Job Ladder and Dynamic Monopsony in Production Networks*

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Abstract

This paper examines how workers' outside options are concentrated within the supply chains of their employers. Using Belgian data on firm-to-firm sales relationships merged with a matched employer-employee dataset, we find that workers are connected through the sparse networks of employers and frequently move to buyers or suppliers of their current employers. Movers within these production networks climb up job ladders at a faster pace but with smaller immediate earnings gains than other movers. Motivated by these findings, we develop and estimate an equilibrium model of firm-to-firm trade and on-the-job search. We estimate a higher job-finding rate along production networks, which enables well-connected firms to suppress wages. Accounting for the network search channel increases workers' exposure to correlated shocks in production networks.

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1 Introduction

Firms operate in a complex network of buyer-supplier relationships with other firms in the product market. Meanwhile, firms also interact with workers in the labor market, hiring job seekers from unemployment and from other firms. In the presence of labor market frictions, the movement of workers between employers accounts for a large fraction of aggregate worker flows (see, e.g., Haltiwanger et al., 2018 and Moscarini and Postel-Vinay, 2018), creating a complex network of current employers and potential next employers (Nimczik, 2023). While we can find well-known anecdotes of workers finding job opportunities through their business contacts, such as management consultants or temporary agency employees, the prevalence of such a search channel at the aggregate level is relatively less known. Yet, an overlap between these networks in the product market and labor market is key for understanding how workers' outside options are diversified against shocks in the product market. That is, if a large fraction of potential employers buys from or sells to the current employers, the shocks to production networks affect not only the current employers but also the majority of potential employers.

The aim of this paper is to examine the roles of firm-to-firm networks in explaining workers' movements between employers and assess their contributions to aggregate labor market flows. To this end, we first combine several administrative datasets from Belgium to study the prevalence and characteristics of worker reallocation along the supply chain. The VAT transaction database covers the universe of domestic firm-to-firm sales relationships, and we merge it with a matched employer-employee dataset based on social security records. The merged dataset allows us to observe worker mobility along firm-to-firm linkages over the period 2003-2014.

Equipped with the data, we provide several pieces of motivating evidence on the interaction between the Belgian labor market and production networks. First, Belgian workers are well connected through the firm-to-firm linkages of their employers. We compute the employment-based *labor market connectedness* of each firm—the share of total employment accounted for by the firms with which it is directly connected in the production network. Even though Belgian production networks are sparse, with the average firm having only 51 buyers and suppliers out of a total of around 100,000 firms, the average Belgian worker is connected to around 23 percent of total employment through the direct links of their employer. This difference arises because larger firms with more employment tend to be connected with more buyers and suppliers.

Next, we find that a sizable fraction of Belgian workers find their next employers among the buyers and suppliers of their current employers. The movements of workers along the firm-to-firm linkages, *B2B moves*, account for around 42 percent of job-to-job transitions. A part of these B2B moves can be rationalized just by chance, as movers at the firms with a

high level of labor market connectedness are likely to move within networks by construction. However, this alone does not fully account for the disproportionately high share of B2B moves among Belgian workers. A simple statistical random benchmark suggests that only 20 percent of movers would move within networks if they were to be randomly matched with hiring firms.[1] Furthermore, these job-to-job transitions along supply chains are common whether workers move within or across the industries and geographic regions of their current employers, and regardless of their gender and worker types.

Workers may choose to move within networks if they find the buyers and suppliers of their current employers either more attractive or easier to move to than the other firms. Therefore, we then perform a movers analysis to examine the consequences of B2B moves on the earnings of workers. We find that, while both B2B movers and non-B2B movers experience earnings gains upon moving, those who move along firm-to-firm linkages do not gain relatively more than those who find their next employers outside the networks. In fact, the earnings gains for B2B movers are slightly lower, and this difference is robust with respect to including various sets of controls that account for market-specific time trends and firm fixed effects.

Motivated and guided by these empirical findings, we construct an equilibrium model that features both firm-to-firm trade and on-the-job search. The goal of the model is to quantify the contribution of firm-to-firm linkages in the product market to aggregate worker flows. In order to incorporate these features of the product and labor markets into our model, we borrow from the models of firm dynamics with nonlinear production technologies and random on-the-job search, such as Elsby and Gottfries (2022) and Bilal et al. (2022), and present a parsimonious way to incorporate firm-to-firm trade in such models.

One important feature of our model is that workers can be matched with vacancies through two search channels. These channels are similar in spirit to the models of Carrillo-Tudela et al. (2022) and Lester et al. (2021) in which workers face multiple job-finding rates through different channels. However, we incorporate a novel channel by considering their interactions with firm-to-firm linkages in the product market. In our model, the standard constant returns to scale matching function allows all workers to meet all vacancies randomly according to the vacancy distribution (*market search*), while the employed workers can also meet vacancies at a constant rate if the vacancy poster is connected to their current employers in the product market (*network search*). The introduction of network search alters the pattern of worker flows in two ways compared to the standard on-the-job search model. First, the overall job-finding rate is now specific to each firm, as the additional job-finding rate from network search is proportional to the vacancy-based labor market connectedness of the

[1]This random benchmark corresponds to the mover-level average of labor market connectedness based on job-to-job hires next period without any additional controls. See Section 3.1 for further discussion.

firm. The more closely the current employers of workers are connected to the other vacancy-posting firms, the more likely they are to meet some vacancies through their search channels. Second, network search directs part of workers' search effort into a set of connected firms in production networks. Therefore, the workers at different firms no longer meet a certain vacancy at the same rate, which makes the outside options of workers more concentrated and less diversified. This creates an overlap between the labor market and product market.

We take our model to the data with the goal of quantifying the contribution of the network search channel to worker flows and explaining the worker impacts of productivity shocks to production networks. The estimation of our labor market and product market parameters follows a two-step procedure. We first estimate all the labor market parameters using the simulated method of moments to match the observed characteristics of the Belgian labor market. In our model, firms' own productivity, intermediate input costs, and demand shifter in the product market all proportionally affect the marginal returns to hiring an additional worker. This allows us to estimate the labor market and product market parameters separately. In the second step, we then estimate the remaining product market parameters using the estimated labor market parameters. Conditional on the labor market parameters and employment distribution, we can consider the within-period decisions of firms in the product market as static. Therefore, we can apply the identification arguments similar to the ones of Bernard et al. (2022) and Huneeus et al. (2021) to the cross section of the Belgian economy and estimate the parameters in the product market. This two-step process allows the estimation of our model to be computationally feasible.

Our estimates suggest that workers utilize both market search and network search to find their next employers and that the standard search-and-matching technology of market search is not enough to generate a disproportionately high share of B2B moves that we observe in the Belgian economy. We estimate the parameter for the network search premium to be around 0.18, which, taken together with the labor market connectedness of each firm, implies that around 30 percent of workers' job search is directed toward the buyers and suppliers of their current employers on average. This suggests a considerable overlap between the set of potential employers in the labor market and firm-to-firm linkages in the product market.

In order to examine how the presence of this network search channel alters the patterns of worker flows, we also consider a counterfactual economy with no additional job finding through the firm-to-firm linkages, similar to the standard labor search model. When matching the same overall quarterly employment-to-employment rate of 5 percent, we find that this counterfactual economy can generate job-to-job transitions along firm-to-firm linkages of only around 17 percent. This number is even lower than the statistical random benchmark reported earlier because the absence of the network search channel contributes to the lower worker flows into and out of well-connected firms, where the B2B moves are more likely to

happen.

Lastly, we take our estimated model to analyze the worker impacts of a 5 percent decline in productivity among Belgian manufacturing firms. The propagation of shocks in the production networks results in an immediate wage reduction of around 6 percent among manufacturing workers and 1 percent among non-manufacturing workers. Furthermore, these shocks also affect the wages of the potential employers. When matched with firms through the market search channel, workers find the average wage of the matched firms to be lower by around 2 percent. On the other hand, when matched with firms through the network search channel, workers at different firms find that the average wage decline of the matched firms ranges from 2 percent to 5 percent. We also find a positive correlation of around 0.6 between the wage decline at the current employer and the average wage decline of the firms matched through the network search channel. Therefore, our results suggest that the network search channel reduces the diversification of workers' outside options against shocks to the production networks.

Related literature. This paper contributes to several strands of literature. First, we contribute to the literature that analyzes the roles of networks in job search. A seminal work by Montgomery (1991) lays out a model in which social networks reduce search frictions. Since then, a large number of papers have provided both empirical evidence and theoretical frameworks in which workers utilize their connection with other workers in their job search process and have analyzed the consequences of such connections (see, e.g., Dustmann et al., 2016, Lester et al., 2021, Glitz, 2017, Arbex et al., 2019, and Caldwell and Harmon, 2019). A recent work by Carrillo-Tudela et al. (2022) studies firms' and workers' use of multiple search channels and also finds that the networks of personal contacts play an important role in the matching process. Compared to these previous works that focus on worker-level networks, we emphasize the roles of employer-level linkages in production networks and investigate how the overlap between two networks in the labor market and product market affects aggregate labor market flows.

Next, this paper joins a set of recent papers that combine a firm-to-firm transaction database with matched employer-employee information from social security records. For instance, Adão et al. (2022) study how international trade affects earnings inequality in Ecuador, while Demir et al. (2021) use data from Turkish manufacturing firms and workers to document the positive assortative matching of skills between buyers and suppliers. Alfaro-Ureña et al. (2021) estimate the effects of foreign multinationals on Costa Rican workers, and Huneeus et al. (2021) show that firm heterogeneity arising from firm-to-firm linkages in production networks plays a substantial role in explaining the volatility of earnings among Chilean workers.

The closest in this literature to our empirical analysis is the work by Cardoza et al. (2023), which analyzes worker mobility along domestic supply chains using firm-to-firm trade data and matched employer-employee data from the Dominican Republic. They reach a similar conclusion that workers move disproportionately more into the suppliers or buyers of their current employers. Several differences are worth highlighting. First, Belgium is an advanced economy with smaller informal sector employment. As pointed out by Donovan et al. (2023), labor market flows are systematically different between advanced economies and developing economies, where the transitions to and from self-employment and informal sector employment play a major role. Second, while they find a positive wage premium upon moving within networks in the Dominican Republic, which they rationalize by the supply-chain-specific human capital, we find that this channel is less evident in the Belgian labor market. Given smaller wage gains for those who move within networks, our results are rather supportive of reduced search frictions along the supply chains in Belgium. Lastly, we construct a structural model of production networks and worker mobility to quantify the value of firm-to-firm linkages in the labor market.

This paper also contributes to the theory of firm-to-firm trade by incorporating labor market frictions and on-the-job search. A limited number of papers incorporate imperfect competition in the labor market into the models of domestic production networks. For instance, Huneeus et al. (2021) develop a structural model of heterogeneous firms and workers to quantify the contribution of network linkages to earnings inequality in Chile. Using the same Belgian firm-to-firm transactions data as ours, Dhyne et al. (2022a) find that accounting for an upward-sloping labor supply curve and fixed overhead costs in labor substantially alters the aggregate implications of foreign demand shocks to production networks. Both papers introduce monopsony power of employers through workers' idiosyncratic preferences over workplaces, such as in Card et al. (2018) and Lamadon et al. (2022). Compared to these studies, we construct a model of dynamic monopsony in the spirit of Burdett and Mortensen (1998), which allows employers to set lower wages because of search frictions.[2] Constructing a dynamic model in the labor market is important in our objective to study the job-to-job transitions and outside options of workers.

Similarly, we add to the theory of on-the-job search and the job ladder by incorporating firm heterogeneity arising from production networks. The majority of job ladder models that build on the seminal work of Burdett and Mortensen (1998) assume a linear production technology in labor as well as the production of a single homogeneous final good. These assumptions make it difficult to introduce production networks into such models. A small set of exceptions propose a model of firm dynamics with nonlinear production technologies and on-the-job search, such as Schaal (2017), Elsby and Gottfries (2022), and Bilal et al.

[2]See Manning (2021) for the review and taxonomy of different monopsony models.

(2022). While their models are able to characterize the rich dynamics of firms and workers, they still abstract away from the intermediate input market and assume a single homogeneous good.[3] We propose a parsimonious way to add the intermediate input market to this class of models, and, to the best of our knowledge, we provide the first model that features both firm-to-firm trade and on-the-job search.

Outline. The remainder of the paper proceeds as follows. Section 2 describes our data, and Section 3 presents motivating empirical facts on the Belgian labor market and production networks. Motivated by those findings, we construct an equilibrium model of firm-to-firm trade and job-to-job transitions in Section 4. We discuss in Section 5 the estimation strategies to bring our model to the data. In Section 6, we perform a counterfactual exercise to quantify the contribution of firm-to-firm linkages to labor market dynamics. Section 7 concludes.

2 Data

Our analyses draw on several administrative datasets from Belgium over the period 2003-2014. These datasets allow us to combine the firm-to-firm linkages in the product market with worker flows in the labor market through the same firm identifiers. In this section, we briefly discuss data sources and the construction of our analysis sample; additional details are provided in Appendix A.

2.1 Data on firm-to-firm linkages

We draw information on the Belgian domestic production networks from the Business-to-Business (B2B) transaction database provided by the National Bank of Belgium (NBB). As further explained in Dhyne et al. (2015), all firms in Belgium are assigned unique identifiers for the purpose of collecting value-added taxes (VAT). In each year, all VAT-liable firms in Belgium are legally required to report the amount of annual sales to each of their VAT-liable buyers, provided that the amount to a given buyer exceeds 250 euro. This allows us to accurately measure the firm-to-firm transaction linkages in the product market.

We then merge this dataset with firms' annual accounts in order to supplement the firm-level information. The annual accounts provide detailed information on firm-level sales, value added, cost of labor inputs, and ownership shares in other VAT-liable firms. In addition, we observe each firm's number of full-time equivalent (FTE) employees as well as its industry code at the NACE four-digit level and the postal code of its main economic activity. For

[3] Other papers construct a model of a frictional labor market with firms producing unique differentiated products, such as Coşar et al. (2016) and Kaas and Kimasa (2021), but they do not consider on-the-job search.

the firms that operate in multiple geographic locations, we also have a list of sub-provinces (*arrondissements* of Belgium, at the NUTS two-digit level) where they have establishments.

It is important to note that all the information described above is recorded at the VAT identifier level. The VAT identifiers in Belgium do not always correspond to the notion of firms or establishments, as some firms may have several VAT identifiers for accounting or tax purposes. In these cases, we follow Dhyne et al. (2021) and aggregate all VAT identifiers into a firm identifier using information about their ownership structure.[4] See Appendix A.1 for further details on the aggregation procedure.

2.2 Data on worker flows

In order to observe worker mobility along firm-to-firm linkages in the product market, we then link information on the employment histories of individual workers using the matched employer-employee data for the period 2003-2014. The employer-employee data are based on the social security records provided by the Crossroads Bank for Social Security (CBSS) and then merged with our firm-level data by NBB. See Appendix A.2 for details on the merging procedure.

The data consist of a quarterly panel for the sample of 500,000 workers, drawn from the population of workers who have worked at least once at the non-financial private-sector firms that have 10 or more FTE employees during the period 2003-2014. For each employer-employee-quarter pair, we observe the status of the worker (blue collar or white collar) and a fraction of the FTE quarter she worked at the employer. When workers work at the non-financial private-sector firms that have 10 or more FTE employees, we also observe their quarterly earnings.[5]

2.3 Constructing the sample of movers

Our merged dataset allows us to observe both firm-to-firm linkages in the product market and employment histories in the labor market through the same firm identifiers. When analyzing worker flows along the supply chain, we impose a few restrictions on firms and workers to construct a suitable sample of movers. On the firms' side, we restrict our analysis to the non-financial private-sector firms that have at least one FTE employee and report

[4]While the linkages and worker flows within the same firm may be of interest, this paper primarily focuses on inter-firm linkages and worker flows. See, for example, Giroud and Mueller (2019) and Huitfeldt et al. (2023) for discussions on firms' internal networks and internal labor market.

[5]Because of the restrictions imposed by the Belgian social security administration, we cannot observe the earnings of workers when they work at small firms. These observations account for around 9 percent of employer-employee matches. However, we can still track the entire employment histories of our sampled workers regardless of the size and industry of their employers.

positive sales, value added, and labor costs. We refer to these firms as the firms in the main analysis sample.

For workers, we first identify their main employers. In each quarter, workers can be observed at multiple firms. This happens when workers hold multiple jobs simultaneously or switch their employers in the middle of the quarter. In our main analysis, we define a worker's main employer in a given quarter as the firm where she spends the highest fraction of the FTE quarter.[6] We restrict our analysis to the workers who are currently employed at the firms in the main analysis sample.

Combining these, we define a *job-to-job mover* in a given quarter as the worker whose main employer changes to another firm in the main analysis sample in the next quarter. We impose two additional restrictions to construct our baseline sample of job-to-job movers. First, we drop the job-to-job movers who return to their previous employers within one quarter. This restriction excludes the workers who only temporarily move to another firm as well as the multiple job holders whose main employers keep switching every quarter. Second, we exclude a group of movers from the baseline sample if more than 500 workers in a given firm move to the same firm in the same quarter. These massive movements could be driven by the mere changes in employer identifiers, such as those due to outsourcing events (see, e.g., Goldschmidt and Schmieder, 2017), and may not necessarily reflect the actual job-to-job transitions of workers. Our analyses in the following sections are not significantly altered when we consider different thresholds or do not impose these restrictions.

Taken together, our sample selections lead to around 100,000 firms in the main analysis sample each year and a total of 446,343 job-to-job movements over the period 2003-2014. In Table 5 of Appendix A.3, we present some summary statistics on the firms and workers in our main analysis sample.

3 Motivating empirical facts

Equipped with the data described in the previous section, we next provide several pieces of motivating evidence on the interaction between the Belgian labor market and production networks.

[6]Note that because of the limited availability of earnings information, we do not define workers' main employers based on their highest quarterly earnings. While it is more common in the existing works to look at the employers with the highest earnings, such as in Haltiwanger et al. (2018) and Lamadon et al. (2022), our definition based on the hours does not seem to be a major concern. Conditional on observing earnings, the employers with the longest hours correspond to the ones with the highest earnings more than 99.7 percent of the time.

3.1 Connectedness of workforce through firm-to-firm linkages

We first document how connected the Belgian labor market is through the linkages of firms in the product market. As reported in Table 5 of Appendix A.3, the Belgian product market can be described as the sparse networks of buyer-supplier relationships. In 2012, the average Belgian firm has only 51 buyers and suppliers out of a total of around 100,000 firms. Therefore, the average share of firms accounted for by the connected firms in production networks is less than 0.1 percent.

