The Informational Role of an Elevated Standard of Proof*

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Abstract

This paper shows that an exacting standard of proof in criminal proceedings aligns prosecutors' information-gathering incentives with society's preferences. A conviction-oriented prosecutor must collect a random evidentiary signal before deciding whether to press charges against a suspect. Elevating the standard of proof above society's ex post optimal standard induces the prosecutor to collect a more informative signal, which reduces society's ex ante expected costs of errors. A higher standard of proof not only reduces the incidence of wrongful convictions but may also increase the rate of rightful convictions.

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Contents

1 Introduction **3**

2 Related Literature **7**

2.1 Criminal standard of proof 7

2.2 Optimality of ex post suboptimal mechanisms 8

2.3 Bayesian persuasion 8

3 Model **9**

3.1 Setup 9

3.2 Probativeness 11

3.3 Discussion of key assumptions 14

4 Optimal Standard of Proof **16**

4.1 The prosecutor's problem 16

4.2 Society's problem 18

4.3 Examples 22

5 Symmetric Signal **23**

5.1 Optimal standard of proof 24

5.2 Type II error probability 29

5.3 Example 30

6 Concluding Comments **31**

6.1 Prosecutorial powers and incentives 32

6.2 Plea bargains 32

6.3 Admissibility of prior acts evidence 33

Appendix: Proofs **35**

A: Propositions 1-4 35

B: Examples 1–3 41

References **52**

"Every man looks through the eyes of his prejudices."

—Samuel Silas Curry

1 Introduction

A mainstay principle of law requires a defendant's guilt to be established beyond a reasonable doubt: "The reasonable doubt standard plays a vital role in the American scheme of criminal procedure."[1] Along with a host of pro-defendant evidentiary and procedural safeguards, the stringent standard of proof in criminal proceedings is commonly lauded as an essential protection against wrongful conviction. "Something about the [reasonable doubt] standard," observed Alec Walen, "rings true in most liberal democracies" (Walen 2015, p. 364).

The requirement of an exacting standard of proof for criminal conviction dates back centuries. English jurist William Blackstone wrote in the 18th century, "[As the] law holds, it is better that ten guilty persons escape than that one innocent suffer" (Blackstone 1769, p. 352). Blackstone's celebrated maxim relays a legal and moral duty to weigh wrongful convictions more heavily than wrongful acquittals. An elevated standard of proof accordingly reflects a societal preference to err on the side of the innocent, balancing out concerns for fairness, retribution and deterrence.

The prudence and relevance of demanding a higher standard of proof for finding of guilt, however, have drawn a fair share of criticism. Jeremy Bentham, Blackstone's contemporary and an iconoclast of rules of evidence, warned against "sentimental exaggerations which tend to give crime impunity, under the pretext of insuring the safety of innocence" (Bentham 1825, p. 198). Following in Bentham's footsteps, modern critics argue that the concern for the innocent overprotects criminals to the detriment of potential victims (Laudan 2011). Others maintain that a higher standard of proof was historically associated with harsher penalties involving higher costs of wrongful conviction (e.g. Epps 2015, p. 1083 ["there is good reason to think that the principle was uniquely tied to the death penalty context"]). Federico Picinali recently noted that attacks "on the reasonable doubt standard are becoming more frequent and beguiling" (Picinali 2018).

The central argument of this paper is that a heightened standard of proof in criminal cases ameliorates an agency problem between society and law enforcement agents concerning the search for information. Rather than (necessarily or merely) reflecting a greater intolerance of type I error, a higher standard of proof mitigates disincentives to ferret out

[1] *In re Winship*, (1970) 397 U.S. 358, 363. The Supreme Court in *In re Winship* anchored the requirement of proof beyond a reasonable doubt in the Due Process Clauses of the United States Constitution's Bill of Rights. The term "reasonable doubt" itself, however, does not feature in the Constitution. (The Fifth and Fourteenth Amendments, directed, respectively, at the federal government and the states, require "due process of law" for depriving "any person" of "life, liberty or property" [U.S. Const. amend. V; amend. XIV, § 1].)

informative evidence. Relative to an evidentiary standard that reflects society's ex post preferences, a heightened standard of proof provides conviction-oriented police and prosecutors incentives to collect more accurate evidence and thereby reduces society's ex ante costs of errors. The higher standard of proof in criminal proceedings thus aligns law enforcement agents' ambition to secure convictions with society's interest is truth-finding.[2] In eschewing the debate over the relative costs of false positives versus false negatives, the rationale offered here provides an error costs-neutral justification for an elevated standard of proof for establishing guilt.[3]

To show the information-forcing role of the standard of proof, we consider a five-period model of evidence generation, collection, and realization. In the *first period*, society chooses a standard of proof: a minimum posterior probability of guilt required for conviction. In the *second period*, a random binary evidentiary signal concerning a suspect's guilt—such as forensic, eyewitness, or documentary evidence—is realized according to two commonly-known continuous probability distributions one for each suspect type. A prosecutor in the *third period* must collect a signal bounded by the realized evidentiary signal; that is, the prosecutor may collect a signal that is as or less informative than the realized signal. Based on the standard of proof, the prosecutor must decide whether to drop the case or file charges against the suspect. If the prosecutor decides to bring charges, then in the *fourth period*, a trier of fact observes whether the realized evidence is inculpatory or exculpatory. For example, a witness testimony who survived cross examination amounts to incriminating evidence, whereas self-incriminating police statement refuted by objective evidence amounts to exculpatory evidence. Comparison of physical evidence—such as biometric identifiers—to other evidence in the record can similarly establish guilt or innocence. Finally, in the *last period*, the trier of fact must acquit or convict the defendant based upon the realized evidence and the standard of proof.

We show that under a conviction-oriented prosecutor, the ex ante optimal standard of proof is strictly higher than the ex post optimal standard (Proposition 1). Compared to the most informative feasible signal—the realized signal—a conviction-oriented prosecutor has incentives to collect a signal that is more likely to incriminate an innocent suspect. That is, the prosecutor has (conscience or sub-conscience) motivation to collect a signal that maximizes the probability of conviction subject to meeting the standard of proof. An elevated standard of proof induces the prosecutor to collect a more informative signal in better alignment with society's preferences.

More specifically, by elevating the standard of proof, the prosecution is forced to drop

[2]To be sure, we do *not* suggest prosecutors are always or solely motivated by career concerns. Rather, our analysis proceeds from the assumption that the set of *material* rewards offered to prosecutors inevitably misalignes their self-interest and society's interest in truth-finding. We elaborate on prosecutors' power and incentives in the conclusion section.

[3]Laudan and Saunders (2008), for example, criticize the "persisting [unfounded] belief . . . that any plausible assignment of utilities [to trial outcomes] will inevitably result in a very high standard of proof."

charges against both innocent and guilty suspects for whom the evidentiary signal does not meet the higher standard of proof. Under a standard of proof that exceeds society's ex post optimal standard, the lower costs of wrongful convictions resulting from such dropped charges are outweighed by the corresponding higher costs of wrongful acquittals. At the same time, an elevated standard of proof *decreases* society's ex ante expected costs of errors by prompting prosecutors to search for more informative signals about defendants whose case proceeds to trial. Cases ending up in court, therefore, more often result in rightful acquittal. Overall, a higher standard of proof trades off higher expected costs of type II error (resulting from fewer cases that could proceed to trial) for lower expected costs of type I error (resulting from fewer wrongful convictions of cases that do proceed to trial).

The ex ante optimal standard of proof depends on the distribution of the evidentiary signal (Proposition 2). The more likely the (realized) evidence is to be correctly exculpatory—in the sense of hazard rate dominance—the higher the ex ante optimal standard of proof.[4] The reason is that the ratio between the marginal benefit and cost of a higher standard of proof increases with the ratio between the probability the realized evidence meets the standard of proof and its corresponding density. The probability the realized evidence meets the standard of proof is associated with the marginal benefit of a higher standard (resulting from a higher rate of rightful acquittals), whereas the corresponding density determines the marginal cost of an elevated standard (resulting from a higher rate of wrongful acquittals). By contrast, the more likely the evidence is to be correctly inculpatory (in the sense of first-order stochastic dominance), the lower or higher is the ex ante optimal standard of proof. Although the marginal benefit of raising the standard of proof is higher under the dominant distribution (because the probability the realized evidence meets the standard of proof is higher under such distribution), the corresponding marginal cost is lower (because the probability the realized evidence correctly incriminates the defendant is lower under the dominated distribution).

A higher standard of proof, moreover, may lower the incidence of both wrongful convictions and wrongful acquittals when the error rates of inculpatory and exculpatory evidence are correlated (Proposition 3-4). Under society's ex post optimal standard of proof, a conviction-bent prosecutor may trade off a lower probability of rightful conviction for a higher probability of wrongful conviction by collecting a less informative signal. A higher standard of proof induces the prosecution to collect a more informative signal concerning both innocent and guilty suspects. An elevated standard of proof thus trades off higher expected costs of type II error (resulting from fewer cases of guilty suspects that could proceed to trial) for lower expected costs of both type I and II errors (resulting from fewer wrong-

[4]The hazard rate is equal to the ratio between the probability density and the survival function (one less the cumulative distribution function). Hazard rate dominance is a stronger form of first-order stochastic dominance. For our purposes, if one distribution hazard-rate dominates the other, a signal drawn from the former distribution is more informative (on average) than a signal drawn from the latter.

ful decisions of cases of guilty and innocent suspects that proceed to trial). Elevating the standard of proof may accordingly reduce the probabilities of *both* wrongful acquittals and wrongful convictions.

The main insight of the paper extends to other circumstances in which interested parties wish to introduce a change to the status quo. Consider, for example, a pharmaceutical company that submits a new drug for FDA's approval. Not only does such a company typically have better access to evidence than does a government agency, but it also has some measure of control over the production of evidence. Whereas a desirable change to the status quo (approval of a useful drug) benefits both society and the applying company, society cares more about an undesirable change (approval of a harmful drug). Adopting a higher standard of proof need not necessarily reflect an aversion to change (conservatism), but rather a means to incentivize a more balanced production of evidence.

Although the magnitude of sanctions also affects prosecutors' incentives to search for evidence, a higher standard of proof has a more pronounced effect on prosecutorial behavior. An informational rationale for a heightened standard of proof is therefore consistent with the fact that sentencing standards, unlike the standard of proof, are subject to fairly frequent changes. By the same token, the information-forcing effect of a higher standard of proof helps to explain the uniformity of the reasonable doubt standard—as well as its notorious imprecision—across the criminal spectrum.[5]

Adjusting the standard of proof to the distribution of raw evidence, however, could increase social welfare. More specifically, lowering the standard of proof in pre-defined categories of cases in which the prosecution is typically unable to seek highly-informative evidence could produce a better balance between society's ex ante and ex post costs of errors. A lower standard of proof could similarly increase social welfare in cases in which the costs of acquiring highly-probative evidence is exorbitant.[6]

The paper proceeds as follows. The next section surveys related literature. Section Three sets up the model and explicates its main assumptions. Section Four shows that the optimal standard of proof for conviction-oriented prosecutors is strictly higher than the ex post optimal standard and presents comparative statics. Section Five extends the analysis to symmetric evidentiary signals, showing in particular that an ex ante optimal standard of proof may reduce the probability of both error types. Section Six contains concluding comments. All the proofs are in the Appendix.

[5]Posner (1999) cites findings that judges' probabilistic definitions of the reasonable doubt standard range from 0.75 to 0.90 (p. 1506).

[6]Distinguishing between "excusable" and "strict" preference for evidential completeness, Nance (2016) similarly argues the requirement to maximize the Keynesian weight - or degree of completeness - of the evidence should depend on the costs of obtaining additional evidence and the stakes of the dispute (pp. 132–135).

2 Related Literature

2.1 Criminal standard of proof

A voluminous literature drawing on diverse methodologies explores the justification for a higher standard of proof in criminal cases.[7] Our paper belongs to a strand of literature that examines the incentive effects of the standard of proof on parties to a criminal case.

A handful of papers study the effects of a standard of proof on litigants' incentives to produce costly evidence (Rubinfeld and Sappington(1987) [defendant]; Yilankaya 2002 [prosecution and defendant]; Miceli 1990 [prosecution]).[8] Our paper is closer to Miceli (1990) who proposes, as we do, an agency-cost rationale for a higher standard of proof: "[T]he court responds to [prosecutorial supra-optimal effort] by adjusting its standard of proof upward . . . such that the prosecutor's effort moves closer to its optimal level" (p. 201). This paper focuses on an opposite manifestation of the agency problem between society and prosecutors: the more pernicious, and often subconscious, disincentive to gather informative evidence. This agency problem is exacerbated by, but does not stem from, the presence of costs of collecting evidence.

Other papers study the agency problem between society and prosecutors. Hylton & Khanna (2007) argue that pro-defendant protections, including an elevated standard of proof, "serve primarily as constraints on the costs associated with rent seeking [involving malevolent conduct] in the law enforcement." In a different variation on this theme, Garoupa (2017) following Dharmapala et al. (2016) propose that "[b]ecause enforcers with punitive preferences might be overzealous, an effective way to mitigate such effect is to raise the standard of proof." (For strategic extensions of Garoupa's model, see Wickelgren 2017.) Most recently, Obidzinski & Oytana (2020) show in a cradle-to-grave model that a presumption of innocence (rather than guilt) improves partially self-serving prosecutors' incentives to exert costly investigative efforts, which could in turn improve deterrence.

Similar to Hylton and Khanna, we show the disciplinary effect of a higher standard of proof on prosecutorial behavior. We focus, however, on prosecutors' incentives to gather informative evidence. Unlike Garoupa's model where prosecutors may misrepresent the actual evidence upon incurring a cost of lying, here prosecutors may only present less informative evidence than the (actually or potentially) available evidence. Prosecutors may not, however, outrightly lie. In contrast to Obidzinski and Oytana's paper, we focus on society's ex ante expected costs of errors (which may incorporate a concern for deterrence) rather than society's expected costs of crime.

[7] For a selected bibliography, see Picinali 2018, fns. 2-3. For studies of the historical origin of the reasonable doubt standard, see Waldman 1959; Whitman 2008 (standard intended to abate jurors' reluctance to convict in face of uncertainty).

[8] Stephenson (2008) similarly considers the effects of evidentiary standards on a government agency's information-gathering incentives.

Further related to our paper are Mungan (2011) and Rizzolli and Saraceno (2013), each rationalizing a higher standard of proof by arguing that type I error is costlier than type II error. Both papers assume the two error types undermine deterrence to the same degree, but false convictions alone inspire precautionary activities (Mungan) or give rise to punishment costs (Rizzolli and Margherita). Here, by contrast, we proceed from an agnostic assumption on the relative costs of each error type. Instead, we show a higher standard of proof is warranted even if the two errors are equally costly.

2.2 Optimality of ex post suboptimal mechanisms

More broadly, this paper is part of a literature that studies the use of ex post suboptimal mechanisms to strengthen incentives to acquire information. Li (2001) shows that an elevated standard of proof in such settings as drugs approval and committee hiring reduces ex ante expected costs of errors when a group of decision-makers—such as members of a jury—is required to make a collective decision. An ex post supra- or infra-optimal standard, Li argues, prompts decision-makers to acquire costly information they would otherwise fail to ferret out due to a collective action problem. The information-gathering effect produced by an elevated standard of proof in this paper applies to similar settings but does not hinge on the cost of information acquisition.

Taylor & Yildirim (2011) explore the trade-off between effort and selection effects of blind (uniform) versus informed (variable) acceptance standards. Although an informed standard that considers the agent type is ex post optimal, Tyler and Yildrim identify conditions under which blind review produces higher overall payoff for an evaluator. The argument here revolves around a similar trade-off between ex ante incentives and ex post optimality, but the ex ante effect involves production of information rather than exertion of effort.

Szalay (2005) shows that a principal may benefit from restricting the agent's set of actions by eliminating intermediate actions (as opposed to extreme ones). Although the principal may prefer ex post the intermediate actions, excluding such actions may provide the agent incentives to acquire costly information. The standard of proof here can be similarly considered as a commitment mechanism that eliminates a subset of ex post optimal actions (conviction under the ex post optimal standard). Unlike Szalay's mechanism, the standard of proof does not apply to the agent's choice of action directly, but rather to his ability to secure conviction through the presentation of evidence.

2.3 Bayesian persuasion

The main idea of this paper is further related to the literature on information design. Initiated by Kamenica and Gentzkow (2011), this literature has investigated the optimal in-

formation design by which a sender can influence a receiver to take an action that benefits the sender. The literature on Bayesian persuasion commonly assumes the sender's choice of evidentiary signals is unrestricted: a sender is able to *endogenously* send any signal. In contrast, a primary assumption underlying the model here is that the evidentiary signal is generated *exogenously* according to known probability distributions. The prosecution may therefore only choose from a limited, stochastically-determined set of signals. A possible interpretation of this assumption is that any signal that is more informative than the realized signal is infinitely costly to collect.

The restrictive assumption here on the set of available signals fits circumstances in which the sender has limited control over the production of evidentiary signals. Because law enforcement agents begin their investigation after a crime has been committed, police and prosecutors can only choose from the set of evidentiary signals left on the crime scene. In cases in which the sender is able to independently generate a signal—say a pharmaceutical company applying for an FDA approval—a sender may have greater latitude to design an evidentiary experiment. The standard of proof is a policy instrument that society (as a receiver) may use to affect law enforcement agencies' (as a sender) choice of information experiment. In this respect, this paper explores the design of information design.

3 Model

3.1 Setup

Players.—There are four players: a social designer whose goal is to maximize social welfare; a (static) suspect/dependent who is implicated in a crime; a prosecutor whose task is to collect relevant evidence and decide whether to file charges against the suspect; and a (non-strategic) fact-finder who must decide whether to acquit or convict the defendant based upon the evidence at trial and the standard of proof.

Timing and Actions.—The game unfolds over a maximum of five periods (see Figure 1). In **period 0,** society chooses a *standard of proof* $s \in [\frac{1}{2}, 1)$ reflecting the minimum posterior probability of guilt required for conviction. A fact-finder who finds the posterior probability of guilt to be (weakly) greater than s must convict a defendant and otherwise must acquit him.

In **period 1**, the suspect is arrested for committing a crime. The suspect's private type is $t \in \{I, \neg I\}$, where I represents an innocent suspect and $\neg I$ a guilty suspect. The initial evidence implicating the suspect in the crime entails a prior probability of guilt of $\pi \in (0, s)$.

A pair of independent continuous random variables, $(p, q) \in X \equiv [0, 1]^2$, represent the raw evidence concerning the suspect's guilt, where (p_r, q_r) is the *realized signal* (we shall use the term "signal" and "raw evidence" interchangeably). The realized signal is drawn

from commonly-known independent distribution functions $G(\cdot)$ and $F(\cdot)$, respectively, with corresponding densities $g(\cdot)$ and $f(\cdot)$. As either p_r or q_r increases, the signal becomes more informative, where $(1,1)$ represents a fully-informative signal.[9]

In **period 2**, the prosecutor costlessly collects a signal (p,q), where $p \leq p_r$ and $q \leq q_r$. That is, the prosecutor can recover all or part of the realized signal (p_r, q_r), but cannot be privy to a more informative signal. Let

$$X_f(p_r, q_r) = \{p, q : p \leq p_r, q \leq q_r\} \subseteq X$$

denote the set of *feasible signals*; i.e., signals that the prosecutor may collect and present at trial (see Figure 2; middle panel). Based upon the standard of proof s, the prosecutor must decide whether to bring charges against the suspect.

