Optimal Income Taxation under Wage Rigidity

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Abstract

This paper revisits two classic questions in public finance: (i) Should capital income taxes be eliminated? (ii) Is uniform labor income taxation optimal? I examine these questions in the context of wage rigidity, a feature prevalent in economies with strong labor protections or centralized wage-setting institutions. I demonstrate that wage adjustment frictions significantly alter the design of optimal income tax policies. Specifically, I show that, under wage rigidity and sufficiently high TFP, capital taxes become welfare-neutral, making strictly positive capital tax rates optimal. Additionally, while optimal capital tax rates remain constant over time, optimal labor tax rates may vary. I show how the evolution of TFP and wage dynamics determine whether labor taxes should be front-loaded or back-loaded.

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1 Introduction

There is an extensive literature in public finance that has applied the theory of Ramsey (1927) to the study of the structure of optimal capital and labor income taxation.[1] Two questions that have received the most attention in this literature are: (i) Should the tax on capital income be eliminated? (ii) Is uniform labor income taxation optimal? In this paper, I revisit these classic questions in a setting with wage rigidity. In many economies—especially those with strong labor protections or centralized wage-setting institutions—wages exhibit short-term stickiness. I argue that these wage adjustment frictions carry important implications for the design of tax policy.

Several studies have documented wage rigidity across various microeconomic datasets (Card and Hyslop, 1997; Kahn, 1997; Lebow et al., 2003; Bauer et al., 2007; Dickens et al., 2007; Knoppik and Beissinger, 2009; Kaur, 2019; Ehrlich and Montes, 2024). Additionally, micro-level evidence links price adjustment frequency to wage rigidity: firms or product categories with a higher share of labor in total costs tend to adjust prices less frequently, likely reflecting the slower adjustment of wages compared to other input prices (Klenow and Malin, 2010).

Wage and price rigidity are also foundational in New Keynesian models, where they are considered essential for capturing the short-term effects of monetary policy interventions on output and inflation (see, for instance, Galí (2010) and Bilbiie and Melitz (2020)). Similarly, as argued by Hall (2005) and Shimer (2005), the textbook search and matching model requires some form of wage stickiness to align with observed employment fluctuations.[2,3] However, despite their importance in explaining macroeconomic fluctuations, studies on fiscal policy have largely overlooked wage adjustment frictions.

Greulich et al. (2023) argue that long-run results on optimal policy do not provide good guidance for policy in the short and medium run. This lack of guidance is particularly relevant in the context of wage rigidity, as the temporary nature of wage stickiness implies that the design of tax policy must account for transitional dynamics. Motivated by these concerns, I analyze a two-class economy of capitalists and workers in a finite-

[1] See, for instance, Diamond and Mirrlees (1971) and Atkinson and Stiglitz (2015).

[2] Hall (2005) argues that wage rigidity could be a consequence of matching frictions, addressing the critique that wage rigidity never results in an allocation of labor that is inefficient from the joint perspective of worker and employer.

[3] The central issue is that wage determination in the standard Mortensen-Pissarides model, based on period-by-period Nash bargaining between firms and workers, generates excessive wage volatility. This exaggerated procyclical wage movement, in turn, suppresses firms' cyclical hiring incentives. To address this puzzle, Menzio (2005) and Kennan (2010) provide an axiomatic foundation for wage rigidity, deriving it explicitly from assumptions about the information structure. Others have explored flexible wage alternatives, e.g., Hagedorn and Manovskii (2008), Mortensen and Nagypal (2007), and Rotemberg and Trigari (2006).

period setting. Wages do not fully adjust to changes in supply and demand. In contrast, capital markets are perfectly competitive, with returns that flexibly adjust to economic conditions and policy shifts. This aligns with empirical evidence showing that financial markets, through interest rates, respond more rapidly to shocks than labor markets, where wage adjustments are constrained by long-term contracts and institutional factors. I assume capitalists have quasi-linear utility in consumption, based on findings in the household finance literature that wealthier individuals demonstrate higher risk tolerance (Guiso et al., 1996; King and Leape, 1998; Carroll, 2002; Calvet et al., 2007; Calvet and Sodini, 2014). As over 90% of U.S. equity wealth is held by the wealthiest 10% (Saez and Zucman, 2016), focusing on such agents ensures that my model reflects real-world distributions, where capital income taxes primarily impact affluent households.

Consistent with the literature that emphasizes the effects of taxation on capital accumulation (see Chari and Kehoe (1999), for an excellent survey), the government faces a dynamic problem: it needs to fund a stream of purchases and transfers over time using linear taxes on capital and labor income. The government is benevolent, and its objective is to choose the optimal tax rates to maximize a weighted sum of the utilities of capitalists and workers. For much of the analysis, I neglect problems of distributive justice and instead concentrate on the implications of taxation for efficiency.

The analysis unfolds in three steps. First, I demonstrate the welfare neutrality of capital taxes under wage rigidity. This result stems from the endogenous adjustment of the rate of return on capital in response to capital taxes, which offsets their impact on household income. Under rigid wages, this adjustment preserves welfare. In contrast, if wages are flexible, capital taxes affect welfare by reducing investment, which decreases the marginal product of labor and, ultimately, household income. Thus, the welfare neutrality of capital taxes relies on wage rigidity, as flexible wages would link capital taxes to wage levels through firms' capital allocation decisions.

Building on the neutrality result, I analyze the implications for optimal capital and labor tax policies. I show that the optimal capital tax rate is positive and constant over time when total factor productivity (TFP) is sufficiently large. Intuitively, when TFP rises but wages do not immediately adjust to increases in the marginal product of labor, total expenditure falls short of total production, and resource constraints become slack. When resource constraints are slack, we can decouple the problems of determining optimal capital and labor taxes. In equilibrium, welfare and labor income tax revenue are independent of capital taxes. Given that welfare decreases with labor income taxes, the planner finds it optimal to reduce the labor tax burden by maximizing capital tax revenues. These results highlight how the flexibility of capital returns, combined with wage rigidity, supports the

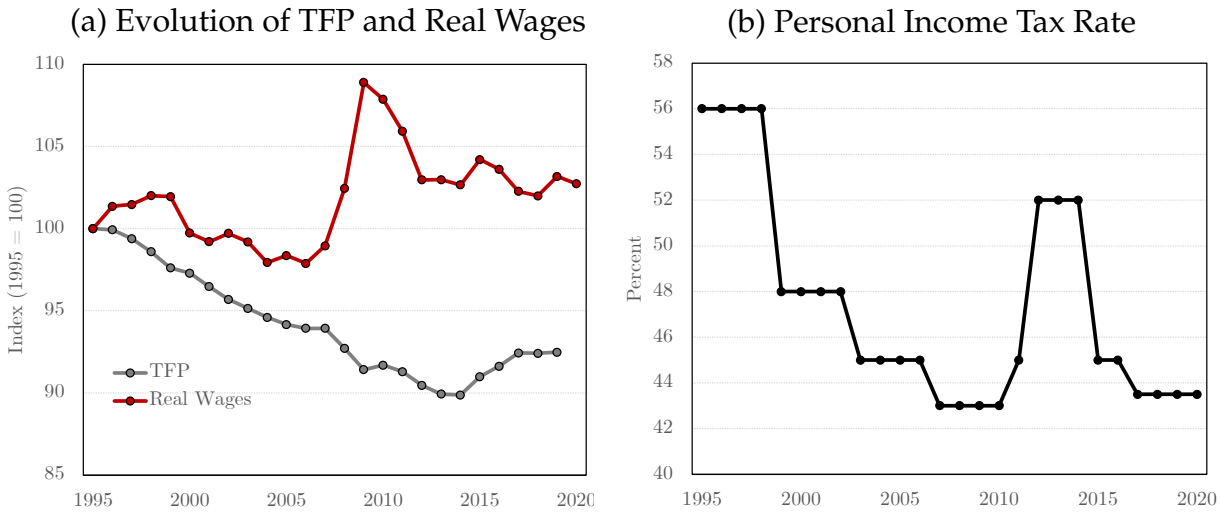
rationale for a positive capital tax rate, as adjustments in the rate of return generate fiscal revenue without distorting household welfare.

Lastly, I examine optimal labor taxation, focusing on the timing of these taxes. Specifically, I show that when labor supply is elastic, the optimal structure of labor income taxes hinges on the evolution of TFP and wages. I consider a production function of the following form where the output (Y_t) is produced using capital (K_t) and labor (L_t): $Y_t = F_t(K_t, L_t) = AZ_tK_t^{\alpha}L_t^{1-\alpha}$. Here, A and Z_t capture the time-invariant and time-varying components of TFP, respectively. When wages are rigid and A is sufficiently high, capital tax revenue outweighs labor tax revenue, making the impact of labor income taxes on capital tax revenue more substantial than their direct effect on labor tax revenue. An increase in labor taxes reduces labor supply, leading to a decrease in equilibrium labor allocations. As equilibrium labor decreases, firms experience a decline in the marginal product of capital because labor and capital are complementary inputs in production. The reduction in the marginal product of capital diminishes firms' incentive to demand capital, resulting in a lower equilibrium level of capital. Consequently, capital tax revenues decline. Since labor income taxes reduce capital tax revenue by distorting capital allocations, it becomes optimal for the planner to set labor taxes across periods to balance the marginal loss in capital tax revenue resulting from labor taxation when TFP is sufficiently high. Additionally, capital allocations respond to changes in the time-variant component of productivity, which affects capital tax revenues. If Z_t is (weakly) decreasing over time, marginal changes in capital tax revenue from labor taxes will also decrease over time under uniform labor taxation, violating the optimality condition discussed above. In this case, back-loading labor taxes can enhance welfare while maintaining budget balance. Conversely, if Z_t rises sufficiently over time, front-loading labor taxes becomes optimal.

Moreover, if wages in the initial period are much higher than in subsequent periods, the case for front-loading labor income taxes grows stronger. The initially high wages allow the government to achieve revenue targets more efficiently by taxing early. However, if wages are expected to decrease over time, then it becomes optimal to back-load labor taxes. I derive an explicit condition that links the ratio of the time-varying component of TFP between two periods and the ratio of wages over the same periods to the ratio of optimal labor income taxes in those periods.

Spain offers an illustrative case. TFP growth rates in Spain were exceptionally high between 1950 and 1995, exceeding 3% per year. This period saw the Spanish manufacturing sector adopt mass production techniques, along with a significant labor reallocation from agriculture to industry and services (de la Escosura and Rosés, 2009). By 1995, TFP levels

in Spain were among the highest in the world, comparable to other advanced economies such as the US (Feenstra et al., 2015). Additionally, both direct and indirect evidence points to real wage rigidity in Spanish labor markets. First, a survey conducted across 15 European Union countries between late 2007 and early 2008 highlights Spain's high degree of real wage rigidity. Babecký et al. (2010) report that 55 percent of firms surveyed in Spain employed automatic wage indexation mechanisms, a key indicator of real wage stickiness. Second, empirical trends reinforce this rigidity. Since 1995, while TFP in Spain has declined, real wages have remained largely unchanged (Panel (a) of Figure 1). Under flexible wages, one would expect real wages to fall alongside TFP, making this evidence strongly indicative of downward real wage rigidity. This makes Spain a strong candidate for applying the main results of this paper, which hold when TFP is sufficiently high and wages are rigid. Given that wages are downward rigid, our theory suggests that the optimal policy for Spain would be to increase labor income tax rates as TFP falls.[4] However, the actual policy trajectory has diverged from this recommendation. Since 1995, personal income tax rates in Spain have generally declined, reflecting a form of frontloading rather than backloading, as implied by the model (Panel (b) of Figure 1).[5]



Notes: In panel (a), data on total factor productivity (in constant prices) are obtained from FRED. Real wage data are obtained from the OECD and correspond to average annual wages (in constant prices) per employee in full-time equivalent unit in the total economy. I convert these time series into indices by rebasing to 100 in 1995. In panel (b), I plot the top statutory personal income tax rate that applies to a single person without dependents.

Figure 1: Trends in TFP, Real Wages, and Labor Income Taxes in Spain

The theoretical results rely on three key assumptions: (i) the time-invariant component

[4]This assumes that the gross interest rate on Spanish debt is greater than equal to the inverse of the rate of time preference.

[5]The one notable exception occurred between 2012 and 2014, when taxes were temporarily increased as part of a fiscal consolidation effort in response to the eurozone debt crisis.

of TFP approaches infinity, (ii) capitalists' preferences are quasi-linear in consumption, and (iii) worker transfers are non-redistributive (i.e., tax revenues from workers are rebated to them lump-sum). For robustness, I numerically demonstrate under plausible parameter values that the main results of this paper are not highly sensitive to these assumptions.

Related Literature. This paper contributes to the extensive literature on optimal capital and labor income taxation. Atkinson and Stiglitz (1976) and Feldstein (1978) analyze taxes on capital and labor income in a finite period model, concluding that the optimal capital income tax rate is zero when preferences are separable. Chamley (1986) and Judd (1985) revisit this issue in a general equilibrium model with infinite lives, finding that, for separable utility functions, the optimal capital income tax rate should be constant for a finite length of time and zero thereafter. Following these seminal papers, several scholars have shown different kinds of zero capital tax results (Lucas Jr, 1990; Jones et al., 1993; Zhu, 1992; Judd, 1992; Chari et al., 1994; Atkeson et al., 1999; Chari and Kehoe, 1999; Werning, 2007; Greulich et al., 2023).[6] In contrast, I find that when wages are rigid, optimal capital taxes should remain positive and constant, even under separability conditions.[7]

In the literature on dynamic Ramsey taxation, some scholars have argued that uniform labor income taxation is optimal (Barro, 1979; Kydland and Prescott, 1980; Werning, 2007). As argued by Zhu (1992), there is no theoretical presumption that uniform labor income taxation is an optimal policy in stochastic environments. Chari et al. (1994) explore this issue in a quantitative model calibrated to the U.S. economy, showing that labor tax rates are roughly constant over the business cycle in a flexible-price setting. The results of this paper are more applicable to other economies, where wage rigidity due to institutional factors plays a more significant role. While much of the literature assumes flexible wages set in competitive markets, I show that wage rigidity can significantly alter optimal fiscal policy, particularly affecting the timing and structure of both capital and labor income taxes.[8]

[6]Some recent results by Reinhorn (2019) and Straub and Werning (2020) show that some previous proofs treated Lagrange multipliers incorrectly and that a correct proof delivers positive capital tax rates for some parameter values. Lansing (1999), Bassetto and Benhabib (2006), and Benhabib and Szoőke (2021) provide more examples along these lines.

[7]As such, this paper is part of a strand of papers that find positive capital taxes can be optimal (Aiyagari, 1995; Golosov et al., 2003; Erosa and Gervais, 2002; Conesa et al., 2009; Straub and Werning, 2020).

[8]Erosa and Gervais (2002) examine Ramsey taxation in a life-cycle economy. Proposition 4.2 in their paper identifies conditions under which labor taxes should increase over the life cycle. However, their result depends on the optimal timing of an endogenous variable: labor allocations. In contrast, this paper characterizes the optimal timing of labor taxes entirely in terms of model primitives.

Wolf (2020) and Belchior and Reis (2024) also study optimal taxation in an environment with wage rigidities. Their work differs from this paper in several ways. First, their model assumes that wage adjustment frictions result in labor supply exceeding labor demand, leading firms to uniquely determine the quantity of labor. In contrast, I focus on a scenario where labor demand exceeds labor supply, a situation particularly relevant in recent periods. For example, during his post-FOMC press conference in November 2022, Chairman Powell noted that "the broader picture is of an overheated labor market where demand substantially exceeds supply." While Belchior and Reis also find that uniform labor income taxes may not be optimal under wage rigidity, their mechanism—manipulating taxes to clear the labor market—differs from the mechanism I highlight in this paper, which emphasizes the interaction between labor tax rates and capital tax revenues. Finally, under the general class of preferences they consider, it is challenging to assess how productivity changes influence the trajectory of taxes, prompting their reliance on a quantitative approach. By contrast, I show that in the limiting quasi-linear economy, this question can be addressed analytically.

The literature on optimal fiscal policies under sticky wages is relatively limited, though there is substantial research on optimal policies under price stickiness, primarily in monetary models. Much of this strand of the literature builds on Chari et al. (1991), who show that when governments finance stochastic spending by taxing labor income and issuing one-period debt, state-contingent returns on that debt enable tax rates to remain roughly constant. In monetary models, this tax smoothing can occur even without state-contingent nominal returns; by adjusting the price level in response to shocks, the government can achieve state-contingent, ex-post real returns. For instance, generating inflation following a positive spending shock allows the government to reduce its real liabilities by lowering the value of outstanding nominal debt, thereby attenuating the tax increase needed to satisfy the present value budget constraint. Chari et al. (1991) derive this result under flexible prices.

Several studies have examined whether the insights of Chari et al. (1991) hold under sticky prices. Benigno and Woodford (2003), Siu (2004) and Schmitt-Grohé and Uribe (2004) argue that, with sticky prices, it is not optimal for inflation to completely offset fiscal stress; instead, substantial adjustment should come from changes in tax rates. Their work differs from this study in three key ways. First, the mechanism driving the optimality of non-uniform labor taxes in our model is distinct from the mechanisms in this literature, which emphasize a trade-off between inflation's tax-smoothing benefits and its distortionary impact on prices. Second, while prior studies do not specifically address the influence of price stickiness on the optimal timing of capital and labor income taxes,

I focus on this interaction as a central part of my analysis. Finally, whereas prior studies characterize optimal fiscal policies under sticky prices with optimally chosen monetary policy, I characterize optimal fiscal policies independently of monetary policy. This distinction is potentially significant, as the set of implementable allocations under optimal monetary policy is generally a strict subset of all implementable allocations when prices are sticky.

