# Transition to Green Technology along the Supply Chain

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**Abstract**

We analyze a model of green technological transition along a supply chain. In each layer, a good is produced with a dirty technology, or, if the required "greenification" innovation has occurred, with a clean technology which uses the immediate upstream good. We show that the economy is characterized by a single equilibrium but multiple steady-states, and that even in the presence of Pigouvian environmental taxation, a targeted industrial policy is generally necessary to implement the social optimum. We also show that: (i) small, targeted industrial policy may bring large welfare gains; (ii) a government which is constrained in its policy instruments, either unable to subsidize greenification in more than one sector or price carbon at its true social cost, should primarily target downstream sectors; and (iii) when extending the model so as to allow for supply chains also for the dirty technology, overinvesting in greenification in the wrong upstream branch may derail the overall transition towards greenification. Finally, we illustrate our model with a calibration to decarbonization of heavy duty transportation (trucking, aviation, etc.) via hydrogen, and show that, absent industrial policy, the economy can get stuck in a "wrong" steady-state with CO2 emissions vastly above the social optimum even with a carbon price in place.

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# 1 Introduction

There is a growing consensus worldwide of the need to speed up the transition away from fossil fuel energy so as to slow down and eventually curb global warming. However, there is all but a unanimity among policy leaders when it comes to the best choice of climate policy instrument. In Europe, carbon price and carbon taxes appear to hold the upper hand, whereas in the US or China, priority has been given to industrial policy as illustrated by the Inflation Reduction Act.

However, until recently most contributions in the economics of climate change have emphasized the carbon tax as the way to solve the climate problem without paying much attention to other instruments. In particular, despite long-standing recognition that a broader portfolio of policies may be required to address market failures in technological development and adoption, formalizations of such policy portfolios have been comparatively limited.[1] One influential recent attempt at introducing a second leg in the design of climate policy design was Acemoglu et al. (2012), which developed a growth model with directed innovation where both, the carbon tax and subsidies to green innovation, are needed to minimize the social cost of redirecting firms' innovation towards clean technologies. While some authors might already consider green innovation subsidies as a particular instance of industrial policy, no serious attempt so far has been made at rationalizing the use of vertically targeted subsidies to fight climate change. By "vertically targeted subsidies" we mean subsidies that target particular sectors, and/or that treat different sectors differently.

In this paper, we develop a model of technological transition along a supply chain to make the case for sector-specific industrial policy to best address the energy transition problem. In our model, a final consumption good (or service) can be produced either with a dirty or with a clean technology, where the dirty technology directly generates pollution. For example, dirty cars use oil while clean cars use electricity. However, the clean technology uses an upstream input which itself can be produced using a clean or a dirty technology. Again in the car example, the production of electric batteries can be CO2-intensive or not. And, even if the input production uses a clean technology, that technology itself requires a more upstream input which itself can be produced using a cleaner or a dirtier technology. And so, we keep moving upstream to increasingly more basic inputs, where at each stage the choice can be made between the dirty technologies which directly generate pollution, and clean technologies which do not directly generate pollution, but require a more basic input which itself may be produced using either cleaner or dirtier technologies.

More specifically, each layer in the supply chain produces a "good" which is a Cobb-Douglas aggregate of a mass of industrial processes or varieties. Each variety of that good can be produced

[1] For early examples see, e.g., Jaffe and Stavins (1995) or Fischer and Newell (2008).

either in a dirty way using labor only or, if that process has been "greenified", in a clean way using labor and the immediate upstream good. To move from dirty to clean, a variety in a given sector—i.e. a given layer in the supply chain—needs to undergo "greenification".[2] We assume heterogeneous fixed costs of greenification across varieties within a sector, so that some varieties may be greenified at one point in time while others still have to greenify. We also assume that once the social cost of carbon has been taken into account, producing using the clean technology is cheaper than producing using the dirty technology, so that a variety producer will always choose the former once the variety has been greenified, provided that a Pigouvian carbon tax is in place. A producer may choose to incur the fixed greenification cost in exchange for a one-period exclusive right of using the clean technology and Bertrand-compete with competitive producers that use the dirty technology. One period after the greenification of a variety, the clean technology becomes available to all producers, and the production of that variety becomes competitive.

Under these assumptions, the incentive to greenify, for any variety producer in any sector on the supply chain, depends both on the degree of greenification downstream (more greenification downstream, which uses the variety as inputs, increases the demand for greenifying the variety) and on greenification upstream (more greenification upstream, which supplies inputs to the variety, reduces the cost of producing the variety once it has been greenified). The resulting cross-sectoral strategic complementarities in greenification generate a coordination problem: namely, insufficient greenification in sectors downstream or upstream of a given sector reduces the private incentives for variety producers in that sector to greenify. While our model is set in the context of the green transition, its insights generalize to any situation where the economy may switch from one technology to another superior one, that requires the development of its own supply chain: cross-sectoral strategic complementarities may lead to insufficiently low adoption without policy intervention.[3]

We characterize the equilibrium in the absence of industrial policy. We show that the equilibrium achieved at any particular date is unique for given initial conditions, but that the cross-sectoral strategic complementarities in greenification generally lead to a multiplicity of steady-states.

We then characterize the social optimum, and show that it generally differs from the decentralized solution even once emissions are optimally priced through a Pigouvian tax. Abstracting from the difference in the time horizon of the social planner and private agents, the key source of inefficiency is that the decentralized economy may be stuck in a local steady-state which is

[2]Greenification is a stand-in here for a process that allows replacing fossil fuels as a production inputs with other inputs that do not directly generate emissions. In practice, this often but not always involve replacing fossil fuels with electricity or hydrogen.

[3]For instance, one could re-cast our model from being about the green transition to being about the adoption of a modern technology that relies on complex inputs versus a traditional technology that uses a simpler production process. In that sense, our model introduces supply chains in Zeira (1998).

not the steady-state corresponding to the global optimum. As a result, the social optimum may differ from the equilibrium with Pigouvian emission taxes even though the private incentives to greenify are locally in line with the social incentives. Yet we show that the socially optimal steady-state is uniquely implemented through the combination of a Pigouvian tax with a whole set of time-varying sector specific subsidies.

Even though we have an environment that displays complementarities as in other models, e.g. the Big Push model of Murphy et al. (1989), the fact that our complementarities occur along the supply chain leads to drastically different insights.

First, our model generates the possibility that a small and temporary subsidy to greenification that targets key-sectors can be sufficient to achieve large welfare gains by moving the decentralized equilibrium just a little out of an inefficient steady-state and then relying on market forces to complete the transition towards the socially efficient steady-state: then, large, sustained interventions across all sectors are not needed.

Second, our framework has implications for how to prioritize public intervention between upstream and downstream. In particular, a government which is constrained to focus its subsidies to greenification in a vertical—or more generally an acyclical—supply chain on one particular sector, should primarily target downstream sectors, the reason being that greenification propagates more easily upstream (through the demand channel) than downstream (through the cost channel). This stems from the fact that, while increasing greenification in a downstream sector generates proportional gains in greenification incentives in more upstream sectors (the demand channel), increasing greenification in an upstream sector instead generates less than proportional gains in greenification incentives in more downstream sectors (the cost channel).[4] In particular, we develop an example showing that a one-shot subsidy to greenification that targets the most upstream sector in a supply chain with at least three layers, does not allow greenification to propagate to more downstream sectors.

Furthermore, a second realistic policy restriction, where carbon prices are set below the true social cost of carbon, provides additional justification for targeting downstream greenification. We show how to adjust industrial policy in the face of suboptimal carbon prices and find that greenification subsidies should incorporate the effect of greenification on emissions. This emission-reduction motive for greenification favors downstream greenification because low initial greenification prevents upstream sectors from influencing the level of emissions in equilibrium. When downstream sectors are not greenified, greenifying upstream sectors will make little difference as they will face no demand for their newly greenified goods. Thus, our model provides a robust set of reasons for prioritizing downstream greenification in the face of restricted policy instruments.

[4]This in turn is due to the fact that the clean technology at all layers in the supply chain does not only use the more upstream input but also labor which captures a positive share of the benefit from the cost reduction induced by the initial greenification shock.

Third, we rationalize the possibility of horizontally misdirected clean industrial policies, with resulting welfare losses. We illustrate this point in an extended version of our model with two layers, where, on top of using labor, the dirty technology for producing varieties in the downstream sector also uses inputs from another upstream sector that can be greenified. In that context, greenification in the two upstream sectors—for the dirty input and for the clean input to downstream varieties—are strategic substitutes. Over-investing in the greenification of the upstream sector associated with the dirty technology downstream may derail the overall transition towards greenification downstream.

Although this is primarily an applied theory paper, in the last section we provide a very preliminary attempt at mapping theory with data. More specifically, we present a numerical application of our model to global iron and steel production, a sector that accounts for 7-9% of global carbon dioxide emissions (Kim et al. 2022). One of the most promising options for deep decarbonization in this sector is hydrogen-based direct reduction-electric arc furnace ("H2-DR-EAF") production (Delvin et al. 2023; BloombergNEF 2021). Hydrogen, in turn, is currently produced mainly from fossil fuels (IEA 2021) but can also be produced from electrolysis powered by renewable energy. We thus model the interplay between upstream hydrogen and downstream iron and steel production. The benchmark calibration confirms the quantitative importance of the multiplicity of steady states: A uniform carbon price of $25/tCO2, for example, is consistent with three steady states: (0% clean H2, 0% clean steel), (59% clean H2, 54% clean steel), and (82% clean H2, 84% clean steel). The stakes of being stuck in the "wrong" steady state are enormous: even at current levels of global steel production, the difference in annual emissions between the (0%, 0%) and (82%, 84%) scenarios is upwards of 2.4 billion tons of CO2 per year, close to the entirety of the European Union's CO2 emissions from all sectors (2.8 billion tons in 2022, EDGAR 2023). The difference between the (59%, 54%) and (82%, 84%) scenarios is similarly large at around1 billion tons of CO2 per year.

Our paper relates to several strands of literature. First, on the macroeconomics of climate change, starting with the so-called "Integrated assessment models" literature (IAMs) initiated by Nordhaus (1994), and pursued more recently, e.g. by Golosov, Hassler, Krusell, and Tsyvinski (2014). However, this literature often takes technological change as given, and their emphasis is on the optimal design of carbon tax policies. More closely related to our analysis is the literature on directed technical change and the environment, in particular Acemoglu, Aghion, Bursztyn, and Hémous (2012), henceforth AABH, who show that the optimal climate policy in the presence of endogenous directed innovation, amounts to the combination between a carbon tax and a green research subsidy. The model in AABH relies on knowledge externalities, not on cross-sectoral strategic complementarities and coordination, as it does not model clean innovations along the

supply chain.[5]

Second, a set of recent contributions study the positive properties of environmental policies in static production networks with carbon emissions, a setting where the optimal intervention remains a uniform carbon tax (e.g., see King, Tarbush, and Teytelboym 2019; Devulder and Lisack 2020).[6] Even though our model also features a production network with environmental externalities, our analysis differs fundamentally from these papers because our setting features endogenous technological change, and the key policy issue is to address the dynamic strategic complementarity of technology adoption along the supply chain, and not the static environmental externalities. In this regard, we relate to papers on strategic complementarities in technology adoption, including the seminal work of Murphy et al. (1989), recent work by Sturm (2023), and in an environmental setting, Greaker and Midttømme (2016), Dugoua and Dumas (2021, 2024), and Smulders and Zhou (2024). While these papers study static models featuring multiple equilibria, which poses a difficulty for analyzing the impact of policy interventions, our dynamic model has multiple steady-states but a unique equilibrium path; our model therefore has unambiguous predictions on industrial policy's equilibrium impact.[7] This feature enables our model to maintain tractability—despite rich strategic interactions along the supply chain—and generate new insights on how industrial policy could target key sectors to aid the technological transition.

A third related strand is the literature on industrial policy.[8] A first rationale for industrial policy is the infant industry argument, recently modeled by Greenwald and Stiglitz (2006), which emphasizes "learning by doing" externalities between the nascent domestic industry and the agricultural sector. Another justification is the existence of cross-sectoral demand externalities: here, a classical reference is Murphy et al (1989) who model Rosenstein-Rodan (1943)'s Big Push idea, namely, investments in increasing returns technologies in some markets or sectors, generates additional income which increases the incentives for industrialization in other sectors through a demand channel. However, none of these papers considers supply chains and the associated motives for sectoral policies.

Closer to our paper is Liu (2019), who analyzes the role of sector-specific policy interventions in a production network setting with market imperfections and finds that targeting upstream

[5] Acemoglu et al. (2016) present another model of directed technical change in the environmental context, which is perhaps closer to ours. In their model, intermediates are aggregated in a Cobb-Douglas fashion and can be produced with a clean or a dirty input (as in our model), however, there is no supply chain or coordination issues.

[6] See Martin, Muuls, and Stoerk (2023) for an empirical evaluation of carbon pricing's effect along supply chains.

[7] In our input-output setting, demand externalities travel in one direction (upstream's demand becomes upstream's market size), whereas cost externalities travel in the other direction (upstream's lower prices becomes downstream's lower costs). Because of this distinction, our one-period Bertrand pricing assumption (i.e., producers who greenify enjoys one period profit) ensures that the externalities travel only through one direction within a period, i.e., through demand and market size, instead of traveling through both directions. This is key for having a unique equilibrium in our setting. Also see Crouzet, Gupta, and Mezzanotti (2023) for a dynamic model of strategic complementarity in the context of adopting electronic payments.

[8] For an illuminating recent survey on the industrial policy literature, see Juhasz, Lane, and Rodrik (2023).

sectors—as was done in South Korea and China—could lead to aggregate welfare gains. Relatedly, Liu and Ma (2023) investigate the optimal cross-sector allocation of R&D in the context of an endogenous growth framework that incorporates an innovation network, featuring knowledge spillovers across technologies. Donald (2023) embeds the innovation network à la Liu and Ma (2023) into AABH's model of directed innovation between clean and dirty technologies and studies the policy implications. Contemporaneous work by Buera and Trachter (2024) studies the role of industrial policy in a static production network model with endogenous technology adoption. Compared to all of these papers, the unique feature of our environment is the strategic complementarity of technology adoption *across* different sectors along the supply chain. As we show, such strategic complementarity has strong implications for the efficacy and targeting of industrial policy, the analysis of which is the focus of this paper.

The remaining part of the paper is organized as follows. Section 2 outlines our baseline model of supply chain and greenification. Section 3 derives the equilibrium equations and establishes both, the uniqueness of an equilibrium for given initial conditions and the possibility of multiple steady-states. Section 4 derives the equations for the social optimum. It compares the social optimum with the steady-states of an economy without industrial policy. It then shows how the social optimum can be implemented by combining a Pigouvian carbon tax with suitably a chosen time-varying set of sector-specific subsidies. Section 5 analyzes the propagation of an exogenous one-sector greenification shock, and how this propagation depends upon whether the selected sector is more downstream or upstream. Section 6 considers how industrial policy should adjust when carbon prices fail to provide a full Pigouvian correction and shows how greenification in different sectors vary in their effect on carbon emissions. Section 7 analyses the possibility of horizontally misguided—and consequently backfiring—industrial policy, using an extension of our baseline model where the dirty technology for producing the consumption good also uses an upstream input that can be produced with a clean or a dirty technology. Section 8 presents the quantitative application. Section 9 concludes.

# 2 Model

In this section, we present our baseline model of a vertical supply chain in a green transition.

## 2.1 Preferences and production technology

Time is discrete and denoted by $t$. The consumer demand side of the economy consists of a continuum of mass one of agents with the same intertemporal utility

$$\sum_{t=0}^{\infty} \beta^t \left( \ln c_t - \ell_t - a_t \right) \tag{1}$$

where $c_t$ denotes the consumption flow, $\ell_t$ denotes the representative individual's labor supply, $a_t$ denotes the disutility of pollution, at time $t$, and $\beta$ is the discount factor.

The production side is a vertical supply chain consisting of $N$ layers or "sectors" which we rank from the most upstream, namely $i$ =1, to the most downstream, $i = N$, which corresponds to the consumption good. Production $y_{it}$ in each sector $i$ at time $t$ is the aggregate outcome of a continuum of mass one of sector-specific varieties ($\nu$), according to:

$$\ln y_{it} = \int_0^1 \ln y_{it}(\nu) \, \mathrm{d}\nu.$$

In turn each variety $\nu$ in any sector $i$ can be produced using either a dirty or a clean technology. Three features distinguish clean and dirty technologies. First, the dirty technology is associated with pollution while the clean one is not. Second, the dirty technology is always available, while the clean technology is only available for a variety $\nu$ in sector $i$ that is "greenified"—greenification occurs through a process described below. And third, the dirty technology only uses labor (one for one), while the clean technology uses good $i-1$ as an intermediate input together with labor in a Cobb-Douglas fashion. More specifically, we assume that the most upstream input is produced according to:

$$y_{1t}(\nu) = \ell_{d1t}(\nu) + \gamma_{1t}(\nu) \, e^z \ell_{c1t}(\nu),$$

and that for all $i > 1$ :

$$y_{it}(\nu) = \ell_{dit}(\nu) + \gamma_{it}(\nu) \left( \frac{e^z \ell_{cit}(\nu)}{\alpha_i} \right)^{\alpha_i} \left( \frac{m_{it}(\nu)}{1-\alpha_i} \right)^{1-\alpha_i}, \tag{2}$$

where: (i) $\gamma(\nu)$ is an indicator function, equal to 1 for greenified varieties and to 0 for non-greenified varieties; (ii) $\alpha_i \in (0,1)$ for all $i > 1$; (iii) for all $i$, $\ell_{dit}(\nu)$ denotes the labor input used by variety $\nu$ in sector $i$ using the dirty technology; (iv) for all $i$, $\ell_{cit}(\nu)$ and $m_{it}(\nu)$ denote respectively the labor input and the amount of intermediate input from sector $i-1$ used for producing variety $\nu$ with the clean technology; (v) $e^z$ is the relative (labor-augmenting) productivity of using the clean rather than the dirty technology.

We assume that producing each unit of output using the dirty technology generates $\xi$ units of disutility due to pollution. The total disutility of pollution is thus $a_t = \xi \ell_{dt}$, where $\ell_{dt}$ is the total labor input used for dirty production in the economy, and $\xi$ is the social unit cost of pollution.[9]

[9]This formulation is equivalent to one where production with the dirty technology uses a free and inexhaustible

Our initial focus is therefore on a vertical supply chain with strategic complementarities in greenification across sectors: this case is both theoretically interesting and empirically relevant, as switching toward green technologies requires thinking specifically about how really clean the inputs of clean technologies are (as illustrated by the debate around the high emission intensity of EV batteries). Yet, as we shall see below, our analysis and results apply beyond vertical supply chains. First, we can relax the simplifying assumption that the relative productivities $z$ and emission rates $\xi$ are the same for all sectors as shown in our numerical exercise in Section 8 below. Second, our insights carry over to more complex networks for the green production process as analyzed in Sections 5.3 and 6.4. Third, our assumption that only the clean technology uses a supply chain is less stringent than it may appear at first sight: our model is isomorphic to one where there is a separate complex network for the dirty production process without greenification, and in fact, our qualitative insights still carry through if the dirty production process shares some inputs with the clean production process but benefits less from upstream greenification.[10] Instead, greenification across some sectors becomes strategic substitutes if the greenification of some inputs benefits dirty production more than clean production, a case we analyze in Section 7.

## 2.2 Market clearing

Labor market clearing requires that, at any time $t$ :

$$\ell_{ct} = \sum_{i=1}^{N} \ell_{cit} = \sum_{i=1}^{N} \int_0^1 \ell_{cit}(\nu)\, \mathrm{d}\nu, \quad \ell_{dt} = \sum_{i=1}^{N} \ell_{dit} = \sum_{i=1}^{N} \int_0^1 \ell_{dit}(\nu)\, \mathrm{d}\nu, \quad \ell_t = \ell_{ct} + \ell_{dt} + \ell_{et},$$

where: $\ell_{ct}$ and is the total labor used for clean production (summing across all sectors $i = 1, \ldots N$ and integrating across varieties within each sector); $\ell_{dt}$ is the total labor used for dirty production; and $\ell_{et}$ is the total labor employed for greenification (we specify the greenification technology below); and $\ell_{cit}$ and $\ell_{dit}$ are the total labor inputs used for clean and dirty usage in sector $i$.

Market clearing in the upstream sectors $i = 1, \ldots, N-1$, requires that for each $i$ the total output of sector $i$ be equal to the total use of it as intermediate inputs across varieties in sector

fossil fuel resource together with labor in a Leontief fashion. Alternatively, the model can accommodate extraction costs for the resource if $\xi$ includes both environmental damages and the extraction costs and the carbon tax $\tau$ introduced below includes both the tax itself and the extraction cost.

[10] With a separate dirty network that features no greenification (i.e. $\gamma(v)$ is constant for all sectors that are input in the dirty production process), the different dirty sectors can simply be aggregated along the value chain and this economy is de facto identical to one where only labor is used for the dirty technology (potentially with heterogeneous $z$ and $\xi$). If some inputs which can be greenified are common to the clean and the dirty production process, greenification across sectors remains strategic complement, delivering similar insights to the ones we develop here, as long as the dirty production process relies less on these common inputs.

$i+1$:

$$y_{i,t} = \int_0^1 m_{i+1,t}(\nu)\ \mathrm{d}\nu$$

whereas market clearing for the downstream consumption good $N$ simply boils down to:

$$c_{Nt} = y_{Nt}.$$

## 2.3 Greenification and market power

To operate the clean technology, the variety must be "greenified". For any variety $\nu$ in any sector $i$, the producer of that variety must incur a one-time sunk cost which is sector-variety specific to greenify production of the variety. We order varieties in any sector $i$ by increasing cost of greenification and therefore we let $\phi_i(s)$ denote the cost of greenification associated with the variety quantile $s$ in sector $i$, where that cost is expressed in labor units.[11]

We let $F_i(\cdot)$ denote the CDF cost distribution, that is, for any cost $\phi$, $F_i(\phi)$ is the measure of the set of variety quantiles $s$ in sector $i$ with costs $\phi_i(s)$ less than $\phi$. A special case we shall consider is the case where the distribution of greenification costs across varieties in sector $i$ is a mass point at some cost level $\phi_i$. Let $\chi_{it}$ denote the fraction of greenified varieties at time $t$ in sector $i$, which also corresponds to the cut-off quantile $s$ beyond which varieties cease to be greenified. The sum of all greenification fixed costs up to $\chi_{it}$ is given by $\mathcal{F}_i(\chi_{it}) \equiv \int_0^{\chi_{it}} \phi_i(s)\ \mathrm{d}s$.[12] The collection of $\chi_{it}$'s across sectors form the key state variables of the economy. Starting from an initial condition $\{\chi_{i0}\}_{i=1}^N$ at $t = 0$, greenification raises $\chi_{it}$'s monotonically over time, until $\chi_{it}$'s converge to a steady-state.

We assume that the dirty technology is operated competitively, but that, if she greenifies her variety, the producer of that variety earns a one-period monopoly profit upon greenification, with fringe producers operating the dirty technology as competitors. Starting from the subsequent period, the clean technology becomes freely available to all producers so that the variety is again competitively produced.

