International Trade in an Uncertain World*

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Abstract

We develop a tractable approach to characterize the distributions of the endogenous variables in quantitative trade models under aggregate uncertainty and incomplete markets. We allow agents to make *ex ante* investments in import and export capacity that determine resilience in the face of this aggregate uncertainty. Equilibrium *ex ante* investments affect *ex post* bilateral trade shares, and these *ex ante* investments depend not only on the expected costs of sourcing goods, but also on the variance and covariance of these costs across countries. Therefore a country's risk profile becomes a key determinant of its comparative advantage, terms of trade, and welfare. Trade liberalization affects welfare through both first moments (expected real income) and second moments (the variance of real income), with endogenous adjustments in *ex ante* investments playing a key role in mediating these effects. Changes in the covariance structure of global shocks have heterogeneous effects across countries, depending on the extent to which they raise or reduce countries' abilities to hedge against aggregate uncertainty through their *ex ante* investments.

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1 Introduction

Recent years have seen growing concerns about uncertainty and resilience. The aftermath of the COVID-19 pandemic highlighted the potential for large-scale transport disruptions. The resurgence of economic nationalism has heightened trade policy uncertainty. Increased geopolitical tensions have raised national security concerns about access to critical goods and minerals. Each of these sources of uncertainty involve aggregate shocks that have general equilibrium effects. We develop a tractable approach to characterize the distributions of the endogenous variables in quantitative trade models under such aggregate uncertainty and incomplete markets. We allow agents to make *ex ante* investments in import and export capacity that determine resilience in the face of this uncertainty. Equilibrium *ex ante* investments affect *ex post* bilateral trade shares, and these *ex ante* investments depend not only on the expected costs of sourcing goods, but also on the variance and covariance of costs across countries. Therefore a country's risk profile becomes a key determinant of its comparative advantage, terms of trade, and welfare. Trade liberalization affects welfare through both first moments (expected real income) and second moments (the variance of real income), with endogenous adjustments in *ex ante* investments playing a key role in mediating these effects. Changes in the covariance structure of global shocks have heterogeneous effects across countries, depending on the extent to which they raise or reduce countries' abilities to hedge against aggregate uncertainty through their *ex ante* investments.

A key challenge in introducing aggregate uncertainty into quantitative trade models is that the general equilibrium conditions of these models are high-dimensional and non-linear, which makes it challenging to obtain analytical results for the mapping from the distribution of exogenous shocks to the distribution of endogenous variables. For this reason, most previous research based on quantitative trade models assumes that agents have perfect knowledge of the realizations of all aggregate variables when making import and export decisions. However, in the presence of aggregate uncertainty, agents make decisions based on the *ex ante* expectation over general equilibrium variables, where the *ex post* realized values of these variables can differ from their expected values. Furthermore, in the presence of risk aversion, agents care not only about the means of general equilibrium variables, but also their variances and covariances.

Modelling aggregate uncertainty matters because much of the debate about resilience involves thinking about aggregate shocks that are likely to have general equilibrium effects. For example, the substantial recent changes in trade policy in the United States are likely to lead to a reallocation of international trade between the United States, Canada, Mexico, China and other countries, with implications for wages and prices in these countries. Similarly, the main source of concern about the dependence of the United States on Taiwan for supplies of advanced semiconductors is about the aggregate costs of losing access to this source of supply in the face of

growing geopolitical tensions between China and the United States.

Our first main contribution is to develop a tractable approach to incorporate aggregate uncertainty into the class of quantitative trade models with a constant trade elasticity. We assume that markets are incomplete and countries are in financial autarky, such that agents cannot directly insure against this aggregate uncertainty. In general, models with aggregate uncertainty and incomplete markets can be intractable, and hence researchers often consider stylized settings with two countries. In contrast, we provide an analytical characterization of the distribution of the endogenous variables in this class of trade models, allowing for many asymmetric countries, an arbitrary network of bilateral trade costs, and a general distribution of aggregate shocks. Our characterization holds up to a second-order approximation for any distribution of the random model fundamentals, including productivities and trade costs.

Our second main contribution is to allow agents to make endogenous *ex ante* investments in import and export capacity that determine resilience in the face of this aggregate uncertainty. Import and export capacity capture bilateral investments in business services, logistics, transportation and customer capital. On the one hand, concentrating investments in import capacity towards the partner with the lowest *ex ante* expected costs reduces the expected cost of importing. On the other hand, dispersing investments in import capacity across a wider range of trade partners enhances the ability to import from partners with high *ex ante* expected costs but low *ex post* realizations of costs.

In the special case of no uncertainty, our model is isomorphic to a conventional constant elasticity trade model. In the presence of uncertainty, each country's aggregate employments in import and export capacity across all trade partners are constant shares of its labor endowment. Given these aggregate allocations, bilateral employments in import and export capacity are proportional to the expectation of export revenue weighted by the marginal utility of income. Up to a second-order approximation, we show that importer n invests more in import capacity with exporter i if i's goods are cheap (i) in expectation; (ii) when importer n's marginal utility is high; (iii) when importer n has high purchasing power; and (iv) when other goods in importer n's consumption bundle are expensive. Analogous results hold for investments in export capacity: exporter i invests more in export capacity with importer n if n has higher expenditure on i (i) in expectation; (ii) when exporter i's marginal utility is high; (iii) when exporter i's CPI is low (i.e., when revenue is valuable in real terms); and (iv) when exporter i's cost of production is low. Therefore, investments in import and export capacity depend not only on means but also variances and covariances of prices, expenditures, and income across trade partners.

We characterize the general equilibrium of our model by taking a second-order Taylor-series expansion in the system of general equilibrium conditions for country wages around the point at which the random variables—log productivities and log bilateral trade costs—are equal to their

respective mean values. We provide a closed-form solution for the Jacobian and Hessian of this general equilibrium system in terms of model parameters and expenditure shares. We show that the Jacobian and Hessian are sufficient statistics for how the first and second moments of productivities and trade costs translate into the first and second moments of the distribution of the model's endogenous variables. The general equilibrium of the model is therefore a fixed point at which investments in import and export capacity based on these first and second moments generate expenditure shares and wages that are consistent with these first and second moments.

In a deterministic constant elasticity trade model, trade liberalization necessarily raises welfare through a higher level of real income (first moment). In the presence of uncertainty, risk version and incomplete markets, trade liberalization now also affects welfare through the volatility of real income (second moment). We find that a modified version of the Arkolakis et al. (2012) (henceforth ACR) formula holds for the first moment effects, modified to take account of endogenous changes in domestic import and export capacity between the closed and open economies. Depending on the variance-covariance matrix of country productivities and the geography of bilateral trade costs, the second moment effects of trade on welfare can be either positive or negative, depending on whether the volatility of real income is higher or lower in the open economy than in the closed economy. Furthermore, some countries can experience an increase in real income volatility following the opening of trade, while other countries can experience a decrease in real income volatility.

We implement our quantitative analysis using panel data on 106 countries from 1993-2017. Given data on international trade and production, we recover the variance-covariance matrix of productivity shocks and bilateral trade costs. We provide a lower-dimensional representation of the variance-covariance matrix of country productivities using a principal components analysis. We find that a global factor (first principal component) accounts for around 43 percent of the variance of productivity shocks, in line with the empirical literature on a global factor in productivity fluctuations. We find that countries with low bilateral trade frictions have higher bilateral productivity correlations, such that productivity uncertainty is systematically correlated with other determinants of trade.

We show how our model can be used to undertake counterfactuals for the impact of changes in global uncertainty. We demonstrate that adjustments in import and export capacity play a key role in mediating the impact of these changes in global uncertainty. Eliminating the correlation in country productivities (by setting the off-diagonal terms in the variance-covariance matrix to zero and holding variances and mean productivity constant) raises welfare in countries that load positively on the global factor in the initial equilibrium. These countries experience an expansion in their ability to diversify risk through international trade, reallocate their investments away from the domestic market to foreign markets, and reduce their domestic expenditure shares. In

contrast, removing the correlation in country productivities reduces welfare in countries that load negatively on the global factor. These countries experience a reduction in their ability to diversify risk through international trade, reallocate their investments away from foreign markets to the domestic market, and increase their domestic expenditure shares.

A proportional increase in global uncertainty (an increase in all variances and covariances holding constant mean productivity) reduces welfare in all countries, because the representative consumer is risk averse. The resulting decline in welfare is larger in countries with more volatile output in the initial equilibrium, because a proportional increase in global uncertainty leads to a larger increase in the variance of productivity in these countries. When sourcing goods in the initial equilibrium, countries trade off sourcing from low-cost suppliers in expectation (often from the domestic market where no trade costs are incurred) against diversifying risk by sourcing goods from abroad (where trade costs are incurred but foreign productivity is imperfectly correlated with domestic productivity). As global uncertainty increases, this increases the incentive for risk diversification, which leads to a reallocation of investments away from the domestic market to foreign markets, and a reduction in domestic trade shares.

Changes in uncertainty in individual countries, such as the United States, propagate through the network of international trade. We consider an increase in the variance of U.S. productivity, holding constant mean U.S. productivity and the cross-country correlation structure. We find that countries with a positive covariance with the U.S. in the initial equilibrium experience a decline in consumption-equivalent welfare. In these countries, the increase in the dispersion of U.S. productivity makes trade with the U.S. less attractive for risk diversification. In contrast, countries with a negative covariance with the U.S. in the initial equilibrium experience a rise in consumption-equivalent welfare. In these countries, the increase in the dispersion of U.S. productivity makes trade with the U.S. more attractive for risk diversification. The resulting changes in demand for U.S. goods, and reallocations of investment across trade partners, lead to endogenous changes in the terms of trade between countries.

Our paper is related to a number of existing lines of research. First, our paper contributes to research on quantitative trade models, including Eaton and Kortum (2002), Arkolakis et al. (2012), and Caliendo and Parro (2015).[1] This research typically abstracts from aggregate uncertainty in fundamentals, such as productivity and trade costs, because the system of general equilibrium conditions in these models is high-dimensional and non-linear in these fundamentals. Caliendo et al. (2025) uses a Ricardian trade model with aggregate uncertainty to study the impact of tariff shocks on trade deficits. Fan and Luo (2025) develops an extension of Ricardian trade models, in which the representative consumer sources varieties after observing the comparative advan-

[1]Studies of trade reallocation in response to unanticipated changes in trade policy (without aggregate uncertainty) include Alfaro and Chor (2023), Grossman et al. (2023), Fajgelbaum et al. (2024), and Adão et al. (2025).

tage of a country for each variety, but before observing the realization of the country's aggregate productivity and bilateral trade costs. In contrast, we introduce aggregate uncertainty and endogenous investments in resilience into the class of trade models with a constant trade elasticity. We provide an analytical characterization of the impact of a country's risk profile on bilateral patterns of trade, the terms of trade, and welfare.

Second, our paper is related to the wider literature on trade and uncertainty. On the one hand, Newbery and Stiglitz (1984) shows that trade can reduce welfare in the presence of uncertainty by inducing greater volatility in prices in the open economy than in the closed economy. On the other hand, Caselli et al. (2020) argues that trade can reduce (rather than increase) volatility if country-level shocks are more important than sector-level shocks.[2] Allen and Atkin (2022) examine the effect of trade on welfare through first and second moments using a model of agricultural land specialization for Indian districts. Relative to this research, we examine the implications of aggregate uncertainty in the class of trade models with a constant trade elasticity, in a setting in which agents can make endogenous investments in resilience.

Third, our work connects with the international macroeconomics literature on international risk diversification. Cole and Obstfeld (1991) shows that trade in goods can contribute to international risk diversification. Obstfeld and Rogoff (2000) argues that the costs of trading goods play a central role in explaining the six major puzzles in international macroeconomics. Following Backus et al. (1992) a large literature analyzes the degree of international risk sharing, including Backus and Smith (1993) and Aguiar et al. (2025). Once trade across states of the world and time periods is taken into account, Fitzgerald (2025) argues that the welfare gains from trade are larger than in conventional models of trade in goods alone. We abstract from international risk diversification, by assuming incomplete markets and financial autarky, in order to highlight the role of country risk as a source of comparative advantage.

Methodologically, the techniques that we use to characterize the effects of uncertainty on bilateral trade patterns are related to those used in the international portfolio literature (see, e.g., Heathcote and Perri 2013 and Coeurdacier and Gourinchas 2016) and in heterogeneous agent models in Auclert et al. (2024) and Bhandari et al. (2023). We use these techniques to study the role of risk in comparative advantage and goods trade. By starting from the class of trade models with a constant trade elasticity, and making minimal departures, we develop an approach that remains tractable in a many-country setting, and lends itself to quantitative analysis.

Fourth, we connect with research in macroeconomics and international trade that examines the propagation of shocks through networks, including Acemoglu et al. (2012), Baqaee and Farhi (2019), Liu (2019), Carvalho et al. (2021), Baqaee and Farhi (2024), Nikolakoudis (2024), and Huo

[2]Other research examines the impact of uncertainty in models of investment with sunk costs, including Dixit and Pindyck (1994), Bloom (2009), Handley and Limao (2017), and Alessandria et al. (2024).

et al. (2025). Within this literature, Taschereau-Dumouchel (2024) examines supply chain uncertainty in a Cobb-Douglas production network, while Liu et al. (2024) develops a model of supplier capital, in which supply chain disruptions have heterogenous effects across firms. In contrast, our focus is on the general equilibrium implications of aggregate uncertainty in a model of international trade between countries.

Fifth, an emerging strand of research within the networks literature has examined network fragility and resilience. Elliott et al. (2022) allow firms to multisource inputs and strategically invest to strengthen relationships, trading off the cost of investment against the benefits of increased robustness. Grossman et al. (2024) analyze optimal resilience in vertical supply chains, in which firms choose investments in protective capabilities and whether to form supply links, and undertake sequential bargaining over quantities and payments. Khanna et al. (2022) and Balboni et al. (2024) provide empirical evidence on the determinants of firm resilience to supply chain shocks. Castro-Vincenzi et al. (2025) provide theory and evidence on firm organization of supply chains in the face of climate risk. Elliott and Golub (2022) reviews this emerging literature on network fragility. While much of this research focuses on the decision problem of the firm in response to uncertainty, we analyze the implications of aggregate uncertainty and investments in resilience in a multi-country, quantitative, general equilibrium model of trade.

Sixth, our work connects with a growing body of research on geoeconomics or the international political economy of trade policy, including Hirschman (1945), Tinbergen (1962), Broner et al. (2024), Clayton et al. (2023), Thoenig (2023), Becko and O'Connor (2024), Clayton et al. (2024), Kleinman et al. (2024), and Liu and Yang (2024). Whereas most of this research is concerned with first moment shocks, our analysis of aggregate uncertainty allows us to also consider second moment shocks, such as an increase in global uncertainty from rising geopolitical tensions between China and the United States.

