follow, we group establishments to various size groups based on the employment of their (parent) firms and present summary statistics as well as the experience-tenure profile of firms with different sizes. To keep consistency with the previous robustness checks, we continue to throw away young establishments and to keep workers who are at most 60 years old. Tables 3 and 4 show that larger *firms* tend to have younger workers who have longer tenure on average. Figures 2 and 3 substantiate that the steeper experience-tenure profile continues to hold at the firm level. Therefore, the stylized facts documented above are drive by size heterogeneity across firms.

[11]The tenure information reported in the dataset is at the firm level. Therefore, it is also ideal to aggregate establishments to firms in order to study how the size of the employer affects average tenure of its workers.

Table 3: Worker Characteristics and Firm Size: 1998-2001 and 2004

Size Bin	Experience	Tenure	Wage
1	23.37	10.55	30424.5
2	21.60	12.04	34780.7
3	19.95	14.02	39917.4
4	20.08	16.23	46639.9

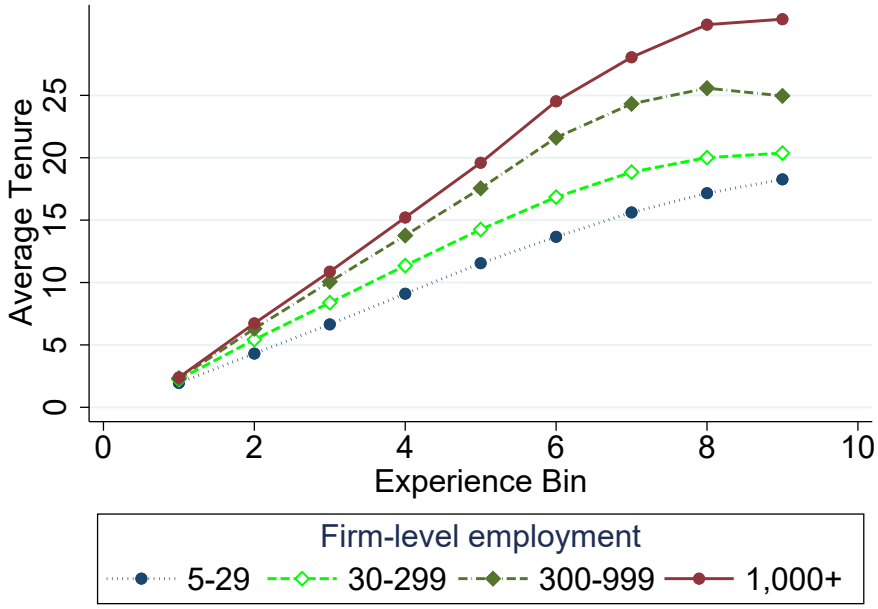
Time period: 1998-2001 and 2004. *exp*: experience (years of working). Unit of *wage*: one hundred yen (or one USD). Establishments founded after 1975 are excluded, and workers older than 60 are excluded. We divide firms to four size bins. Size bin 1: emp $\in$ [5,29]; Size bin 2: emp $\in$ [30, 299]; Size bin 3: emp $\in$ [300, 999]; Size bin 4: emp $\geq$ 1000.

Table 4: Worker Characteristics and Firm Size: 2005-2011

Size Bin	Experience	Tenure	Wage
1	22.94	10.04	27686.5
2	21.61	11.28	31746.4
3	20.22	12.81	36279.1
4	19.91	13.98	42235.4

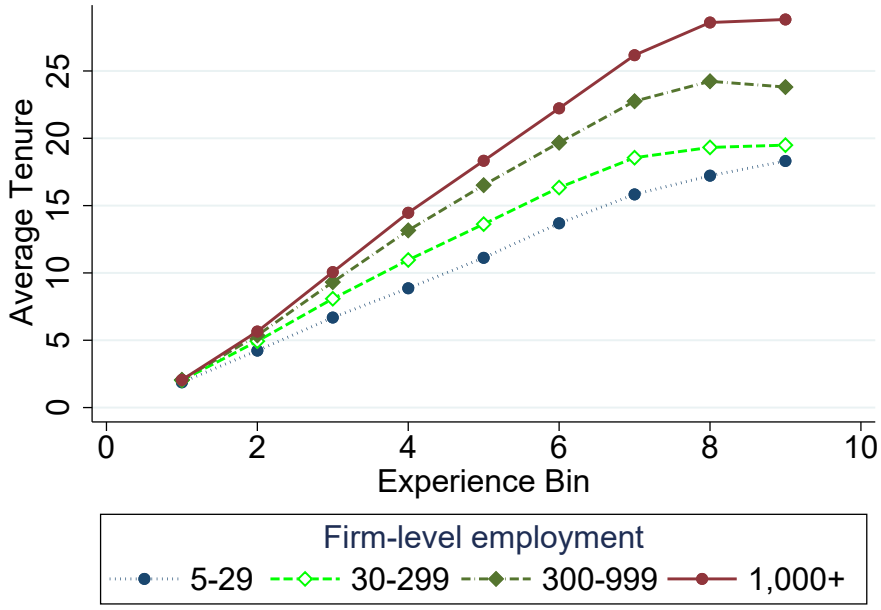
Time period: 1998-2001 and 2004. *exp*: experience (years of working). Unit of *wage*: one hundred yen (or one USD). Establishments founded after 1985 are excluded, and workers older than 60 are excluded. We divide firms to four size bins. Size bin 1: emp $\in$ [5,29]; Size bin 2: emp $\in$ [30, 299]; Size bin 3: emp $\in$ [300, 999]; Size bin 4: emp $\geq$ 1000.

Figure 2: Experience-Tenure Profile at the firm level: 1998-2001 and 2004



Note: Establishments founded after 1975 are excluded, and workers older than 60 are excluded. Experience = age - (length of education + 6). Experience bin 1: 1-5 years; ... Experience bin 8: 36-40 years; Experience bin 9: 41-45 years.

Figure 3: Experience-Tenure Profile at the firm level: 2005-2011



Note: Establishments founded after 1985 are excluded, and workers older than 60 are excluded. Experience = age - (length of education + 6). Experience bin 1: 1-5 years; ... Experience bin 8: 36-40 years; Experience bin 9: 41-45 years.

3 Model: Efficient Worker Composition

This section develops a framework to understand the efficient worker composition and its allocation to firms. We first examine a static model to analyze how changes in labor supply composition across worker groups impact labor allocations and aggregate production. Using Japanese data on employment and wages by age and firm size, we infer worker-firm-specific productivities. Then, we extend the model to a dynamic setting, incorporating the costs of reallocating labor across firms, and we explore how labor market frictions affect allocations and efficiency. The dynamic model also generates implications for worker tenure within firms.

3.1 Static Framework

We consider an economy with a continuum of heterogeneous firms characterized by productivity $z \sim \Phi(z)$. Firms produce a unique consumption good by hiring a portfolio of labor inputs from different *types* of workers. At any point in time, workers belong to a group defined by characteristics $g \in \mathcal{G}$, such as human capital level, experience, and other productivity-relevant attributes. Importantly, markets clear at the g-level and the space of worker groups is finite ($|\mathcal{G}| = G < \infty$), with all worker characteristics fully observable to all agents in the economy.

A firm with productivity z takes wage w_g as given and solves:

$$\max_{l_g \geq 0} F\left(\{l_g\}; z\right) - \sum_g w_g l_g$$

where $\{l_g\} = (l_1, ..., l_G)$ denotes the set of labor allocations for the firm. The first-order conditions are:

$$[l_g(z)]: \ \frac{\partial F(\{l_g\}; z)}{\partial l_g} \leq w_g \quad \forall z, g \tag{1}$$

The existence of a unique solution is guaranteed if the production function $F(\cdot)$ is increasing and jointly concave in $\{l_g\}$. Let $l_g^c(z)$ denote the unique solution to firm z's problem.

Suppose a total mass of L workers endowed with one unit of labor, which they supply inelastically, and let L_g denote the supply of type g workers. All G labor markets are

competitive, with firms and workers taking wages w_g as given. The labor market clearing conditions for each group of workers g are:

$$\int l_g^c(z)\ d\Phi(z) = L_g \quad \forall g. \tag{2}$$

Competitive Equilibrium: A competitive equilibrium is characterized by a set of labor allocations $\{l_g^c(z)\}$ for each firm z and type of worker g, and a set of wages w_g such that firms optimize (1) and markets clear (2).

Planner's Problem: Changing the Labor Supply Across Groups

We now consider a problem where the planner can change the labor supply across different worker groups while keeping the total amount of labor fixed.[12] This is reflects the outcome of potential policies that would would change the age or skill structure of an economy, such as incentivizing changes in fertility, retirement, immigration, or human capital formation.

To characterize the outcome based on how much flexibility the planner has to reallocate labor across groups, let $\mathcal{P}$ denote a partition of $\mathcal{G}$, where each element $P_s \in \mathcal{P}$ is a set of groups g that the planner can adjust its labor supply. If $g, g' \in P_s$, it means the planner can modify the labor allocation between the two demographic groups. Let L_s denote the total amount of labor in the partition element P_s, where the sum over all the elements of the partition must equal the aggregate total labor in the economy, $\sum_s L_s = L$.

Taking $\mathcal{P}$ as given, the planner's problem is,

$$\max_{l_g(z)\geq 0} \int F\left(\{l_g\}; z\right) \phi(z) dz \quad s.t \int \left(\sum_{g \in P_s} l_g(z) \right) \phi(z) dz = L_s \ \forall s$$

The associated Lagrangian is,

$$\mathcal{L} = \int F\left(\{l_g\}; z\right) \phi(z) dz + \sum_s \left(L_s - \mu_s \int \left(\sum_{g \in P_s} l_g(z) \right) \phi(z) dz \right)$$

The first-order conditions imply that the marginal product of labor is equalized within each

[12] If the production function has constant returns to scale in all the labor inputs, we can normalize by total labor, and the analysis focuses on the share of the worker groups in the total population.

labor-adjustable partition,

$$[l_g(z)]: \ \frac{\partial F\left(\{l_g\}; z\right)}{\partial l_g} \leq \mu_s \quad \forall z, g \in P_s, \forall s \tag{3}$$

Let $Y_s \coloneqq \sum_{g \in P_s} \int F\left(\{l_g\}; z\right) \phi(z) dz$ denote the total output from the labor for groups in element s. Recall $\{l_g^c(z)\}$ denotes the competitive labor allocation and let $\{l_g^p(z)\}$ be the planner's optimal labor allocation, with Y^c and Y^p's denoting the corresponding total output. We can show the planner's problem implies output gains from the reallocation of labor as long as the elements of the partition are not exactly the singletons $\mathcal{G}$.

Proposition 1. *If the number of elements in partition $\mathcal{P}$ is smaller than the total number of group $S < G$, the planner's output is larger than in a competitive equilibrium, $Y^p > Y^c$.*

Moreover, if there are two demographic groups in the same element of the partition, and their wages differ, then the planner optimally allocates relatively more labor to the higher wage group.

Proposition 2. *If $g, g' \in P_s$ with $w_g > w_{g'}$, then $\frac{l_g^p}{l_{g'}^p} > \frac{l_g^c}{l_{g'}^c}$.*

Intuitively, higher wages in a competitive market imply higher marginal productivity. As the planner optimally equalize the marginal productivity across groups within an element of the partition, this implies that the labor supply of the higher marginal productivity group must increase.

Moreover, we can rank the potential gains from labor supply reallocation.

Definition 1. *Let $\mathcal{P}$ and $\mathcal{P}'$ be partitions of $\mathcal{G}$. $\mathcal{P}$ is a **refinement** of $\mathcal{P}'$ if every element of $\mathcal{P}$ is a subset of some element of $\mathcal{P}'$. We say $\mathcal{P}$ is 'finer- than' $\mathcal{P}'$ and it is written as $\mathcal{P} \leq \mathcal{P}'$.*

Note that the "finer-than" relationship on the set of partitions of $\mathcal{G}$ is a partial order. Let $Y^p(\mathcal{P})$ denote the total output associated to partition $\mathcal{P}$. We can show that a refinement always decreases the total output:

Proposition 3. *If $\mathcal{P}$ is a refinement of $\mathcal{P}'$, then $Y^p(\mathcal{P}') > Y^p(\mathcal{P})$.*

In particular, this implies that the maximal partition, i.e., that partition consisting of the whole set $\mathcal{P}^{max} = \{\mathcal{G}\}$ yields the maximal potential output gains from labor supply reallocation and the minimal partition, i.e., consisting of the singletons of $\mathcal{G}$, $\mathcal{P}^{min} = \{\{g\} : g \in \mathcal{G}\}$, is equivalent to the competitive equilibrium.

Let $Y^p(\mathcal{P})$ denote the aggregate output of partition $\mathcal{P}$. Proposition 3 implies that total aggregate output is monotone in the partition partial order,

Corollary 1. *$Y^p(\mathcal{P}^{max}) > Y^p(\mathcal{P})$ for any proper partition $\mathcal{P}$, and $Y^p(\mathcal{P}^{min}) = Y^c$.*

This framework gives us the basis to understand the efficiency tradeoffs in changing the labor supply across groups. We can parameterize the production function and use it to infer the worker-firm specific productivities by matching key moments from the empirical observations described in Section 2.

3.2 Static Gains of Labor Supply Reallocation in Japan

To further characterize the equilibrium and illustrate the use of the static framework, we can study the optimal labor supply across groups using a parameterized version of the model. We use our static model and the Japanese establishment-level data to achieve two goals: to back out the worker group-firm productivities and to quantify the static gains from labor supply reallocation. Invoking Corollary 1, we explore the maximal partition $\mathcal{P}^{max} = \{\mathcal{G}\}$ to understand the upper bound of this static gain.

Suppose the production function $F(\cdot)$ is a standard CES aggregator with decreasing returns to scale,

$$Y = \int y(z) d\Phi(z) = \int \left(\sum_g A_g(z)^{\frac{1}{\sigma}} l_g^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\alpha\sigma}{\sigma-1}} d\Phi(z)$$

where $y(z)$ denotes the output of firm z and $A_g(z)$ the worker-firm specific productivity, with $\sigma \in [0, \infty)$ and $\alpha \in (0, 1]$. The two key assumptions here are (1) a decreasing-returns-to-scale technology to generate revenue at the firm level denoted by α, and (2) imperfectly substitutable labor inputs in the production function denoted by the elasticity of substitution

$\sigma < \infty$.[13] The first-order conditions for the firm imply:

$$l_g(z) = \frac{\alpha^\sigma}{w_g^\sigma} y(z)^{\sigma - \frac{\sigma-1}{\alpha}} A_g(z) \;\; \forall g, z.$$

The labor market condition implies the wage rate of group g increases in the marginal productivity of the group and decreases on its labor supply,

$$w_g = \alpha \left(\frac{\int y(z)^{\sigma - \frac{\sigma-1}{\alpha}} A_g(z) d\Phi(z)}{L_g} \right)^{1/\sigma}.$$

Replacing the wage rate in the labor demand we obtain,

$$l_g(z) = \frac{y(z)^{\sigma - \frac{\sigma-1}{\alpha}} A_g(z)}{\int y(z')^{\sigma - \frac{\sigma-1}{\alpha}} A_g(z') d\Phi(z')} L_g.$$

Intuitively, the optimal labor demand for group g at firm z is proportional to the total supply of labor of that group, L_g, scaled by the weighted productivity of that group at firm z, relative to the weighted productivity of the group across all firms z'.

Aggregating labor across groups, we obtain a set of equations that implicitly determines output for each firm z,

$$y(z) = \left(\sum_g \frac{A_g(z) L_g^{\frac{\sigma-1}{\sigma}}}{\left(\int y(z')^{\sigma - \frac{\sigma-1}{\alpha}} A_g(z') d\Phi(z') \right)^{\frac{\sigma-1}{\sigma}}} \right)^{\frac{\alpha}{(\sigma-1)(1-\alpha)}} \quad \forall z.$$

Planner's Problem

Now, we turn to the planner's problem. Let's first consider the problem assuming that the planner can substitute across any group, so the partition $\mathcal{P}$ corresponds to the entire set $\mathcal{G}$. From Corollary 1, this gives the maximal output gain from labor reallocation. The planner

[13] The first assumption is adopted in many canonical models such as Hopenhayn (1992) and Melitz (2003). The second assumption is empirically supported by findings from Card and Lemieux (2001), Borjas (2003) and Ottaviano and Peri (2012).

solves,

$$\max_{l_g(z)\geq 0} \int \left(\sum_g A_g(z)^{\frac{1}{\sigma}} l_g^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\alpha\sigma}{\sigma-1}} d\Phi(z) \quad s.t \quad \int \sum_g l_g(z) d\Phi(z) = L.$$

The first-order conditions equalize the marginal productivity of labor across all groups and firms,

$$\alpha l_g^{-1/\sigma} A_g(z)^{\frac{1}{\sigma}} y(z)^{\frac{\alpha\sigma-(\sigma-1)}{\alpha\sigma}} = \lambda \implies l_g(z) = \frac{\alpha^\sigma}{\lambda^\sigma} y(z)^{\sigma-\frac{\sigma-1}{\alpha}} A_g(z) \ \forall g, z,$$

where $\lambda > 0$ denotes the Lagrange multiplier of the restriction. Adding up the labor allocation across groups using the labor resource constraint implies:

$$l_g^*(z) = \frac{A_g(z) y(z)^{\sigma-\frac{\sigma-1}{\alpha}}}{\sum_{g'} \int y^*(z')^{\sigma-\frac{\sigma-1}{\alpha}} A_{g'}(z') d\Phi(z')} L.$$

Computing the output of firm z leads to

$$y^*(z) = \left(\frac{\sum_g A_g(z) L^{\frac{\sigma-1}{\sigma}}}{\left(\sum_{g'} \int y^*(z')^{\sigma-\frac{\sigma-1}{\alpha}} A_{g'}(z') d\Phi(z') \right)^{\frac{\sigma-1}{\sigma}}} \right)^{\frac{\alpha}{(\sigma-1)(1-\alpha)}}.$$

As implied by Proposition 3, we can verify the total output of the competitive equilibrium is smaller than the output in the planner's solution.[14] In particular, we can write out the explicit solution to the social planner's problem when the production function takes the Cobb-Douglas form (i.e., $\sigma = \frac{1}{1-\alpha} = 2.5$).[15]

Note that in the static problem, there is no labor misallocation across firms since the labor market clears at the g level, firms are competitive, and there are no labor market frictions. Later, we extend the model to a dynamic setting where labor market frictions are crucial and could lead to misallocation across firms.

Numerical Application

To operationalize our framework, we consider firms indexed by productivity z_k $(k = 1, \ldots, K)$

[14] This can be proven as $\frac{\sigma-1}{\sigma} < 1$ and using Jensen's inequality.

[15] See Appendix B.1 for details.

and workers defined by human capital h_i ($i = 1, \ldots, I$) and experience x_j ($j = 1, \ldots, J$). We observe firm types and experiences directly while human capital is unobserved. Experience is split into nine bins: 1-5, 6-10, 11-15, 16-20, 21-25, 31-35, 36-40, and 40+. Firms are categorized as large (over 50 workers) or small (under 50 workers). We assume two unobserved human capital levels: high and low. Let ξ_j denote the high-skill worker proportion in bin j, varying across experience based on the benchmark assumption: $\xi_j = [0.3, 0.3, 0.4, 0.4, 0.5, 0.5, 0.3, 0.3, 0.3]$. Moreover, for our baseline results, we assume $\sigma = \frac{1}{1-\alpha} = 2.5$ (i.e., $\alpha = 0.6$), so the production function is Cobb-Douglas. As robustness checks, we also examine alternative values of σ and α.[16]

The first objective of this numerical exercise is to back out the worker-firm specific productivities implied by the combination of the static model and the data. To do so, we target the observed labor allocations l_{jk} and wages w_{jk} at the worker experience-firm type level. We normalize employment so that it adds up to one, and divide wages by the average wage in the economy. To identify the productivities A_{ijk}, we impose wages to be monotonic in human capital, $w_{i+1,j} \geq w_{i,j}$ $\forall j$. This amounts to assuming that high-skill workers can also work as low-skill workers.

