Attention and the Retail Alignment Puzzle*

Daniel Graves†

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Abstract

This paper establishes a new empirical finance puzzle, the retail alignment puzzle: aggregate retail trader purchases and sales are nearly perfectly correlated across time and in the cross section of equities despite retail traders representing a small fraction of exchange volumes and being commonly represented as displaying lopsided flow patterns. Consistent with this puzzle, retail purchases and sales in the cross section are linearly predicted by the same two attention-associated factors, recent return salience and recent volume, with regressions on purchases and sales possessing almost identical coefficients. Using both directly measured attention through Google Trends search volumes and common indirect measures of attention such as volumes and extreme returns, I show that surges in retail attention consistently generate both large trading volumes and proportionally limited net trading. I then use an equilibrium disagreement model to show analytically and through simulations that while positive shocks to retail attention, sentiment, and disagreement all increase price, only shifts in attention are capable of reproducing empirical volume and return patterns. This paper's results suggest that attention is one of the core drivers of retail volume in common stocks.

JEL Codes: G12, G11, G40, G41, G23, C60

Keywords: attention, retail trader, buy sell imbalance, volume, turnover, disagreement, asset pricing, consideration set, sentiment

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†Yale University. Email: d.graves@yale.edu

1 Introduction

While asset price dynamics have long challenged the canonical models of financial economics, perhaps no financial variable has proved more vexing to economists than volume. As proved by Milgrom and Stokey (1982), under minimal assumptions Rational Expectations Equilibria are incompatible with any trade because, in essence, price changes reveal private information. Behavioral finance has provided alternative frameworks that allow for volume but has also struggled to explain volume data despite progress in explaining price patterns. The meme stock phenomenon of 2021 and the various cryptocurrency booms and busts are merely two of the latest in a centuries-long string of similar trading frenzies that call for a comprehensive explanation of retail investor volume patterns. Accentuating the challenges posed by volume to popular models, these sudden ebbs and flows in volume are not limited to history's most pathological financial episodes, with substantial daily volume fluctuations in many stocks. Understanding the primary drivers of volume fluctuations and price changes, as well as the connection between them, is crucial to a comprehensive understanding of financial markets. The unique attributes of households make the nexus between prices and retail trading particularly salient.

A common finance narrative posits a direct connection between retail volumes and prices, namely that equity mispricings are driven by imbalanced flows from uninformed retail traders who act on heightened sentiment or extrapolative beliefs. This paper casts doubt on this view by empirically establishing what I will refer to as the Retail Alignment Puzzle: in the cross section on single days and over time in commonly traded stocks, aggregate retail purchases and sales are nearly perfectly correlated with minimal imbalances despite the fact that retail traders only comprise around 10% of total volume. Using the recent methodological innovations of Boehmer, Jones, Zhang and Zhang (2021) (henceforth BJZZ) and Barber et al. (2024) (henceforth BHJOS) to identify and sign retail trades via off-exchange subpenny trades in TAQ (Trade and Quote, a dataset of all bids, offers, and trades from stocks listed on US exchanges) data, I show that daily retail purchases and sales in stocks such as Tesla or Microsoft cluster around the 45° line despite extreme variation in levels over time. As an illustration, Figure 1 shows the extraordinary alignment of aggregate daily retail purchases and sales in Tesla, the most commonly traded stock by households from 2013 to 2021. I further show that even during the height of the meme stock phenomenon, GameStop purchases and sales followed each other on parallel tracks with minimal percentage imbalances. This pattern emerges again with a straightforward regression analysis of the cross-sectional distribution of retail trades. Recent return salience and recent volumes are highly significant explanatory variables for retail purchases, but the coefficients are virtually identical when the analysis is repeated with retail sales instead

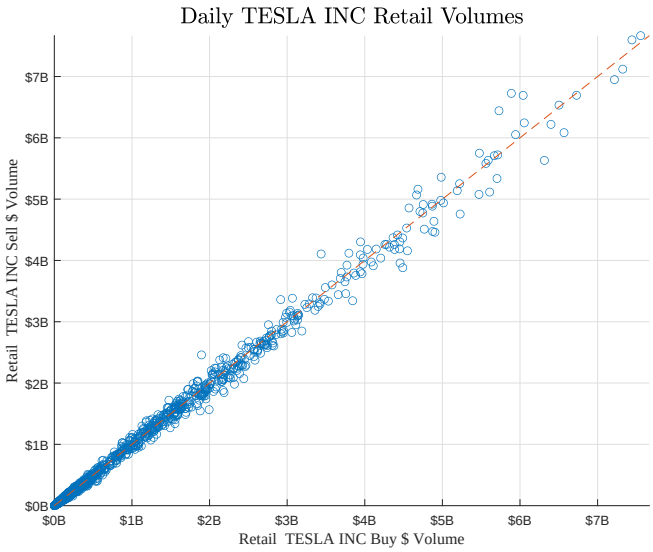


Figure 1: Tesla Retail Purchase and Sale Dollar Volumes over Time

Note: This figure plots the total daily retail dollar purchases and sales in Tesla stock over the nine year period 2013-2021 along with the 45° line. All trades are identified via the methodology of Boehmer, Jones, Zhang and Zhang (2021) and signed via the midpoint methodology of Barber et al. (2024).

of purchases. Sales have a modestly attenuated but nearly identical association with two variables commonly associated with attention.

After establishing and analyzing the Retail Alignment Puzzle, this paper shows both empirically and theoretically that attention shocks can generate extreme volumes, price shocks, and retail alignment. I first study two commonly used indirect proxies for high attention from the literature, high volumes and extreme returns, and show that attention shocks yield a statistically significant increase in net purchases but that these changes are small in magnitude. Next, using daily Google Trends search intensity as a direct measure of attention, I show that attention shocks generate significantly larger turnover — volume divided by shares outstanding — but minimally higher net retail purchases. Finally, I show that in an equilibrium disagreement model with both institutions and households, shocks to household attention are consistent with minimal net trading and high volumes, whereas shocks to average household sentiment result in strong directional imbalances.

That total purchases and total sales are perfectly aligned is a mere tautology because every trade has two parties. Once we separate buyers and sellers into groups I (institutions) and H (households) with different information and behavior via

$$\sum_{i\in\mathcal{I}} \tau_i^{Buy} + \sum_{h\in\mathcal{H}} \tau_h^{Buy} = \sum_{i\in\mathcal{I}} \tau_i^{Sell} + \sum_{h\in\mathcal{H}} \tau_h^{Sell}, \tag{1}$$

where τ is the dollar value of a trade, imbalances become directly related to group size. In

the modern era institutional investors comprise about 90% of exchange volumes with limited time variation and therefore any imbalances within institutional investors must be minimal on a percentage basis: with the standard Buy Sell Imbalance (BSI) calculation of Boehmer, Jones, Zhang and Zhang (2021), a maximally extreme daily institutional imbalance in the average stock would be of the order of $BSI_I = \frac{\$Buys_I - \$Sells_I}{\$Buys_I + \$Sells_I} = \frac{100-80}{180} = \frac{2}{9}$.[1] Retail traders, however, could in principle achieve either $+1$ or -1 with exclusively purchase or sale orders, respectively.

We should generally expect groups with different trading motives, private information, skill, risk aversion, and biases to generate net order imbalances in aggregate. If hedge funds, for example, had near zero daily BSI magnitudes, it would imply that the hedge fund industry collectively holds a particular time-invariant portfolio of stocks. 13F replication portfolios would then serve as perfect facsimiles of the industry's gross performance without incurring fees. As shown in Brunnermeier and Nagel (2004), however, hedge funds do in fact engage in rapid portfolio changes versus the market portfolio in aggregate. Given that households have been shown in the behavioral literature to be overconfident and drawn to purchase attention-grabbing stocks (Barber and Odean (2008)), we have ample reason to suspect that households might in aggregate display rapid oscillations in positions like the hedge funds of Brunnermeier and Nagel (2004).

Yet they do not. A stock experiencing a large surge in retail purchase orders will almost always experience a nearly perfectly matched surge in retail sale orders. I design a simple but novel metric, the Volume Weighted Adjusted Imbalance (VWAI), to evaluate a group's trade imbalances that (1) weights days proportionally to total volume, (2) removes volume parity trivially connected to the volume accounting identity, and (3) takes values on the interval $[0, 1]$ where 0 is evenly matched purchases and sales and 1 is all purchases or all sales. Looking at retail trades identified and signed via the methodology of BJZZ/BHJOS over the period 2013-2021 and weighting each stock by its total retail volume over the dataset, I find a composite VWAI value of merely 0.0826, close to evenly matched. Applying this methodology to weekly CFTC imbalance data for firms responsible for a fraction of futures trading activity similar to that of retail traders in common stocks, I find large imbalances comparable to drawing daily purchases and sales at random from the unit interval. The imbalance dynamics of retail trader activity are thus an unusual feature, not a universal phenomenon.

The same puzzle emerges upon analysis of the cross-sectional distribution of retail volume. I take the percentage of cross-sectional dollar retail purchase volume concentrated in stock X on day t to be the dependent variable and use three types of explanatory variables: (i) stock X's fraction of total CRSP dollar volume in common stocks at different lags, (ii) the

[1] As we will see in Section 2, daily retail fraction of trading has a limited range.

salience of stock X's return over recent time windows, and (iii) the market capitalization of stock X as a fraction of the total market capitalization of the stock market. The salience of a stock's returns over different horizons is computed via the rank-dependent Salience Theory approach of Bordalo, Gennaioli and Shleifer (2012) where returns far from the mean are the most salient. Volume lags and return salience are found to both be highly significant predictors of fractional retail volume with nearly identical coefficients for purchases and sales. A stock's contemporaneous return shifting from the least salient to the most salient is associated with a 139% increase in retail purchase volume, but also a 125% increase in retail sale volume. Subdividing orders into odd lots and "even" lots,[2] the exact same pattern remains, with both small and large retail traders displaying an almost equal affinity for purchasing and selling attention-grabbing stocks, though with a slight imbalance toward purchases as evidenced by the higher coefficients on salience ranking.

Retail traders' penchant for purchasing and selling stocks that attract attention, which is strongly implied by the coefficients of the regression analyses, argues for a unified explanation of fluctuations in retail purchases, retail sales, and net retail order flow. While previous work has often suggested a connection between attention and either net retail order flow or retail purchases, this paper is the first to provide evidence that attention can explain purchase levels, sale levels, the direction of net flows, and the limited magnitude of net flows. This evidence is provided through two forms of analysis on the connection between order imbalances, turnover, and attention: an analysis through attention proxies using the full data set and an analysis through directly measured attention in a subset of large stocks.

I first analyze the BSIs of stocks experiencing extreme returns or abnormal volume, both proxies for high retail attention. For both the return analysis and volume analysis, extreme days are associated with modestly higher BSIs: extreme return days have a 0.012 (t-statistic of 9.46) higher BSI while extreme volume days have a 0.016 (7.16) higher BSI. This pattern is entirely consistent with this paper's cross-sectional analysis of retail volume, which finds modestly attenuated coefficients for sales versus purchases on volume and extreme return variables. These results provide new insight into the dynamics observed by Barber and Odean (2008): whereas Barber and Odean (2008) looks at the impact of today's extreme returns on *tomorrow's* retail imbalances and uses trades as opposed to dollars as the basis for imbalance calculation, I study the *contemporaneous* relationship between extreme returns and *dollar* imbalances.

Directly measured attention to stocks provides sharper quantitative results but illustrates precisely the same message: attention shocks create contemporaneous surges

[2]Odd lots, per standard terminology, are all orders of less than 100 shares. Throughout this paper I will use "even lot" as shorthand for "non-odd lot" since it is less cumbersome.

in volume with only small positive BSIs. Using daily Search Volume Index (SVI) data extracted from Google Trends, I isolate "high attention" days for each of 71 large stocks. For each stock, the SVI data for its ticker symbol is extracted from Google Trends. The top 5% of days by normalized SVI for a given stock are categorized as "high attention" days and I discard all stock $\times$ day pairs that coincide with an earnings announcement. I find that high attention days have a statistically significantly higher BSI, but only by 0.008 (t-statistic of 3.85) while the difference in the average magnitude of BSIs is statistically indistinguishable from zero. On the other hand, high attention days have approximately 170% higher retail turnover (14.0) and nearly twice the average return magnitude. The increase in order imbalances on high attention days is entirely consistent with the longstanding concept of attention shocks triggering net retail buying because of short sale constraints, but the small size of the effect, particularly in comparison to the extreme effect on retail trading levels, indicates that attention ignites both retail purchases and sales. Moreover, retail traders comprise a statistically significantly larger fraction of exchange volumes on high attention days. This phenomenon of limited imbalance differentials under shocks is not, however, universal to non-attention mechanisms. Earnings report days have 0.0581 higher retail BSIs (21.1) versus non-earnings days, whereas high attention days see a mere 0.008 increase in BSIs. Meanwhile, turnover is 95% elevated on earnings report days but 170% elevated on high attention days. Attention triggers limited imbalances but high volume whereas earnings news triggers larger retail imbalances but a more modest volume shock.

To explore the differential effects of attention, disagreement, and sentiment shocks on retail order imbalances, I build a dynamic single risky asset equilibrium disagreement model with heterogeneous institutions and investors. While all three types of shock apply positive price pressure, they create markedly different volume imbalance patterns. The theory section's main results, given analytically in Propositions 1, 2, and 3, are (i) that attention shocks can create small BSIs when institutional return expectations are low compared to household expectations; (ii) that positive sentiment shocks generate consistently higher positive BSIs than attention shocks; and (iii) that changes in disagreement can generate limited BSIs just like attention shocks but can also generate large imbalances depending on the parameters. The basic intuition for these results is that, whereas sentiment and disagreement shocks directly increase the expected returns of all or many households, respectively, attention shocks do not alter any investor's expected return conditional on price, instead bringing in a new mass of traders, some of whom are optimistic and others who are pessimistic but short sale constrained. When the equilibrium price increases because of the attention shock, the existing household holders of the asset sell some or all of their holdings while the more optimistic of the newly attentive traders buy. This is a direct result of downward sloping demand with respect to price and a rebalancing motive. The

marginal asset holder after a positive retail sentiment shock is less optimistic versus their peers than the marginal holder from before the shock. When an attention shock hits, the marginal holder shifts to being a more extreme optimist, with aggregate trading yielding a transfer of the risky asset from existing holders of the asset to the most optimistic of the newly attentive traders. Attention is thus unique in that it creates a dynamic where new and old retail traders trade in opposite directions. Although retail traders do purchase in aggregate from institutions under positive attention shocks, the percentage imbalance is small in stocks with a large fraction of household ownership, precisely the stocks most traded by households.

To synthesize the empirical results and theory, I undertake a case study of the most infamous case of extreme retail trading: GameStop in January and February of 2021. Using TAQ data, CRSP data, and institutional ownership and net short data from Refinitiv, I show how the model's main predictions hold. Negative institutional sentiment resulted in *negative* retail BSIs during the episode despite odd lot retail BSIs being consistently positive in accordance with the common narrative of small trader purchases. The theory predicts that the extreme attention shock would generate a neutral to negative BSI because of a major rebalancing channel: as newly attentive optimistic retail traders herd into GameStop and drive up the price, institutions with short positions need to rebalance through purchasing back shares.[3] Existing retail traders — in this case larger traders — sold into the rally, disposing of the shares at a gain. Although the model does not account for extrapolative expectations or heterogeneity in trader sophistication, both of which were likely at work during this episode, it does capture the crucial importance of attention in triggering a rebalancing channel and an "old versus new trader" dynamic within the retail trader population. This dynamic has significant implications for our understanding of overtrading and its importance to household wealth creation or destruction.

The finding of retail alignment also serves to rationalize the industrial organization of equity markets. "Payment for order flow" and "price improvement" have been intensely debated in the public policy sphere and studied in the literature (see e.g. Schwarz et al. (2022)); wholesalers such as Citadel Securities and Virtu Financial pay retail brokerage firms like Schwab and Robinhood per-share fees in exchange for orders being routed their way. That wholesalers are willing to offer price improvement on the vast majority of retail trades while simultaneously relinquishing a fraction of the bid-ask spread to brokerage firms, all for the mere privilege of internalizing orders, indicates that retail trades are sufficiently aligned that market makers can absorb the flows while accruing minimal inventory risk. If

[3]This is the infamous "short squeeze" that engulfed several large hedge funds like Melvin Capital. Reporting in the WSJ and elsewhere revealed that these firms purchased back GameStop shares en masse in response to the price shock. Short data from Refinitiv starkly illustrates this pattern, with shorts plummeting from over 100% of the market cap to under 20%.

retail order flows were typically lopsided and "toxic" in the sense of Easley, López de Prado and O'Hara (2012), then market makers would build unacceptably large inventory through internalization and be forced to dispose of inventory at a loss. With retail alignment, however, we can readily rationalize the record $\$7.5B$ in net 2022 trading revenue reaped by Citadel Securities, the largest wholesaler, despite payment for order flow and ubiquitous price improvement.

Related Literature on Volume, Retail Trading, and Attention This paper is most closely related to past work on volume patterns, retail trading and imbalances, and attention. A long literature has studied volume patterns in equities, particularly in the context of return predictability. Lee and Swaminathan (2000) and Datar, Y Naik and Radcliffe (1998) find that stocks with low turnover outperform stocks with high turnover, while on the opposite end, Gervais, Kaniel and Mingelgrin (2001) find that over short time horizons, stocks experiencing a surge in volume tend to appreciate, with the opposite for those experiencing a decline. Despite the seeming contradictions implicit in these findings, they are all supported by this paper's results. I find that attention shocks and BSIs are both strongly autocorrelated, supporting the Gervais, Kaniel and Mingelgrin (2001) contention that shifts in visibility can lead to modest short-term return predictability through an attention mechanism in the spirit of Miller (1977). Moreover, stocks with elevated turnover over a longer period of time as in Lee and Swaminathan (2000) or Datar, Y Naik and Radcliffe (1998) will have already experienced an attention-driven (non-fundamental) shock to prices and therefore will be fundamentally overvalued versus equities that have not seen a surge in retail trader interest. All of these arguments are contingent on the existence of disagreement in the presence of short sale constraints, a combination emphasized by Hong and Stein (2007) as crucial for explaining many of the volume patterns in equities. Another combination used to explain volume is extrapolative beliefs plus wavering as proposed in Barberis, Greenwood, Jin and Shleifer (2018), whereby extrapolators see their weighting on extrapolative beliefs versus value motives oscillate slightly.

This paper is connected to a long literature on retail trading and imbalance patterns. Boehmer, Jones, Zhang and Zhang (2021) and Kelley and Tetlock (2013) find that marketable retail order imbalances predict returns, while Barber and Odean (2000) find that retail traders earn consistently poor returns. As recently argued in Barber, Shengle and Odean (2023), these facts are not necessarily in opposition because net retail trades concentrate in the most attention-grabbing names and are not equally distributed across stocks.[4] Whereas equal-weighted portfolios based on retail imbalances have positive return

[4]See Barber, Shengle and Odean (2023) for a more extensive review of the literature that makes these arguments about the positive or negative collective wisdom of retail traders.

predictability, trade-weighted portfolios perform exceptionally poorly. Barber, Shengle and Odean (2023) further argue that trades smaller than $500 display concentration of purchases in attention grabbing stocks but do not display the same degree of concentration on the sell side. This paper has an entirely different focus: the near perfect alignment of retail purchases and sales on a *dollar* basis. I find that retail traders do concentrate sales in attention-grabbing stocks, with larger traders displaying more attention-driven selling behavior than smaller traders.

Extensive previous work has discussed the concept of attention and retail trades, though never in the context of retail alignment. Barber and Odean (2008) find that retail traders tend to purchase stocks with large volumes, large price movement, or news, all proxies for attention. Engelberg, Sasseville and Williams (2012) find strong support for this argument, showing that Jim Cramer recommending a stock on his show *Mad Money* has a positive impact on price overnight, with a subsequent reversal over the following year. They find that the effects of a recommendation increase when the recommendation occupies a greater fraction of the show or when the recommendation occurs during an episode with elevated viewership. Da, Engelberg and Gao (2011) similarly find that attention shocks predict higher returns in the short term but with a reversal back over a one year horizon, but they do so in the full cross section of Russell 3000 stocks from 2004 to 2008 using weekly Google Search Volume Index (SVI) data. They also find that higher SVI leads to higher retail trading activity at a monthly level. This paper uses the direct attention measurement via SVI concept of Da, Engelberg and Gao (2011), but does so with daily instead of weekly SVI and daily retail transaction data, employs a different SVI normalization, covers a different time period, and employs attention data in service of studying retail imbalances as opposed to return predictability.

Another paper to note large attention effects on retail trading is Engelberg and Parsons (2011), who find that differential newspaper exposure among retail traders generates different trading patterns. Specifically, when a local paper in city X reports on a company's earnings but the local paper in city Y does not, traders located in the city X region actively trade the stock whereas traders in city Y trade far less versus X's traders even when the company is headquartered outside of both regions. The observation most closely related to this paper's findings is that in the analysis of Engelberg and Parsons (2011), the surge in local trading activity accompanying media exposure features an imbalance toward purchases but only a small one. I show that this tendency is pervasive across the entire stock market and applies in the modern era, but more importantly I find that the tendency is a general unconditional feature of stocks, implying that attention might be the primary driver of retail purchases and sales.

Finally, models of disagreement are also closely related to this paper. This paper's model

is related to the disagreement model described by Hong and Stein (2007), who show that stock market volumes can in part be explained by disagreement in the presence of short sale constraints. This paper's model relies on the existence of disagreement but argues that fluctuations in attention, as opposed to changes in disagreement itself, most plausibly explain retail volume patterns.

Outline The remaining sections of this paper establish the retail alignment puzzle, show empirically that attention shocks generate minimal BSIs, develop a theoretical model of volume, and then illustrate the results in a case study using GameStop. Section 2 examines the cross-sectional and time series relationship of dollar retail purchases, dollar retail sales, and total dollar volume and shows that both retail purchases and sales are predicted by salient returns and high volumes. Section 3 provides both indirect and direct evidence that attention shocks generate modest BSIs despite significant surges in trading activity. Section 4 develops a model of attention, sentiment, and disagreement shocks and derives analytic and simulation results for retail volume changes. Section 5 uses GameStop as an order imbalance case study. Section 6 concludes.

2 The Retail Alignment Puzzle: Empirical Facts

This section establishes that retail purchases and sales are aligned in the cross section and in single stocks over time and that this alignment is unusual given institutional subgroup non-alignment within CFTC data.

2.1 Data and Retail Trade Identification Methodology

I use the NYSE TAQ database from January 1, 2013 through December 31, 2021 to identify retail purchases and sales via a hybrid of the methodologies of Boehmer, Jones, Zhang and Zhang (2021) (henceforth BJZZ) and Barber et al. (2024) (henceforth BHJOS). Central to both methodologies is the concept of subpenny price improvement whereby wholesalers internalize retail order flow — absorbing or crossing the orders — at non-full penny prices. Institutionally-initiated trades, unlike retail trades, may only occur at full or half pennies because of Regulation NMS, an SEC regulation adopted in 2005. Following the BJZZ methodology, I identify as "retail" all trades that occur at non-whole or half penny prices. The signing of trades as retail purchases or sales is based on the simple midpoint rule advocated by BHJOS and originated by Lee and Ready (1991): all subpenny trades above midpoint are signed as retail purchases and all subpenny trades below midpoint are signed as retail sales.

Table 1: TAQ Subpenny Retail Trade Summary Statistics

	Equities w Order	Avg Daily $ Trds	Std Dev Daily $ Trds	Tot Trad Days
Buys (All)	3208	$8.4B	$4.7B	2267
Sells (All)	3228	$8.3B	$4.7B	2267
Buys (Odd)	2689	$1B	$0.92B	2031
Sells (Odd)	2704	$0.89B	$0.77B	2031
Buys (Even)	3089	$7.5B	$3.8B	2267
Sells (Even)	3126	$7.5B	$3.9B	2267

Note: Table 1 contains summary statistics for the set of subpenny retail trades from 2013-2021 obtained by applying the Boehmer, Jones, Zhang and Zhang (2021)/Barber et al. (2024) methodology to the TAQ database. Only trades that occur at non-midpoint subpenny prices are included.

As remarked upon in BJZZ, the inability of non-retail traders to trade at subpenny non-half penny prices guarantees that identified orders are indeed retail trader-initiated; meanwhile, the midpoint signing rule was shown in BHJOS to have a high level of accuracy even in one lot orders, the type of order most likely to be signed incorrectly. Using the full TAQ database from 2013 to 2021, I employ the outlined identification and signing methodology to generate marketable retail dollar buy orders and sell orders by date and symbol. Moreover, I separately aggregate the daily retail "odd" (1-99 shares) and "even" (100+ shares) lot purchases and sales. Table 1 provides summary statistics for the resulting retail trade database. The full sample is a nine year window, January 1, 2013 through December 31, 2021.[5]

Closely following the BJZZ methodology on this paper's extended time window, I merge the TAQ data with CRSP data, exclude all stocks that do not have "common stock" codes 10 or 11 in CRSP, and exclude all trades that are under $1/share or that do not have the TAQ condition "D" (the off-exchange condition code).[6] The TAQ-derived database of retail dollar purchases and sales is merged with return and volume data from CRSP. Furthermore, because this paper is focused on the concept of "attention" as opposed to instrument-specific microstructure, I aggregate dollar volume data across instruments with the same CRSP PERMCO identifier; aggregation prevents companies with multiple share classes

[5]Previous studies such as BJZZ and BHJOS have excluded the period from October 3, 2016 through September 28, 2018 because of changes in SEC regulations (the Tick Size Pilot Program or TSPP) that altered subpenny price improvement rules over that period; I choose to keep this period in because no material differences are found between that period and the prior or subsequent periods. Some analyses exclude the period 1/1/2013-12/8/2013 because odd lot trades were not required to be reported on TAQ until December 9, 2013.

[6]As detailed in BJZZ, retail trades are almost always executed off-exchange; all off-exchange retail trades are reported via one of the Trade Reporting Facilities (TRFs), which are coded with TAQ condition code "D". Trades under $1 are not subject to the Reg NMS sub-penny rules and therefore the identification method only works for trades over $1 per share.

from appearing repeatedly within cross-sectional analyses. Returns for merged entities are taken to be market cap-weighted averages of the component equity returns.

2.2 Retail Alignment

An extensive literature has already discussed how imbalances in retail order flow anticipate future returns and news (see Boehmer, Jones, Zhang and Zhang (2021), Kumar and Lee (2006), and Kelley and Tetlock (2013), for example). This section focuses not on the differences between household purchases and sales but rather on the similarities. The two sides of retail order flow covary both cross-sectionally and over time with near perfect correlation.

Let us first turn our focus to the daily aggregation of subpenny-identified dollar purchases and sales for common stocks. Plotting daily aggregate common stock retail purchases versus common stock sales gives Figure 2, with daily retail buy-sell pairs consistently clustering near the 45° line. Although many obvious deviations from parity occur, with dollar imbalances ranging from $-\$2.13B$ to $+\$2.24B$, the 99.73% correlation indicates that retail order flows to common stocks nearly cancel out in percentage terms. If we define the Buy Sell Imbalance as

$$BSI_t = \frac{DVBuy_t - DVSell_t}{DVBuy_t + DVSell_t} \in [-1, 1] \tag{2}$$

consistent with BJZZ, where $DVBuy_t$ and $DVSell_t$ are the day t dollar buy and sell volume respectively, then the mean value of $|BSI_t|$ is a mere 0.0175 and the median is 0.014. The single largest magnitude value of BSI_t is 0.116 and occurs on June 24, 2016, the first US trading session after the Brexit result announcement on the evening of June 23rd.[7] The most extreme negative imbalance is only -0.0885. Analyzing aggregate data, however, only allows us to conclude that retail traders tend to buy and sell approximately equal dollar amounts of equities each day. Aggregate alignment does not imply stock level alignment if retail traders preferentially sell certain stocks to "substitute" for other stocks.[8]

Retail alignment at the security level is clear from scrutiny of the daily pattern across time for large cap stocks in different industries and with different share price levels.[9] Daily retail buy and sell volume are plotted against one another for Microsoft and Apple, for instance, in Figures 3a and 3b. In Appendix I, I repeat the exercise for all current Dow Jones

[7] US equity markets declined significantly in the overnight session and whipsawed during the US trading session amidst declines of $3-4\%$ in major indices, with retail traders rushing in to "buy the dip."

[8] The substitution concept, at least at the level of daily stock data and in the context of retail trades, is suspect. During this paper's sample period, securities settle on day T+2 so the cash receipts for a purchase do not arrive until two days after a sale.

[9] Different price levels allows for microstructure variation so the results are not attributable to the idiosyncrasies of BJZZ identification in a single equity.

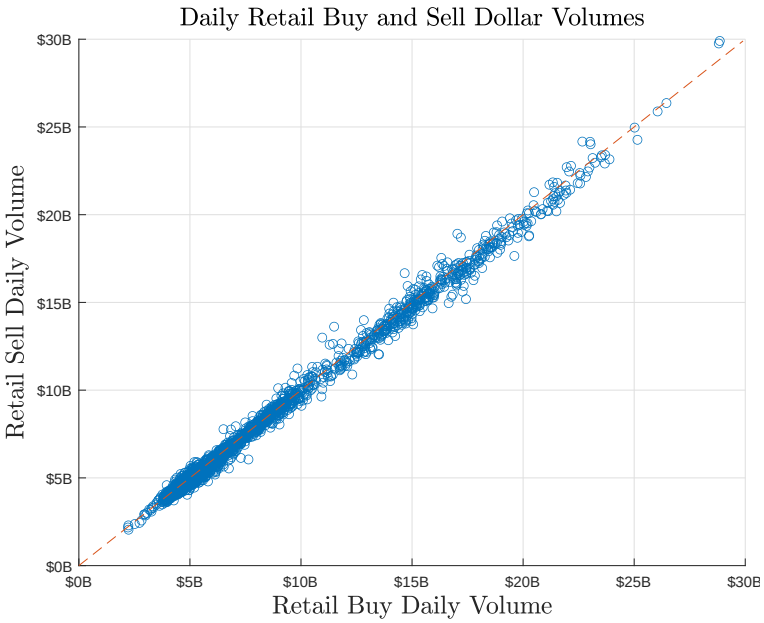


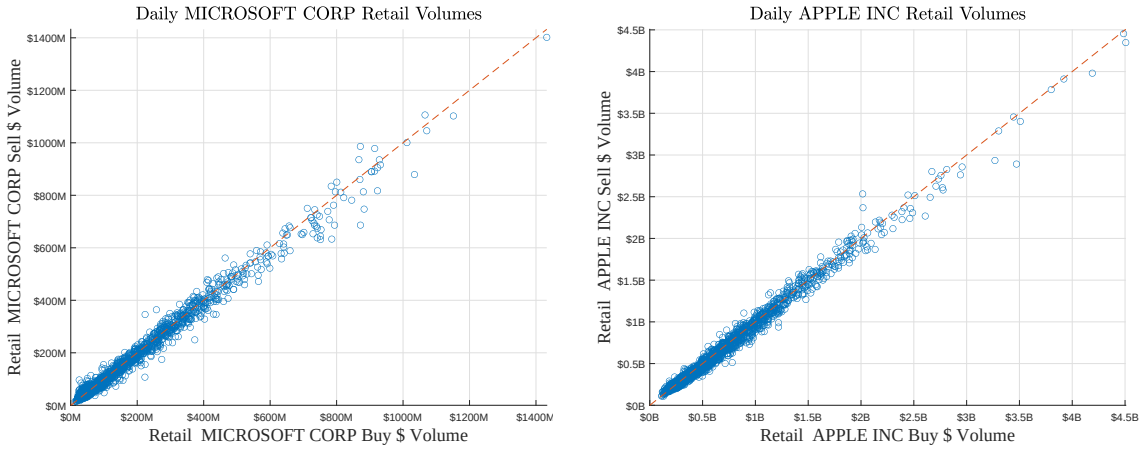
Figure 2: Aggregate Retail Buy and Sell Orders through Time

Note: This figure plots the total daily retail dollar purchases and sales in all common stocks within the CRSP-TAQ dataset over the nine year period 2013-2021. All trades are signed via the midpoint methodology of Barber et al. (2024) and identified via the methodology of Boehmer, Jones, Zhang and Zhang (2021).

Industrial Average components and an assortment of so-called "meme stocks." Once again, the same striking pattern reappears over time in each single stock. Although significant variation is visible, the less visually striking plot of Coca Cola volumes, with its high correlation of 87.3%, only appears weak when compared to its Dow Jones peers like Microsoft (99.3%) and American Airlines (99%). Retail traders trade in and out of various equities, but purchases and sales in common stocks align over time.

The cross section of stocks on single days shows an equally stark pattern. Figure 4a plots the correlation of retail buy and sell dollar volume in the cross section of stocks on each day t within the sample, which is to say the correlation of $(\$RetBuys_{i,t}, \$RetSells_{i,t})$ across i fixing t. One would expect a strong correlation for the simple reason that large market capitalization stocks should have larger dollar volumes of both buy and sell orders, yet this relationship is robust to truncation at arbitrary "small cap" and "large cap" levels and the median correlation is 99.6%. The one extreme value occurs on September 29, 2014 and is solely attributable to McDonald's stock possessing an extreme outlier negative retail imbalance. This unusual institutional purchasing activity was contemporaneously remarked upon in an article in the *Chicago Tribune*.

Using turnover to eliminate the effects of market capitalization on the correlation analysis, I find the same pattern of extreme alignment. I compute the correlation of retail



(a) Microsoft (b) Apple

Figure 3: Retail Alignment in Two Large Stocks

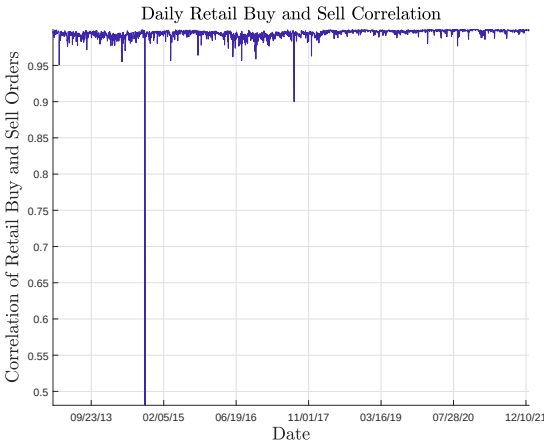
Note: This figure plots the total daily retail dollar purchases and sales in Microsoft and Apple stock over the nine year period 2013-2021. All trades are signed via the midpoint methodology of Barber et al. (2024) and identified via the methodology of Boehmer, Jones, Zhang and Zhang (2021).

purchase turnover and sale turnover in the cross section, which is to say the correlation of $\left(\frac{\$RetBuys_{i,t}}{MktCap_{i,t}}, \frac{\$RetSells_{i,t}}{MktCap_{i,t}}\right)$ across i fixing t. Using only stocks with market caps over $\$200M$ and with daily volume of at least $\$20M$ to avoid small sample issues, I obtain an average buy/sell turnover correlation of 98.6%. Figure 4b plots the daily levels, which are consistently close to unity.

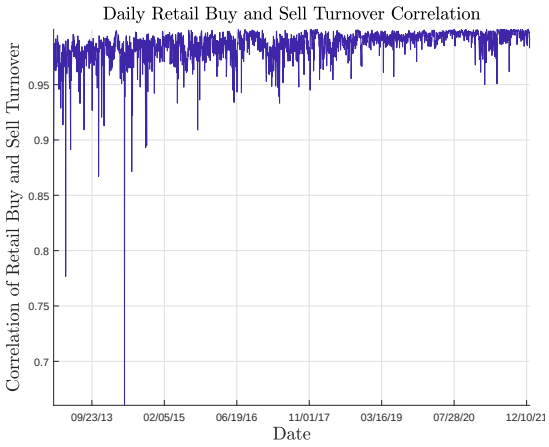
2.3 Retail Volume as Fraction of Total Volume

Given that total dollar purchases and sales are tautologically equal in all financial markets, a subgroup of market participants' deviation from buy-sell parity is only surprising if it both is a small fraction of total volume and possesses distinct attributes. A large behavioral literature has already documented how household trading behavior and preferences differ substantially from those of institutions;[10] therefore, accounting for retail's fractional portion of trading activity is of critical importance. The BJZZ methodology does not allow for the precise recovery of retail fractional volume because it only identifies a subset of retail trades, marketable orders executed with subpenny price improvement that take place away from half pennies. If we assume that executions at midpoint, executions at full penny increments, and passive order executions are a nearly constant fraction of total retail order flow over time, then the time series of aggregate BJZZ-identified retail dollar volumes divided by CRSP

[10] A small sample of such tendencies would include preference for idiosyncratic skew (Barberis and Huang (2008)), overconfidence (Barber and Odean (2000)), extrapolative expectations, limited attention, belief in the Gambler's Fallacy, and many other subjects of behavioral finance.



(a) Daily Cross-Sectional Correlation of Buy and Sell Retail Dollar Volume



(b) Daily Cross-Sectional Correlation of Buy and Sell Turnover

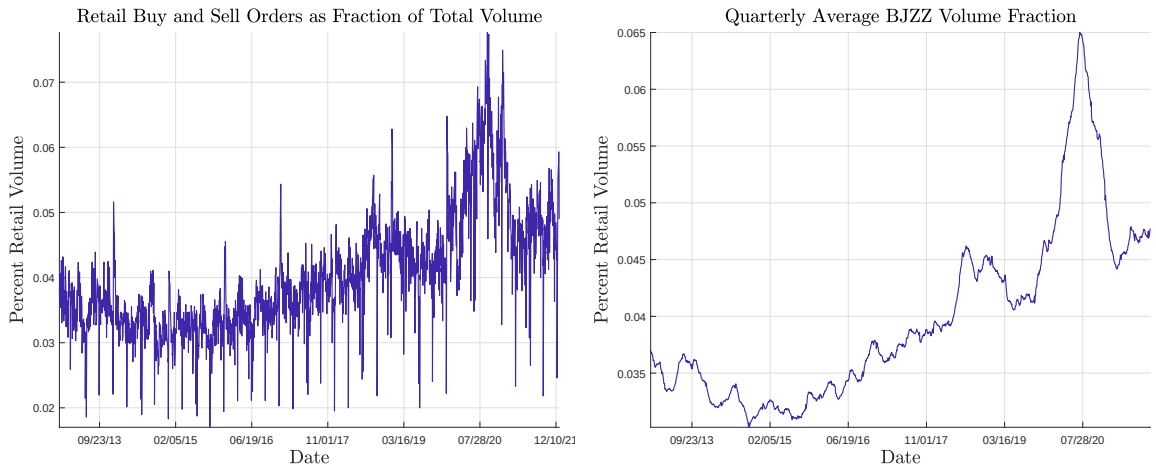
Figure 4: Daily Cross-Sectional Correlations

Note: Figure 4a plots the daily cross-sectional correlations between dollar retail purchases and sales for all equities with more than \$10,000 in total retail trades. The spike down on September 29, 2014 is the result of one extreme point, with a correlation of 0.9948 when McDonalds is excluded and 0.4831 when McDonalds is included. This point does not appear to be the result of a data flaw or methodological issue, as the Chicago Tribune and others reported on anomalous institutional buying of McDonald's stock on that exact day, with speculation as to the identity of the large purchaser. Figure 4b plots the daily cross-sectional correlation over all i for fixed t of $\frac{DollarRetailPurchases_{i,t}}{MarketCap_{i,t}}$ and $\frac{DollarRetailSales_{i,t}}{MarketCap_{i,t}}$ for stocks with market caps over $\$200M$ and daily volume over $\$20M$. This normalization removes spurious correlation driven by market cap effects, still finding a strong cross-sectional relationship between retail purchases and sales.

dollar volumes provides a direct way of studying time variation in retail trades' fractional share of total volume; it should be approximately proportional to the true fraction of total retail trading volume up to a scaling factor. Kelley and Tetlock (2013) use a comprehensive proprietary dataset from OTC market centers that includes approximately one third of all US retail trades from 2003-2007 but includes the non-marketable limit orders missed by the BJZZ methodology. While their database includes 178 million marketable retail orders, it only includes 47 million executed limit orders, a ratio of $3.8:1$, implying that the missing limit orders are likely to be a small fraction of retail volume.

Figure 5a shows the time series of identified dollar retail trading volume as a fraction of total dollar volume, where the fraction is computed as $\frac{\sum RetailBuys+\sum RetailSells}{\sum TotalBuys+\sum TotalSells}$, while Figure 5b presents the quarterly smoothed retail fraction of total dollar volume.[11] Two

[11] A common way of calculating retail volume in the literature and financial press is $\frac{\sum DollarRetailTrades}{DollarVolume}$, which is a peculiar method because of the contrast between single and double counted volume. If retail trades are 60% of purchases and 60% of sales, this would imply that retail traders are either 120% or 100% of total volume depending on how trades are tabulated (unique trades versus counting both parties if both are retail traders). As a thought experiment, if 5% of both purchases and sales are by retail traders, then 10% of marketable orders are attributable to retail traders, but only 5% of volume is attributable to retail traders since another (non-retail) party is always involved in the transactions.



(a) Daily Marketable Retail Fraction of Total Volume (b) Smoothed Marketable Retail Volume Fraction

Figure 5: Retail Volume over Time, 2013-2021

Note: This figure plots the daily dollar retail volume as a fraction of total exchange dollar volume and its smooth quarterly average, where all volumes are identified via the method of Boehmer, Jones, Zhang and Zhang (2021) and signed via the midpoint method of Barber et al. (2024). In Sub-Figure 5a quarterly downward spikes are visible on each of the so-called "quadruple witching" days where institutional traders engage in increased hedging and position unwinding activity due to simultaneous expiry of options and futures. In Sub-Figure 5b volumes display a gradual uptrend after 2015, the start of Robinhood, with a large upward spike at the beginning of the "work from home" period of the pandemic.

facts are visually clear. First, and most importantly, the time series is stable over short periods of time but with a general upward trend and a large increase during the beginning of the pandemic as retail traders began working from home. The second fact, readily apparent even by visual inspection, is that retail volume has cyclical quarterly one-day declines relative to total volume. This cyclicity is not in fact driven by retail trading but rather by quarterly surges in institutional orders, which as previously noted do not receive subpenny non-midpoint price improvement and are therefore not flagged as retail orders by the BJZZ methodology. Each spike down corresponds to the third Friday of March, June, September, and December. These so-called "quadruple witching days" see the expiry of a plethora of futures and options contracts, with institutions engaging in substantial hedging activity and position unwinding. Retail traders, by contrast, with limited futures or options positions and a lack of sophisticated hedging strategies, do not participate in this frenzy of activity. Such cyclical variation in institutional trading is orthogonal to this paper's objects of interest. Of primary interest to this paper is that, even after adjusting for the smaller fraction of limit orders, retail trading is consistently a small fraction of total exchange volumes.