In **period 3**, if the prosecutor brought charges against the suspect, he presents to a fact-finder his collected signal. The *realized evidence* is a binary random variable $e = \{i, \neg i\}$, where i represents exculpatory evidence and $\neg i$ incriminating evidence. The conditional probabilities the evidence is incriminating given the defendant's type are

$$\begin{pmatrix} \Pr(\neg i; \neg I) & \Pr(\neg i; I) \\ \Pr(i; \neg I) & \Pr(i; I) \end{pmatrix} = \begin{pmatrix} \frac{1}{2}(1+p) & \frac{1}{2}(1-q) \\ \frac{1}{2}(1-p) & \frac{1}{2}(1+q) \end{pmatrix}, \tag{1}$$

where $\frac{1}{2}(1+p) \geq \frac{1}{2} \geq \frac{1}{2}(1-p)$ and $\frac{1}{2}(1+q) \geq \frac{1}{2} \geq \frac{1}{2}(1-q)$. This signal technology reflects the fact that exculpatory (inculpatory) evidence is more likely to arise if the defendant is innocent (guilty).

Finally, in **period 4**, after observing the realized evidence and consulting the standard of proof, the fact-finder must either convict or acquit the defendant.

Payoffs.—Society's costs of wrongful conviction and wrongful acquittal are kD and $k(1-D)$, respectively, where $D \in [\frac{1}{2}, 1)$ is a weight parameter and k a scaling constant we shall normalize to 1. If $D = \frac{1}{2}$, society is equally intolerable of wrongful convictions and wrongful acquittals, whereas if $D \to 1$ society is infinitely intolerable of wrongful convictions. We normalize society's payoff of a rightful decision to zero. An unbiased prosecutor's payoff is identical to society's, whereas a conviction-oriented prosecutor obtains a normalized payoff of 1 if the defendant is convicted and 0 if the defendant is acquitted irrespective of the defendant's type. Throughout the analysis we shall accordingly focus on conviction-oriented prosecutors.

[9] This expansive set of signals simplifies the analysis. Relaxing this assumption—by restricting the set of signals—would imply an even higher optimal standard of proof.

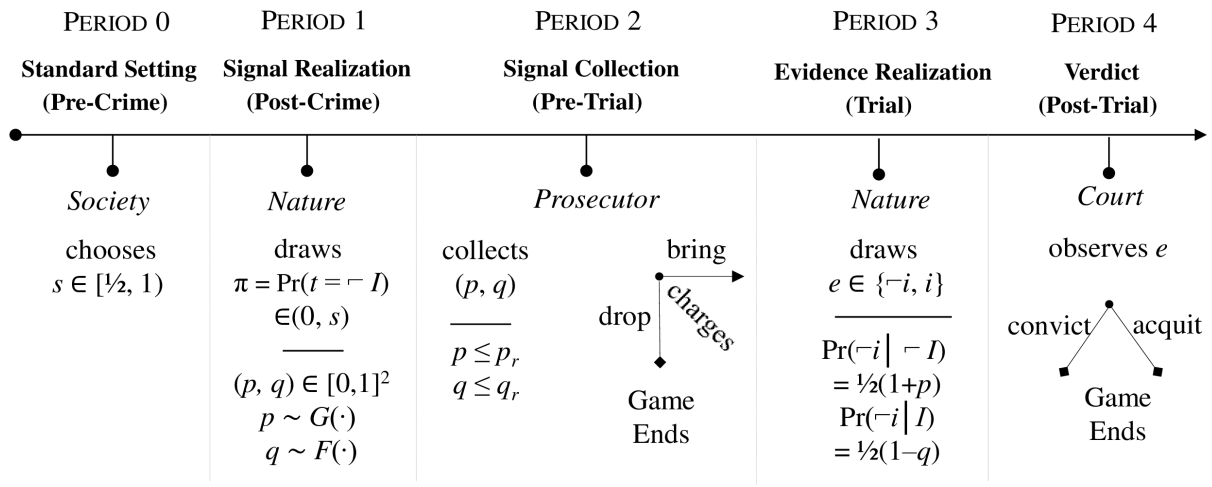


Figure 1: Timeline. Society chooses a standard of proof s. Nature draws a prior probability of guilt, π, and a signal that consists of a pair of independent continuous random variables (p, q). A prosecutor collects a signal (p, q) weakly less informative than the realized signal. Based on the standard of proof s, the prosecutor either drops the case or brings charges. If the prosecutor brings charges, he presents to the court his collected signal. After observing the realization of the evidence, e, the court acquits or convicts.

3.2 Probativeness

We now define the notion of probative signals and its implications. Before doing so, we introduce the *conviction threshold*.

Definition 1 (conviction threshold) *The* conviction threshold *is* $Z(s, \pi) \equiv \frac{\pi}{1-\pi} \div \frac{s}{1-s} \in (0, 1)$.

The conviction threshold reflects the strength of the evidence required for conviction: the *lower* the conviction threshold, the *stronger* the (incriminating) evidence required to convict. If $s \to 1$ or $\pi \to 0$, then $Z(s, \pi) \to 0$ (very hard to convict); if $\pi \to s$, then $Z(s, \pi) \to 1$ (very easy to convict); and if $s \to \frac{1}{2}$, then $Z(s, \pi) \to \frac{\pi}{1-\pi}$ (the prior odds). If $s = D$, the conviction threshold is equal to the ratio of society's expected cost of wrongful acquittal and wrongful conviction: $\frac{\pi(1-D)}{(1-\pi)D}$.

Now, let $LR(p, q) \equiv \frac{1+p}{1-q}$ denote the likelihood ratio of guilt versus innocence given incriminating evidence. The prosecutor meets the standard of proof upon presenting incriminating evidence if $\frac{1}{LR(p,q)}$ is less than or equal to the conviction threshold:

$$\frac{1}{LR(p, q)} \leq Z(s, \pi). \tag{2}$$

This inequality means that the posterior odds ratio is (weakly) greater than the threshold posterior odds required for conviction: $\frac{\pi}{1-\pi}\frac{1+p}{1-q} \geq \frac{1-s}{s}$. By writing the conviction condition as a relationship between the reciprocal likelihood ratio of guilt versus innocence and the

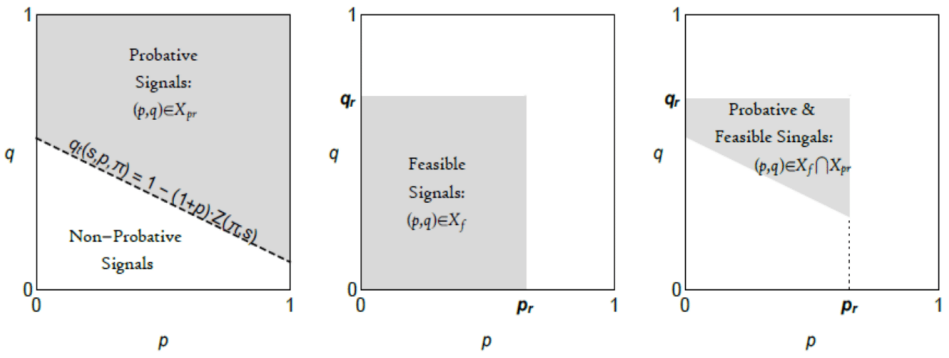


Figure 2: Probative and feasible signals. The left panel depicts the set of probative signals: signals for which a fact-finder convicts upon observing incriminating evidence; the middle panel depicts the set of feasible signals: signals that are available for police and prosecutors; the right panel depicts the set of probative and feasible signals: the intersection of the sets of probative and feasible signals.

conviction threshold, each side of the condition takes value in the unit interval.

Building on Eq. (1), the next definition defines the notion of *probativeness.*

Definition 2 (probative signals) *A signal* (p, q) *is* probative *if and only if the fact-finder convicts upon observing incriminating evidence* $e = \neg i$ *.*

A signal is probative if and only if incriminating evidence distinguishes between a guilty and innocent defendant given the standard of proof; or, equivalently, if $\frac{1}{LR(p,q)} \leq Z(s, \pi)$. Probativeness is a property of the evidentiary signal in relation to the prior probability of guilt and the standard of proof irrespective of society's relative costs of errors. (For explanation of probative value in terms of likelihood ratio, see Nance and Morris 2002; Finkelstein and Levin 2003.)

Building on Definition 2, the next lemma considers the set of probative signals.

Lemma 1A (existence of probative signal) *For* $\pi \in (0, s)$*,* $p \in [0, 1]$ *and* $p_m \equiv \frac{1}{Z(s,\pi)}$*, there exists a* threshold probative signal

$$q_t(s, p, \pi) \equiv \begin{cases} 1 - (1 + p)\, Z(s, \pi) > 0 & \text{if} \quad p < p_m \\ 0 & \text{if} \quad p \geq p_m, \end{cases} \tag{3}$$

such that a signal $(p, q : q \in [q_t(s, p, \pi), 1])$ *is probative for any* $p \in [0, 1]$*, where* $q_t(s, p, \pi)$ *is increasing in* s *and (weakly) decreasing in* p *and* π*.*[10]

Lemma 1A states that for every prior probability of guilt less than the standard of proof,

[10]Lemma 1A can be analogously expressed in terms of a cutoff $p_t(q, \pi, s)$ as follows: For $\pi \in (0, s)$ and $q \in (0, 1]$, there exists a cutoff $p_t(q, \pi, s) \equiv \max\{0, (1 - q)/Z(\pi, s) - 1\} \in [0, 1)$, such that a signal $(p, q : p \in [p_t(q, \pi, s), 1])$ is probative for any $q \in [0, 1]$. As will be evident in the next section, the threshold $q_t(p, \pi, s)$ is more useful for purposes of deriving the paper's main propositions.

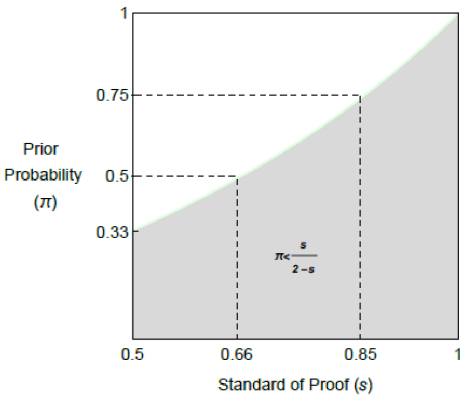


Figure 3: Prior probabilities satisfying Assumption 1.

there exists a non-empty set of probative signals,

$$X_{pr}(s,\pi) \equiv \{(p,q) : p \in [0,1], q \in [q_t(s,p,\pi),1]\}.$$

Furthermore, the threshold probative signal $q_t(p,\pi,s)$ is strictly positive for a sufficiently small p ($p < p_m$) and equals zero for a sufficiently large p ($p \geq p_m$) (see Figure 2; left panel).[11] That is, the higher p is, the lower is $q_t(p,\pi,s)$.

Building on Lemma 1A, the following lemma presents a necessary and sufficient condition for the threshold probative signal $q_t(s,p,\pi)$ to be strictly positive for *every* p (rather than for $p < p_m$).

Lemma 1B (set of probative signals) $q_t(s,p,\pi) > 0$ *for* every $p \in [0,1]$ *if and only if* $\pi \in (0, \frac{s}{2-s})$.

By Lemma 1A, $q_t(s,p,\pi) > 0$ if and only if $p < p_m$. It follows that $q_t(s,p,\pi) > 0$ for *any* $p \in (0,1)$ if and only if $p_m > 1$; i.e., $\frac{1}{Z(s,\pi)} - 1 > 1$ or $Z(s,\pi) < \frac{1}{2}$. Plugging in $\frac{\pi}{1-\pi} \div \frac{s}{1-s}$ into $Z(s,\pi)$ and rearranging gives $\pi \in (0, \frac{s}{2-s})$.[12]

Assumption 1 (range of prior probability) $\pi \in (0, \frac{s}{2-s})$.

Assumption 1 implies that any signal $(p,0)$ is non-probative; i.e., given a standard of proof s, the minimum value of q for which the signal $(1,q)$ is probative is strictly positive. Assumption 1 simplifies the analysis but none of the paper's results hinges upon it. Figure 3 presents the upper bound of the prior probability of guilt as a function of the standard of proof for which Assumption 1 holds for $s \in (\frac{1}{2}, 1)$.

[11] Note that if $q_t(p,s,\pi) = 0$, the conviction condition is satisfied as a strict inequality at $(p,0)$; whereas if $q_t(p,s,\pi) > 0$, the conviction condition is satisfied as an equality at the threshold probative signal, $(p, q_t(p,s,\pi))$.

[12] Note that $Z(\frac{s}{2-s}, s) = \frac{1}{2}$ and that $\frac{\partial}{\partial \pi} Z(\pi, s) < 0$.

3.3 Discussion of key assumptions

3.3.1 Evidentiary signal

Any relevant piece of evidence in a criminal case is endowed with a likelihood ratio: the ratio between the conditional probabilities that such evidence arises given that the suspect is guilty and innocent ($\frac{1+p}{1-q}$ for incriminating evidence and $\frac{1-p}{1+q}$ for exculpatory evidence, where $\frac{1+p}{1-q} > 1 > \frac{1-p}{1+q}$). For example, DNA evidence left on the crime scene that implicates the suspect in the crime is associated with a high likelihood ratio of guilt versus innocence. In contrast, an alibi that establishes that the suspect was away from the crime scene at the relevant time is associated with a low likelihood ratio of guilt versus innocence.

The realized evidentiary signal here is an *aggregate* measure of different pieces of evidence throughout the entire body of potential evidence concerning the suspect's guilt. As a summary of the available evidence, the evidentiary signal depends on such stochastic case-specific factors as the perpetrator's detection-avoidance efforts; the presence or absence of eye witnesses or surveillance cameras; and the quality of the victim's recollection. Two cases involving identical legal and factual claims would therefore likely be endowed with different realized evidentiary signals.

The assumption that the conditional probabilities generating the evidentiary signal are independent is not without loss of generality. For example, a high probability that conclusive DNA evidence is left at the crime scene might entail a low probability of other exculpatory evidence. In other cases, however, different pieces of evidence pulling in different directions may be independently available. For instance, in a homicide case, one piece of evidence concerning the suspect's motive may indicate culpability, whereas another piece of evidence involving the victim's carelessness may point to the possibility of an accident. In Section Five, we consider the case in which the error rates of incriminating and exculpatory evidence are perfectly correlated: the case in which $p = q$.

The informativeness of the evidentiary signal is subject to two interpretations. According to one interpretation, a less informative signal involves evidence of lesser *quality*. For example, eyewitness testimony is generally less accurate than camera surveillance footage, whereas the latter is typically less informative than DNA evidence. An alternative interpretation is that a less informative evidentiary signal involves evidence of lesser *quantity*. For example, one eyewitness testimony is less informative than two eyewitness testimony, whereas the latter is less informative than two eyewitness testimony confirmed by camera surveillance footage. Under both a qualitative and quantitative interpretation, the prosecution has discretionary latitude in collecting the evidentiary signal to be presented at court.

3.3.2 Evidence collection and disclosure

The assumption that the prosecutor may collect any feasible signal at no social or private cost is admittedly unrealistic. This assumption does not affect, however, the paper's essential argument. At the heart of the model is a comparison exercise between the optimal standard of proof in a first- best world where prosecutors are faithful agents of society and in a world in which prosecutors are bent on conviction. Introducing costly evidence would reduce the optimal standard of proof in the former case, but would still imply a relatively higher optimal standard in the latter case (because of conviction-oriented prosecutors' self-serving incentives to obfuscate the evidentiary signal). Relaxing the assumption that evidence can be costlessly obtained, moreover, would bolster the paper's results by providing prosecutors additional disincentive to acquire personally-costly supra-probative signal.[13]

The assumption that the prosecution presents the collected evidentiary signal at trial is subject to two interpretations. One interpretation, in keeping with the Supreme Court's ruling in *Brady v. Maryland*, is that "suppression by the prosecution of evidence favorable to the accused . . . violates due process [under the U.S. Constitution] where the evidence is material either to guilt or to punishment".[14] Under this interpretation, the prosecution abides by its duty to disclose to the defendant all exculpatory information at the prosecution's possession. An alternative interpretation is that the prosecution may violate its *Brady* duty by presenting at trial an evidentiary signal that is less informative than its collected signal. For example, the prosecution may not disclose that a witness interviewed by the police corroborates the defendant's alibi or that it possesses documentary evidence that helps the defense. More generally, the prosecution may present certain pieces of inculpatory information while suppressing other pieces of exculpatory information.

The latter interpretation is consistent with the common perception that the *Brady* duties are difficult to enforce (Sklansky (2018) ["It is widely believed . . . that reported cases drastically understate the frequency of *Brady* violations"]). A main reason for the ineffective enforcement of the *Brady* requirements is that their violations rarely come to light because the "system . . . is not well designed to ensure compliance" (Wines 2018, quoting Jennifer Laurin). Furthermore, even "well-meaning prosecutors may simply misjudge the importance of evidence for a defendant's case, or even be unaware that it is important at all" (*id.*)

3.3.3 Prosecutors' objective

The assumption that prosecutors care solely about securing convictions is admittedly a simplification of the complex set of motivations that underlie law enforcement agents' behavior.

[13] I thank Dale Nunce for prompting me to clarify the innocuousness of the assumption that evidence is costless to obtain.

[14] *Brady v. Maryland*, 373 U.S. 83, 87 (1963). Subsequent case law has expanded the scope of information and circle of agents subject to the disclosure duty under *Brady* (*Giglio v. United States*, 405 U.S. 150 (1972); *Kyles v. Whitley*, 514 U.S. 419, 437 (1995)).

Like other public servants, police and prosecutors are driven by a mixture of public- and self-serving motivations (Glaeser et al. 2000). As Miceli (1990, p. 197) points out, however, both psychological and institutional reasons favor the assumption that prosecutors are inclined to maximize the probability of conviction. The adversarial nature of criminal trials, in the first place, pits prosecutors against defendants in an "all-or-nothing" context. Prosecutors' sense of accomplishment and self-esteem is therefore tied to their ability to secure convictions. The institutional context of criminal trials further provides prosecutors' material incentives to secure convictions because their salary and promotion prospects both depend on their conviction record.

Further justification for assuming that prosecutors seek to maximize conviction rates is the presence of cognitive biases that cloud law enforcement agents' judgment. Like most individuals, law enforcement agents suffer from tunnel vision: an unconscious tendency to dismiss evidence that run counter to one's preconceived beliefs, while seeking evidence that comport with them (Findley, 2012). "[T]unnel vision [and associated biases]," explain Findley & Scott (2006, p. 292), "is more often the product of the human condition as well as institutional and cultural pressures, than of maliciousness or indifference." These biases not only distort police' course of investigation "where it can be most damaging," but also drives prosecutors "to act in ways that subvert justice, whether intentionally or, as is more often the case, unintentionally" (*id.*, p. 295). Assuming that prosecutors' primary motivation is securing convictions reflects that cognitive biases inevitably affect law-enforcement agents' decision-making process.

4 Optimal Standard of Proof

4.1 The prosecutor's problem

Consider the evidence-gathering policy of a prosecutor driven by career concerns. Such a prosecutor seeks to minimize the probability of acquittal of *both* innocent and guilty defendants:

$$\begin{aligned} &\min_{(p,q)} \tfrac{1}{2}(1-\pi)(1+q) + \tfrac{1}{2}\pi(1-p) \\ &\text{subject to} \\ &(p,q) \in X_f(p_r, q_r) \cap X_{pr}(s,\pi), \end{aligned} \tag{4}$$

where the constraint ensures the signal is both feasible and probative.

