Monetary Policy Rules and Environmental Fiscal Policy in a Two-Sector DSGE Model

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Abstract

This paper examines how environmental externalities and fiscal policies aimed at reducing emissions affect the conduct of monetary policy in a two-sector (clean and dirty) environmental dynamic stochastic general equilibrium (E-DSGE) model. We analyze the optimal response of monetary policy to sector-specific inflationary shocks under different environmental policy regimes and compare the performance of standard and non-standard monetary policy rules. Our findings highlight three key results. First, the existence of environmental policies introduces distortions in relative prices, which in turn affect the conduct of (Ramsey) optimal monetary policy. Second, compared to a standard Taylor rule targeting aggregate inflation, a non-standard monetary rule that accounts for sector-specific inflation can mitigate more effectively these policy-induced distortions, reducing output, inflation and emissions volatility. Third, while a non-standard monetary policy rule improves welfare across all environmental policy scenarios, it leads to higher emissions except under a cap policy, where emissions remain unchanged by design. These results highlight the complex interactions between monetary and environmental fiscal policies and the trade-offs involved in designing optimal policy frameworks.

JEL: E32, E52, E58, Q54, Q58

Keywords: Climate change, environmental policy, inflation, business cycle stabilization, monetary policy.

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1 Introduction

Since the Paris Agreement of 2015, climate policies have become central to the global economic agenda. In addition to governments and (supra)national public bodies, monetary authorities have also begun to express concern about the economic consequences of climate change, and they see the need for their proactive involvement. Speeches given in 2015 by Mark Carney, former governor of the Bank of England, and in 2019 by Mary C. Daly, president of the Federal Reserve Bank of San Francisco, are examples of this: both warned that the green transition poses severe risks to economic development and financial stability, and could ultimately affect the conduct of monetary policy to achieve the goals of full employment and price stability. More recently (July 2021), the European Central Bank approved a new monetary policy strategy, claiming that central banks should commit to including the impact of climate change in their policy frameworks and be supportive of policy initiatives addressing environmental issues.

What effects do these developments have on monetary policy, particularly when fiscal environmental policies target specific sectors and introduce distortions in relative prices? Many central banks around the world have recognized the implications that climate change may have for financial stability, envisioning macroprudential policy measures. However, less attention has been given to how environmental fiscal policies, such as carbon taxation or emission caps, which affect clean and dirty sectors differently, impact business-cycle stabilization and the design of monetary policy rules.

In standard real business cycle (RBC) models, monetary policy plays no role. In RBC models with an environmental externality (Heutel 2012), which are often used to describe the impact of climate change on the economic system, an optimal environmental policy is designed to offset the negative externality. This usually takes the form of a carbon tax, as making emissions costly is considered the most efficient way to reduce them (Nordhaus 1977). In an RBC model with an environmental externality and nominal frictions (Annicchiarico and Di Dio 2015), monetary policy is introduced to address nominal rigidities. In multi-sector New Keynesian models, monetary policy faces additional trade-offs due to sectoral heterogeneity. Rubbo (2023) introduces the "Divine Coincidence index," an inflation measure that eliminates these trade-offs by exploiting differences in technology, price stickiness, and sectoral relevance. In light of this existing literature, this paper examines how monetary policy

should respond to standard shocks (e.g., cost-push shocks) in the presence of sub-optimal (exogenous) environmental policies. We consider a setting with two symmetric sectors, identical in technology, nominal price rigidity, and aggregate weight. In this setting, it is not structural asymmetries but rather environmental fiscal policies that create distortions in relative prices and sectoral dynamics and thus affect the optimal conduct of monetary policy.

Our analysis begins with a Ramsey optimal policy benchmark, which reveals that environmental fiscal policies – such as carbon taxes and emission caps – introduce relative price distortions that alter the optimal monetary policy, in response to sectoral cost-push shocks. In light of this, we compare a standard Taylor rule – targeting aggregate inflation – with a non-standard monetary rule that accounts for sector-specific inflation. We show that the non-standard rule more effectively mitigates these policy-induced distortions, reducing output, inflation and emissions volatility and improving welfare. While previous studies have studied the policy outcome of distinguishing between sectoral inflation (see Del Negro et al. (2023) and Rubbo (2023)), we extend this analysis to a setting where asymmetries arise primarily from environmental fiscal policies rather than from structural differences. We further explore whether monetary policy can balance the trade-off between welfare maximization and emission reduction when environmental policies create sectoral imbalances.

To address these questions, we develop an environmental dynamic stochastic general equilibrium (E-DSGE) model featuring a clean (green) and a dirty (polluting) sector. The model incorporates fiscal policies such as emission caps and carbon taxes, which distort relative prices by altering production costs, affecting the transmission of monetary policy. Within a Taylor-rule-based framework (Taylor 1993, 1999), we allow the central bank to differentiate between clean and dirty good inflation when setting its policy. Our analysis shows that the optimal calibration of monetary rule parameters depends on the type of environmental policy in place and on the sector affected by the shock, highlighting the extent to which monetary policy can mitigate the distortions induced by environmental fiscal policies.

In this paper, we want to contribute to two main strands of the literature. The first is that of environmental macroeconomics, particularly work employing environmental DSGE (E-DSGE) models that combine business cycle analysis with environmental issues. The second is that related to optimal monetary policy and the interaction with (environmental) fiscal policies. The bridge in the literature between general equilibrium models and the environment is the integrated assessment model (IAM). The pioneering work of William Nordhaus

(Nordhaus 1977, 2010; Nordhaus and Sztorc 2013), with his *Dynamic Integrated model of Climate and the Economy* (DICE), can be considered the forerunner of the wide strand of literature developed in recent decades around IAMs. DICE is an analytical model, designed as a policy optimization tool, that tries to represent the interconnection between the climate and the global economic system. In addition to the neoclassical economic growth theory, on which it is grounded, the "negative natural capital" of carbon concentrations is included (Nordhaus 2010). Environmental policies that aim to reduce anthropogenic emissions are therefore intended as investments to reduce this negative capital. A strand of literature combining environmental issues with the real business cycle theory is also based on the early work of Nordhaus. Heutel (2012) formally stresses the importance of business cycles in driving public policies: his idea is that, in order to design focused interventions to address climate change, it is important to build a model in which climate policies are explicitly integrated with macroeconomic fluctuations. He develops an E-DSGE model in which pollution appears as a stock variable that negatively affects the economy.

The early E-DSGE literature focused primarily on the different effects of specific public environmental policies on the business cycle. Extensions of the model constructed by Heutel (2012) have been created by adding uncertainty (Hassler and Krusell 2018), New Keynesian nominal rigidities (Annicchiarico and Di Dio 2015; Annicchiarico and Dio 2017) and financial (Carattini et al. 2023) or labor-market frictions and migration (Chan 2019; Gibson and Heutel 2023). More recently, attention has shifted to the interaction between monetary and environmental fiscal policies and the possible extension of the tools available to central banks. These works extend the broader literature on the relationship between optimal monetary and fiscal policies (see for example Chari and Kehoe (1999) or Galí and Monacelli (2008)). Examples of this are the works by Chen et al. (2020) and Chan (2020), who develop a climate-augmented monetary policy rule by adding an emission target to the standard Taylor equation. They find that such a monetary policy can create a conflict between welfare and climate objectives, as emission reductions can be recessive. Nakov and Thomas (2023) analyze the implication of what they define as a climate-conscious (Ramsey optimal) monetary policy when the carbon tax is initially set at a sub-optimal level, showing that Central Banks' conventional interest rate policy is an inefficient instrument for reducing CO_2 emissions. Del Negro et al. (2023) show how climate policies can create a trade-off between inflation and output stabilization in the presence of a shock to the carbon taxation, and how this result

depends on the difference in price flexibility between a dirty and a green sector. Economides and Xepapadeas (2025) highlight the interplay between climate shocks, energy prices, and productivity in a New Keynesian DSGE model with a climate module. While their approach emphasizes climate-induced productivity shocks and optimal Ramsey policies, this paper focuses on the macroeconomic effects of carbon taxation and the monetary policy responses required to manage these distortions, employing a specific monetary policy rule. Other scholars instead focus on the composition of central banks' balance sheets, developing models with financial frictions in which clean-quantitative easing programs are enforced (Diluiso et al. 2021; Ferrari and Nispi Landi 2023).

We contribute to this literature by showing how monetary policy should be conducted when environmental policy and climate frictions affect the business cycle, in the presence of standard shocks. We also address the question of whether monetary policy should actually be concerned with aggregate price stability or whether central banks' optimal responses to price changes in clean and dirty sectors should be asymmetric.

Our main results are as follows: first, the existence of environmental policies introduces distortions in relative prices, which in turn affect the conduct of (Ramsey) optimal monetary policy. Second, compared to a standard Taylor rule targeting aggregate inflation, a non-standard monetary rule that accounts for sector-specific inflation can mitigate more effectively these policy-induced distortions, reducing output, inflation and emissions volatility. Third, while a non-standard monetary policy rule improves welfare across all environmental policy scenarios, it leads to higher emissions except under a cap policy, where emissions remain unchanged by design.

The paper is organized as follows: in section 2, we design our E-DSGE model by integrating a two-sector DSGE model with climate damage and introducing environmental fiscal policies. In section 3, we conduct an analysis of the impulse response functions (IRFs) in the E-DSGE model: first, we simulate the optimal Ramsey monetary policy in different environmental policy scenarios; then, we restrict our attention to a single environmental policy (tax policy) and simulate the model under different monetary regimes[1]. In section 4, we analyze the optimal monetary policy rule parameters in response to inflationary shocks, considering both symmetric and asymmetric cases. In the first step (subsection 4.1), we compare in-

[1]For completeness, the results of simulations conducted under alternative environmental regimes are shown in the appendix.

flation, output gap, and emissions volatility under the standard and non-standard monetary policy rules when exogenous shocks affect the economy simultaneously. In the next step (subsection 4.2), we examine optimal monetary policy rule coefficients, welfare loss, and emission variation when shocks impact the economy asymmetrically. The last section concludes the paper.

2 Environmental-DSGE model

In this section, we describe our two-sector DSGE model augmented with environmental externalities. Building on the work of Annicchiarico and Di Dio (Annicchiarico and Di Dio 2015; Annicchiarico and Dio 2017) and Ferrari and Nispi Landi (2023), we integrate the Environmental-DSGE model of Heutel (2012) with New Keynesian nominal rigidities and an intermediate firm level. In this model, three types of firms operate in the economy: a final-consumption-good producer, which bundles "clean" and "dirty" outputs and operates in a perfectly competitive market; two intermediate firms aggregating, respectively, "clean" and "dirty" differentiated goods using a CES aggregator technology; and the "dirty" polluting firms[2] and "clean" non-polluting firms operating in a monopolistically competitive regime. The latter employ capital and labor as production factors. A sketch of the model is shown in Figure 1.

2.1 Households

Households are all identical and indexed by $i \in [0, 1]$, they consume a consumption good and supply differentiated labor and capital. The utility of the representative infinitely lived household is:

$$U = \mathbb{E}_t \sum_{t=0}^{\infty} \beta^t \left(\frac{c_t^{1-\varphi_c} - 1}{1 - \varphi_c} - \psi \frac{l_t^{1+\varphi_l}}{1 + \varphi_l} \right), \tag{1}$$

where l_t represents working hours and c_t is per-capita consumption (at time t); the period utility is characterized by a constant relative risk aversion (CRRA), where φ_c is the inverse elasticity of intertemporal substitution; ψ weighs the disutility of working; φ_l is the inverse of the Frisch elasticity; and β is the intertemporal discount factor. Households maximize the lifetime utility (equation (1)) subject to the budget constraint. For the budget of households,

[2] Although in Heutel (2012) all firms pollute, in our model, only dirty firms pollute.

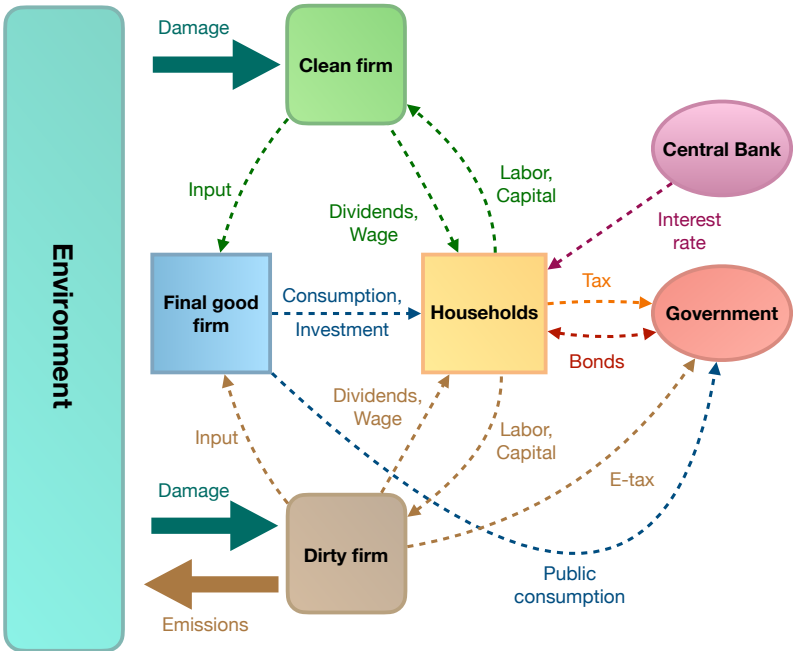


Figure 1: Scheme of the E-DSGE model.

we differentiate capital, labor and wage by sector such that

$$b_t(i) + c_t(i) + i_t(i) = b_{t-1}(i)\frac{r_{t-1}}{\pi_t} + \sum_j \left(r_t^{k,j}(i)k_{t-1}^j(i)\right) + \sum_j \left(w_t^j(i)l_t^j(i)\right) - t_t + \mathcal{T}_t, \quad (2)$$

where $j = \{C, D\}$ stands for clean and dirty sectors. Here, b_t represents bond holdings, i_t represents investment, $b_{t-1}r_{t-1}$ denotes revenues from holding bonds, π_t is the consumer price index (CPI) inflation rate; $r_t^{k,j}k_{t-1}^j$ is the income from sector-specific capital service and $w_t^j l_t^j$ is the income from sector-specific labor; t_t is a lump-sum tax and $\mathcal{T}_t$ represents profits from firms ownership (equal to zero because of the perfect competition regime in which the firms operate). All variables are expressed in real terms[3]. Following the example of Christiano et al. (2005), we also consider the existence of implicit adjustment costs in investment[4], which makes adjusting the investment level in response to a departure of capital from its optimal level costly. By doing this, investment is smoothed over time. Households can also choose whether to invest in green or dirty sector capital. The law of motion of capital with quadratic

[3]Lowercase letters denote real variables, uppercase letters nominal variables.