However, this does not necessarily mean that these sparse networks do not connect the Belgian workforce. In Figure 6 of Appendix B.1, we show that the Belgian firms with a greater number of buyers and suppliers tend to be larger on average, consistent with the evidence from other countries (e.g., Bernard et al., 2019, for Japan and Arkolakis et al., 2023, for Chile). This implies that it is more likely for two randomly selected workers to find their employers to be connected in production networks than for two randomly selected firms to be connected.

To quantify the degree of connectedness from the perspective of workers, we define the labor market connectedness of each firm. The *employment-based labor market connectedness* of firm j, denoted by $\mathbb{C}_j^e$, is the share of total employment accounted for by the firms that firm j is directly connected to in production networks. Let Ω be the set of all firms and Ω_j^B and Ω_j^S be the sets of firm j's buyers and suppliers, respectively. Then, firm j's employment-based labor market connectedness can be computed as follows:

$$\mathbb{C}_j^e = \frac{\sum_{i \in \Omega_j^B \cup \Omega_j^S} n_i}{\sum_{i \in \Omega \setminus \{j\}} n_i}, \tag{1}$$

where n_i is the employment of firm i.[7] This measure ranges between 0 and 1 and captures the degree of connectedness between workers at a given firm with workers at the other firms.[8]

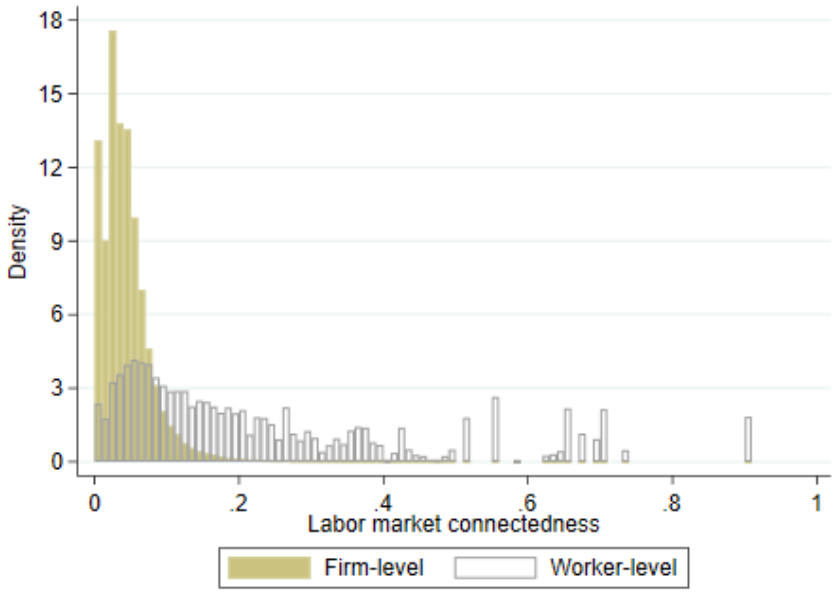
Figure 1 plots the distribution of employment-based labor market connectedness in the Belgian economy. In 2012, an average firm is connected to around 4 percent of total employment through its firm-to-firm linkages. This number is significantly higher when we consider an average worker. An average Belgian worker is connected to around 23 percent of total employment through the direct links of their employer.

One potential concern with this measure is that our results might be driven by a certain set of firms that have small transactions with the majority of firms. For instance, some large

[7] In a similar fashion, we can also define the hiring-based and vacancy-based labor market connectedness of firm j, denoted by $\mathbb{C}_j^h$ and $\mathbb{C}_j^v$, respectively. In Appendix B.2, we plot the distribution of hiring-based labor market connectedness in the Belgian economy.

[8] This measure is essentially a weighted *degree centrality* in the network science, where each node (firm) is weighted by its size (employment).

Figure 1: Distribution of labor market connectedness



Notes: This figure shows the distribution of employment-based labor market connectedness. The employment-based labor market connectedness of firm j, denoted by $\mathbb{C}_j^e$ and defined in equation (1), is the share of total employment accounted for by the firms that firm j is directly connected to in production networks. The white bars represent the distribution of the firm-level measure of labor market connectedness in which one weights the firms by the number of workers at each firm. The figure is based on the main analysis sample of 98,599 private-sector firms in Belgium in 2012 (see Section 2.3 for details).

wholesalers and utility companies supply to thousands of customer firms, each of which might have a small transaction volume. To address this concern, we also compute the labor market connectedness of firms after dropping the links from retailers, wholesalers, and utility companies to their buyers.[9] As shown in Figure 8 of Appendix B.3, we find that an average Belgian worker is still connected to around 20 percent of total employment through the other links. Thus, we conclude that Belgian workers are well connected through the sparse networks of buyer-supplier relationships in the product market.

3.2 Frequency of job-to-job transitions along the supply chain

We next describe the prevalence and characteristics of worker mobility along the supply chain in Belgium. To do so, we first compute the share of job-to-job transitions where the origin and destination firms are connected in production networks, which we call *B2B moves*. We

[9]This procedure drops around 45 percent of firm-to-firm linkages and 36 percent of transaction volume.

can define the *share of B2B moves*, denoted by $\mathbb{B}$, as follows:

$$\mathbb{B} = \frac{\sum_i \mathbf{1}_{\left\{j(i,t+1) \in \Omega^S_{j(i,t),t} \cup \Omega^B_{j(i,t),t}\right\}}}{\sum_i \mathbf{1}_{\{j(i,t+1) \neq j(i,t)\}}}, \tag{2}$$

where we denote the employer of worker i at time t as firm $j(i,t)$. The denominator corresponds to the number of all job-to-job movers, while the numerator computes the number of movers who find their next employers to be in buyer-supplier relationships with their previous employers. In the baseline specification, we pool all job-to-job movers throughout our sample period 2003-2014 and compute the share of B2B moves in the Belgian economy.

Table 1 reports the shares of B2B moves out of all job-to-job movers in Belgium. As reported in the first column, we find that around 32 percent of movers find their next employers among the buyers of their current employers. Similarly, around 23 percent of movers move to the direct suppliers of current employers, and taken together, the movements to the directly connected firms of current employers account for around 42 percent of all job-to-job transitions. This suggests that a sizable fraction of Belgian workers move along the firm-to-firm linkages of their current employers.

A potential concern is that this high share of B2B moves can be fully rationalized by coincidence or other factors that are orthogonal to buyer-supplier relationships. As we discussed in Section 3.1, the Belgian labor market is well connected through firm-to-firm linkages, which makes it more likely for the randomly selected employer to be connected to the current employer. To alleviate this concern, we first plot in Figure 2 the share of B2B moves by the percentiles of labor market connectedness of current employers. As one would expect, the share of B2B moves increases as the firm-to-firm linkages of their current employers account for a larger share of total employment. Nonetheless, the observed shares of B2B moves are significantly higher than the 45 degree line. This finding is suggestive that workers move systematically and disproportionately more into the buyers and suppliers of their current employers.

To formalize this argument, we now conduct a simple simulation exercise to compute the statistical random benchmarks that we can compare our numbers to. The goal of this simulation is to compute the share of B2B moves if movers were to be randomly matched with hiring firms. We take the set of observed movers and hiring firms and randomize the matches between them, so that we can compute the share of job-to-job transitions that happened to be between two firms connected in the product market. We compute the average share after repeating this exercise 100,000 times.[10] The full details of this exercise are provided in

[10]Without any additional controls, this share converges to the mover-level average of labor market connectedness based on the employment-to-employment hires next period, which we report in the last row of Table 6 in Appendix B.2.

Table 1: Share of B2B moves

	Share of moves	
	Data	Random benchmark
All movers		
all B2B moves	0.42	0.20
moves to buyers	0.32	0.14
moves to suppliers	0.23	0.12
moves through key B2B links	0.11	0.02

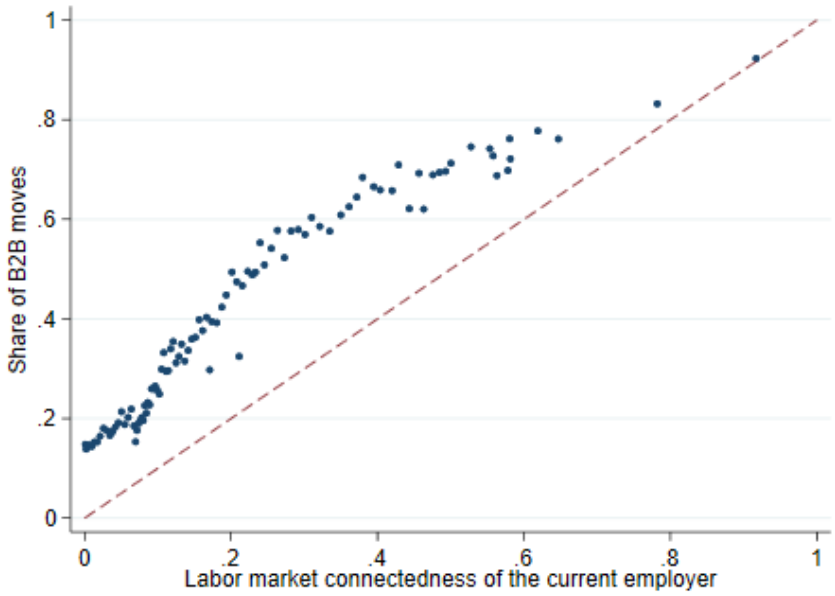
	Share of movers	Share of B2B moves	
		Data	Random benchmark
Industries			
movers within NACE 2	0.27	0.38	0.25
movers across NACE 2	0.73	0.43	0.18
Locations			
movers within NUTS 2	0.37	0.38	0.23
movers across NUTS 2	0.63	0.44	0.18
Markets (NACE 2×NUTS 2)			
movers within market	0.12	0.36	0.24
movers across market	0.88	0.42	0.19
Worker types			
male movers	0.66	0.42	0.19
female movers	0.34	0.41	0.21
blue-collar movers	0.53	0.43	0.16
white-collar movers	0.37	0.49	0.21

Notes: This table reports the shares of B2B moves among different sets of job-to-job movers. B2B moves refer to the job-to-job transitions where the origin and destination firms are connected in production networks, and the share of B2B moves is defined in equation (2). In the first panel, we compute the shares of B2B moves among all movers. We report the observed shares in the data in the first column, while, in the second column, we report the results from simulation exercises to compute the statistical random benchmarks. The random benchmarks compute the share of B2B moves if movers were to be randomly matched with hiring firms (see Appendix D.1 for details). In the last row, we only consider the firm-to-firm linkages with transactions that exceed 5 percent of buyers' network purchases and/or suppliers' network sales. In the second panel, we compute the shares of B2B moves among different groups of movers. We also report the share of movers for each group in an additional column. The shares of blue-collar movers and white-collar movers do not add up to one because we did not include the movers who changed their worker classes upon moving. This table is based on the main analysis sample of 467,194 movers in Belgium over the period 2003-2014 (see Section 2.3 for details).

Appendix D.1.

The second column of Table 1 reports the simulation results. We find that only around 20 percent of job-to-job movers would move within networks if they were randomly matched with hiring firms. This result is in stark contrast with the B2B share of 42 percent that we observe in the data, implying that the random matching of all firms and workers alone

Figure 2: Share of B2B moves vs. labor market connectedness



Notes: This figure shows the share of B2B moves by the percentiles of labor market connectedness of the employers at the origin of job-to-job transitions. For each percentile of job-to-job movers, sorted by the employment-based labor market connectedness of their employers at the origin, we compute the share of B2B moves among those movers. The employment-based labor market connectedness is defined in equation (1), and the share of B2B moves is defined in equation (2). The dashed red line represents the 45 degree line. This figure is based on the main analysis sample of 467,194 movers in Belgium over the period 2003-2014 (see Section 2.3 for details).

cannot justify the movements of workers along the supply chain.[11]

Another concern is that the existence of firm-to-firm transactions may not necessarily reflect significant business ties between two firms in production networks. For instance, as discussed above, some wholesalers and utility companies may report small transactions with many customer firms, each of which neither buyers nor suppliers consider to be an important connection. Therefore, we address this concern by considering the job-to-job moves along the *key links* in production networks, which at least one party considers to be an important part of its business. To be precise, we only consider the firm-to-firm linkages with transactions that exceed 5 percent of buyers' network purchases and/or suppliers' network sales. This procedure yields the subset of production networks, which accounts for 13 percent of firm-to-firm linkages and 78 percent of transaction volume in Belgium. As reported in the fourth row of Table 1, this restriction indeed does reduce the share of B2B moves to 11 percent. Nonetheless, this also cuts the random benchmark further down to 2 percent, suggesting an

[11] Admittedly, this simple simulation exercise does not take into account the endogenous mobility decision of workers and the hiring decision of firms. We will revisit this result using a full structural model in Section 6.

even stronger role of firm-to-firm linkages in explaining job-to-job transitions.

In the second panel of Table 1, we also consider the heterogeneity of firms and workers in the shares of B2B moves. In the first six rows, we compute the shares of B2B moves after splitting all the job-to-job movers into those who moved within or across the industries and geographic regions of their current employers. We find that around 38 percent of movers who stayed in the same two-digit industries moved along firm-to-firm linkages, while 43 percent of movers who switched industries moved within networks. Comparing these numbers with the random benchmarks, we find that both types of movers are still disproportionately more likely to move within networks compared to random moves, though the industry switchers are more likely to move within the linkages of their employers.[12] We find a similar result for the movers within or across the two-digit geographic locations: those who change their work location are more likely to move along the linkages of their current employers.[13] In the last four rows, we also show that our results are robust regardless of workers' gender and their status as blue-collar or white-collar workers.

Table 1 also reveals that a large share of movers switches their industries and work locations: only around 12 percent of movers stay in the same narrowly defined market of two-digit industries and regions. Our results suggest that firm-level linkages play an important role in shaping the patterns of worker flows beyond the conventional boundaries of the labor market in terms of industries and regions. These patterns are consistent with worker flows found in other countries. For instance, Bjelland et al. (2011) find that around 60 percent of job-to-job transitions in the United States are across broadly defined 11 NAICS super-sectors, and Nimczik (2023) finds that industries do not serve as a good predictor of data-driven boundaries in the Austrian labor market. Similar to our setting, Cardoza et al. (2023) find that around one-fifth of job-to-job movers in the Dominican Republic move within production networks. While the magnitude of B2B moves is different, as two countries can differ in how connected their labor markets are, they reach a similar conclusion that workers move disproportionately more into the suppliers or buyers of their current employers.

[12]One might be concerned that our results for the industry switchers are driven by selected industries, such as consultants at the consulting firms moving to their former client firms or workers at temporary employment agencies moving to their customers. In Table 7 of Appendix B.4, we report the B2B shares excluding movers from consulting firms and temporary employment agencies. While the movers from those firms have a higher share of movements to buyers, our result among the other movers is still robust in comparison to the random benchmark

[13]Importantly, we do not observe the exact workplace of each worker if firms have multiple establishments. Therefore, the movers whose current and next employers report different geographic regions may not necessarily move across regions. To alleviate this concern, we checked whether two firms have any establishments that operate in the same regions. As reported in Table 7 of Appendix B.4, our results are robust for the movers between two firms that have no overlapping business coverage.

3.3 Consequence of job-to-job transitions on earnings

The previous discussions point toward Belgian workers systematically moving to the firms that trade with their current employers. A natural question that arises from this observation is why they are more likely to move within networks. Intuitively, workers may be likely to move within networks if they find the buyers and suppliers of their current employers either more attractive or easier to move to than the other firms. In order to shed light on the potential mechanism and guide our theoretical analysis in the coming sections, we now perform a movers analysis to examine the consequences of B2B moves on the earnings of workers.

We consider a sample of movers who switch their main employers between $t-1$ and t and have tenures of at least eight quarters at both the origin and destination firms. In order to track the trajectories of earnings, we also restrict our sample to be the employer-employee matches with observed earnings information. We then use the balanced panel of movers from $t-8$ to $t+7$ and estimate the effects of moving within or across networks by running the following regression:

$$\log w_{i,s} = \psi_i + \sum_{j\in\{0,1\}} \sum_{k=-8}^{7} \tau_k^j \mathbf{1}_{\{k=s,T(i)=j\}} + \epsilon_{i,s}, \tag{3}$$

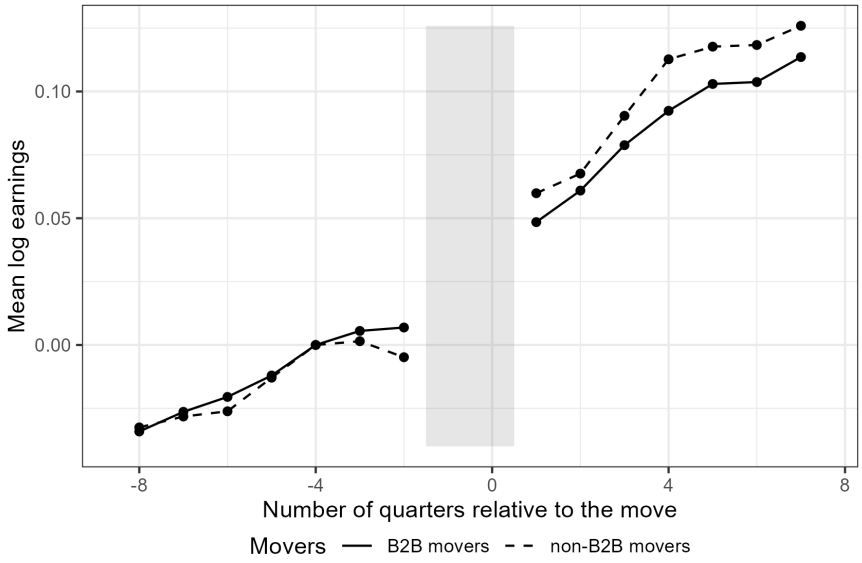
where $\log w_{i,s}$ denotes mover i's log quarterly earnings in quarter s relative to the quarter of the move, $T(i)$ is an indicator for the move along firm-to-firm linkages in the product market, and ψ_i is a worker fixed effect. In order to ensure that our estimates are not contaminated by partial-quarter employment spells in a given firm, we drop the observations in quarters $t-1$ and t.

Panels (a) and (b) of Figure 3 present a graphical representation of the gains in movers' earnings after the move. In Panel (a), we first show the trajectories of average log quarterly earnings from our data, normalized at $t=-2$. This figure shows that both B2B movers and non-B2B movers experience gains in their earnings, although the gains seem to be larger for non-B2B movers. To assess this difference in earnings gains, we then present in Panel (b) the results from our movers analysis. This figure tracks the quarter-by-quarter difference in earnings effects, denoted by $(\tau_k^1 - \tau_k^0)$ in equation (3). Our findings support common trends prior to the move and relatively smaller earnings gains for B2B movers.

