Global Investors in Local-Currency Bond Markets: Implications for Bond Yields and Exchange Rates*

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March 10, 2025

Abstract

The comovement between long-term bond yields and exchange rates differs between advanced and emerging economies. In emerging economies, lower bond term premia are accompanied by lower currency premia. We attribute this feature to the prevalence of global investors in local-currency bond markets. Using transaction-level data from Colombia's bond and foreign exchange markets, we document that any foreign investors' bond transaction is simultaneously associated with a corresponding transaction in the spot foreign exchange market. We incorporate these *correlated inflows* into a portfolio-balance model alongside short-term interest rate risk. The model explains the comovement in bond yields and exchange rates, the patterns of positions and returns in bond and foreign exchange markets, the effects of quantitative easing, and their differences between advanced and emerging economies.

JEL Classification: E43, E52, F31, G12

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1 Introduction

Long-term bond yields and exchange rates influence borrowing, saving, and international trade in open economies, and thus play a central role in the transmission of monetary policy and asset purchases. In this paper, we document fundamental differences in the joint behavior of long-term bond yields and exchange rates between advanced and emerging economies. We argue that these differences can be traced to the varying importance of global investors in local-currency sovereign bond markets, as they influence the properties of capital flows, bond and currency risk premia, and the transmission of monetary policy.

We document that the comovement between bond yields and exchange rates differs between advanced and emerging economies. While high local-currency bond excess returns correlate with low foreign exchange excess returns in advanced economies (Lustig et al., 2019), they correlate with high foreign exchange excess returns in emerging economies, both across countries and over time. We also document that default risk, proxied by credit default swaps (CDS) spreads, does not significantly contribute to this comovement pattern. This evidence challenges leading portfolio balance models, originally developed for advanced economies, which predict a negative comovement due to the inherent exposure of long-term bonds and currencies to short-term interest rate risk (Greenwood et al., 2023; Gourinchas et al., 2022).

We attribute the difference in comovement patterns to the dominant role of *global investors*—investors domiciled outside the country—in emerging economies' local-currency bond markets. Global investors' participation in local-currency sovereign bond markets has grown significantly over time, exhibiting far greater volatility than domestic investors and their counterparts in advanced economies. A key feature of global investors is that every purchase or sale of a country's local-currency bond is simultaneously associated with a purchase or sale of its currency—a feature we term *correlated inflows*. Using transaction-level data from Colombia's sovereign bond and foreign exchange markets, we provide direct evidence of correlated inflows. Whenever global investors enter the local-currency bond market, they simultaneously purchase the domestic currency to obtain bonds. Likewise, they sell domestic currency to liquidate local-currency sovereign bonds. Crucially, we show that the inflows into bond and spot foreign exchange markets are *not* offset by activity in the forward exchange market, indicating that these inflows are not systematically hedged.

We develop a formal portfolio balance model where global investors exert correlated inflows in long-term bond and currency markets. The structure of our small open economy model is otherwise similar to Greenwood et al. (2023) and Gourinchas et al. (2022). A set of

local intermediaries absorbs flows in both bond and currency markets, where part of these flows reflect correlated demand by global investors. Bond and currency returns are exposed to two main sources of risk. First, short-term interest rate risk affects bond and currency returns in opposite fashion. An unexpected increase in local short-term rates reduces the price of long-term bonds and appreciates the value of the currency. This induces a negative correlation in returns. Second, correlated inflows lead to an increase in bond price and a currency appreciation, as it induces local intermediaries to offload sovereign bonds and domestic currency, and thus demand lower term premia and currency premia—inducing a positive correlation in returns. Overall, the covariance in returns, and thus the risk profile of bonds and currencies, will depend on which of these two forces dominates.

Correlated inflows alter the risk profile of bonds and foreign exchange, and rationalize the evidence from emerging economies. When correlated inflows dominate short-term interest rate risk, the overall correlation between bond and currency returns is positive. Moreover, when local intermediaries intermediate both bonds and foreign currency, correlated inflows modify the exchange rate response also to *idiosyncratic* flows in the bond market (*e.g.*, flows due to domestic investors or the central bank). This occurs because correlated inflows make the U.S. dollar a hedge for local intermediaries holding local-currency sovereign bonds in emerging economies. As a result, whenever their sovereign bond holdings decline, they reduce their required returns on home currency, leading to home currency appreciation. Accordingly, the model sheds light on the evidence that central banks' quantitative easing leads to currency appreciation in emerging economies (Rebucci et al., 2022), but currency depreciation in advanced ones (Bhattarai and Neely, 2022).

We draw several implications of the model on returns and positions of different market participants, and confront them with transaction-level data from Colombia's sovereign bond and foreign exchange markets. To begin with, we provide evidence that *local* intermediaries, rather than global banks, act as the "financiers" that absorb imbalances in these markets. Exploiting a rebalancing in a major local-currency government bond index that led to foreign purchases of long-term government bonds, we show that local intermediaries absorbed the rebalancing-driven demand of local-currency bonds in both bond and foreign exchange markets. During this episode, foreign investors simultaneously buy bonds and home currency (Colombian pesos), supplied by local primary dealers in bond markets and local foreign exchange market intermediaries. In most instances, the same local financial entity operates simultaneously as primary dealer in bond market and foreign exchange intermediary. Besides, we measure the returns in both markets conditional on a change in positions of local

intermediaries, from 2012 to 2019. Consistent with the model, movements in excess bond or currency returns are systematically positively correlated with movements in positions of these local financial intermediaries—that is, dealers in bond and foreign exchange markets earn positive and significant returns in their trades in bond and currency markets. Relatedly, purchases of both foreign investors and other domestic investors (not intermediaries) are associated with a decline in subsequent excess returns. Last, to speak to the relevance of correlated inflows, we document that the change in local intermediaries' bond and foreign exchange positions display a positive correlation.

We stress that our findings have broader implications beyond emerging economies, as these insights extend to any economy where foreign investors become significant in local-currency government bond markets.

Related literature We document that bond and currency returns are positively correlated in emerging economies, in contrast to Lustig et al.'s (2019) evidence for advanced economies.[1] Taking a portfolio-balance approach, Greenwood et al. (2023) and Gourinchas et al. (2022) propose that bonds and currencies exposure to interest rate risk implies a negative correlation in bond and currency returns.[2] We show that correlated inflows into bond and currency markets—characteristic of global investors' behavior—instead force a positive correlation in bond and currency returns in otherwise standard models of portfolio balance, and rationalizes micro-level patterns of flows in currency and bond markets as well as the observed effects of QE in these economies (Rebucci et al., 2022).

We propose a new channel through which global investors in local-currency bond markets shape the dynamics of bond yields and exchange rates. Carstens and Shin (2019) highlight that local-currency bonds shift currency mismatch from emerging market borrowers to foreign lenders, a property referred to as "original sin redux." In this context, some evidence shows that broad-based dollar appreciations around advanced economies' monetary policy announcements are associated with an increase in emerging market bond spreads and real activity (Hofmann et al., 2020) and a reduction in emerging market bond holdings by foreign investment funds (Hofmann et al., 2022b).[3] This literature attributes these dynamics to a so-called "US dollar exchange rate risk-taking channel," by which U.S. dollar appreciations alter the risk-taking capacity of constrained global investors that evaluate returns in U.S.

[1] See also Lloyd and Marin (2024).
[2] Zou (2024) examines the role of time-varying convenience yields in this class of models.
[3] In the context of equity markets, Bruno et al. (2022) show that exchange rate fluctuations also tend to amplify stock market returns in emerging market economies, once expressed in U.S. dollar terms.

dollar terms (see, *e.g.*, Hofmann et al., 2022a).[4] In contrast, we emphasize that local-currency bond markets change the patterns of inflows that local intermediaries must absorb, thereby influencing the dynamics of bond yields and exchange rates through term and currency premia. This mechanism formalizes how flows influence currencies in the thinking of market participants and policymakers, and operates whether or not global investors are constrained or whether or not they evaluate returns in U.S. dollars. These factors are thus not essential here, but can amplify the quantitative role of the channel we propose.

We present transaction-level evidence showing that global investors exert correlated pressure on bond and currency markets, intermediated by local intermediaries, and argue that this feature shapes the joint dynamics of bond yields and exchange rates. This paper thus speaks to a broader literature on the role of global investors in emerging economies' real and financial fluctuations (Calvo et al., 1993), the determinants of short-term market rates (De Leo et al., 2024a), deviations from covered interest parity (De Leo et al., 2024b), deviations from uncovered interest parity (Kalemli-Ozcan, 2019; di Giovanni et al., 2021; Cormun and De Leo, 2024), sovereign borrowing costs (Zhou, 2024), the co-movement of a country's long-term yields with global bond markets (Xu, 2024), among others.

The paper also adds evidence to the role of flows in determining bond and currency values. Pandolfi and Williams (2019) examines how capital flows driven by mechanical rebalancings of benchmark indexes impact government bond prices, liquidity, and exchange rates. Williams (2018) uses Colombia's inclusion in *J.P. Morgan*'s emerging markets debt index to study how increased foreign access to sovereign debt markets boosts private credit availability.

2 Bond Yields and Exchange Rates in Emerging Economies

Lustig et al. (2019) document that the correlation between bond returns and foreign exchange returns diminishes as bond maturities increase. This pattern points to a negative correlation between long-term bond excess returns and currency excess returns in these economies. In contrast to advanced economies, this section shows that emerging markets (EMs) feature a positive correlation between long-term bond and currency excess returns.

[4] In a related, but distinct, literature Ottonello and Perez (2019), Du and Schreger (2022), and Engel and Park (2022) develop models that study the determinants and dynamics of a country's sovereign debt currency composition.

2.1 Correlation between bond and currency excess returns

Definitions We denote $P_t^{(n)}$ as the price of a zero-coupon bond of maturity n in local-currency terms at time t, with the continuously compounding yield on this bond given by $y_t^{(n)} = -\frac{1}{n}\log P_t^{(n)}$. The short-term risk-free rate r_t^f is the yield on a one-period bond, and the local-currency bond excess return on the domestic zero-coupon bond in local currency $rx_{t+1}^{(n)}$ is:

$$rx_{t+1}^{(n)} = -(n-1)y_{t+1}^{(n-1)} + ny_t^{(n)} - r_t^f.$$

In what follows, Q_t represents the nominal spot exchange rate in terms of domestic currency per U.S. dollar, with domestic currency depreciation expressed as $\Delta q_{t+1} = \log\left(\frac{Q_{t+1}}{Q_t}\right)$. The excess return on home currency, rx_{t+1}^{FX}, is

$$rx_{t+1}^{\text{FX}} = r_t^f - r_t^{f,US} - \Delta q_{t+1}.$$

Data We select all emerging economies with bond benchmark indexes on *Datastream*. We use monthly data from January 2006 to December 2021.[5] The dataset includes spot exchange rates, 10-year government bond total return indexes, and 3-month deposit rates.[6] Our sample includes 8 countries that have available data for the above variables. These countries are Czech Republic, India, Indonesia, Mexico, Poland, South Africa, South Korea and Thailand.[7]

For comparison, when appropriate, we also include advanced economies, selected following the same criteria as emerging economies. Our sample of advanced economies includes Australia, Canada, Denmark, the Eurozone, Japan, New Zealand, Sweden, Switzerland, and the United Kingdom.

Analysis Table 1 report the estimated slope coefficients in a set of regressions of foreign exchange excess returns, $rx_{j,t+1}^{\text{FX}}$, on local-currency bond excess returns, $rx_{j,t+1}^{(10y)}$. We report the results for three specifications: no fixed effects, month fixed effects, currency fixed effects. Across all specifications, there is a negative association between local currency excess bond returns and currency excess returns for advanced economies, while there is a positive relationship in emerging economies. In emerging economies high excess bond returns are associated with high excess currency returns both in the cross-section of currencies and in

[5]While we can technically cover some countries starting in December 1999, given that data is sparse before 2006, we start our analysis in 2006. We exclude data beyond December 2021 to avoid incorporating recent inflation dynamics and potential distortions.
[6]Note that the long maturity bond return data from *Datastream* may pertain to coupon government bonds.
[7]Datastream also has data on China. Given capital controls in China can distort foreign flows and correlated inflows, we exclude China from our analysis. All of our results are similar when including China to our sample.

Table 1
Foreign exchange and local-currency bond excess returns

	FX rx: Advanced economies			FX rx: Emerging Markets		
	(1)	(2)	(3)	(4)	(5)	(6)
Annualized LC bond return	-0.4195***	-0.7281***	-0.4178***	0.4667***	0.2863***	0.4762***
(in LC)	(0.0649)	(0.0929)	(0.0647)	(0.0671)	(0.0592)	(0.0678)
Month Fe	No	Yes	No	No	Yes	No
Currency Fe	No	No	Yes	No	No	Yes
N	1,719	1,719	1,719	1,161	1,161	1,161
R2	0.049	0.070	0.049	0.083	0.044	0.085

Note: Columns (1) and (4) of the table report the estimated slope coefficient for the following baseline regression $rx^{\text{FX}}_{j,t+1} = \alpha + \beta rx^{(10y)}_{j,t+1} + \epsilon_{j,t+1}$. Columns (2) and (5) add month fixed effects. Columns (3) and (6) add currency fixed effects. Standard errors are in parentheses. In all panels, HAC standard errors were used, allowing for 12-month autocorrelation. Significance stars follow conventional levels: *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$

the time series.

2.2 Does default risk explain the comovement in bond-currency returns?

A possible explanation for the positive association between bond and currency excess returns in emerging markets is default risk. An increase in default risk can lead to lower excess returns on both bonds and currencies, driven by currency depreciation and lower bond prices.

Using credit default swaps (CDS) to measure default risk, we show that default risk is not the primary driver of these relationship between excess bond and currency returns in emerging economies. First, Table 2 incorporates CDS spreads as a control in the baseline regression analysis. The positive correlation between bond and currency returns remains robust, whether or not one includes month fixed effects or country fixed effects. While multicollinearity could be a potential concern, the low correlations between excess bond returns and CDS spreads ($\rho = -0.03$) and between CDS spreads and excess currency returns ($\rho = 0.07$) mitigate this issue.

