Carrot and Stick Zoning*

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Abstract

We construct a structural model describing the optimal timing and intensity of capital investment in response to fiscal and regulatory policies. Our model focuses on real estate development and incorporates a government that can simultaneously relax land-use restrictions, offer property tax exemptions, and mandate public good contributions within developments. Using New York City's affordable housing mandate to calibrate our model, we show that both regulatory and fiscal incentives, with or without affordability mandate, can stimulate housing production, though through different mechanisms. Tax exemptions accelerate development by raising the probability of development on close-to-redevelopment parcels, whereas density bonuses increase capital intensity, specifically on high-quality parcels. We also quantify the public costs associated with each policy, highlighting the trade-offs between expected tax revenue losses and infrastructure costs to support increased density.

JEL classification: H4, H7, R1, R3, R5
Keywords: Capital investment under uncertainty, Land-use regulations, Tax credits, Real estate development, Incentive zoning, Affordable housing, Public-private partnership

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Housing affordability is currently one of the most salient social challenges, particularly in large and economically successful cities (Albouy et al., 2016; Gyourko et al., 2013). Many national and local governments have responded to this problem by enacting policies designed to stimulate housing production and expand affordable housing supply (Favilukis et al., 2023; Saiz, 2023). What complicates the decision of what policies to implement in tandem with one another is the durable nature of real estate development, as this durability implies a lasting impact on the urban environment (Glaeser and Gyourko, 2018), and the economy (Duranton and Puga, 2023; Hsieh and Moretti, 2019). To evaluate these policies, it is thus imperative to anticipate developers' choices regarding the optimal timing and intensity of investment. In this study, we construct and use such a forward-looking model to evaluate how public policies that combine regulatory and fiscal incentives influence real estate investment decisions and, in turn, the built environment.

To construct our model, we build on classic models of irreversible investment under uncertainty (Capozza and Li, 1994; McDonald and Siegel, 1986; Paddock et al., 1988). In our model, a real estate developer inherits a developed parcel that generates returns based on its location and existing building attributes. Each parcel is also subject to a specific land-use regulatory regime, whereby each regime determines the land use that, in turn, defines the return, the volatility, and the maximum allowable density of development. Developers start construction at the time and intensity that maximizes the present value of their property subject to realistic demolition and non-instantaneous construction costs. However, this optimal decision rule depends on the regulatory environment. In line with recently enacted policies, we assume that governments can offer property tax exemptions and density bonuses (*Carrots*) to compensate the developer for in-kind provisions of public goods (*Stick*). These public goods can take several forms, including infrastructure, environmental improvements, or social housing provisions.[1] We solve our model analytically, characterizing expressions for parcel value, the probability of redevelopment at different time horizons, and optimal density under plausible policy combination scenarios.

We calibrate our model using data from the New York City (NYC) real estate market, with a focus on the affordable housing mandate. First, we parametrize the return and cost to density such that they both increase with height, reflecting additional utility or production ef-

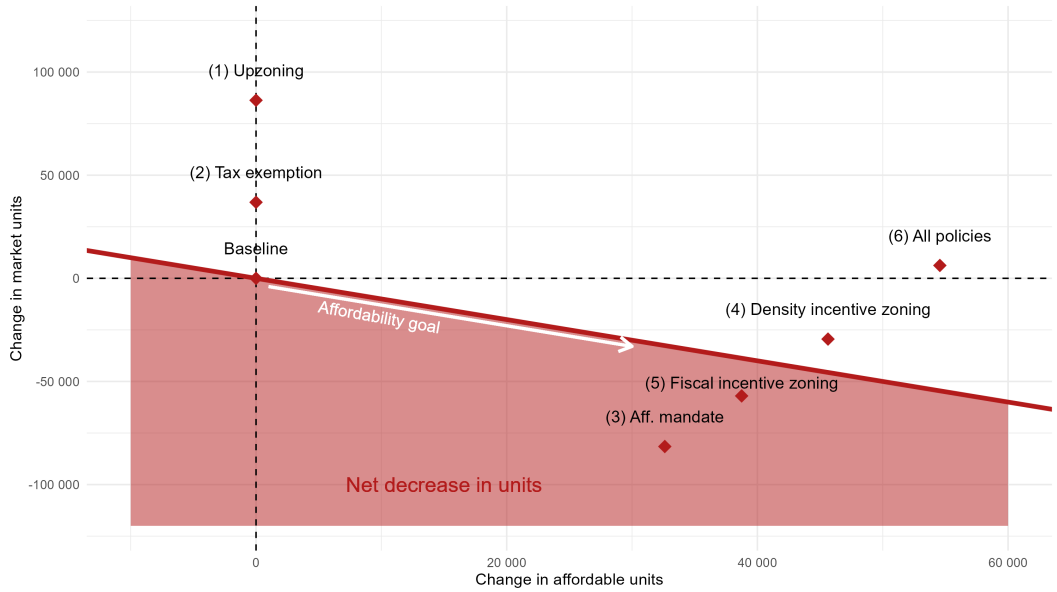
[1]This ability of governments to recapture the value increases resulting from public actions, often referred to as "land value capture" in an urban context (Lincoln Institute of Land Policy, 2022), is a central concept in the current urban policy debate. According to Homsy and Kang (2023), 41% of cities in the United States use some form of incentive zoning. Canadian cities also commonly use development transfers as incentives for obtaining privately provided public amenities (Moore, 2013). The German federal legislation obligates developers to compensate for land development by protecting natural areas (Druckenbrod and Beckmann, 2018). Similar policies are expected to be implemented across all European countries under the "No Net Land Take by 2050" policy.

ficiency at higher floors (Ahlfeldt and Barr, 2022; Danton and Himbert, 2018; Liu et al., 2018), and convex construction costs (Ahlfeldt et al., 2023; Ahlfeldt and McMillen, 2018; Eriksen and Orlando, 2022). We then calibrate the elasticities with respect to the share of affordable units in a building to reduced form estimates. We find that a mandate increase from 10 to 20% reduces property values by 4.36% but also lowers construction costs by 2.03%. Next, we estimate market-specific parameters within each NYC neighborhood. To quantify property values and returns, we combine Census data on rents and historical property return data from Costar. We then match our model's expected housing production to observed housing construction in each neighborhood to estimate implied volatility levels that best align with actual development patterns, following the standard implied volatility estimation for financial options (Black and Scholes, 1973; Dumas et al., 1998). Finally, to account for parcel heterogeneity, we calibrate each NYC parcel with its current density, maximum allowable density, estimated obsolescence, and unobserved time-invariant quality, which captures locational advantages such as proximity to transit or amenities.

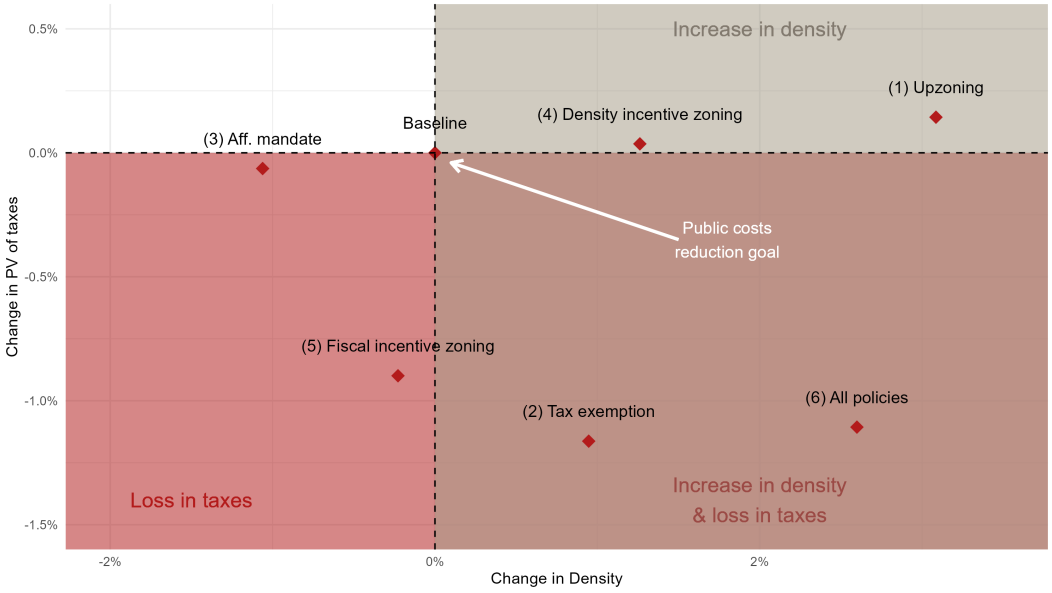
Before analyzing policy counterfactuals, we assess how our baseline calibration predicts key development outcomes, including the probability and intensity of redevelopment for each parcel, the expected number of housing units constructed in each neighborhood, and the projected future density and change in property tax revenues. Our baseline estimates of 211,000 forecasted units over a decade closely align with the observed construction trend. We observe that development is expected to concentrate in high-demand areas, particularly western Brooklyn and Queens. The structural nature of our model explicitly captures the endogenous relation between property values and zoning policies–a link that reduced-form estimates often struggle to capture due to well-documented issues of endogeneity (McMillen and McDonald, 1991; Shertzer et al., 2018). We observe that existing density, time-invariant quality, and obsolescence play a central role in shaping redevelopment incentives. To validate our model's fit, we compare our price predictions to transaction prices in Costar, finding a strong positive correlation.

We next use our model to examine how different combinations of public policies influence the built environment. Specifically, we evaluate three policy instruments and their interactions: a 20% affordable housing mandate, a 25% density bonus, and a 30-year property tax exemption. We summarize the results in Figure 1. Panel A shows that both upzoning (Policy [1]) and tax exemption (Policy [2]) policies increase overall housing supply relative to the baseline scenario. However, these policies operate through distinct channels. Tax exemptions stimulate development by increasing the probability of parcel redevelopment. In contrast, density bonuses raise the optimal density of new developments without necessarily

Figure 1: Summary of Policy Counterfactuals Benefits and Costs



A. Benefits – Housing Production



B. Costs – Public Expenses

Note: This chart summarizes the policy counterfactual results. Panel A shows the expected change in affordable (x-axis) and market-rate units (y-axis). Panel B shows the expected additional costs for the city, including the change in density (x-axis) and the change in expected tax revenue (y-axis). Policy (1) Upzoning corresponds to an increase in allowable density of 25%, Polity (2) Tax exemption corresponds to a 30-year property tax exemption, and Policy (3) corresponds to a 20% affordability mandate. Policies (4), (5), and (6) are combinations of these. All the outcomes are computed as the difference between the policy counterfactual results and the baseline scenario over a 10-year horizon.

accelerating the timing of redevelopment. Adding an affordability mandate to these policies (Policies [4] and [5]) unambiguously reduces the number of expected market-rate units. However, an increase in affordable units partially offsets this reduction. Notably, the overall housing supply increases when both incentives are offered together (Policy [6]), though this comes at a cost to municipal finances and infrastructure planning.

Panel B of Figure 1 underscores these potential costs. Tax exemptions (Policies [2] and [5]) significantly reduce the city's fiscal base, potentially constraining the funding of essential public services. In contrast, density bonuses (Policies [1] and [4]) lead to a higher expected density, requiring greater investment in transportation, utilities, and public amenities. Combining policies (Policy [6]) yields the largest supply increases but also the largest fiscal and infrastructure burdens. These results highlight the inherent trade-offs that local governments must balance when designing policies.

We finally explore the heterogeneous effects of these *Carrot & Stick* policies across different locations. We show that blanket policies produce heterogeneous outcomes across the city. For example, parcels closer to redevelopment thresholds respond more strongly to fiscal incentives, whereas high-quality locations benefit primarily from density bonuses. This result motivates our last exercise. To incorporate developers' incentive compatibility constraints, we allow them to opt into the new regulatory regime or remain under the status quo. Our findings indicate that *voluntary* adoption delivers limited affordable housing production compared to *mandatory* policies except when both *Carrots* are available to developers. This outcome stems from selective developer participation, where only the most viable projects opt in. We note that only 14.0% of parcels participate in *voluntary* Density or Fiscal Incentive Zoning. Adoption rises, however, to 38.6% when both density and fiscal incentives are bundled (Policy [6]). Aligning policy incentives with developer interests thus requires a high level of private benefits to warrant *voluntary* participation.

Our paper primarily contributes to two strands of literature. First, it contributes to the literature on the determinants of real estate supply (Baum-Snow and Han, 2024; Glaeser and Gyourko, 2018). Prior work emphasizes the role of market structure and productivity in explaining the slow growth of housing supply (D'Amico et al., 2024; Goolsbee and Syverson, 2023), or the advancement in construction technology in explaining the emergence of high-rise buildings (Ahlfeldt and Barr, 2022; Ahlfeldt et al., 2023; Ahlfeldt and McMillen, 2018). Another stream of literature focuses on the roles of physical and regulatory constraints to quantify the spatially heterogeneous supply elasticity (Albouy and Ehrlich, 2018; Brueckner and Singh, 2020; Gyourko et al., 2021; Saiz, 2010). Our study is more specifically related to

the literature encompassing forward-looking developers in dynamic models (Cunningham, 2006; Hsiao, 2023; Murphy, 2018; Peng, 2023).[2] Like these studies, we incorporate forward-looking agents who respond to market and regulatory conditions. However, the distinguishing feature of our model is that we explicitly incorporate the interaction between fiscal and regulatory incentives and include parcels that generate heterogeneous returns that depend on the built environment. Consequently, we provide a model that integrates private developer incentives with public policy goals, offering new insights into the optimal design of policies.

Second, our study complements our understanding of the effects of urban regulatory and fiscal policies (Gyourko and Molloy, 2015; Quigley and Rosenthal, 2005).[3] Previous studies have investigated the effects of various forms of supply-side policies including tax incentives in opportunity zones (Chen et al., 2023; Corinth and Feldman, 2024), upzoning (Büchler and Lutz, 2024; Freemark, 2020, 2023; Peng, 2023), affordable housing mandate (Favilukis et al., 2023), inclusionary housing mandate with density bonuses (Schuetz et al., 2009, 2011), or with fiscal incentives (Baum-Snow and Marion, 2009; Eriksen and Rosenthal, 2010; Soltas, 2023, 2022). Interestingly, Krimmel (2021) shows that the affordability constraint attached to upzoning in Seattle led to an increase in development outside of the designated areas, questioning the balance between incentives and constraints. Our paper provides a structural framework that can encompass combinations of fiscal and regulatory policies to perform counterfactual predictions.

Our study proceeds as follows. In Section, 1, we describe our structural model highlighting developers' decision rules and the city's policy instruments. In Section 2, we provide numerical analysis results for a representative parcel, describing the behavior of the model. In Section 3, we detail the empirical setting that we use to parametrize and calibrate the model. In Section 4, we show the baseline results of our models and the aggregate outcomes of interest. In Section 5, we evaluate counterfactual policy results. In Section 6, we conclude.

[2]Theoretically we contribute to the long history of real options models (McDonald and Siegel, 1986; Myers, 1977) used to characterize the real estate development decision (Arnott and Lewis, 1979; Capozza and Li, 1994; Grenadier, 1996; Quigg, 1993; Titman, 1985). These studies recognize that uncertainty acts as a deterrent to real estate development (Bulan et al., 2009), competition increases the exercise of the development option (Grenadier, 2002), and that the redeployment to another real estate use is also a valuable option (Benmelech et al., 2005; Büchler et al., 2023). Building specifically on the real options development framework of Lebret and Liu (2023), we extend this class of models for the evaluation of policy counterfactuals.

[3]Our study also relates to the literature on the causes of land-use restrictions (Clingermayer, 1993; Fischel, 2002; Helpman and Pines, 1977; Hilber and Robert-Nicoud, 2013; McMillen and McDonald, 2002; Stull, 1974) presenting an alternative rationale for implementing ex-ante land-use regulations. These regulations enable regulators to engage in subsequent *Carrot & Stick* negotiations with private stakeholders for the provision of public goods. Although urban planners and political scientists have investigated this mechanism (Freemark, 2020; Homsy and Kang, 2023; Kim, 2020; Korngold, 2022; Moore, 2013), no paper to our knowledge provides an economic analysis of this tool.

1 A Real Estate Development Model with Policy Instruments

In constructing our model, we explicitly recognize that government has regulatory power over urban parcels. For each parcel, the government considers transitioning from current zoning with regulatory environment $\mathcal{P}_0$ to a different regulatory environment $\mathcal{P}$. This new environment $\mathcal{P}$ that might include changes in land use, variation in density (ϕ), a mandate to include public goods on the site (α), and a tax exemption on completed buildings of τ years[4]. Because private developers factor each policy instrument into the timing and intensity of their building construction, we incorporate these policies into our structural model of real estate development. We first describe the developers' optimal decision rules and then lay out the mechanisms through which each policy instrument affects the built environment.

1.1 The Real Estate Developer Decision Rule

We build upon the stochastic financial models of Merton (1974) and Black and Cox (1976), with a particular focus on the timing of investment decisions as explored in Majd and Pindyck (1987). We assume that (i) the public and private sectors have access to complete information, (ii) properties are continuously traded within arbitrage-free and complete markets, (iii) both public and private sectors have access to borrowing and lending at a risk-free rate r, (iv) private developers aim to maximize wealth, and (v) the asset value remains independent of capital structure choices.[5]

We further assume that the market-rate value $(V_t^0)_{t\geq 0}$ expressed as per unit of density under zoning $\mathcal{P}_0$ is governed, under the risk-neutral measure $\mathbb{Q}$, by:

$$dV_t^0 = (r - \delta_0)V_t^0 d_t + \sigma_0 V_t^0 dW_t \tag{1}$$

where δ_0 and σ_0 are, respectively, the unlevered before-tax payout rate and the constant volatility under zoning $\mathcal{P}_0$, and $(W_t)_{t\geq 0}$ is a standard Brownian motion defined on the probability space $(\Omega, \mathcal{F}, \mathbb{Q})$. We thus allow the stochastic process describing real estate demand to vary by zoning. Zoning allows for a mixture of land uses on the same plot of land to reduce σ_0 through diversification. Denoting V as the value at $t = 0$, we can rewrite the value of a market rent per unit of density at $t \geq 0$ as $V_t^0 = Ve^{(r-\delta_0-\frac{\sigma_0^2}{2})t+\sigma_0 W_t} = Ve^{\sigma_0 Z_t^0}$, where $Z_t^0 = b_0 t + W_t$, and

[4] It follows that $\mathcal{P}_0 = \{\alpha = 0, \phi = 0, \tau = 0\}$ and zoning $\mathcal{P} \neq \mathcal{P}_0$.

[5] Implicitly, our model also assumes that the amount of supply in the market does not impact the asset value of the underlying property. Although this assumption is innocuous in the case of a price-taker individual developer, it might create bias when the model applies to the entire city in the sort of counterfactual analyses performed. We discuss in more detail this potential issue in Section 5.5.

$b_0 = \frac{1}{\sigma_0}(r - \delta_0 - \frac{\sigma_0^2}{2})$.

Before discussing development optionality, we discuss two adjustments to obtain the market value of a property that we denote $\pi_0(V, F_B, o_B)$. First, we assume the government imposes a tax proportional to the rental value of the developed properties. This tax effectively reduces the payout to the developer by ψ. Second, we define $\Gamma(F, o, \alpha)$, the function that scales the market value of a unit of density V_t, based on the characteristics of the built property, to yield the stochastic rental value of a developed property. In particular, this value depends on the property density (F), the property obsolescence rate (o) and the share of built space dedicated to public goods (α). For example, $\Gamma(1, 0, 0) = 1$ corresponds to a property with a density of 1 ($F_B = 1$), with no operational efficiency losses due to obsolescence ($o = 0$), and no mandate to include public goods on the site ($\alpha = 0$). We revert to this parametrization of $\Gamma(F, o, \alpha)$ in Section 3. Consequently, the value of the developed property without its development option is

$$\pi_0(V, F_B; \Theta) = \mathbf{E}_{\mathbb{Q}} \left\{ \int_0^{\infty} e^{-ru} \left(\delta_0 - \psi\right) \Gamma(F_B, o_B, 0) V_u^0 \mathrm{d}u \right\} \tag{2}$$

where Θ is the vector of parameters.

Development Decision Under Current Regulatory Environment Private developers have the option to replace the current building with a new one constructed at density $F \leq \bar{F}_0$. Exercising this option entails an instantaneous demolition cost $\gamma(F_B)$, and an annual construction cost during the construction period of d years, denoted as $\kappa(F, \alpha)$. Construction costs depend on the density of the new construction F, and the type of asset built.