In Figure 4, we plot the moments in the data and the model, almost perfectly matching employment and wage schedules. Both the data and the model show that large firms have more workers at all experience levels but have disproportionately more younger workers, as highlighted in the empirical section. The wage-experience profiles for both types of firms are increasing at a higher level and with a steeper slope in large firms, reflecting differences in the composition of the labor force across both types of firms and across the worker's life cycle.[17]

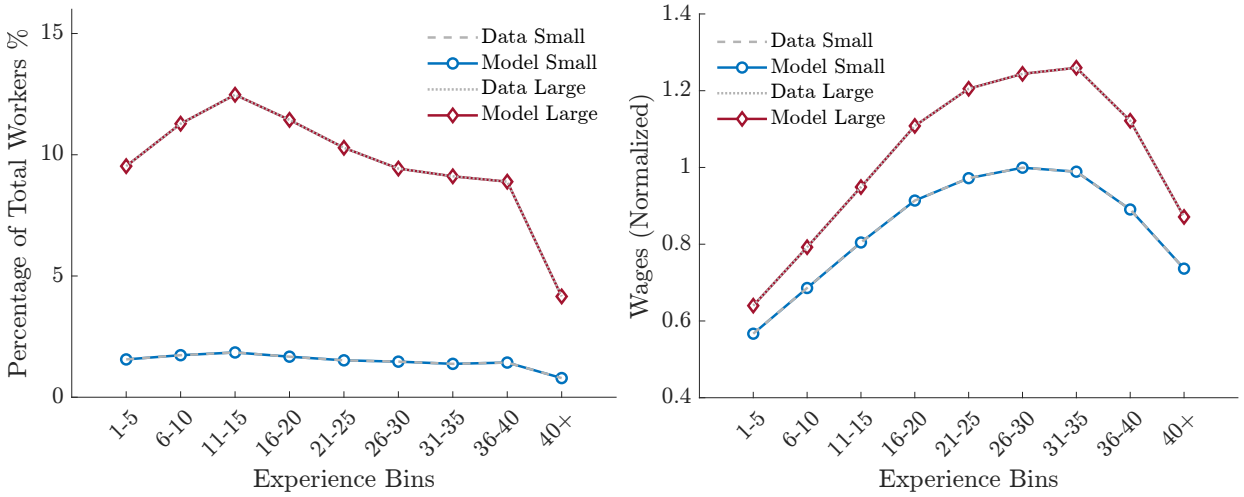
Figure 5 and 6 present the two main results of this section for the benchmark calibration: the estimated worker-firm productivities, A_{ijk}, and compares the labor allocation and output for the equilibrium and the social planner.

Figure 5 shows that both types of workers are more productive in large firms, which explains why those firms are larger in the first place. Less immediate is how high-skill workers are

[16] In Appendix C.1, we show that all our results are robust to assuming that the proportion of high-skill workers is fixed at the median for all experience levels, $\xi_j = \xi = 0.5$ $\forall j$, and show similar results for various values of σ and α.

[17] The hump-shaped life-cycle wage profile is not driven by the variation in working hours at different ages. We obtain the same hump-shaped pattern by using the hourly wage instead of the annual wage (as the data contain information on total working hours at the worker level).

Figure 4: Moments Fit



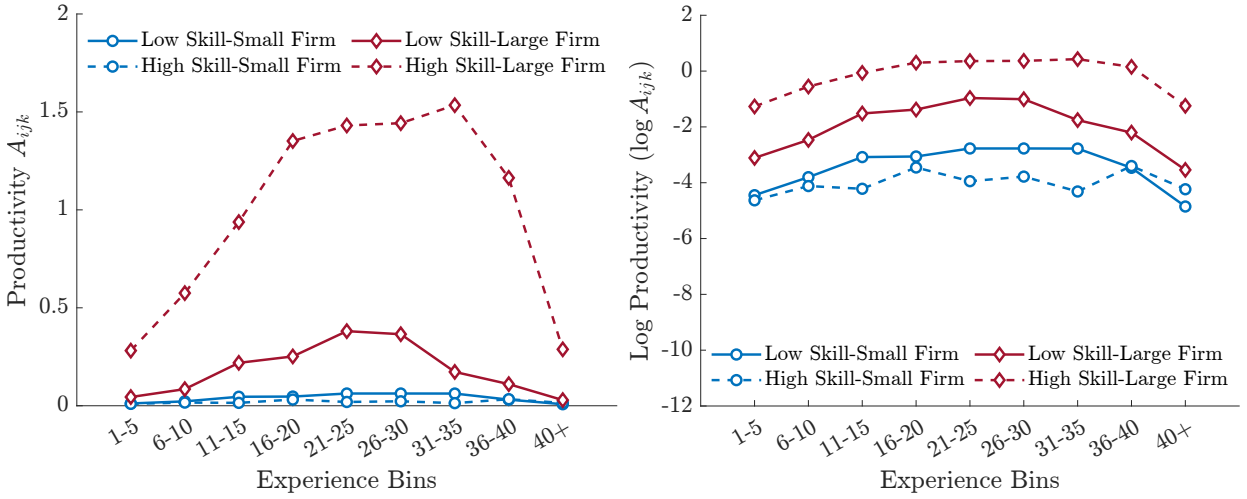
(a) Labor by Experience

(b) Wages by Experience

more productive in large firms compared to low-skill workers. These differences peak for workers with 25 to 35 years of experience. In small firms, however, high-skill workers are not necessarily more productive than lower-skill workers.[18] These findings suggest that high-skill workers are particularly productive when they work for large (and productive) firms.

[18] As the productivities are in log scales, the differences between the productivities of high-skill workers and low-skill workers in small firms are negligible.

Figure 5: Estimated Labor Productivities A_{ijk}

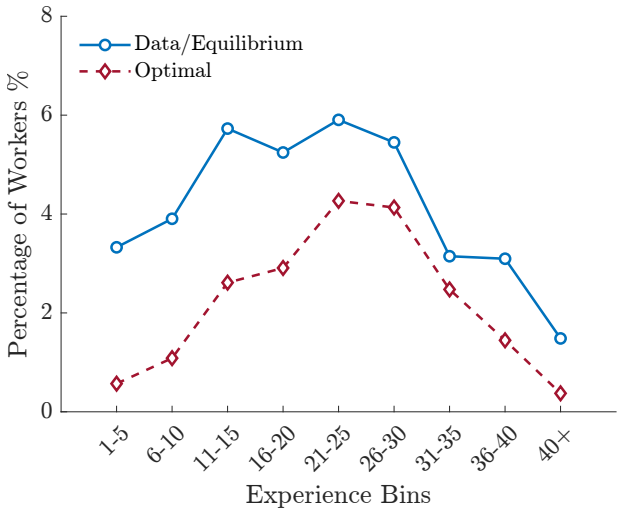


(a) Worker-Firm Labor Productivity

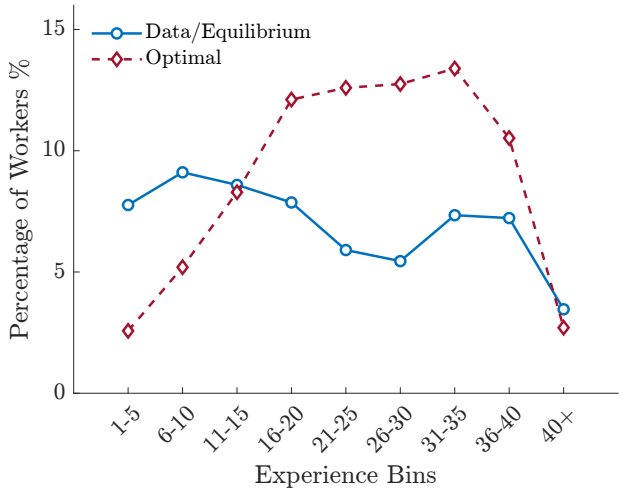
(b) Log Worker-Firm Labor Productivity

Finally, Figure 6 compares equilibrium labor allocation and output to the optimal labor composition. Panels 6a and 6b show labor reallocation for low- and high-skill workers, respectively. The results indicate an oversupply of low-skill workers relative to the optimal composition, especially among younger experience groups, while experienced high-skill workers are in short supply, except for those with over 40 years of experience. Panel 6c presents output in competitive equilibrium and under the planner's allocation, decomposed by experience levels as defined by the CES production function: $Y_j := \sum_{i,k} A_{ijk}^{\frac{1}{\sigma}} l_{ijk}^{\frac{\sigma-1}{\sigma}}$. Consistent with labor allocations, output is higher for experienced workers, except for those with over 40 years of experience, while the optimal supply of younger workers is lower than in equilibrium. Aggregating across experience groups, labor reallocation yields a total output gain of $\Delta\%Y = \frac{Y^* - Y^c}{Y^c} = 5.3\%$.

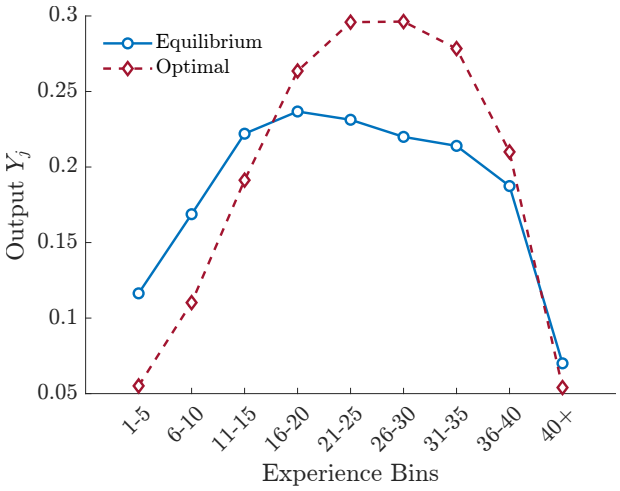
Figure 6: Labor Allocation and Output



(a) Labor Low-Skilled $L_{\ell j}$



(b) Labor High-Skilled L_{hj}



(c) Output by Experience Y_j

While the static model is useful for assessing the potential gains from labor reallocation, it falls short in incorporating some of the crucial considerations when studying the dynamic consequences of labor supply changes. Some dimensions of the problem, such as tenure, experience, and human capital level, have inherent dynamics that constitute restrictions for firms if labor markets are frictional. In light of these considerations, we extend the model to a dynamic setting, where we spell out these restrictions to evaluate the dynamic effects of labor supply reallocation.

3.3 Dynamic Model

Setup

In this section, we extend our static model to study how dynamic considerations affect optimal labor supply allocation and the consequences of population aging on worker allocations across firms and on aggregate outcomes. We explicitly consider two dimensions in which worker groups differ: experience and human capital. To illustrate the effect of these dynamic considerations on the optimal labor composition, we focus on adding firing costs, which are also empirically relevant in the Japanese context. However, the central insights of the dynamic model can also be applied to other frictions.

In the model, heterogeneous firms (with different productivities) hire workers who differ in both experience and human capital. Importantly, firms hire a *portfolio* of workers (i.e., across various experience-human capital bins), and the portfolio of workers evolves within firms in different ways (deterministic vs. stochastic). The key assumption we make is that productive firms have a comparative advantage of using workers with higher human capital rather than using workers with more experience. Leveraging this assumption, our model rationalizes the key empirical finding for Section 2 that the experience-tenure profile is steeper in larger establishments.

Workers

The model economy consists of overlapping generations of workers who participate in the labor market with experience $X := (x_1, x_2, ..., x_J)$. Workers of experience x_J retire and are replaced by a cohort of incoming workers at experience $j = 1$.[19] Workers with less experience

[19] In the calibrated model, we group workers into various five-year bins (21-25, 26-30,...,40+).

than J face random shocks (e.g., death or sickness, marriage, and giving birth to a baby) to their labor market participation. As a result, they exit the labor market with probability δ every period.

Throughout their lives, workers accumulate work experience and general human capital that is useful for all firms.[20] We assume that workers obtain work experience *deterministically* and human capital *stochastically* over their lives. Specifically, we assume that the number of working years equals the amount of work experience that a worker has. Thus, older workers are necessarily more experienced than younger workers. To the contrary, we assume the human capital evolves throughout workers' lives in a ladder-like fashion *stochastically*. Assume that the human capital vector takes the following form: $h \in H := \{h_1, h_2, ..., h_M\}$ where $h_{m+1} - h_m = h_0 > 0 \ \forall \ m \in \{1, 2, ..., M-1\}$. A worker with human capital h_i $(M-1 \geq i \geq 2)$ and experience x_j, moves up to h_{i+1} with probably $p_j(1-q_j)$ and moves down to h_{i-1} with probably $q_j(1-p_j)$ where p_j and q_j are learning and depreciation rates of human capital. Since both events happen independently, the worker stays at his current human capital level h_i with probably $1-p_j-q_j+2p_jq_j$ next period.[21] In other words, the speeds of human capital appreciation and depreciation are all experience-specific and not impacted by the firms they work for.

When workers are born, the distribution of their human capital stock is assumed to be $g_0(h_i)$, where $g_0(h_i)$ denotes the measure of workers' human capital level h_i when entering the economy.

Firms

Every period, a continuum of firms with measure one make time-invariant productivity draws, z when entering the economy. Firms exit randomly with probability δ_f every period. A firm with the productivity draw of z hires workers from various human capital-experience bins to produce a homogeneous good according to the following production function:

$$y(z, h_i, x_j, l_{ij}) = \int_{h_i \in H, x_j \in X} \left[l_{ij}^{\frac{\sigma-1}{\sigma}} \left(z h_i + z^{\nu} x_j \right) \right] dh_i dx_j, \tag{4}$$

where h_i and x_j are the human capital level and experience of workers hired in the human

[20] See Altonji and Shakotko (1987) for the importance of general human capital for the determination of wages.

[21] For workers with the highest human capital level h_M, they either keep their human capital or drop to h_{M-1} next period. For workers with the lowest human capital level h_1, they either keep their human capital or jump to h_2 next period.

capital-experience bin (i, j), and l_{ij} is the number of workers hired from that bin. The key feature of this production function is that we assume $\nu \in [0, 1)$, which implies the complementarity between firm productivity and worker's experience is weaker than that between firm productivity and human capital. In the calibration, ν is the key parameter that makes the model generate divergent relationships between firm size and workers' average experience and between firm size and workers' average tenure.[22]

There are two types of job separations in the model. First, the firm chooses workers to fire (or lay off) every period. For this type of job separation, employment protection comes in the form of the firing cost. Specifically, the firm has to pay a firing cost for every laid-off worker that is $\tau(> 0)$ fraction of the worker's wage in a given period. This type of job separation is the focus of our model. Second, a randomly selected $1 - \eta$ fraction of firms' incumbent workers are separated from their employers every period. This reflects the fact that workers leave their jobs for exogenous reasons, such as job-to-job transitions. We further assume that the firm does not know which workers would be separated from the firm due to exogenous reasons, when it makes the firing decision.[23]

Experience, Human Capital and Labor Submarkets in Equilibrium

As learning (i.e., the accumulation of human capital) is not firm- and coworker-specific, we can characterize the distribution of human capital and worker experience in the steady state analytically. Based on the assumptions above, the distribution of the worker's experience is $1, \delta, \delta^2, \delta^3, ..., \delta^{J-2}, \frac{\delta^{J-1}}{1-\delta}$, which implies that the total measure of workers is $\frac{1}{1-\delta}$. For the distribution of the human capital and experience in the steady state, the inflow has to be equal to the outflow for each human capital-experience bin $(h_i,\ x_j)$ of the steady-state distribution of human capital and experience, $g(h_i, x_j)$, where $g(.,.)$ is the mass or measure of workers that belong to the human capital-experience bin $(h_i,\ x_j)$. Specifically, we have

[22] In Appendix B.2, we present a general production function that is similar to the function defined in equation (4).

[23] In reality, workers who choose to leave their employers are probably not the type of workers firms want to lay off. In particular, those departing workers usually move to better jobs (i.e., job-to-job transitions) or often quit their jobs due to non-performance-related reasons such as, marriage or having babies.

the following law of motion for workers at experience x_2:

$$
\begin{aligned}
g(h_1, x_2) &= \delta\left[(1-p_1)g_0(h_1) + (1-p_1)q_1 g_0(h_2)\right] \\
g(h_i, x_2) &= \delta\left[p_1(1-q_1)g_0(h_{i-1}) + (1-p_1-q_1+2p_1q_1)g_0(h_i) + q_1(1-p_1)g_0(h_{i+1})\right], \quad \forall M-1 \geq i \geq 2, \\
g(h_M, x_2) &= \delta\left[p_1(1-q_1)g_0(h_{M-1}) + (1-q_1)g_0(h_M)\right].
\end{aligned}
$$

For $j \in [3, ..., T]$, we have:

$$
\begin{aligned}
g(h_1, x_j) &= \delta\left[(1-p_{j-1})g(h_1, x_{j-1}) + (1-p_{j-1})q_{j-1}g(h_2, x_{j-1})\right], \\
g(h_i, x_j) &= \delta\left[p_{j-1}(1-q_{j-1})g(h_{i-1}, x_{j-1}) + (1-p_{j-1}-q_{j-1}+2p_{j-1}q_{j-1})g(h_i, x_{j-1})\right. \\
&\quad \left. + q_{j-1}(1-p_{j-1})g(h_{i+1}, x_{j-1})\right], \ \forall 2 \leq i \leq M-1, \\
g(h_M, x_j) &= \delta\left[p_{j-1}(1-q_{j-1})g(h_{M-1}, x_{j-1}) + (1-q_{j-1})g(h_M, x_{j-1})\right].
\end{aligned}
$$

We assume that all workers retire at experience x_J.

Labor submarkets in this economy are denoted by (i, j) where i refers to the human capital bin h_i and j represents the experience bin x_j. In equilibrium, labor demand has to equal labor supply in each labor submarket:

$$
\int_{z_{min}}^{\infty} l_{ij}(z)\frac{d\Phi(z)}{\delta_f} = g(h_i, x_j); \quad \forall\ (i, j), \tag{5}
$$

where $l_{ij}(z)$ is the number of workers with human capital h_i and at experience x_j employed by the establishment with the productivity draw of z. Note that as active establishments exit randomly with probability δ_f and are replaced by entrants with measure one, the mass of establishments in equilibrium is $\frac{1}{\delta_f}$. Equation (5) determines submarket-specific wage w_{ij}.

The product market equilibrium condition states that total revenue of establishments equals the total income of this economy which consists of workers' wage income and the aggregate profit:

$$
\begin{aligned}
&\int_{z_{min}}^{\infty}\int_{h_i \in H, x_j \in X}\left[l_{ij}^{\frac{\sigma-1}{\sigma}}(zh_i + z^{\nu}x_j)\right]dh_i dx_j \frac{d\Phi(z)}{\delta_f} = \int_{h_i \in H, x_j \in X} w_{ij}g(h_i, x_j)dh_i dx_j \\
+ &\left(\int_{z_{min}}^{\infty}\int_{h_i \in H, x_j \in X}\left(\left[l_{ij}^{\frac{\sigma-1}{\sigma}}(zh_i + z^{\nu}x_j)\right] - w_{ij}l_{ij}\right)dh_i dx_j \frac{d\Phi(z)}{\delta_f}\right),
\end{aligned} \tag{6}
$$

where we choose the product price as the numeraire thanks to the Walras' law. Note that

the second term of the right hand side of the above equation is the aggregate profit. It is straightforward to observe that if equation (5) holds for every labor submarket, then equation (6) is satisfied automatically by Walras' law.

3.4 Equilibrium Without Firing Cost

We first consider the model without the firing cost to illustrate some key insights from the dynamic model. Note this is a special case of the static model developed in Section 3.1. Using this more concrete version, we can derive analytical results with respect to two key dimensions of the labor allocation: the sorting of workers into firms by characterizing the log-supermodularity of the production function and the modeling of population aging, which is central to our research question. The model without the firing cost implies the firm-level labor demand as

$$l_{ij}(z) = \left(\frac{\sigma - 1}{\sigma} \frac{(z^{1-\nu} h_i + x_j)}{w_{ij}} \right)^{\sigma}. \tag{7}$$

As a result, the labor submarket clearing condition in equation (5) can be restated as

$$\int_{z_{min}}^{\infty} \left[\frac{\frac{\sigma-1}{\sigma} (z h_i + z^{\nu} x_j)}{w_{ij}} \right]^{\sigma} \frac{\phi(z)}{\delta_f} dz = g(h_i, x_j) =: g_{i,j}, \tag{8}$$

Based on equation (8), we can back-out the wage w_{ij} in each labor submarket.