The first term in the prosecutor's objective function is the joint probability the defendant is innocent and the realized evidence is *correctly* exculpatory; the second term is the joint probability the defendant is guilty and the realized evidence is *incorrectly* exculpatory.

Since the first expression is *increasing* in q and the second is *decreasing* in p irrespective of π, a conviction-oriented prosecutor has incentives to collect the threshold probative signal $(p_r, q_t(s, p_r, \pi))$.

Now, under a conviction-oriented prosecutor, the ex ante probability of *type I error* is

$$\alpha(s, \pi) = \tfrac{1}{2} \iint_{(p,q) \in X_{pr}(s,\pi)} (1 - q_t(s, p, \pi)) dF(q) dG(p), \tag{5}$$

where we integrate with respect to each distribution function. A conviction-oriented prosecutor collects the threshold probative signal, $(p_r, q_t(s, p, \pi))$, which maximally obfuscates an innocent defendant's innocence. The probability of type I error for a specific pair (p, q) is therefore equal to the joint probability the signal is probative and the realized evidence is incorrectly inculpatory. Integrating over the domain of feasible signals gives the ex ante probability of type I error. Observe that under an unbiased prosecutor, the ex ante probability of type I error is identical to Eq. (5) with q replacing $q_t(s, p, \pi)$.

The ex ante probability of *type II error* is

$$\beta(s, \pi) = 1 - \tfrac{1}{2} \iint_{(p,q) \in X_{pr}(s,\pi)} (1 + p) dF(q) dG(p), \tag{6}$$

where the subtrahend is the probability of *rightful conviction*: the joint probability that the realized signal is probative and the realized evidence is *correctly* inculpatory. That is, a guilty suspect or defendant is wrongfully acquitted for a fixed signal (p, q) if the realized signal is non-probative *or* if the realized evidence is incorrectly exculpatory. Integrating over the domain of feasible signals and subtracting from one gives the ex ante probability of type II error.

Lemma 2 (value of information | conviction-oriented prosecutor)

Under a conviction-oriented prosecutor and the ex post optimal standard of proof, society's payoff is equal to its uninformed payoff of $\pi(1 - D)$.

Society's expected costs of errors under a conviction-oriented prosecutor given the ex post optimal standard of D is

$$EC(D, \pi) = (1 - \pi) \cdot \alpha(D, \pi) \cdot D + \pi \cdot \beta(D, \pi) \cdot (1 - D).$$

The first expression is society's expected costs of wrongful convictions: the prior probability the defendant is innocent times the probability of type I error times the cost of type I error. The second expression is society's expected costs of wrongful acquittals: the prior probability the suspect or defendant is guilty times the probability of type II error times the cost of type II error.

Now, substituting $1-(1+p)Z(D,\pi)$ for $q_t(D,p,\pi)$ from Lemma 1A and plugging in $\frac{\pi}{1-\pi} \div \frac{D}{1-D}$ into $Z(D,\pi)$ from Definition 1, the integrand of $\alpha(D)$ becomes $\frac{1}{2}(1+p)\cdot\frac{\pi}{1-\pi}\cdot\frac{1-D}{D}$. Therefore:

$$\begin{aligned} EC(D,\pi) &= \pi(1-D)\tfrac{1}{2}\iint_{(p,q)\in X_{pr}(D,\pi)}(1+p)dF(q)dG(p) \\ &\quad +\pi(1-D)\left(1-\tfrac{1}{2}\iint_{(p,q)\in X_{pr}(D,\pi)}(1+p)dF(q)dG(p)\right) \\ &= \pi(1-D), \end{aligned} \tag{7}$$

where $\pi(1-D)$ is the probability the suspect is guilty times society's cost of wrongful conviction. Note that society's net expected benefit from trials is nil: the expected costs of wrongful convictions exactly offset the expected benefits of rightful convictions.

A conviction-oriented prosecutor's information-gathering policy produces no benefit to society relative to an uninformed decision. Given that the prosecutor collects the threshold probative signal associated with the ex post optimal standard of proof, society's payoff is identical to its payoff of always acquitting the defendant given a probative signal. This is because, given the prosecutor's policy, society is indifferent between acquitting and convicting the defendant if the realized evidence is inculpatory and strictly prefers to acquit the defendant if the realized evidence is exculpatory. Since the suspect is (effectively) acquitted in the absence of information, society's payoff under a conviction-oriented prosecutor is identical to its uninformed payoff.

4.2 Society's problem

We proceed by showing that a commitment to a higher standard of proof above society's ex post optimal standard decreases society's ex ante expected costs of error. Using (5) and (6), we can write society's ex ante expected costs of errors as

$$EC(s,\pi,D) = (1-\pi)\cdot\alpha(s)\cdot D + \pi\cdot\beta(s)\cdot(1-D). \tag{8}$$

The first expression is the expected costs of type I error and the second term the expected costs of type II error.

Partially differentiating the right-hand side of (8) with respect to s, equating to zero, and rearranging, the optimal standard of proof satisfies

$$-\frac{\frac{\partial}{\partial s}\alpha(s,\pi)}{\frac{\partial}{\partial s}\beta(s,\pi)} = Z(D,\pi). \tag{9}$$

The left-hand side is the ratio of the marginal decrease and increase, respectively, in the

probabilities of type I and type II errors resulting from a higher standard of proof; the right-hand side is the conviction threshold at $s = D$; i.e., the ratio of the expected costs of type I and type II errors. Under the optimal standard of proof, the marginal decrease in the expected costs of wrongful convictions resulting from a higher standard of proof is equal to the corresponding marginal increase in the expected costs of wrongful acquittals.

Now, a marginal increase in the standard of proof *decreases* the ex ante probability of type II error by

$$\begin{aligned}\frac{\partial}{\partial s}\alpha(s,\pi) &= -\tfrac{1}{2}\int\limits_{p\in[0,1]}(1-q_t(s,p,\pi))F'(q_t(s,p,\pi))dG(p)\\ &\quad -\tfrac{1}{2}\int\limits_{p\in[0,1]}(1-F(q_t(s,p,\pi))q_t'(s,p,\pi)dG(p). \qquad (10)\end{aligned}$$

An increase in the standard of proof has a twofold *negative* effect on the probability of type I error. First, a higher standard increases the probability the realized signal is non-probative, in which case an innocent suspect is never wrongly convicted. More specifically, for a fixed pair (p, q), a higher standard of proof decreases the probability of wrongful conviction by the marginal change in the cumulative distribution of q at the threshold probative signal times the conditional probability the evidence is incorrectly incriminating at the threshold probative signal $(f(q_t(s))\cdot\frac{1}{2}(1-q_t(s)))$. Second, increasing the standard of proof carries the additional benefit of inducing the prosecutor to collect a more informative signal. The marginal (negative) effect of a higher standard of proof on the ex ante probability of type I error for a particular value of p is equal to the probability that the signal is probative times the marginal increase in the threshold signal resulting from a higher standard $((1-F(q_t(s))\cdot q_t'(s))$.

Substituting $(1+p)Z(s,\pi)$ for $1-q_t(s)$ (from Lemma 1A), we get:

$$\begin{aligned}\frac{\partial}{\partial s}\alpha(s,\pi) &= -\tfrac{1}{2}Z(s,\pi)\int\limits_{p\in[0,1]}(1+p)F'(q_t(s,p,\pi))dG(p)\\ &\quad -\tfrac{1}{2}\int\limits_{p\in[0,1]}(1-F(q_t(s))q_t'(s,p,\pi)dG(p). \qquad (11)\end{aligned}$$

Taking the derivative of $\beta(s)$ with respect to s, a marginal increase in the standard of proof *increases* the ex ante probability of type II error by

$$\frac{\partial}{\partial s}\beta(s,\pi) = \tfrac{1}{2}\int\limits_{p\in[0,1]}(1+p)F'(q_t(s,p,\pi))dG(p). \qquad (12)$$

An increase in the standard of proof increases the incidence of type II error by increasing

the probability that the signal is non-probative and thereby requiring the prosecutor to drop the charges against a guilty suspect. More specifically, holding fixed the value of p, a higher standard of proof increases the probability of wrongful acquittal by the marginal change in the cumulative distribution of q at the threshold probative signal ($F'(q_t(s)) = f(q_t(s))\cdot q_t'(s)$) times the conditional probability the evidence is correctly incriminating ($\frac{1}{2}(1+p)$).

With a slight abuse of notation, let $\alpha_1'(s,\pi)$ and $\alpha_2'(s,\pi)$ denote the first and second integrals in (11). Substituting $Z(s,\pi)$ for $-\frac{\alpha_1'(s)}{\beta'(s)}$ and rearranging, the optimal standard of proof satisfies

$$-\frac{\alpha_2'(s,\pi)}{\beta'(s,\pi)} = Z(D,\pi) - Z(s,\pi). \tag{13}$$

Observe that under an unbiased prosecutor, the left-hand side is nil because for such a prosecutor $\alpha_2'(s,\pi) = 0$. In this case, therefore, the ex ante optimal standard is identical to the ex post optimal standard.

The next proposition summarizes the foregoing analysis.

Proposition 1 (optimal standard of proof | general) *The standard of proof that minimizes society's ex ante expected costs of error under a conviction-oriented prosecutor is strictly higher than the ex post optimal standard:* $s^{ea} \in (D,1) > s^{ep} = D$.

Proof. See Appendix A.1.

Increasing the standard of proof under a conviction-oriented prosecutor has two opposite effects on society's expected costs of errors. A standard of proof exceeding the ex post optimal standard results in misdistribution of errors relative to society's preferences. But increasing the standard of proof also reduces the set of probative signals and thereby induces a conviction-oriented prosecutor to collect a more informative threshold probative signal. The latter information-forcing effect decreases society's expected costs of errors and compensates for the higher costs of errors resulting from the application of an ex post suboptimal standard. A marginal increase in the standard of proof therefore reduces society's expected costs of wrongful convictions by more than the corresponding increase in the expected costs of wrongful acquittals.

Under an unbiased prosecutor, by contrast, the ex ante optimal standard of proof is equal to the ex post optimal standard. Because an unbiased prosecutor collects and presents the realized signal, the standard of proof that reflects society's relative costs of errors equates the marginal increase in society's ex ante costs of errors of a higher standard of proof (more wrongful acquittals) and the corresponding marginal decrease in these costs (fewer wrongful convictions). Since the ratio between the expected marginal error rates is equal to the conviction threshold, the optimal standard of proof trades off society's costs of wrongful convictions against its costs of wrongful acquittals.

How does the optimal standard of proof vary with the distribution of the evidentiary signal? Inspection of Eqs. (11) and (12) shows the ratio $\alpha_2'(s) \div \beta'(s)$ depends on the inverse hazard rate of $F(\cdot)$ at $q = q_t$, where the hazard rate is given by $h_F(q) \equiv \frac{f(q)}{1-F(q)}$, as well as on the cumulative distribution of p, $G(\cdot)$.

Proposition 2 (optimal standard of proof | comparative statics)

(1) If $F_1(\cdot)$ hazard rate dominates $F_2(\cdot)$, the ex ante optimal standard of proof under $F_1(\cdot)$ is higher relative to $F_2(\cdot)$.

(2) If $G_1(\cdot)$ first-order stochastically dominates $G_2(\cdot)$, the ex ante optimal standard of proof under $G_1(\cdot)$ is higher than, lower than or equal to the ex ante optimal standard under $G_2(\cdot)$.

Proof. See Appendix A.2.

If $F_1(\cdot)$ hazard-rate dominates $F_2(\cdot)$, then $\frac{f_1(q)}{1-F_1(q)} < \frac{f_2(q)}{1-F_2(q)}$ for $q \in [0,1]$. Hazard rate dominance is a *stronger* form of first-order stochastic dominance: the former implies the latter but not vice versa.[15] Similar to first-order stochastic dominance, hazard-rate dominance reflects that the dominant distribution has a larger mass of high values relative to the dominated distribution. A distribution with a lower hazard rate is therefore more informative.[16]

Now, $f(q_t) \cdot q_t'(s)$ captures the marginal decrease in the probability the signal is probative at $q = q_t$ resulting from a higher threshold probative signal, whereas $1 - F(q_t)$ is the probability the signal is probative at $q = q_t$. The product $(1+p) \cdot f(q_t) \cdot q_t'(s)$ represents the (cost-increasing) positive marginal effect of a higher standard of proof on the probability of wrongful acquittals (stemming from a higher probability of a non-probative signal given a correct incriminatory signal). By contrast, the product $(1 - F(q_t)) \cdot q_t'(s)$ represents the corresponding (cost-decreasing) negative marginal effect of a higher standard of proof on the probability of wrongful convictions (because *given* that the signal is probative, it is more likely to be correct). For a particular realization of p, the hazard rate $h_F(q_t)$ is thus proportional to the ratio between the marginal cost and benefit of a higher standard of proof at the threshold probative signal. Therefore, if $F_1(\cdot)$ hazard-rate dominates $F_2(\cdot)$, the marginal benefit of raising the standard of proof is higher and the marginal cost lower for $F_1(\cdot)$ relative to $F_2(\cdot)$.

If $G_1(\cdot)$ first-order stochastically dominates $G_2(\cdot)$, then $G_1(p) < G_2(p)$ for $p \in [0,1]$. The optimal standard of proof is either higher or lower under $G_1(\cdot)$ relative to $G_2(\cdot)$ (unless the inverse hazard rate $h_F(q)^{-1} = \frac{1-F(q)}{f(q)}$ at $q = q_t(s,p,\pi)$ is a product of $1+p$, as discussed below). To see why, note the threshold probative signal $q_t(s,p)$ is a decreasing function of

[15] Because $F_1(q) = 1 - \exp \int_0^q h_1(y)dy < 1 - \exp \int_0^q h_2(y)dy = F_2(q)$, where $h_i(y)$ is the hazard rate of $F_i(\cdot)$ for $i = 1, 2$.

[16] Intuitively, given that the cumulative distribution function is smaller than a certain threshold, a marginal increase in the argument of the distribution is smaller under the hazard-rate dominant distribution.

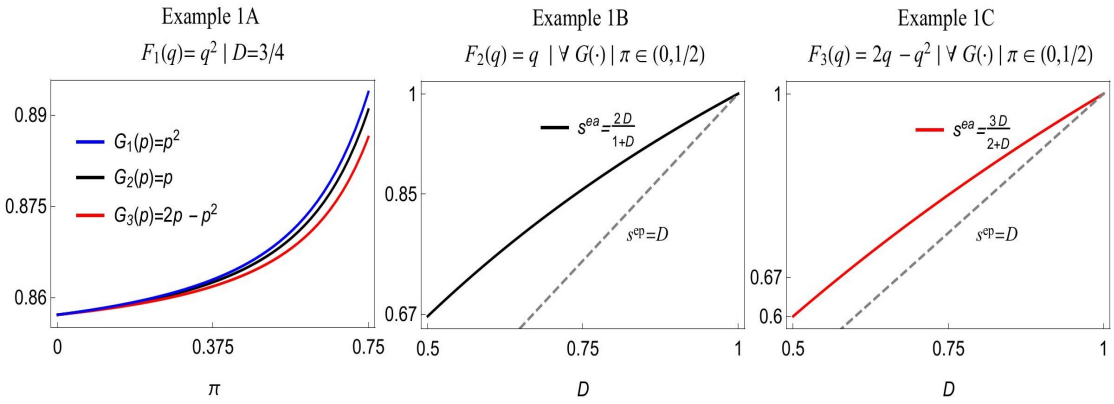


Figure 4: Optimal standard of proof. The *left panel* depicts the optimal standard of proof as a function of $\pi \in (0, \frac{3}{4})$ for $F_1(q) = q^2$ and $D = \frac{3}{4}$ for three probability distributions of $G(\cdot)$: convex (G_1); uniform (G_2); and concave (G_3), where $G_1(\cdot) \overset{FOSD}{\succcurlyeq} G_1(\cdot) \overset{FOSD}{\succcurlyeq} G_3(\cdot)$. The *middle and left panels* respectively depict the optimal standard of proof as a function of $D \in [\frac{1}{2}, 1)$ any any $G(\cdot)$ and π for $F_2(q) = q$ (uniform) and $F_3(q) = 2q - q^2$.

p. As a higher probability mass is shifted to higher values of p, the probability the signal is non-probative, $1 - F(q_t(s, p))$, decreases. The marginal benefit of a higher standard of proof, $[1 - F(q_t(s, p))]q_t'(s)$, is therefore higher under $G_1(\cdot)$ than under $G_2(\cdot)$. The marginal cost of a higher standard of proof, however, is higher under $G_1(\cdot)$ than under $G_2(\cdot)$. This is because a higher standard of proof changes the marginal cost of type II error by $(1+p)\cdot f(q_t(p))\cdot q_t'(s)$. As a higher probability mass is shifted to higher values of p, the expected probability that the incriminate evidence is correct ($\frac{1}{2}\int(1+p)dG(p)$) is higher under $G_2(\cdot)$ relative to $G_1(\cdot)$. The expected costs of wrongful acquittals is therefore lower $G_1(\cdot)$ than under $G_2(\cdot)$.

If $h_F(q_t(s, p, \pi))^{-1}$ is a product of $1 + p$, the optimal standard of proof under $G_1(p)$ is equal to that under $G_2(p)$. In this case, the ratio between the error rates depends solely on a multiple of the conviction threshold. The marginal increase in the probability the evidentiary signal is non-probative resulting from a higher standard of proof is a fixed multiple of the corresponding marginal decrease in the probability the realized evidence is incorrectly inculpatory. The marginal net benefit of a higher standard of proof is consequently independent of both π and $G(\cdot)$, as illustrated in the examples below.

4.3 Examples

The following examples illustrate Propositions 1 and 2 by considering the following closed-form distributions of the evidentiary signal:

$$F_1(q) = q^2,\ F_2(q) = q,\ \text{and}\ F_3(q) = 2q - q^2,$$

where $F_1(\cdot) \overset{HRD}{\succ} F_2(\cdot) \overset{HRD}{\succ} F_3(\cdot)$ (HRD stands for Hazard Rate Dominance); and

$$G_1(p) = p^2, G_2(p) = p, \text{ and } G_3(p) = 2p - 2p^2,$$

where $G_1(\cdot) \overset{FOSD}{\succ} G_2(\cdot) \overset{FOSD}{\succ} G_3(\cdot)$. The ex ante optimal standard of proof for $F_1(q) = q^2$ depends both on the prior probability of guilt, π, and on $G(\cdot)$. By contrast, the ex ante optimal standard of proof for $F_2(q) = q$ and $F_3(q) = 2q - q^2$ depends only on society's relative costs of error but not on π or $G(\cdot)$.

The left panel of Figure 4 presents the ex ante optimal standard of proof as a function the prior probability of guilt for $F_2(q) = q^2$ and $D = \frac{3}{4}$ for $G_1(p) = p^2$, $G_2(p) = p$, and $G_3(p) = 2p - 2p^2$ (Example 1A). The ex ante optimal standard of proof is strictly higher than the ex post optimal standard (D), increases with the prior probability of guilt and decreases in i for $G_i(p)$, where $i = 1, 2, 3$.

The middle panel of Figure 4 depicts the optimal standard of proof for a uniform distribution $F_2(q) = q$ as a function of D (the optimal standard is independent of both π and $G(\cdot)$). The optimal standard increases at a decreasing rate as the relative cost of type I error increases, ranging from $\frac{2}{3}$ to 1 (for $D = \frac{1}{2}$ and $D \to 1$, respectively).