The remainder of the paper is structured as follows. Section 2 briefly reviews the properties of deterministic trade models with a constant trade elasticity. Section 3 introduces aggregate uncertainty into this class of models. Section 4 characterizes the properties of the competitive equilibrium under aggregate uncertainty. To illustrate our approach as clearly as possible, we focus in our baseline specification on the case of a single sector. Section 5 shows that our approach generalizes to the case of multiple sectors. Section 6 introduces our data. Section 7 provides evidence on our model's predictions. Section 8 summarizes our conclusions.

2 Deterministic Trade Model

For expositional simplicity, we focus on an Armington (1969) model of trade, in which goods are differentiated by country of origin. But all of our results hold throughout the class of quantitative

trade models with a constant trade elasticity, including Eaton and Kortum (2002), Krugman (1980), and Melitz (2003) and Chaney (2008) with a Pareto firm productivity distribution.

We consider a world of many countries indexed by $n, i \in \{1, \ldots, N\}$. The representative agent in country n has labor endowment $\overline{\ell}_n$. The aggregate consumption index in country n ($\mathcal{C}_n$) is defined over the consumption of the good produced by each country i (c_{ni}) and is assumed to take the constant elasticity of substitution (CES) form:

$$\mathcal{C}_n = \left(\sum_{i=1}^{N} c_{ni}^{\frac{\omega-1}{\omega}} \right)^{\frac{\omega}{\omega-1}}, \qquad \omega > 1, \tag{1}$$

where $\omega > 1$ is the elasticity of substitution between country varieties.

Country i's good is produced using labor according to a constant returns to scale production technology with productivity z_i. There are iceberg bilateral costs of trade, such that $\tau_{ni} \geq 1$ units of a good must be shipped from country i in order for one unit to arrive in country n, with $\tau_{ni} > 1$ for $n \neq i$ and $\tau_{nn} = 1$. Therefore consumption (c_{ni}) by importer n of the good produced by exporter i is:

$$c_{ni} = z_i \ell_{ni} / \tau_{ni}, \tag{2}$$

where ℓ_{ni} is the quantity of labor in exporter i allocated to importer n.

Labor market clearing requires that the sum of labor allocated to each importer n is equal to the labor endowment in each exporter i:

$$\overline{\ell}_i = \sum_{n=1}^{N} \ell_{ni}. \tag{3}$$

The market equilibrium in this deterministic Armington model takes the conventional form. Markets are assumed to be perfectly competitive. Income accounting requires that income in each exporter i equals the sum across importers n of expenditure on the goods produced by that exporter. Using cost minimization, zero profits and expenditure minimization, we can write this income accounting equation as:

$$w_i \overline{\ell}_i = \sum_{n=1}^{N} w_n \overline{\ell}_n \frac{\left(w_i \tau_{ni} / z_i\right)^{-\theta}}{\sum_{j=1}^{N} \left(w_j \tau_{nj} / z_j\right)^{-\theta}}, \tag{4}$$

where w_i is the wage in country i and the trade elasticity ($\theta = \omega - 1$) is determined by the elasticity of substitution between country varieties.

General equilibrium in this class of trade models reduces to solving for the N wages in each country (w_n) such that the system of N equations (4) holds. Given this solution for wages in each country, we can recover the equilibrium values of all other endogenous variables. Since country wages are gross substitutes in the system of equations (4), there exists a unique wage vector (up

to a choice of numeraire) that solves this system of equations, as shown in Alvarez and Lucas (2007) and Allen et al. (2020). We choose the convenient numeraire that world gross domestic product (GDP) is equal to one.

3 Aggregate Uncertainty

The key fundamentals that enter the general equilibrium system (4) are country productivities $\{z_i\}$ and bilateral trade costs $\{\tau_{ni}\}$. In the previous section, we assumed that these fundamentals were deterministic. In this section, we assume that these fundamentals are random variables. The realizations of these random variables affect endogenous country wages $\{w_i\}$ through the general equilibrium system (4), thereby giving rise to aggregate uncertainty.

We assume that markets are incomplete and countries are in financial autarky, such that agents cannot directly insure against this aggregate uncertainty. Consequently, trade must be balanced for each realization of the random variables. Agents, however, can make endogenous *ex ante* investments in developing bilateral trade relationships, before observing the *ex post* realizations of productivity and trade costs. These *ex ante* investments determine the resilience of the economy, in the sense of the response of the endogenous variables to these exogenous shocks. We allow the distribution of productivities to differ across countries and the distribution of bilateral trade costs to differ across countries and partners.

The representative agent in country n has labor endowment $\bar{\ell}_n$ and constant relative risk aversion preferences ($\mathcal{U}_n$) over the aggregate consumption index ($\mathcal{C}_n$):

$$\mathcal{U}_n = \mathbb{E}\left[\frac{\mathcal{C}_n^{1-\gamma}}{1-\gamma}\right], \tag{5}$$

where γ is the coefficient of relative risk aversion and $\mathbb{E}$ denotes the expectations operator with respect to the distributions of country productivities.

This aggregate consumption index ($\mathcal{C}_n$) is defined over the consumption of the good produced by each country i (c_{ni}) and again takes the constant elasticity of substitution (CES) form:

$$\mathcal{C}_n = \left(\sum_{i=1}^{N} c_{ni}^{\frac{\omega-1}{\omega}}\right)^{\frac{\omega}{\omega-1}}, \qquad \omega > 1, \tag{6}$$

where $\omega > 1$ is the elasticity of substitution between country varieties.

Country i's good is produced using labor according to a constant returns to scale production technology with productivity z_i. There are iceberg bilateral costs of trade, such that $\tau_{ni} \geq 1$ units of a good must be shipped from country i in order for one unit to arrive in country n, with $\tau_{ni} > 1$ for $n \neq i$ and $\tau_{nn} = 1$.

Consuming a trade partner's good requires both imports and import capacity, where import capacity captures business services, logistics, transportation and customer capital. If country n employs ι_{ni} workers in developing import capacity with exporter i, and sources x_{ni} units of that exporters's good net of variable trade costs, c_{ni} units can be consumed:

$$c_{ni} = \iota_{ni}^{\alpha} x_{ni}^{1-\alpha}, \qquad 0 \leq \alpha < 1. \tag{7}$$

Similarly, supplying a country's good to a trade partner requires exports and export capacity. If country i employs h_{ni} workers in developing export capacity with importer n, and produces $z_i \ell_{ni}/\tau_{ni}$ units for that importer net of variable trade costs, x_{ni} units are supplied:

$$x_{ni} = h_{ni}^{1-\widetilde{\delta}} \left(z_i \ell_{ni}/\tau_{ni}\right)^{\widetilde{\delta}}, \qquad 0 < \widetilde{\delta} < 1. \tag{8}$$

Combining the importing technology (7) and the exporting technology (8), consumption c_{ni} by importer n of the good produced by exporter i is related to importer capacity (ι_{ni}), export capacity (h_{ni}) and production net of variable trade costs ($z_i \ell_{ni}/\tau_{ni}$) as follows:

$$c_{ni} = \iota_{ni}^{\alpha} h_{ni}^{\beta} \left(z_i \ell_{ni}/\tau_{ni}\right)^{\delta}, \qquad \alpha + \beta + \delta = 1, \tag{9}$$

where $\beta \equiv (1-\alpha)\left(1-\widetilde{\delta}\right)$; $\delta \equiv (1-\alpha)\,\widetilde{\delta}$; $0 < \beta < 1$; and $0 < \delta < 1$.

In this formulation (9), countries have access to a constant returns to scale trading technology in import capacity, export capacity and production net of variable trade costs. Import capacity (ι_{ni}) and export capacity (h_{ni}) correspond to a form non-traded services that are supplied to the production goods sector in order to enable these goods to be traded. We assume a Cobb-Douglas functional form for this trading technology (9) for tractability and for three main reasons. First, this specification captures extensive margin decisions of whether to participate in trade relationships (as in the networks literature on multi-sourcing), because positive investments in both import and export capacity are required in order for positive trade to occur. Second, this specification captures intensive margin decisions of how much to invest in trade relationships conditional on positive trade, because because larger investments in import and export capacity increase the productivity of the trading technology. Third, in the absence of uncertainty, this specification is isomorphic to the conventional deterministic Armington in the previous subsection, up to an adjustment to trade elasticity, as shown formally below.[3]

We assume that import and export capacity are chosen *ex ante* before observing the realizations of the stochastic fundamentals—country productivities $\{z_i\}$ and bilateral trade costs $\{\tau_{ni}\}$. Labor allocated to import or export capacity with a given trade partner cannot be reallocated to

[3]Notwithstanding these advantages, our specification for import and export capacity can be generalized to the CES functional form or a nested CES functional form, in which case import capacity, export capacity and production net of variable trade costs can be either complements or substitutes.

another trade partner, moved between import and export capacity, or used for production. After observing the realizations of the fundamentals, the remaining production labor is allocated *ex post* across trade partners. Labor market clearing requires that the sum across trade partners of labor allocated to import capacity, export capacity and production is equal to the economy's labor endowment:

$$\overline{\ell}_i = \sum_{j=1}^{N} \iota_{ij} + \sum_{n=1}^{N} \left(\ell_{ni} + h_{ni} \right). \tag{10}$$

We now show that in the absence of uncertainty the competitive market equilibrium of our model is isomorphic to that in the deterministic Armington model in the previous section, up to an adjustment to the trade elasticity. We assume that there is a competitive production sector that uses labor to produce a country's good, a competitive importing sector that uses labor to develop import capacity, and a competitive exporting sector that uses labor to develop export capacity. Using the income accounting relationship between a country's income and expenditure on its goods, we obtain the following result for the market equilibrium of our model.

Proposition 1. ***(Trade Capacity Without Uncertainty)*** *In the absence of uncertainty, the market equilibrium in our model of trade capacity is isomorphic to that in the deterministic Armington model. The system of general equilibrium conditions takes the same form in both models, with an appropriate adjustment to the trade elasticity ($\theta = \alpha + \beta + \delta\omega - 1 < \omega - 1$):*

$$w_i \overline{\ell}_i = \sum_{n=1}^{N} w_n \overline{\ell}_n \frac{\left(\tau_{ni} w_i / z_i \right)^{-\theta}}{\sum_{j=1}^{N} \left(\tau_{nj} w_j / z_j \right)^{-\theta}}. \tag{11}$$

Proof. See the Online Appendix. □

We interpret the trade elasticity in the special case of our model without uncertainty as the *long-run* trade elasticity ($\theta^{LR} = \alpha + \beta + \delta\omega - 1$) , because it captures the full response of trade with respect to variable trade costs (including adjustments in import and export capacity) after the resolution of all uncertainty. A key implication of Proposition 1 is that the novel predictions of our model below do not arise from the introduction of import and export capacity *per se*, but are rather driven by aggregate uncertainty and its interaction with import and export capacity, as characterized in the next section.

4 Trade Equilibrium Under Uncertainty

We now solve for the market equilibrium of our model under aggregate uncertainty and incomplete markets, assuming that firms in the importing, exporting and production sectors are owned by the representative consumer. We begin by considering *ex ante* investments in import and export capacity, before turning to the *ex post* equilibrium given these investments.

4.1 *Ex Ante* Stage

The representative consumer must be indifferent in equilibrium between allocating labor to import and export capacity *ex ante* and allocating labor to production across trade partners *ex post* given the realizations of country productivities. Therefore, the *ex ante* wage (w_i^-) is equal to the expectation of the wage times the marginal utility of income *ex post* ($w_n \mathcal{U}'(\mathcal{C}_n)/\mathcal{P}_n$):

$$w_n^- = \mathbb{E}\left[\frac{U'(\mathcal{C}_n)}{\mathcal{P}_n} w_n\right] \tag{12}$$

where the expectation $\mathbb{E}[\cdot]$ is taken across states of the world (across the productivity and trade cost distributions) and $\mathcal{P}_n$ is the price index dual to consumption index (6) for a given realization for country productivities.

Firms choose *ex ante* investments to maximize profits, using the consumer's marginal utility of income as the stochastic discount factor, where firm profits or losses are rebated back to the domestic consumer. Firms in the importing sector in country n choose import capacity (ι_{ni}) to solve the following problem for each exporter i, taking prices and wages as given:

$$\max_{\iota_{ni}} \left\{ \mathbb{E}\left[\frac{\mathcal{U}'(\mathcal{C}_n)}{\mathcal{P}_n} \max_{x_{ni}} \left(p_{ni}^c c_{ni} - p_{ni}^x x_{ni}\right)\right] - w_n^- \iota_{ni} \right\}, \tag{13}$$

where p_{ni}^c is the price of consuming the good supplied by exporter i in importer n (after investments in both import and export capacity), which corresponds the dual of equation (9); and p_{ni}^x is the price of the good supplied by exporter i to importer n on world markets (after investments in export capacity but before investments in import capacity), which corresponds to the dual of equation (8).

Firms in the exporting sector in country i choose export capacity (h_{ni}) to solve the following problem for each importer n, taking prices and wages as given:

$$\max_{h_{ni}} \left\{ \mathbb{E}\left[\frac{\mathcal{U}'(\mathcal{C}_i)}{\mathcal{P}_i} \max_{h_{ni}} \left(p_{ni}^x x_{ni} - w_i \ell_{ni}\right)\right] - w_i^- h_{ni} \right\}, \tag{14}$$

where since markets are incomplete, the marginal utility of income can differ between the importer n and exporter i: $\mathcal{U}'(\mathcal{C}_n)/\mathcal{P}_n \neq \mathcal{U}'(\mathcal{C}_i)/\mathcal{P}_i$ for $n \neq i$.

4.2 *Ex Post* Stage

After the *ex ante* investments in import and export capacity have been incurred, agents observe the *ex post* realizations for country productivities and trade costs. The representative consumer in importer n chooses consumption of the good produced by each exporter i to maximize utility, given these realizations for country productivities $\{z_i\}$ and trade costs $\{\tau_{ni}\}$, and the *ex ante*

investments in import capacity $\{\iota_{ni}\}$ and export capacity $\{h_{ni}\}$:

$$\max_{\{c_{ni}\}} \left\{ \left(\sum_{i=1}^{N} c_{ni}^{\frac{\omega-1}{\omega}} \right)^{\frac{\omega}{\omega-1}} \right\}, \tag{15}$$

$$\text{s.t.} \qquad \sum_{i=1}^{N} p_{ni}^{c} c_{ni} = \mathcal{W}_n, \tag{16}$$

where $\mathcal{W}_n$ denotes total expenditure by importer n, which is equal to its total income.