2 Model

I start with the two-class economy laid out in Judd (1985). Capitalists save and derive income from returns on capital and wages, while workers live hand-to-mouth, consuming their entire wage income plus any transfers. Labor is inelastically supplied. The government taxes returns to capital and wages to fund government purchases and transfers targeted to workers. Since wage rigidity is a temporary phenomenon, I focus on a finite-period setting. Time and discrete, with periods labeled $t = 1, \ldots, T$.

2.1 Environment

Preferences. There is a continuum of capitalists of unit measure. Consumption by capitalists is denoted by c, and labor supply by capitalists is denoted by L. The flow utility of the representative capitalist is given by:

$$u(c) = \frac{c^{1-\sigma}}{1-\sigma},$$

where $\sigma \geq 0$. Throughout the theoretical analysis, I focus on the limiting economy where $\sigma \to 0$.

The timing of the decisions of these agents is as follows: capital in period 1, K_1, is chosen at the beginning of the first period and liquidated at the end of the first period; capital in period 2, K_t, is chosen at the end of period $t-1$ and liquated at the end of the period t; labor supply decisions (L_t) are made at the beginning of each period; consumption (c_t) takes place at the end of each period. Note that K_1 can be interpreted as a short-term investment, as it yields returns within the same period in which the investment is made. The possibility of a short-term investment opportunity generates variation in the rates of return to capital across periods even when capital tax rates remain stable. This allows us to illustrate that optimal labor taxes are generally non-uniform not only due to differences in capital allocations across periods but also due to variations in their rates of return. The standard timing for capital investment, consistent across periods, can be recovered in my framework by imposing the restriction $K_1 = 0$. Importantly, the main results of this paper remain valid under this restriction.

Given wages $\{w_t\}_{t\in\{1,...,T\}}$, the return on capital $\{r_t\}_{t\in\{1,...,T\}}$, and tax policies $\{\tau_t^L, \tau_t^K\}_{t\in\{1,...,T\}}$, the representative capitalist solves the following problem:

$$\max_{\{c_t,K_t\}_{t\in\{1,...,T\}}} \sum_{t\in\{1,...,T\}} \beta^{t-1}u(c_t)$$
$$\text{s.t. } c_1 + K_2 - (1-\delta)K_1 + K_1 = w_1L_1(1-\tau_1^L) + r_1K_1(1-\tau_1^K)$$
$$c_t + K_{t+1} - K_t(1-\delta) = w_tL_t(1-\tau_t^L) + r_tK_t(1-\tau_t^K)\ \forall t \in \{2,...,T-1\}$$
$$c_T = w_TL_T(1-\tau_T^L) + r_TK_T(1-\tau_T^K) + K_T(1-\delta)$$

Here β denotes the rate of time preference, and δ denotes the rate of depreciation of capital. I assume $\beta \in (0,1)$ and $\delta \in (0,1)$.

There is a continuum of workers of unit measure. I denote workers' consumption and labor allocations by c^w and L^w, respectively. The flow utility of the representative worker is given by $v(c^w)$, where v is increasing, concave, continuous, and satisfies the Inada condition: $\lim_{x\to 0} v'(x) = \infty$. Workers inelastically supply labor. Workers are hand-to-mouth and cannot access capital markets. Given wages, w_t, labor tax policies, τ_t^L, and transfers, $\mathfrak{T}_t$, the representative worker's period-t consumption allocation is given by

$$c_t^w = w_t(1-\tau_t^L)L_t^w + \mathfrak{T}_t.$$

Technology. There is a continuum of firms of unit measure. Output is produced using labor L_t^d and capital K_t^d with Cobb-Douglas production function: $Y_t = AZ_t(K_t^d)^\alpha(L_t^d)^{1-\alpha}$, where A and Z_t represent the fixed and time-varying components of total factor productivity, respectively. I assume A and $\{Z_t\}_{t\in\{1,...,T\}}$ are strictly positive, and $\alpha \in (0,1)$. In each period, given $\{w_t, r_t\}$, the representative firm solves the following problem:

$$\max_{L_t^d,K_t^d} Y_t - w_tL_t^d - r_tK_t^d$$

Market Clearing Conditions. Capital markets are perfectly competitive. This assumption reflects the fact that financial markets adjust interest rates relatively quickly in response to changes in economic conditions, monetary policy, or shifts in demand and supply for credit. Formally, $\forall t \in \{1,...,T\}$:

$$K_t = K_t^d. \tag{1}$$

Wages, on the other hand, tend to be more rigid due to factors like long-term contracts, collective bargaining agreements, and institutional norms. Given wages w_t, and

assuming capitalists and workers are of equal measure, equilibrium labor allocations in period t are given by $\min\{L_t^d, L_t + L_t^w\}$. I primarily focus on cases where A is sufficiently large. Observe that as long as this condition is met, and wages remain finite, labor demand will exceed labor supply. Consequently, labor allocations are governed by labor supply conditions.

In addition, the following resource constraints hold:

$$c_1 + c_1^w + G_1 + K_2 - \delta K_1 \leq Y_1, \tag{2a}$$

$$c_t + c_t^w + G_t + K_{t+1} - K_t(1-\delta) \leq Y_t \; \forall t \in \{2, ..., T-1\}, \tag{2b}$$

$$c_T + c_T^w + G_T - K_T(1-\delta) \leq Y_T. \tag{2c}$$

Government Budget Constraints. Let B_t denote government debt accrued in period t and maturing in one period $t+1$, and r_B denote the interest rate on this debt. The government must finance an exogenous stream of purchases, $\{G_t\}_{t\in\{1,...,T\}}$, and transfers, $\{\mathfrak{T}_t\}_{t\in\{1,...,T\}}$, through taxes, $\{\tau_t^L, \tau_t^K\}_{t\in\{1,...,T\}}$, and borrowing, $B_{t\in\{1,...,T-1\}}$, given prices, $\{r_B, \{w_t, r_t\}_{t\in\{1,...,T\}}\}$, resulting in the following budget constraints:

$$G_t + \mathfrak{T}_t + (1+r_B)B_{t-1} = r_t K_t \tau_t^K + w_t(L_t + L_t^w)\tau_t^L + B_t \; \forall t \in \{1, ..., T\},$$

where $B_0 = B_T = 0$.

2.2 *Wage Determination and Sticky Wage Equilibria*

I assume wages are determined through a Nash (1953) demand game or auction, as in Hall (2005).[9] In the auction, the employee (who could be a capitalist or a worker) and the employer know each other's reservation values. In period t, the employee proposes a (real) wage, w_{Lt}, and the corresponding firm, without knowing the employee's proposal, makes its own proposal, w_{Ht}. If $w_{Lt} \leq w_{Ht}$, the match is made (or continues), and the wage is agreed to be $w_t = \kappa w_{Lt} + (1-\kappa)w_{Ht}$ with $0 < \kappa < 1$. This auction has the property that any wage in the bargaining set $[\underline{W}_t, \bar{W}_t]$ is a Nash equilibrium, where $\underline{W}_t$ and $\bar{W}_t$ denote the employee's and employer's reservation wages, respectively.[10] Believing the employee is bidding w_{Lt}, the firm will bid w_{Lt} as well, provided $w_{Lt} \leq \bar{W}_t$. Similarly, believing the firm is bidding w_{Ht}, the worker will bid w_{Ht}, provided $\underline{W}_t \leq w_{Ht}$. I consider rules that take the form of assigning a different wage in each time period.[11]

[9] When the production technology exhibits constant returns to scale and labor markets are frictionless, free entry and exit drive profits to zero. However, with wage-setting frictions, firms can earn positive profits when TFP is sufficiently high. I assume these profits are retained by firms and not rebated to households.

[10] These reservation wages can be interpreted as the income derived from non-market activities.

[11] This is similar to Hall (2005) in that he considers rules that assign a different wage in each state.

Let $\mathcal{T} = \{\tau_t^L, \tau_t^K\}_{t\in\{1,...,T\}}$ denote the tax policy vector.

Definition 1. *Given* $\{\{w_t, L_t, L_t^w, \mathfrak{T}_t\}_{t\in\{1,...,T\}}, \mathcal{T}\}$*, a sticky wage equilibrium is given by sequences of allocations* $\{c_t, c_t^w, K_t, L_t^d, K_t^d\}_{t\in\{1,...,T\}}$ *and interest rates* $\{r_t\}_{t\in\{1,...,T\}}$ *that satisfy the following conditions:*

1. *Given* $\{\{w_t, r_t\}_{t\in\{1,...,T\}}, \mathcal{T}\}$, $\{c_t, K_t\}_{t\in\{1,...,T\}}$ *solve the capitalists problem.*
2. *Given* $\{w_t, r_t\}$, $\{L_t^d, K_t^d\}$ *solve the firms problem* $\forall t \in \{1, ..., T\}$.
3. *The capital market clearing condition (1) holds.*
4. *Given wages* $\{w_t\}$*, equilibrium labor allocations in period t are given by* $\min\{L_t^d, L_t + L_t^w\}$.
5. *Resource constraints (2a)–(2c) hold.*

Let sticky wage equilibria be denoted by

$$\mathcal{C}^*(\mathcal{T}) \equiv \{c_t^*(\mathcal{T}), c_t^{w*}(\mathcal{T}), K_t^*(\mathcal{T}), L_t^{d*}(\mathcal{T}), K_t^{d*}(\mathcal{T}), r_t^*(\mathcal{T})\},$$

where the equilibrium functions map from $[0,1]^{2T}$ to $\mathbb{R}_+$.

2.3 Planning Problem

Let the indirect social welfare function be denoted by

$$\text{SWF} \equiv \sum_{t\in\{1,...,T\}} \beta^{t-1}\{\mu u(c_t^*(\mathcal{T})) + (1-\mu)v(c_t^{w*}(\mathcal{T}))\}.$$

The government aims to maximize a weighted sum of utilities with a weight μ on capitalists. By varying μ, we can trace out points on the Pareto frontier. Given a sequence of government expenditures $\{G_t, \mathfrak{T}_t\}_{t\in\{1,...,T\}}$ and the interest rate on government borrowing r_B, optimal capital and labor taxes are given by the solution to the following planning problem:

$$\max_{\mathcal{T},B} \text{SWF}$$
$$\text{s.t. } G_t + \mathfrak{T}_t + (1+r_B)B_{t-1} = r_t^*(\mathcal{T})K_t^*(\mathcal{T})\tau_t^K + \tau_t^L w_t(L_t + L_t^w) + B_t \;\forall t \in \{1, ...T\}$$

I assume that the present discounted stream of government expenditures, $\sum_1^T (G_t + \mathfrak{T}_t)/(1+r_B)^{t-1}$, is large enough such that it cannot be financed solely by capital income taxes. Let the solution to the above planning problem be denoted by $\mathcal{T}^* \equiv \{\tau_t^{L*}, \tau_t^{K*}\}_{t\in\{1,...,T\}}$. A *sticky wage Ramsey equilibrium* is given by $\mathcal{C}^*(\mathcal{T}^*)$.

In much of the analysis, I focus on *non-redistributive transfer policies* where

$$\mathfrak{T}_t = w_t \tau_t^L L_t^w \; \forall t \in \{1, ..., T\}.$$

3 Optimal Taxation with Inelastic Labor Supply

I first derive closed-form solutions for sticky wage equilibria.

Lemma 1. *If $\tau_t^K < 1 \; \forall t \in \{1, ..., T\}$, and A is sufficiently large, then a unique sticky wage equilibrium exists. Specifically:*

(i) Capitalists' allocations are given by:

$$c_1^*(\mathcal{T}) = w_1 L_1 (1 - \tau_1^L) - (L_2 + L_2^w) \left\{ \frac{\alpha A Z_1 \beta (1 - \tau_2^K)}{1 - \beta(1 - \delta)} \right\}^{\frac{1}{1-\alpha}},$$

$$c_t^*(\mathcal{T}) = w_t L_t (1 - \tau_t^L) + \frac{1}{\beta}(L_t + L_t^w) \left\{ \frac{\alpha A Z_t \beta (1 - \tau_t^K)}{1 - \beta(1 - \delta)} \right\}^{\frac{1}{1-\alpha}}$$
$$- (L_{t+1} + L_{t+1}^w) \left\{ \frac{\alpha A Z_{t+1} \beta (1 - \tau_{t+1}^K)}{1 - \beta(1 - \delta)} \right\}^{\frac{1}{1-\alpha}} \; \forall t \in \{2, ..., T-1\},$$

$$c_T^*(\mathcal{T}) = w_T L_T (1 - \tau_T^L) + \frac{1}{\beta}(L_T + L_T^w) \left\{ \frac{\alpha A Z_T \beta (1 - \tau_T^K)}{1 - \beta(1 - \delta)} \right\}^{\frac{1}{1-\alpha}},$$

$$K_1^*(\mathcal{T}) = K_1^{d*}(\mathcal{T}) = (L_1 + L_1^w) \left\{ \frac{\alpha A Z_1 (1 - \tau_1^K)}{\delta} \right\}^{\frac{1}{1-\alpha}}, \textit{ and}$$

$$K_t^*(\mathcal{T}) = K_t^{d*}(\mathcal{T}) = (L_t + L_t^w) \left\{ \frac{\alpha A Z_t \beta (1 - \tau_t^K)}{1 - \beta(1 - \delta)} \right\}^{\frac{1}{1-\alpha}} \; \forall t \in \{2, ..., T\}.$$

(ii) The equilibrium rates of return on capital are given by:

$$r_1^*(\mathcal{T}) = \frac{\delta}{1 - \tau_1^K}, \textit{ and}$$
$$r_t^*(\mathcal{T}) = \frac{1 - \beta(1 - \delta)}{\beta(1 - \tau_t^K)} \; \forall t \in \{1, ..., T\}.$$

Lemma 1 demonstrates that in all sticky wage equilibria, period-t labor allocations depend exclusively on the contemporaneous labor tax (τ_t^L), and are independent of any capital taxes or labor taxes from other periods. Additionally, consumption allocations in period 1 are unaffected by the capital tax in period 1 (τ_1^K), but they are sensitive to labor taxes in both periods 1 and 2 ($\{\tau_t^L\}_{t \in \{1,2\}}$), as well as the capital tax in period 2 (τ_2^K). Consumption allocations in period $t \in \{2, ..., T-1\}$ are affected by labor and capital income

taxes in periods t and $t+1$. Moreover, consumption allocations in period T are influenced only by the taxes in period T and exhibit no dependency on the taxes in previous periods. Finally, capital allocations in period t respond solely to the labor and capital taxes in that same period ($\{\tau_t^L, \tau_t^K\}$), with no dependence on taxes from earlier or future periods.

Remark. *Proposition 2 implies that the assumption of $\tau_t^K < 1 \; \forall t \in \{1, ..., T\}$ in Lemma 1 is not required in a sticky wage Ramsey equilibrium.*

I now present two results regarding the design of capital taxes. I first derive a set of neutrality results relating to the impact of capital taxes on welfare and tax revenues (Proposition 1). Using these results, I then derive the optimal capital tax (Proposition 2).

Proposition 1. *If A is sufficiently large, then in all sticky wage equilibria:*
(i) SWF is independent of $\{\tau_t^K\}_{t\in\{1,...,T\}}$,
(ii) Period-t capital income tax revenue, $r_t^*(\mathcal{T})K_t^*(\mathcal{T})\tau_t^K$, is independent of $\{\tau_t^K\}_{t\in\{1,...,T\}\setminus t}$.

Proposition 1 implies that, from a welfare standpoint, government revenues can be financed costlessly using capital taxes. In the model, this occurs because the rate of return on capital, r_t, endogenously increases as the capital tax, τ_t^K, is increased. Under the assumed preferences and technology, this change in r_t perfectly offsets the change in τ_t^K, leaving household income unchanged. As household income remains unaffected by capital taxes, welfare is also independent of these taxes. This stark result hinges on the assumption that wages are sticky. To see this, note that if wages are flexible, then w_t will be linked to K_t via the firm optimality condition $w_t = \partial Y_t / \partial L_t$. Additionally, as K_t is affected by τ_t^K, w_t would also be impacted by these taxes. Consequently, because w_t appears in SWF, capital taxes will indirectly impact welfare.

To determine optimal capital tax rates, it is useful to establish two additional results. The first result shows that when the time-invariant component of TFP is sufficiently high, resource constraints do not bind. This occurs because positive capital allocations (ensured when capital taxes are less than one) allow production to increase with higher TFP. However, due to wage rigidity, the rise in TFP does not translate into higher wages. As a result, total expenditure in the economy grows more slowly than total production, causing the resource constraint to slacken.