2.4 Accounting Identities and Measuring Retail Alignment

The fact that retail traders are only a small fraction of total volume, established in the last subsection but also in Kelley and Tetlock (2013), assuages concerns about the accounting identity of Equation (1) leading to the spurious appearance of retail alignment when studying imbalances, but retail fractional volume fluctuates over time and in the cross section. We thus require an imbalance metric that accounts for retail traders' fraction of trading volume in each stock on each day. Furthermore, we need the metric to account for time heterogeneity in trading: a day with 1 share traded by retail traders should be weighted near zero versus a day with $1,000,000$ shares traded.

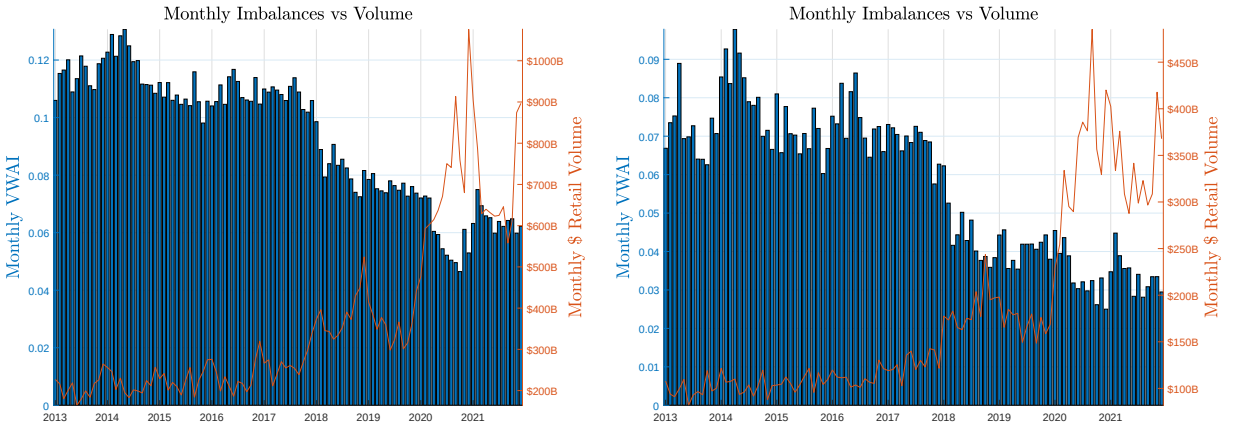
I therefore design a simple but novel metric to evaluate a group's trade imbalances that (i) weights stock $\times$ day pairs proportionally to total retail volume, (ii) conservatively shrinks volumes uniformly toward zero to account for retail traders mechanically trading with each other, and (iii) takes values on the unit interval $[0, 1]$ with 0 implying perfect parity and 1 signifying all purchase orders or all sales. Taking each of these attributes in turn, (i) is accomplished by applying a weight $T_t / \left(\sum_\tau T_\tau\right)$ to each daily imbalance, where T_t is the total retail volume on day t. (ii) is accomplished by shrinking retail dollar purchases and sales by $Frac_t^2 \times T\$V_t$, where $Frac_t$ is retail's fraction of total volume on day t and $T\$V_t$ is total dollar exchange volume on day t. $Frac_t^2 \times T\$V_t$ is the maximal dollar volume with retail traders on both sides of the transaction if buy-sell matching is random and $Frac_t$ of volume is attributable to retail traders.[12] Finally, (iii) is accomplished by computing for each day the classic BSI measure with the shrunken purchase and sale values and taking the absolute value. The Volume Weighted Adjusted Imbalance (VWAI) for a given asset is then

$$VWAI = \sum_t \frac{T_t}{\sum_\tau T_\tau} \left| \frac{\max\left(\$Buys_t - Frac_t^2 \times T\$V_t, 0\right) - \max\left(\$Sells_t - Frac_t^2 \times T\$V_t, 0\right)}{\max\left(\$Buys_t - Frac_t^2 \times T\$V_t, 0\right) + \max\left(\$Sells_t - Frac_t^2 \times T\$V_t, 0\right)} \right|. \quad (3)$$

Computing this value over the entire $2013 - 2021$ dataset and weighting each stock by its total retail volume over the nine year period, I compute a value of 0.0826. If we exclude stocks with missing data, stocks with no retail volume on one or more dates, stocks with no exchange volume on one or more dates, and stocks in the bottom 95% of median market cap over the sample period, then the weighted value is a mere 0.0655. The remaining liquid stocks represent 56.8% of total marketable retail volume in the dataset and illustrate the basic role of daily sample size in order imbalance metrics: a stock with fewer orders will have larger $|BSI|$ values due to the law of large numbers.

Yet extremely low VWAI values in months with active retail trading indicate the existence of another mechanism that underlies the connection between high volumes and low VWAI. Figure 6a plots the monthly VWAI for the entire stock market against monthly

[12] If we have \$100 purchased and \$100 sold with retail traders representing 10% of purchases and sales, then random matching would lead to $0.1^2 \times \$100 = \$1$ of self-matched volume.



(a) Monthly VWAI and Retail $ Volume

(b) Monthly VWAI and Retail $ Volume, High Volume Stocks

Figure 6: Retail VWAI and $ Volumes by Month, 2013-2021

Note: Figure 6a plots the monthly retail Volume Weighted Adjusted Imbalance (VWAI) and monthly retail dollar volume over the nine year period 2013-2021, while Figure 6b plots the monthly retail Volume Weighted Adjusted Imbalance (VWAI) and monthly retail dollar volume over the nine year period 2013-2021 for the n stocks that comprise at least 50% of retail trading in a given month for the smallest feasible n. All trades are signed via the midpoint methodology of Barber et al. (2024) and identified via the methodology of Boehmer, Jones, Zhang and Zhang (2021).

retail dollar volumes while Figure 6b plots the monthly aggregate VWAI for the n stocks in a given month that comprise 50% of retail volume for the smallest feasible n. Included stocks are clearly not subject to small sample bias and the accompanying volume-VWAI relationship, yet the time trend is stark: later years in the sample have seen a proliferation of retail trading along with unprecedented retail trader alignment. Indeed, by September 2020 after the work from home retail trading craze, a mere six stocks comprised 51.7% of retail trading activity with a VWAI over those six stocks of only 0.0324. The VWAI measurements reveal that a full understanding of the retail alignment puzzle requires the establishment of mechanisms that can reproduce the concurrence of spikes in retail volume and lower adjusted imbalances.

As shown in Appendices D and E, the extreme level of alignment in equities is unusual. Using CFTC trading data, I show that subsets of professional managers in futures markets display significantly larger VWAIs than retail traders in equities despite representing a similar fraction of overall exchange volumes. Money manager futures imbalances in markets such as the S&P 500 are comparable to those generated by randomly drawing daily purchase and sale numbers from a uniform distribution.

2.5 Predicting Retail Order Flow

Past literature has established that extreme returns draw household attention to stocks, thus triggering positive order flow (see Barber and Odean (2008)), but has not set out to separately characterize the drivers of retail purchases and sales. This section uses the subpenny-identified trades over the time period 2013-2021 to study the drivers of the daily distribution of retail trades in the cross section. I show that retail purchases and sales exhibit nearly identical daily cross-sectional patterns, with both order types disproportionately concentrating in high-volume stocks with extreme contemporaneous or recent returns.

The dependent variables are the fraction of total day t retail purchases and the fraction of total day t retail sales that occur in equity n, represented as $RetFrac_{t,n}^{Buys}$ and $RetFrac_{t,n}^{Sells}$. The first of the explanatory variables is equity n's day $t-1$ market cap as a fraction of the total stock market capitalization of the stock universe on day $t-1$, given by $fracMktCap_{n,t}$. Size is clearly related to attention and, in turn, total daily dollar volume, as large companies are on average more pervasive in media coverage, have larger consumer bases, and generate more analyst and institutional attention. Normalization by aggregate market capitalization makes the explanatory variable relative instead of absolute market capitalization, as the latter is impacted by aggregate stock market fluctuations.

The second set of explanatory variables measures how attention-grabbing prior returns are. One model that captures the degree to which a decision maker's attention is drawn to a lottery is that of Bordalo, Gennaioli and Shleifer (2012). Their Salience Theory explains phenomena like the Allais paradox via a weighting function that distorts payoff weighting in favor of more "salient" states of the world, those that are most different from the norm. To specify a functional form for attention with respect to return variables, I leverage the insights of the salience literature and take as my main salience model a parsimonious function that embeds shifting attention as a function of recent price changes with rank dependence. This function is directly analogous to the generalized many-lottery salience weightings found in the Online Appendix of Bordalo, Gennaioli and Shleifer (2012), only here we have just one state and we are ranking price changes from the past instead of a prospective lottery. For a given cross-sectional vector of returns $x \in \mathbb{R}^N$ define the salience function

$$\sigma(x^n, x^{-n}) = \frac{|x^n - m\left(x^{-n}\right)|}{|x^n| + |m\left(x^{-n}\right)| + \theta} \tag{4}$$

where $m\left(x^{-n}\right) = \frac{1}{N-1}\sum_{i \neq n} x^i$ is simply the average return for all other assets. Let x be a vector of returns with j_n the salience ranking of the return for asset n, found by sorting all

values of $\sigma(x^n, x^{-n})$ with 1 the most salient and N the least salient. Define the function

$$v\left(x\right)_n = \frac{\delta^{j_n}}{\sum_r \delta^{j_r}} \tag{5}$$

where $\delta \in (0, 1]$. To make the model more parsimonious, I do not allow for heterogeneity in δ or θ with respect to the specific price change vector x_j, and simply take $\theta = 0.01$ while δ drops out of the regression equations as shown later in this section.

Ex-ante, one should expect to see a sharp decay in the association between past daily returns r_{t-k} and today's volume as the lag k increases. With that in mind, and to avoid overparameterization, I create only five return variables. Define x_0^R to be contemporaneous (same day) returns, define x_1^R to be the previous day's return, x_2^R to be the return two days ago, x_3^R to be the return from the week prior to the penultimate day, and x_4^R to be the return from the month (defined as 21 trading days) prior to the period covered by x_3^R. The combined salience function for all return data is then

$$\prod_{k=0}^{4} \left(\frac{\delta^{j_{n,k}}}{\sum_q \delta^{j_{q,k}}} \right)^{\alpha_k^R} \tag{6}$$

where $\alpha_k^R \in [0, 1]$ is a parameter to be estimated that captures the relative importance of the different returns and $j_{n,k}$ is the salience ranking of stock n for return lag k. This salience specification is strictly increasing in how "unusual" a stock's recent return history is versus its peers.

Turning to the final set of variables, those related to volume, I define $WV_t \in [0, 1]^N$ to be the vector of each asset's fraction of total common stock dollar volume on day t. Define x_1^V to be WV_{t-1}, x_2^V to be WV_{t-2}, x_3^V to be $\left(\sum_{\tau=3}^{\tau=7} WV_{t-\tau}\right)/5$, and x_4^V to be $\left(\sum_{\tau=8}^{\tau=28} WV_{t-\tau}\right)/21$.

The main model of household attention is then given by

$$RetFrac_{t,n}^Z = c \times \prod_{k=1}^{4} \left(x_{k,n}^V\right)^{\alpha_{k,Z}^V} \times \prod_{k=0}^{4} \left(\frac{\delta^{j_{n,k}}}{\sum_q \delta^{j_{q,k}}} \right)^{\alpha_{k,Z}^R} \times \left(fracMktCap_{n,t}\right)^{\alpha_{5,Z}} e^{\epsilon_{t,n,Z}} \tag{7}$$

where c is a positive constant, $\alpha_5 \in [0, 1]$ is the geometric weighting of relative market capitalization, $e^{\epsilon_{t,n}}$ is an error term, and $Z = Buys$ or $Z = Sells$. Taking logs, this becomes

$$\log\left(RetFrac_{t,n}^Z\right) = \log\left(c\right) + \sum_{k=1}^{4} \alpha_{k,Z}^V \log\left(x_{k,n}^V\right)$$

Table 2: Predicting Fractional Dollar Volume of Retail Buy Orders

	β (Buys)	β (Sells)
Yest Volume	0.4 (91.04)	0.39 (91.27)
2 Day Ago Volume	0.07 (23.34)	0.071 (24.12)
Avg Volume 3-7 days ago	0.22 (71.31)	0.22 (74.05)
Avg Volume 8-28 days ago	0.26 (45.56)	0.25 (46.12)
Sal. Rank of Today Rets	0.87 (123.49)	0.81 (119.99)
Sal. Rank of Yesterday Rets	0.35 (48.7)	0.32 (45.85)
Sal. Rank of Two Days Ago Rets	0.29 (38.03)	0.26 (35.09)
Sal. Rank of Last Week Rets	0.21 (34)	0.19 (30.45)
Sal. Rank of Last Month Rets	0.18 (27.78)	0.16 (25.24)
Frac Mkt Cap	-0.088 (-5.66)	-0.087 (-5.74)

Note: This table shows the results of regression (9) where fractional volume of retail buy and sell orders are the dependent variables. All standard errors use the correction for clustering proposed by Liang and Zeger (1986) to account for intertemporal correlation in the residuals and independent variables by equity.

$$+\sum_{j=0}^{4}\alpha_{k,Z}^{R}\left(\log\left(\delta^{j_{n,k}}\right)-\log\left(\sum_{q}\delta^{j_{q,k}}\right)\right)+\alpha_5\log\left(fracMktCap_{n,t}\right)+\epsilon_{t,n} \tag{8}$$

Because the number of equities exceeds 3000 throughout the sample, $\sum_q \delta^{j_q} \approx \frac{1}{1-\delta} - 1$ for each of the variables $\alpha_{k,Z}^{R}$ and so the regression equation can easily be rewritten as

$$\log\left(RetFrac_{t,n}^{Z}\right)=c+\sum_{k=1}^{4}\alpha_{k,Z}^{V}\log\left(x_{k,n}^{V}\right)+\sum_{k=0}^{4}\eta_{k}^{R}j_{n,k}+\alpha_5\log\left(fracMktCap_{n,t}\right)+\epsilon_{t,n}. \tag{9}$$

I normalize $j_{n,k}$, the salience ranking of stock n within return window k, to always lie in the interval $[0,1]$ with 1 the most salient return and 0 the least. This model can be estimated via standard OLS, with the parameter vector to be estimated being $\beta^Z = \left(\left\{\alpha_{i,Z}^{V}\right\}_{i=1}^{4}, \left\{\eta_{i,Z}^{R}\right\}_{i=1}^{5}, \alpha_{5,Z}\right) \in \mathbb{R}^{10}$.

Running this estimation twice with the dependent variable set to fractional retail dollar buy volume and dollar sell volume, I find that unusual return patterns and greater recent dollar volume in a given stock are both predictive of future high dollar volume compared to other equities. As shown in Table 2, on a given day the equities likely to have the greatest fractional share of dollar volume on the exchanges are those with extreme returns and high recent dollar volume.

What stands out most is not the coefficients, but rather how similar the analysis remains when retail buy orders are replaced with retail sell orders. Indeed, the coefficients are virtually indistinguishable. In other words, the same factors that appear to predict waves of

retail purchases - high recent exchange volumes and salient returns - also predict retail sales. Moreover, this process is slow-moving: active exchange-wide trading yesterday in stock X strongly predicts higher fractional retail volume today, but also higher retail volumes well into next month. Market capitalization, meanwhile, carries a negative and comparatively insignificant coefficient: any effects that size has on dollar volume are conveyed through the lagged recent dollar volume explanatory variables, leaving size as a negative predictor of fractional volume in the multivariate regression.[13] In Appendix C I show that these parallel patterns for purchases and sales are almost the same when the cross-sectional distributions of small and large retail trades are studied separately. Small traders only display a preference for trading stocks with extreme contemporaneous returns and day prior returns, whereas large traders are additionally drawn to extreme movement going back over a month.

2.6 Advantages and Limitations of Identification Approach and Related Approaches

The results showing retail alignment among marketable retail orders are contingent on proper identification and signing of retail trades. In this section I explore the advantages and limitations of this paper's approach.

This paper's use of the BJZZ/Barber et al. (2024) (BHJOS) retail order identification and signing approach offers two key advantages. In using TAQ data instead of private brokerage data, I am able to examine retail orders that originate from every retail brokerage firm and that execute on every venue. This mitigates the potential selection bias from using exclusively discount brokerage data, wholesale brokerage data, or a single venue's proprietary data. The second major advantage is consistency: although the percentage of trades with subpenny price improvement has increased over time, such time variation had largely ceased by the time range of 2013-2021 used in this paper as the markets settled into their post-Reg NMS equilibrium (see Boehmer, Jones, Zhang and Zhang (2021) as well as their online appendix).

One disadvantage of the BJZZ approach, explored at length in BHJOS, is that non-penny spreads result in a straightforward problem with the BJZZ methodology. I circumvent this problem by adopting the Lee and Ready (1991) modification to the BJZZ methodology outlined in BHJOS. This misidentification problem of the BJZZ approach, and the relevant improvement outlined in BHJOS, are less relevant to this paper's study of retail dollar volume, which differs from many past studies that have focused on the number of marketable retail buy and sell orders. This issue is discussed at greater length in Appendix F.

The main disadvantage of this paper's approach, common to both BJZZ and BHJOS,

[13] Naturally, a series of univariate regressions with each of the same explanatory variables plus a constant reveals identical signs except in the case of market cap, where a large positive coefficient appears instead.

is that non-marketable limit orders are not included. These orders, however, are precisely the ones most likely to resemble the "noise traders" or "liquidity providers" of canonical microstructure, asset pricing, and behavioral finance models, with no predictive power with respect to future cash flow news in contrast to marketable orders (see Kelley and Tetlock (2013)). These noise trader and liquidity provision properties of limit order trades suggest that the paper's finding of close alignment of retail purchases and sales would likely be strengthened instead of weakened if all executed trades were identifiable. Moreover, the vast majority of trades are marketable orders and thus subject to price improvement and identification by this paper. Finally, a story in which the most aggressive, optimistic, sentiment-driven retail traders disproportionately place passive resting orders in the limit order book versus their less aggressive peers is inherently contradictory, as perceived information advantage directly influences the immediacy implicit in the selection of marketable versus non-marketable orders.

3 Attention Shocks with Limited BSIs and High Retail Volumes

This section's analysis shows that attention shocks, whether measured indirectly through attention proxies or directly through Google search volume, lead to an increase in trading activity but only marginal increases in BSIs. In subsection 3.1 I find that for stock $\times$ day pairs with indirectly measured high attention, BSIs are minimal. Subsection 3.2 documents the same pattern with directly measured attention via Google search while subsection 3.3 shows that, unlike on high attention days, on earnings announcement days net retail flows increase substantially.

3.1 BSIs, Returns, and Volume

We have now seen that the daily cross-sectional distribution of retail purchases and sales is highly predictable based on recent return salience, intraday return salience, market cap, and volume history. Furthermore, the best linear model for predicting retail purchases has virtually identical coefficients to the best linear model for predicting retail sales. Although these patterns suggest a puzzling connection between purchases and sales, they do not reveal the joint patterns of stocks' BSIs, returns, and retail volume levels. Evaluating and modeling the retail alignment puzzle requires a simultaneous understanding of whether volumes and returns correlate with BSIs, as different mechanisms that generate nonzero BSIs will cause different comovement among the three variables.

If attention drives volume changes, then following the arguments of Miller (1977)

regarding disagreement in the presence of short sale constraints, we should anticipate higher BSIs on high volume days or days with extreme returns; greater stock visibility draws in more potential household buyers and sellers, but the sellers are short sale constrained and therefore only the newly attentive optimists see their views incorporated into prices through purchase orders. Extreme return days will display asymmetry, however, as the slope of retail traders' demand curves will cause differential reaction to increases and decreases in price. With downward sloping demand, poor performance will be met by increased retail demand whereas large increases in price will be met by selling; attention dynamics universally act to shift aggregate retail trader demand upward when returns are extreme. Likewise, a stock with high volumes will draw attention, thus generating a larger BSI than on days that do not experience an influx of attention. This effect should be most pronounced for orders from those most prone to attention shifts, those of small and unsophisticated traders (odd lots).

To test these hypotheses in the data, I compare BSIs on extreme and non-extreme return days as well as high and non-high volume days. Because retail trading clusters in a small subset of stocks (see Barber, Shengle and Odean (2023)), I analyze only the top 5% of stocks by median market cap over the time period 2013-2021. For each of these 266 high market cap equities, I compute the daily BSIs by order size for high return, low return, and medium return days, where "high" and "low" are the top and bottom 5% of returns from CRSP over the covered time period and "medium" return days include all others. BSIs are compared via a straightforward difference of means t-test, with "high," "low," and "medium" day mean BSIs calculated for each stock and the averages of those means across stocks computed to form the composite measures. Table 3 provides the category averages and difference of means test results, which precisely reflect the overlap of the attention channel and the price response channel.

Days with salient negative returns have statistically significantly higher BSIs among all order types compared to days with salient positive returns. This asymmetry is generated by liquidity-providing behavior, with demand downward sloping in price. Retail traders respond to extreme low returns with net purchases but respond to extreme high returns with net sales. This effect, however, is highly asymmetric, as is anticipated by the attention mechanism: the increase in BSIs on salient low return days versus non-salient days is seven times the magnitude of the decline in BSIs on salient low return days. Increased attention on extreme return days generates the positive BSIs suggested by Miller (1977), but this effect is overwhelmed by the slope of household's demand curves in the case of high return days.

A similar pattern should occur with respect to volume: high volume days are indicative of increased attention from retail traders, which generates positive BSIs and larger returns.

Table 3: BSIs for Extreme Returns

	BSI All	BSI Even	BSI Odd
Low Ret Days	0.019	0.0065	0.088
High Ret Days	-0.012	-0.021	0.029
Non-HL Days	-0.0083	-0.018	0.036
L-H Difference	0.031 (12.99)	0.028 (10.79)	0.059 (17.78)
L-M Difference	0.027 (14.82)	0.024 (12.4)	0.052 (17.45)
M-H Difference	0.004 (2.53)	0.0037 (2.26)	0.0072 (3.6)
LH-M Difference	0.012 (9.46)	0.01 (8.16)	0.022 (11.67)

Note: This table compares retail trade Buy Sell Imbalances (BSIs) on high return days versus low return days. First, the top 5% of stocks by median market cap over the time period 2013-2021 are selected and stocks with missing data are excluded. Next, for each stock the days with the worst 5% and best 5% of daily returns above daily cross-sectional market average are identified as well as the middle 90%. For each of these three categories, BSIs are computed for all orders, even lot orders, and odd lot orders. Paired t-tests are conducted to compare Low and High (L-H), Low and Medium (L-M), Medium and High (M-H), and Extreme versus Non-Extreme (LH-M).

Table 13 in the Appendix details results for odd-even BSI correlations on high and low volume days as well as $|BSI|$ and shows that odd lot and even lot BSIs are significantly more correlated on days with high volume.[14] Meanwhile, Table 4 shows that while the magnitude of retail BSIs are minimally different across high and non-high volume days, the difference in levels is statistically significant, with high volume days possessing higher BSIs across all order size categories. Volume surges are therefore just like extreme return events, another attention proxy, in generating the positive BSI shifts previously observed in the literature, yet the magnitude of these shifts is far smaller than one would anticipate.

Extreme volumes and returns have been used throughout the literature as proxies for attention, but this subsection's analysis does not conclusively tie shocks to attention to small retail BSIs, BSI shifts, or volume changes. The next subsection will explicitly show such mechanisms using direct measurement of retail trader attention.

3.2 BSIs on High Search Days vs Low Search Days

Da, Engelberg and Gao (2011) have already shown how to obtain a direct measure of attention using the Search Volume Index (SVI) from Google Trends. Google makes available a historical dataset of normalized search intensity, with 100 the maximum SVI value over any retrieved time window and 0 the minimum (representing approximately zero searches). Da, Engelberg and Gao (2011) obtain weekly SVIs for the time period 2004-2008 and use

[14]The correlation difference can be attributed in part to a sample size effect (high volume days have numerous odd and even lot orders, so the law of large numbers reduces noise).

Table 4: BSIs for Extreme Volume Days

	BSI All	BSI Odd	BSI Even	Ret-Mkt
High Volume Days	0.0076	0.066	0.0018	0.0013
Non-High Volume Days	-0.0079	0.037	-0.018	0.00068
H-NH Volume Diff	0.016	0.029	0.019	0.00062
H-NH t-stat	(7.16)	(8.8)	(9.06)	(2.46)

Note: This table compares the average value of retail trade Buy Sell Imbalances (BSIs) on high volume days versus low volume days. First, the top 5% of stocks by median market cap over the time period 2013-2021 are selected and stocks with missing data are excluded. Next, for each stock the days with the highest 5% and bottom 95% of daily retail volumes are identified. For high volume days and "normal" volume days, BSIs are computed for all orders, even lot orders, and odd lot orders; odd/even lot correlations and average returns are also calculated. A paired t-test is conducted to compare all five average statistics by stock for high and normal volume days.

the variable "Abnormal Search Volume Index" (ASVI) as their primary attention change proxy, defined as the log of current SVI minus the log of the median SVI over the past eight weeks. As is extensively remarked upon in Da, Engelberg and Gao (2011), the SVI-based attention measures lead other well established attention proxies such as news or extreme returns and have low correlations with all other extant attention proxies.

This paper similarly uses Google's SVI to generate an attention proxy, but with three major differences. First, I exclusively use daily data as opposed to weekly or monthly data, thus enabling finer separation of earnings announcement-based trading and attention-based trading. Second, this paper does not study return predictability or monthly trading levels but rather daily retail order imbalances and volumes, neither of which have previously been studied in the context of search intensity to the best of the author's knowledge. Finally, I separately study earnings announcement days and non-earnings "high attention" days to distinguish information and disagreement-driven BSIs from attention-driven BSIs.

I begin by assembling a universe of 71 large cap stocks formed by joining the list of all current DJIA constituents with all four letter ticker symbols from the list of high median market cap stocks from subsection 3.1. All "noisy tickers" in the spirit of Da, Engelberg and Gao (2011) are eliminated.[15] Because the time period 2013-2021 yields only monthly data when queried via Google Trends, I create 18 overlapping windows starting with January 1, 2013-August 31, 2013 and ending with July 1, 2021-December 31, 2021; using 8 month or smaller windows yields daily search data. The first 17 windows are 8 months duration while the last window is only 6 months, but all adjacent windows overlap by either 59 or 62

[15]For example, "FAST," "CAT," and "T" are eliminated because of being either words or letters, any of which are far more likely to be searched outside the context of stock research. I retain only four letter symbols for non-DJIA components because they are similarly less likely to be searched for non-trading reasons such as misspelling or unrelated acronyms.

days. The median ratios of SVI values for overlapping time periods are used to rescale and concatenate the data into a single, daily nine-year SVI dataset for each of the 71 tickers using the 1,278 Google Trends query results.

Google search intensity displays cyclical patterns and can vary mechanically over long periods of time based on market capitalization. To distinguish daily attention inflows from low frequency changes, I normalize by three month average SVI to generate a Search Volume Index Ratio Difference (SVIRD), calculated as

$$SVIRD_t = \frac{SVI_t}{\frac{1}{60}\sum_{\tau=1}^{60} SVI_{t-\tau}} - 1.$$

Over the time period $2013 - 2021$, the top 5% of days for each stock when rank ordered by SVIRD are labeled "high attention" while all others are labeled "low attention," thus providing 110 "high attention" days per stock.[16] Variables of interest are computed separately for high attention days and low attention days, and the t-statistics for differences between means are computed using date-based clustering in the manner of Liang and Zeger (1986), following the suggested approach of Abadie, Athey, Imbens and Wooldridge (2017) for when the sample includes the full universe of viable ticker symbols. All day $\times$ stock pairs that coincide with an earnings report are excluded for both the high attention and low attention groups.

Table 5 provides the results of this analysis, with a clear pattern of higher returns, marginally higher BSIs, and far higher volumes on high attention days compared to low attention days. A stock experiencing high attention sees 0.08% higher daily returns, 0.00778 higher BSIs, 2.7 times the turnover (dollar volume divided by market cap), and 1.12 times higher retail trading as a fraction of exchange volume. The combination of higher but moderate BSIs and higher turnover is visible across both odd lot orders and even lot orders, with the greatest surge in turnover and volume coming from even lot orders.

These results sharply illustrate the role of attention in resolving the retail alignment puzzle, as it drives large changes in retail trading while simultaneously increasing BSIs only marginally. When retail investors start to consider a stock for the first time, none of them will go short because of short sale constraints, but some optimistic fraction of the newly attentive traders will make purchases. This pattern has been remarked upon by Miller (1977), Engelberg and Parsons (2011), and Barber and Odean (2008), among others. What is novel in this paper's analysis is that although attention induces large volumes of retail purchases, it also induces large volumes of retail sales. Attention therefore applies upward pressure to prices and increases volumes, but nonetheless has the effect of inducing far smaller net retail flows than have previously been envisioned in the literature or popular

[16] The first 60 days of the 2013-2021 window are excluded.

Table 5: Returns, Volume, Turnover, and BSIs: High vs Low Google Searches, No Earnings

	All Lots	Odd Lots	Even Lots
High Att Ret	0.00162		
Low Att Ret	0.000803		
H/L Att Ret Diff	0.000818(1.47)		
High Att |Ret|	0.0244		
Low Att |Ret|	0.0121		
H/L Att |Ret| Diff	0.0123(27.7)		
High Att Retail Turn	0.00103	7.91e-05	0.000949
Low Att Retail Turn	0.000381	3.52e-05	0.000346
H/L Att Retail Turn Diff	0.000647(14)	4.39e-05(11.3)	0.000603(13.8)
High Att Turn	0.0167		
Low Att Turn	0.00849		
H/L Att Turn Diff	0.00824(18.8)		
High Att Rtl Volume	110000000	14500000	95400000
Low Att Rtl Volume	46900000	6760000	40200000
H/L Att Rtl Volume Diff	63000000(12.3)	7790000(7.37)	55200000(13.1)
High Att Volume	1610000000		
Low Att Volume	823000000		
H/L Att Volume Diff	790000000(15)		
High Att BSI	0.000178	0.0563	-0.00861
Low Att BSI	-0.0076	0.0353	-0.0167
H/L Att BSI Diff	0.00778(3.85)	0.021(8.23)	0.00805(3.7)
High Att |BSI|	0.109	0.156	0.115
Low Att |BSI|	0.107	0.125	0.118
H/L Att |BSI| Diff	0.00226(1.71)	0.0311(18.5)	-0.00292(-1.99)
High Att Ret Volume Frac	0.037	0.00367	0.0334
Low Att Ret Volume Frac	0.0331	0.00399	0.0292
H/L Att Ret Diff Volume Frac	0.0039(11.4)	-0.000322(-6.73)	0.00422(13.1)

Note: This table characterizes the differences in returns, volumes, turnover, and Buy Sell Imbalances (BSIs) on non-earnings days where Google search volume is high versus low. The universe of 71 stocks is formed by taking the highest 5% of stocks by median market cap in the merged 2013-2021 CRSP-TAQ database and then eliminating all stocks with tickers of string length less than four. Stocks in the DJIA with "non-noisy tickers" in the spirit of Da, Engelberg and Gao (2011) are added. Google search volume indices (SVIs) are obtained from the Google Trends interface and rescaled into a single nine-year time series for each ticker. For each of the 71 stocks, the top 5% of days by SVI change are classified as "high attention" while the bottom 95% are classified as "low attention." Earnings report days are subsequently removed using I/B/E/S data. Each line of the table provides the mean value for a different variable or difference of variables, with t-statistics in parenthesis for all comparison tests. Standard errors are clustered at the day level following the guidance provided in Abadie, Athey, Imbens and Wooldridge (2017).

media. As we will see in the next subsection, earnings announcements can generate larger retail imbalances than attention, albeit with more moderate volume levels.

3.3 BSIs on Earnings Days vs Non-Earnings Days

For the same set of stocks as the last subsection, we now study the return, volume, turnover, and BSI differences across earnings and non-earnings days. Table 6 details the results, which show that earnings announcement days trigger fundamentally different behavior among retail traders versus non-earnings days that feature high attention. Compared to a stock-day with no reported earnings, a stock-day with an earnings announcement sees 0.14% higher daily returns, 0.0581 higher BSIs, 1.95 times higher retail turnover, and 1.11 times higher retail trading as a fraction of exchange volume. The BSI change induced by earnings announcement days is nearly 7.5 times larger than the BSI change induced by high attention, while the increase in retail turnover is 40% smaller on earnings days. Earnings days, with their substantial news flows, provide an entirely different basis for trading. If all participants have identical preferences and update their beliefs in the same way, we would anticipate a price change but almost no trading activity. When participants disagree about expected returns and update their beliefs in conjunction with private information or using biased processes, we can observe a flurry of trading activity as beliefs change asymmetrically. For example, if retail investors are more prone to optimistically biased updating than institutional investors, one would expect retail traders to be net purchasers on announcement days. Because the biased updating impacts beliefs, the shift in sentiment will outweigh the decreased demand caused by the accompanying price increase and negative household demand slope.

The seemingly discordant combination of limited BSI changes and extreme increases in trading activity on high search volume days is underscored by the earnings report day findings. Were every news event, attention shift, or sentiment swing to cause limited BSIs then we would suspect either a mechanical reason for the small BSIs or a selection bias issue, e.g. if the marketable subpenny-executed orders are more likely to be balanced than non-marketable limit orders or round penny executions. Because almost all retail executions are at subpenny prices, the latter explanation is implausible. The fact that earnings announcement days have larger BSIs demonstrates that small BSI responses to shocks are not a mechanical fact for large equities, but rather are a conditional phenomenon. Earnings announcement days, with their large flows of information and substantial capacity to provoke or resolve disagreement among traders, violate the requisite conditions for limited BSI shifts. In section 4 I show that this accords with a theoretical disagreement model: attention uniquely generates limited BSIs whereas disagreement and sentiment channels, those most active on earnings announcement days, generate larger BSIs. If

Table 6: Returns, Volume, Turnover, and BSIs: Earnings vs No Earnings

	All Lots	Odd Lots	Even Lots
Earn Day Ret	0.00227		
Non Earn Day Ret	0.000837		
Earn/No Earn Ret Diff	0.00144(2)		
Earn Day |Ret|	0.0226		
Non Earn Day |Ret|	0.0127		
Earn/No Earn |Ret| Diff	0.00997(19.2)		
Earn Day Retail Turn	0.000796	5.85e-05	0.000737
Non Earn Day Retail Turn	0.000407	3.7e-05	0.00037
Earn/No Earn Retail Turn Diff	0.000388(9.97)	2.15e-05(8.5)	0.000367(9.78)
Earn Day Turn	0.0169		
Non Earn Day Turn	0.00883		
Earn/No Earn Turn Diff	0.00805(17.1)		
Earn Day Rtl Volume	74900000	9560000	65300000
Non Earn Day Rtl Volume	49500000	7080000	42400000
Earn/No Earn Rtl Volume Diff	25400000(6.4)	2480000(2.58)	22900000(7.1)
Earn Day Volume	1340000000		
Non Earn Day Volume	855000000		
Earn/No Earn Volume Diff	481000000(9.35)		
Earn Day BSI	0.0508	0.0982	0.0438
Non Earn Day BSI	-0.00728	0.0362	-0.0163
Earn/No Earn BSI Diff	0.0581(21.1)	0.062(17.2)	0.0602(20.9)
Earn Day |BSI|	0.106	0.158	0.108
Non Earn Day |BSI|	0.107	0.127	0.118
Earn/No Earn |BSI| Diff	-0.000669(-0.349)	0.0312(12.6)	-0.0101(-4.98)
Earn Day Ret Volume Frac	0.0371	0.00339	0.0337
Non Earn Day Ret Volume Frac	0.0333	0.00398	0.0293
Earn/No Earn Ret Diff Volume Frac	0.00384(8.6)	-0.000585(-8.22)	0.00442(10.5)

Note: This table characterizes the differences in returns, volumes, turnover, and Buy Sell Imbalances (BSIs) on days where earnings are reported versus not reported. The universe of 71 stocks is formed by taking the highest 5% of stocks by median market cap in the merged 2013-2021 CRSP-TAQ database and then eliminating all stocks with tickers of string length less than four. Stocks in the DJIA with "non-noisy tickers" in the spirit of Da, Engelberg and Gao (2011) are added. Earnings days and non-earnings days are isolated using I/B/E/S data. Each line of the table provides the mean value for a different variable or difference of variables, with t-statistics in parenthesis for all comparison tests. Standard errors are clustered at the day level following the guidance provided in Abadie, Athey, Imbens and Wooldridge (2017).



Figure 7: Retail BSIs Following Earnings

Note: This figure plots the average retail BSI on days 0 (earnings report day) through +62 following earnings for a universe of 71 large cap stocks. The universe of 71 stocks is formed by taking the highest 5% of stocks by median market cap in the merged 2013-2021 CRSP-TAQ database and then eliminating all stocks with tickers of string length less than four. Stocks in the DJIA with "non-noisy tickers" in the spirit of Da, Engelberg and Gao (2011) are added. Earnings days and non-earnings days are isolated using I/B/E/S data. 95% confidence intervals are provided for each day.

earnings announcement days instantaneously resolve uncertainty for short-sale constrained institutional investors but not for short-sale constrained retail traders, we would expect a negative BSI for the short-sale constrained institutions and therefore an amplified and positive BSI for retail traders, at least until they eventually process the information.

In Figure 7 we see precisely this pattern: retail BSIs are elevated on earnings days but flip to being statistically significantly negative within several days of the earnings report, bottoming out at -0.0257 from the high of 0.0508 on the day of earnings reports. The pattern is consistent with delayed processing of information and thus delayed reduction in uncertainty versus short-sale constrained institutions. As one might surmise given the retail alignment puzzle, this phenomenon is not an artifact of outlier positive BSIs; 68.9% of BSIs are positive in this sample of 2246 stock×earnings day pairs. Moreover, 57 out of 70 stocks with non-missing data have majority positive earnings day BSIs versus only 9 out of 70 with majority negative earnings day BSIs.

4 A Disagreement Model of Retail Volume Imbalances

The common narrative of large net retail flows driving mispricings finds its intellectual underpinnings in Miller (1977) and noise trader models: increases in retail sentiment,

disagreement, or attention will apply positive price pressure because when households disagree with one another, only the optimists can express their views through trade. In this section I show via a straightforward disagreement model that while the price pressure intuition of this literature is readily formalized and confirmed, the seemingly obvious volume corollary — that price increases correspond to lopsided retail flows — does not generally hold.

4.1 Environment

Two assets are available for trading, a risky asset with total supply of one infinitely divisible share and a riskless asset in perfectly elastic supply that without loss of generality has a 0% net return each period. The price of the risky asset, determined endogenously by market clearing, is denoted P_t.

Institutions i are the first agent type and behave as price takers but are non-atomistic. i possesses investable assets worth $A_{i,t}$ at time t and has log return variance expectations of σ_t^2 in period t and log expected return given by

$$\mu_{i,t} = \hat{\beta}_{i,t} P_t + s_{i,t} \tag{10}$$

where $s_{i,t}$ is the private return belief of institution i at time t (the "sentiment" of i) and $\hat{\beta}_{i,t}$ (< 0 for the "normal" downward-sloping demand case) captures the extent to which log return expectations are sensitive to price. Institutions have log utility over terminal wealth and therefore optimize myopically each period.[17] The risky asset weighting of institution i is thus approximately given by

$$w_{i,t} \approx \sigma_{i,t}^{-2} \left(\hat{\beta}_{i,t} P_t + s_{i,t} \right) \tag{11}$$

with a total of $A_{i,t} w_{i,t}$ dollars invested in the risky asset.

Households, the second agent type, are atomistic and represented by a continuum of measure 1 indexed by $h \in [0, 1]$. Households in aggregate hold $A_{H,t}$ of investable capital in period t and have log return variance expectations of $\sigma_{H,t}^2$ and log expected returns given by

$$\mu_{h,t} = \bar{\beta}_t P_t + \xi_t + \sigma_{s,t} s_{h,t}, \tag{12}$$

where $\bar{\beta}_t < 0$ is the slope of each household's return expectation with respect to price, ξ_t is the average sentiment of households at time t, $\sigma_{s,t}$ is the standard deviation of household

[17] As shown in Campbell and Viceira (2002) a first order approximation of their demand, or exact in the continuous time limit, under lognormally distributed returns, is mean-variance utility with log expected return and log return covariance (in this case just variance) as the two determining factors of portfolio demand.

return beliefs at time t, and $s_{h,t} \sim N(0,1)$ is the private sentiment value of household h. Household h's risky asset weighting is given by $w_{h,t} = \sigma_{H,t}^{-2} \max\{0, \mu_{h,t}\}$ because they cannot sell short.[18]

Each household possesses two distinct, independent attributes: the aforementioned private sentiment $s_{h,t}$ which covers the degree of disagreement versus the household average sentiment ξ_t, and a binary attention variable that can take either 1 or 0. A household that is paying attention has the risky asset in its consideration set and therefore behaves like a standard rational agent. Households with attention set to 0 are only able to invest in the riskless asset. The probability that a household is paying attention at time t is given by $g_t \in [0,1]$, which represents the measure of households paying attention to the risky asset at time t.

Total household dollar demand is thus

$$X(P_t) = A_{H,t} g_t \int \max\left\{0, \sigma_{H,t}^{-2}\left(\bar{\beta}_t P_t + \xi_t + \sigma_{\epsilon,t} s_{h,t}\right)\right\} dh. \tag{13}$$

The distribution of household return expectations thus has three components: (1) a price-based return component that is decreasing in price, (2) a shared "retail sentiment" component that shifts the average of household return expectations as it changes, and (3) a "disagreement" component $\sigma_{s,t}$ that governs the dispersion of household expectations around the average of $\bar{\beta}_t P_t + \xi_t$.

4.2 Equilibrium at time t

Using the fact that the asset supply is always 1 to obtain the left hand side and summing over institutional dollar demands and integrating over household dollar demands, we obtain

$$P_t = \sum_i A_{i,t} \sigma_t^{-2}\left(\hat{\beta}_{0,i,t} P_t + s_{i,t}\right) + A_{H,t} g_t \int \max\{0, w_{h,t}\}\, dh. \tag{14}$$

Equilibrium exists and is unique under limited parameter restrictions, as shown in Appendix A.