To isolate returns from default risk, we regress currency excess returns on CDS spreads to derive residuals representing currency returns net of default risk, and similarly regress bond excess returns on CDS spreads to obtain bond returns net of default risk. Figure A.3 depicts the relationship between currency and bond excess returns after accounting for default risk. The scatterplot of these residuals confirms that the positive correlation between bond and currency excess returns persists, reinforcing the conclusion that default risk does not account for the bulk of this relationship.

We further investigate the relationship between excess bond and currency returns by categorizing CDS spreads into low, medium, and high groups based on relative CDS levels within each country. Specifically, Panel A of Table 3 sorts each country's excess returns

Table 2
Correlation of excess returns controlling for CDS spreads in EM

	EM without CDS			EM with CDS		
	(1)	(2)	(3)	(4)	(5)	(6)
Annualized LC bond return (in LC)	0.4667*** (0.0671)	0.2863*** (0.0592)	0.4762*** (0.0678)	0.5080*** (0.0688)	0.3060*** (0.0710)	0.5055*** (0.0686)
CDS (%)				4.5874*** (1.7162)	2.9362** (1.3380)	7.3135** (2.9987)
Month Fe	No	Yes	No	No	Yes	No
Currency Fe	No	No	Yes	No	No	Yes
N	1,161	1,161	1,161	942	942	942
R2	0.083	0.044	0.085	0.104	0.052	0.107

Note: The first three columns show the slope of baseline regression $rx^{\text{FX}}_{j,t+1} = \alpha + \beta rx^{(10y)}_{j,t+1} + \epsilon_{j,t+1}$ using different sets of fixed effects. The last three columns add to the baseline regression credit default swaps as controls and also reports those coefficients for various sets of fixed effects. Standard errors are in parentheses. In all panels, HAC standard errors were used, allowing for 12-month autocorrelation. Significance stars follow conventional levels: *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$

into CDS buckets according to its own historical CDS spreads over time. The positive correlation between bond and currency excess returns persists even during periods when countries experience low CDS levels relative to their own history. This relationship appears robust and is not driven by specific dates or countries. Across all CDS environments (low, medium, and high relative to a country's history), the positive correlation remains significant, even after controlling for month and country (currency) fixed effects.

Panel B of Table 3 extends this analysis by categorizing CDS spreads at the monthly level. In this framework, countries are sorted into low, medium, and high CDS groups within each month, enabling a cross-sectional analysis where all countries are compared relative to one another during the same time period. The positive correlation between bond and currency excess returns persists. Similar to the results obtained when sorting CDS spreads by country, these correlations are not attributable to specific countries or particular dates.

3 Global Investors and Correlated Inflows

A potential explanation for the comovement documented in Section 2 hinges on the important role of global investors in local-currency bond markets of emerging economies. To purchase local currency bonds, these investors purchase domestic currency, giving rise to positively correlated flows into bond and foreign exchange spot markets.

3.1 Global investors in local-currency bond markets

There is extensive evidence on the increased share of foreign investors in emerging economies' local-currency bond markets (*e.g.*, Carstens and Shin, 2019). In this section, we present

Table 3
Correlation of excess returns sorted by CDS

Panel A: Correlation of excess returns: CDS sorted by country

	Low CDS			Medium CDS			High CDS		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Annualized LC bond return (in LC)	0.5686*** (0.1437)	0.2905* (0.1547)	0.5747*** (0.1453)	0.5453*** (0.1302)	0.1547 (0.1499)	0.5439*** (0.1286)	0.7896*** (0.1201)	0.4924** (0.2032)	0.7982*** (0.1303)
Month Fe	No	Yes	No	No	Yes	No	No	Yes	No
Currency Fe	No	No	Yes	No	No	Yes	No	No	Yes
N	263	229	263	216	159	216	199	139	199
R2	0.128	0.039	0.130	0.156	0.016	0.154	0.195	0.096	0.198

Panel B: Correlation of excess returns: CDS sorted each month

	Low CDS			Medium CDS			High CDS		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Annualized LC bond returns (in LC)	0.2145** (0.1037)	0.1876 (0.1454)	0.2353** (0.1047)	0.5916*** (0.1374)	0.2750* (0.1552)	0.5757*** (0.1382)	0.8148*** (0.1024)	0.5087** (0.2364)	0.8227*** (0.1066)
Month Fe	No	Yes	No	No	Yes	No	No	Yes	No
Currency Fe	No	No	Yes	No	No	Yes	No	No	Yes
N	336	292	336	242	180	242	267	230	267
R2	0.021	0.015	0.026	0.129	0.049	0.125	0.267	0.111	0.271

Note: Panel A and Panel B show the correlation between the FX excess returns and local currency bond excess returns for emerging market countries, sorted by either their own sample-average CDS spreads (Panel A) or by its CDS magnitude across countries within the same month (Panel B).
Standard errors are in parenthesis. In all panels, HAC standard errors were used, allowing for 12-month autocorrelation. Significance stars follow conventional levels: *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$.

evidence on their importance relative to other domestic investors, especially comparing emerging and advanced economies. Using the data on emerging and advanced market debt from Arslanalp and Tsuda (2014) and Arslanalp and Tsuda (2012), respectively, we compute the share of foreign investors in local-currency government debt securities. Additionally, we construct the share of non-bank domestic investors (excluding both banks and central bank holdings).[8] We focus on non-bank domestic investors, as we later show that domestic banks act as dealers in long-term bond and currency markets. Then, for each country in our sample, we calculate the standard deviation of both the foreign and non-bank domestic investors, which we present in Table 4.

Table 4 highlights the larger relative volatility of foreign investors compared to domestic investors in emerging economies. While there is considerable variation across emerging economies, the average (median) ratio of the standard deviation of the foreign to non-bank domestic share of local-currency sovereign debt is 1.33 (1.27). In contrast, in advanced economies, non-bank domestic investors tend to be more volatile than foreign investors, with an average (median) ratio of 0.7 (0.74). Overall, foreign investors appear to play a more

[8]For emerging economies, we construct these shares based on the share of non-bank domestic investors for all sovereign debt since there is no breakdown for local-currency government debt securities.

Table 4
Volatility of Foreign and Domestic Investors in Local-Currency Bond Markets

Country	Foreigners (Std. Dev. Share)	Domestic Non-Bank (Std. Dev. Share)	Ratio (1)/(2)
Emerging Markets (median)	***8.19%***	***6.72%***	***1.27***
Argentina	10.49%	8.33%	1.26
Brazil	3.99%	3.52%	1.13
Chile	4.73%	10.81%	0.44
China	4.02%	1.46%	2.76
Colombia	7.87%	8.68%	0.91
Czech Republic	6.90%	4.10%	1.69
Egypt	11.16%	8.09%	1.38
Hungary	11.60%	6.70%	1.73
India	1.12%	4.63%	0.24
Indonesia	11.68%	6.63%	1.76
Korea	5.37%	11.68%	0.46
Malaysia	5.03%	3.94%	1.28
Mexico	12.44%	8.85%	1.40
Peru	13.71%	4.90%	2.80
Philippines	3.66%	4.94%	0.74
Poland	8.51%	7.47%	1.14
Romania	4.50%	7.67%	0.59
Russia	9.55%	9.39%	1.02
Slovenia	14.46%	11.84%	1.22
South Africa	9.89%	6.22%	1.59
Thailand	4.22%	3.79%	1.11
Turkey	6.94%	3.68%	1.89
Ukraine	10.36%	6.75%	1.53
Uruguay	13.09%	7.47%	1.75
Advanced Economies (median)	***6.71%***	***8.94%***	***0.74***
Australia	6.97%	10.74%	0.65
Canada	3.27%	6.15%	0.53
Denmark	6.45%	6.05%	1.06
Japan	3.50%	11.80%	0.30
New Zealand	8.74%	8.31%	1.05
Norway	9.38%	10.78%	0.87
Sweden	8.81%	9.05%	0.97
Switzerland	7.07%	8.83%	0.80
United Kingdom	2.96%	24.28%	0.12
United States	3.91%	5.82%	0.67

significant role in terms of overall volatility in emerging economies compared to non-bank domestic investors. Relative volatility is the relevant measure of global investors' relevance in the model we outline in Section 4.

3.2 Correlated inflows in bond and currency markets

Given the significant role of foreign investor inflows in local-currency bond markets, we dig deeper into the characteristics of these inflows. To this end, we leverage transaction-level data from Colombia's government bond and foreign exchange markets, which provide insights into foreign investors' behavior in both arenas. Colombia offers a representative example of a small open economy with levels of foreign participation in its sovereign bond market comparable to those of other emerging markets.

3.2.1 Institutional setup and data

Government bonds (*Títulos de Tesorería*, TES) Since the 2000s, the majority of Colombia's sovereign debt is denominated in domestic currency (COP). For instance, in 2020,

the total fiscal debt reached $177 billion USD, equivalent to approximately 65% of GDP. Two-thirds of total debt was denominated in COP, comprising 20% in inflation-adjusted bonds and 46% in standard COP-denominated instruments.

The *Ministry of Finance* annually publishes rankings of financial sector participants competing for inclusion in the "primary dealers" (PD) program for TES. Due to limited membership, only institutions ranked above a specified threshold—10^{th} place prior to 2022, and 15^{th} place thereafter—qualify as primary dealers. On average, the primary market issues approximately 300 billion COP (less than $100 million USD) worth of TES bonds daily. In contrast, the secondary market for TES, detailed below, experiences a significantly higher daily turnover of around 2 trillion COP ($500 million USD).

Colombia foreign exchange market (FX) The COP-USD spot and forward interdealer market in Colombia is highly centralized. A single electronic trading platform, *SET-ICAP FX*, accounts for approximately 95% of total market volume. Offshore trading is restricted by regulatory measures. Transactions in the foreign exchange market are restricted to authorized dealers (*Intermediarios del Mercado Cambiario*, IMC). Consequently, all market participants must conduct transactions through one of these 50 authorized dealers. These intermediaries include banking institutions, financial corporations, financial cooperatives, and stock brokerage firms. On average, the daily turnover for the spot market is $1.5 billion USD, while the forward market sees a turnover of $4 billion USD.

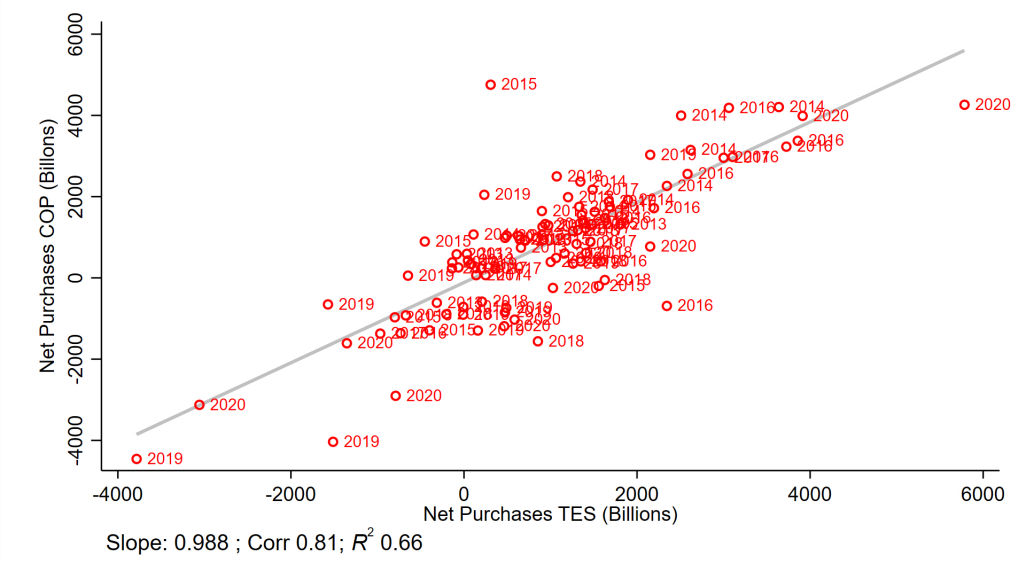
Colombia micro-level data We use several datasets to perform our analysis. Transactions involving purchases and sales of TES in the secondary market take place on two trading platforms: (i) the Colombian Electronic Market (MEC), operated by the Colombian Stock Exchange, and (ii) the Electronic Trading System (SEN), managed by the Central Bank of Colombia. All trades executed on these platforms are registered in the Central Securities Depository (DCV), overseen by the Central Bank of Colombia. To identify trades involving foreign investors, we use data from the "Declarations of Foreign Exchange Transactions" (*Declaraciones de Cambio*), which records all transactions involving foreign exchange and is compiled by the Technical and Economic Information Department of the Central Bank of Colombia. Additionally, we analyze FX Spot, Next-Day, and Forward markets in the interdealer market using data from *SET-ICAP FX*.[9]

[9]A challenge we face is that, while TES trades can be directly attributed to individual foreign investors, FX transactions may be conducted either directly by these investors or through intermediaries, obscuring the identity of the ultimate buyer. Consequently, it is not possible to establish a precise link between FX transactions and TES trades at the individual transaction level.

3.2.2 Evidence on correlated inflows in bond and currency markets

Central to our analysis is the simple observation that when foreign investors purchase local-currency government bonds, they simultaneously need to purchase COP through the foreign exchange market. Figure 1 plots the monthly-level foreign investor purchases of COP (in exchange for foreign currency) through the FX market (y-axis) together with the foreign purchases of TES through the secondary market for government bonds (x-axis). Purchases in both markets from foreign investors align tightly around the 45-degree line. A regression of COP purchases by foreigners on TES purchases by foreigners has a slope that is statistically not different from 1, with an R-squared of 65%. Overall, this evidence suggests that foreign inflows to local-currency sovereign bond markets occur simultaneously with inflows to the spot FX market. In addition, it suggests that a large portion of spot FX market transactions by foreign investors is due to their purchase or sale of Colombia sovereign bonds. We denote the occurrence of these simultaneous flows as "correlated inflows."