Finally, the right panel of Figure 4 depicts the ex ante optimal standard of proof for the concave distribution $F_3(q) = 2q - q^2$. The optimal standard lies below the optimal standard for $F_2(q) = q$ and similarly increases at a decreasing rate with D, ranging from $\frac{3}{5}$ (for $D = \frac{1}{2}$) to 1 (for $D \to 1$). The proofs of all three examples are in Appendix B.1.

5 Symmetric Signal

We have assumed the set of feasible signals includes any signal that is less informative than the realized signal (see Figure 3). In this section, we replace the assumption that the prosecutor may collect any signal that is less informative than the realized signal by assuming instead that $p = \sigma = q$:

$$\begin{pmatrix} \Pr(\neg i; \neg I) & \Pr(\neg i; I) \\ \Pr(i; \neg I) & \Pr(i; I) \end{pmatrix} = \begin{pmatrix} \frac{1}{2}(1+\sigma) & \frac{1}{2}(1-\sigma) \\ \frac{1}{2}(1-\sigma) & \frac{1}{2}(1-\sigma) \end{pmatrix}. \tag{14}$$

That is, the conditional probabilities that the signal is correct are equal for a guilty and an innocent defendant. The precision of the evidentiary signal thus does not depend on the defendant's type. We assume $\sigma \in [0, 1]$ is drawn from a distribution F with a corresponding density f.

Our purpose in this section is twofold. First, given this more restricted distribution

of the evidentiary signal, we can express the optimal standard of proof under a conviction-oriented prosecutor as a function of the ex post optimal standard, thereby obtaining a crisper result than in the benchmark case (Proposition 3). Second, we show that given a symmetric signal, an elevated standard of proof may reduce the probability of *both* wrongful convictions and wrongful acquittals (Proposition 4). This result stands in contrast to the baseline model in which an elevated standard of proof reduces the probability of wrongful conviction but inevitably increases the probability of wrongful acquittals.

Throughout this section, we make the following assumption.

Assumption 2 (range of prior probability) $\pi \in (0, \frac{1}{2})$.

According to Assumption 2, the prior probability of guilt is strictly less than the prior probability of innocence. As will be clear soon, this assumption is essential for a conviction-oriented prosecutor's incentives to diverge from society's interest in truth finding.

5.1 Optimal standard of proof

By Definition 2, a signal $\sigma \in (0, 1]$ is *probative* if $\frac{1}{LR(\sigma)} \geq Z(s, \pi)$, where $LR(\sigma) = \frac{1+\sigma}{1-\sigma}$ is the likelihood ratio of incriminating evidence and $Z(s, \pi) = \frac{\pi}{1-\pi} \div \frac{s}{1-s}$ the conviction threshold. The next lemma defines (i)) the threshold probative signal as a function of the standard of proof and the prior probability of guilt, and (ii) the standard of proof as a function of the threshold probative signal and the prior probability of guilt.

Lemma 3 (threshold probative signal and SoP | symmetric signal)

(1) The threshold probative signal under a standard of proof s and a prior probability of guilt π is $\sigma_t(s, \pi) = \frac{1-Z(s,\pi)}{1+Z(s,\pi)}$, where $\sigma_t(s, \pi)$ is increasing in s and decreasing in π; and

(2) The standard of proof that implements a threshold probative signal σ_t given a prior probability of guilt π is $s(\sigma_t, \pi) = \frac{\pi(1+\sigma_t)}{1-\sigma_t(1-2\pi)}$, where $s(\sigma_t, \pi)$ is increasing in σ_t and decreasing in π.

The threshold probative signal is the value of σ that satisfies the equality $\frac{1}{LR(\sigma)} = Z(s, \pi)$ (see Eq. (2)). The corresponding standard of proof is the inverse function of the threshold probative signal. Note that the ex post optimal threshold probative signal reflects society's relative costs of wrongful convictions and wrongful acquittals: $\sigma_t^{ep}(\pi, D) = \frac{1-Z(D,\pi)}{1+Z(D,\pi)}$. Accordingly, the corresponding standard of proof is $s^{ep} = D$.

Consider society's optimal choice of a threshold probative signal under a conviction-oriented prosecutor. Such a prosecutor seeks to minimize the probability of acquittal as a function of σ:

$$\min_{\sigma} \tfrac{1}{2}(1-\pi)(1-\sigma) + \tfrac{1}{2}\pi(1+\sigma)$$
$$\text{subject to } \sigma \geq \sigma_t \tag{15}$$

where the first term in the prosecutor's objective function is the probability of wrongful acquittal and the second the probability of rightful acquittal. Taking the derivative with respect to σ gives $\frac{1}{2}(2\pi - 1)$, which is strictly negative for $\pi \in (0, \frac{1}{2})$. A conviction-oriented prosecutor therefore collects the threshold probative signal, σ_t, for $\pi \in (0, \frac{1}{2})$. Recall that in the baseline model the prosecutor collects the threshold probative signal to minimize the probability of wrongful conviction. In the case of a symmetric signal, by contrast, a conviction-oriented prosecutor who collects the least informative probative signal, σ_t, trades off a lower probability of rightful convictions for a higher probability of wrongful convictions.

Exploiting the symmetry between society's costs of wrongful acquittal and wrongful conviction and the likelihoods of incorrect evidence, the probability of type I error under a conviction-oriented prosecutor is

$$\alpha(\sigma_t) = \tfrac{1}{2}(1 - F(\sigma_t))\cdot(1 - \sigma_t), \tag{16}$$

which is the joint probability that the realized signal is probative $(1 - F(\sigma_t))$ and that the evidence at trial is incorrectly inculpatory $(\frac{1}{2}(1 - \sigma_t))$.

The probability of type II error is

$$\beta(\sigma_t) = F(\sigma_t) + \alpha(\sigma_t), \tag{17}$$

which is the probability that the signal is not probative $(F(\sigma_t))$–in which case a guilty suspect is effectively acquitted–plus the probability that the signal is probative and the evidence at trial is incorrectly exculpatory, which is equal to the probability of type I error $(\alpha(\sigma_t))$.

Taking the derivative of $\alpha(\sigma_t)$ with respect to σ_t gives

$$\alpha'(\sigma_t) = -\tfrac{1}{2}[f(\sigma_t)(1 - \sigma_t) + (1 - F(\sigma_t)]. \tag{18}$$

The first term is the marginal benefit of a higher threshold probative signal resulting from fewer cases involving innocent suspects that proceed to trial. The second term is the corresponding marginal benefit resulting from more correct decisions for innocent defendants whose case proceeds to trial.

Taking the derivative of $\beta(\sigma_t)$ with respect to σ_t similarly gives

$$\beta'(\sigma_t) = \tfrac{1}{2}[f(\sigma_t)(1 + \sigma_t) - (1 - F(\sigma_t)], \tag{19}$$

where the first term is the marginal cost of a higher threshold probative signal resulting from fewer cases involving guilty suspects that proceed to trial and the second term is the corresponding marginal benefit resulting from more correct decisions of guilty defendants whose case proceeds to trial.

The ex ante optimal threshold probative signal therefore satisfies

$$-\frac{\alpha'(\sigma_t)}{\beta'(\sigma_t)} = \frac{f(\sigma_t)(1-\sigma_t)+(1-F(\sigma_t))}{f(\sigma_t)(1+\sigma_t)-(1-F(\sigma_t))} = Z(D,\pi). \tag{20}$$

The next proposition expresses the optimal threshold signal under a conviction-oriented prosecutor as a function of the ex post optimal threshold signal.

Proposition 3 (optimal threshold probative signal | symmetric signal)

(1) The ex ante optimal threshold probative signal *under a conviction-oriented prosecutor satisfies the equality*

$$\sigma_t^{ea} - \sigma_t^{ep} = \frac{1-F(\sigma_t^{ea})}{f(\sigma_t^{ea})}, \tag{21}$$

where $\sigma_t^{ep} = \frac{1-Z(D,\pi)}{1+Z(D,\pi)}$ *is the ex post optimal threshold probative signal.*

(2) If $F_1(\cdot)$ *hazard rate dominates* $F_2(\cdot)$*, then the ex ante optimal* standard of proof *is higher under* $F_1(\cdot)$ *relative to* $F_2(\cdot)$*.*

Proof. See Appendix A.3.

The difference between the ex ante and ex post optimal threshold probative signal is increasing in the inverse hazard rate $h_F(\sigma_t)^{-1} = \frac{1-F(\sigma_t)}{f(\sigma_t)}$. As in the benchmark model, the marginal benefit of a higher threshold probative signal is increasing in the probability the signal is probative $(1-F(\sigma_t))$, whereas the corresponding marginal cost is increasing in the marginal probability the signal is non-probative $(f(\sigma_t))$. As the inverse hazard rate increases, the expected reduction in error costs of a higher threshold probative signal (higher probability of non-probative signal) is relatively high compared to the corresponding expected cost (higher marginal probability of non-probative signal). The more informative the signal is (in the sense of hazard rate dominance), the higher the ex ante optimal threshold signal and the corresponding standard of proof.

The next example illustrates Proposition 3 in the case in which the signal distribution is uniform.

Example 2 (optimal standard of proof | symmetric signal | uniform distribution) *If* $\sigma \sim U[0,1]$*, the ex ante optimal threshold signal under a conviction-oriented prosecutor is* $\sigma_t^{ea}(D,\pi) = \frac{1}{1+Z(D,\pi)}$ *with a corresponding standard of proof of* $s^{ea}(D,\pi) = \frac{\pi(2+Z(D,\pi))}{2\pi+Z(D,\pi)} > D = s^{ep}(D)$*.*

Proof. See Appendix B.2.

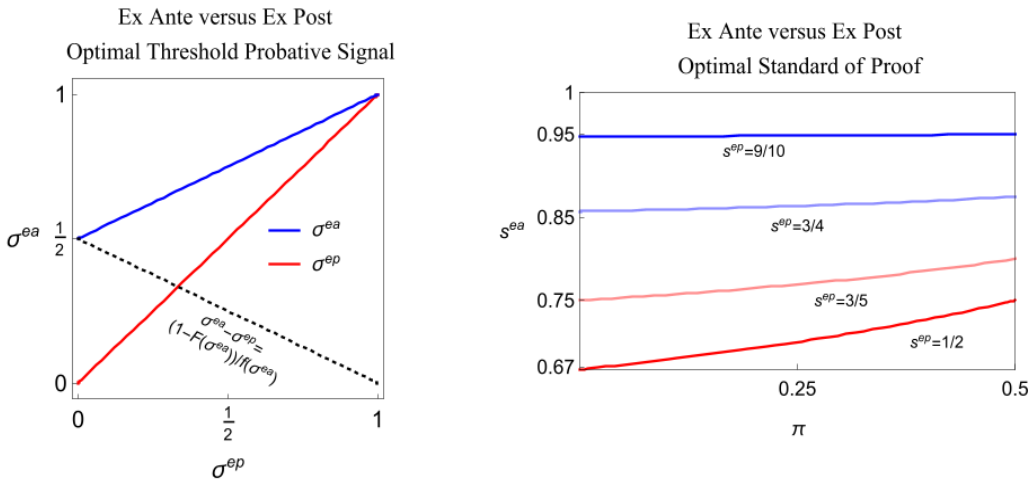


Figure 5: Ex ante versus ex post optimal threshold probative signal and standard of proof for $\sigma \sim U[0,1]$. The *left panel* compares the ex ante and ex post optimal threshold probative signals for a uniform distribution of σ and prior probability of guilt $\pi \in (0, \frac{1}{2})$. The *right panel* compares the corresponding standards of proof for $e^{ep} = \frac{1}{2}, \frac{3}{5}, \frac{3}{4}$ and $\frac{9}{10}$.

Under a uniform distribution, $h(\sigma) = \frac{1}{1-\sigma}$. From Proposition 2, the ex ante optimal threshold signal satisfies the equality $s(\sigma_t, \pi) = \frac{\pi(1+\sigma_t)}{1-\sigma_t(1-2\pi)}$, where $1-\sigma_t^{ea} = \frac{1}{h_F(\sigma_t^{ea})}$. Substituting $\frac{1-Z(D,\pi)}{1+Z(D,\pi)}$ for σ_t^{ep} and solving for σ_t^{ea} gives $\sigma_t^{ea}(\pi, D) = \frac{1}{1+Z(D,\pi)}$. Plugging in $\frac{1}{1+Z(D,\pi)}$ into $s(\sigma_t, \pi) = \frac{\pi(1+\sigma_t)}{1-\sigma_t(1-2\pi)}$ gives the ex ante optimal standard of proof $s^{ea}(D, \pi) = \frac{\pi(2+Z(D,\pi))}{2\pi+Z(D,\pi)} > D = s^{ep}(D)$.[17]

The left panel of Figure 5 presents the relations between the ex ante and ex post optimal threshold probative signals, whereas the right panel presents the ex ante and ex post optimal standard of proof for different values of D. As this figure shows, the ex ante optimal standard of proof is increasing in D and π, where its slope with respect to π is decreasing in D.

Consider the case in which society ascribes equal cost to wrongful convictions and wrongful acquittals ($D = \frac{1}{2}$). Substituting $\frac{\pi}{1-\pi}$ for $Z(D, \pi) = \frac{\pi}{1-\pi}\frac{1-D}{D}$ in the ex ante optimal standard of proof $\sigma_t^{ea}(\pi) = \frac{1}{1+Z(D,\pi)}$ gives $\sigma_t^{ea}(\pi) = 1 - \pi$. The ex ante optimal standard of proof associated with $\sigma_t^{ea}(\pi)$ is $s^{ea}(\pi) = \frac{2-\pi}{3-2\pi} > \frac{1}{2}$, which is increasing in π, ranging from $\frac{2}{3}$ to $\frac{3}{4}$ (for $\pi \to 0$ and $\pi \to \frac{1}{2}$, respectively). The ex post optimal threshold signal, by contrast, is $\sigma_t^{ep}(\pi) = \frac{1-Z(\pi)}{1+Z(\pi)} = 1 - 2\pi$ with a corresponding standard of proof of $s^{ep} = \frac{1}{2}$.

Figure 6 compares error probabilities for a conviction-oriented prosecutor under the ex post versus ex ante optimal standards of proof: $s^{ep} = \frac{1}{2}$ versus $s^{ea}(\pi) = \frac{2-\pi}{3-2\pi}$. An optimally elevated standard of proof trades off a higher rate of type II error for a lower rate of type I error (left panel).

The probability of type I error under the ex post optimal standard of proof is $2\pi^2$

[17] The optimal standard of proof is increasing in π because the proportional increase in π is greater than the corresponding proportional decrease in the optimal threshold probative signal $(1 - \pi)$.

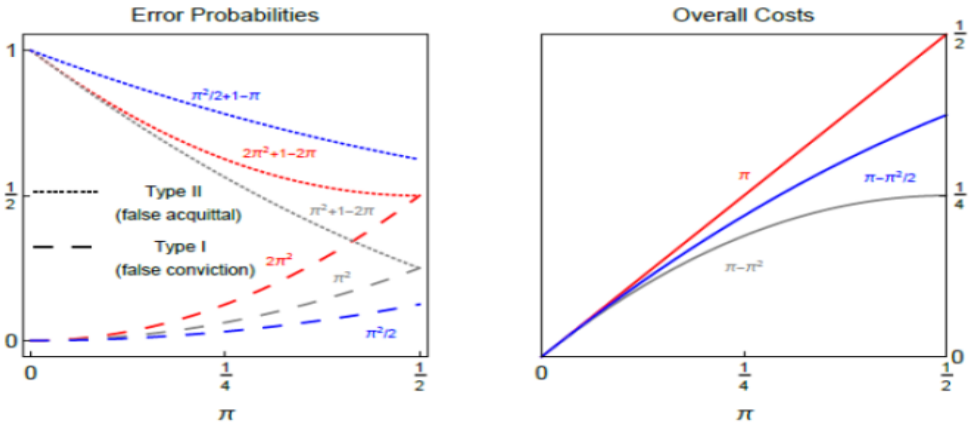


Figure 6: Error probabilities and costs for $\sigma \sim U[0,1]$ and $D = \frac{1}{2}$. The *left panel* shows the probabilities of type I and type II error for a uniform distribution of the evidentiary signal under the ex ante optimal standard of proof (blue); the ex post optimal standard of proof (red); and the first best (gray). The *right panel* shows society's corresponding expected costs of error.

$[= \frac{1}{2}(1-\sigma_t^{ep}(\pi))^2$, where $\sigma_t^{ep}(\pi) = 1-2\pi]$. The corresponding probability under the ex ante optimal standard of proof is $\frac{1}{2}\pi^2$ $[= \frac{1}{2}(1-\sigma_t^{ea}(\pi))^2$, where $\sigma_t^{ea}(\pi) = 1-\pi]$. Relative to the ex post optimal standard, the probability of type I error under the ex ante optimal standard is *lower* by $\frac{3}{2}\pi^2$. The corresponding reduction in society's expected costs of error is accordingly $(1-\pi) \times \frac{3}{2}\pi^2$.

On the other hand, the probability of type II error under the ex post optimal standard is $1-2\pi+2\pi^2$ $[= \sigma_t^{ep}(\pi) + \frac{1}{2}(1-\sigma_t^{ep}(\pi))^2]$. The corresponding probability under the ex ante optimal standard is $1-\pi+\frac{1}{2}\pi^2$ $[= \sigma_t^{ea}(\pi) + \frac{1}{2}(1-\sigma_t^{ea}(\pi))^2]$. Relative to the ex post optimal standard, the probability of type II error under the ex ante optimal standard is *lower* by $\pi - \frac{3}{2}\pi^2$. The corresponding increase in society's expected error costs is $\pi(\pi - \frac{3}{2}\pi^2)$.

Now, under the ex post optimal standard, society's expected costs of error are equal to its uninformed payoff of π (because society is indifferent between acquittal and conviction given probative incriminating evidence). Under the ex ante optimal standard, by contrast, society's expected costs of errors are $\pi \times \frac{1}{2}\pi^2 + (1-\pi) \times (\frac{1}{2}\pi^2 + 1 - \pi)$, which simplifies to $\pi - \frac{1}{2}\pi^2$. Overall, society's expected costs of error decrease from π under the ex post optimal standard to $\pi - \frac{1}{2}\pi^2$ under the ex ante optimal standard, a saving of $\frac{1}{2}\pi^2$.

Finally, in the first best outcome the prosecutor collects the realized signal (as opposed to the threshold probative signal). The conditional expectation of a probative signal is therefore $1-\pi$ $(= E[\sigma]_{\sigma \geq \sigma_t^{ep}(\pi)} = \frac{(1-2\pi)+1}{2})$. The probability of an incorrect (incriminating or exculpatory) evidence is $\frac{\pi}{2}$ $(= \frac{1-(1-\pi)}{2})$. The probability of type I error is accordingly π^2: the probability the signal is probative $(2\pi = 1-\sigma_t^{ep}(\pi))$ times the probability of an incorrect incriminating evidence $(\frac{\pi}{2})$. The probability of type II error is higher by the probability the signal is non-probative $(\sigma_t^{ep}(\pi) = 1-2\pi)$ and equals $\pi^2 + 1 - 2\pi$. Hence, society's expected

costs of error are $(1-\pi) \times \pi^2 + \pi \times (\pi^2 + 1 - 2\pi)$, which simplifies to $\pi(1-\pi)$. Society's expected costs of error under the ex ante optimal standard of proof thus equal the average of society's corresponding costs under the ex post optimal standard of proof and the first best ($\pi - \frac{1}{2}\pi^2 = \frac{1}{2}(\pi + \pi - \pi^2)$).