Developers exercise the option to expand their property under $\mathcal{P}_0$ only when favorable market conditions prevail. We denote V_0^e, the predetermined level (i.e., expansion barrier) at which they initiate construction of a building of height F, and $T_{V_0^e}$, the time this barrier is reached for the first time. Hence, to complete the property development at density F, developers need a robust market that persists for d years. We denote $H^+_{V_0^e,d}$, the *Parisian* time at which the property is fully developed.[6]

Given the financial market assumptions, the two fundamental theorems of asset pricing ensure the existence of a unique probability measure $\mathbb{Q}$, under which the price of any asset, discounted at the risk-free rate, behaves as a martingale. As a result, we can express the value

[6]Formally these stopping times are defined as $T_{V_0^e}(W) = \inf\{t \geq 0 : W_t = V_0^e\}$, and $H^+_{V_0^e,d}(W) = \inf\{t \geq 0 : (t - g_t^{V_0^e}(W)) \geq d, W_t \geq V_0^e\}$ with $g_t^{V_0^e}(W) = \sup\{s \leq t : W_s = V_0^e\}$ (Chesney et al., 1997).

of the property under $\mathcal{P}_0$ inclusive of the development option as follows:

$$\pi_0(V, F_B, F; \Theta) = \underbrace{\mathbf{E}_{\mathbb{Q}}\left\{ \int_0^{T_{V_0^e}} e^{-ru}\ (\delta_0 - \psi)\ \Gamma(F_B, o_B, 0) V_u^0 \mathbf{1}_{V_u^0 < V_0^e}\, \mathrm{d}u \right\}}_{\text{Value before development}} - \underbrace{\mathbf{E}_{\mathbb{Q}}\left\{ e^{-rT_{V_0^e}} \gamma(F_B) \right\}}_{\text{Demolition costs}}$$
$$- \underbrace{\mathbf{E}_{\mathbb{Q}}\left\{ \int_0^{H^+_{V_0^e,d}} e^{-ru} \kappa(F, \alpha) \mathbf{1}_{V_u^0 > V_0^e}\, \mathrm{d}u \right\}}_{\text{Development costs}} + \underbrace{\mathbf{E}_{\mathbb{Q}}\left\{ \int_{H^+_{V_0^e,d}}^{\infty} e^{-ru}\, (\delta_0 - \psi)\, \Gamma(F, 0, \alpha) V_u^0 \mathrm{d}u \right\}}_{\text{Value after development}} \quad (3)$$

where $\mathbf{1}_x$ represents the indicator function for the event x. Equation (3) consists of the sum of four present-value terms. These sequential terms represent the cash flows from the existing building until construction starts, demolition cost, costs incurred during the property's construction period, and the cash flows from the new development.

Adding Policy Instruments Into the Development Decision Rule We model three distinct policy instruments that local governments can use independently or together. First, the government can mandate that the developer allocate α percent of the development to the public. These public goods have a payout rate δ_α, constant volatility of asset return σ_α, and covariance with the market's rent process of $\sigma_{\alpha,0}$. Increasing α also impacts the value of the property and the construction cost through $\Gamma(.)$ and $\kappa(.)$, respectively. The market value per unit of density under the new regulatory regime $\mathcal{P}$ is thus governed by the process

$$dV_t = (r - \delta)\, V_t d_t + \sigma V_t dW_t \quad (4)$$

where we assume that $\delta = \alpha\delta_\alpha + (1-\alpha)\delta_0$ and $\sigma^2 = \alpha^2\sigma_\alpha^2 + (1-\alpha)^2\sigma_0^2 + 2\alpha(1-\alpha)\sigma_{\alpha,0}$.[7] Second, the government can provide additional density bonuses allowing an additional height ϕ, so the developer chooses F provided that $F \leq \bar{F}$ where $\bar{F} = \bar{F}_0(1+\phi)$. Finally, developers can receive tax exemptions for τ years once construction is completed. The policy instrument vector is thus $\mathcal{P} = \{\alpha, \phi, \tau\}$.

[7] The sign and magnitude of the covariance $\sigma_{\alpha,0}$ give rise to several scenarios. If $\sigma_{\alpha,0} < 0$, the public good provides risk reduction through diversification benefits, potentially enhancing the property's value due to *within-building* positive externalities. This scenario could occur, for instance, with investments in infrastructure that improve access to public transportation. Conversely, if $\sigma_{\alpha,0} > 0$, the public good amplifies risk, which may negatively impact the property's value. An example of this could be the introduction of affordable housing, where well-documented *within-building* negative externalities, such as NIMBY (Not In My Backyard) concerns, may decrease property values.

The value of a property under the new regulatory environment is:

$$\pi(V, F_B, F, \alpha, \phi, \tau; \Theta) = \underbrace{\mathbf{E}_{\mathbb{Q}}\left\{ \int_0^{T_{V^e}} e^{-ru}\ (\delta_0 - \psi)\ \Gamma(F_B, o_B, 0) V_u\, \mathbf{1}_{V_u < V^e}\, \mathrm{d}u \right\}}_{\text{Value before completed development}}$$
$$- \underbrace{\mathbf{E}_{\mathbb{Q}}\left\{ e^{-rT_{V^e}} \gamma(F_B) \right\}}_{\text{Demolition costs}} - \underbrace{\mathbf{E}_{\mathbb{Q}}\left\{ \int_0^{H^+_{V^e,d}} e^{-ru} \kappa(F, \alpha)\, \mathbf{1}_{V_u > V^e}\, \mathrm{d}u \right\}}_{\text{Development costs}}$$
$$+ \underbrace{\mathbf{E}_{\mathbb{Q}}\left\{ \int_{H^+_{V^e,d}}^{\infty} e^{-ru} \delta\, \Gamma(F, 0, \alpha) V_u\, \mathrm{d}u \right\}}_{\text{Value after development gross of tax}} - \underbrace{\mathbf{E}_{\mathbb{Q}}\left\{ \int_{H^+_{V^e,d}+\tau}^{\infty} e^{-ru} \psi\, \Gamma(F, 0, \alpha) V_u\, \mathrm{d}u \right\}}_{\text{Property taxes after } \tau \text{ years of exemption}} \quad (5)$$

Similarly to Equation (3), the property value in $\mathcal{P}$ consists of the sum of present values with an additional final term that captures the impact of postponing property taxes for τ years after the development has been completed.

1.2 Analytical Solutions

To derive analytical solutions to (5), we modify the value of the property prior to the completion of development (first term in Equation [5]) as follows

$$\mathbf{E}_{\mathbb{Q}}\left\{ \int_0^{T_{V^e}} e^{-ru}\ (\delta_0 - \psi)\ \Gamma(F_B, o_B, 0)\, V\, \mathbf{1}_{V_u < V^e} \mathrm{d}u \right\}$$

assuming that the rental income $(\delta_0 - \psi)\ \Gamma(F_B, o_B, 0)\, V$ is constant until the demolition.

Proposition 1 *In the presence of the expansion threshold V^e, the property value $\pi(.)$ satisfies*

$$\pi(V, V^e, F_B, F, \alpha, \phi, \tau; \Theta) =$$
$$\underbrace{\Gamma(F_B, o_B, 0) \frac{(\delta_0 - \psi)\ V}{r} \left[1 + \left[\frac{r}{\lambda(b - \lambda)} - 1 \right] \left(\frac{V^e}{V} \right)^{\xi} \right]}_{\textit{Value before completed development}} - \underbrace{\gamma(F_B) \left(\frac{V^e}{V} \right)^{\xi}}_{\textit{Demolition costs}}$$
$$- \underbrace{\frac{\kappa(F, \alpha)}{r} \Big(B(d) - A(d) \Big) \left(\frac{V^e}{V} \right)^{\xi}}_{\textit{Development costs}} + \underbrace{\Gamma(F, 0, \alpha) V^e C(d) \left(\frac{V^e}{V} \right)^{\xi}}_{\textit{Value after development}} - \underbrace{\Gamma(F, 0, \alpha) \frac{\psi h}{\delta} V^e C(d) \left(\frac{V^e}{V} \right)^{\xi}}_{\textit{Property taxes after } \tau \textit{ years of tax credits}} \quad (6)$$

with $b = (1/\sigma)(r - \delta - \sigma^2/2)$, $\lambda = \sqrt{2r + b^2}$, $h = e^{-\left[(b+\sigma)^2+\lambda^2\right]\frac{\tau}{2}}$ *and* $\xi = (1/\sigma)(b - \lambda)$ *and where also*

$$A(d) = \frac{\Phi(b\sqrt{d})}{\Phi(\lambda\sqrt{d})},\ B(d) = \frac{b+\lambda}{2\lambda} - \frac{b-\lambda}{2\lambda}\frac{\Phi(-\lambda\sqrt{d})}{\Phi(\lambda\sqrt{d})},\ \textit{and} \quad C(d) = \frac{\Phi\left((\sigma+b)\sqrt{d}\right)}{\Phi(\lambda\sqrt{d})}$$

with $\Phi(x) = \int_0^{+\infty} ze^{zx-\frac{z^2}{2}}\,\mathrm{d}z = 1 + x\sqrt{2\pi}e^{\frac{x^2}{2}}\mathcal{N}(x)$, $\mathcal{N}(.)$ *being the standard normal cumulative distribution function.*

The proof is in Appendix B.1.

Although explicitly controlled by developers, the timing of development is endogenous to the vector of policy choices $\mathcal{P}$. Conditional on the current market and policy conditions, developers determine the most favorable time to initiate construction based on the stochastic process reaching a specific threshold V^{e*}.

Proposition 2 *When developers select the optimal timing for development under regulatory regime* $\mathcal{P}$*, the expansion threshold is given by*

$$V^{e*} = \frac{\xi}{1+\xi}\frac{1}{\vartheta}\left[\gamma(F_B) + \frac{\kappa(F,\alpha)}{r}\Big(B(d) - A(d)\Big) - \Gamma(F_B, o_B, 0)\frac{\delta_0 - \psi}{r}V\left[\frac{r}{\lambda(b-\lambda)} - 1\right]\right] \tag{7}$$

with $\vartheta = \Gamma(F, 0, \alpha)\left(1 - \frac{\psi h}{\delta}\right)C(d)$.

The proof is in Appendix B.2.

From Proposition 2, we note that as the expansion barrier increases as a result of rising construction and demolition costs, developers delay development. Recalling that $T^{V^{e*}}(V) = \inf\{t \geq 0 | V_t = V^{e*}\}$ represents the first time the value of an unlevered unit of density reaches the expansion threshold, we can derive the probability that development begins before time t.

Proposition 3 *Given the expansion threshold* V^{e*}*, the probability that development starts before time* t *under regulatory regime* $\mathcal{P}$ *satisfies*

$$\begin{aligned}\mathbf{P}\left(T^{V^{e*}} \leq t\right) &= \int_0^t \mathbf{P}\left(T^{V^{e*}} \in \mathrm{d}s\right) \\ &= \int_0^t \frac{a}{\sqrt{2\pi s^3}}\exp\left(-\frac{a^2}{2s}\right)\mathrm{d}s \end{aligned} \tag{8}$$

where $a = \ln(V^{e*}/V)/\sigma$ *and* $\mathbf{P}(.)$ *is the probability under the new measure* $\mathbb{P}$*, such that* $(Z_t)_{t\geq 0}$[8] *becomes a Brownian motion under* $\mathbb{P}$*.*

The proof is in Appendix B.3

1.3 Optimal Density

In the analytical solutions presented above, developers chose an optimal timing of construction conditional on construction density $F \leq \bar{F}$. To numerically derive the optimal density F^*, we maximize the value of the property subject to the maximum allowable density and optimal barrier V^{e*}:

$$\begin{aligned} F^* = \arg\max_{F} \quad & \pi(V, V^{e*}, F_B, F, \alpha, \phi, \tau; \Theta) \\ \text{s.t.} \quad & 0 \leq F \leq \bar{F} \end{aligned} \tag{9}$$

This numerical optimization allows us to derive the property value $\pi^*(.)$, the optimal barrier V^{e**}, and the development probability $\mathbf{P}^*$ that corresponds to the developer's decision rule considering both the optimal timing and intensity of development.

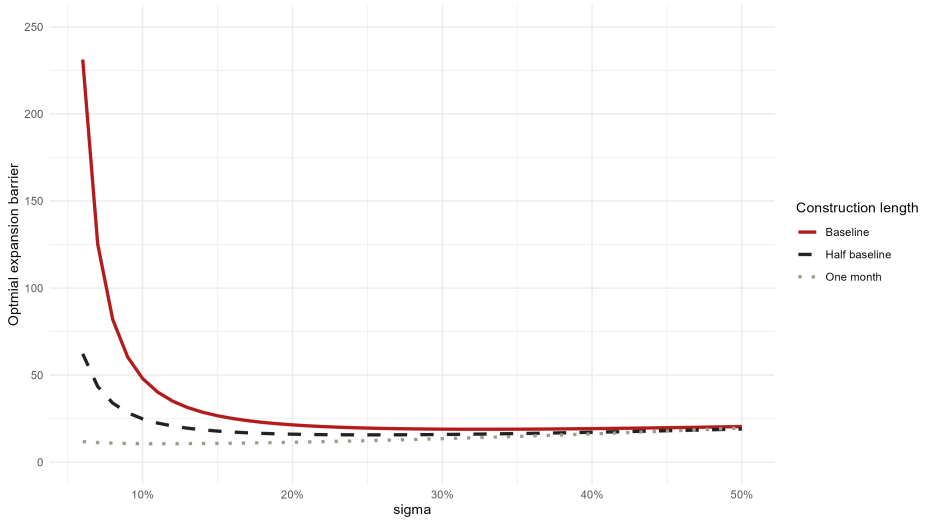
The general solutions in Equations (6), (7) and (8) combined with the maximization function over density F in Equation (9) provide expressions for the property value and the probability of development under a flexible policy framework. We can now evaluate all policy outcomes embedded in this setup. For example, we can derive the property value and development probability under commonly used regulatory environments such as (1) current zoning $\mathcal{P}_0 = \{\alpha = 0, \phi = 0, \tau = 0\}$, (2) upzoning $\mathcal{P} = \{\alpha = 0, \phi > 0, \tau = 0\}$, or (3) mandatory inclusionary programs with density bonuses $\mathcal{P} = \{\alpha > 0, \phi > 0, \tau = 0\}$, or with tax incentives $\mathcal{P} = \{\alpha > 0, \phi = 0, \tau > 0\}$. In addition to these three policy instruments at the core of our study, we can also study changes from one land use (e.g., office) to another land use (e.g., residential), if we modify the underlying stochastic process as well as the functions $\Gamma()$ and $\kappa()$ in addition to the other changes implied by a change of $\mathcal{P}$.

1.4 Importance of Non-instantaneous Construction Duration

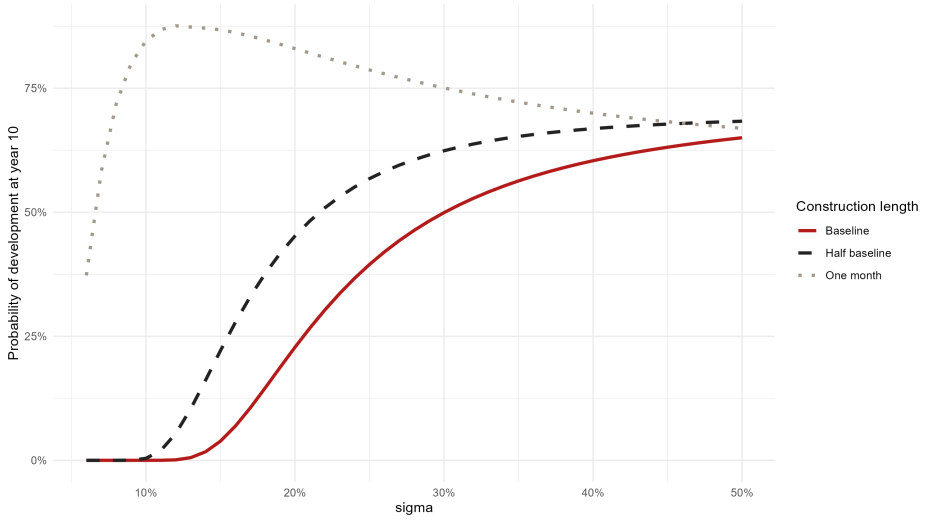
The impact of uncertainty on investment behavior changes fundamentally when construction is non-instantaneous. In the classical real options framework with instantaneous investment, uncertainty always delays investment by raising the expansion barrier. Figure 2, Panel A, confirms this result for short construction durations ($d \to 0$), where greater uncertainty increases V^{e*}. Developers require a higher expected return before initiating construc-

[8] $Z_t = bt + W_t$

Figure 2: Uncertainty, Construction Length, and Probability of Development



A. Optimal Expansion Barrier - V^{e*}



B. Probability of Development - $\mathbf{P}\left(T^{V^{e*}} \leq 10\right)$

Note: This chart shows the effects of varying the underlying volatility σ_i on the optimal expansion barrier (panel A) and the probability of development within 10 years (panel B). Each line shows this effect for different assumptions regarding the length of construction (baseline: $d = 2.64$ years. These figures are obtained for a parcel of land built at 25% of its capacity with $F_B = 2$ and $\bar{F} = 8$, with a rental return of $\delta_0 = .05$, and an obsolescence rate of 50% ($o_B = 0.5$). The assumption of the stochastic processes as follows: $\delta = 0.10$, $\sigma = 0.25$, $\delta_p = 0.09$, and $\sigma_p = 0.125$. All other parameters and elasticities are the baseline case shown in Table 2.

tion. However, the effect of uncertainty is non-monotonic in low-volatility markets: greater uncertainty lowers the expansion barrier, allowing earlier investment. This divergence from Capozza and Li (1994) occurs because, with a longer construction period, developers consider not only when to invest but whether conditions remain favorable throughout construction.

Figure 2, Panel B, shows a second key contrast: the probability of development rises with uncertainty as in classical models, but the threshold effect is stronger when d is long. For short d, higher uncertainty steadily increases the development probability because future prices are more likely to reach profitable levels. For long d, developers hesitate at moderate uncertainty levels waiting for clearer signals to occur. Once uncertainty is high enough, however, the probability of investment rises sharply. This pattern differs from the *incremental* investment framework in Bar-Ilan and Strange (1999), where uncertainty lowers investment intensity at every stage by discouraging the addition of capital. Unlike *lumpy* investment, where uncertainty affects the timing and size of a one-time decision, in *incremental* investment, firms make many small decisions over time. Each of those decisions is negatively impacted by uncertainty. This continuous adjustment means that investment is always scaled down in response to uncertainty. With *lumpy and lengthy* investment, as is the case here, uncertainty alters the timing but does not reduce intensity once investment occurs. This analysis highlights a key limitation of models that assume instantaneous construction or incremental investments. Such models would tend to overestimate the likelihood of development, leading to inflated forecasts of new housing supply. Our model yields more realistic estimates of expected development outcomes, enhancing the accuracy of policy evaluations.

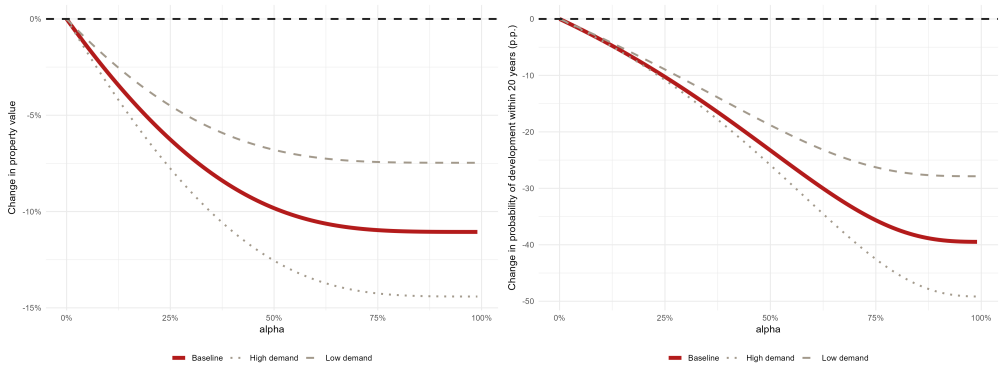
2 Parcel-level Numerical Analysis

As a necessary prelude to analyzing policies from the city perspective, we first provide evidence that our model captures the trade-off between carrots (ϕ and τ) and sticks (α) both in the valuation of the parcel and the probability of development. We use a representative parcel close to redevelopment to create those comparative statics that we relax below when incorporating heterogeneous city parcels. We use the parametrization and the calibration described in more detail in Section 3. In short, this calibration not only captures return and cost to density but also considers a mandate α that reduces the value, the construction cost, the return, and the return volatility

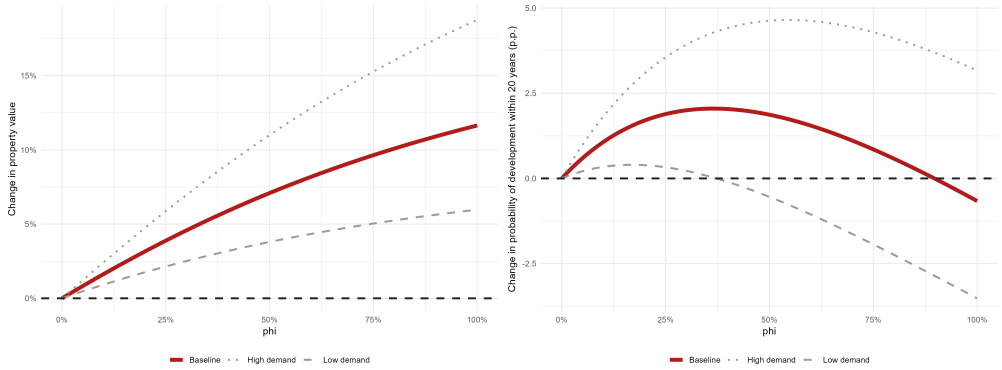
We first show the impact of each policy instrument on property values (left panels) and the probability of development within 10 years (right panels) in Figure 3. Figure 3 also illustrates how these effects vary when initial demand is higher or lower than the baseline. Figure A1 further separates the effects of the policy instruments on the property value by highlighting the effects on the value of the developed property and the value of the redevelopment option (i.e., the final four terms in Equation [6]).[9] Table 1 shows the sensitivity of these results

[9]Marginal effects with respect to other parameters are illustrated in Figure A2.