Lemma 2. *If A is sufficiently large, and $\tau_t^K \in [0, 1) \; \forall t \in \{1, ..., T\}$, then resource constraints (2a)–(2c) are slack in any sticky wage Ramsey equilibrium.*

The second result concerns the shape of the capital income tax Laffer curve. Specifically, in sticky price equilibria, Laffer curves are not globally concave when the capital

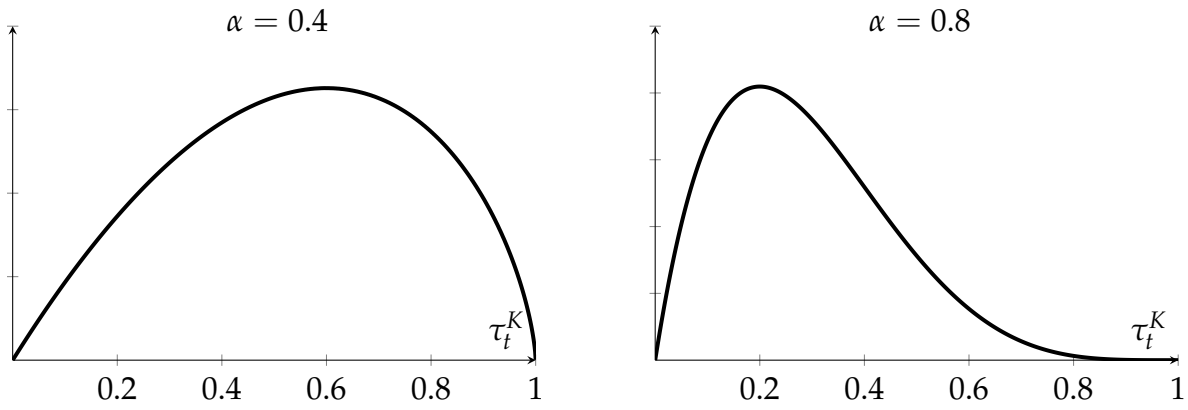


Figure 2: Dependence of Shape of Capital Income Tax Laffer Curves on α

share exceeds 0.5. This is evident in Figure 2, which illustrates the shape of the Laffer curve for two different capital shares. However, the figures also demonstrate that the Laffer curves are locally concave at their maximum. More generally, one can show that this local concavity holds in any sticky wage equilibrium. Consequently, the first-order conditions can be used to identify the global maximum of capital tax revenues.

Lemma 3. *The period-t capital income tax Laffer curve is single-peaked and locally concave in* τ_t^K *at its peak* $1 - \alpha$ *in any sticky wage Ramsey equilibrium.*

Building on these results, I now demonstrate that when TFP is sufficiently high, optimal capital tax rates are uniform and equal to the labor share.

Proposition 2. *If A is sufficiently large, then* $\tau_t^{K*} = 1 - \alpha \; \forall t \in \{1, ..., T\}$.

I provide a sketch of the proof of the above result before discussing its implications. The proof employs a guess-and-verify approach, conjecturing that when TFP is sufficiently large, resource constraints become slack. Under this conjecture, the problem simplifies to maximizing social welfare subject only to the government budget constraint. To proceed, I show that capital and labor taxes can be decoupled. Proposition 1(i) establishes that welfare is independent of capital income taxes, implying that capital taxes affect the relaxed problem solely through the government budget constraint. Additionally, note that labor tax revenue is independent of capital taxes when labor is inelastically supplied. If the resource constraint is slack, the solution to the relaxed problem minimizes the burden on labor taxes—which reduce welfare—by maximizing capital tax revenues. This allows us to reduce the problem of determining optimal capital taxes to one of maximizing total capital tax revenue. Furthermore, because capital tax revenue in period t is unaffected by taxes in period $t' \neq t$ (Proposition 1(ii)), the problem can be solved by pointwise maximization of capital tax revenue in each period; here, I leverage Lemma

3, which demonstrates that the period-t capital income tax Laffer curve is single-peaked and locally concave at its peak. Finally, I verify that the capital tax rate that maximizes revenue is strictly less than one, which turns out to be a sufficient condition for resource constraints to be slack when TFP is sufficiently large. This is because, as long as capital allocations remain strictly positive (ensured when capital taxes are less than one), production rises with increases in TFP. However, since wages are rigid, the increase in TFP does not translate into higher wages. Consequently, total expenditure in the economy grows more slowly than total production, causing the resource constraint to become slack. This validates the initial conjecture.

Proposition 2 shows that capital income taxes should be positive and constant over time when wages are sticky and the rate of return on capital is flexible. This contrasts with the classic result of Atkinson and Stiglitz (1976), who demonstrate that, in their setting, optimal capital income taxes should be zero. In both my model and that of Atkinson and Stiglitz, preferences are separable in consumption and leisure, which implies that the marginal utility of consumption and the marginal disutility of labor are independent. However, a key difference in my environment lies in the treatment of the return on capital. While Atkinson and Stiglitz assume that the return on capital is exogenous, my model endogenizes the return on capital. Specifically, I allow for the rate of return on capital to adjust in response to capital taxes. This flexibility in the rate of return, combined with wage rigidity, generates the rationale for a positive optimal capital tax in my setting, as it leads to an equilibrium adjustment that preserves household welfare while generating fiscal revenue.

Substituting optimal capital taxes obtained in Proposition 2 into the government's lifetime budget constraint, I find that under non-redistributive transfer policies:

$$\sum_{t=1}^{T} \frac{G_t}{(1+r_B)^{t-1}} = \sum_{t=1}^{T} \frac{\tilde{r}_t^*(\mathcal{T})\tilde{K}_t^*(\mathcal{T})(1-\alpha) + w_t L_t^*(\mathcal{T})\tau_t^L}{(1+r_B)^{t-1}}, \tag{3}$$

where $\tilde{x}_t(.)$ caputres the variable $x_t(.)$ when $\tau_t^K = \tau_t^{K*}$. Leveraging the fact that welfare is independent of capital taxes in any sticky wage equilibrium (Proposition 1(i)) and that resource constraints are slack when capital taxes are optimally set (see Lemma 2 and Proposition 2), optimal labor taxes can then be obtained as the solution to the following problem:

$$\max_{(\{\tau_t^L\}_{t\in\{1,\dots,T\}})\in[0,1]^T} \text{SWF s.t. } (3). \tag{P-L}$$

The solution to problem (P-L) coincides with the solution to the planning problem presented in Section 2.3 when the conditions stated in Propositions 1 and 2 are met.

Lemma 4. *A solution to problem* (P-L) *exists.*

Lemma 5. *Suppose A is sufficiently large, and transfer policies are non-redistributive. Then $\tau_t^{L*} < 1 \; \forall t \in \{1, ..., T\}$.*

The next result shows that, although the optimal level of labor taxes remains difficult to pin down, one can derive meaningful insights into their relative levels over time.

Proposition 3. *Suppose A is sufficiently large, transfer policies are non-redistributive, $\beta(1 + r_B) = 1$, and wages and labor supply are stagnant. Then uniform labor income taxation is optimal.*

In the next section, I argue that the uniformity of optimal labor taxes under sticky wages critically depends on labor supply being inelastic.

4 Optimal Taxation with Elastic Labor Supply

In this section, I allow the labor supply of capitalists and workers to be elastic. I show that all the previously stated results on optimal capital taxes continue to hold. However, unlike the case where capitalists and workers supply labor inelastically, optimal labor income taxes are non-uniform and respond to changes in TFP when labor supply is elastic.

I assume the flow utility of the representative capitalist is additively separable in consumption (c) and labor supply (L):

$$u(c, L) = \frac{c^{1-\sigma}}{1-\sigma} - \chi \frac{L^{1+\varphi}}{1+\varphi},$$

where $\chi > 0$, $\varphi > 0$, and $\sigma \geq 0$. The problem for the representative capitalist remains unchanged aside from the fact that capitalists now choose how much labor to supply given wages $\{w_t\}_{t \in \{1,...,T\}}$, the return on capital $\{r_t\}_{t \in \{1,...,T\}}$, and tax policies $\{\tau_t^L, \tau_t^K\}_{t \in \{1,...,T\}}$.

I denote workers' consumption and labor allocations by c^w and L^w, respectively. The flow utility of the representative worker is additively separable in c^w and L^w and given by

$$u^w(c^w, L^w) = v(c^w) - \chi^w \frac{(L^w)^{1+\varphi}}{1+\varphi},$$

where $\chi^w \geq 0$. In addition to the assumptions imposed on v in the model with inelastic labor supply, I assume v has a satiation point. Specfically, there exists a finite level of consumption, $\bar{c}$, such that $\forall c > \bar{c} : v(c) = v(\bar{c})$ and $\forall c < \bar{c} : v(c) < v(\bar{c})$.

Given wages, w_t, labor tax policies, τ_t^L, and transfers, $\mathfrak{T}_t$, the representative worker solves the following problem in period t:

$$\max_{c_t^w, L_t^w} u^w(c_t^w, L_t^w) \quad \text{s.t.} \quad c_t^w = w_t(1-\tau_t^L)L_t^w + \mathfrak{T}_t.$$

Lemma 6. *Suppose $\tau_t^K < 1\ \forall t \in \{1, ..., T\}$ and A is sufficiently large. Then*
(i) $\mathcal{C}^*(\mathcal{T})$ exists and is unique.
(ii) $\frac{\partial L_t^*(\mathcal{T})}{\partial \tau_t^L} \neq 0, \frac{\partial L_t^*(\mathcal{T})}{\partial \tau_s^i} = 0\ \forall s \neq t\ \forall i \in \{L, K\}$.
(iii) $\frac{\partial c_t^*(\mathcal{T})}{\partial \tau_s^i} \neq 0\ \forall s \in \{t, t+1\}\ \forall i \in \{L, K\}, \frac{\partial c_t^*(\mathcal{T})}{\partial \tau_s^i} = 0\ \forall s \notin \{t, t+1\}\ \forall i \in \{L, K\}$.
(iv) $\frac{\partial K_t^*(\mathcal{T})}{\partial \tau_t^i} \neq 0\ \forall i \in \{L, K\}, \frac{\partial K_t^*(\mathcal{T})}{\partial \tau_s^i} = 0\ \forall s \neq t\ \forall i \in \{L, K\}$.

Closed-form solutions for key equilibrium objects are not obtainable when labor supply is elastic. However, using Lemma 6, it can be shown that welfare and government revenues remain neutral to capital taxation under elastic labor supply (i.e., Proposition 1); this is formally shown in Proposition 5 in the Appendix. Moreover, the proofs of Lemma 2 and Lemma 3 go through even when labor supply is elastic. Using these results and observing that labor income tax revenues are independent of capital taxes in equilibrium (Proposition 5(ii)), one can show that Proposition 2 holds as well when labor supply is elastic.

I now characterize optimal labor income taxes under elastic labor supply. As before, optimal labor taxes can be derived as the solution to problem (P-L). The arguments establishing the existence and interiority of the solution (i.e., Lemmas 4 and 5) remain unchanged from the baseline setup with inelastic labor supply.

Assumption 1. $\frac{\varphi w_t - \tilde{r}_t^*(\mathcal{T})(1-\alpha)[\alpha A Z_t(\tilde{r}_t^*(\mathcal{T}))^{-1}]^{\frac{1}{1-\alpha}}}{w_t(1+\varphi)} \in (0,1)\ \forall t$.[12]

Assumption 1 ensures that the peak of government revenues as a function of labor taxes is less than one. Under this assumption, workers' labor supply becomes insensitive to tax policy changes when their satiation point approaches zero. Without imposing assumption 1, optimal labor taxes depend on the shape of v. For now, I switch off this channel and investigate it numerically in Section 6.3. Moreover, in Section 5.1, I show that the main results of this paper go through without imposing this assumption when labor markets are segmented. The next result provides a sharp characterization of the trajectory of optimal labor taxes under elastic labor supply.

[12]Note that this condition can be cast purely in terms of model primitives as $\tilde{r}_1^*(\mathcal{T}) = \delta/\alpha$ and $\tilde{r}_t^*(\mathcal{T}) = \frac{1-\beta(1-\delta)}{\beta\alpha}\ \forall t \in \{2, ..., T\}$.

Lemma 7. *Suppose assumption 1 holds, χ^w and $\bar{c}$ are sufficiently small, A is sufficiently large, and transfer policies are non-redistributive. Then optimal labor income taxes can be approximated by $\{\tau_t^L\}_{t\in\{1,...,T\}}$ that satisfy equation (3) and*

$$\varphi - \frac{ac_1 - \tau_1^L}{1-\tau_1^L} = \frac{1}{\{\beta(1+r_B)\}^{t-1}}\left[\varphi - \frac{b_t c_t - \tau_t^L}{1-\tau_t^L}\right] \forall t \in \{2,...,T\}, \tag{4}$$

where $a \equiv \frac{1}{w_1}\left\{\frac{1}{\delta}\right\}^{\frac{\alpha}{1-\alpha}}$, $b_t \equiv \frac{1}{w_t}\left\{\frac{\beta}{1-\beta(1-\delta)}\right\}^{\frac{\alpha}{1-\alpha}}$, *and* $c_t \equiv \frac{(1-\alpha)(\alpha^2 A Z_t)^{\frac{1}{1-\alpha}}}{\alpha}$.

Using Lemma 7, I now demonstrate how the optimal timing of labor taxes depends on the evolution of wages and firm productivity.

Proposition 4. *Suppose assumption 1 holds, χ^w and $\bar{c}$ are sufficiently small, and transfer policies are non-redistributive. Then*

$$(i)\ \lim_{A\to\infty}\tau_1^{L*} \lesseqgtr \lim_{A\to\infty}\tau_t^{L*} \text{ if } \frac{w_1}{w_t} \lesseqgtr \{\beta(1+r_B)\}^{t-1}\left(\frac{Z_1}{Z_t}\right)^{\frac{1}{1-\alpha}}\left\{\frac{1-\beta(1-\delta)}{\beta\delta}\right\}^{\frac{\alpha}{1-\alpha}}, \textit{and}$$

$$(ii)\ \forall t,t' \geq 2, t\neq t\ \lim_{A\to\infty}\tau_t^{L*} \lesseqgtr \lim_{A\to\infty}\tau_{t'}^{L*} \text{ if } \frac{w_t}{w_{t'}} \lesseqgtr \{\beta(1+r_B)\}^{t'-t}\left(\frac{Z_t}{Z_{t'}}\right)^{\frac{1}{1-\alpha}}.$$

The proof of this result uses Lagrangian methods. To gain some intuition for this result, below I sketch out an alternative perturbation argument for the case where $T = 2$, $\beta(1+r_B) = 1$, and Z_t is time-invariant. Note that the wage ratio threshold, $\bar{w} \equiv \left\{\frac{1-\beta(1-\delta)}{\beta\delta}\right\}^{\frac{\alpha}{1-\alpha}}$, below which labor income taxes should be back-loaded is strictly greater than 1. In particular, if wages are stagnant, the optimal policy is to back-load labor income taxes when A is sufficiently large. Now note that when A is sufficiently large, capital tax revenue outweighs labor tax revenue as equilibrium capital allocations increase with A, whereas equilibrium labor allocations and rates of return on capital remain unaffected by changes in A. Consequently, for government revenue requirements, the effect of labor taxation on capital tax revenue becomes more significant than its effect on labor tax revenue when A is high enough. However, changes in labor taxation do impact welfare, potentially leading to some welfare loss when adjusting the timing of labor taxes. Yet, if labor taxes are initially constant over time, one can design an alternative back-loaded labor tax policy that enhances welfare while satisfying the government budget constraint. To see this, suppose optimal labor tax policies satisfy $\tau_1^L = \tau_2^L$. Now note

that welfare in a sticky price equilibrium can be written as

$$\begin{aligned}\text{SWF} =& \frac{\mu}{\chi^{1/\varphi}}\left[1-\frac{\chi}{1+\varphi}\right]\left[\{w_1(1-\tau_1^L)\}^{1/\varphi+1}+\beta\{w_2(1-\tau_2^L)\}^{1/\varphi+1}\right] \\ &+(1-\mu)\sum_{t\in\{1,2\}}\beta^{t-1}u^w(c_t^{w*}(\tau_t^L),L_t^{w*}(\tau_t^L)).\end{aligned}$$

Since the change in worker utility due to a marginal change in labor taxes is approximately zero in equilibrium when χ^w is small enough, by increasing τ_2^L by $\epsilon > 0$ and reducing τ_1^L by ϵ, the perturbed labor tax profile yields higher welfare (given $\beta < 1$ and $w_1 = w_2$). Additionally, as $\epsilon \to 0$, this perturbation decreases period 2 capital tax revenue by $\tilde{r}_2^*\tau_2^{K*}\partial\tilde{K}_2^*/\partial\tau_L^2$ and increases period 1 capital tax revenue by $\tilde{r}_1^*\tau_1^{K*}\partial\tilde{K}_1^*/\partial\tau_L^1$. Since $\tau_1^{K*} = \tau_2^{K*}$ (per Proposition 2), this allocation will increase government revenue if $\tilde{r}_2^*\partial\tilde{K}_2^*/\partial\tau_L^2 < \tilde{r}_1^*\partial\tilde{K}_1^*/\partial\tau_L^1$. One can verify that this condition always holds when wages and capital taxes are constant over time, and workers' labor supply is not highly sensitive to changes in labor taxes. This occurs because, in an interior sticky wage equilibrium in which capital taxes are constant over time, the return on capital in period 2 must be higher than the return on capital in period 1, ensuring that the marginal utility of investment is equalized across periods. It is worth noting that even when the social welfare function is convex in labor income taxes, it can be shown—using the perturbation argument outlined above—that uniform labor taxation is never optimal when productivity is sufficiently high and wages are rigid.

If the ratio of wages in period 1 to wages in period 2 exceeds $\bar{w}$, then as productivity tends to infinity, optimal labor income taxes should be front-loaded. The intuition behind this result is as follows: increasing τ_1^L has two effects when wages in period 1 rise. First, the increase in w_1 directly improves welfare, and raising τ_1^L reduces this gain. However, increasing τ_1^L also relaxes the government's budget constraint, and this effect is stronger when w_1 is higher. If the latter effect dominates, it justifies a front-loading of labor taxes, as the government can achieve its revenue goals more efficiently by taxing higher wages in the earlier period. In contrast, when wages are declining over time, the optimal policy is to back-load labor income taxes as productivity increases. Also note that as δ or β tends to zero, $\bar{w}$ tends to infinity. This implies that when δ or β are small enough, then no matter how wages evolve, as long as they remain finite, labor income taxes should be back-loaded as productivity tends to infinity.

