Sophisticated Borrowing Constraints and Macroeconomic Dynamics*

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Abstract

Classic macro-finance models feature "hard" borrowing constraints, where creditors cut credit indiscriminately to maintain borrowing limits and financial acceleration ensues. We study "sophisticated" borrowing constraints featured in the debt contracts of U.S. nonfinancial firms. They use covenants to specify borrowing limits linked to operating earnings. Moreover, following covenant violation, creditors obtain control rights and improve firm performance by reducing agency problems, so earnings increase although borrowing and investment decrease. We construct a quantitative model with these key features. At the micro level, our model matches the patterns of firms' borrowing and investment in the data, in normal times and after covenant violation. At the macro level, our model produces less financial acceleration, consistent with the resilience of U.S. nonfinancial corporations during recessions.

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1 Introduction

In classic macro-finance models, firms' debt contracts impose hard borrowing constraints (Kiyotaki and Moore, 1997): the enforcement of these constraints forces firms that reach their borrowing limits to cut borrowing and investment indiscriminately, resulting in sharp declines of output and earnings. At the macro level, the presence of these hard borrowing constraints leads to "financial acceleration": firms' adjustment further tightens borrowing constraints and precipitates larger economic contractions in response to aggregate shocks.

In this paper, we analyze "sophisticated" borrowing constraints that approximate common features of debt contracts among U.S. nonfinancial firms, and study their macroeconomic implications. These contracts have legally binding financial covenants that set borrowing limits based on operating earnings. Unlike hard borrowing constraints, when covenants are violated, creditors do not indiscriminately enforce borrowing limits. Instead, creditors obtain control rights over firms' operations. Importantly, they use the control rights to reduce managerial agency problems and firm performance improves: earnings increase, even though firms reduce borrowing and capital expenditures.

We begin by developing a model of sophisticated borrowing constraints. The model features a contingent transfer of control rights, as in Aghion and Bolton (1992), consistent with the interpretation of covenant violation in previous work (e.g., Chava and Roberts, 2008; Roberts and Sufi, 2009; Nini, Smith, and Sufi, 2012). Before covenant violation, managers have control rights and make decisions to maximize their value, but they are susceptible to agency problems and value some unproductive use of capital. After covenant violation, creditors gain control rights, make decisions to maximize their value, and alleviate managerial agency problems, as they do not value the unproductive use of capital.

We embed this framework of sophisticated borrowing constraints into an otherwise standard heterogeneous firms New Keynesian model, as in Ottonello and Winberry (2020). Heterogeneous production firms finance their production and investment expenditures through defaultable long-term debt or costly equity issuance. Their borrowing is subject to sophisticated borrowing constraints, defined in terms of the debt-to-earnings ratio, as documented by contractual evidence in Lian and Ma (2021). We contrast this framework with a minimally different model that instead features hard borrowing constraints, where managers always retain control rights and must strictly adhere to borrowing limits based on the debt-to-earnings ratio.

Our sophisticated borrowing constraint quantitatively matches the empirical patterns of firms' borrowing and investment around covenant violation, which build on data and evidence from Nini, Smith, and Sufi (2012). Specifically, firms exhibit positive capital growth and declining earnings before covenant violation, but experience negative capital growth and rising earnings afterwards. This

empirical pattern is inconsistent with the standard hard borrowing constraints model, which predicts that capital growth and earnings growth should move in the same direction. Our sophisticated borrowing constraint model replicates this pattern because managerial agency frictions before covenant violation are curbed by creditor control after the covenant violation. Our model also matches unconditional moments related to firms' borrowing, such as the fraction of firms newly violating a financial covenant each quarter, the total fraction of firms in the violation state, and the average firm-level earnings-to-capital ratio.

We then study the macroeconomic implications of sophisticated borrowing constraints, particularly the importance of financial acceleration in determining macroeconomic outcomes. We find muted financial acceleration under sophisticated borrowing constraints. Tightening of sophisticated borrowing constraints triggers creditor control for covenant violators, which would reduce managerial agency problems and improve TFP. This contrasts with significant financial acceleration under hard borrowing constraints. After the tightening of hard constraints, borrowing and investment must be reduced indiscriminately to satisfy borrowing limits, which further worsens economic outcomes and generating financial acceleration. Together, sophisticated borrowing constraints provide a form of stabilizer for nonfinancial firms during recessions and is consistent with their resilience during downturns (Gertler and Gilchrist, 2018).

Related literature Our paper connects to three strands of literature. First, we contribute to the literature on the role of financial frictions in the macroeconomy. A core topic in this literature is financial acceleration: adverse economic outcomes exacerbate financial frictions, which in turn worsen economic outcomes (e.g., Bernanke and Gertler, 1989; Kiyotaki and Moore, 1997; Bernanke, Gertler, and Gilchrist, 1999). We show that with sophisticated borrowing constraints, which are central to the debt contracts among large U.S. nonfinancial firms, financial acceleration becomes muted. This result is in line with the observation that financial frictions played a modest role for nonfinancial firms compared to banks and households during the Great Recession Gertler and Gilchrist (2018).

Second, we amplify insights from theoretical and empirical research on debt contracting. A number of theories propose that contingent control by creditors could address agency problems (Diamond, 1984; Jensen, 1986; Aghion and Bolton, 1992; Dewatripont and Tirole, 1994a; Holmstrom and Tirole, 1997). Moreover, empirical work of debt contracts has documented the importance of creditor control in practice, which follows violation of debt covenants (Chava and Roberts, 2008; Roberts and Sufi, 2009; Nini, Smith, and Sufi, 2012). In particular, the data show that firms' performance (e.g., earnings, profitability, and equity valuation) improves after covenant violation trigger creditor control, while they cut back on borrowing and capital expenditures. This observation has not been integrated into macroeconomic analysis. Our paper fills this gap and demonstrates its relevance for

macroeconomic research.

Third, our paper adds to the research agenda on connecting firms' micro evidence with macro models to study business cycles (e.g., Khan and Thomas, 2013; Ottonello and Winberry, 2020, 2024). A growing stream of recent research examines firms' debt contracts in practice to provide new perspectives on their features and macro effects (Lian and Ma, 2021; Chodorow-Reich and Falato, 2022; Ivashina, Laeven, and Moral-Benito, 2022; Drechsel, 2023; Greenwald, 2019; Caglio, Darst, and Kalemli-Ozcan, 2024). Our work highlights how the enforcement of contractual constraints affects macroeconomic outcomes. Our results echo the view that institutions for debt enforcement influence macroeconomic fluctuations (e.g., Corbae and D'Erasmo, 2021; Jordà et al., 2022; Bornstein and Castillo-Martinez, 2023; Kornejew et al., 2024).

As mentioned above, our focus is nonfinancial firms. Debt contracts for financial institutions, such as deposits, commercial paper, asset-backed securities, repos, or bonds, generally do not enforce creditor control the same way (Ma and Scheinkman, 2019). One possibility is that payment default of financial institutions is messy (given their short-term liabilities), so creditors cannot credibly use payment acceleration as a threat to implement creditor control. Financial regulation also restricts the scope of creditor control. Many debt contracts for financial institutions (e.g., margin loans) enforce hard borrowing constraints tied to the market value of financial securities, and the accompanying financial acceleration has been well studied in the literature (Brunnermeier and Pedersen, 2009; Geanakoplos, 2010; Brunnermeier and Sannikov, 2014). Debt contracts for households cannot easily implement creditor control either, but they also do not continuously enforce compliance with borrowing limits after origination. In general, resolution of households' financial distress is challenging, which has contributed to persistent downturns (Mian, Rao, and Sufi, 2013).