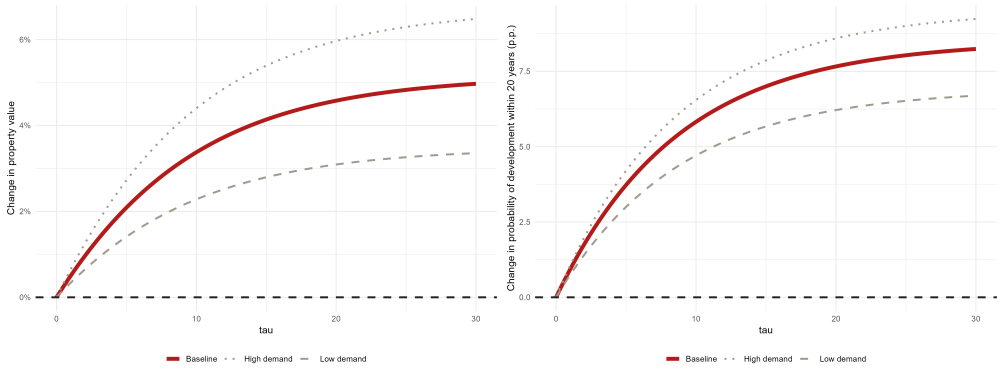
Figure 3: Property Value and Probability of Development with Respect to Policy Instruments



A. Mandatory Share for Public Goods (α)



B. Density Bonuses (ϕ)



C. Length of Tax Exemptions (τ)

Note: These figures show the effects of changing the policy instruments on the change in the property value (left panels) and the change in the probability of development starting within 10 years (right panels). These figures are obtained for a parcel of land built at 25% of its capacity with $F_B = 2$ and $\bar{F} = 8$, with a rental return of $\delta_0 = .05$, and an obsolescence rate of 50% ($o_B = 0.5$). The assumption of the stochastic processes as follows: $\delta = 0.10$, $\sigma = 0.25$, $\delta_p = 0.09$, and $\sigma_p = 0.125$. All other parameters and elasticities are the baseline case shown in Table 2.

with respect to key elasticities.

Table 1: Parcel Level Comparative Statics and Sensitivity Analyses

This table reports the change in property value and the change in the probability of development within 10 years with respect to the policy instruments. These figures are obtained for a parcel of land built at 25% of its capacity with $F_B = 2$ and $\bar{F} = 8$, with a rental return of $\delta_0 = .05$, and an obsolescence rate of 50% ($o_B = 0.5$). The assumption of the stochastic processes as follows: $\delta = 0.10$, $\sigma = 0.25$, $\delta_p = 0.09$, and $\sigma_p = 0.125$. All other parameters and elasticities are the baseline case shown in Table 2.

	Change in Property Value (%)	Change in Prob. Dev. (p.p.)
Mandating 20% of Affordable Housing ($\alpha = 0.20$)		
Baseline	-5.22	-7.97
Higher demand ($V = 2V$)	-8.96	-8.23
Higher affordable rental discount ($\epsilon_\alpha = 2\epsilon_\alpha$)	-6.11	-10.87
Higher affordable construction discount ($\eta_\alpha = 2\eta_\alpha$)	-4.84	-6.42
Increase Maximum Allowed Density by 50% ($\phi = 0.50$)		
Baseline	7.09	1.87
Higher demand ($V = 2V$)	24.14	12.06
Higher value from height ($\epsilon_F = 1.5\epsilon_F$)	9.65	3.34
Higher cost from height ($\eta_F = 1.5\eta_F$)	0.34	-4.04
20-years Property Tax Exemption ($\tau = 20$)		
Baseline	4.58	7.66
Higher demand ($V = 2V$)	9.31	9.88
Higher property tax rate ($\psi = 1.5\psi$)	8.37	12.13

Affordable Housing Mandates (α) Panel (a) of Figure 3 shows that increasing the affordable unit requirement consistently decreases both property values and development probabilities. For instance, Table 1 indicates that a 20% mandate leads to a 5.22% drop in property values and a 7.97 percentage point reduction in the probability of development. Figure A1 confirms that the loss in property value from increasing α is due to a reduction in the redevelopment value. This lower development probability increases the present value of the existing building, given the longer expected cash flows.

In Table 1, we see that the same 20% mandate in parcels with higher demand (V up by 100%) causes a larger decline in property value (8.96%) and a sharper drop in development probability (8.23 percentage points). This result illustrates the greater impact that affordable housing mandates have in high-demanded areas. If the rental discount is doubled for affordable units (ϵ_α) then the negative effects on property value (-6.11%) and 10-years development probability (-10.87 percentage points) are larger. Finally, if affordable housing devel-

opment costs are 100% higher, the 20% mandate would decrease property values by -4.84% and reduce the likelihood of development (-6.42 percentage points). This sensitivity analysis demonstrates that cost escalation can make development less feasible with a higher negative impact on property value.

Density Bonuses (ϕ) Panel (b) of Figure 3 shows that increasing the allowed density increases property value. However, it can also reduce the probability of development at high values of ϕ. The analysis clearly indicates that property values and development probabilities are not necessarily in sync with one another. Increasing ϕ raises construction costs through $\kappa()$ but also boosts income through greater density. Because developers can exercise the option to build at higher density or not, higher density always increases property value. However, for high values of ϕ, developers may delay construction, strategically reducing the observed development probability. Thus, higher property values result from the increased present value of both the developed property and the redevelopment option (Figure A1).

Table 1 shows that a 50% increase in allowable density (setting $\phi = 0.5$) increases property values by 7.09% and development probabilities by 1.87 percentage points. If the increase is in high-demand areas, this same density bonus magnifies property values. A property value increase of 24.14% and development probability increase of 12.06 percentage points is realized when areas are targeted that have twice the underlying rental value. Increasing the marginal benefits of height (ϵ_F) shows that a $\phi = 0.5$ marginally raises the property value and development probability by 9.65% and 3.34 percentage points, respectively, compared to our baseline case. Although a consensus exists on the average height elasticity of construction cost, the elasticity also depends on the bedrock depth, which impacts the elasticity by factors of 10 (Ahlfeldt et al., 2023). We show that increasing the height elasticity of construction costs by only 50% results in a modest property value increase (0.34%) and a negative impact on development probability (-4.04 percentage points). Consequently, this result highlights the role of construction technology in the real estate development industry.

Tax Exemptions (τ) Offering tax exemptions unambiguously increases property value and accelerates development (Panel c of Figure 3). The increase in property value comes solely from the redevelopment option, as the tax exemption applies to the newly developed building (Panel c of Figure A1). For example, offering a 20-year tax credit raises property value by 4.58% and increases development probability by 7.66 percentage points for this representative parcel. Offering identical tax exemption to high-demand parcels (doubling V) boosts property value by 9.31% and raises development probability substantially (+9.88 percentage points). As expected, higher property tax rates amplify the benefits of tax credits. Doubling

the property tax rate, for example, increases the impact of a 20-year tax credit, raising property value and development probability compared to the baseline case.

3 Model Calibration

We focus on NYC affordable housing mandate as a case study to calibrate our model. The calibration process is conducted in two stages. First, we define the functional forms of the three key functions embedded in the model—$\Gamma(.)$, $\gamma(.)$, and $\kappa(.)$—and then estimate their respective elasticities using either micro-based evidence or values from the existing literature. Next, we incorporate market-specific parameters and parcel-level heterogeneity. A detailed description of all parameters is provided in Table 2.

3.1 Model Parametrization

3.1.1 The Density and Mandated Public Good Share Elasticities of Value (ϵ_F and ϵ_α)

In our model, the underlying market value of an unlevered unit of density V is scaled by the function $\Gamma()$ to reflect the built space of the parcel. We parametrize this scaling function as

$$\Gamma(F, o, \alpha) = F^{(1+\epsilon_F)}(1-\alpha)^{\epsilon_\alpha}(1-o) \tag{10}$$

where $\epsilon_F > 0$ is the density elasticity of market-rate value, and $\epsilon_\alpha > 0$ denotes the non-market-unit share elasticity of value. We calibrate ϵ_F to 0.07 following the results in the literature on the economics of skyscrapers (Ahlfeldt and Barr, 2022; Ahlfeldt et al., 2023; Danton and Himbert, 2018).[10] This positive ϵ_F captures the amenity value associated with better views and lower noise at higher floors which translates into increased market-rate value.

Given our focus on affordable housing mandates in the counterfactual analysis and the lack of existing elasticities, we use reduced-form estimates to calibrate the value elasticity with respect to α. Rearranging Equation (10) and assuming that a unit of density (F) is an apartment unit, one can estimate ϵ_α by regressing the log price per apartment (PPU_i) on $(\log(1 - AffShare_i))$

$$\log PPU_i = \beta + \epsilon_F \log Unit_i + \epsilon_\alpha \log\left(1 - AffShare_i\right) + \nu_i \tag{11}$$

[10]Using the elasticity of rent instead of value presents no issues unless the capitalization rate varies by building height, which could happen if there is a risk or growth rate differential between taller and shorter buildings. We also abstract from the value increase associated with ground-floor properties (Danton and Himbert, 2018; Liu et al., 2018), since our model focuses on additional floor space ϕ, which inherently refers to construction above the ground floor.

Table 2: Baseline Model Elasticities and Parameters

This table reports the baseline parameters of the vector Θ. The sources, calibrations, and estimations are detailed in Sections 3.

Parameters	Description	Value	Calibrated method & main source
	The Price function		
ϵ_F	Density elasticity of value	0.07	Literature
ϵ_α	Non-market units share elasticity of price	0.17	Estimation – Table A1
	The demolition cost function		
η_D	Demolition cost elasticity of density	0.72	Target: Fixed cost demolition (55%)
k_d	Baseline demolition cost per FAR	11.36	Target: Share of demolition cost (15%)
	The construction cost function		
η_F	Density elasticity of construction cost	0.60	Literature
η_α	α elasticity of construction cost	0.20	Estimation - RCLO report
k_c	Baseline construction cost per FAR	64.09	Target: Land-value ratio (25.26%)
	Others		
ψ	Property tax rate (%)	1.08	Estimation - DoF & Costar
d	Construction periods (years)	2.64	Estimation – DCP
r	Risk free rate (%)	3.79	Estimation – FED
	Market-level parameters		
V_j	Market-value per unit of density (000's)	[330.30-918.80]	Estimation – Census Rent
δ_j	Drift rate of stochastic process	[0.080-0.14]	Estimation – Census Rent
σ_j	Volatility of stochastic process	[0.091-0.50]	Target: observed housing production
$\delta_{\alpha(j)}$	Affordable housing return	[0.07-0.13]	Estimation – Census Rent
$\sigma_{\alpha(j)}$	Affordable housing return volatility	[0.046-0.25]	Target: observed housing production
	Parcel-level parameters		
F_i	Built density (units per 2,500 sqft of parcel)	[0.00-70.22]	PLUTO (standardized)
$\bar{F}_i$	Maximum density (units per 2,500 sqft of parcel)	[1.00-70.22]	PLUTO (standardized)
o_i	Obsolescence cost	[0.00-0.95]	Estimation – PLUTO (building age)
q_i	Parcel time-invariant quality	[0.13-6.90]	j specific random draw – Census Rent

where $\beta \approx \log(V)$.[11] We can estimate this regression with real estate transactions using ordinary least squares (OLS) if $(1 - AffShare_i)$ is randomly assigned to buildings. Because it is highly unlikely that the allocation of affordable units in buildings is randomly assigned, we limit the transactions to properties that include both affordable and market-rate units. We thus concentrate on the extensive margin of variation in α. We merge the transaction of apartment buildings from Costar with the list of properties having affordable units from the Affordable Housing Production by Building dataset obtained from the NYC Department of Housing Preservation and Development. To control for the impact of location and the timing of sales on apartment prices, we include borough and time-fixed effects in our regression, in addition to the log of apartment units.

[11] This rearrangement implicitly assumes that the observed building prices do not incorporate the option value of redevelopment. To align with this assumption, we focus on transactions involving buildings less than 50 years old, as these are likely to have negligible redevelopment option value.

The results in Table A1 indicate that the elasticity of price with respect to $(1-\alpha)$ ranges from 0.198 to 0.273. Under reasonable discount rate assumptions, an ϵ_α of 0.2 is equivalent to a rental discount on affordable units of 20.4-25.9% compared to market-rate units, which is in the range of what Favilukis et al. (2023) use to calibrate their rental discounts for rent-stabilized units in NYC. We also observe heterogeneity across boroughs. The lowest and highest discounts occur in the Bronx and in Manhattan, respectively. We recognize that the level of the rental discount associated with affordable housing is likely within the policy choice set $\mathcal{P}$. To shed light on how this can change the incentive of developers, we provide an evaluation of how our main results change with respect to this parameter in Section 5.

3.1.2 The Demolition Cost Function

To account for the expenses during the demolition phase, we parametrize the demolition cost function as

$$\gamma(F) = k_d F^{\eta_D} \tag{12}$$

Here, $0 < \eta_D < 1$ represents the density elasticity of demolition costs, while k_d is a cost shifter associated with demolishing one unit of density. We calibrate these parameters based on insights from discussions with real estate developers. Specifically, developers noted that fixed costs in demolition projects typically account for 50-60%, with the remainder attributed to the structure itself. Developers also indicated that demolition costs generally account for 13-20% of development costs. The first statistic is matched to the mean fixed costs predicted by our model resulting in a calibration of $\eta_D = 0.72$. The second statistic is used to establish the relation between k_d and k_c, as discussed in Section 3.1.4.[12]

3.1.3 The Construction Cost Function

To account for construction costs that are convex in height as well as the potential differences in costs associated with increasing α, we define the annual cost of construction as

$$\kappa(F, \alpha, d) = \frac{1}{d} k_c F^{(1+\eta_F)}(1 - \alpha\ \eta_\alpha) \tag{13}$$

where k_c is a scalar capturing the baseline cost of construction of a single unit of density, $\eta_F > 1$ is the density elasticity of construction cost, and η_α captures the discount/premium in construction cost for space dedicated to the public good α. We use the average estimates

[12]The probability of development and the property value are homogeneous functions of degree zero and one, respectively, with respect to the level parameters V, k_d, and k_c. As a result, their relations can be expressed in relative terms rather than absolute levels without affecting the validity of our counterfactual analysis.

from the literature to calibrate the height elasticity of construction cost. For instance, Ahlfeldt and McMillen (2018) estimate a 0.61 height elasticity of cost of construction for residential buildings that have more than 5 floors, Ahlfeldt et al. (2023) calibrate their residential height elasticity of construction cost at 0.55, and Ahlfeldt and Barr (2022) estimate an elasticity of 0.56 for tall residential buildings. We thus set $\eta_F = 0.60$ to reflect increases in construction cost due to height.

The second elasticity, η_α, reflects the cost reduction resulting from changes in the quality of fixtures and finishes associated with non-market units. To calibrate η_α, we rely on data from the National Apartment Association's Income and Expenses Report as cited in the RCLO report on The Affordable Housing Asset Class. This report offers detailed insights into capital expenditures per square foot for market-rate and subsidized housing units from 2012 to 2021. On average, subsidized units receive 19.9% less capital expenditure per square foot compared to market-rate units. Acknowledging that capital expenditures differ from construction costs, we assume that the variation in capital intensity serves as a reasonable proxy for the construction cost differential. Based on this, we set $\eta_\alpha = 0.199$.

3.1.4 Baseline Construction and Demolition Costs

There are three parameters that govern the levels of our model, the value of a unit of non-obsolete density V, the demolition cost k_d, and construction costs k_c. All three parameters are expressed per unit of density. Because the probability of development and the property value are homogeneous functions of degree zero and one, respectively, with respect to these, we set V to outside data (see following section) that will vary across parcels. We calibrate the other two parameters using the method of moments. We assume that they are constant across NYC neighborhoods given that labor, material, and regulatory costs are likely similar within the city (Glaeser and Gyourko, 2018).

We use our model to match two sets of moments, using the baseline elasticities and the current built space in NYC as shown in Table 2. First, we use PLUTO information regarding the ratio of land value compared to the value of the property. We match this average to our model moment by setting the current built space to $F_i = 0$: $E[\pi(F_i = 0; \Theta)/\pi_i(F_i = F_i; \Theta) - 1] = 0.25$.Second, the demolition cost shifter is explicitly matched using the fact that demolition costs range between 13-20% of the total development costs. This calibration yields $k_c = 64.09$ and $k_d = 11.36$. To put this into perspective, note that the mean value of V_i in our model is 562.2. This implies implies that the cost of construction of a unit of density is around 11% the value of that density.

3.2 Calibration of Other Parameters

Construction Length (d) We calibrate the length of the construction period using data from the DCP Housing Database Project-Level Files. We compute the duration from permit issuance to building completion. We restrict the sample to muti-family unit buildings that private for-profit developers construct. As the baseline calibration for d, we use 2.44 years, the median time between permit issuance and building completion.[13]

Risk-free Rate (r) To calibrate r, we use the average 3-month market yield on U.S. treasury securities from the Board of Governors of the Federal Reserve System (FRED). This series averages 3.79% over the available period (September 1981 to December 2023).

Property Tax Rate (ψ) To compute the *effective* property tax rate on residential properties, we combine several datasets. First, we link property assessment data for multi-family properties from the Primary Land Use Tax Lot Output (PLUTO) with the statutory tax rates (NYC Department of Finance) to calculate property tax bills for all parcels in NYC since 2000. We then merge the tax bills with 36,000 transactions of multi-family properties from Costar. Finally, we divide the tax bills by the corresponding sale prices to derive the effective tax rates. Since 2000, this rate has averaged 1.34%, with an interquartile range of 0.56% to 1.8%.

3.3 Market-level Parameters

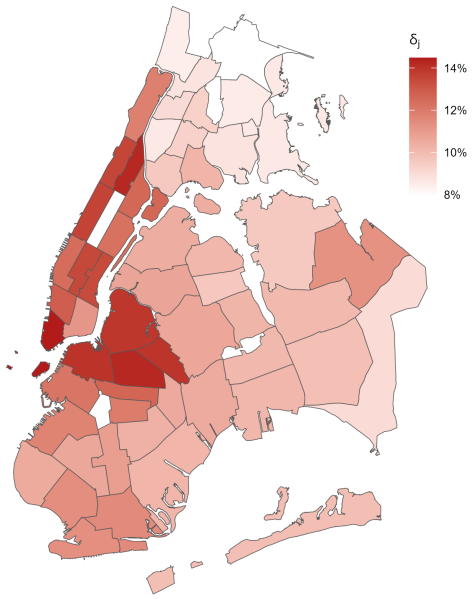
We define market $j = \{1, 2, ..., J\}$ as the aggregation of residential parcels i located within a neighborhood, with n_j representing the total number of parcels. We chose community districts as our spatial unit of neighborhoods given that zoning changes typically require approval at this local level. Each market is characterized by its return δ_j, return volatility σ_j, and the average market value per unit of density V_j.

