Reconciling Wage and Price Phillips Curves [*]

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Abstract

Contrary to the traditional Phillips Curve, the data suggests that the Price Phillips Curve steepens in excess of the Wage Phillips Curve during especially tight labor markets. I provide evidence for this excess steepening on US aggregate and cross-sectional data and in a cross-section of developed countries. This excess steepening is correlated with market concentration in both country and sectoral data. I attribute the excess steepening to a User Cost of Labor based decision to price out demand rather than raise wages and provide evidence for this mechanism on firm survey data. I introduce the mechanism into a NK-SAM framework with monopsony power and downward nominal wage rigidity, leading to the non-linear steepening of the Price Phillips Curve in tight labor markets. The excess steepening of the Price Phillips Curve is important for monetary policy due to its implications for a painless disinflation. Additionally, the resulting decrease in real wage growth during especially tight labor markets yields a labor market target region for policy makers.

Keywords: Phillips Curve, Downward Nominal Wage Rigidity, User Cost of Labor, Profit Margins, Market Concentration, Monetary Policy.
JEL-Classification: E24, E31, E52, E6.

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1 Introduction

The Phillips Curve has been a central component of macroeconomics since its discovery and refinement by Phillips (1958) and Samuelson and Solow (1960) respectively. They explain the relation between tight labor markets and inflation with faster wage growth due to higher bargaining power for workers. This view still informs policy, as evidenced by Federal Reserve Governor Kugler (March 2024): "In a context of strong aggregate demand, firms will likely grant the wage increases to retain their workers — then pass through the resulting cost increase as higher prices". Most researchers follow this conventional view by quoting Phillips' original argument to justify their Phillips Curve estimations (Benigno and Eggertsson (2023)). Some even make it explicit in their models (Bernanke and Blanchard (2023)) by allowing labor market tightness to affect prices only through wages.[1] These models imply that labor market tightness impacts price inflation at most as much as wage growth.

However, recent results are inconsistent with this view. The Price Phillips Curve (PPC) is found to steepen non-linearly (Ball et al. (2022), Benigno and Eggertsson (2023), Hooper et al. (2020)), while the Wage Phillips Curve (WPC) remains linear over the same data (Ball et al. (2022), Bernanke and Blanchard (2023), Hooper et al. (2020)).[2] The goal of this paper is to highlight and explain this puzzling divergence of the Wage and Price Phillips Curves. I reconcile the divergence with the conventional view by introducing a channel through which labor market tightness affects inflation directly for given wage growth. This implies a reduced form Phillips Curve that includes both the conventional wage and the new direct channel.

The direct impact is significant on a range of US data. First, the channel is verified on various inflation measures by explicitly controlling for current and past wage growth, import price pressures, and labor productivity, as well as excluding the pandemic period. Despite these controls, the non-linear excess steepening of the PPC persists. The contrasting linear nature of the WPC indicates that the direct impact channel is the driver of Phillips Curve non-linearity. A similar steepening is observed in profit margins in response to labor market tightness, with the reverse effect seen in real wage growth.

Furthermore, I find that the excess steepening holds in a cross-section of U.S. service sectors, and across countries. A country grouping exercise using LASSO estimation further confirms that all but a handful of countries in my panel exhibit a similar excess steepening of the PPC as the U.S. Additionally, I show that market concentration, as measured by the Herfindahl-Hirschman Index, plays a crucial role in driving the excess steepening across countries and service sectors. Lower concentration mutes the direct channel, while higher concentration leads to a greater excess steeping of the PPC during tight labor markets. I

[1] Labor market tightness is defined as the ratio of job vacancies and the number of unemployed, thus combining measures of labor demand and labor supply to one coherent labor market measure.

[2] Hooper et al. (2020) test the PPC on MSA and the WPC on state level data.

further confirm my US profit margin results in the country cross-section.

Based on the profit margin and market concentration results I conclude that this steepening is driven by optimal price setting. I postulate that firms prioritize pricing out demand over raising wages when faced with transitory upward wage pressure due to downward nominal wage rigidity (DNWR). Thus, firms price out a demand overhang if fulfilling the demand would raise wages above expected future wages, similar to Kudlyak (2014)'s User Cost of Labor, which leads to the excess steepening of the Price Phillips Curve.

I validate the channel using firm survey data from the National Federation of Independent Business (NFIB), which provides information on expected real demand growth and the difficulty of filling open positions. The interaction between these two variables negatively impacts inflation, consistent with my proposed channel. I incorporate this channel into an illustrative model, which features a monopolist firm facing iso-elastic labor supply and demand schedules. Introducing a receding positive demand or negative labor supply shock results in price increases that exceed the typical markup if wages are downwardly rigid.

Finally, I incorporate the channel into a NK-SAM framework by giving firms wage setting power and enforcing downward nominal wage rigidity (DNWR). The combination of firms setting wages and labor market frictions creates monopsony power (Bloesch et al. (2024)). I discipline wage-setting power through on-the-job search, and requiring firms to pay the same wage to all workers. I model general nominal frictions using Rotemberg adjustment costs. Additionally, I enforce absolute DNWR. Firms experiencing idiosyncratic and non-permanent nominal demand shocks incorporate the UCL into their optimal price-setting behavior. By introducing a Lucas-style informational friction, where firms imperfectly distinguish between aggregate and idiosyncratic demand shocks, I introduce the UCL channel into the aggregate PPC.

In summary, this paper provides substantial evidence for an excess steepening of the PPC, along with a viable explanation supported by firm survey data. Additionally, this paper is the first to estimate V/U-based Phillips Curves across cross-sections of countries and sectors, as well as the first to incorporate the user cost of labor into a NK-SAM framework.

The UCL channel provides a more robust explanation for the low cost of disinflation observed over the past two years. As prices rise faster than wages under the UCL channel, there is more room for prices to decline without hitting the downward nominal wage rigidity (DNWR) constraint. This dynamic leads to smaller increases in unemployment and, consequently, a smaller reduction in real output. Moreover, the UCL channel establishes a labor market target region that fosters real wage growth. Under traditional Phillips Curve assumptions (Benigno and Eggertsson (2023)), real wage growth continues to rise and plateaus as labor market tightness increases. However, the UCL channel leads to declining real wage growth once a certain level of labor market tightness is breached. In this way, the

channel provides a labor market tightness target region to maximize real wage growth.

This paper contributes to the expansive literature on the Phillips Curve. Its closest relation is to three particular strands of this literature. First and foremost, it contributes to the rapidly expanding literature on the non-linear nature of the Phillips Curve (Babb and Detmeister (2017), Gagnon and Collins (2019), Cristini and Ferri (2021)). It provides evidence for a greater non-linearity of the Price Phillips Curve than in the Wage Phillips Curve, as indicated by the findings of Hooper et al. (2020), and Ball et al. (2022). This paper thus validates the indication of Dao et al. (2024) that Ball et al. (2022)'s cubic Phillips Curve is robust to controlling for current wage growth. Furthermore, the paper explores the underlying cause of this non-linearity. The most commonly cited reason for the curve's non-linearity is the repetition of Phillips (1958)' original argument of rapidly rising wages during tight labor markets (Benigno and Eggertsson (2023), Gitti (2024)). Other explanations include heterogeneous downward nominal wage rigidity for individual labor varieties (Schmitt-Grohé and Uribe (2022)), and the amplification of cost-push shocks through quasi-kinked demand for goods (Harding et al. (2023)). I contribute to this literature by showing that the Phillips Curve non-linearity is primarily driven by optimal price setting instead of increasing wages or other cost-push shocks.

Second, this paper adds to the literature that explains Phillips Curve non-linearity through nominal downward wage rigidity. The idea that downward rigid wages bend the Phillips Curve is not new (Schmitt-Grohé and Uribe (2022), Gagnon and Collins (2019), Donayre and Panovska (2016), Gitti (2024)). However, this literature has so far focused on how downward wage rigidity flattens the Phillips Curve during slack labor markets. I extend this strand by showing that downward nominal wage rigidity also leads to a steepening of the Phillips Curve during especially tight labor markets. I explain this steep increase of the Phillips Curve with optimal price setting reacting to downward nominal wage rigidity during tight labor markets.

Finally, this paper extends the literature on the pandemic inflation surge, and the importance of the labor market in explaining it. Part of this literature (Bernanke and Blanchard (2023)) find only a limited role of the labor market. However, they estimate its impact solely through the conventional wage growth channel, by including wage growth instead of labor market tightness in their inflation estimation. In contrast, other papers (Ball et al. (2022)) find a dominant role of labor market tightness in the inflation narrative, when estimating the impact of labor market tightness on inflation. The direct impact channel reconciles these two dissonant findings. Bernanke and Blanchard (2023) only capture the small impact of the traditional Phillips Curve channel, and miss the non-linear rise in inflation stemming from the direct impact channel. For further details see Osten (2025b).

The rest of the paper is structured as follows. Section 2 highlights the puzzle in the data. Section 3 provides the empirical evidence for the direct impact channel on inflation, wage

growth, real wage growth and profit margins. Section 4 quantifies the direct impact channel during the pandemic inflation surge. Section 5 and 6 present the evidence across countries and US service sectors, respectively. Section 7 illustrates the mechanism of the direct impact channel in a simple framework. Section 8 lays out the New Keynesian Framework incorporating the channel, and Section 9 concludes.

2 Diverging Wage and Price Phillips Curves - A Puzzle

This section presents the divergence of the Wage and Price Phillips Curves found by the previous literature. In particular, Hooper et al. (2020) and Ball et al. (2022) find a greater steepening of the Price than the Wage Phillips Curve during tight labor markets on cross-sectional and aggregate US data respectively. I use Ball et al. (2022)'s approach on aggregate data to illustrate these findings of diverging Wage and Price Phillips Curves. I simplify Ball et al. (2022)'s equations to the following Wage and Price Phillips Curves[3]:

$$\pi_t - \pi_t^e = \alpha_1(V/U)_t + \alpha_2(V/U)_t^2 + \alpha_3(V/U)_t^3 + \beta \text{Prod}_t + e_t \tag{1}$$

$$\omega_t - \pi_t^e = \acute{\alpha}_1(V/U)_t + \acute{\alpha}_2(V/U)_t^2 + \acute{\alpha}_3(V/U)_t^3 + \acute{\beta} \text{Prod}_t + \acute{e}_t \tag{2}$$

where π is Median CPI inflation, π^e is the median 10 year inflation forecast from the Survey of Professional Forecasters, V/U is the ratio of job openings to unemployed, Prod is an eight quarter moving average of productivity growth and ω is wage growth, based on ECI wages and salaries. I show estimation results of equations 1 and 2 on 1985Q1 to 2023Q4 US aggregate data in Table 1.

Note the discrepancy between the Price and the Wage Phillips Curve, with median inflation reacting significantly cubic, while wage growth exhibits merely a linear relation to labor market tightness. Figure 1 graphs the cubic labor market functions of Table 1. The divergence of Wage and Price Phillips Curves is also found under different functional forms such as quadratic, log-linear threshold, or spline models (Hooper et al. (2020)).

The divergence of the Wage and Price Phillips Curves under tight labor markets constitutes a puzzle under the conventional Phillips Curve view of rising wages causing inflation. The divergence gives rise to two hypothesis. First, tight labor markets exert an influence on inflation above and beyond their influence on wage growth. Second, this direct impact is a key driver of Phillips Curve non-linearity. I provide evidence on these hypothesis in the ensuing section.

[3] Ball et al. (2022) further include linear, quadratic and cubic "HeadShocks" which they define as the difference of headline inflation and median inflation.

Table 1: Diverging Impacts on Inflation and Wage Growth

	π	ω
V/U	5.99^{***}	4.44^{*}
	(2.08)	(2.33)
$(V/U)^2$	-6.52^{***}	-2.47
	(2.46)	(2.74)
$(V/U)^3$	2.72^{***}	0.78
	(0.82)	(0.90)
Prod	-0.09^{*}	0.22^{***}
	(0.05)	(0.06)
Constant	-1.62^{***}	-2.20^{***}
	(0.49)	(0.54)
R^2	0.81	0.54
N	156	156

Standard errors in parentheses

* $p < .10$, ** $p < .05$, *** $p < .01$

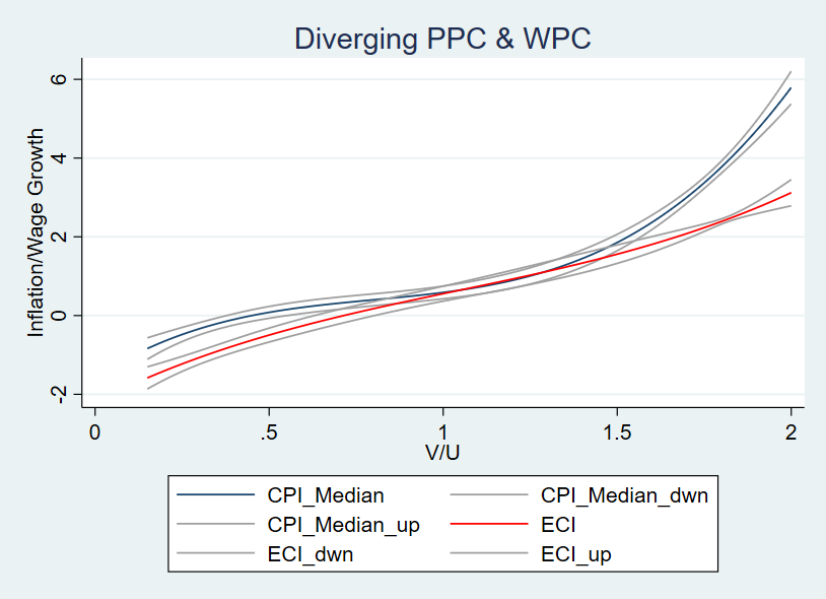


Figure 1: Comparing the cubic Wage and Price Phillips Curves

3 Evidence on Aggregate Data

I resolve the puzzle of diverging Wage and Price Phillips Curves by introducing a direct effect from labor market tightness on inflation. I provide empirical evidence for this direct impact in five parts. First, I show that labor market tightness exerts significant influence on

price inflation, even when wage growth is accounted for. Second, I explore the non-linearity of the direct impact to test if it contributes to Phillips Curve non-linearity (Smith et al. (2023), Benigno and Eggertsson (2023)). Third, I test the impact of labor market tightness under equivalent conditions. Fourth, I test the direct impact on real wage growth. Fifth and finally, I explore the origin of the direct impact by testing it on profit margin data, which allows me to control more broadly for marginal cost. This section provides the evidence for each step in turn.

3.1 Evidence on Inflation Data

I test a similar Phillips Curve as in Section 2 but include further controls for wage growth, energy inflation, and import prices.

The price equation:

$$\pi_t = \phi_1\omega_t + \phi_2\omega_{t-1} + \acute{\alpha}_1(V/U)_t + \acute{\alpha}_2(V/U)_t^2 + \acute{\alpha}_3(V/U)_t^3 + \delta\pi_t^e + \rho X_t + c_\pi + \epsilon_t \quad (3)$$

where π is inflation, ω wage growth (measured by ECI wages and salaries), V/U labor market tightness (job openings to unemployment ratio), π^e inflation expectations (10-year SPF median forecast), and X_t a vector of controls containing an eight quarter moving average of labor productivity growth, current and lagged energy inflation, as well as Yellen (2015)'s relative import price pressure[4].

The price equation obeys the following homogeneity constraint:

$$\phi_1 + \phi_2 + \delta = 1$$

I estimate my price equation on US aggregate data from 1985Q1 - 2023Q4. I test multiple inflation measures, and present results on CPI headline, traditional core CPI and median CPI inflation in Table 2.

Models 1-3 test the price equation on CPI headline, CPI excluding food and energy, and median CPI inflation[5]. All models exhibit a significant impact of labor market tightness on inflation as shown by the Wald test p-value in the third to last row ("No Impact"). The direct impact is also highly non-linear as shown by the Wald test in the second to last row ("Linear Impact"). The Null-Hypothesis here is that the higher order coefficients on labor market tightness are jointly zero. This direct impact on inflation holds even though past and current wage growth as well as inflation expectations, labor productivity, energy shocks, and import

[4]Yellen (2015) excludes petroleum products, natural gas, computers, and semiconductors from the import price index. I merely take out petroleum products as I control for energy inflation separately.

[5]I omit reporting the coefficients of the control variables. These can be found in Appendix A.1. For the PCE version of Table 2 see Appendix A.4, for the results on monthly data see Appendix A.5 and for density graphs see Appendix A.6

Table 2: Direct Impact on Inflation

	CPI	XFE	Median
(V/U)	4.69*	4.99*	4.11**
	(2.67)	(2.53)	(1.66)
$(V/U)^2$	-5.04	-5.08	-5.11***
	(3.41)	(3.35)	(1.93)
$(V/U)^3$	2.11*	2.09*	2.20***
	(1.18)	(1.18)	(0.63)
π^e	0.85***	1.02***	0.63***
	(0.10)	(0.09)	(0.07)
ω	0.05	0.03	0.18***
	(0.06)	(0.05)	(0.04)
L.ω	0.10	-0.05	0.19***
	(0.08)	(0.08)	(0.05)
Constant	-1.52**	-1.76***	-0.79*
	(0.63)	(0.58)	(0.43)
R^2	0.84	0.68	0.84
N	156	156	156
No Impact (p-value)	0.0000	0.0000	0.0000
Linear Impact (p-value)	0.0060	0.0085	0.0000
Partial R^2	0.19	0.25	0.42
Controls	✓	✓	✓

Standard errors in parentheses

* $p < .10$, ** $p < .05$, *** $p < .01$

Notes: "CPI" uses headline Consumer Price Index inflation as dependent variable. "XFE", and "Median" do the same with CPI excluding food and energy, and median CPI inflation respectively. The "No Impact" Wald test has the hypothesis of all V/U coefficients equal to zero. The "Linear Impact" tests the higher order V/U coefficients being zero. Partial R^2 reports the variation explained by V/U and its higher order terms. The results are from estimating Equation 3 on US quarterly data from 1985Q1 - 2023Q4.

price shocks are controlled for.

I base the number of wage growth lags on the Bayesian Information Criterion. I further explore the robustness of the direct impact to inclusions of additional wage lags in Appendix A.2. The coefficients are almost unchanged by the inclusion of four lags of wage growth, while the Wald test resolutely rejects both the non-existence and linearity of the direct impact. Additionally, I test the price equation on a plethora of other inflation measures, and all but the Producer Price Index evince the same cubic pattern[6]. The failure of the PPI is unsurprising as it is almost exclusively based on manufacturing which is not labor-intensive. I also want to point to the robustness of measuring inflation with core services excluding shelter due to the ongoing discussion on the role of shelter in the CPI (Adams et al. (2024)). The results are similar to those on median CPI inflation.

Similarly, using different wage growth measures leads to the same significantly cubic direct impact of labor market tightness on inflation[7]. I further test robustness of my results in Table 2 to winsorizing the labor market tightness and inflation measures to their respective 99th percentile. Winsorizing labor market tightness and inflation measures leaves the estimated coefficients essentially unchanged.

Finally, I constrain the estimation to the pre-pandemic period (1985Q1 - 2019Q4). Using purely pre-pandemic data turns the Wald tests on headline CPI inflation insignificant. However, core, Median and core services less housing inflation still provide evidence for the cubic direct impact (see Appendix A.3). The noise in headline inflation complicates the identification of the direct impact. This is particularly true during the pre-pandemic estimation period (1985Q1 - 2019Q4), in which especially tight labor markets were rare. The scarcity of meaningful labor market tightness over the pre-pandemic period is conjectured to have caused the flatter Phillips Curve estimates over the same period (Gagnon and Collins (2019)). The direct impact can explain this observation, as it is almost non-existent under average labor market tightness. Furthermore, the evidence for the direct impact on core inflation measures shows that the estimation is not singularly driven by the pandemic inflation surge.

In summary, Table 2 provides evidence for a direct impact and its contribution to the non-linearity of the Phillips Curve. My price equation also synthesizes the approaches of Ball et al. (2022) and Bernanke and Blanchard (2023) and the empirical evidence for the direct impact reconciles the divergent findings of high and low labor market importance for the post-pandemic inflation in these papers[8]. However, wage cost is still likely to add to the Phillips Curve and potentially to its non-linearity. It is therefore prudent to test the direct

[6]CPI Trimmed Mean, CPI Services exc. Shelter, CPI Services exc. Energy Services, CPI Services exc. Energy Services and Shelter, PCE, PCEXFE, PCE Median, PCE Trimmed Mean, PPI, and the GDP Deflator.

[7]Average Hourly Earnings, Average Hourly Compensation, Unit Labor Cost, and ECI Total Compensation.

[8]See Osten (2025b) for details.

effect and non-linearity of the Wage Phillips Curve.

3.2 Evidence on Wage Growth

This section provides evidence against the non-linearity of the underlying Wage Phillips Curve relation, once inflation and expected inflation are controlled for. It concludes that the direct impact is the sole driver of the Phillips Curve non-linearity found by in the literature (Benigno and Eggertsson (2023), Gitti (2024)). I estimate a wage equation similar to my price equation, by simply switching the role of inflation and wage growth as dependent and independent variables. This is a slight divergence from Bernanke and Blanchard (2023) as they only use controls for past inflation. However, leaving out contemporary inflation has no influence on the results.

The wage equation:

$$\omega_t = \tilde{\phi}_1 \pi_t + \tilde{\phi}_2 \pi_{t-1} + \tilde{\alpha}_1 (V/U)_t + \tilde{\alpha}_2 (V/U)_t^2 + \tilde{\alpha}_3 (V/U)_t^3 + \tilde{\delta} \pi_t^e + \tilde{\rho} X_t + c_\omega + \tilde{\epsilon}_t \qquad (4)$$

where ω is wage growth, π inflation, V/U labor market tightness, π^e 10-year ahead expected inflation, and X_t is the same vector of controls containing an eight quarter moving average of labor productivity growth, current and lagged energy inflation, as well as Yellen (2015)'s relative import price pressure. The wage equation obeys the following homogeneity constraint:

$$\tilde{\phi}_1 + \tilde{\phi}_2 + \tilde{\delta} = 1$$

Table 3 explores the possibility of a non-linear Wage Phillips Curve as found by part of the literature (Nalewaik (2016)). I estimate the Wage Phillips Curve on US aggregate data from 1985Q1-2023Q4 for a one-to-one comparison with the non-linearity of the price equation. I measure wage growth using the Employment Cost Index (ECI) for wages and salaries in Table 3.

Model 1 estimates the cubic form of the wage equation while model 2 estimates a linear version of the same. I omit reporting the coefficients on the control variables besides labor productivity. The estimates yield no evidence for non-linearity. The higher order terms are close to zero and the Wald Test ("Linear Impact") fails to reject their joint insignificance. Hence the non-linearity found by Ball et al. (2022), Benigno and Eggertsson (2023) and others seems entirely driven by the direct impact.[9]

[9]Some contributions to the literature (Nalewaik (2016), Smith et al. (2023)) find a non-linear Wage Phillips Curve. These papers do not control for price inflation and expected inflation however. Thus, they are likely picking up the non-linearity of the Price Phillips Curve, which affects wage growth due to a desire to preserve real wages. To test this I exclude the controls for inflation and inflation expectations. The Wage Phillips Curve is significantly non-linear when excluding these controls. This explains non-linear Wage Phillips Curve estimations in the literature. However, once these controls are included, the Wage Phillips Curve is clearly linear in nature. Hence, the direct impact is the source of the non-linear Phillips Curve as found by Ball et al. (2022) and others.

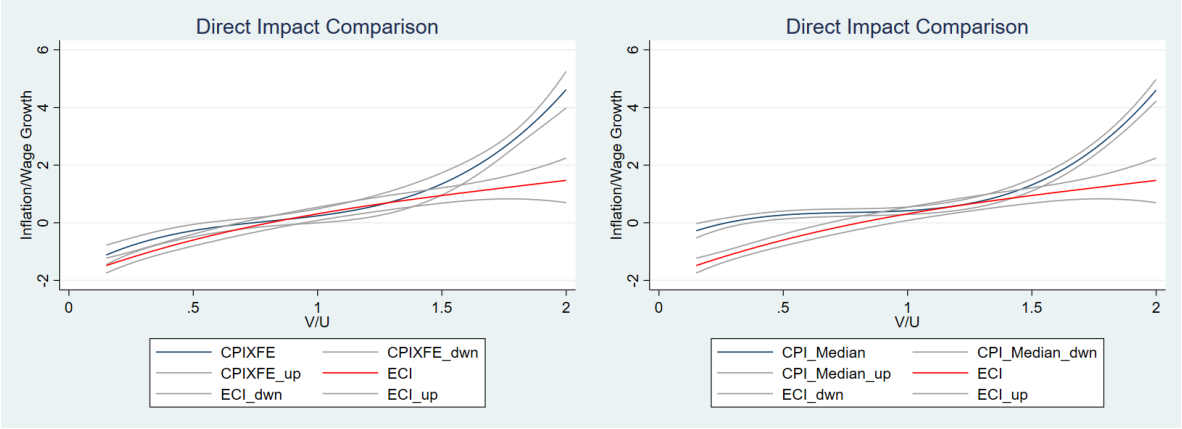
Table 3: Impact on Wage Growth

	Cubic	Linear
V/U	3.03^{*}	1.73^{***}
	(1.83)	(0.26)
$(V/U)^2$	-0.99	
	(2.10)	
$(V/U)^3$	0.13	
	(0.73)	
π	0.06	0.03
	(0.06)	(0.06)
L.π	0.21^{*}	0.17^{*}
	(0.11)	(0.09)
π^e	0.74^{***}	0.80^{***}
	(0.11)	(0.09)
Prod	0.24^{***}	0.24^{***}
	(0.05)	(0.06)
Constant	-1.84^{***}	-1.48^{***}
	(0.45)	(0.26)
R^2	0.53	0.52
N	156	156
No Impact (p-value)	0.0000	0.0000
Linear Impact (p-value)	0.1551	$-$
Partial R^2	0.24	0.22
Controls	✓	✓

Standard errors in parentheses

* $p < .10$, ** $p < .05$, *** $p < .01$

Notes: "Cubic" estimates Equation 4 while "Linear" reduces the functional form of V/U to a linear form. Both are estimated on US quarterly data from 1985Q1 - 2023Q4. The "No Impact" Wald test has the hypothesis of all V/U coefficients equal to zero. The "Linear Impact" tests the higher order V/U coefficients being zero. Partial R^2 reports the variation explained by V/U and its higher order terms.



(a) CPIXFE

(b) Median CPI

Figure 2: Comparison of the direct impact on wage and price growth.
Subfigure a) shows the labor market's cubic functional form on CPI inflation excluding food and energy. Subfigure b) does the same for Median CPI inflation.

However, labor market tightness exerts a significant influence on wage growth, as the Wald test on the "No Impact"-Hypothesis shows. The direct impact channel and the conventional Phillips Curve channel are thus complementary rather than mutually exclusive. However, the impact of labor market tightness on wage growth is linear in nature and fails to explain the steep rise of inflation during hot labor markets.

The results are robust to alternative wage measures[10]. All measures fail to give significant non-linearity, except for Average Hourly Earnings (AHE), which features a significant but negative third order labor market tightness coefficient. The negative non-linearity is likely due to composition effects, which cause the AHE infamously to spike upwards during sizeable labor market downturns. Furthermore, the results of Table 3 are robust to limiting the estimation period to pre-pandemic and winsorizing the data. Finally, I exclude the contemporaneous inflation control to match the lag structure of Bernanke and Blanchard (2023). The coefficients and the verdict against non-linearity are essentially unchanged by these robustness checks.

I compare the estimated functional forms of the wage and price equations in Figure 2. The functions are plotted over the range of V/U in the data: $[0.15, 2.0]$.

Figure 2 highlights the linear nature of the relation between labor market tightness and wage growth, once expectations and inflation are accounted for. Even though I include the higher order terms, the function is essentially linear over the range of observed labor market tightness. The direct impact on inflation and wage growth is similar for a majority of the observed V/U values, hinting at stable real wage gains under normal labor market

[10] Average Hourly Earnings, Average Hourly Compensation, Unit Labor Cost, and ECI Total Compensation.

conditions. However, from a V/U ratio of 1.25 onwards the price equation lifts off while the wage equation remains linear. These findings suggest an increasingly rapid decline in real wage growth. I explore this indication in the ensuing section.

To summarize the empirical evidence so far: There is substantial evidence for a non-linear Price Phillips Curve. This non-linearity survives when controlling for wage growth, expected inflation, labor productivity, energy shocks and import price pressure. The impact of labor market tightness on wage growth on the other hand is linear, once inflation and inflation expectations are controlled for. The non-linear reaction of inflation and the linear reaction of wage growth to labor market tightness imply that real wage growth also reacts non-linearly to labor market tightness. Specifically, it should decline rapidly during hot labor markets.

3.3 Evidence on Real Wage Growth

Real wage growth gives a direct comparison of the relative impact of labor market tightness on wages and prices. Therefore, I provide evidence for the direct impact on a real wage version of the previous equations in this section.

My real wage equation:

$$(\omega_t - \pi_t) = \bar{\alpha}_1(V/U)_t + \bar{\alpha}_2(V/U)_t^2 + \bar{\alpha}_3(V/U)_t^3 + \bar{\delta}\pi_t^e + \bar{\phi}(\omega_{t-1} - \pi_{t-1}) + \bar{\rho}X_t + c_{rwg} + \bar{\epsilon}_t \quad (5)$$

I measure real wage growth as the difference between ECI and CPI annualized growth rates[11]. I stop enforcing the homogeneity constraint in the real wage equation as inflation expectations impact both inflation and wage growth in the steady state.

Table 4 provides the coefficient estimates of equation 5. I successively introduce the linear, quadratic and cubic labor market tightness of equation 5 in Models 1 to 3. I again omit the coefficients of energy inflation, and relative import prices in the table. The linear impact is entirely rejected by the Wald test, while the cubic functional form adds next to nothing in explanatory power. Thus, the data supports a quadratic functional form for real wage growth. Figure 3 shows the quadratic functional form on real wage growth.

Real wage growth is subdued during slack labor markets and rises with increasing tightness to form a maximum during moderately tight labor markets. After this point the direct impact leads to declining real wage growth, as inflation acceleration increasingly surpasses additional wage growth. Thus, the direct impact delivers a region of labor market tightness which favors real wage growth.

The negative and non-linear impact of labor market tightness on real wages is robust to the use of winsorized and pre-pandemic (1985Q1-2019Q4) data. Finally, the results

[11] Using Average Real Hourly Compensation yields similar results.

Table 4: Real Wage Growth

	Linear	Quadratic	Cubic
π^e	-0.48***	-0.58***	-0.58***
	(0.15)	(0.15)	(0.16)
Prod	0.30***	0.29***	0.29***
	(0.09)	(0.07)	(0.07)
L.RWG	-0.15	-0.23***	-0.23***
	(0.09)	(0.08)	(0.08)
V/U	0.19	3.43***	3.92*
	(0.43)	(0.65)	(2.20)
$(V/U)^2$		-1.80***	-2.41
		(0.32)	(2.53)
$(V/U)^3$			0.21
			(0.82)
Constant	1.28*	0.47	0.38
	(0.72)	(0.62)	(0.72)
R^2	0.72	0.74	0.74
N	156	156	156
No Impact (p-value)	0.6669	0.0000	0.0000
Linear Impact (p-value)	-	0.0000	0.0000
Partial R^2	0.00	0.09	0.09
Controls	✓	✓	✓

Standard errors in parentheses

* $p < .10$, ** $p < .05$, *** $p < .01$

Notes: "Cubic" estimates Equation 5 while "Quadratic and "Linear" reduce the functional form of V/U to a quadratic and linear form, respectively. All three are estimated on US quarterly data from 1985Q1 - 2023Q4. The "No Impact" Wald test has the hypothesis of all V/U coefficients equal to zero. The "Linear Impact" tests the higher order V/U coefficients being zero. Partial R^2 reports the variation explained by V/U and its higher order terms.

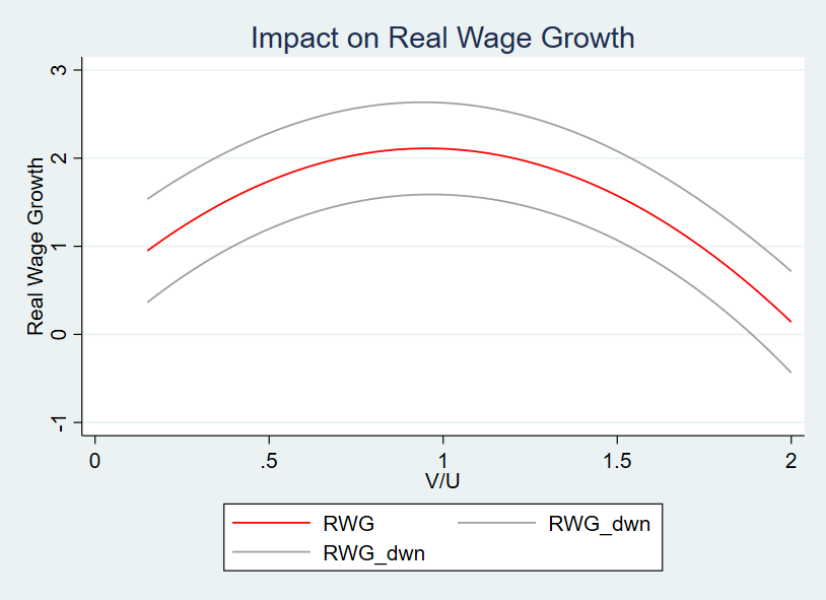


Figure 3: Functional Form on Real Wage Growth

are robust to including additional auto-regressive lags into the real wage equation. The resulting non-linear function is roughly inverse to the functional form estimated on price inflation. Thus, the direct impact on inflation dominates the impact on wage growth during tight labor markets.[12]

I further test my price, wage and real wage equations' robustness to different measures of inflation expectations. The non-linear direct impact on inflation, the limited linear impact on wage growth, and the negative non-linear impact on real wage growth are all robust to using the Cleveland Fed's measure for 5, 10, 20, and 30 year ahead inflation expectations, instead of the median 10 year inflation forecast from the Survey of Professional Forecasters.

3.4 Evidence on Profit Margins

There are two plausible explanations for my empirical findings so far. Increases in marginal cost other than wages would lead to a rise in inflation independent of wage growth. Alternatively, optimal price increases in response to tight labor markets would also raise prices in excess of increases in wages. I estimate the response of profit margins to labor market tightness to distinguish between these two explanations. A shock to marginal cost

[12]The negative effect on real wage growth reconciles Blanchflower et al. (2022) criticism of labor market tightness as the measure of labor market slack with its superior explanatory power (Domash and Summers (2022)). Blanchflower et al. (2022) argue that the concomitant fall in real wages and the rise of labor market tightness shows its unsuitability as measure of labor market slack. However, the direct impact reconciles the strong explanatory power of labor market tightness on inflation and the fall in real wage growth during especially tight labor markets. I thus contribute to the literature concerned with the question why labor market tightness outperformed traditional slack measures over the pandemic period (Furman and Powell (2021)).

other than wages would reduce profit margins in the absence of additional demand. On the other hand, increasing prices that are detached from marginal cost would lead to rising profit margins. Thus, I test the relation between profit margins and labor market tightness to shed light on the source of the direct impact.

The existing literature already documents the pro-cyclicality of profit margins and mark-ups (Small (1997), Macallan et al. (2008), Nekarda and Ramey (2013), Nekarda and Ramey (2020)). Okun's law suggests that this pro-cyclicality translates into a positive correlation between profit margins and labor market tightness. However, the relation between profit margins and labor market tightness has not yet been explored. Furthermore, if optimal price setting is behind the direct impact, then the profit to labor market tightness relation should show a degree of non-linearity, which is new to the pro-cyclical profits and markup literature.