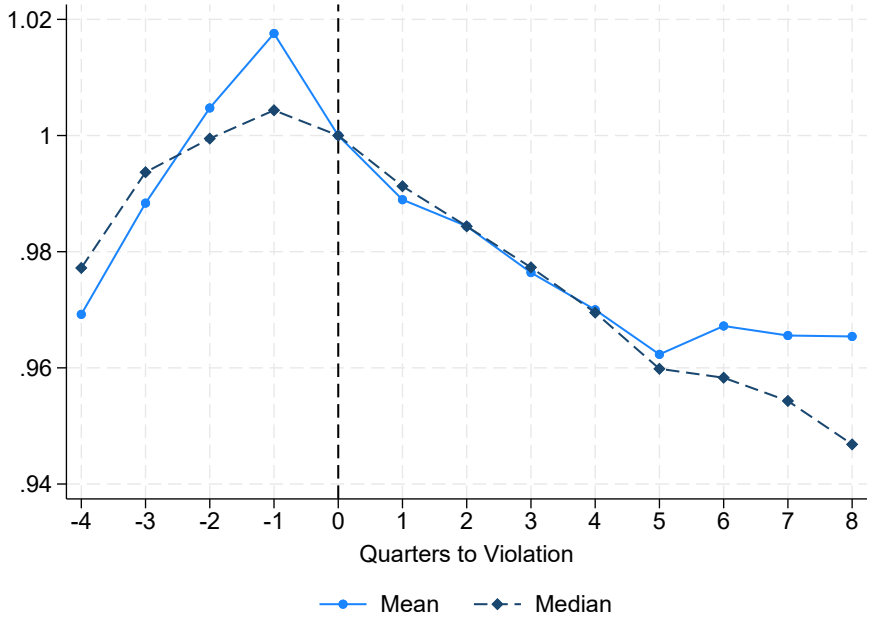
2 Enforcement of Corporate Borrowing Constraints in Practice

Corporate debt contracts in the US commonly impose restrictions on borrowers using financial covenants, which are legally binding requirements that restrict firms' financial ratios. The most widely used set of financial covenants restrict total debt (or debt payments) as a function of operating earnings in the form of EBITDA (earnings before interest, taxes, depreciation, and amortization) (Lian and Ma, 2021). Violation of financial covenants triggers "technical default," in which case creditors have legal power to accelerate payments. In practice, creditors rarely pursue acceleration, but use the power from this threat to impose creditor control and influence firms' decisions. Financial covenants are especially common in loans, which assess covenant compliance on a quarterly basis.[1]

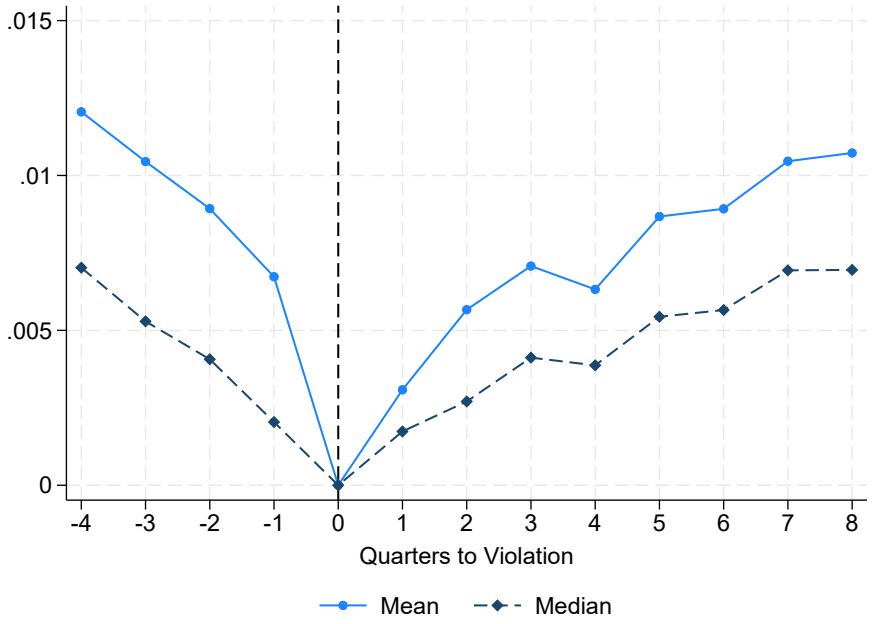
[1]Covenants in bonds are assessed whenever borrowers take significant actions (e.g., issuing new debt). Moreover, for large U.S. nonfinancial firms, monitoring-intensive syndicated loans typically become substantial when debt is high

Figure 1: Firm Outcomes around Covenant Violation

Panel A. Cumulative Asset Changes $(k_{t+h} - k_t)/k_t$



Panel B. Earnings Change $(EBITDA_{t+h} - EBITDA_t)/k_t$



Notes: Panel A shows firms' assets in quarter $t+h$ relative to quarter t when they initially report a covenant violation (normalized by assets in quarter t so that the differences over time come from the numerator rather than the denominator). Panel B shows firms' quarterly earnings in quarter $t+h$ relative to quarter t (normalized by assets in quarter t). The solid line with triangles shows the mean value for each quarter $t+h$ and the dashed line with diamonds shows the median value for each quarter $t+h$. The sample includes US Compustat firms that have debt to EBITDA covenant in quarter t (according to DealScan data). Covenant violation data are kindly shared by Greg Nini and cover 1996 to 2016.

Several important papers collect data on covenant violation and analyze firm outcomes before and after covenant violation (Chava and Roberts, 2008; Roberts and Sufi, 2009; Nini, Smith, and Sufi, 2012). We highlight a striking pattern in Figure 1, using data from Nini, Smith, and Sufi (2012) and following their event-time study approach. In particular, Panel A shows firms' assets in quarter $t+h$ relative to quarter t when they initially report a covenant violation (normalized by assets in quarter t so that the differences over time come from the numerator rather than the denominator). Panel B shows firms' quarterly earnings in quarter $t+h$ relative to quarter t (normalized by assets in quarter t).

The striking pattern in Figure 1 is that before covenant violation, firms have positive asset growth and declining earnings; after covenant violation, firms have negative asset growth and rising earnings. This pattern is consistent with the evidence from Nini, Smith, and Sufi (2012) Figure 4 (asset dynamics around covenant violation) and Figure 7 (earnings dynamics around covenant violation). In addition, Nini, Smith, and Sufi (2012) Figures 8 and 9 show that stock returns also improve after covenant violation.

Why would firms grow assets while their earnings decline prior to covenant violation? And why do earnings rise when they slim down assets after covenant violation? In fact, productivity shocks generally predict that asset growth and earnings growth should move in the same direction, rather than in the opposite direction. Standard models of financial acceleration also predict that when financial constraints bind, shocks should affect asset growth and earnings growth in the same direction.

To understand the empirical evidence, a natural interpretation is that substantial asset growth despite declining earnings prior to covenant violation represents excessive expansion due to managerial agency problems (Jensen, 1986). After covenant violation, creditors step in to cut down excessive expansion and improve operating efficiency (e.g., Chava and Roberts, 2008; Roberts and Sufi, 2009; Nini, Smith, and Sufi, 2012). Doing so is in the interest of the creditors: improving firm performance reduces the prospects of payment default, or increases creditors' recovery rates even if the firm defaults (especially given that creditors' payoffs are determined by the continuation value of the firm after restructuring in the case of Chapter 11 bankruptcy). Why do shareholders not eliminate agency problems before the covenant violation? Shareholders may not care about improving firms' downside, and the excessive expansion prior to covenant violation could also come from risk shifting by equity (Jensen and Meckling, 1976; Dewatripont and Tirole, 1994b).

enough to be close to covenant violation.

3 Model

We now develop a heterogeneous firm New Keynesian model featuring sophisticated borrowing constraints, and study its macroeconomic implications. The model has two blocks: a production firm block, which captures key features of corporate borrowing constraint enforcement in practice (Section 2), and a macro block following Ottonello and Winberry (2020).

