Corporate Finance Through Loyalty Programs*

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Abstract

Loyalty programs (LPs) are widely prevalent in practice and typically analyzed in economic research for their role in boosting business. This paper uncovers a novel role of LPs as financing instruments. The rewards issued to and redeemed by consumers cause shifts in firms' present and future cash flows, effectively enabling firms to borrow from their consumers. We document three stylized facts about LPs in the airline and hotel industries: 1) LPs serve as significant financing sources, with cobranded credit card programs contributing substantially and sales directly to consumers minimally; 2) rewards are issued through broad consumption but are redeemed predominantly for consumption related to the issuing firm; 3) LP financing helps smooth cash flows. We then build a dynamic model of LP financing. The model features convenient rewards that do not impose restrictions on redemption. Due to convenience, consumers would demand a lower discount rate, which creates gains from trades. But the funds raised through LPs emerge endogenously in equilibrium as a result of the interplay between reward issuance and redemption. The model suggests that 1) firms supplying high-value, low-frequency services can leverage LPs more effectively for financing; 2) LP financing helps smooth cash flows and is thus attractive to firms with highly volatile business; 3) firms should aim to decouple reward issuance from their business; 4) firms should limit consumers' discretion to purchase or transfer rewards.

Keywords: loyalty programs, corporate finance, rewards, tokens, cobranded credit cards, financial resilience

JEL codes: G32, D21, D22, G51, L83, L93

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1 Introduction

Loyalty programs (LPs) are widely prevalent in practice. According to 2017 COLLOQUY Loyalty Census, the total number of loyalty program memberships in the U.S. reached 3.8 billion. The distribution of these memberships spans various industries with 42% in retail, 29% in travel and hospitality, and 17% in financial services. Certain sectors have even embraced loyalty programs as a standard business practice. Indeed, almost all major airlines, hotel chains, banks, and top retailers offer some form of loyalty programs (Sun and Zhang, 2019). A common and crucial feature of LPs is their provision of rewards to consumers. These rewards, usually presented as points, complimentary nights, or discounts on subsequent purchases, represent promises for future services.

Economists have offered several theories to rationalize the emergence of LPs that offer rewards. Rewards may provide psychological benefits or engender price misperceptions. They might also enable price discrimination based on quantity or purchasing history. In a market with multiple competing brands, rewards can heighten consumers' switching costs, thereby weakening competition. More recent research suggests that rewards can be used to capitalize on agency conflicts between employers and employees. Essentially, all these theories revolve around the central theme of boosting income.

The finance behind LPs has long been ignored by the academia. In general, when a firm issues rewards to consumers, it usually receives advance payments from consumers or some other benefits, boosting the firm's present cash flows. These rewards are recorded as the firm's liabilities, as it has yet to provide the promised services. Later, consumers will redeem these rewards for the firm's services at discounted prices or free of charge, reducing the firm's future cash flows. These shifts in the firm's present and future cash flows constitute de facto borrowings from consumers. Recently, the finance behind LPs has gained increasing attention. U.S. Department of Transportation wrote in its inquiry letter to airlines,

> "Airline rewards programs have become a meaningful part of the U.S. economy and a major part of the airline business model, by some measures developing a financial significance that rivals or even outweighs the operation of flights... and many Americans reportedly have point balances substantial enough that they amount to part of their savings."

Some observers contend that airlines function similarly to banks, or even central banks, by issuing their own forms of currency to consumers.[1]

[1] See for example,
https://www.theatlantic.com/ideas/archive/2023/09/airlines-banks-mileage-programs/675374/,
https://fepfinanceclub.org/2024/04/30/article-airlines-are-actually-banks/,
https://www.youtube.com/watch?v=ggUduBmvQ_4.

In this paper, we aim to uncover the finance behind LPs both empirically and theoretically. We first document three stylized facts about LPs in the airline and hotel industries. Our focus on these two sectors is due to the central role their LPs play in business operations and their significant influence compared to other industries. The information presented here is mainly sourced from firms' 10-K reports and the 8-K reports of the three major airlines, Unite Airlines, Delta Air Lines, and American Airlines.[2]

First, LPs serve as significant financing sources, with cobranded credit card (CCC) programs contributing substantially and sales directly to consumers minimally. Prior to the Covid-19 pandemic, the total LP liabilities of the two industries reached 40 billion dollars, amounting to 30% of their respective debt and lease obligations. Whether intentionally or not, LPs have become de facto significant financing sources for both industries. Further, airlines and hotels issue a large fraction of rewards through their CCC programs. To incentivize consumers to sign up for and use CCCs, bank partners purchase rewards from airlines and hotels and offer them to consumers. For instance, in 2019, approximately 70% of the cash flows of Delta's LP, SkyMiles, came from its bank partner, American Express. Conversely, sales directly to consumers accounted for only a tiny fraction of the cash flows, largely because airlines and hotels sell rewards at prices considerably higher than their actual value.

Second, rewards are issued through broad consumption, but are redeemed predominantly for purchases related to the issuing firm. American households extensively use CCCs for a broad spectrum of consumption, much of which bears no direct relation to the issuing firm. Moreover, firms strategically allow consumers to earn rewards on purchases made with their partners, including those in unrelated sectors such as retail and dining. When it comes to the redemption of these rewards, the majority are used for purchases made with the issuing firm, thereby ensuring that the rewards are ultimately directed back towards the issuing firm.

Third, LP financing can help smooth cash flows. Reward redemption is closely tied to the issuing firm's business while reward issuance is linked to broad consumption. For firms with volatile business, such as those in the travel industry, reward issuance tends to be less volatile than reward redemption. This pattern was particularly evident for airlines and hotel chains during the Covid-19 pandemic. As a result, firms' LP liabilities grow rapidly in the bad times of their business. This cyclical nature of LP financing naturally enables firms to borrow more from consumers in bad times, thereby smoothing overall cash flows.

Our findings indicate that airlines and hotels systematically issue rewards through broad consumption in addition to the purchases made with them, but not through sales directly to consumers. This issuance pattern is not suggested by classical theories of LPs. However, we demonstrate that

[2]The three airlines raised money in senior secured financing collateralized by their LPs during the Covid-19 pandemic. They made additional disclosure about their LPs.

it can be well explained by the finance theory of LPs.

Building on these stylized facts, we develop a model of LP financing. Similar to other forms of financing, LP financing hinges on the assumption that consumers discount the value of rewards at lower rates than the firm. While the model offers some rationale for why this might be the case, we acknowledge that it remains largely an empirical question that cannot be fully addressed within the model. Instead of answering why LP financing can arise, the model assumes its existence and illustrates its features. The power of the model lies in its ability to generate predictions that align closely with real-world observations, particularly those that classical theories of LPs cannot explain.

The model is formulated as an infinite-horizon continuous-time game with consumers, a single firm, and banks. All consumers are homogeneous, with an exogenous demand for a service that the firm monopolizes. This setting naturally eliminates classical motives for LPs such as price discrimination and competition. The firm has the option to finance its operations through debt and a LP. By adopting an LP, the firm issues rewards to consumers and receives advance payments, which are referred to as LP liabilities. For consumers, the rewards are convenient in that there are no restrictions on the timing of their redemption—a consumer can purchase a service anytime he wishes, using any combination of cash and rewards they can afford. Due to the convenience of rewards, consumers discount their value at a lower interest rate than they would for an investment. In practice, this differnential could be further amplified by the "coarse thinking" of consumers, who perceive rewards as analogous to demand deposits and, as a result, do not require a substantial interest rate (Mullainathan, Schwartzstein, and Shleifer, 2008). The differential creates gains from trade and gives rise to the financing role of LPs.[3] However, rewards are less liquid than cash, as they can only be redeemed for the service. Hence, consumers weakly prefer holding cash to holding rewards. Additionally, redeeming rewards is more cumbersome than using cash and incurs a cost each time a consumer redeems. For simplicity, we assume that rewards have a constant valuation.

We consider two types of LPs motivated by Fact 1: consumption-based LPs (CLPs) and purchase-based LPs (PLPs). In adopting a CLP , the firm collaborates with a partner bank to issue a co-branded credit card (CCC). Since banks act competitively, it is essentially the firm that issues rewards to consumers for their consumption charged to the CCC and receives the associated interchange fees. In adopting a PLP, the firm issues rewards to consumers based on their purchases made directly with the firm. LPs are static in the sense that, once adopted, their design remains unchanged over time.

[3]Some other factors may also affect consumers' discount rates. The model does not take a particular stance on how consumers' discount rates are determined, aside from acknowledging that convenience plays an important role. Please see Section 3.5 for more discussion.

One might wonder how LP financing differs from other forms of financing. In general, effective financing should allow a firm to raise sufficient funds at low costs. Typical debt financing stipulates a fixed repayment schedule, which does not accommodate investors' liquidity demands. Consequently, debt financing is associated with low convenience yields or high required returns. Meanwhile, with the fixed repayment schedule, the firm can directly determine the raised funds. Conversely, LP financing, similar to demand deposits, allow financiers to control the redemption. This convenience results in high convenience yields and low required returns. However, what is also inherent in this convenience is that the firm loses direct control over the raised funds. They emerge endogenously as a result of the interplay between issuance and redemption. Issuance is determined by the firm, and redemption is determined by the consumption of the particular service supplied by the firm. Then two questions naturally arise: how should firms issue rewards, and how does the nature of the service affect financing? The model primarily addresses the two questions. However, these considerations are less relevant to demand deposits, as issuance and redemption are exogenously determined by consumers' total income and total consumption.

In the baseline setup, the game is stationary, and we focus on the steady-state equilibrium paths, in which each consumer behaves optimally and the aggregate behavior of all consumers remains unchanged. The model solution starts with the characterization of consumers' optimal behavior. Due to the convenience of rewards, consumers control the redemption process. Specifically, when a consumer needs a service, hewill purchase it using rewards alone if his reward holdings are sufficient; otherwise, he will use cash alone. Despite having the option, a consumer never opts for partial redemption for a service, as full redemption minimizes redemption costs. Additionally, a consumer does not desire to stockpile rewards, as cash is generally preferred over rewards.

The funds raised by the firm are equal to the product of the reward balance of all consumers and the unit price of rewards. We then explicitly determine the reward balance when either type of LPs is adopted. The reward balance consists of two parts. The first part stems from that a consumer never redeems rewards before accumulating a sufficient amount for a service, so it equals half the value of a service. However, a consumer may also overaccumulate—hold rewards more than sufficient for a service, which constitutes the second part of the reward balance. Under a CLP, overaccumulation arises due to the asynchrony between reward issuance and reward redemption: a consumer consistently earns rewards through the CCC, even during periods when he does not need a service. In contrast, such asynchrony is absent under a PLP, as both issuance and redemption are directly linked to the purchases of services and mutually exclusive. Instead, overaccumulation arises under a PLP due to the lumpiness of reward earning. Each time a consumer purchases a service with cash, he receives a chunk of rewards equivalent to a fraction of the service's value.

The analysis of the baseline setup generates three novel insights into LP financing. First, firms supplying high-value, low-frequency services can leverage LPs more effectively for financing, as

these firms can accumulate a high reward balance under both types of LPs. This model prediction aligns well with empirical observations across various industries. In particular, it elucidates why airlines and hotels are the most proactive in developing LPs and expanding the issuance of rewards.

Second, to leverage LPs more effectively for financing, a firm should weaken the linkage of reward issuance to the firm's business. We obtain this implication by comparing CLPs and PLPs. We analytically show that the asynchrony mechanism under a CLP is more powerful in generating overaccumulation than the lumpiness mechanism under a PLP. We also numerically analyze the case where both types are adopted and find that the reward balance increases as the CLP gains more importance relative to the PLP. This implication resonates with Fact 2 that rewards are issued through broad consumption, including that unrelated to firms' business.

Third, firms should not allow consumers to freely purchase rewards from the firm or other consumers. The driving force of the result is consumers' preference for holding cash over holding rewards. If consumers are given flexible control over reward issuance or reward holdings, they will attempt to accelerate the redemption of rewards and minimize their reward holdings, as long as their overall payoffs remain the same. This result is consistent with Fact 1 that firms do not intend to issue rewards through direct sales, and observations that they discourage transactions of rewards by imposing high fees or restricting the scopes and frequencies of transactions.

Lastly, we examine the cyclical nature of LPs and their impact on a firm's financial resilience. We consider a scenario where the economy unexpectedly enters an adverse state, leading to a significant decline in consumer demand for the firm's services compared to the normal state. This decline results in negative cash flows for the firm, forcing it to utilize its liquidity buffers to remain operational. If the adverse state persists for an extended period, the firm will deplete its liquidity buffers and face bankruptcy. We investigate how the firm's bankruptcy risk varies with its choice of LP. We find that both types of LPs help stabilize the firm's cash flows, thereby enhancing its financial resilience. This cyclical nature of LP financing is especially appealing to firms with highly volatile business such as airlines, hotels, and durable goods retailers.

A common mechanism that stabilizies cash flows under both types of LPs is the lower financing cost of LPs in bad times. LP financing is costly as consumers use rewards, which are issued at a discount, to purchase services instead of cash. Thus, an LP effectively operates as a revenue-sharing arrangement where a fixed fraction of revenue is shared with consumers in the form of reward discounts. Consequently, when revenue decreases, the shared revenue follows suit. Additionally, a CLP also helps the firm raise more funds from consumers in the adverse state. The key is that when reward issuance is linked to broad consumption, it becomes less volatile than reward redemption. This mechanism is clearly revealed by Fact 3. A general implication of this mechanism is that to strengthen the desirable cyclical nature of a LP, the firm should weaken the correlation of reward issuance to its business, which resonates with Fact 2.

The rest of this paper is organized as follows. In the rest of the introduction, we review the related literature. Section 2 documents three stylized facts about LPs in the airline and the hotel industries. Section 3 outlines the baseline setup of the model. Section 4 derives the equilibrium path in the steady states of the baseline setup. Section 5 derives the implications about LP financing. Section 6 examines the cyclical nature of LPs and its impact on the firm's financial resilience. Section 7 concludes. All proofs that are not provided in the main text are relegated to the Appendix.

Related literature

Our paper proposes a novel finance motive for LPs, thereby contributing to the extensive literature that seeks to rationalize LPs. Behavioral theories posit that LPs can yield psychological benefits or create pricing misconceptions. For instance, Lim, Chun, and Satopää (2021) discovers that certain consumers attribute more value to LP points than to actual money. In the literature of industrial organization, a traditional viewpoint is that LPs implement price discrimination based on purchase history or quantity. Cremer (1984) demonstrates that rewards can be used to differentiate between first-time and repeat buyers. Sun and Zhang (2019) postulate that the use of limited reward expiration terms could be driven by the intent to discriminate between frequent and infrequent customers. In a multi-brand environment, LPs can increase consumer switching costs, thereby weakening competition (Klemperer, 1987, 1995). Particularly, Banerjee and Summers (1987) and Caminal and Matutes (1990) are the first to analyze rewards as a form of endogenous switching cost. Kim, Shi, and Srinivasan (2001) shows that by offering the incentives for repeat purchases, LPs increase a firm's cost to attract competing firms' current customers. Ke, Shin, and Zhu (2024) analyzes LPs from the perspective of product search. In their framework, switching costs originate from search costs, and firms use LPs to make consumers prioritize their products when doing sequential search. However, whether switching costs have real impacts in practice is ambiguous (Hartmann and Viard, 2008; Orhun, Guo, and Hagemann, 2022; Rossi and Chintagunt, 2023). Basso, Clements, and Ross (2009) argue that LPs, such as airlines' frequent-flier programs, can exploit agency conflicts between employers and employees. Chod and Lyandres (2023) argues that pricing output in tokens provides a firm with a de facto second-mover advantage. Several recent studies explore firms' optimal decisions in the context of LPs (Kim, Shi, and Srinivasan, 2004; Chun and Ovchinnikov, 2019; Chun, Iancu, and Trichakis, 2020). Taken together, the existing literature predominantly interprets LPs as instruments to boost business. Our paper documents a novel fact: rewards are issued through broad consumption, with a large fraction not directly related to issuing firms, and are not issued systematically through direct sales. This issuance pattern does not align well with classical theories on LPs, but it is consistent with the financing role of LPs.[4]

[4]Please see Section 2.5 for more detailed discussion on why the existing theories are not able to explain the documented fact well.

To our knowledge, our paper is the first to empirically document the financing role of LPs and to theoretically scrutinize its nature.