Log Supermodularity/Submodularity

We can use the model to prove some properties of the equilibrium labor allocation. An important observation in the data is that the average experience of the labor force decreases with firm size. The goal is to establish the result that the experience distribution moves to the right in the first-order stochastic dominance (FOSD) sense when we increase z. The following lemma establishes the relative change in the labor demand of (i, j) when z increases.

Lemma 1. *The proportional increase (with respect to firm productivity) in labor demand is smaller for more experienced workers,* $x_j > x_{j'}$, $\frac{\partial^2 \ln(l_{ij}(z))}{\partial x_j \partial z} < 0$*; while larger for higher human capital workers,* $h_i > h_{i'}$, $\frac{\partial^2 \ln(l_{ij}(z))}{\partial h_i \partial z} > 0$.

Using Lemma 1, we can derive the following proposition that establishes a link between the change in average human capital and experience when we increase z.

Proposition 4. *For a given human capital group, average experience decreases with firm productivity. For a given experience group, average human capital increases with firm productivity.*

Population Aging

Population aging in our model refer to a change in the distribution of workers' experience in the labor force. Specifically, we assume that $dg_{i,j} > 0$ for $j \geq \bar{j}(i)$ and $dg_{i,j} < 0$ for $j < \bar{j}(i)$. Moreover, we assume that the marginal distribution of skills (i.e., human capital) is unchanged to facilitate the analysis:

$$\sum_j g_{i,j} = g_i,$$

which is unchanged after population aging. After the change, the labor force becomes more productive overall, as experience always contributes positively to production. The question is whether aggregate output/productivity always increases. We are going to show that the answer is no.

First, we solve for equilibrium wage. Using the labor submarket clearing condition (8), we obtain

$$w_{ij} = \frac{\sigma - 1}{\sigma} \left(\frac{A_{ij}}{g_{ij}} \right)^{\frac{1}{\sigma}}, \tag{9}$$

where

$$A_{ij} := \int_{z_{min}}^{\infty} \left(z h_i + z^{\nu} x_j \right)^{\sigma} M \phi(z) dz,$$

where M is the mass of firms in the stationary equilibrium and $\phi(z)$ is the PDF of the productivity distribution. One observation is that when there is a $x\%$ increase (decrease) in the supply of workers in the submarket (i, j), there will be a $\frac{x\%}{\sigma}$ decrease (increase) in w_{ij}. As a result, every firm increases (decreases) its labor demand in submarket (i, j) by $x\%$. This implies that an increase in the *total* labor supply (without a compositional change) leads to no change in market concentration.

When there is a compositional change in the labor force, the market concentration changes. Population aging leads to less (more) supply of young (old), which makes all firms hire fewer (more) young (old) workers. Since productive firms hire disproportionate more young workers, a given percentage decrease in the employment of young workers reduces productive

firms' (total) employment disproportionately more than the same percentage decrease in the employment of young workers for unproductive firms. Conversely, a given percentage increase in the employment of old workers increases unproductive firms' (total) employment disproportionately more than the same percentage increase in the employment of old workers for productive firms. As a result, the market concentration of productive (i.e., large) firms has to go down under population aging. We will see this prediction continue to hold when there is a firing cost in Section 4.3.

Now, we discuss how population aging (without changing the distribution of workers across human capital bins) affects aggregate output. The aggregate output produced by (h_i, x_j)-type workers is

$$Y_{ij} := \int_{z_{min}}^{\infty} l_{i,j}^{\frac{\sigma-1}{\sigma}}(z)\left(zh_i + z^{\nu}x_j\right) M\phi(z)dz.$$

Based on equations (7) and (9), we can rewrite it as

$$Y_{ij} = A_{ij}^{\frac{1}{\sigma}} g_{ij}^{\frac{\sigma-1}{\sigma}}.$$

Therefore, total output equals

$$Y = \sum_{i,j} A_{ij}^{\frac{1}{\sigma}} g_{ij}^{\frac{\sigma-1}{\sigma}}.$$

Although we know $A_{i,j_1} > A_{i,j_2}$ for $j_i > j_2$, a shift of workforce toward old workers can lead to a *decline* in the total output by shifting employment toward unproductive firms.[24]

3.5 Equilibrium with Firing Cost

Having discussed the key insights of the model without the firing cost, we proceed to include the dynamic model, where firms make their labor choices facing firing costs. The employment problem faced by the firm now becomes dynamic, and we have to solve the value function of the firm. First, the state variables of the firm include its productivity draw z and a series of labor stocks L_{ijs} of human capital level $i \in \{h_1, ..., h_M\}$, experience $j \in \{x_1, ..., x_J\}$ and tenure $s \in \{1, ..., S\}$ at the beginning of each period. The firm chooses the labor l_{ijs} to maximize

[24] See Appendix B.3 for a simple numerical example.

the value function:

$$V(z,\{L_{ijs}\}) = \max_{\{l_{ijs}\}} \sum_i \sum_j l_{i,j}^{\frac{\sigma-1}{\sigma}}(zh_i + z^{\nu}x_j) - \sum_i \sum_j w_{ij}l_{ij} \\ - \sum_i \sum_j \eta\tau w_{ij}\mathbb{1}(L_{ij} > l_{ij}) \sum_s (L_{ijs} - l_{ijs}) + \beta\mathbb{E}V(z,\{\tilde{L}_{i,j,s}\}), \tag{10}$$

where $\sum_s l_{ijs} = l_{ij}$, $\sum_s L_{ijs} = L_{ij}$ and $\tilde{L}_{ijs}$ is the stock of workers for next period.[25] The stock of labor evolves as follows,

$$\mathbb{E}(\tilde{L}_{ijs}) = \delta\left[p_{j-1}(1-q_{j-1})l_{i-1,j-1,s-1} + (1-p_{j-1}-q_{j-1}+2p_{j-1}q_{j-1})l_{i,j-1,s-1} + (1-p_{j-1})q_{j-1}l_{i+1,j-1,s-1}\right],$$

where $j \geq 2$ and $2 \leq s \geq 2$ and $2 \leq i \leq M-1$. For cases where $i = 1$ or M, we have

$$\mathbb{E}(\tilde{L}_{1,j,s}) = \delta\left[(1-p_{j-1})l_{1,j-1,s-1} + (1-p_{j-1})q_{j-1}l_{2,j-1,s-1}\right],$$

and

$$\mathbb{E}(\tilde{L}_{M,j,s}) = \delta\left[p_{j-1}(1-q_{j-1})l_{M-1,j-1,s-1} + (1-q_{j-1})l_{M,j-1,s-1}\right],$$

Note that all workers retire after reaching experience x_J, and there is no employment stock for workers with the least experience (x_1). As workers with different tenures in the same human capital-experience bin have identical contributions to production and the same firing cost, we assume the firm fires workers randomly in a given human capital-experience bin if it decides to do so.

Firms face uncertainty concerning the accumulation of human capital and the exits/separations of their workers. This matters to the extent that they cannot freely adjust labor each

[25] As we assume that a randomly selected $1-\eta$ fraction of incumbent workers are exogenously separated from their employers, the firm only pays a η fraction of the total firing cost it would have to pay in the case without these separations. The key is that the firm does not know who is going to leave the firm voluntarily when making the firing/hiring decisions. As a result, the firm cannot target these workers who are going to leave the firm voluntarily when making the firing decision.

period. The first order conditions (FOCs) of the optimization problem (10) are,

$$\frac{\sigma-1}{\sigma}\left(\sum_s l_{ijs}\right)^{-1/\sigma}(zh_i+z^\nu x_j)=w_{ij}\left(1-\eta\tau\mathbb{1}[L_{ij}>l_{ij}]\right)$$
$$-\beta\left[\delta(1-p_i-q_i+2p_iq_i)\mathbb{E}\left(\frac{\partial V}{\partial\tilde{L}_{ijs}}\right)+\delta q_i(1-p_i)\mathbb{E}\left(\frac{\partial V}{\partial\tilde{L}_{i-1,j,s}}\right)+\delta p_i(1-q_i)\mathbb{E}\left(\frac{\partial V}{\partial\tilde{L}_{i+1,j,s}}\right)\right]$$

The Envelope Condition in the current and next periods implies that

$$\frac{\partial V}{\partial L_{ijs}}=-\eta\tau w_{ij}\mathbb{1}[L_{ij}>l_{ij}];\quad \mathbb{E}\left[\frac{\partial V}{\partial\tilde{L}_{ijs}}\right]=\mathbb{E}\left[-\eta\tau w_{ij+1}\mathbb{1}[\tilde{L}_{ij+1}>l_{ij+1}]\right].$$

In total, we can rewrite the FOCs as

$$\frac{\sigma-1}{\sigma}l_{ij}^{-1/\sigma}(zh_i+z^\nu x_j)=w_{ij}\left(1-\eta\tau\mathbb{1}[L_{ij}>l_{ij}]\right)+\beta\eta\tau\delta\Bigg[(1-p_i-q_i+2p_iq_i)w_{ij+1}\mathbb{E}\left[\mathbb{1}[\tilde{L}_{i,j+1}>l_{i,j+1}]\right]$$
$$+p_i(1-q_i)w_{i+1,j+1}\mathbb{E}\mathbb{1}[\tilde{L}_{i+1,j+1}>l_{i+1,j+1}]+q_i(1-p_i)w_{i-1,j+1}\mathbb{E}\left[\mathbb{1}[\tilde{L}_{i-1,j+1}>l_{i-1,j+1}]\right]\Bigg].\tag{11}$$

Comparing equation (11) with equation (7), we find that there are two more benefits/costs of hiring an additional worker when there is the firing cost. First, the firm saves on paying the firing cost, if its employment choice (l_{ij}) is below its employment stock (L_{ij}). Second, the firm faces a potential firing cost next period, when its optimal employment next period, ($l_{i,j+1}$), is below its employment stock ($\tilde{L}_{i,j+1}$) of next period. Since there is uncertainty concerning the accumulation of human capital, the firing cost the firm faces next period is an expected cost (thanks to the law of large numbers).

Now, we derive the optimal hiring/firing strategies, which imply an inaction region (given its employment stock L_{ij}) in terms of the employment. Specifically, if the firm decides to hire workers, the FOC for worker group (i, j) becomes

$$\frac{\sigma-1}{\sigma}l_{ij}^{-1/\sigma}(zh_i+z^\nu x_j)=w_{ij}+\beta\eta\tau\Bigg[\delta(1-p_i-q_i+2p_iq_i)w_{ij+1}\mathbb{E}\left[\mathbb{1}[\tilde{L}_{i,j+1}>l_{i,j+1}]\right]+$$
$$\delta p_i(1-q_i)w_{i+1,j+1}\mathbb{E}\left[\mathbb{1}[\tilde{L}_{i+1,j+1}>l_{i+1,j+1}]\right]+\delta q_i(1-p_i)w_{i-1,j+1}\mathbb{E}\left[\mathbb{1}[\tilde{L}_{i-1,j+1}>l_{i-1,j+1}]\right]\Bigg],$$

which leads to a unique value of l_{ij} denoted as l_{ij}^{lb}. Next, if the firm decides to fire workers,

the equation is,

$$\frac{\sigma-1}{\sigma} l_{ij}^{-1/\sigma}\left(z h_i + z^{\nu} x_j\right) = w_{ij}(1-\tau) + \beta\eta\tau\Bigg[\delta(1-p_i-q_i+2p_iq_i)w_{ij+1}\mathbb{E}\left[\mathbb{1}\left[\tilde{L}_{i,j+1} > l_{i,j+1}\right]\right] +$$

$$\delta p_i(1-q_i)w_{i+1,j+1}\mathbb{E}\left[\mathbb{1}\left[\tilde{L}_{i+1,j+1} > l_{i+1,j+1}\right]\right] + \delta q_i(1-p_i)w_{i-1,j+1}\mathbb{E}\left[\mathbb{1}\left[\tilde{L}_{i-1,j+1} > l_{i-1,j+1}\right]\right]\Bigg],$$

which leads to a unique value of l_{ij} denoted as l_{ij}^{ub}. Clearly, $l_{ij}^{ub} > l_{ij}^{lb}$. In total, we can define an inaction region of $[l_{ij}^{lb}, l_{ij}^{ub}]$ such that the firm neither fires nor hires when the employment stock $L_{ij} \in [l_{ij}^{lb}, l_{ij}^{ub}]$. When the employment stock, L_{ij}, is above (below) l_{ij}^{ub} (l_{ij}^{lb}), the firm fires (hires) workers until its employment reaches l_{ij}^{ub} (l_{ij}^{lb}). For future use, we define the expected firing cost for worker group (i, j) as

$$\begin{aligned} FC_{ij}(z): \quad = \quad & \eta\tau\delta\Bigg[(1-p_i-q_i+2p_iq_i)w_{ij+1}\mathbb{E}\left[\mathbb{1}\left[\tilde{L}_{i,j+1} > l_{i,j+1}\right]\right] \\ & + p_i(1-q_i)w_{i+1,j+1}\mathbb{E}\left[\mathbb{1}\left[\tilde{L}_{i+1,j+1} > l_{i+1,j+1}\right]\right] + q_i(1-p_i)w_{i-1,j+1}\mathbb{E}\left[\mathbb{1}\left[\tilde{L}_{i-1,j+1} > l_{i-1,j+1}\right]\right]\Bigg], \end{aligned}$$

which simplifies the optimal hiring/firing strategies as

$$\frac{\sigma-1}{\sigma} l_{ij}^{-1/\sigma}\left(z h_i + z^{\nu} x_j\right) = w_{ij} + \beta\, FC_{ij}(z), \tag{12}$$

and

$$\frac{\sigma-1}{\sigma} l_{ij}^{-1/\sigma}\left(z h_i + z^{\nu} x_j\right) = w_{ij}(1-\eta\tau) + \beta\, FC_{ij}(z). \tag{13}$$

In general, firm's labor choice dictated by equations (12) and (13) requires information on employment chosen last period and employment the firm will choose next period. In addition, employment choice l_{ij} depends not only on the current wage rate of w_{ij}, but also on the future wage rates (of the same and adjacent worker groups) the firm pays (as they matter for the expected firing cost). Since we focus on the steady state, the wage rates are constant over time and the firm only need to form (correct) expectations of the prevailing wage rates in the current period (and the expected firing probability in the future).

In Appendix C.2, we develop an algorithm to solve for the equilibrium. The key idea is to solve for the "right" expectations of the employment choices for the next experience group,

as these expectations matter for the current employment choices and, thus, the equilibrium in each labor submarket. In equilibrium, the expectations of the employment choices for the next experience group have to be consistent with the actual employment choices for the next experience group chosen by the firm in equilibrium.

4 Quantitative Exercises

In this section, we take our dynamic model to the Japanese worker-establishment data. We start by describing the calibration procedure and the moments we target. We then use the model to show how firms' hiring and firing strategies respond to demographic shifts and labor market policies, emphasizing the distributional effects of firing costs and population aging on firm performance, wage inequality, and market concentration.

4.1 Calibration *(In Progress)*

We consider eight sets of moments by pooling data from 2005-2007: (1) the tenure-experience profile by establishment groups, (2) the employment-tenure distribution by establishment groups, (3) the wage-experience profile (normalized) by establishment groups, (4) the employment-exp distribution by establishment groups, (5) average tenure by establishment groups, (6) average experience by establishment groups, (7) average wage by firm (normalized) by establishment groups, and (8) the economy-wide firing rate.

We match 8 experience bins (0-5, 6-10, ..., 36-40) and 8 tenure bins (0-5, 6-10, ..., 36-40) and split establishments into two groups, above 50 workers and below 50 workers. We normalize the wages to the overall average wage. We assume there are six human capital levels and normalize the value of the first step in the human capital ladder to one.

For the first four sets of moments, there are $8 \times 4 - 16 = 48$ data moments in total.[26] For the middle three set of moments, there are $3 \times 2 = 6$ data moments in total. Since the firing rate is for the entire economy, we have $55 = 48 + 6 + 1$ data moments to match in our calibration exercise. We use these moments to estimate 8 parameters, summarized in Table

[26] Although there are $8 \times 4 \times 2 = 64$ data moments for two types of establishments in principle, 8 moments in each of the second and fourth sets of data moments are redundant. I.e., the employment-tenure (experience) shares of the two types of establishments add up to one in a given tenure (experience) bin.

5, by minimizing the squared distance between the data and model-generated moments,

$$\min_{\Theta} \sum_{j} \frac{\left(data_j - model_j(\Theta)\right)^2}{data_j^2 + model_j(\Theta)^2}.$$

We externally calibrate the reminding parameters with the discount rate equal to $\beta = 0.815$ (four-year periodicity), an elasticity of substitution of $\sigma = 3$, and a uniform initial distribution of human capital, $g_0(h_i) = 1/6$ for the six human capital level.

Table 5: Parameters

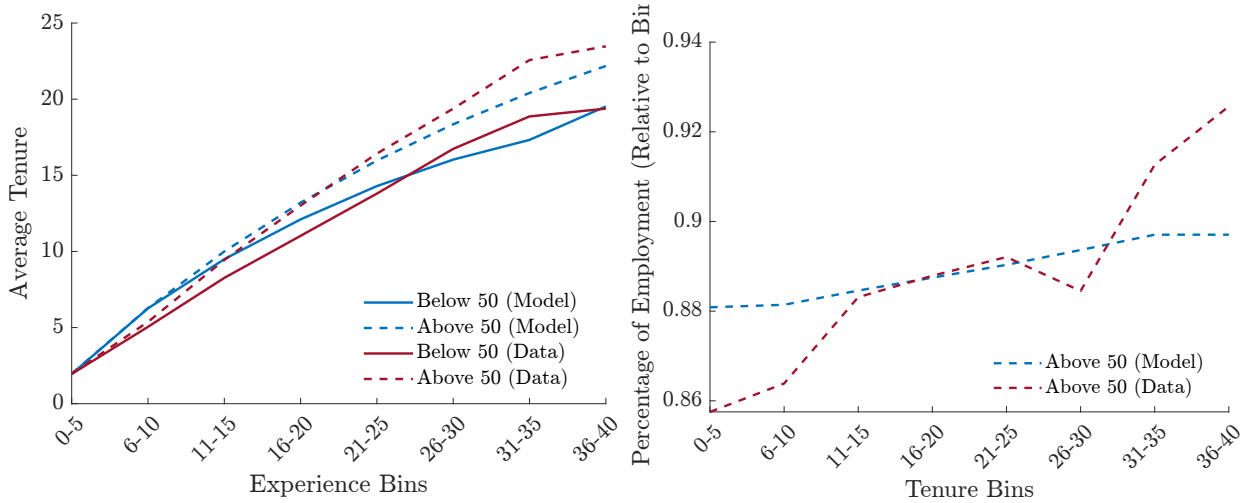
Parameter	**Description**	**Value**
Externally Calibrated		
β	Discount rate (4 years)	0.815
σ	Elasticity of Substitution Inputs	3
g_0	Initial Distribution of Human Capital	[1/6, 1/6, 1/6, 1/6, 1/6, 1/6]
Simulated Method of Moments		
τ	Firing Costs	0.2
ν	Elasticity of Output Relative to Experience	0.006
δ	Survival Rate of Cohort	0.938
η	No Exogenous Separation	0.862
z	Firm Productivity	[8.293 22.926]
Δh	Human Capital Ladder Step	1.62
p	Learning Probability	0.217
δ_h	Human Capital Depreciation by Experience	0.216

Table 6 and Figure 7 present the fit of the model. Overall, the model closely fits the moments, capturing well the difference in tenure and experience profile across the different firm types and the total firing rate.