3.1 The Production Firm Block

A unit mass of heterogeneous production firms produce and invest in capital, subject to sophisticated borrowing constraints. Time is discrete and infinite. There is no aggregate uncertainty; in Section 5 below, we study the transition path in response to unexpected aggregate shocks. Each firm $j \in [0,1]$ produces an undifferentiated good with relative price p_t (in terms of the final good, described below) using the production function $y_{jt} = z_{jt}\left[\left(1-\tau_{jt}\right)k_{jt}\right]^{\alpha}\ell_{jt}^{\nu}$, where z_{jt} is an idiosyncratic productivity shock, k_{jt} is the firm's capital stock, $1-\tau_{jt}$ is the fraction of capital used for production, ℓ_{jt} is the firm's labor input, and $\alpha+\nu<1$. The idiosyncratic productive shock follows a log-AR(1) process $\log z_{jt+1} = \rho_z \log z_{jt} + \epsilon_{jt+1}^{z}$, where $\epsilon_{jt+1}^{z} \sim \mathcal{N}\left(0,\sigma_z^2\right)$. The production firm generates operating earnings

$$\pi_t\left(z_{jt},\epsilon_{jt},\left(1-\tau_{jt}\right)k_{jt}\right) \equiv \max_{\ell_{jt}} p_t z_{jt}\left[\left(1-\tau_{jt}\right)k_{jt}\right]^{\alpha}\ell_{jt}^{\nu} - w_t\ell_{jt} - f_c - \epsilon_{jt},$$

where w_t is the real wage, f_c is the average real fixed operating cost, and $\epsilon_{jt} \sim \mathcal{N}\left(0,\sigma_\epsilon^2\right)$ is an i.i.d. shock to the operating cost. Capital depreciates at rate δ, and firms can purchase new capital $k_{jt+1} - (1-\delta)k_{jt}$ at the relative price of capital $q_{k,t}$, subject to the adjustment costs

$$AC\left(k_{jt},k_{jt+1}\right) = \psi_{ac}\left(\frac{k_{jt+1}-(1-\delta)k_{jt}}{k_{jt}}\right)^2 k_{jt}.$$

The production firm can finance its expenditures by borrowing externally through a long-term, real debt where the payment declines at a factor $\gamma \in [0,1]$. We use $b_{jt} \geq 0$ to denote the payment of long-term debt due at period t. The firm can issue new debt $b_{jt+1} - \gamma b_{jt}$, where lenders offer a competitive price schedule $q_{b,t}(\cdots)$ for this debt (we derive its price schedule below). This issuance of new debt is subject to the sophisticated borrowing constraint, which introduces three different states for the allocation of control rights. In the normal state ($\omega_{jt} = n$), managers make decisions to maximize their value, subject to agency frictions described below. In the violation state ($\omega_{jt} = v$), creditors make decisions to maximize their value without any agency frictions. Finally, in the liquidation state ($\omega_{jt} = \ell$), the firm is liquidated, and creditors' and managers' payoffs are determined by the liquidation value of capital. Following the contractual evidence of Lian and Ma (2021), the sophisticated borrowing constraint governs the evolution of this state based on the ratio of the face

value of debt and EBITDA ratio and takes the form of

$$q_{b,t}^{f} b_{jt+1} \leq \phi \pi_t \left(z_{jt}, \epsilon_{jt}, \left(1-\tau_{jt}\right) k_{jt}\right), \tag{1}$$

where $q_{b,t}^{f} = \sum_{k=0}^{\infty} \gamma^k \left(\prod_{i=0}^{k} \frac{\Pi_{t+i+1}}{R_{t+i}^{\text{nom}}}\right)$ is the face value of debt, ϕ captures the tightness of the constraint, R_t^{nom} is the nominal risk-free interest rate, and Π_{t+1} is the inflation between period t and $t+1$. A firm currently in the normal state remains in the normal state in the next period (barring an exogenous exit shock) if (1) is satisfied but transitions to the violation state in the next period if (1) is violated. The complete transition of firm states will be specified below.

The dividend payment of the production firm is denoted by d_{jt}. Negative dividend payments capture equity issuance, which is an alternative source of financing and is costly. Specifically, motivated by the fact that around 90% of Compustat firms make zero dividend payments,[2] we use $AD\left(d_{jt}\right) = \left(1+\psi_e \mathbb{I}\left(d_{jt}<0\right)\right)\psi_d d_{jt}^2$ to capture the costs associated with dividends, where $\psi_e > 0$ represents the cost of equity insurance, and $\psi_d > 0$, a small number, capture costs associated with adjusting dividends and maintains the concavity of the problem discussed below.

Motivated by the empirical evidence in Section 2, we introduce managerial agency frictions. Specifically, managers derive utility $e_{jt} g\left(\tau_{jt} k_{kt}\right)$ by diverting a fraction $\tau_{jt} \in [0,1]$ of capital for unproductive use, where $g(x) = x^{\tilde{\alpha}}$ with $\tilde{\alpha} \in (0,1)$ and e_{jt} captures idiosyncratic shock to its attractiveness to managers. It follows a log-AR(1) process $\log e_{jt+1} = \left(1-\rho_e\right)\log\left(\mu_e\right) + \rho_e \log e_{jt} + \epsilon_{jt+1}^{e}$, where $\epsilon_{jt+1}^{e} \sim \mathcal{N}\left(0, \sigma_e^2\right)$. On the other hand, creditors do not value the unproductive use of capital.

The timing of events within each period t is as follows. At the beginning of each period, with probability π_d, each production firm receives an i.i.d. exit shock and must exit the economy by directly switching to the liquidation state. New firms enter the economy, and the mass of new entrants is equal to the mass of firms being liquidated. Each new entrant starts in the normal state, and its initial borrowing, capital, productivity, and attractiveness to unproductive use of capital are drawn from the ergodic distribution of (b,k,z,e). Idiosyncratic shocks to existing firms' productivity and attractiveness to unproductive use of capital, e_{jt} and z_{jt}, also occur. Each new and existing firm (excluding liquidated ones) then decides on its borrowing and investment. The idiosyncratic operating cost shock ϵ_{jt} occurs after those decisions, generating some randomness in the evolution of firm states and preserve the smoothness of the problem. After that, each production firm hires labor ℓ_{jt} from a competitive labor market, sells output to retailers in a competitive market, and pays dividends.

We now write the optimization problem in each state recursively. The relevant state variables for each firm are (ω, b, k, z, e), and we omit subscripts for notational simplicity.

The nominal state $\omega = n$ with manager control. In the normal state where the sophisticated bor-

[2]The sample of all calibration targets consists of Compustat firms that have debt/EBITDA covenants, the same as in 1. For those firms, the fraction of firms with zero quarterly dividends is 90%. For all Compustat firms, the fraction of firms with zero quarterly dividends is 87%.

rowing constraint is not violated, the manager of each production firm holds control rights and makes borrowing, investment, and diversion decisions to maximize the manager's value:

$$V_{m,t}(n,b,k,z,e) = \max_{\tau,b'\geq 0,k'} \mathbb{E}_\epsilon [d - AD(d)] + eg(\tau k) + \mathbb{E}\left[\Lambda_{t+1} V_{m,t+1}\left(\omega', b', k', z', e'\right) \mid z, e\right], \tag{2}$$

where Λ_{t+1} is the stochastic discount factor (SDF), further specified below, and is subject to the flow budget constraint:

$$d = \pi_t(z,\epsilon,(1-\tau)k) + q_{b,t}\left(n,b',k',z,e,\tau\right)\left(b' - \gamma b\right) - q_{k,t}\left(k' - (1-\delta)k\right) - AC\left(k,k'\right) - b, \tag{3}$$

and the transition of firm states:

$$\omega' = \begin{cases} n & \text{with probability } 1-\pi_d \quad \text{and} \quad q^f_{b,t} b' \leq \phi \pi_t(z,\epsilon,(1-\tau)k) \\ v & \text{with probability } 1-\pi_d \quad \text{and} \quad q^f_{b,t} b' > \phi \pi_t(z,\epsilon,(1-\tau)k) \\ \ell & \text{with probability } \pi_d, \end{cases} \tag{4}$$

which is governed by the sophisticated borrowing constraint (1) and the exogenous exit risk. Given the timing assumption above, borrowing, investment, and diversion decisions are made before the realization of the idiosyncratic operating cost shock ϵ, and the dividend adjusts to maintain budget balance in (3). The creditor's value in the normal state is determined passively where ω',b', and k' are determined by manager decisions in (2)–(4):

$$V_{c,t}(n,b,k,z,e) = b - q_{b,t}\left(n,b',k',z,e,\tau\right)\left(b' - \gamma b\right) + \mathbb{E}\left[\Lambda_{t+1} V_{c,t+1}\left(\omega', b', k', z', e'\right) \mid z, e\right]. \tag{5}$$

The violation state $\omega = v$ with creditor control. In the violation state, the creditor obtains control rights and makes investment and diversion decisions to maximize the creditor's value.

$$V_{c,t}(n,b,k,z,e) = \max_{\tau,k'} \mathbb{E}_\epsilon \left[\min\left\{ \underbrace{\pi_t(z,\epsilon,(1-\tau)k) - q_{k,t}\left(k' - (1-\delta)k\right) - AC\left(k,k'\right)}_{\text{available cash flow}}, b \right\} \right] \tag{6}$$

$$+ \mathbb{E}\left[\Lambda_{t+1} V_{c,t+1}\left(\omega', \gamma b, k', z', e'\right) \mid z, e\right], \tag{7}$$

subject to the transition of firm states:

$$\omega' = \begin{cases} n & \text{with probability } \lambda(1-\pi_d) \\ v & \text{with probability } (1-\lambda)(1-\pi_d) \text{and} \quad V_{c,t+1}\left(v,\gamma b,k',z',e'\right) \geq V_{c,t+1}\left(\ell,\gamma b,k',z',e'\right) \\ \ell & \text{with probability} \pi_d \text{or probability } (1-\lambda) \text{ and} \quad V_{c,t+1}\left(v,\gamma b,k',z',e'\right) < V_{c,t+1}\left(\ell,\gamma b,k',z',e'\right), \end{cases} \tag{8}$$

where $\lambda \in (0,1)$ captures the exogenous probability of transitioning back to the normal state, conditional on not being hit by the exit shock. The firm transitions to the liquidation state either when it is hit by an exogenous exit shock or when the manager deems liquidation more valuable than continuing to operate the firm.