[4]In their work, Christiano et al. (2005) do not specify a functional form for the investment adjustment cost; instead, they specify some properties that the function should have. The functional form we choose is in line with those properties.

adjustment costs is

$$k_t^j(i) = (1-\delta)k_{t-1}^j(i) + i_t^j(i)\left[1 - \frac{\phi_i}{2}\left(\frac{i_t^j(i)}{i_{t-1}^j(i)} - 1\right)^2\right], \tag{3}$$

where $\phi_i > 0$ denotes the investment cost parameter. Investment and labor supply for the two sectors aggregate according to the following equations:

$$l_t(i) = \left[\left(l_t^D(i)\right) + \left(l_t^C(i)\right)\right], \tag{4}$$

$$i_t(i) = \left[i_t^D(i) + i_t^C(i)\right], \tag{5}$$

Individual labor supply varieties are aggregated using the Dixit-Stiglitz function (Dixit and Stiglitz 1977).

$$l_t^j = \left(\int_0^1 \left(l_t^j(i)\right)^{\frac{\xi_W - 1}{\xi_W}} di\right)^{\frac{\xi_W}{\xi_W - 1}}. \tag{6}$$

where ξ_W is the elasticity of substitution between individual labor supply. The latter is defined as a function of individual to aggregate wage share

$$l_t^j(i) = \left(\frac{w_t^j(i)}{w_t^j}\right)^{-\xi^W} l_t^j, \tag{7}$$

such that the aggregate intermediate wage reads

$$w_t^j = \left(\int_0^1 w_t(i)^{1-\xi_W} di\right)^{\frac{1}{1-\xi_W}}. \tag{8}$$

Additionally we introduce sectoral nominal wage rigidity in the form of quadratic adjustment costs à la Rotemberg (1983)[5]:

$$AC_{W,t}^j(i) = \frac{\phi_W}{2}\left(\frac{W_t^j(i)}{W_{t-1}^j(i)} - \bar{\pi}\right)^2 l_t^j W_t^j, \tag{9}$$

which is added to the left-hand side of the budget constraint (2). Here ϕ_W represents the nominal wage adjustment cost parameter. Households' utility maximization problem yields

[5]As an alternative, we could have employed the model of pricing of Calvo (1983), but we opted for Rotemberg's model because it is a more parsimonious model. Moreover, Rotemberg and Calvo models deliver equivalent dynamics when they are log-linearized around a zero-inflation steady state (Ascari and Rossi 2012).

the following first-order conditions (FOCs) with respect to $c_t(i)$, $w_t^j(i)$ and $k_t^j(i)$:

$$
\begin{aligned}
c_t^{-\varphi_c} =& \lambda_t, && (10)\\
mrs^j{}_t =& \frac{\psi\, l_t^{\phi_l-\phi_h} lj_t^{\phi_h}}{\lambda_t} && (11)\\
\pi^j{}_{W,t}\left(\pi^j{}_{W,t}-\bar{\pi}\right) =& \beta\frac{\lambda_{t+1}}{\lambda_t}\frac{l^j{}_{t+1}}{l^j{}_t}\frac{w^j{}_{t+1}}{w^j{}_t}\pi^j{}_{W,t+1}\left(\pi^j{}_{W,t+1}-\bar{\pi}\right)\\
&+\frac{\xi_W}{\phi_W}\left(\frac{mrs^j{}_t}{w^j{}_t}-\frac{\xi_W-1}{\xi_W}\right), && (12)\\
q_t^j =& \mathbb{E}_t\left\{\frac{\lambda_{t+1}}{\lambda_t}\beta\left((1-\delta)q_{t+1}^j+r_{t+1}^{k,j}\right)\right\}, && (13)
\end{aligned}
$$

where:

$$
\begin{aligned}
\pi^j{}_{W,t} &= \frac{W_t^j}{W_{t-1}^j}\\
&= \frac{W_t^j}{p_t}\frac{p_t}{p_{t-1}}\frac{p_{t-1}}{W_{t-1}^j} && (14)\\
&= \frac{w_t^j}{w_{t-1}^j}\pi_t.
\end{aligned}
$$

and mrs_t^j is the inverse of the marginal rate of substitution between consumption and leisure; $\pi^j{}_{W,t}$ is the sectoral wage inflation; q_t measures the marginal value of capital with respect to consumption and is known as Tobin's q. From the FOC for b_t, we obtain the Euler equation:

$$
\lambda_t = \beta\mathbb{E}_t\left(\lambda_{t+1}\frac{r_t}{\pi_{t+1}}\right), \tag{15}
$$

where inflation π_{t+1} is defined as

$$
\pi_{t+1} = \frac{p_{t+1}}{p_t}. \tag{16}
$$

Lastly, the FOC with respect to i_t^j yields

$$
\begin{aligned}
1 =& q_t^j\left[1-\frac{\phi_i}{2}\left(\frac{i_t^j}{i_{t-1}^j}-1\right)^2-\phi_i\left(\frac{i_t^j}{i_{t-1}^j}-1\right)\frac{i_t^j}{i_{t-1}^j}\right]\\
&+\beta\mathbb{E}_t q_{t+1}^j\left[\frac{\lambda_{t+1}}{\lambda_t}\phi_i\left(\frac{i_{t+1}^j}{i_t^j}-1\right)\left(\frac{i_{t+1}^j}{i_t^j}\right)^2\right]. && (17)
\end{aligned}
$$

From equation (15), we can write the real interest rate r_t^r as

$$
r_t^r = \frac{r_t}{\pi_{t+1}}. \tag{18}
$$

2.2 Firms

2.2.1 Final-good firms

Final good firms employ clean (y_t^C) and dirty (y_t^D) intermediate goods as inputs in a constant elasticity of substitution (CES) aggregator function:

$$y_t = \left[(1-\Delta)^{\frac{1}{\epsilon}} \left(y_t^C\right)^{\frac{\epsilon-1}{\epsilon}} + \Delta^{\frac{1}{\epsilon}} \left(y_t^D\right)^{\frac{\epsilon-1}{\epsilon}}\right]^{\frac{\epsilon}{\epsilon-1}}, \tag{19}$$

where y_t^C and y_t^D are goods from industry C and industry D, respectively, Δ is a weighting parameter and ϵ represents the elasticity of substitution between the two[6]. The final-good firm solves an intratemporal maximization problem to determine the optimal input combination:

$$\max_{y_t^C, y_t^D} \quad p_t y_t - \left[P_t^C y_t^C + P_t^D y_t^D\right],$$

where P_t^C and P_t^D are sector-specific prices and p_t represents the aggregate price index (CPI). This problem yields the following demand functions:

$$y_t^C = y_t(1-\Delta)\left(\frac{P_t^C}{p_t}\right)^{-\epsilon}, \tag{20}$$

$$y_t^D = y_t\Delta\left(\frac{P_t^D}{p_t}\right)^{-\epsilon}. \tag{21}$$

The aggregate price is

$$p_t = \left[(1-\Delta)\left(P_t^C\right)^{1-\epsilon} + \Delta\left(P_t^D\right)^{1-\epsilon}\right]^{\frac{1}{1-\epsilon}}. \tag{22}$$

We define $p_t^C \equiv \frac{P_t^C}{p_t}$ and $p_t^D \equiv \frac{P_t^D}{p_t}$ as the price of clean and dirty goods in terms of the aggregate CPI.

Both demand functions are increasing in terms of the final good production and price and decreasing in terms of their own price. Both demand functions depend on the composition of y, which is given by the exogenous value of Δ. For simplicity, we set $\Delta = 0.5$, such that the economy is split into two perfectly symmetric sectors. We define intermediate goods C and D as an aggregation of intermediate inputs $y_t^{C,D}(i)$ produced by a continuum of monopolistically

[6] A unitary elasticity of substitution yields the classic Cobb-Douglas function; instead, we choose a value $\epsilon > 1$, which makes the two inputs imperfect substitutes.

competitive firms indexed with i, with constant elasticity of substitution:

$$y_t^j = \left(\int_0^1 \left(y_t^j(i) \right)^{\frac{\xi_t^j - 1}{\xi_t^j}} di \right)^{\frac{\xi_t^j}{\xi_t^j - 1}}. \tag{23}$$

where ξ_t^j is the sector-specific elasticity of substitution parameter and y_t^j is the Dixit-Stiglitz aggregator. ξ_t^j is also a stochastic process that describes a markup shock to the inflation equation, as defined by Smets and Wouters (2003). It evolves following an AR(1) process:

$$\ln \xi_t^j = \rho_\xi \ln \xi_{t-1}^j + (1 - \rho_\xi) \ln \overline{\xi^j} - e_{\xi,t}^j, \tag{24}$$

with $0 < \rho_\xi < 1$ and $e_{\xi,t}^j \sim i.i.d.N\left(0, \sigma_{j,M}^2\right)$, where $\overline{\xi}$ is the steady-state level of the elasticity parameter and $e_{\xi,t}$ is an exogenous markup shock. The intermediate input demand is

$$y_t^j(i) = \Delta \left(\frac{P_t^j(i)}{P_t^j} \right)^{-\xi_t^j} y_t^j. \tag{25}$$

By substituting the values of $y_t^{C,D}$ from equations (20) and (21) into the intermediate input demand function (equation (25)), we obtain

$$y_t^C(i) = \left(\frac{P_t^C(i)}{P_t^C} \right)^{-\xi_t^C} y_t(1 - \Delta) \left(\frac{P_t^C}{p_t} \right)^{-\epsilon}, \tag{26}$$

$$y_t^D(i) = \left(\frac{P_t^D(i)}{P_t^D} \right)^{-\xi_t^D} y_t \Delta \left(\frac{P_t^D}{p_t} \right)^{-\epsilon}. \tag{27}$$

The zero-profit condition for intermediate-good firms then requires the aggregate intermediate price index to be

$$P_t^j = \left(\int_0^1 P_t(i)^{1 - \xi_t^j} di \right)^{\frac{1}{1 - \xi_t^j}}. \tag{28}$$

2.2.2 Clean and dirty intermediate-input firms

Clean and dirty firms employ sector-specific labor and capital in their production; additionally, the TFP in both industries is negatively affected by pollution. Dirty-good firms differ from clean-good producers in that they face a trade off between paying a tax on their negative externalities and using only a fraction of their production to abate their polluting emissions.

The production function of the two sectors is

$$y_t^j(i) = A_t^j \left(k_{t-1}^j(i)\right)^{\alpha} \left(l_t^j(i)\right)^{1-\alpha}, \tag{29}$$

$$\text{and} \quad A_t^j = (1 - D_t(x))\, a_t^j, \tag{30}$$

where A_t^j is the total factor productivity (TFP) net of the damage function D_t, and a_t^j is the industry-specific technology factor; α represents the input share parameters. Technology in both sectors evolves following an AR(1) process:

$$log(a_t^j) = (1 - \rho_a) log(\bar{a}^j) + \rho_a log(a_{t-1}^j) + e_a^j, \tag{31}$$

where $\bar{a}^j$ is the steady-state level of technology and e_a^j is a sector-specific exogenous productivity shock[7].

The linkage between production and the climate is expressed via the damage function and abatement spending. Here, we define the functions that make up the climate module:

$$D_t(x_t) = d_0 + d_1 x_t + d_2 x_t^2, \tag{32}$$

$$x_t = \eta x_{t-1} + e_t + e_t^{ROW}, \tag{33}$$

$$e_t = (1 - \mu_t) h(y_t^D(i)), \quad \text{with} \quad \mu_t \in [0, 1], \tag{34}$$

$$z_t = g(\mu_t) y_t^D(i). \tag{35}$$

The damage D_t is a quadratic function of the pollution stock x_t. The pollution stock is a function of domestic emissions e_t and emissions from the rest of the world e_t^{ROW}; η is a parameter describing the decay rate of atmospheric pollution. Domestic emissions are a function of polluting firms' production and abatement; μ_t is the fraction of emissions abated; and z_t represents the total abatement spending. In addition, we define two auxiliary functions,

$$h\left(y_t^D(i)\right) = \gamma_1 \left(y_t^D(i)\right)^{1-\gamma_2}, \quad \text{with} \quad \gamma_2 < 0 < \gamma_1 \leq 1, \tag{36}$$

$$g(\mu_t) = \theta_1 \mu_t^{\theta_2}, \tag{37}$$

such that industrial emissions are an increasing and convex function of the output.

[7] This is the standard expression for TFP in DSGE models; however, in this paper we abstract from technology shocks.