As changes in shares demanded by a household h are dependent on prices, I use the properties of normal distributions to find an implicit formula for price without an integral; all proofs from this section are contained in Appendix A.

Let us define $\alpha_t = \left(1 - \sum_i A_{i,t} \sigma_t^{-2} \hat{\beta}_{i,t}\right)$ as the risk-capital weighted demand slope of

[18] Note that the fact that demand is downward sloping with respect to current price does not preclude extrapolative expectations, which generally are a function of past price changes as opposed to today's price (e.g. returns from 12 months ago through last month). Additional asset characteristics, such as recent price changes, could be readily added to form a random coefficients model with no changes to this section's main conclusions.

institutions, $h_t = -\sigma_{s,t}^{-1}\left(\bar{\beta}_t P_t + \xi_t\right)$ as the "marginal household" that desires a precisely 0 share position in the risky asset and is not short sale constrained, $\omega_t = \sum_i A_{i,t}\sigma_t^{-2}s_{i,t}$ as the risk capital-weighted sentiment of institutions, $\eta_t = (1 - \Phi(h_t))\, A_{H,t}\sigma_{H,t}^{-2}g_t$ as the attention and risk-adjusted household capital, and $\vartheta_t = \left(\alpha_t - \bar{\beta}_t\eta_t\right)^{-1}$ as the inelasticity of demand with respect to price.

The risky asset's price in period t can be expressed as

$$P_t = \vartheta_t\omega_t + \vartheta_t\eta_t\sigma_{s,t}\lambda(h_t) + \vartheta_t\eta_t\xi_t \tag{15}$$

where $\lambda(t)$ is the inverse Mills ratio $\frac{\phi(t)}{1-\Phi(t)}$. Price is equal to inelasticity times the sum of three terms: risk-weighted institutional sentiment, a disagreement-short sale constraint interaction term, and risk and attention-weighted household sentiment. Appendix A contains Lemma 1, which details the relative price changes induced by changes to attention g_t, sentiment ξ_t, and household disagreement $\sigma_{s,t}$. As expected from the arguments of Miller (1977) for attention and disagreement and straightforward logic for sentiment, all three partial derivatives are positive.

4.3 Dynamic Model of Retail Volume

We now study trading volume between two arbitrary periods, t and $t+1$. If we hold fixed household assets and variance expectations, then a given household h will change its shareholdings between t and $t+1$ for one of four reasons: (i) it begins to pay attention to, or ceases to pay attention to, the risky asset; (ii) the risky asset price changes, thus altering shares required to achieve a given level of dollar risk; (iii) aggregate retail sentiment ξ_t changes, causing h's return expectation to shift in turn; or (iv) disagreement $\sigma_{s,t}$ changes to $\sigma_{s,t+1}$, increasing or decreasing h's private sentiment. I assume that each household maintains its relative position in the continuum as an optimist or pessimist, with the attrition parameter a_t, explored in the next paragraph, governing the volume induced by wavering in attention and sentiment.

A small, randomly drawn measure of households $a_t \times \min(g_t, g_{t+1})$, with $a_t \in (0,1)$, whom I will call attritional agents, will be forced to sell at time $t+1$ and will be replaced by agents with precisely the same private sentiment and attention level. Turnover among these agents can be viewed as the result of external drivers of asset demand shifts such as taxes, births, and deaths, or as the result of internal shifts through attention and sentiment wavering among individuals. If $g_{t+1} > g_t$, then $(1-a_t)\, g_t$ is the measure of agents who previously paid attention and are still attentive, $a_t g_t$ agents sell any holdings, and $(g_{t+1} - g_t) + a_t g_t$ agents buy any desired holdings for the first time. Attrition is an important technical component of the model as otherwise leaving all variables the same

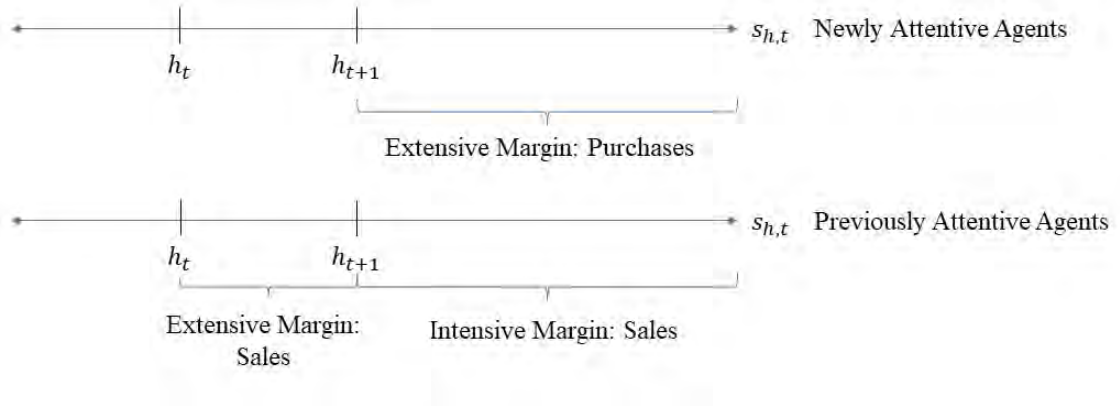


Figure 8: Intensive and Extensive Margin with Attention Changes

across t and $t+1$ results in no household trading volume.

The next three subsections consider changes in attention $g_{t+1} > g_t$, sentiment $\xi_{t+1} > \xi_t$, and disagreement $\sigma_{s,t+1} > \sigma_{s,t}$ and evaluate the induced retail purchase and sale volume as well as how order imbalances change.

4.4 Volume under Increases in Attention

A straightforward intuition argues for large positive Buy Sell Imbalances (BSIs) when household attention increases: new traders will be presented with the asset and only the optimists will trade because of the short sale constraint. Yet this intuition is incomplete and leaves out two critical channels. While an extensive margin of newly attentive traders will indeed purchase the asset, an extensive margin of previously attentive households will sell all of their holdings and an intensive margin will sell some of their holdings. Figure 8 illustrates these changes on the sentiment continuum graphically. When new traders become attentive, the price of the asset increases, thus (i) reducing the number of shares needed to achieve a given level of risk and (ii) decreasing the expected return of the asset because of downward sloping demand as given by $\bar{\beta}_t < 0$. Attention shocks therefore set off a baton-passing phenomenon among retail traders: new optimists enter and purchase the asset, thus driving the price of the asset up and increasing the degree of optimism required to hold the risky asset. All previously attentive households partially or fully dispose of the asset, passing their holdings on to the new optimists. Institutions are also subject to these channels, therefore the degree of retail imbalance is directly related to the relative strength of the rebalancing and demand slope channels among households and institutions.

The object of greatest interest to this paper is $\frac{dV_{g\uparrow}^{Retail,Buy}}{dg_{t+1}} / \frac{dV_{g\uparrow}^{Retail,Sell}}{dg_{t+1}}$, the ratio of the

total derivatives of purchase and sale volumes with respect to attention changes. The following proposition, whose proof is given in Appendix A, shows how the household and institutional rebalancing and demand slope channels interact.

Proposition 1. *(Buy Sell Imbalances under Attention Changes)*

When attention increases from g_t to g_{t+1}, the ratio of the rate of change of purchases and sales evaluated at $g_{t+1} = g_t$ is given by

$$\frac{dV_{g\uparrow}^{Retail,Buy}}{dg_{t+1}} \Big/ \frac{dV_{g\uparrow}^{Retail,Sell}}{dg_{t+1}} = 1 + \frac{\omega_t}{(1-a_t)\,\eta_t\,\{\xi_t + \sigma_{s,t}\lambda\,(h_t)\}}. \tag{16}$$

This expression does not, at first glance, appear to offer a compelling explanation for retail alignment, as a positive value for institutional sentiment will yield positive retail imbalances whenever institutional sentiment $\omega_t > 0$. But we are not looking to show neutral or negative BSIs as the base case under attention shocks: the prior literature and this paper's results both show that attention shocks are in fact accompanied by positive imbalances; the puzzle lies in the small magnitude of imbalances when weighted by dollar volume.

Even with the empirical results illustrating the retail alignment puzzle, we should not be surprised to observe extreme imbalances in stocks that retail traders are inactive in. As a thought experiment, if a stock has no attentive households and then experiences a single day surge in attention, retail traders will purchase the stock with nearly perfect positive imbalance because the short sale constraint prevents newly attentive households from selling. But this is precisely the type of stock downweighted by the VWAI and largely irrelevant to aggregate household trading behavior: such stocks have low market caps, no retail holdings, and limited retail trading even with a surge in attention. On the other hand, the stocks with the highest concentration of household positions, greatest household sentiment, and largest dollar trading volumes are precisely those for which the model predicts attention shocks will generate limited imbalances. Retail dollar volumes tend to be high only in stocks with low $\frac{\omega_t}{(1-a_t)\eta_t\{\xi_t+\sigma_{s,t}\lambda(h_t)\}}$, thus retail alignment is a natural consequence of attention shocks. As we will see in subsection 4.7, simulation results illustrate precisely this pattern.

4.5 Volume under Increases in Sentiment

Unlike increases in attention, changes to average household sentiment create significant buy-sell imbalances among retail traders in aggregate; in this regard positive sentiment shocks generate large net flows. Whereas attention shocks generate purchase volume by introducing new optimists to the equilibrium, positive sentiment shocks increase the return expectation of all traders, thus driving an increase in the desired number of shares for most previous

holders of the asset while simultaneously inducing some previously attentive non-holders to purchase. Some retail traders who were attentive but with insufficiently high private return expectations ($s_{h,t} < h_t$) to hold a long position at time t will now make purchases: this interval (h_{t+1}, h_t) is the extensive margin for purchases.[19] Meanwhile, with $\xi_{t+1} > \xi_t$ we can find a marginal trader h_t^S, with limit $\lim_{\xi_{t+1} \downarrow \xi_t} h_t^S = h_t + \sigma_{s,t}^{-1} \alpha_t \eta_t^{-1} P_t = h_t^{LS}$, who holds precisely the same number of shares at times t and $t+1$.[20] All traders in the interval interval $\left(h_t, h_t^S\right)$ purchase shares; this is the intensive margin of purchases. Meanwhile, in an entirely counterintuitive dynamic, the extreme optimists, with private sentiment in the interval $\left(h_t^S, +\infty\right)$, dispose of a fraction of their shares.

The selling by extreme optimists is the most surprising dynamic under sentiment shocks. The academic literature and popular media often conflate holding an asset and purchasing an asset, yet in many cases the most optimistic asset holders will in fact be the sellers of the asset. This phenomenon is well appreciated in the context of the fire sale and collateral literatures, but here no constraint is forcing the optimists out of a desired position. When a uniform shift in expectations triggers a price increase, the rebalancing channel exceeds the expectations increase channel, causing share sales. Although an optimist's optimal dollar position increases, her optimal number of shares declines.

Putting together these channels and using the derivations found in Appendix A, we arrive at the following proposition.

Proposition 2. *(Buy Sell Imbalances under Sentiment Changes)*

(a) [Ratio of Volume Changes] When sentiment increases from ξ_t to ξ_{t+1}, the ratio of the rate of change of purchases and sales evaluated at $\xi_{t+1} = \xi_t$ is given by

$$\frac{dV_{sentiment,\uparrow t+1}^{Retail,Buy}}{d\xi_{t+1}} \Big/ \frac{dV_{sentiment,\uparrow t+1}^{Retail,Sell}}{d\xi_{t+1}} = 1 + \left(\frac{1}{1-a_t}\right) \frac{1-\Phi\left(h_t\right)}{1-\Phi\left(h_t^{LS}\right)} \frac{\omega_t}{\eta_t \sigma_{s,t} \left(\lambda\left(h_t^{LS}\right) - \lambda\left(h_t\right)\right) - \omega_t}. \tag{18}$$

(b) [Sentiment versus Attention Imbalances] Marginal increases in sentiment cause greater retail imbalances than marginal increases in attention: either

$$\frac{dV_{sentiment,\uparrow t+1}^{Retail,Buy}}{d\xi_{t+1}} \Big/ \frac{dV_{sentiment,\uparrow t+1}^{Retail,Sell}}{d\xi_{t+1}} > \frac{dV_{attention,\uparrow,t+1}^{Retail,Buy}}{dg_{t+1}} \Big/ \frac{dV_{attention,\uparrow,t+1}^{Retail,Sell}}{dg_{t+1}}$$

[19] Indeed, $h_{t+1} < h_t$ because

$$\frac{dh_t}{d\xi_t} = -\frac{\alpha_t \left(\alpha_t - \bar{\beta}_{0,t}\eta_t\right)^{-1}}{\sigma_{\epsilon,t}} < 0. \tag{17}$$

[20] This marginal trader is given by

$$h_t^S = \sigma_{s,t}^{-1} \left(-\xi_t + P_t \frac{\xi_{t+1} - \xi_t}{P_{t+1} - P_t}\right).$$

or else $\frac{dV^{Retail,Buy}_{sentiment,\uparrow^{t+1}}}{d\xi_{t+1}} > 0$ *and* $\frac{dV^{Retail,Sell}_{sentiment,\uparrow^{t+1}}}{d\xi_{t+1}} < 0$.

Part (b) shows the main result of the proposition, namely that sentiment shifts cause more lopsided retail imbalances than attention shifts. Whereas attention shifts result in purchases by new traders and sales by existing traders, upward sentiment shifts cause purchases by new traders and old traders alike with only the extreme optimists selling a small number of shares despite an increase in return expectations. Part (a) of Proposition 2 captures the primary drivers of trade imbalance under sentiment shifts. Institutional sentiment ω_t arises again because greater institutional sentiment magnifies the institutional rebalancing channel, which results in sales by institutions but no purchases as price goes up. The second major variable is the belief wedge between the time t marginal trader h_t and the marginal purchaser at time $t+1$, h_t^{LS}. The largest sentiment-induced imbalances will arise in equities with a small wedge between h_t and h_t^{LS}, which is to say small value of $\sigma_{s,t}^{-1}\alpha_t\eta_t^{-1}P_t$: more attention (embedded in $\eta_t^{-1}P_t$) and greater dispersion (lower $\sigma_{s,t}^{-1}$) increase the pool of households to be impacted by a sentiment increase in the presence of short sale constraints.

4.6 Volume under Increases in Dispersion

The basic intuition for changes in disagreement, which is to say shifts in $\sigma_{s,t}$, is straightforward: when beliefs are intensified, optimists generally become more optimistic and make purchases while pessimists, being short sale constrained, can do nothing with their newfound intense pessimism. While appealing in its simplicity, this story does not capture the full range of possibilities under dispersion changes, which can cause either lopsided imbalances like sentiment or near perfect alignment like attention.

Suppose that retail and institutional sentiment are both positive, but that institutional sentiment is vastly higher than retail sentiment so that the high price of the asset leaves most retail traders short sale constrained. If retail trader disagreement increases — the standard deviation of private sentiment $\sigma_{s,t}$ goes up to $\sigma_{s,t+1} > \sigma_{s,t}$ — then the pessimists will still hold no shares while the optimists intensify their beliefs and hold more of the asset than before. This will result in a wave of purchases but no new sales beyond those of attritional traders, consistent with the basic story.

If we suppose, however, that retail and institutional sentiment are positive but that retail sentiment vastly exceeds institutional sentiment, then a large measure of attentive retail traders are asset holders at time t. When disagreement intensifies, the distribution of retail trader return expectations widens about its mean, with less sanguine holders of the asset selling while the most optimistic buy. Some pessimistic traders exit the asset entirely, other pessimists sell some of their shares, and some optimists make purchases. Under the

first hypothetical, the extensive and intensive margin of traders were exclusively on the purchase side, while in the second hypothetical with high retail sentiment the extensive margin is on the sell side while an intensive margin of both buyers and sellers trades the asset.[21]

Let h^D be the marginal trader who neither buys nor sells after $\sigma_{s,t}$ changes to $\sigma_{s,t+1} > \sigma_{s,t}$, and let h_t^{LD} be the limit of h^D as $\sigma_{\epsilon,t+1} \downarrow \sigma_{\epsilon,t}$, with equations given in Appendix A.5. If we assume that $\omega_t, \xi_t > 0$, then we must determine which of the above two cases applies. Looking at the derivative of h_t with respect to $\sigma_{s,t}$ shows that the sign of $\bar{\beta}_t\omega_t + \xi_t\alpha_t$, which is to say the relative weighted sentiment of institutions and households, determines the case, precisely as the above logic dictates.[22]

We thus arrive at the following proposition, with the proof of a more general proposition and formulas for volume given in Appendix A.

Proposition 3. *(Buy Sell Imbalances under Dispersion Changes)*

Suppose that $\omega_t, \xi_t > 0$.

(a) If $\bar{\beta}_{0,t}\omega_t + \xi_t\alpha_t < 0$ *then when dispersion increases from* $\sigma_{s,t}$ *to* $\sigma_{s,t+1}$*, retail buying increases while no additional selling takes place, creating imbalances.*

(b) If $\bar{\beta}_{0,t}\omega_t + \xi_t\alpha_t > 0$ *then when dispersion increases from* $\sigma_{s,t}$ *to* $\sigma_{s,t+1}$ *, the ratio of the rate of change of purchases and sales evaluated at* $\sigma_{s,t} = \sigma_{s,t+1}$ *is given by*

$$\frac{dV^{Retail,Buy}_{dispersion,\uparrow^{t+1}}}{d\sigma_{s,t+1}} \Big/ \frac{dV^{Retail,Sell}_{dispersion,\uparrow^{t+1}}}{d\sigma_{s,t+1}} = \tag{20}$$
$$1 + \frac{\omega_t\phi\left(h_t\right) + \frac{a_t}{1-a_t}\left(1-\Phi\left(h_t\right)\right)P_t\lambda\left(h_t\right)\left(\alpha_t + \xi_t\eta_t\right)}{\omega_t\phi\left(h_t^{LD}\right) - \omega_t\phi\left(h_t\right) + \left(1-\Phi\left(h_t^{LD}\right)\right)\eta_t\xi_t\left(\lambda\left(h_t^{LD}\right) - \lambda\left(h_t\right)\right)} > 1$$

where the inequality holds whenever $\frac{dV^{Retail,Sell}_{dispersion,\uparrow^{t+1}}}{d\sigma_{s,t+1}} > 0$ *and otherwise* $\frac{dV^{Retail,Sell}_{dispersion,\uparrow^{t+1}}}{d\sigma_{s,t+1}} < 0$ *and* $\frac{dV^{Retail,Buy}_{dispersion,\uparrow^{t+1}}}{d\sigma_{s,t+1}} > 0$.

Proposition 3(a) shows that if household sentiment is outweighed by institutional sentiment, the sentiment shock generates a wave of purchases and no sales. When retail sentiment is sufficiently low, however, retail traders will not hold much of the risky asset and

[21] Two other unusual scenarios are also feasible when institutional or retail sentiment are allowed to be negative, as the rebalancing channel flips signs and the short sale constraint's interaction with private sentiment becomes more important. These two peculiar cases are detailed as (b) and (d) of Proposition 5 in Appendix A.

[22] The derivative of the marginal trader's sentiment with respect to dispersion is given by

$$\frac{\partial h_t}{\partial \sigma_{s,t}} = \sigma_{\epsilon,t}^{-2}\vartheta_t\left(\bar{\beta}_t\omega_t + \xi_t\alpha_t\right), \tag{19}$$

where $\sigma_{s,t}^{-2}\vartheta_t > 0, \alpha_t > 0$, and $\bar{\beta}_t < 0$ so that $\bar{\beta}_t\omega_t + \xi_t\alpha_t$ can be either sign.

therefore the extreme imbalance scenario is associated with low absolute levels of trading volume. Proposition 3(b) captures a scenario where institutional and household beliefs are more in alignment or where households are exceedingly optimistic and hold large positions: here intensification of beliefs triggers a wave of both purchases and sales, but with purchases outweighing sales.

Shifts in disagreement can therefore be associated with either extreme imbalances or modest imbalances depending on the relationship of institutional sentiment and retail sentiment. Unlike with attention, where high levels of volume necessarily require large household positions and therefore low imbalances due to the relationship of equation (16), dispersion changes allow for perfectly lopsided trading patterns. As we shall see in the next subsection, the differences between dispersion and attention become more stark when the full parameter space is explored.

4.7 Simulation Results

In this subsection I show via simulation that only attention shifts are consistent with the empirical results. In particular, only attention can generate both retail alignment and the relationship between volumes and returns. While the results of Propositions 1, 2, and 3 clearly illustrate major differences in volume patterns through analytic derivations, the contrasts are illustrated even more sharply through simulation. Simulations offer us the ability to observe the model at work for a wide variety of parameter values, which in turn vary the market capitalization of the risky asset and the ownership level of households, two primary contributors to dollar volume. Table 7 provides the values and distributions used in the three simulations, which each draw $10,000$ random risky asset environments and find the equilibrium retail dollar trading levels under both no shocks and shocks, with the three types of shocks being attention, sentiment, and disagreement shocks, respectively. Although the model is not a many-asset environment, under narrow framing this simulation exercise can be viewed as similar to drawing a cross section of risky assets.

Figure 9 plots the retail buy and sell dollar volumes for the three different types of shocks in the cross section. Attention shocks, seen in Sub-Figure 9a, generate dollar buy-sell pairs that lie in a wide cone when near the origin but a narrow cone as we move along the 45° line, consistent with the retail alignment puzzle. The points near the origin are for stocks that have high institutional ownership relative to household ownership, which is to say low household sentiment relative to institutional sentiment. These stocks see low dollar retail volume but wider percentage dispersion. Expensive stocks with high retail sentiment generate the largest retail trading volume under attention shocks; points representing such stocks can be found in a narrow cone lying close to the 45° line as we move up.

Sub-Figure 9b starkly illustrates Proposition 2, namely that sentiment shifts are almost

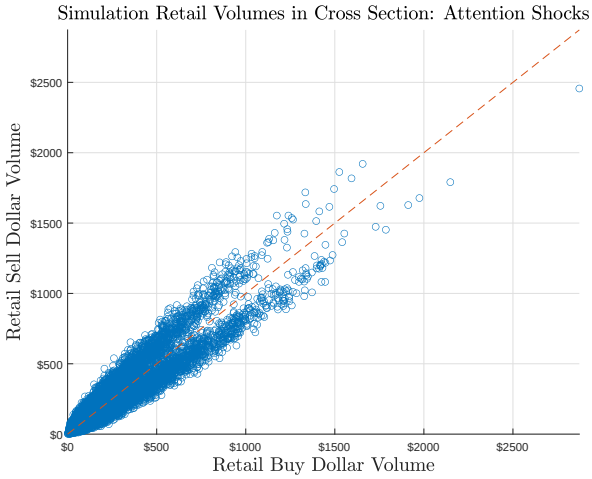
Table 7: Simulation Parameters

	Distribution or Value
ξ (Household Sentiment)	N(0.08,0.04^2), Jointly Truncated at 0 with $\mu(s_i)$
$\mu(s_i)$ (Avg Institutional Sentiment)	N(0.08,0.04^2), Jointly Truncated at 0 with ξ
ρ =Corr(ξ,$\mu(s_i)$)	0.5
$\sigma_{s,h}$	N(0,0.05^2)
Num Households (Discretization)	1,000,000
Num Stocks (Simulations per Shock Type)	10,000
a (Attrition)	0.005
Cross Sectional Attention Distribution	U[0,0.2]
Household Assets	$100,000
Institutional Total Assets	$6,000
Num Institutions	10,000
Institutional Asset % Change	U[-0.1,0.1]
$\hat{\beta} = \bar{\beta}$	U[0,1e-6]
$\sigma_i^2 = \sigma_H^2$	0.04^2
$\frac{g_{t+1}}{g_t}$ (Attention Shocks)	U[0.975,1.025]
$\Delta\ \xi_{t+1}$ (Sentiment Shocks)	U[-0.005,0.005]
$\frac{\sigma_{t+1}}{\sigma_t}$ (Disagreement Shocks)	U[0.9,1.1]
Institution i Sentiment Deviation from Average	N(0,0.01^2)
Institution i Sentiment Change from t to $t+1$	N(0,0.02^2)

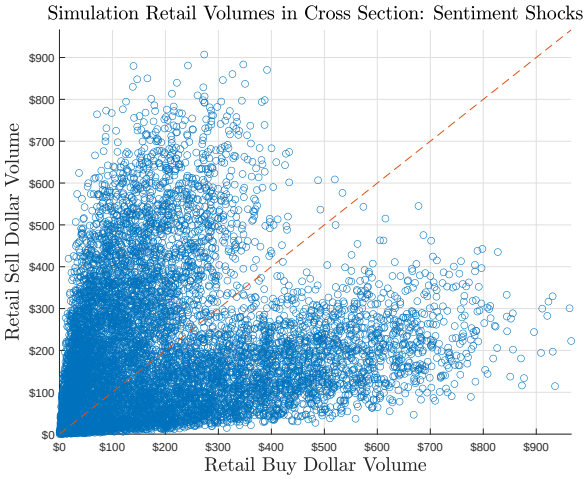
Note: This table provides the parameter values used for the simulations.

universally associated with significant imbalances. Instead of clustering around the 45° line, nearly all points are pushed away from the line, thus producing a figure that looks to be the opposite of the empirical facts. On the other hand, Sub-Figure 9c shows that dispersion changes generate retail alignment for the most part but with some extreme outliers. Disagreement shocks are capable of generating large imbalances but rarely do so for stocks actively traded by retail traders because such stocks tend to feature high levels of retail ownership and therefore high retail sentiment levels. When disagreement increases in a high sentiment stock, existing holders of the asset increase or decrease their positions but with nearly opposing quantities because the short sale constraint is only active for a small measure of investors.

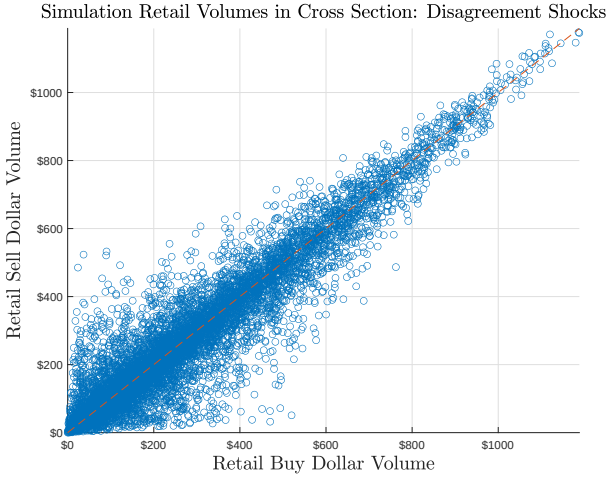
Disagreement might appear to be a plausible second explanation for the retail alignment puzzle, but the return-volume relationship generated by the simulation illustrates why this explanation is inconsistent with the data. Figure 10 provides a different perspective on the simulation results by plotting the relationship between total retail volumes and stock returns under each of the three types of shock simulations. Sub-Figures 10a and 10b show a "V" shaped graph where the highest retail trading coincides with the most extreme returns, with such days typically entailing large attention or sentiment shocks in high retail sentiment



(a) Dollar Purchases and Sales in the Cross Section: Attention Shocks



(b) Dollar Purchases and Sales in the Cross Section: Sentiment Shocks



(c) Dollar Purchases and Sales in the Cross Section: Disagreement Shocks

Figure 9: Simulated Retail Buy and Sell Dollar Volumes under Different Shocks

Note: Using the model of Section 4, I simulate retail volumes in the cross section under attention shocks, sentiment shocks, and disagreement shocks; these figures provide the retail volume simulation results for a cross section of 10,000 stocks with shocks to attention, sentiment, and disagreement in Sub-Figures 9a, 9b, and 9c, respectively. All parameters are set according to the values in Table 7; average retail and institutional return expectations are drawn from a truncated bivariate normal distribution with $\rho = 0.5$ correlation, and a standard deviation of 0.04 with a mean of 0.08 for both retail and institutional return expectations.

stocks. These are empirically supported by the results of Section 2 where we saw that recent and contemporary return salience is strongly associated with high levels of retail stock trading. On the other hand, Sub-Figure 10c provides a clearly counterfactual pattern: the highest volume days are the days where the stock goes nowhere. This simulation result is a direct corollary of Lemma 1 in Appendix A, which shows that the levels of price change under disagreement shocks are proportional to $\lambda(h_t)$, the inverse Mills ratio of the marginal household. When household sentiment ξ_t is very high, which is to say when disagreement shocks cause high retail trading volume, $\lambda(h_t)$ is near zero, making for extremely limited price movement. If households were not short sale constrained, then an increase in disagreement would have no impact on price because the aggregate demand curve would not shift. $\lambda(h_t)$ captures the extent to which the short sale constraint binds the continuum of households and creates price pressure following disagreement shocks.

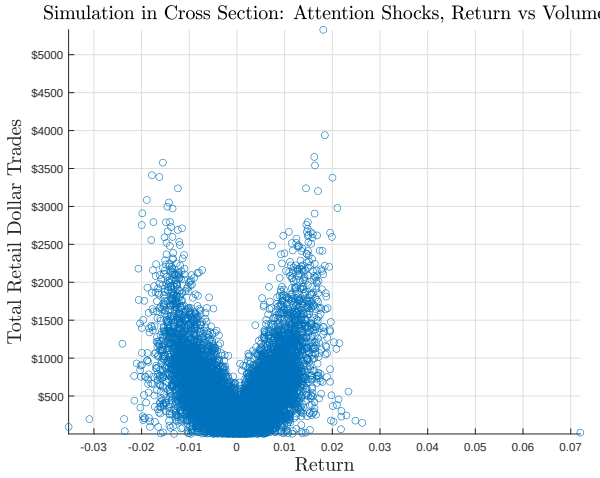
We therefore have that were disagreement shocks the primary driver of retail trading volumes, we would expect to see surges in retail trading accompanied by limited directional movement. Were institutional demand shifts to accompany such a household disagreement shock, we could still observe directional movement accompanied by high levels of retail trading, but the institutional demand shifts and accompanying price shocks would trigger rebalancing and demand slope effects among households, resulting in imbalances. Of all three types of shock, only attention is able to generate the empirical patterns of large volume fluctuations, retail alignment, and a positive contemporaneous relationship between return salience and retail trading.

4.8 Remarks on Mispricing, Imbalances, Short Sale Constraints, and Passive Investing

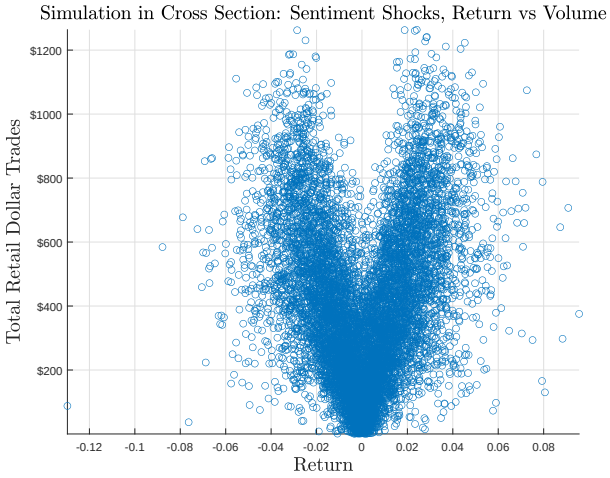
Here I highlight three major points about the model's mechanisms and their relationship to the data.

Remark 1. *(Mispricings and Balanced Flow)* I have shown that mispricings will generally be correlated with retail trade imbalances in that positive shocks to attention, sentiment, and disagreement will all generate positive price shocks and, for many reasonable values of the parameters, positive net retail flow. At the same time, the model indicates that retail-driven mispricings and retail imbalances are not an inseparable pair. Liao, Peng and Zhu (2021) have established an extrapolation and disposition effect-driven channel that creates volume and price changes without retail imbalances, while this paper contributes an attention and rebalancing-driven mechanism that similarly allows for a baton-passing phenomenon: new retail traders purchase while old retail traders dispose of the asset.

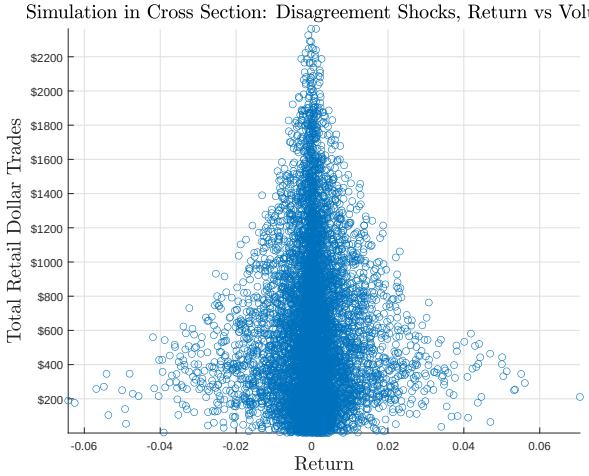
Remark 2. *(Short Sale Constraints)* Throughout this section I have assumed that retail



(a) Dollar Purchases and Sales in the Cross Section: Attention Shocks



(b) Dollar Purchases and Sales in the Cross Section: Sentiment Shocks



(c) Dollar Purchases and Sales in the Cross Section: Disagreement Shocks

Figure 10: Simulated Retail Volumes versus Returns

Note: Using the model of Section 4, I simulate retail volumes in the cross section under attention shocks, sentiment shocks, and disagreement shocks; these figures scatter plot the total retail volumes versus returns for a cross section of 10,000 stocks under shocks to attention, sentiment, and disagreement, given in Sub-Figures 10a, 10b, and 10c, respectively. All parameters are set according to the values in Table 7; average retail and institutional return expectations are drawn from a truncated bivariate normal distribution with $\rho = 0.5$ correlation, and a standard deviation of 0.04 with a mean of 0.08 for both retail and institutional return expectations.

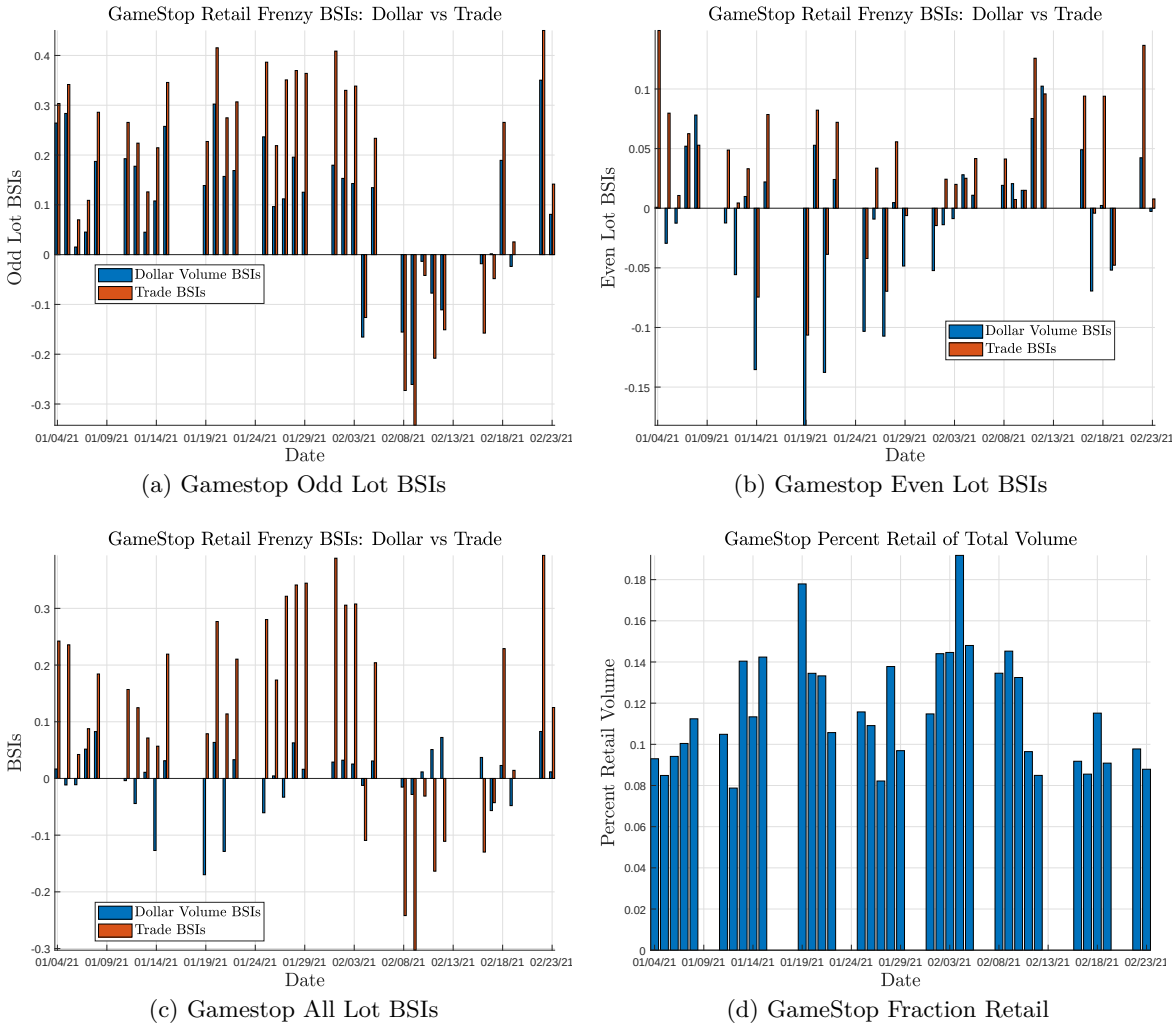
traders are bound by short sale constraints because this assumption is standard in the literature and standard in financial markets. However, this assumption is not essential to most of the model's central conclusions. Without short sale constraints, increases in attention will still trigger surges in trading volume, but the price increase will be attenuated by the short positions of newly attentive pessimists while the retail imbalances will be even smaller because of this extensive margin of new retail short positions. Dispersion changes are where the short sale constraint has the greatest impact. Under no constraints, shocks to household disagreement result in perfectly matched retail sales and purchases with no price pressure.

Remark 3. *(Passive Investing and Household Ownership)* This paper does not explicitly model passive investing, wherein mutual funds and index funds generally hold market capitalization-weighted portfolios of large universes of equities. The model does readily accommodate this, however, as a small, positive, constant $\hat{\beta}_{i,t} = \beta_i$ with neutral sentiment $s_{i,t} = 0$ mirrors the portfolio formation process of an index fund. If $s_{i,t} = 0$ for a large fraction of institutions when weighted by assets, then $\omega_{i,t}$, the risk and asset-weighted institutional sentiment, must be small. In assets with $30 - 60\%$ household ownership, a common range for the stocks actively traded by retail traders and studied in this paper, a small $\omega_{i,t}$ versus household asset-weighted sentiment is to be expected due to actively managed institutional traders representing a small fraction of ownership despite being a large fraction of trading volume. Proposition 1 predicts small retail imbalances under attention shocks for such equities, precisely as shown in this paper's empirical sections.

5 GameStop: a Case Study

The GameStop retail trading frenzy of early 2021 presents an ideal case study that illustrates this paper's main empirical and theoretical points, namely that attention is uniquely capable of driving significant surges in volume while failing to generate large imbalances between retail purchases and sales.

Figures 11a, 11b, and 11c plot the odd lot, even lot, and total retail daily trade and dollar BSIs in GameStop from the beginning of January 2021 through late February 2021, while Figure 11d plots the daily fraction of marketable retail trades out of total volume. The red bars are "trade BSIs," which compute imbalances using counts of executed marketable purchase and sale orders as opposed to total dollars, while the blue bars are the standard dollar BSIs used throughout the rest of the paper. Several patterns are visually obvious: small traders (odd lots) aggressively bought GameStop, in accordance with the popular narrative surrounding the episode. On every single day in January 2021, odd lot trade and dollar BSIs were positive and often extremely high. Aggregate trade BSIs — which count



(a) Gamestop Odd Lot BSIs

(b) Gamestop Even Lot BSIs

(c) Gamestop All Lot BSIs

(d) GameStop Fraction Retail

Figure 11: GameStop BSIs: 2021 Retail Frenzy

Note: The first three figures plot the dollar and trade buy-sell imbalances (BSIs) during the GameStop retail trading frenzy of January and February 2021, while the four plots retail traders' daily fraction of total volume. Dollar BSIs are calculated as $BSI_{DV,t} = \frac{DollarBuys_t - DollarSells_t}{DollarBuys_t + DollarSells_t}$ while trade BSIs are calculated as $BSI_{TV,t} = \frac{\#Buys_t - \#Sells_t}{\#Buys_t + \#Sells_t}$. Figures 11a and 11b plot imbalances for odd lot (1-99 shares) and even lot (100+ shares) respectively, while Figure11c plots this for all trades.

one lot orders the same as $10,000$ lot orders in computing imbalances — followed a similar pattern, with marketable retail purchase orders outnumbering marketable retail sale orders on every day of January.

Yet Figures 11b and 11c run sharply contrary to the popular narrative: in aggregate, retail traders were frequently net sellers of GameStop during the peak of the episode in mid to late January. In lieu of outlier positive BSIs we see outlier negative BSIs and all BSIs in the narrow range of $(-0.17, 0.085)$. Larger retail traders were responsible for this

counterintuitive pattern, as even lot trades had negative BSIs on many days. As argued in the theory section of this paper, attention shocks create purchase orders by newly attentive traders but early, previously attentive holders of the asset dispose of it en masse because of a sensitivity to price as well as rebalancing motives. Moreover, stocks with extreme short interest will see a price magnification effect from short covering as the attention-based rally triggers net purchasing of shares by short institutions to limit risk; this mechanism was infamously at work during the GameStop-related short seller implosions of early 2021. The signed VWAI for GameStop over this period (the corollary to equation (3) with no absolute value sign) is -0.0036 and the VWAI is 0.0431, implying an extreme amount of retail alignment with negative net retail imbalances just as the theory predicts.[23]

While the GameStop episode is a highly relevant illustration of attention-based retail volume surges, the negative correlation between odd and even lot traders is highly anomalous: for virtually every stock and every time window, odd and even lot dollar BSIs are positively correlated. Even lot traders are more price sensitive and less prone to attention shocks, but both are subject to the same general pattern in imbalances. If small and large retail traders were differentially responding to price shocks and trading opposite one another to generate small aggregate BSIs, then we would anticipate consistently negative correlations, precisely the opposite of what we observe in the data. Any satisfactory explanation of retail alignment must capture the behavior of both small and large traders independently and in aggregate, which is to say it must account for minimal imbalances in the presence of attention shocks despite positive imbalance correlations between small and large traders. This paper's model underscores the role of attention, whereby the price sensitivity channel and the rebalancing channel counteract a new wave of optimistic purchasers. The story of GameStop is therefore one of attention drawing in more investors, with the most optimistic purchasing the stock and driving the price up while less optimistic households exit. Essentially, the meme stock phenomenon can be reframed as a story of greater visibility increasing the optimism of the marginal household asset holder and inflating prices; this increase, however, does not imply large net retail flows.

