International Trade in an Uncertain World*

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Abstract

We develop a tractable approach to introducing aggregate uncertainty into the class of quantitative trade models with a constant trade elasticity. We characterize analytically the distribution of endogenous variables induced in general equilibrium by exogenous shocks. We extend this class of models to allow agents make upfront investments in bilateral import capacity that determine resilience to these shocks. We use a portfolio approach to solve for investment in import capacity and import sourcing in general equilibrium. We show that both trade patterns and the welfare gains from trade depend on first and second moments of the distribution of exogenous shocks. Short and long-run trade elasticities differ from one another. Foreign trade policies affect domestic welfare, not only through conventional terms of trade effects, but also through investments in import capacity.

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1 Introduction

Recent years have seen growing concerns about uncertainty and resilience in the global economy. The aftermath of the COVID-19 pandemic highlighted the potential for large-scale transportation disruptions. The resurgence of national protectionist policies has heightened trade policy uncertainty. Increased geopolitical tensions have raised national security concerns about access to critical goods and minerals. Each of these sources of aggregate shocks are likely to have general equilibrium effects. We develop a tractable approach to introducing aggregate uncertainty into the class of quantitative trade models with a constant trade elasticity. We characterize analytically the distribution of the endogenous variables in general equilibrium as a function of the exogenous shocks. We extend this class of models to allow agents make investments in import capacity that determine resilience to these shocks. We show that both trade patterns and the welfare gains from trade depend on first and second moments of the distribution of exogenous shocks. Short and long-run trade elasticities differ from one another. Foreign trade policies affect domestic welfare, not only through conventional terms of trade effects, but also through equilibrium investments in import capacity.

A key challenge in introducing aggregate uncertainty in quantitative trade models is that the general equilibrium conditions of these models are typically non-linear. For this reason, most previous research assumes that agents have perfect foresight for all aggregate variables, and considers the impact of unanticipated shocks (sometimes called "MIT shocks") that are revealed under perfect foresight. However, in the presence of aggregate uncertainty, agents make decisions based on the expected evolution of general equilibrium variables, where the realized values of these variables differ from their expected values. But it is challenging to obtain any analytical results for the expected evolution of these general equilibrium variables, because this involves taking expectations in a non-linear system of general equilibrium conditions.

Modelling uncertainty over endogenous variables that are determined in general equilibrium matters, because much of the debate about resilience involves aggregate uncertainty. For example, the main source of concern about the dependence of the United States on Taiwan for advanced semi-conductors relates to the aggregate consequences of losing access to this source of imports. As in the case of the shortages of semiconductors following the COVID-19 pandemic, it is feared that losing access to these components could lead to widespread disruptions throughout the economy, including to sectors of national security importance.

Our first main contribution is to overcome this challenge by building on techniques from the finance literature. We provide an analytical characterization of the distribution of the general equilibrium variables in this class of quantitative trade models in terms of the distribution of exogenous productivities. We show that this characterization holds up to a second-order ap-

proximation if country productivities are lognormally distributed. Such a lognormal productivity distribution emerges as a limiting distribution in continuous time if country productivity evolves according to Brownian motion. While we focus on uncertainty over productivity for simplicity of exposition, analogous results hold for uncertainty over trade costs.

Our second main contribution is to use our results for the distribution of endogenous variables to allow agents to make investments in bilateral import capacity in the face of aggregate uncertainty. We assume that positive investments in import capacity are required in order to import from a trade partner, and the greater these investments, the lower the marginal cost of importing from that trade partner. Uncertainty affects these equilibrium investments in import capacity. On the one hand, concentrating investments towards the supplier with the lowest expected costs reduces the expected cost of importing. On the other hand, dispersing investments across a wider range of trade partners enhances the ability to adjust import sourcing patterns *ex post* to take advantage of low realizations for costs for other suppliers.

We use a portfolio approach to solve for equilibrium investments in import capacity and import sourcing as a mean-variance problem. A distinctive feature about our setting relative to many finance applications that are partial equilibrium is that our portfolio problem involves solving for the terms of trade in general equilibrium. In contrast to some international finance settings, the solution to this portfolio problem is determinate even in the limiting case without uncertainty, because of consumer love of variety and diminishing marginal returns to investments in import capacity with any one trade partner. Nevertheless, uncertainty plays an important role in shaping these investments in import capacity, because the representative agent is risk averse. We show that our approach remains tractable even in settings with a high-dimensional state space of many countries and industries and a rich geography of bilateral trade costs.

We consider a setting with a risk-averse representative agent in each importing country who decides how much of each exporting country's good to import, taking international prices as given. Goods are differentiated by country of origin. Consuming the good produced by an exporting country requires *ex ante* investments in import capacity, which capture bilateral investments in logistics, transportation networks and business relationships. These investments must be incurred before stochastic shocks to country productivities are realized. These productivity shocks are aggregate shocks in the sense that they affect general equilibrium prices and wages. Financial markets are incomplete, such that the representative agent in each country cannot insure against these shocks. We show that equilibrium trade shares take a similar form as in a conventional deterministic Armington model. However, equilibrium trade shares now depend not only on average prices, but also on the variance and covariance of prices. As a result, model inversion exercises in deterministic Armington models recover biased measures of productivity and trade costs. High implied bilateral trade barriers may in part reflect low investments in im-

port capacity, because of a high variance shocks or a high covariance between shocks for that supplier and other suppliers.

Since the representative agent is risk averse, trade liberalization affects welfare through both first moments (mean prices) and second moments (variance and covariance of prices). The volatility of prices can be either higher or lower in the open economy than in the closed economy, depending on the variance of foreign productivity shocks relative to domestic productivity shocks, the covariance of productivity shocks across countries, and the geography of bilateral trade costs. Endogenous investments in import capacity play a key role in shaping the impact of first and second moment shocks. In response to a multilateral trade liberalization that reduces mean foreign prices, our framework highlights a distinction between the short-run trade elasticity (conditional on existing investments in import capacity) and the long-run trade elasticity (once investments in import capacity have adjusted). We derive closed-form expressions for these elasticities and show that the short-run elasticity is smaller in absolute value than the long-run elasticity. In response to a shift in the geopolitical environment that increases the variance of foreign prices, we show that the representative agent diversifies her sourcing decisions across foreign trade partners, thereby affecting both the volume and cross-sectional pattern of international trade.

We show that the solution to the problem of the representative agent can be decentralized in a competitive market equilibrium in which households own firms. We provide comparative statics for the response of domestic welfare to foreign shocks and policies. As in standard trade models, foreign policies affect domestic welfare through changes in the international terms of trade. Unlike standard trade models, foreign policies also shift domestic equilibrium investments in import capacity. This new channel operates like a change in patterns of comparative advantage across trade partners in the domestic market. But this change in comparative advantage within the domestic market is endogenously determined through equilibrium investments in import capacity. As a result, countries can have geopolitical incentives to adopt trade policies that induce favorable (to them) investments in import capacity in their trade partners. The ability to successfully exert geopolitical influence depends on country size and exogenous variation in comparative advantage based on differences in productivity and trade costs.

Our paper is related to a number of existing lines of research. First, we connect with research on networks in macroeconomics and international trade, including Acemoglu et al. (2012), Baqaee and Farhi (2019), Boehm et al. (2019), Liu (2019), Bigio and La'o (2020), Carvalho et al. (2021), Bachman et al. (2024), Baqaee and Farhi (2024), and Nikolakoudis (2024). Much existing research on networks focuses on the propagation of first moment shocks through exogenous production networks. Oberfield (2018), Arkolakis et al. (2023) and Taschereau-Dumouchel (2025) consider endogenous network formation. Liu and Tsyvinski (2024) models the dynamics of network propagation as a result of adjustment costs. Taschereau-Dumouchel (2024) examines supply chain

uncertainty in a Cobb-Douglas production network. Liu et al. (2024) develops a model of supplier capital, in which supply chain disruptions have heterogenous effects across firms. Our main contribution relative to this research is to develop a tractable portfolio approach that allows us to generalize the class of quantitative trade models with constant elasticity of substitution (CES) import demand systems to incorporate aggregate uncertainty.

Second, an emerging strand of research within the networks literature has examined network fragility and resilience. Elliott et al. (2022) allow firms to multisource inputs and strategically invest to strengthen relationships, trading off the cost of investment against the benefits of increased robustness. Grossman et al. (2024) analyze optimal resilience in vertical supply chains, in which firms choose investments in protective capabilities, and whether to form supply links, and undertake sequential bargaining over quantities and payments. Elliott and Golub (2022) reviews this emerging literature on network fragility. Relative to this existing research, we consider a quantitative, multi-country model of international trade. We use a portfolio approach to provide a tractable characterization of equilibrium investments in bilateral import capacity in the face of aggregate uncertainty.

Third, our paper is related to the wider literature on trade and uncertainty. On the one hand, Newbery and Stiglitz (1984) shows that trade can reduce welfare in the presence of uncertainty by inducing greater volatility in prices in the open economy than in the closed economy. On the other hand, Caselli et al. (2020) argues that trade can reduce (rather than increase) volatility if country-level shocks are more important than sector-level shocks. Using a model of agricultural land specialization for Indian districts, Allen and Atkin (2022) examine the effect of trade on welfare through first and second moments. In a dynamic discrete choice model of migration, Fan et al. (2023) depart from perfect foresight by introducing a specification for beliefs about the future evolution of fundamentals. In contrast, we introduce aggregate uncertainty into the class of quantitative trade models with a constant trade elasticity, and characterize the impact of this aggregate uncertainty on equilibrium allocations and welfare.

Fourth, our paper contributes to research on quantitative trade models, including Eaton and Kortum (2002), Arkolakis et al. (2012), Costinot and Rodríguez-Clare (2014), and Caliendo and Parro (2015).[1] We develop a tractable framework for introducing aggregate uncertainty into this class of quantitative trade models. We show that equilibrium trade shares in our framework depend not only on mean prices but also on the variance and covariance of prices. Trade affects welfare through both the first and second moments of the distribution of prices, where each of these effects is shaped by endogenous investments in import capacity.

Fifth, our work connects with a growing body of research on geoeconomics or the interna-

[1] A related literature examines the trade reallocation in response to unanticipated changes in trade policy (without aggregate uncertainty), including Alfaro and Chor (2023), Grossman et al. (2023), and Fajgelbaum et al. (2024).

tional political economy of trade policy, including Hirschman (1945), Tinbergen (1962), Broner et al. (2024), Clayton et al. (2023), Thoenig (2023), Becko and O'Connor (2024), Clayton et al. (2024), Kleinman et al. (2024), and Liu and Yang (2024). Whereas most of this research is concerned with first moment shocks, we allow agents to solve a portfolio problem in the presence of uncertainty, which allows us to explore the impact of second moment shocks. Each country makes endogenous investments in import capacity in the face of this uncertainty, which implies that countries can have geopolitical incentives to adopt trade policies that induce favorable investments in import capacity in their trade partners.

The remainder of the paper is structured as follows. Section 2 develops our baseline quantitative model. To illustrate our approach as clearly as possible, we focus in this baseline specification on the case of a single sector with a constant elasticity gravity equation. Section 3 shows that our approach generalizes to a specification with multiple sectors and a constant elasticity gravity equation within each sector. Section 4 introduces our data. Section 5 provides evidence on our model's predictions. Section 6 summarizes our conclusions.

2 Theoretical Framework

We begin by introducing our baseline single-sector model. Subsection 2.1 develops the general version of this model with aggregate uncertainty and investments in import capacity. Subsection 2.2 examines the special case with neither uncertainty nor investments in import capacity, which corresponds to a conventional deterministic Armington model. Subsection 2.3 introduces aggregate uncertainty into this conventional framework to consider a stochastic Armington model. Subsection 2.4 shows that without aggregate uncertainty our specification of import capacity is isomorphic to the deterministic Armington model. Finally, Subsection 2.5 characterizes the general version of our model with both aggregate uncertainty and investments in import capacity. Throughout this section, we use bold math font to denote vectors or matrices.

2.1 Baseline Single-Sector Model

We consider a world of many countries indexed by $n, i \in \{1, \ldots, N\}$. Time is discrete and indexed by $t \in \{1, \ldots, \infty\}$. For expositional simplicity, we focus on an Armington model of trade, in which goods are differentiated by country of origin. But all of our results hold throughout the class of quantitative trade models with a constant trade elasticity, including Eaton and Kortum (2002), Krugman (1980), and Melitz (2003) with a Pareto firm productivity distribution. From now onwards, we omit the time subscript to streamline the notation.

2.1.1 Preferences, Technology and Endowments

The representative agent in country n is endowed with a measure of ℓ_n units of labor. We assume that this representative agent has constant relative risk aversion preferences ($\mathcal{U}_n$) over the aggregate consumption index ($\mathcal{C}_n$):

$$\mathcal{U}_n = \mathbb{E}\left[\frac{\mathcal{C}_n^{1-\gamma}}{1-\gamma}\right], \tag{1}$$

where γ is the coefficient of relative risk aversion and $\mathbb{E}$ denotes the expectations operator.

The aggregate consumption index ($\mathcal{C}_n$) is defined over the consumption of the good produced by each country (c_{ni}) and is assumed to take the constant elasticity of substitution (CES) form:

$$\mathcal{C}_n = \left(\sum_{i=1}^{N} c_{ni}^{\frac{\omega-1}{\omega}}\right)^{\frac{\omega}{\omega-1}}, \qquad \omega > 1, \tag{2}$$

where $\omega > 1$ is the elasticity of substitution between country varieties.

Consumption of a trade partner's good requires imports of that good and the import capacity to absorb those imports. If country n employs ι_{ni} workers to develop import capacity with trade partner i, and sources x_{ni} units of that trade partner's good (net of variable trade costs), c_{ni} units of that trade partner's good can be consumed:

$$c_{ni} = \iota_{ni}^{\alpha} x_{ni}^{1-\alpha}, \qquad 0 \leq \alpha < 1. \tag{3}$$

We interpret import capacity (ι_{ni}) as capturing bilateral investments in logistics, transportation networks and business capital. We assume a Cobb-Douglas functional form for import capacity for tractability. This specification has three additional advantages. First, we capture the extensive margin decision of whether to create import relationships (as in the networks literature on multi-sourcing), because strictly positive investments in import capacity are required to import from a trade partner. Second, we incorporate the intensive margin of developing deeper relationships with trade partners, because larger investments in import capacity reduce the cost of importing a trade partner's good. Third, in the absence of uncertainty, we show that our specification with import capacity is isomorphic to a conventional deterministic Armington model, given the appropriate adjustment to the trade elasticity. Therefore, the novel predictions of our framework are not the result of import capacity alone, but are rather driven by aggregate uncertainty and its interaction with import capacity.[2]

We consider the problem of a representative agent who chooses import capacity and imports to maximize utility, taking international prices as given. We show below that the solution to this

[2]Notwithstanding these advantages, our specification for import capacity can be generalized to the CES functional form, in which case import capacity and imports can be either complements or substitutes.

problem corresponds to the competitive equilibrium of a market economy in which the representative household owns firms. We assume the following timing of decisions. At the beginning of each period, the representative agent in each country chooses *ex ante* how much labor to allocate to import capacity with each of its trade partners and how much labor to allocate to production, before uncertainty over country productivities is realized. After making these decisions, and observing the realizations for country productivities, this representative agent chooses *ex post* how much of its good to sell to each trade partner and how much of each trade partner's good to import. Prices and wages adjust *ex post* to clear markets, given the realizations for country productivities. We assume that financial markets are incomplete, such that the representative agent cannot insure against the uncertainty over country productivities.

Output of country i's good (y_i) is produced using labor (ℓ_i^y) according to a constant returns to scale technology with productivity z_i:

$$y_i = z_i \ell_i^y.$$

Productivity (z_i) is drawn stochastically each period. There are iceberg bilateral costs of trade, such that $\tau_{ni} \geq 1$ units of a good must be shipped from country i in order for one unit to arrive in country n, with $\tau_{ni} > 1$ for $n \neq i$ and $\tau_{nn} = 1$. Import capacity (ι_{ni}) is produced using labor (ℓ_{ni}^ι) according to a constant returns to scale technology with a unit input requirement:[3]

$$\iota_{ni} = \ell_{ni}^\iota.$$

Import capacity plays a key role in determining the resilience of each country to shocks, because the more dispersed are *ex ante* investments in import capacity, the easier it is to reallocate imports across trade partners *ex post*, based on the realizations for country productivities.

Goods market clearing requires that the sum across all markets of demand for each country's good (inclusive of variable trade costs) is equal to production of that country's good by workers who are not used for import capacity:

$$\sum_{n=1}^{N} x_{ni} \tau_{ni} = z_i \left(\ell_i - \sum_{j=1}^{N} \iota_{ji} \right). \tag{4}$$

We focus on shocks to country productivities (z_i) as the source of aggregate uncertainty for expositional simplicity. But it is straightforward to also consider uncertainty over bilateral trade costs (τ_{ni}). In each case, these shocks correspond to aggregate shocks, in the sense that they affect general equilibrium variables, such as prices and wages. Therefore, when making *ex ante* decisions about import capacity, the representative agent must take expectations across the distribution of the general equilibrium variables that is induced by the distribution of exogenous

[3]We model import capacity as produced with the same factor as goods for simplicity. But it would be straightforward to introduce a separate factor of production for import capacity that is allocated across trade partners.

productivities. We show below how to characterize the properties of this distribution of the endogenous variables in general equilibrium.