Nominal price rigidities are modeled again by introducing quadratic adjustment costs ($AC_t(i)$) à la Rotemberg (1983), which intermediate firms pay whenever they adjust their price with respect to the steady-state level of inflation:

$$AC_t^j(i) = \frac{\phi_p}{2}\left(\frac{P_t^j(i)}{P_{t-1}^j(i)} - \bar{\pi}\right)^2 y_t^j P_t^j. \tag{38}$$

In both sectors, firms pay the price adjustment cost and maximize their profits, expressed in terms of the CPI level p_t. The profit function of dirty-good producers is as follows:

$$\Pi_t^D(i) = \frac{P_t^D(i)}{p_t} y_t^D(i) - \tau_t^E(i) e_t(i) - z_t(i) - w_t^D L_t^D(i) - r_t^{k,D} k_{t-1}^D(i) \frac{AC_t^D(i)}{p_t}, \tag{39}$$

where $\Pi_t^D(i)$ is the dirty-firm profit and τ_t^E is the carbon tax on industrial emissions that polluting firms pay to the central fiscal authority. From the profit maximization problem for polluting firms, we get the following FOCs with respect to k_t^D, l_t^D and μ_t:

$$r_t^{k,D} = mc_t^D A_t^D \alpha \left(k_{t-1}^D\right)^{\alpha-1} (l_t^D)^{1-\alpha}, \tag{40}$$

$$w_t^D = mc_t^D A_t^D (1-\alpha) \left(k_{t-1}^D\right)^{\alpha} (l_t^D)^{-\alpha}, \tag{41}$$

$$\tau_t^E \gamma_1 \left(y_t^D\right)^{-\gamma_2} = \theta_1 \theta_2 \mu_t^{\theta_2 - 1}, \tag{42}$$

where mc_t^D is the sectoral Lagrangian multiplier related to marginal costs[8]. The dirty Phillips curve is

$$\begin{aligned}\pi_t^D\left(\pi_t^D - \bar{\pi}\right) = \beta \mathbb{E}_t \left[\frac{\lambda_{t+1}}{\lambda_t} \frac{p_{t+1}^D y_{t+1}^D}{p_t^D y_t^D} \pi_{t+1}^D \left(\pi_{t+1}^D - \bar{\pi}\right)\right] \\ + \frac{\xi_t^D}{\phi_p}\left[\frac{mc_t^D}{p_t^D} - \frac{\xi_t^D - 1}{\xi_t^D} + \tau_t^E (1-\mu_t)\gamma_1(1-\gamma_2)\frac{\left(y_t^D\right)^{-\gamma_2}}{p_t^D} + \theta_1 \frac{\mu_t^{\theta_2}}{p_t^D}\right],\end{aligned} \tag{43}$$

[8]Since in symmetric equilibrium all firms within the same sector choose the same price, we can drop the index i

where:

$$\begin{aligned}\pi_t^j &= \frac{P_t^j}{P_{t-1}^j} \\ &= \frac{P_t^j}{p_t}\frac{p_t}{p_{t-1}}\frac{p_{t-1}}{P_{t-1}^j} \\ &= \frac{p_t^j}{p_{t-1}^j}\pi_t.\end{aligned} \tag{44}$$

and p_t^D is the relative price of dirty good. Note that since dirty and clean firms incur different costs, they will not set the same price even in equilibrium; hence, we cannot simplify the price variables P_t^D and p_t. Marginal costs depend on the degree of elasticity between intermediate goods within the same sector.

For clean-good producers, the resource constraint is

$$\Pi_t^C(i) = \frac{P_t^C(i)}{p_t} y_t^C(i) - w_t^C L_t^C(i) - r_t^{k,C} k_{t-1}^C(i) - \frac{AC_t^C(i)}{p_t}. \tag{45}$$

As before, the profit maximization problem yields the following FOCs with respect to k_t^C and l_t^C:

$$r_t^{k,C} = mc_t^C A_t^C \alpha \left(k_{t-1}^C\right)^{\alpha-1} (l_t^C)^{1-\alpha}, \tag{46}$$

$$w_t^C = mc_t^C A_t^C (1-\alpha) \left(k_{t-1}^C\right)^{\alpha} (l_t^C)^{-\alpha}. \tag{47}$$

The clean Phillips curve is

$$\pi_t^C \left(\pi_t^C - \bar{\pi}\right) = \beta \mathbb{E}_t \left[\frac{\lambda_{t+1}}{\lambda_t}\frac{p_{t+1}^C y_{t+1}^C}{p_t^C y_t^C}\pi_{t+1}^C \left(\pi_{t+1}^C - \bar{\pi}\right)\right] + \frac{\xi_t^C}{\phi_p}\left[\frac{mc_t^C}{p_t^C} - \frac{\xi_t^C - 1}{\xi_t^C}\right]. \tag{48}$$

2.3 Public sector and market clearing

Government consumption g_t is financed by both income t_t and emissions taxes. The public sector budget constraint with a zero net supply of bonds is

$$g_t = t_t + \tau_t^E e_t + b_t - (1 + r_{t-1})\, b_{t-1}, \tag{49}$$

$$\text{where} \quad t_t = \omega y_t. \tag{50}$$

t_t is defined as a fixed share of the income y_t, which is determined by the parameter ω. The good-market clearing condition implies that

$$y_t = c_t + g_t + i_t + z_t + \sum_j \left[\frac{\phi_p}{2} \left(\pi_t^j - \pi_t \right)^2 y_t^j p_t^j + \frac{\phi_W}{2} \left(\pi_{W,t}^j - \bar{\pi} \right)^2 l_t^j w_t^j \right]. \tag{51}$$

2.3.1 Standard monetary policy rule

The monetary authority follows a feedback rule of the Taylor rule class[9] to set the nominal interest rate in response to changes in the inflation rate. The standard TR is

$$\frac{r_t}{\bar{r}} = \left(\frac{r_{t-1}}{\bar{r}} \right)^{\rho_m} \left[\left(\frac{\pi_t}{\bar{\pi}} \right)^{\phi_\pi} \left(\frac{y_t}{y_t^*} \right)^{\phi_y} \right]^{1-\rho_m}, \tag{52}$$

where $\bar{r}$ and $\bar{\pi}$ represent the corresponding Ramsey steady state of the nominal interest rate and inflation; y^* is the flexible price output[10]; ρ_m denotes the degree of monetary policy inertia. The ratio $\frac{y_t}{y_t^*}$ is the output gap and represents the deviation of output from its flexible counterpart. ϕ_π and ϕ_y denote the response of the interest rate to inflation and output gap variation.

2.3.2 Non-standard monetary policy rule

Given the structure of our model, we propose an alternative monetary policy rule, in which the inflation target is replaced by two targets, which correspond to the relative inflation of the two sectors. So, while maintaining the usual mandate of price stability, the central bank can set its policy rate taking into account the relative changes of P_t^C and P_t^D instead of the general level of the price p_t. By doing so, the coefficients of the monetary policy rule can be set independently. The non-standard TR reads

$$\frac{r_t}{\bar{r}} = \left(\frac{r_{t-1}}{\bar{r}} \right)^{\rho_m} \left[\left(\frac{\pi_t^C}{\bar{\pi}} \right)^{\phi_\pi^C} \left(\frac{\pi_t^D}{\bar{\pi}} \right)^{\phi_\pi^D} \left(\frac{y_t}{y_t^*} \right)^{\phi_y} \right]^{1-\rho_m}, \tag{53}$$

where ϕ_π^C and ϕ_π^D denote the response of the interest rate to the sector-specific inflation variation.

[9] The choice of a monetary policy rule stems from the fact that this type of rule-based policy is readily implementable and easy to communicate to the public. In contrast, a socially optimal policy designed by a Ramsey planner is not directly operational. Schmitt-Grohé and Uribe (2007) have shown that the outcomes of the two approaches, in terms of welfare maximization, can be extremely close.

[10] The output level obtained in the absence of of wage and price nominal rigidities.

2.4 Environmental dynamics and policies

In RBC studies with an environmental production externality (e.g., Heutel (2012)), an optimal carbon tax is derived that can compensate for this externality. But what if an optimal carbon tax is not feasible? In this work, we define three environmental regimes characterized by different policies:

1. *baseline*: the government does not impose any tax on polluting emissions, nor there is the incentive for firms to abate, such that $\tau_t^E = \mu_t = 0$;

2. *tax policy*: the government levies an emissions tax whose value is fixed such that $\tau_t^e = \bar{\tau}_e$ and abatement is constant[11];

3. *cap policy*: the government set a permanent limit on emissions based on a fixed amount of pollution stock, i.e. $e_t = \bar{x}(1-\eta) - e^{row}$; tax and abatement adjust accordingly. This policy is basically a simplified version of the cap-and-trade system implemented within the EU, in which the carbon budget is determined periodically by the relevant authorities and the price of carbon permits (the carbon tax) is determined by market demand.

In all three cases, the emission tax represents the major instrument used to offset the environmental friction. Note that $\bar{\tau}_e$ is predetermined by the government and does not correspond to an efficient Ramsey tax[12]. This means that it may not completely internalize the environmental externality, as it is not optimized to do so. In this work, we focus on an economy with sub-optimal climate policies, such that the intervention of monetary policy, besides being necessary to offset nominal frictions (as in a standard NK-RBC model), can contribute to offsetting the distortion caused by the environmental damage[13].

When no climate policy is set, we shut down the environmental tax and the abatement effort, such that $\tau_t^E = \mu_t = 0$. In doing this, the public sector is not endowed with any

[11] In Heutel (2012) the carbon tax is procyclical.

[12] Many countries, including the U.S., set their carbon tax equal to the social cost of carbon (SCC), that is, the marginal cost to society (e.g., economic or human health costs) associated with the emission of an extra unit (tonne) of greenhouse gas. However, the carbon price is susceptible to political decisions, as the estimation of the SCC is dependent upon the public administration in charge. This explains why we have seen the SCC in the United States drop from $36 under the Obama administration to $7 during the Trump administration.

[13] In a keynote speech at the ILF conference on Clean Banking and Clean Central Banking, the President of the ECB, Christine Lagarde, stated that "Relying on just one solution, or on one party" (in our case, the environmental tax of the fiscal authority) "will not be enough to avoid a climate catastrophe" (Lagarde 2021).

policy instruments to correct the distortion, and dirty firms do not internalize their negative externalities; they keep producing at the same pace regardless of the amount of CO_2 they emit. However, setting the emission tax equal to 0 and keeping the value of the weighting parameter Δ the same affects the steady-state level of the pollution stock, which increases significantly.

2.5 Calibration and steady state

The model is calibrated at a quarterly frequency in the U.S., mainly relying on the parametrization commonly used in the RBC and New Keynesian literature. The parameter ψ is calibrated such that the steady state labor supply is equal to 0.33 (1/3 of a day or 8 hours). Other specific ratios observed in the data are met, like the investment/output ratio (around 25%) and the government expenditure to GDP ratio (ca. 10%). Climate module parameters are updated according to the most recent available data, following Gibson and Heutel (2023) and Carattini et al. (2023). In their work, Gibson and Heutel (2023) employ the updated version of Nordhaus' DICE 2016R2 model to estimate the parameters of the damage, emissions and abatement cost functions. There is an extensive discussion in the literature about the magnitude of the damage. According to some experts, Nordhaus' damage function underestimates the impact of the pollution stock on the economy (Howard and Sterner 2017). In addition, temperature increases due to environmental degradation would affect different parts of the world asymmetrically and would even benefit some countries in the northern part of the globe (Kalkuhl and Wenz 2020). Nevertheless, investigating the nature of the damage function is beyond the scope of our work, so we follow the standard approach from the E-DSGE literature. The magnitude of the shocks is set to standard levels[14]. We set the standard error of the markup shock to -0.18, to induce a (negative) variation of the parameter ξ by one unit. The calibrated parameters are reported in Table 2. For comparison, we report in Table 1 the steady-state values of some variables of interest in E-DSGE models under the three environmental policy scenarios. We fix $\Delta = 0.5$, such that the intermediate goods sector is equally split between clean and dirty firms[15]. In this way, we want to isolate the transmission

[14] One of the objectives of the paper is to show how the asymmetric monetary policy rule performs with respect to the standard TR, rather than finding the exact response parameters. Hence, although estimated shocks would fit reality better, they would not alter the nature of the results.

[15] According to the U.S. Environmental Protection Agency, the economic sectors that contribute the most to greenhouse gas (GHG) emissions in the United States are transportation, utilities, industry and manufacturing, commercial and residential, and agriculture. The sum of the GDP shares of these sectors for 2021

mechanism of monetary policy from the sector composition. The steady-state value of τ_t^E is numerically determined such that the initial amount of carbon stock is equal to ca. 880 $GtCO_2$ (similar to the level of global atmospheric carbon stock over the last decade, according to the update version of Nordhaus' DICE model) and $\Delta = 0.5$[16]. This value is common to all the environmental regimes considered here. For the baseline scenario only, $\bar{x}$ increases to ca. 1120, in order to keep emissions intensity and the parameter Δ constant. The steady-state value of the emission tax is 0 with no environmental policy, and it is 0.0066 otherwise. It is expressed in the arbitrary units of the model. Total environmental tax revenues to GDP amounts to 0.30% (it was around 0.35% in the US in 2022). When no environmental policy is implemented, production and prices of dirty and clean goods are the same in the two sectors, since they are perfectly identical and face the same marginal costs. On the contrary, in the model with a positive emission tax, dirty production is lower and the price of dirty goods is higher than that of clean goods. The comparison of welfare level reveals that, at the steady state, the introduction of a positive carbon tax increases welfare by 3.46%. This is due to the smaller pollution stock in the presence of environmental policies and, consequently, lower environmental damage/GDP ratio. If we exclude climate damage from the framework, the distorting environmental fiscal policies lower welfare in the steady state by 1.27%.

Table 1: Steady-state variables for the E-DSGE model, baseline (no-policy) vs environmental (tax/cap) policy scenario

Variable	Description	Baseline	Tax/Cap Policy
c/y	Consumption to GDP ratio	65.01%	64.73%
i/y	Investment to GDP ratio	24.99%	24.93%
g/y	Government exp. to GDP ratio	10.00%	10.30%
D/y	Damage to GDP ratio	3.41%	1.76%
e	Emissions	0.6536	0.5133
p^D	Dirty good relative price	1.0	1.0026
p^C	Clean good relative price	1.0	0.9974
$mc^{C,D}$	Clean/Dirty marginal cost	0.8333	0.8312
$\tau_e e$	Environmental tax to GDP ratio	0.0%	0.30%
x	Pollution stock	1120.40	880
W	Welfare	-448.72	-433.18
$W\%$	Welfare variation wrt baseline	/	+3.46%
$(W-D)\%$	Welfare variation (excluding D)	/	-1.27%

(data from the U.S. Bureau of Economic Analysis) amounts to 35%.