Calibration of δ_j – Total Return to Multi-family Housing We compute δ_j as the combined sum of property (asset) and rental (dividend) returns. Property returns are calculated at the borough level using Costar's multi-family asset transaction data from 2004 to 2023.[14] Property annual returns are highest in Manhattan (7.6%), followed by Brooklyn (6.8%), Queens (6.0%),

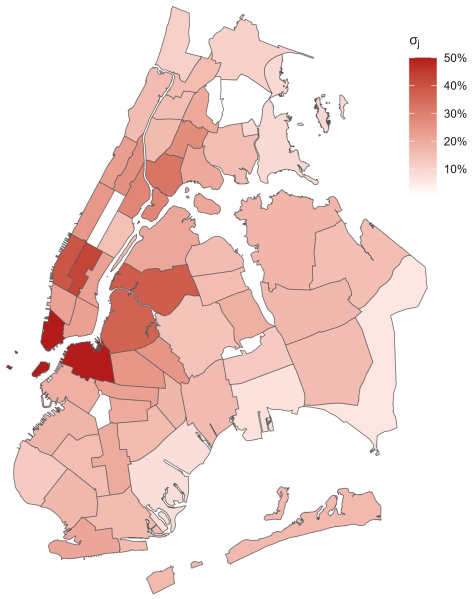
[13] Through informal discussions with developers, we learned that the size of a building surprisingly has only a minor impact on construction duration. For instance, iconic projects such as the The Empire State Building were completed in just 1 year and 45 days, highlighting that factors other than scale often dominate construction timelines.

[14] Due to the limited number of transactions per neighborhood, property returns are computed at the borough level. Staten Island's neighborhood returns are excluded from calibration for the same reason.

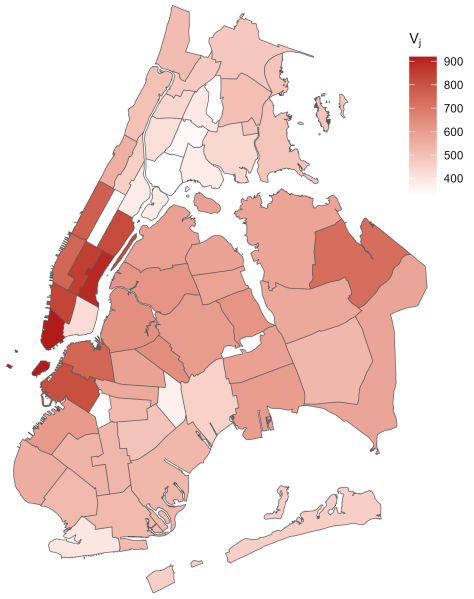
Figure 4: Market-level Parameters Calibration



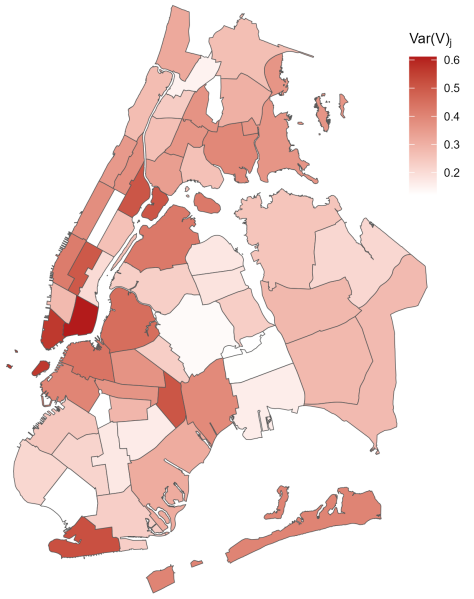
A. Return (δ_j)



B. Return Volatility (σ_j)



C. Market-value per Unit of Density (V_j)



D. Std. Dev. of the Log Residuals of V_j (σ_{V_j})

Note: These maps show the market-level parameters used in the baseline calibration and counterfactual analyses. In Panel A, we show the return on equity (the sum of property and rental returns) of multi-family residential assets. In Panel B, we show the return volatility implied by our model using targeted market-level construction intensity. In Panel C, we show the 2022 calibration of the market-value per unit of density V_j (in \$000's). In Panel D, we show the standard deviation of the log residuals of median rent.

and the Bronx (5.0%). Rental growth is then calculated at the neighborhood level using the average annual increase in median gross rent from 2013 to 2022, collected from the Census at the census block group level. For instance, the Williamsburg neighborhood in Brooklyn experienced an average annual rental growth of 7.4% over the past decade. Combining property and rental returns yields the distribution of δ_j, illustrated in Panel A of Figure 4, with an average return of 10.5%.

Calibration of σ_j – Model's Implied Volatility Conditional on the baseline elasticities and parameters, we determine the return volatility, σ_j, that best aligns our model's predictions with the observed housing production across markets. Specifically, we match the model's predicted number of housing units constructed to the actual housing units built in each neighborhood between 2010 and 2023, using data from NYC Planning to calibrate σ_j. Specifically, calibrating the parcels' characteristics to 2010, we estimate σ_j by minimizing the difference between the model's predicted construction activity and the empirical data for each market j:

$$\arg\min_{\sigma_j} \left[\sum_{i \in j} \underbrace{\mathbf{E}_{2010}\left[\mathbf{P}_i(..., \sigma_j, \Theta | t = 13) F_i^*(..., \sigma_j, \Theta)\right]}_{\text{Model's expected units within 13 years given 2010 data}} - \underbrace{UnitsConstructed_j^{2010-2023}}_{\text{Data}} \right]. \quad (14)$$

We calibrate σ_j using the volatility implied by our model, drawing parallels to the concept of implied volatility introduced by Black and Scholes (1973) for the pricing of corporate securities (Dumas et al., 1998). The resulting average implied volatility is 16.1%, with variations across markets depicted in Panel B of Figure 4.

Calibration of V_j – Mean Value per Unit of Density To calibrate V_j, we use the 2022 Census median gross rent, averaged across census block groups within each neighborhood. Applying a uniform capitalization rate across the city results in an average V_j of \$561,000, with variations illustrated in Panel C of Figure 4. Additionally, we use rent variations within each neighborhood to parameterize parcel-specific V_i (Panel D). The methodology for incorporating these variations into parcel-level parametrization is detailed below.

Affordable Housing Stochastic Process Assumption Unless otherwise specified, we calibrate the returns and volatility of affordable housing units using the market's stochastic processes. Affordable housing, often associated with social impact investment, generally yields lower returns. Thus, we set the return on affordable housing in market j to be 1% lower than the corresponding market-rate return, i.e., $\delta_{\alpha(j)} = \delta_j - 0.01$. However, given the reduced risk associated with high-demand affordable units, we assume that return volatility for affordable

housing is 50% of that for market-rate units, such that $\sigma_{\alpha(j)} = \sigma_j/2$. Since both processes are linked to the same housing market, we assume a perfect positive correlation, setting $\sigma_{\alpha,j} = 1$.

3.4 Parcel-level Parameters

Each parcel i is characterized not only by its location within market j but also by its current density F_i, maximum allowable density $\bar{F}_i$, operational obsolescence o_i, and a time-invariant quality factor q_i. The quality parameter q_i captures above- or below-average location attributes that, in turn, influence the parcel-specific market value per unit of density V_i. The range of these variables is summarized in Table 2. To maintain consistency with our analysis, we restrict the sample to parcels zoned or partially zoned for residential use, excluding those designated for public or other non-residential purposes.[15] In total, we use 672,241 parcels.

Built and Maximum Allowed Density (F_i and $\bar{F}_i$) We calibrate F_i and $\bar{F}_i$ using the existing as-built Floor-Area-Ratio (FAR) and the maximum allowable FAR, because they represent key density measures. To ensure consistency, we normalize these values based on the observed relationship between FAR and the number of residential units, controlling for parcel size. This allows us to express F_i in terms of residential units per a standard parcel size of 2,500 square feet.[16] In our baseline calibration, F_i has an average value of 2.94, while $\bar{F}_i$ averages 4.11, with 35.2% of parcels built to their maximum allowable density.

Operational Obsolescence (o_i) To account for operational inefficiencies, such as energy loss, higher capital reserves, or vacancy, we allow the current asset to yield a lower rental than the market it would absent this inefficiency. We capture this operational obsolescence using the functional form $o_i = 1 - (1 - \mu)^{RenovAge_i}$, where $RenovAge_i$ is the number of years since the last recorded renovation or construction and μ is the annual rental depreciation rate. We set $\mu = 0.015$ based on the results of Lopez and Yoshida (2022) who estimate a 1.5% annual rental depreciation for condominiums that reflects both structural and functional obsolescence. In our baseline calibration, operational obsolescence averages 63%.

Lot-specific Quality Shifter (q_i) To account for within-market heterogeneity in values, we allow the market value per unit of density to vary across parcels. Factors such as proximity

[15] To keep parcels that can accommodate residential units, we, in effect, keep parcels that have an "R-x" zoning classification in either the first, second, third, or fourth zoning classification. In addition, we keep parcels with observed residential units regardless of their zoning classification.

[16] For instance, in the neighborhood encompassing Manhattan's Chelsea and Hudson Yards, a built FAR of one corresponds to 3.46 residential units on a typical 2,500-square-foot parcel, whereas in Queens, this statistic averages 2.53 units.

to metro stations, parks, or waterfront views contribute to these variations. To capture this heterogeneity, we define parcel-specific values as $V_i = V_j q_i$, where q_i is the time-invariant locational quality. We assume q_i is exogenous to other lot-level characteristics (e.g., built density, obsolescence) and draw it from a log-normal distribution with mean zero and standard deviation σ_{V_j}. The standard deviation is computed using the log residuals of median rents in market j, obtained from Census data.[17] This specification enables us to capture the quality variation present in neighborhoods with significant heterogeneity, such as Manhattan's Alphabet City, as illustrated in Panel D of Figure 4.

4 Baseline Model Outcomes

Using the parameterization described in the previous section, we compute key outcomes under our baseline scenario ($\mathcal{P}_0 = \{\alpha = 0, \phi = 0, \tau = 0\}$). These include: (1) $\mathbf{P}_i(\mathcal{P}_0; \Theta|t)$ - the probability that a parcel undergoes redevelopment within a t-year horizon, (2) $\pi_i(\mathcal{P}_0; \Theta)$ - the parcel value accounting for both current use and its option value, and (3) $F_i^*(\mathcal{P}_0; \Theta)$ - the optimal redevelopment density. Our baseline scenario represents what happens in the absence of the policy interventions and serves as our benchmark for evaluating counterfactual policy interventions.

Aggregate Outcomes We analyze several aggregate outcomes either at the city or market level, with a primary focus on the expected number of housing units constructed (or under construction) at the 10-year horizon, $\mathbf{E}[Apt|t = 10]$. Additionally, we compute related measures, including the expected net addition of units ($\mathbf{E}[AptNet|t = 10]$), expected density ($\mathbf{E}[Density|t = 10]$), the present value of all parcels (Π), and the average value of the redevelopment option ($\mathbf{E}[Option|t = 10]$). The market-level outcomes, for instance, are defined as:

$$
\begin{aligned}
\mathbf{E}\left[Apt_j|\mathcal{P}_0, t = 10\right] &= \sum_{i \in j} \mathbf{P}_i\left(\mathcal{P}_0; \Theta|t = 10\right) F_i^*\left(\mathcal{P}_0; \Theta\right) \\
\mathbf{E}[AptNet_j|\mathcal{P}_0, t = 10] &= \sum_{i \in j} \mathbf{P}_i(\mathcal{P}_0; \Theta|t = 10)(F_i^*(\mathcal{P}_0; \Theta) - F_i) \\
\mathbf{E}[Density_j|\mathcal{P}_0, t = 10] &= \frac{1}{n_j} \sum_{i \in j} \mathbf{P}_i(\mathcal{P}_0; \Theta|t = 10) F_i^*(\mathcal{P}_0; \Theta) + (1 - \mathbf{P}_i(\mathcal{P}_0; \Theta|t = 10))\, F_i \\
\mathbf{E}[\Pi_j|\mathcal{P}_0] &= \sum_{i \in j} \pi_i(\mathcal{P}_0; \Theta)
\end{aligned}
$$

[17] Specifically, for each market j we retrieve the log residuals rents $\varepsilon_{b,j}$ of each block group $\log(Rent_{b,j}) = \log(V_j) + \varepsilon_b$, and compute the variance of $\varepsilon_{b,j}$ within j: σ_{V_j}.

$$\mathbf{E}[Option_j|\mathcal{P}_0, t=10] \quad = \quad \frac{\sum_{i\in j}\tilde{\pi}_i(\mathcal{P}_0;\Theta)}{\sum_{i\in j}\pi_i(\mathcal{P}_0;\Theta)} \tag{15}$$

where $\tilde{\pi}_i(\mathcal{P}_0;\Theta)$ is the value of the four last terms of Equation (6).

We present the aggregated city-level outcomes in the first row of Table A2. Our model estimates the construction of 211,996 housing units over a 10-year horizon. This projection is consistent with the historical NYC construction trend of 10,000 to 28,000 new units per year since 2010 (NYC housing production). Accounting for the demolition of existing units, we estimate a net construction of 12,095 units per year over a 10-year period. Figure 5 illustrates the spatial distribution of new housing units (completed or under development) throughout the city in this baseline scenario.[18] The model predicts more development activity in areas of Brooklyn and Queens adjacent to the East River, as well as in Manhattan's Chelsea and Hudson Yards, reflecting the continued demand for housing in these prime locations.

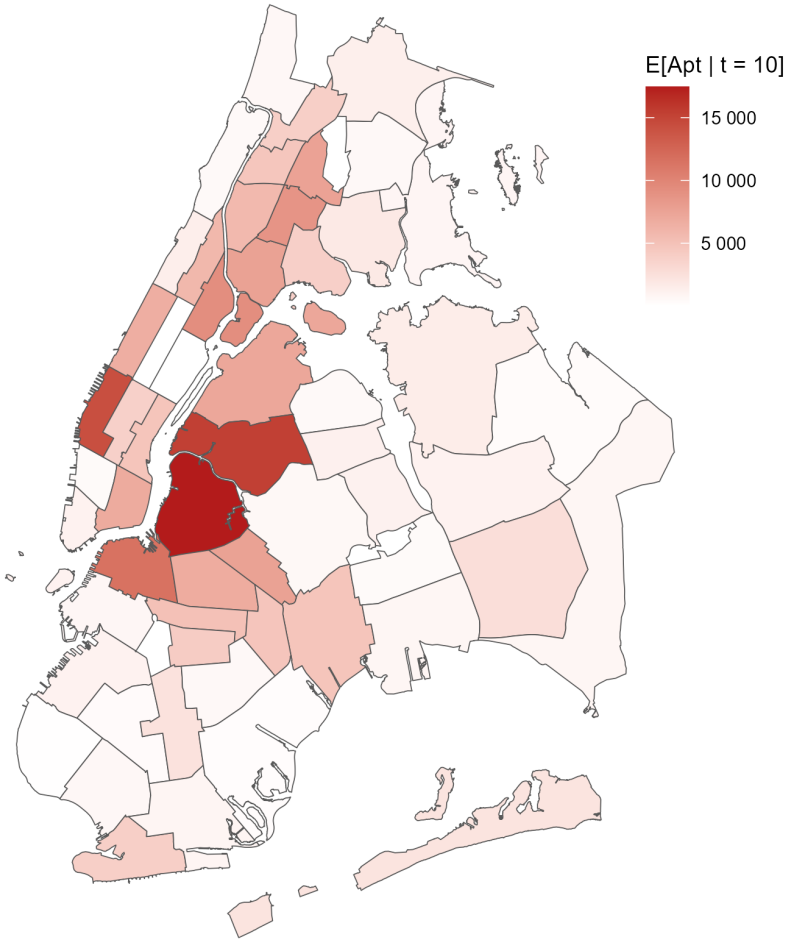
At the 10-year horizon, the model projects an increase in average residential density from 2.88 to 3.11 units per 2,500 square feet of land. As expected, density attenuates with distance from the center of Manhattan (Panel B of Figure A3), consistent with predictions from the monocentric city model (Alonso, 1964; Mills, 1967; Muth, 1969). Furthermore, our model estimates the total value of residential assets in New York City at approximately $960 billion, which is in the same order of magnitude as the valuation reported by the Department of Finance (Tax Lot Assessment Portal) despite the focus on non-residential parcels and the exclusion of Staten Island.

Finally, our model finds that the redevelopment option accounts for 2% of the total value of residential assets, a relatively low proportion that reflects that most of the parcels in the city are already developed. However, as shown in Panel D of Figure A3, some neighborhoods, particularly those with significant redevelopment potential, such as Long Island City in Queens, or those with aging building stock, such as parts of the Bronx, exhibit a higher share of option value, reaching up to 20%.

To further validate our model, we assess whether its predicted property values align with observed real estate prices in New York City. Specifically, we calculate the predicted price per unit for each parcel and average these values within each market, $1/n_j \sum_{i\in j}(\pi_i(\mathcal{P}_0;\Theta)/F_i)$. Figure 6 compares these model-implied prices with the unconditional average price per unit of multi-family transactions in each neighborhood, using Costar transaction data. The strong positive correlation ($\rho = 0.89$) between the predictions of the model and the observed market

[18] Additional maps detailing other outcomes are provided in Figure A3 of the Appendix.

Figure 5: Baseline Calibration – Expected Number of Apartments



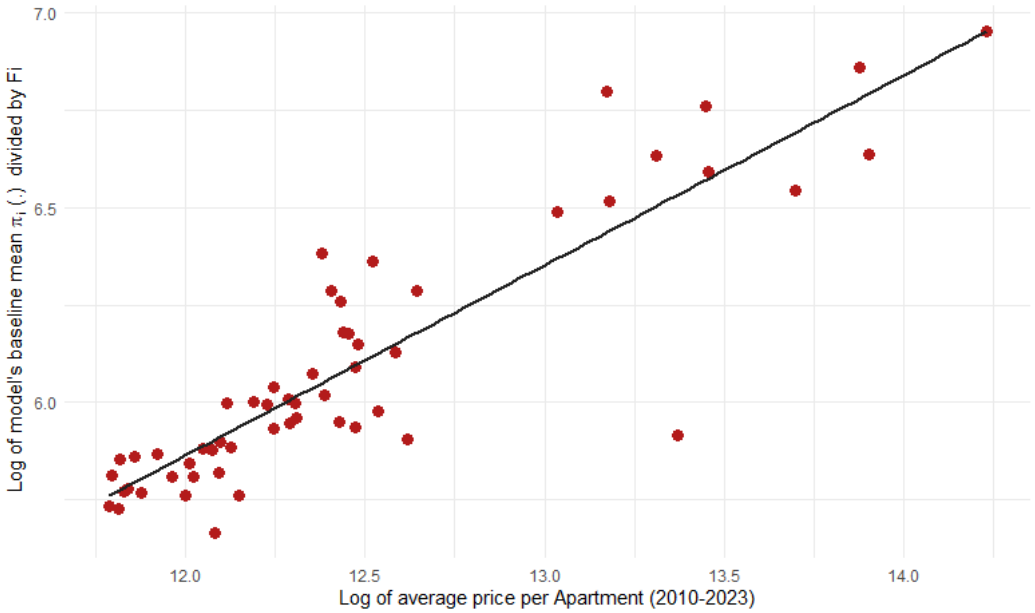
Note: This map shows the expected number of apartments constructed in each community district within a 10-year horizon, $E[Apt_j|\mathcal{P}_0, t = 10] = \sum_{i \in j} \mathbf{P}_i^{dev}(\mathcal{P}_0, \Theta | t = 10) F_i^*(\mathcal{P}_0, \Theta)$ using the model baseline calibration with parameters described in Table 2.

prices underscores the accuracy of our model's calibration.

Parcel Heterogeneity To examine the role of parcel heterogeneity in shaping key real estate outcomes, Table 3 presents an analysis of parcel-level characteristics and their influence on core model outputs. These outputs include the probability of development $\mathbf{P}_i$, property value $\log(\pi_i)$, and optimal redevelopment density F_i^*. We estimate the partial correlations between these outcomes and parcel-level characteristics while controlling for neighborhood fixed effects.

Column (1) shows that the probability that development will occur is positively corre-

Figure 6: Model Fit



Note: This scatter plot shows the relationship between the log of the model's mean price per unit of density $\sum \pi_i / F_i(.)$ *in each NYC community district against the log of the unconditional mean price per apartments of Multi-Family buildings using transaction from CoStar since 2010. .*

lated with parcel quality and obsolescence, while the existing density acts as a deterrent. This result aligns with economic intuition: higher obsolescence lowers the value of current cash flows, making redevelopment more attractive, while taller buildings increase the opportunity cost of redevelopment. Column (2) shows that the optimal density of redevelopment is positively correlated with the quality of the plot. This result suggests that developers tend to build taller buildings when potential cash flows justify the additional cost. However, the most influential predictor is the allowable density as developers tend to build to the maximum allowed density (Brueckner et al., 2024; Büchler and Lutz, 2024).