Our paper documents and illustrates a new form of financing from consumers through LPs. It is particularly related to the literature on raising funds from stakeholders. Trade credit, for instance, raises funds from suppliers (Biais and Gollier 1997; Burkart and Ellingsen 2004; Chod, Lyandres, and Yang 2019; Lehar, Song, and Yuan 2020). The literature argues that suppliers are better able to mitigate the financial frictions faced by external investors, such as asymmetric information and lack of commitment. Inside debt or equity raises funds from executives and could be a superior solution to the agency costs of debt than other forms of executive compensation (Sundaram and Yermack 2007; Edmans and Liu 2011). Crowdfunding and token financing can be used to raise funds from consumers or users. Strausz (2017) considers the benefit of crowdfunding as a way of acquiring information about the eventual payoff of the project if demand is uncertain (see also Astebro, Fernnndez Sierra, Lovo, and Vulkan 2017; Chemla and Tinn 2020; Ellman and Hurkens 2019; Cong and Xiao 2024). Lee and Parlour (2022) find that raising fund from consumers rather than investors can be more efficient, because consumers are able to commit to paying for the consumption benefit ignored by investors. Howell, Niessner, and Yermack (2020) and Lyandres, Palazzo, and Rabetti (2022) empirically examine the factors that determine successful initial coin offerings (ICOs). The theoretical literature on tokens centers the discussion around the network effect, decentralization, and the tradability of tokens in the setting of platforms (Chod and Lyandres 2021; Cong, Li, and Wang 2021; Gryglewicz, Mayer, and Morellec 2021; Cong, Li, and Wang 2022; Rogoff and You 2023; Sockin and Xiong 2023b,a; Li and Mann 2024; Goldstein, Gupta, and Sverchkov 2024; He, Rogoff, and You 2024).

This paper distinguishes itself by characterizing LP financing as a method for firms to raise funds from consumers while simultaneously catering for their liquidity demands. This feature is reminiscent of demand deposits issued by banks. While the existing banking literature predominantly emphasizes the run risks associated with liquidity provision (as highlighted by Diamond and Dybvig, 1983), our theory is built upon a overlooked consequence: liquidity provision drestricts the firm's ability to fully control the amount of raised funds. Moreover, the characteristics of the firm's business and the issuance strategy become crucial to the effectiveness of LP financing. Based on this mechanism, the paper derives several implications regarding the types of firms that are likely to utilize LP financing and the optimal strategies for doing so. These implications are consistent with the observations in practice.

2 Three Facts about Loyalty Programs in the Airline and Hotel Industries

In this section, we present three stylized facts that illustrate the importance, mechanics, and features of loyalty programs as financing instruments in the travel and hospitality sector. We choose to focus on the airline industry and the hotel industry for two reasons. First, loyalty programs are prevalent in the travel and hospitality sector, which account for 29% of all loyalty memberships in the United States.[5] Second, loyalty programs are considered to be an important part of the business in this sector. All major airlines and hotel chains offer some form of loyalty programs, and disclose related information in more details than other sectors.

2.1 Data

Public firms with large loyalty programs disclose loyalty program liabilities in their annual reports, which represent what firms owe to loyalty program members. Different firms may report them using different terms such as loyalty program deferred revenue, contract liability, and gift card balance. Historically, there are two methods to calculate loyalty program liabilities. The first approach, known as the incremental cost method, recognizes a liability for the marginal costs incurred in providing services to eligible customers. On the other hand, the second approach, referred to as the deferred revenue method, recognizes a liability for advance payments received in anticipation of future service delivery. Under the deferred revenue method, loyalty program liabilities represent the amount of cash flows that the firm will forgo to fulfill its promises related to the advance payments and are close to the opportunity cost in economics. Since 2018, IFRS 15 and ASC 606 made it obligatory for companies to adopt the deferred revenue method. In our analysis, loyalty program liabilities refer to the numbers estimated using the deferred revenue method.

Estimating loyalty program liabilities using the deferred revenue method is not simple because it involves the estimation of consumers' redemption behavior. For example, airlines' loyalty program liabilities mainly take the form of miles. To determine the opportunity cost induced by promises related to miles, airlines estimate the equivalent selling price of miles and the amount of miles that will be redeemed. United Airlines describe its method as follows:

> "The Company's estimated selling price of miles is based on an equivalent ticket value, which incorporates the expected redemption of miles, as the best estimate of selling

[5] According to the 2017 COLLOQUY loyalty census, loyalty programs are most prevalent in the retail sector, the travel and hospitality sector, and the financial services sector. They account for 42%, 29%, and 17% of all loyalty memberships in the United States, respectively.

price for these miles. The equivalent ticket value is based on the prior 12 months' weighted average equivalent ticket value of similar fares as those used to settle award redemptions while taking into consideration such factors as redemption pattern, cabin class, loyalty status and geographic region."

"The Company's breakage model is based on the assumption that the likelihood that an account will redeem its miles can be estimated based on a consideration of the account's historical behavior. The Company uses a logit regression model to estimate the probability that an account will redeem its current miles balance. The Company reviews its breakage estimates annually based upon the latest available information. "

We collect information on the stock and the flow of loyalty program liabilities, debt, and revenue from 10-K reports. Regarding debt, we include not only long-term debt and its current maturities, but also financial leases and operating leases.[6] In our within-industry analysis, we use revenue to control for the size of firms' business. To obtain a consistent measure within industries, we use total revenue for airlines and total revenue net of reimbursement costs for hotels.[7] Since firms all adopt the deferred revenue method in 2018, we focus on the sample period spanning from 2018 to 2022. During the Covid-19 pandemic, both the airline industry and the hotel industry experienced a severe downturn, and as a response, their capital structures changed massively. To provide a picture of loyalty programs in normal times, we focus on the year 2019 when conducting cross-sectional analysis.[8] Unless otherwise specified, the scaled version of a variable refers to the ratio of the variable to revenue in 2019.

Information about the operation of airlines' loyalty programs is collected from the 8-K reports of United Airlines, Delta Airlines, and American Airlines in 2020 and 2021. Severely hit by the Covid-19 pandemic, the three airlines raised money in senior secured financing collateralized by their LPs, so they made additional disclosure of LPs.

[6]Disclosure of leases is required by IFRS 16. For more details, please see https://www.pwc.com/gx/en/services/audit-assurance/assets/ifrs-16-new-leases.pdf.

[7]Hotels do not own all the properties they manage. For those purely managed properties, hotels typically cover some operating expenses such as employees' payroll and later receive full reimbursement from property owners. As a result, two reimbursement terms appear in hotels' revenue and expense, respectively, and they are almost equal. Hence, the reimbursement practice affects only hotels' revenue but not their profits and cash flows. Note that the size of the reimbursement depends on the fraction of purely managed properties and the agreement between hotels and property owners, which are heterogeneous across hotels. We think that total revenue net of reimbursement costs is a better measure of hotels' true economic revenue.

[8]Due to IFRS 16, information on leases has been universally available since 2019.

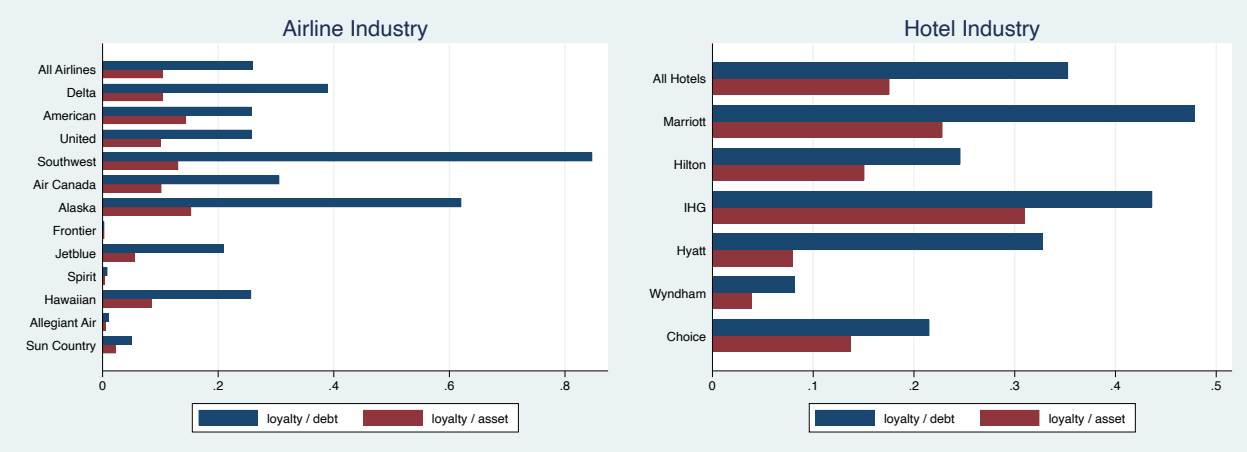


Figure 1: Loyalty program liabilities relative to total debt and total assets

2.2 Fact 1: LPs serve as significant financing sources, with CCC programs contributing substantially and sales directly to consumers minimally.

In 2019, the LP liabilities of the airline industry were approximately $30 billion, while those of the hotel industry were approximately $10 billion. To put the numbers in perspective, we calculate the ratios of LP liabilities to total debt and total assets, which are shown in Figure 1. On average, LP liabilities amounted to approximately 30% of total debt and around 15% of total assets of the two industries. Here, total debt includes debt, operating leases, and finance leases. Whether intentionally or not, LPs are de facto important sources of financing for the two industries.

Given the large size of LP liabilities, one might wonder how firms accumulate them. Figure 2 illustrates the operation of Delta's LP, SkyMiles, in 2019.[9] Every time a customer flies with Delta and chooses to earn Delta miles, Delta purchases miles from SkyMiles and grants to him. This part accounts for 32% of SkyMiles' cash flows. The other 68% of the cash flows are mostly due to sales to Amex, Delta's partner bank. Amex issues a collection of cobranded credit cards with Delta and grants the miles to cardholders when they sign up for or spend with the cards.

Delta provided a more detailed breakdown over time. As shown in the lower half of Figure 2, sales to Amex have been the largest source of SkyMiles' cash flows and increasingly important in recent years, followed by sales to Delta. United and American Airlines are in similar situations. In 2019, sales to third parties accounted for 71% of the loyalty program's cash flows for United and 65% for American. Such breakdown of cash flows is not available for other firms. Nevertheless, we do observe a significantly positive relationship between firms' LP liabilities and the sizes of their CCC programs.[10]

[9] During the COVID-19 pandemic, United, Delta, and American Airlines borrowed against the cash flows of their LPs and disclosed information on the inner workings of LPs.

[10] We use the variety of a firm's CCCs—the number of the types of a firm's CCCs—as a proxy for the sizes for the

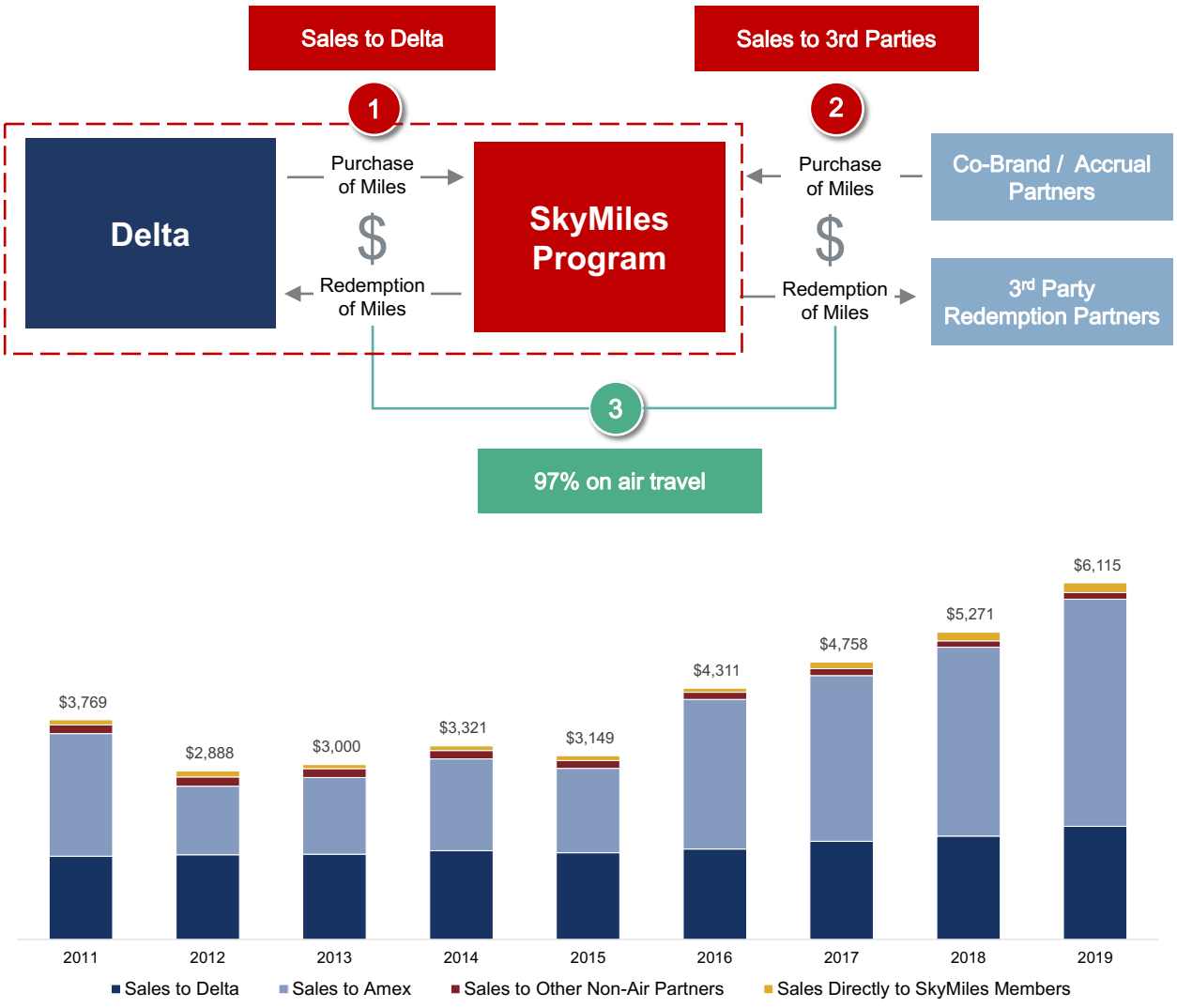


Figure 2: Delta's loyalty program

Another noteworthy observation from Figure 2 is that sales directly to consumers are tiny and do not constitute a way to systematically issue rewards to consumers for Delta. This observation is likely true for other firms in the airline and hotel sector as well. Airlines and hotels do sell rewards directly to consumers, but they typically sell rewards at a premium rather than a discount compared with the actual values of rewards to consumers. Table 1 shows the best selling prices and valuations of rewards of major airlines and hotels as of 2024 November. The best selling prices

CCC program, as we do not observe the number of cardholders or card spend. The rationale is that when a firm's CCC program is larger and has more cardholders, the firm and its partner bank will develop a richer product line to better attract and serve customers. Figure 8 shows that there is a clear positive relationship between the LP liabilities scaled by revenue and the variety of CCCs for both industries. Table 4 confirm that this positive relationship is statistically significant at 5% level.

are collected from the firms' official website,[11] and the valuations are calculated by The Points Guy (TPG).[12] It is widely acknowledged among experienced travelers that directly buying rewards is rarely a good idea for consumers, unless the firm is offering special promotions or a consumer needs only a small amount of additional rewards to reach for a good redemption deal.[13]

Table 1: The selling prices and valuations of rewards as of 2024 November

Airlines	Best selling price (¢)	TPG valuations (¢)	Hotels	Best selling price (¢)	TPG valuations (¢)
Delta	3.5	1.2	Marriott	0.93	0.85
American	2.45	1.6	Hilton	0.55	0.6
United	2	1.35	IHG	1.2	0.5
Southwest	3	1.4	Hyatt	1.92	1.7
Air Canada	2.45	1.5	Wyndham	1.3	1.1
Alaska	1.65	1.4	Choice	0.64	0.6
Frontier	2.5	1.1			
Jetblue	1.43	1.3			
Spirit	1.61	1.1			
Hawaiian	2.5	1.2			

2.3 Fact 2: rewards are issued through broad consumption but are redeemed predominantly for consumption related to the firm

CCCs usually offer the highest rewards for purchases directly made with the issuing firm, so we naturally expect that cardholders will use CCCs a lot for such purchases. In fact, consumers also use them extensively for purchases with other firms. Figure 3 shows the annual transaction volume of Delta's CCCs. The numbers are disclosed by Amex in its 10-K reports because Delta is its most important partner. It was 101 billion dollars and accounted for 12% of Amex US transaction volume in 2019; it later increased to 155 billion dollars and accounted for 14% of Amex US transaction volume. Meanwhile, Delta's annual revenue was close to 50 billion dollars. Also note that a large fraction of Delta's revenue is not billed to its CCCs. Therefore, a dominant fraction of the spend with Delta's CCCs was not related to Delta's business. This pattern is not unique to

[11] the best selling prices may vary across time and consumers, as firms sometimes roll out promotions.

