Multinational Production and Trade Policy*

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Abstract

Multinational enterprises dominate international trade, yet much of the literature on optimal trade policy has overlooked their significance. We show that in 2018-2019 the Trump administration prioritized tariffs on goods from China that belong to industries with a higher related-party share, and that imports from China declined more for goods that belong to such industries. Theoretically, we show that multinational production shapes the aggregate trade elasticity, via the substitution of labor inside multinational enterprises across different countries of production. When markets are competitive and abstracting from entry costs to become a multinational enterprise, we obtain a neutrality result for optimal tariffs: unilaterally optimal second-best tariffs are uniform across sectors and countries, despite variations in multinational activity. In imperfectly competitive markets, governments have an incentive to set optimal tariffs that shift profits toward domestically owned multinationals.

*The views and opinions expressed in this work do not necessarily represent the views of the Federal Reserve Bank of New York or the Federal Reserve System.

1 Introduction

Multinational enterprises (MNEs) carry out most international commerce and account for around a quarter of world GDP. In the U.S., they account for a quarter of private sector employment and around 70 percent of aggregate exports and imports (Antràs et al., 2023). In recent years, trade policy has risen again to the forefront of the economic policy debate. Despite the large importance of MNEs for international trade flows, nearly all existing studies of welfare maximizing trade policies have ignored them. The handbook chapter on trade policy by Caliendo and Parro (2022) provides a recent summary of the literature.

In 2018-2019, the US and China entered into a trade conflict that resulted in both countries imposing significantly higher tariffs on each other's goods. The conflict escalated in waves of tariff increases, with several lists of goods imported from China receiving subsequently higher tariffs. Notably, the US administration appeared to select the initial tariffed goods with multinational firms' activities in mind: products in the first two waves of tariffs had significantly higher shares of related-party imports—trade between parent companies and their foreign affiliates—compared to products in later lists or those not subjected to tariffs at all. Moreover, we show that products from industries with higher related-party trade shares experienced more substantial declines in trade compared to products that are mostly traded at arm's length.

In this paper, we explore how multinational production (MP) shapes the response of bilateral trade flows to tariffs and how this, in turn, influences the optimal tariff rate. To do so, we develop a simple general equilibrium model of trade and horizontal multinational production.

In our model, firms are divided into two categories: multinational companies and local firms. Both types operate under monopolistic competition, and can sell their variety where it is produced or export it. While local firms can only produce at their headquarters, multinational companies can produce globally. The selection into becoming a global firm is endogenized by the presence of fixed costs. Labor is the sole factor of production, but multinational firms face additional bilateral frictions that reduce the marginal product of labor in foreign affiliates.

Trade flows in our model are affected by tariffs and exogenous iceberg costs, which are common across multinationals and local firms. Goods are differentiated by their country of origin (i.e., the origin country of a domestic firm or multinational enterprise). All goods from a given origin country are further differentiated by their location of production. This setup implies that multinational production influences how trade flows respond to tariffs, i.e., the trade elasticity. When all frictions to MP are eliminated, the key factor affecting the trade elasticity is the substitution between locations of production within a firm of a given origin country. On the other hand, when MP frictions are prohibitively high, all production occurs domestically, and the trade elasticity is driven by the substitution between goods from different origin countries. For intermediate levels of MP frictions, we characterize how the trade elasticity depends on two Armington elasticity parameters and observable measures of multinational activity. Since multinational activity responds to tariffs, the trade elasticity is not constant but adjusts according to the tariff rates.

Our model builds on Antràs et al. (2024). As in their paper, our model contains a double CES Armington structure, governing the substitutability between goods originating in different countries as well as the substitutability of labor between countries inside the multinational enterprises. We focus on two types of firms, global and local firms, and incorporate multiple sectors, that each may have heterogeneous technologies and frictions to take ideas abroad for production. When the fixed costs to becoming a global firm are eliminated and all mark-ups are removed, the general equilibrium structure of the model becomes isomorphic to a special case of the model by Ramondo and Rodríguez-Clare (2013). However, unlike in their model, the trade elasticity in our framework can either decrease or increase as the frictions to multinational production are reduced.

We study optimal tariffs in an open economy consisting of a small Home country and multiple large foreign countries. To model the small open economy, we extend the methods from Caliendo and Feenstra (2024) and Demidova et al. (2024) to our quantitative trade framework, which incorporates multinational production. The small-country assumption allows us to hold foreign wages constant and to derive analytical solutions for optimal trade policy using the dual approach from Lucas and Stokey (1983) and Costinot et al. (2015). We have two main theoretical results: First, we characterize analytically the aggregate trade elasticity and relate it to measures of multinational firms' activities. Second, we derive the unilateral second-best optimal import tariffs in a special case of our model without mark-ups and no fixed costs of becoming a global firm. For a small open economy, the optimal import tariff is inversely related to the weighted sum of the trade elasticity for exports of the tariff-imposing country, and is uniform across countries and sectors.

These theoretical findings suggest that the U.S. policy of selecting higher tariffs for certain products with higher shares of intra-firm trade activity was suboptimal. We explore this further in a quantitative analysis of our model, with multiple sectors and countries. This quantitative analysis is still in progress and incomplete. One aspect of the general model that appears to play an important role in the quantitative analysis is the profit-shifting motive: when accounting for imperfect competition we obtain noticably higher optimal import tariffs. Interestingly, imperfect competition would play no role for the optimal tariff level in a special case of our model when the frictions to MP are prohibitive.

Significant progress has been made in the analysis of optimal tariffs for both large and small open economies since the work of Gros (1987). A consistent finding across various microfoundations is that even small countries can exploit their market power abroad to improve their terms of trade by imposing tariffs, with the effectiveness of this strategy closely tied to the trade elasticity (see Caliendo and Parro (2022) for a comprehensive summary). Costinot et al. (2020) show that larger and more productive firms respond more strongly to changes in trade policy, while Antràs and Helpman (2004) highlight how firms' decisions to vertically integrate or outsource abroad shape global trade flows. Perhaps surprisingly, however, the literature has given limited attention to multinational enterprises—the largest and more productive firms—which should play a central role in shaping trade policy outcomes. In a slightly different context, a notable exception is the work by Dyrda et al. (2024), which explores the optimal design of the international *corporate tax* system to

address profit-shifting motives in an economy with multinational firms. While tariffs also generate profit-shifting, they are not considered in the authors' analysis.

It has been widely recognized that tariffs can lead to tariff jumping FDI (e.g., Goldberg (1995)) and that even the threat of tariffs can induce foreign investment (Blonigen and Feenstra (1996)). In a two-country model of multinational production with a proximity concentration trade-off (Helpman et al. (2004)), Cole and Davies (2011) show that Nash equilibrium tariffs are lower when accounting for FDI. Tintelnot (2017) and Flaaen and Pierce (2019) show that tariffs can lead to third-country effects when considering export-platform FDI. Ludema (2002) analyzes preferential trade agreements in the presence of multinational enterprises. Holmes et al. (2015) examine the interaction of technology transfer via multinational production and market access in China. We contribute to this literature by developing a tractable multi-sector and multi-country framework for trade and multinational production, characterizing how multinational production shapes the trade elasticity, and solving for the optimal (second-best) tariff policies.

The rest of the paper is structured as follows. Section 2 describes our empirical findings on the selection of tariffs and the responses of trade flows during the US-China trade war. Section 3 presents our model and Section 4 presents our main theoretical findings. Section 5 describes the calibration of the quantitative model and provides numerical results on optimal import tariffs. Section 6 concludes.

2 Motivating Empirical Facts

In this section, we present empirical facts regarding related party imports and tariffs during the recent U.S.-China trade war to motivate the setup of our model. We first describe the data and then present the empirical findings.

2.1 Data for Descriptive Analyses

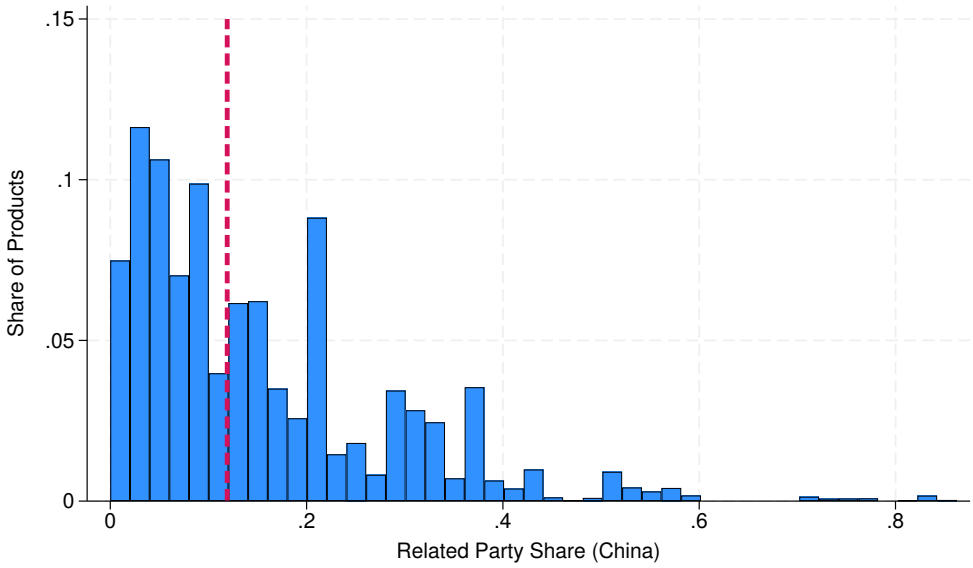
We use three main datasets. First, we obtain monthly trade data on U.S. imports at the 10-digit HS code by country level over the period 2016-2020 from the U.S. Census Bureau.[1] These data contain information on the product code and description, 6-digit NAICS industry, exporting country, value, and quantity, from which we construct the duty-exclusive unit price for each country-product-month triplet. We drop any observations with missing NAICS industry.

Second, we use monthly data on base tariff rates prior to the trade war at the HS10-country level from the U.S. International Trade Commission. In 2019 and 2020, to ensure that we properly capture the additional trade war tariffs, we compare the base tariff rates against USTR documents and update the ad valorem tariffs when necessary to reflect trade war changes.[2] For each country-product-month triplet, the data contain information on the base tariff rate, additional ad valorem

[1] https://id-provider.tco.census.gov/nidp/saml2/sso?id=kerberos/ftdsecure&sid=1&option=credential&sid=1. Raw data can be downloaded from this link with a free account.

[2] See https://dataweb.usitc.gov/tariff/annual and https://ustr.gov/issue-areas/enforcement/section-301-investigations/tariff-actions.

Figure 1: Related Party Share from China



or specific tariffs, and the effective tariff rate.

Our third main dataset is the share of imports from China that is done by *related parties*. We compute the share of related party imports in total imports from China for each 6-digit NAICS industry using data from the U.S. Census Bureau for 2017.[3] We map the industry-level related party shares to their associated HS10 codes in the trade data using the concordance provided by the Census Bureau. Since the related party share is reported as an industry-level statistic, all HS10 products that belong to the same industry will have the same related party share. Our final dataset covers 20,077 HS10 products from 233 countries over the period 2016-2020.

Figure 1 provides a histogram of the distribution of the related party share from China across HS10 products in 2017. For most products, the share of related party imports from China is relatively low. The mean is 0.15 and the median is 0.12 (indicated by the red vertical line). However, there is considerable heterogeneity, and for some products the related party share is as high as 0.8.

2.2 Policy During the Recent U.S.-China Trade War

We now describe the tariff changes during the U.S.-China trade war in 2018-19 and how these changes correlated with related party trade. During the trade war, there were several rounds of Section 301 tariffs placed on a wide variety of products from China. These rounds were announced and implemented throughout 2018 and 2019 in the form of four *lists*. List 1, which was implemented on July 6, 2018, set a tariff of 25% on 1,678 10-digit HS products with a total trade volume of $34

[3] https://www.census.gov/foreign-trade/Press-Release/related_party/index.html. Based on Section 402(e) of the Tariff Act of 1930, related party trade includes import transactions between parties with various types of relationships including "any person directly or indirectly, owning, controlling, or holding power to vote, 6 percent of the outstanding voting stock or shares of any organization".

Table 1: Related party (RP) Share of Imports from China

	All		Intermediates		Final goods	
	Avg. share	Median share	Avg. share	Median share	Avg. share	Median share
Round 1	0.313	0.301	0.335	0.327	0.303	0.284
Round 2	0.272	0.296	0.244	0.204	0.306	0.333
Round 3	0.145	0.117	0.143	0.101	0.143	0.117
Round 4	0.111	0.066	0.135	0.146	0.095	0.048
Non-targeted	0.161	0.130	0.185	0.183	0.114	0.071

Notes: Goods are classified as intermediate or final according to the Broad Economic Categories rev. 5 (BEC5) code corresponding to their HS code. BEC5 codes with a third digit of 1 are classified as intermediate goods and BEC5 codes with a third digit of 2 or 3 (capital goods or goods for consumption) are classified as final consumption goods. Dual use goods are not included in these calculations.

billion.[4] List 2 was implemented on August 23 and set a tariff of 25% on 444 additional products worth $16 billion. These goods included, for example, semiconductors, chemicals, and plastics. List 3, implemented on September 24, set tariffs at an initial rate of 10% for a further 9,064 products with a value of $200 billion. While these tariffs were set to increase to 25% on January 1, 2019, this planned increase was subsequently delayed and only came into effect on May 10, 2019. Finally, list 4 was implemented on September 1, 2019, and set tariffs of 15% for 5,993 products worth $125 billion. These goods included for example footwear, diapers, and food products. Additional tariffs were planned as part of list 4 for December 15, but these tariffs were not implemented. The list 4 tariffs were reduced to 7.5% on February 14, 2020. The remaining 2,898 products in our data were not subject to new tariffs for China.

The Trump administration appears to have taken multinational activities into account when selecting products for the tariff lists. Column 1 of Table 1 shows the simple average of the related party share of trade from China for the products for each of the four tariff lists, and column 2 shows the median. We find that the related party share was highest for the products that were hit with tariffs earlier, on lists 1 and 2. The related party share was significantly lower for lists 3 and 4, and in fact below the share for those products that were not hit with tariffs.

Columns 3-6 present similar statistics by end use category. Goods are classified as intermediate or final according to the Broad Economic Categories rev. 5 (BEC5) code corresponding to their HS code. BEC5 codes with a third digit of 1 are classified as intermediate goods and BEC5 codes with a third digit of 2 or 3 (capital goods or goods for consumption) are classified as final goods. While the Trump administration targeted predominantly intermediate goods on the earlier lists, the related party share pattern holds within both intermediates and final goods.[5]

[4]While the tariffs were imposed at the 8-digit HS level, we report here the number of 10-digit codes hit by the tariff to be consistent with the level of the observations of our dataset.

[5]Appendix A contains an alternative version of the table where capital goods are classified as intermediates. The same patterns hold.

Table 2: Change in U.S. Imports ($bn), 2017-2020

Panel a: Total imports by source						
	Not Tariffed			Tariffed		
	2017	2020	% Change	2017	2020	% Change
China	204.5	208.9	+2.2%	321.7	248.2	−22.8%
Rest of World	297.6	364.9	+22.6%	1,585.8	1,585.4	−0.0%
Panel b: Imports from China by related party status						
	Not Tariffed			Tariffed		
	2017	2020	% Change	2017	2020	% Change
Low related party share	41.9	44.0	+5.0%	120.1	104.0	−13.4%
High related party share	162.6	165.0	+1.4%	201.6	144.3	−28.4%

Source: U.S. Census Bureau and authors' calculations. Figures are in billions of U.S. dollars. Panel a shows the total U.S. imports by source in 2017 and 2020. "Not tariffed" refers to HS10 codes that were not on the tariff lists by the Trump administration. "Tariffed" refers to codes on the tariff lists. Panel b focuses on U.S. imports from China and splits these by tariff status and related party share. Low related party share refers to products with a below-median share of related party imports from China. High related party share refers to products with an above-median share of related party imports from China.

2.3 Aggregate Trade Flows

We next present empirical facts regarding the response of related party imports to changes in tariffs to motivate the setup of our model. Panel a of Table 2 shows the value of U.S. imports from China and the rest of the world, just before the trade war in 2017 and in 2020, by tariff status. Products that were on one of the four tariff lists implemented by the Trump administration are referred to as "tariffed". All other goods are "non-tariffed". The table shows that imports of non-tariffed goods grew by 2.2% for imports from China and by 22.6% for imports from other countries. Imports of tariffed products from China contracted strongly, falling by 22.8%, consistent with previous work that has documented a significant decline in U.S. imports from China following the imposition of tariffs (e.g., Fajgelbaum et al. (2021), Dang et al. (2023)).

Panel b of Table 2 zooms in on China and analyzes the change in U.S. imports by related party share. We refer to HS10 products with an above-median Chinese related party share as "high" related party share products and to all others as "low". The panel shows that for the tariffed products, trade declined by nearly twice as much for products with a high share of related party trade than for products with a low share, providing suggestive evidence that these products are more sensitive to tariffs. We further examine this in the next section.

2.4 Tariff Regressions

We next analyze more formally how heterogeneity in the related party share influences the effect of tariffs on U.S. imports, using similar regressions as Fajgelbaum et al. (2020). We implement reduced-form regressions and analyze the effect of monthly tariff changes on import quantities. Let i index a source country, g an HS10 product, t a month between 2016:m1 and 2020:m12, $s(g)$ the 4-digit NAICS sector of the imported good, and $n(g)$ the 6-digit NAICS industry associated with

Table 3: Tariffs and Related Party Share

	(1) No related share	(2) Baseline	(3) P50 dummy	(4) Quarterly
$\Delta \ln(1+\tau_{igt})$	-0.9890***	-0.7221***	-0.7420***	-0.7593***
	(0.1269)	(0.1388)	(0.1533)	(0.2702)
$\Delta \ln(1+\tau_{igt}) \cdot \phi^{CN}_{n(g)}$		-1.4817***		-3.2956***
		(0.4583)		(0.6171)
$\Delta \ln(1+\tau_{igt}) \cdot \mathbb{I}(\phi^{CN}_{n(g)} > P50)$			-0.3468**	
			(0.1715)	
Product-Time Fixed effects	Yes	Yes	Yes	Yes
Country-Time Fixed effects	Yes	Yes	Yes	Yes
Country-Sector Fixed Effects	Yes	Yes	Yes	Yes
R2	0.1252	0.1252	0.1252	0.1177
Observations	5,537,684	5,537,684	5,537,684	2,601,559

Source: Census Bureau and authors' calculations. Table reports the results of regressing the monthly change in the quantity imported (m_{igt}) on the change in the tariff (τ_{igt}) as well as the tariff change interacted with the related party share. The regressions include product-time, country-time, and country-sector fixed effects. Standard errors, adjusted for clustering by HS8 product code and country are reported below coefficient estimates. ***, **, and * represent statistical significance at the 1, 5 and 10 percent levels.

good g. We run specifications of the form

$$\Delta \ln m_{igt} = \eta_{gt} + \eta_{it} + \eta_{is(g)} + \sigma \Delta \ln(1+\tau_{igt}) + \rho \Delta \ln(1+\tau_{igt}) \cdot \phi^{CN}_{n(g)} + \varepsilon_{igt}, \tag{1}$$

where m_{igt} is the quantity imported from country i of good g in month t, η_{gt}, η_{it}, and $\eta_{is(g)}$ are product-time, country-time, and country-sector fixed effects, respectively, and τ_{igt} is the statutory tariff rate for the product and country. The variable $\phi^{CN}_{n(g)}$ is the share of imports of good g from China that is sourced via a related party in 2017. Our coefficient of interest is ρ, which captures whether the trade elasticity differs for products based on the related party share.[6]

We first run the regression excluding the related party term and report the results in column 1 of panel a of Table 3. Standard errors are two-way clustered at the 8-digit HS code and country level. This regression corresponds to the reduced-form regression run by Fajgelbaum et al. (2020). The results indicate a tariff elasticity of about one, i.e., a one percent increase in tariffs is associated with a one percent decline in quantity imported. This elasticity is slightly lower than in Fajgelbaum et al. (2020) due to the different time period used.

The second column runs our baseline specification (1). We find that the tariff elasticity is significantly higher in absolute value when the related party share is higher: at the mean tariff change (0.08), going from the 25th percentile to the 75th percentile of the related party share from China (from 0.05 to 0.21) is associated with an additional decline in imports of 2 log points.

In column 3, we re-run the regression but replace the continuous related party share for imports from China with a dummy variable that is equal to one if the related party share is above the median (0.12), $\mathbb{I}(\phi^{CN}_{n(g)} > P50)$. The results are consistent with our earlier findings: at the median

[6]Note that we do not need to include $\phi^{CN}_{n(g)}$ on its own since it is absorbed by the product-time fixed effect.

tariff change, going from a related party share below the median to one above the median reduces imports by about 2.5 log points.

Since our regressions are run at the monthly level, our dataset contains many missing observations for cases when a particular country-product is not imported in a given month. This feature of the data may bias our results. In column 4, we therefore aggregate import quantities to the quarterly level, and use the average tariff rate of the quarter. We find similar results, with a stronger significance of the interaction term.

Related party transactions may be subject to transfer pricing, with affiliates lowering the import price after the tariff change to reduce the tariff bill. Since our dependent variable is the quantity traded rather than import values, our coefficients are not due to a mechanical price effect. In Appendix A, we re-run our regressions with the change in price as dependent variable and show that we do not find a significant difference in price setting between industries with a low versus high related-party share.

Another concern with our results so far is that they could be driven by the list number, rather than by the related party share, for example due to anticipatory effects for later lists when the related party share was lower. To address this concern, we re-run our regression with additional controls:

$$\begin{aligned}\Delta \ln m_{igt} = \eta_{gt} + \eta_{it} + \eta_{is(g)} + \sigma \Delta \ln(1+\tau_{igt}) + \rho \Delta \ln(1+\tau_{igt}) \cdot \phi^{CN}_{n(g)} \\ + \sum_k \gamma_k \Delta \ln(1+\tau_{igt}) \cdot \mathbb{I}(\text{List} = k) + \varepsilon_{igt},\end{aligned}$$

where $\mathbb{I}(\text{List} = k)$ is a dummy for whether good g is on tariff list k. This specification conditions on the tariff effects of a given list and identifies the related party effect separately.

Table 4 shows that our results also hold conditional on the tariff list. We repeat columns 2-4 from above in this table and find that the coefficient on the related party share remains negative, although it is insignificant in the monthly regression with the continuous share in column 1. The effect strengthens considerably once we go to the quarterly level in column 3.

Overall, trade appears to be significantly more elastic when a relatively larger share of it is conducted within multinationals. We next develop a theoretical framework that allows us to capture this fact.

3 Theoretical Framework

In our model there are N countries and K sectors. We use the letters i, ℓ, m to denote countries and the letters k or s to denote sectors. Goods are identified by the variety type $\omega \in \Omega_m$, the origin of the good $i \in \mathcal{N}$, the location of production $l \in \mathcal{N}$, and the sector $k \in \mathcal{K}$. If a third country index is used, that denotes the country in which the good is consumed. For example, $q^k_{i,\ell m}(\omega)$ denotes the quantity of variety ω produced in country ℓ and shipped to country m by a firm headquartered in i,

Table 4: Tariff Regressions Conditional on List

	(1) Baseline	(2) P50 dummy	(3) Quarterly
Dep. var.	$\Delta \ln m_{igt}$	$\Delta \ln m_{igt}$	$\Delta \ln m_{igt}$
$\Delta \ln(1+\tau_{igt})$	-0.198	-0.183	-2.026^{*}
	(0.352)	(0.349)	(1.037)
$\Delta \ln(1+\tau_{igt}) \cdot \phi^{CN}_{n(g)}$	-0.552		-1.656^{***}
	(0.601)		0.475)
$\Delta \ln(1+\tau_{igt}) \cdot \mathbb{I}(\phi^{CN}_{n(g)} > P50)$		-0.315^{*}	
		(0.182)	
$\Delta \ln(1+\tau_{igt}) \cdot \mathbb{I}(\text{List}=1)$	-0.937^{***}	-0.821^{**}	0.324
	(0.392)	(0.397)	(1.073)
$\Delta \ln(1+\tau_{igt}) \cdot \mathbb{I}(\text{List}=2)$	-1.610^{***}	-1.502^{***}	-0.656
	(0.536)	(0.548)	(1.030)
$\Delta \ln(1+\tau_{igt}) \cdot \mathbb{I}(\text{List}=3)$	-1.015^{***}	-0.948^{**}	1.460
	(0.375)	(0.376)	(1.026)
$\Delta \ln(1+\tau_{igt}) \cdot \mathbb{I}(\text{List}=4)$	-0.356	-0.226	1.195
	(0.352)	(0.365)	(1.070)
Observations	5,537,684	5,537,684	2,601,559
R-squared	0.125	0.125	0.118
Product × time FE	Y	Y	Y
Country × time FE	Y	Y	Y
Country × sector FE	Y	Y	Y

in sector k.[7]

Within each country, there are two types of firms: local firms and global firms. Local firms produce domestically and serve every country through exporting. Global firms produce domestically and abroad and use all locations of production to serve other countries. While the total measure of firms of a given origin $i \in \mathcal{N}$ is exogenous, the decision to become a global firm is endogenous via the presence of fixed costs of selection into multinational production.

On the demand side, each country m is characterized by a representative household which receives labor income, tariff revenues and profits of local and global firms headquartered in m, and uses its income to consume goods produced by global and local firms from all countries. Finally, in each country m there is a government that sets import tariffs and rebates the proceeds to the household in a lump-sum fashion.

3.1 Preferences and Endowments

The representative household in country m has preference given by:

$$U_m = \prod_{k=1}^{K} (Q_m^k)^{e_m^k},$$

where Q_m^k represents the total quantity of the composite good from sector k consumed in country m, and $e_m^k \in (0,1)$ is country m's expenditure share on sector k. This composite good is a CES

[7]The expressions *headquarters*, *origin-countries* and *origin-ideas* are used interchangeably.

aggregate

$$Q_m^k = \left[\sum_{i=1}^{N} \int_{\omega \in \Omega_i^k} Q_{i,m}^k(\omega)^{\frac{\sigma-1}{\sigma}} d\omega\right]^{\frac{\sigma}{\sigma-1}}, \tag{2}$$

with $Q_{i,m}^k(\omega)$ country m's consumption of variety ω originated in country i. The (exogenous) set of varieties from country i sector k is given by Ω_i^k, and $\sigma > 1$ is the elasticity of substitution between goods from different origin countries.[8]

This preference structure implies a demand system given by

$$P_{i,m}^k(\omega) Q_{i,m}^k(\omega) = \left(\frac{P_{i,m}^k(\omega)}{P_m^k}\right)^{1-\sigma_k} e_m^k E_m \tag{3}$$

where E_m denote the income of the household, and P_m^k the price index of the composite final good given by

$$P_m^k = \left[\sum_{i=1}^{N} \int_{\omega \in \Omega_i^k} P_{i,m}^k(\omega)^{1-\sigma} d\omega\right]^{\frac{1}{1-\sigma}}. \tag{4}$$

Households supply one unit of labor inelastically and labor is assumed to be freely mobile across sectors within a country, implying a single wage in country m, w_m.

3.2 Global and Local Firms

We consider two types of firms, local and multinational firms. For simplicity, we assume multinational firms operate globally, hence we refer to them as global firms. This abstracts from hetergeneity among multinational enterprises in their plant location decisions (e.g., Tintelnot (2017)), which are not the focus of our study. Our modeling decision is motivated by tractability and evidence from many countries that a few superstar firms carry out most of aggregate trade (Bernard et al. (2018)). Due to their size, these superstar firms are likely well approximated as global firms.

Local Firms A local firm produces a unique variety ω in the same country in which it is headquartered, and from there serves all markets through exporting. Output for destination market m increases linearly with labor input,

$$q_{i,im}^k(\omega) = \frac{A_i^k \times z(\omega)}{\tau_{im}^k} \ell_{i,im}^k(\omega), \tag{5}$$

where $\tau_{im}^k > 1$ is the iceberg cost to move sector k's goods from country i to m, and $A_i^k \times z(\omega)$ is firm's total productivity, which is the product of a country-sector specific component and a firm specific component. We assume that iceberg trade costs τ_{im}^k are homogeneous across sectors, allowing us to remove the subscript k for the remainder of the paper, and impose the normalization that $\tau_{ii}^k = 1$.

[8] The parameter σ also controls the substitution of varieties within an origin, however, this will place little role in our analysis.

We do not assume a specific market structure, instead we assume that firms charge prices that are a constant markup μ_k^L over marginal costs[9]

$$P_{i,m}^k(\omega) = \mu_k^L \times \frac{\tau_{im}}{A_i^k \times z(\omega)} w_i. \tag{6}$$

This allow us to nest both monopolistic and perfect competition models.