## 2.4 Policy instruments

We assume that the government can: (i) impose a carbon tax $\tau$ on any variety which uses the dirty technology; (ii) impose a cap-and-trade limit $\bar{\ell}_d$ on the amount of dirty input used by any variety (leading to a carbon price $\tau$); (iii) (industrial policy) implement a set of sector-specific greenification subsidies $\kappa_i$. All of these policy instruments are allowed to be time-varying, and

[11] Our model of greenification is directly inspired by the automation literature where after an innovation or payment of a fixed cost, labor can be replaced by capital in the production process (see e.g., Zeira 1998).

[12] There is an obvious relationship between $F_i$ and $\mathcal{F}_i$. Namely, for any $s$ : $F_i(\phi_i(s)) = s$ or equivalently $\phi_i(s) = F_i^{-1}(s)$. This directly leads to $\mathcal{F}_i(\chi_{it}) = \int_0^{\chi_{it}} F_i^{-1}(s)\,\mathrm{d}s$.

our analysis will focus on showing that the carbon tax and/or the cap-and-trade alone do not implement the optimum, whereas suitable sector-specific greenification subsidies $\kappa_i$ can implement the optimum.

# 3 Equilibrium

In this section we characterize the equilibrium of the economy if it only resorts to a carbon tax.

## 3.1 Equilibrium price

In each period, the representative consumer solves

$$\max_{c_t,\ell_t} \ln c_t - \ell_t - a_t \qquad \text{s.t. } p_t c_t = w_t \ell_t + \pi_t$$

where $p_t$ is the price index of the consumption good, $\pi_t$ is the net transfer from monopolistic producers and the government, and $w_t$ is the wage rate with we normalize to one. Consumer optimization implies that the total expenditure on the consumption good is equal to one, that is:

$$p_t c_t = 1. \tag{3}$$

Given that dirty production uses only labor one-for-one, then, absent policy interventions the marginal cost of the dirty technology is equal to the wage, namely 1. If instead the government taxes pollution (or imposes a binding cap-and-trade limit), the marginal cost of dirty production is $1+\tau$, with $\tau > 0$. We assume that $\tau$ and/or $z$ are large enough to ensure that $1+\tau > e^{-z}$. This condition ensures that a producer always uses the clean technology once her variety has been greenified. For notational simplicity, we define $Z \equiv \ln(1+\tau) + z$, the tax-adjusted relative productivity of clean versus dirty technology, and we assume that $Z > 0$.

The equilibrium price index $p_{it}$ for good $i$ satisfies:

$$\ln p_{it} \equiv \int_0^1 \ln p_{it}(\nu)\, \mathrm{d}\nu, \tag{4}$$

where, for each variety $\nu$ in sector $i$, $p_{it}(\nu)$ is determined as follows. If variety $\nu$ has been greenified in the previous period, then it is priced at the marginal cost associated with the clean technology . If variety $\nu$ has not been greenified in the previous period, then it is priced at the marginal cost of a dirty producer. This is trivial for producers using the dirty technology, but it is also true for a newly greenified variety. The producer of a newly greenified variety is a monopolist for the clean technology facing a fringe that uses the dirty technology. Given the unit demand elasticity, she will charge a price equal to the marginal cost of the fringe.

The market structure implies that in the most upstream sector (sector 1), the price of a variety that has been greenified in the previous period is equal to $p_{1t}(\nu) = e^{-z}$, whereas the price of a

non greenified or of a newly greenified variety is $p_{1t}(\nu) = 1 + \tau$.

Next, in sectors $i > 1$, we have:

$$p_{it}(\nu) = \begin{cases} e^{-\alpha_i z} p_{i-1,t}^{1-\alpha_i} & \text{if the variety has been greenified by time } t-1, \\ 1+\tau & \text{otherwise,} \end{cases} \tag{5}$$

as the marginal cost of producing dirty is $1 + \tau$, while the marginal cost of producing with the clean technology is $e^{-\alpha_i z} p_{i-1,t}^{1-\alpha_i}$.

To get more explicit expressions for these equilibrium prices, it will be helpful to consider, for all $i$, the network adjusted share $\mu_{it}$ of greenified content in the production of any greenified variety in sector $i$. In the most upstream sector 1, we obviously have $\mu_{1t} = 1$. In more downstream sectors $i > 1$, $\mu_{it}$ is recursively determined by:

$$\mu_{it} = \alpha_i + (1 - \alpha_i)\, \chi_{i-1,t} \mu_{i-1,t} \tag{6}$$

In words, the greenified content of an greenified variety in sector $i$, is equal to the direct share of clean labor input $\alpha_i$ plus $(1 - \alpha_i)$ times the aggregate greenified content of input $i - 1$, which in turn is equal to the greenified content $\mu_{i-1,t}$ of each greenified variety in sector $i - 1$ times the fraction $\chi_{i-1,t}$ of greenified varieties for good $i - 1$.

We can now solve for the price index in each sector. For the most upstream sector 1, we can use (4) and (5) to get:

$$p_{1t} = (1 + \tau)\, e^{-\chi_{1,t-1} Z}.$$

Then moving downstream, we get by induction:

$$p_{it} = (1 + \tau)\, e^{-\chi_{i,t-1} \mu_{it-1} Z}. \tag{7}$$

## 3.2 Equilibrium profits

The incentive to greenify depends on the profits that a newly greenified variety obtain. We now derive these profits. The producer of a newly greenified variety charges a price $1 + \tau$ but faces marginal costs equal to $e^{-\alpha_i z} p_{i-1,t}^{1-\alpha_i}$, therefore she charges a mark-up $\theta_{i,t}$ given by

$$\theta_{i,t} = \frac{1+\tau}{e^{-\alpha_i z} p_{i-1,t}^{1-\alpha_i},} = e^{Z \mu_{it-1}},$$

where the second equality uses equations (7) to substitute for $p_{i-1t}$ and (6). The share of revenues going to profits is $1 - \theta_{i,t}^{-1}$ and the share going to input costs is $\theta_{i,t}^{-1}$.

The next step is to derive the equilibrium revenue $r_{it}$ of a producer in sector $i$ at time $t$. In the most downstream sector $N$, which produces the final consumption good, we already know that:

$$r_{Nt} = p_t c_t = 1.$$

From there we move upstream, as revenues trickle up from the most downstream to the upstream sectors. Take as given the revenues $r_{i+1,t}$ of a producer in sector $i+1$ at time $t$. Then, sector $i$'s good is only used as an input by the greenified varieties in sector $i+1$. For any variety in sector $i$,the revenue $r_{it}$ includes both sales to previously greenified varieties and sales to newly greenified varieties. There is a mass $\chi_{i+1,t-1}$ of previous greenified varieties. These are produced competitively, so that a share $1-\alpha_{i+1}$ of their revenues go to sector $i$. There is a mass $\chi_{i+1,t} - \chi_{i+1,t-1}$ of newly greenified varieties, only a share $\theta_{i+1,t}^{-1} = e^{-Z\mu_{i+1.t-1}}$ of their revenues go to the payment of inputs, out of which a share $1-\alpha_{i+1}$ go to sector $i$. We then obtain:

$$\begin{aligned} r_{it} &= \underbrace{\chi_{i+1,t-1} r_{i+1,t} \left(1-\alpha_{i+1}\right)}_{\text{sales to previously greenified varieties}} + \underbrace{\left(\chi_{i+1,t} - \chi_{i+1,t-1}\right) r_{i+1,t} e^{-Z\mu_{i+1,t-1}} \left(1-\alpha_{i+1}\right)}_{\text{sales to newly greenified varieties}} \\ &= \widetilde{\chi}_{i+1,t} r_{i+1,t} \left(1-\alpha_{i+1}\right) \end{aligned}$$

where we define the share of revenues of sector $i+1$ spent on inputs for the clean technology:

$$\widetilde{\chi}_{i+1,t} \equiv \chi_{i+1,t-1} + \left(\chi_{i+1,t} - \chi_{i+1,t-1}\right) e^{-Z\mu_{i+1,t-1}}. \tag{8}$$

This in turn immediately yields the following expression for the equilibrium revenue accruing from downstream to good $i$ production:

$$r_{it} = \prod_{j=i+1}^{N} \left(\widetilde{\chi}_{jt}(1-\alpha_j)\right). \tag{9}$$

The corresponding profit rent from greenification for any variety producer in sector $i$, is then simply equal to the profit share times revenues:

$$\pi_{it} = (1 - e^{-Z\mu_{it-1}}) r_{it} \tag{10}$$

## 3.3 Greenification

A variety producer in sector $i$ greenifies at time $t$ if and only if $\pi_{it}$ is bigger than the cost of greenification of that variety. It then immediately follows that the equilibrium share of greenifying varieties $\chi_{it}$ in sector $i$ at time $t$ satisfies:

$$\chi_{it} = F_i(\pi_{it}) = F_i\left(\left(1 - e^{-Z\mu_{it-1}}\right) \prod_{j=i+1}^{N} \left(\widetilde{\chi}_{jt}(1-\alpha_j)\right)\right) \tag{11}$$

whenever this is greater than $\chi_{it-1}$ and to $\chi_{it-1}$ otherwise, as there is no dis-greenification.

As equation (11) shows very clearly, the incentive of a variety producer in sector $i$ to greenify depends positively upon both upstream and downstream greenification. On the one hand, (past and contemporaneous) downstream greenification increases total revenue to good $r_{it}$ accruing to good $i$ production, and on the other hand (past) upstream greenification increases $\mu_{i,t-1}$ and therefore the rent share $\left[1 - e^{-Z\mu_{it-1}}\right]$ to a newly greenified producer in sector $i$.

Two further remarks can be made at this point. First, there are complementarities in greenification across sectors which will be a source of multiplicity of steady-states. As we shall see below, absent adequate industrial policy nothing guarantees that the economy will converge to the equilibrium with maximum greenification.

Second, note that the incentives to greenify travel upstream contemporaneously whereas they travel downstream with a one period lag. This features ensures that, given any initial condition $\{\chi_{i0}\}$ the model features a unique equilibrium path $\{\chi_{it}\}_{t\geq 0}$. We now explain the timing of how incentives to greenify travel along the supply chain. Moving upstream: as downstream sectors $i+1$ to $N$ greenify they increases the revenue accruing to the immediate upstream sector $i$ which in turn also increases the incentive to greenify in that sector: this is captured by the $\prod_{j=i+1}^{N}\widetilde{\chi}_{jt}$ factor in $r_{it}$. Moving downstream: as variety producers in sector $i$ greenify, the cost of production of good $i$ declines, which in turn leads to the decline in the price of that good next period once the monopoly power of the greenifying producers in sector $i$ has expired. This in turn increases the profit margin of greenifying producers in the more downstream sector $i+1$, thereby inducing more greenification in that sector. The logic extends to sectors which are further downstream.

## 3.4 Equilibrium equations

Our above analysis leads us to the following simple characterization of an equilibrium. Given initial greenification shares $\{\chi_{i0}\}$ at time zero, an equilibrium with a sequence of carbon taxes $\{\tau_t\}$ is a sequence of greenification shares $\{\chi_{it}\}_{t>0}$ such that

$$\mu_{1t}=1, \quad \mu_{it}=\alpha_i+\chi_{i-1,t}\mu_{i-1,t}(1-\alpha_i), \tag{12}$$

$$\widetilde{\chi}_{i+1,t}\equiv\chi_{i+1,t-1}+\left(\chi_{i+1,t}-\chi_{i+1,t-1}\right)\left(e^{z}\left(1+\tau_t\right)\right)^{-\mu_{i+1,t-1}} \tag{13}$$

$$\chi_{it}=\max\left\{\chi_{i,t-1},F_i\left(\left(1-\left[e^{z}\left(1+\tau_t\right)\right]^{-\mu_{it-1}}\right)\prod_{j=i+1}^{N}\left(\widetilde{\chi}_{jt}(1-\alpha_j)\right)\right)\right\}. \tag{14}$$

Despite the complementarities in greenification, the market structure ensures that the equilibrium is unique:

**Proposition 1.** *Given initial condition $\{\chi_{i0}\}_{i=1}^{N}$, the economy with carbon taxes $\{\tau_t\}$ features a unique equilibrium path $\{\chi_{it}\}_{t>0}$.*

*Proof.* We proceed by induction. Suppose we know the sequence of greenification shares $\{\chi_{it-1}\}_{i=1}^{N}$ at time $t-1$. Then, note first that we can compute the sequence $\{\mu_{it}\}_{i=1}^{N}$ recursively from upstream moving downstream using the equations:

$$\mu_{1t}=1, \quad \mu_{it}=\alpha_i+\chi_{i-1,t}\mu_{i-1,t}\left(1-\alpha_i\right)$$

Next, given $\chi_{N,t-1}$ and the $\mu$'s at time $t-1$, one can compute $\chi_{Nt}$ using

$$\chi_{Nt} = \max\left\{\chi_{N,t-1}, F_N\left(\left(1-\left[e^z\left(1+\tau_t\right)\right]^{-\mu_{Nt-1}}\right)\right)\right\}$$

meaning that the measure of greenified varieties at time $t$ cannot decrease, and new varieties are greenified if the cost of doing so is lower than the contemporary profit $\left(1-\left[e^z\left(1+\tau_t\right)\right]^{-\mu_{Nt-1}}\right)$.

Next, for given greenification shares $\chi_{jt}$ for $j \in \{i+1, N\}$ one can compute the equilibrium greenification share $\chi_{it}$, hence moving from downstream to upstream, using the equations

$$\widetilde{\chi}_{j,t} \equiv \chi_{j,t-1} + \left(\chi_{j,t} - \chi_{j,t-1}\right)\left(e^z\left(1+\tau_t\right)\right)^{-\mu_{j,t-1}} \text{ for all } j \in \{i+1, N\}$$

and then

$$\chi_{it} = \max\left\{\chi_{i,t-1}, F_i\left(\left(1-\left[e^z\left(1+\tau_t\right)\right]^{-\mu_{it-1}}\right)\prod_{j=i+1}^{N}\widetilde{\chi}_{jt}\left(1-\alpha_j\right)\right)\right\}.$$

By induction the equilibrium sequence of shares at date $t$, $\{\chi_{it}\}_{i=1}^{N}$, is uniquely pinned down. □

## 3.5 Multiplicity of Steady-States

We next characterize the steady-state(s) of the economy. In a steady-state, there are no newly greenified varieties, so that $\widetilde{\chi}_i = \chi_i$ for all $i$. It then follows that a steady-state with carbon tax $\tau$ is a set of greenification shares $\{\chi_i\}$ such that:

$$\mu_1 = 1, \quad \mu_i = \alpha_i + \chi_{i-1}\mu_{i-1}(1-\alpha_i) \text{ and} \tag{15}$$

$$\chi_i \geq F_i\left(\left(1-\left[e^z\left(1+\tau\right)\right]^{-\mu_i}\right)\prod_{j=i+1}^{N}\chi_j\left(1-\alpha_j\right)\right). \tag{16}$$

Condition (16) is an inequality for the same reason as the "max" notation in (14): technically, because greenification cannot decrease, any $\{\chi_i^{ss}\}$ for which (16) holds as a strict inequality is also a steady-state providing that the economy starts with $\chi_{i0} = \chi_i^{ss}$. These are, however, not very interesting steady-states since starting from $\chi_{i0} < \chi_i^{ss}$, there is no path that the economy can follow to reach these steady-states (without a direct government intervention). We therefore generally ignore them in our analysis, and we focus only on steady-states in which condition (16) holds as an equality:

$$\chi_i = F_i\left(\left(1-\left[e^z\left(1+\tau\right)\right]^{-\mu_i}\right)\prod_{j=i+1}^{N}\chi_j\left(1-\alpha_j\right)\right). \tag{17}$$

While the equilibrium is unique, complementarities in greenification ensure that the economy generally features more than 1 steady-state.

**Proposition 2.** *For a given carbon tax $\tau$, there may exist multiple steady-states over a non-empty open set of parameters whenever $N \geq 2$. There exists a unique steady-state when $N = 1$.*

*Proof.* When $N = 1$, the steady-state equilibrium greenification share $\chi_1$ satisfies the equation:

$$\chi_1 = F_1\left(1 - e^{-Z}\right),$$

which has a unique solution.

Assume now that $N \geq 2$, to establish the result, we simply need to build an example with multiple steady state equilibria. We do so by assuming that the distribution of greenification costs is a mass point at some value $\phi$. We further assume that all the $\alpha_i$ 's are equal to the same $\alpha$ (except for $i = 1$). We derive conditions under which there exist a steady-state where all sectors fully greenify and another steady-states where no sector greenifies.

Consider first that no firm greenifies in any sector. In this case, the profit from greenification in all sectors $i < N$ is zero due to zero demand. The rent from greenifying in the most downstream sector $N$ is $\pi_N = 1 - e^{-\alpha Z}$. Thus there will be no greenification in sector $N$ whenever:

$$1 - e^{-\alpha Z} < \phi.$$

Next, suppose that all but a measure zero of firms greenify in all sectors. In this case, the profit from greenification in sector $i$ is $\pi_i = (1-\alpha)^{N-i}\left[1 - e^{-Z}\right]$. For all firms to have an incentive to greenify, we need that:

$$(1-\alpha)^{N-1}\left[1 - e^{-Z}\right] > \phi$$

Hence, in order for both, full and no greenification to be steady state equilibria, $\phi$ must satisfy:

$$1 - e^{-\alpha Z} < \phi < (1-\alpha)^{N-1}\left[1 - e^{-Z}\right].$$

This is possible as soon as there exist values of $z, \tau$, and $\alpha$ such that:

$$1 - e^{-\alpha Z} < (1-\alpha)^{N-1}\left[1 - e^{-Z}\right].$$

For instance, we note that for small $Z$ ($e^Z \approx 1 + z$) this inequality boils down hat the above inequality boils down to $\alpha < (1-\alpha)^{N-1}$which is satisfied for $\alpha$ sufficiently small. This completes the proof. □

Naturally, the same logic extends to the case where the government implements a cap-and-trade system instead of a carbon tax. In particular, multiple steady-states remain possible: for instance, a steady-state with a high level of greenification and a low price of carbon can exist together with a steady-state with a low-level of greenification and a high price for carbon. Both steady-states may achieve the same level of emissions (if the cap binds) but the low greenification steady-state does it with lower output (see an example in Appendix 10.1).[13]

[13] A cap at 0, however, eliminates the multiplicity since it forces full greenification in all sectors but it will generally not be optimal to force emissions to 0.

# 4 Social optimum

## 4.1 Characterizing the optimum

We now characterize the social optimum, while in Sections 5 and 6 below we shall consider constrained cases where the Planner is limited in their policy instruments: either only able to subsidize greenification in a limited number of sectors or unable to price carbon at its true social cost. Greenification, once the fixed costs are paid, weakly pushes out the production possibility frontier; and, with linear labor disutility, the social costs of greenifying a given varieties are independent of the share of already greenified varieties. As a result, all greenification happens immediately in the optimum: the optimum features instantaneous greenification of (a fraction of) varieties in all sectors in the initial period, and no further greenification in subsequent periods.

Starting from initial condition $\{\chi_{i0}\}_i$, the social planner seeks to maximize the intertemporal utility of consumption minus the labor costs (including for labor hired in greenification) and minus the pollution costs associated with the use of the dirty technology. Keeping in mind that $c_t = y_{Nt}$ and that $\xi$ is the social cost of pollution, the social planner will solve:

$$\max_{\{\{\ell_{dit},\ell_{cit}\},\chi_i\}} \sum_{t=0}^{\infty} \beta^t \left( \ln y_{Nt} - (1+\xi) \sum_i \ell_{dit} - \sum_i \ell_{cit} \right) - \sum_i \left( \mathcal{F}_i\left(\chi_i\right) - \mathcal{F}_i\left(\chi_{i,0}\right) \right).$$

We assume that $(1+\xi)\, e^z > 1$, which ensures that the social planner uses the clean technology whenever it is available. Then, the social planner treats all greenified varieties in a given sector symmetrically and all non-greenified varieties symmetrically as well, which ensures that $\ell_{it}(\nu) = \ell_{cit}(\nu) = \ell_{cit}/\chi_{it}$ and $m_{cit}(\nu) = y_{i-1,t}/\chi_{it}$ if the variety is greenified, and $\ell_{it}(\nu) = \ell_{dit}(\nu) = \ell_{dit}/(1-\chi_{it})$ if the variety is not greenified. We can then write output in sector $i$ as:

$$\ln y_{it} = \chi_{it} \ln \left[ \left( \frac{e^z \ell_{cit}}{\chi_{it}\alpha_i} \right)^{\alpha_i} \left( \frac{y_{i-1,t}}{\chi_{it}(1-\alpha_i)} \right)^{1-\alpha_i} \right] + (1-\chi_{it}) \ln \frac{\ell_{dit}}{1-\chi_{it}}. \tag{18}$$

We can then characterize the social planner problem as follows:

**Proposition 3.** *The social planner's problem can be rewritten as:*

$$\max_{\{\chi_i \geq \chi_{i0}\}} \ln\left((1+\xi)\, e^z\right) \sum_{i=1}^{N} \chi_i \alpha_i \prod_{j=i+1}^{N} \left[\chi_j (1-\alpha_j)\right] - (1-\beta) \sum_i \mathcal{F}_i\left(\chi_i\right). \tag{19}$$

*If the solution $\{\chi_i\}$ is interior, it must satisfy the first-order conditions:*

$$\chi_i = F_i \left( \frac{\ln\left((1+\xi)\, e^z\right)}{1-\beta} \mu_i \prod_{j=i+1}^{N} \left[\chi_j (1-\alpha_j)\right] \right) \tag{20}$$

*Proof.* See Appendix 10.2. □

## 4.2 Steady-states versus the social optimum

At this stage it is worth comparing the steady-state decentralized equilibrium with the social optimum. Obviously, for given greenification shares $\chi's$, the decentralized economy in steady-state follows the optimal allocation provided that the carbon tax is set at its Pigouvian level: $\tau = \xi$.

More interestingly, we can compare greenification in the two economies by comparing equation (17) and the first-order condition for the social optimum (20). There are two differences. First, the factor $\frac{1}{1-\beta}$ on the RHS of (20) is absent from the RHS of (17). This simply captures an intertemporal spillover effect whereby the social gain from greenification carries over to the whole future, whereas private producers benefit from greenification for one period only, given our assumption that patents expire after one period. This inefficiency could potentially be corrected with a uniform subsidy to greenification at rate $\beta$.

A second difference comes from the term $1-\left[e^z\left(1+\xi\right)\right]^{-\mu_i}$ on the RHS of (17) (assuming that the social planner implements a Pigouvian tax: $\tau = \xi$) versus $\mu_i \ln\left(\left(1+\xi\right)e^z\right)$ on the RHS of (20). This difference reflects the fact that the social planner maximizes total surplus including consumer surplus whereas producers maximize their private rent from greenification.[14] The social surplus is generally larger than the private surplus, so that ceteris paribus, there is too little greenification in equilibrium (for $e^z\left(1+\xi\right) > 1$, we get that $1-\left[e^z\left(1+\xi\right)\right]^{-\mu_i} < \mu_i \ln\left(\left(1+\xi\right)e^z\right)$). Nevertheless, if $\left(1+\xi\right)e^z$ is close to one, the difference is negligible: $1-\left[e^z\left(1+\tau\right)\right]^{-\mu_i} \approx \mu_i \ln\left(\left(1+\xi\right)e^z\right)$ so that abstracting from $\frac{1}{1-\beta}$ factor on the RHS of (20), the two equations (17) and (20) become identical.