I utilize the Bureau of Economic Analysis' measure of profits per unit of real gross value added for non-financial corporations, with inventory value adjustments and adjusting for capital cost. The BEA measure stems from NIPA Table 1.15, accounts for material input cost, compensation for employees, depreciation of fixed capital, and taxes on production and imports (less subsidies and transfers). It is the best publicly available profit measure according to Barkai and Benzell (2018), and improves on Palazzo (2023)'s aggregate profit margin measure by adjusting not only for capital cost in terms of interest payments, but also for changes in inventory values, and miscellaneous cost. This results in a raw aggregate operating profit margin before taxes on profits.

I estimate the impact of labor market tightness on profit margins in Table 5 on the period 1985Q1-2023Q3[13]. The empirical model follows a similar pattern as in Equation 5 for real wages. I explore the relation of profit margins to linear, quadratic, and cubic labor market tightness. I further control for wage growth to filter out the impact of the traditional Phillips Curve channel and the eight quarter moving average of labor productivity growth.

The linear estimation gives a significant ("No Impact" Wald test) and sizeable impact (Partial R^2) of labor market tightness on profit margins. This impact increases further when allowing for a quadratic relation between labor market tightness and profit margins. The cubic estimation yields only insignificant coefficients, and adds next to nothing in explanatory power. Hence the quadratic relation is preferred. The notable and non-linear influence of labor market tightness on profit margins implies that the direct impact is not simply a coincidence of rising marginal cost and tight labor markets. The impact of labor market tightness on profit margins is also robust to winsorizing labor market tightness and to limiting the estimation to the pre-pandemic period (1985Q1-2019Q4).

I show the quadratic functional form of profit margins in Figure 4.

[13] 2023Q4 had not been published during my latest data pull.

Table 5: Profit Margins

	Linear	Quadratic	Cubic
ω	-1.59^{***}	-1.37^{***}	-1.24^{***}
	(0.40)	(0.36)	(0.41)
Prod	-0.41	-0.35	-0.44
	(0.62)	(0.57)	(0.58)
V/U	7.66^{***}	-3.96	-15.56
	(1.92)	(4.22)	(12.57)
$(V/U)^2$		6.12^{***}	20.27
		(1.82)	(13.79)
$(V/U)^3$			-4.86
			(4.49)
Constant	10.93^{***}	14.38^{***}	16.79^{***}
	(2.53)	(2.61)	(3.35)
R^2	0.38	0.47	0.48
N	155	155	155
No Impact (p-value)	0.0001	0.0000	0.0000
Linear Impact (p-value)	–	0.0010	0.0025
Partial R^2	0.31	0.42	0.43

Standard errors in parentheses

* $p < .10$, ** $p < .05$, *** $p < .01$

Notes: "Cubic" estimates the impact of a cubic functional form of V/U on aggregate profit margins, while "Quadratic and "Linear" reduce the functional form of V/U to a quadratic and linear form, respectively. All three are estimated on US quarterly data from 1985Q1 - 2023Q3. The "No Impact" Wald test has the hypothesis of all V/U coefficients equal to zero. The "Linear Impact" tests the higher order V/U coefficients being zero. Partial R^2 reports the variation explained by V/U and its higher order terms.

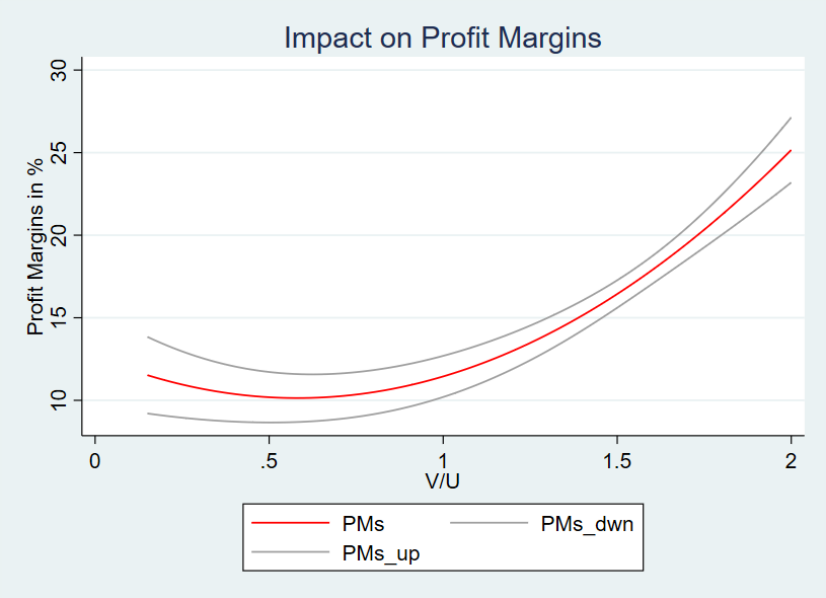


Figure 4: Functional Form on Profit Margins

The profit margin function increases with inflation during the tight labor market. The similarity highlights the common underlying cause of rising profit margins and price inflation. These results indicate a profit maximizing price setting mechanism behind the direct impact. I will discuss this price setting response to a tight labor market in Section 7.[14]

4 Quantification of the Pandemic Inflation Surge

This section quantifies the extend to which the direct impact contributed to the post-pandemic inflation surge, how wage pressure contributed, and how the direct impact does on explaining the fall in real wages and the rise in profit margins. I leverage the findings from the preceding section for this quantification.

In particular I use the estimated coefficients on labor market tightness from Equation 3 as measure for the direct impact. Recall that these coefficients were obtained under controls for past and current wage growth. I use these to compare average 2019 levels ($V/U = 1.19$) with peak labor market tightness in 2022Q2 ($V/U = 1.9$) and use the estimated functional forms to calculate how much of the inflation increase is explained by the direct impact. For example, I use the functional form of the headline CPI based model from Table 2:

[14] I repeat the previous robustness exercises on real wage growth and profit margins, by estimating them on winsorized data, pre-pandemic data and testing the threshold model as alternative functional form. The results remain essentially unchanged. My results on inflation, wage growth, real wage growth, and profit margins are similarly robust to using NFIB's percent measure of small businesses reporting job openings they could not fill in the current period instead of V/U.

$\pi = 3.35 * (V/U) - 3.59 * (V/U)^2 + 1.58 * (V/U)^3$ to calculate the explained rise in inflation. I plug the relevant V/U numbers for 2019 and 2022Q2 into the functional form and calculate the difference between the predicted inflation levels. The difference (2.68 percentage points (pp)) is the estimated contribution of the direct impact to the rise in headline inflation between average 2019 levels and peak labor market tightness in 2022Q2; a rise of 7.61pp.

I present the results of this and similar exercises for core and median inflation in Table 6. The first two rows provide the average 2019 and 2022Q2 peak inflation rates from the data. The difference between the two is provided in row three as the measure of increase in the respective inflation measure. Row four finally provides the estimate of the explained variation in inflation due to the direct impact. In row five I provide the same result when leaving out the controls for wage growth in Equation 3.[15] Hence, the last row provides an estimate of the total impact of labor market tightness between the two periods.

Table 6: Direct Impact Quantification

	CPIXFE	**Median**	**CPI**
Avg 2019	2.33%	2.89%	2.05%
2022Q2	5.87%	6.98%	9.66%
Δ	3.54pp	4.09pp	7.61pp
Predicted Δ	3.12pp	3.1pp	2.68pp
Total Effect	3.03pp	3.74pp	3.01pp

The direct impact accounts for almost all of the rise in core inflation (3.12pp out of 3.54pp), and a good portion of the rise in Median inflation (3.1pp out of 4.09pp). The fit is less good for headline CPI inflation (2.68pp out of 7.61pp). Removing the wage controls (see "Total Effect") further increases the estimated impact of labor market tightness on inflation, although only by a comparatively small amount. Wage pressures added to Median inflation, but seemed to have added little to traditional core inflation. The tight fit of the direct impact with the overall impact of labor market tightness on core inflation is particularly noteworthy.

The quantification highlights the importance of allowing for the direct impact when assessing the labor market impact on inflation. The channel reconciles the discrepancy between the small impact found by Bernanke and Blanchard (2023) and the sizeable impact found by Ball et al. (2022). Bernanke and Blanchard (2023) only measure the impact stemming from wage growth, while Ball et al. (2022) allow for both the traditional wage and the direct impact on inflation. For more details see Osten (2025b) (forthcoming).

I repeat the comparison of average 2019 and 2022Q2 peak levels for wage growth, real

[15] The resulting functional forms are as follows. CPI = 4.12 V/U - 4.09 $(V/U)^2$ + 1.75 $(V/U)^3$, CPIXFE = 4.69 V/U - 4.76 $(V/U)^2$ + 1.96 $(V/U)^3$, Median = 5.68 V/U - 6.10 $(V/U)^2$ + 2.53 $(V/U)^3$.

wage growth and profit margins in Table 7. The direct effect on nominal wage growth explains less than half of the actual increase (1.23pp out of 2.95pp). The rest of the increase in wage growth is likely due to workers' attempt to catch-up with inflation. Similarly, the drop in real wage growth is under-predicted (-1.52pp out of -5.07pp), likely a result of the substantial shocks in food and energy prices. Finally, profit margins are over-predicted by the direct impact (10pp compared to the observed 7pp). Higher input costs from food and energy inflation are again a plausible explanation for the lower than predicted profit margins.

Table 7: Direct Impact Quantification Part 2

	Wage	**Real Wage**	**Profit**
Avg 2019	2.97%	0.91%	13%
2022Q2	5.92%	-4.16%	20%
Δ	2.95pp	-5.07pp	7pp
Predicted Δ	1.23pp	-1.52pp	10pp

5 Cross-Country Evidence

This section tests the direct impact on cross-country panel data from 1985Q1 to 2024Q1. I use job vacancy, unemployment, employment cost, labor productivity, inflation expectations, market concentration, headline and core inflation for 24 developed countries.[16] I find evidence for the existence of the direct impact on both inflation and profit margin data, as well as support for the role of market concentration in the panel data. Furthermore, this paper is the first to test and provide evidence for a non-linear V/U based Phillips Curve on a large panel of countries.

I estimate the following equation on headline and core inflation to test for the direct impact in the panel:

$$\pi_{c,t} = \check{\alpha}_1(V/U)_{c,t} + \check{\alpha}_2(V/U)^2_{c,t} + \check{\alpha}_3(V/U)^3_{c,t} + \check{\beta}\pi^e_{c,t} + \check{\gamma}_1\omega_{c,t} + \check{\gamma}_2\omega_{c,t-1} + \check{\phi}\text{Prod}_{c,t} + \check{\delta}_c + \check{\delta}_t + \check{\epsilon}_{c,t} \tag{6}$$

where π is either headline consumer price or core (less food and energy (in some cases also less alcohol and tobacco)) inflation, V/U is the ratio of job openings to unemployed, π^e is the 5 year forward inflation prediction from the IMF's World Economic Outlook, ω is nominal employment cost growth, and Prod is an eight quarter moving average of labor productivity growth. I thus estimate two-way fixed effects version of equation 3, with the exception of using a shorter inflation expectations horizon (5 vs 10 years), and omitting controls for energy inflation and relative import price pressures.

I test for the rejection of the following homogeneity constraint. I do this to test if common shifts in inflation expectations and time fixed effects are present, which may violate the homogeneity constraint.

$$1 = \check{\beta} + \check{\gamma}_1 + \check{\gamma}_2$$

The homogeneity constraint is not rejected by the Wald test.[17] Thus, I impose the homogeneity constraint on the TWFEs estimation. The labor market tightness coefficients are almost unchanged by the constraint.

The two-way fixed effects allow for a cleaner identification of the channel as time fixed effects filter out aggregate variables such as energy inflation, cost of global imports, global demand and supply shocks. Furthermore, the country fixed effects control for most social and policy differences across countries. Finally, clustering standard errors at time and country level controls for serial correlation of the error terms.

I present the estimation results in Table 8. The panel data provides evidence for the existence of the direct impact outside the US. The joint insignificance of the labor market

[16] Australia, Canada, Belgium, Germany, Estonia, Ireland, Spain, France, Cyprus, Latvia, Lithuania, Luxembourg, Malta, Netherlands, Austria, Poland, Portugal, Slovenia, Slovakia, Finland, Great Britain, Switzerland, Greece, and Japan

[17] p-value for headline based estimation is 0.46, and 0.53 for core inflation.

tightness coefficients is rejected by the Wald test for both headline and core inflation. The joint insignificance of the quadratic and cubic terms is further rejected for core inflation. The same hypothesis is only rejected at the 10% confidence level for headline inflation.

Table 8: Cross-Country Direct Impact Estimation

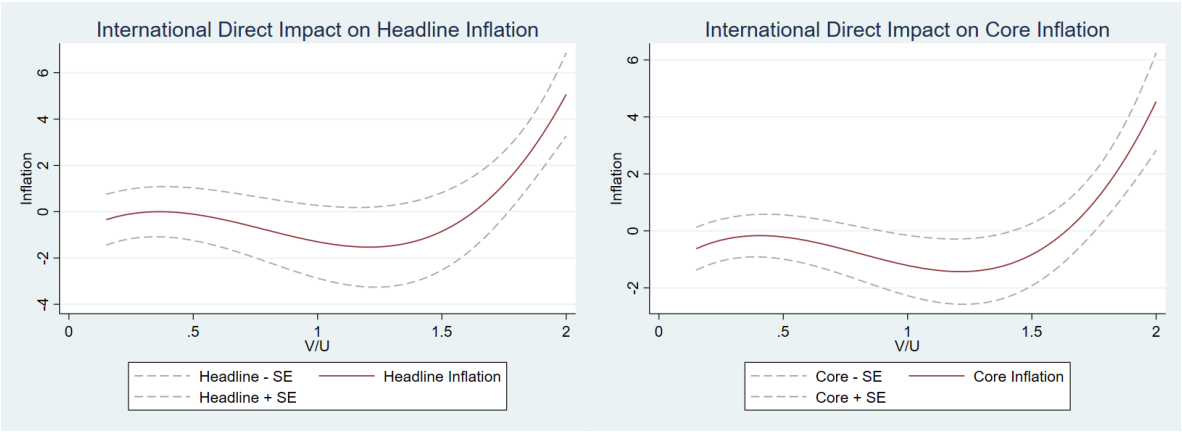
	π	Core
(V/U)	7.17**	7.37**
	(3.30)	(2.82)
$(V/U)^2$	-12.24**	-11.82***
	(4.65)	(3.98)
$(V/U)^3$	5.11**	4.82***
	(1.93)	(1.63)
ω	0.01	0.02*
	(0.01)	(0.01)
L.ω	0.01	0.01
	(0.01)	(0.01)
5Y π^e	0.98***	0.97***
	(0.02)	(0.02)
Prod	0.07	0.03
	(0.08)	(0.04)
R^2	0.69	0.67
N	1412	1412
No Impact p-value	0.0226	0.0423
Linear p-value	0.0593	0.0208

Standard errors in parentheses

* $p < .10$, ** $p < .05$, *** $p < .01$

The direct impact clearly shows up on the cross-section of developed countries, without even including the US data. These results further bolster the growing body of evidence on the non-linearity of the Phillips Curve around the world (Dao et al. (2024), Smith et al. (2024), Ho and Iyke (2019)). This paper is the first to test a non-linear Phillips Curve specification based on the job openings to unemployed ratio on a large group of countries.[18] Furthermore, the evidence for the direct impact and its non-linearity indicates that as in US aggregate data the direct impact is a source of the Phillips Curve's non-linearity.

[18] Dao et al. (2024) test it only for US and Canadian data.



(a) Headline (b) Core

Figure 5: Comparison of the international direct impact on headline (Subfigure a) and core inflation (Subfigure b).

I provide the functional form of the direct impact channel as estimated on headline and core inflation of the cross-country panel in Figure 5. I graph both functional forms on the labor market tightness range observed in US data [0.15,2.0] for comparison with my previous results. Both functional forms are essentially flat for slack and normal labor market tightness, and soar when labor markets get really tight. Thus, the functional forms from my cross-country panel follow a similar shape as the one estimated on US aggregate data, even though the US was excluded from the exercise.

To test if the majority of countries in the panel experiences a similar direct impact as the US I now include the US data and run a linear Classifier-Lasso model to test for differences in the direct impact of labor market tightness on core inflation. The linear Classifier-Lasso model is allowed to both determine the number of distinct groups as well as assigning each single country to a fitting group. The model should provide a single group as the best fit if all countries experience a similar direct impact. Countries with different direct impacts should be sorted into separate groups. Thus, if the US data behaves significantly different than the data from other countries it should end up in its own sub-group.

Classifier-Lasso Linear Model	**Number of obs**	=	1,608			
Post-estimation with 2 groups	**Number of units**	=	25			
Fixed effect estimation with Group 1	**R-squared**	=	0.3190			
Absorbing: country	**Adj R-squared**	=	0.2887			
No. of obs	=	212	**Within R-sq.**	=	0.2758	
No. of units	=	4	**Root MSE**	=	2.1862	
Core	**Coefficient**	**Std. err.**	**z**	**P>|z|**	**[95% conf. interval]**	
V/U	65.85566	88.42338	0.74	0.456	[-107.451, 239.1623]	
$(V/U)^2$	-553.6134	1404.459	-0.39	0.693	[-3306.303, 2199.077]	
$(V/U)^3$	2162.271	5844.606	0.37	0.711	[-9292.946, 13617.49]	
ω	0.1009044	0.0356007	2.83	0.005	[0.0311282, 0.1706805]	
L1.ω	0.0139388	0.0270538	0.52	0.606	[-0.0390857, 0.0669633]	
5Y π^e	0.0090785	0.9808463	0.01	0.993	[-1.913345, 1.931502]	
Constant	-0.5967207	1.631794	-0.37	0.715	[-3.794978, 2.601537]	
Fixed effect estimation with Group 2	**R-squared**	=	0.3880			
Absorbing: country	**Adj R-squared**	=	0.3764			
No. of obs	=	1,396	**Within R-sq.**	=	0.2775	
No. of units	=	21	**Root MSE**	=	1.8364	
Core	**Coefficient**	**Std. err.**	**z**	**P>|z|**	**[95% conf. interval]**	
V/U	12.32452	3.909859	3.15	0.002	[4.661337, 19.9877]	
$(V/U)^2$	-11.34077	4.657104	-2.44	0.015	[-20.46853, -2.213018]	
$(V/U)^3$	3.720736	1.690654	2.20	0.028	[0.4071145, 7.034357]	
ω	0.0326818	0.0195904	1.67	0.095	[-0.0057147, 0.0710783]	
L1.ω	0.0365847	0.0185398	1.97	0.048	[0.0002474, 0.072922]	
5Y π^e	0.7687929	0.3582984	2.15	0.032	[0.0665409, 1.471045]	
Constant	-2.619952	1.053437	-2.49	0.013	[-4.68465, -0.5552529]	

The model determines two groups when estimating it on core inflation, with the majority of countries in group 2. Group 2 includes the US indicating again that the direct impact is not a phenomenon unique to the US. Group 1 consists of Spain, Portugal, Greece, and

Table 9: Country by Automated Group

Group 1	**Group 2**
Spain	Australia
Portugal	Canada
Greece	Japan
Slovakia	Finland
	Slovenia
	Cypress
	Switzerland
	Poland
	United Kingdom
	Malta
	Estonia
	Latvia
	Lithuania
	Belgium
	Germany
	Ireland
	France
	Luxembourg
	Netherlands
	Austria
	United States

Slovakia. Their difference from the rest is likely due to the economic downturn intensity they experienced during the Euro Crisis. My data on the four countries only starts in 2008 at the earliest, and their labor market tightness levels ranges from 0.008 to 0.178 across the time frame. This is small compared with the average labor market tightness of 0.39 for the countries in the automated grouping exercise.[19] Using headline inflation, this group also includes France, Estonia, Latvia, and Finland.

The direct impact is significant on all but 4 countries. These countries (see Group 1) had only poor labor markets over the estimation period. Thus, it is not surprising that they are not providing evidence for the direct impact which is only measurable during tight labor markets.

I also find that the direct impact is significant for the vast majority of single country regressions in my panel and follows a similar functional form as in US data (see Appendix

[19]Spain's V/U range (2010Q1 - 2024Q1): 0.009 - 0.054; Slovakia's range (2008Q1 - 2024Q1): 0.032 - 0.159; Portugal's range (2010Q1 - 2024Q1): 0.012 - 0.178; Greece's range (2014Q1 - 2024Q1): 0.008 - 0.118.

A.8). However, due to international demand, supply, and commodity shocks, as well as policy differences between countries, the two-way fixed effects estimation presented provides cleaner results. Furthermore, I test the importance of market concentration for the direct impact. I do this by interacting the Herfindahl-Hirschman Index with the labor market tightness measure by country. I find evidence that market concentration does play an important role in the direct impact channel. For the estimation results and functional forms see Appendix A.9.

5.1 Unit Profits

Finally, I test if profit margins rise with labor market tightness in country panel data as well. There is already evidence that "extraordinary unit profits"[20] have played a crucial role in the post-pandemic Euro Area inflation surge (Hansen et al. (2023)). Hansen et al. (2023) further find that unit profits in the service sector were a major channel in the inflation surge and that they rose with the growth in real service consumption. I test Hansen's measure of unit profits on Ball et al. (2022)'s cubic labor market tightness in this sub-section.[21]

I use Equation 38 as basis for my empirical approach to profit margins, simply switching out the dependent variable and dropping wage controls. However, the profit margin measure is not available on all the countries in my dataset. I thus now include the US into the estimation of the direct impact on profit margins.[22] Table 10 presents the results of this estimation.

The Wald Test provides evidence for a direct impact of labor market tightness on profit margins, as well as for its non-linear rise. Thus, the direct impact shows up on international profit margin data, highlighting again that the channel is likely driven by an optimal price setting mechanism instead of other input cost. I provide the estimated functional form graphed on the US data range [0.15,2.0] in Figure 6. The functional form is again similar to the estimated functional form on US aggregate profit margins.

[20] Philip Lane.

[21] Hansen uses value added per unit minus unit labor cost as measure of unit profits. He thus uses a similar but less clean measure than the US measure I use in my aggregate results.

[22] Countries with profit margin data: Australia, Canada, Belgium, Germany, Estonia, Ireland, Spain, France, Netherlands, Austria, Poland, Portugal, Finland, the UK, Greece, and the USA. I am missing profit margin data for Switzerland, Japan, Luxembourg, Slovenia, Cypress, Malta, Slovakia, Latvia, and Lithuania.

Table 10: International Direct Impact on Profit Margins

	PMs
(V/U)	2.09
	(3.03)
$(V/U)^2$	-4.41
	(4.10)
$(V/U)^3$	3.14*
	(1.48)
5Y π^e	-1.11
	(0.79)
ω	0.01
	(0.01)
L.ω	0.01
	(0.00)
Prod	-0.02
	(0.02)
Constant	3.55*
	(1.74)
R^2	0.95
N	943
No Impact p-value	0.0000
Linear p-value	0.0000

Standard errors in parentheses

* $p < .10$, ** $p < .05$, *** $p < .01$

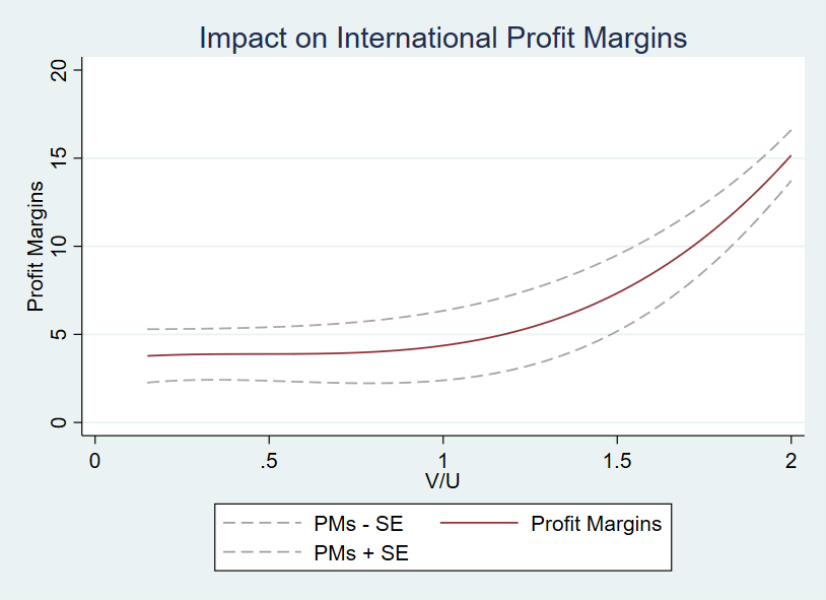


Figure 6: International Direct Impact on Profit Margins.

6 Evidence from US Services Sectors

In this section I test the direct impact on a cross-section of US service sectors to control for endogenous monetary policy with time fixed effects. The time fixed effects also control for potential endogeneity from fiscal policy and inflation expectations as argued by McLeay and Tenreyro (2020) and Hazell et al. (2022). I test my channel across the sectors of Construction, Transportation, Warehousing and Utilities, Finance and Insurance, Real estate and Leasing, Professional Business Services, Arts, Entertainment and Recreation, Accommodation and Food Services, as well as Other services.

Inflation is measured using sectoral CPI, while wage growth is based on sectoral ECI wages and salaries. Job openings stem from the JOLTS survey and unemployment from the Current Population Survey. The CPS' unemployment measure assigns the sector of last employment to an unemployed respondent. It is thus prone to mis-measurement depending on cross-sector labor mobility. In taking them at face value I make the implicit assumption of no labor mobility across sectors. I will later discuss on how to remedy this assumption using a Shift-Share IV approach.

The sectoral CPI data stems from seasonal adjusted quarterly CPI component data from 1996 to early 2024. The component inflation is aggregated up to match the sectors at which V, U and ECI data are available. I use annual Consumption Expenditure spending weights of the pertaining CPI series to create weighted average inflation rates for the two-digit NAICS sectors. For example, I aggregate Food Away From Home, Alcoholic Beverages Away From Home and Lodging Away From Home to get a CPI measure for the Accommodation and Food Services Sector.

The ECI is reported on the same NAICS level as V and U, except for "Transportation and Warehousing" and "Utilities". These are reported separately in the ECI data. I use average 2000-2024 employment levels to aggregate the two ECI series together, where Transportation and Warehousing takes a weight of about 90%.

Another benefit of using cross-sectional data is the greater prevalence of tight labor markets than in aggregate data. Hooper et al. (2020) noted this for MSA level labor market data. I find that this observation holds true in the cross-section of US service sectors as well. Labor market tightness ranges from 0.026 to 3.78 in sectoral data, with a mean of 0.79. 14% of the sectoral labor markets lie on or above the 1.5 level of extremely tight labor markets. Thus, tight labor markets are much more prevalent in sectoral data than in the aggregate US economy, where V/U values on or above 1.5 only constitute about 3.5% of labor market data points.

6.1 Empirical Results

First, I provide an overview of all sectors for which V/U data is available. This includes the Durable and Non-durable goods sectors, as well as the Education, Health Services, and Information sectors. The only sector that is missing is the Mining and Logging sector due to missing wage data. Education and Health Services are excluded from the cross-section due to large government involvement in these sectors.[23] Finally, Information is excluded as Software can be multiplied with no additional manpower cost. Thus, it is unlikely to exhibit the direct impact.

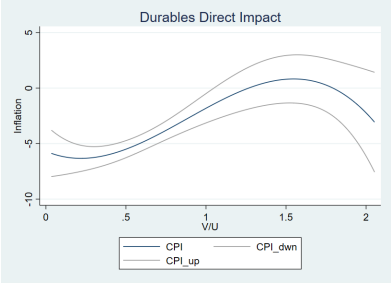
For the single-sector results I estimate the following equation per sector:

$$\text{CPI}_{i,t} = \grave{\alpha}_1(V/U)_{i,t} + \grave{\alpha}_2(V/U)^2_{i,t} + \grave{\alpha}_3(V/U)^3_{i,t} + \grave{\beta}\pi^e_t + \grave{\gamma}_1\omega_{i,t} + \grave{\gamma}_2\omega_{i,t-1} + \grave{c}_i + \grave{\epsilon}_{i,t}$$

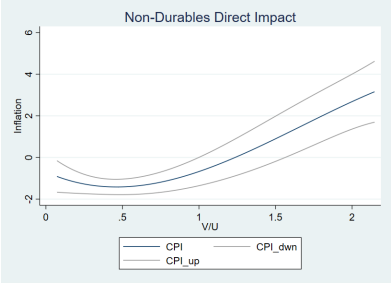
The estimated functional form of each sector is presented in 7.

The single sector estimation provides evidence for the direct impact on most sectors. First, the core services sectors "Accommodation & Food", "Arts, Entertainment, & Recreation", "Health Care", "Other", "Business", "Finance & Insurance", and "Real Estate & Leisure" all show a non-linear rise in inflation beyond wage growth during tight labor markets. Second, sectors that blend services and goods like "Transportation, Warehousing & Utilities" and "Construction" also show a significant rise in inflation beyond wage growth when labor markets are tight. The "Education" and "Information" sectors are not showing much evidence for a direct impact. Finally, even "Durables" and "Non-durables" show a

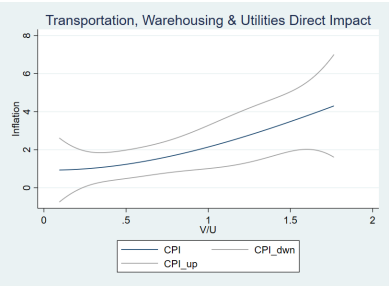
[23] "In 2023, the public sector accounted for 14 percent of annual average employment (21.8 million jobs). Of this 14 percent, two-thirds were mostly concentrated in two sectors: education and healthcare." from Thompson (2024).



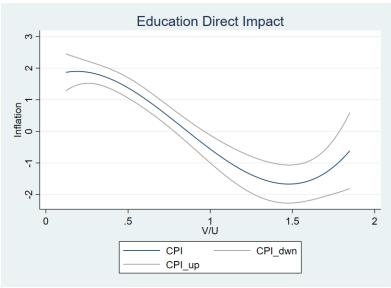
Durable Goods



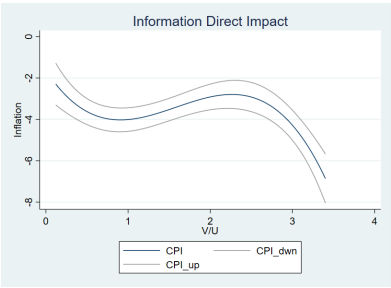
Non-Durable Goods



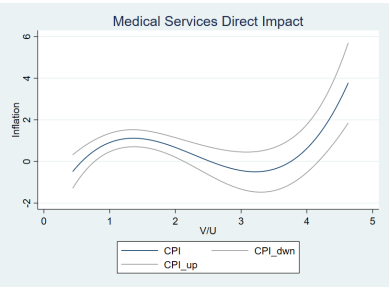
Transportation, Warehousing & Utilities



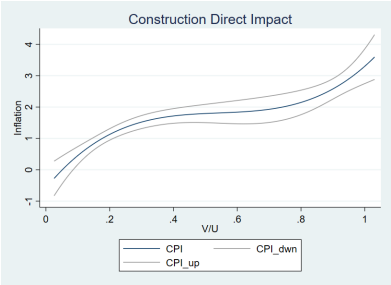
Education



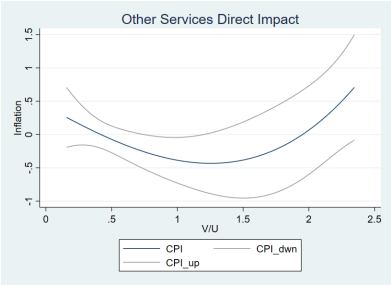
Information



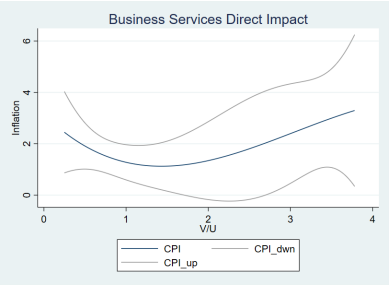
Medical Services



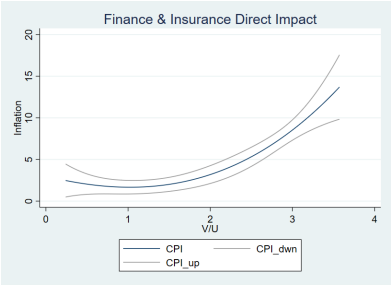
Construction



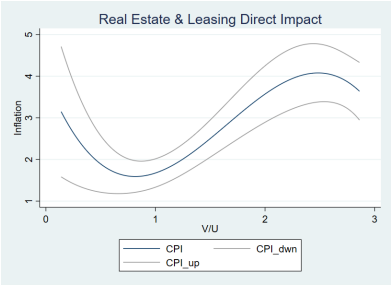
Other Services



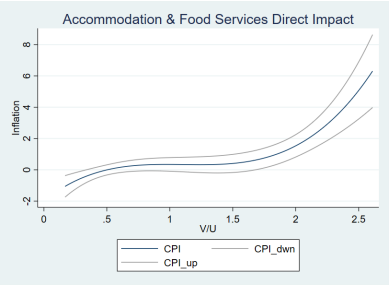
Business Services



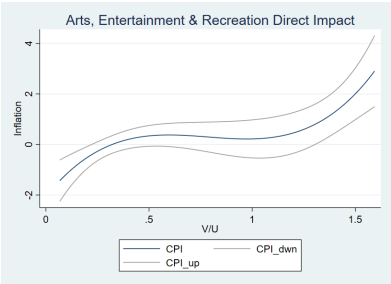
Finance & Insurance



Real Estate & Leasing



Accommodation & Food



Arts, Entertainment & Recreation

Figure 7: Direct Impact by Sector

non-linear rise in inflation beyond wage growth under tight labor markets.

To test the indication of the single-sector analysis, I now provide the evidence from a two-way fixed effects regression with two-way clustered standard errors on the service sectors not dominated by government price setting and excluding the no-marginal cost "Information" sector. I estimate the following empirical model

$$\text{CPI}_{i,t} = \hat{\alpha}_1(V/U)_{i,t} + \hat{\alpha}_2(V/U)^2_{i,t} + \hat{\alpha}_3(V/U)^3_{i,t} + \hat{\gamma}_1\omega_{i,t} + \hat{\gamma}_2\omega_{i,t-1} + \hat{\phi}\text{Prod}_{i,t} + \hat{\delta}_i + \hat{\delta}_t + \hat{\epsilon}_{i,t} \quad (7)$$

I estimate this on all service sectors, barring those dominated by government price setting (Education, and Health), and the low-marginal cost information sector. The estimated sectors are Construction, Finance and Insurance, Real Estate and Leasing, Business Services, Transportation, Warehousing and Utilities, Arts, Entertainment, and Recreation, Accommodation and Food, as well as Other Services. I exclude all four quarters of 2020 from the estimation. The initial estimation fails to reach significance due to CPI volatility in the business services sector which has thin coverage in CPI data. Excluding the business services sector clears the noise and leads to significant cubic coefficients. These are further strengthened when omitting the Transportation and Warehousing sector which exhibits a strong inflation cyclicality outside common seasonality.

The resulting estimates are presented in Table 11. I further test a threshold model specification, with the threshold splitting the range of sectoral V/U values in half. The Wald tests reject the null hypothesis of a linear direct impact for both models, while the non-linear coefficients are also highly significant. The results are again robust to the inclusion of further lags of wage growth.