6 Conclusion

This paper provides both theoretical and empirical evidence that retail trader attention plays a central and underappreciated role in generating price and volume fluctuations in the cross section of equities. In a model with institutional investors as well as short sale constrained retail traders with heterogeneous attention and sentiment, only fluctuations

[23] Using the traditional BSI and $|BSI|$ metrics with retail volume weighting by day, we similarly obtain -0.0031 and 0.0377, respectively.

in attention can consistently generate small imbalances between retail purchases and sales, whereas shifts in sentiment and disagreement often generate large imbalances. Based on this model, empirical evidence suggests that attention changes play a primary role in generating retail volumes. Examining nine years of retail trades identified via the methodology of Boehmer, Jones, Zhang and Zhang (2021), I find nearly perfect alignment of retail purchases and sales in aggregate over time, by stock over time, and in the cross-section of stocks on different trading dates. If a large dollar volume of retail purchases occurs in a stock, a nearly precisely equal dollar volume of retail sales almost certainly occurs in the same stock on the same day.

Although this paper's model exploits multiple dimensions of retail investor heterogeneity, several additional dimensions require substantial research to elucidate the path of retail attention. One dimension is the time horizon of investors; investors differ in their motivations for trading as well as their target investment time horizon, whereas this paper's model assumes that agents engage in approximately mean-variance optimization over their consideration set each period. Another two dimensions are wealth and sophistication. Retail investors differ in their portfolio size, with larger retail investors responsible for a disproportionate share of trading. Understanding the relationship between retail volume patterns, sentiment, price changes, and heterogeneity in investor wealth is beyond the scope of this paper and could provide a rich vein for future research.

My results suggest that understanding how and why retail attention fluctuates is an ambitious and important goal for future asset pricing research. In one such example, Levy, Rossi, Shalev and Zur (2023) use Seeking Alpha watchlists as an attention proxy and find that when stocks are added or taken off of lists, it impacts the stock price. Further research into the neuroscience of financial attention as well as the primary transmission mechanisms or networks for narrative contagion in the spirit of Shiller (2017) would greatly enhance our ability to forecast and understand not just systemically destabilizing financial bubbles but also high volume episodes such as the meme stock phenomenon or cryptocurrencies. While extrapolation and other well known behavioral tendencies are responsible for some fraction of bubble behavior, this paper has illustrated that mere attention can destabilize prices and create the high volumes that are the hallmark of bubbles. One can only purchase tulip bulbs, after all, if one is aware that they are for sale.

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A Proofs from Section 4

Throughout, $s_{i.t}$ is used interchangeably with $\epsilon_{i,t}$ and $\epsilon_{h,t}$ with $s_{h,t}$.

A.1 Proof of Existence and Uniqueness of Equilibrium

The assumptions on parameters are summarized in the following statement, and the subsequent proposition shows the existence of a unique equilibrium under these conditions.

Assumption 1. *$\bar{\beta}_{0,t} < 0$, $\sum_i A_{i,t}\hat{\beta}_{0,i,t}\sigma_{i,t}^{-2} < 0$, and $\sum_i A_{i,t}\hat{\beta}_{0,i,t}\sigma_{i,t}^{-2} + A_{H,t}\sigma_{H,t}^{-2}g_t\bar{\beta}_{0,t} > -1$.*

Proposition 4. *Under the conditions of Assumption 1, equilibrium exists and is unique.*

Proof. Using Proposition 1 of Graves (2022) applied to this paper's model, equilibrium exists and is unique provided that

$$\|\sum_i A_{i,t}\hat{\beta}_{0,i,t}\Sigma_{i,t}^{-1}\|_\infty + \|\eta^{-1}A_{H,t}\int 1_{n\in\mathcal{C}_{h,t}}\Sigma_{H,t}^{-1}\mathbf{1}|\bar{\beta}_{0,t} + \sigma_{0,t}\nu_{0,h,t}|dh\|_\infty < 1 \tag{21}$$

in a multi-asset model. The equivalent condition here is $|\sum_i A_{i,t}\hat{\beta}_{0,i,t}\sigma_{i,t}^{-2}| + A_{H,t}\sigma_{H,t}^{-2}g_t|\bar{\beta}_{0,t}| < 1$, where we have used the facts that $n = 1$ for the dimension and $\nu_{0,h,t} = 0, \forall h$. This condition is then equivalent to $\sum_i A_{i,t}\hat{\beta}_{0,i,t}\sigma_{i,t}^{-2} + A_{H,t}\sigma_{H,t}^{-2}g_t\bar{\beta}_{0,t} > -1$ since both components are strictly negative. □

The condition on downward sloping demand for institutions is unnecessary, as condition 21 is sufficient for existence and uniqueness of equilibrium: a restriction on the degree to which institutional demand slopes downward or upward is required.

A.2 Lemma 1 And Proof 4.3

Using the implicit function theorem, we arrive at the following lemma; all proofs from this section are contained in Appendix A.

Lemma 1. *The partial derivatives of price with respect to g_t (attention), ξ_t (average sentiment), and $\sigma_{\epsilon,t}$ (dispersion) are given by*

$$\frac{\partial P_t}{\partial g_t} = \vartheta_t\eta_t\frac{\lambda'(h_t)}{\lambda(h_t)} > 0 \tag{22}$$

$$\frac{\partial P_t}{\partial \xi_t} = \theta_t\eta_t > 0 \tag{23}$$

$$\frac{\partial P_t}{\partial \sigma_{\epsilon,t}} = \vartheta_t\eta_t\lambda(h_t) > 0. \tag{24}$$

The elasticities of price with respect to these three variables are then given by

$$\frac{\partial \log P_t}{\partial \log g_t} = \vartheta_t \eta_t \frac{\lambda'(h_t)}{\lambda(h_t)} \frac{g_t}{P_t}, g_t > 0. \tag{25}$$

$$\frac{\partial \log P_t}{\partial \log \xi_t} = \vartheta_t \eta_t \frac{\xi_t}{P_t}, \xi_t > 0 \tag{26}$$

$$\frac{\partial \log P_t}{\partial \log \sigma_{\epsilon,t}} = \vartheta_t \eta_t \lambda(h_t) \frac{\sigma_{\epsilon,t}}{P_t}, \sigma_{\epsilon,t} > 0 \tag{27}$$

While we will require the results of the first part of the lemma to compute the effects of variable changes on volume, the elasticity results are of independent interest because they show the differential impact of variable changes on a stock's price within the model. For example,

$$\frac{\partial \log P_t}{\partial \log \xi_t} \Big/ \frac{\partial \log P_t}{\partial \log \sigma_{\epsilon,t}} = \frac{\xi_t}{\sigma_{\epsilon,t} \lambda(h_t)}. \tag{28}$$

Suppose that the average household had return expectations of 10% and that 1% of households are paying attention to the asset. Further suppose that $\bar{\beta}_{0,t}$ is small so that $\bar{\beta}_{0,t} P_t + \xi_t \approx \xi_t$ and the disagreement term is $\sigma_{\epsilon,t} = 0.05$. Then $h_t = -\sigma_{\epsilon,t}^{-1} \left(\bar{\beta}_{0,t} P_t + \xi_t \right) \approx -\sigma_{\epsilon,t}^{-1} \xi_t = -2$ and $\lambda(-2) = 0.0552$ and equation (28) gives $\frac{\partial \log P_t}{\partial \log \xi_t} / \frac{\partial \log P_t}{\partial \log \sigma_{\epsilon,t}} = \frac{0.1}{0.05 \times 0.0552} \approx 36$. The same relative ratio for attention and sentiment gives

$$\frac{\partial \log P_t}{\partial \log \xi_t} \Big/ \frac{\partial \log P_t}{\partial \log g_t} = \frac{\lambda(h_t) \xi_t}{\lambda'(h_t) g_t} = \frac{1}{.20552} \approx 5.$$

For these parameter values, a small percentage change in sentiment results in a large shift in price while changes in disagreement have much smaller impacts on price; attention changes cause moderate price changes.

$$P_t = \sum_i A_{i,t} \sigma_t^{-2} \left(\hat{\beta}_{0,i,t} P_t + \epsilon_{i,t} \right) + A_{H,t} \sigma_{H,t}^{-2} g_t \int \max \left\{ 0, \bar{\beta}_{0,t} P_t + \xi_t + \sigma_{\epsilon,t} \epsilon_{h,t} \right\} dh$$

where $\epsilon_{h,t} \sim N(0,1)$. Using the properties of the normal distribution, we have the formula

$$\int \max \left\{ 0, \bar{\beta}_{0,t} P_t + \xi_t + \sigma_{\epsilon,t} \epsilon_{h,t} \right\} dh = \Pr(X > \alpha) \mathbb{E}[X | X > \alpha]$$

where $\alpha = 0$ and $X \sim N\left(\bar{\beta}_{0,t}P_t + \xi_t, \sigma^2_{\epsilon,t}\right)$. We then see using the formula that

$$\mathbb{E}\left[X|X > \alpha\right] = \mu + \sigma\lambda\left(\alpha_x\right), \lambda\left(\alpha_x\right) = \frac{\phi\left(\alpha_x\right)}{1 - \Phi\left(\alpha_x\right)}, \alpha_x = \frac{\alpha - \mu}{\sigma}$$

gives us

$$\int \max\left\{0, \bar{\beta}_{0,t}P_t + \xi_t + \sigma_{\epsilon,t}\epsilon_{h,t}\right\} dh = \left(1 - \Phi\left(-\frac{\bar{\beta}_{0,t}P_t + \xi_t}{\sigma_{\epsilon,t}}\right)\right)\left[\bar{\beta}_{0,t}P_t + \xi_t + \sigma_{\epsilon,t}\frac{\phi\left(\frac{-\bar{\beta}_{0,t}P_t - \xi_t}{\sigma_{\epsilon,t}}\right)}{1 - \Phi\left(\frac{-\bar{\beta}_{0,t}P_t - \xi_t}{\sigma_{\epsilon,t}}\right)}\right]$$

so then the equilibrium price is defined implicitly by

$$P_t = \sum_i A_{i,t}\sigma_t^{-2}\left(\hat{\beta}_{0,i,t}P_t + \epsilon_{i,t}\right) + A_{H,t}\sigma_{H,t}^{-2}g_t\left(1 - \Phi\left(-\frac{\bar{\beta}_{0,t}P_t + \xi_t}{\sigma_{\epsilon,t}}\right)\right)\left[\bar{\beta}_{0,t}P_t + \xi_t + \sigma_{\epsilon,t}\frac{\phi\left(\frac{-\bar{\beta}_{0,t}P_t - \xi_t}{\sigma_{\epsilon,t}}\right)}{1 - \Phi\left(\frac{-\bar{\beta}_{0,t}P_t - \xi_t}{\sigma_{\epsilon,t}}\right)}\right]$$

or rearranging, $\sum_i A_{i,t}\sigma_t^{-2}\left(\hat{\beta}_{0,i,t}P_t + \epsilon_{i,t}\right) + A_{H,t}\sigma_{H,t}^{-2}g_t\left(1 - \Phi\left(h_t\right)\right)\left[\bar{\beta}_{0,t}P_t + \xi_t + \sigma_{\epsilon,t}\lambda\left(h_t\right)\right] - P_t = 0$. As this defines a market clearing function $F\left(g_t, P_t\left(g_t\right)\right) = 0$, we can use the implicit function theorem to find the derivative of P_t with respect to g_t as (letting $h_t = -\frac{\bar{\beta}_{0,t}P_t + \xi_t}{\sigma_{\epsilon,t}}$) and noting that $\frac{\partial h_t}{\partial P_t} = -\frac{\bar{\beta}_{0,t}}{\sigma_{\epsilon,t}}$

$$\frac{\partial P_t}{\partial g_t} = -\left[\left(\sum_i A_{i,t}\sigma_t^{-2}\hat{\beta}_{0,i,t} - 1\right) + A_{H,t}\sigma_{H,t}^{-2}g_t\bar{\beta}_{0,t}\left(\frac{1}{\sigma_{\epsilon,t}}\phi\left(h_t\right)\left[\bar{\beta}_{0,t}P_t + \xi_t + \sigma_{\epsilon,t}\lambda\left(h_t\right)\right] + \left(1 - \Phi\left(h_t\right)\right)\left[1 - \lambda'\left(h_t\right)\right]\right)\right]^{-1} \times \left[A_{H,t}\sigma_{H,t}^{-2}\left(1 - \Phi\left(h_t\right)\right)\left[\bar{\beta}_{0,t}P_t + \xi_t + \sigma_{\epsilon,t}\lambda\left(h_t\right)\right]\right] > 0$$

where the inequality follows from the assumption that $\sum_i A_{i,t}\sigma_t^{-2}\hat{\beta}_{0,i,t} < 1$, the fact that $1 - \lambda'\left(h_t\right) > 0$ by properties of the inverse Mills ratio, and the fact that $\bar{\beta}_{0,t}P_t + \xi_t + \sigma_{\epsilon,t}\lambda\left(h_t\right) > 0$ by definition of h_t while $\bar{\beta}_{0,t} < 0$. Using what's derived below, this becomes

$$\frac{\partial P_t}{\partial g_t} = -\left[\left(\sum_i A_{i,t}\sigma_t^{-2}\hat{\beta}_{0,i,t} - 1\right) + A_{H,t}\sigma_{H,t}^{-2}g_t\bar{\beta}_{0,t}\left(1 - \Phi\left(h_t\right)\right)\right]^{-1} \times \left[A_{H,t}\sigma_{H,t}^{-2}\left(1 - \Phi\left(h_t\right)\right)\left[\bar{\beta}_{0,t}P_t + \xi_t + \sigma_{\epsilon,t}\lambda\left(h_t\right)\right]\right] > 0$$

which we can rewrite as

$$\frac{\partial P_t}{\partial g_t} = -\left[\left(\sum_i A_{i,t}\sigma_t^{-2}\hat{\beta}_{0,i,t} - 1\right) + A_{H,t}\sigma_{H,t}^{-2} g_t \bar{\beta}_{0,t}\left(1 - \Phi\left(h_t\right)\right)\right]^{-1} \times \left[A_{H,t}\sigma_{H,t}^{-2}\sigma_{\epsilon,t}\left(\phi\left(h_t\right) - \left(1 - \Phi\left(h_t\right)\right)h_t\right)\right] > 0$$

or alternatively,

$$\frac{\partial P_t}{\partial g_t} = -\left[\left(\sum_i A_{i,t}\sigma_t^{-2}\hat{\beta}_{0,i,t} - 1\right) + A_{H,t}\sigma_{H,t}^{-2} g_t \bar{\beta}_{0,t}\left(1 - \Phi\left(h_t\right)\right)\right]^{-1} \times \left[A_{H,t}\sigma_{H,t}^{-2}\sigma_{\epsilon,t}\left(1 - \Phi\left(h_t\right)\right)\frac{\lambda'\left(h_t\right)}{\lambda\left(h_t\right)}\right] > 0.$$

Defining

$$h_t = -\sigma_{\epsilon,t}^{-1}\left(\bar{\beta}_{0,t}P_t + \xi_t\right), \eta_t = \left(1 - \Phi\left(h_t\right)\right)A_{H,t}\sigma_{H,t}^{-2} g_t$$

$$\alpha_t = \left(1 - \sum_i A_{i,t}\sigma_t^{-2}\hat{\beta}_{0,i,t}\right),\ \omega_t = \sum_i A_{i,t}\sigma_t^{-2}\epsilon_{i,t}$$

$$\vartheta_t = \left(\alpha_t - \bar{\beta}_{0,t}\eta_t\right)^{-1}.$$

we can rewrite as

$$\frac{\partial P_t}{\partial g_t} = \vartheta_t\eta_t\frac{\lambda'\left(h_t\right)}{\lambda\left(h_t\right)}.$$

We can similarly look at the derivative of prices with respect to sentiment ξ_t and find it to be

$$\frac{\partial P_t}{\partial \xi_t} = -\left[\left(\sum_i A_{i,t}\sigma_t^{-2}\hat{\beta}_{0,i,t} - 1\right) + A_{H,t}\sigma_{H,t}^{-2} g_t \bar{\beta}_{0,t}\left(1 - \Phi\left(h_t\right)\right)\right]^{-1} \times \left[A_{H,t}\sigma_{H,t}^{-2} g_t\left(\phi\left(h_t\right)\sigma_{\epsilon,t}^{-1}\left[\bar{\beta}_{0,t}P_t + \xi_t + \sigma_{\epsilon,t}\lambda\left(h_t\right)\right] + \left(1 - \Phi\left(h_t\right)\right)\left[1 - \lambda'\left(h_t\right)\right]\right)\right] > 0$$

where once again we have the inequality from the same facts. Furthermore, we can use that

$$\phi\left(h_t\right)\sigma_{\epsilon,t}^{-1}\left[\bar{\beta}_{0,t}P_t + \xi_t + \sigma_{\epsilon,t}\lambda\left(h_t\right)\right] + \left(1 - \Phi\left(h_t\right)\right)\left[1 - \lambda'\left(h_t\right)\right]$$

is equal to

$$\left(1 - \Phi\left(h_t\right)\right)\left\{\left[-\lambda\left(h_t\right)h_t + \lambda^2\left(h_t\right)\right] + 1 - \left(-\lambda\left(h_t\right)h_t + \lambda^2\left(h_t\right)\right)\right\} = \left(1 - \Phi\left(h_t\right)\right)$$

to rewrite as

$$\frac{\partial P_t}{\partial \xi_t} = -\left[\left(\sum_i A_{i,t}\sigma_t^{-2}\hat{\beta}_{0,i,t} - 1\right) + A_{H,t}\sigma_{H,t}^{-2} g_t \bar{\beta}_{0,t}\left(1 - \Phi\left(h_t\right)\right)\right]^{-1} \times \left[\left(1 - \Phi\left(h_t\right)\right) A_{H,t}\sigma_{H,t}^{-2} g_t\right] > 0.$$

or $\frac{\partial P_t}{\partial \xi_t} = \theta_t \eta_t$.

The derivative of prices with respect to dispersion is found to be (using $\frac{\partial h_t}{\partial \sigma_{\epsilon,t}} = \sigma_{\epsilon,t}^{-1}\frac{\bar{\beta}_{0,t}P_t + \xi_t}{\sigma_{\epsilon,t}} = -\sigma_{\epsilon,t}^{-1} h_t$

$$\frac{\partial P_t}{\partial \sigma_{\epsilon,t}} = -\left[\left(\sum_i A_{i,t}\sigma_t^{-2}\hat{\beta}_{0,i,t} - 1\right) + A_{H,t}\sigma_{H,t}^{-2} g_t \bar{\beta}_{0,t}\left(1 - \Phi\left(h_t\right)\right)\right]^{-1} \times$$

$$\left[A_{H,t}\sigma_{H,t}^{-2} g_t \left\{\phi\left(h_t\right)\left(\sigma_{\epsilon,t}^{-1} h_t\right)\left[\bar{\beta}_{0,t}P_t + \xi_t + \sigma_{\epsilon,t}\lambda\left(h_t\right)\right] + \left(1 - \Phi\left(h_t\right)\right)\left[1 + \lambda\left(h_t\right) - \lambda'\left(h_t\right) h_t\right]\right\}\right] > 0$$

where we have used the derivation

$$\begin{aligned}
&\phi\left(h_t\right)\left(\sigma_{\epsilon,t}^{-1} h_t\right)\left[\bar{\beta}_{0,t}P_t + \xi_t + \sigma_{\epsilon,t}\lambda\left(h_t\right)\right] + \left(1 - \Phi\left(h_t\right)\right)\left[1 + \lambda\left(h_t\right) h_t^2 + \lambda\left(h_t\right)\left(1 - \lambda\left(h_t\right) h_t\right)\right] \\
&\quad = \left(1 - \Phi\left(h_t\right)\right)\left[-\lambda\left(h_t\right) h_t^2 + h_t\lambda^2\left(h_t\right) + 1 + \lambda\left(h_t\right) h_t^2 + \lambda\left(h_t\right) - \lambda^2\left(h_t\right) h_t\right] \\
&\quad = \left(1 - \Phi\left(h_t\right)\right)\lambda\left(h_t\right) > 0
\end{aligned}$$

Thus our expression is

$$\frac{\partial P_t}{\partial \sigma_{\epsilon,t}} = -\left[\left(\sum_i A_{i,t}\sigma_t^{-2}\hat{\beta}_{0,i,t} - 1\right) + A_{H,t}\sigma_{H,t}^{-2} g_t \bar{\beta}_{0,t}\left(1 - \Phi\left(h_t\right)\right)\right]^{-1} \times \left[A_{H,t}\sigma_{H,t}^{-2} g_t\left(1 - \Phi\left(h_t\right)\right)\lambda\left(h_t\right)\right] > 0.$$

or

$$\frac{\partial P_t}{\partial \sigma_{\epsilon,t}} = \vartheta_t \eta_t \lambda\left(h_t\right).$$

A.3 Omitted Proofs for Subsection 4.4

A.3.1 Proof of volume expressions

First we must see how changes in attention impact the marginal holder.

$$\frac{dh_t}{dg_t} = -\frac{\bar{\beta}_{0,t}\frac{\partial P_t}{\partial g_t}}{\sigma_{\epsilon,t}} > 0$$

so that $h_t < h_{t+1}$: this implies no extensive margin among existing attentive traders. No intensive margin exists either, as sentiment does not shift but prices increase, reducing return expectations across all households. Summing together demand for attritional traders as well as newly attentive traders, we have

$$V_{attention,\uparrow}^{Retail,Buy} = (\Delta g_{t+1} + a_t g_t) P_{t+1}^{-1} A_{H,t+1} \sigma_{H,t+1}^{-2} (1 - \Phi(h_{t+1})) \left[\bar{\beta}_{0,t} P_{t+1} + \xi_t + \sigma_{\epsilon,t} \lambda(h_{t+1})\right]. \tag{29}$$

For sell volume, we have only those who previously held the asset (which is to say were paying attention). These can be divided into two categories. First, we have the measure $a_t g_t$ of agents who will merely sell off what they already have. This is given by

$$V_{attention,\uparrow}^{Retail,Sell,attrition} = a_t g_t P_t^{-1} A_{H,t} \sigma_{H,t}^{-2} (1 - \Phi(h_t)) \left[\bar{\beta}_{0,t} P_t + \xi_t + \sigma_{\epsilon,t} \lambda(h_t)\right]$$

Everyone who previously held the asset will, under the assumption of constant sentiment and dispersion, be less enthusiastic about the asset as new traders begin paying attention and drive up the price of the asset. Therefore all traders who previously held the asset will decrease their dollar holdings. This is exacerbated by the fact that such traders will desire a decrease in dollar holdings, which at a higher price will cause an even larger decline in share holdings. We must be careful to account for the fact that traders are short sale constrained and therefore cannot sell off more than they already have, but we already have an expression for the aggregate dollar holdings of these traders in the first period. We only need to subtract off the same in the second period, both adjusted for prices to give share changes. Thus

$$\begin{aligned} V_{attention,\uparrow}^{Retail,Sell,priceChange} = {} & (1 - a_t) g_t P_t^{-1} A_{H,t} \sigma_{H,t}^{-2} (1 - \Phi(h_t)) \left[\bar{\beta}_{0,t} P_t + \xi_t + \sigma_{\epsilon,t} \lambda(h_t)\right] - \\ & (1 - a_t) g_t P_{t+1}^{-1} A_{H,t} \sigma_{H,t}^{-2} (1 - \Phi(-h_{t+1})) \left[\bar{\beta}_{0,t} P_{t+1} + \xi_t + \sigma_{\epsilon,t} \lambda(h_{t+1})\right] \end{aligned}$$

where we know that $P_{t+1} \geq P_t$. Summing these two terms together we get

$$\begin{aligned} V_{attention,\uparrow}^{Retail,Sell} = {} & g_t P_t^{-1} A_{H,t} \sigma_{H,t}^{-2} (1 - \Phi(h_t)) \left[\bar{\beta}_{0,t} P_t + \xi_t + \sigma_{\epsilon,t} \lambda(h_t)\right] - \\ & (1 - a_t) g_t P_{t+1}^{-1} A_{H,t} \sigma_{H,t}^{-2} (1 - \Phi(h_{t+1})) \left[\bar{\beta}_{0,t} P_{t+1} + \xi_t + \sigma_{\epsilon,t} \lambda(h_{t+1})\right]. \end{aligned}$$

A.3.2 Proof of Proposition 1 (a)

When computing the changes in volume with respect to attention, sentiment, and sentiment dispersion g_{t+1}, ξ_{t+1}, and $\sigma_{\epsilon,t+1}$, we will repeatedly use the previous derivations of $\frac{\partial P_t}{\partial g_t}$, $\frac{\partial P_t}{\partial \xi_t}$, and $\frac{\partial P_t}{\partial \sigma_{\epsilon,t}}$ via the implicit function theorem from the last subsection. The formula for total

derivative of volume is always

$$\frac{dV_{t+1}}{dx_{t+1}} = \frac{\partial V_{t+1}}{\partial x_{t+1}} + \frac{\partial V_{t+1}}{\partial P_{t+1}} \frac{dP_{t+1}}{dx_{t+1}}$$

for our various $x_t = g_{t+1}, \xi_{,t+1}, \sigma_{\epsilon,t+1}$.

First computing $\frac{\partial V_{attention,\uparrow}^{Retail,Buy}}{\partial P_{t+1}}$ we obtain

$$\begin{aligned}
\frac{\partial V_{attention,\uparrow}^{Retail,Buy}}{\partial P_{t+1}} = &- \left(\Delta g_{t+1} + a_t g_t\right) P_{t+1}^{-2} A_{H,t+1} \sigma_{H,t+1}^{-2} \left(1 - \Phi\left(h_{t+1}\right)\right) \left[\bar{\beta}_{0,t} P_{t+1} + \xi_{t+1} + \sigma_{\epsilon,t+1} \lambda\left(h_{t+1}\right)\right] \\
&+ \left(\Delta g_{t+1} + a_t g_t\right) P_{t+1}^{-1} A_{H,t+1} \sigma_{H,t+1}^{-2} \left(\phi\left(h_{t+1}\right) \left(\frac{\bar{\beta}_{0,t}}{\sigma_{\epsilon,t+1}}\right)\right) \left[\bar{\beta}_{0,t} P_{t+1} + \xi_{t+1} + \sigma_{\epsilon,t+1} \lambda\left(h_{t+1}\right)\right] \\
&\left(\Delta g_{t+1} + a_t g_t\right) P_{t+1}^{-1} A_{H,t+1} \sigma_{H,t+1}^{-2} \left(1 - \Phi\left(h_{t+1}\right)\right) \left[\bar{\beta}_{0,t} + \sigma_{\epsilon,t+1} \lambda'\left(h_{t+1}\right) \left(-\frac{\bar{\beta}_{0,t}}{\sigma_{\epsilon,t+1}}\right)\right]
\end{aligned}$$

which can be further reduced to

$$\begin{aligned}
\frac{\partial V_{attention,\uparrow}^{Retail,Buy}}{\partial P_{t+1}} &= \left(\Delta g_{t+1} + a_t g_t\right) P_{t+1}^{-1} A_{H,t+1} \sigma_{H,t+1}^{-2} \left\{-P_{t+1}^{-1} \left(1 - \Phi\left(h_{t+1}\right)\right) \left[\bar{\beta}_{0,t} P_{t+1} + \xi_t + \sigma_{\epsilon,t} \lambda\left(h_{t+1}\right)\right]\right. \\
&\left. + \left(\phi\left(h_{t+1}\right) \left(\frac{\bar{\beta}_{0,t}}{\sigma_{\epsilon,t+1}}\right)\right) \left[\bar{\beta}_{0,t} P_{t+1} + \xi_t + \sigma_{\epsilon,t} \lambda\left(h_{t+1}\right)\right] + \left(1 - \Phi\left(h_{t+1}\right)\right) \left[\bar{\beta}_{0,t} + \sigma_{\epsilon,t} \lambda'\left(h_{t+1}\right) \left(-\frac{\bar{\beta}_{0,t}}{\sigma_{\epsilon,t+1}}\right)\right]\right\} \\
&= \left(\Delta g_{t+1} + a_t g_t\right) P_{t+1}^{-2} A_{H,t+1} \sigma_{H,t+1}^{-2} \left(1 - \Phi\left(h_{t+1}\right)\right) \left\{-\xi_t - \sigma_{\epsilon,t} \lambda\left(h_{t+1}\right)\right\} \\
&= \left(\Delta g_{t+1} + a_t g_t\right) P_{t+1}^{-1} A_{H,t+1} \sigma_{H,t+1}^{-2} \left(1 - \Phi\left(h_{t+1}\right)\right) \left\{\bar{\beta}_{0,t} - \sigma_{\epsilon,t+1} P_{t+1}^{-1} \frac{\lambda'\left(h_t\right)}{\lambda\left(h_t\right)}\right\} < 0
\end{aligned}$$

where the last inequality follows from the fact that all of the first terms are positive while the $\frac{\lambda'}{\lambda} > 0$ and $\bar{\beta}_{0,t} < 0$.

Now we compute the derivative $\frac{\partial V_{attention,\uparrow}^{Retail,Buy}}{\partial g_{t+1}}$ as

$$\frac{\partial V_{attention,\uparrow}^{Retail,Buy}}{\partial g_{t+1}} = P_{t+1}^{-1} A_{H,t+1} \sigma_{H,t+1}^{-2} \left(1 - \Phi\left(h_{t+1}\right)\right) \sigma_{\epsilon,t+1} \left[\lambda\left(h_{t+1}\right) - h_{t+1}\right]$$

with no other terms because g_{t+1} only appears in the first term. The total derivative is then, letting $\kappa_{t+1} = P_{t+1}^{-1} A_{H,t+1} \sigma_{H,t+1}^{-2} \left(1 - \Phi\left(h_{t+1}\right)\right)$

$$\begin{aligned}
\frac{dV_{attention,\uparrow}^{Retail,Buy}}{dg_{t+1}} = &P_{t+1}^{-1} A_{H,t+1} \sigma_{H,t+1}^{-2} \left(1 - \Phi\left(h_{t+1}\right)\right) \sigma_{\epsilon,t+1} \left[\lambda\left(h_{t+1}\right) - h_{t+1}\right] \\
&+ \left(\Delta g_{t+1} + a_t g_t\right) P_{t+1}^{-1} A_{H,t+1} \sigma_{H,t+1}^{-2} \left(1 - \Phi\left(h_{t+1}\right)\right) \times \\
&\left\{\bar{\beta}_{0,t} - \sigma_{\epsilon,t+1} P_{t+1}^{-1} \frac{\lambda'\left(h_t\right)}{\lambda\left(h_t\right)}\right\} \vartheta_{t+1} \left[A_{H,t} \sigma_{H,t}^{-2} \sigma_{\epsilon,t} \left(1 - \Phi\left(h_t\right)\right) \frac{\lambda'\left(h_t\right)}{\lambda\left(h_t\right)}\right]
\end{aligned}$$

$$\frac{dV_{attention,\uparrow}^{Retail,Buy}}{dg_{t+1}} = V_{attention,\uparrow}^{Retail,Buy}\left(\frac{1}{(\Delta g_{t+1}+a_t g_t)}\right.$$
$$\left.-P_{t+1}^{-1}A_{H,t+1}\sigma_{H,t+1}^{-2}\left(1-\Phi\left(h_{t+1}\right)\right)\sigma_{\epsilon,t+1}\left(\lambda\left(h_{t+1}\right)+\sigma_{\epsilon,t+1}^{-1}\xi_{t+1}\right)\vartheta_{t+1}\right)$$
$$=\left(\frac{1}{(\Delta g_{t+1}+a_t g_t)}-\frac{1}{g_{t+1}}\left(1-P_{t+1}^{-1}\vartheta_{t+1}\omega_{t+1}\right)\right)V_{attention,\uparrow}^{Retail,Buy}$$
$$=\left(\frac{1}{g_{t+1}-(1-a_t)g_t}-\frac{1}{g_{t+1}}\left(1-P_{t+1}^{-1}\vartheta_{t+1}\omega_{t+1}\right)\right)V_{attention,\uparrow}^{Retail,Buy}$$

where the inequality $\frac{dV_{attention,\uparrow}^{Retail,Buy}}{dg_{t+1}} > 0$ arises whenever institutional demand is sufficiently non-negative.

This gives us that the elasticity of retail buy volume with respect to attention is

$$\frac{\partial \log V_{attention,\uparrow}^{Retail,Buy}}{\partial \log g_{t+1}} = \frac{g_{t+1}}{(\Delta g_{t+1}+a_t g_t)} - g_{t+1}P_{t+1}^{-1}A_{H,t+1}\sigma_{H,t+1}^{-2}\left(1-\Phi\left(h_{t+1}\right)\right)\sigma_{\epsilon,t+1}\left(\lambda\left(h_{t+1}\right)+\sigma_{\epsilon,t+1}^{-1}\xi_{t+1}\right)\vartheta_{t+1}.$$

We now look at the same analysis but with retail sales.

$$V_{attention,\uparrow}^{Retail,Sell} = g_t P_t^{-1}A_{H,t}\sigma_{H,t}^{-2}\left(1-\Phi\left(-\frac{\bar{\beta}_{0,t}P_t+\xi_t}{\sigma_{\epsilon,t}}\right)\right)\left[\lambda\left(h_t\right)-h_t\right]\sigma_{\epsilon,t}$$
$$-\left(1-a_t\right)g_t P_{t+1}^{-1}A_{H,t}\sigma_{H,t}^{-2}\left(1-\Phi\left(h_{t+1}\right)\right)\left[\lambda\left(h_{t+1}\right)-h_{t+1}\right]\sigma_{\epsilon,t}.$$

Now we compute the direct effect derivative $\frac{\partial V_{attention,\uparrow}^{Retail,Sell}}{\partial g_{t+1}}$ which is computed trivially as

$$\frac{\partial V_{attention,\uparrow}^{Retail,Sell}}{\partial g_{t+1}} = 0$$

since altering g_{t+1} brings in new traders but not directly cause sales. Instead, it triggers sales through its effect on price, leaving us with $\frac{\partial V_{attention,\uparrow}^{Retail,Sell}}{\partial P_{t+1}}\frac{\partial P_{t+1}}{\partial g_{t+1}}$ where we get using $h_{t+1} = -\frac{\bar{\beta}_{0,t}P_{t+1}+\xi_{t+1}}{\sigma_{\epsilon,t+1}}$

$$\frac{\partial V_{attention,\uparrow}^{Retail,Sell}}{\partial P_{t+1}} = \left(1-a_t\right)g_t P_{t+1}^{-2}A_{H,t}\sigma_{H,t}^{-2}\left(1-\Phi\left(h_{t+1}\right)\right)\left[\lambda\left(h_{t+1}\right)-h_{t+1}\right]\sigma_{\epsilon,t}$$
$$-\bar{\beta}_{0,t}\left(1-a_t\right)g_t P_{t+1}^{-1}A_{H,t}\sigma_{H,t}^{-2}\left(1-\Phi\left(h_{t+1}\right)\right)\lambda'\left(h_{t+1}\right)$$
$$-\bar{\beta}_{0,t}\left(1-a_t\right)g_t P_{t+1}^{-1}A_{H,t}\sigma_{H,t}^{-2}\left(1-\Phi\left(h_{t+1}\right)\right)\left[\left(-\lambda'\left(h_{t+1}\right)+1\right)\right]$$
$$=\left(1-a_t\right)g_t P_{t+1}^{-2}A_{H,t}\sigma_{H,t}^{-2}\left(1-\Phi\left(h_{t+1}\right)\right)\left[\lambda\left(h_{t+1}\right)-h_{t+1}\right]\sigma_{\epsilon,t}$$
$$-\bar{\beta}_{0,t}\left(1-a_t\right)g_t P_{t+1}^{-1}A_{H,t}\sigma_{H,t}^{-2}\left(1-\Phi\left(h_{t+1}\right)\right)$$

$$= (1-a_t)\, g_t P_{t+1}^{-1} A_{H,t} \sigma_{H,t}^{-2} \left(1-\Phi\left(h_{t+1}\right)\right) \left\{ P_{t+1}^{-1} \left[\lambda\left(h_{t+1}\right) - h_{t+1}\right] \sigma_{\epsilon,t} - \bar{\beta}_{0,t} \right\}$$
$$= (1-a_t)\, g_t P_{t+1}^{-1} A_{H,t} \sigma_{H,t}^{-2} \left(1-\Phi\left(h_{t+1}\right)\right) P_{t+1}^{-1} \left\{ \lambda\left(h_{t+1}\right) \sigma_{\epsilon,t} + \xi_{t+1} \right\}$$

Now multiplying gives us, using $\frac{\partial P_{t+1}}{\partial g_{t+1}} = \vartheta_{t+1} \left[A_{H,t} \sigma_{H,t}^{-2} \sigma_{\epsilon,t} \left(1-\Phi\left(h_{t+1}\right)\right) \frac{\lambda'(h_{t+1})}{\lambda(h_{t+1})} \right]$

$$\frac{dV_{attention,\uparrow}^{Retail,Buy}}{dg_{t+1}} = V_{attention,\uparrow}^{Retail,Buy} \left(\frac{1}{\left(\Delta g_{t+1} + a_t g_t\right)} \right.$$
$$\left. - P_{t+1}^{-1} A_{H,t+1} \sigma_{H,t+1}^{-2} \left(1-\Phi\left(h_{t+1}\right)\right) \sigma_{\epsilon,t+1} \left(\lambda\left(h_{t+1}\right) + \sigma_{\epsilon,t+1}^{-1} \xi_{t+1} \right) \vartheta_{t+1} \right)$$

$$\frac{dV_{attention,\uparrow}^{Retail,Sell}}{dg_{t+1}} = (1-a_t)\, g_t P_{t+1}^{-1} A_{H,t} \sigma_{H,t}^{-2} \left(1-\Phi\left(h_{t+1}\right)\right) P_{t+1}^{-1}$$
$$\times \left\{ \lambda\left(h_{t+1}\right) \sigma_{\epsilon,t} + \xi_{t+1} \right\} \vartheta_{t+1} A_{H,t} \sigma_{H,t}^{-2} \sigma_{\epsilon,t} \left(1-\Phi\left(h_{t+1}\right)\right) \frac{\lambda'\left(h_t\right)}{\lambda\left(h_t\right)}$$

We thus have

$$\frac{dV_{attention,\uparrow}^{Retail,Buy}}{dg_{t+1}} \Big/ \frac{dV_{attention,\uparrow}^{Retail,Sell}}{dg_{t+1}}$$
$$= \frac{\partial V_{attention,\uparrow}^{Retail,Buy}}{\partial g_{t+1}} \Big/ \left(\frac{\partial V_{attention,\uparrow}^{Retail,Sell}}{\partial P_{t+1}} \frac{\partial P_{t+1}}{\partial g_{t+1}} \right) + \left(\frac{\partial V_{attention,\uparrow}^{Retail,Buy}}{\partial P_{t+1}} \frac{\partial P_{t+1}}{\partial g_{t+1}} \right) \Big/ \left(\frac{\partial V_{attention,\uparrow}^{Retail,Sell}}{\partial P_{t+1}} \frac{\partial P_{t+1}}{\partial g_{t+1}} \right)$$
$$= \frac{1}{(1-a_t)\, g_t P_{t+1}^{-1} \left\{ \lambda\left(h_{t+1}\right) \sigma_{\epsilon,t} + \xi_{t+1} \right\} \vartheta_{t+1} A_{H,t} \sigma_{H,t}^{-2} \left(1-\Phi\left(h_t\right)\right)} + \frac{\left(-g_{t+1} + (1-a_t)\, g_t\right)}{(1-a_t)\, g_t}$$
$$= 1 + \frac{1}{(1-a_t)\, g_t P_{t+1}^{-1} \left\{ \lambda\left(h_{t+1}\right) \sigma_{\epsilon,t} + \xi_{t+1} \right\} \vartheta_{t+1} A_{H,t} \sigma_{H,t}^{-2} \left(1-\Phi\left(h_{t+1}\right)\right)} + \frac{-g_{t+1}}{(1-a_t)\, g_t}$$
$$= 1 - \frac{g_{t+1}}{(1-a_t)\, g_t} + \frac{-\left(\sum_i A_{i,t+1} \sigma_{t+1}^{-2} \hat{\beta}_{0,i,t+1} - 1\right) - A_{H,t+1} \sigma_{H,t+1}^{-2} g_{t+1} \bar{\beta}_{0,t+1} \left(1-\Phi\left(h_{t+1}\right)\right)}{(1-a_t)\, g_t P_{t+1}^{-1} \left\{ \lambda\left(h_{t+1}\right) \sigma_{\epsilon,t} + \xi_{t+1} \right\} A_{H,t} \sigma_{H,t}^{-2} \left(1-\Phi\left(h_{t+1}\right)\right)}$$
$$= 1 - \frac{g_{t+1}}{(1-a_t)\, g_t} - \frac{\sum_i A_{i,t+1} \sigma_{t+1}^{-2} \hat{\beta}_{0,i,t+1} - 1}{(1-a_t)\, g_t P_{t+1}^{-1} \left\{ \lambda\left(h_{t+1}\right) \sigma_{\epsilon,t} + \xi_{t+1} \right\} A_{H,t} \sigma_{H,t}^{-2} \left(1-\Phi\left(h_{t+1}\right)\right)}$$
$$+ \frac{-g_{t+1} \bar{\beta}_{0,t+1}}{(1-a_t)\, g_t P_{t+1}^{-1} \left\{ \lambda\left(h_{t+1}\right) \sigma_{\epsilon,t} + \xi_{t+1} \right\}}$$
$$= 1 - \frac{g_{t+1}}{(1-a_t)\, g_t} + \frac{P_{t+1}^{-1} \sum_i A_{i,t} \sigma_t^{-2} \epsilon_{i,t} + P_{t+1}^{-1} A_{H,t} \sigma_{H,t}^{-2} g_{t+1} \sigma_{\epsilon,t+1} \left(1-\Phi\left(h_{t+1}\right)\right) \left[\lambda\left(h_{t+1}\right) - h_{t+1}\right]}{(1-a_t)\, g_t P_{t+1}^{-1} \left\{ \sigma_{\epsilon,t} \left(\lambda\left(h_{t+1}\right) - h_{t+1}\right) - \bar{\beta}_{0,t} P_{t+1} \right\} A_{H,t} \sigma_{H,t}^{-2} \left(1-\Phi\left(h_{t+1}\right)\right)}$$
$$+ \frac{-g_{t+1} \bar{\beta}_{0,t+1}}{(1-a_t)\, g_t P_{t+1}^{-1} \left\{ \lambda\left(h_{t+1}\right) \sigma_{\epsilon,t} + \xi_{t+1} \right\}}$$
$$= 1 - \frac{g_{t+1}}{(1-a_t)\, g_t} + \frac{P_{t+1}^{-1} \sum_i A_{i,t} \sigma_t^{-2} \epsilon_{i,t} + P_{t+1}^{-1} A_{H,t} \sigma_{H,t}^{-2} g_{t+1} \sigma_{\epsilon,t+1} \left(1-\Phi\left(h_{t+1}\right)\right) \left[\lambda\left(h_{t+1}\right) - h_{t+1}\right]}{(1-a_t)\, g_t P_{t+1}^{-1} A_{H,t} \sigma_{H,t}^{-2} \left(1-\Phi\left(h_{t+1}\right)\right) \left\{ \sigma_{\epsilon,t} \left(\lambda\left(h_{t+1}\right) - h_{t+1}\right) - \bar{\beta}_{0,t} P_{t+1} \right\}}$$
$$+ \frac{-g_{t+1} \bar{\beta}_{0,t+1}}{(1-a_t)\, g_t P_{t+1}^{-1} \left\{ \lambda\left(h_{t+1}\right) \sigma_{\epsilon,t} + \xi_{t+1} \right\}}$$

$$= 1 - \frac{g_{t+1}}{(1-a_t)\, g_t} + \frac{\sum_i A_{i,t}\sigma_t^{-2}\epsilon_{i,t}}{(1-a_t)\, g_t A_{H,t}\sigma_{H,t}^{-2}\left(1-\Phi\left(h_{t+1}\right)\right)\left\{\xi_{t+1}+\sigma_{\epsilon,t+1}\lambda\left(h_{t+1}\right)\right\}}$$
$$+\frac{g_{t+1}P_{t+1}^{-1}\sigma_{\epsilon,t+1}\left[\lambda\left(h_{t+1}\right)-h_{t+1}\right]-g_{t+1}\bar{\beta}_{0,t+1}}{(1-a_t)\, g_t P_{t+1}^{-1}\left\{\xi_{t+1}+\sigma_{\epsilon,t+1}\lambda\left(h_{t+1}\right)\right\}}$$
$$= 1 - \frac{g_{t+1}}{(1-a_t)\, g_t} + \frac{\sum_i A_{i,t}\sigma_t^{-2}\epsilon_{i,t}}{(1-a_t)\, g_t A_{H,t}\sigma_{H,t}^{-2}\left(1-\Phi\left(h_{t+1}\right)\right)\left\{\xi_{t+1}+\sigma_{\epsilon,t+1}\lambda\left(h_{t+1}\right)\right\}}$$
$$+\frac{g_{t+1}P_{t+1}^{-1}\sigma_{\epsilon,t+1}\left[\lambda\left(h_{t+1}\right)-h_{t+1}\right]+g_{t+1}P_{t+1}^{-1}\sigma_{\epsilon,t+1}\left(h_{t+1}+\sigma_{\epsilon,t+1}^{-1}\xi_{t+1}\right)}{(1-a_t)\, g_t P_{t+1}^{-1}\left\{\xi_{t+1}+\sigma_{\epsilon,t+1}\lambda\left(h_{t+1}\right)\right\}}$$
$$= 1 - \frac{g_{t+1}}{(1-a_t)\, g_t} + \frac{\sum_i A_{i,t}\sigma_t^{-2}\epsilon_{i,t}}{(1-a_t)\, g_t A_{H,t}\sigma_{H,t}^{-2}\left(1-\Phi\left(h_{t+1}\right)\right)\left\{\xi_{t+1}+\sigma_{\epsilon,t+1}\lambda\left(h_{t+1}\right)\right\}} + \frac{g_{t+1}}{(1-a_t)\, g_t}$$
$$= 1 + \frac{\sum_i A_{i,t}\sigma_t^{-2}\epsilon_{i,t}}{(1-a_t)\, g_t A_{H,t}\sigma_{H,t}^{-2}\left(1-\Phi\left(h_{t+1}\right)\right)\left\{\xi_{t+1}+\sigma_{\epsilon,t+1}\lambda\left(h_{t+1}\right)\right\}}.$$

This gives us that

$$\kappa_t = \frac{dV_{attention,\uparrow}^{Retail,Buy}}{dg_{t+1}} \Big/ \frac{dV_{attention,\uparrow}^{Retail,Sell}}{dg_{t+1}} = 1 + \frac{\omega_t}{(1-a_t)\,\eta_t\left\{\xi_t+\sigma_{\epsilon,t}\lambda\left(h_t\right)\right\}}.$$

A.3.3 Proof of Proposition 1 (b)

This follows directly from the expression in 1, plugging in $\omega_t = 0$ to obtain $\kappa_t = 1$.