[16]Here, we follow an approach similar to that taken by Ferrari and Nispi Landi (2023). In their work, the authors fix the initial level of carbon stock and the emission tax such that the value of Δ is 0.35. This value reflects the intermediate input composition in the EU, accounting for the current level of air pollution, which was calculated by Papoutsi et al. (2021).

Table 2: Model parameters

Parameter	Description	Value	Source
β	Discount factor	0.9975	RBC literature
φ_c	Inverse elasticity of intertemporal sub.	2	Carattini et al. (2023)
φ_l	Inverse Frisch elasticity	1	–
ξ_W	CES parameter of labor supply	6	Calibrated
ϕ_W	Wage adjustment cost	59.553	Implied from ξ_W (Ascari and Rossi 2012)
ψ	Disutility of work	9.893	Implied from $\bar{l} = 1/3$
δ	Capital depreciation	0.025	RBC literature
ϕ_i	Investment adjustment cost	10	NK-DSGE literature
ξ	Elasticity of sub. intermediate goods	6	NK-DSGE literature
ϵ	Elasticity of sub. dirty vs clean goods	2	Carattini et al. (2023)
Δ	Weight of dirty good	0.5	Arbitrary
ϕ_p	Price adjustment cost	59.553	Implied from ξ (Ascari and Rossi 2012)
α	Share of capital in production	1/3	RBC literature
d_0	Damage function constant	-0.026	Carattini et al. (2023)
d_1	Damage function linear parameter	3.6613e-5	–
d_2	Damage function quadratic parameter	1.4812e-8	–
θ_1	Abatement cost function coefficient	0.0334	–
θ_2	Abatement cost function exponent	2.6	–
γ_1	Shifter of emissions function	1	Gibson and Heutel (2023)
γ_2	Emissions elasticity	0.4	–
η	Pollution decay rate	0.9965	Allen et al. (2018)
$\phi_\pi^{C,D}$	Mon. pol. param. clean/dirty inflation	1.5	NK-DSGE literature
ϕ_π	Mon. pol. param. inflation	1.5	–
ϕ_y	Mon. pol. param. output gap	0.5	–
ρ_a	Persistence of TFP/markup shock	0.95	–
ρ_ξ	Persistence of markup shock	0.95	–
ρ_g	Public spending persistence	0.8	–
ρ_m	Mon. pol. inertia	0.2	–
$\sigma_M^{C,D}$	SD of sector-specific markup shock	0.18	Calibrated

3 Impulse responses in the E-DSGE model

We simulate our E-DSGE model to track the dynamic response of the endogenous variables to temporary negative sector-specific markup shocks[17]. In what follows, first, we simulate the optimal Ramsey monetary policy in different environmental policy scenarios; then, we restrict our attention to a single environmental policy (tax policy) and simulate the model under different monetary regimes. The same simulation is also conducted with alternative environmental policies, and the results are displayed in Appendix E. The model is solved by computing the first-order Taylor approximation around the deterministic steady state of the linearized model, using the perturbation method. The model is solved using the MATLAB pre-processor Dynare (Adjemian et al. 2024).

3.1 Ramsey Monetary Policy with Alternative Environmental Regimes

Unlike the TR, with Ramsey optimal policy the social planner maximizes the household's utility by choosing the value of a specific policy instrument (in this case r_t), under the equilibrium conditions of the decentralized economy and contingent on the observation of the nature of the shock, but not (necessarily) on the specific variables showing up in the policy rule. It is common in the literature to use simple rules because they are closer to the actual policies of central banks. The use of a Ramsey policy in this section is for illustrative purposes, to show how the optimal response of a social planner changes depending on the shock (the affected sector) and the environmental fiscal policy. In figures 2a and 2b we observe the impulse response of some aggregate variables to a clean and a dirty cost-push shock, respectively. All the three environmental policy regimes are represented here. The central bank interest rate is the key variable (shown in details in figure 3). We observe that the response of r_t on impact varies across environmental regimes and between the two shocks. Regardless of the emissions policy, the interest rate adjustment is more pronounced following an inflationary shock in the clean sector, with the largest decrease – more than double – occurring under an emissions cap. Comparing environmental regimes, a clean shock leads to the most significant interest rate change under an emissions cap, whereas a dirty shock results in the largest adjustment under a tax policy.

[17] A negative shock implies that the elasticity of substitution ξ_t^j between intermediate inputs, within each sector, goes down, meaning the overall markup is higher and increases inflation.

These differing monetary policy responses also have significant implications for the sector not directly affected by the shock. After a clean shock, inflation in the dirty sector rises sharply, particularly under a cap policy. This occurs because the clean shock raises inflation in the clean sector, making clean goods relatively more expensive and shifting demand toward dirty goods. However, under a cap policy, emissions cannot increase, which drives up the marginal cost of producing dirty goods and amplifies their price increase. Conversely, after a dirty shock, inflation in the clean sector is more pronounced under tax or no-policy regimes compared to a cap policy. In this case, the absence of rigid emission constraints allows for a more moderate interest rate response, which, in turn, leads to greater spillover of inflationary pressures from the dirty to the clean sector. Under a cap policy, the tighter monetary stance contains these pressures, resulting in lower inflation in the clean sector.

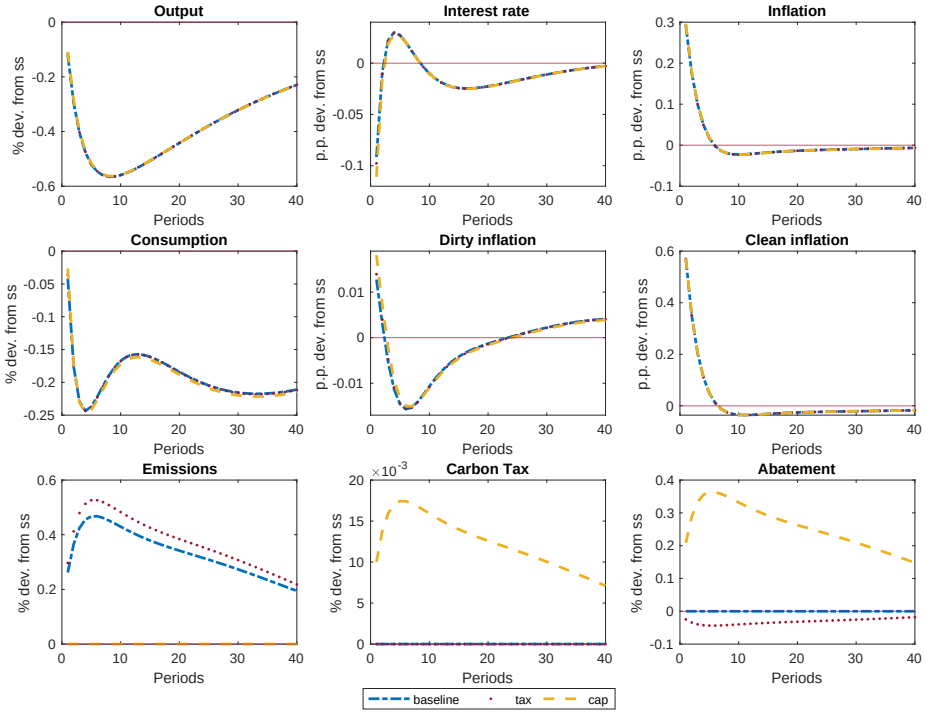
The effects of these dynamics are also evident in consumption. While consumption initially declines more sharply when the dirty sector is affected by the cost shock, it recovers and is slightly higher a decade later, particularly under a cap policy. This is driven by the tighter monetary response under the cap regime, which helps stabilize inflation and output over the long term, thereby supporting higher levels of consumption.

3.2 Tax Policy with Alternative Monetary policy rules

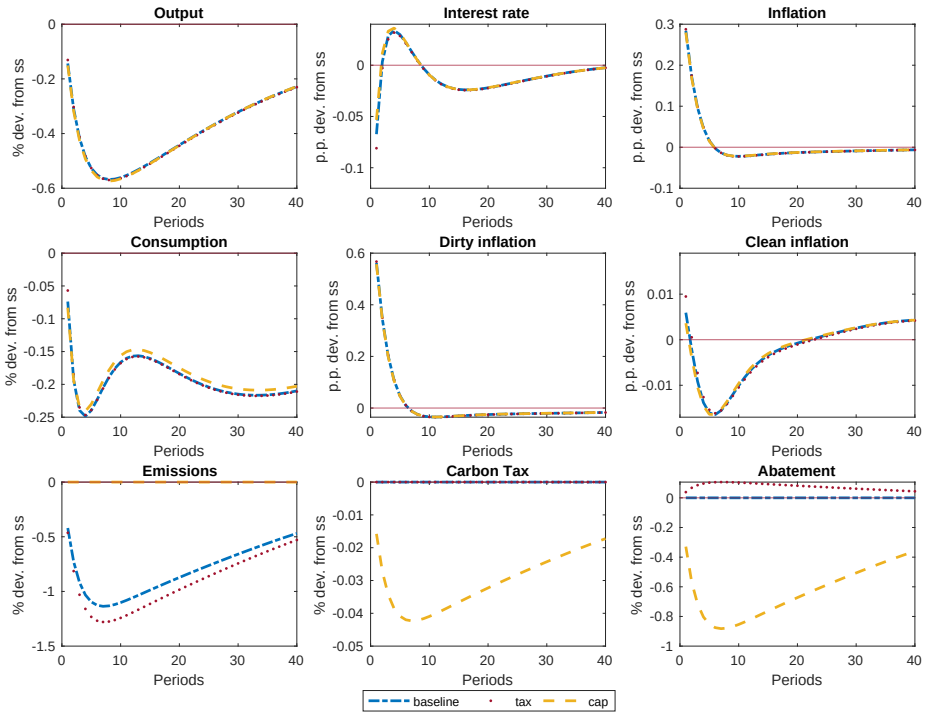
We distinguish four scenarios: in the first one, the central bank sets its monetary policy according to a standard TR with an aggregate inflation target. In the next two scenarios, the monetary policy rule employed is the one with sectoral inflation targets, as described in equation (53): these two scenarios differ in that in one regime, the central bank targets only clean inflation ($\phi_\pi^C = 1.5$, $\phi_\pi^D = 0$), while in the other, it targets only dirty inflation ($\phi_\pi^C = 0$, $\phi_\pi^D = 1.5$). For simplicity, we will call these rules "clean rule" and "dirty rule", respectively. In the last scenario, the Ramsey planner chooses the value of the monetary policy instrument. The IRFs are plotted in figures 4a and 4b.

While the interest rate declines sharply when the policy rate is chosen by the Ramsey planner (as detailed in the previous section), the initial reaction is positive or null when a monetary policy rule is employed. In all scenarios, the interest rate gap nearly closes by the end of the simulation period, except when a non-standard monetary policy rule is applied. Depending on the sector affected by the shock, two opposite outcomes emerge. Under a dirty shock, the dirty rule keeps the interest rate persistently above its steady state. The opposite

Figure 2: Impulse responses across shocks and environmental regimes with a Ramsey Policy



(a) Clean markup shock



(b) Dirty markup shock

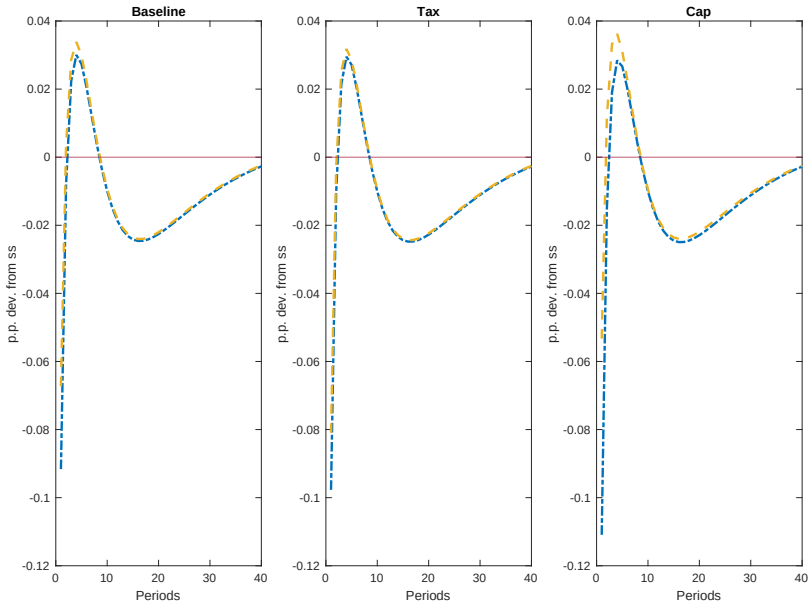


Figure 3: Interest rate across shocks and environmental regimes with a Ramsey Policy

occurs when the shock hits the clean sector.

Regarding the dynamics of output and emissions, the Ramsey policy minimizes output reductions compared to other monetary policy rules due to its welfare-maximizing design. However, as welfare depends on consumption, higher demand for goods implies increased production and consequently higher emissions. Thus, from an emissions control perspective, the optimal Ramsey policy may not be the most effective. In that sense, a non-standard Taylor Rule (TR) outperforms the Ramsey approach. The most significant output contraction occurs when the monetary policy rule specifically targets the inflation of the sector affected by the shock (e.g., a clean rule under a clean markup shock or a dirty rule under a dirty shock). This leads to a sustained positive interest rate, cooling the economy more effectively and reducing emissions.