Proposition 4(i) further implies that, as A approaches infinity while Z_t remains unchanged, factor prices and the capital share become sufficient statistics for determining the optimal timing of taxes. Specifically, the wage threshold beyond which back-loading

of labor taxes becomes optimal can be expressed as $\left\{\frac{\tilde{r}_2}{\tilde{r}_1}\right\}^{\frac{\alpha}{1-\alpha}}$ when $\beta(1+r_B)=1$. This suggests that, even if both factor prices are rising over time, labor taxes should be back-loaded if the rate of return on capital increases significantly more than wages.

In general, the wage threshold for backloading labor taxes also depends on the level of r_B. Specifically, the case for backloading strengthens as r_B rises when $A \to \infty$. A higher r_B reduces the present discounted value of future labor tax revenue. Since this value is already minimal, increasing future labor income tax rates has little impact on government revenue but significantly lowers welfare when β is high. This effect exerts pressure to front-load labor income taxes as r_B rises. However, as r_B increases, marginal changes in the present discounted value of future capital tax revenue due to labor tax adjustments tend to decline. To equalize these marginal changes across periods, the planner increasingly favors backloading labor taxes as r_B increases. The latter effect dominates when $A \to \infty$.

As argued above, under uniform labor taxation, differences in capital tax revenue across periods 1 and 2 can arise due to differences in the rates of return on capital, even if productivity remains constant. However, from period 2 onward, the rate of return on capital stabilizes. To see this, note that part (ii) of Lemma 1 and Proposition 2 together imply that $r_t^*(\mathcal{T}^*) = \frac{1-\beta(1-\delta)}{\beta\alpha}\ \forall t \geq 2$ when capital taxes are optimally set. Consequently, from period 2 onward, any differences in capital tax revenue across periods stem solely from differences in capital allocations due to time-varying productivity when labor tax rates are constant over time. To understand how time-varying productivity affects the optimal timing of labor taxes, note that capital tax revenue is more sensitive to changes in labor tax rates when productivity is high. This implies that as the time-varying component of productivity declines over time, the effect of a marginal change in labor tax rates on capital tax revenue also diminishes. Thus, a uniform labor tax policy becomes suboptimal, and shifting toward a back-loaded labor income tax policy can increase welfare. Conversely, if the time-variant component of productivity rises sufficiently over time, front-loading labor taxes becomes optimal (see part (ii) of Proposition 4).

The key insight here is that, when wages are rigid, high productivity amplifies capital tax revenue relative to labor tax revenue, making the impact of labor taxation on capital tax revenue more significant than its direct effect on labor tax revenue. Since labor income taxes reduce capital tax revenue by distorting capital allocations, it becomes optimal for the planner to set labor taxes across periods to balance the marginal loss in capital tax revenue resulting from labor taxation when productivity is sufficiently high. As the time-varying component of TFP affects these marginal losses, it also influences the timing of optimal labor income taxes.

5 Extensions

5.1 Segemented Labor Markets

In this section, I analyze a setting where capitalists and workers participate in segmented labor markets, with their labor supply assumed to be elastic.

There is a continuum of *capital-intensive* firms of unit measure that hire capitalists. These firms employ the same technology as described in the baseline setup. In each period, given $\{w_t, r_t\}$, the representative capital-intensive firm chooses labor and capital demand, (L_t^d, K_t^d), to maximize profits.

I assume that firms hiring workers are distinct from those hiring capitalists. These firms are *labor-intensive*, with output Y_t^w depending solely on labor allocations L_t^{wd} through the production function $Y_t^w = f(L_t^{wd})$. I assume that $f'(.)$ is bounded below by $\underline{f}$. In each period, given the wage w_t^w, the representative labor-intensive firm solves the following problem:

$$\max_{L_t^{wd}} Y_t^w - w_t^w L_t^{wd}.$$

Resource constraints in this economy are given by:[13]

$$c_1 + c_1^w + G_1 + K_2 - \delta K_1 \leq Y_1 + Y_1^w, \tag{5a}$$

$$c_t + c_t^w + G_t + K_{t+1} - K_t(1-\delta) \leq Y_t + Y_t^w \ \forall t \in \{2, ..., T-1\}, \tag{5b}$$

$$c_T + c_T^w + G_T - K_T(1-\delta) \leq Y_T + Y_T^w. \tag{5c}$$

Definition 2. *Given $\{\{w_t, w_t^w, \mathfrak{T}_t\}_{t\in\{1,...,T\}}, \mathcal{T}\}$, a sticky wage equilibrium under segmented labor markets is given by sequences of allocations $\{c_t, c_t^w, L_t, L_t^w, K_t, L_t^d, L_t^{wd}, K_t^d\}_{t\in\{1,...,T\}}$ and interest rates $\{r_t\}_{t\in\{1,...,T\}}$ that satisfy the following conditions:*

1. *Given $\{\{w_t, r_t\}_{t\in\{1,...,T\}}, \mathcal{T}\}$, $\{c_t, L_t, K_t\}_{t\in\{1,...,T\}}$ solve the capitalists problem.*
2. *Given $\{w_t^w, \mathfrak{T}_t, \tau_t^L\}$, $\{c_t^w, L_t^w\}$ solve the workers problem $\forall t \in \{1, ..., T\}$.*
3. *Given $\{w_t, r_t\}$, $\{L_t^d, K_t^d\}$ solve the capital-intensive firms problem $\forall t \in \{1, ..., T\}$.*
4. *Given w_t^w, L_t^{wd} solves the labor-intensive firms problem $\forall t \in \{1, ..., T\}$.*
5. *The capital market clearing condition (1) holds.*

[13] The main results of this paper remain robust even under the alternative assumption that workers consume only goods produced by labor-intensive firms, while capitalists consume only goods produced by capital-intensive firms, provided workers' wages are sufficiently low. This assumption reflects a segmentation in preferences, where workers may have cultural or practical inclinations toward goods produced by labor-intensive firms, and capitalists may prefer goods from capital-intensive firms. The segmentation is particularly plausible if the goods are distinct in nature—for example, basic necessities versus luxury items.

6. *Given wages* $\{w_t, w_t^w\}$*, equilibrium labor allocations for capitalists and workers in period* t *are given by* $\min\{L_t^d, L_t\}$ *and* $\min\{L_t^{wd}, L_t^w\}$.

7. *Resource constraints (5a)–(5c) hold.*

By imposing the additional condition that $\underline{f}$ is sufficiently large, it can be shown that the main findings on the welfare-neutrality of capital taxes and the characterization of optimal capital tax rates remain valid. Furthermore, under segmented markets, neither Assumption 1 nor the condition that $\bar{c}$ is sufficiently small are necessary to establish the main result on the optimal timing of labor taxes (i.e., Proposition 4), provided $\underline{f}$ is sufficiently large. More details are provided in Appendix B.

5.2 Nominal Wage Stickyness

Here, I argue that the results of this paper hinge on *real* wages being sticky rather than *nominal* ones. To fix ideas, suppose nominal, instead of real, wages are determined through a Nash (1953) demand game. This auction occurs at the start of period 1, where the worker and firm sign a long-term contract for T periods. The contract specifies the nominal wage in period 1, W_1, and a constant wage indexation rate, ι. I assume the price level, P_t, is pinned down by the binding resource constraint. Consequently, the real wage rate $w_t = \frac{W_1(1+\iota)^t}{P_t}$ will vary over time unless the wage indexation rate equals the realized inflation rate.

Following the logic outlined in the proof of Proposition 2, it can be shown that when resource constraints bind:

$$\frac{W_1(1+\iota)^t(1-\tau_t^L)}{P_t} + \frac{G_t(\chi P_t)^{1/\varphi}}{\left[W_1(1+\iota)^t(1-\tau_t^L)\right]^{1/\varphi}} = AZ_t[1-\alpha(1-\tau_t^K)]\left(\frac{\alpha AZ_t\beta(1-\tau_t^K)}{1-\beta(1-\delta)}\right)^{\frac{\alpha}{1-\alpha}}.$$

This condition implies that P_t depends on τ_t^K. As a result, the social welfare function (SWF) also depends on τ_t^K via w_t, invalidating Proposition 1. Since Proposition 1 does not hold, Propositions 2 and 4, which rely on Proposition 1, also fail to hold. This illustrates that my results characterizing the optimal timing of capital and labor taxes depend critically on real wages being sticky rather than nominal ones.

5.3 Time-varying Factor Shares

TFP tends to be more volatile than labor or capital shares, especially in the short run. This is because TFP is sensitive to a range of factors beyond the direct input of labor or capital, such as changes in efficiency, regulatory shifts, and unexpected economic shocks. For instance, Basu and Fernald (2001) find that TFP adjustments often result from cyclical

variations in utilization rates, such as hours worked and capacity usage. Such fluctuations make TFP more volatile relative to factor shares, which tend to shift gradually in response to structural changes, such as automation.

However, over the medium term, factor shares can vary considerably. Using a data set that contains corporate sector-level information on the income structure of 59 countries for various years between 1975 and 2012, Karabarbounis and Neiman (2014) document a global decline in labor share. In addition to TFP changes, such shifts in factor shares can also influence the optimal timing of labor taxes. Specifically, if the capital share, α, is time-variant, then both the return on capital and capital allocations will vary over time. Using the logic of Propositions 1 and 2, it can be shown that when α changes over time, optimal capital taxes will also vary accordingly. This implies that returns on capital and capital allocations will fluctuate with time due to variations in α. In this case, the relative magnitude of optimal labor taxes across periods will depend not only on relative wages and time-varying productivity but also on relative capital shares.

6 Robustness

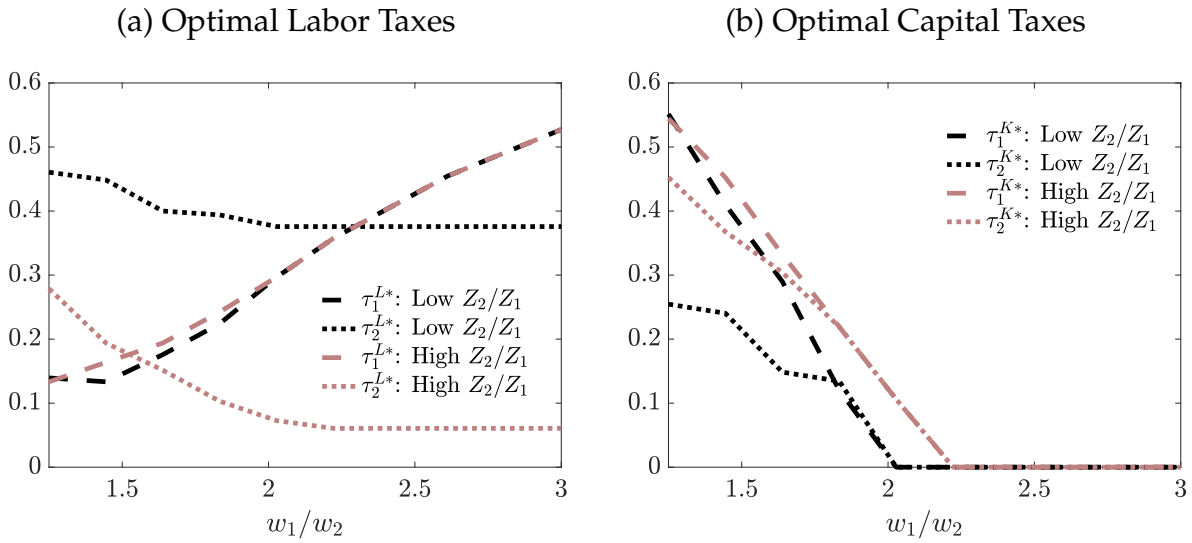
The theoretical results presented thus far rely on three key assumptions: (i) TFP is sufficiently large, (ii) capital owners have quasi-linear preferences, and (iii) transfers are non-redistributive. In this section, I numerically demonstrate under plausible parameter values that our main results are not highly sensitive to these assumptions. Throughout these exercises, I maintain the assumption of segmented labor markets.

6.1 $A < \infty$

To account for potential non-convexities, I use a brute-force approach to solve for the optimal tax policies. The parameters used align with those commonly employed in business cycle literature: the labor disutility coefficient, χ, is set to 1; the Frisch elasticity of labor supply, $1/\varphi$, is 0.5; the capital share, α, is 0.35; the time-preference rate, β, is 0.95; and the capital depreciation rate, δ, is 0.05. The fixed component of total factor productivity, A, the time-varying components of total factor productivity, $\{Z_t\}_{t\in\{1,2\}}$, and the wage in period 2, w_2, are all normalized to 1; $\{w_t^w\}_{t\in\{1,2\}}$ are set to 0; government expenditures, $\{G_t\}_{t\in\{1,2\}}$, are set to 0.5; the Pareto weight, μ, is set to 1, and the return on bonds, r_B, is $1/\beta - 1$.

Figure 3 depicts simulated optimal labor and capital taxes for varying values of the wage ratio w_1/w_2. Panel (a) shows that conditions favoring back-loading of labor taxes become more prominent when Z_2/Z_1 is low, underscoring how variations in the TFP affect optimal labor tax timing, even when productivity levels are finite. Additionally,

when A is finite, resource constraints may bind. In such scenarios, optimal capital taxes might deviate from being constant and/or strictly positive, as reflected in panel (b).[14]



Notes: These figures present simulated optimal labor taxes for different values of the wage ratio, w_1/w_2. Specifically, I vary w_1 keeping w_2 fixed. The low Z_2/Z_1 scenario corresponds to the baseline parameterization where $Z_2 = Z_1 = 1$. In the high Z_2/Z_1 scenario, $Z_2 = 1.1$ and $Z_1 = 1$.

Figure 3: Optimal Tax Profiles: Front-Loading vs. Back-Loading Scenarios

6.2 $\sigma > 0$

Theoretically, the welfare neutrality of capital taxes hinges on the preferences of capitalists being quasi-linear in consumption (i.e., $\sigma \to 0$). In this section, I demonstrate that this neutrality result remains quantitatively robust under reasonable parameter values, even when $\sigma > 0$.

Figure 4 illustrates the elasticity of sticky-price equilibria and welfare with respect to capital income tax rates across different values of σ in a two-period setting. For this analysis, I assume capital taxes are uniform across periods, and fix labor income tax rates in both periods at 0.4—approximately the top personal income tax rate in the U.S. I set $A = 4$ to ensure that resource constraints are slack $\forall \tau_t^K \in (0, 1)$. All other parameters are consistent with those in Section 6.1. The figure reveals that welfare exhibits low sensitivity to changes in capital tax rates when these rates are low (panel (a)). This insensitivity extends to its components, i.e., consumption and labor allocations (panels (b)–(e)). Although capital allocations decline substantially with higher capital income taxes (panels (f)–(g)), the rate of return to capital rises significantly (panels (h)–(i)). As a result, household income

[14] I obtain similar results even when the rates of return on capital are equal across periods (i.e., $r_1 = r_2^*$), and differences in equilibrium allocations across periods arise solely from variations in total factor productivity (i.e., $Z_1 \neq Z_2$).

remains relatively stable, mitigating the impact on welfare. Furthermore, welfare tends to be more insensitive to changes in capital tax rates when σ is lower. Since capital income earners are predominantly wealthier households who typically exhibit low values of σ (even if not exactly zero), these quantitative findings suggest that the theoretical results are not merely capturing a limiting case but rather reflect a broader, more general phenomenon.

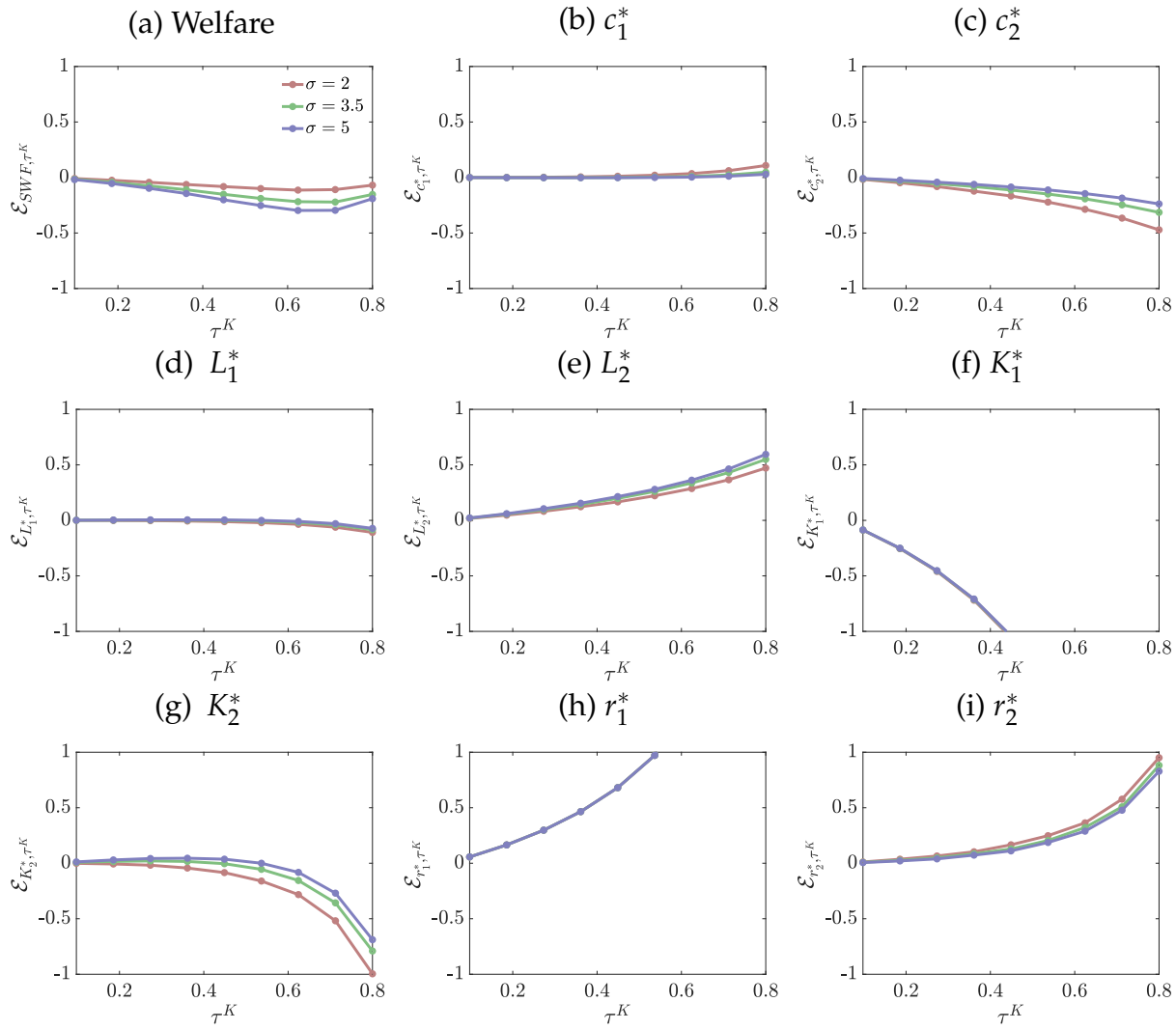


Figure 4: Elasticity of Equilibria and Welfare with Respect to Capital Income Taxes

6.3 Redistributive Concerns

Note that in the theoretical analysis, optimal labor income taxes are derived under the assumption that transfer policies are non-redistributive. In this section, I discuss how redistributive concerns can alter the optimal timing of labor taxes.