Global Firms. Global firms differ from local firms in their ability to produce in multiple locations and serve other countries from these production sites. The output of a global firm destined for market m, produced at location l, is proportional to labor input and follows the production function:

$$q_{i,\ell m}^k(\omega) = \frac{A_i^k \times z(\omega)}{\tau_{\ell m} \times \gamma_{i,\ell}^k} \ell_{i,\ell m}^k(\omega), \tag{7}$$

where $\gamma_{i,l}^k \geq 1$ represents the efficiency loss when producing outside the headquarters location, with the normalization $\gamma_{i,i}^k = 1$. Beyond these variable production costs, multinational production involves a fixed cost f_i^k, expressed in units of headquarters labor, reflecting the overhead required to coordinate production across borders. These fixed costs determine the selection of firms into either local or multinational enterprises.

To supply destination market m, multinationals aggregate output from all production locations using a CES function with elasticity of substitution $\varepsilon > 1$. This yields a bundled good, $Q_{i,m}^k(\omega)$, offered to consumers:

$$Q_{i,m}^k(\omega) = \left[\sum_{\ell=1}^{N} q_{i,\ell m}^k(\omega)^{\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon-1}}. \tag{8}$$

We assume that multinationals sell output from all production locations at cost to the destination market before assembling it into a bundled good and charge a mark-up μ_k^G on the bundled good. The price of the bundled good, $P_{i,m}^k(\omega)$, is given by:

$$P_{i,m}^k(\omega) = \mu_k^G \left[\sum_{\ell=1}^{N} \left(t_{\ell m}^k \frac{\tau_{\ell m} \times \gamma_{\ell,m}^k}{A_i^k \times z(\omega)} w_\ell \right)^{1-\varepsilon} \right]^{\frac{1}{1-\varepsilon}}, \tag{9}$$

with t_{lm}^k the gross add-valorem tariff set by country m.

3.3 Equilibrium

We begin this section by outlining the role of the government, followed by a detailed description of the model's equilibrium.

[9] We use here that $p_{i,im}^k(\omega) = P_{i,m}^k(\omega)$, as local firms produce exclusively from their headquarters location, implying no differentiation in prices across destination markets.

Government. The government in country m collects revenue from import tariffs $t_{\ell m}^k$ on sector k goods imported from countries $\ell \neq m$. These revenues are rebated as lump-sum transfers, T_m, to the representative household in m:

$$T_m = \sum_{k=1}^{K} \sum_{i=1}^{N} \int_{\omega \in \Omega_i^k} \sum_{\ell \in N} \left(t_{\ell m}^k - 1 \right) p_{i,\ell m}^k(\omega) q_{i,\ell m}^k(\omega) d\omega. \tag{10}$$

Consequently, household income E_m is given by

$$E_m = w_m L_m + \Pi_m + T_m, \tag{11}$$

where Π_m represents the aggregate profits of firms headquartered in m:

$$\Pi_m = \sum_{k=1}^{K} \sum_{\ell=1}^{N} \sum_{j=1}^{N} \int_{\omega \in \Omega_m^k} \pi_{m,\ell j}^k(\omega) \, d\omega.$$

Firm Selection and Productivity Thresholds. As in standard models of trade, profits are increasing functions in firm's productivity level. The presence of fixed costs for becoming a multinational enterprise and increasing profits implies that only the most productive firms in the economy become global. This leads to a productivity threshold, $\tilde{z}_m^k$, above which firms headquartered in country m with productivity $z > \tilde{z}_m^k$ operate as multinationals, producing in every location. Letting $G(\cdot)$ denote the ex-ante cumulative distribution of firm productivities, and given a fixed number of firms M_m^k, the number of global firms is given by

$$M_m^{k,G} = M_m^k (1 - G(\tilde{z}_m^k)), \tag{12}$$

where the measure of local firms satisfies the identity

$$M_m^{k,G} + M_m^{k,L} = M_m^k. \tag{13}$$

Equilibrium Definition. An equilibrium, given import tariffs $\mathbf{t} = \{t_{\ell m}^k\}_{\ell,m,k}$, consists of an allocation, prices, firm measures, and productivity thresholds that satisfy the following conditions: (i) households maximize utility subject to their budget constraint, i.e., equations (2) to (4) hold; ii) firms set prices according to equations (6) and (9), respectively; iii) firms select into global and local firms, as stated in equations (12) and (13); iv) the government runs a balanced budget, provided in (10); and (iv) labor markets clear

$$L_m = M_m^{k,G} f_m^k + \sum_{k=1}^{K} \sum_{i=1}^{N} \sum_{j=1}^{N} \int_{\omega \in \Omega_i^k} \ell_{i,mj}^k(\omega) \, d\omega; \tag{14}$$

The definition of equilibrium is standard; however, equation (14) warrants further discussion. The labor market condition equates the total supply of labor to total demand, which includes the

fixed cost payments associated with multinational operations headquartered in country m, the labor used by local firms in m for production, and specific to our model, the labor employed by foreign multinationals operating in m.

Trade Balance. The assumption that profits are redistributed based on firm ownership implies that trade flows need not be balanced. Specifically, Appendix B.1 shows that country m features a trade surplus equal to the difference between the profits of foreign subsidiaries located in m and the profits of domestic subsidiaries abroad:

$$NX_m = \sum_{k=1}^{K} \sum_{j=1}^{N} \sum_{o=1}^{N} \left[\int_{\omega \in \Omega_o^k} \pi_{o,mj}^k(\omega) d\omega - \int_{\omega \in \Omega_m^k} \pi_{m,oj}^k(\omega) d\omega \right], \tag{15}$$

These imbalances play a dual role in the subsequent analysis of optimal tariffs, providing an additional channel for profit-shifting effects from import tariffs.

4 Optimal unilateral import tariffs

A central contribution of this paper is to study the optimal import tariffs in the presence of multinational enterprises, and how these multinationals alter the forces that determine optimal tariffs in trade models.

The problem of the domestic planner is to maximize domestic welfare, subject to the equilibrium conditions, and the import tariffs of the rest of the world. To build intuition for the mechanisms and to highlight the primitive economic forces that shape the optimal tariffs, we present simplified versions of our model.

4.1 Optimal tariffs for a Small Open Economy with perfect competition

We consider first a small open economy under perfect competition, where firms set prices equal to marginal cost, and markups are therefore equal to one, i.e., $\mu_k^L = \mu_k^G = 1$. To maintain perfect competition, we eliminate firm selection into global production by setting the fixed costs f_m^k to zero, which implies that all firms are global. The absence of profits guarantees that trade is balanced.

In Appendix C.1, we show that the equilibrium in this model reduces to finding the vector of wages that satisfies the trade balance condition for all countries:

$$w_i\left(\boldsymbol{t}_m, \boldsymbol{t}_{-m}\right) L_i = \sum_{j \in \mathcal{N}} \Lambda_{ij}\left(\boldsymbol{t}_m, \boldsymbol{t}_{-m}\right) w_j\left(\boldsymbol{t}_m, \boldsymbol{t}_{-m}\right) L_j \quad \forall i \in \mathcal{N} \tag{16}$$

where Λ_{ij} represent the share of country j's income spent on goods from country i. Equation (16) explicitly highlights how each component of the trade balance condition depends on the entire set of import tariffs.

We build on Alvarez and Lucas Jr (2007) and, more recently, Caliendo and Feenstra (2024) and Demidova et al. (2024) to model a small open economy. We adopt the following parametrization

for population, trade costs, and multinational production frictions across all destination countries j and all sector k, and analyze the limit of the sequence of equilibria indexed by n, as country m becomes arbitrarily small, i.e., as $n \to 0$:

$$\begin{aligned}
\tilde{L}_m &= n^{\frac{\sigma-1}{\varepsilon-1}} \times L_m \\
\tilde{\tau}_{mj} &= n^{-1/(\varepsilon-1)} \times \tau_{mj} \\
\tilde{\gamma}^k_{m,j} &= n^{-1/(\varepsilon-1)} \times \gamma^k_{m,j}, \quad \forall j \text{ and } k
\end{aligned}$$

This parameterization implies that as the country becomes smaller, imports shrink, and, through trade balance, exports contract as well. For a sequence with constant iceberg costs for exporting, the value of exports reduces to zero, which can only be sustained as an equilibrium with wages that rise to infinity. In a pure trade model, rising iceberg costs for exports alleviate this pressure on wages, resulting in a finite equilibrium wage with non-zero import shares—suitable for quantitative analysis. However, with finite wages and increasing export cost, domestic firms serve foreign markets primarily through multinational production and export platforms. The rising MP frictions ensure that production does not entirely relocate outside the domestic economy.

In Appendix C.4, we show that as country m becomes arbitrarily small, there exists an equilibrium in which the vector of wages w_{-m} can be determined independently of country m's variables (including country m's import tariffs), and the equilibrium wage w_m remains finite. Therefore, in the limit country m is a small open, and the domestic planner chooses its import tariffs to maximize welfare subject a single constraint, country m's trade balance condition

$$w_m\left(\boldsymbol{t_m}\right) L_m \left[1 - \tilde{\Lambda}_{mm}\left(\boldsymbol{t_m}, w_m\left(\boldsymbol{t_m}\right)\right)\right] = \sum_{j \neq m} \tilde{\Lambda}_{mj}\left(w_m\left(\boldsymbol{t_m}\right)\right) w_j L_j \tag{17}$$

where we use $\tilde{x}$ to denote the limit of x for $n \to 0$.

Equation (17) reveals the domestic planner's incentives to impose import tariffs. Tariffs manipulates the terms of trade by influencing relative wages but introduce distortions in relative prices. We summarize the main result of this subsection in the following proposition:

Proposition 1. In a Small Open Economy with perfect competition, country m's optimal import tariffs are homogeneous across countries and sectors, and equal to:

$$t^k_{\ell m} - 1 = t_m - 1 = 1/\sum_{k=1}^{k} \sum_{\ell \neq m} \omega^k_{m\ell} \theta^k_{m\ell} \tag{18}$$

where $\omega^k_{m\ell}$ is the ratio between exports from country m to ℓ in sector k and total exports, and $\theta^k_{m\ell}$ is the trade elasticity with respect to wages:

$$\theta^k_{m\ell} \equiv -\frac{\partial \log\left(\lambda^k_{m\ell}/\lambda^k_{\ell\ell}\right)}{\partial \log w_m}$$

with $\lambda^k_{m\ell}$ country ℓ's trade share in sector k goods.

The optimal tariff balances two effects: manipulating the terms of trade by raising domestic wages and the distortionary impact of tariffs on relative prices. Since the terms of trade only depend on relative wages and the household preferences imply no substitution effects across sectors, it would be suboptimal for country m to manipulate terms of trade bilaterally, because that would further introduce distortions in relative import prices. This leads to homogeneous tariffs across countries and sectors.

The level of the import tariff balances the terms-of-trade manipulation with the distortionary effect between domestic and foreign prices. By increasing the domestic wage, import tariffs make domestic production more costly. This affects both domestic firms as well as foreign firms with multinational operations in the Home country, ultimately affecting the export capacity of the country. The trade elasticity to wages captures exactly how those exports are affected ($\theta^k_{m\ell}$), and these responses are weighted by how important are exports in that sector over all exports ($\omega^k_{m\ell}$). This weighted sum constitutes the export supply elasticity for country m.

Unlike standard trade models in which the trade elasticity is constant (a summary of such models is in Caliendo and Parro (2022)), the presence of multinational production makes these elasticities endogenous. Specifically, we show:

$$\theta^k_{m\ell} = (\sigma - 1) + (\varepsilon - \sigma)\left(1 - \lambda^k_{m,m\ell}\right) \tag{19}$$

where $\lambda^k_{m,m\ell}$ is the share of m's total sales to l originating in m. If multinational production is absent, trade flows depend solely on $\sigma - 1$, the standard trade elasticity in Armington models. When MNEs are active, the elasticity depends on the size of the MP activity but also on the relative magnitudes of the elasticity of labor across location of productions governed by ε, and the elasticity of substitution across goods governed by σ. Notably, when the former is larger than the latter, it is easier for firms to serve different locations by relocating production across affiliates, reducing the benefits of terms-of-trade manipulation. In this case, optimal tariffs are lower relative to standard trade-only models.

Proposition 1 allows for a final re-interpretation of the results related with those in Gros (1987). In models with perfect competition and differentiated varieties, the domestic planner uses import tariffs to exert market power abroad by reducing production and therefore increasing its price. In Armington models the optimal import tariff is therefore related to the elasticity of demand which also governs the trade elasticity. Our optimal import tariff has a similar interpretation. The domestic planner exerts its market power abroad and it does so by setting import tariffs that are functions of the elasticity of demand. In our nested CES structure with global firms, the elasticity of demand is given by equation (19), which is the elasticity of demand in models like Atkeson and Burstein (2008).

4.1.1 Optimal tariffs for a Large Open Economy with perfect competition

We extend the analysis to large open economies and show that the main insights from the small open economy case remain valid. For analytical tractability, we consider a two-country, single-sector setup. In Section 5.3, we show numerically that our formula approximates well the optimal policy in an environment with multiple countries and sectors.

Proposition 2. In a large open economy, the optimal import tariff is given by:

$$t_{\ell m} - 1 = \frac{1}{\Lambda_{\ell\ell}\theta_{m\ell}}$$

where $\Lambda_{\ell\ell}$ is Foreign's own consumption share, and $\theta_{m\ell}$ is the trade elasticity:

$$\theta_{m\ell} = (\varepsilon - 1) - (\varepsilon - \sigma) \times \sum_{i=1}^{N} (\lambda_{i,m\ell} \times \lambda_{i,\ell}) \times \left(\frac{\lambda_{i,m\ell}}{\lambda_{m\ell}} - \frac{\lambda_{i,\ell\ell}}{\lambda_{\ell\ell}} \right),$$

As in the SOE case, multinational activity reduces the optimal tariff, particularly when $\varepsilon > \sigma$, as firms can flexibly reallocate production across affiliates in response to domestic wage changes. Our results underscore how multinational production alters the optimal tariff structure by endogenizing trade elasticities. Compared to standard trade models, the presence of MNEs reduces both the gains and distortions from tariff imposition, leading to lower optimal tariffs when production relocation is highly elastic.

4.1.2 Optimal tariffs for a Small Open Economy with imperfect competition

[TBD]

5 Quantitative Results for a Large Open Economy

In section 5.1 we describe the data used for the calibration. In section 5.2, we describe the calibration strategy. Finally, in section 5.3 we present the quantitative results.

5.1 Data

Tariff Data. Quantification of our large open economy model requires tariff data from all countries. We combine data on bilateral tariffs at the HS6 level from UNCTAD-TRAINS and the WTO-IDB dataset.

Output and Trade Shares. The OECD AMNE Analytical Database provides data on output that contains information on the industry, the location of production, and the country of ownership. Output of domestic firms is further split into output by MNEs and output by non-MNEs, using data from the OECD AMNE Analytical Database - Domestic MNEs. Industry-level trade flows come from the OECD AMNE Inter-country Input-Output tables.

Measure of Local and Global Firms. We use Orbis Historical Data to compute the measure of global and local firms for each country and industry.

Labor Force We set the size of the labor force proportional to the size of manufacturing employment, computed from the ILO database.

Countries and Sectors We group countries into ten regions and the United States. We group manufacturing industries into two sectors, based on the related party share of US imports from China. The high related party share sector includes chemicals, minerals, and automobiles. The low related party share sector includes agriculture, apparel, machinery, materials, and metals.

More details are contained in Appendix H.

5.2 Calibration

We set $\varepsilon = 6$ and $\sigma = 4$. The remaining parameters that need to be calibrated are: the fixed costs of selection into MP, f_m^k, country-sector specific productivity, A_m^k, trade costs, $\tau_{\ell i}$, and MP frictions, $\gamma_{m,\ell}^k$. In principle, these parameters could be estimated through model inversion. However, a critical moment required for the model inversion are the trade flows associated with affiliates of global firms, which are not available from the AMNE Database.[10] Standard methods of imputation relying on proportionality assumptions are inconsistent with the model, unless one is willing to impose the restriction that $\sigma = \varepsilon$. For this reason, we resort to moments matching as our baseline calibration strategy.

The key moments we target are: i) the MNE share of the value of output produced by firms headquartered in m in country m, in each sector k; ii) the share of output in location ℓ, produced by firms headquartered in m in sector k; iii) the bilateral trade shares; iv) the share of global firms by country of ownership and sector; v) relative income across countries.

To reduce the dimensionality of the problem, we parameterize trade costs as a function of distance (from CEPII data), $\log \tau_{ij} = \alpha^d + \beta^d dist_{ij}$, and we parametrize MP frictions as a function of distance, letting the coefficient depend on the sector, $\log \gamma_{ij}^k = \alpha^\gamma + \beta_k^\gamma dist_{ij}$. I

In Figure I.1 in Appendix I, we show the fit between the model-simulated moments and the data.

5.3 Results

We compute optimal unilateral tariffs for the US in the following three scenarios: i) perfect competition (i.e. $\mu_k^G = \mu_k^L = 1$); ii) imperfect competition, with no selection into MP; iii) benchmark calibration, featuring imperfect competition and selection into MP. For scenarios i) and ii), we

[10]The AMNE Database only contains information on the output of affiliates by country of origin, without providing information on where the output is sold.

assume that all firms are global (i.e. we set $f_m^k = 0$), and recalibrate the remaining parameters after dropping the share of global firms by country of origin from the set of targeted moments.

Figure 2 reports the numerical results from this exercise. In each scenario, the yellow bar shows the optimal import tariff, calculated under the restriction of uniform tariffs across all countries and sectors and under the assumption of no foreign retaliation. We contrast the optimal import tariff with the formula derived in Proposition 1.[11] While this formula was derived under restrictive assumptions, it remains a good approximation for the optimal tariffs in large open economies with perfect competition.

Since the optimal tariff with imperfect competition in the small open economy is still work in progress, we also use Proposition 1 as point of comparisson for the cases with imperfect competition. Imperfect competition adds a new motive for the planner to impose import tariff on its trade partners, to shift profits back Home. Figure 2 reveals that the *profit shifting motive* leads to an increase in the optimal import tariff and it is quantitatively important. Intuitively, an increase in import tariffs reallocates demand from goods produced abroad to goods produced domestically. Given the presence of MP frictions, domestic-owned affiliates located abroad face a stronger incentive to relocate production to the domestic country, relative to foreign-owned affiliates. As a result, on top of the terms of trade improvement, an import tariff reallocates profits from foreign-owned to domestic-owned companies.

Importantly, the profit shifting motive would be absent in a version of the model with positive profits but no MNEs, as all domestic-owned firms would already be producing in the domestic country. To show this point, we compute optimal import tariffs in a version of the model with prohibitively high MP frictions, which is isomorphic to a standard Armington model with no MP. Figure I.3 and I.4 show that results in the no MP case are unaffected by the precense of profits.

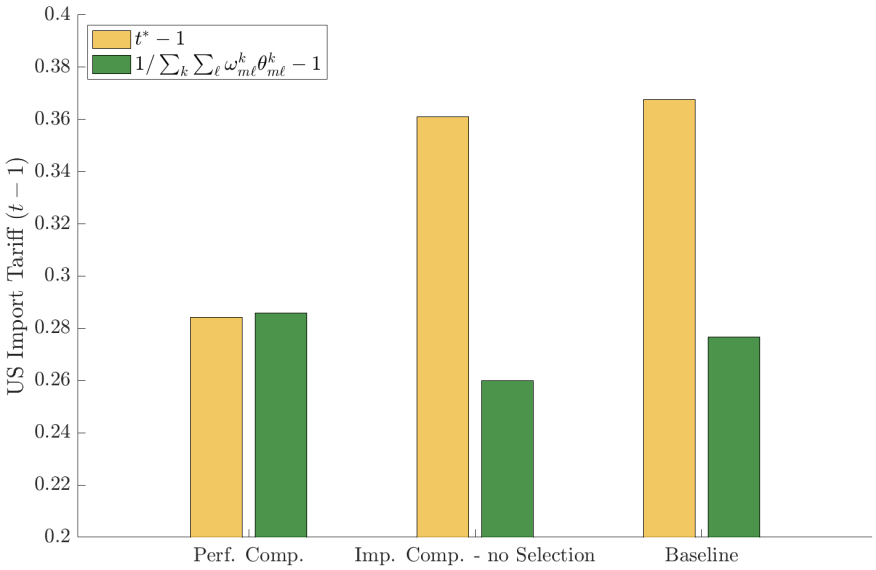
Finally, the third scenario suggests that the *profit shifting motive* mostly operates along the intensive margin. While an increase in the profitability of domestic production and an increase in the domestic wage reduces the incentives for domestic firms to become global firms, the introduction of the extensive margin of selection into MP has a limited impact on the optimal import tariff.

Figure I.2 reports the welfare gains associated with optimal import tariffs. In line with this class of models, we find relatively small welfare gains in the perfect competition benchmark ($\sim 0.63\%$), which rise to around 0.95% in the imperfect competition case.

Sensitivity. The quantitative relevance of the profit shifting motive is sensitive to the assumptions on markups. For example, in Figure I.5, we recompute optimal import tariffs under the assumption that markups are 0.15 (against a value of 0.33 in the benchmark calibration) and find an optimal import tariff close to 33% (compared to 36% in the benchmark calibration).

[11] The derivation of the trade elasticity in the general model with local and global firms is reported in Appendix G.

Figure 2: Optimal US Unilateral Import Tariff and Trade Elasticity



Notes: The yellow column represents the value of the optimal US unilateral import tariff. The green column reports the reciprocal of the average trade elasticity, defined in Proposition 1. The perfect competition scenario imposes that all firms are global and markups are equal to 1. In the imperfect competition scenario, we maintain that all firms are global and set markups to $\mu_k = \sigma/(\sigma - 1)$. The baseline scenario is the calibrated model with selection into MP and positive markups.

6 Conclusions

TBA

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Multinational Production and Trade Policy

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Online Appendix (Not for Publication)

A Additional Descriptive Results

Table A.1 presents an alternative version of Table 1 where capital goods are classified as intermediates. We still find that the related party share was lower in earlier tariff lists. We note that only one 10-digit HS code was classified as final consumption good on list 1, and no final consumption goods were on list 2.

Table A.2 presents analogous regressions to Table 3 but using the change in the price as dependent variable. Price is computed as the unit value, i.e., value exclusive of duties divided by quantity. We do not find a significant difference between the price changes of industries with low versus high related-party share.

Table A.1: Related party (RP) Share of Imports from China - Alternative Classification of Goods

	Intermediates		Final goods	
	Avg. share	Median share	Avg. share	Median share
Round 1	0.314	0.300	0.548	0.548
Round 2	0.262	0.296	–	–
Round 3	0.150	0.120	0.113	0.099
Round 4	0.152	0.146	0.083	0.048
Non-targeted	0.197	0.215	0.090	0.066

Notes: Goods are classified as intermediate or final according to the Broad Economic Categories rev. 5 (BEC5) code corresponding to their HS code. BEC5 codes with a third digit of 1 or 2 (intermediate and capital final use) are classified as intermediate goods and BEC5 codes with a third digit of 3 are classified as final consumption goods.

Table A.2: Price Setting and Related Party Share

	(1)	(2)	(3)	(4)
	No related share	Baseline	P50 dummy	Quarterly
$\Delta \ln(1+\tau_{igt})$	-0.003703	-0.08375	-0.02128	0.03158
	(0.05326)	(0.06821)	(0.09595)	(0.07117)
$\Delta \ln(1+\tau_{igt}) \cdot \phi^{CN}_{n(g)}$		0.4445		0.06367
		(0.2792)		(0.2952)
$\Delta \ln(1+\tau_{igt}) \cdot \mathbb{I}(\phi^{CN}_{n(g)} > P50)$			0.02470	
			(0.08967)	
Product-Time Fixed effects	Yes	Yes	Yes	Yes
Country-Time Fixed effects	Yes	Yes	Yes	Yes
Country-Sector Fixed Effects	Yes	Yes	Yes	Yes
R2	0.1058	0.1058	0.1058	0.08934
Observations	5,537,684	5,537,684	5,537,684	2,601,559

Source: Census Bureau and authors' calculations. Table reports the results of regressing the monthly change in the unit value (p_{igt}) on the change in the tariff (τ_{igt}) as well as the tariff change interacted with the related party share. The regressions include product-time, country-time, and country-sector fixed effects. Standard errors, adjusted for clustering by HS8 product code and country are reported below coefficient estimates. ***, **, and * represent statistical significance at the 1, 5 and 10 percent levels.

B Theoretical Results

B.1 Equilibrium Conditions

First, we restate the equilibrium conditions:

1. Labor market clearing conditions:

$$L_m = \sum_{k=1}^{K}\sum_{i=1}^{N}\int_0^{M_i^k}\sum_{j=1}^{N}\ell_{i,mj}^k(\omega)d\omega + \sum_{k=1}^{K}M_m^{k,G}f_m^k \tag{B.1}$$

2. Budget constraint:

$$E_m = w_m L_m + \Pi_m + T_m \tag{B.2}$$

3. Cutoff rule for selection into global firms:

$$\pi_m^{k,G}(z_m^{k*}) - w_m f_m^k \geq \pi_m^{k,L}(z_m^{k*}) \tag{B.3}$$

where a strict inequality at $z_m^{k*} = 1$ implies that all firms are global. More in details:

$$z_m^{k*} = \max\left\{\left(\frac{w_m f_m^k}{\bar{\pi}_m^{k,G} - \bar{\pi}_{m,j}^{k,L}}\right)^{\frac{1}{\sigma-1}}, 1\right\}$$

where $\bar{\pi}_m^{k,G}, \bar{\pi}_m^{k,L}$ denotes the component of profits that does not depend on productivity.