A few features of the creditor's problem in the violation state are worth mentioning. First, the creditor does not derive value from the unproductive use of capital. As a result, the creditor endogenously chooses not to divert any capital ($\tau = 0$), capturing the empirical evidence in Section 2 that creditor control mitigates managerial agency frictions and improves firm performance (e.g., increases earnings post-covenant violation). Second, consistent with practice (new debt issuance or acceleration of debt payment during covenant violation is rare) and empirical evidence on debt dynamics post-covenant violation in Appendix Figure C.1, debt naturally matures, $b' = \gamma b$, instead of being actively chosen after the violation. Third, transitioning back to the normal state is exogenous, reflecting that problems caused by managerial agency frictions (e.g., inefficient acquisitions, pet projects, bureaucracies) may take a nontrivial and unpredictable amount of time to mitigate. If transitioning back to the normal state is instead determined by whether the constraint on the ratio of the face value of debt to EBITDA in (1) is satisfied, almost all violating firms will transition back immediately, as creditors significantly improve earnings by not diverting any capital ($\tau = 0$), inconsistent with the practice.

The manager's value in the violation state is determined passively where ω' and k' are determined by creditor decisions in (6)–(8):

$$V_{m,t}(v,b,k,z,e) = \underbrace{\max\left\{\pi_t(z,\epsilon,(1-\tau)k) - q_{k,t}\left(k' - (1-\delta)k\right) - AC\left(k,k'\right), 0\right\}}_{\equiv \text{dividend positive only if debt obligation repayed}}$$
$$+ \mathbb{E}\left[\Lambda_{t+1} V_{m,t+1}\left(\omega', \gamma b, k', z', e'\right) \mid z, e\right].$$

Consistent with the practice that new equity issuance during covenant violation is rare, equity issuance (negative dividend) is not allowed during covenant violation. More broadly, there are no costs associated with adjusting dividends (no $AD\left(d_{jt}\right)$ term), as dividends cease to play a signaling role after a covenant violation, which itself serves as a signal.

The liquidation state $\omega = \ell$. In the case of liquidation, the creditor recovers a fraction ξ of the firm's capital stock, where $\xi \in [0,1]$ captures liquidation value per unit of capital.[3] The creditor's value is hence given by

$$V_{c,t}(\ell,b,k,z,e) = \min\left\{\xi k, \tilde{q}^f_{b,t} b\right\},$$

[3]Following Ottonello and Winberry (2020), the fraction of a liquidated firm's capital not recovered by its creditors is transferred lump-sum to the representative household. This prevents capital destruction during liquidation and avoids complications in the dynamics of aggregate capital (10). The representative household also distributes capital to new entrants and acquires or sells capital to balance the difference between the capital transferred to them and the capital distributed to entrants at the competitive price $q_{k,t}$.

where $\tilde{q}^{\text{f}}_{b,t} = 1 + \sum_{k=1}^{\infty} \gamma^k \left(\prod_{i=0}^{k-1} \frac{\Pi_{t+i+1}}{R^{\text{nom}}_{t+i}} \right)$.[4] The manager's value is given by

$$V_{m,t}(\ell, b, k, z, e) = \max\left\{ \xi k - \tilde{q}^{f}_{b,t} b, 0 \right\}.$$

Creditors and the price schedule of debt. There is a representative financial intermediary that lends resources from the representative household to production firms at the firm-specific price schedule $q_{b,t}\left(\omega, e, z, b', k', \tau\right)$. The debt price schedule competitively accounts for the risk of partial debt payment in the violation and liquidation states. See Appendix A for the detailed formula for the pricing schedule.

3.2 The Macro Block and Equilibrium

We follow Ottonello and Winberry (2020) to parsimoniously generate a New Keynesian Phillips curve that relates nominal variables to the real economy by introducing sticky-price retailers. There is a unit mass of retailers $i \in [0,1]$. Each retailer i produces a differentiated variety $\tilde{y}_{it}$ using the production firms' undifferentiated good as its input: $\tilde{y}_{it} = y_{it}$, where y_{it} is the amount of the undifferentiated good demanded by retailer i. Retailers set a nominal price for their variety $\tilde{p}_{it}$ but must pay a quadratic it price adjustment cost $\frac{\varphi}{2}\left(\frac{\tilde{p}_{it}}{\tilde{p}_{it-1}} - 1\right)^2 Y_t$.

A representative, competitive final good producers purchases differentiated varieties from retailers and uses a CES production technology that incorporates retailers' varieties as inputs, with an elasticity of substitution γ_{CES}: $Y_t = \left(\int \tilde{y}_{it}^{\frac{\gamma_{CES}-1}{\gamma_{CES}}} di \right)^{\frac{\gamma_{CES}}{\gamma_{CES}-1}}$. Together, this gives rise to a (linearized for simplicity) New Keynesian Phillips curve:

$$\log \Pi_t = \frac{\gamma_{CES} - 1}{\varphi} \times \log \frac{p_t}{p_{ss}} + \beta \log \Pi_{t+1}, \tag{9}$$

where $p_{ss} = \frac{\gamma_{CES}-1}{\gamma_{CES}}$ is the steady state relative price of the heterogeneous production firm output. The Phillips Curve links the New Keynesian block to the investment block through the relative price p_t. When aggregate demand increases, sticky-price retailers expand production of their differentiated goods and raise demand for the heterogeneous firms' produced goods, y_{it}, which increases their relative price, p_t, and generates inflation through (9).

A representative, competitive capital goods producer produces new capital goods using the technology $\Phi\left(\frac{I_t}{K_t}\right) K_t$, where I_t is units of the final good used to produce capital, $K_t = \int k_{jt} dj$ is the aggregate start-of-period capital, and $\Phi(x) = \frac{\delta^{\frac{1}{\eta}}}{1-1/\eta}(x)^{1-1/\eta} - \frac{\delta}{\eta-1}$. Its optimality pins down the relative price

[4] Note that $\tilde{q}^{\text{f}}_{b,t}$ here slightly differs from $q^{f}_{b,t}$ in (1), as $\tilde{q}^{\text{f}}_{b,t}$ captures the period-t face value of debt that starts payment at t, whereas $q^{f}_{b,t}$ captures the period-t face value of debt that starts payment at $t+1$.

of capital as $q_{k,t} = \left(\frac{I_t/K_t}{\delta}\right)^{1/\eta}$. The evolution of aggregate capital is then given by

$$K_{t+1} = \Phi\left(\frac{I_t}{K_t}\right) K_t + K_t(1-\delta) \tag{10}$$

The monetary authority sets the nominal risk-free interest rate R_t^{nom} according to the Taylor rule

$$\log R_t^{\text{nom}} = \log \frac{1}{\beta} + \varphi_\pi \log \Pi_t + \epsilon_t^m, \tag{11}$$

where ϵ_t^m is the monetary policy shock and $\varphi_\pi > 1$ is the Taylor coefficient.

There is a representative household with preferences over consumption C_t and labor supply L_t with life-time expected utility

$$\mathbb{E}_0\left[\sum_{t=0}^{\infty} \beta^t \left(\log(C_t) - \Psi L_t\right)\right],$$

where β is the discount factor and Ψ parametrizes the disutility of labor. The household owns all firms in the economy. The stochastic discount factor and nominal interest rate are linked through the Euler equation for bonds, $\Lambda_t = \beta \frac{C_t}{C_{t+1}} = \frac{\Pi_{t+1}}{R_t^{nom}}$.