[12] The Points Guy is an American travel website and blog that produces sponsored news and stories on travel, means of accumulating and using airline points and miles, politics, and credit cards. The valuations of rewards are updated on a monthly basis at https://thepointsguy.com/loyalty-programs/monthly-valuations/.

[13] For example, see https://www.nerdwallet.com/article/travel/when-does-it-make-sense-to-buy-points-miles

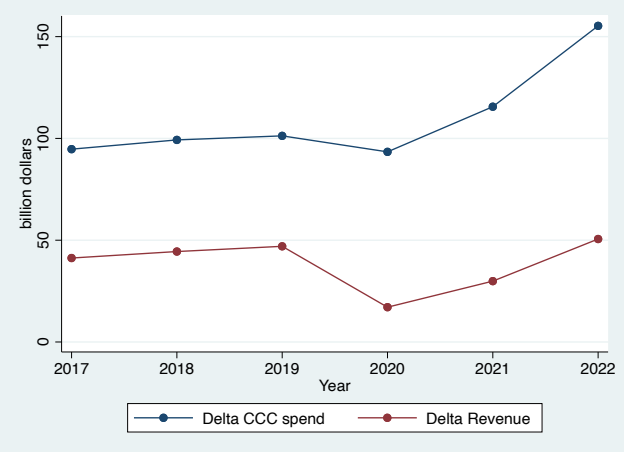


Figure 3: Delta's CCCs

Delta. In 2019, the spend with American's CCCs was 109 billion dollars and accounted for 12% of Mastercard U.S. volume, while American's annual revenue was 45.8 billion dollars. Given the importance of CCC programs to LPs indicated by Fact 1, big U.S. airlines actually issue a large fraction of rewards through broad consumption, which is not necessarily related to their business. This statement will be further confirmed in the next subsection. There, we will see that the issuance of airline miles exhibit much lower correlation with the airline business than the redemption.

One may think that such issuance strategy is simply a by-product of CCCs and not intended by firms. A more direct indication of firms' intention is that they build extensive networks of partners and allow consumers to earn rewards on purchases with their partners. For example, consumers can earn Delta miles even without Delta CCCs, when they book hotel stays and car rental on Delta's website, shop at over 1000 retailers on SkyMiles Shopping, and dine out at over 10000 restaurants listed on SkyMiles Dining. Although issuance through hotel stays and car rental can potentially be ascribed to the synergy between different travel products, shopping and dining are not connected to Delta' business in particular ways. In fact, U.S. airlines clearly describe this issuance strategy as a key strength of their loyalty programs. For example, in its investors presentation, Delta states,[14]

> "SkyMiles has an extensive network of longstanding partner relationships, allowing members to earn miles on car rental, hotel, retail, dining and other partners."

As for redemption of miles, around 95% of miles are redeemed for air travel for the three airlines. United further disclosed that it provided around 90% of the air travel that its miles are redeemed for.[15] The concentration of redemption on air travel is not surprising, as the value of miles is set to be the highest when they are redeemed in this way. For the same reason, we speculate that most of

[14] United and American also made similar statements.

[15] The number increased from 86% in 2018 to 92% in 2022.

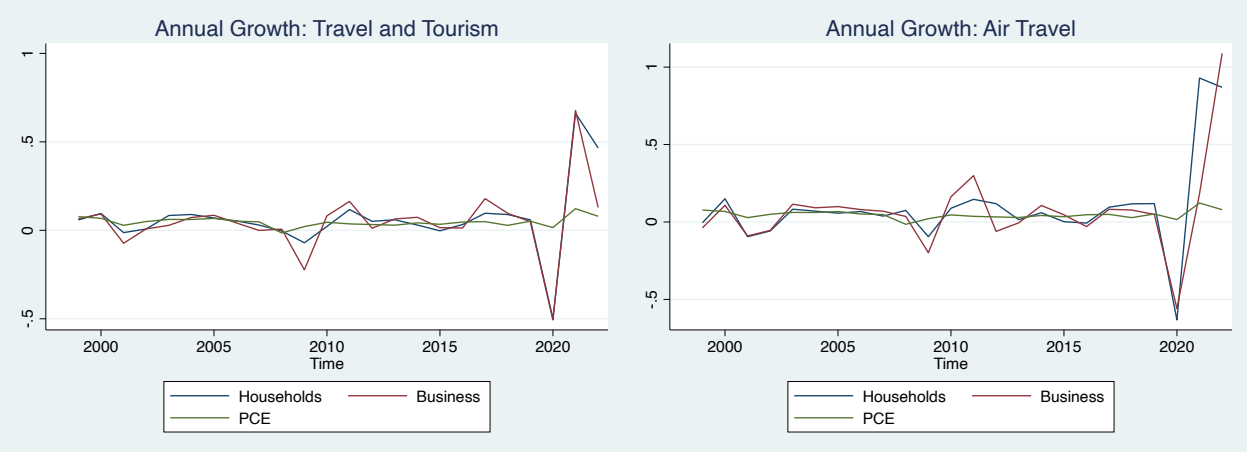


Figure 4: Volatility of travel demand and aggregate consumption

hotel points are redeemed for hotel stays.[16]

2.4 Fact 3: LP financing helps smooth cash flows

Travel consumption is highly volatile, more than most consumption categories. Figure 4 demonstrates the time series of the annual growth rates of US travel demand and personal consumption expenditures. US travel demand is obtained from Tourism Satellite Accounts Data of Bureau of Economic Analysis, and personal consumption expenditures are obtained from St. Louis Fed. The left panel plots the demand for the whole travel and tourism sector, while the right panel plots the demand for air travel. Clearly, both households' travel demand (blue) and business' (red) are more volatile than the aggregate consumption (green), and business' demand is even more volatile than households'.

On the one hand, most rewards are redeemed for travel services, meaning that reward redemption tends to move in tandem with travel consumption, which is highly volatile. On the other hand, reward issuance through CCC programs and partners tends to move in tandem with broad consumption, which is relatively stable. Taken together, for firms with highly volatile business such as airlines and hotels, reward issuance is less volatile than reward redemption. In fact, United reported that during 2008-2009 recession, its revenue declined 19% while its loyalty program revenue only declined 2%. Due to Covid-19, Delta's revenue dropped by 63.6% in 2020, but the transaction volume of its CCCs dropped only by 7.8%, as shown in Figure 3. The different cyclical nature of reward issuance and reward redemption implies that LPs help smooth cash flows: in the bad states of their business, firms can receive more cash flows through issuance than those foregone due to redemption. Notably, what drives the cyclical nature is not that firms have higher sales in

[16] In many cases, hotel rewards are directly free nights instead of hotel points that may be redeemed for other things.

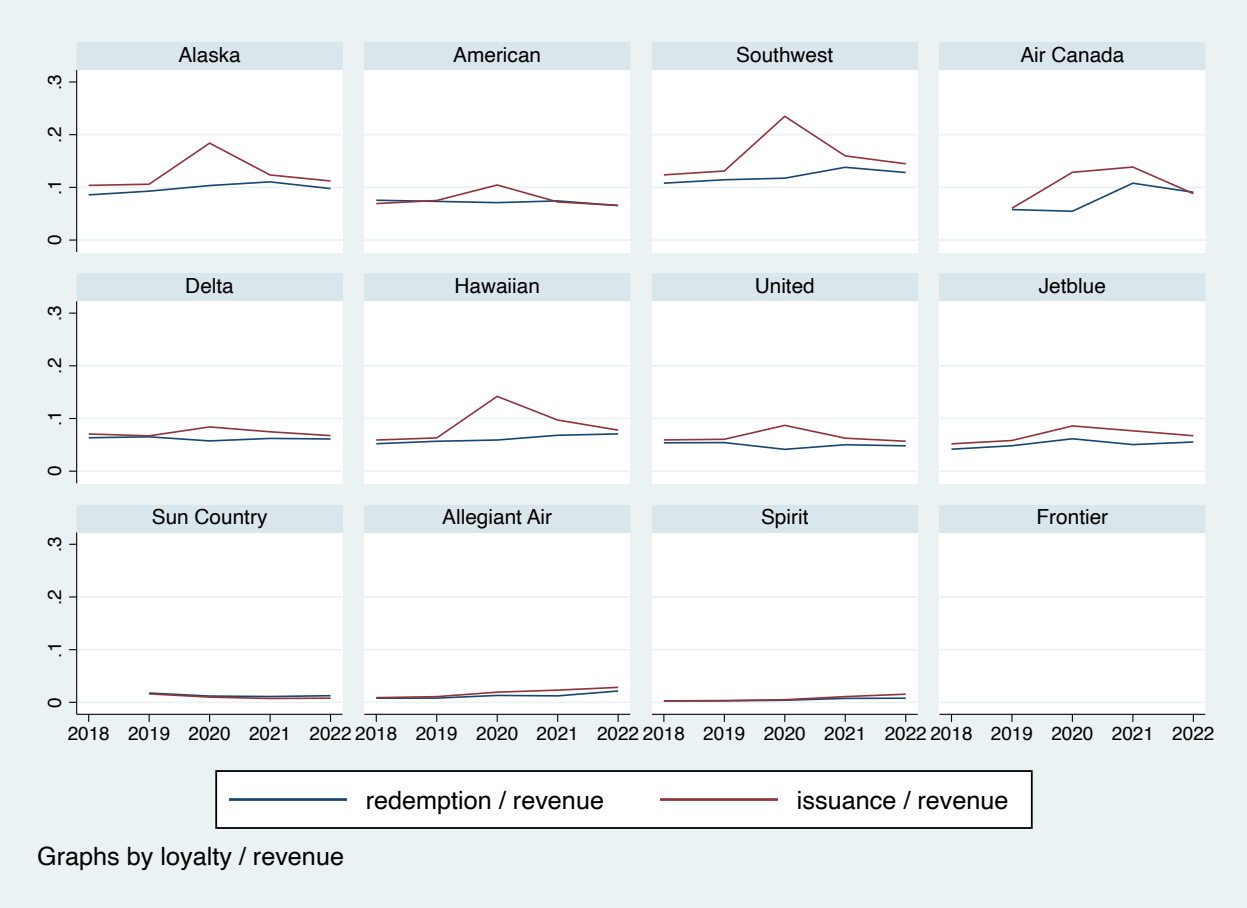


Figure 5: The time series of issuance and redemption

bad times but that they raise more funds through LP financing, which is reflected by increases in LP liabilities.

To see more direct evidence of the cyclical nature of LP financing, we investigate how Covid-19 affects its flows and balances. Figure 5 presents the time series of the issuance and redemption of airline miles scaled by the revenue in the same year from 2018 to 2022. The airlines are placed in descending order of scaled LP liabilities in 2019, where Alaska Airlines has the largest scaled LP liabilities. For most of the 12 airlines depicted, the scaled redemption remains stable, implying that redemption of miles closely follows the overall airline business. Meanwhile, the scaled issuance experienced a pronounced spike in 2020, for the airlines with large scaled LP liabilities, e.g. the first seven airlines from Alaska to United. The logic here is that a firm issuing rewards in more diverse ways will have larger LP liabilities relative to its business, and its reward issuance is less correlated with its business.

Figure 6 plots the time series of LP liabilities. To control for the cross-sectional variations in the size of business and demonstrate the change in LP liabilities over time, we scale them by

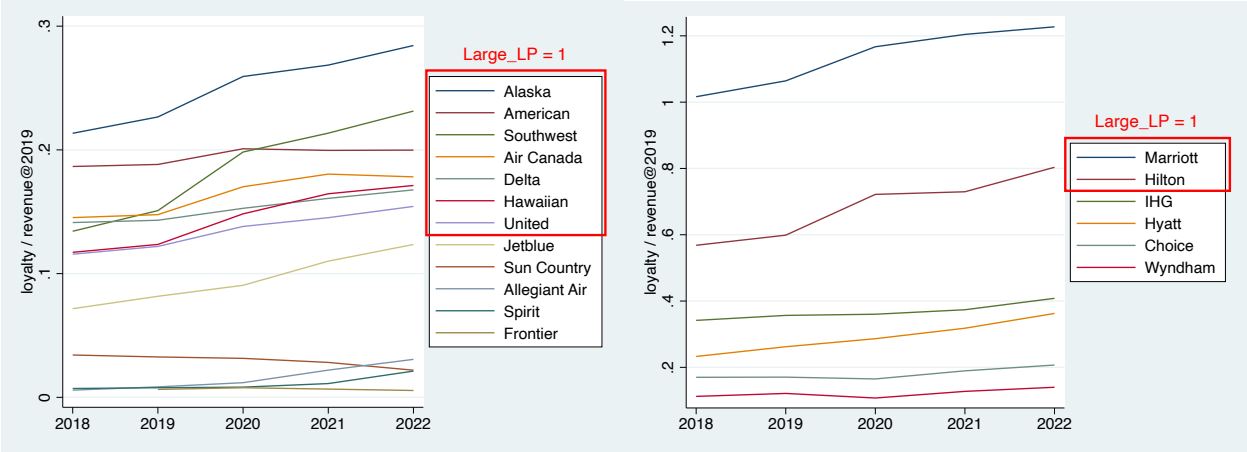


Figure 6: The time series of scaled loyalty program liabilities

revenue in 2019. LP liabilities experienced a significant increase in 2020 for the seven airlines with large scaled LP liabilities, while this trend was not observed for other airlines. Similarly, Marriott and Hilton, which have the largest scaled LP liabilities among hotels, also saw a dramatic increase in their LP liabilities in 2020. We run OLS regressions to test this hypothesis in a more formal manner. We construct an indicator of large loyalty programs that equals 1 for the seven airlines from Alaska to United, Marriott, and Hilton, and 0 otherwise. We regress the change in scaled LP liabilities from 2019 to 2020 on the indicator. We also add the same change from 2018 to 2019 to control for potential time trends. As shown in Table 5, the coefficient of the indicator is positive at 5% significance level for both industries. Note that this analysis is essentially a joint test of two statements: 1) firms issuing rewards in more diverse ways have larger LP liabilities compared with its business, and 2) their LP liabilities increase in the bad times of their business.

One may wonder how the industries view these observations. Do they know, and do they care? Delta Airlines described one strength of its LP to investors as follows:

> "SkyMiles offers significant diversity of cash flows with a long-term track record of stable and growing performance through cycles."

United and American have almost the same description. Clearly, big U.S. airlines realize that more diverse and thus more stable issuance of miles can help them better survive business cycle fluctuation. They attempt to achieve it by expanding their partnership networks and allow consumers to earn miles on unrelated consumption.

2.5 Discussion

Traditionally, economists tend to characterize loyalty programs as issuing rewards through purchases with the issuing firm. However, it turns out that for airlines and hotels, rewards are issued

through broad consumption with a large fraction not directly related to issuing firms. In addition, it is completely feasible to issue rewards through direct sales, but firms choose to discourage this type of issuance by setting high selling prices for rewards.

In fact, this issuance pattern does not align well with classical theories on loyalty programs. Under the theory of price discrimination, rewards serve as a summary of a consumer's purchase history that is informative about the consumer's relevant characteristics. A firm should not issue rewards based on broad consumption beyond purchases with the firm, unless the former contains more relevant information than the latter, which is a bit far-fetched. Under the theory of switching cost, rewards are used to generate or take advantage of switching cost and soften competition. The theory does not have particular implications for how rewards should be issued, so it does not explain why rewards should be issued through broad consumption and should not through direct sales at the same time. The agency theory would suggest that rewards should be issued through business consumption instead of personal consumption. Later, we show that this issuance strategy is implied by the finance theory of loyalty programs. Specifically, to leverage the financing function of LPs, a firm would find it more desirable to issue rewards through broad consumption than through purchase with the firm, and undesirable to issue rewards through direct sales.