For visualization I plot the cubic functional form of model 1 in Figure 8. The sectoral direct impact follows the aggregate impact closely. The direct impact is non-existent under slack to mildly tight labor markets, while it soars during especially tight labor markets.

I repeat my aggregate wage and real wage growth estimation on sectoral data. Table 12 provides wage growth estimates in the cross-section with two-way fixed effects and two-way clustered standard errors. All labor market coefficients fail to reach significance in the two non-linear models 1 (cubic) and 2 (threshold). Even a linear specification fails significance, while the Wald tests resolutely fail to reject the no impact hypothesis.

This is puzzling as the aggregate data gave a significant, albeit linear, impact of labor market tightness on wage growth. It may be that wage growth is a much more aggregate phenomenon than price changes, such that the impact on wage growth is taken up by the time fixed effects. I am not aware of any paper exploring the correlation of wage growth across sectors, but the question of why wage growth moves in accord across sectors would be an interesting one for future research. I suspect that labor mobility is greater across

Table 11: Sectoral Direct Impact

	Cubic	Threshold
(V/U)	2.12	-1.01
	(2.90)	(0.57)
$(V/U)^2$	-2.67	
	(1.73)	
$(V/U)^3$	0.81**	
	(0.31)	
Prod	-0.01	-0.01
	(0.04)	(0.04)
ECI	0.18**	0.18**
	(0.05)	(0.06)
L.ECI	0.11	0.11
	(0.08)	(0.08)
tight=1 $\times$ (V/U)		4.93***
		(0.97)
tight=1		-9.22***
		(1.84)
Constant	1.92	2.90***
	(1.14)	(0.67)
R^2	0.59	0.58
N	436	436
No Impact (p-value)	0.0004	0.0103
Linear Impact (p-value)	0.0002	0.0038

Standard errors in parentheses

* $p < .10$, ** $p < .05$, *** $p < .01$

Table 12: Sectoral Wage Impact

	W_Cubic	W_Linear	RWG_C	RWG_Q
(V/U)	1.20	-0.25	-0.89	4.95*
	(4.25)	(0.47)	(5.89)	(2.40)
$(V/U)^2$	-0.21		2.21	-1.64*
	(2.43)		(3.44)	(0.79)
$(V/U)^3$	-0.07		-0.78	
	(0.43)		(0.60)	
Prod	0.04	0.05	0.03	0.04
	(0.08)	(0.09)	(0.08)	(0.08)
CPI	0.28*	0.25		
	(0.13)	(0.13)		
L.CPI	-0.04	-0.05		
	(0.16)	(0.16)		
L.RWG			-0.12	-0.11
			(0.10)	(0.11)
Constant	1.59	2.51***	-0.36	-2.38*
	(1.73)	(0.41)	(2.21)	(1.16)
R^2	0.43	0.42	0.40	0.39
N	442	442	436	436
No Impact (p-value)	0.6074	0.6201	0.0052	0.2063
Linear Impact (p-value)	0.4367	-	0.0027	0.0916

Standard errors in parentheses

* $p < .10$, ** $p < .05$, *** $p < .01$

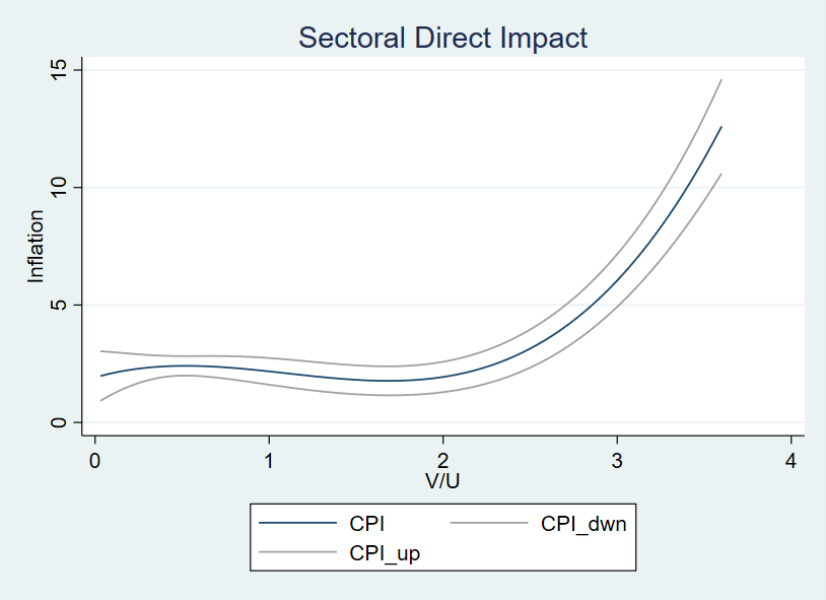


Figure 8: Sectoral Cubic Model

Note: Estimated cubic model of the direct impact on sectoral data. V/U ranges from 0.03 to 3.6 in the sectors used in Table 11.

sectors than previously assumed (Alonso-Carrera and Raurich (2018)).

Further, I test the impact wage-price differentials in the sector panel data. I subtract sector specific CPI inflation from the sector's ECI wage growth and estimate the impact of labor market tightness on this differential. The results are presented in Models 3 and 4. The non-linear terms fail significance at the 5% level in Model 3 but indicate the same direction as on aggregate data. Furthermore, the Wald test rejects a purely linear impact. In model 4 the highest order coefficient is significantly negative at the ten percent level. Using a cubic term instead of a quadratic term in model 4 leads to a fully significantly negative coefficient. Thus, the results are broadly in line with those on aggregate US data.

Finally, I test the implied importance for market concentration from my cross-country results on the sectoral cross-section. I use 2017 Census firm survey data to construct sectoral Herfindahl-Hirschman Indexes (HHIs). The Census data provides brackets of firm size by revenue in the industry as well as total revenue in the sector. From these I impute the average revenue share per firm in each bracket and applying the appropriate weights I then construct the HHI for each sector. I interact these HHIs with labor market tightness to test if concentration increases or dampens the direct impact during tight labor markets. The results are presented in Table 13.

The cubic interaction model provides a negative highest order coefficient that is nonetheless dominated by the positive quadratic interaction term. Reducing the functional form to quadratic delivers a positive highest order interaction coefficient. I show in Figure 9 that the resulting functional forms under maximal HHI in the estimated data are almost identical for

Table 13: HHI Direct Impact Interaction

	Cubic	Quadratic
V/U	7.23**	0.35
	(2.34)	(1.68)
$(V/U)^2$	-6.54**	-0.33
	(2.09)	(0.56)
$(V/U)^3$	1.57**	
	(0.57)	
$V/U \times$ HHI	-15.54**	-7.68*
	(4.69)	(3.63)
$(V/U)^2 \times$ HHI	12.66**	3.48**
	(3.43)	(1.17)
$(V/U)^3 \times$ HHI	-2.54**	
	(0.92)	
Prod	0.02	0.01
	(0.04)	(0.04)
ω	0.20***	0.19**
	(0.05)	(0.05)
L.ω	0.14	0.14
	(0.08)	(0.08)
Constant	1.05	2.67**
	(0.95)	(0.71)
R^2	0.61	0.59
N	436	436
No Interaction (p-value)	0.0424	0.0490
Linear Interaction (p-value)	0.0290	0.0308

Standard errors in parentheses

* $p < .10$, ** $p < .05$, *** $p < .01$

the cubic and quadratic estimations. The Wald test rejects the hypothesis that the interaction terms are jointly zero or only linear in both specifications. The positive quadratic interaction term hints at a strengthening of the direct impact in concentrated industries under tight labor markets. The negative linear interaction term on the other hand hints at lower inflation rates for concentrated industries under slack and normal labor markets. This is likely due to already elevated prices and margins in concentrated sectors. This finding fits with the thin existing literature on Phillips Curves and market concentration (Firat (2024)).

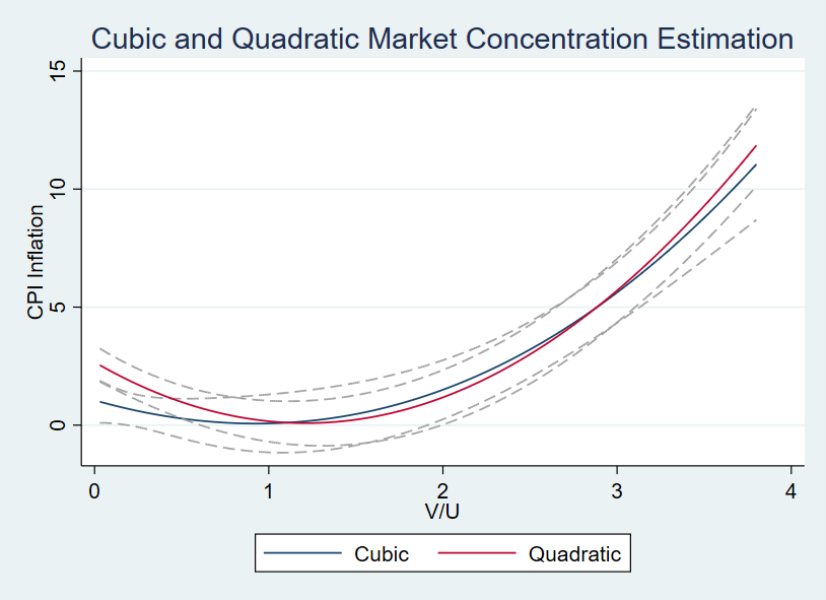


Figure 9: Sectoral Cubic Model

Note: Comparison of the cubic and quadratic market concentration interaction models. The comparison is done under the assumption of maximal level of market concentration; HHI = 0.56 to better visualize the difference in the impact of the interaction terms.

I highlight the role of market concentration in the direct impact channel by comparing the quadratic functional form with minimal market concentration (HHI = 0.03), mean market concentration (HHI = 0.2), and maximal market concentration (HHI = 0.56) as observed in the estimated services sectors. The resulting functional forms are presented in Figure 10. The figure highlights the importance of market concentration for the direct impact channel. Under minimal market concentration the impact is at most flat, and even shows a slight indication for a decline under tight labor markets. Under normal market concentration the direct impact steepens slightly during tight labor markets, and soars under high market concentration.

By using time fixed effects I control for aggregate supply shocks, the endogeneity of monetary and fiscal policy (Fitzgerald et al. (2020), McLeay and Tenreyro (2020)) as well as long-run inflation expectations informed by the monetary policy regime (Hazell et al. (2022)). However, the problem of sectoral supply shocks remains. One way to control for disaggregate supply shocks is to use shift-share innovations in employment to proxy for labor demand shocks to US regions (Bartik (1991), Gitti (2024)).

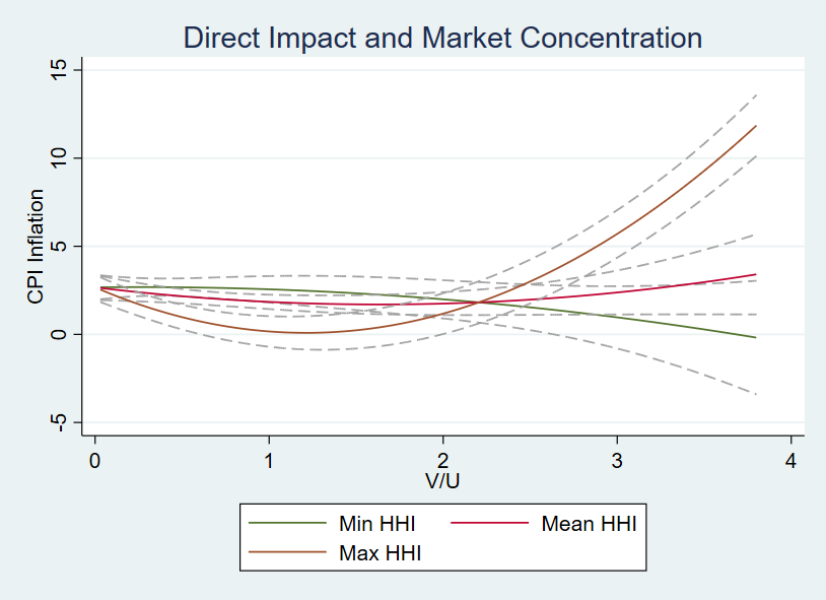


Figure 10: Sectoral Cubic Model

Note: Comparison of direct impact functional forms under different levels of market concentration, with minimal market concentration (HHI = 0.03), mean market concentration (HHI = 0.2), and maximal market concentration (HHI = 0.56) as observed in the estimated services sectors.

Furthermore, there is potential measurement error in the calculation of sectoral unemployment. The number of unemployed by sector come from the Current Population Survey. Sectoral affiliation of an unemployed respondent are assigned as the sector the unemployed worked in last. However, workers often switch between sectors when transitioning from unemployment back to employment. Thus, the sectoral unemployment measure is imperfect and likely introduces noise into the measurement of sectoral V/U.

A Shift-Share instrumental variable (SSIV) can reduce the measurement error in an independent variable and deliver an identification independent of sector specific supply shocks. Thus, SSIV offers an improved estimation of the underlying relation between sectoral inflation and labor market tightness.

I design the SSIV based on state level economic activity shocks, by using the deviation of annual state employment growth from national employment growth. I.e. if state A has an economic boom relative to the rest of the country this will be reflected in faster employment growth than the rest of the nation. I then use the employment share that the shocked state has in the sector to translate these regional economic activity shocks into sectoral labor market variation. This translates the regional activity shocks into sectoral labor demand shocks via the variation in expenditure baskets across states.

The bundle of services purchased varies quite a bit between states (e.g. New Yorkers spend on average 1.3% of their income on public transport, whereas Texans only spend 0.5%, Californians spend 1.9% on Arts and Entertainment services, while New Jersians

only spend 0.7%. Similarly, Lodging away form home is bigger in California (1.8%) than in Texas (0.9%), while both Californians and Texans spend big for outside meals (5 & 4.9% respectively), while this is more muted in New Jersey and Florida (3.3 & 3.4%). Utility services take up more of household income in less densely populated states (CA: 5.8%, TX: 5.6%; NJ:4.1%, NY:4.3%). Health Care Services: CA:1.4% vs NJ: 0.9%. Personal Care: CA 1.3% vs TX or FL:0.9%.[24]

Because of this variation in the consumption bundles across states I can use a shift share instrument to estimate V/U shocks in the service sectors. An economic shock to a state with a high share of sector i in spending will increase sector i's overall demand relative to other sectors. Thus, the shift-share instrument is cleaned of unemployment measurement errors and sectoral supply shocks.

I multiply the exposure share with the regional economic shock and finally sum up over all fifty states plus the District of Columbia (DC). This gives:

$$SSIV_{i,t} = \sum_{s=1}^{51} \text{Empl. Share}_{i,s,t}[g_{s,t} - g_{National,t}]$$

where $\text{Empl. Share}_{i,s,t}$ is the employment share that industry i has in state s at time t, and $g_{s,t} - g_{National,t}$ is the difference between annual employment growth rates of the state and the nation as a whole.

The first stage regression results are solid (see F-test p-values in Table 14). However, the instruments are weak according to the Cragg-Donald-Stock-Yogo test. The second stage TWFEs results are presented in Table 14. Model 1 reports the estimation of the cubic model, Model 2 reports the results of the quadratic model, and Model 3 presents the threshold model.

The coefficients and Wald test results all support a positive non-linearity in the direct impact as in the non-IV panel estimation. To summarize, the direct impact is alive and well in sectoral data. Sectoral inflation increases non-linearly above and beyond wage growth in response to tightening sectoral labor markets, causing wage-price differentials to suffer during especially tight labor markets. This excess steepening is stronger for concentrated sectors, hinting at an optimal price setting mechanism behind the impact. Furthermore, taking out aggregate effects obliterates the traditional wage channel. The latter finding indicates that other cross-sectional Phillips Curve estimations may be driven by the direct impact channel rather than the traditional wage channel.

[24]Source: BLS Research Tables for CA, FL, NJ, NY, TX: (State CE Tables). I used the all consumer expenditure weights from the most recent available research tables (2021-2022); except for NJ (2019-2020).

Table 14: Sectoral Phillips Curves

	Cubic	Quad	Thresh
V/U	1.46	-4.81***	-2.89*
	(4.36)	(1.56)	(1.54)
$(V/U)^2$	-3.35	1.81***	
	(3.41)	(0.53)	
$(V/U)^3$	1.17		
	(0.76)		
ω	0.26***	0.32***	0.18***
	(0.07)	(0.06)	(0.04)
L.ω	0.25***	0.30***	0.12***
	(0.07)	(0.06)	(0.04)
Prod	0.06	0.04	
	(0.05)	(0.05)	
V/U_tight			1.60**
			(0.73)
R^2	0.24	0.18	0.04
N	436	436	436
No Impact (p-value)	0.0021	0.0033	0.0854
Linear Impact (p-value)	0.0007	-	-
First Stage F-Test (p-values)	0.0000	0.0000	0.0000

Standard errors in parentheses

* $p < .10$, ** $p < .05$, *** $p < .01$

7 Source of the Direct Impact

This section discusses the potential source of the direct impact, given the evidence on inflation, real wages and profit margins provided earlier. It concludes that the greater stickiness of wages than prices and associated optimal pricing behavior under transitory demand or labor supply shocks is the most likely driver of the direct impact channel.

Given the response of inflation, real wages (reverse sign) and profit margins to labor market tightness, it is evident that a profit maximizing price setting element is behind the relation between inflation and tight labor markets. Although, there are many possible explanations for the direct impact, there is only one simple theory that explains all the empirical findings in a straight-forward manner.

I propose higher downward-stickiness for wages than for prices, as found by Spencer (1998), paired with transitory demand (or labor supply) shocks as the explanation for the direct impact. A transitory positive demand shock will lead to production expansion during a slack labor market, when wages do not need to rise much to hire the necessary workers. Similarly, a transitory negative labor supply shock amidst a loose labor market will not lead to difficulties in keeping positions filled.

During a tight labor market however, it is necessary to raise wages steeply to hire the workers needed to full-fill extra demand. If the demand shock is transitory, or at least perceived as such, then firms will resist raising wages as they will stay high beyond the current demand shock. It is thus more profitable to raise prices in the short run than raising wages and output. This mechanism gives rise to a natural non-linearity as prices do not rise during slack labor markets but are the main equalizer between demand and supply during a tight labor market.

The transitory negative labor supply shock is analogous. With supply and demand in disequilibrium at long-run wages, the firm has an incentive to price out the superfluous demand created by newly vacant positions instead of raising wages to bring supply and demand into short run equilibrium.

The ensuing subsection highlights this mechanism in a simple model. The primary goal of the model is to explain the existence of the direct impact channel. It demonstrates that downward nominal wage rigidity leads to excess price increases under transitory positive demand or negative labor supply shocks. The non-linearity of the direct impact is explained by the asymmetry between upward wage pressure and downward wage pressure shocks. The direct impact only operates on shocks that increase wage pressure, such as positive demand or negative labor supply shocks.

Thus, this paper extends the literature relating downward nominal wage rigidity to

Phillips Curve non-linearity from its focus on recessions (Schmitt-Grohé and Uribe (2022), Gagnon and Collins (2019), Donayre and Panovska (2016), Gitti (2024)) to how DNWR affects Phillips Curve non-linearity during boom phases. The only other paper concerned with non-linearity during tight labor markets is Benigno and Ricci (2011). They argue that workers voluntarily reduce their wage demands during the boom to prevent getting fired during the bust. However, this leads to a flatter Phillips Curve during boom phases, which is contrary to the evidence (Hooper et al. (2020) , Ball et al. (2022), Benigno and Eggertsson (2023), Gitti (2024)). In contrast, I incorporate the evidence that the steep increase in the Phillips Curve slope during tight labor markets is due to optimal price setting in my model in the ensuing section.

Empirical evidence for firms' resistance to wage increases due to DNWR comes from Elsby (2009). Elsby (2009) argues that firms will "compress" wages under sizeable DNWR to prevent getting stuck at the DNWR constraint. Elsby (2009) uses micro employment data from British establishment data and shows that firms resist wage increases when inflation is low, and they are more likely to get stuck at the DNWR constraint in future periods. Schaefer and Singleton (2022) update the evidence and Daly and Hobijn (2014) show that the wage compression is evident in a fitted model with DNWR during negative demand shocks. My proposed channel applies the same logic to the boom phase. When firms experience such a high level of demand that it cannot last, they also resist raising wages too far above the level they expect to be sufficient to fill their positions in the future.

The mechanism is also in line with recent findings from Caldwell et al. (2024). The paper analysis German firm survey data and finds that labor market conditions explain how firms negotiate wages with workers rather than firm specific characteristics. I postulate that firms similarly change how they think about price increases depending on how tight their relevant labor market is.

7.1 Illustrative Model

This section delineates a simple framework that highlights the direct impact channel via transitory demand shocks and downward sticky wages. I first describe the optimal behavior of a firm with power both in the goods and labor market in a flexible-wage two period maximization problem. I then introduce downward rigid wages to compare how a transitory demand shock affects optimal price setting behavior.

I begin by stating the assumptions on my simple economy. First, labor is the only factor of production:

$$Y_t = L_t \text{ for } t \in \{1, 2\}$$

Second, the monopolist also has monopsonist power and can choose how much labor to

employ according to an upward sloping wage schedule:

$W_t = (\eta_t L_t)^{1/\beta}$ derived from iso-elastic labor supply: $L_t = \frac{1}{\eta_t} W_t^{\beta}$ where $\beta > 1$ for $t \in \{1, 2\}$

and η_t is a labor supply shifter with $\eta_t > 1$ leading to a negative labor supply shock, and $\eta_t < 1$ leading to a positive labor supply shock.

The monopolist further faces a downward sloping demand schedule, with a demand multiplier x_t:

$$D(P_t, x_t) = x_t P_t^{-\epsilon} \text{ where } \epsilon > 1 \text{ for } t \in \{1, 2\}$$

and chooses the optimal price to maximize its profits:

$$\max_{P_1, P_2} \Pi = \max_{P_1, P_2} P_1 D_1 - W_1 L_1 + P_2 D_2 - W_2 L_2$$

It suffices to maximize profits with respect to $P_t,\ t \in \{1, 2\}$ as in equilibrium $L_t = Y_t = D(P_t^*, x_t)$ and thus P_t^* determines output, labor input and wages, given a value for the demand-shifter x_t. I.e. the maximization problem is:

$$\max_{P_1, P_2} \Pi = \max_{P_1, P_2} P_1 D(P_1, x_1) - \eta_1^{\frac{1}{\beta}} (D(P_1, x_1))^{\frac{1+\beta}{\beta}} + P_2 D(P_2, x_2) - \eta_2^{\frac{1}{\beta}} (D(P_2, x_2))^{\frac{1+\beta}{\beta}}$$

In the case of totally flexible prices and wages the two time periods are independent of each other and the maximization with respect to each price $P_t,\ t \in \{1, 2\}$ yields the same First Order Condition:

$$0 = D(P_t, x_t) + P_t \frac{\partial D(P_t, x_t)}{\partial P_t} - \frac{1+\beta}{\beta} D(P_t, x_t)^{1/\beta} \frac{\partial D(P_t, x_t)}{\partial P_t} \eta_t^{\frac{1}{\beta}}$$

Solving for the optimal price P_t^* gives:

$$P_t^* = \left[\frac{\epsilon}{\epsilon - 1} \frac{1+\beta}{\beta} (\eta_t x_t)^{1/\beta} \right]^{\frac{\beta}{\beta+\epsilon}}$$

This is true for both periods as they are independent of each other. It further implies a steady state wage of:

$$W_t^* = (\eta_t L_t)^{1/\beta} = (\eta_t D(P_t, x_t))^{1/\beta} = (\eta_t x_t)^{1/\beta} (P_t^*)^{-\frac{\epsilon}{\beta}} = (\eta_t x_t)^{1/\beta} \left[\frac{\epsilon}{\epsilon - 1} \frac{1+\beta}{\beta} (\eta_t x_t)^{1/\beta} \right]^{\frac{-\epsilon}{\beta+\epsilon}}$$

The results yield the constant markup over marginal cost typical for iso-elastic demand functions:

$$\frac{P_t^*}{W_t^*} = \frac{P_t^*}{(\eta_t x_t)^{1/\beta} (P_t^*)^{-\frac{\epsilon}{\beta}}} = (P_t^*)^{\frac{\beta+\epsilon}{\beta}} (\eta_t x_t)^{-1/\beta} = \frac{\epsilon}{\epsilon - 1} \frac{1+\beta}{\beta}$$

Hence, any shock to the demand shifter x_t or the labor supply shifter η_t does not change the markup, and thus wages and prices have the same growth rate with respect to demand and labor supply shocks. It is nonetheless useful to calculate how P_1^* and W_1^* react to a demand shock in the flexible wage case. To this end I use comparative statics on the optimal first

period price (P_1^*) and wage (W_1^*). I linearize the impact of a small demand shock around the long-run demand shifter, which I pick as $x_1 = x_2 = 1$ for simplicity. The derivation for a negative labor supply shock is identical.

The optimal period one price in the flexible wage case reacts as follows:

$$\frac{\partial P_1^*}{\partial x_1} = \frac{1}{\beta + \epsilon} \left[\frac{\epsilon}{\epsilon - 1} \frac{1+\beta}{\beta} \right]^{\frac{\beta}{\beta+\epsilon}}$$

The optimal period one wage in the flexible wage case reacts as follows:

$$\frac{\partial W_1^*}{\partial x_1} = \frac{1}{\beta + \epsilon} \left[\frac{\epsilon}{\epsilon - 1} \frac{1+\beta}{\beta} \right]^{-\frac{\epsilon}{\beta+\epsilon}}$$

Those responses to the demand shifter do yield the constant markup again:

$$\frac{\partial P_1^*/\partial x_1}{\partial W_1^*/\partial x_1} = \frac{\epsilon}{\epsilon - 1} \frac{1+\beta}{\beta}$$

Hence, inflation and wage growth increase in lockstep following a positive demand or negative labor supply shock in the flexible wage case. With the solutions to optimal price setting in the flexible wage case, I now move on to the case with absolute downward wage rigidity. I.e. I assume that the following constraint has to hold:

$$W_2 = \begin{cases} W(L_2) & \text{if } W(L_2) \geq W_1 \\ W_1 & \text{if } W(L_2) < W_1 \end{cases}$$

This downward nominal wage rigidity (DNWR) constraint modifies the total profit function to

$$\Pi = P_1 D_1 - W_1 L_1 + P_2 D_2 - W_2 L_2 \, \mathbb{1}_{W_2 \geq W_1} - W_1 L_2 \, \mathbb{1}_{W_2 < W_1}$$

Let us now assume a greater demand or labor supply shifter in period one than in period 2; i.e. $x_1 > x_2$ or $\eta_1 > \eta_2$. Then the downward wage rigidity constraint should bite and the optimization problem can be written as:

$$\max_{P_1, P_2} \Pi = \max_{P_1, P_2} P_1 D(P_1, x_1) - \eta_1^{\frac{1}{\beta}} (D(P_1, x_1))^{\frac{1+\beta}{\beta}} + P_2 D(P_2, x_2) - (\eta_1 D(P_1, x_1))^{\frac{1}{\beta}} D(P_2, x_2)$$

This yields the following first order condition with respect to P_1

$$0 = D_1 + P_1 \frac{\partial D_1}{\partial P_1} - \frac{1+\beta}{\beta} (\eta_1 D_1)^{1/\beta} \frac{\partial D_1}{\partial P_1} - \frac{1}{\beta} \eta_1^{\frac{1}{\beta}} D_1^{\frac{1-\beta}{\beta}} \frac{\partial D_1}{\partial P_1} D_2$$

Plugging in the demand functions, while assuming $x_2 = 1$, $x_1 = x > 1$, $\eta_2 = 1$, and $\eta_1 = \eta > 1$ for simplicity, this yields:

$$0 = x - \epsilon x + \frac{1+\beta}{\beta} \epsilon \eta^{\frac{1}{\beta}} x^{\frac{1+\beta}{\beta}} P_1^{-\frac{\epsilon+\beta}{\beta}} + \frac{\epsilon}{\beta} (\eta x)^{1/\beta} P_1^{\frac{\epsilon\beta - (\epsilon+\beta)}{\beta}} P_2^{-\epsilon} \tag{8}$$

which is not further solvable without optimal P_2. Hence, I solve for the FOC with respect to P_2:

$$0 = D_2 + P_2 \frac{\partial D_2}{\partial P_2} - \eta^{\frac{1}{\beta}} D_1^{1/\beta} \frac{\partial D_2}{\partial P_2}$$

which simplifies to:

$$P_2 = \frac{\epsilon}{(\epsilon-1)}(\eta x)^{1/\beta} P_1^{-\frac{\epsilon}{\beta}}$$

Plugging this into equation 8 and simplifying yields

$$\frac{1+\beta}{\beta}\epsilon\eta^{\frac{1}{\beta}}x^{\frac{1+\beta}{\beta}} = P_1^{\frac{\epsilon+\beta}{\beta}}\left[x(\epsilon-1) - \frac{\epsilon}{\beta}\left(\frac{\epsilon-1}{\epsilon}\right)^{\epsilon}(\eta x)^{\frac{1-\epsilon}{\beta}} P_1^{(\epsilon-1)\frac{\beta+\epsilon}{\beta}}\right]$$

which is not solvable without knowing what ϵ and β are. Additionally, the observation that ϵ cannot be 1 can be gleaned from the expression.

As unwieldy as the expression for P_1 under downward nominal wage rigidity (DNWR) is, we can use it to derive the impact of a demand or labor supply shock on the optimal first period price level under DNWR. I conduct comparative statics on the expression for the optimal P_1 to this end.

Comparative statics with respect to x, assuming no other shocks; i.e. $\eta = 1$ and linearizing around the equilibrium demand shifter $x = 1$ gives the following expression after simplification:

$$\frac{\partial P_1}{\partial x} = \frac{\partial P_1^*}{\partial x}\frac{1+\beta+\left[\frac{\epsilon}{\epsilon-1}\frac{1+\beta}{\beta}\right]^{\epsilon-1}\left(\epsilon-1-\frac{\epsilon}{\beta}\left(\frac{\epsilon-1}{\epsilon}\right)^{\epsilon}\right)}{1-\frac{(1+\beta)^{\epsilon-1}}{\beta^{\epsilon}}+\left[\frac{\epsilon}{\epsilon-1}\frac{1+\beta}{\beta}\right]^{\epsilon-1}\left(\epsilon-1-\frac{\epsilon}{\beta}\left(\frac{\epsilon-1}{\epsilon}\right)^{\epsilon}\right)} \tag{9}$$

where P_1^* is the optimal price set in steady state and $\frac{\partial P_1^*}{\partial x}$ is the derivative of the optimal price level in the flexible wage case with respect to the demand shock. Both were derived in the first part of the model.

I.e.

$$\begin{aligned} &\frac{\partial P_1}{\partial x} > \frac{\partial P_1^*}{\partial x} \\ \iff \quad &\underbrace{\beta}_{>0} > \underbrace{-\frac{(1+\beta)^{\epsilon-1}}{\beta^{\epsilon}}}_{<0} \end{aligned}$$

Thus, under the assumption that the DNWR constraint binds the linearized impact of the demand shock on first period prices is bigger than under flexible wages. This is also true if ϵ or β or both are allowed to grow towards infinity (see Appendix B.1).

These findings imply that prices under DNWR rise faster than wages in the first period under a positive demand shock, above and beyond the rise needed to keep the markup constant. To show this more rigorously remember that the wage in period one can be written as:

$$W_1 = x^{\frac{1}{\beta}} P_1^{-\frac{\epsilon}{\beta}}$$

which leads to a response to a demand shock, linearized around $x = 1$, of:

$$\frac{\partial W_1}{\partial x} = \frac{1}{\beta}(P_1^*)^{-\frac{\epsilon}{\beta}} - \frac{\epsilon}{\beta}(P_1^*)^{-\frac{(\epsilon+\beta)}{\beta}}\frac{\partial P_1}{\partial x}$$

This expression is true for both the case with DNWR and the flexible wage case. Thus, as $\frac{\partial P_1}{\partial x} > \frac{\partial P_1^*}{\partial x}$ we get:

$$\frac{\partial W_1}{\partial x} < \frac{\partial W_1^*}{\partial x}$$

and thus

$$\frac{\partial P_1/\partial x}{\partial W_1/\partial x} > \frac{\partial P_1^*/\partial x}{\partial W_1^*/\partial x} = \frac{\epsilon}{\epsilon - 1}\frac{1+\beta}{\beta}$$

I.e. the price in period one reacts above and beyond wage growth following a transitory demand shock if wages are downward sticky, while they just react with the increase in wages, to retain the optimal constant markup, in the flexible wage case.

A parallel effect can be derived for negative labor supply shocks, which were sizeable during the pandemic (Goda and Soltas (2023)). See Appendix B.2 for the derivation of the direct impact channel under a transitory negative labor supply shock.

There is one final necessary check: does the DNWR constraint actually bind? I.e.

$$W(L_1) = L_1^{1/\beta} \overset{?}{>} L_2^{1/\beta} = W(L_2)$$

This can be shown by contradiction. Assume that it is optimal to set production and thus wages higher in period two: $L_2 \geq L_1 \iff W(L_2) \geq W(L_1)$, when faced with a greater positive demand shock in period 1 than in period 2 ($x_1 > x_2$). Then the DNWR constraint is not binding, and the firm follows the price setting behavior of the flexible wage case. This case has been solved at the beginning of the Toy model and yielded an optimal wage of:

$$W_t^* = L_t^{1/\beta} = D(P_t, x_t)^{1/\beta} = x_t^{1/\beta}(P_t^*)^{-\frac{\epsilon}{\beta}} = x_t^{1/\beta}\left[\frac{\epsilon}{\epsilon-1}\frac{1+\beta}{\beta}x_t^{1/\beta}\right]^{\frac{-\epsilon}{\beta+\epsilon}}$$

Simplifying further yields

$$W_t^* = x_t^{\frac{1}{\beta}-\frac{\epsilon}{\beta(\beta+\epsilon)}}\left[\frac{\epsilon}{\epsilon-1}\frac{1+\beta}{\beta}\right]^{\frac{-\epsilon}{\beta+\epsilon}} = x_t^{\frac{\beta+\epsilon-\epsilon}{\beta(\beta+\epsilon)}}\left[\frac{\epsilon}{\epsilon-1}\frac{1+\beta}{\beta}\right]^{\frac{-\epsilon}{\beta+\epsilon}} = x_t^{\frac{1}{\beta+\epsilon}}\left[\frac{\epsilon}{\epsilon-1}\frac{1+\beta}{\beta}\right]^{\frac{-\epsilon}{\beta+\epsilon}}$$

The exponent on the demand multiplier is clearly positive for $[\beta > 0;\ \epsilon > 0]$. Hence, the optimal wage set in the flexible wage case increases with the demand multiplier. Therefore, assuming that the DNWR does not bind leads to $W_1 > W_2$ as $x_1 > x_2$, and thus contradicts the assumption. Therefore, the DNWR constraint has to bind following a positive transitory demand shock.

How does this explain the existence of a direct impact of labor market tightness on inflation? A wage pressure shock will create vacancies no matter if it is driven by labor supply, demand a combination of both shocks. A negative labor supply shock creates

vacancies by quitting employees, while a demand shock increases vacancies through the desire to expand supply. If the supply of job seekers falls short of the demand then vacancies will remain unfilled, leading to a rising ratio of job vacancies to unemployed.