4. Measure of local firms:

$$M_m^{k,L} = G(z_m^{k*})M_m^k \tag{B.4}$$

Prices The endogenous variables that enter the previous equations can be expressed as a function of wages w_m, expenditures E_m and the share of global firms $M_m^{k,G}$. To see why, first recall that prices of local and global firms headquartered in m that sell goods to country j in sector k are:

$$P_{m,i}^{k,G}(\omega) = \left[\sum_{\ell=1}^{N}(t_{\ell i}^k)^{1-\varepsilon}p_{m,\ell i}^{k,G}(\omega)^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}}$$
$$P_{m,i}^{k,L}(\omega) = t_{mi}^k p_{m,mi}^{k,L}(\omega)$$

where

$$p_{m,\ell i}^{k,G}(\omega) = \mu_k^G \times \tau_{\ell i} \times \gamma_{m,\ell}^k \frac{w_\ell}{A_m^k \times z_m^k(\omega)}$$
$$p_{m,i}^{k,L}(\omega) = p_{m,mi}^{k,L}(\omega) = \mu_k^L \times \tau_{mi} \times \frac{w_m}{A_m^k \times z_m^k(\omega)}$$

Since productivity is common across locations of production, we can rewrite the price index of origin-goods m sold to country i in sector k by global firms as:

$$P_{m,i}^{k,G}(\omega) = \frac{\mu_k^G}{z_m^k(\omega)} \left[\sum_{\ell=1}^{N} (t_{\ell i}^k)^{1-\varepsilon} \left(\tau_{\ell i} \times \gamma_{m,\ell}^k \frac{w_\ell}{A_m^k} \right)^{1-\varepsilon} \right]^{\frac{1}{1-\varepsilon}} = \frac{1}{z_m^k(\omega)} \bar{\pi}_{m,i}^{k,G}$$

where we use the notation $\bar{(\cdot)}$ to denote the component that is common to all firms. Then, the price index in location i can be expressed as:

$$P_{k,i} = \left[\sum_{o=1}^{N} M_o^k \left(\left(\mu_k^L \times \tau_{oi} \times \frac{w_o}{A_o^k} \right)^{1-\sigma} \int_1^{z_o^{o*}} (z_o^k)^{\sigma-1} g(z) dz + \left(\mu_k^G \times \tau_{\ell i} \times \gamma_{o,\ell}^k \frac{w_\ell}{A_o^k} \right)^{1-\sigma} \int_{z_o^{o*}}^{\infty} (z_o^k)^{\sigma-1} g(z) dz \right) \right]^{\frac{1}{1-\sigma}}$$

Profits and Threshold Rule for Selection into Multinationals Once prices have been determined, we can compute profits by producer m in selling from location ℓ to location i in sector k as:

$$\begin{aligned} \pi_{m,\ell i}^{k,G}(z(\omega)) &= \left(1 - \frac{1}{\mu_k^G}\right) \frac{1}{t_{\ell i}^k} \left(p_{m,\ell i}^{k,G}(z(\omega)) \right)^{1-\varepsilon} \left(p_{mk,i}^G(z(\omega)) \right)^{\varepsilon-\sigma} (P_i^k)^{\sigma-1} e_i^k E_i \\ &= \left(1 - \frac{1}{\mu_k^G}\right) \frac{1}{t_{\ell i}} \left(\bar{p}_{m,\ell j}^{k,G} \right)^{1-\varepsilon} \left(\bar{P}_{m,i}^{k,G} \right)^{\varepsilon-\sigma} z(\omega)^{\sigma-1} (P_i^k)^{\sigma-1} e_i^k E_i \end{aligned}$$

where we notice once again that profits can be expressed as the product of a component common to all firms and an increasing function of idiosyncratic productivity. Aggregating across locations of production and destination countries, gross profits of a firm headquartered in i of productivity z in sector k are:

$$\pi_m^{k,G}(z(\omega)) = z(\omega)^{\sigma-1} \left(1 - \frac{1}{\mu_k^G}\right) \sum_j \left(\bar{P}_{m,j}^{k,G} \right)^{\varepsilon-\sigma} (P_j^k)^{\sigma-1} e_j^k E_j \sum_\ell \frac{1}{t_{\ell j}^k} \left(\bar{p}_{m,\ell j}^{k,G} \right)^{1-\varepsilon}$$

For local firms:

$$\pi_m^{k,L}(z(\omega)) = z(\omega)^{\sigma-1} \left(1 - \frac{1}{\mu_k^L}\right) \sum_j \frac{1}{t_{mj}^k} \left(\bar{p}_{m,mi}^{k,L} \right)^{1-\sigma} (P_j^k)^{\sigma-1} e_j^k E_j$$

Therefore, the productivity threshold above which a firm becomes global can be found as:

$$z_m^{k*} = \max \left\{ \left(\frac{w_m f_m^k}{\bar{\pi}_m^{k,G} - \bar{\pi}_m^{k,L}} \right)^{\frac{1}{\sigma-1}}, 1 \right\}$$

Labor Demand Labor demand by global and local firms, net of labor used for payments of fixed costs, is:

$$\begin{aligned} \ell_{m,\ell j}^{k,G}(z) &= \left(\frac{1}{\mu_k^G}\right) \frac{1}{t_{\ell j}^K} \left(\bar{p}_{o,\ell j}^{k,G} \right)^{1-\varepsilon} \left(\bar{P}_{o,j}^{k,G} \right)^{\varepsilon-\sigma} z^{\sigma-1} (P_j^k)^{\sigma-1} \frac{e_j^k E_j}{w_\ell}, \quad \text{for } \ell = 1, \cdots, N \\ \ell_{m,mj}^{k,L}(z) &= \left(\frac{1}{\mu_k^L}\right) \frac{1}{t_{mj}^k} \left(\bar{p}_{m,mi}^{k,L} \right)^{1-\sigma} (P_j^k)^{\sigma-1} z^{\sigma-1} \frac{e_j^k E_j}{w_m} \end{aligned}$$

Then, labor used in production in country m in sector k is:

$$\ell_\ell^m = \sum_o M_o^k \left(\frac{1}{\mu_k^G}\right) \frac{\int_{z_o^{k*}}^{\infty} z^{\sigma-1} g(z)dz}{w_m} \sum_j \frac{1}{t_{mj}^k} \left(\bar{p}_{o,mj}^{k,G}\right)^{1-\varepsilon} \left(\bar{P}_{o,j}^{k,G}\right)^{\varepsilon-\sigma} (P_j^k)^{\sigma-1} e_j^k E_j$$
$$+ M_o^k \frac{\left(\frac{1}{\mu_k^L}\right) \int_1^{z_o^{k*}} z^{\sigma-1} g(z)dz}{w_m} \sum_j \frac{1}{t_{mj}^k} \left(\bar{P}_{o,j}^{k,L}\right)^{1-\sigma} (P_j^k)^{\sigma-1} e_j^k E_j$$

Then, labor market clearing requires:

$$L_m = \sum_k \ell_m^k + \sum_k M_m^{k,G} f_m^k$$

Tariff Revenues Similarly, tariff revenues generated from the sales of a global or local firm i of productivity z to country m in sector k:

$$T_{i,\ell m}^{k,G}(z) = \frac{t_{\ell m}^k - 1}{t_{\ell m}^k} \left(\bar{p}_{i,\ell m}^{k,G}\right)^{1-\varepsilon} \left(\bar{P}_{i,m}^{k,G}\right)^{\varepsilon-\sigma} z^{\sigma-1} (P_m^k)^{\sigma-1} e_j^k E_j$$
$$T_{i,im}^{k,L}(z) = \frac{t_{im}^k - 1}{t_{im}^k} \left(\bar{p}_{i,im}^{k,L}\right)^{1-\sigma} (P_j^k)^{\sigma-1} z^{\sigma-1} e_j^k E_j$$

Therefore, total tariff revenues in country m from sector k are:

$$T_m^k = \sum_o M_o^k \int_{z_o^{k*}}^{\infty} z^{\sigma-1} g(z)dz \sum_\ell \frac{t_{\ell m}^k - 1}{t_{\ell m}^k} \left(\bar{p}_{o,\ell m}^{k,G}\right)^{1-\varepsilon} \left(\bar{P}_{o,m}^{k,G}\right)^{\varepsilon-\sigma} (P_m^k)^{\sigma-1} e_m^k E_m$$
$$+ \sum_o M_o^k \int_1^{z_o^{k*}} z^{\sigma-1} g(z)dz \frac{t_{om}^k - 1}{t_{om}^k} \left(\bar{P}_{o,m}^{k,L}\right)^{1-\sigma} (P_m^k)^{\sigma-1} e_m^k E_m$$

Expenditures and Trade Balance We can now express aggregate expenditures in country m as:

$$
\begin{aligned}
E_m &= w_m L_m + T_m + \Pi_m \\
&= \sum_k \sum_i M_i^k \left(\frac{1}{\mu_k^G}\right) \int_{z_i^{k*}}^{\infty} z^{\sigma-1} g(z) dz \sum_j \frac{1}{t_{mj}^k} \left(\bar{p}_{i,mj}^{k,G}\right)^{1-\varepsilon} \left(\bar{P}_{i,j}^{k,G}\right)^{\varepsilon-\sigma} (P_j^k)^{\sigma-1} e_j^k E_j + M_m^{k,G} f_m^k w_m \\
&+ \sum_k M_m^k \left(\frac{1}{\mu_k^L}\right) \int_1^{z_i^{k*}} z^{\sigma-1} g(z) dz \sum_j \frac{1}{t_{mj}^k} \left(\bar{P}_{m,j}^{k,L}\right)^{1-\sigma} (P_j^k)^{\sigma-1} e_j^k E_j \\
&+ \sum_k \sum_i M_i^k \int_{z_{ik}^*}^{\infty} z^{\sigma-1} g(z) dz \sum_m \frac{t_{m\ell}-1}{t_{m\ell}} \left(\bar{p}_{ik,m\ell}^{G}\right)^{1-\varepsilon} \left(\bar{P}_{ik,\ell}^{G}\right)^{\varepsilon-\sigma} P_{k,\ell}^{\sigma-1} e_k P_\ell Q_\ell \\
&+ \sum_k \sum_i M_i^k \int_1^{z_i^{k*}} z^{\sigma-1} g(z) dz \frac{t_{im}^k - 1}{t_{m\ell}^k} \left(\bar{P}_{i,m}^{k,L}\right)^{1-\sigma} (P_m^k)^{\sigma-1} e_m^k E_m \\
&+ \sum_k M_m^k \int_{z_m^{k*}}^{\infty} z^{\sigma-1} g(z) dz \left(1 - \frac{1}{\mu_k^G}\right) \sum_j \left(\bar{P}_{m,j}^{k,G}\right)^{\varepsilon-\sigma} (P_j^k)^{\sigma-1} e_j^k E_j \sum_\ell \frac{1}{t_{mj}^k} \left(\bar{p}_{m,\ell j}^{k,G}\right)^{1-\varepsilon} - M_m^{k,G} f_m^k w_m \\
&+ \sum_k M_m^k \int_1^{z_m^{k*}} z^{\sigma-1} g(z) dz \left(1 - \frac{1}{\mu_k^L}\right) \sum_j \frac{1}{t_{mj}^k} \left(\bar{p}_{m,mj}^{k,L}\right)^{1-\sigma} (P_j^k)^{\sigma-1} e_j^k E_j \\
&= \sum_k \sum_i \sum_j (X_{i,mj}^{k,G}) + \sum_k \sum_j X_{m,mj}^{k,L} + \left(1 - \frac{1}{\mu_k^G}\right) \sum_j \left[\sum_\ell (X_{m,\ell j}^G) - \sum_i (X_{i,mj}^G)\right] + E_m - IM_m
\end{aligned}
$$

where the second equality obtains after adding and subtracting

$$
\sum_k \sum_i M_i^k \left(1 - \frac{1}{\mu_k^G}\right) \int_{z_i^{k*}}^{\infty} z^{\sigma-1} g(z) dz \sum_j \frac{1}{t_{mj}^k} \left(\bar{p}_{i,mj}^{k,G}\right)^{1-\varepsilon} \left(\bar{P}_{i,j}^{k,G}\right)^{\varepsilon-\sigma} (P_j^k)^{\sigma-1} e_j^k E_j
$$

and where we used the fact that:

$$
\begin{aligned}
E_m &= \sum_k \sum_i \sum_\ell t_{\ell m}^k \left(X_{i,\ell m}^{k,G} + X_{i,\ell m}^{k,L}\right) \\
&= \sum_k \sum_i M_i^k \int_{z_i^{k*}}^{\infty} z^{\sigma-1} g(z) dz \sum_\ell \left(\bar{p}_{i,\ell m}^{k,G}\right)^{1-\varepsilon} \left(\bar{P}_{i,m}^{k,G}\right)^{\varepsilon-\sigma} (P_m^k)^{\sigma-1} e_m^k E_m \\
&+ \sum_k \sum_i M_i^k \int_1^{z_i^{k*}} z^{\sigma-1} g(z) dz \left(\bar{P}_{i,m}^{k,L}\right)^{1-\sigma} (P_m^k)^{\sigma-1} e_m^k E_m \\
IM_m &= \sum_k \sum_i \sum_\ell \left(X_{i,\ell m}^{k,G}\right) - \sum_k \sum_\ell X_{\ell,\ell m}^{k,L} \\
&= \sum_k \sum_i M_i^k \int_{z_i^{k*}}^{\infty} z^{\sigma-1} g(z) dz \sum_\ell \frac{1}{t_{\ell m}^k} \left(\bar{p}_{i,\ell m}^{k,G}\right)^{1-\varepsilon} \left(\bar{P}_{i,m}^{k,G}\right)^{\varepsilon-\sigma} (P_m^k)^{\sigma-1} e_m^k E_m \\
&+ \sum_k \sum_i M_i^k \frac{1}{t_{im}^k} \int_1^{z_i^{k*}} z^{\sigma-1} g(z) dz \left(\bar{P}_{i,m}^{k,L}\right)^{1-\sigma} (P_m^k)^{\sigma-1} e_m^k E_m
\end{aligned}
\tag{B.5}
$$

Therefore, rearranging the previous expression, we can express the trade balance as:

$$\begin{aligned} NX_m &\equiv EX_m - IM_m \\ &= \sum_k \sum_i \sum_j \left(X_{i,mj}^{k,G}\right) + \sum_k \sum_j X_{m,mj}^{k,L} - \sum_k \sum_i \sum_j \left(X_{i,jm}^{k,G}\right) - \sum_k \sum_j X_{mk,jm}^{L} \\ &= \sum_k \left(1 - \frac{1}{\mu_k}\right) \sum_j \left[\sum_i \left(X_{i,mj}^{k,G}\right) - \sum_\ell \left(X_{m,\ell j}^{k,G}\right)\right] \end{aligned} \tag{B.6}$$

From this expression it is clear that trade imbalances are given by the difference between the net profits of domestic firms generated abroad and the net profits of firms headquartered in foreign countries generated in the domestic market. Consistently with the balance of payment identity, this equation is stating that a country spends more than the value of its production, if it receives positive net income from abroad.

Compact Equilibrium In this section, we show how the equilibrium can be expressed as a system of equations in: $\{E_m, w_m, \{M_m^{k,G}\}_{k\in K}\}_{m\in\mathcal{N}}$:

First, let's define the assembly share of affiliate j over the total sales of global firm m in sector k to country i.

$$\lambda_{m,ji}^{k,G} \equiv \frac{t_{ji}^k P_{m,ji}^{k,G} Q_{m,ji}^{k,G}}{\sum_{\ell\in\mathcal{N}} t_{k,\ell i} P_{m,\ell i}^{k,G} Q_{m,\ell i}^{k,G}} = \frac{\left(t_{ji}^k \tau_{ji} \gamma_{m,j}^k w_j\right)^{1-\varepsilon}}{\sum_{\ell\in\mathcal{N}} \left(t_{\ell i}^k \tau_{\ell i} \gamma_{m,\ell}^k w_\ell\right)^{1-\varepsilon}} \tag{B.7}$$

Note that these shares do not depend on idiosyncratic productivity, given that z is common across affiliates in different locations of production. Then, let's define an auxiliary price index of global variety m:

$$P_{m,i}^{k,G}(z) = \mu_k^G (z_m^k)^{-1} (A_m^k)^{-1} \left(\sum_{\ell\in\mathcal{N}} \left(t_{\ell i}^k \tau_{\ell i} \gamma_{m,\ell}^k w_\ell\right)^{1-\varepsilon}\right)^{\frac{1}{1-\varepsilon}} = w_m t_{mi}^k \tau_{mi} \mu_k^G (z_m^k)^{-1} (A_m^k)^{-1} (\lambda_{m,mi}^{k,G})^{-\frac{1}{1-\varepsilon}} \tag{B.8}$$

Then, the ideal price index of origin-goods m, aggregating across global and local varieties is:

$$\begin{aligned} (P_{m,ki})^{1-\sigma} &= M_m^k \Big[(w_m t_{mi}^k \tau_{mi} \mu_k (A_m^k)^{-1})^{1-\sigma} \int_1^{z_m^{k*}} (z_m^k)^{\sigma-1} g(z) dz \\ &\quad + (w_m t_{mi}^k \tau_{mi} \mu_k (A_m^k)^{-1} \gamma_{m,m}^k)^{1-\sigma} (\tilde{\lambda}_{m,mi}^{k,G})^{-\frac{1-\sigma}{1-\varepsilon}} \int_{z_m^{k*}}^{\infty} (z_m^k)^{\sigma-1} g(z) dz \Big] \\ &= M_m^k (w_m t_{mi}^k \tau_{mi} \mu_k (A_m^k)^{-1})^{1-\sigma} \left[\int_1^{z_m^{k*}} (z_m^k)^{\sigma-1} g(z) dz + (\gamma_{m,m}^k)^{1-\sigma} (\tilde{\lambda}_{m,mi}^{k,G})^{-\frac{1-\sigma}{1-\varepsilon}} \int_{z_m^{k*}}^{\infty} (z_m^k)^{\sigma-1} g(z) dz \right] \end{aligned} \tag{B.9}$$

where we have further assumed that the markup is constant across global and local firms. Then, the expenditure

share of origin-goods o in country i is:

$$\begin{aligned}\lambda^k_{m,i} &\equiv \frac{\sum_{\ell\in\mathcal{N}} t^k_{\ell i} P^{k,G}_{m,\ell i} Q^{k,G}_{m,\ell i} + t^k_{mi} P^{k,L}_{m',m'i} Q^{k,L}_{m',m'i}}{\sum_{m'\in\mathcal{N}}[\sum_{j\in\mathcal{N}} t^k_{ji} P^{k,G}_{m',ji} Q^{k,G}_{m',ji} + t^k_{mi} P^{k,L}_{m',m'i} Q^{k,L}_{m',m'i}]} \\ &= \left(\frac{P^k_{om,i}}{P^k_i}\right)^{1-\sigma} \\ &= \frac{M^k_m (w_m t^k_{mi} \tau_{mi} \mu_k (A^k_m)^{-1})^{1-\sigma} \left[\int_1^{z^{k*}_m} (z^k_m)^{\sigma-1} g(z) dz + (\gamma^k_{m,m})^{1-\sigma} (\tilde{\lambda}^{k,G}_{m,mi})^{-\frac{1-\sigma}{1-\varepsilon}} \int_{z^{k*}_m}^{\infty} (z^k_m)^{\sigma-1} g(z) dz\right]}{\sum_{m'} M^k_{m'} (w'_m t^k_{m'i} \tau_{m'i} \mu_k (A^k_{m'})^{-1})^{1-\sigma} \left[\int_1^{z^{k*}_{m'}} (z^k_m)^{\sigma-1} g(z) dz + (\gamma^k_{m',m'})^{1-\sigma} (\tilde{\lambda}^{k,G}_{m',m'i})^{-\frac{1-\sigma}{1-\varepsilon}} \int_{z^{k*}_{m'}}^{\infty} (z^k_{m'})^{\sigma-1} g(z) dz\right]}\end{aligned} \tag{B.10}$$

Finally, trade shares (in sector k) are given by:

$$\begin{aligned}\lambda^k_{ji} &\equiv \frac{\sum_o t^k_{ji} P^{k,G}_{ok,ji} Q^{k,G}_{o,ji} + t^k_{ji} P^{k,L}_{j,ji} Q^L_{j,ji}}{\sum_{o'\in\mathcal{N}}[\sum_{j\in\mathcal{N}} t^k_{ji} P^{k,G}_{o',ji} Q^{k,G}_{o',ji} + t^k_{o'i} P^{k,L}_{o',o'i} Q^{k,L}_{o',o'i}]} \\ &= \frac{\sum_o \tilde{\lambda}^{k,G}_{o,ji} \sum_{\ell\in\mathcal{N}} t_{k,\ell i} P^{k,G}_{o,\ell i} Q^{k,G}_{o,\ell i} + t^k_{ji} P^{k,L}_{j,ji} Q^{k,L}_{j,ji}}{\sum_{o'\in\mathcal{N}}[\sum_{j\in\mathcal{N}} t^k_{ji} P^{k,G}_{o',ji} Q^{k,G}_{o',ji} + t^k_{oi} P^{k,L}_{o',o'i} Q^{k,L}_{o',o'i}]} \\ &= \sum_o \lambda^{k,G}_{o,ji} \lambda^{k,G}_{o,i} + \lambda^{k,L}_{o,i}\end{aligned} \tag{B.11}$$

Then, we can express expenditures as:

$$\begin{aligned}E_m &\equiv \sum_k \sum_o \sum_\ell t^k_{\ell m} (X^{k,G}_{o,\ell m} + X^{k,L}_{o,om}) \\ &\equiv \sum_k \sum_o \sum_\ell t^k_{\ell m} (P^{k,G}_{o,\ell m} Q^{k,G}_{o,\ell m} + P^{k,L}_{o,\ell m} Q^{k,L}_{o,\ell m}) \\ &= \sum_k \sum_o \sum_\ell t^k_{k,\ell m} \frac{(P^{k,G}_{o,\ell m} Q^{k,G}_{o,\ell m} + P^{k,L}_{o,\ell m} Q^{k,L}_{o,\ell m})}{\sum_{o'} \sum_{\ell'} \sum_{k'} \frac{t^{k'}_{\ell' m}}{t^{k'}_{\ell' m}} (P^{k',G}_{o',l'm} Q^{k',G}_{o',\ell' m} + P^{k',L}_{\ell',\ell' m} Q^{k',L}_{\ell',\ell' m})} \left[\sum_k \sum_\ell P^k_{m\ell} Q^k_{m\ell} - N X_m\right] \\ &= \sum_k \sum_o \sum_\ell t^k_{\ell m} \frac{(P^{k,G}_{o,\ell m} Q^{k,G}_{o,\ell m} + P^{k,L}_{o,\ell m} Q^{k,L}_{o,\ell m})}{\sum_{\ell'} \sum_{k'} \frac{1}{t^{k'}_{\ell' m}} \lambda^{k'}_{\ell' m} e^k_m E_m} \left[\sum_k w_m \ell^k_m + \sum_k \sum_\ell \left(1 - \frac{1}{\mu_k}\right) P^k_{m\ell} Q^k_{m\ell} - N X_m\right] \\ &= \left(\sum_{\ell'} \sum_{k'} \frac{1}{t^{k'}_{\ell' m}} e_k \lambda^{k'}_{\ell' m}\right)^{-1} \left[\sum_k w_m \ell^k_m + \Pi^{host}_m - N X_m\right] \\ &= \left(\sum_{\ell'} \sum_{k'} \frac{1}{t^{k'}_{\ell' m}} e^k \lambda^{k'}_{\ell' m}\right)^{-1} \left[\sum_k \mu_k w_m \ell^k_m - N X_m\right]\end{aligned}$$

where the first and second line use the definition of expenditures, the third line divides and multiplies by $E_m = X_m - NX_m$, the fourth line uses the definition of expenditure shares and the last line splits the value of output into labor costs and profits. The superscript *host* is used to denote profits that are generated in a

location of production, independently of firm ownership. Recall from the labor market clearing condition:

$$\begin{aligned}
\sum_k \mu_k w_\ell \ell_\ell^k &= \sum_k \sum_o \sum_j \sum_\ell P_{o,\ell j}^k Q_{o,\ell j}^k \\
&= \sum_k \sum_o \sum_j \frac{t_{\ell j}^k}{t_{\ell j}^k} P_{o,\ell j}^k Q_{o,\ell j}^k \\
&= \sum_k \sum_o \sum_j \frac{1}{t_{\ell j}^k} \frac{t_{\ell j}^k P_{o,\ell j}^k Q_{o,\ell j}^k}{\sum_o t_{\ell j}^k P_{o,\ell j}^k Q_{o,\ell j}^k} \frac{\sum_o t_{\ell j}^k P_{o,\ell j}^k Q_{o,\ell j}^k}{e_j^k E_j} e_j^k E_j \\
&= \sum_k \sum_o \sum_j \frac{1}{t_{\ell j}^k} \frac{t_{\ell j}^k P_{o,\ell j}^k Q_{o,\ell j}^k}{\sum_o t_{\ell j}^k P_{o,\ell j}^k Q_{o,\ell j}^k} \lambda_{\ell j}^k e_j^k E_j \\
&= \sum_k \sum_j \frac{1}{t_{\ell j}^k} \lambda_{\ell j}^k e_j^k E_j \underbrace{\sum_o \frac{t_{\ell j}^k P_{o,\ell j}^k Q_{o,\ell j}^k}{\sum_o t_{\ell j}^k P_{o,\ell j}^k Q_{o,\ell j}^k}}_{=1} \\
&= \sum_k \sum_j \frac{\lambda_{\ell j}^k}{t_{\ell j}^k} e_j^k \left(\sum_{\ell'} \sum_{k'} \frac{1}{t_{\ell' j}^{k'}} e_j^k \lambda_{\ell' j}^{k'} \right)^{-1} \left[\sum_k \mu_k w_j \ell_j^k - N X_j \right] \\
&= \sum_j \Lambda_{\ell j} \left[\sum_k \mu_k w_j \ell_j - N X_j \right]
\end{aligned}$$

where

$$\Lambda_{\ell j} = \frac{\sum_k \frac{e_k \lambda_{k,\ell j}}{t_{k,\ell j}}}{\left(\sum_{\ell'} \sum_{k'} \frac{1}{t_{k',\ell' j}} e_k \lambda_{k',\ell' j} \right)}$$

Note that ℓ_ℓ^k is labor used in production in country ℓ, net of labor used for the payment of fixed costs. The left hand side can then be rewritten as:

$$\sum_k \mu_k \left(w_\ell L_\ell^k - w_\ell M_\ell^{k,G} f_\ell^k \right) = \sum_j \Lambda_{\ell j} \left[\sum_k \mu_k w_j \ell_j - N X_j \right]$$

Then note that net imports are:

$$-N X_j = \sum_k \left[\left(1 - \frac{1}{\mu_k} \right) \sum_m \left(\sum_\ell X_{j,lm}^{k,G} - \sum_o X_{o,jm}^{k,L} \right) \right]$$

We can then express expenditures as:

$$E_m = \left(\sum_{\ell'} \sum_{k'} \frac{1}{t_{\ell' m}^{k'}} e_m^{k'} \lambda_{\ell' m}^{k'} \right)^{-1} \left[w_m L_m + \sum_k \left(1 - \frac{1}{\mu_k} \right) \sum_j \left(\sum_\ell \left(X_{m,\ell j}^{k,G} \right) + \left(X_{m,\ell j}^{k,L} \right) \right) - \sum_k M_m^{k,G} w_m f_m^k \right]$$

And thus we can rewrite the labor market clearing condition as:

$$\begin{aligned}
&w_\ell L_\ell - w_\ell M_\ell^{k,G} f_\ell^k + \sum_k \left(1 - \frac{1}{\mu_k} \right) \sum_m \left(\sum_i \left(X_{\ell,im}^{k,G} \right) + \left(X_{\ell,im}^{k,L} \right) \right) \\
&= \sum_j \Lambda_{\ell j} \left[w_j L_j + \left(1 - \frac{1}{\mu_k} \right) \sum_m \left(\sum_i \left(X_{j,im}^{k,G} \right) + \left(X_{j,in}^{k,L} \right) \right) - M_j^{k,G} w_j f_j^k \right]
\end{aligned}$$

These expressions coincide with (B.16) and (B.17). One can further substitute profits and obtain:

$$E_m = \left(\sum_{\ell'}\sum_{k'} \frac{1}{t^{k'}_{\ell' m}} e^{k'}_m \lambda^{k'}_{\ell' m}\right)^{-1} \left[w_m L_m + \sum_k \left(1 - \frac{1}{\mu_k}\right) \sum_j \left(\left(\sum_\ell \frac{1}{t^k_{\ell j}} \tilde{\lambda}^{k,G}_{m,\ell j} + \tilde{\lambda}^{k,L}_{m,mj}\right) e^k_j E_j\right) - \sum_k M^{k,G}_m w_m f^k_m\right]$$

and analogously for the labor market clearing condition. The notation $\tilde{\lambda}$ is used to indicate expenditure shares as opposed to assembly shares. We now have to express the rule determining selection into multinationals as a functions of exogenous parameters and the endogenous variables we are solving for.

If the productivity threshold is interior for all countries and sectors, the cutoff rule that determines selection into multinational production is given by:

$$z^{k*}_o = \left(\frac{w_o f^k_o}{\bar{\pi}^{k,G}_o - \bar{\pi}^{k,L}_o}\right)^{\frac{1}{\sigma-1}} \tag{B.12}$$

Profits of the marginal firm can then be expressed as a function of average profits, using the relationship: $\frac{\pi^k_o(z')}{\pi^k_o(z)} = \left(\frac{z'}{z}\right)^{\sigma-1}$. First note that aggregate profits are:

$$\Pi^G_{ok} = \int_{z^{k*}_o}^{\infty} M^k_o \bar{\pi}^{k,G}_o z^{\sigma-1} g(z) dz = M^k_o \frac{\vartheta_k}{\vartheta_k - \sigma + 1} (z^{k*}_o)^{\sigma-\vartheta_k-1} \bar{\Pi}^{k,G}_o = M^{k,G}_o \frac{\vartheta_k}{\vartheta_k - \sigma + 1} (z^{k*}_o)^{\sigma-1} \bar{\Pi}^{k,G}_o$$

$$\Pi^{k,L}_o = M^k_o \int_1^{z^{k*}_o} \bar{\Pi}^{k,L}_o z^{\sigma-1} g(z) dz = M^k_o \left(\frac{\vartheta_k \left(\left(z^{k*}_o\right)^{\sigma-\vartheta_k-1} - 1\right)}{(\sigma - \vartheta_k - 1)}\right) \bar{\Pi}^{k,L}_o$$

Then we can express profits of the marginal firms as a function of aggregate profits. For global firms, we find:

$$(z^{k*}_k)^{\sigma-1} \bar{\Pi}^{k,G}_o = \frac{\vartheta_k - \sigma + 1}{\vartheta_k} \frac{\Pi^{k,G}_o}{M^{k,G}_o} \tag{B.13}$$

For local firms, we note that:

$$\begin{aligned} (z^{k*}_o)^{\sigma-\vartheta_k-1} \bar{\Pi}^{k,L}_o &= \bar{\Pi}^{k,L}_o - \frac{\vartheta_k - \sigma + 1}{\vartheta_k} \frac{\Pi^{k,L}_o}{M^k_o} \\ \implies (z^{k*}_o)^{\sigma-1} \bar{\Pi}^{k,L}_o &= \frac{M^k_o}{M^{k,G}_o} \bar{\Pi}^{k,L}_o - \frac{1}{M^{k,G}_o} \frac{\vartheta_k - \sigma + 1}{\vartheta_k} \left(\Pi^{k,L}_o\right) \end{aligned} \tag{B.14}$$

where the last equality uses $\frac{M^{k,G}_o}{M^k_o} = (z^{k*}_o)^{-\vartheta_k}$. The note that profits of the marginal local company can be written as:

$$M^k_o \left(\bar{\Pi}^{k,L}_o\right) = \frac{\sigma - \vartheta_k - 1}{\vartheta_k \left(\left(z^{k*}_o\right)^{\sigma-\vartheta_k-1} - 1\right)} \Pi^{k,L}_j$$

and that z^{k*}_o can be expressed as a function of $M^{k,G}_o$ using the properties of the Pareto distribution:

$$z^{k*}_o = \left(\frac{M^{k,G}_o}{M^k_o}\right)^{-\frac{1}{\vartheta_k}} \implies M^k_o (z^{k*}_o)^{-\vartheta_k} = M^{k,G}_o$$

Putting things together:

$$M_\ell^{k,G} w_\ell f_\ell^k = \frac{\vartheta_k - \sigma + 1}{\vartheta_k}(\Pi_\ell^{k,G} + \Pi_\ell^{k,L}) + \frac{\vartheta_k - \sigma + 1}{\vartheta_k}\left(\frac{1}{\left(\frac{M_\ell^{k,G}}{M_\ell^k}\right)^{-\frac{\sigma-\vartheta_k-1}{\vartheta_k}} - 1}\Pi_\ell^{k,L}\right)$$

where aggregate profits of local and global firms can then be expressed as a function of the previously computed expenditure shares, wages and expenditures.