Is it the case then, that a uniform subsidy $\beta$ to greenification coupled with a Pigouvian carbon tax is enough to decentralize the optimum in steady-state when the overall advantage of the clean technology is small (i.e. $\left(1+\xi\right)e^z$ is close to 1)? The answer is no, which reveals the fundamental reason why industrial policy is warranted in our set-up: cross-sectoral strategic complementarities in greenification. That is, insufficient greenification in sectors downstream and/or upstream to sector $i$, reduces private incentives to greenify in sector $i$ (see (20)), typically leading to multiple steady-state equilibria as showed above.[15] Then, starting from initially low levels of greenification in all sectors, the economy may end up being stuck in a steady-state which also features low-level of greenification, even though the optimum might involve a high level of greenification in all

[14] To see this, suppose there is just one sector, and that the carbon tax is set equal to the social cost of pollution. The model is then equivalent to having a (dirty) technology with productivity $\frac{1}{1+\xi}$ and a clean technology with productivity $e^z$. Under Cobb-Douglas preferences, the demand curve is $p = 1/q$. Under competitive production, the consumer surplus of improving productivity of a variety from $\frac{1}{1+\xi}$ to $e^z$ is $\int_{\frac{1}{1+\xi}}^{e^z} \frac{1}{q}\,\mathrm{d}q = \ln\left(1+\xi\right)e^z$, which differs from the private rent from such productivity improvement.

[15] This also implies that the first order conditions (20) are generally not sufficient to identify the global optimum.

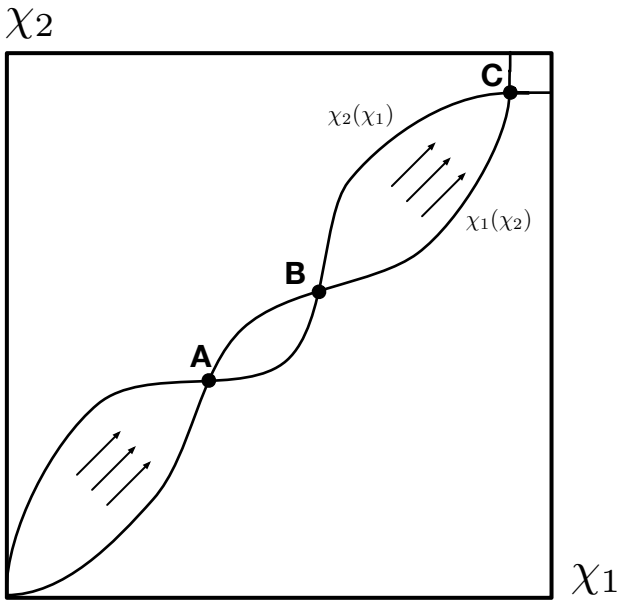



**Figure 1.** First order conditions of the social planner problem and the decentralized economy with a subsidy $\beta$

sectors. The following example with Pigouvian taxation on emissions illustrates this point.[16]

**Example 1.** We consider a two sector supply chain, hence $N = 2$. We set $z = 0$ for simplicity and assume that $\xi$ is small. In addition we assume that greenification is uniformly subsidized at rate $\beta$ , so that firms only face the greenification cost $(1 - \beta)\,\phi$. Under these conditions, both the steady-state equilibrium equations and the first order conditions for the social optimum can be written as:

$$\chi_1 = F_1\left(\frac{\xi\chi_2\,(1-\alpha_2)}{1-\beta}\right) \text{ and} \tag{21}$$

$$\chi_2 = F_2\left(\frac{\xi\,(\alpha_2+\chi_1\,(1-\alpha_2))}{1-\beta}\right). \tag{22}$$

where $F_1$ and $F_2$ are chosen so that the two curves 1 and 2 in Figure 1 intersect three times, at A, B, and C, all of which correspond to a steady-state of the decentralized economy. As Figure 1 shows, the two steady-states A and C are both stable whereas B is unstable. For $\beta$ sufficiently large, the social optimum will correspond to point C, whereas a decentralized economy starting from initially low or no greenification will end up being stuck at the low-greenification steady-state A.

Without the uniform subsidy $\beta$, the decentralized steady-states satisfy:

$$\chi_1 = F_1\left(\xi\chi_2\,(1-\alpha_2)\right) \text{ and} \chi_2 = F_2\left(\xi\,(\alpha_2+\chi_1\,(1-\alpha_2))\right).$$

These two equations correspond to the two dash lines in Figure 2, which also intersect three times

[16]Of course, a sufficiently high carbon tax can remove the multiplicity of steady-states, but implementing such a tax above its Pigovian value $\xi$ is suboptimal.

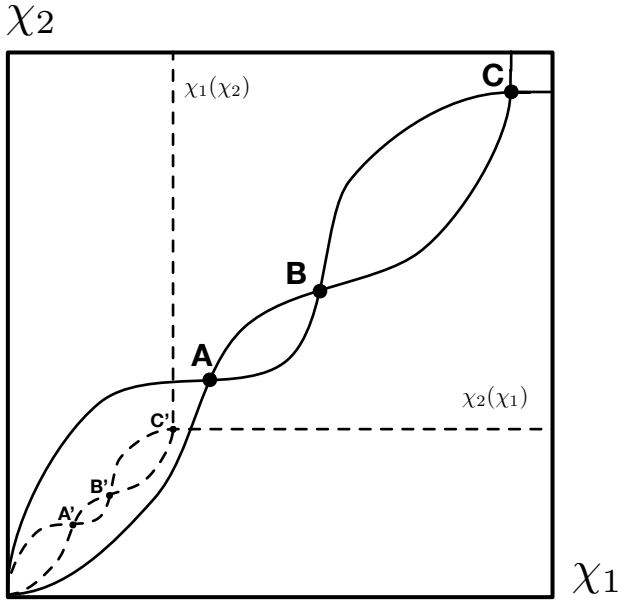



**Figure 2.** First order conditions of the social planner problem and the decentralized economy without a subsidy $\beta$

at the steady-state equilibria A', B' and C'. Then an economy with low or no initial greenification will end up being stuck at the—even lower greenification—steady-state A'. The uniform subsidy $\beta$ allows an economy with initially very low or no greenification to converge to A instead of A' but yet sector specific subsidies are required on top of the uniform subsidy to make the economy converge to C.

## 4.3 Implementing the social optimum

Yet, starting from the same initial conditions with low levels of greenification in all sectors, the social planner can achieve the optimum through a set of temporary subsidies to greenification—given that the equilibrium for any given policy and given initial conditions, is unique as shown in the previous section. More precisely, we can easily establish:

**Proposition 4.** *The optimal steady-state can be implemented through a carbon price together with a whole set of time-varying sector specific subsidies.*

*Proof.* Consider any initial allocation $\{\chi_{i,0}\}$ and assume that the social planner imposes a Pigouvian tax $\tau_t = \xi$, and a set of sector specific subsidies $\{q_{i,t}\}$. Then, the equilibrium level of greenification at time $t$ is given by

$$\chi_{i,t} = F_i\left(\frac{\left(1-\left[e^z\left(1+\xi\right)\right]^{-\mu_{i,t-1}}\right)}{1-q_{i,t}}\prod_{j=i+1}^{N}\left(\widetilde{\chi}_{j,t}(1-\alpha_j)\right)\right).$$

For sector $N$ at time 1, we can always set $q_{N,1}$ such that

$$\chi_N^{SP} = F_N\left(\frac{\left(1-\left[e^z\left(1+\xi\right)\right]^{-\mu_{N,0}}\right)}{1-q_{N,1}}\right),$$

where $\chi_N^{SP}$ is the social planner level of greenification in sector $N$ and $\mu_{N,0}$ is predetermined. Assume now that the social planner uses a set of sector specific subsidies $\{q_{j,1}\}$ for $j > i$, in order to implement the social planner level of greenification $\chi_j^{SP}$ for $j > i$ at $t = 1$. Then for sector $i$, the social planner can choose $q_{i,1}$ such that

$$\chi_i^{SP} = F_i\left(\frac{\left(1-\left[e^z\left(1+\xi\right)\right]^{-\mu_{i,0}}\right)}{1-q_{i,1}}\prod_{j=i+1}^{N}\left(\widetilde{\chi}_{j,1}(1-\alpha_j)\right)\right)$$

since $\mu_{i,0}$ is again pre-determined and $\widetilde{\chi}_{j,1} = \chi_{j,0} + \left(\chi_j^{SP} - \chi_{j,0}\right)\left(e^z\left(1+\xi\right)\right)^{-\mu_{j,0}}$ is also given. Therefore, by induction, the social planner can implement the socially optimal levels of greenification $\chi_i^{SP}$ in all sectors from the most downstream to the most upstream at time $t = 1$.

At time $t = 2$, there is no more incentives to greenify when $q_{i,2} = 0$, because if $\chi_{j,2} = \chi_j^{SP}$, we get that:

$$\chi_i^{SP} = F_i\left(\frac{\ln\left(\left(1+\xi\right)e^z\right)}{1-\beta}\mu_i^{SP}\prod_{j=i+1}^{N}\left(\chi_j^{SP}(1-\alpha_j)\right)\right) > F_i\left(\left(1-\left[e^z\left(1+\xi\right)\right]^{-\mu_{i,1}}\right)\prod_{j=i+1}^{N}\left(\widetilde{\chi}_{j,2}(1-\alpha_j)\right)\right),$$

where the equality stems from the fact that $\chi_i^{SP}$ is the optimum, while the inequality uses that $\mu_{i,1} = \mu_i^{SP}$, $\widetilde{\chi}_{j,2} = \chi_j^{SP}$ if there is no further greenification, and that $\frac{\ln((1+\xi)e^z)}{1-\beta}\mu_i^{SP} > \left(1-\left[e^z\left(1+\xi\right)\right]^{-\mu_i^{SP}}\right)$. The overall inequality implies that there is no further incentive to greenify in the decentralized economy. Since, there is no greenification, there are no profits either and the social optimum is implemented from $t = 2$ onward. This completes the proof. □

Note that Proposition 4 is about decentralizing the optimal steady-state but not the full optimal allocation.[17] In the full optimum, users of intermediate inputs should buy newly greenified inputs at marginal costs, which requires the implementation of subsidies for the use of intermediates. However, with such instruments (a Pigouvian carbon tax, sector specific subsidies for greenification and subsidies to the use of intermediates), the equilibrium will generally not be unique and a social planner would have to use the type of instruments analyzed by Sturm (2023) to ensure uniqueness.[18]

[17] In the proof above, at $t = 1$, the optimal environmental tax and the optimal greenification levels are implemented, but newly greenified varieties are priced monopolistically and therefore inefficiently under-supplied.

[18] The logic extends to the case where carbon pricing is enforced through a cap-and-trade system instead of a tax. As long as there are multiple steady-states, sector specific subsidies to greenification remain necessary, on top of cap-and-trade, to ensure that the economy converges toward the optimal steady-state.

## 4.4 Small subsidies can make a big difference

Are large subsidies—big-push policies—always necessary to move the economy away from a low-greenification steady-state? To conclude this section, we provide an example where small sector-specific subsidies are enough to move the economy from an initial low greenification steady-state towards a high greenification steady-state, which dominates it in terms of welfare (and may be the optimum).

We consider again a two-sector supply chain, but now assume mass point distributions of greenification costs across varieties in the two sectors. We build the example such that there are three steady-state equilibria, respectively with no greenification, full greenification and in-between an interior, unstable steady-state. Provided that consumers are sufficiently patient, the full greenification steady-state dominates the other two and may even correspond to the optimum. Temporary subsidies can then ensure that the economy moves away from the no greenification steady-state to a little beyond the interior one. Afterwards, the economy will move to the full greenification steady-state on its own without the use of any subsidy. We show that if the downstream cost of greenification is just high enough that the no greenification steady-state exists but not much higher, then a very small greenification subsidy is enough to make the economy eventually converge to the full adoption steady-state.

**Example 2.** We assume that the distributions of fixed costs $F_1$ and $F_2$ have mass points at $\phi_1$ and $\phi_2$ respectively. As before, we denote $Z = z + \ln(1+\tau)$. We first derive conditions under which there are three steady-states characterized by no greenification, full greenification and an interior level of greenification. We then derive conditions under which moving from the no greenification to the interior level of greenification only requires a small intervention.

First, assume that the economy features no greenification, then there are no incentive to greenify downstream (sector 2) provided that:

$$1 - e^{-\alpha_2 Z} < \phi_2.$$

n that case, there is no market for sector 1 and no greenification upstream either as long as $\phi_1 > 0$. Because these are inequalities, there are still no incentives to greenify for $\chi$'s slightly different from $0$, so that the steady-state is stable.

Second, full greenification is also a steady-state provided that:

$$\left(1 - e^{Z}\right)(1 - \alpha_2) > \phi_1$$

which ensures full greenification upstream, and

$$\left(1 - e^{-Z}\right) > \phi_2,$$

which ensures full greenification downstream. For the same reason as before, this steady-state is

also stable.

Therefore, no-greenification and full-greenification steady-states can coexist provided that:

$$1-e^{-\alpha_2 Z}<\phi_2<1-e^{-Z} \text{ and } 0<\phi_1<\left(1-e^{-Z}\right)(1-\alpha_2).$$

Third, an interior steady-state equilibrium $(\chi_1^*,\chi_2^*)$ must satisfy:

$$\left(1-e^{-Z}\right)\chi_2^*(1-\alpha_2) = \phi_1 \tag{23}$$
$$1-e^{-\left(\alpha_2+\chi_1^*(1-\alpha_2)\right)Z} = \phi_2 \tag{24}$$

Given that $1-e^{-\alpha_2 Z}<\phi_2<1-e^{-Z}$, there always exists a $\chi_1^*\in(0,1)$ which satisfies the second equation. Similarly, given that $\left(1-e^{-Z}\right)(1-\alpha_2)>\phi_1>0$, there also always exists a $\chi_2^*\in(0,1)$ that satisfies the first equation. Since the left hand side of (23) is increasing in $\chi_2$ and the left-hand side of (24) is increasing in $\chi_1$ while the right-hand sides are fixed at $\phi_1$ and $\phi_2$, this interior steady-state is necessarily unstable. Therefore, a small increase in $\chi_1$ and/or $\chi_2$ starting from $(\chi_1^*,\chi_2^*)$ will lead to further greenification.

Next, we derive a set of subsidies sufficient to ensure that the economy move from the no greenification to the interior steady-states at time 1. For greenification in sector 1 to be interior, we need a subsidy $q_{1,1}$ which satisfies:

$$\frac{1-e^{-Z}}{1-q_{1,1}}\widetilde{\chi}_{2,1}(1-\alpha_2)=\phi_1 \text{ with } \widetilde{\chi}_{2,1}=\chi_2^* e^{-\alpha_2 Z},$$

which, using (23), yields $q_{1,1}=1-e^{-\alpha_2 Z}$. Similarly, for greenification in sector 2 to be interior, we need a subsidy $q_{2,1}$ such that:

$$\frac{1-e^{-\alpha_2 Z}}{1-q_{2,1}}=\phi_2 \Longrightarrow q_{2,1}=1-\frac{1-e^{-\alpha_2 Z}}{\phi_2}$$

Overall, the total amount of subsidies to move the economy to the interior unstable steady-state, is given by

$$Q_1=\chi_1^* q_{1,1}+\chi_2^* q_{2,1}=\chi_1^*\left(1-e^{-\alpha_2 Z}\right)+\chi_2^*\left(1-\frac{1-e^{-\alpha_2 Z}}{\phi_2}\right)$$

If $\phi_2$ is above but close to $1-e^{-\alpha_2 Z}$, then $q_{2,1}$ is small. In addition, in that case, (24) implies that $\chi_1^*$ is small too, which ensure that the total amount spent $Q_1$ is small. This establishes the result described above: if greenification costs are just a little too high downstream, then a small intervention is enough to push the economy toward the interior steady-state and eventually the full greenification one.[19]

[19]This logic does not extend to the case where greenification costs are just a little too high upstream: If $\phi_1$ is positive but close to 0, then $\chi_2^*$ is close to 0 and subsidies spent for sector 2 are small. However, there is no guarantee that subsidies spent for greenification in sector 1 are small without additional assumptions.

# 5 Propagation and appropriate industrial policy

Our analysis so far has considered a government that could and generally would intervene in several sectors at the same time – since implementing the optimum generically require such a multi-faceted intervention. In practice, however, it may be that a government is constrained to focus on one or a few key sectors at a time. This raises a number of questions that our framework can shed light upon: What can be achieved when the government is constrained to greenify at most one sector at a time? How do greenification incentives propagate along the supply chain starting from a particular sector? In other words, what are the effects of an exogenous change in the greenification share $\chi_k$ in sector $k$ at a given time—for instance due to targeted policy—on the equilibrium greenification incentive in all other sectors in the supply chain?

We first analyze these questions in our general framework before providing more definitive answers on which sectors should be targeted by the government in an example.

## 5.1 Basic intuition

In presence of a subsidy to greenification $q_i$ in sector $i$, a steady-state satisfies:

$$\chi_i = F_i\left(\frac{\pi_i}{1-q_i}\right) \text{ with } \pi_i \equiv \underbrace{\left(1-e^{-\mu_i Z}\right)}_{\text{productivity gain}} \underbrace{\prod_{j=i+1}^{N} \chi_j\left(1-\alpha_j\right)}_{\text{demand from downstream}},$$

$$\text{and } \mu_1 = 1, \quad \mu_i = \alpha_i + \chi_{i-1}\mu_{i-1}(1-\alpha_i) \text{ for } i \in \{2, ...N\},$$

where with Pigouvian taxes $e^Z = e^z(1+\xi)$. The above expression for $\pi_i$ suggests that, a priori, the incentives to greenify could propagate in both directions: more greenification in a sector $k$ which is downstream from $i$ ($k > i$) affects the demand component of $\pi_i$, whereas more greenification in a sector $k$ which is upstream relative to $i$ ($k < i$) affects the productivity gain component of $\pi_i$.

Yet we argue below that the same perturbation in the greenification rate $\chi_k$ in sector $k$, always generates more greenification incentives in sectors $i < k$, i.e., those that are more upstream than $k$. That is, the impact of greenification in $k$ working through the demand components, are always stronger than the impacts working through the productivity components. In fact, the latter is close to zero in a steady-state with low greenification rates $\chi$'s. In other words, starting from a steady-state with low greenification across all sectors, the bang-for-the-buck is generically higher if policy can induce greenification first in more downstream sectors.

To see this more formally, note that the elasticity of greenification in sector $i$ to a subsidy in sector $k$ can be decomposed into

$$\frac{\partial \ln \chi_i}{\partial \ln q_k} = \frac{\partial \ln \chi_i}{\partial \ln \pi_i}\frac{\partial \ln \pi_i}{\partial \ln \chi_k}\frac{\partial \ln \chi_k}{\partial \ln q_k}.$$

The first and third terms on the right depend on the cost distribution of sectors $i$ and $k$ respectively. In contrast, the middle term, which represents the elasticity of greenification incentives in $i$ (i.e. profits) with respect to greenification in sector $k$, only depends on the structure of the network and is therefore the object of our attention. We show in Appendix 10.3 that

$$\frac{\partial \ln \pi_i}{\partial \ln \chi_k} = \begin{cases} 1 & k > i \\ \frac{\mu_i Z e^{-\mu_i Z}}{1-e^{-\mu_i Z}} \frac{\left(\prod_{j=0}^{i-k-1}(1-\alpha_{i-j})\chi_{i-j-1}\right)\mu_k}{\mu_i} < 1 & k < i \end{cases}. \tag{25}$$

This in turn leads us to make the following two observations. First, a 1% increase in greenification in a sector $k$ which is downstream to sector $i$ (i.e., with $k > i$), always induces a 1% proportional increase in the steady-state greenification incentive in sector $i$. Intuitively, greenification downstream increases the market for greenified varieties upstream 1 for 1. Second, a 1% increase in greenification in a sector $k$ which is upstream to sector $i$ (i.e. with $k < i$) always provides lower marginal incentives for greenification in sector $i$ than a 1% increase in greenification downstream from sector $i$. This is because both $\frac{\mu_i Z e^{-\mu_i Z}}{1-e^{-\mu_i Z}} < 1$ and $\frac{\left(\prod_{j=0}^{i-k-1}(1-\alpha_{i-j})\chi_{i-j-1}\right)\mu_k}{\mu_i} < 1$, where in fact the latter expression decreases as $k$ becomes smaller (that is as $k$ is further upstream from $i$). Intuitively, greenification upstream reduces the cost of greenified varieties downstream but less than proportionately, because at each stage of the production process, the costs of producing a given intermediate depends on the costs of the more upstream intermediates – but only in proportion to how much greenification has occurred – and also on labor costs. To summarize, exogenously more greenification in downstream sectors generate proportional gains in greenification incentives in all upstream sectors. However exogenously more greenification upstream generates less than proportional gains in greenification incentives in downstream sectors, and the incentives gets smaller and smaller the further downstream we go.[20]

The contrast in how the incentives propagate is particularly stark when at least one sector features low greenification, as the additional downstream incentives resulting from upstream greenification can become arbitrarily small. To see this, note that $\mu_i$ (the network-adjusted share of greenified content for producing an greenified variety in sector $i$) is bounded away from 0 so that for any sector $k$ upstream to $i$ ($k < i$), the additional incentive in sector $i$ generated by greenification in sector $k$ $\frac{\partial \ln \pi_i}{\partial \ln \chi_k}$ goes to 0 whenever one of the $\chi_j$ tends to 0 for any $j$ in between $k$ and $i$. In other words, the downstream propagation of greenification incentives from $k$ to $i$ is only as strong as the weakest link along the chain between $k$ and $i$. Therefore, starting from a steady-state with low greenification in at least some sectors ($\chi_i \approx 0$ for some $i$), the downstream propagation of incentives from targeting upstream sectors is close to zero, and industrial policy should first

[20]Here we are examining the propagation of greenification, but given that the end goal of green industrial policy will be to reduce carbon emissions, we consider in Section 6 how changes in greenification in different sectors affects equilibrium emissions.

target downstream sectors[21] and leverage the equilibrium upstream propagation of incentives.

Note that our intuitive reasoning in this subsection is incomplete in two respects. First, it considers the change in incentives in sector $i$ while perturbing $\chi_k$, holding all other $\chi_j$'s constant. However, in equilibrium, $\chi_j$ would respond for all $j$, generating further changes in incentives. Second, it focuses on perturbations in a steady-state, while ignoring the sequence of changes in incentives along the transition. But the intuitions continue to extend when the path of $\chi_{jt}$'s respond to the exogenous shock in $\chi_k$. We shall now focus on a special case where a more formal statement can be made.

## 5.2 A Proposition

In this subsection, we now consider a special case of an economy starting in a no-greenification steady-state. Should the government only be able to intervene in one sector, then greenification will only propagate contemporaneously if it targets the most downstream sector and with a lag if it targets the sector immediately above. Formally, we prove the following proposition:

**Proposition 5.** *Consider a supply chain with $N \geq 3$ sectors, and suppose that the basic parameters and the $F_i$'s are such that no greenification in all sectors is a steady-state. Suppose also that initially the economy is stuck in this no-greenification steady-state and that the government can directly greenify a positive mass of varieties in one sector only. Then, provided that greenification costs are not too high, (i) the one-sector exogenous greenification will start to propagate immediately—creating incentives for other sectors to greenify—only if the government intervenes in the most downstream sector $N$; (ii) the one-sector greenification will start propagating with a one-period delay if the government intervenes in sector $N-1$; (iii) the one-sector greenification will not propagate at all if the government intervenes in a sector which is more upstream than sector $N-1$.*

*Proof.* Part (i):

Suppose the government starts greenifying in sector $N$ at the beginning of time $t$, so that $\chi_{N,t} > 0$. Then, at the end of time $t$, greenification in sector $N-1$ is given by

$$\chi_{N-1.t} = F_{N-1}\left(\left(1 - e^{-\alpha_{N-1}Z}\right)\chi_{N,t}e^{-\alpha_{N-1}Z}\left(1-\alpha_{N-1}\right)\right),$$

With $\chi_{N,t} > 0$, we get $\chi_{N-1,t} > 0$ (as long as greenification costs at $N-1$ are not too high: $F_{N-1}\left(\left(1 - e^{-\alpha_{N-1}Z}\right)\chi_{N,t}e^{-\alpha_{N-1}Z}\left(1-\alpha_{N-1}\right)\right) > 0$). In contrast at $t-1$, pre-intervention, we had $\chi_{N,t-1} = \chi_{N-1,t-1} = 0$.