Employers could offer a higher wage to incentivize more marginally attached workers into the labor force, or try and poach them from competitors. And this is observed in the flexible wage case, where wages do rise following the wage pressure shock. But if the shock is seen as transitory then employers with market power will resist bidding up wages too far above their expected future value to prevent higher marginal cost in the future.

Under the assumption of market clearing this leaves firms the option to price away part of demand to lower their need to raise wages. Lower demand leads to lower needed supply, and thus fewer workers needed to produce it. Thus, increasing prices forgoes part of the otherwise necessary wage increase. Due to DNWR this is profit maximizing due to the reduction of future marginal cost.

Therefore, my simple model is sufficient to explain the existence of the direct impact. Furthermore, it explains the non-linearity of the direct impact. Positive demand shocks of a certain size will make the DNWR constraint bind, while smaller or even negative shocks will not. Thus, the direct impact channel is only active for part of the spectrum of observed demand shocks, leading to the non-linearity in the data.

This on-off nature of the direct impact channel justifies the threshold approach to the non-linear Phillips Curve taken in the literature (Smith et al. (2023)). The observed smoothness of the direct impact under the cubic specification used in this paper is likely due to the varying levels of expected future wages, as well as varying labor supply and demand elasticities across firms.

Thus, Table 5 demonstrates a substantial influence of labor market tightness on profit margins and exhibits a significant non-linear pattern. These findings imply that the direct impact can explain the concomitant rise in inflation and profit margins. Therefore, another contribution of this paper is to substitute a more satisfying explanation to the concomitant rise in inflation and profit margins than the postulated increase in corporate greed purported under the popular "Greedflation" hypothesis (Vinod (2022)) or its close cousin "sellers' inflation" in which firms use shocks as signals for tacit collusion (Weber and Wasner (2023)). The results are also in line with the greater impact of profits on the post-pandemic inflation in Europe found by Hansen et al. (2023). Their results hint that my findings may not be limited to the US.

Finally, the paper contributes another angle on the documented pro-cyclical nature of profit margins and markups (Macallan et al. (2008), Anderson et al. (2018), Qiu and Ríos-Rull (2022)). The pro-cyclicality is heretofore explained by the pro-cyclicality of productivity

(Qiu and Ríos-Rull (2022)) and demand (Anderson et al. (2018)). Other colloquial reasons are pro-cyclical cost reduction due to the declining ratio of fixed cost to revenue, and financial leverage. The literature has explored neither these latter channels nor my proposed direct impact channel, driven by downward nominal wage rigidity. Therefore, offering wage-rigidity-driven price setting in response to labor market tightness as the driver of profit margin pro-cyclicality is another contribution of this paper.

7.2 NFIB Firm Expectations

Did firms really face difficulties recruiting new employees, despite modest wage growth and declining real wages for many workers during the post-pandemic recovery? The National Federation of Independent Business (NFIB)[25] poses the question: "Do you have any job openings that you are unable to fill right now?" The share of businesses answering "yes" rose sharply during the post-pandemic inflationary period (and slightly preceded it). As Kudlyak and Miskanic (2024) observed, this increase outpaced the rise in the commonly used vacancy-to-unemployment (V/U) ratio. I plot the two series for comparison in Figure 11.

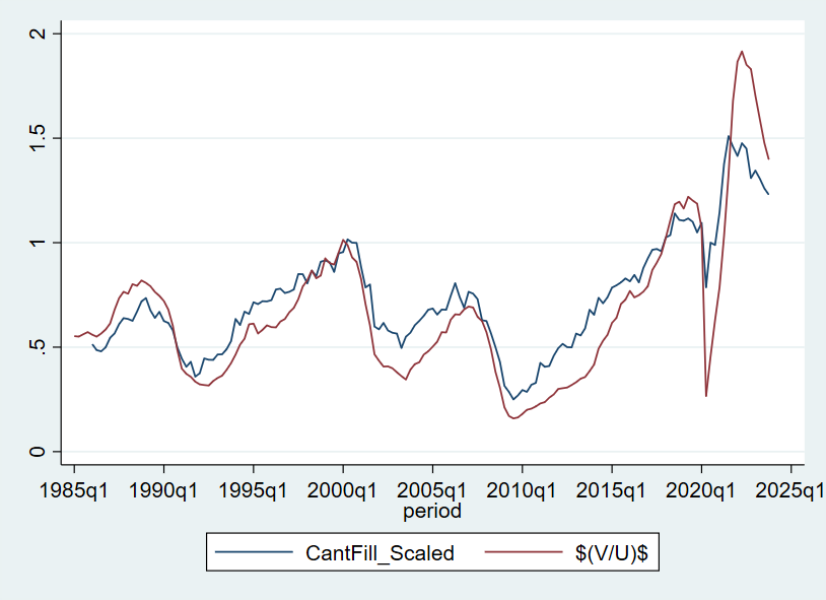


Figure 11: NFIB & BLS Labor Market Tightness

Note: Comparing BLS' V/U measure with the NFIB fraction of firms reporting unfillable positions, scaled by a factor of three to roughly match the range in the V/U data.

I use this measure as a robustness check for the V/U ratio. This is fitting as my channel operates through the difficulty of expanding production via hiring. I find that the NFIB metric for unfillable positions performs equally well in predicting higher inflation, alongside

[25]The NFIB surveys small businesses with mostly less than 40 employees from all Census Regions. The three major sectors by respondent type were Retail (22.3%), Construction (18.2%), and Services (16.6%) in 2019.

subdued wage growth (see Table 15). This suggests that firms' difficulty in filling positions and consequently meeting demand may indeed be the key driver of the V/U-inflation relationship extending beyond the role of wage growth.

Models 1–4 in Table 15 present the estimates of the cubic NFIB "Cannot Fill Vacancy" measure on median, core, headline, and ECI wages and salaries inflation. The results are robust to restricting the estimation to the pre-pandemic period. The impact on inflation mirrors the pattern observed with the V/U ratio, while its effect on wage growth (Model 4) is again linear. This suggests that firms' reported hiring difficulties align closely with labor market tightness, as highlighted by Kudlyak and Miskanic (2024). Furthermore, this may explain why inflation correlates strongly with labor market tightness, even when controlling for wage growth.

The results of a horse race between cubic V/U ratio and the cubic NFIB measure are mixed. For median CPI inflation, the NFIB measure is highly significant, while the V/U ratio becomes insignificant when both are included. Conversely, for traditional core inflation, the V/U ratio remains significant, while the NFIB measure loses significance. On headline inflation, both measures become insignificant.

Furthermore, the NFIB provides data to test my proposed mechanism. This mechanism is based on businesses' reluctance to raise offered wages to fill the positions they describe as "unfillable." I hypothesize that this reluctance stems from an expectation of declining demand. Diminishing demand could reduce the need for additional workers and risk locking firms into uncompetitively high wages, which would be difficult to adjust downward when demand falls.

Unfortunately, the NFIB does not ask firms about their expectations for future wages. But the NFIB asks "Do you think that six months from now general business condition will be better than they are now, about the same, or worse?" (Q5: http://www.nfib-sbet.org/micro-data/).

The NFIB constructs an indicator for expected business conditions by subtracting the percentage of firms answering "worse" from those answering "better." This indicator declined sharply in 2021 and continued downward, reaching its lowest point in June 2022, coinciding with the inflation peak. However, this measure may partially reflect the impact of inflation, as rising costs affect business outlooks. The primary evidence suggesting that these business perceptions influence inflation rather than being a result of it is that the decline in the indicator preceded the rise in inflation by a few months.

To further explore firm expectations, I use the NFIB measure on expected real sales growth over the next three months (Q8; http://www.nfib-sbet.org/micro-data/) "What do you expect to happen to the real volume (number of units) of goods and/or services that

Table 15: NFIB Cannot Fill Vacancy Impact

	Median	XFE	CPI	ECI
CantFill	31.46***	41.14***	53.90***	12.03
	(9.06)	(12.80)	(15.12)	(13.57)
CantFill2	-125.50***	-176.18***	-221.90***	-14.32
	(33.98)	(48.00)	(56.68)	(52.01)
CantFill3	166.36***	247.73***	294.38***	13.65
	(39.63)	(55.98)	(66.10)	(62.06)
π^e	0.60***	1.00***	0.83***	0.82***
	(0.08)	(0.12)	(0.14)	(0.10)
ω	0.16***	0.03	0.05	
	(0.06)	(0.08)	(0.10)	
L.ω	0.24***	-0.03	0.12	
	(0.06)	(0.08)	(0.09)	
Prod	-0.25***	-0.09	-0.15**	0.16***
	(0.04)	(0.05)	(0.06)	(0.05)
Energy	0.00	0.00	0.10***	0.00
	(0.00)	(0.00)	(0.01)	(0.01)
L.Energy	-0.00	0.00	0.00	-0.01
	(0.00)	(0.00)	(0.01)	(0.01)
Import	-0.06	-0.04	0.38	-0.27
	(0.16)	(0.22)	(0.26)	(0.22)
L.Import	-0.03	-0.13	-0.17	-0.21
	(0.16)	(0.22)	(0.26)	(0.22)
π				0.03
				(0.07)
L.π				0.16**
				(0.07)
Constant	-2.16***	-3.25***	-4.22***	-2.40**
	(0.80)	(1.13)	(1.33)	(1.14)
R^2	0.81	0.71	0.86	0.62
N	152	152	152	152

Standard errors in parentheses

* $p < .10$, ** $p < .05$, *** $p < .01$



Figure 12: NFIB Business Condition Expectations

Note: Fraction of firms expecting business conditions to improve minus fraction of firms expecting conditions to worsen. Source: http://www.nfib-sbet.org/indicators/

you will sell during the next 3 months?'

The NFIB calculates the real sales expectations indicator by subtracting the percentage of respondents answering "lower" from those answering "higher." Outside of recessions, this indicator typically ranges between 15% and 20%. However, it declined steadily as the labor market tightened and inflation surged to a low of almost -30%.

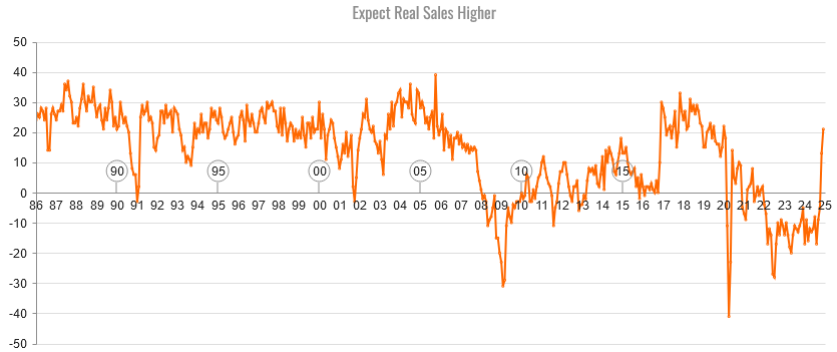


Figure 13: NFIB Real Sales Growth Expectations

Note: Fraction of firms expecting real sales volume to increase minus fraction of firms expecting real sales volume to decrease. Source: http://www.nfib-sbet.org/indicators/

While not conclusive, this suggests that firms were anticipating a decline in sales volume even as the economy was booming and inflation was surging. The NFIB measures on expected business conditions in six months and expected real sales growth over the next three months allow me to test if the proposed expected decline in future demand contributes to inflation when labor markets are tight. The standard New Keynesian (NK) framework suggests the opposite: firms' inflation expectations typically rise with increasing aggregate demand leading to price increases today. Similarly, expectations of higher idiosyncratic demand in the future will lead firms to raise prices today. Thus, the coincidence of these two should lead to even greater inflationary pressure.

In the first case, firms raise their prices in anticipation of future inflation due to the price rigidities at the core of the NK framework. In the second case, firms raise prices to smooth the path to the optimal future price level when the higher demand manifests. Additionally, tight labor markets increase inflation due to rising wages. When both tight labor markets and rising sales expectations come together the NK model suggests either a simple adding of the two factors (when linearized), or a positive interaction as rising demand will increase already high labor demand and will thus raise wages even more.

In contrast, my model predicts that firms raise their prices by more when tight labor markets coincide with an expected **decline** in future demand for their goods and services. Firms do so to evade cementing high wage cost into the future, when demand is expected to flag again.

To test between these two models I interact the business conditions and the real sales growth expectations with the NFIB's measure of hiring difficulty:

$$\pi = \alpha V/U_{NFIB,t} + \beta V/U_{NFIB,t} Z_t + \gamma_1 \omega_t + \gamma_2 \omega_{t-1} + \epsilon_t$$

where Z_t is a place-holder for the two different expected demand measures of the NFIB. β should be positive if the traditional NK logic holds. If my channel holds, it should be negative instead.[26] Table 16 presents the results for expected business conditions in six month and for three month expected real sales growth on Median CPI inflation, respectively. The results are similar for headline and traditional core inflation.

Both measures deliver a highly significant and negative β coefficient. These results are robust to real GDP, and sales growth controls. This supports my channel over the traditional NK view. Incidentally, this finding rules out a whole class of models, from the basic NK framework to the diminishing productivity approach of Bunn et al. (2024).

8 New Keynesian Model

This section introduces the higher future marginal cost channel highlighted in my illustrative model into a New Keynesian DSGE framework. The model differs from the mainline New Keynesian framework with search and matching primarily through using wage posting instead of wage bargaining, on-the-job-search, and downward nominal wage rigidity. Wage posting is introduced to give firms power over wages relative to the surplus of a match. On-the-job search is introduced to create competition for workers among firms, and thus raise the wage significantly above worker's outside option.

[26] See derivation of negative interaction from the illustrative model in Appendix B.5

Table 16: NFIB Business and Real Sales Expectations

	ImprovBCs	ImprovRSales
CantFill	5.03***	6.60***
	(0.93)	(0.88)
ImprovBCs	2.04***	
	(0.68)	
CantFill × ImprovBCs	-11.36***	
	(2.19)	
π^e	0.83***	0.77***
	(0.09)	(0.08)
ω	0.06	0.10*
	(0.06)	(0.06)
L.ω	0.11*	0.13**
	(0.06)	(0.06)
ImprovRSales		3.97***
		(0.93)
CantFill × ImprovRSales		-22.85***
		(3.42)
Constant	-1.16***	-1.32***
	(0.22)	(0.21)
R^2	0.74	0.76
N	152	152

Standard errors in parentheses

* $p < .10$, ** $p < .05$, *** $p < .01$

The standard New Keynesian framework is unable to accommodate my channel as its bargaining based wage setting leads to wages and prices growing in lock-step.[27] To incorporate my channel I need a framework in which firms decide both on prices and wages. Thus, I build on a family of New Keynesian models with simultaneous wage and price setting by firms pioneered by Sveen and Weinke (2009), and Thomas (2011). My model extends the newest generation of these models, who use wage posting instead of wage bargaining to provide firms with monopsony power (Bloesch and Larsen (2023), de la Barrera i Bardalet (2024), Bloesch et al. (2024)).[28]

However, monopsony power is not enough to explain the price increase in excess of marginal cost found in the data. Monopsony power alone does not lead to an increasing profit share as labor markets tighten (see Bloesch and Larsen (2023) for details). I introduce my future marginal cost channel into the model via downward nominal wage rigidity. Downward nominal wage rigidity (DNWR) introduces a forward looking wage cost component into the optimal price setting behavior under transitory demand and labor supply shocks. The forward looking wage cost stems from the rise of wages beyond future expected wages. Due to DNWR these higher wages are then cemented into the future, leading to higher future marginal cost. The firm takes these additional cost into account when maximizing its profits and raises prices beyond the optimal level under flexible wages.

The forward looking wage costs introduced by DNWR are similar to the User Cost of Labor as defined by Kudlyak (2014). The User Cost of Labor is defined as the present discounted value of the difference in wages for a worker hired today instead of tomorrow. The User Cost of Labor is thus only playing a role during boom phases of the economy, when expected wages necessary to hire an additional worker are lower in the future than today's wage rate. This increases the cost of hiring an additional worker beyond the traditional wage and vacancy cost.

The true wage cost of an additional worker in Kudlyak (2014) is

$$w_{i,t} + E_t \left[\sum_{k=1}^{\infty} (\beta(1-\bar{\delta}))^k (w_{i,t} - w_{i,t+k}) \right]$$

if $w_{i,t} > E_t[w_{i,t+k}]$, and simply the wage $w_{i,t}$ if the expected future wage is greater or equal to the current wage. β is the discount rate and $\bar{\delta}$ is the separation rate for employees, which together deliver the effective discount factor for the expected future wage differentials. An additional piece to the total user cost of labor from Kudlyak (2014) is the difference between

[27] This is true even for the non-linear PC model of Benigno and Eggertsson (2023) or Gitti (2024). Their non-linearity is simply driven by a non-linearity in wages. Price setting still follows a constant markup over marginal cost behavior.

[28] For a good survey of the literature on monopsony power in the labor market see Ashenfelter et al. (2010).

hiring costs today and tomorrow. This brings the total cost of a worker to:

$$w_{i,t} + E_t\left[\sum_{k=1}^{\infty}(\beta(1-\bar{\delta}))^k(w_{i,t}-w_{i,t+k})\right] + \frac{\kappa_v}{a_t\left(\frac{w_{i,t}}{W_t}\right)} - \beta(1-\bar{\delta})E_t\left[\frac{\kappa_v}{a_{t+1}\left(\frac{w_{i,t+1}}{W_{t+1}}\right)}\right]$$

where $a_t\left(\frac{w_{i,t}}{W_t}\right)$ is the probability of turning a vacancy into an employee and κ_v is the cost of a vacancy.

I begin by describing the simple household block of the model. Then I will describe the dynamics of the labor market, and state the firm's maximization problem, which sits at the heart of the model.

8.1 Households

I begin the model with the household block, which consists of a representative household maximizing life time utility based on aggregate consumption C_{t+k}, subject to the discount factor $\beta_t = \bar{\beta} + \rho_\beta(\beta_{t-1} - \bar{\beta}) + \eta_t^\beta$. Where η_t^β is a shock and ρ_β the autocorrelation of the discount factor. Life time utility is described by

$$\mathcal{U}_t = E_t\left[\sum_{k=0}^{\infty}\beta_t^k u(C_{t+k})\right] \tag{10}$$

which is maximized under the following intertemporal budget constraint

$$\int_0^1 p_{i,t}c_{i,t}\,di + Q_tB_t = \int_0^1 w_{i,t}n_{i,t}\,di + B_{t-1} + \Pi_t \tag{11}$$

where $p_{i,t}$, and $c_{i,t}$ are the price and the consumption of variety $i \in [0,1]$. Thus, $\int_0^1 p_{i,t}c_{i,t}\,di$ is the total consumption expenditure of the household in period t. Q_t is the price of a one period savings bond B_t. Hence, Q_tB_t is the amount the household is saving in period t. $w_{i,t}$ is the wage earned by workers $n_{i,t}$ producing variety i. Therefore, $\int_0^1 w_{i,t}n_{i,t}\,di$ is the total amount of labor income of the household in period t. B_{t-1} is the amount of savings plus interest received from savings in the previous period $t-1$. Finally, Π_t are firms' profits distributed to households via dividends. Thus, households use their labor income, their savings, and the proceeds from stock ownership to fund consumption and savings.

I follow the standard assumption of Constant Elasticity of Substitution (CES) preferences over consumption varieties. Households trade off consuming less of one variety for more of another depending on how much of each variety the household already has in its consumption basket. The elasticity of substitution is subject to AR(1)-shocks and evolves as follows: $\epsilon_t = \bar{\epsilon} + \rho_\epsilon(\epsilon_{t-1} - \bar{\epsilon}) + \eta_t^\epsilon$. With the CES preferences we can write the expression of the aggregate consumption bundle of the representative household.

$$C_t = \left(\int_0^1 (c_{i,t})^{\frac{\epsilon_t - 1}{\epsilon_t}}\,di\right)^{\frac{\epsilon_t}{\epsilon_t - 1}} \tag{12}$$

The expression of the aggregate consumption bundle tells us how the household prefers to split its consumption budget into the single varieties, but not how much of each variety is bought relative to all other varieties. To determine this, the household has to take the price differences between varieties into account. Recall that the overall consumption expenditure is split among varieties in the following way:

$$\text{Consumption Expenditure} = \int_0^1 p_{i,t} c_{i,t} \, di \tag{13}$$

This equation represents the cost of the aggregate consumption bundle with quantities $c_{i,t}$ for each of the consumption varieties $i \in [0, 1]$. Minimizing the cost of the optimal consumption bundle C_t yields household demand for a single variety:

$$\left(\frac{p_{i,t}}{P_t} \right)^{-\epsilon_t} C_t \tag{14}$$

This expression finally delivers the relative consumption weights of each consumption variety. Incidentally, these weights can be used to derive the price index P_t of the aggregate consumption bundle C_t. The corresponding aggregate price index is

$$P_t = \left(\int_0^1 p_{i,t}^{1-\epsilon_t} \, di \right)^{\frac{1}{1-\epsilon_t}} \tag{15}$$

The final step in solving the household's optimal behavior is to determine the split of market resources between savings and consumption. Optimizing the households life time utility function under the budget constraint, with respect to bond holdings B_t, yields the household's Euler Equation.

$$Q_t = \beta E_t \left[\frac{u'(C_{t+1})}{u'(C_t)} \frac{P_t}{P_{t+1}} \right] \tag{16}$$

This concludes the household side of the economy. We have derived how the household splits its income from wages, savings and dividends into optimal savings and consumption. Further, we derived how the preferences over varieties determine the households relative demand for each variety depending on its relative price to all other varieties. This is crucial in determining the demand each firm faces for its variety in response to its set price $p_{i,t}$. I now move on to describe the labor market which will set the constraints under which firms choose the optimal wage to post on their vacancies and the amount of vacancies to post.

8.2 Labor Market

At the beginning of period t a fraction $\bar{\delta} \in (0, 1)$ of the previously employed workers get separated from their employer exogenously. Thus, at the beginning of period t, the total pool of unemployed workers consists of the unemployed at the end of the previous period and the newly separated workers.

$$U_t^{start} = U_{t-1} + \bar{\delta} N_{t-1} \tag{17}$$

In this expression U_t^{start} and U_t are the number of unemployed workers at the beginning and end of period t, respectively.[29] $\bar{\delta}$ is the exogenous separation rate, and N_t the number of employees in period t. In the labor market, both employed and unemployed workers browse existing job postings V_t. The employed do so with a relative search intensity $s \leq 1$. Search intensity is a measure of time available for employed workers to search for an alternative job, relative to the time unemployed workers have for job hunting. For example employed workers may only search for jobs during the weekend, while unemployed workers can search for job postings 24/7. This would give a relative search intensity of $s = 2/7$. The resulting effective amount of job seekers in the economy is

$$S_t = U_{t-1} + \bar{\delta} N_{t-1} + s(1 - \bar{\delta}) N_{t-1} \tag{18}$$

The amount of effective job seekers describes the amount of labor ready for hire in the labor market. Unfilled labor demand in the model is described by the total amount of open jobs in the economy

$$V_t = \int_0^1 v_{i,t} \, di \tag{19}$$

Labor market tightness in the model is the ratio of job postings V_t to the effective job searchers S_t.[30] Labor market tightness is a measure of how many workers firms are trying to hire relative to the available workers free for hiring. If the ratio is large, then there is a lot of labor demand relative to available labor supply and firms will have a hard time filling a given vacancy. If the ratio is small, then there is ample labor supply and firms will find it relatively easy to fill a vacancy. On the flip side, if the ratio is high, unemployed workers are likely to find a job and currently employed workers are likely to get an offer which they can compare to their current wage and job conditions. If the ratio is small, unemployed workers are more likely to stay unemployed and employed workers are less likely to get a competing offer and are thus less likely to transition from their current position to a new one.

$$\theta_t = \frac{V_t}{U_{t-1} + \bar{\delta} N_{t-1} + s(1 - \bar{\delta}) N_{t-1}} =: \frac{V_t}{S_t} \tag{20}$$

The likelihood that a current job seeker is matched to a current job posting is determined by how efficient the labor market works. I introduce labor market frictions via a standard search and matching process. Job seekers are matched to an available job vacancy with a standard Cobb-Douglas matching function

$$m(V_t, S_t) = \kappa_\theta S_t^\alpha V_t^{(1-\alpha)} \;\; , \; \alpha \in (0, 1) \tag{21}$$

[29] U_t^{start} evolves according to exogenous separation and the job finding rate of the unemployed: $U_t^{start} = U_{t-1} + \bar{\delta} N_{t-1} = U_{t-1}^{start} - p_{t-1}^U q(\theta_{t-1}) V_{t-1} + \bar{\delta} N_{t-1}$. Where V_t is the amount of vacancies, p_t^U the likelihood of encountering an unemployed worker, and $q(\theta_t)$ the probability of filling a vacancy given labor market tightness θ_t.

[30] This indicates that Abraham et al. (2020)'s measure of labor market tightness based on effective job searchers would be a better measurement for my empirical work. However, they construct their measure only up to 2019. Furthermore their measure is closely correlated with the standard V/U measure. Thus, using their measure in future renditions is unlikely to change my results.

The more job seekers S_t are in the labor market, the easier it is for a vacancy to be matched to a fitting candidate. But the more job seekers are in the labor market, the less likely it is for any given job seeker to be matched to a fitting vacancy. Similarly, the more vacancies V_t are out there, the lower the probability of filling one of them from the available pool of job seekers.

The matching function determines the matching probability for a given job vacancy

$$q(\theta_t) = m(\theta_t^{-1}, 1) = \frac{m(V_t, S_t)}{V_t} = \kappa_\theta \theta^{-\alpha} \tag{22}$$

As $\alpha > 0$, the tighter the job market is (i.e. the higher θ is) the lower the probability of finding a worker to fill a given vacancy.

Once a job seeker is matched to a vacancy, the worker has to decide whether to accept the offered job or not. The likelihood of accepting the offered job potentially differs a lot between unemployed job seekers and those who are searching on-the-job. The probability that a matched worker is already employed is the ratio of effective on-the-job seekers relative to all job seekers.

$$\mathfrak{p}_t^E = \frac{s(1-\bar{\delta})N_{t-1}}{U_{t-1} + \bar{\delta}N_{t-1} + s(1-\bar{\delta})N_{t-1}} \tag{23}$$

Similarly, the probability that the matched worker is unemployed is the ratio of all unemployed job seekers to all effective job seekers.

$$\mathfrak{p}_t^U = \frac{U_{t-1} + \bar{\delta}N_{t-1}}{U_{t-1} + \bar{\delta}N_{t-1} + s(1-\bar{\delta})N_{t-1}} \tag{24}$$

For simplicity, I assume that there are no unemployment benefits, and an unemployed job seeker always accepts a job offer. The employed workers compare the offered salary with their current wages. The employed worker also receives a taste shock $\psi \sim F_\psi$ for the offered job relative to its current position. Think of this taste shock as a preference for location, fringe benefits like health insurance, childcare etc, or simply lower work hours, free coffee or nicer colleagues.

A simple specification for F_ψ is a normal distribution with mean one and standard deviation σ_ψ. This taste shock is compared with the relative wage between the current and the offered job. If the taste shock is smaller than the ratio of the current wage to the offered wage, the worker's comparative preference for the new job is too small and the worker will decline the offer. On the other hand, if the taste shock is larger than the ratio of the current wage to the offered wage, the worker accepts the offered position and quits his current job. Thus, offering a higher wage than the going wage rate increases the likelihood of poaching a worker from a competitor, while it lowers the likelihood of losing employees to a competitor. On average, the worker accepts an offered job with wage $w_{i,t}$ relative to the going wage rate W_t with probability

$$1 - F_\psi\left(\frac{W_t}{w_{i,t}}\right) \tag{25}$$

If the taste shock ψ is smaller than the ratio of its current wage (W_t in the aggregate) to the offered wage $w_{i,t}$, then the taste shock is too small for the worker to be willing to switch. This can be the case even if the offered wage is greater than the worker's current wage. This would mean that the worker values the overall job package of their current job more than the overall package, including the higher wage, of the offered position. If the taste shock is larger than the relative wage offer then the overall job package is better at the new position and the worker will switch to the new position.

The importance of taste shocks in the acceptance decision depends crucially on the variance of the taste shocks. If the variance goes to zero, a worker will always choose the higher paying job. On the other hand if the variance increases, the payoff from raising the offered wage diminishes as worker decisions for or against a job offering become increasingly random. The ex-ante likelihood of a matched worker accepting an offered job is one for unemployed workers, and $1 - F_\psi\left(\frac{W_t}{w_{i,t}}\right)$ for employed workers. Hence the overall acceptance probability is

$$\mathfrak{p}_t^U + \mathfrak{p}_t^E(1 - F_\psi\left(\frac{W_t}{w_{i,t}}\right)) = \mathfrak{p}_t^U + \mathfrak{p}_t^E - \mathfrak{p}_t^E F_\psi\left(\frac{W_t}{w_{i,t}}\right) = 1 - \mathfrak{p}_t^E F_\psi\left(\frac{W_t}{w_{i,t}}\right) \tag{26}$$

Combining the likelihood of matching a vacancy to a worker given labor market tightness θ_t, and the likelihood of being accepted by an unemployed or employed worker leads to the ex-ante acceptance probability of a posted vacancy depending on the wage posted with the vacancy $w_{i,t}$ relative to the going wage rate in the labor market W_t.

$$a_t\left(\frac{w_{i,t}}{W_t}\right) = q(\theta_t)\left(1 - \mathfrak{p}_t^E F_\psi\left(\frac{W_t}{w_{i,t}}\right)\right) \tag{27}$$

The ex-ante probability of filling a vacancy is the product of the probability of matching a vacancy to a worker in the current labor market described by tightness θ_t and the probability that a matched vacancy is accepted by the matched worker.

The job-to-job transition from on-the-job search leads to an endogenous worker-firm separation additional to the exogenous separation at the beginning of the period. This endogenous separation depends on the relative wage of the firm to the going wage rate, as it determines the likelihood of its employees to accept an offered job from a competitor. The endogenous separation rate is described by

$$\delta_t\left(\frac{w_{i,t}}{W_t}\right) = \frac{V_t q(\theta_t)\mathfrak{p}_t^E\left(1 - F_\psi\left(\frac{w_{i,t}}{W_t}\right)\right)}{(1-\bar{\delta})N_{t-1}} \tag{28}$$

Endogenous firm separation is the product of the likelihood of matching a vacancy to an existing employee $q(\theta_t)\mathfrak{p}_t^E$, the probability that this employee would accept the outside offer with wage W_t given the wage $w_{i,t}$ at its current employer firm i, $(1 - F_\psi\left(\frac{w_{i,t}}{W_t}\right))$, times the number of total vacancies. This product is divided by the number of total employees still employed after exogenous separation $(1-\bar{\delta})N_{t-1}$. Thus, the numerator describes the

number of vacancies matched to employed workers times the probability that an employed worker of firm i would accept an outside option. Dividing the first by the amount of still employed workers gives the probability that a worker of firm i has of being matched with a vacancy. Hence, the expression is the probability of a single employed worker being matched to a vacancy times the probability that a worker of firm i would accept the outside job. This delivers the probability of a firm i employee leaving the firm for an outside job.

The total amount of workers leaving firm i, given its relative wage, is thus the likelihood that a single worker of firm i is matched to and accepts a job vacancy elsewhere times the amount of workers still working at firm i after exogenous separation has taken place.

$$\delta_t \left(\frac{w_{i,t}}{W_t} \right) (1 - \bar{\delta}) n_{i,t-1} \tag{29}$$

On-the-job search is a disciplining device for wage setting. Firms with low wages constantly bleed employees to other firms and thus incur elevated vacancy cost every period to replace their lost employees. Thus, firms can only put their offered wage so low before they are losing more money on recruiting than they save on paid wages. Bloesch et al. (2024) show that this disciplining device leads to equilibrium wages way above the outside option, which is non-zero in their version of the model. Their finding is in line with the empirical evidence that unemployment is not the credible outside threat for employees, but other job offers (see Jäger et al. (2020)).

With the endogenous separation rate we can write out the law of motion for firm i's employment

$$n_{i,t} = \left[1 - \left(\bar{\delta} + (1 - \bar{\delta}) \delta_t \left(\frac{w_{i,t}}{W_t} \right) \right) \right] n_{i,t-1} + a_t \left(\frac{w_{i,t}}{W_t} \right) v_{i,t} \tag{30}$$

Employment $n_{i,t}$ for firm i, posting $v_{i,t}$ vacancies and setting its wage to $w_{i,t}$, equals its last period employment $n_{i,t-1}$ subject to exogenous $\bar{\delta}$ and endogenous δ_t separation plus the amount of newly hired employees, which is the product of the likelihood of filling a vacancy $a_t \left(\frac{w_{i,t}}{W_t} \right)$ times the amount of posted vacancies $v_{i,t}$.

The aggregate law of motion for employment is independent of the endogenous separation rate and only depends on the amount of matches with unemployed workers relative to the exogenous separation rate. The exogenous separation rate determines the flow of workers from employment to unemployment, while the level of vacancies times its matching rate with unemployed workers determines the flow from unemployment to employment.

$$N_t = (1 - \bar{\delta}) N_{t-1} + V_t q(\theta_t) \mathfrak{p}_t^U \tag{31}$$

I briefly describe the timing of the labor market to illustrate the employment law of motion. First, firms inherit their labor from last period $n_{i,t-1}$ subject to the exogenous separation rate $\bar{\delta}$. Second, firms decide on the optimal wage $w_{i,t}$ and job postings $v_{i,t}$ for

this period to reach optimal employment $n_{i,t}$. Job vacancies are matched to employed and unemployed workers. If matched to an employed worker the firm poaches this worker from its current employer with probability $\delta_t\left(\frac{w_{i,t}}{W_t}\right)$, depending on the relative wage offered. This job-to-job transition leads to an endogenous separation on top of the exogenous separation. Unemployed workers accept the offered job with probability one, irrespective of the offered wage. The overall vacancy filling rate $a_t\left(\frac{w_{i,t}}{W_t}\right)$ from employed and unemployed workers for firm i also depends on its relative wage. Finally, firm i realizes its employment $n_{i,t}$ and produces accordingly.

For simplicity I assume that labor is the only input to production, and all firms have the same production technology. The production function of variety $i \in [0,1]$ is

$$y_{i,t} = A_t f(n_{i,t}) \tag{32}$$

where A_t is an aggregate productivity multiplier and $f(n_{i,t})$ the production function depending on the amount of labor input.

8.3 Firm's Problem

The firm producing variety i maximizes its expected present-discounted value of current and future profits. The firm's state variables are employment $n_{i,t}$, price $p_{i,t}$, wage $w_{i,t}$, and job postings $v_{i,t}$. The firm faces two major trade-offs in deciding the optimal level of each of these four decision variables.

First, increasing the price of its output is increasing profit per unit, but reducing demand due to households partial substitution by other varieties. Second, raising employment raises output and consequently profits due to a positive markup. But in order to increase employment the firm has to either increase vacancies, which cost κ_v to post, or raise its wage to shelter its employee base from competitors while making job offers to workers at other firms more attractive. However, raising the wage raises marginal cost.