3 The Baseline Setup

In this section, we build a dynamic model of LP financing. It is helpful to clarify that the main purpose of the model. Conceptually, LP financing hinges on that consumers discount the value of rewards at lower rates than the firm. While the model offers some rationale for why this might be the case, we acknowledge that it remains largely an empirical question that cannot be fully addressed within the model. Instead of answering why LP financing can arise, the model assumes its existence and illustrates its features. The power of the model lies in its ability to generate predictions that align closely with real-world observations, particularly those that classical theories of LPs cannot explain.

3.1 Players

We consider an infinite-horizon continuous-time game. The two main players in the game are consumers and a firm, while banks play a minor role. Players are all risk-neutral. The return on investment is uniformly r_0, and there are no taxes.

Consumers are infinitesimal and homogeneous, with their total mass being normalized to 1. Each consumer needs a service, such as a flight or a hotel night, at the rate of q_0. The occurence of the need is independent across consumers and over time. A service is worth p_0 to a consumer

when it is needed and 0 otherwise. Taken together, the demand for the service is $p_0 q_0$. In addition to the service, each consumer also engages in exogenous consumption of other goods. Each consumer's total consumption, including both the service and other goods, is completely smoothed out, resulting in a constant consumption flow of C_0 per unit of time.

The firm monopolizes the supply of the service that consumers need, thus the model abstracts from competition. To extract the entire consumer surplus, the firm sets the price of a service to p_0. To provide services, the firm needs to hold an illiquid asset, which entails a one-time investment cost of I_0 and a flow operating cost of O_0 per unit of time. In practice, the flow cost contains equipment depreciation, employee compensation, and other operational costs. Since the variable cost of providing one service must be smaller than consumers' reservation value p_0, it does not affect the firm's decision in the model. Hence, we assume it is zero.

In addition to the illiquid asset, the firm holds liquidity buffers such as cash and short-term investments, denoted as L_0, which earn interest at the rate of r_0. These liquidity buffers are used to support daily operation as well as to provide a financial cushion to absorb temporary operating losses during adverse economic states.

The firm finances its operations through debt, which carries a coupon rate of r_0, and potentially through LPs.[17] Bascially, we illustrate the features of LP financing by comparing it with typical debt financing. To simplify the illustration, we assume that 1) the firm has zero equity,2) the parameters I_0, O_0, L_0, and r_0 are exogenously given, and 3) the firm pays out its positive cash flows as dividend and the balance sheet remains unchanged.

The financial cost of maintaining the liquidity buffers is offset by the interest income they generate. Therefore, the firm's total operating expense in the baseline setup equals the sum of the operating cost and the financing cost of the illiquid asset:

$$OE_0 = O_0 + r_0 I_0.$$

As a benchmark, we first consider the case where the firm adopts no LP and consumers can make purchases only with cash. In this case, a consumer will buy one service whenever it is needed, and the firm will receive p_0 for each purchase.

Proposition 1. *If no loyalty program is adopted, a consumers buys a service with cash whenever it is needed, and the firm's cash flows are*

$$CF_0 \triangleq p_0 q_0 - O_0 - r_0 I_0. \tag{1}$$

Banks play passive roles in the game. Each bank can issue a bank credit card (BCC) to con-

[17] We assume equal interest rates on liquidity buffers and debt purely for simplicity. All results remain if they are different.

sumers and earn interchange fees paid by merchants. Interchange fees are a fraction ϕ of the consumption amount.[18] For simplicity, we assume that banks do not charge any fee to consumers.[19] To competitively attract consumers' usage of their BCCs, banks offer each consumer cashback as much as the fees they earns from him.

3.2 Loyalty programs

The firm can choose to adopt a LP and issue rewards. Given the assumptions of homogeneous consumers and a monopolistic supplier, the model inherently excludes the typical functions of LPs discussed in economics, marketing and operation literature, leaving financing as the sole potential function.

In line with observed practices, we assume that rewards are convenient to use. First, although a service is indivisible, a consumer can purchase it with any combination of cash and rewards that are worth one service. Second, unlike debt, which has a fixed payout schedule, consumers can redeem rewards at their discretion. This substantial convenience makes holding rewards similar to holding demand deposits rather than making investments. To capture the benefits of convenience, we assume that consumers treat rewards like a checking account term and do not discount their future redemption value. We do not consider the possibility that the firm issues non-convenient rewards, as consumers would require more compensation for holding such rewards, rendering LP financing unattractive.

Rewards are not as desirable as cash. This is primarily because cash can be used for a wide range of consumption purposes, whereas rewards are restricted to specific services. Consequently, it is reasonable to assume that consumers have a weak preference for holding cash over rewards, which eliminates their motive to stockpile rewards. Furthermore, the process of redeeming rewards is more cumbersome compared to using cash, as each redemption incurs a *redemption cost*, denoted by ε, which is smaller than the price of the service, p_0. Since the firm needs to compensate consumers for the use costs, rewards must be issued at a discount in equilibrium.

We assume one reward is always worth $1/p_0$ of one service. Hence, a consumer can purchase one service with x rewards and $p_0 - x$ cash for any $x \in [0, p_0]$. Essentially, the real value of one reward is 1, since one service is worth p_0 when a consumer needs it. The exact value of one reward does not matter here, as the firm determines the amount of rewards issued to consumers. The critical aspect of this assumption is that the value of rewards remains constant over time.

[18] Interchanges fees are roughly 2% to 3% of the consumption in the U.S. and lower in other countries. The exact fraction varies with consumption categories and merchants.

[19] In practice, banks may issue upscale credit cards that provide perks for consumers while charging them annual fees. In these cases, banks usually claim that perks are worth at least as much as the annual fees.

3.3 CLPs and PLPs

Motivated by Fact 1, we consider two types of LPs: consumption-based LPs (CLPs) and purchase-based LPs (PLPs) . CLPs issue rewards through CCCs, while PLPs issue rewards through purchases made with the firm. We assume that consumers cannot purchase rewards from the firm or transfer rewards to other consumers. In Section 5.3, we explore the impact of these kinds of transactions and discuss the observations in practice.

In the case of a CLP, the firm collaborates with a bank to issue a cobranded credit card (CCC). For the CCC, the bank purchases rewards from the firm and offer them to consumers as an alternative to cashback. Due to competition among banks for such partnerships, the firm receives all fees earned by the bank from this CCC and determines the rewards offered to consumers. The rewards structure of the CCC is determined by the scope of the reward category, γ, and the reward rate, a_c. The firm classifies a fraction γ of consumption as eligible for rewards. For each dollar of consumption in the reward category charged to the CCC, a consumer receives a_c rewards, whereas no rewards are given for other types of consumption. In practice, a CCC typically offers multiple tiers of rewards to cardholders. For example, Delta SkyMiles Gold Amex Card offers

- 2 miles per dollar for purchases made directly with Delta,
- 2 miles per dollar for purchases at restaurants and U.S. supermarkets, and
- 1 miles per dollar for other purchases.

The firm can determine γ by selecting the categories with high rewards, which are Delta, restaurants and supermarkets in this case. The firm can determine a_c by specifying the amount of miles consumers earn for consumption in these categories, which is 2 miles per dollar of consumption in this case. For each dollar of consumption in the reward category, consumers receive a_c rewards instead of ϕ cash. Hence, the price of a reward can be calculated as:

$$\theta_c \triangleq \phi / a_c.$$

Note that a consumer does not use the CCC for consumption not in the reward category. If he opts to use the CCC for all consumption in the reward category, he will earn rewards at the rate of $A_c \triangleq \gamma C_0 a_c$. Our focus is on scenarios where $\gamma C_0 a_c < p_0 q_0$. This condition ensures that consumers do not accumulate excessive rewards that may go unused. It generally holds in practice, as it is common for consumers to still use cash for certain transactions.[20]

[20] On the one hand, the issuance rate of rewards of a CCC is typically small because it is related to the rate of interchange fees and the consumption paid for with credit cards. Interchange fees are roughly 2% to 3% of the consumption. A large fraction of consumption such as rents for housing is usually required to be paid for with cash in order to avoid interchange fees. On the other hand, consumers in the model are basically frequent users of the service and thus have high expected demand.

Adopting a PLP, the firm offers a consumer the option to buy rewards when making purchases. For each dollar spent on services, a consumer can choose to buy π rewards at a unit price of θ_p. In practice, it is more common to see that the firm requires consumers to buy rewards by selling a bundle of services and rewards. For example, an air ticket is a bundle that includes both a flight and some miles. In this context, bundling and offering an option are equivalent.[21] We use the language of offering an option to draw a clearer parallel between the two types of LPs. We focus on the cases where $\pi \leq 1$, since $\pi = 1$ is already equivalent to a "buy one, get one free" offer. In practice, consumers usually need to accumulate rewards through multiple purchases to get one service for free.

We say a consumer fully earn rewards, if he charges all consumption in the reward category to the CCC under a CLP or always exercises the option to buy rewards under a PLP. When we do not distinguish between the two types of LPs, we use θ to represent the unit price of rewards and A to represent the issuance rate of rewards, assuming the consumer fully earns and optimally redeems rewards. Under a CLP, $A = A_c$. For a PLP, Lemma 1 shows that a consumer will optimally redeem almost all his rewards. If a consumer fully earns rewards, then cash will be used to cover a fraction $\frac{1}{\pi+1}$ of the total costs, and rewards will cover the remaining fraction, $\frac{\pi}{\pi+1}$. This implies that under a PLP, $A = A_p \triangleq \frac{p_0 q_0}{1/\pi+1}$. Given that it is either a CLP or a PLP, an LP is fully characterized by θ and A. The firm can determine these parameters through the specific design features of each type of LPs, as outlined below.

	A: the issuance rate of rewards	θ: the unit price of rewards
CLP	$\gamma C_0 a_c$	ϕ / a_c
PLP	$\frac{p_0 q_0}{1/\pi+1}$	θ_p

3.4 The steady state

For the baseline setup, we consider that the firm can choose to adopt either a CLP or a PLP, but not both. This simplification allows for analytical solutions of the model.[22] Additionally, we focus on a static LP—once chosen, its design remains unchanged over time. Given an infinite time horizon and the assumption that consumers do not discount future payoffs, a consumer cares about his average payoff flow. Note that a consumer may not redeem rewards immediately after earning them. As a result, a consumer may accumulate a substantial amount of rewards. Since the LP is static and does not respond to consumers' behavior, it is without loss of generality to

[21] To see this, consider any bundling under which a consumer can earn π' rewards for each dollar spent on services. Due to the rewards, the firm sets the price of a service to p', which is higher than p_0. The firm can always replicate this bundling by offering an option with $\theta_p = (1 - p_0/p')/\pi'$ and $\pi = \pi' p'/p_0$.

[22] In Section 5.2, we numerically demonstrate the case when both types are adopted.

assume that consumers play Markov strategies that depends only on their current reward holdings. Furthermore, due to the homogeneity of consumers, the state of the game can be summarized by the distribution of all consumers' reward holdings. Therefore, we define the steady state under a static LP as follows.

Definition 1. In a steady state under a static LP, the following conditions hold:

- Each consumer's decision maximizes his expected average payoff flow.
- The distribution of consumers' reward holdings remains unchanged over time.

For the baseline setup, we focus on equilibrium paths in steady states. In such a state, the aggregate behavior of all consumers remains constant, and the firm's cash flows are constant. Our analysis centers on how an LP affects the constant steady-state cash flows.

3.5 Discussion about model assumptions

Constant total consumption of a consumer. The essence of this assumption is that a consumer's overall consumption expenditure remains stable and is not perfectly correlated with his idiosyncratic demand for the service. This characteristic is generally observed in practice. Furthermore, the widespread use of consumer credit products—such as credit card loans and buy-now-pay-later services—helps smooth out fluctuations in consumption expenditure. For simplicity, we assume that total consumption remains constant.

Zero discount rate for rewards. The essence of this assumption lies in the notion that consumers discount the payment of rewards at a lower rate than the firm does. This mechanism serves as a straightforward method to generate gains from trade and is widely employed in the finance literature. A critical question arises: why do consumers and firms have different discount rates? The model suggests a narrative of convenience yield: consumers are willing to accept a lower rate because the firm makes rewards convenient to use, similar to demand deposits. This effect could be further amplified by the "coarse thinking" of consumers, who perceive rewards as analogous to demand deposits and, as a result, do not require a substantial interest rate (Mullainathan, Schwartzstein, and Shleifer, 2008). Some other factors may also affect consumers' discount rates. For instance, consumers and the firm may face different investment opportunities (Rogoff and You, 2023); some individuals may derive positive utility from earning and holding rewards (Lim, Chun, and Satopää, 2021). The model does not take a particular stance on how consumers' discount rates are determined, aside from acknowledging that convenience plays an important role. For simplicity of illustration, we assume an exactly zero discount rate rather than a merely lower one. With a zero discount rate, a consumer will redeem for a service only when he owns at least p_0 rewards,

as shown in Lemma 1. If the discount rate is positive, a consumer may redeem even when he owns slightly fewer than p_0 rewards. This difference does not affect the qualitative features of LP financing, but complicates the illustration.

Constant valuation of rewards. The model assumes constant valuation of rewards for simplicity. In practice, however, the values of rewards are set by firms and can change over time. Many consumers have complained that airlines and hotels devalue their rewards. Firms benefit from devaluation as it reduces the actual payment to the existing reward holders, akin to the dilution of creditors. In a model that incorporates devaluation, consumers would demand a lower unit price of rewards to offset the loss incurred by devaluation. Additionally, consumers would accelerate the redemption of rewards to avoid being adversely affected by devaluation, leading to smaller reward balances. Both effects reduce the funds raised from consumers, thereby making LP financing less effective. In practice, several forces may restrain firms from rampant devaluation. First, devaluation harms a firm's reputation for customer care and particularly enrages frequent customers, who tend to accumulate sizable balances. Second, devaluation diminishes the appeal of cobranded credit cards, negatively impacting both the firm's and its bank partner's interests. Third, devaluation at the expense of consumers could attract regulatory scrutiny. For instance, in response to complaints about the devaluation of airline miles, the U.S. Department of Transportation (DOT) initiated an investigation into airlines' rewards practices in September 2024.

Redemption cost. The model posits that each redemption entails a fixed cost for the consumer. The basic role of this assumption is to generate an incentive for consumers to accumulate rewards. As shown in Lemma 1, a consumer will redeem for a service only when he owns at least p_0 rewards. In practice, many firms either prohibit partial redemptions or discourage them by setting unfavorable pricing structures. Consequently, consumers are compelled to accumulate a sufficient number of rewards before redeeming them. By incorporating this assumption, the model effectively captures this real-world scenario. It is important to note that the existence of a redemption cost, rather than its magnitude, is the critical factor driving consumer behavior in this context.

3.6 How LP financing differs from other financings

One might wonder how LP financing differs from other forms of financing. In general, effective financing should allow a firm to raise sufficient funds at low costs. Typical debt financing stipulates a fixed repayment schedule, which does not accommodate investors' liquidity demands. Consequently, debt financing is associated with low convenience yields or high required returns. Meanwhile, with the fixed repayment schedule, the firm can directly determine the raised funds.

Conversely, LP financing, similar to demand deposits, allow financiers to control the redemption. This convenience results in high convenience yields and low required returns. However, what is also inherent in this convenience is that the firm loses direct control over the raised funds. They emerge endogenously as a result of the interplay between issuance and redemption.

For LP financing, issuance is determined by the firm, and redemption is determined by the consumption of the particular service supplied by the firm. Then two questions naturally arise: how should firms issue rewards, and how does the nature of the service affect financing? The model primarily addresses the two questions. However, these considerations are less relevant to demand deposits, as issuance and redemption are exogenously determined by consumers' total income and total consumption.

4 Model Solution

In this section, we characterize the steady state under a static CLP or PLP.