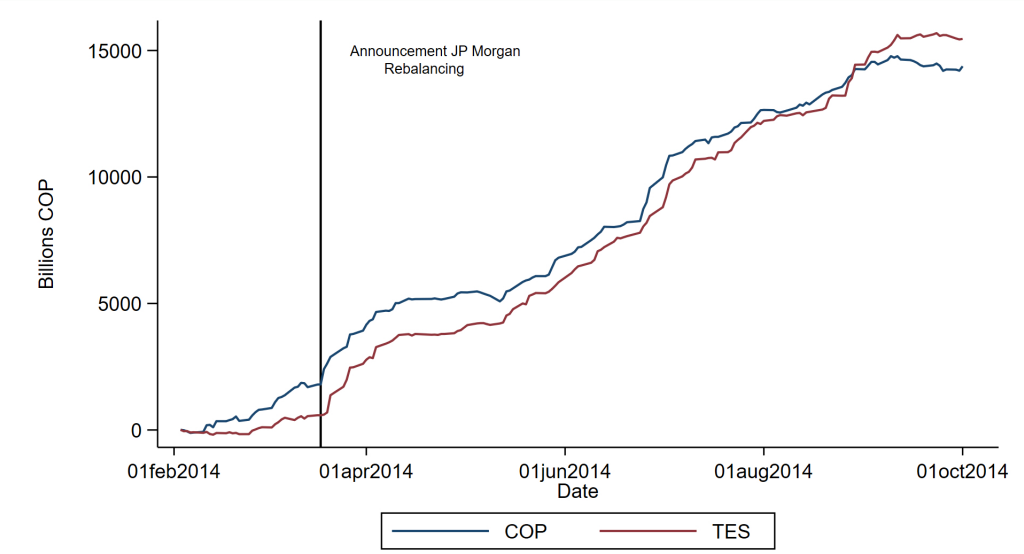
Figure 1
Correlated Inflows in TES and FX Market



Note: This figure displays the monthly-level foreign purchases of TES bonds in billions COP (x-axis) and the monthly-level foreign purchases of COP in the spot market (y-axis). Slope is the slope of a regression of the y-axis variable on the x-axis variable, with the corresponding R-squared.

Correlated inflows during episode of large foreign purchases of local-currency bonds We provide further evidence of these correlated inflows through an episode of significant foreign investor purchases of TES. In March 2014, J.P. Morgan announced the inclusion of five Colombian local-currency government bonds in its flagship benchmark index for emerging market local-currency government bonds. This resulted in massive purchases

Figure 2
Purchases of TES and COP in J.P. Morgan Rebalancing



Note: This figure displays the evolution of the daily accumulated foreign purchases of of TES bonds together with the daily accumulated foreign purchases of COP in the spot market during the J.P. Morgan rebalancing in 2014. The black vertical line denotes 19th of March 2014, the day the rebalancing was announced.

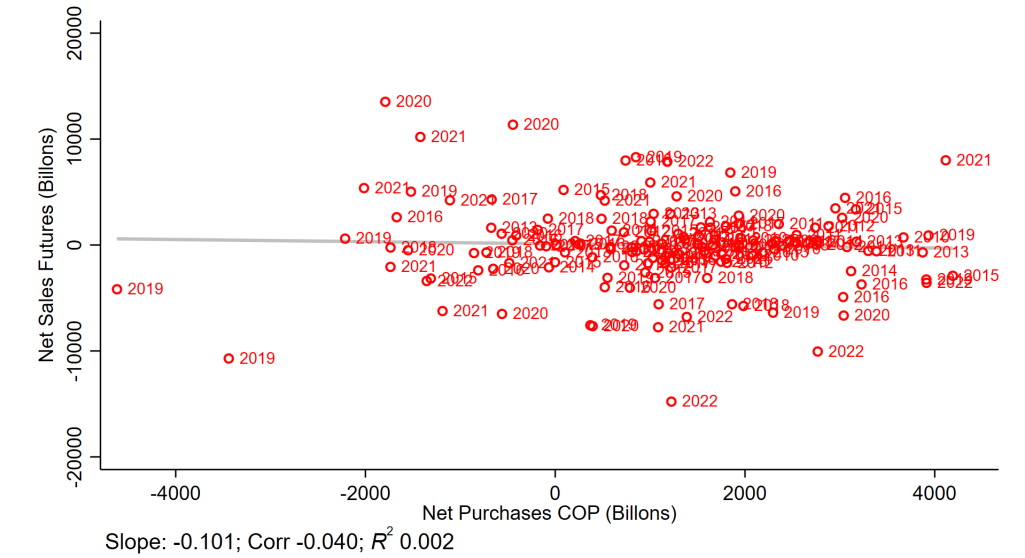
by index-tracking investors, who acquired nearly 10% of the outstanding TES bonds from the announcement until the full inclusion was effective in October 2014 (Williams, 2018). Figure 2 illustrates foreign investor purchases of both TES and COP (in the spot market) during this period. The two lines track each other closely, with the volume of purchases in both markets being very similar. This shock plausibly originates in the local-currency bond market and spills over into the FX market through correlated inflows from foreign investors.

3.3 Do global investors hedge emerging market currency risk?

Correlated inflows from foreign investors may have limited implications for currency markets overall if foreign investors systematically hedge their domestic currency exposure by selling it in forward markets to offset their spot market purchases of COP. As a result, their net exposure to the local currency—and that of other investors—may not necessarily change with inflows into the government bond market. To investigate this further, we analyze data from Colombia's forward markets.

Figure 3 reports foreign investor purchases of COP in the spot market (on the y-axis) and their sales of COP in the forward market (on the x-axis). If foreign investor purchases in the spot market were fully hedged in the forward market, we would observe the data points to align along the 45-degree line, while a partial hedge would result in a positive relationship between the two variables. However, we observe no consistent relationship, indicating that

Figure 3
Purchases of COP Spot and Sales in Future Markets



Note: This figure displays the monthly-level foreign purchases of COP in the spot market (x-axis) and the monthly-level foreign sales of COP in the forward market (y-axis). Slope is the slope of a regression of the y-axis variable on the x-axis variable, with the corresponding R-squared.

foreign investors are not systematically hedging their positions.[10]

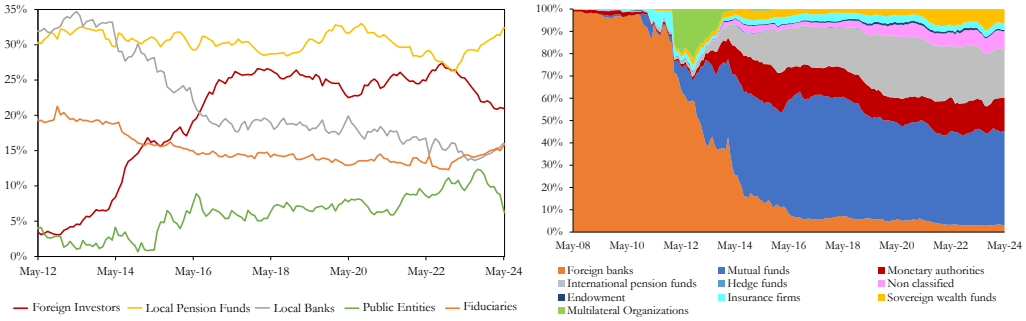
Our evidence from Colombia suggests that foreigners tend to produce correlated inflows in bond and currency markets. Additionally, the extent of currency hedging from foreigners that purchase local-currency government bonds seem negligible. We next explore the extent of currency hedging by foreign investors exploring data beyond Colombia.

EM currency risk management of foreign investors First, we document the type of foreign investors operating in Colombia's sovereign bond market. As shown in Figure 4a, foreign participation in Colombia's bond market was limited before 2012 but increased significantly thereafter. This growth was partly driven by regulatory reforms implemented between 2010 and 2013, which facilitated investment through local managers and simplified tax reporting for fixed-income securities. Furthermore, a 2012 tax reform reduced income tax on TES for non-resident investors from 33% to 14%. The country's investment-grade credit rating, awarded in 2011 by credit rating agencies such as *S&P*, *Moody's*, and *Fitch Ratings*, and its increased weight in *J.P. Morgan*'s emerging market debt indices in 2014, also played a key role in attracting international capital.

Figure 4b shows the share of bond holding across foreign investors. Before 2014 foreign

[10]This evidence, including the observation that foreign investors' transactions in the forward exchange market are around three times larger than their transactions in the spot exchange market, aligns with De Leo et al.'s (2024b) analysis, which suggests that foreign investors primarily utilize the forward market for currency speculation rather than for hedging purposes.

Figure 4
Participation in the Colombian TES market



(a) Total bond holdings by entity

(b) Bond holdings across foreign investors

Note: The data presented correspond to all trades registered with the Central Securities Depository (DCV), which operates under the supervision of the Central Bank of Colombia. Panel A shows the TES participation (percentage of total outstanding TES volume) across major entities, while Panel B focuses specifically on the share held by foreign investors, detailing the distribution among different types of foreign investors within the total foreign share.

banks held the largest share of sovereign bonds among foreign investors, although their overall market participation was relatively low. Since 2014, mutual funds have become the dominant investors, followed by international pension funds and monetary authorities, collectively holding about 80% of the foreign investor base, which is now more significant relative to the total outstanding debt.

Overall, foreign investors that participate in the TES market in Colombia are primarily institutional investors such as mutual and pension funds. We thus explore the extent of currency hedging by global mutual funds that are present in emerging economies local-currency bond markets. We use data from *Morningstar Direct* to collect information on hedging patterns of international mutual funds investing in fixed income. Table 5 displays summary statistics for foreign funds investing in advanced and emerging economies fixed income such as government bonds. For advanced economies (left panel), we observe a significant amount of hedging for these funds with a total hedging of 27%. This is largely consistent with the evidence in Sialm and Zhu (2024).

We also provide a similar analysis for international mutual funds and ETFs that invest in local-currency emerging market government bonds (right panel). The hedging ratios for these fixed income funds are substantially smaller than those that invest in advanced economies. The total hedging ratio is slightly below 5%, with the median fund almost not hedging at all. This evidence is consistent with the little amount of currency hedging by foreign investors that purchase local treasury bonds in Colombia, and suggests that currency hedging by institutional investors that target local-currency emerging market bonds is limited, in line

Table 5
Hedging for Foreign Mutual Funds and ETFs

		Holdings of AEs' FI				Holdings of EMs' FI (local currency)	
Top-20 AEFI holdings	***Type***	***USD bn***	***% hedged***	***Top-20 EMFI holdings***	***Type***	***USD bn***	***% hedged***
iShares	ETF	112.59	25.6%	PIMCO	ETF	4.67	0.0%
JPMorgan	OEF	31.45	18.7%	Colchester	OEF	4.28	2.1%
AllianceBernstein	OEF	28.78	13.6%	BlackRock	OEF	3.93	7.0%
UBS	OEF	20.45	46.9%	Degroof Petercam	OEF	3.68	0.0%
Fidelity International	OEF	15.94	16.4%	State Street	OEF	3.59	0.0%
PIMCO	OEF	15.76	27.9%	State Street	OEF	3.28	0.0%
Vanguard	ETF	12.76	32.3%	iShares	ETF	3.27	0.0%
State Street	ETF	10.92	3.0%	Legal & General	ETF	3.17	0.0%
BlackRock	OEF	10.38	50.8%	VanEck	OEF	2.77	0.0%
AXA IM	OEF	10.26	34.7%	Pictet	OEF	2.20	10.2%
Lord Abbett	OEF	9.63	7.3%	HSBC	OEF	2.09	7.2%
Xtrackers	ETF	8.29	51.8%	Deka	ETF	2.03	0.0%
Amundi	OEF	7.46	28.9%	Ninety One	OEF	1.95	10.0%
Neuberger Berman	OEF	7.45	45.9%	LGT	OEF	1.82	25.0%
Nomura	OEF	7.40	24.7%	Aktia	OEF	1.81	0.0%
Amundi	ETF	6.97	42.5%	Franklin Templeton	ETF	1.81	6.7%
Nordea	OEF	6.79	86.8%	Barings	OEF	1.72	1.4%
JPMorgan	ETF	6.71	51.3%	Capital Group	ETF	1.64	0.1%
Goldman Sachs	OEF	6.16	12.2%	Ashmore	OEF	1.43	29.0%
UBS	ETF	6.13	48.7%	abrdn	ETF	1.34	0.0%
Total Top-20		342.31	28.3%	Total Top-20		52.49	3.7%
All other		120.24	23.1%	All other		24.70	7.4%
TOTAL		***462.55***	***26.9%***	***TOTAL***		***77.76***	***4.9%***
Mean (Top-50)		*8.36*	*28.3%*	*Mean (Top-50)*		*1.48*	*6.7%*
Standard Deviation (Top-50)		*16.24*	*26.1%*	*Standard Deviation (Top-50)*		*1.15*	*11.4%*
25th Percentile (Top-50)		*2.02*	*6.3%*	*25th Percentile (Top-50)*		*0.61*	*0.0%*
Median (Top-50)		*3.58*	*25.1%*	*Median (Top-50)*		*1.04*	*0.2%*
75th Percentile (Top-50)		*7.46*	*46.7%*	*75th Percentile (Top-50)*		*1.92*	*8.4%*

Notes: The table reports information on hedging practices by global mutual funds and exchange-traded funds that invest in emerging and advanced economies' fixed-income securities. The data sources is *Morningstar Direct*. For emerging economies, we include only funds that belong to 'local currency bonds' category of *Morningstar*. We exclude funds that invest in their country of origin.

with the evidence in Chen and Zhou (2025).

In summary, global investors play a significant role in the local-currency bond markets of emerging economies, and, notably, they exhibit correlated inflows in both bond and currency markets, with minimal to no hedging in the forward exchange market. We next incorporate these dynamics into a portfolio-balance model.

4 Bond and Currency Premia in a Model of Correlated Inflows

Our baseline model extends the portfolio-balance models of Greenwood et al. (2023) and Gourinchas et al. (2022) to a setting with correlated inflows into bond and foreign exchange market, consistent with the micro-level evidence from Section 3.[11]

In our small-open economy model, sovereign bond and foreign-exchange markets are integrated with one another but segmented from other financial markets. A group of local

[11] Greenwood et al. (2023) and Gourinchas et al. (2022) build on Vayanos and Vila (2021) and Gabaix and Maggiori (2015).

intermediaries trades in both of these markets, conducting both the "yield-curve trade"—the trade that borrows short-term and lends long-term in domestic currency—and the "UIP trade"—the trade that borrows short-term in foreign currency and lends short-term in domestic currency. These two distinct trades are exposed to two sources of risk: interest rate risk, as in Greenwood et al. (2023) and Gourinchas et al. (2022), and risk associated with correlated inflows due to global investors in local-currency bond markets.