These major trade-offs are incorporated in the firm's Bellman equation, and its constraints. The value optimized by the firm in the Bellman equation depends on current economic conditions such as total employment N_t, total production Y_t and expectations about the future. It further depends on the state the firm inherits from the previous period $x_{i,t-1} = (n_{i,t-1}, p_{i,t-1}, w_{i,t-1})$.

$$J_t(x_{i,t-1}) = \max_{n_{i,t},p_{i,t},w_{i,t},v_{i,t}} \frac{p_{i,t}}{P_t} D_t\left(\frac{p_{i,t}}{P_t}\right) - \frac{w_{i,t}}{P_t} n_{i,t} \tag{33}$$

$$- \kappa_v v_{i,t} - \frac{\kappa_p}{2}\left(\frac{p_{i,t}}{p_{i,t-1}} - 1\right)^2 Y_t - \frac{\kappa_w}{2}\left(\frac{w_{i,t}}{w_{i,t-1}} - 1\right)^2 N_t + E_t\left[\Lambda_{t,t+1} J_{t+1}(x_{i,t})\right] \tag{34}$$

The firm's Bellman equation increases in the firm's current and expected future revenue, and decreases in its current and expected future wage, vacancy and price and wage adjustment

costs. $\Lambda_{t,t+1}$ is the stochastic discount factor the firm is using to discount the future between period t and $t+1$. The easiest way is to think of the firm's discount factor as equivalent to the household's discount factor β.

The optimization is constrained by absolute downward nominal wage rigidity

$$w_{i,t} \geq w_{i,t-1}$$

the employment law of motion

$$\begin{aligned} n_{i,t} &= \left[1 - \left(\bar{\delta} + (1-\bar{\delta})\delta_t\left(\frac{w_{i,t}}{W_t}\right)\right)\right] n_{i,t-1} + a_t\left(\frac{w_{i,t}}{W_t}\right) v_{i,t} \\ &=: \left[1 - \tilde{\delta}\left(\frac{w_{i,t}}{W_t}\right)\right] n_{i,t-1} + a_t\left(\frac{w_{i,t}}{W_t}\right) v_{i,t} \end{aligned}$$

where I simplify the expression by defining the overall separation function $\tilde{\delta}$. Otherwise this is the employment law of motion introduced in the labor market section. The Bellman equation is further constrained by the market clearing condition

$$D_t\left(\frac{p_{i,t}}{P_t}\right) = Y_{i,t} = A_t f(n_{i,t})$$

which equalizes the optimal household demand for firm i's variety with firm i's output $Y_{i,t}$ described by its production function and labor input. The productivity multiplier A_t is subject to AR(1)-shocks: $A_t = \bar{A} + \rho_A(A_{t-1} - \bar{A}) + \eta_t^A$.

The firm's optimization delivers four first order conditions. The first order condition with respect to the optimal job posting $v_{i,t}$ provides an expression for the value of an additional worker $\mu_{i,t}$, which is the Lagrange-multiplier of the employment law of motion. The second FOC, with respect to the optimal wage posted $w_{i,t}$ delivers an expression for the Wage Phillips Curve. Third, the FOC with respect to optimal employment $n_{i,t}$ delivers an expression for marginal cost $\lambda_{i,t}$, which is the Lagrange-multiplier of the market clearing condition. Finally, the FOC with respect to the optimal price $p_{i,t}$ delivers the Price Phillips Curve of the model. I will provide these FOCs and the derivation of the value of an additional worker $\mu_{i,t}$, marginal cost $\lambda_{i,t}$, and the Wage and Price Phillips Curves in the following section after closing the model.

8.4 Closing of the Model

To close the model monetary policy is described by a Taylor Rule:

$$i_t = e^{m_t}(\pi_t^p)^{\phi_\pi} \tag{35}$$

where i_t is the interest paid on the savings bond B_t, π_t^p is aggregate price inflation, e^{m_t} is the monetary policy shock multiplier ($m_t = \bar{m} + \rho_m(m_{t-1} - \bar{m}) + \eta_t^m$), and ϕ_π is the Taylor Coefficient. In words, the price of the price of the bond falls with inflation, which raises the

bond's rate of return. This is the usual way to model an increase in the Fed's funds rate in New Keynesian models **?**. The Taylor coefficient determines how much the policy rate reacts to inflation.

Finally, market clearing is imposed.

$$Y_t = C_t + \kappa_v V_t + \frac{\kappa_p}{2}\left(\frac{p_{i,t}}{p_{i,t-1}} - 1\right)^2 Y_t + \frac{\kappa_w}{2}\left(\frac{w_{i,t}}{w_{i,t-1}} - 1\right)^2 N_t + \mathcal{D}_t \tag{36}$$

The vacancy, price, and wage adjustment costs are sunk into the sea, and $\mathcal{D}_t = \rho_D \mathcal{D}_{t-1} + \epsilon_t^D$ is an MIT shock to aggregate demand, where $\epsilon_t^D \sim N(0, \delta^D)$.

8.5 Optimization and the Steady State

In this section I derive the flexible wage steady state, the optimal behavior under shocks leading to a binding DNWR constraint in the current period, as well as shocks leading to binding DNWR in expectation in the future. All the complexity is centered in the firms' optimization problem as they decide on optimal wages, prices, employment, and thus production. The household problem is standard and described by the Euler equation and the demand for a single variety of the consumption good already described at the beginning of the model. The solution to the household problem are found in equations 14-16, consisting of the demand for a single variety, the aggregate price index and the Euler Equation.

All that is left to do is to solve the optimization problem of the firm producing variety $i \in [0, 1]$. However, due to DNWR there are essentially three cases. First, the flexible wage case. Second, the case where DNWR binds in period t to period $t + \tau - 1$, reverting back to the flexible wage case in period $t + \tau$. This would be the case under a shock that leads to downward pressure on wages, such as a negative demand or a positive labor supply shock. Third, the case where DNWR does not bind in period t but is expected to bind in period $t + k$ for all $0 < k < \tau$. This would be the case for a sufficiently strong but transitive positive demand or negative labor supply shock.

The question of sufficient shock size depends on two things. First, steady state inflation. If the shock leads to less wage growth in period t than wages would grow from period t to $t + 1$ in steady state then DNWR is not expected to bind in period $t + 1$. Second, the size of the shock determines the amount of time necessary to revert back to steady state τ. For simplicity I will assume that steady state inflation is zero. The size of τ will have to be determined when deriving the dynamics of the model following a shock of a given size. I thus, refer the reader to the (future) section on the model's dynamics.

Of the three cases I will focus on the optimal behavior in the third case. Its FOCs nest the flexible wage case, while the case with binding DNWR in period t is a degenerate version

of it. Thus, I now turn to the derivation of the optimal firm behavior under future expected binding DNWR for $\tau - 1$ periods.

8.6 Optimal Firm Behavior under Future DNWR

This subsection solves the optimal behavior of a firm experiencing a shock leading wages to hit the DNWR constraint in periods $t+1$ to $t+\tau-1$, only reverting back to the flexible wage case in period $t+\tau$. The optimization problem faced by the firm follows the following Bellman Equation:

$$J(x_{i,t-1}) = \max_{n_{i,t},p_{i,t},w_{i,t},v_{i,t}} \frac{p_{i,t}}{P_t} D_t\left(\frac{p_{i,t}}{P_t}\right) - \frac{w_{i,t}}{P_t} n_{i,t} \\ - \kappa_v v_{i,t} - \frac{\kappa_p}{2}\left(\frac{p_{i,t}}{p_{i,t-1}} - 1\right)^2 Y_t - \frac{\kappa_w}{2}\left(\frac{w_{i,t}}{w_{i,t-1}} - 1\right)^2 N_t + E_t\left[\Lambda_{t,t+1} J_{t+1}(x_{i,t})\right]$$

I highlight the terms introduced by the expected future biting of the DNWR constraint in red, to differentiate it from the nested flexible wage case in black. The optimization problem is constrained by downward nominal wage rigidity:

$$w_{i,t} \geq w_{i,t-1}$$

as well as the law of motion for employment:

$$n_{i,t} = \left[1 - \left(\bar{\delta} + (1-\bar{\delta})\delta_t\left(\frac{w_{i,t}}{W_t}\right)\right)\right] n_{i,t-1} + a_t\left(\frac{w_{i,t}}{W_t}\right) v_{i,t} =: \left[1 - \tilde{\delta}\left(\frac{w_{i,t}}{W_t}\right)\right] n_{i,t-1} + a_t\left(\frac{w_{i,t}}{W_t}\right) v_{i,t}$$

and the market clearing condition:

$$D_t\left(\frac{p_{i,t}}{P_t}\right) = Y_{i,t} = A_t f(n_{i,t})$$

The constraints are introduced into the optimization problem via Lagrange multipliers $\phi_{i,t}$, $\mu_{i,t}$ and $\lambda_{i,t}$ respectively. The first order condition with respect to the number of posted vacancies $v_{i,t}$ gives:

$$\mu_{i,t} = \frac{\kappa_v}{a_t\left(\frac{w_{i,t}}{W_t}\right)}$$

Before I provide the derivation of the Wage and Price Phillips Curves as well as marginal cost it is helpful to state and proof a proposition on optimal employment under the DNWR constraint. Under downward nominal wage rigidity firms get stuck with an employment and wage level above their optimum. I.e. firms would like to sacrifice some employment to lower wages, but cannot lower wages due to DNWR. This leads to degenerate employment dynamics. Employment simply declines following exogenous separation until the employment level is low enough so that it is optimal for the firm to raise wages again. I state this formally in Proposition 1.

Under DNWR employment $n_{i,t} \approx (1-\bar{\delta}) n_{i,t-1}$ if workers' preference shocks over employers are small $\sigma_\epsilon << (w_{i,t-1} - w_{i,t}^{flex})$ relative to the gap between past wages and flexible wages.

The first derivative of the Law of Motion of Employment with respect to $w_{i,t}$ is:

$$-(1-\bar{\delta})\delta_t'\left(\frac{w_{i,t}}{W_t}\right)\frac{n_{i,t-1}}{W_t}+a_t'\left(\frac{w_{i,t}}{W_t}\right)\frac{v_{i,t}}{W_t}$$

This is unambiguously positive as the derivative of the endogenous separation function with respect to $w_{i,t}$: $\delta_t' < 0$ and the derivative of the acceptance function $a_t' > 0$. Thus, employment rises and falls with the wage. I.e. if the wage is stuck above the optimal wage chosen under flexible wages, then firms would want to trade lower employment for lower wages, but cannot. Thus, employment is higher than under flexible wages $n_{i,t}^{flex} < n_{i,t}$. This is true regardless of the level of $v_{i,t}$.
Thus, optimal $v_{i,t} = 0$ under the DNWR constraint, which implies:

$$n_{i,t}=\left[1-\left(\bar{\delta}+(1-\bar{\delta})\delta_t\left(\frac{w_{i,t-1}}{W_t}\right)\right)\right]n_{i,t-1}$$

Firms would rather have lower wages and pay the price of a smaller employment base, than having higher wages and a larger employment base. Given that the wage is stuck at an elevated level it is still not optimal to fire workers. Optimal firing equals minus the value of an additional worker, which is greater equal to zero. Thus, firms simply seize hiring and let their employment base shrink due to separation. This is similar to the phenomenon of lingering employment when economic activity declines, also known as labor hoarding.

Given that by symmetry all other firms will also post no vacancies $V_t = 0$, and thus $\delta_t = 0$ as job-to-job turns zero. This gives:

$$n_{i,t}=(1-\bar{\delta})n_{i,t-1}$$

Above we showed that the proposed relation holds when DNWR binds in period t. All there is to show is that it is also true in expectations. I.e. if the firm expects DNWR to bind in period $t+k$, does it also expect to have employment at $(1-\bar{\delta})n_{i,t+k-1}$? This is certainly true if the firm expects every other firm to be subject to DNWR as well. If it expects to be the only one subject to the DNWR constraint in the future, then the expected employment loss due to endogenous job-to-job transition depends on the distribution of the taste shocks. Using the endogenous separation function, while enforcing DNWR at $w_{i,t+k-1}$ gives:

$$\delta_{t+k}\left(\frac{w_{i,t+k-1}}{W_{t+k}}\right)=\frac{V_{t+k}q(\theta_{t+k})p_{t+k}^{E}(1-F_\epsilon\left(\frac{w_{i,t+k-1}}{W_{t+k}}\right))}{(1-\bar{\delta})N_{t+k-1}}$$

If $\sigma_\epsilon << (w_{i,t+k-1}-w_{i,t+k}^{flex})$, then $F_\epsilon\left(\frac{w_{i,t+k-1}}{W_{t+k}}\right)\approx 1$ and thus $\delta_{t+k}\left(\frac{w_{i,t+k-1}}{W_{t+k}}\right)\approx 0$, which leads to

$$E_t[n_{i,t+k}]\approx(1-\bar{\delta})n_{i,t+k-1}$$

With Proposition 1 under the belt we can now derive the optimal wage setting behavior of the firm.

8.7 Wage Phillips Curve Derivation

The first order condition with respect to $w_{i,t}$, assuming that the firm returns to steady state in τ periods after a shock triggered DNWR, gives:

$$0 = -\frac{n_{i,t}}{P_t} - \kappa_w \left(\frac{w_{i,t}}{w_{i,t-1}} - 1\right)\frac{N_t}{w_{i,t-1}} + E_t\left[\Lambda_{t,t+1}\kappa_w \left(\frac{w_{i,t+1}}{w_{i,t}} - 1\right)\frac{w_{i,t+1}}{w_{i,t}^2}N_{t+1}\right]$$
$$+ \mu_{i,t}\left[a_t'\left(\frac{w_{i,t}}{W_t}\right)\frac{v_{i,t}}{W_t} - \tilde{\delta}_t'\left(\frac{w_{i,t}}{W_t}\right)\frac{n_{i,t-1}}{W_t}\right] + \phi_{i,t} - \sum_{k=1}^{\tau} E_t\left[\Lambda_{t,t+k}\phi_{i,t+k}\right]$$

if the DNWR starts binding in period $t+1$. If DNWR starts binding in period t, due to a negative MIT demand shock for example, then the FOC with respect to $w_{i,t}$ is simply zero as the Bellman equation will only depend on $w_{i,t-1}$.

Returning to the case with expected DNWR binding in $t+1$ to $t+\tau-1$. DNWR is not binding in period t and period $t+\tau$ as the optimal wage is higher than the wage paid in the preceding period. Thus, $\phi_t = \phi_{t+\tau} = 0$. The shadow-benefit of relaxing the DNWR constraint by one unit is zero in these periods as the DNWR constraint is non-binding. In all interim periods $\phi_{i,t+k} \geq 0$, $k \in [1, \tau-1]$. $a_t'\left(\frac{w_{i,t}}{W_t}\right)$ is the outer derivative of $a_t\left(\frac{w_{i,t}}{W_t}\right)$ with respect to $w_{i,t}$. Similar for $\tilde{\delta}_t'\left(\frac{w_{i,t}}{W_t}\right)$.

By symmetry across firms $i \in [0,1]$ the individual firm's decision reflects aggregate decision variables $x_{i,t} = X_t \; \forall t$, where x is a place holder for employment n, wages w, posted vacancies v, and prices p. Similarly, all Lagrange multipliers are equal across firms (e.g. $\mu_{j,t} = \mu_t \; \forall t, \; \forall i$). Thus, enforcing symmetry and multiplying the FOC by W_t/N_t yields:

$$\frac{W_t}{P_t} + \kappa_w\left(\frac{W_t}{W_{t-1}} - 1\right)\frac{W_t}{W_{t-1}} - E_t\left[\Lambda_{t,t+1}\kappa_w\left(\frac{W_{t+1}}{W_t} - 1\right)\frac{W_{t+1}}{W_t}\frac{N_{t+1}}{N_t}\right]$$
$$= \mu_t\left[a_t'(1)\frac{V_t}{N_t} - \tilde{\delta}_t'(1)\frac{N_{t-1}}{N_t}\right] - \frac{W_t}{N_t}\sum_{k=1}^{\tau-1} E_t\left[\Lambda_{t,t+k}\phi_{t+k}\right]$$

From this we can derive the Wage Phillips Curve of the model. I define $\Pi_t^w := \frac{W_t}{W_{t-1}}$.

$$\frac{W_t}{P_t} + \kappa_w\left(\Pi_t^w - 1\right)\Pi_t^w - E_t\left[\Lambda_{t,t+1}\kappa_w\left(\Pi_{t+1}^w - 1\right)\Pi_{t+1}^w\frac{N_{t+1}}{N_t}\right]$$
$$= \mu_t\left[a_t'(1)\frac{V_t}{N_t} - \tilde{\delta}_t'(1)\frac{N_{t-1}}{N_t}\right] - \frac{W_t}{N_t}\sum_{k=1}^{\tau-1} E_t\left[\Lambda_{t,t+k}\phi_{t+k}\right]$$

Reordering the expression yields the Wage Phillips Curve:

$$(\Pi_t^w - 1)\,\Pi_t^w = -\frac{1}{\kappa_w}\frac{W_t}{P_t} + \frac{1}{\kappa_w}\mu_t\left[a_t'(1)\,\frac{V_t}{N_t} - \tilde{\delta}_t'(1)\,\frac{N_{t-1}}{N_t}\right]$$
$$-\frac{1}{\kappa_w}\sum_{k=1}^{\tau-1} E_t\left[\Lambda_{t,t+k}\frac{W_t}{N_t}\phi_{t+k}\right] + E_t\left[\Lambda_{t,t+1}\left(\Pi_{t+1}^w - 1\right)\Pi_{t+1}^w\frac{N_{t+1}}{N_t}\right]$$

Plugging in the expressions for

$$a_t'(1) = q(\theta_t)\mathfrak{p}_t^E f_\psi(1)\frac{1}{W_t}$$

$$\tilde{\delta}_t'(1) = -\frac{V_t q(\theta_t)\mathfrak{p}_t^E}{N_{t-1}} f_\psi(1)\frac{1}{W_t}$$

$$\mu_t = \frac{\kappa_v}{a_t\,(1)}$$

provides the Wage Phillips Curve in terms of fundamentals and the shadow benefit of loosening the DNWR constraint ϕ:

$$(\Pi_t^w - 1)\,\Pi_t^w = \frac{1}{\kappa_w}\left(\frac{\kappa_v}{a_t\,(1)}2\frac{V_t}{N_t}q(\theta_t)\mathfrak{p}_t^E f_\psi(1)\right)$$
$$-\frac{1}{\kappa_w}\left(\frac{W_t}{P_t} + \sum_{k=1}^{\tau-1} E_t\left[\Lambda_{t,t+k}\frac{W_t}{N_t}\phi_{t+k}\right]\right) + E_t\left[\Lambda_{t,t+1}\left(\Pi_{t+1}^w - 1\right)\Pi_{t+1}^w\frac{N_{t+1}}{N_t}\right]$$

To solve for the shadow-benefit ϕ we need to solve the FOC with respect to $w_{i,t}$ under DNWR in the pertaining period; e.g. t. The FOC with respect to $w_{i,t}$ under DNWR in period t is:

$$0 = -\frac{n_{i,t}}{P_t} - \kappa_w\left(\frac{w_{i,t}}{w_{i,t-1}} - 1\right)\frac{N_t}{w_{i,t-1}} + E_t\left[\Lambda_{t,t+1}\kappa_w\left(\frac{w_{i,t+1}}{w_{i,t}} - 1\right)\frac{w_{i,t+1}}{w_{i,t}^2}N_{t+1}\right]$$
$$+\,\mu_{i,t}\left[a_t'\left(\frac{w_{i,t}}{W_t}\right)\frac{v_{i,t}}{W_t} - \tilde{\delta}_t'\left(\frac{w_{i,t}}{W_t}\right)\frac{n_{i,t-1}}{W_t}\right] + \phi_{i,t}$$

The Rotemberg adjustment cost term drops out as $w_{i,t} = w_{i,t-1}$. Reordering provides the shadow cost of the DNWR constraint:

$$\phi_{i,t} = \frac{n_{i,t}}{P_t} - E_t\left[\Lambda_{t,t+1}\kappa_w\left(\frac{w_{i,t+1}}{w_{i,t}} - 1\right)\frac{w_{i,t+1}}{w_{i,t}^2}N_{t+1}\right] - \mu_{i,t}\left[a_t'\left(\frac{w_{i,t}}{W_t}\right)\frac{v_{i,t}}{W_t} - \tilde{\delta}_t'\left(\frac{w_{i,t}}{W_t}\right)\frac{n_{i,t-1}}{W_t}\right]$$

By symmetry across firms:

$$\phi_t = \frac{N_t}{P_t} - E_t\left[\Lambda_{t,t+1}\kappa_w\left(\Pi_{t+1}^w - 1\right)\frac{\Pi_{t+1}^w}{W_t}N_{t+1}\right] - \mu_t\left[a_t'\,(1)\,\frac{V_t}{W_t} - \tilde{\delta}_t'\,(1)\,\frac{N_{t-1}}{W_t}\right]$$

Plugging the values for ϕ_{t+k} $k \in [1, \tau-1]$, into the Wage Phillips Curve yields:

$$\begin{aligned}
&\left(\Pi_t^w - 1\right)\Pi_t^w = \\
&+\frac{1}{\kappa_w}\left(\frac{\kappa_v}{a_t\left(1\right)}2\frac{V_t}{N_t}q(\theta_t)\mathfrak{p}_t^E f_\psi(1)\right) - \frac{1}{\kappa_w}\left(\frac{W_t}{P_t} + \sum_{k=1}^{\tau-1} E_t\left[\Lambda_{t,t+k}\frac{W_t}{P_{t+k}}\frac{N_{t+k}}{N_t}\right]\right) \\
&+\sum_{k=1}^{\tau-1} E_t\left[\Lambda_{t,t+k+1}\left(\Pi_{t+k+1}^w - 1\right)\Pi_{t+k+1}^w\frac{W_t}{W_{t+k+1}}\frac{N_{t+k+1}}{N_t}\right] + E_t\left[\Lambda_{t,t+1}\left(\Pi_{t+1}^w - 1\right)\Pi_{t+1}^w\frac{N_{t+1}}{N_t}\right] \\
&+\frac{1}{\kappa_w}\sum_{k=1}^{\tau-1} E_t\left[\Lambda_{t,t+k}\mu_{t+k}\left[a'_{t+k}\left(1\right)\frac{V_{t+k}}{N_t}\frac{W_t}{W_{t+k}} - \tilde{\delta}'_{t+k}\left(1\right)\frac{W_t}{W_{t+k}}\frac{N_{t+k-1}}{N_t}\right]\right]
\end{aligned}$$

I now use the fact that by symmetry and the binding DNWR constraint in periods $t+1, \ldots, t+\tau-1$: $W_{t+k-1} = W_{t+k}$ $\forall k \in [1, \tau-1]$. This also implies that wage inflation is zero in periods $t+1, \ldots, t+\tau-1$. Furthermore, according to Proposition 1 $N_{t+k} = (1-\bar{\delta})^k N_t$ for $k \in [1, \tau-1]$. Finally, I put in the expressions for $a'_{t+k}\left(1\right)$, $\tilde{\delta}'_{t+k}\left(1\right)$ as well as μ_{t+k}. This yields the Wage Phillips Curve purely in terms of fundamentals:

$$\begin{aligned}
&\left(\Pi_t^w - 1\right)\Pi_t^w = \\
&E_t\left[\Lambda_{t,t+\tau}\left(\Pi_{t+\tau}^w - 1\right)\Pi_{t+\tau}^w\frac{W_t}{W_{t+\tau}}\frac{N_{t+\tau}}{N_t}\right] - \frac{1}{\kappa_w}\left(\frac{W_t}{P_t} + \sum_{k=1}^{\tau-1} E_t\left[\Lambda_{t,t+k}\frac{W_t}{P_{t+k}}(1-\bar{\delta})^k\right]\right) \\
&+\frac{1}{\kappa_w}\left(\frac{\kappa_v}{a_t\left(1\right)}2\frac{V_t}{N_t}q(\theta_t)\mathfrak{p}_t^E f_\psi(1)\right) + \frac{1}{\kappa_w}\sum_{k=1}^{\tau-1} E_t\left[\Lambda_{t,t+k}\frac{\kappa_v}{a_{t+k}\left(1\right)}2q(\theta_{t+k})p_{t+k}^E f_\psi(1)\frac{V_{t+k}}{N_t}\right]
\end{aligned}$$

Furthermore, as hiring is zero in periods in which the DNWR constraint binds, labor market tightness and the probability of matching a vacancy $q(\theta_t)$ is zero in these periods. Thus, the Wage Phillips Curve simplifies to:

$$\begin{aligned}
&\left(\Pi_t^w - 1\right)\Pi_t^w = \\
&E_t\left[\Lambda_{t,t+\tau}\left(\Pi_{t+\tau}^w - 1\right)\Pi_{t+\tau}^w\frac{W_t}{W_{t+\tau}}\frac{N_{t+\tau}}{N_t}\right] - \frac{1}{\kappa_w}\left(\frac{W_t}{P_t} + \sum_{k=1}^{\tau-1} E_t\left[\Lambda_{t,t+k}\frac{W_t}{P_{t+k}}(1-\bar{\delta})^k\right]\right) \\
&+\frac{1}{\kappa_w}\left(\frac{\kappa_v}{a_t\left(1\right)}2\frac{V_t}{N_t}q(\theta_t)\mathfrak{p}_t^E f_\psi(1)\right)
\end{aligned}$$

For ease I define the following term

$$\xi_t = 2\frac{\kappa_v}{a_t\left(1\right)}\frac{V_t}{N_t}q(\theta_t)\mathfrak{p}_t^E f_\psi(1)$$

ξ_t describes the cost of filling a job vacancy with an already employed worker. It also equals the cost of protecting the firms own employee base in symmetric equilibrium. These two are the same in symmetric equilibrium, which provides the factor of 2 in the equation. As we will later see, ξ_t describes the equilibrium real wage in the zero inflation steady state. Thus, the Wage Phillips Curve above depends on expected wage growth in the future, and the deviation of the real wage from its steady state value.

A first order approximation delivers the Wage Phillips Curve in its common form:

$$\pi_t^w = E_t\left[\Lambda_{t,t+\tau}\pi_{t+\tau}^w\right] - \frac{1}{\kappa_w}\left(\hat{w}_t - \hat{p}_t + \sum_{k=1}^{\tau-1} E_t\left[\Lambda_{t,t+k}(1-\bar{\delta})^k(\hat{w}_t - \hat{p}_{t+k})\right]\right) + \frac{1}{\kappa_w}\underbrace{(\hat{\mu}_t + \hat{v}_t - \hat{n}_t + \hat{q}_t(\theta) + \hat{p}_t^E)}_{\text{Labor Market Changes}}$$

8.8 Price Phillips Curve

The first order condition with respect to the price of variety i is:

$$\frac{D_t\left(\frac{p_{i,t}}{P_t}\right)}{P_t} + \frac{p_{i,t}}{P_t}D_t'\left(\frac{p_{i,t}}{P_t}\right) - \kappa_p\left(\frac{p_{i,t}}{p_{i,t-1}} - 1\right)\frac{Y_t}{p_{i,t-1}} + E_t\left[\Lambda_{t,t+1}\kappa_p\left(\frac{p_{i,t+1}}{p_{i,t}} - 1\right)\frac{p_{i,t+1}}{p_{i,t}^2}Y_{t+1}\right] - \lambda_t D_t'\left(\frac{p_{i,t}}{P_t}\right)\frac{1}{P_t} = 0$$

where $D_t'\left(\frac{p_{i,t}}{P_t}\right)$ is the derivative of $D_t\left(\frac{p_{i,t}}{P_t}\right)$, and λ_t is the Lagrange multiplier of the market clearing condition and thus represents marginal cost. Multiplying the FOC by P_t/Y_t and symmetry across firms yields:

$$1 - \epsilon - \kappa_p(\Pi_t^p - 1)\Pi_t^p + E_t\left[\Lambda_{t,t+1}\kappa_p\left(\Pi_{t+1}^p - 1\right)\Pi_{t+1}^p\frac{Y_{t+1}}{Y_t}\right] + \epsilon\lambda_t = 0$$

where $\Pi_t^p = P_t/P_{t-1}$. Rearrange to derive the model's Price Phillips Curve:

$$(\Pi_t^p - 1)\Pi_t^p = \frac{1}{\kappa_p}(1 - \epsilon + \epsilon\lambda_t) + E_t\left[\Lambda_{t,t+1}\kappa_p\left(\Pi_{t+1}^p - 1\right)\Pi_{t+1}^p\frac{Y_{t+1}}{Y_t}\right]$$

The Price Phillips Curve is determined by a markup depending on the elasticity of demand ϵ over marginal cost λ_t, plus a discounted term for expected inflation.

The linear approximation once more provides the usual form of the Price Phillips Curve:

$$\pi_t^p = \frac{1}{\kappa_p}(\hat{\mathcal{M}}_t + \hat{\lambda}_t) + E_t\left[\Lambda_{t,t+1}\pi_{t+1}^p\right]$$

However, the Price Phillips Curve is incomplete without the measure of marginal cost λ_t. Marginal cost can be derived from the FOC with respect to optimal employment $n_{i,t}$.

8.9 Marginal Cost

Marginal cost λ_t is obtained from the first order condition with respect to employment $n_{i,t}$. I again use Proposition 1 to characterize expected employment in the periods where the DNWR constraint is expected to hold. The FOC is thus

$$0 = -\frac{w_{i,t}}{P_t} - \mu_{i,t} + \lambda_{i,t}Af'(n_{i,t}) - \sum_{k=1}^{\tau-1} E_t\left[\Lambda_{t,t+k}(1-\bar{\delta})^k\frac{w_{i,t+k}}{P_{t+k}}\right] + \sum_{k=1}^{\tau} E_t\left[\Lambda_{t,t+k}(1-\bar{\delta})^{k-1}(1-\tilde{\delta}_{t+k})\mu_{i,t+k}\right]$$

Due to the binding of the DNWR constraint $w_{i,t+k} = w_{i,t}\ \forall k \in [1, \tau-1]$. Using this and reordering yields the marginal cost of firm i

$$\lambda_{i,t} = \frac{\frac{w_{i,t}}{P_t} + \mu_{i,t} + \sum_{k=1}^{\tau-1} E_t\left[\Lambda_{t,t+k}(1-\bar{\delta})^k\frac{w_{i,t}}{P_{t+k}}\right] - \sum_{k=1}^{\tau} E_t\left[\Lambda_{t,t+k}(1-\bar{\delta})^{k-1}(1-\tilde{\delta}_{t+k})\mu_{i,t+k}\right]}{Af'(n_{i,t})}$$

Using symmetry marginal cost in period t is:

$$\lambda_t = \frac{\frac{W_t}{P_t} + \sum_{k=1}^{\tau-1} E_t\left[\Lambda_{t,t+k}(1-\bar{\delta})^k \frac{W_t}{P_{t+k}}\right] + \mu_t - \sum_{k=1}^{\tau} E_t\left[\Lambda_{t,t+k}(1-\bar{\delta})^{k-1}(1-\tilde{\delta}_{t+k})\mu_{t+k}\right]}{Af'(N_t)}$$

Finally, I use the fact that aggregate vacancies $V_{t+k} = 0, \forall k = 1, \ldots, \tau - 1$. This implies a zero shadow benefit of an additional employee.

$$\mu_{t+k} = \frac{\kappa_v}{a_{t+k}(1)} = \kappa_v \frac{\theta^\alpha}{\kappa_\theta(1 - p^E_{t+k} F_\psi(1))} = \kappa_v \frac{\left(\frac{V_{t+k}}{S_{t+k}}\right)^\alpha}{\kappa_\theta(1 - p^E_{t+k} F_\psi(1))} = 0\ , \forall k = 1, \ldots, \tau - 1$$

This simplifies the marginal cost expression to

$$\lambda_t = \frac{\frac{W_t}{P_t} + \sum_{k=1}^{\tau-1} E_t\left[\Lambda_{t,t+k}(1-\bar{\delta})^k \frac{W_t}{P_{t+k}}\right] + \mu_t - E_t\left[\Lambda_{t,t+\tau}(1-\bar{\delta})^{\tau-1}(1-\tilde{\delta}_{t+\tau})\mu_{t+\tau}\right]}{Af'(N_t)}$$

The terms introduced by the DNWR constraint are almost exactly Kudlyak's User Cost of Labor. The difference is that Kudlyak subtracts the expected flexible wage in periods $t+1$ to $t+\tau-1$. This difference is driven by Kudlyak's differing assumptions that firms can pay new hires less than existing employees, and that firms have to hire either today or tomorrow. Thus, I do not get the expected difference between today's and tomorrow's wage, but the full discounted wage of an additional employee at the wage chosen in period t.[31]

Linear approximation of the marginal cost expression provides

$$\begin{aligned}\hat{\lambda}_t =& \hat{w}_t - \hat{p}_t - \hat{A}_t + \sum_{k=1}^{\tau-1} E_t\left[\Lambda_{t,t+k}(1-\bar{\delta})^k(\hat{w}_t - \hat{p}_{t+k})\right] \\ &+ \hat{\mu}_t - E_t\left[\Lambda_{t,t+\tau}(1-\bar{\delta})^{\tau-1}(1-\tilde{\delta}_{t+\tau})\hat{\mu}_{t+\tau}\right]\end{aligned}$$

Plugging the linear approximation of marginal cost into the linear approximation of the Price Phillips Curve provides the ultimate Price Phillips Curve of the model in its usually reported form.

$$\begin{aligned}\pi^p_t =& \frac{\epsilon}{\kappa_p}\left(\hat{w}_t - \hat{p}_t - \hat{A}_t + \sum_{k=1}^{\tau-1} E_t\left[\Lambda_{t,t+k}(1-\bar{\delta})^k(\hat{w}_t - \hat{p}_{t+k})\right] + \hat{\mu}_t - E_t\left[\Lambda_{t,t+\tau}(1-\bar{\delta})^{\tau-1}(1-\tilde{\delta}_{t+\tau})\hat{\mu}_{t+\tau}\right]\right) \\ &+ E_t\left[\Lambda_{t,t+1}\pi^p_{t+1}\right]\end{aligned}$$

[31] I do get Kudlyak's difference between the current and future expected wages when I allow firms to separate the wage offered to new employees from the wage paid to existing employees. Then DNWR does not bind for additional hires, leading to Kudlyak's trade-off. See Appendix B.10.

8.10 Steady State Derivation

To derive the zero inflation steady state I start with the FOC with respect to $p_{i,t}$. In steady state $P_t = P_{t-1}$, thus it simplifies to

$$\begin{aligned}
0 =& \frac{1}{\kappa_p}(1 - \epsilon + \epsilon\lambda_t) \\
1 =& \left(\frac{\epsilon}{\epsilon - 1}\right)\lambda_t = \mathcal{M}_p\lambda_t \\
1 =& \mathcal{M}_p \frac{\frac{W_t}{P_t} + \frac{\kappa_v}{a_t(1)} - E_t\left[\Lambda_{t,t+1}(1 - \tilde{\delta}_{t+1}(1))\frac{\kappa_v}{a_{t+1}(1)}\right]}{Af'(N_t)}
\end{aligned}$$

In equilibrium demand is equal across periods. Thus, the stochastic discount factor $\Lambda_{t,t+1} = \beta\frac{U(C_{t+1})}{U(C_t)}$ equals the discount factor β. Further substituting $a_t(1) = a_{t+1}(1) =: a$ as well as $\tilde{\delta}_{t+1}(1) =: \tilde{\delta}$, and generally dropping t subscripts gives:

$$1 = \mathcal{M}_p \frac{\frac{W}{P} + \frac{\kappa_v}{a} - \beta(1 - \tilde{\delta})\frac{\kappa_v}{a}}{Af'(N)} = \mathcal{M}_p \frac{\frac{W}{P} + \frac{\kappa_v}{a}(1 - \beta(1 - \tilde{\delta}))}{Af'(N)}$$

I use the derived Wage Phillips Curve, omitting all the terms introduced by expecting the DNWR to bind in the future (terms in red), to derive the wage setting behavior in equilibrium:

$$\left(\Pi_t^w - 1\right)\Pi_t^w = \frac{1}{\kappa_w}\left(-\frac{W_t}{P_t} + \left(\frac{\kappa_v}{a_t(1)}2\frac{V_t}{N_t}q(\theta_t)\mathfrak{p}_t^E f_\psi(1)\right)\right) + E_t\left[\Lambda_{t,t+1}\left(\Pi_{t+1}^w - 1\right)\Pi_{t+1}^w\frac{N_{t+1}}{N_t}\right]$$

The ratio $\Pi_t^w = \frac{W_t}{W_{t-1}} = 1$ in equilibrium for all t. Thus,

$$\begin{aligned}
0 =& \frac{1}{\kappa_w}\left(-\frac{W_t}{P_t} + \left(\frac{\kappa_v}{a_t(1)}2\frac{V_t}{N_t}q(\theta_t)\mathfrak{p}_t^E f_\psi(1)\right)\right) \\
\iff \frac{W_t}{P_t} =& \frac{\kappa_v}{a_t(1)}2\frac{V_t}{N_t}q(\theta_t)\mathfrak{p}_t^E f_\psi(1)
\end{aligned}$$

In equilibrium the real wage equals the recruitment cost per employee. This makes intuitive sense. If the real wage was higher than recruitment cost for a new employee, firms would increase vacancies instead of raising wages in order to increase their employee base. Only when the two are equal are firms indifferent in raising wages or vacancies when looking to hire more workers.