A.3.4 Proof of Proposition 1 (c)

Let us take the derivative of this ratio with respect to ω_t. First, we can compute the derivative of P_t with respect to ω_t via the implicit function theorem:

$$\frac{\partial P_t}{\partial \omega_t} = \theta_t.$$

Where we used the expression

$$P_t = \sum_i A_{i,t}\sigma_t^{-2}\left(\hat{\beta}_{0,i,t}P_t+\epsilon_{i,t}\right)+A_{H,t}\sigma_{H,t}^{-2}g_t\left(1-\Phi\left(-\frac{\bar{\beta}_{0,t}P_t+\xi_t}{\sigma_{\epsilon,t}}\right)\right)\left[\bar{\beta}_{0,t}P_t+\xi_t+\sigma_{\epsilon,t}\frac{\phi\left(\frac{-\bar{\beta}_{0,t}P_t-\xi_t}{\sigma_{\epsilon,t}}\right)}{1-\Phi\left(\frac{-\bar{\beta}_{0,t}P_t-\xi_t}{\sigma_{\epsilon,t}}\right)}\right]$$

Next we see that

$$\frac{d\kappa_t}{d\omega_t} = \left(\left(1-a_t\right)\eta_t\left\{\xi_t+\sigma_{\epsilon,t}\lambda\left(h_t\right)\right\}\right)^{-1}$$
$$-\omega_t\left(\left(1-a_t\right)\eta_t\left\{\xi_t+\sigma_{\epsilon,t}\lambda\left(h_t\right)\right\}\right)^{-2}\left\{\left(1-a_t\right)A_{H,t}\sigma_{H,t}^{-2}g_t\left\{\left(-\sigma_{\epsilon,t}^{-1}\bar{\beta}_{0,t}P_t\bar{\beta}_{0,t}\phi\left(h_t\right)\right)\right\}\right\}\theta_t$$
$$= \left(\left(1-a_t\right)\eta_t\left\{\xi_t+\sigma_{\epsilon,t}\lambda\left(h_t\right)\right\}\right)^{-1}$$

$$+\theta_t\omega_t\left(\left(1-a_t\right)\eta_t\left\{\xi_t+\sigma_{\epsilon,t}\lambda\left(h_t\right)\right\}\right)^{-2}\left\{\left(1-a_t\right)A_{H,t}\sigma_{H,t}^{-2}g_t\left\{\left(\sigma_{\epsilon,t}^{-1}\bar{\beta}_{0,t}P_t\bar{\beta}_{0,t}\phi\left(h_t\right)\right)\right\}\right\}$$
$$=\left(\left(1-a_t\right)\eta_t\left\{\xi_t+\sigma_{\epsilon,t}\lambda\left(h_t\right)\right\}\right)^{-1}\left[1+\sigma_{\epsilon,t}^{-1}\frac{\bar{\beta}_{0,t}^2}{\alpha_t-\beta_t\eta_t}\omega_t\left(\frac{P_t}{\xi_t/\lambda\left(h_t\right)+\sigma_{\epsilon,t}}\right)\right]>0$$

provided that $\xi_t+\sigma_{\epsilon,t}\lambda\left(h_t\right)>0$.

A.3.5 Proof of 1 (d)

Similar computation shows

$$\frac{d\kappa_t}{d\xi_t}=\frac{-\omega_t}{\left(1-a_t\right)A_{H,t}\sigma_{H,t}^{-2}g_t}\left\{\left(1-\Phi\left(h_t\right)\right)\xi_t+\sigma_{\epsilon,t}\phi\left(h_t\right)\right\}^{-2}\left\{1-\Phi\left(h_t\right)-\sigma_{\epsilon,t}^{-1}\bar{\beta}_{0,t}P_t\phi\left(h_t\right)\right\}<0.$$

A.4 Omitted Proofs for Subsection 4.5

A.4.1 Proof of Proposition 2 (a)

Consider a shift in sentiment $\xi_{t+1}>\xi_t$ with all other values left alone. As before, this will trigger an increase in price $P_{t+1}>P_t$ that decreases demand among some traders. Unlike with a shift in sentiment dispersion, however, this will trigger a universal shift up in return expectations because every agent's return expectation decreases by $\bar{\beta}_{0,t}\Delta P_{t+1}$ since $\bar{\beta}_{0,t}<0$ and increases by $\xi_{t+1}-\xi_t>|\bar{\beta}_{0,t}\Delta P_{t+1}|$. Three types of purchases will now occur: (1) attritional purchases, (2) new purchases by existing holders, and (3) new purchases by agents who are were previously attentive but without sufficiently high sentiment to make a purchase. When we look at a change in share demands for a given agent h who did not previously purchase the asset, we see that an upward shift occurs provided that $\bar{\beta}_{0,t}P_t+\xi_t+\sigma_{\epsilon,t}\epsilon_{h,t}<0$ and $\bar{\beta}_{0,t+1}P_{t+1}+\xi_{t+1}+\sigma_{\epsilon,t}\epsilon_{h,t}>0$ (new purchases), in which case $\epsilon_{h,t}<-\sigma_{\epsilon,t}^{-1}\left(\bar{\beta}_{0,t}P_t+\xi_t\right)$ and $\epsilon_{h,t}>-\sigma_{\epsilon,t}^{-1}\left(\bar{\beta}_{0,t+1}P_{t+1}+\xi_{t+1}\right)$ while since $\bar{\beta}_{0,t+1}P_{t+1}+\xi_{t+1}>\bar{\beta}_{0,t}P_t+\xi_t$ such agents always exist. Thus we have some measure of new traders. Meanwhile, for an existing attentive agent to increase holdings we require that

$$P_t^{-1}\left(\bar{\beta}_{0,t}P_t+\xi_t+\sigma_{\epsilon,t}\epsilon_{h,t}\right)<P_{t+1}^{-1}\left(\bar{\beta}_{0,t}P_{t+1}+\xi_{t+1}+\sigma_{\epsilon,t+1}\epsilon_{h,t+1}\right)$$

which holds true when

$$\epsilon_{h,t}<\sigma_{\epsilon,t}^{-1}\frac{P_{t+1}^{-1}\xi_{t+1}-P_t^{-1}\xi_t}{\left(P_t^{-1}-P_{t+1}^{-1}\right)}=\sigma_{\epsilon,t}^{-1}\frac{P_t\xi_{t+1}-P_{t+1}\xi_t}{P_{t+1}-P_t}$$
$$\sigma_{\epsilon,t}^{-1}\frac{P_{t+1}^{-1}\left(\xi_{t+1}-\xi_t+\xi_t\right)-P_t^{-1}\xi_t}{\left(P_t^{-1}-P_{t+1}^{-1}\right)}=-\sigma_{\epsilon,t}^{-1}\xi_t+\sigma_{\epsilon,t}^{-1}\frac{P_{t+1}^{-1}\left(\xi_{t+1}-\xi_t\right)}{\left(P_t^{-1}-P_{t+1}^{-1}\right)}$$

$$= -\sigma_{\epsilon,t}^{-1}\xi_t + \sigma_{\epsilon,t}^{-1}\frac{(\xi_{t+1}-\xi_t)}{(P_{t+1}/P_t-1)} = \sigma_{\epsilon,t}^{-1}\left(-\xi_t + \frac{(\xi_{t+1}-\xi_t)}{(P_{t+1}/P_t-1)}\right).$$
$$= \sigma_{\epsilon,t}^{-1}\left(-\xi_t + P_t\frac{\xi_{t+1}-\xi_t}{P_{t+1}-P_t}\right) = h_t^S.$$

If we take the limit as $\xi_{t+1} \downarrow \xi_t$ then we find the limit

$$\lim_{\xi_{t+1}\downarrow\xi_t} h_t^S = \lim_{k\downarrow 0}\sigma_{\epsilon,t}^{-1}\left(-\xi_t + P_t\frac{\xi_t+k-\xi_t}{P_t+k\left(\theta_t\eta_t\right)-P_t}\right) = \sigma_{\epsilon,t}^{-1}\left(-\xi_t + \frac{P_t}{\theta_t\eta_t}\right)$$
$$= \sigma_{\epsilon,t}^{-1}\left(-\xi_t - P_t\bar{\beta}_{0,t} + P_t\eta_t^{-1}\alpha_t\right) = h_t + \sigma_{\epsilon,t}^{-1}\alpha_t\eta_t^{-1}P_t = h_t^{LS}.$$

The attritional agents will demand

$$V_{sentiment,\uparrow}^{Retail,Buy,attrition} = a_t g_t P_{t+1}^{-1}A_{H,t+1}\sigma_{H,t+1}^{-2}\left(1-\Phi\left(h_{t+1}\right)\right)\left[\bar{\beta}_{0,t}P_{t+1} + \xi_{t+1} + \sigma_{\epsilon,t+1}\lambda\left(h_{t+1}\right)\right]$$

similar to previous expressions.

Agents $h \in \left[-\sigma_{\epsilon,t}^{-1}\left(\bar{\beta}_{0,t}P_t+\xi_t\right), \sigma_{\epsilon,t}^{-1}\left(-\xi_t + \frac{(\xi_{t+1}-\xi_t)}{(P_{t+1}/P_t-1)}\right)\right] = \left[h_t, h_t^S\right]$ will increase their holdings and generate volumes of

$$V_{sentiment,\uparrow}^{Retail,Buy,intensive} = \left(1-a_t\right)g_t P_{t+1}^{-1}A_{H,t}\sigma_{H,t}^{-2}\left(\Phi\left(h_t^S\right)-\Phi\left(h_t\right)\right)\left[\bar{\beta}_{0,t}P_{t+1}+\xi_{t+1}+\sigma_{\epsilon,t}\frac{\phi\left(h_t\right)-\phi\left(h_t^S\right)}{\Phi\left(h_t^S\right)-\Phi\left(h_t\right)}\right]$$
$$-\left(1-a_t\right)g_t P_t^{-1}A_{H,t}\sigma_{H,t}^{-2}\left(\Phi\left(h_t^S\right)-\Phi\left(h_t\right)\right)\left[\bar{\beta}_{0,t}P_t+\xi_t+\sigma_{\epsilon,t}\frac{\phi\left(h_t\right)-\phi\left(h_t^S\right)}{\Phi\left(h_t^S\right)-\Phi\left(h_t\right)}\right],$$

or alternatively,

$$V_{sentiment,\uparrow}^{Retail,Buy,intensive} = \left(1-a_t\right)g_t P_{t+1}^{-1}A_{H,t}\sigma_{H,t}^{-2}$$
$$\times\left[\left(\Phi\left(h_t^S\right)-\Phi\left(h_t\right)\right)\left(\xi_{t+1}P_{t+1}^{-1}-\xi_t P_t^{-1}\right)+\sigma_{\epsilon,t}\left(\phi\left(h_t\right)-\phi\left(h_t^S\right)\right)\left(P_{t+1}^{-1}-P_t^{-1}\right)\right].$$

Meanwhile those who did not previously hold a position, $h \in \left[-\sigma_{\epsilon,t}^{-1}\left(\bar{\beta}_{0,t}P_{t+1}+\xi_{t+1}\right), -\sigma_{\epsilon,t}^{-1}\left(\bar{\beta}_{0,t}P_t+\xi_t\right)\right] = \left[h_{t+1}, h_t\right]$, but now do generate volumes totaling the amount of their new holdings, namely

$$V_{sentiment,\uparrow}^{Retail,Buy,extensive} = \left(1-a_t\right)g_t P_{t+1}^{-1}A_{H,t}\sigma_{H,t}^{-2}$$
$$\times\left[\left(\bar{\beta}_{0,t}P_{t+1}+\xi_{t+1}\right)\left(\Phi\left(h_t\right)-\Phi\left(h_{t+1}\right)\right)+\sigma_{\epsilon,t}\left(\phi\left(h_{t+1}\right)-\phi\left(h_t\right)\right)\right].$$

Total sentiment-based buying volume is then

$$V_{sentiment,\uparrow}^{Retail,Buy} = \left(1-a_t\right)g_t A_{H,t}\sigma_{H,t}^{-2}\left[\left(\Phi\left(h_U^s\right)-\Phi\left(h_t\right)\right)\left(\xi_{t+1}P_{t+1}^{-1}-\xi_t P_t^{-1}\right)+\sigma_{\epsilon,t}\left(\phi\left(h_t\right)-\phi\left(h_U^s\right)\right)\left(P_{t+1}^{-1}-P_t^{-1}\right)\right]$$

$$+(1-a_t)\,g_t A_{H,t}\sigma_{H,t}^{-2}\left[\left(\bar{\beta}_{0,t}+P_{t+1}^{-1}\xi_{t+1}\right)\left(\Phi\left(h_t\right)-\Phi\left(h_{t+1}\right)\right)+P_{t+1}^{-1}\sigma_{\epsilon,t}\left(\phi\left(h_{t+1}\right)-\phi\left(h_t\right)\right)\right]$$
$$+a_t g_t A_{H,t+1}\sigma_{H,t+1}^{-2}\left(1-\Phi\left(h_{t+1}\right)\right)\left[\bar{\beta}_{0,t}+P_{t+1}^{-1}\xi_{t+1}+P_{t+1}^{-1}\sigma_{\epsilon,t+1}\lambda\left(h_{t+1}\right)\right].$$

Computing the partial with respect to sentiment ξ_{t+1} (using $\frac{\partial h_{t+1}}{\partial \xi_{t+1}}=-\sigma_{\epsilon,t+1}^{-1}$,$h_t^S=\sigma_{\epsilon,t}^{-1}\frac{P_t\xi_{t+1}-P_{t+1}\xi_t}{P_{t+1}-P_t}=\sigma_{\epsilon,t}^{-1}\left(-\xi_t+\frac{(\xi_{t+1}-\xi_t)}{(P_{t+1}/P_t-1)}\right)=\sigma_{\epsilon,t}^{-1}\left(-\xi_t+\frac{(\xi_{t+1}-\xi_t)P_t}{P_{t+1}-P_t}\right)$ $\frac{\partial h_U^S}{\partial \xi_{t+1}}=\sigma_{\epsilon,t}^{-1}\frac{1}{(P_{t+1}/P_t-1)}=\sigma_{\epsilon,t}^{-1}\upsilon_{t+1}$ and $\frac{\partial h_U^S}{\partial P_{t+1}}=-\sigma_{\epsilon,t}^{-1}\left(\xi_{t+1}-\xi_t\right)P_t\left(P_{t+1}-P_t\right)^{-2}$ as well as $\xi_{t+1}P_{t+1}^{-1}-\xi_t P_t^{-1}=\frac{\xi_{t+1}P_t-\xi_t P_{t+1}}{P_{t+1}P_t}=P_{t+1}^{-1}P_t^{-1}\left(\xi_{t+1}P_t-\xi_t P_{t+1}\right)$ and $P_{t+1}^{-1}-P_t^{-1}=\frac{P_t-P_{t+1}}{P_{t+1}P_t}=P_{t+1}^{-1}P_t^{-1}\left(P_t-P_{t+1}\right)$ and $\upsilon_{t+1}=P_t\left(P_{t+1}-P_t\right)^{-1}$, plus $\frac{\partial h_{t+1}}{\partial P_{t+1}}=-\sigma_{\epsilon,t+1}^{-1}\bar{\beta}_{0,t}$) gives us

$$\begin{aligned}\frac{\partial V_{sentiment,\uparrow}^{Retail,Buy}}{\partial \xi_{t+1}}&=(1-a_t)\,g_t A_{H,t}\sigma_{H,t}^{-2}\left[P_{t+1}^{-1}\left(\Phi\left(h_U^s\right)-\Phi\left(h_t\right)\right)\right]\\&+(1-a_t)\,g_t A_{H,t}\sigma_{H,t}^{-2}\left[P_{t+1}^{-1}\left(\Phi\left(h_t\right)-\Phi\left(h_{t+1}\right)\right)\right]\\&+a_t g_t A_{H,t+1}\sigma_{H,t+1}^{-2}\left(1-\Phi\left(h_{t+1}\right)\right)P_{t+1}^{-1}\\&=(1-a_t)\,g_t A_{H,t}\sigma_{H,t}^{-2}\left[P_{t+1}^{-1}\left(\Phi\left(h_U^s\right)-\Phi\left(h_{t+1}\right)\right)\right]\\&+a_t g_t A_{H,t+1}\sigma_{H,t+1}^{-2}\left(1-\Phi\left(h_{t+1}\right)\right)P_{t+1}^{-1}\end{aligned}$$

while

$$\begin{aligned}\frac{\partial V_{sentiment,\uparrow}^{Retail,Buy}}{\partial P_{t+1}}&=(1-a_t)\,g_t A_{H,t}\sigma_{H,t}^{-2}P_{t+1}^{-2}\left[-\xi_{t+1}\left(\Phi\left(h_t^S\right)-\Phi\left(h_{t+1}\right)\right)-\sigma_{\epsilon,t}\left(\phi\left(h_t\right)-\phi\left(h_t^S\right)\right)\right]\\&+a_t g_t A_{H,t+1}\sigma_{H,t+1}^{-2}\left(1-\Phi\left(h_{t+1}\right)\right)P_{t+1}^{-2}\left[-\xi_{t+1}-\sigma_{\epsilon,t+1}\lambda\left(h_{t+1}\right)\right]\end{aligned}$$

Plugging in these as well as $\frac{\partial P_t}{\partial \xi_t}=\theta_t\eta_t$ into

$$\frac{dV_{sentiment,\uparrow}^{Retail,Buy}{}_{t+1}}{d\xi_{t+1}}=\frac{\partial V_{sentiment,\uparrow}^{Retail,Buy}{}_{t+1}}{\partial \xi_{t+1}}+\frac{\partial V_{sentiment,\uparrow}^{Retail,Buy}{}_{t+1}}{\partial P_{t+1}}\frac{dP_{t+1}}{d\xi_{t+1}}$$

gives us, taking the limit as $\xi_{t+1}\downarrow\xi_t$,

$$\begin{aligned}\frac{dV_{sentiment,\uparrow}^{Retail,Buy}{}_{t+1}}{d\xi_{t+1}}&=(1-a_t)\,g_t A_{H,t}\sigma_{H,t}^{-2}\left[P_{t+1}^{-1}\left(\Phi\left(h_t^S\right)-\Phi\left(h_{t+1}\right)\right)\right]+a_t g_t A_{H,t+1}\sigma_{H,t+1}^{-2}\left(1-\Phi\left(h_{t+1}\right)\right)P_{t+1}^{-1}\\&+(1-a_t)\,g_t A_{H,t}\sigma_{H,t}^{-2}P_{t+1}^{-2}\left[-\theta_t\eta_t\xi_{t+1}\left(\Phi\left(h_t^S\right)-\Phi\left(h_{t+1}\right)\right)-\theta_t\eta_t\sigma_{\epsilon,t}\left(\phi\left(h_t\right)-\phi\left(h_t^S\right)\right)\right]\\&+a_t g_t A_{H,t+1}\sigma_{H,t+1}^{-2}\left(1-\Phi\left(h_{t+1}\right)\right)P_{t+1}^{-2}\left[-\theta_t\eta_t\xi_{t+1}-\theta_t\eta_t\sigma_{\epsilon,t+1}\lambda\left(h_{t+1}\right)\right]\\&=a_t\eta_t P_t^{-2}\vartheta_t\omega_t\\&+(1-a_t)\,g_t A_{H,t}\sigma_{H,t}^{-2}P_{t+1}^{-2}\left(\left(\vartheta_t\omega_t+\vartheta_t\eta_t\sigma_{\epsilon,t}\lambda\left(h_t\right)\right)\left(\Phi\left(h_t^S\right)-\Phi\left(h_t\right)\right)-\theta_t\eta_t\sigma_{\epsilon,t}\left(\phi\left(h_t\right)-\phi\left(h_t^S\right)\right)\right)\end{aligned}$$

Meanwhile, summing together the attritional sales and the sentiment-based sales gives

us

$$V^{Retail,Sell}_{sentiment,\uparrow} = a_t P_t^{-1}\eta_t\left[\bar{\beta}_{0,t}P_t + \xi_t + \sigma_{\epsilon,t}\lambda\left(h_t\right)\right]$$
$$+\left(1-a_t\right)g_t A_{H,t+1}\sigma^{-2}_{H,t+1}\left(1-\Phi\left(h^s_U\right)\right)\left[\left(P_t^{-1}\xi_t - P_{t+1}^{-1}\xi_{t+1}\right) + \sigma_{\epsilon,t+1}\left(P_t^{-1}-P_{t+1}^{-1}\right)\lambda\left(h^s_U\right)\right].$$

Differentiating, we find

$$\frac{\partial V^{Retail,Sell}_{sentiment,\uparrow}}{\partial \xi_{t+1}} = -\left(1-a_t\right)g_t A_{H,t+1}\sigma^{-2}_{H,t+1}\left[\left(-\sigma^{-1}_{\epsilon,t}\upsilon_{t+1}\phi\left(h_t^S\right)\right)\left(P^{-1}_{t+1}\xi_{t+1}-P_t^{-1}\xi_t\right)+\left(1-\Phi\left(h_t^S\right)\right)\left(P^{-1}_{t+1}\right)\right]$$
$$-\left(1-a_t\right)g_t A_{H,t+1}\sigma^{-2}_{H,t+1}\left[-h_t^S\sigma_{\epsilon,t+1}\left(P^{-1}_{t+1}-P_t^{-1}\right)\sigma^{-1}_{\epsilon,t}\upsilon_{t+1}\phi\left(h_t^S\right)\right]$$
$$= -\left(1-a_t\right)g_t A_{H,t+1}\sigma^{-2}_{H,t+1}P^{-1}_{t+1}\left(1-\Phi\left(h_t^S\right)\right)$$

Turning to the second term, we must compute $\frac{\partial V^{Retail,Sell}_{sentiment,\uparrow^{t+1}}}{\partial P_{t+1}}$ since we have already computed $\frac{dP_{t+1}}{d\xi_{t+1}}$. $\frac{\partial h^S_U}{\partial P_{t+1}} = -\sigma^{-1}_{\epsilon,t}\left(\xi_{t+1}-\xi_t\right)P_t\left(P_{t+1}-P_t\right)^{-2})$

$$\frac{\partial V^{Retail,Sell}_{sentiment,\uparrow^{t+1}}}{\partial P_{t+1}} = -\left(1-a_t\right)g_t A_{H,t+1}\sigma^{-2}_{H,t+1}$$
$$\times\left[\left(\sigma^{-1}_{\epsilon,t}\left(\xi_{t+1}-\xi_t\right)P_t\left(P_{t+1}-P_t\right)^{-2}\phi\left(h_t^S\right)\right)\left(P^{-1}_{t+1}\xi_{t+1}-P_t^{-1}\xi_t\right)-\left(1-\Phi\left(h_t^S\right)\right)\left(P^{-2}_{t+1}\xi_{t+1}\right)\right]$$
$$-\left(1-a_t\right)g_t A_{H,t+1}\sigma^{-2}_{H,t+1}$$
$$\times\left[\sigma_{\epsilon,t+1}\left(-P^{-2}_{t+1}\right)\phi\left(h_t^S\right)+h_t^S\sigma^{-1}_{\epsilon,t}\left(\xi_{t+1}-\xi_t\right)P_t\left(P_{t+1}-P_t\right)^{-2}\sigma_{\epsilon,t+1}\left(P^{-1}_{t+1}-P_t^{-1}\right)\phi\left(h_t^S\right)\right]$$

$$\frac{\partial V^{Retail,Sell}_{sentiment,\uparrow^{t+1}}}{\partial P_{t+1}} = \left(1-a_t\right)g_t A_{H,t+1}\sigma^{-2}_{H,t+1}P^{-2}_{t+1}\left(1-\Phi\left(h_t^S\right)\right)\left[\sigma_{\epsilon,t+1}\lambda\left(h_t^S\right)+\xi_{t+1}\right]$$

And we also recall $\frac{\partial P_t}{\partial \xi_t} = \theta_t\eta_t$ to give us, evaluating at $\xi_{t+1}=\xi_t$,

$$\frac{dV^{Retail,Sell}_{sentiment,\uparrow^{t+1}}}{d\xi_{t+1}} = \left(1-a_t\right)g_t A_{H,t}\sigma^{-2}_{H,t}P_t^{-2}\left(1-\Phi\left(h_t^S\right)\right)\left(-\vartheta_t\omega_t+\vartheta_t\eta_t\sigma_{\epsilon,t}\left(\lambda\left(h_t^S\right)-\lambda\left(h_t\right)\right)\right)$$

We can now evaluate

$$\frac{dV^{Retail,Buy}_{sentiment,\uparrow^{t+1}}}{d\xi_{t+1}} \Big/ \frac{dV^{Retail,Sell}_{sentiment,\uparrow}}{d\xi_{t+1}} =$$
$$\frac{-\left(1-\Phi\left(h_t^S\right)\right)\left(1-\Phi\left(h_t\right)\right)\omega_t+\eta_t\sigma_{\epsilon,t}\phi\left(h_t^S\right)-\eta_t\sigma_{\epsilon,t}\phi\left(h_t^S\right)\Phi\left(h_t\right)-\eta_t\sigma_{\epsilon,t}\phi\left(h_t\right)+\eta_t\sigma_{\epsilon,t}\Phi\left(h_t^S\right)\phi\left(h_t\right)+\frac{1}{1-a_t}\left(1-\Phi\left(h_t\right)\right)^2\omega_t}{-\left(1-\Phi\left(h_t^S\right)\right)\left(1-\Phi\left(h_t\right)\right)\omega_t+\eta_t\sigma_{\epsilon,t}\phi\left(h_t^S\right)-\eta_t\sigma_{\epsilon,t}\phi\left(h_t^S\right)\Phi\left(h_t\right)-\eta_t\sigma_{\epsilon,t}\phi\left(h_t\right)+\eta_t\sigma_{\epsilon,t}\Phi\left(h_t^S\right)\phi\left(h_t\right)}$$
$$1+\left(\frac{1}{1-a_t}\right)\frac{1-\Phi\left(h_t\right)}{1-\Phi\left(h_t^S\right)}\frac{\omega_t}{\eta_t\sigma_{\epsilon,t}\left(\lambda\left(h_t^S\right)-\lambda\left(h_t\right)\right)-\omega_t}$$

A.4.2 Proof of Proposition 2 (b)

Same as proof of Proposition 1 (b).

A.4.3 Proof of Proposition 2 (c)

Suppose $\eta_t\sigma_{\epsilon,t}\left(\lambda\left(h_t^S\right)-\lambda\left(h_t\right)\right)-\omega_t>0$ so that both ratios are positive.

We wish to compare $\frac{dV^{Retail,Buy}_{sentiment,\uparrow^{t+1}}}{d\xi_{t+1}}/\frac{dV^{Retail,Sell}_{sentiment,\uparrow^{t+1}}}{d\xi_{t+1}}=1+\left(\frac{1}{1-a_t}\right)\frac{1-\Phi(h_t)}{1-\Phi\left(h_t^S\right)}\frac{\omega_t}{\eta_t\sigma_{\epsilon,t}\left(\lambda\left(h_t^S\right)-\lambda(h_t)\right)-\omega_t}$ and $\frac{dV^{Retail,Buy}_{attention,\uparrow}}{dg_{t+1}}/\frac{dV^{Retail,Sell}_{attention,\uparrow}}{dg_{t+1}}=1+\frac{\omega_t}{(1-a_t)\eta_t\{\xi_t+\sigma_{\epsilon,t}\lambda(h_t)\}}$, which we will do by comparing the second part of each term. Dividing through, we find

$$\begin{aligned}&\frac{\left(\frac{1}{1-a_t}\right)\frac{1-\Phi(h_t)}{1-\Phi\left(h_t^S\right)}\frac{\omega_t}{\eta_t\sigma_{\epsilon,t}\left(\lambda\left(h_t^S\right)-\lambda(h_t)\right)-\omega_t}}{\frac{\omega_t}{(1-a_t)\eta_t\{\xi_t+\sigma_{\epsilon,t}\lambda(h_t)\}}}\\&=\frac{1-\Phi\left(h_t\right)}{1-\Phi\left(h_t^S\right)}\frac{\eta_t\xi_t+\eta_t\sigma_{\epsilon,t}\lambda\left(h_t\right)}{\eta_t\sigma_{\epsilon,t}\left(\lambda\left(h_t^S\right)-\lambda\left(h_t\right)\right)-\omega_t}\end{aligned}$$

where $\frac{1-\Phi(h_t)}{1-\Phi\left(h_t^S\right)}>1$ since $h_t^S>h_t$. We know that λ is convex and $0<\lambda'<1$ by properties of the inverse Mills ratio, which means that $\lambda\left(h_t^S\right)-\lambda\left(h_t\right)<h_t^S-h_t=\sigma_{\epsilon,t}^{-1}\alpha_t\eta_t^{-1}P_t$ so then

$$\eta_t\sigma_{\epsilon,t}\left(\lambda\left(h_t^S\right)-\lambda\left(h_t\right)\right)-\omega_t<\alpha_t\left(\alpha_t^{-1}\omega_t+\alpha_t^{-1}\eta_t\sigma_{\epsilon,t}\left(\lambda\left(h_t\right)-h_t\right)\right)-\omega_t$$

$$\eta_t\sigma_{\epsilon,t}\left(\lambda\left(h_t^S\right)-\lambda\left(h_t\right)\right)-\omega_t<\eta_t\sigma_{\epsilon,t}\left(\lambda\left(h_t\right)-h_t\right)=\eta_t\sigma_{\epsilon,t}\lambda\left(h_t\right)+\eta_t\bar{\beta}_{0,t}P_t+\eta_t\xi_t$$

but $\eta_t\bar{\beta}_{0,t}P_t<0$ so

$$\eta_t\sigma_{\epsilon,t}\left(\lambda\left(h_t^S\right)-\lambda\left(h_t\right)\right)-\omega_t<\eta_t\xi_t+\eta_t\sigma_{\epsilon,t}\lambda\left(h_t\right)$$

and where we have used the equilibrium condition

$$P_t=\alpha_t^{-1}\omega_t+\alpha_t^{-1}\eta_t\sigma_{\epsilon,t}\left(\lambda\left(h_t\right)-h_t\right).$$

This completes the proof.

A.4.4 Proof of Proposition 2 (d)

This equation can be rewritten as

$$\frac{dV^{Retail,Buy}_{sentiment,\uparrow^{t+1}}}{d\xi_{t+1}}/\frac{dV^{Retail,Sell}_{sentiment,\uparrow^{t+1}}}{d\xi_{t+1}}=1+$$

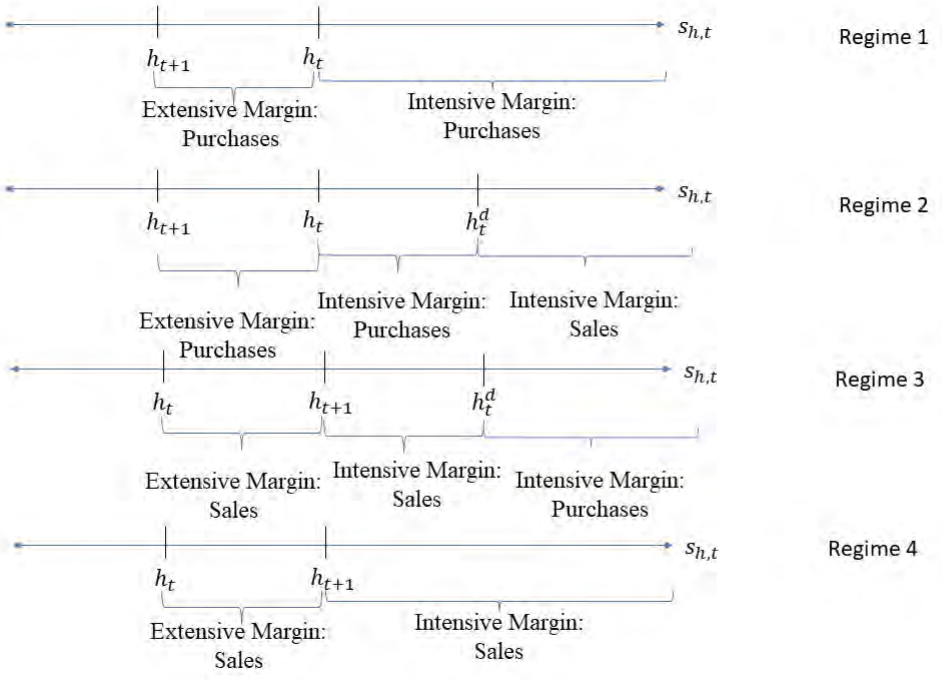


Figure 12: Intensive and Extensive Margin with Dispersion Changes

$$\left(\frac{1}{1-a_t}\right)\frac{1-\Phi\left(h_t\right)}{A_{H,t}\sigma_{H,t}^{-2}g_t\omega_t^{-1}\sigma_{\epsilon,t}\left(\left(1-\Phi\left(h_t\right)\right)\phi\left(h_t^{LS}\right)-\left(1-\Phi\left(h_t^{LS}\right)\right)\phi\left(h_t\right)\right)-\left(1-\Phi\left(h_t\right)\right)}$$

where $1/\sqrt{2\pi} \geq \left(\left(1-\Phi\left(h_t\right)\right)\phi\left(h_t^{LS}\right)-\left(1-\Phi\left(h_t^{LS}\right)\right)\phi\left(h_t\right)\right) > 0$. This directly shows either $\frac{dV_{sentiment,\uparrow}^{Retail,Buy\,t+1}}{d\xi_{t+1}}/\frac{dV_{sentiment,\uparrow}^{Retail,Sell\,t+1}}{d\xi_{t+1}} < 0$ or $\frac{dV_{sentiment,\uparrow}^{Retail,Buy\,t+1}}{d\xi_{t+1}}/\frac{dV_{sentiment,\uparrow}^{Retail,Sell\,t+1}}{d\xi_{t+1}} > 1$.

A.5 Omitted Proofs for Subsection 4.6 and Expanded Version of Proposition 3

Proposition 5. *(Buy Sell Imbalances under Dispersion Changes)*

(a) [Ratio of Volume Changes, Regime 1] Suppose $\bar{\beta}_{0,t}\omega_t + \xi_t\alpha_t < 0$ and $\omega_t + \eta_t\xi_t > 0$. When dispersion increases from $\sigma_{s,t}$ to $\sigma_{s,t+1}$, retail buying increases while no additional selling takes place, creating imbalances.

(b) [Ratio of Volume Changes, Regime 2] Suppose $\bar{\beta}_{0,t}\omega_t + \xi_t\alpha_t < 0$ and $\omega_t + \eta_t\xi_t < 0$. When dispersion increases from $\sigma_{s,t}$ to $\sigma_{s,t+1}$, the ratio of the rate of change of purchases and sales evaluated at $\sigma_{s,t} = \sigma_{s,t+1}$ is given by

$$\frac{dV_{dispersion,\uparrow}^{Retail,Buy\,t+1}}{d\sigma_{s,t+1}}/\frac{dV_{dispersion,\uparrow}^{Retail,Sell\,t+1}}{d\sigma_{s,t+1}} = 1 + \frac{a_t\omega_t}{\left(1-a_t\right)g_tA_{H,t}\sigma_{H,t}^{-2}\left(1-\Phi\left(h_t^D\right)\right)\left[\xi_t + \sigma_{s,t}\lambda\left(h_t^D\right)\right]} \quad (30)$$

$$+\frac{\eta_t^{-1}\omega_t\left(\phi\left(h_t\right)/\phi\left(h_t^D\right)\right)-\left(\eta_t^{-1}\omega_t+\sigma_{s,t}\lambda\left(h_t\right)+\xi_t\right)}{\xi_t\left(\lambda\left(h_t\right)/\lambda\left(h_t^D\right)\right)+\sigma_{s,t}\lambda\left(h_t\right)}$$

(c) [Ratio of Volume Changes, Regime 3] Suppose $\bar{\beta}_{0,t}\omega_t+\xi_t\alpha_t>0$ *and* $\omega_t+\eta_t\xi_t>0$. *When dispersion increases from* $\sigma_{s,t}$ *to* $\sigma_{s,t+1}$ *, the ratio of the rate of change of purchases and sales evaluated at*$\sigma_{s,t}=\sigma_{s,t+1}$ *is given by*

$$\frac{dV_{dispersion,\uparrow}^{Retail,Buy}{}^{t+1}}{d\sigma_{s,t+1}}/\frac{dV_{dispersion,\uparrow}^{Retail,Sell}{}^{t+1}}{d\sigma_{s,t+1}}=1+\frac{\omega_t\phi\left(h_t\right)+\frac{a_t}{1-a_t}\left(1-\Phi\left(h_t\right)\right)P_t\lambda\left(h_t\right)\left(\alpha_t+\xi_t\eta_t\right)}{\omega_t\phi\left(h_t^{LD}\right)-\omega_t\phi\left(h_t\right)+\left(1-\Phi\left(h_t^{LD}\right)\right)\eta_t\xi_t\left(\lambda\left(h_t^{LD}\right)-\lambda\left(h_t\right)\right)} \tag{31}$$

(d) [Ratio of Volume Changes, Regime 4] Suppose $\bar{\beta}_{0,t}\omega_t+\xi_t\alpha_t>0$ *and* $\omega_t+\eta_t\xi_t<0$. *When dispersion increases from* $\sigma_{s,t}$ *to* $\sigma_{s,t+1}$*, no retail purchases occur outside of attritional trading, while retail sales increase, thus creating an imbalance.*

(1) Consider first the case where $\bar{\beta}_{0,t}\omega_t+\xi_t\alpha_t<0$ but $\omega_t+\eta_t\xi_t>0$. This gives us $\frac{\partial h_t}{\partial\sigma_{\epsilon,t}}<0$ and therefore purchases are made by all traders such that $\epsilon_{h,t}>h_{t+1}$ and $\epsilon_{h,t}>h_t^D$. We have that $h_{t+1}<h_t$ and so traders between h_{t+1} and h_t are the extensive margin. Therefore $h_t^D<h_{t+1}$ or else we would have a contradiction. Shares are purchased for the first time by agents in the interval (h_{t+1},h_t) and additional shares are purchased by all traders in the interval $(h_t,+\infty)$. On the other hand, no traders other than the attritional traders sell off their shares because the decrease in return expectations caused by the price increase P_{t+1}/P_t and downward sloping demand is overcome by the increase in expectations of $\epsilon_{h,t}\left(\sigma_{e,t+1}-\sigma_{e,t+1}\right)$ for all traders holding positions. In other words, under these conditions all holders of the risky asset are extreme optimists and belief intensification triggers a wave of purchasing by the optimists while bringing some moderate optimists into the fold.