An equilibrium is a set of value functions $V_{m,t}(\omega, b, k, z, e)$ and $V_{c,t}(\omega, b, k, z, e)$; decision rules $b_t'(n, b, k, z, e)$, $\tau_t(n, b, k, z, e)$, $k_t'(n, b, k, z, e)$, $\tau_t(v, b, k, z, e)$, $k_t'(v, b, k, z, e)$; measure of firms $\mu_t(\omega, b, k, z, e)$; debt price schedule $q_{b,t}\left(n, b', k', z, e, \tau\right)$; and prices w_t, p_t, $q_{k,t}$, Π_t, R_t^{nom}, and Λ_{t+1} such that (i) managers optimize in the normal state and creditors optimize in the violation state, (ii) creditors price competitively, (iii) the household optimizes, (iv) the measure of firms is consistent with decision rules, (v) all markets clear, (vi) monetary policy follows (11). Appendix A precisely defines an equilibrium of our model.

3.3 The Hard-constraint Model

We will contrast the above model with sophisticated borrowing constraints against a model with hard borrowing constraints that minimally deviates from the above model. The only difference is the production firm's problem, which does not have three different states and always respects the hard borrowing constraint:

$$V_t^{\text{hard}}(b, k, z, e) = \max_{\tau, b' \geq 0, k'} \mathbb{E}_\epsilon\left[d - AD(d)\right] + eg(\tau k) + \mathbb{E}\left[\Lambda_{t+1} V_{t+1}^{\text{hard}}\left(b', k', z', e'\right) \mid z, e\right]$$

subject to the flow budget constraint

$$d = \pi_t(z, \epsilon, (1-\tau)k) + q_{b,t}^{\text{hard}}\left(b', k', z, e, \tau\right)\left(b' - \gamma b\right) - q_{k,t}\left(k' - (1-\delta)k\right) - AC\left(k, k'\right) - b, \tag{12}$$

and the hard borrowing constraint

$$q_{b,t}^f b_{jt+1} \leq \mathbb{E}_\epsilon\left[\pi_t\left(z_{jt}, \epsilon_{jt}, \left(1-\tau_{jt}\right)k_{jt}\right)\right]. \tag{13}$$

The enforcement of hard borrowing constraints differs from that of sophisticated borrowing constraints which follows the evidence in Section 2. When a firm hits hard borrowing constraints, it must cut credit indiscriminately to remain within borrowing limits.

4 Matching Micro Evidence on Firm Behavior around Covenant Violation

We calibrate the model in two steps. First, we exogenously assign a subset of parameters. Second, we choose the remaining parameters in order to match moments in the data. Table 1 lists the parameters that we assign. The model period is one quarter, so we set discount rate $\beta = 0.99$. We set ϕ, the capacity of earnings-based borrowing, to 14, following the contractual evidence in Lian and Ma (2021).[5] The capital share and labor share are $\alpha = 0.24$ and $\nu = 0.61$, respectively, from Karabarbounis and Neiman (2014). Capital depreciates at a quarterly rate $\delta = 2.5\%$, a standard value. The quarterly exogenous exit rate is 0.5%, also a standard value. The disutility of labor, $\Psi = 1$, is chosen such that $L = 0.6$ in steady state, consistent with the standard labor market participation rate. The elasticity of substitution between differentiated retailer goods is 10, consistent with a markup of 11%. The elasticity of aggregate investment with the relative price of capital goods is $\eta = 4$, from Bernanke, Gertler, and Gilchrist (1999). The slope of NKPC, $\left(\gamma_{CES} - 1\right)/\varphi = 0.06$, from Hazell et al. (2022).[6]

We then choose the parameters listed in Table 2 to match both unconditional empirical moments related to firms' borrowing and firm outcomes around covenant violation in Figure 1. The first set of parameters govern key features related to the sophisticated borrowing constraint, the decay rate of debt payment γ, and the probability of transitioning back from the violation state to the normal state, λ. The second set of parameters (ρ_z and σ_z) govern idiosyncratic productivity shocks. The third set (ρ_e, σ_e, and μ_e) govern idiosyncratic shocks to the attractiveness of unproductive use of capital. The fourth set ($\psi_{as,}$ ψ_d, and ψ_e) governs production firms' adjustment costs of capital and dividends. The fifth set (f_c and σ_ε) governs fixed and random operating costs.

We target leverage at the 10th, 30th, 50th, 70th, and 90th percentiles in Figure 2. We also target the fraction of firms newly violating a financial covenant each quarter (2.5%), the total fraction of firms in the violation state, and the average earnings-to-capital ratio across firms (0.32). The sample of all calibration targets is Compustat firms that have debt/EBITDA covenants, the same as Figure 1. For firm outcomes around covenant violation, we match the dynamics of capital and earnings changes in Figure 1, normalized by capital at the first quarter of covenant violation t, corresponding

[5] Lian and Ma (2021) find that the capacity of earnings-based borrowing, measured as the annual earnings-to-debt ratio, is approximately 3.5. This translates to a quarterly earnings-to-debt ratio of 14.

[6] See Appendix A for how to translate their estimates of the slope of NKPC to parameters of our model.

Table 1: Assigned Parameters

β	0.990	Discount Factor	4% Annual Interest Rate
α	0.240	Capital Share	Karabarnouni and Neiman (2014)
ν	0.610	Labor Share	Karabarnounis and Neiman (2014)
δ	0.025	Depreciation Rate	10% Annual Rate
ξ	0.350	Recovery in Liq.	Kermani and Ma (2023)
ϕ	14.000	Debt to Earnings in Normal	Lian and Ma (2021)
π_d	0.005	Exit Prob.	Standard
Ψ	1.000	Disutility of Labor	$L = 0.6$
γ_{CES}	10.000	Elasticity of Substitution	11% Markup
η	4.000	Investment Elasticity	Bernanke et al. (1999)
$(\gamma_{CES} - 1)/\varphi$	0.006	PC Slope	Hazell et al. (2022)

Table 2: Fitted Parameters

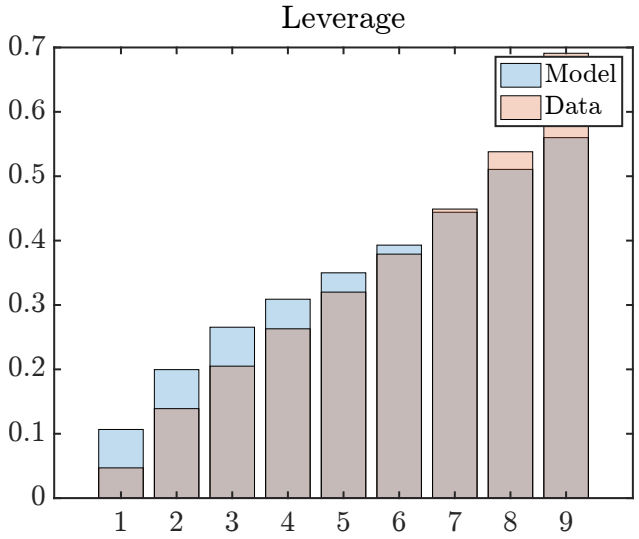
γ	0.938	Coupon Decay Rate
λ	0.090	Prob. of Exit to Normal in Violation
ρ_z	0.796	Persistence of productivity
σ_z	0.041	STD of productivity
ρ_e	0.803	Persistence of diversion
σ_e	0.230	STD of diversion
μ_e	0.047	Mean of diversion
ψ_{ac}	0.069	Scale Adj Cost of Capital
ψ_d	0.242	Divident Payout Cost
ψ_e	1.440	Equity Issuance Cost
f_c	0.036	Operating Cost
σ_ϵ	0.080	STD of Earnings Shock

Table 3: Calibration Targets and Model Fit

	Model	Data
New firms in violation	0.027	0.025
Total firms in violation	0.228	0.200
Earnings to capital ratio	0.049	0.032

to $\left(k_{jt+h}-k_{jt}\right)/k_{jt}$ and $\left(\pi_{jt+h}-\pi_{jt}\right)/k_{jt}$ in the model, for $h \in \{-4,-3,\cdots,8\}$. Finally, we align capital in the model with assets in the data. One alternative is to align capital in the model with the narrowly defined PP&E in the data, but this would result in an empirical distribution of leverage far greater than that in the standard calibration of heterogeneous firm models with financial constraints.