I illustrate this numerically in Figure 5. I consider transfer policies of the form:

$$T_t = v w_t^w \tau_t L_t^w.$$

A negative income tax on workers corresponds to $v > 1$, implying redistribution from capitalists to workers. I set v to 1.5, which corresponds to a 50 percent negative income tax (NIT) on workers.

I assume $v(c) = \frac{c^{1-\sigma^w}}{1-\sigma^w}$, and set the coefficient of relative risk aversion for workers, σ^w, to 1.5. I assume $f(L) = L$. I adopt the utilitarian case by setting $\mu = 0.5$; set the labor disutility coefficient for workers, χ^w, to 1; set the time-invariant component of TFP, A, to 1.1; and normalize $\{w_t^w\}_{t\in\{1,2\}}$ to 1. Other parameters remain unchanged from Section 6.1.

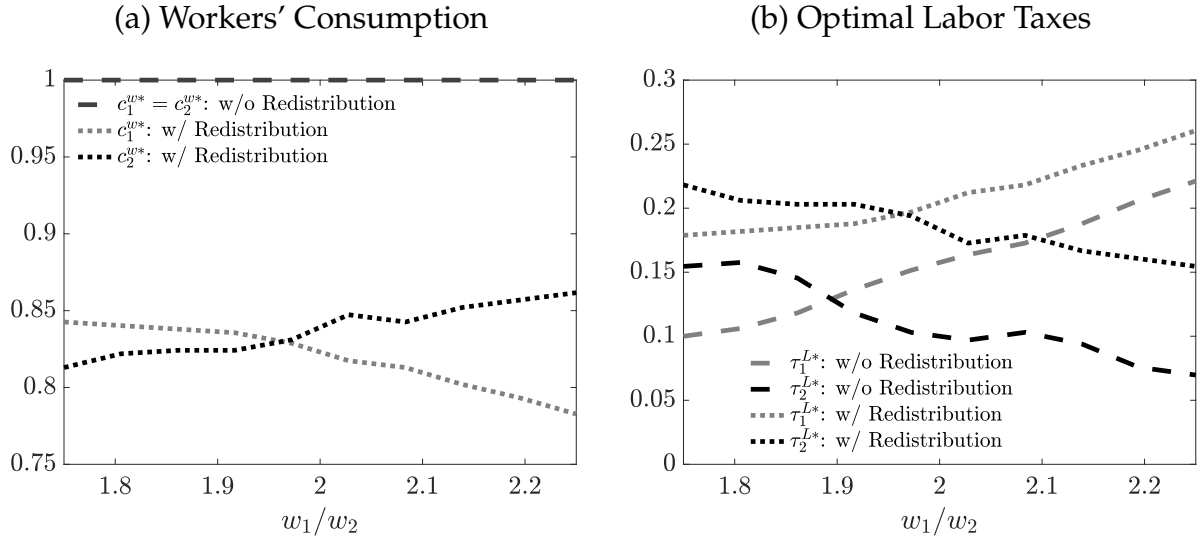
Panel (a) of Figure 5 shows that under non-redistributive policies ($v = 1$) and stagnant workers' wages, workers' consumption allocations are uniform across time and states.

However, under redistributive policies, there is no presumption that $\{w_t^w, \tau_t^L\}_{t\in\{1,2\}}$ are such that workers' labor supply, $\{L_t^w\}_{t\in\{1,2\}}$, is constant across time. Thus, workers' consumption allocations, $\{c_t^w\}_{t\in\{1,2\}}$, will also generally be non-uniform. When w_1 is high (low), labor taxes tend to be front-loaded (back-loaded), resulting in back-loaded (front-loaded) labor supply and consumption.

Since v is concave, relative to the non-redistributive case, optimal taxation under redistribution increases taxes by more in periods with higher consumption, shifting the wage ratio threshold below which labor income taxes should be backloaded (Panel (b) of Figure 5). In the current example, this threshold is higher under redistributive policies than under non-redistributive ones.

7 Conclusion

This paper explores the implications of wage rigidity for optimal capital and labor income taxation. I show that under wage rigidity, capital income taxes tend to be welfare-neutral, a result that contrasts with the negative welfare effects typically associated with capital taxation in settings with flexible wages. This implies that the optimal capital tax rate is positive when TFP is sufficiently high, challenging the conventional wisdom that capital taxes should be zero. In terms of labor taxation, I find that the optimal structure depends on the evolution of TFP and wages. Under flexible wages, optimal labor income taxes tend to remain uniform over the business cycle, in contrast to the time-varying optimal labor taxes that arise under wage rigidity. My analysis provides insights into the optimal timing of labor taxes under wage rigidity, illustrating when front-loading or back-loading labor taxes are desirable.



Notes: These figures compare optimal labor taxes under non-redistributive policies ($v = 1$) with those under redistribution ($v = 1.5$).

Figure 5: Optimal Labor Tax Profiles under Redistibution

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A Omitted Proofs

A.1 Proof of Lemma 1

First note that we can re-write the capitalists' problem as an unconstrained maximization problem:

$$\begin{aligned}\max_{\{K_t\}_{t\in\{1,...,T\}}} & \frac{\left(w_1L_1(1-\tau_1^L)+r_1K_1(1-\tau_1^K)-K_2-\delta K_1\right)^{1-\sigma}}{1-\sigma}-\chi\frac{L_1^{1+\varphi}}{1+\varphi}\\ &+\sum_{t=1}^{T-1}\beta^{t-1}\left\{\frac{\left(w_tL_t(1-\tau_t^L)+r_tK_t(1-\tau_t^K)+K_t(1-\delta)-K_{t+1}\right)^{1-\sigma}}{1-\sigma}-\chi\frac{L_t^{1+\varphi}}{1+\varphi}\right\}\\ &+\beta^{T-1}\left\{\frac{\left(w_TL_T(1-\tau_T^L)+r_TK_T(1-\tau_T^K)+K_T(1-\delta)\right)^{1-\sigma}}{1-\sigma}-\chi\frac{L_T^{1+\varphi}}{1+\varphi}\right\}\end{aligned}$$

The first-order condition w.r.t. K_1 pins down the rate of return on capital in period 1:

$$r_1^*(\mathcal{T})=\frac{\delta}{1-\tau_1^K}.$$

Similarly, the first-order condition w.r.t. K_2 pins down the rate of return on capital in period 2:

$$r_t^*(\mathcal{T})=\frac{1-\beta(1-\delta)}{\beta(1-\tau_t^K)}\ \forall t\in\{1,...,T\}.$$

Next, substituting in the closed-form solutions for $r_t^*(\mathcal{T})$ into firm profit maximization condition (i.e., FOC w.r.t. K_t^d) and using the market clearing condition (1), we obtain a closed-form solution for $K_t^*(\mathcal{T})$:

$$K_t^*(\mathcal{T})=L_t^*(\mathcal{T})\left(\frac{\alpha AZ_t}{r_t^*(\mathcal{T})}\right)^{\frac{1}{1-\alpha}}\ \forall t\in\{1,...,T\}.$$

Specifically,

$$K_1^*(\mathcal{T})=(L_1+L_1^w)\left\{\frac{\alpha AZ_1(1-\tau_1^K)}{\delta}\right\}^{\frac{1}{1-\alpha}},$$

and

$$K_t^*(\mathcal{T})=(L_t+L_t^w)\left\{\frac{\alpha AZ_t\beta(1-\tau_t^K)}{1-\beta(1-\delta)}\right\}^{\frac{1}{1-\alpha}}\ \forall t\in\{2,...,T\}.$$

Lastly, we obtain closed-form solutions for consumption allocations using the per-period budget constraints. Specifically,

$$\begin{aligned} c_1^*(\mathcal{T}) &= w_1 L_1(1-\tau_1^L) + K_1^*(\mathcal{T}) \overbrace{\{r_1^*(\mathcal{T})(1-\tau_1^K(\mathcal{T})) - \delta\}}^{=0} - K_2^*(\mathcal{T}) \\ &= w_1 L_1(1-\tau_1^L) - (L_2 + L_2^w)\left\{\frac{\alpha A Z_2 \beta(1-\tau_2^K)}{1-\beta(1-\delta)}\right\}^{\frac{1}{1-\alpha}}. \end{aligned}$$

Moreover,

$$\begin{aligned} c_T^*(\mathcal{T}) &= w_T L_T(1-\tau_T^L) + K_T^*(\mathcal{T})\{r_T^*(\mathcal{T})(1-\tau_T^K) + 1 - \delta\} \\ &= w_T L_T(1-\tau_T^L) + (L_T + L_T^w)\left\{\frac{\alpha A Z_T \beta(1-\tau_T^K)}{1-\beta(1-\delta)}\right\}^{\frac{1}{1-\alpha}} \left\{\frac{1-\beta(1-\delta)}{\beta(1-\tau_T^K)}(1-\tau_T^K) + 1 - \delta\right\} \\ &= w_T L_T(1-\tau_T^L) + \frac{1}{\beta}(L_T + L_T^w)\left\{\frac{\alpha A Z_T \beta(1-\tau_T^K)}{1-\beta(1-\delta)}\right\}^{\frac{1}{1-\alpha}}. \end{aligned}$$

Similarly,

$$\begin{aligned} c_t^*(\mathcal{T}) = w_t L_t(1-\tau_t^L) &+ \frac{1}{\beta}(L_t + L_t^w)\left\{\frac{\alpha A Z_t \beta(1-\tau_t^K)}{1-\beta(1-\delta)}\right\}^{\frac{1}{1-\alpha}} \\ &- (L_{t+1} + L_{t+1}^w)\left\{\frac{\alpha A Z_{t+1} \beta(1-\tau_{t+1}^K)}{1-\beta(1-\delta)}\right\}^{\frac{1}{1-\alpha}} \quad \forall t \in \{2, ..., T-1\}. \end{aligned}$$

Note that capital allocations are finite-valued as $\delta > 0$; and the rates of return on capital are finite-valued as $\beta > 0$ and $\tau_t^K < 1 \; \forall t \in \{1, ..., T\}$. This fully characterizes the solution to the capitalists' problem. Also, note that the functional forms for capitalists' allocations and equilibrium prices in sticky wage equilibria are explicit, which implies that they are unique. □

A.2 Proof of Proposition 1

Using Lemma 1, we can express welfare as

$$\begin{aligned} \text{SWF} = \mu\Bigg(& w_1 L_1(1-\tau_1^L) - (L_2 + L_2^w)\left\{\frac{\alpha A Z_2 \beta(1-\tau_2^K)}{1-\beta(1-\delta)}\right\}^{\frac{1}{1-\alpha}} \\ & + \sum_{t=2}^{T-1} \beta^{t-1}\Bigg[w_t L_t(1-\tau_t^L) + \frac{1}{\beta}(L_t + L_t^w)\left\{\frac{\alpha A Z_t \beta(1-\tau_t^K)}{1-\beta(1-\delta)}\right\}^{\frac{1}{1-\alpha}} \end{aligned}$$

$$- (L_{t+1} + L_{t+1}^w)\left\{\frac{\alpha A Z_{t+1}\beta(1-\tau_{t+1}^K)}{1-\beta(1-\delta)}\right\}^{\frac{1}{1-\alpha}}\Bigg] + \beta^{T-1}\Bigg[w_T L_T(1-\tau_T^L)$$
$$+ \frac{1}{\beta}(L_T + L_T^w)\left\{\frac{\alpha A Z_T \beta(1-\tau_T^K)}{1-\beta(1-\delta)}\right\}^{\frac{1}{1-\alpha}} - \frac{\chi}{1+\varphi}\left\{\frac{w_T(1-\tau_T^L)}{\chi}\right\}^{\frac{1+\varphi}{\varphi}}\Bigg]\Bigg)$$
$$+ (1-\mu)\sum_{t\in\{1,2\}}\beta^{t-1}v(c_t^{w*}(\tau_t^L)).$$

when $\sigma \to 0$. Canceling like terms, one can show that SWF is independent of $\{\tau_t^K\}_{t\in\{1,\ldots,T\}}$ as $\{w_t\}_{t\in\{1,\ldots,T\}}$ is fixed. This proves part (i). To see part (ii), note that Lemma 1 implies $\{K_t^*(\mathcal{T})\}$ and $\{r_t^*(\mathcal{T})\}$ are unaffected by taxes in periods $t' \neq t$. □

A.3 Proof of Lemma 2

I show that if $\tau_t^K \in [0,1)$, then for any given sequence of real wages, $\{w_t\}$, the resource constaint (2b),

$$c_t + c_t^w + G_t + K_{t+1} - (1-\delta)K_t \leq Y_t,$$

is satisfied if A is sufficiently large. To see this, recall the capitalist's period-t budget constraint:

$$c_t + K_{t+1} - (1-\delta)K_t = r_t K_t(1-\tau_t^K) + w_t L_t(1-\tau_t^L).$$

Aggregating, the resource constraint can be written as:

$$w_t L_t(1-\tau_t^L) + r_t K_t(1-\tau_t^K) + c_t^w + G_t \leq A Z_t K_t^\alpha (L_t + L_t^w)^{1-\alpha}.$$

Substituting in equilibrium capital allocations (see Lemma 1 (i)), we get:

$$w_t L_t(1-\tau_t^L) + r_t(L_t + L_t^w)\left(\frac{\alpha A Z_t}{r_t}\right)^{\frac{1}{1-\alpha}}(1-\tau_t^K) + c_t^w + G_t$$
$$\leq A Z_t\left[(L_t + L_t^w)\left(\frac{\alpha A Z_t}{r_t}\right)^{\frac{1}{1-\alpha}}\right]^\alpha (L_t + L_t^w)^{1-\alpha}.$$

Dividing $L_t + L_t^w > 0$ from both sides:

$$\frac{w_t L_t(1-\tau_t^L)}{L_t + L_t^w} + r_t\left(\frac{\alpha A Z_t}{r_t}\right)^{\frac{1}{1-\alpha}}(1-\tau_t^K) + \frac{c_t^w}{L_t + L_t^w} + \frac{G_t}{L_t + L_t^w} \leq A Z_t\left(\frac{\alpha A Z_t}{r_t}\right)^{\frac{\alpha}{1-\alpha}}.$$

Substituting in equilibrium rates of return on capital (see Lemma 1 (ii)), we get:

$$\frac{w_t L_t(1-\tau_t^L)}{L_t+L_t^w}+\frac{1-\beta(1-\delta)}{\beta(1-\tau_t^K)}\left(\frac{\alpha A Z_t \beta(1-\tau_t^K)}{1-\beta(1-\delta)}\right)^{\frac{1}{1-\alpha}}(1-\tau_t^K)+\frac{c_t^w}{L_t+L_t^w}+\frac{G_t}{L_t+L_t^w}$$
$$\leq AZ_t\left(\frac{\alpha A Z_t \beta(1-\tau_t^K)}{1-\beta(1-\delta)}\right)^{\frac{\alpha}{1-\alpha}}.$$

If $\tau_t^K < 1$, then we can cancel $(1-\tau_t^K)$ in the above equation and rearrange terms to get:

$$\frac{w_t L_t(1-\tau_t^L)}{L_t+L_t^w}+\frac{c_t^w}{L_t+L_t^w}+\frac{G_t}{L_t+L_t^w}\leq AZ_t\Psi_t^{\frac{\alpha}{1-\alpha}}-\frac{1-\beta(1-\delta)}{\beta}\Psi_t^{\frac{1}{1-\alpha}},$$

where $\Psi_t \equiv \frac{\alpha A Z_t \beta(1-\tau_t^K)}{1-\beta(1-\delta)}$. Dividing both sides by $AZ_t\Psi_t^{\frac{\alpha}{1-\alpha}}$, we find:

$$\frac{w_t L_t(1-\tau_t^L)}{(L_t+L_t^w)AZ_t\Psi_t^{\frac{\alpha}{1-\alpha}}}+\frac{c_t^w}{(L_t+L_t^w)AZ_t\Psi_t^{\frac{\alpha}{1-\alpha}}}+\frac{G_t}{(L_t+L_t^w)AZ_t\Psi_t^{\frac{\alpha}{1-\alpha}}}\leq 1-\alpha(1-\tau_t^K).$$