Now suppose greenification did propagate to sectors $j > i$ (i.e. $\chi_{j,t} > 0$). Then it also propa-

[21] This argument requires the mild condition that the relative marginal costs of greenification across sectors are bounded when the levels of greenification is low.

gates to sector $i$ since:

$$\chi_{it} = F_i\left(\left(1-e^{-\alpha_i Z}\right)\prod_{j=i+1}^{N}\chi_{j,t}e^{-\alpha_j Z}\left(1-\alpha_j\right)\right) \text{ for } i<N,$$

which is positive if greenification costs are not too high. Hence greenification propagates all the way from the most downstream sector $N$ to the most upstream sector 1 at time $t$.

At time $t+1$, we have:

$$\chi_{i,t+1} = F_i\left(\left(1-e^{-\mu_{i,t}Z}\right)\prod_{j=i+1}^{N}\left(\widetilde{\chi}_{j,t+1}\left(1-\alpha_j\right)\right)\right)$$

with $\widetilde{\chi}_{j,t+1}$ given by (8) and $\mu_{it}$ by (6). Then, $\mu_{it}\geq\alpha_i$ and $\widetilde{\chi}_{j,t+1}\geq\chi_{j,t}>\chi_{j,t}e^{-\alpha_j Z}$, which implies that further greenification occurs in all sectors at time $t+1$ (as long as $F_i$ has positive mass around the relevant range) and this continues in subsequent periods until we reach a steady-state with positive greenification in all sectors.

Part (ii):

Suppose now that the government starts greenifying in sector $N-1$. Then we get that $\mu_{N,t-1}=\alpha_N$ (i.e. the pre-intervention value) so that $\chi_{Nt}$ must satisfy:

$$\chi_{Nt} = F_N\left(1-e^{-\alpha_N Z}\right) = \chi_{N,t-1} = 0.$$

In other words, greenifying first in sector $N-1$ at time $t$ does not immediately propagate to the most downstream sector $N$.

Consider now sector $N-2$. We have:

$$\begin{aligned}\chi_{N-2.t} &= F_{N-2}\left(\left(1-e^{-\alpha_{N-2}Z}\right)\widetilde{\chi}_{N-1.t}\left(1-\alpha_{N-1}\right)\widetilde{\chi}_{Nt}\left(1-\alpha_N\right)\right)\\ &= F_{N-2}\left(0\right)=\chi_{N-2,t-1}=0\end{aligned}$$

And given that $\widetilde{\chi}_{Nt}=0$, greenification incentives in sector $N$ are the same as at time $t-1$, i.e. there is no greenification in sector $N$ at time $t$. The same logic applies to any sector $i<N-1$. In other words, there is no propagation of the exogenous greenification in sector $N-1$ to other sectors at time $t$.

Consider now time $t+1$. In sector $N$, we get: $\mu_{N,t}=\alpha_N+\chi_{N-1,t}\alpha_{N-1}\left(1-\alpha_N\right)>\alpha_N$ and greenification incentives in sector $N$ obey:

$$\chi_{N,t+1} = F_N\left(1-e^{-\mu_{N,t}Z}\right).$$

Since $\mu_{N,t}>\alpha_N$, then provided that greenification costs are not too high, we can have $F_N\left(1-e^{-\mu_{N,t}Z}\right)>F_N\left(1-e^{-\alpha_N Z}\right)=0$, such that $\chi_{N,t+1}>0$ in which case greenification propagates to sector $N$.

Moving back to sector $N-1$, we have:

$$\chi_{N-1,t+1} = \max\left\{\chi_{N-1,t}, F_{N-1}\left(\left(1-e^{-\alpha_{N-1}Z}\right)\widetilde{\chi}_{N,t+1}\left(1-\alpha_N\right)\right)\right\}$$

with $\widetilde{\chi}_{N,t+1} = \chi_{N,t+1} \left(e^z \left(1+\tau_t\right)\right)^{-\mu_{N,t}}$.

Now moving to sector $N-2$, we get that:

$$\chi_{N-2,t+1} = F_i \left( \left(1 - e^{-\alpha_{N-2} Z^Z}\right) \widetilde{\chi}_{N-1,t+1} \left(1-\alpha_{N-1}\right) \widetilde{\chi}_{N,t+1} \left(1-\alpha_N\right) \right),$$

with $\widetilde{\chi}_{N-1,t+1} = \chi_{N-1,t} + \left(\chi_{N-1,t+1} - \chi_{N-1,t}\right) e^{-\mu_{N-1,t} Z} \geq \chi_{N-1.t}$ and $\widetilde{\chi}_{N,t+1} > 0$ if $\chi_{N,t+1} > 0$. This in turn implies that greenification will also propagates to sector $N-2$ – provided that the distribution of fixed costs $F_i$ has positive mass in the relevant range. The logic extends to all sectors $j > N-1$ so that greenification propagates to all sectors at time $t+1$.

And greenification intensifies in subsequent periods until we reach a steady-state with positive greenification in all sectors.

Part (iii):

Now suppose that the government starts greenifying in a sector $j$ which more upstream than $N-1$,i.e. $j < N-1$, at time $t$. Consider first sector $N$ at time $t$. Given that greenification incentives in that sector only depend upon $\mu_{N,t-1}$, they are the same as pre-intervention so that $\chi_{N,t} = 0$. Next, consider any sector $k \neq N, j$ : greenification incentives in sector $k$ depend multiplicatively upon on $\widetilde{\chi}_{N,t} = 0$, thus profits from greenifying in sector $j$ are equal to zero as it was pre-intervention. We then have that $\chi_{k,t} = 0$ for all $k \neq j$.

Consider now sector $N$ at time $t+1$. Incentives to greenify in that sector hinge upon:

$$\mu_{N,t} = \alpha_N + \chi_{N-1,t} \mu_{N-1,t} \left(1-\alpha_N\right) = \alpha_N = \mu_{N,t-1}$$

since $\chi_{N-1,t} = 0$. Therefore the incentives to greenify in sector $N$ remain the same as in period $t$, i.e. $\chi_{N,t+1} = 0$. It follows that we also have $\chi_{k,t+1} = 0$ for all $k \neq j$. And the same reasoning carries over to all periods $t+h$ with $h > 1$. Hence, the initial greenification push in sector $j < N-1$ never propagates to the other sectors of the supply chain. This establishes the proposition. □

The finding that industrial policy should first target the most downstream sector, hinges heavily, first on industrial policy being one-shot and focused on one sector, and second on having a single vertical supply chain. In particular, to induce greenification in production structures involving a several number of supply chains with the same most upstream sector, a government with the ability to intervene in several sectors, might find it optimal to also intervene in the most upstream sector. This leads us to consider the case of parallel supply chains in the next section.

## 5.3 Generalizations

How general is the intuition that a social planner should worry in priority about the greenification of downstream sectors? In this subsection, we extend our set-up beyond that of a vertical supply chain, but still assuming that greenification innovations can only occur in input sectors of the clean production process.

**Two downstream sectors** One argument to target upstream instead of downstream sectors for greenification is that a single upstream sector may generate spillovers for several downstream sectors. To allow for this, we now consider a simple network with 2 layers ($N = 2$), but two downstream sectors $2a$ and $2b$. For simplicity, we assume that the two sectors share the same labor share $\alpha_2$. We assume that consumption is Cobb-Douglas between the two sectors with consumption shares denoted $\beta_a$ and $\beta_b$. The revenues of the two downstream sectors are then given by $r_{2a} = \beta_a$ and $r_{2b} = \beta_b$, while the revenues of the upstream sector in steady-state are given by:

$$r_1 = (1 - \alpha_2) (\beta_a \chi_{2a} + \beta_b \chi_{2b}) .$$

Following the same steps as in the baseline model, we obtain that a steady-state is characterized by $\chi_i = F_i (\pi_i)$ for $i \in \{1, 2a, 2b\}$, where steady-state profits are given by:

$$\pi_1 = \left(1 - e^{-Z}\right) r_1, \ \pi_{2k} = \left(1 - e^{-Z\mu_2}\right) \beta_k \text{ for } k \in \{a, b\} \text{ with } \mu_2 = \alpha_2 + (1 - \alpha_2) \chi_1.$$

We can then immediately derive how (in steady-state) greenification incentives in one sector directly depend on the level of greenification in other sectors as:

$$\frac{\partial \ln \pi_1}{\partial \ln \chi_{2k}} = \frac{\chi_{2k}\beta_k}{\chi_{2a}\beta_a + \chi_{2b}\beta_b} \text{ for k} \in \{a, b\} \text{ and } \frac{\partial \ln \pi_{2k}}{\partial \ln \chi_1} = \frac{\mu_2 Z e^{-\mu_2 Z}}{1 - e^{-\mu_2 Z}} \frac{(1 - \alpha) \chi_1}{\alpha + (1 - \alpha) \chi_1}. \tag{26}$$

Upstream propagation of greenification incentives from sectors $2a$ and $2b$ on greenification in sector $1$ now occurs in proportion to the importance of each of the two downstream sectors for sector 1's output, whereas the expression for downstream propagation of greenification remains the same as with a single downstream sector.

Even though it is no longer the case in this setting that the elasticities for downstream propagation are always smaller than those for upstream propagation, it is still the case that for low levels of greenification, it is better to target downstream sectors. To see this, note that as $\chi_1 \to 0$, $\frac{\partial \ln \pi_{2k}}{\partial \ln \chi_1} \to 0$, so that it is always strictly better to first target at least one of the two downstream sectors. In contrast, if $\chi_{2a}, \chi_{2b} \to 0$ (but keeping the ratio $\chi_{2a}/\chi_{2b}$ constant), then $\frac{\partial \ln \pi_1}{\partial \ln \chi_{2k}} \nrightarrow 0$, so greenification still propagates upstream. Empirically, one example where one might thus expect stronger potential for downstream propagation is for electricity as electrification shares are already sizable in many sectors. To explore this issue further, the Appendix presents two stylized quantifications of (26) for the US economy with electricity generation as upstream sector and downstream sectors based on both the broad US economy (industry, commercial, residential, transportation) or the manufacturing sector (durable, nondurable). While we find that the downstream elasticity *can* indeed be larger in this setting, this is not generally the case even with the higher greenification shares observed in this setting.

**General Network Structure** We now analyze the propagation of greenification incentives under a general network structure. Consider production functions for varieties in sectors $i, j = 1, \ldots, N$:

$$y_{it}(\nu) = \ell_{dit}(\nu) + \gamma_{it}(\nu) \left( \frac{e^z \ell_{cit}(\nu)}{\alpha_i} \right)^{\alpha_i} \prod_j \left( \frac{m_{ijt}(\nu)}{\sigma_{ij}} \right)^{\sigma_{ij}},$$

where $\sigma_{ij} \in [0, 1)$ is the intermediate elasticity input $j$ in the production of clean varieties of good $i$, and $\sum_j \sigma_{ij} + \alpha_i = 1$. The network structure encapsulated in the intermediate elasticities may take a general form. For expositional simplicity, we assume the last sector $N$ is a final goods sector that supplies only to the consumer ($\sigma_{iN} = 0 \forall i$), and that the consumer purchases only from sector $N$. The market clearing condition is

$$y_{jt} = \sum_{i=1}^{N} \int_0^1 m_{ijt}(\nu) \, \mathrm{d}\nu \qquad \text{for } j < N,$$

whereas market clearing for the final good $N$ is $c_t = y_{Nt}$.

We now derive, for a pair of sectors $i$ and $j$ in a steady-state of this environment, the effect of a marginal greenification in each sector $j$ on the profits of a newly greenified variety in sector $i$. The analysis demonstrates how the incentives of greenification propagates in a general network structure.

The price of a variety in sector $i$ is

$$p_{it}(\nu) = \begin{cases} e^{-\alpha_i z} \prod_j p_{jt}^{\sigma_{ij}} & \text{if the variety has been greenified by time } t-1 \\ 1+\tau & \text{otherwise} \end{cases}$$

where $p_j$ (without the variety index) is the price of the sectoral bundle. As before we denote by $\mu_{it}$ the greenified content for the production of a clean variety of of good $i$. We then get the updated recursive formulation:

$$\mu_{it} = \alpha_i + \sum_{j=1}^{N} \sigma_{ij} \chi_{jt} \mu_{jt}. \tag{27}$$

With this updated expression for $\mu$ sectoral prices are still given by equation (7).

Following the same logic as in the baseline model, sectoral revenues are given by $r_{jt} = \sum_{i=1}^{N} \sigma_{ij} \tilde{\chi}_{it} r_{it}$,with $\tilde{\chi}_{it}$ still defined as in (8). In steady-state $\tilde{\chi}_i = \chi_i$ are constant, and revenues are given by

$$r_j = \sum_{i=1}^{N} \sigma_{ij} \chi_i r_i. \tag{28}$$

The profit of a newly greenified variety is the product between profit margin and revenues:

$$\pi_{it} = \left( 1 - \frac{e^{-\alpha_i z}}{1+\tau} \prod_j p_{jt}^{\sigma_{ij}} \right) r_{it} = \left( 1 - e^{-\mu_{i,t-1} Z} \right) r_{it},$$

with the revenue in the final sector being $r_N = 1$, exactly as in the baseline case.

We then focus on a steady-state. We can derive the marginal impact of additional greenification in sector $j$ on the profit of a newly greenified variety in sector $i$ as

$$\frac{\partial \ln \pi_i}{\partial \ln \chi_k} = \frac{Ze^{-\mu_i Z}}{1-e^{-\mu_i Z}} \frac{\mathrm{d}\mu_i}{\mathrm{d}\ln \chi_k} + \frac{\mathrm{d}\ln r_i}{\mathrm{d}\ln \chi_k}.$$

The first term captures the effect of a marginal greenification in sector $k$ on the profit margin. The second term captures the impact on the revenue.

We now solve for the two channels as a function of the production network. Let $\hat{\mathbf{\Sigma}} \equiv [\sigma_{ij}\chi_j]$, and Let $Diag(\boldsymbol{\mu})$ denote the diagonal matrix with entries $\{\mu_i\}$. In matrix notation, equation (27) becomes

$$\boldsymbol{\mu} = \left(\boldsymbol{I} - \hat{\mathbf{\Sigma}}\right)^{-1} \boldsymbol{\alpha}.$$

Totally differentiating with respect to $\chi_k$, we can solve for the matrix $\Gamma \equiv \left[\frac{\mathrm{d}\mu_i}{\mathrm{d}\ln \chi_k}\right]$ as

$$\begin{aligned} \mathbf{\Gamma} &= \hat{\mathbf{\Sigma}} Diag(\boldsymbol{\mu}) + \hat{\mathbf{\Sigma}}\mathbf{\Gamma} \\ &= \hat{\mathbf{\Sigma}}\left(\boldsymbol{I} - \hat{\mathbf{\Sigma}}\right)^{-1} Diag(\boldsymbol{\mu}). \end{aligned}$$

To analyze the network impact of greenification on revenue, let $\mathbf{\Omega}$ be the matrix whose $ji$-th entry captures $\frac{\sigma_{ij} r_i \chi_i}{r_j}$, the fraction of revenue sector $j$ derives from selling to sector $i$ in steady-state. Totally differentiating (28), we have

$$\mathrm{d}\ln r_j = \sum_{i=1}^{N} \frac{\sigma_{ij}\chi_i r_i}{r_j} \left(\mathrm{d}\ln \chi_i + \mathrm{d}\ln r_i\right)$$

Or, in vector form,

$$\mathrm{d}\ln \boldsymbol{r} = \left(\boldsymbol{I} - \mathbf{\Omega}\right)^{-1} \mathbf{\Omega}\, \mathrm{d}\ln \boldsymbol{\chi}.$$

We are now ready to derive the network impact of greenification on profits. The matrix $\mathbf{\Psi} \equiv \left[\frac{\partial \ln \pi_i}{\partial \ln \chi_k}\right]$ is

$$\begin{aligned} \mathbf{\Psi} &= \underbrace{Diag\left(\frac{Ze^{-\mu_i Z}}{1-e^{-\mu_i Z}}\right)\hat{\mathbf{\Sigma}}\left(\boldsymbol{I} - \hat{\mathbf{\Sigma}}\right)^{-1} Diag(\boldsymbol{\mu})}_{\text{input cost channel}} + \underbrace{\left(\boldsymbol{I} - \mathbf{\Omega}\right)^{-1}\mathbf{\Omega}}_{\text{market size channel}} \\ &= \underbrace{Diag\left(\frac{Ze^{-\mu_i Z}}{1-e^{-\mu_i Z}}\right)\left(\hat{\mathbf{\Sigma}} + \hat{\mathbf{\Sigma}}^2 + \cdots\right) Diag(\boldsymbol{\mu})}_{\text{input cost channel}} + \underbrace{\left(\mathbf{\Omega} + \mathbf{\Omega}^2 + \cdots\right)}_{\text{market size channel}} \end{aligned}$$

The matrix power series $\hat{\mathbf{\Sigma}} + \hat{\mathbf{\Sigma}}^2 + \cdots$ and $\mathbf{\Omega} + \mathbf{\Omega}^2 + \cdots$ respectively captures the network transmission of how greenification in one sector $k$ affects the profits and thus the incentive to greenify in another sector $i$. The first power is the direct effect between sector pairs $i$ and $k$; each successive term captures the indirect transmission through an additional layer of input-output

linkages.

We make three observations. First, as in our baseline model, the input cost channel transmits *forwards*, from input supplier to input users. This is evident as the effect transmits through the power series of the matrix $\hat{\mathbf{\Sigma}}$, the $ik$-th entry of which scales with $\sigma_{ik}$, the cost share sector $i$'s clean production on input $k$. The two diagonal matrices sandwiching the matrix power series demonstrate that the effect is stronger if the clean share of value-added is higher in sector $i$ and sector $k$. By contrast, the market size channel transmits *backwards*, from input users to input suppliers. This is evident as the effect transmits through the power series of the matrix $\mathbf{\Omega}$, the $ik$-th entry of which is $\frac{\sigma_{ki}\chi_k r_k}{r_i}$, capturing the equilibrium fraction of output from sector $i$ that is sent to sector $k$ as inputs.

Second, we note that for a focal intermediate producer $i$, the sum of input cost effects from all of its suppliers is always less than one ($\sum_j \sigma_{ij}\chi_j < 1 - \alpha_i < 1$), whereas by the market clearing condition (28), the sum of the market size effect is always equal to one ($\sum_i \frac{\sigma_{ij}\chi_i r_i}{r_j} = 1$). Moreover, because the input cost effects scale with the degree of greenification in the input-supplying sector $\chi_j$, in an economy where there is little greenification across all sectors, the total network effect of the input cost channel coming from a marginal greenification in all of sector $i$'s suppliers can become arbitrarily small relative to even the first round, direct market size effect coming from each of buyers of input $i$. Formally, denote the degree of greenification in an original steady-state $\chi_i = \epsilon x_i$ for all sectors $i$, with $x_i > 0$. In the limit as $\epsilon \to 0$, for each sector $i$, and for each sector $j$ such that $\sigma_{ij} > 0$, we have that

$$\lim_{\epsilon \to 0} [\mathbf{\Omega}]_{ij} > \lim_{\epsilon \to 0} \sum_{k=1}^{N} \left[ \hat{\mathbf{\Sigma}} \left( \boldsymbol{I} - \hat{\mathbf{\Sigma}} \right)^{-1} \right]_{ik} = 0.$$

Our third observation is a corollary of the second. Continue to denote the degree of greenification in an original steady-state as $\chi_i = \epsilon x_i$. If the network is acyclic (meaning that if inputs flow from sector $i$ to $j$, directly or indirectly, then inputs do not flow from $j$ to $i$; formally, for any sector pairs $i, j$, $\left[\Sigma^k\right]_{ij} > 0$ for some $k \in \mathbb{Z}_{>0}$ implies $\left[\Sigma^k\right]_{ji} = 0$ for all $k$), then for a sector $j$ upstream to $i$ (i.e., $\left[\Sigma^k\right]_{ij} > 0$ for some $k \in \mathbb{Z}_{>0}$), as $\epsilon \to 0$, the asymmetric propagation of incentives to greenify imply that $\frac{\mathrm{d} \ln \pi_i}{\mathrm{d} \ln \chi_j} \leq \frac{\mathrm{d} \ln \pi_j}{\mathrm{d} \ln \chi_i}$.

We conclude that our insights from the baseline model of a vertical production chain continues to hold in this general network environment. Greenification is strategically complementary across sectors, through an input cost channel which propagates from input-suppliers to input-buyers and a market size channel which propagates from input-buyers to input-suppliers. Moreover, when the degree of greenification is low across the economy, the market size channel always dominates the input cost channel; in an acyclic network, this means that targeting downstream sectors create more incentives across the economy to greenify.

# 6 Industrial Policy with Incomplete Carbon Prices

So far, we have assumed that carbon prices are Pigouvian. Unfortunately, global carbon prices are likely far below the true social cost of carbon, and some countries, notably the US, are explicitly trying to use industrial policy as a substitute for a carbon price, rather than as a supplement. For this reason, we will now relax the assumption of Pigouvian carbon prices and consider optimal industrial policy in the face of suboptimal carbon prices.

## 6.1 Planner's Problem

First, we consider a version of the Planner's problem where the carbon price is restricted. That is, the Planner has a complete set of greenification subsidies, but due to some political restriction, the carbon price $\tau$ is set to an exogenous level below the true social cost $\xi$.[22] In this case, the policy problem requires selecting a level of greenification knowing that production decisions will be made in a distorted equilibrium. To keep the analysis otherwise comparable to that of Section 4, we allow the Planner to subsidize the monopoly markup, and as before, all greenification will happen in the initial period, allowing us to drop time subscripts from the problem. To consider the equilibrium relationship between the allocation of labor and the level of greenification, we define functions $\{\ell_{di}(\{\chi_j\}), \ell_{ci}(\{\chi_j\})\}$ that satisfy

$$\{\ell_{di}(\{\chi_j\}), \ell_{ci}(\{\chi_j\})\} = \text{argmax}\Big[p_N y_N - \sum_i \Big((1+\tau)\ell_{di} + \ell_{ci}\Big)\Big], \tag{29}$$

where the final good price $p_N$ is set according to the household condition (3). Equation (29) stipulates that the allocation of labor must be consistent with profit maximization, and because the economy is production efficient, it is as if there is a single, price-taking firm solving the above problem. Note that this firm must pay the numeraire wage for labor as well as the carbon price of $\tau$ for dirty labor.