2.2 Deterministic Armington

We begin by considering the special case of our framework with no uncertainty and no investment in import capacity ($\alpha = 0$). This special case corresponds to the conventional deterministic Armington model. With no investments in import capacity, the aggregate consumption index in country n ($\mathcal{C}_n$) is now defined solely over imports from each trade partner i (x_{ni}):

$$\mathcal{C}_n = \left(\sum_{i=1}^{N} x_{ni}^{\frac{\omega-1}{\omega}} \right)^{\frac{\omega}{\omega-1}}, \qquad \omega > 1. \tag{5}$$

The goods market clearing condition equating the sum of imports (inclusive of variable trade costs) to production now can be written as:

$$\sum_{n=1}^{N} \tau_{ni} x_{ni} = z_i \ell_i. \tag{6}$$

Markets are assumed to be competitive. Profit maximization and zero profits imply that price in each market n of the good produced in country i (p_{ni}) is equal to the marginal cost of supplying that market: $p_{ni} = \tau_{ni} w_i / z_i$, where w_i denotes the wage. Expenditure minimization implies that the price index ($\mathcal{P}_n$) dual to the consumption index ($\mathcal{C}_n$) is:

$$\mathcal{P}_n = \left[\sum_{i=1}^{N} p_{ni}^{1-\omega} \right]^{\frac{1}{1-\omega}}. \tag{7}$$

Using Shephard's Lemma, the share of expenditure in country n on the good produced by country i (S_{ni}) satisfies a constant elasticity gravity equation:

$$S_{ni} = \frac{\left(\tau_{ni} w_i / z_i\right)^{-\theta^{DA}}}{\sum_{j=1}^{N} \left(\tau_{nj} w_j / z_j\right)^{-\theta^{DA}}}, \tag{8}$$

where $\theta^{DA} \equiv (\omega - 1)$ is the trade elasticity and the superscript "DA" indicates the deterministic Armington model.

Income accounting requires that each country's income is equal to expenditure on its goods by all countries. Using the expenditure share (8), this income accounting relationship can be written as follows:

$$w_i \ell_i = \sum_{n=1}^{N} \frac{\left(\tau_{ni} w_i / z_i\right)^{-\theta^{DA}}}{\sum_{j=1}^{N} \left(\tau_{nj} w_j / z_j\right)^{-\theta^{DA}}} w_n \ell_n, \tag{9}$$

where we define country gross domestic product (GDP) as $q_i \equiv w_i \ell_i$.[4]

General equilibrium in this deterministic Armington model (and the broader class of trade models with a constant elasticity gravity equation) reduces to solving for the N wages in each country (w_n) such that the system of N equations (9) holds. Given this solution for wages in each country, we can recover the equilibrium values of all other endogenous variables. Since country wages are gross substitutes in the system of equations (9), there exists a unique wage vector (up to a choice of numeraire) that solves this system of equations, as shown in Alvarez and Lucas (2007) and Allen et al. (2020). We choose the convenient numeraire that world GDP is equal to one: $\sum_{i=1}^{N} q_i = 1$. Defining $\boldsymbol{Q}$ as the matrix with the nominal income row vector $\boldsymbol{q'}$ stacked N times, this choice of numeraire implies $\boldsymbol{Q} d \ln \boldsymbol{q} = 0$.

In this class of constant elasticity trade models, we have an analytical characterization in terms of sufficient statistics for the elasticity of the wage in each country with respect to productivity in each country, which corresponds to the Jacobian of the system of equations (9) with respect to productivity shocks. We also have an analytical characterization in terms of sufficient statistics for the super elasticity of the wage in each country with respect to productivity in each country (the elasticity of the elasticity of wages with respect to productivity), which corresponds to the Hessian of the system of equations (9) with respect to productivity shocks.

Proposition 1. ***(Jacobian and Hessian)*** *The elasticity of the wage in each country i with respect to productivity in each country j ($W_{ij} \equiv \frac{d \ln w_i}{d \ln z_j}$) has a sufficient statistics representation in terms of expenditure shares (S_{ni}), income shares (T_{in}), factor incomes (q_i) and the trade elasticity (θ^{DA}):*

$$d \ln \boldsymbol{w} = \boldsymbol{W} d \ln \boldsymbol{z},$$

$$\boldsymbol{W} \equiv \left(\boldsymbol{I} - \boldsymbol{T} - \theta^{DA} \left(\boldsymbol{T} \boldsymbol{S} - \boldsymbol{I} \right) + \boldsymbol{Q} \right)^{-1} \theta^{DA} \left(\boldsymbol{T} \boldsymbol{S} - \boldsymbol{I} \right),$$

$$T_{in} = \frac{S_{ni} w_n \ell_n}{w_i \ell_i}, \qquad \boldsymbol{Q} d \ln \boldsymbol{q} = 0.$$

The super elasticity of the wage in each country i with respect to productivity in each country j ($H^i_{jk} \equiv d_k W_{ij} \equiv \frac{d^2 \ln w_i}{d \ln z_j d \ln z_k}$) has an analogous sufficient statistics representation as:

$$d_k \boldsymbol{W} = \left(\boldsymbol{I} - \boldsymbol{Q} \right) \left\{ \boldsymbol{Z}^{-1} \left(d_k \boldsymbol{T} \right) \left(\left(\boldsymbol{I} + \theta^{DA} \boldsymbol{S} \right) \boldsymbol{W} - \theta \boldsymbol{S} \right) + \theta^{DA} \boldsymbol{Z}^{-1} \boldsymbol{T} \left(d_k \boldsymbol{S} \right) \left(\boldsymbol{W} - \boldsymbol{I} \right) \right\},$$

$$\boldsymbol{Z} \equiv \boldsymbol{I} - \boldsymbol{T} - \theta^{DA} \left(\boldsymbol{T} \boldsymbol{S} - \boldsymbol{I} \right) + \boldsymbol{Q}.$$

Proof. See Online Appendix C. □

[4] We focus for simplicity on the case of balanced trade. But it is straightforward to introduce exogenous trade imbalances, as in the quantitative international trade literature.

We use these results for the Jacobian and Hession in Proposition 1 to characterize the distribution of the endogenous variables in general equilibrium under aggregate uncertainty in the next section. The elements (W_{ij}) of the Jacobian ($\boldsymbol{W}$) capture the first-order general equilibrium response of the wage in country i with respect to productivity in country j: $\frac{d \ln w_i}{d \ln z_j}$. The expenditure shares (S_{ni}), income shares (T_{in}) and factor incomes (q_i) are all observed in the initial equilibrium in the data. The elements (H^i_{jk}) of the Hessian ($d_k \boldsymbol{W}$) capture how this first-order general equilibrium response changes with productivity: $\frac{d^2 \ln w_i}{d \ln z_j d \ln z_k}$. While the results for the Jacobian in Proposition 1 are in Kleinman et al. (2024), those for the Hessian are new. We use both results to characterize the distribution of the endogenous variables under uncertainty below.

2.3 Stochastic Armington

To provide a benchmark for our later analysis of import capacity, we now introduce aggregate uncertainty over country productivities into the conventional Armington model, but continue in this subsection to abstract from investments in import capacity ($\alpha = 0$). The key feature of this aggregate uncertainty is that the realizations for country productivities affect wages (and all other endogenous variables of the model) in general equilibrium. We now characterize the properties of this distribution of the model's endogenous variables in terms of the properties of the distribution of exogenous productivities.

In this special case with no investments in import capacity, all of the representative agent's decisions are made *after* observing the realizations for country productivities. Therefore, aggregate uncertainty does not directly affect these decisions. Nevertheless, the stochastic shocks to productivities affect the distribution of the model's endogenous variables. In the closed economy, each country is only exposed to its own domestic productivity. In contrast, in the open economy, each country is exposed to both domestic and foreign productivities. Furthermore, the general equilibrium mapping from exogenous productivity to endogenous variables is different in the closed and open economies. Since the representative agent is risk averse, this change in the distribution of the model's endogenous variables between the closed and open economies implies that trade affects welfare through second moments as well as first moments.

In the next subsection, we consider our general model with import capacity ($0 < \alpha < 1$), in which investments in import capacity are made before observing the realizations for country productivities. In this general case, uncertainty directly affects the representative agent's decisions, because investing *ex ante* in import capacity influences the option of importing *ex post* from a trade partner based on realized productivities.

Before considering that general case in the next subsection, we now characterize the distribution of the model's endogenous variables without import capacity ($\alpha = 0$) in three steps. First, we assume that country productivities are lognormally distributed, such that the mean and variance

are sufficient statistics for the productivity distribution.

Assumption 1. ***(Productivities)*** *Country productivities are lognormally distributed:*

$$\ln \boldsymbol{z} \sim \mathcal{N}\left(\boldsymbol{\mu_z}, \boldsymbol{\Sigma_z}\right).$$

We allow the mean (μ_{z_i}) and variance (σ_i^2) of the productivity distribution to differ across countries i. We assume a general variance-covariance matrix with off-diagonal elements σ_{ij},such that the productivities of a pair of countries can be either positively or negatively correlated. We provide empirical evidence below that this assumption of a lognormal distribution provides a good approximation to empirical estimates of country productivities. Such a lognormal distribution emerges as the limiting distribution if countries' productivities evolve according to geometric Brownian motion in continuous time, as shown in Online Appendix B.

Second, we use a result from the finance literature in Campbell and Viceira (2001) to characterize the properties of the log of a weighted sum of lognormally distributed random variables.

Lemma 1. ***(Campbell and Viceira 2001)*** *Let $\boldsymbol{x}$ denote a $N \times 1$ vector of lognormally distributed random variables such that $\ln x \sim \mathcal{N}\left(\boldsymbol{\mu_x}, \boldsymbol{\Sigma_x}\right)$. Let $\boldsymbol{a}$ denote a $N \times 1$ deterministic vector. Then $\boldsymbol{a}'\boldsymbol{x}$ is approximately lognormally distributed (up to second-order) with the following mean and variance:*

$$\mathbb{E}\left[\ln \boldsymbol{a}'\boldsymbol{x}\right] \approx \ln \sum_i a_i \exp\left(\mu_{x_i}\right) + \frac{1}{2}\sum_i m_{x_i}\sigma_i^2 - \frac{1}{2}\boldsymbol{m_x'}\boldsymbol{\Sigma_x}\boldsymbol{m_x},$$

$$\mathbb{V}\left[\ln \boldsymbol{a}'\boldsymbol{x}\right] \approx \boldsymbol{m_x'}\boldsymbol{\Sigma_x}\boldsymbol{m_x}.$$

where $m_{x_i} \equiv \frac{a_i \exp\left(\mu_{x_i}\right)}{\sum_j a_j \mathbb{E}\left[\mu_{x_j}\right]}$; $\mathbb{E}\left[\cdot\right]$ denotes the expectations operator; and $\mathbb{V}\left[\cdot\right]$ denotes the variance operator.

Proof. See Online Appendix D. □

The first part of Lemma 1 states that a weighted sum of lognormal random variables is approximately lognormally distributed. This result is used widely in applied mathematics, including for example Marlow (1967), Leipnik (1991), Mehta et al. (2007), and Dufresne (2004). This approximation becomes exact as the variance of the random variables becomes small, or as the number of random variables becomes sufficiently large, and these random variables are not too correlated with one another (a form of law of large numbers result).

The second part of Lemma 1 computes the mean and variance of this weighted sum using a second-order Taylor-series expansion. As in the widely-used Delta method in econometrics, the mean and variance of a non-linear function of a random variable can be computed using a second-order approximation. We undertake this second-order Taylor-series expansion around the mean of the lognormal random variable ($\boldsymbol{\mu_x}$).

Third, we rewrite the equality between a country's income and expenditure on its goods (9) to express the log wage in each country as the log of a weighted sum of the wage in all countries.

Lemma 2. ***(Income Accounting)*** *The equality between a country's income and expenditure on its goods in the class of quantitative trade models with a constant trade elasticity implies that the log wage in each country equals the log of a weighted sum of wages in all countries:*

$$\ln w_i = \ln \sum_n \xi_{ni} w_n, \qquad \xi_{ni} \equiv \frac{\ell_n \left(\tau_{ni} w_i / z_i\right)^{-\theta}}{\sum_{k=1}^{N} \ell_i \left(\tau_{ni} w_k / z_k\right)^{-\theta}}, \tag{10}$$

where θ is the trade elasticity.

Proof. This lemma follows immediately from taking logarithms in the equality between a country's income and expenditure on its goods (9) and using the definition of ξ_{ni} above. □

In re-writing this income accounting relationship in Lemma 2, we absorb country endowments (ℓ_i) into the definition of the weights (ξ_{ni}), such that this equality can be expressed solely in terms of wages and the weights.

Combining the above three steps, we obtain our main theoretical result for the stochastic Armington model that the distribution of wages is approximately lognormal (up to second-order).

Proposition 2. ***(Stochastic Armington)*** *Under Assumption 1, the distribution of wages is approximately lognormally distributed (up to second order) with the following mean and variance:*

$$\mathbb{E}\left[\ln w_i\right] \approx F_i\left(\boldsymbol{\mu_z}\right) + \frac{1}{2}\sum_j \sum_k H_{jk}^i \left[\boldsymbol{\Sigma_z}\right]_{jk},$$

$$\mathbb{V}\left[\ln \boldsymbol{w}\right] \approx \boldsymbol{W}\boldsymbol{\Sigma_z}\boldsymbol{W'},$$

where $F_i\left(\boldsymbol{\mu_z}\right)$ is the log-wage of country i when log productivities are equal to their expected values, $\boldsymbol{W}$ is the Jacobian with elements $W_{ij} \equiv \frac{d \ln w_i}{d \ln z_j}$, and $\boldsymbol{H}$ is the Hessian with elements $H_{jk}^i \equiv \frac{d^2 \ln w_i}{d \ln z_j d \ln z_k}$) from Proposition 1.

Proof. See Online Appendix E. □

We focus on the distribution of wages in Proposition 2, because general equilibrium in the Armington model reduces to the system of equations (9) in wages. To establish the first part of this proposition, we first conjecture that wages are approximately lognormally distributed, before verifying that this conjecture is indeed satisfied. From Assumption 1, productivity is lognormally distributed. Using this assumption and our conjecture, it follows that the weights (ξ_{ni}) in Lemma 2 are lognormally distributed, as can be established from the properties of their numerator and

denominator. Under our conjecture, the numerator is the product of lognormal distributions, which is lognormally distributed. Additionally, the denominator is a weighted sum of lognormal distributions, which is approximately lognormally distributed (applying Lemma 1 from Campbell and Viceira 2001). Using these results for the numerator and denominator, the weights (ξ_{ni}) are the ratio of two lognormal distributions, which is lognormally distributed.

From Lemma 2, wages are equal to the sum of the weights (ξ_{ni}) times wages (w_i). Under our conjecture, both wages and expenditure shares are lognormally distributed. Since the product of two lognormal distributions is lognormally distributed, it follows that the terms in wages times expenditure shares inside the summation sign are lognormally distributed. Finally, the sum of lognormal distributions is approximately lognormally distributed (applying Lemma 1 from Campbell and Viceira 2001 again). Therefore, we have verified our conjecture that wages are approximately lognormally distributed.

Applying a similar argument, the price index ($\mathcal{P}_n$) in equation (7) is a weighted sum of wages and productivities, and hence is approximately lognormally distributed. Finally, the aggregate consumption index ($\mathcal{C}_n$) is equal to the ratio of wages (w_n) and price indexes ($\mathcal{P}_n$), which are both approximately lognormally distributed. Since the ratio of two lognormal distributions is lognormally distributed, it follows that aggregate consumption index ($\mathcal{C}_n$) is also approximately lognormally distributed.

To compute the mean and variance of the lognormal distribution of wages in the second part of Proposition 2, we take a second-order Taylor-series expansion of the equality between income and expenditure in equation (10) around mean productivity (μ_z), following an analogous approach as in Lemma 1 above. In implementing this Taylor-series expansion for the first and second moments of wages, we use our analytical characterization of the Jacobian and Hessian of the equality between income and expenditure from Proposition 1.

In this characterization of the distribution of wages in general equilibrium in the stochastic Armington model, we assume that productivities are lognormally distributed. We provide empirical evidence below that this assumption provides a good approximation to the data. Nevertheless, in macroeconomics, it is common to write down general utility functions and random variables, derive first-order conditions, and Taylor expand those first-order conditions to second order, such that uncertainty (second moment) matters. The mathematics of our results would remain identical under that alternative interpretation. The main advantage of assuming a lognormal productivity distribution is that it ensures that the first two moments are sufficient statistics for the distribution and it provides a good approximation to the data.

In this stochastic Armington model, as in the conventional deterministic case, the opening of the closed economy to trade leads to welfare gains from higher mean real income (first moment). Unlike the conventional deterministic case, the opening of the closed economy to trade now af-

fects the volatility of real income (second moment). Depending on the variance and covariance of productivity across countries, and the geography of bilateral trade costs, the volatility of real income can be either higher or lower in the open economy than in the closed economy. Furthermore, some countries can experience an increase in the volatility of their real income from the opening of trade, while other countries can experience a decrease in the volatility of their real income. Under our assumption that consumers are risk averse, a reduction in the volatility of real income magnifies the welfare gains from trade, whereas an increase in the volatility of real income provides a source of welfare losses from trade.