5.2 Type II error probability

Example 2 shows that if the evidentiary signal is distributed uniformly, a higher standard of proof trades off a lower probability of type II error (wrongful acquittal) for a higher probability of type I error (wrongful conviction). As we show next, however, an elevated standard of proof may reduce the probability of *both* error types depending on the distribution of the evidentiary signal.

Proposition 4 (probability of type II error) *If $D = \frac{1}{2}$ and $F(\sigma_t^{ea}(\frac{1}{2})) > \frac{\sigma_t^{ea}(\frac{1}{2})}{1+\sigma_t^{ea}(\frac{1}{2})}$, there exists $\underline{\pi} \in (0, \frac{1}{2})$ such that for $\pi \in (\underline{\pi}, \frac{1}{2})$, the probability of type II error under a conviction-oriented prosecutor is lower under the ex ante than under the ex post optimal standard of proof.*

Proof. See Appendix A.4.

Under the ex ante optimal threshold probative signal, the probability of type II error is decreasing in the prior probability of guilt. If it were not, then increasing the threshold probative signal would reduces both types of errors and decrease society's expected costs of error. Under the ex post optimal threshold signal, by contrast, the probability of type II error may be either decreasing or increasing in π, depending on the probability distribution of σ and the value of π. The reason the probability of type II error may be increasing in π for a sufficiently large π under the ex post optimal standard is that, as π increases, the ex post optimal threshold signal decreases. As π increases and the threshold probative signal decrease, (i) the probability the realized signal is non-probative decreases, which reduces the probability of type II error; and (ii) the probability the signal is probative and the evidence is incorrectly exculpatory increases, which increases the probability of type II error. If the hazard rate is sufficiently large, the latter effect dominates the former, in which case the probability of type II error is increasing in π.

Now, for any distribution of σ, the probability of type II error under a conviction-oriented prosecutor and the ex post optimal standard standard approaches $\frac{1}{2}$ as π approaches $\frac{1}{2}$. This is because the ex post optimal threshold signal, $\sigma_t^{ep}(\pi) = \frac{1-Z(\pi,\frac{1}{2})}{1+Z(\pi,\frac{1}{2})}$, approaches 0 as π approaches $\frac{1}{2}$. A conviction-oriented prosecutor therefore presents the minimum signal required for conviction ($\sigma_t^{ep} \to 0$), resulting in guilty defendant (virtually) always being put to trial and wrongfully acquitted with probability approaching $\frac{1}{2}$ (because $\frac{1}{2}(1 - \sigma_t^{ep}(\pi))\big|_{\pi \to \frac{1}{2}} =$

$\frac{1}{2}$). Under the ex ante optimal threshold signal, by contrast, the probability of type II error as π approaches $\frac{1}{2}$ approaches $F(\sigma_t^{ea}(\frac{1}{2})) + \frac{1}{2}(1 - F(\sigma_t^{ea}(\frac{1}{2}))(1 - \sigma_t^{ea}(\frac{1}{2}))$. If $F(\sigma_t^{ea}(\frac{1}{2})) > \frac{\sigma_t^{ea}(\frac{1}{2})}{1+\sigma_t^{ea}(\frac{1}{2})}$, then $\frac{1}{2}(1 - \sigma^{ea}(\pi))\big|_{\pi \to \frac{1}{2}} > \frac{1}{2}$ (because if $F(\sigma_t^{ea}(\frac{1}{2})) \to \frac{\sigma_t^{ea}(\frac{1}{2})}{1+\sigma_t^{ea}(\frac{1}{2})}$, then $F(\sigma_t^{ea}(\frac{1}{2})) \to 1/2$ from above). Thus, depending on the distribution of the evidentiary signal, the probability of type II error under a conviction-oriented prosecutor for π sufficiently close to $\frac{1}{2}$ is higher under the ex post than under the ex ante optimal standard of proof.

Intuitively, both the ex ante optimal standard of proof and the associated threshold probative signal decrease as π increases. A conviction-oriented prosecutor under the ex post optimal standard of proof presents the threshold probative signal (rather than the realized signal), thereby trading off lower probability of rightful conviction—in case the signal is probative and the evidence is incorrectly exculpatory—for a higher probability of wrongful conviction—in case the signal is probative and the evidence is incorrectly incriminating. Under the ex ante optimal standard of proof, by contrast, a conviction-oriented prosecutor is prompted to present a higher threshold probative signal. A higher threshold probative signal, in turn, increases the probability of rightful conviction in case the signal is probative, while increasing the probability of wrongful acquittal in case the signal is non-probative. For a sufficiently informative signal distribution, the decrease in the rate of type II error of cases that proceed to trial outweighs the increase in type II error of cases that do no proceed to trial. A higher standard of proof thus not only reduces society's overall expected costs of error but may also decrease the probability of *both* wrongful convictions and wrongful acquittals.

5.3 Example

To illustrate Proposition 4, consider a closed-form class of signal distributions: exponential distributions with parameter λ. The parameter λ determines the relative probability mass of lower- versus higher-value signals: as λ increases, the mass of higher-value signals increases whereas the mass of lower-value signals decreases (note that $\sigma^{\lambda_1} \overset{HRD}{\succeq} \sigma^{\lambda_0}$ for $\lambda_1 > \lambda_0$). Other things being equal, therefore, the ex ante probability the signal is more informative is increasing in λ.

Example 3 (type II error pr. | symmetric signal | exponential distribution)
Suppose $\sigma \sim F(\sigma, \lambda) = \sigma^\lambda$ for $\lambda > 0$ and that $D = \frac{1}{2}$. There exist cutoffs $\underline{\lambda} \in (1, 2)$ and $\underline{\pi}(\lambda) \in (0, \frac{1}{2})$, where $\underline{\pi}(\lambda)$ is decreasing in λ, such that for $\lambda > \underline{\lambda}$ and $\pi \in (\underline{\pi}(\lambda), \frac{1}{2})$, the probability of type II error under a conviction-oriented prosecutor is lower under the ex ante relative to the ex post optimal standard of proof.

Proof. See Appendix B.3.

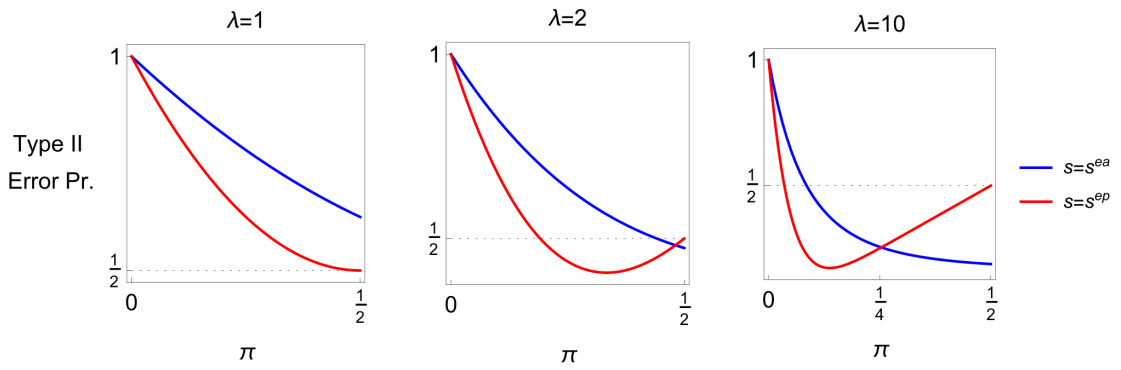


Figure 7: Probability of type II error under the ex ante and ex post optimal standards of proof. In the *left panel* ($F(\sigma) = \sigma$; uniform distribution), the probability of type II error is higher under the ex ante optimal standard for $\pi \in (0, \frac{1}{2})$. In the *middle panel* ($F(\sigma) = \sigma^2$), the probability of type II error is lower under the ex ante optimal standard for π sufficiently close to $\frac{1}{2}$. Finally, in the *right panel* ($F(\sigma) = \sigma^{10}$), the probability of type II error is higher under the ex ante optimal standard for $\pi \in (\frac{1}{4}, \frac{1}{2})$.

The ex post optimal standard of proof for $\lambda > 1$ given a conviction-oriented prosecutor is decreasing and then increasing in the threshold probative signal, attaining a lower minimum value as λ increases. Under the ex ante optimal threshold signal, by contrast, the probability of type II error is decreasing in the prior probability of guilt. The graphs of the probability of type II error under the ex ante and ex post optimal standards of proof either do not intersect (for a sufficiently low λ; left panel of Figure 7) or intersect once along the increasing portion of the graph of the probability of type II error under the ex post optimal standard (for a sufficiently high λ; middle and right panels of Figure 7). As λ increases, the probability of type II error decreases under both standards of proof. Hence, as λ increases, the graphs intersect at a lower prior probability of guilt. For a sufficiently high λ and a sufficiently large π (depending on λ), an optimally-elevated standard of proof reduces the probability of both wrongful convictions and wrongful acquittals.

To stress the intuition, recall that a marginal increase in the standard of proof has two opposite effects on the ex ante probability of type II error. As the standard of proof increases (i) fewer cases involving guilty suspects proceed to trial, but (ii) guilty suspects who do stand trial are more often rightfully convicted. As the signal distribution becomes tilted toward higher-value signals, the former effect wanes and the latter waxes. For a sufficiently skewed distribution, a higher standard of proof not only reduces the incidence of wrongful convictions, but also and increases the rate of rightful convictions.

6 Concluding Comments

The central theme of this paper is that a primary role of the standard of proof in criminal proceedings is to guide the prosecution's search for evidence. Elevating the standard of proof

above society's ex post optimal standard provides prosecutors incentives to seek out more informative evidence. A commitment to an ex post supra-optimal standard in turn reduces society's ex ante expected costs of errors. The standard of proof under this account functions as an exclusionary rule, which precludes lesser-quality probative evidence from serving as a basis for conviction.[18]

We conclude by considering some broader justifications for and implications of the paper's main idea.

6.1 Prosecutorial powers and incentives

An agency-cost rationale for a higher standard of proof in criminal proceedings is of a piece with the broad powers possessed by prosecutors. Then-Attorney General of the United States, Robert H. Jackson (who would later become Supreme Court Justice), cautioned some 80 years ago:

> The prosecutor can order arrests, present cases to the grand jury in secret session, and on the basis of his *one-sided presentation of the facts*, can cause the citizen to be indicted and held for trial.

(Jackson 1940, p. 3; emphasis added).

The broad powers granted to law enforcement agents reflect the practical impossibility of specifying precisely the variable circumstances encountered by police and prosecutors in the course of investigating and prosecuting crimes. Such broad powers, however, inevitably entail the possibility of conscious or subconscious abuse of discretion.

The nature of prosecutorial incentives similarly calls for a higher standard of proof. As Posner (1999, p. 1505) notes, "[E]conomic theory, as well as common sense and observation, suggests that the desire to win . . . is the most important argument in the prosecutorial utility function." The desire to solve a case and secure a conviction may accordingly cloud police and prosecutors' professional judgment. Requiring guilt to be established beyond a reasonable doubt counteracts such incentives by guiding police and prosecutors' search efforts.

6.2 Plea bargains

The vast majority of criminal cases settle (Devers 2011 ["90 to 95[] percent[] of cases result in plea bargaining"]). Multiple factors—including risk-aversion, subjective costs of a criminal trial, and trust in the judicial system—affect suspects' willingness to sign a plea deal.

[18] Nunce (2017, pp. 186-227) similarly argues that a primary role of a host of legal rules regulating the burden of production—such as rules governing admissibility and disclosure of evidence—is the optimization of "Keynesian weight"; i.e., the body of evidence upon which a jury must reach its verdict by applying the appropriate standard of proof.

Restricting attention to the prosecution's ability to impel a suspect to enter a plea deal, a higher standard of proof reduces society's expected costs of errors given conviction-oriented prosecutors. To have a credible threat to press charges against a suspect, law enforcement agents must obtain raw evidence that meets the standard of proof. A higher standard of proof, therefore, provides police and prosecutors incentives to collect more probative evidence to support their threat of putting a suspect on trial. In the presence of plea bargains, too, a higher standard of proof provides incentives to collect more informative evidence.

In a screening model 'a la Baker and Mezzetti (2000), prosecutors may wish to acquire a supra-probative evidentiary signal to improve their bargaining position, increase the probability a guilty suspect enters a plea deal, and save on trial costs. Improving the accuracy of trial outcomes, in turn, improves the separation of suspect types in the plea bargain stage. The main trade-off between the ex ante and ex post error costs resulting from a higher standard of proof, however, continues to hold in the presence of plea bargains. Prosecutors could make higher plea offers, which are more likely to be accepted by guilty suspects. And innocent suspects who refuse a plea deal are more likely to be acquitted under a higher standard of proof.

6.3 Admissibility of prior acts evidence

Federal Rules of Evidence 413–415 establish a lower admissibility standard for evidence of previous similar acts in sexual-assault cases. Enacted by Congress in 1995 against a fierce opposition by the legal profession and the judiciary,[19] rules 413–414 allow a court in sexual-assault and child-molestation criminal cases to "admit evidence that the defendant committed any other sexual assault" (rule 415 applies the same standard in civil cases). The rules mark a sharp departure from the enshrined prohibition on the use of character evidence for proving a propensity to commit crime. A large number of states have since enacted similar laws (Hathorn 2014, p. 69.)[20]

A primary objection to Congress's initiative is that the curve-out rules were not based on evidence of relative rates of recidivism associated with different crimes (Mueller et al. 2018, fn. 10 and associated text). There is no empirical basis, goes the argument, for assuming a higher probative value of prior acts evidence in sexual-assault cases relative to other crimes. Some scholars went on to suggest that the different evidentiary treatment of sexual-assault crimes "reinforces the [wrong] notion that those who rape are unusually dangerous recidivists against whom society must be on constant guard" (Wells and Motley 2001, p. 130).

Leaving aside the epistemological and sociological criticism, the rationale offered here

[19] See Hathorn 2014, p. 28, fn. 21. "That an 'overwhelming majority' of legal scholars objected to the rules is probably an understatement" (*id.*).

[20] Federal Rule of Evidence 404(b)(1) prohibits the use of evidence of other crimes, wrongs, or acts "to prove that on a particular occasion the person acted in accordance with the character or trait [implied by the evidence]." Rule 404(b)(2) allows using such evidence for limited purposes such as "proving motive, opportunity, intent, preparation, [and] plan."

for an elevated standard of proof can similarly justify the laxer evidentiary standard in sexual-assault cases. Victims in such cases are often reluctant to "relive a painful part of their lives [through a trial testimony and] submit to questions [in cross examination] that are designed to cast doubt on their credibility" (North 2020). In the absence of third-party witnesses or physical evidence, the prosecution in sexual-assault cases is typically reluctant to bring charges against alleged wrongdoers.[21] Potential perpetrators, furthermore, may take advantage of the stricter evidentiary standards to prey on victims with impunity.[22]

Relaxation of evidentiary standards when evidence is scarce or exceedingly costly would be consistent with an essential role of rules of evidence as an instrument for guiding prosecutorial search for information. Similar to an elevated standard of proof, exclusionary rules provide incentives for prosecutors to search for higher-quality evidence. When the prosecution could not obtain more probative evidence, however, a lower admissibility standard may aid justice and improve deterrence without compromising prosecutors' incentives.

[21] According to Ransom (2021), "[P]rosecutors [of the Manhattan district attorney's office] dropped 49 percent of sexual assault cases in 2019 — among the highest rates in the city, and an increase from 37 percent in 2017."

[22] The use of prior acts evidence in cases in which evidence is relatively scarce was considered by Congress when enacting Rule 414. See 140 Cong. Rec. H8991-92 (August 21, 1994) (statement of Rep. Molinari) (child molestation cases "require reliance on child victims whose credibility can readily be attacked in the absence of substantial corroboration"). See also *United States v. LeMay (2001)* (holding the district judge "did not abuse his discretion in finding that [the defendant's] prior acts of child molestation were not so prejudicial as to outweigh their probative value").

Appendix: Proofs

A. Propositions 1, 2, 3 and 4

A.1 Proof of Proposition 1

Proposition 1 (optimal standard of proof | General) *The standard of proof that minimizes society's ex ante expected costs of error under a conviction-oriented prosecutor is strictly higher than the ex post optimal standard:* $s^{ea} \in (D,1) > s^{ep} = D$.

Proof. We begin by partially differentiating the error rates in Eqs. (5) and (6) with respect to s. From Eq. (5), the probability of type I error under a conviction-oriented prosecutor is

$$\alpha(s,\pi) = \tfrac{1}{2}\int_0^1 \int_{q_t(s,p,\pi)}^1 (1 - q_t(s,p,\pi)) f(q) g(p) dq dp. \tag{A1}$$

Using Leibniz integral rule, the partial derivative with respect to s is

$$\begin{aligned} \frac{\partial}{\partial s}\alpha(s,\pi) &= -\tfrac{1}{2}\int\limits_{p\in[0,1]} [(1 - q_t(s,p,\pi))\cdot f(q_t(s,p,\pi))\cdot \frac{\partial}{\partial s} q_t(s,p,\pi)] dG(p) \\ &\quad -\tfrac{1}{2}\int_0^1 [(1 - F(q_t(s,p,\pi))\cdot \frac{\partial}{\partial s} q_t(s,p,\pi)] dG(p), \end{aligned} \tag{A2}$$

where the first line follows by the fundamental theorem of calculus and the second by differentiating under the integral sign and substituting $1 - F(q_t(s,p,\pi)$ for $\int_{q_t(s,p,\pi)}^1 f(q)dq$.

Plugging in $1 - (1+p)\,Z(s,\pi)$ into $q_t(s,p,\pi)$ from Lemma 1A we get

$$\begin{aligned} \frac{\partial}{\partial s}\alpha(s,\pi) &= -\tfrac{Z(s,\pi)^2}{2s(1-s)}\int_0^1 [(1+p)\cdot f(q_t(s,p,\pi))] dG(p) \\ &\quad -\tfrac{Z(s,\pi)}{2s(1-s)}\int_0^1 [(1+p)\cdot(1 - F(q_t(s,p,\pi))] dGp), \end{aligned} \tag{A3}$$

because $\frac{\partial}{\partial s} q_t(s,p,\pi) = \frac{\partial}{\partial s}[1 - (1+p)\,Z(s,\pi)] = -(1+p)\cdot\frac{\partial}{\partial s} Z(s,\pi)$, where $\frac{\partial}{\partial s} Z(s,\pi) = -\frac{\pi}{1-\pi}\frac{1}{s^2} = -\frac{Z(s,\pi)}{s(1-s)}$.

Now, let

$$I(s,p,\pi) \equiv (1+p)^2\cdot f(q_t(s,p,\pi)) \tag{A4}$$

and

$$II(s,p,\pi) \equiv (1+p)\cdot(1 - F(q_t(s,p,\pi))), \tag{A5}$$

denote the first and second integrands in (A3).