To summarize, the system of equilibrium conditions is:

$$M_m^{k,G} w_m f_m^k = \frac{\vartheta^k - \sigma + 1}{\vartheta^k}\left[\Pi_m^{k,G} - \frac{\left(\frac{M_m^{k,G}}{M_m^k}\right)^{\frac{\vartheta^k-\sigma+1}{\vartheta^k}}}{1 - \left(\frac{M_m^{k,G}}{M_m^k}\right)^{\frac{\vartheta^k-\sigma+1}{\vartheta^k}}}\Pi_m^{k,L}\right] \tag{B.15}$$

$$E_m = \left(\sum_{\ell'}\sum_s \frac{1}{t_{\ell' i}^s} e_m^s \lambda_{l' m}^s\right)^{-1}\left[w_m L_m + \sum_k \left(\Pi_m^{k,G} + \Pi_m^{k,L}\right) - \sum_k M_m^{k,G} w_m f_m^k\right] \tag{B.16}$$

$$w_m L_m = \sum_j \Lambda_{\ell j}\left[w_j L_j + \sum_k \left(\Pi_j^{k,G} + \Pi_j^{k,L}\right) - \sum_k M_j^{k,G} w_j f_j^k\right] - \sum_k \left(\Pi_m^{k,G} + \Pi_m^{k,L}\right) + \sum_k M_m^{k,G} w_m f_m^k \tag{B.17}$$

where

$$\Lambda_{\ell j} = \frac{\sum_k \frac{e_j^k \lambda_{\ell j}^k}{t_{\ell j}^k}}{\left(\sum_{\ell'}\sum_{k'} \frac{1}{t_{\ell' m}^{k'}} e_j^k \lambda_{\ell' j}^{k'}\right)}, \quad \text{with} \quad \lambda_{\ell j}^k = \frac{\sum_i t_{\ell j}^k P_{i,\ell j}^{k,G} Q_{i,\ell j}^{k,G} + t_{\ell j}^k P_{\ell,\ell j}^{k,L} Q_{\ell,\ell j}^{k,L}}{e_j^k E_j} \tag{B.18}$$

$$\Pi_m^{k,G} = \sum_j \sum_\ell \left(1 - \frac{1}{\mu_k^G}\right)\frac{1}{t_{\ell j}^k}\tilde{\lambda}_{m,\ell j}^{k,G} e_j^k E_j, \quad \text{with} \quad \tilde{\lambda}_{m,\ell j}^{k,L} = \frac{t_{\ell j}^k P_{m,\ell j}^{k,G} Q_{m,\ell j}^{k,G}}{e_j^k E_j} \tag{B.19}$$

$$\Pi_m^{k,L} = \sum_j \left(1 - \frac{1}{\mu_k^L}\right)\frac{1}{t_{mj}^k}\tilde{\lambda}_{m,mj}^{k,L} e_j^k E_j, \quad \text{with} \quad \tilde{\lambda}_{m,mj}^{k,L} = \frac{t_{mj}^k P_{m,mj}^{k,L} Q_{m,mj}^{k,L}}{e_j^k E_j} \tag{B.20}$$

C Perfect Competition Benchmark

This section focuses on the benchmark with perfect competition and no fixed costs. In Section C.1, we show how the system of equilibrium conditions can be simplified in the perfect competition benchmark. In C.2, we compute the gains from trade, in Section C.3 we characterize the trade elasticity and in Section C.4 we prove the existence of an equilibrium.

C.1 Equilibrium Conditions

In this section, we describe some of the model properties in a benchmark with two simplifying assumptions. First, we set fixed costs f_m^k to zero, which implies that all firms are global, and normalize the exogenous measure of firms to one in all countries. As an implication, we can drop

the threshold condition (B.15) which determines selection into multinational companies. Second, we assume perfect competition, i.e. $\mu_k^L = \mu_k^G = 1$ and drop profits from constraints (B.16) and (B.17). The absence of profits implies that trade is balanced. Combining the trade balance condition with the labor market clearing condition, we can express (B.16) and (B.17) as a single constraint in terms of the vector of wages only. As a result, the equilibrium conditions reduce to a system of equations for the vector of wages:

$$w_i\left(\boldsymbol{t}_m, \boldsymbol{t}_{-m}\right) L_i = \sum_{j \in \mathcal{N}} \Lambda_{ij}\left(\boldsymbol{t}_m, \boldsymbol{t}_{-m}\right) w_j\left(\boldsymbol{t}_m, \boldsymbol{t}_{-m}\right) L_j \quad \forall i \in \mathcal{N} \tag{C.1}$$

where the definition of Λ_{ij} is unchanged from (B.18).

C.2 Gains from trade

As in standard trade models, in the decentralized equilibrium the utility of the household equals its real income, $U_m = E_m/P_m$. Because the country collects tariff revenue the income of the household is not equal to wage income and so

$$\frac{U_m}{L_m} = \frac{W_m/P_m}{\sum_{\ell \in \mathcal{N}} \sum_k \left(e_m^k \lambda_{\ell m}^k / \left(t_{\ell m}^k\right)\right)}. \tag{C.2}$$

The term in the denominator equals the ratio of country m's total expenditure to its wage income. When country m collects import tariff the two differ in equilibrium exactly by the tariff revenue. Demidova et al. (2022) refer to this term as country m's tariff multiplier.

We now show that our model implies that the household welfare allows for a sufficient statistic representation given by

$$\frac{U_m}{L_m} = \prod_k \left[e_m^k A_m^k \times \frac{\left(\lambda_{mm}^k\right)^{1/(1-\sigma)}}{\sum_{\ell \in \mathcal{N}} \sum_{k \in \mathcal{K}} \left(e_m^k \lambda_{\ell m}^k / \left(t_{\ell m}^k\right)\right)} \frac{\left(\lambda_{m,mm}^k\right)^{\frac{1}{1-\varepsilon}} \times \left(\lambda_{m,m}^k\right)^{\frac{1}{1-\sigma}}}{\left(\lambda_{mm}^k\right)^{\frac{1}{1-\sigma}}} \right]^{e_m^k}. \tag{C.3}$$

where λ_{mm}^k is country mj's trade share in sector k, $\lambda_{m,m}^k$ is country m's expenditure share in goods originated in m in sector k, and $\lambda_{m,mm}^k$ is country m's assembly share from location m of good originated in m in sector k. The first term measures country $m's$ welfare in autarky $U_m/L_m = \prod_k A_m^k$, which implies that equation (C.3) captures the gains from trade and multinational production relative to autarky, i.e. the welfare gains from openness (Ramondo and Rodríguez-Clare (2013)). As explained in section 3.3 the assumption of Cobb-Douglas preferences implies a log-additive structure that allows us to ignore cross elasticities of substitution between sectors.

Proposition 1. Welfare in country j has a sufficient statistic representation given by:

$$\frac{U_m}{L_m} = \prod_k e_m^k (GO_m^k)^{e_m^k} = \prod_k e_m^k \frac{(\lambda_{mm}^k)^{1/(1-\sigma)}}{\sum_{k\in\mathcal{K}} \sum_{\ell\in\mathcal{N}} \left(e_m^k \lambda_{\ell m}^k / \left(t_{\ell m}^k\right)\right)} \frac{\left(\lambda_{m,mm}^k\right)^{\frac{1}{1-\varepsilon}} \times \left(\lambda_{m,m}^k\right)^{\frac{1}{1-\sigma}}}{\left(\lambda_{mm}^k\right)^{\frac{1}{1-\sigma}}}, \qquad \text{for all} \quad m \in \mathcal{N}. \tag{C.4}$$

Proof. We start by noticing that the utility of the household equals its real income:

$$E_m = \sum_{i\in\mathcal{I}} \sum_{\ell\in\mathcal{L}} \sum_{m\in\mathcal{N}} \left(t_{\ell m}^k\right) P_{i,\ell m}^k Q_{i,\ell m}^k. \tag{C.5}$$

Welfare can then be expressed as,

$$\frac{U_m}{L_m} = \frac{W_m}{P_m} \times \left(\frac{W_m L_m}{\sum_k \sum_{i\in\mathcal{I}} \sum_{\ell\in\mathcal{L}} \left(t_{\ell m}^k\right) P_{i,\ell m}^k Q_{i,\ell m}^k} \right)^{-1}. \tag{C.6}$$

Starting from the labor market clearing condition and using the pricing equation, we can express country m's value added as

$$W_m L_m = \sum_{i\in\mathcal{I}} \sum_{\ell\in\mathcal{N}} L_{i,m\ell} W_m = \sum_{i\in\mathcal{I}} \sum_{\ell\in\mathcal{N}} \sum_{k\in K} P_{i,m\ell}^k Q_{i,m\ell}^k.$$

Note that in the right-hand-side of this equation we have country $m's$ total exports, and therefore through the trade balance condition, we get,

$$W_m L_m = \sum_{i\in\mathcal{I}} \sum_{\ell\in\mathcal{N}} \sum_{k\in K} P_{i,\ell m}^k Q_{i,\ell m}^k. \tag{C.7}$$

Equations (C.5) to (C.7) combined imply,

$$\frac{U_m}{L_m} = \frac{W_m}{P_m} \times \left(\sum_{i\in\mathcal{I}} \sum_{m\in\mathcal{N}} \sum_{k\in K} \frac{1}{\left(t_{mj}^k\right)} \frac{\left(t_{mj}^k\right) P_{i,\ell m}^k Q_{i,\ell m}^k}{\sum_k \sum_{i\in\mathcal{I}} \sum_{\ell\in\mathcal{L}} \left(t_{\ell m}^k\right) P_{i,\ell m}^k Q_{i,\ell m}^k} \right)^{-1}$$

Note that:

$$\frac{\sum_{i\in\mathcal{I}} \left(t_{\ell m}^k\right) P_{i,\ell m}^k Q_{i,\ell m}^k}{\sum_k \sum_{i\in\mathcal{I}} \sum_{\ell'\in\mathcal{L}} \left(t_{\ell' j}^k\right) P_{i,\ell' m}^k Q_{i,\ell' m}^k} = \frac{\sum_{i\in\mathcal{I}} \left(t_{\ell m}^k\right) P_{i,\ell m}^k Q_{i,\ell m}^k}{\sum_{i\in\mathcal{I}} \sum_{\ell'\in\mathcal{L}} \left(t_{\ell' j}^k\right) P_{i,\ell' m}^k Q_{i,\ell' m}^k} \frac{\sum_{i\in\mathcal{I}} \sum_{\ell'\in\mathcal{L}} \left(t_{\ell' j}^k\right) P_{i,\ell' m}^k Q_{i,\ell' m}^k}{\sum_k \sum_{i\in\mathcal{I}} \sum_{\ell'\in\mathcal{L}} \left(t_{\ell' j}^k\right) P_{i,\ell' m}^k Q_{i,\ell' m}^k} = e_m^k \lambda_{\ell m}^k$$

is country m- sector k good'ss import share from country ℓ. Then, country m's welfare can be re-cast as:

$$\frac{U_m}{L_m} = \frac{W_m / P_m}{\sum_{\ell\in\mathcal{N}} \sum_{k\in K} \left(e_m^k \lambda_{\ell m}^k / \left(t_{\ell m}^k\right)\right)}. \tag{C.8}$$

We now relate a country's real wage income W_m/P_m to MP shares. For that we first define two new shares. First, we let $\lambda_{i,\ell m}^k$ denote country m's assembly share from location ℓ of good origin i

in sector k, defined as

$$\lambda_{i,\ell m}^{k} = \frac{\left(t_{\ell n}^{k}\right) P_{i,\ell m}^{k} Q_{i,\ell m}^{k}}{\sum_{j\in\mathcal{L}} \left(t_{jm}^{k}\right) P_{i,jm}^{k} Q_{i,jm}^{k}}. \tag{C.9}$$

Second, we let $\lambda_{i,m}^{k}$ denote country m's expenditure share in goods originated in i out of total expenditures on sector k goods, defined as

$$\lambda_{i,m}^{k} = \frac{\sum_{\ell\in\mathcal{L}} \left(t_{\ell m}^{k}\right) P_{i,\ell m}^{k} Q_{i,\ell m}^{k}}{\sum_{o\in\mathcal{I}} \sum_{j\in\mathcal{L}} \left(1+t_{jm}^{k}\right) P_{o,jm}^{k} Q_{o,jm}^{k}}. \tag{C.10}$$

Note that equations (C.9) and (C.10) imply

$$\lambda_{jm}^{k} = \sum_{i\in\mathcal{I}} \lambda_{i,jm}^{k} \times \lambda_{i,m}^{k}. \tag{C.11}$$

The next step is to use the expressions for prices and combine them with the definition in equations (C.10) and (C.11).

We first combine equation (C.10) with the price of an Armington variety to express assembly shares as a function of countries wage'

$$\lambda_{i,\ell m} = \frac{\left(\left(t_{\ell m}^{k}\right) \tau_{\ell m} W_{\ell} \gamma_{i\ell}^{k} / A_{i}^{k}\right)^{1-\varepsilon}}{\sum_{\ell\in\mathcal{L}} \left(t_{jm}^{k} \tau_{jm} W_{j} \gamma_{ij}^{k} / A_{i}^{k}\right)^{1-\varepsilon}}. \tag{C.12}$$

Since the price index in country j for goods originated in i is given by $(P_{im}^{k})^{1-\varepsilon} = \sum_{\ell\in\mathcal{L}} \left(\left(t_{\ell m}^{k}\right) \tau_{\ell m} W_{k} \gamma_{i\ell}^{k} / A_{i}^{k}\right)^{1-\varepsilon}$ we can express the price index in terms of the assembly share $\lambda_{i,\ell m}^{k}$ as

$$P_{im}^{k} = \frac{\left(t_{\ell m}^{k}\right) \tau_{\ell m} W_{m} \gamma_{i\ell}^{k} / A_{i}^{k}}{\left(\lambda_{i,\ell m}^{k}\right)^{1/(1-\varepsilon)}}, \qquad \text{for all } \ell \in \mathcal{L}. \tag{C.13}$$

It is important to note that equation (C.13) holds for any location ℓ from which country m is buying from. In particular, it holds for purchases from country m itself and so

$$P_{im}^{k} = \left(\lambda_{i,mm}^{k}\right)^{-\frac{1}{1-\varepsilon}} \times \left(W_{m} \gamma_{im}^{k} / A_{i}^{k}\right). \tag{C.14}$$

Plugging this into the ideal price index in sector k, we find:

$$P_{m}^{k} = \left[\sum_{i\in\mathcal{I}} \left(\lambda_{i,mm}^{k}\right)^{-\frac{1-\sigma}{1-\varepsilon}} \times \left(\gamma_{im}^{k} A_{i}^{k}\right)^{-(1-\sigma)}\right]^{\frac{1}{1-\sigma}} W_{m}. \tag{C.15}$$

The last step is to re-write these prices in terms of the MP shares. We start with equation (C.10)

and use the zero profit condition for assemblers to get:

$$\lambda_{i,m} = \frac{P_{in}Q_{in}}{\sum_{a\in\mathcal{I}} P_{an}X_{an}}.$$

Combining with the demand system that implies $\lambda^k_{i,n} = \left(\frac{P^k_{im}}{P^k_m}\right)^{1-\sigma}$, we can write the real wage in terms of sector k good as:

$$\frac{W_m}{P^k_m} = (\lambda^k_{i,m})^{\frac{1}{1-\sigma}} \left(\lambda^k_{i,mm}\right)^{\frac{1}{1-\varepsilon}} \gamma^k_{im} A^k_i, \tag{C.16}$$

and we note as before that this must hold for any origin good i, in particular for those originated at country m. Finally, note that:$P_m = \prod_k \left(\frac{P^k_m}{e^k_m}\right)^{e^k_m}$. Then:

$$\frac{W_m}{P_m} = \prod_k \left[e^k_m (\lambda^k_{m,m})^{\frac{1}{1-\sigma}} \left(\lambda^k_{m,mm}\right)^{\frac{1}{1-\varepsilon}} A^k_m\right]^{e^k_m}, \tag{C.17}$$

Putting together equations (C.8) and (C.17) we get:

$$\frac{U_m}{L_m} = \prod_k \left[e^k_m \frac{A^k_m (\lambda^k_{mm})^{1/(1-\sigma)}}{\sum_{\ell\in\mathcal{N}}\sum_{k\in\mathcal{K}} \left(e^k_{\ell m}\lambda^k_{\ell m}/\left(t^k_{\ell m}\right)\right)} \frac{(\lambda^k_{m,mm})^{\frac{1}{1-\varepsilon}} \times (\lambda^k_{m,m})^{\frac{1}{1-\sigma}}}{(\lambda^k_{mm})^{\frac{1}{1-\sigma}}}\right]^{e^k_m}. \tag{C.18}$$

□

C.3 Trade elasticity

Trade elasticity. We define the trade elasticity as the partial equilibrium response of imports from ℓ relative to domestic consumption due to a permanent change in wages. Given the multiplicative nature of trade costs, wages and tariffs in the Armington framework, it would be equivalent to define the trade elasticity with respect to changes in trade costs or tariffs applied to imports. In this section, we exploit the log-additive nature of our Cobb-Douglas preference system and focus on the trade elasticity in a single sector. For this reason, we remove the superscript k.

$$\theta_{m\ell} \equiv -\frac{\partial \log\left(\lambda_{m\ell}/\lambda_{mm}\right)}{\partial \log \tau_{m\ell}}. \tag{C.19}$$

In the next proposition, we show that the trade elasticity is affected by the same determinants as welfare: (i) the two elasticity of substitution (σ and ε), and (ii) the trade, MP, and assembly shares ($\lambda_{m\ell}, \lambda_{i,m}$,and $\lambda_{i,\ell m}$).

Proposition 2. The trade elasticity between country m and country ℓ is

$$\theta_{m\ell} = (\varepsilon - 1) - (\varepsilon - \sigma) \times \sum_{i\in\mathcal{I}} (\lambda_{i,m\ell} \times \lambda_{i,\ell}) \times \left(\frac{\lambda_{i,m\ell}}{\lambda_{m\ell}} - \frac{\lambda_{i,\ell\ell}}{\lambda_{\ell\ell}}\right). \tag{C.20}$$

Proof. We showed in Proposition 1 that trade, MP, and assembly shares can be written as:

$$\lambda_{m\ell} = \sum_{i \in \mathcal{I}} \lambda_{i,m\ell} \times \lambda_{i,\ell}, \tag{C.21}$$

$$\lambda_{i,m\ell} = \frac{\lambda_{i,jj}^{-\frac{1-\sigma}{1-\varepsilon}} A_{i\ell}^{\sigma-1}}{\sum_{a \in \mathcal{I}} \lambda_{a,\ell\ell}^{-\frac{1-\sigma}{1-\varepsilon}} A_{a\ell}^{\sigma-1}}, \tag{C.22}$$

$$\lambda_{i,m\ell} = \frac{\left(\left(t_{m\ell}\right) \tau_{m\ell} W_m / A_{im}\right)^{1-\varepsilon}}{\sum_{\ell \in \mathcal{L}} \left(\left(t_{\ell m}\right) \tau_{\ell m} W_\ell / A_{i\ell}\right)^{1-\varepsilon}}. \tag{C.23}$$

where the double subscript, e.g. A_{im}, in productivity is used to denote the product between a country i's productivity A_i and the MP frictions γ_{im}. We start by substituting equations (C.22) and (C.23) into equation (C.21) to express trade shares as functions of equilibrium wages:

$$\lambda_{m\ell} = \frac{\sum_{i \in \mathcal{I}} \left[\left(\left(t_{m\ell}\right) \tau_{m\ell} W_m / A_{im}\right)^{1-\varepsilon} \times \left(\sum_{j \in \mathcal{L}} \left(\left(t_{j\ell}\right) \tau_{j\ell} W_j / A_{ij}\right)^{1-\varepsilon}\right)^{\frac{\varepsilon-\sigma}{1-\varepsilon}}\right]}{\sum_{a \in \mathcal{I}} \left(\sum_{j \in \mathcal{L}} \left(\left(t_{j\ell}\right) \tau_{j\ell} W_j / A_{aj}\right)^{1-\varepsilon}\right)^{\frac{1-\sigma}{1-\varepsilon}}}.$$

It follows that,

$$\frac{\lambda_{m\ell}}{\lambda_{\ell\ell}} = \left(\left(t_{m\ell}\right) \tau_{m\ell} \frac{W_m}{W_\ell}\right)^{1-\varepsilon} \times \frac{\sum_{i \in \mathcal{I}} \left(A_{im}\right)^{\varepsilon-1} \times \left(\sum_{j \in \mathcal{L}} \left(\left(t_{j\ell}\right) \tau_{j\ell} W_j / A_{ij}\right)^{1-\varepsilon}\right)^{\frac{\varepsilon-\sigma}{1-\varepsilon}}}{\sum_{a \in \mathcal{I}} \left(A_{a\ell}\right)^{\varepsilon-1} \times \left(\sum_{j \in \mathcal{L}} \left(\left(t_{j\ell}\right) \tau_{j\ell} W_j / A_{aj}\right)^{1-\varepsilon}\right)^{\frac{\varepsilon-\sigma}{1-\varepsilon}}}. \tag{C.24}$$

The last step is to compute the partial derivative of equation (C.24) with respect to $\tau_{m\ell}$ keeping wages fixed. Define,

$$\mathbb{A}_{m\ell} \equiv \sum_{i \in \mathcal{I}} \left(A_{im}\right)^{\varepsilon-1} \times \left(\sum_{j \in \mathcal{L}} \left(\left(t_{j\ell}\right) \tau_{j\ell} W_j / A_{ij}\right)^{1-\varepsilon}\right)^{\frac{\varepsilon-\sigma}{1-\varepsilon}}.$$

Then,

$$\theta_{m\ell} = (\varepsilon - 1) - \frac{1}{\mathbb{A}_{m\ell}} \frac{\partial \mathbb{A}_{m\ell}}{\partial \log \tau_{m\ell}} + \frac{1}{\mathbb{A}_{\ell\ell}} \frac{\partial \mathbb{A}_{\ell\ell}}{\partial \log \tau_{m\ell}},$$

with

$$\frac{1}{\mathbb{A}_{m\ell}} \frac{\partial \mathbb{A}_{m\ell}}{\partial \log \tau_{m\ell}} = (\varepsilon - \sigma) \times \left(\sum_{i \in \mathcal{I}} \lambda_{i,m\ell} \times \frac{\lambda_{i,m\ell} \times \lambda_{i,\ell}}{\lambda_{m\ell}}\right),$$

$$\frac{1}{\mathbb{A}_{\ell\ell}} \frac{\partial \mathbb{A}_{\ell\ell}}{\partial \log \tau_{m\ell}} = (\varepsilon - \sigma) \times \left(\sum_{a \in \mathcal{I}} \lambda_{a,m\ell} \times \frac{\lambda_{a,\ell\ell} \times \lambda_{a,\ell}}{\lambda_{\ell\ell}}\right).$$

Therefore, the trade elasticity between country m and ℓ equals

$$\theta_{m\ell} = (\varepsilon - 1) - (\varepsilon - \sigma) \times \left[\sum_{i \in \mathcal{I}} \left(\lambda_{i,m\ell} \times \lambda_{i,\ell}\right) \times \left(\frac{\lambda_{i,m\ell}}{\lambda_{m\ell}} - \frac{\lambda_{i,\ell\ell}}{\lambda_{\ell\ell}}\right)\right].$$

□

In general, the trade elasticity is not constant and is shape by the amount of trade and MP in the economy. However, there are two extreme cases in which $\theta_{m\ell}$ becomes constant and only depends on either the elasticity of substitution across varieties (σ or ε) as in standard models of trade.

Corollary 1. When either MP is free ($\gamma_{ij} = 1$ for all i and ℓ) or prohibitively costly ($\gamma_{ij} = \infty$ for all i and ℓ) the trade elasticity between country m and country ℓ is constant and equals to $\theta_{m\ell} = \varepsilon - 1$ and $\theta_{m\ell} = \sigma - 1$, respectively.

Proof. In the first case, it is easy to check that $\lambda_{i,mn} = \lambda_{a,mn} = \lambda_{mn}$. Substituting this into equation (C.20) yields

$$\theta_{m\ell} = (\varepsilon - 1) - (\varepsilon - \sigma) \times \sum_{i \in \mathcal{I}} (\lambda_{i,m\ell} \times \lambda_{i,\ell}) \times \left(\frac{1}{\sum_{a \in \mathcal{I}} \lambda_{i,\ell}} - \frac{1}{\sum_{a \in \mathcal{I}} \lambda_{i,\ell}} \right) = (\varepsilon - 1) .$$

For the other case, we note that if $\gamma_{i\ell} = \infty$ for all countries i and locations ℓ trade shares between any two countries become

$$\frac{\lambda_{m\ell}}{\lambda_{\ell\ell}} = \frac{\left((t_{m\ell})\, \tau_{m\ell} W_m / A_{mm} \right)^{1-\sigma}}{\left(W_\ell / A_{\ell\ell} \right)^{1-\sigma}},$$

and clearly $\theta_{m\ell} = \sigma - 1$. □

Corollary 2. If the elasticity across location of production is higher than the elasticity across goods, i.e., if $\varepsilon > \sigma$, then the trade elasticity is higher in the presence of MP.

Proof. Remember from Corollary 1 when MP becomes prohibitively costly the trade elasticity between country m and country j is constant and equal to $\theta_{m\ell} = \sigma - 1$. Then, it suffices to show that $\theta_{m\ell} > \sigma - 1$. From Proposition 1 we have

$$\theta_{m\ell} - (\sigma - 1) > (\varepsilon - \sigma) \left[1 - \sum_{i \in \mathcal{I}} (\lambda_{i,m\ell} \times \lambda_{i,\ell}) \times \left(\frac{\lambda_{i,mj}}{\lambda_{m\ell}} - \frac{\lambda_{i,\ell\ell}}{\lambda_{\ell\ell}} \right) \right].$$

Then $\theta_{m\ell} > \sigma - 1$ if and only if the term in squared brackets is non-negative, which is true since

$$1 - \sum_{i \in \mathcal{I}} (\lambda_{i,m\ell} \times \lambda_{i,\ell}) \times \left(\frac{\lambda_{i,m\ell}}{\lambda_{m\ell}} - \frac{\lambda_{i,\ell\ell}}{\lambda_{\ell\ell}} \right) = \sum_{i \in \mathcal{I}} (\lambda_{i,m\ell} \times \lambda_{i,\ell}) \left(\frac{1 - \lambda_{i,m\ell}}{\lambda_{m\ell}} + \frac{\lambda_{i,\ell\ell}}{\lambda_{\ell\ell}} \right) > 0.$$

□

C.4 Existence of Equilibrium

In this section, we focus on the equilibrium of the perfect competition model with one sector. The proof generalizes to multiple sectors. We showed how trade, assembly, and MP shares can all be expressed as functions of the wages in different countries. We use this in this section to show

that the equilibrium can be defined as the solution to a non-linear system on the vector of wages $\mathbf{W} = \{W_j\}_{j\in\mathcal{N}}$. Remember these share are each given by,

$$\lambda_{i,mj} = \frac{\left(\left(t_{mj}\right)\tau_{mj}W_m/A_{im}\right)^{1-\varepsilon}}{\sum_{\ell\in\mathcal{L}}\left(\left(t_{\ell j}\right)\tau_{\ell j}W_\ell/A_{i\ell}\right)^{1-\varepsilon}}, \tag{C.25}$$

$$\lambda_{i,j} = \frac{\lambda_{i,jj}^{-\frac{1-\sigma}{1-\varepsilon}}A_{ij}^{\sigma-1}}{\sum_{a\in\mathcal{I}}\lambda_{a,jj}^{-\frac{1-\sigma}{1-\varepsilon}}A_{aj}^{\sigma-1}}, \tag{C.26}$$

$$\lambda_{mj} = \sum_{i\in\mathcal{I}}\lambda_{i,mj}\times\lambda_{i|j}. \tag{C.27}$$

Labor market clearing along with the trade balance condition imply,

$$W_jL_j = \sum_{m\in\mathcal{N}}\Lambda_{jm}W_mL_m, \tag{C.28}$$

where,

$$\Lambda_{jm} = \frac{\lambda_{jm}/\left(t_{jm}\right)}{\sum_{n\in\mathcal{N}}\lambda_{nm}/\left(t_{nm}\right)}.$$

Therefore, equation (C.25) to (C.28) encode all the information for an equilibrium and so an a equilibrium is a vector of wages $\mathbf{W}$ such that equation (C.28) holds for all country $j\in\mathcal{N}$. In Proposition (3), we establish that an equilibrium given tariff always exists, and it is unique.