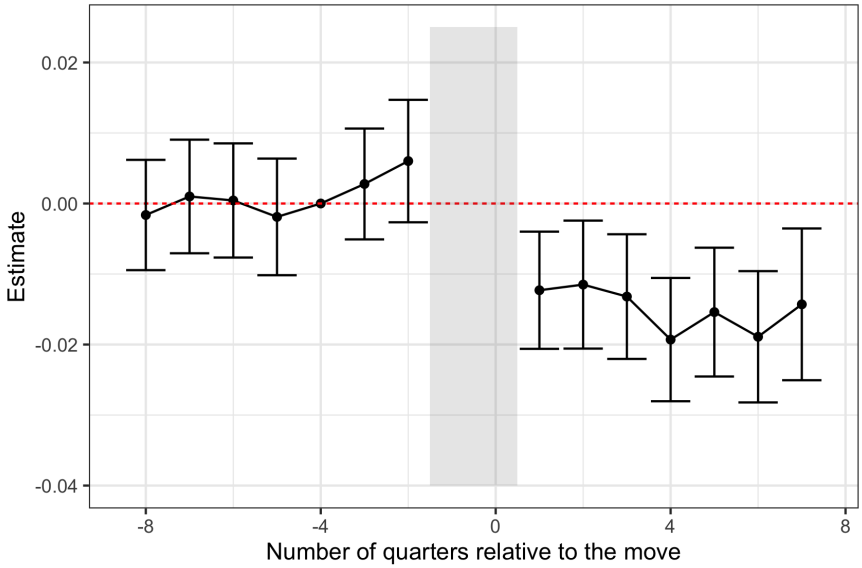
We also consider the sensitivity of our movers analysis in Table 2. We pool the quarter-specific coefficients τ_k^j in equation (3) into a single coefficient τ_{post}^j for all post-move periods $t \geq 1$ and consider the average effect. The difference is robust with respect to including various sets of controls that account for market-specific time trends and firm fixed effects. We find that, while both B2B movers and non-B2B movers experience gains in their earnings

Figure 3: Trajectories of quarterly earnings before and after the move

(a) Raw quarterly earnings



(b) Movers analysis for the difference



Notes: These figures report the trajectories of job-to-job movers' quarterly earnings before and after the move for B2B movers and non-B2B movers. B2B moves refer to the job-to-job transitions where the origin and destination firms are connected in production networks. In Panel (a), we show the trajectories of average log quarterly earnings from the data, normalized at $t = -2$. Panel (b) shows the results from a movers regression in equation (3). For each quarter k relative to the quarter of the move, we report the difference in earnings effects, denoted by $(\tau_k^1 - \tau_k^0)$. In both panels, we drop the observations in quarters $t-1$ and t to avoid the contamination by partial-quarter employment spells in a given firm. The figures are based on the balanced panel of 26,846 movers who have at least two years of tenure at both the origin and destination firms and whose quarterly earnings are observed throughout four years (see Sections 2.3 and 3.3 for details).

upon moving, those who move along the firm-to-firm linkages do not gain relatively more than those who find their next employers outside the networks.

Table 2: Sensitivity of earnings differences

	(1)	(2)	(3)	(4)
Post×B2B move $\left(\tau^1_{post}\right)$	0.0973***	0.0133***	0.0121***	0.0135***
	(0.00204)	(0.00281)	(0.00271)	(0.00332)
Post×non-B2B move $\left(\tau^0_{post}\right)$	0.114***	0.0293***	0.0287***	0.0251***
	(0.00211)	(0.00305)	(0.00296)	(0.00378)
Difference $\left(\tau^1_{post} - \tau^0_{post}\right)$	-0.0163***	-0.0160***	-0.0165***	-0.0117***
	(0.00263)	(0.00257)	(0.00231)	(0.00341)
Worker FE	Yes	Yes	Yes	Yes
Calendar time FE		Yes		
Market×calendar time FE			Yes	Yes
Firm FE				Yes

Notes: This table reports the results from a movers regression in equation (3). For each column, we pool the quarter-specific indicators $\mathbf{1}_{\{k=s,T(i)=j\}}$ and the corresponding coefficients τ^j_k into a single post-move indicator $\mathbf{1}_{\{s>0,T(i)=j\}}$ and a coefficient τ^j_{post}. B2B moves refer to the job-to-job transitions where the origin and destination firms are connected in production networks. In columns (3) and (4), market fixed effects are included at the interaction of NACE two-digit and NUTS two-digit level. The table is based on the balanced panel of 26,846 movers who have at least two years of tenure at both the origin and destination firms and whose quarterly earnings are observed throughout four years. We drop the observations in quarters $t-1$ and t to avoid the contamination by partial-quarter employment spells in a given firm (see Sections 2.3 and 3.3 for details). * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

Our results suggest that the observed patterns of job-to-job transitions along the supply chains in Belgium are, on average, not driven by the immediate gains in earnings compared to other movers. It is interesting to note that these patterns do not necessarily hold in any economy and can be at odds with the findings in other countries. For instance, Cardoza et al. (2023) find a positive wage premium upon moving within networks in the Dominican Republic, which they rationalize by the supply-chain-specific human capital. On the other hand, we find that this channel is less evident in the Belgian labor market. Given smaller gains in earnings for those who move within networks, our results are rather supportive of reduced search frictions in finding more job opportunities along the supply chains in Belgium.

Taken together, our findings point toward the important roles of firm-to-firm linkages in the product market in explaining the patterns of job-to-job transitions in the labor market. While these findings suggest that workers are disproportionately more likely to find job opportunities within the production networks, we have not yet taken into full considerations the endogenous hiring decisions of firms and the mobility decisions of workers. Therefore, in the next section, we will introduce a structural model of firm-to-firm trade and job-to-job transitions.

4 Model

Motivated by the empirical findings in the previous section, we now construct an equilibrium model that features both firm-to-firm trade and job-to-job transitions. The model serves to quantify the contribution of firm-to-firm linkages in the product market to aggregate worker flows. In order to introduce these features in the product market and labor market, we borrow from the models of firm dynamics with nonlinear production technologies and random on-the-job search, such as Elsby and Gottfries (2022) and Bilal et al. (2022), and present a parsimonious way to incorporate firm-to-firm trade in such models.

One important feature of our model is that workers can be matched with vacancies through two search channels, similar to the models of Carrillo-Tudela et al. (2022) and Lester et al. (2021). This class of models allows workers to face multiple job-finding rates through different channels, and we incorporate a novel channel by considering their interactions with firm-to-firm linkages in the product market. In addition to the constant returns to scale matching technology, which is standard in the models of job-to-job transitions, we allow workers to meet vacancies through their employers' firm-to-firm linkages in the product market. This creates an overlap between the labor market and product market, as we elaborate more in the coming sections.

In what follows, we first describe the model environment and introduce the firms' problem in product market. We then discuss the dynamic problems of firms and workers in the labor market and define our equilibrium. Lastly, we characterize the aggregate steady state where the distribution of workers remains unchanged.

4.1 Model environment

Time is continuous, and the economy consists of a mass L of households and a set of firms denoted by Ω. The set Ω has a measure of one, and the firm views itself as infinitesimal. Each firm produces a unique differentiated product by combining labor inputs and intermediate inputs from an exogenous set of suppliers, denoted by Ω_j^S for firm j. Within each period, the firm acts monopolistically competitively in the product market and sells its product to the households and customer firms in an exogenous set of buyers Ω_j^B. The firm takes as given the prices of its intermediate inputs and purchases them in a spot market, whereas hiring workers is subject to search frictions and wage bargaining. We now describe the details of each problem below, starting from the static problems in the product market and moving on to the dynamic problems involving the labor market. For notational convenience, we suppress the dependence on the aggregate state until the end of this subsection.

Final product demand. All households are risk-neutral, discount the future at rate ρ, and have the same preference for goods. The instantaneous utility from consuming the final goods, denoted by C, is given by the following constant elasticity of substitution (CES) aggregate of each firm's goods, $\{q_{jH}\}_{j\in\Omega}$:

$$C = \left(\int_{\Omega} \left(\beta_{jH} q_{jH}\right)^{\frac{\sigma-1}{\sigma}} dj \right)^{\frac{\sigma}{\sigma-1}}, \tag{4}$$

where $\sigma > 1$ is the elasticity of substitution parameter, and $\beta_{jH} \geq 0$ denotes the weights that the households place on firm j's product. Given the CES structure that is common to all households, we can write the aggregate final consumer demand for firm j's product as follows:

$$q_{jH} = \beta_{jH}^{\sigma-1} \frac{p_{jH}^{-\sigma}}{P^{1-\sigma}} E, \tag{5}$$

where $P = \left(\int_{\Omega} \beta_{kH}^{\sigma-1} p_{kH}^{1-\sigma} dk \right)^{\frac{1}{1-\sigma}}$ is the aggregate price index, and E denotes the aggregate income.

Production technology. Each firm has a firm-specific technology to produce a unique differentiated product. Firm j produces its product q_j by converting labor inputs and intermediate inputs from each of its suppliers, denoted by n_j and $\{q_{ij}\}_{i\in\Omega_j^S}$, respectively. We write the production technology of firm j as the following Cobb-Douglas production function between labor inputs and the CES intermediate input bundle:

$$q_j = \phi_j n_j^{\alpha} m_j^{1-\alpha} \tag{6}$$

$$m_j = \left(\int_{\Omega_j^S} \left(\gamma_{ij} q_{ij}\right)^{\frac{\sigma-1}{\sigma}} di \right)^{\frac{\sigma}{\sigma-1}}, \tag{7}$$

where ϕ_j denotes firm j's own total factor productivity, and $\gamma_{ij} > 0$ governs the saliency of firm i's product in the production of firm j.[14]

Intermediate input market. Within each period, firm j takes as given the stock of its employed workers n_j and decides how much to purchase from each of its suppliers in a spot market. We assume that the suppliers, who produce unique differentiated products, have

[14]Note that we assume the elasticity of substitution parameter σ to be the same in both the household utility and production function. This assumption is common in recent literature on production networks, such as Huneeus et al. (2021), Demir et al. (2021), Arkolakis et al. (2023), and Dhyne et al. (2022b), and implies that the demand elasticity does not depend on whether firms sell their outputs to the households or other customer firms.

the power to set prices and that the buyers take the prices of intermediate inputs as given.[15] With the CES structure of the intermediate input bundle in equation (7), we can solve the cost minimization problem of firm j to obtain the following demand function for supplying firm i's product given the level of intermediate input bundle m:

$$q_{ij}(m) = \gamma_{ij}^{\sigma-1} \frac{p_{ij}^{-\sigma}}{z_j^{-\sigma}} m \tag{8}$$

$$z_j = \left(\int_{\Omega_j^S} \gamma_{kj}^{\sigma-1} p_{kj}^{1-\sigma} dk \right)^{\frac{1}{1-\sigma}}, \tag{9}$$

where we call z_j the intermediate input cost index of firm j, which it takes as exogenous.

Output market. Firms sell their unique differentiated products to the households as well as to each of their customer firms in a monopolistically competitive fashion. Taking as given the demand curves from the household and customer firms described in equations (5) and (8), respectively, firm j sets the output prices $\{p_{jk}\}$. As we formally claim in Appendix C.1, the common demand elasticity across all firms and households implies that firms optimally charge the same price to all of their customers:

$$p_{jk} = p_j, \tag{10}$$

for all $k \in \Omega_j^B \cup \{H\}$. As a result, we can aggregate the demand curves across all customers and write the demand curve for firm j's product as follows:

$$q_j \equiv \int_{\Omega_j^B \cup \{H\}} q_{jk} dk = \chi_j p_j^{-\sigma}, \tag{11}$$

where

$$\chi_j = \int_{\Omega_j^B} (\gamma_{jk} z_k)^{\sigma-1} z_k m_k dk + (\beta_{jH} P)^{\sigma-1} E. \tag{12}$$

Firms maximize their profits facing this demand curve and the costs of intermediate inputs. In the presence of labor market frictions, the costs paid to workers are sunk at the time when firms decide how much to produce and how much to buy from their suppliers. Therefore, each firm maximizes its instantaneous revenue net of intermediate input costs

[15] The price-setting power of buyers is often assumed in the models of production networks, such as in Lim (2018) and Bernard et al. (2022), but other approaches are not uncommon. See, for example, Alviarez et al. (2020) for the model of buyer-supplier bargaining and Dhyne et al. (2023) for the model that features buyers' full bargaining power with their suppliers. In our setting, the presence of labor market frictions makes the static marginal cost increasing in intermediate inputs, which makes the problem of the buyer's price setting considerably complicated.

given the level of current employment. Within each period, firm j with n workers solves the following static problem by choosing its output price and demand for the intermediate input bundle:

$$R_j(n) = \max_{p,m} (pq_j - z_j m) \tag{13}$$

such that equations (6) and (11) hold. Solving this problem, we can write firm j's optimal revenue net of intermediate input costs as follows:[16]

$$R_j(n) = \Phi_j n^{\frac{\alpha(\sigma-1)}{1+\alpha(\sigma-1)}}, \tag{14}$$

where

$$\Phi_j \propto z_j^{-\frac{(1-\alpha)(\sigma-1)}{1+\alpha(\sigma-1)}} \phi_j^{\frac{\sigma-1}{1+\alpha(\sigma-1)}} \chi_j^{\frac{1}{1+\alpha(\sigma-1)}}. \tag{15}$$

As shown in equation (14), the revenue net of intermediate input costs exhibits decreasing returns to scale in labor, and its extent depends on both the saliency of labor inputs in the Cobb-Douglass production function (α) and the substitutability of goods in the product market (σ). Furthermore, equation (15) implies that the firm-specific labor productivity Φ_j—one of the key determinants of the firm size distribution—is not only determined by its own productivity ϕ_j but also depends on who it buys from and sells to in the production network. The latter information is summarized by the cost index z_j and demand shifter χ_j.

Labor market. Having described the within-period static problems in the product market, we now turn our attention to the labor market. Firm j with current employment n_j faces search frictions when increasing or decreasing its employment. It posts vacancies v_j when hiring additional workers and loses a part of its current workers through exogenous separation at a constant rate at δ_0 and endogenous separation. Posting a vacancy incurs a flow per-vacancy cost of c, and each vacancy can be matched with workers who search for vacancies while employed and unemployed. The unemployed workers search for vacancies while enjoying a flow payoff of b, whereas the employed workers at firm j receive a flow wage w_j, which is endogenously determined by a bargaining game discussed later. The matching between workers and vacancies can occur through two search channels, which we call *market search* and *network search*, and are described below.

Market search. In the *market search* channel, all job seekers and vacancies meet randomly through the standard search-and-matching procedure. The mass $u = L - \int_\Omega n_j dj$ of unemployed workers dedicates all of their search efforts into market search, while the employed workers $\{n_j\}_{j\in\Omega}$ search for vacancies through market search with exogenous relative

[16] See Appendix C.1 for the derivation. We also provide the full expression of Φ_j in equation (48).

search intensity ζ. The number of meetings between workers and vacancies through market search is governed by the following constant returns to scale aggregate matching function:

$$M(\tilde{u}, V) = A\tilde{u}^{\xi}V^{1-\xi}, \tag{16}$$

where $\tilde{u} = u + \zeta \int_{\Omega} n_k dk$ is the total mass of effective searchers, and $V = \int_{\Omega} v_j dj$ is the total mass of vacancies. We denote the matching efficiency and matching elasticity by A and ξ, respectively.

The constant returns to scale matching technology implies that the job-finding rate for unemployed workers through market search, denoted by λ^m, can be written as follows:

$$\lambda^m(\theta) = M(1, \theta) = \frac{M(\tilde{u}, V)}{u + \zeta \int_{\Omega} n_k dk}, \tag{17}$$

where $\theta = V/\tilde{u}$ denotes the labor market tightness in market search. Similarly, the job-finding rate for employed workers can be written as $\zeta\lambda^m(\theta)$. It is important to note that the job-finding rate through the market search channel is common across all employed workers regardless of their current employers. To ease notation, we make the dependence on the labor market tightness θ implicit in the coming sections.

Network search. In the *network search* channel, the employed workers have an additional chance to meet vacancies at a constant rate if the vacancy poster is connected to their current employers in the product market. To formalize the idea, let $\bar{\lambda}$ be the constant relative search premium from the network search, which is common across all employed workers. Then, the rate for a worker at firm j to meet a vacancy through network search, denoted by λ_j^n, is given by

$$\begin{aligned} \lambda_j^n &= \int_{\Omega} \bar{\lambda} \mathbf{1}_{\{k \in \Omega_j^B \cup \Omega_j^S\}} \tilde{v}_k dk \\ &= \bar{\lambda} \mathbb{C}_j^v, \end{aligned} \tag{18}$$

where $\tilde{v}_k$ is the density of vacancy distribution, and $\mathbb{C}_j^v$ is the vacancy-based labor market connectedness of firm j as defined in Section 3.1.

The introduction of network search alters the patterns of job search behavior in two ways compared to the models with market search alone. First, the overall job-finding rate, which combines the job finding rates from two search channels ($\zeta\lambda^m$ and λ_j^n), is now specific to each firm. The more closely the firms are connected to the other vacancy-posting firms, the more likely their workers are to meet vacancies through network search. Second, network search directs part of workers' search effort into a set of firms that buy from or sell to their

current employers in the product market. This makes the distributions of potential employers different from the perspective of workers at different firms.

Separation rates. We next define the separation rate for each firm. The matches between firms and workers can be dissolved for three reasons. First, workers can be separated into unemployment at an exogenous separate rate δ_0. Second, workers may voluntarily quit and move to another firm when meeting its vacancy through one of the search channels and accepting its offer. Lastly, firms can also implement additional separations at no cost. We then define the separation rate for firm j, denoted by δ_j, as the rate at which workers are separated from firm j for the first or second reasons:

$$\delta_j = \delta_0 + \int_{k \in \Omega_j^A} \lambda_{jk} \tilde{v}_k dk, \tag{19}$$

where

$$\lambda_{jk} = \zeta \lambda^m + \bar{\lambda} \mathbf{1}_{\{k \in \Omega_j^B \cup \Omega_j^S\}}, \tag{20}$$

and Ω_j^A denotes the endogenous set of firms that workers at firm j are willing to accept the offers from when they meet the vacancies. The separation rate δ_j is specific to each firm due to the set of acceptable firms Ω_j^A as well as heterogeneous job-finding rates through network search, which is captured by the second term of equation (19).