The Planner then maximizes household utility with industrial policy that anticipates this equilibrium relationship. They solve

$$\max_{\{\chi_j\}} \frac{1}{1-\beta}\Big(\ln(y_N) - (1+\xi)\sum_i \ell_{di} - \sum_i \ell_{ci}\Big) - \sum_i \mathcal{F}_i(\chi_i). \tag{30}$$

Optimal industrial policy in this second-best setting is characterized as follows:

**Proposition 6.** *In the absence of a Pigouvian carbon price, optimal greenification satisfies*

[22] We retain the assumption that $1+\tau > e^{-z}$ so that there is an incentive to greenify in the equilibrium.

$$\chi_i = F_i\left(\frac{\ln\left((1+\tau)\,e^z\right)}{1-\beta}\mu_i\prod_{j>i}\left[\chi_j\left(1-\alpha_j\right)\right]+\frac{\xi-\tau}{1-\beta}\left(-\frac{\partial \ell_d}{\partial \chi_i}\right)\right). \tag{31}$$

*Proof.* See Appendix 10.6. □

Proposition 6 shows how to adjust industrial policy in the face of suboptimal carbon prices. The first term mirrors that of Equation (20) in that it reflects the direct, perpetual benefit of greenification through expanding production. The only difference is that equilibrium output is determined by the carbon price $\tau$, rather than $\xi$. The second term reflects the wedge between the carbon price and its true social cost, and it pushes industrial policy to consider how greenification influences the equilibrium level of emissions. Now, greenification in a given sector should be higher in so far as it generates a reduction in equilibrium emissions $-\frac{\partial \ell_d}{\partial \chi_i}$, and the emphasis on emission reductions should scale with the carbon price wedge.[23] Naturally, we recover the first-best whenever carbon prices are Pigouvian.

The parsimony of Equation (31) comes from the fact that private producer optimization makes all of the equilibrium effects of greenification envelope out, except insofar as those equilibrium effects interact with the carbon price wedge. In principle, the Planner must consider the effect of changes in the entire labor allocation – clean and dirty in every sector – on their objective, but Equation (29) implies that these changes have only a second-order effect on consumption utility net of labor disutility. However, the effect on disutility from emissions is first-order because the price that producers face for polluting is incorrect.[24]

## 6.2 Equilibrium Emissions

Proposition 6 raises a natural question similar to that considered in Section 5: where along the supply chain will greenification most effectively reduce emissions? Interestingly, it is again downstream greenification that generates the greatest equilibrium response when the level of greenification elsewhere is low.[25]

First, consider that equilibrium emissions follow

$$\ell_d = \frac{1}{1+\tau}\sum_i r_i(1-\chi_i).$$

[23] Note that there is a negative on the emissions derivative to assign a positive value to a *reduction*.

[24] As in Proposition 4, the Planner can implement the second-best steady-state as an equilibrium with sector-specific greenification subsidies. The same caveat applies where greenification subsidies alone only implement the second-best allocation after the initial period because the monopoly markup will distort the use of newly greenified inputs in the initial period.

[25] Downstream's greater emission reductions hold in addition to downstream's greater propagation of greenification because in this section we will hold greenification elsewhere constant.

We can see that a sector's emissions equal downstream demand for inputs – reflected in revenue – times the share of intermediates in one's own sector not greenified, all over one plus the carbon price. This provides us a mapping from greenification to equilibrium emissions that allows us to consider the emission reductions of Equation (31).

Differentiating, we have that emission reductions follow

$$-\frac{\partial \ell_d}{\partial \chi_i} = \frac{1}{1+\tau} r_i \Big[1 - (1-\alpha_i) \sum_{i>q} \prod_{i>k>q} \chi_k (1-\alpha_k)(1-\chi_q)\Big]. \tag{32}$$

From this, we can see that the impact of greenification on emissions depends on the level of greenification both downstream and upstream. The market size of a sector is determined by downstream demand, so upstream greenification will be less effective at reducing emissions when downstream sectors are less greenified. The term in the square brackets represents the direct displacement of dirty production – the one – minus the offsetting increase in emissions coming from increased demand for dirty inputs in all upstream sectors. The fact that $\chi_j, \alpha_j \in (0,1)\ \forall j$ implies that this offsetting term from upstream demand must be less than one, so the offsetting effect is always weaker than the direct effect, guaranteeing that emissions (weakly) reduce.[26]

In the previous sections, we have argued that there is strategic complementarity in the adoption of greenification, and now we can see that there is a complementarity in emission reductions: greenification in each sector is more effective when the others are more greenified. And as before, this complementarity implies that, for low levels of greenification, targeting the downstream sector will be more effective. To see this, consider again the case where $\chi_j = \epsilon x_j$. Then for any $i < j$, we have that

$$\lim_{\epsilon \to 0} \frac{-\partial \ell_d / \partial \chi_i}{-\partial \ell_d / \partial \chi_j} = \lim_{\epsilon \to 0} \prod_{i \geq q > j} \chi_q (1-\alpha_q) \left[\frac{1-(1-\alpha_i)}{1-(1-\alpha_j)}\right] = 0.$$

That is, the more downstream sector $j$ will achieve greater emission reductions when greenification goes to zero. This is because emission reductions for a given sector go to zero as greenification in its downstream sectors goes to zero, and the rate of reduction is of the same order as the number of downstream sectors. However, input shares less than one imply that emission reductions will not go to zero as greenification in upstream sectors goes to zero. Thus, the effectiveness of greenification at reducing emissions is more "vulnerable" to low greenification downstream, rather than upstream. This means that, for low levels of greenification, policymakers should fo-

[26] To see this, note that $(1-\alpha_i)\sum_{i>q}\prod_{i>k>q}\chi_k(1-\alpha_k)(1-\chi_q) \leq \sum_{i>q}\prod_{i>k>q}\chi_k(1-\chi_q) \leq \sum_{i>q}\prod_{i>k>q}\chi_k(1-\chi_q) + \prod_{i>q}\chi_q = 1$. The final equality comes from the fact that one can interpret the preceding sum as a sum of probabilities representing the number of coin flips before achieving a tails, when one is flipping $i-1$-many coins with heads probability $\chi_q$. This argument also implies that $-\frac{\partial^2 \ell_d}{\partial \chi_i \partial \chi_k} \geq 0$, formalizing the fact that greenification is more effective at reducing emissions when other sectors are more greenified.

cus on downstream to reduce emissions as this sort of greenification is not made ineffective by the lack of greenification elsewhere.

Input shares also play a role in the sectoral heterogeneity of emission reductions. Low input shares downstream proportionally dampen emission reductions independent of the level of greenification elsewhere, but low input shares upstream – as well as in one's own sector – enhance emission reductions by reducing the offsetting effect. Therefore, an economy characterized by low input shares throughout the supply chain will favor downstream greenification as these low input shares will penalize upstream sectors by reducing their market size but reward downstream sectors by reducing their total demand for dirty inputs.

## 6.3 Two Sector Example

The relative effectiveness of downstream greenification is well-illustrated in an example economy with two sectors: downstream and upstream. In that case, Equation (32) implies

$$-\frac{\partial \ell_d}{\partial \chi_2} = \frac{1-(1-\alpha_2)(1-\chi_1)}{1+\tau} \text{ and } -\frac{\partial \ell_d}{\partial \chi_1} = \frac{\chi_2(1-\alpha_2)}{1+\tau}.$$

From this, we can see that downstream greenification always reduces emissions, but upstream greenification only reduces emissions insofar as downstream is already greenified ($\chi_2 > 0$). While greenification in each sector is more effective when the other is more greenified, the upstream sector is more "vulnerable" to low greenification of its counterpart. For example, greenifying the productions of car batteries has little effect if no one buys electric cars.

We can better understand this asymmetry by sketching the line where the two sectors are equally effective at reducing emissions:

$$\chi_2 = \frac{\alpha_2}{1-\alpha_2} + \chi_1.$$

When $\chi_2$ is greater than this line, upstream greenification will be more effective at reducing emissions, whereas downstream will be more effective if $\chi_1$ is greater than this line. Note that the slope is always one, but the intercept is decreasing in the downstream input share. This is because a larger input share strengthens the offsetting effect for downstream, while raising the market size for upstream. In the extreme case of an input share equal to one, the situation is symmetric and greenifying the lagging sector is more effective. But as the input share shrinks below one, the space where upstream greenification is better at reducing emissions shrinks. Once the input share reaches one half, downstream greenification will always be (weakly) more effective.

## 6.4 General Supply Chain

We have argued that downstream greenification is more effective at reducing emissions, so we will again examine the generality of this argument by considering a general supply chain network like the one described in Section 5.3. Note that the optimal policy results of Proposition 6 carry over to this more general environment. The substantive difference will be in the equilibrium emission reductions: $-\frac{\partial \ell_d}{\partial \chi_i}$.

Total emissions in this more general setting are given by

$$\ell_d = \frac{1}{1+\tau} \boldsymbol{r}'(\vec{\mathbf{1}} - \boldsymbol{\chi}).$$

That is, emissions equal equilibrium revenue for each sector multiplied by its dirty share. If we define the revenue flow matrix $\Theta_{ij} \equiv \chi_i \sigma_{ij}$, we can again ask how emissions will be affected by greenification in a given sector, which is given by

$$-\frac{\partial \ell_d}{\partial \chi_i} = \frac{1}{1+\tau} \boldsymbol{r}' \Big[ \boldsymbol{e_i} - \frac{\partial \boldsymbol{\Theta}}{\partial \chi_i} \big(\mathbf{I} - \boldsymbol{\Theta}\big)^{-1} (\vec{\mathbf{1}} - \boldsymbol{\chi}) \Big], \tag{33}$$

where $\boldsymbol{e_i}$ is a standard unit vector for the $i$th dimension.[27] Note that $\frac{\partial \boldsymbol{\Theta}}{\partial \chi_i}$ will be all zeros but for the $i$th row composed of the input shares $\sigma_{ij}$. Therefore, Equation (33) has a very similar interpretation to Equation (32), with a term reflecting a sector's total demand out front and square brackets that include both a direct reduction in emissions as well as an offsetting increase in emissions due to demand for the inputs of other sectors.

As before, the effectiveness of upstream greenification for reducing emissions is more "vulnerable" to low levels of greenification. In particular, let $n(j)$ denote the minimum number of steps through the network needed before the goods from sector $j$ are used in the final good. That is, $n(N) = 0$, $n(j) = 1$ for any sector that supplies to final good ($\sigma_{Nj} > 0$), $n(j) = 2$ for any sector that supplies the one-step sectors (but not the final good), and so on. In this more general setting, $n$ provides us with a measure of a sector's upstreamness. If we again consider $\chi_j = \epsilon x_j$ and take $\epsilon$ to zero, then the term in square brackets will converge to $[1 - (1 - \alpha_i)]\boldsymbol{e_i}$. Moreover, the revenue of a sector will be $\mathcal{O}(\epsilon^{n(j)})$ because revenue will have to pass through $n(j)$-many steps before it reaches sector $j$. Thus, as in the vertical network case, we have that more upstream sectors are less effective at reducing emissions for low levels of greenification.

[27] To see this, note that $\boldsymbol{r}' = \boldsymbol{e}'_{\boldsymbol{N}}(\mathbf{I} - \boldsymbol{\Theta})^{-1}$ and $\frac{\partial (\mathbf{I} - \boldsymbol{\Theta})^{-1}}{\partial \chi_i} = (\mathbf{I} - \boldsymbol{\Theta})^{-1} \frac{\partial \boldsymbol{\Theta}}{\partial \chi_i} (\mathbf{I} - \boldsymbol{\Theta})^{-1}$. Combining gives us Equation (33).

# 7 Horizontal misallocation

In Sections 4 and 7, we showed that industrial policy can bring large welfare gains. Yet, misdirected green industrial policies can also bring welfare losses if it "picks the wrong winner". We already saw in the previous section that in supply chains with more than two sectors, a one-time-one-sector intervention should focus on the most downstream sectors. In this section, we entertain the possibility of "horizontal" misallocations of public greenification investment. More specifically, we consider an extended version of our basic model with two layers, where on top of using labor, the downstream dirty industrial process also uses inputs from an upstream sector that can be also greenified.[28] In that context, greenification in the two upstream sectors are strategic substitutes, and over-investing in the greenification of the upstream sector associated with the dirty downstream process may derail the overall transition towards a clean economy.

## 7.1 A simple two-legs model

We consider a simple two-layer network, with a single downstream sector and two upstream sectors denoted by $1a$ and $1b$. The downstream sector uses input $1a$ when producing using the dirty technology and it uses input $1b$ when producing using the clean technology. More formally, a non-greenified downstream variety is produced according to:

$$y_2(v) = \left(\frac{l_{2t}^d(v)}{\alpha}\right)^{\alpha} \left(\frac{m_{1at}(\nu)}{1-\alpha}\right)^{1-\alpha}$$

whereas an greenified downstream variety is produced according to:

$$y_2(v) = \left(\frac{l_{2t}^d(v)}{\alpha}\right)^{\alpha} \left(\frac{m_{1at}(\nu)}{1-\alpha}\right)^{1-\alpha} + \left(\frac{e^z l_{2t}^c(v)}{\alpha}\right)^{\alpha} \left(\frac{m_{1bt}(\nu)}{1-\alpha}\right)^{1-\alpha}$$

Varieties in the upstream sectors are produced according to:

$$y_{1j}(v) = \begin{cases} l_{1j}^d(v) & \text{if not greenified} \\ l_{1j}^d(v) + e^z l_{1j}^c(v) & \text{if greenified} \end{cases} \quad \text{for } j \in \{a, b\}.$$

greenification involves the fixed cost distributions $F_2$, $F_{1a}$ and $F_{1b}$.[29]

[28] For instance, fossil fuel engines for vehicles can be produced in a cleaner way or become more fuel efficient.

[29] A generalization would involve the clean and dirty industrial processes using both upstream goods with Cobb-Douglas shares: $\sigma_a^d$ and $\sigma_b^d$ denoting respectively the use of sector $1a$ and $1b$ in the dirty downstream process, and $\sigma_a^c$ and $\sigma_b^c$ denoting the corresponding shares for the clean process. Then the case with $\sigma_a^d = \sigma_b^d = 0$ corresponds to the original model and its results based on strategic complementarity extend directly for $\sigma_a^d < \sigma_a^c$ and $\sigma_b^d < \sigma_b^c$. If $\sigma_a^d = \sigma_a^c$ and $\sigma_b^d = \sigma_b^c$, then there will be no strategic complementarity. If $\sigma_a^d > \sigma_a^c$ and $\sigma_b^d < \sigma_b^c$ (as here), there is strategic substitutability.

## 7.2 Equilibrium

We solve for the equilibrium following similar steps to Section 3.

**Prices.** As before, in the upstream sectors, the price of a variety greenified in the previous period is $p_{1jt}(\nu) = e^{-z}$ for $j \in \{a, b\}$,whereas the price of a newly greenified variety or of a non-greenified variety is $p_{1jt}(\nu) = (1+\tau_t)$. We then get that in the downstream sector 2, the price of a variety is given by:

$$p_{2t}(\nu) = \begin{cases} \min\left[(1+\tau)^{\alpha} p_{1,a,t}^{1-\alpha}, e^{-\alpha z} p_{1,b,t}^{1-\alpha}\right] & \text{if sector 2 is greenified by time } t-1, \\ (1+\tau)^{\alpha} p_{1,a,t}^{1-\alpha} & \text{otherwise.} \end{cases} \tag{34}$$

We still assume that $e^{-z} < 1+\tau$, so that the upstream sectors use the clean production process when greenified. Therefore, the price of the upstream input $1j$ at date $t$ follows the same formula as in the baseline model, namely: $p_{1jt} = (1+\tau)\, e^{-Z\chi_{1j,t-1}}$ with $Z = z + \ln(1+\tau) > 0$.

The assumption that $e^{-z} < 1+\tau$ is no longer sufficient to ensure that the downstream sector always uses the clean production process since the dirty input may be cheaper than the clean one if it is itself sufficiently greenified. Instead, the downstream sector will use the clean production process if and only if

$$e^{-\alpha z} p_{1,b,t}^{1-\alpha} < (1+\tau)^{\alpha} p_{1,a,t}^{1-\alpha}$$

or equivalently if and only if

$$\mu_{2,t-1} \equiv \alpha - \chi_{1a,t-1}(1-\alpha) + \chi_{1b,t-1}(1-\alpha) > 0, \tag{35}$$

where we adjusted the definition of $\mu_{2,t}$ relative to the baseline model. For expositional purposes we henceforth assume that $\alpha > 1/2$ so that the above condition is always satisfied.

**Equilibrium revenues and profits.** As in our baseline model, the profit margin is equal to $1 - e^{-Z}$ in the upstream sectors $1a$ and $1b$. In the downstream sector 2, the profit margin is given by:

$$1 - \frac{e^{-\alpha z} p_{1,b,t}^{1-\alpha}}{(1+\tau_t)^{\alpha} p_{1,a,t}^{1-\alpha}} = 1 - e^{-Z\mu_{2,t-1}},$$

which is positive since by assumption, we always have $\mu_{2,t} > 0$ for all $t$. This is again the same expression as in the baseline model.

We now derive the revenue accruing to each variety. In the downstream sector 2, we have, as before that revenues obey $r_{2t} = p_t c_t = 1$.

Consider now the upstream sector $1b$. Note that $r_{1bt}$ follows exactly the same logic as the

upstream sector in the baseline model, namely:

$$r_{1bt} = \underbrace{\chi_{2,t-1} r_{2,t} (1-\alpha)}_{\text{sales to previously greenified varieties}} + \underbrace{(\chi_{2,t} - \chi_{2,t-1}) r_{2,t} e^{-Z\mu_{2,t-1}} (1-\alpha)}_{\text{sales to newly greenified varieties}}$$

which we again rewrite as $r_{1bt} = \widetilde{\chi}_{2,t} (1-\alpha)$ with $\widetilde{\chi}_{2,t} \equiv \chi_{2,t-1} + (\chi_{2,t} - \chi_{2,t-1}) e^{-Z\mu_{2,t-1}}$ as before.

Consider now the upstream sector $1a$ : all the revenues accruing to greenified varieties in that sector come from still non-greenified varieties in sector 2, so that:

$$r_{1at} = (1 - \chi_{2,t}) r_{2,t} (1-\alpha) = (1 - \chi_{2,t}) (1-\alpha).$$

The greenification rents in the three sectors $2, 1a, 1b$ are then given by the profit margins times the revenues, namely, they follow (10) for $i \in \{2, 1a, 1b\}$ ,with $\mu_{1a,t} = \mu_{1b,t} = 1$ as in the baseline model.

**Greenification levels.** As in the baseline model, the producer of a variety in sector $i$ will greenify at time $t$ if and only if the rents from greenification $\pi_{it}$ are bigger than her fixed cost of greenification. We then obtain the equilibrium conditions for the greenification levels as:[30]

$$\chi_{1at} = F_{1a}\left(\left(1 - e^{-Z}\right)(1 - \chi_{2,t})(1-\alpha)\right), \tag{36}$$

$$\chi_{1bt} = F_{1b}\left(\left(1 - e^{-Z}\right)\widetilde{\chi}_{2,t}(1-\alpha)\right), \tag{37}$$

$$\chi_{2t} = F_2\left(1 - e^{-Z\mu_{2,t-1}}\right). \tag{38}$$

This system is recursive: given that $\mu_{2,t-1}$ is predetermined, the third equation determines $\chi_{2t}$, from which we uniquely determine $\chi_{1at}$ and $\chi_{1bt}$ using the first two equations. Thus, as in the baseline model, the equilibrium is unique given initial greenification shares in the three sectors.

**Steady-states.** In a steady-state the greenification levels are constant over time, so that steady-states are now defined by:[31]

$$\chi_{1a} = F_{1a}\left(\left(1 - e^{-Z}\right)(1 - \chi_2)(1-\alpha)\right)$$

$$\chi_{1b} = F_{1b}\left(\left(1 - e^{-Z}\right)\chi_2(1-\alpha)\right)$$

$$\chi_2 = F_2\left(1 - e^{-Z\mu_2}\right)$$

[30] Technically, greenification levels can never decrease so they are given by the minimum of the right-hand side and the levels in the previous period exactly as in the baseline model.

[31] Again we focus on strict steady-states where the three conditions below hold as equalities; the actual set of steady-states is larger since whenever the left-hand side is greater than the right-hand side we also have a steady-state as greenification levels cannot decrease.

with $\mu_2 = \alpha - (1-\alpha)\,\chi_{1a} + \chi_{1b}\,(1-\alpha)$. The first equation establishes that $\chi_{1a}$ is a decreasing function of $\chi_2$. The second equation establishes that $\chi_{1b}$ is increasing $\chi_2$ and the third equation establishes that $\chi_2$ is increasing in $\chi_{1b}$, but decreasing in $\chi_{1a}$. Therefore greenifications in $1b$ and $2$ are strategic complement but greenifications in $1a$ versus $1b$ and $2$ are strategic substitutes. In other words, greenifying sector $1a$ diverts efforts away from greenifying sectors $2$ and $1b$, since it reduces the cost of the dirty industrial process in the downstream sector compared to the clean industrial process.

The strategic substitutability between $1a$ and $2$ plus $1b$ will have three important implications: First, an industrial policy that favors $1a$ may backfire and halt the greenification that would have otherwise happened in $2$ and $1b$ without government intervention — thereby reducing long-term welfare compared to no intervention. Second, the laissez-faire equilibrium may involve too much greenification of $1a$ compared to the ex-ante optimum. But, given the history dependence in the dynamics of greenification shares over time, delaying greenification in $2$ plus $1b$ in turn leads to irreversible long-term consequences. We illustrate these two possibilities in the remaining part of this section. Third, we note that strategic substitutability provides another argument to target in priority greenification in the downstream sector: An increase in greenification in the downstream sector, incentivizes greenification in the "right" upstream sector ($1b$). In contrast, and as just argued, a policy that targets the wrong upstream sector ($1a$) discourages greenification downstream.

## 7.3 Backfiring industrial policy

Here we build an example where an industrial policy initially focused on the upstream sector $1a$ backfires: namely, in this example, given initial conditions and the greenification cost functions, the laissez-faire is associated with full greenification in sectors $2$ and $1b$, whereas a (misguided) industrial policy focusing on sector $1a$ in the initial period, reduces long-run welfare by preventing full greenification in sectors $2$ and $1b$. Key in our example is the fact that greenification in sector $1a$ reduces the cost-advantage of greenified varieties in sector $2$, which reduces the incentives to greenify both sector $2$ and its upstream sector, 1b.

**Example 3.** We assume that the economy initially features no-greenification in all three sectors ($\chi_{i,0} = 0$ for $i \in \{2, 1a, 1b\}$). We consider two potential policies: In the first one, the government does not subsidize greenification. In the second one, the government subsidizes greenification in sector $1a$ at time $t = 1$,such that $\chi_{1a,1} = 1$. In both cases, the government implements a Pigouvian carbon tax: $\tau = \xi$.

Consider first the case with no government intervention. Then greenification at time $1$ in sector $2$ follows from (38): with $\mu_{2,0} = \alpha$, $\chi_{2,1}$ is given by $\chi_{2,1} = F_2\left(1 - e^{-Z\alpha}\right)$. In addition,

following (8), $\widetilde{\chi}_{2,1} = \chi_{2,1} e^{-Z\alpha}$,so that, using (37), $\chi_{1b,1}$ satisfies:

$$\chi_{1b,1} = F_{1b}\left(\left(1 - e^{-Z}\right)\chi_{2,1} e^{-Z\alpha}\left(1-\alpha\right)\right).$$

We assume that this is positive, i.e. the smallest fixed cost of greenification in sector $1b$ lies below $(1 - e^{-Z})\chi_{2,1} e^{-Z\alpha}\left(1-\alpha\right)$. For sector $1a$, we then get:

$$\chi_{1a,1} = F_{1a}\left(\left(1 - e^{-Z}\right)\left(1-\chi_{2,1}\right)\left(1-\alpha\right)\right),$$

which we also assume to be equal to zero. This in turn will be the case if the smallest fixed cost of greenification in sector $1a$ is greater than $\left(1 - e^{-Z}\right)\left(1-\alpha\right)$.