Proposition 3. There exists an equilibrium.

Proof. **Existence.** The equilibrium system can be rewritten as

$$z_j\left(\mathbf{W}\right) = 0, \qquad \text{for all } j\in\mathcal{N}$$

where $z_j\left(\mathbf{W}\right)$ is country j's excess labor demand function implied by

$$z_j\left(\mathbf{W}\right) = \left(\sum_{m\in\mathcal{N}}\Lambda_{jm}W_mL_m - W_jL_j\right)/W_j.$$

We first show that the excess labor demand function $\mathbf{z}\left(\mathbf{W}\right)\equiv\left(z_1\left(\mathbf{W}\right),z_2\left(\mathbf{W}\right),\ldots,z_N\left(\mathbf{W}\right)\right)$ defined for all $\mathbf{W}\gg 0$ satisfies the properties in Proposition 17.B.2 of Mas-Colell et al. (1995): (i) $\mathbf{z}\left(\mathbf{W}\right)$ is continuous; (ii) $\mathbf{z}\left(\mathbf{W}\right)$ is homogeneous of degree zero; (iii) $\mathbf{W}\cdot\mathbf{z}\left(\mathbf{W}\right)=0$ for all $\mathbf{W}$; (iv) there exists $s>0$ such that $z_j\left(\mathbf{W}\right)>-s$ for all j and for all $\mathbf{W}$; (v) if $\mathbf{W}^n\to\mathbf{W}^0$, where $\mathbf{W}^0\neq 0$ and $W_i^0=0$ for some i, then $\max\left\{\mathbf{z}\left(\mathbf{W}^n\right)\right\}\to\infty$.

Properties (i) and (iii) are obvious, while property (ii) follows from $\lambda_{i,mj}$ being homogeneous of degree zero when consider a function of $\mathbf{W}$. We get property (iv) by setting $s=\max_k L_k$ since for al country j we have

$$z_j\left(\mathbf{W}\right)+s = \frac{\sum_{m\in\mathcal{N}}\Lambda_{jm}W_mL_m}{W_j} - L_j + \max_k L_k = \frac{\sum_{m\in\mathcal{N}}\Lambda_{jm}W_mL_m}{W_j} + \left(\max_k L_k - L_j\right) > 0.$$

For property (iv), consider a wage vector $\mathbf{W}^0$, where $W_k^0 = 0$ for some k, and $W_j^0 > 0$ for all $j \neq i$. Then, from equation (C.25) and equation (C.26) follows that

$$\lim_{\mathbf{W}^n \to \mathbf{W}^0} \lambda_{i,km} = 1, \qquad \lim_{\mathbf{W}^n \to \mathbf{W}^0} \lambda_{i,jm} = 0, \qquad \lim_{\mathbf{W}^n \to \mathbf{W}^0} \lambda_{i|m} = \frac{A_{ik}^{\sigma-1}}{\sum_{a \in \mathcal{I}} A_{ak}^{\sigma-1}}. \tag{C.29}$$

Using this in equation (C.27) give us that

$$\lim_{\mathbf{W}^n \to \mathbf{W}^0} \lambda_{km} = \lim_{\mathbf{W}^n \to \mathbf{W}^0} \sum_{i \in \mathcal{I}} \lambda_{i,km} \times \lambda_{i|m} = \lim_{\mathbf{W}^n \to \mathbf{W}^0} \sum_{i \in \mathcal{I}} \frac{A_{ik}^{\sigma-1}}{\sum_{a \in \mathcal{I}} A_{ak}^{\sigma-1}} = 1, \tag{C.30}$$

and

$$\lim_{\mathbf{W}^n \to \mathbf{W}^0} \lambda_{nm} = \lim_{\mathbf{W}^n \to \mathbf{W}^0} \sum_{i \in \mathcal{I}} \lambda_{i,nm} \times \lambda_{i|m} = 0. \tag{C.31}$$

Putting together what we've learned from equations (C.29) to (C.31) give us that

$$\lim_{\mathbf{W}^n \to \mathbf{W}^0} \Lambda_{km} = \lim_{\mathbf{W}^n \to \mathbf{W}^0} \frac{\lambda_{km} / (1 + t_{jm})}{\sum_{n \in \mathcal{N}} \lambda_{nm} / (1 + t_{nm})} = 1, \qquad \text{for all } m \in \mathcal{N}. \tag{C.32}$$

Take now the excess labor demand for country k,

$$\lim_{\mathbf{W}^n \to \mathbf{W}^0} z_k(\mathbf{W}^n) = \lim_{\mathbf{W}^n \to \mathbf{W}^0} \left(\Lambda_{kk} L_m + \sum_{m \neq k} \Lambda_{km} \frac{W_m^n}{W_k^n} L_m - L_k \right) = +\infty,$$

which verifies property (v). Since Proposition 17.B.2 constitutes a set of sufficient conditions for Proposition 17.C.1 of Mas-Colell et al. (1995), there exists an equilibrium wage vector $\mathbf{W} \gg 0$ such that $\mathbf{z}(\mathbf{W}) = 0$.

□

Proposition 4. If import tariff satisfy that $t_{\ell m} \in \left[0, \frac{\sigma-1}{\varepsilon-\sigma}\right]$, the equilibrium is unique.

Proof. **Uniqueness.** We next show that $\mathbf{z}(\mathbf{W})$ satisfies the gross substitute property in Definition 17.F.2 of Mas-Colell et al. (1995): if $\mathbf{W}'$ and $\mathbf{W}$ are such that $W_\ell' > W_\ell$ for some ℓ and $W_j' = W_j$ for all $j \neq l$, $z_j(\mathbf{W}') > z_j(\mathbf{W})$ for all $j \neq l$. With differentiability, which we have, $\partial z_j(\mathbf{W}) / \partial W_\ell > 0$ for all j and ℓ with $\ell \neq j$ ensures the gross substitute property. Direct calculations of these derivatives gives,

$$\frac{\partial z_j(\mathbf{W})}{\partial W_\ell} = \left(\Lambda_{j\ell} L_\ell + \sum_{m \in \mathcal{N}} \frac{\partial \Lambda_{jm}}{\partial W_\ell} W_m L_m \right) \frac{L_j}{W_j L_j},$$

where

$$\begin{aligned}
\frac{\partial \Lambda_{jm}}{\partial W_\ell} &= \Lambda_{jm}\left(\frac{1}{\lambda_{jm}}\frac{\partial \lambda_{jm}}{\partial W_\ell} - \sum_{n\in\mathcal{N}} \Lambda_{nm}\frac{1}{\lambda_{nm}}\frac{\partial \lambda_{nm}}{\partial W_\ell}\right), \qquad m \in \mathcal{N} \\
\frac{\partial \lambda_{nm}}{\partial W_\ell} &= \sum_{i\in\mathcal{I}}\left(\frac{\partial \lambda_{i,nm}}{\partial W_\ell} \times \lambda_{i|m} + \lambda_{i,nm} \times \frac{\partial \lambda_{i|m}}{\partial W_\ell}\right) \qquad n, m \in \mathcal{N} \\
\frac{\partial \lambda_{i|m}}{\partial W_\ell} &= -\left(\frac{\sigma-1}{\varepsilon-1}\right)\lambda_{i|m}\left[\frac{1}{\lambda_{i,mm}}\frac{\partial \lambda_{i,mm}}{\partial W_\ell} - \sum_{a\in\mathcal{I}} \lambda_{a|m}\frac{1}{\lambda_{a,mm}}\frac{\partial \lambda_{a,mm}}{\partial W_\ell}\right], \qquad m \in \mathcal{N} \\
\frac{\partial \lambda_{a,nm}}{\partial W_\ell} &= \begin{cases} \frac{1}{W_\ell}(\varepsilon-1)\lambda_{a,nm}\lambda_{a,\ell m} & n \neq l, m \in \mathcal{N} \\ -(\varepsilon-1)\frac{1}{W_\ell}\lambda_{a,\ell m}(1-\lambda_{a,\ell m}) & n = l, m \in \mathcal{N} \end{cases}.
\end{aligned}$$

Therefore,

$$\frac{\partial \lambda_{nm}}{\partial W_\ell} = \begin{cases} (\varepsilon-\sigma)\sum_{i\in\mathcal{I}}\frac{\lambda_{i,\ell m}\lambda_{i|m}\lambda_{i,nm}}{W_\ell} + (\sigma-1)\frac{\lambda_{\ell m}\lambda_{nm}}{W_\ell} & n \neq l, m \in \mathcal{N} \\ -(\varepsilon-\sigma)\sum_{i\in\mathcal{I}}\frac{\lambda_{i,\ell m}\lambda_{i|m}(1-\lambda_{i,\ell m})}{W_\ell} - (\sigma-1)\frac{\lambda_{\ell m}(1-\lambda_{\ell m})}{W_\ell} & n = l, m \in \mathcal{N} \end{cases}.$$

□

and

$$\frac{1}{\lambda_{jm}}\frac{\partial \lambda_{jm}}{\partial W_\ell} - \sum_{n\in\mathcal{N}}\Lambda_{nm}\frac{1}{\lambda_{nm}}\frac{\partial \lambda_{nm}}{\partial W_\ell} = \frac{\Lambda_{\ell m}}{W_\ell}(\varepsilon-1) + \sum_{n\in\mathcal{N}}\frac{\Lambda_{nm}}{W_\ell}(\varepsilon-\sigma)\left(\sum_{i\in\mathcal{I}}\lambda_{i,\ell m}\left(\frac{\lambda_{i,jm}\lambda_{i|m}}{\lambda_{jm}} - \frac{\lambda_{i,nm}\lambda_{i|m}}{\lambda_{nm}}\right)\right).$$

This give us:

$$\begin{aligned}
\sum_{m\in\mathcal{N}}\frac{\partial \Lambda_{jm}}{\partial W_\ell}W_m L_m &= (\varepsilon-1)\sum_{m\in\mathcal{N}}\frac{\Lambda_{jm}\Lambda_{\ell m}}{W_\ell}W_m L_m \\
&\quad + (\varepsilon-\sigma)\sum_{m\in\mathcal{N}}\sum_{n\in\mathcal{N}}\sum_{i\in\mathcal{I}}\frac{\Lambda_{jm}\Lambda_{nm}}{W_\ell}\lambda_{i,\ell m}\left(\frac{\lambda_{i,jm}\lambda_{i|m}}{\lambda_{jm}} - \frac{\lambda_{i,nm}\lambda_{i|m}}{\lambda_{nm}}\right)W_m L_m.
\end{aligned} \tag{C.33}$$

Note that we cannot still sign this derivative since the last term could be either positive or negative. The last step is to bound by below this last term.

First, we note that

$$\Lambda_{nj} = \left(\frac{W_j L_j}{\sum_{i\in\mathcal{I}}\sum_{\ell\in\mathcal{L}}(1+t_{\ell j})P_{i,\ell j}Q_{i,\ell j}}\right)^{-1}\frac{\lambda_{nj}}{(1+t_{nj})} \equiv \Phi_j\lambda_{nj}/(1+t_{nj}). \tag{C.34}$$

and so

$$\frac{\Lambda_{jm}\Lambda_{nm}}{W_\ell} = \frac{1}{W_\ell}\Phi_m^2\frac{\lambda_{jm}}{(1+t_{jm})}\frac{\lambda_{nm}}{(1+t_{nm})}.$$

Now,

$$\sum_{m\in\mathcal{N}}\sum_{n\in\mathcal{N}}\sum_{i\in\mathcal{I}}\frac{\Lambda_{jm}\Lambda_{nm}}{W_\ell}\lambda_{i,\ell m}\left(\frac{\lambda_{i,jm}\lambda_{i|m}}{\lambda_{jm}}-\frac{\lambda_{i,nm}\lambda_{i|m}}{\lambda_{nm}}\right)W_mL_m$$
$$=\sum_{m\in\mathcal{N}}\sum_{n\in\mathcal{N}}\sum_{i\in\mathcal{I}}\frac{1}{W_\ell}\Phi_m^2\frac{\lambda_{jm}}{(1+t_{jm})}\frac{\lambda_{nm}}{(1+t_{nm})}\lambda_{i,\ell m}\left(\frac{\lambda_{i,jm}\lambda_{i|m}}{\lambda_{jm}}-\frac{\lambda_{i,nm}\lambda_{i|m}}{\lambda_{nm}}\right)W_mL_m$$
$$=\frac{1}{W_\ell}\sum_{m\in\mathcal{N}}\Phi_m^2\frac{\lambda_{jm}}{(1+t_{jm})}W_mL_m\left[\sum_{n\in\mathcal{N}}\sum_{i\in\mathcal{I}}\frac{\lambda_{nm}}{(1+t_{nm})}\lambda_{i,\ell m}\left(\frac{\lambda_{i,jm}\lambda_{i|m}}{\lambda_{jm}}-\frac{\lambda_{i,nm}\lambda_{i|m}}{\lambda_{nm}}\right)\right]$$
$$=\frac{1}{W_\ell}\sum_{m\in\mathcal{N}}\Phi_m^2\frac{\lambda_{jm}}{(1+t_{jm})}W_mL_m\left[\left(\frac{\sum_{i\in\mathcal{I}}\lambda_{i,\ell m}\lambda_{i,jm}\lambda_{i|m}}{\lambda_{jm}}\right)\left(\sum_{n\in\mathcal{N}}\frac{\lambda_{nm}}{(1+t_{nm})}\right)-\sum_{i\in\mathcal{I}}\lambda_{i,\ell m}\lambda_{i|m}\sum_{n\in\mathcal{N}}\frac{\lambda_{i,nm}}{(1+t_{nm})}\right].$$

We focus in the case where $t_{nm}\geq 0$ for all countries. Then,

$$\sum_{m\in\mathcal{N}}\sum_{n\in\mathcal{N}}\sum_{i\in\mathcal{I}}\frac{\Lambda_{jm}\Lambda_{nm}}{W_\ell}\lambda_{i,\ell m}\left(\frac{\lambda_{i,jm}\lambda_{i|m}}{\lambda_{jm}}-\frac{\lambda_{i,nm}\lambda_{i|m}}{\lambda_{nm}}\right)W_mL_m$$
$$>\sum_{m\in\mathcal{N}}\frac{W_mL_m}{W_\ell}\Phi_m^2\frac{\lambda_{jm}}{1+t_{jm}}\left[\left(\frac{\sum_{i\in\mathcal{I}}\lambda_{i,\ell m}\lambda_{i,jm}\lambda_{i|m}}{\lambda_{jm}}\right)\left(\sum_{n\in\mathcal{N}}\frac{\lambda_{nm}}{1+t_{nm}}\right)-\sum_{i\in\mathcal{I}}\lambda_{i,\ell m}\lambda_{i|m}\sum_{n\in\mathcal{N}}\lambda_{i,nm}\right]$$
$$=\sum_{m\in\mathcal{N}}\frac{W_mL_m}{W_\ell}\Phi_m^2\frac{\lambda_{jm}}{1+t_{jm}}\left[\left(\frac{\sum_{i\in\mathcal{I}}\lambda_{i,\ell m}\lambda_{i,jm}\lambda_{i|m}}{\lambda_{jm}}\right)\left(\sum_{n\in\mathcal{N}}\frac{\lambda_{nm}}{1+t_{nm}}\right)-\sum_{i\in\mathcal{I}}\lambda_{i,\ell m}\lambda_{i|m}\right]$$
$$=\sum_{m\in\mathcal{N}}\frac{W_mL_m}{W_\ell}\Phi_m^2\frac{\lambda_{jm}}{1+t_{jm}}\left[\left(\frac{\sum_{i\in\mathcal{I}}\lambda_{i,\ell m}\lambda_{i,jm}\lambda_{i|m}}{\lambda_{jm}}\right)\left(\sum_{n\in\mathcal{N}}\frac{\lambda_{nm}}{1+t_{nm}}\right)-\lambda_{\ell m}\right]$$
$$=\sum_{m\in\mathcal{N}}W_mL_m\frac{\Lambda_{jm}\Lambda_{\ell m}}{W_\ell}\frac{1}{\frac{\lambda_{\ell m}}{(1+t_{\ell m})}}\left[\left(\frac{\sum_{i\in\mathcal{I}}\lambda_{i,\ell m}\lambda_{i,jm}\lambda_{i|m}}{\lambda_{jm}}\right)\left(\sum_{n\in\mathcal{N}}\frac{\lambda_{nm}}{1+t_{nm}}\right)-\lambda_{\ell m}\right]$$

where in the last equality we used equation (C.34) to construct inside the summation the term $\frac{\Lambda_{jm}\Lambda_{\ell m}}{W_\ell}W_mL_m$.

So now,

$$\begin{aligned}\sum_{m\in\mathcal{N}}\frac{\partial\Lambda_{jm}}{\partial W_\ell}W_mL_m &> (\varepsilon-1)\sum_{m\in\mathcal{N}}\frac{\Lambda_{jm}\Lambda_{\ell m}}{W_\ell}W_mL_m-\sum_{m\in\mathcal{N}}(\varepsilon-\sigma)W_mL_m\frac{\Lambda_{jm}}{W_\ell}\frac{\lambda_{\ell m}}{\left(\sum_{n\in\mathcal{N}}\frac{\lambda_{nm}}{1+t_{nm}}\right)}\\&\quad+\frac{(\varepsilon-\sigma)}{W_\ell}\sum_{m\in\mathcal{N}}\frac{W_mL_m\Phi_m}{(1+t_{jm})}\left(\sum_{i\in\mathcal{I}}\lambda_{i,\ell m}\lambda_{i,jm}\lambda_{i|m}\right)\\&=\sum_{m\in\mathcal{N}}\frac{W_mL_m\Lambda_{jm}\Lambda_{\ell m}}{W_\ell}\left[(\varepsilon-1)-(\varepsilon-\sigma)(1+t_{\ell m})\right]\\&\quad+\frac{(\varepsilon-\sigma)}{W_\ell}\sum_{m\in\mathcal{N}}\frac{W_mL_m\Phi_m}{(1+t_{jm})}\left(\sum_{i\in\mathcal{I}}\lambda_{i,\ell m}\lambda_{i,jm}\lambda_{i|m}\right)\\&>0,\end{aligned}$$

where the last inequality follows from $t_{\ell m} \leq \frac{\sigma-1}{\varepsilon-\sigma}$. Hence,

$$\frac{\partial z_j(\mathbf{W})}{\partial W_\ell} \geq 0, \qquad \text{for all } j, l \in \mathcal{N} \text{with } \ell \neq j.$$

This concludes the proof.

D The Small Open Economy Limit (SOE)

From now on we focus on country 0. Population, MP-frictions, and iceberg costs are -re-parameterized such that for country $j = 0$ (Home) we have:

$$\begin{aligned} \tilde{L}_0 &= n^{\frac{\sigma-1}{\varepsilon-1}} L_0 \\ \tilde{\tau}_{0j} &= n^{-1/(\varepsilon-1)} \tau_{0j}, \qquad j = 1, 2, \ldots, N \\ \tilde{\gamma}_{0j} &= n^{-1/(\varepsilon-1)} \gamma_{0,j}, \qquad j = 1, 2, \ldots, N. \end{aligned} \tag{D.1}$$

We take labor of country N as the numeraire: $\tilde{w}_N = 1$. In this section, we show that as country 0 becomes arbitrary small (i.e., as $n \to 0$), the equilibrium converges to the one in which $(\tilde{w}_1, \tilde{w}_2, \ldots, \tilde{w}_N)$ can be solved independently of country $0's$ variables, and that $\tilde{w}_0$ is finite. Since in equilibrium Foreign prices are independent of country 0's variables we see this limit as an equilibrium where country 0 is a small open economy.[1]

In this section we proceed in two steps. First we re-write the relevant equations in terms of the new parameterization equation (D.1). We then take the limit with $n \to 0$ and establish the main result.

[1] For now we don't address the problem of showing that an equilibrium exists and that is unique for all n. This step is important since it allow us to conclude that there is a well defined equilibrium in the limit.

1. Working with the equilibrium conditions. We first define:

$$\tilde{\lambda}^k_{o,ij} = \frac{\left(t^k_{ij}\tau_{ij}\gamma_{o,i}\tilde{w}_i\right)^{1-\varepsilon}}{\sum_{\ell=1}^N \left(t^k_{\ell j}\tau_{\ell j}\gamma_{o,\ell}\tilde{w}_\ell\right)^{1-\varepsilon} + n\left(t^k_{0j}\tau_{0j}\gamma_{o,0}\tilde{w}_0\right)^{1-\varepsilon}} \equiv \lambda^k_{o,ij}, \qquad o,i,j = 1,2,\ldots,N \tag{D.2}$$

$$\tilde{\lambda}^k_{0,ij} = \frac{\left(t^k_{ij}\tau_{ij}\gamma_{0,i}\tilde{w}_i\right)^{1-\varepsilon}}{\sum_{\ell=1}^N \left(t^k_{\ell j}\tau_{\ell j}\gamma_{0,\ell}\tilde{w}_\ell\right)^{1-\varepsilon} + \left(t^k_{0j}\tau_{0j}\tilde{w}_0\right)^{1-\varepsilon}} \equiv \lambda^k_{0,ij}, \qquad i,j = 1,2,\ldots,N$$

$$\tilde{\lambda}^k_{o,0j} = n\frac{\left(t^k_{0j}\tau_{0j}\gamma_{o,0}\tilde{w}_0\right)^{1-\varepsilon}}{\sum_{\ell=1}^N \left(t^k_{\ell j}\tau_{\ell j}\gamma_{o,\ell}\tilde{w}_\ell\right)^{1-\varepsilon} + n\left(t^k_{0j}\tau_{0j}\gamma_{o,0}\tilde{w}_0\right)^{1-\varepsilon}} \equiv n\lambda^k_{o,0j}, \qquad o,j = 1,2,\ldots,N$$

$$\tilde{\lambda}^k_{0,0j} = \frac{\left(t^k_{0j}\tau_{0j}\tilde{w}_0\right)^{1-\varepsilon}}{\sum_{\ell=1}^N \left(t^k_{\ell j}\tilde{\tau}_{\ell j}\gamma_{0,\ell}\tilde{w}_\ell\right)^{1-\varepsilon} + \left(t^k_{0j}\tau_{0j}\tilde{w}_0\right)^{1-\varepsilon}} \equiv \lambda^k_{0,0j}, \qquad j = 1,2,\ldots,N$$

$$\tilde{\lambda}^k_{o,i0} = \frac{\left(t^k_{i0}\tau_{i0}\gamma_{o,i}\tilde{w}_i\right)^{1-\varepsilon}}{\sum_{\ell=1}^N \left(t^k_{\ell 0}\tau_{l0}\gamma_{o,\ell}\tilde{w}_\ell\right)^{1-\varepsilon} + \left(t^k_{00}\gamma_{o,0}\tilde{w}_0\right)^{1-\varepsilon}} \equiv \lambda^k_{o,i0}, \qquad o,i = 1,2,\ldots,N$$

$$\tilde{\lambda}^k_{0,i0} = n\frac{\left(t^k_{i0}\tau_{i0}\gamma_{0,i}\tilde{w}_i\right)^{1-\varepsilon}}{n\sum_{\ell=1}^N \left(t^k_{\ell 0}\tau_{\ell 0}\gamma_{0,\ell}\tilde{w}_\ell\right)^{1-\varepsilon} + \left(\tilde{w}_0\right)^{1-\varepsilon}} \equiv n\lambda^k_{0,i0}, \qquad i = 1,2,\ldots,N$$

$$\tilde{\lambda}^k_{o,00} = \frac{\left(\gamma_{o,0}\tilde{w}_0\right)^{1-\varepsilon}}{\sum_{\ell=1}^N \left(t^k_{l0}\tau_{\ell 0}\gamma_{o,\ell}\tilde{w}_\ell\right)^{1-\varepsilon} + \left(\gamma_{o,0}\tilde{w}_0\right)^{1-\varepsilon}} \equiv \lambda^k_{o,00}, \qquad o = 1,2,\ldots,N$$

$$\tilde{\lambda}^k_{0,00} = \frac{\left(\tilde{w}_0\right)^{1-\varepsilon}}{n\sum_{\ell=1}^N \left(t^k_{\ell 0}\tau_{\ell 0}\gamma_{0,\ell}\tilde{w}_\ell\right)^{1-\varepsilon} + \left(\tilde{w}_0\right)^{1-\varepsilon}} \equiv \lambda_{0,00}.$$

Note that:

$$n \times \lambda^k_{o,0j} + \sum_{i=1}^N \lambda^k_{o,ij} = 1, \ o,j \geq 1$$

$$\lambda^k_{0,0j} + \sum_{i=1}^N \lambda^k_{0,ij} = 1, \ j \geq 1$$

$$\lambda^k_{o,00} + \sum_{i=1}^N \lambda^k_{o,i0} = 1, \ o \geq 1$$

$$\lambda^k_{0,00} + n \times \sum_{i=1}^N \lambda^k_{0,i0} = 1.$$

We can then define:

$$\tilde{\lambda}_{o,i}^k = \frac{(\gamma_{o,i})^{1-\sigma}\left(A_o^k\right)^{\sigma-1}\left(\lambda_{o,ii}^k\right)^{-\frac{\sigma-1}{\varepsilon-1}}}{\sum_{o'=1}^N(\gamma_{o',i})^{1-\sigma}\left(A_{o'}^k\right)^{\sigma-1}\left(\lambda_{o',ii}^k\right)^{-\frac{\sigma-1}{\varepsilon-1}}+n^{\frac{\sigma-1}{\varepsilon-1}}(\gamma_{0,i})^{1-\sigma}\left(A_0^k\right)^{\sigma-1}\left(\lambda_{0,ii}^k\right)^{-\frac{\sigma-1}{\varepsilon-1}}}\equiv\lambda_{o,i}^k,\qquad o,i\geq 1 \tag{D.3}$$

$$\tilde{\lambda}_{0,i}^k = n^{\frac{\sigma-1}{\varepsilon-1}}\frac{(\gamma_{0,i})^{1-\sigma}\left(A_0^k\right)^{\sigma-1}\left(\lambda_{0,ii}^k\right)^{-\frac{\sigma-1}{\varepsilon-1}}}{\sum_{o'=1}^N\left(\tilde{\gamma}_{o',i}\right)^{1-\sigma}\left(A_{o'}^k\right)^{\sigma-1}\left(\tilde{\lambda}_{o',ii}^k\right)^{-\frac{\sigma-1}{\varepsilon-1}}+n^{\frac{\sigma-1}{\varepsilon-1}}(\gamma_{0,i})^{1-\sigma}\left(A_0^k\right)^{\sigma-1}\left(\lambda_{0,ii}^k\right)^{-\frac{\sigma-1}{\varepsilon-1}}}\equiv n^{\frac{\sigma-1}{\varepsilon-1}}\lambda_{0,i}^k,\qquad i\geq 1$$

$$\tilde{\lambda}_{o,0}^k = \frac{(\gamma_{o,0})^{1-\sigma}\left(A_o^k\right)^{\sigma-1}\left(\lambda_{o,00}^k\right)^{-\frac{\sigma-1}{\varepsilon-1}}}{\sum_{o'=1}^N(\gamma_{o',0})^{1-\sigma}\left(A_{o'}^k\right)^{\sigma-1}\left(\lambda_{o',00}^k\right)^{-\frac{\sigma-1}{\varepsilon-1}}+\left(A_0^k\right)^{\sigma-1}\left(\lambda_{0,00}^k\right)^{-\frac{\sigma-1}{\varepsilon-1}}}\equiv\lambda_{o,0}^k,\qquad o\geq 1$$

$$\tilde{\lambda}_{0,0}^k = \frac{\left(A_0^k\right)^{\sigma-1}\left(\lambda_{0,00}^k\right)^{-\frac{\sigma-1}{\varepsilon-1}}}{\sum_{o'=1}^N(\gamma_{o',0})^{1-\sigma}\left(A_{o'}^k\right)^{\sigma-1}\left(\lambda_{o',00}^k\right)^{-\frac{\sigma-1}{\varepsilon-1}}+\left(A_0^k\right)^{\sigma-1}\left(\lambda_{0,00}^k\right)^{-\frac{\sigma-1}{\varepsilon-1}}}\equiv\lambda_{0,0}^k.$$

Note that:

$$n^{\frac{\sigma-1}{\varepsilon-1}}\lambda_{0,i}^k+\sum_{o=1}^N\lambda_{o,j}^k=1,\quad \lambda_{0,0}^k+\sum_{o=1}^N\lambda_{o,0}^k=1.$$