Vacancy-filling rates. We can also define the vacancy-filling rate for each firm in a similar manner. When firm j post vacancies, each of its vacancies can be matched with unemployed workers through market search or workers at the other firms through one of the search channels, who then decide whether to accept the offer. Hence, the vacancy-filling rate for firm j, denoted by μ_j, can be written as follows:

$$\mu_j = \frac{1}{V} \left[\lambda^m u + \int_{j \in \Omega_i^A} \lambda_{ij} n_i di \right]. \tag{21}$$

Similar to the separation rate, the vacancy-filling μ_j is also specific to each firm due to the endogenous mobility decision of workers, which is captured by the set of acceptable firms $\{\Omega_i^A\}_{i \in \Omega}$, as well as the heterogeneous job-finding rates at the other firms $\{\lambda_{ij}\}_{i \in \Omega}$.

Wage setting. Lastly, we characterize the wage-setting protocol adopted by firms and workers. When firms and workers form successful matches through one of the search channels described above, the matches generate positive surplus. We model the wage to be determined by the bargaining game between a firm and its workers to split such surplus. We follow the

model of Elsby and Gottfries (2022) that builds on the bargaining games of Stole and Zwiebel (1996) and Brügemann et al. (2019), such that wages are determined after all the successful matches are formed. In other words, a firm pays the same wage to all of its workers once employed.

We now describe the bargaining process in detail. In each period, a firm and each of its workers sequentially engage in bilateral bargaining over the marginal surplus to determine its flow wage, in which all workers have the bargaining power of $\eta \in (0,1)$. As in Elsby and Gottfries (2022), we assume that an unsuccessful negotiation leads to a temporary disruption of the match, during which a worker receives the flow payoff of w^e and the firm pays the flow per-worker cost of w^f. This implies that the wage is determined by splitting the marginal flow surplus between a firm and each of its workers, and the flow wage at firm j solves the following differential equation:

$$(1-\eta)\left(w_j - w^e\right) = \eta\left(\frac{dR_j}{dn} - w_j - \frac{dw_j}{dn}n + w^f\right), \tag{22}$$

where R_j is firm j's optimal revenue net of intermediate input costs that we obtained in equation (14). Solving this differential equation, we can write the flow wage paid to workers at firm j that has n workers as follows:

$$w_j(n) = \frac{\eta}{1-\eta\left(1-\frac{\alpha(\sigma-1)}{1+\alpha(\sigma-1)}\right)}\Phi_j \frac{\alpha(\sigma-1)}{1+\alpha(\sigma-1)} n^{\frac{\alpha(\sigma-1)}{1+\alpha(\sigma-1)}-1} + \bar{w}, \tag{23}$$

where $\bar{w} \equiv \eta w^f + (1-\eta)w^e$.

4.2 Equilibrium and dynamic problems of firms and workers

Given the model environment described above, we now characterize the dynamic problems of firms and workers and define the equilibrium of this economy. From here on, we assume for simplicity that there is no shock to firms' own productivity $\{\phi_j\}_{j\in\Omega}$:

Assumption 1. *A firm's own productivity is time-invariant such that $\phi_{jt} = \phi_j$ for all $j \in \Omega$.*

Note that a firm's labor productivity Φ_j is still time-variant under this assumption, as it also depends on the cost index z_j and demand shifter χ_j, which can evolve according to the evolution of employment at other firms. For notational convenience, we follow Ahn et al. (2018) and use the time-dependent notation with respect to the distributions of labor productivity $\{\Phi_j\}_{j\in\Omega}$ and workers $\{n_j\}_{j\in\Omega}$.

Firms' value. We first state the dynamic problem of firms in the labor market. In a given moment, firms take as given their separation rate and vacancy-filling rate and decide the

optimal amount of vacancies and additional separations, denoted by v and s, respectively. The dynamic problem for firm j that currently has n workers can be characterized by the following Hamilton-Jacobi-Bellman equation:

$$\rho\Pi_{jt}(n) = \max_{v\geq 0, s\geq 0}\left[R_{jt}(n) - w_{jt}(n)n - cv + (\mu_{jt}v - s - \delta_{jt}n)\frac{\partial\Pi_{jt}}{\partial n} + \frac{\mathbb{E}_t\left[d\Pi_{jt}(n)\right]}{dt}\right], \quad (24)$$

where $\mathbb{E}_t\left[dW_{jt}\right]/dt \equiv \lim_{\Delta t\downarrow 0}\mathbb{E}_t\left[W_{jt+\Delta t} - W_{jt}\right]/\Delta t$. The first three terms correspond to firm j's flow profits, which can be obtained from its revenue net of intermediate input costs, wages paid to its workers, and vacancy costs. The fourth term captures the gains and losses from the evolution of its own employment, while the last term summarizes the changes in the firm's value with respect to the evolutions of labor productivity and employment distributions.

Solving this problem, we obtain an optimal level of additional separation as:

$$s^*_{jt}\frac{\partial\Pi_{jt}}{\partial n} = 0, \quad (25)$$

whereas the optimal amount of vacancies is determined by

$$v^*_{jt}\left(c - \mu_{jt}\frac{\partial\Pi_{jt}}{\partial n}\right) = 0. \quad (26)$$

When firm j has excess employment, it dissolves the matches until its marginal value of labor becomes zero. On the other hand, when it is in need of additional workers, it posts vacancies until the marginal value of hiring an additional worker is equal to its marginal cost, which is determined by the ratio between vacancy cost c and vacancy-filling rate μ_{jt}.

Workers' value. The value for a worker employed at firm j with the level of current employment n, denoted by $W_j(n)$, is given by

$$\begin{aligned}\rho W_{jt}(n) = \max\Bigg\{ & w_{jt}(n) + \int_\Omega \lambda_{jkt}\tilde{v}_{kt}\left(W_{kt} - W_{jt}\right)^+ dk + \left(\delta_0 + \frac{s^*_{jt}}{n}\right)\left(U_t - W_{jt}\right) \\ & + \left(\mu_{jt}v^*_{jt} - s^*_{jt} - \delta_{jt}n\right)\frac{\partial W_{jt}}{\partial n} + \frac{\mathbb{E}_t\left[dW_{jt}(n)\right]}{dt}, \rho U_t\Bigg\}, \end{aligned} \quad (27)$$

where U denotes the value for an unemployed worker such that

$$\rho U_t = b + \lambda^m_t\int_\Omega \tilde{v}_{kt}(W_{kt} - U_t)^+ dk + \frac{\mathbb{E}_t\left[dU_t\right]}{dt}. \quad (28)$$

Using these expressions, we can rewrite the separation rate at firm j in equation (21) as

$$\delta_{jt} = \delta_0 + \int_\Omega \lambda_{jkt} \tilde{v}_{kt} \mathbb{1}_{\{W_{kt} > W_{jt}\}} dk \tag{29}$$

and the vacancy-filling rate at firm j in equation (19) as

$$\mu_{jt} = \frac{1}{V_t} \left[\lambda_t^m u_t + \int_\Omega \lambda_{ijt} n_{it} \mathbb{1}_{\{W_{jt} > W_{it}\}} di \right]. \tag{30}$$

General equilibrium. To close the model, we define the aggregate income E_t. The aggregate income of the economy is the sum of firm profits and wages paid to workers, and firm profits are given by their revenue net of intermediate input costs, wage payments, and vacancy costs. We make the following assumption on how firms pay their vacancy costs:

Assumption 2. *The vacancy costs are paid in final goods C defined in equation (4).*

This assumption states that the amount spent by firms to post vacancies remains in the aggregate income of the economy. Therefore, given the distribution of workers $\{n_{jt}\}_{j\in\Omega}$, the aggregate income E_t can be computed as follows:

$$E_t = \int_\Omega R_{jt}(n_{jt}) dj. \tag{31}$$

We can now define the general equilibrium of this economy. As an intermediate step, we first define the within-period product market equilibrium given the realized distribution of workers $\{n_{jt}\}_{j\in\Omega}$.

Definition 1 (Product market equilibrium)**.** *Given the distribution of workers $\{n_{jt}\}_{j\in\Omega}$, a product market equilibrium at time t is the set of prices $\{p_{jt}\}_{j\in\Omega}$ that satisfies equations (4)-(15) and (31).*

Because the problem in the product market is static, a product market equilibrium in each period is characterized independently of the evolution of the employment distribution. Furthermore, we claim that a product market equilibrium can be characterized as a solution to the following system of equations:

Claim 1. *Let $\tilde{p}_j \equiv p_j^{1-\sigma}$, $\tilde{z}_j \equiv z_j^{1-\sigma}$, and $\tilde{m}_j \equiv z_j m_j / p_j^{1-\sigma}$. Under Assumption 2, a product*

market equilibrium is characterized by the following system of equations:

$$\tilde{z}_j = \int_\Omega f_{jk,Z}\left(\tilde{p}_j\right) dk = \int_\Omega \mathbb{1}_{\{k\in\Omega_j^S\}} \gamma_{kj}^{\sigma-1} \tilde{p}_k dk \tag{32}$$

$$\begin{aligned}\tilde{m}_j &= \int_\Omega f_{jk,M}\left(\tilde{z}_k, \tilde{m}_k, \tilde{p}_j\right) dk \\ &= \int_\Omega \left(\frac{1+\alpha(\sigma-1)}{\sigma} + \frac{(1-\alpha)(\sigma-1)}{\sigma} \mathbb{1}_{\{k\in\Omega_j^B\}} \gamma_{jk}^{\sigma-1} \tilde{z}_k^{-1} \right) \tilde{p}_k \tilde{m}_k dk \end{aligned} \tag{33}$$

$$\begin{aligned}\tilde{p}_j &= f_{j,P}\left(\tilde{z}_j, \tilde{m}_j\right) \\ &= \left(\frac{(1-\alpha)(\sigma-1)}{\sigma} \phi_j n_j^\alpha \right)^{\frac{(\sigma-1)}{1+\alpha(\sigma-1)}} \tilde{z}_j^{\frac{1-\alpha}{1+\alpha(\sigma-1)}} \tilde{m}_j^{-\frac{\alpha(\sigma-1)}{1+\alpha(\sigma-1)}} \end{aligned} \tag{34}$$

Derivations of these equations are presented in Appendix C.2.

Lastly, the general equilibrium of this economy features paths of prices $\{p_{jt}\}_{j\in\Omega}$, employment distributions $\{n_{jt}\}_{j\in\Omega}$, vacancy-posting decisions $\{v_{jt}\}_{j\in\Omega}$, firing decisions $\{s_{jt}\}_{j\in\Omega}$, and workers' mobility decisions such that (i) a set of prices in each period is a product market equilibrium given the employment distribution, (ii) firms and workers satisfy their HJB equations, and (iii) markets clear.

4.3 Aggregate steady state

Next, we characterize the aggregate steady state of the model, in which the distribution of workers remains unchanged. Under Assumptions 1 and 2, Claim 1 implies that the firm's labor productivity $\{\Phi_j\}_{j\in\Omega}$ remains constant under the stationary distribution of workers. Therefore, we can characterize the stationary distribution of workers using the distribution of labor productivity instead of the underlying product market equilibrium that generates this distribution:

Claim 2. *Under Assumptions 1 and 2, the stationary distribution of workers at the aggregate steady state can be characterized by the set of employment $\{n_j\}_{j\in\Omega}$ and labor productivity $\{\Phi_j\}_{j\in\Omega}$. In particular, the employment at firm j satisfies the following:*

$$n_j = \left(\frac{c}{\mu_j} + \frac{\bar{w}}{\rho+\delta_j} \right)^{\frac{1}{\tilde{\alpha}-1}} \left(\frac{\Phi_j \tilde{\alpha}}{\rho+\delta_j\tilde{\alpha}} \right)^{-\frac{1}{\tilde{\alpha}-1}} \left[1 - \frac{\eta}{1-\eta(1-\tilde{\alpha})} \tilde{\alpha} \right]^{-\frac{1}{\tilde{\alpha}-1}}, \tag{35}$$

where $\tilde{\alpha} \equiv \frac{\alpha(\sigma-1)}{1+\alpha(\sigma-1)}$.

The vacancy-filling rate μ_j and separation rate δ_j in equation (35) follow the expressions in equations (30) and (29) and are determined by firms' vacancy-posting decisions $\{v_j\}_{j\in\Omega}$. Firms' vacancy-posting decisions are then characterized by the set of employment $\{n_j\}_{j\in\Omega}$

and labor productivity $\{\Phi_j\}_{j\in\Omega}$. We provide the derivation and further discussions in Appendix C.3.

Combining these results, we can now characterize the aggregate steady state of the model. Based on Claims 1 and 2, we characterize the aggregate steady state of the model as follows:[17]

Claim 3. *Under Assumptions 1 and 2, the aggregate steady state of the model is characterized by the set of prices* $\{p_j\}_{j\in\Omega}$ *and employment* $\{n_j\}_{j\in\Omega}$ *such that*

1. $\{p_j\}_{j\in\Omega}$ *is a product market equilibrium given the distribution of workers* $\{n_j\}_{j\in\Omega}$*;*
2. $\{n_j\}_{j\in\Omega}$ *is a stationary distribution of workers given the distribution of labor productivity* $\{\Phi_j\}_{j\in\Omega}$ *implied by the product market equilibrium.*

5 Estimation strategies

We now take our model to the data with the goal of quantifying the contribution of the network search channel to worker flows and explaining the worker impacts of productivity shocks to production networks. Taking advantage of Claims 1-3, we estimate the labor market and product market in a sequential manner: we first estimate the labor market parameters using the simulated method of moments to match the observed characteristics of the Belgian labor market. We then apply Claim 1 and estimate the product market parameters using the cross section of the Belgian economy. This two-step procedure allows the estimation of our model to be computationally feasible. The list of parameters and estimation strategies are summarized in Table 3.

5.1 Parameters calibrated outside the model

We first calibrate several parameters outside the model. We set the time span to be quarterly, and we take the year 2012 as our baseline year. We assume that the economy is at its steady state. We set the discount rate $\rho = 0.01$ to match the quarterly interest rate of 1 percent (or, equivalently, the annual interest rate of 4 percent). The total mass of workers L is fixed at 20.2 to match the average firm size of 18.6 under the steady-state unemployment rate of 8 percent. We set the substitutability parameter $\sigma = 4$ following a common choice in the prior literature (see, e.g, Antras et al., 2017, and Oberfield and Raval, 2021). We calibrate the returns to scale parameter for labor α to match the average labor cost share of 0.37.

[17] We cannot formally establish the uniqueness of the steady state. The difficulty rises from the intractability of workers' mobility decisions in closed form. Nonetheless, we confirm numerically that the employment distribution converges to the unique distribution for a wide range of parameters and for different initial guesses.

Lastly, we set the matching elasticity $\xi = 0.5$ as in the literature (see, e.g., Petrongolo and Pissarides, 2001).

Given the calibration for σ and α, we can directly compute each firm's labor productivity $\{\Phi_j\}_{j\in\Omega}$ in equation (15) using the observed levels of value added and employment. Figure 9 in Appendix E.1 plots the distribution of $\log \Phi_j$ in the Belgian economy. We also normalize labor productivity such that the average of $\log \Phi_j$ is set to be zero.

5.2 Labor market parameters

Given the calibrated parameters, we then estimate the other parameters of the model. As we know the distribution of firms' labor productivity $\{\Phi_j\}_{j\in\Omega}$, Claim 3 implies that we can solve for the stationary distribution of workers without knowing the product market parameters and product market equilibrium. Hence, we first estimate the set of labor market parameters by the method of simulated moments and move on to estimating the product market parameters in Section 5.3.

To proceed with solving for the stationary distribution of workers and estimating the labor market parameters, we first discretize the set of firms. Furthermore, as it is computationally infeasible to evaluate the mobility decisions of workers for every firm pair among 100,000 firms, we cluster firms into *firm groups*. We construct firm groups by first splitting firms based on their NACE two-digit industries and then clustering them by the quantiles of labor productivity $\{\Phi_j\}$ and labor market connectedness $\{\mathbb{C}_j^e\}$. This procedure results in clustering the Belgian firms into around 1,000 firm groups.

We assume that each firm is infinitesimal within its firm group and that all firms are homogeneous within their firm groups. For the network structure, we make the following assumption:

Assumption 3. *For a given pair of firm groups (J, K), a fraction ω_{JK} of firm-pairs (j, k), where $j \in J$ and $k \in K$, is randomly matched in the production network.*

Under Assumption 3, we replace the indicator functions for the firm-to-firm linkages with the continuous measure $\{\omega_{JK}\}$. For a given origin firm $j \in J$, a pair-specific job-finding rate λ_{jk} in equation (20) is now rewritten as $\lambda^{jk} = \zeta\lambda^m + \bar{\lambda}\omega_{JK}$ for all destination firms $k \in K$.[18]

Based on the network structure of firm groups obtained from the data, we now estimate the labor market parameters. Seven parameters are relevant in solving for the stationary equilibrium of workers. We first normalize the matching efficiency of market search A at

[18] With these groupings, it is now possible for a worker to find a vacancy posted by the other firms in the same firm group. In this small probability event, I assume that workers decide randomly with an equal probability whether to stay or move to the matched firm. This choice is not quantitatively important to our findings.

one[19]. We then let $\Theta = \{\bar{\lambda}, \zeta, \delta_0, c, \bar{w}, \eta\}$ be the vector of the remaining six labor market parameters to be estimated. We estimate Θ using the method of simulated moments.

In order to estimate six labor market parameters, we select the six moments to be targeted. While we use all six moments jointly, each moment corresponds to and is more informative about different parameters. We target the share of B2B moves of 0.42 to identify the network search premium λ. Market search intensity ζ corresponds to the average quarterly employment-to-employment (EE) rate of 5 percent, while the exogenous separation rate δ_0 is set to target the average quarterly employment-to-unemployment (EU) rate of 4 percent. We estimate the vacancy cost c and constant in wage equation $\bar{w}$ by targeting the ratio of firm sizes between the 25th and 75th percentiles as well as the steady-state unemployment rate of 8 percent. Lastly, we estimate the worker bargaining power η to target the average wage gains of movers at 0.02, which corresponds to the average gains in detrended earnings reported in column (2) of Table 2. The details for solving for the stationary distribution of workers and computing the model counterparts of these moments are provided in Appendix D.2.