We now consider period $t = 2$. Since $\chi_{2,t}$ is non-decreasing, then incentives to greenify sector 1a are weakly smaller, so that we still have $\chi_{1a,2} = 0$. Using (35), we get $\mu_{2,1} = \alpha + \chi_{1b,1}\left(1-\alpha\right)$. Therefore, following (38), we get that greenification in sector 2 is given by:

$$\chi_{2,2} = F_{2b}\left(1 - e^{-Z\mu_{2,1}}\right),$$

which we take to be equal to 1. That is all greenification costs in sector 2 are below $1 - e^{-Z\mu_{2,1}}$. We now get $\widetilde{\chi}_{2,2} = \chi_{2,1} + \left(1-\chi_{2,1}\right)e^{-Z\mu_{2,1}}$, so that using (37), $\chi_{1b,2}$ satisfies:

$$\chi_{1b,2} = F_{1b}\left(\left(1 - e^{-Z}\right)\left(\chi_{2,1} + \left(1-\chi_{2,1}\right)e^{-Z\mu_{2,1}}\right)\left(1-\alpha\right)\right).$$

Again, we take this to be equal to 1, namely all greenification costs in sector $1b$ are below the term at which $F_{1b}$ is evaluated in the expression above. With full greenification in sectors 1b and 2, the economy has reached a steady-state with $\chi^*_{1a} = 0$, $\chi^*_{1b} = \chi^*_2 = 1$. Not greenifying sector $1a$ in this context comes at no cost, since the input from that sector is used only for a dirty sector 2 which disappears from $t = 3$ onwards. The corresponding utility flow is:[32]

$$\ln y_2 - 1 = z - 1.$$

Now, suppose instead that starting from no-greenification in all sectors, the government decides to fully greenify the upstream sector $1a$ at time $t = 1$, i.e. sets $\chi^\dagger_{1a,1} = 1$ (we add $^\dagger$ to denote variables under this alternative scenario). Since greenifications incentives only move downstream with a lag, this does not change $\chi_{2,1}$ and $\chi_{1b,1}$, which remain the same as without the policy: $\chi^\dagger_{2,1} = \chi_{2,1}$ and $\chi^\dagger_{1b,1} = \chi_{1b,1} = 0$.

Consider now time $t = 2$. Using (35), we get that: $\mu^\dagger_{2,1} = \alpha - \left(1-\chi_{1b,t-1}\right)\left(1-\alpha\right) < \mu_{2,0}$. As a result, there is no further greenification in sector 2, namely: $\chi^\dagger_{2,2} = \widetilde{\chi}^\dagger_{2,2} = \chi^\dagger_{2,1} = \chi_{2,1}$. In sector $1b$, using (37), we get:

$$\chi^\dagger_{1b,2} = F_{1b}\left(\left(1 - e^{-Z}\right)\chi_{2,1}\left(1-\alpha\right)\right).$$

[32]In Appendix 10.4, we show that in steady-state, the utility flow is given by

$$\ln y_2 - \sum_{i\in\{1a,1b,2\}}\left(\left(1+\xi\right)\ell_{di} + \ell_{ci}\right) = \left[\chi_2\alpha + \chi_2\left(1-\alpha\right)\chi_{1b} + \left(1-\chi_2\right)\left(1-\alpha\right)\chi_{1a}\right]Z - \ln\left(1+\xi\right) - 1.$$

We assume that this is still equal to $\chi_{1b,1}$: That is the distribution of greenification costs in sector $1b$ is such that, a positive mass of varieties have fixed costs below $\left(1-e^{-Z}\right)\chi_{2,1}e^{-Z\alpha}\left(1-\alpha\right)$, no varieties has fixed costs in the interval $\left(1-e^{-Z}\right)\left(1-\alpha\right)\times\left(\chi_{2,1}e^{-Z\alpha},\chi_{2,1}\right)$, and all remaining varieties have their fixed costs in the interval $\left(1-e^{-Z}\right)\left(1-\alpha\right)\times\left(\chi_{2,1},\left(\chi_{2,1}+\left(1-\chi_{2,1}\right)e^{-Z\mu_{2,1}}\right)\right)$.[33] At this point, the economy has reached a steady-state and no further greenification occurs. This gives a flow utility

$$\ln y_2 - 1 = \left[\chi_{2,1}\alpha + \chi_{2,1}\left(1-\alpha\right)\chi_{1b,1} + \left(1-\chi_{2,1}\right)\left(1-\alpha\right)\right]Z - \ln\left(1+\xi\right) - 1.$$

As long as $\chi_{2,1}$ is sufficiently small – which is certainly feasible –, this is strictly less than the flow utility $z-1$ under laissez-faire. This establishes the result described at the beginning of the subsection.

Note however that industrial policy cannot backfire when the decentralized economy is initially already stuck in a steady-state. Since the steady-state utility flow is weakly increasing in all the $\chi's$, the only cost associated with industrial policy in that case are the costs of the subsidies themselves (i.e. the costs associated with the corresponding increase in the $\mathcal{F}_i\left(\chi_i\right)$'s).

## 7.4 Excessive greenification and path dependence

We now build an example where, given initial conditions and the greenification cost functions, the laissez-faire is associated with positive greenification in sector $1a$ and no greenification in sectors $2$ and $1b$, whereas thee optimal policy would involve less greenification in sector $1a$ but greenification in sectors $2$ and $1b$. In other words: the laissez-faire economy exhibits excessive greenification in sector $1a$.

**Example 4.** We still assume that a Pigouvian carbon tax is in place. We choose the cost functions so that there exists a steady-state equilibrium $(\chi_{1a}^*,\chi_{1b}^*,\chi_2^*)$ with $\chi_{1a}^*>0$ and $\chi_{1b}^*=\chi_2^*=0$. That is:

$$\chi_{1a}^* = F_{1a}\left(\left(1-e^{-Z}\right)\left(1-\alpha\right)\right)$$
$$\chi_2^* = 0 = F_2\left(1-e^{-Z\left(\alpha-(1-\alpha)\chi_{1a}^*\right)}\right)$$

(the latter condition requires that the lowest fixed cost in sector 2 lie above $1-e^{-Z\left(\alpha-(1-\alpha)\chi_{1a}^*\right)}$.

Suppose now that initially at time $t=0$ the economy starts with greenification shares $\chi_{1a,0}=\chi_{1a}^*-\varepsilon$ (with $\varepsilon>0$ but small) and $\chi_{1b,0}=\chi_{2,0}=0$. Then, we get that $\chi_{1a,1}$ and $\chi_{2,1}$ solve:

$$\chi_{1a,1} = F_{1a}\left(\left(1-e^{-Z}\right)\left(1-\alpha\right)\right) = \chi_{1a}^* \text{ and } \chi_{2,1} = F_2\left(1-e^{-Z(\alpha-(1-\alpha)\chi_{1a,0})}\right) = 0,$$

[33] Of course, this is a very specific case, but our goal here is simply to build an example and that one certainly exists.

if $\varepsilon$ is sufficiently small and the smallest fixed greenification cost in sector 2 lies significantly above $1-e^{-Z\left(\alpha-(1-\alpha)\chi_{1a}^{*}\right)}$ relative to $\varepsilon$. Then the laissez-faire economy will reach the steady-state $(\chi_{1a}^{*}, \chi_{1b}^{*}=0, \chi_{2}^{*}=0)$.

Let us compare this laissez-faire equilibrium with the social optimum. We note first that the social planner always wants to implement the socially optimal steady-state immediately (the Pigouvian tax $\tau=\xi$). Then, provided that greenification costs are bounded above, a sufficiently patient social planner will seek to maximize steady-state utility flow, which is maximized for $\chi_{1b}=1$ and $\chi_{2}=1$ and is independent of $\chi_{1a}$ – see footnote 32. Therefore, the social planner will immediately set $(\chi_{1a,0}, 1, 1)$. The corresponding $\chi_{1a}$ is lower than under laissez-faire, which in turn establishes that there is excessive greenification of $1a$ in the laissez-faire equilibrium compared to the social optimum.

# 8 Numerical example

This section presents a quantitative illustration of our model from the transportation sector, which is responsible for around a quarter of global energy-related $CO_2$ emissions. Within this sector, we focus on long-range and heavy duty (LRHD) transportation, which we define as trucking,[34] aviation, shipping, and rail, which account for around 53% of global transport-related emissions (Ritchie 2020). While the vast majority of this sector still runs on fossil fuels, we consider "greenification" via a switch to hydrogen (H2, either in fuel cells or direct combustion). In contrast to light duty vehicles such as passenger cars, battery electrification is often less viable for LRHD applications due to, e.g., the weight of the batteries and the high charging loads that would be required (e.g., Gross 2020; Tiwari et al. 2024).[35] Even for rail transport, where electrification is advanced in some parts of the world,[36] the high capital cost of catenary system expansions can favor hydrogen as a way of decarbonizing diesel locomotive areas already at current conditions (Ahluwalia et al. 2021). We thus focus on hydrogen, which is generally considered a key potential technology to help decarbonize LRHD transportation (IPCC 2024) due to advantages such as its abundance, high gravimetric energy density, and the fact that its combustion releases only water vapor. Indeed, hydrogen-based LRHD transportation models are emerging both on the market and under commercial development for varieties ranging from trucks (e.g., Hyundia XCIENT, Nikola) and trains (e.g., Alstom Coradia iLint) to passenger aircraft (e.g., Airbus ZEROe, Embraer Energia), although their market share (and thus the initial value of $\chi$) remains close to zero. At the same time how-

[34] We include in "heavy duty" some trucks that would technically be categorized as "medium duty" under the US Department of Transportation classification based on gross vehicle weight ratings.

[35] Of course we acknowledge that, in reality, there is also crossover across technologies in light- and heavy-duty applications, as there are both some hydrogen-powered passenger vehicles and electric trucks on the market.

[36] In the United States, diesel fuels still account for over 95% of railway energy consumption (Davis and Boundy 2020).

ever, hydrogen can itself be produced using both fossil fuels or (green) electricity. Currently, less than 1% of the world's hydrogen production is "green" even if fossil-based production with carbon capture is included in this measure (IEA 2024). Consequently, it is precisely the greenification of the full value chain that is required in this context. Industry observers also frequently note that a core challenge to broader deployment of hydrogen LRHD vehicles lies in the hydrogen value chain (e.g., Gross 20200), further highlighting the relevance of our model application to this context.

We calibrate the model to the US economy with a 2-sector structure: gaseous hydrogen (GH2) production and distribution as upstream sector 1, and LRHD transportation as downstream sector 2. The quantitative model generalizes slightly the benchmark theoretical framework to allow for heterogeneity in the relative input efficiency parameter $z_i$ and the emission rate $\xi_i$ across sectors and in adding total factor productivity (TFP) parameters $A_i$ to each sector $i$'s production function, so that (2) becomes:

$$y_{it}(\nu) = A_i \left[ \ell_{dit}(\nu) + 1_{elec}(v,i) \left( \frac{e^{z_i} \ell_{cit}(\nu)}{\alpha_i} \right)^{\alpha_i} \left( \frac{m_{it}(\nu)}{1-\alpha_i} \right)^{1-\alpha_i} \right].$$

We also introduce a labor disutility parameter into the household's utility function (1) to better match observed wage and LRHD transportation employment data. Appendix 10.7 presents the quantitative model structure in further detail and describes our quantification of initial wages and labor supplies (to match US data).

## 8.1 Calibration - LRHD Transportation

Our downstream sector differentiates 8 varieties: trucks - class 7-8, trucks - class 4-6, air - narrow-body/short-medium range, air - wide-body/long-range, rail - passenger, rail- freight, water - passenger, and water - freight. Among these varieties, the majority of both value-added and carbon emissions are due to trucking (71% and 62%, respectively) and aviation (21% and 24%, respectively).[37] Our general approach to the calibration is to rely on engineering and industry estimates to quantify production function and cost parameters. Conceptually, we include three types of costs in the fixed or one-time costs $\phi_2(.)$ : technology development costs (e.g., of hydrogen aircraft), certain infrastructure adjustment costs (e.g., of refueling stations to accommodate hydrogen storage and dispensation), and the *excess costs* of current vs. future hydrogen-fueled technologies due to anticipated learning-by-doing and other technological developments. Importantly, this means that the production function parameters $\alpha_2$ and $z_2$ should be quantified based on projected *future* (post-fixed cost) levelized costs of producing LRHD transportation with hydrogen fuels.

First, we quantify the cost share of delivered gaseous hydrogen as $1 - \alpha_2$= 0.18 based on

[37] Based on 2022 data from the Bureau of Transportation Statistics (BTS 2024) and Environmental Protection Agency (EPA 2024).

the value added-share weighted average across 26 estimates of future levelized costs (LCOs) of producing LRHD transportation with hydrogen fuels. [38,39] Second, we use pairs of LCO estimates using fossil fuels and hydrogen from the same study to calculate the value of $z_2$ implied by the relevant marginal cost ratio in the model (see Appendix for details), yielding a weighted average value of $z_2$=-2.58 (across 21 estimates). Third, we set $A_2$ to match calibration year (2022) data on prices and effective taxes in the LRHD sector, and calculate the baseline emissions intensity based on data on aggregate emissions, output, and prices (see Appendix). Finally, the quantification of fixed costs $\phi_2(.)$ can be summarized as follows (with details presented in the Appendix):

*Aviation*: We take $H_2$ aircraft development costs to be \$55 billion (\$2022) for the narrow-body short- to medium range market (NB/SMR, e.g., Airbus A320) and \$145 for the wide-body long-range market (WB/LR, e.g., Boeing 777) based on industry estimates and US aviation market data. For airport infrastructure adjustments, we assume a cost increase of +9.1% per revenue passenger kilometer during the adjustment period, in line with other estimates.[40]

*Trucking, Rail, and Water - Passenger*: We quantify the excess costs of initial clean technology adoption based on pairs of LCO estimates for current vs. future hydrogen-fueled transportation within each variety. For refueling infrastructure, we infer (capital) cost shares from estimates in Bracci et al. (2024) and Reddi et al. (2017), and apply these to the relevant estimates of *dispensed* H2 costs underlying each LCO estimate from the literature.

*Water - Freight:* For freight water transport, we lack sufficient data to quantify fixed costs analogously to the other varieties. Given the additional technical challenges associated with the use of H2 as a fuel in long-distance and cargo shipping (EMSA 2023; Ahluwalia et al. 2020), we thus set the fixed costs for this variety as double the fixed costs of passenger water travel (ferries).

Figure 3 showcases our benchmark estimates of fixed costs across sectors, normalized as the percent increase in per unit costs relative to current fossil fuel-based per-unit costs (over an assumed 25-year model period). Encouragingly, the estimates appear consistent with the empirically observed reality that H2 development so far has focused more on lower-cost varieties such as passenger rail (e.g., Alstom Coradia iLint, Siemens Mireo Plus H), trucks (e.g., Hyundia XCIENT, Nikola), and narrow-body aircraft (e.g., Airbus ZEROe, Embraer Energia) compared to the higher-

[38]Given that we define Sector 1 output as *delivered* GH2 - rather than *dispensed* H2 - we are careful to adjust any LCO estimates that use a "dispensed" H2 cost measure to reallocate the relevant refueling infrastructure capital costs from the hydrogen input to the fixed costs for the relevant variety, as described below.

[39]Sources for these estimates include Ahluwalia et al. 2021, 2020; Hoelzen et al. 2023 ; Ledna et al. 2024 ; Gilleon et al. 2022; Burnham et al. 2021; Hunter et al. 2021; Steer 2023.

[40]Hoelzen et al. (2023) estimate that airport refueling system adjustment and liquefaction plant costs will add around \$0.48 to the levelized cost per kg of dispensed liquid hydrogen (LH2) at a typical airport. We combine this estimate with projected future hydrogen aircraft fuel economy (ICCT 2022) and reference aircraft operating costs (Hoelzenen et al. 2022) to calculate the corresponding cost increase per passenger kilometer. For comparison, Steer (2023) project an 11% future increase in airport charges in a typical flight due to the investment costs required to accommodate hydrogen aircraft.

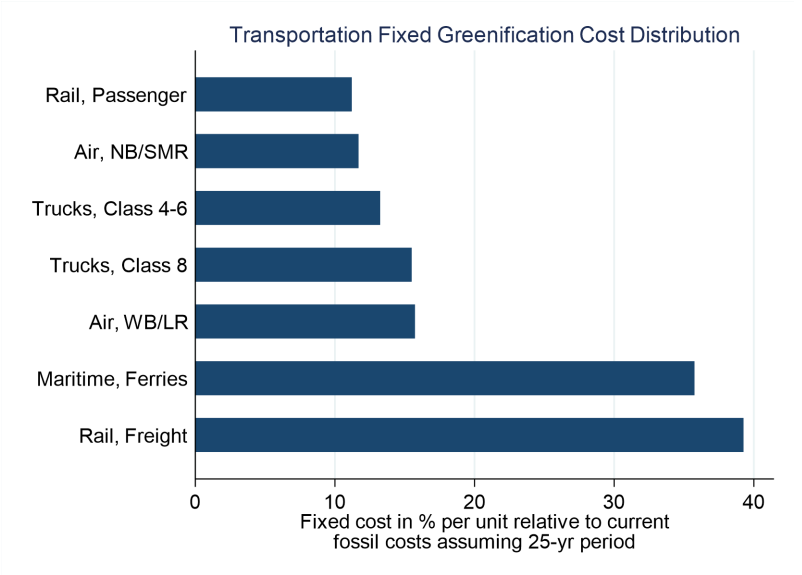



**Figure 3.** Distribution of fixed greenification costs in the downstream sector

cost varieties of freight rail, wide-body aircraft, or maritime transport.

## 8.2 Calibration - Hydrogen Production and Distribution

Our upstream sector encompasses the production of (gaseous) hydrogen and its distribution across 6 U.S. regions.[41] Our approach to the calibration again relies on engineering and industry estimates of clean and polluting production costs for each variety. For H2 production, we consider electrolysis from dedicated renewable energy production[42] as "green" and steam-methane reforming (SMR) with natural gas as polluting benchmark technology. While green H2 may become cost-competitive in the future due to learning-by-doing and technological advancements in electrolyzers and other relevant technologies (e.g., BloombergNEF 2023), at present it is associated with substantially higher production costs. We again consider these *excess initial costs* as a non-recurring cost that must be incurred to switch from fossil fuels-based to green hydrogen production. For hydrogen distribution, we consider pipelines as the "clean" option that can be accessed after incurment of a fixed cost, and tube-trailer trucks as the "dirty" option.[43] This modeling choice is motivated by the facts that (i) pipelines reduce H2 distribution carbon emissions by around 85-95% (Demir et al. 2018; diLullo et al. 2022; Frank et al. 2021) and that (ii) the initial development -

[41]The motivation for modeling hydrogen production and distribution as having some substitutability is that hydrogen can be produced locally (such as near refueling stations) with lower efficiency, or produced at scale and in preferable locations (with, e.g., higher solar potential) but then requiring more distribution.

[42]While technically possible, grid-based electrolysis is generally estimated to be more expensive than with dedicated (or "islanded") renewable electricity (European Hydrogen Observatory 2023).

[43]We disregard the possibility of liquefied H2 distribution via truck due to its limited scalability (e.g., Steer 2023). For aviation, we include liquefaction in the downstream costs of switching to and operating H2 aircraft.

including both technological and legal aspects - and construction of a hydrogen pipeline network constitutes an important fixed cost which will enable access to a more cost-effective and cleaner technology.[44]

For the production function parameters, $\alpha_1 = 1$ is given and we set $z_1 = 1.608$ based on a weighted average across paired estimates of clean and dirty upstream costs from BloombergNEF and the Argonne National Laboratory's Hydrogen Delivery Scenario Analysis Model (HDSAM v4.5, see Appendix for details). The productivity term $A_1$ is set to match calibration year (2022) data on prices, wages, and effective taxes, whereas the emissions rate is calculated based on estimates from the IEA (2023) and the HDSAM. Finally, fixed costs $\phi_1(.)$ for hydrogen production are based on *excess initial cost* values implied by BloombergNEF estimates, whereas we estimate fixed costs for hydrogen distribution pipelines in each region based on the HDSAM model (Brown et al. 2022). The Appendix showcases the resulting fixed cost distribution and provides further details.

## 8.3 Results

This section highlights three main results that arise from our quantitative application. First, we find that the multiplicity of steady-states is empirically relevant. Figure 4 presents an empirical counterpart similar to Figure 2 for a uniform carbon tax values corresponding to the current effective average rate in the US ($13/tCO2, OECD 2023). Already at this low tax value, there are multiple steady-states, including the current stable low steady-state of ($\chi_{1,0} \sim 0, \chi_{2,0} \sim 0$), an unstable intermediate steady-state, and a high steady-state with greenification rates around 90% in both sectors.

Second, it is difficult to escape the low initial greenification steady-state. Figure 5 shows that even with a 15-fold increase of the current US carbon price to a level consistent with recent US EPA estimates of the social cost of carbon emissions ($213/tCO2 in $2023 based on values reported in EPA 2022) is not sufficient to eliminate (0,0) as a steady-state.[45] Indeed, we estimate that, in the absence of clean technology subsidies, a carbon price of around $440/tCO2 would be required to eliminate the multiplicity and ensure a high greenification steady-state.

Third, we confirm that a temporary downstream subsidy may be sufficient to induce greenification and preferable over an upstream subsidy. We specifically model the economy's transition path from the current (0,0) steady-state over five 25-year time periods, where we assume that a temporary greenification subsidy is provided either downstream or upstream. Given the longer-term nature of these simulations, we also assume that carbon prices rise to a future level

[44] We generally assume greenfield hydrogen pipeline development. Our model could, however, also accommodate the possibility of retrofitting existing natural gas pipelines by incorporating their availability in the regional heterogeneity in pipeline network development costs.

[45] A simulation of the transitional dynamics also confirms that the imposition of a uniform $213/tCO2 carbon price is not sufficient to transition LRHD transportation to a clean value chain.