Table 6: Moments

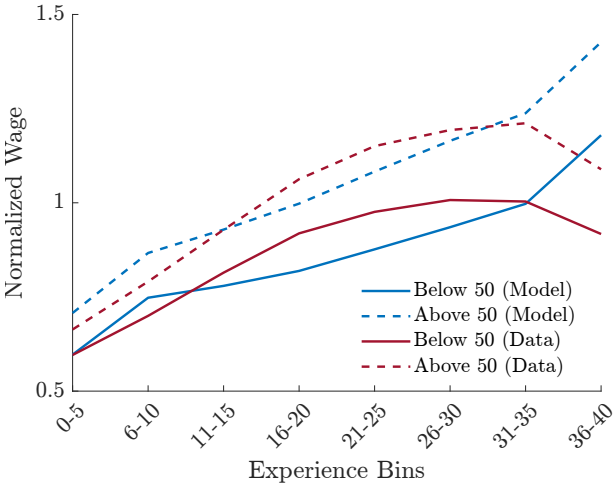
Moment	**Data**	**Model**
1. Tenure-Experience Profile	See Figure 7a	See Figure 7a
2. Employment-Tenure Distribution	See Figure 7b	See Figure 7b
3. Wage-Experience Profile (normalized)	See Figure 7c	See Figure 7c
4. Employment-Experience Distribution	See Figure 7d	See Figure 7d
5. Average Tenure	[11.1 13]	[12;12.5]
6. Average Experience	[21.3 20.9]	[22.2 20.7]
7. Average Wages (Normalized)	[0.872 1.018]	[0.861;1.018]
8. Firing Rate	0.44%	0.44%

Figure 7: Fit Moments

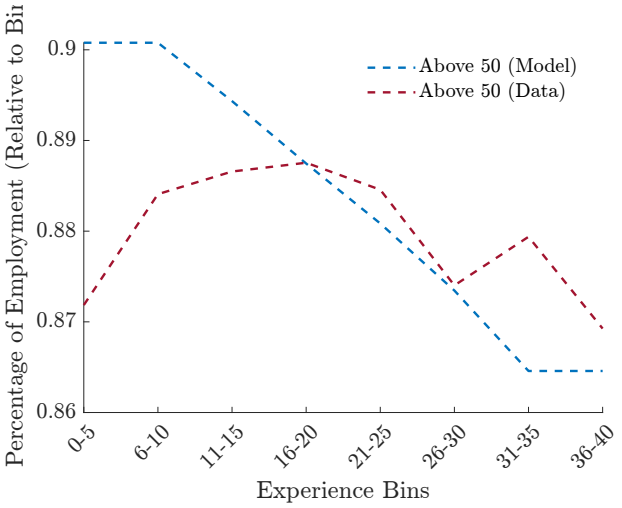


(a) Tenure-Experience

(b) Employment by Tenure



(c) Normalized Wage-Experience

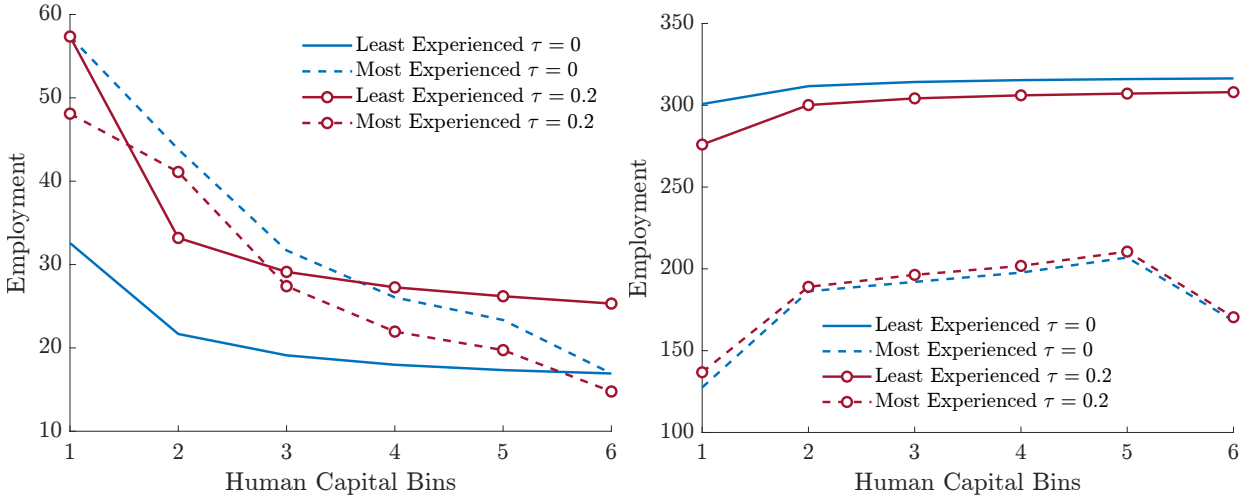


(d) Employment by Experience

4.2 Firms' Hiring and Firing Strategies

We use our calibrated model first to study the different hiring and firing strategies used by firms of different sizes, showing that large/productive firms disproportionately employ young workers. We contrast these findings with evidence using the Japanese establishment data.

Figure 8: Different Employment Across Firms



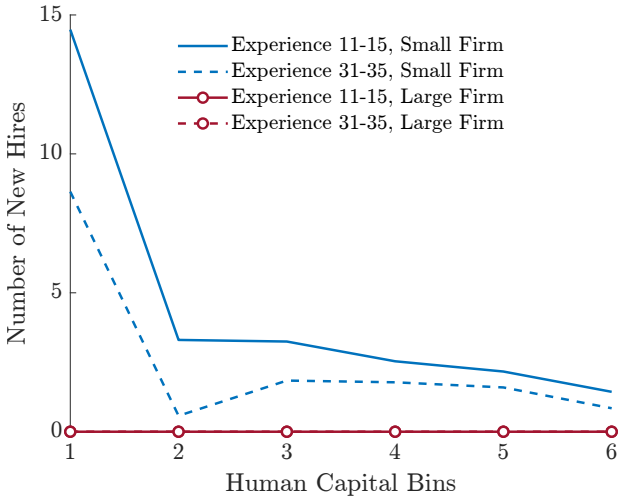
(a) Employment in Small Firms

(b) Employment in Large Firms

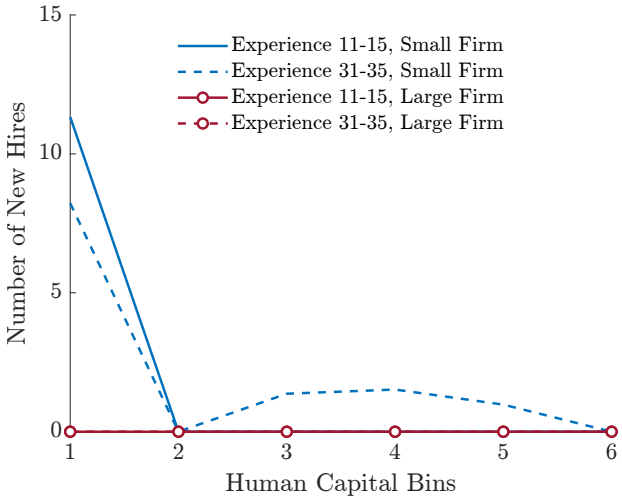
Figure 8 shows that high-productivity establishments hire more young, high-talent workers while shedding middle-aged and older workers who fail to upgrade their human capital. In contrast, low-productivity establishments hire fewer young workers, who tend to be less talented on average, and instead fill vacancies with middle-aged and older workers separated from more productive establishments. Reasonably, an increase in the firing cost makes these differences in the hiring/firing strategies less pronounced.

Figure 9 supports these observations by examining the hiring and firing strategies of large and small firms across various human capital levels. We focus on two experience bins: 11-15 years and 31-35 years. The results reveal a clear distinction in hiring and firing practices. Small firms handle the hiring of workers with more experience, while large firms are responsible for the majority of firings. Large, productive firms specialize in hiring younger workers and promoting talent, resembling an "up or out" dynamic, whereas smaller, less productive firms tend to hire individuals who have been let go by larger firms.

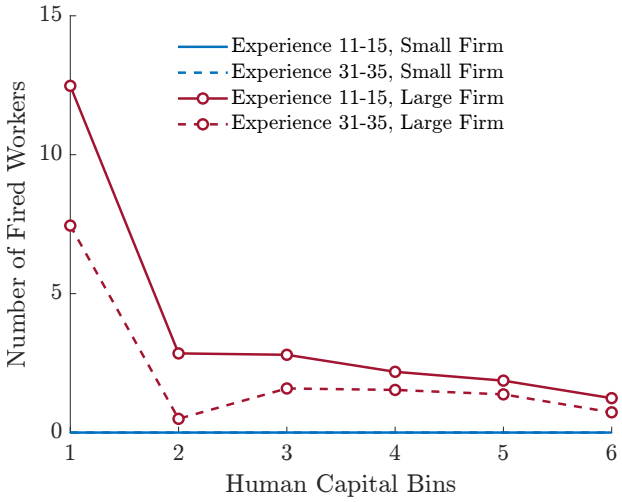
Figure 9: Different Hiring and Firing Strategies Across Firms



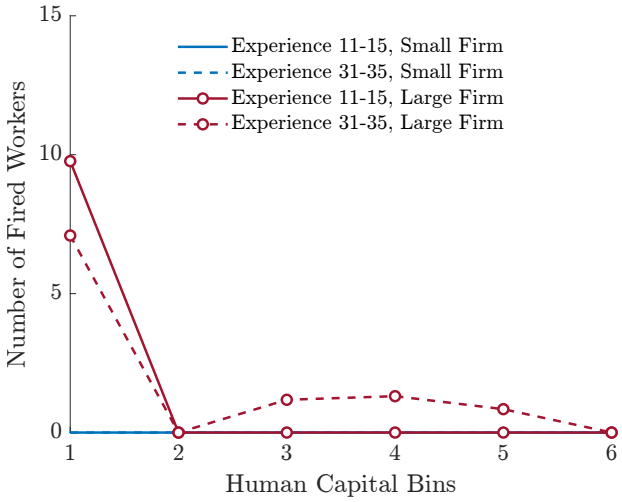
(a) Hiring with No Firing Costs $\tau = 0$



(b) Hiring with Firing Costs $\tau = 0.2$



(c) Firing with No Firing Costs $\tau = 0$



(d) Firing with Firing Costs $\tau = 0.2$

Evidence: Establishments' Differential Hiring Strategies

In the calibrated model, we observe that heterogeneous firms (establishments) use different hiring strategies. Specifically, the productive ones hire disproportionately more young workers and especially job market entrants. Fortunately, we have data on the number of hired job market entrants at the establishment-year level. Moreover, we can calculate the (net) change in employment for a given establishment when it appears in the data in two consecutive years. As a result, we obtain the *net* change in the number of incumbent workers which is the difference between the employment change and the number of hired job market entrants. The two specific empirical investigations we are interested in are (1) do larger establishments hire disproportionately more job-market entrants (the intensive margin)? and (2) are larger establishments more likely to hire job-market entrants than job-market incumbents (the extensive margin)? We use the merged data between 1998-2001 and between 2004 and 2007 to answer these two questions.

Table 7 presents the results for the intensive margin regression. Clearly, larger establishments tend to hire more job market entrants (column 1) and disproportionately more of them relative to total employment (column 2) and relative to the amount of net hiring (column 3). Additionally, Table 8 presents the results for the extensive margin regression, showing that larger establishments are more likely to hire job market entrants rather than the incumbents (or to hire job market entrants but cut the employment of job market incumbents). In summary, the empirical results lend support to our model's predictions on heterogeneous hiring strategies of large and small establishments.[27]

4.3 Counterfactual Analysis

Heterogeneity in firm's comparative advantage of using worker's experience/human capital is the key to understanding various counterfactual results. Productive firms want to have many young workers and can tolerate higher skill (i.e., human capital) premiums, as their comparative advantage is to use workers' human capital rather than experience.

[27] On average, establishments appear twice in our 8-year sample. Therefore, it is hard to include the establishment fixed effects in the above regressions.

Table 7: Cross-sectional regression of hiring shares: 1998-2001 and 2004-2007

Dep.Var:	(1) $log(hired\ fresh)$	(2) $\frac{hired\ fresh}{empl.}$	(3) $\frac{hired\ fresh}{net\ hire}$
log(employment)	0.539***	0.005***	0.135***
	(0.004)	(0.000)	(0.002)
plant age	-0.002***	-0.000***	0.002***
	(0.000)	(0.000)	(0.000)
constant	-1.248***	-0.002***	-0.173***
	(0.016)	(0.000)	(0.008)
City FE	Yes	Yes	Yes
Industry-year FE	Yes	Yes	Yes
Prefecture-year FE	Yes	Yes	Yes
N	70587	303625	54285
R^2	0.462	0.129	0.260

* 0.10 ** 0.05 *** 0.01. Standard errors are clustered at the firm level. $hired\ fresh$ denotes the number of hired job market entrants, while *net hire* is the change in employment over two consecutive years (i.e., net hiring or firing). $\frac{hired\ fresh}{net\ hire}$ is one (or missing), if the net hiring of job-market incumbents is negative (or the hiring of job-market entrants is zero and the net hiring of job-market incumbents is non-positive).

Table 8: Cross-sectional regression of hiring new v.s. hiring old: 1998-2001 and 2004-2007

	(1)	(2)	(3)	(4)
Dependent Variable:	$Hiring\ New > 0$ - $Hiring\ Old > 0$			
log(employment)	0.125***	0.124***	0.124***	0.124***
	(0.002)	(0.002)	(0.002)	(0.002)
Plant Age		0.003***		0.002***
		(0.000)		(0.000)
Constant	-0.506***	-0.569***	-0.503***	-0.566***
	(0.006)	(0.007)	(0.006)	(0.008)
Year FE	Yes	Yes	No	No
City FE	Yes	Yes	Yes	Yes
Prefecture FE	Yes	Yes	No	No
Industry FE	Yes	Yes	No	No
Industry-year FE	No	No	Yes	Yes
Prefecture-year FE	No	No	Yes	Yes
N	135918	108371	135716	108247
R^2	0.099	0.103	0.130	0.133

Notes: * 0.10 ** 0.05 *** 0.01. Standard errors are clustered at the firm level. *hiring new* > 0 (*hiring old* > 0) is an indicator of (net) hiring of job-market entrants (incumbents).

Labor Market Friction: Firing Costs

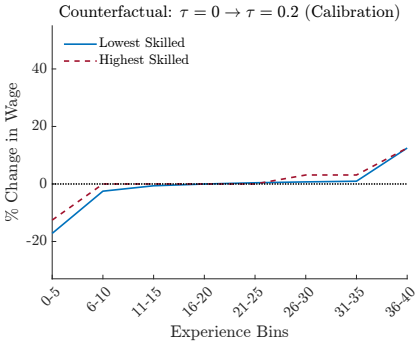
In this counterfactual exercise, we increase the firing cost τ from zero to to 0.2 (the calibrated value), 0.3 and 0.5 . The focus of the analysis is the distributional effect of the firing cost on both workers and establishments.[28] Note that all establishments are subject to the same firing cost.

Figure 10 shows that the young workers' (especially the job market entrants') wages are negatively affected by the firing cost and vice versa for the old workers. Moreover, it is the young workers with little human capital that are most negatively affected by the firing cost. Establishments' heterogeneous firing/hiring strategies are the keys to understanding these results. In the calibrated model, unproductive establishments do not fire workers while the productive ones hire young workers disproportionately more and fire some of them when they become older.[29] Thus, an increase in the firing cost makes firing middle-aged/old workers more costly for productive establishments. As a result, they fire fewer middle-aged/old workers *and* hire fewer young workers, implying that the fraction of young (middle-aged/old) workers who work for productive establishments decreases (increases). Since the group-specific wage of workers is largely determined by the demand from the productivity establishments, young workers lose and old workers gain. Moreover, productive establishments reduce the hiring of young workers with less human capital *more* than those with more human capital, which explains why wages of young workers with little human capital are the most affected.

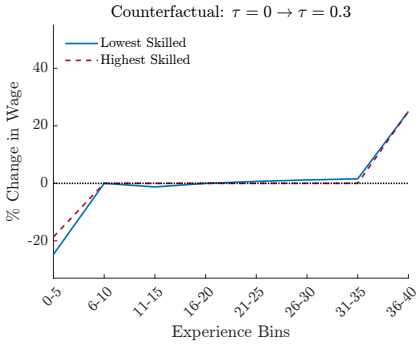
[28]The overall loss in output/productivity due to the firing cost is tiny, as the productivity of the establishment is time-invariant and the mass of establishments is fixed (i.e., no free entry condition).

[29]Figures are available upon request.

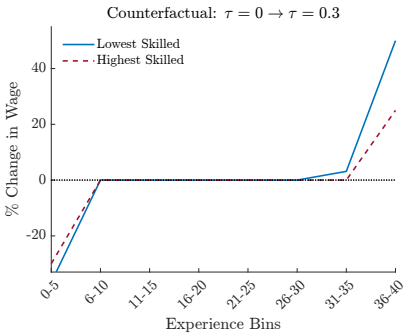
Figure 10: Distributional Effects on Workers Across Experience and Human Capital Bins



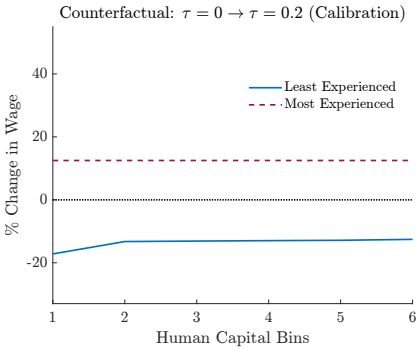
(a) Experience: $\tau = 0 \to 0.2$



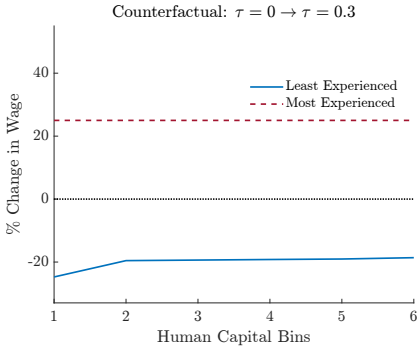
(b) Experience: $\tau = 0 \to 0.3$



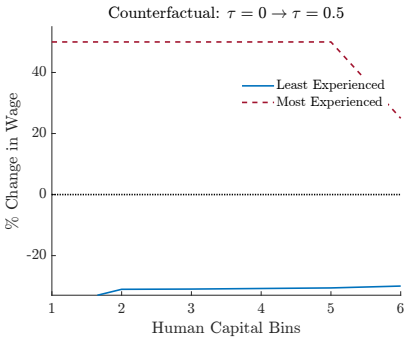
(c) Experience: $\tau = 0 \to 0.5$



(d) Human Capital: $\tau = 0 \to 0.2$



(e) Human Capital: $\tau = 0 \to 0.3$



(f) Human Capital: $\tau = 0 \to 0.5$

Now, let us turn to the distributional impact on establishments. Table 9 shows that as productive establishments hire fewer young workers and are more constrained when firing, their employment/output shares in the market shrink. Interestingly, Table 9 also substantiates that their profitability actually increases (or increases relative to the unproductive ones) despite that they are the ones that fire workers, which is quite puzzling. The key to understanding this is the (indirect) feedback effect from the labor markets on establishments. The direct negative effect does reduce productive establishments' profitability more, as they are more constrained now. However, since young workers' wages drop substantially and the productive establishments (still) employ disproportionately such workers after the increase of the firing cost, they actually gain in terms of the reduced wage costs. On the contrary, as old workers' wages increase and the unproductive establishments hire them disproportionately, the unproductive establishments lose due to higher wage costs. In total, this *heterogeneous feedback effect in general equilibrium* dominates the partial equilibrium (negative) effect on (productive) establishments, which leads to a puzzling effect documented in Table 9.[30]

Table 9: Distributional Effects on Firms: Changing τ

Variable	$\tau = 0$	**Calibration** $\tau = 0.2$	$\tau = 0.3$	$\tau = 0.5$
–Moments				
Firing Rate	1.17%	0.44%	0.25%	0%
Average Tenure	[14.2;15.4]	[15;15.5]	[15.3;15.6]	[15.6;15.6]
Average Experience	[24 20.4]	[22.2 20.7]	[21.5 20.7]	[20.8 20.8]
Average Wages (Normalized)	[0.871;1.016]	[0.861;1.018]	[0.876;1.019]	[0.89;1.028]
–Efficiency and Concentration				
Normalized Output	1	1	0.999	0.992
Labor Productivity ϕ^h	22.307	22.398	22.588	23.369
Labor Productivity ϕ^l	19.135	18.529	17.728	15.536
Firm Profits ϕ^h	1.02	1	1.007	0.989
Firm Profits ϕ^l	1.043	1	0.974	0.706
Output Concentration ϕ^h (%)	90.5 %	90.3 %	89.1 %	85.5 %
Employment Concentration ϕ^h (%)	89.1 %	88.5 %	86.6 %	79.6 %

One point worth mentioning is that higher product market concentration (thanks to a lower firing cost) *does not* necessarily imply widening wage inequality and fewer worker movements

[30]Interestingly, the table also shows that productive establishments also have increasing productivity when the firing cost goes up. The reason is that although the overall employment shrinks, the *share* of workers with high levels of human capital (within each experience bin) goes up in productive establishments after an increase in the firing cost (as shown by Figure 10). In other words, the sorting of workers with high levels of human capital into high-productivity firms becomes stronger when the firing cost increases.

across firms in our model. In fact, this leads to more worker movements (i.e., more workers being fired) and decreasing wage inequality (thanks to higher wages earned by young workers) in our model, as shown by Table 9.[31] Admittedly, our model does not include a frictional labor market that features search/matching frictions. Thus, the predictions mentioned above cannot be directly contrasted with reality. However, the lesson here is that simply looking at the correlation between product market concentration and worker movements/wage inequality *without identifying the underlying causes* is flawed. Additionally, increasing product market concentration may *not* be the cause of decreasing worker mobility, as both of them can be driven by a confounding fact such as the firing cost.