Given the choice of these six moments, we estimate the parameters Θ as follows. We denote the vector of targeted moments in the data by $\hat{y}$, and let $y(\Theta)$ be the vector of model-implied moments at the parameter value Θ. We assume that the following moment condition holds at the true parameter value Θ^*:

$$E[\hat{y} - y(\Theta^*)] = 0 \tag{36}$$

Then, we estimate the labor market parameters by minimizing the following objective function:

$$\hat{\Theta} = \arg\min_{\Theta} [\hat{y} - y(\Theta^*)]'[\hat{y} - y(\Theta^*)]. \tag{37}$$

5.3 Product market parameters

Equipped with the labor market parameters estimated above, we proceed to estimating the remaining product market parameters.

Saliency in intermediate input production. We first describe our identification strategies for the saliency parameters in intermediate input production $\{\gamma_{jk}\}$. Following the discussions in Bernard et al. (2022) and Huneeus et al. (2021), we assume that γ_{jk} can be decomposed as follows:

[19] This affects the level of total vacancies and vacancy-filling rates but does not alter the patterns of worker flows when the other parameters are appropriately scaled.

Assumption 4. *The saliency of firm j's good in the intermediate input production of firm k, denoted by γ_{jk}, takes the following functional form:*

$$\log \gamma_{jk} = \log \gamma_j + \log \gamma_k + \log \tilde{\gamma}_{jk}, \tag{38}$$

where $\tilde{\gamma}_{jk}$ is independent across all firm pairs.

Assumption 4 states that γ_{jk} can be decomposed in a log-additive manner into the *relationship capability* of each firm as well as the firm-pair-specific *relationship residual*. It is useful to observe that we allow the relationship residuals to be asymmetric, such that $\log \tilde{\gamma}_{jk} \neq \log \tilde{\gamma}_{kj}$.

Under Assumption 4, rearranging equation (8) yields the following relationship:

$$\begin{aligned}
\log p_j q_{jk} =& (\sigma - 1) \log \gamma_{jk} + (1 - \sigma) \log p_j + \sigma \log z_k + \log m_k \\
=& \underbrace{(\sigma - 1) \log \gamma_j + (1 - \sigma) \log p_j}_{\equiv \log \Gamma_j^S} + \underbrace{(\sigma - 1) \log \gamma_k + \sigma \log z_k + \log m_k}_{\equiv \log \Gamma_k^B} \\
&+ \underbrace{(\sigma - 1) \log \tilde{\gamma}_{jk}}_{\equiv \log \tilde{\Gamma}_{jk}}.
\end{aligned} \tag{39}$$

Equation (39) gives the structural interpretations to the buyer and supplier fixed effects in the decomposition of firm-to-firm transactions. Practically, we take the observed log sales of firm j to firm k ($\log p_j q_{jk}$) and regress them on firm j and firm k fixed effects, recovering the *supplier fixed effect*, *buyer fixed effect*, and *buyer-supplier residual*, denoted by $\log \Gamma_j^S$, $\log \Gamma_k^B$, and $\log \tilde{\Gamma}_{jk}$, respectively. As further discussed in Bernard et al. (2022), the identifications of these two-way fixed effects can be achieved through cross-sectional variations alone when firms have more than one buyer and supplier.

Using the estimated buyer and supplier fixed effects and buyer-supplier residual, we then estimate the relationship capability γ_j of each firm. Rearranging equation (9), we show that the relationship capability must satisfy the following relationship with the intermediate input cost index implied by the product market equilibrium:

Proposition 1. *The product of firm j's relationship capability γ_j and its intermediate input cost index z_j is identified up to normalization, given by*

$$\gamma_j^{\sigma-1} = \frac{z_j^{1-\sigma}}{\int_{\Omega_j^S} \Gamma_k^S \tilde{\Gamma_{kj}} dk}. \tag{40}$$

In what follows, we estimate the relationship capability and solve for the product market equilibrium simultaneously. Further details for this procedure are provided in Appendix D.3.

Saliency in households' preference. We next provide our identification strategies for the saliency parameters in households' preference $\{\beta_{jH}\}$. We identify β's through the observed variations in firms' share of network sales out of their revenues. We define firm j's share of network sales, denotec by r_j^{net}, as

$$r_j^{net} \equiv \frac{p_j \int_{\Omega_j^B} q_{jk} dk}{p_j q_j} = \frac{\chi_j - (\beta_{jH} P)^{\sigma-1} E}{\chi_j}, \tag{41}$$

where the last equality follows from equations (10) and (12). We can then show that the ratio between the firm's relationship capability and the saliency of its goods in households' preference is as follows:

Proposition 2. *The ratio between firm j's relationship capability γ_j and the saliency of its goods in households' preference β_{jH} is identified up to normalization, given by*

$$\frac{\beta_{jH}^{\sigma-1}}{\gamma_j^{\sigma-1}} = E^{-1} \left(\frac{1 - r_j^{net}}{r_j^{net}} \right) \int_{\Omega_j^B} \Gamma_j^B \tilde{\Gamma}_{ij} dk. \tag{42}$$

Propositions 1 and 2 imply that the saliency parameters $\{\gamma_{jk}\}$ and $\{\beta_{jH}\}$ are jointly determined by the same normalization. Intuitively, if we scale all γ's and β's by the same amount, it affects the levels of labor productivity $\{\Phi_j\}_{j \in \Omega}$ but does not alter the shape of distribution. Therefore, we normalize the saliency parameters $\{\gamma_{jk}\}$ and $\{\beta_{jH}\}$ such that the average of log γ_{jk} is set at zero.

Firms' own productivity. Lastly, we estimate firms' own productivity $\{\phi_j\}$ to match their labor productivity $\{\Phi_j\}$ that we computed in Section 5.1. Equations (15) and (48) suggest that we can find a firm's own productivity ϕ_j that exactly fits its labor productivity Φ_j, using its cost index z_j and demand shifter χ_j implied by a product market equilibrium. Therefore, we iteratively estimate $\{\phi_j\}$ while solving for the product market equilibrium at the steady state. We provide further explanations of our estimation procedures in Appendix D.3.

6 Application

We now use the estimated model to perform several counterfactual analyses. The goals of these exercises are to quantify the contributions of job search through firm-to-firm networks to labor market dynamics and to understand how the introduction of such a search channel would alter the impacts of shocks to production networks on workers.

Table 3: List of model parameters

(a) Externally set parameters

Parameter		Value	Reason
Discount rate	ρ	0.01	Annual interest rate of 4 percent
Returns to scale for labor	α	0.37	Average labor share
Substitutability of goods	σ	4	e.g., Antras et al. (2017)
Total mass of workers	L	20.2	Average firm size of 18.6
Matching elasticity	ξ	0.5	e.g., Petrongolo and Pissarides (2001)

(b) Labor market parameters

Parameter		Value	Target	Data	Model
Matching efficiency	A	1	Normalization		
Network search premium	$\bar{\lambda}$	0.18	Share of B2B moves	0.42	0.42
Market search intensity	ζ	0.07	Average EE rate	0.05	0.05
Exogenous separation rate	δ_0	0.04	Average EU rate	0.04	0.04
Vacancy cost	c	3.85	Firm size ratio 25/75	0.22	0.12
Constant in wage equation	$\bar{w}$	0.16	Unemployment rate	0.08	0.08
Worker bargaining power	η	0.45	Average $\Delta \log w$ for movers	0.02	0.02

(c) Product market parameters

Parameter		Value	Identification strategy
Saliency in interm. input production	$\{\gamma_{jk}\}$	—	See Proposition 1
Saliency in households' preference	$\{\beta_{jH}\}$	Figure 10a	See Proposition 2
Firm's own productivity	$\{\phi_j\}$	Figure 10b	Match $\{\Phi_j\}$ in equation (48)

Notes: In this table, we report the list of model parameters and provide their estimated values as well as estimation strategies. In Panel (a), we list the parameters that are calibrated outside the model and report the rationale for the choice of each parameter value. See Section 5.1 for further discussions. Panel (b) shows the labor market parameters that we estimate using the method of simulated moments. For each parameter, we list the corresponding moment to be targeted and provide its value in the data and in the model. See Section 5.2 for further discussions. Panel (c) lists the product market parameters and their identification strategies. See Figure 10 in Appendix E.2 for the estimated values and Section 5.3 for further discussions.

6.1 Contribution of network search channel

We first quantify the contributions of the network search channel to aggregate worker flows. By doing so, we revisit the discussions in Section 3.2 and characterize the patterns of job-to-job transitions if movers did not take into account the firm-to-firm linkages.

We first solve for the steady-state worker flows in our baseline model using the parameters estimated in Section 5. As reported in Table 4, the estimated value for the network search premium $\bar{\lambda}$ is 0.18. Taken together with the vacancy-based labor market connectedness of each firm, this implies that the job-finding rate through network search in equation (18) is around 0.04 on average. We solve for the stationary distribution of workers and worker flows under this economy. The procedure to solve for the steady state is explained in detail in Appendix D.4.

We report the results in the first column of Table 4. The average quarterly employment-

Table 4: Contribution of network search channel

	Baseline	No Network Search
Network search premium $\bar{\lambda}$	0.18	0
Average EE rate		
overall	0.050	0.050
market search	0.035	0.050
network search	0.015	0
B2B moves	0.021	0.008
non-B2B moves	0.029	0.042
Share of network search	0.30	0
Share of B2B moves	0.42	0.17

Notes: In this table, we report the contribution of the network search channel to the worker flows in the baseline economy and counterfactual economy with no network search. In the first column, we provide the steady-state worker flows in the baseline economy using the parameters estimated in Section 5. The share of network search is computed as the ratio between the average employment-to-employment (EE) rate through network search and the overall average EE rate, and the share of B2B moves is defined in equation (2). In the second column, we report the steady-state worker flows in the economy with no market search. We reestimate the model parameters by setting $\bar{\lambda} = 0$ while targeting the same set of moments as in the baseline economy (including the overall average EE rate of 0.05). The procedure to solve for the steady state is explained in detail in Appendix D.4.

to-employment rate is 5 percent, out of which the job-to-job transitions through market search account for 3.5 percent. The remaining 1.5 percent comes from the matches through network search. Some of the matches through market search also result in the movement between buyers and supplier. Hence, the average employment-to-employment rate of B2B moves is 2.1 percent, while the rate of non-B2B moves is 2.9 percent, which yield the B2B move share of 42 percent. Taken together, we find that around 30 percent of the job-to-job transitions happen through the network search channel, suggesting a considerable overlap between the set of potential employers in the labor market and firm-to-firm linkages in the product market.

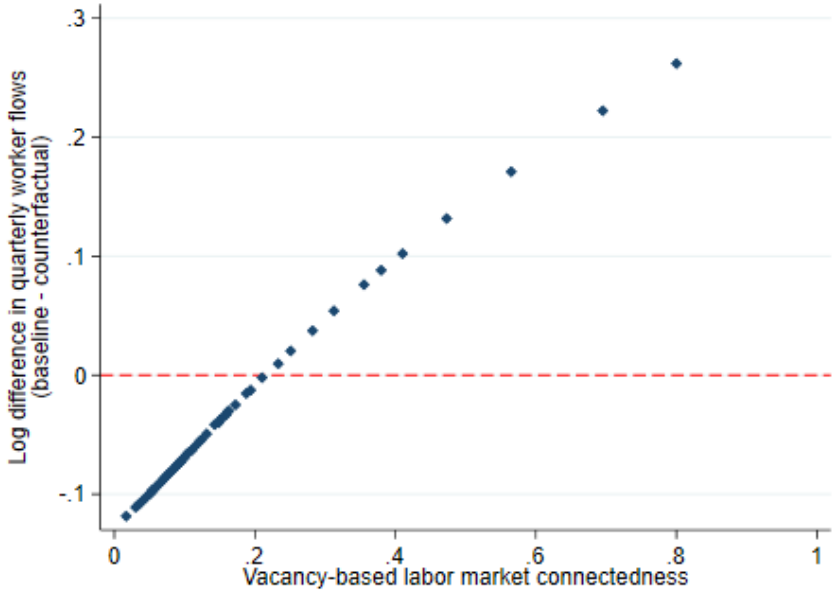
We then consider what would happen to the worker flows if we consider the class of models in which there is no network search. To match the overall EE rate, we reestimate the parameters in a counterfactual economy by setting $\bar{\lambda} = 0$ while targeting the same set of moments. We then consider the worker flows in this counterfactual Belgian economy with alternative parameterization.

The second column of Table 4 summarizes aggregate worker flows in the counterfactual economy with $\bar{\lambda} = 0$. In this counterfactual economy with no network search channel, all matches are formed through market search. Therefore, the B2B moves happen only if workers are randomly matched with the buyers or suppliers of their current employers through market search. The average employment-to-employment rate of B2B moves without network search is 0.8 percent, implying that only around 17 percent of job-to-job transitions would be

considered movements along the supply chain.

It is worth noting that the share of B2B moves in this counterfactual economy is smaller than the statistical random benchmark for the B2B moves reported in Table 1 of Section 3.2. This difference comes from the fact that we now take into account the endogenous vacancy-posting decisions of firms as well as the mobility decisions of workers. Intuitively, in the absence of the network search channel, well-connected firms face both lower job-finding rates and lower separation rates. This is because workers at the other firms are less likely to be matched with the vacancies posted by these well-connected firms, while concurrently, their own workers are less likely to be poached by the other firms. Therefore, the absence of the network search channel contributes to the lower worker flows into and out of well-connected firms, where the B2B moves are more likely to happen.

Figure 4: Quarterly worker flows: baseline vs. no network search



Notes: In this figure, we report the log differences in quarterly worker flows between the baseline economy and the counterfactual economy with no network search. We use the parameters estimated in Section 5 and solve for the steady state in the baseline economy (see Appendix D.4 for details). In the counterfactual economy with no network search, we reestimate the model parameters by setting $\bar{\lambda} = 0$ while targeting the same set of moments as in the baseline economy. In each economy, we use the steady-state employment and separation rate to compute the quarterly worker flows for each firm group. For each bin of firm groups, sorted by the percentiles of the vacancy-based labor market connectedness in the baseline economy, we compute the average log differences in quarterly worker flows between two economies. The vacancy-based labor market connectedness is defined analogously to the employment-based labor market connectedness in equation (1).

In order to see this point that the absence of the network search channel alters the patterns of worker flows, in Figure 4, we compare the worker flows in the baseline and counterfactual economies based on the labor market connectedness of firms. To do so, we first use the

steady-state employment and separation rate to compute the quarterly worker flows for each firm group in both economies. We then present the binscatter plot of log differences in quarterly worker flows based on the vacancy-based labor market connectedness in the baseline economy. Figure 4 reveals that there is a positive relationship between the labor market connectedness and the differences between the two economies. For the majority of the firms that have low levels of labor market connectedness, their worker flows are smaller in the baseline economy, whereas the firms with high levels of labor market connectedness exhibit greater worker flows, up to more than 20 percent in the baseline economy compared to the counterfactual economy with no network search. Thus, we find that not accounting for the network search channel would understate the worker flows in well-connected firms while overstating the worker flows at less-connected firms.

6.2 Wage change in response to the productivity shocks in production networks

Lastly, we use the estimated model to analyze the wage changes in response to the productivity shocks in production networks. Throughout this section, we consider a 5 percent reduction in firms' own productivity $\{\phi_j\}$ for all manufacturing firms in the Belgian economy. We then examine how the shocks propagated in production networks differentially affect workers at different firms.

When computing the worker impacts of a productivity decline among Belgian manufacturing firms, we start from the steady state of our baseline economy and only alter their own productivity $\{\phi_j\}$ while taking as given all the other model primitives and parameters estimated in Section 5. We then solve for the new distribution of labor productivity $\{\Phi_j\}$, for both manufacturing firms and non-manufacturing firms, following the procedure in Appendix D.4. In order to consider the impacts on workers, we primarily focus on the instantaneous responses of firms' labor productivity and workers' wages.[20]

Panel (a) of Figure 5 shows the instantaneous responses in log labor productivity Φ_j. Even though only manufacturing firms are directly affected by their own productivity decline of 5 percent, the total decline in their labor productivity $\{\Phi_j\}$ can be larger than 5 percent, and non-manufacturing firms are also affected indirectly because of the propagation of shocks in the production networks. On average, we find that a manufacturing firm experiences a decline in its labor productivity of around 9.4 percent, while the labor productivity of a non-manufacturing firm drops by around 1.7 percent.

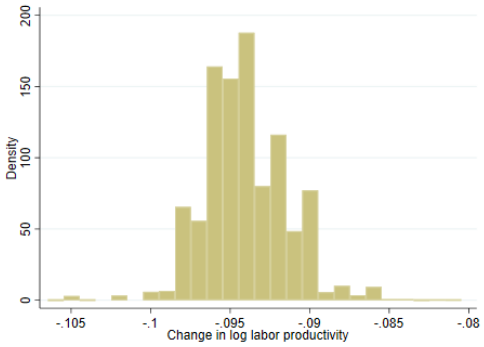
We then examine how these declines in labor productivity affect the wages of workers.

[20] In Appendix E.3, we also provide the long-run response of firms by comparing them to the new steady state.

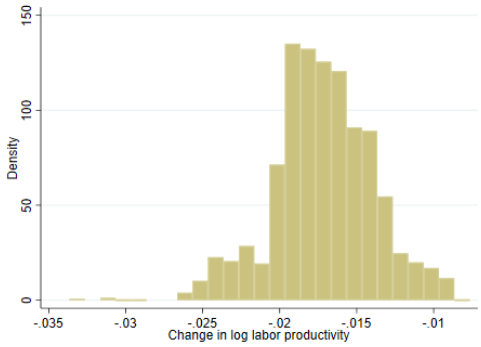
Figure 5: Instantaneous response to 5 percent reduction in manufacturing productivity

(a) Changes in log labor productivity

Manufacturing firms



Non-manufacturing firms

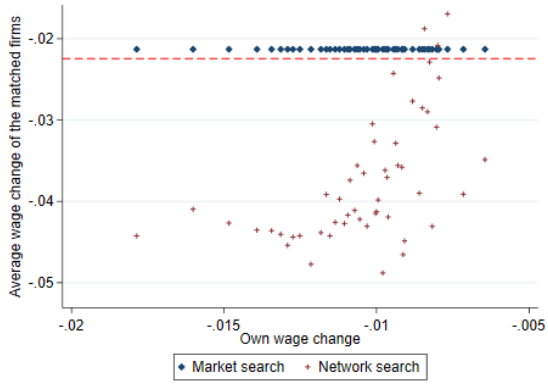


(b) Changes in log wage: own vs matched firms

Manufacturing firms



Non-manufacturing firms



Notes: In this figure, we report the changes in log labor productivity and log wage due to a 5 percent reduction in manufacturing firms' own productivity $\{\phi_j\}$. In Panel (a), we show the distributions of instantaneous changes in log labor productivity $\{\Phi_j\}$ for both manufacturing firms and non-manufacturing firms, following the procedure in Appendix D.4. In Panel (b), we present the relationship between firms' own wage changes and the average wage changes of the firms with which workers are matched through the market search and network search channels. We first use equation (23) to compute the change in wage w_j for each firm. For each bin of firm group, sorted by the percentiles of the own wage changes, we then compute the average wage changes of the matched firms, weighted by the likelihood of the matching. The blue diamonds represent the average wage changes of the firms matched through the market search channel, whereas the red markers represent the average wage changes of the firms matched through the network search channel. The dashed red lines represent the employment-weighted average of the wage changes in the entire economy.