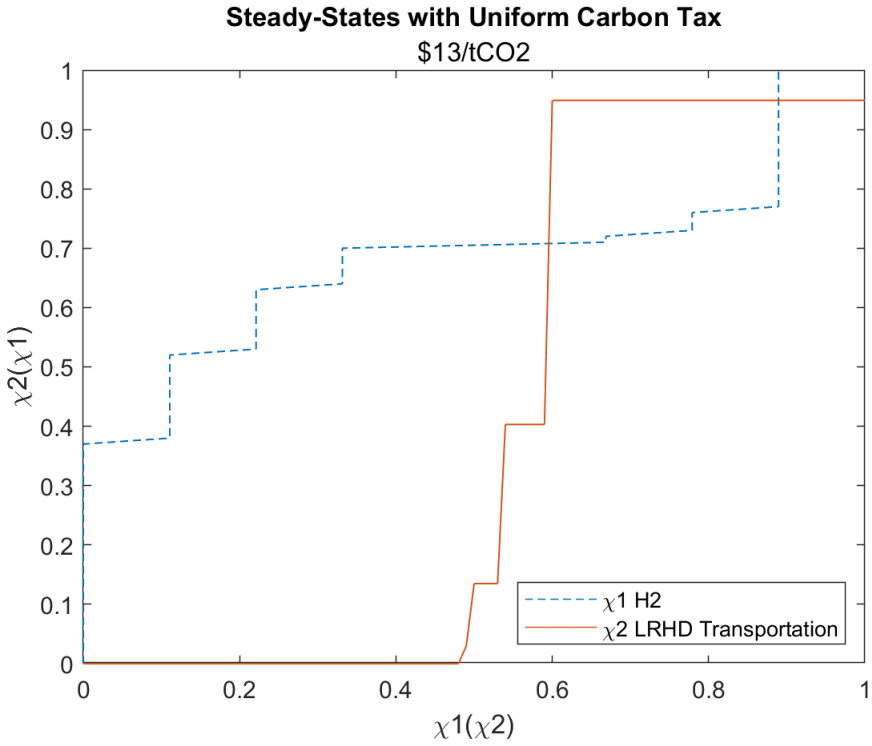



**Figure 4.** Steady-state greenification shares consistent with current carbon price ($13/tCO2)

of $300/tCO2 over this time period to account for the convexity of climate damages.[46] Figure 6 shows that a two-period 50% downstream greenification subsidy succeeds in triggering greenification in both sectors. In contrast, Figure 7 shows that this is not the case for a comparable upstream subsidy. In terms of welfare, we further estimate that the stakes are large: Over the next 50 years alone, adding the downstream greenification subsidy to a carbon price decreases emissions by almost 16 billion metric tons CO2 (undiscounted total) compared to the (ineffective) upstream subsidy scenario. Assuming an annual utility discount factor of $\beta_{yr} = 0.99$, the present value equivalent variation welfare gain associated with the downstream (vs. upstream) subsidy is estimated to be $2.8 trillion. Finally, we also compare the downstream subsidy + carbon tax policy package to a "carbon tax only" scenario that seeks to induce full greenification without a downstream subsidy.

# 9 Conclusion

In this paper we analyzed a model of green technological transition along a supply chain, in which each layer produces a good which is an aggregate of varieties, each of which can be produced us-

[46]For example, in the DICE-2023 model, the social cost of carbon increases by around 300% from 2025-2150 even if TFP growth is set to zero (calculated using replication files from ).

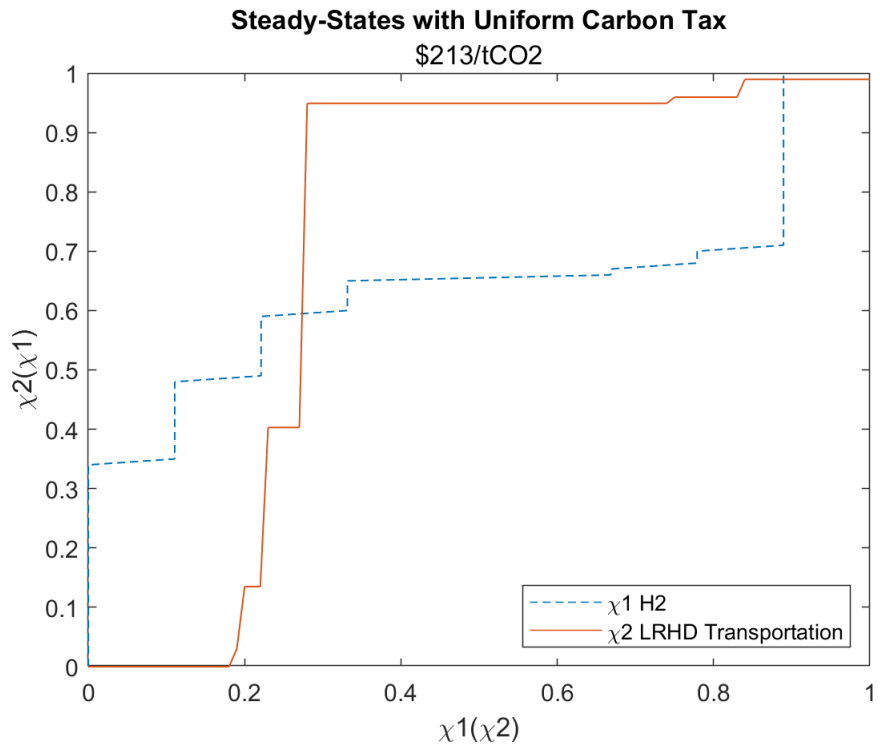



**Figure 5.** Steady-state greenification shares consistent with Pigouvian carbon tax ($213/tCO2)

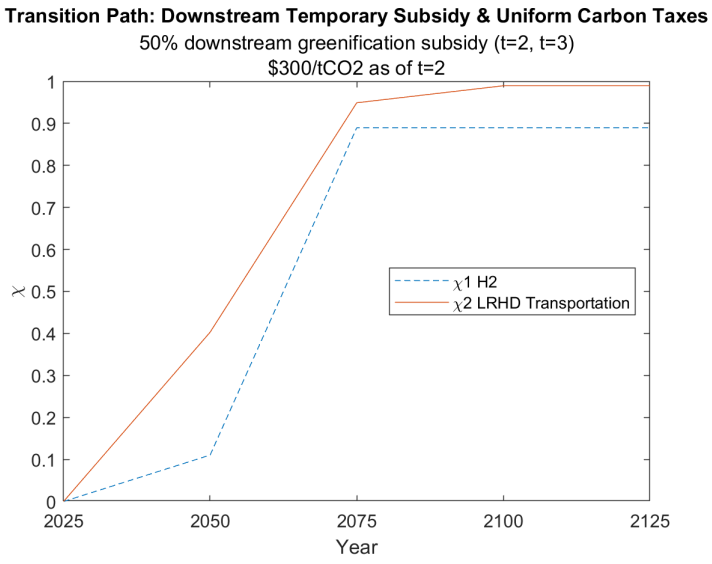



**Figure 6.** Transition dynamics with temporary 50% downstream subsidy

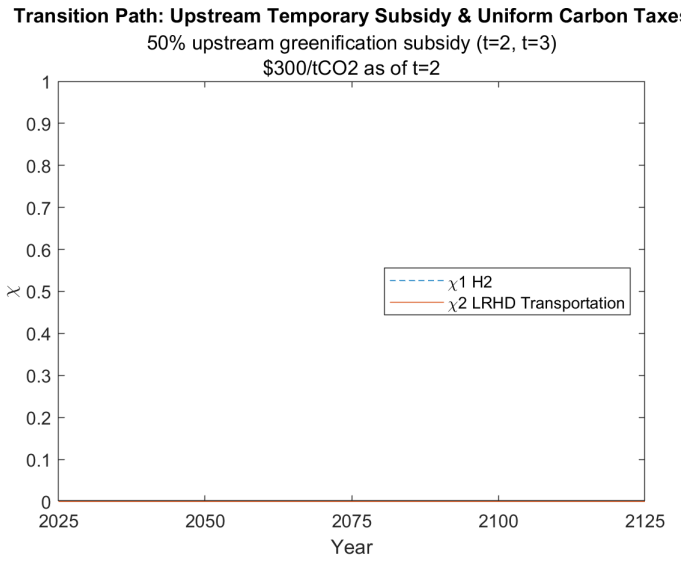



**Figure 7.** Transition dynamics with temporary 50% upstream subsidy

ing either a dirty technology which uses only labor, or a clean, "greenified" technology which uses labor and the good produced by the next upstream layer in the chain. We assumed heterogeneous fixed costs of greenification across varieties within any layer, and that producing a variety using the clean technology once the variety has been greenified, is cheaper than producing the variety using the dirty technology. A producer may choose to incur the fixed greenification cost in exchange for a one-period exclusive right of using the clean technology and Bertrand-compete with competitive producers that use the dirty technology.

Under these assumptions, we first showed that the resulting cross-sectoral strategic complementarities in greenification across layers, generally lead to a multiplicity of steady states, and that the social optimum generally differs from the decentralized solution even when allowing for a Pigouvian tax. Second, we showed that our model generates the possibility that a small and temporary subsidy to greenification that targets key-sectors can be sufficient to achieve large welfare gains by moving the decentralized equilibrium just a little out of an inefficient steady-state. Third, we argued that a government which is constrained in its policy instruments should primarily target downstream sectors. Fourth, in an extension of our model where the dirty technology also uses inputs from another upstream sector that can also be greenified, we showed overinvesting in greenification in the wrong upstream branch may derail the overall transition towards greenification downstream. Finally, we provided a very first attempt at mapping our model with data. We used a two-layer version of our basic model to iron and steel production, and showed that, despite using a uniform carbon price, the economy could get stuck in a "wrong" steady-state with CO2 emissions way above the social optimum.

Our model and analysis in this paper could be extended in several interesting directions. A

first and primary extension would be to develop a quantitative macroeconomic model of green transition of the overall economy seen as a grand network comprising multiple parallel supply chains. Another extension would be to look at coordination and multiple steady states not only on across sectors and layers within a country but also along international value chains. A third extension would be to explore the extent to which allowing for vertical integration between different layers in the production chain affect our main conclusions. A fourth extension would be to use our framework to compute the overall elasticities of substitution between clean and dirty inputs to produce final goods once the supply chains involved in these technologies are fully taken into account. We know from previous work (e.g. see Acemoglu et. al., 2012, 2023; Donald 2023) that these elasticities play a major role in the design of optimal policies, yet rigorous methodologies to compute these elasticities remain to be found. These and other extensions are left for future research.

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# 10 Appendix

## 10.1 Example with multiple steady-states in the cap-and-trade case

In this section, we show that in the presence of a cap-and-trade system with a cap $\bar{\ell}_d$, there exist multiple steady-states over a non-empty open set of parameters whenever $N \geq 2$, while there is a unique steady-state when $N = 1$.

Revenues of the dirty production process get allocated to the payment of labor in these processes and emission permits. Given a price on emissions $\tau_t$, and using that revenues of each sector are given by (9), we then get that

$$(1+\tau_t)\,\ell_{dt} = \sum_{i=1}^{N} (1-\chi_{it}) \prod_{j=i+1}^{N} \widetilde{\chi}_{jt}\,(1-\alpha_j)\,, \tag{39}$$

under the maintained assumption that $(1+\tau_t)\,e^z > 1$. If the cap does not bind, then $\tau_t = 0$ and if it binds, $\ell_{dt} = \bar{\ell}_d$ and the previous equation determines uniquely the price of emissions for given technology levels (noting that $\widetilde{\chi}_{jt}$ decreases in $\tau_t$).

With $N = 1$, a steady-state is then characterized by

$$\chi_1 = F_1\left(1 - \frac{e^{-z}}{1+\tau}\right) \text{ and } (1+\tau_t)\,\ell_{dt} = 1-\chi_1,$$

which defines the steady-state uniquely.

To show that there can be multiple steady-states for $N \geq 2$, we build an example with 2 sectors. We consider parameter values for which the cap always binds. A steady-state is then a pair $\{\chi_1, \chi_2\}$ and a price on emissions $\tau$, which satisfy (15), (17) and (39) such that

$$(1+\tau)\,\bar{\ell}_d = (1-\chi_1)\,\chi_2\,(1-\alpha_2) + (1-\chi_2)\,, \tag{40}$$

$$\chi_1 = F_1\left[\left(1 - \frac{e^{-z}}{1+\tau}\right)\chi_2\,(1-\alpha_2)\right] \tag{41}$$

$$\chi_2 = F_2\left(1 - \left[\frac{e^{-z}}{1+\tau}\right]^{\mu_2}\right) \text{ with } \mu_2 = \alpha_2 + \chi_1\,(1-\alpha_2)\,. \tag{42}$$

We construct non-knife edge examples where one steady-state features $\chi_1 = 0$, $\chi_2 > 0$ and the other features $\chi_1^\dagger = 1$, $\chi_2^\dagger > \chi_2$.

In the first steady-state, $\mu_2 = \alpha_2$ and given (40), $1+\tau = (1-\alpha_2\chi_2)\,/\bar{\ell}_d$, hence (41) and (42) give:

$$0 = F_1\left[\left(1 - \frac{e^{-z}\bar{\ell}_d}{1-\alpha_2\chi_2}\right)\chi_2\,(1-\alpha_2)\right], \tag{43}$$

$$\chi_2 = F_2\left(1 - \left[\frac{e^{-z}\bar{\ell}_d}{1-\alpha_2\chi_2}\right]^{\alpha_2}\right). \tag{44}$$

In the second steady-state, $\mu_2^\dagger = 1$ and given (40), $1+\tau = \left(1-\chi_2^\dagger\right)/\bar{\ell}_d$, hence (41) and (42) give:

$$1 = F_1\left[\left(1-\frac{e^{-z}\bar{\ell}_d}{1-\chi_2^\dagger}\right)\chi_2^\dagger\left(1-\alpha_2\right)\right] \tag{45}$$

$$\chi_2^\dagger = F_2\left(1-\frac{e^{-z}\bar{\ell}_d}{1-\chi_2^\dagger}\right). \tag{46}$$

The right-hand sides of (44) and (46) are decreasing in $\chi_2$ and $\chi_2^\dagger$ respectively. For a sufficiently low cap $\bar{\ell}_d$, we necessarily have that $1-\left[\frac{e^{-z}\bar{\ell}_d}{1-\alpha_2\chi_2}\right]^{\alpha_2} < 1-\frac{e^{-z}\bar{\ell}_d}{1-\chi_2^\dagger}$, in that case, (44) and (46) imply that $\chi_2^\dagger > \chi_2$, and one can build $F_2$ such that there is a large gap between $\chi_2^\dagger$ and $\chi_2$. Again, for a sufficiently low cap, we can then obtain that $\left(1-\frac{e^{-z}\bar{\ell}_d}{1-\alpha_2\chi_2}\right)\chi_2 < \left(1-\frac{e^{-z}\bar{\ell}_d}{1-\chi_2^\dagger}\right)\chi_2^\dagger$. Building $F_1$such that all the mass of the distribution is between these two values, we can satisfy both (43) and (45). This shows that multiple steady-states are possible.

## 10.2 Proof of Proposition 3

The proof proceeds in several steps: 1) We write down output as a function of labor allocation and technology, 2) we derive the optimal labor allocation, 3) we establish that disutility from labor and pollution is a constant, 4) we derive output as a function of technology only and the simplified social planner problem (equation (19)), and 5) we derive the first-order conditions.

Step 1: Under the assumption that $(1+\xi)e^z > 1$, the social planner will use the greenified production process whenever it is available. As argued in the text, all greenification in the optimum happens immediately, so that the share of greenified varieties $\{\chi_{it}\}$ is constant over time. In that case, the allocation of labor to production is also constant over time, which means that we can drop time subscript. We then get that for any $k < N$,

$$y_{k+1} = \left(\frac{\ell_{d(k+1)}}{1-\chi_{k+1}}\right)^{1-\chi_{k+1}} \frac{1}{\chi_{k+1}^{\chi_{k+1}}}\left(\frac{e^z\ell_{c(k+1)}}{\alpha_{k+1}}\right)^{\chi_{k+1}\alpha_{k+1}}\left(\frac{y_k}{1-\alpha_{k+1}}\right)^{\chi_{k+1}(1-\alpha_{k+1})}. \tag{47}$$

Assume that for sector $k$, we have:

$$\ln y_k = \sum_{i=1}^{k}\prod_{j=i+1}^{k}\chi_j\left(1-\alpha_j\right)\left[\begin{array}{c} -\left(\chi_i\ln\chi_i+\left(1-\chi_i\right)\ln\left(1-\chi_i\right)\right)+\chi_i\alpha_i\ln\left(\frac{e^z\ell_{ci}}{\alpha_i}\right) \\ -\chi_i\left(1-\alpha_i\right)\ln\left(1-\alpha_i\right)+\left(1-\chi_i\right)\ln\ell_{di}\end{array}\right].$$

Then taking the log of (47), it is immediate that sector $k+1$ follows the same formula. Therefore, denoting by $\omega_i \equiv \prod_{j=i+1}^{N}\chi_j\left(1-\alpha_j\right)$, we can rewrite output as:

$$\ln y_N = \sum_{i=1}^{N}\omega_i\left[-\left(\chi_i\ln\chi_i+\left(1-\chi_i\right)\ln\left(1-\chi_i\right)\right)+\chi_i\alpha_i\ln\left(\frac{e^z\ell_{ci}}{\alpha_i}\right)-\chi_i\left(1-\alpha_i\right)\ln\left(1-\alpha_i\right)+\left(1-\chi_i\right)\ln\ell_{di}\right] \tag{48}$$

where, to economize notations, we keep $\left(1-\alpha_1\right)\ln\left(1-\alpha_1\right)$ in the sum above but treat it as a

zero (since this term is not there for sector 1 which features $\alpha_1 = 1$).

Step 2: The planner's problem can thus be re-written as

$$\max_{\{\ell_{di},\ell_{ci},\chi_i\}} \frac{1}{1-\beta}\left(\ln y_N - (1+\xi)\sum_i \ell_{di} - \sum_i \ell_{ci}\right) - \sum_i \mathcal{F}_i(\chi_i) \tag{49}$$

Taking first-order conditions with respect to clean and dirty labor inputs, we immediately get that

$$\ell_{ci} = \chi_i \alpha_i \omega_i, \qquad \ell_{di} = \frac{(1-\chi_i)\omega_i}{1+\xi} \tag{50}$$

Step 3: Note that the total disutility of labor in production + pollution is always equal to one: Indeed, using (50), we get:

$$(1+\xi)\ell_{d1} + \ell_{c1} = \omega_1.$$

Assume that

$$\sum_{j=1}^{i}\left[(1+\xi)\ell_{dj} + \ell_{cj}\right] = \omega_i.$$

Then, we get for $i < N$ :

$$\begin{aligned}\sum_{j=1}^{i+1}\left[(1+\xi)\ell_{dj} + \ell_{cj}\right] &= \chi_{i+1}\alpha_{i+1}\omega_{i+1} + (1-\chi_i)\omega_{i+1} + \omega_i \\ &= \left[\chi_{i+1}\alpha_{i+1} + (1-\chi_i) + \chi_{i+1}(1-\alpha_{i+1})\right]\omega_{i+1} \\ &= \omega_{i+1}.\end{aligned}$$

This implies the stated result, namely:

$$\sum_{j=1}^{N}\left[(1+\xi)\ell_{dj} + \ell_{cj}\right] = \omega_N = 1.$$

Step 4: Plugging the labor allocation equations, (50), into (48) we can rewrite output as:

$$\begin{aligned}\ln y_N &= \sum_{i=1}^{N}\omega_i\left[-(1-\alpha_i)\chi_i\ln\chi_i + \chi_i\alpha_i\ln(e^z\omega_i) - \chi_i(1-\alpha_i)\ln(1-\alpha_i) + (1-\chi_i)\ln\frac{\omega_i}{1+\xi}\right]. \\ &= \left(\sum_{i=1}^{N}\omega_i\chi_i\alpha_i\right)z - \left(\sum_{i=1}^{N}\omega_i(1-\chi_i)\right)\ln(1+\xi) \\ &\quad + \sum_{i=1}^{N}\omega_i\left[-(1-\alpha_i)\chi_i\ln(\chi_i(1-\alpha_i)) + (1-\chi_i+\chi_i\alpha_i)\ln\omega_i\right].\end{aligned} \tag{51}$$

We note that

$$\begin{aligned}\sum_{i=1}^{N}(1-\chi_i)\,\omega_i+\sum_{i=1}^{N}\omega_i\chi_i\alpha_i &= \sum_{i=1}^{N}\omega_i-\sum_{i=1}^{N}\omega_i\chi_i\,(1-\alpha_i)\\ &= \sum_{i=1}^{N}\omega_i-\sum_{i=2}^{N}\omega_{i-1}\\ &= \omega_N=1.\end{aligned}$$

Moreover,

$$\begin{aligned}&\sum_{i=1}^{N}\omega_i\left[-(1-\alpha_i)\,\chi_i\ln\left(\chi_i\,(1-\alpha_i)\right)+(1-\chi_i+\chi_i\alpha_i)\ln\omega_i\right]\\ =\;&\sum_{i=1}^{N}\omega_i\ln\omega_i-\sum_{i=1}^{N}\omega_i\,(1-\alpha_i)\,\chi_i\ln\left(\chi_i\,(1-\alpha_i)\,\omega_i\right)\\ =\;&\sum_{i=1}^{N}\omega_i\ln\omega_i-\sum_{i=1}^{N-1}\omega_i\ln\omega_i=0.\end{aligned}$$

Using both relationships in (51), we can rewrite

$$\ln y_N=\left(\sum_{i=1}^{N}\omega_i\chi_i\alpha_i\right)(z+\ln(1+\xi))-\ln(1+\xi)\,. \tag{52}$$

Dropping constants and multiplying the original problem by $1-\beta$, we then get that the social planner solves the simplified social planner problem given Equation (19).

Step 5: If the solution $\{\chi_i\}$ is interior, it must satisfy the first-order conditions given by:

$$\frac{\ln\left((1+\xi)\,e^z\right)}{1-\beta}\sum_{j=1}^{i}\frac{\omega_j\alpha_j\chi_j}{\chi_i}=\mathcal{F}_i'(\chi_i)\,.$$

We note that $\sum_{j=1}^{i}\frac{\omega_j\alpha_j\chi_j}{\chi_i}=\mu_i\omega_i$, which then delivers (20). To see this, note that for $i>1$,

$$\begin{aligned}\chi_i\omega_i\mu_i &= \chi_i\omega_i\left(\alpha_i+(1-\alpha_i)\,\chi_{i-1}\mu_{i-1}\right)\\ &= \chi_i\omega_i\alpha_i+\chi_{i-1}\mu_{i-1}\omega_{i-1}\\ &= \sum_{j=1}^{i}\omega_j\alpha_j\chi_j.\end{aligned}$$

This establishes Step 5 and thus completes the proof of Proposition 3. □

## 10.3 Derivation of Equation (25)

To derive the result, we first derive $\frac{\partial\mu_i}{\partial\chi_k}$. We immediately note that $\frac{\partial\mu_i}{\partial\chi_k}=0$ for $k>i$. Further, we have that $\frac{\partial\mu_i}{\partial\chi_{i-1}}=(1-\alpha_i)\,\mu_{i-1}$, while for $k<i-1$, we get: $\frac{\partial\mu_i}{\partial\chi_k}=\chi_{i-1}\,(1-\alpha_i)\,\frac{\partial\mu_{i-1}}{\partial\chi_k}$. Iterating

on $j$ such that $i-j$ goes down to $k+1$, we then obtain:

$$\frac{\partial \mu_i}{\partial \chi_k} = \begin{cases} \frac{\left(\prod_{j=0}^{i-k-1}(1-\alpha_{i-j})\chi_{i-j-1}\right)\mu_k}{\chi_k} & k < i \\ 0 & \text{otherwise} \end{cases}. \tag{53}$$

For $k > i$, it is then immediate that $\frac{\partial \ln \pi_i}{\partial \ln \chi_k} = 1$. While for $k < i$, we get

$$\frac{\partial \ln \pi_i}{\partial \ln \chi_k} = \frac{\mu_i Z e^{-\mu_i Z}}{1-e^{-\mu_i Z}} \frac{\partial \ln \mu_i}{\partial \ln \chi_k},$$

which combined with (53) gives (25).