From the workers' period-t budget constraint, $c_t^w = w_t L_t^w(1-\tau_t^L)+\mathfrak{T}_t$. Thus, the above condition becomes:

$$\frac{w_t L_t(1-\tau_t^L)}{(L_t+L_t^w)AZ_t\Psi_t^{\frac{\alpha}{1-\alpha}}}+\frac{w_t L_t^w(1-\tau_t^L)+\mathfrak{T}_t}{(L_t+L_t^w)AZ_t\Psi_t^{\frac{\alpha}{1-\alpha}}}+\frac{G_t}{(L_t+L_t^w)AZ_t\Psi_t^{\frac{\alpha}{1-\alpha}}}\leq 1-\alpha(1-\tau_t^K).$$

Note that the above condition holds when $A\to\infty$ as long as $\tau_t^K \geq 0$. Similarly, it can be shown that (2a) and (2c) are also slack. □

A.4 Proof of Lemma 3

Capital tax revenue in period t can be written as $\text{CTR}(\tau_t^K) = \Omega\tau_t^K(1-\tau_t^K)^{\frac{1}{1-\alpha}-1}$, where $\Omega > 0$. Note that

$$\frac{\partial \text{CTR}(\tau_t^K)}{\partial \tau_t^K} = \Omega(1-\tau_t^K)^{\frac{1}{1-\alpha}-2}\left[1-\frac{\tau_t^K}{1-\alpha}\right].$$

Thus, the sign of $\frac{\partial \text{CTR}(\tau_t^K)}{\partial \tau_t^K}$ depends on the sign of $1-\frac{\tau_t^K}{1-\alpha}$. If $\tau_t^K < 1-\alpha$, then $\frac{\partial \text{CTR}(\tau_t^K)}{\partial \tau_t^K} > 0$; if $\tau_t^K > 1-\alpha$, then $\frac{\partial \text{CTR}(\tau_t^K)}{\partial \tau_t^K} < 0$. This shows that the capital tax Laffer curve has a single peak at $1-\alpha$. Lastly, note that

$$\frac{\partial^2 \text{CTR}(\tau_t^K)}{\partial \tau_t^K}\Big|_{\tau_t^K=1-\alpha} = -\frac{\Omega}{1-\alpha}\alpha^{\frac{1}{1-\alpha}-2} < 0. \quad \square$$

A.5 Proof of Proposition 2

I guess that resource constraints (2a)–(2c) are slack at the optimum. By Proposition 1(i), welfare is independent of capital income taxes, implying that capital taxes influence the relaxed problem only via the government budget constraint. Moreover, since labor is inelastically supplied, labor tax revenue is invariant to capital taxes. When the resource constraint is slack, the relaxed problem minimizes the welfare-reducing burden of labor taxes by maximizing capital tax revenue. Consequently, determining optimal capital taxes reduces to maximizing total capital tax revenue. Moreover, by Proposition 1(ii), equilibrium capital allocations, $\{K_t^*(\mathcal{T})\}_{t=1}^T$, and returns to capital, $\{r_t^*(\mathcal{T})\}_{t=1}^T$, are unaffected by taxes in other periods. Therefore, capital tax revenue in period t, $r_t^*(\mathcal{T})K_t^*(\mathcal{T})\tau_t^K$, depends only on the capital tax rate τ_t^K for that period. Thus, the problem reduces to pointwise maximization of capital tax revenue in each period. That is, given τ_t^L, the optimal capital tax in period t solves:

$$\max_{\tau_t^K \in [0,1]} \; r_t^*(\mathcal{T})K_t^*(\mathcal{T})\tau_t^K. \tag{P-K}$$

The first-order condition w.r.t. to τ_t^K can be reduced to $\tau_t^K = 1 - \alpha$. As the capital tax Laffer curve is locally concave at its unique peak (Lemma 3), first-order conditions are necessary and sufficient in isolating the global maximum. Now note that as $\alpha \in (0,1)$, $\tau_t^{K*} \in (0,1)$. Thus, Lemma 2 implies that the resource constraints are slack, verifying our guess. □

A.6 Proof of Lemma 4

First, note that the constraint set of problem (P-L) is bounded as $0 \leq \tau_t^L \leq 1$. Moreover, the constraint set is closed. To see this, note that the set of labor income taxes that satisfy the government budget constraint can be written as $\mathcal{S} \equiv \{(\{\tau_t^L\}) \in [0,1]^T : g(\{\tau_t^L\}) = 0\}$, where $g : [0,1]^T \mapsto \mathbb{R}$ is a continuous function. Then note that $\mathcal{S}$ is the preimage of the closed set $\{0\}$ under the continuous function g. Since the preimage of a closed set under a continuous function is also closed, $\mathcal{S}$ is closed in $[0,1]^T$. Since $\mathcal{S}$ is in Euclidean space, by Heine-borel, it is also compact. Lastly, since the objective function of (P-L), SWF, is continuous in $\{\tau_t^L\}_{t\in\{1,\ldots,T\}}$, by Berge's Maximum Theorem, a solution to (P-L) exists. □

A.7 Proof of Lemma 5

Suppose $\exists t : \tau_t^{L*} = 1$. By Lemma 1, this implies that $c_t^{w*}(\mathcal{T}) = 0$ under non-redistributive transfer policies. Now note that the perturbation that reduces τ_t^L by $\epsilon > 0$

strictly increases SWF as $\lim_{x\to 0} v'(x) = \infty$ and generates more revenue, relaxing the government budget constraint; a contradiction. $\square$

A.8 Proof of Proposition 3

Let the Lagrange multiplier on the government's lifetime budget constraint be denoted by λ. Note that $\lambda > 0$; else reducing labor taxes can increase welfare, violating tax optimality conditions. When transfer policies are non-redistributive and $\sigma \to 0$, then the optimal labor tax in period t is given by τ_t^L which solves the following equation:

$$\beta^{t-1}[\mu w_t L_t(1-\tau_t^L) + (1-\mu)v'(w_t L_t^w(1-\tau_t^L) + \mathfrak{T}_t)w_t L_t^w] = \frac{\lambda w_t(L_t + L_t^w)}{(1+r_B)^{t-1}}.$$

Now note that $\forall t, t'$ $\tau_t^L = \tau_{t'}^L$ when $\beta(1+r_B) = 1$, $w_t = w_{t'}$, $L_t = L_{t'}$, and $L_t^w = L_{t'}^w$. $\square$

A.9 Proof of Lemma 6

First note that we can re-write the capitalists' problem as an unconstrained maximization problem in $2T$ choice variables:

$$\begin{aligned}\max_{\{L_t,K_t\}_{t\in\{1,\dots,T\}}} & \frac{(w_1L_1(1-\tau_1^L) + r_1K_1(1-\tau_1^K) - K_2 - \delta K_1)^{1-\sigma}}{1-\sigma} - \chi\frac{L_1^{1+\varphi}}{1+\varphi} \\ & + \sum_{t=1}^{T-1}\beta^{t-1}\left\{\frac{(w_tL_t(1-\tau_t^L) + r_tK_t(1-\tau_t^K) + K_t(1-\delta) - K_{t+1})^{1-\sigma}}{1-\sigma} - \chi\frac{L_t^{1+\varphi}}{1+\varphi}\right\} \\ & + \beta^{T-1}\left\{\frac{(w_TL_T(1-\tau_T^L) + r_TK_T(1-\tau_T^K) + K_T(1-\delta))^{1-\sigma}}{1-\sigma} - \chi\frac{L_T^{1+\varphi}}{1+\varphi}\right\}\end{aligned}$$

Consider the optimality conditions of the above problem as $\sigma \to 0$. The first-order condition w.r.r. L_t yields a closed-form solution for labor allocations:

$$L_t^*(\mathcal{T}) = \left\{\frac{w_t(1-\tau_t^L)}{\chi}\right\}^{1/\varphi} \forall t \in \{1,\dots,T\}.$$

Now note that the first-order condition w.r.t. K_1 pins down the rate of return on capital in period 1:

$$r_1^*(\mathcal{T}) = \frac{\delta}{1-\tau_1^K}.$$

Similarly, the first-order condition w.r.t. K_2 pins down the rate of return on capital in period 2:

$$r_t^*(\mathcal{T}) = \frac{1-\beta(1-\delta)}{\beta(1-\tau_t^K)} \forall t \in \{1,\dots,T\}.$$

Next, substituting in the closed-form solutions for $L_t^*(\mathcal{T})$ and $r_t^*(\mathcal{T})$ into firm profit maximization condition (i.e., FOC w.r.t. K_t^d) and using the market clearing condition (1), we obtain a closed-form solution for $K_t^*(\mathcal{T})$:

$$K_t^*(\mathcal{T}) = [L_t^*(\mathcal{T}) + L_t^{w*}(\mathcal{T})]\left(\frac{\alpha A Z_t}{r_t^*(\mathcal{T})}\right)^{\frac{1}{1-\alpha}} \quad \forall t \in \{1, ..., T\}.$$

Specifically,

$$K_1^*(\mathcal{T}) = \left[\left\{\frac{w_1(1-\tau_1^L)}{\chi}\right\}^{1/\varphi} + L_1^{w*}(\mathcal{T})\right]\left\{\frac{\alpha A Z_1(1-\tau_1^K)}{\delta}\right\}^{\frac{1}{1-\alpha}},$$

and

$$K_t^*(\mathcal{T}) = \left[\left\{\frac{w_t(1-\tau_t^L)}{\chi}\right\}^{1/\varphi} + L_t^{w*}(\mathcal{T})\right]\left\{\frac{\alpha A Z_t \beta(1-\tau_t^K)}{1-\beta(1-\delta)}\right\}^{\frac{1}{1-\alpha}} \quad \forall t \in \{2, ..., T\}.$$

Lastly, we obtain closed-form solutions for consumption allocations using the per-period budget constraints. Specifically,

$$\begin{aligned} c_1^*(\mathcal{T}) &= w_1 L_1^*(\mathcal{T})(1-\tau_1^L) + K_1^*(\mathcal{T})\overbrace{\{r_1^*(\mathcal{T})(1-\tau_1^K(\mathcal{T})) - \delta\}}^{=0} - K_2^*(\mathcal{T}) \\ &= \frac{\{w_1(1-\tau_1^L)\}^{1/\varphi+1}}{\chi^{1/\varphi}} - \left[\left\{\frac{w_2(1-\tau_2^L)}{\chi}\right\}^{1/\varphi} + L_2^{w*}(\mathcal{T})\right]\left\{\frac{\alpha A Z_2 \beta(1-\tau_2^K)}{1-\beta(1-\delta)}\right\}^{\frac{1}{1-\alpha}}. \end{aligned}$$

Moreover,

$$\begin{aligned} c_T^*(\mathcal{T}) &= w_T L_T^*(\mathcal{T})(1-\tau_T^L) + K_T^*(\mathcal{T})\{r_T^*(\mathcal{T})(1-\tau_T^K) + 1 - \delta\} \\ &= \frac{\{w_T(1-\tau_T^L)\}^{1/\varphi+1}}{\chi^{1/\varphi}} + \left[\left\{\frac{w_T(1-\tau_T^L)}{\chi}\right\}^{1/\varphi} + L_T^{w*}(\mathcal{T})\right]\left\{\frac{\alpha A Z_T \beta(1-\tau_T^K)}{1-\beta(1-\delta)}\right\}^{\frac{1}{1-\alpha}} ... \\ &\quad ...\left\{\frac{1-\beta(1-\delta)}{\beta(1-\tau_T^K)}(1-\tau_T^K) + 1 - \delta\right\} \\ &= \frac{\{w_T(1-\tau_T^L)\}^{1/\varphi+1}}{\chi^{1/\varphi}} + \frac{1}{\beta}\left[\left\{\frac{w_T(1-\tau_T^L)}{\chi}\right\}^{1/\varphi} + L_T^{w*}(\mathcal{T})\right]\left\{\frac{\alpha A Z_T \beta(1-\tau_T^K)}{1-\beta(1-\delta)}\right\}^{\frac{1}{1-\alpha}}. \end{aligned}$$

Similarly,

$$c_t^*(\mathcal{T}) = \frac{\{w_t(1-\tau_t^L)\}^{1/\varphi+1}}{\chi^{1/\varphi}} + \frac{1}{\beta}\left[\left\{\frac{w_t(1-\tau_t^L)}{\chi}\right\}^{1/\varphi} + L_t^{w*}(\mathcal{T})\right]\left\{\frac{\alpha A Z_t \beta(1-\tau_t^K)}{1-\beta(1-\delta)}\right\}^{\frac{1}{1-\alpha}}$$

$$- \left[\left\{\frac{w_{t+1}(1-\tau_{t+1}^L)}{\chi}\right\}^{1/\varphi} + L_{t+1}^{w*}(\mathcal{T})\right]\left\{\frac{\alpha A Z_{t+1}\beta(1-\tau_{t+1}^K)}{1-\beta(1-\delta)}\right\}^{\frac{1}{1-\alpha}} \; \forall t \in \{2,...,T-1\}.$$

Note that capital allocations are finite-valued as $\delta > 0$; and the rates of return on capital are finite-valued as $\beta > 0$ and $\tau_t^K < 1 \; \forall t \in \{1,...,T\}$. This fully characterizes the solution to the capitalists' problem. Also, note that the functional forms for capitalists' allocations and equilibrium prices in sticky wage equilibria are explicit, which implies that they are unique. To complete the proof, note that the solution to the workers' problem is given by the following optimality condition as $u^w(.)$ is concave and differentiable:

$$v'(w_t(1-\tau_t^L)L_t^w + \mathfrak{T}_t)\{w_t(1-\tau_t^L)\} = \chi^w (L_t^w)^\varphi.$$

Moreover, as $u^w(.)$ is strictly concave, the optimal labor allocation of workers is unique. As $c_t^w = w_t(1-\tau_t^L)L_t^w + \mathfrak{T}_t$, this implies that the optimal consumption allocation of workers is unique as well. Lastly, since $u^w(.)$ is continuous and the constraint set of the workers' problem is compact, a solution to this problem exists. □

A.10 Proposition 5

Proposition 5. *Suppose A is sufficiently large and labor supply is elastic. Then in all sticky wage equilibria:*
(i) SWF is independent of $\{\tau_t^K\}_{t\in\{1,...,T\}}$,
(ii) The present discounted value of total labor income tax revenue, $\sum_{t=1}^T \frac{(w_t L_t^*(\mathcal{T}) + w_t^w L_t^{w*}(\mathcal{T}))\tau_t^L}{(1+r_B)^{t-1}}$, *is independent of* $\{\tau_t^K\}_{t\in\{1,...,T\}}$, *and*
(iii) Period-t capital income tax revenue, $r_t^*(\mathcal{T})K_t^*(\mathcal{T})\tau_t^K$, *is independent of* $\{\tau_t^K\}_{t\in\{1,...,T\}\setminus t}$.

Proof. Using arguments outlined in the proof of Lemma 6, we can express welfare as

$$\begin{aligned}
\text{SWF} =& \mu\Bigg(\frac{\{w_1(1-\tau_1^L)\}^{1/\varphi+1}}{\chi^{1/\varphi}} - \left[\left\{\frac{w_2(1-\tau_2^L)}{\chi}\right\}^{1/\varphi} + L_2^{w*}(\mathcal{T})\right]\left\{\frac{\alpha A Z_2\beta(1-\tau_2^K)}{1-\beta(1-\delta)}\right\}^{\frac{1}{1-\alpha}} \\
&- \frac{\chi}{1+\varphi}\left\{\frac{w_1(1-\tau_1^L)}{\chi}\right\}^{\frac{1+\varphi}{\varphi}} + \sum_{t=2}^{T-1}\beta^{t-1}\Bigg[\frac{\{w_t(1-\tau_t^L)\}^{1/\varphi+1}}{\chi^{1/\varphi}} \\
&+ \frac{1}{\beta}\left[\left\{\frac{w_t(1-\tau_t^L)}{\chi}\right\}^{1/\varphi} + L_t^{w*}(\mathcal{T})\right]\left\{\frac{\alpha A Z_t\beta(1-\tau_t^K)}{1-\beta(1-\delta)}\right\}^{\frac{1}{1-\alpha}} \\
&- \left[\left\{\frac{w_{t+1}(1-\tau_{t+1}^L)}{\chi}\right\}^{1/\varphi} + L_{t+1}^{w*}(\mathcal{T})\right]\left\{\frac{\alpha A Z_{t+1}\beta(1-\tau_{t+1}^K)}{1-\beta(1-\delta)}\right\}^{\frac{1}{1-\alpha}}\Bigg] \\
&+ \beta^{T-1}\Bigg[\frac{\{w_T(1-\tau_T^L)\}^{1/\varphi+1}}{\chi^{1/\varphi}} + \frac{1}{\beta}\left[\left\{\frac{w_T(1-\tau_T^L)}{\chi}\right\}^{1/\varphi} + L_T^{w*}(\mathcal{T})\right]\left\{\frac{\alpha A Z_T\beta(1-\tau_T^K)}{1-\beta(1-\delta)}\right\}^{\frac{1}{1-\alpha}}
\end{aligned}$$

$$- \frac{\chi}{1+\varphi}\left\{\frac{w_T(1-\tau_T^L)}{\chi}\right\}^{\frac{1+\varphi}{\varphi}}\Bigg]\Bigg) + (1-\mu)\sum_{t\in\{1,2\}}\beta^{t-1}u^w(c_t^{w*}(\tau_t^L), L_t^{w*}(\tau_t^L)).$$

when $\sigma \to 0$. Cancelling like terms:

$$\text{SWF} = \frac{\mu}{\chi^{1/\varphi}}\left[1-\frac{\chi}{1+\varphi}\right]\left[\sum_{t=1}^{T}\beta^{t-1}\{w_t(1-\tau_t^L)\}^{1/\varphi+1}\right] + (1-\mu)\sum_{t\in\{1,2\}}\beta^{t-1}u^w(c_t^{w*}(\tau_t^L), L_t^{w*}(\tau_t^L)).$$