We can now define the following expenditure shares:

$$\tilde{\lambda}_{ij}^k = n^{\frac{\sigma-1}{\varepsilon-1}}\lambda_{0,ij}^k\lambda_{0,j}^k+\sum_{o=1}^N\lambda_{o,ij}^k\lambda_{o,j}^k\equiv\lambda_{ij}^k\qquad i,j=1,2,\ldots,N \tag{D.4}$$

$$\tilde{\lambda}_{0j}^k = n^{\frac{\sigma-1}{\varepsilon-1}}\left(\lambda_{0,0j}^k\lambda_{0,j}^k+n^{\frac{\varepsilon-\sigma}{\varepsilon-1}}\sum_{o=1}^N\lambda_{o,0j}^k\lambda_{o,j}^k\right)\equiv n^{\frac{\sigma-1}{\varepsilon-1}}\lambda_{0j}^k,\qquad j=1,2,\ldots,N$$

$$\tilde{\lambda}_{i0}^k = n\lambda_{0,i0}^k\lambda_{0,0}^k+\sum_{o=1}^N\lambda_{o,i0}^k\lambda_{o,0}^k\equiv\lambda_{i0}^k,\qquad i=1,2,\ldots,N$$

$$\tilde{\lambda}_{00}^k = \lambda_{0,00}^k\lambda_{0,0}^k+\sum_{o=1}^N\lambda_{o,00}^k\lambda_{o,0}^k\equiv\lambda_{00}^k.$$

Note that:

$$n^{\frac{\sigma-1}{\varepsilon-1}}\lambda_{0j}^k+\sum_{i=1}^N\lambda_{ij}^k=1,\quad \lambda_{00}^k+\sum_{i=1}^N\lambda_{i0}^k=1.$$

Finally, for any $k \in \mathcal{K}$ we define:

$$\tilde{\Lambda}_{ij} = \frac{\sum_{k=1}^{K} \lambda_{ij}^{k} \alpha_k / t_{ij}^{k}}{\sum_{\ell=1}^{N} \sum_{s=1}^{K} \left(\alpha_s \lambda_{\ell j}^{s} / t_{\ell j}^{s} \right) + n^{\frac{\sigma-1}{\varepsilon-1}} \sum_{s=1}^{K} \left(\alpha_s \lambda_{0j}^{s} / t_{0j}^{s} \right)} \equiv \Lambda_{ij}, \qquad i,j = 1,2,\ldots,N \tag{D.5}$$

$$\tilde{\Lambda}_{0j} = n^{\frac{\sigma-1}{\varepsilon-1}} \frac{\sum_{k=1}^{K} \lambda_{0j}^{k} \alpha_k / t_{0j}^{k}}{\sum_{\ell=1}^{N} \sum_{s=1}^{K} \left(\alpha_s \lambda_{\ell j}^{s} / t_{\ell j}^{s} \right) + n^{\frac{\sigma-1}{\varepsilon-1}} \sum_{s=1}^{K} \left(\alpha_s \lambda_{0j}^{s} / t_{0j}^{s} \right)} \equiv n^{\frac{\sigma-1}{\varepsilon-1}} \Lambda_{0j}, \qquad j = 1,2,\ldots,N$$

$$\tilde{\Lambda}_{i0} = \frac{\sum_{k=1}^{K} \lambda_{i0}^{k} \alpha_k / t_{i0}^{k}}{\sum_{\ell=1}^{N} \sum_{s=1}^{K} \left(\alpha_s \lambda_{\ell 0}^{s} / t_{\ell 0}^{s} \right) + \sum_{s=1}^{K} \left(\alpha_s \lambda_{00}^{s} \right)} \equiv \Lambda_{i0}, \qquad i = 1,2,\ldots,N$$

$$\tilde{\Lambda}_{00} = \frac{\sum_{k=1}^{K} \lambda_{00}^{k} \alpha_k}{\sum_{\ell=1}^{N} \sum_{s=1}^{K} \left(\alpha_s \lambda_{\ell 0}^{s} / t_{\ell 0}^{s} \right) + \sum_{s=1}^{K} \left(\alpha_s \lambda_{00}^{s} \right)} \equiv \Lambda_{00}.$$

Note,

$$n^{\frac{\sigma-1}{\varepsilon-1}} \Lambda_{0j} + \sum_{i=1}^{N} \Lambda_{ij} = 1, \qquad \Lambda_{00} + \sum_{i=1}^{N} \Lambda_{i0} = 1.$$

2. The Labor Market clearing condition. We take equation (16) and use equations (D.1) and (D.5) to get:

$$\tilde{w}_I L_I = \sum_{J=0}^{N} \tilde{\Lambda}_{IJ} \tilde{w}_J L_J,$$

$$\tilde{w}_i L_i = \Lambda_{i0} \tilde{w}_0 n^{\frac{\sigma-1}{\varepsilon-1}} L_0 + \sum_{j=1}^{N} \Lambda_{ij} \tilde{w}_j L_j, \qquad i = 1,2,\ldots,N \tag{D.6}$$

$$\tilde{w}_0 L_0 = \Lambda_{00} \tilde{w}_0 L_0 + \sum_{j=1}^{N} \Lambda_{0j} \tilde{w}_j L_j. \tag{D.7}$$

3. The limit economy. We can now characterize the limit equilibrium as $n \to 0$. Suppose that $w_0 \in (0, \infty)$, which will be verified later. Then, our previous equation implies that our variables without tildes converge to positive values, and so the system (D.6)-(D.7) reduces to:

$$w_i L_i = \sum_{j=1}^{N} \Lambda_{ij} w_j L_j, \qquad i = 1,2,\ldots,N \tag{D.8}$$

$$w_0 L_0 = \Lambda_{00} w_0 L_0 + \sum_{j=1}^{N} \Lambda_{0j} w_j L_j. \tag{D.9}$$

Focus for a moment in equation (D.8), which we copy again below

$$
\begin{aligned}
w_i L_i &= \sum_{j=1}^{N} \tilde{\Lambda}_{ij} w_j L_j, \qquad i = 1, 2, \ldots, N \\
\Lambda_{ij} &= \frac{\sum_{k=1}^{K} \lambda_{ij}^{k} \alpha_k / t_{ij}^{k}}{\sum_{\ell=1}^{N} \sum_{s=1}^{K} \left(\alpha_s \lambda_{\ell j}^{s} / t_{\ell j}^{s} \right)} \qquad i, j = 1, 2, \ldots, N \\
\lambda_{ij}^{k} &= \sum_{o=1}^{N} \lambda_{o,ij}^{k} \lambda_{o,j}^{k}, \qquad i, j = 1, 2, \ldots, N, \forall k \\
\lambda_{o,j}^{k} &= \frac{\left(\gamma_{o,j}\right)^{1-\sigma} \left(A_o^k\right)^{\sigma-1} \left(\lambda_{o,jj}^{k}\right)^{-\frac{\sigma-1}{\varepsilon-1}}}{\sum_{o'=1}^{N} \left(\gamma_{o',j}\right)^{1-\sigma} \left(A_{o'}^{k}\right)^{\sigma-1} \left(\lambda_{o',jj}^{k}\right)^{-\frac{\sigma-1}{\varepsilon-1}}}, \qquad o, j = 1, 2, \ldots, N, \forall k \\
\lambda_{o,ij}^{k} &= \frac{\left(t_{ij}^{k} \tau_{ij} \gamma_{o,i} w_i\right)^{1-\varepsilon}}{\sum_{\ell=1}^{N} \left(t_{\ell j}^{k} \tau_{\ell j} \gamma_{o,\ell} w_\ell\right)^{1-\varepsilon}}, \qquad , o, ij \geq 1, \forall k.
\end{aligned}
\tag{D.10}
$$

This system does not depend on country 0's wage (w_0) nor on the import tariff that country 0 imposes to the rest of the countries. Then, given the import tariff set by all countries, system (D.10) defines an autonomous system for $\mathbf{w}_{-0} \equiv (w_1, w_2, \ldots, w_{N-1}, 1)$. Per our preview results an equilibrium exists and it is unique. Furthermore, $\mathbf{w}_{-0}$ does not depend on country $0's$ import tariff.

Proposition 5. The equilibrium of the SOE economy limit has a unique equilibrium wage vector $\mathbf{w}_{-0}$ that is independent of country 0's trade policy. The wage vector $\mathbf{w}_{-0}$ solves system (D.10).

We now move to characterize the equilibrium value for w_0 and show that $w_0 \in (0, \infty)$. Country 0's wage is pin down by the equation:

$$
\sum_{j=1}^{N} \Lambda_{0j} w_j L_j = w_0 L_0 \left(1 - \Lambda_{00}\right).
$$

Given the unique equilibrium vector $\mathbf{w}_{-0}$, the left hand side of the previous equation is decreasing in w_0, approaches infinity as $w_0 \to 0$, and approaches zero as $w_0 \to \infty$ since:

$$
\begin{aligned}
\Lambda_{0j}\left(w_0\right) &= \frac{\sum_{k=1}^{K}\lambda_{0j}^{k}\left(w_0\right)\alpha_k/t_{0j}^{k}}{\sum_{\ell=1}^{N}\sum_{s=1}^{K}\alpha_s\lambda_{\ell j}^{s}/t_{\ell j}^{s}}\\
\lambda_{0j}^{k}\left(w_0\right) &= \lambda_{0,0j}^{k}\left(w_0\right)\lambda_{0,j}^{k}\left(w_0\right)\\
\lambda_{0,j}^{k}\left(w_0\right) &= \frac{\left(\gamma_{0,j}\right)^{1-\sigma}\left(A_0^k\right)^{\sigma-1}\left(\lambda_{0,jj}^{k}\left(w_0\right)\right)^{-\frac{\sigma-1}{\varepsilon-1}}}{\sum_{o'=1}^{N}\left(\gamma_{o',j}\right)^{1-\sigma}\left(A_{o'}^k\right)^{\sigma-1}\left(\lambda_{o',jj}^{k}\right)^{-\frac{\sigma-1}{\varepsilon-1}}}\\
\lambda_{0,0j}^{k}\left(w_0\right) &= \frac{\left(t_{0j}^{k}\tau_{0j}w_0\right)^{1-\varepsilon}}{\sum_{\ell=1}^{N}\left(t_{\ell j}^{k}\tau_{\ell j}\gamma_{0,\ell}w_\ell\right)^{1-\varepsilon}+\left(t_{0j}^{k}\tau_{0j}w_0\right)^{1-\varepsilon}}\\
\lambda_{0,jj}^{k}\left(w_0\right) &= \frac{\left(\gamma_{0,j}w_j\right)^{1-\varepsilon}}{\sum_{\ell=1}^{N}\left(t_{\ell j}^{k}\tau_{\ell j}\gamma_{0,\ell}w_\ell\right)^{1-\varepsilon}+\left(t_{0j}^{k}\tau_{0j}w_0\right)^{1-\varepsilon}}\\
\lambda_{0j}^{k}\left(w_0\right) &\propto \frac{\left(t_{0j}^{k}\tau_{0j}w_0\right)^{1-\varepsilon}}{\left[\sum_{\ell=1}^{N}\left(t_{\ell j}^{k}\tau_{\ell j}\gamma_{0,\ell}w_\ell\right)^{1-\varepsilon}+\left(t_{0j}^{k}\tau_{0j}w_0\right)^{1-\varepsilon}\right]^{\frac{\varepsilon-\sigma}{\varepsilon-1}}}.
\end{aligned}
$$

The right-hand side, on the other hand approaches zero as $w_0 \to 0$, and approaches infinity as $w_0 \to \infty$. This follows from the fact that $\left(1-\tilde{\Lambda}_{00}\right)$ is bounded below by 0 and above by 1 for all values of w_0. Therefore, the two curves cross at least once and in those points $w_0 \in (0,\infty)$. We note that we cannot rule out that the curves cross multiple times.

Proposition 6. The equilibrium of the SOE economy limit has a unique equilibrium wage vector $\mathbf{w}_{-0}$ that is independent of country 0's trade policy. The wage vector $\mathbf{w}_{-0}$ solves system (D.8). Furthermore, there is a unique finite equilibrium value for w_0 which solves equation (D.9).

4. Deriving the trade elasticity. We define the trade elasticity in sector k as

$$
\theta_{0j}^{k} \equiv -\frac{\partial \log\left(\lambda_{0j}^{k}/\lambda_{jj}^{k}\right)}{\partial \log\left(t_{0j}^{k}\right)}.
$$

We note that

$$
\theta_{0j}^{k} = -\frac{\partial\lambda_{0j}^{k}}{\partial w_0}\frac{w_0}{\lambda_{0j}^{k}} = (\varepsilon-1)-(\varepsilon-\sigma)\,\lambda_{0,0j}^{k}\left(w_0\right).
$$

Hence, the trade elasticity is **not constant,** and it depends on the size of MP-costs.

E Optimal Import Tariff in LOE, Two Countries, One Sector

We denote the total expenditure of country n by E_n and the expenditure (inclusive of tariffs) on goods from the rest of the world i as E_{in}. The share of the total expenditure in country n that is spent on imported goods is given by $\lambda_{in} = E_{in}/E_n$. Denote by t_{in} the one plus ad-valorem import tariff applied by country n to goods imported from country i.

Note that we can write tariff revenue as:

$$T_n = (t_{in} - 1)\frac{E_{in}}{t_{in}} = (t_{in} - 1)\frac{\lambda_{in}}{t_{in}}E_n,$$

and so total expenditure is given by:

$$E_n = w_n L_n + (t_{in} - 1)\frac{\lambda_{in}}{t_{in}}E_n.$$

We use this equation to solve for per-capita expenditure and obtain:

$$\frac{E_n}{L_n} = w_n \frac{t_{in}}{t_{in} - (t_{in} - 1)\lambda_{in}}.$$

Now use that $\lambda_{in} + \lambda_{nn} = 1$ to get:

$$\frac{E_n}{L_n} = w_n \frac{t_{in}}{1 + (t_{in} - 1)\lambda_{nn}}. \tag{E.1}$$

Trade is balanced, and therefore the value of exports equals the value of imports:

$$\frac{\lambda_{in}}{t_{in}}E_n = \frac{\lambda_{ni}}{t_{ni}}E_i, \tag{E.2}$$

where the tariff that the rest of the world applies to goods from n is t_{ni}.

Welfare per capita in country n equals country's real income per worker,

$$U_n = \frac{E_n}{L_n P_n} = \frac{w_n}{P_n}\frac{t_{in}}{1 + (t_{in} - 1)\lambda_{nn}}. \tag{E.3}$$

Fully differentiating equation E.3, to obtain

$$\hat{U}_n = \left(\frac{\hat{w_n}}{P_n}\right) + \frac{(1 - \lambda_{nn})}{1 + (t_{in} - 1)\lambda_{nn}}\hat{t}_{in} - \frac{(t_{in} - 1)\lambda_{nn}}{1 + (t_{in} - 1)\lambda_{nn}}\hat{\lambda}_{nn}, \tag{E.4}$$

where a variable with 'hats' refers to the log difference, namely $\hat{x} \equiv d\ln(x)$.

We notice now that all endogenous variables in this economy are functions of the relative wage $\bar{w}$ and the tariff t_{in}, while the relative wage $\bar{w}$ is only a function of the tariff t_{in} and is the quantity that solves equation E.2. We will use this to derive expressions for $\hat{\lambda}_{nn}$ and $\left(\frac{\hat{w_n}}{P_n}\right)$ as functions of $\hat{t}_{in}$.

First we note that $\hat{\lambda}_{nn}$ equals

$$\hat{\lambda}_{nn} = \left[\frac{\partial\lambda_{nn}}{\partial t_{in}}\frac{t_{in}}{\lambda_{nn}}\hat{t}_{in} + \frac{\partial\lambda_{nn}}{\partial\bar{w}}\frac{\bar{w}}{\lambda_{nn}}\hat{\bar{w}}\right] = \left[\frac{\partial\lambda_{nn}}{\partial t_{in}}\frac{t_{in}}{\lambda_{nn}} + \left(\frac{\partial\lambda_{nn}}{\partial\bar{w}}\frac{\bar{w}}{\lambda_{nn}}\right) \times \left(\frac{\partial\bar{w}}{\partial\tau_{in}}\frac{t_{in}}{\bar{w}}\right)\right]\hat{t}_{in}.$$

We note that $-\frac{\partial\lambda_{nn}}{\partial\bar{w}}\frac{\bar{w}}{\lambda_{nn}} = \frac{\partial\lambda_{nn}}{\partial t_{in}}\frac{t_{in}}{\lambda_{nn}}$ and therefore,

$$\hat{\lambda}_{nn} = \frac{\partial\lambda_{nn}}{\partial t_{in}}\frac{t_{in}}{\lambda_{nn}}\left(1 - \frac{\partial\bar{w}}{\partial t_{in}}\frac{t_{in}}{\bar{w}}\right)\hat{t}_{in}.$$

Now we use that $\lambda_{nn} + \lambda_{in} = 1$ and the definition of the trade elasticity θ_{in} to obtain:

$$\lambda_{in}\theta_{in} = \frac{\partial \lambda_{nn}}{\partial t_{in}} \frac{t_{in}}{\lambda_{nn}}.$$

Therefore,

$$\hat{\lambda}_{nn} = \lambda_{in}\theta_{in}\left(1 - \frac{\partial \bar{w}}{\partial t_{in}} \frac{t_{in}}{\bar{w}}\right)\hat{t}_{in}. \tag{E.5}$$

The last two elements we need then are the response of relative wages to tariffs $\left(\hat{\frac{w_n}{P_n}}\right)$, and the general equilibrium response of relative wages $\hat{\bar{w}}$. Depending on the specification of MP frictions we have different general equilibrium responses for relative wages $\frac{\partial \bar{w}}{\partial t_{in}} \frac{t_{in}}{\bar{w}}$, and values for assembly and trade shares. We explore now some special cases and derive the optimal policy before moving to the more general case.

Case I – No-MP case. First, we assume that $\gamma_{in} \to \infty$ for all i and n. In this case, $\lambda_{n,in} = 0$, $\theta_{in} \to (\sigma - 1)$, and relative wages are given by

$$\frac{w_n}{P_n} = \lambda_{n,n}^{\frac{1}{1-\sigma}} A_{nn} = \lambda_{nn}^{\frac{1}{1-\sigma}} A_{nn}.$$

Then, the cahnge in the real wage is given $\left(\hat{\frac{w_n}{P_n}}\right) = \frac{1}{1-\sigma}\hat{\lambda}_{nn}$, while the total change in welfare is

$$\hat{U}_n = \left[\left(\frac{1}{1-\sigma} - \frac{(t_{in}-1)\,\lambda_{nn}}{1+(t_{in}-1)\,\lambda_{nn}}\right)\lambda_{in}\theta_{in}\left(1 - \frac{\partial \bar{w}}{\partial t_{in}} \frac{t_{in}}{\bar{w}}\right) + \frac{(1-\lambda_{nn})}{1+(t_{in}-1)\,\lambda_{nn}}\right]\hat{t}_{in}, \tag{E.6}$$

where we use both equation E.4 along with equation E.5. The optimal tariff is then the solution to $\hat{U}_n = 0$ which implies,

$$(t_{in} - 1) = \frac{1}{\sigma\lambda_{nn}}\left[\frac{1-\lambda_{nn}}{\lambda_{in}}\left(1 - \frac{\partial \bar{w}}{\partial t_{in}} \frac{t_{in}}{\bar{w}}\right)^{-1} - 1\right]. \tag{E.7}$$

The last step is to compute the full general equilibrium response of relative wages to taxes $\frac{\partial \bar{w}}{\partial t_{ni}} \frac{t_{ni}}{\bar{w}}$ which is determined by the trade balance condition

$$\frac{\lambda_{ni}}{t_{ni}} X_n = \frac{\lambda_{in}}{t_{in}} X_i,$$

along with the equation for trade shares

$$\lambda_{ni} = \frac{(t_{ni}w_i/A_i)^{(1-\sigma)}}{(w_n/A_n)^{(1-\sigma)} + (t_{ni}w_i/A_i)^{(1-\sigma)}},$$

and it is given by:

$$\frac{\partial \bar{w}}{\partial t_{in}} \frac{t_{in}}{\bar{w}} = \frac{\sigma\tilde{\lambda}_{nn}}{1 + (\sigma - 1)\left[\tilde{\lambda}_{ii} + \tilde{\lambda}_{nn}\right]}, \tag{E.8}$$

where $\tilde{\lambda}_{ii} \equiv \frac{t_{ni}\lambda_{ii}}{1+(t_{ni}-1)\lambda_{ii}}$ is the share of production sold to domestic consumers in country i.

Combining equations E.7 and E.8it is easy to show that the optimal tariff is given by

$$(t_{in} - 1) = \frac{1}{(\sigma - 1)\,\tilde{\lambda}_{ii}}. \tag{E.9}$$

Note that if $\tau_{in} = 1$, i.e., the foreign country place no tariff on Home's goods, then $\tilde{\lambda}_{ii} = \frac{t_{in}\lambda_{ii}}{1+(t_{in}-1)\lambda_{ii}} = \lambda_{ii}$ and we recover the standard formula.

Case II – Free-MP case. First, we assume that $\gamma_{in} = 1$ for all i and n. In this case, $\lambda_{n,in} = \lambda_{in}$, the trade elasticity collapses to $\theta_{in} \to (\varepsilon - 1)$, and relative wages are given by

$$\frac{w_n}{P_n} = (\lambda_{n,nn})^{\frac{1}{1-\varepsilon}} A_{nn} = (\lambda_{nn})^{\frac{1}{1-\varepsilon}} A_{nn}.$$

In this case, the change in the real wage is given by $\left(\hat{\frac{w_n}{P_n}}\right) = \frac{1}{1-\varepsilon}\hat{\lambda}_{nn}$, and so following similar steps as before it is easy to show that the optimal tariff implied by the model is in this case

$$(\tau_{in} - 1) = \frac{1}{\varepsilon\lambda_{nn}}\left[\frac{1-\lambda_{nn}}{\lambda_{ni}}\left(1 - \frac{\partial\bar{w}}{\partial t_{in}}\frac{t_{in}}{\bar{w}}\right)^{-1} - 1\right]. \tag{E.10}$$

We note that the general equilibrium response of relative wages is still determine by the trade balance condition with the difference that now trade shares are given by:

$$\lambda_{in} = \frac{(t_{in}w_i/A_i)^{(1-\varepsilon)}}{(w_n/A_n)^{(1-\varepsilon)} + (t_{in}w_i/A_i)^{(1-\varepsilon)}}.$$

Comparing the equations to those we had in the previous case it is straightforward to conclude that in this case equation E.10 reduces to

$$(t_{in} - 1) = \frac{1}{(\varepsilon - 1)\,\tilde{\lambda}_{ii}}.$$

Case III – Arbitrary MP costs. With arbitrary MP-costs relative wages can be express as

$$\frac{w_n}{P_n} = \lambda_{n|n}^{\frac{1}{1-\sigma}} (\lambda_{n,nn})^{\frac{1}{1-\varepsilon}} A_{nn}.$$

Fully differentiating this expression we obtain,

$$\left(\hat{\frac{w_n}{P_n}}\right) = \frac{1}{1-\sigma}\hat{\lambda}_{n,n} + \frac{1}{1-\varepsilon}\hat{\lambda}_{n,nn}. \tag{E.11}$$

Now remember that $\lambda_{n,n}$ and $\lambda_{n,nn}$ are each given by

$$\lambda_{i|j} = \frac{\lambda_{i,jj}^{-\frac{1-\sigma}{1-\varepsilon}} A_{ij}^{\sigma-1}}{\sum_{a\in\mathcal{I}} \lambda_{a,jj}^{-\frac{1-\sigma}{1-\varepsilon}} A_{aj}^{\sigma-1}}, \qquad \lambda_{i,mj} = \frac{\left(t_{mj} W_m / A_{im}\right)^{1-\varepsilon}}{\sum_{\ell\in\mathcal{L}} \left(t_{\ell j} W_\ell / A_{i\ell}\right)^{1-\varepsilon}},$$

and so

$$\begin{aligned}
\hat{\lambda}_{n,n} &= -\frac{1-\sigma}{1-\varepsilon}\left(\hat{\lambda}_{n,nn} - \hat{\lambda}_{i,nn}\right)\left(1-\lambda_{n,n}\right), \\
\hat{\lambda}_{n,nn} &= (1-\varepsilon)\left(\frac{\hat{\bar{w}}}{\hat{t}_{in}} - 1\right)\hat{t}_{in}\left(1-\lambda_{n,nn}\right), \\
\hat{\lambda}_{i,nn} &= (1-\varepsilon)\left(\frac{\hat{\bar{w}}}{\hat{t}_{in}} - 1\right)\hat{t}_{in}\left(1-\lambda_{i,nn}\right).
\end{aligned} \tag{E.12}$$

Plugging this back into equation E.4 and using equation E.5, we obtain that the log change in welfare is equal to:

$$\hat{U}_n = \left[\left(\frac{\left(\lambda_{i,nn}-\lambda_{n,nn}\right)\left(1-\lambda_{n|n}\right) - \left(1-\lambda_{n,nn}\right)}{\theta_{in}\lambda_{in}} - \frac{\left(t_{in}-1\right)\lambda_{nn}}{1+\left(t_{in}-1\right)\lambda_{nn}}\right)\lambda_{in}\theta_{in}\left(1 - \frac{\partial\bar{w}}{\partial t_{in}}\frac{t_{in}}{\bar{w}}\right) + \frac{\left(1-\lambda_{nn}\right)}{1+\left(t_{in}-1\right)\lambda_{nn}}\right] \hat{t}$$

(E.13)

It is easy to show that when MP costs are prohibitively, i.e., $\gamma_{in} \to \infty$ for all i and n, $\lambda_{n,n} = 1$ and $\theta_{in} = \sigma - 1$ and so equation E.13 collapses into equation E.6. Similarly, when MP is free, i.e., $\gamma_{in} = 1$ for all i and n, $\lambda_{i,nn} = \lambda_{n,nnn}$ and $\theta_{in} = \varepsilon - 1$, and so equation E.13 collapses into equation E.10.

The optimal import tariff in this case solves $\hat{U}_n = 0$, hence

$$\left(t_{in}-1\right) = \frac{1}{\lambda_{nn}\left(\theta_{in}+1\right)}\left[\left(1 - \frac{\partial\log\bar{w}}{\partial\log t_{in}}\right)^{-1} - 1\right]. \tag{E.14}$$

The structure of the optimal tariff is the same as before, what makes the value different are: (i) the trade elasticity θ_{in} which depends on the size of the MP costs; (ii) the general equilibrium response of relative wages to changes in tariffs $\frac{\partial\log\bar{w}}{\partial\log t_{in}}$. We have characterized the former, we now move to characterizing the latter.