In detail, a clean markup shock increases the price of clean inputs, reducing their demand while boosting demand for cheaper dirty goods, regardless of the monetary policy. This raises emissions as dirty sector production expands, while leading to increased dirty inflation. Given the imperfect substitutability between the two goods, a clean sector supply shock translates into a demand shock for the dirty sector. Except Ramsey's, the largest emissions increase occurs when a dirty rule is applied. In this scenario, the central bank's moderate reaction (owing to negative dirty inflation) results in weak interest rates, initially mitigating output contraction but sustaining higher emissions and associated damage. When the shock originates in the dirty sector, differences in emissions across monetary regimes are less pro-

nounced. Targeting dirty inflation causes a persistent interest rate rise, reducing production and GHG emissions.

Overall, monetary policy plays a crucial role in shaping sectoral markup shock dynamics. Targeting inflation in the affected sector leads to stronger output and emissions reductions but compromises welfare. Instead, since Ramsey optimal policy maximizes welfare, it may conflict with emissions reduction goals. As discussed in the next section, although emissions are included in the welfare calculation through their impact on firm productivity (via the damage function), welfare gains from lower emissions are offset by the higher production costs and lower income (due to the carbon taxation). This trade-off highlights the misalignment between welfare maximization and emissions minimization objectives.

4 Optimal monetary policy rule

Following the approach proposed by Schmitt-Grohé and Uribe (2007), we compute the optimal monetary policy rule (equations (52) and (53)) that maximizes the expected welfare of a representative household, conditional on it being in the steady state[18]. In the first part (section 4.1), we analyze the case in which a clean and a dirty (independent) markup shock occur simultaneously[19]. In the second part (section 4.2), we will see what happens to the optimized TR parameters when the shock hits only one of the two sectors. In section C of the appendix, we perform a sensitivity analysis to assess the impact of key parameters on the optimal TR.

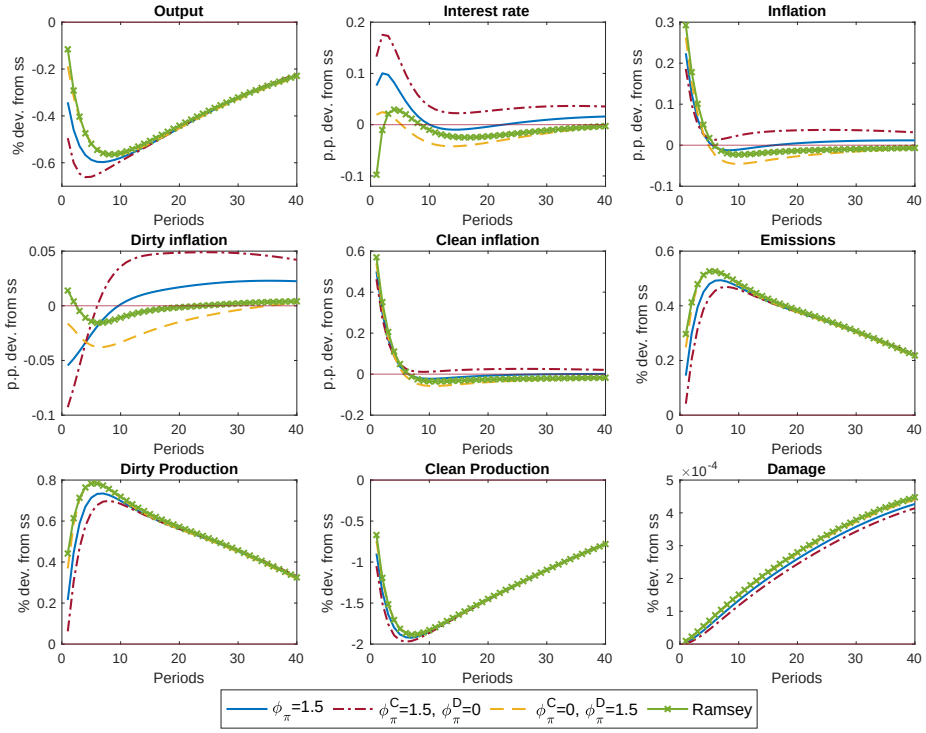
4.1 Symmetric shocks

In the symmetric shock case, both sectors are hit by uncorrelated cost-push shocks occurring simultaneously. We optimize the parameters of both standard and non-standard TRs against welfare maximization, and compare the results under the three environmental policy

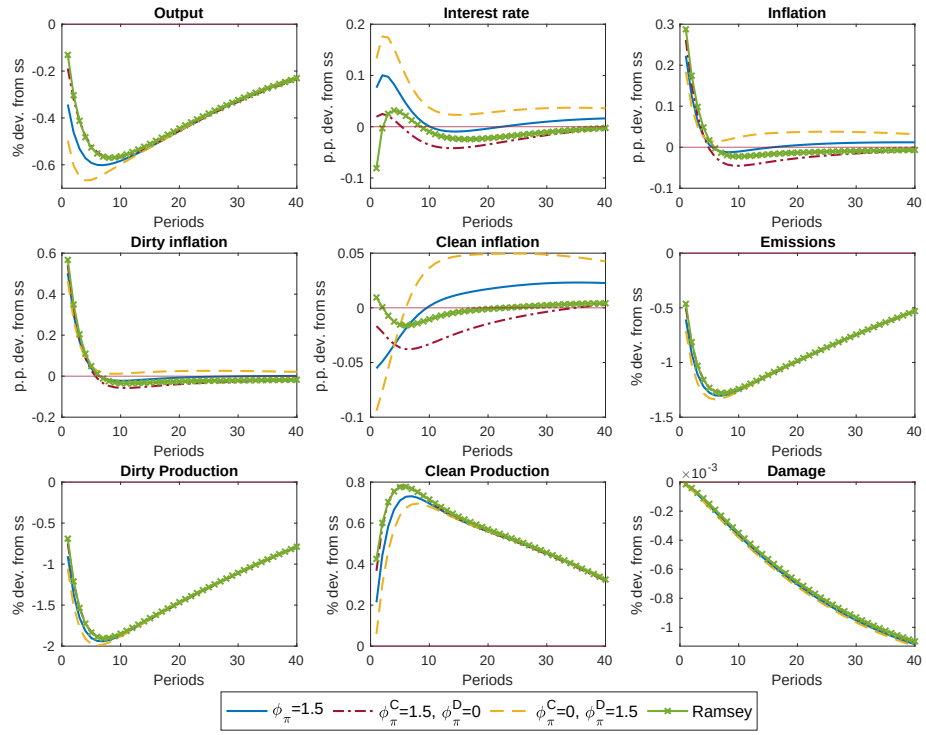
[18]For the simulations, we derive a second-order Taylor approximation to the policy function of the model around its deterministic steady state, using the perturbation method. The model is again solved with Dynare. For numerical optimization from here on, we employ a finite element grid with steps of 0.1, and 3 as the upper bound of the policy rule coefficients, as in Schmitt-Grohé and Uribe (2007).

[19]As emphasized earlier, the inflation target for both sectors is the same (which is also the aggregate inflation target). It is determined by the equilibrium conditions of the model, such that in the steady state prices do not vary (zero trend inflation). Our objective is to observe how the optimal coefficients of the monetary policy rule for each sector vary in response to fluctuations in the level of sectoral inflation. Setting a different inflation target for each sector would not change our experiment, since the coefficients would still be determined by the percentage change in the price level (which would not change) and not by the starting point.

Figure 4: Impulse responses under different monetary policy rules



(a) Clean markup shock



(b) Dirty markup shock

scenarios. In figure 5 we plot the trade-off between inflation and output gap volatility.

In each panel, the red dots represents pairs of standard deviation (SD) values of inflation and output gap for all combinations of output and inflation-gap coefficients[20]. The blue dots indicate all pairs of SDs corresponding to possible combinations of the non-standard TR.

The area below the red dots indicates that there are combinations of the non-standard TR parameters for which both inflation and output gap volatility are reduced. The non-standard TR performs better than the standard one, achieving a reduction in output gap volatility of around 1.78% when comparing their lowest achievable values across all three environmental scenarios (slightly lower with a cap policy). In terms of inflation volatility, the reduction ranges from 4.83% (tax policy) to 5.11% (cap policy) when comparing the lowest inflation volatility achieved with the standard TR to the minimum value achievable with the non-standard TR at the same level of output gap reduction[21]. Alternatively, if we compare the lowest inflation volatility achieved with the standard TR to the minimum value achievable with the non-standard TR, the reduction ranges from 22.85% (baseline) to 22.89% (tax policy). However, it should be noted that the non-standard TR allows for a broader range of parameter configurations, which may contribute to this improved performance.

In figure 6 we show the trade-off between inflation and emissions volatility[22]. In the baseline and tax policy scenario, the emission volatility is reduced by 1.71% in the baseline case and by 1.53% in the case of a fixed carbon tax, when monetary policy targets sectoral inflation separately. With a carbon tax policy, the volatility of emissions is zero (by construction), regardless of the TR employed. Thus, in general, a non-standard TR may be superior in terms of inflation, output gap, and emissions volatility when welfare is maximized, even though there are no significant differences between existing environmental fiscal policies[23].

But what are the optimal parameter values for the two TRs across the three environmental

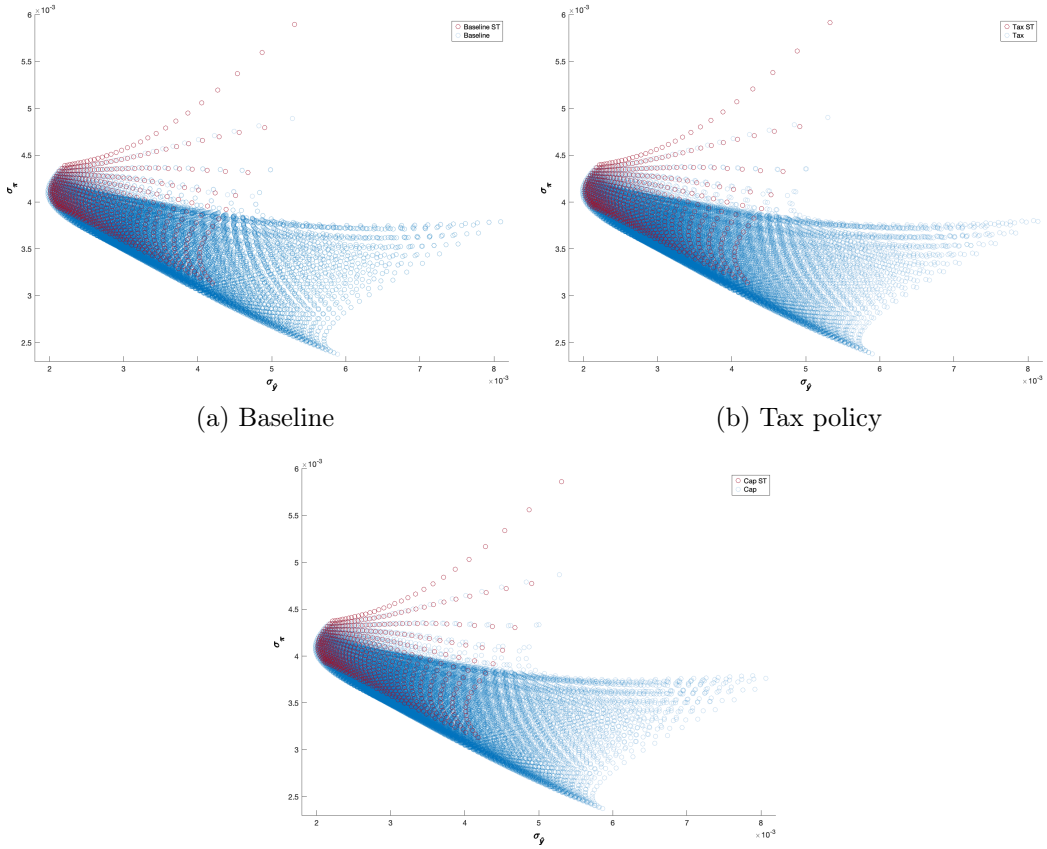
[20]The optimization exercise is performed by testing all the finite combinations of the inflation and output gap coefficients, which we recall have an upper limit of 3, so as to obtain the corresponding values of inflation, output, and emission volatility.

[21]To calculate it, simply take the lowest value of std_π with standard TR (in red) and calculate the corresponding (vertical) distance with the lowest point of std_π with non-standard TR (in blue).

[22]Although emissions minimization is not an explicit goal of central banks, examining the trade-off between inflation and emissions volatility is justified for two reasons. First, as mentioned in the introduction, central banks have a clear interest in climate issues, as it can hinder economic and financial stability. Second, emissions are a by-product of output, which aligns with the traditional trade-off between inflation and output gap that central banks consider when formulating their policy.

[23]It is important to note that different environmental regimes involve different dynamics in emissions and outputs. However, the deviation of actual output from flexible output (the output gap) does not vary. This is because y_t^* represents the optimal output (at flexible prices and wages), but not efficient (monopoly power is not cleared), and is affected by the same cost-push shocks as y_t.