Column (3) reveals that the quality of the parcel (positive), obsolescence (negative), and current density (positive) are the primary drivers of property value. Even after controlling for building size (Column 5), taller buildings exhibit higher values, consistent with the findings on positive returns to height (Ahlfeldt and McMillen, 2018). Meanwhile, Column (4) highlights that the option value is negatively correlated with the existing as-built density and positively correlated with obsolescence. This finding reflects the tendency for redevelopment to occur on underutilized, outdated properties. Overall, our findings emphasize the critical role of the existing built environment in shaping redevelopment activity and the importance of incorporating these factors into policy design.

Table 3: Partial Correlations Between Inputs and Outcomes of Baseline Model

*This table reports the partial correlations between key inputs (rows) and outcomes (columns) of our baseline model results. The partial correlations are conditional on community districts as well as all other inputs listed in the rows. It uses the outcomes of the model with horizon 10-years for the 672,241 city under the baseline calibration described in Table 2. In Column (1), the outcome is the estimated probability of redevelopment decision $\mathbf{P}_i^{\mathbb{P}}(\mathcal{P}_0;\Theta)$. In Column (2), the present value of the parcel $\pi_i(\mathcal{P}_0;\Theta)$. In Column (3), the option value share of total value $\tilde{\pi}_i(\mathcal{P}_0;\Theta)/\pi_i(\mathcal{P}_0;\Theta)$. In Column (4), the log of the total value per unit of built units $\pi_i(\mathcal{P}_0;\Theta)/F_i$ computed for parcels with positive construction. Pearsons standard errors around the correlation estimates are presented in parentheses. Estimates followed by ***, **, and * are statistically significant at the 1%, 5%, and 10% levels, respectively.*

	Baseline Outcome				
	Prob	F opt	log(PV)	Option	log(PV/F)
	All parcels				*Built parcels*
	(1)	(2)	(3)	(4)	(5)
Time-invariant quality (q_i)	0.179^{***}	0.043^{***}	0.556^{***}	0.106^{***}	0.686^{***}
	(0.005)	(0.005)	(0.003)	(0.005)	(0.003)
Current obsolescence (o_i)	0.151^{***}	0.016^{***}	-0.421^{***}	0.085^{***}	-0.512^{***}
	(0.005)	(0.005)	(0.004)	(0.005)	(0.004)
Current density (F_i)	-0.332^{***}	0.148^{***}	0.480^{***}	-0.336^{***}	0.013^{***}
	(0.004)	(0.005)	(0.004)	(0.004)	(0.005)
Maximum allowed density ($\bar{F}_i$)	0.235^{***}	0.974^{***}	0.062^{***}	0.253^{***}	0.058^{***}
	(0.004)	(0.0002)	(0.005)	(0.004)	(0.005)
Non-built parcel ($\mathbf{1}_{F_i=0}$)	0.918^{***}	-0.395^{***}	-0.686^{***}	0.969^{***}	
	(0.001)	(0.004)	(0.002)	(0.0003)	

Robustness to Underlying Parameters Table A2 evaluates the sensitivity of aggregate outcomes to key elasticity assumptions, demonstrating how our model captures trade-offs between development costs and returns. Shorter construction durations increase expected apartment production, density, and property values, primarily by enhancing the value of the redevelopment options. In contrast, a greater height elasticity in construction costs leads to a significant reduction in new development and lower option values. This result highlights the importance of construction technology in shaping urban development (Ahlfeldt et al., 2023).

The third row shows that when developers are constrained to build at the allowable density, aggregate outcomes remain largely unchanged. In a city like NYC, where zoning regulations are often binding (Brueckner et al., 2024), developers always build to the regulatory limit. Adjusting assumptions about the value per unit of density (V_i) reveals that increasing

returns to height raises expected density and property values. However, unlike changes to construction costs, this effect does not translate into higher redevelopment option values.

5 Policy Counterfactuals

This section quantifies the impact of implementing policy instruments on urban development. How do these policies change the benefits and costs relative to our baseline outcomes that involve no policy intervention? These policies counterfactuals introduce a combination of incentives (*Carrots*) and constraints (*Sticks*) that immediately shape developers' decisions regarding the optimal timing and density of development.

To compute counterfactuals, we parameterize each city parcel as outlined in Section 3, using the parameter estimates in Table 2, unless otherwise stated. We evaluate policy interventions that modify the regulatory framework at time $t = 0$, moving from the policy environment $\mathcal{P}_0$ to $\mathcal{P} = \{\alpha \geq 0, \phi \geq 0, \tau \geq 0\}$. We denote counterfactual outcome deviations from the baseline outcomes with Δ. For instance, the market-level change in the expected number of units constructed under policy $\mathcal{P}$ is expressed as:

$$\begin{aligned}\Delta\mathbf{E}[Apt_j|\mathcal{P}, t=10] = & \quad \mathbf{E}[Apt_j|\mathcal{P}, t=10] & - \quad & \mathbf{E}[Apt_j|\mathcal{P}_0, t=10] \\ = & \quad \textstyle\sum_{i\in j} \mathbf{P}_i(\mathcal{P}; \Theta|t=10) F_i^*(\mathcal{P}; \Theta) & - & \sum_{i\in j} \mathbf{P}_i(\mathcal{P}_0; \Theta|t=10) F_i^*(\mathcal{P}_0; \Theta)\end{aligned}$$

Thus, all policy counterfactual results are computed by measuring deviations from baseline outcomes, expressed as percentage changes for monetary outcomes. One additional aggregate outcome, not considered in the baseline case, is the number of expected affordable units, $\Delta\mathbf{E}[\text{Aff}_j|\mathcal{P}, t=10] = \mathbf{E}[\text{Aff}_j|\mathcal{P}, t=10]$ given that $\mathbf{E}[\text{Aff}_j|\mathcal{P}_0, t=10] = 0$ since $\alpha = 0$ in $\mathcal{P}_0$.

In addition to the expected number of total and affordable units constructed within a given time horizon, we also consider potential costs associated with policy implementation. Specifically, we evaluate changes in development density, the present value of tax revenues, and the total present value of NYC residential assets. This last metric is particularly relevant as it may introduce political risks—either from residents if values rise too sharply or from real estate industry stakeholders if values decline significantly. However, explicitly quantifying these monetary costs falls outside the scope of our study.

5.1 Policy Counterfactuals Benefits and Costs

Figure 1 in the introduction presents the core findings of our counterfactual policy analysis, illustrating the trade-offs between benefits and costs associated with various policy interventions. Panel A depicts the benefits, measured in terms of expected changes in affordable and market-rate housing production. Panel B shows two primary public costs: changes to overall density and changes in expected tax revenue.

Our analysis considers six distinct policy interventions: (1) upzoning, which increases allowable density; (2) tax exemptions, which provide financial incentives for new development; (3) affordability mandates, which require a share of units to be affordable; (4) incentive zoning, combining affordability mandates with density bonuses; (5) fiscal incentives, pairing affordability mandates with tax exemptions; and (6) combined incentives, which incorporate all three mechanisms. Sections 5.2 and 5.3 contain a detailed description of these policies and their implications.

5.1.1 Panel A: Benefits of Policy Interventions

We assume that an optimal policy to address the affordability crisis seeks to enhance affordable supply while mitigating adverse impacts on overall market-rate housing production. This optimal policy is depicted by the -45 degrees red line whereby a shift as far to the right reflects an optimal expansion of affordability benefits. This approach inherently results in a net substitution effect. Affordable housing units replace market-rate units, striking a balance between density control and housing accessibility.

Our results indicate that substantial variation exists across policies. Upzoning (Policy [1]) generates the largest expected increase in market-rate housing but does not contribute to affordability. It results in the highest density increase, moving away from the affordability goal. Tax exemptions (Policy [2]) increase overall housing supply with minimal impact on density but do not contribute to affordability, all new units built are market-rate. The Affordability Mandate (Policy [3]) directly replaces market-rate units with affordable housing generating a strong substitution effect. Although it aligns with the affordability goal, it suppresses overall housing production making it less effective in increasing total supply. Incentive Zoning (Policy [4]), which combines affordability mandates with density bonuses, produces affordable units while keeping overall supply unchanged. Fiscal Incentives (Policy [5]), which pair affordability mandates with tax exemptions, increase affordable units at the cost of reducing overall housing production. Finally, the Combined Incentives policy (Policy [6]), which incorporates affordability mandates, density bonuses, and tax exemptions, achieves the highest

number of affordable units but increases overall supply over the baseline case.

5.1.2 Panel B: Costs of Policy Interventions

Panel B of Figure 1 captures the financial and urban planning trade-offs associated with each policy. The two primary cost dimensions include (i) the fiscal burden, measured by changes in expected tax revenue (y-axis), and (ii) the increase in overall density (x-axis)[19]. A cost-neutral policy (relative to the baseline) is positioned as close to the origin as possible, minimizing both fiscal losses and density increases.

Our results show that policies impose varying costs on local governments and urban infrastructure. Upzoning (Policy [1]) generates a substantial increase in density but has a negligible effect on tax revenues. Although it successfully expands housing supply, the strain on infrastructure and public services intensifies due to higher population density. Tax Exemptions (Policy [2]) substantially reduce tax revenues while causing a moderate rise in density, prioritizing housing supply expansion at the expense of fiscal sustainability. The Affordability Mandate (Policy [3]) leads to a modest decline in tax revenue but reduces expected density, as affordability requirements disincentivize new development. Incentive Zoning (Policy [4]) results in notable density increases while keeping tax revenue losses relatively controlled, striking a balance between supply growth and affordability mandates. Fiscal Incentives (Policy [5]) cause the most significant reduction in tax revenues yet have a minimal effect on density, making affordability gains highly dependent on sustained public sector investment. The Combined Incentives policy (Policy 6) presents the highest costs across both dimensions, substantially diminishing tax revenues and significantly increasing density. Although this policy achieves the strongest affordability outcomes, it places the greatest financial burden on the public sector.

Our results highlight the trade-offs policymakers must navigate when designing zoning and fiscal incentives for stimulating housing construction. Upzoning (Policy [1]) expands market-rate housing supply with minimal fiscal cost but does not guarantee affordability. In contrast, fiscal and density incentives (Policies [4] and [5]) support affordability objectives but impose significant costs in terms of forgone tax revenue or increased urban density. A balanced approach may be found in combined incentive zoning (Policy [6]), which integrates multiple policy tools to expand affordability at a higher cost on both margins. Ultimately, the

[19]Higher density imposes costs on public infrastructure and essential services, increasing demand for roads, transit, utilities, schools, and emergency response. It also contributes to congestion, environmental strain, and higher maintenance expenses. These factors must be weighed when assessing the net impact of density-enhancing policies.

Table 4: Counterfactual Results

This table reports the aggregate outcomes of our counterfactual policy analyses. It reports the change of various outcomes described in columns from a counterfactual regulatory environment $\mathcal{P}$, described in the rows, to the baseline calibration $\mathcal{P}_0$. All results are computed with a 10-year horizon. In the first column, we show the change in the expected number of apartments constructed or under construction. The second column shows the same but net of demolished units. In the third and fourth columns, we separate between affordable and market-rent net expected units built. Column (5), shows the average change in expected density in percentage, with density computed as the expected number of apartments per parcel of 2,500 sqft. In Columns (6) and (7), we show the percentage change in the present value of property taxes, and total residential assets, respectively. The last three rows show the aggregate changes if the policy bundle $\mathcal{P}$ is a non-obligatory policy. In these counterfactuals, developers of each parcel select to used the policy bundle as opposed to the status-quo baseline policy $\mathcal{P}_0$ if $\pi_i(\mathcal{P}) \geq \pi_i(\mathcal{P}_0)$.

		Built Environment				Public Potential Costs		
		$\Delta E[Apt]$ # of Apt.	$\Delta E[AptNet]$ # of Apt	$\Delta E[\text{Aff}Net]$ # of Apt.	$\Delta E[MarketNet]$ # of Apt	$\Delta E[Density]$ (%)	ΔTax (%)	ΔPV (%)
		(1)	(2)	(3)	(4)	(5)	(6)	(7)
				Unilateral Policy				
(1)	*Upzoning* - $\phi = 25\%$	$86,309$	$64,589$	0	$64,589$	3.09	0.14	0.46
(2)	*Tax abatement* - $\tau = 30$	$36,869$	$19,821$	0	$19,821$	0.95	-1.16	0.29
(3)	*Affordable mandate* - $\alpha = 20\%$	$-48,931$	$-22,218$	$32,613$	$-54,831$	-1.06	-0.06	-0.42
				Mandatory Carrot & Stick Zoning				
(4)	*Density Incentive Zoning* - $\alpha = 20\%$ & $\phi = 25\%$	$16,169$	$26,417$	$45,633$	$-19,216$	1.26	0.04	-0.12
(5)	*Fiscal Incentive Zoning* - $\alpha = 20\%$ & $\tau = 30$	$-18,278$	$-4,767$	$38,743$	$-43,510$	-0.23	-0.90	-0.21
(6)	*All* - $\alpha = 20\%$, $\phi = 25\%$, & $\tau = 30$	$60,793$	$54,401$	$54,558$	-157	2.60	-1.11	0.17
				Voluntary Carrot & Stick Zoning				
(7)	*Density Incentive Zoning* - $\alpha = 20\%$ & $\phi = 25\%$	$4,922$	$4,729$	$4,657$	72	0.23	0.01	0.01
(8)	*Fiscal Incentive Zoning* - $\alpha = 20\%$ & $\tau = 30$	$1,578$	$1,507$	$5,472$	$-3,966$	0.07	-0.14	0.01
(9)	*All* - $\alpha = 20\%$, $\phi = 25\%$, & $\tau = 30$	$48,081$	$41,353$	$38,487$	$2,866$	1.98	-0.97	0.18
				Carrot & Stick Zoning with Higher Cost of Mandate ($\epsilon_\alpha = 0.3$)				
(10)	*Density Incentive Zoning* - $\alpha = 20\%$ & $\phi = 25\%$	$7,252$	$20,893$	$43,850$	$-22,957$	1.00	0.02	-0.18
(11)	*Fiscal Incentive Zoning* - $\alpha = 20\%$ & $\tau = 30$	$-25,528$	$-8,745$	$37,293$	$-46,038$	-0.42	-0.86	-0.26
(12)	*All* - $\alpha = 20\%$, $\phi = 25\%$, & $\tau = 30$	$50,093$	$47,836$	$52,418$	$-4,582$	2.29	-1.06	0.10

choice of policy depends on how jurisdictions trade-off affordability, fiscal sustainability, and urban planning considerations.

5.2 Unilateral Policies

We outline in the the first panel of Table 4, the evaluations of three key standalone policies: Upzoning (Density Bonus of 25%), Tax Exemptions (30-year exemption), and an Affordability Mandate (20% affordable). We note that upzoning increases total housing supply more than any other policy, adding 64,589 net new units, but does not create dedicated affordable housing. In contrast, an Affordability Mandate generates 32,613 affordable units by shifting production away from market-rate supply, which declines by 54,831 units. Tax exemptions incentivize development but with more limited supply effects, adding 19,821 net new units.

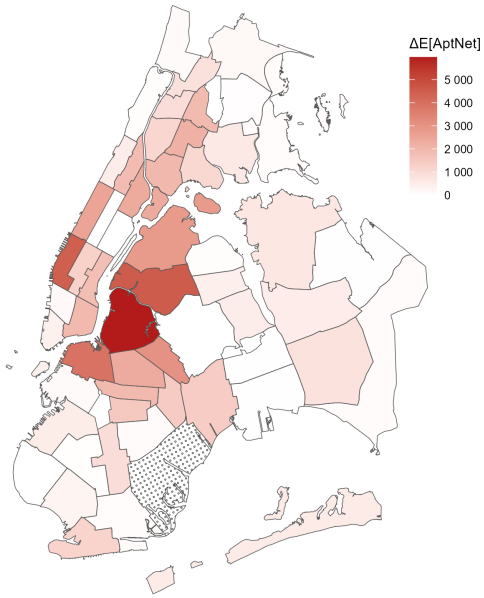
From a public cost perspective, Upzoning significantly increases density while moderately increasing tax revenues. The Affordability Mandate minimizes density growth (-1.06%) but does not expand total supply; market-rate units are substituted with affordable ones. As expected Tax Exemptions carry the largest fiscal cost, reducing the present value of tax revenue by 1.16%.

5.3 Mandatory Carrot & Stick Zoning

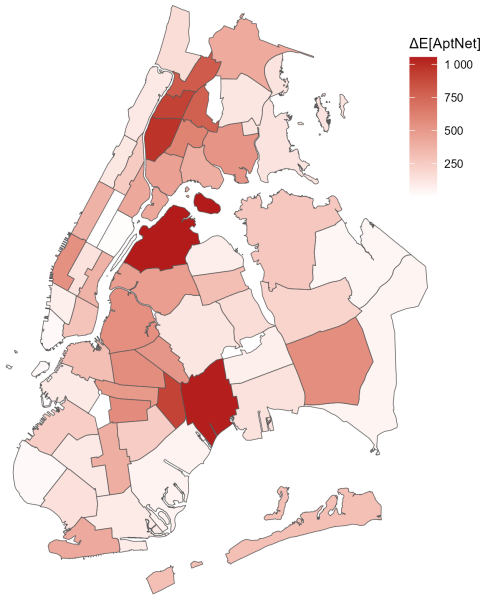
We next evaluate policies that combine affordability mandates with either zoning and fiscal incentives that we show in the second panel of Table 4. Specifically, we compare three key interventions: Density Incentive Zoning (affordability mandate & density bonus), Fiscal Incentive Zoning (affordability mandate & tax exemption), and the Combined Incentive Policy (density bonus, tax exemption, & affordability mandate). These policies, referred to as *Carrot & Stick Zoning*, attempt to mitigate the trade-offs associated with standalone policies. The goal is to maximize affordability gains while minimizing increased public costs in terms of density and fiscal losses.

The highest affordability gains come from the Combined Incentive Policy, that generates an expected 54,558 affordable units compared to 45,633 and 38,743 affordable units from the Density Incentive Zoning and Fiscal Incentive Zoning, respectively. However, Density Incentive Zoning achieves affordability by shifting market-rate production, reducing it by 19,216 units while adding 26,417 net new units. In contrast, Fiscal Incentive Zoning limits total new supply, producing 4,767 fewer net units than the baseline, while imposing a 0.90% tax revenue loss. The Combined Incentive Policy minimizes the decline in market-rate housing to just 157

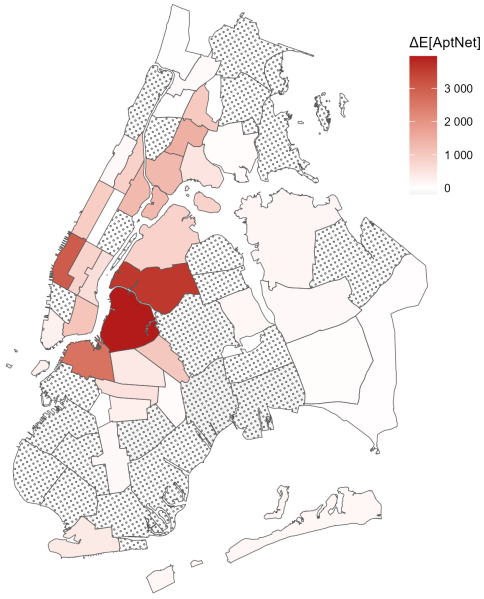
Figure 7: Counterfactuals – Net Change in Housing Units



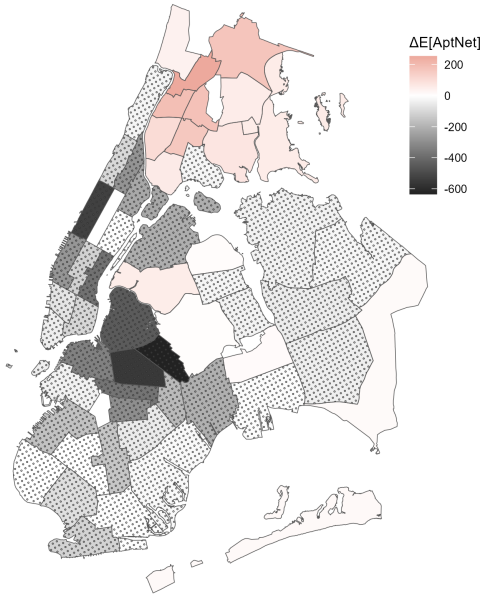
A. Upzoning ($\phi = .25$)



B. Tax Exemptions ($\tau = 30$)



C. Density Incentive Zoning ($\alpha = 0.2$ & $\phi = .25$)



D. Fiscal Incentive Zoning ($\alpha = 0.2$ & $\tau = 30$)

Note: These maps show the expected net change in the number of housing units constructed for different counterfactual policy analyses. It shows for each market the change compared to the baseline outcomes with horizon 10-year as shown in Figure 5.

units but results in the largest fiscal and density costs, with a 1.11% expected tax loss and a 2.60% density increase.