2.4 Import Capacity Without Uncertainty

To inform our later analysis of import capacity and uncertainty, we next consider another special case of our model, with import capacity but without uncertainty. In the absence of uncertainty, we show that our specification with import capacity is isomorphic to a conventional deterministic Armington model up to an adjustment to the trade elasticity.

Consumption in country n of the good produced by country i now depends on both imports (x_{ni}) and the capacity to process those imports (ι_{ni}), as in equation (3). Consumption is supplied by a competitive importing sector. Competitive importers in country n choose imports (x_{ni}) and import capacity (ι_{ni}) for each country i to maximize their profits, taking prices (p^c_{ni}, p^x_{ni}) and wages (w_n) as given:

$$\max_{\{\iota_{ni},x_{ni}\}} \left\{ p^c_{ni} \iota^{\alpha}_{ni} x^{1-\alpha}_{ni} - w_n \iota_{ni} - p^x_{ni} x_{ni} \right\}, \tag{11}$$

where p^c_{ni} is the price of a unit of consumption after import processing; p^x_{ni} is the price of imports before import processing but inclusive of variable trade costs; x_{ni} correspond to the quantity of imports before import processing (net of variable trade costs); and we assume that variable trade costs are paid by producers.

Competitive producers in each country i choose employment for each market n (ℓ^y_{ni}) to maximize profits, taking prices (p^x_{ni}) and wages (w_i) as given:

$$\max_{\ell^y_{ni}} \left\{ p^x_{ni} \frac{z_i \ell^y_{ni}}{\tau_{ni}} - w_i \ell^y_{ni} \right\}, \tag{12}$$

where a firm that produces $y_{ni} = z_i \ell^y_{ni}$ units of output receives payment from importers for $x_{ni} = y_{ni}/\tau_{ni}$ units of imports at a price of p^x_{ni} per unit imported.

From the first-order conditions to producers' problem (12), the equilibrium price for a unit of imports (p^x_{ni}) is a constant markup over marginal cost inclusive of variable trade costs:

$$p^x_{ni} = \frac{\tau_{ni} w_i}{z_i}. \tag{13}$$

From the first-order conditions to importers' problem (11), our assumption of Cobb-Douglas importing technology implies that the total measure of workers employed in import capacity across all trade partners is a constant share of the economy's labor endowment (ℓ_n):

$$\sum_{n=1}^{N} \iota_{ni} = \alpha \ell_n. \tag{14}$$

From the first-order conditions to importers' problem (11) for two different trade partners, our assumption of a Cobb-Douglas importing technology and a CES import demand system implies that relative import capacity (ι_{ni}/ι_{nj}) equals the relative value of imports ($\left(p^x_{ni} x_{ni}\right) / \left(p^x_{nj} x_{nj}\right)$), which equals the relative value of consumption ($\left(p^c_{ni} c_{ni}\right) / \left(p^c_{nj} c_{nj}\right)$), which in turn depends on the relative marginal utilities from consuming the two trade partners' goods:

$$\frac{\iota_{ni}}{\iota_{nj}} = \frac{p^x_{ni} x_{ni}}{p^x_{nj} x_{nj}} = \frac{p^c_{ni} c_{ni}}{p^c_{nj} c_{nj}} = \frac{\left(\iota^{\alpha}_{ni} x^{1-\alpha}_{ni}\right)^{\frac{\omega-1}{\omega}}}{\left(\iota^{\alpha}_{nj} x^{1-\alpha}_{nj}\right)^{\frac{\omega-1}{\omega}}}. \tag{15}$$

From these equalities in equation (15), equilibrium relative import capacity, relative import values and relative consumption are all a constant elasticity function of the relative prices of the two trade partners' goods:

$$\frac{\iota_{ni}}{\iota_{nj}} = \frac{p^x_{ni} x_{ni}}{p^x_{nj} x_{nj}} = \frac{p^c_{ni} c_{ni}}{p^c_{nj} c_{nj}} = \left(\frac{p^x_{ni}}{p^x_{nj}}\right)^{-\theta^{DI}}, \tag{16}$$

where the trade elasticity is now $\theta^{DI} \equiv \alpha + (1-\alpha)\,\omega - 1 < \omega - 1$ and the superscript "DI" denotes deterministic import capacity.

The trade elasticity in equation (16) is smaller in absolute value than in the deterministic Armington model ($\theta^{DA} > \theta^{DI}$, since $\omega > 1$). The reason is that the cost of consumption now includes the cost of both imports and import capacity. Therefore, reductions in variable trade costs only directly reduce part of the overall cost of consumption, and hence have less impact on overall consumption than in the deterministic Armington model without import capacity.

In conventional models of international finance, portfolio allocations of capital across countries typically become indeterminate in the limiting case of no uncertainty (e.g., Devereux and Sutherland 2011). In contrast, the portfolio allocation for import capacity in equation (16) remains determinate even with no uncertainty. The reason is consumer love of variety, which provides an incentive to invest in import capacity with a range of trade partners.

We next use this result for relative import capacity (16) together with the CES expenditure share (8), the goods market clearing condition (4), the pricing rule (13), and our solution for aggregate investments in import capacity (14). Using these relationships in the equality between a country's income and expenditure on its goods (9), we obtain the following result for this special case of our import capacity model with no uncertainty.

Proposition 3. ***(Import Capacity Without Uncertainty)*** *In the absence of uncertainty our model of import capacity is isomorphic to the deterministic Armington model, in the sense that the system of general equilibrium conditions takes exactly the same form, with the appropriate adjustment to the trade elasticity:*

$$w_i \ell_i = \sum_{n=1}^{N} \frac{\left(\tau_{ni} w_i / z_i\right)^{-\theta^{DI}}}{\sum_{j=1}^{N} \left(\tau_{nj} w_j / z_j\right)^{-\theta^{DI}}} w_n \ell_n. \tag{17}$$

Proof. See Online Appendix F. □

Proposition 3 implies that in the absence of uncertainty our model of import capacity is isomorphic to the deterministic Armington model. Therefore, the novel predictions of our model under uncertainty below are driven by this uncertainty and its interaction with import capacity, rather than by effects of import capacity alone.

2.5 Import Capacity With Uncertainty

We now examine the implications of introducing uncertainty into our model of import capacity. Uncertainty provides an additional force beyond love of variety for the dispersion of import capacity across a range of trade partners, in order to be able to reallocate imports across trade partners based on the realizations for country productivities. We first solve the representative agent's *ex post* problem for import sourcing after observing country productivities. We next turn to the representative agent's *ex ante* problem of how much labor to allocate to import capacity with each trade partner, given the probability distribution for country productivities.

2.5.1 *Ex Post* Problem

After incurring the sunk investments in import capacity and observing country productivities, the representative agent in country n chooses imports from each trade partner i (x_{ni}), subject to the budget constraint that expenditure on imports equals income from exports, and taking import capacities (ι_{ni}) and international prices (p^x_{ni}) as given:

$$\max_{\{x_{ni}\}} \left\{ \left(\sum_{i=1}^{N} \left(\iota_{ni}^{\alpha} x_{ni}^{1-\alpha} \right)^{\frac{\omega-1}{\omega}} \right)^{\frac{\omega}{\omega-1}} \right\}, \tag{18}$$

$$\text{s.t.} \qquad \sum_{i=1}^{N} p^x_{ni} x_{ni} = w_n \left(\ell_n - \sum_{j=1}^{N} \iota_{ji} \right) \equiv \mathcal{W}_n, \tag{19}$$

where we use $\mathcal{W}_n$ to denote the value of country n's income at international prices; no-arbitrage and zero profits imply $p^x_{ni} = \tau_{ni} w_i / z_i$ and $w_i = p^x_{ii} z_i$, where we have used $\tau_{ii} = 1$.

From the first-order conditions to the representative agent's problem (18), *ex post* expenditure shares take a similar gravity equation form as in the deterministic Armington model:

$$s_{ni} = \frac{\iota_{ni}^{-\frac{\alpha}{1-\alpha}(1-\widetilde{\omega})} \left(\tau_{ni} w_i / z_i\right)^{1-\widetilde{\omega}}}{\sum_{j=1}^{N} \iota_{nj}^{-\frac{\alpha}{1-\alpha}(1-\widetilde{\omega})} \left(\tau_{nj} w_j / z_j\right)^{1-\widetilde{\omega}}} \mathcal{W}_n, \tag{20}$$

$$\widetilde{\omega} \equiv \frac{\omega}{\omega - (1-\alpha)(\omega - 1)} = \frac{1}{\alpha\omega + (1-\alpha)} \omega < \omega$$

There are two main differences from the expenditure shares in the deterministic Armington model in equation (8). First, the *ex post* trade elasticity in our model of import capacity is smaller in absolute value than that in the deterministic Armington model: $\widetilde{\omega} - 1 < \omega - 1$. The reason is that import capacity is fixed *ex post* and cannot adjust to the variation in relative prices across import partners induced by the realizations for country productivities.

Second, the *ex ante* investments in import capacity enter expenditure shares (20) like a demand shifter ($\iota_{ni}^{-\frac{\alpha}{1-\alpha}(1-\widetilde{\omega})}$) that shifts expenditure shares conditional on prices. The larger the *ex ante* investment in import capacity for a given trade partner, the higher the expenditure share for that trade partner for a given value of *ex post* prices induced by the realizations for country productivities. This property complicates approaches to measuring productivity using observed expenditure shares and inverting the model's equilibrium relationships, because this model inversion needs to take into account these endogenous *ex ante* investments in import capacity.

Combining the first-order condition for utility maximization (20) with the definition of country income at international prices ($\mathcal{W}_n$) from the budget constraint (19), the equilibrium price index satisfies:

$$\mathcal{P}_n = \frac{\mathcal{W}_n}{\mathcal{C}_n} = \mathcal{W}_n^{\alpha} \left[\left(\sum_i \iota_{ni}^{\frac{\omega-\widetilde{\omega}}{\omega}} \left(\tau_{ni} w_i / z_i\right)^{1-\widetilde{\omega}} \right)^{\frac{\omega}{\omega-\widetilde{\omega}}} \right]^{-\alpha}. \tag{21}$$

Again this expression takes a similar form as in the deterministic Armington model in equation (7), but with a different demand elasticity across country varieties, and a demand shifter that depends on endogenous *ex ante* investments in import capacity.

Given realized productivities, *ex post* equilibrium wages are determined by the requirement that a country's income from production equals expenditure by all countries on its goods:

$$w_i \left(\ell_i - \sum_{j=1}^{N} \iota_{ji} \right) = \sum_{n=1}^{N} \frac{\iota_{ni}^{-\frac{\alpha}{1-\alpha}(1-\widetilde{\omega})} \left(\tau_{ni} w_i / z_i\right)^{1-\widetilde{\omega}}}{\sum_{j=1}^{N} \iota_{nj}^{-\frac{\alpha}{1-\alpha}(1-\widetilde{\omega})} \left(\tau_{nj} w_j / z_j\right)^{1-\widetilde{\omega}}} w_n \left(\ell_n - \sum_{j=1}^{N} \iota_{ji} \right). \tag{22}$$

which provides a system of N equations that determines unique values for the N *ex post* equilibrium wages (w_i) given the realizations for country productivities.

Comparing equation (22) with the income accounting relationship in the deterministic Armington model (9), the *ex post* trade elasticity is smaller in absolute value ($\widetilde{\omega} - 1 < \omega - 1$); import

capacity enters like an *ex post* demand shifter; and income and expenditure in production are now equal to total labor payments minus payments to labor used for import capacity.

2.5.2 *Ex Ante* Problem

Having characterized the *ex post* equilibrium conditional on investment in import capacity and the realizations for country productivities, we now solve the representative agent's *ex ante* investments in import capacity, given the distribution for country productivities.

The representative agent in country n chooses import capacity with trade partner i (ι_{ni}) to maximize the following unconstrained utility maximization problem, taking into account that labor used for import capacity cannot be used for production, and forming rational expectations about the *ex post* distribution of international prices (p_{ni}):

$$\max_{\{\iota_{ni}\}} \mathbb{E}\left[\frac{1}{1-\gamma}\left(p_{nn}z_n\left(\ell_n - \sum_i \iota_{ni}\right)\right)^{(1-\alpha)(1-\gamma)}\left[\left(\sum_i \iota_{ni}^{\frac{\omega-\widetilde{\omega}}{\omega}} p_{ni}^{1-\widetilde{\omega}}\right)^{\frac{\omega}{\omega-\widetilde{\omega}}}\right]^{\alpha(1-\gamma)}\right], \tag{23}$$

where recall $\widetilde{\omega} \equiv \omega/\left(\omega - (1-\alpha)(\omega-1)\right) < \omega$.

From the first-order conditions to the representative agent's *ex ante* problem (23), the total measure of workers employed in import capacity across all trade partners is a constant share of total employment (ℓ_n), as in equation (14) without uncertainty in Section 2.4 above:

$$\sum_{n=1}^{N} \iota_{ni} = \alpha \ell_i. \tag{24}$$

Taking the ratio of the first-order conditions to the representative agent's *ex ante* problem, we find that relative investments in import capacity depend on the expected value of relative import values evaluated at the marginal utility of consumption:

$$\frac{\iota_{ni}}{\iota_{nj}} = \frac{\mathbb{E}\left[\mathcal{C}_n^{1-\gamma}\frac{\iota_{ni}^{\frac{\omega-\widetilde{\omega}}{\omega}} p_{ni}^{1-\widetilde{\omega}}}{\sum_l \iota_{nl}^{\frac{\omega-\widetilde{\omega}}{\omega}} p_{nl}^{1-\widetilde{\omega}}}\right]}{\mathbb{E}\left[\mathcal{C}_n^{1-\gamma}\frac{\iota_{nj}^{\frac{\omega-\widetilde{\omega}}{\omega}} p_{nj}^{1-\widetilde{\omega}}}{\sum_l \iota_{nl}^{\frac{\omega-\widetilde{\omega}}{\omega}} p_{nl}^{1-\widetilde{\omega}}}\right]} = \frac{\mathbb{E}\left[\frac{U'(\mathcal{C}_n)}{\mathcal{P}_n} p_{ni}x_{ni}\right]}{\mathbb{E}\left[\frac{U'(\mathcal{C}_n)}{\mathcal{P}_n} p_{nj}x_{nj}\right]}. \tag{25}$$

Comparing equation (25) to the case without uncertainty in equation (16), relative investments in import capacity now depend on expected rather than realized values, and relative import values are now weighted by the marginal utility of income, which depends on the realizations for country productivities.

The representative agent in each country faces two interconnected trade-offs. First, there is an aggregate trade-off. The greater the total amount of labor allocated to import capacity *ex ante*,

the less labor is available for production *ex post*. Second, there is a bilateral trade-off. On the one hand, by concentrating import capacity towards the supplier with the expected lowest price, the representative agent can obtain a greater expected quantity of imports. On the other hand, by diversifying import capacity across suppliers, the representative agent expands the range of varieties that can be consumed, and enhances the ability to source from suppliers who have high expected prices *ex ante* but low realized prices *ex post* given realized productivities.

Using the first-order conditions of the representative agent and our result from Proposition 2 that wages are approximately lognormally distributed, we can re-write equilibrium investments in import capacity in terms of the following four terms:

$$\begin{aligned}\iota_{ni} \propto & \underbrace{\mathbb{E}\left[\left(p_{ni}^{x}\right)^{-(1-\alpha)(\omega-1)}\right]}_{\text{mean prices}} \underbrace{\exp\left(-\frac{\alpha\left(\omega-1\right)}{2\left(\omega-\left(1-\alpha\right)\left(\omega-1\right)\right)}\mathbb{V}\left[\ln\left(p_{ni}^{x}\right)^{-(1-\alpha)(\omega-1)}\right]\right)}_{\text{price volatility}} \\ & \times \underbrace{\exp\left(\text{Cov}\left[\ln\left(p_{ni}^{x}\right)^{-(1-\alpha)(\omega-1)}, \ln \mathcal{W}_{n}^{(1-\alpha)(1-\gamma)}\right]\right)}_{\text{price comovement with factor income}} \\ & \times \underbrace{\exp\left(\text{Cov}\left[\ln\left(p_{ni}^{x}\right)^{-(1-\alpha)(\omega-1)}, \ln\left(\sum_{i} \iota_{ni}^{\frac{\alpha(\omega-1)}{\omega-(1-\alpha)(\omega-1)}}\left(p_{ni}^{x}\right)^{\frac{-(1-\alpha)(\omega-1)}{\omega-(1-\alpha)(\omega-1)}}\right)^{(1-\gamma)\frac{\omega-(1-\alpha)(\omega-1)}{\omega-1}-1}\right]\right)}_{\text{price comovement with the cost of other inputs}},\end{aligned} \tag{26}$$

where recall $p_{ni} = \tau_{ni} w_i / z_i$ and $\text{Cov}\left[\cdot\right]$ denotes the covariance operator.

The first term captures first moment effects, and is similar to the case without uncertainty, but now depends on expected rather than realized prices. The representative agent prefers sourcing from trade partners with lower expected prices, and hence invests more in import capacity with such trade partners, other things equal.