To characterize the general equilibrium response of wages we start with the trade balance condition, equation E.2, and our equation for aggregate spending equation E.1 to get

$$\bar{w}\frac{L_n}{L_i}\frac{\lambda_{in}}{1+\left(t_{in}-1\right)\lambda_{nn}} = \frac{\lambda_{ni}}{1+\left(t_{ni}-1\right)\lambda_{ii}}.$$

Fully differentiating this expression and using that $\hat{t}_{ni} = 0$, we obtain

$$\hat{\bar{w}} = \tilde{\lambda}_{nn}\frac{1}{\lambda_{in}}\hat{\lambda}_{nn} - \tilde{\lambda}_{ii}\frac{1}{\lambda_{ni}}\hat{\lambda}_{ii} + \tilde{\lambda}_{nn}\hat{t}_{in}. \tag{E.15}$$

where we defined $\tilde{\lambda}_{ii} \equiv \frac{t_{ni}\lambda_{ii}}{1+(t_{ni}-1)\lambda_{ii}}$. Substituting in equation E.5, and solving for $\frac{\partial \log \bar{w}}{\partial \log t_{in}}$ we get:

$$\hat{\bar{w}}\left(1+\tilde{\lambda}_{nn}\theta_{in}\right) = \tilde{\lambda}_{nn}\left(\theta_{in}+1\right)\hat{t}_{in} - \frac{\tilde{\lambda}_{ii}}{\lambda_{ni}}\hat{\lambda}_{ii}. \tag{E.16}$$

Finally, note that λ_{ii} is a function of $\bar{w}$ only. Therefore,

$$\hat{\lambda}_{ii} = \frac{\partial \lambda_{ii}}{\partial \bar{w}}\frac{\bar{w}}{\lambda_{ii}}\hat{\bar{w}} = \frac{\partial \lambda_{ii}}{\partial t_{ni}}\frac{t_{ni}}{\lambda_{ii}}\hat{\bar{w}} = \lambda_{ni}\theta_{ni}\hat{\bar{w}}. \tag{E.17}$$

Combining equations E.16 and E.17, and solving for $\hat{\bar{w}}$, we obtain

$$\hat{\bar{w}} = \frac{\tilde{\lambda}_{nn}\left(\theta_{in}+1\right)}{1+\tilde{\lambda}_{nn}\theta_{in}+\tilde{\lambda}_{ii}\theta_{ni}}\hat{t}_{in}. \tag{E.18}$$

Finally, combining equations E.14 and E.18 we get:

$$\left(t_{in}-1\right) = \frac{\tilde{\lambda}_{nn}}{\lambda_{nn}}\frac{1}{1-\tilde{\lambda}_{nn}+\tilde{\lambda}_{ii}\theta_{ni}}.$$

The last step is to remember that $\tilde{\lambda}_{nn} = \frac{t_{in}\lambda_{nn}}{1+(t_{in}-1)\lambda_{nn}}$, and so:

$$t_{in}-1 = \frac{1}{\tilde{\lambda}_{ii}\theta_{ni}}. \tag{E.19}$$

F Restricted Gravity Equations

Following Arkolakis et al. (2018), we first define a restricted gravity equation for trade flows of affiliates of multinational companies. The trade flows from an affiliate j of a firm headquartered in m to country i are given by:

$$\begin{aligned} P^{k,G}_{m,ji}(z)Q^{k,G}_{m,ji}(z) &= \left(\frac{P^k_{m,ji}(z)}{P^k_{m,i}(z)}\right)^{1-\varepsilon}\left(\frac{P^k_{m,i}(z)}{P^k_i}\right)^{1-\sigma} e_{k,i}E_i \\ &= \left(\mu^G_k t^k_{ji}\tau_{ji}w_j(A_m z)^{-1}\gamma^k_{m,j}\right)^{1-\varepsilon}\left(\mu^G_k(z)^{-1}\left(A^k_m\right)^{-1}\left(\sum_{\ell\in\mathcal{N}}\left(t^k_{\ell i}\tau_{\ell i}\gamma^k_{m,\ell}w_\ell\right)^{1-\varepsilon}\right)^{\frac{1}{1-\varepsilon}}\right)^{\varepsilon-\sigma}(P^k_i)^{\sigma-1}e_{k,i}E_i \end{aligned}$$

Aggregating across affiliates of firms headquartered in m, we find:

$$\begin{aligned} P^{k,G}_{m,ji}(z)Q^{k,G}_{m,ji}(z) &= \int_{z^{k*}_m}^{\infty}\left(\frac{P^k_{m,ji}(z)}{P^k_{m,i}(z)}\right)^{1-\varepsilon}\left(\frac{P^k_{m,i}(z)}{P^k_i}\right)^{1-\sigma} e_{k,i}E_i g(z)dz \\ &= \left(t^k_{ji}\tau_{ji}w_j\gamma^k_{m,j}\right)^{1-\varepsilon}\left(\left(\sum_{\ell\in\mathcal{N}}\left(t^k_{\ell i}\tau_{\ell i}\gamma^k_{m,\ell}w_\ell\right)^{1-\varepsilon}\right)^{\frac{1}{1-\varepsilon}}\right)^{\varepsilon-\sigma}(P^k_i)^{\sigma-1}e_{k,i}E_i \times \left(\mu^G_k(A_m)^{-1}\right)^{1-\sigma}\int_{z^{k*}_m}^{\infty}(z)^{\sigma-1}g(z)dz \end{aligned}$$

We can then define a country m-location ℓ fixed effect and a country m-market i fixed effect as follows:

$$\eta_{mj}^k = (w_j \gamma_{m,j}^k)^{1-\varepsilon} (\mu_k^G (A_m)^{-1})^{1-\sigma} \int_{z_m^{k*}}^{\infty} (z)^{\sigma-1} g(z) dz$$

$$\alpha_{mi}^k = \left(\left(\sum_{\ell \in \mathcal{N}} \left(t_{\ell i}^k \tau_{\ell i} \gamma_{m,\ell}^k w_\ell \right)^{1-\varepsilon} \right)^{\frac{1}{1-\varepsilon}} \right)^{\varepsilon-\sigma} (P_i^k)^{\sigma-1} e_{k,i} E_i$$

And express the restricted gravity equation as:

$$\ln P_{m,ji}^{k,G} Q_{m,ji}^{k,G} = \eta_{mj}^k + \alpha_{mi}^k + (1-\varepsilon) \ln t_{ji}^k + (1-\varepsilon) \ln \tau_{ji}$$

Similarly, total sales of local firms in j to country i are given given by:

$$\begin{aligned} P_{j,ji}^{k,L}(z) Q_{j,ji}^{k,L}(z) &= \int_1^{z_j^*} \left(\frac{P_{j,ji}^k(z)}{P_i^k} \right)^{1-\sigma} e_{k,i} E_i g(z) d(z) \\ &= (\mu_k^L t_{ji}^k \tau_{ji} w_j (A_j)^{-1})^{1-\sigma} (P_i^k)^{\sigma-1} e_{k,i} E_i \int_1^{z_j^*} z^{\sigma-1} g(z) d(z) \end{aligned}$$

Hence sales of local firms follow a restricted gravity equation where the relevant trade elasticity is σ:

$$\ln P_{j,ji}^{k,L} Q_{j,ji}^{k,L} = \eta_j + \alpha_i + (1-\sigma) \ln t_{ji}^k + (1-\sigma) \ln \tau_{ji}$$

where $\eta_j = (\mu_k^L w_j (A_j)^{-1})^{1-\sigma}$ and $\alpha_i = (P_i^k)^{\sigma-1} e_{k,i} E_i$.

G Derivation of the Trade Elasticity in Quantitative Model with Global and Local Firms

In this section, we derive the trade elasticity in the general model with profits and selection into MP.

First, let's define the assembly share of affiliate j over the total sales of global firm m in sector k to country i.

$$\lambda_{m,ji}^{k,G} \equiv \frac{t_{ji}^k P_{m,ji}^{k,G} Q_{m,ji}^{k,G}}{\sum_{\ell \in \mathcal{N}} t_{k,\ell i} P_{m,\ell i}^{k,G} Q_{m,\ell i}^{k,G}} = \frac{\left(t_{ji}^k \tau_{ji} \gamma_{m,j}^k w_j \right)^{1-\varepsilon}}{\sum_{\ell \in \mathcal{N}} \left(t_{li}^k \tau_{\ell i} \gamma_{m,\ell}^k w_\ell \right)^{1-\varepsilon}} \tag{G.1}$$

Note that these shares do not depend on idiosyncratic productivity, given that z is common across affiliates in different locations of production, nor do they depend on the markup. Then, let's define an auxiliary price index for the assembler located in country i:

$$P_{m,i}^{k,G}(z) = \mu_k^G (z_m^k)^{-1} (A_m^k)^{-1} \left(\sum_{\ell \in \mathcal{N}} \left(t_{\ell i}^k \tau_{\ell i} \gamma_{m,\ell}^k w_\ell \right)^{1-\varepsilon} \right)^{\frac{1}{1-\varepsilon}} = w_m t_{mi}^k \tau_{mi} \mu_k^G (z_m^k)^{-1} (A_m^k)^{-1} (\lambda_{m,mi}^{k,G})^{-\frac{1}{1-\varepsilon}} \tag{G.2}$$

The price index for a local firm is instead equal to:

$$P_{m,i}^{k,L}(z) = \mu_k^L (z_m^k)^{-1} (A_m^k)^{-1} t_{mi}^k \tau_{mi} w_m \tag{G.3}$$

Under the assumption that the elasticity of substitution between local and global varieties is σ, the price index of origin-good m sold in country i is then:

$$(P_{m,i}^k)^{1-\sigma} = \int_1^{\infty} (P_{m,i}^k(z))^{1-\sigma} = \int_1^{z_m^{k*}} (P_{m,i}^{k,L})^{1-\sigma} + \int_{z_m^{k*}}^{\infty} (P_{m,i}^{k,G})^{1-\sigma} \tag{G.4}$$

Plugging in the expression for the price indices of local and global firms:

$$\begin{aligned}(P_{m,i}^k)^{1-\sigma} &= M_m^k \left[(w_m t_{mi}^k \tau_{mi} \mu_k (A_m^k)^{-1})^{1-\sigma} \int_1^{z_m^{k*}} (z_m^k)^{\sigma-1} g(z) dz + (w_m t_{mi}^k \tau_{mi} \mu_k (A_m^k)^{-1} \gamma_{m,m}^k)^{1-\sigma} (\lambda_{m,mi}^{k,G})^{-\frac{1-\sigma}{1-\varepsilon}} \int_{z_m^{k*}}^{\infty} (z_m^k)^{\sigma-1} g(z) dz \right] \\ &= M_m^k (w_m t_{mi}^k \tau_{mi} \mu_k (A_m^k)^{-1})^{1-\sigma} \left[\int_1^{z_m^{k*}} (z_m^k)^{\sigma-1} g(z) dz + (\gamma_{m,m}^k)^{1-\sigma} (\lambda_{m,mi}^{k,G})^{-\frac{1-\sigma}{1-\varepsilon}} \int_{z_m^{k*}}^{\infty} (z_m^k)^{\sigma-1} g(z) dz \right]\end{aligned} \tag{G.5}$$

where we have further assumed that the markup is constant across global and local firms. Then, the expenditure share on origin-goods m in country i is:

$$\begin{aligned}\lambda_{m,i}^k &\equiv \frac{\sum_{\ell \in \mathcal{N}} t_{\ell i}^k P_{m,\ell i}^{k,G} Q_{m,\ell i}^{k,G} + t_{mi}^k P_{m',m'i}^{k,L} Q_{m',m'i}^{k,L}}{\sum_{m' \in \mathcal{N}} [\sum_{j \in \mathcal{N}} t_{ji}^k P_{m',ji}^{k,G} Q_{m',ji}^{k,G} + t_{mi}^k P_{m',m'i}^{k,L} Q_{m',m'i}^{k,L}]} \\ &= \left(\frac{P_{m,i}^k}{P_i^k} \right)^{1-\sigma} \\ &= \frac{M_m^k (w_m t_{mi}^k \tau_{mi} \mu_k (A_m^k)^{-1})^{1-\sigma} \left[\int_1^{z_m^{k*}} (z_m^k)^{\sigma-1} g(z) dz + (\gamma_{m,m}^k)^{1-\sigma} (\lambda_{m,mi}^{k,G})^{-\frac{1-\sigma}{1-\varepsilon}} \int_{z_m^{k*}}^{\infty} (z_m^k)^{\sigma-1} g(z) dz \right]}{\sum_{m'} M_{m'}^k (w'_m t_{m'i}^k \tau_{m'i} \mu_k (A_{m'}^k)^{-1})^{1-\sigma} \left[\int_1^{z_{m'}^{k*}} (z_m^k)^{\sigma-1} g(z) dz + (\gamma_{m',m'}^k)^{1-\sigma} (\lambda_{m',m'i}^{k,G})^{-\frac{1-\sigma}{1-\varepsilon}} \int_{z_{m'}^{k*}}^{\infty} (z_{m'}^k)^{\sigma-1} g(z) dz \right]}\end{aligned} \tag{G.6}$$

Finally, trade shares (in sector k) are given by:

$$\begin{aligned}\lambda_{ji}^k &\equiv \frac{\sum_o t_{ji}^k P_{ok,ji}^{k,G} Q_{o,ji}^{k,G} + t_{ji}^k P_{j,ji}^{k,L} Q_{j,ji}^L}{\sum_{o' \in \mathcal{N}} [\sum_{j \in \mathcal{N}} t_{ji}^k P_{o',ji}^{k,G} Q_{o',ji}^{k,G} + t_{o'i}^k P_{o',o'i}^{k,L} Q_{o',o'i}^{k,L}]} \\ &= \frac{\sum_o \lambda_{o,ji}^{k,G} \sum_{\ell \in \mathcal{N}} t_{k,\ell i} P_{o,\ell i}^{k,G} Q_{o,\ell i}^{k,G} + t_{ji}^k P_{j,ji}^{k,L} Q_{j,ji}^{k,L}}{\sum_{o' \in \mathcal{N}} [\sum_{j \in \mathcal{N}} t_{ji}^k P_{o',ji}^{k,G} Q_{o',ji}^{k,G} + t_{oi}^k P_{o',o'i}^{k,L} Q_{o',o'i}^{k,L}]} \\ &= \sum_o \lambda_{o,ji}^{k,G} \lambda_{o,i}^{k,G} + \lambda_{j,i}^{k,L} \\ &= \sum_o \lambda_{o,ji}^{k,G} \lambda_{o,i}^{k,G} + \lambda_{j,i}^{k,L}\end{aligned} \tag{G.7}$$

where

$$\lambda_{m,i}^{k,L} = \frac{M_m^k (w_m t_{mi}^k \tau_{mi} \mu_k (A_m^k)^{-1})^{1-\sigma} \left[\int_1^{z_m^{k*}} (z_m^k)^{\sigma-1} g(z) dz \right]}{\sum_{m'} M_{m'}^k (w'_m t_{m'i}^k \tau_{m'i} \mu_k (A_{m'}^k)^{-1})^{1-\sigma} \left[\int_1^{z_{m'}^{k*}} (z_m^k)^{\sigma-1} g(z) dz + (\gamma_{m',m'}^k)^{1-\sigma} (\tilde{\lambda}_{m',m'i}^{k,G})^{-\frac{1-\sigma}{1-\varepsilon}} \int_{z_{m'}^{k*}}^{\infty} (z_{m'}^k)^{\sigma-1} g(z) dz \right]}$$

$$\lambda_{m,i}^{k,G} = \frac{M_m^k (w_m t_{mi}^k \tau_{mi} \mu_k (A_m^k)^{-1})^{1-\sigma} \left[(\gamma_{m,m}^k)^{1-\sigma} (\tilde{\lambda}_{m,mi}^{k,G})^{-\frac{1-\sigma}{1-\varepsilon}} \int_{z_m^{k*}}^{\infty} (z_m^k)^{\sigma-1} g(z) dz \right]}{\sum_{m'} M_{m'}^k (w'_m t_{m'i}^k \tau_{m'i} \mu_k (A_{m'}^k)^{-1})^{1-\sigma} \left[\int_1^{z_{m'}^{k*}} (z_m^k)^{\sigma-1} g(z) dz + (\gamma_{m',m'}^k)^{1-\sigma} (\tilde{\lambda}_{m',m'i}^{k,G})^{-\frac{1-\sigma}{1-\varepsilon}} \int_{z_{m'}^{k*}}^{\infty} (z_{m'}^k)^{\sigma-1} g(z) dz \right]}$$

The trade elasticity is defined as:

$$\theta_{m\ell}^k \equiv -\frac{\partial \log \left(\lambda_{m\ell}^k / \lambda_{\ell\ell}^k \right)}{\partial \log \tau_{m\ell}} \tag{G.8}$$

Let's simplify the ratio $\left(\lambda_{m\ell}^k / \lambda_{\ell\ell}^k \right)$. Also, in what follows I assume that the measure of firms in each country is equal to 1 and denote the integrals of $z^{\sigma-1}$ with $\overline{Z}$ and $\underline{Z}$, respectively for the integration above and below the productivity threshold. First:

$$\begin{aligned}
&\frac{\sum_m \frac{\left(t_{ji}^k \tau_{ji} \gamma_{m,j}^k w_j \right)^{1-\varepsilon}}{\sum_{\ell \in \mathcal{N}} \left(t_{\ell i}^k \tau_{\ell i} \gamma_{m,\ell}^k w_\ell \right)^{1-\varepsilon}} (w_m t_{mi}^k \tau_{mi} \mu_k (A_m^k)^{-1})^{1-\sigma} \left[(\tilde{\lambda}_{m,mi}^{k,G})^{-\frac{1-\sigma}{1-\varepsilon}} \overline{Z} \right] + (w_j t_{ji}^k \tau_{ji} \mu_k (A_j^k)^{-1})^{1-\sigma} \underline{Z}}{\sum_m \frac{\left(t_{ji}^k \tau_{ji} \gamma_{m,j}^k w_j \right)^{1-\varepsilon}}{\sum_{\ell \in \mathcal{N}} \left(t_{\ell i}^k \tau_{\ell i} \gamma_{m,\ell}^k w_\ell \right)^{1-\varepsilon}} (w_m t_{mi}^k \tau_{mi} \mu_k (A_m^k)^{-1})^{1-\sigma} \left[(\tilde{\lambda}_{m,mi}^{k,G})^{-\frac{1-\sigma}{1-\varepsilon}} \overline{Z} \right] + (w_j t_{ji}^k \tau_{ji} \mu_k (A_j^k)^{-1})^{1-\sigma} \underline{Z}} \\
&= \frac{\left(t_{ji}^k \tau_{ji} w_j \right)^{1-\varepsilon}}{\left(t_{ii}^k \tau_{ii} w_i \right)^{1-\varepsilon}} \frac{\sum_m \frac{(\gamma_{m,j}^k)^{1-\varepsilon} (w_m t_{mi}^k \tau_{mi} \mu_k (A_m^k)^{-1})^{1-\sigma}}{\sum_{\ell \in \mathcal{N}} \left(t_{\ell i}^k \tau_{\ell i} \gamma_{m,\ell}^k (w_\ell) \right)^{1-\varepsilon}} \left[(\tilde{\lambda}_{m,mi}^{k,G})^{-\frac{1-\sigma}{1-\varepsilon}} \overline{Z} \right] + (w_j t_{ji}^k \tau_{ji} \mu_k (A_j^k)^{-1})^{\varepsilon-\sigma} \int_1^{z_j^{k*}} (z_j^k)^{\sigma-1} g(z) dz}{\sum_m \frac{(\gamma_{m,j}^k)^{1-\varepsilon} (w_m t_{mi}^k \tau_{mi} \mu_k (A_m^k)^{-1})^{1-\sigma}}{\sum_{\ell \in \mathcal{N}} \left(t_{li}^k \tau_{\ell i} \gamma_{m,\ell}^k (w_\ell) \right)^{1-\varepsilon}} \left[(\tilde{\lambda}_{m,mi}^{k,G})^{-\frac{1-\sigma}{1-\varepsilon}} \overline{Z} \right] + (w_j t_{ji}^k \tau_{ji} \mu_k (A_j^k)^{-1})^{\varepsilon-\sigma} \underline{Z}} \\
&= \frac{\left(t_{ji}^k \tau_{ji} w_j \right)^{1-\varepsilon}}{\left(t_{ii}^k \tau_{ii} w_i \right)^{1-\varepsilon}} \frac{\sum_m \left(\frac{\gamma_{m,j}^k}{A_m} \right)^{1-\varepsilon} (w_m t_{mi}^k \tau_{mi} \mu_k (A_m^k)^{-1})^{\varepsilon-\sigma} \left[(\tilde{\lambda}_{m,mi}^{k,G})^{-\frac{\varepsilon-\sigma}{1-\varepsilon}} \overline{Z} \right] + (w_j t_{ji}^k \tau_{ji} \mu_k (A_j^k)^{-1})^{\varepsilon-\sigma} ((A_j^k)^{-1})^{1-\varepsilon} \underline{Z}}{Den} \\
&= \frac{\left(t_{ji}^k \tau_{ji} w_j \right)^{1-\varepsilon}}{\left(t_{ii}^k \tau_{ii} w_i \right)^{1-\varepsilon}} \frac{\sum_m \left(\frac{\gamma_{m,j}^k}{A_m} \right)^{1-\varepsilon} (\sum_\ell w_\ell t_{\ell i}^k \tau_{\ell i} \gamma_{m,\ell} \mu_k (A_m^k)^{-1})^{\frac{\varepsilon-\sigma}{1-\varepsilon}} \overline{Z} + (w_j t_{ji}^k \tau_{ji} \mu_k (A_j^k)^{-1})^{\varepsilon-\sigma} ((A_j^k)^{-1})^{1-\varepsilon} \underline{Z}}{Den}
\end{aligned} \tag{G.9}$$

Let's now focus on:

$$\frac{\partial \log \sum_m \left(\frac{\gamma_{m,j}^k}{A_m} \right)^{1-\varepsilon} (\sum_\ell w_\ell t_{\ell i}^k \tau_{\ell i} \gamma_{m,\ell} \mu_k (A_m^k)^{-1})^{\frac{\varepsilon-\sigma}{1-\varepsilon}} \overline{Z} + (w_j t_{ji}^k \tau_{ji} \mu_k (A_j^k)^{-1})^{\varepsilon-\sigma} ((A_j^k)^{-1})^{1-\varepsilon} \underline{Z}}{\partial \tau_{ji}} \tau_{ji} \tag{G.10}$$

We find:

$$(\varepsilon-\sigma)\frac{\left[\sum_m\left(\frac{\gamma^k_{m,j}}{A_m}\right)^{1-\varepsilon}\left(\sum_\ell\left(w_\ell t^k_{\ell i}\tau_{\ell i}\gamma_{m,\ell}\mu_k(A^k_m)^{-1}\right)^{1-\varepsilon}\right)^{\frac{\varepsilon-\sigma}{1-\varepsilon}-1}\left(w_j t^k_{ji}\tau_{ji}\gamma_{m,\ell}\mu_k(A^k_m)^{-1}\right)^{1-\varepsilon}\overline{Z}+(w_j t^k_{ji}\tau_{ji}\mu_k(A^k_j)^{-1})^{\varepsilon-\sigma}((A^k_j)^{-1})^{1-\varepsilon}\underline{Z}\right]}{\sum_m\left(\frac{\gamma^k_{m,j}}{A_m}\right)^{1-\varepsilon}(\sum_\ell w_\ell t^k_{\ell i}\tau_{\ell i}\gamma_{m,\ell}\mu_k(A^k_m)^{-1})^{\frac{\varepsilon-\sigma}{1-\varepsilon}}\overline{Z}+(w_j t^k_{ji}\tau_{ji}\mu_k(A^k_j)^{-1})^{\varepsilon-\sigma}((A^k_j)^{-1})^{1-\varepsilon}\underline{Z}}$$

$$(\varepsilon-\sigma)\frac{\left[\sum_m\left(\frac{\gamma^k_{m,j}}{A_m}\right)^{1-\varepsilon}\left(\sum_\ell\left(w_\ell t^k_{\ell i}\tau_{\ell i}\gamma_{m,\ell}\mu_k(A^k_m)^{-1}\right)^{1-\varepsilon}\right)^{\frac{1-\sigma}{1-\varepsilon}-1}\lambda^{k,G}_{m,ji}\overline{Z}+(w_j t^k_{ji}\tau_{ji}\mu_k(A^k_j)^{-1})^{\varepsilon-\sigma}((A^k_j)^{-1})^{1-\varepsilon}\underline{Z}\right]}{\sum_m\left(\frac{\gamma^k_{m,j}}{A_m}\right)^{1-\varepsilon}(\sum_\ell w_\ell t^k_{\ell i}\tau_{\ell i}\gamma_{m,\ell}\mu_k(A^k_m)^{-1})^{\frac{\varepsilon-\sigma}{1-\varepsilon}}\overline{Z}+(w_j t^k_{ji}\tau_{ji}\mu_k(A^k_j)^{-1})^{\varepsilon-\sigma}((A^k_j)^{-1})^{1-\varepsilon}\underline{Z}}$$

$$(\varepsilon-\sigma)\frac{\left[\sum_m\lambda^{k,G}_{m,ji}(w_j\tau_{ji}\mu_k t_{ji})^{-(1-\varepsilon)}\left(P^{k,G}_{m,i}\right)^{1-\sigma}\lambda^{k,G}_{m,ji}+\left(P^{k,L}_{m,i}\right)^{1-\sigma}(w_j t^k_{ji}\tau_{ji}\mu_k)^{-(1-\varepsilon)}\right]}{\sum_m\lambda^{k,G}_{m,ji}(w_j\tau_{ji}\mu_k t_{ji})^{-(1-\varepsilon)}\left(P^{k,G}_{m,i}\right)^{1-\sigma}+\left(P^{k,G}_{m,i}\right)^{1-\sigma}(w_j t^k_{ji}\tau_{ji}\mu_k)^{-(1-\varepsilon)}} \tag{G.11}$$

Now, divide numerator and denominator by $(P^k_i)^{1-\sigma}$ and note that the term $(w_j\tau_{ji}\mu_k t_{ji})^{-(1-\varepsilon)}$ simplifies.

$$(\varepsilon-\sigma)\left[\frac{\sum_m\lambda^{k,G}_{m,ji}\lambda^{k,G}_{m,ji}\lambda^{k,G}_{m,i}+\lambda^{k,L}_{j,i}}{\sum_m\lambda^{k,G}_{m,ji}\lambda^{k,G}_{m,i}+\lambda^{k,L}_{j,i}}\right]=(\varepsilon-\sigma)\left[\frac{\sum_m\lambda^{k,G}_{m,ji}\lambda^{k,G}_{m,ji}\lambda^{k,G}_{m,i}+\lambda^{k,L}_{j,i}}{\lambda^k_{ji}}\right] \tag{G.12}$$

Then, note that in the term denoted as *Den*, trade costs only show up in the price index of global producers. As a result, the partial derivative of the numerator with respect to τ_{ji} is:

$$(\varepsilon-\sigma)\left[\frac{\sum_m\lambda^{k,G}_{m,ji}\lambda^{k,G}_{m,ji}\lambda^{k,G}_{m,i}+\lambda^{k,L}_{j,i}}{\sum_m\lambda^{k,G}_{m,ji}\lambda^{k,G}_{m,i}+\lambda^{k,L}_{j,i}}\right]=(\varepsilon-\sigma)\left[\frac{\sum_m\lambda^{k,G}_{m,ii}\lambda^{k,G}_{m,ji}\lambda^{k,G}_{m,i}}{\lambda^k_{ii}}\right] \tag{G.13}$$

Therefore, the trade elasticity is:

$$\theta^k_{ji}=(\varepsilon-1)-(\varepsilon-\sigma)\sum_m\lambda^{k,G}_{m,ji}\lambda^{k,G}_{m,i}\left(\frac{\lambda^{k,G}_{m,ji}}{\lambda^k_{mi}}-\frac{\lambda^{k,G}_{m,ii}}{\lambda^k_{ii}}\right)-(\varepsilon-\sigma)\frac{\lambda^{k,L}_{j,ji}}{\lambda^k_{ji}} \tag{G.14}$$

H Data

H.1 Tariff Data.

Our baseline measure of tariff rate is the ad-valorem effectively applied rate, which is based on the UNCTAD estimate of the ad-valorem rate and already accounts for preferential trade agreements. In practice, this variable is often missing or reported with error.[2] For this reason, we fill missing values and replace outliers with tariff rates from the WTO-IDB dataset. After this cleaning procedure, there are still values of tariff rates above 100%, which are replaced with the average MFN tariff rate.

To compute the industry-level average tariff rates, we map HS6 Codes into our industry

[2]There are cases in which the estimated value of the tariff rate is the midpoint of the bounds on the tariff lines included in a given HS code.

classification and we take a weighted average of tariff rates, using the volume of imports in 2016 as weights.

H.2 OECD AMNE Database

We combine data from the OECD AMNE Database to construct a matrix of industry-level trade flows and a matrix of bilateral output, which divides output produced in a given country and sector by country of ownership of the firm. Output of domestic firms is then further differentiated into output by global firms and output by local firms. We use the following datasets:

Matrix of Bilateral Output (MBO): this matrix reports gross output produced in each country, by industry and country of ownership. We use this dataset to compute the share of sector k's output in country ℓ produced by firms headquartered in i, for all $l, i = 1, \ldots, N$. Data is available from 2000 to 2020 and we calibrate the steady state of the model to 2016.

Analytical Database - Domestic MNEs (AAMNE - Domestic) : This dataset reports gross output, export, imports and gross value added produced by domestic companies, distinguishing between MNEs and non-MNEs. We use this dataset to compute the total output shares produced by domestic MNEs and non-MNEs. Data is available until 2016.

Analytical Database (AAMNE) This datasets splits gross output, value added, exports and imports by location of production, industry and whether the firm is domestic or foreign-owned.

AMNE - ICIO This dataset expands the $(N \times K) \times (N \times K + 1)$[3] Inter Country Input Output (ICIO) matrices of trade flows from the WIOD Database into a $(2N \times K) \times (2N \times K + 1)$ matrix, where trade flows between industry-country pairs are further split into transactions involving domestically or foreign-owned firms. More precisely, the trade flows $X_{\ell m}^{k \to k'}$ from industry k in country ℓ to industry k' in country m are decomposed into: $X_{\ell \to m, \ell m}^{k \to k'}$, $\sum_{i \neq \ell} X_{i \to m, \ell m}^{k \to k'}$, $\sum_{i \neq m} X_{\ell \to i, \ell m}^{k \to k'}$, $\sum_{i \neq l} \sum_{j \neq m} X_{i \to j, \ell m}^{k \to k'}$. In other words, the matrix does not report the full matrix of trade flows by country of ownership and rather aggregates foreign affiliates into a single category. Since our model does not feature intermediate inputs, we sum exports of intermediate inputs to exports of final goods to construct trade flows from country ℓ to country m in industry k. As a result, the AMNE-ICIO matrix collapses to a $(2N \times K) \times 1$ matrix where an entry corresponds to: $X_{\ell, \ell m}^{k} = \sum_{k'} \sum_{i} X_{\ell \to i, \ell m}^{k \to k'} + \sum_{i} X_{\ell \to i, \ell m}^{k \to F}$ for domestically-owned trade flows and $\sum_{j \neq \ell} X_{j, \ell m}^{k} = \sum_{j} \sum_{k'} \sum_{i} X_{\ell \to i, \ell m}^{k \to k'} + \sum_{j \neq \ell} \sum_{i} X_{\ell \to i, \ell m}^{k \to F}$ for trade flows by foreign subsidiaries located in country ℓ.

[3]The rows correspond to the country-industry source and the columns correspond to the country-industry destination or final demand. More correctly, final demand is further divided into Household Final Consumption Expenditure, Non-Profit Institutions Serving Households, General Government Final Consumption, Gross Fixed Capital Formation, Inventories, and Final Consumption Expenditure of Resident Households Abroad, which have been aggregated into a single final demand category.