Figure 2: Deciles of Firm Leverage

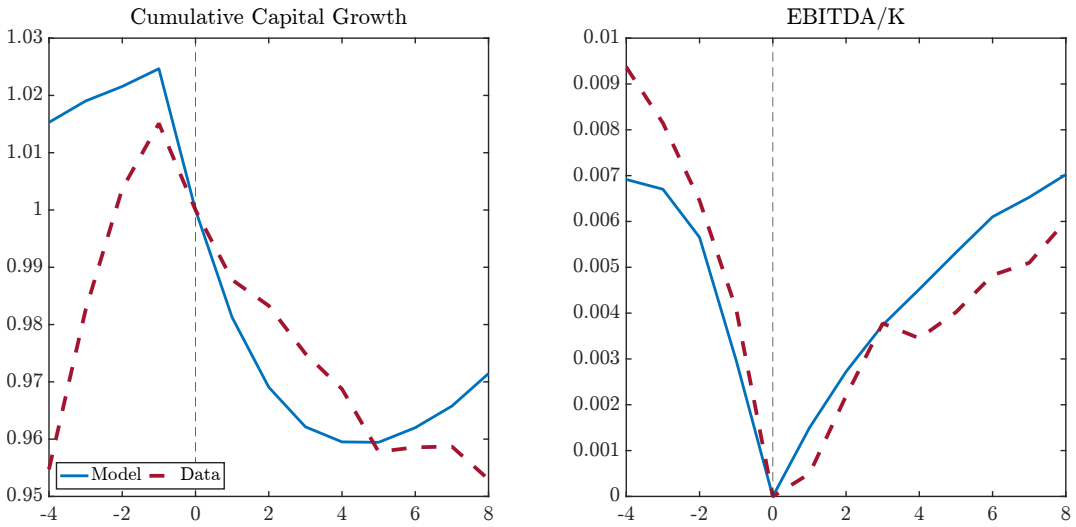


Notes: The red bar plots the deciles of firm leverage in the data, using a sample of US Compustat firms that have a debt-to-EBITDA covenant according to Dealcan. The blue bar plots the deciles of firm leverage in the model, defined as b/k.

Despite the model being severely overidentified, in the sense that we have 12 fitted parameters and 3 + 5 + 12 * 2 = 32 moments, our model matches the targeted moments reasonably well. In terms of unconditional moments in Table 2, our model matches the fraction of firms newly violating a financial covenant each quarter and the total fraction of firms in the violation state. Our model generates a somewhat higher average earnings-to-capital ratio compared to the data (0.049 vs. 0.032). Figure 2 shows that our model also closely matches the distribution of leverage in the data. Figure 3 shows that our model also matches the dynamics of capital and earnings changes around covenant violation, with a somewhat smaller increase in capital before a covenant violation. The calibrated parameters in Table 2 are broadly comparable to existing estimates in the literature. For example, the average maturity of debt $\frac{1}{1-\gamma} \approx 16$ quarters and the average duration of covenant violation $\frac{1}{\lambda} \approx 11$ quarters are close to their empirical counterparts for Compustat firms.

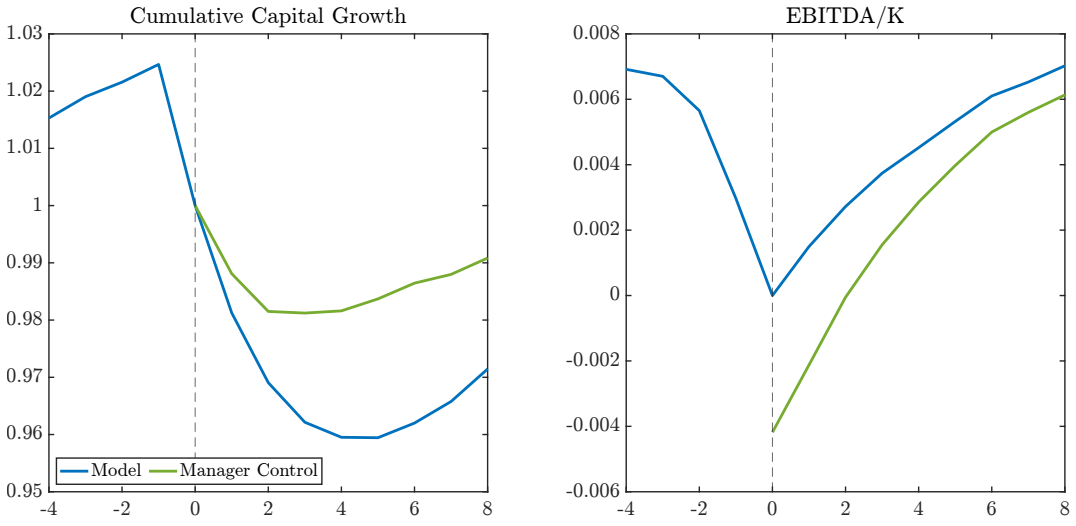
The essence of the sophisticated borrowing constraint model is that after violation, creditor control reduces managerial agency problems and improves firm performance. It does so by avoiding the unproductive use of capital (set $\tau = 0$), thereby reducing capital and increasing earnings. To further illustrate how creditor control impacts the dynamics of capital and earnings in the model, Figure 4 compares the actual dynamics of capital and earnings with the hypothetical dynamics of capital and

Figure 3: Model Fit: Capital & Earnings around Covenant Violation



Notes: The blue solid line in the left panel shows firms' capital in quarter $t+h$ relative to quarter t when they initially violate the covenant (the last period they are in the normal state), normalized by capital at t, i.e., $(k_{t+h}-k_t)/k_t$ in the model. The blue solid line in the right panel shows firms' earnings in quarter $t+h$ relative to earnings at t, normalized by capital at t, i.e., $(\pi_{t+h}-\pi_t)/k_t$ in the model. The red dashed lines in both panels correspond to their empirical counterparts, i.e., the mean lines in Figure 1.

Figure 4: Creditor vs Manager Control Post Covenant Violation



Notes: The blue solid line in the left panel shows firms' capital in quarter $t+h$ relative to quarter t when they initially violate the covenant (the last period they are in the normal state), normalized by capital at t, i.e., $(k_{t+h}-k_t)/k_t$ in the model. The blue solid line in the right panel shows firms' earnings in quarter $t+h$ relative to earnings at t, normalized by capital at t, i.e., $(\pi_{t+h}-\pi_t)/k_t$ in the model. The green solid lines in both panels plot the hypothetical counterparts of normalized capital and earnings dynamics if managers retain control rights.

earnings after a covenant violation if managers retain control rights.[7] Indeed, creditor control reduces capital and increases earnings compared to manager control by avoiding the unproductive use of capital.

Finally, for the hard borrowing constraint model in Section 3.3, we recalibrate the discount rate β so that the fraction of firms with binding hard borrowing constraints matches the fraction of firms in the violation state in the sophisticated borrowing constraint model, keeping all other parameters fixed. This facilitates a comparison between the hard and sophisticated borrowing constraint models with minimal differences between them.

5 Macro Implications of Sophisticated Borrowing Constraints

We now study the macroeconomic implications of sophisticated borrowing constraints, particularly the importance of financial acceleration in determining macroeconomic outcomes. We then contrast these implications with those under hard borrowing constraints. By financial acceleration (e.g., Kiyotaki and Moore, 1997 and Bernanke, Gertler, and Gilchrist, 1999), we refer to the general equilibrium feedback mechanism in which adverse economic outcomes tighten borrowing constraints, thereby further worsen the economic outcomes.[8]

To gauge the importance of financial acceleration, a useful exercise is to study the macroeconomic impact of shocks to the tightness of the constraint ϕ, as examined in Khan and Thomas (2013). This directly informs the second step of the financial acceleration feedback (how tightening financial constraints further worsens economic outcomes) and is thus useful for understanding the role of financial acceleration in shaping the macroeconomic impact of other aggregate shocks (e.g., monetary policy shocks), which we will analyze later.

Specifically, we consider the model specified in Section 3 and allow the tightness of borrowing constraints, ϕ_t, to be time varying. We then study the responses of key aggregate variables to an unexpected tightening of borrowing constraints at period 0, with a magnitude of $\Delta\phi_0 = -1\% \times \phi_{ss}$ and a persistence of 0.8. We compute the perfect foresight transition path of the economy as it converges back to steady state.