In this section, we show that interest rate risk and correlated inflows imply opposite correlations between bond and currency returns. If advanced and emerging economies are differently exposed to these sources of risk, they would present different comovement patterns, consistent with Section 2. In Section 5 we present additional testable implications of the model and confront it with data from Colombia's long-term local-currency bond and foreign exchange markets. In Section 6 we present the model's implication for the exchange rate effects of quantitative easing policies.

4.1 Baseline Model

The discrete-time small-open economy model includes four financial assets: domestic and foreign short- and long-term bonds, denominated in their respective currency. There are two types of agents: local and global bond investors and local intermediaries. Bond investors have a preference— resulting in inelastic demand—for assets of specific currencies and maturities. Local intermediaries, on the other hand, are specialized investors who absorb the net supply of domestic long-term bonds and foreign exchange resulting from exogenous demand shocks. Consistent with the small open economy setting, the returns on foreign short- and long-term bonds, such as U.S. dollar treasuries, follow an exogenous process. We follow Greenwood et al. (2023) and express returns in logs.

Short-term bonds Short-term bonds in both currencies are supplied perfectly elastically, and short-term interest rates are determined exogenously according to AR(1) processes with potentially correlated shocks:

$$i_{t+1} = \bar{\iota} + \phi_i(i_t - \bar{\iota}) + \varepsilon_{i_{t+1}}; \tag{1}$$

$$i^\star_{t+1} = \bar{\iota} + \phi_i(i^\star_t - \bar{\iota}) + \varepsilon_{i^\star_{t+1}}, \tag{2}$$

where $\bar{\iota} > 0$, $\phi_i \in (0,1)$, $\text{var}_t[\varepsilon_{i_{t+1}}] = \text{var}_t[\varepsilon_{i^\star_{t+1}}] = \sigma_i^2 > 0$, and $\text{corr}[\varepsilon_{i_{t+1}}, \varepsilon_{i^\star_{t+1}}] = \rho \in [0,1]$.

Long-term bonds Long-term bonds are default-free perpetuities whose payments decline geometrically. In each period, domestic long-term bonds are available in a given net supply, denoted as s_t^y, which is equal to their gross issuance minus the demand from bond investors. The net supply of domestic long-term bonds follow AR(1) process:

$$s_{t+1}^y = \bar{s}^y + \phi_{s^y}(s_t^y - \bar{s}^y) + \varepsilon_{s_{t+1}^y}. \tag{3}$$

where $\bar{s}^y > 0$, $\phi_{s^y} \in [0, 1)$, $\varepsilon_{s_{t+1}^y} = \eta_{t+1} + \eta_{t+1}^\star$, $\text{var}_t[\varepsilon_{s_{t+1}^y}] = \sigma_\eta^2 + \sigma_{\eta^\star}^2 \geq 0$. We distinguish here between demand shocks coming from local bond investors, η_{t+1}, and global bond investors, $\eta_{t+1}^\star$, and we allow them to have different stochastic properties.

The domestic currency return on long-term domestic bonds from t to $t+1$ is

$$r_{t+1}^y = y_t - \frac{\delta}{1-\delta}(y_{t+1} - y_t) - g_t, \tag{4}$$

where y_t is the log yield-to-maturity on domestic long-term bonds, and parameter $\delta \in (0, 1)$ is the bond's payment decline rate.[12] Equation (4) expresses the bond's return as consisting of three components: (i) a carry component, y_t; (ii) a capital gain component, $\frac{\delta}{1-\delta}(y_{t+1} - y_t)$; and (iii) a stochastic, time-varying tax to bond returns, g_t. This last term introduces a time-varying wedge in the price of domestic long-term bonds without directly influencing the fundamental value of the exchange rate. It can be interpreted as a time-varying tax on local-currency bond returns, or a time-varying convenience yield, and does not play an essential role in the analysis. It follows:

$$g_t = \phi_g g_{t-1} + \varepsilon_{g_t}, \tag{5}$$

where $\phi_g \in [0, 1)$, and $\text{var}_t[\varepsilon_{g_t}] = \sigma_g^2 \geq 0$

Let rx_{t+1}^y denote the excess return on domestic bonds, that is the return on the "yield-curve trade"—the trade that borrows short-term and lends long-term in domestic currency. Iterating equation (4) forward and taking expectations, one obtains:

$$y_t = (1-\delta)\sum_{j=0}^{\infty} \delta^j \, \text{E}_t[i_{t+j} + rx_{t+j+1}^y + g_{t+j}], \tag{6}$$

which indicates that the domestic long-term yield can be decomposed into the standard expectations hypothesis and term premium components, as well as a term that accounts for all current and future expected taxes on bond returns.

[12] Equation (4) is derived in Appendix B.1 and uses Campbell and Shiller (1988) approximation for log returns.

The yield on foreign long-term bond is determined as

$$y_t^\star = (1-\delta)\sum_{j=0}^{\infty}\delta^j \,\mathrm{E}_t[i_{t+j}^\star + rx_{t+j+1}^{y,\star}], \tag{7}$$

where $rx_{t+j+1}^{y,\star}$ is exogenous from the perspective of the small open economy. For simplicity, we assume that local intermediaries do not hold foreign long-term bonds.

Foreign exchange Let the net supply of home currency (that is, net of the demand from bond investors) follow a stochastic process such that

$$s_{t+1}^q = \phi_{s^q} s_t^q + \varepsilon_{s_{t+1}^q}, \tag{8}$$

where $\phi_{s^q} \in [0,1)$, $\varepsilon_{s_{t+1}^q} = \gamma_{t+1} + \gamma_{t+1}^\star$, and $\mathrm{var}_t[\varepsilon_{s_{t+1}^q}] = \sigma_\gamma^2 + \sigma_{\gamma^\star}^2 \geq 0$, distinguishing between demand shocks coming from local FX investors, γ_{t+1}, and global FX investors, $\gamma_{t+1}^\star$. Let q_t denote the log nominal exchange rate, expressed as units of home currency per unit of foreign currency. The log excess return on home currency, i.e. on the "UIP trade"—the trade that borrows short-term in foreign currency and lends short-term in domestic currency, is:

$$rx_{t+1}^q = (i_t - i_t^\star) - (q_{t+1} - q_t). \tag{9}$$

Iterating this expression forward and taking expectations yields:

$$q_t = \sum_{j=0}^{\infty}\mathrm{E}_t[(i_{t+j}^\star - i_{t+j}) + rx_{t+j+1}^q] + \mathrm{E}_t\, q_{t+\infty}. \tag{10}$$

Equation (10) expresses the exchange rate as the sum of three components: (i) the expected future sum of interest rate differentials, i.e. the UIP component; (ii) a foreign exchange risk premium; and (iii) the expected long-run nominal exchange rate. We assume that the long-term nominal exchange rate follows a random walk. That is:

$$q_{t+\infty} = \varepsilon_{q_t}, \tag{11}$$

where $\mathrm{var}_t[\varepsilon_{q_t}] = \sigma_q^2 \geq 0$. Time variation in the long-term value of the nominal exchange rate introduces an additional, yet independent, source of risk for the UIP trade. While it is unmodeled here, time variation in the value of the long-term nominal exchange rate arises naturally in open-economy macroeconomic models with differential inflation rates across countries, while maintaining real exchange rate stationarity.

Global investors in local currency bond markets Departing from the framework of Greenwood et al. (2023) and Gourinchas et al. (2022), we allow for a non-zero covariance

between the inelastic demand of domestic long-term bond and foreign exchange by global bond investors. In particular, consistent with the micro evidence from Section 3, we assume that any purchase of domestic long-term bonds by foreign bond investors is accompanied by a simultaneous purchase of home currency—of same local-currency amount. That is:

$$\text{corr}_t(\eta^\star_{t+1}, \gamma^\star_{t+1}) = 1. \tag{12}$$

This correlation arises naturally, as global investors intending to purchase domestic long-term bonds must inevitably acquire a corresponding amount of the home currency.

To the contrary, bond and currency demand of *domestic* investors are uncorrelated:

$$\text{corr}_t(\eta_{t+1}, \gamma_{t+1}) = 0. \tag{13}$$

As a result, the overall correlation in net supply is (using eqs. (3), (8), (12) and (13)):

$$\text{corr}_t(s^y_{t+1}, s^q_{t+1}) = \frac{\text{cov}_t[s^y_{t+1}, s^q_{t+1}]}{\sqrt{\text{var}_t(s^y_{t+1})}\sqrt{\text{var}_t(s^q_{t+1})}} = \frac{\sigma_{\eta^\star}\sigma_{\gamma^\star}}{\sqrt{(\sigma^2_\eta + \sigma^2_{\eta^\star})}\sqrt{(\sigma^2_\gamma + \sigma^2_{\gamma^\star})}} \equiv \Omega. \tag{14}$$

The cross-market correlation in global investors' inflows generates correlation of net supply across bond and foreign exchange markets in proportion to the relative importance of foreign inflows into the domestic bond and foreign exchange markets. Equation (14) highlights that the relative importance of global investors in local-currency bond markets can be measured in terms of their relative volatility.

Local intermediaries Local intermediaries are the marginal investors in domestic long-term bond and foreign exchange market. They maximize their next-period wealth through mean-variance preferences with a risk tolerance parameter τ. Defining $\mathbf{d}_t \equiv [d^y_t, d^q_t]'$ as the domestic-currency market value of intermediaries' holdings of long-term domestic bonds and home currency, they solve:

$$\max_{\mathbf{d}_t} \left\{ \mathbf{d}'_t \, \text{E}_t[\mathbf{rx}_{t+1}] - \frac{1}{2\tau} \mathbf{d}'_t \, \text{var}_t[\mathbf{rx}_{t+1}] \mathbf{d}_t \right\}, \tag{15}$$

where $\mathbf{rx}_{t+1} \equiv [rx^y_{t+1}, rx^q_{t+1}]'$ is a vector of log excess returns. Taking first-order condition yields the optimality condition faced by local intermediaries:

$$\text{E}_t[\mathbf{rx}_{t+1}] = \tau^{-1} \, \text{var}_t[\mathbf{rx}_{t+1}] \mathbf{d}_t. \tag{16}$$

Equation (16) links the intermediaries' expected excess returns on different assets to their asset holdings. The variance-covariance matrix of excess returns governs the equilibrium relationship of excess returns across the yield-curve trade and the UIP trade.

Market clearing The market clearing condition is:

$$\mathbf{s}_t = \mathbf{d}_t, \tag{17}$$

where $\mathbf{s}_t = [s_t^y, s_t^q]'$ denote a vector of net supplies.

Equilibrium Equilibrium expected excess returns must satisfy the intermediaries' optimality condition (16) as well as the market clearing condition (17), implying:

$$\mathrm{E}_t[\mathbf{rx}_{t+1}] = \tau^{-1} \operatorname{var}_t[\mathbf{rx}_{t+1}]\mathbf{s}_t. \tag{18}$$

To pin down equilibrium bond yields and exchange rates, y_t and q_t, we follow Greenwood et al. (2023) and conjecture that prices are linear functions of the state vector $\mathbf{z}_t$, which contains all stochastic processes, $\mathbf{z}_t = [i_t - \bar{\iota}, i_t^* - \bar{\iota}, g_t, q_{t+\infty}, s_t^y - \bar{s}^y, s_t^q]'$. Appendix B.2 contains the full mathematical solution, provides a characterization of equilibrium bond yield and exchange rate, as well as a discussion on equilibrium multiplicity and selection.

4.2 Comovement of excess bond and currency returns

To characterize the correlation between bond and foreign exchange excess returns, we first present analytical results for a special case of the model, and then turn to a numerical analysis for its general version. Consider the following special case of the model:

Assumption 1. *Short-term interest rates are deterministic ($\sigma_i = 0$), asset-specific shocks are transitory ($\phi_g = 0$) and feature same volatility ($\sigma_g = \sigma_q$), and net supply shocks are transitory ($\phi_{s_y} = \phi_{s_q} = 0$) with same investor-specific volatilities ($\sigma_\eta = \sigma_\gamma$, and $\sigma_{\eta^\star} = \sigma_{\gamma^\star}$). Besides, the long-term bonds has near-infinite duration ($\delta \to 1$), and agents are sufficiently risk-tolerant (τ is large enough).*

Under these restrictions, we highlight the following result:

Proposition 1. *Under Assumption 1,* $\operatorname{corr}_t(rx_{t+1}^y, rx_{t+1}^q) > 0$ *if and only if* $\Omega > 0$.

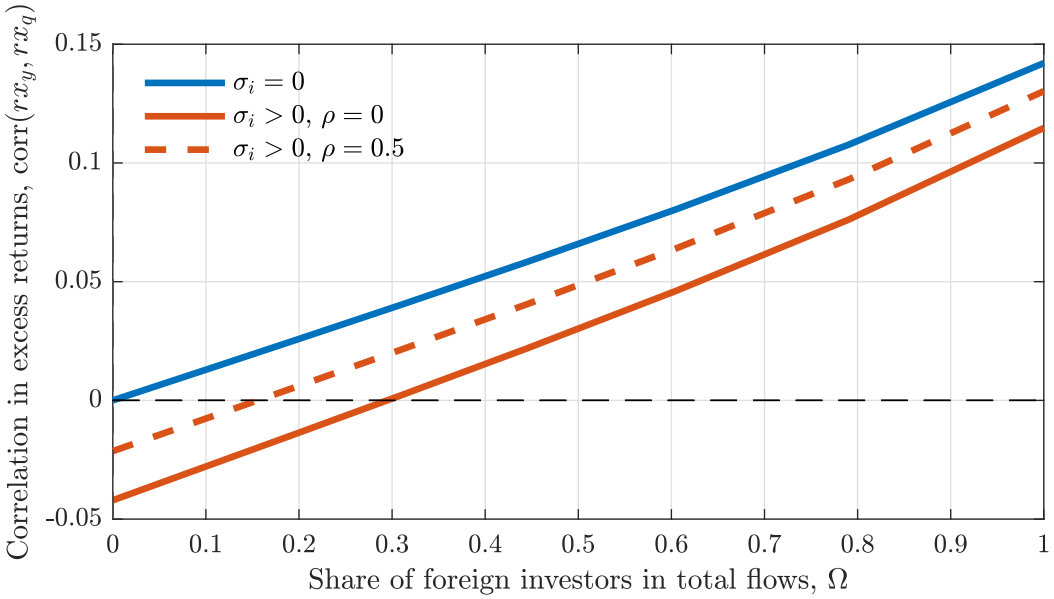
Proof. See Appendix B.3. □

Proposition 1 demonstrates that, in a simplified version of the model without interest rate risk, the correlation between bond and foreign exchange returns is positive whenever global investors operate in the local-currency bond market, thus inducing a positive correlation in net supplies (see eq. (14)). Without interest rate shocks, the fundamental sources of risk are asset-specific independent shocks (fluctuations in the bond return wedge and long-term

nominal exchange rate). If net supply fluctuations were independent, bond and FX premia would also be uncorrelated. However, when net supply fluctuations are correlated, their comovement is reflected in the joint distribution of premia.