Figure 5: Inflation – Output Gap Volatility Trade-Off, Standard vs Non-standard Taylor Rule



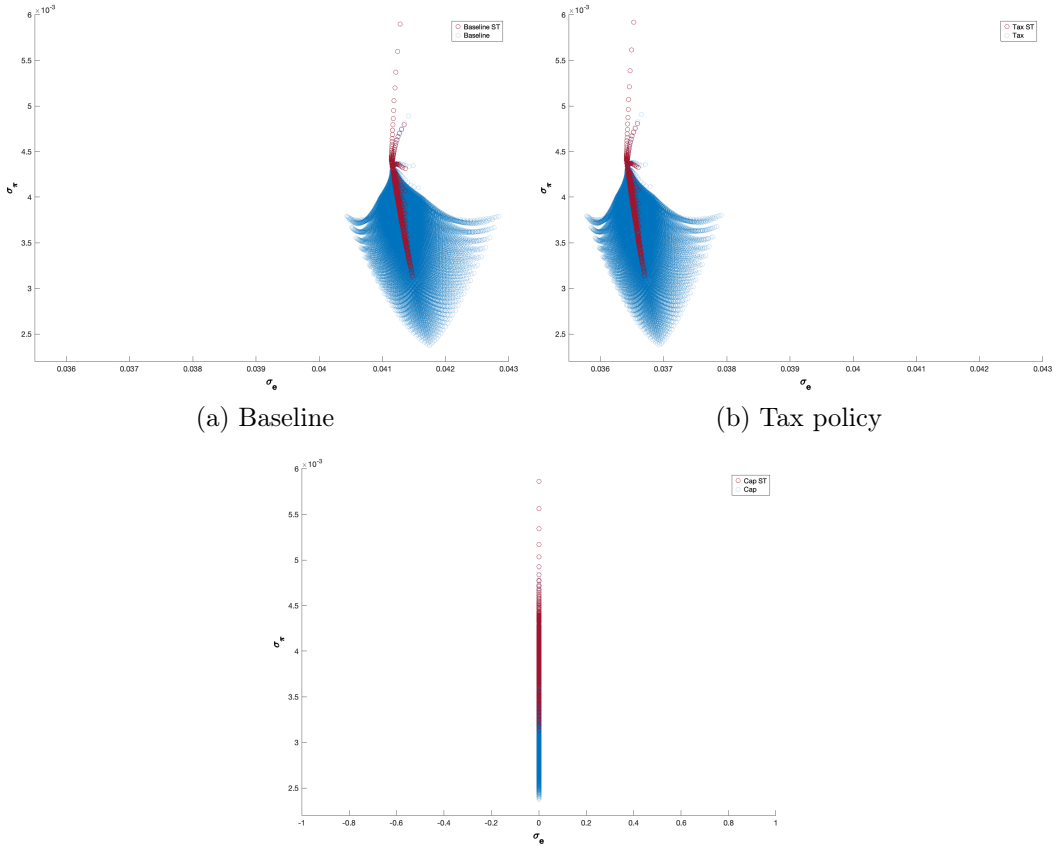
(a) Baseline

(b) Tax policy

(c) Cap policy

Note: trade off between inflation and output gap volatility, standard (red) vs non-standard (blues) TR; different environmental policy scenario with simultaneous markup shocks.

Figure 6: Inflation Gap – Emissions Volatility Trade-Off, Standard vs Non-standard Taylor Rule



(a) Baseline

(b) Tax policy

(c) Cap policy

Note: trade off between inflation gap and emissions volatility, standard (red) vs non-standard (blues) TR; different environmental policy scenarios with simultaneous markup shocks.

policy regimes? Table 3 reports these coefficients. Notably, the optimal parameters of the standard TR are almost unchanged across the different environmental regimes. However, for the non-standard TR, the coefficients exhibit asymmetry especially under the cap policy regime. Specifically, the output gap parameter (ϕ_y) is lower (1.1 versus 1.25), and the sum of the inflation parameters (ϕ_π^C and ϕ_π^D) is reduced (1.9 versus 1.95), with the clean inflation parameter (ϕ_π^C) being the smaller of the two.

This difference arises due to the stricter emission constraint under the cap policy. By limiting the total emissions, the cap policy reduces the room for economic adjustments in response to shocks affecting the entire economy (stemming from simultaneous sectoral shocks). A lower ϕ_y reflects the reduced sensitivity of the interest rate to the output gap, as the cap already imposes a significant restriction on output. Similarly, the lower weight on clean inflation (ϕ_π^C) may stem from the fact that the cap policy amplifies the relative price increases in the dirty sector, reducing the effectiveness of targeting clean inflation for stabilizing the economy.

In terms of welfare, the highest level is achieved with an optimal non-standard TR under a cap policy[24]. Nonetheless, when comparing the optimal standard TR with the optimal non-standard TR within each environmental policy regime, the percentage variation in welfare remains negligible. However, the percentage reduction in emissions is more pronounced, highlighting the environmental benefit of sector-specific inflation targeting.

Table 3: Optimal monetary policy rule coefficients with symmetric shocks

Env. Policy	Standard TR		Non-standard TR			W Δ%	E Δ%
	ϕ_π	ϕ_y	ϕ_π^C	ϕ_π^D	ϕ_y		
Baseline	1.95	1.25	0.95	1.0	1.25	-6.64e-05	-0.0013
Tax policy	1.95	1.2	0.95	1.0	1.2	-6.58e-05	-0.0015
Cap policy	1.95	1.2	0.9	1.05	1.2	5.98e-05	0

First column specifies the environmental regime. From second to sixth column, optimized parameter of the standard and non-standard TR are displayed. Last two columns on the right represents welfare (W) and emissions (E) percentage variation (Δ%). The welfare variation quantifies the benefit of adopting the two-sector inflation-targeting rule compared to the standard TR. This variation is expressed in terms of consumption equivalent (CE), which represents the percentage increase in consumption that would be required in the baseline scenario to achieve the same level of welfare as in the alternative (non-standard) scenario. A positive value indicates that welfare (emissions level) is higher when a non-standard TR is implemented.

[24]Heat-map chart showing the level of welfare for each triplet of policy parameter values is available in Appendix D, see figure D.7

4.2 Asymmetric shocks

So far, we have analyzed the optimal response of monetary policy, through the lens of a monetary policy rule, when both (intermediate) economic sectors are hit by an inflationary markup shock. However, it is plausible to expect that not all sectors are affected in the same way or at the same time. We have already seen in 3.1 that the optimal interest rate response changes with the sector affected by the shock and the environmental policy in place. This was evident when monetary policy was decided by a Ramsey social planner. Now, as in the previous section, we analyze what happens to the parameters of the TRs when the shock hits the two intermediate sectors asymmetrically.

The analysis is conducted as follows: first, a sectoral shock is simulated (i.e., affecting only one of the two sectors) and the optimal two-inflation-target TR coefficients are calculated. The procedure is repeated for each sector and each environmental policy scenario. Then, we perform the same procedure by replacing the previous monetary policy rule with a standard TR with a general inflation target. Finally, we compare the levels of welfare and emission resulting from the implementation of the two optimized rules (see Appendix A.3 for a full derivation). Households' welfare variation is expressed as a percentage of the steady-state consumption variation, or consumption equivalent (CE). A positive value indicates that the implementation of a non-standard TR results in higher welfare (or emissions). The resulting optimized parameters, welfare and emission cost are displayed in Table 4. Here, we focus on the extreme cases in which the shock hits only one of the two intermediate sectors at a time.

When a standard TR is employed, the monetary authority reacts positively to both inflation and output gap. The value of the output gap coefficient varies significantly between environmental scenarios and (sectoral) shocks, while the inflation gap parameter is more stable. Even in the baseline scenario, coefficients are not the same regardless of the sector affected by the shock. This suggests that the presence of the environmental externality (damage) alone can still induce an asymmetric reaction of the monetary policy rule to sectoral inflation. With a tax or a cap policy scenario, and thus when a positive emission tax is introduced in the model, the asymmetric reaction to sectoral shocks is even stronger (the strongest asymmetry is observed with a cap policy regime). In addition to the environmental damage, the presence of an emission tax distorts also the relative prices of the two goods (clean and dirty) and consequently the level of production, calling for a different reaction of

Table 4: Optimal monetary policy rule coefficients with asymmetric shocks

Env. Policy	Shock	Standard TR		Non-standard TR			W Δ%	E Δ%
		ϕ_π	ϕ_y	ϕ_π^C	ϕ_π^D	ϕ_y		
Baseline	C-Markup	1.95	1.3	0	2.65	0	0.0466	0.0947
	D-Markup	1.95	1.2	2.75	0	0	0.0429	0.1061
Tax policy	C-Markup	1.95	1.3	0	2.65	0	0.0466	0.0947
	D-Markup	1.95	1.15	2.7	0	0	0.0445	0.1289
Cap policy	C-Markup	2.0	1.45	0	2.6	0	0.0473	0
	D-Markup	1.9	1.0	2.85	0	0	0.0404	0

First column specifies the environmental regime. The letters C (clean) and D (dirty) in the shock column indicate in which sector the shock occurs. From third to seventh column, optimized parameter of the standard and non-standard TR are displayed. Last two columns on the right represents welfare (W) and emissions (E) percentage variation (Δ%). The welfare variation quantifies the benefit of adopting the two-sector inflation-targeting rule compared to the standard TR. This variation is expressed in terms of consumption equivalent (CE), which represents the percentage increase in consumption that would be required in the baseline scenario to achieve the same level of welfare as in the alternative (non-standard) scenario. A positive value indicates that welfare (emissions level) is higher when a non-standard TR is implemented.

monetary policy depending on the sector affected by the shock.

The stronger asymmetry observed in the output gap coefficient is mainly due to the nature of the sectoral shocks and their impact on output, together with the design of the standard TR. A clean shock increases the price of clean goods, making them relatively more expensive and shifting demand towards dirty goods. This leads to significant distortions in production levels between the two sectors, amplified by environmental policies. For example, a cap policy introduces a binding limit on emissions, raising the marginal cost of producing dirty goods and increasing the output distortion. Similarly, a tax policy increases production costs in the dirty sector, further distorting relative output levels. Instead, aggregate inflation is less directly affected by these dynamics. As a result, the monetary authority responds more strongly to stabilize the output gap, leading to the observed asymmetry in the reaction coefficients.

When a non-standard TR is implemented, the reaction to the output gap is muted across all scenarios. The central bank instead reacts solely to the inflation of the sector not directly affected by the shock. For instance, when a clean markup shock occurs, only the dirty inflation parameter (ϕ_π^D) differs from zero. The intuition behind this is as follows: a clean markup shock increases inflation and reduces output in the clean sector. In response, final-good firms demand more dirty intermediate goods, driving up dirty sector inflation. This dynamic creates a supply shock for the clean sector but a demand shock for the dirty sector.

Since monetary policy generally responds more strongly to demand-driven rather than supply-driven inflation, the optimal coefficient on dirty inflation is higher.

Similarly, in the case of a dirty markup shock, only the clean inflation parameter (ϕ_π^C) differs from zero. The coefficients are symmetric with respect to the sector in which the shock originates, except under the cap policy scenario, where the coefficient for a dirty shock is higher. This occurs because the cap policy imposes a rigid emission limit, raising the marginal cost of producing dirty goods and amplifying the distortion in relative prices and output levels.

By allowing the monetary policy rule to differentiate between sectors based on the origin of the shock, the reaction to the output gap can be effectively neutralized. This approach stabilizes the distortions caused by sectoral shocks more efficiently and leads to an overall improvement in welfare compared to a standard TR (see column 8 of table 4). The welfare improvement arises from the ability of the non-standard TR to target sector-specific dynamics, particularly price distortions and production imbalances caused by environmental policies and damage.

We now evaluate the percentage variation of emissions and welfare from the non-standard to the standard optimal TRs. The non-standard TR consistently achieves higher welfare levels compared to the standard TR, with welfare gains ranging between 0.0404 and 0.0473% in terms of consumption equivalent. Although the size of these gains depends strongly on the size of the shock, the positive sign shows how the mechanism of a two-sector TR can improve welfare by allowing the central bank to discriminate between clean and dirty inflation.

However, the effects on emissions vary. Contrary to the symmetric shock case, the non-standard TR leads to higher emissions compared to the standard TR in all scenarios except under the cap policy, where emissions remain constant by design. The increase in emissions ranges between 0.0947 and 0.1289%, reflecting the trade-off between higher production (necessary for welfare gains) and environmental outcomes. Intuitively, achieving higher welfare requires higher consumption, which in turn demands an expansion in production. Since the two intermediate goods are imperfect substitutes, this expansion involves both clean and dirty inputs, leading to an overall rise in emissions.

In summary, while the non-standard TR enhances welfare across all regimes and maximizes the potential of monetary policy, its implementation – apart from the cap policy scenario – results in a modest increase in emissions due to higher production levels.

5 Conclusions

The relevance of climate change for economic stability has prompted all actors, including central banks, to review their policies concerning growth and business cycle stabilization. In this paper, we have investigated the interaction between monetary policy and environmental fiscal policies in the context of a two-sector E-DSGE model.

Our results highlight that fiscal environmental policies, such as carbon taxes and emission caps, alter the optimal conduct of monetary policy. In our Ramsey benchmark, we observe that these policies introduce relative price distortions that affect the optimal calibration of monetary policy. Building on this insight, we compare a standard Taylor rule-targeting aggregate inflation-with a non-standard rule that distinguishes between clean and dirty inflation. We find that the non-standard rule improves output, inflation and emissions stabilization by responding asymmetrically to clean and dirty inflation (being closer to an optimal Ramsey policy). The asymmetry arises in particular from the distortionary fiscal policies, while environmental damage itself plays a minor role (although not negligible). While environmental damage depends on the long-run pollution stock, monetary policy operates as a short-run tool for business-cycle stabilization, limiting its influence on long-term environmental dynamics. Thus, improvements are due to the ability to mitigate distortions caused by environmental policies.

Instead, welfare gains are almost exclusively observed in scenarios where shocks affect only one sector (clean or dirty), rather than when both sectors are affected simultaneously. In particular, when a fixed cap on emissions is applied, the non-standard rule provides greater welfare without increasing emissions, while reducing fluctuations in prices and output.

Although our analysis focuses on environmental fiscal policies, similar distortions could arise from other fiscal measures targeting specific sectors, potentially amplifying asymmetries in monetary policy responses. Further studies along these lines would be desirable. Future research should also study how heterogeneous agents influence the transmission of monetary policy, the welfare-emissions trade-off, and the coordination between monetary and fiscal policies during transitions. Exploring the role of redistributive fiscal policies and green innovations in this framework could further enhance our understanding.

While many countries remain cautious about adopting strong environmental policies, cap policies have been widely implemented in Europe and some U.S. states, proving effective in

reducing emissions. An asymmetric monetary policy rule that differentiates between price changes in distinct sectors could thus offer additional benefits in stabilizing inflation and output without compromising environmental objectives. As Harold James has noted, "Not all price increases are the same, and some are desirable" (James 2021). Although the practical implementation of such a rule depends on the central bank's ability to accurately identify the source of price changes, our findings suggest that a non-standard monetary policy would provide additional support to stabilize inflation, output and emissions within the business cycle. This responds to the need for central banks to integrate the effects of climate change and environmental policies into monetary policy, thereby reinforcing their commitment to economic stability in an evolving environment.