From the first-order conditions to this problem, the representative consumer's CES preferences (6) imply that each importer's relative expenditure shares across exporters are a constant elasticity function of relative prices:

$$\frac{p_{ni}^{c} c_{ni}}{\sum_{j=1}^{N} p_{nj}^{c} c_{nj}} = \frac{\left(p_{ni}^{c}\right)^{1-\omega}}{\sum_{j=1}^{N} \left(p_{nj}^{c}\right)^{1-\omega}} = \frac{c_{ni}^{\frac{\omega-1}{\omega}}}{\sum_{j=1}^{N} c_{nj}^{\frac{\omega-1}{\omega}}}. \tag{17}$$

The Cobb-Douglas trading technology (9) implies that bilateral expenditure on production labor ($w_i \ell_{ni}$) is a constant share of bilateral consumption expenditure ($p_{ni}^{c} c_{ni}$):

$$w_i \ell_{ni} = \delta p_{ni}^{c} c_{ni}. \tag{18}$$

This trading technology also implies that the *ex post* revenues of exporters and importers are constant shares of bilateral consumption expenditure. The resulting *ex post* quasi-rents of exporters and importers (which can be either positive or negative) are rebated to the representative consumer as:

$$\text{(exporter quasi-rents)} \qquad \pi_{ni}^{exporter} = \beta p_{ni}^{c} c_{ni} - w_i^{-} h_{ni},$$

$$\text{(importer quasi-rents)} \qquad \pi_{ni}^{importer} = \alpha p_{ni}^{c} c_{ni} - w_n^{-} \iota_{ni}.$$

Therefore, the total income of the representative consumer in country i is:

$$\begin{aligned} \mathcal{W}_i &= w_i \sum_n \ell_{ni} + w_i^{-} \left(\sum_n h_{ni} + \sum_k \iota_{ik} \right) + \sum_n \pi_{ni}^{exporter} + \sum_k \pi_{ik}^{importer}, \\ &= (1-\alpha) \sum_n p_{ni}^{c} c_{ni} + \alpha \sum_k p_{ik}^{c} c_{ik}. \end{aligned} \tag{19}$$

Since total income is equal to total expenditure ($\mathcal{W}_i = \sum_{k=1}^{N} p_{ik}^{c} c_{ik}$), equation (19) implies that $\sum_{n=1}^{N} p_{ni}^{c} c_{ni} = \sum_{k=1}^{N} p_{ik}^{c} c_{ik}$, which yields the following result that total income is proportional to total expenditure on production labor:

$$\mathcal{W}_i = \sum_n p_{ni}^{c} c_{ni} = w_i \sum_n \ell_{ni} / \delta.$$

The income accounting relationship between the income of each exporter and expenditure on the goods produced by that exporter now takes the following modified form:

$$w_i\left(\bar{\ell}_i-\sum_{k=1}^{N}\iota_{ik}-\sum_{n=1}^{N}h_{ni}\right)=\sum_{n=1}^{N}\frac{\left(\iota_{ni}^{-\frac{\alpha}{\delta}}h_{ni}^{-\frac{\beta}{\delta}}\tau_{ni}w_i/z_i\right)^{-\theta}}{\sum_{j=1}^{N}\left(\iota_{nj}^{-\frac{\alpha}{\delta}}h_{nj}^{-\frac{\beta}{\delta}}\tau_{nj}w_j/z_j\right)^{-\theta}}w_n\left(\bar{\ell}_n-\sum_{k=1}^{N}\iota_{nk}-\sum_{m=1}^{N}h_{mn}\right), \tag{20}$$

where the ex post trade elasticity is $\theta=\widetilde{\omega}-1$ and $\widetilde{\omega}\equiv\frac{\omega}{\omega-\delta(\omega-1)}$. We interpret this *ex post* trade elasticity as the *short-run* elasticity of trade with respect to variable trade costs ($\theta^{SR}=\frac{\delta(\omega-1)}{(1-\delta)\omega+1}$), because it holds constant bilateral investments in import and export capacity.

The system of *ex post* general equilibrium conditions (20) takes a similar form to the deterministic Armington model, with three modifications. First, the *ex post* trade elasticity $\theta=\frac{\delta}{(1-\delta)\omega+1}(\omega-1)<(\omega-1)$ is smaller in absolute value than in the deterministic Armington model, because traded goods are only part of consumption expenditure (which also includes payments for import and export capacity). Second, the *ex ante* investments in import and export capacity act like demand shifters in the *ex post* expenditure shares. These *ex ante* investments enter the *ex post* expenditure shares inversely to bilateral trade costs and can be absorbed into a composite measure of *ex post* bilateral trade costs: $\widetilde{\tau}_{ni}\equiv\iota_{ni}^{-\alpha/\delta}h_{ni}^{-\beta/\delta}\tau_{ni}$. Third, revenue from sales in all markets net of variable trade costs is equal to payments to production labor, which equals the economy's labor endowment minus the labor used for *ex ante* investments in import and export capacity. We provide a further characterization of these equilibrium *ex ante* allocations of labor to import and export capacity in the next section.

4.3 Properties of the *Ex Ante* Equilibrium

Under our assumption that productivities and trade costs are random variables, the wages determined by the system of general equilibrium conditions (20) are random variables. Since all other endogenous variables of the model can be recovered from wages, they are also random variables. We now characterize equilibrium *ex ante* investments in import and export capacity, which depend on expectations over the distributions of these random variables.

These *ex ante* investments are shaped by two sets of forces. First, there is an aggregate trade-off. The greater the total amount of labor allocated to import and export capacity *ex ante*, the less labor is available for production *ex post*. Second, there is a bilateral trade-off. On the one hand, if import capacity is concentrated towards the supplier with the expected lowest price, a greater expected quantity of imports is obtained. Similarly, if export capacity is concentrated towards the buyer with the expected largest market, a greater volume of exports is attained. On the other hand, if import capacity is diversified across suppliers, there is a greater ability to source from suppliers who have high expected prices *ex ante* but low realized prices *ex post*, given realized

productivities. Similarly, if export capacity is diversified across buyers, there is greater scope to sell to buyers who have low expected market size *ex ante* but high realized market size *ex post*, given realized productivities.

We now characterize this trade-off in the competitive equilibrium. Aggregate employments in import and export capacity across all trade partners are constant shares of each country's labor endowment. Bilateral employments in import and export capacity are proportional to the expectation of export sales times the marginal utility of income, where this marginal utility of income in general differs between the importer and exporter, because of incomplete markets. We summarize these results in the following proposition.

Proposition 2. *The ex ante allocation of labor to import and export capacity in the competitive equilibrium satisfies:*

$$\sum_{i=1}^{N} \iota_{ni} = \alpha \bar{\ell}_n, \quad \sum_{k=1}^{N} h_{ki} = \beta \bar{\ell}_i,$$

$$\iota_{ni} \propto_n \mathbb{E}\left[\frac{\mathcal{U}'(\mathcal{C}_n)}{\mathcal{P}_n} p_{ni}^x x_{ni}\right] = \mathbb{E}\left[\mathcal{C}_n^{1-\gamma}\left(\iota_{ni}^{-\frac{\alpha}{\delta}} \ell_{ni}^{-\frac{\beta}{\delta}} \tau_{ni} w_i / z_i\right)^{1-\widetilde{\omega}} \mathcal{Q}_n^{-(1-\widetilde{\omega})}\right],$$

$$h_{ni} \propto_i \mathbb{E}\left[\frac{\mathcal{U}'(\mathcal{C}_i)}{\mathcal{P}_i} p_{ni}^x x_{ni}\right] = \mathbb{E}\left[\frac{\mathcal{C}_i^{1-\gamma}}{w_i}\left(\iota_{ni}^{-\frac{\alpha}{\delta}} h_{ni}^{-\frac{\beta}{\delta}} \tau_{ni} w_i / z_i\right)^{1-\widetilde{\omega}} \mathcal{Q}_n^{-(1-\widetilde{\omega})} w_n \bar{\ell}_n\right],$$

$$\mathcal{Q}_n \equiv \left(\sum_{j=1}^{N}\left(\iota_{nj}^{-\frac{\alpha}{\delta}} h_{nj}^{-\frac{\beta}{\delta}} \tau_{nj} w_j / z_j\right)^{1-\widetilde{\omega}}\right)^{\frac{1}{1-\widetilde{\omega}}},$$

$$\mathcal{W}_n = w_n \bar{\ell}_n, \qquad \mathcal{C}_n = \mathcal{Q}_n^{-\delta} \mathcal{W}_n^{\delta} \delta^{\delta}, \qquad \mathcal{P}_n = \frac{\mathcal{W}_n}{\mathcal{C}_n} = \mathcal{Q}_n^{\delta} \mathcal{W}_n^{1-\delta} \delta^{-\delta}.$$

Proof. See the Online Appendix. □

Intuitively, the property that aggregate employments in import and export capacity are a constant share of a country's labor endowment is an implication of our assumption of a Cobb-Douglas trading technology (9). Since import and export capacity are non-traded services supplied to the traded production sector, this functional form implies constant aggregate employment shares in these two economic activities. In the absence of uncertainty, this property ensures that our model nests the deterministic Armington model in Proposition 1 above. More generally, this property ensures that aggregate *ex post* income ($\mathcal{W}_n$) equals the *ex post* wage (w_n) times the economy's labor endowment ($\bar{\ell}_n$) in Proposition 2.

In the absence of uncertainty, relative *ex ante* investments in import and export capacity are proportional to relative trade volumes ($p_{ni}^x x_{ni}$). In contrast, in the presence of uncertainty, these relative *ex ante* investments are proportional to the relative value of the expectations of

export values adjusted for the marginal utility of income. From Proposition 2, these expectations depend on the *ex post* values of a price index that incorporates import and export capacity ($\mathcal{Q}_n$) and aggregate income ($\mathcal{W}_n$). Given these variables, we can recover the *ex post* values of the consumption index ($\mathcal{C}_n$) and the dual consumption price index ($\mathcal{P}_n$).

Taking a second-order approximation around the mean vector of log productivities in each country ($\ln \boldsymbol{z} = \boldsymbol{\mu_z}$), we can obtain a further characterization of relative *ex ante* investments in import and export capacity in terms of means, variances and covariances.

Proposition 3. *As a second-order approximation.*

$$\begin{aligned}\iota_{ni} \propto_n \quad & \mathbb{E}\left[\left(\iota_{ni}^{-\frac{\alpha}{\delta}}\ell_{ni}^{-\frac{\beta}{\delta}}\tfrac{w_i\tau_{ni}}{z_i}\right)^{1-\widetilde{\omega}}\right] \\ & \times \exp\left(\mathit{Cov}\left(\ln \mathcal{C}_n^{-\gamma}, \ln\left(\tfrac{w_i\tau_{ni}}{z_i}\right)^{1-\widetilde{\omega}}\right)\right) \\ & \times \exp\left(\mathit{Cov}\left(\ln w_n/\mathcal{P}_n, \ln\left(\tfrac{w_i\tau_{ni}}{z_i}\right)^{1-\widetilde{\omega}}\right)\right) \\ & \times \exp\left(-\mathit{Cov}\left(\ln \mathcal{Q}_n^{1-\widetilde{\omega}}, \ln\left(\tfrac{w_i\tau_{ni}}{z_i}\right)^{1-\widetilde{\omega}}\right)\right),\end{aligned}$$

$$\begin{aligned}h_{ni} \propto_i \quad & \mathbb{E}\left[\left(\iota_{ni}^{-\frac{\alpha}{\delta}}\ell_{ni}^{-\frac{\beta}{\delta}}\tau_{ni}\right)^{1-\widetilde{\omega}}\mathcal{Q}_n^{-(1-\widetilde{\omega})}w_n\bar{\ell}_n\right] \\ & \times \exp\left(\mathit{Cov}\left(\ln \mathcal{C}_i^{-\gamma}, \ln\left(\tfrac{\tau_{ni}}{\mathcal{Q}_n}\right)^{1-\widetilde{\omega}}w_n\right)\right) \\ & \times \exp\left(-\mathit{Cov}\left(\ln \mathcal{P}_i, \ln\left(\tfrac{\tau_{ni}}{\mathcal{Q}_n}\right)^{1-\widetilde{\omega}}w_n\right)\right) \\ & \times \exp\left(\mathit{Cov}\left(\ln\left(\tfrac{w_i}{z_i}\right)^{1-\widetilde{\omega}}, \ln\left(\tfrac{\tau_{ni}}{\mathcal{Q}_n}\right)^{1-\widetilde{\omega}}w_n\right)\right).\end{aligned}$$

Proof. See the Online Appendix. □

The characterization of equilibrium *ex ante* investments in this proposition has an intuitive interpretation. Importer n invests more in bilateral import capacity with exporter i if exporter i's goods are cheap (i) in expectation (first term); (ii) when importer n's marginal utility is high (second term); (iii) when importer n has high purchasing power (third term); (iv) when other goods in importer n's consumption bundle are expensive (fourth term). Similarly, exporter i invests more in bilateral export capacity with importer n if importer n has higher expenditure on good i (i) in expectation (first term); (ii) when exporter i's marginal utility is high (second term); (iii) when exporter i's consumption price index is low, such that revenue is valuable in real terms (third term); (iv) when exporter i's cost of production is low (fourth term).

Uncertainty fundamentally changes the determinants of bilateral trade between countries. In the absence of uncertainty, bilateral trade depends solely on the realized values of production costs inclusive of bilateral trade costs, where these realized values equal their expected values. In contrast, in the presence of uncertainty, there is a hedging motive for bilateral *ex ante* investments, which now depend on variances and covariances in addition to expectations. In the absence of uncertainty, bilateral expenditure shares satisfy the independence of irrelevant alternatives (IIA) property, such that the relative value of importer n's trade with two exporters i and j depends solely on the characteristics of those exporters, and is invariant with respect to the characteristics of other exporters $m \neq i, j$. In contrast, in the presence of uncertainty, bilateral expenditure shares no longer satisfy this IIA property. The relative value of importer n's trade with two

exporters i and j depends on whether or not exporter $m \neq i, j$ is also included in the choice set, since countries i and j can have different covariances with country m.

Proposition 3 uses a second-order Taylor-series expansion of random variables to provide a characterization of the *ex ante* investments that is first-order accurate in terms of second moments (covariances). A key remaining challenge is to characterize the properties of the distributions of the endogenous random variables, such as wages, given the distribution of exogenous variables—productivities and trade costs—using the non-linear mapping from these exogenous variables to wages in the system of general equilibrium conditions (20).

We now turn to this challenge. For expositional simplicity, we first assume trade costs $\{\tau_{ni}\}$ are deterministic and that only productivities are random. Proposition 4 characterizes the first and second moments of log wages given the mean and variance of log productivities. This result is a special case of Proposition 5 below, which characterizes the first and second moments of log wages when both productivities and trade costs are random variables.