Population Aging

For the exercise of population aging, we fix the overall size of the labor force and vary the exit rate from the labor market δ. Specifically, a lower δ implies a labor force with more old workers. When we reduce δ, we also decrease the total mass of the job market entrants such that the overall size of the labor force is fixed.

Table 10 presents the results. An older labor force is associated with workers that have more experience but not more human capital. This change favors unproductive establishments, as their comparative advantage is to utilize workers' human capital. The opposite story happens to the productive establishments. As a result, their output/employment shares go down.[32] This new reallocation force (between establishments) points out another potential channel through which population aging negatively affects aggregate productivity. That is, population aging drags down aggregate productivity, as it reduces market shares accounted for by productive firms.[33]

[31] In a labor market featuring search/matching frictions, higher (product) market concentration would increase firms' bargaining/monopsony power and reduce workers' outside options. As a result, worker movements should become less frequent, and wage inequality should increase.

[32] There is an analogy between the effect of population aging on firms in our model and the Rybczynski theorem studied in international trade. Unproductive and productive firms in our model are likely labor-intensive and capital-intensive industries in a Rybczynski economy. Supply of worker experience and skill in our model is like the supply of labor and capital in a Rybczynski economy. The Rybczynski theorem says that an increase in the supply of workers (i.e., an increase in the supply of worker experience due to population aging in our model) leads to an expansion of labor-intensive sector (an expansion of unproductive firms in our model) and a contraction of capital-intensive sector (a contraction of productive firms in our model), as the labor-intensive sector (i.e., unproductive firms in our model) have comparative advantage of using labor (i.e., worker experience in our model) instead of capital (i.e., worker skill in our model).

[33] Some seminal papers emphasize different channels through which population aging negatively affects aggregate productivity such as lower innovation rates (e.g., Peters and Walsh (2021) and Aksoy et al. (2019))

Table 10: Outcomes Changing δ (survival rate): fixing L

Variable	$\delta = 0.8$	**Calibration** $\delta = 0.94$	$\delta = 1$
—*Firing Cost* $\tau = 0$			
Output ϕ^h (%)	91.1 %	90.5 %	90.3 %
Employment ϕ^h (%)	90.2 %	89.1 %	88.7 %
—*Firing Cost* $\tau = 0.2$ *(Calibration)*			
Output ϕ^h (%)	91 %	90.3 %	90 %
Employment ϕ^h (%)	89.5 %	88.5 %	88.1 %
—*Firing Cost* $\tau = 0.5$			
Output ϕ^h (%)	86.9 %	85.5 %	84.8 %
Employment ϕ^h (%)	81.9 %	79.6 %	78.6 %

Evidence: Population Growth/Aging and Market Concentration

In this subsection, we provide evidence for our model's implication of population aging on market concentration. As more productive firms have comparative advantage of using worker skill (as opposed to experience), a shift of distribution in labor supply toward older workers (and not toward more skilled workers) hurts more productive firms, leading to lower concentration of market shares accounted for by the largest firms. We exploit differential changes in the growth rate of working-age population across (47) Japanese prefectures to investigate how they affect market shares of top 5% or 10% establishments at both the prefecture-year level and the prefecture-industry-year level. We choose this within-country and across-region analysis for two reasons. First, there are substantial differences in the growth rate of working-age population across Japanese prefectures during our sample period.[34] Second, we believe this analysis can alleviate potential concerns that apply to cross-country studies, as different regions in the same country (especially in Japan) are largely subject to the same government policies, institutions and culture.

Several things are worth mentioning before proceeding. For the prefecture-year level analysis, we calculate the sales/employment shares of top 5% and 10% establishments in a given

and lower labor productivity (e.g., Maestas et al. (2016)).

[34]The average growth rate of working-age population across Japanese prefectures had dwindled from 0.7% in 1985 to −1.3% in 2018, while its standard deviation had only decreased from 0.8% in 1985 to 0.6% in 2018. In particular, places like Tokyo had had little reduction in the growth rate of working-age population during 1985-2018 (0.3%), while other places such as Saitama prefecture and Nara prefecture had had substantial reductions in the growth rate of working-age population during same period (over 3%).

prefecture and in a given year. For the prefecture-industry-year level analysis, we calculate the sales/employment shares of top 5% and 10% establishments in a given industry of a prefecture and in a given year. We prefer the prefecture-industry-year level analysis, as it teases out the potential industry-level compositional effect. I.e., some prefectures can have increasing shares of industries that are intrinsically more concentrated in their economies over time. However, this comes at a cost, as some of the prefecture-industry-year cells have few observations which disenable us to calculate the market share of top firms. As a result, we present evidence at both levels and restrict our analysis at the prefecture-industry-year level to prefecture-industry-year cells that have at least one hundred establishments.[35]

In what follows, we use a confidential establishment-level dataset and official statistics of prefecture-level population growth/average experience of population from Japan to implement our empirical analysis. The establishment-level dataset is called the Census of Manufacturers (time span: 1985-2018) which is mandatory too all manufacturing establishments, and we obtain the access through the Japanese Ministry of Economy, Trade, and Industry. It surveys all manufacturing establishments with more than three employees every year, generating a panel dataset of over thirty years with roughly 300,000-350,000 establishments per year.[36] Based on this dataset, we calculate the sales and employment shares of top 5% or 10% establishments. Next, we obtain the growth rate of working-age population/average experience of population at the prefecture-year level (1985-2018) from the National Population Census conducted by the Japanese Ministry of Internal Affairs and Communications. We then merge the two datasets and run the following panel regression:

$$concentration_{p,y}^{top\ firm} = \beta_0 + \beta_1 working\ gr_{p,y} + \delta_p + \delta_y + \epsilon_{p,y}, \tag{14}$$

where $concentration_{p,y}^{top\ firm}$ is the sales/employment shares of top 5% (or 10%) establishments in prefecture p and year y at either the prefecture-year level or the prefecture-industry-year level. The variable of $working\ gr_{p,y}$ is the annual growth rate of working experience population for prefecture p and year y. Next, we always include the prefecture fixed effects δ_p and the year fixed effects δ_y into the prefecture-year regression and the industry-prefecture fixed effects δ_{ip} and the year fixed effects δ_y into the prefecture-industry-year level regression. As we always include the unit fixed effects, we are comparing prefectures/industry-prefecture pairs with different growth rates of working experience population in the same

[35]The empirical results barely change, if we change this threshold to fifty or so.

[36]In years ending with 0, 2, 5 and 8, the Census of Manufacturers surveys all manufacturing establishments.

time period. The Standard errors are clustered at the year level and at the prefecture-level for the prefecture-year level regressions (i.e., two-way clustering) and at the prefecture-year level for the prefecture-industry-year level regressions.

Table 11: Growth of working-age population and employment concentration of the largest establishments

	pref.-year level		ind.-pref.-year level	
	(1)	(2)	(3)	(4)
	$empl\ share^{top\ 5\%}_{pref,y}$	$empl\ share^{top\ 10\%}_{pref,y}$	$empl\ share^{top\ 5\%}_{ind,pref,y}$	$empl\ share^{top\ 10\%}_{ind,pref,y}$
$working\ gr_{p,y}$	1.458***	1.211***	0.464***	0.391***
	(0.427)	(0.372)	(0.108)	(0.102)
Prefecture FE	Yes	Yes	No	No
Industry-prefecture FE	No	No	Yes	Yes
Year FE	Yes	Yes	Yes	Yes
N	1598	1598	19913	19913
adj. R^2	0.894	0.902	0.730	0.733

Notes: Census of manufacturing plants (1985-2018) is used. The estimate of the constant is suppressed. Standard errors are clustered at the year level and at the prefecture level for columns 1-2 (i.e., two-way clustering) and at the prefecture-year level for columns 3-4. For columns 3-4, ind.-pref.-year obs. with at least 100 establishments are included. * 0.10 ** 0.05 *** 0.01.

The regressions results reported in Table 11 support our model's key prediction. Specifically, the coefficient of β_1 is estimated to be positively significant in all columns, suggesting that the employment share of top establishments increases more when the growth rate of working-age population in that prefecture increases more (compared to other prefectures during the same time period). In other words, a *slow-down* in the growth rate of working-age population leads to lower market concentration in a given prefecture or an industry in a given prefecture. This finding is robust to the share of firms at the top we focus on and the measure of concentration we use (i.e., sales/employment share), as shown by Table 12. In addition, using the average experience of the population (which is an imperfect measure of the average experience of the working-age population) at the prefecture-year level yields qualitatively the same result, as shown by Table 13. Namely, an increase in the average experience of the population reduces the market share accounted for by the largest establishments in a given industry of a prefecture. In summary, this empirical finding is new and directly supports our model's key implication that slower population growth/aging population reduce market concentration and market shares accounted for by top firms.

Table 12: Growth of working-age population and sales concentration of the largest establishments

	pref.-year level		ind.-pref.-year level	
	(1)	(2)	(3)	(4)
	$sales\ share^{top\ 5\%}_{pref,y}$	$sales\ share^{top\ 10\%}_{pref,y}$	$sales\ share^{top\ 5\%}_{ind,pref,y}$	$sales\ share^{top\ 10\%}_{ind,pref,y}$
$working\ gr_{p,y}$	1.722***	1.229***	0.309**	0.250**
	(0.522)	(0.381)	(0.145)	(0.122)
Prefecture FE	Yes	Yes	No	No
Industry-prefecture FE	No	No	Yes	Yes
Year FE	Yes	Yes	Yes	Yes
N	1598	1598	19913	19913
adj. R^2	0.871	0.876	0.692	0.702

Notes: Census of manufacturing plants (1985-2018) is used. The estimate of the constant is suppressed. Standard errors are clustered at the year level and at the prefecture level for columns 1-2 (i.e., two-way clustering) and at the prefecture-year level for columns 3-4. For columns 3-4, ind.-pref.-year obs. with at least 100 establishments are included. * 0.10 ** 0.05 *** 0.01.

Table 13: Average age of population and concentration of the largest establishments

	ind.-pref.-year level			
	(1)	(2)	(3)	(4)
	$sales\ share^{top\ 5\%}_{ind,pref,y}$	$sales\ share^{top\ 10\%}_{ind,pref,y}$	$empl\ share^{top\ 5\%}_{ind,pref,y}$	$empl\ share^{top\ 10\%}_{ind,pref,y}$
$average\ age_{p,y}$	-0.00723***	-0.00607***	-0.00484***	-0.00511***
	(0.00217)	(0.00201)	(0.00140)	(0.00131)
Industry-prefecture FE	Yes	Yes	Yes	Yes
Year FE	Yes	Yes	Yes	Yes
N	19913	19913	19913	19913
adj. R^2	0.692	0.702	0.733	0.733

Notes: Census of manufacturing plants (1985-2018) is used. The estimate of the constant is suppressed. Standard errors are clustered at prefecture-year level. For all columns, ind.-pref.-year obs. with at least 100 establishments are included. * 0.10 ** 0.05 *** 0.01.

Confounding Factors: Establishment Age and Worker Migration

Establishment/firm age is shown to be positively correlated it market share, as *surviving* establishment/firms grow. Thus, average establishment age is a confounding factor in our regression results presented above. Population aging/slow-down of working-age population growth leads to firm aging which have higher market shares. Therefore, firm aging would imply that a drop in the working-age population growth leads to a higher levels of market concentration, which is the opposite to our empirical finding. Although we do not have information on establishment age, the confounding factor of firm aging actually works against our empirical finding. If we were able to control for this factor in our regressions, the positive (negative) effect of population growth (aging) on market concentration probably would become even stronger.

Since workers can move across space, the growth rate of the working-age population suffers from the endogeneity concern. In particular, if some local shocks in a prefecture that induce the largest firms to grow even more (and thus higher market concentration) also incentivize young/working-age workers to immigrate to that prefecture (thanks to more job opportunities), the documented positive relationship between the working-age population growth and the market concentration would suffer from the reversed causality problem. In order to isolate some exogenous variation in the growth of the working-age population/average experience of the population that is not related to firm dynamics, we follow Maestas et al. (2016) and Karahan et al. (2019) to use the 20-year lagged birth rate at the prefecture-year level as an instrument variable (IV) for the annual growth rate of working-age population. The result is presented in Table 14 and confirms our finding presented in Table 11.[37] Moreover, the quantitative magnitude of the effect is sizable. The second (last) column of Table 14 imply that a 0.86% decline in the working-age population growth rate (i.e., the standard deviation of this variable in our sample) leads to a 3.68% (1.40%) decrease in the employment share accounted by top 10% establishments at the prefecture-year (industry-prefecture-year) level (standard deviation: 5.21% and 13.0%).

[37]The correlation between the growth rate of the working-age population and the 20-year lagged birth rate is 0.87 in our sample, which is quite high.

Table 14: Lagged birth rate and employment concentration of the largest establishments: IV regressions

	first stage	pref.-year level		ind.-pref.-year level	
	(1)	(2)	(3)	(4)	(5)
		$empl\ share^{top\ 5\%}_{pref,y}$	$empl\ share^{top\ 10\%}_{pref,y}$	$empl\ share^{top\ 5\%}_{ind,pref,y}$	$empl\ share^{top\ 10\%}_{ind,pref,y}$
$working\ gr_{p,y}$		5.007***	4.325***	1.607***	1.645***
		(1.754)	(1.492)	(0.258)	(0.239)
$birth\ rate_{p,y-20}$	0.124***				
	(0.042)				
F-test	284.1				
Prefecture FE	Yes	Yes	Yes	No	No
Industry-prefecture FE	No	No	No	Yes	Yes
Year FE	Yes	Yes	Yes	Yes	Yes
N	1588	1588	1588	19873	19873
R^2	0.908	0.866	0.876	0.744	0.747
adj. R^2	0.903	0.859	0.869	0.729	0.733

Notes: Census of manufacturing plants (1985-2018) is used. We use 20-year lagged birth rate to instrument for the growth rate of working-age population at the prefecture-year level. Both the first and second stage of the IV regressions include the prefecture fixed effects and the year fixed effects. The estimate of the constant is suppressed. Standard errors are clustered at the year level and at the prefecture level for columns 1-2 (i.e., two-way clustering) and at the prefecture-year level for columns 3-4. For columns 3-4, ind.-pref.-year obs. with at least 100 establishments (with non-missing sales and employment) are included. * 0.10 ** 0.05 *** 0.01.

5 Conclusion

The world is becoming older, making it crucial to understand how demographic shifts affect firm performance, inequality, and aggregate productivity. This paper develops a framework to analyze the optimal allocation of workers across firms. Using Japanese establishment-worker data, we show that the optimal labor composition features a larger share of mid-career workers than Japan's current workforce. Strong complementarities between firm productivity and worker's human capital play a key role in shaping productivity and wage outcomes. Our analysis suggests that reallocating workers toward the optimal composition would increase Japan's GDP by 5.3%. Extending the model to a dynamic setting, we show how firms' hiring and firing strategies respond to demographic shifts and labor market policies, highlighting the distributional consequences of the firing cost and population aging on workers and firms.

Future work could deepen these insights by modeling endogenous technological responses to demographic change, incorporating worker-firm bargaining, or extending the analysis to entertain other labor market frictions. Additionally, exploring immigration as a margin of adjustment in aging societies would yield valuable policy implications. More broadly, our findings underscore the need for labor-market policies that facilitate efficient worker reallocation while accounting for firm heterogeneity and dynamic labor-market considerations.

References

Aksoy, Yunus, Henrique S Basso, Ron P Smith, and Tobias Grasl, "Demographic structure and macroeconomic trends," *American Economic Journal: Macroeconomics*, 2019, *11* (1), 193–222.

Altonji, Joseph G and Robert A Shakotko, "Do wages rise with job seniority?," *The Review of Economic Studies*, 1987, *54* (3), 437–459.

Alvarez, Fernando and Marcelo Veracierto, "Severance payments in an economy with frictions," *Journal of monetary Economics*, 2001, *47* (3), 477–498.

Bagga, Sadhika, "Internal and External Labor Markets and Declining Dynamism," 2022.

Bentolila, Samuel and Giuseppe Bertola, "Firing costs and labour demand: how bad is eurosclerosis?," *The Review of Economic Studies*, 1990, *57* (3), 381–402.

Berger, Mark C, "The Effect of Cohort Size on Earnings Growth: A Reexamination of the Evidence," *Journal of Political Economy*, June 1985, *93* (3), 561–573.

Bianchi, Nicola and Matteo Paradisi, "Countries for Old Men: An Analysis of the Age Pay Gap," Technical Report, National Bureau of Economic Research 2024.

Borjas, George J, "The labor demand curve is downward sloping: Reexamining the impact of immigration on the labor market," *The quarterly journal of economics*, 2003, *118* (4), 1335–1374.

Card, David and Thomas Lemieux, "Can falling supply explain the rising return to college for younger men? A cohort-based analysis," *The quarterly journal of economics*, 2001, *116* (2), 705–746.

Cardona, Carolina, "How a Vast Demographic Shift Will Reshape the World," *The New York Times*, 2023.

Engbom, Niklas et al., "Firm and worker dynamics in an aging labor market," Technical Report 2019.

Frey, William H., "Immigration drives the nation's healthy post-pandemic population growth, new census data show," Technical Report, Brookings Insitution 2025.

Fujimoto, Makoto, "What is Japanese Long-Term Employment System? Has it Vanished?," *Japan Labor Issues*, 2017, *1* (1), 22–25.

Gregory, Victoria, "Firms as learning environments: Implications for earnings dynamics and job search," *FRB St. Louis Working Paper*, 2020, (2020-036).

Hayashi, Fumio, "The Root Cause of Japan's 30-year Stagnation: Implications for the US, Europe, and East Asia," *Europe, and East Asia (August 28, 2023)*, 2023.

Hopenhayn, Hugo A, "Entry, exit, and firm dynamics in long run equilibrium," *Econometrica: Journal of the Econometric Society*, 1992, pp. 1127–1150.

Hopenhayn, Hugo and Richard Rogerson, "Job turnover and policy evaluation: A general equilibrium analysis," *Journal of political Economy*, 1993, *101* (5), 915–938.

_ , Julian Neira, and Rish Singhania, "From population growth to firm demographics: Implications for concentration, entrepreneurship and the labor share," *Econometrica*, 2022, *90* (4), 1879–1914.

IMF, "Japan: Demographic Shift Opens Door to Reforms," Technical Report, IMF Country Focus 2020.

Karahan, Fatih, Benjamin Pugsley, and Ayşegül Şahin, "Demographic origins of the startup deficit," Technical Report, National Bureau of Economic Research 2019.

Kawaguchi, Daiji and Tetsushi Murao, "Labor-market institutions and long-term effects of youth unemployment," *Journal of Money, Credit and Banking*, 2014, *46* (S2), 95–116.

Kim, Seula, "Workers' Job Prospects and Young Firm Dynamics," Technical Report, Working Paper 2023.