Plugging in $I(s,p,\pi)$ and $II(s,p,\pi)$ in (A3) and simplifying gives

$$\frac{\partial}{\partial s}\alpha(s,\pi) = -Z(s,\pi)\left(\tfrac{Z(s,\pi)}{2s(1-s)}\int_0^1 I(s,p,\pi)dG(p) + \int_0^1 II(s,p,\pi)dG(p)\right). \tag{A6}$$

From Eq. (6), the probability of type II error is

$$\beta(s,\pi) = 1 - \tfrac{1}{2}\int_0^1 \int_{q_t(s,p,\pi)}^1 (1+p)f(q)g(p)dqdp. \tag{A7}$$

Partially differentiating with respect to s gives

$$\begin{aligned}\frac{\partial}{\partial s}\beta(s,\pi) &= \tfrac{1}{2}\int_0^1 [(1+p)\cdot f(q_t(s,p,\pi))\cdot\frac{\partial}{\partial s}q_t(s,p,\pi)]dG(p) \\ &\equiv \tfrac{Z(s,\pi)}{2s(1-s)}\int_0^1 I(s,p,\pi)dG(p).\end{aligned} \tag{A8}$$

Society's marginal expected costs of a higher standard of proof are therefore

$$\begin{aligned}\frac{\partial}{\partial s}EC(s,\pi,D) &= \\ &\frac{\partial}{\partial s}[\alpha(s,\pi)\cdot(1-\pi)D + \beta(s,\pi)\cdot\pi(1-D)],\end{aligned} \tag{A9}$$

where the first and second expressions are society's expected benefit and cost from a higher standard of proof.

Dividing through by $(1-\pi)D$ and substituting $Z(D,\pi)$ for $\frac{\pi}{1-\pi}\div\frac{D}{1-D}$ from Definition 1 we get

$$\frac{\partial}{\partial s}EC(s,\pi,D) \propto \frac{\partial}{\partial s}[\alpha(s,\pi) + \beta(s,\pi)Z(D,\pi)]. \tag{A10}$$

Substituting the right-hand sides of Eqs. (A6) and (12) for $\frac{\partial}{\partial s}\alpha(s,p,\pi)$ and $\frac{\partial}{\partial s}\beta(s,p,\pi)$, respectively, gives

$$\frac{\partial}{\partial s}EC(s,\pi,D) \propto [Z(D,\pi) - Z(s,\pi)] \times \int_0^1 I(s,p,\pi)dp - \int_0^1 II(s,p,\pi)dp, \tag{A11}$$

where

$$\frac{\partial}{\partial s}EC(s,\pi,D)\Big|_{s=D} \propto -\int_0^1 II(D,p,\pi)dp < 0. \tag{A12}$$

Under an unbiased prosecutor, by contrast, the second term of (A11) ($\int_0^1 II(s,p,\pi)dp$) vanishes and the optimal standard is $s = D$. □

A.2 Proof of Proposition 2

Proposition 2 (optimal standard of proof | comparative statics)

(1) If $F_1(\cdot)$ hazard rate dominates $F_2(\cdot)$, the ex ante optimal standard of proof under $F_1(\cdot)$ is higher than under $F_2(\cdot)$.

(2) If $G_1(\cdot)$ first-order stochastically dominates $G_2(\cdot)$, the ex ante optimal standard of proof under $G_1(\cdot)$ is higher than, lower than or equal to the ex ante optimal standard under $G_2(\cdot)$.

Proof. (1) Suppose $F_1(\cdot)$ hazard rate dominates $F_2(\cdot)$. Using the properties of the derivative of a logarithm, we can write the inequality $\frac{f_1(q_t)}{1-F_1(q_t)} < \frac{f_2(q_t)}{1-F_2(q_t)}$ as

$$-\frac{d}{dq_t}\log(1-F_1(q_t)) < -\frac{d}{dq_t}\log(1-F_2(q_t)). \tag{A13}$$

Integrating both sides with respect to $G(\cdot)$ and factoring out the derivative operator we get

$$-\frac{d}{dq_t}\int \log(1-F_1(q_t))dGp < -\frac{d}{dq_t}\int \log(1-F_1(q_t))dGp. \tag{A14}$$

Commuting the integral and log operators on both sides gives

$$-\frac{d}{dq_t}\log\int (1-F_1(q_t))dGp < -\frac{d}{dq_t}\log\int (1-F_1(q_t))dGp, \tag{A15}$$

which implies that

$$\frac{\int f_1(q_t))dGp}{\int (1-F_1(q_t))dGp} < \frac{\int f_2(q_t))dGp}{\int (1-F_2(q_t))dGp}. \tag{A16}$$

Multiplying through the upper and lower integrands by $q_t'(s,p,\pi) = \frac{\partial}{\partial s}Z(s,\pi)(1+p)$ and the lower integrands by $1+p$ and integrating from 0 to 1 we get

$$\begin{aligned} -\left.\frac{\frac{\partial}{\partial s}\alpha(s,\pi)}{\frac{\partial}{\partial s}\beta(s,\pi)}\right|_{q\sim F_1} &= \\ Z(s,\pi)+\left.\frac{\int_0^1 II(s,p,\pi)d(p)}{\int_0^1 I(s,p,\pi)d(p)}\right|_{q\sim F_1} &> Z(s,\pi)+\left.\frac{\int_0^1 II(s,p,\pi)d(p)}{\int_0^1 I(s,p,\pi)d(p)}\right|_{q\sim F_2} \\ &= -\left.\frac{\frac{\partial}{\partial s}\alpha(s,\pi)}{\frac{\partial}{\partial s}\beta(s,\pi)}\right|_{q\sim F_2}. \end{aligned} \tag{A17}$$

Because ratio of the marginal error rates of a higher standard of proof is higher under $F_1(\cdot)$ relative to $F_2(\cdot)$, the optimal standard of proof is higher under $F_1(\cdot)$ relative to $F_2(\cdot)$.

(2) To prove this part, we begin by proving an ancillary claim.

Claim A1 *Suppose $G_1(\cdot)$ first-order stochastically dominates $G_2(\cdot)$ and let $y(\cdot)$ be a monotone function defined on the unit interval. Then:*
(1) $\int_0^1 y(x)g_1dx > \int_0^1 y(x)g_2dx$ *if* $\frac{dy}{dx} > 0$*; and*
(2) $\int_0^1 y(x)g_1dx < \int_0^1 y(x)g_2dx$ *if* $\frac{dy}{dx} < 0$.

Proof. Let $G(y)$ be a distribution function and $y(x)$ a monotonic function. Integration by parts gives

$$\begin{aligned}\int_0^1 y(x)g(x)dx &= G(x)y(x)|_0^1 - \int_0^1 y'(x)G(x)d(x) \\ &= y(1) - \int_0^1 y'(x)G(x)d(x). \end{aligned} \tag{A18}$$

It follows that

$$\int_0^1 y(x)dG_1(x) - \int_0^1 y(x)dG_2(x) = \int_0^1 y'(x)(G_1(x) - G_2(x))dx, \tag{A19}$$

whose sign depends on the sign $y'(x)$: strictly positive if $\frac{dy}{dx} > 0$ and strictly negative if $\frac{dy}{dx} < 0$. ∥

Now, because $\frac{\partial}{\partial p}II(s,p,\pi) = \frac{\partial}{\partial p}(1+p)\cdot(1-F(q_t(s,p,\pi))) < 0$, it follows from Claim A1 that

$$\int_0^1 II(s,p,\pi)dG_1(p) < \int_0^1 II(s,p,\pi)dG_2(p), \tag{A20}$$

and therefore the marginal *benefit* of a higher standard of proof is higher $G_1(\cdot)$ relative to $G_2(\cdot)$.

For the same reason, whether the marginal cost of a higher standard of proof is higher or lower under $G_1(\cdot)$ relative to $G_2(\cdot)$ depends on whether $I(s,p,\pi) \equiv (1+p)^2\cdot f(q_t(p,s,\pi))$ increases or decreases with p, where

$$\tfrac{Z(s,\pi)}{2s(1-s)}\int_0^1 I(s,p,\pi)dG_1(p) > \tfrac{Z(s,\pi)}{2s(1-s)}\int_0^1 I(s,p,\pi)dG_2(p) \tag{A21}$$

if and only if $\frac{\partial}{\partial p}I(s,p,\pi) > 0$. Because $\frac{\partial}{\partial p}I(s,p,\pi) > 0$, the marginal cost of a higher standard of proof is higher under $G_1(\cdot)$ relative to $G_2(\cdot)$.

Finally, if both $1 - F(q_t(s,p,\pi))$ and $f(q_t(s,p,\pi))$ are a product of $(1+p)$, the *ratio* between the marginal benefit of a higher standard of proof and the corresponding marginal benefit is independent of $G(\cdot)$. To see this, note that if both $1 - F(q_t(s,p,\pi)) \propto 1 -$

$q_t(s,p,\pi)$ and $f(q_t(s,p,\pi))) \propto (1-q_t(s,p,\pi))^i$, where $i=0,1$ then

$$\begin{aligned} -\frac{\alpha'(s,\pi)}{\beta'(s,\pi)} &= \\ & Z(s,\pi) + \frac{\int_0^1[(1-F_1(q_t(s,p,\pi)))\cdot\frac{\partial}{\partial s}q_t(s,p,\pi)]dG(p)}{\int_0^1[(1+p)\cdot f_1(q_t(s,p,\pi)))\cdot\frac{\partial}{\partial s}q_t(s,p,\pi)]dG(p)} \\ &= C\cdot Z(s,\pi), \end{aligned} \tag{A22}$$

for some positive constant $C > D$. Equating $Z(s,\pi)(1+C)$ to $Z(D,\pi)$ and solving for s we get

$$s^{ea}(D) = \frac{C(1+D)}{1+CD} > D, \tag{A23}$$

which is independent of $G(\cdot)$ and π. □

A.3 Proof of Proposition 3

Proposition 3 (optimal threshold probative signal | symmetric signal)

(1) The ex ante optimal threshold probative signal *under a conviction-oriented prosecutor satisfies the equality*

$$\sigma_t^{ea} - \sigma_t^{ep}(\pi) = \frac{1-F(\sigma_t^{ea})}{f(\sigma_t^{ea})}, \tag{A24}$$

where $\sigma_t^{ep}(\pi) = 1-2\pi$ *is the ex post optimal threshold of proof.*

(2) If $F_1(\cdot)$ *hazard rate dominates* $F_2(\cdot)$*, then the ex ante optimal* standard of proof *is higher under* $F_1(\cdot)$ *than under* $F_2(\cdot)$*.*

Proof. Using Eqs. (18) and (19), the derivative of society's expected costs of errors with respect to the threshold probative signal is

$$\begin{aligned} \frac{\partial}{\partial\sigma_t}EC(\sigma_t,\pi,D) &= -[f(\sigma_t)(1-\sigma_t)+(1-F(\sigma_t))]\cdot(1-\pi)D \\ &\quad +[f(\sigma_t)(1+\sigma_t)-(1-F(\sigma_t))]\cdot\pi(1-D), \end{aligned} \tag{A25}$$

where the first expression is the marginal cost of wrongful convictions and the second the marginal cost of wrongful acquittals.

Dividing through by $(1-\pi)D$, substituting $Z(D,\pi)$ for $\frac{\pi}{1-D} \div \frac{D}{1-\pi}$ from Definition 1 and rearranging we get

$$\begin{aligned} \frac{\partial}{\partial\sigma_t}EC(\sigma_t,\pi,D) &\propto \\ & f(\sigma)[Z(D,\pi)-1+\sigma(1+Z(D,\pi))] - [1-F(\sigma_t)]\cdot[1+Z(D,\pi)]. \end{aligned} \tag{A26}$$

Equating to zero and solving for σ_t, the ex ante optimal threshold probative signal satisfies

$$\sigma_t = \frac{1 - Z(D,\pi)}{1 + Z(D,\pi)} + \frac{1 - F(\sigma_t)}{f(\sigma_t)}. \tag{A27}$$

Now, the value of σ_t that satisfies Eq. (A27) increases as $\frac{1-F(\sigma_t)}{f(\sigma_t)}$ increases. Therefore, the ex ante optimal standard of proof is higher under $F_1(\cdot)$ relative to $F_2(\cdot)$ if $F_1(\cdot)$ hazard rate dominates $F_2(\cdot)$.

We conclude by verifying the second-order condition for a minimum is satisfied. Partially differentiating Eq. (A26) with respect to σ_t gives

$$\begin{aligned} \frac{\partial^2}{\partial \sigma_t^2} EC(\sigma_t, \pi, D) \quad &\propto \\ & f'(\sigma_t)\cdot[Z(D,\pi) - 1 + \sigma(1 + Z(D,\pi))] + f(\sigma_t)(1 + 2Z(D,\pi)) > 0, \end{aligned} \tag{A28}$$

for $\sigma_t > \sigma_t^{ep} = \frac{1-Z(D,\pi)}{1+Z(D,\pi)}$ if $f'(\cdot) \geq 0$ and for σ_t sufficiently close to σ_t^{ep} if $f'(\cdot) < 0$. □

A.4 Proof of Proposition 4

Proposition 4 (probability of type II error) *If $D = \frac{1}{2}$ and $F(\sigma_t^{ea}(\frac{1}{2})) > \frac{\sigma_t^{ea}(\frac{1}{2})}{1+\sigma_t^{ea}(\frac{1}{2})}$, there exists $\underline{\pi} \in (0, \frac{1}{2})$ such that for $\pi \in (\underline{\pi}, \frac{1}{2})$, the probability of type II error under a conviction-oriented prosecutor is lower under the ex ante than under the ex post optimal standard of proof.*

Proof. From Eqs. (16) and (17), the probability of type II error under a conviction-oriented prosecutor as a function of the threshold probative signal is

$$\beta(\sigma_t) = F(\sigma_t) + \tfrac{1}{2}(1 - F(\sigma_t))\cdot(1 - \sigma_t). \tag{A29}$$

The proof proceeds along the next five steps.

Step 1 $\lim_{\pi \to \frac{1}{2}} \beta(\sigma_t^{ep}(\pi)) = \frac{1}{2}$.

Because $\beta(\sigma_t^{ep}(\pi))$ is a continuous function of π, $\lim_{\pi \to \frac{1}{2}} \beta(\sigma_t^{ep}(\pi)) = \beta(\sigma_t^{ep}(\pi))|_{\pi=\frac{1}{2}}$. Plugging in $\pi = \frac{1}{2}$ in $\sigma_t^{ep} = 1 - 2\pi$ gives $\lim_{\pi \to \frac{1}{2}} \beta(\sigma_t^{ep}(\pi)) = \beta(0) = \frac{1}{2}$.

Step 2 $\frac{d}{d\sigma_t}\beta(\sigma_t) > 0$ *if and only if* $\sigma_t < \frac{1}{h(\sigma_t)} - 1$, *where* $h(\sigma_t) = \frac{f(\sigma_t)}{1-F(\sigma_t)}$.

The derivative of $\beta(\sigma_t)$ with respect to σ_t is

$$\beta'(\sigma_t) = \tfrac{1}{2}[f(\sigma_t(\pi))\cdot(1 + \sigma_t(\pi)) - (1 - F(\sigma_t)], \tag{A30}$$

which is positive iff $\sigma_t(\pi) < \frac{1}{h(\sigma_t(\pi))} - 1$.

Step 3 $\frac{\partial}{\partial \pi}\beta(\sigma_t^{ea}(\pi)) < 0$ *for* $\pi \in (0, \frac{1}{2})$

From Step 2 it suffices to show that $\sigma_t^{ea}(\pi) < \frac{1}{h(\sigma_t^{ea}(\pi))} - 1$ for $\pi \in (0, \frac{1}{2})$. Now, by Proposition 3, $\sigma_t^{ea}(\pi) = \sigma_t^{ep}(\pi) + \frac{1}{h(\sigma_t^{ea}(\pi))} > \frac{1}{h(\sigma_t^{ea}(\pi))} - 1$. Because $\sigma_t^{ea}(\pi) > \frac{1}{h(\sigma_t^{ea}(\pi))} - 1$, it follows that $\frac{\partial}{\partial \pi}\beta(\sigma_t^{ea}(\pi)) < 0$.

Step 4 *If* $\sigma_t(\pi) > \frac{1}{h(\sigma_t(\pi))} - 1$ *then* $\frac{d}{d\pi}\beta(\sigma_t^{ea}(\pi)) > \frac{d}{d\pi}\beta(\sigma_t^{ep}(\pi))$.

If $\sigma_t(\pi) > \frac{1}{h(\sigma_t(\pi))} - 1$ then $\frac{d}{d\sigma_t}\beta(\sigma_t(\pi)) = f(\sigma_t(\pi))(1 + \sigma_t(\pi)) - (1 - F(\sigma_t) > 0$. Because $\sigma_t^{ep}(\pi) < \sigma_t^{ea}(\pi)$ by Proposition 3, it follows $\beta(\sigma_t^{ea}(\pi)) > \beta(_t^{ep}(\pi))$.

Step 5 *If* $D = \frac{1}{2}$ *and* $F(\sigma_t^{ea}(\frac{1}{2})) > \frac{\sigma_t^{ea}(\frac{1}{2})}{1+\sigma_t^{ea}(\frac{1}{2})}$, *there exists* $\underline{\pi} \in (0, \frac{1}{2})$ *such that* $\beta(\sigma_t^{ea}(\pi)) < \beta(\sigma_t^{ep}(\pi))$ *for* $\pi \in (\underline{\pi}, \frac{1}{2})$.

If $F(\sigma_t^{ea}(\frac{1}{2})) > \frac{\sigma_t^{ea}(\frac{1}{2})}{1+\sigma_t^{ea}(\frac{1}{2})}$, then $F(\sigma_t^{ea}(\frac{1}{2})) > [1 - F(\sigma_t^{ea}(\frac{1}{2}))]\sigma_t^{ea}$. Adding 1 to both sides we get

$$F(\sigma_t^{ea}(\tfrac{1}{2})) + 1 > [1 - F(\sigma_t^{ea}(\tfrac{1}{2}))]\sigma_t^{ea}(\tfrac{1}{2}) + 1. \tag{A31}$$

Substituting $2F(\sigma_t^{ea}(\pi(\frac{1}{2}))) - F(\sigma_t^{ea}(\frac{1}{2}))$ in the left-hand side gives

$$2F(\sigma_t^{ea}(\tfrac{1}{2})) + (1 - F(\sigma_t^{ea}(\tfrac{1}{2})) > (1 - F(\sigma_t^{ea}(\tfrac{1}{2})))\sigma_t^{ea}(\tfrac{1}{2}) + 1. \tag{A32}$$

Subtracting $[1 - F(\sigma_t^{ea}(\frac{1}{2}))]\sigma_t^{ea}$ from both sides, the inequality becomes

$$2F(\sigma_t^{ea}(\tfrac{1}{2})) + [1 - F(\sigma_t^{ea}(\tfrac{1}{2}))]\cdot[1 - \sigma_t^{ea}(\tfrac{1}{2})] > 1. \tag{A33}$$

Finally, dividing through by 2 we get

$$F(\sigma_t^{ea}(\tfrac{1}{2})) + [1 - F(\sigma_t^{ea}(\tfrac{1}{2}))]\cdot[(1 - \sigma_t^{ea}(\tfrac{1}{2})] > \tfrac{1}{2}. \tag{A34}$$

The left-hand side is the probability of type II error under the ex ante optimal standard of proof at $\pi = \frac{1}{2}$, whereas the right-hand side is the corresponding probability under the ex post optimal standard of proof (by Step 1). It follows there exists $\underline{\pi} \in (0, \frac{1}{2})$ such that $\beta(\sigma_t^{ea}(\pi)) < \beta(\sigma_t^{ep}(\pi))$ for $\pi \in (\underline{\pi}, \frac{1}{2})$.