Proposition 4. *Suppose* $\mathbb{E}[\ln \boldsymbol{z}] = \boldsymbol{\mu}_z$, $\mathbb{V}[\ln \boldsymbol{z}] = \boldsymbol{\Sigma}_z$, *and trade costs* $\{\tau_{ni}\}$ *are deterministic. Define* $\boldsymbol{F}$ *as the equilibrium correspondence in the deterministic Armington model (equation (4) that maps log productivities* ($\ln \boldsymbol{z}$) *into log wages* ($\ln \boldsymbol{w}$) *given the trade elasticity* (θ)*:* $\ln \boldsymbol{w} = \boldsymbol{F}(\ln \boldsymbol{z}; \theta)$.
(A) *To second order,*

$$\mathbb{E}[\ln w_i] \approx F_i(\boldsymbol{\mu}_z) + \frac{1}{2}\sum_{j=1}^{N}\sum_{k=1}^{N} \boldsymbol{H}^i_{jk} [\boldsymbol{\Sigma}_z]_{jk}, \qquad Var[\ln \boldsymbol{w}] \approx \boldsymbol{W}\boldsymbol{\Sigma}_z\boldsymbol{W}'$$

where $\boldsymbol{W}$ *and* $\boldsymbol{H}$ *are the Jacobian and Hessian of* $\boldsymbol{F}$ *evaluated at* $\ln \boldsymbol{z} = \boldsymbol{\mu}_z$.
(B) *The bilateral expenditure share matrix* $\boldsymbol{S}$ *is a sufficient statistic for the Jacobian and Hessian:*

$$S_{ni} \equiv \left.\frac{(\tau_{ni} w_i / z_i)^{-\theta}}{\sum_{k=1}^{N} (\tau_{nk} w_k / z_k)^{-\theta}}\right|_{\ln \boldsymbol{z} = \boldsymbol{\mu}_z}, \qquad T_{in} \equiv \left.\frac{w_n \bar{\ell}_n S_{ni}}{w_i \bar{\ell}_i}\right|_{\ln \boldsymbol{z} = \boldsymbol{\mu}_z}.$$

$$\boldsymbol{W} \equiv \left[\frac{\mathrm{d} \ln w_i}{\mathrm{d} \ln z_j}|_{\ln \boldsymbol{z} = \boldsymbol{\mu}_z}\right] = -\theta \boldsymbol{X}^{-1}(\boldsymbol{T}\boldsymbol{S} - \boldsymbol{I}), \quad \boldsymbol{X} \equiv \boldsymbol{I} - \boldsymbol{T} - \theta(\boldsymbol{T}\boldsymbol{S} - \boldsymbol{I}) + (1+\theta)\boldsymbol{Q},$$

$$\boldsymbol{H}^i_{jk} \equiv \partial_k \boldsymbol{W}_{ij}, \qquad \partial_k \boldsymbol{W} = (\boldsymbol{I} - \boldsymbol{Q})\left\{\boldsymbol{X}^{-1}(\partial_k \boldsymbol{T})((\boldsymbol{I} + \theta \boldsymbol{S})\boldsymbol{W} - \theta \boldsymbol{S}) + \theta \boldsymbol{X}^{-1}\boldsymbol{T}(\partial_k \boldsymbol{S})(\boldsymbol{W} - \boldsymbol{I})\right\},$$

$$(\partial_k \boldsymbol{S})_{ni} = \theta \boldsymbol{S}_{ni}(\mathbf{1}_{i=k} - \boldsymbol{W}_{ik} - \boldsymbol{S}_{nk} + (\boldsymbol{S}\boldsymbol{W})_{nk}),$$

$$(\partial_k \boldsymbol{T})_{in} = \boldsymbol{T}_{in}[\boldsymbol{W}_{nk} - \boldsymbol{W}_{ik} + \theta(\mathbf{1}_{i=k} - \boldsymbol{W}_{ik} - \boldsymbol{S}_{nk} + (\boldsymbol{S}\boldsymbol{W})_{nk})].$$

The matrix $\boldsymbol{Q} \equiv \mathbf{1}\boldsymbol{q}'$ *captures the choice of numeraire, where* $q_i = w_i \bar{\ell}_i$ *and our choice of numeraire implies* $\sum_{i=1}^{N} q_i = 1$.

Proof. See the Online Appendix. □

Proposition 4 provides an analytical characterization of the first and second moments of the distribution of endogenous wages up to a second-order approximation. We undertake this approximation around around the mean log productivity in each country. These first and second moments depend on the Jacobian ($\boldsymbol{W}$) and Hessian ($\boldsymbol{H}$) of the system of general equilibrium conditions (4) that links log wages and log productivities in the deterministic Armington model. Both matrices can be expressed in terms of the sufficient statistics of bilateral expenditure shares (S_{ni}) and a constant trade elasticity (θ). Having recovered these first and second moments for wages (w_n), we can recover the corresponding moments for all other endogenous variables, including price indexes ($\mathcal{P}_n$) and real income ($w_n/\mathcal{P}_n$).

Although our model incorporates endogenous *ex ante* investments in import and export capacity, the Jacobian ($\boldsymbol{W}$) and Hessian ($\boldsymbol{H}$) in Proposition 4 take exactly the same form as in the deterministic Armington model, up to an adjustment to trade elasticity. This isomorphism reflects three key properties of our model: (i) aggregate investments in import and export capacity are a constant share of each country's labor endowment; (ii) bilateral investments in import and export capacity enter *ex post* expenditure shares isomorphically to bilateral trade costs; (iii) the trade elasticity in these *ex post* expenditure shares is equal to a constant. Therefore, Proposition 4 can be used to characterize the moments of the distribution of endogenous wages throughout the entire class of quantitative trade models with a constant trade elasticity, including not only Armington (1969) models, but also Ricardian models following Eaton and Kortum (2002), new trade models following Krugman (1980), and heterogeneous firm models with a Pareto productivity distribution following Melitz (2003) and Chaney (2008).

While the Jacobian ($\boldsymbol{W}$) and Hessian ($\boldsymbol{H}$) in Proposition 4 take the same form throughout this class of models, our model implies that *ex post* bilateral trade costs ($\widetilde{\tau}_{ni} \equiv \iota_{ni}^{-\alpha/\delta} h_{ni}^{-\beta/\delta} \tau_{ni}$) are endogenously determined, through *ex ante* investments in import and export capacity. Therefore, bilateral trade costs are not themselves structural parameters that are invariant with respect to changes in the policy environment that affect uncertainty. Instead, they respond endogenously to these policy changes, as in the Lucas Critique in the macroeconomics literature, and as explored in our counterfactuals below.

In general, models with aggregate uncertainty and incomplete markets are notoriously intractable. For this reason, researchers often consider stylized settings with two countries. In contrast, our sufficient statistics representation of the first and second moments of the distribution of the endogenous variables in Proposition 4 holds for many asymmetric countries and an arbitrary network of bilateral trade costs. Therefore, our approach remains tractable and amenable to quantitative analysis in empirically-realistic settings, and can be taken directly to observed data on countries' trade and production.

4.4 General Equilibrium

The general equilibrium of our model is referenced by the model parameters and country fundamentals. The model parameters include: the elasticity of substitution between country varieties (ω); the share of import capacity in expenditure (α); and the share of export capacity in expenditure (β). The country fundamentals are: the vector of labor endowments in each country ($\bar{\boldsymbol{\ell}}$); the vector of mean log country productivities ($\boldsymbol{\mu_z}$); the variance-covariance matrix of log country productivities ($\boldsymbol{\Sigma_z}$); and the matrix of bilateral variable trade costs ($\boldsymbol{\tau}$).

Given assumed values for model parameters and country fundamentals, we solve for general equilibrium by starting with an initial guess for *ex ante* investments in import capacity (ι_{ni}^0) and export capacity (h_{ni}^0). Given these initial guesses, we solve for wages (w_n) and expenditure shares (S_{ni}) in the *ex post* equilibrium with the log of country productivities are equal to their mean value ($\ln z_n = \mu_{zn}$). Using these solutions for wages (w_n) and expenditure shares (S_{ni}), we derive the first and second moments of the distribution of the endogenous variables using the Jacobian and Hessian of the system of general equilibrium conditions in Proposition 4. Using these first and second moments of the distribution of the endogenous variables, we compute equilibrium import capacity (ι_{ni}) and export capacity (h_{ni}) using Proposition 3. If these solutions for equilibrium import and export capacity are not equal to our initial guesses, we update these initial guesses, and iterate until convergence.

We use this solution method to undertake counterfactuals for changes in country fundamentals. We examine how the presence of aggregate uncertainty affects the results of conventional counterfactuals, such as the opening of the closed economy to trade. We also consider novel counterfactuals, such as an increase in global uncertainty (an increase in the dispersion of country productivities), or a change in the correlation structure of country productivities, and evaluate the impact on bilateral trade patterns, the terms of trade, and welfare.

Our solution method uses a second-order approximation of the model's system of general equilibrium conditions. A key advantage of this approach is that we can provide a sharp analytical characterization of how the covariance structure of country productivities affects bilateral trade, wages and welfare using Propositions 3 and 4. We thus obtain analytical results for the general equilibrium impact of aggregate uncertainty and the role of endogenous investments in resilience in shaping this impact. Such analytical results are not possible in the non-linear model, because the endogenous variables are high-dimensional and non-linear functions of the exogenous variables, which implies that the non-linear model only can be solved numerically for simulated productivity distributions. In our empirical application below, we provide Monte Carlo evidence that our analytical results from our second-order Taylor-series expansion provide a close approximation to the numerical solution of the non-linear model for simulated productivity distributions.

4.5 Welfare Gains from Trade

Uncertainty also has fundamental implications for the welfare effects of international trade. In a deterministic constant elasticity trade model, trade liberalization necessarily raises welfare through a higher level of real income (first moment). In the presence of uncertainty, risk aversion and incomplete markets, trade liberalization now also affects welfare through the volatility of real income (second moment). We define certainty-equivalent welfare as the deterministic level of welfare that is equal to the expected value of uncertain welfare:

$$\mathcal{W}_n \equiv \mathcal{U}^{-1}\left(\mathbb{E}\left[\mathcal{U}\left(\mathcal{C}_n\right)\right]\right) = \mathbb{E}\left[\mathcal{C}_n^{1-\gamma}\right]^{\frac{1}{1-\gamma}}. \tag{21}$$

To second-order approximation, the log of certainty-equivalent welfare can be expressed in terms of the mean and variance of the log consumption index:

$$\ln \mathcal{W}_n \approx \mathbb{E}\left[\ln \mathcal{C}_n\right] + \frac{1-\gamma}{2}\mathbb{V}\left[\ln \mathcal{C}_n\right]. \tag{22}$$

Taking log differences between the open and closed-economy equilibria, the welfare gains from opening the closed economy to trade are:

$$\begin{aligned} \ln \mathcal{W}_n^{open}/\mathcal{W}_n^{aut} &\approx \mathbb{E}\left[\ln \mathcal{C}_n^{open} - \ln \mathcal{C}_n^{aut}\right] + \frac{1-\gamma}{2}\left(\mathbb{V}\left[\ln \mathcal{C}_n^{open}\right] - \mathbb{V}\left[\ln \mathcal{C}_n^{aut}\right]\right), \\ &= \mathbb{E}\left[\alpha \ln\left(\frac{\iota_{nn}^{open}}{\alpha\bar{\ell}_n}\right) + \beta \ln\left(\frac{h_{nn}^{open}}{\beta\bar{\ell}_n}\right) + \delta \ln\left(S_{nn}^{open}\right)^{\frac{1}{1-\tilde{\omega}}}\right] \\ &\quad + \frac{1-\gamma}{2}\left(\mathbb{V}\left[\ln w_n^{open}/\mathcal{P}_n^{open}\right] - \delta^2\mathbb{V}\left[\ln z_n\right]\right), \end{aligned} \tag{23}$$

where the second and third lines use the equilibrium relationship between the consumption index ($\mathcal{C}_n$), aggregate expenditure ($\mathcal{W}_n$), the consumption price index ($\mathcal{P}_n$) and the domestic expenditure share (S_{nn}) in the closed and open economies.

In the deterministic Armington model, the welfare gains from opening the closed-economy to trade are summarized by the ACR formula in terms of the domestic trade share (S_{nn}) and the trade elasticity ($\theta = \omega - 1$). In our model of aggregate uncertainty, we find that a version of this formula holds for the first moment effects of trade on welfare (second line), with two modifications. First, the trade elasticity is no longer the elasticity of substitution between country varieties, but also depends on the shares of import and export capacity in consumer expenditure ($\theta = \frac{\delta(\omega-1)}{(1-\delta)\omega+1}$). Second, domestic investments in import capacity and export capacity differ between the closed and open economies, and hence the ACR formula must be adjusted to take account of these endogenous changes in investments. This adjustment implies that the first moment effects of trade on welfare are smaller in our model than implied by the ACR formula, because the observed domestic trade share in the open economy equilibrium is based on lower domestic investments than in the closed economy ($\iota_{nn}^{open} < \alpha\bar{\ell}_n$ and $h_{nn}^{open} < \beta\bar{\ell}_n$ in equation (23)).

In our model of aggregate uncertainty, the opening of the closed economy to trade also affects welfare through changes in the second moment of real income (third line). In the closed economy, the variance in log consumption is proportional to the variance in domestic productivity ($\mathbb{V}\left[\ln \mathcal{C}_n^{aut}\right] = \delta^2 \mathbb{V}\left[\ln z_n\right]$). In contrast, in the open economy, the variance in log consumption ($\mathbb{V}\left[\ln \mathcal{C}_n^{open}\right] = \mathbb{V}\left[\ln w_n^{open}/\mathcal{P}_n^{open}\right]$) depends on the variances and covariances of domestic and foreign productivities. Therefore, depending on the variance-covariance matrix of country productivities and the matrix of bilateral trade costs, the volatility of real income can be either higher or lower in the open economy than in the closed economy. Furthermore, some countries can experience an increase in the volatility of their real income from the opening of trade, while others experience a decrease in the volatility of their real income. Under our assumptions that consumers are risk averse and markets are incomplete, a reduction in the volatility of real income magnifies the welfare gains from trade, whereas an increase in this volatility reduces the magnitude of these welfare gains.

4.6 Alternative Interpretation

The competitive equilibrium in our model has the same allocations as the equilibrium of a game played by the collection of domestic, national planners—one in each country—who choose within-country allocations of labor between import capacity, export capacity and production, taking as given world prices and other countries' *ex ante* investments.

This alternative interpretation helps to make explicit the hedging motive for *ex ante* investments in import and export capacity, because the domestic planners choose these *ex ante* investments akin to solving a mean-variance portfolio problem, taking world prices as given. We now note two key features of the domestic planner's portfolio choice over trade partners and compare with standard portfolio problems in the finance literature. First, due to trade costs, consumers across countries face different consumption bundles, so the distribution of *ex post* wealth influences the returns to import and export capacities. This resembles a portfolio choice problem with heterogeneous agents, where wealth distribution affects asset returns in general equilibrium—rendering the model analytically challenging (see in particular Devereux and Sutherland 2011; Bhandari et al. 2023; and Auclert et al. 2024).