Maestas, Nicole, Kathleen J Mullen, and David Powell, "The effect of population aging on economic growth, the labor force and productivity," Technical Report, National Bureau of Economic Research 2016.

Melitz, Marc J, "The impact of trade on intra-industry reallocations and aggregate industry productivity," *econometrica*, 2003, *71* (6), 1695–1725.

Ottaviano, Gianmarco IP and Giovanni Peri, "Rethinking the effect of immigration on wages," *Journal of the European economic association*, 2012, *10* (1), 152–197.

Ouimet, Paige and Rebecca Zarutskie, "Who works for startups? The relation between firm age, employee age, and growth," *Journal of financial Economics*, 2014, *112* (3), 386–407.

Peters, Michael and Conor Walsh, "Population growth and firm dynamics," Technical Report, National Bureau of Economic Research 2021.

Poschke, Markus, "Employment protection, firm selection, and growth," *Journal of Monetary Economics*, 2009, *56* (8), 1074–1085.

Qi, Liyan, "China Sees a Fresh Decline in Population, Despite a Rise in Births," *The Wall Street Journal*, 2025.

Toossi, Mitra and Torpey Elka, "Older workers: Labor force trends and career options," Technical Report, Bureau of Labor Statistics 2017.

Veracierto, Marcelo, "Employment flows, capital mobility, and policy analysis," *Interna-*

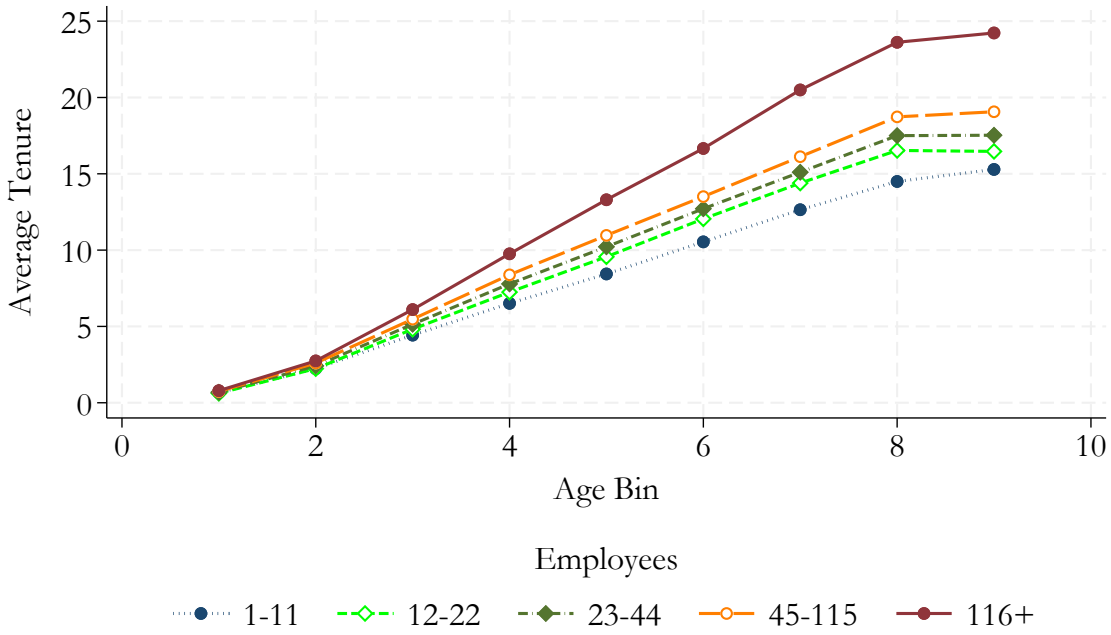
tional Economic Review, 2001, *42* (3), 571–596.

Welch, Finis, "Effects of Cohort Size on Earnings: The Baby Boom Babies' Financial Bust," *Journal of Political Economy*, 1979, *87* (5), S65–97.

Appendix

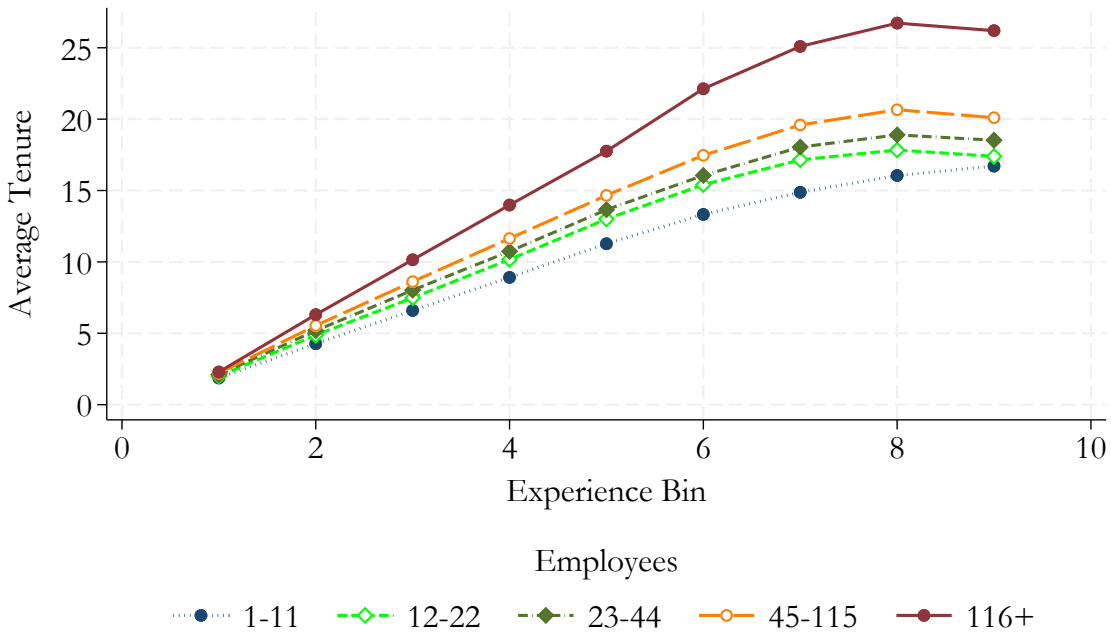
A Empirical Findings: Additional Tables and Plots

Figure A.1: Age-tenure profile (pooled cross-section: 1998-2003)



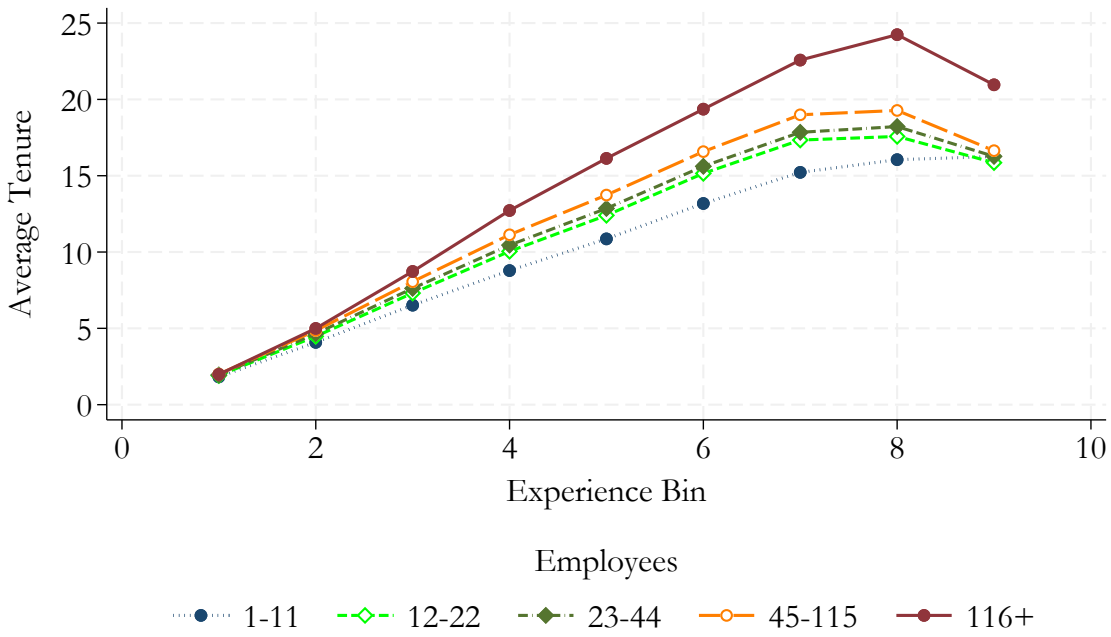
Note: Age bin 1: 16-20 years old; Age bin 2: 21-25 years old; ... Age bin 9: 56-60 years old. We cut the age bin at 60 years old. Size bin 1: emp ≥ 116. Size bin: 1 emp ≤ 11; Size bin 2: emp $\in [12, 22]$; Size bin 3: emp $\in [23, 44]$; Size bin 4: emp $\in [45, 115]$; Size bin 1: emp ≥ 116.

Figure A.2: Experience-tenure profile (pooled cross-section: 1998-2003): worker's age ≤ 60



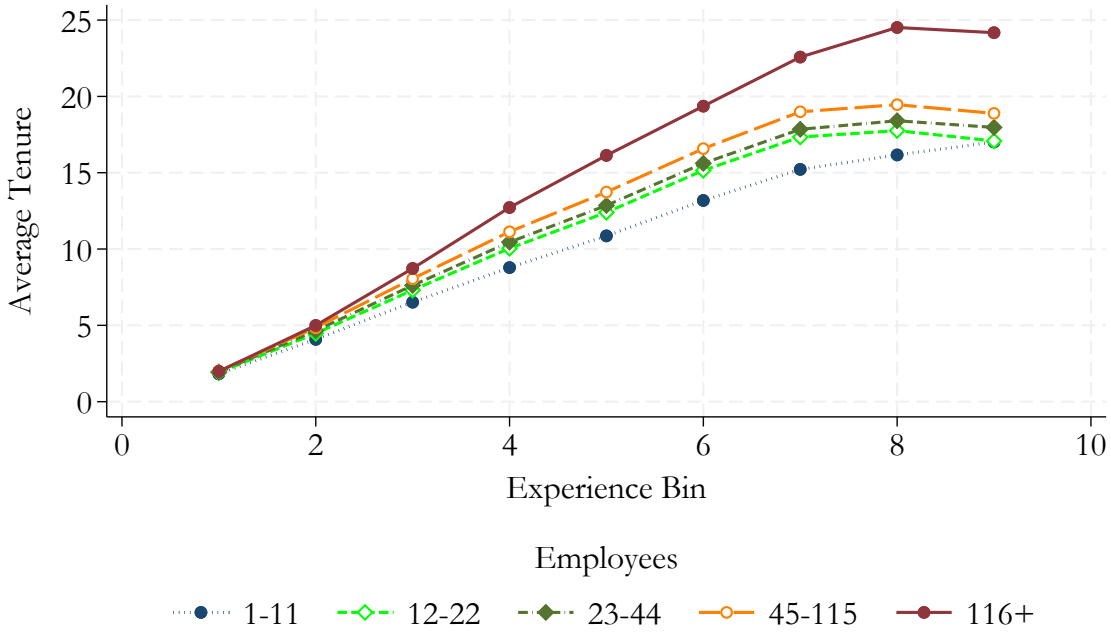
Note: Experience = age - (length of education + 6). Experience bin 1: 1-5 years; ... Experience bin 8: 36-40 years; Experience bin 9: 41-45 years. We cut the experience bin at 45 years. Size bin: 1 emp ≤ 11; Size bin 2: emp $\in [12, 22]$; Size bin 3: emp $\in [23, 44]$; Size bin 4: emp $\in [45, 115]$; Size bin 1: emp ≥ 116.

Figure A.3: Experience-tenure profile (pooled cross-section: 2005-2011)



Note: Experience = age - (length of education + 6). Experience bin 1: 1-5 years; ... Experience bin 8: 36-40 years; Experience bin 9: 41-45 years. We cut the experience bin at 45 years. Size bin: 1 emp ≤ 11; Size bin 2: emp $\in [12, 22]$; Size bin 3: emp $\in [23, 44]$; Size bin 4: emp $\in [45, 115]$; Size bin 1: emp ≥ 116.

Figure A.4: Experience-tenure profile (pooled cross-section: 2005-2011): worker's age ≤ 60



Note: Experience = age - (length of education + 6). Experience bin 1: 1-5 years; ... Experience bin 8: 36-40 years; Experience bin 9: 41-45 years. We cut the experience bin at 45 years. Size bin: 1 emp ≤ 11; Size bin 2: emp $\in [12, 22]$; Size bin 3: emp $\in [23, 44]$; Size bin 4: emp $\in [45, 115]$; Size bin 1: emp ≥ 116.

We show that the above patterns are not specific to the specific sample and/or the time period we focus on. First, since there is mandatory retirement at the experience of 60 in Japan, we restrict our sample to workers who are at most sixty years old and report various statistics in Table A.1. Moreover, we plot the experience-tenure profiles by restricting the sample to workers who are at most 60 years old in Figure A.2. The two key findings are unchanged.

Table A.1: Average Worker Characteristics by Establishment Size Bin: worker age ≤ 60

Size Bin	Employment	Age	Experience	Tenure	Wage	Education
1	8.53	38.88	20.33	9.56	32547.41	12.54
2	16.84	38.15	19.29	10.40	36176.99	12.86
3	32.16	38.31	19.40	11.03	37249.1	12.92
4	72.53	38.33	19.31	11.90	38577.94	13.02
5	683.28	38.00	18.58	14.19	44775.83	13.42

Time period: 1998-2003. Unit of average wage: one hundred yen (or one USD). We divide establishments to five size equal-sized bins. Size bin 1: employment ≤ 11; Size bin 2: employment $\in [12, 22]$; Size bin 3: employment $\in [23, 44]$; Size bin 4: employment $\in [45, 115]$; Size bin 5: employment ≥ 116.

Next, we present our facts and the experience-tenure profiles using a different time period: 2005-2011 (as the survey was redesigned in 2005). Table A.2 and Figures A.3 and A.4 show that the empirical patterns we just uncovered hold even around the financial crisis.

Table A.2: Average Worker Characteristics by Establishment Size Bin (2005-2011): full sample

Size Bin	Employment	Age	Experience	Tenure	Wage	Education
1	8.50	42.73	23.81	9.27	25042.02	12.77
2	16.80	40.18	21.56	8.75	27329.76	13.21
3	31.69	40.35	21.68	9.11	28197.94	13.24
4	72.87	41.74	21.79	10.22	30915.00	13.31
5	609.21	41.25	20.83	12.11	36747.11	13.58

Note: Time period: 2005-2011. Unit of average wage: one hundred yen (or one USD). We divide establishments to five size equal-sized bins. Size bin 1: employment ≤ 11; Size bin 2: employment $\in [12, 22]$; Size bin 3: employment $\in [23, 44]$; Size bin 4: employment $\in [45, 115]$; Size bin 5: employment ≥ 116.

Table A.3: Cross-sectional regression of experience, age and tenure: 1998-2001 and 2004-2007

Dep.Var:	(1) ave. experience	(2) ave. age	(3) ave. tenure
plant age	0.101***	0.093***	0.121***
	(0.001)	(0.001)	(0.001)
log(employment)	-1.487***	-1.263***	0.335***
	(0.017)	(0.016)	(0.013)
constant	24.720***	43.380***	5.570***
	(0.066)	(0.060)	(0.046)
City FE	Yes	Yes	Yes
Industry-year FE	Yes	Yes	Yes
Prefecture-year FE	Yes	Yes	Yes
N	298970	303625	303625
R^2	0.342	0.348	0.357

* 0.10 ** 0.05 *** 0.01. Standard errors are clustered at the firm level.

Table A.4: Cross-sectional regression of experience, age and tenure: 2004-2011

Dep.Var:	(1) ave. experience	(2) ave. age	(3) ave. tenure
plant age	0.092***	0.091***	0.112***
	(0.001)	(0.001)	(0.001)
log(employment)	-1.377***	-1.185***	0.150***
	(0.018)	(0.017)	(0.013)
constant	24.788***	43.642***	6.067***
	(0.065)	(0.061)	(0.045)
City FE	Yes	Yes	Yes
Industry-year FE	Yes	Yes	Yes
Prefecture-year FE	Yes	Yes	Yes
N	309312	317156	317156
R^2	0.319	0.355	0.383

* 0.10 ** 0.05 *** 0.01. Standard errors are clustered at the firm level.

Table A.5: Average Experience and Tenure: 1998-2001 and 2008-2011 (worker age ≤ 60 and establishment age ≥ 22)

	1998-2001			2008-2011		
Size Bin	Employment	Experience	Tenure	Employment	Experience	Tenure
1	8.44	23.28	11.34	8.47	23.22	10.99
2	16.88	21.80	11.72	16.78	20.82	10.78
3	31.30	21.55	12.18	31.91	21.12	10.98
4	73.01	21.19	12.95	73.69	21.33	11.95
5	747.00	20.09	15.12	705.20	20.50	13.62

Unit of wage: one hundred yen (or one USD). We divide establishments to five size equal-sized bins. Size bin 1: employment ≤ 11; Size bin 2: employment $\in [12, 22]$; Size bin 3: employment $\in [23, 44]$; Size bin 4: employment $\in [45, 115]$; Size bin 5: employment ≥ 116.