Correlated inflows generate correlated returns through two related channels. First, correlated inflows directly link the net supply in bond and currency markets that local intermediaries must absorb, and thus in the premia they require. This effect occurs even in models where intermediaries managing bonds and currency are distinct entities. Second, whenever local intermediaries operate across both markets, correlated inflows create an additional effect: an intermediary active in both markets demands a premium in one asset whenever it holds more of the other, further amplifying the first effect. In Section 6, we highlight that this second effect is essential to rationalize the observed effects of quantitative easing policies across countries.

Figure 5
Correlation in bond and currency excess returns



Note: The figure depicts the equilibrium correlation in bond and currency excess returns for different levels of correlation in net supplies, $\Omega \equiv \text{corr}_t(s^y_{t+1}, s^q_{t+1})$ (achieved by different relative importance of foreign to home standard deviations of supply shocks, see eq. (14)), standard deviation of the short-term interest rate, σ_i, and correlation in home and foreign short-term interest rates, ρ. The rest of parameters are set according to Table B.1.

To examine the role of short-term interest rate risk, we resort to numerical simulations and extend the analysis beyond the special case in Assumption 1. Figure 5 depicts the correlation between excess bond and currency returns under an illustrative calibration of the model, with and without short-term interest rate risk. Two key insights emerge. First, the correlation in excess returns increases with the relevance of foreign investors, governed by

Ω (see eq. (14)). A higher variance of foreign investors' flows (for a given total variance of flows) increases the correlation in bond and currency net supply movements and thus the correlation of bond and currency returns (the relative variance of foreign to home inflows, Ω, is around 80% in Colombia micro data). Second, all else equal, interest rate risk lowers the correlation in excess returns—a result originally presented in Greenwood et al. (2023) and Gourinchas et al. (2022). Interest rate risk, in fact, affects the yield-curve trade and the UIP trade in opposite directions. An increase in domestic short-term rates results in lower domestic long-term bond prices, which is detrimental to the yield-curve trade, but it simultaneously appreciates the domestic currency, benefiting the UIP trade.

It is important to note that interest rate risk only influences the correlation in returns if short-term interest rates are not perfectly correlated across countries. Correlated interest rate fluctuations, in fact, limit the variation in the interest rate *differential*, thereby reducing the impact of interest rate risk on the UIP trade, as depicted in Figure 5. As short-term interest rate movements in emerging economies are synchronized with those in the U.S. (this correlation is around 60% for Colombia *vis a vis* the U.S.), it weakens the relative importance of interest rate risk in determining the bond-currency return correlation in these economies.

5 Positions and Returns of Global Investors and Local Intermediaries

Our model has two broad implications that speak to positions and returns of intermediaries in bond and currency markets. First, intermediaries earn positive excess returns in both markets on average, as we outline in Proposition 2 below. Second, if correlated inflows are a dominant force driving returns in both markets, the positions of intermediaries in both markets should be positively correlated, as reflected in equation (14). In this section, we outline these implications and confront them with micro-level data from Colombia.

The following proposition outlines the properties of the model with respect to the returns accrued from the positions held by local intermediaries.

Proposition 2. *Under Assumption 1, local intermediaries gain positive excess returns, on average, on both their bond and currency positions. That is,* $\mathbb{E}(d_t^y rx_{t+1}^y) \geq 0$ *and* $\mathbb{E}(d_t^q rx_{t+1}^q) \geq 0$*, where* $\mathbb{E}$ *is the unconditional expectation operator.*

Proof. Using equation (16), one can show that

$$\mathbb{E}(d_t^y rx_{t+1}^y) = \tau^{-1}\left[\text{var}_t(rx_{t+1}^y)\mathbb{E}\left((d_t^y)^2\right) + \text{cov}_t(rx_{t+1}^y, rx_{t+1}^q)\mathbb{E}\left(d_t^q d_t^y\right)\right]; \tag{19}$$

$$\mathbb{E}\left(d_t^q rx_{t+1}^q\right) = \tau^{-1}\left[\text{var}_t(rx_{t+1}^q)\mathbb{E}\left((d_t^q)^2\right) + \text{cov}_t(rx_{t+1}^y, rx_{t+1}^q)\mathbb{E}\left(d_t^y d_t^q\right)\right]. \tag{20}$$

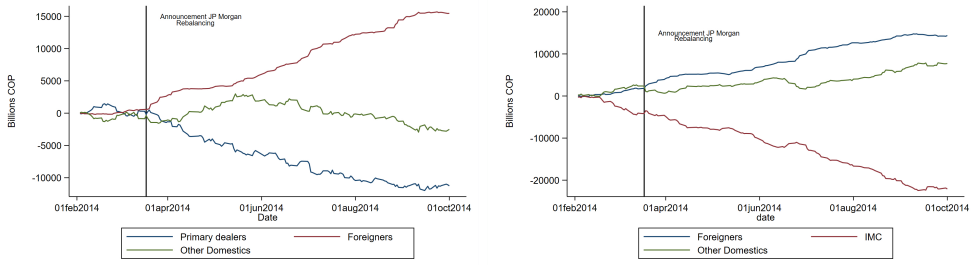
Using the market clearing condition (17), the processes of net supplies, (3) and (10), the properties of foreign and home investors' flows, (12) and (13), as well as Proposition 1, one can show that $\mathbb{E}(d_t^y rx_{t+1}^y) \geq 0$ and $\mathbb{E}(d_t^q rx_{t+1}^q) \geq 0$.

Furthermore, equation (17) then implies that inelastic demand of local and foreign investor delivers negative excess returns, on average. □

Local intermediaries play a central role in these dynamics. In Colombia, designated dealers in both markets are the natural counterparties absorbing bond and currency demand shocks. We refer to the designated dealers in the treasury market as PDs and those in the foreign exchange market as IMCs.

To provide suggestive evidence that these financial institutions act as intermediaries in both markets, we analyze their behavior during the *J.P. Morgan* rebalancing described in Section 3. In principle, intermediaries absorb demand shocks from other investors. Generally, it is difficult to encounter in the data arguably exogenous demand shocks. In the case of the *J.P. Morgan* rebalancing, one can argue that its announcement triggers a portfolio rebalancing that is exclusive to foreigners and affects the treasury (TES) market directly. Additionally, through the simultaneous purchase of domestic currency, it also affects the foreign exchange market as highlighted in Figure 2. In Figure 6 we expand the analysis to the behavior of both PDs (TES market), IMCs (FX market), and other domestic investors, during this episode. In both bond and foreign exchange markets, designated dealers are the ones largely absorbing the demand shock from foreign investor, consistent with them being the relevant "arbitrageurs" in Colombia's bond and foreign exchange markets.

Figure 6
Behavior of Intermediaries during *J.P. Morgan* Rebalancing



(a) TES Market (b) FX Spot Market

Note: This figure displays the evolution of the daily accumulated purchases of of TES bonds (Panel A) and the daily accumulated purchases of COP in the spot market (Panel B) during the J.P. Morgan rebalancing in 2014 for different investor groups. PDs and IMCs are designated dealers in the TES and FX spot market, respectively. The black vertical line denotes 19th of March 2014, the day the rebalancing was announced.

Next, we test whether these intermediaries earn positive excess returns in both markets. To do so, we construct monthly-level returns for different investor groups in the two markets.

More specifically, for the returns from currency trade we compute:

$$\text{Curr Trade Ret}_{t,h}^{j} = X_t^{Curr,j} \times rx_{t+h}^{FX} \qquad h = 1, 3, 6, 12. \tag{21}$$

where $X_t^{Curr,j}$ is the flow of investor j associated with the sale of USD in exchange for COP (measured in USD). rx_{t+h}^{FX} are the currency returns described in Section 2 for a holding period of h months. For the yield curve return we compute:

$$\text{Yield Trade Ret}_{t,h}^{j} = X_t^{Bond,j} \times rx_{t+h}^{(n)} \qquad h = 1, 3, 6, 12. \tag{22}$$

where $X_t^{Bond,j}$ is the flow of investor j associated with the purchase of long-term government bonds in domestic currency. $rx_{t+h}^{(n)}$ are the yield trade return described in Section 2. For simplicity, we compute these returns using the 10-year local-currency government bond and we measure returns in domestic currency. In both markets we consider three investor groups: local intermediaries (either IMCs or PDs), foreign investors, and other domestic intermediaries. After calculating the trade returns for various holding periods, we perform mean t-Tests to evaluate whether the trade returns are significantly different from zero, with the null hypothesis being that the returns are equal to zero.

We present results of these analyses in Table 6. In general, we find that currency (yield) trade returns are positive for IMCs (PDs), and negative both for foreigners and other domestics. Results are noisier for the 1 and 3-month holding period, but in the medium run (6 and 12-month holding periods) all average returns (both positive and negative) are statistically different from zero. These findings support the model's implication outlined in Proposition 2, as well as our conjecture about the investor group acting as intermediaries in Colombia.

Next, we explore another implication of the model concerning the positions of intermediaries in both markets. Specifically, for the group of PDs and IMCs, we compute their purchases of TES and COP (in the FX spot market), respectively, measured in domestic currency. We aggregate these purchases over different horizons, computing the 1, 3, 6 and 12-month totals in both markets, in order to analyze both short-term correlations as well as medium-run trends. To test the relationship between these variables, we estimate the following regression:

$$\text{COP}_t^{h,IMC} = \theta_y + \theta_m + \beta \text{TES}_t^{h,PD} + \varepsilon_t \qquad h = 1, 3, 6, 12. \tag{23}$$

where $\text{COP}_t^{h,IMC}$ are the h-month purchases of COP in the FX spot market by IMCs, and $\text{TES}_t^{h,PD}$ are the h-month purchases of TES by PDs. θ_y and θ_m are year and month fixed effects that control for long-run trends and seasonality effects.

Table 6
Currency and Yield Trade Returns

Panel A: Trade Returns FX				
	1M	3M	6M	12M
Intermediaries	3.393	15.251**	38.845***	74.420***
	(2.397)	(7.049)	(11.839)	(21.646)
Foreigners	0.929	-5.264	-19.877*	-46.449**
	(2.264)	(5.031)	(10.240)	(17.715)
Others	-4.322**	-9.988**	-18.968***	-27.972**
	(1.960)	(4.725)	(6.883)	(13.473)
Observations	83	81	78	72
Panel B: Trade Returns Yield Curve				
	1M	3M	6M	12M
Intermediaries	4.540	13.224**	18.645***	24.924***
	(5.181)	(6.141)	(6.992)	(8.157)
Foreigners	-0.427	-1.847	-3.828	-9.814***
	(1.830)	(1.988)	(2.691)	(3.639)
Others	-4.113	-11.377**	-14.817**	-15.110*
	(4.422)	(5.531)	(6.104)	(7.659)
Observations	120	120	120	120

Note: This table reports results from mean t-Tests of currency (Panel A) and yield curve (Panel B) trade returns for different investor groups. Currency trade returns are computed following equation (21) and yield curve trade returns are computed using equation (22). The 1M, 3M, 6M, and 12M columns denote test for average returns with holding periods of 1, 3, 6, and 12-month, respectively. The null hypothesis is that average returns are equal to zero. *, **, and *** denote statistically significant at the 10%, 5%, and 1% level, respectively.

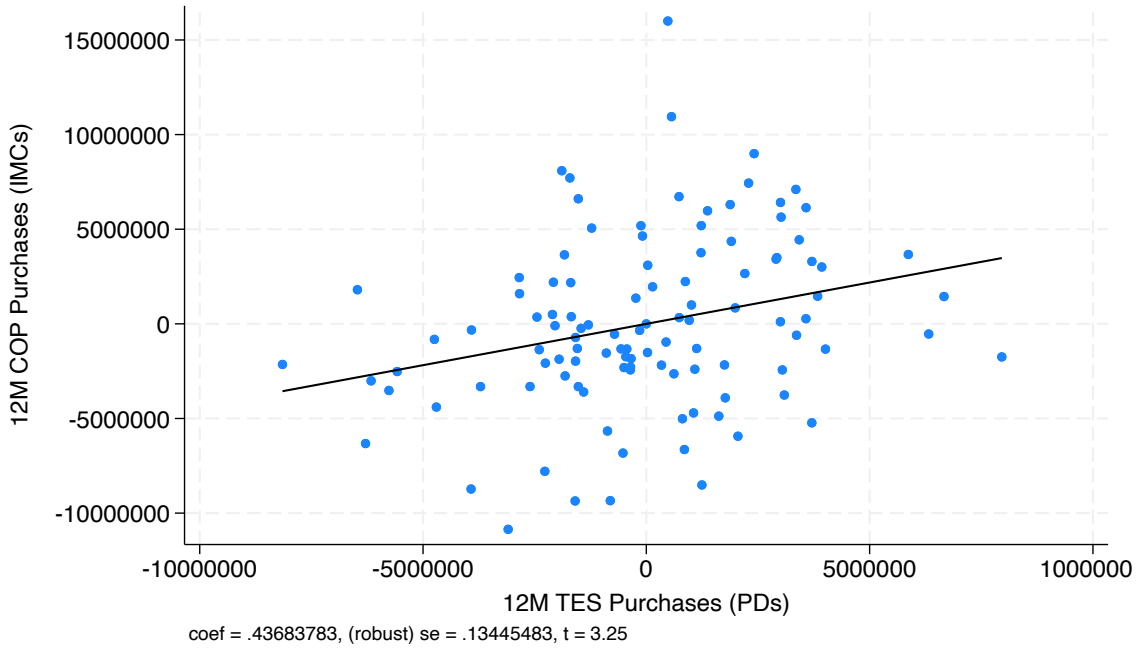
Table 7
Correlation in Intermediaries Position Changes

Dependent Variable: COP Purchases by IMC								
	1M		3M		6M		12M	
TES Purchases by PD	0.156	0.168	0.200**	0.212	0.245**	0.411**	0.361***	0.437***
	(0.106)	(0.110)	(0.098)	(0.135)	(0.107)	(0.157)	(0.123)	(0.134)
Year FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Month FE	No	Yes	No	Yes	No	Yes	No	Yes
Observations	120	120	118	118	115	115	108	108
R-Squared	0.28	0.41	0.55	0.62	0.66	0.70	0.82	0.83

Note: This table presents OLS estimations of COP purchases in the spot market by IMCs on TES purchases in the bond market by PDs with different types of fixed effects. The different columns displays the h-month accumulated purchases for $h = 1, 3, 6, 12$. Robust standard errors are presented in parenthesis. *, **, and *** denote statistically significant at the 10%, 5%, and 1% level, respectively.