Finally, I use the fact that $\epsilon_a = \epsilon_{\tilde{\delta}} = \frac{q(\theta)p^E f_\psi(1)}{a(1)}$ and $\frac{V}{N} = \frac{\tilde{\delta}}{a}$ in steady state to derive:

$$\frac{W_t}{P_t} = \frac{\kappa_v}{a(1)}2\tilde{\delta}\epsilon_{\tilde{\delta}}$$

I combine the price and wage setting behavior to derive the following real wage in steady state:

$$\frac{W_t}{P_t} = \frac{1}{\mathcal{M}_p}\frac{2\tilde{\delta}\epsilon_{\tilde{\delta}}}{2\tilde{\delta}\epsilon_{\tilde{\delta}} + 1 - \beta(1 - \tilde{\delta})}Af'(N)$$

The real wage is the marginal product of labor less the price markup stemming from pricing power in the product market, and subject to a wage mark-down stemming from

monopsony power. The pricing power of firms is due to the imperfect substitution of consumption between consumption varieties. Monopsony power stems from the firms' power in setting wages and vacancies in a labor market with search and matching frictions.

To conclude, this model introduces a user cost of labor channel into a New Keynesian Search and Matching framework with monopsony power. The combination of monopsony power and downward nominal wage rigidity introduces the possibility for an excess steepening of the Price Phillips curve during tight labor markets. Furthermore, the User Cost of Labor is not incurred in the current period, consequently raising the measured profit margin and thus explains the non-linear rise of profit margins with labor market tightness.

8.11 Micro Evidence

The model matches key micro evidence on labor market dynamics. Firstly, the well-identified evidence on the sensitivity of recruiting and quitting to changes in wages (recruiting & separations elasticities) estimated in monopsony literature: e.g., Hirsch et al. (2022); Azar et al. (2022); Datta (2023). Second, wage growth predicted by job-to-job transitions: e.g.,Faberman and Justiniano (2015); Moscarini and Postel-Vinay (2017); Karahan et al. (2022). Third, wages unresponsive to flow benefit of unemployment Jäger et al. (2020). Fourth, wage posting is more common than wage bargaining. The firm's current wage affects past wages: e.g., Di Addario et al. (2023). Finally, the model matches the growing evidence for substantial and almost absolute DNWR (e.g.Grigsby et al. (2021), Lebow et al. (2003)).

Evidence for the pro-cyclicality of the User Cost of labor: Kudlyak (2014) & Bils et al. (2023).

8.12 Potential Extensions

1. Introduce the outside option for unemployed workers in form of a real payment bP_t, (following Bloesch et al. (2024)).
2. Introduce labor force non-participation (following Benigno & Eggertson (2023))
3. Could drop the Rotemberg adjustment cost to wages and only use DNWR. Calvo wage setting would create more realistic dynamics but is impractical for computation.

9 Conclusion

Recent findings suggest that inflation surges in excess of wage growth during especially tight labor markets. This contradicts the traditional view of the Phillips Curve, which sees rising wages as the sole driver of the link between labor market tightness and inflation. I resolve this puzzle by introducing a channel through which labor market tightness affects

inflation directly. I provide evidence for this channel on inflation, real wage growth, profit margins, on OECD country and US service sector data. The impact is highly non-linear and drives Phillips Curve non-linearity. Furthermore, I show that market concentration is key for the strength of the direct impact.

I quantify the direct impact on 2019 to 2022 data. The impact explains more than a third of the rise in headline and more than 80% of the rise in core inflation from average 2019 levels to the mid-2022 inflation peak. Combined with the impact on wage growth it explains almost two thirds of the rise in headline inflation. Thus, this paper ways heavily in favor of the importance of the labor market during the pandemic inflation surge.

I provide a simple mechanism for the direct impact based on a price setting response to downward nominal wage rigidity similar to Kudlyak's User Cost of Labor. The paper thus extends the literature on wage rigidity and Phillips Curve non-linearity from its focus on recessions to the boom part of the cycle. I highlight the mechanism in an illustrative model, show its consistency with firm survey data from the NFIB, and introduce the channel into a New Keynesian framework with monopsony power. The model explains the rise of inflation in excess of wage growth during especially tight labor markets, the accompanying fall in real wage growth and rise in profit margins, as well as the non-linear nature of the channel.

The key policy implication of the channel is a painless disinflation following a transitory demand or labor supply shock.[32]. Due to the channel prices rise in excess of wages. Thus, inflation can reverse without real wages getting stuck at an elevated level, which lowers the cost in terms of unemployment when monetary policy reduces demand. A second policy implication of the direct impact is a target region of labor market tightness to foster real wage growth. Due to the channel real wages suffer not only during downturns but also during especially tight labor markets.

[32]thus resolving the "Missing Unemployment Puzzle" e.g. Schmitt-Grohe Uribe (2024); Foundations of a Nonlinear Phillips Curve.

Appendix:
Reconciling Wage and Price Phillips Curves

David Osten

March 14, 2025

A Robustness Checks

A.1 Full Flagship Table

This section reports the full extend of the coefficients in the flagship table testing for a direct impact of labor market tightness on inflation.

A.2 Robustness to Lagged Wage Growth

Table 18 below shows the robustness of the direct impact to the inclusion of further wage growth lags on CPI headline inflation. I again omit reporting the coefficients on the control variables.

Table 18 shows that the direct impact measured in Table 2 is not due to a negligence of further lags of wage growth. Including further wage lags leads to only miniscule changes in the estimated labor market tightness coefficients. Further the Wald tests resolutely reject the insignificance and linearity of the direct impact of labor market tightness on inflation in all specifications. Similar results are obtained when including further wage lags in the CPIXFE and Median CPI estimations.

A.3 Pre-pandemic Results (1985Q1 - 2019Q4)

This section repeats the flagship table 2 on pre-pandemic data to show that the direct impact is not purely driven by the pandemic inflation surge. Although headline inflation now delivers insignificant results, the general shape of the cubic direct impact is still apparent. Furthermore, CPI excluding food and energy still delivers significant rejection of both non-existence and linearity of the labor market tightness coefficients, even though the single coefficients are insignificant. Finally, Median CPI inflation provides both significant labor market tightness coefficients as well as significant Wald tests on both existence and non-linearity. I again omit reporting the coefficients on the control variables.

Thus, even though the direct impact is slightly less significant on pre-pandemic data, Table 19 shows that the direct impact is robust to excluding the inflation surge during the pandemic recovery phase. The slightly less significant results on the 1985-2019 data is likely due to the limited range of labor market tightness in that time frame. Previous contributions

Table 17: V/U Direct Impact

	CPI	XFE	Median	CS_XS
V/U	3.35	4.80*	4.12**	9.01***
	(2.37)	(2.50)	(1.64)	(2.74)
$(V/U)^2$	−3.59	−4.85	−5.11***	−11.48***
	(3.02)	(3.34)	(1.91)	(3.19)
$(V/U)^3$	1.58	2.00*	2.20***	4.33***
	(1.04)	(1.18)	(0.62)	(1.07)
ω	0.06	0.02	0.17***	−0.11
	(0.06)	(0.05)	(0.04)	(0.09)
L.ω	0.12	−0.05	0.19***	−0.07
	(0.07)	(0.08)	(0.05)	(0.11)
π^e	0.82***	1.02***	0.64***	1.17***
	(0.09)	(0.09)	(0.07)	(0.14)
Prod	−0.09	−0.01	−0.17***	0.00
	(0.07)	(0.08)	(0.05)	(0.10)
Energy	0.09***	0.01	0.00	0.02**
	(0.01)	(0.01)	(0.00)	(0.01)
L.Energy	0.00	0.00	−0.00	0.00
	(0.01)	(0.00)	(0.00)	(0.00)
Import	0.13	−0.23	−0.13	−0.57
	(0.31)	(0.23)	(0.14)	(0.34)
L.Import	0.19	0.15	0.16	0.45*
	(0.25)	(0.27)	(0.14)	(0.23)
Constant	−1.42**	−1.75***	−0.79*	−1.50*
	(0.57)	(0.57)	(0.43)	(0.78)
R^2	0.87	0.69	0.84	0.60
N	156	156	156	155
No Impact (p-value)	0.0000	0.0000	0.0000	0.0000
Linear Impact (p-value)	0.0050	0.0118	0.0000	0.0000
Partial R^2	0.17	0.25	0.42	0.16

Standard errors in parentheses

* $p < .10$, ** $p < .05$, *** $p < .01$

Table 18: Robustness of Headline CPI Direct Impact to Further Wage Lags

	L0	L1	L2	L3	L4
V/U	3.83	3.35	3.16	3.19	3.20
	(2.52)	(2.37)	(2.44)	(2.42)	(2.35)
$(V/U)^2$	−3.93	−3.59	−3.42	−3.46	−3.48
	(3.19)	(3.02)	(3.07)	(3.05)	(2.98)
$(V/U)^3$	1.70	1.58	1.51	1.53	1.55
	(1.10)	(1.04)	(1.07)	(1.06)	(1.03)
π^e	0.93***	0.82***	0.78***	0.80***	0.84***
	(0.06)	(0.09)	(0.08)	(0.08)	(0.09)
ω	0.07	0.06	0.05	0.06	0.06
	(0.06)	(0.06)	(0.06)	(0.06)	(0.06)
L.ω		0.12	0.11	0.12	0.15*
		(0.07)	(0.07)	(0.08)	(0.09)
L2.ω			0.07	0.07	0.10
			(0.07)	(0.07)	(0.08)
L3.ω				−0.06	−0.05
				(0.06)	(0.05)
L4.ω					−0.11
					(0.10)
Constant	−1.67***	−1.42**	−1.34**	−1.35**	−1.38**
	(0.60)	(0.57)	(0.60)	(0.59)	(0.57)
R^2	0.86	0.87	0.87	0.87	0.87
N	156	156	156	156	156
No Impact (p-value)	0.0000	0.0000	0.0000	0.0000	0.0000
Linear Impact (p-value)	0.0029	0.0050	0.0086	0.0043	0.0023
Partial R^2	0.25	0.17	0.14	0.15	0.15

Standard errors in parentheses

* $p < .10$, ** $p < .05$, *** $p < .01$

Table 19: Pre-Pandemic Direct Impact

	CPI	XFE	Median	CSXS
V/U	4.03	5.50*	9.18***	12.52**
	(3.81)	(2.79)	(2.20)	(5.21)
$(V/U)^2$	−4.12	−5.72	−12.86***	−17.95**
	(5.77)	(4.50)	(3.31)	(8.45)
$(V/U)^3$	1.38	1.99	5.72***	7.53*
	(2.64)	(2.12)	(1.50)	(4.00)
ω	0.07	−0.01	0.18***	−0.06
	(0.06)	(0.04)	(0.04)	(0.07)
L.ω	0.10	−0.06	0.15***	−0.01
	(0.07)	(0.05)	(0.05)	(0.08)
π^e	0.84***	1.06***	0.67***	1.07***
	(0.09)	(0.07)	(0.07)	(0.13)
Constant	−1.48*	−1.72***	−1.79***	−2.12**
	(0.85)	(0.53)	(0.45)	(1.01)
R^2	0.86	0.75	0.77	0.75
N	140	140	140	139
No Impact (p-value)	0.1301	0.0000	0.0000	0.0477
Linear Impact (p-value)	0.2710	0.0184	0.0008	0.0194
Partial R^2	0.05	0.11	0.13	0.07

Standard errors in parentheses

* $p < .10$, ** $p < .05$, *** $p < .01$

to the literature have already argued that this lack of tight labor markets leads to flatter Phillips Curve estimations (Gagnon and Collins (2019)). Even during this period of relative labor market tranquility the Median and traditional core inflation measures still provide evidence for both the existence and non-linearity of the direct impact.

A.4 PCE Estimation

This section repeats Table 2 on PCE data. Using PCE data instead of CPI data leads to insignificant coefficients on headline and traditional core inflation. However, the Wald tests still reject the non-existence or mere linearity of the direct impact. Furthermore, using Median PCE inflation still leads to highly significant labor market tightness coefficients. I thus conclude that the direct impact is also robust to using PCE measures of inflation. Similar is also true for other sub-components of both CPI and PCE inflation, such as Trimmed Mean PCE and CPI, CPI core services, and CPI services excluding shelter, as well as for the GDP deflator. I again omit reporting the coefficients on the control variables.

A.5 MoM CPI Estimation

This section provides evidence for the robustness of the direct impact to using monthly instead of quarterly data. Due to the relative noise in monthly data the significance of the coefficients is subdued for all three measures of inflation. The noise further obscures the non-linearity of the direct impact on headline inflation. However, the Wald test rejects non-existence of the direct impact even on headline inflation. Furthermore, both traditional core and median CPI inflation yield significant rejections of both non-existence as well as mere linearity of the direct impact, and median inflation still gives significant labor market tightness coefficients. Thus, the direct impact is robust to measuring inflation on a monthly instead of a quarterly basis. Controls for labor productivity and relative import prices are omitted in the monthly estimation due to data availability issues on the monthly frequency.

A.6 Density Graphs

This section provides data density graphs. I cleaned the data from the influence of the control variables, taking the homogeneity into account, and plot this against the fitted Phillips Curve. First on the quarterly then on the monthly data.

The annualized month-over-month inflation is rather noisy and thus almost drowns out the functional form. Using 12 month moving averages of monthly inflation data gives a clearer picture. In this last figure I plot 12 month inflation averages against 12 month averages of V/U.

Table 20: PCE Direct Impact

	PCE	XFE	Median
V/U	1.08	1.70	4.40***
	(2.05)	(2.11)	(1.40)
$(V/U)^2$	−1.74	−2.18	−5.48***
	(2.56)	(2.76)	(1.64)
$(V/U)^3$	1.05	1.19	2.29***
	(0.90)	(0.98)	(0.54)
ω	0.15**	0.15**	0.15***
	(0.07)	(0.07)	(0.05)
L.ω	0.12**	−0.00	0.11**
	(0.05)	(0.06)	(0.04)
π^e	0.73***	0.85***	0.75***
	(0.08)	(0.09)	(0.08)
Constant	−0.94*	−1.09**	−1.23***
	(0.51)	(0.49)	(0.37)
R^2	0.85	0.68	0.80
N	156	156	156
No Impact (p-value)	0.0000	0.0000	0.0000
Linear Impact (p-value)	0.0005	0.0010	0.0000
Partial R^2	0.21	0.21	0.41

Standard errors in parentheses

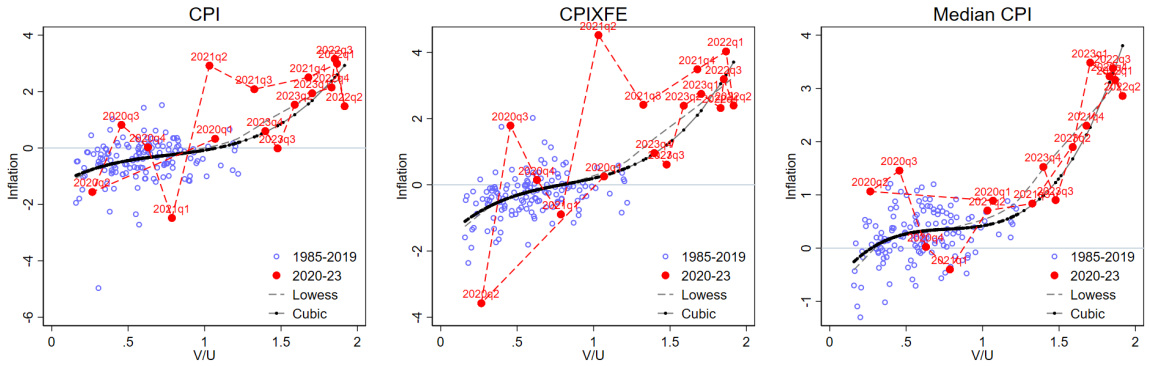
* $p < .10$, ** $p < .05$, *** $p < .01$

Table 21: V/U Direct Impact on MoM Data

	CPI	CPIXFE	Median	CoreServ_XS
V/U	1.95	3.72	4.67**	7.77***
	(3.28)	(2.55)	(2.13)	(2.81)
$(V/U)^2$	−0.76	−3.41	−4.70*	−10.11***
	(3.89)	(3.21)	(2.42)	(3.36)
$(V/U)^3$	0.41	1.42	1.99**	3.82***
	(1.28)	(1.09)	(0.79)	(1.13)
ω	0.08*	0.07	0.04	−0.02
	(0.05)	(0.04)	(0.03)	(0.06)
L.ω	−0.03	0.01	0.05*	0.03
	(0.04)	(0.04)	(0.03)	(0.07)
π^e	0.91***	0.93***	0.90***	0.995***
	(0.06)	(0.05)	(0.04)	(0.09)
Energy	0.07***	−0.00	0.00***	0.00
	(0.01)	(0.00)	(0.00)	(0.00)
L.Energy	0.01**	0.01***	−0.00	0.01
	(0.00)	(0.00)	(0.00)	(0.00)
Constant	−1.76**	−1.50**	−1.55***	−1.20*
	(0.86)	(0.59)	(0.55)	(0.67)
R^2	0.76	0.44	0.64	0.32
N	468	468	468	467
No Impact (p-value)	0.0000	0.0000	0.0000	0.0000
Linear Impact (p-value)	0.5558	0.0428	0.0000	0.0004
Partial R^2	0.16	0.26	0.52	0.08

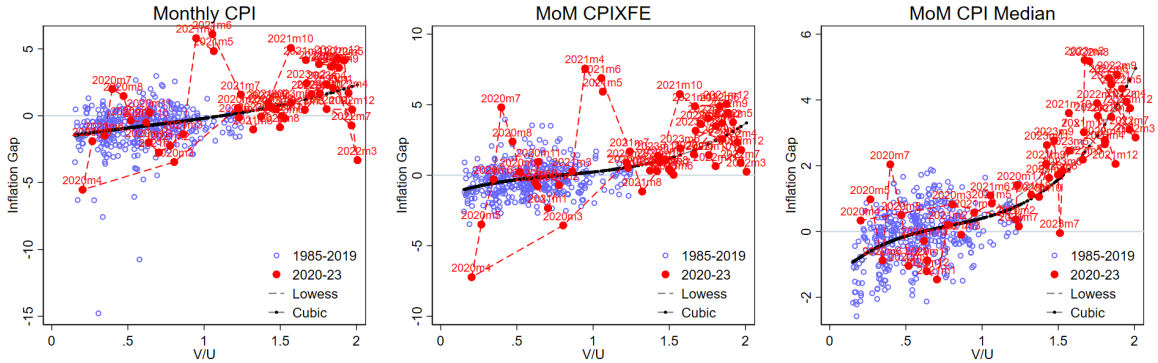
Standard errors in parentheses

* $p < .10$, ** $p < .05$, *** $p < .01$



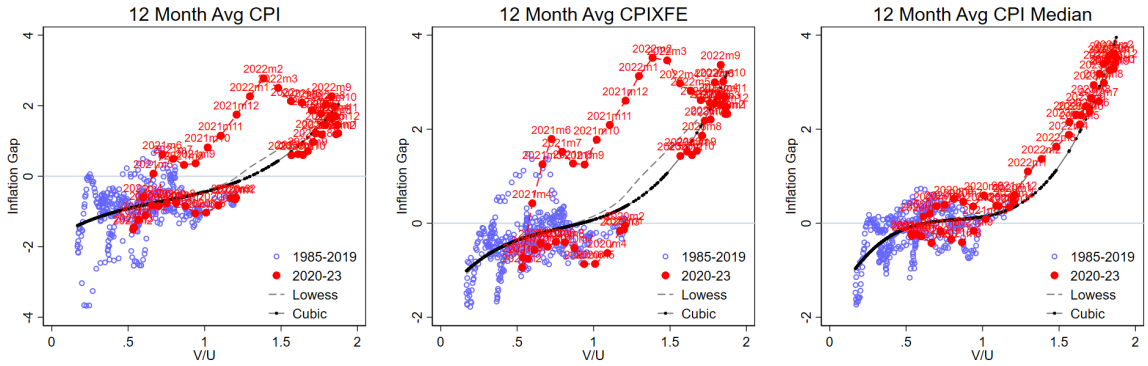
(a) CPI (b) CPIXFE (c) Median CPI

Figure 14: Density graphs on annualized quarterly inflation.
Subfigure a) shows the cubic functional form of the annualized quarterly CPI inflation.
Subfigure b), and c) do the same for CPIXFE, Median CPI inflation, respectively.



(a) CPI (b) CPIXFE (c) Median CPI

Figure 15: Density graphs on annualized month over month inflation.
Subfigure a) shows the cubic functional form of the annualized monthly CPI inflation.
Subfigure b), and c) do the same for CPIXFE, and Median CPI inflation, respectively.



(a) CPI (b) CPIXFE (c) Median CPI

Figure 16: Density graphs on 12 month moving average of annualized month over month inflation.

Subfigure a) shows the cubic functional form of the 12 month average annualized monthly CPI inflation. Subfigure b), and c) do the same for CPIXFE, and Median CPI inflation, respectively.

A.7 Historical Perspective: 1960 - 2023

I re-do the analysis from Benigno and Eggertsson (2023), but control for the wage growth they claim is driving the non-linear rise in inflation. Benigno and Eggertsson (2023) perform this analysis for the sub-periods 1960-1969, 1970-1987, 1987-2008, and 2008-2022. I simplify their approach by focusing on the periods 1960-1969, 1970-1985, and 1985-2023. I estimate the following equation:

$$\pi_t = \alpha log(V/U) + \beta log(V/U)\mathbb{1}_{V/U>1} + \gamma X_t + e_t \tag{37}$$

where X_t is a vector of controls including current and lagged wage growth, energy inflation and labor productivity.

The direct impact is significant across all periods, except for the 1970s, where the major oil price shocks caused a decline in labor market tightness alongside rising inflation. Furthermore, I test Benigno and Eggertsson (2023)'s hypothesis on wage growth, which is at best linear across all sub-periods. I also examine real wage growth (RWG) and profit margins (PMs). RWG falls non-linearly with labor market tightness in all sub-periods except for the 1970s. As for PMs, they fail to exhibit the non-linear rise in the pre-1985 periods. This failure is likely due to the oil price shocks in the 1970s. However, I am still unsure why PMs fail to rise in the 1960s, despite the hot labor markets.

Finally, my 1985-2023 results hold when estimating them over the entire period 1960-2023. This is true whether using Benigno and Eggertsson (2023)'s threshold model or my quadratic and cubic specifications from previous sections. However, the threshold estimation of RWG is only significant at the 10% level. Therefore, the quadratic specification is preferred.

In Table 22 I present the results of equation 37 for the 1960Q1 - 1969Q4 period. The steepening of the Price Phillips Curve remains even when controlling for current and lagged wage growth, labor productivity and energy inflation.

Table 23 shows that in 1960Q1 - 1969Q4 real wage growth and even nominal wage growth were suffering during the especially tight labor market. The results on profit margins are insignificant.

In Table 24 I present the results of equation 37 for the 1970Q1 - 1984Q4 period. During the 70s there is no detectable steepening of the Price Phillips Curve.

Table 25 shows that in 1970Q1 - 1984Q4 real and nominal wage growth were not suffering during tight labor markets. However, profit margins were suffering during the 70s as high energy cost and wage indexing hit profits.

Table 22: V/U Direct Impact 1960-1969

	CPI	XFE	Median
log(V/U)	0.54	-0.03	0.09
	(0.36)	(0.53)	(1.01)
$V/U \geq 1$	3.68**	6.50***	7.58**
	(1.67)	(2.04)	(3.65)
ω	-0.01	0.11	0.11*
	(0.11)	(0.07)	(0.06)
L.ω	0.09	-0.10	0.07
	(0.13)	(0.07)	(0.09)
π^e	0.78***	0.82***	0.99***
	(0.10)	(0.08)	(0.12)
Energy	-0.05	-0.11**	0.03
	(0.08)	(0.05)	(0.06)
L.Energy	-0.02	-0.03	0.03
	(0.04)	(0.05)	(0.03)
Prod	-0.24*	-0.07	-0.28
	(0.14)	(0.08)	(0.17)
Constant	1.15**	0.42	-0.24
	(0.52)	(0.55)	(1.25)
R^2	0.64	0.75	0.76
N	40	40	40

Standard errors in parentheses

* $p < .10$, ** $p < .05$, *** $p < .01$

Table 23: V/U Wage and Profit Impact 1960-1969

	W	RWG	PMs
log(V/U)	0.69	0.06	0.04
	(0.47)	(0.70)	(0.05)
$V/U \geq 1$	-5.66*	-5.97**	-0.16
	(3.13)	(2.68)	(0.29)
π	0.17		
	(0.31)		
π	0.54**		
	(0.26)		
π^e	0.29	0.73	
	(0.49)	(0.71)	
Prod	0.19	0.25	0.04**
	(0.23)	(0.22)	(0.02)
Energy	-0.23**	-0.24**	-0.03*
	(0.09)	(0.09)	(0.02)
L.Energy	-0.08	-0.05	-0.01
	(0.09)	(0.11)	(0.01)
L4.RWG		-0.21***	
		(0.07)	
W			0.01
			(0.01)
L.W			-0.02
			(0.02)
Constant	2.04**	1.33	0.01
	(0.85)	(1.56)	(0.10)
R^2	0.27	0.28	0.41
N	40	39	40

Standard errors in parentheses

* $p < .10$, ** $p < .05$, *** $p < .01$

Table 24: V/U Direct Impact 1970-1984

	CPI	XFE	Median
log(V/U)	3.88***	2.61*	3.54***
	(0.94)	(1.37)	(1.09)
$V/U \geq 1$	-19.57	-18.86	-19.13
	(13.23)	(15.68)	(13.74)
ω	0.13	0.09	0.10
	(0.12)	(0.13)	(0.14)
L.ω	0.10	0.06	0.10
	(0.13)	(0.16)	(0.14)
π^e	0.76***	0.77***	0.70***
	(0.22)	(0.26)	(0.24)
Energy	0.11***	0.02	0.05***
	(0.02)	(0.01)	(0.02)
L.Energy	-0.02	0.01	-0.03
	(0.03)	(0.03)	(0.03)
Prod	-0.59***	-0.82***	-0.87***
	(0.17)	(0.30)	(0.24)
Constant	4.52***	4.16***	4.60***
	(1.02)	(1.33)	(1.04)
R^2	0.74	0.56	0.62
N	60	60	60
Nixon Controls	✓	✓	✓

Standard errors in parentheses

* $p < .10$, ** $p < .05$, *** $p < .01$

Table 25: V/U Wage and Profit Impact 1970-1984

	W	RWG	PMs
log(V/U)	0.70	-2.71***	-0.10
	(0.79)	(0.59)	(0.12)
$V/U \geq 1$	-7.16	-23.41	-5.68**
	(13.31)	(18.17)	(2.36)
π	0.09		
	(0.17)		
L.π	0.15*		
	(0.09)		
π^e	0.72***	-0.42**	
	(0.15)	(0.19)	
Energy	-0.03	-0.11***	-0.00
	(0.03)	(0.02)	(0.00)
L.Energy	0.01	0.04	-0.01*
	(0.03)	(0.04)	(0.00)
Prod	0.25	0.38*	0.00
	(0.29)	(0.22)	(0.03)
π^e		-0.54***	
		(0.20)	
L4.RWG		-0.01	
		(0.10)	
ω			0.02
			(0.02)
L.ω			-0.01
			(0.02)
Constant	0.81	0.45	0.05
	(0.94)	(1.52)	(0.20)
R^2	0.34	0.62	0.17
N	60	60	60
Nixon Controls	✓	✓	✓

Standard errors in parentheses

* $p < .10$, ** $p < .05$, *** $p < .01$

In Table 26 I present the results of equation 37 for the 1985Q1 - 2023Q4 period. The table merely provides a robustness check to the threshold model of the results presented before.

Table 27 provides a robustness check of my 1985Q1 - 2023Q4 results to using a threshold model. Wages are still failing non-linearity, while real wage growth falls and profit margins soar when labor markets get very tight.

In Table 28 I present the results of equation 37 for the entire period: 1960Q1 - 2023Q4. Due to the immense energy shocks in the 70s I control for raw CPI energy inflation instead of the difference between energy inflation and headline inflation. Due to likely regime changes I also shorten the length of the Newey-West SE autocorrelation to 4 quarters.

My previous findings remain mostly the same, with Median inflation's non-linearity only significant at the 10% level, but core and headline inflation still providing significant evidence for the non-linearity of the direct impact of V/U on inflation.

Table 29 provides the wage, real wage and profit margin results on the entire 1960-2023 period. Wage growth remains linear at best, while real wage growth suffers under very tight labor markets alongside rising profit margins.

Table 26: V/U Direct Impact

	CPI	XFE	Median
log(V/U)	0.53*	0.65**	0.25
	(0.27)	(0.25)	(0.17)
$V/U \geq 1$	3.96***	3.95***	3.75***
	(0.74)	(0.64)	(0.55)
ω	0.05	0.02	0.16***
	(0.07)	(0.06)	(0.05)
L.ω	0.10	-0.05	0.19***
	(0.08)	(0.07)	(0.06)
π^e	0.83***	1.03***	0.65***
	(0.11)	(0.09)	(0.08)
Energy	0.10***	0.01	0.00
	(0.01)	(0.01)	(0.00)
L.Energy	0.00	0.00	-0.00
	(0.01)	(0.00)	(0.00)
Prod	-0.10	-0.02	-0.19***
	(0.09)	(0.09)	(0.05)
RIMP	0.27	-0.14	-0.10
	(0.36)	(0.22)	(0.14)
L.RIMP	0.17	0.13	0.15
	(0.27)	(0.25)	(0.15)
Constant	0.19	0.18	0.42***
	(0.26)	(0.20)	(0.16)
R^2	0.83	0.50	0.79
N	154	154	154

Standard errors in parentheses

* $p < .10$, ** $p < .05$, *** $p < .01$

Table 27: V/U Wage and Profit Impact 1985 - 2023

	W	RWG	PMs
log(V/U)	1.11***	0.59***	-0.19*
	(0.18)	(0.21)	(0.11)
$V/U \geq 1$	1.31*	-3.03***	1.02**
	(0.70)	(0.53)	(0.41)
π	0.01		
	(0.05)		
L.π	0.14		
	(0.09)		
Energy	0.01	-0.10***	0.01*
	(0.01)	(0.01)	(0.01)
L.Energy	-0.01	-0.01	0.00
	(0.01)	(0.01)	(0.01)
Prod	0.22***	0.21***	0.09
	(0.05)	(0.07)	(0.06)
π^e	0.42***	-0.64***	
	(0.09)	(0.12)	
L.RWG		-0.01	
		(0.02)	
ω			-0.03
			(0.07)
L.ω			-0.07
			(0.05)
Constant	0.38*	2.34***	0.05
	(0.21)	(0.39)	(0.14)
R^2	0.50	0.70	0.12
N	154	154	154

Standard errors in parentheses

* $p < .10$, ** $p < .05$, *** $p < .01$

Table 28: V/U Direct Impact 1960-2023

	CPI	XFE	Median
log(V/U)	1.06***	0.86***	1.05***
	(0.28)	(0.28)	(0.29)
$V/U \geq 1$	2.12**	2.39**	1.93*
	(0.93)	(1.15)	(1.08)
ω	0.07***	0.04	0.07**
	(0.03)	(0.04)	(0.03)
L.ω	0.06**	0.06**	0.04
	(0.03)	(0.03)	(0.03)
π^e	0.87***	0.90***	0.89***
	(0.05)	(0.08)	(0.12)
Energy	0.10***	0.02***	0.02***
	(0.01)	(0.01)	(0.01)
L.Energy	0.01	0.01*	0.01
	(0.01)	(0.01)	(0.01)
Prod	-0.16*	-0.26**	-0.43***
	(0.10)	(0.12)	(0.11)
Constant	0.55*	1.04***	1.44***
	(0.30)	(0.36)	(0.35)
R^2	0.76	0.46	0.51
N	254	254	254
Nixon Controls	✓	✓	✓

Standard errors in parentheses

* $p < .10$, ** $p < .05$, *** $p < .01$

Table 29: V/U Wage and Profit Impact 1960-2023

	W	RWG	PMs
log(V/U)	0.59	-0.28	-0.34***
	(0.46)	(0.51)	(0.08)
$V/U \geq 1$	-1.45	-3.33*	0.84***
	(1.43)	(1.69)	(0.32)
π	0.13		
	(0.10)		
L.π	0.34*		
	(0.18)		
π^e	0.42***	-0.41***	
	(0.13)	(0.10)	
Prod	0.73***	0.62***	0.07**
	(0.22)	(0.23)	(0.03)
Energy	-0.04*	-0.13***	0.01*
	(0.03)	(0.03)	(0.01)
L.Energy	-0.01	0.01	0.00
	(0.01)	(0.01)	(0.00)
L4.RWG		-0.02	
		(0.06)	
W			-0.03**
			(0.01)
L.W			0.05
			(0.03)
Constant	-0.13	0.83	-0.42**
	(0.62)	(0.67)	(0.19)
R^2	0.14	0.36	0.19
N	254	253	254
Nixon Controls	✓	✓	✓

Standard errors in parentheses

* $p < .10$, ** $p < .05$, *** $p < .01$

A.8 Single Country Regression Results

This Appendix tests the direct impact in single country regressions.[33] I estimate the following equation per country to test for the direct impact in the respective country:

$$\text{Core}_{c,t} = \check{\alpha}_1(V/U)_{c,t} + \check{\alpha}_2(V/U)^2_{c,t} + \check{\alpha}_3(V/U)^3_{c,t} + \check{\beta}\pi^e_{c,t} + \check{\gamma}_1\omega_{c,t} + \check{\gamma}_2\omega_{c,t-1} + \check{\phi}\text{Prod}_{c,t} + \check{c} + \check{\epsilon}_{c,t} \tag{38}$$

where Core is inflation of the consumer price index less food and energy (in some cases also less alcohol and tobacco), V/U is the ratio of job openings to unemployed, π^e is the 5 year forward inflation prediction from the IMF's World Economic Outlook, ω is nominal wage growth, and Prod is an eight quarter moving average of labor productivity growth. I thus estimate the same equation as on US aggregate data before, with the exception of using a shorter inflation expectations horizon (5 vs 10 years), and omitting controls for energy inflation and relative import price pressures.