(2) Suppose $\bar{\beta}_{0,t}\omega_t+\xi_t\alpha_t<0$ and $\omega_t+\eta_t\xi_t<0$. Shares are purchased for the first time by agents in the interval (h_{t+1},h_t) (extensive margin) and additional shares are purchased by all traders in the interval $\left(h_t,h_t^D\right)$. All traders with sentiment greater than h_t^D are sellers. This gives us

$$\begin{aligned}V_{dispersion,\uparrow}^{Retail,Buy}&=a_tP_{t+1}^{-1}A_{H,t}\sigma_{H,t}^{-2}g_t\left(1-\Phi\left(h_{t+1}\right)\right)\left[\bar{\beta}_{0,t}P_{t+1}+\xi_t+\sigma_{\epsilon,t+1}\lambda\left(h_{t+1}\right)\right]\\&+\left(1-a_t\right)g_tP_{t+1}^{-1}A_{H,t}\sigma_{H,t}^{-2}\left[\left(\Phi\left(h_t\right)-\Phi\left(h_{t+1}\right)\right)\left(\bar{\beta}_{0,t}P_{t+1}+\xi_t\right)+\sigma_{\epsilon,t+1}\left(\phi\left(h_{t+1}\right)-\phi\left(h_t\right)\right)\right]\end{aligned} \tag{32}$$

$$+\left(1-a_t\right)g_tA_{H,t}\sigma_{H,t}^{-2}\left\{\left(\Phi\left(h_t^D\right)-\Phi\left(h_t\right)\right)\left\{P_{t+1}^{-1}\xi_{t+1}-P_t^{-1}\xi_t\right\}+\left(P_{t+1}^{-1}\sigma_{\epsilon,t+1}-P_t^{-1}\sigma_{\epsilon,t}\right)\left(\phi\left(h_t\right)-\phi\left(h_t^D\right)\right)\right\}.$$

and

$$V_{dispersion,\uparrow}^{Retail,Sell} = -\left(1-a_t\right) g_t A_{H,t} \sigma_{H,t}^{-2} \left(1-\Phi\left(h_t^D\right)\right) \left[\left(P_{t+1}^{-1}-P_t^{-1}\right)\xi_t + \left(P_{t+1}^{-1}\sigma_{\epsilon,t+1} - P_t^{-1}\sigma_{\epsilon,t}\right)\lambda\left(h_t^D\right)\right] + a_t g_t P_t^{-1} A_{H,t}\sigma_{H,t}^{-2}\left(1-\Phi\left(h_t\right)\right)\left[\bar{\beta}_{0,t}P_t + \xi_t + \sigma_{\epsilon,t}\lambda\left(h_t\right)\right]. \tag{33}$$

The purchases by those purchasing for the first time are given by

$$\begin{aligned} V_{dispersion,\uparrow}^{Retail,Buy,extensive} &= \left(1-a_t\right) g_t P_{t+1}^{-1} A_{H,t}\sigma_{H,t}^{-2}\left(\Phi\left(h_t\right)-\Phi\left(h_{t+1}\right)\right)\left[\bar{\beta}_{0,t}P_{t+1} + \xi_t + \sigma_{\epsilon,t+1}\frac{\phi\left(h_{t+1}\right)-\phi\left(h_t\right)}{\Phi\left(h_t\right)-\Phi\left(h_{t+1}\right)}\right] \\ &= \left(1-a_t\right) g_t P_{t+1}^{-1} A_{H,t}\sigma_{H,t}^{-2}\left[\left(\Phi\left(h_t\right)-\Phi\left(h_{t+1}\right)\right)\left(\bar{\beta}_{0,t}P_{t+1}+\xi_t\right) + \sigma_{\epsilon,t+1}\left(\phi\left(h_{t+1}\right)-\phi\left(h_t\right)\right)\right] \end{aligned}$$

while those purchasing additional shares are given by

$$\begin{aligned} V_{dispersion,\uparrow}^{Retail,Sell,intensive} &= \left(1-a_t\right) g_t A_{H,t+1}\sigma_{H,t+1}^{-2} \\ &\times\left[\left(\Phi\left(h_t^D\right)-\Phi\left(h_t\right)\right)\left(\bar{\beta}_{0,t}+P_{t+1}^{-1}\xi_{t+1}\right) + P_{t+1}^{-1}\sigma_{\epsilon,t+1}\left(\phi\left(h_t\right)-\phi\left(h_t^D\right)\right)\right] \\ &-\left(1-a_t\right) g_t A_{H,t}\sigma_{H,t}^{-2} \\ &\times\left[\left(\Phi\left(h_t^D\right)-\Phi\left(h_t\right)\right)\left(\bar{\beta}_{0,t}+P_t^{-1}\xi_t\right) + P_t^{-1}\sigma_{\epsilon,t}\left(\phi\left(h_t\right)-\phi\left(h_t^D\right)\right)\right]. \\ &= \left(1-a_t\right) g_t A_{H,t}\sigma_{H,t}^{-2} \\ &\times\left\{\left(\Phi\left(h_t^D\right)-\Phi\left(h_t\right)\right)\left\{P_{t+1}^{-1}\xi_{t+1}-P_t^{-1}\xi_t\right\} + \left(P_{t+1}^{-1}\sigma_{\epsilon,t+1}-P_t^{-1}\sigma_{\epsilon,t}\right)\left(\phi\left(h_t\right)-\phi\left(h_t^D\right)\right)\right\} \end{aligned}$$

Meanwhile, attritional traders will purchase

$$V_{dispersion,\uparrow}^{Retail,Buy,attrition} = a_t P_{t+1}^{-1} A_{H,t}\sigma_{H,t}^{-2} g_t\left(1-\Phi\left(h_{t+1}\right)\right)\left[\bar{\beta}_{0,t}P_{t+1}+\xi_t+\sigma_{\epsilon,t+1}\lambda\left(h_{t+1}\right)\right].$$

This leads to total purchase volume of

$$\begin{aligned} V_{dispersion,\uparrow}^{Retail,Buy} &= a_t P_{t+1}^{-1} A_{H,t}\sigma_{H,t}^{-2} g_t\left(1-\Phi\left(h_{t+1}\right)\right)\left[\bar{\beta}_{0,t}P_{t+1}+\xi_t+\sigma_{\epsilon,t+1}\lambda\left(h_{t+1}\right)\right] \\ &+\left(1-a_t\right) g_t P_{t+1}^{-1} A_{H,t}\sigma_{H,t}^{-2}\left[\left(\Phi\left(h_t\right)-\Phi\left(h_{t+1}\right)\right)\left(\bar{\beta}_{0,t}P_{t+1}+\xi_t\right) + \sigma_{\epsilon,t+1}\left(\phi\left(h_{t+1}\right)-\phi\left(h_t\right)\right)\right] \\ +\left(1-a_t\right) g_t A_{H,t}\sigma_{H,t}^{-2}&\left\{\left(\Phi\left(h_t^D\right)-\Phi\left(h_t\right)\right)\left\{P_{t+1}^{-1}\xi_{t+1}-P_t^{-1}\xi_t\right\} + \left(P_{t+1}^{-1}\sigma_{\epsilon,t+1}-P_t^{-1}\sigma_{\epsilon,t}\right)\left(\phi\left(h_t\right)-\phi\left(h_t^D\right)\right)\right\}. \end{aligned}$$

with partial derivatives given by

$$\begin{aligned} \frac{\partial V_{dispersion,\uparrow}^{Retail,Buy}}{\partial\sigma_{\epsilon,t+1}} &= a_t g_t A_{H,t}\sigma_{H,t}^{-2}\left(1-\Phi\left(h_{t+1}\right)\right)P_{t+1}^{-1}\lambda\left(h_{t+1}\right) \\ &+\left(1-a_t\right) g_t A_{H,t}\sigma_{H,t}^{-2}\left\{P_{t+1}^{-1}\left(\phi\left(h_t\right)-\phi\left(h_t^D\right)\right)\right\} \end{aligned}$$

and

$$\frac{\partial V_{dispersion,\uparrow}^{Retail,Buy\,t+1}}{\partial P_{t+1}} = -a_t P_{t+1}^{-2} A_{H,t} \sigma_{H,t}^{-2} g_t \left(1 - \Phi\left(h_{t+1}\right)\right) \left[\xi_t + \sigma_{\epsilon,t+1} \lambda\left(h_{t+1}\right)\right]$$
$$- \left(1 - a_t\right) g_t P_{t+1}^{-2} A_{H,t} \sigma_{H,t}^{-2} \left[\left(\Phi\left(h_t\right) - \Phi\left(h_{t+1}\right)\right) \xi_t + \sigma_{\epsilon,t+1} \left(\phi\left(h_{t+1}\right) - \phi\left(h_t\right)\right)\right]$$
$$+ \left(1 - a_t\right) g_t P_{t+1}^{-2} A_{H,t} \sigma_{H,t}^{-2} \left\{- \left(\Phi\left(h_t^D\right) - \Phi\left(h_t\right)\right) \xi_t - \sigma_{\epsilon,t+1} \left(\phi\left(h_t\right) - \phi\left(h_t^D\right)\right)\right\}$$

while we have already established that $\frac{dP_{t+1}}{d\sigma_{\epsilon,t+1}} = \vartheta_t \eta_t \lambda\left(h_{t+1}\right)$. The total derivative evaluated at $\sigma_{\epsilon,t+1} = \sigma_{\epsilon,t}$ is then

$$\frac{dV_{dispersion,\uparrow}^{Retail,Buy\,t+1}}{d\sigma_{\epsilon,t+1}} = a_t \eta_t P_{t+1}^{-1} \lambda\left(h_t\right) + \left(1 - a_t\right) g_t A_{H,t} \sigma_{H,t}^{-2} P_{t+1}^{-1} \left\{\left(\phi\left(h_t\right) - \phi\left(h_t^D\right)\right)\right\}$$
$$- a_t P_{t+1}^{-2} \eta_t \left[\xi_t + \sigma_{\epsilon,t+1} \lambda\left(h_{t+1}\right)\right] \vartheta_t \eta_t \lambda\left(h_t\right)$$
$$+ \left(1 - a_t\right) g_t P_{t+1}^{-2} A_{H,t} \sigma_{H,t}^{-2} \left\{- \left(\Phi\left(h_t^D\right) - \Phi\left(h_t\right)\right) \xi_t - \sigma_{\epsilon,t} \left(\phi\left(h_t\right) - \phi\left(h_t^D\right)\right)\right\} \vartheta_t \eta_t \lambda\left(h_t\right)$$

$$\frac{dV_{dispersion,\uparrow}^{Retail,Buy\,t+1}}{d\sigma_{\epsilon,t+1}} = a_t \eta_t P_{t+1}^{-1} \lambda\left(h_t\right) + \left(1 - a_t\right) g_t A_{H,t} \sigma_{H,t}^{-2} P_{t+1}^{-1} \left\{\left(\phi\left(h_t\right) - \phi\left(h_t^D\right)\right)\right\}$$
$$- a_t P_{t+1}^{-2} \eta_t \left[\xi_t + \sigma_{\epsilon,t+1} \lambda\left(h_{t+1}\right)\right] \vartheta_t \eta_t \lambda\left(h_t\right)$$
$$+ \left(1 - a_t\right) g_t P_{t+1}^{-2} A_{H,t} \sigma_{H,t}^{-2} \left\{- \left(\Phi\left(h_t^D\right) - \Phi\left(h_t\right)\right) \xi_t - \sigma_{\epsilon,t} \left(\phi\left(h_t\right) - \phi\left(h_t^D\right)\right)\right\} \vartheta_t \eta_t \lambda\left(h_t\right)$$

which can be simplified to

$$\frac{dV_{dispersion,\uparrow}^{Retail,Buy\,t+1}}{d\sigma_{\epsilon,t+1}} = P_{t+1}^{-2} a_t \eta_t \vartheta_t \omega_t \lambda\left(h_t\right)$$
$$+ \left(1 - a_t\right) g_t A_{H,t} \sigma_{H,t}^{-2} P_{t+1}^{-2} \vartheta_t \left\{\left(\omega_t + \eta_t \xi_t\right) \left\{\left(\phi\left(h_t\right) - \phi\left(h_t^D\right)\right)\right\} - \eta_t \lambda\left(h_t\right) \xi_t \left(\Phi\left(h_t^D\right) - \Phi\left(h_t\right)\right)\right\}$$

Attritional sales are given by

$$V_{dispersion,\uparrow}^{Retail,Sell,attrition} = a_t g_t P_t^{-1} A_{H,t} \sigma_{H,t}^{-2} \left(1 - \Phi\left(h_t\right)\right) \left[\bar{\beta}_{0,t} P_t + \xi_t + \sigma_{\epsilon,t} \lambda\left(h_t\right)\right].$$

The intensive margin of sales is given by

$$V_{dispersion,\uparrow}^{Retail,Sell,intensive} = - \left(1 - a_t\right) g_t P_{t+1}^{-1} A_{H,t+1} \sigma_{H,t+1}^{-2} \left(1 - \Phi\left(h_t^D\right)\right) \left[\bar{\beta}_{0,t} P_{t+1} + \xi_{t+1} + \sigma_{\epsilon,t+1} \lambda\left(h_t^D\right)\right]$$
$$+ \left(1 - a_t\right) g_t P_t^{-1} A_{H,t} \sigma_{H,t}^{-2} \left(1 - \Phi\left(h_t^D\right)\right) \left[\bar{\beta}_{0,t} P_t + \xi_t + \sigma_{\epsilon,t} \lambda\left(h_t^D\right)\right]$$

$$= -\left(1-a_t\right) g_t A_{H,t} \sigma_{H,t}^{-2} \left(1-\Phi\left(h_t^D\right)\right)\left[\left(P_{t+1}^{-1}-P_t^{-1}\right)\xi_t + \left(P_{t+1}^{-1}\sigma_{\epsilon,t+1}-P_t^{-1}\sigma_{\epsilon,t}\right)\lambda\left(h_t^D\right)\right]$$

with total volumes given by

$$\begin{aligned} V_{dispersion,\uparrow}^{Retail,Sell} = &-\left(1-a_t\right) g_t A_{H,t} \sigma_{H,t}^{-2} \left(1-\Phi\left(h_t^D\right)\right)\left[\left(P_{t+1}^{-1}-P_t^{-1}\right)\xi_t + \left(P_{t+1}^{-1}\sigma_{\epsilon,t+1}-P_t^{-1}\sigma_{\epsilon,t}\right)\lambda\left(h_t^D\right)\right] \\ &+a_t g_t P_t^{-1} A_{H,t}\sigma_{H,t}^{-2}\left(1-\Phi\left(h_t\right)\right)\left[\bar{\beta}_{0,t}P_t + \xi_t + \sigma_{\epsilon,t}\lambda\left(h_t\right)\right] \end{aligned}$$

$$\begin{aligned} \frac{\partial V_{dispersion,\uparrow}^{Retail,Sell}}{\partial \sigma_{\epsilon,t+1}} = &-\left(1-a_t\right) g_t A_{H,t}\sigma_{H,t}^{-2}\left[\left(-\frac{\partial h_t^D}{\partial \sigma_{\epsilon,t+1}}\phi\left(h_t^D\right)\right)\frac{P_t\xi_t - P_{t+1}\xi_t}{P_{t+1}P_t} + \frac{\xi_t\left(P_t-P_{t+1}\right)}{P_{t+1}P_t}\frac{\partial h_t^D}{\partial \sigma_{\epsilon,t+1}}\phi\left(h_t^D\right)\right] \\ &= 0 \end{aligned}$$

Meanwhile

$$\begin{aligned} \frac{\partial V_{dispersion,\uparrow}^{Retail,Sell}}{\partial P_{t+1}} = &-\left(1-a_t\right) g_t A_{H,t}\sigma_{H,t}^{-2}\left[\left(-\frac{\partial h_t^D}{\partial P_{t+1}}\phi\left(h_t^D\right)\right)\left(P_{t+1}^{-1}-P_t^{-1}\right)\xi_t - \left(1-\Phi\left(h_t^D\right)\right)\left(P_{t+1}^{-2}\right)\xi_t\right. \\ &\left.-P_{t+1}^{-2}\sigma_{\epsilon,t+1}\phi\left(h_t^D\right) - h_t^D\left(P_{t+1}^{-1}\sigma_{\epsilon,t+1}-P_t^{-1}\sigma_{\epsilon,t}\right)\frac{\partial h_t^D}{\partial P_{t+1}}\phi\left(h_t^D\right)\right] \\ &= \left(1-a_t\right) g_t A_{H,t}\sigma_{H,t}^{-2}P_{t+1}^{-2}\left[\left(1-\Phi\left(h_t^D\right)\right)\xi_t + \sigma_{\epsilon,t+1}\phi\left(h_t^D\right)\right] \end{aligned}$$

so putting this together we get

$$\frac{dV_{dispersion,\uparrow}^{Retail,Sell}}{d\sigma_{\epsilon,t+1}} = \left(1-a_t\right) g_t A_{H,t}\sigma_{H,t}^{-2}P_t^{-2}\left[\left(1-\Phi\left(h_t^D\right)\right)\xi_t + \sigma_{\epsilon,t}\phi\left(h_t^D\right)\right]\vartheta_t\eta_t\lambda\left(h_t\right)$$

when evaluated at $\sigma_{\epsilon,t+1} = \sigma_{\epsilon,t}$.

Looking at the ratio, we find

$$\begin{aligned} &\frac{dV_{dispersion,\uparrow}^{Retail,Buy}}{d\sigma_{\epsilon,t+1}} \Big/ \frac{dV_{dispersion,\uparrow}^{Retail,Sell}}{d\sigma_{\epsilon,t+1}} = \\ &\frac{a_t\omega_t}{\left(1-a_t\right) g_t A_{H,t}\sigma_{H,t}^{-2}\left[\left(1-\Phi\left(h_t^D\right)\right)\xi_t + \sigma_{\epsilon,t}\phi\left(h_t^D\right)\right]} \\ &+\frac{\left\{\left(\omega_t+\eta_t\xi_t\right)\left\{\left(\phi\left(h_t\right)-\phi\left(h_t^D\right)\right)\right\} + \xi_t\eta_t\lambda\left(h_t\right)\left(1-\Phi\left(h_t^D\right)\right) - \eta_t\lambda\left(h_t\right)\xi_t\left(1-\Phi\left(h_t\right)\right)\right\}}{\left(1-\Phi\left(h_t^D\right)\right)\xi_t\eta_t\lambda\left(h_t\right) + \sigma_{\epsilon,t}\eta_t\lambda\left(h_t\right)\phi\left(h_t^D\right)} \\ &= 1 + \frac{a_t\omega_t}{\left(1-a_t\right) g_t A_{H,t}\sigma_{H,t}^{-2}\left(1-\Phi\left(h_t^D\right)\right)\left[\xi_t + \sigma_{\epsilon,t}\lambda\left(h_t^D\right)\right]} \\ &+\frac{\eta_t^{-1}\omega_t\left(\phi\left(h_t\right)/\phi\left(h_t^D\right)\right) - \left(\eta_t^{-1}\omega_t + \sigma_{\epsilon,t}\lambda\left(h_t\right) + \xi_t\right)}{\xi_t\left(\lambda\left(h_t\right)/\lambda\left(h_t^D\right)\right) + \sigma_{\epsilon,t}\lambda\left(h_t\right)} \end{aligned}$$

(3) If we suppose that $\bar{\beta}_{0,t}\omega_t + \xi_t\alpha_t > 0$ and $\omega_t + \eta_t\xi_t > 0$ then $\frac{\partial h_t}{\partial \sigma_{\epsilon,t}} > 0$ and purchases are made by all traders such that $\epsilon_{h,t} > h_t^D$. We have that $h_{t+1} > h_t$ and so there is no extensive margin. Therefore $h_t^D > h_{t+1}$ or else we would have a contradiction. Shares are not purchased for the first time by any agents and additional shares are purchased by all traders in the interval $\left(h_t^D, +\infty\right)$. Shares are sold completely off by all agents in the interval (h_t, h_{t+1}) and sold partially by agents in the interval $\left(h_{t+1}, h_t^D\right)$. In this case purchases volumes are given by

$$\begin{aligned} V_{dispersion,\uparrow}^{Retail,Buy} = (1-a_t)\, g_t A_{H,t}\sigma_{H,t}^{-2}\left(1-\Phi\left(h_t^D\right)\right)\left[\left(P_{t+1}^{-1}-P_t^{-1}\right)\xi_t + \left(P_{t+1}^{-1}\sigma_{\epsilon,t+1}-P_t^{-1}\sigma_{\epsilon,t}\right)\lambda\left(h_t^D\right)\right] \\ +a_t P_{t+1}^{-1} A_{H,t}\sigma_{H,t}^{-2} g_t\left(1-\Phi\left(h_{t+1}\right)\right)\left[\bar{\beta}_{0,t}P_{t+1}+\xi_t+\sigma_{\epsilon,t+1}\lambda\left(h_{t+1}\right)\right]. \end{aligned} \tag{34}$$

while sales are given by

$$\begin{aligned} V_{dispersion,\uparrow}^{Retail,Sell} = a_t P_t^{-1}\eta_t\left[\bar{\beta}_{0,t}P_t+\xi_t+\sigma_{\epsilon,t}\lambda\left(h_t\right)\right] \\ +(1-a_t)\, g_t P_t^{-1} A_{H,t}\sigma_{H,t}^{-2}\left[\left(\Phi\left(h_{t+1}\right)-\Phi\left(h_t\right)\right)\left(\bar{\beta}_{0,t}P_t+\xi_t\right)+\sigma_{\epsilon,t}\left(\phi\left(h_t\right)-\phi\left(h_{t+1}\right)\right)\right] \\ -(1-a_t)\, g_t A_{H,t}\sigma_{H,t}^{-2}\left\{\left(\Phi\left(h_t^D\right)-\Phi\left(h_{t+1}\right)\right)\left\{P_{t+1}^{-1}\xi_{t+1}-P_t^{-1}\xi_t\right\}+\left(P_{t+1}^{-1}\sigma_{\epsilon,t+1}-P_t^{-1}\sigma_{\epsilon,t}\right)\left(\phi\left(h_{t+1}\right)-\phi\left(h_t^D\right)\right)\right\}. \end{aligned} \tag{35}$$

Newly attentive traders (due to attention turnover) will have no previous position but now acquire a position if their sentiment exceeds $-\sigma_{\epsilon,t+1}^{-1}\left(\bar{\beta}_{0,t+1}P_{t+1}+\xi_{t+1}\right)$. Shares purchased by such traders will equal

$$V_{dispersion,\uparrow}^{Retail,Buy,attrition} = a_t P_{t+1}^{-1} A_{H,t}\sigma_{H,t}^{-2} g_t\left(1-\Phi\left(h_{t+1}\right)\right)\left[\bar{\beta}_{0,t}P_{t+1}+\xi_t+\sigma_{\epsilon,t+1}\lambda\left(h_{t+1}\right)\right]$$

just as in the case where attention increases, but with the increased standard deviation $\sigma_{\epsilon,t+1}$ taking the place of the old $\sigma_{\epsilon,t}$.

$$\begin{aligned} V_{dispersion,\uparrow}^{Retail,Buy,intensive} &= (1-a_t)\, g_t P_{t+1}^{-1} A_{H,t+1}\sigma_{H,t+1}^{-2}\left(1-\Phi\left(h_t^D\right)\right)\left[\bar{\beta}_{0,t}P_{t+1}+\xi_{t+1}+\sigma_{\epsilon,t+1}\lambda\left(h_t^D\right)\right] \\ &\quad -(1-a_t)\, g_t P_t^{-1} A_{H,t}\sigma_{H,t}^{-2}\left(1-\Phi\left(h_t^D\right)\right)\left[\bar{\beta}_{0,t}P_t+\xi_t+\sigma_{\epsilon,t}\lambda\left(h_t^D\right)\right] \\ &= (1-a_t)\, g_t A_{H,t}\sigma_{H,t}^{-2}\left(1-\Phi\left(h_t^D\right)\right)\left[\left(P_{t+1}^{-1}-P_t^{-1}\right)\xi_t+\left(P_{t+1}^{-1}\sigma_{\epsilon,t+1}-P_t^{-1}\sigma_{\epsilon,t}\right)\lambda\left(h_t^D\right)\right] \end{aligned}$$

Putting these different terms together and using $h_{t+1} = -\frac{\bar{\beta}_{0,t}P_{t+1}+\xi_{t+1}}{\sigma_{\epsilon,t+1}}$ we get

$$\begin{aligned} V_{dispersion,\uparrow}^{Retail,Buy} = (1-a_t)\, g_t A_{H,t}\sigma_{H,t}^{-2}\left(1-\Phi\left(h_t^D\right)\right)\left[\left(P_{t+1}^{-1}-P_t^{-1}\right)\xi_t+\left(P_{t+1}^{-1}\sigma_{\epsilon,t+1}-P_t^{-1}\sigma_{\epsilon,t}\right)\lambda\left(h_t^D\right)\right] \\ +a_t P_{t+1}^{-1} A_{H,t}\sigma_{H,t}^{-2} g_t\left(1-\Phi\left(h_{t+1}\right)\right)\left[\bar{\beta}_{0,t}P_{t+1}+\xi_t+\sigma_{\epsilon,t+1}\lambda\left(h_{t+1}\right)\right]. \end{aligned}$$

First, let us take the partial derivative with respect to $\sigma_{\epsilon,t+1}$, noting that $h_t^D = \frac{\xi_t(P_t - P_{t+1})}{\sigma_{\epsilon,t}P_{t+1} - P_t\sigma_{\epsilon,t+1}}$ and

$$\frac{\partial h_t^D}{\partial \sigma_{\epsilon,t+1}} = P_t\xi_t\left(P_t - P_{t+1}\right)\left(\sigma_{\epsilon,t}P_{t+1} - P_t\sigma_{\epsilon,t+1}\right)^{-2} = \left(\sigma_{\epsilon,t}P_{t+1} - P_t\sigma_{\epsilon,t+1}\right)^{-1} P_t h_t^D$$

and

$$\frac{\partial h_{t+1}}{\partial \sigma_{\epsilon,t+1}} = \sigma_{\epsilon,t+1}^{-2}\left(\bar{\beta}_{0,t}P_{t+1} + \xi_{t+1}\right)$$

$$\begin{aligned}\frac{\partial h_t^D}{\partial P_{t+1}} &= -\xi_t\left(\sigma_{\epsilon,t}P_{t+1} - P_t\sigma_{\epsilon,t+1}\right)^{-1} - \sigma_{\epsilon,t}\xi_t\left(P_t - P_{t+1}\right)\left(\sigma_{\epsilon,t}P_{t+1} - P_t\sigma_{\epsilon,t+1}\right)^{-2} \\ &= -\xi_t\left(\sigma_{\epsilon,t}P_{t+1} - P_t\sigma_{\epsilon,t+1}\right)^{-1} - \sigma_{\epsilon,t}\left(\sigma_{\epsilon,t}P_{t+1} - P_t\sigma_{\epsilon,t+1}\right)^{-1} h_t^D \\ &= \left(-\xi_t - \sigma_{\epsilon,t}h_t^D\right)\left(\sigma_{\epsilon,t}P_{t+1} - P_t\sigma_{\epsilon,t+1}\right)^{-1}\end{aligned}$$

$$\frac{\partial h_{t+1}}{\partial P_{t+1}} = -\frac{\bar{\beta}_{0,t}}{\sigma_{\epsilon,t+1}}$$

where we are only partially differentiating despite the connection.

$$\begin{aligned}\frac{\partial h_t^D}{\partial P_{t+1}} &= -\xi_t\left(\sigma_{\epsilon,t}P_{t+1} - P_t\sigma_{\epsilon,t+1}\right)^{-1} - \sigma_{\epsilon,t}\xi_t\left(P_t - P_{t+1}\right)\left(\sigma_{\epsilon,t}P_{t+1} - P_t\sigma_{\epsilon,t+1}\right)^{-2} \\ &= -\xi_t\left(\sigma_{\epsilon,t}P_{t+1} - P_t\sigma_{\epsilon,t+1}\right)^{-1} - \sigma_{\epsilon,t}\left(\sigma_{\epsilon,t}P_{t+1} - P_t\sigma_{\epsilon,t+1}\right)^{-1} h_t^D\end{aligned}$$

Using the chain rule repeatedly, we find

$$\begin{aligned}\frac{\partial V_{dispersion,\uparrow^{t+1}}^{Retail,Buy}}{\partial \sigma_{\epsilon,t+1}} &= \left(1-a_t\right)g_t A_{H,t}\sigma_{H,t}^{-2}\left(-\left(\sigma_{\epsilon,t}P_{t+1} - P_t\sigma_{\epsilon,t+1}\right)^{-1} P_t h_t^D \phi\left(h_t^D\right)\right) \\ &\quad\times\left[\left(P_{t+1}^{-1} - P_t^{-1}\right)\xi_t + \left(P_{t+1}^{-1}\sigma_{\epsilon,t+1} - P_t^{-1}\sigma_{\epsilon,t}\right)\lambda\left(h_t^D\right)\right] \\ &\quad+\left(1-a_t\right)g_t A_{H,t}\sigma_{H,t}^{-2}\left(1-\Phi\left(h_t^D\right)\right)\left[P_{t+1}^{-1}\lambda\left(h_t^D\right)\right] \\ &\quad+\left(1-a_t\right)g_t A_{H,t}\sigma_{H,t}^{-2}\left(1-\Phi\left(h_t^D\right)\right) \\ &\quad\times\left[\left(P_{t+1}^{-1}\sigma_{\epsilon,t+1} - P_t^{-1}\sigma_{\epsilon,t}\right)\lambda'\left(h_t^D\right)\left(\sigma_{\epsilon,t}P_{t+1} - P_t\sigma_{\epsilon,t+1}\right)^{-1} P_t h_t^D\right] \\ &\quad+a_t P_{t+1}^{-1}A_{H,t}\sigma_{H,t}^{-2}g_t\left(-\sigma_{\epsilon,t+1}^{-2}\left(\bar{\beta}_{0,t}P_{t+1} + \xi_{t+1}\right)\phi\left(h_{t+1}\right)\right)\left[\bar{\beta}_{0,t}P_{t+1} + \xi_t + \sigma_{\epsilon,t+1}\lambda\left(h_{t+1}\right)\right] \\ &\quad+a_t P_{t+1}^{-1}A_{H,t}\sigma_{H,t}^{-2}g_t\left(1-\Phi\left(h_{t+1}\right)\right)\left[\lambda\left(h_{t+1}\right) + \sigma_{\epsilon,t+1}\sigma_{\epsilon,t+1}^{-2}\left(\bar{\beta}_{0,t}P_{t+1} + \xi_{t+1}\right)\lambda'\left(h_{t+1}\right)\right]\end{aligned}$$

which can be simplified (repeatedly using $P_{t+1}^{-1} - P_t^{-1} = P_t^{-1}P_{t+1}^{-1}\left(P_t - P_{t+1}\right)$ and $P_{t+1}^{-1}\sigma_{\epsilon,t+1} - P_t^{-1}\sigma_{\epsilon,t} = \frac{\sigma_{\epsilon,t+1}P_t}{P_{t+1}P_t} - \frac{\sigma_{\epsilon,t}P_{t+1}}{P_tP_{t+1}} = \frac{\sigma_{\epsilon,t+1}P_t - \sigma_{\epsilon,t}P_{t+1}}{P_{t+1}P_t}$ and properties of the inverse

Mills ratio) to

$$\frac{\partial V^{Retail,Buy}_{dispersion,\uparrow^{t+1}}}{\partial \sigma_{\epsilon,t+1}} = (1-a_t)\, g_t A_{H,t} \sigma_{H,t}^{-2} \left(1-\Phi\left(h_t^D\right)\right) P_{t+1}^{-1} \lambda\left(h_t^D\right) + a_t g_t A_{H,t} \sigma_{H,t}^{-2} \left(1-\Phi\left(h_{t+1}\right)\right) P_{t+1}^{-1} \lambda\left(h_{t+1}\right).$$

Using the chain rule once again along with the same identities, we find

$$\begin{aligned}\frac{\partial V^{Retail,Buy}_{dispersion,\uparrow^{t+1}}}{\partial P_{t+1}} &= (1-a_t)\, g_t A_{H,t} \sigma_{H,t}^{-2} \left(-\left(-\xi_t - \sigma_{\epsilon,t} h_t^D\right)\left(\sigma_{\epsilon,t} P_{t+1} - P_t \sigma_{\epsilon,t+1}\right)^{-1} \phi\left(h_t^D\right)\right) \\ &\times \left[\left(P_{t+1}^{-1} - P_t^{-1}\right)\xi_t + \left(P_{t+1}^{-1}\sigma_{\epsilon,t+1} - P_t^{-1}\sigma_{\epsilon,t}\right)\lambda\left(h_t^D\right)\right] \\ &+ (1-a_t)\, g_t A_{H,t} \sigma_{H,t}^{-2} \left(1-\Phi\left(h_t^D\right)\right) \\ &\times \left[-P_{t+1}^{-2}\xi_t - P_{t+1}^{-2}\sigma_{\epsilon,t+1}\lambda\left(h_t^D\right) + \left(-\xi_t - \sigma_{\epsilon,t} h_t^D\right)\left(\sigma_{\epsilon,t} P_{t+1} - P_t \sigma_{\epsilon,t+1}\right)^{-1}\left(P_{t+1}^{-1}\sigma_{\epsilon,t+1} - P_t^{-1}\sigma_{\epsilon,t}\right)\sigma_{\epsilon,t}\lambda'\left(h_t^D\right)\right] \\ &+ a_t P_{t+1}^{-1} A_{H,t} \sigma_{H,t}^{-2} g_t \left(\frac{\bar{\beta}_{0,t}}{\sigma_{\epsilon,t+1}}\phi\left(h_{t+1}\right)\right)\left[\bar{\beta}_{0,t} P_{t+1} + \xi_t + \sigma_{\epsilon,t+1}\lambda\left(h_{t+1}\right)\right] \\ &+ a_t P_{t+1}^{-1} A_{H,t} \sigma_{H,t}^{-2} g_t \left(1-\Phi\left(h_{t+1}\right)\right)\left[\bar{\beta}_{0,t} + \xi_t - \frac{\bar{\beta}_{0,t}}{\sigma_{\epsilon,t+1}}\sigma_{\epsilon,t+1}\lambda'\left(h_{t+1}\right)\right]\end{aligned}$$

which can be further reduced to

$$\begin{aligned}\frac{\partial V^{Retail,Buy}_{dispersion,\uparrow^{t+1}}}{\partial P_{t+1}} &= (1-a_t)\, g_t A_{H,t} \sigma_{H,t}^{-2} \left(1-\Phi\left(h_t^D\right)\right)\left[-P_{t+1}^{-2}\xi_t - P_{t+1}^{-2}\sigma_{\epsilon,t+1}\lambda\left(h_t^D\right)\right] \\ &+ a_t A_{H,t} \sigma_{H,t}^{-2} g_t \left(1-\Phi\left(h_{t+1}\right)\right)\left[P_{t+1}^{-1}\bar{\beta}_{0,t} + P_{t+1}^{-1}\xi_t\right]\end{aligned}$$

and we have already found that $\frac{dP_{t+1}}{d\sigma_{\epsilon,t+1}} = \vartheta_t \eta_t \lambda\left(h_{t+1}\right)$.

Putting these derivations together, we obtain

$$\begin{aligned}\frac{dV^{Retail,Buy}_{dispersion,\uparrow}}{d\sigma_{\epsilon,t+1}} &= \frac{\partial V^{Retail,Buy}_{dispersion,\uparrow^{t+1}}}{\partial \sigma_{\epsilon,t+1}} + \frac{\partial V^{Retail,Buy}_{dispersion,\uparrow^{t+1}}}{\partial P_{t+1}}\frac{dP_{t+1}}{d\sigma_{\epsilon,t+1}} \\ \frac{dV^{Retail,Buy}_{dispersion,\uparrow}}{d\sigma_{\epsilon,t+1}} &= (1-a_t)\, g_t A_{H,t} \sigma_{H,t}^{-2} \left(1-\Phi\left(h_t^D\right)\right) P_{t+1}^{-1} \lambda\left(h_t^D\right) \\ &+ a_t g_t A_{H,t} \sigma_{H,t}^{-2} \left(1-\Phi\left(h_{t+1}\right)\right) P_{t+1}^{-1} \lambda\left(h_{t+1}\right) \\ &+ \left((1-a_t)\, g_t A_{H,t} \sigma_{H,t}^{-2} \left(1-\Phi\left(h_t^D\right)\right)\left[-P_{t+1}^{-2}\xi_t - P_{t+1}^{-2}\sigma_{\epsilon,t+1}\lambda\left(h_t^D\right)\right] + a_t A_{H,t}\sigma_{H,t}^{-2} g_t \left(1-\Phi\left(h_{t+1}\right)\right)\left[P_{t+1}^{-1}\bar{\beta}_{0,t} + P_{t+1}^{-1}\xi_t\right]\right) \\ &\times \vartheta_{t+1}\eta_t \lambda\left(h_{t+1}\right).\end{aligned}$$

This is an expression valid for any $\sigma_{\epsilon,t+1} > \sigma_{\epsilon,t}$. Taking the limit as $\sigma_{\epsilon,t+1} \downarrow \sigma_{\epsilon,t+1}$ we have that this equals, letting $\eta_t^D = (1-a_t)\, g_t A_{H,t} \sigma_{H,t}^{-2} \left(1-\Phi\left(h_t^{LD}\right)\right)$

$$\frac{dV^{Retail,Buy}_{dispersion,\uparrow}}{d\sigma_{\epsilon,t+1}} = \eta_t^{LD} P_t^{-1} \lambda\left(h_t^{LD}\right) + \frac{a_t}{1-a_t}\eta_t P_t^{-1}\lambda\left(h_t\right)$$

$$+\left(\eta_t^{LD}\left[-P_t^{-2}\xi_t - P_t^{-2}\sigma_{\epsilon,t}\lambda\left(h_t^{LD}\right)\right] + \frac{a_t}{1-a_t}\eta_t\left[P_t^{-1}\bar{\beta}_{0,t} + P_t^{-1}\xi_t\right]\right)\vartheta_t\eta_t\lambda\left(h_t\right)$$
$$= \eta_t^{LD}P_t^{-2}\left(P_t\lambda\left(h_t^{LD}\right) - \xi_t\vartheta_t\eta_t\lambda\left(h_t\right) - \sigma_{\epsilon,t}\lambda\left(h_t^{LD}\right)\vartheta_t\eta_t\lambda\left(h_t\right)\right)$$
$$+\frac{a_t}{1-a_t}\eta_t P_t^{-1}\left(\lambda\left(h_t\right) + \bar{\beta}_{0,t}\vartheta_t\eta_t\lambda\left(h_t\right) + \xi_t\vartheta_t\eta_t\lambda\left(h_t\right)\right)$$
$$= \eta_t^{LD}P_t^{-2}\left(\vartheta_t\omega_t\lambda\left(h_t^{LD}\right) + \xi_t\vartheta_t\eta_t\left(\lambda\left(h_t^{LD}\right) - \lambda\left(h_t\right)\right)\right)$$
$$+\frac{a_t}{1-a_t}\eta_t P_t^{-1}\lambda\left(h_t\right)\left(1 + \bar{\beta}_{0,t}\vartheta_t\eta_t + \xi_t\vartheta_t\eta_t\right)$$

where we have used $P_t = \vartheta_t\omega_t + \vartheta_t\eta_t\sigma_{\epsilon,t}\lambda\left(h_t\right) + \vartheta_t\eta_t\xi_t$.

Attritional selling is given by

$$V_{dispersion,\uparrow}^{Retail,Sell,attrition} = a_t g_t P_t^{-1}A_{H,t}\sigma_{H,t}^{-2}\left(1-\Phi\left(h_t\right)\right)\left[\bar{\beta}_{0,t}P_t + \xi_t + \sigma_{\epsilon,t}\lambda\left(h_t\right)\right].$$

The range of sentiment that triggers selling is $\left(h_t, h_t^D\right)$: high enough to have initially held a position but sufficiently low that either the price increase or the change in intensity of opinion result in a desire to sell shares.