The remaining three terms are distinctive to the case with uncertainty and capture second moment effects. Each of these terms depends on the coefficient of relative risk aversion (γ). We focus on the empirically-relevant case, in which the representative agent is risk averse, with a coefficient of relative risk aversion greater than one ($\gamma > 1$). From the second term, risk aversion implies that she prefers sourcing from trade partners with lower volatility of prices, and hence she invests more in import capacity with such trade partners, other things equal.

From the third term, risk aversion also implies that she prefers sourcing from trade partners whose prices are low when factor income is low. Finally, from the fourth term, risk aversion implies that she prefers sourcing from trade partners whose prices are low when the prices of other trade partners are high. Therefore, combining these two terms, the representative agent invests more in import capacity for trade partners with lower covariances of prices with factor income and/or the prices of other trade partners, other things equal.

2.5.3 Decentralization

We have so far focused on the problem of a representative agent who chooses import capacity and sourcing, taking international prices as given. We now briefly discuss that the solution to this problem can be decentralized as a market equilibrium, in which a representative household owns firms in competitive import-processing and production sectors.

We distinguish between the *ex ante* and *ex post* problem of importing firms. At the beginning of each period, and before observing country productivities, each importing firm chooses import capacity to maximize the expected value of profits ($\pi_{ni}(\iota_{ni})$) weighted by the marginal utility of income ($U'(\mathcal{C}_n)/\mathcal{P}_n$), given the distribution of country productivities, the equilibrium distribution of international prices, and the *ex ante* wage (w_n^-):

$$\max_{\iota_{ni}} \left\{ \mathbb{E}\left[\frac{U'(\mathcal{C}_n)}{\mathcal{P}_n} \pi_{ni}(\iota_{ni}) \right] - w_n^- \iota_{ni} \right\}. \tag{27}$$

After these sunk investments in import capacity have been made, each importing firm observes country productivities, and chooses imports from each country to maximize *ex post* profits, taking international prices as given:

$$\max_{x_{ni}} \{\pi_{ni}(\iota_{ni}, x_{ni})\} = \max_{x_{ni}} \left\{ p_{ni}^c \iota_{ni}^{\alpha} x_{ni}^{1-\alpha} - p_{ni}^x x_{ni} \right\}. \tag{28}$$

Given the stochastic discount factor $\frac{U'(\mathcal{C}_n)}{\mathcal{P}_n}$, workers are indifferent between earning wages w_n^- *ex ante* or earning $\left[\frac{U'(\mathcal{C}_n)}{\mathcal{P}_n} w_n\right]$ *ex post*:

$$w_n^- = \mathbb{E}\left[\frac{U'(\mathcal{C}_n)}{\mathcal{P}_n} w_n \right]. \tag{29}$$

Firms in the production sector maximize *ex post* profits, given the observed realizations for country productivities, and taking international prices as given:

From the first-order conditions to the firm problem in the import-processing and production sectors, the equilibrium allocations in this decentralized market economy correspond to the solution of the problem of the representative agent characterized above.

3 Multiple Sectors

In our baseline model, we assume a single final goods sector for expositional simplicity. In this section, we show that our approach accommodates multiple sectors. In this augmented specification, what matters for import sourcing and welfare is the joint distribution of productivities across countries and industries. Again we begin by solving the problem of a representative agent in each country, who takes international prices as given, before showing that the solution to this problem can be decentralized as a competitive market equilibrium.

3.1 Import Capacity and Sourcing

We assume that the representative agent in country n has constant relative risk aversion preferences ($\mathcal{U}_n$) over the aggregate consumption index ($\mathcal{C}_n$), as in equation (1). We assume that this aggregate consumption index ($\mathcal{C}_n$) is a Cobb-Douglas function of the sectoral consumption index for each sector k ($\mathbb{C}_n^k$):

$$\mathcal{C}_n = \prod_{k=1}^{N} \left(\mathbb{C}_n^k\right)^{\beta_n^k}, \quad 0 < \beta_n^k < 1, \quad \sum_{k=1}^{K} \beta_n^k = 1. \tag{30}$$

This sectoral consumption index ($\mathbb{C}_n^k$) is defined over the consumption of each country's good (c_{nit}^k), according to the CES specification in equation (2). The specification of import capacity and production within each sector is the same as in equation (3) in our single-sector model. For simplicity, we assume the same elasticity of substitution across countries within each sector (ω), and the same expenditure share for import capacity (α) across sectors, but is straightforward to relax both assumptions. While we focus on a multi-sector Armington model, our results hold through the class of multi-sector quantitative trade models characterized by a constant elasticity gravity equation within sectors, including Costinot et al. (2012).

The representative agent is endowed with a measure of ℓ_n units of labor. At the beginning of each period, the representative agent in each country n chooses *ex ante* how much labor to allocate for production for each sector k (ℓ_n^k) or for import capacity for each sector k and trade partner i (ι_{ni}^k), before observing the realizations for productivities in each country and sector. The total mass of production workers for country n is $\sum_{k=1}^{K} \ell_n^k$. The total mass of workers employed in import capacity in country n and sector k is $\sum_{i=1}^{N} \iota_{ni}^k$. The total mass of workers employed in import capacity in country n is $\sum_{k=1}^{K} \sum_{i=1}^{N} \iota_{ni}^k$. Labor market clearing in country n requires:

$$\sum_{k=1}^{K} \ell_n^k + \sum_{k=1}^{K} \sum_{i=1}^{N} \iota_{ni}^k = \ell_n \tag{31}$$

After deciding how much labor to allocate to production and import capacity within each sector, the representative agent observes the realizations for country productivities, and chooses *ex post* how much of its good to export to each partner in each sector, and how much of each trade partner's good to import in each sector.

From the first-order conditions to the representative agent's problem, we find that total employment in production across sectors and total employment in import capacity across trade partners within each sector are constant shares of the economy's labor endowment:

$$\sum_{k=1}^{K} \ell_n^k = (1-\alpha)\,\ell_n, \qquad \sum_{i=1}^{N} \iota_{ni}^k = \alpha \beta_n^k \ell_n, \tag{32}$$

where these shares are determined by the Cobb-Douglas share of import capacity in consumption expenditure within each sector (α) and the Cobb-Douglas share of each sector in overall consumption expenditure (β_n^k).

From the first-order conditions to the representative agent's problem, we find that the allocation of labor across sectors equates the expected wage evaluated at the marginal utility of consumption:

$$\frac{\mathbb{E}\left[\frac{U'(\mathcal{C}_n)}{\mathcal{P}_n} w_n^k\right]}{\mathbb{E}\left[\frac{U'(\mathcal{C}_n)}{\mathcal{P}_n} w_n^l\right]} = \frac{\mathbb{E}\left[\mathcal{C}_n^{1-\gamma} \frac{w_n^k \ell_n^k}{\sum_{h=1}^N w_n^h \ell_n^h}\right]}{\mathbb{E}\left[\mathcal{C}_n^{1-\gamma} \frac{w_n^l \ell_n^l}{\sum_{h=1}^N w_n^h \ell_n^h}\right]} = 1, \tag{33}$$

and relative investments in import capacity again depend on the expected value of relative import values evaluated at the marginal utility of consumption:

$$\frac{\iota_{ni}^k}{\iota_{nj}^k} = \frac{\mathbb{E}\left[\mathcal{C}_n^{1-\gamma} \frac{\left(\iota_{ni}^k\right)^{\frac{\omega-\hat{\omega}}{\omega}} \left(p_{ni}^k\right)^{1-\hat{\omega}}}{\sum_{l=1}^N \left(\iota_{nl}^k\right)^{\frac{\omega-\hat{\omega}}{\omega}} \left(p_{nl}^k\right)^{1-\hat{\omega}}}\right]}{\mathbb{E}\left[\mathcal{C}_n^{1-\gamma} \frac{\left(\iota_{nj}^k\right)^{\frac{\omega-\hat{\omega}}{\omega}} \left(p_{nj}^k\right)^{1-\hat{\omega}}}{\sum_{l=1}^N \left(\iota_{nl}^k\right)^{\frac{\omega-\hat{\omega}}{\omega}} \left(p_{nl}^k\right)^{1-\hat{\omega}}}\right]} = \frac{\mathbb{E}\left[\frac{U'(\mathcal{C}_n)}{\mathcal{P}_n} p_{ni}^k x_{ni}^k\right]}{\mathbb{E}\left[\frac{U'(\mathcal{C}_n)}{\mathcal{P}_n} p_{nj}^k x_{nj}^k\right]}. \tag{34}$$

Comparing these allocations to those in a multi-sector model without uncertainty, relative investments in import capacity now depend on expected rather than realized values, and relative import values are weighted by the marginal utility of income. Comparing these allocations in equation (34) to those in the single-sector model with uncertainty (25), the presence of uncertainty now affects the allocation of resources across sectors (depending on the joint distribution of productivities across sectors and countries), as well as the allocation of resources between import capacity and production within sectors.

3.2 Implications of Import Capacity and Aggregate Uncertainty

We now briefly summarize some of the novel implications from incorporating import capacity and aggregate uncertainty into quantitative trade models with a constant trade elasticity.

First, the welfare gains from trade now depend on both first and second moments for the empirically-relevant case of risk aversion. In the closed economy, the representative agent is only exposed to domestic productivity shocks. In contrast, in the open economy, she is exposed to both domestic and foreign shocks. Additionally, the mapping from productivity to general equilibrium prices and wages is different in the closed and open economies. Depending on the volatility of each country's productivity shocks, the covariance of these productivity shocks across countries, and the geography of bilateral trade costs, these second moment effects can be source of either welfare gains or welfare losses from international trade.

Second, cross-section patterns of international trade are systematically different in the presence of uncertainty. In addition to mean prices, the variance and covariance of prices matter

for the representative agent's decisions import capacity and import sourcing from each trade partner. Increases in uncertainty lead to more dispersed investments in import capacity across trade partners, as it becomes more valuable to be able to adjust import sourcing *ex post* based on the realizations for country productivities. These more dispersed investments in import capacity reallocate resources away from the trade partner with the lowest expected prices.

Third, the elasticity of trade flows with respect to permanent changes in trade costs is smaller in the short-run than in the long-run. In the short-run, investments in import capacity are fixed, and act like a quasi-fixed factor in dampening the response of trade flows to permanent changes in trade costs. In contrast, in the long-run, investments in import capacity are variable and can adjust to permanent changes in trade costs.

Fourth, our multi-sector model can be extended to incorporate input-output linkages with a Cobb-Douglas production technology, as in Caliendo and Parro (2015). In this case, the impact of exogenous shocks to productivity on marginal costs is transformed by the network of input-output relationships between countries and industries.

Fifth, the presence of endogenous investments in import capacity provides a geopolitical motive for activist trade policies. If countries can commit to trade policies before investments in import capacity are made, countries can choose trade policies to induce their trade partners to make investments in import capacity that are favorable to them.

Finally, the presence of aggregate uncertainty and incomplete markets provides a potential motive for state-contingent trade policies that are conditional on the realizations for country productivities. More generally, state-contingent trade agreements provide a potential mechanism for member countries to share risk between themselves, with potentially adverse consequences for welfare in non-member countries.

4 Data

We use two main data sources in our empirical analysis. First, we use data on international trade and tariffs for the United States from 2016-2019, immediately preceding and during the first Trump administration. We combine data from the Census Bureau, U.S. Trade Representative (USTR) and U.S. International Trade Commission (USITC).

From U.S. customs data, we obtain import values and quantities by source country. We observe these data for approximately 16,000 ten-digit Harmonized Tariff Schedule (HTS10) categories. Dividing the import values by the quantities, we construct unit values or prices for each disaggregated category (e.g., "baseball and softball gloves and mitts made in China"). These unit values are computed before tariffs are applied and hence they correspond to foreign export prices. Multiplying these unit values by the duty rates from the US International Trade Commission, we

compute U.S. import prices inclusive of tariffs.[5]

Second, we use international trade and national accounts data from 2001-2010 for the 50 largest countries in terms of GDP in the year 2005. We obtain international trade data from the International Monetary Fund (IMF) direction of trade statistics (DOTs). We combine these trade data with national accounts data from the Penn World Tables (PWT) on gross domestic product (GDP), population and price indexes.

5 Empirical Results

We begin in Section 5.1 by presenting some motivating evidence in support of a first key prediction of our model of import capacity, namely that the trade elasticity is smaller in the short-run than in the long-run. We next provide evidence in Section 5.2 in support of a second key prediction of our model that endogenous log wages are approximately lognormally distributed if productivity is lognormally distributed.

5.1 Short Versus Long-run Trade Elasticities

In this subsection, we provide evidence on the evolution of the trade elasticity across alternative time horizons. While there are other potential reasons why the trade elasticity could be smaller in the short-run than the long-run, our model of import capacity provides an intuitive mechanism and a tractable formalization of this property of trade elasticities.

5.1.1 Tariffs of the First Trump Administration 2018-19

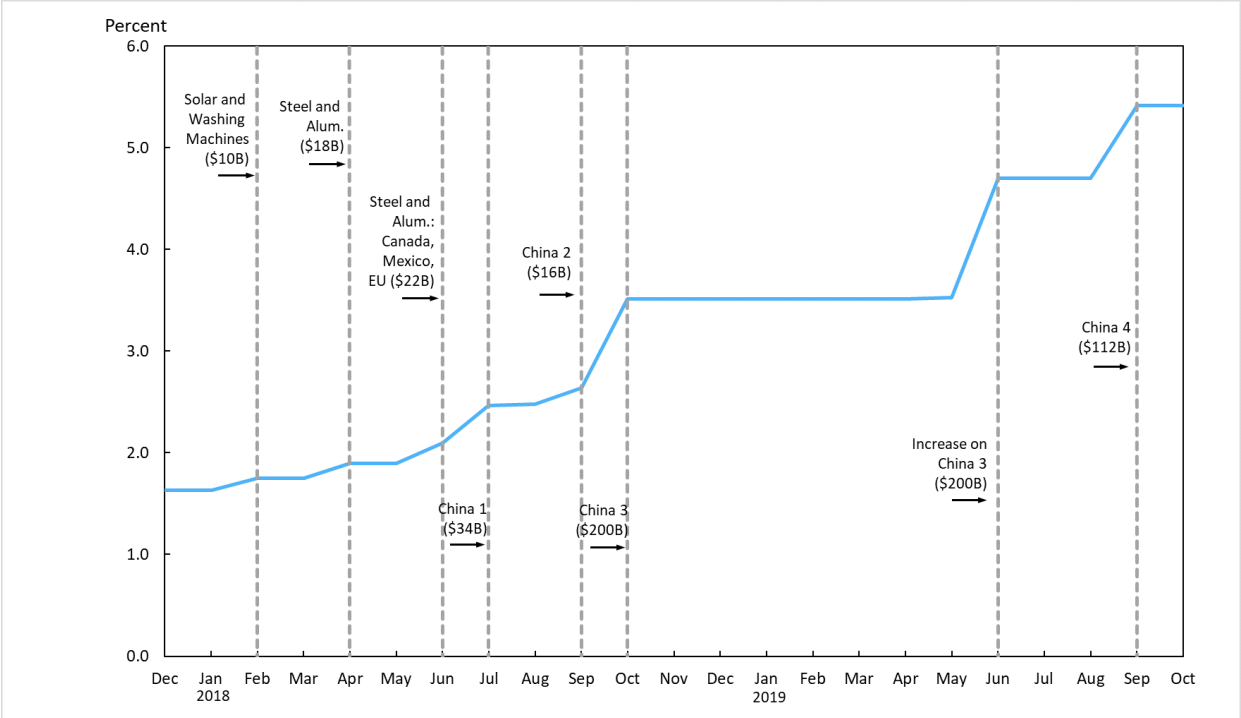
We use the tariffs introduced by the First Trump Administration in 2018-19 as a natural experiment to examine the dynamic response of trade values to unexpected changes in trade policy. Since President Trump's first electoral term was a surprise to many observers, it is unlikely that the tariffs on the affected products were anticipated. Although the extent to which the tariffs on each product were a surprise could have changed over time as the trade war unfolded, Fajgelbaum et al. (2020) find little evidence of pre-trends for the affected products. In addition, the tariffs introduced by first Trump administration are sufficiently large for their effects to be discernible using readily-available bilateral trade data.

We focus on the period from January 2016 through October 2019 (such that our sample period ends before COVID-19). Beginning in February 2018, the first Trump administration introduced eight waves of U.S. tariffs, as discussed in Amiti et al. (2019) and Amiti et al. (2020). In Figure 1 below, we show the evolution of the trade-weighted average U.S. import tariff during 2018-19.

[5]These U.S. tariff data are continuously reported and hence are not subject to the missing data and imputation problems of the United Nations TRAINS data that are highlighted in Teti (2024).

In January 2018, the first wave of tariffs imposed import duties of 30 percent on solar panels and duties of 20–50 percent on washing machines. These two product categories accounted for approximately $10 billion of imports and created a modest increase in US average tariff rates, as shown in the figure. The second wave of tariffs was implemented in March 2018 on $18 billion of steel and aluminum imports. In this wave, aluminum imports were hit with 10 percent tariffs, and a 25 percent tariff was applied to steel imports. The low value of imports covered by the second wave stems from the fact that at least initially, many countries, such as Canada, Mexico, and the countries in the European Union, were exempt. This exemption ended as the third wave of tariffs was imposed on $22 billion of imports from these countries in June 2018.[6]

Figure 1: Average U.S. Tariffs by Wave of the 2018-2019 Trade War



Note: Source: Amiti et al. (2020). Dashed vertical lines indicate the implementation of each of the eight major waves of new tariffs during 2018-2019; tariffs implemented after the fifteenth of the month counted for the subsequent month. Four tranches of tariffs were imposed on China, designated by 1, 2, 3, and 4. Import values associated with each line correspond to headline numbers, not 2017 values, which are a little lower. Based on data from the US Census Bureau, US Trade Representative (USTR), US International Trade Commission; tariffs on HTS10 product code by country, weighted by 2017 annual import value.