Second, whereas returns on financial assets are typically taken as given, the domestic planners in our setting take prices as given but internalize the diminishing returns to investing in import and export capacity with each partner. This property leads to a key difference between standard finance settings and our setting. In a standard finance setting, if there is no uncertainty, returns are equalized across assets, and the optimal international portfolio becomes indeterminate. In contrast, equilibrium *ex ante* investments in import and export capacity remain determinate in our model even in the absence of uncertainty—they are proportional to the relative trade volumes

($p_{ni}^x x_{ni}$)—because countries' goods are horizontally differentiated from one another. Therefore, agents have an incentive to invest positive import and export capacity for all trade partners with finite variable trade costs, even in the absence of uncertainty.

To characterize the solution of the domestic planners' portfolio problem, we begin by summarizing a result from the finance and trade literatures following Campbell and Viceira (2001) and Allen and Atkin (2022).

Lemma 1. ***(Campbell and Viceira 2001, Allen and Atkin 2022)*** *Let $\boldsymbol{x}$ denote a $N \times 1$ lognormal random vector, $\ln \boldsymbol{x} \sim \mathcal{N}(\boldsymbol{\mu}_x, \boldsymbol{\Sigma}_x)$, and $\boldsymbol{a}$ is an $N \times 1$ deterministic vector. Then $\boldsymbol{a}'\boldsymbol{x}$ is approximately lognormally distributed (up to second-order) with the following moments:*

$$\mathbb{E}\left[\ln \boldsymbol{a}'\boldsymbol{x}\right] \approx \ln \sum_{i=1}^{N} a_i \exp\left(\mu_{x_i}\right) + \frac{1}{2}\sum_{i=1}^{N} m_i \sigma_i^2 - \frac{1}{2}\boldsymbol{m}'\boldsymbol{\Sigma}_{\boldsymbol{x}}\boldsymbol{m},$$

$$\mathbb{V}\left[\ln \boldsymbol{a}'\boldsymbol{x}\right] \approx \boldsymbol{m}'\boldsymbol{\Sigma}_{\boldsymbol{x}}\boldsymbol{m}, \qquad m_i \equiv \frac{a_i \exp\left(\mu_{x_i}\right)}{\sum_{j=1}^{N} a_j \exp\left(\mu_{x_j}\right)}.$$

Proof. See the Online Appendix. □

The first part of Lemma 1 states that a weighted sum of lognormal random variables is approximately lognormally distributed. This result is used widely in applied mathematics, including for example Marlow (1967), Leipnik (1991), Mehta et al. (2007), and Dufresne (2004). This approximation becomes exact as the variance of the random variables becomes small, or as the number of random variables becomes sufficiently large, and these random variables are not too correlated with one another (a form of law of large numbers result). The second part of Lemma 1 computes the mean and variance of this weighted sum using a second-order Taylor-series expansion around the mean of the log normal vector ($\ln \boldsymbol{x} = \boldsymbol{\mu}_x$).

Our theoretical results in Propositions 3-4 above hold independently of this Lemma, simply by virtue of taking a second-order approximation in the system of general equilibrium conditions that determines endogenous wages given exogenous productivities. But Lemma 1 provides an alternative interpretation and implies that the distribution of log wages is approximately lognormally distributed if productivity is log normally distributed. To demonstrate this result, we begin by taking logarithms in the income accounting relationship (4), which yields an expression for the log wage in each country as the log of a weighted sum of wages in all countries:

$$\ln w_i = \ln \sum_n \xi_{ni} w_n, \qquad \xi_{ni} \equiv \frac{\ell_n \left(\tau_{ni} w_i / z_i\right)^{-\theta}}{\sum_{k=1}^{N} \ell_i \left(\tau_{ni} w_k / z_k\right)^{-\theta}}. \tag{24}$$

We next assume that productivity is log normally distributed ($\ln \boldsymbol{z} \sim \mathcal{N}(\boldsymbol{\mu}_{\boldsymbol{z}}, \boldsymbol{\Sigma}_{\boldsymbol{z}})$), where we allow the mean and variance of the productivity distribution to differ across countries. Using this

assumption, we next conjecture that wages are approximately lognormally distributed, and then verify this conjecture by applying Lemma 1 iteratively as a fixed point.

Under our assumption that productivity is lognormally distributed and our conjecture that wages are log normally distributed, the weights (ξ_{ni}) in equation (24) are lognormally distributed, as can be established from the properties of the numerator and denominator of these weights. Under our conjecture, the numerator is the product of lognormal distributions, which is lognormally distributed. Additionally, the denominator is a weighted sum of lognormal distributions, which is approximately lognormally distributed from applying Lemma 1. Using these results for the numerator and denominator, it follows that the weights (ξ_{ni}) are the ratio of two lognormal distributions, which is lognormally distributed.

Using this result for the weights (ξ_{ni}), log wages in equation (24) are equal to the log of the sum of the weights (ξ_{ni}) times wages (w_i). Under our conjecture, both the weights (ξ_{ni}) and wages (w_i) are lognormally distributed. Since the product of two lognormal distributions is lognormally distributed, it follows that the terms in wages multiplied by the weights ($\xi_{ni} w_i$) inside the summation sign in equation (24) are lognormally distributed. Finally, applying Lemma 1 for a further time, the sum of lognormal distributions is approximately lognormally distributed. Therefore, we have verified our conjecture that wages are approximately lognormally distributed.

Although the competitive equilibrium of our model coincides with the solution of the problem of domestic planners in each country who take world prices as given, recall that this competitive equilibrium is not efficient, because markets are incomplete and the relative marginal utility of income across across realizations of random variables is not equated across countries (i.e., $\frac{\mathcal{U}'(\mathcal{C}_n(s))/\mathcal{P}_n(s)}{\mathcal{U}'(\mathcal{C}_m(s))/\mathcal{P}_m(s)}$ is not constant across states of the world). Additionally, if a country is large relative to world markets, its domestic planner could internalize that its policies affect world prices, which gives rise to the conventional terms of trade externality between countries (e.g., Bagwell and Staiger 1999). In this case, the domestic planner would have have an incentive to introduce an import tariff or export tax to improve its terms of trade relative to its trade partners. In our model, there is a further source of externalities between countries, because markets are incomplete and the marginal utility of income differs across countries. Each country's trade policy transfers income between itself and its trade partners, where these transfers differ across states of nature, and the marginal utility of income differs across countries and states of nature.

4.7 Stochastic Trade Costs

To extend Proposition 4 to incorporate stochastic trade costs, let $\mathbf{\Sigma}$ denote the $K \times K$ covariance matrix of K mean-zero random variables (factors) $\{\epsilon_k\}_{k=1}^{K}$. Let $\boldsymbol{B_z}$ denote the $N \times K$ matrix

capturing the factor loading of productivities, so that log-productivity $\ln z_i$ follows:

$$\ln z_i = \mu_{z_i} + \sum_{k=1}^{N} \left[\boldsymbol{B_z}\right]_{i,k} \epsilon_k, \tag{25}$$

where this representation allows for dimension reduction ($K < N$), but is without loss of generality, because we incorporate the possibility of as many factors as random variables. Likewise, let $\boldsymbol{B_\tau}$ denote the $N^2 \times K$ matrix capturing the factor loading of iceberg trade costs. The covariance matrix of productivities and trade costs are, respectively,

$$\boldsymbol{\Sigma_z} = \boldsymbol{B_z \Sigma B_z'}, \qquad \boldsymbol{\Sigma_\tau} = \boldsymbol{B_\tau \Sigma B_\tau'}. \tag{26}$$

We also have that $Cov\left(z_i, \tau_{mj}\right)$ is the (i, mj)-th entry of $\boldsymbol{\Sigma_{z\tau}} \equiv \boldsymbol{B_z \Sigma B_\tau'}$.

Proposition 5. *Define $\boldsymbol{F}$ as the equilibrium correspondence in the deterministic Armington model (equation (4) that maps log productivities ($\ln \boldsymbol{z}$) and log trade costs ($\ln \boldsymbol{\tau}$) into log wages ($\ln \boldsymbol{w}$) given the trade elasticity (θ): $\ln \boldsymbol{w} = \boldsymbol{F}\left(\ln \boldsymbol{z}, \ln \boldsymbol{\tau}; \theta\right)$.*
***(A)** To second order,*

$$\begin{aligned}\mathbb{E}\left[\ln w_k\right] \approx\ & F_k\left(\boldsymbol{\mu}_z, \boldsymbol{\mu}_\tau\right) + \frac{1}{2}\sum_{ni,mj} \frac{\mathrm{d}^2 \ln \boldsymbol{F}_k}{\mathrm{d}\ln \tau_{ni}\, \mathrm{d}\ln \tau_{mj}} \left[\boldsymbol{\Sigma}_\tau\right]_{ni,mj} \\ & + \sum_{i,j} \frac{\mathrm{d}^2 \ln \boldsymbol{F}_k}{\mathrm{d}\ln z_i\, \mathrm{d}\ln z_j} \left[\boldsymbol{\Sigma}_z\right]_{i,j} + \frac{1}{2}\sum_{i,mj} \frac{\mathrm{d}^2 \ln \boldsymbol{F}_k}{\mathrm{d}\ln z_i\, \mathrm{d}\ln \tau_{mj}} \left[\boldsymbol{\Sigma}_{z\tau}\right]_{i,mj}\end{aligned}$$

$$V\left[\ln \boldsymbol{w}\right] \approx \boldsymbol{W \Sigma_z W'} + \boldsymbol{M \Sigma_\tau M'} + \boldsymbol{W \Sigma_{z\tau} M'} + \boldsymbol{M \Sigma_{z\tau}' W}$$

where the Jacobian matrices $\boldsymbol{W} \equiv \left[\frac{\mathrm{d}\ln w_k}{\mathrm{d}\ln z_i}\right]$ and $\boldsymbol{M} \equiv \left[\frac{\mathrm{d}\ln w_k}{\mathrm{d}\ln \tau_{ni}}\right]$ and the Hessians (the second derivatives) are evaluated at $\left(\ln \boldsymbol{z}, \ln \boldsymbol{\tau}\right) = \left(\boldsymbol{\mu}_z, \boldsymbol{\mu}_\tau\right)$.
***(B)** The bilateral expenditure share matrix $\boldsymbol{S}$ is a sufficient statistic for the Jacobian and Hessian:*

$$S_{ni} \equiv \left.\frac{\left(\tau_{ni} w_i / z_i\right)^{-\theta}}{\sum_{k=1}^{N}\left(\tau_{nk} w_k / z_k\right)^{-\theta}}\right|_{\ln \boldsymbol{z} = \boldsymbol{\mu}_z}.$$

Proof. See the Online Appendix. □

Therefore, we can also use our approach to study the effects of changes in the variance or covariance structure of trade costs (e.g., transport disruptions) on patterns of trade, the terms of trade and welfare. Again endogenous adjustments in *ex ante* investments play a key role in shaping these effects of uncertainty over trade costs. A caveat is that trade costs here take the form of real resource costs (e.g., transport costs) and that changes in revenue would need to be taken into account in order to study uncertainty over trade policies such as tariffs.

5 Multiple Sectors

In our baseline model, we assume a single final goods sector for expositional simplicity. In this section, we show that our approach generalizes to accommodate multiple sectors. In this augmented specification, aggregate uncertainty affects comparative advantage across countries and sectors, as well as the terms of trade and welfare.

We again assume that the representative agent in country n has constant relative risk aversion preferences ($\mathcal{U}_n$) over the aggregate consumption index ($\mathcal{C}_n$), as in equation (5). We now assume that this aggregate consumption index ($\mathcal{C}_n$) is a Cobb-Douglas function of the sectoral consumption index for each sector k ($\mathbb{C}_n^k$):

$$\mathcal{C}_n = \prod_{k=1}^{N} \left(\mathbb{C}_n^k\right)^{\mu_n^k}, \quad 0 < \mu_n^k < 1, \quad \sum_{k=1}^{K} \mu_n^k = 1. \tag{27}$$

This sectoral consumption index in importer n ($\mathbb{C}_n^k$) is defined over the consumption of the good produced by each exporter i (c_{ni}^k), according to the CES specification in equation (6). The trading technology within each sector takes the same form as in equation (9). For simplicity, we assume the same elasticity of substitution (ω) and the same expenditure shares for import capacity (α) and export capacity (β) across sectors, but it is straightforward to allow these parameters to differ across sectors. While we focus on a multi-sector Armington model in this section, our results hold throughout the class of multi-sector trade models with a constant trade elasticity, including Costinot et al. (2012).

The representative agent in country n again has labor endowment $\overline{\ell}_n$. Import and export capacity in each sector are chosen *ex ante* before observing the realizations for countries' productivities in each sector $\{z_n^k\}$. Labor allocated to import or export capacity with a given trade partner in a given sector cannot be reallocated to another trade partner or another sector, moved between import and export capacity, or used for production. After observing the realizations for country productivities in each sector $\{z_n^k\}$, the remaining production labor is allocated *ex post* across trade partners and sectors. Labor market clearing requires that the sum across trade partners and sectors of labor allocated to import capacity, export capacity and production is equal to the economy's labor endowment:

$$\overline{\ell}_i = \sum_{k=1}^{K} \sum_{j=1}^{N} \iota_{ij}^k + \sum_{k=1}^{K} \sum_{n=1}^{N} \left(\ell_{ni}^k + h_{ni}^k\right) \tag{28}$$

The *ex ante* equilibrium in this multi-sector extension exhibits similar properties as in our baseline single-sector model. From cost minimization and zero profits, total employment in im-

port and export capacity across all sectors are constant shares of the economy's labor endowment:

$$\sum_{k=1}^{K}\sum_{j=1}^{N}\iota_{ij}^{k}=\alpha\mu_{i}^{k}\overline{\ell}_{i},\qquad\qquad\sum_{k=1}^{K}\sum_{n=1}^{N}h_{ni}^{k}=\beta\mu_{i}^{k}\overline{\ell}_{i}. \tag{29}$$

where these shares are determined by the Cobb-Douglas shares of import capacity (α) and export capacity (β) in sectoral expenditure and the shares of sectors in aggregate expenditure (μ_n^k).