Figure 5 plots the aggregate output, investment, and TFP responses to a tightening of borrowing constraints under both hard and sophisticated borrowing constraints, where aggregate TFP is defined as $Z_t = \frac{Y_t}{K_t^{\alpha} L_t^{\gamma}}$, where $Y_t = \int y_{jt} dj$, $K_t = \int k_{jt} dj$, and $L_t = \int \ell_{jt} dj$. Under hard borrowing constraints, we observe a large decline in aggregate output, investment, and TFP, especially in the initial period

[7] That is, the hypothetical path of capital and earnings post-covenant violation follows the decision rules for capital, debt, and diversion, given by their counterparts $b'_t(n,b,k,z,e)$, $\tau_t(n,b,k,z,e)$, and $k'_t(n,b,k,z,e)$ under manager control.

[8] Similarly, the mechanism also applies to positive economic outcomes, which relax borrowing constraints.

when the constraints bind. After the tightening of hard borrowing constraints, borrowing and investment must be reduced indiscriminately to satisfy the borrowing constraints. This further leads to a misallocation of capital across production firms and hence lower aggregate TFP. Together, constraint tightening leads to large and negative macroeconomic responses.

Under sophisticated borrowing constraints, we instead observe a small decline in aggregate output and investment and a mild increase in TFP, which is more clearly illustrated in Figure 8 with a scale tailored to the responses in the sophisticated borrowing constraint model. The tightening of the sophisticated borrowing constraint leads to more firms transitioning toward violation states. However, a transfer of control from managers to creditors helps alleviate managerial agency frictions, particularly by reducing the unproductive use of capital. This improves firms' performance and increases aggregate TFP. This breaks the second step of the financial acceleration feedback mentioned above. Tightening financial constraints no longer worsens economic outcomes (and may even improve them by reducing managerial agency frictions).

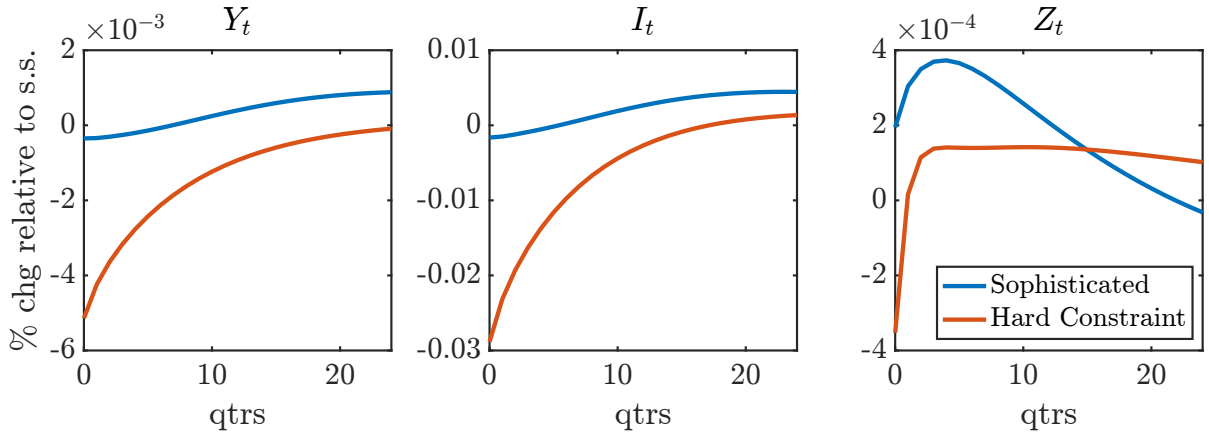
We now turn to the macroeconomic impact of a monetary policy shock. To gauge the importance of financial acceleration in determining the aggregate responses to the monetary policy shock and to connect it to the previously studied responses to shocks to the tightness of the constraint, we will compare the benchmark aggregate responses, incorporating the full general equilibrium financial acceleration feedback, to counterfactual aggregate responses where the feedback is shut down. In this counterfactual, borrowing constraints are governed by the steady-state earnings function $\pi_{ss}(\cdots)$, instead of the one subject to general equilibrium feedback, $\pi_t(\cdots)$ (e.g., being influenced by changes in the relative prices of production firms' goods, p_t). For example, in the sophisticated constraint case, the evolution of firm states will be governed by the counterpart of (1):

$$q^f_{b,t} b_{jt+1} \leq \phi \pi_{ss}\left(z_{jt}, \epsilon_{jt}, \left(1-\tau_{jt}\right) k_{jt}\right). \tag{14}$$

Because the counterfactual aggregate responses shut down general equilibrium constraint tightening and thus financial acceleration, their differences from the benchmark aggregate responses incorporating general equilibrium feedback can be viewed as the contributions of financial acceleration. These differences are also closely linked to the responses to shocks to the tightness of the constraint studied above.

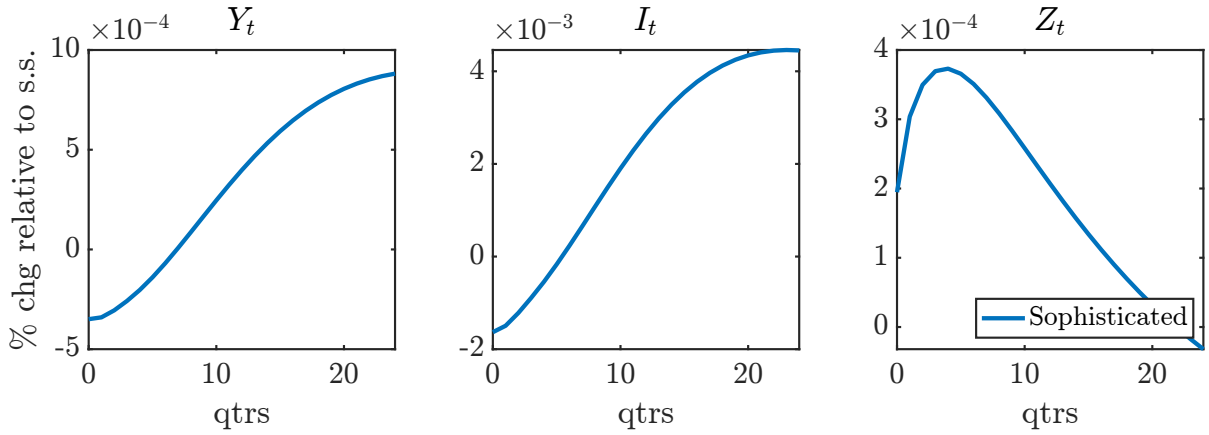
We now study the aggregate output, investment, and TFP responses to a 30 bps monetary tightening ($\epsilon^m_0 = 0.3\%$) with a persistence of 0.90. The persistence is designed to capture the fact that the magnitude of nominal interest rate changes after a monetary policy shock halves over a one-year horizon, as observed in Caravello, McKay, and Wolf (2024). We compute the perfect foresight transition path of the economy as it converges back to steady state. Figure 7 plots both the benchmark aggregate responses incorporating financial acceleration and the counterfactual responses shutting down

Figure 5: Macro Impacts of A Constraint Tightness Shock: Hard vs Sophisticated Borrowing Constraints



Notes: The figure plots aggregate responses to a tightening of borrowing constraints ($\Delta\phi = -1\% \times \phi_{ss}$) with a persistence of 0.8, under both hard and sophisticated borrowing constraints. It is computed as the perfect foresight transition in response to an unexpected shock from the steady state.

Figure 6: Macro Impacts of Constraint Tightness Shock: Sophisticated Borrowing Constraints



Notes: The figure plots aggregate responses to a tightening of borrowing constraints ($\Delta\phi = -1\% \times \phi_{ss}$) with a persistence of 0.8 under sophisticated borrowing constraints. It is computed as the perfect foresight transition in response to an unexpected shock from the steady state.

financial acceleration for the case of hard borrowing constraints. Figure (8) does so for the case of sophisticated borrowing constraints.