H.3 ORBIS Historical Data

We use Orbis Historical Data to compute the measure of global and local firms for each country and industry. We first download information on the industry of the firm from the Firm Description Data, which contains the firm identifier (BVDID) and its industry code according to three different classifications: NACE Rev.2 (4 digit), the NAICS 2012 (4 digit) and the USSIC Code (3 digit). Since firms can own activities in multiple industries, each firm identifier can be assigned different codes corresponding to the core industry, the primary industry and secondary industries. Our model does not feature multi-sector firms, for this reason we only assign firms to their core industry.

We then merge the Firm Description data with the Links files in the Ownership Data. Each row of this dataset represents a link between a target firm and its shareholders, for a given vintage year. The possible types of relation are:

- SHH: single shareholder of first level
- CTP: first level shareholder identified via the Calculated Total Percentage
- ISH: shareholder at first level who is the immediate shareholder
- HQ: when the target company is a branch or foreign company, its single shareholder is its headquarter
- DUO 25 : domestic ultimate owner with a definition min 25% ownership stake
- GUO 25: global ultimate owner with a definition min 25% ownership stake
- DUO 50 : domestic ultimate owner with a definition min 50% ownership stake
- GUO 50 : global ultimate owner with a definition min 50% ownership stake
- DUO 50 C: domestic ultimate owner with a definition min 50% ownership stake, only with owner (shareholder) types B, C, A and F
- GUO 50 C: global ultimate owner with a definition min 50% ownership stake, only with owner (shareholder) types B, C, A, and F (Banks, Financial Companies, Insurance Companies and Industrial Companies)
- GUO 25 C: global ultimate owner with a definition min 25% ownership stake, only with owner (shareholder) types B, C, A, and F

We define a firm as independent if it corresponds to one of the following categories: *Individuals and Families*, *Public authorities/State*, *Employees/Managers/Directors* or if it has no shareholder with more than 50% of ownership stake.[4] We then restrict the sample to independent firms and to

[4]We therefore use a loose definition of independence, which includes both firms with no shareholders with more than 25% and firms with a B independence indicator, i.e. no shareholder above 50%.

the links between a subsidiary and their global ultimate owner (GUO 50).[5] Whenever the nationality of the firm owning the GUO 50 does not coincide with that of the GUO 50 C, we only maintain the former link.

In order to compute the number of local and global firms for each country of origin, we perform the following steps to map the moments from the data into their model analogues. There are three dimensions along which the predictions of the model might be inconsistent with the data.

First, the model does not feature multi-establishment firms, while in the data a shareholder can hold multiple subsidiaries in the same country and industry. In the baseline calibration, we count all subsidiaries in the same country and industry as a single establishment.

Second, a related issue arises when a shareholder owns subsidiaries in different industries. In the baseline calibration, we assign all subsidiaries to the industry of the shareholder and count them as a single unit.

Finally, a third issue arises in the count of foreign affiliates. We first define a firm as local if it does not own subsidiaries or if it only owns subsidiaries in the country of ownership. We then classify a shareholder as global if it owns at least one subsidiary in a different country from the country of ownership. However, the model implies that a global firm owns a subsidiary in each location of production, a feature that is clearly absent in the data. Therefore, we compute the cumulative number of affiliates in country ℓ owned by country i in industry k in three different ways:

1. we introduce fictitious subsidiaries in each location of production for each global shareholder included in the dataset.

2. we compute the average number of affiliates owned by global firms headquartered in i.

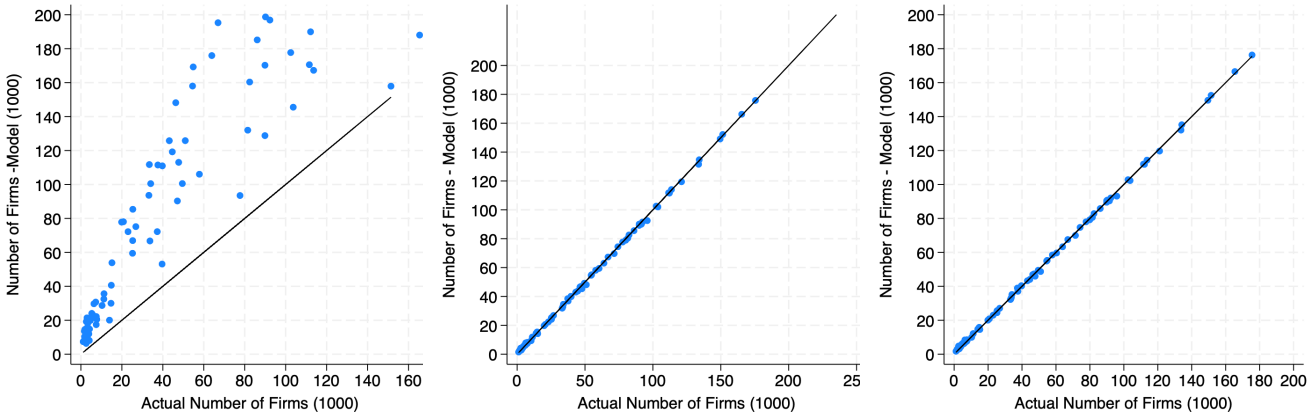
3. we compute the number of affiliates as the median number of affiliates owned by global firms headquartered in i

While option 1 leads to an overestimation of the number of firms in each country, it does not alter the ratio between the number of global firms and local firms for each country of origin which, along with the corresponding output levels, is the key moment affecting the calibration of the fixed costs. To clarify this with a numerical example, suppose that there are three countries, and there are ten global firms in country A, all of them owning a subsidiary in country B and zero subsidiaries in country C. The first approach introduces 10 fictitious subsidiaries in country C. Since the output share of A's subsidiaries in country C is zero, the calibration would still match the intensive margin through prohibitively high MP frictions. In the second and third approach instead (which coincide in this abstract case), the number of global firms, and hence the number of affiliates in each foreign country, would be reduced to 5. Figures H.1a, H.1b and H.1c report the correlation between the

[5] An implication of this choice is that if there is a chain of linkages, e.g. a domestic affiliate is owned by a domestic owner which is owned by a foreign global ultimate owner, we are treating both firms as being owned by the foreign owners. We then follow two alternative approaches: in the baseline approach, we treat these two firms as a single foreign-owned affiliate, consistently with the interpretation that the model does not feature multi-establishment firms; in the second, we maintain both firms in the sample and effectively count them as owned by two different foreign owners.

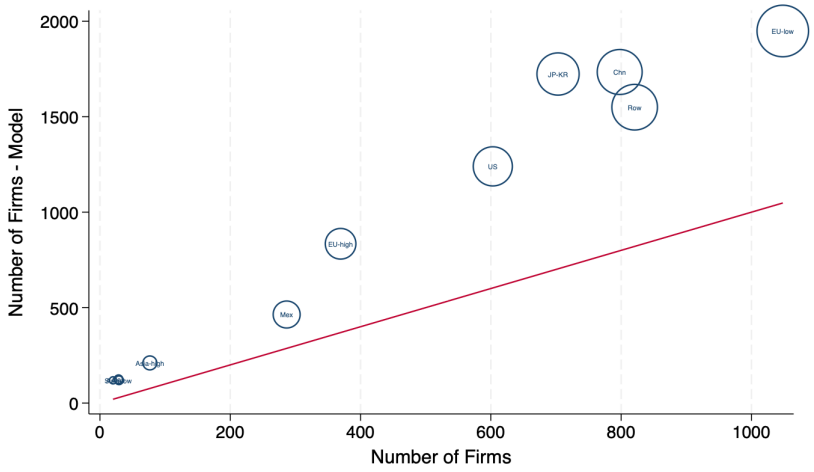
model implied number of firms and the actual number of firms in the data. These figures are produced using the industry and country classification described in the following section.

Figure H.1: Comparison model implied number of firms and actual number of firms



(a) Max **(b)** Median **(c)** Mean

Figure H.2: Comparison model implied number of firms and actual number of firms - aggregate



H.4 Classification of Countries and Sectors

Country Classification. We classify countries based on two criteria. We first group countries into the following geographic areas: North America, Central America, South America, Europe, Africa, Middle East, South-East Asia, Eastern Asia. Within Europe and Asia, we then classify countries based on the share of national output produced by US-owned affiliates and by their relevance for US imports. In particular, we split Europe into terciles and assign the first tercile to a residual Rest of the World group. The medium tercile contains the largest European US trade partners, including France,

Germany, Italy, Spain and most of Central and Northern Europe (Austria, Bulgaria, Czech Republic, Denmark, Finland, Iceland, Norway, Portugal, Romania, Slovakia, Sweden), while the highest tercile contains export-oriented countries, such as Belgium, Netherlands, and the United Kingdom, and countries featuring a high share of multinational companies due to either low manufacturing costs (Poland, Hungary) or high tax incentives (Cyprus, Switzerland, Luxembourg, Ireland). Asian countries are divided into the following groups: China; Japan and Korea; countries with high US MP (Singapore, Thailand, Malaysia, Hong Kong and Taiwan); countries with low US MP (India, Vietnam, Indonesia and Philippines). The residuals country-groups are: Canada; US; Mexico; South America (Brazil, Chile, Colombia and Argentina) and Rest of the World, which includes the remaining European countries in the sample, New Zealand, Australia, Middle Eastern countries (Israel and Saudi Arabia), and African countries (Morocco and South Africa).

Table H.1 contains relevant summary statistics for each country-group.

Table H.1: MNE and Trade Shares by Country Group - 11 regions

Country Group	**MNE Share GO (%)**	**MNE Share EXGR (%)**	**US Output Share (%)**	**China Output Share (%)**	**Domestic Output Share (%)**	**Exports to US (%)**	**Share US imports (%)**	**Relative Expenditures**	M^G **Share**	**Relative L**
EU - Med	40.68	56.56	6.10	0.54	82.46	17.24	14.19	88.41	5.56	1.68
EU - High	37.07	49.80	18.47	0.46	62.40	18.51	7.44	25.27	5.02	0.33
Canada	30.25	42.24	23.65	0.10	57.27	69.33	9.56	9.89	12.46	0.11
US	53.95	62.99	75.34	0.28	75.34	0	0	100.00	1.49	1.00
S Am	26.48	33.97	7.91	0.44	79.35	23.66	2.32	24.26	0.23	0.73
Mexico	38.68	57.88	14.23	0.50	70.69	78.84	13.71	10.65	1.51	0.44
Jap, Kor	51.13	67.71	1.83	0.17	95.34	19.35	10.79	59.48	1.90	0.91
Asia - High	37.73	51.91	9.09	3.21	69.69	12.22	4.54	22.79	8.65	1.06
Asia - Low	29.34	46.54	1.52	1.35	88.27	25.61	6.56	48.50	6.33	2.64
China	52.12	67.41	1.29	91.96	91.96	26.75	27.53	208.91	0.53	3.44
ROW	28.11	39.24	3.57	0.74	83.36	13.45	3.36	39.99	1.43	1.28

The first column reports the share of total output produced domestically by domestic multinational companies. The second column reports the share of exports accounted for by multinational companies. The third and fourth column reports the share of domestic gross output produced by US-owned and Chines-owned companies. The fifth column reports the share of output that is sold domestically. The sixth column reports the ratio between exports to the United states and total exports. The seventh column reports the ratio between exports from the country reported in the row to the US and the total US imports (the entries of the column sum to 100). The eighth column reports relative (to the US) expenditures. The ninth column reports the share of global firms. The tenth column reports relative (to the US) manufacturing employment.

Industry Classification. We follow Fajgelbaum et al. (2020) and construct the following industry-groups: Agriculture, Apparel, Chemicals, Machinery, Materials, Metals, Minerals, Transport, Miscellaneous. The mapping between Product Classifications and Industry Classifications into our Industry Classifications is provided in Table H.2. Summary statistics at the industry-level for the US are provided in Table H.3.

Table H.2: Industry Model Summary Statistics

Industry	**HS 2 Digit**	**AMNE Database**
Agriculture	1-24	A01T03
Apparel	41-43, 50-67	C13T15
Chemicals	28-38	C20-C21
Machinery	84-85	C31T33
Materials	39-40, 44-49, 68-71	C16, C17T18, C22
Metals	72-83	C24, C25
Minerals	25-27	C19, D35_E36T39
Transport	86-89	C29, C30
Miscellaneous	90-99	C26, C27

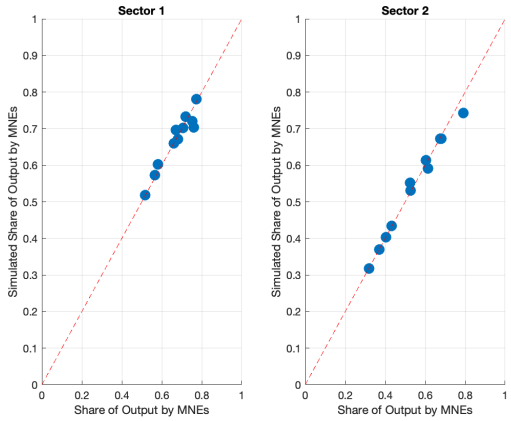
Table H.3: US Industry Model Summary Statistics

Industry	**Exp/Y**	**Imp/Y**	**Imp China (%)**	**RP Share**	**RP Share China**	**MNE Out. Share**	**MNE Exp Share**	**Average Tariff**	**Import Tariff China**	M^G **Share**
Agriculture	8.07	9.34	0.78	27.15	12.56	45.03	69.03	5.19	10.72	0.19
Apparel	6.59	158.10	32.33	19.49	6.25	91.03	96.55	4.66	8.73	0.61
Chemicals	12.50	24.54	2.02	63.38	21.38	76.80	82.85	1.02	2.23	2.39
Machinery	23.41	35.71	12.39	38.93	18.53	90.53	94.23	0.65	0.60	0.44
Materials	8.76	11.51	3.04	40.74	15.86	63.77	87.52	1.90	3.31	1.22
Metals	21.98	26.00	7.14	40.77	18.55	62.84	79.47	1.19	2.14	0.64
Minerals	16.74	14.07	1.17	47.62	20.86	50.60	73.17	2.49	1.63	0.71
Transport	20.93	37.39	2.04	74.28	28.86	79.86	76.17	1.46	1.68	0.47
Miscellaneous	21.21	65.80	23.65	54.02	36.88	85.53	88.03	0.74	1.11	2.37

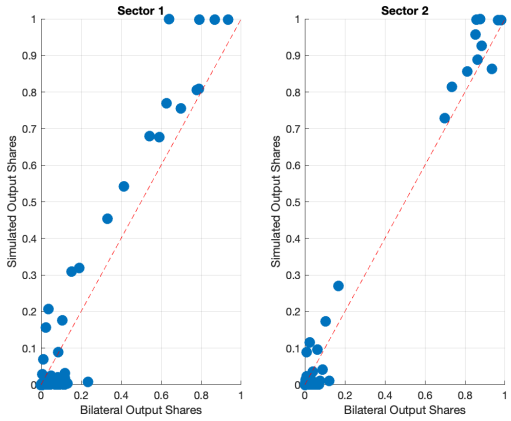
The first (second) column reports the share of exports (imports) over total production. The third column reports the share of imports from China. The fourth and fifth column report the average RP share from all countries and the RP share from China. The sixth and seventh column report the share of output and exports produced by MNEs. The eighth and ninth column report the average import tariff from all countries and from China. The last column report the share of global companies.

I Additional Figures

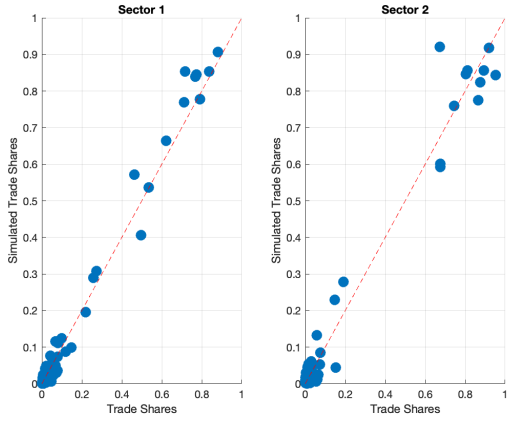
Figure I.1: Correlation between model-simulated moments and empirical moments



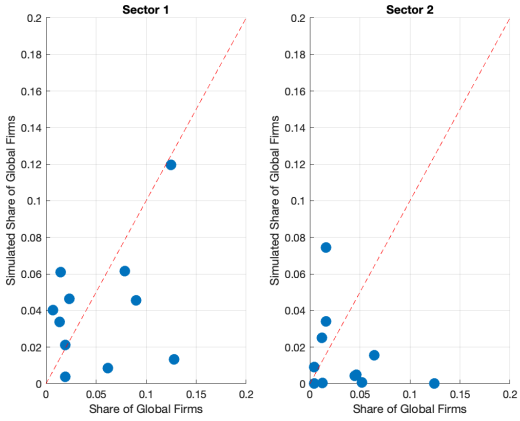
(a) Share of Output by MNEs



(b) Bilateral Output Shares

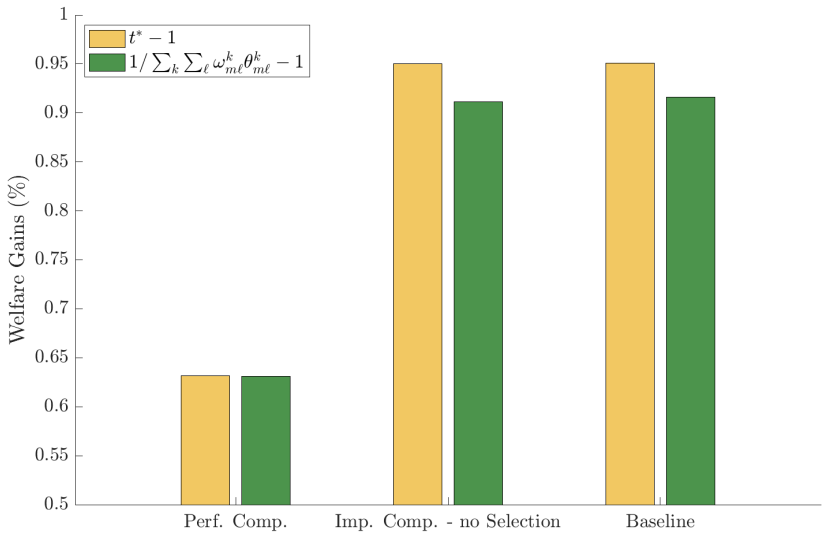


(c) Trade Shares



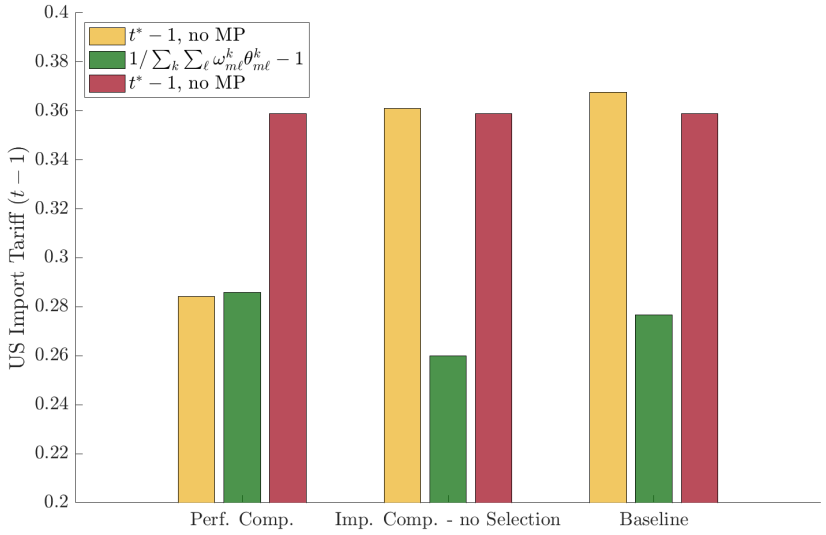
(d) Share Global Firms

Figure I.2: Welfare Gains from Unilateral Optimal Import Tariffs



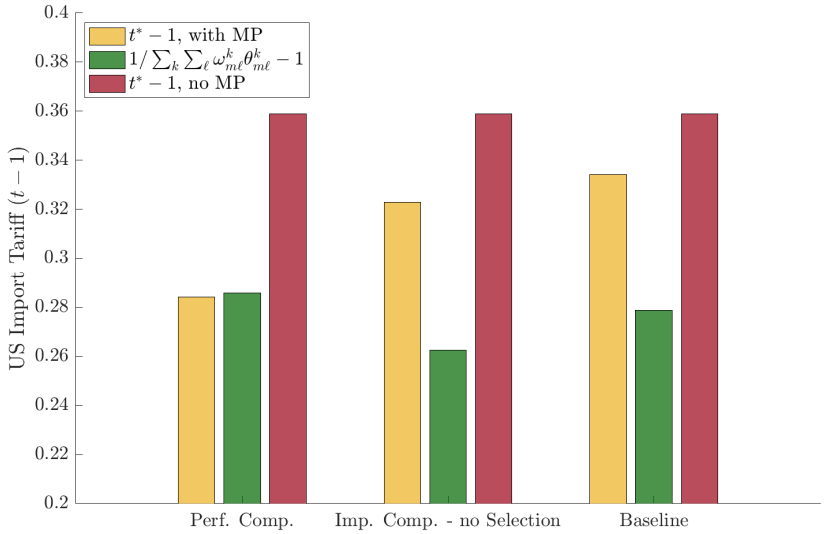
The yellow column represents the welfare gains from imposing the optimal US unilateral import tariff on all countries. The green column reports the welfare gains from imposing an import tariff on all countries equal to the reciprocal of the average trade elasticity. The perfect competition scenario imposes that all firms are global and markups are equal to 1. In the imperfect competition scenario, we maintain that all firms are global and set markups to $\mu_k = \sigma/(\sigma - 1)$. The baseline scenario is the calibrated model with selection into MP and positive markups.

Figure I.3: Optimal Unilateral Import Tariff, in Baseline and Armington Model with no MP



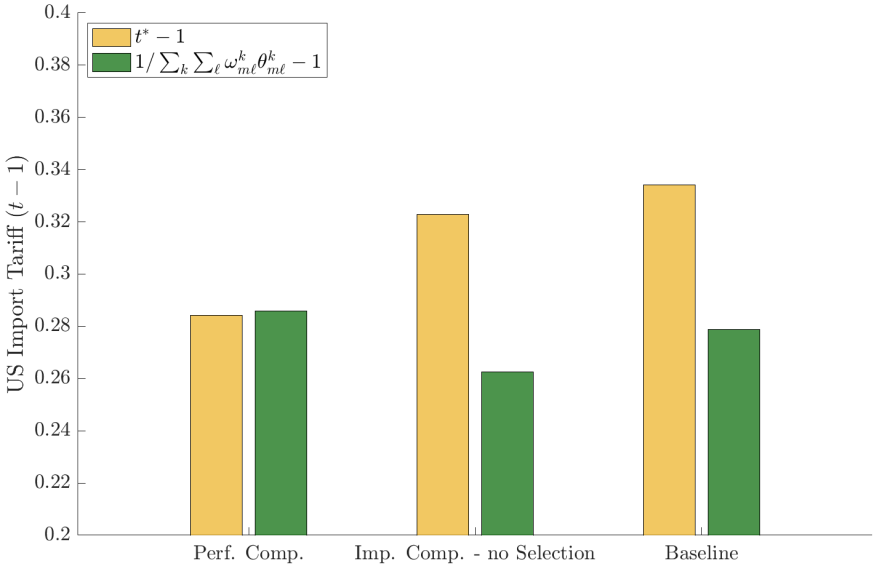
The yellow column represents the optimal US unilateral import tariff on all countries. The green column reports the reciprocal of the average trade elasticity. The red column represents the optimal unilateral import tariff in a model with no MP. The perfect competition scenario imposes that all firms are global and markups are equal to 1. In the imperfect competition scenario, we maintain that all firms are global and set markups to $\mu_k = \sigma/(\sigma - 1)$. The baseline scenario is the calibrated model with selection into MP and positive markups.

Figure I.4: Optimal Unilateral Import Tariff, in Baseline and Armington Model with no MP, low markups



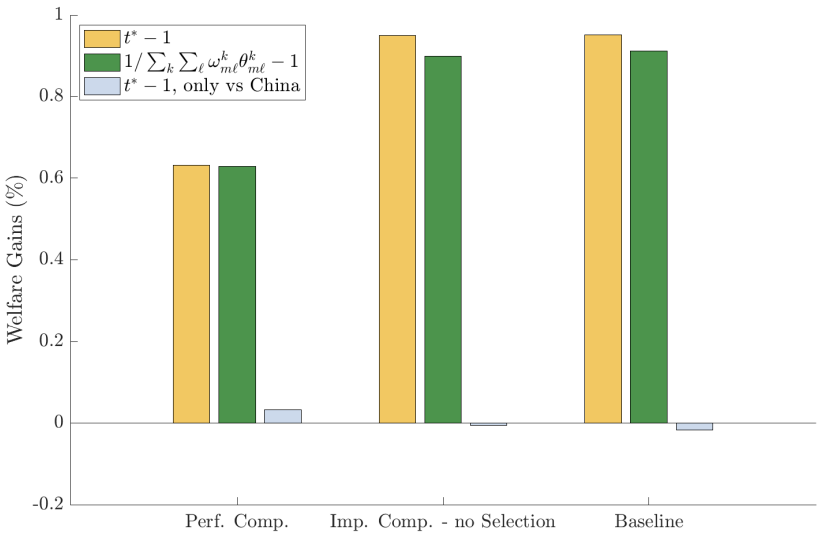
The yellow column represents the optimal US unilateral import tariff on all countries. The green column reports the reciprocal of the average trade elasticity. The red column represents the optimal unilateral import tariff in a model with no MP. The perfect competition scenario imposes that all firms are global and markups are equal to 1. In the imperfect competition scenario, we maintain that all firms are global and set markups to $\mu_k = 0.15$. The baseline scenario is the calibrated model with selection into MP and positive markups.

Figure I.5: Optimal Import Tariffs, Low Markups



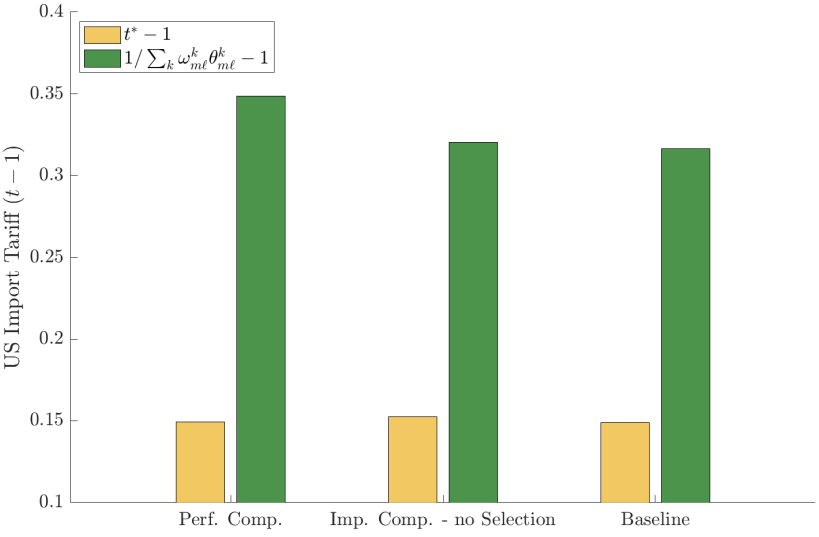
The yellow column represents the optimal US unilateral import tariff on all countries. The green column reports the reciprocal of the average trade elasticity. The red column represents the optimal unilateral import tariff in a model with no MP. The perfect competition scenario imposes that all firms are global and markups are equal to 1. In the imperfect competition scenario, we maintain that all firms are global and set markups to $\mu_k = 0.15$. The baseline scenario is the calibrated model with selection into MP and positive markups.

Figure I.6: Welfare Gains from Bilateral Import Tariffs on China



The yellow column represents the welfare gains from imposing the optimal US unilateral import tariff on all countries. The green column reports the welfare gains from imposing an import tariff on all countries equal to the reciprocal of the average trade elasticity. The light-blue column represents the welfare changes from imposing the optimal unilateral import tariff only on imports from China. The perfect competition scenario imposes that all firms are global and markups are equal to 1. In the imperfect competition scenario, we maintain that all firms are global and set markups to $\mu_k = \sigma/(\sigma - 1)$. The baseline scenario is the calibrated model with selection into MP and positive markups.

Figure I.7: Optimal Bilateral Import Tariffs on China



The yellow column represents the value of the optimal US unilateral import tariff. The green column reports the reciprocal of the bilateral trade elasticity of US exports to China. The perfect competition scenario imposes that all firms are global and markups are equal to 1. In the imperfect competition scenario, we maintain that all firms are global and set markups to $\mu_k = \sigma/(\sigma - 1)$. The baseline scenario is the calibrated model with selection into MP and positive markups.