The sales by those fully exiting are given by

$$V_{dispersion,\uparrow}^{Retail,Sell,extensive} = \left(1-a_t\right)g_t P_t^{-1}A_{H,t}\sigma_{H,t}^{-2}\left(\Phi\left(h_{t+1}\right) - \Phi\left(h_t\right)\right)\left[\bar{\beta}_{0,t}P_t + \xi_t + \sigma_{\epsilon,t}\frac{\phi\left(h_t\right) - \phi\left(h_{t+1}\right)}{\Phi\left(h_{t+1}\right) - \Phi\left(h_t\right)}\right]$$
$$= \left(1-a_t\right)g_t P_t^{-1}A_{H,t}\sigma_{H,t}^{-2}\left[\left(\Phi\left(h_{t+1}\right) - \Phi\left(h_t\right)\right)\left(\bar{\beta}_{0,t}P_t + \xi_t\right) + \sigma_{\epsilon,t}\left(\phi\left(h_t\right) - \phi\left(h_{t+1}\right)\right)\right]$$

while those selling some of their shares are given by

$$V_{dispersion,\uparrow}^{Retail,Sell,intensive} = -\left(1-a_t\right)g_t A_{H,t+1}\sigma_{H,t+1}^{-2}$$
$$\times\left[\left(\Phi\left(h_t^D\right) - \Phi\left(h_{t+1}\right)\right)\left(\bar{\beta}_{0,t} + P_{t+1}^{-1}\xi_{t+1}\right) + P_{t+1}^{-1}\sigma_{\epsilon,t+1}\left(\phi\left(h_{t+1}\right) - \phi\left(h_t^D\right)\right)\right]$$
$$+\left(1-a_t\right)g_t A_{H,t}\sigma_{H,t}^{-2}$$
$$\times\left[\left(\Phi\left(h_t^D\right) - \Phi\left(h_{t+1}\right)\right)\left(\bar{\beta}_{0,t} + P_t^{-1}\xi_t\right) + P_t^{-1}\sigma_{\epsilon,t}\left(\phi\left(h_{t+1}\right) - \phi\left(h_t^D\right)\right)\right]$$
$$= -\left(1-a_t\right)g_t A_{H,t}\sigma_{H,t}^{-2}$$
$$\times\left\{\left(\Phi\left(h_t^D\right) - \Phi\left(h_{t+1}\right)\right)\left\{P_{t+1}^{-1}\xi_{t+1} - P_t^{-1}\xi_t\right\} + \left(P_{t+1}^{-1}\sigma_{\epsilon,t+1} - P_t^{-1}\sigma_{\epsilon,t}\right)\left(\phi\left(h_{t+1}\right) - \phi\left(h_t^D\right)\right)\right\}$$

We then have total retail sell volume of

$$V_{dispersion,\uparrow}^{Retail,Sell} = a_t g_t P_t^{-1}A_{H,t}\sigma_{H,t}^{-2}\left(1-\Phi\left(h_t\right)\right)\left[\bar{\beta}_{0,t}P_t + \xi_t + \sigma_{\epsilon,t}\lambda\left(h_t\right)\right]$$
$$+\left(1-a_t\right)g_t P_t^{-1}A_{H,t}\sigma_{H,t}^{-2}\left[\left(\Phi\left(h_{t+1}\right) - \Phi\left(h_t\right)\right)\left(\bar{\beta}_{0,t}P_t + \xi_t\right) + \sigma_{\epsilon,t}\left(\phi\left(h_t\right) - \phi\left(h_{t+1}\right)\right)\right]$$
$$-\left(1-a_t\right)g_t A_{H,t}\sigma_{H,t}^{-2}$$
$$\times\left\{\left(\Phi\left(h_t^D\right) - \Phi\left(h_{t+1}\right)\right)\left\{P_{t+1}^{-1}\xi_{t+1} - P_t^{-1}\xi_t\right\} + \left(P_{t+1}^{-1}\sigma_{\epsilon,t+1} - P_t^{-1}\sigma_{\epsilon,t}\right)\left(\phi\left(h_{t+1}\right) - \phi\left(h_t^D\right)\right)\right\}.$$

We compute the total derivative via

$$\frac{dV^{Retail,Sell}_{dispersion,\uparrow}}{d\sigma_{\epsilon,t+1}} = \frac{\partial V^{Retail,Sell}_{dispersion,\uparrow t+1}}{\partial \sigma_{\epsilon,t+1}} + \frac{\partial V^{Retail,Sell}_{dispersion,\uparrow t+1}}{\partial P_{t+1}} \frac{dP_{t+1}}{d\sigma_{\epsilon,t+1}}.$$

The first term is, using $h_t^D = \frac{\xi_t(P_t - P_{t+1})}{\sigma_{\epsilon,t}P_{t+1} - P_t\sigma_{\epsilon,t+1}}$

$$\begin{aligned}\frac{\partial V^{Retail,Sell}_{dispersion,\uparrow t+1}}{\partial \sigma_{\epsilon,t+1}} = (1-a_t)\, g_t P_t^{-1} A_{H,t}\sigma_{H,t}^{-2}\left[\sigma_{\epsilon,t+1}^{-2}\sigma_{\epsilon,t}\left(\bar{\beta}_{0,t}P_{t+1} + \xi_t\right)\phi\left(h_{t+1}\right)\left(h_{t+1} - h_t\right)\right] \\ - (1-a_t)\, g_t A_{H,t}\sigma_{H,t}^{-2} P_{t+1}^{-1} P_t^{-1} \\ \times\left\{P_t\left(\phi\left(h_{t+1}\right) - \phi\left(h_t^D\right)\right) + \left(\left(\sigma_{\epsilon,t+1}P_t - \sigma_{\epsilon,t}P_{t+1}\right)\sigma_{\epsilon,t+1}^{-1} h_{t+1}\left(h_{t+1} - h_t^D\right)\phi\left(h_{t+1}\right)\right)\right\}\end{aligned}$$

If we take the limit as $\sigma_{\epsilon,t+1} \downarrow \sigma_{\epsilon,t+1}$ then we have

$$\frac{\partial V^{Retail,Sell}_{dispersion,\uparrow t+1}}{\partial \sigma_{\epsilon,t+1}}|_{\sigma_{\epsilon,t+1}=\sigma_{\epsilon,t}} = \frac{\phi\left(h_t^D\right) - \phi\left(h_t\right)}{1 - \Phi\left(h_t\right)}\eta_t P_t^{-1}.$$

Next computing the partial derivative with respect to next period's price, we find

$$\begin{aligned}\frac{\partial V^{Retail,Sell}_{dispersion,\uparrow t+1}}{\partial P_{t+1}} = (1-a_t)\, g_t P_t^{-1} A_{H,t}\sigma_{H,t}^{-2}\left[-\frac{\bar{\beta}_{0,t}}{\sigma_{\epsilon,t+1}}\phi\left(h_{t+1}\right)\left(\bar{\beta}_{0,t}P_t + \xi_t\right) - h_{t+1}\bar{\beta}_{0,t}\phi\left(h_{t+1}\right)\right] \\ - (1-a_t)\, g_t A_{H,t}\sigma_{H,t}^{-2}\left\{\left(\left(-\xi_t - \sigma_{\epsilon,t}h_t^D\right)\left(\sigma_{\epsilon,t}P_{t+1} - P_t\sigma_{\epsilon,t+1}\right)^{-1}\phi\left(h_t^D\right) + \frac{\bar{\beta}_{0,t}}{\sigma_{\epsilon,t+1}}\phi\left(h_{t+1}\right)\right)\right. \\ \left\{P_{t+1}^{-1}\xi_t - P_t^{-1}\xi_t\right\} + \left(\Phi\left(h_t^D\right) - \Phi\left(h_{t+1}\right)\right)\left\{-P_{t+1}^{-2}\xi_{t+1}\right\}\} \\ - (1-a_t)\, g_t A_{H,t}\sigma_{H,t}^{-2}\left\{\left(-P_{t+1}^{-2}\sigma_{\epsilon,t+1}\right)\left(\phi\left(h_{t+1}\right) - \phi\left(h_t^D\right)\right)\right. \\ \left.\left.+\left(P_{t+1}^{-1}\sigma_{\epsilon,t+1} - P_t^{-1}\sigma_{\epsilon,t}\right)\left(h_{t+1}\frac{\bar{\beta}_{0,t}}{\sigma_{\epsilon,t+1}}\phi\left(h_{t+1}\right) + h_t^D\left(-\xi_t - \sigma_{\epsilon,t}h_t^D\right)\left(\sigma_{\epsilon,t}P_{t+1} - P_t\sigma_{\epsilon,t+1}\right)^{-1}\phi\left(h_t^D\right)\right)\right\}\end{aligned}$$

which can be further reduced to

$$\begin{aligned}\frac{\partial V^{Retail,Sell}_{dispersion,\uparrow t+1}}{\partial P_{t+1}} = -(1-a_t)\, g_t A_{H,t}\sigma_{H,t}^{-2} \\ \times\left\{\left(P_{t+1}^{-1}P_t^{-1}\frac{\bar{\beta}_{0,t}}{\sigma_{\epsilon,t+1}}\phi\left(h_{t+1}\right)\left(P_t\xi_t - P_{t+1}\xi_t\right)\right) + \left(\Phi\left(h_t^D\right) - \Phi\left(h_{t+1}\right)\right)\left\{-P_{t+1}^{-2}\xi_{t+1}\right\}\right\} \\ - (1-a_t)\, g_t A_{H,t}\sigma_{H,t}^{-2} \\ \times\left\{\left(-P_{t+1}^{-2}\sigma_{\epsilon,t+1}\right)\left(\phi\left(h_{t+1}\right) - \phi\left(h_t^D\right)\right) + P_{t+1}^{-1}P_t^{-1}\frac{\bar{\beta}_{0,t}}{\sigma_{\epsilon,t+1}}\phi\left(h_{t+1}\right)h_{t+1}\left(\sigma_{\epsilon,t+1}P_t - \sigma_{\epsilon,t}P_{t+1}\right)\right\}.\end{aligned}$$

Taking the limit again as $\sigma_{\epsilon,t+1} \downarrow \sigma_{\epsilon,t+1}$ this reduces to

$$\frac{\partial V^{Retail,Sell}_{dispersion,\uparrow t+1}}{\partial P_{t+1}} = \frac{\eta_t P_{t+1}^{-2}}{1 - \Phi\left(h_t\right)}\left\{\sigma_{\epsilon,t}\left(\phi\left(h_t\right) - \phi\left(h_t^{LD}\right)\right) + \xi_t\left(\Phi\left(h_t^{LD}\right) - \Phi\left(h_t\right)\right)\right\}.$$

Putting this all together we find

$$\frac{dV^{Retail,Sell}_{dispersion,\uparrow}}{d\sigma_{\epsilon,t+1}} = -\frac{\phi\left(h_t\right)-\phi\left(h_t^{LD}\right)}{1-\Phi\left(h_t\right)}\eta_t P_t^{-1}$$
$$+\frac{\eta_t P_{t+1}^{-2}}{1-\Phi\left(h_t\right)}\left\{\sigma_{\epsilon,t}\left(\phi\left(h_t\right)-\phi\left(h_t^{LD}\right)\right)+\xi_t\left(\Phi\left(h_t^{LD}\right)-\Phi\left(h_t\right)\right)\right\}\vartheta_t\eta_t\lambda\left(h_t\right)$$
$$= -\frac{\eta_t P_t^{-2}\vartheta_t}{1-\Phi\left(h_t\right)}\left(\omega_t\left(\phi\left(h_t\right)-\phi\left(h_t^{LD}\right)\right)+\eta_t\xi_t\left(1-\Phi\left(h_t^{LD}\right)\right)\left(\lambda\left(h_t\right)-\lambda\left(h_t^{LD}\right)\right)\right)$$
$$= \eta_t P_t^{-2}\vartheta_t\left(1-\Phi\left(h_t^{LD}\right)\right)$$
$$\times\left(\omega_t\left(\frac{\lambda\left(h_t^{LD}\right)}{\left(1-\Phi\left(h_t\right)\right)}-\frac{\lambda\left(h_t\right)}{\left(1-\Phi\left(h_t^{LD}\right)\right)}\right)+\frac{1}{1-\Phi\left(h_t\right)}\eta_t\xi_t\left(\lambda\left(h_t^{LD}\right)-\lambda\left(h_t\right)\right)\right)$$

We can now look at, recalling $\eta_t^{LD} = (1-a_t)\, g_t A_{H,t}\sigma_{H,t}^{-2}\left(1-\Phi\left(h_t^{LD}\right)\right)$,

$$\frac{dV^{Retail,Buy}_{dispersion,\uparrow}}{d\sigma_{\epsilon,t+1}}\Big/\frac{dV^{Retail,Sell}_{dispersion,\uparrow}}{d\sigma_{\epsilon,t+1}} =$$
$$\left(\eta_t^{LD}P_t^{-2}\left(\vartheta_t\omega_t\lambda\left(h_t^{LD}\right)+\xi_t\vartheta_t\eta_t\left(\lambda\left(h_t^{LD}\right)-\lambda\left(h_t\right)\right)\right)+\frac{a_t}{1-a_t}\eta_t P_t^{-1}\lambda\left(h_t\right)\left(1+\bar{\beta}_{0,t}\vartheta_t\eta_t+\xi_t\vartheta_t\eta_t\right)\right)/$$
$$\eta_t P_t^{-2}\vartheta_t\left(1-\Phi\left(h_t^{LD}\right)\right)\left(\omega_t\left(\frac{\lambda\left(h_t^{LD}\right)}{\left(1-\Phi\left(h_t\right)\right)}-\frac{\lambda\left(h_t\right)}{\left(1-\Phi\left(h_t^{LD}\right)\right)}\right)+\frac{1}{1-\Phi\left(h_t\right)}\eta_t\xi_t\left(\lambda\left(h_t^{LD}\right)-\lambda\left(h_t\right)\right)\right)$$
$$= \left(\left(\omega_t\phi\left(h_t^{LD}\right)+\left(1-\Phi\left(h_t^{LD}\right)\right)\xi_t\eta_t\left(\lambda\left(h_t^{LD}\right)-\lambda\left(h_t\right)\right)\right)+\frac{a_t}{1-a_t}\left(1-\Phi\left(h_t\right)\right)P_t\lambda\left(h_t\right)\left(\alpha_t+\xi_t\eta_t\right)\right)/$$
$$\left(\omega_t\phi\left(h_t^{LD}\right)-\omega_t\phi\left(h_t\right)+\left(1-\Phi\left(h_t^{LD}\right)\right)\eta_t\xi_t\left(\lambda\left(h_t^{LD}\right)-\lambda\left(h_t\right)\right)\right)$$
$$= \left(\left(\omega_t\phi\left(h_t^{LD}\right)+\left(1-\Phi\left(h_t^{LD}\right)\right)\xi_t\eta_t\left(\lambda\left(h_t^{LD}\right)-\lambda\left(h_t\right)\right)-\omega_t\phi\left(h_t\right)\right)+\omega_t\phi\left(h_t\right)+\frac{a_t}{1-a_t}\left(1-\Phi\left(h_t\right)\right)P_t\lambda\left(h_t\right)\left(\alpha_t+\xi_t\eta_t\right)\right)/$$
$$\left(\omega_t\phi\left(h_t^{LD}\right)-\omega_t\phi\left(h_t\right)+\left(1-\Phi\left(h_t^{LD}\right)\right)\eta_t\xi_t\left(\lambda\left(h_t^{LD}\right)-\lambda\left(h_t\right)\right)\right)$$
$$= 1+\frac{\omega_t\phi\left(h_t\right)+\frac{a_t}{1-a_t}\left(1-\Phi\left(h_t\right)\right)P_t\lambda\left(h_t\right)\left(\alpha_t+\xi_t\eta_t\right)}{\omega_t\phi\left(h_t^{LD}\right)-\omega_t\phi\left(h_t\right)+\left(1-\Phi\left(h_t^{LD}\right)\right)\eta_t\xi_t\left(\lambda\left(h_t^{LD}\right)-\lambda\left(h_t\right)\right)} > 1.$$

where recall $\xi_t\frac{\eta_t\lambda(h_t)}{\omega_t+\eta_t\xi_t} = h_t^{LD}$.

To prove the inequality, note that the numerator is trivially greater than 0 because ξ_t and ω_t are both greater than zero. If the total derivative of volume with respect to dispersion is non-negative, then the denominator is non-negative. Since the denominator is proportional to the total derivative of sale volume, whenever the denominator is negative the numerator is positive by the above derivation of $\frac{dV^{Retail,Buy}_{dispersion,\uparrow}}{d\sigma_{\epsilon,t+1}}\Big/\frac{dV^{Retail,Sell}_{dispersion,\uparrow}}{d\sigma_{\epsilon,t+1}}$.

(4) The final of the four inequalities, $\bar{\beta}_{0,t}\omega_t + \xi_t\alpha_t > 0$, $\omega_t + \eta_t\xi_t < 0$, implies $-\alpha_t^{-1}\bar{\beta}_{0,t}\omega_t < \xi_t < -\eta_t^{-1}\omega_t$ and $\omega_t < 0$. This captures a peculiar case in which households collectively hold more than one share of the asset despite only having moderate sentiment levels, while institutions are net short. At the same time, price elasticity with respect to dispersion is so large than an increase in prices triggers institutions to buy back shares

because their short positions become too large in dollar terms for their desired level of risk. Retail traders in the interval (h_t, h_{t+1}) sell all of their holdings, while traders in the interval $(h_{t+1}, +\infty)$ sell some of their holdings. Only attritional retail traders purchase the assets, while sales increase significantly.

B Additional Tables: Returns, Volumes and BSIs on Earnings Days and Days with High Google Searches

Here I provide additional analysis of the relationship between Google SVI and returns, turnover, volume, and BSIs. Table 8 includes three additional "meme" stocks in the analysis and does not exclude earnings dates. Table 9 does not include any "meme" stocks and confines itself to non-earnings days, but using an alternative classification of "high" versus "low" attention using a 99% threshold instead of 95%. Table 10 includes several "meme" stocks in the earnings day analysis.

C Supplemental Tables: Predicting Even and Odd Lot Retail Fraction

This section provides the even and odd lot analogues of Table 2 from the main text. Results are similar, but with odd lot traders displaying less of an affinity for stocks with salient returns in the more distant past. Odd lot and even lot traders concentrate both sales and purchases in high volume and extreme return stocks. Table 11 contains the estimates for even lots, while Table 12 contains the estimates for odd lots.

D Non-Alignment: Asset Managers in CFTC Data

One might hypothesize that the retail alignment puzzle is generated from either (1) the non-existence of between group differences, in which case alignment is tautological, or (2) an unknown mechanism that generates alignment for all small but sufficiently large and heterogeneous groups. Therefore, before analyzing the retail alignment puzzle in more detail, it is a useful exercise to compare retail traders to another subgroup of traders: asset managers. As we shall see, asset managers display high levels of heterogeneity with far higher VWAI than retail traders.

The CFTC has made public a unique weekly dataset for January 2009 to May 2011 that shows the daily average net position changes of different traders as well as total exchange

Table 8: Returns, Volume, Turnover, and BSIs: High vs Low Google Searches

	All Lots	Odd Lots	Even Lots
High Att Ret	0.00266		
Low Att Ret	0.000839		
H/L Att Ret Diff	0.00182(2.85)		
High Att |Ret|	0.0263		
Low Att |Ret|	0.0127		
H/L Att |Ret| Diff	0.0136(24.9)		
High Att Retail Turn	0.00189	0.000144	0.00174
Low Att Retail Turn	0.00047	3.53e-05	0.000435
H/L Att Retail Turn Diff	0.00142(9.72)	0.000109(6.24)	0.00131(9.75)
High Att Turn	0.0281		
Low Att Turn	0.0098		
H/L Att Turn Diff	0.0183(10.1)		
High Att Rtl Volume	102000000	13200000	88900000
Low Att Rtl Volume	42600000	5850000	36700000
H/L Att Rtl Volume Diff	59500000(15.4)	7360000(9.04)	52100000(16.4)
High Att Volume	1530000000		
Low Att Volume	758000000		
H/L Att Volume Diff	776000000(19.2)		
High Att BSI	0.00524	0.0648	-0.00222
Low Att BSI	-0.00909	0.0337	-0.0169
H/L Att BSI Diff	0.0143(8.28)	0.0311(13.4)	0.0147(8.02)
High Att |BSI|	0.112	0.159	0.116
Low Att |BSI|	0.11	0.128	0.12
H/L Att |BSI| Diff	0.00118(1.01)	0.0312(20.4)	-0.00444(-3.54)
High Att Ret Volume Frac	0.039	0.0033	0.0357
Low Att Ret Volume Frac	0.034	0.00351	0.0304
H/L Att Ret Diff Volume Frac	0.005(15.8)	-0.000212(-4.93)	0.00521(17.2)

Note: This table characterizes the differences in returns, volumes, turnover, and Buy Sell Imbalances (BSIs) on days where Google search volume is high versus low. The universe of 74 stocks is formed by taking the highest 5% of stocks by median market cap in the merged 2013-2021 CRSP-TAQ database and then eliminating all stocks with tickers of string length less than four. Stocks in the DJIA with "non-noisy tickers" in the spirit of Da, Engelberg and Gao (2011) are added along with three of the most popular "meme stocks" from the meme stock frenzy of 2021. Google search volume indices (SVIs) are obtained from the Google Trends interface and rescaled into a single nine-year time series. For each of the 74 stocks, the top 5% of days by SVI change are classified as "high attention" while the bottom 95% are classified as "low attention." Each line of the table provides the mean value for a different variable or difference of variables, with t-statistics in parenthesis for all comparison tests. Standard errors are clustered at the day level following the guidance provided in Abadie, Athey, Imbens and Wooldridge (2017).

Table 9: Returns, Volume, Turnover, and BSIs: High vs Low Google Searches, No Earnings

	All Lots	Odd Lots	Even Lots
High Att Ret	0.00399		
Low Att Ret	0.000895		
H/L Att Ret Diff	0.00309(1.97)		
High Att |Ret|	0.0361		
Low Att |Ret|	0.0124		
H/L Att |Ret| Diff	0.0237(18.9)		
High Att Retail Turn	0.00158	0.000101	0.00148
Low Att Retail Turn	0.000402	3.29e-05	0.000369
H/L Att Retail Turn Diff	0.00118(9.76)	6.83e-05(6.5)	0.00111(9.66)
High Att Turn	0.0241		
Low Att Turn	0.00892		
H/L Att Turn Diff	0.0152(12.5)		
High Att Rtl Volume	129000000	13300000	115000000
Low Att Rtl Volume	46100000	6320000	39800000
H/L Att Rtl Volume Diff	82500000(9.36)	7010000(4.23)	75500000(10.1)
High Att Volume	1920000000		
Low Att Volume	812000000		
H/L Att Volume Diff	1100000000(11.6)		
High Att BSI	0.0131	0.0718	0.00627
Low Att BSI	-0.00896	0.0368	-0.0171
H/L Att BSI Diff	0.0221(5.69)	0.0351(6.29)	0.0233(5.7)
High Att |BSI|	0.112	0.177	0.113
Low Att |BSI|	0.109	0.127	0.119
H/L Att |BSI| Diff	0.00205(0.742)	0.0505(14)	-0.006(-2.05)
High Att Ret Volume Frac	0.0417	0.00329	0.0384
Low Att Ret Volume Frac	0.033	0.00357	0.0294
H/L Att Ret Diff Volume Frac	0.00871(12.4)	-0.000278(-3.22)	0.00899(13.4)

Note: This table characterizes the differences in returns, volumes, turnover, and Buy Sell Imbalances (BSIs) on non-earnings days where Google search volume is high versus low. The universe of 71 stocks is formed by taking the highest 5% of stocks by median market cap in the merged 2013-2021 CRSP-TAQ database and then eliminating all stocks with tickers of string length less than four. Stocks in the DJIA with "non-noisy tickers" in the spirit of Da, Engelberg and Gao (2011)are added. Google search volume indices (SVIs) are obtained from the Google Trends interface and rescaled into a single nine-year time series for each ticker. For each of the 71 stocks, the top 1% of days by SVI change are classified as "high attention" while the bottom 99% are classified as "low attention." Earnings days are subsequently removed using I/B/E/S data. Each line of the table provides the mean value for a different variable or difference of variables, with t-statistics in parenthesis for all comparison tests. Standard errors are clustered at the day level following the guidance provided in Abadie, Athey, Imbens and Wooldridge (2017).

Table 10: Returns, Volume, Turnover, and BSIs: High vs Low Google Searches on Earnings Days

	All Lots	Odd Lots	Even Lots
High Att Ret	0.00202		
Low Att Ret	0.0022		
H/L Att Ret Diff	-0.000184(-0.173)		
High Att |Ret|	0.0238		
Low Att |Ret|	0.0214		
H/L Att |Ret| Diff	0.00238(3.06)		
High Att Retail Turn	0.00123	7.21e-05	0.00115
Low Att Retail Turn	0.000676	4.77e-05	0.000628
H/L Att Retail Turn Diff	0.000551(8.45)	2.43e-05(4.16)	0.000527(8.49)
High Att Turn	0.022		
Low Att Turn	0.0167		
H/L Att Turn Diff	0.00532(6.11)		
High Att Rtl Volume	107000000	13500000	93800000
Low Att Rtl Volume	32400000	3680000	28700000
H/L Att Rtl Volume Diff	74800000(13.1)	9780000(6.9)	65100000(14)
High Att Volume	1830000000		
Low Att Volume	685000000		
H/L Att Volume Diff	1150000000(15.1)		
High Att BSI	0.0635	0.117	0.0565
Low Att BSI	0.0416	0.0793	0.0359
H/L Att BSI Diff	0.0219(5.14)	0.0377(7.01)	0.0206(4.58)
High Att |BSI|	0.102	0.164	0.1
Low Att |BSI|	0.115	0.152	0.12
H/L Att |BSI| Diff	-0.0137(-4.69)	0.012(3.27)	-0.0196(-6.3)
High Att Ret Volume Frac	0.0439	0.00362	0.0403
Low Att Ret Volume Frac	0.0301	0.00251	0.0276
H/L Att Ret Diff Volume Frac	0.0138(21.4)	0.0011(10.6)	0.0127(20.8)

Note: This table characterizes the differences in returns, volumes, turnover, and Buy Sell Imbalances (BSIs) on earnings days where Google search volume is high versus low. Earnings days are isolated using I/B/E/S data. The universe of 71 stocks is formed by taking the highest 5% of stocks by median market cap in the merged 2013-2021 CRSP-TAQ database and then eliminating all stocks with tickers of string length less than four. Stocks in the DJIA with "non-noisy tickers" in the spirit of Da, Engelberg and Gao (2011) are added along with three of the most popular "meme stocks" from the meme stock frenzy of 2021. Google search volume indices (SVIs) are obtained from the Google Trends interface and rescaled into a single nine-year time series. For each of the 71 stocks, the top 5% of days by SVI change are classified as "high attention" while the bottom 95% are classified as "low attention." "High attention" earnings days are those with top 5% SVIs over the nine year period while "low attention" earnings days are those that fall in the bottom 95% of SVI over all days (including non-earnings days). Each line of the table provides the mean value for a different variable or difference of variables, with t-statistics in parenthesis for all comparison tests. Standard errors are clustered at the day level following the guidance provided in Abadie, Athey, Imbens and Wooldridge (2017).

Table 11: Predicting Fractional Dollar Volume of Even Lot Retail Buy Orders

	β (Buys, Even)	β (Sells, Even)
Yest Volume	0.41 (90.73)	0.4 (91.02)
2 Day Ago Volume	0.072 (22.9)	0.073 (23.86)
Avg Volume 3-7 days ago	0.22 (70.92)	0.22 (73.59)
Avg Volume 8-28 days ago	0.27 (45.97)	0.26 (46.31)
Sal. Rank of Today Rets	0.91 (124.63)	0.84 (120.83)
Sal. Rank of Yesterday Rets	0.38 (49.46)	0.35 (46.57)
Sal. Rank of Two Days Ago Rets	0.31 (38.87)	0.27 (35.9)
Sal. Rank of Last Week Rets	0.23 (34.74)	0.2 (31.26)
Sal. Rank of Last Month Rets	0.2 (28.11)	0.17 (25.65)
Frac Mkt Cap	-0.14 (-8.48)	-0.13 (-8.21)

Note: This table shows the results of regression (9) where fractional volume of retail even lot buy and sell orders are the dependent variables. All standard errors use correction for clustering proposed by Liang and Zeger (1986) to account for intertemporal correlation in the residuals and independent variables by equity.

Table 12: Predicting Fractional Dollar Volume of Odd Lot Retail Buy Orders

	β (Buys, Even)	β (Sells, Even)
Yest Volume	0.33 (76.48)	0.31 (74.74)
2 Day Ago Volume	0.077 (25.87)	0.072 (24.87)
Avg Volume 3-7 days ago	0.17 (50.91)	0.18 (54.9)
Avg Volume 8-28 days ago	0.16 (22.92)	0.18 (26.59)
Sal. Rank of Today Rets	0.47 (54.28)	0.39 (47.28)
Sal. Rank of Yesterday Rets	0.081 (9.21)	0.06 (6.95)
Sal. Rank of Two Days Ago Rets	0.027 (3)	0.013 (1.53)
Sal. Rank of Last Week Rets	0.0079 (1.01)	-0.0096 (-1.26)
Sal. Rank of Last Month Rets	-0.0085 (-1)	-0.022 (-2.64)
Frac Mkt Cap	0.34 (20.56)	0.32 (20.29)

Note: This table shows the results of regression (9) where fractional volume of retail odd lot buy and sell orders are the dependent variables. All standard errors use correction for clustering proposed by Liang and Zeger (1986) to account for intertemporal correlation in the residuals and independent variables by equity.

volumes.[24] For example, if the data shows a daily average net "buy" of 500 and a daily average net "sell" of 600 for "Asset Managers," these numbers are the result of (1) separating managers for each day into those who had net purchases versus sales, (2) summing separately across all net purchasers and all net sellers for each day, and finally (3) averaging these daily numbers across days in the week.

To compare the data's institutional subgroup imbalance patterns to those of households, we must account for two differences in the data. First, the data contains only daily net changes and therefore excludes any intraday trading, that is positions opened and closed within the same session by a given trader. Second, the data is weekly instead of daily and therefore will understate imbalances through aggregation. The exclusion of intraday trading is only an issue if the subgroup we examine regularly engages in intraday trading activity. To minimize this issue, I select the "Asset Manager" subgroup in the CFTC's taxonomy, which includes pension funds, mutual funds, endowments, and similar institutions but not the Commodity Trading Advisors or Commodity Pool Operators commonly associated with intraday hedge fund or proprietary trading. The second issue guarantees a downward bias: aggregation to weekly from daily data necessarily shrinks all imbalance metrics and I therefore conduct a similar aggregation exercise for retail traders. Whereas the daily VWAI over the 2013-2021 sample is 0.0826 the weekly statistic a mere 0.0548, making the daily number slightly more than 50% larger.

The daily net average contract purchases and sales for Asset Managers are shown in Figure 13, which provides a stark contrast with Figures 1 and 3. As shown in Appendix E, this pattern of relative non-alignment is not unique to equity index futures, with interest rates, precious metals, grains, and energies displaying similar patterns. The weekly VWAI values are 0.136, 0.143, and 0.134 for Ten Year Treasure Note futures, the E-mini S&P 500 futures, and the T-Bond futures respectively. Asset Managers comprise a similar fraction of futures exchange volumes within this dataset to retail traders in equity markets with total dollar trade values comparable to those of the largest megacap stocks: the data is given in contracts so the extreme value in Figure 13 of around $80,000$ contracts is roughly equivalent to $80,000 \times \$50,000 = \$4B$ of volume, while the smallest average daily values are in the hundreds of millions of dollars. If we independently draw net purchases and sales as random values from the uniform distribution over the unit interval, aggregate the daily figures into weekly averages, and assume a group fraction of 5% of total trading volume, the VWAI computation is approximately 0.1595, a similar value to those observed in futures markets. The CFTC data strongly implies that the patterns observed among retail traders are unexpected and require a non-mechanical explanation.

[24] This dataset, including explanatory notes, is available on the CFTC's website at https://www.cftc.gov/MarketReports/NetPositionChangesData/index.htm.

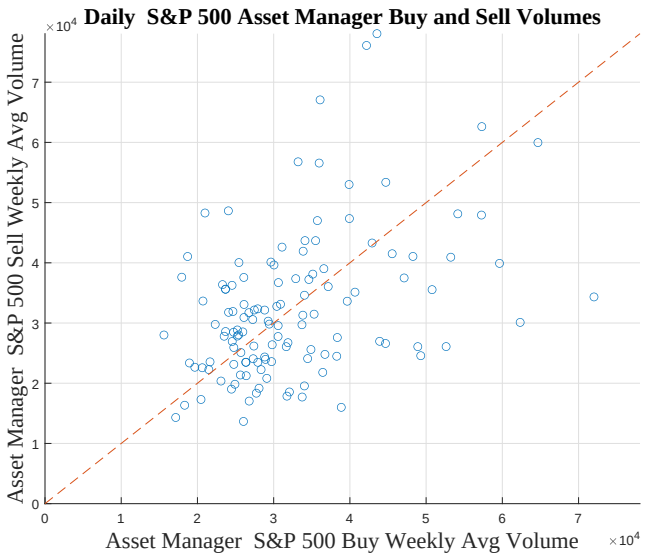


Figure 13: E-mini S&P 500 Asset Manager Weekly Purchase and Sale Contract Volumes over Time

Note: This figure plots the weekly (aggregated by CFTC from daily) net position changes by "Asset Managers" from January 2009 through May 2011 along with the 45° line. The CFTC classifies pension funds, mutual funds, endowments and other similar institutions as "asset managers" whereas proprietary trading firms and other managers such as Commodity Trading Advisors and Commodity Pool Operators are not classified as "Asset Managers" in the report.

E CFTC Asset Manager Alignment for Other Markets

Here, in Figures 14, 15, 16, and 17, I scatterplot average daily purchases and sales for T-Bonds, West Texas Intermediate Crude Oil, Ten Year T-Notes, and Gold futures contracts, respectively. For WTI Crude and Gold futures, the "Asset Manager" category is replaced by the "Managed Money" category, which includes CPOs and CTAs.

F BHJOS vs BJZZ and Dollar Volume vs Transaction Volume

Recall that the BJZZ methodology struggles to sign trades when the spread is wider than a penny: if a bid-ask spread is 3¢, for instance, then price improvement of 0.8¢ is below the midpoint and therefore more likely to be a sale than a purchase whereas the BJZZ methodology would identify it as a purchase because $0.5 < 0.8 < 1.0$. Using predominantly one lot orders, BHJOS show that for their experiment, 28% of trades are incorrectly signed. As remarked upon in BHJOS, this leads to downward bias in the differences between retail purchases and sales.

Yet unlike the BHJOS critique of BJZZ, the focus of this paper is not on individual

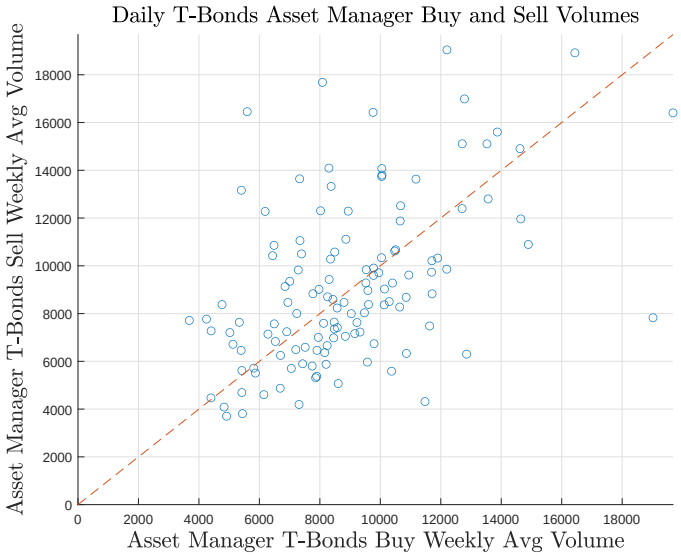


Figure 14: T-Bonds Asset Manager Weekly Purchase and Sale Contract Volumes over Time
Note: This figure plots the weekly (aggregated by CFTC from daily) net position changes by "Asset Managers" from January 2009 through May 2011 along with the 45° line. The CFTC classifies pension funds, mutual funds, endowments and other similar institutions as "asset managers" whereas proprietary trading firms and other managers such as Commodity Trading Advisors and Commodity Pool Operators are not classified as "Asset Managers" in the report.

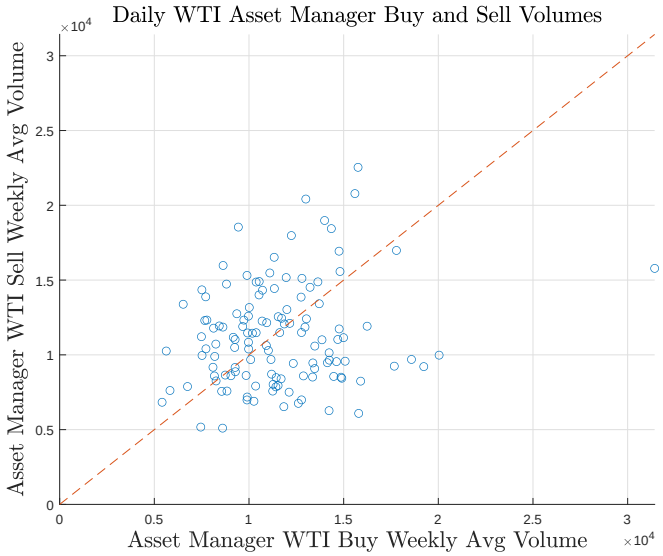


Figure 15: WTI Crude Oil Asset Manager Weekly Purchase and Sale Contract Volumes over Time
Note: This figure plots the weekly (aggregated by CFTC from daily) net position changes by "Managed Money" from January 2009 through May 2011 along with the 45° line. The CFTC classifies trading firms such as Commodity Trading Advisors and Commodity Pool Operators as "Managed Money" in the report.

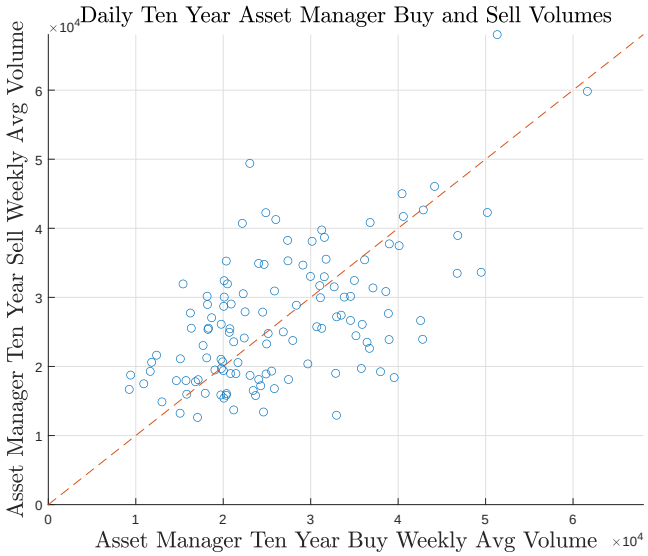


Figure 16: Ten Year T-Notes "Managed Money" Weekly Purchase and Sale Contract Volumes over Time

Note: This figure plots the weekly (aggregated by CFTC from daily) net position changes by "Asset Managers" from January 2009 through May 2011 along with the 45° line. The CFTC classifies pension funds, mutual funds, endowments and other similar institutions as "asset managers" whereas proprietary trading firms and other managers such as Commodity Trading Advisors and Commodity Pool Operators are not classified as "Asset Managers" in the report.

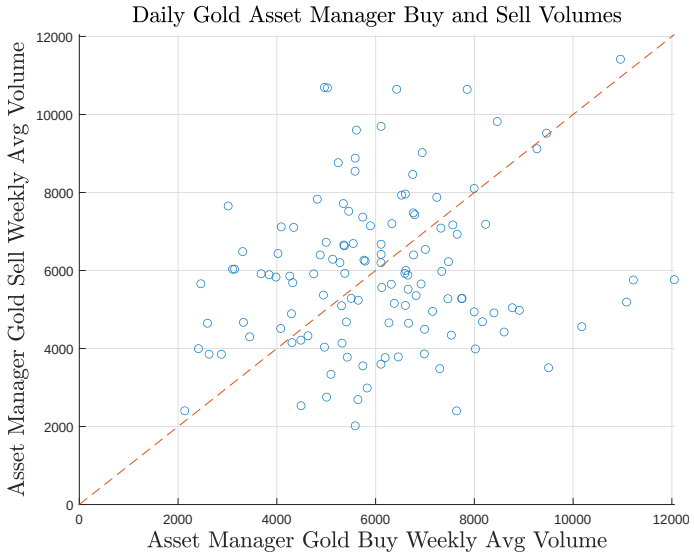


Figure 17: Gold "Managed Money" Weekly Purchase and Sale Contract Volumes over Time

Note: This figure plots the weekly (aggregated by CFTC from daily) net position changes by "Managed Money" from January 2009 through May 2011 along with the 45° line. The CFTC classifies trading firms such as Commodity Trading Advisors and Commodity Pool Operators as "Managed Money" in the report.

trades or trade imbalances, but rather on the relationship between retail *dollar* purchases and retail *dollar* sales. Dollar versus trade volume are profoundly different measures because of the sharp right skew observed in order sizes. If 500 one share trades are signed with an accuracy rate of 72% and one trade of $5,000$ shares is signed correctly, then the BHJOS approach finds an accuracy rate of approximately 72% whereas this paper would find an accuracy rate of over 97% when weighted by volume. For the trade and dollar approaches to be comparable one must assume that the BJZZ methodology has equal mis-signing rates for large versus small orders; publicly available execution data from Schwab (2023) suggests that order size-based heterogeneity in mis-signing is extreme. For retail orders executed on Schwab, $1-99$ share odd lot orders received an average improvement of 3.42¢ per share whereas orders of size $2,000-4,999$ received price improvement of only 0.95¢ per share. This data indicates that the BJZZ methodology is most accurate for larger orders and least accurate for the minimum one share odd lot orders used by BHJOS. Nonetheless, the BJZZ approach invariably generates a downward bias for buy-sell imbalances and is less accurate for stocks with the largest spreads. Although stocks with large spreads and limited liquidity tend to be those with smaller market capitalizations and trading volumes and therefore are those least likely to influence this section's findings, I nonetheless circumvent this problem by adopting the Lee and Ready (1991) modification to the BJZZ methodology outlined in BHJOS.

G Additional Analysis of High Volume Days

Table 13: $|BSI|$ for Extreme Volume Days

	|BSI| All	|BSI| Odd	|BSI| Even	Corr(Odd,Even)	|Ret-Mkt|
High Volume Days	0.13	0.16	0.13	0.45	0.025
Non-High Volume Days	0.12	0.13	0.14	0.31	0.012
H-NH Volume Diff	0.0046	0.032	-0.0068	0.14	0.014
H-NH t-stat	(2.18)	(14.82)	(-3.1)	(17.01)	(25.2)

Note: This table compares the average absolute value of retail trade Buy Sell Imbalances (BSIs) on high volume days versus low volume days. First, the top 5% of stocks by median market cap over the time period 2013-2021 are selected and stocks with missing data are excluded. Next, for each stock the days with the highest 5% and bottom 95% of daily retail volumes are identified. For high volume days and "normal" volume days, BSI magnitudes are computed for all orders, even lot orders, and odd lot orders; odd/even lot correlations and average return magnitudes are also calculated. A paired t-test is conducted to compare all five average statistics by stock for high and normal volume days.