4.1 Consumer behavior and the firm's cash flows

We begin by characterizing consumer behavior. ince consumers are concerned with maximizing their average payoff flow, their strategies can be generally characterized as earning rewards at the rate of z and redeeming x rewards for a service at the rate of y on average. Here, $0 < x \leq p_0$, as a service takes at most p_0 rewards, and $xy \leq z$ due to the budget constraint on rewards. For a service he buys with only cash, he pays p_0 cash. For a service he buys with a positive amount of rewards, he pays the redemption cost ε and $p_0 - x$ cash in addition to the x rewards on average. In addition, the opportunity cost of earning rewards at rate z is $z\theta$. Taken together, a consumer's average payoff flow is

$$(q_0 - y)(p_0 - p_0) + y[p_0 - \varepsilon - (p_0 - x)] - z\theta = y(x - \varepsilon) - z\theta. \tag{2}$$

For a given z, the average payoff flow is maximized when $x = p_0$ and $y = z/p_0$. The intuition is as follows. Each redemption incurs a redemption cost ε. Given that a consumer uses a fixed amount of rewards, he would like to use more rewards in each redemption and redeem less frequently to minimize the total redemption cost. Since the maximum number of rewards that can be redeemed for a single service is p_0, the optimal strategy is to redeem p_0 rewards whenever a redemption occurs.

On the other hand, a consumer weakly prefers holding cash over holding rewards. Consequently, when a consumer has at least p_0 rewards and needs a service, he will always choose to redeem p_0 rewards. The two observations lead to the following lemma.

Lemma 1. *A consumer redeems p_0 rewards for a service when he needs a service and have at least p_0 rewards, and does not redeem rewards otherwise.*

Lemma 1 implies that if a consumer fully earns rewards under a PLP, he will optimally redeem nearly all rewards, because he never holds more than $p_0 + p_0\pi$ rewards. We then readily obtain the issuance rate of a LP, as summarized in Section 3.3.

Lemma 2. *If a consumer fully earns rewards, he receives rewards at a rate of A on average, where A follows the characterization in Section 3.3.*

Combining Lemmas 1 and 2, Proposition 2 characterizes consumer behavior.

Proposition 2. *Suppose that the firm adopts a LP with a unit price of rewards θ and an issuance rate A. If $\theta \leq 1 - \frac{\varepsilon}{p_0}$, each consumer fully earns rewards, and when needing a service, he redeems p_0 rewards if he has at least p_0 rewards and pays p_0 in cash otherwise. If $\theta > 1 - \frac{\varepsilon}{p_0}$, a consumer does not earn rewards and pays p_0 in cash when needing a service.*

Under consumers' optimal redemption strategy, each redemption always involves p_0 rewards. After accounting for the redemption cost ε, the average benefit of a reward is $1 - \frac{\varepsilon}{p_0}$. Therefore, a consumer would like to earn and redeem rewards if and only if this average benefit is at least as high as the unit price of rewards, θ. This condition can be expressed as:

$$\theta \leq 1 - \frac{\varepsilon}{p_0}. \tag{CC}$$

This condition is referred to as the consumer constraint.

Next, we calculate the firm's cash flows in the steady state, assuming that the consumer constraint holds. In aggregate, consumers redeem p_0 rewards for a service at a rate of A/p_0 and buy services using purely cash at a rate of $q_0 - A/p_0$. The firm receives cash flows from two sources: 1) $A\theta$ from issuing rewards, and 2) $(q_0 - A/p_0)\, p_0$ from selling services to consumers. Since the distribution of consumers' reward holdings remains unchanged in the steady state, the aggregate reward balance held by consumers, denoted by B, is constant. Effectively, the firm pre-sells this amount of rewards to consumers and receives $B\theta$ in cash in advance. This cash can be used to reduce debt financing or increase investment, generating additional cash flows of $r_0 B\theta$. Taken together, the firm's cash flows are

$$CF \triangleq A\theta + \left(q_0 - \frac{A}{p_0}\right) p_0 + r_0 B\theta - OE_0,$$

which can be rewritten as:

$$CF = \underbrace{r_0 B\theta}_{\text{interest from reward balance}} - \underbrace{A(1-\theta)}_{\text{discount of rewards}} + CF_0. \tag{3}$$

Although the funds raised through the LP do not incur the typical interest cost, they are not cost-free because the firm needs to issue rewards at a discount to compensate consumers for their use costs. The effective financing cost per unit of funds is

$$\frac{A(1-\theta)}{B\theta} = \frac{A}{B} \cdot \frac{1-\theta}{\theta}. \tag{4}$$

Whether LP financing is desirable hinges on the discount at which the firm issues rewards, $1-\theta$, and the balance-to-flow ratio, B/A. LP financing becomes more effective if the firm can accumulate a greater reward balance relative to the issuance rate. While A and θ are directly chosen by the firm, B emerges endogenously in equilibrium as a result of the interplay between reward issuance and redemption. Next, I characterize the steady-state distribution of consumers' reward holdings and derive B.

4.2 Reward balance under a CLP

Suppose that the firm adopts a CLP. Let $\Gamma(N)$ be the mass of consumers holding rewards fewer than N, and $\gamma(N)$ be the density of consumers holding N rewards. Lemma 3 characterizes $\gamma(\cdot)$ in the steady state under a CLP.

Lemma 3. *In the steady state under a CLP, the density of consumers' reward holdings must satisfy the equation system*

$$\gamma'(N) = \begin{cases} \gamma(N+p_0)\frac{q_0}{A}, & \text{if } N \in [0, p_0), \\ -\gamma(N)\frac{q_0}{A} + \gamma(N+p_0)\frac{q_0}{A}, & \text{if } N \in [p_0, +\infty), \end{cases} \tag{5}$$

and three boundary conditions

$$\lim_{x \uparrow N} \gamma(x) = \lim_{x \downarrow N} \gamma(x), \forall N \tag{6}$$

$$\gamma(N) = 0, \text{ if } N < 0 \tag{7}$$

and

$$\int_0^{+\infty} \gamma(x)\, dx = 1. \tag{8}$$

The steady-state distribution of consumers' reward holdings consists of two parts. For N smaller than p_0, a consumer holding N rewards does not redeem. The density is increasing in

N, since a consumer holding $N+p_0$ rewards may redeem once and end up holding N rewards. For N greater than or equal to p_0, a consumer holding N rewards will redeem if he needs a service. This introduces an attrition term into the equation, reflecting the reduction in reward holdings due to redemptions. The boundary conditions stem from three properties: 1) the distribution is continuous everywhere, 2) no consumer holds a negative number of rewards, and 3) the total mass of consumers sums to 1.

By solving the system of equations outlined in Lemma 3, we explicitly characterize the steady-state distribution $\gamma(\cdot)$ and determine the reward balance B through

$$B = \int_0^\infty N\gamma(N)\,dN.$$

Proposition 3. *The steady-state distribution of consumers' reward holdings is*

$$\gamma(N) = \begin{cases} 0, & \text{if } N \in (-\infty, 0), \\ \frac{1}{p_0} \cdot \left(1 - e^{-N/\lambda}\right), & \text{if } N \in [0, p_0), \\ \frac{1}{p_0\left(\lambda \frac{q_0}{A} - 1\right)} e^{-N/\lambda}, & \text{if } N \in [p_0, +\infty), \end{cases} \tag{9}$$

where λ is the unique positive root of

$$\left(1 - e^{-p_0/\lambda}\right) \cdot \lambda = \frac{A}{q_0}. \tag{10}$$

Further, the reward balance is

$$B = \frac{p_0}{2} + \lambda, \tag{11}$$

The reward balance B consists of two components. The first component is simply half of p_0. This arises because consumers are unwilling to redeem rewards partially. Consider the case that consumers always need services. In this scenario, their reward holdings follow a cycle pattern, as shown in the top left panel of Figure 7. Starting from 0, a consumer's reward holdings increase at a constant rate until they reach p_0. At this point, the consumer redeems the rewards, causing their holdings to drop back to 0. Over this cycle, the average reward holdings are half of p_0.

The second component stems from the fact that consumers do not always need services but always earn rewards at a constant rate. Since the issuance rate is smaller than the expected demand, i.e., $A = \gamma C_0 a_c < p_0 q_0$, consumers are never concerned about leaving rewards unused. As a result, they fully earn rewards and accumulate them at the rate A. Importantly, the issuance of rewards to a consumer is constant and asynchronous with his redemption of services. This asynchrony allows a consumer to accumulate substantially more than p_0 rewards if he does not need services for an extended period. The top right panel of Figure 7 shows a possible trajectory of a consumer's

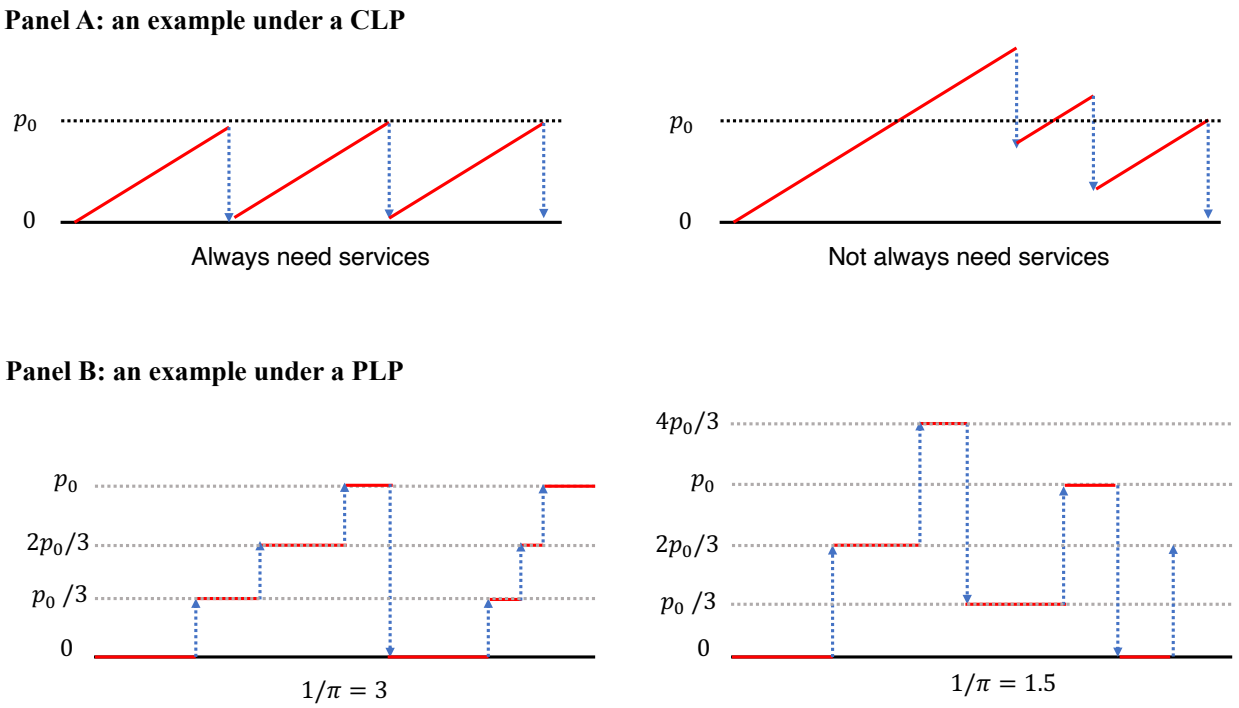


Figure 7: The dynamics of reward balance under CLPs

rewards holdings under this scenario. The second component is referred to as overaccumulation. Put intuitively, the accumulation of reward balance relies on both consumers' unwillingness to redeem rewards partially and the asynchrony between reward issuance and reward redemption in equilibrium.

4.3 Reward balance under a PLP

Suppose that the firm adopts a PLP. In this case, a consumer's reward holdings change only when he buys a service. We can then represent a consumer's reward holdings as a sequence $\{h_i\}_{i\in\mathbb{N}}$, where h_i is the amount between his ith purchase and his $(i+1)$th purchase. By Lemma 1, if a consumer holds fewer than p_0 rewards, he will pay p_0 in cash for the next purchase and earn $p_0\pi$ rewards; if he holds at least p_0 rewards, he will redeem p_0 rewards for the next purchase. That means,

$$h_i = \begin{cases} 0, & \text{if } i = 0, \\ h_{i-1} + p_0\pi, & \text{if } h_{i-1} < p_0, \\ h_{i-1} - p_0, & \text{if } h_{i-1} \geq p_0. \end{cases}$$

Notice that a redemption can be considered as that a consumer still earns $p_0\pi$ rewards but meanwhile gives up $p_0 + p_0\pi$ rewards. This allows us to express the sequence $\{h_i\}_{i\in\mathbb{N}}$ in modular

arithmetic as follows:

$$h_i = i \cdot p_0\pi \pmod{(p_0 + p_0\pi)}.$$

Since the need for a service arrives independently over time, a consumer stays at each point of the sequence for the same expected length of time. This implies that the reward balance in the steady state equals the average value of the sequence. Bases on this formulation, Proposition 4 characterizes the steady-state distribution of consumers' reward holdings and the reward balance.

Proposition 4.

- *If $1/\pi$ is a rational number, let m and n be the positive coprime integers such that $\mu/n = (1+\pi)/\pi$. Consumers are uniformly distributed over the set $\left\{\frac{i}{\mu}(p_0 + p_0\pi)\right\}_{i=0}^{\mu-1}$ in terms of reward holdings.*

- *If $1/\pi$ is an irrational number, consumers are uniformly distributed over the interval $[0, p_0 + p_0\pi)$ in terms of reward holdings.*

- *Let μ be $+\infty$, if $1/\pi$ is an irrational number. The reward balance is*

$$B = \frac{\mu - 1}{2\mu}(1+\pi)\, p_0, \tag{12}$$

which must be in the interval $[p_0/2, p_0(1+\pi)/2]$

The intuition behind Proposition 4 can be understood as follows. We begin by considering the case where $1/\pi$ is an integer. As demonstrated in the bottom left panel of Figure 7, a consumer's reward holdings follow a cycle pattern:

- Starting with zero rewards, a consumer pays for the next $1/\pi$ purchases with cash. During this phase, his reward holdings consistently increase with each purchase until he accumulates p_0 rewards.

- Once the consumer reaches p_0 rewards, he redeems all rewards for the next purchase, causing his reward holdings to drop back to 0.

In the steady state, consumers are uniformly distributed over $\{0, p_0\pi, 2p_0\pi, \ldots, p_0 - p_0\pi, p_0\}$ in terms of reward holdings, and the reward balance is $p_0/2$. This steady-state distribution is driven by two forces. First, a consumer is not willing to redeem rewards partially. Second, his reward holdings never exceed p_0. Since $1/\pi$ is an integer, his reward holdings will exactly reach p_0 when he accumulates or exceeds this amount, prompting them to redeem rewards for the next purchase rather than earning additional rewards.

When $1/\pi$ is not a integer, the situation becomes more complicated. Under the consumers' optimal strategy, reward earning occur in lump sums of $p_0\pi$. Due to the lumpiness, a consumer's

reward holdings may exceed p_0. For example, as demonstrated in the bottom right panel of Figure 7, if $\pi = 2/3$, a consumer's reward holdings are $4p_0/3$ when they first reach or exceed p_0. Furthermore, when the consumer redeems p_0 rewards for the next purchase, his reward holdings decrease to a positive amount rather than dropping to 0. To characterize the sequence of reward holdings in a general way, the formal proof employs simple number theory.

The reward balance is generally characterized by equation (12). Similar to the case under CLPs, the reward balance is at least $p_0/2$, due to consumers' unwillingness to redeem rewards partially. Overaccumulation arises when $1/\pi$ is not a integer, due to the lumpiness of reward earnings discussed above. The size of this overaccumulation is capped at $p_0\pi/2$, which represents half of each reward earning.

5 Implications About LP Financing

As discussed above, the convenience feature of rewards makes the reward balance B endogenous, and the desirability of LP financing hinges on the balance-to-flow ratio, B/A. In this section, we derive the implications about LP financing based on this feature.

5.1 The firms' characteristics

In the model, the firm is characterized by the value of a service, denoted as p_0, and the frequency of the need for a service, denoted as q_0. We examine how the effectiveness of LP financing depends on these firm-specific characteristics. To control for the the overall scale of the firm's business, we hold the total demand for the service, p_0q_0, constant. By doing so, we systematically vary p_0 and q_0 inversely, allowing us to compare firms that offer high-value, low-frequency services with those that provide low-value, high-frequency services.

Proposition 5. *Given the demand for the service and the issuance rate, the reward balance is greater under both CLPs and PLPs if when the value of the service, p_0, is higher and the frequency of the need for the service, q_0, is lower.*

For both types of LPs, the reward balance comprises two components: half of the service value, p_0, and overaccumulation. Under a CLP, the overaccumulation, denoted as λ, is determined by equation (10). It decreases with p_0, as each redemption reduces the balance by p_0. It decreases with q_0, as a lower frequency of demand implies a higher likelihood that that a consumer does not need a service for an extended period, thereby allowing the balance to grow. When the total demand for the service, p_0q_0, is held constant, a higher p_0 is associated with a lower q_0. In this scenario, the impact of q_0 dominates, leading to greater overaccumulation under CLPs.