Figure 7 shows the expected changes in net housing production across neighborhoods in response to unilateral zoning policies versus *Carrot & Stick Zoning* approaches. The introduction of an affordable housing mandate leads to an overall decline in expected housing production across most NYC neighborhoods. However, the spatial distribution of development responses varies significantly between Density Incentive Zoning and Fiscal Incentive Zoning. Specifically, neighborhoods with parcels near redevelopment thresholds—such as those in the Bronx—exhibit a stronger response to fiscal incentives. In contrast, high-valued locations, including Western Queens and Brooklyn, primarily benefit from zoning relaxations. These findings underscore that uniform policies yield heterogeneous effects across different urban areas, highlighting the importance of spatially targeted policy interventions.

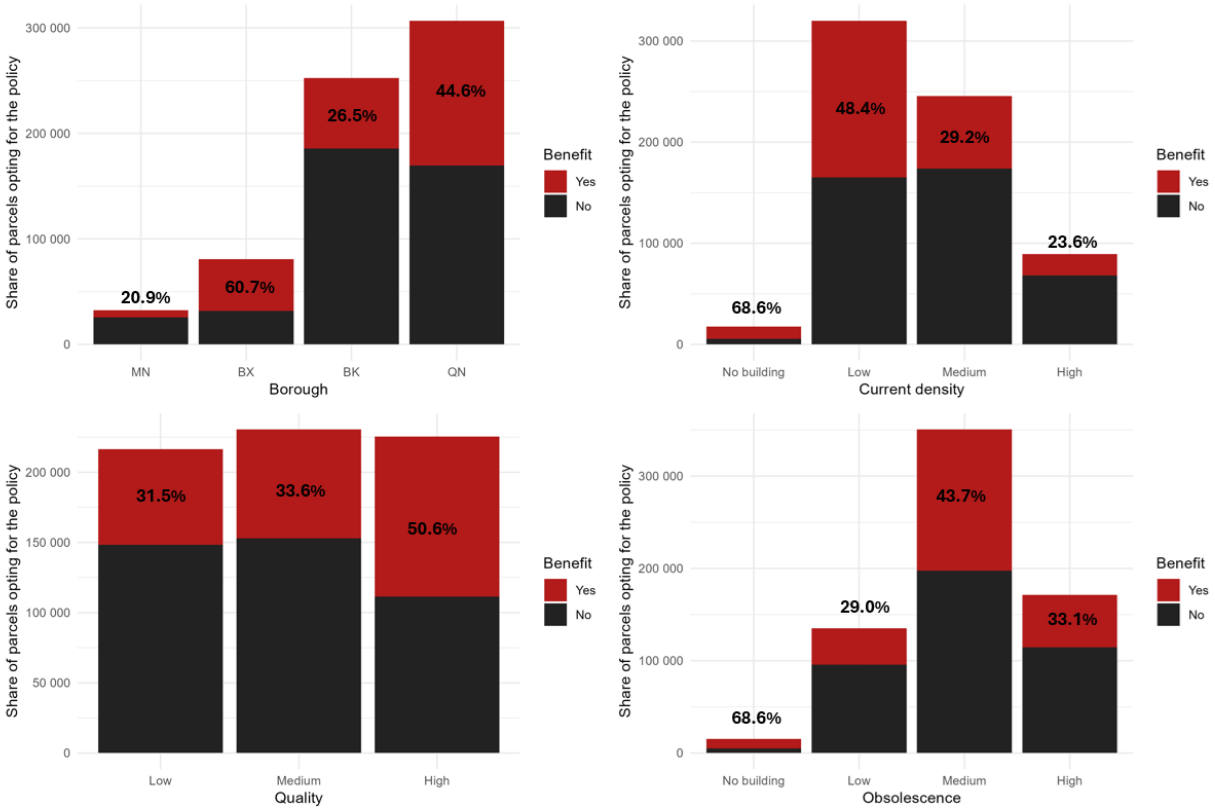
5.4 Voluntary Carrot & Stick Zoning

Given that developers respond heterogeneously to these policies, we present in the third Panel of Table 4, the expected outcomes of *voluntary* Carrot & Stick Zoning policies. Under voluntary adoption, developers choose to opt into a new policy regime $\mathcal{P}$ only if it provides higher property value than the status quo, i.e., if $\pi_i(\mathcal{P}) \geq \pi_i(\mathcal{P}_0)$. This decision rule ensures that only parcels where incentives outweigh the regulatory burden participate, effectively self-targeting affordability incentives to the most economically viable projects. As a result, voluntary policies avoid the inefficiencies of blanket mandates aligning developer incentives with affordability goals.

The results of this exercise clearly outline that Voluntary adoption reduces the supply of affordable units. We note that the voluntary Combined Incentive Policy produces 38,487 affordable units or 70% of the 54,558 affordable units achieved under the mandatory policy. In contrast, the reduction is substantial for the Voluntary Density Incentive Zoning and Fiscal Incentive Zoning that result in circa 5,000 affordable units, a reduction of 90% and 86%, respectively. This suggests that developers are not strongly incentivized to participate given the levels of *Carrots* and *Sticks* considered.

This reduction in affordable unit productions comes, however, with an expected reduction in public costs. We observe that tax revenue losses drop significantly across all policies, with voluntary Density Incentive Zoning keeping tax revenue unchanged and voluntary Fiscal Incentive Zoning reducing tax losses to just 0.14%, compared to 0.90% under the mandatory version. Likewise, the voluntary Combined Incentive Policy cuts tax losses, from 1.11%

Figure 8: Voluntary Adoption of Incentive Zoning Policies



Note: These bar graphs show the share of parcels that are better-off ($\pi_i(\mathcal{P}) \geq \pi_i(\mathcal{P}_0)$) under the policy counterfactual that combine density bonuses ($\phi = 0.25$), tax exemptions ($\tau = 30$), and a public good mandate ($\alpha = .2$), compared to the status-quo policies.

to 0.97%. Costs associated with greater density also decline sharply. For instance, under voluntary adoption, the Combined Incentive Policy leads to a density increase of just 1.98%, compared to 2.60% under mandatory participation. This moderation in density growth alleviates pressure on infrastructure.

The limited response in the voluntary adoption stems from developer selectivity that we report in Table A3. Approximately 14.0% of parcels opt into either the voluntary Density Incentive Zoning or Fiscal Incentive Zoning. We, however, show that the adoption jumps to 38.6% when both density and fiscal incentives are bundled. For example, we observe that 24.6% of the parcels would switch from adopting the policy when the fiscal incentive is added to the density incentive zoning policy. Figure 8 further decomposes the share of parcels that would endogenously choose this policy bundle based on their locations and building attributes. We particularly note higher participation for parcels with low density, high quality, and those located in the Bronx. Thus, although the effects on aggregate affordable produc-

tion are limited, this selective participation helps concentrate affordable units in areas where private development interest aligns with policy goals, reducing the need for costly citywide mandates.

5.5 Limitations of the Counterfactual Analysis

A key assumption underlying our model is the exogeneity of underlying asset values and thus the assumption that real estate developers act as price-takers and are too small individually to influence citywide price levels. Specifically, our model does not explicitly incorporate general equilibrium effects that could arise from a significant expansion of the housing supply nor potential crowd-out effects from affordable supply (Eriksen and Rosenthal, 2010). However, in scenarios where housing supply increases substantially—such as under large-scale upzoning—the assumption of exogenous asset values may not hold, potentially resulting in an upward bias in our counterfactual results. In a market such as New York City, where aggregate housing supply constraints have historically driven high property values, an increase of 86,000 new units—representing a 3.9% increase over baseline levels—may not substantially impact citywide rental prices. However, if the additional supply is highly localized, price adjustments could be more pronounced, potentially altering the incentives for development (Asquith et al., 2023).

Another potential limitation is that our model does not account for behavioral responses from developers beyond the decision to redevelop. In reality, developers may strategically time their investments based on anticipated regulatory changes or adjust the quality of construction in response to policy incentives. Future iterations of this analysis could incorporate a broader set of responses, including strategic delays in development, negotiations with the public sector, or adaptive re-use.

Finally, our counterfactuals focus on broad citywide policy interventions. However, a more refined analysis could assess the effects of targeted policies applied to specific submarkets or parcels with high redevelopment potential. Future research can extend our framework to analyze more spatially targeted policies in order to provide additional insights into the heterogeneous effects of reforms across different neighborhoods.

6 Conclusion

We explore the tradeoffs between using *Carrots* (tax exemptions and density bonuses) and *Stick* (public good provision mandates) policies as catalysts in shaping developers' deci-

sions regarding the optimal timing and density of development. Using a model based on a real options approach, we apply this framework to the redevelopment of existing buildings in urban areas. We calibrate our model using the NYC affordable housing mandate as an application to incorporate key aspects of urban development. These factors include the cost and return to height, uncertainty in future market conditions, and spatial heterogeneity in land values and returns. The results of our baseline model are consistent with the observed development patterns in the city. Our study thus offers an alternative rationale for implementing strict ex-ante land-use regulations which involves an economic analysis of the use of *Carrots* and *Sticks* in negotiations with developers for the provision of public goods.

Our analysis highlights the importance of considering the existing built environment and heterogeneity across parcels when designing public policies. For example, parcels that have either a high current density or a low redevelopment potential respond differently to policy interventions compared to parcels associated with a low density or significant redevelopment options. As such, blanket "one size fits all" *Carrot & Stick* policies produce heterogeneous outcomes across the city. The insight resulting from this spatial variability is that targeted zoning reforms or voluntary incentive programs should maximize housing production while minimizing public costs.

Despite our focus on the affordable housing mandate in this study, our model is flexible in accounting for other policy considerations, including but not limited to changes in land use (e.g., office-to-residential conversions) and incentive zoning for other public goods such as public plazas or infrastructure. Future research could extend our framework to incorporate these additional considerations, especially given the rising importance of adaptive reuse in urban redevelopment.

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Appendix

A Additional Tables & Figures

Table A1: Elasticity of Real Estate Value with Respect to the Share of Non-market Rent Units

*This table reports the estimates of $log(PPU)_i = \beta + \epsilon_F log(Unit_i) + \epsilon_\alpha log(1 - Aff.Share_i) + \nu_i$, where $log(PPU)$ is the log of the building price divided by the number of apartment units, $Aff.Share$ is the share of units that are affordable identified in the data of The Department of Housing Preservation and Development (HPD). The sample consists of all building transactions from 2000 in New York City (excluding Staten Island) that contain affordable units. The time fixed effects are defined as pre-2007, 2007-2010, 2011-2015, 2016-2019, and 2020-2023. Estimates followed by ***, **, and * are statistically significant at the 1%, 5%, and 10% levels, respectively.*

	Dependent variable: log(PPU)			
	(1)	(2)	(3)	(4)
log(1-Aff.Share) - ϵ_α	0.273***	0.199***	0.198***	
	(0.057)	(0.057)	(0.043)	
... x Bronx				0.097*
				(0.058)
... x Brooklyn				0.278**
				(0.107)
... x Queens				0.372
				(0.307)
... x Manhattan				0.348***
				(0.083)
log(Units) - ϵ_F	0.074*	0.091**	0.016	0.007
	(0.042)	(0.041)	(0.031)	(0.031)
Borough Fixed effects		X	X	X
Time Fixed effects			X	X
Observations	203	203	203	203
R^2	0.114	0.196	0.568	0.583
Adjusted R^2	0.105	0.175	0.548	0.557

Table A2: Baseline Calibration Outcomes and Robustness to Parameter Change

This table reports the aggregate outcomes of our baseline calibration and additional sensitivity analyses performed by varying underlying model's parameter assumptions. The first column shows the 10-year horizon expected number of units constructed (or under construction), the second shows expected net addition, the third the expected density in 10 years, the fourth the sum of the residential parcel present values, and the fifth the average value of the redevelopment option. The first line shows the results using the baseline calibration described in Table 2, and the other lines show the same outcomes by varying one of the underlying model assumptions or parameters as described.

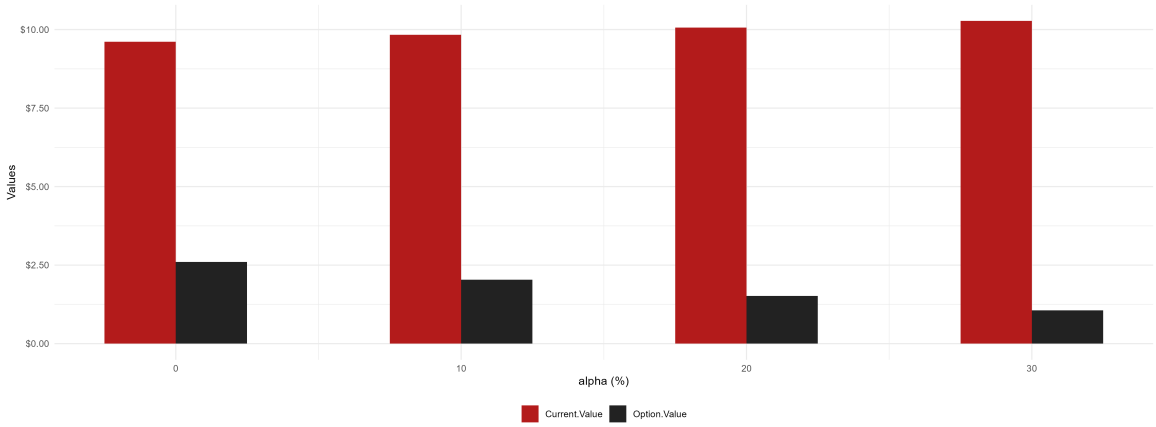
	$E[Apt\|t=10]$ # of Apt.	$E[AptNet\|t=10]$ # of Apt	Avg. Density per parcel	PV(taxes) ($billion)	PV ($billion)	Option Value (%)
	(1)	(2)	(3)	(4)	(5)	(6)
Baseline	$211,996$	$120,955$	3.114	271.83	959.38	2.00
	Varying underlying model's assumptions					
Shorter duration time ($d = d/2$)	$362,793$	$189,266$	3.216	272.55	971.08	4.05
Higher cost from height ($\eta_F = 1.5\eta_F$)	$131,471$	$70,854$	3.039	271.77	956.76	1.31
Higher value from height ($\epsilon_F = 2\epsilon_F$)	$246,017$	$143,445$	3.147	305.53	$1,088.45$	2.34
No endogenous density choice	$217,529$	$126,494$	3.122	271.83	959.27	1.98

Table A3: Share of Parcels that Opt-in *Voluntary* Policy Interventions

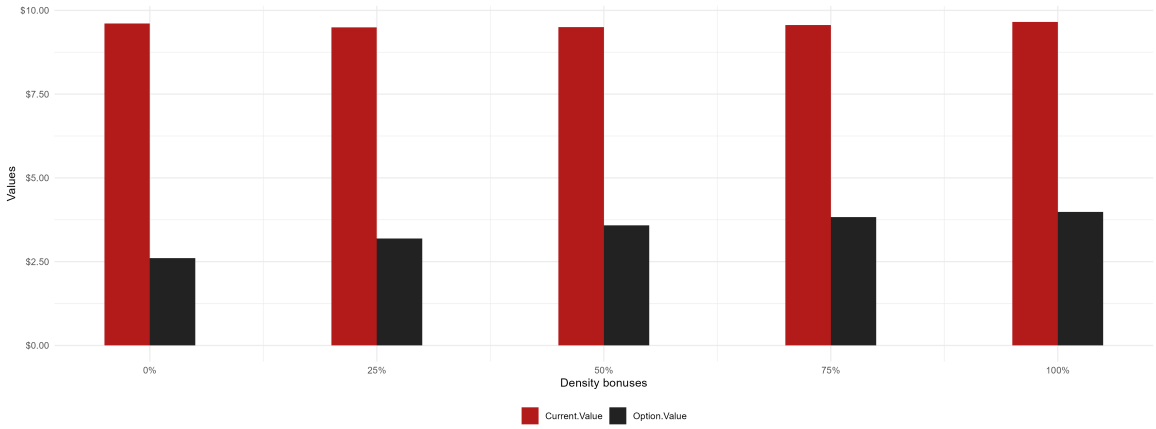
This table reports the number and the share of parcels that would opt-in the policy bundle $\mathcal{P}$ under voluntary adoptions. These parcels are defined as those that are better off under the new policy as in the baseline policy ($\pi_i(\mathcal{P}) \geq \pi_i(\mathcal{P}_0)$).

	Number of Parcels	Share (%)
All Parcels	672,241	100.0
(7) Density Incentive Zoning ($\alpha = 0.2$ & $\phi = 0.25$)	94,172	14.0
but not in fiscal IZ	9,282	1.4
(8) Fiscal Incentive Zoning ($\alpha = 0.2$ & $\tau = 30$)	95,679	14.2
but not in density IZ	10,789	1.6
(9) All Policies	259,571	38.6
but not in density IZ (i.e., switchers with τ)	165,501	24.6
but not in fiscal IZ (i.e., switchers with ϕ)	163.989	24.4

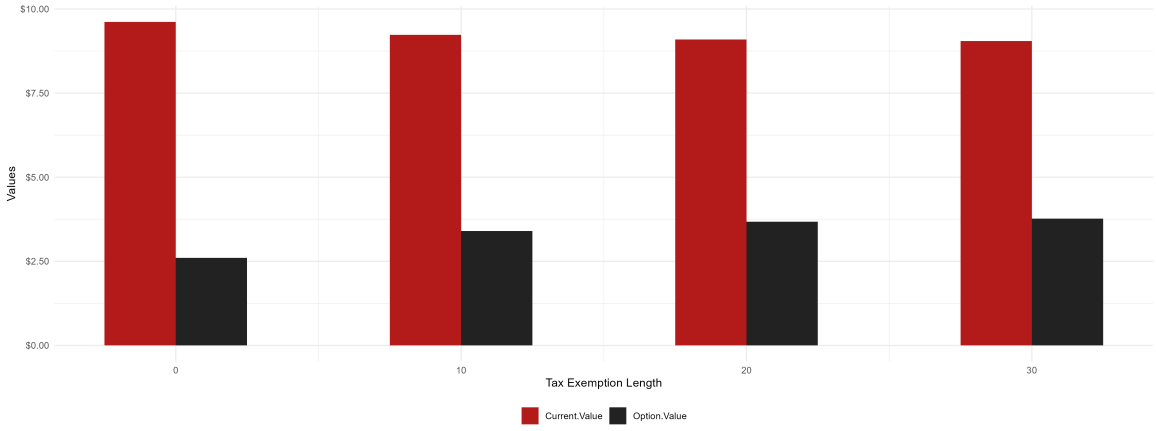
Figure A1: Policy Instruments and Change in Option Value to Redevelop



A. Mandatory Share for Public Goods (α)



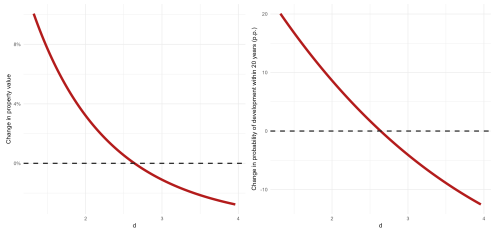
B. Density Bonuses (ϕ)



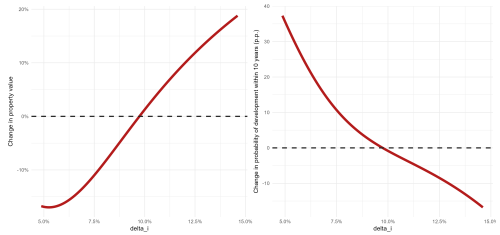
C. Length of Tax Exemptions (τ)

Note: These figures show the effects of changing the policy instruments on the present value of the current building and the present values of the redevelopment option. This option value is calculated by summing the four last elements shown in Equation (5). These figures are obtained for a parcel of land built at 25% of its capacity with $F_B = 2$ and $\bar{F} = 8$, with a rental return of $\delta_0 = .05$, and an obsolescence rate of 50% ($o_B = 0.5$). The assumption of the stochastic processes as follows: $\delta = 0.10$, $\sigma = 0.25$, $\delta_p = 0.09$, and $\sigma_p = 0.125$. All other parameters and elasticities are the baseline case shown in Table 2.