Figure A.5: Average tenure and experience (excluding young establishments): 2008-2011

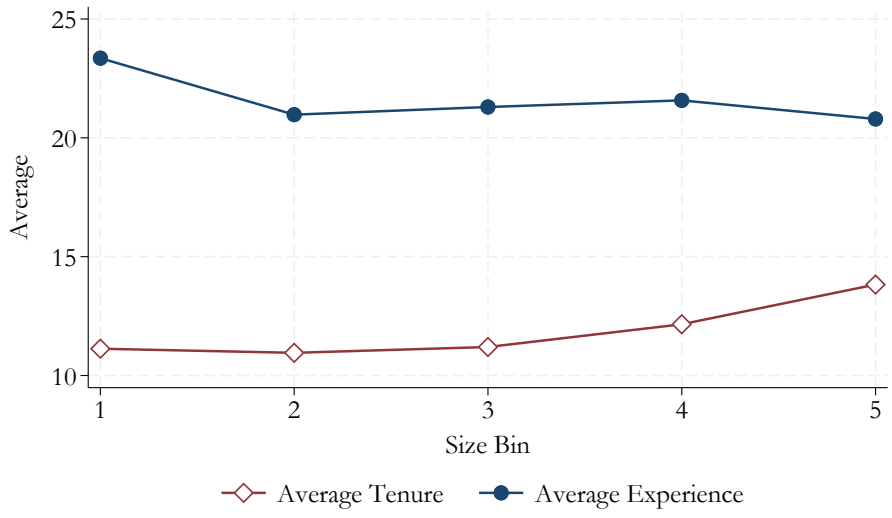


Figure A.6: Average experience by establishment age and size: 1998-2001

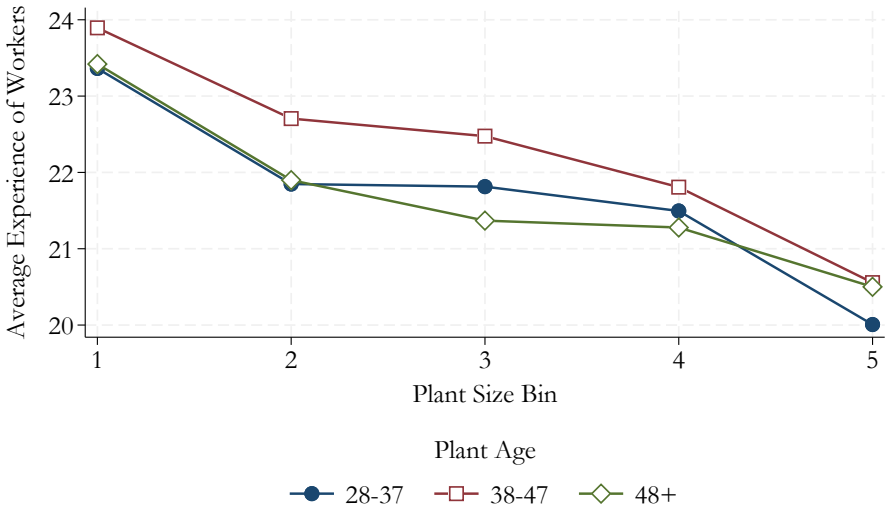


Figure A.7: Average tenure by establishment age and size: 1998-2001

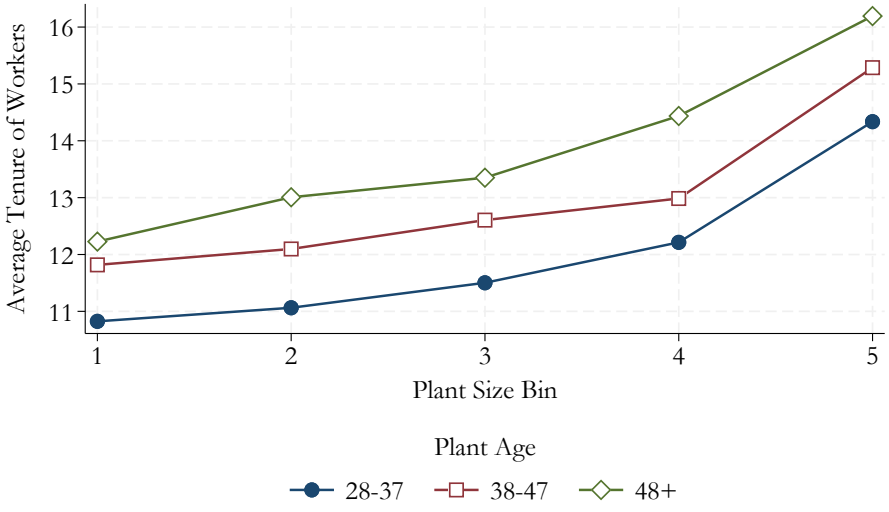
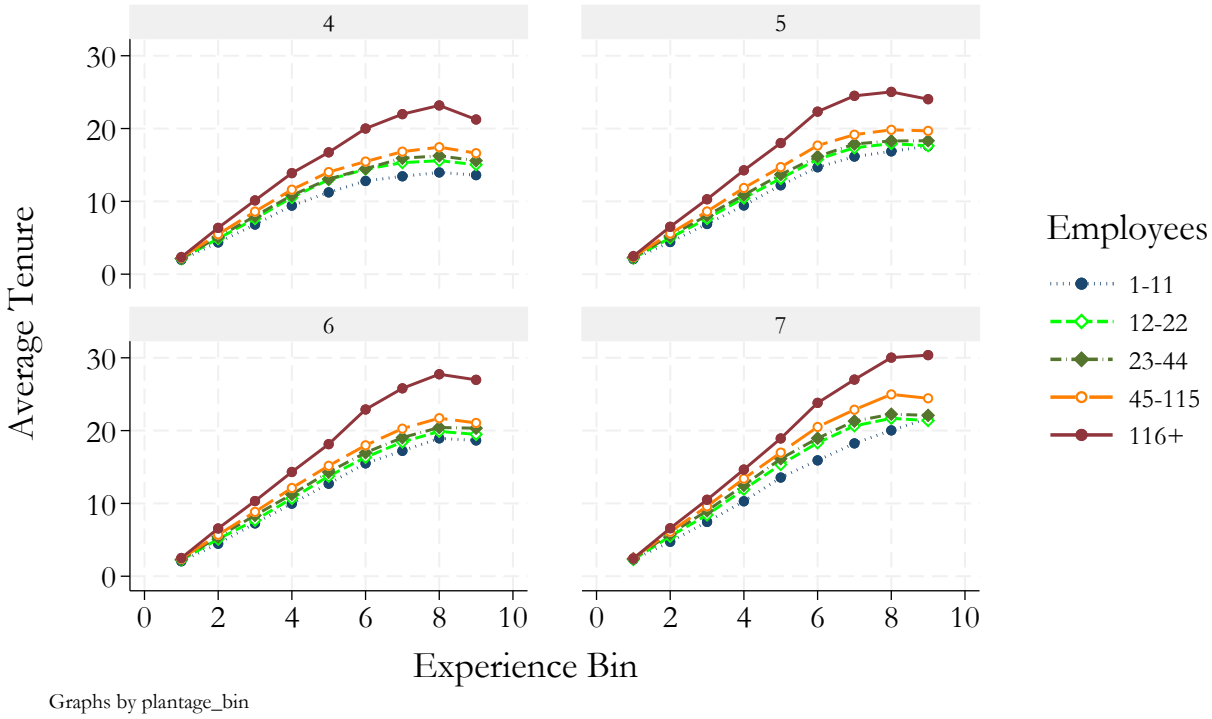
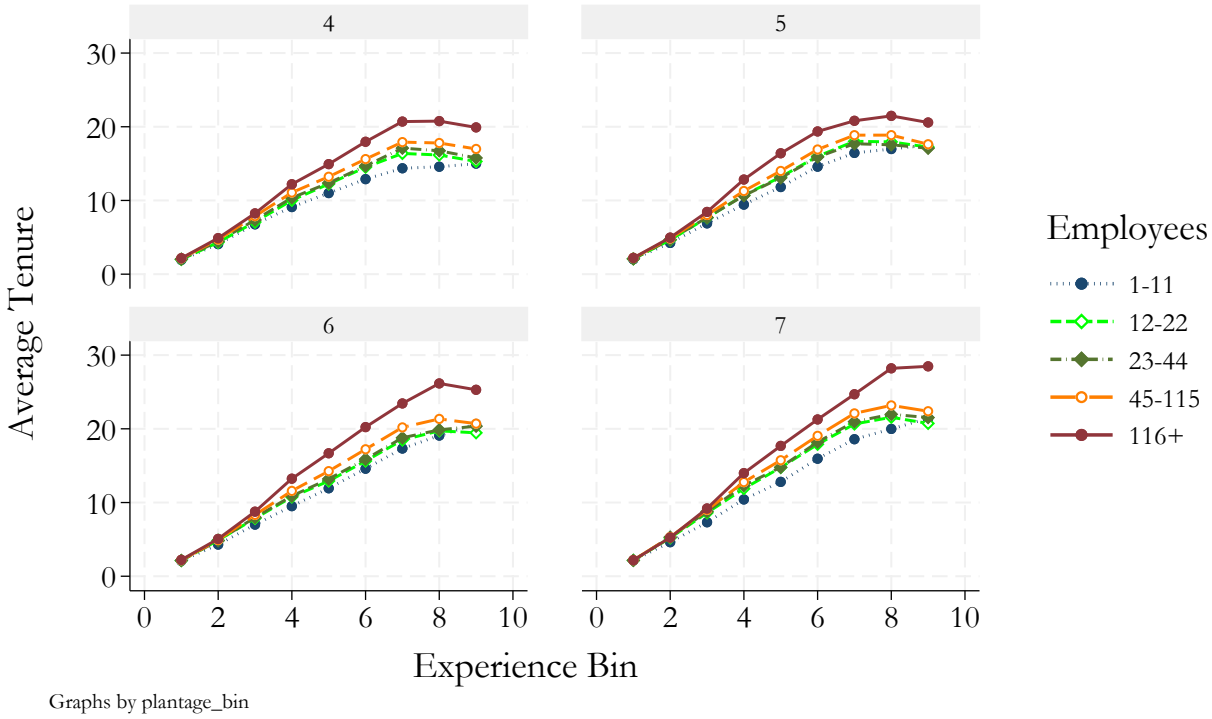


Figure A.8: Experience-tenure profile by establishment age (worker age: 20-60): 1998-2001



Note: Experience = age - (length of education + 6). Experience bin 1: 1-5 years; ... Experience bin 9: 41-45 years. We cut the experience bin at 45 years. Age bin 4: establishment age 18-27; Age bin 5: establishment age 28-37; Age bin 6: establishment age 38-47; Age bin 7: establishment age 48+

Figure A.9: Experience-tenure profile by establishment age (worker age: 20-60): 2008-2011



Note: Experience = age - (length of education + 6). Experience bin 1: 1-5 years; ... Experience bin 9: 41-45 years. We cut the experience bin at 45 years. Age bin 4: establishment age 18-27; Age bin 5: establishment age 28-37; Age bin 6: establishment age 38-47; Age bin 7: establishment age 48+

B Model

B.1 Static Model: Cobb-Douglass Case

Suppose a simple example where $F(\cdot)$ is the sum of Cobb-Douglas functions in labor and firm-worker match productivity $A_g(z)$,

$$F(\{l_g\}; z) = \sum_g l_g^\alpha A_g(z)^{1-\alpha}.$$

Firm z solves,

$$\max_{l_g} \sum_g l_g^\alpha A_g(z)^{1-\alpha} - \sum_g w_g l_g$$

From the FOC,

$$l_g^c(z) = \left(\frac{\alpha}{w_g}\right)^{1/(1-\alpha)} A_g(z)$$

Let $\bar{A}_g := \int A_g(z) d\Phi(z)$. From the market clearing conditions,

$$\int l_g^c(z)\, d\Phi(z) = L_g \implies w_g = \alpha \left(\frac{\bar{A}_g}{L_g}\right)^{1-\alpha}$$

So the labor demand for firm z is proportional to the productivity of the firm relative to the aggregate/average productivity,

$$l_g^c(z) = L_g \frac{A_g(z)}{\bar{A}_g}$$

Total firm labor is,

$$l^c(z) := \sum_g l_g^c(z) = \sum_g L_g \frac{A_g(z)}{\bar{A}_g}$$

Output for firm z is,

$$y^c(z) = \sum_g \left(\frac{L_g}{\bar{A}_g}\right)^{\alpha} A_g(z)$$

Aggregate output for the whole economy is,

$$Y^c := \int y^c(z) d\Phi(z) = \int \sum_g \left(\frac{L_g}{\bar{A}_g}\right)^{\alpha} A_g(z) d\Phi(z) = \sum_g L_g^{\alpha} \bar{A}_g^{1-\alpha} \tag{B.15}$$

Planner's Problem

Suppose the planner can substitute across any group, so the partition $\mathcal{P}$ corresponds to the entire $\mathcal{G}$. By Corollary 1 this gives the maximal gain from labor reallocation

The planner's problem is,

$$\max_{l_g(z)\geq 0} \int \sum_g l_g^{\alpha} A_g(z)^{1-\alpha} d\Phi(z) \quad s.t \quad \int \sum_g l_g(z) d\Phi(z) = L$$

From the FOCs,

$$[l_g(z)]: \ \alpha l_g^{\alpha-1} A_g(z)^{1-\alpha} = \lambda \quad \forall g, z$$

This implies marginal productivity is equalized across all worker groups and firm types,

$$\alpha l_g^{\alpha-1} A_g(z)^{1-\alpha} = \alpha l_{g'}^{\alpha-1} A_{g'}(z')^{1-\alpha} \implies l_{g'}(z') = \frac{A_{g'}(z')}{A_g(z)} l_g(z) \ \forall g, g', z, z'.$$

Substituting in the resource constraint,

$$\begin{aligned} \sum_{g'} l_{g'}(z') d\Phi(z') &= L \\ l_g(z) \sum_{g'} \int \frac{A_{g'}(z')}{A_g(z)} d\Phi(z') &= L \\ \implies l_g^p(z) &= \frac{A_g(z)}{\bar{A}} L, \end{aligned}$$

where $\bar{A} := \int \sum_{g'} A_{g'}(z') d\Phi(z')$

Aggregating across firms implies that the optimal labor supply allocation is,

$$l_g^p = \frac{\bar{A}_g}{\bar{A}} L.$$

The total aggregate output is,

$$Y^p = \int \sum_g l_g^p(z)^\alpha A_g(z)^{1-\alpha} d\Phi(z) = L^\alpha \bar{A}^{1-\alpha}$$

So the maximal output gain from labor supply reallocation is,

$$\Delta\%Y := \frac{Y^p}{Y^c} - 1 = \frac{L^\alpha \bar{A}^{1-\alpha}}{\sum_g L_g^\alpha \bar{A}_g^{1-\alpha}} - 1$$

Notice the gain is non-monotonic as $\alpha = 0$ or $\alpha = 1$, implies $\Delta\%Y = 0$.

B.2 Dynamic Model: Generalized Production Function

We can make the production function more general as

$$y(z, h_i, x_j, l_{ij}) = \int_{h_i \in H, x_j \in X} \left[l_{ij}^{\frac{\sigma-1}{\sigma}} f(z, h_i, x_j; \nu_h, \nu_x) \right] dh_i dx_j, \tag{B.16}$$

where the group specific output $f_{ij} := f(z, h_i, x_j; \nu_h, \nu_x)$ has to satisfy the property that it is log super-modular w.r.t z, h_i and log sub-modular w.r.t z, x_j. In particular, parameters ν_h (and ν_x) govern the complementary between z and h_i (and z and x_j) respectively. We need $\nu_h > \nu_a$. Practically, the production function can include polynomial terms of h_i and x_j:

$$y(z, h_i, x_j, l_{ij}) = \int_{h_i \in H, x_j \in X} \left[l_{ij}^{\frac{\sigma-1}{\sigma}} \left(z^{\nu_h}(b_0 + b_1 h_i + b_2 h_i^2) + z^{\nu_a}(d_0 + d_1 x_j + d_2 x_j^2) \right) \right] dh_i dx_j, \tag{B.17}$$

where d_2 can be negative, meaning that experience (age) contributes to output negatively when it exceeds a threshold.

B.3 Population Aging and Aggregate Output: a Simple Example

Now, we discuss the impact of population aging on total output. Here is a simple example. Suppose $\nu = 0$, $\sigma = 2$ and the distribution of z follows a Pareto distribution with the shape parameter of $k = 3(> \sigma)$ and $z_{min} = 1$. Then, we have

$$A_{ij}^{\frac{1}{\sigma}} = \sqrt{M}\sqrt{3h_i^2 + 3h_i x_j + x_j^2}.$$

Furthermore, let us assume $h_i = 5$ and $x_j = 1, 2$. As a result, we have

$$A_{i1}^{\frac{1}{\sigma}} = \sqrt{M}\sqrt{91} = 9.54\sqrt{M}, A_{i2}^{\frac{1}{\sigma}} = \sqrt{M}\sqrt{109} = 10.44\sqrt{M}.$$

Suppose the initial distribution of g_{ij} is $0.1, 0.1$. After population aging, the distribution of g_{ij} becomes $0.05, 0.15$. The total output for workers in human capital group i equals

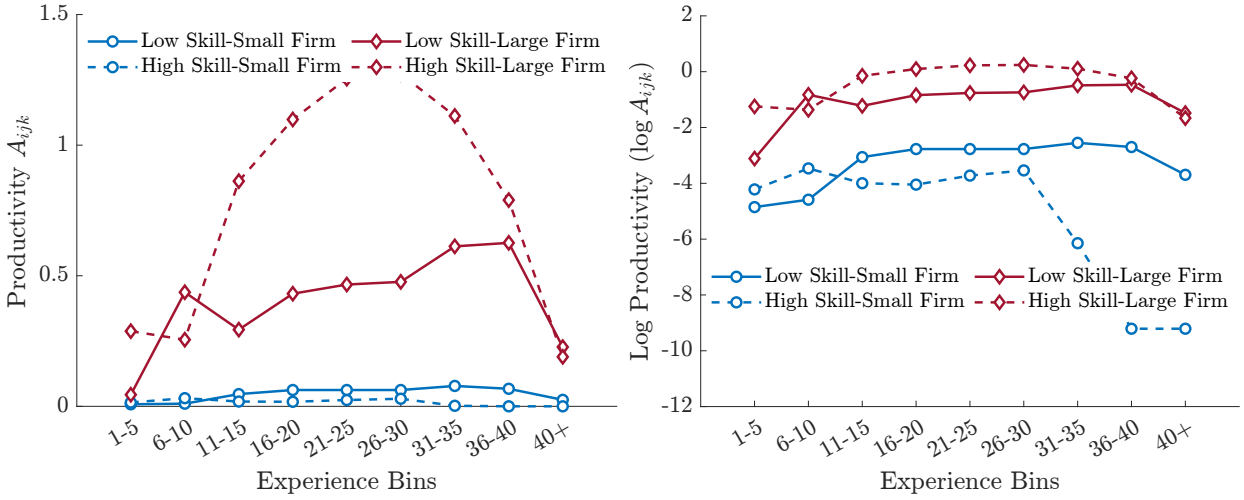
$$\sqrt{M}\Big[(\sqrt{0.05} * 9.54 + \sqrt{0.15} * 10.44) - \sqrt{0.1} * (9.54 + 10.44)\Big] = \sqrt{M}(6.1766 - 6.3182) < 0.$$

Due to the decreasing returns to scale in the production function, the total output of workers with human capital level i actually shrinks, even though the overall labor force becomes more experienced. The aggregated output across all human capital bins also decrease, if the above example holds for sufficiently many human capital bins.

C Quantitative Exercises

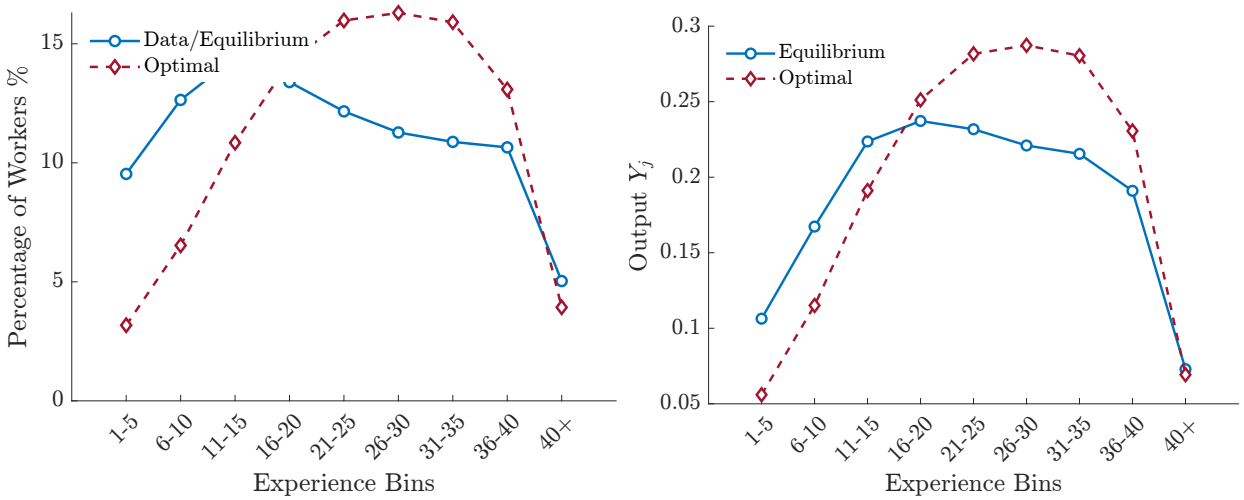
C.1 Robustness Checks in the Static Model

Figure C.1: Estimated Labor Productivities and Optimal Labor Composition $\xi = 0.5$