Firms in 2019	B (deferred revenue*)	A (deferral of revenue*)	B/A (balance-to-flow ratios)
Marriott	5718	2468	2.32
Delta	6728	3156	2.13
Macy's	839	554	1.51
Target	935	729	1.28
GAP	226	187	1.21
ULTA Beauty	230	206.7	1.11
TJX	501	1690	0.30
Starbucks	1114	10984	0.10

*in million dollars

Table 2: The balances and flows of LPs

Under a PLP, the overaccumulation arises from and increases with the lumpiness of reward earning, which is $p_0\pi$. By $A = \frac{p_0 q_0}{1/\pi+1}$, π is fixed given $p_0 q_0$ and A. Hence, a higher p_0 leads to greater lumpiness.

In fact, Proposition 5 aligns well with empirical observations across various industries. Table 2 shows deferred revenue and deferral of revenue, which are essentially balances and flows of LP liabilities, for firms in different industries. Firms such as Marriott and Delta, which provide upscale travel services, exhibit the highest balance-to-flow ratios. In contrast, retailers like Macy's, Target, GAP, and ULTA Beauty, which sell middle-end goods, demonstrate lower but still significant balance-to-flow ratios. Meanwhile, TJX, an off-price retailer, and Starbucks, which primarily sells low-value, high-frequency items such as coffee and bakery products, have very low balance-to-flow ratios. These patterns are consistent with the theoretical predictions of the model.

Proposition 5 suggests that firms supplying high-value, low-frequency services are better positioned to utilize LPs effectively for financing. Such firms can borrow more from consumers without undermining the convenience of rewards or incurring excessive redemption costs. In practice, industries such as airlines, hotels, and department stores, which typically provide high-value, low-frequency services, are the most active in developing LPs and expanding their reward issuance programs.

Remark 1. The firm's characteristics may also influence the redemption cost, which in turn affects the attractiveness of LP financing. A lower redemption cost allows for a higher price of rewards, making LP financing more attractive. However, empirical data on redemption costs across firms is limited, making it difficult to draw definitive conclusions. A reasonable conjecture, supported by common cost structures in economics, is that the redemption cost is concave in p_0. This concavity

can arise if the redemption cost comprises a fixed component and a variable component that is proportional to the value of the service. Under this conjecture, ε/p_0—where ε represents the redemption cost—decreases with p_0. This implies that the consumer constraint becomes more relaxed when p_0 is higher and q_0 is lower, further enhancing the feasibility and attractiveness of LP financing for firms offering high-value, low-frequency services.

5.2 Issuance strategies

We compare the two types of LPs with respect to the reward balance.

Proposition 6. *Given the issuance rate, a CLP results in a greater reward balance than a PLP.*

Under a CLP, overaccumulation arises due to the asynchrony between reward issuance and redemption. Specifically, a consumer consistently earns rewards even during the period that he does not need a service. In contrast, such asynchrony is missing under a PLP, because issuance and redemption are both tied to purchases of services. As a result, opportunities to earn rewards and redeem rewards are synchronous and exclusive. This explains why overaccumulation is zero when $1/\pi$ is an integer: a consumer with p_0 rewards will redeem rewards instead of earning more. When $1/\pi$ is not an integer, overaccumulation arises due to the lumpiness of reward earnings and can reach $p_0\pi/2$ at maximum. Proposition 6 rests on the finding that the asynchrony inherent in CLPs is more effective in generating overaccumulation than the lumpiness associated with PLPs.

Numerical analysis further supports the superiority of CLPs over PLPs, even when the firm adopts both types of LPs simultaneously. Suppose that the issuance rates under the CLP and the PLP are A_c and A_p respectively. Similar to Lemma 3, the steady-state distribution of consumers' reward holdings satisfies the following system of equations

$$\gamma'(N) = \begin{cases} -\gamma(N)\frac{q_0}{A_c} + \gamma(N+p_0)\frac{q_0}{A_c}, & \text{if } N \in [0, p_0\pi), \\ -\gamma(N)\frac{q_0}{A_c} + \gamma(N+p_0)\frac{q_0}{A_c} + \gamma(N-p_0\pi)\frac{q_0}{A_c}, & \text{if } N \in [p_0\pi, p_0+p_0\pi), \\ -\gamma(N)\frac{q_0}{A_c} + \gamma(N+p_0)\frac{q_0}{A_c}, & \text{if } N \in [p_0+p_0\pi, +\infty), \end{cases}$$

along with the three boundary conditions specified in Lemma 3. We numerically calculate the steady-state distribution and the reward balance. For the numerical analysis, we set p_0 to 300, q_0 to 4, and the total issuance rate to 200. Table 3 presents the reward balance for different combinations of A_c and A_p. The results clearly demonstrate that, for a given total issuance rate, the reward balance increases as more rewards are issued through the CLP and fewer through the PLP.

Combining the insights from Proposition 6 and the numerical analysis, we obtain a general implication regarding issuance strategies. Since redemption occurs only when a consumer purchases a service, it is inherently tied to the firm's business. A weaker linkage between reward issuance

A_p	200	199	150	120	100	80	50	0
A_c	0	1	50	80	100	120	150	200
B	150	180	184	186.6	188	191	194	200

Table 3: The reward balance when both types are adopted

This table presents the reward balance predicted by the model when both types are adopted. A_c and A_p are the issuance rates under the CLP and the PLP respectively. For all calculation, $p_0 = 300$, $q_0 = 4$, and the total issuance rate is set to 200.

and the firm's business reduces the synchrony between issuance and redemption. Therefore, to leverage LP financing more effectively —evidenced by a higher balance-to-flow ratio—the firm should strategically weaken the linkage of reward issuance to the firm's business. This implication aligns with Fact 2 in Section 2, which observes that, in practice, a large fraction of rewards are issued through broad consumption, including expenditures unrelated to the firm's business.

5.3 Transactions of rewards

For the baseline setup, we assume that consumers can only obtain rewards through the two types of LPs—either directly from the firm or from other consumers. In this subsection, we extend the analysis by considering the impact of alternative ways to obtain rewards and discuss the observations in practice. Specifically, we analyze two benchmark cases.

1. Free purchases: In this scenario, consumers can purchase any amount of rewards directly from the firm whenever they want. The unit price of rewards under free purchase follows a fixed price determined by the firm, $\hat{\theta}$. This setup allows consumers to acquire rewards independently of their engagement with the firm's business or cobranded credit cards.

2. Free Transfer: In this scenario, consumers can transfer any amount of their rewards to others at a mutually agreed price, and such transfers incur no additional costs. This introduces a secondary market for rewards, enabling consumers to trade rewards among themselves without restrictions.

Proposition 7. *Free purchase is always undesirable for the firm.*

Proposition 7 rests on two points. First, the option of free purchase increases the total discount of rewards that consumers can avail themselves of. A consumer will opt for free purchase only when the associated price, $\hat{\theta}$, is so low that redeeming the rewards purchased through free purchase is a better deal than using cash. In that case, a consumer will redeem rewards for all needed services. This channel always diminishes the firm's cash flows.

Second, free purchase does not increase the raised funds and actually decreases them when a LP is in place. Given the consumer's preference for cash over rewards, a consumer will purchase rewards through free purchase only when he needs a service and holds fewer than p_0 rewards, and he redeems the rewards immediately. Consequently, the firm does not raise funds from the rewards sold via free purchase. If no LP is adopted, or if a LP is adopted but consumers choose not to earn any reward from the LP, the reward balance will be exactly 0. If consumers choose to earn rewards from the LP, free purchase will accelerate the redemption process: when needing a service, a consumer with fewer than p_0 rewards will acquire the remainder through free purchase and redeem all rewards promptly. The more frequent redemption impedes the accumulation of a consumer's reward holdings.

Taken together, permitting free purchase is always undesirable for the firm. This prediction is consistent with Fact 1 presented in Section 2, which notes that firms discourage consumers from directly purchasing rewards by charging high prices, and that direct sales to consumers do not constitute a substantial way to issue rewards.

Proposition 8. *Free transfer renders the reward balance 0 and is always undesirable for the firm.*

Proposition 8 follows a simple observation. If free transfer is permitted, the rewards issued at each instant will be transferred to the consumers who need services and will be redeemed immediately. This type of transactions is desirable for consumers, as it reduces consumers' reward holdings without compromising their payoffs. Consequently, under free transfer, a consumer's average reward balance will effectively be zero. That means, free transfer will entirely undermines the financing function of LPs. This result resonates with Rogoff and You (2023). In the context of platforms issuing tokens, they demonstrate that platforms can benefit from making tokens non-tradable unless the tokens can generate a sufficiently high convenience yield outside the platform.

In practice, airlines and hotels generally discourage the transfer of rewards between consumers, limiting such transfers to family members and friends. Some firms directly impose pecuniary costs on transfers. For instance, for each transfer of miles, Delta charges 1 cent per mile plus a fixed processing fee of $30, and United charges 1.5 cent per mile plus a fixed processing fee of $30. That means, the cost of transferring miles is actually close to the redemption value of the transferred miles shown in Table 1. Some firms restrict the scope or the frequency of transfers. For example, Cathay Pacific Airlines allows a consumer to nominate up to five individuals to his Redemption Group and prohibits him from transfering miles to or redeem his rewards for individuals outside this group. Marriott imposes restrictions on the frequency of transfers. Some firms explicitly prohibit the use of transfers for commercial purposes. For instance, the terms and conditions of American Airlines' AAdvantage program include the following content:

"At no time may AAdvantage mileage credit or award tickets be purchased, sold or

bartered (including but not limited to transferring, gifting, or promising mileage credit or award tickets in exchange for support of a certain business, product or charity and/or participation in an auction, sweepstakes, raffle or contest). Any such mileage or tickets are void if transferred for cash or other consideration. Violators (including any passenger who uses a purchased or bartered award ticket) may be liable for damages and litigation costs, including American Airlines attorneys' fees incurred in enforcing this rule."

6 The Cyclical Nature of LP Financing

In this section, we examine the cyclical nature of LPs and its impact on the firm's financial resilience. We refer to the state in the baseline setup as the normal state. The economy may enter an adverse state, in which consumers' demand for the service drops dramatically. The firm's cash flows may fall short of covering its operational expenses, forcing it to rely on liquidity buffers to sustain its business. If these liquidity buffers are depleted before the economy returns to the normal state, the firm will go bankrupt and shut down the business. Given that bankruptcy is highly value-destructive, the firm's ability to maintain financial resilience during adverse states is of critical importance.

In practice, firms may raise funds from external investors to obtain additional liquidity. However, such emergency financing is far from perfect, due to various financial frictions, particularly in adverse economic conditions. These limitations create an opportunity for LP financing to play a meaningful role in enhancing financial resilience. We will illustrate that LP financing can help smooth the firm's cash flows across economic cycles, thereby improving its financial resilience.

6.1 The adverse state

Suppose the economy is initially in the normal state, and the firm operates in a steady state, no matter whether it adopts a LP. At $t = 0$, the economy is hit by an unexpected shock, for example the 9/11 attacks or the Covid-19 pandemic, and enters an adverse state. A fraction σ of consumers are affected by the shock: they do not need the service, and their overall consumption across all categories drops to δC_0, where $\delta \in (0,1]$. The remaining consumers are unaffected and continue their normal consumption patterns. Here, σ represents the magnitude of the shock. The economy will return to the normal state at some unknown future points. This assumption about the adverse state implies that the firm's business is more volatile than the aggregate consumption: the demand for the service decreases by a fraction σ, while the aggregate consumption decreases by a fraction $\sigma(1-\delta)$.

The shock σ is sufficiently severe that the firm's cash flows turn negative in all scenarios under consideration. Upon entering the adverse state, the firm relies on its liquidity buffers L_0 to sustain operations. If these buffers are depleted before the economy recovers, the firm will face bankruptcy. To avoid the high costs associated with bankruptcy, the firm suspends dividend payments and rolls over all existing debt. To isolate the impact of the LP and simplify the analysis, we assume that the firm's ability to respond to the shock is limited. Specifically, the firm cannot adjust its operating expenses or modify the design of its LP, reflecting operational inflexibility. Additionally, while the firm can roll over existing debt, it cannot issue new debt, reflecting financial constraint.

Consumers' behavior remains unchanged and continues to align with the characterization outlined in Proposition 2. This assumption is motivated by empirical observations during the Covid-19 pandemic: despite significant reductions in air travel and substantial losses for airlines, consumers did not significantly alter their use of airline co-branded credit cards (CCCs). There are two potential reasons for the absence of changes. First, consumers may not be overly concerned about the firm's bankruptcy. Historically, airline bankruptcies have been relatively common, and major airlines often emerge from Chapter 11 bankruptcy with their loyalty programs intact. As a result, LP members may continue to engage with the firm, anticipating minimal disruption to their rewards. Second, consumers may perceive the adverse state as temporary and expect their demand for air travel to rebound once the shock subsides. This expectation reduces their incentive to change their behavior regarding the use of rewards or CCCs.

6.2 The impact of LP financing

We show that LP financing makes the firm's cash flows less volatile, thereby enhancing the firm's financial resilience. The formal result is stated as follows.

Proposition 9. *If adopting an LP leads to weakly higher cash flows than adopting no LP in the normal state, it also leads to a strictly lower probability of bankruptcy than adopting no LP in the adverse state.*

We first analyze the case where the firm adopts no LP. The firm borrows $I_0 + L_0$ in debt, and the debt coupon is

$$r_0 (I_0 + L_0).$$

In the adverse state, the firm's cash flows from sales dwindle to

$$(1-\sigma) p_0 q_0.$$

The firm's negative cash flows translate into decreases in the liquidity buffers. Denote the liquidity buffers at t by $L_0(t;\sigma)$. Then their dynamics are characterized by the following differential

equation:

$$\frac{dL_0(t;\sigma)}{dt} = (1-\sigma)\,p_0q_0 - O_0 - r_0(I_0+L_0) + r_0L_0(t;\sigma),$$

where $r_0L_0(t;\sigma)$ is the interest income on the liquidity buffers at t. With the boundary condition $L_0(0;\sigma) = L_0$, we obtain the liquidity buffers at t as follows:

$$L_0(t;\sigma) = L_0e^{r_0t} + \frac{e^{r_0t}-1}{r_0}\left[(1-\sigma)\,p_0q_0 - O_0 - r_0(I_0+L_0)\right].$$

We then analyze the case where the firm adopts a LP. To incorporate all cases in one framework, we consider that the firm adopts a CLP and a PLP simultaneously. Their issuance rates are A_c and A_p respectively, the unit price of rewards is θ, and the combined reward balance is B. When either of A_c and A_p is zero, the LP degenerates to a pure CLP or a pure PLP.

The firm receives $B\theta$ in cash through the LP and borrows $I_0+L_0-B\theta$ in debt, so the debt coupon is

$$r_0(I_0+L_0-B\theta).$$

In the adverse state, the firm's cash flows from sales dwindle to

$$(1-\sigma)\left[p_0q_0 - (A_c+A_p)(1-\theta)\right] + \sigma\delta A_c\theta.$$

Only the consumers not affected by the shock buy services, which generates the cash flows equal to the first term. The consumers affected by the shock still use the CCC and earn rewards at the rate of δA_c, which generates cash flows equal to the second term. Denote the liquidity buffers at t by $L(t;\sigma)$. Then their dynamics are characterized by the following differential equation:

$$\begin{aligned}\frac{dL(t;\sigma)}{dt} =& (1-\sigma)\left[p_0q_0 - (A_c+A_p)(1-\theta)\right] + \sigma\delta A_c\theta \\ &- O_0 - r_0(I_0+L_0-B\theta) + r_0L(t;\sigma),\end{aligned}$$

We then obtain the liquidity buffers at t as follows:

$$L(t;\sigma) = L_0(t;\sigma) + \frac{e^{r_0t}-1}{r_0}\left[r_0B\theta - (1-\sigma)(A_c+A_p)(1-\theta) + \sigma\delta A_c\theta\right].$$

Note that the difference in the cash flows between adopting the LP and not in the normal state is

$$CF - CF_0 = r_0B\theta - (A_c+A_p)(1-\theta).$$

We can rewrite the liquidity buffers at t as

$$L(t;\sigma) = \frac{e^{r_0 t}-1}{r_0}\sigma(A_c + A_p)(1-\theta) + \frac{e^{r_0 t}-1}{r_0}\sigma\delta A_c\theta + \frac{e^{r_0 t}-1}{r_0}(CF - CF_0) + L_0(t;\sigma). \tag{13}$$

The first two terms in the right-hand side of equation (13) capture the two mechanisms that affect the cyclical nature of LP financing. The first term reflects the fact that the cost of LP financing is lower in the adverse state. LP financing is costly because consumers use rewards to buy services instead of cash, and these rewards are issued at a discount. Given the unit price of rewards, The total discount enjoyed by consumers is proportional to the redemption rate of rewards, which decreases in the adverse state due to reduced demand for the firm's services. An alternative way to view this is that an LP functions as a revenue-sharing arrangement: a fixed fraction of the firm's revenue is shared with consumers in the form of discounts on rewards. When revenue drops, the amount of revenue shared with consumers also decreases.