B: Examples 1A, 1B, 1C, 2 and 3.

B.1 Proof of Example 1A–1C

Throughout the proofs of Examples 1A-1C, we shall use the first-order condition from Eq. (A11):

$$Z(s, \pi) + \frac{\int_0^1 II(s, p, \pi)dp}{\int_0^1 I(s, p, \pi)dp} = Z(D, \pi), \tag{B1}$$

where the left-hand side equals $-\frac{\partial}{\partial s}\alpha(s,\pi) \div \frac{\partial}{\partial s}\beta(s,\pi)$, $I(s,p,\pi) \equiv (1+p)^2 \cdot f(q_t(s,p,\pi)) \cdot g(p)$ and $II(s,p,\pi) \equiv (1+p) \cdot (1 - F(q_t(s,p,\pi))) \cdot g(p)$.

Before proceeding, we pause to derive the second-order condition for a minimum.

Claim B1 *The second-order condition for a minimum is satisfied if $s > D$ and (i) $f'(\cdot) \geq 0$ or (ii) $f'(\cdot) < 0$ and s sufficiently close to D.*

Proof. Taking the derivative of Eq. (A9) with respect to s gives

$$\begin{aligned} \frac{\partial^2}{\partial s^2} EC(s,\pi,D) \quad \propto \quad & \left(\frac{\partial}{\partial s} \int_0^1 I(s,p,\pi) dp \times [Z(D,\pi) - Z(s,\pi)] - \frac{\partial}{\partial s} Z(s,\pi) \int_0^1 I(s,p,\pi) dp) \right) \\ & - \frac{\partial}{\partial s} \int_0^1 II(s,p,\pi) dp, \end{aligned} \tag{B2}$$

where

$$\frac{\partial}{\partial s} \int_0^1 I(s,p,\pi) dp = -\frac{\partial}{\partial s} Z(s,\pi) \int_0^1 [(1+p)^3 f'(q_t(s,p,\pi))] dG(p). \tag{B3}$$

and

$$\frac{\partial}{\partial s} \int_0^1 II(s,p,\pi) dp = \frac{\partial}{\partial s} Z(s,\pi) \int_0^1 I(s,p,\pi) dp. \tag{B4}$$

Because $\frac{\partial}{\partial s} Z(s,\pi) < 0$, we have

$$\begin{aligned} \frac{\partial^2}{\partial s^2} EC(s,\pi,D) \quad \propto \quad & \\ & [Z(D,\pi) - Z(s,\pi)] \times \int_0^1 [(1+p)^3 f'(q_t(s,p,\pi))] dG(p)) \\ & + 2 \int_0^1 II(s,p,\pi) dp, \end{aligned} \tag{B5}$$

which is strictly positive if $s > D$ and (i) $f'(\cdot) \geq 0$ or (ii) $f'(\cdot) < 0$ and s sufficiently close to D. ‖

B.1.1 Proof of Example 1A We begin by considering the case in which $F_1(q) = q^2$.

Example 1A (optimal SoP | convex F)

If $F_1(q) = q^2$, the optimal standard of proof is $s^{ea} \in (\frac{2D}{D+1}, 1)$, which is increasing in π and in the curvature of $G(\cdot)$.

Proof. Substituting $(1-q_t(s,p,\pi))\cdot(1+q_t(s,p,\pi))$ for $1-F(q_t(s,p,\pi)$ and $2q_t(s,p,\pi)$ for $f(q_t(s,p,\pi))$ in Eq. (B1) we obtain

$$\begin{aligned}
\frac{\frac{\partial}{\partial s}\alpha(s,p,\pi)}{\frac{\partial}{\partial s}\beta(s,p,\pi)} &= Z(s,\pi)+\frac{Z(s,\pi)\int_0^1(1+p)^2\cdot(1+q_t(s,p,\pi))\cdot g(p)dp}{2\int_0^1(1+p)^2\cdot q_t(s,p,\pi))\cdot g(p)dp} \\
&= Z(s,\pi)+\frac{Z(s,\pi)\int_0^1(1+p)^2 g(p)dp+Z(s,\pi)\int_0^1(1+p)^2 q_t(s,p,\pi))\cdot g(p)dp}{2\int_0^1(1+p)^2\cdot q_t(s,p,\pi))\cdot g(p)dp.} \\
&= \frac{3Z(s,\pi)}{2}-\frac{1}{2}\frac{Z(s,\pi)\int_0^1(1+p)^2 g(p)dp}{\int_0^1(1+p)^2\cdot g(p)dp-Z(s,\pi)\int_0^1(1+p)^2(1+p)]g(p)dp} \\
&= \frac{3Z(s,\pi)}{2}+\frac{1}{2}\frac{Z(s,\pi)\int_0^1(1+p)^2 g(p)dp}{\int_0^1(1+p)^2\cdot g(p)dp-Z(s,\pi)\int_0^1(1+p)^3 g(p)dp}. \qquad \text{(B6)}
\end{aligned}$$

The second line follows by splitting the numerator of the second summand; the third by dividing the numerator and denominator; and the forth by plugging in $1-(1+p)Z(s,\pi)$ for $q_t(s,p,\pi)$ from Lemma 1A and splitting the corresponding integral.

Collecting terms and equating to $Z(D,\pi)$ gives

$$\frac{Z(s,\pi)}{2}\left(3+\frac{X(p,2)}{X(p,2)-Z(s,\pi)X(p,3)}\right)=Z(D,\pi), \qquad \text{(B7)}$$

where $X(p,2)\equiv\int_0^1(1+p)^2 g(p)dp$ and $X(p,3)\equiv\int_0^1(1+p)^3 g(p)dp$.

Rearranging, the optimal standard of proof is the value of s that solves

$$\begin{aligned}
Z(\pi,s) &= \\
&\quad Z(D,\pi)+\frac{2X(p,2)}{3X(p,3)} \\
&\quad -\frac{1}{3X(p,3)}\sqrt{4X(p,2)^2-2X(p,3)X(p,2)Z(D,\pi)+X(p,3)^2Z(D,\pi)^2} \\
&\equiv Y(\pi,D). \qquad \text{(B8)}
\end{aligned}$$

Plugging in $\frac{\pi}{1-\pi}\div\frac{D}{1-D}$ into $Z(\pi,s)$ and solving for s the ex ante optimal standard of proof is

$$s^{ea}(D,\pi)=\frac{\pi}{Y(\pi,D)(1-\pi)+\pi}. \qquad \text{(B9)}$$

Case 1 (uniform F) $G(p)=p$.

Plugging in $X(p,2)=\int_0^1(1+p)^2dp=\frac{7}{3}$ and $X(p,3)=\int_0^1(1+p)^3 g(p)dp=\frac{15}{4}$ in Eq. (B8), Eq. (B9) yields $s^{ea}(D,\pi)=\frac{A(D,\pi)+\sqrt{A(D,\pi)^2-540\pi D\cdot B(D,\pi)}}{2B(D,\pi)}$, where $A(D,\pi)\equiv 112D-90\pi-68\pi D$ and $B(D,\pi)\equiv 56+56D+34\pi-11D\pi$.

Case 2 (concave F) $G(p)=2p-p^2$.

Plugging in $X(p,2) = 2\int_0^1 (1+p)^2(1-p)dp = \frac{11}{6}$ and $X(p,3) = 2\int_0^1 (1+p)^3(1-p)g(p)dp = \frac{13}{5}$ in Eq. (B8), Eq. (B9) yields $s^{ea}(D,\pi) = \frac{A'(\pi,D)+\sqrt{A'(\pi,D)^2-468D\pi\cdot B'(\pi,D)}}{2B'(D,\pi)}$, where $A'(D,\pi) \equiv 110D - 78\pi - 46\pi D$ and $B'(D,\pi) \equiv 55 + 55D + 23\pi - 16\pi D$.

Case 3 (convex F) $G(p) = p^2$.

Plugging in $X(p,2) = 2\int_0^1 (1+p)pdp = \frac{17}{6}$ and $X(p,3) = 2\int_0^1 (1+p)^3 pdp = \frac{49}{10}$ in Eq. (B8), Eq. (B9) yields $s^{ea}(D,\pi) = \frac{A''(D,\pi)+\sqrt{A''(D,\pi)^2-1764D\pi\cdot B''(D,\pi)}}{2B''(D,\pi)}$, where $A''(\pi,D) \equiv 340D + 294\pi + 248\pi D$ and $B''(D,\pi) \equiv 170 + 170D + 124\pi - 23\pi D$. □

B.1.2 Proof of Example 1B

Example 1B (optimal standard of proof | concave F)

If $F_3(q) = 2q - q^2$, the optimal standard of proof is $s^{ea}(D) = \frac{3D}{2+D}$ irrespective of π and $G(\cdot)$.

If $F_3 = 2q - q^2$, then $h_{F_3}(q) = \frac{2}{1-q} < \frac{1}{1-q} = h_{F_2}(q_t)$. Substituting $1-(1+p)Z(s,\pi)$ for q_t gives $h_F(q_t)^{-1} = \frac{1}{2}(1+p)Z(s,\pi)$, which is a product of $1+p$. The ratio between the marginal error rates due to prosecutorial bias is therefore equal to half the ratio between the marginal error rates under an unbiased prosecutor: $-\frac{1}{2}\frac{\alpha_2'(s)}{\beta'(s)} = \frac{1}{2}Z(s,\pi) = \frac{\alpha_1'(s)}{\beta'(s)}$. This implies that the *overall* ratio of the marginal error rates under a conviction-oriented prosecutor is one-and-a-half as large as under an unbiased prosecutor. The ex ante optimal standard of proof therefore satisfies $\frac{3}{2}Z(s,\pi) = Z(D,\pi)$, which implies $s^{ea}(D) = \frac{3D}{2+D}$ (because $\frac{3}{2}\frac{1-s^{ea}}{s^{ea}} = \frac{1-D}{D}$). As in the case of $F_2(\cdot)$, the ratio between the marginal error probabilities is proportional to the conviction threshold. Hence, the ex ante optimal standard is independent of both π and $G(\cdot)$.

Example 2B (optimal SoP | uniform F)

If $F_2(q) = q$, the optimal standard of proof is $s^{ea} = \frac{2D}{1+D}$ irrespective of π and $G(\cdot)$.

Proof. Substituting $1 - q_t(s,p,\pi)$ for $1 - F(q_t(s,p,\pi))$ and 1 for $f(q_t(s,p,\pi))$ and $1-(1+p)Z(s,\pi)$ into $q_t(s,p,\pi)$, Eq. (B1) becomes

$$\begin{aligned} Z(s,\pi) &= Z(D,\pi) - \frac{Z(s,\pi)\int_0^1 (1+p)^2 g(p)dp}{\int_0^1 (1+p)^2 g(p)dp} \\ &= Z(D,\pi) - Z(s,\pi). \end{aligned} \tag{B10}$$

It follows $2Z(s,\pi) = Z(D,\pi)$. Solving for s gives $s^{ea} = \frac{2D}{1+D}$. □

B.1.3 Proof of Example 1C

Example 1C (optimal SoP | concave F)

If $F_3(q) = 2q - q^2$, the optimal standard of proof is $s^{ea} = \frac{3D}{2+D} \in (D, \frac{2D}{D+1})$ irrespective of π and $G(\cdot)$.

Proof. Substituting $(1 - q_t(s, p, \pi))^2$ for $1 - F(q_t(s, p, \pi))$, $2(1 - q_t(s))$ for $f(q_t(s))$ and $1 - (1 + p)Z(s, \pi)$ for $q_t(s, p, \pi)$, Eq. (B1) becomes

$$\begin{aligned} Z(s,\pi) &= Z(D,\pi) - \frac{Z(s,\pi)^2 \int_0^1 (1+p)^3 g(p)dp}{2Z(s,\pi)\int_0^1 (1+p)^3 g(p)dp} \\ &= Z(D,\pi) - \tfrac{1}{2}Z(s,\pi). \end{aligned} \tag{B11}$$

Rearranging gives $\frac{3}{2}Z(s, \pi) = Z(D, \pi)$ and therefore $s^{ea} = \frac{3D}{2+D}$.

To verify the second-order condition for a minimum is satisfied at $s^{ea} = \frac{3D}{2+D}$, we substitute -2 for $f'(q_t(s))$ in Eq. (B5):

$$\begin{aligned} \frac{\partial^2}{\partial s^2} EC(s,\pi,D) &\propto -2[Z(D,\pi) - Z(s,\pi)] \times \\ &\qquad \int_0^1 [(1+p)^3 dG(p) + 2Z(s,\pi)^2 \int_0^1 (1+p)^3 dG(p) \\ &= 2\int_0^1 [(1+p)^3 dG(p) \times [Z(s,\pi) - Z(D,\pi) + Z(s,\pi)^2]. \end{aligned} \tag{B12}$$

Therefore,

$$\frac{\partial^2}{\partial s^2} EC(s,\pi,D) \propto Z(s,\pi) - Z(D,\pi) + Z(s,\pi)^2. \tag{B13}$$

Plugging in $\frac{3D}{2+D}$ for s and simplifying gives

$$\frac{\partial^2}{\partial s^2} EC(s,\pi,D) \propto \frac{\pi}{1-\pi} \times \frac{2D-1}{3D} + Z(s,\pi)^2 > 0 \tag{B14}$$

for $D \in [\frac{1}{2}, 1)$. □

B.2 Proof of Example 2

Example 2 (optimal standard of proof | symmetric signal | uniform distribution) *If $\sigma \sim U[0, 1]$, the ex ante optimal threshold signal under a conviction-oriented prosecutor is $\sigma_t^{ea}(D, \pi) = \frac{1}{1+Z(D,\pi)}$ with a corresponding standard of proof of $s^{ea}(D, \pi) = \frac{\pi(2+Z(D,\pi))}{2\pi+Z(D,\pi)} > D = s^{ep}(D)$.*

Proof. From Eq. (20), the ratio of the marginal error terms with respect to the threshold

probative signal is

$$-\frac{\alpha'(\sigma_t)}{\beta'(\sigma_t)} = \frac{f(\sigma_t)(1-\sigma_t)+(1-F(\sigma_t))}{f(\sigma_t)(1+\sigma_t)-(1-F(\sigma_t))}. \tag{B15}$$

Substituting 1 for $f(\sigma_t)$ and $1-\sigma_t$ for $1-F(\sigma_t)$ and simplifying, we get

$$-\frac{\alpha'(\sigma_t)}{\beta'(\sigma_t)} = \frac{1-\sigma_t}{\sigma_t}. \tag{B16}$$

Equating to $Z(D,\pi)$ and solving for σ_t gives $\sigma_t^{ea}(D,\pi) = \frac{1}{1+Z(D,\pi)}$. To recover the optimal standard of proof, plug in σ_t^{ea} into $s(\sigma_t,\pi) = \frac{\pi(1-\sigma_t)}{1-\sigma_t(1-2\pi)}$ (from Lemma 3), which yields $s^{ea}(D,\pi) = \frac{\pi(2+Z(D,\pi))}{2\pi+Z(D,\pi)}$.

To show that $s(\sigma_t,\pi) > D$, plug in $\frac{\pi/(1-\pi)}{D/1-D)}$ into $Z(\pi,D)$ in $s^{ea}(D,\pi)$. Subtracting D gives $(1-D)(\pi-D(2\pi-1))$. This expression is strictly positive for $\pi \leq \frac{1}{2}$ (because for such values of π, $D(2\pi-1)) \leq 0$. For $\pi > \frac{1}{2}$, the supremum of $D(2\pi-1)$ is $2\pi-1$ for $D=1$ and $\pi=1$. But $\pi-(2\pi-1) = 1-\pi > 0$ for $\pi < 1$. Therefore, $s^{ea}(D,\pi) > D$.

B.3 Proof of Example 3

Example 3 (type II error pr. | symmetric signal | exponential distribution) *Suppose $\sigma \sim F(\sigma,\lambda) = \sigma^\lambda$ for $\lambda > 0$ and that $D = \frac{1}{2}$. There exist cutoffs $\underline{\lambda} \in (1,2)$ and $\underline{\pi}(\lambda) \in (0,\frac{1}{2})$, where $\underline{\pi}(\lambda)$ is decreasing in λ, such that for $\lambda > \underline{\lambda}$ and $\pi \in (\underline{\pi}(\lambda),\frac{1}{2})$, the probability of type II error under a conviction-oriented prosecutor is lower under the ex ante than under the ex post optimal standard of proof.*

Proof. The proof is divided into four parts.

Part I shows that the probability of type II error is decreasing in π under the ex ante optimal standard of proof and is non-monotone in π for $\lambda \geq 1$ under the ex post optimal standard, where the minimum probability of type II error is decreasing in λ.

Part II shows that the probability of type II error under both the ex ante and ex post optimal standards is decreasing in λ.

Part III shows that the probability of type II error is lower under the ex post than under the ex ante optimal standard of proof for $\lambda = 1$; and is higher under the ex post than under the ex ante optimal standard of proof for $\lambda = 2$ and π sufficiently close to $\frac{1}{2}$.

Part IV concludes the proof by showing that there exists a cutoff $\underline{\lambda} \in (1,2)$ such that for $\lambda > \underline{\lambda}$, the probability of type II error is higher under the ex post than under the ex ante optimal standard of proof for π sufficiently close to $\frac{1}{2}$ depending on λ.

Now, from Eqs. (16) and (17), the probability of type II error under a conviction-oriented prosecutor as a function of the threshold probative signal is

$$\beta(\sigma,\lambda) = \sigma^\lambda + \tfrac{1}{2}(1-\sigma^\lambda)(1-\sigma).$$

We shall denote, respectively, the probability of type II error under the ex post and ex ante optimal standards of proof as

$$\beta^{ep}(\pi, \lambda) \equiv \beta(\sigma_t^{ep}(\pi), \lambda),$$

and

$$\beta^{ea}(\pi, \lambda) \equiv \beta(\sigma_t^{ea}(\pi, \lambda), \lambda),$$

where $\sigma_t^{ep}(\pi) = 1 - 2\pi$ and $\sigma_t^{ea}(\pi, \lambda) > \sigma_t^{ep}(\pi)$ is the (positive and real) solution to the equation $\sigma_t = \sigma_t^{ep}(\pi) + \frac{1-\sigma_t^{\lambda}}{\lambda \sigma_t^{\lambda-1}}$ (see Proposition 3).

Before proceeding to the proof, we present an ancillary claim.