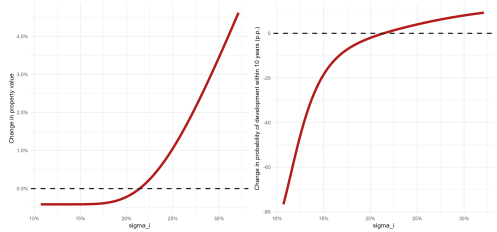
Figure A2: Marginal Effects of Parameter Changes



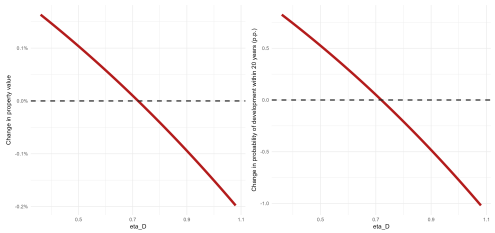
A. Construction Duration – d



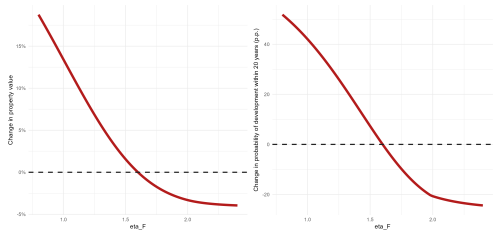
B. Return on Equity – δ_i



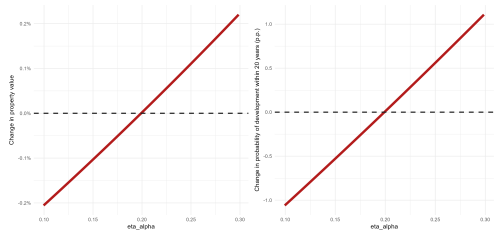
C. Volatility of Return – σ_i



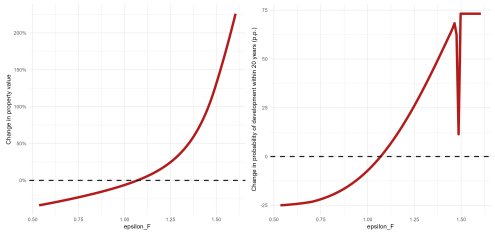
D. Height Elasticity of Demolition – η_D



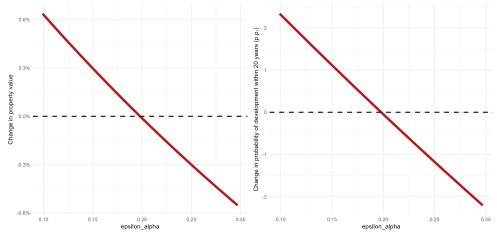
E. Height Elasticity of Construction – η_F



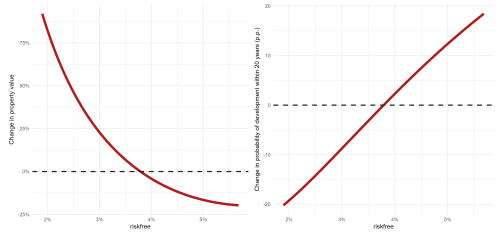
F. α Elasticity of Construction – η_α



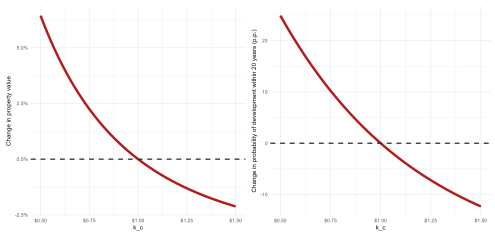
G. Height Elasticity of Rental Value – ϵ_F



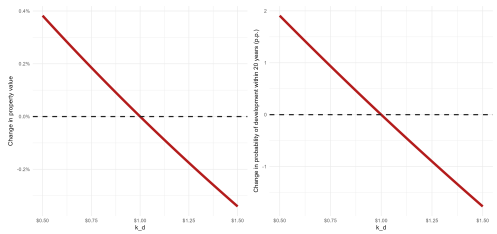
H. α Elasticity of Rental Value – ϵ_α



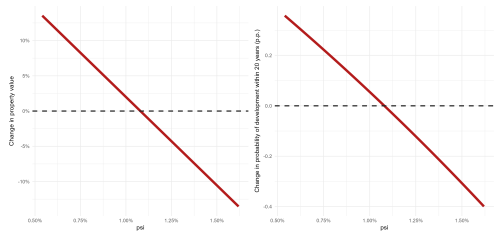
I. Risk-free Rate – r



J. Baseline Construction Cost – k_c



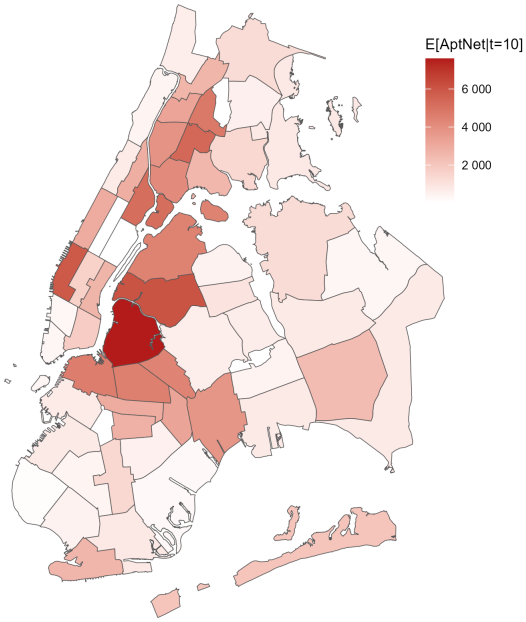
K. Baseline Demolition Cost – k_d



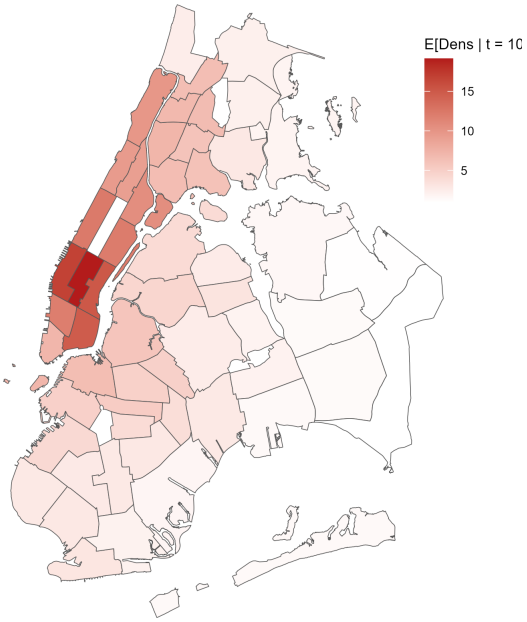
L. Property Tax Rate – ψ

Note: These figures show the effects of changing the baseline parameters on the change in the property value (left panels) and the change in the probability of development starting within 10 years (right panels). These figures are obtained for a parcel of land built at 40% of its capacity with $F = 2$ and $\bar{F} = 5$, with a net rental return after the obsolescence of 3% and the assumption of the stochastic processes as follows: $\delta = 0.08$, $\sigma = 0.20$, $\delta_p = 0.07$, and $\sigma_p = 0.10$. All other parameters and elasticities are the baseline case shown in Table 2. We assume a mandate to include public goods of 30% ($alpha = 0.3$) to generate comparative statics with that includes such variations.

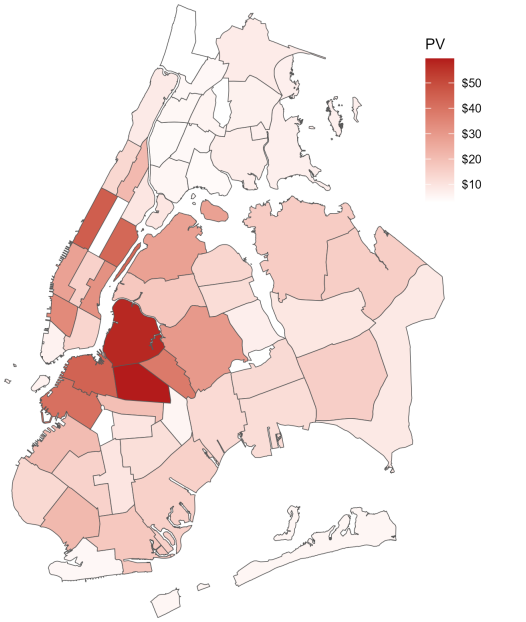
Figure A3: Baseline - Market-level Aggregated Outcomes



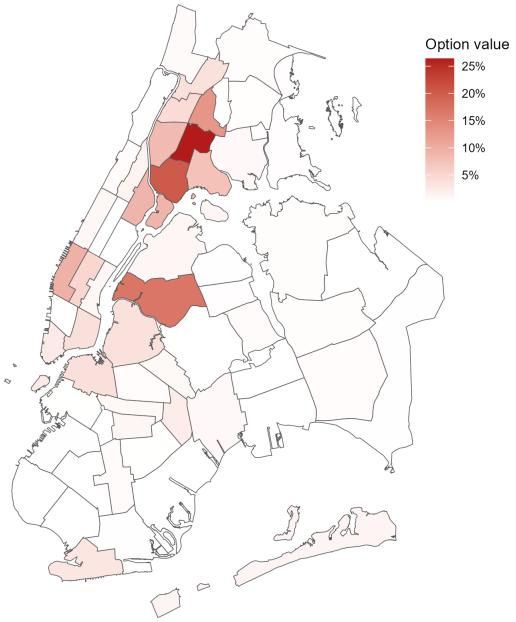
A. Expected Net Units



B. Expected Density



C. Present Value of Residential Parcels



D. Share of Redevelopment Option Value

Note: These maps show the market-level aggregated outcomes of our baseline simulation as described in Table 2. Panel A showsthe expected net units addition, Panel B shows the expected density in 10 years, Panel C shows the sum of the residential parcel present values, and panel D the average value of the redevelopment option.

B Proofs

B.1 Proof of Proposition 1.

Value before completed development We start the proof by working on the first integral in (5). This becomes

$$\mathbf{E}_{\mathbb{Q}}\left\{\int_0^{T_{V^e}} e^{-ru}\ (\delta_0-\psi)\ \Gamma(F_B,o_B,0)V\mathbf{1}_{V_u<V^e}\mathrm{d}u\right\}= (\delta_0-\psi)\ \Gamma(F_B,o_B,0)V\,\mathbf{E}_{\mathbb{Q}}\left\{\int_0^{T_a} e^{-ru}\mathbf{1}_{Z_u<a}\mathrm{d}u\right\} \quad \text{(B.1)}$$

with $a=(1/\sigma)\ln(V^{e*}/V)$. This last integral becomes further

$$\mathbf{E}_{\mathbb{Q}}\left\{\int_0^{T_a} e^{-ru}\mathbf{1}_{Z_u<a}\mathrm{d}u\right\}=\mathbf{E}_{\mathbb{Q}}\left\{\int_0^{\infty} e^{-ru}\mathbf{1}_{Z_u<a}\mathrm{d}u\right\}-\mathbf{E}_{\mathbb{Q}}\left\{\int_{T_a}^{\infty} e^{-ru}\mathbf{1}_{Z_u<a}\mathrm{d}u\right\}$$

Recall that $(W_t)_{t\geq 0}$ is a Brownian motion under the risk neutral measure $\mathbb{Q}$. We wish to change the probability measure to a new measure $\mathbb{P}$ so that $(Z_t)_{t\geq 0}$ becomes a Brownian motion under $\mathbb{P}$. From the equality $W_t=Z_t-bt=Z_t+\int_0^t(-b)\,\mathrm{d}s$ we can apply the Girsanov theorem and derive the appropriate density

$$\begin{aligned}\forall t\geq 0,\quad \left.\frac{\mathrm{d}\mathbb{Q}}{\mathrm{d}\mathbb{P}}\right|_{\mathcal{F}_t} &= \exp\left\{-\int_0^t(-b)\,\mathrm{d}Z_s-\frac{1}{2}\int_0^t(-b)^2\,\mathrm{d}s\right\}\\ &= \exp\left\{bZ_t-\frac{1}{2}b^2t\right\} \qquad \text{(B.2)}\end{aligned}$$

Under this change of measure, we obtain

$$\mathbf{E}_{\mathbb{Q}}\left\{\int_0^{T_a} e^{-ru}\mathbf{1}_{Z_u<a}\,\mathrm{d}u\right\}=\mathbf{E}_{\mathbb{P}}\left\{\int_0^{\infty} e^{-\frac{\lambda^2}{2}u}e^{bZ_u}\mathbf{1}_{Z_u<a}\,\mathrm{d}u\right\}-\mathbf{E}_{\mathbb{P}}\left\{\int_{T_a}^{\infty} e^{-\frac{\lambda^2}{2}u}e^{bZ_u}\mathbf{1}_{Z_u<a}\,\mathrm{d}u\right\} \quad \text{(B.3)}$$

where $\lambda=\sqrt{2r+b^2}$

To continue, we make good use of the following result that can be found in Karatzas and Shreve (1991), p. 272.

Lemma 1 *If $\phi:\mathbb{R}\to\mathbb{R}$ is a piecewise continuous function with*

$$\int_{-\infty}^{\infty} e^{-|y|\sqrt{2\alpha}}|\phi(x+y)|\,\mathrm{d}y<\infty \quad ,\forall x\in\mathbb{R}$$

*for some constant $\alpha>0$ and $(W_t)_{t\geq 0}$ is a standard Brownian motion, the **resolvent operator** of Brownian*

motion $K_\alpha(\phi)$ is defined by

$$K_\alpha(\phi) = \mathbf{E}\left\{\int_0^\infty e^{-\alpha t}\phi(W_t)\,\mathrm{d}t\right\} = \frac{1}{\sqrt{2\alpha}}\int_{-\infty}^{\infty} e^{-|y|\sqrt{2\alpha}}\phi(y)\,\mathrm{d}y$$

We obtain therefore,

$$\begin{aligned}\mathbf{E}_{\mathbb{P}}\left\{\int_0^\infty e^{-\frac{\lambda^2}{2}u}e^{bZ_u}\mathbf{1}_{Z_u<a}\,\mathrm{d}u\right\} &= \frac{1}{\lambda}\int_{-\infty}^{a} e^{by}e^{-|y|\lambda}\,\mathrm{d}y\\ &= \frac{-2}{b^2-\lambda^2}+\frac{e^{(b-\lambda)a}}{\lambda(b-\lambda)}\end{aligned}$$

Additionally notice that $b^2-\lambda^2=-2r$ and that

$$\begin{aligned}e^{(b-\lambda)a} &= e^{(b-\lambda)\frac{1}{\sigma}\ln(\frac{V^e}{V})}\\ &= \left(\frac{V^e}{V}\right)^{\xi}\end{aligned} \tag{B.4}$$

with $\xi=\frac{b-\lambda}{\sigma}$.
It follows that

$$\mathbf{E}_{\mathbb{Q}}\left\{\int_0^\infty e^{-ru}\mathbf{1}_{Z_u<a}\mathrm{d}u\right\} = \frac{1}{r}+\frac{1}{\lambda(b-\lambda)}\left(\frac{V^e}{V}\right)^{\xi} \tag{B.5}$$

We now turn our attention to the second integral in (B.3). We can write

$$\begin{aligned}\mathbf{E}_{\mathbb{P}}\left\{\int_{T_a}^\infty e^{-\frac{\lambda^2}{2}u}e^{bZ_u}\mathbf{1}_{Z_u<a}\,\mathrm{d}u\right\} &= \mathbf{E}_{\mathbb{P}}\left\{e^{-\frac{\lambda^2}{2}T_a}\int_0^\infty e^{-\frac{\lambda^2}{2}u}e^{b(Z_u+a)}\mathbf{1}_{Z_u<0}\,\mathrm{d}u\right\}\\ &= e^{ba}\mathbf{E}_{\mathbb{P}}\left\{e^{-\frac{\lambda^2}{2}T_a}\right\}\mathbf{E}_{\mathbb{P}}\left\{\int_0^\infty e^{-\frac{\lambda^2}{2}u}e^{bZ_u}\mathbf{1}_{Z_u<0}\,\mathrm{d}u\right\}\\ &= e^{(b-\lambda)a}\mathbf{E}_{\mathbb{P}}\left\{\int_0^\infty e^{-\frac{\lambda^2}{2}u}e^{bZ_u}\mathbf{1}_{Z_u<0}\,\mathrm{d}u\right\}\\ &= \left(\frac{V^e}{V}\right)^{\xi}\mathbf{E}_{\mathbb{P}}\left\{\int_0^\infty e^{-\frac{\lambda^2}{2}u}e^{bZ_u}\mathbf{1}_{Z_u<0}\,\mathrm{d}u\right\}\end{aligned} \tag{B.6}$$

Note that the Laplace transform of the distribution of the first passage time of a Brownian motion $(Z_t)_{t\geq 0}$ to a level a yields

$$\begin{aligned}\mathbf{E}_{\mathbb{P}}\left\{e^{-\frac{\lambda^2}{2}T_a}\right\} &= e^{-|a|\sqrt{2\lambda^2/2}}\\ &= e^{-\lambda a} \quad \text{since } a>0\end{aligned}$$

From (**??**), it follows that

$$\mathbf{E}_{\mathbb{P}}\left\{\int_0^\infty e^{-\frac{\lambda^2}{2}u}e^{bZ_u}\mathbf{1}_{Z_u<0}\,\mathrm{d}u\right\} = \frac{1}{\lambda}\int_{-\infty}^{0} e^{by}e^{-|y|\lambda}\,\mathrm{d}y$$

$$= \frac{1}{r} \tag{B.7}$$

and so it follows that

$$\mathbf{E}_{\mathbb{P}}\left\{\int_{T_a}^{\infty} e^{-\frac{\lambda^2}{2}u} e^{bZ_u} \mathbf{1}_{Z_u<a}\,\mathrm{d}u\right\} = \frac{1}{r}\left(\frac{V^e}{V}\right)^{\xi} \tag{B.8}$$

We combine results in (B.3), (**??**), (B.6), (B.4), (B.7), (**??**) and (**??**) to conclude

$$\mathbf{E}_{\mathbb{Q}}\left\{\int_0^{T_{V^e}} e^{-ru}\ (\delta_0 - \psi)\ \Gamma(F_B, o_B, 0) V \mathbf{1}_{V_u<V^e}\mathrm{d}u\right\} = \Gamma(F_B, o_B, 0)\frac{(\delta_0-\psi)\ V}{r}\left[1 + \left[\frac{r}{\lambda(b-\lambda)} - 1\right]\left(\frac{V^e}{V}\right)^{\xi}\right] \tag{B.9}$$

Demolition costs We solve the second expectation in (5) as follows:

$$\mathbf{E}_{\mathbb{Q}}\left\{e^{-rT_{V^e}}\gamma(F_B)\right\} = \gamma(F_B)\left(\frac{V^e}{V}\right)^{\xi} \tag{B.10}$$

where we use consecutively the Girsanov theorem to change to the new measure $\mathbb{P}$ delineated above and the Laplace transform of the distribution of the first passage time of a Brownian motion $(Z_t)_{t\geq 0}$ to a level a with a and ξ defined in Proposition 1.

Development costs We can now move to the third integral in (5). This is equal to

$$\mathbf{E}_{\mathbb{Q}}\left\{\int_0^{H^+_{V^e,d}} e^{-ru}\kappa(F,\alpha)\mathbf{1}_{V_u>V^e}\,\mathrm{d}u\right\} = \kappa(F,\alpha)\mathbf{E}_{\mathbb{Q}}\left\{\int_0^{H^+_{V^e,d}} e^{-ru}\mathbf{1}_{V_u>V_e}\,\mathrm{d}u\right\}$$

we can rewrite this last integral as

$$\mathbf{E}_{\mathbb{Q}}\left\{\int_0^{H^+_{V^e,d}} e^{-ru}\mathbf{1}_{V_u>V^e}\,\mathrm{d}u\right\} = \mathbf{E}_{\mathbb{Q}}\left\{\int_0^{H^+_{V^e,d}} e^{-ru}\,\mathrm{d}u\right\} - \mathbf{E}_{\mathbb{Q}}\left\{\int_0^{H^+_{V^e,d}} e^{-ru}\mathbf{1}_{V_u<V^e}\,\mathrm{d}u\right\} \tag{B.11}$$

To solve the first integral in (B.11) we recall the following lemma which derives the Laplace transform for the Parisian time $H^+_{0,d}$ due to Chesney et al. (1997).

Lemma 2 *If $(W_t)_{t\geq 0}$ is a Brownian motion starting from zero, let ρ be a positive number, the Laplace transform of the first time a positive Brownian excursion lasts more than d is*

$$\mathbf{E}_{\mathbb{Q}}\left[e^{-\rho H^+_{0,d}}\right] = \frac{1}{\Phi(\sqrt{2\rho d})} \quad \textit{for all } \rho > 0 \tag{B.12}$$

with

$$\Phi(x) = \int_0^{+\infty} z\exp(zx - \frac{z^2}{2})\,\mathrm{d}z = 1 + \sqrt{2\pi}x\exp\left(\frac{x^2}{2}\right)\mathcal{N}(x)$$

with $\mathcal{N}(.)$ the standard normal cumulative distribution function.