H Autocorrelation Patterns in Buy Sell Imbalances and Attention

H.1 Autocorrelation Structure of Odd Lot and Even Lot Autocorrelation

As shown in Figure 18, retail BSIs for the largest 5% of stocks by market cap exhibit strong persistence, with odd lot imbalances extraordinarily persistent. Median odd lot BSI autocorrelation only decays to less than 0.10 after 100 days and is 0.47 at a lag of one day. Such abnormal patterns are difficult to explain with sentiment shifts or disagreement. For sentiment to generate persistent BSIs, shocks to retail sentiment would have to be autocorrelated at lags of many months, display differential persistence across large and small retail traders, and only decay to zero after nearly a year. Such persistent shocks would manifest in retail traders repeatedly purchasing the same stocks over many months. For disagreement to generate persistent BSIs in the spirit of Miller (1977), an increase today in disagreement among retail traders would need to be positively correlated with an increase in disagreement on every day over the next year. The attention mechanism, explicitly examined in Section 3, is far more straightforward: a stock that draws more attention today is more likely to draw a flow of additional attention in the near future, even out to longer lags.

H.2 Parallel Autocorrelation Patterns: BSIs and Attention

If attention shocks are largely responsible for changes in BSIs, even if small on average, then we should expect similar decay patterns in the autocorrelation of BSIs and the Google SVIs. Figure 19 shows that is indeed the case. Local cyclical peaks at approximately 63 days, 126 days, and 189 days occur for both SVI and BSIs due to the cyclical variation in earnings-induced attention and the cyclical positive relationship in earnings-related BSIs, but earnings days do not explain the parallel curves as they are both a small fraction of total trading days and responsible for only a small fraction of BSI and SVI shocks. We saw earlier that odd lot BSI autocorrelation is

I Additional Major Equities: Scatter Plots

This appendix contains scatter plots featuring daily retail dollar purchase (x-axis) and sale (y-axis) volume for various different equities. All current Dow Jones Industrial Average components are plotted in addition to several so-called "meme stocks" such as Bed Bath and Beyond, AMC, GameStop, and XPresSpa. These scatterplots depict a wide variety of relationships, from near perfect correlation in the case of most meme stocks and popular

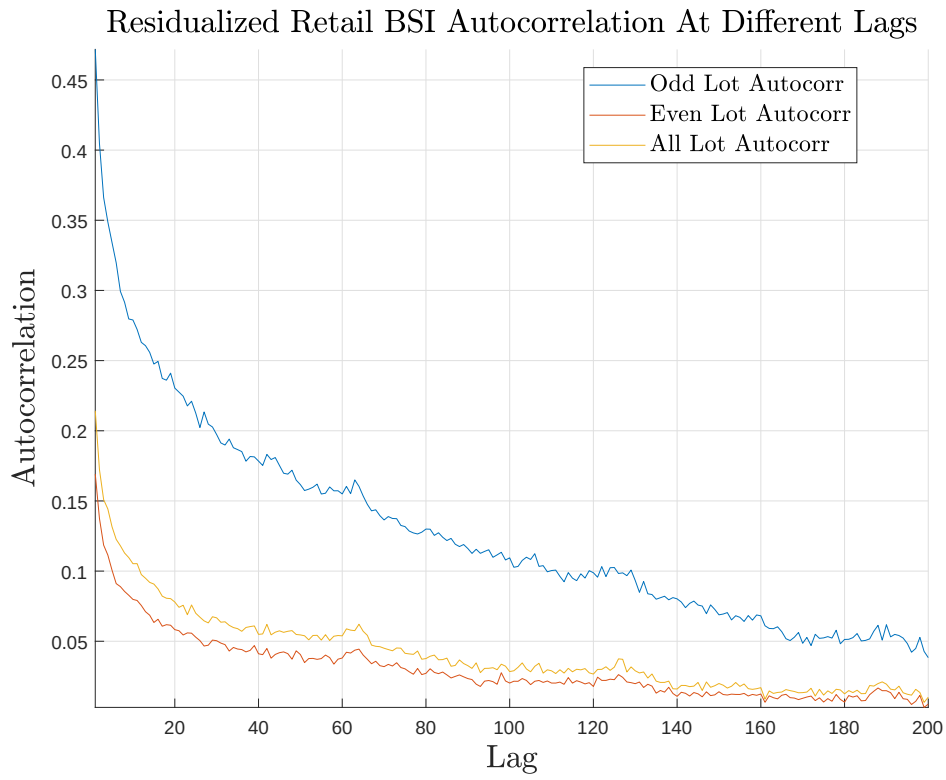


Figure 18: Autocorrelation of Residualized Retail BSIs

Note: This figure plots the median autocorrelation structure of "residualized" odd lot, even lot, and total buy-sell imbalances by stock, which are computed as $BSI_{DV,t} = \frac{DollarBuys_t - DollarSells_t}{DollarBuys_t + DollarSells_t}$. First, a group of the top 5% of stocks by market capitalization are selected and all stocks from the subgroup with missing data are eliminated. Next, the retail BSIs for the aggregate stock market are computed. Finally, residualized BSIs are formed by taking the BSIs for each of the remaining large cap stocks and projecting them onto the space orthogonal to the aggregate stock market retail BSIs. The figure plots the median autocorrelation of the residualized BSIs for all lags from 1 day to 200 days.

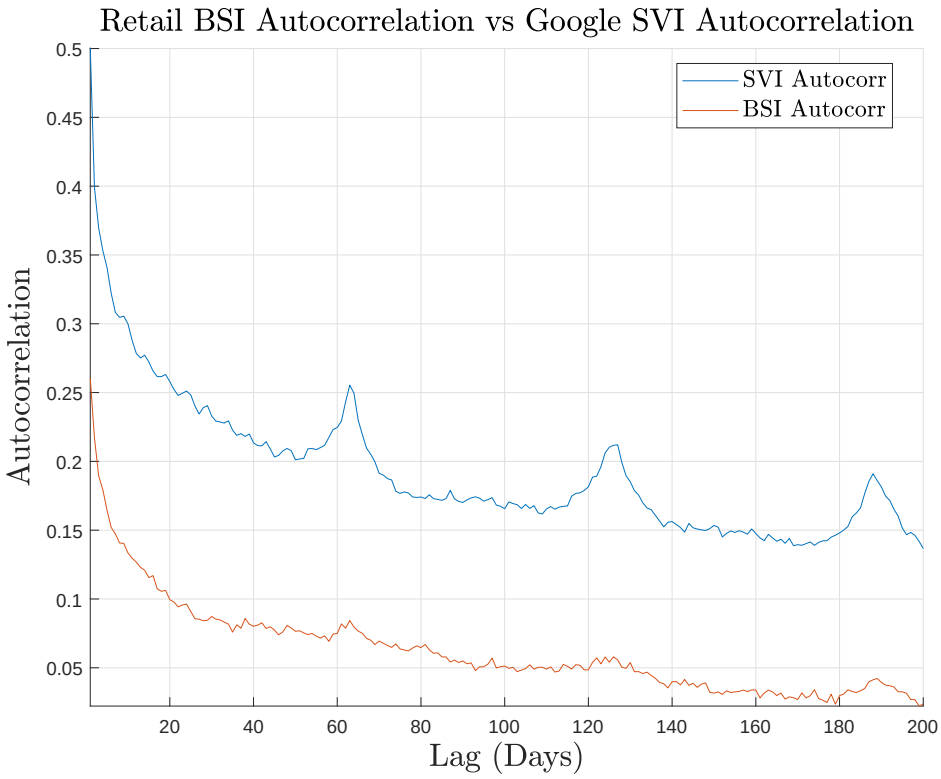


Figure 19: Autocorrelation of Residualized Retail BSIs vs SJuVIs

Note: This figure plots the average autocorrelation structure of retail imbalances and SVIs by stock, with BSIs computed as $BSI_{DV,t} = \frac{DollarBuys_t - DollarSells_t}{DollarBuys_t + DollarSells_t}$ and rescaled SVIs obtained from Google Trends using the methodology of Table 8. First, a group of 74 high market cap stocks and meme stocks are selected using the method detailed in Table 8. Next, the retail BSIs for the stocks are computed along with the autocorrelation structure for each equity. The figure plots the median autocorrelation of the residualized BSIs for all lags from 1 day to 200 days.

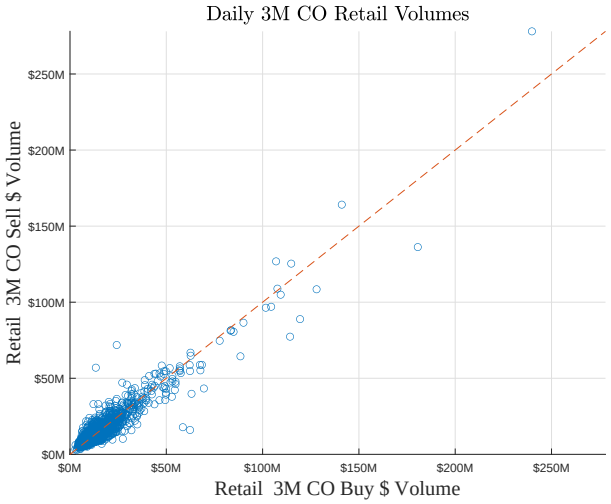


Figure 20: 3M

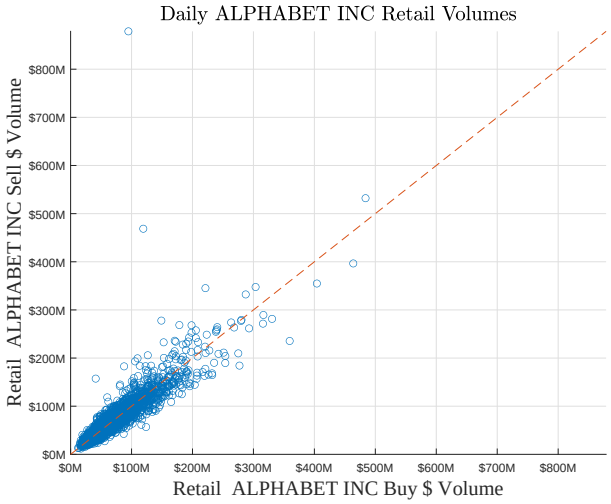


Figure 21: Alphabet



Figure 22: American Express

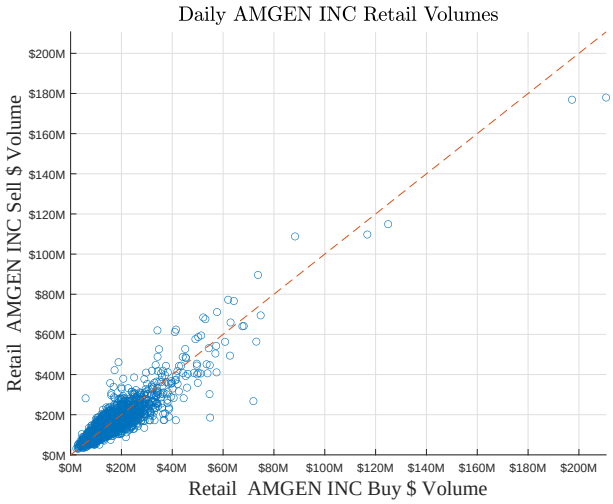


Figure 23: Amgen

retail trades such as Tesla to imperfect but nonetheless strong correlation in a subset of old companies such as Coca Cola or McDonalds.

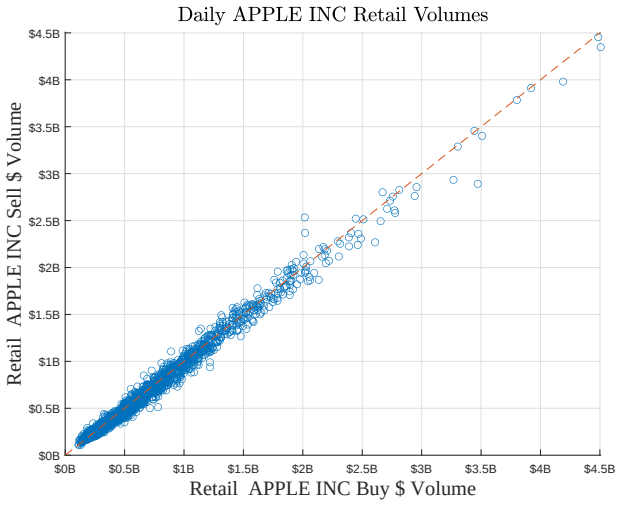


Figure 24: Apple

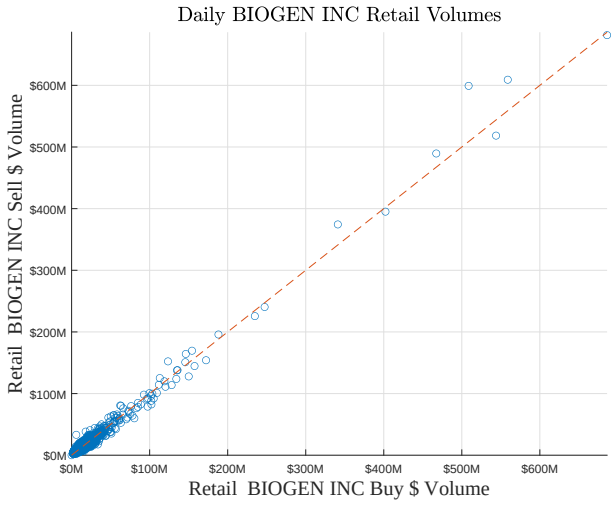


Figure 25: Biogen

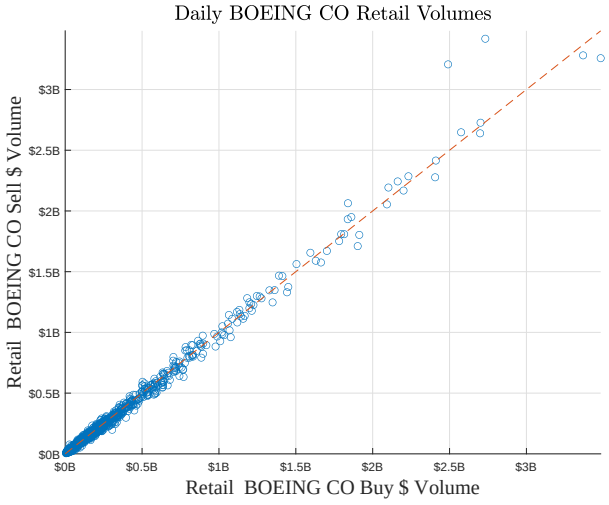


Figure 26: Boeing

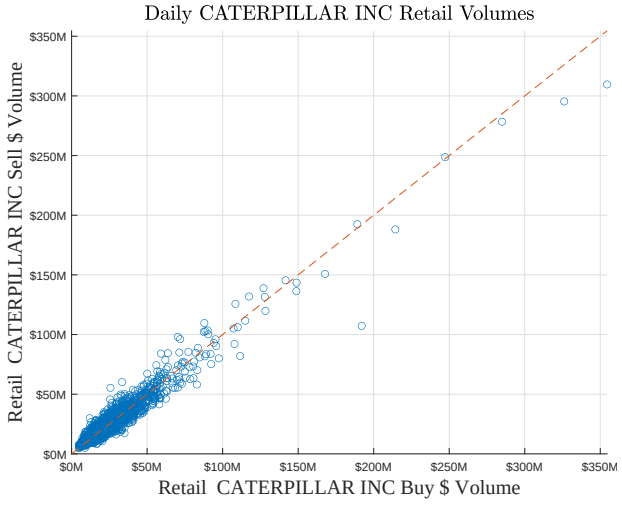


Figure 27: Caterpillar

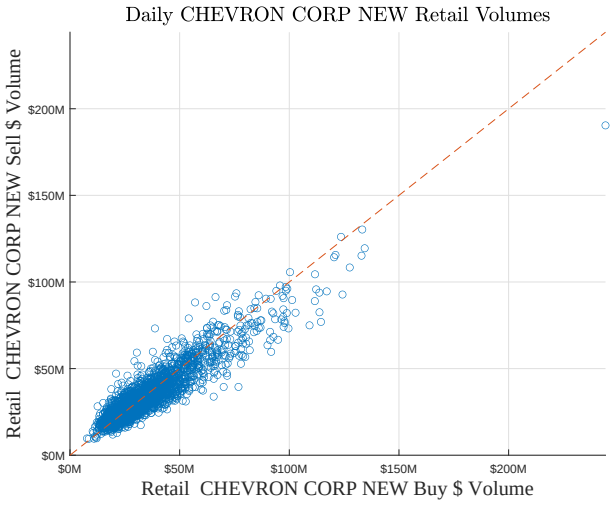


Figure 28: Chevron

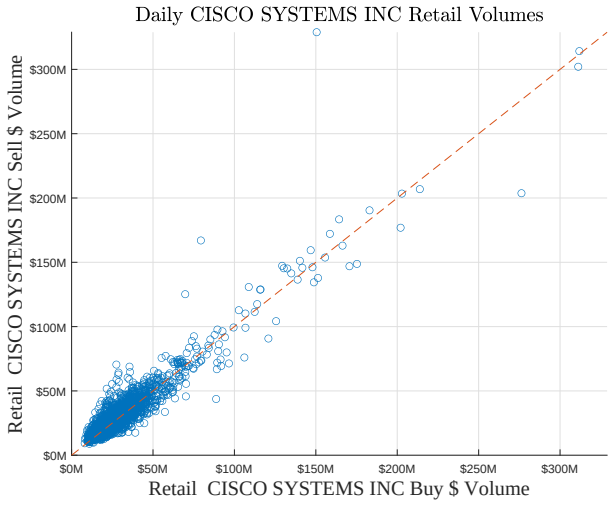


Figure 29: Cisco

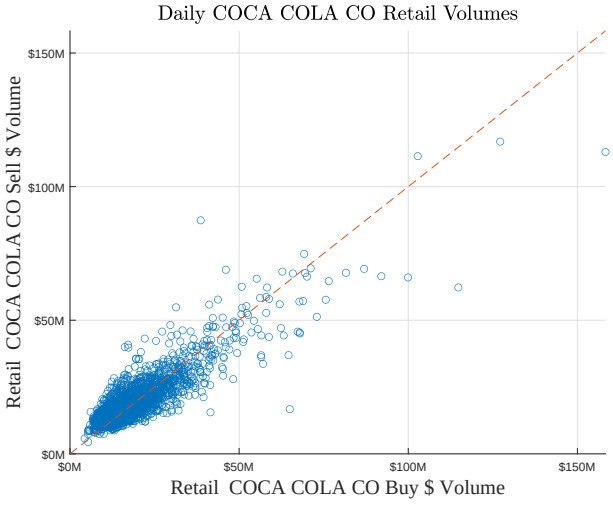


Figure 30: Coca Cola

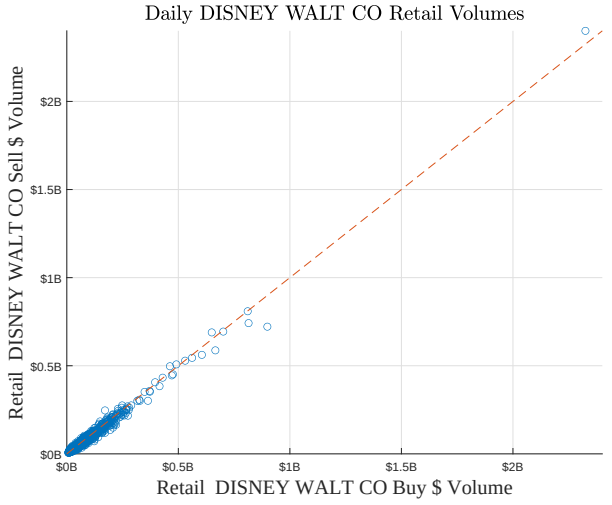


Figure 31: Disney

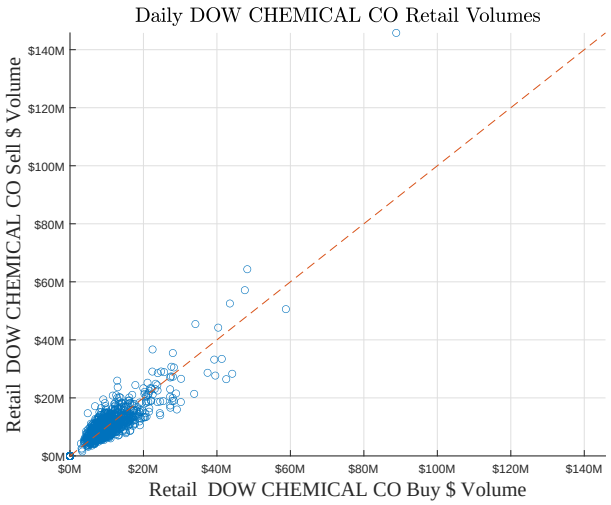


Figure 32: Dow

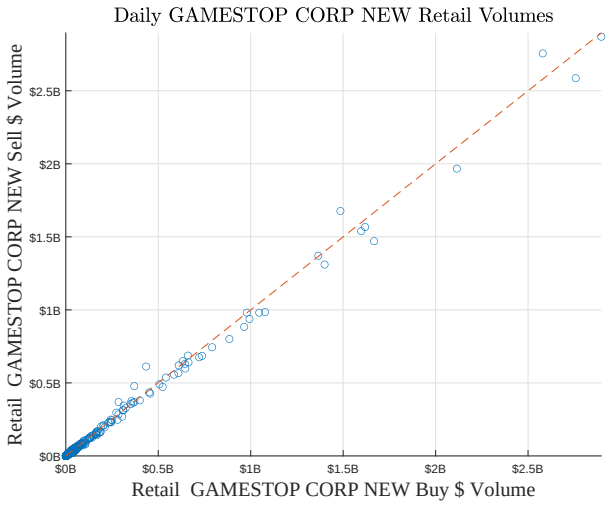


Figure 33: Gamestop

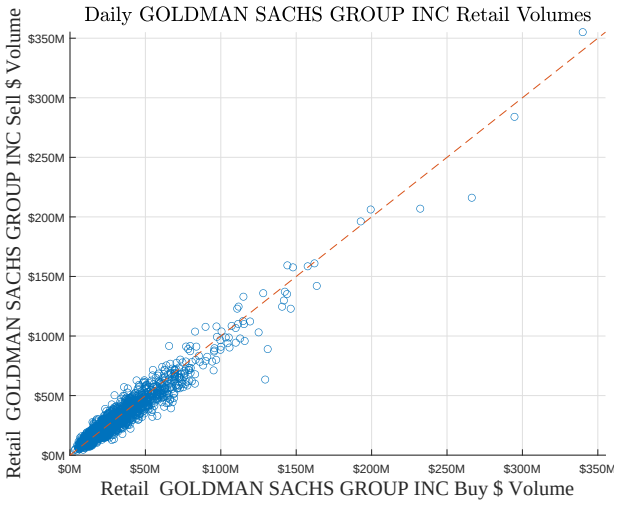


Figure 34: Goldman

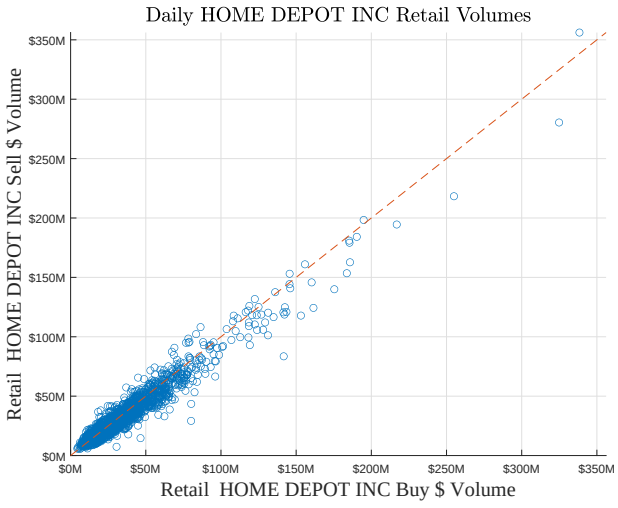


Figure 35: Home Depot

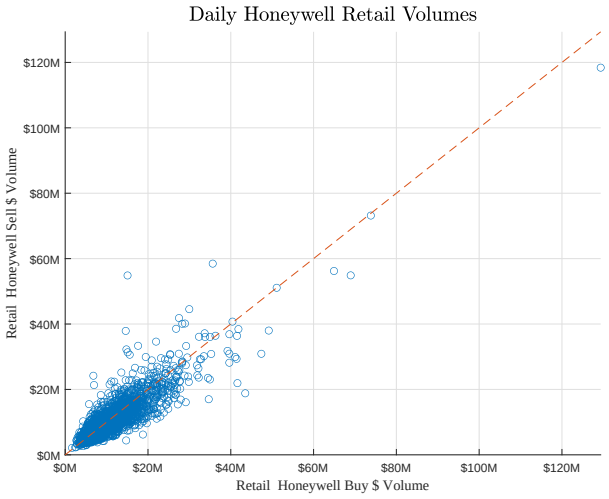


Figure 36: Honeywell

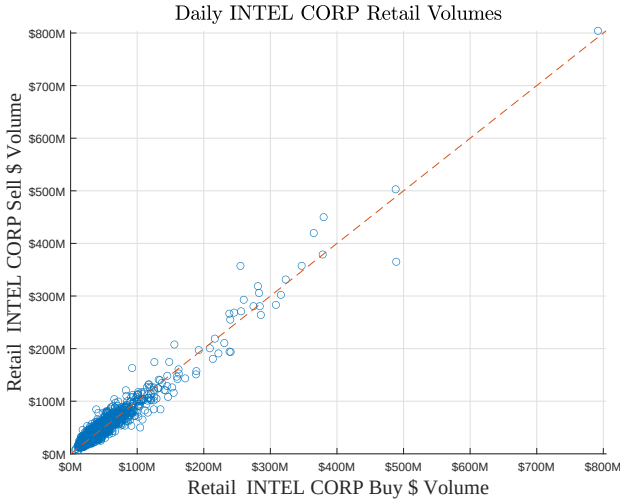


Figure 37: Intel

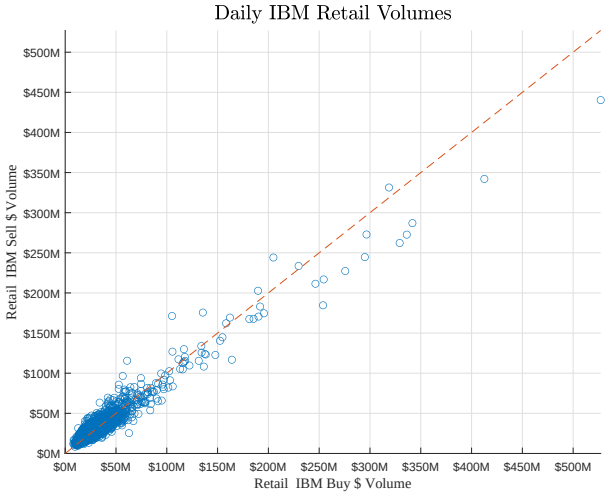


Figure 38: IBM

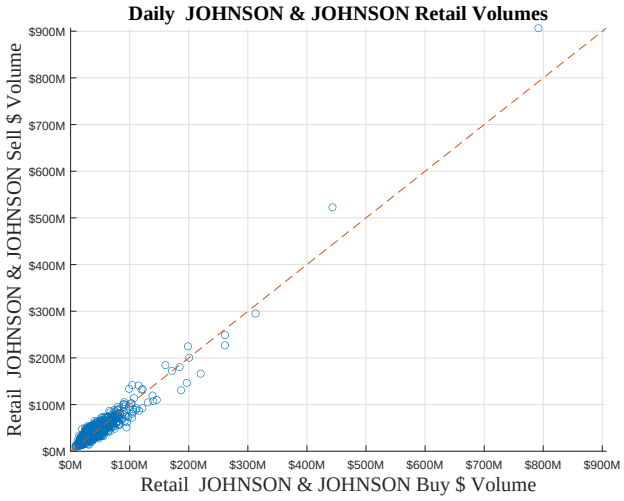


Figure 39: J&J

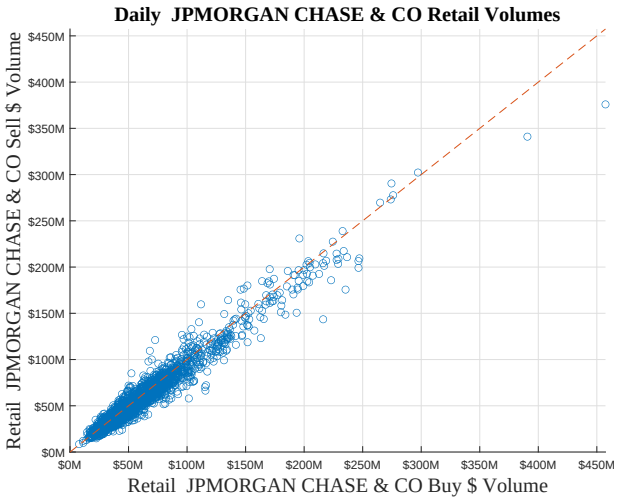


Figure 40: JP Morgan Chase

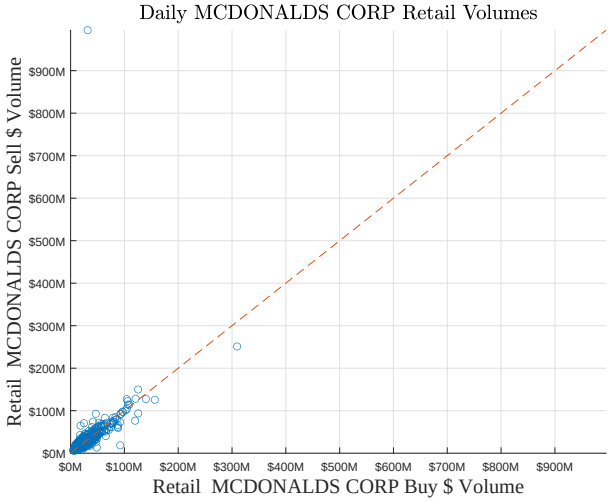


Figure 41: McDonalds

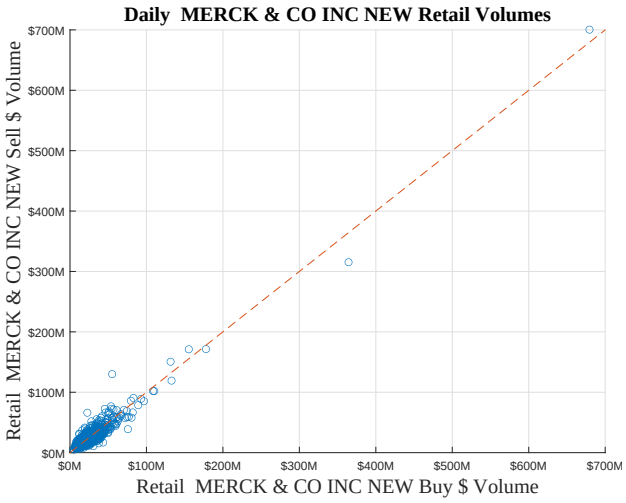


Figure 42: Merck

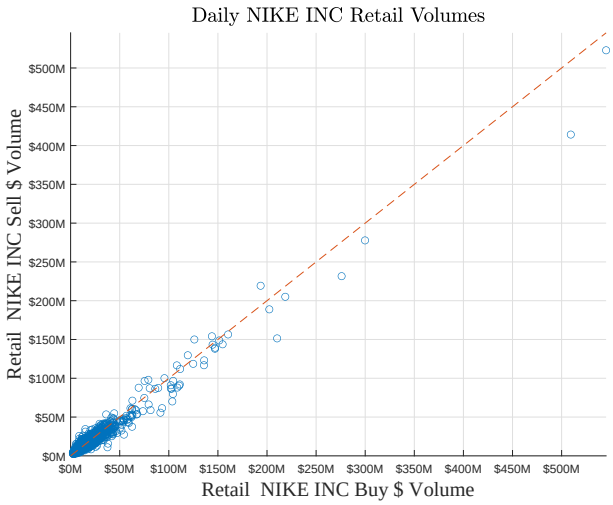


Figure 43: Nike

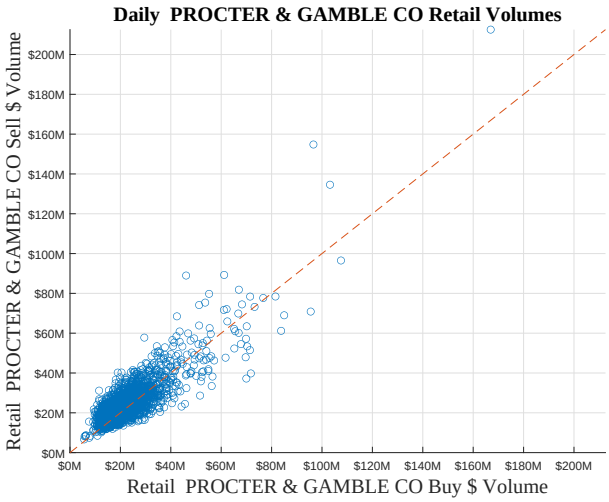


Figure 44: Procter & Gamble

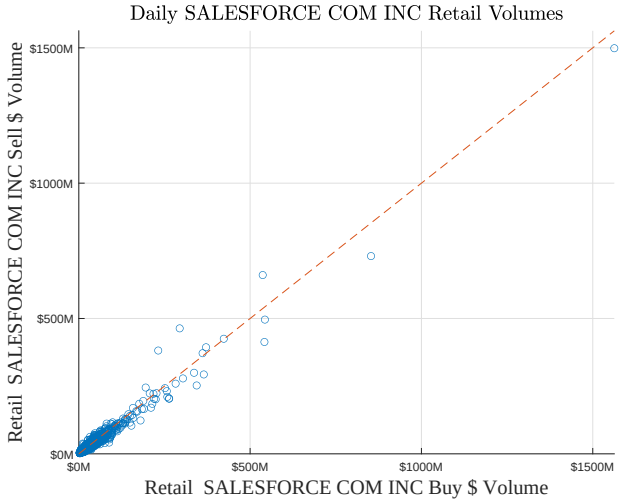


Figure 45: Salesforce

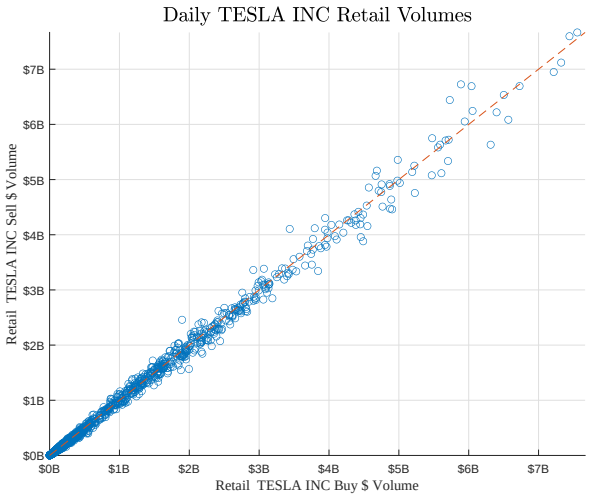


Figure 46: Tesla

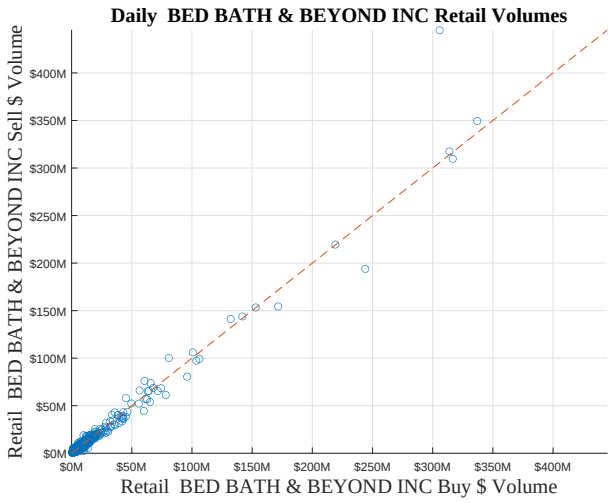


Figure 47: Bed Bath and Beyond

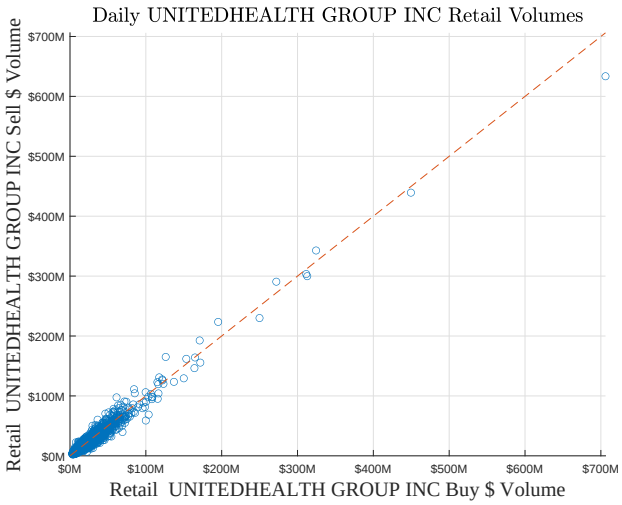


Figure 48: UnitedHealth

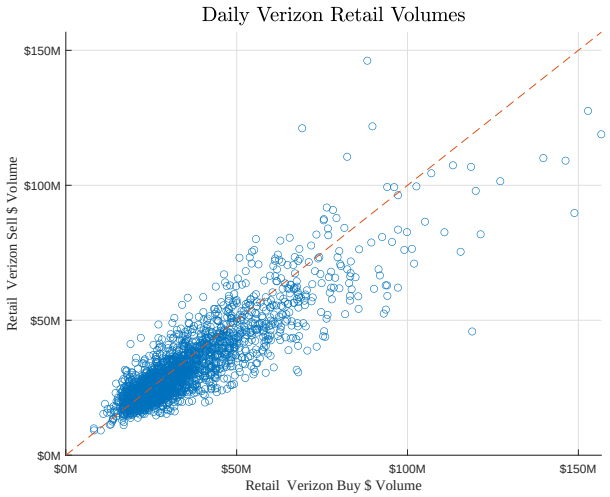


Figure 49: Verizon

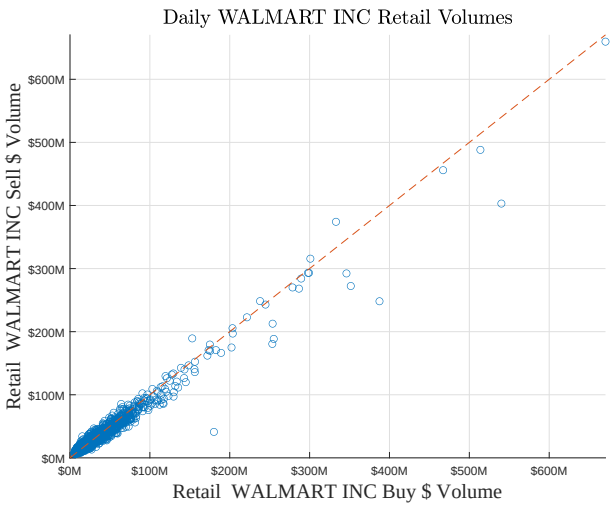


Figure 50: Walmart

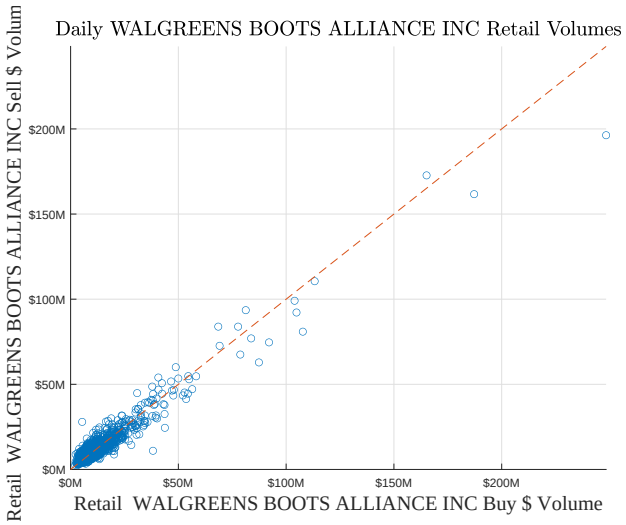


Figure 51: Walgreens

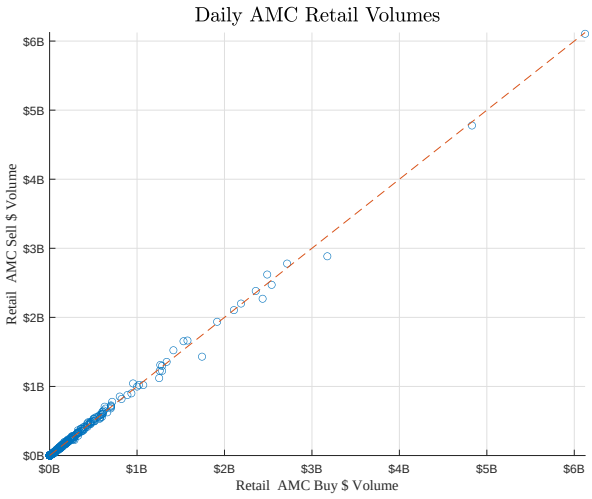


Figure 52: AMC

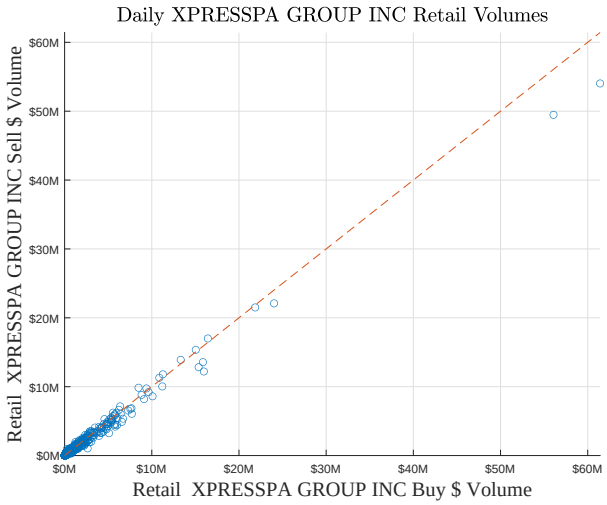


Figure 53: XPresSpa