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Appendix

A E-DSGE Model Derivations

A.1 Households

The Lagrangian function for the household utility maximization problem is

$$\begin{aligned}\mathcal{L} = \mathbb{E}_t \sum_{t=0}^{\infty} \beta^t \Bigg\{ & \left(\frac{c_t^{1-\varphi_c} - 1}{1-\varphi_c} - \psi \frac{\left[\left((l_t^D) + (l_t^C) \right) \right]^{1+\varphi_t}}{1+\varphi_t} \right) \\ & + \lambda_t \left[b_{t-1} \frac{r_{t-1}}{\pi_t} + k_{t-1} r_t^k + (w_t^C l_t^C + w_t^D l_t^D) - t_t + \mathcal{T}_t - b_t - c_t - i_t \right] + \\ & + q_t \left[(1-\delta) k_{t-1} + i_t \left[1 - \frac{\phi_i}{2} \left(\frac{i_t}{i_{t-1}} - 1 \right)^2 \right] - k_t \right] \Bigg\}, \end{aligned} \tag{A.1}$$

A.2 Intermediate Firms: Dirty and Clean Phillips curves

From the Lagrangian function of a dirty sector

$$\begin{aligned}\mathcal{L} = & \mathbb{E}_t \sum_{t=0}^{\infty} \beta^t \lambda_t \Bigg\{ \frac{P_t^D(i)}{p_t} y_t \Delta \left(\frac{P_t^D}{p_t} \right)^{-\epsilon} \left(\frac{P_t^D(i)}{P_t^D} \right)^{-\xi_t^D} + \\ & - \tau_t^E(i) \left(1 - \mu_t(i) \right) \gamma_1 \left[y_t \Delta \left(\frac{P_t^D}{p_t} \right)^{-\epsilon} \left(\frac{P_t^D(i)}{P_t^D} \right)^{-\xi_t^D} \right]^{1-\gamma_2} + \\ & - \theta_1 \mu_t(i)^{\theta_2} \left[y_t \Delta \left(\frac{P_t^D}{p_t} \right)^{-\epsilon} \left(\frac{P_t^D(i)}{P_t^D} \right)^{-\xi_t^D} \right] + \\ & - w_t^D l_t^D(i) - r_t^{k,D} k_{t-1}^D(i) - \frac{\phi_p}{2} \left(\frac{P_t^D(i)}{P_{t-1}^D(i)} - \bar{\pi} \right)^2 \frac{y_t^D(i) P_t^D(i)}{p_t} + \\ & + mc_t^D(i) \left[A_t^D(i) \left(k_{t-1}^D(i) \right)^{\alpha} \left(l_t^D(i) \right)^{1-\alpha} - y_t \Delta \left(\frac{P_t^D}{p_t} \right)^{-\epsilon} \left(\frac{P_t^D(i)}{P_t^D} \right)^{-\xi_t^D} \right] \Bigg\}, \end{aligned} \tag{A.2}$$

we derive the dirty Phillips curve by taking the FOC with respect to $P_t^D(i)$:

$$\begin{aligned}\frac{\partial L}{\partial P_t^D(i)} : \lambda_t \Bigg\{ & \frac{y_t^D}{p_t} (1 - \xi_t^D) \left(\frac{P_t^D(i)}{P_t^D} \right)^{-\xi_t^D} - \frac{\phi_p}{P_{t-1}^D(i)} \left[\frac{P_t^D(i)}{P_{t-1}^D(i)} - \bar{\pi} \right] \frac{y_t^D P_t^D}{p_t} + \\ & + \xi_t^D mc_t^D(i) \frac{y_t^D}{P_t^D} \left(\frac{P_t^D(i)}{P_t^D} \right)^{-\xi_t^D - 1} - \theta_1 \mu_t(i)^{\theta_2} \frac{y_t^D}{P_t^D} (-\xi_t^D) \left(\frac{P_t^D(i)}{P_t^D} \right)^{-\xi_t^D - 1} + \\ & - \tau_t^E(i) (1 - \mu_t(i)) \gamma_1 \frac{(y_t^D)^{1-\gamma_2}}{P_t^D} \xi_t^D (\gamma_2 - 1) \left[\frac{P_t^D(i)}{P_t^D} \right]^{\xi_t^D(\gamma_2 - 1) - 1} \Bigg\} + \\ & + \lambda_{t+1} \beta \mathbb{E}_t \left\{ \phi_p \frac{y_{t+1}^D P_{t+1}^D}{p_{t+1}} \left[\frac{P_{t+1}^D(i)}{P_t^D(i)} - \bar{\pi} \right] \frac{P_{t+1}^D(i)}{P_t^D(i)^2} \right\} = 0. \end{aligned}$$

At this point, we can drop the index i, since we are in a symmetric equilibrium, and simplify the equation:

$$(1-\xi_t^D)-\frac{\phi_p}{P_{t-1}^D}\left(\pi_t^D-\bar{\pi}\right)P_t^D+\xi_t^D mc_t^D\frac{p_t}{P_t^D}+\xi_t^D\tau_t^E(1-\gamma_2)(1-\mu_t)\gamma_1\frac{(t_t^D)^{-\gamma_2}p_t}{P_t^D}+$$
$$+\theta_1\mu_t^{\theta_2}\xi_t^D\frac{p_t}{P_t^D}+\frac{\lambda_{t+1}}{\lambda_t}\beta\mathbb{E}_t\left[\phi_p\frac{y_{t+1}^D P_{t+1}^D p_t}{p_{t+1}y_t^D}\left(\pi_{t+1}^D-\bar{\pi}\right)\frac{P_{t+1}^D}{P_t^{D2}}\right]=0$$

By rearranging the terms and employing the definition of relative inflation as in equation 44, we obtain the dirty Phillips curve:

$$\pi_t^D\left(\pi_t^D-\bar{\pi}\right)=\beta\mathbb{E}_t\left[\frac{\lambda_{t+1}}{\lambda_t}\frac{p_{t+1}^D y_{t+1}^D}{p_t^D y_t^D}\pi_{t+1}^D\left(\pi_{t+1}^D-\bar{\pi}\right)\right]+$$
$$+\frac{\xi_t^D}{\phi_p}\left[\frac{mc_t^D}{p_t^D}-\frac{\xi_t^D-1}{\xi_t^D}+\tau_t^E(1-\mu_t)\gamma_1(1-\gamma_2)\frac{\left(y_t^D\right)^{-\gamma_2}}{p_t^D}+\theta_1\frac{\mu_t^{\theta_2}}{p_t^D}\right]. \tag{A.3}$$

We follow the same procedure to derive the clean Phillips curve. We write the clean Lagrangian function as

$$\mathcal{L}=\mathbb{E}_t\sum_{t=0}^{\infty}\beta^t\lambda_t\left\{\frac{P_t^C(i)}{p_t}y_t(1-\Delta)\left(\frac{P_t^C}{p_t}\right)^{-\epsilon}\left(\frac{P_t^C(i)}{P_t^C}\right)^{-\xi_t^C}+\right.$$
$$-w_t^C l_t^C(i)-r_t^{k,C}k_{t-1}^C(i)-\frac{\phi_p}{2}\left(\frac{P_t^C(i)}{P_{t-1}^C(i)}-\bar{\pi}\right)^2\frac{y_t^C(i)P_t^C(i)}{p_t}+$$
$$\left.+mc_t^C(i)\left[A_t^C(i)\left(k_{t-1}^C(i)\right)^{\alpha}\left(l_t^C(i)\right)^{1-\alpha}-y_t(1-\Delta)\left(\frac{P_t^C}{p_t}\right)^{-\epsilon}\left(\frac{P_t^C(i)}{P_t^C}\right)^{-\xi_t^C}\right]\right\}. \tag{A.4}$$

Next, we take the FOC with respect to $P_t^C(i)$:

$$\frac{\partial L}{\partial P_t^C(i)}:\lambda_t\left\{\frac{y_t^C}{p_t}(1-\xi_t^C)\left(\frac{P_t^C(i)}{P_t^C}\right)^{-\xi_t^C}-\frac{\phi_p}{P_{t-1}^C(i)}\left[\frac{P_t^C(i)}{P_{t-1}^C(i)}-\bar{\pi}\right]\frac{y_t^C P_t^C}{p_t}+\right.$$
$$\left.+\xi_t^C mc_t^C(i)\frac{y_t^C}{P_t^C}\left(\frac{P_t^C(i)}{P_t^C}\right)^{-\xi_t^C-1}\right\}+$$
$$+\lambda_{t+1}\beta\mathbb{E}_t\left\{\phi_p\frac{y_{t+1}^C P_{t+1}^C}{p_{t+1}}\left[\frac{P_{t+1}^C(i)}{P_t^C(i)}-\bar{\pi}\right]\frac{P_{t+1}^C(i)}{P_t^C(i)^2}\right\}=0.$$

Then, we drop the index i:

$$(1-\xi_t^C)-\frac{\phi_p}{P_{t-1}^C}\left(\pi_t^C-\bar{\pi}\right)P_t^C+\xi_t^C mc_t^C\frac{p_t}{P_t^C}+\frac{\lambda_{t+1}}{\lambda_t}\beta\mathbb{E}_t\left[\phi_p\frac{y_{t+1}^C P_{t+1}^C p_t}{p_{t+1}y_t^C}\left(\pi_{t+1}^C-\bar{\pi}\right)\frac{P_{t+1}^C}{P_t^{C2}}\right]=0$$

After rearranging the equation, we obtain the clean Phillips curve as follows:

$$\pi_t^C\left(\pi_t^C-\bar{\pi}\right)=\beta\mathbb{E}_t\left[\frac{\lambda_{t+1}}{\lambda_t}\frac{p_{t+1}^C y_{t+1}^C}{p_t^C y_t^C}\pi_{t+1}^C\left(\pi_{t+1}^C-\bar{\pi}\right)\right]+\frac{\xi_t^C}{\phi_p}\left[\frac{mc_t^C}{p_t^G}-\frac{\xi_t^C-1}{\xi_t^C}\right]. \tag{A.5}$$

A.3 Optimal monetary policy: consumption equivalent variation

The optimal monetary policy is computed by maximizing households' utility function, and the welfare variation is measured in terms of the consumption equivalent as follows:

$$W_t^o=\mathbb{E}_t\sum_{t=0}^{\infty}\beta^t U((1-\Upsilon)c_t,l_t), \tag{A.6}$$

$$W_t^o-W_t^b=\mathbb{E}_t\sum_{t=0}^{\infty}\beta^t\left[U((1-\Upsilon)c_t,l_t)-U(c_t,l_t)\right],$$

$$W_t^o-W_t^b=\mathbb{E}_t\sum_{t=0}^{\infty}\beta^t\left[\frac{((1-\Upsilon)c_t)^{1-\varphi_c}-1}{1-\varphi_c}-\psi\frac{l_t^{1+\varphi_l}}{1+\varphi_l}-\frac{c_t^{1-\varphi_c}-1}{1-\varphi_c}+\psi\frac{l_t^{1+\varphi_l}}{1+\varphi_l}\right],$$

$$\Upsilon=1-\left[\left(W_t^o-W_t^b\right)(1-\varphi_c)(1-\beta)(c)^{\varphi_c-1}+1\right]^{\frac{1}{1-\varphi_c}}. \tag{A.7}$$

Here, Υ stands for the welfare cost of implementing a specific policy rule – denoted as optimal (o) – vs the baseline (b) policy, in terms of the CE.

B E-DSGE steady state

Output

$$y=\bar{y} \tag{B.1}$$

Capital stock

$$x=\bar{x} \tag{B.2}$$

Dirty relative price

$$p^D=\overline{p^D} \tag{B.3}$$

Technology

$$a^{C,D}=\bar{a} \tag{B.4}$$

Cost-push/elasticity of substitution

$$\xi^{C,D}=\bar{\xi} \tag{B.5}$$

Carbon tax

$$\tau^E = \overline{\tau^E} \tag{B.6}$$

Income tax

$$t = \omega \bar{y} \tag{B.7}$$

Public consumption

$$g = \tau^E + t \tag{B.8}$$

Inflation

$$\pi = \pi^{C,D} = \bar{\pi} \tag{B.9}$$

Nominal interest rate

$$r = \frac{\bar{\pi}}{\beta} \tag{B.10}$$

Real interest rate

$$r^r = \frac{1}{\beta} \tag{B.11}$$

Tobin's q

$$q^C = q^D = 1 \tag{B.12}$$

Rental rate of capital

$$r^k = \frac{1}{\beta} - (1 - \delta) \tag{B.13}$$

Aggregate price (numeraire)

$$p = 1 \tag{B.14}$$

Emissions

$$e = \frac{x(1-\eta)}{6} \tag{B.15}$$

Abatement cost

$$\mu = \left[\frac{\tau^E \gamma_1 \left(\Delta y \left(p^D \right)^{-\epsilon} \right)^{-\gamma_2}}{\theta_1 \theta_2} \right]^{\frac{1}{\theta_2 - 1}} \tag{B.16}$$

Dirty production

$$y^D = \left[\frac{e}{\gamma_1 (1 - \mu)} \right]^{\frac{1}{1-\gamma_2}} \tag{B.17}$$

Damage function

$$D = \left[d_2(x)^2 + d_1(x) + d_0\right] \tag{B.18}$$

Clean TFP

$$A^C = (1 - D)a^C \tag{B.19}$$

Dirty TFP

$$A^D = (1 - D)a^D \tag{B.20}$$

Clean relative price

$$p^C = \left[\frac{1 - \Delta(p^D)^{1-\epsilon}}{1 - \Delta}\right]^{\frac{1}{1-\epsilon}} \tag{B.21}$$