The estimates show a positive relationship between both variables (Table 7). This relationship is noisier when we consider the 1-month change in positions, being positive but not statistically different from zero. However, as we consider change in the positions of the intermediaries over longer horizons, the relationship becomes stronger. The magnitude of the estimated coefficients increases and they are statistically different from zero at conventional significance levels.

Figure 7
Change in Positions of Intermediaries in TES and FX market



Note: This figure presents a partial correlation scatter plot of 12-month COP purchases by IMCs in the FX spot market on the 12-month TES purchases by PDs with both year and month fixed effects. Both y and x-axis are in billions COP.

We complement the regressions with a partial correlation scatter plot of the 12-month level purchases from intermediaries in both markets (Figure 7). We observe a strong positive relationship along the fitted positive slope, with limited outlier observations. Additionally, we compute the correlations between both variables after residualizing for year and month fixed effects. We obtain correlations of 0.16 (1-month), 0.17 (3-month), 0.24 (6-month), and 0.28 (12-month). All of these are statistically different from 0 at the 10% significance level. The overall positive relationship in the positions of intermediaries lends support to the idea that "correlated inflows" are a significant driver of flows and premia in both the bond and currency markets.

6 Quantitative Easing in Local-Currency Bond Markets

In this section, we show that the model can rationalize the differential effects of quantitative easing (QE) policies across emerging and advanced economies. Using a high-frequency identification approach, Rebucci et al. (2022) document that QE, consisting in the purchase of long-term government bonds by the central bank, led to currency appreciation in emerging economies during COVID-19 (see also Arslan et al., 2020). This evidence is contrary to the one in advanced economies where QE is associated with currency depreciations (Bauer and Neely, 2014; Neely, 2015; Swanson, 2021; Bhattarai and Neely, 2022), which, in part, motivated the analysis in Greenwood et al. (2023) and Gourinchas et al. (2022).

When foreign investors are prevalent in local-currency bond markets, the exchange rate appreciates following a central bank's purchase of domestic long-term bond. We formalize this result in the next Proposition.

Proposition 3. *A domestic central bank's purchase of local-currency long-term bonds, modeled as a reduction in η_t in equation (3), causes a home currency appreciation if and only if $\text{cov}_t(rx^y_{t+1}, rx^q_{t+1}) > 0$.*

Proof. Consider a unit decrease in η_t in (3), which implies a reduction in bond net supply s_y without a corresponding movement in currency net supply (i.e. $s_q = 0$). By the equilibrium solution of the exchange rate, eq. (B.12) derived in Appendix B.2, such impulse causes a currency appreciation if and only if $\text{cov}_t(rx^y_{t+1}, rx^q_{t+1}) > 0$. □

Proposition 3 highlights that the exchange rate effects of QE depend on two key features of our small open economy model. First, local intermediaries intermediate both bond and currency. For this reason, a reduction in net supply of bonds brings them to require different premia on both local-currency bonds and foreign currency holdings. Second, the effect on currency premia depend upon the stochastic properties of bonds and currencies, embodied in the covariance of returns, and in turn determined by the relative importance of correlated foreign inflows vs interest rate shocks (see Proposition 1 and Figure 5). If correlated inflows dominate, then the UIP trade and the yield-curve trade are exposed to positively correlated risks. As a result, offloading domestic bonds induces local intermediaries to demand lower term premia as well as lower currency premia. This results in an equilibrium currency appreciation.

We stress that the currency appreciation occurs notwithstanding the fact that the central bank's asset purchase is confined to the local-currency bond market, *i.e.* it is not associated

with a simultaneous intervention in the currency market.[13]

7 Conclusion

The composition of investors holding local-currency bonds shapes the equilibrium dynamics of a country's exchange rate. When local intermediaries face portfolio-balance constraints, the flows they must absorb influence bond prices and exchange rates by affecting term premia and currency premia. Using micro-level data from Colombia's local-currency sovereign bond market, along with spot and forward exchange markets, we document that global investor inflows into the bond market are systematically linked to corresponding inflows into the foreign exchange spot market, but not into the forward market. Incorporating these features into an equilibrium model, we show that correlated inflows increase the co-movement between bond and currency premia, consistent with macro-level evidence on excess bond and currency returns as well as micro-level evidence on investor-specific returns in these markets.

The stochastic properties of a country's exchange rates, in turn, determine the impact of policies that alter the net supply of bonds or foreign currency. For instance, quantitative easing (QE) policies that reduce the net supply of sovereign bonds mediated by local intermediaries, lead to a simultaneous increase in bond prices *and* the value of the home currency, a prediction supported by the data. Beyond providing insights into positions and returns in bond and currency markets, as well as the effects of QE, the model also offers implications for the price effects of sovereign debt auctions and central banks' foreign currency purchases—questions we aim to explore in future iterations of this paper.

[13]To be clear, the correct way to interpret QE in the model is a central bank purchase of bond under the variance-covariance structure of implied by the properties of all exogenous processes, not the effect of an asset purchase in a counterfactual world in which *only* asset purchases exist.

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Appendix

A Appendix to Section 2

In the cross section, Figure A.1 shows that it has a positive correlation for EM while a negative correlation for AE. For this plot, we have used foreign exchange excess returns using a 3-month deposit rate, but the plot is virtually the same if we use currency excess returns using forwards. In the time series, Figure A.2 shows the 3-month moving average of the cross-country averages of excess returns for both currency and bonds.[14] The time series correlation within country remains positive for emerging economies while in advanced economies one observes the opposite.

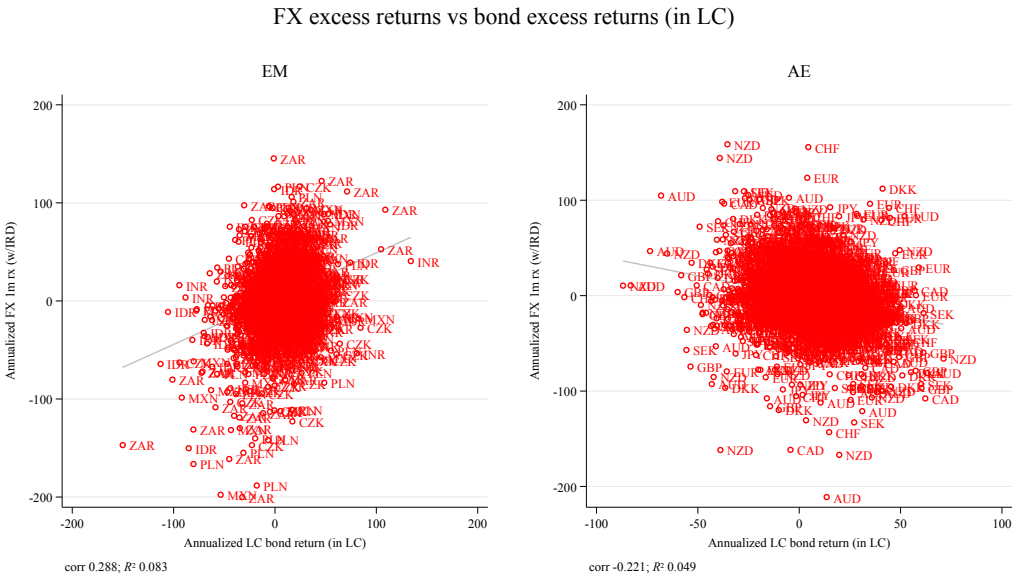


Figure A.1
Cross-section:Excess returns bonds and currencies (local currency)

[14] We applied a 3-month moving average (including two lags and the current month) to present smoother monthly returns. However, the findings remain consistent when using 1-month excess returns.

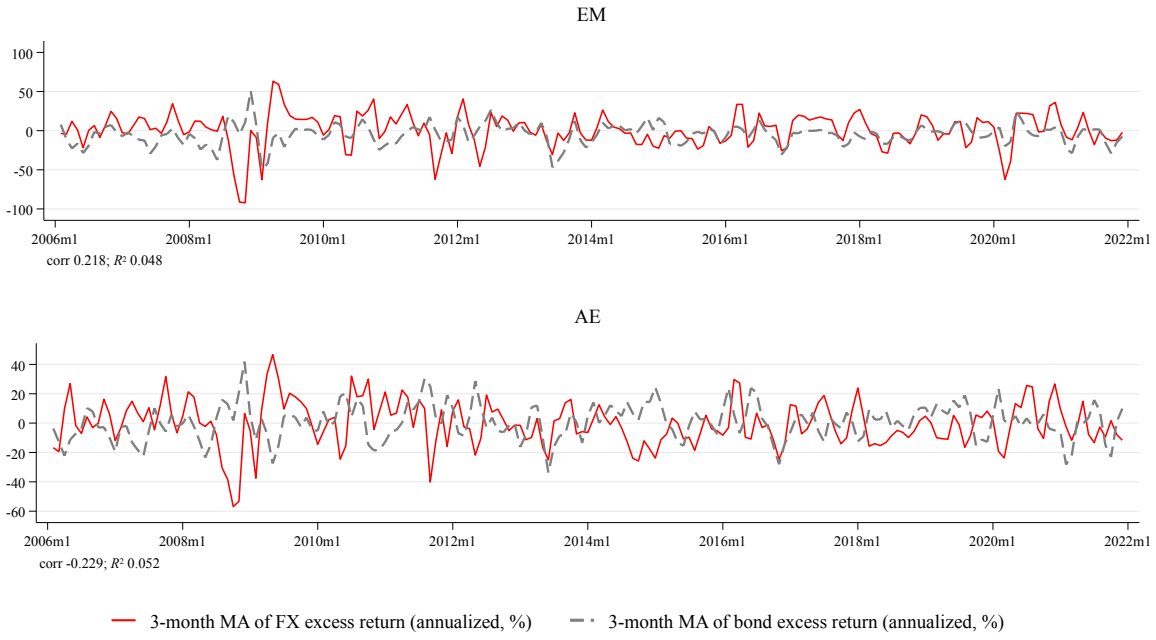


Figure A.2
Time Series: Excess returns bonds and currencies (local currency)

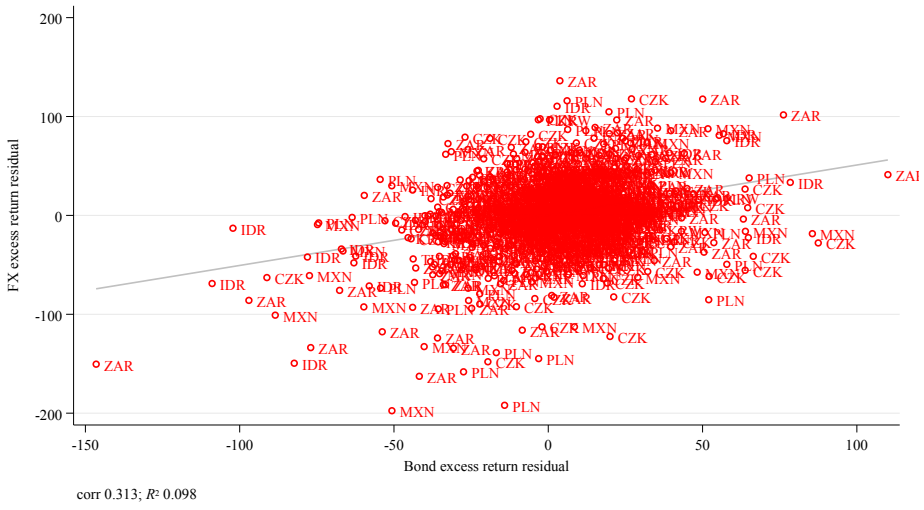


Figure A.3
Correlation of residualized excess returns

B Appendix to Section 4

B.1 Return of a Perpetuity: Campbell-Shiller Approximation

We use the Campbell and Shiller (1988) log-linear approximation to model the before-tax return on default-free long-term bonds. We assume that agents take as given the exogenous tax described in the model section. These bonds are self-amortizing perpetuities whose payments decline geometrically, are free of default risk, and have a face value of 1 at time t. Let P_t^y be the price and Y_t the yield-to-maturity of these long-term bonds at time t. At $t+1$, this instrument will offer a coupon payment of C, a principal repayment of $1-\kappa$ for some $\kappa \in [0,1]$, and κ units of the asset. Here, κ is the amortization rate. The gross before-tax return on long-term bonds from t to $t+1$ is thus

$$1 + R_{t+1}^y = \frac{C + 1 - \kappa + \kappa P_{t+1}^y}{P_t^y}, \tag{B.1}$$

where

$$P_t^y = \sum_{j=1}^{\infty} \frac{\kappa^{j-1}(1-\kappa+C)}{(1+Y_t)^j} = \frac{1+C-\kappa}{1+Y_t-\kappa}. \tag{B.2}$$

Taking a log-linear approximation of the long-term bond's return around the point where it is trading at par at $t+1$ obtains

$$r_{t+1}^y \approx \theta + \delta p_{t+1}^y - p_t^y, \tag{B.3}$$

where $\theta \equiv \ln(1+C)$ and $\delta \equiv \kappa/(1+C)$ are parameters. We can iterate this equation forward and apply this approximation to Y_t to get

$$p_t^y \approx \frac{1}{1-\delta}\theta - \frac{1}{1-\delta} y_t. \tag{B.4}$$

We plug equation (B.4) into (B.3) to get the approximate one-period log return on the long-term bond

$$r_{t+1}^y \approx \frac{1}{1-\delta} y_t - \frac{\delta}{1-\delta} y_{t+1}, \tag{B.5}$$

where $D = (1-\delta)^{-1}$ is the Macaulay duration when the instrument is trading at par.