These early waves of tariffs were small relative to the tariffs on China that began in July and were rolled out in five waves. In July and September 2018, average tariffs of 25 percent were imposed on $34 billion and $16 billion of U.S. imports, respectively. In October 2018 and June 2019, average tariffs of around 10 percent were applied to $200 and $200 billion of U.S. imports,

[6]The reported import values in this and the next paragraph are headline values when tariffs were imposed.

respectively. Finally, in September 2019, average tariffs of 15 percent were introduced on $112 billion of U.S. imports.

We examine the dynamic response of U.S. import values to these tariffs using an event-study specification following Amiti et al. (2020). We denote source country (exporter) by i, 10-digit Harmonized Tariff Product by j, and month by t. We define treatment months for each country and product based on time relative to the last month before a tariff is imposed (month zero). We construct the log change in tariffs between treatment month s and the last untreated month as $\ln\left[\left(1+\tau_{ijs}\right)/\left(1+\tau_{ij0}\right)\right]$. We regress the log of import values ($\ln x_{ijt}$) on interactions between indicator variables for treatment months (1_{ijs}) and the log change in tariffs in the following event-study specification:

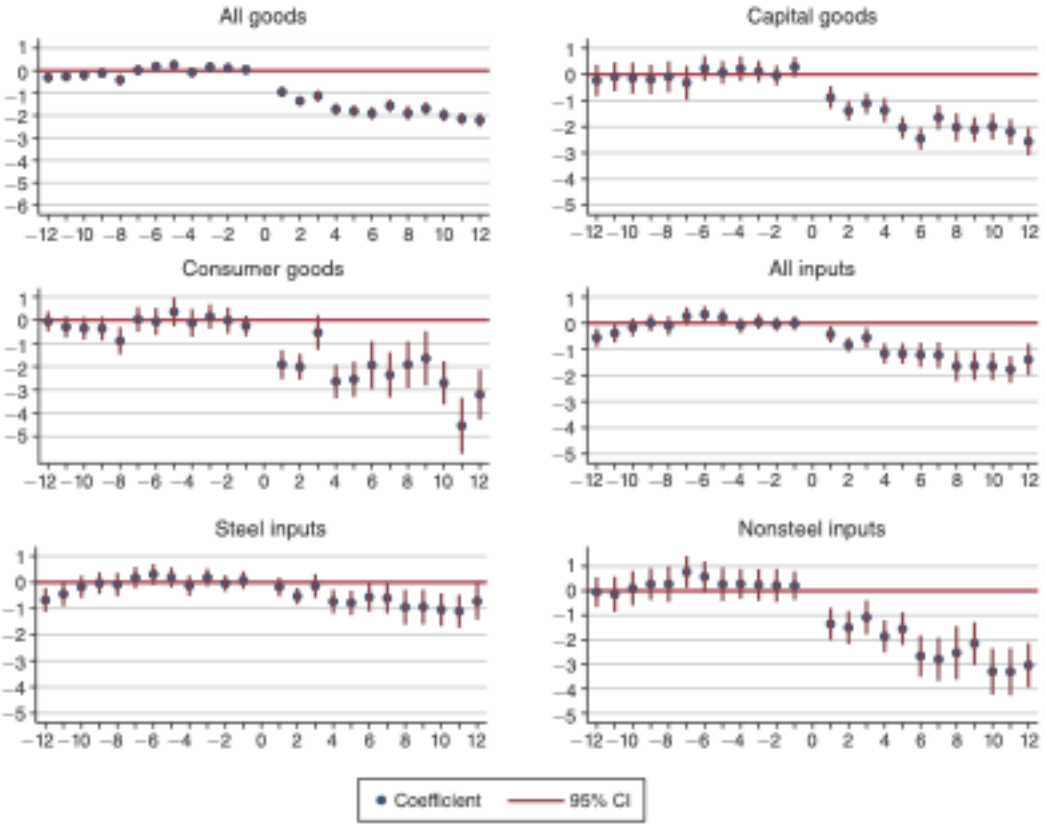
$$\ln x_{ijt} = \eta_{ij} + \sum_{s=-T}^{T} \beta_s \left(1_{ijs} \times \ln\left(\frac{1+\tau_{ijs}}{1+\tau_{ij0}}\right)\right) + \delta_{jt} + \mu_{it} + u_{ijt}, \tag{35}$$

where the coefficients β_s are elasticities estimated over different time horizons s. The excluded category is the last untreated month (i.e., $\beta_0 = 0$). We include country-product fixed effects (η_{ij}) to control for the level of import values in the last untreated month and capture differences in quality or comparative advantage across countries and products. The country-time fixed effects (δ_{jt}) capture time-varying factors that affect import values (e.g., exchange rates). The product-time fixed effects (μ_{it}) allow for time-varying forces that affect import values for a product across all countries (e.g., common technological change).

Figure 2 displays the estimated event-study coefficients. We show results for all goods in the top-left panel, for capital goods in the top-right panel, for consumer goods in the middle-left panel, for all inputs in the middle-right panel, for steel inputs in the bottom-left panel, and for non-steel inputs in the bottom-right panel. Across all panels, we find little evidence of pre-trends, and substantial changes in U.S. import values in response to the Trump tariffs. In the twelve months leading up to a tariff wave, we find coefficients that are typically small in magnitude and often statistically insignificant. In contrast, in the months immediately after a tariff wave, we find large, negative and statistically significant coefficients.

Consistent with the predictions of our import capacity specification, we find that the long-run response of import values to the changes in tariffs is substantially greater than the short-run response. For all goods, the estimated trade elasticity approximately doubles between one month after the tariffs (around minus one) and twelve months after the tariffs (around minus two). Since the event-study specification (35) is estimated in logs, zeros import values are dropped from the regression. Therefore, the estimated coefficients capture the intensive margin response of import values to the tariffs conditional on goods continuing to be imported. As a result, the full response of imports to the tariffs, including the extensive margin for goods that cease to be imported, could be even larger than these estimated elasticities.

Figure 2: Event-Study Estimates for the Impact of the First Trump Administration Tariffs on Import Values



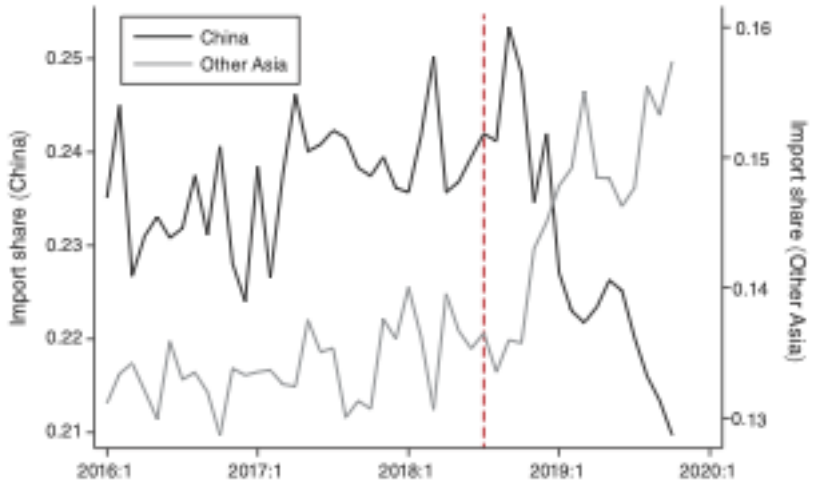
Note: Source: Amiti et al. (2020). Coefficient estimates (circles) and 95 percent confidence intervals (vertical bars) from estimating the event-study specification in equation (35) for import values. Coefficients corresponds to the elasticities of import values with respect to tariffs (β_s). Horizontal axis denotes months relative to the tariff increase. Zero corresponds to the last month before the tariff increase.

We next provide evidence of a reallocation of U.S. imports away from China and towards Other Asian countries in the aftermath of the U.S. tariffs on China, following Grossman et al. (2023). In Figure 3, we show the shares of China and other Asian countries in the total value of U.S. imports. We define other Asian countries as Bangladesh, Cambodia, Hong Kong, India, Indonesia, Malaysia, Pakistan, Philippines, Singapore, Sri Lanka, Taiwan, Thailand, and Vietnam, which are identified by Kearney (2020) as traditional offshoring trade partners besides China. We seasonally adjust these import shares by removing month fixed effects, where we normalize these month fixed effects to sum to zero.

After the first wave of tariffs on China in July 2018 (marked by the dashed vertical line), we find a sharp decline in China's share of U.S. imports of around 3 percent (left scale), and a corresponding rise in Other Asia's share of U.S. imports of a similar magnitude (right scale). The similarity between the decline for China and the rise for Other Asia's import share is consistent with a relocation of import sourcing from China to Other Asia in response to the Trump tariffs.

We find that this relocation of import sourcing takes time to occur, which is consistent with our specification of import capacity, in which the trade elasticity is smaller in the short-run than in the long-run. Alfaro and Chor (2023) provide further empirical evidence of a gradual reallocation of U.S. import sourcing away from China and towards Other Asia over time.

Figure 3: Reallocation of U.S. Imports Away from China and Towards Other Asia



Note: Source: Grossman et al. (2023). Note: Black solid line shows share of US imports from China; gray solid line shows share of US imports from Other Asia (Bangladesh, Cambodia, Hong Kong, India, Indonesia, Malaysia, Pakistan, the Philippines, Singapore, Sri Lanka, Taiwan, Thailand, and Vietnam); dashed vertical line shows the date of the first Trump tariff wave on China; both series seasonally adjusted by removing month fixed effects.

5.2 Monte Carlo Results

We next provide monte carlo evidence in support of the prediction of our model in Proposition 2 that wages are approximately lognormal if productivity is lognormal.

5.2.1 Monte Carlo Setup

We use cross-country data on international trade and national income to construct an empirically-realistic monte carlo. We invert the equilibrium relationships of the deterministic Armington model to recover empirical estimates of productivity and bilateral trade costs from these observed data. This exercise differs from an inversion of our model of import capacity and aggregate uncertainty, because it does not take into account endogenous investments in import capacity or the distinction between the expected and realized values of variables. Nonetheless, it allows us to implement our monte carlo using data that capture features of the real-world economy.

We observe bilateral international trade, GDP, population and price indexes for the 50 largest countries in 2005. We invert the equilibrium conditions of the deterministic Armington model to obtain measures of bilateral trade costs in 2005 and productivity in each country and year from

2001-2010. We focus on a relatively narrow time window of 10 years to abstract from shifts in the productivity distribution over long time horizons. We use our estimates of country log productivities to compute mean log productivity in each country ($\boldsymbol{\mu_z}$) and the variance-covariance matrix of country log productivities ($\boldsymbol{\Sigma_z}$).

Using these estimates for the mean and variance-covariance matrix of log productivities and our assumption that log productivities are normally distributed, we simulate $nreps = 1,000$ random vectors of log productivities from a normal distribution: $\ln \widehat{\boldsymbol{z}} \sim \mathcal{N}(\boldsymbol{\mu_z}, \boldsymbol{\Sigma_z})$. Given each simulated productivity draw, $\ln \widehat{\boldsymbol{z}}^{(\nu)}$, $\nu \in \{1, \ldots, nreps\}$, we solve for log wages ($\ln \widehat{\boldsymbol{w}}^{(\nu)}$) using the income accounting relationship (9) in the non-linear model. From these solutions for log wages, all other endogenous variables of the model can be recovered, including the matrices of expenditure shares ($\widehat{\boldsymbol{S}}^{(\upsilon)}$) and income shares ($\widehat{\boldsymbol{T}}^{(\upsilon)}$).

We thus obtain a simulated distribution of log wages ($\ln \widehat{\boldsymbol{w}}^{(\nu)}$) from the solution of the non-linear model. We compare this simulated distribution to the predicted log normal wage distribution from Proposition 2, using our results for the Jacobian and Hessian from Proposition 1, the simulated productivity draws ($\ln \widehat{\boldsymbol{z}}^{(\nu)}$), and our solutions for the matrices of expenditure shares ($\widehat{\boldsymbol{S}}^{(\nu)}$) and income shares ($\widehat{\boldsymbol{T}}^{(\nu)}$) from above.

We assess the quality of the approximation in Proposition 2 in three main ways. First, we examine the absolute magnitude of the in the mean ($\widehat{\boldsymbol{\mu}}_{\boldsymbol{w}}$) and variance ($\widehat{\boldsymbol{\Sigma}}_{\boldsymbol{w}}$) of log wages ($\ln \widehat{\boldsymbol{w}}^{(\nu)}$) between the simulated distribution in the non-linear model and the lognormal distribution from Proposition 2. Second, we test the null hypothesis that the simulated distribution of log wages ($\ln \widehat{\boldsymbol{w}}^{(\nu)}$) in the non-linear model is normally distributed using a Kolmogorov-Smirnov (KS) test. Third, we compare non-parametric plots of the simulated distribution of log wages ($\ln \widehat{\boldsymbol{w}}^{(\nu)}$) in the non-linear model to the lognormal distribution from Proposition 2.

5.2.2 Model Inversion

We now provide further details of our inversion of the equilibrium relationships in the deterministic Armington model. We observe nominal GDP (y_i), population (ℓ_i), bilateral expenditure shares (s_{ni}) and price indexes (p_i). We set nominal wages equal to GDP per capita (which corresponds to an interpretation of labor as equipped labor):

$$w_i = \frac{y_i}{\ell_i}. \tag{36}$$

We recover country productivities (z_i) using the above solutions for nominal wages (w_i), together with observed bilateral expenditure shares (s_{ni}) and price indexes (p_i). From the definition of real income (w_i/p_i) and expenditure shares (8) and price indexes (7), we have the following

relationship in the deterministic Armington model:

$$\frac{w_i}{p_i} = z_i \left(\frac{1}{s_{ii}} \right)^{\frac{1}{\omega - 1}}, \tag{37}$$

from which we recover our solutions for country productivities (z_i).

Finally, we recover bilateral trade costs (τ_{ni}) using our solutions for wages (w_i) and productivities (z_i), together with observed expenditure shares (s_{ni}) and price indexes (p_i). From expenditure shares (8) and price indexes (7), we have the following relationship in the deterministic Armington model:

$$s_{ni} = \left(\frac{\tau_{ni} w_i}{z_i p_i} \right)^{1-\omega}, \tag{38}$$

from which we recover our solutions for bilateral trade costs (τ_{ni}).

Using the resulting solutions for country productivities (z_i) from 2001-2010, we compute mean log productivity for each country ($\boldsymbol{\mu_z}$) and the variance-covariance matrix of country log productivities ($\boldsymbol{\Sigma_z}$). We thus allow mean log productivity to differ across countries and allow for a general variance-covariance matrix, such that productivities in a pair of countries can be either positively or negatively correlated.

5.2.3 Taylor-series Approximation

We now examine the quality of our Taylor-series approximation in Proposition 2 for the mean and variance-covariance matrix of log wages. We begin by comparing the mean of the simulated distribution of log wages in the non-linear model ($\mathbb{E}\left[\ln \hat{w}_i\right]$) to the predictions of our Taylor-series approximation ($\ln \widetilde{w}_i$). We find that the absolute magnitude of the largest deviation between these mean vectors across our monte carlo replications is negligible:

$$\frac{||\ln \widetilde{\boldsymbol{w}}_{\boldsymbol{i}} - \mathbb{E}\left[\ln \widehat{\boldsymbol{w}}_{\boldsymbol{i}}\right]||}{||\mathbb{E}\left[\ln \widehat{\boldsymbol{w}}_{\boldsymbol{i}}\right]||} = 0.00002\%. \tag{39}$$

We next compare the variance-covariance matrix of the simulated distribution of log wages in the non-linear model ($\widehat{\boldsymbol{\Sigma}}_{\boldsymbol{w}}$) to the predictions of our Taylor-series approximation ($\boldsymbol{W \Sigma_z W'}$). We find that the absolute magnitude of the deviations between these matrices is larger than for the mean vectors above. Nonetheless, the absolute magnitude of the largest deviation across our monte carlo replications remains relatively small:

$$\frac{\left|\left|\boldsymbol{W \Sigma_z W'} - \hat{\boldsymbol{\Sigma}}_{\boldsymbol{w}}\right|\right|}{\left|\left|\hat{\boldsymbol{\Sigma}}_{\boldsymbol{w}}\right|\right|} = 0.94\%. \tag{40}$$

Therefore, we find that our Taylor-series expansion in Proposition 2 provides a good approximation to the mean and variance of the simulated distribution of log wages.

5.2.4 Lognormality

We now provide evidence on the extent to which the prediction of Proposition 2 of an approximate lognormal distribution for wages provides a good approximation to the simulated distribution of wages ($\boldsymbol{w}^{(\boldsymbol{\nu})}$) in the non-linear model.