Equation (23) allows us to compute how the change in labor productivity Φ_j can be translated into the change in wages w_j. We find that the shocks to manufacturing firms result in an immediate wage reduction of around 6.3 percent and 1.2 percent among manufacturing workers and non-manufacturing workers, respectively.

It is useful to note that computing the wage response at the current employers does not

necessarily require the model of on-the-job search. Our model further allows us to compute the wage changes at the firms that workers are matched with. In Panel (b) of Figure 5, we present binscatter plots to show the relationship between firms' own wage changes and the average wage changes of the matched firms. Importantly, we compute the average wage changes of the firms matched through each of two search channels, so that the average changes are represented by a blue diamond for market search and a red marker for network search, respectively.

Several interesting observations arise. First, the average wage change of the firms matched through market search is constant regardless of their own wage change. This is because the composition of matched firms is determined solely by the vacancy distribution and does not depend on the identity of the current employers. When matched with firms through the market search channel, workers find the average wage of the matched firms to be lower by around 2.1 percent.

On the other hand, the average wage decline is no longer homogeneous once we take into account the network search. When matched with firms through the network search channel, workers at different firms find that the average wage decline of the matched firms ranges from 1.8 percent to 5.2 percent. Quantitatively, we find that the average wage decline of the firms matched through the network search channel tends to be greater than that of the firms matched through the market search channel. This greater wage decline arises when well-connected firms tend to be hit harder by the shocks to production networks.

Furthermore, Panel (b) of Figure 5 also reveals a positive correlation between the wage decline at the current employer and the average wage decline of the firms matched through the network search channel. When workers experience a larger decline in their wages at the current employers, they also tend to meet the other firms that are hit harder by the productivity shocks. We find a positive correlation of 0.69 for manufacturing firms and 0.56 for non-manufacturing firms.

It is worth noting that this positive correlation does not necessarily appear in any economy. For instance, suppose there exists an economy whose production networks are complete, such that all firms are directly connected with all the other firms. In this economy, workers at any given firm could meet all the other firms through the network search channel, and thus, the average wage change of the firms matched through the network search channel coincides with that of the firms matched through the market search channel. The positive correlation between firms' own wage decline and the average wage decline of the matched firms appears when well-connected firms that are hit harder by the productivity shocks are more likely to trade with the other well-connected firms. Our estimated model implies that the structure of the Belgian production networks is such that it exhibits a positive correlation in wage responses among the current and potential employers. Therefore, our results suggest that

the network search channel reduces the diversification of workers' outside options against productivity shocks to production networks.

7 Conclusion

The aim of this paper was to examine the prevalence of job-to-job transitions along the supply chain and quantify its contribution to the dynamics of the Belgian labor market. We find that a sizable fraction of Belgian workers find their next employers among the buyers and suppliers of their current employers, suggesting that the linkages of firms in the product market play a significant role in explaining workers' job search behaviors in the labor market. Quantifying the contribution of firm-to-firm linkages in aggregate worker flows through the lens of an equilibrium model, we show a considerable overlap between the set of potential employers in the labor market and the firm-to-firm linkages in the product market. Our results suggest that shocks to production networks are likely to affect wages not only at the current employers but also at the future employers.

While our results shed new light on the interaction between the production network and labor market, our model is still parsimonious and restrictive in several dimensions. For instance, we have not taken into account the endogenous link formations in buyer-supplier relationships as well as firm entry and exit. Incorporating the endogenous responses in the extensive margins of firm-to-firm trade could be important depending on the nature of shocks researchers are interested in. We have also abstracted away from worker heterogeneity in skills and preferences, which can introduce another source of labor market imperfection. Lastly, future works may incorporate the business cycle into our model and analyze the responses to temporary shocks.

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A Data appendix

A.1 Aggregation of VAT identifiers into firms

As we describe in Section 2.1, all the information from the B2B datasets is reported at the level of VAT identifiers. Because some firms may have several VAT identifiers for accounting or tax purposes, the VAT identifiers do not necessarily correspond to the notion of firms or establishments. In order to focus on the firm-level analyses, we follow the same procedure as in Dhyne et al. (2021) to aggregate multiple VAT identifiers into the firm identifiers.

In order to collect multiple VAT identifiers that belong to the same firm, we proceed as follows. First, we determine whether a pair of VAT identifiers can be aggregated into the same firm based on their ownership structure. We use the information from ownership filings in the annual accounts as well as the Balance of Payments survey and collect multiple VAT identifiers that are linked with at least 50 percent of ownership. We also aggregate multiple VAT identifiers into the same firm if at least 50 percent of their shares are held by the same foreign parent firm. The foreign parent firms are recorded by their names, so we apply a "fuzzy string matching" method to compare all possible pairs of foreign firms' names and determine the foreign parent firm of a given VAT identifier. Lastly, we also link a pair of VAT identifiers if they are linked one year before and one year after, so that we avoid potential misreporting.

After following the aggregation procedure above, we select the "most representative" VAT, or "head" VAT, identifiers among each collection of VAT identifiers. See Appendix C.4 of Dhyne et al. (2021) for the selection criteria. We use these head VAT identifiers as the identifiers of the firms and sum up all the variables across VAT identifiers to the firm level. For some variables that cannot be added up, such as firms' primary industry and the location of their main economic activities, we take those of the head VAT identifiers. For variables such as total sales and inputs, we further adjust them by the amount of B2B sales among the pairs of VAT identifiers that belong to the same head VAT identifier, so that we correct for the double counting of transactions within the same firm.

A.2 Merging procedures for the NBB and CBSS datasets

The information on the employers from the matched employer-employee data is recorded at the level of Banque Carrefour des Entreprises (Crossroads Bank for Enterprises, BCE) identifiers. All businesses in Belgium are assigned the unique identifiers upon their registration with the BCE. Because these businesses are required to register with the BCE when they pay VAT, their BCE identifiers can be easily converted to VAT identifiers. When we merge the matched employer-employee data from the CBSS datasets with the NBB datasets, we

first convert all BCE identifiers into VAT identifiers. We then follow the same aggregation procedure explained in Appendix A.1 and aggregate multiple VAT identifiers into firms.

A.3 Descriptive statistics on the merged sample

In Section 2.3, we construct our main analysis sample of firms and workers. Table 5 reports the descriptive statistics of firm characteristics and worker characteristics in the main analysis sample in 2012. For firm characteristics, we report both firm-level and worker-level averages. For instance, the average firm in 2012 buys from and sells to around 51 firms, whereas the employment-weighted averages reveal that the employer of the average worker is connected to thousands of firms.

Table 5: Descriptive statistics

	Firm-level average	Employment-weighted average
Firm characteristics in 2012		
Value added (in million euro)	1.68	412
Labor cost (in million euro)	1.02	259
Employment (FTE)	18.6	4,827
Number of buyers	51.4	3,648
Number of suppliers	51.4	1,190

	All	Blue-collar	White-collar	Male	Female
Worker characteristics in 2012Q1					
Share of all workers	1.00	0.51	0.49	0.66	0.34
Average quarterly earnings (FTE)	8,000	6,026	10,010	8,477	7,044

Notes: This table shows the descriptive statistics of firm characteristics and worker characteristics. The table is based on the main analysis sample of 98,599 private-sector firms in Belgium in 2012 (see Section 2.3 for details).

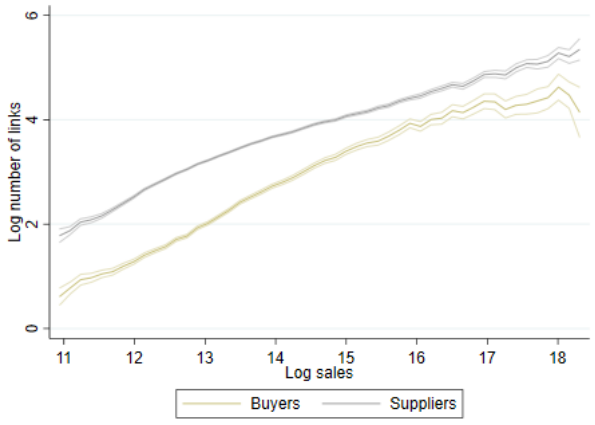
B Additional empirical results

B.1 Firm size and number of buyers and suppliers

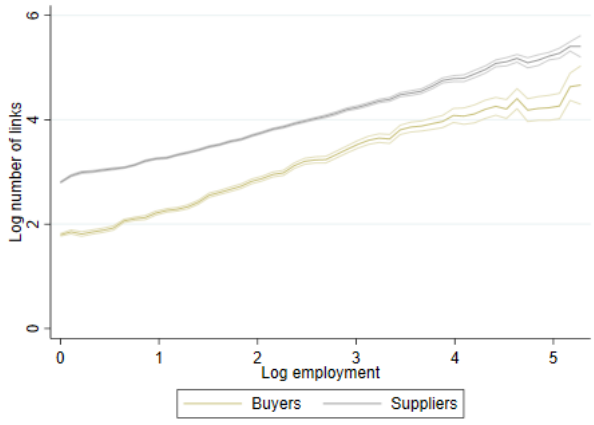
In Section 3.1, we show that the worker-level average of labor market connectedness is higher than the firm-level average. In this section, we show that the Belgian firms with a greater number of buyers and suppliers tend to be larger on average. To do so, we use local polynomial regressions to non-parametrically estimate the relationships between log firm size and log number of links. In Figure 6, we show that larger firms, measured in terms of both log sales and log employment, have a greater number of buyers and suppliers. These relationships further corroborate the finding in Table 5 of Appendix A.3 that the employer of the average worker in Belgium has more connections than the average firm.

Figure 6: Relationship between firm size and number of buyers and suppliers

(a) Sales and number of links



(b) Employment and number of links



Notes: The figures display the relationship between firm-level sales, employment, and number of links, using the smoothed values of kernel-weighted local polynomial regression estimates with 95 percent confidence intervals. We use the Epanechnikov kernel function with kernel bandwidth of 0.05. Log sales are trimmed at the top and bottom 1 percentiles. The figures are based on the main analysis sample of 98,599 private-sector firms in Belgium in 2012 (see Section 2.3 for details).

B.2 Alternative measures of labor market connectedness

In Section 3.1, we define the employment-based labor market connectedness and present its distribution in the Belgian economy. In this section, we consider several alternative measures of labor market connectedness. First, we define the *hiring-based labor market connectedness*, which captures the share of total gross hires accounted for by the firms that firm j is directly connected to in production networks. Firm j's hiring-based labor market connectedness at

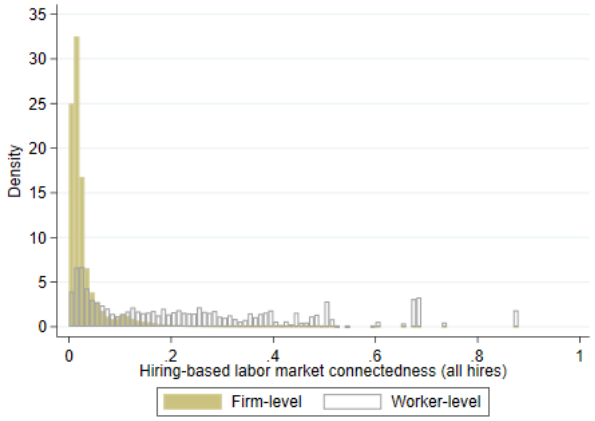
time t, denoted by $\mathbb{C}^h_{j,t}$, can be computed as follows:

$$\mathbb{C}^h_{j,t} = \frac{\sum_{i \in \Omega^B_{j,t} \cup \Omega^S_{j,t}} (\text{gross hires})_{i,t}}{\sum_{i \in \Omega \setminus \{j\}} (\text{gross hires})_{i,t}}, \tag{43}$$

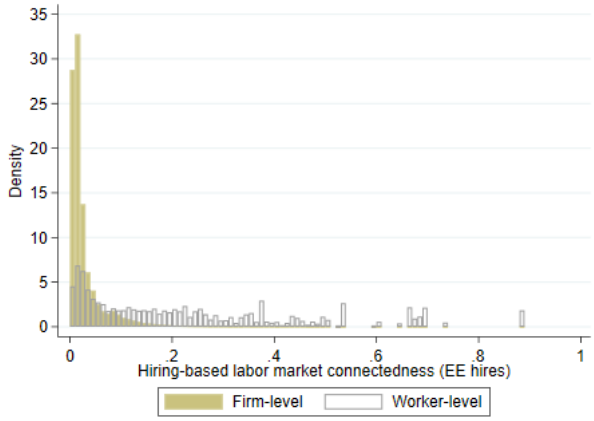
where the term $(\text{gross hires})_{i,t}$ is computed from the worker-level data, which captures the number of workers employed by firm i at time t but not at $t-1$.

Figure 7: Distribution of hiring-based labor market connectedness

(a) Based on all hires



(b) Based on EE hires



Notes: These figures show the distributions of hiring-based labor market connectedness. The hiring-based labor market connectedness of firm j, denoted by $\mathbb{C}^h_j$ and defined in equation (43), is the share of total gross hires accounted for by the firms that firm j is directly connected to in production networks. The white bars represent the distribution of the firm-level measure of labor market connectedness in which one weights the firms by the number of workers at each firm. Panel (a) displays the hiring-based labor market connectedness based on all hires. In Panel (b), we only consider the employment-to-employment (EE) hires, in which the new hire is required to be employed at another firm in the previous period. The figures are based on the main analysis sample of 98,599 private-sector firms in Belgium in 2012 (see Section 2.3 for details).

In Figure 7, we present the distributions of hiring-based labor market connectedness. In 2012, an average Belgian firm is connected to around 3 percent of all hires, whereas an average worker is connected to around 24 percent of all hires through the direct links of their employer.

We summarize different measures of labor market connectedness in Table 6. In addition to the firm-level and mover-level averages of labor market connectedness in 2012, we also report the mover-level average of each measure as well as the average over the entire sample period 2003-2014. We also report the average labor market connectedness based on the employment-to-employment hires next period. It is worthwhile to note that the mover-level average of labor market connectedness based on the employment-to-employment hires next period, which we report in the last row of Table 6, corresponds to the statistical random

benchmark discussed in Section 3.2 without any additional controls.

Table 6: Average labor market connectedness

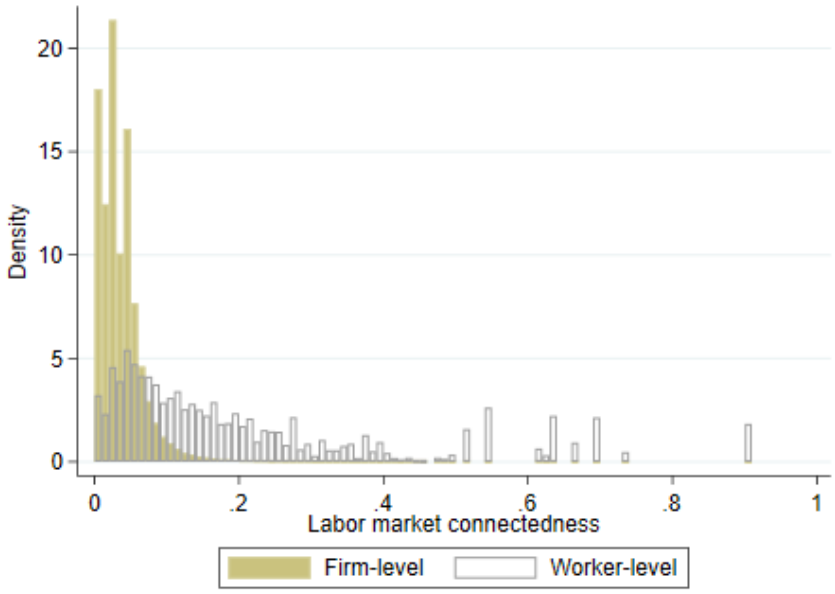
	Average		
	Firm-level	Worker-level	Mover-level
Labor market connectedness in 2012			
Employment-based	0.04	0.23	0.22
Hiring-based (all hires)	0.03	0.24	0.23
Hiring-based (EE hires)	0.03	0.23	0.23
Hiring-based (EE hires next period)	0.03	0.23	0.23
Labor market connectedness in 2003-2014			
Employment-based	0.05	0.23	0.22
Hiring-based (all hires)	0.03	0.23	0.21
Hiring-based (EE hires)	0.03	0.22	0.20
Hiring-based (EE hires next period)	0.03	0.22	0.20

Notes: This table reports the averages of different measures of labor market connectedness. The employment-based labor market connectedness is defined in equation (1) and measures the share of total employment accounted for by the firms directly connected in production networks. The hiring-based labor market connectedness is defined in equation (43) and measures the share of total gross hires accounted for by the firms directly connected in production networks. In the third and fourth rows, we only consider the employment-to-employment (EE) hires, in which the new hire at time t or $t+1$ is required to be employed at another firm in the previous period (at time $t-1$ or t, respectively). The table is based on the main analysis sample of 98,599 private-sector firms in Belgium in 2012 (see Section 2.3 for details).

B.3 Labor market connectedness without links from retailers, wholesalers, and utility companies

In Section 3.1, we compute the employment-based labor market connectedness using all the firm-to-firm linkages observed in the B2B datasets. One potential concern is that our results might be driven by a certain set of firms that have small transactions with the majority of firms. In this section, we show the robustness of labor market connectedness when only considering the subset of firm-to-firm linkages. In Figure 8, we show the distribution of labor market connectedness, computed after dropping the links from retailers, wholesalers, and utility companies to their buyers. This procedure drops around 45 percent of firm-to-firm linkages and 36 percent of transaction volume. In this alternative measure, an average Belgian worker is still connected to around 20 percent of total employment through the other links.