Lastly, one can subtract the tax g_t from this return to get expression (4).

B.2 Solving the Baseline Model

In this subsection, we derive the system of equations necessary to solve the baseline model presented in Section 4.1. We follow Greenwood et al. (2023) by conjecturing that equilibrium prices are a linear function of a state vector of shocks and arrive at a system of three equations with three unknowns. The resulting equations can be studied to derive qualitative implications about the model. We close out Appendix B by proving the main propositions of the paper.

B.2.1 Equilibrium Conjecture and Properties

Equilibrium Conjecture We conjecture that the two prices that we need to pin down in equilibrium, y_t and q_t, are a linear function of a state vector of z_t

$$y_t = \alpha_0^y + \alpha_1^{y'} z_t;$$
$$q_t = \alpha_0^q + \alpha_1^{q'} z_t,$$

where the 6×1 state vector $\mathbf{z}_t = [i_t - \bar{\iota}, i_t^* - \bar{\iota}, g_t, q_{t+\infty}, s_t^y - \bar{s}^y, s_t^q]'$ follows a VAR(1) process $\mathbf{z}_{t+1} = \Phi \mathbf{z}_t + \varepsilon_{t+1}$, with $\text{var}_t[\varepsilon_{t+1}] = \Sigma$ and $\Phi = \text{diag}(\phi_i, \phi_i, \phi_g, 0, \phi_{s^y}, \phi_{s^q})$. In vector form the two prices yield $\mathbf{y}_t + \mathbf{a} + \mathbf{A}\mathbf{z}_t$, where $\mathbf{y}_t = [y_t, q_t]'$, $\mathbf{a} = [\alpha_0^y, \alpha_0^q]'$, and $\mathbf{A} = [\alpha_1^y, \alpha_1^q]'$.

Rational Expectations Equilibrium Let $f(\alpha_0)$ be an operator that gives the price-impact coefficients that clear the markets for long-term bonds and FX when agents conjecture that $\alpha = \alpha_0$, where $\alpha = vec(\text{A})$. We say that a rational expectations equilibrium in this model is a fixed point

$$\alpha^* = f(\alpha^*). \tag{B.7}$$

Within this context, local intermediaries must form beliefs—specifically, price-impact coefficients—about how the net supplies of long-term bonds and foreign exchange, s_t^y and s_t^q, influence equilibrium asset prices, y_t and q_t. A rational expectations equilibrium is therefore a fixed point of a specific operator involving these price-impact coefficients.

Equilibrium Properties The presence of supply risk in this model makes the risk tolerance of investors τ a key variable. If agents are not risk-tolerant enough, then an equilibrium does not exist. However, for high levels of risk tolerance, stochastic supply shocks generate multiple equilibria. The different equilibria correspond to different self-fulfilling beliefs (price-impact coefficients) that investors might have about the risk of holding the different assets. For example, if investors believe that supply shocks barely affect prices, they will perceive these assets as less risky. Consequently, investors will absorb large supply shocks and will not

require large compensations through a fall in asset prices. However, if investors believe supply shocks will have greater impact on prices, they demand a large decline in prices as compensation for absorbing these shocks.

Despite multiple equilibria, there is always a unique stable equilibrium. Denoting by $\{\lambda_i\}$ the eigenvalues of the Jacobian $D_\alpha f(\alpha^*)$, we see that α^* is stable if $|\lambda_i| < 1$. The importance of a unique stable equilibrium lies in the fact that we can infer qualitative implications from our model. It also implies that comparative statics on our equilibrium price-impact coefficients α^* offer an easy and informative interpretation.

B.2.2 Equilibrium Solution

The first-order condition of the local intermediaries that we derived is written again for completeness

$$\mathrm{E}_t[\mathbf{rx}_{t+1}] = \tau^{-1}\,\mathrm{var}_t[\mathbf{rx}_{t+1}]\mathbf{d}_t. \tag{B.8}$$

This, coupled with the usual market clearing condition that supply equals demand ($\mathbf{d}_t = \mathbf{s}_t$), and letting $\mathbf{V} = \mathrm{var}_t[\mathbf{rx}_{t+1}]$, we get

$$\mathrm{E}_t[\mathbf{rx}_{t+1}] = \tau^{-1}\mathbf{V}\mathbf{s}_t, \tag{B.9}$$

with

$$\mathbf{V} = \mathrm{var}_t[\mathbf{rx}_{t+1}] = \begin{bmatrix} V_y & C_{y,q} \\ C_{y,q} & V_q \end{bmatrix}.$$

We can write out equation (B.8), yielding individual excess return equations:

$$\mathrm{E}_t\left[rx^y_{t+1}\right] = \frac{1}{\tau}\left[V_y \cdot s^y_t + C_{y,q} \cdot s^q_t\right]; \tag{B.10a}$$

$$\mathrm{E}_t\left[rx^q_{t+1}\right] = \frac{1}{\tau}\left[C_{y,q} \cdot s^y_t + V_q \cdot s^q_t\right], \tag{B.10b}$$

where $V_y \equiv \mathrm{var}_t[rx^y_{t+1}]$, $V_q \equiv \mathrm{var}_t[rx^q_{t+1}]$, and $C_{y,q} \equiv \mathrm{cov}_t[rx^y_{t+1}, rx^q_{t+1}]$. Note that $\mathbf{V}$ is constant in equilibrium and we hereafter drop the time subscripts.
Using these equilibrium excess return equations, along with asset prices equations (6) and (10), as well as the exogenous processes, we can characterize equilibrium bond yields and foreign exchange prices:

$$y_t = \left\{\bar{\iota} + \frac{1-\delta}{1-\delta\phi_i}\cdot(i_t - \bar{\iota})\right\} + \frac{1-\delta}{1-\delta\phi_g}g_t + \left\{\tau^{-1}V_y \cdot \bar{s}^y\right\} + \tau^{-1}\left\{\frac{1-\delta}{1-\delta\phi_{s^y}}V_y \cdot (s^y_t - \bar{s}^y) + \frac{1-\delta}{1-\delta\phi_{s^q}}C_{y,q} \cdot s^q_t\right\}; \tag{B.11}$$

$$q_t = \left\{\frac{1}{1-\phi_i}\cdot(i^\star_t - i_t)\right\} + \mathrm{E}_t\, q_{t+\infty} + \tau^{-1}\left\{C_{y,q}\bar{s}^y + \frac{1}{1-\phi_{s^y}}C_{y,q} \cdot (s^y_t - \bar{s}^y) + \frac{1}{1-\phi_{s^q}}V_q \cdot s^q_t\right\}. \tag{B.12}$$

We now focus on the fixed-point problem. The vector of excess returns is

$$\mathbf{rx}_{t+1} \equiv \begin{bmatrix} rx^y_{t+1} \\ rx^q_{t+1} \end{bmatrix} = \begin{bmatrix} \frac{1}{1-\delta} y_t - \frac{\delta}{1-\delta} y_{t+1} - i_t - g_t \\ i_t - i^\star_t - (q_{t+1} - q_t) \end{bmatrix} = \mathbf{B}_0 \mathbf{y}_t + \mathbf{B}_1 \mathbf{y}_{t+1} + \mathbf{R}_1 \mathbf{z}_t + \mathbf{r}_0.$$

where I have used equations (1), (2), (4), (9), and the fact that $rx^y_{t+1} \equiv r^y_{t+1} - i_t$. Additionally,

$$\mathbf{B}_0 = \begin{bmatrix} \frac{1}{1-\delta} & 0 \\ 0 & 1 \end{bmatrix}, \quad \mathbf{B}_1 = \begin{bmatrix} -\frac{\delta}{1-\delta} & 0 \\ 0 & -1 \end{bmatrix}, \quad \mathbf{R}_1 = \begin{bmatrix} -1 & 0 & -1 & 0 & 0 & 0 \\ 1 & -1 & 0 & 0 & 0 & 0 \end{bmatrix}, \mathbf{r}_0 = \begin{bmatrix} -\bar{\iota} \\ 0 \end{bmatrix}.$$

Note that when conjecturing the equilibrium, we defined $\mathbf{y}_t = \mathbf{a} + \mathbf{A}\mathbf{z}_t$. Iterating this expression one period forward and using that $\mathbf{z}_{t+1} = \Phi \mathbf{z}_t + \varepsilon_{t+1}$, one can obtain that

$$\mathbf{y}_{t+1} = \mathbf{a} + \mathbf{A}\mathbf{z}_{t+1} = \mathbf{a} + \mathbf{A}\Phi \mathbf{z}_t + \mathbf{A}\varepsilon_{t+1}. \tag{B.13}$$

Recall that Φ is a diagonal matrix with the AR(1) coefficients of the 6 different exogenous processes. Going back to the equation for $\mathbf{rx}_{t+1}$ I just derived yields

$$\mathbf{rx}_{t+1} = [\mathbf{B}_0\mathbf{a} + \mathbf{B}_1\mathbf{a} + \mathbf{r}_0] + [\mathbf{B}_0\mathbf{A} + \mathbf{B}_1\mathbf{A}\Phi + \mathbf{R}_1]\mathbf{z}_t + [\mathbf{B}_1\mathbf{A}]\varepsilon_{t+1}, \tag{B.14}$$

which implies that

$$E_t[\mathbf{rx}_{t+1}] = [\mathbf{B}_0\mathbf{a} + \mathbf{B}_1\mathbf{a} + \mathbf{r}_0] + [\mathbf{B}_0\mathbf{A} + \mathbf{B}_1\mathbf{A}\Phi + \mathbf{R}_1]\mathbf{z}_t; \tag{B.15}$$

$$\mathbf{V} \equiv \text{var}_t[\mathbf{rx}_{t+1}] = \mathbf{B}_1\mathbf{A}\Sigma\mathbf{A}'\mathbf{B}_1'. \tag{B.16}$$

Going back to the market-clearing condition in equation (B.9), $\mathbf{s}_t = [s^y_t, s^q_t]'$ can be written as $\mathbf{s}_t = \mathbf{s}_0 + \mathbf{S}_1\mathbf{z}_t$, where

$$\mathbf{S}_1 \equiv \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}, \text{ and } \mathbf{s}_0 = \begin{bmatrix} \bar{s}^y \\ 0 \end{bmatrix},$$

which allows us to write equation (B.9) as

$$[\mathbf{B}_0\mathbf{a} + \mathbf{B}_1\mathbf{a} + \mathbf{r}_0] + [\mathbf{B}_0\mathbf{A} + \mathbf{B}_1\mathbf{A}\Phi + \mathbf{R}_1]\mathbf{z}_t = \tau^{-1}(\mathbf{B}_1\mathbf{A}\Sigma\mathbf{A}'\mathbf{B}_1')(\mathbf{s}_0 + \mathbf{S}_1\mathbf{z}_t). \tag{B.17}$$

Equation (B.17) is the main equation which will be used to solve the fixed-point problem. We first separate the terms that contain $\mathbf{z}_t$ from the terms that do not. For the constant terms we find that

$$(\mathbf{B}_0 + \mathbf{B}_1)\mathbf{a} = \left[\tau^{-1}\mathbf{B}_1\mathbf{A}\Sigma\mathbf{A}'\mathbf{B}_1'\mathbf{s}_0 - \mathbf{r}_0\right]. \tag{B.18}$$

Recall how $\mathbf{B}_0$ and $\mathbf{B}_1$ look like. The last row of the sum only contains zeros. Therefore, the domestic long-term bond yield is pinned down in equilibrium - but the constant for the exchange rate is not.

For the terms containing $\mathbf{z}_t$, note that $\mathbf{B}_0, \mathbf{B}_1$, and Φ are diagonal, and thus it follows that

$$[\mathbf{B}_0 \mathrm{A} + \mathbf{B}_1 \mathrm{A}\Phi] = \mathbf{A} \circ [\mathbf{B}_0\mathbf{E} + \mathbf{B}_1\mathbf{E}\Phi], \tag{B.19}$$

where $\circ$ is the element-wise matrix multiplication and $\mathbf{E}$ is a 3×5 matrix of 1s. Thus, we get

$$[\mathbf{B}_0\mathbf{E} + \mathbf{B}_1\mathbf{E}\Phi] = \begin{bmatrix} \frac{1-\delta\phi_i}{1-\delta} & \frac{1-\delta\phi_i}{1-\delta} & \frac{1-\delta\phi_g}{1-\delta} & \frac{1}{1-\delta} & \frac{1-\delta\phi_{s^y}}{1-\delta} & \frac{1-\delta\phi_{s^q}}{1-\delta} \\ 1-\phi_i & 1-\phi_i & 1-\phi_g & 1 & 1-\phi_{s^y} & 1-\phi_{s^q} \end{bmatrix}.$$

Using this, the terms containing $\mathbf{z}_t$ have to equate on both sides. That is,

$$[\mathbf{A} \circ (\mathbf{B}_0\mathbf{E} + \mathbf{B}_1\mathbf{E}\Phi) + \mathbf{R}_1]\mathbf{z}_t = \tau^{-1}(\mathbf{B}_1\mathbf{A}\Sigma\mathbf{A}'\mathbf{B}_1')\mathbf{S}_1\mathbf{z}_t. \tag{B.20}$$

Solving for the $\mathbf{A}$ in the LHS yields

$$\mathbf{A} = [\tau^{-1}\mathbf{B}_1\mathbf{A}\Sigma\mathbf{A}'\mathbf{B}_1'\mathbf{S}_1 - \mathbf{R}_1] \oslash [\mathbf{B}_0\mathbf{E} + \mathbf{B}_1\mathbf{E}\Phi], \tag{B.21}$$

where $\oslash$ is element-wise matrix division. To further characterize the solution to the problem in (B.21), we can partition $\mathbf{z}_t$ as $\mathbf{z}_t = [\mathbf{z}_{1,t}', \mathbf{z}_{2,t}', \mathbf{z}_{3,t}']'$, where $\mathbf{z}_{1,t} = [i_t - \bar{\iota}, i_t^* - \bar{\iota}, g_t]'$, $\mathbf{z}_{2,t} = q_{t+\infty}$, and $\mathbf{z}_{3,t} = [s_t^y - \bar{s}^y, s_t^q]'$. Similarly, we partition $\mathbf{A} = [\mathbf{A}_1, \mathbf{A}_2, \mathbf{A}_3]$, where $\mathbf{A}_1$ is the 2×3 matrix of loadings on $\mathbf{z}_{1,t}$, $\mathbf{A}_2$ is the 2×1 matrix of loadings on $\mathbf{z}_{2,t}$, and $\mathbf{A}_3$ is the 2×2 matrix of loadings on $\mathbf{z}_{3,t}$.