The second term stems from the fact that the firm raises more funds from consumers in the adverse state. Under the assumption that consumers do not change their behavior, reward issuance remains tied to consumers' aggregate consumption, while reward redemption is linked to their demand for the service. For firms with volatile business analyzed here, reward issuance is less volatile than reward redemption. As a result, in the adverse state, reward issuance exceeds reward redemption, leading to an increase in the firm's reward balance at a rate of $\sigma\delta A_c$. Such divergence between reward issuance and redemption means that the firm effectively raises more funds from consumers during downturns. This mechanism aligns with Fact 3 in Section 2, which highlights that LP financing helps smooth firms' cash flows.

If the difference in cash flows between adopting an LP and not adopting an LP in the normal state is nonnegative, i.e., $CF - CF_0 \geq 0$, the liquidity buffers under the LP are strictly higher than those under no LP. This implies that adopting an LP leads to a strictly lower probability of bankruptcy, as formalized in Proposition 9. This cyclical nature makes LP financing particularly attractive to firms with highly volatile business such as airlines, hotels, upscale department stores, and durable goods retailers. Specially, airlines feature heavy assets, high financial expenses, and low profit margins. Highly volatile business plus financial weakness makes airlines particularly eager to utilize LP financing.

Proposition 10. *Given the total issuance rate $A_c + A_p$, the higher the issuance rate under the CLP, the lower the sensitivity of the firm's liquidity buffers to the magnitude of the shock; i.e.,*

$$\frac{\partial^2 L(t;\sigma)}{\partial\sigma\partial A_c} > \frac{\partial^2 L(t;\sigma)}{\partial\sigma\partial A_p}$$

Proposition 10 suggests that, for a given total issuance rate, more issuance through the CLP strengthens the cyclical nature of LP financing. This effect arises because the second mechanism—where the firm raises more funds from consumers during adverse states—is unique to the CLP. Unlike the PLP, the issuance under the CLP is not directly tied to purchases of the service. As a result, CLP issuance remains stable even when demand for the firm's services declines, whereas PLP issuance decreases in tandem with reduced demand. A broader implication of this finding is that firms can enhance the cyclical nature of LP financing by decoupling reward issuance from their core business operations. This implication aligns closely with Fact 2 in Section 2, which observes that, in practice, rewards are also issued through broad consumption, including spending categories unrelated to firms' business.

7 Conclusion

This paper demonstrates that loyalty programs can be used and are de facto used by some firms to raise funds from consumers. With this starting point, the paper mainly addresses two questions. First, what firms should do LP financing? They are firms supplying high-value, low-frequency services and firms with highly volatile business. Second, How can firms leverage LP financing more effectively? They should have more diverse issuance channels and weaken the linkage of reward issuance to the firm's business, and they should not give consumers too much freedom in purchasing rewards from the firm or from other consumers. These theoretical predictions are consistent with observations in practice.

Rewards programs have become an integral component of American economic life, warranting rigorous academic scrutiny. As noted by Pete Buttigieg, the former U.S. Transportation Secretary,

> "Points systems like frequent flyer miles and credit card rewards have become such a meaningful part of our economy that many Americans view their rewards points balances as part of their savings."

This observation underscores the growing significance of rewards programs, not only as a consumer incentive mechanism but also as a quasi-financial asset that influences individual financial behavior and decision-making.

This paper adopts a corporate finance perspective to examine the implications of rewards programs. However, the study of rewards programs is inherently multifaceted, and future research could productively explore their effects from a household finance perspective. Such an approach would complement the corporate finance lens by examining how rewards programs influence individual savings behavior, budgeting, and financial planning, thereby providing a more comprehensive understanding of their role in the broader economic landscape.

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Appendix

A Additional Tables & Figures

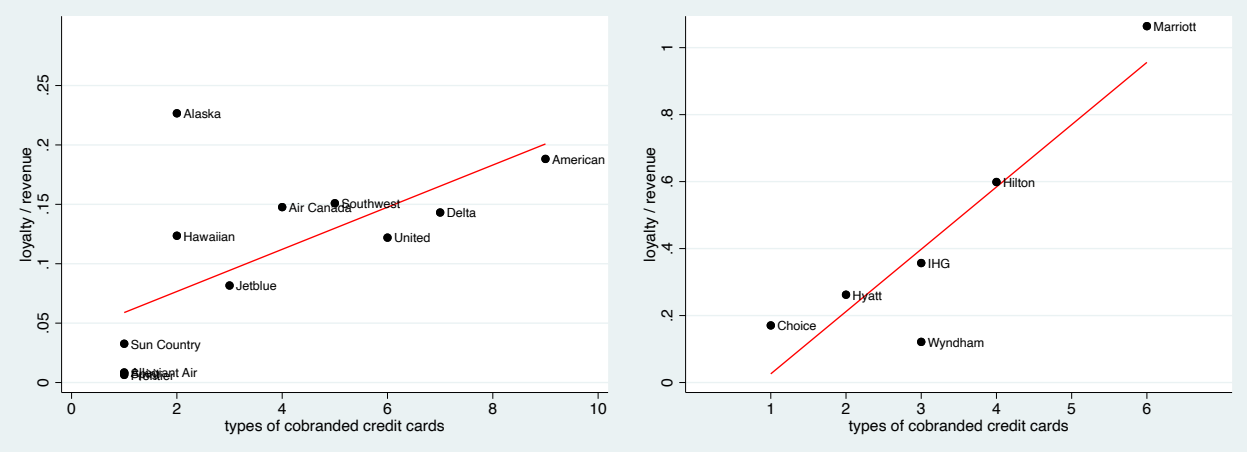


Figure 8: LP liabilities and varieties of CCCs

Table 4: LP liabilities and varieties of CCCs

	loyalty / revenue	
	(1) Airlines	(2) Hotels
Varieties of CCCs	0.018**	0.186**
	(0.007)	(0.044)
Constant	0.041	-0.160
	(0.029)	(0.155)
Observations	12	6
R-squared	0.411	0.819

Table 5: The change in LP liabilities during Covid

	Change in loyalty / revenue from 2019 to 2020			
	(1)	(2)	(3)	(4)
	Airlines	Airlines	Hotels	Hotels
Large_LP	0.021***	0.014***	0.111***	0.096**
	(0.006)	(0.004)	(0.014)	(0.022)
Change in loyalty / revenue		1.727***		0.590
from 2018 to 2019		(0.315)		(0.669)
Constant	0.003	-0.002	0.002	-0.006
	(0.005)	(0.003)	(0.008)	(0.012)
Observations	12	11	6	6
R-squared	0.546	0.896	0.942	0.954

B Proofs

Proof of Lemma 1

Suppose that a consumer does not follow the strategy in Lemma 1 and use a combination of cash and rewards with positive probability. That implies $y > 0$ and $x < p_0$. If $x \leq \varepsilon$, then $y(x-\varepsilon) - z\theta < 0$, so the consumer is better off if he does not join the LP and buy a service with only cash. If $x > \varepsilon$, consider an alternative strategy that he still receives rewards at the rate of z, and when needing a service, he buys it with only rewards if he has at least p_0 rewards and with only cash otherwise. Under this strategy, he redeems p_0 rewards for a service at the rate of z/p_0. Then his average payoff flow is

$$\frac{z}{p_0}(p_0 - \varepsilon) - z\theta.$$

By $\varepsilon < x < p_0$ and $xy \leq z$,

$$\frac{z}{p_0}(p_0 - \varepsilon) - z\theta > \frac{z}{x}(x-\varepsilon) - z\theta \geq y(x-\varepsilon) - z\theta.$$

Hence, the consumer is better off by using the alternative strategy under which he buys a service with either only cash or only rewards.

Proof of Lemma 3

Denote the probability that a consumer needs exactly one service over t units of time as $P_1(t)$ and the probability that a consumer needs at least one service over t units of time as $P_0(t)$. We first show that $P_1(t) = q_0 t e^{-q_0 t}$, $P_0(t) = 1 - e^{-q_0 t}$, and $P_0(t) - P_1(t) = o(t)$.

$P_0(t)$ satisfies

$$\begin{aligned} P_0(t+dt) &= P_0(t) + (1 - P_0(t)) P_0(dt) \\ \Rightarrow -\frac{d\ln(1-P_0(t))}{dt} &= \frac{P_0(dt)}{dt} = P_0'(0) = q_0. \end{aligned}$$

Combining with the boundary condition $P_0(0) = 0$, we obtain $P_0(t) = 1 - e^{-q_0 t}$. $P_1(t)$ satisfies

$$\begin{aligned} P_1(t+dt) &= P_1(t)(1 - P_0(dt)) + (1 - P_0(t)) P_1(dt) \\ \Rightarrow P_1(t+dt) - P_1(t) &= -P_1(t) q_0 dt + e^{-q_0 t} P_1(dt) \\ \Rightarrow \frac{dP_1(t)}{dt} &= -P_1(t) q_0 + e^{-q_0 t} q_0 \\ \Rightarrow \frac{de^{q_0 t} P_1(t)}{dt} &= q_0 \end{aligned}$$

Combining with the boundary condition $P_1(0) = 0$, we obtain $P_1(t) = q_0 t e^{-q_0 t}$. $P_0(t) - P_1(t)$ satisfies

$$\begin{aligned} P_0(t) - P_1(t) &= 1 - e^{-q_0 t} - q_0 t e^{-q_0 t} \\ &= 1 - (1 + q_0 t)\left[1 - q_0 t + \sum_{k=2}^{+\infty} \frac{1}{k!} (-q_0 t)^k\right] \\ &= (q_0 t)^2 - (1 + q_0 t) \sum_{k=2}^{+\infty} \frac{1}{k!} (-q_0 t)^k . \end{aligned}$$

Next, we prove the equation system. For a small τ, let $\Delta\Gamma(N + A\tau) \triangleq \Gamma(N + A\tau) - \Gamma(N)$ be the mass of consumers holding rewards between N units and $N + A\tau$ units at t. These consumers consist of three groups as follows.

- The first group is the consumers who do not redeem rewards for any service during the past τ units of time. Since they are always accumulating rewards at the rate of A, they must hold rewards between $N - A\tau$ units and N units at $t - \tau$.
- The second group is the consumers who redeem rewards for one service during the past τ units of time. They must hold rewards between $N + p_0 - A\tau$ units and $N + p_0$ units at $t - \tau$.
- The third group is the consumers who redeem rewards for more than one service during the past τ units of time and end up holding rewards between N units and $N + A\tau$ units at t.

For $N \in [0, p_0 - A\tau)$, consumers holding rewards between $N - A\tau$ units and N units at $t - \tau$ do not have sufficient rewards to redeem for one service by t, so all of them belong to the first group. On the other hand, consumers holding rewards between $N + p_0 - A\tau$ units and $N + p_0$ units at $t - \tau$ have needs for services with probability $1 - e^{-q_0 \tau}$. Since $N + p_0 + A\tau < 2p_0$, these consumers have sufficient rewards to redeem for only one service, so all the fraction $1 - e^{-q_0 \tau}$ of them belong to the second group. The mass of the third group is of a higher order than τ. Hence, we obtain the following equation:

$$\Delta\Gamma(N + A\tau) = \Delta\Gamma(N) + \Delta\Gamma(N + p_0)\left(1 - e^{-q_0 \tau}\right) + o(\tau) .$$

For $N \in [p_0 + A\tau, +\infty)$, consumers holding rewards between $N - A\tau$ units and N units at $t - \tau$ have sufficient rewards to redeem for one service, so only those of them having no needs for services, which make up the fraction $e^{-q_0 \tau}$, belong to the first group. On the other hand, consumers holding rewards between $N + p_0 - A\tau$ units and $N + p_0$ units at $t - \tau$ have sufficient rewards to redeem for more than one service, so only those of them having needs for exactly one service by t, which

make up the fraction $q_0\tau e^{-q_0\tau}$, belong to the second group. The mass of the third group is of a higher order than τ. Hence, we obtain the following equation:

$$\Delta\Gamma(N+A\tau)=\Delta\Gamma(N)\,e^{-q_0\tau}+\Delta\Gamma(N+p_0)\,q_0\tau e^{-q_0\tau}+o(\tau).$$

Taking τ to 0, we obtain equation (5). The boundary conditions (6), (7) and (8) stem from that the distribution is continuous everywhere, no consumer holds a negative number of rewards, and the total mass of consumers is 1, respectively.

Proof of Proposition 3

The equation system in Lemma 3 is an advance differential equation system. We conjecture

$$\gamma(N)=e^{-N/\lambda}\gamma_0 \tag{14}$$

for $N\in(p_0,+\infty)$, where $\lambda>0$. Then according to equation (5), we have

$$\begin{aligned}-\frac{1}{\lambda}e^{-N/\lambda}\gamma_0+e^{-N/\lambda}\gamma_0\frac{q_0}{A}-e^{-N/\lambda}e^{-p_0/\lambda}\gamma_0\frac{q_0}{A}&=0\\ \Leftrightarrow\left(1-e^{-p_0/\lambda}\right)\cdot\lambda&=\frac{A}{q_0}.\end{aligned}$$

Plugging the conjectured distribution (14) into equation (5), we obtain that for $N\in[0,p_0)$,

$$\begin{aligned}\gamma(N)&=\int_0^N\gamma(x+p_0)\frac{A}{q_0}dx+\gamma(0)=\int_0^N e^{-x/\lambda}e^{-p_0/\lambda}\cdot\gamma_0\frac{q_0}{A}dx\\&=\lambda e^{-p_0/\lambda}\cdot\gamma_0\frac{q_0}{A}\cdot\left(1-e^{-N/\lambda}\right).\end{aligned}$$

Note that the boundary condition (6) naturally holds:

$$\lim_{N\uparrow p_0}\gamma(N)=\lambda e^{-p_0/\lambda}\cdot\gamma_0\frac{q_0}{A}\cdot\left(1-e^{-p_0/\lambda}\right)=e^{-p_0/\lambda}\cdot\gamma_0=\gamma(p_0).$$

The boundary conditions (7) and (8) require

$$\int_0^{+\infty}\gamma(N)\,dN=1.$$

Since

$$\begin{aligned}\int_0^{+\infty}\gamma(N)\,dN =&\lambda e^{-p_0/\lambda}\cdot\gamma_0\frac{q_0}{A}\cdot\int_0^{p_0}\left(1-e^{-N/\lambda}\right)dN+\gamma_0\int_{p_0}^{+\infty}e^{-N/\lambda}dN\\ =&\lambda e^{-p_0/\lambda}\cdot\gamma_0\frac{q_0}{A}\cdot\left(p_0+\lambda e^{-p_0/\lambda}-\lambda\right)+\gamma_0\lambda e^{-p_0/\lambda}\\ =&\lambda e^{-p_0/\lambda}\cdot\gamma_0\frac{q_0}{A}\cdot\left(p_0-\frac{A}{q_0}\right)+\gamma_0\lambda e^{-p_0/\lambda}\\ =&\gamma_0\lambda e^{-p_0/\lambda}\frac{p_0q_0}{A}=\gamma_0p_0\left(\lambda\frac{q_0}{A}-1\right)\end{aligned}$$

So,

$$\gamma_0=\frac{1}{p_0\left(\lambda\frac{q_0}{A}-1\right)}.$$

And for $N\in[0,p_0)$,

$$\begin{aligned}\gamma(N)&=\lambda e^{-p_0/\lambda}\cdot\gamma_0\frac{q_0}{A}\cdot\left(1-e^{-N/\lambda}\right)\\ &=\left(\lambda\frac{q_0}{A}-1\right)\cdot\frac{1}{p_0\left(\lambda\frac{q_0}{A}-1\right)}\left(1-e^{-N/\lambda}\right)\\ &=\frac{1}{p_0}\left(1-e^{-N/\lambda}\right).\end{aligned}$$

Next, we show that the equation

$$\left(1-e^{-p_0/\lambda}\right)\cdot\lambda=\frac{A}{q_0}$$

has a unique positive root λ. First, since

$$\lim_{\lambda\to0}\left(1-e^{-p_0/\lambda}\right)\cdot\lambda=0<\frac{A}{q_0}$$

and

$$\lim_{\lambda\to+\infty}\left(1-e^{-p_0/\lambda}\right)\cdot\lambda=\lim_{\lambda\to+\infty}\frac{1-e^{-p_0/\lambda}}{p_0/\lambda}\cdot p_0=\lim_{x\to0}e^{-x}\cdot p_0=p_0>\frac{A}{q_0},$$

the equation must have positive roots. Second, since

$$\frac{d\left[1-e^{-x}-e^{-x}x\right]}{dx}=e^{-x}x>0,$$

we have for $\lambda>0$,

$$\frac{d\left[\left(1-e^{-p_0/\lambda}\right)\cdot\lambda\right]}{d\lambda}=1-e^{-p_0/\lambda}-e^{-p_0/\lambda}\frac{p_0}{\lambda}>1-e^{-0}-e^{-0}\cdot0=0.$$

This implies that $\left(1-e^{-p_0/\lambda}\right)\cdot\lambda$ is strictly increasing in λ for $\lambda>0$. Therefore, the equation has a unique positive root.