Figure 8: Distribution of employment-based labor market connectedness, excluding the links from retailers, wholesalers, and utility companies



Notes: This figure shows the distribution of employment-based labor market connectedness, in which we exclude the firm-to-firm linkages with retailers, wholesalers, and utility companies as the suppliers. The employment-based labor market connectedness of firm j, denoted by $\mathbb{C}_j^e$ and defined in equation (1), is the share of total employment accounted for by the firms that firm j is directly connected to in production networks. The white bars represent the distribution of the firm-level measure of labor market connectedness in which one weights the firms by the number of workers at each firm. The figure is based on the main analysis sample of 98,599 private-sector firms in Belgium in 2012 (see Section 2.3 for details).

B.4 Additional results for the share of B2B moves

In Section 3.2, we show that workers move disproportionately more into the buyers and suppliers of their current employers. In this section, we provide several additional evidence for the shares of B2B moves, reported in Table 7.

In the first panel of Table 7, we consider the locations of establishments. Importantly, we do not observe the exact workplace of each worker if firms have multiple establishments. Therefore, the movers whose current and next employers report different geographic regions may not necessarily moved across regions. To alleviate this concern, we consider whether two firms have any establishments that operate in the same regions. Our results are robust for the movers between two firms that have no overlapping business coverage.

In the second panel, we split the movers based on the industries of their current employers and report the B2B shares excluding movers from consulting firms and temporary employment agencies. One might be concerned that our results for the industry switchers are driven by selected industries, such as consultants at the consulting firms moving to their

former client firms or workers at temporary employment agencies moving to their customers. While the movers from those firms have a higher share of movements to buyers, our result among the other movers is still robust in comparison to the random benchmark.

Table 7: Share of B2B moves: additional results

	Share of movers	Share of B2B moves	
		Data	Random benchmark
Locations of establishments			
movers between firms with overlaps	0.85	0.46	0.25
movers between firms with no overlaps	0.15	0.16	0.05

	Share of movers	Share of moves to buyers	
		Data	Random benchmark
Consulting firms and temp agencies			
movers from consulting firms	0.03	0.34	0.12
movers from temp agencies	0.35	0.52	0.25
movers from other firms	0.62	0.21	0.07
movers from any firm	1.00	0.32	0.14

Notes: This table reports the shares of B2B moves among different sets of job-to-job movers. B2B moves refer to the job-to-job transitions where the origin and destination firms are connected in production networks, and the share of B2B moves is defined in equation (2). In the first panel, we split the movers into two groups based on whether their origin and destination firms have any establishments that operate in the same NUTS two-digit regions. We report the share of movers for each group in the first column. In the second and third columns, we report the observed shares in the data and the results from simulation exercises to compute the statistical random benchmarks, respectively. The random benchmarks compute the share of B2B moves if movers were to be randomly matched with hiring firms (see Appendix D.1 for details). In the second panel, we split the movers based on the industries of their current employers and report the B2B shares excluding movers from consulting firms and temporary employment agencies. This table is based on the main analysis sample of 467,194 movers in Belgium over the period 2003-2014 (see Section 2.3 for details).

C Model appendix

C.1 Derivations of optimal revenue net of intermediate input costs

In this section, we provide a formal argument to the derivations of firms' optimal revenue net of intermediate input costs discussed in Section 4.1. Formally, the static problem of firm j with n workers to maximize its revenue net of intermediate input costs can be written as follows:

$$R_j(n) = \max_{\substack{\{p_{jk}\}_{k\in\Omega_j^B\cup\{H\}} \\ \{q_{jk}\}_{k\in\Omega_j^B\cup\{H\}} \\ \{q_{ij}\}_{i\in\Omega_j^S}}} \left(\int_{\Omega_j^B\cup\{H\}} p_{jk}q_{jk}dk - \int_{\Omega_j^S} p_{ij}q_{ij}di \right),$$

subject to the following constraints:

$$\begin{aligned}
q_j &= \int_{\Omega_j^B\cup\{H\}} q_{jk}dk \\
q_j &= \phi_j n^{\alpha} m_j^{1-\alpha} \\
m_j &= \left(\int_{\Omega_j^S} (\gamma_{ij}q_{ij})^{\frac{\sigma-1}{\sigma}} di \right)^{\frac{\sigma}{\sigma-1}} \\
q_{jk} &= \gamma_{jk}^{\sigma-1} \frac{p_{jk}^{-\sigma}}{z_k^{-\sigma}} m_k \quad \text{for all } k \in \Omega_j^B \\
q_{jH} &= \beta_{jH}^{\sigma-1} \frac{p_{jH}^{-\sigma}}{P^{1-\sigma}} E.
\end{aligned}$$

As discussed in the main texts, the cost minimization problem of firm j yields equations (8) and (9), implying that the following relationship holds true for the optimizing firm:

$$z_j m_j = \int_{\Omega_j^S} p_{ij}q_{ij}di.$$

Plugging this in, we rewrite firm j's problem, suppressing the constraints, as follows:

$$\begin{aligned}
R_j(n) &= \max_{\{p_{jk}\}_{k\in\Omega_j^B\cup\{H\}},m} \left(\int_{\Omega_j^B\cup\{H\}} p_{jk}q_{jk}(p_{jk})dk - z_j m_j \right) \\
&= \max_{\{p_{jk}\}_{k\in\Omega_j^B\cup\{H\}}} \left[\int_{\Omega_j^B\cup\{H\}} p_{jk}q_{jk}(p_{jk})dk - z_j \phi_j^{-\frac{1}{1-\alpha}} n^{-\frac{\alpha}{1-\alpha}} \left(\int_{\Omega_j^B\cup\{H\}} q_{jk}(p_{jk})dk \right)^{\frac{1}{1-\alpha}} \right].
\end{aligned}$$

The first order condition with respect to p_{jk} yields:

$$p_{jk}\frac{dq_{jk}}{dp_{jk}} + q_{jk} - \frac{1}{1-\alpha} z_j \phi_j^{-\frac{1}{1-\alpha}} n^{-\frac{\alpha}{1-\alpha}} q_j^{\frac{\alpha}{1-\alpha}} \frac{dq_{jk}}{dp_{jk}} = 0.$$

Notice that for all customers $k \in \Omega_j^B \cup \{H\}$, the shape of demand curve implies that $dq_{jk}/dp_{jk} = -\sigma q_{jk}/p_{jk}$. Therefore, the optimal price p_{jk}^* satisfies:

$$p_{jk}^* = \frac{\sigma}{\sigma-1}\frac{1}{1-\alpha} z_j \phi_j^{-\frac{1}{1-\alpha}} n^{-\frac{\alpha}{1-\alpha}} q_j^{\frac{\alpha}{1-\alpha}}. \tag{44}$$

This expression holds true for all $k \in \Omega_j^B \cup \{H\}$, and thus, firms optimally charge the same price to all of their customers.[21]

Now we derive the exact expressions for equations (14) and (15). Using equations (6) and (11), we rewrite the static problem of firms as follows:

$$R_j(n) = \max_m \left(\phi_j^{\frac{\sigma-1}{\sigma}} \chi_j^{\frac{1}{\sigma}} n^{\frac{\alpha(\sigma-1)}{\sigma}} m^{\frac{(1-\alpha)(\sigma-1)}{\sigma}} - z_j m \right).$$

The first order condition with respect to m yields:

$$\frac{(1-\alpha)(\sigma-1)}{\sigma} \phi_j^{\frac{\sigma-1}{\sigma}} \chi_j^{\frac{1}{\sigma}} n^{\frac{\alpha(\sigma-1)}{\sigma}} m^{*\frac{-1-\alpha(\sigma-1)}{\sigma}} = z_j$$

Solving this equation for m^*, we obtain:

$$m^* = \left[\frac{(1-\alpha)(\sigma-1)}{\sigma} z_j^{-1} \phi_j^{\frac{\sigma-1}{\sigma}} \chi_j^{\frac{1}{\sigma}} n^{\frac{\alpha(\sigma-1)}{\sigma}} \right]^{\frac{\sigma}{1+\alpha(\sigma-1)}} \tag{45}$$

Using this expression, we have:

$$\begin{aligned} p^* &= \phi_j^{-\frac{1}{\sigma}} \chi_j^{\frac{1}{\sigma}} n^{-\frac{\alpha}{\sigma}} m^{-\frac{1-\alpha}{\sigma}} \\ &= \left[\left(\frac{\sigma}{(1-\alpha)(\sigma-1)} \right)^{1-\alpha} z_j^{1-\alpha} \phi_j^{-1} \chi_j^{\alpha} n^{-\alpha} \right]^{\frac{1}{1+\alpha(\sigma-1)}}, \end{aligned} \tag{46}$$

[21] This result is not unique to our setting and holds in a wide range of production network models that assume a common and constant demand elasticity across all customers. See, for example, Huneeus et al. (2021).

and:

$$\begin{aligned} R_j(n) &= \left(\phi_j^{\frac{\sigma-1}{\sigma}} \chi_j^{\frac{1}{\sigma}} n^{\frac{\alpha(\sigma-1)}{\sigma}} m^{*\frac{-1-\alpha(\sigma-1)}{\sigma}} - z_j\right) m^* \\ &= \frac{1+\alpha(\sigma-1)}{(1-\alpha)(\sigma-1)} z_j m^* \\ &= \Phi_j n^{\frac{\alpha(\sigma-1)}{1+\alpha(\sigma-1)}}, \end{aligned} \tag{47}$$

where

$$\Phi_j = \frac{1+\alpha(\sigma-1)}{(1-\alpha)(\sigma-1)} \left[\frac{(1-\alpha)(\sigma-1)}{\sigma}\right]^{\frac{\sigma}{1+\alpha(\sigma-1)}} z_j^{-\frac{(1-\alpha)(\sigma-1)}{1+\alpha(\sigma-1)}} \phi_j^{\frac{\sigma-1}{1+\alpha(\sigma-1)}} \chi_j^{\frac{1}{1+\alpha(\sigma-1)}}. \tag{48}$$

C.2 Derivations of product market equilibrium in Claim 1

In this section, we provide the derivation of equations (32)-(34) which characterize the product market equilibrium in Claim 1. First, the derivation of equation (32) immediately follows from equation (9). Second, in order to derive equation (33), we use the expression for the optimal revenue net of intermediate input costs in equation (47), which yield

$$p_j q_j - z_j m_j = \frac{1+\alpha(\sigma-1)}{(1-\alpha)(\sigma-1)} z_j m_j.$$

Rearranging this equation and using equation (11), we obtain the following:

$$\begin{aligned} z_j m_j &= \frac{(1-\alpha)(\sigma-1)}{\sigma} p_j q_j \\ &= \frac{(1-\alpha)(\sigma-1)}{\sigma} p_j^{1-\sigma} \chi_j, \end{aligned}$$

which implies

$$\tilde{m}_j = \frac{(1-\alpha)(\sigma-1)}{\sigma} \chi_j. \tag{49}$$

Now we can use equations (31) and (47) to rewrite the expression for χ_j in equation (12) as follows:

$$\begin{aligned} \chi_j &= \beta_{jH}^{\sigma-1} \int_\Omega R_k(n_k) dk + \int_{\Omega_j^B} (\gamma_{jk} z_k)^{\sigma-1} z_k m_k dk \\ &= \beta_{jH}^{\sigma-1} \int_\Omega \frac{1+\alpha(\sigma-1)}{(1-\alpha)(\sigma-1)} z_k m_k dk + \int_\Omega \mathbb{1}_{\{k \in \Omega_j^B\}} \gamma_{jk}^{\sigma-1} z_k^{\sigma-1} z_k m_k dk \\ &= \int_\Omega \left(\frac{1+\alpha(\sigma-1)}{(1-\alpha)(\sigma-1)} \beta_{jH}^{\sigma-1} + \mathbb{1}_{\{k \in \Omega_j^B\}} \gamma_{jk}^{\sigma-1} \tilde{z}_k^{-1} \right) \tilde{p}_k \tilde{m}_k dk. \end{aligned} \tag{50}$$

Substituting equation (50) into equation (49) yields equation (33).

Lastly, we derive equation (34) using the optimal pricing equation (44). Substituting equation (6) into equation (44), we obtain the following:

$$\begin{aligned} p_j &= \frac{\sigma}{(1-\alpha)(\sigma-1)} z_j \phi_j^{-\frac{1}{1-\alpha}} n_j^{-\frac{\alpha}{1-\alpha}} \left(\phi_j n_j^{\alpha} m_j^{1-\alpha}\right)^{\frac{\alpha}{1-\alpha}} \\ &= \frac{\sigma}{(1-\alpha)(\sigma-1)} \phi_j^{-1} n_j^{-\alpha} z_j^{1-\alpha} \tilde{m}_j^{\alpha} p_j^{\alpha(1-\sigma)}. \end{aligned} \tag{51}$$

Solving equation (51) for $\tilde{p}_j \equiv p_j^{1-\sigma}$ yields equation (34).

It is useful to observe that $\tilde{p}_j$ in equation (34) only depends on variables at firm j. Therefore, we can further reduce the number of equations by substituting equation (34) into equations (32) and (33). We can rewrite the system of equations in Claim 1 as follows:

$$\begin{aligned} x_{j1} &= \sum_{k\in\Omega} f_{jk2}(x_{k1}, x_{k2}) \\ &= \sum_{k\in\Omega} \omega_{kj} \gamma_{kj}^{\sigma-1} \left(\frac{(1-\alpha)(\sigma-1)}{\sigma} \phi_k n_k^{\alpha}\right)^{\frac{(\sigma-1)}{1+\alpha(\sigma-1)}} x_{k1}^{\frac{1-\alpha}{1+\alpha(\sigma-1)}} x_{k2}^{-\frac{\alpha(\sigma-1)}{1+\alpha(\sigma-1)}} \\ x_{j2} &= \sum_{k\in\Omega} f_{jk1}(x_{k1}, x_{k2}) \\ &= \sum_{k\in\Omega} \left(\frac{1+\alpha(\sigma-1)}{\sigma} \beta_{jH}^{\sigma-1} + \frac{(1-\alpha)(\sigma-1)}{\sigma} \omega_{jk} \gamma_{jk}^{\sigma-1} x_{k1}^{-1}\right) \\ &\quad \times \left(\frac{(1-\alpha)(\sigma-1)}{\sigma} \phi_k n_k^{\alpha}\right)^{\frac{(\sigma-1)}{1+\alpha(\sigma-1)}} x_{k1}^{\frac{1-\alpha}{1+\alpha(\sigma-1)}} x_{k2}^{\frac{1}{1+\alpha(\sigma-1)}}. \end{aligned}$$

C.3 Derivations of employment distribution in Claim 2

In this section, we provide the derivation of equation (35) which characterizes the steady-state employment in Claim 2. In the steady state where the distributions of labor productivity Φ_j and workers n_j remain unchanged, firm j's value Π_j in equation (24) has a closed-form solution. Using equations (14) and (23), we obtain the following expression for Π_j:

$$\Pi_j(n) = \frac{\Phi_j}{\rho + \delta_j \tilde{\alpha}} \left[1 - \frac{\eta}{1 - \eta(1-\tilde{\alpha})} \tilde{\alpha}\right] n^{\tilde{\alpha}} - \frac{\bar{w}}{\rho + \delta_j} n.$$

Taking its derivative with respect to employment, we find that the following expression holds to satisfy the optimal vacancy-posting decision in equation (26):

$$\frac{c}{\mu_j} = \frac{\Phi_j \tilde{\alpha}}{\rho + \delta_j \tilde{\alpha}} \left[1 - \frac{\eta}{1 - \eta(1-\tilde{\alpha})} \tilde{\alpha}\right] n_j^{\tilde{\alpha}-1} - \frac{\bar{w}}{\rho + \delta_j} \tag{52}$$

Rearranging equation (52) for n_j yields equation (35).

It is useful to observe that the vacancy-filling rate μ_j and separation rate δ_j in equation (35) are determined by firms' vacancy-posting decisions $\{v_j\}_{j\in\Omega}$, which are then characterized by the set of employment $\{n_j\}_{j\in\Omega}$ and labor productivity $\{\Phi_j\}_{j\in\Omega}$. First notice that the law of motion for employment implies that the following relationship holds in the steady state:

$$v_j = \frac{\delta_j}{\mu_j} n_j. \tag{53}$$

Next, we can rewrite the value for an unemployed worker in equation (27) as follows in the steady state:

$$U = \frac{b}{\rho + \lambda^m} + \frac{\lambda^m}{\rho + \lambda^m} \int_\Omega \tilde{v}_k W_k dk. \tag{54}$$

The value for a worker employed at firm j in equation (27) yields

$$(\rho + \delta_0) W_j = w_j + \int_\Omega \lambda_{jk} \tilde{v}_k \left(W_k - W_j\right)^+ dk + \delta_0 U. \tag{55}$$

We can then characterize μ_j and δ_j using equations (30) and (29), respectively.

D Estimation and computation details

D.1 Computing the random benchmark for B2B moves

In this section. we describe the procedures to compute the statistical random benchmarks for B2B moves presented in Section 3.2. The goal of this exercise is to compute the share of B2B moves if movers were to be matched with hiring firms randomly. We compute the random benchmark for the share of B2B moves as follows:

1. Randomly draw a mover from the set of all movers. Denote the quarter and employer before move as time $\hat{t}$ and firm $\hat{j}$, respectively.
2. Create a list of firms that satisfies the following conditions:
   (a) firms exist at time $\hat{t}+1$ and are different from firm $\hat{j}$: $k \in \Omega_{\hat{t}+1} \setminus \{\hat{j}\}$.
   (b) we observe at least one new hire at time $\hat{t}+1$: there exists worker i such that $k = j(i, \hat{t}+1)$ and $k \neq j(i, \hat{t})$.
3. Randomly draw a destination firm from the list. Call it firm $\hat{k}$.
4. Check if the destination firm $\hat{k}$ is connected to firm $\hat{j}$ in production network at time $\hat{t}$ and record the value of $\mathbf{1}_{\left\{\hat{k} \in \Omega^S_{\hat{j},t} \cup \Omega^B_{\hat{j},t}\right\}}$.
5. Repeat Steps (1)-(4) 100,000 times and report the average value of $\mathbf{1}_{\left\{\hat{k} \in \Omega^S_{\hat{j},t} \cup \Omega^B_{\hat{j},t}\right\}}$.

When computing the random benchmarks with additional controls reported in the second panel of Table 1 as well as Table 7, we introduce additional conditions to Steps (1) and (2). For instance, we only draw from the set of movers who are blue-collar workers at the origin firms when computing the random benchmark for blue-collar movers. Similarly, when we compute the random benchmark for movers within industries, we require the destination firm $\hat{k}$ to be in the same NACE two-digit industry as firm $\hat{j}$.