We also recall the law of $Z_{H^+_{0,d}}$. This can be inferred from Jeanblanc et al. (2009) p.249

Lemma 3 *Let $(W_t)_{t\geq 0}$ be a Brownian motion and $H^+_d(W) = \inf\{t \geq 0 : (t - g_t(W)) \geq d, W_t \geq 0\}$. The random variables H^+_d and $W_{H^+_d}$ are independent and*

$$\mathbb{P}(W_{H^+_d} \in \mathrm{d}x) = \frac{x}{d} e^{-\frac{x^2}{2d}} \mathbf{1}_{x>0} \in \mathrm{d}x \tag{B.13}$$

We proceed to solving the first integral in (B.11) with a similar line of reasoning as in the previous calculation. This yields

$$\begin{aligned}
\mathbf{E}_{\mathbb{Q}}\left\{ \int_0^{H^+_{V^e,d}} e^{-ru}\,\mathrm{d}u \right\} &= \mathbf{E}_{\mathbb{Q}}\left\{ \int_0^{\infty} e^{-ru}\,\mathrm{d}u \right\} - \mathbf{E}_{\mathbb{Q}}\left\{ \int_{H^+_{V^e,d}}^{\infty} e^{-ru}\,\mathrm{d}u \right\} \\
&= \frac{1}{r}\left[1 - \frac{\Phi(b\sqrt{d})}{\Phi(\lambda\sqrt{d})} \left(\frac{V^e}{V}\right)^{\xi} \right]
\end{aligned} \tag{B.14}$$

The last integral in (B.11) can also be solved to obtain successively

$$\begin{aligned}
\mathbf{E}_{\mathbb{Q}}\left\{ \int_0^{H^+_{V^e,d}} e^{-ru}\mathbf{1}_{V_u<V^e}\,\mathrm{d}u \right\} &= \mathbf{E}_{\mathbb{Q}}\left\{ \int_0^{H^+_{a,d}} e^{-ru}\mathbf{1}_{Z_u<a}\,\mathrm{d}u \right\} \\
&= \mathbf{E}_{\mathbb{Q}}\left\{ \int_0^{\infty} e^{-ru}\mathbf{1}_{Z_u<a}\,\mathrm{d}u \right\} - \mathbf{E}_{\mathbb{Q}}\left\{ \int_{H^+_{a,d}}^{\infty} e^{-ru}\mathbf{1}_{Z_u<a}\,\mathrm{d}u \right\}
\end{aligned}$$

Using the Girsanov theorem to change the measure delineated in (B.2)

$$\begin{aligned}
\mathbf{E}_{\mathbb{Q}}\left\{ \int_0^{H^+_{V^e,d}} e^{-ru}\mathbf{1}_{V_u<V^e}\,\mathrm{d}u \right\} = \mathbf{E}_{\mathbb{P}}\Bigg\{ &\int_0^{\infty} e^{-\frac{\lambda^2}{2}u} e^{bZ_u} \quad \mathbf{1}_{Z_u<a}\,\mathrm{d}u \Bigg\} \\
&- \mathbf{E}_{\mathbb{P}}\left\{ \int_{H^+_{a,d}}^{\infty} e^{-\frac{\lambda^2}{2}u} e^{bZ_u}\mathbf{1}_{Z_u<a}\,\mathrm{d}u \right\}
\end{aligned} \tag{B.15}$$

with $\lambda = \sqrt{2r + b^2}$. We obtain next with lemma 1,

$$\mathbf{E}_{\mathbb{P}}\left\{ \int_0^{\infty} e^{-\frac{\lambda^2}{2}u} e^{bZ_u}\mathbf{1}_{Z_u<a}\,\mathrm{d}u \right\} = \frac{1}{\lambda}\int_{-\infty}^{a} e^{by} e^{-|y|\lambda}\,\mathrm{d}y$$

which after simplification becomes

$$\mathbf{E}_{\mathbb{P}}\left\{ \int_0^{\infty} e^{-\frac{\lambda^2}{2}u} e^{bZ_u}\mathbf{1}_{Z_u<a}\,\mathrm{d}u \right\} = \frac{1}{r} + \frac{1}{\lambda(b-\lambda)}\left(\frac{V^e}{V}\right)^{\xi} \tag{B.16}$$

We can now work on the last term

$$\begin{aligned}\mathbf{E}_{\mathbb{P}}\left\{\int_{H^+_{a,d}}^{\infty} e^{-\frac{\lambda^2}{2}u}e^{bZ_u}\mathbf{1}_{Z_u<a}\,\mathrm{d}u\right\} &= \mathbf{E}_{\mathbb{P}}\left\{e^{-\frac{\lambda^2}{2}T_a}\int_{H^+_{0,d}}^{\infty} e^{-\frac{\lambda^2}{2}u}e^{b(Z_u+a)}\mathbf{1}_{Z_u<0}\,\mathrm{d}u\right\}\\ &= e^{ba-|a|\lambda}\mathbf{E}_{\mathbb{P}}\left\{e^{-\frac{\lambda^2}{2}H^+_{0,d}}\int_0^{\infty} e^{bZ_{u+H^+_{0,d}}}e^{-\frac{\lambda^2}{2}u}\mathbf{1}_{Z_{u+H^+_{0,d}}<0}\,\mathrm{d}u\right\}\end{aligned}$$

Which is equal to

$$\frac{1}{\Phi(\lambda\sqrt{d})}\left(\frac{V^e}{V}\right)^{\xi}\int_0^{\infty}\mathbb{P}(Z_{H^+_{0,d}}\in\,\mathrm{d}x)e^{bx}\mathbf{E}_{\mathbb{P}}\left\{\int_0^{\infty} e^{-\frac{\lambda^2}{2}u}e^{bZ_u}\mathbf{1}_{Z_u<-x}\,\mathrm{d}u\right\} \tag{B.17}$$

but

$$\mathbf{E}_{\mathbb{P}}\left\{\int_0^{\infty} e^{-\frac{\lambda^2}{2}u}e^{bZ_u}\mathbf{1}_{Z_u<-x}\,\mathrm{d}u\right\} = \frac{1}{\lambda}\int_{-\infty}^{-x} e^{by}e^{-|y|\lambda}\,\mathrm{d}y$$

since $x>0$ then $y<0$. It follows that

$$\begin{aligned}\frac{1}{\lambda}\int_{-\infty}^{-x} e^{by}e^{-|y|\lambda}\,\mathrm{d}y &= \frac{1}{\lambda}\int_{-\infty}^{-x} e^{(b+\lambda)y}\,\mathrm{d}y\\ &= \frac{e^{-(b+\lambda)x}}{\lambda(b+\lambda)}\end{aligned} \tag{B.18}$$

Combining (B.17) and (B.18) yields

$$\begin{aligned}\mathbf{E}_{\mathbb{P}}\left\{\int_{H^+_{a,d}}^{\infty} e^{-\frac{\lambda^2}{2}u}e^{bZ_u}\mathbf{1}_{Z_u<a}\,\mathrm{d}u\right\} &= \frac{1}{\Phi(\lambda\sqrt{d})}\left(\frac{V^e}{V}\right)^{\xi}\int_0^{\infty}\frac{x}{d}e^{-\frac{x^2}{2d}}e^{bx}\frac{e^{-(b+\lambda)x}}{\lambda(b+\lambda)}\,\mathrm{d}x\\ &= \frac{1}{\lambda(b+\lambda)}\left(\frac{V^e}{V}\right)^{\xi}\frac{1}{\Phi(\lambda\sqrt{d})}\int_0^{\infty}\frac{x}{d}e^{-\frac{x^2}{2d}-\lambda x}\,\mathrm{d}x\end{aligned}$$

We proceed next with a change of variables $x=\sqrt{d}y$ and conclude

$$\begin{aligned}\mathbf{E}_{\mathbb{P}}\left\{\int_{H^+_{a,d}}^{\infty} e^{-\frac{\lambda^2}{2}u}e^{bZ_u}\mathbf{1}_{Z_u<a}\,\mathrm{d}u\right\} &= \frac{1}{\lambda(b+\lambda)}\left(\frac{V^e}{V}\right)^{\xi}\frac{1}{\Phi(\lambda\sqrt{d})}\int_0^{\infty} ye^{-\frac{y^2}{2}-\lambda\sqrt{d}y}\,\mathrm{d}y\\ &= \frac{1}{\lambda(b+\lambda)}\left(\frac{V^e}{V}\right)^{\xi}\frac{\Phi(-\lambda\sqrt{d})}{\Phi(\lambda\sqrt{d})}\end{aligned} \tag{B.19}$$

Finally combining (B.16) and (B.19) yields the last integral from (B.11),

$$\mathbf{E}_{\mathbb{Q}}\left\{\int_0^{H^+_{V^e,d}} e^{-ru}\mathbf{1}_{V_u<V^e}\,\mathrm{d}u\right\} = \frac{1}{r}\left[1-\left[\frac{b+\lambda}{2\lambda}-\frac{b-\lambda}{2\lambda}\frac{\Phi(-\lambda\sqrt{d})}{\Phi(\lambda\sqrt{d})}\right]\left(\frac{V^e}{V}\right)^{\xi}\right] \tag{B.20}$$

Combining (B.11), (B.14) and (B.20) yields

$$\mathbf{E}_{\mathbb{Q}}\left\{\int_0^{H^+_{V^e,d}} e^{-ru}\mathbf{1}_{V_u>V^e}\,\mathrm{d}u\right\} = \frac{1}{r}\left[\frac{b+\lambda}{2\lambda} - \frac{b-\lambda}{2\lambda}\frac{\Phi(-\lambda\sqrt{d})}{\Phi(\lambda\sqrt{d})} - \frac{\Phi(b\sqrt{d})}{\Phi(\lambda\sqrt{d})}\right]\left(\frac{V^e}{V}\right)^{\xi} \tag{B.21}$$

Value after development The expectation related to the present value of the proceeds after the stopping time $H^+_{V^e,d}$ in (5) becomes

$$\mathbf{E}_{\mathbb{Q}}\left\{\int_{H^+_{V^e,d}}^{\infty} e^{-ru}\delta\,\Gamma(F,\alpha)V_u\,\mathrm{d}u\right\} = \delta\,\Gamma(F,\alpha)\mathbf{E}_{\mathbb{Q}}\left\{\int_{H^+_{a,d}}^{\infty} e^{-ru}Ve^{\sigma Z_u}\,\mathrm{d}u\right\}$$

After the change of measure delineated in (B.2) the last expectation yields

$$\delta\,\Gamma(F,\alpha)\mathbf{E}_{\mathbb{Q}}\left\{\int_{H^+_{a,d}}^{\infty} e^{-ru}Ve^{\sigma Z_u}\,\mathrm{d}u\right\} = \delta\,\Gamma(F,\alpha)V\,\mathbf{E}_{\mathbb{P}}\left\{\int_{H^+_{a,d}}^{\infty} e^{-\frac{\lambda^2}{2}u}e^{(b+\sigma)Z_u}\,\mathrm{d}u\right\}$$

We combine results from (B.6), (B.4), (B.7), (**??**) and (**??**) to solve this expectation.

$$\delta\,\Gamma(F,\alpha)V\,\mathbf{E}_{\mathbb{P}}\left\{\int_{H^+_{a,d}}^{\infty} e^{-\frac{\lambda^2}{2}u}e^{(b+\sigma)Z_u}\,\mathrm{d}u\right\} = \Gamma(F,\alpha)\frac{\Phi\left((\sigma+b)\sqrt{d}\right)}{\Phi(\lambda\sqrt{d})}\left(\frac{V^e}{V}\right)^{\xi}V^e \tag{B.22}$$

Property taxes after τ years of tax credits The last expectation related to the present value of the property taxes after τ years of tax exemptions in (5) is solved as follows,

$$\mathbf{E}_{\mathbb{Q}}\left\{\int_{H^+_{V^e,d}+\tau}^{\infty} e^{-ru}\psi\,\Gamma(F,\alpha)V_u\,\mathrm{d}u\right\} = \psi\,\Gamma(F,\alpha)\mathbf{E}_{\mathbb{Q}}\left\{\int_{H^+_{a,d}+\tau}^{\infty} e^{-ru}Ve^{\sigma Z_u}\,\mathrm{d}u\right\}$$

After the change of measure delineated in (B.2) the last expectation becomes

$$\psi\,\Gamma(F,\alpha)\mathbf{E}_{\mathbb{Q}}\left\{\int_{H^+_{a,d}+\tau}^{\infty} e^{-ru}Ve^{\sigma Z_u}\,\mathrm{d}u\right\} = \psi\,\Gamma(F,\alpha)V\,\mathbf{E}_{\mathbb{P}}\left\{\int_{H^+_{a,d}}^{\infty} e^{-\frac{\lambda^2}{2}(u+\tau)}e^{(b+\sigma)Z_{u+\tau}}\,\mathrm{d}u\right\}$$

Thanks to the equality in law between $H^+_{a,d}$ and $H^+_{0,d}+T_a$ for two independent copies, the independence of T_a and $H^+_{0,d}$, the Laplace transform of the distribution of the first passage time of a Brownian motion $(Z_t)_{t\geq 0}$ to a level a, and the Independence between $H^+_{0,d}$ and $Z_{H^+_{0,d}}$ we can write

$$\mathbf{E}_{\mathbb{P}}\left\{\int_{H^+_{a,d}}^{\infty} e^{-\frac{\lambda^2}{2}(u+\tau)}e^{(b+\sigma)Z_{u+\tau}}\,\mathrm{d}u\right\} =$$
$$e^{(b+\sigma)a}\mathbf{E}_{\mathbb{P}}\left\{e^{-\frac{\lambda^2}{2}T_a}\right\}\mathbf{E}_{\mathbb{P}}\left\{e^{-\frac{\lambda^2}{2}H^+_{0,d}}\right\}\mathbf{E}_{\mathbb{P}}\left\{e^{(b+\sigma)Z_{H^+_{0,d}}}\right\}\mathbf{E}_{\mathbb{P}}\left\{\int_0^{\infty} e^{-\frac{\lambda^2}{2}(u+\tau)}e^{(b+\sigma)Z_{u+\tau}}\,\mathrm{d}u\right\}$$

and the last integral becomes

$$\mathbf{E}_{\mathbb{P}}\left\{\int_0^{\infty} e^{-\frac{\lambda^2}{2}(u+\tau)}e^{(b+\sigma)Z_{u+\tau}}\,\mathrm{d}u\right\} = e^{-\lambda^2\frac{\tau}{2}}\mathbf{E}_{\mathbb{P}}\left\{e^{(b+\sigma)Z_\tau}\right\}\mathbf{E}_{\mathbb{P}}\left\{\int_0^{\infty} e^{-\frac{\lambda^2}{2}u}e^{(b+\sigma)Z_u}\,\mathrm{d}u\right\}$$

Z_τ is a Brownian motion under $\mathbb{P}$ distributed $\mathcal{N}(0,\tau)$. We recall the following result from Borodin and Salminen (2015) p. 153

Lemma 4 *Let $(W_t)_{t\geq 0}$ be a Brownian motion. The expectation of $e^{\beta W_t}$ at time T with the process $(W_t)_{t\geq 0}$ started at time x is*

$$\mathbf{E}_x\left\{e^{\beta W_T}\right\} = e^{\beta x - \frac{\beta^2 T}{2}}$$

it follows that

$$\mathbf{E}_{\mathbb{P}}\left\{\int_0^{\infty} e^{-\frac{\lambda^2}{2}(u+\tau)}e^{(b+\sigma)Z_{u+\tau}}\,\mathrm{d}u\right\} = e^{-\lambda^2\frac{\tau}{2}}e^{-(b+\sigma)^2\frac{\tau}{2}}\mathbf{E}_{\mathbb{P}}\left\{\int_0^{\infty} e^{-\frac{\lambda^2}{2}u}e^{(b+\sigma)Z_u}\,\mathrm{d}u\right\}$$

Combining result from (B.6), (B.4), (B.7), (**??**) and (**??**) yields

$$\mathbf{E}_{\mathbb{Q}}\left\{\int_{H^+_{V^e,d}+\tau}^{\infty} e^{-ru}\psi\,\Gamma(F,\alpha)V_u\,\mathrm{d}u\right\} = \Gamma(F,\alpha)\frac{\psi h}{\delta}\frac{\Phi\left((\sigma+b)\sqrt{d}\right)}{\Phi(\lambda\sqrt{d})}\left(\frac{V^e}{V}\right)^{\xi}V^e$$

with $h = e^{-\left[(b+\sigma)^2+\lambda^2\right]\frac{\tau}{2}}$. This concludes the proof.

B.2 Proof of Proposition 2.

We start by rearranging the property value in equation (6) in the following way:

$$\pi(V,V^e,F_B,F,\alpha,\phi,\tau;\Theta) = \Gamma(F_B,o_B,0)\frac{\delta_0-\psi}{r}V + \left[\Gamma(F_B,o_B,0)\frac{\delta_0-\psi}{r}V\left[\frac{r}{\lambda(b-\lambda)}-1\right]\right.$$
$$\left. + \vartheta V^e - \gamma(F_B) - \frac{\kappa(F,\alpha)}{r}\Big(B(d)-A(d)\Big)\right]\left(\frac{V^e}{V}\right)^{\xi}$$

with $\vartheta = \Gamma(F,0,\alpha)\frac{\delta-\psi h}{\delta}C(d)$.

We establish the expansion threshold under regulatory environment $\mathcal{P}$, denoted as V^{e*}, as the parameter that optimizes the property value. In other words,

$$V^{e*} = \arg\max \pi(V,V^e,F_B,F,\alpha,\phi,\tau;\Theta)$$

It follows that V^{e*} must satisfy the first order conditions and the second order conditions. The partial derivative of $\pi(V,V^e,F_B,F,\alpha,\phi,\tau;\Theta)$ with respect to V^e is equal to

$$\frac{\partial\pi(V,V^e,F_B,F,\alpha,\phi,\tau;\Theta)}{\partial V^e} = \vartheta\left(\frac{V^e}{V}\right)^{\xi} + \frac{\xi}{V}\left[\Gamma(F_B,o_B,0)\frac{\delta_0-\psi}{r}V\left[\frac{r}{\lambda(b-\lambda)}-1\right]\right.$$

$$+\vartheta V^e-\gamma(F_B)-\frac{\kappa(F,\alpha)}{r}\Big(B(d)-A(d)\Big)\Bigg]\left(\frac{V^e}{V}\right)^{\xi-1}$$

Which can be simplified to

$$\left(\frac{V^e}{V}\right)^{\xi}\Bigg[\vartheta+\frac{\xi}{V^e}\Bigg[\Gamma(F_B,o_B,0)\frac{\delta_0-\psi}{r}V\left[\frac{r}{\lambda(b-\lambda)}-1\right]$$
$$+\vartheta V^e-\gamma(F_B)-\frac{\kappa(F,\alpha)}{r}\Big(B(d)-A(d)\Big)\Bigg]$$

We know that $V^e > V$ so $V^e \neq 0$. This implies that first order conditions are satisfied if and only if

$$\vartheta+\frac{\xi}{V^e}\left[\Gamma(F_B,o_B,0)\frac{\delta_0-\psi}{r}V\left[\frac{r}{\lambda(b-\lambda)}-1\right]+\vartheta V^e-\gamma(F_B)-\frac{\kappa(F,\alpha)}{r}\Big(B(d)-A(d)\Big)\right]=0$$

some further simplification yields V^{e*}.

$$V^{e*}=\frac{\xi}{1+\xi}\frac{1}{\vartheta}\left[\gamma(F_B)+\frac{\kappa(F,\alpha)}{r}\Big(B(d)-A(d)\Big)-\Gamma(F_B,o_B,0)\frac{\delta_0-\psi}{r}V\left[\frac{r}{\lambda(b-\lambda)}-1\right]\right]$$

B.3 Proof of Proposition 3.

Recall that $V_t = Ve^{\sigma Z_t}$, where $Z_t = bt + W_t$, and $b = \frac{1}{\sigma}(r-\delta-\frac{\sigma^2}{2})$. This implies that $T^{V^{e*}}(V) = \inf\{t \geq 0 : Z_t = a\}$ with $a = (1/\sigma)\ln(V^{e*}/V)$. Recall that $(W_t)_{t\geq 0}$ is a Brownian motion under the risk neutral measure $\mathbb{Q}$ and that after we change the probability measure to a new measure $\mathbb{P}$, $(Z_t)_{t\geq 0}$ becomes a Brownian motion under $\mathbb{P}$. We recall the law of the hitting time $T^{V^{e*}}$ of the level V^{e*} from Jeanblanc et al. (2009) p.140

Lemma 5 *Let W_t be a Brownian motion and for any $x > 0$ define $T_x(W) = \inf\{t \geq 0 | W_t = x\}$. The density of the random variable T_x is given by $\mathbf{P}(T_x \in \mathrm{d}t) = \frac{x}{\sqrt{2\pi t^3}}\exp\left(-\frac{x^2}{2t}\right)\mathrm{d}t$*

We use Lemma 5 to conlude the proof.