We begin by performing a Kolmogorov-Smirnov (KS) test of the null hypothesis that $\left\{\ln w_i^{(\nu)}\right\}_{\nu=1}^{nreps}$ is normally distributed for each country i across replications $\nu \in \{1, \ldots, nreps\}$ for each monte carlo. We then repeat this exercise for 100 monte carlos. Table 1 reports the distribution of the number of countries for which we can reject normality at the 5 percent level of significance across these 100 monte carlos. In 44 percent of cases, we are unable to reject normality for any country at conventional levels of statistical significance. For 83 percent of cases, we can only reject normality for one or fewer countries at conventional levels of statistical significance. Therefore our result from Proposition 2 that wages are approximately lognormally distributed if productivity is lognormally distributed is statistically successful in capturing the general equilibrium distribution of wages in the nonlinear model.

Table 1: Number of Countries for Which Lognormality is Rejected using a Kolmogorov-Smirnov (KS) Test Across 100 Monte Carlos

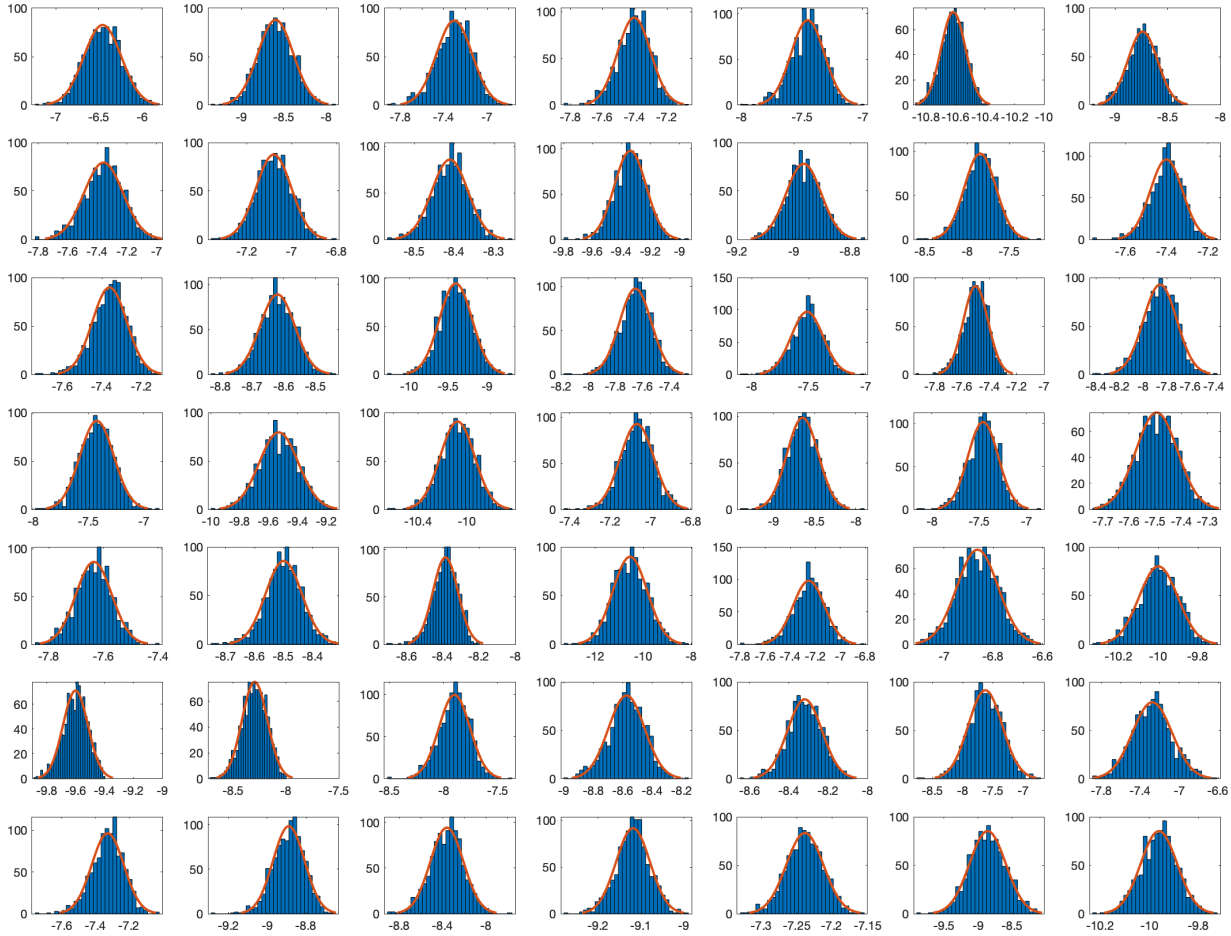
Number Countries Rejected for Normality	Number of s (out of 100)
0	44
1	39
2	10
3	3
4	2
8	1
9	1

Notes: Results from 100 monte carlos. For each monte carlo, we undertake 1,000 replications ($\nu \in \{1, \ldots, nreps\}$). For each replication ν, we draw random vectors of country productivities from a log normal distribution $\ln \widehat{\boldsymbol{z}}^{(\boldsymbol{\nu})} \sim N\left(\boldsymbol{\mu_z}, \boldsymbol{\Sigma_z}\right)$, where $\boldsymbol{\mu_z}$ and $\boldsymbol{\Sigma_z}$ are the mean vector and variance-covariance matrix of country productivities from our inversion of the deterministic Armington model. For each replication ν, we use these random vectors of country productivities ($\ln \widehat{\boldsymbol{z}}^{(\boldsymbol{\nu})}$) and our solutions for bilateral trade costs (τ_{ni}) from inverting the deterministic Armington model to solve for country wages in the non-linear model ($\ln \widehat{\boldsymbol{w}}^{(\boldsymbol{\nu})}$). For each country i, we perform a Kolmogorov-Smirnov (KS) test of the null hypothesis that $\left\{\ln w_i^{(\nu)}\right\}_{\nu=1}^{nreps}$ is normally distributed across the replications. We undertake these Kolmogorov-Smirnov (KS) tests for each country across replications for each of the 100 monte carlos. The Table reports the number of countries for which the null hypothesis that log wages are normally distributed is rejected at the 5 percent level of statistical significance across the 100 monte carlos.

In addition to statistical significance, we examine the economic magnitude of the departures from a lognormal distribution of wages. In Figure 4, we display the simulated distribution of log wages in the non-linear model, based on solving for general equilibrium using the random pro-

ductivity draws from a lognormal distribution. Each panel of the figure shows the distribution for a separate country across replications and monte carlos. The blue bars correspond to histograms of the distribution of log wages for each country in the non-linear model. The red line shows the predictions of a normal distribution of log wages with the same mean and variance for each country. As apparent from the figure, we find that the distribution of log wages in the non-linear model lies relatively close to the predictions of a normal distribution.

Figure 4: Distributions of Log Wages ($\ln w_n$) for Each Country

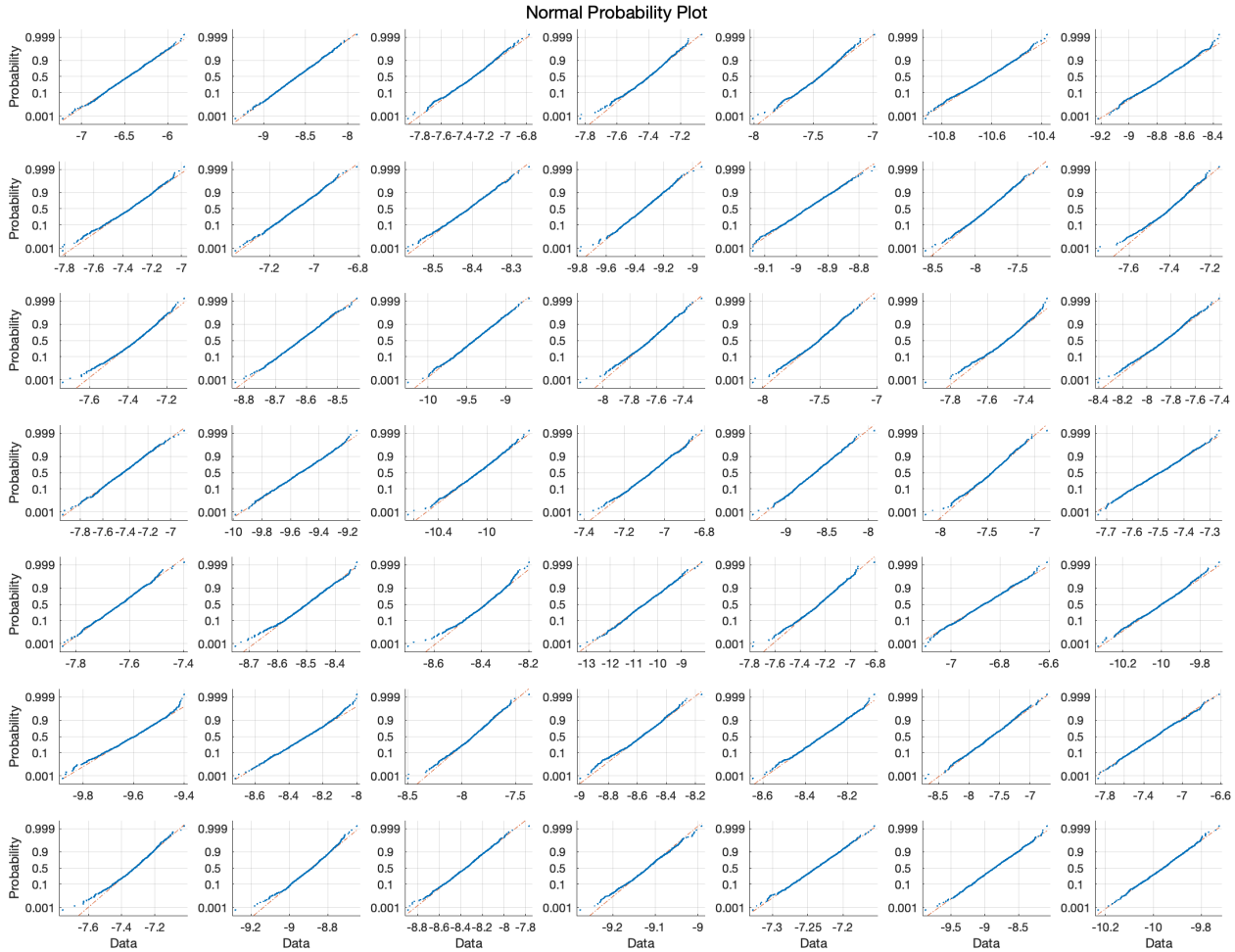


Note: simulated distribution of log wages based on solving for general equilibrium in the non-linear model using random productivity draws from a lognormal distribution. Each panel shows the distribution for a separate country across replications and monte carlos. Blue bars correspond to histograms of the simulated distribution of log wages for each country. The red line shows the predictions of a normal distribution of log wages with the same mean and variance for each country.

In Figure 5, we provide further evidence on the quality of the approximation in Proposition 2 using a QQ plot of the simulated distribution of log wages in the non-linear model. Each panel of the figure shows the distribution for a separate country across replications and monte carlos. The vertical axis displays cumulative probabilities. The horizontal axis displays quantiles of log

wages. The blue circles denote the empirical quantiles of the simulated distribution of log wages in the the non-linear model. The red dashed line denotes the theoretical quantiles of a normal distribution of log wages with the same mean and variance for each country. Throughout most of the distribution, we find that a lognormal distribution provides a good approximation to the simulated distribution of wages in the non-linear model, as reflected in the blue dots lying close to the red dashed line. Only in the tails of the distribution, do we find relatively small departures from the predictions of a lognormal distribution.

Figure 5: QQ Plot of Empirical Probability Distribution of Log Wages ($\ln w_n$) Versus Theoretical Log Normal Probability Distribution



Note: QQ plot of the simulated distribution of log wages based on solving for general equilibrium in the non-linear model using random productivity draws from a lognormal distribution. Each panel shows the distribution for a separate country across replications and monte carlos. Vertical axis displays cumulative probabilities. Horizontal axis displays quantiles of log wages. Blue circles denote the empirical quantiles of the simulated distribution of log wages in the non-linear model. Red dashed line denotes the theoretical quantiles of a normal distribution of log wages with the same mean and variance for each country.

Taken together, these monte carlo results suggest that the predictions of Proposition 2 that

wages are approximately lognormally distributed if productivity is lognormally distributed are a good approximation to the distribution of wages in the non-linear model. These monte carlo results also suggest that the mean and variance-covariance matrix of log wages from our Taylor-series expansion in Proposition 2 are a good approximation to the first and second moments of wages in the non-linear model. Both sets of results validate our use of Proposition 2 to analytically characterize the impact of aggregate uncertainty on cross-section patterns of trade, the welfare gains from trade and the impact of trade policies.

6 Conclusions

Recent years have seen growing concerns about uncertainty and resilience in the global economy. Despite the prominence of these issues in the public policy debate, most prior research using quantitative trade models has abstracted from aggregate uncertainty, because of the challenge of solving for the distribution of the endogenous variables from the non-linear system of general equilibrium conditions in these models.

Our first main contribution is to overcome this challenge by building on techniques from the finance literature. We provide an analytical characterization of the distribution of endogenous variables that is induced in general equilibrium by the exogenous distribution of productivity. We show that this characterization holds up to a second-order approximation if country productivities are lognormally distributed. We focus on uncertainty over productivity for simplicity of exposition. But we show that analogous results hold for uncertainty over trade costs.

Our second main contribution is to use our results for the distribution of endogenous variables to allow agents to make upfront investments in import capacity, which determine the resilience of the economy to shocks. We use a portfolio approach to solve for equilibrium investments in import capacity and import sourcing as a mean-variance problem. A distinctive feature about our setting relative to many finance applications is that our portfolio problem involves solving for the terms of trade in general equilibrium. We show that our approach remains tractable even in settings with a high-dimensional state space of many countries and industries and a rich geography of bilateral trade costs.

We show that equilibrium trade shares take a similar form as in a conventional deterministic trade model. However, in the empirically-relevant case in which agents are risk averse, equilibrium trade shares depend not only on average prices, but also on the variance and covariance of prices. As a result, model inversion exercises in deterministic Armington models recover biased measures of productivity and trade costs. High implied trade barriers may in part reflect low investments in import capacity, because of a high variance shocks or a high covariance between shocks for that supplier and other suppliers.

When agents are risk averse, trade liberalization affects welfare through both first moments (mean equilibrium prices) and second moments (variance and covariance of equilibrium prices). In general, the volatility of equilibrium prices can be either higher or lower in the open economy than in the closed economy, depending on the variance of foreign shocks relative to domestic shocks, the covariance of shocks across countries, and the geography of bilateral trade costs. Endogenous investments in import capacity play a key role in shaping the impact of both first and second moment shocks.

In response to a permanent reduction in trade costs, our framework highlights a distinction between the short-run trade elasticity (conditional on existing investments in import capacity) and the long-run trade elasticity (once investments in import capacity have adjusted). In response to a shift in the geopolitical environment that increases the variance of foreign prices, the representative agent diversifies her sourcing decisions across foreign trade partners, thereby affecting both the volume and cross-sectional pattern of international trade.

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Online Appendix for "Uncertainty and Resilience in the Global Economy" (Not for Publication)

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A Introduction

This Online Appendix reports the derivations of results in the main paper and provides the proof of Propositions.

B Geometric Brownian Motion

In this section of the Online Appendix, we summarize the classic result that the limiting distribution for a geometric Brownian motion process is a lognormal distribution. Assume that country productivity evolves according to geometric Brownian motion with drift:

$$z_i(t) = z_i(0)\, e^{\chi(t)}; \tag{B.1}$$

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where $z_i(0)$ is the initial condition for productivity for country i:

$$\chi(t) = \sigma B(t) + \mu t; \tag{B.2}$$

$B(t)$ is a Wiener process with normally-distributed increments that satisfy $B(t+u) - B(t) \sim \mathcal{N}(0,u)$; σ is a dispersion parameter; and μ is a drift parameter. Taking logarithms in equation (B.1), we have:

$$\ln(z_i(t)) = \ln z_i(0) + \sigma B(t) + \mu t. \tag{B.3}$$

Since $B(t)$ is a Wiener process with normally-distributed increments, it follows immediately that $\ln z_i(t)$ is normally distributed with mean $\mu t + \ln z_i(0)$ and variance $\sigma^2 t$.

C Proof of Proposition 1 (Jacobian and Hessian)

Proposition. *The elasticity of wages with respect to productivity in constant elasticity trade models (the Jacobian $W_{ij} \equiv \frac{d\ln w_i}{d\ln z_j}$) has a sufficient statistics representation in terms of expenditure shares (S_{ni}), income shares (T_{in}), factor incomes (q_i) and the trade elasticity (θ^{DA}):*

$$d\ln \boldsymbol{w} = \boldsymbol{W} d\ln \boldsymbol{z},$$

$$\boldsymbol{W} \equiv \left(\boldsymbol{I} - \boldsymbol{T} - \theta^{DA}(\boldsymbol{TS} - \boldsymbol{I}) + \boldsymbol{Q}\right)^{-1} \theta^{DA}(\boldsymbol{TS} - \boldsymbol{I}),$$

$$T_{in} = \frac{S_{ni} w_n \ell_n}{w_i \ell_i}, \qquad \boldsymbol{Q} d\ln \boldsymbol{q} = 0.$$

The super elasticity of wages with respect to productivity in constant elasticity trade models (the Hessian $H^i_{jk} \equiv d_k W_{ij} \equiv \frac{d^2 \ln w_i}{d\ln z_j d\ln z_k}$) has an analogous sufficient statistics representation as:

$$d_k \boldsymbol{W} = (\boldsymbol{I} - \boldsymbol{Q})\left\{\boldsymbol{Z}^{-1}(d_k \boldsymbol{T})\left(\left(\boldsymbol{I} + \theta^{DA}\boldsymbol{S}\right)\boldsymbol{W} - \theta^{DA}\boldsymbol{S}\right) + \theta^{DA}\boldsymbol{Z}^{-1}\boldsymbol{T}(d_k \boldsymbol{S})(\boldsymbol{W} - \boldsymbol{I})\right\},$$

where $\boldsymbol{Z}$ is the matrix with the productivity vector $\boldsymbol{z}'$ stacked N times.