The proof concludes with the observation that SWF is independent of $\{\tau_t^K\}_{t\in\{1,\dots,T\}}$ as $\{w_t\}_{t\in\{1,\dots,T\}}$ is fixed. This proves part (i). To see part (ii), note that Lemma 6 also implies that $\{L_t^*(\mathcal{T})\}_{t\in\{1,\dots,T\}}$ remain unaffected by capital taxes as they are derived by labor supply conditions when A is large enough. Similarly, part (iii) follows from the observation that $\{K_t^*(\mathcal{T})\}$ and $\{r_t^*(\mathcal{T})\}$ are unaffected by taxes in periods $t' \neq t$. □

A.11 Proof of Lemma 7

Before characterizing the trajectory of optimal labor taxes, I first prove the following:

Sublemma 1. *Suppose assumption 1 holds, χ^w is sufficiently small, and transfers are non-redistributive. Then in any sticky wage Ramsey equilibrium:*

$$\lim_{\bar{c}\to 0}\frac{\partial L_t^{w*}(\mathcal{T})}{\partial \tau_t^L} \approx 0 \;\forall t.$$

Proof of Sublemma 1: Note that worker optimality requires $f(L_t^{w*}(\mathcal{T}), \tau_t^L) = 0$ where $f(L_t^w(\mathcal{T}), \tau_t^L) = v'(w_t(1-\tau_t^L)L_t^w(\mathcal{T}) + \mathfrak{T}_t)w_t(1-\tau_t^L) - \chi^w(L_t^w(\mathcal{T}))^\varphi$. Thus, by the Implicit Function Theorem,

$$\frac{\partial L_t^{w*}(\mathcal{T})}{\partial \tau_t^L} = \frac{-\partial f(L_t^w(\mathcal{T}), \tau_t^L)/\partial \tau_t^L}{\partial f(L_t^w(\mathcal{T}), \tau_t^L)/\partial L_t^w(\mathcal{T})} = \frac{-w_t[v'(c_t^w(\mathcal{T})) + v''(c_t^w(\mathcal{T}))w_t L_t^w(\mathcal{T})(1-\tau_t^L)]}{v''(c_t^w(\mathcal{T}))[w_t(1-\tau_t^L)]^2 - \chi^w\varphi(L_t^w(\mathcal{T}))^{\varphi-1}}.$$

When χ^w is small enough, worker optimality implies that $v'(c_t^w)$ is close to zero. Also note that under non-redistributive transfers, $L_t^w = \frac{\bar{c}}{w_t}$ at the satiation point. Thus, the above condition reduces to:

$$\frac{\partial L_t^{w*}(\mathcal{T})}{\partial \tau_t^L} \approx \frac{-\bar{c}}{w_t(1-\tau_t^L)}.$$

I now guess and verify $\lim_{\bar{c}\to 0}\frac{-\bar{c}}{w_t(1-\tau_t^L)} = 0$ in any sticky wage Ramsey equilibrium. Note that when this limiting condition and assumption 1 hold, revenue-maximizing labor tax

rates are less than one. To see this, note when $\bar{c}$ and $\lim_{\bar{c}\to 0} \frac{\partial L_t^{w*}}{\partial \tau_t^L}$ are close to zero, then the period-t labor tax rate that maximizes government revenues is approximately given by τ_t^L that solves:

$$w_t\left(\frac{w_t(1-\tau_t^L)}{\chi}\right)^{1/\varphi} = r_t\left\{\frac{w_t}{\chi\varphi}\left(\frac{w_t(1-\tau_t^L)}{\chi}\right)^{1/\varphi-1}[\alpha A Z_t r_t^{-1}]^{\frac{1}{1-\alpha}}\right\}(1-\alpha) + \frac{w_t^2\tau_t^L}{\chi\varphi}\left(\frac{w_t(1-\tau_t^L)}{\chi}\right)^{1/\varphi-1}.$$

Dividing by $\frac{w_t}{\chi\varphi}\left(\frac{w_t(1-\tau_t^L)}{\chi}\right)^{1/\varphi-1} > 0$ on both sides we find that, keeping all else constant, the revenue-maximizing period-t labor tax rate is given by $\frac{\varphi w_t - r_t(1-\alpha)[\alpha A Z_t r_t^{-1}]^{\frac{1}{1-\alpha}}}{w_t(1+\varphi)}$. Note that this value is less than one as per assumption 1, thus it must be that $\lim_{\bar{c}\to 0}\tau_t^{L*} \neq 1$. Thus, in any sticky wage Ramsey equilibrium,

$$\lim_{\bar{c}\to 0}\frac{\partial L_t^{w*}(\mathcal{T})}{\partial \tau_t^L} \approx 0 \;\forall t. \quad \square$$

Now now that when χ^w is small enough and transfer policies are non-redistributive, then the optimal labor tax in period 1 is given by τ_1^L which solves the following equation as $\sigma \to 0$:

$$\begin{aligned}
&\frac{\mu(w_1)^{1/\varphi+1}(1-\tau_1^L)^{1/\varphi}}{\chi^{1/\varphi}}\left[\frac{1+\varphi-\chi}{\varphi}\right] + (1-\mu)v'(c_1^{w*})\left[-w_1^w L_1^{w*} + w_1(1-\tau_1^L)\frac{\partial L_1^{w*}}{\partial \tau_1^L}\right] \\
&= \lambda\left\{\frac{\delta(1-\alpha)}{\alpha}\left\{\frac{\alpha^2 A Z_1}{\delta}\right\}^{\frac{1}{1-\alpha}}\left[-\frac{1}{\varphi}\left\{\frac{w_1}{\chi}\right\}^{1/\varphi}(1-\tau_1^L)^{1/\varphi-1} + \frac{\partial L_1^w}{\partial \tau_1^L}\right]\right. \\
&\left. + \frac{w_1^{1/\varphi+1}}{\chi^{1/\varphi}}\left((1-\tau_1^L)^{1/\varphi} - \frac{\tau_1^L}{\varphi}(1-\tau_1^L)^{1/\varphi-1}\right)\right\},
\end{aligned}$$

where $\lambda > 0$ denotes the Lagrange multiplier on the government's lifetime budget constraint.

Note that as v has a satiation point, L_1^{w*} has a finite upper bound: $\bar{c}/w_1^w$. Moreover, $\tau_1^{L*} < 1$ (by Lemma 5). Thus, if χ^w is small enough, worker optimality implies $v'(c_1^{w*})$ is close to zero. Additionally, since $L_t^{w*}(\mathcal{T})$ is continuous, Lemma 1 implies that $\frac{\partial L_t^{w*}(\mathcal{T})}{\partial \tau_t^L} \approx 0 \;\forall t$ when 1 holds, χ^w and $\bar{c}$ are sufficiently small, and transfer policies are non-redistributive. Thus, the optimal labor tax in period 1 can be approximated by τ_1^L

that solves:

$$\frac{\mu(w_1)^{1/\varphi+1}(1-\tau_1^L)^{1/\varphi}}{\chi^{1/\varphi}}\left[\frac{1+\varphi-\chi}{\varphi}\right]=\lambda\Bigg\{-\frac{\delta(1-\alpha)}{\alpha\varphi}\left\{\frac{\alpha^2AZ_1}{\delta}\right\}^{\frac{1}{1-\alpha}}\left\{\frac{w_1}{\chi}\right\}^{1/\varphi}(1-\tau_1^L)^{1/\varphi-1}$$
$$+\frac{w_1^{1/\varphi+1}}{\chi^{1/\varphi}}\left((1-\tau_1^L)^{1/\varphi}-\frac{\tau_1^L}{\varphi}(1-\tau_1^L)^{1/\varphi-1}\right)\Bigg\}.$$

Canceling $\frac{w_1^{1/\varphi+1}(1-\tau_1^L)^{1/\varphi}}{\chi^{1/\varphi}}>0$, we find that the optimal labor tax in period 1 solves:

$$\frac{\mu(1+\varphi-\chi)}{\lambda\varphi}=1-\left\{\frac{\delta(1-\alpha)}{\alpha\varphi w_1(1-\tau_1^L)}\right\}\left\{\frac{\alpha^2AZ_1}{\delta}\right\}^{\frac{1}{1-\alpha}}-\frac{\tau_1^L}{\varphi(1-\tau_1^L)}. \tag{6}$$

Similarly, the optimal labor tax in period t can be approximated by τ_t^L that solves the following equation:

$$\frac{\mu\beta^{t-1}(w_t)^{1/\varphi+1}(1-\tau_t^L)^{1/\varphi}}{\chi^{1/\varphi}}\left[\frac{1+\varphi-\chi}{\varphi}\right]=\frac{\lambda}{(1+r_B)^{t-1}}\Bigg\{-\left\{\frac{(1-\beta(1-\delta))(1-\alpha)}{\alpha\beta\varphi}\right\}...$$
$$...\left\{\frac{\alpha^2AZ_t\beta}{1-\beta(1-\delta)}\right\}^{\frac{1}{1-\alpha}}\left\{\frac{w_t}{\chi}\right\}^{1/\varphi}(1-\tau_t^L)^{1/\varphi-1}+\frac{w_t^{1/\varphi+1}}{\chi^{1/\varphi}}\left((1-\tau_t^L)^{1/\varphi}-\frac{\tau_t^L}{\varphi}(1-\tau_t^L)^{1/\varphi-1}\right)\Bigg\}.$$

Canceling $\frac{w_t^{1/\varphi+1}(1-\tau_t^L)^{1/\varphi}}{\chi^{1/\varphi}}>0$, we find that the optimal labor tax in period t solves:

$$\frac{\mu(1+\varphi-\chi)}{\lambda\varphi}=\frac{1}{\{\beta(1+r_B)\}^{t-1}}\Bigg[1-\left\{\frac{(1-\beta(1-\delta))(1-\alpha)}{\alpha\beta\varphi w_t(1-\tau_t^L)}\right\}\left\{\frac{\alpha^2AZ_t\beta}{1-\beta(1-\delta)}\right\}^{\frac{1}{1-\alpha}}$$
$$-\frac{\tau_t^L}{\varphi(1-\tau_t^L)}\Bigg]. \tag{7}$$

Substituting the value of $\frac{\mu(1+\varphi-\chi)}{\lambda\varphi}$ from equation (7) into equation (6), we find that $\forall t\in\{2,...,T\}$:

$$1-\left\{\frac{\delta(1-\alpha)}{\alpha\varphi w_1(1-\tau_1^L)}\right\}\left\{\frac{\alpha^2AZ_1}{\delta}\right\}^{\frac{1}{1-\alpha}}-\frac{\tau_1^L}{\varphi(1-\tau_1^L)}$$
$$=\frac{1}{\{\beta(1+r_B)\}^{t-1}}\Bigg[1-\left\{\frac{(1-\beta(1-\delta))(1-\alpha)}{\alpha\beta\varphi w_t(1-\tau_t^L)}\right\}\left\{\frac{\alpha^2AZ_t\beta}{1-\beta(1-\delta)}\right\}^{\frac{1}{1-\alpha}}-\frac{\tau_t^L}{\varphi(1-\tau_t^L)}\Bigg]. \tag{8}$$

Since $\varphi > 0$, the above condition can be re-written as:

$$\varphi - \frac{ac_1 - \tau_1^L}{1 - \tau_1^L} = \frac{1}{\{\beta(1+r_B)\}^{t-1}}\left[\varphi - \frac{b_t c_t - \tau_t^L}{1 - \tau_t^L}\right]. \quad \square$$

A.12 Proof of Proposition 4

First, note that optimal labor taxes are strictly less than unity (see Lemma 5). Now note that as A becomes large, the terms ac_t and $b_t c_t$ dominate over τ_1^{L*} and τ_t^{L*} in the numerators of the optimality condition (4). Thus, condition (4) approximates:

$$\frac{ac_1}{1 - \tau_1^{L*}} \approx \frac{b_t c_t}{\{\beta(1+r_B)\}^{t-1}(1 - \tau_t^{L*})}.$$

$$\implies aZ_1^{\frac{1}{1-\alpha}}(1 - \tau_t^{L*}) \approx \frac{b_t Z_t^{\frac{1}{1-\alpha}}(1 - \tau_1^{L*})}{\{\beta(1+r_B)\}^{t-1}}.$$

Now note that $aZ_1^{\frac{1}{1-\alpha}} > b_t Z_t^{\frac{1}{1-\alpha}} / \{\beta(1+r_B)\}^{t-1}$ when

$$\frac{w_1}{w_t} < \bar{w}_t \equiv \{\beta(1+r_B)\}^{t-1}\left(\frac{Z_1}{Z_t}\right)^{\frac{1}{1-\alpha}}\left\{\frac{1 - \beta(1-\delta)}{\beta\delta}\right\}^{\frac{\alpha}{1-\alpha}}.$$

Thus, it must be that $\lim_{A\to\infty} \tau_1^{L*} < \lim_{A\to\infty} \tau_t^{L*}$ when $w_1/w_t < \bar{w}_t$. Similarly, it can be shown that $\lim_{A\to\infty} \tau_1^{L*} = \lim_{A\to\infty} \tau_t^{L*}$ when $w_1/w_t = \bar{w}_t$, and $\lim_{A\to\infty} \tau_1^{L*} > \lim_{A\to\infty} \tau_t^{L*}$ when $w_1/w_t > \bar{w}$. This proves part (i).

To see part (ii), note that when c is large, condition (4) approximates:

$$\frac{b_t c_t}{\{\beta(1+r_B)\}^{t-1}(1 - \tau_t^{L*})} = \frac{b_{t'} c_{t'}}{\{\beta(1+r_B)\}^{t'-1}(1 - \tau_{t'}^{L*})}$$

As $A \to \infty$, it must be that:

$$\frac{b_t Z_t^{\frac{1}{1-\alpha}}}{\{\beta(1+r_B)\}^{t-1}(1 - \lim_{A\to\infty} \tau_t^{L*})} = \frac{b_{t'} Z_{t'}^{\frac{1}{1-\alpha}}}{\{\beta(1+r_B)\}^{t'-1}(1 - \lim_{A\to\infty} \tau_{t'}^{L*})}$$

Thus, when $\frac{w_t}{w_{t'}} < \{\beta(1+r_B)\}^{t'-t}\left(\frac{Z_t}{Z_{t'}}\right)^{\frac{1}{1-\alpha}}$, then $\lim_{A\to\infty} \tau_{t'}^{L*} > \lim_{A\to\infty} \tau_t^{L*}$. This proves part (ii). $\square$

B Details of Model with Segmented Labor Markets

Lemma 8. *If $\tau_t^K < 1\ \forall t \in \{1, ..., T\}$, and A and $\underline{f}$ are sufficiently large, then a unique sticky wage equilibrium exists. Specifically:*

(i) Capitalists' allocations have the following closed-form solutions:

$$
\begin{aligned}
c_1^*(\mathcal{T}) &= \frac{\{w_1(1-\tau_1^L)\}^{1/\varphi+1}}{\chi^{1/\varphi}} - \left\{\frac{w_2(1-\tau_2^L)}{\chi}\right\}^{1/\varphi}\left\{\frac{\alpha A Z_1 \beta(1-\tau_2^K)}{1-\beta(1-\delta)}\right\}^{\frac{1}{1-\alpha}}, \\
c_t^*(\mathcal{T}) &= \frac{\{w_t(1-\tau_t^L)\}^{1/\varphi+1}}{\chi^{1/\varphi}} + \frac{1}{\beta}\left\{\frac{w_t(1-\tau_t^L)}{\chi}\right\}^{1/\varphi}\left\{\frac{\alpha A Z_t \beta(1-\tau_t^K)}{1-\beta(1-\delta)}\right\}^{\frac{1}{1-\alpha}} \\
&\quad - \left\{\frac{w_{t+1}(1-\tau_{t+1}^L)}{\chi}\right\}^{1/\varphi}\left\{\frac{\alpha A Z_{t+1} \beta(1-\tau_{t+1}^K)}{1-\beta(1-\delta)}\right\}^{\frac{1}{1-\alpha}} \forall t \in \{2, ..., T-1\}, \\
c_T^*(\mathcal{T}) &= \frac{\{w_T(1-\tau_T^L)\}^{1/\varphi+1}}{\chi^{1/\varphi}} + \frac{1}{\beta}\left\{\frac{w_T(1-\tau_T^L)}{\chi}\right\}^{1/\varphi}\left\{\frac{\alpha A Z_T \beta(1-\tau_T^K)}{1-\beta(1-\delta)}\right\}^{\frac{1}{1-\alpha}}, \\
L_t^*(\mathcal{T}) &= L_t^{d*}(\mathcal{T}) = \left\{\frac{w_t(1-\tau_t^L)}{\chi}\right\}^{1/\varphi} \forall t \in \{1, ..., T\}, \\
K_1^*(\mathcal{T}) &= K_1^{d*}(\mathcal{T}) = \left\{\frac{w_1(1-\tau_1^L)}{\chi}\right\}^{1/\varphi}\left\{\frac{\alpha A Z_1(1-\tau_1^K)}{\delta}\right\}^{\frac{1}{1-\alpha}}, \textit{ and} \\
K_t^*(\mathcal{T}) &= K_t^{d*}(\mathcal{T}) = \left\{\frac{w_t(1-\tau_t^L)}{\chi}\right\}^{1/\varphi}\left\{\frac{\alpha A Z_t \beta(1-\tau_t^K)}{1-\beta(1-\delta)}\right\}^{\frac{1}{1-\alpha}} \forall t \in \{2, ..., T\}.
\end{aligned}
$$