Bilateral employments in import and export capacity within each sector are proportional to the expectations of of export sales within that sector weighted by the marginal utility of income. Therefore, these bilateral employments in import and export capacity depend not only on expected values but also on variances and covariances across countries and sectors:

$$\begin{aligned}\iota_{ni}^{k}\propto_{n,k}\;&\mathbb{E}\left[\left(\left(\iota_{ni}^{k}\right)^{-\frac{\alpha}{\delta}}\left(h_{ni}^{k}\right)^{-\frac{\beta}{\delta}}\frac{w_{i}\tau_{ni}^{k}}{z_{i}^{k}}\right)^{1-\widetilde{\omega}}\right]\\&\exp\left(\mathrm{Cov}\left(\ln\mathcal{C}_{n}^{-\gamma},\ln\left(\frac{w_{i}\tau_{ni}^{k}}{z_{i}^{k}}\right)^{1-\widetilde{\omega}}\right)\right)\\&\times\exp\left(\mathrm{Cov}\left(\ln\frac{w_{n}}{\mathcal{P}_{n}},\ln\left(\frac{w_{i}\tau_{ni}^{k}}{z_{i}^{k}}\right)^{1-\widetilde{\omega}}\right)\right)\\&\times\exp\left(-\mathrm{Cov}\left(\ln\left(\mathcal{Q}_{n}^{k}\right)^{1-\widetilde{\omega}},\ln\left(\frac{w_{i}\tau_{ni}^{k}}{z_{i}^{k}}\right)^{1-\widetilde{\omega}}\right)\right),\end{aligned}\qquad\begin{aligned}h_{ni}^{k}\propto_{i,k}\;&\mathbb{E}\left[\left(\frac{\left(\iota_{ni}^{k}\right)^{-\alpha/\delta}\left(h_{ni}^{k}\right)^{-\beta/\delta}\tau_{ni}^{k}}{\mathcal{Q}_{n}^{k}}\right)^{1-\widetilde{\omega}}w_{n}\overline{\ell}_{n}\mu_{n}^{k}\right]\\&\times\exp\left(Cov\left(\ln\mathcal{C}_{i}^{-\gamma},\ln\left(\frac{\tau_{ni}^{k}}{\mathcal{Q}_{n}^{k}}\right)^{1-\widetilde{\omega}}w_{n}\right)\right)\\&\times\exp\left(-Cov\left(\ln\mathcal{P}_{i},\ln\left(\frac{\tau_{ni}^{k}}{\mathcal{Q}_{n}^{k}}\right)^{1-\widetilde{\omega}}w_{n}\right)\right)\\&\times\exp\left(Cov\left(\ln\left(\frac{w_{i}}{z_{i}^{k}}\right)^{1-\widetilde{\omega}},\ln\left(\frac{\tau_{ni}^{k}}{\mathcal{Q}_{n}^{k}}\right)^{1-\widetilde{\omega}}w_{n}\right)\right)\end{aligned}$$

These *ex ante* investments in import and export capacity in turn determine *ex post* bilateral trade shares. In the absence of uncertainty, these bilateral trade shares depend solely on the level of relative costs across countries and sectors (first moment). In contrast, in the presence of uncertainty, they also depend on variances and covariances across countries and sectors (second moment). Therefore, in this multi-sector environment, uncertainty shapes the allocation of resources across both sectors and trade partners.

6 Data and Parameterization

We first introduce our data sources and parameterize our model. We next use our parameterized model to provide evidence on the impact of aggregate uncertainty on patterns of trade, the terms of trade and welfare.

6.1 Data Sources

We obtain international trade data from the IMF's Direction of Trade Statistics (DOTS). We combine these bilateral trade data with national accounts data on GDP, population, and price indexes from the Penn World Tables (PWT) and the Global Macro Database (GMD). We obtain data on gross output from the World Input-Output Database (WIOD). We exclude small countries (those with populations below 0.05% of the world total), and several conflict-affected countries

(Afghanistan, Venezuela, Iraq, Syria, Yemen, Zimbabwe, and Liberia), which have particularly volatile output. The resulting panel covers 103 countries over the period 1993–2017.

The DOTS data on bilateral trade flows do not include countries' expenditures on domestically-produced output. We compute domestic expenditure by subtracting exports from total gross output. Gross output data are available from the World Input-Output Database (WIOD) for a subset of countries and years. For observations not covered by WIOD, we impute gross output by predicting the ratio of gross output to GDP based on country characteristics in the WIOD sample. We then apply these predicted ratios to GDP data to estimate gross output for the remaining observations.

6.2 Model Parameters

We calibrate model parameters using standard values from the existing literature and moments of our data. In the macro literature with CRRA utility, the parameter γ is typically set to 2, balancing the intertemporal elasticity of substitution (EIS) and risk aversion, which are inversely related. In contrast, the finance literature, using Epstein-Zin preferences that separate the EIS and risk aversion, often calibrates γ well above 2 in order to match asset pricing moments. Since we abstract from intertemporal savings decisions and focus on risk aversion, we adopt a value of $\gamma = 10$ following the finance literature (see, e.g., Collin-Dufresne et al. 2016).

We focus in our baseline specification on *ex ante* investments in import capacity by setting the expenditure share for *ex ante* investments in export capacity equal to zero ($\beta = 0$). We calibrate the expenditure share for import capacity (α) and the elasticity of substitution (ω) using our model's predictions for short and long-run trade elasticities. We assume $\alpha = 0.5$ and $\omega = 9$. Together these parameter values imply a long-run trade elasticity of $\theta^{LR} = \alpha + \beta + \delta\omega - 1 = 4$, which is a standard value in the international trade literature. They also imply a short-run trade elasticity of $\theta^{SR} = \frac{\delta(\omega-1)}{(1-\delta)\omega+1} = 0.8$, in line with the estimates in Boehm et al. (2023).

To obtain empirical estimates of the variance-covariance matrix for country productivities and the matrix of bilateral trade costs, we invert the equilibrium conditions of the deterministic Armington model. We use this deterministic specification for this exercise, because the mapping from the observed endogenous variables to model fundamentals is more transparent, and the only role of this exercise is to obtain empirically-reasonable estimates for these model fundamentals. Using the indirect utility function and expenditure shares, we recover productivity for country i in year t as $z_{it} = s_{iit}^{1/\theta} y_{it}$, where $y_{it} = w_{it}/\mathcal{P}_{it}$ denotes real income per capita, s_{iit} is the domestic expenditure share, and θ is the trade elasticity. We measure real income per capita using the PWT; we impose the normalization $\tau_{iit} = 1$; and we assume a standard parameter value of $\theta = 4$. To abstract from secular trends in country productivities, we detrend the productivity series for each country using a log quadratic trend, and generate the residuals. We compute the variance-

covariance matrix of productivities over time (Σ_z) using these residuals.

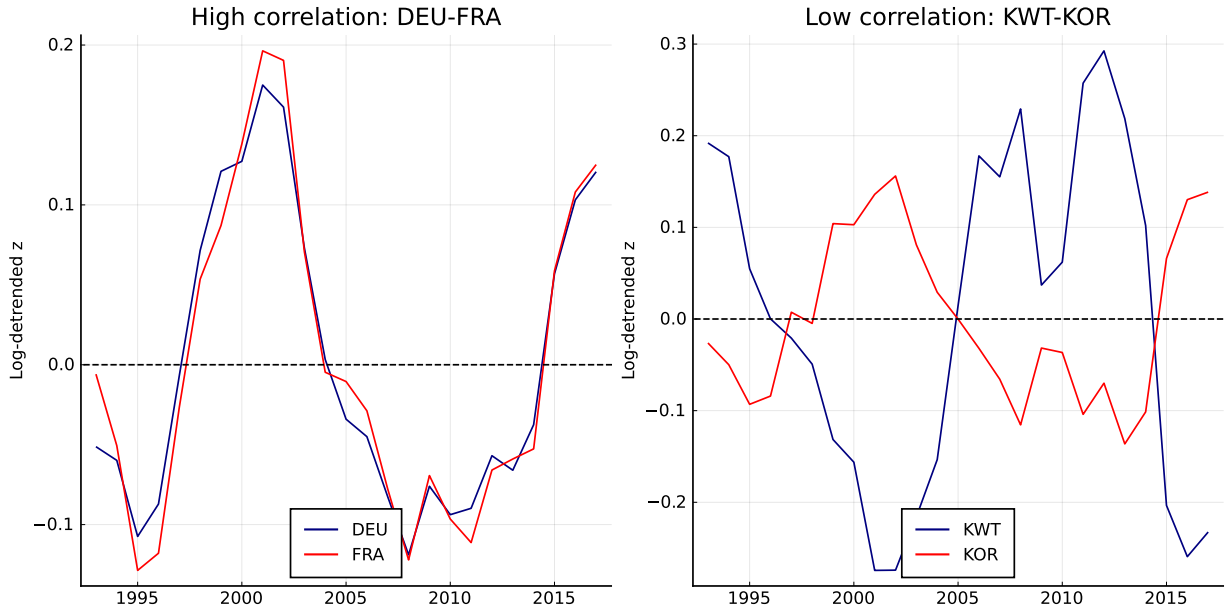
Given these solutions for country productivities and expenditure shares, we recover bilateral trade costs for importer n, exporter i and year t as $\tau_{nit} = s_{nit}^{-\frac{1}{\theta}} z_{it}/y_{it}$. Since we focus for most of our analysis on uncertainty in country productivities, we set bilateral trade costs for importer n and exporter i as equal to their value in our benchmark year of 2017 (τ_{ni}). We also set country labor endowments ($\bar{\ell}_n$) equal to observed populations in that benchmark year.

6.3 Properties of Country Productivities

As a check on our estimates of country productivities, Figure 1 displays de-trended productivity over time for two example pairs of countries. In the left panel, we show values for France and Germany. Consistent with these countries being economically and geographically close, we find a high correlation across years. In the right panel, we show values for Kuwait and South Korea. In line with these countries being economically and geographically remote from one another, we find a low correlation across years.

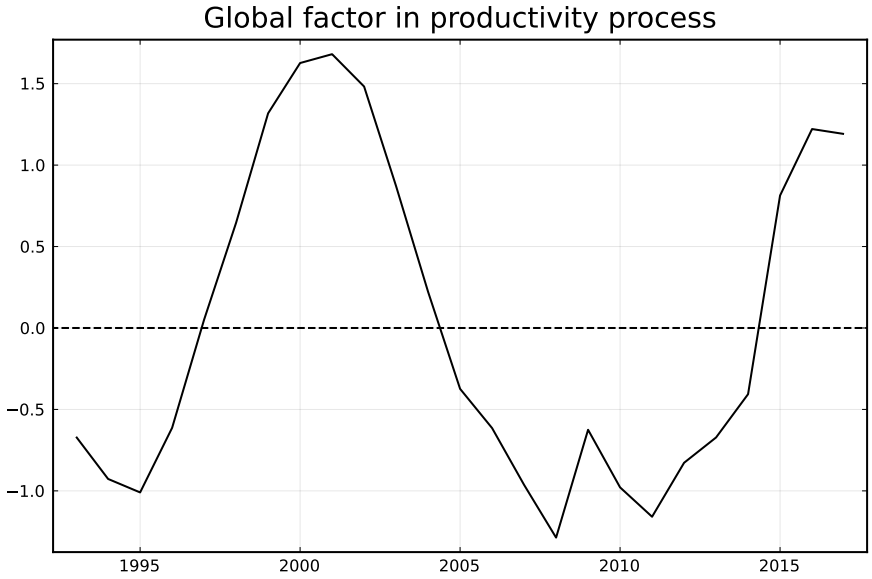
A key characteristic of our variance-covariance matrix for country productivities is the presence of a dominant global factor that explains much of the variance in de-trended productivities across countries, consistent with findings for business-cycle comovements in the international macroeconomics and international finance literatures. Specifically, we implement a principal components decomposition of our variance-covariance matrix, and evaluate the contribution of each component to the overall variance. In Figure 2, we show the time series of the contribution of the global factor (first principal component) to country productivities. We find that this global factor accounts for over 43% of the variance in country productivities. The first five factors collectively account for approximately 80% of the variance in country productivities.

Figure 1: Examples of Country Pairs with High and Low Productivity Correlations



Note: Productivities over time for two example pairs of countries; productivity estimates are based on inverting the equilibrium conditions in the deterministic Armington model; the productivity series for each country is detrended using a log quadratic trend; left-panel shows values for France (FRA) and Germany (DEU) as two countries with a high correlation; right panel shows values for Kuwait and South Korea as two countries with a low correlation.

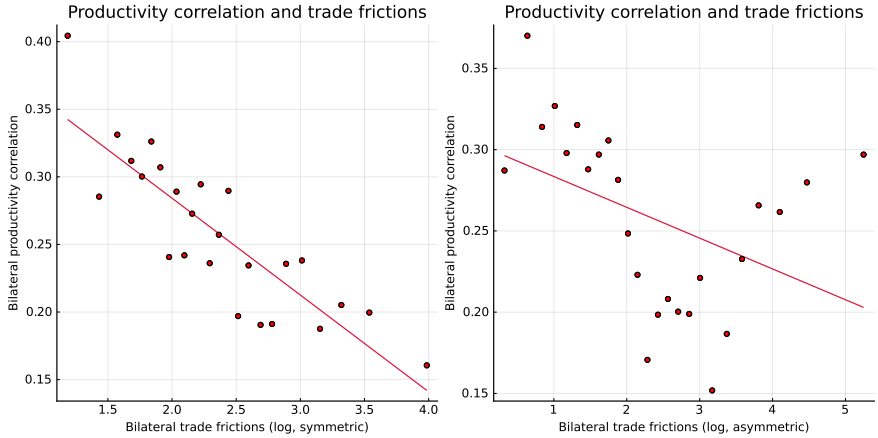
Figure 2: Contribution of the Global Factor to Productivity over Time



Note: Productivity estimates are based on inverting the equilibrium conditions of the deterministic Armington model; the productivity series for each country is detrended using a log quadratic trend; we compute the variance-covariance matrix of these detrended country productivities over time; we implement a principal components decomposition of this variance-covariance matrix; the figure shows the contribution of the global factor (first principal component) to country productivities over time.

Another salient feature of our variance-covariance matrix for country productivities is that we find a higher covariance between pairs of countries with lower bilateral trade costs, consistent with the idea that geographically and economically proximate countries have both lower trade frictions with each other and more integrated economies. Figure 3 displays a binscatter of the relationship between the bilateral productivity correlation of country productivities and our estimates of bilateral trade costs for our benchmark year. Since the productivity correlation is symmetric, the left panel shows this relationship using a measure of symmetric trade costs ($\tau_{ni}^{in} = (\tau_{ni}\tau_{in})^{1/2}$), and the right panel uses asymmetric trade costs (τ_{ni}). In both cases, we find a negative and statistically significant relationship, although this relationship is less strong for asymmetric trade costs, in part because the asymmetry is especially pronounced for bilateral pairs with the highest trade costs. Since these measures of trade costs are based on inverting the equilibrium conditions of the deterministic Armington model, they include the effects of import and export capacity from which that model abstracts. Therefore, we find that bilateral trade costs incorporating export and import capacity are negatively related to the bilateral correlation between countries productivities. This pattern of results is consistent with the predictions of our model, in which countries can hedge by investing greater import and export capacity (thereby lowering trade costs) for partners with lower productivity correlations.

Figure 3: Productivity Correlation and Trade Frictions



Note: Productivity and trade cost estimates based on inverting the equilibrium conditions of the deterministic Armington model; the productivity series for each country is detrended using a log quadratic trend; we compute the variance-covariance matrix of these detrended country productivities over time; trade cost estimates are for our benchmark year of 2017; figure shows a binscatter of the bilateral productivity correlation against bilateral trade costs; blue dots show the ventiles of this relationship; red line shows the linear regression relationship; since the bilateral productivity correlation is symmetric, left panel uses a measure of symmetric trade costs ($\tau_{in}^{ni} = (\tau_{ni}\tau_{ni})^{1/2}$); right panel uses asymmetric trade costs (τ_{ni}).