Proof. We begin by deriving the Jacobian: $W_{ij} \equiv \frac{d\ln w_i}{d\ln z_j}$. Totally differentiating the market clearing condition (9) around an initial equilibrium, holding constant labor endowments (ℓ_n) and trade costs (τ_{ni}), and allowing country productivities (z_n) to change, we obtain:

$$\mathrm{d}\ln w_i = \sum_{n=1}^{N} t_{in}\,\mathrm{d}\ln w_n + \theta^{DA}\left(\sum_{h=1}^{N}\sum_{n=1}^{N} t_{in} s_{nh}\left[\mathrm{d}\ln w_h - \mathrm{d}\ln z_h\right] - \left[\mathrm{d}\ln w_i - \mathrm{d}\ln z_i\right]\right),$$

where t_{in} is the share of exporter i's income from market n:

$$t_{in} \equiv \frac{s_{ni} w_n \ell_n}{w_i \ell_i}, \qquad s_{nit} = \frac{(\tau_{ni} w_i / z_i)^{1-\omega}}{\sum_{j=1}^{N}(\tau_{nj} w_j / z_j)^{1-\omega}}.$$

This market clearing condition has the following matrix representation:

$$\mathrm{d}\ln \mathbf{w} = \boldsymbol{T}\,\mathrm{d}\ln \mathbf{w} + \theta^{DA}(\boldsymbol{TS} - \boldsymbol{I})(\mathrm{d}\ln \mathbf{w} - \mathrm{d}\ln \mathbf{z}),$$

which can be re-written as:

$$\left(\boldsymbol{I}-\boldsymbol{T}-\theta^{DA}\left(\boldsymbol{TS}-\boldsymbol{I}\right)\right)\,\mathrm{d}\ln\mathbf{w}=-\theta^{DA}\left(\boldsymbol{TS}-\boldsymbol{I}\right)\,\mathrm{d}\ln\mathbf{z}.$$

To solve for wages we use our choice of world GDP as numeraire, which implies $\boldsymbol{Q}\,\mathrm{d}\ln\mathbf{w}=0$. Using this choice of numeraire in the market clearing condition above, we have:

$$\left(\boldsymbol{I}-\boldsymbol{T}-\theta^{DA}\left(\boldsymbol{TS}-\boldsymbol{I}\right)+\boldsymbol{Q}\right)\,\mathrm{d}\ln\mathbf{w}=-\theta^{DA}\left(\boldsymbol{TS}-\boldsymbol{I}\right)\,\mathrm{d}\ln\mathbf{z}.$$

which can be re-written as:

$$\mathrm{d}\ln\mathbf{w}=\mathbf{W}\,\mathrm{d}\ln\mathbf{z},$$

$$\mathbf{W}\equiv-\left(\boldsymbol{I}-\boldsymbol{T}-\theta^{DA}\left(\boldsymbol{TS}-\boldsymbol{I}\right)+\boldsymbol{Q}\right)^{-1}\theta^{DA}\left(\boldsymbol{TS}-\boldsymbol{I}\right).$$

To derive the Hessian, i.e., $H^i_{jk}\equiv\frac{d^2\ln w_i}{d\ln z_j d\ln z_k}$, we need to take derivative of Jacobian matrix ($\boldsymbol{W}$) with respect to $d\ln z_k$. Define $\boldsymbol{Z}\equiv\boldsymbol{I}-\boldsymbol{T}-\theta^{DA}\left(\boldsymbol{TS}-\boldsymbol{I}\right)+\left(1+\theta^{DA}\right)\boldsymbol{Q}$. Note that:

$$d\boldsymbol{Z}=-d\boldsymbol{T}-\theta^{DA}\left(d\boldsymbol{T}\boldsymbol{S}+\boldsymbol{T}d\boldsymbol{S}\right).$$

Using this result, we have:

$$\begin{aligned}
dW&=\left(\boldsymbol{I}-\boldsymbol{Q}\right)\left[\boldsymbol{Z}^{-1}\left(d\boldsymbol{Z}\right)\boldsymbol{Z}^{-1}\theta^{DA}\left(\boldsymbol{TS}-\boldsymbol{I}\right)-\theta^{DA}\boldsymbol{Z}^{-1}d\left(\boldsymbol{TS}-\boldsymbol{I}\right)\right],\\
&=\left(\boldsymbol{I}-\boldsymbol{Q}\right)\left[-\boldsymbol{Z}^{-1}\left(d\boldsymbol{Z}\right)\boldsymbol{W}-\theta^{DA}\boldsymbol{Z}^{-1}\left(d\boldsymbol{T}\boldsymbol{S}+\boldsymbol{T}d\boldsymbol{S}\right)\right],\\
&=\left(\boldsymbol{I}-\boldsymbol{Q}\right)\left[\boldsymbol{Z}^{-1}\left(d\boldsymbol{T}+\theta^{DA}\left(d\boldsymbol{T}\boldsymbol{S}+\boldsymbol{T}d\boldsymbol{S}\right)\right)\boldsymbol{W}-\theta^{DA}\boldsymbol{Z}^{-1}\left(d\boldsymbol{T}\boldsymbol{S}+\boldsymbol{T}d\boldsymbol{S}\right)\right]\\
&\left(\boldsymbol{I}-\boldsymbol{Q}\right)\left[\boldsymbol{Z}^{-1}d\boldsymbol{T}\left[\left(\boldsymbol{I}+\theta^{DA}\boldsymbol{S}\right)\boldsymbol{W}-\theta\boldsymbol{S}\right]+\theta^{DA}\boldsymbol{Z}^{-1}\boldsymbol{T}d\boldsymbol{S}\left(\boldsymbol{W}-\boldsymbol{I}\right)\right].
\end{aligned}$$

Therefore Hessian ($H^i_{jk}\equiv d_kW\equiv\frac{d^2\ln w_i}{d\ln z_j d\ln z_k}$) of wages to productivities is:

$$d_k\boldsymbol{W}=\left(\boldsymbol{I}-\boldsymbol{Q}\right)\left\{\boldsymbol{Z}^{-1}\left(d_k\boldsymbol{T}\right)\left(\left(\boldsymbol{I}+\theta^{DA}\boldsymbol{S}\right)\boldsymbol{W}-\theta^{DA}\boldsymbol{S}\right)+\theta^{DA}\boldsymbol{Z}^{-1}\boldsymbol{T}\left(d_k\boldsymbol{S}\right)\left(\boldsymbol{W}-\boldsymbol{I}\right)\right\},$$

$$\left(d_k\boldsymbol{S}\right)_{ni}=\theta^{DA}\boldsymbol{S}_{ni}\left(\boldsymbol{\delta}_{ik}-\boldsymbol{J}_{ik}-\boldsymbol{S}_{nk}+\left(\boldsymbol{SJ}\right)_{nk}\right),$$

$$\left(d_k\boldsymbol{T}\right)_{in}=\boldsymbol{T}_{in}\left[\boldsymbol{J}_{nk}-\boldsymbol{J}_{ik}+\theta^{DA}\left(\boldsymbol{\delta}_{ik}-\boldsymbol{J}_{ik}-\boldsymbol{S}_{nk}+\left(\boldsymbol{SJ}\right)_{nk}\right)\right],$$

where δ_{ik} is the Kronecker delta:

$$\delta_{ik}=\begin{cases}1 & \text{if } i=k\\ 0 & \text{if } i\neq k\end{cases}.$$

□

D Proof of Lemma 1 (Campbell and Viceira 2001)

In this section of the Online Appendix, we summarize a result for linear combinations of lognormally distributed random variables from the finance literature that is used in the proofs of our propositions below.

Lemma. ***(Campbell and Viceira 2001)*** *Let $\boldsymbol{x}$ denote a $N \times 1$ vector of lognormally distributed random variables such that $\ln x \sim \mathcal{N}(\mu_x, \boldsymbol{\Sigma_x})$. Let $\boldsymbol{a}$ denote a $N \times 1$ deterministic vector. Then $\boldsymbol{a}'\boldsymbol{x}$ is approximately lognormally distributed (up to second-order) with the following mean and variance:*

$$\mathbb{E}\left[\ln \boldsymbol{a}'\boldsymbol{x}\right] \approx \ln \sum_i a_i \exp\left(\mu_{x_i}\right) + \frac{1}{2}\sum_i m_{x_i}\sigma_i^2 - \frac{1}{2}\boldsymbol{m_x'}\boldsymbol{\Sigma_x}\boldsymbol{m_x},$$

$$\mathbb{V}\left[\ln \boldsymbol{a}'\boldsymbol{x}\right] \approx \boldsymbol{m_x'}\boldsymbol{\Sigma_x}\boldsymbol{m_x},$$

where $m_{x_i} \equiv \frac{a_i \exp\left(\mu_{x_i}\right)}{\sum_j a_j \exp\left(\mu_{x_j}\right)}$.

Proof. Define $\epsilon_i \equiv \ln x_i - \mu_{x_i}$. Note that $\mathbb{E}\left[\epsilon_{x_i}\right] = -\frac{1}{2}\sigma_i^2$ from the properties of the lognormal distribution. We have

$$\begin{aligned}
\ln \boldsymbol{a}'\boldsymbol{x} &\equiv \ln \sum_i a_i x_i, \qquad \text{(D.1)}\\
&= \ln \sum_i a_i \exp\left(\mu_{x_i}\right) \frac{x_i}{\exp\left(\mu_{x_i}\right)},\\
&= \ln \sum_i a_i \exp\left(\mu_{x_i}\right) + \ln \sum_i \frac{a_i \exp\left(\mu_{x_i}\right)}{\sum_j a_j \exp\left(\mu_{x_j}\right)} \frac{x_i}{\exp\left(\mu_{x_i}\right)},\\
&= \ln \sum_i a_i \exp\left(\mu_{x_i}\right) + \ln \sum_i m_{x_i} \exp\left(\ln x_i - \mu_{x_i}\right),
\end{aligned}$$

where we have defined

$$m_{x_i} \equiv \frac{a_i \exp\left(\mu_{x_i}\right)}{\sum_j a_j \exp\left(\mu_{x_j}\right)}$$

Note that $\ln \boldsymbol{a}'\boldsymbol{x}$ in equation (D.1) is a linear combination of lognormally distributed random variables. Therefore, this linear combination of lognormally distributed random variables is approximately lognormally distribution, as shown in Marlow (1967), Leipnik (1991), Dufresne (2004) and Mehta et al. (2007). We now derive the mean and variance-covariance matrix of this approximate lognormal distribution. Taking a second-order approximation around $\epsilon_i = 0$, equation (D.1) is approximately equal to the following expression:

$$\begin{aligned}
\ln \boldsymbol{a}'\boldsymbol{x} &\approx \ln \sum_i a_i \exp\left(\mu_{x_i}\right) + \ln \sum_i m_{x_i}\left(1 + \epsilon_i + \frac{1}{2}\epsilon_i^2\right), \qquad \text{(D.2)}\\
&\approx \ln \sum_i a_i \exp\left(\mu_{x_i}\right) + \sum_i m_{x_i}\left(\epsilon_i + \frac{1}{2}\epsilon_i^2\right) - \frac{1}{2}\sum_i \sum_j m_{x_i} m_{x_j} \epsilon_i \epsilon_j.
\end{aligned}$$

Taking expectations in equation (D.2) we have:

$$\mathbb{E}\left[\ln \boldsymbol{a}'\boldsymbol{x}\right] \approx \ln \sum_i a_i \exp\left(\mu_{x_i}\right) + \frac{1}{2}\sum_i m_{x_i}\sigma_i^2 - \frac{1}{2}\boldsymbol{m}_{\boldsymbol{x}}'\boldsymbol{\Sigma}_{\boldsymbol{x}}\boldsymbol{m}_{\boldsymbol{x}}, \tag{D.3}$$

$$\mathbb{V}\left[\ln \boldsymbol{a}'\boldsymbol{x}\right] \approx \boldsymbol{m}_{\boldsymbol{x}}'\boldsymbol{\Sigma}_{\boldsymbol{x}}\boldsymbol{m}_{\boldsymbol{x}}. \tag{D.4}$$

□

E Proof of Proposition 2 (Stochastic Armington)

Proposition. ***(Stochastic Armington)*** *Under Assumption 1, the distribution of wages is approximately lognormally distributed (up to second order) with the following mean and variance:*

$$\mathbb{E}\left[\ln w_i\right] \approx F_i\left(\mu_z\right) + \frac{1}{2}\sum_j\sum_k H_{jk}^i\left[\Sigma_z\right]_{jk},$$

$$\mathbb{V}\left[\ln \boldsymbol{w}\right] \approx W\Sigma_z W',$$

where $F_i\left(\mu_z\right)$ is the log-wage of country i when log-productivities are equal to their expected values, $\boldsymbol{W}$ is the Jacobian and $\boldsymbol{H}$ is the Hessian from Proposition 1.

Proof. We establish the proposition in three steps.

Step 1: Recall Lemma 1 (Campbell and Viceira 2001) from the previous subsection of this Online Appendix. Let $\boldsymbol{x}$ denote a $N \times 1$ vector of lognormally distributed random variables such that $\ln x \sim \mathcal{N}\left(\mu_x, \boldsymbol{\Sigma}_{\boldsymbol{x}}\right)$. Let $\boldsymbol{a}$ denote a $N \times 1$ deterministic vector. Then $\boldsymbol{a}'\boldsymbol{x}$ is approximately lognormally distributed (up to second-order) with the following mean and variance:

$$\mathbb{E}\left[\ln \boldsymbol{a}'\boldsymbol{x}\right] \approx \ln \sum_i a_i \exp\left(\mu_{x_i}\right) + \frac{1}{2}\sum_i m_{x_i}\sigma_i^2 - \frac{1}{2}\boldsymbol{m}_{\boldsymbol{x}}'\boldsymbol{\Sigma}_{\boldsymbol{x}}\boldsymbol{m}_{\boldsymbol{x}},, \tag{E.1}$$

$$\mathbb{V}\left[\ln \boldsymbol{a}'\boldsymbol{x}\right] \approx \boldsymbol{m}_{\boldsymbol{x}}'\boldsymbol{\Sigma}_{\boldsymbol{x}}\boldsymbol{m}_{\boldsymbol{x}}, \tag{E.2}$$

where $m_{x_i} \equiv \frac{a_i \exp\left(\mu_{x_i}\right)}{\sum_j a_j \exp\left(\mu_{x_j}\right)}$. We begin by rewriting this result in an equivalent form. Note that:

$$\frac{\mathrm{d}\ln \boldsymbol{a}'\boldsymbol{x}}{\mathrm{d}\ln x_i} = \frac{a_i x_i}{\boldsymbol{a}'\boldsymbol{x}}.$$

Evaluating this ratio at the mean value of x_i, we have:

$$\frac{a_i \exp\left(\mu_{x_i}\right)}{\sum_j a_j \exp\left(\mu_{x_j}\right)} = m_{x_i}.$$

Note also that:

$$\begin{aligned} H_{ij} &\equiv \frac{\mathrm{d}^2 \ln \boldsymbol{a}'\boldsymbol{x}}{\mathrm{d}\ln x_i \, \mathrm{d}\ln x_j}, \\ &= \frac{\mathrm{d}\frac{a_i x_i}{\boldsymbol{a}'\boldsymbol{x}}}{\mathrm{d}\ln x_j}, \\ &= \frac{\boldsymbol{a}'\boldsymbol{x} a_i x_i \mathbf{1}_{i=j} - a_i x_i a_j x_j}{\left(\boldsymbol{a}'\boldsymbol{x}\right)^2}. \end{aligned}$$

Evaluating this expression at the mean value of x_i, we have:

$$H_{ij} = m_{x_i}\mathbf{1}_{i=j} - m_{x_i}m_{x_j}.$$

Using these results, we can rewrite the mean and variance in Lemma 1 in the following equivalent form

$$\mathbb{E}\left[\ln \boldsymbol{a}'\boldsymbol{x}\right] \approx \ln \sum_i a_i \exp\left(\mu_{x_i}\right) + \frac{1}{2}\sum_i \sum_j H_{ij}\left[\Sigma_x\right]_{ij}, \tag{E.3}$$

$$\mathbb{V}\left[\ln \boldsymbol{a}'\boldsymbol{x}\right] \approx \boldsymbol{m}_{\boldsymbol{x}}'\boldsymbol{\Sigma}_{\boldsymbol{x}}\boldsymbol{m}_{\boldsymbol{x}}. \tag{E.4}$$

Step 2: We next use Lemma 2 to rewrite the equality between a country's income and expenditure on its goods in equation (9) in the following form:

$$\ln w_i = \ln \sum_n \xi_{ni} w_n, \qquad \xi_{ni} \equiv \frac{\ell_n \left(\tau_{ni} w_i / z_i\right)^{-\theta}}{\sum_{k=1}^N \ell_i \left(\tau_{ni} w_k / z_k\right)^{-\theta}}. \tag{E.5}$$

Step 3: We now use this representation and Lemma 1 (Campbell and Viceira 2001) to show that wages are approximately lognormally distributed. We first conjecture that wages are approximately lognormally distributed, before verifying that this conjecture is indeed satisfied. From Assumption 1, productivity is lognormally distributed. Using this assumption and our conjecture, it follows that the weights (ξ_{ni}) are lognormally distributed, as can be established from the properties of their numerator and denominator. Under our conjecture, the numerator is the product of lognormal distributions, which is lognormally distributed. Additionally, the denominator is a weighted sum of lognormal distributions, which is approximately lognormally distributed (applying Lemma 1 for a first time). Using these results for the numerator and denominator, the weights (ξ_{ni}) are the ratio of two lognormal distributions, which is lognormally distributed. Note that wages are equal to the sum of these weights (ξ_{ni}) times wages (w_i). Under our conjecture, both wages and expenditure shares are lognormally distributed. Since the product of two lognormal distributions is lognormally distributed, it follows that the terms in wages times expenditure shares inside the summation sign in equation (E.5) are lognormally distributed. Finally, the sum of lognormal distributions is approximately lognormally distributed (applying Lemma 1 for a second time). Therefore, we have verified our conjecture that wages in equation (E.5) are

approximately lognormally distributed.