Claim B2 *The hazard rate $h(\sigma, \lambda) = \frac{\lambda \sigma^{\lambda-1}}{1-\sigma^{\lambda}}$ is (1) increasing in σ and (2) decreasing in λ.*

Proof. (1) Partially differentiating $h(\sigma, \lambda)$ with respect to σ gives

$$\frac{\partial}{\partial \sigma} h(\sigma, \lambda) = \frac{\lambda \sigma^{\lambda-2}(\sigma^{\lambda} + \lambda - 1)}{(1-\sigma^{\lambda})^2} > 0, \tag{B17}$$

because $\sigma^{\lambda} + \lambda - 1 > 0$ for $\lambda > 1$.
(2) Partially differentiating $h(\sigma, \lambda)$ with respect to λ gives

$$\frac{\partial}{\partial \lambda} h(\sigma, \lambda) = \frac{\sigma^{\lambda-1}}{(1-\sigma^{\lambda})^2}(1 - \sigma^{\lambda} + \lambda \log \sigma) \tag{B18}$$

whose sign depends on the sign of $\phi(\sigma, \lambda) \equiv 1 - \sigma^{\lambda} + \lambda \log \sigma$. Now, $\frac{\partial}{\partial \lambda}\phi(\sigma, \lambda) = (1 - \sigma^{\lambda}) \log(\sigma) \leq 0$, whereas $\frac{\partial}{\partial \sigma}\phi(\sigma, \lambda) = \frac{\lambda(1-\sigma^{\lambda})}{\sigma} > 0$. Because $\phi(\sigma, \lambda)|_{\sigma=1, \lambda=1} = 0$, it follows that $\phi(\sigma, \lambda) < 0$ for $\lambda > 1$. ∥

Part I *$\beta^{ea}(\pi, \lambda)$ is monotonically decreasing in π irrespective of λ, whereas $\beta^{ep}(\pi, \lambda)$ is non-monotone in π for $\lambda \geq 1$.*

Step 1 *$\frac{\partial}{\partial \pi}\beta(\sigma_t, \lambda) < 0$ if and only if $\sigma_t < \frac{1}{h(\sigma_t, \lambda)} - 1$ or, equivalently, $h(\sigma_t, \lambda) < \frac{1}{1+\sigma_t}$.*

The partial derivative of $\beta(\sigma_t, \lambda)$ with respect to σ_t is

$$\frac{\partial}{\partial \sigma}\beta(\sigma_t, \lambda) = \tfrac{1}{2}[f(\sigma_t, \lambda) \cdot (1 + \sigma_t) - (1 - F(\sigma_t, \lambda)], \tag{B19}$$

which is positive iff $\sigma_t < \frac{1}{h(\sigma_t, \lambda)} - 1$. Multiplying through by $\frac{h(\sigma_t, \lambda)}{\sigma_t}$ and rearranging gives $h(\sigma_t, \lambda) < \frac{1}{1+\sigma_t}$.

Step 2 *$\frac{\partial}{\partial \pi}\beta^{ep}(\pi, \lambda) < 0$ for $\pi \in (0, \frac{1}{2})$ irrespective of λ.*

From Step 1, $\beta^{ea}(\pi,\lambda)$ is decreasing in π if and only if

$$\sigma_t(\pi) < \frac{1}{h(\sigma_t(\pi),\lambda)} - 1. \tag{B20}$$

Now, $\sigma_t^{ea}(\pi) = \sigma_t^{ep} + \frac{1}{h(\sigma_t^{ea}(\pi))} > \frac{1}{h(\sigma_t^{ea}(\pi),\lambda)} - 1$, where the equality follows from

Step 3 *There exists a cutoff prior probability* $\pi_{\min}(\lambda) \in (0, \frac{1}{2})$ *such that* $\frac{\partial}{\partial\pi}\beta^{ep}(\pi,\lambda) < 0$ *for* $\pi \in [0, \pi_{\min}(\lambda))$ *and* $\frac{\partial}{\partial\pi}\beta^{ep}(\pi,\lambda) > 0$ *for* $\pi \in (\pi_{\min}(\lambda), \frac{1}{2})$.

From Step 1, $\beta^{ep}(\pi,\lambda)$ is decreasing in π if and only if $h(\sigma_t^{ep}(\pi),\lambda) < \frac{1}{1+\sigma_t^{ep}(\pi)}$. Let $\pi_{\min}(\lambda)$ denote the value of π that satisfies Eq. (B20) as an equality. For $\lambda = 1$ we have

$$h(\sigma_t^{ep}(\pi),\lambda)|_{\lambda=1} = \frac{1}{1-\sigma_t^{ep}(\pi)}.$$

Equating to $\frac{1}{1+\sigma_t^{ep}(\pi)}$ and solving for π gives $\pi = \frac{1}{2}$. Now, $\frac{\partial}{\partial\sigma}h(\sigma,\lambda) > 0$ From Claim B2 and $\frac{d}{d\sigma}\sigma_t^{ep}(\pi) = -2$. Therefore, $\frac{\partial}{\partial\sigma}\, h(\sigma_t(\pi))|_{\sigma_t=\sigma_t^{ep}} = \frac{h(\sigma_t)}{\partial\sigma_t}\frac{d\sigma_t}{d\pi}\Big|_{\sigma_t=\sigma_t^{ep}} < 0$. Because $\frac{1}{1+\sigma_t^{ep}(\pi)}$ is increasing in π, it follows that $\beta(\sigma_t^{ep}(\pi))$ is decreasing in π for $\pi \in (0, \pi_{\min}(\lambda))$.

Step 4 $\frac{d}{\partial\lambda}\pi_{\min}(\lambda) < 0$.

From Step 3, $\pi_{\min}(\lambda)$ is the value of π that satisfies Eq. (B20) as an equality. By the implicit function theorem

$$\frac{d\pi_{\min}(\lambda)}{d\lambda} = -\frac{\frac{\partial}{\partial\sigma_t}h(\sigma_t(\pi),\lambda)\frac{d\sigma_t}{d\pi}\Big|_{\sigma_t=\sigma_t^{ep}}}{\frac{\partial}{\partial\pi}h(\sigma_t(\pi),\lambda)\Big|_{\sigma_t=\sigma_t^{ep}}} < 0 \tag{B21}$$

because $\frac{\partial}{\partial\sigma_t}h(\sigma_t(\pi),\lambda) > 0$ and $\frac{\partial}{\partial\pi}h(\sigma_t(\pi),\lambda) < 0$ from Claim B2 and $\frac{d\sigma_t^{ep}(\pi)}{d\pi} = -2$.

Step 5 $\beta^{ea}(\pi,\lambda) > \beta^{ep}(\pi,\lambda)$ *for* $\pi \in (0, \pi_{\min}(\lambda))$.

From Step 4, both $\beta^{ep}(\pi,\lambda)$ and $\beta^{ep}(\pi,\lambda)$ are decreasing in π for $\pi \in (0, \pi_{\min}(\lambda))$. Because, from Proposition 3 and Step 2, respectively, $\sigma_t^{ea}(\pi) > \sigma_t^{ep}(\pi)$ and $\beta'(\sigma_t) < 0$ for $\pi \in (0, \pi_{\min}(\lambda))$, it follows that $\beta^{ea}(\pi,\lambda) > \beta^{ep}(\pi,\lambda)$ for $\pi \in (0, \pi_{\min}(\lambda))$

Part II *Both* $\beta^{ep}(\pi,\lambda)$ *and* $\beta^{ea}(\pi,\lambda)$ *are decreasing in* λ.

Step 1 $\frac{\partial}{\partial\lambda}\beta^{ep}(\pi,\lambda) < 0$.

Plugging in $\sigma_t^{ea}(\pi) = 1 - 2\pi$ into σ_t and simplifying gives

$$\begin{aligned}\beta(\sigma_t,\lambda)|_{\sigma_t=1-2\pi} &= (1-2\pi)^\lambda + \tfrac{1}{2}[1-(1-2\pi)^\lambda](2\pi) \\ &= \pi + (1-2\pi)^\lambda(1-\pi),\end{aligned} \tag{B22}$$

which is decreasing in λ for $\pi \in (0, \frac{1}{2})$.

Step 2 $\frac{\partial}{\partial \lambda}\beta^{ea}(\pi, \lambda) < 0$.

Taking the total derivative of $\beta(\sigma_t^{ea}(\lambda, \pi), \lambda)$ with respect to λ gives

$$\frac{d\beta(\sigma_t^{ea}(\lambda, \pi), \lambda)}{d\lambda} = \frac{\partial \beta}{\partial \sigma_t}\frac{d\sigma_t^{ea}}{d\lambda} + \frac{\partial \beta}{\partial \lambda}. \tag{B23}$$

We proceed to show that (i) $\frac{\partial \beta}{\partial \sigma_t}\Big|_{\sigma_t = \sigma_t^{ea}} < 0$, (ii) $\frac{d\sigma_t^{ea}}{d\lambda} > 0$ and (iii) $\frac{\partial \beta}{\partial \lambda} < 0$.

(i) $\frac{\partial \beta}{\partial \sigma_t}\Big|_{\sigma_t = \sigma_t^{ea}} < 0$.

Taking the partial derivative of $\beta(\sigma_t, \lambda)$ with respect to σ_t gives

$$\begin{aligned} \frac{\partial \beta}{\partial \sigma_t} &= \frac{\partial}{\partial \sigma_t}[\sigma_t^\lambda + \tfrac{1}{2}(1 - \sigma_t^\lambda)(1 - \sigma_t)] \\ &= \lambda \sigma_t^{\lambda - 1} - \tfrac{1}{2}\lambda \sigma_t^{\lambda - 1}(1 - \sigma_t) - \tfrac{1}{2}(1 - \sigma_t^\lambda), \end{aligned} \tag{B24}$$

which is strictly negative if $\sigma_t > \frac{1}{h(\sigma_t, \lambda)} - 1$. Because $\sigma_t^{ea} = \sigma_t^{ep} + \frac{1}{h(\sigma_t^{ea}, \lambda)} > \frac{1}{h(\sigma_t^{ea}, \lambda)} - 1$ from Proposition 3, it follows that $\frac{\partial \beta}{\partial \sigma_t}\Big|_{\sigma_t = \sigma_t^{ea}} < 0$.

(ii) $\frac{d\sigma_t^{ea}}{d\lambda} > 0$.

By Proposition 3, the ex ante optimal signal is the value of σ that solves the equality $\varphi(\sigma_t, \lambda) \equiv \sigma_t^{ep} + \frac{1}{h(\sigma_t, \lambda)} - \sigma_t = 0$. By the implicit function theorem

$$\frac{d\sigma_t^{ea}}{d\lambda} = -\frac{\frac{\partial}{\partial \lambda}\varphi(\sigma_t, \lambda)}{\frac{\partial}{\partial \sigma_t}\varphi(\sigma_t, \lambda)} = \frac{\partial}{\partial \lambda}\frac{1}{h(\sigma_t, \lambda)},$$

where the second equality follows because $\frac{\partial}{\partial \sigma_t}\varphi(\sigma_t, \lambda) = -1$. Because $\frac{\partial}{\partial \lambda}h(\sigma, \lambda) < 0$ by Claim B2, it follows that $\frac{d\sigma_t^{ea}}{d\lambda}$.

(iii) $\frac{\partial \beta}{\partial \lambda} < 0$.

The partial derivative of $\beta(\sigma_t, \lambda)$ with respect to λ is

$$\begin{aligned} \frac{\partial \beta}{\partial \lambda} &= \frac{\partial}{\partial \lambda}[\sigma_t^\lambda + \tfrac{1}{2}(1 - \sigma_t^\lambda)(1 - \sigma_t)] \\ &= \tfrac{1}{2}\log \sigma_t \sigma_t^\lambda (1 + \sigma_t) \\ &< 0 \end{aligned} \tag{B25}$$

because the first factor is strictly negative and the second strictly positive.

Part III *$\beta^{ea}(\pi, \lambda)|_{\lambda=1} > \beta^{ep}(\pi, \lambda)|_{\lambda=1}$ for $\pi \in (0, \frac{1}{2})$, whereas $\beta^{ea}(\pi, \lambda)|_{\lambda=2} < \beta^{ep}(\pi, \lambda)|_{\lambda=2}$ for π sufficiently close to $\frac{1}{2}$.*

Step 1 *$\beta^{ea}(\pi,\lambda)|_{\lambda=1} > \beta^{ep}(\pi,\lambda)|_{\lambda=1}$ for $\pi \in (0,\frac{1}{2})$.*

The ex post and ex ante optimal threshold signals for $\lambda = 1$ are $\sigma_t^{ep}(\pi) = 1-2\pi$ and $\sigma_t^{ea}(\pi) = 1-\pi$. The corresponding probabilities of type II error are

$$\begin{aligned}\beta(\sigma_t^{ep}(\pi),\lambda)|_{\lambda=1} &= (1-2\pi)+2\pi^2 \\ &> (1-\pi)+\tfrac{1}{2}\pi^2 = \beta(\sigma_t^{ea}(\pi,\lambda),\lambda)|_{\lambda=1}\,, \end{aligned} \tag{B26}$$

where the inequality follows because $\pi - \frac{3}{2}\pi^2 > 0$ for $\pi \in (0,\frac{1}{2})$.

Step 2 $\lim_{\pi\to\frac{1}{2}} \beta^{ea}(\pi,\lambda)|_{\lambda=2} > \lim_{\pi\to\frac{1}{2}} \beta^{ea}(\pi,\lambda)|_{\lambda=2} = \frac{1}{2}$.

From Proposition 3, the ex ante optimal threshold probative signal for $\lambda = 2$ is the positive solution to the equality

$$\sigma_t = \sigma_t^{ep}(\pi) + \frac{1-\sigma_t^2}{2\sigma_t}, \tag{B27}$$

where $\sigma_t^{ea}(\pi) \to \frac{1}{\sqrt{3}}$ as $\pi \to \frac{1}{2}$. The limit of the corresponding probability of type II error as $\pi \to \frac{1}{2}$ is therefore

$$\lim_{\pi\to\frac{1}{2}} \beta^{ea}(\pi,\lambda)|_{\lambda=2} = \tfrac{1}{3} + \tfrac{1}{2}\left(1-\tfrac{1}{3}\right)\left(1-\tfrac{1}{\sqrt{3}}\right) < \tfrac{1}{2}, \tag{B28}$$

because the left-hand side simplifies to $\frac{2}{3} - \frac{1}{3}\cdot\frac{1}{\sqrt{3}} < \frac{2}{3} - \frac{1}{3}\cdot\frac{1}{2} = \frac{1}{2}$. Under the ex post optimal standard of proof, by contrast, $\lim_{\pi\to\frac{1}{2}} \beta(\sigma_t^{ep}(\pi)) = \frac{1}{2}$ irrespective of λ by Step 1 of Proposition 4. ‖

Step 3 *There exists a cutoff $\widehat{\pi} \in (\pi_{\min}(\lambda),\frac{1}{2})$ such that*

$$\beta^{ep}(\pi,\lambda)|_{\lambda=2} > \beta^{ea}(\pi,\lambda)|_{\lambda=2} \text{ if and only if } \pi \in (\widehat{\pi},\tfrac{1}{2}).$$

From Part I, (i) $\beta^{ep}(\pi,\lambda)$ is decreasing in π for $\pi \in (0,\pi_{\min}(\lambda))$ and increasing in π for $\pi \in (\pi_{\min},\frac{1}{2})$; (ii) $\beta^{ea}(\pi,\lambda)$ decreasing in π throughout $\pi \in (0,\frac{1}{2})$ irrespective of λ; and (iii) $\beta^{ea}(\pi,\lambda) > \beta^{ep}(\pi,\lambda)$ for $\pi \in (0,\pi_{\min}(\lambda))$ and $\lambda > 1$.

Now, $\beta^{ea}(\pi,\lambda) < \beta^{ep}(\pi,\lambda)$ as $\pi \to \frac{1}{2}$ for $\lambda = 2$. Because $\beta^{ep}(\pi,\lambda)|_{\lambda=2}$ is decreasing and then increasing on $\pi \in (0,\frac{1}{2})$ and $\beta^{ea}(\pi,\lambda)|_{\lambda=2}$ is strictly decreasing on $\pi \in (0,\frac{1}{2})$, there exists $\widehat{\pi} \in (\pi_{\min}(\lambda),\frac{1}{2})$ such that $\beta^{ep}(\widehat{\pi},\lambda)|_{\lambda=2} = \beta^{ea}(\widehat{\pi},\lambda)|_{\lambda=2}$ and therefore $\beta^{ep}(\pi,\lambda)|_{\lambda=2} > \beta^{ea}(\pi,\lambda)|_{\lambda=2}$ if and only if $\pi \in (\widehat{\pi},\frac{1}{2})$. ‖

Part IV *There exist cutoffs $\underline{\lambda} \in (1,2)$ and $\underline{\pi}(\lambda) \in (0,\frac{1}{2})$, where $\underline{\pi}(\lambda)$ is decreasing in λ, such that $\beta^{ep}(\pi,\lambda) > \beta^{ea}(\pi,\lambda)$ for $\lambda > \underline{\lambda}$ and $\pi \in (\underline{\pi}(\lambda),\frac{1}{2})$.*

Step 1 *There exists a cutoff* $\underline{\lambda} \in (1,2)$ *such that*

$$\lim_{\pi \to \frac{1}{2}} \beta^{ea}(\pi, \lambda)|_{\lambda=\underline{\lambda}} = \tfrac{1}{2} = \lim_{\pi \to \frac{1}{2}} \beta^{ea}(\pi, \lambda)|_{\lambda=\underline{\lambda}}.$$

From Part I, the graphs of $\beta^{ep}(\pi, \lambda)$ and $\beta^{ea}(\pi, \lambda)$ intersect at most once along the increasing portion of $\beta^{ep}(\pi, \lambda)$. by Part II, $\beta^{ep}(\pi, \lambda) > \beta^{ea}(\pi, \lambda)$ for $\pi \in (0, \frac{1}{2})$ and $\lambda = 1$, whereas $\beta^{ep}(\pi, \lambda) < \beta^{ea}(\pi, \lambda)$ for $\pi \in (\widehat{\pi}, \frac{1}{2})$ and $\lambda = 2$, where $\widehat{\pi} \in (\pi_{\min}(\lambda), \frac{1}{2})$. Since, by Part III, both $\beta^{ep}(\pi, \lambda)$ and $\beta^{ea}(\pi, \lambda)$ are decreasing in λ, there exists $\underline{\lambda} \in (1,2)$ such that for $\lambda = \underline{\lambda}$, the probability of type II error under both the ex post and ex ante optimal standards of proof converge to $\frac{1}{2}$ as $\pi \to \frac{1}{2}$. ‖

Step 2 *There exists a cutoff* $\underline{\pi}(\lambda) \in (0, \frac{1}{2})$*, where* $\underline{\pi}'(\lambda) < 0$*, such that*

$$\beta^{ep}(\pi, \lambda) > \beta^{ea}(\pi, \lambda) \text{ for } \lambda > \underline{\lambda} \text{ and } \pi \in (\underline{\pi}(\lambda), \tfrac{1}{2}).$$

From the previous step, there exists a cutoff $\underline{\lambda} = \sup\{\lambda : \beta^{ep}(\pi, \lambda) > \beta^{ea}(\pi, \lambda) \in (1,2)\}$ for some $\pi \in (0, \frac{1}{2})$. Now, by Part I, (i) $\beta^{ep}(\pi, \lambda)$ is increasing in π for $\pi \in (\pi_{\min}(\lambda), \frac{1}{2})$ for $\lambda > 1$, where $\pi'_{\min}(\lambda) < 0$; and (iii) $\beta^{ea}(\pi, \lambda)$ is decreasing in π for $\pi \in (0, \frac{1}{2})$ irrespective of λ. It follows that for $\lambda > \underline{\lambda}$, $\beta^{ea}(\pi, \lambda)$ and $\beta^{ea}(\pi, \lambda)$ intersect once at a prior probability of guilt $\underline{\pi}(\lambda) \in (\pi_{\min}(\lambda), \frac{1}{2})$, where ${}^{ep}(\pi, \lambda) > \beta^{ea}(\pi, \lambda)$ for $\lambda > \underline{\lambda}$ and $\pi \in (\underline{\pi}(\lambda), \frac{1}{2})$. Finally, because both $\beta^{ea}(\pi, \lambda)$ and $\beta^{ea}(\pi, \lambda)$ are decreasing in λ by Part II, $\underline{\pi}(\lambda)$ is decreasing in λ. □

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