7 Empirical Results

Given our calibrated model fundamentals, we now undertake counterfactuals to evaluate the effects of aggregate uncertainty and the role of endogenous investments in import and export capacity in shaping these effects. We begin in Section 7.1 by studying the impact of global changes in the covariance structure of productivity. We next proceed in Section 7.2 to examine the implications of a higher productivity variance in individual countries. Finally, Section 7.3 provides Monte Carlo evidence using a simulated productivity distribution that our analytical results from our second-order Taylor-series expansion provide a close approximation to the distribution of the endogenous variables from the numerical solution of the non-linear model.

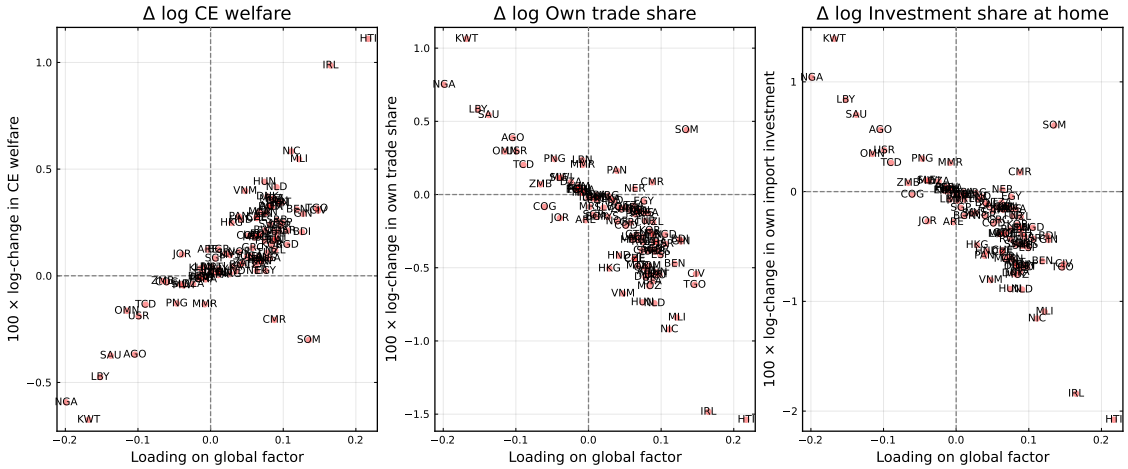
7.1 Global Shocks

Our first counterfactual examines the implications of eliminating the correlation in the global productivity structure, in order to highlight the importance of cross-country correlations for global trade patterns. We set the off-diagonal terms in the productivity variance-covariance matrix equal to zero, holding constant mean productivity and the variance of productivity. In Figure 4, we show the counterfactual changes in certainty-equivalent welfare (left panel); the change in the domestic trade share (middle panel); and the change in domestic investment in trade capacity (right panel). We display these counterfactual changes for each country (using ISO 3-digit country codes) against each country's loading on the global productivity factor in the initial equilibrium.

We find that welfare increases in countries that load positively on the global factor and decreases in countries that load negatively on the global factor. Intuitively, countries that load positively on the global factor have limited opportunities for hedging through international trade in the initial equilibrium, because their productivity is positively correlated with the global factor. Once we eliminate this positive correlation, these countries experience an expansion in their hedging opportunities through international trade, which leads to a reallocation of investments away from the domestic market (right panel), a decrease in the domestic trade share (middle panel), and an increase in welfare (left panel).

In contrast, countries that load negatively on the global factor have substantial opportunities for hedging through international trade in the initial equilibrium, because their productivity is negative correlated with the global factor. Once we eliminate this negative correlation, these countries experience a reduction in their hedging opportunities through international trade, which leads to a reallocation of investments towards the domestic market (right panel), an increase in the domestic trade share (middle panel), and a reduction in welfare (left panel).

Figure 4: Eliminating all Bilateral Productivity Correlations

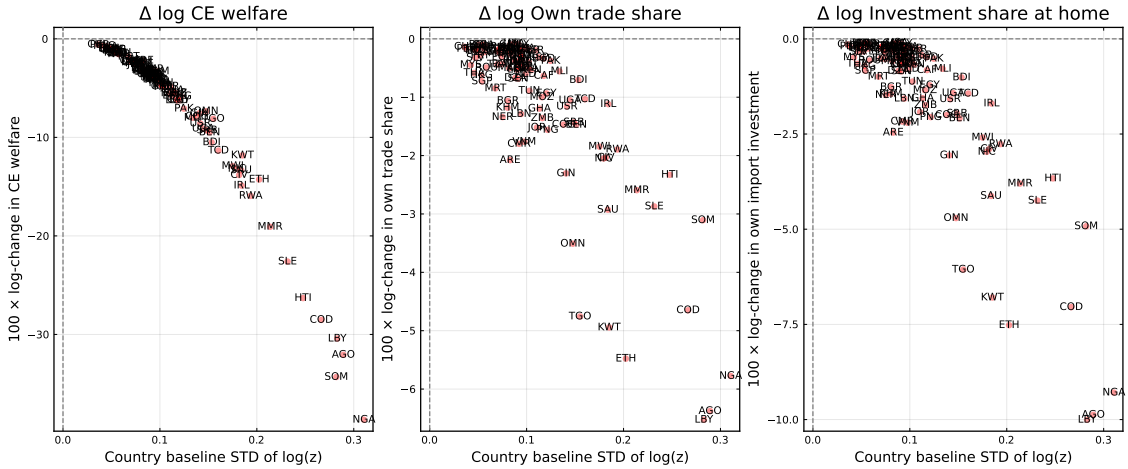


Note: We start from an initial equilibrium with our calibrated variance-covariance matrix of country productivities (Σ_z), bilateral trade costs (τ_{ni}) and labor endowments ($\overline{\ell}_n$). We solve for a counterfactual equilibrium in which we set the off-diagonal terms of the variance-covariance matrix equal to zero, holding constant the mean and variance of productivity for each country. The panels of the figure show changes between the counterfactual and initial equilibria in certainty equivalent welfare (left panel); domestic trade share (middle panel); and domestic investment in trade capacity (right panel). Each panel displays the counterfactual changes in these variables against each country's loading on the global productivity factor (first principal component) in the initial equilibrium.

Our second counterfactual examines the implications of an increase in global uncertainty. We multiply the productivity variance-covariance matrix by 2 and apply a variance correction to keep mean productivity constant. Therefore, this counterfactual isolates the effects of higher global uncertainty, holding constant the mean and correlation structure of productivity. In Figure 5, we show the counterfactual changes in certainty-equivalent welfare (left panel); the change in the domestic trade share (middle panel); and the change in domestic investment in trade capacity (right panel). We display these counterfactual changes for each country (using ISO 3-digit country codes) against each country's standard deviation of productivity in the initial equilibrium.

Since the representative consumer in each country is risk averse, this increase in global uncertainty reduces certainty-equivalent welfare in all countries. The resulting decline in welfare is larger in countries with more volatile output in the initial equilibrium, because a proportional increase in global uncertainty leads to a larger increase in the variance of productivity in these countries (left panel). When sourcing goods in the initial equilibrium, countries trade off sourcing from low-cost suppliers (often from the domestic market where no trade costs are incurred) against diversifying risk by sourcing goods from abroad (where trade costs are incurred but foreign productivity is imperfectly correlated with domestic productivity). As global uncertainty increases, this raises the incentive for risk diversification, which leads to a reduction in domestic investments (right panel) and a reduction in domestic trade shares (middle panel).

Figure 5: Mean-Preserving Increase in the Variance of the Global Productivity Distribution



Note: We start from an initial equilibrium with our calibrated variance-covariance matrix of country productivities (Σ_z), bilateral trade costs (τ_{ni}) and labor endowments ($\overline{\ell}_n$). We solve for a counterfactual equilibrium in which we double the variances and covariances of productivities and apply a variance correction to keep mean productivity for each country constant. The panels of the figure show changes between the counterfactual and initial equilibria in certainty equivalent welfare (left panel); domestic trade share (middle panel); and domestic investment in trade capacity (right panel). Each panel displays the counterfactual changes in these variables against each country's standard deviation of productivity in the initial equilibrium.

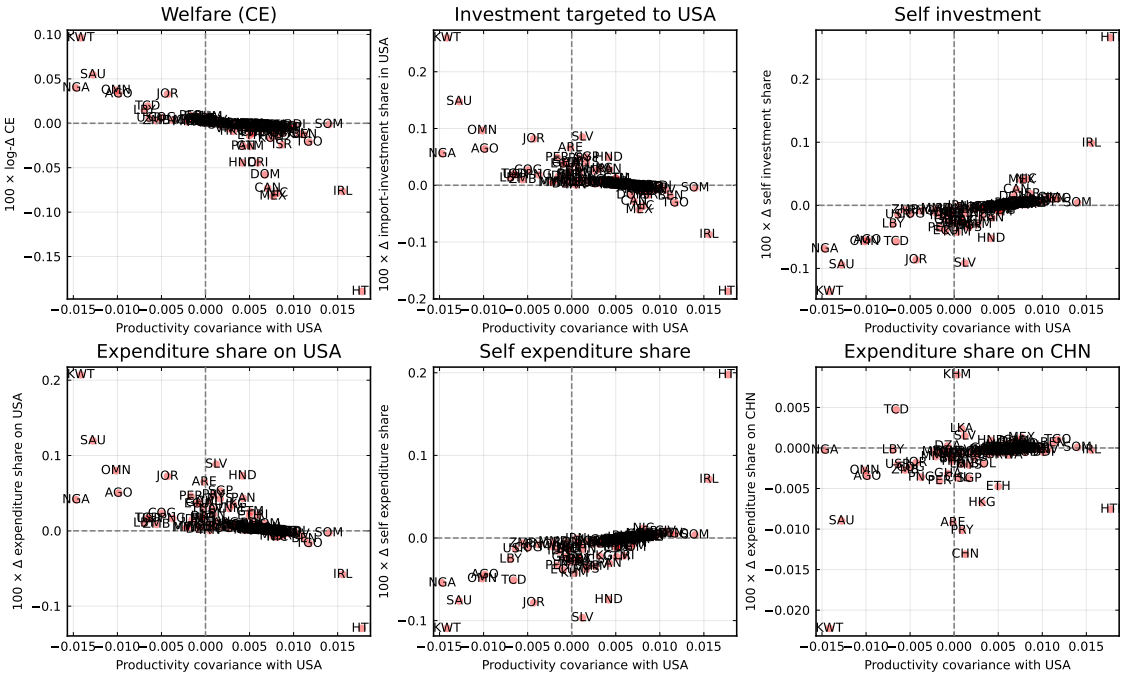
7.2 An Increase in Country-specific Risk

We next examine the implications of changes in uncertainty for individual countries. Our third counterfactual considers an increase in the variance of U.S. productivity, holding constant mean U.S. productivity and the cross-country correlation structure of productivity. Specifically, we double the U.S. variance, and multiply all the covariance terms between each country and the U.S. by the square root of 2 in order to hold the cross-country correlations constant. When productivity takes the form of a factor model as in equation (25) above, this counterfactual can be interpreted as a uniform increase in the U.S. productivity loading across all factors. In Figure 6, we show the results from this counterfactual, where we display the counterfactual change in each variable against each country's productivity covariance with the U.S. in the initial equilibrium.

We find that countries with a positive covariance with the U.S. in the initial equilibrium—such as Ireland, Mexico, and Canada—experience a decline in consumption-equivalent welfare, because this increase in the dispersion of U.S. productivity makes trade with the U.S. less attractive for risk diversification (top-left panel). Consequently, these positive-covariance countries reduce investment towards the U.S. (top-middle panel) and increase domestic investment (top-right panel). This reallocation of investment tends to reduce the expenditure share on the U.S. (bottom-left panel) and increase the domestic expenditure share (bottom-middle panel) for these positive-covariance countries. Finally, while most of the adjustment occurs through reallocations towards the domestic market, we find an increase in the expenditure share on some third

countries such as China (bottom-right panel) for these positive-covariance countries.

Figure 6: Mean-Preserving Increase in the Variance of the U.S. Productivity Distribution



Note: We start from an initial equilibrium with our calibrated variance-covariance matrix of countries productivities (Σ_z), bilateral trade costs (τ_{ni}) and labor endowments ($\bar{\ell}_n$). We solve for a counterfactual equilibrium in which we double the variance of the U.S. productivity distribution, and raise all bilateral covariance terms vis-a-vis the U.S. by the square root of two, to keep bilateral correlations constant. We also apply a correction to the expected value of U.S. log productivity to keep its expected value in levels constant. The panels of the figure show changes between the counterfactual and initial equilibria in certainty equivalent welfare (top-left panel); investment in trade capacity with the U.S. (middle-left panel); investment in domestic trade capacity (top-right panel); expenditure share on the U.S. (bottom-left panel); domestic expenditure share (bottom-middle panel); and expenditure share on China (bottom-right panel). Each panel displays the counterfactual changes in these variables against each country's productivity covariance with the U.S. in the initial equilibrium.

In contrast, countries with a negative covariance with the U.S. in the initial equilibrium—such as Kuwait and Nigeria—experience a rise in consumption-equivalent welfare, because this increase in the dispersion of U.S. productivity makes trade with the U.S. more attractive for risk diversification (top-left panel). Consequently, these negative-covariance countries raise investment towards the U.S. (top-middle panel) and decrease domestic investment (top-right panel). This reallocation of investment tends to increase the expenditure share on the U.S. (bottom-left panel) and decrease the domestic expenditure share (bottom-middle panel) for these negative-covariance countries . Finally, while most of the adjustment again occurs through reallocations with the domestic market, we find a decrease in the expenditure share on some third countries such as China (bottom-right panel) for these negative-covariance countries.