Step 4: We next compute the mean and variance of this approximate lognormal distribution of wages. We begin with the equality between a country's income and expenditure on its goods in equation (E.5). We use our analytical characterization of the Jacobian and Hessian for the response of wages to productivities from Proposition 1. Using these analytical results for the Jacobian and Hessian in the equivalent representation of Lemma 1 (Campbell and Viceira 2001) in equations (E.3) and (E.4), we obtain the following mean and variance of the approximate lognormal distribution of wages:

$$\begin{aligned}\mathbb{E}\left[\ln w_i\right] &\approx F_i\left(\mu_z\right)+\mathbb{E}\left[\sum_j W_{ij}\left(\ln z_j-\mu_{z_j}\right)\right]+\frac{1}{2}\mathbb{E}\left[\sum_j\sum_k H^i_{jk}\left(\ln z_j-\mu_{z_j}\right)\left(\ln z_k-\mu_{z_k}\right)\right],\\ &= F_i\left(\mu_z\right)+\frac{1}{2}\sum_j\sum_k H^i_{jk}\left[\Sigma_z\right]_{jk},\end{aligned}$$

$$\mathbb{V}\left[\ln \boldsymbol{w}\right]\approx W\Sigma_z W',$$

which establishes the proposition. □

F Proof of Proposition 3 (Import Capacity Without Uncertainty)

In this section of the Online Appendix, we report the full derivations for our model of import capacity without uncertainty in Section 2.4 of the paper, including the proof of Proposition 3. In the absence of uncertainty, we show that our model of import capacity is isomorphic to a deterministic Armington model, with an appropriate adjustment to the trade elasticity.

F.1 Preferences, Endowments and Technology

We begin by summarizing our assumptions about preferences, endowments and technology.

F.1.1 Preferences

The representative consumer in each country is endowed with a measure of $\ell_n$units of labor. The consumption index ($\mathcal{C}_n$) is defined over the consumption of the good produced by each country (c_{ni}) and is assumed to take the following constant elasticity of substitution form:

$$\mathcal{C}_n=\left(\sum_{i=1}^{N} c_{ni}^{\frac{\omega-1}{\omega}}\right)^{\frac{\omega}{\omega-1}},\qquad \omega>1, \tag{F.1}$$

where $\omega>1$ is the elasticity of substitution between country varieties.

F.1.2 Import Capacity

Consumption in country n of the good produced by country i (c_{ni}) requires both imports net of variable trade costs (x_{ni}) and investments in import capacity (ι_{ni}) to process those imports:

$$c_{ni} = \iota_{ni}^{\alpha} x_{ni}^{1-\alpha}, \qquad 0 < \alpha < 1. \tag{F.2}$$

F.1.3 Production

Each country's good is produced using labor according to a constant returns to scale production technology with productivity z_i. There are iceberg bilateral costs of trade, such that $\tau_{ni} \geq 1$units of a good must be shipped from country i in order for one unit to arrive in country n, with $\tau_{ni} > 1$ for $n \neq i$ and $\tau_{nn} = 1$. Market clearing requires that the sum across all markets n of the quantity of imports gross of variable trade costs from country i ($\tau_{ni} x_{ni}$) is equal to production of country i's good:

$$\sum_{n=1}^{N} \tau_{ni} x_{ni} = z_i \left(\ell_i - \sum_{j=1}^{N} \iota_{ji} \right), \tag{F.3}$$

where labor used in production in country i equals its endowment (ℓ_i) minus labor used for import capacity.

F.2 Market Equilibrium

We now summarize the market equilibrium. Both the import sector and the production sector are assumed to be perfectly competitive.

F.2.1 Consumer Expenditure Minimization

The representative consumer in country n solves the following problem:

$$\mathcal{L} = \left(\sum_{i=1}^{N} c_{ni}^{\frac{\omega-1}{\omega}} \right)^{\frac{\omega}{\omega-1}} - \lambda \left[\sum_{i=1}^{N} p_{ni}^{c} c_{ni} - w_n \ell_n \right], \tag{F.4}$$

where p_{ni}^{c} is the price in country n of consuming goods good from country i, taking into account the cost of import processing according to the technology (F.2); w_i is the wage; and λ is the Lagrange Multiplier on the consumer's budget constraint.

From the first-order conditions for utility maximization, we have:

$$c_{ni}^{\frac{\omega-1}{\omega}} \left(\sum_{i=1}^{N} c_{ni}^{\frac{\omega-1}{\omega}} \right)^{\frac{\omega}{\omega-1}-1} = p_{ni}^{c} c_{ni} \tag{F.5}$$

$$\sum_{i=1}^{N} p_{ni}^{c} c_{ni} = w_n \ell_n. \tag{F.6}$$

Combining these relationships, we obtain the following conventional results for expenditure shares and the price index with CES preferences:

$$\frac{p_{ni}^c c_{ni}}{w_i \ell_i} = \frac{p_{ni}^{1-\omega}}{\sum_j p_{nj}^{1-\omega}}, \tag{F.7}$$

$$\mathcal{P}_n = \left[\sum_{i=1}^{N} p_{ni}^{1-\omega} \right]^{\frac{1}{1-\omega}}. \tag{F.8}$$

F.2.2 Importers

Competitive importers in country n choose imports (x_{ni}) and import capacity (ι_{ni}) for each country i to maximize their profits, taking prices (p_{ni}^c, p_{ni}^x) and wages (w_n) as given:

$$\max_{\{\iota_{ni}, x_{ni}\}} \left\{ p_{ni}^c \iota_{ni}^{\alpha} x_{ni}^{1-\alpha} - w_n \iota_{ni} - p_{ni}^x x_{ni} \right\}, \tag{F.9}$$

where p_{ni}^c is the price of a unit of consumption after import processing; p_{ni}^x is the price of imports before import processing; x_{ni} correspond to the quantity of imports before import processing (net of variable trade costs); and we assume that variable trade costs are paid by producers.

From the first-order conditions for profit maximization, equilibrium choices of import capacity and imports satisfy:

$$w_n \iota_{ni} = \alpha p_{ni}^c c_{ni}, \tag{F.10}$$

$$p_{ni}^x x_{ni} = (1-\alpha)\, p_{ni}^c c_{ni}. \tag{F.11}$$

Production Competitive producers in each country i choose employment for each market n (ℓ_{ni}) to maximize profits, taking prices (p_{ni}^x) and wages (w_i) as given:

$$\max_{\ell_{ni}} \left\{ p_{ni}^x \frac{z_i \ell_{ni}}{\tau_{ni}} - w_i \ell_{ni} \right\}, \tag{F.12}$$

where if firm produces $y_{ni} = z_i \ell_{ni}$ units of output, it receives payment from importers for $x_{ni} = y_{ni}/\tau_{ni}$ units of imports at a price of p_{ni}^x per unit imported.

From the first-order conditions for profit maximization, the price charged by producers equals their marginal cost inclusive of variable trade costs:

$$p_{ni}^x = \frac{\tau_{ni} w_i}{z_i}. \tag{F.13}$$

F.2.3 Market Clearing

Multiplying both sides of the goods market clearing condition for country i by wages (w_i), dividing both sides by productivity (z_i), and using profit maximization and zero profits (F.13), we can re-write the goods market clearing condition (F.3) in value terms:

$$\sum_{n=1}^{N} p_{ni}^{x} x_{ni} = w_i \left(\ell_i - \sum_{j=1}^{N} \iota_{ji} \right), \tag{F.14}$$

where the left-hand side is the sum across all markets n of expenditure on imports from country i at the point of consumption; the right-hand side is the income of workers employed in producing output in country i at the point of production.

F.2.4 Equilibrium Import Capacity and Imports

From the first-order condition for import capacity (F.10), we have:

$$\sum_{n=1}^{N} w_n \iota_{ni} = \alpha \sum_{n=1}^{N} p_{ni}^{c} c_{ni}.$$

Using the consumer budget constraint (F.6), we can write this first-order condition as:

$$\sum_{n=1}^{N} w_n \iota_{ni} = \alpha w_n \ell_n.$$

which implies that investments in import capacity are a constant fraction of the labor force:

$$\sum_{n=1}^{N} \iota_{ni} = \alpha \ell_n. \tag{F.15}$$

From the first-order condition for import capacity (F.10) for a given importing country and two different exporting countries, we have:

$$\frac{\iota_{ni}}{\iota_{nj}} = \frac{p_{ni}^{c} c_{ni}}{p_{nj}^{c} c_{nj}}.$$

From the first-order condition for imports (F.11) for a given importing country and two different exporting countries, we have:

$$\frac{p_{ni}^{x} x_{ni}}{p_{nj}^{x} x_{nj}} = \frac{p_{ni}^{c} c_{ni}}{p_{nj}^{c} c_{nj}}.$$

From the first-order condition to the consumer's problem (F.5) for a given importing country and two different exporting countries, we also have:

$$\frac{c_{ni}^{\frac{\omega-1}{\omega}}}{c_{nj}^{\frac{\omega-1}{\omega}}} = \frac{p_{ni}^{c} c_{ni}}{p_{nj}^{c} c_{nj}}.$$

Combining these three relationships, and using the import technology (F.2), we have:

$$\frac{\iota_{ni}}{\iota_{nj}} = \frac{p^x_{ni}x_{ni}}{p^x_{nj}x_{nj}} = \frac{p^c_{ni}c_{ni}}{p^c_{nj}c_{nj}} = \frac{\left(\iota^\alpha_{ni}x^{1-\alpha}_{ni}\right)^{\frac{\omega-1}{\omega}}}{\left(\iota^\alpha_{nj}x^{1-\alpha}_{nj}\right)^{\frac{\omega-1}{\omega}}}. \tag{F.16}$$

From these three equalities (F.16), we have:

$$\frac{\iota_{ni}}{\iota_{nj}} = \frac{\left(\iota^\alpha_{ni}x^{1-\alpha}_{ni}\right)^{\frac{\omega-1}{\omega}}}{\left(\iota^\alpha_{nj}x^{1-\alpha}_{nj}\right)^{\frac{\omega-1}{\omega}}},$$

which yields the following relationship between relative import capacity and relative imports:

$$\frac{\iota_{ni}}{\iota_{nj}} = \left(\frac{x_{ni}}{x_{nj}}\right)^{\frac{(1-\alpha)(\omega-1)}{\omega-\alpha(\omega-1)}}. \tag{F.17}$$

From these three equalities (F.16), we also have:

$$\frac{p^x_{ni}x_{ni}}{p^x_{nj}x_{nj}} = \left(\frac{\iota_{ni}}{\iota_{nj}}\right)^{\frac{\alpha(\omega-1)}{\omega}} \left(\frac{x_{ni}}{x_{nj}}\right)^{\frac{(1-\alpha)(\omega-1)}{\omega}}.$$

Using equation (F.17) to substitute for the terms in import capacity, we obtain the following relationship between relative imports and relative prices:

$$\frac{x_{ni}}{x_{nj}} = \left(\frac{p^x_{ni}}{p^x_{nj}}\right)^{-(\omega-\alpha(\omega-1))}. \tag{F.18}$$

From equations (F.17) and (F.18), we obtain the following relationship between relative import capacity and relative prices:

$$\frac{\iota_{ni}}{\iota_{nj}} = \left(\frac{p^x_{ni}}{p^x_{nj}}\right)^{-(1-\alpha)(\omega-1)}.$$

Returning to our three equalities (F.16), we have:

$$\frac{\iota_{ni}}{\iota_{nj}} = \frac{p^x_{ni}x_{ni}}{p^x_{nj}x_{nj}} = \frac{\left(\iota^\alpha_{ni}x^{1-\alpha}_{ni}\right)^{\frac{\omega-1}{\omega}}}{\left(\iota^\alpha_{nj}x^{1-\alpha}_{nj}\right)^{\frac{\omega-1}{\omega}}}.$$

Using equations (F.17) and (F.18) to substitute for relative import capacity and relative imports, we obtain the following relationship between relative import capacity, relative imports and relative prices:

$$\frac{\iota_{ni}}{\iota_{nj}} = \frac{p^x_{ni}x_{ni}}{p^x_{nj}x_{nj}} = \frac{p^c_{ni}c_{ni}}{p^c_{nj}c_{nj}} = \left(\frac{p^x_{ni}}{p^x_{nj}}\right)^{-\theta^{DI}}, \tag{F.19}$$

where we have defined the trade elasticity as $\theta^{DI} \equiv \alpha+(1-\alpha)\,\omega-1 < \omega-1$. This trade elasticity is smaller in absolute value in the deterministic Armington model without import capacity (since $\omega > 1$). The reason is that the cost of consumption now includes both the cost of imports and the cost of import capacity. Therefore, reductions in variable trade costs only directly reduce part of the overall cost on consumption, and hence have less impact on overall consumption than in the deterministic Armington model without import capacity.

F.2.5 General Equilibrium

Substituting our solution for aggregate investments in import capacity (F.10) into the goods market clearing condition (F.14), the equality between expenditure on the goods produced by a country and its income can be written as:

$$\sum_{n=1}^{N} p_{ni}^{x} x_{ni} = w_i \left(1-\alpha\right) \ell_i, \tag{F.20}$$

where the left-hand side is the sum across all markets n of expenditure on imports from country i; and the right-hand side is the income of workers employed in production in i (including those used by producers to incur variable trade costs).

From the import technology (F.2), together with cost minimization and zero profits for importers, expenditure on imports is a constant share of overall consumption expenditure:

$$p_{ni}^{x} x_{ni} = \left(1-\alpha\right) p_{ni}^{c} c_{ni}. \tag{F.21}$$

Combining equations (F.20) and (F.21), we can re-write the equality between income and expenditure on the goods produced by a country:

$$w_i \ell_i = \sum_{n=1}^{N} p_{ni}^{c} c_{ni}. \tag{F.22}$$

From relative expenditures (F.19), expenditure shares satisfy:

$$s_{ni} = \frac{p_{ni}^{c} c_{ni}}{\sum_{j=1}^{N} p_{nj}^{c} c_{nj}} = \frac{\left(p_{ni}^{x}\right)^{-\theta^{DI}}}{\sum_{j=1}^{N} \left(p_{nj}^{x}\right)^{-\theta^{DI}}}. \tag{F.23}$$

Combining this expression for expenditure shares (F.23) with the consumer's budget constraint (F.6), expenditure by an importing country n on each exporting country i is:

$$p_{ni}^{c} c_{ni} = \frac{\left(p_{ni}^{x}\right)^{-\theta^{DI}}}{\sum_{j=1}^{N} \left(p_{nj}^{x}\right)^{-\theta^{DI}}} w_n \ell_{n.}. \tag{F.24}$$

Using this expression for expenditure (F.24), we can re-write the market clearing condition (F.22) as:

$$w_i \ell_i = \sum_{n=1}^{N} \frac{\left(p_{ni}^{x}\right)^{-\theta^{DI}}}{\sum_{j=1}^{N} \left(p_{nj}^{x}\right)^{-\theta^{DI}}} w_n \ell_{n.}. \tag{F.25}$$

Substituting for equilibrium prices (p_{ni}^{x}) using equation (F.13), we obtain the following system of N general equilibrium conditions that determines the N wages in each country:

$$w_i \ell_i = \sum_{n=1}^{N} \frac{\left(\tau_{ni} w_i / z_i\right)^{-\theta^{DI}}}{\sum_{j=1}^{N} \left(\tau_{nj} w_j / z_j\right)^{-\theta^{DI}}} w_n \ell_n. \tag{F.26}$$

We have thus established Proposition 3. In the absence of uncertainty, our model of import capacity is isomorphic to our baseline Armington model, given an adjustment to the trade elasticity (θ), which now reflects not only the elasticity of substitution (ω) but also the share of import capacity in consumption (α).

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