We note that (for $Z > 0$)

$$\begin{aligned} \frac{\mu_i Z e^{-\mu_i Z}}{1-e^{-\mu_i Z}} < 1 \quad &\Longleftrightarrow \quad \mu_i Z e^{-\mu_i Z} < 1 - e^{-\mu_i Z} \\ &\Longleftrightarrow \quad 1 + \mu_i Z < e^{\mu_i Z}, \end{aligned}$$

which is true for $Z > 0$. Next, for $k = i-1$, we get using (6) that:

$$\begin{aligned} \frac{\left(\prod_{j=0}^{i-k-1}(1-\alpha_{i-j})\chi_{i-j-1}\right)\mu_k}{\mu_i} &= \frac{(1-\alpha_i)\chi_{i-1}\mu_{i-1}}{\mu_i} \\ &= \frac{\mu_i - \alpha_i}{\mu_i} < 1. \end{aligned}$$

In addition, for any $k < i-1$, then, using again (6), we get:

$$\begin{aligned} \frac{\left(\prod_{j=0}^{i-k-1}(1-\alpha_{i-j})\chi_{i-j-1}\right)\mu_k}{\mu_i} &= \frac{\left(\prod_{j=0}^{i-(k+1)-1}(1-\alpha_{i-j})\chi_{i-j-1}\right)(\mu_{k+1}-\alpha_{k+1})}{\mu_i} \\ &< \frac{\left(\prod_{j=0}^{i-(k+1)-1}(1-\alpha_{i-j})\chi_{i-j-1}\right)\mu_{k+1}}{\mu_i}. \end{aligned}$$

Therefore we get that $\frac{\left(\prod_{j=0}^{i-k-1}(1-\alpha_{i-j})\chi_{i-j-1}\right)\mu_k}{\mu_i}$ is increasing in $k$ for $k \leq i-1$ and by induction $\frac{\left(\prod_{j=0}^{i-k-1}(1-\alpha_{i-j})\chi_{i-j-1}\right)\mu_k}{\mu_i} < 1$ for all $k \leq i-1$.

## 10.4 Utility flow in the extended model

The utility flow is given by $\ln y_{2t} - \sum_{i\in\{1a,1b,2\}}\left((1+\xi)\,\ell_{dit} + \ell_{cit}\right)$. In steady-state, all inputs are priced at marginal costs. Since downstream production is Cobb-Douglas between the clean and dirty production process, we get that in steady-state output in sector 2 is given by:

$$y_2 = \frac{\left(e^{\alpha z}\ell_{c2}^{\alpha} y_{1b}^{1-\alpha}\right)^{\chi_2}\left(\ell_{d2}^{\alpha} y_{1a}^{1-\alpha}\right)^{1-\chi_2}}{\alpha^{\alpha}\left(1-\alpha\right)^{1-\alpha}\chi_2^{\chi_2}\left(1-\chi_2\right)^{1-\chi_2}}, \tag{54}$$

where we kept the assumption that downstream greenified producers prefer to use the clean production process. In sector $1a$ and $1b$, we similarly have

$$y_{1k} = \frac{\left(e^{z}\ell_{c1k}\right)^{\chi_{1k}}\ell_{d1k}^{1-\chi_{1k}}}{\chi_{1k}^{\chi_{1k}}\left(1-\chi_{1k}\right)^{1-\chi_{1k}}} \text{ for } k \in \{a,b\}\,. \tag{55}$$

Since wages and revenues are both equal to 1, then clean labor commands income shares equal to its overall factor share in production, namely we have:

$$\ell_{c2} = \chi_2\alpha,\ \ell_{c1b}{=}\chi_2\left(1-\alpha\right)\chi_{1b} \text{ and } \ell_{c1a}{=}\left(1-\chi_2\right)\left(1-\alpha\right)\chi_{1a}. \tag{56}$$

With Pigouvian taxation, the cost of using dirty labor is $1+\xi$, such that we obtain:

$$\left(1+\xi\right)\ell_{d2} = \left(1-\chi_2\right)\alpha,\ \left(1+\xi\right)\ell_{d1b}{=}\chi_2\left(1-\alpha\right)\left(1-\chi_{1b}\right) \text{ and } \left(1+\xi\right)\ell_{d1a}{=}\left(1-\chi_2\right)\left(1-\alpha\right)\left(1-\chi_{1a}\right). \tag{57}$$

Using these expressions, we get that the total disutility of labor in production plus pollution is always equal to one:

$$\left(1+\xi\right)\sum_i \ell_{di} + \sum_i \ell_{ci} = 1.$$

Plugging (55) into (54) and taking logs we can express log output as:

$$\begin{aligned}
\ln y_2 \;=\;& -\left(\chi_2\ln\chi_2 + \left(1-\chi_2\right)\ln\left(1-\chi_2\right)\right) - \alpha\ln\alpha - \left(1-\alpha\right)\ln\left(1-\alpha\right)\\
&+\chi_2\alpha z + \chi_2\alpha\ln\ell_{c2} + \left(1-\chi_2\right)\alpha\ln\ell_{d2}\\
&+\chi_2\left(1-\alpha\right)\left[\chi_{1b}z + \chi_{1b}\ln\ell_{c1b} - \chi_{1b}\ln\chi_{1b} + \left(1-\chi_{1b}\right)\ln\ell_{d1bt} - \left(1-\chi_{1b}\right)\ln\left(1-\chi_{1b}\right)\right]\\
&+\left(1-\chi_2\right)\left(1-\alpha\right)\left[\chi_{1a}z + \chi_{1a}\ln\ell_{c1a} - \chi_{1a}\ln\chi_{1a} + \left(1-\chi_{1a}\right)\ln\ell_{d1a} - \left(1-\chi_{1a}\right)\ln\left(1-\chi_{1a}\right)\right]
\end{aligned}$$

Substituting the labor allocations, (56) and (57), into the previous expression gives:

$$\begin{aligned}
\ln y_2 \;=\;& -\left(\chi_2\ln\chi_2 + \left(1-\chi_2\right)\ln\left(1-\chi_2\right)\right) - \alpha\ln\alpha - \left(1-\alpha\right)\ln\left(1-\alpha\right)\\
&+\chi_2\alpha z + \chi_2\alpha\ln\left(\chi_2\alpha\right) + \left(1-\chi_2\right)\alpha\ln\frac{\left(1-\chi_2\right)\alpha}{\left(1+\xi\right)}\\
&+\chi_2\left(1-\alpha\right)\left[\chi_{1b}z + \ln\chi_2 + \ln\left(1-\alpha\right) - \left(1-\chi_{1b}\right)\ln\left(1+\xi\right)\right]\\
&+\left(1-\chi_2\right)\left(1-\alpha\right)\left[\chi_{1a}z + \ln\left(1-\chi_2\right) + \ln\left(1-\alpha\right) - \left(1-\chi_{1a}\right)\ln\left(1+\xi\right)\right].
\end{aligned}$$

Rearranging terms, we then obtain:

$$\begin{aligned}
\ln y_2 \;=\;& \chi_2\alpha z - \left(1-\chi_2\right)\alpha\ln\left(1+\xi\right)\\
&+\chi_2\left(1-\alpha\right)\left[\chi_{1b}z - \left(1-\chi_{1b}\right)\ln\left(1+\xi\right)\right]\\
&+\left(1-\chi_2\right)\left(1-\alpha\right)\left[\chi_{1a}z - \left(1-\chi_{1a}\right)\ln\left(1+\xi\right)\right].
\end{aligned}$$

We then get that the utility flow in steady state is given by

$$\begin{aligned}
&\ln y_2 - \textstyle\sum_{i\in\{1a,1b,2\}}\left(\left(1+\xi\right)\ell_{di} + \ell_{ci}\right)\\
=\;& \left[\chi_2\alpha + \chi_2\left(1-\alpha\right)\chi_{1b} + \left(1-\chi_2\right)\left(1-\alpha\right)\chi_{1a}\right]Z - \ln\left(1+\xi\right) - 1.
\end{aligned}$$

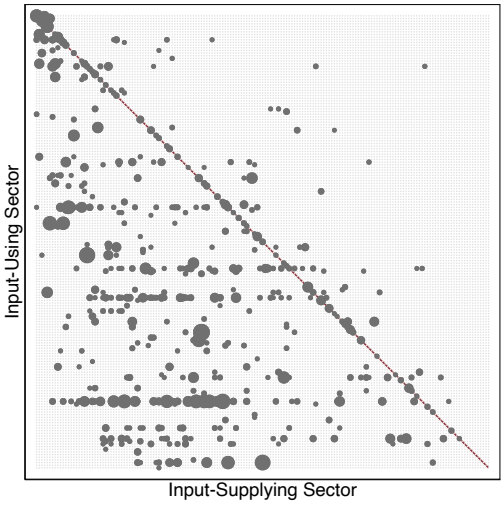



**Figure 8.** The US IO Table is Largely Acyclic

## 10.5 The U.S. Input-Output Table is Largely Acyclic

Figure 8, taken from Liu and Tsyvinski (2024), shows the U.S. input-output matrix, with sectors sorted by descending upstreamness as defined in that paper. Each entry shows the fraction of output in the column sector that is supplied to the row sector, with larger entries representing stronger input-output linkages. Entries are truncated below at 4%. A completely acyclic network is lower-triangular. The figure shows that downstream sectors purchase heavily from the upstream ones—but not the reverse—as the figure is dense below the diagonal and sparse above. The figure indicates that the U.S. production network is largely acyclic, as the bulk of input flows are one directional. This near-acyclical feature of the U.S. production network is echoed in other economies as well: Liu (2019) notes similar features in China and South Korea, and Dhyne et al. (2022) show that Belgium production network can be well-approximated by an acyclic network.

## 10.6 Proof of Proposition 6

Applying Step 1 of Appendix 10.2 gives us a mapping from technology and the labor allocation to output via Equation (48). This mapping implies that the profit maximization conditions governing the allocation of labor follow

$$\ell_{ci} = \chi_i \alpha_i \omega_i, \qquad \ell_{di} = \frac{(1-\chi_i)\,\omega_i}{1+\tau},$$

where we have used the fact that household optimization implies that $p_N \frac{\partial y_N}{\partial x} = \frac{\partial \ln(y_N)}{\partial x}$. Step 3 then implies that total labor costs, inclusive of the carbon price, equal one, while Step 4 implies that the Planner's problem can be rewritten as

$$\max_{\{\chi_j\}} \ln\left((1+\tau)\, e^z\right) \sum_i \omega_i \chi_i \alpha_i - (\xi - \tau) \sum_i \ell_{di} - (1-\beta) \sum_i \mathcal{F}_i(\chi_i).$$

This gives us the first-order condition

$$\frac{\ln\left((1+\xi)\, e^z\right)}{1-\beta} \sum_{j \leq i} \frac{\omega_j \alpha_j \chi_j}{\chi_i} + \frac{\xi - \tau}{1-\beta}\Big( - \frac{\partial \ell_d}{\partial \chi_i}\Big) = \mathcal{F}_i'\left(\chi_i\right).$$

Step 5 implies that $\sum_{j \leq i} \frac{\omega_j \alpha_j \chi_j}{\chi_i} = \mu_i \omega_i$, which gives us Equation (31) of Proposition 6. □

## 10.7 Details on the calibration

### 10.7.1 Calibrated model

We briefly present the model that we calibrate in Section 8. We modify the model such that each sector may feature heterogeneity in the relative productivity of clean and dirty technologies $z_i$, the emission rate associated with the use of the dirty production process $\xi_i$ and a TFP parameter $A_i$. We present the model for $N \geq 2$ – though we will have $N = 2$ in the calibration. Therefore a variety $\nu$ in sector $i$ is now produced according to

$$y_{it}(\nu) = A_i \left[ \ell_{dit}(\nu) + 1_{elec}(v,i) \left( \frac{e^{z_i} \ell_{cit}(\nu)}{\alpha_i} \right)^{\alpha_i} \left( \frac{m_{it}(\nu)}{1-\alpha_i} \right)^{1-\alpha_i} \right],$$

where $1_{elec}(v,i)$ is an index function which takes value 1 if and only if production process $\nu$ in sector $i$ has been greenified.

Following the same logic as in the baseline model, the price of production process $\nu$ in sector $i$ is given by:

$$p_{it}(\nu) = \begin{cases} \min\left(e^{-\alpha_i z_i} p_{i-1,t}^{1-\alpha_i}, 1+\tau_{it}\right)/A_i & \text{if greenified by time } t-1, \\ (1+\tau_{it})/A_i & \text{otherwise,} \end{cases} \tag{58}$$

where for sector 1, $\alpha_1 = 1$ and the term $p_{i-1,t}^{1-\alpha_i}$ drops from the previous expression.

Empirically, we get that, even when $\tau_{it} = 0$, the clean production process is cheaper than the dirty one (i.e. $e^{-\alpha_2 z_2} p_{1,t}^{1-\alpha_2} < 1$ and $z_1 > 0$). Assuming that this is the case, we can then solve for the price index in each sector as:

$$p_{1t} = \frac{1}{A_1} (1+\tau_{1t})^{1-\chi_{1,t-1}} e^{-\chi_{1,t-1} z_1} \tag{59}$$

$$\text{and} p_{it} = \frac{1}{A_i} (1+\tau_{it})^{1-\chi_{i,t-1}} e^{-\chi_{i,t-1} \alpha_i z_i} p_{i-1,t}^{(1-\alpha_i)\chi_{i,t-1}} \text{for } i > 1. \tag{60}$$

Because sectors are heterogeneous in $z$ and $\tau$, we cannot introduce the variable $\mu_i$ that allowed for closed form solutions as before. However, this does not affect the logic of the model.

Following the same steps as in the baseline model, we get that if a variety from sector $i$ is greenified at time $t$, the innovator obtains a profit margin given by $1-\frac{e^{-\alpha_i z_i}p_{i-1,t}^{(1-\alpha_i)}}{1+\tau_{it}}$ (or $1-\frac{e^{-z_1}}{1+\tau_{1t}}$ for sector 1). That is, an innovator obtains profits given by

$$\pi_{it}(\nu)=r_{it}\left[1-\frac{e^{-\alpha_i z_i}p_{i-1,t}^{(1-\alpha_i)}}{1+\tau_{it}}\right],$$

where, as before, $r_{it}$ denote the revenue of a variety in sector $i$ at time $t$. We still get that in sector $N$, $r_{Nt}=p_t c_t=1$, while we can obtain revenues for sector $i<N$ recursively from:

$$r_{it}=\prod_{j=i+1}^{N}\widetilde{\chi}_{jt}(1-\alpha_j),$$

where $\widetilde{\chi}_{j,t}$ is now defined as

$$\widetilde{\chi}_{j,t}\equiv\chi_{j,t-1}+(\chi_{j,t}-\chi_{j,t-1})\frac{e^{-\alpha_j z_j}p_{j-1,t}^{(1-\alpha_j)}}{1+\tau_{jt}} \tag{61}$$

. For completeness, we introduce the possibility that the government subsidizes greenification in sector $i$ at rate $s_{it}$ (for the calibration, we assume that $s_{it}=\beta$). Then get that an equilibrium is defined as follows:

**Definition.** Given initial greenification shares $\{\chi_{i0}\}$, an equilibrium with a sequence of carbon taxes $\{\tau_{it}\}$ and greenification subsidies $\{s_{it}\}$ is the sequence $\{\chi_{it},\widetilde{\chi}_{it},p_{it}\}_{t>0}$ such that $p_{1t}$ and $p_{it}$ obey (59) and (60), $\widetilde{\chi}_{it}$ is given by (61), and $\chi_{it}$ is given by

$$\chi_{it}=\max\left\{F_i\left(\left[1-\frac{e^{-\alpha_i z_i}p_{i-1,t}^{(1-\alpha_i)}}{1+\tau_{it}}\right]\frac{\prod_{j=i+1}^{N}\widetilde{\chi}_{jt}(1-\alpha_j)}{1-s_{it}}\right),\chi_{i,t-1}\right\}. \tag{62}$$

As before, the equilibrium is unique. In turn, a steady-state is characterized by:

**Definition.** For given carbon taxes $\{\tau_i\}$ and greenification subsidies $\{s_i\}$, a steady-state is a vector of greenification shares and prices $\{\chi_i,p_i\}$ such that

$$p_1=\frac{1}{A_1}(1+\tau_1)^{1-\chi_1}e^{-\chi_1 z_1},\quad p_i=\frac{(1+\tau_i)^{1-\chi_i}e^{-\chi_i\alpha_i z_i}p_{i-1}^{(1-\alpha_i)\chi_i}}{A_i}, \tag{63}$$

$$\chi_i\geq F_i\left(\left(1-\frac{e^{-\alpha_i z_i}p_{i-1}^{(1-\alpha_i)}}{1+\tau_i}\right)\frac{\prod_{j=i+1}^{N}\chi_j(1-\alpha_j)}{1-s_i}\right). \tag{64}$$

As before, the inequality in (64) results from the fact that greenification can never decrease and we focus on interesting steady-states where (64) holds with equality.

To derive output in each sector, we start from the output for each variety, which using the Cobb-Douglas aggregator must obey: $y_{it}(\nu)=r_{it}/p_{it}(\nu)$. We then get that output of a non-greenified or newly greenified variety is $y_{it}(\nu)=A_i(1+\tau_{it})^{-1}r_{it}$, while the employment of dirty labor for non-greenified varieties is $(1+\tau_{it})^{-1}r_{it}$. The output of a previously greenified

variety is $y_{it}(\nu) = \frac{A_i r_{it}}{e^{-\alpha_i z_i} p_{i-1,t}^{1-\alpha_i}}$. We then get that the output of sectoral good $i$—combining all varieties—is

$$y_{it} = A_i \left(e^{-\alpha_i z_i} p_{i-1,t}^{1-\alpha_i}\right)^{-\chi_{i,t-1}} (1+\tau_{it})^{-(1-\chi_{i,t-1})} r_{it},$$

with the disutility from emissions in sector $i$ is given by:

$$a_{it} = \xi_i \frac{1-\chi_{it}}{1+\tau_{it}} r_{it}.$$

### 10.7.2 Calibration details

**Initial wages, prices, and productivity:** The initial wage $w_0$equals the value-added per worker-year in LRHD transportation services in 2022 (= $362,220 based on initial output of $833.1 billion (BTS 2024) and employment of 2.3 million worker-years (BLS 2024 counting air, rail, water, and truck transportation (NAICS 481-484)). The initial price of transportation services $p_{20} = \$0.23$ is calculated as a Cobb-Douglas price index over the revenue-share weighted average freight price per ton-mile of $0.26 and the revenue-share weighted average passenger price per passenger-mile of $0.13 in 2022 (both calculated for air, truck, railroad, and water transport from BTS Tables 1-50, 3-21, 1-40, and 3-20), where the expenditure share of freight (vs. passenger) miles is set to 0.81 in line with its value for 2022 in the data. Transportation TFP $A_2$=1.5914e+06 is calculated from (60) where we set $\tau_{20}$ based on the average effective carbon price in the United States in 2021 ($13/tCO2, OECD 2023) and the sector's emissions intensity (0.0008 tCO2e/$ calculated from total emissions data from EPA 2024 and output data from BTS 2024) and assume that $\chi_{20} = 0$. The initial price of delivered $GH_2$ is set to $p_{10} = \$4.65$/kg based on the current (2023) estimated US fossil (SMR) hydrogen production cost of $1.45/kg from BloombergNEF (2023) plus current fossil (truck) distribution costs of $3.20/kg estimated using the Argonne National Laboratory's Hydrogen Delivery Scenario Analysis Model (HDSAM v4.5, Elgowainy et al. 2024, as described further below) and converted to $2022 (using US GDP deflator data from FRED). Hydrogen TFP $A_1 = 8.8779e+04$ is calculated from (59) analogous to the calculation of $A_2$ but with$\tau_{10}$ based on an estimated emissions intensity of 10.75 $kgCO_2/kgH_2$ with 9 $kgCO_2/kgH_2$ from natural gas-based hydrogen production (median value from IEA 2023) plus 1.75 $kgCO_2/kgH_2$from distribution (from the HDSAM). Here we also assume that $\chi_{10} \approx 0$ based on estimates from the IEA (2024).

**Transportation productivity parameter** $z_2$: For each literature paired estimate $i$ of the levelized costs of producing a transportation service with fossil vs. hydrogen fuels, we infer the implied value of $z_{2,i}$ from the associated ratio of clean vs. dirty marginal costs:

$$\frac{mc_{c,i}}{mc_{d,i}} = \frac{(w_0)^{\alpha_2} e^{-\alpha_2 z_{2,i}} (p_{1,i})^{1-\alpha_2}/A_2}{w_0(1+\widetilde{\tau}_{2,i})/A_2}$$

with$p_{1,i}$ based on the hydrogen input price assumed in each study $i$ (with adjustments where

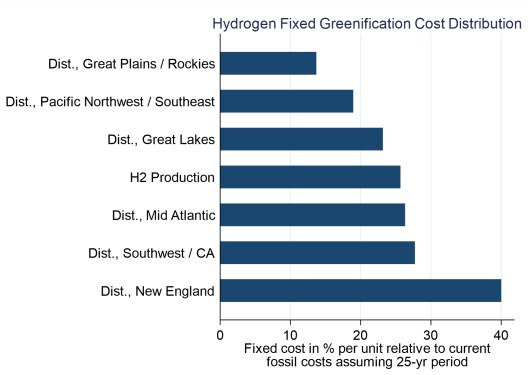



**Figure 9.** Distribution of fixed greenification costs in the upstream sector

necessary as described in Footnote 33),$\widetilde{\tau}_{2,i}$ based on the tax rates assumed, and the other parameters are set as described above. We then average across $z_{2,i}$'s for each variety and compute the value added-share weighted average across varieties.

**Aviation Fixed Costs:** For aircraft development costs, we first take an industry estimate of $16.4 billion (Steer 2023) for a $H_2$ narrow-body short- to medium range (NB/SMR) aircraft similar to the Airbus A320 and scale it up based on typical NB/SMR aircraft family expenditure shares to yield an overall cost of $55 billion ($2022).[47] For the wide-body long-range (WB/LR) market, development costs are generally expected to be higher as the lower volumetric energy density of hydrogen necessitates additional changes in aircraft designs to accommodate increased fuel storage requirements (ICAO 2022a). We thus adjust our per-aircraft family development cost estimate upwards based on estimated hydrogen- gas turbines and blended wing- body aircraft development costs (from Ballesteros et al. 2022) to yield a per-aircraft family cost of $21.8 billion, and scale this figure up based on typical aircraft family market shares for WB/LR aircraft, which we take to be 15%.[48]

**Hydrogen Sector Fixed Costs:** Figure 9 showcases the estimated distribution of fixed greenification costs (in percent relative to current per unit fossil costs) in the hydrogen production and distribution sector. Fixed pipeline distribution costs are lowest in the Great Plains and highest in New England due to, e.g., the relatively flat vs. rocky nature of the terrain ().

[47]Using US BTS Air Carrier Financial (Schedule P-5.2) operating expenditure data - which include both passenger and cargo operations - for 2023 we calculate an NB/SMR market expenditure share of 25.2% for the A320 family (including both A318 and A319 aircraft but not the A321/LR due to its distinctly larger size and range). Given Airbus' smaller market share in the United States, we also consider the Boeing 737 (again excluding longer range variants such as the 737-900ER) for which the corresponding expenditure share is 45.6%. We thus assume that the $16.4 billion development costs cover 30% of the NB/SMR market.

[48]BTS data suggest WB/LR expenditure shares range from, e.g., 9.2% for older models such as the B757 to 18.1% for the B777 family.