I graph the functional form of the V/U coefficients and the constant together with error bands by country, against the V/U range of the respective country. I compile the 24 country graphs in Figure 17. Most countries yield a non-linear upward sloping functional form of the direct impact. These results are similar to those on US data. I include a US based version of the graph at the end for better comparison.

Most countries evince a non-linear impact of labor market tightness on core inflation, providing evidence for the direct impact as an international phenomenon. The results of the per country estimation indicate that the direct impact is not restricted to the US. However, global demand and supply shocks, commodity price shocks, as well as correlation of monetary policy between advanced economies can confound the estimation of Phillips Curve channels. Thus, a two-way fixed effects panel estimation as presented in the body of the paper allows for a cleaner identification of the channel. Furthermore, the country fixed effects control for most social and policy differences across countries. Finally, clustering standard errors at the country level controls for serial correlation of the error terms.

A.9 Market Concentration

I am testing the role of market power on the direct impact in this section. My results on US profit margins suggest a price setting channel, for which market power is likely important. This section finds that this is indeed the case. In section 9 I am proposing an optimal price setting channel that takes future employment cost implications of current wage setting into account, which needs both price and wage setting power. In Appendix B.3 I further show how the direct impact vanishes in my illustrative model as the price elasticity of demand

[33] Australia, Canada, Belgium, Germany, Estonia, Ireland, Spain, France, Cyprus, Latvia, Lithuania, Luxembourg, Malta, Netherlands, Austria, Poland, Portugal, Slovenia, Slovakia, Finland, Great Britain, Switzerland, Greece, and Japan



Figure 17: Direct Impact Estimation by Country

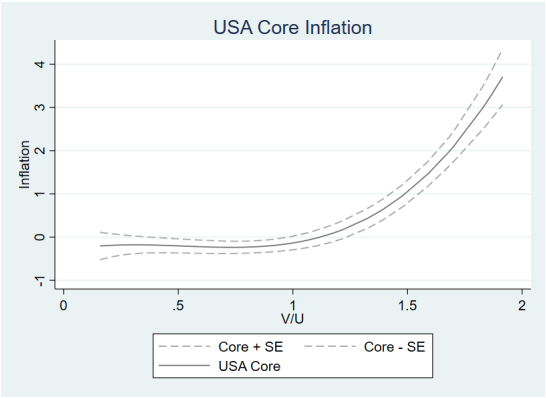


Figure 18: US Direct Impact Functional Form

goes to infinity.

To test the importance of market power I use the Herfindahl-Hirschman Index (HHI) of market concentration. The HHI is a measure of market power in both the goods and services market as well as the labor market. It thus functions as an indicator for the degree of both monopoly and monopsony power. I interact the HHI by country and year with labor market tightness. I estimate equation 38, but now include the following terms:

$$\tilde{\alpha}_1(V/U)_{c,t} * \text{HHI}_{c,t} + \tilde{\alpha}_2(V/U)^2_{c,t} * \text{HHI}_{c,t} + \tilde{\alpha}_3(V/U)^3_{c,t} * \text{HHI}_{c,t} + \tilde{\beta}\text{HHI}_{c,t}$$

Table 30 reports the estimation results. The coefficients on current and lagged employment cost growth are very small and insignificant, and I thus omit them from the table.

The reported p-values in the table are now from the Wald tests on the joint insignificance of the interaction terms instead of the simple labor market tightness coefficients. They indicate that market concentration leads to higher inflation during tight labor markets. Furthermore, in Osten (2025a) I show that market concentration and international labor mobility flatten the wage Phillips Curve. This is particularly true during tight labor markets. These results are in line with my proposed direct impact (see Section 9). Higher market power allows companies to resist upward wage pressure and price out the resulting demand overhang. This leads to slower wage growth and higher inflation during tight labor markets then under lower market power.

In Figure 19 I show the empirical effect of market concentration on core inflation. To this end I choose the mean HHI plus minus one SD of HHI in my international data to illustrate the change in the functional form between a country with elevated or low market concentration. Note that the direct impact is gone when market concentration is falling close to zero, but lifts inflation during tight labor markets in countries with sizeable market concentration.

Table 30: Market Concentration

	Core	P
(V/U)	5.48	2.04
	(4.81)	(5.38)
$(V/U)^2$	-4.96	1.23
	(6.47)	(7.11)
$(V/U)^3$	0.54	-2.22
	(2.45)	(2.70)
$(V/U) \times$ HHI	0.37	34.30
	(52.58)	(60.66)
$(V/U)^2 \times$ HHI	-39.54	-102.87
	(70.44)	(78.68)
$(V/U)^3 \times$ HHI	32.06	59.32*
	(27.37)	(30.21)
HHI	-2.24	-8.89
	(10.13)	(12.08)
5Y π^e	1.15***	1.33**
	(0.36)	(0.52)
Prod	0.03	0.08
	(0.04)	(0.07)
Constant	-1.01	-0.19
	(1.19)	(1.34)
R^2	0.66	0.69
N	1513	1513
No Impact p-value	0.0004	0.0025
Linear p-value	0.0001	0.0030

Standard errors in parentheses

* $p < .10$, ** $p < .05$, *** $p < .01$

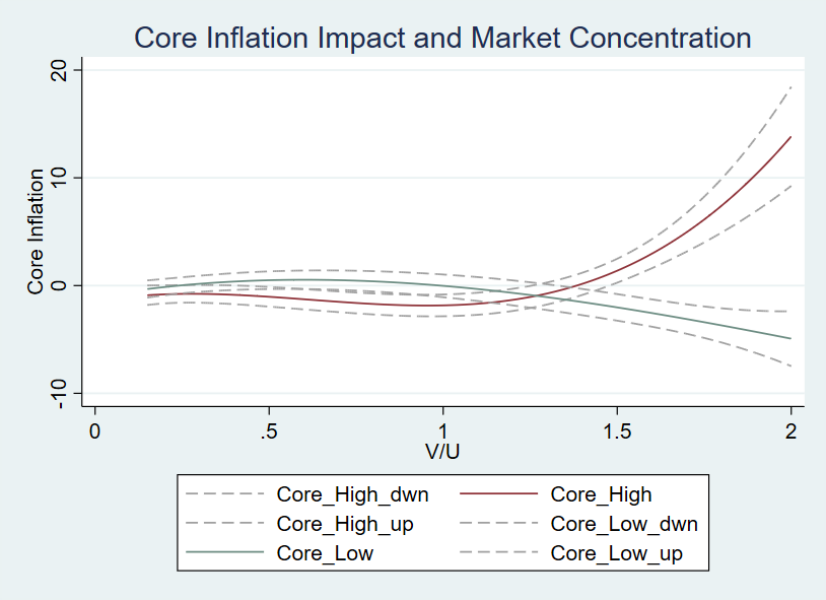


Figure 19: Direct Impact and Market Concentration

Note: Comparing the Direct Impact on Core Inflation under high (0.2) and low (0.01) market concentration as measured via the Herfindahl-Hirschman Index.

B Dual Impact of Labor Mobility and Market Concentration on Wages

This appendix estimates the impact of international labor mobility and market concentration on the Wage Phillips Curve. I add interactions of labor market tightness with market power to the Foreign Born Population interaction equation **??**. The goal of this exercise is to see if the level of international labor mobility and labor productivity can explain the linearity of the wage channel in the US. I thus allow myself to now include the US in the estimation as well. The estimation results are presented in Table 31. I omit reporting the coefficients of control variables such as inflation, expected inflation and labor productivity to simplify the table.

The US has an average FBP of 12.57% and an average market concentration of 0.066 over the estimation period. Thus, the relevant labor market tightness coefficients for the US are given by $\omega = 12.31V/U - 16.87(V/U)^2 + 4.95(V/U)^3$. These are all within one standard error of the basic V/U function $(22.65, 40.04, 18.98)$ from zero. The function is also rather flat on the US range of labor market tightness data (0.15,2.0), and declines rather than rises for tighter labor markets in the range. I provide a graphical comparison to the function estimated on aggregate US data in the body of the paper. The cross-sectional results indicate that international labor mobility and US levels of market concentration can explain the missing non-linearity in the US Wage Phillips Curve once inflation expectations are controlled for.

Table 31: FBP and HHI Impact on Wage Growth

	Wage Growth
(V/U)	69.65***
	(22.65)
$(V/U)^2$	-109.52**
	(40.04)
$(V/U)^3$	48.94**
	(18.98)
$(V/U) \times$ HHI	-630.74**
	(296.76)
$(V/U)^2 \times$ HHI	1038.25*
	(504.97)
$(V/U)^3 \times$ HHI	-511.52*
	(248.83)
HHI	83.79*
	(41.79)
$(V/U) \times$ FBP	-1.25
	(0.73)
$(V/U)^2 \times$ FBP	1.92
	(1.27)
$(V/U)^3 \times$ FBP	-0.81
	(0.64)
FBP	-0.17
	(0.42)
Constant	-7.50
	(6.46)
R^2	0.23
N	1183

Standard errors in parentheses

* $p < .10$, ** $p < .05$, *** $p < .01$

B.1 Extreme Demand and Labor Supply Elasticities in the Model

This Appendix checks if the direct impact holds in the Toy Model even when either the elasticity of demand ϵ or the elasticity of labor supply β go to infinity.

As a reminder, I reprint the comparison of the two optimal reactions to demand shifter x derived with comparative statics in section 7.1 above:

$$\frac{\partial P_1}{\partial x} > \frac{\partial P_1^*}{\partial x} \iff$$

$$1+\beta+\left[\frac{\epsilon}{\epsilon-1}\frac{1+\beta}{\beta}\right]^{\epsilon-1}\left(\epsilon-1-\frac{\epsilon}{\beta}\left(\frac{\epsilon-1}{\epsilon}\right)^{\epsilon}\right) > 1-\frac{(1+\beta)^{\epsilon-1}}{\beta^{\epsilon}} \quad +\left[\frac{\epsilon}{\epsilon-1}\frac{1+\beta}{\beta}\right]^{\epsilon-1}\left(\epsilon-1-\frac{\epsilon}{\beta}\left(\frac{\epsilon-1}{\epsilon}\right)^{\epsilon}\right)$$

$$\iff \quad 1+\beta > 1-\frac{(1+\beta)^{\epsilon-1}}{\beta^{\epsilon}}$$

$$\iff \quad \underbrace{\beta}_{>0} > \underbrace{-\frac{(1+\beta)^{\epsilon-1}}{\beta^{\epsilon}}}_{<0}$$

I start with the case of perfectly flexible demand $\epsilon \to \infty$:

$$\lim_{\epsilon\to\infty}\left[\beta \overset{?}{>} -\frac{(1+\beta)^{\epsilon-1}}{\beta^{\epsilon}}\right]$$

$$\to \beta > -\frac{(1+\beta)^{\epsilon\to\infty}}{\beta}$$

The RHS is still clearly negative, while $\beta > 1$; thus the response of the optimal price to a positive demand shock is still larger under DNWR than under flexible wages.

Second case: $\beta \to \infty$:

$$\lim_{\beta\to\infty}\left[\beta \overset{?}{>} -\frac{(1+\beta)^{\epsilon-1}}{\beta^{\epsilon}}\right]$$

LHS still bigger than RHS, as RHS is still negative.

Third case: $\beta, \epsilon \to \infty$:

$$\lim_{\beta,\epsilon\to\infty}\left[\beta \overset{?}{>} -\frac{(1+\beta)^{\epsilon-1}}{\beta^{\epsilon}}\right]$$

The limit of the RHS is -1 and thus smaller than the LHS which goes to infinity.

Similar one can show that the DIC exists if ϵ and/or β go to zero:

$$\begin{aligned}\beta \to 0: \quad & 0 > -\infty \\ \epsilon \to 0: \quad & \beta > -\frac{1}{1+\beta} \\ \epsilon, \beta \to 0: \quad & 0 > -1\end{aligned}$$

Finally, the DIC also exists if product markets become perfectly flexible, and the labor market perfectly inflexible (i.e. $\epsilon \to \infty;\ \beta \to 0$). The LHS of the key equation goes to zero while the RHS goes to negative infinity. Hence, the impact of additional demand on prices is still larger in the DNWR constrained case than in the flexible wage case.

B.2 Labor Supply Shock

This section gives the parallel results of the comparative statics exercise for the transitory negative labor supply shock. The comparative statics assume again that there are no other shocks, i.e. $x = 1$. I repeat the expression derived for optimal price setting under these conditions below:

$$\frac{1+\beta}{\beta}\epsilon\eta^{\frac{1}{\beta}} = P_1^{\frac{\epsilon+\beta}{\beta}}\left[(\epsilon-1) - \frac{\epsilon}{\beta}\left(\frac{\epsilon-1}{\epsilon}\right)^{\epsilon}\eta^{\frac{1-\epsilon}{\beta}}P_1^{(\epsilon-1)\frac{\beta+\epsilon}{\beta}}\right]$$

Getting the first derivative of the expression w.r.t. η yields:

$$\begin{aligned}\frac{1+\beta}{\beta^2}\epsilon\eta^{\frac{1-\beta}{\beta}} = \frac{\epsilon+\beta}{\beta}P_1^{\frac{\epsilon}{\beta}}\frac{\partial P_1}{\partial \eta}\left[(\epsilon-1) - \frac{\epsilon}{\beta}\left(\frac{\epsilon-1}{\epsilon}\right)^{\epsilon}\eta^{\frac{1-\epsilon}{\beta}}P_1^{(\epsilon-1)\frac{\beta+\epsilon}{\beta}}\right] \\ -\frac{\epsilon}{\beta}P_1^{\frac{\epsilon+\beta}{\beta}}\left(\frac{\epsilon-1}{\epsilon}\right)^{\epsilon}\left[\frac{1-\epsilon}{\beta}\eta^{\frac{1-\epsilon-\beta}{\beta}}P_1^{(\epsilon-1)\frac{\beta+\epsilon}{\beta}} + \eta^{\frac{1-\epsilon}{\beta}}(\epsilon-1)\frac{\beta+\epsilon}{\beta}P_1^{(\epsilon-1)\frac{\beta+\epsilon}{\beta}-\frac{\beta}{\beta}}\frac{\partial P_1}{\partial \eta}\right]\end{aligned}$$

For simplicity I linearize around small shocks to the labor supply shifter η, such that when I plug the steady state value of $\eta = 1$ into the expression I get a first order approximation for small negative labor supply shocks:

$$\begin{aligned}\frac{1+\beta}{\beta^2}\epsilon = \frac{\epsilon+\beta}{\beta}P_1^{\frac{\epsilon}{\beta}}\frac{\partial P_1}{\partial \eta}\left[(\epsilon-1) - \frac{\epsilon}{\beta}\left(\frac{\epsilon-1}{\epsilon}\right)^{\epsilon}P_1^{(\epsilon-1)\frac{\beta+\epsilon}{\beta}}\right] \\ -\frac{\epsilon}{\beta}P_1^{\frac{\epsilon+\beta}{\beta}}\left(\frac{\epsilon-1}{\epsilon}\right)^{\epsilon}\left[\frac{1-\epsilon}{\beta}P_1^{(\epsilon-1)\frac{\beta+\epsilon}{\beta}} + (\epsilon-1)\frac{\beta+\epsilon}{\beta}P_1^{(\epsilon-1)\frac{\beta+\epsilon}{\beta}-\frac{\beta}{\beta}}\frac{\partial P_1}{\partial \eta}\right]\end{aligned}$$

Which ultimately simplifies to:

$$\frac{\partial P_1}{\partial \eta} = \frac{\partial P_1^*}{\partial \eta}\frac{1 - \frac{(\epsilon-1)}{\beta}\left(\frac{1+\beta}{\beta}\right)^{\epsilon-1}}{1 - \frac{(\epsilon-1)}{\beta}\left(\frac{1+\beta}{\beta}\right)^{\epsilon-1} - \frac{1}{\beta}\left(\frac{\epsilon}{\epsilon-1}\right)^{1-\epsilon}\left[\frac{\epsilon}{\epsilon-1}\frac{1+\beta}{\beta}\right]^{\frac{\beta(\epsilon-1)}{\epsilon}}}$$

Thus,

$$\begin{aligned}
&\frac{\partial P_1}{\partial \eta} > \frac{\partial P_1^*}{\partial \eta} \\
\iff 1 - \frac{(\epsilon-1)}{\beta}\left(\frac{1+\beta}{\beta}\right)^{\epsilon-1} &> 1 - \frac{(\epsilon-1)}{\beta}\left(\frac{1+\beta}{\beta}\right)^{\epsilon-1} - \frac{1}{\beta}\left(\frac{\epsilon}{\epsilon-1}\right)^{1-\epsilon}\left[\frac{\epsilon}{\epsilon-1}\frac{1+\beta}{\beta}\right]^{\frac{\beta(\epsilon-1)}{\epsilon}} \\
\iff \qquad 0 &> -\frac{1}{\beta}\left(\frac{\epsilon}{\epsilon-1}\right)^{1-\epsilon}\left[\frac{\epsilon}{\epsilon-1}\frac{1+\beta}{\beta}\right]^{\frac{\beta(\epsilon-1)}{\epsilon}}
\end{aligned}$$

The right hand side is less than zero as $\epsilon > 1$ and $\beta > 1$. Hence, the price is rising faster following a transitory negative labor supply shock if wages are downward nominal sticky. I.e. inflation goes above and beyond wage growth under these circumstances, leading to the puzzle in the data. The Toy model thus explains the existence of the direct impact channel, in face of transitory negative labor supply shocks as well as transitory positive demand shocks.

B.3 Market Power and the Direct Impact Channel

This section explores the relation between market power in the goods and services market and the size of the direct impact channel. To show the importance of market power I first quantify the direct impact channel at reasonable values for the price elasticity of demand $\epsilon = 2$ and the wage elasticity of labor supply $\beta = 2$.

The direct impact channel can be described as the difference between $\frac{\partial P_1}{\partial x}$ and $\frac{\partial P_1^*}{\partial x}$, but it is still easier to use Equation 9:

$$\frac{\partial P_1}{\partial x} = \frac{\partial P_1^*}{\partial x}\frac{1+\beta+\left[\frac{\epsilon}{\epsilon-1}\frac{1+\beta}{\beta}\right]^{\epsilon-1}\left(\epsilon-1-\frac{\epsilon}{\beta}\left(\frac{\epsilon-1}{\epsilon}\right)^{\epsilon}\right)}{1-\frac{(1+\beta)^{\epsilon-1}}{\beta^{\epsilon}}+\left[\frac{\epsilon}{\epsilon-1}\frac{1+\beta}{\beta}\right]^{\epsilon-1}\left(\epsilon-1-\frac{\epsilon}{\beta}\left(\frac{\epsilon-1}{\epsilon}\right)^{\epsilon}\right)}$$

where $\beta > 1$ and $\epsilon > 1$. Plugging in the chosen values for $\beta = 2$ and $\epsilon = 2$ yields:

$$\frac{\partial P_1}{\partial x} = \frac{\partial P_1^*}{\partial x}\frac{3+3(1-1/4)}{1-(3/4)+3(1-1/4)} = \frac{\partial P_1^*}{\partial x}2.1$$

I.e. under the given parameter values the direct impact channel more than doubles the price response to a positive demand shock.

In the following I want to quantify how important market power is for the direct impact channel. Market power in the goods market is inversely related to the price elasticity of

demand. Thus, market power goes to zero as $\epsilon \to \infty$.

$$
\begin{aligned}
\lim_{\epsilon\to\infty} \frac{\partial P_1}{\partial x} = & \\
&= \lim_{\epsilon\to\infty} \frac{\partial P_1^*}{\partial x} \frac{1+\beta+\left[\frac{\epsilon}{\epsilon-1}\frac{1+\beta}{\beta}\right]^{\epsilon-1}\left(\epsilon-1-\frac{\epsilon}{\beta}\left(\frac{\epsilon-1}{\epsilon}\right)^{\epsilon}\right)}{1-\frac{(1+\beta)^{\epsilon-1}}{\beta^{\epsilon}}+\left[\frac{\epsilon}{\epsilon-1}\frac{1+\beta}{\beta}\right]^{\epsilon-1}\left(\epsilon-1-\frac{\epsilon}{\beta}\left(\frac{\epsilon-1}{\epsilon}\right)^{\epsilon}\right)} \\
&= \lim_{\epsilon\to\infty} \frac{\partial P_1^*}{\partial x} \frac{\left[\frac{1+\beta}{\beta}\right]^{\epsilon}\left(\epsilon-\frac{\epsilon}{\beta}\right)}{-\left[\frac{1+\beta}{\beta}\right]^{\epsilon}+\left[\frac{1+\beta}{\beta}\right]^{\epsilon}\left(\epsilon-\frac{\epsilon}{\beta}\right)} \\
&= \lim_{\epsilon\to\infty} \frac{\partial P_1^*}{\partial x} \frac{\left[\frac{1+\beta}{\beta}\right]^{\epsilon}\left(\epsilon\frac{\beta-1}{\beta}\right)}{\left[\frac{1+\beta}{\beta}\right]^{\epsilon}\left(\epsilon\frac{\beta-1}{\beta}-1\right)} \\
&\overset{\beta>1}{=} \lim_{\epsilon\to\infty} \frac{\partial P_1^*}{\partial x} \frac{\left[\frac{1+\beta}{\beta}\right]^{\epsilon}\epsilon}{\left[\frac{1+\beta}{\beta}\right]^{\epsilon}\epsilon} = \frac{\partial P_1^*}{\partial x}
\end{aligned}
$$

I.e. words the direct impact channel vanishes and the reaction to a positive demand shock is equal to the optimal price setting under flexible prices.

B.4 DNWR and optimal price setting under negative demand shocks

This section solves for the optimal price setting under DNWR and negative demand shocks in the illustrative model. The optimization problem facing the firm is:

$$\max_{P_1} P_1 D(P_1) - W^* D(P_1) = \max_{P_1} x P_1^{1-\epsilon} - W^* x P^{-\epsilon}$$

where the wage is now independent of the amount produced and thus stuck at steady state level W^*. The resulting first order condition is

$$\begin{aligned}(1-\epsilon)xP^{-\epsilon} + W^* x \epsilon P^{-(\epsilon+1)} &\stackrel{!}{=} 0 \\ \iff \epsilon - 1 &= W^* \epsilon P^{-1} \\ \iff P_1 &= W^* \frac{\epsilon}{\epsilon - 1}\end{aligned}$$

Note that the new steady state price is lower than in the flexible case as the labor supply part of the markup is gone.

B.5 Interaction of Future Demand and Labor Market Tightness

I derive the interaction term of expected demand growth and labor market tightness on the direct impact channel in the illustrative model (see Section 7 for the model). I use comparative statics to derive the interaction, and show that it is negative under reasonable price elasticity of demand and labor supply elasticity with respect to wages. I start with a brief reminder of the model.

The model is a simple monopolistic firm, facing iso-elastic demand and labor supply, with labor the only input into production, and a constant returns to scale technology. Optimizing the firm's profits yields the usual constant markup over marginal cost under flexible wages. I want to start by showing that under the flexible wage case; i.e. without the direct impact channel the percent increase in wages and prices is the same. Under the flexible wage assumption the response of the optimal period one price level to a period one demand shock was derived as (see illustrative model section 7):

$$\frac{\partial P_1}{\partial x_1}|_{flex} = \frac{1}{\beta+\epsilon}\frac{P_1|_{flex}}{x_1}$$

Thus, the rate of change of the optimal price with respect to the demand shock is

$$\frac{\partial P_1/\partial x_1}{P_1}|_{flex} = \frac{\partial \ln(P_1)}{\partial x_1}|_{flex} = \frac{1}{\beta+\epsilon}\frac{1}{x_1}$$

Now I will derive the percent increase in wages with respect to the demand shock. The optimal level of wages is:

$$W_1 = (L_1)^{1/\beta} = (D(P_1, x_1))^{1/\beta} = x_1^{1/\beta}(P_1)^{-\frac{\epsilon}{\beta}}$$

Taking the derivative with respect to x_1:

$$\frac{\partial W_1}{\partial x_1}|_{flex} = \frac{1}{\beta}x_1^{(1-\beta)/\beta}(P_1)^{-\frac{\epsilon}{\beta}} - \frac{\epsilon}{\beta}x_1^{1/\beta}P_1^{-\frac{\epsilon+\beta}{\beta}}\frac{\partial P_1}{\partial x_1}|_{flex}$$

Using the definition of W_1 this can be re-expressed as

$$\frac{\partial W_1}{\partial x_1}|_{flex} = \frac{1}{\beta}x_1^{-1}W_1 - \frac{\epsilon}{\beta}W_1P_1^{-1}\frac{\partial P_1}{\partial x_1}|_{flex}$$

$$\implies \frac{\partial \ln(W_1)}{\partial x_1}|_{flex} = \frac{1}{\beta}x_1^{-1} - \frac{\epsilon}{\beta}\frac{\partial \ln(P_1)}{\partial x_1}|_{flex}$$

Plugging in the expression for $\frac{\partial \ln(P_1)}{\partial x_1}|_{flex}$ gives

$$\frac{\partial \ln(W_1)}{\partial x_1}|_{flex} = \frac{1}{\beta}x_1^{-1} - \frac{\epsilon}{\beta}\frac{1}{\beta+\epsilon}x_1^{-1} = \frac{1}{\beta+\epsilon}\frac{1}{x_1} = \frac{\partial \ln(P_1)}{\partial x_1}|_{flex}$$

In other words, under flexible wages the percent increase in prices and wages in response to a positive demand shock is the same. This is the traditional Phillips Curve logic.

However, when I introduce downward nominal wage rigidity

$$W_2 = \begin{cases} W(L_2) & \text{if } W(L_2) \geq W_1 \\ W_1 & \text{if } W(L_2) < W_1 \end{cases}$$

the direct impact channel comes into play, leading to a positive increase of prices above and beyond the increase in wages. With the downward nominal wage rigidity (DNWR) constraint the optimization problem becomes

$$\max_{P_1,P_2} \Pi = \max_{P_1,P_2,W_1,W_2} P_1D_1 - W_1L_1 + P_2D_2 - W_2L_2 = \max_{P_1,P_2,W_1,W_2} x_1P_1^{1-\epsilon} - W_1x_1P_1^{-\epsilon} + x_2P_2^{1-\epsilon} - W_2x_2P_2^{-\epsilon}$$

under the following constraints. Downward Nominal Wage Rigidity:

$$W_2 \geq W_1$$

Period one labor supply:

$$W_1 \geq (x_1P_1^{-\epsilon})^{\frac{1}{\beta}}$$

Period two labor supply:

$$W_2 \geq (x_2P_2^{-\epsilon})^{\frac{1}{\beta}}$$

Using a Lagrangian approach with multipliers ϕ_1 through ϕ_3:

$$\max_{P_1,P_2,W_1,W_2} x_1P_1^{1-\epsilon} - W_1x_1P_1^{-\epsilon} + x_2P_2^{1-\epsilon} - W_2x_2P_2^{-\epsilon} + \phi_1(W_2 - W_1) + \phi_2(W_1 - (x_1P_1^{-\epsilon})^{\frac{1}{\beta}}) + \phi_3(W_2 - (x_2P_2^{-\epsilon})^{\frac{1}{\beta}})$$

The Lagrangian has the following Karush-Kuhn-Tucker conditions, which include the above constraints and the first order conditions.

$$\begin{aligned}
\phi_1, \phi_2, \phi_3 &\geq 0 \\
\phi_1(W_2 - W_1) &= 0 \\
\phi_2(W_1 - (x_1P_1^{-\epsilon})^{\frac{1}{\beta}}) &= 0 \\
\phi_3(W_2 - (x_2P_2^{-\epsilon})^{\frac{1}{\beta}}) &= 0 \\
W_2 &\geq W_1 \\
W_1 &\geq (x_1P_1^{-\epsilon})^{\frac{1}{\beta}} \\
W_2 &\geq (x_2P_2^{-\epsilon})^{\frac{1}{\beta}} \\
(1-\epsilon)P_1^{-\epsilon}x_1 + \epsilon P_1^{-(\epsilon+1)}x_1W_1 + \phi_2\frac{\epsilon}{\beta}P_1^{-\frac{\epsilon+\beta}{\beta}}x_1^{\frac{1}{\beta}} &= 0 \\
(1-\epsilon)P_2^{-\epsilon}x_2 + \epsilon P_2^{-(\epsilon+1)}x_2W_2 + \phi_3\frac{\epsilon}{\beta}P_2^{-\frac{\epsilon+\beta}{\beta}}x_2^{\frac{1}{\beta}} &= 0 \\
-x_1P_1^{-\epsilon} - \phi_1 + \phi_2 &= 0 \\
-x_2P_2^{-\epsilon} + \phi_1 + \phi_3 &= 0
\end{aligned}$$

These imply that $\phi_1 = 0$ when $W_2 > W_1$. Similar for the other conditions.

To solve the optimization problem we will have to go through all the different cases of the three constraints binding or not. Due to the KKT conditions, whenever a constraint is not

binding the pertaining Lagrange multiplier must be zero. Thus, written in tuples, the case where no constraint is binding can be written as $(\phi_1, \phi_2, \phi_3) = (0, 0, 0)$. Remember that ϕ_1 is associated with the DNWR constraint, ϕ_2 with the labor supply schedule in period one, and ϕ_3 with the labor supply schedule in period two. The eight cases are thus

$$
\begin{aligned}
&1.(0,0,0) \rightarrow \text{no constraint binds} \\
&2.(0,>0,0) \rightarrow W_1 = (x_1 P_1^{-\epsilon})^{\frac{1}{\beta}} \\
&3.(0,0,>0) \rightarrow W_2 = (x_2 P_2^{-\epsilon})^{\frac{1}{\beta}} \\
&4.(0,>0,>0) \rightarrow W_1 = (x_1 P_1^{-\epsilon})^{\frac{1}{\beta}} \ \& \ W_2 = (x_2 P_2^{-\epsilon})^{\frac{1}{\beta}} \\
&5.(>0,0,0) \rightarrow W_2 = W_1 \\
&6.(>0,>0,0) \rightarrow W_2 = W_1 \ \& \ W_1 = (x_1 P_1^{-\epsilon})^{\frac{1}{\beta}} \\
&7.(>0,0,>0) \rightarrow W_2 = W_1 \ \& \ W_2 = (x_2 P_2^{-\epsilon})^{\frac{1}{\beta}} \\
&8.(>0,>0,>0) \rightarrow W_2 = W_1 \ \& \ W_1 = (x_1 P_1^{-\epsilon})^{\frac{1}{\beta}} \ \& \ W_2 = (x_2 P_2^{-\epsilon})^{\frac{1}{\beta}}
\end{aligned}
$$

There are some trivial cases. For example when DNWR and one of the other constraints are not binding. In this case $\phi_1 = 0$ and on of the other multipliers are also equal to zero. Using the FOCs with respect to the wages

$$
\begin{aligned}
-x_1 P_1^{-\epsilon} - \phi_1 + \phi_2 = 0 \\
-x_2 P_2^{-\epsilon} + \phi_1 + \phi_3 = 0
\end{aligned}
$$

this gives

$$
\begin{aligned}
x_i P_i^{-\epsilon} = 0 \\
\implies P_i = \infty
\end{aligned}
$$

for $i = 1$ when both ϕ_1 and ϕ_2 equal to zero, and for $i = 2$ if both ϕ_1 and ϕ_3 equal to zero. This makes cases 1-3 redundant. Similar reasoning rules out any case where $W_1 > (x_1 P_1^{-\epsilon})^{\frac{1}{\beta}}$. This leaves cases 4, 6, and 8.