D.2 Estimating labor market parameters

In this section, we describe the procedures to estimate the labor market parameters discussed in Section 5.2. The goal is to estimate the vector of six labor market parameters $\Theta = \{\bar{\lambda}, \zeta, \delta_0, c, \bar{w}, \eta\}$ by the method of simulated moments. Recall that we estimate the labor market parameters by minimizing the following objective function:

$$\hat{\Theta} = \arg\min_{\Theta} [\hat{y} - y(\Theta^*)]' [\hat{y} - y(\Theta^*)].$$

where $\hat{y}$ and $y(\Theta)$ denote the vectors of targeted moments in the data and model-implied moments at the parameter value Θ, respectively. In what follows, we explain the algorithm to compute $y(\Theta)$ given the choice of parameter values at Θ.

In the estimation step, we take as given the parameters we externally set in Section 5.1, namely $\{\rho, \alpha, \sigma, L, \xi\}$. We use the distribution of data-implied labor productivity $\{\Phi_j\}$ presented in Section E.1 as well as the network structures $\{\omega_{IJ}\}$ as inputs. We then compute the vector of moments $y(\Theta)$ for a given choice of parameters Θ as follows:

1. guess $\{n_j\}$, $\{\delta_j\}$, and $\{\mu_j\}$
2. compute $\{w_j\}$ using equation (23) and guess $\{W_j\}$ and U
3. compute $\{v_j\}$, $\{s_j\}$, and $\{\lambda_{jk}\}$ using equations (35), (52), (25), and (20)
4. update $\{\delta_j\}$ and $\{\mu_j\}$ using equations (29) and (30)
5. update $\{W_j\}$ and U using equations (55) and (54)
6. update $\{n_j\}$ using the law of motion $n_j^{new} = n_j + (\mu_j^{new} v_j - s_j - \delta_j^{new} n_j)$
7. repeat Steps 3-6 until $\{n_j\}$ converges
8. compute the moments $y(\Theta)$ given the steady-state distribution of employment $\{n_j\}$ and the mobility decisions of workers

D.3 Estimating product market parameters

In this section, we describe the procedures to estimate the product market parameters discussed in Section 5.3. The goal is to estimate the product market parameters for each firm group while concurrently solving for the product market equilibrium.

In the estimation step, we take as given the parameters we externally set in Section 5.1, namely $\{\rho, \alpha, \sigma, L, \xi\}$. We use the distribution of data-implied labor productivity $\{\Phi_j\}$ presented in Section E.1, the network structures $\{\omega_{IJ}^B, \omega_{IJ}^S\}$, network sales shares $\{r_j^{net}\}$, and the estimated supplier fixed effect, buyer fixed effect, and buyer-supplier residual $\{\log \Gamma_j^S, \log \Gamma_k^B, \log \tilde{\Gamma}_{jk}\}$ as inputs. We also take as given the employment $\{n_j\}$ and the labor market parameters estimated in Section 5.2. We then estimate the product market parameters and solve for the product market equilibrium as follows:

1. guess $\{z_j\}$ and $\{\chi_j\}$
2. estimate $\{\gamma_{jk}\}$ and $\{\beta_{jH}\}$ using Propositions 1 and 2
3. compute $\{\phi_j\}$ using equation (48)

4. compute $\{m_j\}$ and $\{p_j\}$ using equations (45) and (46)

5. update $\{z_j\}$ and $\{\chi_j\}$ using equations (41), (9), and (12)

6. repeat Steps 3-5 until $\{z_j\}$ converges

7. estimate $\{\gamma_{jk}\}$ and $\{\beta_{jH}\}$ using Propositions 1 and 2

8. go back to Step 3 and repeat until $\{\beta_{jH}\}$ converges

D.4 Solving for the steady state

In this section, we describe the procedures to solve for the steady state distributions of labor productivity $\{\Phi_j\}$ and employment $\{n_j\}$. In doing so, we take advantage of Claim 3 and solve for the product market equilibrium and the stationary distribution of workers in a sequential manner. Within each step, the procedures to solve for the product market and labor market resemble the estimation procedures described in Appendices D.2 and D.3.

Given the initial guess for the distributions of labor productivity $\{\Phi_j\}$ and employment $\{n_j\}$, we proceed as follows. We first solve for the product market equilibrium and update the labor productivity given the distribution of employment:

1. guess $\{z_j\}$ and $\{\chi_j\}$

2. compute $\{\phi_j\}$ using equation (48)

3. compute $\{m_j\}$ and $\{p_j\}$ using equations (45) and (46)

4. update $\{z_j\}$ and $\{\chi_j\}$ using equations (41), (9), and (12)

5. repeat Steps 2-4 until $\{z_j\}$ converges

6. update $\{\Phi_j\}$ using equation (48)

Using the updated distribution of labor productivity $\{\Phi_j\}$, we then solve for the new distribution of employment $\{n_j\}$:

1. guess $\{n_j\}$, $\{\delta_j\}$, and $\{\mu_j\}$

2. compute $\{w_j\}$ using equation (23) and guess $\{W_j\}$ and U

3. compute $\{v_j\}$, $\{s_j\}$, and $\{\lambda_{jk}\}$ using equations (35), (52), (25), and (20)

4. update $\{\delta_j\}$ and $\{\mu_j\}$ using equations (29) and (30)

5. update $\{W_j\}$ and U using equations (55) and (54)

6. update $\{n_j\}$ using the law of motion $n_j^{new} = n_j + (\mu_j^{new} v_j - s_j - \delta_j^{new} n_j)$

7. repeat Steps 3-6 until $\{n_j\}$ converges

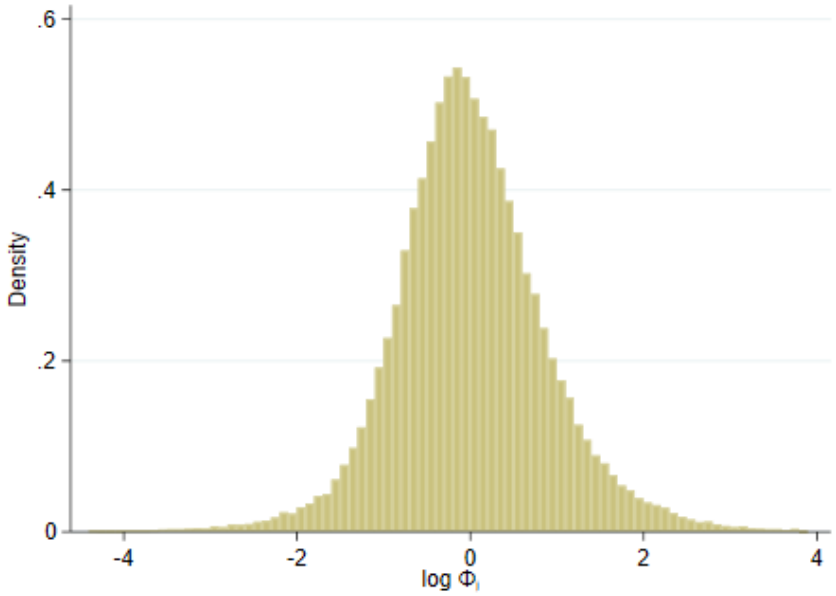
If the new distribution of employment is not close enough to the initial distribution, we construct the new distribution of employment as a linear combination of the previous guess and the computed values and go back to the first step. We iterate between these two steps until $\{n_j\}$ converges.

E Estimation and computation results

E.1 Steady-state distribution of labor productivity Φ_j

In Figure 9, we present the distribution of log labor productivity Φ_j. For each firm, we compute its labor productivity Φ_j based on equation (14), using the observed levels of value added and employment. The return to scale parameter for labor α and substitutability parameter for goods σ in equation (14) are set externally at 0.37 and 4, respectively, following the discussions in Section 5.1. We also normalize labor productivity such that the average of $\log \Phi_j$ is set to be zero.

Figure 9: Distribution of log labor productivity Φ_j



Notes: This figure shows the distribution of log labor productivity Φ_j. For each firm, we compute its labor productivity Φ_j based on equation (14), using the observed levels of value added and employment. The return to scale parameter for labor α and substitutability parameter for goods σ in equation (14) are set externally at 0.37 and 4, respectively, following the discussions in Section 5.1. We normalize labor productivity such that the average of $\log \Phi_j$ is set to be zero. Top and bottom 1 percent of the estimated parameters are trimmed in the figures for illustrative purposes. The figure is based on the main analysis sample of 98,599 private-sector firms in Belgium in 2012 (see Section 2.3 for details).

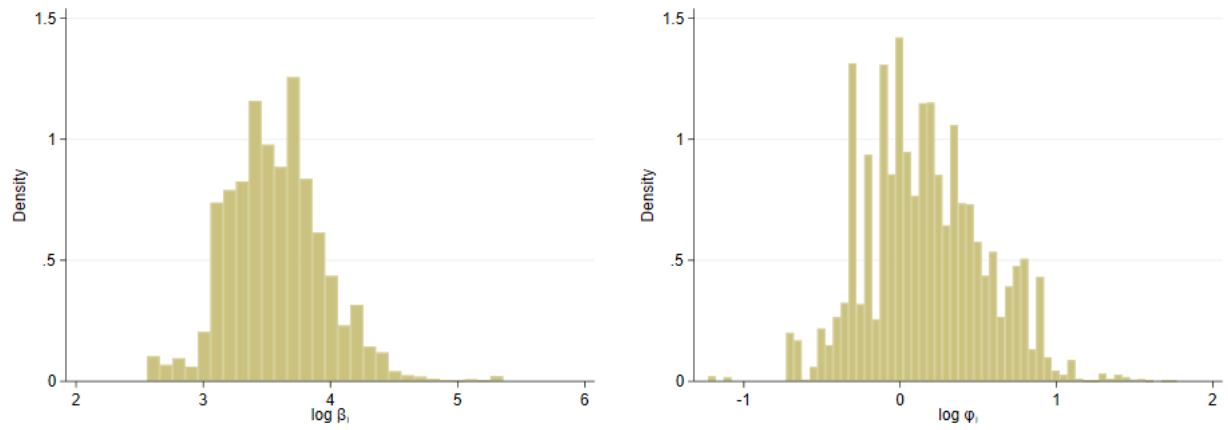
E.2 Estimated product market parameters

In Section 5.3, we provide the identification arguments for estimating the product market parameters in our model. In Figure 10, we present the distributions of the estimated saliency parameters in households' preference $\{\beta_{jH}\}$ as well as firm's own productivity $\{\phi_j\}$. Fol-

lowing the procedure explained in Section 5.2, we first cluster firms into firms groups and estimate the parameter values for each firm group. In Figure 10, we then present the firm-level distributions by weighting the firm groups by the number of firms within each firm group.

Figure 10: Distribution of estimated product market parameters

(a) $\log \beta_{jH}$ (b) $\log \phi_j$



Notes: These figures show the distributions of the estimated product market parameters. Panel (a) displays the distribution of the estimated saliency parameters in households' preference $\{\beta_{jH}\}$, and Panel (b) shows the distribution of the estimated firm's own productivity $\{\phi_j\}$. The identification strategies for estimating these parameters are discussed in Section 5.3. For both parameters, we estimate their values for each firm group, which is constructed by following the procedure explained in Section 5.2. In each panel, we present the firm-level distribution by weighting the firm groups by the number of firms within each firm group. Top and bottom 1 percent of the estimated parameters are trimmed in the figures for illustrative purposes.

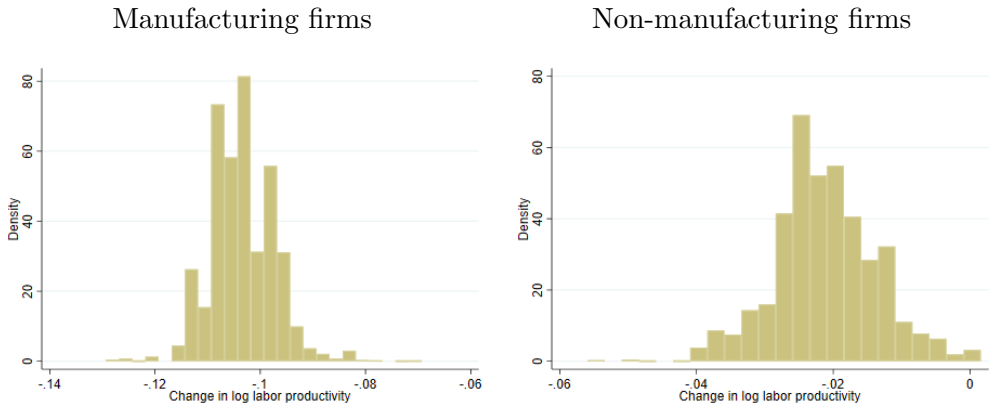
E.3 Long-run response to productivity shocks

In Section 6.2, we presented the instantaneous responses of labor productivity and wages to a 5 percent reduction in manufacturing productivity. In this section, we consider the long-run response of firms by comparing the initial steady state to the new steady state after the shock. Compared to the analysis in the main text, where we focus on the instantaneous impacts on workers, we now consider the firms' responses after taking into account the endogenous reallocation of employment.

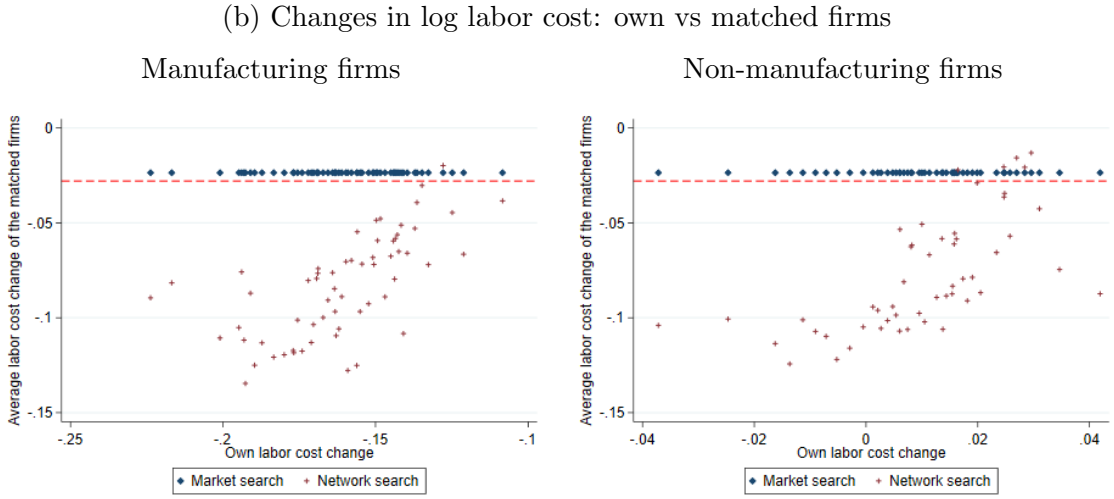
Panel (a) of Figure 11 shows the long-run responses in log labor productivity Φ_j. As in Figure 5, both manufacturing firms and non-manufacturing firms are affected by the decline in manufacturing productivity. Nonetheless, a small fraction of non-manufacturing firms now experience positive changes in their labor productivity relative to the initial steady state. This is possible in the long run because some non-manufacturing firms experience gains in their employment, which is also displayed in Figure 12.

Figure 11: Long-run response to 5 percent reduction in manufacturing productivity

(a) Changes in log labor productivity



(b) Changes in log labor cost: own vs matched firms



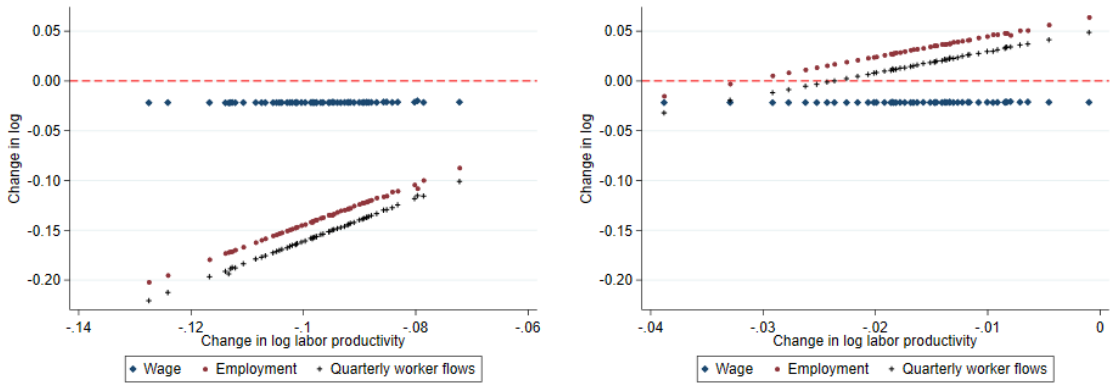
Notes: In this figure, we report the long-run changes in log labor productivity and log labor cost due to a 5 percent reduction in manufacturing firms' own productivity $\{\phi_j\}$. In Panel (a), we show the distributions of long-run changes in log labor productivity $\{\Phi_j\}$ for both manufacturing firms and non-manufacturing firms. We compute the long-run changes by solving for the new steady state after the shock, following the procedure in Appendix D.4. In Panel (b), we present the relationship between firms' own labor cost changes and the average labor cost changes of the firms with which workers are matched through the market search and network search channels. For each bin of firm group, sorted by the percentiles of the own labor cost changes, we compute the average labor cost changes of the matched firms, weighted by the likelihood of the matching. The blue diamonds represent the average labor cost changes of the firms matched through the market search channel, whereas the red markers represent the average labor cost changes of the firms matched through the network search channel. The dashed red lines represent the employment-weighted average of the labor cost changes in the entire economy.

In Panel (b) of Figure 11, we show the relationship between firms' own labor cost changes and the average labor cost changes of the matched firms. Because we now compare different workers at two steady states, we present the changes in firm-level labor costs instead of the changes in average wage. The overall patterns are similar to the findings in 5. While the

average decline in labor cost among the firms that workers meet through the market search does not depend on the current employers, the firms that are hit harder by the productivity shocks are more likely to be connected to other firms with larger labor cost declines through the network search channel.

Figure 12: Long-run response to 5 percent reduction in manufacturing productivity

(a) Changes in log wage, employment, and quarterly worker flows



Notes: In this figure, we report the long-run changes in log wage, employment, and quarterly worker flows due to a 5 percent reduction in manufacturing firms' own productivity $\{\phi_j\}$. We compute the long-run changes by solving for the new steady state after the shock, following the procedure in Appendix D.4. For each bin of firm group, sorted by the percentiles of the changes in their log labor productivity $\{\Phi_j\}$, we compute the average change in each variable.