(a) Worker-Firm Labor Productivity

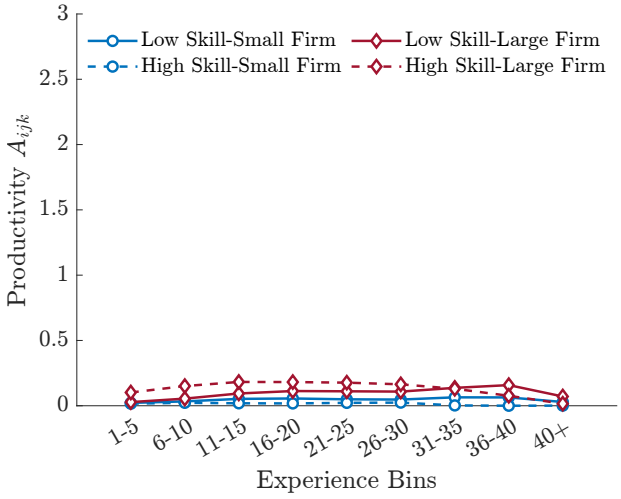
(b) Log Worker-Firm Labor Productivity



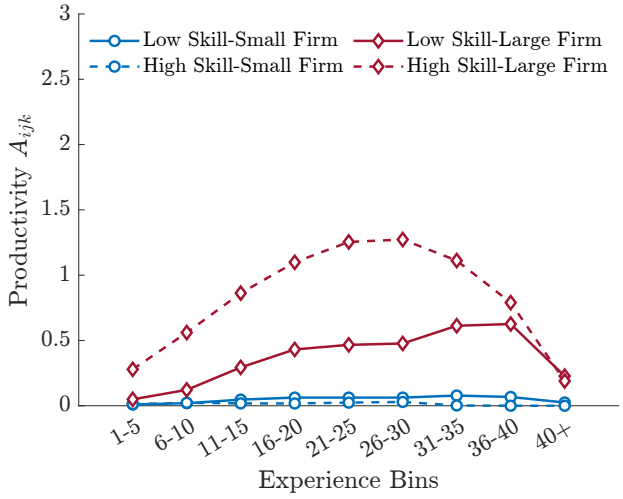
(c) Labor Supply

(d) Output

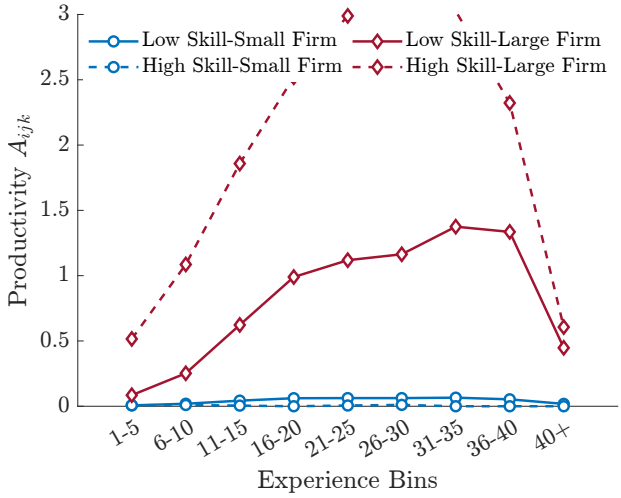
Figure C.2: Worker-Firm Productivity Changing σ



(a) $\sigma = 1$: Productivity A_{ijk}

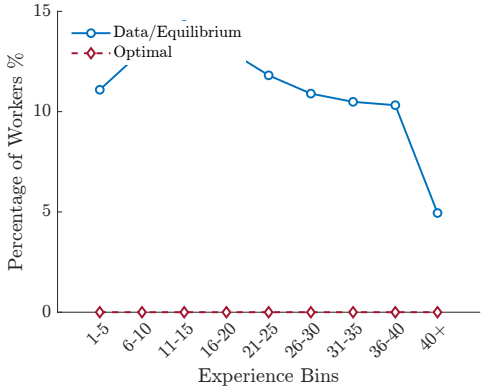


(b) $\sigma = 2.5$: Productivity A_{ijk}

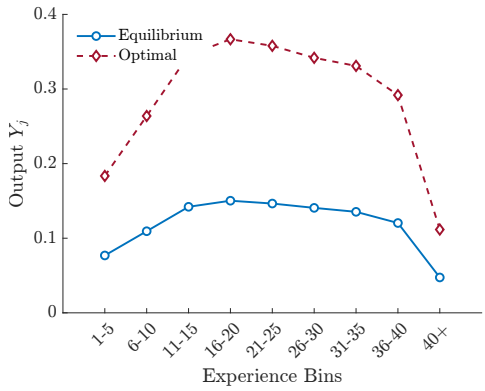


(c) $\sigma = 3$: Productivity A_{ijk}

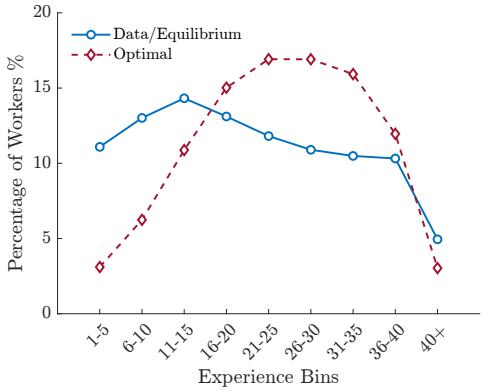
Figure C.3: Optimal Labor Composition and Output Changing σ



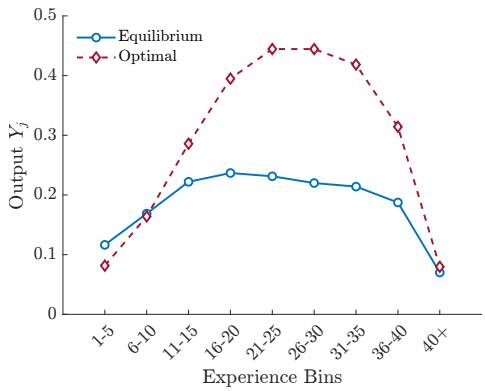
(a) $\sigma = 1$: Labor Supply L_j



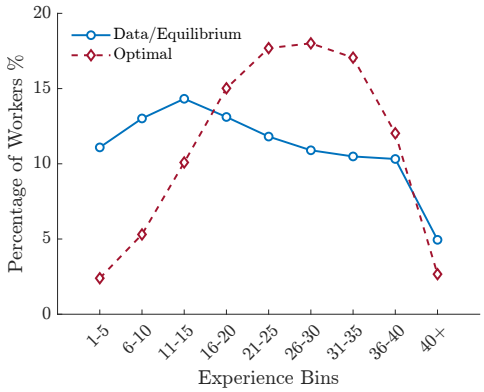
(b) $\sigma = 1$: Output



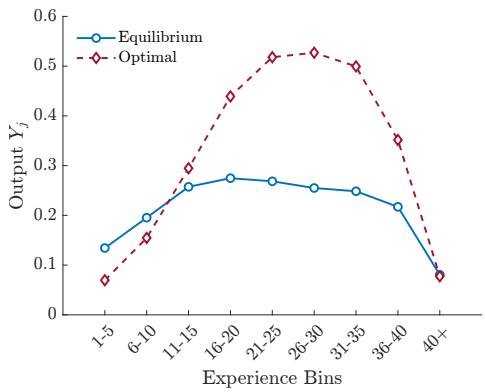
(c) $\sigma = 2.5$: Labor Supply L_j



(d) $\sigma = 2.5$: Output

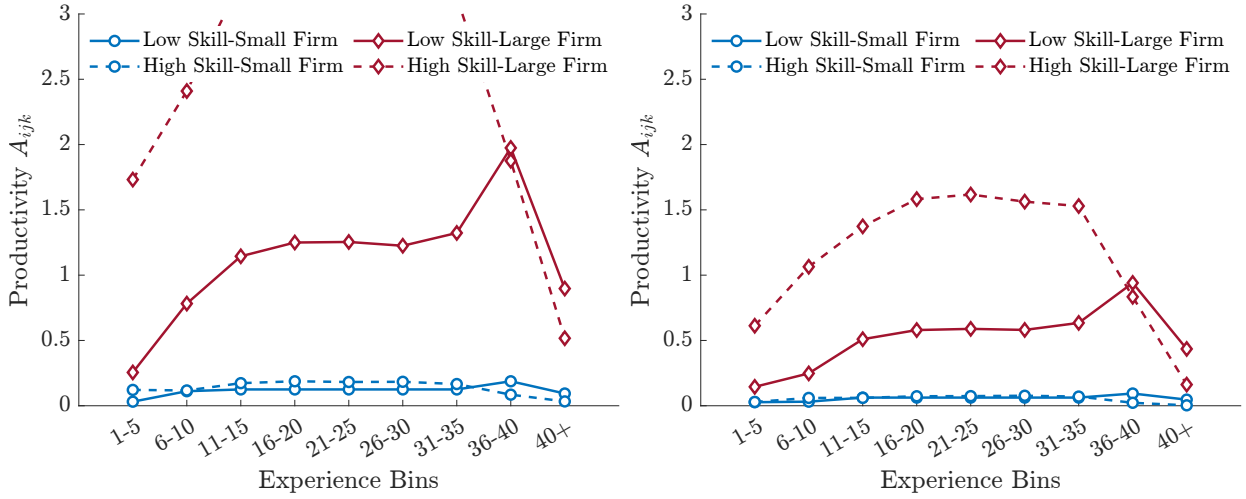


(e) $\sigma = 3$: Labor Supply L_j



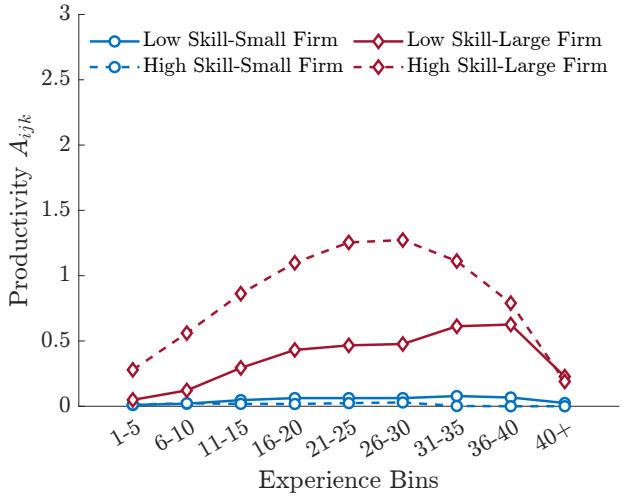
(f) $\sigma = 3$: Output

Figure C.4: Worker-Firm Productivity Changing α

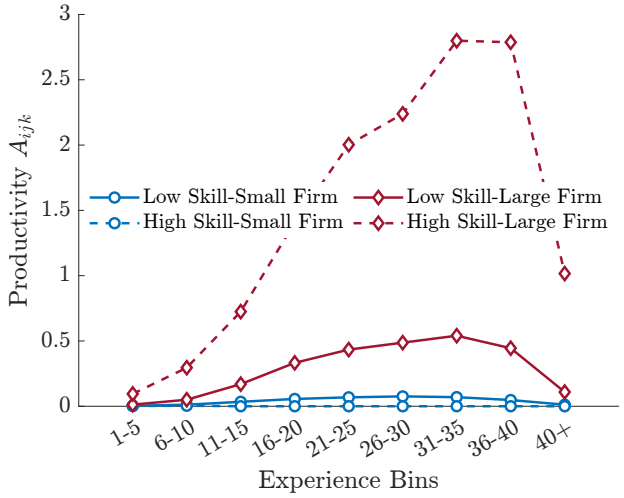


(a) $\alpha = 0.1$: Productivity A_{ijk}

(b) $\alpha = 0.3$: Productivity A_{ijk}

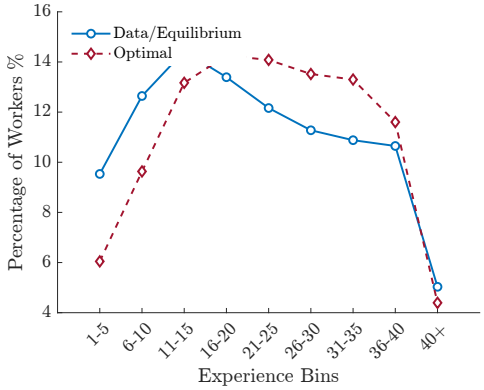


(c) $\alpha = 0.6$: Productivity A_{ijk}

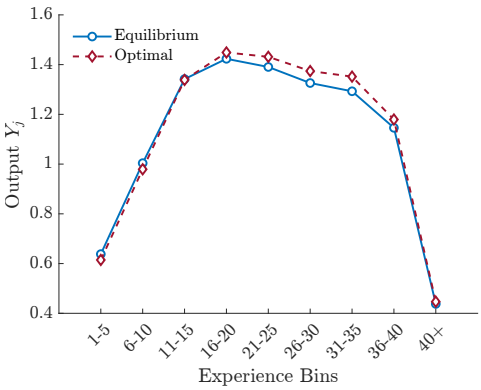


(d) $\alpha = 0.8$: Productivity A_{ijk}

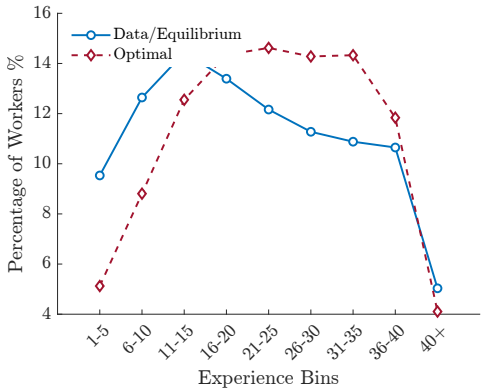
Figure C.5: Optimal Labor Composition Changing α



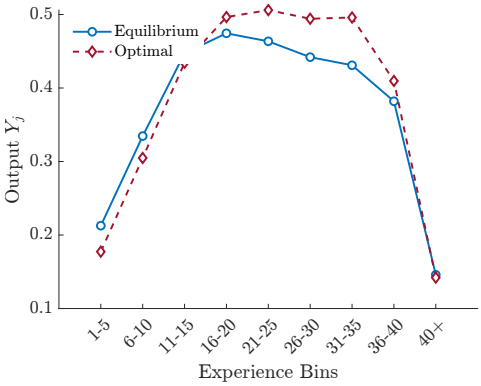
(a) $\alpha = 0.1$: Labor Supply L_j



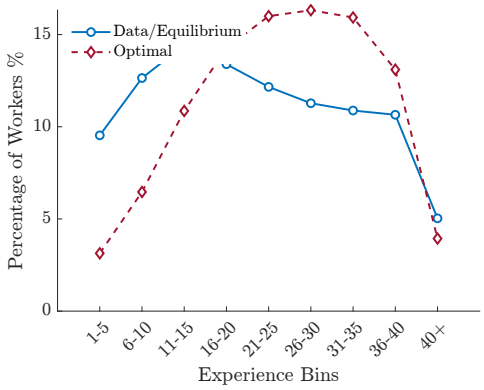
(b) $\alpha = 0.1$: Output



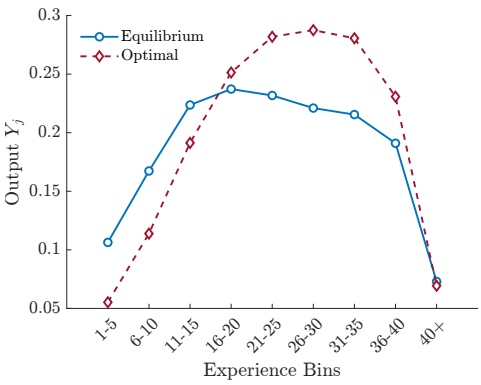
(c) $\alpha = 0.3$: Labor Supply L_j



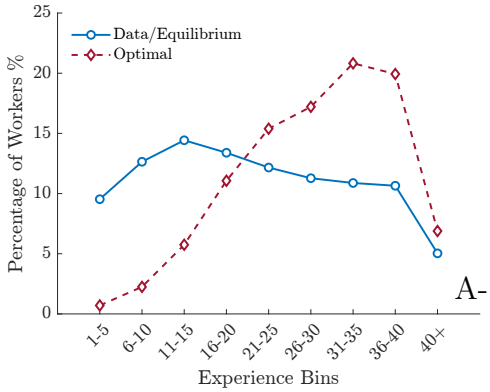
(d) $\alpha = 0.3$: Output



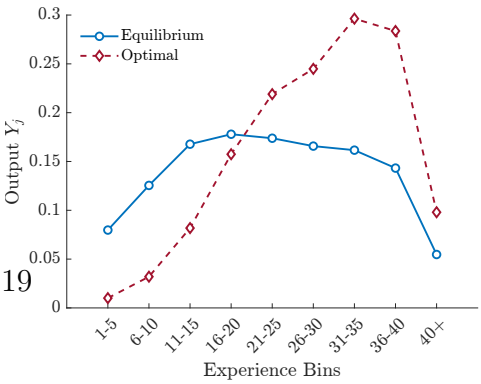
(e) $\alpha = 0.6$: Labor Supply L_j



(f) $\alpha = 0.6$: Output



(g) $\alpha = 0.8$: Labor Supply L_j



(h) $\alpha = 0.8$: Output

C.2 Algorithm of Solving the Equilibrium

In this section, we describe the algorithm we use to solve the dynamic model from Section 3.5.

In our setting, two special experience bins do not require employment information in two periods when solving for the current employment choice. First, the firm does not pay the firing cost when choosing employment of workers with the least experience, as there is no employment stock for this group of workers. Second, for bins with the most experience (i.e., the oldest workers), the firm does not need to take into account the firing cost it would have to pay next period as they all retire by the end of the period. Therefore, we can solve the employment choices for the youngest workers first and proceed to solve the employment choices *sequentially* from experience group x_j to experience group x_{j+1} where $j \in [1, J-1]$. Other than the wage schedule, key equilibrium objects needed for solving the employment choices are the expected firing probabilities of a worker with h_i and x_j hired this period (i.e., $\mathbb{E}\big[\mathbb{1}[\tilde{L}_{i,j+1} > l_{i,j+1}]\big]$ and $\mathbb{E}\big[\mathbb{1}[\tilde{L}_{i+1,j+1} > l_{i+1,j+1}]\big]$).

Practically, we solve the labor market equilibrium using the following steps.

1. First, we solve the model without the firing cost and obtain the wage schedule $w^{no}_{i,j}$ and firms' employment choices, $l^{no}(z)_{ij}$ ($j \in [1, 2, 3, ..., J]$), and employment, stocks $\tilde{L}^{no}_{ij}$ ($j \in [2, 3, ..., J]$). Based on these solved equilibrium objects, we construct the expected firing costs that show up in the right hand side of equations (12) and (13) denoted by FC^0_{ij}.

2. Next, we solve for $w_{i,1}$ that equalizes the labor supply in the submarket ($g(h_i, x_1)$) to the aggregated labor demand across all active firms whose firm-level labor demand is denoted by $l_{i,1}(z)$. For this step, we assume each firm uses the expected firing cost constructed from the previous step. Having solved $w_{i,1}$, we calculate firms' employment choices of the youngest workers, $l_{i,1}(z)$, and their employment stocks of the second youngest experience bins $\tilde{L}_{i,2}(z)$.

3. Third, we solve for $w_{i,j}$ (where $j \in [2, T]$) that equalizes the labor supply in the submarket ($g(h_i, x_j)$) to the aggregated labor demand across all active firms *sequentially* (i.e., from experience group x_{j-1} to experience group x_j). Importantly, we use the *derived* $\tilde{L}_{i,j}(z)$ calculated from the employment choice made for the experience group x_{j-1}. For this step, we also assume each firm uses the expected firing cost constructed

from the first step. Note that when $j = T$, there is no need to calculate the expected firing cost as all workers retire at the end of experience x_J.

Using the above three steps, we derive the wage schedule, $w_{i,j}$, employment choices and stocks, $l_{i,j}(z)$ and $\tilde{L}_{i,j}(z)$, which lead to the newly calculated values of the expected firing cost:

$$\begin{aligned} FC^1_{ij}(z): \quad = \quad & \eta\tau\delta\Bigg[(1-p_i-q_i+2p_iq_i)w_{ij+1}\mathbb{E}\left[\mathbb{1}\left[\tilde{L}_{i,j+1} > l_{i,j+1}\right]\right] \\ & +p_i(1-q_i)w_{i+1,j+1}\mathbb{E}\left[\mathbb{1}\left[\tilde{L}_{i+1,j+1} > l_{i+1,j+1}\right]\right] + q_i(1-p_i)w_{i-1,j+1}\mathbb{E}\left[\mathbb{1}\left[\tilde{L}_{i-1,j+1} > l_{i-1,j+1}\right]\right]\Bigg], \end{aligned}$$

where $w_{i,j}$, $l_{i,j}(z)$ and $\tilde{L}_{i,j}(z)$ are derived in the second and third steps. In the final step, we update the expected firing cost by taking a weighted average of the assumed expected firing cost in previous steps and newly calculated expected firing cost:

$$FC^2_{ij} = \zeta FC^1_{ij} + (1-\zeta)FC^0_{ij},$$

where $\zeta \in (0,1)$ is the adjustment factor. We then iterate our procedure by using FC^2_{ij} to replace FC^0_{ij} and redoing steps two to four. We stop until the newly calculated wage schedule and employment choices are the same as the ones calculated from the previous iteration.[38]

[38] In such a case, the newly calculated expected firing cost is also the same as the one calculated from the previous iteration.