B.6 Flexible Wage Case

Case 4, where $\phi_1 = 0$, but the other constraints are binding (i.e. $W_i = (x_i P_i^{-\epsilon})^{\frac{1}{\beta}}$, yields the flexible wage solution.

$$
\begin{aligned}
\phi_2 = x_1 P_1^{-\epsilon} - \phi_1 = x_1 P_1^{-\epsilon} \\
\phi_3 = x_2 P_2^{-\epsilon} + \phi_1 = x_2 P_2^{-\epsilon}
\end{aligned}
$$

Plugging these into the FOCs from price optimization gives

$$
\begin{aligned}
&(1-\epsilon)P_i^{-\epsilon}x_i + \epsilon P_i^{-(\epsilon+1)}x_i W_i + x_i P_i^{-\epsilon}\frac{\epsilon}{\beta}P_i^{-\frac{\epsilon+\beta}{\beta}}x_i^{\frac{1}{\beta}} = 0 \\
\iff &(\epsilon-1)P_i^{-\epsilon}x_i = \epsilon P_i^{-(\epsilon+1)}x_i\left((x_i P_i^{-\epsilon})^{\frac{1}{\beta}}\right) + \frac{\epsilon}{\beta}P_i^{-\frac{\epsilon+\beta}{\beta}-\epsilon}x_i^{\frac{1+\beta}{\beta}} \\
\iff &\frac{\epsilon-1}{\epsilon}P_i = x_i^{\frac{1}{\beta}}P_i^{-\frac{\epsilon}{\beta}} + \frac{1}{\beta}P_i^{-\frac{\epsilon}{\beta}}x_i^{\frac{1}{\beta}} \\
\iff &P_i^{\frac{\epsilon+\beta}{\beta}} = \frac{\epsilon}{\epsilon-1}\frac{1+\beta}{\beta}x_i^{\frac{1}{\beta}} \\
\iff &P_i^* = \left[\frac{\epsilon}{\epsilon-1}\frac{1+\beta}{\beta}x_i^{\frac{1}{\beta}}\right]^{\frac{\beta}{\epsilon+\beta}}
\end{aligned}
$$

B.7 All Constraints Binding

Case 8, where all three constraints are binding, is also straight forward. It follows the math that I submitted to you last week. If you want to skip it go to page xxx. A short reminder of the FOCs:

$$(1-\epsilon)P_1^{-\epsilon}x_1 + \epsilon P_1^{-(\epsilon+1)}x_1 W_1 + \phi_2\frac{\epsilon}{\beta}P_1^{-\frac{\epsilon+\beta}{\beta}}x_1^{\frac{1}{\beta}} = 0 \tag{39}$$

$$(1-\epsilon)P_2^{-\epsilon}x_2 + \epsilon P_2^{-(\epsilon+1)}x_2 W_2 + \phi_3\frac{\epsilon}{\beta}P_2^{-\frac{\epsilon+\beta}{\beta}}x_2^{\frac{1}{\beta}} = 0 \tag{40}$$

$$-x_1 P_1^{-\epsilon} - \phi_1 + \phi_2 = 0 \tag{41}$$

$$-x_2 P_2^{-\epsilon} + \phi_1 + \phi_3 = 0 \tag{42}$$

Reordering equations 41 & 42 to get expressions for ϕ_1 and then setting these equal gives an expression of ϕ_2 in terms of ϕ_3.

$$
\begin{aligned}
-x_1 P_1^{-\epsilon} + \phi_2 &= x_2 P_2^{-\epsilon} - \phi_3 \\
\phi_2 &= x_1 P_1^{-\epsilon} + x_2 P_2^{-\epsilon} - \phi_3
\end{aligned}
$$

Plugging this into equation 39 gives

$$(1-\epsilon)P_1^{-\epsilon}x_1 + \epsilon P_1^{-(\epsilon+1)}x_1 W_1 + [x_1 P_1^{-\epsilon} + x_2 P_2^{-\epsilon} - \phi_3]\frac{\epsilon}{\beta}P_1^{-\frac{\epsilon+\beta}{\beta}}x_1^{\frac{1}{\beta}} = 0 \tag{43}$$

$$\frac{\epsilon-1}{\epsilon}P_1^{-\epsilon}x_1 = P_1^{-(\epsilon+1)}x_1 W_1 + \frac{1}{\beta}P_1^{-\frac{\epsilon+\beta}{\beta}}x_1^{\frac{1}{\beta}}[x_1 P_1^{-\epsilon} + x_2 P_2^{-\epsilon} - \phi_3] \tag{44}$$

$$\frac{\epsilon-1}{\epsilon}P_1 = W_1 + \frac{1}{\beta}P_1^{\epsilon-\frac{\epsilon}{\beta}}x_1^{\frac{1}{\beta}}[P_1^{-\epsilon} + \frac{x_2}{x_1}P_2^{-\epsilon} - \frac{\phi_3}{x_1}] \tag{45}$$

As in this case all three constraints are binding; i.e. $\phi_1, \phi_2, \phi_3 > 0$, $W_2 = W_1$, $W_1 = (x_1 P_1^{-\epsilon})^{\frac{1}{\beta}}$, and $W_2 = (x_2 P_2^{-\epsilon})^{\frac{1}{\beta}}$, this leads to

$$(x_1 P_1^{-\epsilon})^{\frac{1}{\beta}} = (x_2 P_2^{-\epsilon})^{\frac{1}{\beta}}$$

In the case of finite β this is equivalent to

$$\begin{aligned} x_1 P_1^{-\epsilon} &= x_2 P_2^{-\epsilon} \\ \iff P_2 &= \left(\frac{x_1}{x_2}\right)^{-\frac{1}{\epsilon}} P_1 \end{aligned}$$

Plugging this and $(x_1 P_1^{-\epsilon})^{\frac{1}{\beta}}$ into 45 for P_2 and $W_1 = W_2$ respectively yields

$$\begin{aligned} \frac{\epsilon-1}{\epsilon} P_1 &= (x_1 P_1^{-\epsilon})^{\frac{1}{\beta}} + \frac{1}{\beta} P_1^{\epsilon-\frac{\epsilon}{\beta}} x_1^{\frac{1}{\beta}} [P_1^{-\epsilon} + \frac{x_2}{x_1}\left(\frac{x_1}{x_2}\right)^{-\frac{1}{\epsilon}} P_1^{-\epsilon} - \frac{\phi_3}{x_1}] \\ &= x_1^{\frac{1}{\beta}} P_1^{-\frac{\epsilon}{\beta}} + \frac{1}{\beta} P_1^{\epsilon-\frac{\epsilon}{\beta}} x_1^{\frac{1}{\beta}} [2P_1^{-\epsilon} - \frac{\phi_3}{x_1}] \\ &= P_1^{-\frac{\epsilon}{\beta}} x_1^{\frac{1}{\beta}} [1 + \frac{2}{\beta}] - \frac{1}{\beta} P_1^{\epsilon-\frac{\epsilon}{\beta}} x_1^{\frac{1}{\beta}} \frac{\phi_3}{x_1}] \\ \iff \frac{\epsilon-1}{\epsilon} P_1^{\frac{\epsilon+\beta}{\beta}} x_1^{-\frac{1}{\beta}} &= \frac{\beta+2}{\beta} - \frac{1}{\beta} P_1^{\epsilon} \frac{\phi_3}{x_1} \\ \implies \phi_3 &= (\beta+2) P_1^{-\epsilon} x_1 - \beta x_1^{\frac{\beta-1}{\beta}} \frac{\epsilon-1}{\epsilon} P_1^{\frac{\epsilon+\beta}{\beta}-\epsilon} \end{aligned}$$

Plugging the expression for ϕ_3, as well as for W_1, W_2, and P_2 into equation 40

$$\begin{aligned} &(1-\epsilon) P_1^{-\epsilon} x_1 + \epsilon \left(\left(\frac{x_1}{x_2}\right)^{-\frac{1}{\epsilon}} P_1\right)^{-(\epsilon+1)} x_2 (x_1 P_1^{-\epsilon})^{\frac{1}{\beta}} \\ &+ \left[(\beta+2) P_1^{-\epsilon} x_1 - \beta x_1^{\frac{\beta-1}{\beta}} \frac{\epsilon-1}{\epsilon} P_1^{\frac{\epsilon+\beta}{\beta}-\epsilon}\right] \frac{\epsilon}{\beta} \left(\left(\frac{x_1}{x_2}\right)^{-\frac{1}{\epsilon}} P_1\right)^{-\frac{\epsilon+\beta}{\beta}} x_2^{\frac{1}{\beta}} = 0 \\ \iff (\epsilon-1) P_1^{-\epsilon} x_1 = \epsilon \left(\frac{x_1}{x_2}\right)^{\frac{\epsilon+1}{\epsilon}} &P_1^{-(\epsilon+1)} x_2 x_1^{\frac{1}{\beta}} P_1^{-\frac{\epsilon}{\beta}} + P_1^{-\epsilon} x_1 \left[\beta + 2 - \beta x_1^{-\frac{1}{\beta}} \frac{\epsilon-1}{\epsilon} P_1^{\frac{\epsilon+\beta}{\beta}}\right] \frac{\epsilon}{\beta} \left(\frac{x_1}{x_2}\right)^{\frac{\epsilon+\beta}{\beta\epsilon}} P_1^{-\frac{\epsilon+\beta}{\beta}} x_2^{\frac{1}{\beta}} \\ \iff \frac{\epsilon-1}{\epsilon} &= \left(\frac{x_1}{x_2}\right)^{\frac{1}{\epsilon}} x_1^{\frac{1}{\beta}} P_1^{-\frac{\epsilon+\beta}{\beta}} + \left[\beta + 2 - \beta x_1^{-\frac{1}{\beta}} \frac{\epsilon-1}{\epsilon} P_1^{\frac{\epsilon+\beta}{\beta}}\right] \frac{1}{\beta} \left(\frac{x_1}{x_2}\right)^{\frac{1}{\epsilon}} P_1^{-\frac{\epsilon+\beta}{\beta}} x_1^{\frac{1}{\beta}} \\ \iff P_1^{\frac{\epsilon+\beta}{\beta}} &= \frac{\epsilon}{\epsilon-1} \left(\frac{x_1}{x_2}\right)^{\frac{1}{\epsilon}} x_1^{\frac{1}{\beta}} + \frac{\epsilon}{\epsilon-1} \frac{2+\beta}{\beta} \left(\frac{x_1}{x_2}\right)^{\frac{1}{\epsilon}} x_1^{\frac{1}{\beta}} - P_1^{\frac{\epsilon+\beta}{\beta}} \left(\frac{x_1}{x_2}\right)^{\frac{1}{\epsilon}} \\ \iff P_1^{\frac{\epsilon+\beta}{\beta}} &\left[1 + \left(\frac{x_1}{x_2}\right)^{\frac{1}{\epsilon}}\right] = \frac{\epsilon}{\epsilon-1} \left(\frac{x_1}{x_2}\right)^{\frac{1}{\epsilon}} x_1^{\frac{1}{\beta}} \left[1 + \frac{2+\beta}{\beta}\right] \\ \iff P_1^{\frac{\epsilon+\beta}{\beta}} &\left[1 + \left(\frac{x_1}{x_2}\right)^{\frac{1}{\epsilon}}\right] = \frac{\epsilon}{\epsilon-1} \left(\frac{x_1}{x_2}\right)^{\frac{1}{\epsilon}} x_1^{\frac{1}{\beta}} \left[2\frac{1+\beta}{\beta}\right] \end{aligned}$$

If $x_1 = x_2$ then this yields the same solution for optimal P_1 as under flexible wage optimization

$$P_1 = \left[\frac{2\frac{\epsilon}{\epsilon-1}\frac{1+\beta}{\beta}}{1+1} x_1^{\frac{1}{\beta}}\right]^{\frac{\beta}{\beta+\epsilon}} = \left[\frac{\epsilon}{\epsilon-1}\frac{1+\beta}{\beta} x_1^{\frac{1}{\beta}}\right]^{\frac{\beta}{\beta+\epsilon}}$$

In general the optimal period one price is

$$P_1^* = \left[2\frac{\epsilon}{\epsilon-1}\frac{1+\beta}{\beta}x_1^{\frac{1}{\beta}}\frac{\left(\frac{x_1}{x_2}\right)^{\frac{1}{\epsilon}}}{1+\left(\frac{x_1}{x_2}\right)^{\frac{1}{\epsilon}}} \right]^{\frac{\beta}{\beta+\epsilon}}$$

This case leads to a positive direct impact, but also a positive cross derivative with respect to both x_1 and x_2. I commented these sections out for now so we can focus on the three non-degenerate cases.

B.8 DNWR leads to non-binding Labor Supply Constraint in Period 2

Case 6, where $W_2 = W_1$ and $W_1 = (x_1 P_1^{-\epsilon})^{\frac{1}{\beta}}$, but $W_2 > (x_2 P_2^{-\epsilon})^{\frac{1}{\beta}}$ is a tougher nut to crack. Setting $\phi_3 = 0$ changes the FOCs to

$$\begin{aligned}
(1-\epsilon)P_1^{-\epsilon}x_1 + \epsilon P_1^{-(\epsilon+1)}x_1 W_1 + \phi_2\frac{\epsilon}{\beta}P_1^{-\frac{\epsilon+\beta}{\beta}}x_1^{\frac{1}{\beta}} &= 0 \\
(1-\epsilon)P_2^{-\epsilon}x_2 + \epsilon P_2^{-(\epsilon+1)}x_2 W_2 &= 0 \\
-x_1 P_1^{-\epsilon} - \phi_1 + \phi_2 &= 0 \\
-x_2 P_2^{-\epsilon} + \phi_1 &= 0
\end{aligned}$$

This yields:

$$\begin{aligned}
\phi_1 &= x_2 P_2^{-\epsilon} \\
\phi_2 &= x_1 P_1^{-\epsilon} + x_2 P_2^{-\epsilon} \\
P_2 &= \frac{\epsilon}{\epsilon-1}W_2
\end{aligned}$$

Putting all this and the equalities from the binding constraints into the FOC with respect to P_1 simplifies as follows

$$
\begin{aligned}
(1-\epsilon)P_1^{-\epsilon}x_1 + \epsilon P_1^{-(\epsilon+1)}x_1 W_1 + \phi_2 \frac{\epsilon}{\beta} P_1^{-\frac{\epsilon+\beta}{\beta}} x_1^{\frac{1}{\beta}} &= 0 \\
\frac{\epsilon-1}{\epsilon}P_1^{-\epsilon}x_1 = x_1^{\frac{1+\beta}{\beta}} P_1^{-(\epsilon+1)-\frac{\epsilon}{\beta}} + \frac{1}{\beta}P_1^{-\frac{\epsilon+\beta}{\beta}} x_1^{\frac{1}{\beta}} \left[x_1 P_1^{-\epsilon} + x_2 P_2^{-\epsilon}\right] \\
\frac{\epsilon-1}{\epsilon}P_1 = x_1^{\frac{1}{\beta}} P_1^{-\frac{\epsilon}{\beta}} + \frac{1}{\beta}P_1^{\epsilon-\frac{\epsilon}{\beta}} x_1^{\frac{1}{\beta}} \left[P_1^{-\epsilon} + \frac{x_2}{x_1}P_2^{-\epsilon}\right] \\
\frac{\epsilon-1}{\epsilon}P_1^{\frac{\epsilon+\beta}{\beta}} = x_1^{\frac{1}{\beta}} \left(1 + \frac{1}{\beta}\left[1 + P_1^{\epsilon}\frac{x_2}{x_1}\left(\frac{\epsilon}{\epsilon-1}\right)^{-\epsilon} W_2^{-\epsilon}\right]\right) \\
\frac{\epsilon-1}{\epsilon}P_1^{\frac{\epsilon+\beta}{\beta}} = x_1^{\frac{1}{\beta}} \left(1 + \frac{1}{\beta}\left[1 + P_1^{\epsilon}\frac{x_2}{x_1}\left(\frac{\epsilon}{\epsilon-1}\right)^{-\epsilon} W_2^{-\epsilon}\right]\right) \\
\frac{\epsilon-1}{\epsilon}P_1^{\frac{\epsilon+\beta}{\beta}} = x_1^{\frac{1}{\beta}} \left(1 + \frac{1}{\beta}\left[1 + P_1^{\epsilon}\frac{x_2}{x_1}\left(\frac{\epsilon}{\epsilon-1}\right)^{-\epsilon} \left((x_1 P_1^{-\epsilon})^{\frac{1}{\beta}}\right)^{-\epsilon}\right]\right) \\
\frac{\epsilon-1}{\epsilon}P_1^{\frac{\epsilon+\beta}{\beta}} = x_1^{\frac{1}{\beta}} \left(1 + \frac{1}{\beta}\left[1 + P_1^{\epsilon}\frac{x_2}{x_1}\left(\frac{\epsilon}{\epsilon-1}\right)^{-\epsilon} x_1^{-\frac{\epsilon}{\beta}} P_1^{\frac{\epsilon^2}{\beta}}\right]\right) \\
\frac{\epsilon-1}{\epsilon}P_1^{\frac{\epsilon+\beta}{\beta}} = x_1^{\frac{1}{\beta}} \left(\frac{1+\beta}{\beta} + \frac{1}{\beta}\frac{x_2}{x_1}\left(\frac{\epsilon}{\epsilon-1}\right)^{-\epsilon} x_1^{-\frac{\epsilon}{\beta}} P_1^{\epsilon\frac{\epsilon+\beta}{\beta}}\right)
\end{aligned}
$$

which is the same implicit function we derived before, when we were simply plugging in W_1 for W_2.

B.9 When is which Case optimal?

These three cases hold under different conditions. For example the flexible wage case (0,>0,>0), where $W_1 = (x_1 P_1^{-\epsilon})^{\frac{1}{\beta}}$ & $W_2 = (x_2 P_2^{-\epsilon})^{\frac{1}{\beta}}$ the KKT conditions still specify that $W_2 \geq W_1$. Otherwise, the DNWR constraint is binding and we go to either case 8: (>0,>0,>0), where all three conditions bind, or case 6: (>0,>0,0), where DNWR binds and lifts period two wage above the level needed to satisfy the labor supply constraint $W_2 \geq (x_2 P_2^{-\epsilon})^{\frac{1}{\beta}}$.

But when is DNWR binding? This can be derived by looking at the flexible wage case, where optimal price setting is characterized by

$$P_i = \left[\frac{\epsilon}{\epsilon-1}\frac{1+\beta}{\beta}x_i^{\frac{1}{\beta}}\right]^{\frac{\beta}{\epsilon+\beta}}$$

For the DNWR constraint to be non-binding, we need $W_2 \geq W_1$. Given that the labor

supply constraints are binding this is equivalent to

$$\begin{aligned}
(x_2 P_2^{-\epsilon})^{\frac{1}{\beta}} &\geq (x_1 P_1^{-\epsilon})^{\frac{1}{\beta}} \\
\iff x_2 P_2^{-\epsilon} &\geq x_1 P_1^{-\epsilon} \\
\iff x_2 \left[\frac{\epsilon}{\epsilon-1}\frac{1+\beta}{\beta} x_2^{\frac{1}{\beta}}\right]^{-\frac{\epsilon\beta}{\epsilon+\beta}} &\geq x_1 \left[\frac{\epsilon}{\epsilon-1}\frac{1+\beta}{\beta} x_1^{\frac{1}{\beta}}\right]^{-\frac{\epsilon\beta}{\epsilon+\beta}} \\
\iff x_2^{\frac{\beta}{\epsilon+\beta}} &\geq x_1^{\frac{\beta}{\epsilon+\beta}} \\
\iff x_2 &\geq x_1
\end{aligned}$$

In other words, DNWR is binding if $x_1 > x_2$.

The question of which non-degenerate DNWR case (6 or 8) is binding is trickier. This depends on if the labor supply constraint is going to be binding under optimal price setting or not. We have to start from Case 6, where the labor supply constraint is not binding, as it is binding in Case 8 by assumption. Remember that Case 6 gave us an implicit function for optimal period one price P_1.

$$\frac{\epsilon-1}{\epsilon} P_1^{\frac{\epsilon+\beta}{\beta}} = x_1^{\frac{1}{\beta}} \left(\frac{1+\beta}{\beta} + \frac{1}{\beta}\frac{x_2}{x_1}\left(\frac{\epsilon}{\epsilon-1}\right)^{-\epsilon} x_1^{-\frac{\epsilon}{\beta}} P_1^{\epsilon\frac{\epsilon+\beta}{\beta}}\right)$$

It also gave us optimal P_2

$$P_2 = \frac{\epsilon}{\epsilon-1} W_2$$

and of course period one wage W_1 by constraint.

$$W_1 = (x_1 P_1^{-\epsilon})^{\frac{1}{\beta}}$$

Among the KKT conditions is the specification for the non-binding of period two's labor supply constraint.

$$W_2 \geq (x_2 P_2^{-\epsilon})^{\frac{1}{\beta}}$$

Given the binding of DNWR in Case 6 ($W_2 = W_1$), we can bring these together to gain a condition on P_1.

$$\begin{aligned}
x_1^{\frac{1}{\beta}} P_1^{-\frac{\epsilon}{\beta}} = W_1 = W_2 \geq (x_2 P_2^{-\epsilon})^{\frac{1}{\beta}} = x_2^{\frac{1}{\beta}} \left(\frac{\epsilon}{\epsilon-1} W_2\right)^{-\frac{\epsilon}{\beta}} = x_2^{\frac{1}{\beta}} \left(\frac{\epsilon}{\epsilon-1}\right)^{-\frac{\epsilon}{\beta}} x_1^{-\frac{\epsilon}{\beta^2}} P_1^{\frac{\epsilon^2}{\beta^2}} \\
\iff \left(\frac{x_1}{x_2}\right)^{\frac{1}{\beta}} P_1^{-\frac{\epsilon}{\beta}\left(1+\frac{\epsilon}{\beta}\right)} \geq \left(\frac{\epsilon}{\epsilon-1}\right)^{-\frac{\epsilon}{\beta}} x_1^{-\frac{\epsilon}{\beta^2}} \\
\iff P_1^{\frac{\epsilon}{\beta}\left(\frac{\epsilon+\beta}{\beta}\right)} \leq \left(\frac{x_1}{x_2}\right)^{\frac{1}{\beta}} \left(\frac{\epsilon}{\epsilon-1}\right)^{\frac{\epsilon}{\beta}} x_1^{\frac{\epsilon}{\beta^2}} \\
\iff P_1 \leq \left[\frac{x_1}{x_2}\left(\frac{\epsilon}{\epsilon-1}\right)^{\epsilon} x_1^{\frac{\epsilon}{\beta}}\right]^{\frac{\beta}{\epsilon(\epsilon+\beta)}} \\
\iff P_1 \leq \left[\left(\frac{x_1}{x_2}\right)^{\frac{1}{\epsilon}} \frac{\epsilon}{\epsilon-1} x_1^{\frac{1}{\beta}}\right]^{\frac{\beta}{\epsilon+\beta}}
\end{aligned}$$

I.e. for period two labor supply constraint to remain non-binding in Case 6, the optimal period one price cannot be larger than a level that depends on the relative size of period one and two demand, as well as parameter values ϵ and β. If the optimal period one price were to be higher than this constraint, then Case 6 would never occur under an optimizing price setter. Thus, we need to check under what parameter values this constraint is congruent with the implicit function describing optimal pricing behavior in Case 6.

The implicit function describing optimal period one price setting under Case 6 is

$$\frac{\epsilon-1}{\epsilon}P_1^{\frac{\epsilon+\beta}{\beta}} = x_1^{\frac{1}{\beta}}\left(\frac{1+\beta}{\beta}+\frac{1}{\beta}\frac{x_2}{x_1}\left(\frac{\epsilon}{\epsilon-1}\right)^{-\epsilon}x_1^{-\frac{\epsilon}{\beta}}P_1^{\epsilon\frac{\epsilon+\beta}{\beta}}\right)$$

It is not immediately clear how the upper limit on P_1 is affecting the implicit function. Thus, I re-order the expression to have all P_1 terms on the LHS

$$P_1^{\frac{\epsilon+\beta}{\beta}} - x_1^{\frac{1}{\beta}}\frac{1}{\beta}\frac{x_2}{x_1}\left(\frac{\epsilon}{\epsilon-1}\right)^{1-\epsilon}x_1^{-\frac{\epsilon}{\beta}}P_1^{\epsilon\frac{\epsilon+\beta}{\beta}} = x_1^{\frac{1}{\beta}}\frac{1+\beta}{\beta}\frac{\epsilon}{\epsilon-1}$$

Does the LHS increase or decrease in P_1? To check this I derive the first derivative of the LHS with respect to P_1.

$$\begin{aligned}
&\frac{\epsilon+\beta}{\beta}P_1^{\frac{\epsilon}{\beta}} - x_1^{\frac{1-\beta-\epsilon}{\beta}}\frac{\epsilon}{\beta}x_2\left(\frac{\epsilon}{\epsilon-1}\right)^{1-\epsilon}\frac{\epsilon+\beta}{\beta}P_1^{\epsilon\frac{\epsilon+\beta}{\beta}-1} \overset{?}{>} 0\\
\iff &\frac{\epsilon+\beta}{\beta}P_1^{\frac{\epsilon}{\beta}}\left[1 - x_1^{\frac{1-\beta-\epsilon}{\beta}}\frac{\epsilon}{\beta}x_2\left(\frac{\epsilon}{\epsilon-1}\right)^{1-\epsilon}P_1^{\epsilon\frac{\epsilon+\beta}{\beta}-\frac{\epsilon+\beta}{\beta}}\right] \overset{?}{>} 0\\
\iff &1 \overset{?}{>} x_1^{\frac{1-\beta-\epsilon}{\beta}}\frac{\epsilon}{\beta}x_2\left(\frac{\epsilon}{\epsilon-1}\right)^{1-\epsilon}P_1^{(\epsilon-1)\frac{\epsilon+\beta}{\beta}}
\end{aligned}$$

Evaluating this at the maximal P_1 which is still consistent with the period two labor supply constraint remaining non-binding, and thus remaining in Case 6, gives

$$\begin{aligned}
1 &\overset{?}{>} x_1^{\frac{1-\beta-\epsilon}{\beta}}\frac{\epsilon}{\beta}x_2\left(\frac{\epsilon}{\epsilon-1}\right)^{1-\epsilon}\left[\left(\frac{x_1}{x_2}\right)^{\frac{1}{\epsilon}}\frac{\epsilon}{\epsilon-1}x_1^{\frac{1}{\beta}}\right]^{(\epsilon-1)}\\
\iff 1 &\overset{?}{>} \frac{\epsilon}{\beta}\left(\frac{x_1}{x_2}\right)^{-\frac{1}{\epsilon}}\\
\iff \left(\frac{x_1}{x_2}\right)^{\frac{1}{\epsilon}} &\overset{?}{>} \frac{\epsilon}{\beta}
\end{aligned}$$

This means that the LHS of the reordered implicit function, evaluated at maximal P_1 for Case 6, is increasing in P_1 if the relative demand shock is large enough compared to the relative size of ϵ and β. In the case where the LHS, evaluated at maximal P_1, is increasing in P_1, there exists a solution as long as the LHS under maximal P_1 is greater or equal to the RHS.

$$P_1^{\frac{\epsilon+\beta}{\beta}} - x_1^{\frac{1}{\beta}}\frac{1}{\beta}\frac{x_2}{x_1}\left(\frac{\epsilon}{\epsilon-1}\right)^{1-\epsilon}x_1^{-\frac{\epsilon}{\beta}}P_1^{\epsilon\frac{\epsilon+\beta}{\beta}} \geq x_1^{\frac{1}{\beta}}\frac{1+\beta}{\beta}\frac{\epsilon}{\epsilon-1}$$

If the LHS were less than the RHS, and the LHS increases in P_1 when evaluated at maximal P_1, then there would be no solution to the implicit function that is less than the maximal P_1 still consistent with Case 6. Thus, we would have to leave Case 6 to go to Case 8, where period two's labor supply constraint is binding.

Similarly, if the LHS were decreasing in P_1; i.e. if $\left(\frac{x_1}{x_2}\right)^{\frac{1}{\epsilon}} < \frac{\epsilon}{\beta}$, then the LHS would need to be less than or equal to the RHS for there to be an optimal solution consistent with Case 6.

Thus, we have three sub-cases to Case 6:

$$\left(\frac{x_1}{x_2}\right)^{\frac{1}{\epsilon}} > \frac{\epsilon}{\beta} \tag{6.1}$$

$$\left(\frac{x_1}{x_2}\right)^{\frac{1}{\epsilon}} = \frac{\epsilon}{\beta} \tag{6.2}$$

$$\left(\frac{x_1}{x_2}\right)^{\frac{1}{\epsilon}} < \frac{\epsilon}{\beta} \tag{6.3}$$

I will describe each sub-case in turn.

B.9.1 Case 6.1

In this case the LHS is increasing in P_1 when evaluated at the maximal P_1 consistent with Case 6. Thus, if there is a solution the LHS must be greater or equal to the RHS, when evaluating at the maximal P_1.

$$\begin{aligned}
\left[\left(\frac{x_1}{x_2}\right)^{\frac{1}{\epsilon}} \frac{\epsilon}{\epsilon-1} x_1^{\frac{1}{\beta}}\right] - x_1^{\frac{1}{\beta}} \frac{1}{\beta} \frac{x_2}{x_1} \left(\frac{\epsilon}{\epsilon-1}\right)^{1-\epsilon} x_1^{-\frac{\epsilon}{\beta}} \left[\left(\frac{x_1}{x_2}\right)^{\frac{1}{\epsilon}} \frac{\epsilon}{\epsilon-1} x_1^{\frac{1}{\beta}}\right]^{\epsilon} &\geq x_1^{\frac{1}{\beta}} \frac{1+\beta}{\beta} \frac{\epsilon}{\epsilon-1} \\
\iff \left(\frac{x_1}{x_2}\right)^{\frac{1}{\epsilon}} \frac{\epsilon}{\epsilon-1} x_1^{\frac{1}{\beta}} - x_1^{\frac{1}{\beta}} \frac{1}{\beta} \frac{\epsilon}{\epsilon-1} &\geq x_1^{\frac{1}{\beta}} \frac{1+\beta}{\beta} \frac{\epsilon}{\epsilon-1} \\
\iff \frac{\epsilon}{\epsilon-1} x_1^{\frac{1}{\beta}} \left[\left(\frac{x_1}{x_2}\right)^{\frac{1}{\epsilon}} - \frac{1}{\beta}\right] &\geq x_1^{\frac{1}{\beta}} \frac{1+\beta}{\beta} \frac{\epsilon}{\epsilon-1} \\
\iff \left(\frac{x_1}{x_2}\right)^{\frac{1}{\epsilon}} - \frac{1}{\beta} &\geq \frac{1+\beta}{\beta} \\
\iff \left(\frac{x_1}{x_2}\right)^{\frac{1}{\epsilon}} &\geq \frac{1+\beta}{\beta} + \frac{1}{\beta} \\
\iff \left(\frac{x_1}{x_2}\right)^{\frac{1}{\epsilon}} &\geq \frac{2+\beta}{\beta}
\end{aligned}$$

Thus, for an optimal Case 6.1 solution to exist we need $\left(\frac{x_1}{x_2}\right)^{\frac{1}{\epsilon}} > \frac{\epsilon}{\beta}$ and $\left(\frac{x_1}{x_2}\right)^{\frac{1}{\epsilon}} \geq \frac{2+\beta}{\beta}$.

If β is very large, and wages are not very responsive to labor demand then we are in Case 6.1.

B.9.2 Case 6.2

In this case the LHS of the reordered implicit function is not changing in the level of P_1 and the case only has an optimal solution if the maximal Case 6 P_1 is the solution to the implicit function. Following the same math as above, this is the case if $\left(\frac{x_1}{x_2}\right)^{\frac{1}{\epsilon}} = \frac{\epsilon}{\beta}$ and $\left(\frac{x_1}{x_2}\right)^{\frac{1}{\epsilon}} = \frac{2+\beta}{\beta}$. The derivation is similar as in Case 6.1.

This case is very narrow, as $\epsilon = 2 + \beta$ is necessary.

B.9.3 Case 6.3

In case 6.3 the LHS of the reordered implicit function falls in the level of P_1. Thus, a solution only exists if the LHS is less than or equal to the RHS. Similar math as in Case 6.1 leads to $\left(\frac{x_1}{x_2}\right)^{\frac{1}{\epsilon}} \leq \frac{2+\beta}{\beta}$, combined with the original Case 6.3 condition $\left(\frac{x_1}{x_2}\right)^{\frac{1}{\epsilon}} < \frac{\epsilon}{\beta}$.

If β is rather small, and wages are responsive to labor demand then we are in Case 6.3.

B.10 Kudlyak's User Cost of Labor

This Appendix shows how Kudlyak (2014)'s full User Cost of Labor can be achieved in the model. For its derivation we have to start at the FOC with respect to employment again:

$$0 = -\frac{w_{i,t}}{P_t} - \mu_{i,t} + \lambda_{i,t} A f'(n_{i,t}) - \sum_{k=1}^{\tau-1} \Lambda_{t,t+k}(1-\bar{\delta})^k E_t \left[\frac{w_{i,t+k}}{P_{t+k}} - \lambda_{t+k} f'((1-\bar{\delta})^k n_{i,t}) A_{t+k} \right]$$
$$+ \sum_{k=1}^{\tau} E_t \left[\Lambda_{t,t+k}(1-\bar{\delta})^{k-1}(1-\tilde{\delta}_{t+k})\mu_{i,t+k} \right]$$

I omitted the $-\lambda_{t+k} f'((1-\bar{\delta})^k n_{i,t}) A_{t+k}$ previously as it is zero under the assumption of equal wages across existing and new employees in the DNWR case. To see this check the FOC of the firm's problem under DNWR with respect to $n_{i,t}$. As employment is described by $(1-\bar{\delta})n_{i,t-1}$ under DNWR, the wage is not entering λ_t. Similarly, $\mu_t = 0$ under the binding DNWR constraint. Thus, $\lambda_t = 0$ under DNWR.

However, if the firm can hire at a different wage than the wage for existing employees, i.e. hire a secondary workforce at a flexible wage $w_{i,t}^{flex}$ then

$$\lambda_{i,t} = \frac{\frac{w_{i,t}^{flex}}{P_t} + \mu_{i,t}}{A_{i,t} f'(n_{i,t})}$$

or in aggregate:

$$\lambda_t = \frac{\frac{W_t^{flex}}{P_t} + \mu_t}{A_t f'(N_t)}$$

Plugging this in for the λ_{t+k}s in the FOC above yields:

$$0 = -\frac{w_{i,t}}{P_t} - \mu_{i,t} + \lambda_{i,t} A f'(n_{i,t}) - \sum_{k=1}^{\tau-1} \Lambda_{t,t+k}(1-\bar{\delta})^k E_t \left[\frac{w_{i,t+k}}{P_{t+k}} - \frac{\frac{W_{t+k}^{flex}}{P_{t+k}} + \mu_{t+k}}{A_{t+k} f'(N_{t+k})} f'((1-\bar{\delta})^k n_{i,t}) A_{t+k} \right]$$
$$+ \sum_{k=1}^{\tau} E_t \left[\Lambda_{t,t+k}(1-\bar{\delta})^{k-1}(1-\tilde{\delta}_{t+k})\mu_{i,t+k} \right]$$

Using symmetry across firms, note that $N_{t+k} = (1-\bar{\delta})^k) N_t$ and that $\mu_{t+k} = 0$ in the periods where DNWR is expected to bind. This finally gives

$$\lambda_t = \frac{\frac{W_t}{P_t} + \sum_{k=1}^{\tau-1} \Lambda_{t,t+k}(1-\bar{\delta})^k E_t \left[\frac{W_t}{P_{t+k}} - \frac{W_{t+k}^{flex}}{P_{t+k}} \right] + \mu_t - E_t \left[\Lambda_{t,t+\tau}(1-\bar{\delta})^{\tau-1}(1-\tilde{\delta}_{t+\tau})\mu_{t+\tau} \right]}{A f'(N_t)}$$

which gives Kudlyak (2014)'s User Cost of Labor as marginal cost.

B.11 Downward Bias from the Inclusion of Short-Term Expectations

It is well documented (e.g. Weber et al. (2022)) that inflation causes inflation expectations to rise. This is particularly true for short-term expectations, while long-term expectations stay more anchored. Furthermore, short-term inflation expectations are strongly driven by slack variables, such as labor market tightness. Hence, the structural equation of expected inflation is described as follows:

$$\pi^e_{ST} = \gamma\pi + \phi(V/U) + \psi\pi^e_{LT} + \epsilon_1$$

If the relation of interest is between labor market tightness (or any other slack variable) and inflation, while the latter is also influenced by long-term inflation expectations:

$$\pi = \alpha(V/U) + \beta\pi^e_{LT} + \epsilon_2$$

then the inclusion of short-term instead of long-term expectations leads to an endogeneity bias in $\hat{\alpha}$, for two reasons. The first is mentioned but not further explained by Gagnon and Collins (2019). It is an endogeneity bias due to the impact of inflation on short-term inflation expectations. The second bias stems from the fact that labor market tightness also drives π^e. This leads to a second endogeneity bias, as I will show below. If one naively estimates the structural inflation equation, while using short-term instead of long-term expectations:

$$\pi = \hat{\alpha}(V/U) + \hat{\beta}\pi^e_{ST} + \hat{\epsilon}$$

then $\hat{\alpha}$ is biased due to the endogeneity of π^e_{ST} to both π and V/U. I.e. plugging in the structural equation of π^e_{ST} gives:

$$\pi = \hat{\alpha}(V/U) + \hat{\beta}[\gamma\pi + \phi(V/U) + \psi\pi^e_{LT} + \epsilon_1] + \hat{\epsilon}$$

or

$$(1 - \hat{\beta}\gamma)\pi = \hat{\alpha}(V/U) + \hat{\beta}\phi(V/U) + \hat{\beta}\psi\pi^e_{LT} + \hat{\beta}\epsilon_1 + \hat{\epsilon}$$

Inserting the true relation between inflation, labor market tightness and expected inflation on the lhs yields

$$(1 - \hat{\beta}\gamma)[\alpha(V/U) + \beta\pi^e + \epsilon_2] = \hat{\alpha}(V/U) + \hat{\beta}\phi(V/U) + \hat{\beta}\psi\pi^e_{LT} + \hat{\beta}\epsilon_1 + \hat{\epsilon}$$

Hence the estimated labor market tightness coefficient $\hat{\alpha}$ has the following form:

$$\hat{\alpha} = (1 - \hat{\beta}\gamma)\alpha - \hat{\beta}\phi$$

I.e. the estimated impact is biased downwards by two endogeneity biases. The first one, stemming from the endogeneity of inflation expectations to actual inflation, yields a downward bias in proportion to the correlation between inflation and inflation expectations. The second endogeneity bias, derived from the impact of labor market tightness on expectations, gives a downward bias that is one to one with the impact of labor market tightness on inflation expectations, and its impact on inflation.

As short-term expectations are much more responsive to movements in inflation than long-term expectations during inflation surges, their endogeneity bias on the labor market coefficient is more sizeable. Further, the impact of slack variables, such as V/U, is much greater on short-term expectations than on long-term ones. Hence, it is advisable to use long-term expectations.

Furlanetto and Lepetit (2024) make a similar point emphasizing the importance of the auto-correlation of the forcing variable (V/U here). If the variable has a high auto-correlation then a shock to it will influence inflation expectations into the near future. If the desired outcome is to measure the total impact of a shock to the forcing variable then controlling for short-term inflation expectations is ill-advised, as these expectations will control for all the future inflationary pressure of the forcing variable. This explains why Beaudry et al. (2024) or Del Negro et al. (2020) find only a very flat Phillips Curve, as they control for one year inflation expectations.

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