The changes in hedging opportunities induced by the increase in the variance of U.S. productivity, and the resulting reallocations of investment across markets, have general equilibrium ef-

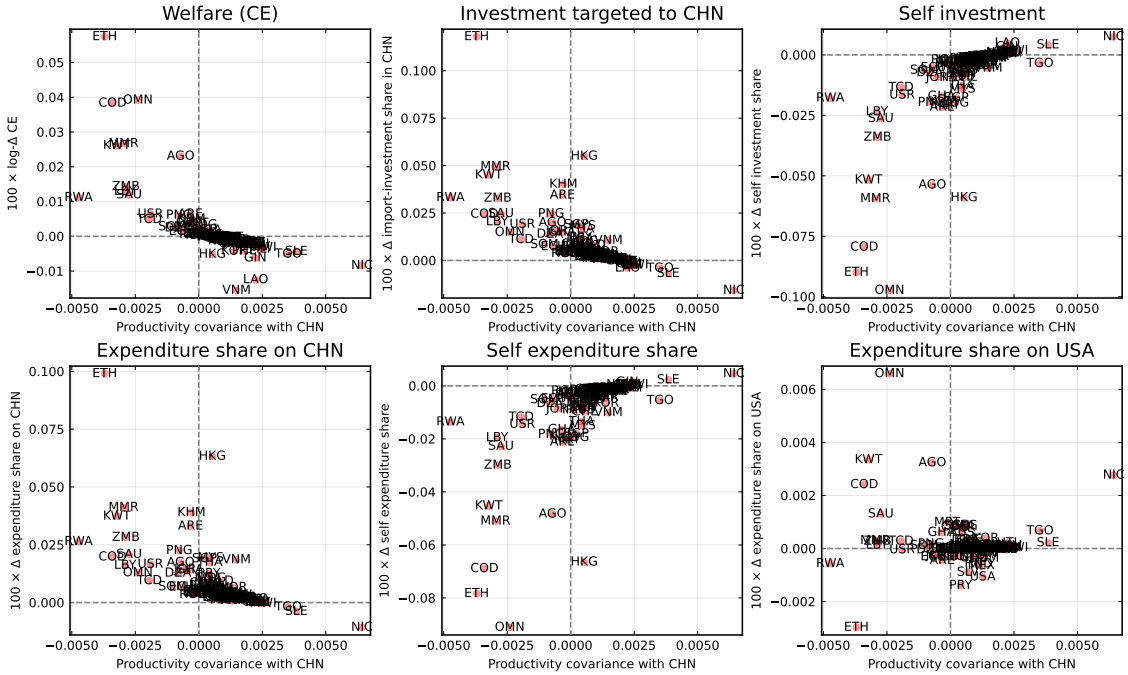
fects for the terms of trade. We find that the lower demand for U.S. goods from positive-covariance countries dominates the higher demand for U.S. goods from negative-covariance countries, because the countries with a positive covariance with the U.S. in the initial equilibrium account for a larger share of world GDP than those with a negative covariance with the U.S.. As result, the relative price of U.S. goods falls in the new equilibrium, which tends to increase the expenditure share on the U.S. for all countries, such that the unweighted average change in the expenditure share on the U.S. is above zero in the bottom-left panel.

Our fourth counterfactual considers a similar mean-preserving spread in China's productivity distribution by doubling its variance, holding constant mean productivity in China and the cross-country correlation structure of productivity. In Figure 7, we show the results from this counterfactual, where we display the counterfactual change in each variable against each country's productivity covariance with China in the initial equilibrium.

We find an analogous pattern of results as for the United States in the previous figure. Countries with a positive covariance with China in the initial equilibrium experience a decline in consumption-equivalent welfare, because this increase in the dispersion of Chinese productivity makes trade with China less attractive for risk diversification (top-left panel). Consequently, these positive-covariance countries reduce investment towards China (top-middle panel) and increase domestic investment (top-right panel). This reallocation of investment tends to reduce the expenditure share on China (bottom-left panel) and increase the domestic expenditure share (bottom-middle panel) for these positive-covariance countries. Finally, while most of the adjustment occurs through reallocations towards the domestic market, we find an increase in the expenditure share on some third countries such as the U.S. (bottom-right panel) for these positive-covariance countries.

In contrast, countries with a negative covariance with China in the initial equilibrium experience a rise in consumption-equivalent welfare, because this increase in the dispersion of Chinese productivity makes trade with China more attractive for risk diversification (top-left panel). Consequently, these negative-covariance countries raise investment towards China (top-middle panel) and decrease domestic investment (top-right panel). This reallocation of investment tends to increase the expenditure share on China (bottom-left panel) and decrease in the domestic expenditure share (bottom-middle panel) for these negative-covariance countries. Finally, while most of the adjustment again occurs through reallocations with the domestic market, we find a decrease in the expenditure share on some third countries such as the U.S. (bottom-right panel) for these negative-covariance countries.

Figure 7: Mean-Preserving Increase in the Variance of the Chinese Productivity Distribution



Note: We start from an initial equilibrium with our calibrated variance-covariance matrix of country productivities (Σ_z), bilateral trade costs (τ_{ni}) and labor endowments ($\bar{\ell}_n$). We solve for a counterfactual equilibrium in which we double the variance of the Chinese productivity distribution, and raise all bilateral covariance terms vis-a-vis the China by the square root of two, to keep bilateral correlations constant. We also apply a correction to the expected value of China's log productivity to keep its expected value in levels constant. The panels of the figure show changes between the counterfactual and initial equilibria in certainty equivalent welfare (top-left panel); investment in trade capacity with China (middle-left panel); investment in domestic trade capacity (top-right panel); expenditure share on China (bottom-left panel); domestic expenditure share (bottom-middle panel); and expenditure share on USA (bottom-right panel). Each panel displays the counterfactual changes in these variables against each country's productivity covariance with China in the initial equilibrium.

The changes in hedging opportunities induced by the increase in the variance of Chinese productivity, and the resulting reallocations of investment across markets, again have general equilibrium effects for the terms of trade. We find that the lower demand for Chinese goods from positive-covariance countries dominates the higher demand for Chinese goods from negative-covariance countries, because the countries with a positive covariance with China in the initial equilibrium account for a larger share of world GDP than those with a negative covariance with China. As result, the relative price of Chinese goods falls in the new equilibrium, which tends to increase the expenditure share on China for all countries, such that the unweighted average change in the expenditure share on China is above zero in the bottom-left panel.

Across each of these counterfactuals, we find that changes in the global structure of uncertainty lead to changes in bilateral trade and welfare, with endogenous investments in resilience playing an important role in shaping these effects on trade and welfare.

7.3 Monte Carlo

We now provide Monte Carlo evidence using a simulated productivity distribution that our analytical results for the properties of the distribution of the endogenous variables under aggregate uncertainty provide a close approximation to the numerical solution of the non-linear model.

We use our inversion of the deterministic Armington model to recover productivity (z_i) and bilateral trade costs (τ_{ni}). We compute mean country log productivity over time ($\boldsymbol{\mu_z}$) and the variance-covariance matrix of country log productivity over time ($\boldsymbol{\Sigma_z}$). We assume that productivity is lognormally distributed: $\ln \widehat{\boldsymbol{z}} \sim \mathcal{N}(\boldsymbol{\mu_z}, \boldsymbol{\Sigma_z})$. Using our estimates for the mean ($\boldsymbol{\mu_z}$) and variance-covariance matrix ($\boldsymbol{\Sigma_z}$) of log productivities, we simulate $nreps = 1,000$ random vectors of productivities for each country from this lognormal distribution.

Given each simulated productivity draw, $\ln \widehat{\boldsymbol{z}}^{(\nu)}$, $\nu \in \{1, \ldots, nreps\}$, we solve for log wages ($\ln \widehat{\boldsymbol{w}}^{(\nu)}$) using the system of general equilibrium conditions (4) in the deterministic Armington model. From these solutions for log wages, all other endogenous variables of the deterministic Armington model can be recovered, including the matrices of expenditure shares ($\widehat{\boldsymbol{S}}^{(\nu)}$) and income shares ($\widehat{\boldsymbol{T}}^{(\nu)}$). We thus obtain a simulated distribution of log wages ($\ln \widehat{\boldsymbol{w}}^{(\nu)}$) from the numerical solution of the non-linear model. We compare this simulated distribution to our analytical results for the distribution of the endogenous variables of the model from our second-order approximation in Proposition 4.

First, we examine the absolute magnitude of the deviations between the first and second moments of the distribution of endogenous wages from the solution of the non-linear model and our second-order approximation. We find negligible deviations between the mean of the simulated distribution ($\mathbb{E}[\ln \hat{w}_i]$) and our second-order approximation ($\ln \widehat{w}_i$):

$$\frac{||\ln \widehat{\boldsymbol{w}}_{\boldsymbol{i}} - \mathbb{E}[\ln \widehat{\boldsymbol{w}}_{\boldsymbol{i}}]||}{||\mathbb{E}[\ln \widehat{\boldsymbol{w}}_{\boldsymbol{i}}]||} = 0.00002\%. \tag{30}$$

Similarly, we find small deviations between the variance-covariance matrix of the simulated distribution ($\boldsymbol{W}\boldsymbol{\Sigma_z}\boldsymbol{W'}$) and our second-order approximation ($\widehat{\boldsymbol{\Sigma}}_{\boldsymbol{w}}$):

$$\frac{\left|\left|\boldsymbol{W}\boldsymbol{\Sigma_z}\boldsymbol{W'} - \hat{\boldsymbol{\Sigma}}_{\boldsymbol{w}}\right|\right|}{\left|\left|\hat{\boldsymbol{\Sigma}}_{\boldsymbol{w}}\right|\right|} = 0.94\%. \tag{31}$$

Second, we test the null hypothesis that the simulated distribution of log wages ($\ln \widehat{\boldsymbol{w}}^{(\nu)}$) in the non-linear model is normally distributed using a Kolmogorov-Smirnov (KS) test. We use the simulated distribution of log wages ($\ln \widehat{\boldsymbol{w}}^{(\nu)}$) for each country across our 1,000 Monte Carlo replications to test the null hypothesis of log normality for each country. To generate a distribution of test statistics across countries, we repeat this exercise 100 times in a seeded test with 1,000 Monte Carlo replications each time. Table 1 reports the distribution of the number of countries

for which we can reject normality at the 5 percent level of significance. In 44 percent of cases, we are unable to reject normality for any country at conventional levels of statistical significance. For 83 percent of cases, we can only reject normality for one or fewer countries at conventional levels of statistical significance. Even in the few cases when we can reject normality statistically, we find that the empirical quantiles of the simulated distribution of log wages ($\ln \widehat{\boldsymbol{w}}^{(\boldsymbol{\nu})}$) in the non-linear model lie close to the theoretical quantiles of a normal distribution.

Table 1: Number of Countries for Which Lognormality is Rejected using a Kolmogorov-Smirnov (KS) Test Across 100 Monte Carlos

Number Countries Rejected for Normality	Number of Cases (out of 100)
0	44
1	39
2	10
3	3
4	2
8	1
9	1

Notes: For each monte carlo, we undertake 1,000 replications ($\nu \in \{1, \ldots, nreps\}$). For each replication ν, we draw random vectors of country productivities from a log normal distribution $\ln \widehat{\boldsymbol{z}}^{(\boldsymbol{\nu})} \sim N(\boldsymbol{\mu_z}, \boldsymbol{\Sigma_z})$, where $\boldsymbol{\mu_z}$ and $\boldsymbol{\Sigma_z}$ are the mean vector and variance-covariance matrix of log country productivities from our inversion of the deterministic Armington model. For each replication ν, we use these random vectors of country productivities ($\ln \widehat{\boldsymbol{z}}^{(\boldsymbol{\nu})}$) and our solutions for bilateral trade costs (τ_{ni}) from inverting the deterministic Armington model to solve for country wages in the non-linear model ($\ln \widehat{\boldsymbol{w}}^{(\boldsymbol{\nu})}$). For each country i, we perform a Kolmogorov-Smirnov (KS) test of the null hypothesis that $\left\{\ln w_i^{(\nu)}\right\}_{\nu=1}^{nreps}$ is normally distributed across the replications. To generate a distribution of test statistics across countries, we repeat this exercise 100 times in a seeded test with 1,000 replications each time. The first column reports the number of countries for which the null hypothesis of normality is rejected. The second column reports the frequency distribution of these rejections of the null hypothesis of normality across the 100 monte carlos.

A key advantage of our second-order Taylor-series expansion is that we can provide a sharp analytical characterization of the properties of quantitative trade models in the presence of aggregate uncertainty. Such an analytical characterization is impossible in the non-linear model, because the endogenous variables are high-dimensional and non-linear functions of the exogenous variables, which implies that the non-linear model only can be solved numerically for simulated productivity distributions. From the Monte Carlo evidence in this section, we find that our second-order Taylor-series expansion provides a close approximation to the distributions of the endogenous variables from these numerical solutions for simulated productivity distributions. This pattern of results is consistent with findings from the macroeconomics literature that perturbation methods provide a close approximation to the solution of high-dimensional and non-linear heterogeneous agent models.

8 Conclusions

Recent years have seen growing concerns about uncertainty and resilience. Despite the prominence of these issues in the public policy debate, most prior research using quantitative trade models has abstracted from aggregate uncertainty. The main reason is that the system of general equilibrium conditions in these models is high-dimensional and non-linear, which makes it challenging to obtain analytical results for the mapping from the distribution of exogenous shocks to the distribution of endogenous variables.

Our first main contribution is to develop a tractable approach to incorporating aggregate uncertainty into the class of trade models with a constant trade elasticity. We provide an analytical characterization of the distribution of the endogenous variables in this class of constant elasticity trade models, allowing for many asymmetric countries, an arbitrary network of bilateral trade costs, and a general distribution of aggregate shocks. Our characterization holds up to a second-order approximation for any distribution of the random variables.

Our second main contribution is to allow agents to make endogenous *ex ante* investments in import and export capacity that determine resilience in the face of this aggregate uncertainty. Import and export capacity capture bilateral investments in business services, logistics, transportation and customer capital. On the one hand, concentrating investments in import capacity towards the partner with the lowest *ex ante* expected costs reduces the expected cost of importing. On the other hand, dispersing investments in import capacity across a wider range of trade partners enhances the ability to import from partners with high *ex ante* expected costs but low *ex post* realizations of costs.

Equilibrium *ex ante* investments affect *ex post* bilateral trade shares. Equilibrium *ex post* trade shares depend not only on the expected costs of sourcing goods, but also on the variance and covariance of these costs across countries. Therefore a country's risk profile becomes a key determinant of its comparative advantage, terms of trade, and welfare. In a deterministic constant elasticity trade model, trade liberalization necessarily raises welfare through a higher level of real income (first moment). In our setting with aggregate uncertainty, trade liberalization now also affects welfare through the volatility of real income (second moment). Depending in the variance-covariance matrix of country productivities and the geography of bilateral trade costs, these second moment welfare effects of trade can be either positive or negative for each country, and can be positive for some countries and negative for others.

We show how to quantify our model using readily-available international trade and production data. We find that a global factor (first principal component) accounts for around 43 percent of the variance of country productivities. We show that changes in the covariance structure of country productivities have heterogeneous effects, depending on the extent to which they raise

or reduce countries' ability to hedge against aggregate uncertainty through *ex ante* investments in import and export capacity.

We find that eliminating the correlation structure in country productivities raises welfare in countries that load positively on the global factor in the initial equilibrium, because eliminating this positive correlation raises the ability of these countries to diversify risk through international trade. In contrast, countries that load negatively on the global factor in the initial equilibrium experience a reduction in their welfare, because eliminating this negative correlation reduces the ability of these countries to diversify risk through international trade. Changes in *ex ante* investments in resilience play an important role in shaping the impact of these changes in the correlation structure of country productivities.

Overall, we provide a tractable methodology that can be used to explore a wide range of counterfactual questions in international trade that involve issues of aggregate uncertainty that have general equilibrium effects on trade patterns, the terms of trade and welfare.

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