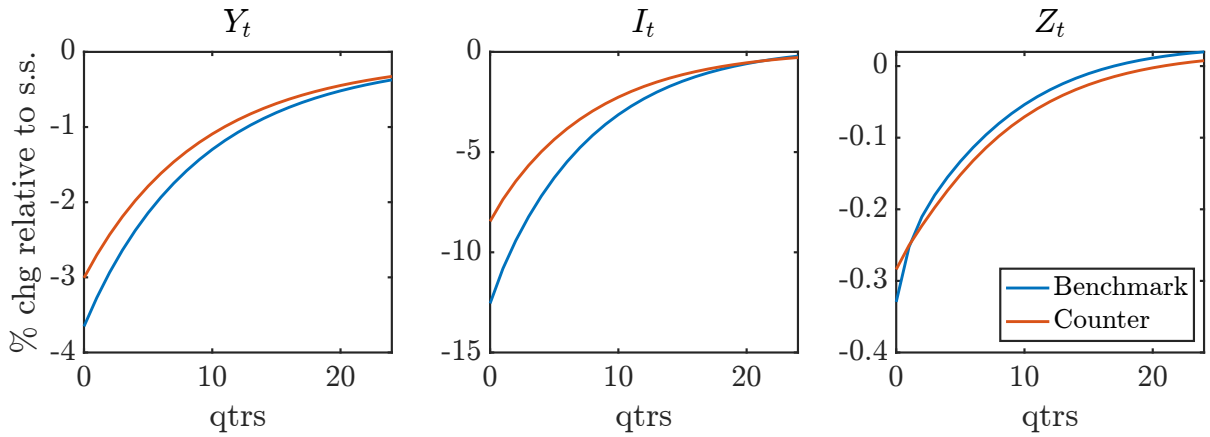
There are several key observations. First, the benchmark aggregate output, investment, and TFP responses are much more negative in the hard borrowing constraint model than in the sophisticated borrowing constraint model. Second, in the hard borrowing constraint case, the counterfactual aggregate output and investment responses are significantly dampened compared to the benchmark responses, indicating that a nontrivial degree of financial acceleration worsens aggregate outcomes. This relates to the significant macro impact of shocks to constraint tightness under hard borrowing constraints: after the tightening of hard borrowing constraints, borrowing and investment must be reduced indiscriminately to satisfy borrowing constraints, which weakens firm performance and lowers aggregate TFP. Third, in the sophisticated borrowing constraint case, the counterfactual aggregate output and investment responses are similar to the benchmark responses, indicating little scope for financial acceleration. This relates to the muted macro impact of shocks to constraint tightness under sophisticated borrowing constraints: after the tightening of sophisticated constraints, more firms transition toward violation states, and creditors obtain control rights. These creditors help alleviate managerial agency frictions and improve firm performance. This channel is reflected in the fact that the benchmark aggregate TFP responses, which incorporate the general equilibrium transfers of control rights to creditors due to constraint tightening, are higher than the counterfactual TFP responses, which shut down those transfers.

Together, we find muted financial acceleration under sophisticated borrowing constraints, in contrast to the significant financial acceleration under hard borrowing constraints. Violation of sophisticated borrowing constraints triggers creditor control, which reduces managerial agency problems and improves firm performance. This provides a form of automatic stabilizer for nonfinancial firms during recessions and is consistent with their resilience during downturns (Gertler and Gilchrist, 2018).

6 Conclusion

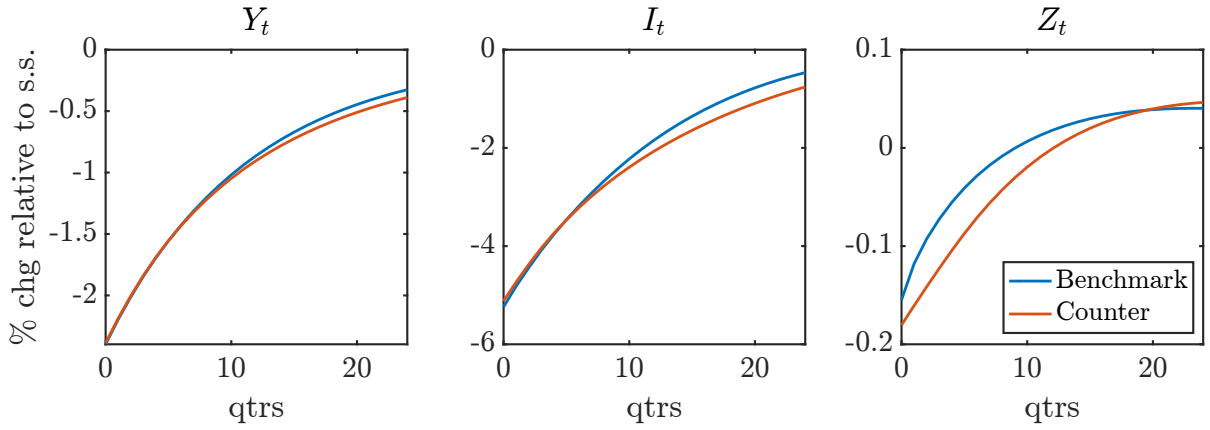
Classic macro-finance models feature "hard" borrowing constraints tied to the liquidation value of physical assets, where creditors cut credit indiscriminately to maintain borrowing limits and financial acceleration ensues. In this paper, we study "sophisticated" borrowing constraints in the debt contracts of U.S. nonfinancial firms. They use covenants to specify borrowing limits linked to operating earnings. Moreover, following covenant violation, creditors obtain control rights and improve firm performance by reducing agency problems, so earnings increase although borrowing and investment decrease. We construct a quantitative model with these key features. At the micro level, our model matches the patterns of firms' borrowing and investment in the data, in normal times and af-

Figure 7: Macro Impacts of A Monetary Policy Shock: Hard Borrowing Constraints



Notes: The figure plots aggregate responses to a 30 bps monetary tightening ($\epsilon_0^m = 0.3\%$) with a persistence of 0.90 under hard borrowing constraints. It is computed as the perfect foresight transition in response to an unexpected shock from the steady state.

Figure 8: Macro Impacts of A Monetary Policy Shock: Sophisticated Borrowing Constraints



Notes: The figure plots aggregate responses to a 30 bps monetary tightening ($\epsilon_0^m = 0.3\%$) with a persistence of 0.90 under sophisticated borrowing constraints. It is computed as the perfect foresight transition in response to an unexpected shock from the steady state.

ter covenant violation. At the macro level, our model produces less financial acceleration, consistent with the resilience of U.S. nonfinancial corporations during recessions.

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Appendices for:
Sophisticated Borrowing Constraints and Macroeconomic Dynamics

This Appendix contains further material for the article "Sophisticated Borrowing Constraints and Macroeconomic Dynamics". Any references to equations, figures, tables, assumptions, propositions, lemmas, or sections that are not preceded by "A."—"C." refer to the main article.

A Supplementary Theoretical Details

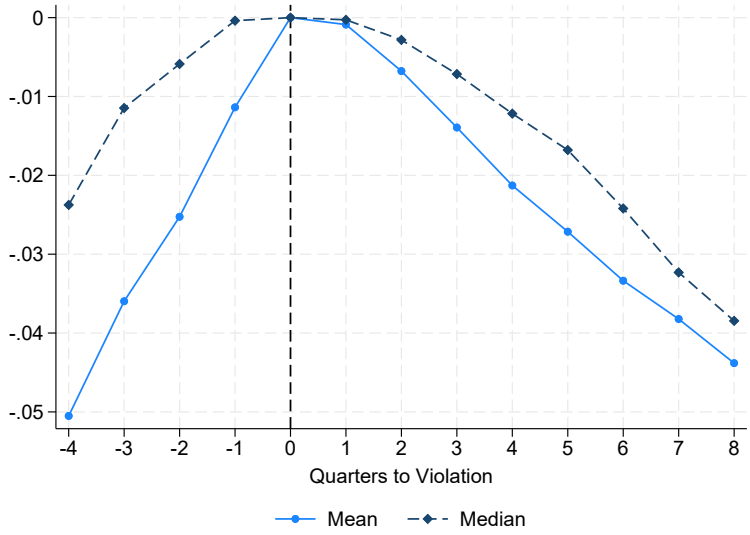
[Under Construction]

B Additional Results for Quantitative Analysis

[Under Construction]

C Additional Empirical Results

Figure C.1: Debt Dynamics around Covenant Violation



Notes: The figure shows firms' debt in quarter $t+h$ relative to quarter t when they initially report a covenant violation (normalized by assets in quarter t so that the differences over time come from the numerator rather than the denominator). The sample includes US Compustat firms that have debt to EBITDA covenant in quarter t (according to DealScan data). Covenant violation data are kindly shared by Greg Nini and cover 1996 to 2016.