Finally, we derive the reward balance.

$$\begin{aligned}
B &= \int_0^{+\infty}\gamma(N)NdN \\
&= \frac{1}{p_0}\cdot\int_0^{p_0}\left(1-e^{-N/\lambda}\right)NdN+\frac{1}{p_0\left(\lambda\frac{q_0}{A}-1\right)}\cdot\int_{p_0}^{+\infty}e^{-N/\lambda}NdN \\
&= \frac{1}{p_0}\cdot\frac{1}{2}p_0^2-\frac{1}{p_0}\cdot\int_0^{p_0}e^{-N/\lambda}NdN+\frac{1}{p_0\left(\lambda\frac{q_0}{A}-1\right)}\cdot\int_{p_0}^{+\infty}e^{-N/\lambda}NdN \\
&= \frac{p_0}{2}-\frac{1}{p_0}\frac{\lambda\frac{q_0}{A}}{\lambda\frac{q_0}{A}-1}\cdot\int_0^{p_0}e^{-N/\lambda}NdN+\frac{1}{p_0\left(\lambda\frac{q_0}{A}-1\right)}\cdot\int_0^{+\infty}e^{-N/\lambda}NdN \\
&= \frac{p_0}{2}-\frac{1}{p_0}\frac{\lambda\frac{q_0}{A}}{\lambda\frac{q_0}{A}-1}\cdot\lambda^2\left[e^{-p_0/\lambda}\left(-\frac{p_0}{\lambda}-1\right)+1\right]+\frac{1}{p_0\left(\lambda\frac{q_0}{A}-1\right)}\cdot\lambda^2 \\
&= \frac{p_0}{2}+\frac{1}{p_0}\frac{1}{\lambda\frac{q_0}{A}-1}\cdot\lambda^2\left[e^{-p_0/\lambda}\left(\frac{p_0}{\lambda}+1\right)\lambda\frac{q_0}{A}-\lambda\frac{q_0}{A}+1\right].
\end{aligned}$$

By $\left(1-e^{-p_0/\lambda}\right)\cdot\lambda=\frac{A}{q_0}$, we have

$$\begin{aligned}
B &= \frac{p_0}{2}+\frac{1}{p_0}\frac{1}{\lambda\frac{q_0}{A}-1}\cdot\lambda^2\left[\left(\lambda\frac{q_0}{A}-1\right)\left(\frac{p_0}{\lambda}+1\right)-\lambda\frac{q_0}{A}+1\right] \\
&= \frac{p_0}{2}+\frac{1}{p_0}\frac{1}{\lambda\frac{q_0}{A}-1}\cdot\lambda^2\left(\lambda\frac{q_0}{A}-1\right)\frac{p_0}{\lambda} \\
&= \frac{p_0}{2}+\lambda.
\end{aligned}$$

Proof of Proposition 4

When $1/\pi$ is a rational number, there exists a unique pair of positive coprime integers μ and n such that $\mu/n=(1+\pi)/\pi$. We then obtain

$$\frac{h_i}{(p_0+p_0\pi)/\mu}=i\cdot n\pmod{\mu}.$$

Since μ and n are coprime, $\frac{h_i}{(p_0+p_0\pi)/\mu}$ follows a cycle, and in one cycle, $\frac{h_i}{(p_0+p_0\pi)/\mu}$ hits each element in $\{0,1,2,\ldots,\mu-2,\mu-1\}$ once. That means, consumers are uniformly distributed over the set $\left\{\frac{i}{\mu}(p_0+p_0\pi)\right\}_{i=0}^{\mu-1}$ in terms of reward holdings. Further, the reward balance is

$$B=\frac{1}{\mu}(p_0+p_0\pi)\sum_{i=0}^{\mu-1}i\cdot\frac{1}{\mu}=\frac{\mu-1}{2\mu}(1+\pi)p_0.$$

When $1/\pi$ is an irrational number, the economic forces are basically the same. Conceptually, we can construct an infinite sequence of rational numbers that converges to $1/\pi$. During this process, μ will go to infinity, and the set $\left\{\frac{i}{\mu}(p_0+p_0\pi)\right\}_{i=0}^{\mu-1}$ will get closer to the interval $[0, p_0+p_0\pi)$. Formally, by Weyl's equidistribution theorem, if $\frac{\pi}{1+\pi}$ is an irrational number and

$$\frac{h_i}{p_0+p_0\pi} = i\cdot\frac{\pi}{1+\pi} \pmod 1,$$

h_i is uniformly distributed over the interval $[0, p_0+p_0\pi)$. Hence, the reward balance is $(p_0+p_0\pi)/2$.

The reward balance is generally characterized by equation (12). Since n is a positive integer and must be at least 1, μ is greater than or equal to $(1+\pi)/\pi$. That means

$$\frac{1}{2}(1+\pi)p_0 \geq B \geq \frac{\frac{1+\pi}{\pi}-1}{2\frac{1+\pi}{\pi}}(1+\pi)p_0 = \frac{p_0}{2}.$$

The left equality is attained when $1/\pi$ is an irrational number, and the right equality is attained when $1/\pi$ is an integer.

Proof of Proposition 5

For any CLP, let $\hat{\lambda} \triangleq \lambda/p_0$. Then by equation (10), $\hat{\lambda}$ satisfies

$$\left(1-e^{-1/\hat{\lambda}}\right)\cdot\hat{\lambda} = \frac{A}{p_0q_0},$$

and is thus fixed given p_0q_0 and A. The reward balance is

$$B = \left(\frac{1}{2}+\hat{\lambda}\right)p_0,$$

which is greater if p_0 is higher and q_0 is lower.

Under any PLP, by $A = \frac{p_0q_0}{1/\pi+1}$, π is fixed given p_0q_0 and A. So is μ, as it is completely determined by π. The reward balance is

$$B = \frac{\mu-1}{2\mu}(1+\pi)p_0$$

which is greater if p_0 is higher and q_0 is lower.

Proof of Proposition 6

Under a PLP with π, the issuance rate is

$$A = \frac{p_0 q_0}{1/\pi + 1},$$

and the reward balance is no greater than

$$\frac{p_0}{2} + \frac{p_0 \pi}{2}.$$

On the other hand, for the same issuance rate, the reward balance under a CLP is

$$\frac{p_0}{2} + \lambda,$$

where

$$\left(1 - e^{-p_0/\lambda}\right) \cdot \lambda = \frac{A}{q_0}.$$

It is easy to see

$$\lambda > \frac{A}{q_0} = \frac{p_0 \pi}{1+\pi} \geq \frac{p_0}{2}\pi.$$

The last inequality follows $\pi \leq 1$.

Proof of Proposition 7

For convenience of illustration, we stipulate that consumers do free purchase or free transfer, as long as it is at least weakly beneficial. Following the argument of Lemma 1, each redemption must involve p_0 rewards.

Case I: no LP is adopted.

The average benefit of a reward obtained through free purchase is $1 - \frac{\varepsilon}{p_0}$. If $\hat{\theta} > 1 - \frac{\varepsilon}{p_0}$, a consumer never obtains rewards through free purchase, so free purchase makes no difference. If $\hat{\theta} \leq 1 - \frac{\varepsilon}{p_0}$, a consumer purchases p_0 rewards whenever he needs a service, and he redeems them immediately. Hence, consumers enjoy positive discounts of rewards, and the reward balance is 0. The firm's cash flows become

$$p_0 q_0 \hat{\theta} - OE_0,$$

which is lower than CF_0.

Case II: a PLP is adopted.

Exercising the option to buy rewards in a cash purchase, a consumer spends $p_0 + p_0\pi\theta_p$ cash and receives a payoff of p_0 and $p_0\pi$ rewards. Instead, if a consumer obtains $p_0 + p_0\pi$ rewards through free purchase and redeem p_0 rewards for a service, he receives a payoff of $p_0 - \varepsilon$ and $p_0\pi$ rewards. A consumer prefers the latter if and only if

$$\begin{aligned} & p_0 - \varepsilon - (p_0 + p_0\pi)\,\hat{\theta} \geq p_0 - (p_0 + p_0\pi\theta_p) \\ \Leftrightarrow\ & \hat{\theta} \leq \frac{\pi}{1+\pi}\theta_p + \frac{1}{1+\pi}\left(1 - \frac{\varepsilon}{p_0}\right). \end{aligned}$$

It is easy to see that this inequality implies that a cash purchase without exercising the option is also dominated. Therefore, if $\hat{\theta} \leq \frac{\pi}{1+\pi}\theta_p + \frac{1}{1+\pi}\left(1 - \frac{\varepsilon}{p_0}\right)$, a consumer does not use cash purchases: he purchases p_0 rewards whenever he needs a service, and he redeems them immediately. Similar to the case when no LP is adopted, the firm's cash flows become

$$p_0 q_0 \hat{\theta} - OE_0.$$

Without free purchase, the firm's cash flows under a PLP are

$$\begin{aligned} & r_0 B\theta_p - \frac{p_0 q_0}{1/\pi + 1}(1 - \theta_p) + p_0 q_0 - OE_0 \\ >& -\frac{p_0 q_0}{1/\pi + 1} + \frac{p_0 q_0}{1/\pi + 1}\theta_p + p_0 q_0 - OE_0 \\ \geq& -\frac{p_0 q_0}{1/\pi + 1} + p_0 q_0\left[\hat{\theta} - \frac{1}{1+\pi}\left(1 - \frac{\varepsilon}{p_0}\right)\right] + p_0 q_0 - OE_0 \\ =& q_0 \frac{1}{1+\pi}\varepsilon + p_0 q_0 \hat{\theta} - OE_0 \\ >& p_0 q_0 \hat{\theta} - OE_0. \end{aligned}$$

Therefore, free purchase with $\hat{\theta} \leq \frac{\pi}{1+\pi}\theta_p + \frac{1}{1+\pi}\left(1 - \frac{\varepsilon}{p_0}\right)$ reduces the firm's cash flows.

If $\hat{\theta} > \frac{\pi}{1+\pi}\theta_p + \frac{1}{1+\pi}\left(1 - \frac{\varepsilon}{p_0}\right)$, a consumer prefers using cash to buy the bundle of a service and rewards to free purchase. A consumer never obtains rewards through free purchase, so free purchase makes no difference in this case.

Case III: a CLP is adopted.

If $\hat{\theta} > 1 - \frac{\varepsilon}{p_0}$, a consumer never obtains rewards through free purchase, so free purchase makes no difference. If $\hat{\theta} \leq \theta_c$, a consumer will not use the CCC to earn rewards or use cash purchases: he purchases p_0 rewards whenever he needs a service, and he redeems them immediately. Similar to

the case when no LP is adopted, the firm's cash flows are

$$p_0 q_0 \hat{\theta} - OE_0.$$

Without free purchase, the firm's cash flows under a CLP are

$$\begin{aligned} & r_0 B\theta_c - A(1-\theta_c) + p_0 q_0 - OE_0 \\ >& -A\left(1-\hat{\theta}\right) + p_0 q_0 - OE_0 \\ >& -p_0 q_0 \left(1-\hat{\theta}\right) + p_0 q_0 - OE_0 \\ =& p_0 q_0 \hat{\theta} - OE_0. \end{aligned}$$

Therefore, free purchase with $\hat{\theta} \leq \theta_c$ reduces the firm's cash flows.

If $1 - \frac{\varepsilon}{p_0} \geq \hat{\theta} > \theta_c$, a consumer will fully use the CCC to earn rewards but do not use cash purchases: he earns rewards through the CCC at the rate of A, and if he needs a service but does not have p_0 rewards, he get the rest through free purchase and redeem all rewards immediately. Similar to Section 4.2, we derive the steady-state distribution of consumers' reward holdings. Note that holding fewer than p_0 rewards does not impede reward redemption. The steady-state distribution of consumers' reward holdings is

$$\gamma(N) = \begin{cases} 0, & \text{if } N \in (-\infty, 0), \\ \frac{1}{\lambda} e^{-N/\lambda}, & \text{if } N \in [0, +\infty), \end{cases} \tag{15}$$

where λ is still the one defined in Proposition 3 as follows:

$$\left(1 - e^{-p_0/\lambda}\right) \cdot \lambda = \frac{A}{q_0}.$$

We then obtain that the reward balance is

$$\int_0^{+\infty} N \cdot \frac{1}{\lambda} e^{-N/\lambda} dN = \lambda,$$

which is smaller than the reward balance without free purchase. Taken together, the firm's cash flows are

$$\begin{aligned} & r_0 \lambda \theta_c + A\theta_c + (p_0 q_0 - A)\hat{\theta} - OE_0 \\ <& r_0 \left(\frac{p_0}{2} + \lambda\right) \theta_c - A(1-\theta_c) + p_0 q_0 - OE_0. \end{aligned}$$

Therefore, free purchase with $1 - \frac{\varepsilon}{p_0} \geq \hat{\theta} > \theta_c$ reduces the firm's cash flows.

Proof of Proposition 8

Assume free transfer is allowed. A consumer who need a service would like to buy a total of p_0 rewards from consumers at a price no greater than $1-\frac{\varepsilon}{p_0}$. On the other hand, a consumer who has rewards and does not need a service would like to sell rewards at a price no smaller than $1-\frac{\varepsilon}{p_0}$, due to the preference for holding cash.

If the reward balance is positive, then each consumers who needs a service can get enough rewards to redeem for a service. Hence, rewards are redeemed at the rate of p_0q_0 in total. The maximum issuance rate under a CLP is $\gamma C_0 a_c$, and that under a PLP is $p_0q_0\pi$. Both are smaller than p_0q_0, so the reward balance must decrease. In the steady state, the reward balance must be 0. In the steady state, the rewards issued at each instant will be transferred to the consumers who need services and redeemed immediately.

Proof of Proposition 10

The liquidity buffer at t is decreasing in the magnitude of the shock σ:

$$\frac{\partial L(t;\sigma)}{\partial \sigma}=\frac{e^{r_0 t}-1}{r_0}(A_c+A_p)(1-\theta)+\frac{e^{r_0 t}-1}{r_0}\delta A_c\theta-\frac{e^{r_0 t}-1}{r_0}p_0q_0<0.$$

Since

$$\frac{\partial^2 L(t;\sigma)}{\partial \sigma \partial A_c}=\frac{e^{r_0 t}-1}{r_0}(1-\theta)+\frac{e^{r_0 t}-1}{r_0}\delta\theta>\frac{\partial^2 L(t;\sigma)}{\partial \sigma \partial A_p}=\frac{e^{r_0 t}-1}{r_0}(1-\theta),$$

an increase in A_c weakens the sensitivity of the liquidity buffer to the magnitude of the shock more than an increase in A_p. The difference $\frac{e^{r_0 t}-1}{r_0}\delta\theta$ stems from the second mechanism.