For an arbitrary matrix $\mathbf{X}$, denote $\mathbf{X}^{[n-m]}$ for $n < m$ be the submatrix of $\mathbf{X}$ consisting of columns $n, n+1, ..., m-1, m$. Therefore, given the form of $\mathbf{R}_1$ and $\mathbf{S}_1$ ($\mathbf{S}_1^{[1-3]} = \mathbf{0}_{2\times 3}$) we can define submatrix $\mathbf{A}_1$ as

$$\mathbf{A}_1 = -\mathbf{R}_1^{[1-3]} \oslash [\mathbf{B}_0\mathbf{E} + \mathbf{B}_1\mathbf{E}\Phi]^{[1-3]} = \begin{bmatrix} \frac{1-\delta}{1-\delta\phi_i} & 0 & \frac{1-\delta}{1-\delta\phi_g} \\ -\frac{1}{1-\phi_i} & \frac{1}{1-\phi_i} & 0 \end{bmatrix}.$$

This matrix displays the price-impact coefficients of the domestic short-term rate (first column), the foreign short-term rate (second column), and the bond-specific shock (third column), on the domestic long-term yield (first row), and on FX (second row).

For $\mathbf{A}_2$, which contains the FX-specific shock, we can already see from the equilibrium price in equation (B.12) that $\mathbf{A}_2 = [0, 1]'$. In other words, the specific shock on the price of the exchange rate has no impact on the long-term bond price, while affecting the FX rate one-for-one.

Lastly, we now move to the supply shocks; that is, pinning down $\mathbf{A}_3$. Due to the

orthogonality of the different shocks, the variance-covariance matrix Σ can be partitioned as

$$\Sigma = \begin{bmatrix} \Sigma_1 & 0_{3\times1} & 0_{3\times2} \\ 0_{1\times3} & \Sigma_2 & 0_{1\times2} \\ 0_{2\times3} & 0_{2\times1} & \Sigma_3 \end{bmatrix} \quad \text{where} \quad \Sigma_1 = \begin{bmatrix} \sigma_i^2 & \rho\sigma_i^2 & 0 \\ \rho\sigma_i^2 & \sigma_i^2 & 0 \\ 0 & 0 & \sigma_g^2 \end{bmatrix}, \quad \Sigma_3 = \begin{bmatrix} \sigma_{s^y}^2 & \sigma_{\eta^*}\sigma_{\gamma^*} \\ \sigma_{\eta^*}\sigma_{\gamma^*} & \sigma_{s^q}^2 \end{bmatrix},$$

and $\Sigma_2 = \sigma_q^2$. The variance-covariance matrix of excess returns ($\mathbf{V} \equiv \text{var}_t[\mathbf{rx}_{t+1}]$) becomes

$$\mathbf{V} = (\mathbf{B}_1\mathbf{A}_1\Sigma_1\mathbf{A}_1'\mathbf{B}_1') + (\mathbf{B}_1\mathbf{A}_2\Sigma_2\mathbf{A}_2'\mathbf{B}_1') + (\mathbf{B}_1\mathbf{A}_3\Sigma_3\mathbf{A}_3'\mathbf{B}_1'). \tag{B.22}$$

Making use of the form $\mathbf{S}_1$ and $\mathbf{R}_1$ ($\mathbf{R}_1^{[5-6]} = \mathbf{0}_{2\times2}$), the following fixed-point problem involving $\mathbf{A}_3$ is obtained

$$\mathbf{A}_3 = \mathbf{F}_3(\mathbf{A}_3) \equiv \tau^{-1}[(\mathbf{B}_1\mathbf{A}_1\Sigma_1\mathbf{A}_1'\mathbf{B}_1')+(\mathbf{B}_1\mathbf{A}_2\Sigma_2\mathbf{A}_2'\mathbf{B}_1')+(\mathbf{B}_1\mathbf{A}_3\Sigma_3\mathbf{A}_3'\mathbf{B}_1')]\oslash[\mathbf{B}_0\mathbf{E}+\mathbf{B}_1\mathbf{E}\Phi]^{[5-6]}. \tag{B.23}$$

The operator $\mathbf{F}_3(\mathbf{A}_3)$ gives the price function $\mathbf{y}_t = \text{g}(\mathbf{A}_3) + \mathbf{A}_1\mathbf{z}_{1,t} + \mathbf{A}_2\mathbf{z}_{2,t} + \mathbf{F}_3(\mathbf{A}_3)\mathbf{z}_{3,t}$ that will clear the markets for long-term bonds and FX when agents conjecture that the risk of holding of assets is determined by the price function $\mathbf{y}_{t+1} = \mathbf{a}_0+\mathbf{A}_1\mathbf{z}_{1,t+1}+\mathbf{A}_2\mathbf{z}_{2,t+1}+\mathbf{A}_3\mathbf{z}_{3,t+1}$.

From equation (B.23), and using $\mathbf{V}$ in its matrix form, the equilibrium price-impact coefficients must satisfy

$$\begin{bmatrix} \alpha_{s^y}^y & \alpha_{s^q}^y \\ \alpha_{s^y}^q & \alpha_{s^q}^q \end{bmatrix} = \tau^{-1} \begin{bmatrix} \frac{1-\delta}{1-\delta\phi_{s^y}} V_y & \frac{1-\delta}{1-\delta\phi_{s^y}} C_{y,q} \\ \frac{1-\delta}{1-\delta\phi_{s^y}} C_{q,y} & \frac{1-\delta}{1-\delta\phi_{s^y}} V_q \end{bmatrix}. \tag{B.24}$$

The var.-cov. matrix in the absence of supply risk is $[(\mathbf{B}_1\mathbf{A}_1\Sigma_1\mathbf{A}_1'\mathbf{B}_1') + (\mathbf{B}_1\mathbf{A}_2\Sigma_2\mathbf{A}_2'\mathbf{B}_1')] =$

$$\begin{bmatrix} \left(\frac{\delta}{1-\delta\phi_i}\right)^2 \sigma_i^2 + \left(\frac{\delta}{1-\delta\phi_g}\right)^2 \sigma_g^2 & -\frac{\delta}{1-\delta\phi_i}\frac{1}{1-\phi_i}\sigma_i^2(1-\rho) \\ -\frac{\delta}{1-\delta\phi_i}\frac{1}{1-\phi_i}\sigma_i^2(1-\rho) & \left(\frac{1}{1-\phi_i}\right)^2 2\sigma_i^2(1-\rho) + \sigma_q^2 \end{bmatrix}. \tag{B.25}$$

Solving for the contribution of supply risk to the variance-covariance matrix, one can additionally find $\mathbf{B}_1\mathbf{A}_3\Sigma_3\mathbf{A}_3'\mathbf{B}_1'$. Note that one can recast the fixed-point problem in terms of the variance-covariance matrix, instead of using the 2×2 matrix $\mathbf{A}_3$. This is convenient because $\mathbf{V}$ is symmetric, effectively reducing the fixed-point problem to one involving three unknowns instead of four. One needs to find a fixed point in the form $\mathbf{V} = \mathbf{G}(\mathbf{V})$. Plugging the α's of equation (B.24) in the contribution of supply risk to the variance-covariance matrix, and using this and (B.25) in (B.22), along with denoting the constants

$$g_y \equiv \tau^{-1}\frac{\delta}{1-\delta\phi_{s^y}}\sigma_\eta, \quad g_y^* \equiv \tau^{-1}\frac{\delta}{1-\delta\phi_{s^y}}\sigma_{\eta^*}, \quad g_q \equiv \tau^{-1}\frac{\delta}{1-\delta\phi_{s^q}}\sigma_\gamma, \quad g_q^* \equiv \tau^{-1}\frac{\delta}{1-\delta\phi_{s^q}}\sigma_{\gamma^*}, \tag{B.26}$$

$$h_y \equiv \tau^{-1}\frac{1}{1-\phi_{s^y}}\sigma_\eta,\;\; h_y^* \equiv \tau^{-1}\frac{1}{1-\phi_{s^y}}\sigma_{\eta^*},\;\; h_q \equiv \tau^{-1}\frac{1}{1-\phi_{s^q}}\sigma_\gamma,\;\; h_q^* \equiv \tau^{-1}\frac{1}{1-\phi_{s^q}}\sigma_{\gamma^*}.$$

we get that $\mathbf{V}$ must satisfy the following system of three equations in three unknowns:

$$V_y = \left(\frac{\delta}{1-\delta\phi_i}\right)^2\sigma_i^2 + \left(\frac{\delta}{1-\delta\phi_g}\right)^2\sigma_g^2 + (V_y)^2(g_y^2+g_{y^*}^2) + (C_{y,q})^2(g_q^2+g_{q^*}^2) + 2g_{y^*}g_{q^*}V_yC_{y,q}; \tag{B.27a}$$

$$V_q = \left(\frac{1}{1-\phi_i}\right)^2 2\sigma_i^2(1-\rho) + \sigma_q^2 + (C_{y,q})^2(h_y^2+h_{y^*}^2) + (V_q)^2(h_q^2+h_{q^*}^2) + 2h_{y^*}h_{q^*}V_qC_{y,q}; \tag{B.27b}$$

$$C_{y,q} = -\frac{\delta}{1-\delta\phi_i}\frac{1}{1-\phi_i}\sigma_i^2(1-\rho) + V_yC_{y,q}(g_yh_y+g_{y^*}h_{y^*}) + V_qC_{y,q}(g_qh_q+g_{q^*}h_{q^*}) \tag{B.27c}$$

$$+ (C_{y,q})^2g_{q^*}h_{y^*} + V_yV_qg_{y^*}h_{q^*}.$$

where $C_{y,q} = C_{q,y}$ and

$$\mathbf{V} = \text{var}_t[\mathbf{rx}_{t+1}] = \begin{bmatrix} V_y & C_{y,q} \\ C_{y,q} & V_q \end{bmatrix}.$$

which completes the full write-down of the solution method. One must now combine these three equations to find qualitative properties of the three unknowns. Instead, in the next subsection we prove Proposition 1, which effectively reduces this system of equations into a more manageable set that yields qualitatively siimlar implications.

B.3 Proof of Proposition 1

Proof of Proposition 1. The assumptions spelled out imply that $V_y = V_q$, which simplifies our system of equations to the following

$$V_y = \sigma_g^2 + (V_y)^2\tau^{-2}(\sigma_\eta^2+\sigma_{\eta^*}^2) + (C_{y,q})^2\tau^{-2}(\sigma_\eta^2+\sigma_{\eta^*}^2) + 2V_yC_{y,q}\tau^{-2}\sigma_{\eta^*}^2; \tag{B.28a}$$

$$C_{y,q} = 2V_yC_{y,q}\tau^{-2}(\sigma_\eta^2+\sigma_{\eta^*}^2) + (C_{y,q})^2\tau^{-2}\sigma_{\eta^*}^2 + (V_y)^2\tau^{-2}\sigma_{\eta^*}^2. \tag{B.28b}$$

We can use equation (14) to rewrite this system as

$$V_y = \sigma_g^2 + \tau^{-2}(\sigma_\eta^2+\sigma_{\eta^*}^2)\left[(V_y)^2 + (C_{y,q})^2 + 2\Omega V_yC_{y,q}\right]; \tag{B.29a}$$

$$C_{y,q} = \tau^{-2}(\sigma_\eta^2+\sigma_{\eta^*}^2)\left[2V_yC_{y,q} + (C_{y,q})^2\Omega + (V_y)^2\Omega\right]. \tag{B.29b}$$

First, setting $\Omega = 0$, it can be readily seen from equations (B.29a)-(B.29b) that the only real solution implies $C_{y,q} = 0$. Second, differentiating both sides of this equation with respect to

Ω, we achieve

$$\frac{d(C_{y,q})}{d\Omega} = \tau^{-2}(\sigma_{\eta}^2 + \sigma_{\eta^*}^2)\left[2V_y\frac{d(C_{y,q})}{d\Omega} + 2C_{y,q}\frac{d(C_{y,q})}{d\Omega}\Omega + C_{y,q}^2 + V_y^2\right]. \tag{B.30}$$

Next, collecting all terms that contain $\frac{dC_{y,q}}{d\Omega}$ on the left-hand side yields

$$\frac{d(C_{y,q})}{d\Omega} = \frac{\tau^{-2}(\sigma_{\eta}^2 + \sigma_{\eta^*}^2)\left[C_{y,q}^2 + V_y^2\right]}{1 - \tau^{-2}(\sigma_{\eta}^2 + \sigma_{\eta^*}^2)\left(2V_y + 2C_{y,q}\Omega\right)}. \tag{B.31}$$

To ensure that the denominator is positive, we require agents to be risk-tolerant enough. That is, for τ sufficiently large, we get that $\frac{d(C_{y,q})}{d\Omega} > 0$, which completes our proof.

□

B.4 Calibration

Parameter	Interpretation	Value
τ	Risk-tolerance	13
δ	Maturity of long-term bonds	0.75
ϕ_i	Persistence of interest rate shocks	0
ϕ_g	Persistence of bond-specific shocks	0
ϕ_{s^y}	Persistence of bond supply shocks	0.25
ϕ_{s^q}	Persistence of foreign exchange supply shocks	0.25
σ_i	Std. dev. of interest rate shocks	0; 0.4
σ_g	Std. dev. of bond-specific shocks	2
σ_q	Std. dev. of long-term q shocks	2
σ_η	Std. dev. of domestic bond-supply shocks	[0,. . . , 2]
σ_γ	Std. dev. of domestic FX-supply shocks	[0,. . . , 2]
$\sigma_{\eta^\star}$	Std. dev. of foreign bond-supply shocks	[0,. . . , 2]
$\sigma_{\eta^\star}$	Std. dev. of foreign FX-supply shocks	[0,. . . , 2]

Table B.1
Baseline parameterization for numerical illustrations