(ii) The equilibrium rates of return on capital are given by:

$$
\begin{aligned}
r_1^*(\mathcal{T}) &= \frac{\delta}{1-\tau_1^K}, \textit{ and} \\
r_t^*(\mathcal{T}) &= \frac{1-\beta(1-\delta)}{\beta(1-\tau_t^K)} \forall t \in \{1, ..., T\}.
\end{aligned}
$$

(iii) Workers' period-t labor allocations, $L_t^{w*}(\mathcal{T})$, are given by the solution to the following implicit function:

$$v'(w_t^w(1-\tau_t^L)L_t^{w*}(\mathcal{T}) + \mathfrak{T}_t)\{w_t^w(1-\tau_t^L)\} = \chi^w(L_t^{w*}(\mathcal{T}))^\varphi.$$

Workers' consumption allocations are given by:

$$c_t^{w*}(\mathcal{T}) = w_t^w(1-\tau_t^L)L_t^{w*}(\mathcal{T}) + \mathfrak{T}_t \ \forall t \in \{1, ..., T\}.$$

Proof. The derivations of $L_t^*(\mathcal{T})$ and $r_t^*(\mathcal{T})$ are unchanged from the baseline environment without labor market segmentation. Substituting in the closed-form solutions for $L_t^*(\mathcal{T})$ and $r_t^*(\mathcal{T})$ into firm profit maximization condition (i.e., FOC w.r.t. K_t^d) and using the market clearing condition (1), we obtain a closed-form solution for $K_t^*(\mathcal{T})$:

$$K_t^*(\mathcal{T}) = L_t^*(\mathcal{T})\left(\frac{\alpha A Z_t}{r_t^*(\mathcal{T})}\right)^{\frac{1}{1-\alpha}} \ \forall t \in \{1, ..., T\}.$$

Specifically,

$$K_1^*(\mathcal{T}) = \left\{\frac{w_1(1-\tau_1^L)}{\chi}\right\}^{1/\varphi}\left\{\frac{\alpha A Z_1(1-\tau_1^K)}{\delta}\right\}^{\frac{1}{1-\alpha}},$$

and

$$K_t^*(\mathcal{T}) = \left\{\frac{w_t(1-\tau_t^L)}{\chi}\right\}^{1/\varphi}\left\{\frac{\alpha A Z_t \beta(1-\tau_t^K)}{1-\beta(1-\delta)}\right\}^{\frac{1}{1-\alpha}} \ \forall t \in \{2, ..., T\}.$$

Lastly, we obtain closed-form solutions for consumption allocations using the per-period budget constraints. Specifically,

$$\begin{aligned} c_1^*(\mathcal{T}) &= w_1 L_1^*(\mathcal{T})(1-\tau_1^L) + K_1^*(\mathcal{T})\overbrace{\{r_1^*(\mathcal{T})(1-\tau_1^K(\mathcal{T})) - \delta\}}^{=0} - K_2^*(\mathcal{T}) \\ &= \frac{\{w_1(1-\tau_1^L)\}^{1/\varphi+1}}{\chi^{1/\varphi}} - \left\{\frac{w_2(1-\tau_2^L)}{\chi}\right\}^{1/\varphi}\left\{\frac{\alpha A Z_2 \beta(1-\tau_2^K)}{1-\beta(1-\delta)}\right\}^{\frac{1}{1-\alpha}}. \end{aligned}$$

Moreover,

$$\begin{aligned} c_T^*(\mathcal{T}) &= w_T L_T^*(\mathcal{T})(1-\tau_T^L) + K_T^*(\mathcal{T})\{r_T^*(\mathcal{T})(1-\tau_T^K) + 1 - \delta\} \\ &= \frac{\{w_T(1-\tau_T^L)\}^{1/\varphi+1}}{\chi^{1/\varphi}} + \left\{\frac{w_T(1-\tau_T^L)}{\chi}\right\}^{1/\varphi}\left\{\frac{\alpha A Z_T \beta(1-\tau_T^K)}{1-\beta(1-\delta)}\right\}^{\frac{1}{1-\alpha}} ... \\ &\quad ...\left\{\frac{1-\beta(1-\delta)}{\beta(1-\tau_T^K)}(1-\tau_T^K) + 1 - \delta\right\} \\ &= \frac{\{w_T(1-\tau_T^L)\}^{1/\varphi+1}}{\chi^{1/\varphi}} + \frac{1}{\beta}\left\{\frac{w_T(1-\tau_T^L)}{\chi}\right\}^{1/\varphi}\left\{\frac{\alpha A Z_T \beta(1-\tau_T^K)}{1-\beta(1-\delta)}\right\}^{\frac{1}{1-\alpha}}. \end{aligned}$$

Similarly,

$$c_t^*(\mathcal{T}) = \frac{\{w_t(1-\tau_t^L)\}^{1/\varphi+1}}{\chi^{1/\varphi}} + \frac{1}{\beta}\left\{\frac{w_t(1-\tau_t^L)}{\chi}\right\}^{1/\varphi}\left\{\frac{\alpha A Z_t \beta(1-\tau_t^K)}{1-\beta(1-\delta)}\right\}^{\frac{1}{1-\alpha}}$$

$$- \left\{ \frac{w_{t+1}(1-\tau_{t+1}^L)}{\chi} \right\}^{1/\varphi} \left\{ \frac{\alpha A Z_{t+1} \beta (1-\tau_{t+1}^K)}{1-\beta(1-\delta)} \right\}^{\frac{1}{1-\alpha}} \forall t \in \{2, ..., T-1\}.$$

Note that capital allocations are finite-valued as $\delta > 0$; and the rates of return on capital are finite-valued as $\beta > 0$ and $\tau_t^K < 1 \; \forall t \in \{1, ..., T\}$. This fully characterizes the solution to the capitalists' problem. Also, note that the functional forms for capitalists' allocations and equilibrium prices in sticky wage equilibria are explicit, which implies that they are unique. To complete the proof, note that the solution to the workers' problem is given by the following optimality condition as $u^w(.)$ is concave and differentiable:

$$v'(w_t^w(1-\tau_t^L)L_t^w + \mathfrak{T}_t)\{w_t^w(1-\tau_t^L)\} = \chi^w(L_t^w)^\varphi.$$

Moreover, as $u^w(.)$ is strictly concave, the optimal labor allocation of workers is unique. As $c_t^w = w_t^w(1-\tau_t^L)L_t^w + \mathfrak{T}_t$, this implies that the optimal consumption allocation of workers is unique as well. Lastly, since $u^w(.)$ is continuous and the constraint set of the workers' problem is compact, a solution to this problem exists. □

Using Lemma 8, it can be shown that Proposition 1 holds in segmented markets with the additional restriction that $\underline{f}$ is sufficiently large. The proof is omitted, as the logic is identical to the baseline case.

Lemma 9. *If A and $\underline{f}$ are sufficiently large, and $\tau_t^K \in [0,1) \; \forall t \in \{1, ..., T\}$, then resource constraints (2a)–(2c) are slack in any sticky wage Ramsey equilibrium.*

Proof. I show that if $\tau_t^K \in [0,1)$, then for any given sequence of real wages, $\{w_t, w_t^w\}$, the resource constaint (5b),

$$c_t + c_t^w + G_t + K_{t+1} - (1-\delta)K_t \leq Y_t + Y_t^w,$$

is satisfied if A is sufficiently large. To see this, recall the capitalist's period-t budget constraint:

$$c_t + K_{t+1} - (1-\delta)K_t = r_t K_t(1-\tau_t^K) + w_t L_t(1-\tau_t^L).$$

Aggregating, the resource constraint can be written as:

$$w_t L_t(1-\tau_t^L) + r_t K_t(1-\tau_t^K) + c_t^w + G_t \leq A Z_t K_t^\alpha L_t^{1-\alpha} + f(L_t^w).$$

Substituting in equilibrium capital allocations (see Lemma 8 (i)), we get:

$$w_t L_t(1-\tau_t^L) + r_t L_t\left(\frac{\alpha A Z_t}{r_t}\right)^{\frac{1}{1-\alpha}}(1-\tau_t^K) + c_t^w + G_t \leq A Z_t\left[L_t\left(\frac{\alpha A Z_t}{r_t}\right)^{\frac{1}{1-\alpha}}\right]^{\alpha} L_t^{1-\alpha} + f(L_t^w).$$

Dividing $L_t > 0$ from both sides:

$$w_t(1-\tau_t^L) + r_t\left(\frac{\alpha A Z_t}{r_t}\right)^{\frac{1}{1-\alpha}}(1-\tau_t^K) + \frac{c_t^w}{L_t} + \frac{G_t}{L_t} \leq A Z_t\left(\frac{\alpha A Z_t}{r_t}\right)^{\frac{\alpha}{1-\alpha}} + \frac{f(L_t^w)}{L_t}.$$

Substituting in equilibrium rates of return on capital (see Lemma 8 (ii)), we get:

$$\begin{aligned} &w_t(1-\tau_t^L) + \frac{1-\beta(1-\delta)}{\beta(1-\tau_t^K)}\left(\frac{\alpha A Z_t \beta(1-\tau_t^K)}{1-\beta(1-\delta)}\right)^{\frac{1}{1-\alpha}}(1-\tau_t^K) + \frac{c_t^w}{L_t} + \frac{G_t}{L_t} \\ &\leq A Z_t\left(\frac{\alpha A Z_t \beta(1-\tau_t^K)}{1-\beta(1-\delta)}\right)^{\frac{\alpha}{1-\alpha}} + \frac{f(L_t^w)}{L_t}. \end{aligned}$$

If $\tau_t^K < 1$, then we can cancel $(1-\tau_t^K)$ in the above equation and rearrange terms to get:

$$w_t(1-\tau_t^L) + \frac{c_t^w}{L_t} + \frac{G_t}{L_t} \leq A Z_t \Psi_t^{\frac{\alpha}{1-\alpha}} - \frac{1-\beta(1-\delta)}{\beta}\Psi_t^{\frac{1}{1-\alpha}} + \frac{f(L_t^w)}{L_t}.$$

where $\Psi_t \equiv \frac{\alpha A Z_t \beta(1-\tau_t^K)}{1-\beta(1-\delta)}$. Dividing both sides by $A Z_t \Psi_t^{\frac{\alpha}{1-\alpha}}$, we find:

$$\frac{w_t(1-\tau_t^L)}{A Z_t \Psi_t^{\frac{\alpha}{1-\alpha}}} + \frac{c_t^w}{L_t A Z_t \Psi_t^{\frac{\alpha}{1-\alpha}}} + \frac{G_t}{L_t A Z_t \Psi_t^{\frac{\alpha}{1-\alpha}}} \leq 1 - \alpha(1-\tau_t^K) + \frac{f(L_t^w)}{L_t A Z_t \Psi_t^{\frac{\alpha}{1-\alpha}}}.$$

From the workers' period-t budget constraint, $c_t^w = w_t^w L_t^w(1-\tau_t^L) + \mathfrak{T}_t$. Thus, the above condition becomes:

$$\frac{w_t(1-\tau_t^L)}{A Z_t \Psi_t^{\frac{\alpha}{1-\alpha}}} + \frac{w_t^w L_t^w(1-\tau_t^L) + \mathfrak{T}_t}{L_t A Z_t \Psi_t^{\frac{\alpha}{1-\alpha}}} + \frac{G_t}{L_t A Z_t \Psi_t^{\frac{\alpha}{1-\alpha}}} \leq 1 - \alpha(1-\tau_t^K) + \frac{f(L_t^w)}{L_t A Z_t \Psi_t^{\frac{\alpha}{1-\alpha}}}.$$

Since L_t is strictly positive, and L_t and L_t^w are independent of A (see parts (i) and (iii) of Lemma 8) in any sticky price equilibrium, the above condition holds when $A \to \infty$ as long as $\tau_t^K \geq 0$. Similarly, it can be shown that (5a) and (5c) are also slack. □

Note that the proofs of Lemmas 3–4, and Proposition 2 go through under segmented labor markets.

Lemma 10. *Suppose χ^w is sufficiently small, A and $\underline{f}$ are sufficiently large, and transfer policies*

are non-redistributive. Then optimal labor income taxes can be approximated by $\{\tau_t^L\}_{t\in\{1,\dots,T\}}$ *that satisfy the equations stated in Lemma 7.*

Proof. When χ^w is small enough and transfer policies are non-redistributive, then the optimal labor tax in period 1 is given by τ_1^L which solves the following equation as $\sigma \to 0$:

$$\frac{\mu(w_1)^{1/\varphi+1}(1-\tau_1^L)^{1/\varphi}}{\chi^{1/\varphi}}\left[\frac{1+\varphi-\chi}{\varphi}\right] + (1-\mu)v'(c_1^{w*})\left[-w_1^w L_1^{w*} + w_1^w(1-\tau_1^L)\frac{\partial L_1^{w*}}{\partial \tau_1^L}\right]$$
$$= \lambda\Bigg\{-\frac{\delta(1-\alpha)}{\alpha\varphi}\left\{\frac{\alpha^2 A Z_1}{\delta}\right\}^{\frac{1}{1-\alpha}}\left\{\frac{w_1}{\chi}\right\}^{1/\varphi}(1-\tau_1^L)^{1/\varphi-1} + \frac{w_1^{1/\varphi+1}}{\chi^{1/\varphi}}\left((1-\tau_1^L)^{1/\varphi} - \frac{\tau_1^L}{\varphi}(1-\tau_1^L)^{1/\varphi-1}\right)\Bigg\},$$

where $\lambda > 0$ denotes the Lagrange multiplier on the government's lifetime budget constraint. Note that as v has a satiation point, L_1^{w*} has a finite upper bound: $\bar{c}/w_1^w$. Moreover, $\tau_1^{L*} < 1$ (by Lemma 5). Thus, if χ^w is small enough, worker optimality implies $v'(c_1^{w*})$ is close to zero. Thus, the optimal labor tax in period 1 can be approximated by τ_1^L that solves:

$$\frac{\mu(w_1)^{1/\varphi+1}(1-\tau_1^L)^{1/\varphi}}{\chi^{1/\varphi}}\left[\frac{1+\varphi-\chi}{\varphi}\right] = \lambda\Bigg\{-\frac{\delta(1-\alpha)}{\alpha\varphi}\left\{\frac{\alpha^2 A Z_1}{\delta}\right\}^{\frac{1}{1-\alpha}}\left\{\frac{w_1}{\chi}\right\}^{1/\varphi}(1-\tau_1^L)^{1/\varphi-1} + \frac{w_1^{1/\varphi+1}}{\chi^{1/\varphi}}\left((1-\tau_1^L)^{1/\varphi} - \frac{\tau_1^L}{\varphi}(1-\tau_1^L)^{1/\varphi-1}\right)\Bigg\}.$$

Canceling $\frac{w_1^{1/\varphi+1}(1-\tau_1^L)^{1/\varphi}}{\chi^{1/\varphi}} > 0$, we find that the optimal labor tax in period 1 solves:

$$\frac{\mu(1+\varphi-\chi)}{\lambda\varphi} = 1 - \left\{\frac{\delta(1-\alpha)}{\alpha\varphi w_1(1-\tau_1^L)}\right\}\left\{\frac{\alpha^2 A Z_1}{\delta}\right\}^{\frac{1}{1-\alpha}} - \frac{\tau_1^L}{\varphi(1-\tau_1^L)}. \tag{9}$$

Similarly, the optimal labor tax in period t can be approximated by τ_t^L which solves the following equation:

$$\frac{\mu\beta^{t-1}(w_t)^{1/\varphi+1}(1-\tau_t^L)^{1/\varphi}}{\chi^{1/\varphi}}\left[\frac{1+\varphi-\chi}{\varphi}\right] = \frac{\lambda}{(1+r_B)^{t-1}}\Bigg\{-\left\{\frac{(1-\beta(1-\delta))(1-\alpha)}{\alpha\beta\varphi}\right\}\dots$$
$$\dots\left\{\frac{\alpha^2 A Z_t\beta}{1-\beta(1-\delta)}\right\}^{\frac{1}{1-\alpha}}\left\{\frac{w_t}{\chi}\right\}^{1/\varphi}(1-\tau_t^L)^{1/\varphi-1} + \frac{w_t^{1/\varphi+1}}{\chi^{1/\varphi}}\left((1-\tau_t^L)^{1/\varphi} - \frac{\tau_t^L}{\varphi}(1-\tau_t^L)^{1/\varphi-1}\right)\Bigg\}.$$

Canceling $\frac{w_t^{1/\varphi+1}(1-\tau_t^L)^{1/\varphi}}{\chi^{1/\varphi}} > 0$, we find that the optimal labor tax in period t solves:

$$\frac{\mu(1+\varphi-\chi)}{\lambda\varphi} = \frac{1}{\{\beta(1+r_B)\}^{t-1}}\left[1-\left\{\frac{(1-\beta(1-\delta))(1-\alpha)}{\alpha\beta\varphi w_t(1-\tau_t^L)}\right\}\left\{\frac{\alpha^2 AZ_t\beta}{1-\beta(1-\delta)}\right\}^{\frac{1}{1-\alpha}} - \frac{\tau_t^L}{\varphi(1-\tau_t^L)}\right]. \tag{10}$$

Substituting the value of $\frac{\mu(1+\varphi-\chi)}{\lambda\varphi}$ from Equation (10) into Equation (9) yields the conclusion. □

The proof of Proposition 4 remains valid under the conditions stated in Lemma 10.