Dirty demand

$$y^D = y\Delta\left(\frac{P^D}{p}^{-\epsilon}\right) \tag{B.22}$$

Clean demand

$$y^C = y(1 - \Delta)\left(\frac{P^C}{p}^{-\epsilon}\right) \tag{B.23}$$

Dirty marginal cost

$$mc^D = \frac{\xi^D - 1}{\xi^D}p^D - \left[\theta_1\mu^{\theta_2} + \gamma_1\tau^E(y^D)^{-\gamma_2}(1 - \gamma_2)(1 - \mu)\right] \tag{B.24}$$

Clean marginal cost

$$mc^C = \frac{\xi^C - 1}{\xi^C}p^C \tag{B.25}$$

Dirty capital demand

$$k^D = \frac{\alpha y^D}{r_k^D}mc^D \tag{B.26}$$

Clean capital demand

$$k^C = \frac{\alpha y^C}{r_k^C}mc^C \tag{B.27}$$

Dirty labor demand

$$l^D = \left(\frac{y^D}{A^D(k^D)^\alpha}\right)^{\frac{1}{1-\alpha}} \tag{B.28}$$

Clean labor demand

$$l^C = \left(\frac{y^C}{A^C(k^C)^\alpha}\right)^{\frac{1}{1-\alpha}} \tag{B.29}$$

Dirty wage

$$w^D = \frac{(1-\alpha)y^D}{l^D} mc^D \tag{B.30}$$

Clean wage

$$w^C = \frac{(1-\alpha)y^C}{l^C} mc^C \tag{B.31}$$

Aggregate capital

$$k = k^C + k^D \tag{B.32}$$

Aggregate labor

$$l = \left[(l^C)^{(1+\phi_h)} + (l^D)^{(1+\phi_h)} \right]^{\frac{1}{1+\phi_h}} \tag{B.33}$$

Investment C

$$i^C = \delta k^C \tag{B.34}$$

Investment D

$$i^D = \delta k^D \tag{B.35}$$

Abatement spending

$$z = y^D \theta_1 \mu^{\theta_2} \tag{B.36}$$

Households' consumption

$$c = y - i - \bar{g} - z \tag{B.37}$$

Dirty labor marginal rate of substitution

$$mrs^D = \frac{\xi_W - 1}{\xi_W} w^D \tag{B.38}$$

Clean labor marginal rate of substitution

$$mrs^C = \frac{\xi_W - 1}{\xi_W} w^C \tag{B.39}$$

Dirty labor supply

$$mrs^d = \frac{\psi l^{\phi_l - \phi_h} l_d^{\phi_h}}{\lambda} \tag{B.40}$$

Clean labor supply

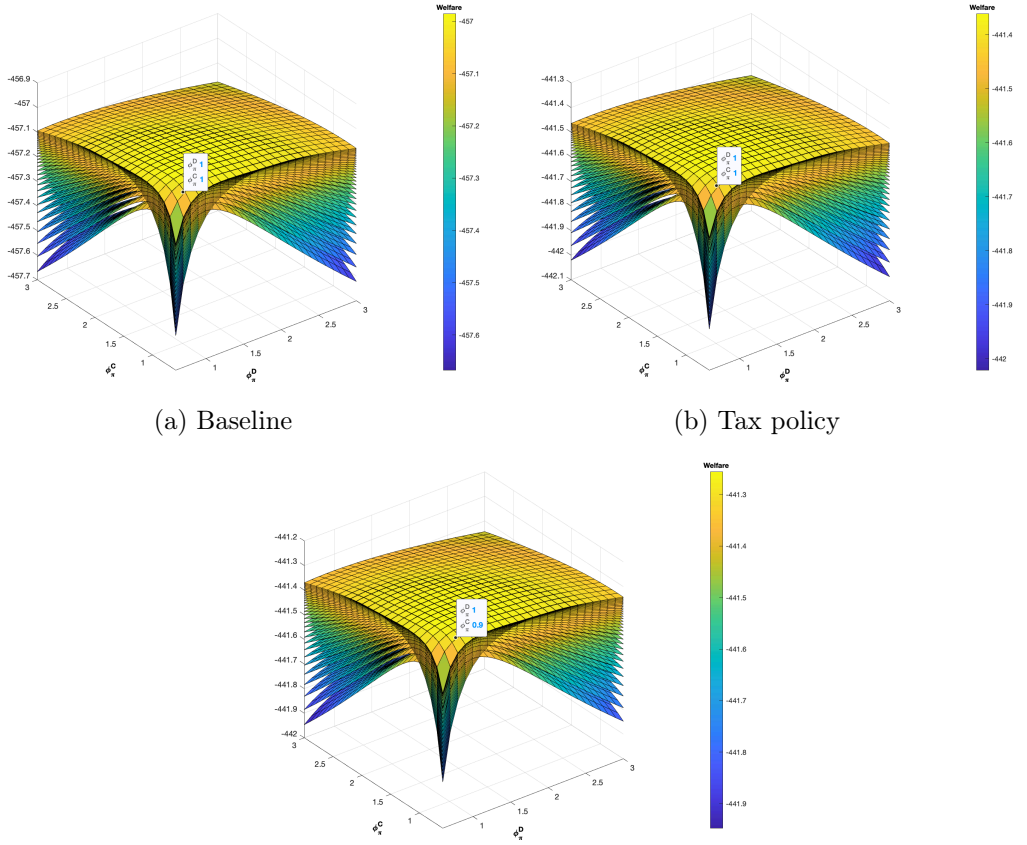
$$mrs^c = \frac{\psi l^{\phi_l - \phi_h} l_c^{\phi_h}}{\lambda} \tag{B.41}$$

C Sensitivity analysis

UNDER REVIEW

D Welfare

Figure D.7: Optimal Welfare level with Non-standard Taylor Rule



(a) Baseline

(b) Tax policy

(c) Cap policy

Note: Welfare level (color bar) of each possible combination of non-standard TR parameters; different environmental policy scenarios with simultaneous markup shocks. The values of the parameters ϕ_π^D and ϕ_π^C are reported on the x and y axes respectively (range between 0.5 and 3). Each layer of the surface plot is associated with a value of ϕ_y (range between 0 and 3), with the lowest at the bottom.

E Additional IRF plots on comparison of monetary policies: cap and baseline regimes

Cap policy

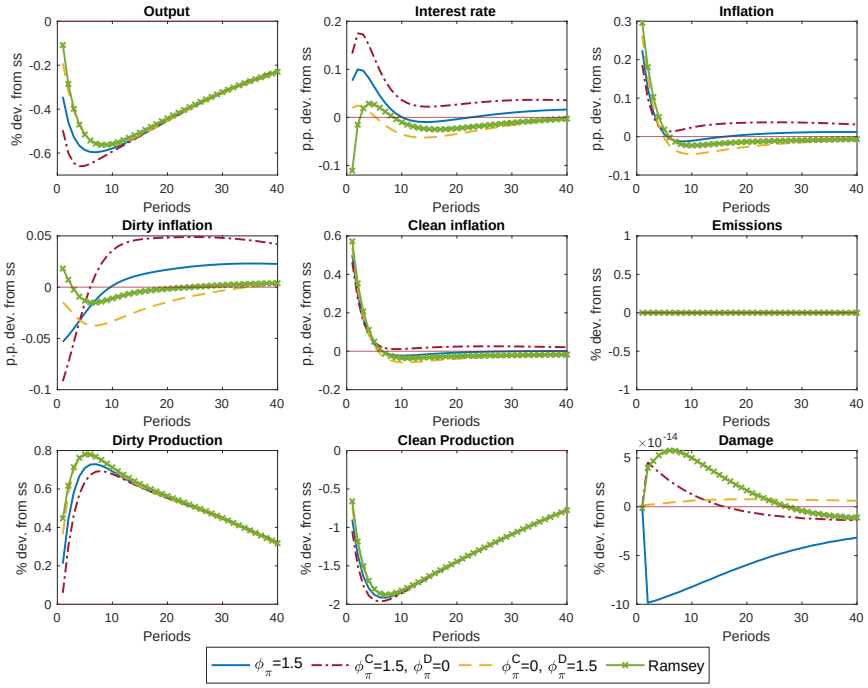


Figure E.8: Clean markup shock – impulse response function under different monetary policy rules.

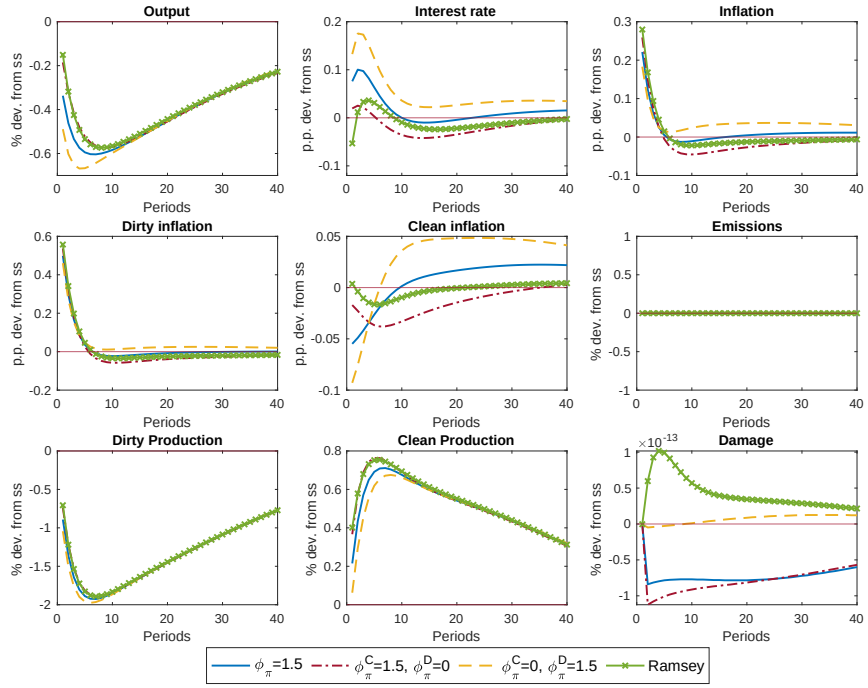


Figure E.9: Dirty markup shock – impulse response function under different monetary policy rules.

Baseline

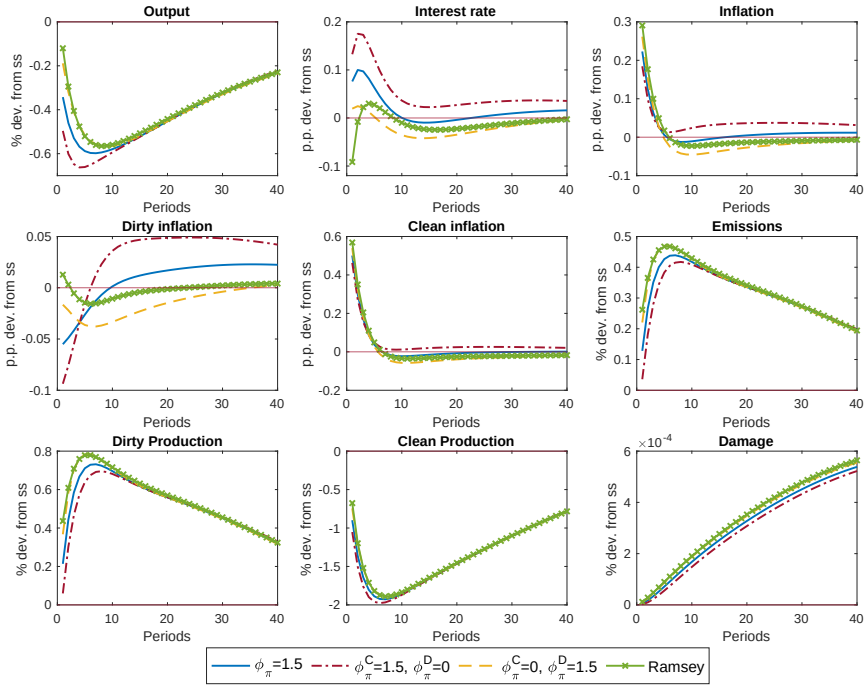


Figure E.10: Clean markup shock – impulse response function under different monetary policy rules.

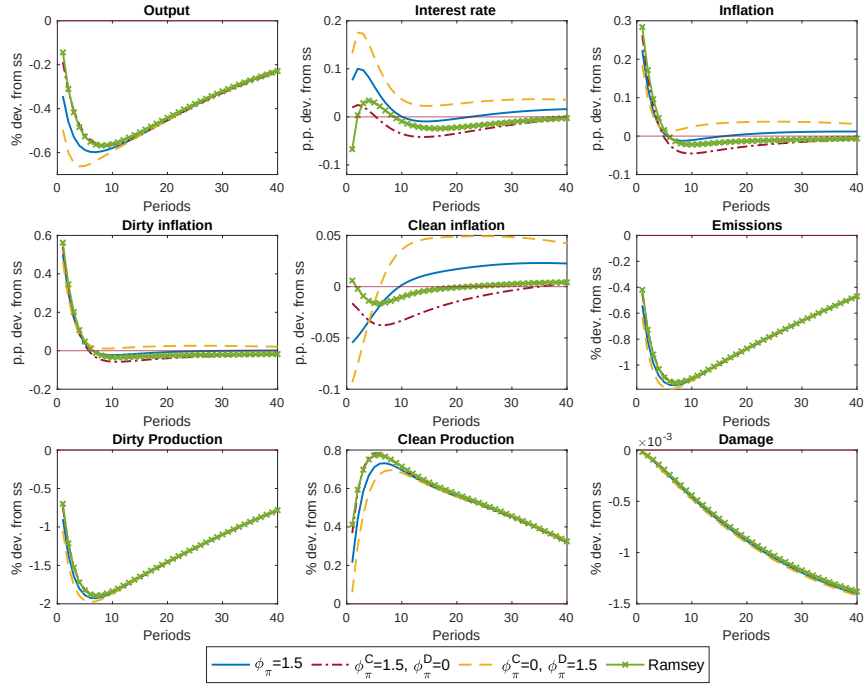


Figure E.11: Dirty markup shock – impulse response function under different monetary policy rules.